Computer algebra independent integration tests

0-Independent-test-suites/Timofeev-Problems

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [705]. This is test number [10].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (705)	% 0. (0)
Mathematica	% 100. (705)	% 0. (0)
Maple	% 91.77 (647)	% 8.23 (58)
Maxima	% 76.88 (542)	% 23.12 (163)
Fricas	% 92.2 (650)	% 7.8 (55)
Sympy	% 59.86 (422)	% 40.14 (283)
Giac	% 80. (564)	% 20. (141)

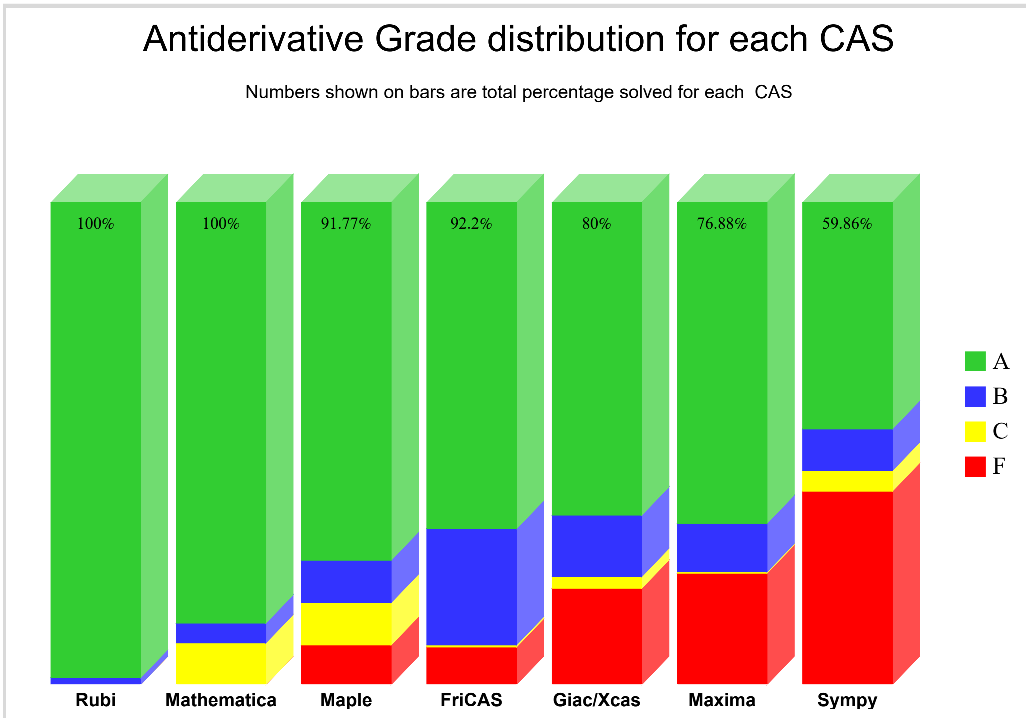
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

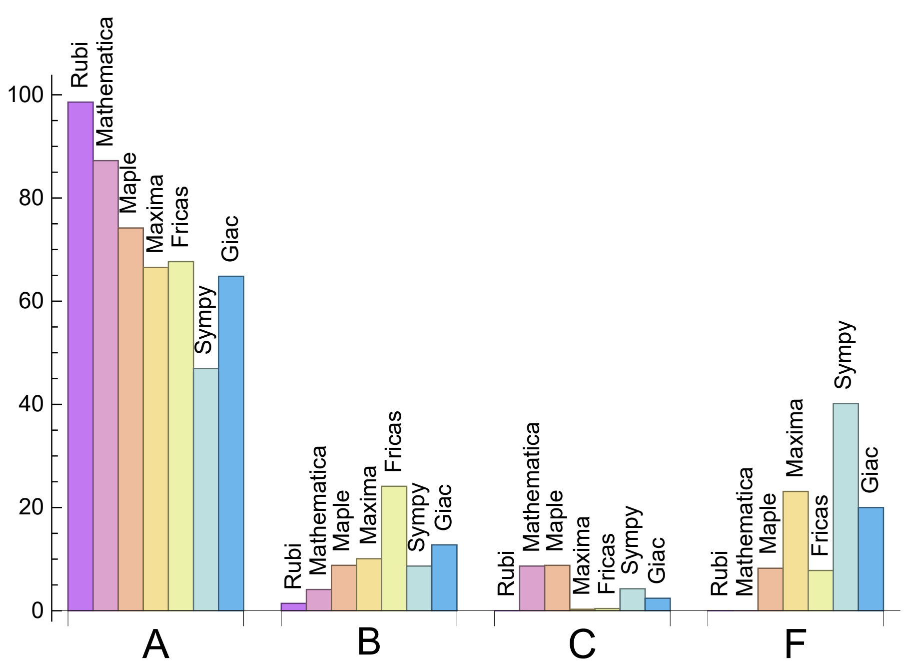
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	98.58	1.42	0.	0.
Mathematica	87.23	4.11	8.65	0.
Maple	74.18	8.79	8.79	8.23
Maxima	66.52	10.07	0.28	23.12
Fricas	67.66	24.11	0.43	7.8
Sympy	46.95	8.65	4.26	40.14
Giac	64.82	12.77	2.41	20.

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.09	51.59	1.05	38.	1.
Mathematica	0.25	81.2	1.52	35.	1.
Maple	0.05	640.3	8.23	35.	0.94
Maxima	1.19	80.37	1.99	45.	1.23
Fricas	2.84	214.88	5.	115.	3.23
Sympy	4.57	84.01	1.98	34.	1.04
Giac	1.09	81.79	1.96	46.	1.24

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {222, 417, 686, 687, 688, 689, 691}

Mathematica {113, 137, 138, 143, 144, 193, 198, 220, 222, 227, 228, 265, 317, 324, 327, 417, 435, 438, 446, 506, 511, 592, 657, 665, 674, 675, 678, 683, 689, 698, 705}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

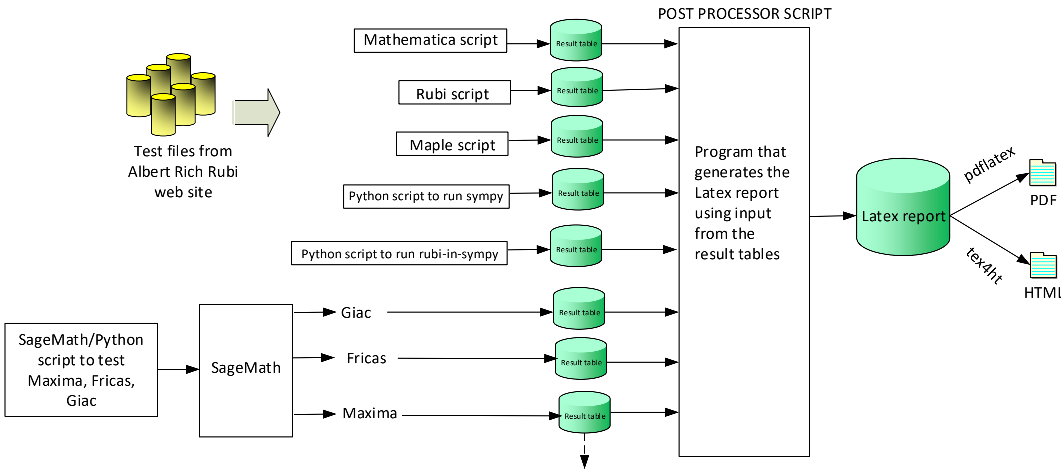
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705 }

B grade: { 226, 228, 232, 306, 335, 377, 413, 416, 447, 695 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 218, 219, 220, 223, 224, 225, 227, 229, 230, 233, 234, 235, 236, 237, 238, 239, 240, 241, 243, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 293, 295, 296, 297, 299, 300, 301, 307, 308, 309, 310, 313, 314, 315, 316, 318, 319, 322, 323, 326, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 342, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 358, 359, 360, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 385, 386, 387, 388, 390, 391, 392, 393, 394, 395, 396, 398, 400, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 418, 419, 420, 421, 422, 423, 425, 427, 428, 429, 430, 431, 432, 433, 435, 436, 437, 441, 442, 447, 449, 450, 453, 455, 457, 458, 459, 460, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 556, 558, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 575, 576, 577, 578, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 704 }

B grade: { 3, 4, 41, 63, 76, 99, 311, 338, 357, 361, 438, 439, 444, 445, 451, 452, 456, 488, 553, 554, 555, 557, 559, 574, 579, 622, 623, 689, 705 }

C grade: { 37, 113, 193, 198, 215, 216, 217, 221, 222, 226, 228, 231, 232, 242, 244, 245, 246, 247, 248, 249, 260, 283, 292, 294, 298, 302, 303, 304, 305, 306, 312, 317, 320, 321, 324, 325, 327, 343, 384, 389, 397, 399, 401, 411, 416, 417, 424, 426, 434, 440, 443, 446, 448, 454, 461, 476, 526, 592, 677, 678, 703 }

F grade: { }

2.1.3 Maple

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 135, 140, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 218, 220, 223, 224, 225, 227, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 243, 244, 245, 250, 251, 252, 254, 255, 256, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 307, 310, 312, 318, 322, 323, 326, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 356, 357, 358, 360, 361, 362, 364, 365, 366, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 387, 388, 389, 390, 391, 393, 394, 395, 396, 397, 398, 399, 401, 412, 419, 420, 421, 422, 424, 425, 426, 428, 430, 437, 440, 441, 448, 451, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 488, 489, 490, 491, 493, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 534, 535, 536, 537, 538, 539, 540, 541, 542, 544, 545, 546, 547, 558, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 579, 580, 581, 583, 584, 585, 590, 591, 593, 596, 597, 599, 600, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 633, 634, 635, 636, 637, 638, 639, 640, 645, 646, 647, 648, 649, 650, 651, 652, 654, 655, 659, 660, 663, 664, 665, 666, 668, 669, 670, 671, 673, 675, 676, 677, 678, 679, 680, 681, 682, 689, 695, 696, 697, 698, 700, 701, 702 }

B grade: { 1, 13, 35, 79, 128, 141, 196, 197, 213, 217, 219, 240, 241, 242, 246, 247, 248, 249, 253, 257, 260, 299, 300, 311, 335, 355, 359, 363, 367, 374, 383, 400, 423, 429, 431, 432, 433, 438, 439, 447, 456, 487, 548, 556, 561, 563, 578, 586, 587, 588, 595, 598, 601, 602, 642, 644, 657, 661, 662, 672, 674, 683 }

C grade: { 81, 136, 137, 138, 139, 142, 143, 144, 301, 302, 303, 304, 305, 306, 313, 316, 320, 321, 324, 325, 327, 392, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 413, 416, 417, 427, 434, 435, 436, 457, 492, 543, 589, 592, 594, 641, 643, 653, 656, 658, 667, 684, 685, 686, 687, 688, 690, 691, 692, 693, 694, 704 }

F grade: { 67, 126, 133, 145, 193, 198, 221, 222, 226, 228, 232, 308, 309, 314, 315, 317, 319, 328, 329, 352, 414, 415, 418, 442, 443, 444, 445, 446, 449, 450, 452, 453, 454, 455, 500, 506, 511, 516, 521, 529, 532, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 632, 699, 703, 705 }

2.1.4 Maxima

A grade: { 2, 3, 6, 7, 9, 10, 11, 12, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 49, 50, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 83, 84, 85, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 229, 235, 237, 243, 250, 251, 252, 253, 254, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 282, 284, 285, 286, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 310, 312, 313, 316, 326, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 370, 371, 372, 375, 376, 377, 378, 379, 380, 381, 382, 392, 396, 397, 398, 400, 401, 412, 414, 415, 418, 419, 420, 422, 424, 428, 439, 444, 445, 448, 450, 453, 458, 459, 460, 461, 462, 464, 465, 466, 467, 468, 469, 470, 471, 472, 475, 476, 477, 478, 479, 480, 481, 483, 484, 485, 486, 490, 496, 497, 498, 499, 501, 502, 503, 504, 505, 507, 508, 509, 510, 512, 513, 514, 515, 517, 518, 519, 520, 522, 524, 525, 526, 528, 530, 531, 532, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 585, 590, 591, 598, 600, 602, 604, 605, 606, 607, 610, 611, 612, 613, 614, 615, 616, 617, 620, 622, 623, 624, 628, 629, 630, 631, 632, 633, 634, 635, 636, 638, 639, 640, 641, 642, 644, 645, 646, 647, 648, 649, 651, 652, 653, 654, 655, 656, 658, 659, 660, 661, 662, 663, 664, 666, 667, 669, 670, 671, 673, 676, 677, 678, 679, 680, 681, 682, 685, 686, 687, 690, 692, 694, 697, 698, 700, 702, 705

B grade: { 1, 4, 5, 8, 13, 19, 31, 41, 62, 76, 81, 82, 86, 128, 159, 236, 240, 241, 257, 311, 318, 322, 323, 369, 373, 374, 384, 385, 386, 387, 388, 389, 393, 394, 399, 423, 425, 429, 430, 431, 433, 437, 449, 451, 456, 474, 482, 488, 489, 491, 492, 493, 577, 578, 579, 580, 581, 584, 588, 589, 593, 594, 596, 597, 599, 601, 643, 691, 695, 696, 704 }

C grade: { 421, 426 }

F grade: { 48, 51, 52, 53, 68, 69, 89, 126, 133, 135, 145, 149, 174, 193, 194, 195, 196, 197, 198, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 238, 239, 242, 244, 245, 246, 247, 248, 249, 255, 256, 258, 259, 260, 261, 262, 263, 279, 281, 283, 287, 288, 289, 290, 291, 306, 307, 308, 309, 314, 315, 317, 319, 320, 321, 324, 325, 327, 328, 329, 352, 383, 390, 391, 395, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 413, 416, 417, 427, 432, 434, 435, 436, 438, 440, 441, 442, 443, 446, 447, 452, 454, 455, 457, 463, 473, 487, 494, 495, 500, 506, 511, 516, 521, 523, 527, 529, 533, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 583, 586, 587, 592, 595, 603, 608, 609, 618, 619, 621, 625, 626, 627, 637, 650, 657, 665, 668, 672, 674, 675, 683, 684, 688, 689, 693, 699, 701, 703 }

2.1.5 FriCAS

A grade: { 1, 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 58, 59, 60, 61, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 134, 140, 141, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 178, 179, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 194, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 225, 227, 230, 231, 233, 234, 238, 241, 243, 250, 251, 252, 258, 260, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 282, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 308, 309, 310, 312, 313, 316, 318, 320, 325, 326, 327, 330, 331, 332, 333, 334, 336, 339, 341, 344, 345, 346, 347, 348, 349, 350, 351, 356, 359, 360, 362, 364, 366, 368, 370, 373, 374, 375, 376, 378, 380, 381, 385, 386, 387, 395, 396, 398, 409, 412, 414, 415, 418, 419, 420, 422, 425, 428, 436, 437, 439, 440, 441, 448, 450, 454, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 492, 493, 494, 495, 496, 497, 498, 499, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 532, 534, 535, 536, 537, 538, 539, 540, 541, 542, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 570, 571, 576, 583, 585, 590, 598, 599, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 622, 623, 624, 626, 627, 628, 629, 630, 631, 632, 633, 634, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 658, 659, 660, 663, 664, 666, 667, 668, 669, 670, 671, 673, 676, 677, 679, 680, 681, 682, 684, 685, 686, 687, 688, 690, 691, 692, 693, 694, 695, 696, 697, 699, 700, 705 }

B grade: { 3, 4, 5, 19, 30, 31, 36, 37, 41, 53, 54, 55, 56, 57, 62, 63, 76, 99, 135, 149, 159, 161, 165, 174, 180, 187, 195, 196, 197, 202, 221, 222, 223, 224, 226, 228, 229, 232, 235, 236, 237, 239, 240, 242, 244, 245, 246, 247, 249, 253, 254, 255, 256, 257, 259, 268, 269, 276, 281, 283, 284, 303, 311, 314, 315, 317, 319, 321, 322, 323, 324, 328, 335, 337, 338, 340, 342, 343, 353, 354, 355, 357, 358, 361, 363, 365, 367, 369, 371, 372, 377, 379, 382, 383, 384, 388, 389, 390, 391, 392, 393, 394, 397, 399, 400, 402, 403, 404, 405, 406, 407, 408, 410, 411, 416, 423, 424, 427, 429, 430, 431, 432, 433, 434, 435, 438, 443, 447, 451, 452, 456, 457, 490, 491, 505, 510, 572, 573, 574, 575, 577, 578, 579, 580, 581, 582, 584, 586, 587, 588, 589, 591, 592, 593, 594, 595, 596, 597, 600, 601, 602, 603, 625, 635, 661, 662, 701, 702, 703, 704 }

C grade: { 177, 401, 421 }

F grade: { 126, 133, 136, 137, 138, 139, 142, 143, 144, 145, 193, 198, 248, 307, 329, 352, 413, 417, 426, 442, 444, 445, 446, 449, 453, 455, 500, 506, 511, 516, 521, 529, 533, 543, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 616, 657, 665, 672, 674, 675, 678, 683, 689, 698 }

2.1.6 Sympy

A grade: { 3, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 22, 23, 24, 25, 26, 27, 28, 29, 32, 33, 34, 35, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 60, 63, 65, 67, 68, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 120, 121, 124, 128, 129, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 199, 200, 201, 202, 203, 204, 205, 206, 208, 209, 210, 212, 240, 241, 250, 251, 292, 293, 300, 322, 323, 330, 331, 332, 333, 334, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 355, 356, 357, 358, 361, 365, 366, 367, 368, 371, 372, 377, 385, 387, 441, 458, 459, 460, 463, 464, 465, 466, 470, 471, 472, 476, 479, 480, 483, 484, 485, 486, 494, 496, 497, 498, 499, 501, 502, 503, 504, 507, 508, 509, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 530, 531, 535, 537, 538, 539, 541, 542, 543, 544, 545, 546, 564, 565, 566, 567, 568, 569, 570, 571, 577, 578, 583, 584, 585, 596, 597, 598, 599, 603, 610, 611, 612, 613, 614, 615, 617, 618, 619, 620, 621, 628, 629, 630, 631, 632, 634, 635, 637, 638, 641, 642, 644, 646, 647, 648, 651, 652, 653, 654, 656, 659, 660, 661, 662, 664, 666, 667, 668, 669, 671, 676, 677, 679, 680, 681, 699, 700 }

B grade: { 1, 4, 8, 19, 21, 30, 31, 36, 41, 62, 114, 149, 159, 194, 195, 196, 197, 211, 214, 215, 252, 253, 254, 295, 297, 299, 312, 335, 340, 353, 354, 364, 369, 370, 376, 378, 379, 382, 383, 467, 468, 475, 477, 487, 488, 489, 493, 547, 548, 572, 573, 576, 580, 589, 590, 591, 602, 604, 655, 670, 697 }

C grade: { 2, 118, 119, 122, 123, 125, 126, 127, 130, 132, 133, 134, 145, 175, 207, 216, 294, 296, 298, 301, 302, 303, 304, 305, 313, 316, 461, 462, 536, 616 }

F grade: { 54, 55, 56, 57, 58, 59, 61, 64, 66, 69, 86, 193, 198, 213, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 242, 243, 244, 245, 246, 247, 248, 249, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 306, 307, 308, 309, 310, 311, 314, 315, 317, 318, 319, 320, 321, 324, 325, 326, 327, 328, 329, 352, 359, 360, 362, 363, 373, 374, 375, 380, 381, 384, 386, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 469, 473, 474, 478, 481, 482, 490, 491, 492, 495, 500, 505, 506, 510, 511, 516, 521, 528, 529, 532, 533, 534, 540, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 574, 575, 579, 581, 582, 586, 587, 588, 592, 593, 594, 595, 600, 601, 605, 606, 607, 608, 609, 622, 623, 624, 625, 626, 627, 633, 636, 639, 640, 643, 645, 649, 650, 657, 658, 663, 665, 672, 673, 674, 675, 678, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 698, 701, 702, 703, 704, 705 }

2.1.7 Giac

A grade: { 2, 6, 7, 8, 10, 11, 12, 18, 20, 21, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 54, 56, 57, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 227, 229, 230, 235, 236, 237, 241, 250, 251, 252, 253, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 278, 282, 284, 285, 286, 288, 289, 290, 292, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 309, 312, 313, 330, 331, 332, 333, 334, 335, 336, 337, 339, 340, 341, 344, 345, 346, 347, 348, 349, 350, 351, 354, 355, 356, 358, 359, 360, 361, 362, 363, 364, 366, 368, 369, 370, 373, 375, 376, 377, 378, 379, 380, 381, 382, 385, 387, 394, 397, 398, 400, 401, 419, 420, 422, 423, 424, 425, 429, 430, 433, 440, 441, 442, 443, 450, 451, 452, 454, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 492, 493, 496, 497, 498, 499, 501, 502, 507, 512, 513, 514, 515, 517, 518, 519, 520, 522, 523, 524, 525, 526, 527, 528, 530, 531, 532, 534, 535, 536, 538, 539, 541, 542, 543, 544, 545, 546, 547, 548, 556, 558, 561, 563, 564, 565, 566, 567, 568, 569, 574, 583, 584, 585, 587, 589, 593, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 609, 610, 611, 612, 613, 614, 617, 618, 619, 621, 622, 624, 627, 628, 629, 630, 631, 633, 634, 637, 638, 639, 640, 641, 642, 643, 644, 646, 647, 651, 652, 653, 654, 655, 656, 659, 660, 661, 662, 663, 664, 667, 668, 669, 670, 671, 676, 677, 686, 687, 688, 691, 696, 697, 700, 701, 705 }

B grade: { 1, 3, 4, 5, 9, 13, 14, 15, 16, 17, 19, 22, 23, 52, 55, 58, 99, 128, 197, 220, 238, 239, 240, 242, 243, 244, 254, 255, 256, 257, 258, 259, 265, 274, 279, 280, 281, 283, 287, 308, 338, 342, 343, 353, 357, 365, 367, 371, 372, 374, 383, 384, 386, 388, 389, 431, 447, 449, 456, 474, 476, 487, 488, 489, 570, 571, 572, 573, 575, 576, 577, 578, 579, 580, 581, 586, 588, 590, 591, 596, 597, 620, 635, 645, 658, 666, 685, 690, 702, 704 }

C grade: { 79, 245, 246, 247, 421, 426, 438, 457, 494, 495, 503, 504, 505, 508, 509, 510, 537 }

F grade: { 64, 86, 126, 133, 145, 154, 177, 193, 198, 221, 222, 223, 224, 225, 226, 228, 231, 232, 233, 234, 248, 249, 291, 296, 306, 307, 310, 311, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 352, 390, 391, 392, 393, 395, 396, 399, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 427, 428, 432, 434, 435, 436, 437, 439, 444, 445, 446, 448, 453, 455, 473, 490, 491, 500, 506, 511, 516, 521, 529, 533, 540, 549, 550, 551, 552, 553, 554, 555, 557, 559, 560, 562, 582, 592, 594, 595, 608, 615, 616, 623, 625, 626, 632, 636, 648, 649, 650, 657, 665, 672, 673, 674, 675, 678, 679, 680, 681, 682, 683, 684, 689, 692, 693, 694, 695, 698, 699, 703 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	32	42	55	20	45
normalized size	1	1.	1.	2.29	3.	3.93	1.43	3.21
time (sec)	N/A	0.008	0.003	0.007	0.931	1.991	0.131	1.059

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	28	26	19
normalized size	1	1.	1.	1.07	1.36	2.	1.86	1.36
time (sec)	N/A	0.005	0.003	0.003	1.41	1.954	0.125	1.053

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	37	18	23	70	29	54
normalized size	1	1.	2.85	1.38	1.77	5.38	2.23	4.15
time (sec)	N/A	0.004	0.008	0.003	0.922	2.032	0.099	1.049

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	23	15	26	88	22	31
normalized size	1	1.	2.09	1.36	2.36	8.	2.	2.82
time (sec)	N/A	0.003	0.006	0.006	0.919	2.082	0.098	1.061

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	21	36	90	22	39
normalized size	1	1.	1.	1.4	2.4	6.	1.47	2.6
time (sec)	N/A	0.003	0.006	0.006	0.929	2.074	0.198	1.058

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	5	14	3	5
normalized size	1	1.	1.	1.5	2.5	7.	1.5	2.5
time (sec)	N/A	0.006	0.001	0.004	0.925	1.993	0.061	1.056

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	8	15	5	8
normalized size	1	1.	1.	1.25	2.	3.75	1.25	2.
time (sec)	N/A	0.006	0.002	0.	0.923	1.934	0.062	1.055

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	36	16	7	5
normalized size	1	1.	1.	0.83	6.	2.67	1.17	0.83
time (sec)	N/A	0.024	0.013	0.014	0.932	1.864	0.717	1.055

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	12	28	3	41
normalized size	1	1.	0.67	0.56	1.33	3.11	0.33	4.56
time (sec)	N/A	0.007	0.004	0.	0.928	1.84	0.172	1.06

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	14	30	7	11
normalized size	1	1.	0.67	0.75	1.17	2.5	0.58	0.92
time (sec)	N/A	0.008	0.008	0.	0.936	1.835	0.348	1.058

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	18	30	15	19
normalized size	1	1.	1.	1.08	1.5	2.5	1.25	1.58
time (sec)	N/A	0.023	0.019	0.007	0.931	2.169	0.366	1.047

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	35	31	20
normalized size	1	1.	1.	1.07	1.33	2.33	2.07	1.33
time (sec)	N/A	0.026	0.01	0.013	1.418	2.099	0.716	1.055

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	34	45	70	44	47
normalized size	1	1.	1.	2.27	3.	4.67	2.93	3.13
time (sec)	N/A	0.028	0.009	0.011	0.93	2.16	0.733	1.045

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	23	49	32	104
normalized size	1	1.	1.	1.06	1.35	2.88	1.88	6.12
time (sec)	N/A	0.035	0.009	0.021	0.956	2.171	3.152	1.117

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	27	49	34	104
normalized size	1	1.	1.	1.05	1.42	2.58	1.79	5.47
time (sec)	N/A	0.038	0.009	0.021	0.928	2.235	2.928	1.076

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	22	22	19	24	41	34	130
normalized size	1	1.22	1.22	1.06	1.33	2.28	1.89	7.22
time (sec)	N/A	0.04	0.015	0.016	0.929	2.374	3.034	1.074

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	22	22	19	26	39	32	165
normalized size	1	1.22	1.22	1.06	1.44	2.17	1.78	9.17
time (sec)	N/A	0.042	0.013	0.017	0.925	2.635	2.947	1.094

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	19	14	18	104	61	62
normalized size	1	1.	0.46	0.34	0.44	2.54	1.49	1.51
time (sec)	N/A	0.017	0.025	0.013	1.419	2.372	0.758	1.046

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	20	51	15	22
normalized size	1	1.	1.	1.	3.33	8.5	2.5	3.67
time (sec)	N/A	0.018	0.002	0.	0.927	2.265	0.095	1.069

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	18	3	5
normalized size	1	1.	1.	1.33	1.33	6.	1.	1.67
time (sec)	N/A	0.012	0.004	0.	0.925	2.157	0.088	1.046

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	22	15	4
normalized size	1	1.	1.	1.33	1.33	7.33	5.	1.33
time (sec)	N/A	0.019	0.01	0.001	1.403	2.268	0.128	1.052

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	7	10	9	24	7	30
normalized size	1	1.	0.78	1.11	1.	2.67	0.78	3.33
time (sec)	N/A	0.02	0.009	0.001	0.927	2.294	0.089	1.064

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	26	7	77
normalized size	1	1.	1.	1.11	1.33	2.89	0.78	8.56
time (sec)	N/A	0.018	0.016	0.	0.932	2.277	0.095	1.053

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	30	70	22	31
normalized size	1	1.	1.	0.88	1.2	2.8	0.88	1.24
time (sec)	N/A	0.018	0.013	0.008	0.93	2.187	0.402	1.051

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	21	27	78	31	28
normalized size	1	1.	1.	0.7	0.9	2.6	1.03	0.93
time (sec)	N/A	0.094	0.017	0.006	0.927	2.207	5.277	1.064

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	27	8	15
normalized size	1	1.	1.	1.1	1.4	2.7	0.8	1.5
time (sec)	N/A	0.005	0.002	0.001	0.942	2.071	0.073	1.05

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	24	49	105	42	50
normalized size	1	1.	1.	0.51	1.04	2.23	0.89	1.06
time (sec)	N/A	0.023	0.028	0.002	1.409	2.031	0.107	1.071

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	24	31	77	27	28
normalized size	1	1.	1.	0.8	1.03	2.57	0.9	0.93
time (sec)	N/A	0.008	0.013	0.002	1.408	2.157	0.104	1.068

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	18	77	22	18
normalized size	1	1.	1.	0.67	0.86	3.67	1.05	0.86
time (sec)	N/A	0.008	0.011	0.027	0.925	2.412	0.539	1.046

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	78	26	15
normalized size	1	1.	1.	0.8	1.	5.2	1.73	1.
time (sec)	N/A	0.008	0.006	0.	0.939	2.128	0.523	1.063

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	251	261	138	24
normalized size	1	1.	1.	0.86	11.95	12.43	6.57	1.14
time (sec)	N/A	0.034	0.077	0.036	1.477	2.097	1.312	1.092

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	38	10	14
normalized size	1	1.	1.	0.79	1.	2.71	0.71	1.
time (sec)	N/A	0.005	0.002	0.	0.961	2.004	0.062	1.064

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	36	10	14
normalized size	1	1.	1.	0.79	1.	2.57	0.71	1.
time (sec)	N/A	0.006	0.002	0.	0.96	2.067	0.07	1.055

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	20	7	8
normalized size	1	1.	1.	0.88	1.	2.5	0.88	1.
time (sec)	N/A	0.012	0.001	0.003	0.925	1.931	0.058	1.06

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	32	19	62	14	19
normalized size	1	1.	1.	2.91	1.73	5.64	1.27	1.73
time (sec)	N/A	0.015	0.007	0.006	0.962	1.941	0.087	1.061

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	6	15	12	47	12	12
normalized size	1	1.	0.86	2.14	1.71	6.71	1.71	1.71
time (sec)	N/A	0.024	0.007	0.	0.96	1.822	0.07	1.054

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	28	14	16	72	19	24
normalized size	1	1.	2.	1.	1.14	5.14	1.36	1.71
time (sec)	N/A	0.006	0.013	0.003	1.46	2.016	0.069	1.064

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	26	19	16	61	17	30
normalized size	1	1.	1.62	1.19	1.	3.81	1.06	1.88
time (sec)	N/A	0.011	0.011	0.001	1.417	2.001	0.141	1.094

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	16	50	10	16
normalized size	1	1.	1.	1.1	1.6	5.	1.	1.6
time (sec)	N/A	0.028	0.016	0.014	1.465	2.045	0.393	1.055

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	12	15	19	89	10	19
normalized size	1	1.	0.86	1.07	1.36	6.36	0.71	1.36
time (sec)	N/A	0.063	0.007	0.013	1.475	2.049	1.118	1.07

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	25	13	38	88	34	16
normalized size	1	1.	2.27	1.18	3.45	8.	3.09	1.45
time (sec)	N/A	0.022	0.039	0.011	1.444	1.963	0.538	1.063

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	21	13	31	45	8	16
normalized size	1	1.	1.31	0.81	1.94	2.81	0.5	1.
time (sec)	N/A	0.026	0.023	0.01	1.47	1.801	0.48	1.051

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	29	24	70	19	24
normalized size	1	1.	1.	1.16	0.96	2.8	0.76	0.96
time (sec)	N/A	0.025	0.013	0.005	0.956	1.915	0.098	1.053

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	18	46	12	20
normalized size	1	1.	1.	0.67	0.86	2.19	0.57	0.95
time (sec)	N/A	0.004	0.003	0.004	0.967	1.868	0.09	1.065

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	36	10	14
normalized size	1	1.	1.	0.79	1.	2.57	0.71	1.
time (sec)	N/A	0.016	0.004	0.003	1.44	1.84	0.105	1.071

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	47	29	47	128	42	59
normalized size	1	1.	0.92	0.57	0.92	2.51	0.82	1.16
time (sec)	N/A	0.016	0.026	0.002	1.48	1.813	0.105	1.044

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	47	34	86	29	34
normalized size	1	1.	1.	1.42	1.03	2.61	0.88	1.03
time (sec)	N/A	0.01	0.012	0.008	0.956	1.879	0.115	1.047

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	19	0	163	94	24
normalized size	1	1.	1.	0.66	0.	5.62	3.24	0.83
time (sec)	N/A	0.014	0.007	0.008	0.	2.018	1.583	1.052

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	17	30	51	37	26
normalized size	1	1.	0.74	0.63	1.11	1.89	1.37	0.96
time (sec)	N/A	0.01	0.006	0.004	1.475	1.984	0.664	1.051

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	24	53	31	14
normalized size	1	1.	1.	0.86	1.09	2.41	1.41	0.64
time (sec)	N/A	0.009	0.003	0.01	1.488	1.761	1.064	1.077

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	41	0	53	24	27
normalized size	1	1.	1.	1.86	0.	2.41	1.09	1.23
time (sec)	N/A	0.013	0.004	0.003	0.	1.964	1.087	1.049

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	37	0	45	24	58
normalized size	1	1.	1.	1.61	0.	1.96	1.04	2.52
time (sec)	N/A	0.014	0.004	0.005	0.	1.88	1.076	1.072

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	35	0	90	7	50
normalized size	1	1.	1.	1.67	0.	4.29	0.33	2.38
time (sec)	N/A	0.013	0.003	0.004	0.	1.869	1.016	1.065

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	7	11	77	0	8
normalized size	1	1.	1.	0.58	0.92	6.42	0.	0.67
time (sec)	N/A	0.006	0.005	0.004	1.431	1.781	0.	1.063

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	15	22	111	0	45
normalized size	1	1.	0.95	0.79	1.16	5.84	0.	2.37
time (sec)	N/A	0.012	0.008	0.003	1.553	2.036	0.	1.079

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	12	7	8	39	0	8
normalized size	1	1.	1.5	0.88	1.	4.88	0.	1.
time (sec)	N/A	0.003	0.009	0.003	1.431	1.807	0.	1.06

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	28	111	0	28
normalized size	1	1.	1.	0.81	1.04	4.11	0.	1.04
time (sec)	N/A	0.01	0.006	0.007	1.427	1.918	0.	1.072

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	25	45	111	0	96
normalized size	1	1.	1.	0.78	1.41	3.47	0.	3.
time (sec)	N/A	0.011	0.006	0.003	1.441	1.922	0.	1.1

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	23	21	16	23	43	0	38
normalized size	1	1.1	1.	0.76	1.1	2.05	0.	1.81
time (sec)	N/A	0.005	0.004	0.003	1.415	1.743	0.	1.056

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	54	23	45	147	22	42
normalized size	1	1.	1.93	0.82	1.61	5.25	0.79	1.5
time (sec)	N/A	0.129	0.06	0.048	0.959	1.978	0.795	1.079

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	48	17	14	18	108	0	62
normalized size	1	1.3	0.46	0.38	0.49	2.92	0.	1.68
time (sec)	N/A	0.021	0.049	0.012	1.433	1.963	0.	1.077

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	21	11	63	101	27	15
normalized size	1	1.	1.5	0.79	4.5	7.21	1.93	1.07
time (sec)	N/A	0.034	0.015	0.047	0.954	2.041	1.642	1.077

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	23	4	20	139	15	23
normalized size	1	1.	2.09	0.36	1.82	12.64	1.36	2.09
time (sec)	N/A	0.031	0.007	0.007	0.943	1.923	0.148	1.085

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	19	19	15	19	41	0	0
normalized size	1	1.06	1.06	0.83	1.06	2.28	0.	0.
time (sec)	N/A	0.047	0.017	0.015	0.941	2.425	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	47	17	35
normalized size	1	1.	1.	0.83	1.06	2.61	0.94	1.94
time (sec)	N/A	0.045	0.019	0.02	0.945	2.158	2.574	1.151

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	46	0	20
normalized size	1	1.	1.	0.94	1.18	2.71	0.	1.18
time (sec)	N/A	0.013	0.004	0.007	1.445	2.099	0.	1.088

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	0	24	47	7	22
normalized size	1	1.	1.	0.	1.2	2.35	0.35	1.1
time (sec)	N/A	0.026	0.004	0.021	0.966	1.978	0.618	1.15

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	42	15	16
normalized size	1	1.	1.	1.08	0.	3.5	1.25	1.33
time (sec)	N/A	0.023	0.004	0.003	0.	2.074	4.109	1.112

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	38	0	92	0	16
normalized size	1	1.	1.	0.9	0.	2.19	0.	0.38
time (sec)	N/A	0.065	0.04	0.047	0.	1.997	0.	1.27

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	23	7	8
normalized size	1	1.	1.	0.88	1.	2.88	0.88	1.
time (sec)	N/A	0.025	0.005	0.004	1.406	1.902	2.797	1.102

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	23	61	22	30
normalized size	1	1.	1.	0.82	0.82	2.18	0.79	1.07
time (sec)	N/A	0.01	0.001	0.001	0.935	1.946	0.096	1.058

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	35	15	18
normalized size	1	1.	1.	0.82	1.06	2.06	0.88	1.06
time (sec)	N/A	0.007	0.001	0.002	0.934	1.68	0.092	1.052

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	38	38	53	38	80	26	39
normalized size	1	1.06	1.06	1.47	1.06	2.22	0.72	1.08
time (sec)	N/A	0.026	0.006	0.012	0.94	1.887	0.118	1.057

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	23	17	20	58	17	20
normalized size	1	1.	1.21	0.89	1.05	3.05	0.89	1.05
time (sec)	N/A	0.008	0.002	0.	0.933	1.901	0.06	1.049

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	26	24	81	31	30
normalized size	1	1.	0.88	0.76	0.71	2.38	0.91	0.88
time (sec)	N/A	0.034	0.008	0.	0.936	1.868	0.059	1.067

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	71	26	57	227	46	51
normalized size	1	1.	2.73	1.	2.19	8.73	1.77	1.96
time (sec)	N/A	0.012	0.006	0.031	0.933	1.921	0.135	1.054

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	14	18	15	54	17	15
normalized size	1	1.	0.61	0.78	0.65	2.35	0.74	0.65
time (sec)	N/A	0.008	0.012	0.004	0.938	2.159	0.46	1.051

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	26	65	26	26
normalized size	1	1.	0.81	0.81	0.96	2.41	0.96	0.96
time (sec)	N/A	0.009	0.035	0.004	0.931	2.324	0.331	1.056

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	71	32	61	107	444
normalized size	1	1.	0.65	2.29	1.03	1.97	3.45	14.32
time (sec)	N/A	0.01	0.018	0.017	0.949	2.212	1.061	1.092

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	53	15	18
normalized size	1	1.	1.	0.82	0.82	3.12	0.88	1.06
time (sec)	N/A	0.003	0.002	0.	0.933	2.189	0.37	1.054

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	61	127	68	15	16
normalized size	1	1.	1.	5.08	10.58	5.67	1.25	1.33
time (sec)	N/A	0.019	0.015	0.037	1.428	1.868	135.385	1.077

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	144	66	19	31
normalized size	1	1.	1.	1.33	9.6	4.4	1.27	2.07
time (sec)	N/A	0.014	0.018	0.009	1.428	2.029	0.161	1.072

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	45	105	22	51
normalized size	1	1.	1.	0.95	2.05	4.77	1.	2.32
time (sec)	N/A	0.015	0.002	0.003	1.42	2.222	1.652	1.06

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	68	22	31
normalized size	1	1.	1.	0.96	1.24	2.72	0.88	1.24
time (sec)	N/A	0.032	0.006	0.019	1.426	1.852	0.177	1.053

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	32	68	19	26
normalized size	1	1.	1.	0.87	1.39	2.96	0.83	1.13
time (sec)	N/A	0.047	0.014	0.006	1.451	2.027	0.381	1.055

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	57	49	44	105	85	0	0
normalized size	1	1.5	1.29	1.16	2.76	2.24	0.	0.
time (sec)	N/A	0.031	0.059	0.012	1.426	2.008	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	50	22	28
normalized size	1	1.	1.	0.88	1.12	2.	0.88	1.12
time (sec)	N/A	0.007	0.002	0.	0.929	1.571	0.055	1.048

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	30	39	72	34	39
normalized size	1	1.	1.	0.77	1.	1.85	0.87	1.
time (sec)	N/A	0.016	0.001	0.	0.931	1.596	0.059	1.063

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	29	0	55	75	31
normalized size	1	1.	1.	1.26	0.	2.39	3.26	1.35
time (sec)	N/A	0.008	0.01	0.043	0.	1.938	55.273	1.059

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	27	23	30	63	20	31
normalized size	1	1.	0.9	0.77	1.	2.1	0.67	1.03
time (sec)	N/A	0.011	0.006	0.003	0.93	1.778	0.07	1.058

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	35	27	35	112	34	35
normalized size	1	1.	0.85	0.66	0.85	2.73	0.83	0.85
time (sec)	N/A	0.012	0.01	0.003	1.417	1.763	0.092	1.054

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	20	49	14	23
normalized size	1	1.	1.	0.76	0.95	2.33	0.67	1.1
time (sec)	N/A	0.006	0.003	0.005	0.934	1.771	0.09	1.049

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	29	38	89	36	38
normalized size	1	1.	0.97	0.88	1.15	2.7	1.09	1.15
time (sec)	N/A	0.017	0.009	0.003	1.416	1.819	0.101	1.054

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	68	22	28
normalized size	1	1.	1.	0.88	1.12	2.72	0.88	1.12
time (sec)	N/A	0.016	0.004	0.003	1.407	1.752	0.097	1.045

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	47	39	51	115	46	51
normalized size	1	1.	1.	0.83	1.09	2.45	0.98	1.09
time (sec)	N/A	0.062	0.016	0.004	1.419	1.858	0.103	1.047

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	23	62	17	27
normalized size	1	1.	1.	0.72	0.92	2.48	0.68	1.08
time (sec)	N/A	0.037	0.006	0.008	0.932	1.885	0.117	1.06

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	26	34	77	26	38
normalized size	1	1.	0.88	0.79	1.03	2.33	0.79	1.15
time (sec)	N/A	0.054	0.014	0.008	0.939	1.82	0.371	1.057

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	61	37	26	34	92	29	39
normalized size	1	1.65	1.	0.7	0.92	2.49	0.78	1.05
time (sec)	N/A	0.037	0.006	0.01	0.929	1.948	0.157	1.058

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	69	26	58	142	60	80
normalized size	1	1.	2.23	0.84	1.87	4.58	1.94	2.58
time (sec)	N/A	0.009	0.018	0.008	1.426	2.123	0.477	1.063

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	26	34	92	26	39
normalized size	1	1.	1.	0.63	0.83	2.24	0.63	0.95
time (sec)	N/A	0.02	0.007	0.008	0.923	2.204	0.154	1.047

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	16	20	30	76	17	24
normalized size	1	1.	0.64	0.8	1.2	3.04	0.68	0.96
time (sec)	N/A	0.011	0.009	0.004	0.926	2.104	0.089	1.047

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	43	117	31	61
normalized size	1	1.	1.	0.92	1.19	3.25	0.86	1.69
time (sec)	N/A	0.017	0.014	0.005	0.921	2.106	0.08	1.05

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	37	28	41	138	31	36
normalized size	1	1.	0.9	0.68	1.	3.37	0.76	0.88
time (sec)	N/A	0.037	0.02	0.007	0.933	2.085	0.124	1.054

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	39	136	26	50
normalized size	1	1.	1.	1.04	1.44	5.04	0.96	1.85
time (sec)	N/A	0.036	0.015	0.009	0.943	2.053	0.125	1.059

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	47	32	39	131	31	47
normalized size	1	1.	1.21	0.82	1.	3.36	0.79	1.21
time (sec)	N/A	0.036	0.017	0.011	0.956	2.199	0.136	1.049

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	35	54	150	37	54
normalized size	1	1.	0.87	0.76	1.17	3.26	0.8	1.17
time (sec)	N/A	0.022	0.016	0.008	0.93	2.117	0.146	1.051

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	31	77	22	32
normalized size	1	1.	1.	0.83	1.07	2.66	0.76	1.1
time (sec)	N/A	0.028	0.007	0.006	1.4	2.082	0.108	1.057

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	22	28	82	22	31
normalized size	1	1.	1.	0.81	1.04	3.04	0.81	1.15
time (sec)	N/A	0.025	0.007	0.004	1.384	2.022	0.157	1.062

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	32	26	34	93	29	36
normalized size	1	1.	1.33	1.08	1.42	3.88	1.21	1.5
time (sec)	N/A	0.01	0.009	0.007	1.43	2.149	0.131	1.058

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	43	142	39	45
normalized size	1	1.	1.	0.8	1.05	3.46	0.95	1.1
time (sec)	N/A	0.087	0.021	0.007	1.43	2.177	0.139	1.052

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	37	49	171	37	84
normalized size	1	1.	0.87	0.8	1.07	3.72	0.8	1.83
time (sec)	N/A	0.204	0.019	0.008	1.426	2.114	0.187	1.061

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	37	34	49	171	36	43
normalized size	1	1.	0.79	0.72	1.04	3.64	0.77	0.91
time (sec)	N/A	0.043	0.024	0.006	1.454	2.106	0.149	1.053

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	73	54	72	180	70	72
normalized size	1	1.	1.09	0.81	1.07	2.69	1.04	1.07
time (sec)	N/A	0.038	0.058	0.005	1.435	2.073	0.163	1.055

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	67	46	73	190	172	86
normalized size	1	1.	1.4	0.96	1.52	3.96	3.58	1.79
time (sec)	N/A	0.053	0.018	0.007	1.415	2.215	1.323	1.072

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	46	41	58	203	46	53
normalized size	1	1.	0.79	0.71	1.	3.5	0.79	0.91
time (sec)	N/A	0.055	0.024	0.008	1.422	2.159	0.167	1.072

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	47	36	47	146	44	47
normalized size	1	1.	0.92	0.71	0.92	2.86	0.86	0.92
time (sec)	N/A	0.284	0.024	0.009	1.413	2.238	0.357	1.061

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	34	45	105	32	45
normalized size	1	1.	1.	0.83	1.1	2.56	0.78	1.1
time (sec)	N/A	0.311	0.008	0.01	0.921	2.173	0.162	1.06

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	52	52	66	124	73	68
normalized size	1	1.	0.93	0.93	1.18	2.21	1.3	1.21
time (sec)	N/A	0.03	0.012	0.008	1.405	2.054	0.12	1.053

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	50	52	66	122	71	68
normalized size	1	1.	0.89	0.93	1.18	2.18	1.27	1.21
time (sec)	N/A	0.028	0.005	0.005	1.399	2.084	0.114	1.052

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	27	8	15
normalized size	1	1.	1.	0.92	1.17	2.25	0.67	1.25
time (sec)	N/A	0.003	0.002	0.	0.918	2.061	0.092	1.058

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	34	31	51	19	30
normalized size	1	1.	1.	1.55	1.41	2.32	0.86	1.36
time (sec)	N/A	0.008	0.004	0.005	0.922	2.018	0.179	1.055

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	60	60	77	147	83	78
normalized size	1	1.	0.95	0.95	1.22	2.33	1.32	1.24
time (sec)	N/A	0.037	0.014	0.007	1.398	2.333	0.299	1.049

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	68	60	77	161	80	78
normalized size	1	1.	1.05	0.92	1.18	2.48	1.23	1.2
time (sec)	N/A	0.039	0.015	0.008	1.404	2.221	0.323	1.053

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	43	42	77	29	54
normalized size	1	1.	1.	1.3	1.27	2.33	0.88	1.64
time (sec)	N/A	0.02	0.005	0.006	0.925	2.037	0.391	1.053

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	74	67	89	178	90	90
normalized size	1	1.	1.01	0.92	1.22	2.44	1.23	1.23
time (sec)	N/A	0.044	0.013	0.009	1.411	2.088	0.376	1.073

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	0	92	0
normalized size	1	1.	0.98	0.	0.	0.	2.	0.
time (sec)	N/A	0.01	0.01	0.041	0.	0.	4.574	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	38	33	43	70	37	46
normalized size	1	1.	1.41	1.22	1.59	2.59	1.37	1.7
time (sec)	N/A	0.008	0.004	0.008	1.401	2.129	0.131	1.049

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	30	39	59	24	41
normalized size	1	1.	1.	2.	2.6	3.93	1.6	2.73
time (sec)	N/A	0.007	0.003	0.004	0.922	1.912	0.137	1.059

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	41	34	53	19	35
normalized size	1	1.	1.	1.71	1.42	2.21	0.79	1.46
time (sec)	N/A	0.01	0.005	0.008	0.917	1.694	0.212	1.044

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	46	41	54	93	44	57
normalized size	1	1.	1.31	1.17	1.54	2.66	1.26	1.63
time (sec)	N/A	0.014	0.007	0.007	1.413	1.977	0.33	1.061

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	50	43	50	89	34	51
normalized size	1	1.	1.92	1.65	1.92	3.42	1.31	1.96
time (sec)	N/A	0.012	0.007	0.006	0.924	1.82	0.347	1.07

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	48	41	54	112	48	57
normalized size	1	1.	1.3	1.11	1.46	3.03	1.3	1.54
time (sec)	N/A	0.012	0.007	0.007	1.406	1.845	0.369	1.049

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	44	0	0	0	95	0
normalized size	1	1.	0.98	0.	0.	0.	2.11	0.
time (sec)	N/A	0.008	0.009	0.042	0.	0.	0.879	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	34	29	18
normalized size	1	1.	1.	0.93	1.2	2.27	1.93	1.2
time (sec)	N/A	0.006	0.003	0.003	1.413	1.858	0.127	1.072

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	79	101	0	651	19	154
normalized size	1	1.	0.72	0.93	0.	5.97	0.17	1.41
time (sec)	N/A	0.062	0.028	0.005	0.	1.944	0.126	1.059

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	204	101	385	0	39	239
normalized size	1	1.	1.01	0.5	1.92	0.	0.19	1.19
time (sec)	N/A	0.314	0.15	0.011	1.422	0.	0.127	1.058

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-1)	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	204	97	358	0	41	239
normalized size	1	1.	1.01	0.48	1.78	0.	0.2	1.19
time (sec)	N/A	0.271	0.057	0.005	1.42	0.	0.127	1.087

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-1)	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	204	101	358	0	41	239
normalized size	1	1.	1.01	0.5	1.78	0.	0.2	1.19
time (sec)	N/A	0.317	0.077	0.007	1.425	0.	0.139	1.079

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	204	97	385	0	39	239
normalized size	1	1.	1.01	0.48	1.92	0.	0.19	1.19
time (sec)	N/A	0.313	0.046	0.007	1.421	0.	0.134	1.072

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	27	8	15
normalized size	1	1.	1.	0.92	1.17	2.25	0.67	1.25
time (sec)	N/A	0.003	0.002	0.001	0.929	1.798	0.102	1.059

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	49	31	51	19	30
normalized size	1	1.	1.	2.23	1.41	2.32	0.86	1.36
time (sec)	N/A	0.008	0.004	0.007	0.927	1.9	0.225	1.067

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	172	109	398	0	48	250
normalized size	1	1.	0.82	0.52	1.9	0.	0.23	1.2
time (sec)	N/A	0.329	0.162	0.008	1.427	0.	0.331	1.085

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-1)	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	174	105	373	0	51	250
normalized size	1	1.	0.82	0.5	1.77	0.	0.24	1.18
time (sec)	N/A	0.327	0.145	0.008	1.423	0.	0.364	1.09

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-1)	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	175	109	373	0	51	250
normalized size	1	1.	0.83	0.52	1.77	0.	0.24	1.18
time (sec)	N/A	0.288	0.13	0.008	1.43	0.	0.383	1.079

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	45	0	0	0	92	0
normalized size	1	1.	0.98	0.	0.	0.	2.	0.
time (sec)	N/A	0.01	0.009	0.043	0.	0.	34.013	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	21	28	36	39	8	36
normalized size	1	1.	0.6	0.8	1.03	1.11	0.23	1.03
time (sec)	N/A	0.425	0.007	0.024	1.412	2.	0.11	1.046

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	51	52	73	216	63	59
normalized size	1	1.	0.85	0.87	1.22	3.6	1.05	0.98
time (sec)	N/A	0.022	0.028	0.003	1.418	1.98	0.152	1.066

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	30	28	51	132	36	38
normalized size	1	1.	0.7	0.65	1.19	3.07	0.84	0.88
time (sec)	N/A	0.014	0.012	0.006	1.409	2.044	0.135	1.056

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	88	146	0	932	323	124
normalized size	1	1.	0.98	1.62	0.	10.36	3.59	1.38
time (sec)	N/A	0.071	0.087	0.007	0.	1.982	1.111	1.07

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	45	140	31	45
normalized size	1	1.	1.	0.92	1.18	3.68	0.82	1.18
time (sec)	N/A	0.026	0.013	0.006	1.398	1.981	0.129	1.061

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	53	57	171	53	58
normalized size	1	1.	0.86	0.93	1.	3.	0.93	1.02
time (sec)	N/A	0.024	0.023	0.01	1.478	2.09	0.141	1.044

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	29	42	109	32	38
normalized size	1	1.	0.92	0.81	1.17	3.03	0.89	1.06
time (sec)	N/A	0.023	0.016	0.009	1.447	1.945	0.133	1.055

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	78	41	54	122	46	54
normalized size	1	1.	1.59	0.84	1.1	2.49	0.94	1.1
time (sec)	N/A	0.037	0.014	0.006	1.416	2.101	0.13	1.071

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	20	20	17	19	26	14	0
normalized size	1	1.54	1.54	1.31	1.46	2.	1.08	0.
time (sec)	N/A	0.046	0.023	0.013	0.939	2.226	2.511	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	38	35	46	103	42	46
normalized size	1	1.	0.93	0.85	1.12	2.51	1.02	1.12
time (sec)	N/A	0.04	0.01	0.003	1.405	1.973	0.109	1.043

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	25	32	69	24	34
normalized size	1	1.	1.	0.78	1.	2.16	0.75	1.06
time (sec)	N/A	0.025	0.005	0.005	0.926	2.011	0.107	1.059

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	23	54	15	26
normalized size	1	1.	1.	0.72	0.92	2.16	0.6	1.04
time (sec)	N/A	0.017	0.004	0.005	0.928	2.049	0.102	1.059

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	24	27	43	123	29	31
normalized size	1	1.	0.67	0.75	1.19	3.42	0.81	0.86
time (sec)	N/A	0.022	0.014	0.005	0.928	1.991	0.109	1.052

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	24	30	97	196	71	30
normalized size	1	1.	0.53	0.67	2.16	4.36	1.58	0.67
time (sec)	N/A	0.012	0.006	0.005	0.937	1.895	0.161	1.055

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	41	46	65	178	42	74
normalized size	1	1.	0.75	0.84	1.18	3.24	0.76	1.35
time (sec)	N/A	0.039	0.012	0.005	0.934	1.942	0.118	1.062

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	51	155	41	58
normalized size	1	1.	1.06	0.97	1.42	4.31	1.14	1.61
time (sec)	N/A	0.015	0.021	0.008	0.935	1.987	0.125	1.051

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	31	28	42	124	29	54
normalized size	1	1.	0.89	0.8	1.2	3.54	0.83	1.54
time (sec)	N/A	0.014	0.018	0.008	0.928	1.932	0.119	1.065

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	46	42	68	224	51	57
normalized size	1	1.	0.75	0.69	1.11	3.67	0.84	0.93
time (sec)	N/A	0.012	0.022	0.009	0.921	1.942	0.145	1.061

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	36	44	59	180	42	46
normalized size	1	1.	0.71	0.86	1.16	3.53	0.82	0.9
time (sec)	N/A	0.014	0.016	0.003	1.416	2.122	0.152	1.055

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	49	81	306	58	63
normalized size	1	1.	1.	0.91	1.5	5.67	1.07	1.17
time (sec)	N/A	0.026	0.015	0.01	0.929	2.268	0.159	1.053

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	42	53	32	54	136	49	65
normalized size	1	1.17	1.47	0.89	1.5	3.78	1.36	1.81
time (sec)	N/A	0.015	0.035	0.006	1.406	2.072	0.103	1.054

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	40	32	55	166	39	42
normalized size	1	1.	0.69	0.55	0.95	2.86	0.67	0.72
time (sec)	N/A	0.017	0.019	0.007	1.405	1.875	0.141	1.053

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	37	49	140	42	49
normalized size	1	1.	1.	0.86	1.14	3.26	0.98	1.14
time (sec)	N/A	0.021	0.034	0.004	1.409	1.964	0.129	1.066

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	62	37	63	184	58	69
normalized size	1	1.	1.44	0.86	1.47	4.28	1.35	1.6
time (sec)	N/A	0.023	0.037	0.003	1.477	1.986	0.124	1.061

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	25	28	41	92	24	54
normalized size	1	1.	0.68	0.76	1.11	2.49	0.65	1.46
time (sec)	N/A	0.013	0.01	0.007	1.446	1.944	0.149	1.067

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	39	26	29	38	92	26	38
normalized size	1	1.22	0.81	0.91	1.19	2.88	0.81	1.19
time (sec)	N/A	0.026	0.013	0.007	0.95	1.965	0.128	1.058

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	41	20	15
normalized size	1	1.	1.	0.92	1.15	3.15	1.54	1.15
time (sec)	N/A	0.002	0.003	0.	0.922	1.874	0.796	1.058

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	46	49	77	181	51	76
normalized size	1	1.	0.85	0.91	1.43	3.35	0.94	1.41
time (sec)	N/A	0.03	0.024	0.016	0.928	1.971	2.211	1.054

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	134	152	0	987	65	203
normalized size	1	1.	0.85	0.97	0.	6.29	0.41	1.29
time (sec)	N/A	0.101	0.105	0.012	0.	2.116	3.766	1.061

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	75	57	81	173	76	68
normalized size	1	1.	1.17	0.89	1.27	2.7	1.19	1.06
time (sec)	N/A	0.03	0.048	0.013	1.414	2.016	6.353	1.063

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	34	46	112	27	41
normalized size	1	1.	0.85	0.87	1.18	2.87	0.69	1.05
time (sec)	N/A	0.025	0.013	0.009	0.924	1.95	0.171	1.108

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	C	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	293	354	452	6294	37	0
normalized size	1	1.	0.92	1.11	1.42	19.73	0.12	0.
time (sec)	N/A	0.584	0.304	0.101	1.44	145.437	0.27	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	65	39	76	182	58	74
normalized size	1	1.	1.1	0.66	1.29	3.08	0.98	1.25
time (sec)	N/A	0.021	0.064	0.009	1.411	1.782	0.153	1.055

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	29	42	109	32	38
normalized size	1	1.	0.92	0.81	1.17	3.03	0.89	1.06
time (sec)	N/A	0.024	0.006	0.	1.418	1.746	0.132	1.053

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	31	51	154	36	47
normalized size	1	1.	0.84	0.82	1.34	4.05	0.95	1.24
time (sec)	N/A	0.062	0.021	0.01	0.926	1.794	0.126	1.045

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	65	61	68	213	60	81
normalized size	1	1.	1.02	0.95	1.06	3.33	0.94	1.27
time (sec)	N/A	0.073	0.033	0.011	1.412	2.028	0.198	1.057

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	54	80	277	66	97
normalized size	1	1.	1.	0.86	1.27	4.4	1.05	1.54
time (sec)	N/A	0.121	0.035	0.01	1.415	1.991	0.172	1.073

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	36	49	153	34	45
normalized size	1	1.	1.07	0.84	1.14	3.56	0.79	1.05
time (sec)	N/A	0.019	0.019	0.01	0.937	1.865	0.136	1.064

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	24	58	130	37	24
normalized size	1	1.	0.74	0.89	2.15	4.81	1.37	0.89
time (sec)	N/A	0.019	0.007	0.007	0.934	1.747	0.181	1.052

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	39	53	143	34	54
normalized size	1	1.	0.96	0.85	1.15	3.11	0.74	1.17
time (sec)	N/A	0.03	0.013	0.01	0.925	1.705	0.121	1.072

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	50	39	35	50	147	34	58
normalized size	1	1.14	0.89	0.8	1.14	3.34	0.77	1.32
time (sec)	N/A	0.017	0.021	0.009	0.927	1.763	0.128	1.059

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	49	81	306	58	63
normalized size	1	1.	1.	0.91	1.5	5.67	1.07	1.17
time (sec)	N/A	0.018	0.015	0.	0.927	1.756	0.16	1.051

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	43	117	31	61
normalized size	1	1.	1.	0.92	1.19	3.25	0.86	1.69
time (sec)	N/A	0.015	0.014	0.	0.93	1.585	0.08	1.052

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	39	51	107	39	53
normalized size	1	1.	0.93	0.89	1.16	2.43	0.89	1.2
time (sec)	N/A	0.037	0.012	0.003	0.926	1.471	0.058	1.067

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	91	95	123	252	100	132
normalized size	1	1.	0.95	0.99	1.28	2.62	1.04	1.38
time (sec)	N/A	0.104	0.028	0.001	0.933	1.565	0.075	1.058

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	167	237	231	460	189	254
normalized size	1	1.	1.	1.42	1.38	2.75	1.13	1.52
time (sec)	N/A	0.188	0.033	0.001	0.933	1.514	0.088	1.063

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	263	263	363	369	752	313	414
normalized size	1	1.	1.	1.38	1.4	2.86	1.19	1.57
time (sec)	N/A	0.337	0.059	0.001	0.937	1.544	0.108	1.051

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	267	0	0	0	0	0
normalized size	1	1.	1.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.443	0.145	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	66	95	0	446	246	81
normalized size	1	1.	1.02	1.46	0.	6.86	3.78	1.25
time (sec)	N/A	0.042	0.043	0.004	0.	1.75	0.61	1.062

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	88	146	0	964	323	124
normalized size	1	1.	0.99	1.64	0.	10.83	3.63	1.39
time (sec)	N/A	0.045	0.08	0.003	0.	1.848	1.045	1.071

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	127	274	0	2279	622	262
normalized size	1	1.	0.98	2.11	0.	17.53	4.78	2.02
time (sec)	N/A	0.072	0.139	0.004	0.	1.921	2.034	1.074

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	168	405	0	4097	1027	490
normalized size	1	1.	0.97	2.34	0.	23.68	5.94	2.83
time (sec)	N/A	0.11	0.219	0.006	0.	2.103	3.874	1.075

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	264	0	0	0	0	0
normalized size	1	1.	1.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	0.344	0.081	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	31	55	136	46	62
normalized size	1	1.	0.9	0.63	1.12	2.78	0.94	1.27
time (sec)	N/A	0.016	0.021	0.003	1.407	1.672	0.099	1.059

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	70	56	90	251	75	82
normalized size	1	1.	1.15	0.92	1.48	4.11	1.23	1.34
time (sec)	N/A	0.022	0.046	0.002	1.417	1.624	0.147	1.06

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	28	41	131	31	43
normalized size	1	1.	0.92	0.78	1.14	3.64	0.86	1.19
time (sec)	N/A	0.008	0.017	0.008	0.924	1.729	0.114	1.054

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	87	70	122	481	88	84
normalized size	1	1.	1.	0.8	1.4	5.53	1.01	0.97
time (sec)	N/A	0.023	0.025	0.01	0.928	1.824	0.194	1.057

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	70	62	93	315	80	90
normalized size	1	1.	0.86	0.77	1.15	3.89	0.99	1.11
time (sec)	N/A	0.056	0.035	0.013	1.41	1.761	0.202	1.076

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	87	70	122	505	88	84
normalized size	1	1.	0.84	0.67	1.17	4.86	0.85	0.81
time (sec)	N/A	0.074	0.022	0.008	0.935	1.626	0.19	1.056

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	99	80	127	555	90	89
normalized size	1	1.	0.97	0.78	1.25	5.44	0.88	0.87
time (sec)	N/A	0.037	0.051	0.01	0.932	1.747	0.209	1.064

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	35	53	92	37	63
normalized size	1	1.	1.	0.88	1.32	2.3	0.92	1.58
time (sec)	N/A	0.023	0.005	0.007	0.931	1.736	0.378	1.055

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	62	56	112	263	102	78
normalized size	1	1.	0.75	0.67	1.35	3.17	1.23	0.94
time (sec)	N/A	0.039	0.022	0.011	1.54	1.769	35.744	1.062

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	32	42	103	42	42
normalized size	1	1.	1.	0.65	0.86	2.1	0.86	0.86
time (sec)	N/A	0.057	0.022	0.002	0.958	1.687	2.234	1.051

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	55	32	42	143	48	42
normalized size	1	1.	1.	0.58	0.76	2.6	0.87	0.76
time (sec)	N/A	0.238	0.046	0.003	0.941	1.639	2.037	1.073

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	31	24	54	19	24
normalized size	1	1.	1.	1.41	1.09	2.45	0.86	1.09
time (sec)	N/A	0.009	0.008	0.001	0.932	1.663	0.111	1.051

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	19	13	16	31	22	16
normalized size	1	1.	1.27	0.87	1.07	2.07	1.47	1.07
time (sec)	N/A	0.006	0.015	0.002	0.932	1.744	0.751	1.043

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	21	27	59	20	28
normalized size	1	1.	0.96	0.84	1.08	2.36	0.8	1.12
time (sec)	N/A	0.017	0.01	0.002	0.941	1.717	0.139	1.071

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	111	34	84	0	35
normalized size	1	1.	1.	3.36	1.03	2.55	0.	1.06
time (sec)	N/A	0.019	0.021	0.031	0.927	1.724	0.	1.084

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	20	26	69	134	26
normalized size	1	1.	0.83	0.69	0.9	2.38	4.62	0.9
time (sec)	N/A	0.009	0.008	0.002	0.935	1.786	1.33	1.049

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	53	20	73	74	174	3966	66
normalized size	1	1.02	0.38	1.4	1.42	3.35	76.27	1.27
time (sec)	N/A	0.01	0.004	0.012	0.937	1.782	2.695	1.075

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	127	26	157	150	328	971	140
normalized size	1	1.08	0.22	1.33	1.27	2.78	8.23	1.19
time (sec)	N/A	0.038	0.006	0.015	0.937	1.808	15.747	1.066

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	22	158	142	282	0	111
normalized size	1	1.	0.21	1.52	1.37	2.71	0.	1.07
time (sec)	N/A	0.04	0.005	0.015	1.416	1.696	0.	1.072

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	67	39	58	90	0	39
normalized size	1	1.	1.76	1.03	1.53	2.37	0.	1.03
time (sec)	N/A	0.013	0.023	0.006	1.406	1.781	0.	1.054

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	95	115	208	176	165	0	139
normalized size	1	1.03	1.25	2.26	1.91	1.79	0.	1.51
time (sec)	N/A	0.059	0.257	0.019	1.422	1.798	0.	1.091

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	54	54	47	64	0	211	0	100
normalized size	1	1.	0.87	1.19	0.	3.91	0.	1.85
time (sec)	N/A	0.096	0.102	0.023	0.	1.826	0.	1.082

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	319	153	0	0	1611	0	0
normalized size	1	1.05	0.5	0.	0.	5.3	0.	0.
time (sec)	N/A	0.823	0.485	0.049	0.	2.191	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	292	522	348	0	0	2724	0	0
normalized size	1	1.79	1.19	0.	0.	9.33	0.	0.
time (sec)	N/A	1.846	0.676	0.039	0.	2.664	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	29	25	22	0	108	0	0
normalized size	1	1.16	1.	0.88	0.	4.32	0.	0.
time (sec)	N/A	0.013	0.008	0.001	0.	1.778	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	30	25	22	0	169	0	0
normalized size	1	1.2	1.	0.88	0.	6.76	0.	0.
time (sec)	N/A	0.016	0.013	0.003	0.	1.745	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	63	30	27	0	184	0	0
normalized size	1	1.19	0.57	0.51	0.	3.47	0.	0.
time (sec)	N/A	0.02	0.016	0.001	0.	1.673	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	188	49	0	0	331	0	0
normalized size	1	2.81	0.73	0.	0.	4.94	0.	0.
time (sec)	N/A	0.121	0.011	0.008	0.	1.711	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	133	114	118	0	375	0	230
normalized size	1	1.09	0.93	0.97	0.	3.07	0.	1.89
time (sec)	N/A	0.346	0.13	0.018	0.	1.797	0.	1.19

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	150	404	112	0	0	730	0	0
normalized size	1	2.69	0.75	0.	0.	4.87	0.	0.
time (sec)	N/A	0.327	0.067	0.028	0.	1.885	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	27	32	69	159	0	31
normalized size	1	1.	0.63	0.74	1.6	3.7	0.	0.72
time (sec)	N/A	0.006	0.011	0.003	0.941	1.712	0.	1.089

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	45	0	161	0	46
normalized size	1	1.	0.88	1.07	0.	3.83	0.	1.1
time (sec)	N/A	0.043	0.008	0.008	0.	1.729	0.	1.073

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	35	144	0	355	0	0
normalized size	1	1.	0.25	1.04	0.	2.55	0.	0.
time (sec)	N/A	0.127	0.008	0.016	0.	1.744	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	C	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	188	49	0	0	352	0	0
normalized size	1	2.51	0.65	0.	0.	4.69	0.	0.
time (sec)	N/A	0.118	0.011	0.01	0.	1.811	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	23	24	0	59	0	0
normalized size	1	1.	0.79	0.83	0.	2.03	0.	0.
time (sec)	N/A	0.033	0.008	0.002	0.	1.748	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	32	34	0	115	0	0
normalized size	1	1.	0.35	0.37	0.	1.25	0.	0.
time (sec)	N/A	0.085	0.008	0.003	0.	1.628	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	22	93	0	22
normalized size	1	1.	1.	0.79	1.16	4.89	0.	1.16
time (sec)	N/A	0.014	0.01	0.002	1.448	1.752	0.	1.087

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	5	11	74	0	5
normalized size	1	1.	1.	0.62	1.38	9.25	0.	0.62
time (sec)	N/A	0.005	0.006	0.003	1.417	1.791	0.	1.06

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	15	115	0	15
normalized size	1	1.	1.	1.	1.25	9.58	0.	1.25
time (sec)	N/A	0.004	0.007	0.004	1.425	2.039	0.	1.069

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	0	70	0	69
normalized size	1	1.	1.	0.94	0.	2.26	0.	2.23
time (sec)	N/A	0.008	0.008	0.01	0.	2.079	0.	1.089

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	22	0	157	0	77
normalized size	1	1.	1.	0.71	0.	5.06	0.	2.48
time (sec)	N/A	0.01	0.011	0.008	0.	1.98	0.	1.066

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	100	151	126	61	57
normalized size	1	1.	1.	4.17	6.29	5.25	2.54	2.38
time (sec)	N/A	0.019	0.009	0.025	1.536	2.107	3.419	1.074

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	100	136	93	24	27
normalized size	1	1.	1.	4.	5.44	3.72	0.96	1.08
time (sec)	N/A	0.019	0.009	0.027	1.463	1.991	3.302	1.059

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	96	70	0	189	0	100
normalized size	1	1.	2.23	1.63	0.	4.4	0.	2.33
time (sec)	N/A	0.02	0.045	0.018	0.	2.022	0.	1.11

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	61	49	73	211	0	147
normalized size	1	1.	0.98	0.79	1.18	3.4	0.	2.37
time (sec)	N/A	0.036	0.025	0.011	1.463	2.021	0.	1.153

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	159	69	0	479	0	221
normalized size	1	1.	1.94	0.84	0.	5.84	0.	2.7
time (sec)	N/A	0.121	0.31	0.018	0.	2.148	0.	1.116

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	114	53	0	1057	0	228
normalized size	1	1.	1.81	0.84	0.	16.78	0.	3.62
time (sec)	N/A	0.054	0.106	0.011	0.	2.231	0.	1.162

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	80	128	0	1041	0	139
normalized size	1	1.	1.43	2.29	0.	18.59	0.	2.48
time (sec)	N/A	0.047	0.025	0.016	0.	2.615	0.	1.112

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	174	158	0	1110	0	261
normalized size	1	1.	2.49	2.26	0.	15.86	0.	3.73
time (sec)	N/A	0.064	0.418	0.023	0.	2.842	0.	1.126

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	89	94	192	0	0	0	0
normalized size	1	1.11	1.18	2.4	0.	0.	0.	0.
time (sec)	N/A	0.123	0.114	0.02	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	76	94	0	254	0	0
normalized size	1	1.	2.	2.47	0.	6.68	0.	0.
time (sec)	N/A	0.024	0.039	0.01	0.	2.106	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	40	49	65	107	155	49
normalized size	1	1.	0.62	0.75	1.	1.65	2.38	0.75
time (sec)	N/A	0.017	0.022	0.003	1.542	1.799	4.539	1.065

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	28	25	50	74	41	69
normalized size	1	1.	0.57	0.51	1.02	1.51	0.84	1.41
time (sec)	N/A	0.011	0.006	0.003	1.469	1.822	4.003	1.073

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	28	25	50	109	139	35
normalized size	1	1.	0.57	0.51	1.02	2.22	2.84	0.71
time (sec)	N/A	0.008	0.008	0.002	0.959	1.914	75.924	1.091

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	328	14	167	53	15
normalized size	1	1.	1.	27.33	1.17	13.92	4.42	1.25
time (sec)	N/A	0.047	0.026	0.04	0.946	1.976	1.713	1.065

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	86	29	30
normalized size	1	1.	1.	0.92	1.17	7.17	2.42	2.5
time (sec)	N/A	0.013	0.011	0.006	1.426	1.827	6.49	1.08

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	23	0	173	0	86
normalized size	1	1.	1.	0.85	0.	6.41	0.	3.19
time (sec)	N/A	0.011	0.013	0.01	0.	1.717	0.	1.073

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	74	46	0	215	0	136
normalized size	1	1.	1.54	0.96	0.	4.48	0.	2.83
time (sec)	N/A	0.014	0.245	0.014	0.	1.824	0.	1.072

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	100	144	211	0	97
normalized size	1	1.	1.	2.44	3.51	5.15	0.	2.37
time (sec)	N/A	0.022	0.028	0.036	1.478	1.89	0.	1.118

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	85	70	0	122	0	166
normalized size	1	1.	1.81	1.49	0.	2.6	0.	3.53
time (sec)	N/A	0.021	0.149	0.026	0.	1.852	0.	1.087

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	137	82	0	450	0	182
normalized size	1	1.	1.56	0.93	0.	5.11	0.	2.07
time (sec)	N/A	0.239	0.177	0.02	0.	1.981	0.	1.135

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	261	654	0	541	0	238
normalized size	1	1.	1.92	4.81	0.	3.98	0.	1.75
time (sec)	N/A	1.496	0.863	0.044	0.	2.007	0.	1.106

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	72	73	0	108	0	61
normalized size	1	1.	1.12	1.14	0.	1.69	0.	0.95
time (sec)	N/A	0.017	0.11	0.007	0.	1.779	0.	1.085

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	80	91	0	135	0	73
normalized size	1	1.	0.9	1.02	0.	1.52	0.	0.82
time (sec)	N/A	0.03	0.102	0.003	0.	2.146	0.	1.081

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	88	111	0	158	0	85
normalized size	1	1.	0.78	0.98	0.	1.4	0.	0.75
time (sec)	N/A	0.043	0.115	0.005	0.	2.033	0.	1.069

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	64	42	73	170	0	96
normalized size	1	1.	1.31	0.86	1.49	3.47	0.	1.96
time (sec)	N/A	0.031	0.029	0.012	1.464	2.163	0.	1.118

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	72	74	89	158	0	198
normalized size	1	1.	0.91	0.94	1.13	2.	0.	2.51
time (sec)	N/A	0.044	0.056	0.013	1.486	2.082	0.	1.071

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	15	51	0	24
normalized size	1	1.	1.	0.83	1.25	4.25	0.	2.
time (sec)	N/A	0.008	0.006	0.003	1.45	2.026	0.	1.072

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	41	47	65	116	0	53
normalized size	1	1.	0.77	0.89	1.23	2.19	0.	1.
time (sec)	N/A	0.023	0.017	0.005	1.448	2.125	0.	1.076

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	30	90	0	20
normalized size	1	1.	1.	0.84	1.58	4.74	0.	1.05
time (sec)	N/A	0.002	0.004	0.004	0.943	2.242	0.	1.073

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	30	84	0	18
normalized size	1	1.	1.	0.82	1.76	4.94	0.	1.06
time (sec)	N/A	0.004	0.034	0.002	0.959	2.053	0.	1.068

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	48	61	63	184	0	51
normalized size	1	1.	0.86	1.09	1.12	3.29	0.	0.91
time (sec)	N/A	0.024	0.012	0.004	1.434	2.045	0.	1.077

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	46	49	76	132	0	59
normalized size	1	1.	0.71	0.75	1.17	2.03	0.	0.91
time (sec)	N/A	0.024	0.021	0.005	1.437	2.027	0.	1.069

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	46	43	76	134	0	59
normalized size	1	1.	0.84	0.78	1.38	2.44	0.	1.07
time (sec)	N/A	0.014	0.015	0.002	1.434	2.109	0.	1.069

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	56	58	104	176	0	73
normalized size	1	1.	0.76	0.78	1.41	2.38	0.	0.99
time (sec)	N/A	0.019	0.021	0.003	1.447	2.182	0.	1.067

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	31	50	144	0	90
normalized size	1	1.	1.	0.82	1.32	3.79	0.	2.37
time (sec)	N/A	0.013	0.009	0.005	1.45	2.15	0.	1.09

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	43	44	68	169	0	113
normalized size	1	1.	0.75	0.77	1.19	2.96	0.	1.98
time (sec)	N/A	0.024	0.014	0.003	1.438	2.069	0.	1.08

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	50	56	78	258	0	108
normalized size	1	1.	0.81	0.9	1.26	4.16	0.	1.74
time (sec)	N/A	0.027	0.016	0.004	1.462	2.211	0.	1.083

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	65	69	96	284	0	158
normalized size	1	1.	0.82	0.87	1.22	3.59	0.	2.
time (sec)	N/A	0.037	0.013	0.005	1.67	2.114	0.	1.098

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	22	34	86	0	43
normalized size	1	1.	1.	1.	1.55	3.91	0.	1.95
time (sec)	N/A	0.009	0.003	0.003	1.682	2.027	0.	1.097

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	83	69	0	319	0	198
normalized size	1	1.	0.97	0.8	0.	3.71	0.	2.3
time (sec)	N/A	0.278	0.06	0.01	0.	2.164	0.	1.16

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	61	91	82	263	0	204
normalized size	1	1.	0.98	1.47	1.32	4.24	0.	3.29
time (sec)	N/A	0.047	0.038	0.007	1.458	1.901	0.	1.16

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	146	123	0	504	0	317
normalized size	1	1.	1.92	1.62	0.	6.63	0.	4.17
time (sec)	N/A	0.068	0.369	0.024	0.	1.883	0.	1.096

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	26	21	38	58	0	41
normalized size	1	1.	0.72	0.58	1.06	1.61	0.	1.14
time (sec)	N/A	0.128	0.011	0.005	1.469	1.886	0.	1.071

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	134	69	0	455	0	200
normalized size	1	1.	1.54	0.79	0.	5.23	0.	2.3
time (sec)	N/A	0.209	0.147	0.017	0.	1.87	0.	1.08

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	39	38	103	262	0	45
normalized size	1	1.	0.67	0.66	1.78	4.52	0.	0.78
time (sec)	N/A	0.011	0.011	0.003	0.964	1.786	0.	1.098

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	80	197	0	36
normalized size	1	1.	0.7	0.64	1.7	4.19	0.	0.77
time (sec)	N/A	0.007	0.016	0.002	0.962	1.641	0.	1.079

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	80	136	0	53
normalized size	1	1.	0.7	0.64	1.7	2.89	0.	1.13
time (sec)	N/A	0.008	0.011	0.003	0.957	1.842	0.	1.096

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	30	40	0	108	0	81
normalized size	1	1.	1.03	1.38	0.	3.72	0.	2.79
time (sec)	N/A	0.039	0.02	0.007	0.	1.837	0.	1.085

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	59	59	52	0	193	0	89
normalized size	1	1.31	1.31	1.16	0.	4.29	0.	1.98
time (sec)	N/A	0.032	0.03	0.005	0.	1.727	0.	1.09

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	71	59	0	159	0	73
normalized size	1	1.	0.9	0.75	0.	2.01	0.	0.92
time (sec)	N/A	0.136	0.074	0.003	0.	1.865	0.	1.07

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	80	80	0	306	0	142
normalized size	1	1.	1.	1.	0.	3.82	0.	1.78
time (sec)	N/A	0.343	0.11	0.01	0.	1.935	0.	1.142

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	151	140	0	460	0	0
normalized size	1	1.	0.96	0.89	0.	2.91	0.	0.
time (sec)	N/A	0.526	0.277	0.012	0.	1.883	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	22	30	51	82	131	39
normalized size	1	1.	0.54	0.73	1.24	2.	3.2	0.95
time (sec)	N/A	0.009	0.004	0.007	1.442	1.899	2.672	1.075

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	18	15	41	10	15
normalized size	1	1.	1.	1.38	1.15	3.15	0.77	1.15
time (sec)	N/A	0.004	0.005	0.001	0.953	1.78	0.916	1.064

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	26	109	103	238	541	103
normalized size	1	1.	0.37	1.54	1.45	3.35	7.62	1.45
time (sec)	N/A	0.028	0.005	0.013	1.433	1.776	2.13	1.084

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	23	20	38	90	180	66
normalized size	1	1.	0.57	0.5	0.95	2.25	4.5	1.65
time (sec)	N/A	0.007	0.012	0.003	0.959	1.765	1.718	1.078

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	53	70	180	51	0
normalized size	1	1.	1.	1.1	1.46	3.75	1.06	0.
time (sec)	N/A	0.018	0.012	0.007	1.46	1.833	3.489	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	36	46	61	86	3305	85
normalized size	1	1.	0.52	0.67	0.88	1.25	47.9	1.23
time (sec)	N/A	0.02	0.013	0.003	0.955	1.843	2.057	1.079

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	34	130	212	643	44	192
normalized size	1	1.	0.18	0.67	1.1	3.33	0.23	0.99
time (sec)	N/A	0.113	0.008	0.013	1.466	1.895	19.928	1.078

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	37	12	30	26	12
normalized size	1	1.	1.	2.85	0.92	2.31	2.	0.92
time (sec)	N/A	0.003	0.003	0.006	0.937	1.738	0.56	1.06

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	37	12	31	12	12
normalized size	1	1.	1.	2.85	0.92	2.38	0.92	0.92
time (sec)	N/A	0.003	0.003	0.004	0.943	1.779	0.573	1.062

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	84	74	86	217	42	86
normalized size	1	1.	1.42	1.25	1.46	3.68	0.71	1.46
time (sec)	N/A	0.037	0.022	0.058	1.444	1.948	1.035	1.093

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	26	76	89	240	34	90
normalized size	1	1.	0.37	1.09	1.27	3.43	0.49	1.29
time (sec)	N/A	0.038	0.006	0.039	1.455	1.813	1.896	1.09

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	B	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	27	35	112	390	41	112
normalized size	1	1.	0.4	0.51	1.65	5.74	0.6	1.65
time (sec)	N/A	0.021	0.004	0.04	1.468	26.156	1.053	1.083

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	26	42	176	298	41	149
normalized size	1	1.	0.28	0.45	1.89	3.2	0.44	1.6
time (sec)	N/A	0.029	0.005	0.021	1.469	1.867	2.094	1.084

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	43	42	174	302	39	147
normalized size	1	1.	0.46	0.45	1.87	3.25	0.42	1.58
time (sec)	N/A	0.029	0.014	0.02	1.439	1.898	1.6	1.107

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	200	40	15	0	347	0	0
normalized size	1	2.15	0.43	0.16	0.	3.73	0.	0.
time (sec)	N/A	0.144	0.009	0.026	0.	3.469	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	114	83	108	0	0	0	0
normalized size	1	0.9	0.66	0.86	0.	0.	0.	0.
time (sec)	N/A	0.115	0.049	0.01	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	29	0	0	128	0	95
normalized size	1	1.	0.85	0.	0.	3.76	0.	2.79
time (sec)	N/A	0.032	0.013	0.053	0.	2.406	0.	1.138

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	47	0	0	131	0	66
normalized size	1	1.	0.81	0.	0.	2.26	0.	1.14
time (sec)	N/A	0.037	0.017	0.025	0.	2.372	0.	1.076

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	55	60	70	196	0	0
normalized size	1	1.	0.77	0.85	0.99	2.76	0.	0.
time (sec)	N/A	0.052	0.028	0.01	1.469	2.36	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	57	46	70	149	0	0
normalized size	1	1.	2.71	2.19	3.33	7.1	0.	0.
time (sec)	N/A	0.029	0.015	0.017	0.979	2.136	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	75	22	18	35	36	18
normalized size	1	1.	4.41	1.29	1.06	2.06	2.12	1.06
time (sec)	N/A	0.01	0.055	0.005	0.964	1.996	1.249	1.086

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	40	116	46	97	221	46
normalized size	1	1.	0.87	2.52	1.	2.11	4.8	1.
time (sec)	N/A	0.193	0.081	0.067	0.972	2.035	3.088	1.074

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	107	104	0	0	624	0	0
normalized size	1	1.37	1.33	0.	0.	8.	0.	0.
time (sec)	N/A	0.071	0.077	0.042	0.	27.472	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	120	0	0	1062	0	0
normalized size	1	1.	0.85	0.	0.	7.53	0.	0.
time (sec)	N/A	0.06	0.048	0.024	0.	49.475	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	91	29	127	252	71	0
normalized size	1	1.	1.44	0.46	2.02	4.	1.13	0.
time (sec)	N/A	0.011	0.034	0.027	1.471	1.916	1.828	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	41	0	0	668	0	0
normalized size	1	1.	0.55	0.	0.	9.03	0.	0.
time (sec)	N/A	0.024	0.01	0.033	0.	52.759	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	59	26	23	99	111	0	0
normalized size	1	1.23	0.54	0.48	2.06	2.31	0.	0.
time (sec)	N/A	0.012	0.009	0.006	0.959	1.508	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	123	120	0	0	1281	0	0
normalized size	1	1.37	1.33	0.	0.	14.23	0.	0.
time (sec)	N/A	0.082	0.144	0.022	0.	71.299	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	40	112	0	61	0	0
normalized size	1	1.	1.74	4.87	0.	2.65	0.	0.
time (sec)	N/A	0.034	0.096	0.007	0.	1.769	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	36	112	0	111	0	0
normalized size	1	1.	1.57	4.87	0.	4.83	0.	0.
time (sec)	N/A	0.036	0.09	0.006	0.	1.77	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	23	21	18	77	123	14	0
normalized size	1	1.44	1.31	1.12	4.81	7.69	0.88	0.
time (sec)	N/A	0.024	0.017	0.009	1.512	1.56	3.96	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	23	19	18	77	123	14	0
normalized size	1	1.44	1.19	1.12	4.81	7.69	0.88	0.
time (sec)	N/A	0.025	0.014	0.005	1.454	1.497	4.096	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	26	26	522	184	0	119	0	0
normalized size	1	1.	20.08	7.08	0.	4.58	0.	0.
time (sec)	N/A	0.043	1.239	0.157	0.	1.73	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	94	188	0	42	0	0
normalized size	1	1.	6.27	12.53	0.	2.8	0.	0.
time (sec)	N/A	0.039	0.13	0.023	0.	1.768	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	29	30	31	0	0
normalized size	1	1.	1.	1.81	1.88	1.94	0.	0.
time (sec)	N/A	0.018	0.036	0.006	1.153	1.498	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	74	17955	247419	0	633	0	0
normalized size	1	1.	242.64	3343.5	0.	8.55	0.	0.
time (sec)	N/A	0.204	6.46	0.114	0.	3.1	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	24	0	0	149	0	0
normalized size	1	1.	1.09	0.	0.	6.77	0.	0.
time (sec)	N/A	0.064	1.007	0.024	0.	6.079	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	26	0	0	0	0	0
normalized size	1	1.	1.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.081	0.079	0.	0.	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	36	10	14
normalized size	1	1.	1.	0.79	1.	2.57	0.71	1.
time (sec)	N/A	0.007	0.002	0.	1.026	1.541	0.056	1.053

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	15	11	12	36	8	12
normalized size	1	1.	1.36	1.	1.09	3.27	0.73	1.09
time (sec)	N/A	0.007	0.002	0.003	0.928	1.481	0.058	1.044

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	18	22	59	24	22
normalized size	1	1.	0.92	0.75	0.92	2.46	1.	0.92
time (sec)	N/A	0.01	0.002	0.	0.932	1.6	0.057	1.048

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	24	32	82	36	30
normalized size	1	1.	0.88	0.71	0.94	2.41	1.06	0.88
time (sec)	N/A	0.019	0.002	0.005	0.935	1.59	0.057	1.059

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	38	30	41	111	48	38
normalized size	1	1.	0.86	0.68	0.93	2.52	1.09	0.86
time (sec)	N/A	0.021	0.003	0.034	0.94	1.697	0.06	1.072

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	64	21	39	31	112	99	19
normalized size	1	3.2	1.05	1.95	1.55	5.6	4.95	0.95
time (sec)	N/A	0.016	0.017	0.01	0.934	1.671	0.616	1.052

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	23	31	70	39	31
normalized size	1	1.	1.	0.74	1.	2.26	1.26	1.
time (sec)	N/A	0.011	0.016	0.007	0.936	1.567	0.309	1.061

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	27	18	27	112	32	27
normalized size	1	1.	1.29	0.86	1.29	5.33	1.52	1.29
time (sec)	N/A	0.009	0.003	0.028	0.937	1.583	0.058	1.066

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	95	32	73	302	60	151
normalized size	1	1.	2.64	0.89	2.03	8.39	1.67	4.19
time (sec)	N/A	0.02	0.008	0.034	0.923	1.778	0.159	1.057

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	57	37	45	138	66	45
normalized size	1	1.	1.39	0.9	1.1	3.37	1.61	1.1
time (sec)	N/A	0.017	0.004	0.033	0.932	1.609	0.06	1.046

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	40	69	198	388	72
normalized size	1	1.	1.	1.	1.72	4.95	9.7	1.8
time (sec)	N/A	0.012	0.012	0.036	0.93	1.61	1.291	1.062

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	30	19	24	57	31	24
normalized size	1	1.	1.36	0.86	1.09	2.59	1.41	1.09
time (sec)	N/A	0.014	0.004	0.002	1.412	1.526	0.069	1.049

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	16	26	30	116	19	50
normalized size	1	1.	0.8	1.3	1.5	5.8	0.95	2.5
time (sec)	N/A	0.015	0.003	0.007	0.933	1.721	0.101	1.069

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	40	28	41	207	20	72
normalized size	1	1.	1.25	0.88	1.28	6.47	0.62	2.25
time (sec)	N/A	0.014	0.032	0.003	1.427	1.681	0.194	1.091

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	46	42	32	127	56	46
normalized size	1	1.	0.82	0.75	0.57	2.27	1.	0.82
time (sec)	N/A	0.062	0.015	0.008	0.93	1.981	0.061	1.058

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	55	38	34	85	27	34
normalized size	1	1.	1.67	1.15	1.03	2.58	0.82	1.03
time (sec)	N/A	0.03	0.027	0.008	0.934	2.05	0.065	1.049

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	37	54	123	44	51
normalized size	1	1.	1.	0.8	1.17	2.67	0.96	1.11
time (sec)	N/A	0.028	0.015	0.01	0.935	2.128	0.09	1.07

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	53	56	50	174	44	35
normalized size	1	1.	1.29	1.37	1.22	4.24	1.07	0.85
time (sec)	N/A	0.034	0.036	0.011	0.943	1.932	0.067	1.063

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	14	19	14	58	14	14
normalized size	1	1.	0.58	0.79	0.58	2.42	0.58	0.58
time (sec)	N/A	0.027	0.004	0.004	0.931	1.823	0.063	1.061

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	22	36	22	103	31	22
normalized size	1	1.	0.48	0.78	0.48	2.24	0.67	0.48
time (sec)	N/A	0.057	0.007	0.	0.933	1.764	0.067	1.066

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	30	52	32	151	46	30
normalized size	1	1.	0.44	0.76	0.47	2.22	0.68	0.44
time (sec)	N/A	0.085	0.015	0.008	0.938	2.014	0.066	1.052

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	38	68	41	208	61	38
normalized size	1	1.	0.42	0.76	0.46	2.31	0.68	0.42
time (sec)	N/A	0.137	0.015	0.033	0.945	2.004	0.069	1.057

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	58	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.066	0.298	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	45	25	55	207	54	178
normalized size	1	1.	1.41	0.78	1.72	6.47	1.69	5.56
time (sec)	N/A	0.021	0.067	0.019	0.94	1.811	1.743	1.108

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	11	8	50	17	8
normalized size	1	1.	1.	1.38	1.	6.25	2.12	1.
time (sec)	N/A	0.022	0.003	0.01	0.933	1.749	0.063	1.073

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	32	19	62	14	19
normalized size	1	1.	1.	2.91	1.73	5.64	1.27	1.73
time (sec)	N/A	0.016	0.007	0.009	0.944	1.868	0.089	1.082

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	19	7	8
normalized size	1	1.	1.	0.88	1.	2.38	0.88	1.
time (sec)	N/A	0.013	0.005	0.008	0.931	1.827	0.064	1.067

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	71	36	51	221	39	63
normalized size	1	1.	2.73	1.38	1.96	8.5	1.5	2.42
time (sec)	N/A	0.031	0.018	0.012	0.934	1.88	0.133	1.09

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	19	86	14	24
normalized size	1	1.	1.	1.29	1.12	5.06	0.82	1.41
time (sec)	N/A	0.026	0.008	0.008	0.929	1.783	0.096	1.101

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	24	49	26	73	0	31
normalized size	1	1.	0.77	1.58	0.84	2.35	0.	1.
time (sec)	N/A	0.026	0.05	0.092	0.939	1.872	0.	1.1

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	22	26	18	86	0	18
normalized size	1	1.	1.05	1.24	0.86	4.1	0.	0.86
time (sec)	N/A	0.024	0.037	0.259	0.936	2.461	0.	1.093

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	95	52	73	297	56	59
normalized size	1	1.	2.5	1.37	1.92	7.82	1.47	1.55
time (sec)	N/A	0.053	0.019	0.016	0.932	2.56	0.16	1.087

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	74	76	100	252	0	128
normalized size	1	1.	0.97	1.	1.32	3.32	0.	1.68
time (sec)	N/A	0.039	0.074	0.023	0.948	2.653	0.	1.18

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	84	127	186	155	487	0	201
normalized size	1	0.95	1.44	2.11	1.76	5.53	0.	2.28
time (sec)	N/A	0.551	1.78	0.196	1.444	2.782	0.	1.124

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	74	66	73	194	162	73
normalized size	1	1.	1.06	0.94	1.04	2.77	2.31	1.04
time (sec)	N/A	0.153	0.081	0.039	0.942	2.172	1.671	1.1

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	33	49	220	39	101
normalized size	1	1.	0.88	1.	1.48	6.67	1.18	3.06
time (sec)	N/A	0.062	0.023	0.004	1.414	2.151	0.612	1.142

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	20	21	19	80	24	19
normalized size	1	1.	1.25	1.31	1.19	5.	1.5	1.19
time (sec)	N/A	0.043	0.014	0.059	1.431	2.181	110.271	1.066

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	22	28	97	24	31
normalized size	1	1.	1.	1.83	2.33	8.08	2.	2.58
time (sec)	N/A	0.023	0.01	0.021	0.931	2.301	1.324	1.068

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	59	20	18
normalized size	1	1.	1.	0.82	1.06	3.47	1.18	1.06
time (sec)	N/A	0.008	0.006	0.046	0.924	2.205	0.516	1.113

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	31	73	142	75	51
normalized size	1	1.	1.	1.19	2.81	5.46	2.88	1.96
time (sec)	N/A	0.03	0.05	0.023	0.939	2.452	108.838	1.077

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	29	41	99	144	38
normalized size	1	1.	1.	0.76	1.08	2.61	3.79	1.
time (sec)	N/A	0.03	0.01	0.062	0.958	2.475	27.566	1.084

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	35	45	138	22	136
normalized size	1	1.	1.	1.75	2.25	6.9	1.1	6.8
time (sec)	N/A	0.046	0.011	0.048	0.942	2.367	105.548	1.102

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	19	26	78	14	96
normalized size	1	1.	1.	1.58	2.17	6.5	1.17	8.
time (sec)	N/A	0.021	0.009	0.05	0.937	2.245	15.391	1.081

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	65	39	35	281	223	0	101
normalized size	1	1.02	0.61	0.55	4.39	3.48	0.	1.58
time (sec)	N/A	0.078	0.036	0.062	1.508	2.467	0.	1.148

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	17	17	27	124	59	0	32
normalized size	1	1.55	1.55	2.45	11.27	5.36	0.	2.91
time (sec)	N/A	0.033	0.013	0.044	0.956	2.212	0.	1.111

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	12	12	34	0	12
normalized size	1	1.	1.	1.71	1.71	4.86	0.	1.71
time (sec)	N/A	0.039	0.031	0.035	1.423	2.278	0.	1.072

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	83	33	43	36	128	107	105
normalized size	1	1.57	0.62	0.81	0.68	2.42	2.02	1.98
time (sec)	N/A	0.098	0.059	0.046	1.41	2.369	8.793	1.067

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	94	22	20	53	230	39	50
normalized size	1	2.85	0.67	0.61	1.61	6.97	1.18	1.52
time (sec)	N/A	0.074	0.043	0.031	1.411	2.224	1.711	1.151

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	9	8	9	70	148	27
normalized size	1	1.	0.33	0.3	0.33	2.59	5.48	1.
time (sec)	N/A	0.021	0.021	0.026	1.416	2.149	14.474	1.113

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	41	29	38	153	105	39
normalized size	1	1.	1.46	1.04	1.36	5.46	3.75	1.39
time (sec)	N/A	0.041	0.04	0.1	1.421	2.162	0.51	1.117

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	58	39	51	212	0	105
normalized size	1	1.	0.87	0.58	0.76	3.16	0.	1.57
time (sec)	N/A	0.046	0.115	0.063	1.423	2.221	0.	1.103

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	54	29	38	184	0	82
normalized size	1	1.	0.98	0.53	0.69	3.35	0.	1.49
time (sec)	N/A	0.032	0.084	0.038	1.416	2.128	0.	1.087

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	70	43	59	243	253	53
normalized size	1	1.	1.67	1.02	1.4	5.79	6.02	1.26
time (sec)	N/A	0.096	0.217	0.025	1.417	2.242	0.54	1.13

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	20	0	65	76	28
normalized size	1	1.	1.	2.22	0.	7.22	8.44	3.11
time (sec)	N/A	0.022	0.007	0.04	0.	2.192	6.425	1.106

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	174	13	174	97	0	66
normalized size	1	1.	11.6	0.87	11.6	6.47	0.	4.4
time (sec)	N/A	0.014	0.372	0.02	1.449	2.167	0.	1.142

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	28	19	173	96	22	27
normalized size	1	1.	1.65	1.12	10.18	5.65	1.29	1.59
time (sec)	N/A	0.042	0.015	0.037	1.531	2.209	1.233	1.117

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	109	62	0	120
normalized size	1	1.	1.	0.86	5.19	2.95	0.	5.71
time (sec)	N/A	0.041	0.015	0.053	1.432	2.32	0.	1.114

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	34	174	65	17	41
normalized size	1	1.	1.	1.62	8.29	3.1	0.81	1.95
time (sec)	N/A	0.023	0.007	0.051	1.412	2.222	1.175	1.086

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	28	231	158	0	65
normalized size	1	1.	1.	1.08	8.88	6.08	0.	2.5
time (sec)	N/A	0.024	0.023	0.056	1.483	2.065	0.	1.116

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	218	28	231	159	0	65
normalized size	1	1.	8.38	1.08	8.88	6.12	0.	2.5
time (sec)	N/A	0.035	0.316	0.066	1.473	2.033	0.	1.126

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	25	22	0	99	0	0
normalized size	1	1.	1.56	1.38	0.	6.19	0.	0.
time (sec)	N/A	0.009	0.014	0.033	0.	1.805	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	27	31	0	99	0	0
normalized size	1	1.	1.59	1.82	0.	5.82	0.	0.
time (sec)	N/A	0.012	0.014	0.033	0.	1.812	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	47	9	55	161	0	0
normalized size	1	1.	1.74	0.33	2.04	5.96	0.	0.
time (sec)	N/A	0.013	0.014	0.015	1.588	1.852	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	33	17	136	167	0	0
normalized size	1	1.	1.1	0.57	4.53	5.57	0.	0.
time (sec)	N/A	0.013	0.015	0.034	1.613	1.813	0.	0.

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	61	52	585	300	0	80
normalized size	1	1.	1.15	0.98	11.04	5.66	0.	1.51
time (sec)	N/A	0.028	0.118	0.047	1.671	1.94	0.	1.206

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	76	47	0	223	0	0
normalized size	1	1.	1.04	0.64	0.	3.05	0.	0.
time (sec)	N/A	0.033	0.166	0.037	0.	1.887	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	36	42	58	127	0	0
normalized size	1	1.	0.65	0.76	1.05	2.31	0.	0.
time (sec)	N/A	0.151	0.077	0.436	0.929	2.081	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	24	49	108	572	0	108
normalized size	1	1.	0.24	0.5	1.1	5.84	0.	1.1
time (sec)	N/A	0.069	0.012	0.001	1.419	1.889	0.	1.142

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	69	69	53	70	192	0	70
normalized size	1	1.21	1.21	0.93	1.23	3.37	0.	1.23
time (sec)	N/A	0.058	0.05	0.012	1.415	1.87	0.	1.168

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	73	130	4338	1613	0	0
normalized size	1	1.	0.84	1.49	49.86	18.54	0.	0.
time (sec)	N/A	0.113	0.093	0.049	2.499	2.09	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	38	219	41	259	0	42
normalized size	1	1.	0.95	5.48	1.02	6.48	0.	1.05
time (sec)	N/A	0.147	1.201	0.461	0.935	2.078	0.	1.203

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	C	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	133	62	97	158	2503	0	150
normalized size	1	1.58	0.74	1.15	1.88	29.8	0.	1.79
time (sec)	N/A	0.371	0.293	0.024	1.424	11.726	0.	1.213

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	266	0	455	0	0
normalized size	1	1.	1.	8.58	0.	14.68	0.	0.
time (sec)	N/A	0.015	0.038	0.041	0.	2.013	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	29	98	0	455	0	0
normalized size	1	1.	0.94	3.16	0.	14.68	0.	0.
time (sec)	N/A	0.015	0.029	0.048	0.	1.983	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	41	171	0	513	0	0
normalized size	1	1.	0.91	3.8	0.	11.4	0.	0.
time (sec)	N/A	0.03	0.035	0.053	0.	1.972	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	43	443	0	262	0	0
normalized size	1	1.	0.91	9.43	0.	5.57	0.	0.
time (sec)	N/A	0.096	0.062	0.08	0.	1.892	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	50	510	0	594	0	0
normalized size	1	1.	0.82	8.36	0.	9.74	0.	0.
time (sec)	N/A	0.08	0.088	0.105	0.	2.073	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	1108	0	684	0	0
normalized size	1	1.	0.92	18.16	0.	11.21	0.	0.
time (sec)	N/A	0.082	0.074	0.058	0.	2.133	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	508	0	124	0	0
normalized size	1	1.	1.	31.75	0.	7.75	0.	0.
time (sec)	N/A	0.022	0.032	0.049	0.	1.935	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	286	0	100	0	0
normalized size	1	1.	0.65	9.23	0.	3.23	0.	0.
time (sec)	N/A	0.041	0.029	0.05	0.	1.952	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	121	0	130	0	0
normalized size	1	1.	0.83	4.17	0.	4.48	0.	0.
time (sec)	N/A	0.048	0.03	0.039	0.	1.889	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	95	150	761	0	506	0	0
normalized size	1	1.4	2.21	11.19	0.	7.44	0.	0.
time (sec)	N/A	0.856	6.036	0.215	0.	2.118	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	29	19	16	8	55	0	0
normalized size	1	1.53	1.	0.84	0.42	2.89	0.	0.
time (sec)	N/A	0.124	0.007	0.126	1.491	2.127	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	204	66	310	0	0	0	0
normalized size	1	2.22	0.72	3.37	0.	0.	0.	0.
time (sec)	N/A	0.239	0.074	0.141	0.	0.	0.	0.

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	63	0	18	96	0	0
normalized size	1	1.	1.34	0.	0.38	2.04	0.	0.
time (sec)	N/A	0.144	0.146	0.348	1.522	2.872	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	35	0	104	336	0	0
normalized size	1	1.	0.5	0.	1.49	4.8	0.	0.
time (sec)	N/A	0.196	0.054	0.29	1.515	3.102	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	234	105	247	0	2007	0	0
normalized size	1	2.17	0.97	2.29	0.	18.58	0.	0.
time (sec)	N/A	1.54	0.261	0.317	0.	76.053	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-2)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	364	665	475	23475	0	0	0	0
normalized size	1	1.83	1.3	64.49	0.	0.	0.	0.
time (sec)	N/A	4.73	14.528	3.754	0.	0.	0.	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	141	58	0	81	182	0	0
normalized size	1	1.13	0.46	0.	0.65	1.46	0.	0.
time (sec)	N/A	1.019	0.393	0.75	1.738	2.994	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	67	61	56	74	352	0	74
normalized size	1	0.92	0.84	0.77	1.01	4.82	0.	1.01
time (sec)	N/A	0.047	0.138	0.012	1.467	3.394	0.	1.085

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	48	103	72	296	0	55
normalized size	1	1.	0.7	1.49	1.04	4.29	0.	0.8
time (sec)	N/A	0.053	0.073	0.092	1.431	3.08	0.	1.088

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	61	82	49	431	0	41
normalized size	1	1.	1.05	1.41	0.84	7.43	0.	0.71
time (sec)	N/A	0.051	0.075	0.082	1.484	2.789	0.	1.112

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	37	44	58	155	0	41
normalized size	1	1.	0.67	0.8	1.05	2.82	0.	0.75
time (sec)	N/A	0.054	0.092	0.03	0.96	3.544	0.	1.119

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	58	967	302	0	51
normalized size	1	1.	1.	1.49	24.79	7.74	0.	1.31
time (sec)	N/A	0.067	0.094	0.113	1.733	3.22	0.	1.102

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	63	53	93	413	0	54
normalized size	1	1.	1.31	1.1	1.94	8.6	0.	1.12
time (sec)	N/A	0.072	0.304	0.074	1.489	3.007	0.	1.112

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	28	46	259	4	0	45
normalized size	1	1.	0.57	0.94	5.29	0.08	0.	0.92
time (sec)	N/A	0.116	0.083	0.046	1.029	3.177	0.	1.098

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	F(-2)	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	119	338	130	155	0	0	356
normalized size	1	1.07	3.05	1.17	1.4	0.	0.	3.21
time (sec)	N/A	0.575	2.327	0.184	1.511	0.	0.	1.786

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	156	97512	0	632	0	0
normalized size	1	1.	1.39	870.64	0.	5.64	0.	0.
time (sec)	N/A	0.444	0.344	4.062	0.	4.091	0.	0.

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	29	38	34	89	0	0
normalized size	1	1.	0.88	1.15	1.03	2.7	0.	0.
time (sec)	N/A	0.061	0.041	0.101	0.97	3.465	0.	0.

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	61	659	242	0	36
normalized size	1	1.	0.97	1.85	19.97	7.33	0.	1.09
time (sec)	N/A	0.022	0.018	0.056	1.6	3.213	0.	1.115

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	55	1067	329	0	65
normalized size	1	1.	0.89	1.	19.4	5.98	0.	1.18
time (sec)	N/A	0.033	0.095	0.038	1.635	3.43	0.	1.115

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	39	122	109	0	62
normalized size	1	1.	1.	2.44	7.62	6.81	0.	3.88
time (sec)	N/A	0.013	0.036	0.075	1.51	3.198	0.	1.274

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	100	0	365	0	0
normalized size	1	1.	1.	2.04	0.	7.45	0.	0.
time (sec)	N/A	0.057	0.093	0.067	0.	3.844	0.	0.

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	91	62	180	1835	506	0	74
normalized size	1	1.05	0.71	2.07	21.09	5.82	0.	0.85
time (sec)	N/A	0.208	0.201	0.168	2.033	3.799	0.	1.139

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	115	754	0	417	0	0
normalized size	1	1.	1.69	11.09	0.	6.13	0.	0.
time (sec)	N/A	0.055	0.207	0.264	0.	3.727	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	40	40	79	473	0	351	0	0
normalized size	1	1.	1.98	11.82	0.	8.78	0.	0.
time (sec)	N/A	0.03	0.124	0.211	0.	3.39	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	131	975	0	285	0	0
normalized size	1	1.	1.39	10.37	0.	3.03	0.	0.
time (sec)	N/A	0.189	0.513	0.412	0.	4.15	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	36	61	85	101	0	0
normalized size	1	1.	0.92	1.56	2.18	2.59	0.	0.
time (sec)	N/A	0.149	0.078	0.457	0.975	3.441	0.	0.

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	255	1615	0	799	0	231
normalized size	1	1.	3.49	22.12	0.	10.95	0.	3.16
time (sec)	N/A	1.23	2.619	0.276	0.	4.455	0.	1.351

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	116	117	63	252	0	0
normalized size	1	1.	2.04	2.05	1.11	4.42	0.	0.
time (sec)	N/A	0.849	1.815	0.337	1.428	3.266	0.	0.

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	49	61	0	157	0	70
normalized size	1	1.	0.74	0.92	0.	2.38	0.	1.06
time (sec)	N/A	0.071	0.225	0.025	0.	3.067	0.	1.087

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	71	41	0	227	46	49
normalized size	1	1.	1.31	0.76	0.	4.2	0.85	0.91
time (sec)	N/A	0.06	0.681	0.02	0.	3.071	5.403	1.074

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	105	0	0	0	0	251
normalized size	1	1.	0.79	0.	0.	0.	0.	1.89
time (sec)	N/A	0.156	0.22	0.106	0.	0.	0.	1.233

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	42	0	0	360	0	107
normalized size	1	1.	0.61	0.	0.	5.22	0.	1.55
time (sec)	N/A	0.087	0.139	0.05	0.	11.591	0.	1.104

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	256	0	96	0	0	0
normalized size	1	1.	4.92	0.	1.85	0.	0.	0.
time (sec)	N/A	0.089	0.407	0.112	1.422	0.	0.	0.

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	245	0	100	0	0	0
normalized size	1	1.	4.54	0.	1.85	0.	0.	0.
time (sec)	N/A	0.087	0.445	0.103	1.421	0.	0.	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	133	174	4397	0	0	0	0	0
normalized size	1	1.31	33.06	0.	0.	0.	0.	0.
time (sec)	N/A	5.11	56.154	180.	0.	0.	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	208	168	559	0	784	0	265
normalized size	1	2.08	1.68	5.59	0.	7.84	0.	2.65
time (sec)	N/A	1.226	4.594	0.482	0.	3.3	0.	1.344

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	47	137	184	450	0	0
normalized size	1	1.	0.42	1.22	1.64	4.02	0.	0.
time (sec)	N/A	0.169	0.042	0.027	1.427	2.951	0.	0.

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	162	154	0	196	0	0	197
normalized size	1	1.71	1.62	0.	2.06	0.	0.	2.07
time (sec)	N/A	0.26	0.131	0.11	1.417	0.	0.	1.149

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	30	0	50	158	0	50
normalized size	1	1.	0.61	0.	1.02	3.22	0.	1.02
time (sec)	N/A	0.108	0.462	0.61	0.93	3.061	0.	1.086

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	42	26	116	111	0	34
normalized size	1	1.	2.1	1.3	5.8	5.55	0.	1.7
time (sec)	N/A	0.225	0.2	0.031	1.17	2.299	0.	1.127

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	35	58	0	0	281	0	55
normalized size	1	1.3	2.15	0.	0.	10.41	0.	2.04
time (sec)	N/A	0.967	4.103	0.485	0.	16.919	0.	1.137

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	126	89	0	135	0	0	0
normalized size	1	1.25	0.88	0.	1.34	0.	0.	0.
time (sec)	N/A	1.405	0.41	0.742	1.475	0.	0.	0.

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	140	0	0	84	0	34
normalized size	1	1.	5.6	0.	0.	3.36	0.	1.36
time (sec)	N/A	0.061	0.304	0.17	0.	2.429	0.	1.185

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	154	153	0	0	0	0	0
normalized size	1	1.51	1.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	0.117	0.295	0.	0.	0.	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	18	45	88	350	150	0	115
normalized size	1	1.06	2.65	5.18	20.59	8.82	0.	6.76
time (sec)	N/A	0.019	0.069	0.144	1.487	2.875	0.	1.218

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-2)	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	52	242	0	328	0	188
normalized size	1	1.	1.62	7.56	0.	10.25	0.	5.88
time (sec)	N/A	0.039	0.067	0.327	0.	2.67	0.	1.231

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	24	36	84	24	43
normalized size	1	1.	1.	0.77	1.16	2.71	0.77	1.39
time (sec)	N/A	0.015	0.004	0.006	0.932	2.292	0.112	1.15

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	29	41	100	34	41
normalized size	1	1.	1.	0.74	1.05	2.56	0.87	1.05
time (sec)	N/A	0.014	0.014	0.006	1.443	2.089	0.125	1.115

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	40	60	62	201	41	57
normalized size	1	1.	0.69	1.03	1.07	3.47	0.71	0.98
time (sec)	N/A	0.032	0.016	0.012	0.925	2.002	0.146	1.138

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	30	37	45	180	986	47
normalized size	1	1.	0.58	0.71	0.87	3.46	18.96	0.9
time (sec)	N/A	0.023	0.006	0.005	1.405	2.063	5.233	1.077

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	46	57	158	167	62
normalized size	1	1.	0.77	0.75	0.93	2.59	2.74	1.02
time (sec)	N/A	0.032	0.018	0.005	1.456	2.056	9.045	1.067

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	15	15	0	36	19	19
normalized size	1	1.	0.94	0.94	0.	2.25	1.19	1.19
time (sec)	N/A	0.003	0.002	0.002	0.	2.083	0.055	1.075

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	36	33	47	130	32	49
normalized size	1	1.	0.78	0.72	1.02	2.83	0.7	1.07
time (sec)	N/A	0.024	0.013	0.009	0.947	1.865	0.112	1.053

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	23	58	49	85	34	28
normalized size	1	1.	0.57	1.45	1.22	2.12	0.85	0.7
time (sec)	N/A	0.035	0.008	0.01	0.934	1.946	0.129	1.053

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	17	26	62	41	26
normalized size	1	1.	0.74	0.63	0.96	2.3	1.52	0.96
time (sec)	N/A	0.011	0.006	0.004	0.945	1.955	18.786	1.079

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	25	28	38	89	82	35
normalized size	1	1.	0.66	0.74	1.	2.34	2.16	0.92
time (sec)	N/A	0.015	0.009	0.004	0.976	2.029	6.208	1.064

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	23	20	34	74	61	26
normalized size	1	1.	0.7	0.61	1.03	2.24	1.85	0.79
time (sec)	N/A	0.004	0.005	0.001	0.937	2.074	6.525	1.074

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	26	23	69	147	0	28
normalized size	1	1.	0.6	0.53	1.6	3.42	0.	0.65
time (sec)	N/A	0.006	0.014	0.003	0.934	2.111	0.	1.063

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	28	23	47	95	151	84
normalized size	1	1.	0.6	0.49	1.	2.02	3.21	1.79
time (sec)	N/A	0.009	0.005	0.002	1.405	2.048	4.752	1.078

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	19	16	22	43	24	22
normalized size	1	1.	0.68	0.57	0.79	1.54	0.86	0.79
time (sec)	N/A	0.006	0.005	0.005	0.932	2.06	8.785	1.068

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	49	47	72	189	49	78
normalized size	1	1.	0.94	0.9	1.38	3.63	0.94	1.5
time (sec)	N/A	0.028	0.026	0.011	0.942	2.262	0.166	1.075

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	0	61	0	0
normalized size	1	1.	1.	0.88	0.	2.44	0.	0.
time (sec)	N/A	0.057	0.009	0.004	0.	4.553	0.	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	35	28	107	196	0	132
normalized size	1	1.	0.7	0.56	2.14	3.92	0.	2.64
time (sec)	N/A	0.015	0.023	0.032	1.418	2.202	0.	1.166

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	22	28	90	49	39
normalized size	1	1.	1.	0.92	1.17	3.75	2.04	1.62
time (sec)	N/A	0.005	0.016	0.005	1.404	2.149	0.516	1.09

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	24	58	61	109	32	104
normalized size	1	1.	0.6	1.45	1.52	2.72	0.8	2.6
time (sec)	N/A	0.009	0.003	0.005	1.43	2.017	2.569	1.088

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	28	25	50	105	291	47
normalized size	1	1.	0.57	0.51	1.02	2.14	5.94	0.96
time (sec)	N/A	0.007	0.008	0.002	0.925	2.108	85.665	1.084

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	29	28	80	120	0	47
normalized size	1	1.	0.62	0.6	1.7	2.55	0.	1.
time (sec)	N/A	0.007	0.011	0.003	0.933	2.115	0.	1.083

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	33	43	77	34	43
normalized size	1	1.	1.	0.92	1.19	2.14	0.94	1.19
time (sec)	N/A	0.012	0.001	0.001	0.946	1.825	0.055	1.063

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	30	39	74	34	39
normalized size	1	1.	1.	0.77	1.	1.9	0.87	1.
time (sec)	N/A	0.014	0.001	0.001	0.93	1.751	0.057	1.063

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	33	30	80	180	0	36
normalized size	1	1.	0.7	0.64	1.7	3.83	0.	0.77
time (sec)	N/A	0.009	0.047	0.003	0.938	2.012	0.	1.088

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	33	30	103	194	0	55
normalized size	1	1.	0.73	0.67	2.29	4.31	0.	1.22
time (sec)	N/A	0.022	0.112	0.005	0.956	2.028	0.	1.097

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	67	70	74	190	112	74
normalized size	1	1.	0.81	0.84	0.89	2.29	1.35	0.89
time (sec)	N/A	0.094	0.054	0.012	0.961	2.311	3.469	1.079

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	51	57	66	146	92	66
normalized size	1	1.	0.7	0.78	0.9	2.	1.26	0.9
time (sec)	N/A	0.084	0.096	0.023	0.977	2.249	1.953	1.071

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	70	96	89	240	192	89
normalized size	1	1.	0.67	0.91	0.85	2.29	1.83	0.85
time (sec)	N/A	0.11	0.086	0.03	0.943	2.315	5.743	1.062

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	39	32	47	111	53	47
normalized size	1	1.	0.89	0.73	1.07	2.52	1.2	1.07
time (sec)	N/A	0.042	0.122	0.004	0.957	2.251	1.21	1.082

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	125	0	138	507	278
normalized size	1	1.	1.	3.79	0.	4.18	15.36	8.42
time (sec)	N/A	0.054	0.029	0.193	0.	2.025	2.542	1.126

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	104	30	836	154	551	460
normalized size	1	1.	3.47	1.	27.87	5.13	18.37	15.33
time (sec)	N/A	0.041	0.113	0.122	1.512	1.915	1.816	1.529

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	178	47	128	72
normalized size	1	1.	1.	0.81	11.12	2.94	8.	4.5
time (sec)	N/A	0.018	0.016	0.005	0.945	1.797	1.053	1.078

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	57	57	89	432	0	0
normalized size	1	1.	0.92	0.92	1.44	6.97	0.	0.
time (sec)	N/A	0.072	0.015	0.06	1.446	2.075	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	59	288	473	0	0
normalized size	1	1.	0.92	1.	4.88	8.02	0.	0.
time (sec)	N/A	0.061	0.013	0.197	1.56	2.119	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	14	44	105	42	0	14
normalized size	1	1.	1.17	3.67	8.75	3.5	0.	1.17
time (sec)	N/A	0.099	0.214	0.711	1.353	1.951	0.	1.067

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	19	37	93	55	66	53
normalized size	1	1.	0.95	1.85	4.65	2.75	3.3	2.65
time (sec)	N/A	0.034	0.208	0.214	0.955	1.821	2.356	1.067

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	53	42	439
normalized size	1	1.	1.	1.05	0.	2.41	1.91	19.95
time (sec)	N/A	0.026	0.014	0.004	0.	1.915	0.655	1.149

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	41	46	65	0	113	0	589
normalized size	1	1.21	1.35	1.91	0.	3.32	0.	17.32
time (sec)	N/A	0.208	0.05	0.017	0.	1.829	0.	1.309

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	30	7	9
normalized size	1	1.	1.	0.89	1.	3.33	0.78	1.
time (sec)	N/A	0.003	0.003	0.002	0.927	1.688	0.083	1.086

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	19	22	58	17	32
normalized size	1	1.	1.	0.86	1.	2.64	0.77	1.45
time (sec)	N/A	0.018	0.006	0.006	0.926	1.868	0.101	1.058

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	30	24	31	70	24	34
normalized size	1	1.	0.97	0.77	1.	2.26	0.77	1.1
time (sec)	N/A	0.019	0.013	0.005	0.923	1.731	0.126	1.065

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	34	31	38	90	31	49
normalized size	1	1.	0.94	0.86	1.06	2.5	0.86	1.36
time (sec)	N/A	0.026	0.021	0.007	0.929	1.804	0.137	1.064

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	52	45	0	0	0	0	0
normalized size	1	1.08	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.047	0.012	0.127	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	33	56	55	103	54	62
normalized size	1	1.	0.77	1.3	1.28	2.4	1.26	1.44
time (sec)	N/A	0.027	0.032	0.019	0.925	1.917	0.218	1.062

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	28	36	51	29	36
normalized size	1	1.	1.	1.04	1.33	1.89	1.07	1.33
time (sec)	N/A	0.011	0.006	0.003	0.948	1.77	0.286	1.065

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	59	69	138	250	933
normalized size	1	1.	1.	1.11	1.3	2.6	4.72	17.6
time (sec)	N/A	0.078	0.042	0.019	0.922	1.92	2.047	1.157

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	65	90	104	278	665	1395
normalized size	1	1.	0.82	1.14	1.32	3.52	8.42	17.66
time (sec)	N/A	0.1	0.088	0.03	0.942	1.788	30.886	1.206

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	80	109	134	448	0	1835
normalized size	1	1.	0.82	1.11	1.37	4.57	0.	18.72
time (sec)	N/A	0.125	0.091	0.016	0.932	1.907	0.	1.247

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	72	80	73	0	0	0	0	0
normalized size	1	1.11	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.029	0.05	0.	0.	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	38	53	29	38
normalized size	1	1.	1.	1.04	1.36	1.89	1.04	1.36
time (sec)	N/A	0.008	0.006	0.001	0.934	1.873	0.287	1.125

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	59	69	139	248	933
normalized size	1	1.	1.	1.11	1.3	2.62	4.68	17.6
time (sec)	N/A	0.071	0.034	0.012	0.927	1.862	2.033	1.21

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	66	90	104	278	663	1395
normalized size	1	1.	0.84	1.14	1.32	3.52	8.39	17.66
time (sec)	N/A	0.083	0.068	0.026	0.937	1.882	31.378	1.248

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	80	109	134	450	0	1835
normalized size	1	1.	0.82	1.11	1.37	4.59	0.	18.72
time (sec)	N/A	0.098	0.07	0.015	0.932	1.941	0.	1.357

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	74	82	75	0	0	0	0	0
normalized size	1	1.11	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.026	0.048	0.	0.	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	47	15	20
normalized size	1	1.	1.	1.07	1.33	3.13	1.	1.33
time (sec)	N/A	0.006	0.004	0.	0.924	1.827	0.088	1.124

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	31	46	42	74	46	41
normalized size	1	1.	0.94	1.39	1.27	2.24	1.39	1.24
time (sec)	N/A	0.013	0.01	0.008	0.928	1.814	0.111	1.087

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	35	62	62	97	71	54
normalized size	1	1.	0.7	1.24	1.24	1.94	1.42	1.08
time (sec)	N/A	0.016	0.026	0.002	0.935	1.808	0.134	1.09

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	49	78	82	122	88	65
normalized size	1	1.	0.75	1.2	1.26	1.88	1.35	1.
time (sec)	N/A	0.019	0.02	0.002	0.922	1.673	0.156	1.137

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.021	0.014	0.029	0.	0.	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	22	47	19	22
normalized size	1	1.	1.	1.06	1.38	2.94	1.19	1.38
time (sec)	N/A	0.005	0.004	0.001	0.927	1.838	0.091	1.161

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	25	46	42	74	46	41
normalized size	1	1.	0.76	1.39	1.27	2.24	1.39	1.24
time (sec)	N/A	0.013	0.015	0.002	0.931	1.66	0.11	1.073

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	35	62	62	97	71	54
normalized size	1	1.	0.7	1.24	1.24	1.94	1.42	1.08
time (sec)	N/A	0.017	0.026	0.002	0.93	1.936	0.137	1.081

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	49	78	82	122	88	65
normalized size	1	1.	0.75	1.2	1.26	1.88	1.35	1.
time (sec)	N/A	0.019	0.016	0.002	0.93	1.844	0.155	1.108

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.019	0.014	0.032	0.	0.	0.	0.

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	31	31	46	15	32
normalized size	1	1.	1.	1.29	1.29	1.92	0.62	1.33
time (sec)	N/A	0.016	0.006	0.007	0.935	1.842	0.112	1.095

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	123	97	95	0	795	22	157
normalized size	1	1.23	0.97	0.95	0.	7.95	0.22	1.57
time (sec)	N/A	0.094	0.055	0.007	0.	1.874	0.16	1.127

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	14	15	28	8	15
normalized size	1	1.	1.	1.17	1.25	2.33	0.67	1.25
time (sec)	N/A	0.022	0.008	0.005	0.927	1.87	0.078	1.12

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	38	50	127	22	50
normalized size	1	1.	0.94	0.81	1.06	2.7	0.47	1.06
time (sec)	N/A	0.059	0.021	0.007	1.413	1.789	0.134	1.134

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	51	29	38	97	48	39
normalized size	1	1.	1.31	0.74	0.97	2.49	1.23	1.
time (sec)	N/A	0.062	0.042	0.012	1.413	1.949	0.187	1.131

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	33	0	80	94	43
normalized size	1	1.	1.	1.1	0.	2.67	3.13	1.43
time (sec)	N/A	0.038	0.038	0.003	0.	1.811	1.207	1.169

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	57	76	192	0	77
normalized size	1	1.	1.	1.06	1.41	3.56	0.	1.43
time (sec)	N/A	0.022	0.014	0.006	1.424	1.695	0.	1.146

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.021	0.027	0.	0.	0.	0.

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	12	45	31	24
normalized size	1	1.	1.	0.83	0.67	2.5	1.72	1.33
time (sec)	N/A	0.028	0.005	0.01	0.93	1.885	0.63	1.134

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	27	46	31	27
normalized size	1	1.	1.	0.85	1.35	2.3	1.55	1.35
time (sec)	N/A	0.029	0.005	0.009	0.932	1.886	0.648	1.143

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	0	53	154	0	65
normalized size	1	1.	1.	0.	1.32	3.85	0.	1.62
time (sec)	N/A	0.103	0.02	0.019	1.416	1.953	0.	1.201

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	57	54	0	0	0	0	0
normalized size	1	1.54	1.46	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	0.016	0.021	0.	0.	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	44	37	66	101	0	88
normalized size	1	1.	0.6	0.51	0.9	1.38	0.	1.21
time (sec)	N/A	0.047	0.02	0.019	0.932	1.836	0.	1.148

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	23	22	26	55	20	26
normalized size	1	1.	0.52	0.5	0.59	1.25	0.45	0.59
time (sec)	N/A	0.032	0.007	0.002	0.938	1.753	0.085	1.089

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	26	31	9	66	32	36
normalized size	1	1.	0.67	0.79	0.23	1.69	0.82	0.92
time (sec)	N/A	0.033	0.022	0.001	1.025	1.779	1.442	1.111

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	29	28	36	77	39	1115
normalized size	1	1.	0.66	0.64	0.82	1.75	0.89	25.34
time (sec)	N/A	0.022	0.01	0.006	0.942	1.901	0.108	1.183

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	24	23	8	24
normalized size	1	1.	1.	0.83	2.	1.92	0.67	2.
time (sec)	N/A	0.049	0.005	0.002	0.928	1.783	0.086	1.135

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	31	26	34	89	24	54
normalized size	1	1.	0.97	0.81	1.06	2.78	0.75	1.69
time (sec)	N/A	0.055	0.035	0.01	1.435	1.801	0.098	1.154

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	28	27	0	0
normalized size	1	1.	1.	1.33	1.87	1.8	0.	0.
time (sec)	N/A	0.057	0.026	0.005	1.092	2.077	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	26	68	26	26
normalized size	1	1.	0.81	0.81	0.96	2.52	0.96	0.96
time (sec)	N/A	0.011	0.026	0.007	0.947	2.195	0.478	1.109

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	22	53	80	29	53
normalized size	1	1.	0.74	0.63	1.51	2.29	0.83	1.51
time (sec)	N/A	0.111	0.059	0.014	0.93	2.175	0.829	1.083

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	37	55	42	0	70	53
normalized size	1	1.	0.65	0.96	0.74	0.	1.23	0.93
time (sec)	N/A	0.028	0.066	0.05	1.418	0.	3.063	1.14

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	39	45	61	95	265	58
normalized size	1	1.	0.72	0.83	1.13	1.76	4.91	1.07
time (sec)	N/A	0.023	0.035	0.02	0.943	2.086	3.428	1.095

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	64	68	99	165	644	85
normalized size	1	1.	0.78	0.83	1.21	2.01	7.85	1.04
time (sec)	N/A	0.035	0.167	0.023	0.976	1.894	14.677	1.109

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	36	38	36	138	76	45
normalized size	1	1.	0.46	0.48	0.46	1.75	0.96	0.57
time (sec)	N/A	0.045	0.038	0.039	0.951	1.902	2.645	1.111

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	21	28	36	120	70	32
normalized size	1	1.	0.58	0.78	1.	3.33	1.94	0.89
time (sec)	N/A	0.044	0.032	0.007	0.961	1.866	5.18	1.128

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	21	63	36	147	99	32
normalized size	1	1.	0.58	1.75	1.	4.08	2.75	0.89
time (sec)	N/A	0.043	0.038	0.017	0.947	1.919	5.62	1.127

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	85	97	0	0	0	0	0
normalized size	1	1.47	1.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	0.237	0.058	0.	0.	0.	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	90	0	0	0	0	0
normalized size	1	1.	2.	0.	0.	0.	0.	0.
time (sec)	N/A	0.018	0.174	0.083	0.	0.	0.	0.

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	77	66	0	0	0	0	0
normalized size	1	1.51	1.29	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	0.047	0.113	0.	0.	0.	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.031	0.01	0.028	0.	0.	0.	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	84	0	0	0	0	0
normalized size	1	1.	3.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.032	0.073	0.035	0.	0.	0.	0.

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	61	0	0	0	0	0
normalized size	1	1.	2.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.572	0.044	0.	0.	0.	0.

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	61	0	0	0	0	0
normalized size	1	1.	2.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.034	0.623	0.054	0.	0.	0.	0.

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	11	33	30	35	0	14
normalized size	1	1.	0.73	2.2	2.	2.33	0.	0.93
time (sec)	N/A	0.03	0.218	0.047	1.27	2.17	0.	1.141

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	45	100	0	0	0	0	0
normalized size	1	1.1	2.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.209	0.064	0.	0.	0.	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	10	8	30	34	0	9
normalized size	1	1.	0.83	0.67	2.5	2.83	0.	0.75
time (sec)	N/A	0.028	0.18	0.034	1.21	1.951	0.	1.151

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	44	87	0	0	0	0	0
normalized size	1	1.05	2.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	0.213	0.063	0.	0.	0.	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	48	72	0	0	0	0	0
normalized size	1	1.04	1.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	0.701	0.079	0.	0.	0.	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	23	53	30	74	0	27
normalized size	1	1.	1.64	3.79	2.14	5.29	0.	1.93
time (sec)	N/A	0.024	0.075	0.069	1.215	1.819	0.	1.142

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	47	73	0	0	0	0	0
normalized size	1	1.09	1.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	0.186	0.069	0.	0.	0.	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	23	51	30	76	0	28
normalized size	1	1.	1.77	3.92	2.31	5.85	0.	2.15
time (sec)	N/A	0.023	0.063	0.056	1.22	1.851	0.	1.106

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	18	20	23	58	27	20
normalized size	1	1.	0.6	0.67	0.77	1.93	0.9	0.67
time (sec)	N/A	0.039	0.026	0.004	0.94	1.923	0.793	1.116

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	25	27	35	81	48	34
normalized size	1	1.	0.5	0.54	0.7	1.62	0.96	0.68
time (sec)	N/A	0.108	0.036	0.003	0.979	1.741	2.104	1.09

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	38	36	45	120	80	45
normalized size	1	1.	0.51	0.48	0.6	1.6	1.07	0.6
time (sec)	N/A	0.14	0.035	0.006	0.969	1.905	2.228	1.154

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	253	72	78	104	279	202	99
normalized size	1	1.35	0.39	0.42	0.56	1.49	1.08	0.53
time (sec)	N/A	0.478	0.149	0.039	0.99	2.019	12.812	1.087

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	40	40	55	123	85	53
normalized size	1	1.	0.46	0.46	0.63	1.41	0.98	0.61
time (sec)	N/A	0.157	0.078	0.004	0.956	1.758	2.207	1.122

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	76	78	104	282	202	99
normalized size	1	1.	0.41	0.42	0.56	1.52	1.09	0.54
time (sec)	N/A	0.356	0.211	0.024	0.985	2.259	12.776	1.147

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	12	2	15
normalized size	1	1.	1.	1.5	1.5	6.	1.	7.5
time (sec)	N/A	0.003	0.002	0.	0.933	2.058	0.111	1.127

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	12	2	15
normalized size	1	1.	1.	1.5	1.5	6.	1.	7.5
time (sec)	N/A	0.003	0.001	0.002	0.927	1.992	0.111	1.101

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	55	7	15
normalized size	1	1.	1.	1.33	1.33	18.33	2.33	5.
time (sec)	N/A	0.003	0.002	0.001	0.925	2.085	0.119	1.137

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	55	12	16
normalized size	1	1.	1.	1.33	1.33	18.33	4.	5.33
time (sec)	N/A	0.004	0.004	0.001	0.923	2.166	0.29	1.084

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	9	4	4	39	0	7
normalized size	1	1.	3.	1.33	1.33	13.	0.	2.33
time (sec)	N/A	0.003	0.004	0.	0.928	2.082	0.	1.09

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	7	6	7	78	0	19
normalized size	1	1.	1.4	1.2	1.4	15.6	0.	3.8
time (sec)	N/A	0.004	0.004	0.001	0.936	2.123	0.	1.133

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	22	39	24	32
normalized size	1	1.	1.	0.79	1.57	2.79	1.71	2.29
time (sec)	N/A	0.007	0.002	0.006	0.926	2.032	0.184	1.099

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	23	18	47	155	29	50
normalized size	1	1.	1.21	0.95	2.47	8.16	1.53	2.63
time (sec)	N/A	0.011	0.002	0.033	0.932	2.102	1.122	1.107

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	26	51	221	10	35
normalized size	1	1.	1.29	1.86	3.64	15.79	0.71	2.5
time (sec)	N/A	0.011	0.003	0.002	0.948	2.102	0.216	1.132

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	36	11	61	737	0	61
normalized size	1	1.	2.25	0.69	3.81	46.06	0.	3.81
time (sec)	N/A	0.012	0.004	0.036	0.938	2.113	0.	1.091

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	30	21	82	1547	422	81
normalized size	1	1.	1.15	0.81	3.15	59.5	16.23	3.12
time (sec)	N/A	0.017	0.004	0.035	1.415	2.093	3.983	1.161

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	47	863	0	58
normalized size	1	1.	1.	0.94	2.61	47.94	0.	3.22
time (sec)	N/A	0.023	0.006	0.013	1.408	2.151	0.	1.127

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	0	0	1253	0	0
normalized size	1	1.	0.87	0.	0.	40.42	0.	0.
time (sec)	N/A	0.031	0.053	0.046	0.	2.12	0.	0.

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	42	41	36	0	460	126	43
normalized size	1	1.02	1.	0.88	0.	11.22	3.07	1.05
time (sec)	N/A	0.049	0.032	0.01	0.	2.105	10.628	1.169

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	16	16	66	211	14	19
normalized size	1	1.	0.64	0.64	2.64	8.44	0.56	0.76
time (sec)	N/A	0.017	0.011	0.005	0.939	2.001	0.41	1.127

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	29	55	55	108	146	58
normalized size	1	1.	0.74	1.41	1.41	2.77	3.74	1.49
time (sec)	N/A	0.05	0.057	0.018	0.942	2.233	0.662	1.126

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	98	0	743	0	107
normalized size	1	1.	1.	3.16	0.	23.97	0.	3.45
time (sec)	N/A	0.035	0.064	0.047	0.	2.148	0.	1.105

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	74	0	945	0	68
normalized size	1	1.	1.	2.11	0.	27.	0.	1.94
time (sec)	N/A	0.037	0.045	0.024	0.	2.25	0.	1.125

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	55	93	382	0	65
normalized size	1	1.	0.96	2.2	3.72	15.28	0.	2.6
time (sec)	N/A	0.017	0.094	0.021	1.441	2.157	0.	1.166

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	37	78	95	265	102	30
normalized size	1	1.	1.12	2.36	2.88	8.03	3.09	0.91
time (sec)	N/A	0.136	0.129	0.084	1.462	2.227	1.991	1.088

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	57	166	116	65
normalized size	1	1.	1.	0.77	1.9	5.53	3.87	2.17
time (sec)	N/A	0.035	0.01	0.039	0.951	2.085	13.005	1.092

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	57	362	138	65
normalized size	1	1.	1.	0.77	1.9	12.07	4.6	2.17
time (sec)	N/A	0.034	0.01	0.099	0.952	2.207	12.004	1.085

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	69	102	392	609	0	1337	0	0
normalized size	1	1.48	5.68	8.83	0.	19.38	0.	0.
time (sec)	N/A	0.973	29.616	0.217	0.	2.476	0.	0.

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	26	30	169	1585	0	54
normalized size	1	1.	0.7	0.81	4.57	42.84	0.	1.46
time (sec)	N/A	0.045	0.071	0.012	1.035	2.059	0.	1.141

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	21	28	239	548	0	0
normalized size	1	1.	0.72	0.97	8.24	18.9	0.	0.
time (sec)	N/A	0.108	0.054	0.044	1.65	2.161	0.	0.

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	63	0	1636	0	0
normalized size	1	1.	1.	4.2	0.	109.07	0.	0.
time (sec)	N/A	0.019	0.011	0.048	0.	2.308	0.	0.

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	28	66	305	17	69
normalized size	1	1.	1.	1.75	4.12	19.06	1.06	4.31
time (sec)	N/A	0.019	0.021	0.013	1.561	2.188	0.171	1.168

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	28	72	305	22	72
normalized size	1	1.	1.	1.75	4.5	19.06	1.38	4.5
time (sec)	N/A	0.021	0.019	0.013	1.106	2.242	0.907	1.146

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	23	38	18	74	26	15
normalized size	1	1.	1.15	1.9	0.9	3.7	1.3	0.75
time (sec)	N/A	0.067	0.091	0.101	0.972	2.023	0.344	1.1

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	20	16	47	74	12	19
normalized size	1	1.	1.33	1.07	3.13	4.93	0.8	1.27
time (sec)	N/A	0.13	0.075	0.024	0.963	2.123	0.429	1.109

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	20	30	254	0	32
normalized size	1	1.	1.	1.	1.5	12.7	0.	1.6
time (sec)	N/A	0.018	0.015	0.079	0.939	2.142	0.	1.149

Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	52	101	338	0	14
normalized size	1	1.	1.	4.	7.77	26.	0.	1.08
time (sec)	N/A	0.017	0.01	0.043	0.976	2.111	0.	1.083

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	14	8	61	12	8
normalized size	1	1.	1.	1.56	0.89	6.78	1.33	0.89
time (sec)	N/A	0.017	0.003	0.002	0.944	2.028	0.38	1.096

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	19	18	18	0	82	32	22
normalized size	1	1.46	1.38	1.38	0.	6.31	2.46	1.69
time (sec)	N/A	0.029	0.03	0.002	0.	2.115	0.547	1.11

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1	1	1	2	1	4	10	1
normalized size	1	1.	1.	2.	1.	4.	10.	1.
time (sec)	N/A	0.015	0.001	0.018	0.936	1.91	0.353	1.083

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	36	24	22	115	0	23
normalized size	1	1.	1.64	1.09	1.	5.23	0.	1.05
time (sec)	N/A	0.024	0.032	0.01	0.949	1.989	0.	1.111

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	18	18	15	69	0	15
normalized size	1	1.	1.38	1.38	1.15	5.31	0.	1.15
time (sec)	N/A	0.031	0.026	0.012	0.981	2.036	0.	1.123

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	20	18	15	70	0	15
normalized size	1	1.	1.33	1.2	1.	4.67	0.	1.
time (sec)	N/A	0.033	0.029	0.017	0.973	1.988	0.	1.092

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	19	34	0	59	0	0
normalized size	1	1.	0.73	1.31	0.	2.27	0.	0.
time (sec)	N/A	0.01	0.008	0.007	0.	2.086	0.	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	30	61	0	115	0	113
normalized size	1	1.	0.71	1.45	0.	2.74	0.	2.69
time (sec)	N/A	0.024	0.01	0.008	0.	2.134	0.	1.098

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	21	23	30	59	34	30
normalized size	1	1.	0.62	0.68	0.88	1.74	1.	0.88
time (sec)	N/A	0.02	0.007	0.016	0.955	2.158	6.137	1.167

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	29	28	25	34	63	22	32
normalized size	1	1.04	1.	0.89	1.21	2.25	0.79	1.14
time (sec)	N/A	0.011	0.002	0.002	0.99	2.042	0.102	1.097

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	81	72	93	157	71	96
normalized size	1	1.	1.21	1.07	1.39	2.34	1.06	1.43
time (sec)	N/A	0.033	0.022	0.001	0.938	2.046	0.133	1.142

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	49	69	26	31
normalized size	1	1.	1.	1.04	2.13	3.	1.13	1.35
time (sec)	N/A	0.014	0.002	0.002	0.946	2.115	0.102	1.135

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	73	60	53	89	144	51	70
normalized size	1	1.22	1.	0.88	1.48	2.4	0.85	1.17
time (sec)	N/A	0.091	0.004	0.002	0.957	2.064	0.13	1.132

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	37	11	115	32	0
normalized size	1	1.	1.	0.86	0.26	2.67	0.74	0.
time (sec)	N/A	0.059	0.016	0.005	1.054	2.046	0.676	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	30	32	34	0	151	0
normalized size	1	1.	1.03	1.1	1.17	0.	5.21	0.
time (sec)	N/A	0.02	0.002	0.007	0.958	0.	7.306	0.

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	30	51	77	24	49
normalized size	1	1.	0.93	1.03	1.76	2.66	0.83	1.69
time (sec)	N/A	0.013	0.016	0.005	0.932	2.224	0.358	1.096

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	34	15	16
normalized size	1	1.	1.	1.08	0.	2.83	1.25	1.33
time (sec)	N/A	0.017	0.003	0.002	0.	2.408	0.865	1.133

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	0	58	36	26
normalized size	1	1.	1.	1.05	0.	3.05	1.89	1.37
time (sec)	N/A	0.028	0.012	0.001	0.	2.361	1.259	1.146

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	28	8	41
normalized size	1	1.	1.	1.09	1.36	2.55	0.73	3.73
time (sec)	N/A	0.024	0.015	0.001	0.934	2.396	0.113	1.086

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	0	62	71	30
normalized size	1	1.	1.	1.04	0.	2.7	3.09	1.3
time (sec)	N/A	0.032	0.012	0.	0.	2.449	37.215	1.097

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	46	15	12	50	0	24
normalized size	1	1.	2.88	0.94	0.75	3.12	0.	1.5
time (sec)	N/A	0.038	0.022	0.004	0.949	2.495	0.	1.119

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	50	17	27	51	0	0
normalized size	1	1.	2.78	0.94	1.5	2.83	0.	0.
time (sec)	N/A	0.042	0.024	0.007	0.935	2.415	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	12	63	0	14
normalized size	1	1.	1.	0.94	0.67	3.5	0.	0.78
time (sec)	N/A	0.041	0.024	0.006	1.443	2.472	0.	1.385

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	37	0	117	0	0
normalized size	1	1.	1.	1.68	0.	5.32	0.	0.
time (sec)	N/A	0.088	0.017	0.005	0.	2.398	0.	0.

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	39	0	58	0	0
normalized size	1	1.	1.	1.62	0.	2.42	0.	0.
time (sec)	N/A	0.098	0.016	0.007	0.	2.372	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	43	0	65	0	28
normalized size	1	1.	1.	1.87	0.	2.83	0.	1.22
time (sec)	N/A	0.099	0.016	0.005	0.	2.335	0.	1.065

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	39	10	15
normalized size	1	1.	1.	1.09	1.36	3.55	0.91	1.36
time (sec)	N/A	0.007	0.004	0.002	0.95	2.458	0.275	1.062

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	20	76	24	27
normalized size	1	1.	1.	1.05	1.	3.8	1.2	1.35
time (sec)	N/A	0.018	0.006	0.003	0.959	2.412	0.316	1.063

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	30	30	109	36	39
normalized size	1	1.	1.	1.03	1.03	3.76	1.24	1.34
time (sec)	N/A	0.022	0.009	0.002	1.044	2.328	0.356	1.062

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	39	39	147	48	51
normalized size	1	1.	1.	1.03	1.03	3.87	1.26	1.34
time (sec)	N/A	0.026	0.006	0.002	0.981	2.755	0.397	1.068

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	0	39	51	24	0
normalized size	1	1.	1.	0.	1.62	2.12	1.	0.
time (sec)	N/A	0.032	0.017	0.023	1.05	2.777	2.767	0.

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	24	0	7
normalized size	1	1.	1.	1.25	1.25	6.	0.	1.75
time (sec)	N/A	0.02	0.011	0.008	0.937	2.38	0.	1.068

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	10	12	32	7	12
normalized size	1	1.	1.	1.43	1.71	4.57	1.	1.71
time (sec)	N/A	0.047	0.004	0.013	0.938	2.448	2.421	1.065

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	185	10	43
normalized size	1	1.	1.	1.09	1.36	16.82	0.91	3.91
time (sec)	N/A	0.011	0.007	0.004	0.948	2.205	0.939	1.065

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	27	0	0
normalized size	1	1.	1.	0.89	1.	3.	0.	0.
time (sec)	N/A	0.015	0.004	0.008	0.942	2.392	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	23	0	57	20	30
normalized size	1	1.	1.	0.88	0.	2.19	0.77	1.15
time (sec)	N/A	0.005	0.02	0.002	0.	2.365	9.296	1.059

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	27	26	34	68	26	36
normalized size	1	1.	0.77	0.74	0.97	1.94	0.74	1.03
time (sec)	N/A	0.015	0.01	0.011	0.944	2.371	0.128	1.058

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	24	32	27	72	0	27
normalized size	1	1.	0.75	1.	0.84	2.25	0.	0.84
time (sec)	N/A	0.054	0.031	0.016	0.952	2.393	0.	1.061

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	42	43	57	149	0	58
normalized size	1	1.	0.81	0.83	1.1	2.87	0.	1.12
time (sec)	N/A	0.044	0.033	0.023	0.95	2.378	0.	1.071

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	126	34	90	42	35
normalized size	1	1.	1.	4.2	1.13	3.	1.4	1.17
time (sec)	N/A	0.035	0.008	0.035	0.959	2.619	5.091	1.073

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	29	55	34	117	46	35
normalized size	1	1.	0.97	1.83	1.13	3.9	1.53	1.17
time (sec)	N/A	0.058	0.071	0.153	0.944	2.613	160.155	1.066

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	32	164	76	105	0	58
normalized size	1	1.	1.14	5.86	2.71	3.75	0.	2.07
time (sec)	N/A	0.035	0.085	0.163	0.956	2.612	0.	1.087

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	56	106	116	174	107	49
normalized size	1	1.	0.93	1.77	1.93	2.9	1.78	0.82
time (sec)	N/A	0.132	0.148	0.086	1.432	2.491	11.404	1.214

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	52	52	69	119	0	140
normalized size	1	1.	0.8	0.8	1.06	1.83	0.	2.15
time (sec)	N/A	0.109	0.032	0.033	1.429	2.811	0.	1.116

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	42	37	63	109	54	77
normalized size	1	1.	0.69	0.61	1.03	1.79	0.89	1.26
time (sec)	N/A	0.094	0.022	0.023	1.441	2.442	0.597	1.076

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	37	42	59	115	44	55
normalized size	1	1.	0.7	0.79	1.11	2.17	0.83	1.04
time (sec)	N/A	0.116	0.015	0.01	1.447	2.545	0.614	1.082

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	56	48	86	140	53	0
normalized size	1	1.	0.92	0.79	1.41	2.3	0.87	0.
time (sec)	N/A	0.126	0.021	0.017	1.432	2.519	0.901	0.

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	42	56	128	115	0	0
normalized size	1	1.	0.67	0.89	2.03	1.83	0.	0.
time (sec)	N/A	0.067	0.048	0.034	1.668	2.625	0.	0.

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	92	165	0	220	0	0
normalized size	1	1.	0.62	1.11	0.	1.49	0.	0.
time (sec)	N/A	0.147	0.074	0.062	0.	2.703	0.	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	31	41	81	48	36
normalized size	1	1.	0.88	0.91	1.21	2.38	1.41	1.06
time (sec)	N/A	0.029	0.009	0.038	1.433	2.404	18.919	1.082

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	33	41	81	48	36
normalized size	1	1.	0.88	0.97	1.21	2.38	1.41	1.06
time (sec)	N/A	0.03	0.017	0.044	1.424	2.559	19.158	1.068

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	26	134	30	78	37	30
normalized size	1	1.	0.87	4.47	1.	2.6	1.23	1.
time (sec)	N/A	0.031	0.028	0.15	1.418	2.685	1.005	1.087

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	42	58	68	115	53	68
normalized size	1	1.	0.71	0.98	1.15	1.95	0.9	1.15
time (sec)	N/A	0.05	0.034	0.038	1.401	2.562	2.118	1.098

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	37	36	105	63	46
normalized size	1	1.	0.95	1.	0.97	2.84	1.7	1.24
time (sec)	N/A	0.037	0.018	0.037	1.426	2.588	3.544	1.084

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	430	66	138	88	81
normalized size	1	1.	0.77	7.05	1.08	2.26	1.44	1.33
time (sec)	N/A	0.075	0.057	0.28	1.409	2.459	153.986	1.074

Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	119	227	0	0	0	0
normalized size	1	1.	1.25	2.39	0.	0.	0.	0.
time (sec)	N/A	0.158	0.256	0.191	0.	0.	0.	0.

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	36	201	47	113	0	182
normalized size	1	1.	0.88	4.9	1.15	2.76	0.	4.44
time (sec)	N/A	0.059	0.044	0.532	1.452	2.785	0.	1.093

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	32	43	82	26	36
normalized size	1	1.	0.82	0.94	1.26	2.41	0.76	1.06
time (sec)	N/A	0.06	0.011	0.035	1.407	2.591	0.333	1.076

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	43	53	70	115	53	68
normalized size	1	1.	0.7	0.87	1.15	1.89	0.87	1.11
time (sec)	N/A	0.107	0.025	0.044	1.426	2.441	1.295	1.09

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	46	34	123	20	36
normalized size	1	1.	1.	2.42	1.79	6.47	1.05	1.89
time (sec)	N/A	0.038	0.011	0.035	1.405	2.532	4.896	1.078

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	32	47	34	122	20	36
normalized size	1	1.	1.88	2.76	2.	7.18	1.18	2.12
time (sec)	N/A	0.035	0.054	0.038	1.41	2.408	6.044	1.08

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	45	63	65	151	0	73
normalized size	1	1.	0.73	1.02	1.05	2.44	0.	1.18
time (sec)	N/A	0.039	0.088	0.037	1.411	2.539	0.	1.09

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	40	61	61	155	37	54
normalized size	1	1.	1.11	1.69	1.69	4.31	1.03	1.5
time (sec)	N/A	0.069	0.064	0.219	1.445	2.502	15.3	1.089

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	62	62	112	97	0	0	0	0
normalized size	1	1.	1.81	1.56	0.	0.	0.	0.
time (sec)	N/A	0.116	0.181	0.159	0.	0.	0.	0.

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	38	43	57	95	60	128
normalized size	1	1.	0.7	0.8	1.06	1.76	1.11	2.37
time (sec)	N/A	0.091	0.043	0.043	1.41	2.698	114.909	1.104

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	50	158	70	119	78	72
normalized size	1	1.	0.76	2.39	1.06	1.8	1.18	1.09
time (sec)	N/A	0.072	0.053	0.14	1.452	2.522	2.945	1.112

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	60	69	0	151	66	81
normalized size	1	1.	0.82	0.95	0.	2.07	0.9	1.11
time (sec)	N/A	0.155	0.032	0.053	0.	2.459	1.272	1.117

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	21	27	35	55	31	35
normalized size	1	1.	0.66	0.84	1.09	1.72	0.97	1.09
time (sec)	N/A	0.026	0.02	0.004	1.408	2.363	0.687	1.064

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	36	37	53	95	88	46
normalized size	1	1.	0.82	0.84	1.2	2.16	2.	1.05
time (sec)	N/A	0.029	0.02	0.005	1.4	2.436	1.142	1.063

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	32	68	19	26
normalized size	1	1.	1.	0.87	1.39	2.96	0.83	1.13
time (sec)	N/A	0.048	0.015	0.009	1.411	2.409	0.385	1.082

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	57	128	0	0	0	0
normalized size	1	1.	0.85	1.91	0.	0.	0.	0.
time (sec)	N/A	0.094	0.032	0.031	0.	0.	0.	0.

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	28	29	54	80	0	0
normalized size	1	1.	0.82	0.85	1.59	2.35	0.	0.
time (sec)	N/A	0.047	0.026	0.012	1.418	2.526	0.	0.

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	79	79	64	139	0	0	0	0
normalized size	1	1.	0.81	1.76	0.	0.	0.	0.
time (sec)	N/A	0.113	0.067	0.013	0.	0.	0.	0.

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	89	89	70	149	0	0	0	0
normalized size	1	1.	0.79	1.67	0.	0.	0.	0.
time (sec)	N/A	0.234	0.201	0.018	0.	0.	0.	0.

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	28	72	19	34
normalized size	1	1.	1.	1.05	1.27	3.27	0.86	1.55
time (sec)	N/A	0.034	0.006	0.008	1.415	2.312	0.385	1.069

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	59	30	42	74	34	42
normalized size	1	1.	1.9	0.97	1.35	2.39	1.1	1.35
time (sec)	N/A	0.023	0.008	0.008	1.411	2.435	0.86	1.077

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	81	79	96	0	0	0
normalized size	1	1.	1.29	1.25	1.52	0.	0.	0.
time (sec)	N/A	0.084	0.007	0.019	1.575	0.	0.	0.

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	36	92	22	0
normalized size	1	1.	1.	0.89	1.29	3.29	0.79	0.
time (sec)	N/A	0.057	0.006	0.012	1.439	2.505	0.595	0.

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	38	34	49	113	32	0
normalized size	1	1.	0.97	0.87	1.26	2.9	0.82	0.
time (sec)	N/A	0.07	0.019	0.001	1.426	2.509	0.547	0.

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	56	57	96	153	61	0
normalized size	1	1.	0.93	0.95	1.6	2.55	1.02	0.
time (sec)	N/A	0.078	0.032	0.019	1.44	2.429	0.908	0.

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	82	47	78	127	132	0	0
normalized size	1	1.04	0.59	0.99	1.61	1.67	0.	0.
time (sec)	N/A	0.133	0.056	0.025	1.486	2.531	0.	0.

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	116	116	708	0	0	0	0
normalized size	1	1.08	1.08	6.62	0.	0.	0.	0.
time (sec)	N/A	0.154	0.172	0.391	0.	0.	0.	0.

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	133	86	305	0	146	0	0
normalized size	1	1.25	0.81	2.88	0.	1.38	0.	0.
time (sec)	N/A	0.196	0.295	0.322	0.	2.752	0.	0.

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	48	602	36	69	0	101
normalized size	1	1.	1.17	14.68	0.88	1.68	0.	2.46
time (sec)	N/A	0.052	0.053	0.383	1.477	2.338	0.	1.105

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	65	67	67	128	65	198	0	78
normalized size	1	1.03	1.03	1.97	1.	3.05	0.	1.2
time (sec)	N/A	0.031	0.13	0.24	1.182	2.354	0.	1.116

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	51	53	61	121	62	180	0	72
normalized size	1	1.04	1.2	2.37	1.22	3.53	0.	1.41
time (sec)	N/A	0.064	0.117	0.248	1.156	2.644	0.	1.092

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	82	84	72	197	0	186	0	86
normalized size	1	1.02	0.88	2.4	0.	2.27	0.	1.05
time (sec)	N/A	0.09	0.175	0.403	0.	2.629	0.	1.119

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	175	232	383	240	0	0	0	0
normalized size	1	1.33	2.19	1.37	0.	0.	0.	0.
time (sec)	N/A	0.317	1.87	0.372	0.	0.	0.	0.

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	35	186	23	45	0	68
normalized size	1	1.	1.52	8.09	1.	1.96	0.	2.96
time (sec)	N/A	0.048	0.034	0.223	1.472	2.577	0.	1.105

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	F(-1)	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD
size	70	91	79	702	166	223	0	142
normalized size	1	1.3	1.13	10.03	2.37	3.19	0.	2.03
time (sec)	N/A	0.086	0.128	0.475	2.579	2.536	0.	1.13

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	101	76	443	78	117	0	0
normalized size	1	1.36	1.03	5.99	1.05	1.58	0.	0.
time (sec)	N/A	0.179	0.059	0.405	1.507	2.581	0.	0.

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	172	84	327	0	163	0	0
normalized size	1	1.29	0.63	2.46	0.	1.23	0.	0.
time (sec)	N/A	0.201	0.241	0.355	0.	2.398	0.	0.

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	146	92	1153	126	163	0	0
normalized size	1	1.33	0.84	10.48	1.15	1.48	0.	0.
time (sec)	N/A	0.2	0.09	0.588	2.538	2.432	0.	0.

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	118	99	85	139	132	0	0
normalized size	1	2.15	1.8	1.55	2.53	2.4	0.	0.
time (sec)	N/A	0.843	0.144	0.025	1.449	2.535	0.	0.

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	71	64	120	154	0	66
normalized size	1	1.	1.78	1.6	3.	3.85	0.	1.65
time (sec)	N/A	0.043	0.049	0.013	1.417	2.88	0.	1.116

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	32	42	146	153	43
normalized size	1	1.	0.9	0.82	1.08	3.74	3.92	1.1
time (sec)	N/A	0.029	0.028	0.009	1.411	2.799	0.662	1.076

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	122	122	105	102	159	0	0	0
normalized size	1	1.	0.86	0.84	1.3	0.	0.	0.
time (sec)	N/A	0.091	0.03	0.013	1.625	0.	0.	0.

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	0	0	42	27	0
normalized size	1	1.	1.	0.	0.	1.5	0.96	0.
time (sec)	N/A	0.033	0.017	0.056	0.	2.551	2.921	0.

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	26	34	76	26	34
normalized size	1	1.	1.	0.84	1.1	2.45	0.84	1.1
time (sec)	N/A	0.038	0.027	0.007	0.942	2.527	3.328	1.072

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	61	70	0	235	0	78
normalized size	1	1.	1.07	1.23	0.	4.12	0.	1.37
time (sec)	N/A	0.032	0.111	0.006	0.	2.716	0.	1.098

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	140	72	76	157	347	0	266
normalized size	1	1.71	0.88	0.93	1.91	4.23	0.	3.24
time (sec)	N/A	0.078	0.081	0.015	3.699	2.476	0.	1.247

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	66	0	0	1785	0	0
normalized size	1	1.	1.35	0.	0.	36.43	0.	0.
time (sec)	N/A	0.14	0.228	0.046	0.	2.554	0.	0.

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	40	854	73	1422	0	95
normalized size	1	1.	1.11	23.72	2.03	39.5	0.	2.64
time (sec)	N/A	0.12	0.167	0.462	1.448	2.305	0.	1.092

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	28	28	64	0	22	100	0	51
normalized size	1	1.	2.29	0.	0.79	3.57	0.	1.82
time (sec)	N/A	0.077	0.837	0.046	1.647	2.213	0.	1.079

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [397] had the largest ratio of [1.333]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	14	0.071
2	A	1	1	1.	13	0.077
3	A	1	1	1.	5	0.2
4	A	2	2	1.	10	0.2
5	A	1	1	1.	12	0.083
6	A	2	2	1.	5	0.4
7	A	2	2	1.	5	0.4
8	A	2	1	1.	7	0.143
9	A	1	1	1.	6	0.167
10	A	1	1	1.	8	0.125
11	A	2	2	1.	12	0.167
12	A	2	2	1.	17	0.118
13	A	2	2	1.	18	0.111
14	A	3	2	1.	19	0.105
15	A	3	2	1.	20	0.1
16	A	3	2	1.22	19	0.105
17	A	3	2	1.22	20	0.1
18	A	2	2	1.	10	0.2
19	A	2	2	1.	13	0.154
20	A	2	2	1.	8	0.25
21	A	2	1	1.	12	0.083
22	A	2	2	1.	12	0.167
23	A	2	2	1.	14	0.143
24	A	3	2	1.	16	0.125
25	A	4	2	1.	22	0.091
26	A	2	1	1.	13	0.077
27	A	3	2	1.	15	0.133
28	A	3	3	1.	15	0.2
29	A	1	1	1.	9	0.111
30	A	1	1	1.	9	0.111
31	A	4	3	1.	10	0.3
32	A	2	2	1.	4	0.5
33	A	2	2	1.	4	0.5
34	A	2	2	1.	7	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
35	A	2	1	1.	7	0.143
36	A	3	2	1.	9	0.222
37	A	2	2	1.	8	0.25
38	A	2	2	1.	8	0.25
39	A	4	2	1.	9	0.222
40	A	4	4	1.	9	0.444
41	A	2	2	1.	9	0.222
42	A	2	2	1.	11	0.182
43	A	3	2	1.	19	0.105
44	A	3	2	1.	10	0.2
45	A	3	3	1.	16	0.188
46	A	3	2	1.	14	0.143
47	A	4	3	1.	11	0.273
48	A	2	2	1.	13	0.154
49	A	3	2	1.	13	0.154
50	A	3	3	1.	15	0.2
51	A	3	3	1.	17	0.176
52	A	3	3	1.	17	0.176
53	A	3	3	1.	15	0.2
54	A	2	2	1.	12	0.167
55	A	2	2	1.	14	0.143
56	A	2	2	1.	11	0.182
57	A	3	3	1.	18	0.167
58	A	2	2	1.	16	0.125
59	A	1	1	1.1	18	0.056
60	A	7	5	1.	17	0.294
61	A	2	2	1.3	10	0.2
62	A	3	1	1.	11	0.091
63	A	2	1	1.	17	0.059
64	A	3	2	1.06	19	0.105
65	A	3	2	1.	19	0.105
66	A	3	3	1.	11	0.273
67	A	3	3	1.	17	0.176
68	A	1	1	1.	12	0.083
69	A	2	2	1.	24	0.083
70	A	1	1	1.	16	0.062
71	A	2	2	1.	6	0.333
72	A	1	1	1.	6	0.167
73	A	5	4	1.06	12	0.333
74	A	2	1	1.	4	0.25
75	A	4	3	1.	9	0.333
76	A	3	2	1.	4	0.5
77	A	1	1	1.	8	0.125
78	A	1	1	1.	10	0.1

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
79	A	1	1	1.	6	0.167
80	A	1	1	1.	3	0.333
81	A	3	4	1.	8	0.5
82	A	3	3	1.	6	0.5
83	A	4	4	1.	6	0.667
84	A	3	3	1.	4	0.75
85	A	4	4	1.	13	0.308
86	A	6	6	1.5	12	0.5
87	A	2	1	1.	11	0.091
88	A	2	1	1.	16	0.062
89	A	1	1	1.	15	0.067
90	A	2	1	1.	11	0.091
91	A	3	2	1.	13	0.154
92	A	3	2	1.	12	0.167
93	A	4	4	1.	16	0.25
94	A	5	5	1.	14	0.357
95	A	7	6	1.	29	0.207
96	A	3	2	1.	19	0.105
97	A	2	1	1.	39	0.026
98	A	12	5	1.65	21	0.238
99	A	3	2	1.	20	0.1
100	A	2	1	1.	21	0.048
101	A	2	1	1.	11	0.091
102	A	2	1	1.	9	0.111
103	A	2	1	1.	25	0.04
104	A	2	1	1.	24	0.042
105	A	3	2	1.	19	0.105
106	A	2	1	1.	19	0.053
107	A	5	4	1.	19	0.21
108	A	5	4	1.	16	0.25
109	A	3	3	1.	14	0.214
110	A	6	5	1.	33	0.152
111	A	5	4	1.	21	0.19
112	A	5	4	1.	21	0.19
113	A	9	5	1.	10	0.5
114	A	10	9	1.	17	0.529
115	A	6	5	1.	26	0.192
116	A	6	2	1.	29	0.069
117	A	3	2	1.	30	0.067
118	A	6	6	1.	9	0.667
119	A	6	6	1.	11	0.546
120	A	1	1	1.	13	0.077
121	A	4	4	1.	13	0.308
122	A	7	7	1.	13	0.538

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
123	A	7	7	1.	13	0.538
124	A	3	2	1.	13	0.154
125	A	8	7	1.	13	0.538
126	A	1	1	1.	15	0.067
127	A	3	3	1.	11	0.273
128	A	2	2	1.	13	0.154
129	A	4	4	1.	15	0.267
130	A	4	4	1.	15	0.267
131	A	3	3	1.	15	0.2
132	A	4	4	1.	15	0.267
133	A	1	1	1.	17	0.059
134	A	2	2	1.	11	0.182
135	A	9	6	1.	13	0.462
136	A	6	6	1.	9	0.667
137	A	6	6	1.	11	0.546
138	A	6	6	1.	13	0.462
139	A	6	6	1.	13	0.462
140	A	1	1	1.	13	0.077
141	A	4	4	1.	13	0.308
142	A	7	7	1.	13	0.538
143	A	7	7	1.	13	0.538
144	A	7	7	1.	13	0.538
145	A	1	1	1.	15	0.067
146	A	22	8	1.	13	0.615
147	A	4	3	1.	10	0.3
148	A	4	4	1.	16	0.25
149	A	3	3	1.	19	0.158
150	A	5	5	1.	26	0.192
151	A	7	7	1.	7	1.
152	A	4	4	1.	18	0.222
153	A	7	7	1.	9	0.778
154	A	5	4	1.54	18	0.222
155	A	5	5	1.	18	0.278
156	A	5	4	1.	16	0.25
157	A	4	3	1.	16	0.188
158	A	2	1	1.	16	0.062
159	A	2	1	1.	9	0.111
160	A	3	2	1.	15	0.133
161	A	3	2	1.	11	0.182
162	A	2	1	1.	11	0.091
163	A	5	3	1.	10	0.3
164	A	4	3	1.	10	0.3
165	A	2	1	1.	11	0.091
166	A	4	3	1.17	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
167	A	5	3	1.	11	0.273
168	A	3	3	1.	18	0.167
169	A	3	3	1.	18	0.167
170	A	4	4	1.	11	0.364
171	A	3	3	1.22	21	0.143
172	A	1	1	1.	13	0.077
173	A	3	2	1.	13	0.154
174	A	12	8	1.	13	0.615
175	A	5	4	1.	13	0.308
176	A	3	2	1.	13	0.154
177	A	8	8	1.	13	0.615
178	A	5	4	1.	16	0.25
179	A	4	4	1.	18	0.222
180	A	2	1	1.	44	0.023
181	A	7	6	1.	16	0.375
182	A	7	6	1.	22	0.273
183	A	2	1	1.	15	0.067
184	A	3	2	1.	13	0.154
185	A	3	2	1.	13	0.154
186	A	2	1	1.14	11	0.091
187	A	2	1	1.	11	0.091
188	A	2	1	1.	9	0.111
189	A	2	1	1.	17	0.059
190	A	2	1	1.	19	0.053
191	A	2	1	1.	19	0.053
192	A	2	1	1.	19	0.053
193	A	2	2	1.	19	0.105
194	A	4	4	1.	19	0.21
195	A	3	3	1.	19	0.158
196	A	4	4	1.	19	0.21
197	A	5	4	1.	19	0.21
198	A	2	2	1.	21	0.095
199	A	3	2	1.	14	0.143
200	A	4	4	1.	18	0.222
201	A	4	3	1.	14	0.214
202	A	7	3	1.	10	0.3
203	A	7	6	1.	16	0.375
204	A	8	5	1.	14	0.357
205	A	7	5	1.	20	0.25
206	A	3	2	1.	14	0.143
207	A	6	4	1.	13	0.308
208	A	9	4	1.	24	0.167
209	A	5	3	1.	33	0.091
210	A	4	3	1.	11	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
211	A	2	1	1.	13	0.077
212	A	4	3	1.	21	0.143
213	A	5	4	1.	19	0.21
214	A	3	2	1.	17	0.118
215	A	5	3	1.02	11	0.273
216	A	9	3	1.08	13	0.231
217	A	8	5	1.	11	0.454
218	A	3	3	1.	15	0.2
219	A	4	4	1.03	19	0.21
220	A	6	6	1.	27	0.222
221	A	33	16	1.05	52	0.308
222	A	46	21	1.79	56	0.375
223	A	2	2	1.16	15	0.133
224	A	2	2	1.2	15	0.133
225	A	3	3	1.19	15	0.2
226	B	3	3	2.81	13	0.231
227	A	9	9	1.09	19	0.474
228	B	6	6	2.69	17	0.353
229	A	2	2	1.	12	0.167
230	A	4	4	1.	17	0.235
231	A	7	5	1.	17	0.294
232	B	3	3	2.51	17	0.176
233	A	3	3	1.	17	0.176
234	A	5	4	1.	17	0.235
235	A	2	2	1.	14	0.143
236	A	2	2	1.	14	0.143
237	A	2	2	1.	14	0.143
238	A	2	2	1.	19	0.105
239	A	2	2	1.	19	0.105
240	A	3	3	1.	22	0.136
241	A	3	3	1.	22	0.136
242	A	5	4	1.	17	0.235
243	A	5	5	1.	21	0.238
244	A	9	7	1.	20	0.35
245	A	5	5	1.	24	0.208
246	A	5	4	1.	21	0.19
247	A	5	4	1.	30	0.133
248	A	5	4	1.11	32	0.125
249	A	2	2	1.	30	0.067
250	A	4	3	1.	15	0.2
251	A	3	2	1.	13	0.154
252	A	3	2	1.	11	0.182
253	A	3	2	1.	20	0.1
254	A	2	2	1.	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
255	A	4	4	1.	17	0.235
256	A	3	3	1.	17	0.176
257	A	5	5	1.	20	0.25
258	A	4	4	1.	24	0.167
259	A	15	9	1.	33	0.273
260	A	32	14	1.	44	0.318
261	A	4	4	1.	16	0.25
262	A	5	5	1.	18	0.278
263	A	6	5	1.	18	0.278
264	A	5	5	1.	19	0.263
265	A	4	4	1.	24	0.167
266	A	2	2	1.	10	0.2
267	A	4	4	1.	14	0.286
268	A	1	1	1.	10	0.1
269	A	1	1	1.	12	0.083
270	A	4	4	1.	14	0.286
271	A	5	5	1.	14	0.357
272	A	4	3	1.	10	0.3
273	A	5	3	1.	10	0.3
274	A	3	3	1.	14	0.214
275	A	4	4	1.	14	0.286
276	A	4	4	1.	14	0.286
277	A	5	5	1.	14	0.357
278	A	2	2	1.	16	0.125
279	A	10	7	1.	22	0.318
280	A	6	6	1.	18	0.333
281	A	6	6	1.	28	0.214
282	A	4	4	1.	34	0.118
283	A	14	10	1.	24	0.417
284	A	3	2	1.	12	0.167
285	A	2	2	1.	14	0.143
286	A	2	2	1.	14	0.143
287	A	5	4	1.	16	0.25
288	A	3	2	1.31	14	0.143
289	A	7	6	1.	23	0.261
290	A	26	8	1.	29	0.276
291	A	36	8	1.	31	0.258
292	A	4	3	1.	11	0.273
293	A	1	1	1.	15	0.067
294	A	6	6	1.	13	0.462
295	A	2	1	1.	13	0.077
296	A	6	6	1.	17	0.353
297	A	3	2	1.	15	0.133
298	A	13	9	1.	17	0.529

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
299	A	1	1	1.	13	0.077
300	A	1	1	1.	13	0.077
301	A	5	5	1.	15	0.333
302	A	6	6	1.	13	0.462
303	A	5	5	1.	15	0.333
304	A	6	5	1.	15	0.333
305	A	6	6	1.	15	0.4
306	B	12	12	2.15	13	0.923
307	A	8	6	0.9	13	0.462
308	A	3	3	1.	22	0.136
309	A	5	5	1.	16	0.312
310	A	5	5	1.	18	0.278
311	A	3	3	1.	23	0.13
312	A	1	1	1.	23	0.043
313	A	9	4	1.	39	0.103
314	A	7	7	1.37	17	0.412
315	A	10	7	1.	17	0.412
316	A	2	2	1.	15	0.133
317	A	5	5	1.	17	0.294
318	A	3	2	1.23	17	0.118
319	A	9	9	1.37	32	0.281
320	A	2	2	1.	24	0.083
321	A	2	2	1.	24	0.083
322	A	6	6	1.44	18	0.333
323	A	6	6	1.44	18	0.333
324	A	2	2	1.	27	0.074
325	A	2	2	1.	27	0.074
326	A	1	1	1.	21	0.048
327	A	1	1	1.	44	0.023
328	A	2	2	1.	27	0.074
329	A	2	2	1.	31	0.065
330	A	2	2	1.	4	0.5
331	A	2	1	1.	4	0.25
332	A	3	2	1.	4	0.5
333	A	4	2	1.	4	0.5
334	A	5	2	1.	4	0.5
335	B	3	2	3.2	14	0.143
336	A	2	1	1.	14	0.071
337	A	2	1	1.	4	0.25
338	A	4	2	1.	4	0.5
339	A	2	1	1.	4	0.25
340	A	2	2	1.	12	0.167
341	A	4	2	1.	4	0.5
342	A	3	2	1.	4	0.5

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
343	A	3	2	1.	14	0.143
344	A	6	3	1.	9	0.333
345	A	3	2	1.	9	0.222
346	A	4	3	1.	7	0.429
347	A	3	2	1.	9	0.222
348	A	3	3	1.	9	0.333
349	A	5	3	1.	9	0.333
350	A	7	3	1.	9	0.333
351	A	9	3	1.	9	0.333
352	A	1	1	1.	13	0.077
353	A	3	2	1.	23	0.087
354	A	2	2	1.	9	0.222
355	A	2	1	1.	7	0.143
356	A	2	2	1.	7	0.286
357	A	3	3	1.	9	0.333
358	A	3	2	1.	9	0.222
359	A	3	2	1.	11	0.182
360	A	3	2	1.	11	0.182
361	A	4	3	1.	9	0.333
362	A	3	3	1.	29	0.103
363	A	8	5	0.95	22	0.227
364	A	15	7	1.	17	0.412
365	A	4	3	1.	17	0.176
366	A	4	2	1.	9	0.222
367	A	4	3	1.	7	0.429
368	A	1	1	1.	7	0.143
369	A	4	4	1.	9	0.444
370	A	6	2	1.	9	0.222
371	A	4	3	1.	9	0.333
372	A	3	1	1.	9	0.111
373	A	7	6	1.02	11	0.546
374	A	5	5	1.55	9	0.556
375	A	5	4	1.	16	0.25
376	A	3	3	1.57	14	0.214
377	B	3	3	2.85	12	0.25
378	A	2	1	1.	16	0.062
379	A	6	4	1.	10	0.4
380	A	4	3	1.	9	0.333
381	A	3	2	1.	9	0.222
382	A	4	4	1.	21	0.19
383	A	2	1	1.	9	0.111
384	A	2	2	1.	7	0.286
385	A	4	3	1.	9	0.333
386	A	4	3	1.	9	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
387	A	5	5	1.	7	0.714
388	A	4	2	1.	7	0.286
389	A	4	2	1.	9	0.222
390	A	1	1	1.	10	0.1
391	A	1	1	1.	12	0.083
392	A	2	2	1.	10	0.2
393	A	2	2	1.	12	0.167
394	A	3	3	1.	12	0.25
395	A	3	2	1.	14	0.143
396	A	4	2	1.	32	0.062
397	A	11	8	1.	6	1.333
398	A	9	9	1.21	8	1.125
399	A	6	5	1.	12	0.417
400	A	4	2	1.	32	0.062
401	A	19	13	1.58	13	1.
402	A	1	1	1.	11	0.091
403	A	1	1	1.	11	0.091
404	A	2	2	1.	11	0.182
405	A	6	5	1.	16	0.312
406	A	4	3	1.	13	0.231
407	A	4	3	1.	13	0.231
408	A	1	1	1.	13	0.077
409	A	2	2	1.	13	0.154
410	A	3	3	1.	11	0.273
411	A	6	4	1.4	35	0.114
412	A	5	4	1.53	11	0.364
413	B	13	9	2.22	11	0.818
414	A	4	2	1.	13	0.154
415	A	4	2	1.	13	0.154
416	B	27	11	2.17	27	0.407
417	A	66	21	1.83	41	0.512
418	A	13	4	1.13	28	0.143
419	A	5	3	0.92	15	0.2
420	A	5	3	1.	18	0.167
421	A	5	4	1.	20	0.2
422	A	4	3	1.	20	0.15
423	A	3	3	1.	19	0.158
424	A	4	3	1.	22	0.136
425	A	4	3	1.	23	0.13
426	A	18	13	1.07	33	0.394
427	A	27	6	1.	39	0.154
428	A	5	3	1.	17	0.176
429	A	3	3	1.	11	0.273
430	A	5	4	1.	11	0.364

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
431	A	1	1	1.	11	0.091
432	A	6	6	1.	13	0.462
433	A	11	9	1.05	28	0.321
434	A	7	6	1.	12	0.5
435	A	4	4	1.	12	0.333
436	A	10	10	1.	22	0.454
437	A	5	4	1.	23	0.174
438	A	16	12	1.	31	0.387
439	A	10	7	1.	48	0.146
440	A	7	5	1.	15	0.333
441	A	6	5	1.	15	0.333
442	A	6	6	1.	19	0.316
443	A	7	7	1.	15	0.467
444	A	6	6	1.	19	0.316
445	A	6	6	1.	20	0.3
446	A	29	16	1.31	61	0.262
447	B	21	10	2.08	29	0.345
448	A	7	7	1.	20	0.35
449	A	14	10	1.71	15	0.667
450	A	4	3	1.	19	0.158
451	A	2	2	1.	33	0.061
452	A	15	9	1.3	31	0.29
453	A	14	10	1.25	52	0.192
454	A	4	3	1.	15	0.2
455	A	14	10	1.51	15	0.667
456	A	3	3	1.06	11	0.273
457	A	6	5	1.	11	0.454
458	A	3	2	1.	11	0.182
459	A	4	2	1.	11	0.182
460	A	3	2	1.	11	0.182
461	A	5	4	1.	13	0.308
462	A	6	4	1.	13	0.308
463	A	1	1	1.	7	0.143
464	A	3	2	1.	11	0.182
465	A	4	3	1.	19	0.158
466	A	3	2	1.	13	0.154
467	A	3	2	1.	13	0.154
468	A	2	2	1.	11	0.182
469	A	2	2	1.	12	0.167
470	A	3	2	1.	13	0.154
471	A	2	1	1.	13	0.077
472	A	3	2	1.	11	0.182
473	A	3	3	1.	23	0.13
474	A	2	2	1.	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
475	A	2	2	1.	15	0.133
476	A	3	2	1.	15	0.133
477	A	3	2	1.	11	0.182
478	A	2	2	1.	14	0.143
479	A	2	1	1.	12	0.083
480	A	2	1	1.	16	0.062
481	A	2	2	1.	20	0.1
482	A	2	2	1.	25	0.08
483	A	9	4	1.	8	0.5
484	A	8	4	1.	8	0.5
485	A	13	4	1.	8	0.5
486	A	4	4	1.	10	0.4
487	A	6	5	1.	10	0.5
488	A	5	4	1.	8	0.5
489	A	3	3	1.	8	0.375
490	A	8	8	1.	8	1.
491	A	7	7	1.	6	1.167
492	A	2	2	1.	18	0.111
493	A	3	3	1.	15	0.2
494	A	2	2	1.	11	0.182
495	A	9	4	1.21	22	0.182
496	A	3	1	1.	11	0.091
497	A	4	3	1.	13	0.231
498	A	3	2	1.	13	0.154
499	A	4	3	1.	13	0.231
500	A	4	4	1.08	13	0.308
501	A	4	3	1.	15	0.2
502	A	3	1	1.	11	0.091
503	A	6	2	1.	13	0.154
504	A	7	2	1.	13	0.154
505	A	8	2	1.	13	0.154
506	A	2	2	1.11	13	0.154
507	A	3	1	1.	13	0.077
508	A	6	2	1.	15	0.133
509	A	7	2	1.	15	0.133
510	A	8	2	1.	15	0.133
511	A	2	2	1.11	15	0.133
512	A	2	1	1.	7	0.143
513	A	3	2	1.	9	0.222
514	A	3	2	1.	9	0.222
515	A	3	2	1.	9	0.222
516	A	2	2	1.	9	0.222
517	A	2	1	1.	9	0.111
518	A	3	2	1.	11	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
519	A	3	2	1.	11	0.182
520	A	3	2	1.	11	0.182
521	A	2	2	1.	11	0.182
522	A	4	4	1.	11	0.364
523	A	7	7	1.23	15	0.467
524	A	3	2	1.	13	0.154
525	A	5	5	1.	24	0.208
526	A	6	5	1.	29	0.172
527	A	2	2	1.	21	0.095
528	A	6	6	1.	15	0.4
529	A	2	2	1.	15	0.133
530	A	3	3	1.	17	0.176
531	A	3	3	1.	19	0.158
532	A	3	3	1.	39	0.077
533	A	4	4	1.54	17	0.235
534	A	3	2	1.	21	0.095
535	A	4	2	1.	11	0.182
536	A	3	2	1.	11	0.182
537	A	3	2	1.	9	0.222
538	A	5	3	1.	12	0.25
539	A	6	6	1.	13	0.462
540	A	1	1	1.	25	0.04
541	A	1	1	1.	10	0.1
542	A	6	4	1.	21	0.19
543	A	2	2	1.	16	0.125
544	A	2	2	1.	10	0.2
545	A	2	2	1.	10	0.2
546	A	3	3	1.	16	0.188
547	A	4	3	1.	14	0.214
548	A	4	3	1.	22	0.136
549	A	5	3	1.47	10	0.3
550	A	1	1	1.	10	0.1
551	A	2	2	1.51	10	0.2
552	A	2	2	1.	10	0.2
553	A	2	2	1.	12	0.167
554	A	2	2	1.	10	0.2
555	A	2	2	1.	12	0.167
556	A	1	1	1.	18	0.056
557	A	7	6	1.1	16	0.375
558	A	1	1	1.	14	0.071
559	A	7	6	1.05	16	0.375
560	A	7	6	1.04	18	0.333
561	A	1	1	1.	16	0.062
562	A	7	6	1.09	14	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
563	A	1	1	1.	16	0.062
564	A	4	3	1.	7	0.429
565	A	11	5	1.	9	0.556
566	A	11	5	1.	11	0.454
567	A	31	8	1.35	15	0.533
568	A	11	5	1.	13	0.385
569	A	24	6	1.	17	0.353
570	A	1	1	1.	2	0.5
571	A	1	1	1.	2	0.5
572	A	1	1	1.	2	0.5
573	A	1	1	1.	2	0.5
574	A	1	1	1.	2	0.5
575	A	1	1	1.	2	0.5
576	A	2	2	1.	4	0.5
577	A	2	1	1.	4	0.25
578	A	3	2	1.	4	0.5
579	A	2	2	1.	4	0.5
580	A	3	2	1.	4	0.5
581	A	4	3	1.	7	0.429
582	A	3	2	1.	11	0.182
583	A	2	2	1.02	8	0.25
584	A	2	2	1.	6	0.333
585	A	2	2	1.	8	0.25
586	A	2	2	1.	14	0.143
587	A	2	2	1.	15	0.133
588	A	3	3	1.	10	0.3
589	A	5	3	1.	23	0.13
590	A	5	2	1.	11	0.182
591	A	5	2	1.	15	0.133
592	A	8	4	1.48	31	0.129
593	A	3	3	1.	15	0.2
594	A	5	3	1.	21	0.143
595	A	2	2	1.	11	0.182
596	A	3	3	1.	6	0.5
597	A	3	3	1.	6	0.5
598	A	13	7	1.	16	0.438
599	A	8	4	1.	13	0.308
600	A	3	3	1.	10	0.3
601	A	3	3	1.	10	0.3
602	A	2	2	1.	13	0.154
603	A	3	3	1.46	13	0.231
604	A	2	2	1.	11	0.182
605	A	4	3	1.	12	0.25
606	A	3	2	1.	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
607	A	3	2	1.	18	0.111
608	A	1	1	1.	6	0.167
609	A	2	2	1.	8	0.25
610	A	2	2	1.	10	0.2
611	A	2	1	1.04	8	0.125
612	A	4	4	1.	10	0.4
613	A	6	2	1.	14	0.143
614	A	13	8	1.22	16	0.5
615	A	5	3	1.	8	0.375
616	A	2	2	1.	10	0.2
617	A	2	2	1.	10	0.2
618	A	2	2	1.	8	0.25
619	A	2	2	1.	12	0.167
620	A	2	2	1.	12	0.167
621	A	2	2	1.	14	0.143
622	A	3	2	1.	16	0.125
623	A	3	2	1.	18	0.111
624	A	3	2	1.	18	0.111
625	A	4	3	1.	20	0.15
626	A	4	3	1.	22	0.136
627	A	4	3	1.	22	0.136
628	A	1	1	1.	7	0.143
629	A	3	2	1.	9	0.222
630	A	4	2	1.	9	0.222
631	A	5	2	1.	9	0.222
632	A	3	2	1.	9	0.222
633	A	3	3	1.	8	0.375
634	A	3	1	1.	8	0.125
635	A	2	2	1.	6	0.333
636	A	2	3	1.	6	0.5
637	A	2	2	1.	14	0.143
638	A	3	2	1.	8	0.25
639	A	8	6	1.	20	0.3
640	A	6	6	1.	14	0.429
641	A	4	4	1.	8	0.5
642	A	4	3	1.	8	0.375
643	A	4	5	1.	14	0.357
644	A	6	7	1.	12	0.583
645	A	5	5	1.	8	0.625
646	A	5	5	1.	8	0.625
647	A	10	7	1.	8	0.875
648	A	13	8	1.	8	1.
649	A	5	5	1.	8	0.625
650	A	10	5	1.	8	0.625

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
651	A	3	3	1.	14	0.214
652	A	3	3	1.	14	0.214
653	A	2	1	1.	15	0.067
654	A	6	5	1.	14	0.357
655	A	3	2	1.	15	0.133
656	A	4	5	1.	17	0.294
657	A	10	7	1.	17	0.412
658	A	4	3	1.	17	0.176
659	A	3	3	1.	17	0.176
660	A	5	3	1.	17	0.176
661	A	2	2	1.	15	0.133
662	A	2	2	1.	15	0.133
663	A	4	4	1.	14	0.286
664	A	3	5	1.	17	0.294
665	A	8	6	1.	17	0.353
666	A	4	4	1.	17	0.235
667	A	5	4	1.	17	0.235
668	A	6	4	1.	19	0.21
669	A	3	3	1.	11	0.273
670	A	4	3	1.	11	0.273
671	A	4	4	1.	13	0.308
672	A	8	8	1.	13	0.615
673	A	2	2	1.	13	0.154
674	A	8	8	1.	13	0.615
675	A	17	11	1.	13	0.846
676	A	8	8	1.	11	0.727
677	A	3	2	1.	11	0.182
678	A	12	6	1.	13	0.462
679	A	7	7	1.	13	0.538
680	A	8	7	1.	8	0.875
681	A	11	8	1.	13	0.615
682	A	4	4	1.04	15	0.267
683	A	9	7	1.08	15	0.467
684	A	11	8	1.25	15	0.533
685	A	4	4	1.	15	0.267
686	A	4	6	1.03	12	0.5
687	A	4	5	1.04	15	0.333
688	A	5	6	1.02	15	0.4
689	A	16	11	1.33	15	0.733
690	A	2	3	1.	15	0.2
691	A	5	8	1.3	15	0.533
692	A	6	4	1.36	17	0.235
693	A	11	9	1.29	17	0.529
694	A	8	5	1.33	17	0.294

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
695	B	8	7	2.15	16	0.438
696	A	4	4	1.	16	0.25
697	A	5	4	1.	8	0.5
698	A	5	5	1.	13	0.385
699	A	2	2	1.	24	0.083
700	A	2	3	1.	21	0.143
701	A	5	5	1.	12	0.417
702	A	7	6	1.71	10	0.6
703	A	5	6	1.	8	0.75
704	A	6	6	1.	10	0.6
705	A	5	6	1.	7	0.857

Chapter 3

Listing of integrals

3.1 $\int \frac{1}{a^2-b^2x^2}\,dx$

Optimal. Leaf size=14

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{ab}$$

```
[Out] ArcTanh[(b*x)/a]/(a*b)
```

Rubi [A] time = 0.0076569, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {208}

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*x^2)^(-1),x]

[Out] ArcTanh[(b*x)/a]/(a*b)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{a^2-b^2x^2}\,dx=\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Mathematica [A] time = 0.0032388, size = 14, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*x^2)^(-1),x]
```

```
[Out] ArcTanh[(b*x)/a]/(a*b)
```

Maple [B] time = 0.007, size = 32, normalized size = 2.3

$$-\frac{\ln(bx-a)}{2ab}+\frac{\ln(bx+a)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b^2*x^2+a^2),x)
```

```
[Out] -1/2/a/b*ln(b*x-a)+1/2*ln(b*x+a)/a/b
```

Maxima [B] time = 0.931373, size = 42, normalized size = 3.

$$\frac{\log(bx+a)}{2ab}-\frac{\log(bx-a)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b^2*x^2+a^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*x + a)/(a*b) - 1/2*log(b*x - a)/(a*b)
```

Fricas [A] time = 1.99127, size = 55, normalized size = 3.93

$$\frac{\log(bx+a)-\log(bx-a)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b^2*x^2+a^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(b*x + a) - log(b*x - a))/(a*b)
```

Sympy [B] time = 0.130621, size = 20, normalized size = 1.43

$$-\frac{\frac{\log\left(-\frac{a}{b}+x\right)}{2}-\frac{\log\left(\frac{a}{b}+x\right)}{2}}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b**2*x**2+a**2),x)
```

```
[Out] -(log(-a/b + x)/2 - log(a/b + x)/2)/(a*b)
```

Giac [B] time = 1.05949, size = 45, normalized size = 3.21

$$\frac{\log(|bx + a|)}{2\,ab} - \frac{\log(|bx - a|)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b^2*x^2+a^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*x + a))/(a*b) - 1/2*log(abs(b*x - a))/(a*b)
```

3.2 $\int \frac{1}{a^2+b^2x^2} \, dx$

Optimal. Leaf size=14

$$\frac{\tan^{-1}\left(\frac{bx}{a}\right)}{ab}$$

```
[Out] ArcTan[(b*x)/a]/(a*b)
```

Rubi [A] time = 0.0048183, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2*x^2)^(-1),x]
```

```
[Out] ArcTan[(b*x)/a]/(a*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a^2+b^2x^2} \, dx = \frac{\tan^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Mathematica [A] time = 0.0030547, size = 14, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{bx}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2*x^2)^(-1),x]
```

```
[Out] ArcTan[(b*x)/a]/(a*b)
```

Maple [A] time = 0.003, size = 15, normalized size = 1.1

$$\frac{1}{ab}\arctan\left(\frac{bx}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*x^2+a^2),x)
```

```
[Out] arctan(b*x/a)/a/b
```

Maxima [A] time = 1.40985, size = 19, normalized size = 1.36

$$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+a^2),x, algorithm="maxima")
```

```
[Out] arctan(b*x/a)/(a*b)
```

Fricas [A] time = 1.95438, size = 28, normalized size = 2.

$$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+a^2),x, algorithm="fricas")
```

```
[Out] arctan(b*x/a)/(a*b)
```

Sympy [C] time = 0.125087, size = 26, normalized size = 1.86

$$\frac{-\frac{i\log\left(-\frac{ia}{b}+x\right)}{2}+\frac{i\log\left(\frac{ia}{b}+x\right)}{2}}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*x**2+a**2),x)
```

```
[Out] (-I*log(-I*a/b + x)/2 + I*log(I*a/b + x)/2)/(a*b)
```

Giac [A] time = 1.0527, size = 19, normalized size = 1.36

$$\frac{\arctan\left(\frac{bx}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x^2+a^2),x, algorithm="giac")
```

```
[Out] arctan(b*x/a)/(a*b)
```

3.3 $\int \sec(2ax)\,dx$

Optimal. Leaf size=13

$$\frac{\tanh^{-1}(\sin(2ax))}{2a}$$

```
[Out] ArcTanh[Sin[2*a*x]]/(2*a)
```

Rubi [A] time = 0.0037081, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3770}

$$\frac{\tanh^{-1}(\sin(2ax))}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*a*x],x]
```

```
[Out] ArcTanh[Sin[2*a*x]]/(2*a)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(2ax)\,dx = \frac{\tanh^{-1}(\sin(2ax))}{2a}$$

Mathematica [B] time = 0.0082175, size = 37, normalized size = 2.85

$$\frac{\log(\sin(ax)+\cos(ax))}{2a} - \frac{\log(\cos(ax)-\sin(ax))}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*a*x],x]
```

```
[Out] -Log[Cos[a*x] - Sin[a*x]]/(2*a) + Log[Cos[a*x] + Sin[a*x]]/(2*a)
```

Maple [A] time = 0.003, size = 18, normalized size = 1.4

$$\frac{\ln(\sec(2\,ax)+\tan(2\,ax))}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*a*x),x)
```

```
[Out] 1/2/a*ln(sec(2*a*x)+tan(2*a*x))
```

Maxima [A] time = 0.922288, size = 23, normalized size = 1.77

$$\frac{\log\left(\sec\left(2\,ax\right)+\tan\left(2\,ax\right)\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*a*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(sec(2*a*x) + tan(2*a*x))/a
```

Fricas [B] time = 2.03189, size = 70, normalized size = 5.38

$$\frac{\log\left(\sin\left(2\,ax\right)+1\right)-\log\left(-\sin\left(2\,ax\right)+1\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*a*x),x, algorithm="fricas")
```

```
[Out] 1/4*(log(sin(2*a*x) + 1) - log(-sin(2*a*x) + 1))/a
```

Sympy [A] time = 0.099176, size = 29, normalized size = 2.23

$$\begin{cases} \frac{-\frac{\log\left(\sin\left(2ax\right)-1\right)}{2}+\frac{\log\left(\sin\left(2ax\right)+1\right)}{2}}{2a} & \text{for}\ 2a\neq 0\\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*a*x),x)
```

```
[Out] Piecewise(((-log(sin(2*a*x) - 1)/2 + log(sin(2*a*x) + 1)/2)/(2*a), Ne(2*a,
0)), (x, True))
```

Giac [B] time = 1.04893, size = 54, normalized size = 4.15

$$\frac{\log\left(\left|\frac{1}{\sin(2\,ax)}+\sin\left(2\,ax\right)+2\right|\right)-\log\left(\left|\frac{1}{\sin(2\,ax)}+\sin\left(2\,ax\right)-2\right|\right)}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*a*x),x, algorithm="giac")
```

```
[Out] 1/8*(log(abs(1/sin(2*a*x) + sin(2*a*x) + 2)) - log(abs(1/sin(2*a*x) + sin(2
*a*x) - 2)))/a
```

3.4 $\int \frac{1}{4} \csc\left(\frac{x}{3}\right) dx$

Optimal. Leaf size=11

$$-\frac{3}{4} \tanh^{-1}\left(\cos\left(\frac{x}{3}\right)\right)$$

```
[Out] (-3*ArcTanh[Cos[x/3]])/4
```

Rubi [A] time = 0.0032333, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {12, 3770}

$$-\frac{3}{4} \tanh^{-1}\left(\cos\left(\frac{x}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x/3]/4,x]
```

```
[Out] (-3*ArcTanh[Cos[x/3]])/4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4} \csc\left(\frac{x}{3}\right) dx &= \frac{1}{4} \int \csc\left(\frac{x}{3}\right) dx \\ &= -\frac{3}{4} \tanh^{-1}\left(\cos\left(\frac{x}{3}\right)\right)\end{aligned}$$

Mathematica [B] time = 0.0062143, size = 23, normalized size = 2.09

$$\frac{1}{4} \left(3 \log\left(\sin\left(\frac{x}{6}\right)\right) - 3 \log\left(\cos\left(\frac{x}{6}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x/3]/4,x]
```

```
[Out] (-3*Log[Cos[x/6]] + 3*Log[Sin[x/6]])/4
```

Maple [A] time = 0.006, size = 15, normalized size = 1.4

$$\frac{3}{4}\ln\left(\csc\left(\frac{x}{3}\right)-\cot\left(\frac{x}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/4/sin(1/3*x),x)
```

```
[Out] 3/4*ln(csc(1/3*x)-cot(1/3*x))
```

Maxima [B] time = 0.918564, size = 26, normalized size = 2.36

$$-\frac{3}{8}\log\left(\cos\left(\frac{1}{3}x\right)+1\right)+\frac{3}{8}\log\left(\cos\left(\frac{1}{3}x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/4/sin(1/3*x),x, algorithm="maxima")
```

```
[Out] -3/8*log(cos(1/3*x) + 1) + 3/8*log(cos(1/3*x) - 1)
```

Fricas [B] time = 2.08161, size = 88, normalized size = 8.

$$-\frac{3}{8}\log\left(\frac{1}{2}\cos\left(\frac{1}{3}x\right)+\frac{1}{2}\right)+\frac{3}{8}\log\left(-\frac{1}{2}\cos\left(\frac{1}{3}x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/4/sin(1/3*x),x, algorithm="fricas")
```

```
[Out] -3/8*log(1/2*cos(1/3*x) + 1/2) + 3/8*log(-1/2*cos(1/3*x) + 1/2)
```

Sympy [B] time = 0.097576, size = 22, normalized size = 2.

$$\frac{3\log\left(\cos\left(\frac{x}{3}\right)-1\right)}{8}-\frac{3\log\left(\cos\left(\frac{x}{3}\right)+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/4/sin(1/3*x),x)
```

```
[Out] 3*log(cos(x/3) - 1)/8 - 3*log(cos(x/3) + 1)/8
```

Giac [B] time = 1.06149, size = 31, normalized size = 2.82

$$-\frac{3}{8}\log\left(3\cos\left(\frac{1}{3}x\right)+3\right)+\frac{3}{8}\log\left(-3\cos\left(\frac{1}{3}x\right)+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/4/sin(1/3*x),x, algorithm="giac")
```

```
[Out] -3/8*log(3*cos(1/3*x) + 3) + 3/8*log(-3*cos(1/3*x) + 3)
```

3.5 $\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx$

Optimal. Leaf size=15

$$-\frac{1}{2}\tanh^{-1}\left(\sin\left(2x+\frac{\pi}{4}\right)\right)$$

```
[Out] -ArcTanh[Sin[Pi/4 + 2*x]]/2
```

Rubi [A] time = 0.0034344, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3770}

$$-\frac{1}{2}\tanh^{-1}\left(\sin\left(2x+\frac{\pi}{4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[-Sec[Pi/4 + 2*x],x]
```

```
[Out] -ArcTanh[Sin[Pi/4 + 2*x]]/2
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int -\sec\left(\frac{\pi}{4}+2x\right)\,dx=-\frac{1}{2}\tanh^{-1}\left(\sin\left(\frac{\pi}{4}+2x\right)\right)$$

Mathematica [A] time = 0.0058907, size = 15, normalized size = 1.

$$-\frac{1}{2}\tanh^{-1}\left(\sin\left(2x+\frac{\pi}{4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-Sec[Pi/4 + 2*x],x]
```

```
[Out] -ArcTanh[Sin[Pi/4 + 2*x]]/2
```

Maple [A] time = 0.006, size = 21, normalized size = 1.4

$$-\frac{1}{2}\ln\left(\sec\left(\frac{\pi}{4}+2\,x\right)+\tan\left(\frac{\pi}{4}+2\,x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/cos(1/4*Pi+2*x),x)
```

```
[Out] -1/2*ln(sec(1/4*Pi+2*x)+tan(1/4*Pi+2*x))
```

Maxima [B] time = 0.929331, size = 36, normalized size = 2.4

$$-\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)+1\right)+\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/cos(1/4*pi+2*x),x, algorithm="maxima")
```

```
[Out] -1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(sin(1/4*pi + 2*x) - 1)
```

Fricas [B] time = 2.07409, size = 90, normalized size = 6.

$$-\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(\frac{1}{4}\pi+2x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/cos(1/4*pi+2*x),x, algorithm="fricas")
```

```
[Out] -1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(-sin(1/4*pi + 2*x) + 1)
```

Sympy [A] time = 0.198141, size = 22, normalized size = 1.47

$$\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)-1\right)}{2}-\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/cos(1/4*pi+2*x),x)
```

```
[Out] log(tan(x + pi/8) - 1)/2 - log(tan(x + pi/8) + 1)/2
```

Giac [B] time = 1.05812, size = 39, normalized size = 2.6

$$-\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(\frac{1}{4}\pi+2x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/cos(1/4*pi+2*x),x, algorithm="giac")
```

```
[Out] -1/4*log(sin(1/4*pi + 2*x) + 1) + 1/4*log(-sin(1/4*pi + 2*x) + 1)
```

3.6 $\int \sec(x)\tan(x)\,dx$

Optimal. Leaf size=2

$$\sec(x)$$

```
[Out] Sec[x]
```

Rubi [A] time = 0.0058834, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2606, 8}

$$\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Tan[x],x]
```

```
[Out] Sec[x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec(x)\tan(x)\,dx &= \text{Subst}\left(\int 1\,dx, x, \sec(x)\right)\\ &= \sec(x)\end{aligned}$$

Mathematica [A] time = 0.0006184, size = 2, normalized size = 1.

$$\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Tan[x],x]
```

```
[Out] Sec[x]
```

Maple [A] time = 0.004, size = 3, normalized size = 1.5

$$\sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x),x)
```

```
[Out] sec(x)
```

Maxima [A] time = 0.925093, size = 5, normalized size = 2.5

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x),x, algorithm="maxima")
```

```
[Out] 1/cos(x)
```

Fricas [A] time = 1.99299, size = 14, normalized size = 7.

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x),x, algorithm="fricas")
```

```
[Out] 1/cos(x)
```

Sympy [A] time = 0.061183, size = 3, normalized size = 1.5

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x),x)
```

```
[Out] 1/cos(x)
```

Giac [A] time = 1.05607, size = 5, normalized size = 2.5

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x),x, algorithm="giac")
```

```
[Out] 1/cos(x)
```

3.7 $\int \cot(x)\csc(x)\,dx$

Optimal. Leaf size=4

$$-\csc(x)$$

```
[Out] -Csc[x]
```

Rubi [A] time = 0.006253, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2606, 8}

$$-\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Csc[x],x]
```

```
[Out] -Csc[x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cot(x)\csc(x)\,dx &= -\text{Subst}\left(\int 1\,dx, x, \csc(x)\right)\\ &= -\csc(x)\end{aligned}$$

Mathematica [A] time = 0.0016246, size = 4, normalized size = 1.

$$-\csc(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Csc[x],x]
```

```
[Out] -Csc[x]
```

Maple [A] time = 0., size = 5, normalized size = 1.3

$$-\csc(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*csc(x),x)
```

```
[Out] -csc(x)
```

Maxima [A] time = 0.923209, size = 8, normalized size = 2.

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x),x, algorithm="maxima")
```

```
[Out] -1/sin(x)
```

Fricas [A] time = 1.93399, size = 15, normalized size = 3.75

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x),x, algorithm="fricas")
```

```
[Out] -1/sin(x)
```

Sympy [A] time = 0.061743, size = 5, normalized size = 1.25

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x),x)
```

```
[Out] -1/sin(x)
```

Giac [A] time = 1.05541, size = 8, normalized size = 2.

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x),x, algorithm="giac")
```

```
[Out] -1/sin(x)
```

3.8 $\int \csc(2x)\tan(x)\,dx$

Optimal. Leaf size=6

$$\frac{\tan(x)}{2}$$

[Out] Tan[x]/2

Rubi [A] time = 0.0235679, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {8}

$$\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Csc[2*x]*Tan[x],x]

[Out] Tan[x]/2

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \csc(2x)\tan(x)\,dx &= \text{Subst}\left(\int \frac{1}{2}\,dx, x, \tan(x)\right)\\ &= \frac{\tan(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0128772, size = 6, normalized size = 1.

$$\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[2*x]*Tan[x],x]

[Out] Tan[x]/2

Maple [A] time = 0.014, size = 5, normalized size = 0.8

$$\frac{\tan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/sin(2*x),x)
```

```
[Out] 1/2*tan(x)
```

Maxima [B] time = 0.932013, size = 36, normalized size = 6.

$$\frac{\sin(2x)}{\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/sin(2*x),x, algorithm="maxima")
```

```
[Out] sin(2*x)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A] time = 1.86448, size = 16, normalized size = 2.67

$$\frac{1}{2}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/sin(2*x),x, algorithm="fricas")
```

```
[Out] 1/2*tan(x)
```

Sympy [B] time = 0.716762, size = 7, normalized size = 1.17

$$\frac{\sin(x)}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/sin(2*x),x)
```

```
[Out] sin(x)/(2*cos(x))
```

Giac [A] time = 1.05491, size = 5, normalized size = 0.83

$$\frac{1}{2}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/sin(2*x),x, algorithm="giac")
```

```
[Out] 1/2*tan(x)
```

3.9 $\int \frac{1}{1+\cos(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\sin(x)}{\cos(x)+1}$$

```
[Out] Sin[x]/(1 + Cos[x])
```

Rubi [A] time = 0.0067796, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2648}

$$\frac{\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x])^(-1),x]
```

```
[Out] Sin[x]/(1 + Cos[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{1+\cos(x)}$$

Mathematica [A] time = 0.0036655, size = 6, normalized size = 0.67

$$\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])^(-1),x]
```

```
[Out] Tan[x/2]
```

Maple [A] time = 0., size = 5, normalized size = 0.6

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)+1),x)
```

```
[Out] tan(1/2*x)
```

Maxima [A] time = 0.928184, size = 12, normalized size = 1.33

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="maxima")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Fricas [A] time = 1.8403, size = 28, normalized size = 3.11

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Sympy [A] time = 0.17233, size = 3, normalized size = 0.33

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x)
```

```
[Out] tan(x/2)
```

Giac [B] time = 1.06038, size = 41, normalized size = 4.56

$$-\frac{2\,\tan\left(\frac{1}{2}\,x\right)}{\left(x^2+1\right)\left(\frac{x^2-1}{x^2+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="giac")
```

```
[Out] -2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

3.10 $\int \frac{1}{1-\cos(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rubi [A] time = 0.0079056, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2648}

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x])^(-1),x]
```

```
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A] time = 0.007922, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x])^(-1),x]
```

```
[Out] -Cot[x/2]
```

Maple [A] time = 0., size = 9, normalized size = 0.8

$$-\left(\tan\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)),x)
```

```
[Out] -1/tan(1/2*x)
```

Maxima [A] time = 0.936023, size = 14, normalized size = 1.17

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="maxima")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Fricas [A] time = 1.83453, size = 30, normalized size = 2.5

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Sympy [A] time = 0.348173, size = 7, normalized size = 0.58

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x)
```

```
[Out] -1/tan(x/2)
```

Giac [A] time = 1.0581, size = 11, normalized size = 0.92

$$-\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="giac")
```

```
[Out] -1/tan(1/2*x)
```

3.11 $\int \frac{\sin(x)}{a-b\cos(x)}\,dx$

Optimal. Leaf size=12

$$\frac{\log(a-b\cos(x))}{b}$$

```
[Out] Log[a - b*Cos[x]]/b
```

Rubi [A] time = 0.0226925, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2668, 31}

$$\frac{\log(a-b\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a - b*Cos[x]),x]
```

```
[Out] Log[a - b*Cos[x]]/b
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{a-b\cos(x)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+x}\,dx, x, -b\cos(x)\right)}{b}\\ &= \frac{\log(a-b\cos(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.0185615, size = 12, normalized size = 1.

$$\frac{\log(a-b\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a - b*Cos[x]),x]
```

```
[Out] Log[a - b*Cos[x]]/b
```

Maple [A] time = 0.007, size = 13, normalized size = 1.1

$$\frac{\ln\left(a-b\cos\left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a-b*cos(x)),x)
```

```
[Out] ln(a-b*cos(x))/b
```

Maxima [A] time = 0.931421, size = 18, normalized size = 1.5

$$\frac{\log\left(b\cos\left(x\right)-a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a-b*cos(x)),x, algorithm="maxima")
```

```
[Out] log(b*cos(x) - a)/b
```

Fricas [A] time = 2.16931, size = 30, normalized size = 2.5

$$\frac{\log\left(-b\cos\left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a-b*cos(x)),x, algorithm="fricas")
```

```
[Out] log(-b*cos(x) + a)/b
```

Sympy [A] time = 0.366212, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\log\left(-\frac{a}{b}+\cos\left(x\right)\right)}{b} & \text{for}\ b\neq 0 \\ -\frac{\cos\left(x\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a-b*cos(x)),x)
```

```
[Out] Piecewise((log(-a/b + cos(x))/b, Ne(b, 0)), (-cos(x)/a, True))
```

Giac [A] time = 1.04691, size = 19, normalized size = 1.58

$$\frac{\log\left(|b\cos\left(x\right)-a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a-b*cos(x)),x, algorithm="giac")
```

```
[Out] log(abs(b*cos(x) - a))/b
```

3.12 $\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

[Out] ArcTan[(b*Sin[x])/a]/(a*b)

Rubi [A] time = 0.0260214, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {3190, 205}

$$\frac{\tan^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/(a^2 + b^2*Sin[x]^2),x]

[Out] ArcTan[(b*Sin[x])/a]/(a*b)

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{a^2+b^2\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a^2+b^2x^2}\,dx, x, \sin(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}\end{aligned}$$

Mathematica [A] time = 0.0096648, size = 15, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]/(a^2 + b^2*Sin[x]^2),x]

```
[Out] ArcTan[(b*Sin[x])/a]/(a*b)
```

Maple [A] time = 0.013, size = 16, normalized size = 1.1

$$\frac{1}{ab}\arctan\left(\frac{b\sin(x)}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a^2+b^2*sin(x)^2),x)
```

```
[Out] arctan(b*sin(x)/a)/a/b
```

Maxima [A] time = 1.4175, size = 20, normalized size = 1.33

$$\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a^2+b^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(b*sin(x)/a)/(a*b)
```

Fricas [A] time = 2.09921, size = 35, normalized size = 2.33

$$\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a^2+b^2*sin(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(b*sin(x)/a)/(a*b)
```

Sympy [A] time = 0.715625, size = 31, normalized size = 2.07

$$\begin{cases} \frac{\tilde{\infty}}{\sin(x)} & \text{for } a = 0 \wedge b = 0 \\ \frac{\sin(x)}{a^2} & \text{for } b = 0 \\ -\frac{1}{b^2\sin(x)} & \text{for } a = 0 \\ \frac{\operatorname{atan}\left(\frac{b\sin(x)}{a}\right)}{ab} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a**2+b**2*sin(x)**2),x)
```

```
[Out] Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (sin(x)/a**2, Eq(b, 0)), (-1/(
b**2*sin(x)), Eq(a, 0)), (atan(b*sin(x)/a)/(a*b), True))
```

Giac [A] time = 1.05471, size = 20, normalized size = 1.33

$$\frac{\arctan\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a^2+b^2*sin(x)^2),x, algorithm="giac")
```

```
[Out] arctan(b*sin(x)/a)/(a*b)
```

3.13 $\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

```
[Out] ArcTanh[(b*Sin[x])/a]/(a*b)
```

Rubi [A] time = 0.027777, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3190, 208}

$$\frac{\tanh^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a^2 - b^2*Sin[x]^2),x]
```

```
[Out] ArcTanh[(b*Sin[x])/a]/(a*b)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{a^2-b^2\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a^2-b^2x^2}\,dx, x, \sin(x)\right)\\ &= \frac{\tanh^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}\end{aligned}$$

Mathematica [A] time = 0.0092904, size = 15, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{b\sin(x)}{a}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a^2 - b^2*Sin[x]^2),x]
```

```
[Out] ArcTanh[(b*Sin[x])/a]/(a*b)
```

Maple [B] time = 0.011, size = 34, normalized size = 2.3

$$-\frac{\ln(b\sin(x)-a)}{2\,ab}+\frac{\ln(b\sin(x)+a)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a^2-b^2*sin(x)^2),x)
```

```
[Out] -1/2/a/b*ln(b*sin(x)-a)+1/2/a/b*ln(b*sin(x)+a)
```

Maxima [B] time = 0.929799, size = 45, normalized size = 3.

$$\frac{\log(b\sin(x)+a)}{2\,ab}-\frac{\log(b\sin(x)-a)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a^2-b^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(b*sin(x) + a)/(a*b) - 1/2*log(b*sin(x) - a)/(a*b)
```

Fricas [A] time = 2.1602, size = 70, normalized size = 4.67

$$\frac{\log(b\sin(x)+a)-\log(-b\sin(x)+a)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a^2-b^2*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(b*sin(x) + a) - log(-b*sin(x) + a))/(a*b)
```

Sympy [A] time = 0.732941, size = 44, normalized size = 2.93

$$\begin{cases} \frac{\tilde{\infty}}{\sin(x)} & \text{for } a=0 \wedge b=0 \\ \frac{\sin(x)}{a^2} & \text{for } b=0 \\ \frac{1}{b^2\sin(x)} & \text{for } a=0 \\ -\frac{\log\left(-\frac{a}{b}+\sin(x)\right)}{2ab}+\frac{\log\left(\frac{a}{b}+\sin(x)\right)}{2ab} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a**2-b**2*sin(x)**2),x)
```

```
[Out] Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (sin(x)/a**2, Eq(b, 0)), (1/(b
**2*sin(x)), Eq(a, 0)), (-log(-a/b + sin(x))/(2*a*b) + log(a/b + sin(x))/(2
```

```
*a*b), True))
```

Giac [B] time = 1.04545, size = 47, normalized size = 3.13

$$\frac{\log\left(|b\sin\left(x\right)+a|\right)}{2\,ab}-\frac{\log\left(|b\sin\left(x\right)-a|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a^2-b^2*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(b*sin(x) + a))/(a*b) - 1/2*log(abs(b*sin(x) - a))/(a*b)
```

3.14 $\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx$

Optimal. Leaf size=17

$$\frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

```
[Out] Log[a^2 + b^2*Sin[x]^2]/b^2
```

Rubi [A] time = 0.0353553, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {12, 260}

$$\frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/(a^2 + b^2*Sin[x]^2),x]
```

```
[Out] Log[a^2 + b^2*Sin[x]^2]/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(2x)}{a^2+b^2\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{2x}{a^2+b^2x^2}\,dx, x, \sin(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{x}{a^2+b^2x^2}\,dx, x, \sin(x)\right)\\ &= \frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.008749, size = 17, normalized size = 1.

$$\frac{\log\left(a^2+b^2\sin^2(x)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/(a^2 + b^2*Sin[x]^2),x]
```

```
[Out] Log[a^2 + b^2*Sin[x]^2]/b^2
```

Maple [A] time = 0.021, size = 18, normalized size = 1.1

$$\frac{\ln\left(a^{2}+b^{2}\left(\sin\left(x\right)\right)^{2}\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(a^2+b^2*sin(x)^2),x)
```

```
[Out] ln(a^2+b^2*sin(x)^2)/b^2
```

Maxima [A] time = 0.955742, size = 23, normalized size = 1.35

$$\frac{\log\left(b^{2}\sin\left(x\right)^{2}+a^{2}\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] log(b^2*sin(x)^2 + a^2)/b^2
```

Fricas [A] time = 2.17097, size = 49, normalized size = 2.88

$$\frac{\log\left(-b^{2}\cos\left(x\right)^{2}+a^{2}+b^{2}\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x, algorithm="fricas")
```

```
[Out] log(-b^2*cos(x)^2 + a^2 + b^2)/b^2
```

Sympy [A] time = 3.15217, size = 32, normalized size = 1.88

$$2\left(\begin{cases}-\frac{\cos^{2}\left(x\right)}{2a^{2}} & \text{for}\; b^{2}=0\\ \frac{\log\left(a^{2}+b^{2}\sin^{2}\left(x\right)\right)}{2b^{2}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a**2+b**2*sin(x)**2),x)
```

```
[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (log(a**2 + b**2*sin(x)**2)
/(2*b**2), True))
```

Giac [B] time = 1.11654, size = 104, normalized size = 6.12

$$-\frac{2\log\left(-\frac{\cos(x)-1}{\cos(x)+1}+1\right)}{b^2}+\frac{\log\left(\left|a^2-\frac{2a^2(\cos(x)-1)}{\cos(x)+1}-\frac{4b^2(\cos(x)-1)}{\cos(x)+1}+\frac{a^2(\cos(x)-1)^2}{(\cos(x)+1)^2}\right|\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2+b^2*sin(x)^2),x, algorithm="giac")
```

```
[Out] -2*log(-(cos(x) - 1)/(cos(x) + 1) + 1)/b^2 + log(abs(a^2 - 2*a^2*(cos(x) -
1)/(cos(x) + 1) - 4*b^2*(cos(x) - 1)/(cos(x) + 1) + a^2*(cos(x) - 1)^2/(cos
(x) + 1)^2))/b^2
```

3.15 $\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}\,dx$

Optimal. Leaf size=19

$$-\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

```
[Out] -(Log[a^2 - b^2*Sin[x]^2]/b^2)
```

Rubi [A] time = 0.0376124, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {12, 260}

$$-\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/(a^2 - b^2*Sin[x]^2),x]
```

```
[Out] -(Log[a^2 - b^2*Sin[x]^2]/b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(2x)}{a^2-b^2\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{2x}{a^2-b^2x^2}\,dx, x, \sin(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{x}{a^2-b^2x^2}\,dx, x, \sin(x)\right)\\ &= -\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0092193, size = 19, normalized size = 1.

$$-\frac{\log\left(a^2-b^2\sin^2(x)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/(a^2 - b^2*Sin[x]^2),x]
```

```
[Out] -(Log[a^2 - b^2*Sin[x]^2]/b^2)
```

Maple [A] time = 0.021, size = 20, normalized size = 1.1

$$-\frac{\ln\left(a^2 - b^2 (\sin(x))^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(a^2-b^2*sin(x)^2),x)
```

```
[Out] -ln(a^2-b^2*sin(x)^2)/b^2
```

Maxima [A] time = 0.927902, size = 27, normalized size = 1.42

$$-\frac{\log\left(b^2 \sin(x)^2 - a^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] -log(b^2*sin(x)^2 - a^2)/b^2
```

Fricas [A] time = 2.23514, size = 49, normalized size = 2.58

$$-\frac{\log\left(b^2 \cos(x)^2 + a^2 - b^2\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -log(b^2*cos(x)^2 + a^2 - b^2)/b^2
```

Sympy [A] time = 2.92803, size = 34, normalized size = 1.79

$$2\left(\begin{cases} -\frac{\cos^2(x)}{2a^2} & \text{for } b^2 = 0 \\ -\frac{\log\left(a^2 - b^2 \sin^2(x)\right)}{2b^2} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a**2-b**2*sin(x)**2),x)
```

```
[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (-log(a**2 - b**2*sin(x)**2
)/(2*b**2), True))
```

Giac [B] time = 1.07609, size = 104, normalized size = 5.47

$$-\frac{\log\left(a^{2} - \frac{2\,a^{2}(\cos(x)-1)}{\cos(x)+1} + \frac{4\,b^{2}(\cos(x)-1)}{\cos(x)+1} + \frac{a^{2}(\cos(x)-1)^{2}}{(\cos(x)+1)^{2}}\right)}{b^{2}} + \frac{2\,\log\left(-\frac{\cos(x)-1}{\cos(x)+1} + 1\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-b^2*sin(x)^2),x, algorithm="giac")
```

```
[Out] -log(a^2 - 2*a^2*(cos(x) - 1)/(cos(x) + 1) + 4*b^2*(cos(x) - 1)/(cos(x) + 1
) + a^2*(cos(x) - 1)^2/(cos(x) + 1)^2)/b^2 + 2*log(-(cos(x) - 1)/(cos(x) +
1) + 1)/b^2
```

3.16 $\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx$

Optimal. Leaf size=18

$$-\frac{\log\left(a^2+b^2\cos^2(x)\right)}{b^2}$$

```
[Out] -(Log[a^2 + b^2*Cos[x]^2]/b^2)
```

Rubi [A] time = 0.0404996, antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {12, 260}

$$-\frac{\log\left(a^2-b^2\sin^2(x)+b^2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/(a^2 + b^2*Cos[x]^2),x]
```

```
[Out] -(Log[a^2 + b^2 - b^2*Sin[x]^2]/b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(2x)}{a^2+b^2\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{2x}{a^2+b^2-b^2x^2}\,dx, x, \sin(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{x}{a^2+b^2-b^2x^2}\,dx, x, \sin(x)\right)\\ &= -\frac{\log\left(a^2+b^2-b^2\sin^2(x)\right)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0150171, size = 22, normalized size = 1.22

$$-\frac{\log\left(a^2-b^2\sin^2(x)+b^2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/(a^2 + b^2*Cos[x]^2),x]
```

```
[Out] -(Log[a^2 + b^2 - b^2*Sin[x]^2]/b^2)
```

Maple [A] time = 0.016, size = 19, normalized size = 1.1

$$-\frac{\ln\left(a^{2}+b^{2}\left(\cos\left(x\right)\right)^{2}\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(a^2+b^2*cos(x)^2),x)
```

```
[Out] -ln(a^2+b^2*cos(x)^2)/b^2
```

Maxima [A] time = 0.928581, size = 24, normalized size = 1.33

$$-\frac{\log\left(b^{2}\cos\left(x\right)^{2}+a^{2}\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x, algorithm="maxima")
```

```
[Out] -log(b^2*cos(x)^2 + a^2)/b^2
```

Fricas [A] time = 2.37374, size = 41, normalized size = 2.28

$$-\frac{\log\left(b^{2}\cos\left(x\right)^{2}+a^{2}\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x, algorithm="fricas")
```

```
[Out] -log(b^2*cos(x)^2 + a^2)/b^2
```

Sympy [A] time = 3.0342, size = 34, normalized size = 1.89

$$2\left(\begin{cases}-\frac{\cos^{2}\left(x\right)}{2a^{2}} & \text{for}\; b^{2}=0\\-\frac{\log\left(a^{2}+b^{2}\cos^{2}\left(x\right)\right)}{2b^{2}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a**2+b**2*cos(x)**2),x)
```

```
[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (-log(a**2 + b**2*cos(x)**2
)/(2*b**2), True))
```

Giac [B] time = 1.07439, size = 130, normalized size = 7.22

$$-\frac{\log\left(a^2+b^2-\frac{2\,a^2(\cos(x)-1)}{\cos(x)+1}+\frac{2\,b^2(\cos(x)-1)}{\cos(x)+1}+\frac{a^2(\cos(x)-1)^2}{(\cos(x)+1)^2}+\frac{b^2(\cos(x)-1)^2}{(\cos(x)+1)^2}\right)}{b^2}+\frac{2\,\log\left(-\frac{\cos(x)-1}{\cos(x)+1}+1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2+b^2*cos(x)^2),x, algorithm="giac")
```

```
[Out] -log(a^2 + b^2 - 2*a^2*(cos(x) - 1)/(cos(x) + 1) + 2*b^2*(cos(x) - 1)/(cos(
x) + 1) + a^2*(cos(x) - 1)^2/(cos(x) + 1)^2 + b^2*(cos(x) - 1)^2/(cos(x) +
1)^2)/b^2 + 2*log(-(cos(x) - 1)/(cos(x) + 1) + 1)/b^2
```

3.17 $\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}\,dx$

Optimal. Leaf size=18

$$\frac{\log\left(a^2-b^2\cos^2(x)\right)}{b^2}$$

```
[Out] Log[a^2 - b^2*Cos[x]^2]/b^2
```

Rubi [A] time = 0.0423405, antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {12, 260}

$$\frac{\log\left(a^2+b^2\sin^2(x)-b^2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/(a^2 - b^2*Cos[x]^2),x]
```

```
[Out] Log[a^2 - b^2 + b^2*Sin[x]^2]/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(2x)}{a^2-b^2\cos^2(x)}\,dx &= \text{Subst}\left(\int \frac{2x}{a^2-b^2+b^2x^2}\,dx,x,\sin(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{x}{a^2-b^2+b^2x^2}\,dx,x,\sin(x)\right)\\ &= \frac{\log\left(a^2-b^2+b^2\sin^2(x)\right)}{b^2}\end{aligned}$$

Mathematica [A] time = 0.0131116, size = 22, normalized size = 1.22

$$\frac{\log\left(a^2+b^2\sin^2(x)-b^2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/(a^2 - b^2*Cos[x]^2),x]
```

```
[Out] Log[a^2 - b^2 + b^2*Sin[x]^2]/b^2
```

Maple [A] time = 0.017, size = 19, normalized size = 1.1

$$\frac{\ln\left(a^{2}-b^{2}\left(\cos\left(x\right)\right)^{2}\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(a^2-b^2*cos(x)^2),x)
```

```
[Out] ln(a^2-b^2*cos(x)^2)/b^2
```

Maxima [A] time = 0.925176, size = 26, normalized size = 1.44

$$\frac{\log\left(b^{2}\cos\left(x\right)^{2}-a^{2}\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x, algorithm="maxima")
```

```
[Out] log(b^2*cos(x)^2 - a^2)/b^2
```

Fricas [A] time = 2.6345, size = 39, normalized size = 2.17

$$\frac{\log\left(b^{2}\cos\left(x\right)^{2}-a^{2}\right)}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x, algorithm="fricas")
```

```
[Out] log(b^2*cos(x)^2 - a^2)/b^2
```

Sympy [A] time = 2.94719, size = 32, normalized size = 1.78

$$2\left(\begin{cases}-\frac{\cos^{2}\left(x\right)}{2a^{2}} & \text{for } b^{2}=0\\ \frac{\log\left(a^{2}-b^{2}\cos^{2}\left(x\right)\right)}{2b^{2}} & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a**2-b**2*cos(x)**2),x)
```

```
[Out] 2*Piecewise((-cos(x)**2/(2*a**2), Eq(b**2, 0)), (log(a**2 - b**2*cos(x)**2)
/(2*b**2), True))
```

Giac [B] time = 1.09442, size = 165, normalized size = 9.17

$$\frac{(a+b)\log\left(\left|a-b-\frac{a(\cos(x)-1)}{\cos(x)+1}-\frac{b(\cos(x)-1)}{\cos(x)+1}\right|\right)}{ab^2+b^3}+\frac{(a-b)\log\left(\left|-a-b+\frac{a(\cos(x)-1)}{\cos(x)+1}-\frac{b(\cos(x)-1)}{\cos(x)+1}\right|\right)}{ab^2-b^3}-\frac{2\log\left(-\frac{\cos(x)-1}{\cos(x)+1}+1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-b^2*cos(x)^2),x, algorithm="giac")
```

```
[Out] (a + b)*log(abs(a - b - a*(cos(x) - 1)/(cos(x) + 1) - b*(cos(x) - 1)/(cos(x
) + 1)))/(a*b^2 + b^3) + (a - b)*log(abs(-a - b + a*(cos(x) - 1)/(cos(x) +
1) - b*(cos(x) - 1)/(cos(x) + 1)))/(a*b^2 - b^3) - 2*log(-(cos(x) - 1)/(cos
(x) + 1) + 1)/b^2
```

3.18 $\int \frac{1}{4-\cos^2(x)} \, dx$

Optimal. Leaf size=41

$$\frac{x}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+2\sqrt{3}+3}\right)}{2\sqrt{3}}$$

```
[Out] x/(2*Sqrt[3]) + ArcTan[(Cos[x]*Sin[x])/(3 + 2*Sqrt[3] + Sin[x]^2)]/(2*Sqrt[
3])
```

Rubi [A] time = 0.0168743, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3181, 203}

$$\frac{x}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+2\sqrt{3}+3}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 - Cos[x]^2)^(-1),x]
```

```
[Out] x/(2*Sqrt[3]) + ArcTan[(Cos[x]*Sin[x])/(3 + 2*Sqrt[3] + Sin[x]^2)]/(2*Sqrt[
3])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4-\cos^2(x)} \, dx &= -\text{Subst}\left(\int \frac{1}{4+3x^2} \, dx, x, \cot(x)\right)\\ &= \frac{x}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{3+2\sqrt{3}+\sin^2(x)}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0251306, size = 19, normalized size = 0.46

$$\frac{\tan^{-1}\left(\frac{2\tan(x)}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - Cos[x]^2)^(-1),x]
```

```
[Out] ArcTan[(2*Tan[x])/Sqrt[3]]/(2*Sqrt[3])
```

Maple [A] time = 0.013, size = 14, normalized size = 0.3

$$\frac{\sqrt{3}}{6}\arctan\left(\frac{2\sqrt{3}\tan(x)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-cos(x)^2),x)
```

```
[Out] 1/6*3^(1/2)*arctan(2/3*3^(1/2)*tan(x))
```

Maxima [A] time = 1.41861, size = 18, normalized size = 0.44

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(2/3*sqrt(3)*tan(x))
```

Fricas [A] time = 2.37184, size = 104, normalized size = 2.54

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{7\sqrt{3}\cos(x)^2 - 4\sqrt{3}}{12\cos(x)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*arctan(1/12*(7*sqrt(3)*cos(x)^2 - 4*sqrt(3))/(cos(x)*sin(x)))
```

Sympy [A] time = 0.757533, size = 61, normalized size = 1.49

$$\frac{\sqrt{3}\left(\operatorname{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right) + \pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6} + \frac{\sqrt{3}\left(\operatorname{atan}\left(\sqrt{3}\tan\left(\frac{x}{2}\right)\right) + \pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-cos(x)**2),x)
```

```
[Out] sqrt(3)*(atan(sqrt(3)*tan(x/2)/3) + pi*floor((x/2 - pi/2)/pi))/6 + sqrt(3)*
(atan(sqrt(3)*tan(x/2)) + pi*floor((x/2 - pi/2)/pi))/6
```

Giac [A] time = 1.04561, size = 62, normalized size = 1.51

$$\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin\left(2x\right)-2\sin\left(2x\right)}{\sqrt{3}\cos\left(2x\right)+\sqrt{3}-2\cos\left(2x\right)+2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-cos(x)^2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) - 2*sin(2*x))/(sqrt(3)*cos(2*x)
+ sqrt(3) - 2*cos(2*x) + 2)))
```

3.19 $\int \frac{e^x}{-1+e^{2x}}\,dx$

Optimal. Leaf size=6

$$-\tanh^{-1}\left(e^x\right)$$

```
[Out] -ArcTanh[E^x]
```

Rubi [A] time = 0.0181226, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2249, 207}

$$-\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(-1 + E^(2*x)),x]
```

```
[Out] -ArcTanh[E^x]
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{-1+e^{2x}}\,dx &= \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,e^x\right)\\ &= -\tanh^{-1}\left(e^x\right)\end{aligned}$$

Mathematica [A] time = 0.0023773, size = 6, normalized size = 1.

$$-\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(-1 + E^(2*x)),x]
```

```
[Out] -ArcTanh[E^x]
```

Maple [A] time = 0., size = 6, normalized size = 1.

$$-\operatorname{Artanh}(\mathrm{e}^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(-1+exp(2*x)),x)
```

```
[Out] -arctanh(exp(x))
```

Maxima [B] time = 0.927296, size = 20, normalized size = 3.33

$$-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="maxima")
```

```
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Fricas [B] time = 2.26539, size = 51, normalized size = 8.5

$$-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(e^{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Sympy [B] time = 0.095121, size = 15, normalized size = 2.5

$$\frac{\log\left(e^{x}-1\right)}{2}-\frac{\log\left(e^{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-1+exp(2*x)),x)
```

```
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

Giac [B] time = 1.06926, size = 22, normalized size = 3.67

$$-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(\left|e^{x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="giac")
```

```
[Out] -1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

3.20 $\int \frac{1}{x \log(x)} dx$

Optimal. Leaf size=3

$$\log(\log(x))$$

[Out] Log[Log[x]]

Rubi [A] time = 0.0123612, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2302, 29}

$$\log(\log(x))$$

Antiderivative was successfully verified.

[In] Int[1/(x*Log[x]),x]

[Out] Log[Log[x]]

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x \log(x)} dx &= \text{Subst}\left(\int \frac{1}{x} dx, x, \log(x)\right)\\ &= \log(\log(x))\end{aligned}$$

Mathematica [A] time = 0.0041385, size = 3, normalized size = 1.

$$\log(\log(x))$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Log[x]),x]

[Out] Log[Log[x]]

Maple [A] time = 0., size = 4, normalized size = 1.3

$$\ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x),x)
```

```
[Out] ln(ln(x))
```

Maxima [A] time = 0.925341, size = 4, normalized size = 1.33

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="maxima")
```

```
[Out] log(log(x))
```

Fricas [A] time = 2.15694, size = 18, normalized size = 6.

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="fricas")
```

```
[Out] log(log(x))
```

Sympy [A] time = 0.08814, size = 3, normalized size = 1.

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x),x)
```

```
[Out] log(log(x))
```

Giac [A] time = 1.04621, size = 5, normalized size = 1.67

$$\log(|\log(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="giac")
```

```
[Out] log(abs(log(x)))
```

3.21 $\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\log(x))$$

```
[Out] ArcTan[Log[x]]
```

Rubi [A] time = 0.0192723, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {203}

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\log(x)\right)\\ &= \tan^{-1}(\log(x))\end{aligned}$$

Mathematica [A] time = 0.0103559, size = 3, normalized size = 1.

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Maple [A] time = 0.001, size = 4, normalized size = 1.3

$$\arctan(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+ln(x)^2),x)
```

```
[Out] arctan(ln(x))
```

Maxima [A] time = 1.40251, size = 4, normalized size = 1.33

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(log(x))
```

Fricas [A] time = 2.26766, size = 22, normalized size = 7.33

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(log(x))
```

Sympy [B] time = 0.127938, size = 15, normalized size = 5.

$$\operatorname{RootSum}\left(4z^2+1,\left(i\mapsto i\log(2i+\log(x))\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+ln(x)**2),x)
```

```
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

Giac [A] time = 1.0521, size = 4, normalized size = 1.33

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="giac")
```

```
[Out] arctan(log(x))
```

3.22 $\int \frac{1}{x(1-\log(x))} \, dx$

Optimal. Leaf size=9

$$-\log(1-\log(x))$$

```
[Out] -Log[1 - Log[x]]
```

Rubi [A] time = 0.0202668, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2302, 29}

$$-\log(1-\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 - Log[x])),x]
```

```
[Out] -Log[1 - Log[x]]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1-\log(x))} \, dx &= -\text{Subst}\left(\int \frac{1}{x} \, dx, x, 1-\log(x)\right)\\ &= -\log(1-\log(x))\end{aligned}$$

Mathematica [A] time = 0.008645, size = 7, normalized size = 0.78

$$-\log(\log(x)-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 - Log[x])),x]
```

```
[Out] -Log[-1 + Log[x]]
```

Maple [A] time = 0.001, size = 10, normalized size = 1.1

$$-\ln(1-\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1-ln(x)),x)
```

```
[Out] -ln(1-ln(x))
```

Maxima [A] time = 0.926967, size = 9, normalized size = 1.

$$-\log(\log(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-log(x)),x, algorithm="maxima")
```

```
[Out] -log(log(x) - 1)
```

Fricas [A] time = 2.29388, size = 24, normalized size = 2.67

$$-\log(\log(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-log(x)),x, algorithm="fricas")
```

```
[Out] -log(log(x) - 1)
```

Sympy [A] time = 0.0888, size = 7, normalized size = 0.78

$$-\log(\log(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-ln(x)),x)
```

```
[Out] -log(log(x) - 1)
```

Giac [B] time = 1.06416, size = 30, normalized size = 3.33

$$-\frac{1}{2} \log\left(\frac{1}{4} \pi^2(\operatorname{sgn}(x) - 1)^2 + (\log(|x|) - 1)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1-log(x)),x, algorithm="giac")
```

```
[Out] -1/2*log(1/4*pi^2*(sgn(x) - 1)^2 + (log(abs(x)) - 1)^2)
```

3.23 $\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx$

Optimal. Leaf size=9

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

```
[Out] Log[1 + Log[x/a]]
```

Rubi [A] time = 0.0180361, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2302, 29}

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + Log[x/a])),x]
```

```
[Out] Log[1 + Log[x/a]]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1+\log\left(\frac{x}{a}\right)\right)}\,dx &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, 1+\log\left(\frac{x}{a}\right)\right)\\ &= \log\left(1+\log\left(\frac{x}{a}\right)\right)\end{aligned}$$

Mathematica [A] time = 0.0161651, size = 9, normalized size = 1.

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + Log[x/a])),x]
```

```
[Out] Log[1 + Log[x/a]]
```

Maple [A] time = 0., size = 10, normalized size = 1.1

$$\ln\left(1+\ln\left(\frac{x}{a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+ln(x/a)),x)
```

```
[Out] ln(1+ln(x/a))
```

Maxima [A] time = 0.931764, size = 12, normalized size = 1.33

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x/a)),x, algorithm="maxima")
```

```
[Out] log(log(x/a) + 1)
```

Fricas [A] time = 2.27744, size = 26, normalized size = 2.89

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x/a)),x, algorithm="fricas")
```

```
[Out] log(log(x/a) + 1)
```

Sympy [A] time = 0.095479, size = 7, normalized size = 0.78

$$\log\left(\log\left(\frac{x}{a}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+ln(x/a)),x)
```

```
[Out] log(log(x/a) + 1)
```

Giac [B] time = 1.05264, size = 77, normalized size = 8.56

$$\frac{1}{2}\log\left(\frac{1}{4}\left(\pi(\operatorname{sgn}(a)-1)+\pi(\operatorname{sgn}(x)-1)+4\pi\left\lfloor -\frac{\pi(\operatorname{sgn}(a)-1)+\pi(\operatorname{sgn}(x)-1)}{4\pi}+\frac{1}{2}\right\rfloor\right)^2+\left(\log\left(\frac{|x|}{|a|}\right)+1\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x/a)),x, algorithm="giac")
```

```
[Out] 1/2*log(1/4*(pi*(sgn(a) - 1) + pi*(sgn(x) - 1) + 4*pi*floor(-1/4*(pi*(sgn(a
) - 1) + pi*(sgn(x) - 1))/pi + 1/2))^2 + (log(abs(x)/abs(a)) + 1)^2)
```

3.24 $\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2} \, dx$

Optimal. Leaf size=25

$$x-4\sqrt{x}+\frac{4}{\sqrt{x}}-\frac{1}{x}+3\log(x)$$

```
[Out] -x^(-1) + 4/Sqrt[x] - 4*Sqrt[x] + x + 3*Log[x]
```

Rubi [A] time = 0.0184495, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1357, 698}

$$x-4\sqrt{x}+\frac{4}{\sqrt{x}}-\frac{1}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[x] + x)^2/x^2,x]
```

```
[Out] -x^(-1) + 4/Sqrt[x] - 4*Sqrt[x] + x + 3*Log[x]
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-\sqrt{x}+x\right)^2}{x^2} \, dx &= 2\,\text{Subst}\left(\int \frac{\left(1-x+x^2\right)^2}{x^3} \, dx,x,\sqrt{x}\right)\\ &= 2\,\text{Subst}\left(\int \left(-2+\frac{1}{x^3}-\frac{2}{x^2}+\frac{3}{x}+x\right) \, dx,x,\sqrt{x}\right)\\ &= -\frac{1}{x}+\frac{4}{\sqrt{x}}-4\sqrt{x}+x+3\log(x)\end{aligned}$$

Mathematica [A] time = 0.0129274, size = 25, normalized size = 1.

$$x-4\sqrt{x}+\frac{4}{\sqrt{x}}-\frac{1}{x}+3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[x] + x)^2/x^2,x]
```

```
[Out] -x^(-1) + 4/Sqrt[x] - 4*Sqrt[x] + x + 3*Log[x]
```

Maple [A] time = 0.008, size = 22, normalized size = 0.9

$$-x^{-1} + x + 3\ln(x) + 4\frac{1}{\sqrt{x}} - 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x-x^(1/2))^2/x^2,x)
```

```
[Out] -1/x+x+3*ln(x)+4/x^(1/2)-4*x^(1/2)
```

Maxima [A] time = 0.929867, size = 30, normalized size = 1.2

$$x - 4\sqrt{x} + \frac{4\sqrt{x} - 1}{x} + 3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-x^(1/2))^2/x^2,x, algorithm="maxima")
```

```
[Out] x - 4*sqrt(x) + (4*sqrt(x) - 1)/x + 3*log(x)
```

Fricas [A] time = 2.18741, size = 70, normalized size = 2.8

$$\frac{x^2 + 6x\log\left(\sqrt{x}\right) - 4(x-1)\sqrt{x} - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-x^(1/2))^2/x^2,x, algorithm="fricas")
```

```
[Out] (x^2 + 6*x*log(sqrt(x)) - 4*(x - 1)*sqrt(x) - 1)/x
```

Sympy [A] time = 0.402118, size = 22, normalized size = 0.88

$$-4\sqrt{x} + x + 3\log(x) - \frac{1}{x} + \frac{4}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-x**(1/2))**2/x**2,x)
```

```
[Out] -4*sqrt(x) + x + 3*log(x) - 1/x + 4/sqrt(x)
```

Giac [A] time = 1.05138, size = 31, normalized size = 1.24

$$x - 4\sqrt{x} + \frac{4\sqrt{x} - 1}{x} + 3\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-x^(1/2))^2/x^2,x, algorithm="giac")
```

```
[Out] x - 4*sqrt(x) + (4*sqrt(x) - 1)/x + 3*log(abs(x))
```

3.25 $\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx$

Optimal. Leaf size=30

$$-\frac{6x^{7/6}}{7}-\frac{3x^{2/3}}{2}+4\sqrt{x}+2\log(x)$$

```
[Out] 4*Sqrt[x] - (3*x^(2/3))/2 - (6*x^(7/6))/7 + 2*Log[x]
```

Rubi [A] time = 0.0941351, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1584, 1820}

$$-\frac{6x^{7/6}}{7}-\frac{3x^{2/3}}{2}+4\sqrt{x}+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((2 - x^(2/3))*(Sqrt[x] + x))/x^(3/2),x]
```

```
[Out] 4*Sqrt[x] - (3*x^(2/3))/2 - (6*x^(7/6))/7 + 2*Log[x]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2-x^{2/3}\right)\left(\sqrt{x}+x\right)}{x^{3/2}}\,dx &= \int \frac{\left(1+\sqrt{x}\right)\left(2-x^{2/3}\right)}{x}\,dx\\
&= -\left(6\,\text{Subst}\left(\int \frac{\left(1+x^3\right)\left(-2+x^4\right)}{x}\,dx, x, \sqrt[6]{x}\right)\right)\\
&= -\left(6\,\text{Subst}\left(\int \left(-\frac{2}{x}-2x^2+x^3+x^6\right)dx, x, \sqrt[6]{x}\right)\right)\\
&= 4\sqrt{x}-\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}+2\log(x)
\end{aligned}$$

Mathematica [A] time = 0.016938, size = 30, normalized size = 1.

$$-\frac{6x^{7/6}}{7}-\frac{3x^{2/3}}{2}+4\sqrt{x}+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 - x^(2/3))*(Sqrt[x] + x))/x^(3/2),x]

[Out] 4*Sqrt[x] - (3*x^(2/3))/2 - (6*x^(7/6))/7 + 2*Log[x]
```

Maple [A] time = 0.006, size = 21, normalized size = 0.7

$$-\frac{3}{2}x^{\frac{2}{3}}-\frac{6}{7}x^{\frac{7}{6}}+2\ln\left(x\right)+4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x)

[Out] -3/2*x^(2/3)-6/7*x^(7/6)+2*ln(x)+4*x^(1/2)
```

Maxima [A] time = 0.927492, size = 27, normalized size = 0.9

$$-\frac{6}{7}x^{\frac{7}{6}}-\frac{3}{2}x^{\frac{2}{3}}+4\sqrt{x}+2\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x, algorithm="maxima")

[Out] -6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 2*log(x)
```

Fricas [A] time = 2.20734, size = 78, normalized size = 2.6

$$-\frac{6}{7}x^{\frac{7}{6}}-\frac{3}{2}x^{\frac{2}{3}}+4\sqrt{x}+12\log\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x, algorithm="fricas")

[Out] -6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 12*log(x^(1/6))
```

Sympy [A] time = 5.2774, size = 31, normalized size = 1.03

$$-\frac{6x^{\frac{7}{6}}}{7}-\frac{3x^{\frac{2}{3}}}{2}+4\sqrt{x}+4\log\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-x**(2/3))*(x+x**(1/2))/x**(3/2),x)

[Out] -6*x**(7/6)/7 - 3*x**(2/3)/2 + 4*sqrt(x) + 4*log(sqrt(x))
```

Giac [A] time = 1.06412, size = 28, normalized size = 0.93

$$-\frac{6}{7}\,x^{\frac{7}{6}} - \frac{3}{2}\,x^{\frac{2}{3}} + 4\,\sqrt{x} + 2\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-x^(2/3))*(x+x^(1/2))/x^(3/2),x, algorithm="giac")
```

```
[Out] -6/7*x^(7/6) - 3/2*x^(2/3) + 4*sqrt(x) + 2*log(abs(x))
```

3.26 $\int \frac{-1+2x}{3+2x}\,dx$

Optimal. Leaf size=10

$$x-2\log(2x+3)$$

```
[Out] x - 2*Log[3 + 2*x]
```

Rubi [A] time = 0.0050317, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$x-2\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x)/(3 + 2*x),x]

[Out] x - 2*Log[3 + 2*x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-1+2x}{3+2x}\,dx &= \int \left(1-\frac{4}{3+2x}\right)dx\\ &= x-2\log(3+2x)\end{aligned}$$

Mathematica [A] time = 0.0017653, size = 10, normalized size = 1.

$$x-2\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x)/(3 + 2*x),x]

[Out] x - 2*Log[3 + 2*x]
```

Maple [A] time = 0.001, size = 11, normalized size = 1.1

$$x-2\ln(3+2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x-1)/(3+2*x),x)

[Out] x-2*ln(3+2*x)
```

Maxima [A] time = 0.941657, size = 14, normalized size = 1.4

$$x - 2\log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(3+2*x),x, algorithm="maxima")

[Out] x - 2*log(2*x + 3)
```

Fricas [A] time = 2.07079, size = 27, normalized size = 2.7

$$x - 2\log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(3+2*x),x, algorithm="fricas")

[Out] x - 2*log(2*x + 3)
```

Sympy [A] time = 0.072634, size = 8, normalized size = 0.8

$$x - 2\log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(3+2*x),x)

[Out] x - 2*log(2*x + 3)
```

Giac [A] time = 1.04953, size = 15, normalized size = 1.5

$$x - 2\log(|2x + 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x)/(3+2*x),x, algorithm="giac")

[Out] x - 2*log(abs(2*x + 3))
```

3.27 $\int \frac{-5+2x}{-2+3x^2}\,dx$

Optimal. Leaf size=47

$$\frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(3x+\sqrt{6}\right)$$

```
[Out] ((4 - 5*Sqrt[6])*Log[Sqrt[6] - 3*x])/12 + ((4 + 5*Sqrt[6])*Log[Sqrt[6] + 3*
x])/12
```

Rubi [A] time = 0.0227613, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {633, 31}

$$\frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(3x+\sqrt{6}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 2*x)/(-2 + 3*x^2),x]
```

```
[Out] ((4 - 5*Sqrt[6])*Log[Sqrt[6] - 3*x])/12 + ((4 + 5*Sqrt[6])*Log[Sqrt[6] + 3*
x])/12
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{-5+2x}{-2+3x^2}\,dx &= \frac{1}{4}\left(4-5\sqrt{6}\right)\int \frac{1}{-\sqrt{6}+3x}\,dx+\frac{1}{4}\left(4+5\sqrt{6}\right)\int \frac{1}{\sqrt{6}+3x}\,dx\\ &= \frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(\sqrt{6}+3x\right)\end{aligned}$$

Mathematica [A] time = 0.027503, size = 47, normalized size = 1.

$$\frac{1}{12}\left(4-5\sqrt{6}\right)\log\left(\sqrt{6}-3x\right)+\frac{1}{12}\left(4+5\sqrt{6}\right)\log\left(3x+\sqrt{6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 2*x)/(-2 + 3*x^2),x]
```

```
[Out] ((4 - 5*Sqrt[6])*Log[Sqrt[6] - 3*x])/12 + ((4 + 5*Sqrt[6])*Log[Sqrt[6] + 3*
x])/12
```

Maple [A] time = 0.002, size = 24, normalized size = 0.5

$$\frac{\ln\left(3x^2-2\right)}{3}+\frac{5\sqrt{6}}{6}\operatorname{Artanh}\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x-5)/(3*x^2-2),x)
```

```
[Out] 1/3*ln(3*x^2-2)+5/6*6^(1/2)*arctanh(1/2*x*6^(1/2))
```

Maxima [A] time = 1.40894, size = 49, normalized size = 1.04

$$-\frac{5}{12}\sqrt{6}\log\left(\frac{3x-\sqrt{6}}{3x+\sqrt{6}}\right)+\frac{1}{3}\log\left(3x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(3*x^2-2),x, algorithm="maxima")
```

```
[Out] -5/12*sqrt(6)*log((3*x - sqrt(6))/(3*x + sqrt(6))) + 1/3*log(3*x^2 - 2)
```

Fricas [A] time = 2.03127, size = 105, normalized size = 2.23

$$\frac{5}{12}\sqrt{6}\log\left(\frac{3x^2+2\sqrt{6}x+2}{3x^2-2}\right)+\frac{1}{3}\log\left(3x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(3*x^2-2),x, algorithm="fricas")
```

```
[Out] 5/12*sqrt(6)*log((3*x^2 + 2*sqrt(6)*x + 2)/(3*x^2 - 2)) + 1/3*log(3*x^2 - 2
)
```

Sympy [A] time = 0.106597, size = 42, normalized size = 0.89

$$\left(\frac{1}{3}-\frac{5\sqrt{6}}{12}\right)\log\left(x-\frac{\sqrt{6}}{3}\right)+\left(\frac{1}{3}+\frac{5\sqrt{6}}{12}\right)\log\left(x+\frac{\sqrt{6}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(3*x**2-2),x)
```

```
[Out] (1/3 - 5*sqrt(6)/12)*log(x - sqrt(6)/3) + (1/3 + 5*sqrt(6)/12)*log(x + sqrt
(6)/3)
```

Giac [A] time = 1.07114, size = 50, normalized size = 1.06

$$\frac{1}{12}\left(5\sqrt{6}+4\right)\log\left(\left|x+\frac{1}{3}\sqrt{6}\right|\right)-\frac{1}{12}\left(5\sqrt{6}-4\right)\log\left(\left|x-\frac{1}{3}\sqrt{6}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(3*x^2-2),x, algorithm="giac")
```

```
[Out] 1/12*(5*sqrt(6) + 4)*log(abs(x + 1/3*sqrt(6))) - 1/12*(5*sqrt(6) - 4)*log(a
bs(x - 1/3*sqrt(6)))
```

3.28 $\int \frac{-5+2x}{2+3x^2} \, dx$

Optimal. Leaf size=30

$$\frac{1}{3}\log\left(3x^2+2\right)-\frac{5\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

```
[Out] (-5*ArcTan[Sqrt[3/2]*x])/Sqrt[6] + Log[2 + 3*x^2]/3
```

Rubi [A] time = 0.0081567, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {635, 203, 260}

$$\frac{1}{3}\log\left(3x^2+2\right)-\frac{5\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 2*x)/(2 + 3*x^2),x]

[Out] (-5*ArcTan[Sqrt[3/2]*x])/Sqrt[6] + Log[2 + 3*x^2]/3
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{-5+2x}{2+3x^2}\,dx &= 2\int \frac{x}{2+3x^2}\,dx-5\int \frac{1}{2+3x^2}\,dx\\ &= -\frac{5\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}+\frac{1}{3}\log\left(2+3x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0131009, size = 30, normalized size = 1.

$$\frac{1}{3}\log\left(3x^2+2\right)-\frac{5\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 2*x)/(2 + 3*x^2),x]
```

```
[Out] (-5*ArcTan[Sqrt[3/2]*x])/Sqrt[6] + Log[2 + 3*x^2]/3
```

Maple [A] time = 0.002, size = 24, normalized size = 0.8

$$\frac{\ln\left(3x^2+2\right)}{3}-\frac{5\sqrt{6}}{6}\arctan\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x-5)/(3*x^2+2),x)
```

```
[Out] 1/3*ln(3*x^2+2)-5/6*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Maxima [A] time = 1.40806, size = 31, normalized size = 1.03

$$-\frac{5}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)+\frac{1}{3}\log\left(3x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(3*x^2+2),x, algorithm="maxima")
```

```
[Out] -5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*log(3*x^2 + 2)
```

Fricas [A] time = 2.15712, size = 77, normalized size = 2.57

$$-\frac{5}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)+\frac{1}{3}\log\left(3x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(3*x^2+2),x, algorithm="fricas")
```

```
[Out] -5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*log(3*x^2 + 2)
```

Sympy [A] time = 0.104192, size = 27, normalized size = 0.9

$$\frac{\log\left(x^2+\frac{2}{3}\right)}{3}-\frac{5\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(3*x**2+2),x)
```

```
[Out] log(x**2 + 2/3)/3 - 5*sqrt(6)*atan(sqrt(6)*x/2)/6
```

Giac [A] time = 1.06772, size = 28, normalized size = 0.93

$$-\frac{5}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right)+\frac{1}{3}\log\left(x^2+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+2*x)/(3*x^2+2),x, algorithm="giac")
```

```
[Out] -5/6*sqrt(6)*arctan(1/2*sqrt(6)*x) + 1/3*log(x^2 + 2/3)
```

3.29 $\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}\sin\left(\frac{3x}{4}\right)-\frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

```
[Out] (2*Sin[(3*x)/4])/3 - (2*Sin[(5*x)/4])/5
```

Rubi [A] time = 0.0080012, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4282}

$$\frac{2}{3}\sin\left(\frac{3x}{4}\right)-\frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x/4]*Sin[x],x]
```

```
[Out] (2*Sin[(3*x)/4])/3 - (2*Sin[(5*x)/4])/5
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin\left(\frac{x}{4}\right)\sin(x)\,dx=\frac{2}{3}\sin\left(\frac{3x}{4}\right)-\frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

Mathematica [A] time = 0.0108375, size = 21, normalized size = 1.

$$\frac{2}{3}\sin\left(\frac{3x}{4}\right)-\frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x/4]*Sin[x],x]
```

```
[Out] (2*Sin[(3*x)/4])/3 - (2*Sin[(5*x)/4])/5
```

Maple [A] time = 0.027, size = 14, normalized size = 0.7

$$\frac{2}{3}\sin\left(\frac{3x}{4}\right)-\frac{2}{5}\sin\left(\frac{5x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/4*x)*sin(x),x)
```

```
[Out] 2/3*sin(3/4*x)-2/5*sin(5/4*x)
```

Maxima [A] time = 0.924895, size = 18, normalized size = 0.86

$$-\frac{2}{5}\sin\left(\frac{5}{4}x\right)+\frac{2}{3}\sin\left(\frac{3}{4}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4*x)*sin(x),x, algorithm="maxima")
```

```
[Out] -2/5*sin(5/4*x) + 2/3*sin(3/4*x)
```

Fricas [A] time = 2.41178, size = 77, normalized size = 3.67

$$-\frac{16}{15}\left(6\cos\left(\frac{1}{4}x\right)^{4}-7\cos\left(\frac{1}{4}x\right)^{2}+1\right)\sin\left(\frac{1}{4}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4*x)*sin(x),x, algorithm="fricas")
```

```
[Out] -16/15*(6*cos(1/4*x)^4 - 7*cos(1/4*x)^2 + 1)*sin(1/4*x)
```

Sympy [A] time = 0.539427, size = 22, normalized size = 1.05

$$-\frac{16\sin\left(\frac{x}{4}\right)\cos\left(x\right)}{15}+\frac{4\sin\left(x\right)\cos\left(\frac{x}{4}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4*x)*sin(x),x)
```

```
[Out] -16*sin(x/4)*cos(x)/15 + 4*sin(x)*cos(x/4)/15
```

Giac [A] time = 1.04563, size = 18, normalized size = 0.86

$$-\frac{2}{5}\sin\left(\frac{5}{4}x\right)+\frac{2}{3}\sin\left(\frac{3}{4}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/4*x)*sin(x),x, algorithm="giac")
```

```
[Out] -2/5*sin(5/4*x) + 2/3*sin(3/4*x)
```

3.30 $\int \cos(3x)\cos(4x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

```
[Out] Sin[x]/2 + Sin[7*x]/14
```

Rubi [A] time = 0.0082189, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4283}

$$\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]*Cos[4*x],x]
```

```
[Out] Sin[x]/2 + Sin[7*x]/14
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(3x)\cos(4x)\,dx=\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

Mathematica [A] time = 0.0060621, size = 15, normalized size = 1.

$$\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]*Cos[4*x],x]
```

```
[Out] Sin[x]/2 + Sin[7*x]/14
```

Maple [A] time = 0., size = 12, normalized size = 0.8

$$\frac{\sin(x)}{2}+\frac{\sin(7x)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*cos(4*x),x)
```

```
[Out] 1/2*sin(x)+1/14*sin(7*x)
```

Maxima [A] time = 0.939458, size = 15, normalized size = 1.

$$\frac{1}{14}\sin(7x) + \frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="maxima")
```

```
[Out] 1/14*sin(7*x) + 1/2*sin(x)
```

Fricas [B] time = 2.12828, size = 78, normalized size = 5.2

$$\frac{1}{7}\left(32\cos(x)^6 - 40\cos(x)^4 + 12\cos(x)^2 + 3\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="fricas")
```

```
[Out] 1/7*(32*cos(x)^6 - 40*cos(x)^4 + 12*cos(x)^2 + 3)*sin(x)
```

Sympy [B] time = 0.52267, size = 26, normalized size = 1.73

$$-\frac{3\sin(3x)\cos(4x)}{7} + \frac{4\sin(4x)\cos(3x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x)
```

```
[Out] -3*sin(3*x)*cos(4*x)/7 + 4*sin(4*x)*cos(3*x)/7
```

Giac [A] time = 1.06341, size = 15, normalized size = 1.

$$\frac{1}{14}\sin(7x) + \frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="giac")
```

```
[Out] 1/14*sin(7*x) + 1/2*sin(x)
```

3.31 $\int -\tan(a-x)\tan(x)\,dx$

Optimal. Leaf size=21

$$\cot(a)\log(\cos(a-x)) - \cot(a)\log(\cos(x)) - x$$

```
[Out] -x + Cot[a]*Log[Cos[a - x]] - Cot[a]*Log[Cos[x]]
```

Rubi [A] time = 0.033836, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4612, 4610, 3475}

$$\cot(a)\log(\cos(a-x)) - \cot(a)\log(\cos(x)) - x$$

Antiderivative was successfully verified.

```
[In] Int[-(Tan[a - x]*Tan[x]),x]
```

```
[Out] -x + Cot[a]*Log[Cos[a - x]] - Cot[a]*Log[Cos[x]]
```

Rule 4612

```
Int[Tan[(a_.) + (b_.)*(x_)]*Tan[(c_) + (d_.)*(x_)], x_Symbol] :> -Simp[(b*x
)/d, x] + Dist[(b*Cos[(b*c - a*d)/d])/d, Int[Sec[a + b*x]*Sec[c + d*x], x],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c - a*d, 0]
```

Rule 4610

```
Int[Sec[(a_.) + (b_.)*(x_)]*Sec[(c_) + (d_.)*(x_)], x_Symbol] :> -Dist[Csc[
(b*c - a*d)/d], Int[Tan[a + b*x], x], x] + Dist[Csc[(b*c - a*d)/b], Int[Tan
[c + d*x], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b^2 - d^2, 0] && NeQ[b*c
 - a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int -\tan(a-x)\tan(x)\,dx &= -x+\cos(a)\int \sec(a-x)\sec(x)\,dx\\ &= -x+\cot(a)\int \tan(a-x)\,dx+\cot(a)\int \tan(x)\,dx\\ &= -x+\cot(a)\log(\cos(a-x))-\cot(a)\log(\cos(x))\end{aligned}$$

Mathematica [A] time = 0.0772765, size = 21, normalized size = 1.

$$\cot(a)\log(\cos(a-x)) - \cot(a)\log(\cos(x)) - x$$

Antiderivative was successfully verified.

```
[In] Integrate[-(Tan[a - x]*Tan[x]),x]
```

```
[Out] -x + Cot[a]*Log[Cos[a - x]] - Cot[a]*Log[Cos[x]]
```

Maple [A] time = 0.036, size = 18, normalized size = 0.9

$$\frac{\ln(1+\tan(x)\tan(a))}{\tan(a)} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-tan(x)*tan(a-x),x)
```

```
[Out] 1/tan(a)*ln(1+tan(x)*tan(a))-x
```

Maxima [B] time = 1.47707, size = 251, normalized size = 11.95

$$-\frac{\left(\cos(2a)^2 + \sin(2a)^2 - 2\cos(2a) + 1\right)x + \left(\cos(2a)^2 + \sin(2a)^2 - 1\right)\arctan(\sin(2a) + \sin(2x), \cos(2a) + \cos(2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(x)*tan(a-x),x, algorithm="maxima")
```

```
[Out] -((cos(2*a)^2 + sin(2*a)^2 - 2*cos(2*a) + 1)*x + (cos(2*a)^2 + sin(2*a)^2 -
 1)*arctan2(sin(2*a) + sin(2*x), cos(2*a) + cos(2*x)) - (cos(2*a)^2 + sin(2
*a)^2 - 1)*arctan2(sin(2*x), cos(2*x) + 1) - log(cos(2*a)^2 + 2*cos(2*a)*co
s(2*x) + cos(2*x)^2 + sin(2*a)^2 + 2*sin(2*a)*sin(2*x) + sin(2*x)^2)*sin(2*
a) + log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*sin(2*a))/(cos(2*a)^2 +
sin(2*a)^2 - 2*cos(2*a) + 1)
```

Fricas [B] time = 2.09668, size = 261, normalized size = 12.43

$$\frac{(\cos(2a)+1)\log\left(-\frac{(\cos(2a)-1)\tan(x)^2-2\sin(2a)\tan(x)-\cos(2a)-1}{(\cos(2a)+1)\tan(x)^2+\cos(2a)+1}\right) - (\cos(2a)+1)\log\left(\frac{1}{\tan(x)^2+1}\right) - 2x\sin(2a)}{2\sin(2a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(x)*tan(a-x),x, algorithm="fricas")
```

```
[Out] 1/2*((cos(2*a) + 1)*log(-((cos(2*a) - 1)*tan(x)^2 - 2*sin(2*a)*tan(x) - cos
(2*a) - 1)/((cos(2*a) + 1)*tan(x)^2 + cos(2*a) + 1)) - (cos(2*a) + 1)*log(1
/(tan(x)^2 + 1)) - 2*x*sin(2*a))/sin(2*a)
```

Sympy [B] time = 1.31194, size = 138, normalized size = 6.57

$$-\left(\left(\begin{cases}\frac{2x\tan(a)}{2\tan^2(a)+2} - \frac{2\log\left(\tan(x)+\frac{1}{\tan(a)}\right)}{2\tan^2(a)+2} + \frac{\log\left(\tan^2(x)+1\right)}{2\tan^2(a)+2} & \text{for } a \neq 0 \\ \frac{\log\left(\tan^2(x)+1\right)}{2} & \text{otherwise}\end{cases}\right)\tan(a) + \begin{cases}-\frac{2x\tan(a)}{2\tan^3(a)+2\tan(a)} + \frac{2\log\left(\tan(x)+\frac{1}{\tan(a)}\right)}{2\tan^3(a)+2\tan(a)} + \frac{\log}{2} \\ -x+\tan(x)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(x)*tan(a-x),x)
```

```
[Out] -Piecewise((2*x*tan(a)/(2*tan(a)**2 + 2) - 2*log(tan(x) + 1/tan(a))/(2*tan(
a)**2 + 2) + log(tan(x)**2 + 1)/(2*tan(a)**2 + 2), Ne(a, 0)), (log(tan(x)**
2 + 1)/2, True))*tan(a) + Piecewise((-2*x*tan(a)/(2*tan(a)**3 + 2*tan(a)) +
 2*log(tan(x) + 1/tan(a))/(2*tan(a)**3 + 2*tan(a)) + log(tan(x)**2 + 1)*tan
(a)**2/(2*tan(a)**3 + 2*tan(a)), Ne(a, 0)), (-x + tan(x), True))
```

Giac [A] time = 1.0918, size = 24, normalized size = 1.14

$$-x + \frac{\log\left(\left|\tan\left(a\right)\tan\left(x\right) + 1\right|\right)}{\tan\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-tan(x)*tan(a-x),x, algorithm="giac")
```

```
[Out] -x + log(abs(tan(a)*tan(x) + 1))/tan(a)
```

3.32 $\int \sin^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0050332, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2,x]
```

```
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^2(x)\,dx &= -\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0016141, size = 14, normalized size = 1.

$$\frac{x}{2}-\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2,x]
```

```
[Out] x/2 - Sin[2*x]/4
```

Maple [A] time = 0., size = 11, normalized size = 0.8

$$\frac{x}{2} - \frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x)
```

```
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Maxima [A] time = 0.961152, size = 14, normalized size = 1.

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

Fricas [A] time = 2.00429, size = 38, normalized size = 2.71

$$-\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.061932, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2,x)
```

```
[Out] x/2 - sin(x)*cos(x)/2
```

Giac [A] time = 1.06377, size = 14, normalized size = 1.

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

3.33 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0058507, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\,dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0016073, size = 14, normalized size = 1.

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2,x]
```

```
[Out] x/2 + Sin[2*x]/4
```

Maple [A] time = 0., size = 11, normalized size = 0.8

$$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
```

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A] time = 0.959867, size = 14, normalized size = 1.

$$\frac{1}{2}x+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A] time = 2.06738, size = 36, normalized size = 2.57

$$\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.070355, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

Giac [A] time = 1.05503, size = 14, normalized size = 1.

$$\frac{1}{2}x+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

3.34 $\int \cos^3(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$-\frac{1}{4}\cos^4(x)$$

```
[Out] -Cos[x]^4/4
```

Rubi [A] time = 0.0122785, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2565, 30}

$$-\frac{1}{4}\cos^4(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Sin[x],x]
```

```
[Out] -Cos[x]^4/4
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \cos^3(x)\sin(x)\,dx &= -\text{Subst}\left(\int x^3\,dx, x, \cos(x)\right)\\ &= -\frac{1}{4}\cos^4(x) \end{aligned}$$

Mathematica [A] time = 0.0010436, size = 8, normalized size = 1.

$$-\frac{1}{4}\cos^4(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Sin[x],x]
```

```
[Out] -Cos[x]^4/4
```

Maple [A] time = 0.003, size = 7, normalized size = 0.9

$$-\frac{(\cos(x))^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x),x)
```

```
[Out] -1/4*cos(x)^4
```

Maxima [A] time = 0.925397, size = 8, normalized size = 1.

$$-\frac{1}{4}\cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x),x, algorithm="maxima")
```

```
[Out] -1/4*cos(x)^4
```

Fricas [A] time = 1.93062, size = 20, normalized size = 2.5

$$-\frac{1}{4}\cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x),x, algorithm="fricas")
```

```
[Out] -1/4*cos(x)^4
```

Sympy [A] time = 0.057671, size = 7, normalized size = 0.88

$$-\frac{\cos^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x),x)
```

```
[Out] -cos(x)**4/4
```

Giac [A] time = 1.06011, size = 8, normalized size = 1.

$$-\frac{1}{4}\cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x),x, algorithm="giac")
```

```
[Out] -1/4*cos(x)^4
```

3.35 $\int \cot^3(x)\csc(x)\,dx$

Optimal. Leaf size=11

$$\csc(x) - \frac{\csc^3(x)}{3}$$

```
[Out] Csc[x] - Csc[x]^3/3
```

Rubi [A] time = 0.0150474, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2606}

$$\csc(x) - \frac{\csc^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Csc[x],x]
```

```
[Out] Csc[x] - Csc[x]^3/3
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot^3(x)\csc(x)\,dx &= -\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \csc(x)\right)\\ &= \csc(x) - \frac{\csc^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.0065849, size = 11, normalized size = 1.

$$\csc(x) - \frac{\csc^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Csc[x],x]
```

```
[Out] Csc[x] - Csc[x]^3/3
```

Maple [B] time = 0.006, size = 32, normalized size = 2.9

$$-\frac{(\cos(x))^4}{3\,(\sin(x))^3} + \frac{(\cos(x))^4}{3\,\sin(x)} + \frac{\left(2+(\cos(x))^2\right)\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/sin(x)^4,x)
```

```
[Out] -1/3/sin(x)^3*cos(x)^4+1/3/sin(x)*cos(x)^4+1/3*(2+cos(x)^2)*sin(x)
```

Maxima [A] time = 0.96211, size = 19, normalized size = 1.73

$$\frac{3\sin(x)^2 - 1}{3\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

Fricas [A] time = 1.94119, size = 62, normalized size = 5.64

$$\frac{3\cos(x)^2 - 2}{3\left(\cos(x)^2 - 1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*cos(x)^2 - 2)/((cos(x)^2 - 1)*sin(x))
```

Sympy [A] time = 0.086659, size = 14, normalized size = 1.27

$$\frac{3\sin^2(x) - 1}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/sin(x)**4,x)
```

```
[Out] (3*sin(x)**2 - 1)/(3*sin(x)**3)
```

Giac [A] time = 1.06053, size = 19, normalized size = 1.73

$$\frac{3\sin(x)^2 - 1}{3\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

3.36 $\int \csc^2(x)\sec^2(x)\,dx$

Optimal. Leaf size=7

$$\tan(x) - \cot(x)$$

```
[Out] -Cot[x] + Tan[x]
```

Rubi [A] time = 0.0237016, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2620, 14}

$$\tan(x) - \cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2*Sec[x]^2,x]
```

```
[Out] -Cot[x] + Tan[x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(x)\sec^2(x)\,dx &= \text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)dx, x, \tan(x)\right)\\
&= -\cot(x)+\tan(x)
\end{aligned}$$

Mathematica [A] time = 0.0070756, size = 6, normalized size = 0.86

$$-2\cot(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2*Sec[x]^2,x]
```

```
[Out] -2*Cot[2*x]
```

Maple [A] time = 0., size = 15, normalized size = 2.1

$$\frac{1}{\cos(x)\sin(x)} - 2\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^2/sin(x)^2,x)
```

```
[Out] 1/sin(x)/cos(x)-2*cot(x)
```

Maxima [A] time = 0.959902, size = 12, normalized size = 1.71

$$-\frac{1}{\tan(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/tan(x) + tan(x)
```

Fricas [B] time = 1.8216, size = 47, normalized size = 6.71

$$-\frac{2\cos(x)^2 - 1}{\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="fricas")
```

```
[Out] -(2*cos(x)^2 - 1)/(cos(x)*sin(x))
```

Sympy [B] time = 0.069937, size = 12, normalized size = 1.71

$$-\frac{2\cos(2x)}{\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**2/sin(x)**2,x)
```

```
[Out] -2*cos(2*x)/sin(2*x)
```

Giac [A] time = 1.05369, size = 12, normalized size = 1.71

$$-\frac{1}{\tan(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/tan(x) + tan(x)
```

3.37 $\int \cot^2\left(\frac{3x}{4}\right) dx$

Optimal. Leaf size=14

$$-x-\frac{4}{3}\cot\left(\frac{3x}{4}\right)$$

```
[Out] -x - (4*Cot[(3*x)/4])/3
```

Rubi [A] time = 0.0062183, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3473, 8}

$$-x-\frac{4}{3}\cot\left(\frac{3x}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[(3*x)/4]^2,x]
```

```
[Out] -x - (4*Cot[(3*x)/4])/3
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2\left(\frac{3x}{4}\right) dx &= -\frac{4}{3}\cot\left(\frac{3x}{4}\right)-\int 1\,dx\\ &= -x-\frac{4}{3}\cot\left(\frac{3x}{4}\right)\end{aligned}$$

Mathematica [C] time = 0.013271, size = 28, normalized size = 2.

$$-\frac{4}{3}\cot\left(\frac{3x}{4}\right)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2\left(\frac{3x}{4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[(3*x)/4]^2,x]
```

```
[Out] (-4*Cot[(3*x)/4]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[(3*x)/4]^2])/3
```

Maple [A] time = 0.003, size = 14, normalized size = 1.

$$-\frac{4}{3}\cot\left(\frac{3x}{4}\right)+\frac{2\pi}{3}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(3/4*x)^2,x)
```

```
[Out] -4/3*cot(3/4*x)+2/3*Pi-x
```

Maxima [A] time = 1.46041, size = 16, normalized size = 1.14

$$-x-\frac{4}{3\tan\left(\frac{3}{4}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3/4*x)^2,x, algorithm="maxima")
```

```
[Out] -x - 4/3/tan(3/4*x)
```

Fricas [B] time = 2.01595, size = 72, normalized size = 5.14

$$-\frac{3x\sin\left(\frac{3}{2}x\right)+4\cos\left(\frac{3}{2}x\right)+4}{3\sin\left(\frac{3}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3/4*x)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(3*x*sin(3/2*x) + 4*cos(3/2*x) + 4)/sin(3/2*x)
```

Sympy [A] time = 0.068754, size = 19, normalized size = 1.36

$$-x-\frac{4\cos\left(\frac{3x}{4}\right)}{3\sin\left(\frac{3x}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3/4*x)**2,x)
```

```
[Out] -x - 4*cos(3*x/4)/(3*sin(3*x/4))
```

Giac [A] time = 1.0636, size = 24, normalized size = 1.71

$$-x - \frac{2}{3 \tan\left(\frac{3}{8}x\right)} + \frac{2}{3}\tan\left(\frac{3}{8}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(3/4*x)^2,x, algorithm="giac")
```

```
[Out] -x - 2/3/tan(3/8*x) + 2/3*tan(3/8*x)
```

3.38 $\int (1+\tan(2x))^2 \, dx$

Optimal. Leaf size=16

$$\frac{1}{2}\tan(2x)-\log(\cos(2x))$$

```
[Out] -Log[Cos[2*x]] + Tan[2*x]/2
```

Rubi [A] time = 0.0109919, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3477, 3475}

$$\frac{1}{2}\tan(2x)-\log(\cos(2x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[2*x])^2,x]
```

```
[Out] -Log[Cos[2*x]] + Tan[2*x]/2
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (1+\tan(2x))^2 \, dx &= \frac{1}{2}\tan(2x)+2\int \tan(2x) \, dx\\ &= -\log(\cos(2x))+\frac{1}{2}\tan(2x)\end{aligned}$$

Mathematica [A] time = 0.0107909, size = 26, normalized size = 1.62

$$x-\frac{1}{2}\tan^{-1}(\tan(2x))+\frac{1}{2}\tan(2x)-\log(\cos(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[2*x])^2,x]
```

```
[Out] x - ArcTan[Tan[2*x]]/2 - Log[Cos[2*x]] + Tan[2*x]/2
```

Maple [A] time = 0.001, size = 19, normalized size = 1.2

$$\frac{\tan(2x)}{2}+\frac{\ln\left(1+(\tan(2x))^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tan(2*x))^2,x)
```

```
[Out] 1/2*tan(2*x)+1/2*ln(1+tan(2*x)^2)
```

Maxima [A] time = 1.41653, size = 16, normalized size = 1.

$$\log(\sec(2x))+\frac{1}{2}\tan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(2*x))^2,x, algorithm="maxima")
```

```
[Out] log(sec(2*x)) + 1/2*tan(2*x)
```

Fricas [A] time = 2.00127, size = 61, normalized size = 3.81

$$-\frac{1}{2}\log\left(\frac{1}{\tan(2x)^2+1}\right)+\frac{1}{2}\tan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(2*x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*log(1/(tan(2*x)^2 + 1)) + 1/2*tan(2*x)
```

Sympy [A] time = 0.141312, size = 17, normalized size = 1.06

$$\frac{\log\left(\tan^2(2x)+1\right)}{2}+\frac{\tan(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(2*x))**2,x)
```

```
[Out] log(tan(2*x)**2 + 1)/2 + tan(2*x)/2
```

Giac [A] time = 1.09449, size = 30, normalized size = 1.88

$$-\frac{1}{2}\log\left(\frac{4}{\tan(2x)^2+1}\right)+\frac{1}{2}\tan(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(2*x))^2,x, algorithm="giac")
```

```
[Out] -1/2*log(4/(tan(2*x)^2 + 1)) + 1/2*tan(2*x)
```

3.39 $\int(-\cot(x)+\tan(x))^2\,dx$

Optimal. Leaf size=10

$$-4x+\tan(x)-\cot(x)$$

```
[Out] -4*x - Cot[x] + Tan[x]
```

Rubi [A] time = 0.0275772, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {461, 203}

$$-4x+\tan(x)-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-Cot[x] + Tan[x])^2,x]

[Out] -4*x - Cot[x] + Tan[x]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int(-\cot(x)+\tan(x))^2\,dx &= \text{Subst}\left(\int\frac{\left(1-x^2\right)^2}{x^2\left(1+x^2\right)}\,dx,x,\tan(x)\right)\\
&= \text{Subst}\left(\int\left(1+\frac{1}{x^2}-\frac{4}{1+x^2}\right)\,dx,x,\tan(x)\right)\\
&= -\cot(x)+\tan(x)-4\,\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\tan(x)\right)\\
&= -4x-\cot(x)+\tan(x)
\end{aligned}$$

Mathematica [A] time = 0.0155332, size = 10, normalized size = 1.

$$-4x+\tan(x)-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cot[x] + Tan[x])^2,x]
```

```
[Out] -4*x - Cot[x] + Tan[x]
```

Maple [A] time = 0.014, size = 11, normalized size = 1.1

$$-4\,x-\cot\left(x\right)+\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cot(x)+tan(x))^2,x)
```

```
[Out] -4*x-cot(x)+tan(x)
```

Maxima [A] time = 1.46524, size = 16, normalized size = 1.6

$$-4\,x-\frac{1}{\tan\left(x\right)}+\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cot(x)+tan(x))^2,x, algorithm="maxima")
```

```
[Out] -4*x - 1/tan(x) + tan(x)
```

Fricas [A] time = 2.04472, size = 50, normalized size = 5.

$$-\frac{4\,x\tan\left(x\right)-\tan\left(x\right)^{2}+1}{\tan\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cot(x)+tan(x))^2,x, algorithm="fricas")
```

```
[Out] -(4*x*tan(x) - tan(x)^2 + 1)/tan(x)
```

Sympy [A] time = 0.393155, size = 10, normalized size = 1.

$$-4x+\tan\left(x\right)-\frac{1}{\tan\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cot(x)+tan(x))**2,x)
```

```
[Out] -4*x + tan(x) - 1/tan(x)
```

Giac [A] time = 1.05474, size = 16, normalized size = 1.6

$$-4\,x-\frac{1}{\tan\left(x\right)}+\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cot(x)+tan(x))^2,x, algorithm="giac")
```

```
[Out] -4*x - 1/tan(x) + tan(x)
```

3.40 $\int (-\sec(x)+\tan(x))^2\,dx$

Optimal. Leaf size=14

$$-x-\frac{2\cos(x)}{\sin(x)+1}$$

```
[Out] -x - (2*Cos[x])/(1 + Sin[x])
```

Rubi [A] time = 0.0631115, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4391, 2670, 2680, 8}

$$-x-\frac{2\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sec[x] + Tan[x])^2,x]

[Out] -x - (2*Cos[x])/(1 + Sin[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2670

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*m), Int[(g*Cos[e + f*x])^(2*m + p)/(a
 - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2,
 0] && IntegerQ[m] && LtQ[p, -1] && GeQ[2*m + p, 0]
```

Rule 2680

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(2*g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f
*x])^(m + 1))/(b*f*(2*m + p + 1)), x] + Dist[(g^2*(p - 1))/(b^2*(2*m + p +
1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /; F
reeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &&
 NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int (-\sec(x)+\tan(x))^2\,dx &= \int \sec^2(x)(-1+\sin(x))^2\,dx \\
&= \int \frac{\cos^2(x)}{(-1-\sin(x))^2}\,dx \\
&= -\frac{2\cos(x)}{1+\sin(x)} - \int 1\,dx \\
&= -x - \frac{2\cos(x)}{1+\sin(x)}
\end{aligned}
$$

Mathematica [A] time = 0.0067543, size = 12, normalized size = 0.86

$$-x+2\tan(x)-2\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sec[x] + Tan[x])^2,x]
```

```
[Out] -x - 2*Sec[x] + 2*Tan[x]
```

Maple [A] time = 0.013, size = 15, normalized size = 1.1

$$2\tan(x)-x-2(\cos(x))^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-sec(x)+tan(x))^2,x)
```

```
[Out] 2*tan(x)-x-2/cos(x)
```

Maxima [A] time = 1.47501, size = 19, normalized size = 1.36

$$-x-\frac{2}{\cos(x)}+2\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(x)+tan(x))^2,x, algorithm="maxima")
```

```
[Out] -x - 2/cos(x) + 2*tan(x)
```

Fricas [A] time = 2.04946, size = 89, normalized size = 6.36

$$-\frac{(x+2)\cos(x)+(x-2)\sin(x)+x+2}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(x)+tan(x))^2,x, algorithm="fricas")
```

```
[Out] -((x + 2)*cos(x) + (x - 2)*sin(x) + x + 2)/(cos(x) + sin(x) + 1)
```

Sympy [A] time = 1.11767, size = 10, normalized size = 0.71

$$-x + 2\tan(x) - 2\sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(x)+tan(x))**2,x)
```

```
[Out] -x + 2*tan(x) - 2*sec(x)
```

Giac [A] time = 1.06971, size = 19, normalized size = 1.36

$$-x - \frac{4}{\tan\left(\frac{1}{2}x\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-sec(x)+tan(x))^2,x, algorithm="giac")
```

```
[Out] -x - 4/(tan(1/2*x) + 1)
```

3.41 $\int \frac{\sin(x)}{1+\sin(x)}\,dx$

Optimal. Leaf size=11

$$x+\frac{\cos(x)}{\sin(x)+1}$$

```
[Out] x + Cos[x]/(1 + Sin[x])
```

Rubi [A] time = 0.0218378, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2735, 2648}

$$x+\frac{\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(1 + Sin[x]),x]

[Out] x + Cos[x]/(1 + Sin[x])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{1+\sin(x)}\,dx &= x-\int \frac{1}{1+\sin(x)}\,dx\\ &= x+\frac{\cos(x)}{1+\sin(x)}\end{aligned}$$

Mathematica [B] time = 0.0394707, size = 25, normalized size = 2.27

$$x-\frac{2\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(1 + Sin[x]),x]

[Out] x - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Maple [A] time = 0.011, size = 13, normalized size = 1.2

$$2\,(1+\tan(x/2))^{-1}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1+sin(x)),x)
```

```
[Out] 2/(1+tan(1/2*x))+x
```

Maxima [B] time = 1.44419, size = 38, normalized size = 3.45

$$\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}+2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)),x, algorithm="maxima")
```

```
[Out] 2/(sin(x)/(cos(x) + 1) + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [B] time = 1.96256, size = 88, normalized size = 8.

$$\frac{(x+1)\cos(x)+(x-1)\sin(x)+x+1}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)),x, algorithm="fricas")
```

```
[Out] ((x + 1)*cos(x) + (x - 1)*sin(x) + x + 1)/(cos(x) + sin(x) + 1)
```

Sympy [B] time = 0.53787, size = 34, normalized size = 3.09

$$\frac{x\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)+1}+\frac{x}{\tan\left(\frac{x}{2}\right)+1}-\frac{2\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)),x)
```

```
[Out] x*tan(x/2)/(tan(x/2) + 1) + x/(tan(x/2) + 1) - 2*tan(x/2)/(tan(x/2) + 1)
```

Giac [A] time = 1.06276, size = 16, normalized size = 1.45

$$x+\frac{2}{\tan\left(\frac{1}{2}\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)),x, algorithm="giac")
```

```
[Out] x + 2/(tan(1/2*x) + 1)
```

3.42 $\int \frac{\cos(x)}{1-\cos(x)} \, dx$

Optimal. Leaf size=16

$$-x-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -x - Sin[x]/(1 - Cos[x])
```

Rubi [A] time = 0.0263574, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2735, 2648}

$$-x-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(1 - Cos[x]),x]
```

```
[Out] -x - Sin[x]/(1 - Cos[x])
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{1-\cos(x)} \, dx &= -x+\int \frac{1}{1-\cos(x)} \, dx\\ &= -x-\frac{\sin(x)}{1-\cos(x)}\end{aligned}$$

Mathematica [A] time = 0.0231004, size = 21, normalized size = 1.31

$$\frac{2x\sin^2\left(\frac{x}{2}\right)+\sin(x)}{\cos(x)-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(1 - Cos[x]),x]
```

```
[Out] (2*x*Sin[x/2]^2 + Sin[x])/(-1 + Cos[x])
```

Maple [A] time = 0.01, size = 13, normalized size = 0.8

$$-\left(\tan\left(\frac{x}{2}\right)\right)^{-1}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(1-cos(x)),x)
```

```
[Out] -1/tan(1/2*x)-x
```

Maxima [A] time = 1.46981, size = 31, normalized size = 1.94

$$-\frac{\cos(x)+1}{\sin(x)}-2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(x)),x, algorithm="maxima")
```

```
[Out] -(cos(x) + 1)/sin(x) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 1.80118, size = 45, normalized size = 2.81

$$-\frac{x\sin(x)+\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] -(x*sin(x) + cos(x) + 1)/sin(x)
```

Sympy [A] time = 0.480344, size = 8, normalized size = 0.5

$$-x-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(x)),x)
```

```
[Out] -x - 1/tan(x/2)
```

Giac [A] time = 1.05088, size = 16, normalized size = 1.

$$-x-\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1-cos(x)),x, algorithm="giac")
```

```
[Out] -x - 1/tan(1/2*x)
```

3.43 $\int e^{-x/2} \left(-1+e^{x/2}\right)^3 \, dx$

Optimal. Leaf size=25

$$3x+2e^{-x/2}-6e^{x/2}+e^x$$

```
[Out] 2/E^(x/2) - 6*E^(x/2) + E^x + 3*x
```

Rubi [A] time = 0.0246283, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {2248, 43}

$$3x+2e^{-x/2}-6e^{x/2}+e^x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + E^(x/2))^3/E^(x/2),x]
```

```
[Out] 2/E^(x/2) - 6*E^(x/2) + E^x + 3*x
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{-x/2}\left(-1+e^{x/2}\right)^3 \, dx &= 2\,\text{Subst}\left(\int \frac{(-1+x)^3}{x^2} \, dx, x, e^{x/2}\right)\\
&= 2\,\text{Subst}\left(\int \left(-3-\frac{1}{x^2}+\frac{3}{x}+x\right) dx, x, e^{x/2}\right)\\
&= 2e^{-x/2}-6e^{x/2}+e^x+3x
\end{aligned}$$

Mathematica [A] time = 0.0130611, size = 25, normalized size = 1.

$$3x+2e^{-x/2}-6e^{x/2}+e^x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + E^(x/2))^3/E^(x/2),x]
```

```
[Out] 2/E^(x/2) - 6*E^(x/2) + E^x + 3*x
```

Maple [A] time = 0.005, size = 29, normalized size = 1.2

$$\left(e^{\frac{x}{2}}\right)^2 - 6\,e^{x/2} + 6\,\ln\left(e^{x/2}\right) + 2\,\left(e^{x/2}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+exp(1/2*x))^3/exp(1/2*x),x)
```

```
[Out] exp(1/2*x)^2-6*exp(1/2*x)+6*ln(exp(1/2*x))+2/exp(1/2*x)
```

Maxima [A] time = 0.955786, size = 24, normalized size = 0.96

$$3\,x - 6\,e^{\left(\frac{1}{2}\,x\right)} + 2\,e^{\left(-\frac{1}{2}\,x\right)} + e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(1/2*x))^3/exp(1/2*x),x, algorithm="maxima")
```

```
[Out] 3*x - 6*e^(1/2*x) + 2*e^(-1/2*x) + e^x
```

Fricas [A] time = 1.91503, size = 70, normalized size = 2.8

$$\left(3\,xe^{\left(\frac{1}{2}\,x\right)} + e^{\left(\frac{3}{2}\,x\right)} - 6\,e^{x} + 2\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(1/2*x))^3/exp(1/2*x),x, algorithm="fricas")
```

```
[Out] (3*x*e^(1/2*x) + e^(3/2*x) - 6*e^x + 2)*e^(-1/2*x)
```

Sympy [A] time = 0.098204, size = 19, normalized size = 0.76

$$3x - 6e^{\frac{x}{2}} + e^{x} + 2e^{-\frac{x}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(1/2*x))**3/exp(1/2*x),x)
```

```
[Out] 3*x - 6*exp(x/2) + exp(x) + 2*exp(-x/2)
```

Giac [A] time = 1.05295, size = 24, normalized size = 0.96

$$3\,x - 6\,e^{\left(\frac{1}{2}\,x\right)} + 2\,e^{\left(-\frac{1}{2}\,x\right)} + e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(1/2*x))^3/exp(1/2*x),x, algorithm="giac")
```

```
[Out] 3*x - 6*e^(1/2*x) + 2*e^(-1/2*x) + e^x
```

3.44 $\int \frac{1}{5-6x+x^2} dx$

Optimal. Leaf size=21

$$\frac{1}{4}\log(5-x)-\frac{1}{4}\log(1-x)$$

```
[Out] -Log[1 - x]/4 + Log[5 - x]/4
```

Rubi [A] time = 0.0041527, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {616, 31}

$$\frac{1}{4}\log(5-x)-\frac{1}{4}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 6*x + x^2)^(-1),x]
```

```
[Out] -Log[1 - x]/4 + Log[5 - x]/4
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{5-6x+x^2}\,dx &= \frac{1}{4}\int \frac{1}{-5+x}\,dx-\frac{1}{4}\int \frac{1}{-1+x}\,dx\\ &= -\frac{1}{4}\log(1-x)+\frac{1}{4}\log(5-x)\end{aligned}$$

Mathematica [A] time = 0.0028664, size = 21, normalized size = 1.

$$\frac{1}{4}\log(5-x)-\frac{1}{4}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 6*x + x^2)^(-1),x]
```

```
[Out] -Log[1 - x]/4 + Log[5 - x]/4
```

Maple [A] time = 0.004, size = 14, normalized size = 0.7

$$\frac{\ln\left(-5+x\right)}{4}-\frac{\ln\left(-1+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-6*x+5),x)
```

```
[Out] 1/4*ln(-5+x)-1/4*ln(-1+x)
```

Maxima [A] time = 0.967044, size = 18, normalized size = 0.86

$$-\frac{1}{4}\log\left(x-1\right)+\frac{1}{4}\log\left(x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-6*x+5),x, algorithm="maxima")
```

```
[Out] -1/4*log(x - 1) + 1/4*log(x - 5)
```

Fricas [A] time = 1.86827, size = 46, normalized size = 2.19

$$-\frac{1}{4}\log\left(x-1\right)+\frac{1}{4}\log\left(x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-6*x+5),x, algorithm="fricas")
```

```
[Out] -1/4*log(x - 1) + 1/4*log(x - 5)
```

Sympy [A] time = 0.089792, size = 12, normalized size = 0.57

$$\frac{\log\left(x-5\right)}{4}-\frac{\log\left(x-1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-6*x+5),x)
```

```
[Out] log(x - 5)/4 - log(x - 1)/4
```

Giac [A] time = 1.06522, size = 20, normalized size = 0.95

$$-\frac{1}{4}\log\left(\left|x-1\right|\right)+\frac{1}{4}\log\left(\left|x-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-6*x+5),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(x - 1)) + 1/4*log(abs(x - 5))
```

3.45 $\int \frac{x^2}{13-6x^3+x^6} \, dx$

Optimal. Leaf size=14

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{2}\left(x^3-3\right)\right)$$

```
[Out] ArcTan[(-3 + x^3)/2]/6
```

Rubi [A] time = 0.0159955, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 618, 204}

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{2}\left(x^3-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(13 - 6*x^3 + x^6),x]

[Out] ArcTan[(-3 + x^3)/2]/6
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{13-6x^3+x^6} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{13-6x+x^2} \, dx, x, x^3\right)\\
&= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{-16-x^2} \, dx, x, 2\left(-3+x^3\right)\right)\right)\\
&= \frac{1}{6}\tan^{-1}\left(\frac{1}{2}\left(-3+x^3\right)\right)
\end{aligned}$$

Mathematica [A] time = 0.0044996, size = 14, normalized size = 1.

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{2}\left(x^3-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(13 - 6*x^3 + x^6),x]
```

```
[Out] ArcTan[(-3 + x^3)/2]/6
```

Maple [A] time = 0.003, size = 11, normalized size = 0.8

$$\frac{1}{6}\arctan\left(\frac{x^3}{2}-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6-6*x^3+13),x)
```

```
[Out] 1/6*arctan(1/2*x^3-3/2)
```

Maxima [A] time = 1.43972, size = 14, normalized size = 1.

$$\frac{1}{6}\arctan\left(\frac{1}{2}x^3-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-6*x^3+13),x, algorithm="maxima")
```

```
[Out] 1/6*arctan(1/2*x^3 - 3/2)
```

Fricas [A] time = 1.84042, size = 36, normalized size = 2.57

$$\frac{1}{6}\arctan\left(\frac{1}{2}x^3-\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-6*x^3+13),x, algorithm="fricas")
```

```
[Out] 1/6*arctan(1/2*x^3 - 3/2)
```

Sympy [A] time = 0.105266, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^3}{2}-\frac{3}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6-6*x**3+13),x)
```

```
[Out] atan(x**3/2 - 3/2)/6
```

Giac [A] time = 1.07075, size = 14, normalized size = 1.

$$\frac{1}{6} \arctan\left(\frac{1}{2} x^3 - \frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-6*x^3+13),x, algorithm="giac")
```

```
[Out] 1/6*arctan(1/2*x^3 - 3/2)
```

3.46 $\int \frac{2+x}{-1-4x+x^2}\,dx$

Optimal. Leaf size=51

$$\frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(-x-\sqrt{5}+2\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(-x+\sqrt{5}+2\right)$$

```
[Out] ((5 - 4*Sqrt[5])*Log[2 - Sqrt[5] - x])/10 + ((5 + 4*Sqrt[5])*Log[2 + Sqrt[5
] - x])/10
```

Rubi [A] time = 0.0158961, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {632, 31}

$$\frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(-x-\sqrt{5}+2\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(-x+\sqrt{5}+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(-1 - 4*x + x^2),x]
```

```
[Out] ((5 - 4*Sqrt[5])*Log[2 - Sqrt[5] - x])/10 + ((5 + 4*Sqrt[5])*Log[2 + Sqrt[5
] - x])/10
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{-1-4x+x^2}\,dx &= -\left(\frac{1}{10}\left(-5+4\sqrt{5}\right)\int \frac{1}{-2+\sqrt{5}+x}\,dx\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\int \frac{1}{-2-\sqrt{5}+x}\,dx\\ &= \frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(2-\sqrt{5}-x\right)+\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(2+\sqrt{5}-x\right)\end{aligned}$$

Mathematica [A] time = 0.0256704, size = 47, normalized size = 0.92

$$\frac{1}{10}\left(5+4\sqrt{5}\right)\log\left(-x+\sqrt{5}+2\right)+\frac{1}{10}\left(5-4\sqrt{5}\right)\log\left(x+\sqrt{5}-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(-1 - 4*x + x^2),x]
```

```
[Out] ((5 + 4*Sqrt[5])*Log[2 + Sqrt[5] - x])/10 + ((5 - 4*Sqrt[5])*Log[-2 + Sqrt[
5] + x])/10
```

Maple [A] time = 0.002, size = 29, normalized size = 0.6

$$\frac{\ln\left(x^2-4x-1\right)}{2}-\frac{4\sqrt{5}}{5}\mathrm{Artanh}\left(\frac{(2x-4)\sqrt{5}}{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(x^2-4*x-1),x)
```

```
[Out] 1/2*ln(x^2-4*x-1)-4/5*5^(1/2)*arctanh(1/10*(2*x-4)*5^(1/2))
```

Maxima [A] time = 1.48026, size = 47, normalized size = 0.92

$$\frac{2}{5}\sqrt{5}\log\left(\frac{x-\sqrt{5}-2}{x+\sqrt{5}-2}\right)+\frac{1}{2}\log\left(x^2-4x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2-4*x-1),x, algorithm="maxima")
```

```
[Out] 2/5*sqrt(5)*log((x - sqrt(5) - 2)/(x + sqrt(5) - 2)) + 1/2*log(x^2 - 4*x -
1)
```

Fricas [A] time = 1.81323, size = 128, normalized size = 2.51

$$\frac{2}{5}\sqrt{5}\log\left(\frac{x^2-2\sqrt{5}(x-2)-4x+9}{x^2-4x-1}\right)+\frac{1}{2}\log\left(x^2-4x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2-4*x-1),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*log((x^2 - 2*sqrt(5)*(x - 2) - 4*x + 9)/(x^2 - 4*x - 1)) + 1/2*
log(x^2 - 4*x - 1)
```

Sympy [A] time = 0.104519, size = 42, normalized size = 0.82

$$\left(\frac{1}{2}-\frac{2\sqrt{5}}{5}\right)\log\left(x-2+\sqrt{5}\right)+\left(\frac{1}{2}+\frac{2\sqrt{5}}{5}\right)\log\left(x-\sqrt{5}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2-4*x-1),x)
```

```
[Out] (1/2 - 2*sqrt(5)/5)*log(x - 2 + sqrt(5)) + (1/2 + 2*sqrt(5)/5)*log(x - sqrt
(5) - 2)
```

Giac [A] time = 1.04397, size = 59, normalized size = 1.16

$$\frac{2}{5}\sqrt{5}\log\left(\frac{\left|2x - 2\sqrt{5} - 4\right|}{\left|2x + 2\sqrt{5} - 4\right|}\right) + \frac{1}{2}\log\left(\left|x^2 - 4x - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2-4*x-1),x, algorithm="giac")
```

```
[Out] 2/5*sqrt(5)*log(abs(2*x - 2*sqrt(5) - 4)/abs(2*x + 2*sqrt(5) - 4)) + 1/2*lo
g(abs(x^2 - 4*x - 1))
```

3.47 $\int \frac{1}{1+\sqrt[3]{1+x}}\,dx$

Optimal. Leaf size=33

$$\frac{3}{2}(x+1)^{2/3}-3\sqrt[3]{x+1}+3\log\left(\sqrt[3]{x+1}+1\right)$$

```
[Out] -3*(1 + x)^(1/3) + (3*(1 + x)^(2/3))/2 + 3*Log[1 + (1 + x)^(1/3)]
```

Rubi [A] time = 0.0101715, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {247, 190, 43}

$$\frac{3}{2}(x+1)^{2/3}-3\sqrt[3]{x+1}+3\log\left(\sqrt[3]{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + (1 + x)^(1/3))^(-1),x]
```

```
[Out] -3*(1 + x)^(1/3) + (3*(1 + x)^(2/3))/2 + 3*Log[1 + (1 + x)^(1/3)]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\sqrt[3]{1+x}}\,dx &= \text{Subst}\left(\int \frac{1}{1+\sqrt[3]{x}}\,dx,x,1+x\right)\\ &= 3\,\text{Subst}\left(\int \frac{x^2}{1+x}\,dx,x,\sqrt[3]{1+x}\right)\\ &= 3\,\text{Subst}\left(\int \left(-1+x+\frac{1}{1+x}\right)dx,x,\sqrt[3]{1+x}\right)\\ &= -3\sqrt[3]{1+x}+\frac{3}{2}(1+x)^{2/3}+3\log\left(1+\sqrt[3]{1+x}\right)\end{aligned}$$

Mathematica [A] time = 0.0120075, size = 33, normalized size = 1.

$$\frac{3}{2}(x+1)^{2/3}-3\sqrt[3]{x+1}+3\log\left(\sqrt[3]{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (1 + x)^(1/3))^(-1),x]
```

```
[Out] -3*(1 + x)^(1/3) + (3*(1 + x)^(2/3))/2 + 3*Log[1 + (1 + x)^(1/3)]
```

Maple [A] time = 0.008, size = 47, normalized size = 1.4

$$\ln(2+x)+\frac{3}{2}(1+x)^{\frac{2}{3}}+2\ln\left(1+\sqrt[3]{1+x}\right)-\ln\left((1+x)^{\frac{2}{3}}-\sqrt[3]{1+x}+1\right)-3\sqrt[3]{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(1+x)^(1/3)),x)
```

```
[Out] ln(2+x)+3/2*(1+x)^(2/3)+2*ln(1+(1+x)^(1/3))-ln((1+x)^(2/3)-(1+x)^(1/3)+1)-3
*(1+x)^(1/3)
```

Maxima [A] time = 0.955634, size = 34, normalized size = 1.03

$$\frac{3}{2}(x+1)^{\frac{2}{3}}-3(x+1)^{\frac{1}{3}}+3\log\left((x+1)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+x)^(1/3)),x, algorithm="maxima")
```

```
[Out] 3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*log((x + 1)^(1/3) + 1)
```

Fricas [A] time = 1.87864, size = 86, normalized size = 2.61

$$\frac{3}{2}(x+1)^{\frac{2}{3}}-3(x+1)^{\frac{1}{3}}+3\log\left((x+1)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+x)^(1/3)),x, algorithm="fricas")
```

```
[Out] 3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*log((x + 1)^(1/3) + 1)
```

Sympy [A] time = 0.115146, size = 29, normalized size = 0.88

$$\frac{3(x+1)^{\frac{2}{3}}}{2}-3\sqrt[3]{x+1}+3\log\left(\sqrt[3]{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+x)**(1/3)),x)
```

```
[Out] 3*(x + 1)**(2/3)/2 - 3*(x + 1)**(1/3) + 3*log((x + 1)**(1/3) + 1)
```

Giac [A] time = 1.04661, size = 34, normalized size = 1.03

$$\frac{3}{2}\,(x+1)^{\frac{2}{3}} - 3\,(x+1)^{\frac{1}{3}} + 3\,\log\left((x+1)^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+x)^(1/3)),x, algorithm="giac")
```

```
[Out] 3/2*(x + 1)^(2/3) - 3*(x + 1)^(1/3) + 3*log((x + 1)^(1/3) + 1)
```

3.48 $\int \frac{1}{\sqrt{x}(b+ax)}\,dx$

Optimal. Leaf size=29

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.013797, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(b + a*x)),x]

[Out] (2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x}(b+ax)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, \sqrt{x}\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.0074179, size = 29, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(b + a*x)),x]
```

```
[Out] (2*ArcTan[(Sqrt[a]*Sqrt[x])/Sqrt[b]])/(Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.008, size = 19, normalized size = 0.7

$$2\frac{1}{\sqrt{ab}}\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x+b)/x^(1/2),x)
```

```
[Out] 2/(a*b)^(1/2)*arctan(a*x^(1/2)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b)/x^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.01815, size = 163, normalized size = 5.62

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{ax-b-2\sqrt{-ab}\sqrt{x}}{ax+b}\right)}{ab}, -\frac{2\sqrt{ab}\arctan\left(\frac{\sqrt{ab}}{a\sqrt{x}}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b)/x^(1/2),x, algorithm="fricas")
```

```
[Out] [-sqrt(-a*b)*log((a*x - b - 2*sqrt(-a*b)*sqrt(x))/(a*x + b))/(a*b), -2*sqrt
(a*b)*arctan(sqrt(a*b)/(a*sqrt(x)))/(a*b)]
```

Sympy [A] time = 1.58298, size = 94, normalized size = 3.24

$$\begin{cases} \tilde{\infty}\sqrt{x} & \text{for } a = 0 \wedge b = 0 \\ -\frac{2}{a\sqrt{x}} & \text{for } b = 0 \\ \frac{2\sqrt{x}}{b} & \text{for } a = 0 \\ -\frac{i\log\left(-i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{a\sqrt{b}\sqrt{\frac{1}{a}}} + \frac{i\log\left(i\sqrt{b}\sqrt{\frac{1}{a}}+\sqrt{x}\right)}{a\sqrt{b}\sqrt{\frac{1}{a}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b)/x**(1/2),x)
```

```
[Out] Piecewise((zoo*sqrt(x), Eq(a, 0) & Eq(b, 0)), (-2/(a*sqrt(x)), Eq(b, 0)), (
2*sqrt(x)/b, Eq(a, 0)), (-I*log(-I*sqrt(b)*sqrt(1/a) + sqrt(x))/(a*sqrt(b)*
sqrt(1/a)) + I*log(I*sqrt(b)*sqrt(1/a) + sqrt(x))/(a*sqrt(b)*sqrt(1/a)), Tr
ue))
```

Giac [A] time = 1.05216, size = 24, normalized size = 0.83

$$\frac{2\arctan\left(\frac{a\sqrt{x}}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x+b)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(a*sqrt(x)/sqrt(a*b))/sqrt(a*b)
```

3.49 $\int x^3\sqrt{1+x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{5}\left(x^2+1\right)^{5/2}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] -(1 + x^2)^(3/2)/3 + (1 + x^2)^(5/2)/5
```

Rubi [A] time = 0.0101029, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{5}\left(x^2+1\right)^{5/2}-\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[1 + x^2],x]
```

```
[Out] -(1 + x^2)^(3/2)/3 + (1 + x^2)^(5/2)/5
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{1+x^2}\,dx&=\frac{1}{2}\operatorname{Subst}\left(\int x\sqrt{1+x}\,dx,x,x^2\right)\\&=\frac{1}{2}\operatorname{Subst}\left(\int\left(-\sqrt{1+x}+(1+x)^{3/2}\right)dx,x,x^2\right)\\&=-\frac{1}{3}\left(1+x^2\right)^{3/2}+\frac{1}{5}\left(1+x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0059802, size = 20, normalized size = 0.74

$$\frac{1}{15}\left(x^2+1\right)^{3/2}\left(3x^2-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[1 + x^2],x]
```

```
[Out] ((1 + x^2)^(3/2)*(-2 + 3*x^2))/15
```

Maple [A] time = 0.004, size = 17, normalized size = 0.6

$$\frac{3x^2-2}{15}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+1)^(1/2),x)
```

```
[Out] 1/15*(x^2+1)^(3/2)*(3*x^2-2)
```

Maxima [A] time = 1.475, size = 30, normalized size = 1.11

$$\frac{1}{5}\left(x^2+1\right)^{\frac{3}{2}}x^2-\frac{2}{15}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(x^2 + 1)^(3/2)*x^2 - 2/15*(x^2 + 1)^(3/2)
```

Fricas [A] time = 1.98449, size = 51, normalized size = 1.89

$$\frac{1}{15}\left(3x^4+x^2-2\right)\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^4 + x^2 - 2)*sqrt(x^2 + 1)
```

Sympy [A] time = 0.663862, size = 37, normalized size = 1.37

$$\frac{x^4\sqrt{x^2+1}}{5}+\frac{x^2\sqrt{x^2+1}}{15}-\frac{2\sqrt{x^2+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+1)**(1/2),x)
```

```
[Out] x**4*sqrt(x**2 + 1)/5 + x**2*sqrt(x**2 + 1)/15 - 2*sqrt(x**2 + 1)/15
```

Giac [A] time = 1.0514, size = 26, normalized size = 0.96

$$\frac{1}{5}\left(x^2+1\right)^{\frac{5}{2}}-\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*(x^2 + 1)^(5/2) - 1/3*(x^2 + 1)^(3/2)
```

3.50 $\int \frac{x}{\sqrt{a^4-x^4}}\,dx$

Optimal. Leaf size=22

$$\frac{1}{2}\tan^{-1}\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)$$

```
[Out] ArcTan[x^2/Sqrt[a^4 - x^4]]/2
```

Rubi [A] time = 0.0092697, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {275, 217, 203}

$$\frac{1}{2}\tan^{-1}\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a^4 - x^4],x]
```

```
[Out] ArcTan[x^2/Sqrt[a^4 - x^4]]/2
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a^4-x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a^4-x^2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\frac{x^2}{\sqrt{a^4-x^4}}\right)\\ &= \frac{1}{2}\tan^{-1}\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)\end{aligned}$$

Mathematica [A] time = 0.0034013, size = 22, normalized size = 1.

$$\frac{1}{2}\tan^{-1}\left(\frac{x^2}{\sqrt{a^4-x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a^4 - x^4],x]
```

```
[Out] ArcTan[x^2/Sqrt[a^4 - x^4]]/2
```

Maple [A] time = 0.01, size = 19, normalized size = 0.9

$$\frac{1}{2}\arctan\left(x^{2}\frac{1}{\sqrt{a^{4}-x^{4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^4-x^4)^(1/2),x)
```

```
[Out] 1/2*arctan(x^2/(a^4-x^4)^(1/2))
```

Maxima [A] time = 1.48826, size = 24, normalized size = 1.09

$$-\frac{1}{2}\arctan\left(\frac{\sqrt{a^{4}-x^{4}}}{x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4-x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(sqrt(a^4 - x^4)/x^2)
```

Fricas [A] time = 1.76094, size = 53, normalized size = 2.41

$$-\arctan\left(-\frac{a^{2}-\sqrt{a^{4}-x^{4}}}{x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4-x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(-(a^2 - sqrt(a^4 - x^4))/x^2)
```

Sympy [A] time = 1.06362, size = 31, normalized size = 1.41

$$\begin{cases} -\frac{i\operatorname{acosh}\left(\frac{x^{2}}{a^{2}}\right)}{2} & \text{for}\ \frac{|x^{4}|}{|a^{4}|}>1 \\ \frac{\operatorname{asin}\left(\frac{x^{2}}{a^{2}}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**4-x**4)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(x**2/a**2)/2, Abs(x**4)/Abs(a**4) > 1), (asin(x**2/a**2
)/2, True))
```

Giac [A] time = 1.07695, size = 14, normalized size = 0.64

$$\frac{1}{2}\arcsin\left(\frac{x^2}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4-x^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arcsin(x^2/a^2)
```

3.51 $\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx$

Optimal. Leaf size=22

$$\frac{\tan^{-1}\left(\frac{\sqrt{x^2-a^2}}{a}\right)}{a}$$

```
[Out] ArcTan[Sqrt[-a^2 + x^2]/a]/a
```

Rubi [A] time = 0.0125473, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {266, 63, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{x^2-a^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a^2 + x^2]),x]

[Out] ArcTan[Sqrt[-a^2 + x^2]/a]/a
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{-a^2+x}}\,dx, x, x^2\right)\\
&= \text{Subst}\left(\int \frac{1}{a^2+x^2}\,dx, x, \sqrt{-a^2+x^2}\right)\\
&= \frac{\tan^{-1}\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.003801, size = 22, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{x^2-a^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a^2 + x^2]),x]
```

```
[Out] ArcTan[Sqrt[-a^2 + x^2]/a]/a
```

Maple [A] time = 0.003, size = 41, normalized size = 1.9

$$-\ln\left(\frac{1}{x}\left(-2\,a^2+2\,\sqrt{-a^2}\sqrt{-a^2+x^2}\right)\right)\frac{1}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2+x^2)^(1/2),x)
```

```
[Out] -1/(-a^2)^(1/2)*ln((-2*a^2+2*(-a^2)^(1/2)*(-a^2+x^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.96431, size = 53, normalized size = 2.41

$$\frac{2\arctan\left(-\frac{x-\sqrt{-a^2+x^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(-(x - sqrt(-a^2 + x^2))/a)/a
```

Sympy [A] time = 1.08734, size = 24, normalized size = 1.09

$$\begin{cases} \frac{i\operatorname{acosh}\left(\frac{a}{x}\right)}{a} & \text{for}\ \frac{|a^2|}{|x^2|}>1 \\ -\frac{\operatorname{asin}\left(\frac{a}{x}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2+x**2)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(a/x)/a, Abs(a**2)/Abs(x**2) > 1), (-asin(a/x)/a, True))
```

Giac [A] time = 1.04928, size = 27, normalized size = 1.23

$$\frac{\arctan\left(\frac{\sqrt{-a^2+x^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(-a^2 + x^2)/a)/a
```

3.52 $\int \frac{1}{x\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

```
[Out] -(ArcTanh[Sqrt[a^2 - x^2]/a]/a)
```

Rubi [A] time = 0.0141744, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {266, 63, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 - x^2]),x]

[Out] -(ArcTanh[Sqrt[a^2 - x^2]/a]/a)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a^2-x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{a^2-xx}}\,dx,x,x^2\right)\\
&= -\,\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx,x,\sqrt{a^2-x^2}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0038571, size = 23, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-x^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 - x^2]),x]

[Out] -(ArcTanh[Sqrt[a^2 - x^2]/a]/a)
```

Maple [A] time = 0.005, size = 37, normalized size = 1.6

$$-\ln\left(\frac{1}{x}\left(2a^2+2\sqrt{a^2}\sqrt{a^2-x^2}\right)\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2-x^2)^(1/2),x)

[Out] -1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2-x^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2-x^2)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88047, size = 45, normalized size = 1.96

$$\frac{\log\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2-x^2)^(1/2),x, algorithm="fricas")

[Out] log(-(a - sqrt(a^2 - x^2))/x)/a
```

Sympy [A] time = 1.07572, size = 24, normalized size = 1.04

$$\begin{cases} -\frac{\operatorname{acosh}\left(\frac{a}{x}\right)}{a} & \text{for}\ \frac{|a^2|}{|x^2|} > 1 \\ \frac{i\operatorname{asin}\left(\frac{a}{x}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2-x**2)**(1/2),x)
```

```
[Out] Piecewise((-acosh(a/x)/a, Abs(a**2)/Abs(x**2) > 1), (I*asin(a/x)/a, True))
```

Giac [B] time = 1.07154, size = 58, normalized size = 2.52

$$-\frac{\log\left(\left|a+\sqrt{a^2-x^2}\right|\right)}{2\,a}+\frac{\log\left(\left|-a+\sqrt{a^2-x^2}\right|\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(a + sqrt(a^2 - x^2)))/a + 1/2*log(abs(-a + sqrt(a^2 - x^2)))/a
```

3.53 $\int \frac{1}{x\sqrt{a^2+x^2}}\,dx$

Optimal. Leaf size=21

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}$$

```
[Out] -(ArcTanh[Sqrt[a^2 + x^2]/a]/a)
```

Rubi [A] time = 0.0127425, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {266, 63, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + x^2]),x]

[Out] -(ArcTanh[Sqrt[a^2 + x^2]/a]/a)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a^2+x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{a^2+x}}\,dx, x, x^2\right)\\
&= \operatorname{Subst}\left(\int \frac{1}{-a^2+x^2}\,dx, x, \sqrt{a^2+x^2}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0032219, size = 21, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+x^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + x^2]),x]

[Out] -(ArcTanh[Sqrt[a^2 + x^2]/a]/a)
```

Maple [A] time = 0.004, size = 35, normalized size = 1.7

$$-\ln\left(\frac{1}{x}\left(2a^2+2\sqrt{a^2}\sqrt{a^2+x^2}\right)\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2+x^2)^(1/2),x)

[Out] -1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2+x^2)^(1/2))/x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.86862, size = 90, normalized size = 4.29

$$-\frac{\log\left(a-x+\sqrt{a^2+x^2}\right)-\log\left(-a-x+\sqrt{a^2+x^2}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2),x, algorithm="fricas")

[Out] -(log(a - x + sqrt(a^2 + x^2)) - log(-a - x + sqrt(a^2 + x^2)))/a
```

Sympy [A] time = 1.01594, size = 7, normalized size = 0.33

$$-\frac{\operatorname{asinh}\left(\frac{a}{x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+x**2)**(1/2),x)
```

```
[Out] -asinh(a/x)/a
```

Giac [A] time = 1.06523, size = 50, normalized size = 2.38

$$-\frac{\log\left(a+\sqrt{a^2+x^2}\right)}{2\,a}+\frac{\log\left(-a+\sqrt{a^2+x^2}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(a + sqrt(a^2 + x^2))/a + 1/2*log(-a + sqrt(a^2 + x^2))/a
```

3.54 $\int \frac{1}{\sqrt{2+x-x^2}} dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

```
[Out] -ArcSin[(1 - 2*x)/3]
```

Rubi [A] time = 0.0061394, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {619, 216}

$$-\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + x - x^2],x]
```

```
[Out] -ArcSin[(1 - 2*x)/3]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{2+x-x^2}}\,dx &= -\left(\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx, x, 1-2x\right)\right)\\ &= -\sin^{-1}\left(\frac{1}{3}(1-2x)\right)\end{aligned}$$

Mathematica [A] time = 0.0051869, size = 12, normalized size = 1.

$$-\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[2 + x - x^2],x]
```

```
[Out] -ArcSin[(1 - 2*x)/3]
```

Maple [A] time = 0.004, size = 7, normalized size = 0.6

$$\arcsin\left(-\frac{1}{3}+\frac{2x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+x+2)^(1/2),x)
```

```
[Out] arcsin(-1/3+2/3*x)
```

Maxima [A] time = 1.43113, size = 11, normalized size = 0.92

$$-\arcsin\left(-\frac{2}{3}x+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-2/3*x + 1/3)
```

Fricas [B] time = 1.78112, size = 77, normalized size = 6.42

$$-\arctan\left(\frac{\sqrt{-x^2+x+2}(2x-1)}{2\left(x^2-x-2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*sqrt(-x^2 + x + 2)*(2*x - 1)/(x^2 - x - 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-x^2+x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+x+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 + x + 2), x)
```

Giac [A] time = 1.06304, size = 8, normalized size = 0.67

$$\arcsin\left(\frac{2}{3}x-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+x+2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(2/3*x - 1/3)
```

3.55 $\int \frac{1}{\sqrt{5-4x+3x^2}} \, dx$

Optimal. Leaf size=19

$$-\frac{\sinh^{-1}\left(\frac{2-3x}{\sqrt{11}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcSinh[(2 - 3*x)/Sqrt[11]]/Sqrt[3])
```

Rubi [A] time = 0.0119958, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 215}

$$-\frac{\sinh^{-1}\left(\frac{2-3x}{\sqrt{11}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[5 - 4*x + 3*x^2],x]

[Out] -(ArcSinh[(2 - 3*x)/Sqrt[11]]/Sqrt[3])
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{5-4x+3x^2}} \, dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{44}}} \, dx, x, -4+6x\right)}{2\sqrt{33}} \\ &= -\frac{\sinh^{-1}\left(\frac{2-3x}{\sqrt{11}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.0082043, size = 18, normalized size = 0.95

$$\frac{\sinh^{-1}\left(\frac{3x-2}{\sqrt{11}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[5 - 4*x + 3*x^2],x]
```

```
[Out] ArcSinh[(-2 + 3*x)/Sqrt[11]]/Sqrt[3]
```

Maple [A] time = 0.003, size = 15, normalized size = 0.8

$$\frac{\sqrt{3}}{3}\text{Arcsinh}\left(\frac{3\sqrt{11}}{11}\left(x-\frac{2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-4*x+5)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*arcsinh(3/11*11^(1/2)*(x-2/3))
```

Maxima [A] time = 1.55264, size = 22, normalized size = 1.16

$$\frac{1}{3}\sqrt{3}\,\text{arsinh}\left(\frac{1}{11}\sqrt{11}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsinh(1/11*sqrt(11)*(3*x - 2))
```

Fricas [B] time = 2.03638, size = 111, normalized size = 5.84

$$\frac{1}{6}\sqrt{3}\log\left(-2\sqrt{3}\sqrt{3x^2-4x+5}(3x-2)-18x^2+24x-19\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-2*sqrt(3)*sqrt(3*x^2 - 4*x + 5)*(3*x - 2) - 18*x^2 + 24*x
- 19)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^2-4x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-4*x+5)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**2 - 4*x + 5), x)
```

Giac [B] time = 1.07872, size = 45, normalized size = 2.37

$$-\frac{1}{3}\sqrt{3}\log\left(-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-4x+5}\right)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-4*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*log(-sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 - 4*x + 5)) + 2)
```

3.56 $\int \frac{1}{\sqrt{x-x^2}} \, dx$

Optimal. Leaf size=8

$$-\sin^{-1}(1-2x)$$

```
[Out] -ArcSin[1 - 2*x]
```

Rubi [A] time = 0.0031741, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {619, 216}

$$-\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x - x^2],x]

[Out] -ArcSin[1 - 2*x]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{x-x^2}} \, dx &= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} \, dx, x, 1-2x\right)\\ &= -\sin^{-1}(1-2x)\end{aligned}$$

Mathematica [A] time = 0.0087655, size = 12, normalized size = 1.5

$$-2\sin^{-1}\left(\sqrt{1-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x - x^2],x]

[Out] -2*ArcSin[Sqrt[1 - x]]
```

Maple [A] time = 0.003, size = 7, normalized size = 0.9

$$\arcsin(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+x)^(1/2),x)
```

```
[Out] arcsin(2*x-1)
```

Maxima [A] time = 1.43094, size = 8, normalized size = 1.

$$\arcsin(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(2*x - 1)
```

Fricas [B] time = 1.80721, size = 39, normalized size = 4.88

$$-2\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-x^2 + x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 + x), x)
```

Giac [A] time = 1.06003, size = 8, normalized size = 1.

$$\arcsin(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(2*x - 1)
```

3.57 $\int \frac{1+2x}{\sqrt{2+x-x^2}} \, dx$

Optimal. Leaf size=27

$$-2\sqrt{-x^2+x+2}-2\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

```
[Out] -2*Sqrt[2 + x - x^2] - 2*ArcSin[(1 - 2*x)/3]
```

Rubi [A] time = 0.0099554, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {640, 619, 216}

$$-2\sqrt{-x^2+x+2}-2\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/Sqrt[2 + x - x^2],x]
```

```
[Out] -2*Sqrt[2 + x - x^2] - 2*ArcSin[(1 - 2*x)/3]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+2x}{\sqrt{2+x-x^2}} \, dx &= -2\sqrt{2+x-x^2}+2\int \frac{1}{\sqrt{2+x-x^2}} \, dx \\
&= -2\sqrt{2+x-x^2}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}} \, dx,x,1-2x\right) \\
&= -2\sqrt{2+x-x^2}-2\sin^{-1}\left(\frac{1}{3}(1-2x)\right)
\end{aligned}$$

Mathematica [A] time = 0.0063088, size = 27, normalized size = 1.

$$-2\sqrt{-x^2+x+2}-2\sin^{-1}\left(\frac{1}{3}(1-2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/Sqrt[2 + x - x^2],x]
```

```
[Out] -2*Sqrt[2 + x - x^2] - 2*ArcSin[(1 - 2*x)/3]
```

Maple [A] time = 0.007, size = 22, normalized size = 0.8

$$2\arcsin\left(-1/3+2/3x\right)-2\sqrt{-x^2+x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(-x^2+x+2)^(1/2),x)
```

```
[Out] 2*arcsin(-1/3+2/3*x)-2*(-x^2+x+2)^(1/2)
```

Maxima [A] time = 1.42728, size = 28, normalized size = 1.04

$$-2\sqrt{-x^2+x+2}-2\arcsin\left(-\frac{2}{3}x+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(-x^2+x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-x^2 + x + 2) - 2*arcsin(-2/3*x + 1/3)
```

Fricas [B] time = 1.91765, size = 111, normalized size = 4.11

$$-2\sqrt{-x^2+x+2}-2\arctan\left(\frac{\sqrt{-x^2+x+2}(2x-1)}{2\left(x^2-x-2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(-x^2+x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(-x^2 + x + 2) - 2*arctan(1/2*sqrt(-x^2 + x + 2)*(2*x - 1)/(x^2 - x
 - 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{2x+1}{\sqrt{-(x-2)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(-x**2+x+2)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/sqrt(-(x - 2)*(x + 1)), x)
```

Giac [A] time = 1.07244, size = 28, normalized size = 1.04

$$-2\sqrt{-x^2 + x + 2} + 2 \arcsin\left(\frac{2}{3}x - \frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(-x^2+x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*sqrt(-x^2 + x + 2) + 2*arcsin(2/3*x - 1/3)
```

3.58 $\int \frac{1}{x\sqrt{2+x-x^2}}\,dx$

Optimal. Leaf size=32

$$-\frac{\tanh^{-1}\left(\frac{x+4}{2\sqrt{2}\sqrt{-x^2+x+2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTanh[(4 + x)/(2*Sqrt[2]*Sqrt[2 + x - x^2])]/Sqrt[2])
```

Rubi [A] time = 0.0105747, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {724, 206}

$$-\frac{\tanh^{-1}\left(\frac{x+4}{2\sqrt{2}\sqrt{-x^2+x+2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[2 + x - x^2]),x]
```

```
[Out] -(ArcTanh[(4 + x)/(2*Sqrt[2]*Sqrt[2 + x - x^2])]/Sqrt[2])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{2+x-x^2}}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{8-x^2}\,dx, x, \frac{4+x}{\sqrt{2+x-x^2}}\right)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{4+x}{2\sqrt{2}\sqrt{2+x-x^2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0055365, size = 32, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{x+4}{2\sqrt{2}\sqrt{-x^2+x+2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[2 + x - x^2]),x]
```

```
[Out] -(ArcTanh[(4 + x)/(2*Sqrt[2]*Sqrt[2 + x - x^2])]/Sqrt[2])
```

Maple [A] time = 0.003, size = 25, normalized size = 0.8

$$-\frac{\sqrt{2}}{2}\operatorname{Artanh}\left(\frac{(4+x)\sqrt{2}}{4}\frac{1}{\sqrt{-x^2+x+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^2+x+2)^(1/2),x)
```

```
[Out] -1/2*arctanh(1/4*(4+x)*2^(1/2)/(-x^2+x+2)^(1/2))*2^(1/2)
```

Maxima [A] time = 1.44071, size = 45, normalized size = 1.41

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{-x^2+x+2}}{|x|}+\frac{4}{|x|}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(2*sqrt(2)*sqrt(-x^2 + x + 2)/abs(x) + 4/abs(x) + 1)
```

Fricas [A] time = 1.9215, size = 111, normalized size = 3.47

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{4\sqrt{2}\sqrt{-x^2+x+2}(x+4)+7x^2-16x-32}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(4*sqrt(2)*sqrt(-x^2 + x + 2)*(x + 4) + 7*x^2 - 16*x - 32)
/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-(x-2)(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**2+x+2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(x - 2)*(x + 1))), x)
```

Giac [B] time = 1.10022, size = 96, normalized size = 3.

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(2\sqrt{-x^2+x+2}-3\right)}{2x-1}-6\right|}{\left|4\sqrt{2}+\frac{2\left(2\sqrt{-x^2+x+2}-3\right)}{2x-1}-6\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^2+x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*(2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) -
6)/abs(4*sqrt(2) + 2*(2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) - 6))
```

3.59 $\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx$

Optimal. Leaf size=21

$$\frac{2\sqrt{-x^2+x+2}}{3(x-2)}$$

```
[Out] (2*Sqrt[2 + x - x^2])/(3*(-2 + x))
```

Rubi [A] time = 0.0054853, antiderivative size = 23, normalized size of antiderivative = 1.1, number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {650}

$$-\frac{2\sqrt{-x^2+x+2}}{3(2-x)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + x)*Sqrt[2 + x - x^2]),x]
```

```
[Out] (-2*Sqrt[2 + x - x^2])/(3*(2 - x))
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d - b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{1}{(-2+x)\sqrt{2+x-x^2}}\,dx = -\frac{2\sqrt{2+x-x^2}}{3(2-x)}$$

Mathematica [A] time = 0.0042338, size = 21, normalized size = 1.

$$-\frac{2\sqrt{-x^2+x+2}}{6-3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + x)*Sqrt[2 + x - x^2]),x]
```

```
[Out] (-2*Sqrt[2 + x - x^2])/(6 - 3*x)
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$-\frac{2x+2}{3}\frac{1}{\sqrt{-x^2+x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2+x)/(-x^2+x+2)^(1/2),x)
```

```
[Out] -2/3*(1+x)/(-x^2+x+2)^(1/2)
```

Maxima [A] time = 1.41472, size = 23, normalized size = 1.1

$$\frac{2\sqrt{-x^2+x+2}}{3(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(-x^2 + x + 2)/(x - 2)
```

Fricas [A] time = 1.74288, size = 43, normalized size = 2.05

$$\frac{2\sqrt{-x^2+x+2}}{3(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-x^2 + x + 2)/(x - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-2)(x+1)}\,(x-2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(-x**2+x+2)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(x - 2)*(x + 1))*(x - 2)), x)
```

Giac [A] time = 1.05579, size = 38, normalized size = 1.81

$$-\frac{4}{3\left(\frac{2\sqrt{-x^2+x+2}-3}{2x-1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)/(-x^2+x+2)^(1/2),x, algorithm="giac")
```

```
[Out] -4/3/((2*sqrt(-x^2 + x + 2) - 3)/(2*x - 1) + 1)
```

3.60 $\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{1-\cos(x)}-\frac{3\sin(x)}{1-\cos(x)}-\tanh^{-1}(\cos(x))$$

```
[Out] -ArcTanh[Cos[x]] - (1 - Cos[x])^(-1) - (3*Sin[x])/(1 - Cos[x])
```

Rubi [A] time = 0.12947, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {4401, 2648, 2667, 44, 207}

$$-\frac{1}{1-\cos(x)}-\frac{3\sin(x)}{1-\cos(x)}-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]*(2 + 3*Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] -ArcTanh[Cos[x]] - (1 - Cos[x])^(-1) - (3*Sin[x])/(1 - Cos[x])
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)(2+3\sin(x))}{1-\cos(x)}\,dx &= \int \left(-\frac{3}{-1+\cos(x)}-\frac{2\csc(x)}{-1+\cos(x)}\right)dx\\ &= -\left(2\int \frac{\csc(x)}{-1+\cos(x)}\,dx\right)-3\int \frac{1}{-1+\cos(x)}\,dx\\ &= -\frac{3\sin(x)}{1-\cos(x)}+2\,\text{Subst}\left(\int \frac{1}{(-1-x)(-1+x)^2}\,dx,x,\cos(x)\right)\\ &= -\frac{3\sin(x)}{1-\cos(x)}+2\,\text{Subst}\left(\int \left(-\frac{1}{2(-1+x)^2}+\frac{1}{2\left(-1+x^2\right)}\right)dx,x,\cos(x)\right)\\ &= -\frac{1}{1-\cos(x)}-\frac{3\sin(x)}{1-\cos(x)}+\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\cos(x)\right)\\ &= -\tanh^{-1}(\cos(x))-\frac{1}{1-\cos(x)}-\frac{3\sin(x)}{1-\cos(x)}\end{aligned}$$

Mathematica [A] time = 0.060302, size = 54, normalized size = 1.93

$$\frac{1}{2}\csc^2\left(\frac{x}{2}\right)\left(-3\sin(x)+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)+\cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]*(2 + 3*Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] (Csc[x/2]^2*(-1 - Log[Cos[x/2]] + Cos[x]*(Log[Cos[x/2]] - Log[Sin[x/2]]) +
Log[Sin[x/2]] - 3*Sin[x]))/2
```

Maple [A] time = 0.048, size = 23, normalized size = 0.8

$$\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{1}{2}\left(\tan\left(\frac{x}{2}\right)\right)^{-2}-3\,\left(\tan\left(x/2\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*sin(x))/(1-cos(x))/sin(x),x)
```

```
[Out] ln(tan(1/2*x))-1/2/tan(1/2*x)^2-3/tan(1/2*x)
```

Maxima [A] time = 0.958616, size = 45, normalized size = 1.61

$$-\frac{\left(\cos\left(x\right)+1\right)^2}{2\,\sin\left(x\right)^2}-\frac{3\,\left(\cos\left(x\right)+1\right)}{\sin\left(x\right)}+\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*sin(x))/(1-cos(x))/sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(x) + 1)^2/sin(x)^2 - 3*(cos(x) + 1)/sin(x) + log(sin(x)/(cos(x) +
 1))
```

Fricas [A] time = 1.97785, size = 147, normalized size = 5.25

$$-\frac{(\cos(x)-1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)-1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-6\sin(x)-2}{2(\cos(x)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*sin(x))/(1-cos(x))/sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*((cos(x) - 1)*log(1/2*cos(x) + 1/2) - (cos(x) - 1)*log(-1/2*cos(x) + 1
/2) - 6*sin(x) - 2)/(cos(x) - 1)
```

Sympy [A] time = 0.794547, size = 22, normalized size = 0.79

$$\log\left(\tan\left(\frac{x}{2}\right)\right)-\frac{3}{\tan\left(\frac{x}{2}\right)}-\frac{1}{2\tan^2\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*sin(x))/(1-cos(x))/sin(x),x)
```

```
[Out] log(tan(x/2)) - 3/tan(x/2) - 1/(2*tan(x/2)**2)
```

Giac [A] time = 1.07862, size = 42, normalized size = 1.5

$$-\frac{3\tan\left(\frac{1}{2}x\right)^2+6\tan\left(\frac{1}{2}x\right)+1}{2\tan\left(\frac{1}{2}x\right)^2}+\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*sin(x))/(1-cos(x))/sin(x),x, algorithm="giac")
```

```
[Out] -1/2*(3*tan(1/2*x)^2 + 6*tan(1/2*x) + 1)/tan(1/2*x)^2 + log(abs(tan(1/2*x))
)
```

3.61 $\int \frac{1}{2+3\cos^2(x)}\,dx$

Optimal. Leaf size=37

$$\frac{x}{\sqrt{10}}-\frac{\tan^{-1}\left(\frac{3\sin(x)\cos(x)}{3\cos^2(x)+\sqrt{10}+2}\right)}{\sqrt{10}}$$

```
[Out] x/Sqrt[10] - ArcTan[(3*Cos[x]*Sin[x])/(2 + Sqrt[10] + 3*Cos[x]^2)]/Sqrt[10]
```

Rubi [A] time = 0.0213491, antiderivative size = 48, normalized size of antiderivative = 1.3, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3181, 203}

$$\frac{x}{\sqrt{10}}-\frac{\tan^{-1}\left(\frac{\left(\sqrt{\frac{5}{2}}-1\right)\sin(x)\cos(x)}{\left(\sqrt{\frac{5}{2}}-1\right)\cos^2(x)+1}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*Cos[x]^2)^(-1),x]
```

```
[Out] x/Sqrt[10] - ArcTan[((-1 + Sqrt[5/2])*Cos[x]*Sin[x])/(1 + (-1 + Sqrt[5/2])*
Cos[x]^2)]/Sqrt[10]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2+3\cos^2(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{2+5x^2}\,dx, x, \cot(x)\right)\\ &= \frac{x}{\sqrt{10}}-\frac{\tan^{-1}\left(\frac{\left(-1+\sqrt{\frac{5}{2}}\right)\cos(x)\sin(x)}{1+\left(-1+\sqrt{\frac{5}{2}}\right)\cos^2(x)}\right)}{\sqrt{10}}\end{aligned}$$

Mathematica [A] time = 0.0487157, size = 17, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{\frac{2}{5}}\tan(x)\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*Cos[x]^2)^(-1),x]
```

```
[Out] ArcTan[Sqrt[2/5]*Tan[x]]/Sqrt[10]
```

Maple [A] time = 0.012, size = 14, normalized size = 0.4

$$\frac{\sqrt{10}}{10}\arctan\left(\frac{\tan(x)\sqrt{10}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*cos(x)^2),x)
```

```
[Out] 1/10*10^(1/2)*arctan(1/5*tan(x)*10^(1/2))
```

Maxima [A] time = 1.43313, size = 18, normalized size = 0.49

$$\frac{1}{10}\sqrt{10}\arctan\left(\frac{1}{5}\sqrt{10}\tan(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(x)^2),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(10)*arctan(1/5*sqrt(10)*tan(x))
```

Fricas [A] time = 1.96332, size = 108, normalized size = 2.92

$$-\frac{1}{20}\sqrt{10}\arctan\left(\frac{7\sqrt{10}\cos(x)^2-2\sqrt{10}}{20\cos(x)\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(x)^2),x, algorithm="fricas")
```

```
[Out] -1/20*sqrt(10)*arctan(1/20*(7*sqrt(10)*cos(x)^2 - 2*sqrt(10))/(cos(x)*sin(x
)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{3\cos^2(x)+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(x)**2),x)
```

```
[Out] Integral(1/(3*cos(x)**2 + 2), x)
```

Giac [A] time = 1.07651, size = 62, normalized size = 1.68

$$\frac{1}{10}\sqrt{10}\left(x+\arctan\left(-\frac{\sqrt{10}\sin\left(2\,x\right)-2\,\sin\left(2\,x\right)}{\sqrt{10}\cos\left(2\,x\right)+\sqrt{10}-2\,\cos\left(2\,x\right)+2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*cos(x)^2),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(10)*(x + arctan(-(sqrt(10)*sin(2*x) - 2*sin(2*x))/(sqrt(10)*cos(2
*x) + sqrt(10) - 2*cos(2*x) + 2)))
```

3.62 $\int \csc(2x)(1-\tan(x))\,dx$

Optimal. Leaf size=14

$$\frac{1}{2}\log(\tan(x))-\frac{\tan(x)}{2}$$

```
[Out] Log[Tan[x]]/2 - Tan[x]/2
```

Rubi [A] time = 0.0339943, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {12}

$$\frac{1}{2}\log(\tan(x))-\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*x]*(1 - Tan[x]),x]
```

```
[Out] Log[Tan[x]]/2 - Tan[x]/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\begin{aligned}\int \csc(2x)(1-\tan(x))\,dx &= \text{Subst}\left(\int \frac{1}{2}\left(-1+\frac{1}{x}\right)\,dx,x,\tan(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-1+\frac{1}{x}\right)\,dx,x,\tan(x)\right)\\ &= \frac{1}{2}\log(\tan(x))-\frac{\tan(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0148639, size = 21, normalized size = 1.5

$$-\frac{\tan(x)}{2}+\frac{1}{2}\log(\sin(x))-\frac{1}{2}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*x]*(1 - Tan[x]),x]
```

```
[Out] -Log[Cos[x]]/2 + Log[Sin[x]]/2 - Tan[x]/2
```

Maple [A] time = 0.047, size = 11, normalized size = 0.8

$$\frac{\ln(\tan(x))}{2}-\frac{\tan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-tan(x))/sin(2*x),x)
```

```
[Out] 1/2*ln(tan(x))-1/2*tan(x)
```

Maxima [B] time = 0.953807, size = 63, normalized size = 4.5

$$-\frac{\sin(2x)}{\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1}-\frac{1}{4}\log(\cos(2x)+1)+\frac{1}{4}\log(\cos(2x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tan(x))/sin(2*x),x, algorithm="maxima")
```

```
[Out] -sin(2*x)/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - 1/4*log(cos(2*x) + 1
) + 1/4*log(cos(2*x) - 1)
```

Fricas [B] time = 2.04071, size = 101, normalized size = 7.21

$$\frac{1}{4}\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)-\frac{1}{4}\log\left(\frac{1}{\tan(x)^2+1}\right)-\frac{1}{2}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tan(x))/sin(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(tan(x)^2/(tan(x)^2 + 1)) - 1/4*log(1/(tan(x)^2 + 1)) - 1/2*tan(x)
```

Sympy [B] time = 1.64234, size = 27, normalized size = 1.93

$$\frac{\log(\cos(2x)-1)}{4}-\frac{\log(\cos(2x)+1)}{4}-\frac{\sin(x)}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tan(x))/sin(2*x),x)
```

```
[Out] log(cos(2*x) - 1)/4 - log(cos(2*x) + 1)/4 - sin(x)/(2*cos(x))
```

Giac [A] time = 1.07706, size = 15, normalized size = 1.07

$$\frac{1}{2}\log(|\tan(x)|)-\frac{1}{2}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tan(x))/sin(2*x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(tan(x))) - 1/2*tan(x)
```

3.63 $\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{1}{2}\tanh^{-1}(2\sin(x)\cos(x))$$

```
[Out] ArcTanh[2*Cos[x]*Sin[x]]/2
```

Rubi [A] time = 0.0305843, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {206}

$$\frac{1}{2}\tanh^{-1}(2\sin(x)\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[x]^2)/(1 - Tan[x]^2),x]
```

```
[Out] ArcTanh[2*Cos[x]*Sin[x]]/2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+\tan^2(x)}{1-\tan^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tan(x)\right)\\ &= \frac{1}{2}\tanh^{-1}(2\cos(x)\sin(x))\end{aligned}$$

Mathematica [B] time = 0.0066211, size = 23, normalized size = 2.09

$$\frac{1}{2}\log(\sin(x)+\cos(x))-\frac{1}{2}\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[x]^2)/(1 - Tan[x]^2),x]
```

```
[Out] -Log[Cos[x] - Sin[x]]/2 + Log[Cos[x] + Sin[x]]/2
```

Maple [A] time = 0.007, size = 4, normalized size = 0.4

$$\text{Artanh}\left(\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)^2+1)/(1-tan(x)^2),x)
```

```
[Out] arctanh(tan(x))
```

Maxima [A] time = 0.943211, size = 20, normalized size = 1.82

$$\frac{1}{2}\log\left(\tan\left(x\right)+1\right)-\frac{1}{2}\log\left(\tan\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)/(1-tan(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(tan(x) + 1) - 1/2*log(tan(x) - 1)
```

Fricas [B] time = 1.92308, size = 139, normalized size = 12.64

$$\frac{1}{4}\log\left(\frac{\tan\left(x\right)^{2}+2\,\tan\left(x\right)+1}{\tan\left(x\right)^{2}+1}\right)-\frac{1}{4}\log\left(\frac{\tan\left(x\right)^{2}-2\,\tan\left(x\right)+1}{\tan\left(x\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)/(1-tan(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) - 1/4*log((tan(x)^2 - 2*t
an(x) + 1)/(tan(x)^2 + 1))
```

Sympy [A] time = 0.148052, size = 15, normalized size = 1.36

$$-\frac{\log\left(\tan\left(x\right)-1\right)}{2}+\frac{\log\left(\tan\left(x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)**2)/(1-tan(x)**2),x)
```

```
[Out] -log(tan(x) - 1)/2 + log(tan(x) + 1)/2
```

Giac [A] time = 1.08463, size = 23, normalized size = 2.09

$$\frac{1}{2}\log\left(\left|\tan\left(x\right)+1\right|\right)-\frac{1}{2}\log\left(\left|\tan\left(x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tan(x)^2)/(1-tan(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(tan(x) + 1)) - 1/2*log(abs(tan(x) - 1))
```

3.64 $\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx$

Optimal. Leaf size=18

$$\frac{1}{7}\left(a^2-4\cos^2(x)\right)^{7/4}$$

```
[Out] (a^2 - 4*Cos[x]^2)^(7/4)/7
```

Rubi [A] time = 0.0474616, antiderivative size = 19, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {12, 261}

$$\frac{1}{7}\left(a^2+4\sin^2(x)-4\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - 4*Cos[x]^2)^(3/4)*Sin[2*x],x]

[Out] (-4 + a^2 + 4*Sin[x]^2)^(7/4)/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \left(a^2-4\cos^2(x)\right)^{3/4}\sin(2x)\,dx &= \text{Subst}\left(\int 2x\left(-4+a^2+4x^2\right)^{3/4}\,dx, x, \sin(x)\right)\\ &= 2\,\text{Subst}\left(\int x\left(-4+a^2+4x^2\right)^{3/4}\,dx, x, \sin(x)\right)\\ &= \frac{1}{7}\left(-4+a^2+4\sin^2(x)\right)^{7/4}\end{aligned}$$

Mathematica [A] time = 0.0168411, size = 19, normalized size = 1.06

$$\frac{1}{7}\left(a^2+4\sin^2(x)-4\right)^{7/4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - 4*Cos[x]^2)^(3/4)*Sin[2*x],x]

[Out] (-4 + a^2 + 4*Sin[x]^2)^(7/4)/7
```

Maple [A] time = 0.015, size = 15, normalized size = 0.8

$$\frac{1}{7}\left(a^{2}-4\left(\cos\left(x\right)\right)^{2}\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x)
```

```
[Out] 1/7*(a^2-4*cos(x)^2)^(7/4)
```

Maxima [A] time = 0.941172, size = 19, normalized size = 1.06

$$\frac{1}{7}\left(a^{2}-4\cos\left(x\right)^{2}\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="maxima")
```

```
[Out] 1/7*(a^2 - 4*cos(x)^2)^(7/4)
```

Fricas [A] time = 2.42498, size = 41, normalized size = 2.28

$$\frac{1}{7}\left(a^{2}-4\cos\left(x\right)^{2}\right)^{\frac{7}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="fricas")
```

```
[Out] 1/7*(a^2 - 4*cos(x)^2)^(7/4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-4*cos(x)**2)**(3/4)*sin(2*x),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-4*cos(x)^2)^(3/4)*sin(2*x),x, algorithm="giac")
```

```
[Out] Timed out
```

3.65 $\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx$

Optimal. Leaf size=18

$$-\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3}$$

```
[Out] (-3*(a^2 - 4*Sin[x]^2)^(2/3))/8
```

Rubi [A] time = 0.044924, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {12, 261}

$$-\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/(a^2 - 4*Sin[x]^2)^(1/3),x]
```

```
[Out] (-3*(a^2 - 4*Sin[x]^2)^(2/3))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(2x)}{\sqrt[3]{a^2-4\sin^2(x)}}\,dx &= \text{Subst}\left(\int \frac{2x}{\sqrt[3]{a^2-4x^2}}\,dx, x, \sin(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{x}{\sqrt[3]{a^2-4x^2}}\,dx, x, \sin(x)\right)\\ &= -\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3}\end{aligned}$$

Mathematica [A] time = 0.0185391, size = 18, normalized size = 1.

$$-\frac{3}{8}\left(a^2-4\sin^2(x)\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/(a^2 - 4*Sin[x]^2)^(1/3),x]
```

```
[Out] (-3*(a^2 - 4*Sin[x]^2)^(2/3))/8
```

Maple [A] time = 0.02, size = 15, normalized size = 0.8

$$-\frac{3}{8}\left(a^{2}-4\left(\sin\left(x\right)\right)^{2}\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x)
```

```
[Out] -3/8*(a^2-4*sin(x)^2)^(2/3)
```

Maxima [A] time = 0.945078, size = 19, normalized size = 1.06

$$-\frac{3}{8}\left(a^{2}-4\,\sin\left(x\right)^{2}\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x, algorithm="maxima")
```

```
[Out] -3/8*(a^2 - 4*sin(x)^2)^(2/3)
```

Fricas [A] time = 2.15758, size = 47, normalized size = 2.61

$$-\frac{3}{8}\left(a^{2}+4\,\cos\left(x\right)^{2}-4\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/8*(a^2 + 4*cos(x)^2 - 4)^(2/3)
```

Sympy [A] time = 2.57448, size = 17, normalized size = 0.94

$$-\frac{3\left(a^{2}-4\sin^{2}\left(x\right)\right)^{\frac{2}{3}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a**2-4*sin(x)**2)**(1/3),x)
```

```
[Out] -3*(a**2 - 4*sin(x)**2)**(2/3)/8
```

Giac [A] time = 1.15055, size = 35, normalized size = 1.94

$$-\frac{3}{8}\left(a^{2}-\frac{16\,\tan\left(\frac{1}{2}\,x\right)^{2}}{\left(\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)^{2}}\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(a^2-4*sin(x)^2)^(1/3),x, algorithm="giac")
```

```
[Out] -3/8*(a^2 - 16*tan(1/2*x)^2/(tan(1/2*x)^2 + 1)^2)^(2/3)
```

3.66 $\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx$

Optimal. Leaf size=17

$$\frac{\tan^{-1}\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

```
[Out] ArcTan[Sqrt[-1 + a^(2*x)]]/Log[a]
```

Rubi [A] time = 0.0129283, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2282, 63, 203}

$$\frac{\tan^{-1}\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + a^(2*x)],x]
```

```
[Out] ArcTan[Sqrt[-1 + a^(2*x)]]/Log[a]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{-1+a^{2x}}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{-1+xx}}\,dx, x, a^{2x}\right)}{2\log(a)}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+a^{2x}}\right)}{\log(a)}$$
$$= \frac{\tan^{-1}\left(\sqrt{-1+a^{2x}}\right)}{\log(a)}$$

Mathematica [A] time = 0.0044183, size = 17, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + a^(2*x)],x]
```

```
[Out] ArcTan[Sqrt[-1 + a^(2*x)]]/Log[a]
```

Maple [A] time = 0.007, size = 16, normalized size = 0.9

$$\frac{1}{\ln(a)}\arctan\left(\sqrt{-1+a^{2x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+a^(2*x))^(1/2),x)
```

```
[Out] arctan((-1+a^(2*x))^(1/2))/ln(a)
```

Maxima [A] time = 1.445, size = 20, normalized size = 1.18

$$\frac{\arctan\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+a^(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(a^(2*x) - 1))/log(a)
```

Fricas [A] time = 2.09896, size = 46, normalized size = 2.71

$$\frac{\arctan\left(\sqrt{a^{2x}-1}\right)}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+a^(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(a^(2*x) - 1))/log(a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a^{2x}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+a**(2*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a**(2*x) - 1), x)
```

Giac [A] time = 1.08796, size = 20, normalized size = 1.18

$$\frac{\arctan\left(\sqrt{a^{2x}-1}\right)}{\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+a^(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(a^(2*x) - 1))/log(a)
```

3.67 $\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx$

Optimal. Leaf size=20

$$2\tanh^{-1}\left(\frac{e^{x/2}}{\sqrt{e^x-1}}\right)$$

```
[Out] 2*ArcTanh[E^(x/2)/Sqrt[-1 + E^x]]
```

Rubi [A] time = 0.0258523, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2249, 217, 206}

$$2\tanh^{-1}\left(\frac{e^{x/2}}{\sqrt{e^x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(x/2)/Sqrt[-1 + E^x],x]
```

```
[Out] 2*ArcTanh[E^(x/2)/Sqrt[-1 + E^x]]
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{x/2}}{\sqrt{-1+e^x}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, e^{x/2}\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{e^{x/2}}{\sqrt{-1+e^x}}\right)\\
&= 2\tanh^{-1}\left(\frac{e^{x/2}}{\sqrt{-1+e^x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0038406, size = 20, normalized size = 1.

$$2\tanh^{-1}\left(\frac{e^{x/2}}{\sqrt{e^x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(x/2)/Sqrt[-1 + E^x],x]
```

```
[Out] 2*ArcTanh[E^(x/2)/Sqrt[-1 + E^x]]
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int e^{\frac{x}{2}}\frac{1}{\sqrt{-1+e^x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1/2*x)/(-1+exp(x))^(1/2),x)
```

```
[Out] int(exp(1/2*x)/(-1+exp(x))^(1/2),x)
```

Maxima [A] time = 0.966361, size = 24, normalized size = 1.2

$$2\log\left(2\sqrt{e^x-1}+2e^{\left(\frac{1}{2}x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*log(2*sqrt(e^x - 1) + 2*e^(1/2*x))
```

Fricas [A] time = 1.97761, size = 47, normalized size = 2.35

$$-2\log\left(\sqrt{e^x-1}-e^{\left(\frac{1}{2}x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*log(sqrt(e^x - 1) - e^(1/2*x))
```

Sympy [A] time = 0.618287, size = 7, normalized size = 0.35

$$2\operatorname{acosh}\left(e^{\frac{x}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)/(-1+exp(x))**(1/2),x)
```

```
[Out] 2*acosh(exp(x/2))
```

Giac [A] time = 1.14971, size = 22, normalized size = 1.1

$$-2 \log\left(-\sqrt{e^{x}-1}+e^{\left(\frac{1}{2} x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)/(-1+exp(x))^(1/2),x, algorithm="giac")
```

```
[Out] -2*log(-sqrt(e^x - 1) + e^(1/2*x))
```

3.68 $\int \frac{\tan^{-1}(x)^n}{1+x^2} dx$

Optimal. Leaf size=12

$$\frac{\tan^{-1}(x)^{n+1}}{n+1}$$

```
[Out] ArcTan[x]^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0232248, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4884}

$$\frac{\tan^{-1}(x)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x]^n/(1 + x^2),x]
```

```
[Out] ArcTan[x]^(1 + n)/(1 + n)
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}(x)^n}{1+x^2} dx = \frac{\tan^{-1}(x)^{1+n}}{1+n}$$

Mathematica [A] time = 0.0041617, size = 12, normalized size = 1.

$$\frac{\tan^{-1}(x)^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]^n/(1 + x^2),x]
```

```
[Out] ArcTan[x]^(1 + n)/(1 + n)
```

Maple [A] time = 0.003, size = 13, normalized size = 1.1

$$\frac{(\arctan(x))^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)^n/(x^2+1),x)
```

```
[Out] arctan(x)^(1+n)/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^n/(x^2+1),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.07399, size = 42, normalized size = 3.5

$$\frac{\arctan(x)^n \arctan(x)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^n/(x^2+1),x, algorithm="fricas")
```

```
[Out] arctan(x)^n*arctan(x)/(n + 1)
```

Sympy [A] time = 4.10939, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\operatorname{atan}^{n+1}(x)}{n+1} & \text{for } n \neq -1 \\ \log(\operatorname{atan}(x)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)**n/(x**2+1),x)
```

```
[Out] Piecewise((atan(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(atan(x)), True))
```

Giac [A] time = 1.11214, size = 16, normalized size = 1.33

$$\frac{\arctan(x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^n/(x^2+1),x, algorithm="giac")
```

```
[Out] arctan(x)^(n + 1)/(n + 1)
```

3.69 $\int \frac{\sin^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}} \, dx$

Optimal. Leaf size=42

$$\frac{2a\sqrt{1-\frac{x^2}{a^2}}\sin^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

```
[Out] (2*a*Sqrt[1 - x^2/a^2]*ArcSin[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

Rubi [A] time = 0.0654377, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4643, 4641}

$$\frac{2a\sqrt{1-\frac{x^2}{a^2}}\sin^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
```

```
[Out] (2*a*Sqrt[1 - x^2/a^2]*ArcSin[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

Rule 4643

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2], Int[(a + b*ArcSin[c*x])^n
/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e
, 0] &&  !GtQ[d, 0]
```

Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}} \, dx &= \frac{\sqrt{1-\frac{x^2}{a^2}} \int \frac{\sin^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{1-\frac{x^2}{a^2}}} \, dx}{\sqrt{a^2-x^2}}\\ &= \frac{2a\sqrt{1-\frac{x^2}{a^2}}\sin^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}\end{aligned}$$

Mathematica [A] time = 0.0397304, size = 42, normalized size = 1.

$$\frac{2a\sqrt{1-\frac{x^2}{a^2}}\sin^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
```

```
[Out] (2*a*Sqrt[1 - x^2/a^2]*ArcSin[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

Maple [A] time = 0.047, size = 38, normalized size = 0.9

$$\frac{2a}{5}\left(\arcsin\left(\frac{x}{a}\right)\right)^{\frac{5}{2}}\sqrt{\frac{a^2-x^2}{a^2}}\frac{1}{\sqrt{a^2-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x)
```

```
[Out] 2/5*arcsin(x/a)^(5/2)*a/(a^2-x^2)^(1/2)*((a^2-x^2)/a^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arcsin\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\sqrt{a^2-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsin(x/a)^(3/2)/sqrt(a^2 - x^2), x)
```

Fricas [A] time = 1.99734, size = 92, normalized size = 2.19

$$\frac{2}{5}\sqrt{-\arctan\left(-\frac{x}{\sqrt{a^2-x^2}}\right)}\arctan\left(-\frac{x}{\sqrt{a^2-x^2}}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(-arctan(-x/sqrt(a^2 - x^2)))*arctan(-x/sqrt(a^2 - x^2))^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asin}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\sqrt{-(-a+x)(a+x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x/a)**(3/2)/(a**2-x**2)**(1/2),x)
```

```
[Out] Integral(asin(x/a)**(3/2)/sqrt(-(-a + x)*(a + x)), x)
```

Giac [A] time = 1.26984, size = 16, normalized size = 0.38

$$\frac{2}{5}\arcsin\left(\frac{x}{a}\right)^{\frac{5}{2}}\operatorname{sgn}(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*arcsin(x/a)^(5/2)*sgn(a)
```

3.70 $\int \frac{1}{\sqrt{1-x^2}\cos^{-1}(x)^3}\,dx$

Optimal. Leaf size=8

$$\frac{1}{2\cos^{-1}(x)^2}$$

```
[Out] 1/(2*ArcCos[x]^2)
```

Rubi [A] time = 0.024774, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4642}

$$\frac{1}{2\cos^{-1}(x)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*ArcCos[x]^3),x]
```

```
[Out] 1/(2*ArcCos[x]^2)
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}\cos^{-1}(x)^3}\,dx=\frac{1}{2\cos^{-1}(x)^2}$$

Mathematica [A] time = 0.0053038, size = 8, normalized size = 1.

$$\frac{1}{2\cos^{-1}(x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*ArcCos[x]^3),x]
```

```
[Out] 1/(2*ArcCos[x]^2)
```

Maple [A] time = 0.004, size = 7, normalized size = 0.9

$$\frac{1}{2\left(\arccos(x)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccos(x)^3/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2/arccos(x)^2
```

Maxima [A] time = 1.40617, size = 8, normalized size = 1.

$$\frac{1}{2 \arccos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2/arccos(x)^2
```

Fricas [A] time = 1.90228, size = 23, normalized size = 2.88

$$\frac{1}{2 \arccos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2/arccos(x)^2
```

Sympy [A] time = 2.79693, size = 7, normalized size = 0.88

$$\frac{1}{2 \operatorname{acos}^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acos(x)**3/(-x**2+1)**(1/2),x)
```

```
[Out] 1/(2*acos(x)**2)
```

Giac [A] time = 1.10248, size = 8, normalized size = 1.

$$\frac{1}{2 \arccos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccos(x)^3/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2/arccos(x)^2
```

3.71 $\int x \log^2(x)\, dx$

Optimal. Leaf size=28

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rubi [A] time = 0.0096363, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2305, 2304}

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\log^2(x)\,dx &= \frac{1}{2}x^2\log^2(x)-\int x\log(x)\,dx\\ &= \frac{x^2}{4}-\frac{1}{2}x^2\log(x)+\frac{1}{2}x^2\log^2(x)\end{aligned}$$

Mathematica [A] time = 0.0009641, size = 28, normalized size = 1.

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Maple [A] time = 0.001, size = 23, normalized size = 0.8

$$\frac{x^2}{4} - \frac{x^2 \ln(x)}{2} + \frac{x^2 (\ln(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x)^2,x)
```

```
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Maxima [A] time = 0.935202, size = 23, normalized size = 0.82

$$\frac{1}{4}\left(2 \log(x)^2 - 2 \log(x) + 1\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

Fricas [A] time = 1.94557, size = 61, normalized size = 2.18

$$\frac{1}{2} x^2 \log(x)^2 - \frac{1}{2} x^2 \log(x) + \frac{1}{4} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Sympy [A] time = 0.096138, size = 22, normalized size = 0.79

$$\frac{x^2 \log(x)^2}{2} - \frac{x^2 \log(x)}{2} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x)**2,x)
```

```
[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

Giac [A] time = 1.05814, size = 30, normalized size = 1.07

$$\frac{1}{2} x^2 \log(x)^2 - \frac{1}{2} x^2 \log(x) + \frac{1}{4} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

3.72 $\int \frac{\log(x)}{x^5} \, dx$

Optimal. Leaf size=17

$$-\frac{1}{16x^4}-\frac{\log(x)}{4x^4}$$

```
[Out] -1/(16*x^4) - Log[x]/(4*x^4)
```

Rubi [A] time = 0.006899, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2304}

$$-\frac{1}{16x^4}-\frac{\log(x)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/x^5,x]

[Out] -1/(16*x^4) - Log[x]/(4*x^4)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log(x)}{x^5} \, dx = -\frac{1}{16x^4}-\frac{\log(x)}{4x^4}$$

Mathematica [A] time = 0.0009253, size = 17, normalized size = 1.

$$-\frac{1}{16x^4}-\frac{\log(x)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/x^5,x]

[Out] -1/(16*x^4) - Log[x]/(4*x^4)
```

Maple [A] time = 0.002, size = 14, normalized size = 0.8

$$-\frac{1}{16\,x^4}-\frac{\ln(x)}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x^5,x)
```

```
[Out] -1/16/x^4-1/4*ln(x)/x^4
```

Maxima [A] time = 0.934378, size = 18, normalized size = 1.06

$$-\frac{\log(x)}{4\,x^4} - \frac{1}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/4*log(x)/x^4 - 1/16/x^4
```

Fricas [A] time = 1.67994, size = 35, normalized size = 2.06

$$-\frac{4\,\log(x) + 1}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/16*(4*log(x) + 1)/x^4
```

Sympy [A] time = 0.092494, size = 15, normalized size = 0.88

$$-\frac{\log(x)}{4x^4} - \frac{1}{16x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x**5,x)
```

```
[Out] -log(x)/(4*x**4) - 1/(16*x**4)
```

Giac [A] time = 1.05191, size = 18, normalized size = 1.06

$$-\frac{\log(x)}{4\,x^4} - \frac{1}{16\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x^5,x, algorithm="giac")
```

```
[Out] -1/4*log(x)/x^4 - 1/16/x^4
```

3.73 $\int x^2 \log\left(\frac{-1+x}{x}\right) dx$

Optimal. Leaf size=36

$$-\frac{x^2}{6}+\frac{1}{3}x^3\log\left(\frac{x-1}{x}\right)-\frac{x}{3}-\frac{1}{3}\log(x-1)$$

```
[Out] -x/3 - x^2/6 - Log[-1 + x]/3 + (x^3*Log[(-1 + x)/x])/3
```

Rubi [A] time = 0.0255435, antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2461, 2455, 263, 43}

$$-\frac{x^2}{6}+\frac{1}{3}x^3\log\left(1-\frac{1}{x}\right)-\frac{x}{3}-\frac{1}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[(-1 + x)/x],x]
```

```
[Out] -x/3 - x^2/6 + (x^3*Log[1 - x^(-1)])/3 - Log[1 - x]/3
```

Rule 2461

```
Int[((a_.) + Log[(c_.)*(v_)^(p_.)]*(b_.))^(q_.)*((f_.)*(x_))^(m_.), x_Symbo
l] :> Int[(f*x)^m*(a + b*Log[c*ExpandToSum[v, x]^p])^q, x] /; FreeQ[{a, b,
c, f, m, p, q}, x] && BinomialQ[v, x] &&  !BinomialMatchQ[v, x]
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 263

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2 \log\left(\frac{-1+x}{x}\right) dx = \int x^2 \log\left(1-\frac{1}{x}\right) dx$$
$$= \frac{1}{3}x^3 \log\left(1-\frac{1}{x}\right) - \frac{1}{3}\int \frac{x}{1-\frac{1}{x}} dx$$
$$= \frac{1}{3}x^3 \log\left(1-\frac{1}{x}\right) - \frac{1}{3}\int \frac{x^2}{-1+x} dx$$
$$= \frac{1}{3}x^3 \log\left(1-\frac{1}{x}\right) - \frac{1}{3}\int \left(1+\frac{1}{-1+x}+x\right) dx$$
$$= -\frac{x}{3} - \frac{x^2}{6} + \frac{1}{3}x^3 \log\left(1-\frac{1}{x}\right) - \frac{1}{3}\log(1-x)$$

Mathematica [A] time = 0.0056543, size = 38, normalized size = 1.06

$$-\frac{x^2}{6} + \frac{1}{3}x^3 \log\left(\frac{x-1}{x}\right) - \frac{x}{3} - \frac{1}{3}\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[(-1 + x)/x],x]
```

```
[Out] -x/3 - x^2/6 - Log[1 - x]/3 + (x^3*Log[(-1 + x)/x])/3
```

Maple [A] time = 0.012, size = 53, normalized size = 1.5

$$-\frac{x^2}{6} + \frac{1}{3}\ln\left(-x^{-1}\right) - \frac{x}{3} + \frac{x^3}{3}\ln\left(1-x^{-1}\right)\left(1-x^{-1}\right)\left(\left(1-x^{-1}\right)^2 + 3x^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln((-1+x)/x),x)
```

```
[Out] -1/6*x^2+1/3*ln(-1/x)-1/3*x+1/3*ln(1-1/x)*(1-1/x)*((1-1/x)^2+3/x)*x^3
```

Maxima [A] time = 0.940232, size = 38, normalized size = 1.06

$$\frac{1}{3}x^3 \log\left(\frac{x-1}{x}\right) - \frac{1}{6}x^2 - \frac{1}{3}x - \frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log((-1+x)/x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log((x - 1)/x) - 1/6*x^2 - 1/3*x - 1/3*log(x - 1)
```

Fricas [A] time = 1.88659, size = 80, normalized size = 2.22

$$\frac{1}{3}x^3 \log\left(\frac{x-1}{x}\right) - \frac{1}{6}x^2 - \frac{1}{3}x - \frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log((-1+x)/x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*log((x - 1)/x) - 1/6*x^2 - 1/3*x - 1/3*log(x - 1)
```

Sympy [A] time = 0.117856, size = 26, normalized size = 0.72

$$\frac{x^3 \log\left(\frac{x-1}{x}\right)}{3} - \frac{x^2}{6} - \frac{x}{3} - \frac{\log(x-1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln((-1+x)/x),x)
```

```
[Out] x**3*log((x - 1)/x)/3 - x**2/6 - x/3 - log(x - 1)/3
```

Giac [A] time = 1.05709, size = 39, normalized size = 1.08

$$\frac{1}{3}\,x^3 \log\left(\frac{x-1}{x}\right) - \frac{1}{6}\,x^2 - \frac{1}{3}\,x - \frac{1}{3}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log((-1+x)/x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log((x - 1)/x) - 1/6*x^2 - 1/3*x - 1/3*log(abs(x - 1))
```

3.74 $\int \cos^5(x)\,dx$

Optimal. Leaf size=19

$$\frac{\sin^5(x)}{5}-\frac{2\sin^3(x)}{3}+\sin(x)$$

```
[Out] Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Rubi [A] time = 0.0083754, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2633}

$$\frac{\sin^5(x)}{5}-\frac{2\sin^3(x)}{3}+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5,x]
```

```
[Out] Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(x)\,dx &= -\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, -\sin(x)\right)\\ &= \sin(x)-\frac{2\sin^3(x)}{3}+\frac{\sin^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.0019477, size = 23, normalized size = 1.21

$$\frac{5\sin(x)}{8}+\frac{5}{48}\sin(3x)+\frac{1}{80}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5,x]
```

```
[Out] (5*Sin[x])/8 + (5*Sin[3*x])/48 + Sin[5*x]/80
```

Maple [A] time = 0., size = 17, normalized size = 0.9

$$\frac{\sin(x)}{5}\left(\frac{8}{3}+(\cos(x))^4+\frac{4(\cos(x))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5,x)
```

```
[Out] 1/5*(8/3+cos(x)^4+4/3*cos(x)^2)*sin(x)
```

Maxima [A] time = 0.932977, size = 20, normalized size = 1.05

$$\frac{1}{5}\sin(x)^5 - \frac{2}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5,x, algorithm="maxima")
```

```
[Out] 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

Fricas [A] time = 1.9014, size = 58, normalized size = 3.05

$$\frac{1}{15}\left(3\cos(x)^4 + 4\cos(x)^2 + 8\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5,x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)
```

Sympy [A] time = 0.059906, size = 17, normalized size = 0.89

$$\frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5,x)
```

```
[Out] sin(x)**5/5 - 2*sin(x)**3/3 + sin(x)
```

Giac [A] time = 1.04896, size = 20, normalized size = 1.05

$$\frac{1}{5}\sin(x)^5 - \frac{2}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5,x, algorithm="giac")
```

```
[Out] 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

3.75 $\int \cos^4(x)\sin^2(x)\,dx$

Optimal. Leaf size=34

$$\frac{x}{16}-\frac{1}{6}\sin(x)\cos^5(x)+\frac{1}{24}\sin(x)\cos^3(x)+\frac{1}{16}\sin(x)\cos(x)$$

```
[Out] x/16 + (Cos[x]*Sin[x])/16 + (Cos[x]^3*Sin[x])/24 - (Cos[x]^5*Sin[x])/6
```

Rubi [A] time = 0.0339044, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{x}{16}-\frac{1}{6}\sin(x)\cos^5(x)+\frac{1}{24}\sin(x)\cos^3(x)+\frac{1}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4*Sin[x]^2,x]
```

```
[Out] x/16 + (Cos[x]*Sin[x])/16 + (Cos[x]^3*Sin[x])/24 - (Cos[x]^5*Sin[x])/6
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(x)\sin^2(x)\,dx &= -\frac{1}{6}\cos^5(x)\sin(x)+\frac{1}{6}\int \cos^4(x)\,dx\\
&= \frac{1}{24}\cos^3(x)\sin(x)-\frac{1}{6}\cos^5(x)\sin(x)+\frac{1}{8}\int \cos^2(x)\,dx\\
&= \frac{1}{16}\cos(x)\sin(x)+\frac{1}{24}\cos^3(x)\sin(x)-\frac{1}{6}\cos^5(x)\sin(x)+\frac{\int 1\,dx}{16}\\
&= \frac{x}{16}+\frac{1}{16}\cos(x)\sin(x)+\frac{1}{24}\cos^3(x)\sin(x)-\frac{1}{6}\cos^5(x)\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.0081987, size = 30, normalized size = 0.88

$$\frac{x}{16}+\frac{1}{64}\sin(2x)-\frac{1}{64}\sin(4x)-\frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4*Sin[x]^2,x]
```

```
[Out] x/16 + Sin[2*x]/64 - Sin[4*x]/64 - Sin[6*x]/192
```

Maple [A] time = 0., size = 26, normalized size = 0.8

$$-\frac{(\cos(x))^5\sin(x)}{6}+\frac{\sin(x)}{24}\left((\cos(x))^3+\frac{3\cos(x)}{2}\right)+\frac{x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*sin(x)^2,x)
```

```
[Out] -1/6*cos(x)^5*sin(x)+1/24*(cos(x)^3+3/2*cos(x))*sin(x)+1/16*x
```

Maxima [A] time = 0.936026, size = 24, normalized size = 0.71

$$\frac{1}{48}\sin(2x)^3+\frac{1}{16}x-\frac{1}{64}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/48*sin(2*x)^3 + 1/16*x - 1/64*sin(4*x)
```

Fricas [A] time = 1.86811, size = 81, normalized size = 2.38

$$-\frac{1}{48}\left(8\cos(x)^5-2\cos(x)^3-3\cos(x)\right)\sin(x)+\frac{1}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(x)^5 - 2*cos(x)^3 - 3*cos(x))*sin(x) + 1/16*x
```

Sympy [A] time = 0.058952, size = 31, normalized size = 0.91

$$\frac{x}{16}-\frac{\sin(x)\cos^5(x)}{6}+\frac{\sin(x)\cos^3(x)}{24}+\frac{\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4*sin(x)**2,x)
```

```
[Out] x/16 - sin(x)*cos(x)**5/6 + sin(x)*cos(x)**3/24 + sin(x)*cos(x)/16
```

Giac [A] time = 1.06698, size = 30, normalized size = 0.88

$$\frac{1}{16}\,x - \frac{1}{192}\,\sin\left(6\,x\right) - \frac{1}{64}\,\sin\left(4\,x\right) + \frac{1}{64}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/16*x - 1/192*sin(6*x) - 1/64*sin(4*x) + 1/64*sin(2*x)
```

3.76 $\int \csc^5(x)\,dx$

Optimal. Leaf size=26

$$-\frac{3}{8}\tanh^{-1}(\cos(x))-\frac{1}{4}\cot(x)\csc^3(x)-\frac{3}{8}\cot(x)\csc(x)$$

```
[Out] (-3*ArcTanh[Cos[x]])/8 - (3*Cot[x]*Csc[x])/8 - (Cot[x]*Csc[x]^3)/4
```

Rubi [A] time = 0.0124967, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3768, 3770}

$$-\frac{3}{8}\tanh^{-1}(\cos(x))-\frac{1}{4}\cot(x)\csc^3(x)-\frac{3}{8}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^5,x]
```

```
[Out] (-3*ArcTanh[Cos[x]])/8 - (3*Cot[x]*Csc[x])/8 - (Cot[x]*Csc[x]^3)/4
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^5(x)\,dx &= -\frac{1}{4}\cot(x)\csc^3(x)+\frac{3}{4}\int \csc^3(x)\,dx\\ &= -\frac{3}{8}\cot(x)\csc(x)-\frac{1}{4}\cot(x)\csc^3(x)+\frac{3}{8}\int \csc(x)\,dx\\ &= -\frac{3}{8}\tanh^{-1}(\cos(x))-\frac{3}{8}\cot(x)\csc(x)-\frac{1}{4}\cot(x)\csc^3(x)\end{aligned}$$

Mathematica [B] time = 0.0058338, size = 71, normalized size = 2.73

$$-\frac{1}{64}\csc^4\left(\frac{x}{2}\right)-\frac{3}{32}\csc^2\left(\frac{x}{2}\right)+\frac{1}{64}\sec^4\left(\frac{x}{2}\right)+\frac{3}{32}\sec^2\left(\frac{x}{2}\right)+\frac{3}{8}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{3}{8}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^5,x]
```

```
[Out] (-3*Csc[x/2]^2)/32 - Csc[x/2]^4/64 - (3*Log[Cos[x/2]])/8 + (3*Log[Sin[x/2]]
)/8 + (3*Sec[x/2]^2)/32 + Sec[x/2]^4/64
```

Maple [A] time = 0.031, size = 26, normalized size = 1.

$$\left(-\frac{(\csc(x))^{3}}{4}-\frac{3\csc(x)}{8}\right)\cot(x)+\frac{3\ln(\csc(x)-\cot(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^5,x)
```

```
[Out] (-1/4*csc(x)^3-3/8*csc(x))*cot(x)+3/8*ln(csc(x)-cot(x))
```

Maxima [B] time = 0.932535, size = 57, normalized size = 2.19

$$\frac{3\cos(x)^{3}-5\cos(x)}{8\left(\cos(x)^{4}-2\cos(x)^{2}+1\right)}-\frac{3}{16}\log(\cos(x)+1)+\frac{3}{16}\log(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^5,x, algorithm="maxima")
```

```
[Out] 1/8*(3*cos(x)^3 - 5*cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) - 3/16*log(cos(x) +
 1) + 3/16*log(cos(x) - 1)
```

Fricas [B] time = 1.92129, size = 227, normalized size = 8.73

$$\frac{6\cos(x)^{3}-3\left(\cos(x)^{4}-2\cos(x)^{2}+1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+3\left(\cos(x)^{4}-2\cos(x)^{2}+1\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-10\cos}{16\left(\cos(x)^{4}-2\cos(x)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(6*cos(x)^3 - 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*cos(x) + 1/2) + 3*
(cos(x)^4 - 2*cos(x)^2 + 1)*log(-1/2*cos(x) + 1/2) - 10*cos(x))/(cos(x)^4 -
 2*cos(x)^2 + 1)
```

Sympy [A] time = 0.135067, size = 46, normalized size = 1.77

$$\frac{3\cos^{3}(x)-5\cos(x)}{8\cos^{4}(x)-16\cos^{2}(x)+8}+\frac{3\log(\cos(x)-1)}{16}-\frac{3\log(\cos(x)+1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)**5,x)
```

```
[Out] (3*cos(x)**3 - 5*cos(x))/(8*cos(x)**4 - 16*cos(x)**2 + 8) + 3*log(cos(x) -
1)/16 - 3*log(cos(x) + 1)/16
```

Giac [A] time = 1.05432, size = 51, normalized size = 1.96

$$\frac{3\cos(x)^3 - 5\cos(x)}{8\left(\cos(x)^2 - 1\right)^2} - \frac{3}{16}\log(\cos(x) + 1) + \frac{3}{16}\log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^5,x, algorithm="giac")
```

```
[Out] 1/8*(3*cos(x)^3 - 5*cos(x))/(cos(x)^2 - 1)^2 - 3/16*log(cos(x) + 1) + 3/16*
log(-cos(x) + 1)
```

3.77 $\int e^{-x} \sin(x)\,dx$

Optimal. Leaf size=23

$$-\frac{1}{2}e^{-x}\sin(x)-\frac{1}{2}e^{-x}\cos(x)$$

```
[Out] -Cos[x]/(2*E^x) - Sin[x]/(2*E^x)
```

Rubi [A] time = 0.0084535, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4432}

$$-\frac{1}{2}e^{-x}\sin(x)-\frac{1}{2}e^{-x}\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/E^x,x]

[Out] -Cos[x]/(2*E^x) - Sin[x]/(2*E^x)
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-x}\sin(x)\,dx=-\frac{1}{2}e^{-x}\cos(x)-\frac{1}{2}e^{-x}\sin(x)$$

Mathematica [A] time = 0.0123571, size = 14, normalized size = 0.61

$$-\frac{1}{2}e^{-x}(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/E^x,x]

[Out] -(Cos[x] + Sin[x])/(2*E^x)
```

Maple [A] time = 0.004, size = 18, normalized size = 0.8

$$-\frac{e^{-x}\cos(x)}{2}-\frac{e^{-x}\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/exp(x),x)
```

```
[Out] -1/2*exp(-x)*cos(x)-1/2*exp(-x)*sin(x)
```

Maxima [A] time = 0.938448, size = 15, normalized size = 0.65

$$-\frac{1}{2}(\cos(x) + \sin(x))e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/exp(x),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(x) + sin(x))*e^(-x)
```

Fricas [A] time = 2.15851, size = 54, normalized size = 2.35

$$-\frac{1}{2}\cos(x)e^{(-x)} - \frac{1}{2}e^{(-x)}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/exp(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*e^(-x) - 1/2*e^(-x)*sin(x)
```

Sympy [A] time = 0.4605, size = 17, normalized size = 0.74

$$-\frac{e^{-x}\sin(x)}{2} - \frac{e^{-x}\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/exp(x),x)
```

```
[Out] -exp(-x)*sin(x)/2 - exp(-x)*cos(x)/2
```

Giac [A] time = 1.05146, size = 15, normalized size = 0.65

$$-\frac{1}{2}(\cos(x) + \sin(x))e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/exp(x),x, algorithm="giac")
```

```
[Out] -1/2*(cos(x) + sin(x))*e^(-x)
```

3.78 $\int e^{2x} \sin(3x) \, dx$

Optimal. Leaf size=27

$$\frac{2}{13} e^{2x} \sin(3x) - \frac{3}{13} e^{2x} \cos(3x)$$

```
[Out] (-3*E^(2*x)*Cos[3*x])/13 + (2*E^(2*x)*Sin[3*x])/13
```

Rubi [A] time = 0.0093931, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4432}

$$\frac{2}{13} e^{2x} \sin(3x) - \frac{3}{13} e^{2x} \cos(3x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*Sin[3*x],x]
```

```
[Out] (-3*E^(2*x)*Cos[3*x])/13 + (2*E^(2*x)*Sin[3*x])/13
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{2x} \sin(3x) \, dx = -\frac{3}{13} e^{2x} \cos(3x) + \frac{2}{13} e^{2x} \sin(3x)$$

Mathematica [A] time = 0.0349009, size = 22, normalized size = 0.81

$$\frac{1}{13} e^{2x} (2 \sin(3x) - 3 \cos(3x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*Sin[3*x],x]
```

```
[Out] (E^(2*x)*(-3*Cos[3*x] + 2*Sin[3*x]))/13
```

Maple [A] time = 0.004, size = 22, normalized size = 0.8

$$-\frac{3 \, e^{2 \, x} \cos(3 \, x)}{13} + \frac{2 \, e^{2 \, x} \sin(3 \, x)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*sin(3*x),x)

[Out] -3/13*exp(2*x)*cos(3*x)+2/13*exp(2*x)*sin(3*x)
```

Maxima [A] time = 0.931119, size = 26, normalized size = 0.96

$$-\frac{1}{13} (3 \cos(3x) - 2 \sin(3x))e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*sin(3*x),x, algorithm="maxima")

[Out] -1/13*(3*cos(3*x) - 2*sin(3*x))*e^(2*x)
```

Fricas [A] time = 2.32407, size = 65, normalized size = 2.41

$$-\frac{3}{13} \cos(3x) e^{(2x)} + \frac{2}{13} e^{(2x)} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*sin(3*x),x, algorithm="fricas")

[Out] -3/13*cos(3*x)*e^(2*x) + 2/13*e^(2*x)*sin(3*x)
```

Sympy [A] time = 0.331116, size = 26, normalized size = 0.96

$$\frac{2e^{2x} \sin(3x)}{13} - \frac{3e^{2x} \cos(3x)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*sin(3*x),x)

[Out] 2*exp(2*x)*sin(3*x)/13 - 3*exp(2*x)*cos(3*x)/13
```

Giac [A] time = 1.05647, size = 26, normalized size = 0.96

$$-\frac{1}{13} (3 \cos(3x) - 2 \sin(3x))e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*sin(3*x),x, algorithm="giac")

[Out] -1/13*(3*cos(3*x) - 2*sin(3*x))*e^(2*x)
```

3.79 $\int a^x \cos(x)\, dx$

Optimal. Leaf size=31

$$\frac{a^x\sin(x)}{\log^2(a)+1}+\frac{a^x\log(a)\cos(x)}{\log^2(a)+1}$$

```
[Out] (a^x*Cos[x]*Log[a])/(1 + Log[a]^2) + (a^x*Sin[x])/(1 + Log[a]^2)
```

Rubi [A] time = 0.0099713, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4433}

$$\frac{a^x\sin(x)}{\log^2(a)+1}+\frac{a^x\log(a)\cos(x)}{\log^2(a)+1}$$

Antiderivative was successfully verified.

```
[In] Int[a^x*Cos[x],x]
```

```
[Out] (a^x*Cos[x]*Log[a])/(1 + Log[a]^2) + (a^x*Sin[x])/(1 + Log[a]^2)
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int a^x\cos(x)\,dx=\frac{a^x\cos(x)\log(a)}{1+\log^2(a)}+\frac{a^x\sin(x)}{1+\log^2(a)}$$

Mathematica [A] time = 0.0176623, size = 20, normalized size = 0.65

$$\frac{a^x(\log(a)\cos(x)+\sin(x))}{\log^2(a)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^x*Cos[x],x]
```

```
[Out] (a^x*(Cos[x]*Log[a] + Sin[x]))/(1 + Log[a]^2)
```

Maple [B] time = 0.017, size = 71, normalized size = 2.3

$$\left(\frac{\ln(a)\,e^{x\ln(a)}}{1+(\ln(a))^2}+2\,\frac{e^{x\ln(a)}\tan(x/2)}{1+(\ln(a))^2}-\frac{\ln(a)\,e^{x\ln(a)}}{1+(\ln(a))^2}\left(\tan\left(\frac{x}{2}\right)\right)^2\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x*cos(x),x)
```

```
[Out] (1/(1+ln(a)^2)*ln(a)*exp(x*ln(a))+2/(1+ln(a)^2)*exp(x*ln(a))*tan(1/2*x)-1/(
1+ln(a)^2)*ln(a)*exp(x*ln(a))*tan(1/2*x)^2)/(tan(1/2*x)^2+1)
```

Maxima [A] time = 0.948647, size = 32, normalized size = 1.03

$$\frac{a^x \cos(x) \log(a) + a^x \sin(x)}{\log(a)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*cos(x),x, algorithm="maxima")
```

```
[Out] (a^x*cos(x)*log(a) + a^x*sin(x))/(log(a)^2 + 1)
```

Fricas [A] time = 2.21215, size = 61, normalized size = 1.97

$$\frac{(\cos(x)\log(a) + \sin(x))a^x}{\log(a)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*cos(x),x, algorithm="fricas")
```

```
[Out] (cos(x)*log(a) + sin(x))*a^x/(log(a)^2 + 1)
```

Sympy [A] time = 1.06125, size = 107, normalized size = 3.45

$$\begin{cases} \frac{ixe^{-ix}\sin(x)}{2} + \frac{xe^{-ix}\cos(x)}{2} + \frac{ie^{-ix}\cos(x)}{2} & \text{for}\ a = e^{-i} \\ -\frac{ixe^{ix}\sin(x)}{2} + \frac{xe^{ix}\cos(x)}{2} - \frac{ie^{ix}\cos(x)}{2} & \text{for}\ a = e^{i} \\ \frac{a^x\log(a)\cos(x)}{\log(a)^2+1} + \frac{a^x\sin(x)}{\log(a)^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x*cos(x),x)
```

```
[Out] Piecewise((I*x*exp(-I*x)*sin(x)/2 + x*exp(-I*x)*cos(x)/2 + I*exp(-I*x)*cos(
x)/2, Eq(a, exp(-I))), (-I*x*exp(I*x)*sin(x)/2 + x*exp(I*x)*cos(x)/2 - I*ex
p(I*x)*cos(x)/2, Eq(a, exp(I))), (a**x*log(a)*cos(x)/(log(a)**2 + 1) + a**x
*sin(x)/(log(a)**2 + 1), True))
```

Giac [C] time = 1.09182, size = 444, normalized size = 14.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*cos(x),x, algorithm="giac")
```

```
[Out] abs(a)^x*(2*cos(1/2*pi*x*sgn(a) - 1/2*pi*x + x)*log(abs(a))/((pi - pi*sgn(a
) - 2)^2 + 4*log(abs(a))^2) - (pi - pi*sgn(a) - 2)*sin(1/2*pi*x*sgn(a) - 1/
2*pi*x + x)/((pi - pi*sgn(a) - 2)^2 + 4*log(abs(a))^2)) + abs(a)^x*(2*cos(1
/2*pi*x*sgn(a) - 1/2*pi*x - x)*log(abs(a))/((pi - pi*sgn(a) + 2)^2 + 4*log(
abs(a))^2) - (pi - pi*sgn(a) + 2)*sin(1/2*pi*x*sgn(a) - 1/2*pi*x - x)/((pi
- pi*sgn(a) + 2)^2 + 4*log(abs(a))^2)) - 1/2*I*abs(a)^x*(-2*I*e^(1/2*I*pi*x
*sgn(a) - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sgn(a) + 4*log(abs(a)) + 4*I)
 + 2*I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*x - I*x)/(2*I*pi - 2*I*pi*sgn(a) +
4*log(abs(a)) - 4*I)) - 1/2*I*abs(a)^x*(-2*I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*p
i*x - I*x)/(-2*I*pi + 2*I*pi*sgn(a) + 4*log(abs(a)) - 4*I) + 2*I*e^(-1/2*I*
pi*x*sgn(a) + 1/2*I*pi*x + I*x)/(2*I*pi - 2*I*pi*sgn(a) + 4*log(abs(a)) + 4
*I))
```

3.80 $\int \cos(\log(x))\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x\sin(\log(x)) + \frac{1}{2}x\cos(\log(x))$$

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rubi [A] time = 0.0029857, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4476}

$$\frac{1}{2}x\sin(\log(x)) + \frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Log[x]],x]
```

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rule 4476

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(x*
Cos[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] + Simp[(b*d*n*x*Sin[d*(a +
 b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\int \cos(\log(x))\,dx = \frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

Mathematica [A] time = 0.0022366, size = 17, normalized size = 1.

$$\frac{1}{2}x\sin(\log(x)) + \frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Log[x]],x]
```

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Maple [A] time = 0., size = 14, normalized size = 0.8

$$\frac{x\cos(\ln(x))}{2} + \frac{x\sin(\ln(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(ln(x)),x)
```

```
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Maxima [A] time = 0.933118, size = 14, normalized size = 0.82

$$\frac{1}{2}x(\cos(\log(x)) + \sin(\log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x*(cos(log(x)) + sin(log(x)))
```

Fricas [A] time = 2.18866, size = 53, normalized size = 3.12

$$\frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A] time = 0.369784, size = 15, normalized size = 0.88

$$\frac{x\sin(\log(x))}{2} + \frac{x\cos(\log(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(ln(x)),x)
```

```
[Out] x*sin(log(x))/2 + x*cos(log(x))/2
```

Giac [A] time = 1.05382, size = 18, normalized size = 1.06

$$\frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="giac")
```

```
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

3.81 $\int \log(\cos(x)) \sec^2(x) \, dx$

Optimal. Leaf size=12

$$-x+\tan(x)+\tan(x)\log(\cos(x))$$

```
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Rubi [A] time = 0.019235, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3767, 8, 2554, 3473}

$$-x+\tan(x)+\tan(x)\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cos[x]]*Sec[x]^2,x]

[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x] - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x]] /; InverseFunctionFreeQ[u, x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \log(\cos(x))\sec^2(x)\,dx &= \log(\cos(x))\tan(x)+\int \tan^2(x)\,dx\\
&= \tan(x)+\log(\cos(x))\tan(x)-\int 1\,dx\\
&= -x+\tan(x)+\log(\cos(x))\tan(x)
\end{aligned}$$

Mathematica [A] time = 0.015091, size = 12, normalized size = 1.

$$-x+\tan(x)+\tan(x)\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cos[x]]*Sec[x]^2,x]
```

```
[Out] -x + Tan[x] + Log[Cos[x]]*Tan[x]
```

Maple [C] time = 0.037, size = 61, normalized size = 5.1

$$\frac{-2\,i\mathrm{e}^{2\,ix}\ln\left(2\,\cos\left(x\right)\right)}{1+\mathrm{e}^{2\,ix}}+\frac{2\,i}{1+\mathrm{e}^{2\,ix}}+i\ln\left(1+\mathrm{e}^{2\,ix}\right)-\frac{2\,i\ln\left(2\right)}{1+\mathrm{e}^{2\,ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cos(x))*sec(x)^2,x)
```

```
[Out] -2*I/(1+exp(2*I*x))*exp(2*I*x)*ln(2*cos(x))+2*I/(1+exp(2*I*x))+I*ln(1+exp(2
*I*x))-2*I*ln(2)/(1+exp(2*I*x))
```

Maxima [B] time = 1.42825, size = 127, normalized size = 10.58

$$-\frac{2\,\log\left(-\frac{\frac{\sin(x)^2}{(\cos(x)+1)^2}-1}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}\right)\sin\left(x\right)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)\left(\cos\left(x\right)+1\right)}-\frac{2\,\sin\left(x\right)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}-1\right)\left(\cos\left(x\right)+1\right)}-2\,\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="maxima")
```

```
[Out] -2*log(-(sin(x)^2/(cos(x) + 1)^2 - 1)/(sin(x)^2/(cos(x) + 1)^2 + 1))*sin(x)
/((sin(x)^2/(cos(x) + 1)^2 - 1)*(cos(x) + 1)) - 2*sin(x)/((sin(x)^2/(cos(x)
 + 1)^2 - 1)*(cos(x) + 1)) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 1.8679, size = 68, normalized size = 5.67

$$-\frac{x\cos\left(x\right)-\log\left(\cos\left(x\right)\right)\sin\left(x\right)-\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="fricas")
```

```
[Out] -(x*cos(x) - log(cos(x))*sin(x) - sin(x))/cos(x)
```

Sympy [A] time = 135.385, size = 15, normalized size = 1.25

$$-x+\log\left(\cos\left(x\right)\right)\tan\left(x\right)+\frac{\sin\left(x\right)}{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cos(x))*sec(x)**2,x)
```

```
[Out] -x + log(cos(x))*tan(x) + sin(x)/cos(x)
```

Giac [A] time = 1.0774, size = 16, normalized size = 1.33

$$\log(\cos(x))\tan(x) - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(x))*sec(x)^2,x, algorithm="giac")
```

```
[Out] log(cos(x))*tan(x) - x + tan(x)
```

3.82 $\int x \tan^2(x) \, dx$

Optimal. Leaf size=15

$$-\frac{x^2}{2} + x\tan(x) + \log(\cos(x))$$

```
[Out] -x^2/2 + Log[Cos[x]] + x*Tan[x]
```

Rubi [A] time = 0.0143883, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3720, 3475, 30}

$$-\frac{x^2}{2} + x\tan(x) + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Tan[x]^2,x]

[Out] -x^2/2 + Log[Cos[x]] + x*Tan[x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int x\tan^2(x)\,dx &= x\tan(x) - \int x\,dx - \int \tan(x)\,dx \\ &= -\frac{x^2}{2} + \log(\cos(x)) + x\tan(x) \end{aligned}$$

Mathematica [A] time = 0.0181929, size = 15, normalized size = 1.

$$-\frac{x^2}{2} + x\tan(x) + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tan[x]^2,x]
```

```
[Out] -x^2/2 + Log[Cos[x]] + x*Tan[x]
```

Maple [A] time = 0.009, size = 20, normalized size = 1.3

$$x\tan(x) - \frac{x^2}{2} - \frac{\ln\left((\tan(x))^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(x)^2,x)
```

```
[Out] x*tan(x)-1/2*x^2-1/2*ln(tan(x)^2+1)
```

Maxima [B] time = 1.42822, size = 144, normalized size = 9.6

$$-\frac{x^2\cos(2x)^2 + x^2\sin(2x)^2 + 2x^2\cos(2x) + x^2 - \left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right)\log\left(\cos(2x)^2 + \sin(2x\right.}{2\left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x^2*cos(2*x)^2 + x^2*sin(2*x)^2 + 2*x^2*cos(2*x) + x^2 - (cos(2*x)^2
+ sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1
) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A] time = 2.02897, size = 66, normalized size = 4.4

$$-\frac{1}{2}x^2 + x\tan(x) + \frac{1}{2}\log\left(\frac{1}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x*tan(x) + 1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] time = 0.160963, size = 19, normalized size = 1.27

$$-\frac{x^2}{2} + x\tan(x) - \frac{\log\left(\tan^2(x) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)**2,x)
```

```
[Out] -x**2/2 + x*tan(x) - log(tan(x)**2 + 1)/2
```

Giac [A] time = 1.07227, size = 31, normalized size = 2.07

$$-\frac{1}{2}x^2 + x\tan(x) + \frac{1}{2}\log\left(\frac{4}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^2 + x*tan(x) + 1/2*log(4/(tan(x)^2 + 1))
```

3.83 $\int \frac{\sin^{-1}(x)}{x^2}\,dx$

Optimal. Leaf size=22

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sin^{-1}(x)}{x}$$

```
[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

Rubi [A] time = 0.0152534, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4627, 266, 63, 206}

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sin^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]/x^2,x]

[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^{-1}(x)}{x^2}\,dx &= -\frac{\sin^{-1}(x)}{x} + \int \frac{1}{x\sqrt{1-x^2}}\,dx\\ &= -\frac{\sin^{-1}(x)}{x} + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\,dx, x, x^2\right)\\ &= -\frac{\sin^{-1}(x)}{x} - \operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-x^2}\right)\\ &= -\frac{\sin^{-1}(x)}{x} - \tanh^{-1}\left(\sqrt{1-x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0021597, size = 22, normalized size = 1.

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right) - \frac{\sin^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]/x^2,x]
```

```
[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

Maple [A] time = 0.003, size = 21, normalized size = 1.

$$-\frac{\arcsin(x)}{x} - \operatorname{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)/x^2,x)
```

```
[Out] -arcsin(x)/x-arctanh(1/(-x^2+1)^(1/2))
```

Maxima [A] time = 1.41977, size = 45, normalized size = 2.05

$$-\frac{\arcsin(x)}{x} - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2,x, algorithm="maxima")
```

```
[Out] -arcsin(x)/x - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A] time = 2.22189, size = 105, normalized size = 4.77

$$-\frac{x\log\left(\sqrt{-x^2+1}+1\right) - x\log\left(\sqrt{-x^2+1}-1\right) + 2\arcsin(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(x*log(sqrt(-x^2 + 1) + 1) - x*log(sqrt(-x^2 + 1) - 1) + 2*arcsin(x))/
x
```

Sympy [A] time = 1.65224, size = 22, normalized size = 1.

$$\begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \frac{1}{|x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases} - \frac{\operatorname{asin}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)/x**2,x)
```

```
[Out] Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True)) - asin(x)/x
```

Giac [A] time = 1.06031, size = 51, normalized size = 2.32

$$-\frac{\arcsin(x)}{x} - \frac{1}{2}\log\left(\sqrt{-x^2+1}+1\right) + \frac{1}{2}\log\left(-\sqrt{-x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2,x, algorithm="giac")
```

```
[Out] -arcsin(x)/x - 1/2*log(sqrt(-x^2 + 1) + 1) + 1/2*log(-sqrt(-x^2 + 1) + 1)
```

3.84 $\int \sin^{-1}(x)^2 \, dx$

Optimal. Leaf size=25

$$2\sqrt{1-x^2}\sin^{-1}(x) - 2x + x\sin^{-1}(x)^2$$

```
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Rubi [A] time = 0.0323284, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {4619, 4677, 8}

$$2\sqrt{1-x^2}\sin^{-1}(x) - 2x + x\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]^2,x]

[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4677

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p +
 1)), x] + Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \sin^{-1}(x)^2 \, dx &= x\sin^{-1}(x)^2 - 2\int \frac{x\sin^{-1}(x)}{\sqrt{1-x^2}} \, dx \\
&= 2\sqrt{1-x^2}\sin^{-1}(x) + x\sin^{-1}(x)^2 - 2\int 1 \, dx \\
&= -2x + 2\sqrt{1-x^2}\sin^{-1}(x) + x\sin^{-1}(x)^2
\end{aligned}$$

Mathematica [A] time = 0.0058889, size = 25, normalized size = 1.

$$2\sqrt{1-x^2}\sin^{-1}(x) - 2x + x\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]^2,x]
```

```
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Maple [A] time = 0.019, size = 24, normalized size = 1.

$$-2x + x(\arcsin(x))^2 + 2\arcsin(x)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)^2,x)
```

```
[Out] -2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.42581, size = 31, normalized size = 1.24

$$x\arcsin(x)^2 + 2\sqrt{-x^2+1}\arcsin(x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="maxima")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Fricas [A] time = 1.8521, size = 68, normalized size = 2.72

$$x\arcsin(x)^2 + 2\sqrt{-x^2+1}\arcsin(x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="fricas")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Sympy [A] time = 0.177275, size = 22, normalized size = 0.88

$$x\operatorname{asin}^2(x) - 2x + 2\sqrt{1-x^2}\operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)**2,x)
```

```
[Out] x*asin(x)**2 - 2*x + 2*sqrt(1 - x**2)*asin(x)
```

Giac [A] time = 1.05263, size = 31, normalized size = 1.24

$$x\arcsin(x)^2 + 2\sqrt{-x^2+1}\arcsin(x) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="giac")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

3.85 $\int \frac{x^2 \tan^{-1}(x)}{1+x^2} \, dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)^2+x\tan^{-1}(x)$$

```
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Rubi [A] time = 0.0474393, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4916, 4846, 260, 4884}

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)^2+x\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[x])/(1 + x^2),x]
```

```
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \tan^{-1}(x)}{1+x^2} \, dx &= \int \tan^{-1}(x) \, dx - \int \frac{\tan^{-1}(x)}{1+x^2} \, dx \\
&= x\tan^{-1}(x) - \frac{1}{2}\tan^{-1}(x)^2 - \int \frac{x}{1+x^2} \, dx \\
&= x\tan^{-1}(x) - \frac{1}{2}\tan^{-1}(x)^2 - \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0140977, size = 23, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)^2+x\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[x])/(1 + x^2),x]
```

```
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Maple [A] time = 0.006, size = 20, normalized size = 0.9

$$x\arctan(x)-\frac{(\arctan(x))^2}{2}-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x)/(x^2+1),x)
```

```
[Out] x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)
```

Maxima [A] time = 1.45109, size = 32, normalized size = 1.39

$$(x-\arctan(x))\arctan(x)+\frac{1}{2}\arctan(x)^2-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] (x - arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

Fricas [A] time = 2.02677, size = 68, normalized size = 2.96

$$x\arctan(x)-\frac{1}{2}\arctan(x)^2-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

Sympy [A] time = 0.380675, size = 19, normalized size = 0.83

$$x\operatorname{atan}(x)-\frac{\log\left(x^2+1\right)}{2}-\frac{\operatorname{atan}^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x)/(x**2+1),x)
```

```
[Out] x*atan(x) - log(x**2 + 1)/2 - atan(x)**2/2
```

Giac [A] time = 1.05476, size = 26, normalized size = 1.13

$$x\arctan(x) - \frac{1}{2}\arctan(x)^2 - \frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="giac")
```

```
[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

3.86 $\int \cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) dx$

Optimal. Leaf size=38

$$(x+1)\left(\sqrt{\frac{1}{x+1}}\sqrt{\frac{x}{x+1}}+\cos^{-1}\left(\sqrt{\frac{x}{x+1}}\right)\right)$$

```
[Out] (1 + x)*(Sqrt[(1 + x)^(-1)]*Sqrt[x/(1 + x)] + ArcCos[Sqrt[x/(1 + x)]])
```

Rubi [A] time = 0.0305641, antiderivative size = 57, normalized size of antiderivative = 1.5, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4841, 12, 6719, 50, 63, 203}

$$\sqrt{\frac{x}{(x+1)^2}}(x+1)+x\cos^{-1}\left(\sqrt{\frac{x}{x+1}}\right)-\frac{\sqrt{\frac{x}{(x+1)^2}}(x+1)\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[Sqrt[x/(1 + x)]],x]
```

```
[Out] Sqrt[x/(1 + x)^2]*(1 + x) + x*ArcCos[Sqrt[x/(1 + x)]] - (Sqrt[x/(1 + x)^2]*
(1 + x)*ArcTan[Sqrt[x]])/Sqrt[x]
```

Rule 4841

```
Int[ArcCos[u_], x_Symbol] :> Simp[x*ArcCos[u], x] + Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] && !Functio
nOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) dx &= x\cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) + \int \frac{1}{2}\sqrt{\frac{x}{(1+x)^2}}\, dx \\
&= x\cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) + \frac{1}{2}\int \sqrt{\frac{x}{(1+x)^2}}\, dx \\
&= x\cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) + \frac{\left(\sqrt{\frac{x}{(1+x)^2}}(1+x)\right)\int \frac{\sqrt{x}}{1+x}\, dx}{2\sqrt{x}} \\
&= \sqrt{\frac{x}{(1+x)^2}}(1+x) + x\cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) - \frac{\left(\sqrt{\frac{x}{(1+x)^2}}(1+x)\right)\int \frac{1}{\sqrt{x}(1+x)}\, dx}{2\sqrt{x}} \\
&= \sqrt{\frac{x}{(1+x)^2}}(1+x) + x\cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) - \frac{\left(\sqrt{\frac{x}{(1+x)^2}}(1+x)\right)\text{Subst}\left(\int \frac{1}{1+x^2}\, dx, x, \sqrt{x}\right)}{\sqrt{x}} \\
&= \sqrt{\frac{x}{(1+x)^2}}(1+x) + x\cos^{-1}\left(\sqrt{\frac{x}{1+x}}\right) - \frac{\sqrt{\frac{x}{(1+x)^2}}(1+x)\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}
\end{aligned}
$$

Mathematica [A] time = 0.0594106, size = 49, normalized size = 1.29

$$
x\cos^{-1}\left(\sqrt{\frac{x}{x+1}}\right) + \frac{\sqrt{\frac{x}{(x+1)^2}}(x+1)\left(\sqrt{x} - \tan^{-1}\left(\sqrt{x}\right)\right)}{\sqrt{x}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[Sqrt[x/(1 + x)]],x]
```

```
[Out] x*ArcCos[Sqrt[x/(1 + x)]] + (Sqrt[x/(1 + x)^2]*(1 + x)*(Sqrt[x] - ArcTan[Sq
rt[x]]))/Sqrt[x]
```

Maple [A] time = 0.012, size = 44, normalized size = 1.2

$$
x\arccos\left(\sqrt{\frac{x}{1+x}}\right) + \sqrt{x}\sqrt{(1+x)^{-1}}\left(\sqrt{x} - \arctan\left(\sqrt{x}\right)\right)\frac{1}{\sqrt{\frac{x}{1+x}}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos((x/(1+x))^(1/2)),x)
```

```
[Out] x*arccos((x/(1+x))^(1/2))+1/(x/(1+x))^(1/2)*x^(1/2)*(1/(1+x))^(1/2)*(x^(1/2
)-arctan(x^(1/2)))
```

Maxima [B] time = 1.42552, size = 105, normalized size = 2.76

$$-\frac{\arccos\left(\sqrt{\frac{x}{x+1}}\right)}{\frac{x}{x+1}-1}-\frac{\sqrt{-\frac{x}{x+1}+1}}{2\left(\sqrt{\frac{x}{x+1}}+1\right)}-\frac{\sqrt{-\frac{x}{x+1}+1}}{2\left(\sqrt{\frac{x}{x+1}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos((x/(1+x))^(1/2)),x, algorithm="maxima")
```

```
[Out] -arccos(sqrt(x/(x + 1)))/(x/(x + 1) - 1) - 1/2*sqrt(-x/(x + 1) + 1)/(sqrt(x
/(x + 1)) + 1) - 1/2*sqrt(-x/(x + 1) + 1)/(sqrt(x/(x + 1)) - 1)
```

Fricas [A] time = 2.00843, size = 85, normalized size = 2.24

$$(x+1)\arccos\left(\sqrt{\frac{x}{x+1}}\right)+\sqrt{x+1}\sqrt{\frac{x}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos((x/(1+x))^(1/2)),x, algorithm="fricas")
```

```
[Out] (x + 1)*arccos(sqrt(x/(x + 1))) + sqrt(x + 1)*sqrt(x/(x + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acos}\left(\sqrt{\frac{x}{x+1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos((x/(1+x))**(1/2)),x)
```

```
[Out] Integral(acos(sqrt(x/(x + 1))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos((x/(1+x))^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.87 $\int \left(2x+3x^2\right)^3 \, dx$

Optimal. Leaf size=25

$$\frac{27x^7}{7}+9x^6+\frac{36x^5}{5}+2x^4$$

```
[Out] 2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

Rubi [A] time = 0.0071173, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {611}

$$\frac{27x^7}{7}+9x^6+\frac{36x^5}{5}+2x^4$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + 3*x^2)^3,x]
```

```
[Out] 2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rubi steps

$$\begin{aligned}\int \left(2x+3x^2\right)^3 \, dx &= \int \left(8x^3+36x^4+54x^5+27x^6\right) \, dx\\ &= 2x^4+\frac{36x^5}{5}+9x^6+\frac{27x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0016558, size = 25, normalized size = 1.

$$\frac{27x^7}{7}+9x^6+\frac{36x^5}{5}+2x^4$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + 3*x^2)^3,x]
```

```
[Out] 2*x^4 + (36*x^5)/5 + 9*x^6 + (27*x^7)/7
```

Maple [A] time = 0., size = 22, normalized size = 0.9

$$2\,x^4+\frac{36\,x^5}{5}+9\,x^6+\frac{27\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2*x)^3,x)
```

```
[Out] 2*x^4+36/5*x^5+9*x^6+27/7*x^7
```

Maxima [A] time = 0.929313, size = 28, normalized size = 1.12

$$\frac{27}{7}\,x^{7}+9\,x^{6}+\frac{36}{5}\,x^{5}+2\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x)^3,x, algorithm="maxima")
```

```
[Out] 27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

Fricas [A] time = 1.57105, size = 50, normalized size = 2.

$$\frac{27}{7}x^{7}+9x^{6}+\frac{36}{5}x^{5}+2x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x)^3,x, algorithm="fricas")
```

```
[Out] 27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

Sympy [A] time = 0.054514, size = 22, normalized size = 0.88

$$\frac{27x^{7}}{7}+9x^{6}+\frac{36x^{5}}{5}+2x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2*x)**3,x)
```

```
[Out] 27*x**7/7 + 9*x**6 + 36*x**5/5 + 2*x**4
```

Giac [A] time = 1.04843, size = 28, normalized size = 1.12

$$\frac{27}{7}\,x^{7}+9\,x^{6}+\frac{36}{5}\,x^{5}+2\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x)^3,x, algorithm="giac")
```

```
[Out] 27/7*x^7 + 9*x^6 + 36/5*x^5 + 2*x^4
```

3.88 $\int (-1+x)\left(-1+2x+3x^2\right)^2 \, dx$

Optimal. Leaf size=39

$$\frac{3x^6}{2}+\frac{3x^5}{5}-\frac{7x^4}{2}-\frac{2x^3}{3}+\frac{5x^2}{2}-x$$

```
[Out] -x + (5*x^2)/2 - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

Rubi [A] time = 0.0157301, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {631}

$$\frac{3x^6}{2}+\frac{3x^5}{5}-\frac{7x^4}{2}-\frac{2x^3}{3}+\frac{5x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)*(-1 + 2*x + 3*x^2)^2,x]
```

```
[Out] -x + (5*x^2)/2 - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int (-1+x)\left(-1+2x+3x^2\right)^2 \, dx &= \int \left(-1+5x-2x^2-14x^3+3x^4+9x^5\right) \, dx\\ &= -x+\frac{5x^2}{2}-\frac{2x^3}{3}-\frac{7x^4}{2}+\frac{3x^5}{5}+\frac{3x^6}{2}\end{aligned}$$

Mathematica [A] time = 0.0014211, size = 39, normalized size = 1.

$$\frac{3x^6}{2}+\frac{3x^5}{5}-\frac{7x^4}{2}-\frac{2x^3}{3}+\frac{5x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)*(-1 + 2*x + 3*x^2)^2,x]
```

```
[Out] -x + (5*x^2)/2 - (2*x^3)/3 - (7*x^4)/2 + (3*x^5)/5 + (3*x^6)/2
```

Maple [A] time = 0., size = 30, normalized size = 0.8

$$-x+\frac{5x^2}{2}-\frac{2x^3}{3}-\frac{7x^4}{2}+\frac{3x^5}{5}+\frac{3x^6}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)*(3*x^2+2*x-1)^2,x)
```

```
[Out] -x+5/2*x^2-2/3*x^3-7/2*x^4+3/5*x^5+3/2*x^6
```

Maxima [A] time = 0.931304, size = 39, normalized size = 1.

$$\frac{3}{2}x^6 + \frac{3}{5}x^5 - \frac{7}{2}x^4 - \frac{2}{3}x^3 + \frac{5}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(3*x^2+2*x-1)^2,x, algorithm="maxima")
```

```
[Out] 3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

Fricas [A] time = 1.59602, size = 72, normalized size = 1.85

$$\frac{3}{2}x^6 + \frac{3}{5}x^5 - \frac{7}{2}x^4 - \frac{2}{3}x^3 + \frac{5}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(3*x^2+2*x-1)^2,x, algorithm="fricas")
```

```
[Out] 3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

Sympy [A] time = 0.058886, size = 34, normalized size = 0.87

$$\frac{3x^6}{2} + \frac{3x^5}{5} - \frac{7x^4}{2} - \frac{2x^3}{3} + \frac{5x^2}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(3*x**2+2*x-1)**2,x)
```

```
[Out] 3*x**6/2 + 3*x**5/5 - 7*x**4/2 - 2*x**3/3 + 5*x**2/2 - x
```

Giac [A] time = 1.06343, size = 39, normalized size = 1.

$$\frac{3}{2}x^6 + \frac{3}{5}x^5 - \frac{7}{2}x^4 - \frac{2}{3}x^3 + \frac{5}{2}x^2 - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)*(3*x^2+2*x-1)^2,x, algorithm="giac")
```

```
[Out] 3/2*x^6 + 3/5*x^5 - 7/2*x^4 - 2/3*x^3 + 5/2*x^2 - x
```

3.89 $\int x^{-1+k} \left(a+bx^k\right)^n \, dx$

Optimal. Leaf size=23

$$\frac{\left(a+bx^k\right)^{n+1}}{bk(n+1)}$$

```
[Out] (a + b*x^k)^(1 + n)/(b*k*(1 + n))
```

Rubi [A] time = 0.0077646, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {261}

$$\frac{\left(a+bx^k\right)^{n+1}}{bk(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + k)*(a + b*x^k)^n,x]
```

```
[Out] (a + b*x^k)^(1 + n)/(b*k*(1 + n))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^{-1+k} \left(a+bx^k\right)^n \, dx = \frac{\left(a+bx^k\right)^{1+n}}{bk(1+n)}$$

Mathematica [A] time = 0.0100025, size = 23, normalized size = 1.

$$\frac{\left(a+bx^k\right)^{n+1}}{bk(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + k)*(a + b*x^k)^n,x]
```

```
[Out] (a + b*x^k)^(1 + n)/(b*k*(1 + n))
```

Maple [A] time = 0.043, size = 29, normalized size = 1.3

$$\frac{\left(a+bx^k\right)\left(a+bx^k\right)^n}{b\,(1+n)\,k}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+k)*(a+b*x^k)^n,x)
```

```
[Out] (a+b*x^k)/b/(1+n)/k*(a+b*x^k)^n
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+k)*(a+b*x^k)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93785, size = 55, normalized size = 2.39

$$\frac{(bx^k + a)(bx^k + a)^n}{bkn + bk}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+k)*(a+b*x^k)^n,x, algorithm="fricas")
```

```
[Out] (b*x^k + a)*(b*x^k + a)^n/(b*k*n + b*k)
```

Sympy [A] time = 55.2727, size = 75, normalized size = 3.26

$$\begin{cases} \frac{\log(x)}{a} & \text{for}\ b = 0 \wedge k = 0 \wedge n = -1 \\ \frac{a^n x^k}{k} & \text{for}\ b = 0 \\ (a+b)^n \log(x) & \text{for}\ k = 0 \\ \frac{\log\left(\frac{a}{b} + x^k\right)}{bk} & \text{for}\ n = -1 \\ \frac{a(a+bx^k)^n}{bkn+bk} + \frac{bx^k(a+bx^k)^n}{bkn+bk} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+k)*(a+b*x**k)**n,x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(k, 0) & Eq(n, -1)), (a**n*x**k/k, Eq(b,
0)), ((a + b)**n*log(x), Eq(k, 0)), (log(a/b + x**k)/(b*k), Eq(n, -1)), (a*
(a + b*x**k)**n/(b*k*n + b*k) + b*x**k*(a + b*x**k)**n/(b*k*n + b*k), True)
)
```

Giac [A] time = 1.05887, size = 31, normalized size = 1.35

$$\frac{(bx^k + a)^{n+1}}{bk(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+k)*(a+b*x^k)^n,x, algorithm="giac")
```

```
[Out] (b*x^k + a)^(n + 1)/(b*k*(n + 1))
```

3.90 $\int \frac{x^3}{1+2x} \, dx$

Optimal. Leaf size=30

$$\frac{x^3}{6} - \frac{x^2}{8} + \frac{x}{8} - \frac{1}{16}\log(2x+1)$$

```
[Out] x/8 - x^2/8 + x^3/6 - Log[1 + 2*x]/16
```

Rubi [A] time = 0.0113922, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{x^3}{6} - \frac{x^2}{8} + \frac{x}{8} - \frac{1}{16}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + 2*x),x]
```

```
[Out] x/8 - x^2/8 + x^3/6 - Log[1 + 2*x]/16
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1+2x} \, dx &= \int \left(\frac{1}{8} - \frac{x}{4} + \frac{x^2}{2} - \frac{1}{8(1+2x)}\right) dx\\ &= \frac{x}{8} - \frac{x^2}{8} + \frac{x^3}{6} - \frac{1}{16}\log(1+2x)\end{aligned}$$

Mathematica [A] time = 0.0063221, size = 27, normalized size = 0.9

$$\frac{1}{96}\left(16x^3 - 12x^2 + 12x - 6\log(2x+1) + 11\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + 2*x),x]
```

```
[Out] (11 + 12*x - 12*x^2 + 16*x^3 - 6*Log[1 + 2*x])/96
```

Maple [A] time = 0.003, size = 23, normalized size = 0.8

$$\frac{x}{8} - \frac{x^2}{8} + \frac{x^3}{6} - \frac{\ln(1+2x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+2*x),x)
```

```
[Out] 1/8*x-1/8*x^2+1/6*x^3-1/16*ln(1+2*x)
```

Maxima [A] time = 0.929777, size = 30, normalized size = 1.

$$\frac{1}{6}x^3 - \frac{1}{8}x^2 + \frac{1}{8}x - \frac{1}{16}\log(2x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+2*x),x, algorithm="maxima")
```

```
[Out] 1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(2*x + 1)
```

Fricas [A] time = 1.7782, size = 63, normalized size = 2.1

$$\frac{1}{6}x^3 - \frac{1}{8}x^2 + \frac{1}{8}x - \frac{1}{16}\log(2x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+2*x),x, algorithm="fricas")
```

```
[Out] 1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(2*x + 1)
```

Sympy [A] time = 0.069682, size = 20, normalized size = 0.67

$$\frac{x^3}{6} - \frac{x^2}{8} + \frac{x}{8} - \frac{\log(2x+1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+2*x),x)
```

```
[Out] x**3/6 - x**2/8 + x/8 - log(2*x + 1)/16
```

Giac [A] time = 1.05833, size = 31, normalized size = 1.03

$$\frac{1}{6}x^3 - \frac{1}{8}x^2 + \frac{1}{8}x - \frac{1}{16}\log(|2x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+2*x),x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/8*x^2 + 1/8*x - 1/16*log(abs(2*x + 1))
```

3.91 $\int \frac{x^6}{2+3x^2} dx$

Optimal. Leaf size=41

$$\frac{x^5}{15}-\frac{2x^3}{27}+\frac{4x}{27}-\frac{4}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

```
[Out] (4*x)/27 - (2*x^3)/27 + x^5/15 - (4*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x])/27
```

Rubi [A] time = 0.0122719, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {302, 203}

$$\frac{x^5}{15}-\frac{2x^3}{27}+\frac{4x}{27}-\frac{4}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(2 + 3*x^2),x]
```

```
[Out] (4*x)/27 - (2*x^3)/27 + x^5/15 - (4*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x])/27
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{2+3x^2}\,dx &= \int \left(\frac{4}{27}-\frac{2x^2}{9}+\frac{x^4}{3}-\frac{8}{27\left(2+3x^2\right)}\right)dx\\ &= \frac{4x}{27}-\frac{2x^3}{27}+\frac{x^5}{15}-\frac{8}{27}\int \frac{1}{2+3x^2}\,dx\\ &= \frac{4x}{27}-\frac{2x^3}{27}+\frac{x^5}{15}-\frac{4}{27}\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\end{aligned}$$

Mathematica [A] time = 0.0098126, size = 35, normalized size = 0.85

$$\frac{1}{405}\left(27x^5-30x^3+60x-20\sqrt{6}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(2 + 3*x^2),x]
```

```
[Out] (60*x - 30*x^3 + 27*x^5 - 20*Sqrt[6]*ArcTan[Sqrt[3/2]*x])/405
```

Maple [A] time = 0.003, size = 27, normalized size = 0.7

$$\frac{4x}{27} - \frac{2x^3}{27} + \frac{x^5}{15} - \frac{4\sqrt{6}}{81}\arctan\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(3*x^2+2),x)
```

```
[Out] 4/27*x-2/27*x^3+1/15*x^5-4/81*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Maxima [A] time = 1.4167, size = 35, normalized size = 0.85

$$\frac{1}{15}x^5 - \frac{2}{27}x^3 - \frac{4}{81}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right) + \frac{4}{27}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2),x, algorithm="maxima")
```

```
[Out] 1/15*x^5 - 2/27*x^3 - 4/81*sqrt(6)*arctan(1/2*sqrt(6)*x) + 4/27*x
```

Fricas [A] time = 1.76267, size = 112, normalized size = 2.73

$$\frac{1}{15}x^5 - \frac{2}{27}x^3 - \frac{4}{81}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}x\right) + \frac{4}{27}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2),x, algorithm="fricas")
```

```
[Out] 1/15*x^5 - 2/27*x^3 - 4/81*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x) +
4/27*x
```

Sympy [A] time = 0.092286, size = 34, normalized size = 0.83

$$\frac{x^5}{15} - \frac{2x^3}{27} + \frac{4x}{27} - \frac{4\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(3*x**2+2),x)
```

```
[Out] x**5/15 - 2*x**3/27 + 4*x/27 - 4*sqrt(6)*atan(sqrt(6)*x/2)/81
```

Giac [A] time = 1.05415, size = 35, normalized size = 0.85

$$\frac{1}{15}x^5 - \frac{2}{27}x^3 - \frac{4}{81}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}x\right) + \frac{4}{27}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(3*x^2+2),x, algorithm="giac")
```

```
[Out] 1/15*x^5 - 2/27*x^3 - 4/81*sqrt(6)*arctan(1/2*sqrt(6)*x) + 4/27*x
```

3.92 $\int \frac{1}{2-7x+3x^2}\,dx$

Optimal. Leaf size=21

$$\frac{1}{5}\log(2-x)-\frac{1}{5}\log(1-3x)$$

```
[Out] -Log[1 - 3*x]/5 + Log[2 - x]/5
```

Rubi [A] time = 0.0055049, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {616, 31}

$$\frac{1}{5}\log(2-x)-\frac{1}{5}\log(1-3x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 7*x + 3*x^2)^(-1),x]
```

```
[Out] -Log[1 - 3*x]/5 + Log[2 - x]/5
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2-7x+3x^2}\,dx &= \frac{3}{5}\int \frac{1}{-6+3x}\,dx-\frac{3}{5}\int \frac{1}{-1+3x}\,dx\\ &= -\frac{1}{5}\log(1-3x)+\frac{1}{5}\log(2-x)\end{aligned}$$

Mathematica [A] time = 0.0029235, size = 21, normalized size = 1.

$$\frac{1}{5}\log(2-x)-\frac{1}{5}\log(1-3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 7*x + 3*x^2)^(-1),x]
```

```
[Out] -Log[1 - 3*x]/5 + Log[2 - x]/5
```

Maple [A] time = 0.005, size = 16, normalized size = 0.8

$$-\frac{\ln\left(3x-1\right)}{5}+\frac{\ln\left(-2+x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2-7*x+2),x)
```

```
[Out] -1/5*ln(3*x-1)+1/5*ln(-2+x)
```

Maxima [A] time = 0.934454, size = 20, normalized size = 0.95

$$-\frac{1}{5}\log\left(3x-1\right)+\frac{1}{5}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-7*x+2),x, algorithm="maxima")
```

```
[Out] -1/5*log(3*x - 1) + 1/5*log(x - 2)
```

Fricas [A] time = 1.77085, size = 49, normalized size = 2.33

$$-\frac{1}{5}\log\left(3x-1\right)+\frac{1}{5}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-7*x+2),x, algorithm="fricas")
```

```
[Out] -1/5*log(3*x - 1) + 1/5*log(x - 2)
```

Sympy [A] time = 0.089842, size = 14, normalized size = 0.67

$$\frac{\log\left(x-2\right)}{5}-\frac{\log\left(x-\frac{1}{3}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2-7*x+2),x)
```

```
[Out] log(x - 2)/5 - log(x - 1/3)/5
```

Giac [A] time = 1.04913, size = 23, normalized size = 1.1

$$-\frac{1}{5}\log\left(|3x-1|\right)+\frac{1}{5}\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2-7*x+2),x, algorithm="giac")

[Out] -1/5*log(abs(3*x - 1)) + 1/5*log(abs(x - 2))
```

3.93 $\int \frac{-1+3x}{1-x+x^2}\,dx$

Optimal. Leaf size=33

$$\frac{3}{2}\log\left(x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + (3*Log[1 - x + x^2])/2
```

Rubi [A] time = 0.0167352, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {634, 618, 204, 628}

$$\frac{3}{2}\log\left(x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x)/(1 - x + x^2),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) + (3*Log[1 - x + x^2])/2
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+3x}{1-x+x^2}\,dx &= \frac{1}{2}\int \frac{1}{1-x+x^2}\,dx + \frac{3}{2}\int \frac{-1+2x}{1-x+x^2}\,dx\\ &= \frac{3}{2}\log\left(1-x+x^2\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right)\\ &= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{3}{2}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0085975, size = 32, normalized size = 0.97

$$\frac{3}{2}\log\left(x^2-x+1\right) + \frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x)/(1 - x + x^2),x]
```

```
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] + (3*Log[1 - x + x^2])/2
```

Maple [A] time = 0.003, size = 29, normalized size = 0.9

$$\frac{3\,\ln\left(x^2-x+1\right)}{2} + \frac{\sqrt{3}}{3}\arctan\left(\frac{(2\,x-1)\,\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x-1)/(x^2-x+1),x)
```

```
[Out] 3/2*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.41596, size = 38, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{3}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+3*x)/(x^2-x+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*log(x^2 - x + 1)
```

Fricas [A] time = 1.81856, size = 89, normalized size = 2.7

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,x-1)\right) + \frac{3}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+3*x)/(x^2-x+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*log(x^2 - x + 1)
```

Sympy [A] time = 0.100851, size = 36, normalized size = 1.09

$$\frac{3\log\left(x^2-x+1\right)}{2}+\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+3*x)/(x**2-x+1),x)
```

```
[Out] 3*log(x**2 - x + 1)/2 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3
```

Giac [A] time = 1.05432, size = 38, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{3}{2}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+3*x)/(x^2-x+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 3/2*log(x^2 - x + 1)
```

3.94 $\int \frac{x^2}{5+2x+x^2} \, dx$

Optimal. Leaf size=25

$$-\log \left(x^2+2x+5\right)+x-\frac{3}{2}\tan^{-1}\left(\frac{x+1}{2}\right)$$

```
[Out] x - (3*ArcTan[(1 + x)/2])/2 - Log[5 + 2*x + x^2]
```

Rubi [A] time = 0.0159191, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {703, 634, 618, 204, 628}

$$-\log \left(x^2+2x+5\right)+x-\frac{3}{2}\tan^{-1}\left(\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(5 + 2*x + x^2),x]
```

```
[Out] x - (3*ArcTan[(1 + x)/2])/2 - Log[5 + 2*x + x^2]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{5+2x+x^2}\,dx &= x+\int \frac{-5-2x}{5+2x+x^2}\,dx\\ &= x-3\int \frac{1}{5+2x+x^2}\,dx-\int \frac{2+2x}{5+2x+x^2}\,dx\\ &= x-\log\left(5+2x+x^2\right)+6\,\text{Subst}\left(\int \frac{1}{-16-x^2}\,dx,x,2+2x\right)\\ &= x-\frac{3}{2}\tan^{-1}\left(\frac{1+x}{2}\right)-\log\left(5+2x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0036703, size = 25, normalized size = 1.

$$-\log\left(x^2+2x+5\right)+x-\frac{3}{2}\tan^{-1}\left(\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(5 + 2*x + x^2),x]
```

```
[Out] x - (3*ArcTan[(1 + x)/2])/2 - Log[5 + 2*x + x^2]
```

Maple [A] time = 0.003, size = 22, normalized size = 0.9

$$x-\frac{3}{2}\arctan\left(\frac{1}{2}+\frac{x}{2}\right)-\ln\left(x^2+2x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+2*x+5),x)
```

```
[Out] x-3/2*arctan(1/2+1/2*x)-ln(x^2+2*x+5)
```

Maxima [A] time = 1.40652, size = 28, normalized size = 1.12

$$x-\frac{3}{2}\arctan\left(\frac{1}{2}x+\frac{1}{2}\right)-\log\left(x^2+2x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+5),x, algorithm="maxima")
```

```
[Out] x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)
```

Fricas [A] time = 1.75202, size = 68, normalized size = 2.72

$$x-\frac{3}{2}\arctan\left(\frac{1}{2}x+\frac{1}{2}\right)-\log\left(x^2+2x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+5),x, algorithm="fricas")
```

```
[Out] x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)
```

Sympy [A] time = 0.096622, size = 22, normalized size = 0.88

$$x - \log\left(x^2 + 2x + 5\right) - \frac{3\,\mathrm{atan}\left(\frac{x}{2} + \frac{1}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+2*x+5),x)
```

```
[Out] x - log(x**2 + 2*x + 5) - 3*atan(x/2 + 1/2)/2
```

Giac [A] time = 1.04478, size = 28, normalized size = 1.12

$$x - \frac{3}{2}\arctan\left(\frac{1}{2}\,x + \frac{1}{2}\right) - \log\left(x^2 + 2\,x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+5),x, algorithm="giac")
```

```
[Out] x - 3/2*arctan(1/2*x + 1/2) - log(x^2 + 2*x + 5)
```

3.95 $\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx$

Optimal. Leaf size=47

$$x^3-\frac{x^2}{2}+\frac{1}{4}\log\left(2x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{2\sqrt{7}}$$

```
[Out] -x^2/2 + x^3 - ArcTan[(1 - 4*x)/Sqrt[7]]/(2*Sqrt[7]) + Log[1 - x + 2*x^2]/4
```

Rubi [A] time = 0.0623759, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1594, 1628, 634, 618, 204, 628}

$$x^3-\frac{x^2}{2}+\frac{1}{4}\log\left(2x^2-x+1\right)-\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(4*x^2 - 5*x^3 + 6*x^4)/(1 - x + 2*x^2),x]
```

```
[Out] -x^2/2 + x^3 - ArcTan[(1 - 4*x)/Sqrt[7]]/(2*Sqrt[7]) + Log[1 - x + 2*x^2]/4
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{4x^2-5x^3+6x^4}{1-x+2x^2}\,dx &= \int \frac{x^2\left(4-5x+6x^2\right)}{1-x+2x^2}\,dx\\
&= \int \left(-x+3x^2+\frac{x}{1-x+2x^2}\right)dx\\
&= -\frac{x^2}{2}+x^3+\int \frac{x}{1-x+2x^2}\,dx\\
&= -\frac{x^2}{2}+x^3+\frac{1}{4}\int \frac{1}{1-x+2x^2}\,dx+\frac{1}{4}\int \frac{-1+4x}{1-x+2x^2}\,dx\\
&= -\frac{x^2}{2}+x^3+\frac{1}{4}\log\left(1-x+2x^2\right)-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-7-x^2}\,dx,x,-1+4x\right)\\
&= -\frac{x^2}{2}+x^3-\frac{\tan^{-1}\left(\frac{1-4x}{\sqrt{7}}\right)}{2\sqrt{7}}+\frac{1}{4}\log\left(1-x+2x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0159531, size = 47, normalized size = 1.

$$x^3-\frac{x^2}{2}+\frac{1}{4}\log\left(2x^2-x+1\right)+\frac{\tan^{-1}\left(\frac{4x-1}{\sqrt{7}}\right)}{2\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*x^2 - 5*x^3 + 6*x^4)/(1 - x + 2*x^2),x]
```

```
[Out] -x^2/2 + x^3 + ArcTan[(-1 + 4*x)/Sqrt[7]]/(2*Sqrt[7]) + Log[1 - x + 2*x^2]/
4
```

Maple [A] time = 0.004, size = 39, normalized size = 0.8

$$x^3-\frac{x^2}{2}+\frac{\ln\left(2x^2-x+1\right)}{4}+\frac{\sqrt{7}}{14}\arctan\left(\frac{(-1+4x)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x)
```

```
[Out] x^3-1/2*x^2+1/4*ln(2*x^2-x+1)+1/14*7^(1/2)*arctan(1/7*(-1+4*x)*7^(1/2))
```

Maxima [A] time = 1.41874, size = 51, normalized size = 1.09

$$x^3-\frac{1}{2}x^2+\frac{1}{14}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x-1)\right)+\frac{1}{4}\log\left(2x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x, algorithm="maxima")
```

```
[Out] x^3 - 1/2*x^2 + 1/14*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1)) + 1/4*log(2*x^2
- x + 1)
```

Fricas [A] time = 1.85802, size = 115, normalized size = 2.45

$$x^3 - \frac{1}{2}x^2 + \frac{1}{14}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x-1)\right) + \frac{1}{4}\log\left(2x^2 - x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x, algorithm="fricas")
```

```
[Out] x^3 - 1/2*x^2 + 1/14*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1)) + 1/4*log(2*x^2
- x + 1)
```

Sympy [A] time = 0.10346, size = 46, normalized size = 0.98

$$x^3 - \frac{x^2}{2} + \frac{\log\left(x^2 - \frac{x}{2} + \frac{1}{2}\right)}{4} + \frac{\sqrt{7}\operatorname{atan}\left(\frac{4\sqrt{7}x}{7} - \frac{\sqrt{7}}{7}\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x**4-5*x**3+4*x**2)/(2*x**2-x+1),x)
```

```
[Out] x**3 - x**2/2 + log(x**2 - x/2 + 1/2)/4 + sqrt(7)*atan(4*sqrt(7)*x/7 - sqrt
(7)/7)/14
```

Giac [A] time = 1.04706, size = 51, normalized size = 1.09

$$x^3 - \frac{1}{2}x^2 + \frac{1}{14}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x-1)\right) + \frac{1}{4}\log\left(2x^2 - x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^4-5*x^3+4*x^2)/(2*x^2-x+1),x, algorithm="giac")
```

```
[Out] x^3 - 1/2*x^2 + 1/14*sqrt(7)*arctan(1/7*sqrt(7)*(4*x - 1)) + 1/4*log(2*x^2
- x + 1)
```

3.96 $\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}\log(2-x)+\frac{\log(x)}{6}+\frac{1}{3}\log(x+3)$$

```
[Out] Log[2 - x]/2 + Log[x]/6 + Log[3 + x]/3
```

Rubi [A] time = 0.0365624, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1594, 1628}

$$\frac{1}{2}\log(2-x)+\frac{\log(x)}{6}+\frac{1}{3}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x + x^2)/(-6*x + x^2 + x^3),x]
```

```
[Out] Log[2 - x]/2 + Log[x]/6 + Log[3 + x]/3
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x+x^2}{-6x+x^2+x^3}\,dx &= \int \frac{-1+x+x^2}{x\left(-6+x+x^2\right)}\,dx\\ &= \int \left(\frac{1}{2(-2+x)}+\frac{1}{6x}+\frac{1}{3(3+x)}\right)dx\\ &= \frac{1}{2}\log(2-x)+\frac{\log(x)}{6}+\frac{1}{3}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.005858, size = 25, normalized size = 1.

$$\frac{1}{2}\log(2-x)+\frac{\log(x)}{6}+\frac{1}{3}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + x^2)/(-6*x + x^2 + x^3),x]
```

```
[Out] Log[2 - x]/2 + Log[x]/6 + Log[3 + x]/3
```

Maple [A] time = 0.008, size = 18, normalized size = 0.7

$$\frac{\ln(x)}{6}+\frac{\ln(3+x)}{3}+\frac{\ln(-2+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x-1)/(x^3+x^2-6*x),x)
```

```
[Out] 1/6*ln(x)+1/3*ln(3+x)+1/2*ln(-2+x)
```

Maxima [A] time = 0.931829, size = 23, normalized size = 0.92

$$\frac{1}{3}\log(x+3)+\frac{1}{2}\log(x-2)+\frac{1}{6}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(x^3+x^2-6*x),x, algorithm="maxima")
```

```
[Out] 1/3*log(x + 3) + 1/2*log(x - 2) + 1/6*log(x)
```

Fricas [A] time = 1.88495, size = 62, normalized size = 2.48

$$\frac{1}{3}\log(x+3)+\frac{1}{2}\log(x-2)+\frac{1}{6}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(x^3+x^2-6*x),x, algorithm="fricas")
```

```
[Out] 1/3*log(x + 3) + 1/2*log(x - 2) + 1/6*log(x)
```

Sympy [A] time = 0.117081, size = 17, normalized size = 0.68

$$\frac{\log(x)}{6}+\frac{\log(x-2)}{2}+\frac{\log(x+3)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x-1)/(x**3+x**2-6*x),x)
```

```
[Out] log(x)/6 + log(x - 2)/2 + log(x + 3)/3
```

Giac [A] time = 1.06005, size = 27, normalized size = 1.08

$$\frac{1}{3}\log(|x+3|)+\frac{1}{2}\log(|x-2|)+\frac{1}{6}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(x^3+x^2-6*x),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(x + 3)) + 1/2*log(abs(x - 2)) + 1/6*log(abs(x))
```

3.97 $\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx$

Optimal. Leaf size=33

$$\frac{9}{2}\log(a-x)-17\log(2a-x)+\frac{35}{2}\log(3a-x)$$

```
[Out] (9*Log[a - x])/2 - 17*Log[2*a - x] + (35*Log[3*a - x])/2
```

Rubi [A] time = 0.0538914, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2074}

$$\frac{9}{2}\log(a-x)-17\log(2a-x)+\frac{35}{2}\log(3a-x)$$

Antiderivative was successfully verified.

```
[In] Int[(11*a^2 - 7*a*x + 5*x^2)/(-6*a^3 + 11*a^2*x - 6*a*x^2 + x^3),x]
```

```
[Out] (9*Log[a - x])/2 - 17*Log[2*a - x] + (35*Log[3*a - x])/2
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{11a^2-7ax+5x^2}{-6a^3+11a^2x-6ax^2+x^3}\,dx &= \int \left(-\frac{9}{2(a-x)}+\frac{17}{2a-x}-\frac{35}{2(3a-x)}\right)dx\\ &= \frac{9}{2}\log(a-x)-17\log(2a-x)+\frac{35}{2}\log(3a-x)\end{aligned}$$

Mathematica [A] time = 0.014098, size = 29, normalized size = 0.88

$$\frac{35}{2}\log(x-3a)-17\log(x-2a)+\frac{9}{2}\log(x-a)$$

Antiderivative was successfully verified.

```
[In] Integrate[(11*a^2 - 7*a*x + 5*x^2)/(-6*a^3 + 11*a^2*x - 6*a*x^2 + x^3),x]
```

```
[Out] (35*Log[-3*a + x])/2 - 17*Log[-2*a + x] + (9*Log[-a + x])/2
```

Maple [A] time = 0.008, size = 26, normalized size = 0.8

$$\frac{35\ln(x-3a)}{2}+\frac{9\ln(-a+x)}{2}-17\ln(x-2a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x)
```

```
[Out] 35/2*ln(x-3*a)+9/2*ln(-a+x)-17*ln(x-2*a)
```

Maxima [A] time = 0.938507, size = 34, normalized size = 1.03

$$\frac{9}{2}\log\left(-a+x\right)-17\log\left(-2a+x\right)+\frac{35}{2}\log\left(-3a+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x, algorithm="
maxima")
```

```
[Out] 9/2*log(-a + x) - 17*log(-2*a + x) + 35/2*log(-3*a + x)
```

Fricas [A] time = 1.82003, size = 77, normalized size = 2.33

$$\frac{9}{2}\log\left(-a+x\right)-17\log\left(-2a+x\right)+\frac{35}{2}\log\left(-3a+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x, algorithm="
fricas")
```

```
[Out] 9/2*log(-a + x) - 17*log(-2*a + x) + 35/2*log(-3*a + x)
```

Sympy [A] time = 0.371312, size = 26, normalized size = 0.79

$$\frac{35\log\left(-3a+x\right)}{2}-17\log\left(-2a+x\right)+\frac{9\log\left(-a+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((11*a**2-7*a*x+5*x**2)/(-6*a**3+11*a**2*x-6*a*x**2+x**3),x)
```

```
[Out] 35*log(-3*a + x)/2 - 17*log(-2*a + x) + 9*log(-a + x)/2
```

Giac [A] time = 1.05728, size = 38, normalized size = 1.15

$$\frac{9}{2}\log\left(|-a+x|\right)-17\log\left(|-2a+x|\right)+\frac{35}{2}\log\left(|-3a+x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((11*a^2-7*a*x+5*x^2)/(-6*a^3+11*a^2*x-6*a*x^2+x^3),x, algorithm="
giac")
```

```
[Out] 9/2*log(abs(-a + x)) - 17*log(abs(-2*a + x)) + 35/2*log(abs(-3*a + x))
```

3.98 $\int \frac{2-x+x^2}{4-5x^2+x^4} \, dx$

Optimal. Leaf size=37

$$-\frac{1}{3}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{2}{3}\log(x+1)-\frac{2}{3}\log(x+2)$$

```
[Out] -Log[1 - x]/3 + Log[2 - x]/3 + (2*Log[1 + x])/3 - (2*Log[2 + x])/3
```

Rubi [A] time = 0.0370515, antiderivative size = 61, normalized size of antiderivative = 1.65, number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1673, 1161, 616, 31, 1107}

$$\frac{1}{6}\log\left(1-x^2\right)-\frac{1}{6}\log\left(4-x^2\right)-\frac{1}{2}\log(1-x)+\frac{1}{2}\log(2-x)+\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + x^2)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -Log[1 - x]/2 + Log[2 - x]/2 + Log[1 + x]/2 - Log[2 + x]/2 + Log[1 - x^2]/6
 - Log[4 - x^2]/6
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{2-x+x^2}{4-5x^2+x^4}\,dx &= -\int \frac{x}{4-5x^2+x^4}\,dx+\int \frac{2+x^2}{4-5x^2+x^4}\,dx\\ &= \frac{1}{2}\int \frac{1}{2-3x+x^2}\,dx+\frac{1}{2}\int \frac{1}{2+3x+x^2}\,dx-\frac{1}{2}\text{Subst}\left(\int \frac{1}{4-5x+x^2}\,dx,x,x^2\right)\\ &= -\left(\frac{1}{6}\text{Subst}\left(\int \frac{1}{-4+x}\,dx,x,x^2\right)\right)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{-1+x}\,dx,x,x^2\right)+\frac{1}{2}\int \frac{1}{-2+x}\,dx-\frac{1}{2}\int \frac{1}{-1}\\ &= -\frac{1}{2}\log(1-x)+\frac{1}{2}\log(2-x)+\frac{1}{2}\log(1+x)-\frac{1}{2}\log(2+x)+\frac{1}{6}\log\left(1-x^2\right)-\frac{1}{6}\log\left(4-x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0064361, size = 37, normalized size = 1.

$$-\frac{1}{3}\log(1-x)+\frac{1}{3}\log(2-x)+\frac{2}{3}\log(x+1)-\frac{2}{3}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^2)/(4 - 5*x^2 + x^4),x]
```

```
[Out] -Log[1 - x]/3 + Log[2 - x]/3 + (2*Log[1 + x])/3 - (2*Log[2 + x])/3
```

Maple [A] time = 0.01, size = 26, normalized size = 0.7

$$-\frac{2\ln(2+x)}{3}+\frac{2\ln(1+x)}{3}-\frac{\ln(-1+x)}{3}+\frac{\ln(-2+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+2)/(x^4-5*x^2+4),x)
```

```
[Out] -2/3*ln(2+x)+2/3*ln(1+x)-1/3*ln(-1+x)+1/3*ln(-2+x)
```

Maxima [A] time = 0.928802, size = 34, normalized size = 0.92

$$-\frac{2}{3}\log(x+2)+\frac{2}{3}\log(x+1)-\frac{1}{3}\log(x-1)+\frac{1}{3}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] -2/3*log(x + 2) + 2/3*log(x + 1) - 1/3*log(x - 1) + 1/3*log(x - 2)
```

Fricas [A] time = 1.94754, size = 92, normalized size = 2.49

$$-\frac{2}{3}\log(x+2)+\frac{2}{3}\log(x+1)-\frac{1}{3}\log(x-1)+\frac{1}{3}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^4-5*x^2+4),x, algorithm="fricas")
```

```
[Out] -2/3*log(x + 2) + 2/3*log(x + 1) - 1/3*log(x - 1) + 1/3*log(x - 2)
```

Sympy [A] time = 0.15669, size = 29, normalized size = 0.78

$$\frac{\log(x-2)}{3} - \frac{\log(x-1)}{3} + \frac{2\log(x+1)}{3} - \frac{2\log(x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+2)/(x**4-5*x**2+4),x)
```

```
[Out] log(x - 2)/3 - log(x - 1)/3 + 2*log(x + 1)/3 - 2*log(x + 2)/3
```

Giac [A] time = 1.05793, size = 39, normalized size = 1.05

$$-\frac{2}{3}\log(|x+2|) + \frac{2}{3}\log(|x+1|) - \frac{1}{3}\log(|x-1|) + \frac{1}{3}\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+2)/(x^4-5*x^2+4),x, algorithm="giac")
```

```
[Out] -2/3*log(abs(x + 2)) + 2/3*log(abs(x + 1)) - 1/3*log(abs(x - 1)) + 1/3*log(
abs(x - 2))
```

3.99 $\int \frac{-5+2x^2}{6-5x^2+x^4} \, dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTanh[x/Sqrt[2]]/Sqrt[2]) - ArcTanh[x/Sqrt[3]]/Sqrt[3]
```

Rubi [A] time = 0.0090298, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1166, 207}

$$-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 2*x^2)/(6 - 5*x^2 + x^4),x]
```

```
[Out] -(ArcTanh[x/Sqrt[2]]/Sqrt[2]) - ArcTanh[x/Sqrt[3]]/Sqrt[3]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-5+2x^2}{6-5x^2+x^4} \, dx &= \int \frac{1}{-3+x^2} \, dx+\int \frac{1}{-2+x^2} \, dx\\ &=-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [B] time = 0.0178556, size = 69, normalized size = 2.23

$$\frac{1}{12} \left(3 \sqrt{2} \log \left(\sqrt{2}-x\right)+2 \sqrt{3} \log \left(\sqrt{3}-x\right)-3 \sqrt{2} \log \left(x+\sqrt{2}\right)-2 \sqrt{3} \log \left(x+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 2*x^2)/(6 - 5*x^2 + x^4),x]
```

```
[Out] (3*Sqrt[2]*Log[Sqrt[2] - x] + 2*Sqrt[3]*Log[Sqrt[3] - x] - 3*Sqrt[2]*Log[Sq
rt[2] + x] - 2*Sqrt[3]*Log[Sqrt[3] + x])/12
```

Maple [A] time = 0.008, size = 26, normalized size = 0.8

$$-\frac{\sqrt{2}}{2}\mathrm{Artanh}\left(\frac{x\sqrt{2}}{2}\right)-\frac{\sqrt{3}}{3}\mathrm{Artanh}\left(\frac{x\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-5)/(x^4-5*x^2+6),x)
```

```
[Out] -1/2*arctanh(1/2*x*2^(1/2))*2^(1/2)-1/3*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.42559, size = 58, normalized size = 1.87

$$\frac{1}{6}\sqrt{3}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right)+\frac{1}{4}\sqrt{2}\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5)/(x^4-5*x^2+6),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*log((x - sqrt(3))/(x + sqrt(3))) + 1/4*sqrt(2)*log((x - sqrt(2)
)/(x + sqrt(2)))
```

Fricas [B] time = 2.12263, size = 142, normalized size = 4.58

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}x+2}{x^2-2}\right)+\frac{1}{6}\sqrt{3}\log\left(\frac{x^2-2\sqrt{3}x+3}{x^2-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5)/(x^4-5*x^2+6),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^2 - 2*sqrt(2)*x + 2)/(x^2 - 2)) + 1/6*sqrt(3)*log((x^2 -
 2*sqrt(3)*x + 3)/(x^2 - 3))
```

Sympy [A] time = 0.476763, size = 60, normalized size = 1.94

$$\frac{\sqrt{2}\log\left(x-\sqrt{2}\right)}{4}-\frac{\sqrt{2}\log\left(x+\sqrt{2}\right)}{4}+\frac{\sqrt{3}\log\left(x-\sqrt{3}\right)}{6}-\frac{\sqrt{3}\log\left(x+\sqrt{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-5)/(x**4-5*x**2+6),x)
```

```
[Out] sqrt(2)*log(x - sqrt(2))/4 - sqrt(2)*log(x + sqrt(2))/4 + sqrt(3)*log(x - s
qrt(3))/6 - sqrt(3)*log(x + sqrt(3))/6
```

Giac [B] time = 1.06288, size = 80, normalized size = 2.58

$$\frac{1}{6}\sqrt{3}\log\left(\frac{\left|2x-2\sqrt{3}\right|}{\left|2x+2\sqrt{3}\right|}\right)+\frac{1}{4}\sqrt{2}\log\left(\frac{\left|2x-2\sqrt{2}\right|}{\left|2x+2\sqrt{2}\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-5)/(x^4-5*x^2+6),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3))/abs(2*x + 2*sqrt(3))) + 1/4*sqrt(2)*lo
g(abs(2*x - 2*sqrt(2))/abs(2*x + 2*sqrt(2)))
```

3.100 $\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx$

Optimal. Leaf size=41

$$-\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x)-\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)$$

```
[Out] -Log[1 - x]/6 + Log[2 - x]/2 - Log[3 - x]/2 + Log[4 - x]/6
```

Rubi [A] time = 0.0204782, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {180}

$$-\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x)-\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)),x]

[Out] -Log[1 - x]/6 + Log[2 - x]/2 - Log[3 - x]/2 + Log[4 - x]/6
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-4+x)(-3+x)(-2+x)(-1+x)}\,dx &= \int \left(\frac{1}{6(-4+x)}-\frac{1}{2(-3+x)}+\frac{1}{2(-2+x)}-\frac{1}{6(-1+x)}\right)dx\\ &= -\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x)-\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)\end{aligned}$$

Mathematica [A] time = 0.0073356, size = 41, normalized size = 1.

$$-\frac{1}{6}\log(1-x)+\frac{1}{2}\log(2-x)-\frac{1}{2}\log(3-x)+\frac{1}{6}\log(4-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-4 + x)*(-3 + x)*(-2 + x)*(-1 + x)),x]

[Out] -Log[1 - x]/6 + Log[2 - x]/2 - Log[3 - x]/2 + Log[4 - x]/6
```

Maple [A] time = 0.008, size = 26, normalized size = 0.6

$$-\frac{\ln(-1+x)}{6}-\frac{\ln(-3+x)}{2}+\frac{\ln(-2+x)}{2}+\frac{\ln(x-4)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-4)/(-3+x)/(-2+x)/(-1+x),x)
```

```
[Out] -1/6*ln(-1+x)-1/2*ln(-3+x)+1/2*ln(-2+x)+1/6*ln(x-4)
```

Maxima [A] time = 0.923479, size = 34, normalized size = 0.83

$$-\frac{1}{6}\log(x-1)+\frac{1}{2}\log(x-2)-\frac{1}{2}\log(x-3)+\frac{1}{6}\log(x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x, algorithm="maxima")
```

```
[Out] -1/6*log(x - 1) + 1/2*log(x - 2) - 1/2*log(x - 3) + 1/6*log(x - 4)
```

Fricas [A] time = 2.20441, size = 92, normalized size = 2.24

$$-\frac{1}{6}\log(x-1)+\frac{1}{2}\log(x-2)-\frac{1}{2}\log(x-3)+\frac{1}{6}\log(x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x, algorithm="fricas")
```

```
[Out] -1/6*log(x - 1) + 1/2*log(x - 2) - 1/2*log(x - 3) + 1/6*log(x - 4)
```

Sympy [A] time = 0.154118, size = 26, normalized size = 0.63

$$\frac{\log(x-4)}{6}-\frac{\log(x-3)}{2}+\frac{\log(x-2)}{2}-\frac{\log(x-1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x)
```

```
[Out] log(x - 4)/6 - log(x - 3)/2 + log(x - 2)/2 - log(x - 1)/6
```

Giac [A] time = 1.04729, size = 39, normalized size = 0.95

$$-\frac{1}{6}\log(|x-1|)+\frac{1}{2}\log(|x-2|)-\frac{1}{2}\log(|x-3|)+\frac{1}{6}\log(|x-4|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4+x)/(-3+x)/(-2+x)/(-1+x),x, algorithm="giac")
```

```
[Out] -1/6*log(abs(x - 1)) + 1/2*log(abs(x - 2)) - 1/2*log(abs(x - 3)) + 1/6*log(
abs(x - 4))
```

3.101 $\int \frac{1+x^2}{(-1+x)^3} \, dx$

Optimal. Leaf size=25

$$\frac{2}{1-x} - \frac{1}{(1-x)^2} + \log(1-x)$$

```
[Out] -(1 - x)^(-2) + 2/(1 - x) + Log[1 - x]
```

Rubi [A] time = 0.0105989, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {697}

$$\frac{2}{1-x} - \frac{1}{(1-x)^2} + \log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(-1 + x)^3,x]
```

```
[Out] -(1 - x)^(-2) + 2/(1 - x) + Log[1 - x]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{(-1+x)^3} \, dx &= \int \left(\frac{2}{(-1+x)^3} + \frac{2}{(-1+x)^2} + \frac{1}{-1+x}\right) dx \\ &= -\frac{1}{(1-x)^2} + \frac{2}{1-x} + \log(1-x) \end{aligned}$$

Mathematica [A] time = 0.0089271, size = 16, normalized size = 0.64

$$\frac{1-2x}{(x-1)^2} + \log(x-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(-1 + x)^3,x]
```

```
[Out] (1 - 2*x)/(-1 + x)^2 + Log[-1 + x]
```

Maple [A] time = 0.004, size = 20, normalized size = 0.8

$$-(-1+x)^{-2} + \ln(-1+x) - 2(-1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(-1+x)^3,x)
```

```
[Out] -1/(-1+x)^2+ln(-1+x)-2/(-1+x)
```

Maxima [A] time = 0.926384, size = 30, normalized size = 1.2

$$-\frac{2x-1}{x^2-2x+1}+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-1+x)^3,x, algorithm="maxima")
```

```
[Out] -(2*x - 1)/(x^2 - 2*x + 1) + log(x - 1)
```

Fricas [A] time = 2.1045, size = 76, normalized size = 3.04

$$\frac{(x^2-2x+1)\log(x-1)-2x+1}{x^2-2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-1+x)^3,x, algorithm="fricas")
```

```
[Out] ((x^2 - 2*x + 1)*log(x - 1) - 2*x + 1)/(x^2 - 2*x + 1)
```

Sympy [A] time = 0.088809, size = 17, normalized size = 0.68

$$-\frac{2x-1}{x^2-2x+1}+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(-1+x)**3,x)
```

```
[Out] -(2*x - 1)/(x**2 - 2*x + 1) + log(x - 1)
```

Giac [A] time = 1.04723, size = 24, normalized size = 0.96

$$-\frac{2x-1}{(x-1)^2}+\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-1+x)^3,x, algorithm="giac")
```

```
[Out] -(2*x - 1)/(x - 1)^2 + log(abs(x - 1))
```

3.102 $\int \frac{x^5}{(3+x)^2} \, dx$

Optimal. Leaf size=36

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + \frac{243}{x+3} + 405\log(x+3)$$

```
[Out] -108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3 + x) + 405*Log[3 + x]
```

Rubi [A] time = 0.0166641, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + \frac{243}{x+3} + 405\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(3 + x)^2,x]
```

```
[Out] -108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3 + x) + 405*Log[3 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^5}{(3+x)^2} \, dx &= \int \left(-108 + 27x - 6x^2 + x^3 - \frac{243}{(3+x)^2} + \frac{405}{3+x}\right) dx \\ &= -108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405\log(3+x) \end{aligned}$$

Mathematica [A] time = 0.0142851, size = 36, normalized size = 1.

$$\frac{1}{4}\left(x^4 - 8x^3 + 54x^2 - 432x + \frac{972}{x+3} - 2079\right) + 405\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(3 + x)^2,x]
```

```
[Out] (-2079 - 432*x + 54*x^2 - 8*x^3 + x^4 + 972/(3 + x))/4 + 405*Log[3 + x]
```

Maple [A] time = 0.005, size = 33, normalized size = 0.9

$$-108\,x + \frac{27\,x^2}{2} - 2\,x^3 + \frac{x^4}{4} + 243\,(3+x)^{-1} + 405\,\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(3+x)^2,x)
```

```
[Out] -108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)
```

Maxima [A] time = 0.921378, size = 43, normalized size = 1.19

$$\frac{1}{4}x^4 - 2x^3 + \frac{27}{2}x^2 - 108x + \frac{243}{x+3} + 405 \log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3+x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 2*x^3 + 27/2*x^2 - 108*x + 243/(x + 3) + 405*log(x + 3)
```

Fricas [A] time = 2.10633, size = 117, normalized size = 3.25

$$\frac{x^5 - 5x^4 + 30x^3 - 270x^2 + 1620(x+3)\log(x+3) - 1296x + 972}{4(x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3+x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(x^5 - 5*x^4 + 30*x^3 - 270*x^2 + 1620*(x + 3)*log(x + 3) - 1296*x + 97
2)/(x + 3)
```

Sympy [A] time = 0.079906, size = 31, normalized size = 0.86

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + 405\log(x+3) + \frac{243}{x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(3+x)**2,x)
```

```
[Out] x**4/4 - 2*x**3 + 27*x**2/2 - 108*x + 405*log(x + 3) + 243/(x + 3)
```

Giac [A] time = 1.05035, size = 61, normalized size = 1.69

$$-\frac{1}{4}(x+3)^4\left(\frac{20}{x+3} - \frac{180}{(x+3)^2} + \frac{1080}{(x+3)^3} - 1\right) + \frac{243}{x+3} + 405 \log(|x+3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3+x)^2,x, algorithm="giac")
```

```
[Out] -1/4*(x + 3)^4*(20/(x + 3) - 180/(x + 3)^2 + 1080/(x + 3)^3 - 1) + 243/(x +
 3) + 405*log(abs(x + 3))
```

3.103 $\int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4} \, dx$

Optimal. Leaf size=41

$$\frac{407}{16(3-x)} - \frac{133}{8(3-x)^2} + \frac{313}{64}\log(3-x) + \frac{7}{64}\log(x+1)$$

```
[Out] -133/(8*(3 - x)^2) + 407/(16*(3 - x)) + (313*Log[3 - x])/64 + (7*Log[1 + x]
)/64
```

Rubi [A] time = 0.0370383, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2074}

$$\frac{407}{16(3-x)} - \frac{133}{8(3-x)^2} + \frac{313}{64}\log(3-x) + \frac{7}{64}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 5*x^3)/(-27 + 18*x^2 - 8*x^3 + x^4),x]
```

```
[Out] -133/(8*(3 - x)^2) + 407/(16*(3 - x)) + (313*Log[3 - x])/64 + (7*Log[1 + x]
)/64
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned} \int \frac{-2+5x^3}{-27+18x^2-8x^3+x^4} \, dx &= \int \left(\frac{133}{4(-3+x)^3} + \frac{407}{16(-3+x)^2} + \frac{313}{64(-3+x)} + \frac{7}{64(1+x)}\right) dx \\ &= -\frac{133}{8(3-x)^2} + \frac{407}{16(3-x)} + \frac{313}{64}\log(3-x) + \frac{7}{64}\log(1+x) \end{aligned}$$

Mathematica [A] time = 0.019543, size = 37, normalized size = 0.9

$$-\frac{407}{16(x-3)} - \frac{133}{8(x-3)^2} + \frac{313}{64}\log(3-x) + \frac{7}{64}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 5*x^3)/(-27 + 18*x^2 - 8*x^3 + x^4),x]
```

```
[Out] -133/(8*(-3 + x)^2) - 407/(16*(-3 + x)) + (313*Log[3 - x])/64 + (7*Log[1 +
x])/64
```

Maple [A] time = 0.007, size = 28, normalized size = 0.7

$$\frac{7\ln(1+x)}{64}-\frac{133}{8(-3+x)^2}-\frac{407}{-48+16x}+\frac{313\ln(-3+x)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x)
```

```
[Out] 7/64*ln(1+x)-133/8/(-3+x)^2-407/16/(-3+x)+313/64*ln(-3+x)
```

Maxima [A] time = 0.933254, size = 41, normalized size = 1.

$$-\frac{407\,x-955}{16\left(x^2-6\,x+9\right)}+\frac{7}{64}\log(x+1)+\frac{313}{64}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x, algorithm="maxima")
```

```
[Out] -1/16*(407*x - 955)/(x^2 - 6*x + 9) + 7/64*log(x + 1) + 313/64*log(x - 3)
```

Fricas [A] time = 2.08478, size = 138, normalized size = 3.37

$$\frac{7\left(x^2-6\,x+9\right)\log(x+1)+313\left(x^2-6\,x+9\right)\log(x-3)-1628\,x+3820}{64\left(x^2-6\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x, algorithm="fricas")
```

```
[Out] 1/64*(7*(x^2 - 6*x + 9)*log(x + 1) + 313*(x^2 - 6*x + 9)*log(x - 3) - 1628*
x + 3820)/(x^2 - 6*x + 9)
```

Sympy [A] time = 0.123652, size = 31, normalized size = 0.76

$$-\frac{407x-955}{16x^2-96x+144}+\frac{313\log(x-3)}{64}+\frac{7\log(x+1)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**3-2)/(x**4-8*x**3+18*x**2-27),x)
```

```
[Out] -(407*x - 955)/(16*x**2 - 96*x + 144) + 313*log(x - 3)/64 + 7*log(x + 1)/64
```

Giac [A] time = 1.05388, size = 36, normalized size = 0.88

$$-\frac{407\,x-955}{16\,(x-3)^2}+\frac{7}{64}\log(|x+1|)+\frac{313}{64}\log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3-2)/(x^4-8*x^3+18*x^2-27),x, algorithm="giac")
```

```
[Out] -1/16*(407*x - 955)/(x - 3)^2 + 7/64*log(abs(x + 1)) + 313/64*log(abs(x - 3
))
```

3.104 $\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx$

Optimal. Leaf size=27

$$\frac{99}{x+3}+\frac{181}{x+4}+264\log(x+3)-263\log(x+4)$$

```
[Out] 99/(3 + x) + 181/(4 + x) + 264*Log[3 + x] - 263*Log[4 + x]
```

Rubi [A] time = 0.0359, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1620}

$$\frac{99}{x+3}+\frac{181}{x+4}+264\log(x+3)-263\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[(-9 + 3*x - 6*x^2 + x^3)/((3 + x)^2*(4 + x)^2),x]
```

```
[Out] 99/(3 + x) + 181/(4 + x) + 264*Log[3 + x] - 263*Log[4 + x]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{-9+3x-6x^2+x^3}{(3+x)^2(4+x)^2}\,dx &= \int \left(-\frac{99}{(3+x)^2}+\frac{264}{3+x}-\frac{181}{(4+x)^2}-\frac{263}{4+x}\right)dx\\ &= \frac{99}{3+x}+\frac{181}{4+x}+264\log(3+x)-263\log(4+x)\end{aligned}$$

Mathematica [A] time = 0.015453, size = 27, normalized size = 1.

$$\frac{99}{x+3}+\frac{181}{x+4}+264\log(x+3)-263\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-9 + 3*x - 6*x^2 + x^3)/((3 + x)^2*(4 + x)^2),x]
```

```
[Out] 99/(3 + x) + 181/(4 + x) + 264*Log[3 + x] - 263*Log[4 + x]
```

Maple [A] time = 0.009, size = 28, normalized size = 1.

$$99\,(3+x)^{-1}+181\,(4+x)^{-1}+264\,\ln(3+x)-263\,\ln(4+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x)
```

```
[Out] 99/(3+x)+181/(4+x)+264*ln(3+x)-263*ln(4+x)
```

Maxima [A] time = 0.9427, size = 39, normalized size = 1.44

$$\frac{280\,x+939}{x^2+7\,x+12}-263\,\log\left(x+4\right)+264\,\log\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x, algorithm="maxima")
```

```
[Out] (280*x + 939)/(x^2 + 7*x + 12) - 263*log(x + 4) + 264*log(x + 3)
```

Fricas [A] time = 2.05338, size = 136, normalized size = 5.04

$$-\frac{263\,\left(x^2+7\,x+12\right)\log\left(x+4\right)-264\,\left(x^2+7\,x+12\right)\log\left(x+3\right)-280\,x-939}{x^2+7\,x+12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x, algorithm="fricas")
```

```
[Out] -(263*(x^2 + 7*x + 12)*log(x + 4) - 264*(x^2 + 7*x + 12)*log(x + 3) - 280*x
 - 939)/(x^2 + 7*x + 12)
```

Sympy [A] time = 0.125058, size = 26, normalized size = 0.96

$$\frac{280x+939}{x^2+7x+12}+264\log\left(x+3\right)-263\log\left(x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-6*x**2+3*x-9)/(3+x)**2/(4+x)**2,x)
```

```
[Out] (280*x + 939)/(x**2 + 7*x + 12) + 264*log(x + 3) - 263*log(x + 4)
```

Giac [A] time = 1.05885, size = 50, normalized size = 1.85

$$\frac{181}{x+4}-\frac{99}{\frac{1}{x+4}-1}+\log\left(|x+4|\right)+264\,\log\left(\left|-\frac{1}{x+4}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-6*x^2+3*x-9)/(3+x)^2/(4+x)^2,x, algorithm="giac")
```

```
[Out] 181/(x + 4) - 99/(1/(x + 4) - 1) + log(abs(x + 4)) + 264*log(abs(-1/(x + 4)
 + 1))
```

3.105 $\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2} \, dx$

Optimal. Leaf size=39

$$\frac{x+3}{2\left(1-x^2\right)}-\frac{3}{4}\log(1-x)+2\log(x)-\frac{5}{4}\log(x+1)$$

```
[Out] (3 + x)/(2*(1 - x^2)) - (3*Log[1 - x])/4 + 2*Log[x] - (5*Log[1 + x])/4
```

Rubi [A] time = 0.0362482, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1805, 801}

$$\frac{x+3}{2\left(1-x^2\right)}-\frac{3}{4}\log(1-x)+2\log(x)-\frac{5}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 + x^3)/(x*(-1 + x^2)^2),x]
```

```
[Out] (3 + x)/(2*(1 - x^2)) - (3*Log[1 - x])/4 + 2*Log[x] - (5*Log[1 + x])/4
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x^2+x^3}{x\left(-1+x^2\right)^2} \, dx &= \frac{3+x}{2\left(1-x^2\right)}+\frac{1}{2}\int \frac{-4+x}{x\left(-1+x^2\right)} \, dx\\ &= \frac{3+x}{2\left(1-x^2\right)}+\frac{1}{2}\int \left(-\frac{3}{2(-1+x)}+\frac{4}{x}-\frac{5}{2(1+x)}\right) dx\\ &= \frac{3+x}{2\left(1-x^2\right)}-\frac{3}{4}\log(1-x)+2\log(x)-\frac{5}{4}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0165338, size = 47, normalized size = 1.21

$$\frac{1}{4}\left(-\frac{4}{x^2-1}-4\log\left(1-x^2\right)-\frac{2}{x-1}+\log(1-x)+8\log(x)-\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 + x^3)/(x*(-1 + x^2)^2),x]
```

```
[Out] (-2/(-1 + x) - 4/(-1 + x^2) + Log[1 - x] + 8*Log[x] - Log[1 + x] - 4*Log[1
- x^2])/4
```

Maple [A] time = 0.011, size = 32, normalized size = 0.8

$$2\,\ln(x)+\frac{1}{2\,x+2}-\frac{5\,\ln(1+x)}{4}-(-1+x)^{-1}-\frac{3\,\ln(-1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+2)/x/(x^2-1)^2,x)
```

```
[Out] 2*ln(x)+1/2/(1+x)-5/4*ln(1+x)-1/(-1+x)-3/4*ln(-1+x)
```

Maxima [A] time = 0.956347, size = 39, normalized size = 1.

$$-\frac{x+3}{2\left(x^2-1\right)}-\frac{5}{4}\,\log(x+1)-\frac{3}{4}\,\log(x-1)+2\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2)/x/(x^2-1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x + 3)/(x^2 - 1) - 5/4*log(x + 1) - 3/4*log(x - 1) + 2*log(x)
```

Fricas [A] time = 2.19858, size = 131, normalized size = 3.36

$$-\frac{5\left(x^2-1\right)\log(x+1)+3\left(x^2-1\right)\log(x-1)-8\left(x^2-1\right)\log(x)+2\,x+6}{4\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2)/x/(x^2-1)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(5*(x^2 - 1)*log(x + 1) + 3*(x^2 - 1)*log(x - 1) - 8*(x^2 - 1)*log(x)
+ 2*x + 6)/(x^2 - 1)
```

Sympy [A] time = 0.136416, size = 31, normalized size = 0.79

$$-\frac{x+3}{2x^2-2}+2\log(x)-\frac{3\log(x-1)}{4}-\frac{5\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+2)/x/(x**2-1)**2,x)
```

```
[Out] -(x + 3)/(2*x**2 - 2) + 2*log(x) - 3*log(x - 1)/4 - 5*log(x + 1)/4
```

Giac [A] time = 1.04904, size = 47, normalized size = 1.21

$$-\frac{x+3}{2\,(x+1)(x-1)}-\frac{5}{4}\,\log\left(|x+1|\right)-\frac{3}{4}\,\log\left(|x-1|\right)+2\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+2)/x/(x^2-1)^2,x, algorithm="giac")
```

```
[Out] -1/2*(x + 3)/((x + 1)*(x - 1)) - 5/4*log(abs(x + 1)) - 3/4*log(abs(x - 1))
+ 2*log(abs(x))
```

3.106 $\int \frac{1}{x^3-x^4-x^5+x^6}\,dx$

Optimal. Leaf size=46

$$-\frac{1}{2x^2}+\frac{1}{2(1-x)}-\frac{1}{x}-\frac{7}{4}\log(1-x)+2\log(x)-\frac{1}{4}\log(x+1)$$

```
[Out] 1/(2*(1 - x)) - 1/(2*x^2) - x^(-1) - (7*Log[1 - x])/4 + 2*Log[x] - Log[1 +
x]/4
```

Rubi [A] time = 0.0224983, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2058}

$$-\frac{1}{2x^2}+\frac{1}{2(1-x)}-\frac{1}{x}-\frac{7}{4}\log(1-x)+2\log(x)-\frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3 - x^4 - x^5 + x^6)^(-1),x]
```

```
[Out] 1/(2*(1 - x)) - 1/(2*x^2) - x^(-1) - (7*Log[1 - x])/4 + 2*Log[x] - Log[1 +
x]/4
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3-x^4-x^5+x^6}\,dx &= \int \left(\frac{1}{2(-1+x)^2}-\frac{7}{4(-1+x)}+\frac{1}{x^3}+\frac{1}{x^2}+\frac{2}{x}-\frac{1}{4(1+x)}\right)dx\\ &= \frac{1}{2(1-x)}-\frac{1}{2x^2}-\frac{1}{x}-\frac{7}{4}\log(1-x)+2\log(x)-\frac{1}{4}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0155976, size = 40, normalized size = 0.87

$$\frac{1}{4}\left(-\frac{2}{x^2}-\frac{2}{x-1}-\frac{4}{x}-7\log(1-x)+8\log(x)-\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3 - x^4 - x^5 + x^6)^(-1),x]
```

```
[Out] (-2/(-1 + x) - 2/x^2 - 4/x - 7*Log[1 - x] + 8*Log[x] - Log[1 + x])/4
```

Maple [A] time = 0.008, size = 35, normalized size = 0.8

$$-\frac{1}{2\,x^2}-x^{-1}+2\,\ln(x)-\frac{\ln(1+x)}{4}-\frac{1}{2\,x-2}-\frac{7\,\ln(-1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-x^5-x^4+x^3),x)
```

```
[Out] -1/2/x^2-1/x+2*ln(x)-1/4*ln(1+x)-1/2/(-1+x)-7/4*ln(-1+x)
```

Maxima [A] time = 0.930141, size = 54, normalized size = 1.17

$$-\frac{3\,x^2-x-1}{2\left(x^3-x^2\right)}-\frac{1}{4}\,\log\left(x+1\right)-\frac{7}{4}\,\log\left(x-1\right)+2\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^5-x^4+x^3),x, algorithm="maxima")
```

```
[Out] -1/2*(3*x^2 - x - 1)/(x^3 - x^2) - 1/4*log(x + 1) - 7/4*log(x - 1) + 2*log(
x)
```

Fricas [A] time = 2.11719, size = 150, normalized size = 3.26

$$-\frac{6\,x^2+\left(x^3-x^2\right)\log\left(x+1\right)+7\left(x^3-x^2\right)\log\left(x-1\right)-8\left(x^3-x^2\right)\log\left(x\right)-2\,x-2}{4\left(x^3-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^5-x^4+x^3),x, algorithm="fricas")
```

```
[Out] -1/4*(6*x^2 + (x^3 - x^2)*log(x + 1) + 7*(x^3 - x^2)*log(x - 1) - 8*(x^3 -
x^2)*log(x) - 2*x - 2)/(x^3 - x^2)
```

Sympy [A] time = 0.14559, size = 37, normalized size = 0.8

$$2\log\left(x\right)-\frac{7\log\left(x-1\right)}{4}-\frac{\log\left(x+1\right)}{4}-\frac{3x^2-x-1}{2x^3-2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-x**5-x**4+x**3),x)
```

```
[Out] 2*log(x) - 7*log(x - 1)/4 - log(x + 1)/4 - (3*x**2 - x - 1)/(2*x**3 - 2*x**
2)
```

Giac [A] time = 1.05111, size = 54, normalized size = 1.17

$$-\frac{3\,x^2-x-1}{2\left(x-1\right)x^2}-\frac{1}{4}\,\log\left(|x+1|\right)-\frac{7}{4}\,\log\left(|x-1|\right)+2\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^5-x^4+x^3),x, algorithm="giac")
```

```
[Out] -1/2*(3*x^2 - x - 1)/((x - 1)*x^2) - 1/4*log(abs(x + 1)) - 7/4*log(abs(x -
1)) + 2*log(abs(x))
```

3.107 $\int \frac{1+x^4}{-1+x-x^2+x^3} \, dx$

Optimal. Leaf size=29

$$\frac{x^2}{2} - \frac{1}{2}\log\left(x^2+1\right) + x + \log(1-x) - \tan^{-1}(x)$$

```
[Out] x + x^2/2 - ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0282889, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2074, 635, 203, 260}

$$\frac{x^2}{2} - \frac{1}{2}\log\left(x^2+1\right) + x + \log(1-x) - \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(-1 + x - x^2 + x^3),x]
```

```
[Out] x + x^2/2 - ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{-1+x-x^2+x^3} \, dx &= \int \left(1+\frac{1}{-1+x}+x+\frac{-1-x}{1+x^2}\right) dx \\
&= x+\frac{x^2}{2}+\log(1-x)+\int \frac{-1-x}{1+x^2} \, dx \\
&= x+\frac{x^2}{2}+\log(1-x)-\int \frac{1}{1+x^2} \, dx-\int \frac{x}{1+x^2} \, dx \\
&= x+\frac{x^2}{2}-\tan^{-1}(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0072064, size = 29, normalized size = 1.

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)+x+\log(1-x)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(-1 + x - x^2 + x^3),x]
```

```
[Out] x + x^2/2 - ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Maple [A] time = 0.006, size = 24, normalized size = 0.8

$$x+\frac{x^2}{2}-\frac{\ln\left(x^2+1\right)}{2}-\arctan(x)+\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^3-x^2+x-1),x)
```

```
[Out] x+1/2*x^2-1/2*ln(x^2+1)-arctan(x)+ln(-1+x)
```

Maxima [A] time = 1.3999, size = 31, normalized size = 1.07

$$\frac{1}{2}x^2+x-\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^3-x^2+x-1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x - arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

Fricas [A] time = 2.08234, size = 77, normalized size = 2.66

$$\frac{1}{2}x^2+x-\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^3-x^2+x-1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x - arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

Sympy [A] time = 0.107935, size = 22, normalized size = 0.76

$$\frac{x^2}{2}+x+\log(x-1)-\frac{\log\left(x^2+1\right)}{2}-\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**3-x**2+x-1),x)
```

```
[Out] x**2/2 + x + log(x - 1) - log(x**2 + 1)/2 - atan(x)
```

Giac [A] time = 1.0573, size = 32, normalized size = 1.1

$$\frac{1}{2}x^2 + x - \arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^3-x^2+x-1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x - arctan(x) - 1/2*log(x^2 + 1) + log(abs(x - 1))
```

3.108 $\int \frac{1}{x(1+x)(1+x^2)}\,dx$

Optimal. Leaf size=27

$$-\frac{1}{4}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\log(x+1)-\frac{1}{2}\tan^{-1}(x)$$

```
[Out] -ArcTan[x]/2 + Log[x] - Log[1 + x]/2 - Log[1 + x^2]/4
```

Rubi [A] time = 0.0248596, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {894, 635, 203, 260}

$$-\frac{1}{4}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\log(x+1)-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x)*(1 + x^2)),x]

[Out] -ArcTan[x]/2 + Log[x] - Log[1 + x]/2 - Log[1 + x^2]/4
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x(1+x)\left(1+x^2\right)}\,dx &= \int \left(\frac{1}{x} - \frac{1}{2(1+x)} + \frac{-1-x}{2\left(1+x^2\right)}\right) dx \\
&= \log(x) - \frac{1}{2}\log(1+x) + \frac{1}{2}\int \frac{-1-x}{1+x^2}\,dx \\
&= \log(x) - \frac{1}{2}\log(1+x) - \frac{1}{2}\int \frac{1}{1+x^2}\,dx - \frac{1}{2}\int \frac{x}{1+x^2}\,dx \\
&= -\frac{1}{2}\tan^{-1}(x) + \log(x) - \frac{1}{2}\log(1+x) - \frac{1}{4}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0067695, size = 27, normalized size = 1.

$$
-\frac{1}{4}\log\left(x^2+1\right) + \log(x) - \frac{1}{2}\log(x+1) - \frac{1}{2}\tan^{-1}(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x)*(1 + x^2)),x]
```

```
[Out] -ArcTan[x]/2 + Log[x] - Log[1 + x]/2 - Log[1 + x^2]/4
```

Maple [A] time = 0.004, size = 22, normalized size = 0.8

$$
-\frac{\arctan(x)}{2} + \ln(x) - \frac{\ln(1+x)}{2} - \frac{\ln\left(x^2+1\right)}{4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+x)/(x^2+1),x)
```

```
[Out] -1/2*arctan(x)+ln(x)-1/2*ln(1+x)-1/4*ln(x^2+1)
```

Maxima [A] time = 1.38402, size = 28, normalized size = 1.04

$$
-\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log(x+1) + \log(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(x) - 1/4*log(x^2 + 1) - 1/2*log(x + 1) + log(x)
```

Fricas [A] time = 2.02233, size = 82, normalized size = 3.04

$$
-\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2}\log(x+1) + \log(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(x) - 1/4*log(x^2 + 1) - 1/2*log(x + 1) + log(x)
```

Sympy [A] time = 0.157123, size = 22, normalized size = 0.81

$$\log(x) - \frac{\log(x+1)}{2} - \frac{\log(x^2+1)}{4} - \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(x**2+1),x)
```

```
[Out] log(x) - log(x + 1)/2 - log(x**2 + 1)/4 - atan(x)/2
```

Giac [A] time = 1.06179, size = 31, normalized size = 1.15

$$-\frac{1}{2}\arctan(x) - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\log(|x+1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*arctan(x) - 1/4*log(x^2 + 1) - 1/2*log(abs(x + 1)) + log(abs(x))
```

3.109 $\int \frac{x^2}{-2+x^2+x^4} \, dx$

Optimal. Leaf size=24

$$\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{3}\tanh^{-1}(x)$$

```
[Out] (Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - ArcTanh[x]/3
```

Rubi [A] time = 0.0099284, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1130, 203, 207}

$$\frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-2 + x^2 + x^4),x]

[Out] (Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - ArcTanh[x]/3
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{-2+x^2+x^4} \, dx &= \frac{1}{3}\int \frac{1}{-1+x^2} \, dx+\frac{2}{3}\int \frac{1}{2+x^2} \, dx\\ &= \frac{1}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{3}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0093032, size = 32, normalized size = 1.33

$$\frac{1}{6}\left(\log(1-x)-\log(x+1)+2\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-2 + x^2 + x^4),x]
```

```
[Out] (2*Sqrt[2]*ArcTan[x/Sqrt[2]] + Log[1 - x] - Log[1 + x])/6
```

Maple [A] time = 0.007, size = 26, normalized size = 1.1

$$-\frac{\ln(1+x)}{6}+\frac{\ln(-1+x)}{6}+\frac{\sqrt{2}}{3}\arctan\left(\frac{x\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+x^2-2),x)
```

```
[Out] -1/6*ln(1+x)+1/6*ln(-1+x)+1/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.43036, size = 34, normalized size = 1.42

$$\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{6}\log(x+1)+\frac{1}{6}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+x^2-2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(x + 1) + 1/6*log(x - 1)
```

Fricas [A] time = 2.14877, size = 93, normalized size = 3.88

$$\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{6}\log(x+1)+\frac{1}{6}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+x^2-2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(x + 1) + 1/6*log(x - 1)
```

Sympy [A] time = 0.131159, size = 29, normalized size = 1.21

$$\frac{\log(x-1)}{6}-\frac{\log(x+1)}{6}+\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+x**2-2),x)
```

```
[Out] log(x - 1)/6 - log(x + 1)/6 + sqrt(2)*atan(sqrt(2)*x/2)/3
```

Giac [A] time = 1.05814, size = 36, normalized size = 1.5

$$\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{6}\log\left(|x+1|\right)+\frac{1}{6}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+x^2-2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/6*log(abs(x + 1)) + 1/6*log(abs(x - 1
))
```

3.110 $\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4} \, dx$

Optimal. Leaf size=41

$$\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{x+1}-\frac{1}{3}\log(x+1)+\frac{4}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] (1 + x)^(-1) + (4*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - Log[1 + x]/3 + (2*Log[2 +
x^2])/3
```

Rubi [A] time = 0.0870716, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {1594, 2075, 635, 203, 260}

$$\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{x+1}-\frac{1}{3}\log(x+1)+\frac{4}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(6*x + 4*x^2 + x^3)/(2 + 4*x + 3*x^2 + 2*x^3 + x^4),x]
```

```
[Out] (1 + x)^(-1) + (4*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - Log[1 + x]/3 + (2*Log[2 +
x^2])/3
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 2075

```
Int[(P_)^(p_)*(Qm_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegran
d[PP^p*Qm, x], x] /; QuadraticProductQ[PP, x]] /; PolyQ[Qm, x] && PolyQ[P,
x] && ILtQ[p, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{6x+4x^2+x^3}{2+4x+3x^2+2x^3+x^4}\,dx &= \int \frac{x\left(6+4x+x^2\right)}{2+4x+3x^2+2x^3+x^4}\,dx \\ &= \int \left(-\frac{1}{(1+x)^2}-\frac{1}{3(1+x)}+\frac{4(2+x)}{3\left(2+x^2\right)}\right)dx \\ &= \frac{1}{1+x}-\frac{1}{3}\log(1+x)+\frac{4}{3}\int \frac{2+x}{2+x^2}\,dx \\ &= \frac{1}{1+x}-\frac{1}{3}\log(1+x)+\frac{4}{3}\int \frac{x}{2+x^2}\,dx+\frac{8}{3}\int \frac{1}{2+x^2}\,dx \\ &= \frac{1}{1+x}+\frac{4}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{3}\log(1+x)+\frac{2}{3}\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0209546, size = 41, normalized size = 1.

$$\frac{2}{3}\log\left(x^2+2\right)+\frac{1}{x+1}-\frac{1}{3}\log(x+1)+\frac{4}{3}\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(6*x + 4*x^2 + x^3)/(2 + 4*x + 3*x^2 + 2*x^3 + x^4),x]
```

```
[Out] (1 + x)^(-1) + (4*Sqrt[2]*ArcTan[x/Sqrt[2]])/3 - Log[1 + x]/3 + (2*Log[2 +
x^2])/3
```

Maple [A] time = 0.007, size = 33, normalized size = 0.8

$$(1+x)^{-1}-\frac{\ln(1+x)}{3}+\frac{2\ln\left(x^2+2\right)}{3}+\frac{4\sqrt{2}}{3}\arctan\left(\frac{x\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x)
```

```
[Out] 1/(1+x)-1/3*ln(1+x)+2/3*ln(x^2+2)+4/3*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.42998, size = 43, normalized size = 1.05

$$\frac{4}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{x+1}+\frac{2}{3}\log\left(x^2+2\right)-\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/(x + 1) + 2/3*log(x^2 + 2) - 1/3*log(
x + 1)
```

Fricas [A] time = 2.17671, size = 142, normalized size = 3.46

$$\frac{4\sqrt{2}(x+1)\arctan\left(\frac{1}{2}\sqrt{2}x\right)+2(x+1)\log\left(x^2+2\right)-(x+1)\log(x+1)+3}{3(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x, algorithm="fricas")
```

```
[Out] 1/3*(4*sqrt(2)*(x + 1)*arctan(1/2*sqrt(2)*x) + 2*(x + 1)*log(x^2 + 2) - (x
+ 1)*log(x + 1) + 3)/(x + 1)
```

Sympy [A] time = 0.138548, size = 39, normalized size = 0.95

$$-\frac{\log(x+1)}{3}+\frac{2\log\left(x^2+2\right)}{3}+\frac{4\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{3}+\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+4*x**2+6*x)/(x**4+2*x**3+3*x**2+4*x+2),x)
```

```
[Out] -log(x + 1)/3 + 2*log(x**2 + 2)/3 + 4*sqrt(2)*atan(sqrt(2)*x/2)/3 + 1/(x +
1)
```

Giac [A] time = 1.05185, size = 45, normalized size = 1.1

$$\frac{4}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{x+1}+\frac{2}{3}\log\left(x^2+2\right)-\frac{1}{3}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+4*x^2+6*x)/(x^4+2*x^3+3*x^2+4*x+2),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/(x + 1) + 2/3*log(x^2 + 2) - 1/3*log(
abs(x + 1))
```

3.111 $\int \frac{x}{(1+x)(1+2x)^2(1+x^2)}\,dx$

Optimal. Leaf size=46

$$-\frac{7}{100}\log\left(x^2+1\right)+\frac{2}{5(2x+1)}-\frac{1}{2}\log(x+1)+\frac{16}{25}\log(2x+1)+\frac{1}{50}\tan^{-1}(x)$$

```
[Out] 2/(5*(1 + 2*x)) + ArcTan[x]/50 - Log[1 + x]/2 + (16*Log[1 + 2*x])/25 - (7*L
og[1 + x^2])/100
```

Rubi [A] time = 0.204399, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {6725, 635, 203, 260}

$$-\frac{7}{100}\log\left(x^2+1\right)+\frac{2}{5(2x+1)}-\frac{1}{2}\log(x+1)+\frac{16}{25}\log(2x+1)+\frac{1}{50}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(1 + 2*x)^2*(1 + x^2)),x]
```

```
[Out] 2/(5*(1 + 2*x)) + ArcTan[x]/50 - Log[1 + x]/2 + (16*Log[1 + 2*x])/25 - (7*L
og[1 + x^2])/100
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x}{(1+x)(1+2x)^2\left(1+x^2\right)}\,dx = \int \left(-\frac{1}{2(1+x)} - \frac{4}{5(1+2x)^2} + \frac{32}{25(1+2x)} + \frac{1-7x}{50\left(1+x^2\right)}\right) dx$$
$$= \frac{2}{5(1+2x)} - \frac{1}{2}\log(1+x) + \frac{16}{25}\log(1+2x) + \frac{1}{50}\int \frac{1-7x}{1+x^2}\,dx$$
$$= \frac{2}{5(1+2x)} - \frac{1}{2}\log(1+x) + \frac{16}{25}\log(1+2x) + \frac{1}{50}\int \frac{1}{1+x^2}\,dx - \frac{7}{50}\int \frac{x}{1+x^2}\,dx$$
$$= \frac{2}{5(1+2x)} + \frac{1}{50}\tan^{-1}(x) - \frac{1}{2}\log(1+x) + \frac{16}{25}\log(1+2x) - \frac{7}{100}\log\left(1+x^2\right)$$

Mathematica [A] time = 0.0187063, size = 40, normalized size = 0.87

$$\frac{1}{100}\left(-7\log\left(x^2+1\right) + \frac{40}{2x+1} - 50\log(x+1) + 64\log(2x+1) + 2\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*(1 + 2*x)^2*(1 + x^2)),x]
```

```
[Out] (40/(1 + 2*x) + 2*ArcTan[x] - 50*Log[1 + x] + 64*Log[1 + 2*x] - 7*Log[1 + x
^2])/100
```

Maple [A] time = 0.008, size = 37, normalized size = 0.8

$$\frac{2}{5+10x} + \frac{\arctan(x)}{50} - \frac{\ln(1+x)}{2} + \frac{16\ln(1+2x)}{25} - \frac{7\ln\left(x^2+1\right)}{100}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/(1+2*x)^2/(x^2+1),x)
```

```
[Out] 2/5/(1+2*x)+1/50*arctan(x)-1/2*ln(1+x)+16/25*ln(1+2*x)-7/100*ln(x^2+1)
```

Maxima [A] time = 1.42559, size = 49, normalized size = 1.07

$$\frac{2}{5(2x+1)} + \frac{1}{50}\arctan(x) - \frac{7}{100}\log\left(x^2+1\right) + \frac{16}{25}\log(2x+1) - \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] 2/5/(2*x + 1) + 1/50*arctan(x) - 7/100*log(x^2 + 1) + 16/25*log(2*x + 1) -
1/2*log(x + 1)
```

Fricas [A] time = 2.11362, size = 171, normalized size = 3.72

$$\frac{2(2x+1)\arctan(x) - 7(2x+1)\log\left(x^2+1\right) + 64(2x+1)\log(2x+1) - 50(2x+1)\log(x+1) + 40}{100(2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/100*(2*(2*x + 1)*arctan(x) - 7*(2*x + 1)*log(x^2 + 1) + 64*(2*x + 1)*log(
2*x + 1) - 50*(2*x + 1)*log(x + 1) + 40)/(2*x + 1)
```

Sympy [A] time = 0.186602, size = 37, normalized size = 0.8

$$\frac{16\log\left(x+\frac{1}{2}\right)}{25}-\frac{\log\left(x+1\right)}{2}-\frac{7\log\left(x^{2}+1\right)}{100}+\frac{\operatorname{atan}\left(x\right)}{50}+\frac{2}{10x+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(1+2*x)**2/(x**2+1),x)
```

```
[Out] 16*log(x + 1/2)/25 - log(x + 1)/2 - 7*log(x**2 + 1)/100 + atan(x)/50 + 2/(1
0*x + 5)
```

Giac [A] time = 1.06118, size = 84, normalized size = 1.83

$$\frac{2}{5\,(2\,x+1)}+\frac{1}{50}\,\arctan\left(-\frac{5}{2\,(2\,x+1)}+\frac{1}{2}\right)-\frac{7}{100}\,\log\left(-\frac{2}{2\,x+1}+\frac{5}{(2\,x+1)^{2}}+1\right)-\frac{1}{2}\,\log\left(\left|-\frac{1}{2\,x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(1+2*x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 2/5/(2*x + 1) + 1/50*arctan(-5/2/(2*x + 1) + 1/2) - 7/100*log(-2/(2*x + 1)
+ 5/(2*x + 1)^2 + 1) - 1/2*log(abs(-1/(2*x + 1) - 1))
```

3.112 $\int \frac{-2+x+3x^2}{(-1+x)^3(1+x^2)}\,dx$

Optimal. Leaf size=47

$$\frac{3}{4}\log\left(x^2+1\right)+\frac{5}{2(1-x)}-\frac{1}{2(1-x)^2}-\frac{3}{2}\log(1-x)-\tan^{-1}(x)$$

```
[Out] -1/(2*(1 - x)^2) + 5/(2*(1 - x)) - ArcTan[x] - (3*Log[1 - x])/2 + (3*Log[1
+ x^2])/4
```

Rubi [A] time = 0.0428182, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {1629, 635, 203, 260}

$$\frac{3}{4}\log\left(x^2+1\right)+\frac{5}{2(1-x)}-\frac{1}{2(1-x)^2}-\frac{3}{2}\log(1-x)-\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x + 3*x^2)/((-1 + x)^3*(1 + x^2)),x]
```

```
[Out] -1/(2*(1 - x)^2) + 5/(2*(1 - x)) - ArcTan[x] - (3*Log[1 - x])/2 + (3*Log[1
+ x^2])/4
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-2+x+3x^2}{(-1+x)^3\left(1+x^2\right)}\,dx &= \int \left(\frac{1}{(-1+x)^3}+\frac{5}{2(-1+x)^2}-\frac{3}{2(-1+x)}+\frac{-2+3x}{2\left(1+x^2\right)}\right)dx\\
&= -\frac{1}{2(1-x)^2}+\frac{5}{2(1-x)}-\frac{3}{2}\log(1-x)+\frac{1}{2}\int \frac{-2+3x}{1+x^2}\,dx\\
&= -\frac{1}{2(1-x)^2}+\frac{5}{2(1-x)}-\frac{3}{2}\log(1-x)+\frac{3}{2}\int \frac{x}{1+x^2}\,dx-\int \frac{1}{1+x^2}\,dx\\
&= -\frac{1}{2(1-x)^2}+\frac{5}{2(1-x)}-\tan^{-1}(x)-\frac{3}{2}\log(1-x)+\frac{3}{4}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02364, size = 37, normalized size = 0.79

$$
\frac{1}{4}\left(3\log\left(x^2+1\right)-\frac{10}{x-1}-\frac{2}{(x-1)^2}-6\log(x-1)-4\tan^{-1}(x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x + 3*x^2)/((-1 + x)^3*(1 + x^2)),x]
```

```
[Out] (-2/(-1 + x)^2 - 10/(-1 + x) - 4*ArcTan[x] - 6*Log[-1 + x] + 3*Log[1 + x^2]
)/4
```

Maple [A] time = 0.006, size = 34, normalized size = 0.7

$$
\frac{3\ln\left(x^2+1\right)}{4}-\arctan(x)-\frac{1}{2\,(-1+x)^2}-\frac{5}{2x-2}-\frac{3\ln(-1+x)}{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+x-2)/(-1+x)^3/(x^2+1),x)
```

```
[Out] 3/4*ln(x^2+1)-arctan(x)-1/2/(-1+x)^2-5/2/(-1+x)-3/2*ln(-1+x)
```

Maxima [A] time = 1.4535, size = 49, normalized size = 1.04

$$
-\frac{5x-4}{2\left(x^2-2x+1\right)}-\arctan(x)+\frac{3}{4}\log\left(x^2+1\right)-\frac{3}{2}\log(x-1)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x, algorithm="maxima")
```

```
[Out] -1/2*(5*x - 4)/(x^2 - 2*x + 1) - arctan(x) + 3/4*log(x^2 + 1) - 3/2*log(x -
 1)
```

Fricas [A] time = 2.10564, size = 171, normalized size = 3.64

$$
-\frac{4\left(x^2-2x+1\right)\arctan(x)-3\left(x^2-2x+1\right)\log\left(x^2+1\right)+6\left(x^2-2x+1\right)\log(x-1)+10x-8}{4\left(x^2-2x+1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(x^2 - 2*x + 1)*arctan(x) - 3*(x^2 - 2*x + 1)*log(x^2 + 1) + 6*(x^2
 - 2*x + 1)*log(x - 1) + 10*x - 8)/(x^2 - 2*x + 1)
```

Sympy [A] time = 0.149242, size = 36, normalized size = 0.77

$$-\frac{5x-4}{2x^2-4x+2}-\frac{3\log(x-1)}{2}+\frac{3\log\left(x^2+1\right)}{4}-\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+x-2)/(-1+x)**3/(x**2+1),x)
```

```
[Out] -(5*x - 4)/(2*x**2 - 4*x + 2) - 3*log(x - 1)/2 + 3*log(x**2 + 1)/4 - atan(x
)
```

Giac [A] time = 1.05329, size = 43, normalized size = 0.91

$$-\frac{5x-4}{2(x-1)^2}-\arctan(x)+\frac{3}{4}\log\left(x^2+1\right)-\frac{3}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+x-2)/(-1+x)^3/(x^2+1),x, algorithm="giac")
```

```
[Out] -1/2*(5*x - 4)/(x - 1)^2 - arctan(x) + 3/4*log(x^2 + 1) - 3/2*log(abs(x - 1
))
```

3.113 $\int \frac{1}{1+x^2+x^4}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{4}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) - Log[1 - x + x^2]/4 + Log[1 + x + x^2]/4
```

Rubi [A] time = 0.0380003, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{1}{4}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) - Log[1 - x + x^2]/4 + Log[1 + x + x^2]/4
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{1+x^2+x^4}\,dx = \frac{1}{2}\int \frac{1-x}{1-x+x^2}\,dx + \frac{1}{2}\int \frac{1+x}{1+x+x^2}\,dx$$
$$= \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx - \frac{1}{4}\int \frac{-1+2x}{1-x+x^2}\,dx + \frac{1}{4}\int \frac{1}{1+x+x^2}\,dx + \frac{1}{4}\int \frac{1+2x}{1+x+x^2}\,dx$$
$$= -\frac{1}{4}\log\left(1-x+x^2\right) + \frac{1}{4}\log\left(1+x+x^2\right) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-3-\right.$$
$$= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{4}\log\left(1-x+x^2\right) + \frac{1}{4}\log\left(1+x+x^2\right)$$

Mathematica [C] time = 0.0577123, size = 73, normalized size = 1.09

$$\frac{i\left(\sqrt{1-i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)-\sqrt{1+i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)\right)}{\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2 + x^4)^(-1),x]
```

```
[Out] (I*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x)/2] - Sqrt[1 + I*Sqrt[3]]*
ArcTan[((I + Sqrt[3])*x)/2]))/Sqrt[6]
```

Maple [A] time = 0.005, size = 54, normalized size = 0.8

$$-\frac{\ln\left(x^2-x+1\right)}{4}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\ln\left(x^2+x+1\right)}{4}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+x^2+1),x)
```

```
[Out] -1/4*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/4*ln(x^2+x+1)+1/
6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.43509, size = 72, normalized size = 1.07

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

Fricas [A] time = 2.07286, size = 180, normalized size = 2.69

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

Sympy [A] time = 0.162888, size = 70, normalized size = 1.04

$$-\frac{\log\left(x^2-x+1\right)}{4}+\frac{\log\left(x^2+x+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+x**2+1),x)
```

```
[Out] -log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 - s
qrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6
```

Giac [A] time = 1.05511, size = 72, normalized size = 1.07

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

3.114 $\int \frac{3+2x^3}{-9x+x^5}\,dx$

Optimal. Leaf size=48

$$\frac{1}{12}\log\left(9-x^4\right)-\frac{\log(x)}{3}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[x/Sqrt[3]]/Sqrt[3] - ArcTanh[x/Sqrt[3]]/Sqrt[3] - Log[x]/3 + Log[9 -
 x^4]/12
```

Rubi [A] time = 0.0526631, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {1593, 1831, 266, 36, 31, 29, 298, 203, 206}

$$\frac{1}{12}\log\left(9-x^4\right)-\frac{\log(x)}{3}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x^3)/(-9*x + x^5),x]
```

```
[Out] ArcTan[x/Sqrt[3]]/Sqrt[3] - ArcTanh[x/Sqrt[3]]/Sqrt[3] - Log[x]/3 + Log[9 -
 x^4]/12
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{3+2x^3}{-9x+x^5}\,dx &= \int \frac{3+2x^3}{x\left(-9+x^4\right)}\,dx \\
&= \int \left(\frac{3}{x\left(-9+x^4\right)}+\frac{2x^2}{-9+x^4}\right)dx \\
&= 2\int \frac{x^2}{-9+x^4}\,dx+3\int \frac{1}{x\left(-9+x^4\right)}\,dx \\
&= \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{(-9+x)x}\,dx,x,x^4\right)-\int \frac{1}{3-x^2}\,dx+\int \frac{1}{3+x^2}\,dx \\
&= \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{12}\,\text{Subst}\left(\int \frac{1}{-9+x}\,dx,x,x^4\right)-\frac{1}{12}\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^4\right) \\
&= \frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{\log(x)}{3}+\frac{1}{12}\log\left(9-x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.018192, size = 67, normalized size = 1.4

$$\frac{1}{12}\left(\log\left(9-x^4\right)-4\log(x)+2\sqrt{3}\log\left(3-\sqrt{3}x\right)-2\sqrt{3}\log\left(\sqrt{3}x+3\right)+4\sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x^3)/(-9*x + x^5),x]
```

```
[Out] (4*Sqrt[3]*ArcTan[x/Sqrt[3]] - 4*Log[x] + 2*Sqrt[3]*Log[3 - Sqrt[3]*x] - 2*
Sqrt[3]*Log[3 + Sqrt[3]*x] + Log[9 - x^4])/12
```

Maple [A] time = 0.007, size = 46, normalized size = 1.

$$-\frac{\ln(x)}{3}+\frac{\ln(x^2+3)}{12}+\frac{\sqrt{3}}{3}\arctan\left(\frac{x\sqrt{3}}{3}\right)+\frac{\ln(x^2-3)}{12}-\frac{\sqrt{3}}{3}\mathrm{Artanh}\left(\frac{x\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3)/(x^5-9*x),x)
```

```
[Out] -1/3*ln(x)+1/12*ln(x^2+3)+1/3*arctan(1/3*x*3^(1/2))*3^(1/2)+1/12*ln(x^2-3)-
1/3*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.41503, size = 73, normalized size = 1.52

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{6}\sqrt{3}\log\left(\frac{x-\sqrt{3}}{x+\sqrt{3}}\right)+\frac{1}{12}\log\left(x^2+3\right)+\frac{1}{12}\log\left(x^2-3\right)-\frac{1}{3}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3)/(x^5-9*x),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/6*sqrt(3)*log((x - sqrt(3))/(x + sqrt
(3))) + 1/12*log(x^2 + 3) + 1/12*log(x^2 - 3) - 1/3*log(x)
```

Fricas [A] time = 2.21509, size = 190, normalized size = 3.96

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{6}\sqrt{3}\log\left(\frac{x^2-2\sqrt{3}x+3}{x^2-3}\right)+\frac{1}{12}\log\left(x^2+3\right)+\frac{1}{12}\log\left(x^2-3\right)-\frac{1}{3}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3)/(x^5-9*x),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/6*sqrt(3)*log((x^2 - 2*sqrt(3)*x + 3)
/(x^2 - 3)) + 1/12*log(x^2 + 3) + 1/12*log(x^2 - 3) - 1/3*log(x)
```

Sympy [B] time = 1.32339, size = 172, normalized size = 3.58

$$-\frac{\log(x)}{3}+\frac{\log(x^2+3)}{12}+\left(\frac{1}{12}-\frac{\sqrt{3}}{6}\right)\log\left(x-\frac{108000\left(\frac{1}{12}-\frac{\sqrt{3}}{6}\right)^4}{481}+\frac{1368\left(\frac{1}{12}-\frac{\sqrt{3}}{6}\right)^3}{481}+\frac{943\sqrt{3}}{5772}+\frac{4158\left(\frac{1}{12}-\frac{\sqrt{3}}{6}\right)^2}{481}+\frac{17}{11}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3)/(x**5-9*x),x)
```

```
[Out] -log(x)/3 + log(x**2 + 3)/12 + (1/12 - sqrt(3)/6)*log(x - 108000*(1/12 - sq
rt(3)/6)**4/481 + 1368*(1/12 - sqrt(3)/6)**3/481 + 943*sqrt(3)/5772 + 4158*
(1/12 - sqrt(3)/6)**2/481 + 17413/11544) + (1/12 + sqrt(3)/6)*log(x - 10800
0*(1/12 + sqrt(3)/6)**4/481 - 943*sqrt(3)/5772 + 1368*(1/12 + sqrt(3)/6)**3
```

```
/481 + 4158*(1/12 + sqrt(3)/6)**2/481 + 17413/11544) + sqrt(3)*atan(sqrt(3)
*x/3)/3
```

Giac [A] time = 1.0724, size = 86, normalized size = 1.79

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)+\frac{1}{6}\sqrt{3}\log\left(\frac{\left|2x-2\sqrt{3}\right|}{\left|2x+2\sqrt{3}\right|}\right)+\frac{1}{12}\log\left(x^2+3\right)+\frac{1}{12}\log\left(\left|x^2-3\right|\right)-\frac{1}{3}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3)/(x^5-9*x),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*x) + 1/6*sqrt(3)*log(abs(2*x - 2*sqrt(3))/ab
s(2*x + 2*sqrt(3))) + 1/12*log(x^2 + 3) + 1/12*log(abs(x^2 - 3)) - 1/3*log(
abs(x))
```

3.115 $\int \frac{-20+8x+5x^3}{(-4+x)^3(8-4x+x^2)}\,dx$

Optimal. Leaf size=58

$$\frac{45}{32}\log\left(x^2-4x+8\right)+\frac{41}{4(4-x)}-\frac{83}{4(4-x)^2}-\frac{45}{16}\log(4-x)-\frac{3}{16}\tan^{-1}\left(1-\frac{x}{2}\right)$$

```
[Out] -83/(4*(4 - x)^2) + 41/(4*(4 - x)) - (3*ArcTan[1 - x/2])/16 - (45*Log[4 - x
])/16 + (45*Log[8 - 4*x + x^2])/32
```

Rubi [A] time = 0.0550267, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1628, 634, 617, 204, 628}

$$\frac{45}{32}\log\left(x^2-4x+8\right)+\frac{41}{4(4-x)}-\frac{83}{4(4-x)^2}-\frac{45}{16}\log(4-x)-\frac{3}{16}\tan^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-20 + 8*x + 5*x^3)/((-4 + x)^3*(8 - 4*x + x^2)),x]
```

```
[Out] -83/(4*(4 - x)^2) + 41/(4*(4 - x)) - (3*ArcTan[1 - x/2])/16 - (45*Log[4 - x
])/16 + (45*Log[8 - 4*x + x^2])/32
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{-20+8x+5x^3}{(-4+x)^3\left(8-4x+x^2\right)}\,dx = \int \left(\frac{83}{2(-4+x)^3}+\frac{41}{4(-4+x)^2}-\frac{45}{16(-4+x)}+\frac{3(-28+15x)}{16\left(8-4x+x^2\right)}\right)dx$$
$$= -\frac{83}{4(4-x)^2}+\frac{41}{4(4-x)}-\frac{45}{16}\log(4-x)+\frac{3}{16}\int \frac{-28+15x}{8-4x+x^2}\,dx$$
$$= -\frac{83}{4(4-x)^2}+\frac{41}{4(4-x)}-\frac{45}{16}\log(4-x)+\frac{3}{8}\int \frac{1}{8-4x+x^2}\,dx+\frac{45}{32}\int \frac{-4+2x}{8-4x+x^2}\,dx$$
$$= -\frac{83}{4(4-x)^2}+\frac{41}{4(4-x)}-\frac{45}{16}\log(4-x)+\frac{45}{32}\log\left(8-4x+x^2\right)+\frac{3}{16}\operatorname{Subst}\left(\int \frac{1}{-1-\cdots}\right.$$
$$= -\frac{83}{4(4-x)^2}+\frac{41}{4(4-x)}-\frac{3}{16}\tan^{-1}\left(1-\frac{x}{2}\right)-\frac{45}{16}\log(4-x)+\frac{45}{32}\log\left(8-4x+x^2\right)$$

Mathematica [A] time = 0.0244687, size = 46, normalized size = 0.79

$$\frac{1}{32}\left(45\log\left(x^2-4x+8\right)-\frac{328}{x-4}-\frac{664}{(x-4)^2}-90\log(x-4)+6\tan^{-1}\left(\frac{x-2}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-20 + 8*x + 5*x^3)/((-4 + x)^3*(8 - 4*x + x^2)),x]
```

```
[Out] (-664/(-4 + x)^2 - 328/(-4 + x) + 6*ArcTan[(-2 + x)/2] - 90*Log[-4 + x] + 4
5*Log[8 - 4*x + x^2])/32
```

Maple [A] time = 0.008, size = 41, normalized size = 0.7

$$-\frac{83}{4\,(x-4)^2}-\frac{41}{4\,x-16}-\frac{45\,\ln(x-4)}{16}+\frac{45\,\ln\left(x^2-4\,x+8\right)}{32}+\frac{3}{16}\arctan\left(-1+\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^3+8*x-20)/(x-4)^3/(x^2-4*x+8),x)
```

```
[Out] -83/4/(x-4)^2-41/4/(x-4)-45/16*ln(x-4)+45/32*ln(x^2-4*x+8)+3/16*arctan(-1+1
/2*x)
```

Maxima [A] time = 1.42162, size = 58, normalized size = 1.

$$-\frac{41\,x-81}{4\left(x^2-8\,x+16\right)}+\frac{3}{16}\arctan\left(\frac{1}{2}\,x-1\right)+\frac{45}{32}\log\left(x^2-4\,x+8\right)-\frac{45}{16}\log(x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x, algorithm="maxima")
```

```
[Out] -1/4*(41*x - 81)/(x^2 - 8*x + 16) + 3/16*arctan(1/2*x - 1) + 45/32*log(x^2
- 4*x + 8) - 45/16*log(x - 4)
```

Fricas [A] time = 2.15851, size = 203, normalized size = 3.5

$$\frac{6\left(x^2-8x+16\right)\arctan\left(\frac{1}{2}x-1\right)+45\left(x^2-8x+16\right)\log\left(x^2-4x+8\right)-90\left(x^2-8x+16\right)\log\left(x-4\right)-328x+648}{32\left(x^2-8x+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x, algorithm="fricas")
```

```
[Out] 1/32*(6*(x^2 - 8*x + 16)*arctan(1/2*x - 1) + 45*(x^2 - 8*x + 16)*log(x^2 -
4*x + 8) - 90*(x^2 - 8*x + 16)*log(x - 4) - 328*x + 648)/(x^2 - 8*x + 16)
```

Sympy [A] time = 0.167134, size = 46, normalized size = 0.79

$$-\frac{41x-81}{4x^2-32x+64}-\frac{45\log\left(x-4\right)}{16}+\frac{45\log\left(x^2-4x+8\right)}{32}+\frac{3\,\mathrm{atan}\left(\frac{x}{2}-1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**3+8*x-20)/(-4+x)**3/(x**2-4*x+8),x)
```

```
[Out] -(41*x - 81)/(4*x**2 - 32*x + 64) - 45*log(x - 4)/16 + 45*log(x**2 - 4*x +
8)/32 + 3*atan(x/2 - 1)/16
```

Giac [A] time = 1.07188, size = 53, normalized size = 0.91

$$-\frac{41\,x-81}{4\,(x-4)^2}+\frac{3}{16}\arctan\left(\frac{1}{2}x-1\right)+\frac{45}{32}\log\left(x^2-4x+8\right)-\frac{45}{16}\log\left(|x-4|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3+8*x-20)/(-4+x)^3/(x^2-4*x+8),x, algorithm="giac")
```

```
[Out] -1/4*(41*x - 81)/(x - 4)^2 + 3/16*arctan(1/2*x - 1) + 45/32*log(x^2 - 4*x +
 8) - 45/16*log(abs(x - 4))
```

3.116 $\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} dx$

Optimal. Leaf size=51

$$-\frac{1}{12}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{6}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[x/2]/12 + ArcTan[x]/6 - ArcTan[x/Sqrt[2]]/(2*Sqrt[2]) + ArcTan[x/Sq
rt[3]]/(2*Sqrt[3])
```

Rubi [A] time = 0.284273, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {6725, 203}

$$-\frac{1}{12}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{6}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

```
[Out] -ArcTan[x/2]/12 + ArcTan[x]/6 - ArcTan[x/Sqrt[2]]/(2*Sqrt[2]) + ArcTan[x/Sq
rt[3]]/(2*Sqrt[3])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} dx &= \int \left(\frac{1}{6(1+x^2)}-\frac{1}{2(2+x^2)}+\frac{1}{2(3+x^2)}-\frac{1}{6(4+x^2)}\right) dx\\
&= \frac{1}{6}\int \frac{1}{1+x^2} dx-\frac{1}{6}\int \frac{1}{4+x^2} dx-\frac{1}{2}\int \frac{1}{2+x^2} dx+\frac{1}{2}\int \frac{1}{3+x^2} dx\\
&= -\frac{1}{12}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{6}\tan^{-1}(x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{2\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0235758, size = 47, normalized size = 0.92

$$\frac{1}{12}\left(-\tan^{-1}\left(\frac{x}{2}\right)+2\tan^{-1}(x)-3\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+2\sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

```
[Out] (-ArcTan[x/2] + 2*ArcTan[x] - 3*Sqrt[2]*ArcTan[x/Sqrt[2]] + 2*Sqrt[3]*ArcTa
n[x/Sqrt[3]])/12
```

Maple [A] time = 0.009, size = 36, normalized size = 0.7

$$-\frac{1}{12}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{6}-\frac{\sqrt{2}}{4}\arctan\left(\frac{x\sqrt{2}}{2}\right)+\frac{\sqrt{3}}{6}\arctan\left(\frac{x\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x)
```

```
[Out] -1/12*arctan(1/2*x)+1/6*arctan(x)-1/4*arctan(1/2*x*2^(1/2))*2^(1/2)+1/6*arc
tan(1/3*x*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.41324, size = 47, normalized size = 0.92

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{12}\arctan\left(\frac{1}{2}x\right)+\frac{1}{6}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/1
2*arctan(1/2*x) + 1/6*arctan(x)
```

Fricas [A] time = 2.23834, size = 146, normalized size = 2.86

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{12}\arctan\left(\frac{1}{2}x\right)+\frac{1}{6}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/1
2*arctan(1/2*x) + 1/6*arctan(x)
```

Sympy [A] time = 0.356608, size = 44, normalized size = 0.86

$$-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{12}+\frac{\operatorname{atan}(x)}{6}-\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)/(x**2+2)/(x**2+3)/(x**2+4),x)
```

```
[Out] -atan(x/2)/12 + atan(x)/6 - sqrt(2)*atan(sqrt(2)*x/2)/4 + sqrt(3)*atan(sqrt
(3)*x/3)/6
```

Giac [A] time = 1.06052, size = 47, normalized size = 0.92

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)-\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1}{12}\arctan\left(\frac{1}{2}x\right)+\frac{1}{6}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/1
2*arctan(1/2*x) + 1/6*arctan(x)
```

3.117 $\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} \, dx$

Optimal. Leaf size=41

$$\frac{1}{12}\log(x^2+1)-\frac{1}{4}\log(x^2+2)+\frac{1}{4}\log(x^2+3)-\frac{1}{12}\log(x^2+4)$$

```
[Out] Log[1 + x^2]/12 - Log[2 + x^2]/4 + Log[3 + x^2]/4 - Log[4 + x^2]/12
```

Rubi [A] time = 0.311424, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {6694, 180}

$$\frac{1}{12}\log(x^2+1)-\frac{1}{4}\log(x^2+2)+\frac{1}{4}\log(x^2+3)-\frac{1}{12}\log(x^2+4)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/12 - Log[2 + x^2]/4 + Log[3 + x^2]/4 - Log[4 + x^2]/12
```

Rule 6694

```
Int[(u_)*((c_.) + (d_.)*(v_))^(n_.)*((e_.) + (f_.)*(w_))^(p_.)*((a_.) + (b_
.)*(y_))^(m_.)*((g_.) + (h_.)*(z_))^(q_.), x_Symbol] :> With[{r = Derivativ
eDivides[y, u, x]}, Dist[r, Subst[Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(
g + h*x)^q, x], x, y], x] /;  !FalseQ[r]] /; FreeQ[{a, b, c, d, e, f, g, h,
 m, n, p, q}, x] && EqQ[v, y] && EqQ[w, y] && EqQ[z, y]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(1+x^2)(2+x^2)(3+x^2)(4+x^2)} \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1+x)(2+x)(3+x)(4+x)} \, dx, x, x^2\right) \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{6(1+x)}-\frac{1}{2(2+x)}+\frac{1}{2(3+x)}-\frac{1}{6(4+x)}\right) dx, x, x^2\right) \\
&= \frac{1}{12}\log(1+x^2)-\frac{1}{4}\log(2+x^2)+\frac{1}{4}\log(3+x^2)-\frac{1}{12}\log(4+x^2)
\end{aligned}$$

Mathematica [A] time = 0.0083798, size = 41, normalized size = 1.

$$\frac{1}{12}\log(x^2+1)-\frac{1}{4}\log(x^2+2)+\frac{1}{4}\log(x^2+3)-\frac{1}{12}\log(x^2+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x^2)*(2 + x^2)*(3 + x^2)*(4 + x^2)),x]

[Out] Log[1 + x^2]/12 - Log[2 + x^2]/4 + Log[3 + x^2]/4 - Log[4 + x^2]/12
```

Maple [A] time = 0.01, size = 34, normalized size = 0.8

$$\frac{\ln\left(x^2+1\right)}{12}-\frac{\ln\left(x^2+2\right)}{4}+\frac{\ln\left(x^2+3\right)}{4}-\frac{\ln\left(x^2+4\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x)

[Out] 1/12*ln(x^2+1)-1/4*ln(x^2+2)+1/4*ln(x^2+3)-1/12*ln(x^2+4)
```

Maxima [A] time = 0.921042, size = 45, normalized size = 1.1

$$-\frac{1}{12}\log\left(x^2+4\right)+\frac{1}{4}\log\left(x^2+3\right)-\frac{1}{4}\log\left(x^2+2\right)+\frac{1}{12}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="maxima")

[Out] -1/12*log(x^2 + 4) + 1/4*log(x^2 + 3) - 1/4*log(x^2 + 2) + 1/12*log(x^2 + 1
)
```

Fricas [A] time = 2.17329, size = 105, normalized size = 2.56

$$-\frac{1}{12}\log\left(x^2+4\right)+\frac{1}{4}\log\left(x^2+3\right)-\frac{1}{4}\log\left(x^2+2\right)+\frac{1}{12}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="fricas")

[Out] -1/12*log(x^2 + 4) + 1/4*log(x^2 + 3) - 1/4*log(x^2 + 2) + 1/12*log(x^2 + 1
)
```

Sympy [A] time = 0.161699, size = 32, normalized size = 0.78

$$\frac{\log\left(x^2+1\right)}{12}-\frac{\log\left(x^2+2\right)}{4}+\frac{\log\left(x^2+3\right)}{4}-\frac{\log\left(x^2+4\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+1)/(x**2+2)/(x**2+3)/(x**2+4),x)

[Out] log(x**2 + 1)/12 - log(x**2 + 2)/4 + log(x**2 + 3)/4 - log(x**2 + 4)/12
```

<u>Giac</u> [A] time = 1.0602, size = 45, normalized size = 1.1

$$-\frac{1}{12} \log\left(x^2+4\right) + \frac{1}{4} \log\left(x^2+3\right) - \frac{1}{4} \log\left(x^2+2\right) + \frac{1}{12} \log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+2)/(x^2+3)/(x^2+4),x, algorithm="giac")
```

```
[Out] -1/12*log(x^2 + 4) + 1/4*log(x^2 + 3) - 1/4*log(x^2 + 2) + 1/12*log(x^2 + 1
)
```

3.118 $\int \frac{1}{a^3+x^3}\,dx$

Optimal. Leaf size=56

$$-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}+\frac{\log(a+x)}{3a^2}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^2}$$

```
[Out] -(ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^2)) + Log[a + x]/(3*a^2) - Log[a
^2 - a*x + x^2]/(6*a^2)
```

Rubi [A] time = 0.0303337, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}+\frac{\log(a+x)}{3a^2}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^3 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^2)) + Log[a + x]/(3*a^2) - Log[a
^2 - a*x + x^2]/(6*a^2)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a^3+x^3}\,dx &= \frac{\int \frac{1}{a+x}\,dx}{3a^2}+\frac{\int \frac{2a-x}{a^2-ax+x^2}\,dx}{3a^2}\\
&= \frac{\log(a+x)}{3a^2}-\frac{\int \frac{-a+2x}{a^2-ax+x^2}\,dx}{6a^2}+\frac{\int \frac{1}{a^2-ax+x^2}\,dx}{2a}\\
&= \frac{\log(a+x)}{3a^2}-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{a}\right)}{a^2}\\
&= -\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^2}+\frac{\log(a+x)}{3a^2}-\frac{\log\left(a^2-ax+x^2\right)}{6a^2}
\end{aligned}$$

Mathematica [A] time = 0.0123992, size = 52, normalized size = 0.93

$$\frac{-\log\left(a^2-ax+x^2\right)+2\log(a+x)+2\sqrt{3}\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^3 + x^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] + 2*Log[a + x] - Log[a^2 - a*x +
x^2])/(6*a^2)
```

Maple [A] time = 0.008, size = 52, normalized size = 0.9

$$-\frac{\ln\left(a^2-ax+x^2\right)}{6\,a^2}+\frac{\sqrt{3}}{3\,a^2}\arctan\left(\frac{(2\,x-a)\sqrt{3}}{3\,a}\right)+\frac{\ln(a+x)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^3+x^3),x)
```

```
[Out] -1/6*ln(a^2-a*x+x^2)/a^2+1/3/a^2*3^(1/2)*arctan(1/3*(2*x-a)*3^(1/2)/a)+1/3*
ln(a+x)/a^2
```

Maxima [A] time = 1.40532, size = 66, normalized size = 1.18

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2\,x)}{3\,a}\right)}{3\,a^2}-\frac{\log\left(a^2-ax+x^2\right)}{6\,a^2}+\frac{\log(a+x)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^2 - 1/6*log(a^2 - a*x + x^2)
/a^2 + 1/3*log(a + x)/a^2
```

Fricas [A] time = 2.05402, size = 124, normalized size = 2.21

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right) - \log\left(a^2 - ax + x^2\right) + 2\log\left(a + x\right)}{6a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - log(a^2 - a*x + x^2) + 2*
log(a + x))/a^2
```

Sympy [C] time = 0.119672, size = 73, normalized size = 1.3

$$\frac{\frac{\log(a+x)}{3} + \left(-\frac{1}{6} - \frac{\sqrt{3}i}{6}\right)\log\left(3a\left(-\frac{1}{6} - \frac{\sqrt{3}i}{6}\right) + x\right) + \left(-\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)\log\left(3a\left(-\frac{1}{6} + \frac{\sqrt{3}i}{6}\right) + x\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**3+x**3),x)
```

```
[Out] (log(a + x)/3 + (-1/6 - sqrt(3)*I/6)*log(3*a*(-1/6 - sqrt(3)*I/6) + x) + (-
1/6 + sqrt(3)*I/6)*log(3*a*(-1/6 + sqrt(3)*I/6) + x))/a**2
```

Giac [A] time = 1.05349, size = 68, normalized size = 1.21

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^2} - \frac{\log\left(a^2 - ax + x^2\right)}{6a^2} + \frac{\log\left(|a + x|\right)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^3+x^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^2 - 1/6*log(a^2 - a*x + x^2)
/a^2 + 1/3*log(abs(a + x))/a^2
```

3.119 $\int \frac{x}{a^3+x^3} \, dx$

Optimal. Leaf size=56

$$\frac{\log \left(a^2-a x+x^2\right)}{6 a}-\frac{\log (a+x)}{3 a}-\frac{\tan ^{-1}\left(\frac{a-2 x}{\sqrt{3} a}\right)}{\sqrt{3} a}$$

```
[Out] -(ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a)) - Log[a + x]/(3*a) + Log[a^2 -
 a*x + x^2]/(6*a)
```

Rubi [A] time = 0.0277833, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {292, 31, 634, 617, 204, 628}

$$\frac{\log \left(a^2-a x+x^2\right)}{6 a}-\frac{\log (a+x)}{3 a}-\frac{\tan ^{-1}\left(\frac{a-2 x}{\sqrt{3} a}\right)}{\sqrt{3} a}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^3 + x^3),x]
```

```
[Out] -(ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a)) - Log[a + x]/(3*a) + Log[a^2 -
 a*x + x^2]/(6*a)
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a^3+x^3}\,dx &= -\frac{\int \frac{1}{a+x}\,dx}{3a}+\frac{\int \frac{a+x}{a^2-ax+x^2}\,dx}{3a}\\ &= -\frac{\log(a+x)}{3a}+\frac{1}{2}\int \frac{1}{a^2-ax+x^2}\,dx+\frac{\int \frac{-a+2x}{a^2-ax+x^2}\,dx}{6a}\\ &= -\frac{\log(a+x)}{3a}+\frac{\log\left(a^2-ax+x^2\right)}{6a}+\frac{\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{a}\right)}{a}\\ &= -\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a}-\frac{\log(a+x)}{3a}+\frac{\log\left(a^2-ax+x^2\right)}{6a}\end{aligned}$$

Mathematica [A] time = 0.0054318, size = 50, normalized size = 0.89

$$\frac{\log\left(a^2-ax+x^2\right)-2\log(a+x)+2\sqrt{3}\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)}{6a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^3 + x^3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*Log[a + x] + Log[a^2 - a*x +
x^2])/(6*a)
```

Maple [A] time = 0.005, size = 52, normalized size = 0.9

$$\frac{\ln\left(a^2-ax+x^2\right)}{6a}+\frac{\sqrt{3}}{3a}\arctan\left(\frac{(2x-a)\sqrt{3}}{3a}\right)-\frac{\ln(a+x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^3+x^3),x)
```

```
[Out] 1/6*ln(a^2-a*x+x^2)/a+1/3*3^(1/2)/a*arctan(1/3*(2*x-a)*3^(1/2)/a)-1/3*ln(a+
x)/a
```

Maxima [A] time = 1.39883, size = 66, normalized size = 1.18

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a}+\frac{\log\left(a^2-ax+x^2\right)}{6a}-\frac{\log(a+x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a + 1/6*log(a^2 - a*x + x^2)/a
 - 1/3*log(a + x)/a
```

Fricas [A] time = 2.08417, size = 122, normalized size = 2.18

$$\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)+\log\left(a^2-ax+x^2\right)-2\log\left(a+x\right)}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a) + log(a^2 - a*x + x^2) - 2*
log(a + x))/a
```

Sympy [C] time = 0.113897, size = 71, normalized size = 1.27

$$\frac{-\frac{\log\left(a+x\right)}{3}+\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)^2+x\right)+\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)^2+x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**3+x**3),x)
```

```
[Out] (-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(9*a*(1/6 - sqrt(3)*I/6)**2 + x) +
(1/6 + sqrt(3)*I/6)*log(9*a*(1/6 + sqrt(3)*I/6)**2 + x))/a
```

Giac [A] time = 1.05209, size = 68, normalized size = 1.21

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a}+\frac{\log\left(a^2-ax+x^2\right)}{6a}-\frac{\log\left(|a+x|\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^3+x^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a + 1/6*log(a^2 - a*x + x^2)/a
 - 1/3*log(abs(a + x))/a
```

3.120 $\int \frac{x^2}{a^3+x^3} dx$

Optimal. Leaf size=12

$$\frac{1}{3}\log\left(a^3+x^3\right)$$

```
[Out] Log[a^3 + x^3]/3
```

Rubi [A] time = 0.0026214, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {260}

$$\frac{1}{3}\log\left(a^3+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^3 + x^3),x]

[Out] Log[a^3 + x^3]/3
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^2}{a^3+x^3} dx = \frac{1}{3}\log\left(a^3+x^3\right)$$

Mathematica [A] time = 0.0021873, size = 12, normalized size = 1.

$$\frac{1}{3}\log\left(a^3+x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^3 + x^3),x]

[Out] Log[a^3 + x^3]/3
```

Maple [A] time = 0., size = 11, normalized size = 0.9

$$\frac{\ln\left(a^3+x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^3+x^3),x)
```

```
[Out] 1/3*ln(a^3+x^3)
```

Maxima [A] time = 0.917987, size = 14, normalized size = 1.17

$$\frac{1}{3}\,\log\left(a^3 + x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] 1/3*log(a^3 + x^3)
```

Fricas [A] time = 2.06094, size = 27, normalized size = 2.25

$$\frac{1}{3}\,\log\left(a^3 + x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] 1/3*log(a^3 + x^3)
```

Sympy [A] time = 0.092382, size = 8, normalized size = 0.67

$$\frac{\log\left(a^3 + x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**3+x**3),x)
```

```
[Out] log(a**3 + x**3)/3
```

Giac [A] time = 1.05799, size = 15, normalized size = 1.25

$$\frac{1}{3}\,\log\left(\left|a^3 + x^3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^3+x^3),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(a^3 + x^3))
```

3.121 $\int \frac{1}{x(a^3+x^3)}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a^3}-\frac{\log\left(a^3+x^3\right)}{3a^3}$$

```
[Out] Log[x]/a^3 - Log[a^3 + x^3]/(3*a^3)
```

Rubi [A] time = 0.0083182, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a^3}-\frac{\log\left(a^3+x^3\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^3 + x^3)),x]
```

```
[Out] Log[x]/a^3 - Log[a^3 + x^3]/(3*a^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a^3+x^3\right)}\,dx &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{x\left(a^3+x\right)}\,dx,x,x^3\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^3\right)}{3a^3}-\frac{\operatorname{Subst}\left(\int \frac{1}{a^3+x}\,dx,x,x^3\right)}{3a^3}\\ &= \frac{\log(x)}{a^3}-\frac{\log\left(a^3+x^3\right)}{3a^3}\end{aligned}$$

Mathematica [A] time = 0.0044086, size = 22, normalized size = 1.

$$\frac{\log(x)}{a^3}-\frac{\log\left(a^3+x^3\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^3 + x^3)),x]
```

```
[Out] Log[x]/a^3 - Log[a^3 + x^3]/(3*a^3)
```

Maple [A] time = 0.005, size = 34, normalized size = 1.6

$$\frac{\ln(x)}{a^3}-\frac{\ln\left(a^2-ax+x^2\right)}{3\,a^3}-\frac{\ln(a+x)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^3+x^3),x)
```

```
[Out] ln(x)/a^3-1/3/a^3*ln(a^2-a*x+x^2)-1/3*ln(a+x)/a^3
```

Maxima [A] time = 0.922174, size = 31, normalized size = 1.41

$$-\frac{\log\left(a^3+x^3\right)}{3\,a^3}+\frac{\log\left(x^3\right)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] -1/3*log(a^3 + x^3)/a^3 + 1/3*log(x^3)/a^3
```

Fricas [A] time = 2.01846, size = 51, normalized size = 2.32

$$-\frac{\log\left(a^3+x^3\right)-3\,\log(x)}{3\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] -1/3*(log(a^3 + x^3) - 3*log(x))/a^3
```

Sympy [A] time = 0.17946, size = 19, normalized size = 0.86

$$\frac{\log(x)}{a^3}-\frac{\log\left(a^3+x^3\right)}{3a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**3+x**3),x)
```

```
[Out] log(x)/a**3 - log(a**3 + x**3)/(3*a**3)
```

Giac [A] time = 1.05538, size = 30, normalized size = 1.36

$$-\frac{\log\left(\left|a^3 + x^3\right|\right)}{3\,a^3} + \frac{\log\left(|x|\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^3+x^3),x, algorithm="giac")
```

```
[Out] -1/3*log(abs(a^3 + x^3))/a^3 + log(abs(x))/a^3
```

3.122 $\int \frac{1}{x^2(a^3+x^3)}\,dx$

Optimal. Leaf size=63

$$-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}-\frac{1}{a^3x}+\frac{\log(a+x)}{3a^4}+\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^4}$$

```
[Out] -(1/(a^3*x)) + ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^4) + Log[a + x]/(3*
a^4) - Log[a^2 - a*x + x^2]/(6*a^4)
```

Rubi [A] time = 0.0370236, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {325, 292, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}-\frac{1}{a^3x}+\frac{\log(a+x)}{3a^4}+\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^3 + x^3)),x]
```

```
[Out] -(1/(a^3*x)) + ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^4) + Log[a + x]/(3*
a^4) - Log[a^2 - a*x + x^2]/(6*a^4)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\left(a^3+x^3\right)}\,dx &= -\frac{1}{a^3x}-\frac{\int \frac{x}{a^3+x^3}\,dx}{a^3}\\
&= -\frac{1}{a^3x}+\frac{\int \frac{1}{a+x}\,dx}{3a^4}-\frac{\int \frac{a+x}{a^2-ax+x^2}\,dx}{3a^4}\\
&= -\frac{1}{a^3x}+\frac{\log(a+x)}{3a^4}-\frac{\int \frac{-a+2x}{a^2-ax+x^2}\,dx}{6a^4}-\frac{\int \frac{1}{a^2-ax+x^2}\,dx}{2a^3}\\
&= -\frac{1}{a^3x}+\frac{\log(a+x)}{3a^4}-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{a}\right)}{a^4}\\
&= -\frac{1}{a^3x}+\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^4}+\frac{\log(a+x)}{3a^4}-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}
\end{aligned}$$

Mathematica [A] time = 0.013682, size = 60, normalized size = 0.95

$$-\frac{x\log\left(a^2-ax+x^2\right)-2x\log(a+x)+2\sqrt{3}x\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)+6a}{6a^4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^3 + x^3)),x]
```

```
[Out] -(6*a + 2*Sqrt[3]*x*ArcTan[(-a + 2*x)/(Sqrt[3]*a)] - 2*x*Log[a + x] + x*Log
[a^2 - a*x + x^2])/(6*a^4*x)
```

Maple [A] time = 0.007, size = 60, normalized size = 1.

$$-\frac{\ln\left(a^2-ax+x^2\right)}{6\,a^4}-\frac{\sqrt{3}}{3\,a^4}\arctan\left(\frac{(2\,x-a)\sqrt{3}}{3\,a}\right)+\frac{\ln(a+x)}{3\,a^4}-\frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^3+x^3),x)
```

```
[Out] -1/6*ln(a^2-a*x+x^2)/a^4-1/3/a^4*3^(1/2)*arctan(1/3*(2*x-a)*3^(1/2)/a)+1/3*
ln(a+x)/a^4-1/a^3/x
```

Maxima [A] time = 1.39782, size = 77, normalized size = 1.22

$$-\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^4}-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}+\frac{\log\left(a+x\right)}{3a^4}-\frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^4 - 1/6*log(a^2 - a*x + x^2
)/a^4 + 1/3*log(a + x)/a^4 - 1/(a^3*x)
```

Fricas [A] time = 2.33257, size = 147, normalized size = 2.33

$$-\frac{2\sqrt{3}x\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)+x\log\left(a^2-ax+x^2\right)-2x\log\left(a+x\right)+6a}{6a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x*arctan(-1/3*sqrt(3)*(a - 2*x)/a) + x*log(a^2 - a*x + x^2)
 - 2*x*log(a + x) + 6*a)/(a^4*x)
```

Sympy [C] time = 0.298719, size = 83, normalized size = 1.32

$$-\frac{1}{a^3x}+\frac{\frac{\log\left(a+x\right)}{3}+\left(-\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(-\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)^2+x\right)+\left(-\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(-\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)^2+x\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**3+x**3),x)
```

```
[Out] -1/(a**3*x) + (log(a + x)/3 + (-1/6 - sqrt(3)*I/6)*log(9*a*(-1/6 - sqrt(3)*
I/6)**2 + x) + (-1/6 + sqrt(3)*I/6)*log(9*a*(-1/6 + sqrt(3)*I/6)**2 + x))/a
**4
```

Giac [A] time = 1.0489, size = 78, normalized size = 1.24

$$-\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^4}-\frac{\log\left(a^2-ax+x^2\right)}{6a^4}+\frac{\log\left(|a+x|\right)}{3a^4}-\frac{1}{a^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^3+x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^4 - 1/6*log(a^2 - a*x + x^2
)/a^4 + 1/3*log(abs(a + x))/a^4 - 1/(a^3*x)
```

3.123 $\int \frac{1}{x^3(a^3+x^3)}\,dx$

Optimal. Leaf size=65

$$-\frac{1}{2a^3x^2}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}-\frac{\log(a+x)}{3a^5}+\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5}$$

```
[Out] -1/(2*a^3*x^2) + ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^5) - Log[a + x]/(
3*a^5) + Log[a^2 - a*x + x^2]/(6*a^5)
```

Rubi [A] time = 0.0386181, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {325, 200, 31, 634, 617, 204, 628}

$$-\frac{1}{2a^3x^2}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}-\frac{\log(a+x)}{3a^5}+\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^3 + x^3)),x]
```

```
[Out] -1/(2*a^3*x^2) + ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^5) - Log[a + x]/(
3*a^5) + Log[a^2 - a*x + x^2]/(6*a^5)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a^3+x^3\right)}\,dx &= -\frac{1}{2a^3x^2}-\frac{\int \frac{1}{a^3+x^3}\,dx}{a^3}\\ &= -\frac{1}{2a^3x^2}-\frac{\int \frac{1}{a+x}\,dx}{3a^5}-\frac{\int \frac{2a-x}{a^2-ax+x^2}\,dx}{3a^5}\\ &= -\frac{1}{2a^3x^2}-\frac{\log(a+x)}{3a^5}+\frac{\int \frac{-a+2x}{a^2-ax+x^2}\,dx}{6a^5}-\frac{\int \frac{1}{a^2-ax+x^2}\,dx}{2a^4}\\ &= -\frac{1}{2a^3x^2}-\frac{\log(a+x)}{3a^5}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}-\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{a}\right)}{a^5}\\ &= -\frac{1}{2a^3x^2}+\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^5}-\frac{\log(a+x)}{3a^5}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}\end{aligned}$$

Mathematica [A] time = 0.0153109, size = 68, normalized size = 1.05

$$-\frac{1}{2a^3x^2}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}-\frac{\log(a+x)}{3a^5}-\frac{\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)}{\sqrt{3}a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^3 + x^3)),x]
```

```
[Out] -1/(2*a^3*x^2) - ArcTan[(-a + 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^5) - Log[a + x]/
(3*a^5) + Log[a^2 - a*x + x^2]/(6*a^5)
```

Maple [A] time = 0.008, size = 60, normalized size = 0.9

$$\frac{\ln\left(a^2-ax+x^2\right)}{6\,a^5}-\frac{\sqrt{3}}{3\,a^5}\arctan\left(\frac{(2\,x-a)\sqrt{3}}{3\,a}\right)-\frac{1}{2\,a^3x^2}-\frac{\ln(a+x)}{3\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^3+x^3),x)
```

```
[Out] 1/6*ln(a^2-a*x+x^2)/a^5-1/3/a^5*3^(1/2)*arctan(1/3*(2*x-a)*3^(1/2)/a)-1/2/a
^3/x^2-1/3*ln(a+x)/a^5
```

Maxima [A] time = 1.40383, size = 77, normalized size = 1.18

$$-\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^5}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}-\frac{\log\left(a+x\right)}{3a^5}-\frac{1}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^5 + 1/6*log(a^2 - a*x + x^2
)/a^5 - 1/3*log(a + x)/a^5 - 1/2/(a^3*x^2)
```

Fricas [A] time = 2.22055, size = 161, normalized size = 2.48

$$-\frac{2\sqrt{3}x^2\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)-x^2\log\left(a^2-ax+x^2\right)+2x^2\log\left(a+x\right)+3a^2}{6a^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x^2*arctan(-1/3*sqrt(3)*(a - 2*x)/a) - x^2*log(a^2 - a*x +
x^2) + 2*x^2*log(a + x) + 3*a^2)/(a^5*x^2)
```

Sympy [C] time = 0.323452, size = 80, normalized size = 1.23

$$-\frac{1}{2a^3x^2}+\frac{-\frac{\log(a+x)}{3}+\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)\log\left(-3a\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)+x\right)+\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)\log\left(-3a\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)+x\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**3+x**3),x)
```

```
[Out] -1/(2*a**3*x**2) + (-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(-3*a*(1/6 - sqr
t(3)*I/6) + x) + (1/6 + sqrt(3)*I/6)*log(-3*a*(1/6 + sqrt(3)*I/6) + x))/a**
5
```

Giac [A] time = 1.05287, size = 78, normalized size = 1.2

$$-\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^5}+\frac{\log\left(a^2-ax+x^2\right)}{6a^5}-\frac{\log\left(|a+x|\right)}{3a^5}-\frac{1}{2a^3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^3+x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^5 + 1/6*log(a^2 - a*x + x^2
)/a^5 - 1/3*log(abs(a + x))/a^5 - 1/2/(a^3*x^2)
```

3.124 $\int \frac{1}{x^4(a^3+x^3)}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{3a^3x^3}+\frac{\log\left(a^3+x^3\right)}{3a^6}-\frac{\log(x)}{a^6}$$

```
[Out] -1/(3*a^3*x^3) - Log[x]/a^6 + Log[a^3 + x^3]/(3*a^6)
```

Rubi [A] time = 0.0195942, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$-\frac{1}{3a^3x^3}+\frac{\log\left(a^3+x^3\right)}{3a^6}-\frac{\log(x)}{a^6}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^3 + x^3)),x]
```

```
[Out] -1/(3*a^3*x^3) - Log[x]/a^6 + Log[a^3 + x^3]/(3*a^6)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^4\left(a^3+x^3\right)}\,dx &= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x^2\left(a^3+x\right)}\,dx, x, x^3\right)\\ &= \frac{1}{3}\,\text{Subst}\left(\int \left(\frac{1}{a^3x^2}-\frac{1}{a^6x}+\frac{1}{a^6\left(a^3+x\right)}\right)dx, x, x^3\right)\\ &= -\frac{1}{3a^3x^3}-\frac{\log(x)}{a^6}+\frac{\log\left(a^3+x^3\right)}{3a^6}\end{aligned}$$

Mathematica [A] time = 0.0052609, size = 33, normalized size = 1.

$$-\frac{1}{3a^3x^3}+\frac{\log\left(a^3+x^3\right)}{3a^6}-\frac{\log(x)}{a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^3 + x^3)),x]
```

```
[Out] -1/(3*a^3*x^3) - Log[x]/a^6 + Log[a^3 + x^3]/(3*a^6)
```

Maple [A] time = 0.006, size = 43, normalized size = 1.3

$$-\frac{1}{3\,a^{3}x^{3}}-\frac{\ln\left(x\right)}{a^{6}}+\frac{\ln\left(a^{2}-ax+x^{2}\right)}{3\,a^{6}}+\frac{\ln\left(a+x\right)}{3\,a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^3+x^3),x)
```

```
[Out] -1/3/a^3/x^3-ln(x)/a^6+1/3/a^6*ln(a^2-a*x+x^2)+1/3*ln(a+x)/a^6
```

Maxima [A] time = 0.925473, size = 42, normalized size = 1.27

$$\frac{\log\left(a^{3}+x^{3}\right)}{3\,a^{6}}-\frac{\log\left(x^{3}\right)}{3\,a^{6}}-\frac{1}{3\,a^{3}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] 1/3*log(a^3 + x^3)/a^6 - 1/3*log(x^3)/a^6 - 1/3/(a^3*x^3)
```

Fricas [A] time = 2.037, size = 77, normalized size = 2.33

$$\frac{x^{3}\log\left(a^{3}+x^{3}\right)-3\,x^{3}\log\left(x\right)-a^{3}}{3\,a^{6}x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3*log(a^3 + x^3) - 3*x^3*log(x) - a^3)/(a^6*x^3)
```

Sympy [A] time = 0.391197, size = 29, normalized size = 0.88

$$-\frac{1}{3a^{3}x^{3}}-\frac{\log\left(x\right)}{a^{6}}+\frac{\log\left(a^{3}+x^{3}\right)}{3a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**3+x**3),x)
```

```
[Out] -1/(3*a**3*x**3) - log(x)/a**6 + log(a**3 + x**3)/(3*a**6)
```

Giac [A] time = 1.05288, size = 54, normalized size = 1.64

$$\frac{\log\left(\left|a^3+x^3\right|\right)}{3\,a^6}-\frac{\log\left(\left|x\right|\right)}{a^6}-\frac{a^3-x^3}{3\,a^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^3+x^3),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(a^3 + x^3))/a^6 - log(abs(x))/a^6 - 1/3*(a^3 - x^3)/(a^6*x^3)
```

3.125 $\int \frac{1}{x^5(a^3+x^3)}\,dx$

Optimal. Leaf size=73

$$-\frac{1}{4a^3x^4}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}+\frac{1}{a^6x}-\frac{\log(a+x)}{3a^7}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7}$$

```
[Out] -1/(4*a^3*x^4) + 1/(a^6*x) - ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^7) -
Log[a + x]/(3*a^7) + Log[a^2 - a*x + x^2]/(6*a^7)
```

Rubi [A] time = 0.0440198, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {325, 292, 31, 634, 617, 204, 628}

$$-\frac{1}{4a^3x^4}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}+\frac{1}{a^6x}-\frac{\log(a+x)}{3a^7}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(a^3 + x^3)),x]
```

```
[Out] -1/(4*a^3*x^4) + 1/(a^6*x) - ArcTan[(a - 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^7) -
Log[a + x]/(3*a^7) + Log[a^2 - a*x + x^2]/(6*a^7)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(a^3+x^3\right)}\,dx &= -\frac{1}{4a^3x^4}-\frac{\int \frac{1}{x^2\left(a^3+x^3\right)}\,dx}{a^3}\\
&= -\frac{1}{4a^3x^4}+\frac{1}{a^6x}+\frac{\int \frac{x}{a^3+x^3}\,dx}{a^6}\\
&= -\frac{1}{4a^3x^4}+\frac{1}{a^6x}-\frac{\int \frac{1}{a+x}\,dx}{3a^7}+\frac{\int \frac{a+x}{a^2-ax+x^2}\,dx}{3a^7}\\
&= -\frac{1}{4a^3x^4}+\frac{1}{a^6x}-\frac{\log(a+x)}{3a^7}+\frac{\int \frac{-a+2x}{a^2-ax+x^2}\,dx}{6a^7}+\frac{\int \frac{1}{a^2-ax+x^2}\,dx}{2a^6}\\
&= -\frac{1}{4a^3x^4}+\frac{1}{a^6x}-\frac{\log(a+x)}{3a^7}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}+\frac{\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{a}\right)}{a^7}\\
&= -\frac{1}{4a^3x^4}+\frac{1}{a^6x}-\frac{\tan^{-1}\left(\frac{a-2x}{\sqrt{3}a}\right)}{\sqrt{3}a^7}-\frac{\log(a+x)}{3a^7}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}
\end{aligned}$$

Mathematica [A] time = 0.0126279, size = 74, normalized size = 1.01

$$-\frac{1}{4a^3x^4}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}+\frac{1}{a^6x}-\frac{\log(a+x)}{3a^7}+\frac{\tan^{-1}\left(\frac{2x-a}{\sqrt{3}a}\right)}{\sqrt{3}a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(a^3 + x^3)),x]
```

```
[Out] -1/(4*a^3*x^4) + 1/(a^6*x) + ArcTan[(-a + 2*x)/(Sqrt[3]*a)]/(Sqrt[3]*a^7) -
 Log[a + x]/(3*a^7) + Log[a^2 - a*x + x^2]/(6*a^7)
```

Maple [A] time = 0.009, size = 67, normalized size = 0.9

$$-\frac{1}{4\,a^3x^4}+\frac{1}{a^6x}+\frac{\ln\left(a^2-ax+x^2\right)}{6\,a^7}+\frac{\sqrt{3}}{3\,a^7}\arctan\left(\frac{(2\,x-a)\sqrt{3}}{3\,a}\right)-\frac{\ln(a+x)}{3\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(a^3+x^3),x)
```

```
[Out] -1/4/a^3/x^4+1/a^6/x+1/6*ln(a^2-a*x+x^2)/a^7+1/3/a^7*3^(1/2)*arctan(1/3*(2*
x-a)*3^(1/2)/a)-1/3*ln(a+x)/a^7
```

Maxima [A] time = 1.41137, size = 89, normalized size = 1.22

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^7}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}-\frac{\log\left(a+x\right)}{3a^7}-\frac{a^3-4x^3}{4a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^7 + 1/6*log(a^2 - a*x + x^2)
/a^7 - 1/3*log(a + x)/a^7 - 1/4*(a^3 - 4*x^3)/(a^6*x^4)
```

Fricas [A] time = 2.08795, size = 178, normalized size = 2.44

$$\frac{4\sqrt{3}x^4\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)+2x^4\log\left(a^2-ax+x^2\right)-4x^4\log\left(a+x\right)-3a^4+12ax^3}{12a^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] 1/12*(4*sqrt(3)*x^4*arctan(-1/3*sqrt(3)*(a - 2*x)/a) + 2*x^4*log(a^2 - a*x
+ x^2) - 4*x^4*log(a + x) - 3*a^4 + 12*a*x^3)/(a^7*x^4)
```

Sympy [C] time = 0.375662, size = 90, normalized size = 1.23

$$\frac{-a^3+4x^3}{4a^6x^4}+\frac{-\frac{\log(a+x)}{3}+\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(\frac{1}{6}-\frac{\sqrt{3}i}{6}\right)^2+x\right)+\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)\log\left(9a\left(\frac{1}{6}+\frac{\sqrt{3}i}{6}\right)^2+x\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(a**3+x**3),x)
```

```
[Out] (-a**3 + 4*x**3)/(4*a**6*x**4) + (-log(a + x)/3 + (1/6 - sqrt(3)*I/6)*log(9
*a*(1/6 - sqrt(3)*I/6)**2 + x) + (1/6 + sqrt(3)*I/6)*log(9*a*(1/6 + sqrt(3)
*I/6)**2 + x))/a**7
```

Giac [A] time = 1.07286, size = 90, normalized size = 1.23

$$\frac{\sqrt{3}\arctan\left(-\frac{\sqrt{3}(a-2x)}{3a}\right)}{3a^7}+\frac{\log\left(a^2-ax+x^2\right)}{6a^7}-\frac{\log\left(|a+x|\right)}{3a^7}-\frac{a^3-4x^3}{4a^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(a^3+x^3),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(a - 2*x)/a)/a^7 + 1/6*log(a^2 - a*x + x^2)
/a^7 - 1/3*log(abs(a + x))/a^7 - 1/4*(a^3 - 4*x^3)/(a^6*x^4)
```

3.126 $\int \frac{x^{-m}}{a^3+x^3} \, dx$

Optimal. Leaf size=46

$$\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1-m}{3},\frac{4-m}{3},-\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

```
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/3, (4 - m)/3, -(x^3/a^3)])/(a^3*(1
- m))
```

Rubi [A] time = 0.010311, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {364}

$$\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1-m}{3},\frac{4-m}{3},-\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^m*(a^3 + x^3)),x]
```

```
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/3, (4 - m)/3, -(x^3/a^3)])/(a^3*(1
- m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^{-m}}{a^3+x^3} \, dx = \frac{x^{1-m} \, _2F_1\left(1,\frac{1-m}{3};\frac{4-m}{3};-\frac{x^3}{a^3}\right)}{a^3(1-m)}$$

Mathematica [A] time = 0.009845, size = 45, normalized size = 0.98

$$-\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1}{3}-\frac{m}{3},\frac{4}{3}-\frac{m}{3},-\frac{x^3}{a^3}\right)}{a^3(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^m*(a^3 + x^3)),x]
```

```
[Out] -((x^(1 - m)*Hypergeometric2F1[1, 1/3 - m/3, 4/3 - m/3, -(x^3/a^3)])/(a^3*(
-1 + m)))
```

Maple [F] time = 0.041, size = 0, normalized size = 0.

$$\int \frac{1}{x^{m}\left(a^{3}+x^{3}\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^m)/(a^3+x^3),x)
```

```
[Out] int(1/(x^m)/(a^3+x^3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{3}+x^{3}\right)x^{m}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^3+x^3),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^3 + x^3)*x^m), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\left(a^{3}+x^{3}\right)x^{m}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^3+x^3),x, algorithm="fricas")
```

```
[Out] integral(1/((a^3 + x^3)*x^m), x)
```

Sympy [C] time = 4.57421, size = 92, normalized size = 2.

$$-\frac{mxx^{-m}\Phi\left(\frac{x^{3}e^{i\pi}}{a^{3}},1,\frac{1}{3}-\frac{m}{3}\right)\Gamma\left(\frac{1}{3}-\frac{m}{3}\right)}{9a^{3}\Gamma\left(\frac{4}{3}-\frac{m}{3}\right)}+\frac{xx^{-m}\Phi\left(\frac{x^{3}e^{i\pi}}{a^{3}},1,\frac{1}{3}-\frac{m}{3}\right)\Gamma\left(\frac{1}{3}-\frac{m}{3}\right)}{9a^{3}\Gamma\left(\frac{4}{3}-\frac{m}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**m)/(a**3+x**3),x)
```

```
[Out] -m*x*x**(-m)*lerchphi(x**3*exp_polar(I*pi)/a**3, 1, 1/3 - m/3)*gamma(1/3 -
m/3)/(9*a**3*gamma(4/3 - m/3)) + x*x**(-m)*lerchphi(x**3*exp_polar(I*pi)/a*
*3, 1, 1/3 - m/3)*gamma(1/3 - m/3)/(9*a**3*gamma(4/3 - m/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^3 + x^3)x^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^3+x^3),x, algorithm="giac")
```

```
[Out] integrate(1/((a^3 + x^3)*x^m), x)
```

3.127 $\int \frac{1}{a^4-x^4}\,dx$

Optimal. Leaf size=27

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^3}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^3}$$

```
[Out] ArcTan[x/a]/(2*a^3) + ArcTanh[x/a]/(2*a^3)
```

Rubi [A] time = 0.0080693, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^3}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a^4 - x^4)^(-1),x]
```

```
[Out] ArcTan[x/a]/(2*a^3) + ArcTanh[x/a]/(2*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a^4-x^4}\,dx&=\frac{\int \frac{1}{a^2-x^2}\,dx}{2a^2}+\frac{\int \frac{1}{a^2+x^2}\,dx}{2a^2}\\&=\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^3}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^3}\end{aligned}$$

Mathematica [A] time = 0.004278, size = 38, normalized size = 1.41

$$-\frac{\log(a-x)}{4a^3}+\frac{\log(a+x)}{4a^3}+\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^4 - x^4)^(-1),x]
```

```
[Out] ArcTan[x/a]/(2*a^3) - Log[a - x]/(4*a^3) + Log[a + x]/(4*a^3)
```

Maple [A] time = 0.008, size = 33, normalized size = 1.2

$$-\frac{\ln(-a+x)}{4a^3}+\frac{1}{2a^3}\arctan\left(\frac{x}{a}\right)+\frac{\ln(a+x)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^4-x^4),x)
```

```
[Out] -1/4/a^3*ln(-a+x)+1/2*arctan(x/a)/a^3+1/4*ln(a+x)/a^3
```

Maxima [A] time = 1.40099, size = 43, normalized size = 1.59

$$\frac{\arctan\left(\frac{x}{a}\right)}{2a^3}+\frac{\log(a+x)}{4a^3}-\frac{\log(-a+x)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^4-x^4),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x/a)/a^3 + 1/4*log(a + x)/a^3 - 1/4*log(-a + x)/a^3
```

Fricas [A] time = 2.12863, size = 70, normalized size = 2.59

$$\frac{2\arctan\left(\frac{x}{a}\right)+\log(a+x)-\log(-a+x)}{4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^4-x^4),x, algorithm="fricas")
```

```
[Out] 1/4*(2*arctan(x/a) + log(a + x) - log(-a + x))/a^3
```

Sympy [C] time = 0.13063, size = 37, normalized size = 1.37

$$-\frac{\frac{\log(-a+x)}{4}-\frac{\log(a+x)}{4}+\frac{i\log(-ia+x)}{4}-\frac{i\log(ia+x)}{4}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**4-x**4),x)
```

```
[Out] -(log(-a + x)/4 - log(a + x)/4 + I*log(-I*a + x)/4 - I*log(I*a + x)/4)/a**3
```

Giac [A] time = 1.04852, size = 46, normalized size = 1.7

$$\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^3}+\frac{\log\left(|a+x|\right)}{4\,a^3}-\frac{\log\left(|-a+x|\right)}{4\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^4-x^4),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x/a)/a^3 + 1/4*log(abs(a + x))/a^3 - 1/4*log(abs(-a + x))/a^3
```

3.128 $\int \frac{x}{a^4-x^4} dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

```
[Out] ArcTanh[x^2/a^2]/(2*a^2)
```

Rubi [A] time = 0.0065664, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {275, 206}

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^4 - x^4),x]
```

```
[Out] ArcTanh[x^2/a^2]/(2*a^2)
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a^4-x^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{a^4-x^2}\,dx,x,x^2\right)\\ &= \frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.0034625, size = 15, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^4 - x^4),x]
```

```
[Out] ArcTanh[x^2/a^2]/(2*a^2)
```

Maple [B] time = 0.004, size = 30, normalized size = 2.

$$\frac{\ln\left(a^2+x^2\right)}{4\,a^2}-\frac{\ln\left(-a^2+x^2\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^4-x^4),x)
```

```
[Out] 1/4/a^2*ln(a^2+x^2)-1/4/a^2*ln(-a^2+x^2)
```

Maxima [B] time = 0.921784, size = 39, normalized size = 2.6

$$\frac{\log\left(a^2+x^2\right)}{4\,a^2}-\frac{\log\left(-a^2+x^2\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4-x^4),x, algorithm="maxima")
```

```
[Out] 1/4*log(a^2 + x^2)/a^2 - 1/4*log(-a^2 + x^2)/a^2
```

Fricas [A] time = 1.91185, size = 59, normalized size = 3.93

$$\frac{\log\left(a^2+x^2\right)-\log\left(-a^2+x^2\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4-x^4),x, algorithm="fricas")
```

```
[Out] 1/4*(log(a^2 + x^2) - log(-a^2 + x^2))/a^2
```

Sympy [A] time = 0.13667, size = 24, normalized size = 1.6

$$-\frac{\frac{\log\left(-a^2+x^2\right)}{4}-\frac{\log\left(a^2+x^2\right)}{4}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**4-x**4),x)
```

```
[Out] -(log(-a**2 + x**2)/4 - log(a**2 + x**2)/4)/a**2
```

Giac [B] time = 1.05916, size = 41, normalized size = 2.73

$$\frac{\log\left(a^2+x^2\right)}{4\,a^2}-\frac{\log\left(\left|-a^2+x^2\right|\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4-x^4),x, algorithm="giac")
```

```
[Out] 1/4*log(a^2 + x^2)/a^2 - 1/4*log(abs(-a^2 + x^2))/a^2
```

3.129 $\int \frac{1}{x(a^4-x^4)}\,dx$

Optimal. Leaf size=24

$$\frac{\log(x)}{a^4}-\frac{\log\left(a^4-x^4\right)}{4a^4}$$

```
[Out] Log[x]/a^4 - Log[a^4 - x^4]/(4*a^4)
```

Rubi [A] time = 0.0099267, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 36, 31, 29}

$$\frac{\log(x)}{a^4}-\frac{\log\left(a^4-x^4\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^4 - x^4)),x]
```

```
[Out] Log[x]/a^4 - Log[a^4 - x^4]/(4*a^4)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a^4-x^4\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\left(a^4-x\right)x}\,dx,x,x^4\right)\\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{a^4-x}\,dx,x,x^4\right)}{4a^4}+\frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^4\right)}{4a^4}\\ &= \frac{\log(x)}{a^4}-\frac{\log\left(a^4-x^4\right)}{4a^4}\end{aligned}$$

Mathematica [A] time = 0.0054005, size = 24, normalized size = 1.

$$\frac{\log(x)}{a^4}-\frac{\log\left(x^4-a^4\right)}{4a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^4 - x^4)),x]
```

```
[Out] Log[x]/a^4 - Log[-a^4 + x^4]/(4*a^4)
```

Maple [A] time = 0.008, size = 41, normalized size = 1.7

$$\frac{\ln(x)}{a^4}-\frac{\ln(-a+x)}{4\,a^4}-\frac{\ln\left(a^2+x^2\right)}{4\,a^4}-\frac{\ln(a+x)}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^4-x^4),x)
```

```
[Out] ln(x)/a^4-1/4/a^4*ln(-a+x)-1/4/a^4*ln(a^2+x^2)-1/4*ln(a+x)/a^4
```

Maxima [A] time = 0.917133, size = 34, normalized size = 1.42

$$-\frac{\log\left(-a^4+x^4\right)}{4\,a^4}+\frac{\log\left(x^4\right)}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^4-x^4),x, algorithm="maxima")
```

```
[Out] -1/4*log(-a^4 + x^4)/a^4 + 1/4*log(x^4)/a^4
```

Fricas [A] time = 1.69358, size = 53, normalized size = 2.21

$$-\frac{\log\left(-a^4+x^4\right)-4\,\log(x)}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^4-x^4),x, algorithm="fricas")
```

```
[Out] -1/4*(log(-a^4 + x^4) - 4*log(x))/a^4
```

Sympy [A] time = 0.212124, size = 19, normalized size = 0.79

$$\frac{\log(x)}{a^4}-\frac{\log\left(-a^4+x^4\right)}{4a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**4-x**4),x)
```

```
[Out] log(x)/a**4 - log(-a**4 + x**4)/(4*a**4)
```

Giac [A] time = 1.0444, size = 35, normalized size = 1.46

$$\frac{\log\left(x^4\right)}{4\,a^4} - \frac{\log\left(\left|-a^4 + x^4\right|\right)}{4\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^4-x^4),x, algorithm="giac")
```

```
[Out] 1/4*log(x^4)/a^4 - 1/4*log(abs(-a^4 + x^4))/a^4
```

3.130 $\int \frac{1}{x^2\left(a^4-x^4\right)} \, dx$

Optimal. Leaf size=35

$$-\frac{1}{a^4 x}-\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^5}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^5}$$

```
[Out] -(1/(a^4*x)) - ArcTan[x/a]/(2*a^5) + ArcTanh[x/a]/(2*a^5)
```

Rubi [A] time = 0.013661, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {325, 298, 203, 206}

$$-\frac{1}{a^4 x}-\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^5}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^4 - x^4)),x]
```

```
[Out] -(1/(a^4*x)) - ArcTan[x/a]/(2*a^5) + ArcTanh[x/a]/(2*a^5)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^2\left(a^4-x^4\right)}\,dx = -\frac{1}{a^4x} + \frac{\int \frac{x^2}{a^4-x^4}\,dx}{a^4}$$
$$= -\frac{1}{a^4x} + \frac{\int \frac{1}{a^2-x^2}\,dx}{2a^4} - \frac{\int \frac{1}{a^2+x^2}\,dx}{2a^4}$$
$$= -\frac{1}{a^4x} - \frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^5} + \frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^5}$$

Mathematica [A] time = 0.0067181, size = 46, normalized size = 1.31

$$-\frac{1}{a^4x} - \frac{\log(a-x)}{4a^5} + \frac{\log(a+x)}{4a^5} - \frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^4 - x^4)),x]
```

```
[Out] -(1/(a^4*x)) - ArcTan[x/a]/(2*a^5) - Log[a - x]/(4*a^5) + Log[a + x]/(4*a^5
)
```

Maple [A] time = 0.007, size = 41, normalized size = 1.2

$$-\frac{\ln(-a+x)}{4\,a^5} - \frac{1}{2\,a^5}\arctan\left(\frac{x}{a}\right) + \frac{\ln(a+x)}{4\,a^5} - \frac{1}{a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^4-x^4),x)
```

```
[Out] -1/4/a^5*ln(-a+x)-1/2*arctan(x/a)/a^5+1/4*ln(a+x)/a^5-1/a^4/x
```

Maxima [A] time = 1.41269, size = 54, normalized size = 1.54

$$-\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^5} + \frac{\log(a+x)}{4\,a^5} - \frac{\log(-a+x)}{4\,a^5} - \frac{1}{a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^4-x^4),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(x/a)/a^5 + 1/4*log(a + x)/a^5 - 1/4*log(-a + x)/a^5 - 1/(a^4*x)
```

Fricas [A] time = 1.97679, size = 93, normalized size = 2.66

$$-\frac{2\,x\arctan\left(\frac{x}{a}\right) - x\log(a+x) + x\log(-a+x) + 4\,a}{4\,a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^4-x^4),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x*arctan(x/a) - x*log(a + x) + x*log(-a + x) + 4*a)/(a^5*x)
```

Sympy [C] time = 0.329665, size = 44, normalized size = 1.26

$$-\frac{1}{a^4x}-\frac{\frac{\log\left(-a+x\right)}{4}-\frac{\log\left(a+x\right)}{4}-\frac{i\log\left(-ia+x\right)}{4}+\frac{i\log\left(ia+x\right)}{4}}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**4-x**4),x)
```

```
[Out] -1/(a**4*x) - (log(-a + x)/4 - log(a + x)/4 - I*log(-I*a + x)/4 + I*log(I*a
 + x)/4)/a**5
```

Giac [A] time = 1.06125, size = 57, normalized size = 1.63

$$-\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^5}+\frac{\log\left(|a+x|\right)}{4\,a^5}-\frac{\log\left(|-a+x|\right)}{4\,a^5}-\frac{1}{a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^4-x^4),x, algorithm="giac")
```

```
[Out] -1/2*arctan(x/a)/a^5 + 1/4*log(abs(a + x))/a^5 - 1/4*log(abs(-a + x))/a^5 -
 1/(a^4*x)
```

3.131 $\int \frac{1}{x^3(a^4-x^4)}\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^6}-\frac{1}{2a^4x^2}$$

```
[Out] -1/(2*a^4*x^2) + ArcTanh[x^2/a^2]/(2*a^6)
```

Rubi [A] time = 0.0121596, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {275, 325, 206}

$$\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^6}-\frac{1}{2a^4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^4 - x^4)),x]
```

```
[Out] -1/(2*a^4*x^2) + ArcTanh[x^2/a^2]/(2*a^6)
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a^4-x^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2\left(a^4-x^2\right)}\,dx,x,x^2\right)\\ &= -\frac{1}{2a^4x^2}+\frac{\text{Subst}\left(\int \frac{1}{a^4-x^2}\,dx,x,x^2\right)}{2a^4}\\ &= -\frac{1}{2a^4x^2}+\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^6}\end{aligned}$$

Mathematica [A] time = 0.0068284, size = 50, normalized size = 1.92

$$-\frac{1}{2a^4x^2}+\frac{\log\left(a^2+x^2\right)}{4a^6}-\frac{\log(a-x)}{4a^6}-\frac{\log(a+x)}{4a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^4 - x^4)),x]
```

```
[Out] -1/(2*a^4*x^2) - Log[a - x]/(4*a^6) - Log[a + x]/(4*a^6) + Log[a^2 + x^2]/(
4*a^6)
```

Maple [A] time = 0.006, size = 43, normalized size = 1.7

$$-\frac{1}{2\,a^4x^2}-\frac{\ln\left(-a+x\right)}{4\,a^6}+\frac{\ln\left(a^2+x^2\right)}{4\,a^6}-\frac{\ln\left(a+x\right)}{4\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^4-x^4),x)
```

```
[Out] -1/2/a^4/x^2-1/4/a^6*ln(-a+x)+1/4/a^6*ln(a^2+x^2)-1/4*ln(a+x)/a^6
```

Maxima [A] time = 0.924299, size = 50, normalized size = 1.92

$$\frac{\log\left(a^2+x^2\right)}{4\,a^6}-\frac{\log\left(-a^2+x^2\right)}{4\,a^6}-\frac{1}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^4-x^4),x, algorithm="maxima")
```

```
[Out] 1/4*log(a^2 + x^2)/a^6 - 1/4*log(-a^2 + x^2)/a^6 - 1/2/(a^4*x^2)
```

Fricas [A] time = 1.8199, size = 89, normalized size = 3.42

$$\frac{x^2\log\left(a^2+x^2\right)-x^2\log\left(-a^2+x^2\right)-2\,a^2}{4\,a^6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^4-x^4),x, algorithm="fricas")
```

```
[Out] 1/4*(x^2*log(a^2 + x^2) - x^2*log(-a^2 + x^2) - 2*a^2)/(a^6*x^2)
```

Sympy [A] time = 0.347397, size = 34, normalized size = 1.31

$$-\frac{1}{2a^4x^2}-\frac{\frac{\log\left(-a^2+x^2\right)}{4}-\frac{\log\left(a^2+x^2\right)}{4}}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**4-x**4),x)
```

```
[Out] -1/(2*a**4*x**2) - (log(-a**2 + x**2)/4 - log(a**2 + x**2)/4)/a**6
```

Giac [A] time = 1.07005, size = 51, normalized size = 1.96

$$\frac{\log\left(a^2+x^2\right)}{4\,a^6}-\frac{\log\left(\left|-a^2+x^2\right|\right)}{4\,a^6}-\frac{1}{2\,a^4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^4-x^4),x, algorithm="giac")
```

```
[Out] 1/4*log(a^2 + x^2)/a^6 - 1/4*log(abs(-a^2 + x^2))/a^6 - 1/2/(a^4*x^2)
```

3.132 $\int \frac{1}{x^4(a^4-x^4)}\,dx$

Optimal. Leaf size=37

$$-\frac{1}{3a^4x^3}+\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^7}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^7}$$

```
[Out] -1/(3*a^4*x^3) + ArcTan[x/a]/(2*a^7) + ArcTanh[x/a]/(2*a^7)
```

Rubi [A] time = 0.0122327, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {325, 212, 206, 203}

$$-\frac{1}{3a^4x^3}+\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^7}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^7}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^4 - x^4)),x]
```

```
[Out] -1/(3*a^4*x^3) + ArcTan[x/a]/(2*a^7) + ArcTanh[x/a]/(2*a^7)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^4\left(a^4-x^4\right)}\,dx = -\frac{1}{3a^4x^3}+\frac{\int \frac{1}{a^4-x^4}\,dx}{a^4}$$
$$= -\frac{1}{3a^4x^3}+\frac{\int \frac{1}{a^2-x^2}\,dx}{2a^6}+\frac{\int \frac{1}{a^2+x^2}\,dx}{2a^6}$$
$$= -\frac{1}{3a^4x^3}+\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^7}+\frac{\tanh^{-1}\left(\frac{x}{a}\right)}{2a^7}$$

Mathematica [A] time = 0.0068022, size = 48, normalized size = 1.3

$$-\frac{1}{3a^4x^3}-\frac{\log(a-x)}{4a^7}+\frac{\log(a+x)}{4a^7}+\frac{\tan^{-1}\left(\frac{x}{a}\right)}{2a^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a^4 - x^4)),x]
```

```
[Out] -1/(3*a^4*x^3) + ArcTan[x/a]/(2*a^7) - Log[a - x]/(4*a^7) + Log[a + x]/(4*a
^7)
```

Maple [A] time = 0.007, size = 41, normalized size = 1.1

$$-\frac{1}{3\,a^4x^3}-\frac{\ln(-a+x)}{4\,a^7}+\frac{1}{2\,a^7}\arctan\left(\frac{x}{a}\right)+\frac{\ln(a+x)}{4\,a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^4-x^4),x)
```

```
[Out] -1/3/a^4/x^3-1/4/a^7*ln(-a+x)+1/2*arctan(x/a)/a^7+1/4*ln(a+x)/a^7
```

Maxima [A] time = 1.40574, size = 54, normalized size = 1.46

$$\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^7}+\frac{\log(a+x)}{4\,a^7}-\frac{\log(-a+x)}{4\,a^7}-\frac{1}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^4-x^4),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x/a)/a^7 + 1/4*log(a + x)/a^7 - 1/4*log(-a + x)/a^7 - 1/3/(a^4*x
^3)
```

Fricas [A] time = 1.84476, size = 112, normalized size = 3.03

$$\frac{6\,x^3\arctan\left(\frac{x}{a}\right)+3\,x^3\log(a+x)-3\,x^3\log(-a+x)-4\,a^3}{12\,a^7x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^4-x^4),x, algorithm="fricas")
```

```
[Out] 1/12*(6*x^3*arctan(x/a) + 3*x^3*log(a + x) - 3*x^3*log(-a + x) - 4*a^3)/(a^
7*x^3)
```

Sympy [C] time = 0.368821, size = 48, normalized size = 1.3

$$-\frac{1}{3a^4x^3}-\frac{\frac{\log(-a+x)}{4}-\frac{\log(a+x)}{4}+\frac{i\log(-ia+x)}{4}-\frac{i\log(ia+x)}{4}}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**4-x**4),x)
```

```
[Out] -1/(3*a**4*x**3) - (log(-a + x)/4 - log(a + x)/4 + I*log(-I*a + x)/4 - I*lo
g(I*a + x)/4)/a**7
```

Giac [A] time = 1.04898, size = 57, normalized size = 1.54

$$\frac{\arctan\left(\frac{x}{a}\right)}{2\,a^7}+\frac{\log\left(|a+x|\right)}{4\,a^7}-\frac{\log\left(|-a+x|\right)}{4\,a^7}-\frac{1}{3\,a^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^4-x^4),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x/a)/a^7 + 1/4*log(abs(a + x))/a^7 - 1/4*log(abs(-a + x))/a^7 -
1/3/(a^4*x^3)
```

3.133 $\int \frac{x^{-m}}{a^4-x^4}\,dx$

Optimal. Leaf size=45

$$\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1-m}{4},\frac{5-m}{4},\frac{x^4}{a^4}\right)}{a^4(1-m)}$$

```
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/4, (5 - m)/4, x^4/a^4])/(a^4*(1 - m
))
```

Rubi [A] time = 0.0084131, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {364}

$$\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1-m}{4},\frac{5-m}{4},\frac{x^4}{a^4}\right)}{a^4(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^m*(a^4 - x^4)),x]
```

```
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/4, (5 - m)/4, x^4/a^4])/(a^4*(1 - m
))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^{-m}}{a^4-x^4}\,dx = \frac{x^{1-m}\,{}_2F_1\left(1,\frac{1-m}{4};\frac{5-m}{4};\frac{x^4}{a^4}\right)}{a^4(1-m)}$$

Mathematica [A] time = 0.0088513, size = 44, normalized size = 0.98

$$-\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1}{4}-\frac{m}{4},\frac{5}{4}-\frac{m}{4},\frac{x^4}{a^4}\right)}{a^4(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^m*(a^4 - x^4)),x]
```

```
[Out] -((x^(1 - m)*Hypergeometric2F1[1, 1/4 - m/4, 5/4 - m/4, x^4/a^4])/(a^4*(-1
+ m)))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{x^m\left(a^4-x^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^m)/(a^4-x^4),x)
```

```
[Out] int(1/(x^m)/(a^4-x^4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^4-x^4\right)x^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^4-x^4),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^4 - x^4)*x^m), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^4-x^4\right)x^m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^4-x^4),x, algorithm="fricas")
```

```
[Out] integral(1/((a^4 - x^4)*x^m), x)
```

Sympy [C] time = 0.878606, size = 95, normalized size = 2.11

$$-\frac{mxx^{-m}\Phi\left(\frac{x^4e^{2i\pi}}{a^4},1,\frac{1}{4}-\frac{m}{4}\right)\Gamma\left(\frac{1}{4}-\frac{m}{4}\right)}{16a^4\Gamma\left(\frac{5}{4}-\frac{m}{4}\right)}+\frac{xx^{-m}\Phi\left(\frac{x^4e^{2i\pi}}{a^4},1,\frac{1}{4}-\frac{m}{4}\right)\Gamma\left(\frac{1}{4}-\frac{m}{4}\right)}{16a^4\Gamma\left(\frac{5}{4}-\frac{m}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**m)/(a**4-x**4),x)
```

```
[Out] -m*x*x**(-m)*lerchphi(x**4*exp_polar(2*I*pi)/a**4, 1, 1/4 - m/4)*gamma(1/4
- m/4)/(16*a**4*gamma(5/4 - m/4)) + x*x**(-m)*lerchphi(x**4*exp_polar(2*I*p
i)/a**4, 1, 1/4 - m/4)*gamma(1/4 - m/4)/(16*a**4*gamma(5/4 - m/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^4 - x^4)x^m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^4-x^4),x, algorithm="giac")
```

```
[Out] integrate(1/((a^4 - x^4)*x^m), x)
```

3.134 $\int \frac{x}{a^4+x^4}\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

```
[Out] ArcTan[x^2/a^2]/(2*a^2)
```

Rubi [A] time = 0.0055853, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {275, 203}

$$\frac{\tan^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^4 + x^4),x]
```

```
[Out] ArcTan[x^2/a^2]/(2*a^2)
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{a^4+x^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{a^4+x^2}\,dx, x, x^2\right)\\ &= \frac{\tan^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.0034063, size = 15, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a^4 + x^4),x]
```

```
[Out] ArcTan[x^2/a^2]/(2*a^2)
```

Maple [A] time = 0.003, size = 14, normalized size = 0.9

$$\frac{1}{2\,a^{2}}\arctan\left(\frac{x^{2}}{a^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^4+x^4),x)
```

```
[Out] 1/2*arctan(x^2/a^2)/a^2
```

Maxima [A] time = 1.41344, size = 18, normalized size = 1.2

$$\frac{\arctan\left(\frac{x^{2}}{a^{2}}\right)}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4+x^4),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x^2/a^2)/a^2
```

Fricas [A] time = 1.85782, size = 34, normalized size = 2.27

$$\frac{\arctan\left(\frac{x^{2}}{a^{2}}\right)}{2\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4+x^4),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x^2/a^2)/a^2
```

Sympy [C] time = 0.12741, size = 29, normalized size = 1.93

$$\frac{-\frac{i\log\left(-ia^{2}+x^{2}\right)}{4}+\frac{i\log\left(ia^{2}+x^{2}\right)}{4}}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**4+x**4),x)
```

```
[Out] (-I*log(-I*a**2 + x**2)/4 + I*log(I*a**2 + x**2)/4)/a**2
```

Giac [A] time = 1.07166, size = 18, normalized size = 1.2

$$\frac{\arctan\left(\frac{x^2}{a^2}\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^4+x^4),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x^2/a^2)/a^2
```

3.135 $\int \frac{x^2}{a^4+x^4} \, dx$

Optimal. Leaf size=109

$$\frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}-\frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{a}+1\right)}{2\sqrt{2}a}$$

```
[Out] -ArcTan[1 - (Sqrt[2]*x)/a]/(2*Sqrt[2]*a) + ArcTan[1 + (Sqrt[2]*x)/a]/(2*Sqr
t[2]*a) + Log[a^2 - Sqrt[2]*a*x + x^2]/(4*Sqrt[2]*a) - Log[a^2 + Sqrt[2]*a*
x + x^2]/(4*Sqrt[2]*a)
```

Rubi [A] time = 0.0616562, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}-\frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{a}+1\right)}{2\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^4 + x^4),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*x)/a]/(2*Sqrt[2]*a) + ArcTan[1 + (Sqrt[2]*x)/a]/(2*Sqr
t[2]*a) + Log[a^2 - Sqrt[2]*a*x + x^2]/(4*Sqrt[2]*a) - Log[a^2 + Sqrt[2]*a*
x + x^2]/(4*Sqrt[2]*a)
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{a^4+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{a^2-x^2}{a^4+x^4}\,dx\right)+\frac{1}{2}\int \frac{a^2+x^2}{a^4+x^4}\,dx\\ &= \frac{1}{4}\int \frac{1}{a^2-\sqrt{2}ax+x^2}\,dx+\frac{1}{4}\int \frac{1}{a^2+\sqrt{2}ax+x^2}\,dx+\frac{\int \frac{\sqrt{2}a+2x}{-a^2-\sqrt{2}ax-x^2}\,dx}{4\sqrt{2}a}+\frac{\int \frac{\sqrt{2}a-2x}{-a^2+\sqrt{2}ax-x^2}\,dx}{4\sqrt{2}a}\\ &= \frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}-\frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx\right.}{2\sqrt{2}a}\\ &= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}x}{a}\right)}{2\sqrt{2}a}+\frac{\log\left(a^2-\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}-\frac{\log\left(a^2+\sqrt{2}ax+x^2\right)}{4\sqrt{2}a}\end{aligned}$$

Mathematica [A] time = 0.0277268, size = 79, normalized size = 0.72

$$\frac{\log\left(a^2-\sqrt{2}ax+x^2\right)-\log\left(a^2+\sqrt{2}ax+x^2\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}x}{a}\right)+2\tan^{-1}\left(\frac{\sqrt{2}x}{a}+1\right)}{4\sqrt{2}a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^4 + x^4),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*x)/a] + 2*ArcTan[1 + (Sqrt[2]*x)/a] + Log[a^2 - Sqr
t[2]*a*x + x^2] - Log[a^2 + Sqrt[2]*a*x + x^2])/(4*Sqrt[2]*a)
```

Maple [A] time = 0.005, size = 101, normalized size = 0.9

$$\frac{\sqrt{2}}{8}\ln\left(\left(x^2-\sqrt[4]{a^4}x\sqrt{2}+\sqrt{a^4}\right)\left(x^2+\sqrt[4]{a^4}x\sqrt{2}+\sqrt{a^4}\right)^{-1}\right)\frac{1}{\sqrt[4]{a^4}}+\frac{\sqrt{2}}{4}\arctan\left(x\sqrt{2}\frac{1}{\sqrt[4]{a^4}}+1\right)\frac{1}{\sqrt[4]{a^4}}+\frac{\sqrt{2}}{4}\arctan\left(x\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^4+x^4),x)
```

```
[Out] 1/8/(a^4)^(1/4)*2^(1/2)*ln((x^2-(a^4)^(1/4)*x*2^(1/2)+(a^4)^(1/2))/(x^2+(a^
4)^(1/4)*x*2^(1/2)+(a^4)^(1/2)))+1/4/(a^4)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a^
4)^(1/4)*x+1)+1/4/(a^4)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a^4)^(1/4)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^4+x^4),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.94405, size = 651, normalized size = 5.97

$$-\frac{1}{2}\sqrt{2}\frac{1}{a^4}^{\frac{1}{4}}\arctan\left(-\sqrt{2}\frac{1}{a^4}^{\frac{1}{4}}x+\sqrt{2}\sqrt{\sqrt{2}a^4\frac{1}{a^4}^{\frac{3}{4}}x+a^4\sqrt{\frac{1}{a^4}}+x^2}\frac{1}{a^4}^{\frac{1}{4}}-1\right)-\frac{1}{2}\sqrt{2}\frac{1}{a^4}^{\frac{1}{4}}\arctan\left(-\sqrt{2}\frac{1}{a^4}^{\frac{1}{4}}x+\sqrt{2}\sqrt{-\sqrt{2}a^4\frac{1}{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^4+x^4),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*(a^(-4))^(1/4)*arctan(-sqrt(2)*(a^(-4))^(1/4)*x + sqrt(2)*sqrt
(sqrt(2)*a^4*(a^(-4))^(3/4)*x + a^4*sqrt(a^(-4)) + x^2)*(a^(-4))^(1/4) - 1)
 - 1/2*sqrt(2)*(a^(-4))^(1/4)*arctan(-sqrt(2)*(a^(-4))^(1/4)*x + sqrt(2)*sq
rt(-sqrt(2)*a^4*(a^(-4))^(3/4)*x + a^4*sqrt(a^(-4)) + x^2)*(a^(-4))^(1/4) +
 1) - 1/8*sqrt(2)*(a^(-4))^(1/4)*log(sqrt(2)*a^4*(a^(-4))^(3/4)*x + a^4*sqr
t(a^(-4)) + x^2) + 1/8*sqrt(2)*(a^(-4))^(1/4)*log(-sqrt(2)*a^4*(a^(-4))^(3/
4)*x + a^4*sqrt(a^(-4)) + x^2)
```

Sympy [A] time = 0.126486, size = 19, normalized size = 0.17

$$\frac{\operatorname{RootSum}\left(256t^4+1,\left(t\mapsto t\log\left(64t^3a+x\right)\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**4+x**4),x)
```

```
[Out] RootSum(256*_t**4 + 1, Lambda(_t, _t*log(64*_t**3*a + x)))/a
```

Giac [A] time = 1.05912, size = 154, normalized size = 1.41

$$\frac{\sqrt{2}|a|\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}|a|+2x\right)}{2|a|}\right)}{4a^2}+\frac{\sqrt{2}|a|\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}|a|-2x\right)}{2|a|}\right)}{4a^2}-\frac{\sqrt{2}|a|\log\left(\sqrt{2}x|a|+x^2+|a|^2\right)}{8a^2}+\frac{\sqrt{2}|a|\log\left(-\sqrt{2}x|a|+x^2+\right.}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^4+x^4),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*abs(a)*arctan(1/2*sqrt(2)*(sqrt(2)*abs(a) + 2*x)/abs(a))/a^2 +
1/4*sqrt(2)*abs(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*abs(a) - 2*x)/abs(a))/a^2 -
 1/8*sqrt(2)*abs(a)*log(sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^2 + 1/8*sqrt(2
)*abs(a)*log(-sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^2
```

3.136 $\int \frac{1}{a^5+x^5} dx$

Optimal. Leaf size=201

$$-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^4}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^4}+\frac{\log(a+x)}{5a^4}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\Bigg(}{5a^4}$$

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqrt[5]
)]*a)])/(5*a^4) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1
 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^4) + Log[a + x]/(5*a^4) - ((1 - Sqrt[5])
*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^4) - ((1 + Sqrt[5])*Log[a^2
- ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^4)
```

Rubi [A] time = 0.314295, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {201, 634, 618, 204, 628, 31}

$$-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^4}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^4}+\frac{\log(a+x)}{5a^4}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\Bigg(}{5a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a^5 + x^5)^(-1),x]
```

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqrt[5]
)]*a)])/(5*a^4) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1
 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^4) + Log[a + x]/(5*a^4) - ((1 - Sqrt[5])
*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^4) - ((1 + Sqrt[5])*Log[a^2
- ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^4)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[((2*k -
 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r*Int[1
/(r + s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}], x], x]
] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a^5+x^5}\,dx = \frac{2\int \frac{a-\frac{1}{4}(1-\sqrt{5})x}{a^2-\frac{1}{2}(1-\sqrt{5})ax+x^2}\,dx}{5a^4} + \frac{2\int \frac{a-\frac{1}{4}(1+\sqrt{5})x}{a^2-\frac{1}{2}(1+\sqrt{5})ax+x^2}\,dx}{5a^4} + \frac{\int \frac{1}{a+x}\,dx}{5a^4}$$

$$= \frac{\log(a+x)}{5a^4} - \frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}(1-\sqrt{5})a+2x}{a^2-\frac{1}{2}(1-\sqrt{5})ax+x^2}\,dx}{20a^4} - \frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}(1+\sqrt{5})a+2x}{a^2-\frac{1}{2}(1+\sqrt{5})ax+x^2}\,dx}{20a^4} + \frac{\left(5-\sqrt{5}\right)\int \frac{1}{a^2-\frac{1}{2}(1+\sqrt{5})\cdots}}{20a^3}$$

$$= \frac{\log(a+x)}{5a^4} - \frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a^4} - \frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}ax+2x^2\right)}{20a^4} - \frac{\left(5-\cdots\right.}{}$$

$$= -\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{(1-\sqrt{5})a-4x}{\sqrt{2(5+\sqrt{5})}a}\right)}{5a^4} - \frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}\left((1+\sqrt{5})a-4x\right)}{2a}\right)}{5a^4} + \frac{\log(a+x)}{5a^4} - \frac{(1+\cdots}{}$$

Mathematica [A] time = 0.149607, size = 204, normalized size = 1.01

$$-\frac{-\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\sqrt{5}\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+\log\left(a^2-\frac{1}{2}\cdots\right.}{20a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^5 + x^5)^(-1),x]
```

```
[Out] -(-2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 + Sqr
t[5])]*a)] - 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt
[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^
2] - Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 + Sqrt
[5])*a*x)/2 + x^2] + Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^
4)
```

Maple [C] time = 0.011, size = 101, normalized size = 0.5

$$\frac{1}{5a^4}\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+a^2\_Z^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3+2\_R^2a-3\_R a^2+4a^3\right)\ln(x-\_R)}{4\_R^3-3\_R^2a+2\_R a^2-a^3}+\frac{\ln(a+x)}{5a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^5+x^5),x)
```

```
[Out] 1/5/a^4*sum((-_R^3+2*_R^2*a-3*_R*a^2+4*a^3)/(4*_R^3-3*_R^2*a+2*_R*a^2-a^3)*
ln(x-_R),_R=RootOf(_Z^4-_Z^3*a+_Z^2*a^2-_Z*a^3+a^4))+1/5*ln(a+x)/a^4
```

Maxima [A] time = 1.4219, size = 385, normalized size = 1.92

$$\frac{\sqrt{5}\left(\sqrt{5}-1\right)\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}\right)}{10\left(a^5\right)^{\frac{4}{5}}\sqrt{2\sqrt{5}-10}}+\frac{\sqrt{5}\left(\sqrt{5}+1\right)\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}\right)}{10\left(a^5\right)^{\frac{4}{5}}\sqrt{-2\sqrt{5}-10}}-\frac{\left(\sqrt{5}+3\right)\log\left(-\left(a^5\right)\right.}{10\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(5)*(sqrt(5) - 1)*log(((a^5)^(1/5)*(sqrt(5) + 1) - 4*x + (a^5)^(1/
5)*sqrt(2*sqrt(5) - 10))/((a^5)^(1/5)*(sqrt(5) + 1) - 4*x - (a^5)^(1/5)*sqr
t(2*sqrt(5) - 10)))/((a^5)^(4/5)*sqrt(2*sqrt(5) - 10)) + 1/10*sqrt(5)*(sqrt
(5) + 1)*log(((a^5)^(1/5)*(sqrt(5) - 1) + 4*x - (a^5)^(1/5)*sqrt(-2*sqrt(5)
 - 10))/((a^5)^(1/5)*(sqrt(5) - 1) + 4*x + (a^5)^(1/5)*sqrt(-2*sqrt(5) - 10
)))/((a^5)^(4/5)*sqrt(-2*sqrt(5) - 10)) - 1/10*(sqrt(5) + 3)*log(-(a^5)^(1/
5)*x*(sqrt(5) + 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(4/5)*(sqrt(5) + 1)) - 1
/10*(sqrt(5) - 3)*log((a^5)^(1/5)*x*(sqrt(5) - 1) + 2*x^2 + 2*(a^5)^(2/5))/
((a^5)^(4/5)*(sqrt(5) - 1)) + 1/5*log(x + (a^5)^(1/5))/(a^5)^(4/5)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.127278, size = 39, normalized size = 0.19

$$\frac{\frac{\log(a+x)}{5}+\operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log(5ta+x)\right)\right)}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**5+x**5),x)
```

```
[Out] (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda
(_t, _t*log(5*_t*a + x))))/a**4
```

Giac [A] time = 1.05799, size = 239, normalized size = 1.19

$$\frac{\sqrt{2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10\,a^4}+\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10\,a^4}-\frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20\,a^4}+\sqrt{5}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^5+x^5),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
+ 10)))/a^4 + 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a
*sqrt(-2*sqrt(5) + 10)))/a^4 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*x
 + x^2)/a^4 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^4 - 1/2
0*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^4 + 1/5*log(abs(a + x))/a
^4
```

3.137 $\int \frac{x}{a^5+x^5} \, dx$

Optimal. Leaf size=201

$$\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^3}+\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^3}-\frac{\log(a+x)}{5a^3}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{(1-}{\sqrt{}}\right.}{5a^3}$$

```
[Out] (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqrt[5])
]*a)])/(5*a^3) - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1
+ Sqrt[5])*a - 4*x))/(2*a)])/(5*a^3) - Log[a + x]/(5*a^3) + ((1 + Sqrt[5])*
Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^3) + ((1 - Sqrt[5])*Log[a^2 -
 ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^3)
```

Rubi [A] time = 0.271211, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {293, 634, 618, 204, 628, 31}

$$\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^3}+\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^3}-\frac{\log(a+x)}{5a^3}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{(1-}{\sqrt{}}\right.}{5a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a^5 + x^5),x]
```

```
[Out] (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqrt[5])
]*a)])/(5*a^3) - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1
+ Sqrt[5])*a - 4*x))/(2*a)])/(5*a^3) - Log[a + x]/(5*a^3) + ((1 + Sqrt[5])*
Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^3) + ((1 - Sqrt[5])*Log[a^2 -
 ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^3)
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x}{a^5+x^5}\,dx = \frac{2\int \frac{\frac{1}{4}(1-\sqrt{5})a-\frac{1}{4}(-1-\sqrt{5})x}{a^2-\frac{1}{2}(1-\sqrt{5})ax+x^2}\,dx}{5a^3} + \frac{2\int \frac{\frac{1}{4}(1+\sqrt{5})a-\frac{1}{4}(-1+\sqrt{5})x}{a^2-\frac{1}{2}(1+\sqrt{5})ax+x^2}\,dx}{5a^3} - \frac{\int \frac{1}{a+x}\,dx}{5a^3}$$

$$= -\frac{\log(a+x)}{5a^3} + \frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}(1+\sqrt{5})a+2x}{a^2-\frac{1}{2}(1+\sqrt{5})ax+x^2}\,dx}{20a^3} + \frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}(1-\sqrt{5})a+2x}{a^2-\frac{1}{2}(1-\sqrt{5})ax+x^2}\,dx}{20a^3} - \frac{\int \frac{1}{a^2-\frac{1}{2}(1-\sqrt{5})ax+x^2}\,dx}{2\sqrt{5}a^2}$$

$$= -\frac{\log(a+x)}{5a^3} + \frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a^3} + \frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}ax+2x^2\right)}{20a^3} - \frac{\text{Su}}{}$$

$$= \frac{\sqrt{\frac{2}{5(5+\sqrt{5})}}\tan^{-1}\left(\frac{(1-\sqrt{5})a-4x}{\sqrt{2(5+\sqrt{5})}a}\right)}{a^3} - \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}(5+\sqrt{5})}((1+\sqrt{5})a-4x)}{2a}\right)}{5a^3} - \frac{\log(a+x)}{5a^3} + \frac{\left(1-\sqrt{5}\right)}{}$$

Mathematica [A] time = 0.0565078, size = 204, normalized size = 1.01

$$\frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)-\sqrt{5}\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+\log\left(a^2-\frac{1}{2}\left(1\right.\right.}{20a^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(a^5 + x^5),x]
```

```
[Out] (-2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 + Sqrt[
5])]*a)] + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt[
10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2
] + Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 + Sqrt[
5])*a*x)/2 + x^2] - Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^3
)
```

Maple [C] time = 0.005, size = 97, normalized size = 0.5

$$\frac{1}{5a^3}\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+a^2\_Z^2-a^3\_Z+a^4\right)}\frac{\left(\_R^3-2\_R^2a+3\_R\,a^2+a^3\right)\ln(x-\_R)}{4\_R^3-3\_R^2a+2\_R\,a^2-a^3}-\frac{\ln(a+x)}{5a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^5+x^5),x)
```

```
[Out] 1/5/a^3*sum((_R^3-2*_R^2*a+3*_R*a^2+a^3)/(4*_R^3-3*_R^2*a+2*_R*a^2-a^3)*ln(
x-_R),_R=RootOf(_Z^4-_Z^3*a+_Z^2*a^2-_Z*a^3+a^4))-1/5*ln(a+x)/a^3
```

Maxima [A] time = 1.42026, size = 358, normalized size = 1.78

$$-\frac{\log\left(x+\left(a^5\right)^{\frac{1}{5}}\right)}{5\left(a^5\right)^{\frac{3}{5}}}+\frac{\sqrt{5}\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}\right)}{5\left(a^5\right)^{\frac{3}{5}}\sqrt{2\sqrt{5}-10}}-\frac{\sqrt{5}\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}\right)}{5\left(a^5\right)^{\frac{3}{5}}\sqrt{-2\sqrt{5}-10}}-\frac{\log\left(-\left(a^5\right)^{\frac{1}{5}}x\left(\sqrt{5}\right.\right.}{5\left(a^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] -1/5*log(x + (a^5)^(1/5))/(a^5)^(3/5) + 1/5*sqrt(5)*log(((a^5)^(1/5)*(sqrt(
5) + 1) - 4*x + (a^5)^(1/5)*sqrt(2*sqrt(5) - 10))/((a^5)^(1/5)*(sqrt(5) + 1
) - 4*x - (a^5)^(1/5)*sqrt(2*sqrt(5) - 10)))/((a^5)^(3/5)*sqrt(2*sqrt(5) -
10)) - 1/5*sqrt(5)*log(((a^5)^(1/5)*(sqrt(5) - 1) + 4*x - (a^5)^(1/5)*sqrt(
-2*sqrt(5) - 10))/((a^5)^(1/5)*(sqrt(5) - 1) + 4*x + (a^5)^(1/5)*sqrt(-2*sq
rt(5) - 10)))/((a^5)^(3/5)*sqrt(-2*sqrt(5) - 10)) - 1/5*log(-(a^5)^(1/5)*x*
(sqrt(5) + 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(3/5)*(sqrt(5) + 1)) + 1/5*lo
g((a^5)^(1/5)*x*(sqrt(5) - 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(3/5)*(sqrt(5
) - 1))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.126691, size = 41, normalized size = 0.2

$$\frac{-\frac{\log(a+x)}{5}+\operatorname{RootSum}\left(625t^4-125t^3+25t^2-5t+1,\left(t\mapsto t\log\left(-125t^3a+x\right)\right)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**5+x**5),x)
```

```
[Out] (-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t + 1, Lambd
a(_t, _t*log(-125*_t**3*a + x))))/a**3
```

Giac [A] time = 1.08677, size = 239, normalized size = 1.19

$$-\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^3}+\frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^3}-\frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a^3}+\frac{\sqrt{5}\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^5+x^5),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5
) + 10)))/a^3 + 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(
a*sqrt(-2*sqrt(5) + 10)))/a^3 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*
x + x^2)/a^3 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^3 + 1/
20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^3 - 1/5*log(abs(a + x))/
a^3
```

3.138 $\int \frac{x^2}{a^5+x^5}\,dx$

Optimal. Leaf size=201

$$-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^2}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^2}+\frac{\log(a+x)}{5a^2}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\cdots\right.}{5a^2}$$

```
[Out] (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqrt[5])
]*a)])/(5*a^2) - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1
+ Sqrt[5])*a - 4*x))/(2*a)])/(5*a^2) + Log[a + x]/(5*a^2) - ((1 + Sqrt[5])*
Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^2) - ((1 - Sqrt[5])*Log[a^2 -
 ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^2)
```

Rubi [A] time = 0.316646, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {293, 634, 618, 204, 628, 31}

$$-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^2}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^2}+\frac{\log(a+x)}{5a^2}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\cdots\right.}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^5 + x^5),x]
```

```
[Out] (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqrt[5])
]*a)])/(5*a^2) - (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1
+ Sqrt[5])*a - 4*x))/(2*a)])/(5*a^2) + Log[a + x]/(5*a^2) - ((1 + Sqrt[5])*
Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^2) - ((1 - Sqrt[5])*Log[a^2 -
 ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^2)
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^2}{a^5+x^5}\,dx = \frac{2\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)a-\frac{1}{4}\left(1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a^2} + \frac{2\int \frac{\frac{1}{4}\left(-1+\sqrt{5}\right)a-\frac{1}{4}\left(1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a^2} + \frac{\int \frac{1}{a+x}\,dx}{5a^2}$$

$$= \frac{\log(a+x)}{5a^2} - \frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a^2} - \frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a^2} - \frac{\int \frac{1}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{2\sqrt{5}a} +$$

$$= \frac{\log(a+x)}{5a^2} - \frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a^2} - \frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}ax+2x^2\right)}{20a^2} - \text{Subs}$$

$$= \frac{\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{a^2} - \frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^2} + \frac{\log(a+x)}{5a^2} - \left(1-\sqrt{5}\right)$$

Mathematica [A] time = 0.0772716, size = 204, normalized size = 1.01

$$-\frac{\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)-\sqrt{5}\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+\log\left(a^2-\frac{1}{2}\left(\right.\right.}{20a^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/(a^5 + x^5),x]
```

```
[Out] -(2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 + Sqrt[
5])]*a)] - 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt[
10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2
] + Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 + Sqrt[
5])*a*x)/2 + x^2] - Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^2
)
```

Maple [C] time = 0.007, size = 101, normalized size = 0.5

$$\frac{1}{5\,a^2}\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+a^2\_Z^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3+2\_R^2a+2\_R\,a^2-a^3\right)\ln\left(x-\_R\right)}{4\_R^3-3\_R^2a+2\_R\,a^2-a^3}+\frac{\ln\left(a+x\right)}{5\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^5+x^5),x)
```

```
[Out] 1/5/a^2*sum((-_R^3+2*_R^2*a+2*_R*a^2-a^3)/(4*_R^3-3*_R^2*a+2*_R*a^2-a^3)*ln
(x-_R),_R=RootOf(_Z^4-_Z^3*a+_Z^2*a^2-_Z*a^3+a^4))+1/5*ln(a+x)/a^2
```

Maxima [A] time = 1.42501, size = 358, normalized size = 1.78

$$\frac{\log\left(x+\left(a^5\right)^{\frac{1}{5}}\right)}{5\left(a^5\right)^{\frac{2}{5}}}+\frac{\sqrt{5}\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}\right)}{5\left(a^5\right)^{\frac{2}{5}}\sqrt{2\sqrt{5}-10}}-\frac{\sqrt{5}\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}\right)}{5\left(a^5\right)^{\frac{2}{5}}\sqrt{-2\sqrt{5}-10}}+\frac{\log\left(-\left(a^5\right)^{\frac{1}{5}}x\left(\sqrt{5}+\right.\right.}{5\left(a^5\right)^{\frac{2}{5}}\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] 1/5*log(x + (a^5)^(1/5))/(a^5)^(2/5) + 1/5*sqrt(5)*log(((a^5)^(1/5)*(sqrt(5
) + 1) - 4*x + (a^5)^(1/5)*sqrt(2*sqrt(5) - 10))/((a^5)^(1/5)*(sqrt(5) + 1)
 - 4*x - (a^5)^(1/5)*sqrt(2*sqrt(5) - 10)))/((a^5)^(2/5)*sqrt(2*sqrt(5) - 1
0)) - 1/5*sqrt(5)*log(((a^5)^(1/5)*(sqrt(5) - 1) + 4*x - (a^5)^(1/5)*sqrt(-
2*sqrt(5) - 10))/((a^5)^(1/5)*(sqrt(5) - 1) + 4*x + (a^5)^(1/5)*sqrt(-2*sqr
t(5) - 10)))/((a^5)^(2/5)*sqrt(-2*sqrt(5) - 10)) + 1/5*log(-(a^5)^(1/5)*x*(
sqrt(5) + 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(2/5)*(sqrt(5) + 1)) - 1/5*log
((a^5)^(1/5)*x*(sqrt(5) - 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(2/5)*(sqrt(5)
 - 1))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.1393, size = 41, normalized size = 0.2

$$\frac{\frac{\log\left(a+x\right)}{5}+\operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(25t^2a+x\right)\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**5+x**5),x)
```

```
[Out] (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*_t + 1, Lambda
(_t, _t*log(25*_t**2*a + x))))/a**2
```

Giac [A] time = 1.07868, size = 239, normalized size = 1.19

$$-\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a\left(\sqrt{5}-1\right)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^2}+\frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a\left(\sqrt{5}+1\right)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^2}+\frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a^2}-\frac{\sqrt{5}\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^5+x^5),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5
) + 10)))/a^2 + 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(
a*sqrt(-2*sqrt(5) + 10)))/a^2 + 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*
x + x^2)/a^2 - 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^2 - 1/
20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^2 + 1/5*log(abs(a + x))/
a^2
```

3.139 $\int \frac{x^3}{a^5+x^5} \, dx$

Optimal. Leaf size=201

$$\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a}+\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a}-\frac{\log(a+x)}{5a}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{(1\cdots}{\sqrt{\cdots}}\right.}{5a}$$

```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqrt[5]
)]*a)])/(5*a) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1 +
 Sqrt[5])*a - 4*x))/(2*a)])/(5*a) - Log[a + x]/(5*a) + ((1 - Sqrt[5])*Log[a
^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a) + ((1 + Sqrt[5])*Log[a^2 - ((1 +
Sqrt[5])*a*x)/2 + x^2])/(20*a)
```

Rubi [A] time = 0.312836, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {293, 634, 618, 204, 628, 31}

$$\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a}+\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a}-\frac{\log(a+x)}{5a}-\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{(1\cdots}{\sqrt{\cdots}}\right.}{5a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a^5 + x^5),x]
```


```
[Out] -(Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[2*(5 + Sqrt[5]
)]*a)])/(5*a) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*((1 +
 Sqrt[5])*a - 4*x))/(2*a)])/(5*a) - Log[a + x]/(5*a) + ((1 - Sqrt[5])*Log[a
^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a) + ((1 + Sqrt[5])*Log[a^2 - ((1 +
Sqrt[5])*a*x)/2 + x^2])/(20*a)
```


Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^3}{a^5+x^5}\,dx = \frac{2\int \frac{\frac{1}{4}\left(1+\sqrt{5}\right)a-\frac{1}{4}\left(-1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a}+\frac{2\int \frac{\frac{1}{4}\left(1-\sqrt{5}\right)a-\frac{1}{4}\left(-1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a}-\frac{\int \frac{1}{a+x}\,dx}{5a}$$

$$= -\frac{\log(a+x)}{5a}+\frac{1}{20}\left(5-\sqrt{5}\right)\int \frac{1}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx+\frac{1}{20}\left(5+\sqrt{5}\right)\int \frac{1}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}$$

$$= -\frac{\log(a+x)}{5a}+\frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a}+\frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}ax+2x^2\right)}{20a}+\frac{1}{10}$$

$$= -\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a}-\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a}-\frac{\log(a+x)}{5a}+\frac{(1+}{}$$

Mathematica [A] time = 0.0461625, size = 204, normalized size = 1.01

$$\frac{-\sqrt{5}\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\sqrt{5}\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+\log\left(a^2-\frac{1}{2}\right.}{20a}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a^5 + x^5),x]
```

```
[Out] (2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[2*(5 + Sqrt[
5])]*a)] + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt[5])*a) + 4*x)/(Sqrt[1
0 - 2*Sqrt[5]]*a)] - 4*Log[a + x] + Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2]
 - Sqrt[5]*Log[a^2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + Log[a^2 - ((1 + Sqrt[5
])*a*x)/2 + x^2] + Sqrt[5]*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a)
```

Maple [C] time = 0.007, size = 97, normalized size = 0.5

$$\frac{1}{5a}\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+a^2\_Z^2-a^3\_Z+a^4\right)}\frac{\left(\_R^3+3\_R^2a-2\_Ra^2+a^3\right)\ln(x-\_R)}{4\_R^3-3\_R^2a+2\_Ra^2-a^3}-\frac{\ln(a+x)}{5a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^5+x^5),x)
```

```
[Out] 1/5/a*sum((_R^3+3*_R^2*a-2*_R*a^2+a^3)/(4*_R^3-3*_R^2*a+2*_R*a^2-a^3)*ln(x-
_R),_R=RootOf(_Z^4-_Z^3*a+_Z^2*a^2-_Z*a^3+a^4))-1/5*ln(a+x)/a
```

Maxima [A] time = 1.42073, size = 385, normalized size = 1.92

$$\frac{\sqrt{5}\left(\sqrt{5}-1\right)\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}\right)}{10\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}+\frac{\sqrt{5}\left(\sqrt{5}+1\right)\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}\right)}{10\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}+\frac{\left(\sqrt{5}+3\right)\log\left(-\left(a^5\right)\cdots\right.}{10\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(5)*(sqrt(5) - 1)*log(((a^5)^(1/5)*(sqrt(5) + 1) - 4*x + (a^5)^(1/
5)*sqrt(2*sqrt(5) - 10))/((a^5)^(1/5)*(sqrt(5) + 1) - 4*x - (a^5)^(1/5)*sqr
t(2*sqrt(5) - 10)))/((a^5)^(1/5)*sqrt(2*sqrt(5) - 10)) + 1/10*sqrt(5)*(sqrt
(5) + 1)*log(((a^5)^(1/5)*(sqrt(5) - 1) + 4*x - (a^5)^(1/5)*sqrt(-2*sqrt(5)
 - 10))/((a^5)^(1/5)*(sqrt(5) - 1) + 4*x + (a^5)^(1/5)*sqrt(-2*sqrt(5) - 10
)))/((a^5)^(1/5)*sqrt(-2*sqrt(5) - 10)) + 1/10*(sqrt(5) + 3)*log(-(a^5)^(1/
5)*x*(sqrt(5) + 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(1/5)*(sqrt(5) + 1)) + 1
/10*(sqrt(5) - 3)*log((a^5)^(1/5)*x*(sqrt(5) - 1) + 2*x^2 + 2*(a^5)^(2/5))/
((a^5)^(1/5)*(sqrt(5) - 1)) - 1/5*log(x + (a^5)^(1/5))/(a^5)^(1/5)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.133624, size = 39, normalized size = 0.19

$$\frac{-\frac{\log\left(a+x\right)}{5}+\operatorname{RootSum}\left(625t^4-125t^3+25t^2-5t+1,\left(t\mapsto t\log\left(625t^4a+x\right)\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**5+x**5),x)
```

```
[Out] (-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**2 - 5*_t + 1, Lambd
a(_t, _t*log(625*_t**4*a + x))))/a
```

Giac [A] time = 1.07163, size = 239, normalized size = 1.19

$$\frac{\sqrt{2\sqrt{5}+10}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a}+\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a}+\frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a}-\frac{\sqrt{5}\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^5+x^5),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
+ 10)))/a + 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a*s
qrt(-2*sqrt(5) + 10)))/a + 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*x + x
^2)/a - 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a + 1/20*log(ab
s(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a - 1/5*log(abs(a + x))/a
```

3.140 $\int \frac{x^4}{a^5+x^5} dx$

Optimal. Leaf size=12

$$\frac{1}{5}\log\left(a^5+x^5\right)$$

```
[Out] Log[a^5 + x^5]/5
```

Rubi [A] time = 0.002512, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {260}

$$\frac{1}{5}\log\left(a^5+x^5\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(a^5 + x^5),x]
```

```
[Out] Log[a^5 + x^5]/5
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x^4}{a^5+x^5} dx = \frac{1}{5}\log\left(a^5+x^5\right)$$

Mathematica [A] time = 0.0023469, size = 12, normalized size = 1.

$$\frac{1}{5}\log\left(a^5+x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(a^5 + x^5),x]
```

```
[Out] Log[a^5 + x^5]/5
```

Maple [A] time = 0.001, size = 11, normalized size = 0.9

$$\frac{\ln\left(a^5+x^5\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a^5+x^5),x)
```

```
[Out] 1/5*ln(a^5+x^5)
```

Maxima [A] time = 0.929379, size = 14, normalized size = 1.17

$$\frac{1}{5}\log\left(a^5 + x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] 1/5*log(a^5 + x^5)
```

Fricas [A] time = 1.79827, size = 27, normalized size = 2.25

$$\frac{1}{5}\log\left(a^5 + x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] 1/5*log(a^5 + x^5)
```

Sympy [A] time = 0.101514, size = 8, normalized size = 0.67

$$\frac{\log\left(a^5 + x^5\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a**5+x**5),x)
```

```
[Out] log(a**5 + x**5)/5
```

Giac [A] time = 1.0589, size = 15, normalized size = 1.25

$$\frac{1}{5}\log\left(\left|a^5 + x^5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a^5+x^5),x, algorithm="giac")
```

```
[Out] 1/5*log(abs(a^5 + x^5))
```

3.141 $\int \frac{1}{x(a^5+x^5)}\,dx$

Optimal. Leaf size=22

$$\frac{\log(x)}{a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}$$

```
[Out] Log[x]/a^5 - Log[a^5 + x^5]/(5*a^5)
```

Rubi [A] time = 0.0083307, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 36, 29, 31}

$$\frac{\log(x)}{a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^5 + x^5)),x]
```

```
[Out] Log[x]/a^5 - Log[a^5 + x^5]/(5*a^5)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(a^5+x^5\right)}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{1}{x\left(a^5+x\right)}\,dx,x,x^5\right)\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^5\right)}{5a^5}-\frac{\operatorname{Subst}\left(\int \frac{1}{a^5+x}\,dx,x,x^5\right)}{5a^5}\\
&= \frac{\log(x)}{a^5}-\frac{\log\left(a^5+x^5\right)}{5a^5}
\end{aligned}$$

Mathematica [A] time = 0.0039255, size = 22, normalized size = 1.

$$\frac{\log(x)}{a^5} - \frac{\log\left(a^5 + x^5\right)}{5a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^5 + x^5)),x]
```

```
[Out] Log[x]/a^5 - Log[a^5 + x^5]/(5*a^5)
```

Maple [B] time = 0.007, size = 49, normalized size = 2.2

$$-\frac{\ln\left(a^4 - a^3x + a^2x^2 - ax^3 + x^4\right)}{5\,a^5} + \frac{\ln(x)}{a^5} - \frac{\ln(a+x)}{5\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^5+x^5),x)
```

```
[Out] -1/5/a^5*ln(a^4-a^3*x+a^2*x^2-a*x^3+x^4)+ln(x)/a^5-1/5*ln(a+x)/a^5
```

Maxima [A] time = 0.927172, size = 31, normalized size = 1.41

$$-\frac{\log\left(a^5 + x^5\right)}{5\,a^5} + \frac{\log\left(x^5\right)}{5\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] -1/5*log(a^5 + x^5)/a^5 + 1/5*log(x^5)/a^5
```

Fricas [A] time = 1.89974, size = 51, normalized size = 2.32

$$-\frac{\log\left(a^5 + x^5\right) - 5\,\log(x)}{5\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] -1/5*(log(a^5 + x^5) - 5*log(x))/a^5
```

Sympy [A] time = 0.224743, size = 19, normalized size = 0.86

$$\frac{\log(x)}{a^5} - \frac{\log\left(a^5 + x^5\right)}{5a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**5+x**5),x)
```

```
[Out] log(x)/a**5 - log(a**5 + x**5)/(5*a**5)
```

Giac [A] time = 1.06726, size = 30, normalized size = 1.36

$$-\frac{\log\left(\left|a^5+x^5\right|\right)}{5\,a^5}+\frac{\log\left(|x|\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^5+x^5),x, algorithm="giac")
```

```
[Out] -1/5*log(abs(a^5 + x^5))/a^5 + log(abs(x))/a^5
```

3.142 $\int \frac{1}{x^2\left(a^5+x^5\right)}\,dx$

Optimal. Leaf size=209

$$-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^6}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^6}-\frac{1}{a^5x}+\frac{\log(a+x)}{5a^6}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}}{}$$

```
[Out] -(1/(a^5*x)) + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[
2*(5 + Sqrt[5])]*a)])/(5*a^6) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sq
rt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^6) + Log[a + x]/(5*a^6) -
((1 - Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^6) - ((1 + Sqr
t[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^6)
```

Rubi [A] time = 0.329494, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {325, 293, 634, 618, 204, 628, 31}

$$-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^6}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^6}-\frac{1}{a^5x}+\frac{\log(a+x)}{5a^6}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^5 + x^5)),x]
```

```
[Out] -(1/(a^5*x)) + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqrt[
2*(5 + Sqrt[5])]*a)])/(5*a^6) + (Sqrt[(5 - Sqrt[5])/2]*ArcTan[(Sqrt[(5 + Sq
rt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^6) + Log[a + x]/(5*a^6) -
((1 - Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^6) - ((1 + Sqr
t[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^6)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\left(a^5+x^5\right)}\,dx &= -\frac{1}{a^5x}-\frac{\int \frac{x^3}{a^5+x^5}\,dx}{a^5}\\
&= -\frac{1}{a^5x}+\frac{\int \frac{1}{a+x}\,dx}{5a^6}-\frac{2\int \frac{\frac{1}{4}\left(1+\sqrt{5}\right)a-\frac{1}{4}\left(-1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a^6}-\frac{2\int \frac{\frac{1}{4}\left(1-\sqrt{5}\right)a-\frac{1}{4}\left(-1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a^6}\\
&= -\frac{1}{a^5x}+\frac{\log(a+x)}{5a^6}-\frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a^6}-\frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a^6}-\frac{\left(5-\sqrt{5}\right)\int \cdots}{\cdots}\\
&= -\frac{1}{a^5x}+\frac{\log(a+x)}{5a^6}-\frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a^6}-\frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}ax+2x^2\right)}{20a^6}\cdots\\
&= -\frac{1}{a^5x}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{5a^6}+\frac{\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^6}+\frac{\log(a+x)}{5a^6}\cdots\end{aligned}$$

Mathematica [A] time = 0.161731, size = 172, normalized size = 0.82

$$-\frac{-\left(\sqrt{5}-1\right)\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+\frac{20a}{x}-4\log(a+x)+2\sqrt{2\left(5+\sqrt{5}\right)}\cdots}{20a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^5 + x^5)),x]
```

```
[Out] -((20*a)/x + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sqrt[
2*(5 + Sqrt[5])]*a)] + 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[(-((1 + Sqrt[5])*a) +
4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] - 4*Log[a + x] - (-1 + Sqrt[5])*Log[a^2 + ((
-1 + Sqrt[5])*a*x)/2 + x^2] + (1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2
 + x^2])/(20*a^6)
```

Maple [C] time = 0.008, size = 109, normalized size = 0.5

$$\frac{1}{5a^6}\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+a^2\_Z^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3-3\_R^2a+2\_R\,a^2-a^3\right)\ln\left(x-\_R\right)}{4\_R^3-3\_R^2a+2\_R\,a^2-a^3}+\frac{\ln\left(a+x\right)}{5a^6}-\frac{1}{a^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^5+x^5),x)
```

```
[Out] 1/5/a^6*sum((-_R^3-3*_R^2*a+2*_R*a^2-a^3)/(4*_R^3-3*_R^2*a+2*_R*a^2-a^3)*ln
(x-_R),_R=RootOf(_Z^4-_Z^3*a+_Z^2*a^2-_Z*a^3+a^4))+1/5*ln(a+x)/a^6-1/a^5/x
```

Maxima [A] time = 1.42739, size = 398, normalized size = 1.9

$$-\frac{\frac{\sqrt{5}\left(\sqrt{5}-1\right)\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}\right)}{\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}+\frac{\sqrt{5}\left(\sqrt{5}+1\right)\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}\right)}{\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}+\frac{\left(\sqrt{5}+3\right)\log\left(-\left(a^5\right)^{\frac{1}{5}}x\left(\sqrt{5}+1\right)+2x^2+2\left(a^5\right)^{\frac{2}{5}}\right)}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)}+}{10a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] -1/10*(sqrt(5)*(sqrt(5) - 1)*log(((a^5)^(1/5)*(sqrt(5) + 1) - 4*x + (a^5)^(
1/5)*sqrt(2*sqrt(5) - 10))/((a^5)^(1/5)*(sqrt(5) + 1) - 4*x - (a^5)^(1/5)*s
qrt(2*sqrt(5) - 10)))/((a^5)^(1/5)*sqrt(2*sqrt(5) - 10)) + sqrt(5)*(sqrt(5)
 + 1)*log(((a^5)^(1/5)*(sqrt(5) - 1) + 4*x - (a^5)^(1/5)*sqrt(-2*sqrt(5) -
10))/((a^5)^(1/5)*(sqrt(5) - 1) + 4*x + (a^5)^(1/5)*sqrt(-2*sqrt(5) - 10)))
/((a^5)^(1/5)*sqrt(-2*sqrt(5) - 10)) + (sqrt(5) + 3)*log(-(a^5)^(1/5)*x*(sq
rt(5) + 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(1/5)*(sqrt(5) + 1)) + (sqrt(5)
- 3)*log((a^5)^(1/5)*x*(sqrt(5) - 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(1/5)*
(sqrt(5) - 1)) - 2*log(x + (a^5)^(1/5))/(a^5)^(1/5))/a^5 - 1/(a^5*x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.331365, size = 48, normalized size = 0.23

$$-\frac{1}{a^5x}+\frac{\frac{\log(a+x)}{5}+\operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(625t^4a+x\right)\right)\right)}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**5+x**5),x)
```

```
[Out] -1/(a**5*x) + (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2 + 5*
_t + 1, Lambda(_t, _t*log(625*_t**4*a + x))))/a**6
```

Giac [A] time = 1.08505, size = 250, normalized size = 1.2

$$-\frac{\sqrt{2\sqrt{5}+10}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^6}-\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^6}-\frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a^6}+\frac{\sqrt{5}\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^5+x^5),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
 + 10)))/a^6 - 1/10*sqrt(-2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(
a*sqrt(-2*sqrt(5) + 10)))/a^6 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*
x + x^2)/a^6 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^6 - 1/
20*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^6 + 1/5*log(abs(a + x))/
a^6 - 1/(a^5*x)
```

3.143 $\int \frac{1}{x^3\left(a^5+x^5\right)}\,dx$

Optimal. Leaf size=211

$$-\frac{1}{2a^5x^2}+\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^7}+\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^7}-\frac{\log(a+x)}{5a^7}-\frac{\sqrt{\frac{1}{2}\left(5-\right.}}{}$$

```
[Out] -1/(2*a^5*x^2) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqr
t[2*(5 + Sqrt[5])]*a)])/(5*a^7) + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5 +
Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^7) - Log[a + x]/(5*a^7)
+ ((1 + Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^7) + ((1 - S
qrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^7)
```

Rubi [A] time = 0.327494, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {325, 293, 634, 618, 204, 628, 31}

$$-\frac{1}{2a^5x^2}+\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^7}+\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^7}-\frac{\log(a+x)}{5a^7}-\frac{\sqrt{\frac{1}{2}\left(5-\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^5 + x^5)),x]
```

```
[Out] -1/(2*a^5*x^2) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqr
t[2*(5 + Sqrt[5])]*a)])/(5*a^7) + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5 +
Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^7) - Log[a + x]/(5*a^7)
+ ((1 + Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^7) + ((1 - S
qrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^7)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x^3\left(a^5+x^5\right)}\,dx = -\frac{1}{2a^5x^2}-\frac{\int \frac{x^2}{a^5+x^5}\,dx}{a^5}$$

$$= -\frac{1}{2a^5x^2}-\frac{\int \frac{1}{a+x}\,dx}{5a^7}-\frac{2\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)a-\frac{1}{4}\left(1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a^7}-\frac{2\int \frac{\frac{1}{4}\left(-1+\sqrt{5}\right)a-\frac{1}{4}\left(1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a^7}$$

$$= -\frac{1}{2a^5x^2}-\frac{\log(a+x)}{5a^7}+\frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a^7}+\frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a^7}+\frac{\int \frac{1}{a^2-\frac{1}{2}\left(1-\right.}}{2\sqrt{}}$$

$$= -\frac{1}{2a^5x^2}-\frac{\log(a+x)}{5a^7}+\frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a^7}+\frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}ax+\right.}{20a^7}$$

$$= -\frac{1}{2a^5x^2}-\frac{\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{a^7}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^7}-\frac{\log(a+x)}{5a^7}$$

Mathematica [A] time = 0.145139, size = 174, normalized size = 0.82

$$-\frac{\frac{10a^2}{x^2}-\left(1+\sqrt{5}\right)\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+\left(\sqrt{5}-1\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+4\log(a+x)-2\sqrt{10-2\sqrt{5}}\,\mathrm{ta}}{20a^7}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^3*(a^5 + x^5)),x]
```

```
[Out] -((10*a^2)/x^2 - 2*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sq
rt[2*(5 + Sqrt[5])]*a)] + 2*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[5])*a
) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] + 4*Log[a + x] - (1 + Sqrt[5])*Log[a^2 +
 ((-1 + Sqrt[5])*a*x)/2 + x^2] + (-1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*
x)/2 + x^2])/(20*a^7)
```

Maple [C] time = 0.008, size = 105, normalized size = 0.5

$$\frac{1}{5\,a^{7}}\sum_{\_R=\mathrm{RootOf}\left(\_Z^{4}-a\_Z^{3}+a^{2}\_Z^{2}-a^{3}\_Z+a^{4}\right)}\frac{\left(\_R^{3}-2\_R^{2}a-2\_R\,a^{2}+a^{3}\right)\ln\left(x-\_R\right)}{4\_R^{3}-3\_R^{2}a+2\_R\,a^{2}-a^{3}}-\frac{1}{2\,a^{5}x^{2}}-\frac{\ln\left(a+x\right)}{5\,a^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^5+x^5),x)
```

```
[Out] 1/5/a^7*sum((_R^3-2*_R^2*a-2*_R*a^2+a^3)/(4*_R^3-3*_R^2*a+2*_R*a^2-a^3)*ln(
x-_R),_R=RootOf(_Z^4-_Z^3*a+_Z^2*a^2-_Z*a^3+a^4))-1/2/a^5/x^2-1/5*ln(a+x)/a
^7
```

Maxima [A] time = 1.42332, size = 373, normalized size = 1.77

$$-\frac{\frac{\log\left(x+\left(a^{5}\right)^{\frac{1}{5}}\right)}{\left(a^{5}\right)^{\frac{2}{5}}}+\frac{\sqrt{5}\log\left(\frac{\left(a^{5}\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4\,x+\left(a^{5}\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}{\left(a^{5}\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4\,x-\left(a^{5}\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}\right)}{\left(a^{5}\right)^{\frac{2}{5}}\sqrt{2\sqrt{5}-10}}-\frac{\sqrt{5}\log\left(\frac{\left(a^{5}\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4\,x-\left(a^{5}\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}{\left(a^{5}\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4\,x+\left(a^{5}\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}\right)}{\left(a^{5}\right)^{\frac{2}{5}}\sqrt{-2\sqrt{5}-10}}+\frac{\log\left(-\left(a^{5}\right)^{\frac{1}{5}}x\left(\sqrt{5}+1\right)+2\,x^{2}+2\left(a^{5}\right)^{\frac{2}{5}}\right)}{\left(a^{5}\right)^{\frac{2}{5}}\left(\sqrt{5}+1\right)}-\frac{\log}{}}{5\,a^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] -1/5*(log(x + (a^5)^(1/5))/(a^5)^(2/5) + sqrt(5)*log(((a^5)^(1/5)*(sqrt(5)
+ 1) - 4*x + (a^5)^(1/5)*sqrt(2*sqrt(5) - 10))/((a^5)^(1/5)*(sqrt(5) + 1) -
 4*x - (a^5)^(1/5)*sqrt(2*sqrt(5) - 10)))/((a^5)^(2/5)*sqrt(2*sqrt(5) - 10)
) - sqrt(5)*log(((a^5)^(1/5)*(sqrt(5) - 1) + 4*x - (a^5)^(1/5)*sqrt(-2*sqrt
(5) - 10))/((a^5)^(1/5)*(sqrt(5) - 1) + 4*x + (a^5)^(1/5)*sqrt(-2*sqrt(5) -
 10)))/((a^5)^(2/5)*sqrt(-2*sqrt(5) - 10)) + log(-(a^5)^(1/5)*x*(sqrt(5) +
1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(2/5)*(sqrt(5) + 1)) - log((a^5)^(1/5)*x
*(sqrt(5) - 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(2/5)*(sqrt(5) - 1)))/a^5 -
1/2/(a^5*x^2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.363622, size = 51, normalized size = 0.24

$$-\frac{1}{2a^5x^2} + \frac{-\frac{\log(a+x)}{5} + \operatorname{RootSum}\left(625t^4 - 125t^3 + 25t^2 - 5t + 1, \left(t \mapsto t\log\left(25t^2a + x\right)\right)\right)}{a^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**5+x**5),x)
```

```
[Out] -1/(2*a**5*x**2) + (-log(a + x)/5 + RootSum(625*_t**4 - 125*_t**3 + 25*_t**
2 - 5*_t + 1, Lambda(_t, _t*log(25*_t**2*a + x))))/a**7
```

Giac [A] time = 1.09023, size = 250, normalized size = 1.18

$$\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^7} - \frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^7} - \frac{\sqrt{5}\log\left(a^2 - \frac{1}{2}\left(\sqrt{5}a+a\right)x + x^2\right)}{20a^7} + \frac{\sqrt{5}\log\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^5+x^5),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
 + 10)))/a^7 - 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a
*sqrt(-2*sqrt(5) + 10)))/a^7 - 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*x
 + x^2)/a^7 + 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^7 + 1/2
0*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^7 - 1/5*log(abs(a + x))/a
^7 - 1/2/(a^5*x^2)
```

3.144 $\int \frac{1}{x^4(a^5+x^5)} dx$

Optimal. Leaf size=211

$$-\frac{1}{3a^5x^3}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^8}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^8}+\frac{\log(a+x)}{5a^8}-\frac{\sqrt{\frac{1}{2}\left(5-\right.}}{}$$

```
[Out] -1/(3*a^5*x^3) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqr
t[2*(5 + Sqrt[5])]*a)])/(5*a^8) + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5 +
Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^8) + Log[a + x]/(5*a^8)
- ((1 + Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^8) - ((1 - S
qrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^8)
```

Rubi [A] time = 0.288057, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {325, 293, 634, 618, 204, 628, 31}

$$-\frac{1}{3a^5x^3}-\frac{\left(1+\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2\right)}{20a^8}-\frac{\left(1-\sqrt{5}\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)}{20a^8}+\frac{\log(a+x)}{5a^8}-\frac{\sqrt{\frac{1}{2}\left(5-\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a^5 + x^5)),x]
```

```
[Out] -1/(3*a^5*x^3) - (Sqrt[(5 - Sqrt[5])/2]*ArcTan[((1 - Sqrt[5])*a - 4*x)/(Sqr
t[2*(5 + Sqrt[5])]*a)])/(5*a^8) + (Sqrt[(5 + Sqrt[5])/2]*ArcTan[(Sqrt[(5 +
Sqrt[5])/10]*((1 + Sqrt[5])*a - 4*x))/(2*a)])/(5*a^8) + Log[a + x]/(5*a^8)
- ((1 + Sqrt[5])*Log[a^2 - ((1 - Sqrt[5])*a*x)/2 + x^2])/(20*a^8) - ((1 - S
qrt[5])*Log[a^2 - ((1 + Sqrt[5])*a*x)/2 + x^2])/(20*a^8)
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x^4\left(a^5+x^5\right)}\,dx = -\frac{1}{3a^5x^3}-\frac{\int \frac{x}{a^5+x^5}\,dx}{a^5}$$

$$= -\frac{1}{3a^5x^3}+\frac{\int \frac{1}{a+x}\,dx}{5a^8}-\frac{2\int \frac{\frac{1}{4}\left(1-\sqrt{5}\right)a-\frac{1}{4}\left(-1-\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{5a^8}-\frac{2\int \frac{\frac{1}{4}\left(1+\sqrt{5}\right)a-\frac{1}{4}\left(-1+\sqrt{5}\right)x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{5a^8}$$

$$= -\frac{1}{3a^5x^3}+\frac{\log(a+x)}{5a^8}-\frac{\left(1-\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1+\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2}\,dx}{20a^8}-\frac{\left(1+\sqrt{5}\right)\int \frac{-\frac{1}{2}\left(1-\sqrt{5}\right)a+2x}{a^2-\frac{1}{2}\left(1-\sqrt{5}\right)ax+x^2}\,dx}{20a^8}+\frac{\int \frac{1}{a^2-\frac{1}{2}\left(1-\right.}}{2\sqrt{}}$$

$$= -\frac{1}{3a^5x^3}+\frac{\log(a+x)}{5a^8}-\frac{\left(1-\sqrt{5}\right)\log\left(2a^2-ax-\sqrt{5}ax+2x^2\right)}{20a^8}-\frac{\left(1+\sqrt{5}\right)\log\left(2a^2-ax+\sqrt{5}ax+\right.}{20a^8}$$

$$= -\frac{1}{3a^5x^3}-\frac{\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{\left(1-\sqrt{5}\right)a-4x}{\sqrt{2\left(5+\sqrt{5}\right)}a}\right)}{a^8}+\frac{\sqrt{\frac{1}{2}\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(\left(1+\sqrt{5}\right)a-4x\right)}{2a}\right)}{5a^8}+\frac{\log(a+x)}{5a^8}$$

Mathematica [A] time = 0.129991, size = 175, normalized size = 0.83

$$\frac{-\frac{20a^3}{x^3}-3\left(1+\sqrt{5}\right)\log\left(a^2+\frac{1}{2}\left(\sqrt{5}-1\right)ax+x^2\right)+3\left(\sqrt{5}-1\right)\log\left(a^2-\frac{1}{2}\left(1+\sqrt{5}\right)ax+x^2\right)+12\log(a+x)+6\sqrt{10-2}}{60a^8}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(a^5 + x^5)),x]
```

```
[Out] ((-20*a^3)/x^3 + 6*Sqrt[10 - 2*Sqrt[5]]*ArcTan[((-1 + Sqrt[5])*a + 4*x)/(Sq
rt[2*(5 + Sqrt[5])]*a)] - 6*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(-((1 + Sqrt[5])*a
) + 4*x)/(Sqrt[10 - 2*Sqrt[5]]*a)] + 12*Log[a + x] - 3*(1 + Sqrt[5])*Log[a^
2 + ((-1 + Sqrt[5])*a*x)/2 + x^2] + 3*(-1 + Sqrt[5])*Log[a^2 - ((1 + Sqrt[5
])*a*x)/2 + x^2])/(60*a^8)
```

Maple [C] time = 0.008, size = 109, normalized size = 0.5

$$\frac{1}{5\,a^8}\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-a\_Z^3+a^2\_Z^2-a^3\_Z+a^4\right)}\frac{\left(-\_R^3+2\_R^2a-3\_R\,a^2-a^3\right)\ln\left(x-\_R\right)}{4\_R^3-3\_R^2a+2\_R\,a^2-a^3}-\frac{1}{3\,a^5x^3}+\frac{\ln\left(a+x\right)}{5\,a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a^5+x^5),x)
```

```
[Out] 1/5/a^8*sum((-_R^3+2*_R^2*a-3*_R*a^2-a^3)/(4*_R^3-3*_R^2*a+2*_R*a^2-a^3)*ln
(x-_R),_R=RootOf(_Z^4-_Z^3*a+_Z^2*a^2-_Z*a^3+a^4))-1/3/a^5/x^3+1/5*ln(a+x)/
a^8
```

Maxima [A] time = 1.43015, size = 373, normalized size = 1.77

$$\frac{\frac{\log\left(x+\left(a^5\right)^{\frac{1}{5}}\right)}{\left(a^5\right)^{\frac{3}{5}}}-\frac{\sqrt{5}\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}+1\right)-4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}}\right)}{\left(a^5\right)^{\frac{3}{5}}\sqrt{2\sqrt{5}-10}}+\frac{\sqrt{5}\log\left(\frac{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x-\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}{\left(a^5\right)^{\frac{1}{5}}\left(\sqrt{5}-1\right)+4x+\left(a^5\right)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}}\right)}{\left(a^5\right)^{\frac{3}{5}}\sqrt{-2\sqrt{5}-10}}+\frac{\log\left(-\left(a^5\right)^{\frac{1}{5}}x\left(\sqrt{5}+1\right)+2x^2+2\left(a^5\right)^{\frac{2}{5}}\right)}{\left(a^5\right)^{\frac{3}{5}}\left(\sqrt{5}+1\right)}-\frac{\log\left(\left(\cdots\right.\right.}{\cdots}}{5\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] 1/5*(log(x + (a^5)^(1/5))/(a^5)^(3/5) - sqrt(5)*log(((a^5)^(1/5)*(sqrt(5) +
 1) - 4*x + (a^5)^(1/5)*sqrt(2*sqrt(5) - 10))/((a^5)^(1/5)*(sqrt(5) + 1) -
4*x - (a^5)^(1/5)*sqrt(2*sqrt(5) - 10)))/((a^5)^(3/5)*sqrt(2*sqrt(5) - 10))
 + sqrt(5)*log(((a^5)^(1/5)*(sqrt(5) - 1) + 4*x - (a^5)^(1/5)*sqrt(-2*sqrt(
5) - 10))/((a^5)^(1/5)*(sqrt(5) - 1) + 4*x + (a^5)^(1/5)*sqrt(-2*sqrt(5) -
10)))/((a^5)^(3/5)*sqrt(-2*sqrt(5) - 10)) + log(-(a^5)^(1/5)*x*(sqrt(5) + 1
) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(3/5)*(sqrt(5) + 1)) - log((a^5)^(1/5)*x*
(sqrt(5) - 1) + 2*x^2 + 2*(a^5)^(2/5))/((a^5)^(3/5)*(sqrt(5) - 1)))/a^5 - 1
/3/(a^5*x^3)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [A] time = 0.383102, size = 51, normalized size = 0.24

$$-\frac{1}{3a^5x^3}+\frac{\frac{\log(a+x)}{5}+\operatorname{RootSum}\left(625t^4+125t^3+25t^2+5t+1,\left(t\mapsto t\log\left(125t^3a+x\right)\right)\right)}{a^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a**5+x**5),x)
```

```
[Out] -1/(3*a**5*x**3) + (log(a + x)/5 + RootSum(625*_t**4 + 125*_t**3 + 25*_t**2
 + 5*_t + 1, Lambda(_t, _t*log(125*_t**3*a + x))))/a**8
```

Giac [A] time = 1.07923, size = 250, normalized size = 1.18

$$\frac{\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{a(\sqrt{5}-1)+4x}{a\sqrt{2\sqrt{5}+10}}\right)}{10a^8}-\frac{\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{a(\sqrt{5}+1)-4x}{a\sqrt{-2\sqrt{5}+10}}\right)}{10a^8}+\frac{\sqrt{5}\log\left(a^2-\frac{1}{2}\left(\sqrt{5}a+a\right)x+x^2\right)}{20a^8}-\frac{\sqrt{5}\log\left(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a^5+x^5),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(-2*sqrt(5) + 10)*arctan((a*(sqrt(5) - 1) + 4*x)/(a*sqrt(2*sqrt(5)
 + 10)))/a^8 - 1/10*sqrt(2*sqrt(5) + 10)*arctan(-(a*(sqrt(5) + 1) - 4*x)/(a
*sqrt(-2*sqrt(5) + 10)))/a^8 + 1/20*sqrt(5)*log(a^2 - 1/2*(sqrt(5)*a + a)*x
 + x^2)/a^8 - 1/20*sqrt(5)*log(a^2 + 1/2*(sqrt(5)*a - a)*x + x^2)/a^8 - 1/2
0*log(abs(a^4 - a^3*x + a^2*x^2 - a*x^3 + x^4))/a^8 + 1/5*log(abs(a + x))/a
^8 - 1/3/(a^5*x^3)
```

3.145 $\int \frac{x^{-m}}{a^5+x^5}\,dx$

Optimal. Leaf size=46

$$\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1-m}{5},\frac{6-m}{5},-\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

```
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/5, (6 - m)/5, -(x^5/a^5)])/(a^5*(1
- m))
```

Rubi [A] time = 0.0095126, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {364}

$$\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1-m}{5},\frac{6-m}{5},-\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^m*(a^5 + x^5)),x]
```

```
[Out] (x^(1 - m)*Hypergeometric2F1[1, (1 - m)/5, (6 - m)/5, -(x^5/a^5)])/(a^5*(1
- m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{x^{-m}}{a^5+x^5}\,dx = \frac{x^{1-m}\,{}_2F_1\left(1,\frac{1-m}{5};\frac{6-m}{5};-\frac{x^5}{a^5}\right)}{a^5(1-m)}$$

Mathematica [A] time = 0.0093539, size = 45, normalized size = 0.98

$$-\frac{x^{1-m}\text{Hypergeometric2F1}\left(1,\frac{1}{5}-\frac{m}{5},\frac{6}{5}-\frac{m}{5},-\frac{x^5}{a^5}\right)}{a^5(m-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^m*(a^5 + x^5)),x]
```

```
[Out] -((x^(1 - m)*Hypergeometric2F1[1, 1/5 - m/5, 6/5 - m/5, -(x^5/a^5)])/(a^5*(
-1 + m)))
```

Maple [F] time = 0.043, size = 0, normalized size = 0.

$$\int \frac{1}{x^{m} \left(a^{5}+x^{5}\right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^m)/(a^5+x^5),x)
```

```
[Out] int(1/(x^m)/(a^5+x^5),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a^{5}+x^{5}\right) x^{m}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^5+x^5),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^5 + x^5)*x^m), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(a^{5}+x^{5}\right) x^{m}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^5+x^5),x, algorithm="fricas")
```

```
[Out] integral(1/((a^5 + x^5)*x^m), x)
```

Sympy [C] time = 34.013, size = 92, normalized size = 2.

$$-\frac{mxx^{-m}\Phi\left(\frac{x^{5}e^{i\pi}}{a^{5}},1,\frac{1}{5}-\frac{m}{5}\right)\Gamma\left(\frac{1}{5}-\frac{m}{5}\right)}{25a^{5}\Gamma\left(\frac{6}{5}-\frac{m}{5}\right)}+\frac{xx^{-m}\Phi\left(\frac{x^{5}e^{i\pi}}{a^{5}},1,\frac{1}{5}-\frac{m}{5}\right)\Gamma\left(\frac{1}{5}-\frac{m}{5}\right)}{25a^{5}\Gamma\left(\frac{6}{5}-\frac{m}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**m)/(a**5+x**5),x)
```

```
[Out] -m*x*x**(-m)*lerchphi(x**5*exp_polar(I*pi)/a**5, 1, 1/5 - m/5)*gamma(1/5 -
m/5)/(25*a**5*gamma(6/5 - m/5)) + x*x**(-m)*lerchphi(x**5*exp_polar(I*pi)/a
**5, 1, 1/5 - m/5)*gamma(1/5 - m/5)/(25*a**5*gamma(6/5 - m/5))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a^5 + x^5)x^m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^m)/(a^5+x^5),x, algorithm="giac")
```

```
[Out] integrate(1/((a^5 + x^5)*x^m), x)
```

3.146 $\int \frac{1+x^4}{1+x^6} \, dx$

Optimal. Leaf size=35

$$-\frac{1}{3}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{2}{3}\tan^{-1}(x)+\frac{1}{3}\tan^{-1}\left(2x+\sqrt{3}\right)$$

```
[Out] -ArcTan[Sqrt[3] - 2*x]/3 + (2*ArcTan[x])/3 + ArcTan[Sqrt[3] + 2*x]/3
```

Rubi [A] time = 0.425398, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {1876, 209, 634, 618, 204, 628, 203, 295}

$$-\frac{1}{3}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{2}{3}\tan^{-1}(x)+\frac{1}{3}\tan^{-1}\left(2x+\sqrt{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(1 + x^6),x]
```

```
[Out] -ArcTan[Sqrt[3] - 2*x]/3 + (2*ArcTan[x])/3 + ArcTan[Sqrt[3] + 2*x]/3
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{1+x^6}\,dx &= \int \left(\frac{1}{1+x^6}+\frac{x^4}{1+x^6}\right)dx\\
&= \int \frac{1}{1+x^6}\,dx+\int \frac{x^4}{1+x^6}\,dx\\
&= \frac{1}{3}\int \frac{1-\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx+\frac{1}{3}\int \frac{-\frac{1}{2}+\frac{\sqrt{3}x}{2}}{1-\sqrt{3}x+x^2}\,dx+\frac{1}{3}\int \frac{-\frac{1}{2}-\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx+\frac{1}{3}\int \frac{1+\frac{\sqrt{3}x}{2}}{1+\sqrt{3}x+x^2}\,dx+\frac{2}{3}\int\\
&= \frac{2}{3}\tan^{-1}(x)+2\left(\frac{1}{12}\int \frac{1}{1-\sqrt{3}x+x^2}\,dx\right)+2\left(\frac{1}{12}\int \frac{1}{1+\sqrt{3}x+x^2}\,dx\right)\\
&= \frac{2}{3}\tan^{-1}(x)-2\left(\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,-\sqrt{3}+2x\right)\right)-2\left(\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\sqrt{3}+2x\right)\right)\\
&= -\frac{1}{3}\tan^{-1}\left(\sqrt{3}-2x\right)+\frac{2}{3}\tan^{-1}(x)+\frac{1}{3}\tan^{-1}\left(\sqrt{3}+2x\right)
\end{aligned}$$

Mathematica [A] time = 0.0069387, size = 21, normalized size = 0.6

$$\frac{2}{3}\tan^{-1}(x)-\frac{1}{3}\tan^{-1}\left(\frac{x}{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(1 + x^6),x]
```

```
[Out] (2*ArcTan[x])/3 - ArcTan[x/(-1 + x^2)]/3
```

Maple [A] time = 0.024, size = 28, normalized size = 0.8

$$\frac{2\arctan(x)}{3}+\frac{\arctan\left(2x-\sqrt{3}\right)}{3}+\frac{\arctan\left(2x+\sqrt{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^6+1),x)
```

```
[Out] 2/3*arctan(x)+1/3*arctan(2*x-3^(1/2))+1/3*arctan(2*x+3^(1/2))
```

Maxima [A] time = 1.41201, size = 36, normalized size = 1.03

$$\frac{1}{3}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{3}\arctan\left(2x-\sqrt{3}\right)+\frac{2}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^6+1),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(2*x + sqrt(3)) + 1/3*arctan(2*x - sqrt(3)) + 2/3*arctan(x)
```

Fricas [A] time = 1.99997, size = 39, normalized size = 1.11

$$\frac{1}{3}\arctan\left(x^3\right)+\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^6+1),x, algorithm="fricas")
```

```
[Out] 1/3*arctan(x^3) + arctan(x)
```

Sympy [A] time = 0.110474, size = 8, normalized size = 0.23

$$\operatorname{atan}(x)+\frac{\operatorname{atan}\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**6+1),x)
```

```
[Out] atan(x) + atan(x**3)/3
```

Giac [A] time = 1.0464, size = 36, normalized size = 1.03

$$\frac{1}{3}\arctan\left(2x+\sqrt{3}\right)+\frac{1}{3}\arctan\left(2x-\sqrt{3}\right)+\frac{2}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^6+1),x, algorithm="giac")

[Out] 1/3*arctan(2*x + sqrt(3)) + 1/3*arctan(2*x - sqrt(3)) + 2/3*arctan(x)
```

3.147 $\int \frac{1}{\left(5+3x+x^2\right)^3}\,dx$

Optimal. Leaf size=60

$$\frac{3(2x+3)}{121\left(x^2+3x+5\right)}+\frac{2x+3}{22\left(x^2+3x+5\right)^2}+\frac{12\tan^{-1}\left(\frac{2x+3}{\sqrt{11}}\right)}{121\sqrt{11}}$$

```
[Out] (3 + 2*x)/(22*(5 + 3*x + x^2)^2) + (3*(3 + 2*x))/(121*(5 + 3*x + x^2)) + (1
2*ArcTan[(3 + 2*x)/Sqrt[11]])/(121*Sqrt[11])
```

Rubi [A] time = 0.02187, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {614, 618, 204}

$$\frac{3(2x+3)}{121\left(x^2+3x+5\right)}+\frac{2x+3}{22\left(x^2+3x+5\right)^2}+\frac{12\tan^{-1}\left(\frac{2x+3}{\sqrt{11}}\right)}{121\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 3*x + x^2)^(-3),x]
```

```
[Out] (3 + 2*x)/(22*(5 + 3*x + x^2)^2) + (3*(3 + 2*x))/(121*(5 + 3*x + x^2)) + (1
2*ArcTan[(3 + 2*x)/Sqrt[11]])/(121*Sqrt[11])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(5+3x+x^2)^3}\,dx = \frac{3+2x}{22(5+3x+x^2)^2} + \frac{3}{11}\int \frac{1}{(5+3x+x^2)^2}\,dx$$
$$= \frac{3+2x}{22(5+3x+x^2)^2} + \frac{3(3+2x)}{121(5+3x+x^2)} + \frac{6}{121}\int \frac{1}{5+3x+x^2}\,dx$$
$$= \frac{3+2x}{22(5+3x+x^2)^2} + \frac{3(3+2x)}{121(5+3x+x^2)} - \frac{12}{121}\,\text{Subst}\left(\int \frac{1}{-11-x^2}\,dx, x, 3+2x\right)$$
$$= \frac{3+2x}{22(5+3x+x^2)^2} + \frac{3(3+2x)}{121(5+3x+x^2)} + \frac{12\tan^{-1}\left(\frac{3+2x}{\sqrt{11}}\right)}{121\sqrt{11}}$$

Mathematica [A] time = 0.0277894, size = 51, normalized size = 0.85

$$\frac{\frac{11(2x+3)(6x^2+18x+41)}{(x^2+3x+5)^2} + 24\sqrt{11}\tan^{-1}\left(\frac{2x+3}{\sqrt{11}}\right)}{2662}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*x + x^2)^(-3),x]
```

```
[Out] ((11*(3 + 2*x)*(41 + 18*x + 6*x^2))/(5 + 3*x + x^2)^2 + 24*Sqrt[11]*ArcTan[
(3 + 2*x)/Sqrt[11]])/2662
```

Maple [A] time = 0.003, size = 52, normalized size = 0.9

$$\frac{3+2x}{22(x^2+3x+5)^2} + \frac{9+6x}{121x^2+363x+605} + \frac{12\sqrt{11}}{1331}\arctan\left(\frac{(3+2x)\sqrt{11}}{11}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+3*x+5)^3,x)
```

```
[Out] 1/22*(3+2*x)/(x^2+3*x+5)^2+3/121*(3+2*x)/(x^2+3*x+5)+12/1331*arctan(1/11*(3
+2*x)*11^(1/2))*11^(1/2)
```

Maxima [A] time = 1.41801, size = 73, normalized size = 1.22

$$\frac{12}{1331}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}(2x+3)\right) + \frac{12x^3+54x^2+136x+123}{242(x^4+6x^3+19x^2+30x+25)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+5)^3,x, algorithm="maxima")
```

```
[Out] 12/1331*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 3)) + 1/242*(12*x^3 + 54*x^2 +
 136*x + 123)/(x^4 + 6*x^3 + 19*x^2 + 30*x + 25)
```

Fricas [A] time = 1.97968, size = 216, normalized size = 3.6

$$\frac{132\,x^3 + 24\,\sqrt{11}\left(x^4 + 6\,x^3 + 19\,x^2 + 30\,x + 25\right)\arctan\left(\frac{1}{11}\,\sqrt{11}(2\,x + 3)\right) + 594\,x^2 + 1496\,x + 1353}{2662\left(x^4 + 6\,x^3 + 19\,x^2 + 30\,x + 25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+5)^3,x, algorithm="fricas")
```

```
[Out] 1/2662*(132*x^3 + 24*sqrt(11)*(x^4 + 6*x^3 + 19*x^2 + 30*x + 25)*arctan(1/1
1*sqrt(11)*(2*x + 3)) + 594*x^2 + 1496*x + 1353)/(x^4 + 6*x^3 + 19*x^2 + 30
*x + 25)
```

Sympy [A] time = 0.152225, size = 63, normalized size = 1.05

$$\frac{12x^3 + 54x^2 + 136x + 123}{242x^4 + 1452x^3 + 4598x^2 + 7260x + 6050} + \frac{12\sqrt{11}\operatorname{atan}\left(\frac{2\sqrt{11}x}{11} + \frac{3\sqrt{11}}{11}\right)}{1331}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+3*x+5)**3,x)
```

```
[Out] (12*x**3 + 54*x**2 + 136*x + 123)/(242*x**4 + 1452*x**3 + 4598*x**2 + 7260*
x + 6050) + 12*sqrt(11)*atan(2*sqrt(11)*x/11 + 3*sqrt(11)/11)/1331
```

Giac [A] time = 1.06634, size = 59, normalized size = 0.98

$$\frac{12}{1331}\,\sqrt{11}\arctan\left(\frac{1}{11}\,\sqrt{11}(2\,x + 3)\right) + \frac{12\,x^3 + 54\,x^2 + 136\,x + 123}{242\left(x^2 + 3\,x + 5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+5)^3,x, algorithm="giac")
```

```
[Out] 12/1331*sqrt(11)*arctan(1/11*sqrt(11)*(2*x + 3)) + 1/242*(12*x^3 + 54*x^2 +
 136*x + 123)/(x^2 + 3*x + 5)^2
```

3.148 $\int \frac{1+x^2+x^4}{\left(1+x^2\right)^4} \, dx$

Optimal. Leaf size=43

$$\frac{7x}{16\left(x^2+1\right)}-\frac{x}{24\left(x^2+1\right)^2}+\frac{x}{6\left(x^2+1\right)^3}+\frac{7}{16}\tan^{-1}(x)$$

```
[Out] x/(6*(1 + x^2)^3) - x/(24*(1 + x^2)^2) + (7*x)/(16*(1 + x^2)) + (7*ArcTan[x
])/16
```

Rubi [A] time = 0.0135686, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1157, 385, 199, 203}

$$\frac{7x}{16\left(x^2+1\right)}-\frac{x}{24\left(x^2+1\right)^2}+\frac{x}{6\left(x^2+1\right)^3}+\frac{7}{16}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^4)/(1 + x^2)^4,x]
```

```
[Out] x/(6*(1 + x^2)^3) - x/(24*(1 + x^2)^2) + (7*x)/(16*(1 + x^2)) + (7*ArcTan[x
])/16
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2+x^4}{\left(1+x^2\right)^4}\,dx &= \frac{x}{6\left(1+x^2\right)^3}-\frac{1}{6}\int \frac{-5-6x^2}{\left(1+x^2\right)^3}\,dx\\ &= \frac{x}{6\left(1+x^2\right)^3}-\frac{x}{24\left(1+x^2\right)^2}+\frac{7}{8}\int \frac{1}{\left(1+x^2\right)^2}\,dx\\ &= \frac{x}{6\left(1+x^2\right)^3}-\frac{x}{24\left(1+x^2\right)^2}+\frac{7x}{16\left(1+x^2\right)}+\frac{7}{16}\int \frac{1}{1+x^2}\,dx\\ &= \frac{x}{6\left(1+x^2\right)^3}-\frac{x}{24\left(1+x^2\right)^2}+\frac{7x}{16\left(1+x^2\right)}+\frac{7}{16}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0119681, size = 30, normalized size = 0.7

$$\frac{1}{48}\left(\frac{x\left(21x^4+40x^2+27\right)}{\left(x^2+1\right)^3}+21\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^4)/(1 + x^2)^4,x]
```

```
[Out] ((x*(27 + 40*x^2 + 21*x^4))/(1 + x^2)^3 + 21*ArcTan[x])/48
```

Maple [A] time = 0.006, size = 28, normalized size = 0.7

$$\frac{1}{\left(x^2+1\right)^3}\left(\frac{7\,x^5}{16}+\frac{5\,x^3}{6}+\frac{9\,x}{16}\right)+\frac{7\arctan(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)/(x^2+1)^4,x)
```

```
[Out] (7/16*x^5+5/6*x^3+9/16*x)/(x^2+1)^3+7/16*arctan(x)
```

Maxima [A] time = 1.40947, size = 51, normalized size = 1.19

$$\frac{21\,x^5+40\,x^3+27\,x}{48\left(x^6+3\,x^4+3\,x^2+1\right)}+\frac{7}{16}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(21*x^5 + 40*x^3 + 27*x)/(x^6 + 3*x^4 + 3*x^2 + 1) + 7/16*arctan(x)
```

Fricas [A] time = 2.04415, size = 132, normalized size = 3.07

$$\frac{21\,x^5+40\,x^3+21\,\left(x^6+3\,x^4+3\,x^2+1\right)\arctan\left(x\right)+27\,x}{48\,\left(x^6+3\,x^4+3\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(21*x^5 + 40*x^3 + 21*(x^6 + 3*x^4 + 3*x^2 + 1)*arctan(x) + 27*x)/(x^6
 + 3*x^4 + 3*x^2 + 1)
```

Sympy [A] time = 0.13525, size = 36, normalized size = 0.84

$$\frac{21x^5+40x^3+27x}{48x^6+144x^4+144x^2+48}+\frac{7\operatorname{atan}\left(x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)/(x**2+1)**4,x)
```

```
[Out] (21*x**5 + 40*x**3 + 27*x)/(48*x**6 + 144*x**4 + 144*x**2 + 48) + 7*atan(x)
/16
```

Giac [A] time = 1.05616, size = 38, normalized size = 0.88

$$\frac{21\,x^5+40\,x^3+27\,x}{48\,\left(x^2+1\right)^3}+\frac{7}{16}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)^4,x, algorithm="giac")
```

```
[Out] 1/48*(21*x^5 + 40*x^3 + 27*x)/(x^2 + 1)^3 + 7/16*arctan(x)
```

3.149 $\int \frac{B+Ax}{\left(c+2bx+ax^2\right)^2}\,dx$

Optimal. Leaf size=90

$$-\frac{-x(Ab-aB)-Ac+bB}{2\left(b^2-ac\right)\left(ax^2+2bx+c\right)}-\frac{(Ab-aB)\tanh^{-1}\left(\frac{ax+b}{\sqrt{b^2-ac}}\right)}{2\left(b^2-ac\right)^{3/2}}$$

```
[Out] -(b*B - A*c - (A*b - a*B)*x)/(2*(b^2 - a*c)*(c + 2*b*x + a*x^2)) - ((A*b -
a*B)*ArcTanh[(b + a*x)/Sqrt[b^2 - a*c]])/(2*(b^2 - a*c)^(3/2))
```

Rubi [A] time = 0.0709111, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {638, 618, 206}

$$-\frac{-x(Ab-aB)-Ac+bB}{2\left(b^2-ac\right)\left(ax^2+2bx+c\right)}-\frac{(Ab-aB)\tanh^{-1}\left(\frac{ax+b}{\sqrt{b^2-ac}}\right)}{2\left(b^2-ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(B + A*x)/(c + 2*b*x + a*x^2)^2,x]
```

```
[Out] -(b*B - A*c - (A*b - a*B)*x)/(2*(b^2 - a*c)*(c + 2*b*x + a*x^2)) - ((A*b -
a*B)*ArcTanh[(b + a*x)/Sqrt[b^2 - a*c]])/(2*(b^2 - a*c)^(3/2))
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{B+Ax}{\left(c+2bx+ax^2\right)^2}\,dx = -\frac{bB-Ac-(Ab-aB)x}{2\left(b^2-ac\right)\left(c+2bx+ax^2\right)} + \frac{(Ab-aB)\int \frac{1}{c+2bx+ax^2}\,dx}{2\left(b^2-ac\right)}$$

$$= -\frac{bB-Ac-(Ab-aB)x}{2\left(b^2-ac\right)\left(c+2bx+ax^2\right)} - \frac{(Ab-aB)\,\text{Subst}\left(\int \frac{1}{4\left(b^2-ac\right)-x^2}\,dx, x, 2b+2ax\right)}{b^2-ac}$$

$$= -\frac{bB-Ac-(Ab-aB)x}{2\left(b^2-ac\right)\left(c+2bx+ax^2\right)} - \frac{(Ab-aB)\tanh^{-1}\left(\frac{b+ax}{\sqrt{b^2-ac}}\right)}{2\left(b^2-ac\right)^{3/2}}$$

Mathematica [A] time = 0.0867668, size = 88, normalized size = 0.98

$$\frac{\frac{(Ab-aB)\tan^{-1}\left(\frac{ax+b}{\sqrt{ac-b^2}}\right)}{\sqrt{ac-b^2}} + \frac{-aBx+Abx+Ac-bB}{x(ax+2b)+c}}{2\left(b^2-ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B + A*x)/(c + 2*b*x + a*x^2)^2,x]
```

```
[Out] ((-(b*B) + A*c + A*b*x - a*B*x)/(c + x*(2*b + a*x)) + ((A*b - a*B)*ArcTan[(
b + a*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(2*(b^2 - a*c))
```

Maple [A] time = 0.007, size = 146, normalized size = 1.6

$$\frac{(-2\,Ab+2\,Ba)\,x+2\,bB-2\,Ac}{\left(4\,ac-4\,b^2\right)\left(ax^2+2\,bx+c\right)} - 2\,\frac{Ab}{\left(4\,ac-4\,b^2\right)\sqrt{ac-b^2}}\arctan\left(1/2\,\frac{2\,ax+2\,b}{\sqrt{ac-b^2}}\right) + 2\,\frac{Ba}{\left(4\,ac-4\,b^2\right)\sqrt{ac-b^2}}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A*x+B)/(a*x^2+2*b*x+c)^2,x)
```

```
[Out] ((-2*A*b+2*B*a)*x+2*b*B-2*A*c)/(4*a*c-4*b^2)/(a*x^2+2*b*x+c)-2/(4*a*c-4*b^2
)/(a*c-b^2)^(1/2)*arctan(1/2*(2*a*x+2*b)/(a*c-b^2)^(1/2))*A*b+2/(4*a*c-4*b^
2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*a*x+2*b)/(a*c-b^2)^(1/2))*B*a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.98165, size = 932, normalized size = 10.36

$$\left[-\frac{2\,Bb^3+2\,Aac^2-\left(\left(Ba^2-Aab\right)x^2+(Ba-Ab)c+2\left(Bab-Ab^2\right)x\right)\sqrt{b^2-ac}\log\left(\frac{a^2x^2+2\,abx+2\,b^2-ac+2\,\sqrt{b^2-ac}(ax+b)}{ax^2+2\,bx+c}\right)-2\left(Ba\right.}{4\left(b^4c-2\,ab^2c^2+a^2c^3+\left(ab^4-2\,a^2b^2c+a^3c^2\right)x^2+2\left(b^5-2\,ab^3c+a^2bc^2\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*b^3 + 2*A*a*c^2 - ((B*a^2 - A*a*b)*x^2 + (B*a - A*b)*c + 2*(B*a*
b - A*b^2)*x)*sqrt(b^2 - a*c)*log((a^2*x^2 + 2*a*b*x + 2*b^2 - a*c + 2*sqrt
(b^2 - a*c)*(a*x + b))/(a*x^2 + 2*b*x + c)) - 2*(B*a*b + A*b^2)*c + 2*(B*a*
b^2 - A*b^3 - (B*a^2 - A*a*b)*c)*x)/(b^4*c - 2*a*b^2*c^2 + a^2*c^3 + (a*b^4
 - 2*a^2*b^2*c + a^3*c^2)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2*b*c^2)*x), -1/2*(B
*b^3 + A*a*c^2 - ((B*a^2 - A*a*b)*x^2 + (B*a - A*b)*c + 2*(B*a*b - A*b^2)*x
)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(a*x + b)/(b^2 - a*c)) - (B*a*b
 + A*b^2)*c + (B*a*b^2 - A*b^3 - (B*a^2 - A*a*b)*c)*x)/(b^4*c - 2*a*b^2*c^2
 + a^2*c^3 + (a*b^4 - 2*a^2*b^2*c + a^3*c^2)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2
*b*c^2)*x)]
```

Sympy [B] time = 1.11121, size = 323, normalized size = 3.59

$$-\frac{\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}\,(-Ab+Ba)\log\left(x+\frac{-Ab^2+Bab-a^2c^2\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(-Ab+Ba)+2ab^2c\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(-Ab+Ba)-b^4\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(-Ab+Ba)}{-Aab+Ba^2}\right)}{4}+\frac{\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(a*x**2+2*b*x+c)**2,x)
```

```
[Out] -sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a)*log(x + (-A*b**2 + B*a*b - a**2*c**2
*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a) + 2*a*b**2*c*sqrt(-1/(a*c - b**2)**3
)*(-A*b + B*a) - b**4*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a))/(-A*a*b + B*a*
*2))/4 + sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a)*log(x + (-A*b**2 + B*a*b + a
**2*c**2*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a) - 2*a*b**2*c*sqrt(-1/(a*c -
b**2)**3)*(-A*b + B*a) + b**4*sqrt(-1/(a*c - b**2)**3)*(-A*b + B*a))/(-A*a*
b + B*a**2))/4 + (-A*c + B*b + x*(-A*b + B*a))/(2*a*c**2 - 2*b**2*c + x**2*
(2*a**2*c - 2*a*b**2) + x*(4*a*b*c - 4*b**3))
```

Giac [A] time = 1.06969, size = 124, normalized size = 1.38

$$-\frac{(Ba-Ab)\arctan\left(\frac{ax+b}{\sqrt{-b^2+ac}}\right)}{2\left(b^2-ac\right)\sqrt{-b^2+ac}}-\frac{Bax-Abx+Bb-Ac}{2\left(ax^2+2\,bx+c\right)\left(b^2-ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(a*x^2+2*b*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a - A*b)*arctan((a*x + b)/sqrt(-b^2 + a*c))/((b^2 - a*c)*sqrt(-b^2
+ a*c)) - 1/2*(B*a*x - A*b*x + B*b - A*c)/((a*x^2 + 2*b*x + c)*(b^2 - a*c))
```

3.150 $\int \frac{-41+55x-27x^2+5x^3}{\left(5-4x+x^2\right)^2} \, dx$

Optimal. Leaf size=38

$$\frac{1-x}{x^2-4x+5}+\frac{5}{2}\log\left(x^2-4x+5\right)-2\tan^{-1}(2-x)$$

```
[Out] (1 - x)/(5 - 4*x + x^2) - 2*ArcTan[2 - x] + (5*Log[5 - 4*x + x^2])/2
```

Rubi [A] time = 0.0258385, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1660, 634, 618, 204, 628}

$$\frac{1-x}{x^2-4x+5}+\frac{5}{2}\log\left(x^2-4x+5\right)-2\tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(-41 + 55*x - 27*x^2 + 5*x^3)/(5 - 4*x + x^2)^2,x]
```

```
[Out] (1 - x)/(5 - 4*x + x^2) - 2*ArcTan[2 - x] + (5*Log[5 - 4*x + x^2])/2
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-41+55x-27x^2+5x^3}{\left(5-4x+x^2\right)^2}\,dx &= \frac{1-x}{5-4x+x^2}+\frac{1}{4}\int \frac{-32+20x}{5-4x+x^2}\,dx\\
&= \frac{1-x}{5-4x+x^2}+2\int \frac{1}{5-4x+x^2}\,dx+\frac{5}{2}\int \frac{-4+2x}{5-4x+x^2}\,dx\\
&= \frac{1-x}{5-4x+x^2}+\frac{5}{2}\log\left(5-4x+x^2\right)-4\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx,x,-4+2x\right)\\
&= \frac{1-x}{5-4x+x^2}-2\tan^{-1}(2-x)+\frac{5}{2}\log\left(5-4x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0131029, size = 38, normalized size = 1.

$$\frac{1-x}{x^2-4x+5}+\frac{5}{2}\log\left(x^2-4x+5\right)-2\tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-41 + 55*x - 27*x^2 + 5*x^3)/(5 - 4*x + x^2)^2,x]

[Out] (1 - x)/(5 - 4*x + x^2) - 2*ArcTan[2 - x] + (5*Log[5 - 4*x + x^2])/2
```

Maple [A] time = 0.006, size = 35, normalized size = 0.9

$$\frac{1-x}{x^2-4x+5}+2\arctan\left(-2+x\right)+\frac{5\ln\left(x^2-4x+5\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x)

[Out] (1-x)/(x^2-4*x+5)+2*arctan(-2+x)+5/2*ln(x^2-4*x+5)
```

Maxima [A] time = 1.39753, size = 45, normalized size = 1.18

$$-\frac{x-1}{x^2-4x+5}+2\arctan\left(x-2\right)+\frac{5}{2}\log\left(x^2-4x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x, algorithm="maxima")

[Out] -(x - 1)/(x^2 - 4*x + 5) + 2*arctan(x - 2) + 5/2*log(x^2 - 4*x + 5)
```

Fricas [A] time = 1.98051, size = 140, normalized size = 3.68

$$\frac{4\left(x^{2}-4x+5\right)\arctan\left(x-2\right)+5\left(x^{2}-4x+5\right)\log\left(x^{2}-4x+5\right)-2x+2}{2\left(x^{2}-4x+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*(x^2 - 4*x + 5)*arctan(x - 2) + 5*(x^2 - 4*x + 5)*log(x^2 - 4*x + 5)
 - 2*x + 2)/(x^2 - 4*x + 5)
```

Sympy [A] time = 0.128605, size = 31, normalized size = 0.82

$$-\frac{x-1}{x^{2}-4x+5}+\frac{5\log\left(x^{2}-4x+5\right)}{2}+2\operatorname{atan}\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**3-27*x**2+55*x-41)/(x**2-4*x+5)**2,x)
```

```
[Out] -(x - 1)/(x**2 - 4*x + 5) + 5*log(x**2 - 4*x + 5)/2 + 2*atan(x - 2)
```

Giac [A] time = 1.06078, size = 45, normalized size = 1.18

$$-\frac{x-1}{x^{2}-4x+5}+2\arctan\left(x-2\right)+\frac{5}{2}\log\left(x^{2}-4x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^3-27*x^2+55*x-41)/(x^2-4*x+5)^2,x, algorithm="giac")
```

```
[Out] -(x - 1)/(x^2 - 4*x + 5) + 2*arctan(x - 2) + 5/2*log(x^2 - 4*x + 5)
```

3.151 $\int \frac{1}{(-1+x^3)^2} \, dx$

Optimal. Leaf size=57

$$\frac{x}{3(1-x^3)}+\frac{1}{9}\log\left(x^2+x+1\right)-\frac{2}{9}\log(1-x)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] x/(3*(1 - x^3)) + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (2*Log[1 - x]
)/9 + Log[1 + x + x^2]/9
```

Rubi [A] time = 0.0244686, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {199, 200, 31, 634, 618, 204, 628}

$$\frac{x}{3(1-x^3)}+\frac{1}{9}\log\left(x^2+x+1\right)-\frac{2}{9}\log(1-x)+\frac{2\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)^(-2),x]
```

```
[Out] x/(3*(1 - x^3)) + (2*ArcTan[(1 + 2*x)/Sqrt[3]])/(3*Sqrt[3]) - (2*Log[1 - x]
)/9 + Log[1 + x + x^2]/9
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-1+x^3\right)^2}\,dx &= \frac{x}{3\left(1-x^3\right)}-\frac{2}{3}\int \frac{1}{-1+x^3}\,dx\\ &= \frac{x}{3\left(1-x^3\right)}-\frac{2}{9}\int \frac{1}{-1+x}\,dx-\frac{2}{9}\int \frac{-2-x}{1+x+x^2}\,dx\\ &= \frac{x}{3\left(1-x^3\right)}-\frac{2}{9}\log(1-x)+\frac{1}{9}\int \frac{1+2x}{1+x+x^2}\,dx+\frac{1}{3}\int \frac{1}{1+x+x^2}\,dx\\ &= \frac{x}{3\left(1-x^3\right)}-\frac{2}{9}\log(1-x)+\frac{1}{9}\log\left(1+x+x^2\right)-\frac{2}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\ &= \frac{x}{3\left(1-x^3\right)}+\frac{2\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{2}{9}\log(1-x)+\frac{1}{9}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0231122, size = 49, normalized size = 0.86

$$\frac{1}{9}\left(-\frac{3x}{x^3-1}+\log\left(x^2+x+1\right)-2\log(1-x)+2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)^(-2),x]
```

```
[Out] ((-3*x)/(-1 + x^3) + 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] - 2*Log[1 - x] + L
og[1 + x + x^2])/9
```

Maple [A] time = 0.01, size = 53, normalized size = 0.9

$$-\frac{1}{-9+9x}-\frac{2\ln(-1+x)}{9}+\frac{-1+x}{9x^2+9x+9}+\frac{\ln\left(x^2+x+1\right)}{9}+\frac{2\sqrt{3}}{9}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-1)^2,x)
```

```
[Out] -1/9/(-1+x)-2/9*ln(-1+x)+1/9*(-1+x)/(x^2+x+1)+1/9*ln(x^2+x+1)+2/9*arctan(1/
3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.47755, size = 57, normalized size = 1.

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{x}{3\left(x^3-1\right)}+\frac{1}{9}\log\left(x^2+x+1\right)-\frac{2}{9}\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)^2,x, algorithm="maxima")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*x/(x^3 - 1) + 1/9*log(x^2 +
 x + 1) - 2/9*log(x - 1)
```

Fricas [A] time = 2.09021, size = 171, normalized size = 3.

$$\frac{2\sqrt{3}\left(x^3-1\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\left(x^3-1\right)\log\left(x^2+x+1\right)-2\left(x^3-1\right)\log\left(x-1\right)-3x}{9\left(x^3-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)^2,x, algorithm="fricas")
```

```
[Out] 1/9*(2*sqrt(3)*(x^3 - 1)*arctan(1/3*sqrt(3)*(2*x + 1)) + (x^3 - 1)*log(x^2
+ x + 1) - 2*(x^3 - 1)*log(x - 1) - 3*x)/(x^3 - 1)
```

Sympy [A] time = 0.141168, size = 53, normalized size = 0.93

$$-\frac{x}{3x^3-3}-\frac{2\log\left(x-1\right)}{9}+\frac{\log\left(x^2+x+1\right)}{9}+\frac{2\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-1)**2,x)
```

```
[Out] -x/(3*x**3 - 3) - 2*log(x - 1)/9 + log(x**2 + x + 1)/9 + 2*sqrt(3)*atan(2*s
qrt(3)*x/3 + sqrt(3)/3)/9
```

Giac [A] time = 1.04441, size = 58, normalized size = 1.02

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{x}{3\left(x^3-1\right)}+\frac{1}{9}\log\left(x^2+x+1\right)-\frac{2}{9}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)^2,x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*x/(x^3 - 1) + 1/9*log(x^2 +
 x + 1) - 2/9*log(abs(x - 1))
```

3.152 $\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx$

Optimal. Leaf size=36

$$-\frac{25x}{8\left(x^2+1\right)}-\frac{7x}{4\left(x^2+1\right)^2}-\frac{4}{x}-\frac{57}{8}\tan^{-1}(x)$$

```
[Out] -4/x - (7*x)/(4*(1 + x^2)^2) - (25*x)/(8*(1 + x^2)) - (57*ArcTan[x])/8
```

Rubi [A] time = 0.0232351, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1260, 456, 453, 203}

$$-\frac{25x}{8\left(x^2+1\right)}-\frac{7x}{4\left(x^2+1\right)^2}-\frac{4}{x}-\frac{57}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

```
[Out] -4/x - (7*x)/(4*(1 + x^2)^2) - (25*x)/(8*(1 + x^2)) - (57*ArcTan[x])/8
```

Rule 1260

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Sym
bol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^
(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m
*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(-(m/2) + 1)*e^(2*p)*(
q + 1)*(a + c*x^4)^p - ((c*d^2 + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x
^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, c, d, e}, x] && IGtQ[p, 0] && I
LtQ[q, -1] && ILtQ[m/2, 0]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx &= -\frac{7x}{4\left(1+x^2\right)^2}-\frac{1}{4}\int \frac{-16+9x^2}{x^2\left(1+x^2\right)^2}\,dx\\
&= -\frac{7x}{4\left(1+x^2\right)^2}-\frac{25x}{8\left(1+x^2\right)}+\frac{1}{8}\int \frac{32-25x^2}{x^2\left(1+x^2\right)}\,dx\\
&= -\frac{4}{x}-\frac{7x}{4\left(1+x^2\right)^2}-\frac{25x}{8\left(1+x^2\right)}-\frac{57}{8}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{4}{x}-\frac{7x}{4\left(1+x^2\right)^2}-\frac{25x}{8\left(1+x^2\right)}-\frac{57}{8}\tan^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.0160468, size = 33, normalized size = 0.92

$$-\frac{57x^4+103x^2+32}{8x\left(x^2+1\right)^2}-\frac{57}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

```
[Out] -(32 + 103*x^2 + 57*x^4)/(8*x*(1 + x^2)^2) - (57*ArcTan[x])/8
```

Maple [A] time = 0.009, size = 29, normalized size = 0.8

$$-\frac{1}{\left(x^2+1\right)^2}\left(\frac{25\,x^3}{8}+\frac{39\,x}{8}\right)-\frac{57\arctan(x)}{8}-4\,x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4+4)/x^2/(x^2+1)^3,x)
```

```
[Out] -(25/8*x^3+39/8*x)/(x^2+1)^2-57/8*arctan(x)-4/x
```

Maxima [A] time = 1.44727, size = 42, normalized size = 1.17

$$-\frac{57\,x^4+103\,x^2+32}{8\left(x^5+2\,x^3+x\right)}-\frac{57}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(57*x^4 + 103*x^2 + 32)/(x^5 + 2*x^3 + x) - 57/8*arctan(x)
```

Fricas [A] time = 1.94532, size = 109, normalized size = 3.03

$$-\frac{57 x^4 + 103 x^2 + 57 \left(x^5 + 2 x^3 + x\right) \arctan(x) + 32}{8 \left(x^5 + 2 x^3 + x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(57*x^4 + 103*x^2 + 57*(x^5 + 2*x^3 + x)*arctan(x) + 32)/(x^5 + 2*x^3
+ x)
```

Sympy [A] time = 0.133365, size = 32, normalized size = 0.89

$$-\frac{57x^4 + 103x^2 + 32}{8x^5 + 16x^3 + 8x} - \frac{57 \operatorname{atan}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4+4)/x**2/(x**2+1)**3,x)
```

```
[Out] -(57*x**4 + 103*x**2 + 32)/(8*x**5 + 16*x**3 + 8*x) - 57*atan(x)/8
```

Giac [A] time = 1.05531, size = 38, normalized size = 1.06

$$-\frac{25 x^3 + 39 x}{8 \left(x^2 + 1\right)^2} - \frac{4}{x} - \frac{57}{8} \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -1/8*(25*x^3 + 39*x)/(x^2 + 1)^2 - 4/x - 57/8*arctan(x)
```

3.153 $\int \frac{x}{1+x^6} \, dx$

Optimal. Leaf size=49

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{12}\log\left(x^4-x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 + x^2]/6 - Log[1 - x^2 + x
^4]/12
```

Rubi [A] time = 0.0369044, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {275, 200, 31, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{12}\log\left(x^4-x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^6),x]
```

```
[Out] -ArcTan[(1 - 2*x^2)/Sqrt[3]]/(2*Sqrt[3]) + Log[1 + x^2]/6 - Log[1 - x^2 + x
^4]/12
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{1+x^6}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, x^2\right)\\
&= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) + \frac{1}{6}\,\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, x^2\right)\\
&= \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{12}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^2\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^2\right)\\
&= \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{12}\log\left(1-x^2+x^4\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^2\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{2\sqrt{3}} + \frac{1}{6}\log\left(1+x^2\right) - \frac{1}{12}\log\left(1-x^2+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.0140413, size = 78, normalized size = 1.59

$$\frac{1}{12}\left(2\log\left(x^2+1\right) - \log\left(x^2-\sqrt{3}x+1\right) - \log\left(x^2+\sqrt{3}x+1\right) - 2\sqrt{3}\tan^{-1}\left(\sqrt{3}-2x\right) - 2\sqrt{3}\tan^{-1}\left(2x+\sqrt{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^6),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[Sqrt[3] - 2*x] - 2*Sqrt[3]*ArcTan[Sqrt[3] + 2*x] + 2*Log
[1 + x^2] - Log[1 - Sqrt[3]*x + x^2] - Log[1 + Sqrt[3]*x + x^2])/12
```

Maple [A] time = 0.006, size = 41, normalized size = 0.8

$$\frac{\ln\left(x^2+1\right)}{6} - \frac{\ln\left(x^4-x^2+1\right)}{12} + \frac{\sqrt{3}}{6}\arctan\left(\frac{\left(2\,x^2-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^6+1),x)
```

```
[Out] 1/6*ln(x^2+1)-1/12*ln(x^4-x^2+1)+1/6*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))
```

Maxima [A] time = 1.41578, size = 54, normalized size = 1.1

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{12}\log\left(x^4-x^2+1\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/6
*log(x^2 + 1)
```

Fricas [A] time = 2.10148, size = 122, normalized size = 2.49

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{12}\log\left(x^4-x^2+1\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/6
*log(x^2 + 1)
```

Sympy [A] time = 0.129665, size = 46, normalized size = 0.94

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^4-x^2+1\right)}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^2}{3}-\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**6+1),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**4 - x**2 + 1)/12 + sqrt(3)*atan(2*sqrt(3)*x**2/3 -
 sqrt(3)/3)/6
```

Giac [A] time = 1.07139, size = 54, normalized size = 1.1

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2-1\right)\right)-\frac{1}{12}\log\left(x^4-x^2+1\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^6+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) - 1/12*log(x^4 - x^2 + 1) + 1/6
*log(x^2 + 1)
```

3.154 $\int \frac{-1+x^{-1+n}}{-nx+x^n} \, dx$

Optimal. Leaf size=13

$$\frac{\log(x^n - nx)}{n}$$

```
[Out] Log[-(n*x) + x^n]/n
```

Rubi [A] time = 0.0455255, antiderivative size = 20, normalized size of antiderivative = 1.54, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1593, 514, 446, 72}

$$\frac{\log\left(1 - nx^{1-n}\right)}{n} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^(-1 + n))/(-(n*x) + x^n),x]
```

```
[Out] Log[x] + Log[1 - n*x^(1 - n)]/n
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^{-1+n}}{-nx+x^n}\,dx &= \int \frac{x^{-n}\left(-1+x^{-1+n}\right)}{1-nx^{1-n}}\,dx\\ &= \int \frac{1-x^{1-n}}{x\left(1-nx^{1-n}\right)}\,dx\\ &= \frac{\text{Subst}\left(\int \frac{1-x}{x(1-nx)}\,dx,x,x^{1-n}\right)}{1-n}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{x}+\frac{1-n}{-1+nx}\right)dx,x,x^{1-n}\right)}{1-n}\\ &= \log(x)+\frac{\log\left(1-nx^{1-n}\right)}{n}\end{aligned}$$

Mathematica [A] time = 0.0229577, size = 20, normalized size = 1.54

$$\frac{\log\left(1-nx^{1-n}\right)}{n}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^(-1 + n))/(-(n*x) + x^n),x]

[Out] Log[x] + Log[1 - n*x^(1 - n)]/n
```

Maple [A] time = 0.013, size = 17, normalized size = 1.3

$$\frac{\ln\left(nx-e^{n\ln(x)}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x^(-1+n))/(-n*x+x^n),x)

[Out] 1/n*ln(n*x-exp(n*ln(x)))
```

Maxima [A] time = 0.938639, size = 19, normalized size = 1.46

$$\frac{\log\left(nx-x^n\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(-1+n))/(-n*x+x^n),x, algorithm="maxima")

[Out] log(n*x - x^n)/n
```

Fricas [A] time = 2.22595, size = 26, normalized size = 2.

$$\frac{\log\left(-nx+x^n\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(-1+n))/(-n*x+x^n),x, algorithm="fricas")
```

```
[Out] log(-n*x + x^n)/n
```

Sympy [A] time = 2.51136, size = 14, normalized size = 1.08

$$\begin{cases} \frac{\log(-nx+x^n)}{n} & \text{for } n \neq 0 \\ -x + \log(x) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x**(-1+n))/(-n*x+x**n),x)
```

```
[Out] Piecewise((log(-n*x + x**n)/n, Ne(n, 0)), (-x + log(x), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^{n-1}-1}{nx-x^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x^(-1+n))/(-n*x+x^n),x, algorithm="giac")
```

```
[Out] integrate(-(x^(n - 1) - 1)/(n*x - x^n), x)
```

3.155 $\int \frac{x^3}{1-2x^2+3x^4} \, dx$

Optimal. Leaf size=41

$$\frac{1}{12}\log\left(3x^4-2x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-3x^2}{\sqrt{2}}\right)}{6\sqrt{2}}$$

```
[Out] -ArcTan[(1 - 3*x^2)/Sqrt[2]]/(6*Sqrt[2]) + Log[1 - 2*x^2 + 3*x^4]/12
```

Rubi [A] time = 0.0403192, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1114, 634, 618, 204, 628}

$$\frac{1}{12}\log\left(3x^4-2x^2+1\right)-\frac{\tan^{-1}\left(\frac{1-3x^2}{\sqrt{2}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 - 2*x^2 + 3*x^4),x]

[Out] -ArcTan[(1 - 3*x^2)/Sqrt[2]]/(6*Sqrt[2]) + Log[1 - 2*x^2 + 3*x^4]/12
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{1-2x^2+3x^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{1-2x+3x^2}\,dx,x,x^2\right)\\ &= \frac{1}{12}\operatorname{Subst}\left(\int \frac{-2+6x}{1-2x+3x^2}\,dx,x,x^2\right)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx,x,x^2\right)\\ &= \frac{1}{12}\log\left(1-2x^2+3x^4\right)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,2\left(-1+3x^2\right)\right)\\ &= -\frac{\tan^{-1}\left(\frac{1-3x^2}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{1}{12}\log\left(1-2x^2+3x^4\right)\end{aligned}$$

Mathematica [A] time = 0.0096231, size = 38, normalized size = 0.93

$$\frac{1}{12}\left(\log\left(3x^4-2x^2+1\right)+\sqrt{2}\tan^{-1}\left(\frac{3x^2-1}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 - 2*x^2 + 3*x^4),x]
```

```
[Out] (Sqrt[2]*ArcTan[(-1 + 3*x^2)/Sqrt[2]] + Log[1 - 2*x^2 + 3*x^4])/12
```

Maple [A] time = 0.003, size = 35, normalized size = 0.9

$$\frac{\ln\left(3x^4-2x^2+1\right)}{12}+\frac{\sqrt{2}}{12}\arctan\left(\frac{\left(6x^2-2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(3*x^4-2*x^2+1),x)
```

```
[Out] 1/12*ln(3*x^4-2*x^2+1)+1/12*2^(1/2)*arctan(1/4*(6*x^2-2)*2^(1/2))
```

Maxima [A] time = 1.40538, size = 46, normalized size = 1.12

$$\frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(3x^2-1\right)\right)+\frac{1}{12}\log\left(3x^4-2x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^4-2*x^2+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*log(3*x^4 - 2*x^2 + 1)
```

Fricas [A] time = 1.97331, size = 103, normalized size = 2.51

$$\frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(3x^2-1\right)\right)+\frac{1}{12}\log\left(3x^4-2x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^4-2*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*log(3*x^4 - 2*x^2 + 1)
```

Sympy [A] time = 0.109402, size = 42, normalized size = 1.02

$$\frac{\log\left(x^4 - \frac{2x^2}{3} + \frac{1}{3}\right)}{12} + \frac{\sqrt{2}\,\operatorname{atan}\left(\frac{3\sqrt{2}x^2}{2} - \frac{\sqrt{2}}{2}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(3*x**4-2*x**2+1),x)
```

```
[Out] log(x**4 - 2*x**2/3 + 1/3)/12 + sqrt(2)*atan(3*sqrt(2)*x**2/2 - sqrt(2)/2)/
12
```

Giac [A] time = 1.04273, size = 46, normalized size = 1.12

$$\frac{1}{12}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(3\,x^2 - 1\right)\right) + \frac{1}{12}\,\log\left(3\,x^4 - 2\,x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(3*x^4-2*x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*x^2 - 1)) + 1/12*log(3*x^4 - 2*x^2 + 1)
```

3.156 $\int \frac{x^5}{-4+x^2+3x^4} \, dx$

Optimal. Leaf size=32

$$\frac{x^2}{6}+\frac{1}{14}\log\left(1-x^2\right)-\frac{8}{63}\log\left(3x^2+4\right)$$

```
[Out] x^2/6 + Log[1 - x^2]/14 - (8*Log[4 + 3*x^2])/63
```

Rubi [A] time = 0.0245061, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1114, 703, 632, 31}

$$\frac{x^2}{6}+\frac{1}{14}\log\left(1-x^2\right)-\frac{8}{63}\log\left(3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(-4 + x^2 + 3*x^4),x]
```

```
[Out] x^2/6 + Log[1 - x^2]/14 - (8*Log[4 + 3*x^2])/63
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{-4+x^2+3x^4}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{-4+x+3x^2}\,dx, x, x^2\right)\\ &= \frac{x^2}{6}+\frac{1}{6}\operatorname{Subst}\left(\int \frac{4-x}{-4+x+3x^2}\,dx, x, x^2\right)\\ &= \frac{x^2}{6}+\frac{3}{14}\operatorname{Subst}\left(\int \frac{1}{-3+3x}\,dx, x, x^2\right)-\frac{8}{21}\operatorname{Subst}\left(\int \frac{1}{4+3x}\,dx, x, x^2\right)\\ &= \frac{x^2}{6}+\frac{1}{14}\log\left(1-x^2\right)-\frac{8}{63}\log\left(4+3x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0053258, size = 32, normalized size = 1.

$$\frac{x^2}{6}+\frac{1}{14}\log\left(1-x^2\right)-\frac{8}{63}\log\left(3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(-4 + x^2 + 3*x^4),x]
```

```
[Out] x^2/6 + Log[1 - x^2]/14 - (8*Log[4 + 3*x^2])/63
```

Maple [A] time = 0.005, size = 25, normalized size = 0.8

$$\frac{x^2}{6}-\frac{8\ln\left(3x^2+4\right)}{63}+\frac{\ln\left(x^2-1\right)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(3*x^4+x^2-4),x)
```

```
[Out] 1/6*x^2-8/63*ln(3*x^2+4)+1/14*ln(x^2-1)
```

Maxima [A] time = 0.925519, size = 32, normalized size = 1.

$$\frac{1}{6}x^2-\frac{8}{63}\log\left(3x^2+4\right)+\frac{1}{14}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^4+x^2-4),x, algorithm="maxima")
```

```
[Out] 1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(x^2 - 1)
```

Fricas [A] time = 2.01062, size = 69, normalized size = 2.16

$$\frac{1}{6}x^2-\frac{8}{63}\log\left(3x^2+4\right)+\frac{1}{14}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^4+x^2-4),x, algorithm="fricas")
```

```
[Out] 1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(x^2 - 1)
```

Sympy [A] time = 0.107478, size = 24, normalized size = 0.75

$$\frac{x^2}{6} + \frac{\log\left(x^2 - 1\right)}{14} - \frac{8\log\left(x^2 + \frac{4}{3}\right)}{63}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(3*x**4+x**2-4),x)
```

```
[Out] x**2/6 + log(x**2 - 1)/14 - 8*log(x**2 + 4/3)/63
```

Giac [A] time = 1.05862, size = 34, normalized size = 1.06

$$\frac{1}{6}\,x^2 - \frac{8}{63}\,\log\left(3\,x^2 + 4\right) + \frac{1}{14}\,\log\left(\left|x^2 - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3*x^4+x^2-4),x, algorithm="giac")
```

```
[Out] 1/6*x^2 - 8/63*log(3*x^2 + 4) + 1/14*log(abs(x^2 - 1))
```

3.157 $\int \frac{x^2}{9-10x^3+x^6} \, dx$

Optimal. Leaf size=25

$$\frac{1}{24}\log\left(9-x^3\right)-\frac{1}{24}\log\left(1-x^3\right)$$

```
[Out] -Log[1 - x^3]/24 + Log[9 - x^3]/24
```

Rubi [A] time = 0.0171892, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1352, 616, 31}

$$\frac{1}{24}\log\left(9-x^3\right)-\frac{1}{24}\log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(9 - 10*x^3 + x^6),x]
```

```
[Out] -Log[1 - x^3]/24 + Log[9 - x^3]/24
```

Rule 1352

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a,
b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{9-10x^3+x^6} \, dx &= \frac{1}{3}\text{Subst}\left(\int \frac{1}{9-10x+x^2} \, dx,x,x^3\right)\\
&= \frac{1}{24}\text{Subst}\left(\int \frac{1}{-9+x} \, dx,x,x^3\right)-\frac{1}{24}\text{Subst}\left(\int \frac{1}{-1+x} \, dx,x,x^3\right)\\
&= -\frac{1}{24}\log\left(1-x^3\right)+\frac{1}{24}\log\left(9-x^3\right)
\end{aligned}$$

Mathematica [A] time = 0.0038913, size = 25, normalized size = 1.

$$\frac{1}{24}\log\left(9-x^3\right)-\frac{1}{24}\log\left(1-x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(9 - 10*x^3 + x^6),x]
```

```
[Out] -Log[1 - x^3]/24 + Log[9 - x^3]/24
```

Maple [A] time = 0.005, size = 18, normalized size = 0.7

$$-\frac{\ln\left(x^3-1\right)}{24}+\frac{\ln\left(x^3-9\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^6-10*x^3+9),x)
```

```
[Out] -1/24*ln(x^3-1)+1/24*ln(x^3-9)
```

Maxima [A] time = 0.928193, size = 23, normalized size = 0.92

$$-\frac{1}{24}\log\left(x^3-1\right)+\frac{1}{24}\log\left(x^3-9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-10*x^3+9),x, algorithm="maxima")
```

```
[Out] -1/24*log(x^3 - 1) + 1/24*log(x^3 - 9)
```

Fricas [A] time = 2.04915, size = 54, normalized size = 2.16

$$-\frac{1}{24}\log\left(x^3-1\right)+\frac{1}{24}\log\left(x^3-9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-10*x^3+9),x, algorithm="fricas")
```

```
[Out] -1/24*log(x^3 - 1) + 1/24*log(x^3 - 9)
```

Sympy [A] time = 0.10225, size = 15, normalized size = 0.6

$$\frac{\log\left(x^3-9\right)}{24}-\frac{\log\left(x^3-1\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**6-10*x**3+9),x)
```

```
[Out] log(x**3 - 9)/24 - log(x**3 - 1)/24
```

Giac [A] time = 1.05928, size = 26, normalized size = 1.04

$$-\frac{1}{24}\,\log\left(\left|x^3-1\right|\right)+\frac{1}{24}\,\log\left(\left|x^3-9\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^6-10*x^3+9),x, algorithm="giac")
```

```
[Out] -1/24*log(abs(x^3 - 1)) + 1/24*log(abs(x^3 - 9))
```

3.158 $\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx$

Optimal. Leaf size=36

$$\frac{2}{2-x}+\frac{2}{(2-x)^2}-\frac{7}{3(2-x)^3}+\log(2-x)$$

```
[Out] -7/(3*(2 - x)^3) + 2/(2 - x)^2 + 2/(2 - x) + Log[2 - x]
```

Rubi [A] time = 0.0219846, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1850}

$$\frac{2}{2-x}+\frac{2}{(2-x)^2}-\frac{7}{3(2-x)^3}+\log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 4*x^2 + x^3)/(-2 + x)^4,x]
```

```
[Out] -7/(3*(2 - x)^3) + 2/(2 - x)^2 + 2/(2 - x) + Log[2 - x]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{1-4x^2+x^3}{(-2+x)^4}\,dx &= \int \left(-\frac{7}{(-2+x)^4}-\frac{4}{(-2+x)^3}+\frac{2}{(-2+x)^2}+\frac{1}{-2+x}\right)dx\\ &= -\frac{7}{3(2-x)^3}+\frac{2}{(2-x)^2}+\frac{2}{2-x}+\log(2-x)\end{aligned}$$

Mathematica [A] time = 0.0141903, size = 24, normalized size = 0.67

$$\frac{-6x^2+30x-29}{3(x-2)^3}+\log(x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 4*x^2 + x^3)/(-2 + x)^4,x]
```

```
[Out] (-29 + 30*x - 6*x^2)/(3*(-2 + x)^3) + Log[-2 + x]
```

Maple [A] time = 0.005, size = 27, normalized size = 0.8

$$2\,(-2+x)^{-2}+\frac{7}{3\,(-2+x)^3}-2\,(-2+x)^{-1}+\ln(-2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-4*x^2+1)/(-2+x)^4,x)
```

```
[Out] 2/(-2+x)^2+7/3/(-2+x)^3-2/(-2+x)+ln(-2+x)
```

Maxima [A] time = 0.927856, size = 43, normalized size = 1.19

$$-\frac{6\,x^2 - 30\,x + 29}{3\left(x^3 - 6\,x^2 + 12\,x - 8\right)} + \log\left(x - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+1)/(-2+x)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(6*x^2 - 30*x + 29)/(x^3 - 6*x^2 + 12*x - 8) + log(x - 2)
```

Fricas [A] time = 1.99107, size = 123, normalized size = 3.42

$$-\frac{6\,x^2 - 3\left(x^3 - 6\,x^2 + 12\,x - 8\right)\log\left(x - 2\right) - 30\,x + 29}{3\left(x^3 - 6\,x^2 + 12\,x - 8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+1)/(-2+x)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(6*x^2 - 3*(x^3 - 6*x^2 + 12*x - 8)*log(x - 2) - 30*x + 29)/(x^3 - 6*x
^2 + 12*x - 8)
```

Sympy [A] time = 0.108621, size = 29, normalized size = 0.81

$$-\frac{6x^2 - 30x + 29}{3x^3 - 18x^2 + 36x - 24} + \log\left(x - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-4*x**2+1)/(-2+x)**4,x)
```

```
[Out] -(6*x**2 - 30*x + 29)/(3*x**3 - 18*x**2 + 36*x - 24) + log(x - 2)
```

Giac [A] time = 1.05217, size = 31, normalized size = 0.86

$$-\frac{6\,x^2 - 30\,x + 29}{3\left(x - 2\right)^3} + \log\left(|x - 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+1)/(-2+x)^4,x, algorithm="giac")
```

```
[Out] -1/3*(6*x^2 - 30*x + 29)/(x - 2)^3 + log(abs(x - 2))
```

3.159 $\int \frac{x^3}{(-1+x)^{12}}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{8(1-x)^8}+\frac{1}{3(1-x)^9}-\frac{3}{10(1-x)^{10}}+\frac{1}{11(1-x)^{11}}$$

```
[Out] 1/(11*(1 - x)^11) - 3/(10*(1 - x)^10) + 1/(3*(1 - x)^9) - 1/(8*(1 - x)^8)
```

Rubi [A] time = 0.0124213, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$-\frac{1}{8(1-x)^8}+\frac{1}{3(1-x)^9}-\frac{3}{10(1-x)^{10}}+\frac{1}{11(1-x)^{11}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(-1 + x)^12,x]
```

```
[Out] 1/(11*(1 - x)^11) - 3/(10*(1 - x)^10) + 1/(3*(1 - x)^9) - 1/(8*(1 - x)^8)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(-1+x)^{12}}\,dx &= \int \left(\frac{1}{(-1+x)^{12}}+\frac{3}{(-1+x)^{11}}+\frac{3}{(-1+x)^{10}}+\frac{1}{(-1+x)^9}\right)dx\\ &= \frac{1}{11(1-x)^{11}}-\frac{3}{10(1-x)^{10}}+\frac{1}{3(1-x)^9}-\frac{1}{8(1-x)^8}\end{aligned}$$

Mathematica [A] time = 0.0063717, size = 24, normalized size = 0.53

$$\frac{-165x^3+55x^2-11x+1}{1320(x-1)^{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(-1 + x)^12,x]
```

```
[Out] (1 - 11*x + 55*x^2 - 165*x^3)/(1320*(-1 + x)^11)
```

Maple [A] time = 0.005, size = 30, normalized size = 0.7

$$-\frac{3}{10\,(-1+x)^{10}}-\frac{1}{11\,(-1+x)^{11}}-\frac{1}{8\,(-1+x)^{8}}-\frac{1}{3\,(-1+x)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-1+x)^12,x)
```

```
[Out] -3/10/(-1+x)^10-1/11/(-1+x)^11-1/8/(-1+x)^8-1/3/(-1+x)^9
```

Maxima [B] time = 0.936681, size = 97, normalized size = 2.16

$$-\frac{165\,x^3-55\,x^2+11\,x-1}{1320\,\left(x^{11}-11\,x^{10}+55\,x^9-165\,x^8+330\,x^7-462\,x^6+462\,x^5-330\,x^4+165\,x^3-55\,x^2+11\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-1+x)^12,x, algorithm="maxima")
```

```
[Out] -1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x^11 - 11*x^10 + 55*x^9 - 165*x^8 +
330*x^7 - 462*x^6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^2 + 11*x - 1)
```

Fricas [B] time = 1.89486, size = 196, normalized size = 4.36

$$-\frac{165\,x^3-55\,x^2+11\,x-1}{1320\,\left(x^{11}-11\,x^{10}+55\,x^9-165\,x^8+330\,x^7-462\,x^6+462\,x^5-330\,x^4+165\,x^3-55\,x^2+11\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-1+x)^12,x, algorithm="fricas")
```

```
[Out] -1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x^11 - 11*x^10 + 55*x^9 - 165*x^8 +
330*x^7 - 462*x^6 + 462*x^5 - 330*x^4 + 165*x^3 - 55*x^2 + 11*x - 1)
```

Sympy [B] time = 0.161188, size = 71, normalized size = 1.58

$$-\frac{165x^3-55x^2+11x-1}{1320x^{11}-14520x^{10}+72600x^9-217800x^8+435600x^7-609840x^6+609840x^5-435600x^4+217800x^3-72600x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-1+x)**12,x)
```

```
[Out] -(165*x**3 - 55*x**2 + 11*x - 1)/(1320*x**11 - 14520*x**10 + 72600*x**9 - 2
17800*x**8 + 435600*x**7 - 609840*x**6 + 609840*x**5 - 435600*x**4 + 217800
*x**3 - 72600*x**2 + 14520*x - 1320)
```

Giac [A] time = 1.05528, size = 30, normalized size = 0.67

$$-\frac{165\,x^3-55\,x^2+11\,x-1}{1320\,(x-1)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-1+x)^12,x, algorithm="giac")

[Out] -1/1320*(165*x^3 - 55*x^2 + 11*x - 1)/(x - 1)^11
```

3.160 $\int \frac{-3x+x^4}{(1+2x)^5}\,dx$

Optimal. Leaf size=55

$$\frac{1}{8(2x+1)}-\frac{3}{32(2x+1)^2}+\frac{7}{24(2x+1)^3}-\frac{25}{128(2x+1)^4}+\frac{1}{32}\log(2x+1)$$

```
[Out] -25/(128*(1 + 2*x)^4) + 7/(24*(1 + 2*x)^3) - 3/(32*(1 + 2*x)^2) + 1/(8*(1 +
 2*x)) + Log[1 + 2*x]/32
```

Rubi [A] time = 0.0392657, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 1620}

$$\frac{1}{8(2x+1)}-\frac{3}{32(2x+1)^2}+\frac{7}{24(2x+1)^3}-\frac{25}{128(2x+1)^4}+\frac{1}{32}\log(2x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-3*x + x^4)/(1 + 2*x)^5,x]
```

```
[Out] -25/(128*(1 + 2*x)^4) + 7/(24*(1 + 2*x)^3) - 3/(32*(1 + 2*x)^2) + 1/(8*(1 +
 2*x)) + Log[1 + 2*x]/32
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-3x+x^4}{(1+2x)^5}\,dx &= \int \frac{x\left(-3+x^3\right)}{(1+2x)^5}\,dx\\
&= \int \left(\frac{25}{16(1+2x)^5}-\frac{7}{4(1+2x)^4}+\frac{3}{8(1+2x)^3}-\frac{1}{4(1+2x)^2}+\frac{1}{16(1+2x)}\right)dx\\
&= -\frac{25}{128(1+2x)^4}+\frac{7}{24(1+2x)^3}-\frac{3}{32(1+2x)^2}+\frac{1}{8(1+2x)}+\frac{1}{32}\log(1+2x)
\end{aligned}$$

Mathematica [A] time = 0.0116806, size = 41, normalized size = 0.75

$$\frac{384x^3+432x^2+368x+12(2x+1)^4\log(2x+1)+49}{384(2x+1)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3*x + x^4)/(1 + 2*x)^5,x]
```

```
[Out] (49 + 368*x + 432*x^2 + 384*x^3 + 12*(1 + 2*x)^4*Log[1 + 2*x])/(384*(1 + 2*
x)^4)
```

Maple [A] time = 0.005, size = 46, normalized size = 0.8

$$-\frac{25}{128\,(1+2\,x)^{4}}+\frac{7}{24\,(1+2\,x)^{3}}-\frac{3}{32\,(1+2\,x)^{2}}+\frac{1}{8+16\,x}+\frac{\ln(1+2\,x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-3*x)/(1+2*x)^5,x)
```

```
[Out] -25/128/(1+2*x)^4+7/24/(1+2*x)^3-3/32/(1+2*x)^2+1/8/(1+2*x)+1/32*ln(1+2*x)
```

Maxima [A] time = 0.933696, size = 65, normalized size = 1.18

$$\frac{384\,x^{3}+432\,x^{2}+368\,x+49}{384\left(16\,x^{4}+32\,x^{3}+24\,x^{2}+8\,x+1\right)}+\frac{1}{32}\log(2\,x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3*x)/(1+2*x)^5,x, algorithm="maxima")
```

```
[Out] 1/384*(384*x^3 + 432*x^2 + 368*x + 49)/(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1)
 + 1/32*log(2*x + 1)
```

Fricas [A] time = 1.9424, size = 178, normalized size = 3.24

$$\frac{384\,x^{3}+432\,x^{2}+12\left(16\,x^{4}+32\,x^{3}+24\,x^{2}+8\,x+1\right)\log(2\,x+1)+368\,x+49}{384\left(16\,x^{4}+32\,x^{3}+24\,x^{2}+8\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3*x)/(1+2*x)^5,x, algorithm="fricas")
```

```
[Out] 1/384*(384*x^3 + 432*x^2 + 12*(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1)*log(2*x
+ 1) + 368*x + 49)/(16*x^4 + 32*x^3 + 24*x^2 + 8*x + 1)
```

Sympy [A] time = 0.11814, size = 42, normalized size = 0.76

$$\frac{384x^{3}+432x^{2}+368x+49}{6144x^{4}+12288x^{3}+9216x^{2}+3072x+384}+\frac{\log(2x+1)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-3*x)/(1+2*x)**5,x)
```

```
[Out] (384*x**3 + 432*x**2 + 368*x + 49)/(6144*x**4 + 12288*x**3 + 9216*x**2 + 30
72*x + 384) + log(2*x + 1)/32
```

Giac [A] time = 1.06204, size = 74, normalized size = 1.35

$$\frac{1}{8(2x+1)} - \frac{3}{32(2x+1)^2} + \frac{7}{24(2x+1)^3} - \frac{25}{128(2x+1)^4} - \frac{1}{32}\log\left(\frac{|2x+1|}{2(2x+1)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-3*x)/(1+2*x)^5,x, algorithm="giac")
```

```
[Out] 1/8/(2*x + 1) - 3/32/(2*x + 1)^2 + 7/24/(2*x + 1)^3 - 25/128/(2*x + 1)^4 -
1/32*log(1/2*abs(2*x + 1)/(2*x + 1)^2)
```

3.161 $\int \frac{1}{(-1+x)^2(1+x)^3}\,dx$

Optimal. Leaf size=36

$$\frac{1}{8(1-x)}-\frac{1}{4(x+1)}-\frac{1}{8(x+1)^2}+\frac{3}{8}\tanh^{-1}(x)$$

```
[Out] 1/(8*(1 - x)) - 1/(8*(1 + x)^2) - 1/(4*(1 + x)) + (3*ArcTanh[x])/8
```

Rubi [A] time = 0.0151888, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {44, 207}

$$\frac{1}{8(1-x)}-\frac{1}{4(x+1)}-\frac{1}{8(x+1)^2}+\frac{3}{8}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)^2*(1 + x)^3),x]
```

```
[Out] 1/(8*(1 - x)) - 1/(8*(1 + x)^2) - 1/(4*(1 + x)) + (3*ArcTanh[x])/8
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-1+x)^2(1+x)^3}\,dx &= \int \left(\frac{1}{8(-1+x)^2}+\frac{1}{4(1+x)^3}+\frac{1}{4(1+x)^2}-\frac{3}{8\left(-1+x^2\right)}\right)dx\\
&= \frac{1}{8(1-x)}-\frac{1}{8(1+x)^2}-\frac{1}{4(1+x)}-\frac{3}{8}\int \frac{1}{-1+x^2}\,dx\\
&= \frac{1}{8(1-x)}-\frac{1}{8(1+x)^2}-\frac{1}{4(1+x)}+\frac{3}{8}\tanh^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0212087, size = 38, normalized size = 1.06

$$\frac{1}{16}\left(\frac{-6x^2-6x+4}{(x-1)(x+1)^2}-3\log(x-1)+3\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)^2*(1 + x)^3),x]
```

```
[Out] ((4 - 6*x - 6*x^2)/((-1 + x)*(1 + x)^2) - 3*Log[-1 + x] + 3*Log[1 + x])/16
```

Maple [A] time = 0.008, size = 35, normalized size = 1.

$$-\frac{1}{8\,(1+x)^{2}}-\frac{1}{4+4\,x}+\frac{3\,\ln\left(1+x\right)}{16}-\frac{1}{-8+8\,x}-\frac{3\,\ln\left(-1+x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)^2/(1+x)^3,x)
```

```
[Out] -1/8/(1+x)^2-1/4/(1+x)+3/16*ln(1+x)-1/8/(-1+x)-3/16*ln(-1+x)
```

Maxima [A] time = 0.935295, size = 51, normalized size = 1.42

$$-\frac{3\,x^{2}+3\,x-2}{8\,\left(x^{3}+x^{2}-x-1\right)}+\frac{3}{16}\,\log\left(x+1\right)-\frac{3}{16}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(1+x)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*x^2 + 3*x - 2)/(x^3 + x^2 - x - 1) + 3/16*log(x + 1) - 3/16*log(x -
 1)
```

Fricas [B] time = 1.98701, size = 155, normalized size = 4.31

$$-\frac{6\,x^{2}-3\,\left(x^{3}+x^{2}-x-1\right)\log\left(x+1\right)+3\,\left(x^{3}+x^{2}-x-1\right)\log\left(x-1\right)+6\,x-4}{16\,\left(x^{3}+x^{2}-x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(1+x)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(6*x^2 - 3*(x^3 + x^2 - x - 1)*log(x + 1) + 3*(x^3 + x^2 - x - 1)*log
(x - 1) + 6*x - 4)/(x^3 + x^2 - x - 1)
```

Sympy [A] time = 0.125333, size = 41, normalized size = 1.14

$$-\frac{3x^{2}+3x-2}{8x^{3}+8x^{2}-8x-8}-\frac{3\log\left(x-1\right)}{16}+\frac{3\log\left(x+1\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**2/(1+x)**3,x)
```

```
[Out] -(3*x**2 + 3*x - 2)/(8*x**3 + 8*x**2 - 8*x - 8) - 3*log(x - 1)/16 + 3*log(x
 + 1)/16
```

Giac [A] time = 1.05106, size = 58, normalized size = 1.61

$$-\frac{1}{8(x-1)}+\frac{\frac{12}{x-1}+5}{32\left(\frac{2}{x-1}+1\right)^2}+\frac{3}{16}\log\left(\left|-\frac{2}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(1+x)^3,x, algorithm="giac")
```

```
[Out] -1/8/(x - 1) + 1/32*(12/(x - 1) + 5)/(2/(x - 1) + 1)^2 + 3/16*log(abs(-2/(x
 - 1) - 1))
```

3.162 $\int \frac{1}{(5-6x)^2x^2} dx$

Optimal. Leaf size=35

$$\frac{6}{25(5-6x)}-\frac{1}{25x}-\frac{12}{125}\log(5-6x)+\frac{12\log(x)}{125}$$

```
[Out] 6/(25*(5 - 6*x)) - 1/(25*x) - (12*Log[5 - 6*x])/125 + (12*Log[x])/125
```

Rubi [A] time = 0.0142911, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{6}{25(5-6x)}-\frac{1}{25x}-\frac{12}{125}\log(5-6x)+\frac{12\log(x)}{125}$$

Antiderivative was successfully verified.

```
[In] Int[1/((5 - 6*x)^2*x^2),x]
```

```
[Out] 6/(25*(5 - 6*x)) - 1/(25*x) - (12*Log[5 - 6*x])/125 + (12*Log[x])/125
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(5-6x)^2x^2}\,dx &= \int \left(\frac{1}{25x^2}+\frac{12}{125x}+\frac{36}{25(-5+6x)^2}-\frac{72}{125(-5+6x)}\right)dx\\ &= \frac{6}{25(5-6x)}-\frac{1}{25x}-\frac{12}{125}\log(5-6x)+\frac{12\log(x)}{125}\end{aligned}$$

Mathematica [A] time = 0.0176111, size = 31, normalized size = 0.89

$$\frac{1}{125}\left(\frac{30}{5-6x}-\frac{5}{x}-12\log(5-6x)+12\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((5 - 6*x)^2*x^2),x]
```

```
[Out] (30/(5 - 6*x) - 5/x - 12*Log[5 - 6*x] + 12*Log[x])/125
```

Maple [A] time = 0.008, size = 28, normalized size = 0.8

$$-\frac{6}{-125+150\,x}-\frac{12\,\ln(-5+6\,x)}{125}-\frac{1}{25\,x}+\frac{12\,\ln(x)}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5-6*x)^2/x^2,x)
```

```
[Out] -6/25/(-5+6*x)-12/125*ln(-5+6*x)-1/25/x+12/125*ln(x)
```

Maxima [A] time = 0.928173, size = 42, normalized size = 1.2

$$-\frac{12\,x-5}{25\,\left(6\,x^{2}-5\,x\right)}-\frac{12}{125}\,\log\left(6\,x-5\right)+\frac{12}{125}\,\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-6*x)^2/x^2,x, algorithm="maxima")
```

```
[Out] -1/25*(12*x - 5)/(6*x^2 - 5*x) - 12/125*log(6*x - 5) + 12/125*log(x)
```

Fricas [A] time = 1.93243, size = 124, normalized size = 3.54

$$-\frac{12\,\left(6\,x^{2}-5\,x\right)\log\left(6\,x-5\right)-12\,\left(6\,x^{2}-5\,x\right)\log\left(x\right)+60\,x-25}{125\,\left(6\,x^{2}-5\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-6*x)^2/x^2,x, algorithm="fricas")
```

```
[Out] -1/125*(12*(6*x^2 - 5*x)*log(6*x - 5) - 12*(6*x^2 - 5*x)*log(x) + 60*x - 25
)/(6*x^2 - 5*x)
```

Sympy [A] time = 0.118725, size = 29, normalized size = 0.83

$$-\frac{12x-5}{150x^{2}-125x}+\frac{12\log\left(x\right)}{125}-\frac{12\log\left(x-\frac{5}{6}\right)}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-6*x)**2/x**2,x)
```

```
[Out] -(12*x - 5)/(150*x**2 - 125*x) + 12*log(x)/125 - 12*log(x - 5/6)/125
```

Giac [A] time = 1.06476, size = 54, normalized size = 1.54

$$-\frac{6}{25\,(6\,x-5)}+\frac{6}{125\left(\frac{5}{6\,x-5}+1\right)}+\frac{12}{125}\,\log\left(\left|-\frac{5}{6\,x-5}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-6*x)^2/x^2,x, algorithm="giac")
```

```
[Out] -6/25/(6*x - 5) + 6/125/(5/(6*x - 5) + 1) + 12/125*log(abs(-5/(6*x - 5) - 1
))
```

3.163 $\int \frac{1}{(-3-2x+x^2)^3}\,dx$

Optimal. Leaf size=61

$$\frac{3(1-x)}{128\left(-x^2+2x+3\right)}+\frac{1-x}{16\left(-x^2+2x+3\right)^2}+\frac{3}{512}\log(3-x)-\frac{3}{512}\log(x+1)$$

```
[Out] (1 - x)/(16*(3 + 2*x - x^2)^2) + (3*(1 - x))/(128*(3 + 2*x - x^2)) + (3*Log
[3 - x])/512 - (3*Log[1 + x])/512
```

Rubi [A] time = 0.012479, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {614, 616, 31}

$$\frac{3(1-x)}{128\left(-x^2+2x+3\right)}+\frac{1-x}{16\left(-x^2+2x+3\right)^2}+\frac{3}{512}\log(3-x)-\frac{3}{512}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - 2*x + x^2)^(-3),x]
```


```
[Out] (1 - x)/(16*(3 + 2*x - x^2)^2) + (3*(1 - x))/(128*(3 + 2*x - x^2)) + (3*Log
[3 - x])/512 - (3*Log[1 + x])/512
```


Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-3-2x+x^2\right)^3}\,dx &= \frac{1-x}{16\left(3+2x-x^2\right)^2} - \frac{3}{16}\int \frac{1}{\left(-3-2x+x^2\right)^2}\,dx \\ &= \frac{1-x}{16\left(3+2x-x^2\right)^2} + \frac{3(1-x)}{128\left(3+2x-x^2\right)} + \frac{3}{128}\int \frac{1}{-3-2x+x^2}\,dx \\ &= \frac{1-x}{16\left(3+2x-x^2\right)^2} + \frac{3(1-x)}{128\left(3+2x-x^2\right)} + \frac{3}{512}\int \frac{1}{-3+x}\,dx - \frac{3}{512}\int \frac{1}{1+x}\,dx \\ &= \frac{1-x}{16\left(3+2x-x^2\right)^2} + \frac{3(1-x)}{128\left(3+2x-x^2\right)} + \frac{3}{512}\log(3-x) - \frac{3}{512}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0224939, size = 46, normalized size = 0.75

$$\frac{1}{512}\left(\frac{4\left(3x^3-9x^2-11x+17\right)}{\left(x^2-2x-3\right)^2}+3\log(3-x)-3\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 - 2*x + x^2)^(-3),x]
```

```
[Out] ((4*(17 - 11*x - 9*x^2 + 3*x^3))/(-3 - 2*x + x^2)^2 + 3*Log[3 - x] - 3*Log[
1 + x])/512
```

Maple [A] time = 0.009, size = 42, normalized size = 0.7

$$\frac{1}{128\,(1+x)^2}+\frac{3}{256+256\,x}-\frac{3\ln(1+x)}{512}-\frac{1}{128\,(-3+x)^2}+\frac{3}{-768+256\,x}+\frac{3\ln(-3+x)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x-3)^3,x)
```

```
[Out] 1/128/(1+x)^2+3/256/(1+x)-3/512*ln(1+x)-1/128/(-3+x)^2+3/256/(-3+x)+3/512*l
n(-3+x)
```

Maxima [A] time = 0.921371, size = 68, normalized size = 1.11

$$\frac{3\,x^3-9\,x^2-11\,x+17}{128\left(x^4-4\,x^3-2\,x^2+12\,x+9\right)}-\frac{3}{512}\log(x+1)+\frac{3}{512}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-3)^3,x, algorithm="maxima")
```

```
[Out] 1/128*(3*x^3 - 9*x^2 - 11*x + 17)/(x^4 - 4*x^3 - 2*x^2 + 12*x + 9) - 3/512*
log(x + 1) + 3/512*log(x - 3)
```

Fricas [A] time = 1.94209, size = 224, normalized size = 3.67

$$\frac{12\,x^3-36\,x^2-3\left(x^4-4\,x^3-2\,x^2+12\,x+9\right)\log\left(x+1\right)+3\left(x^4-4\,x^3-2\,x^2+12\,x+9\right)\log\left(x-3\right)-44\,x+68}{512\left(x^4-4\,x^3-2\,x^2+12\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-3)^3,x, algorithm="fricas")
```

```
[Out] 1/512*(12*x^3 - 36*x^2 - 3*(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)*log(x + 1) + 3*
(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)*log(x - 3) - 44*x + 68)/(x^4 - 4*x^3 - 2*x
^2 + 12*x + 9)
```

Sympy [A] time = 0.145338, size = 51, normalized size = 0.84

$$\frac{3x^3-9x^2-11x+17}{128x^4-512x^3-256x^2+1536x+1152}+\frac{3\log\left(x-3\right)}{512}-\frac{3\log\left(x+1\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x-3)**3,x)
```

```
[Out] (3*x**3 - 9*x**2 - 11*x + 17)/(128*x**4 - 512*x**3 - 256*x**2 + 1536*x + 11
52) + 3*log(x - 3)/512 - 3*log(x + 1)/512
```

Giac [A] time = 1.06083, size = 57, normalized size = 0.93

$$\frac{3\,x^3-9\,x^2-11\,x+17}{128\left(x^2-2\,x-3\right)^2}-\frac{3}{512}\,\log\left(|x+1|\right)+\frac{3}{512}\,\log\left(|x-3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-3)^3,x, algorithm="giac")
```

```
[Out] 1/128*(3*x^3 - 9*x^2 - 11*x + 17)/(x^2 - 2*x - 3)^2 - 3/512*log(abs(x + 1))
 + 3/512*log(abs(x - 3))
```

3.164 $\int \frac{1}{(13-4x+x^2)^3}\,dx$

Optimal. Leaf size=51

$$-\frac{2-x}{216\left(x^2-4x+13\right)}-\frac{2-x}{36\left(x^2-4x+13\right)^2}+\frac{1}{648}\tan^{-1}\left(\frac{x-2}{3}\right)$$

```
[Out] -(2 - x)/(36*(13 - 4*x + x^2)^2) - (2 - x)/(216*(13 - 4*x + x^2)) + ArcTan[
(-2 + x)/3]/648
```

Rubi [A] time = 0.0140963, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {614, 618, 204}

$$-\frac{2-x}{216\left(x^2-4x+13\right)}-\frac{2-x}{36\left(x^2-4x+13\right)^2}+\frac{1}{648}\tan^{-1}\left(\frac{x-2}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(13 - 4*x + x^2)^(-3),x]
```

```
[Out] -(2 - x)/(36*(13 - 4*x + x^2)^2) - (2 - x)/(216*(13 - 4*x + x^2)) + ArcTan[
(-2 + x)/3]/648
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(13-4x+x^2\right)^3}\,dx = -\frac{2-x}{36\left(13-4x+x^2\right)^2}+\frac{1}{12}\int \frac{1}{\left(13-4x+x^2\right)^2}\,dx$$
$$= -\frac{2-x}{36\left(13-4x+x^2\right)^2}-\frac{2-x}{216\left(13-4x+x^2\right)}+\frac{1}{216}\int \frac{1}{13-4x+x^2}\,dx$$
$$= -\frac{2-x}{36\left(13-4x+x^2\right)^2}-\frac{2-x}{216\left(13-4x+x^2\right)}-\frac{1}{108}\,\text{Subst}\left(\int \frac{1}{-36-x^2}\,dx,x,-4+2x\right)$$
$$= -\frac{2-x}{36\left(13-4x+x^2\right)^2}-\frac{2-x}{216\left(13-4x+x^2\right)}+\frac{1}{648}\tan^{-1}\left(\frac{1}{3}(-2+x)\right)$$

Mathematica [A] time = 0.016078, size = 36, normalized size = 0.71

$$\frac{1}{648}\left(\frac{3(x-2)\left(x^2-4x+19\right)}{\left(x^2-4x+13\right)^2}+\tan^{-1}\left(\frac{x-2}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(13 - 4*x + x^2)^(-3),x]
```

```
[Out] ((3*(-2 + x)*(19 - 4*x + x^2))/(13 - 4*x + x^2)^2 + ArcTan[(-2 + x)/3])/648
```

Maple [A] time = 0.003, size = 44, normalized size = 0.9

$$\frac{2\,x-4}{72\,\left(x^2-4\,x+13\right)^2}+\frac{2\,x-4}{432\,x^2-1728\,x+5616}+\frac{1}{648}\arctan\left(-\frac{2}{3}+\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-4*x+13)^3,x)
```

```
[Out] 1/72*(2*x-4)/(x^2-4*x+13)^2+1/432*(2*x-4)/(x^2-4*x+13)+1/648*arctan(-2/3+1/
3*x)
```

Maxima [A] time = 1.41552, size = 59, normalized size = 1.16

$$\frac{x^3-6\,x^2+27\,x-38}{216\,\left(x^4-8\,x^3+42\,x^2-104\,x+169\right)}+\frac{1}{648}\arctan\left(\frac{1}{3}\,x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+13)^3,x, algorithm="maxima")
```

```
[Out] 1/216*(x^3 - 6*x^2 + 27*x - 38)/(x^4 - 8*x^3 + 42*x^2 - 104*x + 169) + 1/64
8*arctan(1/3*x - 2/3)
```

Fricas [A] time = 2.1216, size = 180, normalized size = 3.53

$$\frac{3\,x^{3} - 18\,x^{2} + \left(x^{4} - 8\,x^{3} + 42\,x^{2} - 104\,x + 169\right) \arctan\left(\frac{1}{3}\,x - \frac{2}{3}\right) + 81\,x - 114}{648\,\left(x^{4} - 8\,x^{3} + 42\,x^{2} - 104\,x + 169\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+13)^3,x, algorithm="fricas")
```

```
[Out] 1/648*(3*x^3 - 18*x^2 + (x^4 - 8*x^3 + 42*x^2 - 104*x + 169)*arctan(1/3*x -
 2/3) + 81*x - 114)/(x^4 - 8*x^3 + 42*x^2 - 104*x + 169)
```

Sympy [A] time = 0.152098, size = 42, normalized size = 0.82

$$\frac{x^{3} - 6x^{2} + 27x - 38}{216x^{4} - 1728x^{3} + 9072x^{2} - 22464x + 36504} + \frac{\operatorname{atan}\left(\frac{x}{3} - \frac{2}{3}\right)}{648}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-4*x+13)**3,x)
```

```
[Out] (x**3 - 6*x**2 + 27*x - 38)/(216*x**4 - 1728*x**3 + 9072*x**2 - 22464*x + 3
6504) + atan(x/3 - 2/3)/648
```

Giac [A] time = 1.05459, size = 46, normalized size = 0.9

$$\frac{x^{3} - 6\,x^{2} + 27\,x - 38}{216\,\left(x^{2} - 4\,x + 13\right)^{2}} + \frac{1}{648}\,\arctan\left(\frac{1}{3}\,x - \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+13)^3,x, algorithm="giac")
```

```
[Out] 1/216*(x^3 - 6*x^2 + 27*x - 38)/(x^2 - 4*x + 13)^2 + 1/648*arctan(1/3*x - 2
/3)
```

3.165 $\int \frac{1}{(2+x)^3(3+x)^4}\,dx$

Optimal. Leaf size=54

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

```
[Out] -1/(2*(2 + x)^2) + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x
) + 10*Log[2 + x] - 10*Log[3 + x]
```

Rubi [A] time = 0.0257839, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + x)^3*(3 + x)^4),x]
```

```
[Out] -1/(2*(2 + x)^2) + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x
) + 10*Log[2 + x] - 10*Log[3 + x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2+x)^3(3+x)^4}\,dx &= \int \left(\frac{1}{(2+x)^3}-\frac{4}{(2+x)^2}+\frac{10}{2+x}-\frac{1}{(3+x)^4}-\frac{3}{(3+x)^3}-\frac{6}{(3+x)^2}-\frac{10}{3+x}\right)dx\\ &= -\frac{1}{2(2+x)^2}+\frac{4}{2+x}+\frac{1}{3(3+x)^3}+\frac{3}{2(3+x)^2}+\frac{6}{3+x}+10\log(2+x)-10\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.0145087, size = 54, normalized size = 1.

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + x)^3*(3 + x)^4),x]
```

```
[Out] -1/(2*(2 + x)^2) + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x
) + 10*Log[2 + x] - 10*Log[3 + x]
```

Maple [A] time = 0.01, size = 49, normalized size = 0.9

$$-\frac{1}{2\left(2+x\right)^{2}}+4\left(2+x\right)^{-1}+\frac{1}{3\left(3+x\right)^{3}}+\frac{3}{2\left(3+x\right)^{2}}+6\left(3+x\right)^{-1}+10\ln\left(2+x\right)-10\ln\left(3+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+x)^3/(3+x)^4,x)
```

```
[Out] -1/2/(2+x)^2+4/(2+x)+1/3/(3+x)^3+3/2/(3+x)^2+6/(3+x)+10*ln(2+x)-10*ln(3+x)
```

Maxima [A] time = 0.928574, size = 81, normalized size = 1.5

$$\frac{60\,x^{4}+630\,x^{3}+2450\,x^{2}+4175\,x+2627}{6\left(x^{5}+13\,x^{4}+67\,x^{3}+171\,x^{2}+216\,x+108\right)}-10\,\log\left(x+3\right)+10\,\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 +
171*x^2 + 216*x + 108) - 10*log(x + 3) + 10*log(x + 2)
```

Fricas [B] time = 2.26756, size = 306, normalized size = 5.67

$$\frac{60\,x^{4}+630\,x^{3}+2450\,x^{2}-60\left(x^{5}+13\,x^{4}+67\,x^{3}+171\,x^{2}+216\,x+108\right)\log\left(x+3\right)+60\left(x^{5}+13\,x^{4}+67\,x^{3}+171\,x^{2}+\right.}{6\left(x^{5}+13\,x^{4}+67\,x^{3}+171\,x^{2}+216\,x+108\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 - 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 21
6*x + 108)*log(x + 3) + 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108)*
log(x + 2) + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108)
```

Sympy [A] time = 0.159292, size = 58, normalized size = 1.07

$$\frac{60x^{4}+630x^{3}+2450x^{2}+4175x+2627}{6x^{5}+78x^{4}+402x^{3}+1026x^{2}+1296x+648}+10\log\left(x+2\right)-10\log\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)**3/(3+x)**4,x)
```

```
[Out] (60*x**4 + 630*x**3 + 2450*x**2 + 4175*x + 2627)/(6*x**5 + 78*x**4 + 402*x*
*3 + 1026*x**2 + 1296*x + 648) + 10*log(x + 2) - 10*log(x + 3)
```

Giac [A] time = 1.05268, size = 63, normalized size = 1.17

$$\frac{60\,x^4 + 630\,x^3 + 2450\,x^2 + 4175\,x + 2627}{6\,(x+3)^3(x+2)^2} - 10\,\log\left(|x+3|\right) + 10\,\log\left(|x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="giac")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/((x + 3)^3*(x + 2)^2) - 1
0*log(abs(x + 3)) + 10*log(abs(x + 2))
```

3.166 $\int \frac{x^6}{\left(-2+x^2\right)^2} dx$

Optimal. Leaf size=36

$$\frac{x^3}{3} - \frac{2x}{x^2-2} + 4x - 5\sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 4*x + x^3/3 - (2*x)/(-2 + x^2) - 5*Sqrt[2]*ArcTanh[x/Sqrt[2]]
```

Rubi [A] time = 0.0147501, antiderivative size = 42, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {288, 302, 207}

$$\frac{x^5}{2\left(2-x^2\right)} + \frac{5x^3}{6} + 5x - 5\sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(-2 + x^2)^2,x]
```

```
[Out] 5*x + (5*x^3)/6 + x^5/(2*(2 - x^2)) - 5*Sqrt[2]*ArcTanh[x/Sqrt[2]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6}{\left(-2+x^2\right)^2} dx &= \frac{x^5}{2\left(2-x^2\right)} + \frac{5}{2}\int \frac{x^4}{-2+x^2} dx \\
&= \frac{x^5}{2\left(2-x^2\right)} + \frac{5}{2}\int \left(2+x^2+\frac{4}{-2+x^2}\right) dx \\
&= 5x + \frac{5x^3}{6} + \frac{x^5}{2\left(2-x^2\right)} + 10\int \frac{1}{-2+x^2} dx \\
&= 5x + \frac{5x^3}{6} + \frac{x^5}{2\left(2-x^2\right)} - 5\sqrt{2}\tanh^{-1}\left(\frac{x}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0354436, size = 53, normalized size = 1.47

$$\frac{x^3}{3}-\frac{2x}{x^2-2}+4x+\frac{5\log\left(\sqrt{2}-x\right)}{\sqrt{2}}-\frac{5\log\left(x+\sqrt{2}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(-2 + x^2)^2,x]
```

```
[Out] 4*x + x^3/3 - (2*x)/(-2 + x^2) + (5*Log[Sqrt[2] - x])/Sqrt[2] - (5*Log[Sqrt
[2] + x])/Sqrt[2]
```

Maple [A] time = 0.006, size = 32, normalized size = 0.9

$$4\,x+\frac{x^3}{3}-2\,\frac{x}{x^2-2}-5\,\mathrm{Artanh}\left(1/2\,x\sqrt{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^2-2)^2,x)
```

```
[Out] 4*x+1/3*x^3-2*x/(x^2-2)-5*arctanh(1/2*x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.4058, size = 54, normalized size = 1.5

$$\frac{1}{3}\,x^3+\frac{5}{2}\,\sqrt{2}\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)+4\,x-\frac{2\,x}{x^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^2-2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 5/2*sqrt(2)*log((x - sqrt(2))/(x + sqrt(2))) + 4*x - 2*x/(x^2 - 2
)
```

Fricas [A] time = 2.07184, size = 136, normalized size = 3.78

$$\frac{2\,x^5+20\,x^3+15\,\sqrt{2}\left(x^2-2\right)\log\left(\frac{x^2-2\,\sqrt{2}x+2}{x^2-2}\right)-60\,x}{6\left(x^2-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^2-2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^5 + 20*x^3 + 15*sqrt(2)*(x^2 - 2)*log((x^2 - 2*sqrt(2)*x + 2)/(x^2
 - 2)) - 60*x)/(x^2 - 2)
```

Sympy [A] time = 0.102646, size = 49, normalized size = 1.36

$$\frac{x^3}{3} + 4x - \frac{2x}{x^2 - 2} + \frac{5\sqrt{2}\log\left(x - \sqrt{2}\right)}{2} - \frac{5\sqrt{2}\log\left(x + \sqrt{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**2-2)**2,x)
```

```
[Out] x**3/3 + 4*x - 2*x/(x**2 - 2) + 5*sqrt(2)*log(x - sqrt(2))/2 - 5*sqrt(2)*lo
g(x + sqrt(2))/2
```

Giac [A] time = 1.054, size = 65, normalized size = 1.81

$$\frac{1}{3}x^3 + \frac{5}{2}\sqrt{2}\log\left(\frac{\left|2x - 2\sqrt{2}\right|}{\left|2x + 2\sqrt{2}\right|}\right) + 4x - \frac{2x}{x^2 - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^2-2)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 5/2*sqrt(2)*log(abs(2*x - 2*sqrt(2))/abs(2*x + 2*sqrt(2))) + 4*x
- 2*x/(x^2 - 2)
```

3.167 $\int \frac{x^8}{\left(4+x^2\right)^4} dx$

Optimal. Leaf size=58

$$-\frac{x^7}{6\left(x^2+4\right)^3}-\frac{7x^5}{24\left(x^2+4\right)^2}-\frac{35x^3}{48\left(x^2+4\right)}+\frac{35x}{16}-\frac{35}{8}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] (35*x)/16 - x^7/(6*(4 + x^2)^3) - (7*x^5)/(24*(4 + x^2)^2) - (35*x^3)/(48*(
4 + x^2)) - (35*ArcTan[x/2])/8
```

Rubi [A] time = 0.0166097, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {288, 321, 203}

$$-\frac{x^7}{6\left(x^2+4\right)^3}-\frac{7x^5}{24\left(x^2+4\right)^2}-\frac{35x^3}{48\left(x^2+4\right)}+\frac{35x}{16}-\frac{35}{8}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(4 + x^2)^4,x]
```

```
[Out] (35*x)/16 - x^7/(6*(4 + x^2)^3) - (7*x^5)/(24*(4 + x^2)^2) - (35*x^3)/(48*(
4 + x^2)) - (35*ArcTan[x/2])/8
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^8}{\left(4+x^2\right)^4}\,dx &= -\frac{x^7}{6\left(4+x^2\right)^3}+\frac{7}{6}\int \frac{x^6}{\left(4+x^2\right)^3}\,dx\\
&= -\frac{x^7}{6\left(4+x^2\right)^3}-\frac{7x^5}{24\left(4+x^2\right)^2}+\frac{35}{24}\int \frac{x^4}{\left(4+x^2\right)^2}\,dx\\
&= -\frac{x^7}{6\left(4+x^2\right)^3}-\frac{7x^5}{24\left(4+x^2\right)^2}-\frac{35x^3}{48\left(4+x^2\right)}+\frac{35}{16}\int \frac{x^2}{4+x^2}\,dx\\
&= \frac{35x}{16}-\frac{x^7}{6\left(4+x^2\right)^3}-\frac{7x^5}{24\left(4+x^2\right)^2}-\frac{35x^3}{48\left(4+x^2\right)}-\frac{35}{4}\int \frac{1}{4+x^2}\,dx\\
&= \frac{35x}{16}-\frac{x^7}{6\left(4+x^2\right)^3}-\frac{7x^5}{24\left(4+x^2\right)^2}-\frac{35x^3}{48\left(4+x^2\right)}-\frac{35}{8}\tan^{-1}\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0190208, size = 40, normalized size = 0.69

$$\frac{x\left(12x^6+231x^4+1120x^2+1680\right)}{12\left(x^2+4\right)^3}-\frac{35}{8}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(4 + x^2)^4,x]
```

```
[Out] (x*(1680 + 1120*x^2 + 231*x^4 + 12*x^6))/(12*(4 + x^2)^3) - (35*ArcTan[x/2]
)/8
```

Maple [A] time = 0.007, size = 32, normalized size = 0.6

$$x-16\,\frac{1}{\left(x^2+4\right)^3}\left(-\frac{29\,x^5}{64}-\frac{17\,x^3}{6}-\frac{19\,x}{4}\right)-\frac{35}{8}\arctan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(x^2+4)^4,x)
```

```
[Out] x-16*(-29/64*x^5-17/6*x^3-19/4*x)/(x^2+4)^3-35/8*arctan(1/2*x)
```

Maxima [A] time = 1.40538, size = 55, normalized size = 0.95

$$x+\frac{87\,x^5+544\,x^3+912\,x}{12\left(x^6+12\,x^4+48\,x^2+64\right)}-\frac{35}{8}\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^2+4)^4,x, algorithm="maxima")
```

```
[Out] x + 1/12*(87*x^5 + 544*x^3 + 912*x)/(x^6 + 12*x^4 + 48*x^2 + 64) - 35/8*arc
tan(1/2*x)
```

Fricas [A] time = 1.87502, size = 166, normalized size = 2.86

$$\frac{24\,x^7+462\,x^5+2240\,x^3-105\,\left(x^6+12\,x^4+48\,x^2+64\right)\arctan\left(\frac{1}{2}\,x\right)+3360\,x}{24\,\left(x^6+12\,x^4+48\,x^2+64\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^2+4)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(24*x^7 + 462*x^5 + 2240*x^3 - 105*(x^6 + 12*x^4 + 48*x^2 + 64)*arctan
(1/2*x) + 3360*x)/(x^6 + 12*x^4 + 48*x^2 + 64)
```

Sympy [A] time = 0.140521, size = 39, normalized size = 0.67

$$x+\frac{87x^5+544x^3+912x}{12x^6+144x^4+576x^2+768}-\frac{35\,\mathrm{atan}\left(\frac{x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(x**2+4)**4,x)
```

```
[Out] x + (87*x**5 + 544*x**3 + 912*x)/(12*x**6 + 144*x**4 + 576*x**2 + 768) - 35
*atan(x/2)/8
```

Giac [A] time = 1.05332, size = 42, normalized size = 0.72

$$x+\frac{87\,x^5+544\,x^3+912\,x}{12\,\left(x^2+4\right)^3}-\frac{35}{8}\arctan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(x^2+4)^4,x, algorithm="giac")
```

```
[Out] x + 1/12*(87*x^5 + 544*x^3 + 912*x)/(x^2 + 4)^3 - 35/8*arctan(1/2*x)
```

3.168 $\int \frac{-4+7x}{(5+2x+3x^2)^2} dx$

Optimal. Leaf size=43

$$-\frac{19x+39}{28(3x^2+2x+5)}-\frac{19\tan^{-1}\left(\frac{3x+1}{\sqrt{14}}\right)}{28\sqrt{14}}$$

```
[Out] -(39 + 19*x)/(28*(5 + 2*x + 3*x^2)) - (19*ArcTan[(1 + 3*x)/Sqrt[14]])/(28*S
qrt[14])
```

Rubi [A] time = 0.0206964, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {638, 618, 204}

$$-\frac{19x+39}{28(3x^2+2x+5)}-\frac{19\tan^{-1}\left(\frac{3x+1}{\sqrt{14}}\right)}{28\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 7*x)/(5 + 2*x + 3*x^2)^2,x]
```

```
[Out] -(39 + 19*x)/(28*(5 + 2*x + 3*x^2)) - (19*ArcTan[(1 + 3*x)/Sqrt[14]])/(28*S
qrt[14])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-4+7x}{(5+2x+3x^2)^2}\,dx &= -\frac{39+19x}{28(5+2x+3x^2)} - \frac{19}{28}\int \frac{1}{5+2x+3x^2}\,dx\\ &= -\frac{39+19x}{28(5+2x+3x^2)} + \frac{19}{14}\,\text{Subst}\left(\int \frac{1}{-56-x^2}\,dx, x, 2+6x\right)\\ &= -\frac{39+19x}{28(5+2x+3x^2)} - \frac{19\tan^{-1}\left(\frac{1+3x}{\sqrt{14}}\right)}{28\sqrt{14}}\end{aligned}$$

Mathematica [A] time = 0.033955, size = 43, normalized size = 1.

$$\frac{-19x-39}{28(3x^2+2x+5)} - \frac{19\tan^{-1}\left(\frac{3x+1}{\sqrt{14}}\right)}{28\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 7*x)/(5 + 2*x + 3*x^2)^2,x]
```

```
[Out] (-39 - 19*x)/(28*(5 + 2*x + 3*x^2)) - (19*ArcTan[(1 + 3*x)/Sqrt[14]])/(28*S
qrt[14])
```

Maple [A] time = 0.004, size = 37, normalized size = 0.9

$$\frac{-38\,x-78}{168\,x^2+112\,x+280} - \frac{19\sqrt{14}}{392}\arctan\left(\frac{(6\,x+2)\sqrt{14}}{28}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4+7*x)/(3*x^2+2*x+5)^2,x)
```

```
[Out] 1/56*(-38*x-78)/(3*x^2+2*x+5)-19/392*14^(1/2)*arctan(1/28*(6*x+2)*14^(1/2))
```

Maxima [A] time = 1.40944, size = 49, normalized size = 1.14

$$-\frac{19}{392}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(3\,x+1)\right) - \frac{19\,x+39}{28\,(3\,x^2+2\,x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+7*x)/(3*x^2+2*x+5)^2,x, algorithm="maxima")
```

```
[Out] -19/392*sqrt(14)*arctan(1/14*sqrt(14)*(3*x + 1)) - 1/28*(19*x + 39)/(3*x^2
+ 2*x + 5)
```

Fricas [A] time = 1.96407, size = 140, normalized size = 3.26

$$-\frac{19\sqrt{14}(3\,x^2+2\,x+5)\arctan\left(\frac{1}{14}\sqrt{14}(3\,x+1)\right)+266\,x+546}{392\,(3\,x^2+2\,x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+7*x)/(3*x^2+2*x+5)^2,x, algorithm="fricas")
```

```
[Out] -1/392*(19*sqrt(14)*(3*x^2 + 2*x + 5)*arctan(1/14*sqrt(14)*(3*x + 1)) + 266
*x + 546)/(3*x^2 + 2*x + 5)
```

Sympy [A] time = 0.128665, size = 42, normalized size = 0.98

$$-\frac{19x+39}{84x^2+56x+140}-\frac{19\sqrt{14}\,\mathrm{atan}\left(\frac{3\sqrt{14}x}{14}+\frac{\sqrt{14}}{14}\right)}{392}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+7*x)/(3*x**2+2*x+5)**2,x)
```

```
[Out] -(19*x + 39)/(84*x**2 + 56*x + 140) - 19*sqrt(14)*atan(3*sqrt(14)*x/14 + sq
rt(14)/14)/392
```

Giac [A] time = 1.06648, size = 49, normalized size = 1.14

$$-\frac{19}{392}\sqrt{14}\arctan\left(\frac{1}{14}\sqrt{14}(3x+1)\right)-\frac{19x+39}{28\left(3x^2+2x+5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4+7*x)/(3*x^2+2*x+5)^2,x, algorithm="giac")
```

```
[Out] -19/392*sqrt(14)*arctan(1/14*sqrt(14)*(3*x + 1)) - 1/28*(19*x + 39)/(3*x^2
+ 2*x + 5)
```

3.169 $\int \frac{5-4x}{\left(-2-4x+3x^2\right)^2} \, dx$

Optimal. Leaf size=43

$$-\frac{18-7x}{20\left(-3x^2+4x+2\right)}-\frac{7\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

```
[Out] -(18 - 7*x)/(20*(2 + 4*x - 3*x^2)) - (7*ArcTanh[(2 - 3*x)/Sqrt[10]])/(20*Sq
rt[10])
```

Rubi [A] time = 0.0230899, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {638, 618, 206}

$$-\frac{18-7x}{20\left(-3x^2+4x+2\right)}-\frac{7\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 4*x)/(-2 - 4*x + 3*x^2)^2,x]
```

```
[Out] -(18 - 7*x)/(20*(2 + 4*x - 3*x^2)) - (7*ArcTanh[(2 - 3*x)/Sqrt[10]])/(20*Sq
rt[10])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{5-4x}{\left(-2-4x+3x^2\right)^2}\,dx = -\frac{18-7x}{20\left(2+4x-3x^2\right)} - \frac{7}{20}\int \frac{1}{-2-4x+3x^2}\,dx$$
$$= -\frac{18-7x}{20\left(2+4x-3x^2\right)} + \frac{7}{10}\,\text{Subst}\left(\int \frac{1}{40-x^2}\,dx, x, -4+6x\right)$$
$$= -\frac{18-7x}{20\left(2+4x-3x^2\right)} - \frac{7\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

Mathematica [A] time = 0.0374513, size = 62, normalized size = 1.44

$$\frac{18-7x}{20\left(3x^2-4x-2\right)} - \frac{7\log\left(-3x+\sqrt{10}+2\right)}{40\sqrt{10}} + \frac{7\log\left(3x+\sqrt{10}-2\right)}{40\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x)/(-2 - 4*x + 3*x^2)^2,x]
```

```
[Out] (18 - 7*x)/(20*(-2 - 4*x + 3*x^2)) - (7*Log[2 + Sqrt[10] - 3*x])/(40*Sqrt[1
0]) + (7*Log[-2 + Sqrt[10] + 3*x])/(40*Sqrt[10])
```

Maple [A] time = 0.003, size = 37, normalized size = 0.9

$$-\frac{14\,x-36}{120\,x^2-160\,x-80} + \frac{7\sqrt{10}}{200}\text{Artanh}\left(\frac{(6\,x-4)\sqrt{10}}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5-4*x)/(3*x^2-4*x-2)^2,x)
```

```
[Out] -1/40*(14*x-36)/(3*x^2-4*x-2)+7/200*10^(1/2)*arctanh(1/20*(6*x-4)*10^(1/2))
```

Maxima [A] time = 1.47678, size = 63, normalized size = 1.47

$$-\frac{7}{400}\sqrt{10}\log\left(\frac{3\,x-\sqrt{10}-2}{3\,x+\sqrt{10}-2}\right) - \frac{7\,x-18}{20\left(3\,x^2-4\,x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5-4*x)/(3*x^2-4*x-2)^2,x, algorithm="maxima")
```

```
[Out] -7/400*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2)) - 1/20*(7*x
 - 18)/(3*x^2 - 4*x - 2)
```

Fricas [A] time = 1.98568, size = 184, normalized size = 4.28

$$\frac{7\sqrt{10}\left(3\,x^2-4\,x-2\right)\log\left(\frac{9\,x^2+2\sqrt{10}(3\,x-2)-12\,x+14}{3\,x^2-4\,x-2}\right) - 140\,x + 360}{400\left(3\,x^2-4\,x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5-4*x)/(3*x^2-4*x-2)^2,x, algorithm="fricas")
```

```
[Out] 1/400*(7*sqrt(10)*(3*x^2 - 4*x - 2)*log((9*x^2 + 2*sqrt(10)*(3*x - 2) - 12*
x + 14)/(3*x^2 - 4*x - 2)) - 140*x + 360)/(3*x^2 - 4*x - 2)
```

Sympy [A] time = 0.124447, size = 58, normalized size = 1.35

$$-\frac{7x-18}{60x^2-80x-40}+\frac{7\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{400}-\frac{7\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{400}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5-4*x)/(3*x**2-4*x-2)**2,x)
```

```
[Out] -(7*x - 18)/(60*x**2 - 80*x - 40) + 7*sqrt(10)*log(x - 2/3 + sqrt(10)/3)/40
0 - 7*sqrt(10)*log(x - sqrt(10)/3 - 2/3)/400
```

Giac [A] time = 1.06143, size = 69, normalized size = 1.6

$$-\frac{7}{400}\sqrt{10}\log\left(\frac{\left|6x-2\sqrt{10}-4\right|}{\left|6x+2\sqrt{10}-4\right|}\right)-\frac{7x-18}{20\left(3x^2-4x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5-4*x)/(3*x^2-4*x-2)^2,x, algorithm="giac")
```

```
[Out] -7/400*sqrt(10)*log(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10) - 4)) -
1/20*(7*x - 18)/(3*x^2 - 4*x - 2)
```

3.170 $\int \frac{x^5}{\left(1+x^4\right)^3} \, dx$

Optimal. Leaf size=37

$$\frac{x^2}{16\left(x^4+1\right)}-\frac{x^2}{8\left(x^4+1\right)^2}+\frac{1}{16}\tan^{-1}\left(x^2\right)$$

```
[Out] -x^2/(8*(1 + x^4)^2) + x^2/(16*(1 + x^4)) + ArcTan[x^2]/16
```

Rubi [A] time = 0.0129613, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {275, 288, 199, 203}

$$\frac{x^2}{16\left(x^4+1\right)}-\frac{x^2}{8\left(x^4+1\right)^2}+\frac{1}{16}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + x^4)^3,x]

[Out] -x^2/(8*(1 + x^4)^2) + x^2/(16*(1 + x^4)) + ArcTan[x^2]/16
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^5}{\left(1+x^4\right)^3}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^3}\,dx, x, x^2\right)$$
$$= -\frac{x^2}{8\left(1+x^4\right)^2} + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2}\,dx, x, x^2\right)$$
$$= -\frac{x^2}{8\left(1+x^4\right)^2} + \frac{x^2}{16\left(1+x^4\right)} + \frac{1}{16}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, x^2\right)$$
$$= -\frac{x^2}{8\left(1+x^4\right)^2} + \frac{x^2}{16\left(1+x^4\right)} + \frac{1}{16}\tan^{-1}\left(x^2\right)$$

Mathematica [A] time = 0.0104496, size = 25, normalized size = 0.68

$$\frac{1}{16}\left(\frac{\left(x^4-1\right)x^2}{\left(x^4+1\right)^2} + \tan^{-1}\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + x^4)^3,x]
```

```
[Out] ((x^2*(-1 + x^4))/(1 + x^4)^2 + ArcTan[x^2])/16
```

Maple [A] time = 0.007, size = 28, normalized size = 0.8

$$\frac{1}{2\left(x^4+1\right)^2}\left(\frac{x^6}{8} - \frac{x^2}{8}\right) + \frac{\arctan\left(x^2\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^4+1)^3,x)
```

```
[Out] 1/2*(1/8*x^6-1/8*x^2)/(x^4+1)^2+1/16*arctan(x^2)
```

Maxima [A] time = 1.44623, size = 41, normalized size = 1.11

$$\frac{x^6-x^2}{16\left(x^8+2x^4+1\right)} + \frac{1}{16}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(x^6 - x^2)/(x^8 + 2*x^4 + 1) + 1/16*arctan(x^2)
```

Fricas [A] time = 1.94351, size = 92, normalized size = 2.49

$$\frac{x^6 - x^2 + \left(x^8 + 2\,x^4 + 1\right)\arctan\left(x^2\right)}{16\left(x^8 + 2\,x^4 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(x^6 - x^2 + (x^8 + 2*x^4 + 1)*arctan(x^2))/(x^8 + 2*x^4 + 1)
```

Sympy [A] time = 0.148692, size = 24, normalized size = 0.65

$$\frac{x^6 - x^2}{16x^8 + 32x^4 + 16} + \frac{\operatorname{atan}\left(x^2\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**4+1)**3,x)
```

```
[Out] (x**6 - x**2)/(16*x**8 + 32*x**4 + 16) + atan(x**2)/16
```

Giac [A] time = 1.06708, size = 54, normalized size = 1.46

$$\frac{x^2 - \frac{1}{x^2}}{16\left(\left(x^2 - \frac{1}{x^2}\right)^2 + 4\right)} + \frac{1}{32}\arctan\left(\frac{x^4 - 1}{2\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1)^3,x, algorithm="giac")
```

```
[Out] 1/16*(x^2 - 1/x^2)/((x^2 - 1/x^2)^2 + 4) + 1/32*arctan(1/2*(x^4 - 1)/x^2)
```

3.171 $\int \frac{x(1+x^2)^3}{(2+2x^2+x^4)^2} \, dx$

Optimal. Leaf size=32

$$\frac{1}{4(x^4+2x^2+2)}+\frac{1}{4}\log\left(x^4+2x^2+2\right)$$

```
[Out] 1/(4*(2 + 2*x^2 + x^4)) + Log[2 + 2*x^2 + x^4]/4
```

Rubi [A] time = 0.0263224, antiderivative size = 39, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1247, 686, 628}

$$\frac{1}{4}\log\left(x^4+2x^2+2\right)-\frac{(x^2+1)^2}{4(x^4+2x^2+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + x^2)^3)/(2 + 2*x^2 + x^4)^2,x]
```

```
[Out] -(1 + x^2)^2/(4*(2 + 2*x^2 + x^4)) + Log[2 + 2*x^2 + x^4]/4
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 686

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Simp[(d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)),
x] - Dist[(d*e*(m - 1))/(b*(p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2
)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Eq
Q[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] && GtQ[m, 1] && Inte
gerQ[2*p]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{x\left(1+x^2\right)^3}{\left(2+2x^2+x^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(1+x)^3}{\left(2+2x+x^2\right)^2}\,dx, x, x^2\right)$$
$$= -\frac{\left(1+x^2\right)^2}{4\left(2+2x^2+x^4\right)} + \frac{1}{2}\,\text{Subst}\left(\int \frac{1+x}{2+2x+x^2}\,dx, x, x^2\right)$$
$$= -\frac{\left(1+x^2\right)^2}{4\left(2+2x^2+x^4\right)} + \frac{1}{4}\log\left(2+2x^2+x^4\right)$$

Mathematica [A] time = 0.0127078, size = 26, normalized size = 0.81

$$\frac{1}{4}\left(\frac{1}{\left(x^2+1\right)^2+1} + \log\left(\left(x^2+1\right)^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + x^2)^3)/(2 + 2*x^2 + x^4)^2,x]
```

```
[Out] ((1 + (1 + x^2)^2)^(-1) + Log[1 + (1 + x^2)^2])/4
```

Maple [A] time = 0.007, size = 29, normalized size = 0.9

$$\frac{1}{4\,x^4+8\,x^2+8} + \frac{\ln\left(x^4+2\,x^2+2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x)
```

```
[Out] 1/4/(x^4+2*x^2+2)+1/4*ln(x^4+2*x^2+2)
```

Maxima [A] time = 0.949625, size = 38, normalized size = 1.19

$$\frac{1}{4\left(x^4+2\,x^2+2\right)} + \frac{1}{4}\,\log\left(x^4+2\,x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 1/4/(x^4 + 2*x^2 + 2) + 1/4*log(x^4 + 2*x^2 + 2)
```

Fricas [A] time = 1.9647, size = 92, normalized size = 2.88

$$\frac{\left(x^4+2\,x^2+2\right)\log\left(x^4+2\,x^2+2\right)+1}{4\left(x^4+2\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((x^4 + 2*x^2 + 2)*log(x^4 + 2*x^2 + 2) + 1)/(x^4 + 2*x^2 + 2)
```

Sympy [A] time = 0.128267, size = 26, normalized size = 0.81

$$\frac{\log\left(x^4+2x^2+2\right)}{4}+\frac{1}{4x^4+8x^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**3/(x**4+2*x**2+2)**2,x)
```

```
[Out] log(x**4 + 2*x**2 + 2)/4 + 1/(4*x**4 + 8*x**2 + 8)
```

Giac [A] time = 1.05774, size = 38, normalized size = 1.19

$$\frac{1}{4\left(x^4+2\,x^2+2\right)}+\frac{1}{4}\,\log\left(x^4+2\,x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^3/(x^4+2*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 1/4/(x^4 + 2*x^2 + 2) + 1/4*log(x^4 + 2*x^2 + 2)
```

3.172 $\int \frac{x^3}{\left(a^4+x^4\right)^3} dx$

Optimal. Leaf size=13

$$-\frac{1}{8\left(a^4+x^4\right)^2}$$

[Out] -1/(8*(a^4 + x^4)^2)

Rubi [A] time = 0.0023815, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{8\left(a^4+x^4\right)^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a^4 + x^4)^3,x]

[Out] -1/(8*(a^4 + x^4)^2)

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^3}{\left(a^4+x^4\right)^3} dx = -\frac{1}{8\left(a^4+x^4\right)^2}$$

Mathematica [A] time = 0.0034365, size = 13, normalized size = 1.

$$-\frac{1}{8\left(a^4+x^4\right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a^4 + x^4)^3,x]

[Out] -1/(8*(a^4 + x^4)^2)

Maple [A] time = 0., size = 12, normalized size = 0.9

$$-\frac{1}{8\left(a^4+x^4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a^4+x^4)^3,x)
```

```
[Out] -1/8/(a^4+x^4)^2
```

Maxima [A] time = 0.921507, size = 15, normalized size = 1.15

$$-\frac{1}{8\left(a^4+x^4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^4+x^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8/(a^4 + x^4)^2
```

Fricas [A] time = 1.87433, size = 41, normalized size = 3.15

$$-\frac{1}{8\left(a^8+2\,a^4x^4+x^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^4+x^4)^3,x, algorithm="fricas")
```

```
[Out] -1/8/(a^8 + 2*a^4*x^4 + x^8)
```

Sympy [A] time = 0.795548, size = 20, normalized size = 1.54

$$-\frac{1}{8a^8+16a^4x^4+8x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a**4+x**4)**3,x)
```

```
[Out] -1/(8*a**8 + 16*a**4*x**4 + 8*x**8)
```

Giac [A] time = 1.0583, size = 15, normalized size = 1.15

$$-\frac{1}{8\left(a^4+x^4\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a^4+x^4)^3,x, algorithm="giac")
```

```
[Out] -1/8/(a^4 + x^4)^2
```

3.173 $\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx$

Optimal. Leaf size=54

$$\frac{1}{4a^8\left(a^4+x^4\right)}+\frac{1}{8a^4\left(a^4+x^4\right)^2}-\frac{\log\left(a^4+x^4\right)}{4a^{12}}+\frac{\log(x)}{a^{12}}$$

```
[Out] 1/(8*a^4*(a^4 + x^4)^2) + 1/(4*a^8*(a^4 + x^4)) + Log[x]/a^12 - Log[a^4 + x
^4]/(4*a^12)
```

Rubi [A] time = 0.0302035, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 44}

$$\frac{1}{4a^8\left(a^4+x^4\right)}+\frac{1}{8a^4\left(a^4+x^4\right)^2}-\frac{\log\left(a^4+x^4\right)}{4a^{12}}+\frac{\log(x)}{a^{12}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a^4 + x^4)^3),x]
```

```
[Out] 1/(8*a^4*(a^4 + x^4)^2) + 1/(4*a^8*(a^4 + x^4)) + Log[x]/a^12 - Log[a^4 + x
^4]/(4*a^12)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(a^4+x^4\right)^3}\,dx &= \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\left(a^4+x\right)^3}\,dx,x,x^4\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \left(\frac{1}{a^{12}x}-\frac{1}{a^4\left(a^4+x\right)^3}-\frac{1}{a^8\left(a^4+x\right)^2}-\frac{1}{a^{12}\left(a^4+x\right)}\right)dx,x,x^4\right)\\ &= \frac{1}{8a^4\left(a^4+x^4\right)^2}+\frac{1}{4a^8\left(a^4+x^4\right)}+\frac{\log(x)}{a^{12}}-\frac{\log\left(a^4+x^4\right)}{4a^{12}}\end{aligned}$$

Mathematica [A] time = 0.0243026, size = 46, normalized size = 0.85

$$\frac{\frac{2a^4x^4+3a^8}{(a^4+x^4)^2}-2\log\left(a^4+x^4\right)+8\log(x)}{8a^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a^4 + x^4)^3),x]
```

```
[Out] ((3*a^8 + 2*a^4*x^4)/(a^4 + x^4)^2 + 8*Log[x] - 2*Log[a^4 + x^4])/(8*a^12)
```

Maple [A] time = 0.016, size = 49, normalized size = 0.9

$$\frac{1}{8\,a^4\left(a^4+x^4\right)^2}+\frac{1}{4\,a^8\left(a^4+x^4\right)}+\frac{\ln(x)}{a^{12}}-\frac{\ln\left(a^4+x^4\right)}{4\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^4+x^4)^3,x)
```

```
[Out] 1/8/a^4/(a^4+x^4)^2+1/4/a^8/(a^4+x^4)+ln(x)/a^12-1/4*ln(a^4+x^4)/a^12
```

Maxima [A] time = 0.927591, size = 77, normalized size = 1.43

$$\frac{3\,a^4+2\,x^4}{8\left(a^{16}+2\,a^{12}x^4+a^8x^8\right)}-\frac{\log\left(a^4+x^4\right)}{4\,a^{12}}+\frac{\log\left(x^4\right)}{4\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^4+x^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*a^4 + 2*x^4)/(a^16 + 2*a^12*x^4 + a^8*x^8) - 1/4*log(a^4 + x^4)/a^12
 + 1/4*log(x^4)/a^12
```

Fricas [A] time = 1.9711, size = 181, normalized size = 3.35

$$\frac{3\,a^8+2\,a^4x^4-2\left(a^8+2\,a^4x^4+x^8\right)\log\left(a^4+x^4\right)+8\left(a^8+2\,a^4x^4+x^8\right)\log(x)}{8\left(a^{20}+2\,a^{16}x^4+a^{12}x^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^4+x^4)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(3*a^8 + 2*a^4*x^4 - 2*(a^8 + 2*a^4*x^4 + x^8)*log(a^4 + x^4) + 8*(a^8
+ 2*a^4*x^4 + x^8)*log(x))/(a^20 + 2*a^16*x^4 + a^12*x^8)
```

Sympy [A] time = 2.21095, size = 51, normalized size = 0.94

$$\frac{3a^4 + 2x^4}{8a^{16} + 16a^{12}x^4 + 8a^8x^8} + \frac{\log(x)}{a^{12}} - \frac{\log(a^4 + x^4)}{4a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**4+x**4)**3,x)
```

```
[Out] (3*a**4 + 2*x**4)/(8*a**16 + 16*a**12*x**4 + 8*a**8*x**8) + log(x)/a**12 -
log(a**4 + x**4)/(4*a**12)
```

Giac [A] time = 1.05378, size = 76, normalized size = 1.41

$$-\frac{\log(a^4 + x^4)}{4\,a^{12}} + \frac{\log(x^4)}{4\,a^{12}} + \frac{6\,a^8 + 8\,a^4x^4 + 3\,x^8}{8\,(a^4 + x^4)^2\,a^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^4+x^4)^3,x, algorithm="giac")
```

```
[Out] -1/4*log(a^4 + x^4)/a^12 + 1/4*log(x^4)/a^12 + 1/8*(6*a^8 + 8*a^4*x^4 + 3*x
^8)/((a^4 + x^4)^2*a^12)
```

3.174 $\int \frac{1}{x^2\left(a^4+x^4\right)^3} dx$

Optimal. Leaf size=157

$$\frac{9}{32a^8x\left(a^4+x^4\right)}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}-\frac{45\log\left(a^2-\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}+\frac{45\log\left(a^2+\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}-\frac{45}{32a^{12}x}+\frac{45\tan^{-1}\left(1-\right.}{64\sqrt{2}a^{13}}$$

```
[Out] -45/(32*a^12*x) + 1/(8*a^4*x*(a^4 + x^4)^2) + 9/(32*a^8*x*(a^4 + x^4)) + (4
5*ArcTan[1 - (Sqrt[2]*x)/a])/(64*Sqrt[2]*a^13) - (45*ArcTan[1 + (Sqrt[2]*x)
/a])/(64*Sqrt[2]*a^13) - (45*Log[a^2 - Sqrt[2]*a*x + x^2])/(128*Sqrt[2]*a^1
3) + (45*Log[a^2 + Sqrt[2]*a*x + x^2])/(128*Sqrt[2]*a^13)
```

Rubi [A] time = 0.100967, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {290, 325, 297, 1162, 617, 204, 1165, 628}

$$\frac{9}{32a^8x\left(a^4+x^4\right)}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}-\frac{45\log\left(a^2-\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}+\frac{45\log\left(a^2+\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}-\frac{45}{32a^{12}x}+\frac{45\tan^{-1}\left(1-\right.}{64\sqrt{2}a^{13}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a^4 + x^4)^3),x]
```

```
[Out] -45/(32*a^12*x) + 1/(8*a^4*x*(a^4 + x^4)^2) + 9/(32*a^8*x*(a^4 + x^4)) + (4
5*ArcTan[1 - (Sqrt[2]*x)/a])/(64*Sqrt[2]*a^13) - (45*ArcTan[1 + (Sqrt[2]*x)
/a])/(64*Sqrt[2]*a^13) - (45*Log[a^2 - Sqrt[2]*a*x + x^2])/(128*Sqrt[2]*a^1
3) + (45*Log[a^2 + Sqrt[2]*a*x + x^2])/(128*Sqrt[2]*a^13)
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{x^2\left(a^4+x^4\right)^3}\,dx = \frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9\int \frac{1}{x^2\left(a^4+x^4\right)^2}\,dx}{8a^4}$$

$$= \frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}+\frac{45\int \frac{1}{x^2\left(a^4+x^4\right)}\,dx}{32a^8}$$

$$= -\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}-\frac{45\int \frac{x^2}{a^4+x^4}\,dx}{32a^{12}}$$

$$= -\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}+\frac{45\int \frac{a^2-x^2}{a^4+x^4}\,dx}{64a^{12}}-\frac{45\int \frac{a^2+x^2}{a^4+x^4}\,dx}{64a^{12}}$$

$$= -\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}-\frac{45\int \frac{\sqrt{2}a+2x}{-a^2-\sqrt{2}ax-x^2}\,dx}{128\sqrt{2}a^{13}}-\frac{45\int \frac{\sqrt{2}a-2x}{-a^2+\sqrt{2}ax-x^2}\,dx}{128\sqrt{2}a^{13}}-\frac{45\int}{}$$

$$= -\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}-\frac{45\log\left(a^2-\sqrt{2}ax+x^2\right)}{128\sqrt{2}a^{13}}+\frac{45\log\left(a^2+\sqrt{2}ax+x\right.}{128\sqrt{2}a^{13}}$$

$$= -\frac{45}{32a^{12}x}+\frac{1}{8a^4x\left(a^4+x^4\right)^2}+\frac{9}{32a^8x\left(a^4+x^4\right)}+\frac{45\tan^{-1}\left(1-\frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}}-\frac{45\tan^{-1}\left(1+\frac{\sqrt{2}x}{a}\right)}{64\sqrt{2}a^{13}}-45\,\text{lo}$$

Mathematica [A] time = 0.105094, size = 134, normalized size = 0.85

$$-\frac{\frac{32a^5x^3}{(a^4+x^4)^2}+\frac{104ax^3}{a^4+x^4}+45\sqrt{2}\log\left(a^2-\sqrt{2}ax+x^2\right)-45\sqrt{2}\log\left(a^2+\sqrt{2}ax+x^2\right)+\frac{256a}{x}-90\sqrt{2}\tan^{-1}\left(1-\frac{\sqrt{2}x}{a}\right)+90\sqrt{2}}{256a^{13}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a^4 + x^4)^3),x]
```

```
[Out] -((256*a)/x + (32*a^5*x^3)/(a^4 + x^4)^2 + (104*a*x^3)/(a^4 + x^4) - 90*Sqr
t[2]*ArcTan[1 - (Sqrt[2]*x)/a] + 90*Sqrt[2]*ArcTan[1 + (Sqrt[2]*x)/a] + 45*
Sqrt[2]*Log[a^2 - Sqrt[2]*a*x + x^2] - 45*Sqrt[2]*Log[a^2 + Sqrt[2]*a*x + x
^2])/(256*a^13)
```

Maple [A] time = 0.012, size = 152, normalized size = 1.

$$-\frac{1}{a^{12}x}-\frac{13x^7}{32a^{12}\left(a^4+x^4\right)^2}-\frac{17x^3}{32a^8\left(a^4+x^4\right)^2}-\frac{45\sqrt{2}}{256a^{12}}\ln\left(\left(x^2-\sqrt[4]{a^4}x\sqrt{2}+\sqrt{a^4}\right)\left(x^2+\sqrt[4]{a^4}x\sqrt{2}+\sqrt{a^4}\right)^{-1}\right)\frac{1}{\sqrt[4]{a^4}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a^4+x^4)^3,x)
```

```
[Out] -1/a^12/x-13/32/a^12/(a^4+x^4)^2*x^7-17/32/a^8/(a^4+x^4)^2*x^3-45/256/a^12/
(a^4)^(1/4)*2^(1/2)*ln((x^2-(a^4)^(1/4)*x*2^(1/2)+(a^4)^(1/2))/(x^2+(a^4)^(
1/4)*x*2^(1/2)+(a^4)^(1/2)))-45/128/a^12/(a^4)^(1/4)*2^(1/2)*arctan(2^(1/2)
/(a^4)^(1/4)*x+1)-45/128/a^12/(a^4)^(1/4)*2^(1/2)*arctan(2^(1/2)/(a^4)^(1/4
)*x-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^4+x^4)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.1156, size = 987, normalized size = 6.29

$$-\frac{256a^8+648a^4x^4+360x^8-180\sqrt{2}\left(a^{20}x+2a^{16}x^5+a^{12}x^9\right)\left(\frac{1}{a^{52}}\right)^{\frac{1}{4}}\arctan\left(-\sqrt{2}a^{12}\left(\frac{1}{a^{52}}\right)^{\frac{1}{4}}x+\sqrt{2}\sqrt{\sqrt{2}a^{40}\left(\frac{1}{a^{52}}\right)^{\frac{3}{4}}x+a^{28}\sqrt{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^4+x^4)^3,x, algorithm="fricas")
```

```
[Out] -1/256*(256*a^8 + 648*a^4*x^4 + 360*x^8 - 180*sqrt(2)*(a^20*x + 2*a^16*x^5
+ a^12*x^9)*(a^(-52))^(1/4)*arctan(-sqrt(2)*a^12*(a^(-52))^(1/4)*x + sqrt(2
)*sqrt(sqrt(2)*a^40*(a^(-52))^(3/4)*x + a^28*sqrt(a^(-52)) + x^2)*a^12*(a^(
-52))^(1/4) - 1) - 180*sqrt(2)*(a^20*x + 2*a^16*x^5 + a^12*x^9)*(a^(-52))^(
1/4)*arctan(-sqrt(2)*a^12*(a^(-52))^(1/4)*x + sqrt(2)*sqrt(-sqrt(2)*a^40*(a
^(-52))^(3/4)*x + a^28*sqrt(a^(-52)) + x^2)*a^12*(a^(-52))^(1/4) + 1) - 45*
sqrt(2)*(a^20*x + 2*a^16*x^5 + a^12*x^9)*(a^(-52))^(1/4)*log(sqrt(2)*a^40*(
a^(-52))^(3/4)*x + a^28*sqrt(a^(-52)) + x^2) + 45*sqrt(2)*(a^20*x + 2*a^16*
x^5 + a^12*x^9)*(a^(-52))^(1/4)*log(-sqrt(2)*a^40*(a^(-52))^(3/4)*x + a^28*
sqrt(a^(-52)) + x^2))/(a^20*x + 2*a^16*x^5 + a^12*x^9)
```

Sympy [A] time = 3.76579, size = 65, normalized size = 0.41

$$-\frac{32a^8+81a^4x^4+45x^8}{32a^{20}x+64a^{16}x^5+32a^{12}x^9}+\frac{\operatorname{RootSum}\left(268435456t^4+4100625,\left(t\mapsto t\log\left(-\frac{2097152t^3a}{91125}+x\right)\right)\right)}{a^{13}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a**4+x**4)**3,x)
```

```
[Out] -(32*a**8 + 81*a**4*x**4 + 45*x**8)/(32*a**20*x + 64*a**16*x**5 + 32*a**12*
x**9) + RootSum(268435456*_t**4 + 4100625, Lambda(_t, _t*log(-2097152*_t**3
*a/91125 + x)))/a**13
```

Giac [A] time = 1.06128, size = 203, normalized size = 1.29

$$-\frac{45\sqrt{2}|a|\arctan\left(\frac{\sqrt{2}(\sqrt{2}|a|+2x)}{2|a|}\right)}{128a^{14}}-\frac{45\sqrt{2}|a|\arctan\left(-\frac{\sqrt{2}(\sqrt{2}|a|-2x)}{2|a|}\right)}{128a^{14}}+\frac{45\sqrt{2}|a|\log\left(\sqrt{2}x|a|+x^2+|a|^2\right)}{256a^{14}}-\frac{45\sqrt{2}|a|\log\left(-\right.}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a^4+x^4)^3,x, algorithm="giac")
```

```
[Out] -45/128*sqrt(2)*abs(a)*arctan(1/2*sqrt(2)*(sqrt(2)*abs(a) + 2*x)/abs(a))/a^
14 - 45/128*sqrt(2)*abs(a)*arctan(-1/2*sqrt(2)*(sqrt(2)*abs(a) - 2*x)/abs(a
))/a^14 + 45/256*sqrt(2)*abs(a)*log(sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^14
 - 45/256*sqrt(2)*abs(a)*log(-sqrt(2)*x*abs(a) + x^2 + abs(a)^2)/a^14 - 1/3
2*(17*a^4*x^3 + 13*x^7)/((a^4 + x^4)^2*a^12) - 1/(a^12*x)
```

3.175 $\int \frac{1}{x^3\left(a^4+x^4\right)^3}\,dx$

Optimal. Leaf size=64

$$\frac{5}{16a^8x^2\left(a^4+x^4\right)}-\frac{15}{16a^{12}x^2}+\frac{1}{8a^4x^2\left(a^4+x^4\right)^2}-\frac{15\tan^{-1}\left(\frac{x^2}{a^2}\right)}{16a^{14}}$$

```
[Out] -15/(16*a^12*x^2) + 1/(8*a^4*x^2*(a^4 + x^4)^2) + 5/(16*a^8*x^2*(a^4 + x^4)
) - (15*ArcTan[x^2/a^2])/(16*a^14)
```

Rubi [A] time = 0.0302262, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {275, 290, 325, 203}

$$\frac{5}{16a^8x^2\left(a^4+x^4\right)}-\frac{15}{16a^{12}x^2}+\frac{1}{8a^4x^2\left(a^4+x^4\right)^2}-\frac{15\tan^{-1}\left(\frac{x^2}{a^2}\right)}{16a^{14}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a^4 + x^4)^3),x]
```

```
[Out] -15/(16*a^12*x^2) + 1/(8*a^4*x^2*(a^4 + x^4)^2) + 5/(16*a^8*x^2*(a^4 + x^4)
) - (15*ArcTan[x^2/a^2])/(16*a^14)
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^3\left(a^4+x^4\right)^3}\,dx=\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(a^4+x^2\right)^3}\,dx,x,x^2\right)$$

$$=\frac{1}{8a^4x^2\left(a^4+x^4\right)^2}+\frac{5\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(a^4+x^2\right)^2}\,dx,x,x^2\right)}{8a^4}$$

$$=\frac{1}{8a^4x^2\left(a^4+x^4\right)^2}+\frac{5}{16a^8x^2\left(a^4+x^4\right)}+\frac{15\,\mathrm{Subst}\left(\int \frac{1}{x^2\left(a^4+x^2\right)}\,dx,x,x^2\right)}{16a^8}$$

$$=-\frac{15}{16a^{12}x^2}+\frac{1}{8a^4x^2\left(a^4+x^4\right)^2}+\frac{5}{16a^8x^2\left(a^4+x^4\right)}-\frac{15\,\mathrm{Subst}\left(\int \frac{1}{a^4+x^2}\,dx,x,x^2\right)}{16a^{12}}$$

$$=-\frac{15}{16a^{12}x^2}+\frac{1}{8a^4x^2\left(a^4+x^4\right)^2}+\frac{5}{16a^8x^2\left(a^4+x^4\right)}-\frac{15\tan^{-1}\left(\frac{x^2}{a^2}\right)}{16a^{14}}$$

Mathematica [A] time = 0.0478722, size = 75, normalized size = 1.17

$$\frac{-\frac{a^2\left(25a^4x^4+8a^8+15x^8\right)}{x^2\left(a^4+x^4\right)^2}+15\tan^{-1}\left(1-\frac{\sqrt{2}x}{a}\right)+15\tan^{-1}\left(\frac{\sqrt{2}x}{a}+1\right)}{16a^{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a^4 + x^4)^3),x]
```

```
[Out] (-((a^2*(8*a^8 + 25*a^4*x^4 + 15*x^8))/(x^2*(a^4 + x^4)^2)) + 15*ArcTan[1 -
 (Sqrt[2]*x)/a] + 15*ArcTan[1 + (Sqrt[2]*x)/a])/(16*a^14)
```

Maple [A] time = 0.013, size = 57, normalized size = 0.9

$$-\frac{1}{2\,a^{12}x^2}-\frac{9\,x^2}{16\,a^8\left(a^4+x^4\right)^2}-\frac{7\,x^6}{16\,a^{12}\left(a^4+x^4\right)^2}-\frac{15}{16\,a^{14}}\arctan\left(\frac{x^2}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a^4+x^4)^3,x)
```

```
[Out] -1/2/a^12/x^2-9/16/a^8/(a^4+x^4)^2*x^2-7/16/a^12/(a^4+x^4)^2*x^6-15/16*arct
an(x^2/a^2)/a^14
```

Maxima [A] time = 1.41359, size = 81, normalized size = 1.27

$$-\frac{8\,a^8+25\,a^4x^4+15\,x^8}{16\left(a^{20}x^2+2\,a^{16}x^6+a^{12}x^{10}\right)}-\frac{15\,\arctan\left(\frac{x^2}{a^2}\right)}{16\,a^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^4+x^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(8*a^8 + 25*a^4*x^4 + 15*x^8)/(a^20*x^2 + 2*a^16*x^6 + a^12*x^10) - 1
5/16*arctan(x^2/a^2)/a^14
```

Fricas [A] time = 2.01592, size = 173, normalized size = 2.7

$$-\frac{8\,a^{10} + 25\,a^{6}x^{4} + 15\,a^{2}x^{8} + 15\left(a^{8}x^{2} + 2\,a^{4}x^{6} + x^{10}\right)\arctan\left(\frac{x^{2}}{a^{2}}\right)}{16\left(a^{22}x^{2} + 2\,a^{18}x^{6} + a^{14}x^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^4+x^4)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(8*a^10 + 25*a^6*x^4 + 15*a^2*x^8 + 15*(a^8*x^2 + 2*a^4*x^6 + x^10)*a
rctan(x^2/a^2))/(a^22*x^2 + 2*a^18*x^6 + a^14*x^10)
```

Sympy [C] time = 6.35304, size = 76, normalized size = 1.19

$$-\frac{8a^{8} + 25a^{4}x^{4} + 15x^{8}}{16a^{20}x^{2} + 32a^{16}x^{6} + 16a^{12}x^{10}} + \frac{\frac{15i\log\left(-ia^{2}+x^{2}\right)}{32} - \frac{15i\log\left(ia^{2}+x^{2}\right)}{32}}{a^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a**4+x**4)**3,x)
```

```
[Out] -(8*a**8 + 25*a**4*x**4 + 15*x**8)/(16*a**20*x**2 + 32*a**16*x**6 + 16*a**1
2*x**10) + (15*I*log(-I*a**2 + x**2)/32 - 15*I*log(I*a**2 + x**2)/32)/a**14
```

Giac [A] time = 1.06292, size = 68, normalized size = 1.06

$$-\frac{9\,a^{4}x^{2} + 7\,x^{6}}{16\left(a^{4} + x^{4}\right)^{2}a^{12}} - \frac{15\,\arctan\left(\frac{x^{2}}{a^{2}}\right)}{16\,a^{14}} - \frac{1}{2\,a^{12}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a^4+x^4)^3,x, algorithm="giac")
```

```
[Out] -1/16*(9*a^4*x^2 + 7*x^6)/((a^4 + x^4)^2*a^12) - 15/16*arctan(x^2/a^2)/a^14
 - 1/2/(a^12*x^2)
```

3.176 $\int \frac{x^{14}}{(3+2x^5)^3} dx$

Optimal. Leaf size=39

$$\frac{3}{20\left(2x^5+3\right)}-\frac{9}{80\left(2x^5+3\right)^2}+\frac{1}{40}\log\left(2x^5+3\right)$$

```
[Out] -9/(80*(3 + 2*x^5)^2) + 3/(20*(3 + 2*x^5)) + Log[3 + 2*x^5]/40
```

Rubi [A] time = 0.0249247, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{20\left(2x^5+3\right)}-\frac{9}{80\left(2x^5+3\right)^2}+\frac{1}{40}\log\left(2x^5+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^14/(3 + 2*x^5)^3,x]
```

```
[Out] -9/(80*(3 + 2*x^5)^2) + 3/(20*(3 + 2*x^5)) + Log[3 + 2*x^5]/40
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^{14}}{\left(3+2x^5\right)^3}\,dx &= \frac{1}{5}\operatorname{Subst}\left(\int \frac{x^2}{(3+2x)^3}\,dx, x, x^5\right)\\ &= \frac{1}{5}\operatorname{Subst}\left(\int \left(\frac{9}{4(3+2x)^3}-\frac{3}{2(3+2x)^2}+\frac{1}{4(3+2x)}\right)dx, x, x^5\right)\\ &= -\frac{9}{80\left(3+2x^5\right)^2}+\frac{3}{20\left(3+2x^5\right)}+\frac{1}{40}\log\left(3+2x^5\right)\end{aligned}$$

Mathematica [A] time = 0.0134275, size = 33, normalized size = 0.85

$$\frac{1}{80}\left(\frac{3\left(8x^5+9\right)}{\left(2x^5+3\right)^2}+2\log\left(2x^5+3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^14/(3 + 2*x^5)^3,x]
```

```
[Out] ((3*(9 + 8*x^5))/(3 + 2*x^5)^2 + 2*Log[3 + 2*x^5])/80
```

Maple [A] time = 0.009, size = 34, normalized size = 0.9

$$-\frac{9}{80\left(2x^{5}+3\right)^{2}}+\frac{3}{40x^{5}+60}+\frac{\ln\left(2x^{5}+3\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^14/(2*x^5+3)^3,x)
```

```
[Out] -9/80/(2*x^5+3)^2+3/20/(2*x^5+3)+1/40*ln(2*x^5+3)
```

Maxima [A] time = 0.923762, size = 46, normalized size = 1.18

$$\frac{3\left(8x^{5}+9\right)}{80\left(4x^{10}+12x^{5}+9\right)}+\frac{1}{40}\log\left(2x^{5}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(2*x^5+3)^3,x, algorithm="maxima")
```

```
[Out] 3/80*(8*x^5 + 9)/(4*x^10 + 12*x^5 + 9) + 1/40*log(2*x^5 + 3)
```

Fricas [A] time = 1.9503, size = 112, normalized size = 2.87

$$\frac{24x^{5}+2\left(4x^{10}+12x^{5}+9\right)\log\left(2x^{5}+3\right)+27}{80\left(4x^{10}+12x^{5}+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(2*x^5+3)^3,x, algorithm="fricas")
```

```
[Out] 1/80*(24*x^5 + 2*(4*x^10 + 12*x^5 + 9)*log(2*x^5 + 3) + 27)/(4*x^10 + 12*x^
5 + 9)
```

Sympy [A] time = 0.171112, size = 27, normalized size = 0.69

$$\frac{24x^{5}+27}{320x^{10}+960x^{5}+720}+\frac{\log\left(2x^{5}+3\right)}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**14/(2*x**5+3)**3,x)
```

```
[Out] (24*x**5 + 27)/(320*x**10 + 960*x**5 + 720) + log(2*x**5 + 3)/40
```

Giac [A] time = 1.10811, size = 41, normalized size = 1.05

$$-\frac{3\left(x^{10}+x^5\right)}{20\left(2x^5+3\right)^2}+\frac{1}{40}\log\left(\left|2x^5+3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^14/(2*x^5+3)^3,x, algorithm="giac")
```

```
[Out] -3/20*(x^10 + x^5)/(2*x^5 + 3)^2 + 1/40*log(abs(2*x^5 + 3))
```

3.177 $\int \frac{x^6}{\left(3+2x^5\right)^3}\,dx$

Optimal. Leaf size=319

$$\frac{x^2}{150\left(2x^5+3\right)}-\frac{x^2}{20\left(2x^5+3\right)^2}+\frac{\left(1+\sqrt{5}\right)\log\left(2^{2/5}x^2-\frac{\sqrt[5]{3}\left(1-\sqrt{5}\right)x}{2^{4/5}}+3^{2/5}\right)}{1000\ 2^{2/5}3^{3/5}}+\frac{\left(1-\sqrt{5}\right)\log\left(2^{2/5}x^2-\frac{\sqrt[5]{3}\left(1+\sqrt{5}\right)x}{2^{4/5}}+3^{2/5}\right)}{1000\ 2^{2/5}3^{3/5}}$$

```
[Out] -x^2/(20*(3 + 2*x^5)^2) + x^2/(150*(3 + 2*x^5)) - (Sqrt[5 + Sqrt[5]]*ArcTan
[Sqrt[(5 + 2*Sqrt[5])/5] - (2*2^(7/10)*x)/(3^(1/5)*Sqrt[5 - Sqrt[5]])])/(25
0*2^(9/10)*3^(3/5)) - (Sqrt[5 - Sqrt[5]]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + (
2*2^(7/10)*x)/(3^(1/5)*Sqrt[5 + Sqrt[5]])])/(250*2^(9/10)*3^(3/5)) - Log[3^
(1/5) + 2^(1/5)*x]/(250*2^(2/5)*3^(3/5)) + ((1 + Sqrt[5])*Log[3^(2/5) - (3^
(1/5)*(1 - Sqrt[5])*x)/2^(4/5) + 2^(2/5)*x^2])/(1000*2^(2/5)*3^(3/5)) + ((1
 - Sqrt[5])*Log[3^(2/5) - (3^(1/5)*(1 + Sqrt[5])*x)/2^(4/5) + 2^(2/5)*x^2])
/(1000*2^(2/5)*3^(3/5))
```

Rubi [A] time = 0.583666, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {288, 290, 293, 634, 618, 204, 628, 31}

$$\frac{x^2}{150\left(2x^5+3\right)}-\frac{x^2}{20\left(2x^5+3\right)^2}+\frac{\left(1+\sqrt{5}\right)\log\left(2^{2/5}x^2-\frac{\sqrt[5]{3}\left(1-\sqrt{5}\right)x}{2^{4/5}}+3^{2/5}\right)}{1000\ 2^{2/5}3^{3/5}}+\frac{\left(1-\sqrt{5}\right)\log\left(2^{2/5}x^2-\frac{\sqrt[5]{3}\left(1+\sqrt{5}\right)x}{2^{4/5}}+3^{2/5}\right)}{1000\ 2^{2/5}3^{3/5}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(3 + 2*x^5)^3,x]
```


```
[Out] -x^2/(20*(3 + 2*x^5)^2) + x^2/(150*(3 + 2*x^5)) - (Sqrt[5 + Sqrt[5]]*ArcTan
[Sqrt[(5 + 2*Sqrt[5])/5] - (2*2^(7/10)*x)/(3^(1/5)*Sqrt[5 - Sqrt[5]])])/(25
0*2^(9/10)*3^(3/5)) - (Sqrt[5 - Sqrt[5]]*ArcTan[Sqrt[(5 - 2*Sqrt[5])/5] + (
2*2^(7/10)*x)/(3^(1/5)*Sqrt[5 + Sqrt[5]])])/(250*2^(9/10)*3^(3/5)) - Log[3^
(1/5) + 2^(1/5)*x]/(250*2^(2/5)*3^(3/5)) + ((1 + Sqrt[5])*Log[3^(2/5) - (3^
(1/5)*(1 - Sqrt[5])*x)/2^(4/5) + 2^(2/5)*x^2])/(1000*2^(2/5)*3^(3/5)) + ((1
 - Sqrt[5])*Log[3^(2/5) - (3^(1/5)*(1 + Sqrt[5])*x)/2^(4/5) + 2^(2/5)*x^2])
/(1000*2^(2/5)*3^(3/5))
```


Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 293

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x]; -(((-r)^(m + 1)*Int[1/(r + s*x), x])/(a*n*s^m
)) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /; Fr
eeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ
[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x^6}{\left(3+2x^5\right)^3}\,dx = -\frac{x^2}{20\left(3+2x^5\right)^2} + \frac{1}{10}\int \frac{x}{\left(3+2x^5\right)^2}\,dx$$

$$= -\frac{x^2}{20\left(3+2x^5\right)^2} + \frac{x^2}{150\left(3+2x^5\right)} + \frac{1}{50}\int \frac{x}{3+2x^5}\,dx$$

$$= -\frac{x^2}{20\left(3+2x^5\right)^2} + \frac{x^2}{150\left(3+2x^5\right)} - \frac{\int \frac{1}{\sqrt[5]{3}+\sqrt[5]{2}x}\,dx}{250\sqrt[5]{2}3^{3/5}} + \frac{\int \frac{\frac{1}{4}\sqrt[5]{3}(1-\sqrt{5})-\frac{(-1-\sqrt{5})x}{2\ 2^{4/5}}}{3^{2/5}-\frac{\sqrt[5]{3}(1-\sqrt{5})x}{2^{4/5}}+2^{2/5}x^2}\,dx}{125\sqrt[5]{2}3^{3/5}} + \frac{\int \frac{\frac{1}{4}\sqrt[5]{3}(1+\sqrt{5})-\frac{(-1+\sqrt{5})}{2\ 2^{4/5}}}{3^{2/5}-\frac{\sqrt[5]{3}(1+\sqrt{5})x}{2^{4/5}}+2^{2/5}}}{125\sqrt[5]{2}3^{3/5}}$$

$$= -\frac{x^2}{20\left(3+2x^5\right)^2} + \frac{x^2}{150\left(3+2x^5\right)} - \frac{\log\left(\sqrt[5]{3}+\sqrt[5]{2}x\right)}{250\ 2^{2/5}3^{3/5}} - \frac{\int \frac{1}{3^{2/5}-\frac{\sqrt[5]{3}(1-\sqrt{5})x}{2^{4/5}}+2^{2/5}x^2}\,dx}{100\sqrt[5]{2}3^{2/5}\sqrt{5}} + \frac{\int \frac{1}{3^{2/5}-\frac{\sqrt[5]{3}(1+\sqrt{5})x}{2^{4/5}}+}}{100\sqrt[5]{2}3^{2/5}}$$

$$= -\frac{x^2}{20\left(3+2x^5\right)^2} + \frac{x^2}{150\left(3+2x^5\right)} - \frac{\log\left(\sqrt[5]{3}+\sqrt[5]{2}x\right)}{250\ 2^{2/5}3^{3/5}} + \frac{\left(1-\sqrt{5}\right)\log\left(2\ 3^{2/5}-\sqrt[5]{6}x-\sqrt{5}\sqrt[5]{6}x+2\ 2\right.}{1000\ 2^{2/5}3^{3/5}}$$

$$= -\frac{x^2}{20\left(3+2x^5\right)^2} + \frac{x^2}{150\left(3+2x^5\right)} - \frac{\tan^{-1}\left(\frac{\sqrt[5]{3}(1+\sqrt{5})-4\sqrt[5]{2}x}{\sqrt[5]{3}\sqrt{2(5-\sqrt{5})}}\right)}{25\ 2^{9/10}3^{3/5}\sqrt{5\left(5-\sqrt{5}\right)}} - \frac{\tan^{-1}\left(\frac{\sqrt[5]{3}\sqrt{3-\sqrt{5}}+2\ 2^{7/10}x}{\sqrt[5]{3}\sqrt{5+\sqrt{5}}}\right)}{25\ 2^{9/10}3^{3/5}\sqrt{5\left(5+\sqrt{5}\right)}} - \frac{\log\left(\sqrt[5]{\right.}{250}$$

Mathematica [A] time = 0.303986, size = 293, normalized size = 0.92

$$\frac{\frac{40x^2}{2x^5+3} - \frac{300x^2}{\left(2x^5+3\right)^2} + 2^{3/5}3^{2/5}\left(1+\sqrt{5}\right)\log\left(2^{2/5}3^{3/5}x^2+\left(\frac{3}{2}\right)^{4/5}\left(\sqrt{5}-1\right)x+3\right) - 2^{3/5}3^{2/5}\left(\sqrt{5}-1\right)\log\left(2^{2/5}3^{3/5}x^2-\left(\frac{3}{2}\right)^{4/5}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(3 + 2*x^5)^3,x]
```

```
[Out] ((-300*x^2)/(3 + 2*x^5)^2 + (40*x^2)/(3 + 2*x^5) - 4*2^(1/10)*3^(2/5)*Sqrt[
5 - Sqrt[5]]*ArcTan[(-3 + 3*Sqrt[5] + 4*2^(1/5)*3^(4/5)*x)/(3*Sqrt[2*(5 + S
qrt[5])])] + 4*2^(1/10)*3^(2/5)*Sqrt[5 + Sqrt[5]]*ArcTan[(-3*(1 + Sqrt[5])
+ 4*2^(1/5)*3^(4/5)*x)/(3*Sqrt[10 - 2*Sqrt[5]])] - 4*2^(3/5)*3^(2/5)*Log[3
+ 2^(1/5)*3^(4/5)*x] + 2^(3/5)*3^(2/5)*(1 + Sqrt[5])*Log[3 + (3/2)^(4/5)*(-
1 + Sqrt[5])*x + 2^(2/5)*3^(3/5)*x^2] - 2^(3/5)*3^(2/5)*(-1 + Sqrt[5])*Log[
3 - (3/2)^(4/5)*(1 + Sqrt[5])*x + 2^(2/5)*3^(3/5)*x^2])/6000
```

Maple [A] time = 0.101, size = 354, normalized size = 1.1

$$4\frac{1}{\left(2x^5+3\right)^2}\left(\frac{x^7}{300}-\frac{3x^2}{400}\right) + \frac{48^{\frac{2}{5}}\ln\left(\sqrt[5]{48}+2x\right)}{\left(150\sqrt{5}-750\right)\left(5+\sqrt{5}\right)} + \frac{48^{\frac{2}{5}}\ln\left(-x\sqrt{5}\sqrt[5]{48}+48^{\frac{2}{5}}-x\sqrt[5]{48}+4x^2\right)}{12000} - \frac{48^{\frac{2}{5}}\ln\left(-x\sqrt{5}\sqrt[5]{\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(2*x^5+3)^3,x)
```

```
[Out] 4*(1/300*x^7-3/400*x^2)/(2*x^5+3)^2+1/150*48^(2/5)/(5^(1/2)-5)/(5+5^(1/2))*
ln(48^(1/5)+2*x)+1/12000*48^(2/5)*ln(-x*5^(1/2)*48^(1/5)+48^(2/5)-x*48^(1/5
)+4*x^2)-1/12000*48^(2/5)*ln(-x*5^(1/2)*48^(1/5)+48^(2/5)-x*48^(1/5)+4*x^2)
*5^(1/2)+1/1500*48^(3/5)/(10*48^(2/5)-2*5^(1/2)*48^(2/5))^(1/2)*arctan(-1/(
10*48^(2/5)-2*5^(1/2)*48^(2/5))^(1/2)*5^(1/2)*48^(1/5)-1/(10*48^(2/5)-2*5^(
1/2)*48^(2/5))^(1/2)*48^(1/5)+8*x/(10*48^(2/5)-2*5^(1/2)*48^(2/5))^(1/2))*5
^(1/2)+1/12000*48^(2/5)*ln(x*5^(1/2)*48^(1/5)-x*48^(1/5)+48^(2/5)+4*x^2)*5^
(1/2)+1/12000*48^(2/5)*ln(x*5^(1/2)*48^(1/5)-x*48^(1/5)+48^(2/5)+4*x^2)-1/1
500*48^(3/5)*5^(1/2)/(10*48^(2/5)+2*5^(1/2)*48^(2/5))^(1/2)*arctan(1/(10*48
^(2/5)+2*5^(1/2)*48^(2/5))^(1/2)*5^(1/2)*48^(1/5)-1/(10*48^(2/5)+2*5^(1/2)*
48^(2/5))^(1/2)*48^(1/5)+8*x/(10*48^(2/5)+2*5^(1/2)*48^(2/5))^(1/2))
```

Maxima [A] time = 1.44039, size = 452, normalized size = 1.42

$$\frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(\sqrt{5}-5\right)\arctan\left(\frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(4\cdot 2^{\frac{2}{5}}x+\sqrt{5}3^{\frac{1}{5}}2^{\frac{1}{5}}-3^{\frac{1}{5}}2^{\frac{1}{5}}\right)}{6\sqrt{2\sqrt{5}+10}}\right)}{750\left(\sqrt{5}3^{\frac{2}{5}}2^{\frac{1}{5}}-3^{\frac{2}{5}}2^{\frac{1}{5}}\right)\sqrt{2\sqrt{5}+10}}+\frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(\sqrt{5}+5\right)\arctan\left(\frac{3^{\frac{4}{5}}2^{\frac{4}{5}}\left(4\cdot 2^{\frac{2}{5}}x-\sqrt{5}3^{\frac{1}{5}}2^{\frac{1}{5}}-3^{\frac{1}{5}}2^{\frac{1}{5}}\right)}{6\sqrt{-2\sqrt{5}+10}}\right)}{750\left(\sqrt{5}3^{\frac{2}{5}}2^{\frac{1}{5}}+3^{\frac{2}{5}}2^{\frac{1}{5}}\right)\sqrt{-2\sqrt{5}+10}}-\frac{1}{1500}\cdot 3^{\frac{2}{5}}2^{\frac{3}{5}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(2*x^5+3)^3,x, algorithm="maxima")
```

```
[Out] 1/750*3^(4/5)*2^(4/5)*(sqrt(5) - 5)*arctan(1/6*3^(4/5)*2^(4/5)*(4*2^(2/5)*x
 + sqrt(5)*3^(1/5)*2^(1/5) - 3^(1/5)*2^(1/5))/sqrt(2*sqrt(5) + 10))/((sqrt(
5)*3^(2/5)*2^(1/5) - 3^(2/5)*2^(1/5))*sqrt(2*sqrt(5) + 10)) + 1/750*3^(4/5)
*2^(4/5)*(sqrt(5) + 5)*arctan(1/6*3^(4/5)*2^(4/5)*(4*2^(2/5)*x - sqrt(5)*3^
(1/5)*2^(1/5) - 3^(1/5)*2^(1/5))/sqrt(-2*sqrt(5) + 10))/((sqrt(5)*3^(2/5)*2
^(1/5) + 3^(2/5)*2^(1/5))*sqrt(-2*sqrt(5) + 10)) - 1/1500*3^(2/5)*2^(3/5)*l
og(2^(1/5)*x + 3^(1/5)) + 1/300*(4*x^7 - 9*x^2)/(4*x^10 + 12*x^5 + 9) - 1/2
50*log(2*2^(2/5)*x^2 - x*(sqrt(5)*3^(1/5)*2^(1/5) + 3^(1/5)*2^(1/5)) + 2*3^
(2/5))/(sqrt(5)*3^(3/5)*2^(2/5) + 3^(3/5)*2^(2/5)) + 1/250*log(2*2^(2/5)*x^
2 + x*(sqrt(5)*3^(1/5)*2^(1/5) - 3^(1/5)*2^(1/5)) + 2*3^(2/5))/(sqrt(5)*3^(
3/5)*2^(2/5) - 3^(3/5)*2^(2/5))
```

Fricas [C] time = 145.437, size = 6294, normalized size = 19.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(2*x^5+3)^3,x, algorithm="fricas")
```

```
[Out] 1/216000*(2880*x^7 - 2*108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5 + 9)*(sqrt(5)
+ I*sqrt(-2*sqrt(5) + 10) + 1)*log(-1/6912*108^(3/5)*(-1)^(2/5)*(108^(4/5)*
(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(1/5)
)*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 1/64*108^(2/5)*(-1)^(3/5)*(sq
rt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^3 - 1/6912*108^(4/5)*(-1)^(1/5)*(108^(
3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 4*108^(3/5)*(-1
)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) + 16*108^(3/5)*(-1)^(2/5))*
(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) + 1/16*108^(2/5)*(-1)^(3/5)*(sqrt(5
) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 1/4*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*
sqrt(-2*sqrt(5) + 10) + 1) + 108^(2/5)*(-1)^(3/5) + 6*x) - 2*108^(4/5)*(-1)
^(1/5)*(4*x^10 + 12*x^5 + 9)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)*log(1/
```

```
384*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^3 + x) + 8
*108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5 + 9)*log(-108^(2/5)*(-1)^(3/5) + 6*x
) - 6480*x^2 + (108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5 + 9)*(sqrt(5) + I*sqr
t(-2*sqrt(5) + 10) + 1) + 108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5 + 9)*(sqrt(
5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*x^5
 + 9) - 24*sqrt(3)*(4*x^10 + 12*x^5 + 9)*sqrt(-1/864*108^(4/5)*(-1)^(1/5)*(
108^(4/5)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*
(-1)^(1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) - 3/16*108^(3/5)*(-1)^(
2/5)*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 3/16*108^(3/5)*(-1)^(2/5)*
(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 + 1/2*108^(3/5)*(-1)^(2/5)*(sqrt(
5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 3*108^(3/5)*(-1)^(2/5)))*log(1/768*108^
(3/5)*(-1)^(2/5)*(108^(4/5)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) +
 1) - 4*108^(4/5)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 + 1
/768*108^(4/5)*(-1)^(1/5)*(108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5
) + 10) + 1)^2 - 4*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10)
+ 1) + 16*108^(3/5)*(-1)^(2/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) - 9
/16*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 + 9/4*10
8^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) + 1/3456*(108^(4
/5)*(-1)^(1/5)*(108^(4/5)*sqrt(3)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) +
 10) + 1) - 4*108^(4/5)*sqrt(3)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) +
10) + 1) - 432*108^(3/5)*sqrt(3)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) +
10) + 1))*sqrt(-1/864*108^(4/5)*(-1)^(1/5)*(108^(4/5)*(-1)^(1/5)*(sqrt(5) -
 I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-
2*sqrt(5) + 10) + 1) - 3/16*108^(3/5)*(-1)^(2/5)*(sqrt(5) + I*sqrt(-2*sqrt(
5) + 10) + 1)^2 - 3/16*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) +
10) + 1)^2 + 1/2*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) +
1) - 3*108^(3/5)*(-1)^(2/5)) + 108*x) + (108^(4/5)*(-1)^(1/5)*(4*x^10 + 12*
x^5 + 9)*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) + 108^(4/5)*(-1)^(1/5)*(4*
x^10 + 12*x^5 + 9)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-
1)^(1/5)*(4*x^10 + 12*x^5 + 9) + 24*sqrt(3)*(4*x^10 + 12*x^5 + 9)*sqrt(-1/8
64*108^(4/5)*(-1)^(1/5)*(108^(4/5)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5)
+ 10) + 1) - 4*108^(4/5)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1
) - 3/16*108^(3/5)*(-1)^(2/5)*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 3
/16*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 + 1/2*10
8^(3/5)*(-1)^(2/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 3*108^(3/5)*(-
1)^(2/5)))*log(1/768*108^(3/5)*(-1)^(2/5)*(108^(4/5)*(-1)^(1/5)*(sqrt(5) -
I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(1/5))*(sqrt(5) + I*sqrt(-2
*sqrt(5) + 10) + 1)^2 + 1/768*108^(4/5)*(-1)^(1/5)*(108^(3/5)*(-1)^(2/5)*(s
qrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 4*108^(3/5)*(-1)^(2/5)*(sqrt(5) -
 I*sqrt(-2*sqrt(5) + 10) + 1) + 16*108^(3/5)*(-1)^(2/5))*(sqrt(5) + I*sqrt(
-2*sqrt(5) + 10) + 1) - 9/16*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt
(5) + 10) + 1)^2 + 9/4*108^(2/5)*(-1)^(3/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) +
10) + 1) - 1/3456*(108^(4/5)*(-1)^(1/5)*(108^(4/5)*sqrt(3)*(-1)^(1/5)*(sqrt
(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*sqrt(3)*(-1)^(1/5))*(sqrt(
5) + I*sqrt(-2*sqrt(5) + 10) + 1) - 432*108^(3/5)*sqrt(3)*(-1)^(2/5)*(sqrt(
5) - I*sqrt(-2*sqrt(5) + 10) + 1))*sqrt(-1/864*108^(4/5)*(-1)^(1/5)*(108^(4
/5)*(-1)^(1/5)*(sqrt(5) - I*sqrt(-2*sqrt(5) + 10) + 1) - 4*108^(4/5)*(-1)^(
1/5))*(sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1) - 3/16*108^(3/5)*(-1)^(2/5)*(
sqrt(5) + I*sqrt(-2*sqrt(5) + 10) + 1)^2 - 3/16*108^(3/5)*(-1)^(2/5)*(sqrt(
5) - I*sqrt(-2*sqrt(5) + 10) + 1)^2 + 1/2*108^(3/5)*(-1)^(2/5)*(sqrt(5) - I
*sqrt(-2*sqrt(5) + 10) + 1) - 3*108^(3/5)*(-1)^(2/5)) + 108*x))/(4*x^10 + 1
2*x^5 + 9)
```

Sympy [A] time = 0.270478, size = 37, normalized size = 0.12

$$\frac{4x^7 - 9x^2}{1200x^{10} + 3600x^5 + 2700} + \operatorname{RootSum}\left(105468750000000t^5 + 1, \left(t \mapsto t\log\left(-281250000t^3 + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(2*x**5+3)**3,x)
```

```
[Out] (4*x**7 - 9*x**2)/(1200*x**10 + 3600*x**5 + 2700) + RootSum(105468750000000
*_t**5 + 1, Lambda(_t, _t*log(-281250000*_t**3 + x)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(2*x^5+3)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.178 $\int \frac{9}{5x^2(3-2x^2)^3}\,dx$

Optimal. Leaf size=59

$$\frac{1}{8x\left(3-2x^2\right)}+\frac{3}{20x\left(3-2x^2\right)^2}-\frac{1}{8x}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3}}x\right)}{4\sqrt{6}}$$

```
[Out] -1/(8*x) + 3/(20*x*(3 - 2*x^2)^2) + 1/(8*x*(3 - 2*x^2)) + ArcTanh[Sqrt[2/3]
*x]/(4*Sqrt[6])
```

Rubi [A] time = 0.0208878, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {12, 290, 325, 206}

$$\frac{1}{8x\left(3-2x^2\right)}+\frac{3}{20x\left(3-2x^2\right)^2}-\frac{1}{8x}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3}}x\right)}{4\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[9/(5*x^2*(3 - 2*x^2)^3),x]
```

```
[Out] -1/(8*x) + 3/(20*x*(3 - 2*x^2)^2) + 1/(8*x*(3 - 2*x^2)) + ArcTanh[Sqrt[2/3]
*x]/(4*Sqrt[6])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{9}{5x^2\left(3-2x^2\right)^3}\,dx &= \frac{9}{5}\int \frac{1}{x^2\left(3-2x^2\right)^3}\,dx \\
&= \frac{3}{20x\left(3-2x^2\right)^2}+\frac{3}{4}\int \frac{1}{x^2\left(3-2x^2\right)^2}\,dx \\
&= \frac{3}{20x\left(3-2x^2\right)^2}+\frac{1}{8x\left(3-2x^2\right)}+\frac{3}{8}\int \frac{1}{x^2\left(3-2x^2\right)}\,dx \\
&= -\frac{1}{8x}+\frac{3}{20x\left(3-2x^2\right)^2}+\frac{1}{8x\left(3-2x^2\right)}+\frac{1}{4}\int \frac{1}{3-2x^2}\,dx \\
&= -\frac{1}{8x}+\frac{3}{20x\left(3-2x^2\right)^2}+\frac{1}{8x\left(3-2x^2\right)}+\frac{\tanh^{-1}\left(\sqrt{\frac{2}{3}}x\right)}{4\sqrt{6}}
\end{aligned}
$$

Mathematica [A] time = 0.0642256, size = 65, normalized size = 1.1

$$
\frac{1}{240}\left(-\frac{12\left(10x^4-25x^2+12\right)}{x\left(3-2x^2\right)^2}-5\sqrt{6}\log\left(\sqrt{6}-2x\right)+5\sqrt{6}\log\left(2x+\sqrt{6}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[9/(5*x^2*(3 - 2*x^2)^3),x]
```

```
[Out] ((-12*(12 - 25*x^2 + 10*x^4))/(x*(3 - 2*x^2)^2) - 5*Sqrt[6]*Log[Sqrt[6] - 2
*x] + 5*Sqrt[6]*Log[Sqrt[6] + 2*x])/240
```

Maple [A] time = 0.009, size = 39, normalized size = 0.7

$$
-\frac{1}{15\,x}-\frac{8}{15\,\left(2\,x^2-3\right)^2}\left(\frac{7\,x^3}{16}-\frac{27\,x}{32}\right)+\frac{\sqrt{6}}{24}\mathrm{Artanh}\left(\frac{x\sqrt{6}}{3}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(9/5/x^2/(-2*x^2+3)^3,x)
```

```
[Out] -1/15/x-8/15*(7/16*x^3-27/32*x)/(2*x^2-3)^2+1/24*arctanh(1/3*x*6^(1/2))*6^(
1/2)
```

Maxima [A] time = 1.41092, size = 76, normalized size = 1.29

$$
-\frac{1}{48}\sqrt{6}\log\left(\frac{2\,x-\sqrt{6}}{2\,x+\sqrt{6}}\right)-\frac{10\,x^4-25\,x^2+12}{20\left(4\,x^5-12\,x^3+9\,x\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(9/5/x^2/(-2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] -1/48*sqrt(6)*log((2*x - sqrt(6))/(2*x + sqrt(6))) - 1/20*(10*x^4 - 25*x^2
+ 12)/(4*x^5 - 12*x^3 + 9*x)
```

Fricas [A] time = 1.78168, size = 182, normalized size = 3.08

$$-\frac{120\,x^4 - 5\,\sqrt{6}\left(4\,x^5 - 12\,x^3 + 9\,x\right)\log\left(\frac{2\,x^2+2\,\sqrt{6}x+3}{2\,x^2-3}\right) - 300\,x^2 + 144}{240\,\left(4\,x^5 - 12\,x^3 + 9\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(9/5/x^2/(-2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/240*(120*x^4 - 5*sqrt(6)*(4*x^5 - 12*x^3 + 9*x)*log((2*x^2 + 2*sqrt(6)*x
 + 3)/(2*x^2 - 3)) - 300*x^2 + 144)/(4*x^5 - 12*x^3 + 9*x)
```

Sympy [A] time = 0.152695, size = 58, normalized size = 0.98

$$-\frac{9\left(10x^4 - 25x^2 + 12\right)}{720x^5 - 2160x^3 + 1620x} - \frac{\sqrt{6}\log\left(x - \frac{\sqrt{6}}{2}\right)}{48} + \frac{\sqrt{6}\log\left(x + \frac{\sqrt{6}}{2}\right)}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(9/5/x**2/(-2*x**2+3)**3,x)
```

```
[Out] -9*(10*x**4 - 25*x**2 + 12)/(720*x**5 - 2160*x**3 + 1620*x) - sqrt(6)*log(x
 - sqrt(6)/2)/48 + sqrt(6)*log(x + sqrt(6)/2)/48
```

Giac [A] time = 1.05502, size = 74, normalized size = 1.25

$$-\frac{1}{48}\,\sqrt{6}\log\left(\frac{\left|4\,x - 2\,\sqrt{6}\right|}{\left|4\,x + 2\,\sqrt{6}\right|}\right) - \frac{14\,x^3 - 27\,x}{60\,\left(2\,x^2 - 3\right)^2} - \frac{1}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(9/5/x^2/(-2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] -1/48*sqrt(6)*log(abs(4*x - 2*sqrt(6))/abs(4*x + 2*sqrt(6))) - 1/60*(14*x^3
 - 27*x)/(2*x^2 - 3)^2 - 1/15/x
```

3.179 $\int \frac{4+3x^4}{x^2(1+x^2)^3}\,dx$

Optimal. Leaf size=36

$$-\frac{25x}{8(x^2+1)}-\frac{7x}{4(x^2+1)^2}-\frac{4}{x}-\frac{57}{8}\tan^{-1}(x)$$

```
[Out] -4/x - (7*x)/(4*(1 + x^2)^2) - (25*x)/(8*(1 + x^2)) - (57*ArcTan[x])/8
```

Rubi [A] time = 0.024039, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1260, 456, 453, 203}

$$-\frac{25x}{8(x^2+1)}-\frac{7x}{4(x^2+1)^2}-\frac{4}{x}-\frac{57}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

```
[Out] -4/x - (7*x)/(4*(1 + x^2)^2) - (25*x)/(8*(1 + x^2)) - (57*ArcTan[x])/8
```

Rule 1260

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_.), x_Sym
bol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^
(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m
*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(-(m/2) + 1)*e^(2*p)*(
q + 1)*(a + c*x^4)^p - ((c*d^2 + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x
^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, c, d, e}, x] && IGtQ[p, 0] && I
LtQ[q, -1] && ILtQ[m/2, 0]
```

Rule 456

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{4+3x^4}{x^2\left(1+x^2\right)^3}\,dx &= -\frac{7x}{4\left(1+x^2\right)^2}-\frac{1}{4}\int \frac{-16+9x^2}{x^2\left(1+x^2\right)^2}\,dx \\
&= -\frac{7x}{4\left(1+x^2\right)^2}-\frac{25x}{8\left(1+x^2\right)}+\frac{1}{8}\int \frac{32-25x^2}{x^2\left(1+x^2\right)}\,dx \\
&= -\frac{4}{x}-\frac{7x}{4\left(1+x^2\right)^2}-\frac{25x}{8\left(1+x^2\right)}-\frac{57}{8}\int \frac{1}{1+x^2}\,dx \\
&= -\frac{4}{x}-\frac{7x}{4\left(1+x^2\right)^2}-\frac{25x}{8\left(1+x^2\right)}-\frac{57}{8}\tan^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.0057465, size = 33, normalized size = 0.92

$$-\frac{57x^4+103x^2+32}{8x\left(x^2+1\right)^2}-\frac{57}{8}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*x^4)/(x^2*(1 + x^2)^3),x]
```

```
[Out] -(32 + 103*x^2 + 57*x^4)/(8*x*(1 + x^2)^2) - (57*ArcTan[x])/8
```

Maple [A] time = 0., size = 29, normalized size = 0.8

$$-\frac{1}{\left(x^2+1\right)^2}\left(\frac{25\,x^3}{8}+\frac{39\,x}{8}\right)-\frac{57\arctan(x)}{8}-4\,x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^4+4)/x^2/(x^2+1)^3,x)
```

```
[Out] -(25/8*x^3+39/8*x)/(x^2+1)^2-57/8*arctan(x)-4/x
```

Maxima [A] time = 1.41802, size = 42, normalized size = 1.17

$$-\frac{57\,x^4+103\,x^2+32}{8\left(x^5+2\,x^3+x\right)}-\frac{57}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(57*x^4 + 103*x^2 + 32)/(x^5 + 2*x^3 + x) - 57/8*arctan(x)
```

Fricas [A] time = 1.74588, size = 109, normalized size = 3.03

$$-\frac{57\,x^{4}+103\,x^{2}+57\,\left(x^{5}+2\,x^{3}+x\right)\arctan\left(x\right)+32}{8\,\left(x^{5}+2\,x^{3}+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(57*x^4 + 103*x^2 + 57*(x^5 + 2*x^3 + x)*arctan(x) + 32)/(x^5 + 2*x^3
+ x)
```

Sympy [A] time = 0.131786, size = 32, normalized size = 0.89

$$-\frac{57x^{4}+103x^{2}+32}{8x^{5}+16x^{3}+8x}-\frac{57\,\mathrm{atan}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**4+4)/x**2/(x**2+1)**3,x)
```

```
[Out] -(57*x**4 + 103*x**2 + 32)/(8*x**5 + 16*x**3 + 8*x) - 57*atan(x)/8
```

Giac [A] time = 1.05319, size = 38, normalized size = 1.06

$$-\frac{25\,x^{3}+39\,x}{8\,\left(x^{2}+1\right)^{2}}-\frac{4}{x}-\frac{57}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^4+4)/x^2/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -1/8*(25*x^3 + 39*x)/(x^2 + 1)^2 - 4/x - 57/8*arctan(x)
```

3.180 $\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx$

Optimal. Leaf size=38

$$\frac{2}{1-x}+\frac{1}{x+1}-\frac{3}{2(1-x)^2}+\log(1-x)-2\log(x+1)$$

```
[Out] -3/(2*(1 - x)^2) + 2/(1 - x) + (1 + x)^(-1) + Log[1 - x] - 2*Log[1 + x]
```

Rubi [A] time = 0.0619045, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2074}

$$\frac{2}{1-x}+\frac{1}{x+1}-\frac{3}{2(1-x)^2}+\log(1-x)-2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 3*x + 6*x^2 + 5*x^3 - x^4)/(-1 + x + 2*x^2 - 2*x^3 - x^4 + x^5),x]
```

```
[Out] -3/(2*(1 - x)^2) + 2/(1 - x) + (1 + x)^(-1) + Log[1 - x] - 2*Log[1 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{5-3x+6x^2+5x^3-x^4}{-1+x+2x^2-2x^3-x^4+x^5}\,dx &= \int \left(\frac{3}{(-1+x)^3}+\frac{2}{(-1+x)^2}+\frac{1}{-1+x}-\frac{1}{(1+x)^2}-\frac{2}{1+x}\right)dx\\ &= -\frac{3}{2(1-x)^2}+\frac{2}{1-x}+\frac{1}{1+x}+\log(1-x)-2\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0206963, size = 32, normalized size = 0.84

$$-\frac{2}{x-1}+\frac{1}{x+1}-\frac{3}{2(x-1)^2}+\log(x-1)-2\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 3*x + 6*x^2 + 5*x^3 - x^4)/(-1 + x + 2*x^2 - 2*x^3 - x^4 + x
^5),x]
```

```
[Out] -3/(2*(-1 + x)^2) - 2/(-1 + x) + (1 + x)^(-1) + Log[-1 + x] - 2*Log[1 + x]
```

Maple [A] time = 0.01, size = 31, normalized size = 0.8

$$(1+x)^{-1}-2\ln(1+x)+\ln(-1+x)-\frac{3}{2(-1+x)^2}-2(-1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x)
```

```
[Out] 1/(1+x)-2*ln(1+x)+ln(-1+x)-3/2/(-1+x)^2-2/(-1+x)
```

Maxima [A] time = 0.925671, size = 51, normalized size = 1.34

$$-\frac{2\,x^2+7\,x-3}{2\left(x^3-x^2-x+1\right)}-2\,\log\left(x+1\right)+\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x, algorithm="
maxima")
```

```
[Out] -1/2*(2*x^2 + 7*x - 3)/(x^3 - x^2 - x + 1) - 2*log(x + 1) + log(x - 1)
```

Fricas [B] time = 1.79386, size = 154, normalized size = 4.05

$$-\frac{2\,x^2+4\left(x^3-x^2-x+1\right)\log\left(x+1\right)-2\left(x^3-x^2-x+1\right)\log\left(x-1\right)+7\,x-3}{2\left(x^3-x^2-x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x, algorithm="
fricas")
```

```
[Out] -1/2*(2*x^2 + 4*(x^3 - x^2 - x + 1)*log(x + 1) - 2*(x^3 - x^2 - x + 1)*log(
x - 1) + 7*x - 3)/(x^3 - x^2 - x + 1)
```

Sympy [A] time = 0.126344, size = 36, normalized size = 0.95

$$-\frac{2x^2+7x-3}{2x^3-2x^2-2x+2}+\log\left(x-1\right)-2\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**4+5*x**3+6*x**2-3*x+5)/(x**5-x**4-2*x**3+2*x**2+x-1),x)
```

```
[Out] -(2*x**2 + 7*x - 3)/(2*x**3 - 2*x**2 - 2*x + 2) + log(x - 1) - 2*log(x + 1)
```

Giac [A] time = 1.04539, size = 47, normalized size = 1.24

$$-\frac{2\,x^2+7\,x-3}{2\,(x+1)(x-1)^2}-2\,\log\left(|x+1|\right)+\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^4+5*x^3+6*x^2-3*x+5)/(x^5-x^4-2*x^3+2*x^2+x-1),x, algorithm="
giac")
```

```
[Out] -1/2*(2*x^2 + 7*x - 3)/((x + 1)*(x - 1)^2) - 2*log(abs(x + 1)) + log(abs(x
- 1))
```

3.181 $\int \frac{1+x^2}{x(1+x^3)^2} dx$

Optimal. Leaf size=64

$$\frac{x(x-x^2)}{3(x^3+1)} - \frac{5}{18}\log(x^2-x+1) + \log(x) - \frac{4}{9}\log(x+1) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (x*(x - x^2))/(3*(1 + x^3)) - ArcTan[(1 - 2*x)/Sqrt[3]]/(3*Sqrt[3]) + Log[x
] - (4*Log[1 + x])/9 - (5*Log[1 - x + x^2])/18
```

Rubi [A] time = 0.0730219, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {1829, 1834, 634, 618, 204, 628}

$$\frac{x(x-x^2)}{3(x^3+1)} - \frac{5}{18}\log(x^2-x+1) + \log(x) - \frac{4}{9}\log(x+1) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(x*(1 + x^3)^2),x]
```

```
[Out] (x*(x - x^2))/(3*(1 + x^3)) - ArcTan[(1 - 2*x)/Sqrt[3]]/(3*Sqrt[3]) + Log[x
] - (4*Log[1 + x])/9 - (5*Log[1 - x + x^2])/18
```

Rule 1829

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q =
Expon[Pq, x]}, Module[{Q = PolynomialQuotient[a*b^(Floor[(q - 1)/n] + 1)*x^
m*Pq, a + b*x^n, x], R = PolynomialRemainder[a*b^(Floor[(q - 1)/n] + 1)*x^m
*Pq, a + b*x^n, x], i}, Dist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[
x^m*(a + b*x^n)^(p + 1)*ExpandToSum[(n*(p + 1)*Q)/x^m + Sum[((n*(p + 1) + i
 + 1)*Coeff[R, x, i]*x^(i - m))/a, {i, 0, n - 1}], x], x], x] - Simp[(x*R*(
a + b*x^n)^(p + 1))/(a^2*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]]] /; FreeQ
[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2}{x\left(1+x^3\right)^2}\,dx &= \frac{x\left(x-x^2\right)}{3\left(1+x^3\right)}-\frac{1}{3}\int \frac{-3-x^2}{x\left(1+x^3\right)}\,dx\\
&= \frac{x\left(x-x^2\right)}{3\left(1+x^3\right)}-\frac{1}{3}\int \left(-\frac{3}{x}+\frac{4}{3(1+x)}+\frac{-4+5x}{3\left(1-x+x^2\right)}\right)dx\\
&= \frac{x\left(x-x^2\right)}{3\left(1+x^3\right)}+\log(x)-\frac{4}{9}\log(1+x)-\frac{1}{9}\int \frac{-4+5x}{1-x+x^2}\,dx\\
&= \frac{x\left(x-x^2\right)}{3\left(1+x^3\right)}+\log(x)-\frac{4}{9}\log(1+x)+\frac{1}{6}\int \frac{1}{1-x+x^2}\,dx-\frac{5}{18}\int \frac{-1+2x}{1-x+x^2}\,dx\\
&= \frac{x\left(x-x^2\right)}{3\left(1+x^3\right)}+\log(x)-\frac{4}{9}\log(1+x)-\frac{5}{18}\log\left(1-x+x^2\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= \frac{x\left(x-x^2\right)}{3\left(1+x^3\right)}-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{3\sqrt{3}}+\log(x)-\frac{4}{9}\log(1+x)-\frac{5}{18}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0330718, size = 65, normalized size = 1.02

$$\frac{1}{18}\left(\frac{6\left(x^2+1\right)}{x^3+1}+\log\left(x^2-x+1\right)-6\log\left(x^3+1\right)+18\log(x)-2\log(x+1)+2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(x*(1 + x^3)^2),x]
```

```
[Out] ((6*(1 + x^2))/(1 + x^3) + 2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] + 18*Log[x]
 - 2*Log[1 + x] + Log[1 - x + x^2] - 6*Log[1 + x^3])/18
```

Maple [A] time = 0.011, size = 61, normalized size = 1.

$$\ln(x)+\frac{2}{9+9x}-\frac{4\ln(1+x)}{9}-\frac{-1-x}{9x^2-9x+9}-\frac{5\ln\left(x^2-x+1\right)}{18}+\frac{\sqrt{3}}{9}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/x/(x^3+1)^2,x)
```

```
[Out] ln(x)+2/9/(1+x)-4/9*ln(1+x)-1/9*(-1-x)/(x^2-x+1)-5/18*ln(x^2-x+1)+1/9*3^(1/
2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.41201, size = 68, normalized size = 1.06

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{x^2+1}{3(x^3+1)}-\frac{5}{18}\log\left(x^2-x+1\right)-\frac{4}{9}\log\left(x+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x/(x^3+1)^2,x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/3*(x^2 + 1)/(x^3 + 1) - 5/18*
log(x^2 - x + 1) - 4/9*log(x + 1) + log(x)
```

Fricas [A] time = 2.02832, size = 213, normalized size = 3.33

$$\frac{2\sqrt{3}(x^3+1)\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+6x^2-5\left(x^3+1\right)\log\left(x^2-x+1\right)-8\left(x^3+1\right)\log\left(x+1\right)+18\left(x^3+1\right)\log\left(x\right)+6}{18\left(x^3+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x/(x^3+1)^2,x, algorithm="fricas")
```

```
[Out] 1/18*(2*sqrt(3)*(x^3 + 1)*arctan(1/3*sqrt(3)*(2*x - 1)) + 6*x^2 - 5*(x^3 +
1)*log(x^2 - x + 1) - 8*(x^3 + 1)*log(x + 1) + 18*(x^3 + 1)*log(x) + 6)/(x^
3 + 1)
```

Sympy [A] time = 0.197632, size = 60, normalized size = 0.94

$$\frac{x^2+1}{3x^3+3}+\log\left(x\right)-\frac{4\log\left(x+1\right)}{9}-\frac{5\log\left(x^2-x+1\right)}{18}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/x/(x**3+1)**2,x)
```

```
[Out] (x**2 + 1)/(3*x**3 + 3) + log(x) - 4*log(x + 1)/9 - 5*log(x**2 - x + 1)/18
+ sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/9
```

Giac [A] time = 1.05661, size = 81, normalized size = 1.27

$$\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{x^2+1}{3\left(x^2-x+1\right)(x+1)}-\frac{5}{18}\log\left(x^2-x+1\right)-\frac{4}{9}\log\left(|x+1|\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x/(x^3+1)^2,x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/3*(x^2 + 1)/((x^2 - x + 1)*(x
 + 1)) - 5/18*log(x^2 - x + 1) - 4/9*log(abs(x + 1)) + log(abs(x))
```

3.182 $\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2} \, dx$

Optimal. Leaf size=63

$$-\frac{5x+7}{3\left(x^2+x+1\right)}+\frac{1}{2}\log\left(x^2+x+1\right)-\frac{2}{x+1}-\log(x+1)-\frac{25\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -2/(1 + x) - (7 + 5*x)/(3*(1 + x + x^2)) - (25*ArcTan[(1 + 2*x)/Sqrt[3]])/(
3*Sqrt[3]) - Log[1 + x] + Log[1 + x + x^2]/2
```

Rubi [A] time = 0.121158, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1646, 1628, 634, 618, 204, 628}

$$-\frac{5x+7}{3\left(x^2+x+1\right)}+\frac{1}{2}\log\left(x^2+x+1\right)-\frac{2}{x+1}-\log(x+1)-\frac{25\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 - 3*x + x^2)/((1 + x)^2*(1 + x + x^2)^2),x]
```

```
[Out] -2/(1 + x) - (7 + 5*x)/(3*(1 + x + x^2)) - (25*ArcTan[(1 + 2*x)/Sqrt[3]])/(
3*Sqrt[3]) - Log[1 + x] + Log[1 + x + x^2]/2
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{-2-3x+x^2}{(1+x)^2\left(1+x+x^2\right)^2}\,dx &= -\frac{7+5x}{3\left(1+x+x^2\right)}+\frac{1}{3}\int \frac{-8-19x-5x^2}{(1+x)^2\left(1+x+x^2\right)}\,dx\\
&= -\frac{7+5x}{3\left(1+x+x^2\right)}+\frac{1}{3}\int \left(\frac{6}{(1+x)^2}-\frac{3}{1+x}+\frac{-11+3x}{1+x+x^2}\right)dx\\
&= -\frac{2}{1+x}-\frac{7+5x}{3\left(1+x+x^2\right)}-\log(1+x)+\frac{1}{3}\int \frac{-11+3x}{1+x+x^2}\,dx\\
&= -\frac{2}{1+x}-\frac{7+5x}{3\left(1+x+x^2\right)}-\log(1+x)+\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{25}{6}\int \frac{1}{1+x+x^2}\,dx\\
&= -\frac{2}{1+x}-\frac{7+5x}{3\left(1+x+x^2\right)}-\log(1+x)+\frac{1}{2}\log\left(1+x+x^2\right)+\frac{25}{3}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,\right.\\
&= -\frac{2}{1+x}-\frac{7+5x}{3\left(1+x+x^2\right)}-\frac{25\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{3\sqrt{3}}-\log(1+x)+\frac{1}{2}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0348852, size = 63, normalized size = 1.

$$-\frac{5x+7}{3\left(x^2+x+1\right)}+\frac{1}{2}\log\left(x^2+x+1\right)-\frac{2}{x+1}-\log(x+1)-\frac{25\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 - 3*x + x^2)/((1 + x)^2*(1 + x + x^2)^2),x]
```

```
[Out] -2/(1 + x) - (7 + 5*x)/(3*(1 + x + x^2)) - (25*ArcTan[(1 + 2*x)/Sqrt[3]])/(
3*Sqrt[3]) - Log[1 + x] + Log[1 + x + x^2]/2
```

Maple [A] time = 0.01, size = 54, normalized size = 0.9

$$-2\,(1+x)^{-1}-\ln(1+x)+\frac{1}{x^2+x+1}\left(-\frac{5\,x}{3}-\frac{7}{3}\right)+\frac{\ln\left(x^2+x+1\right)}{2}-\frac{25\,\sqrt{3}}{9}\arctan\left(\frac{(1+2\,x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x)
```

```
[Out] -2/(1+x)-ln(1+x)+(-5/3*x-7/3)/(x^2+x+1)+1/2*ln(x^2+x+1)-25/9*arctan(1/3*(1+
2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.41475, size = 80, normalized size = 1.27

$$-\frac{25}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{11x^2+18x+13}{3\left(x^3+2x^2+2x+1\right)}+\frac{1}{2}\log\left(x^2+x+1\right)-\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x, algorithm="maxima")
```

```
[Out] -25/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/3*(11*x^2 + 18*x + 13)/(x^3
 + 2*x^2 + 2*x + 1) + 1/2*log(x^2 + x + 1) - log(x + 1)
```

Fricas [A] time = 1.99066, size = 277, normalized size = 4.4

$$-\frac{50\sqrt{3}\left(x^3+2x^2+2x+1\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+66x^2-9\left(x^3+2x^2+2x+1\right)\log\left(x^2+x+1\right)+18\left(x^3+2x^2+2\right.}{18\left(x^3+2x^2+2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(50*sqrt(3)*(x^3 + 2*x^2 + 2*x + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) + 6
6*x^2 - 9*(x^3 + 2*x^2 + 2*x + 1)*log(x^2 + x + 1) + 18*(x^3 + 2*x^2 + 2*x
+ 1)*log(x + 1) + 108*x + 78)/(x^3 + 2*x^2 + 2*x + 1)
```

Sympy [A] time = 0.171504, size = 66, normalized size = 1.05

$$-\frac{11x^2+18x+13}{3x^3+6x^2+6x+3}-\log\left(x+1\right)+\frac{\log\left(x^2+x+1\right)}{2}-\frac{25\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-3*x-2)/(1+x)**2/(x**2+x+1)**2,x)
```

```
[Out] -(11*x**2 + 18*x + 13)/(3*x**3 + 6*x**2 + 6*x + 3) - log(x + 1) + log(x**2
+ x + 1)/2 - 25*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/9
```

Giac [A] time = 1.07256, size = 97, normalized size = 1.54

$$-\frac{25}{9}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(\frac{2}{x+1}-1\right)\right)+\frac{\frac{7}{x+1}-2}{3\left(\frac{1}{x+1}-\frac{1}{(x+1)^2}-1\right)}-\frac{2}{x+1}+\frac{1}{2}\log\left(-\frac{1}{x+1}+\frac{1}{(x+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-3*x-2)/(1+x)^2/(x^2+x+1)^2,x, algorithm="giac")
```

```
[Out] -25/9*sqrt(3)*arctan(-1/3*sqrt(3)*(2/(x + 1) - 1)) + 1/3*(7/(x + 1) - 2)/(1
/(x + 1) - 1/(x + 1)^2 - 1) - 2/(x + 1) + 1/2*log(-1/(x + 1) + 1/(x + 1)^2
+ 1)
```

3.183 $\int \frac{1}{(1-4x)^3(2-3x)}\,dx$

Optimal. Leaf size=43

$$-\frac{3}{25(1-4x)}+\frac{1}{10(1-4x)^2}-\frac{9}{125}\log(1-4x)+\frac{9}{125}\log(2-3x)$$

```
[Out] 1/(10*(1 - 4*x)^2) - 3/(25*(1 - 4*x)) - (9*Log[1 - 4*x])/125 + (9*Log[2 - 3
*x])/125
```

Rubi [A] time = 0.0194159, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {44}

$$-\frac{3}{25(1-4x)}+\frac{1}{10(1-4x)^2}-\frac{9}{125}\log(1-4x)+\frac{9}{125}\log(2-3x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - 4*x)^3*(2 - 3*x)),x]
```

```
[Out] 1/(10*(1 - 4*x)^2) - 3/(25*(1 - 4*x)) - (9*Log[1 - 4*x])/125 + (9*Log[2 - 3
*x])/125
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1-4x)^3(2-3x)}\,dx &= \int \left(\frac{27}{125(-2+3x)}-\frac{4}{5(-1+4x)^3}-\frac{12}{25(-1+4x)^2}-\frac{36}{125(-1+4x)}\right)dx\\ &= \frac{1}{10(1-4x)^2}-\frac{3}{25(1-4x)}-\frac{9}{125}\log(1-4x)+\frac{9}{125}\log(2-3x)\end{aligned}$$

Mathematica [A] time = 0.0187331, size = 46, normalized size = 1.07

$$\frac{120x+18(1-4x)^2\log(8-12x)-18(1-4x)^2\log(4x-1)-5}{250(1-4x)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - 4*x)^3*(2 - 3*x)),x]
```

```
[Out] (-5 + 120*x + 18*(1 - 4*x)^2*Log[8 - 12*x] - 18*(1 - 4*x)^2*Log[-1 + 4*x])/
(250*(1 - 4*x)^2)
```

Maple [A] time = 0.01, size = 36, normalized size = 0.8

$$\frac{9 \ln (-2+3 x)}{125}+\frac{1}{10 (-1+4 x)^{2}}+\frac{3}{-25+100 x}-\frac{9 \ln (-1+4 x)}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-4*x)^3/(2-3*x),x)
```

```
[Out] 9/125*ln(-2+3*x)+1/10/(-1+4*x)^2+3/25/(-1+4*x)-9/125*ln(-1+4*x)
```

Maxima [A] time = 0.936639, size = 49, normalized size = 1.14

$$\frac{24 x-1}{50 \left(16 x^{2}-8 x+1\right)}-\frac{9}{125} \log (4 x-1)+\frac{9}{125} \log (3 x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*x)^3/(2-3*x),x, algorithm="maxima")
```

```
[Out] 1/50*(24*x - 1)/(16*x^2 - 8*x + 1) - 9/125*log(4*x - 1) + 9/125*log(3*x - 2
)
```

Fricas [A] time = 1.8645, size = 153, normalized size = 3.56

$$-\frac{18 \left(16 x^{2}-8 x+1\right) \log (4 x-1)-18 \left(16 x^{2}-8 x+1\right) \log (3 x-2)-120 x+5}{250 \left(16 x^{2}-8 x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*x)^3/(2-3*x),x, algorithm="fricas")
```

```
[Out] -1/250*(18*(16*x^2 - 8*x + 1)*log(4*x - 1) - 18*(16*x^2 - 8*x + 1)*log(3*x
- 2) - 120*x + 5)/(16*x^2 - 8*x + 1)
```

Sympy [A] time = 0.136271, size = 34, normalized size = 0.79

$$\frac{24x-1}{800x^{2}-400x+50}+\frac{9 \log \left(x-\frac{2}{3}\right)}{125}-\frac{9 \log \left(x-\frac{1}{4}\right)}{125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*x)**3/(2-3*x),x)
```

```
[Out] (24*x - 1)/(800*x**2 - 400*x + 50) + 9*log(x - 2/3)/125 - 9*log(x - 1/4)/12
5
```

Giac [A] time = 1.06351, size = 45, normalized size = 1.05

$$\frac{24\,x-1}{50\,(4\,x-1)^2}-\frac{9}{125}\,\log\left(|4\,x-1|\right)+\frac{9}{125}\,\log\left(|3\,x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-4*x)^3/(2-3*x),x, algorithm="giac")
```

```
[Out] 1/50*(24*x - 1)/(4*x - 1)^2 - 9/125*log(abs(4*x - 1)) + 9/125*log(abs(3*x -
 2))
```

3.184 $\int \frac{x^3}{(2-5x^2)^7} dx$

Optimal. Leaf size=27

$$\frac{1}{150\left(2-5x^2\right)^6}-\frac{1}{250\left(2-5x^2\right)^5}$$

```
[Out] 1/(150*(2 - 5*x^2)^6) - 1/(250*(2 - 5*x^2)^5)
```

Rubi [A] time = 0.0191322, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{150\left(2-5x^2\right)^6}-\frac{1}{250\left(2-5x^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(2 - 5*x^2)^7,x]

[Out] 1/(150*(2 - 5*x^2)^6) - 1/(250*(2 - 5*x^2)^5)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\left(2-5x^2\right)^7} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(2-5x)^7} dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{2}{5(-2+5x)^7}-\frac{1}{5(-2+5x)^6}\right) dx, x, x^2\right)\\ &= \frac{1}{150\left(2-5x^2\right)^6}-\frac{1}{250\left(2-5x^2\right)^5}\end{aligned}$$

Mathematica [A] time = 0.0074893, size = 20, normalized size = 0.74

$$\frac{15x^2-1}{750\left(2-5x^2\right)^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(2 - 5*x^2)^7,x]
```

```
[Out] (-1 + 15*x^2)/(750*(2 - 5*x^2)^6)
```

Maple [A] time = 0.007, size = 24, normalized size = 0.9

$$\frac{1}{150\left(5x^2-2\right)^6}+\frac{1}{250\left(5x^2-2\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-5*x^2+2)^7,x)
```

```
[Out] 1/150/(5*x^2-2)^6+1/250/(5*x^2-2)^5
```

Maxima [A] time = 0.934018, size = 58, normalized size = 2.15

$$\frac{15x^2-1}{750\left(15625x^{12}-37500x^{10}+37500x^8-20000x^6+6000x^4-960x^2+64\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-5*x^2+2)^7,x, algorithm="maxima")
```

```
[Out] 1/750*(15*x^2 - 1)/(15625*x^12 - 37500*x^10 + 37500*x^8 - 20000*x^6 + 6000*
x^4 - 960*x^2 + 64)
```

Fricas [A] time = 1.74723, size = 130, normalized size = 4.81

$$\frac{15x^2-1}{750\left(15625x^{12}-37500x^{10}+37500x^8-20000x^6+6000x^4-960x^2+64\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-5*x^2+2)^7,x, algorithm="fricas")
```

```
[Out] 1/750*(15*x^2 - 1)/(15625*x^12 - 37500*x^10 + 37500*x^8 - 20000*x^6 + 6000*
x^4 - 960*x^2 + 64)
```

Sympy [A] time = 0.18098, size = 37, normalized size = 1.37

$$\frac{15x^2-1}{11718750x^{12}-28125000x^{10}+28125000x^8-15000000x^6+4500000x^4-720000x^2+48000}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-5*x**2+2)**7,x)
```

```
[Out] (15*x**2 - 1)/(11718750*x**12 - 28125000*x**10 + 28125000*x**8 - 15000000*x
**6 + 4500000*x**4 - 720000*x**2 + 48000)
```

Giac [A] time = 1.05182, size = 24, normalized size = 0.89

$$\frac{15\,x^2-1}{750\left(5\,x^2-2\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-5*x^2+2)^7,x, algorithm="giac")
```

```
[Out] 1/750*(15*x^2 - 1)/(5*x^2 - 2)^6
```

3.185 $\int \frac{x^7}{(2-5x^2)^3} dx$

Optimal. Leaf size=46

$$-\frac{x^2}{250}-\frac{6}{625(2-5x^2)}+\frac{2}{625(2-5x^2)^2}-\frac{3}{625}\log(2-5x^2)$$

```
[Out] -x^2/250 + 2/(625*(2 - 5*x^2)^2) - 6/(625*(2 - 5*x^2)) - (3*Log[2 - 5*x^2])
/625
```

Rubi [A] time = 0.0299033, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$-\frac{x^2}{250}-\frac{6}{625(2-5x^2)}+\frac{2}{625(2-5x^2)^2}-\frac{3}{625}\log(2-5x^2)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(2 - 5*x^2)^3,x]
```

```
[Out] -x^2/250 + 2/(625*(2 - 5*x^2)^2) - 6/(625*(2 - 5*x^2)) - (3*Log[2 - 5*x^2])
/625
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(2-5x^2)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{(2-5x)^3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{1}{125}-\frac{8}{125(-2+5x)^3}-\frac{12}{125(-2+5x)^2}-\frac{6}{125(-2+5x)}\right)dx, x, x^2\right)\\
&= -\frac{x^2}{250}+\frac{2}{625(2-5x^2)^2}-\frac{6}{625(2-5x^2)}-\frac{3}{625}\log(2-5x^2)
\end{aligned}$$

Mathematica [A] time = 0.0134481, size = 44, normalized size = 0.96

$$-\frac{125x^6-150x^4+6(2-5x^2)^2\log(5x^2-2)+12}{1250(2-5x^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(2 - 5*x^2)^3,x]
```

```
[Out] -(12 - 150*x^4 + 125*x^6 + 6*(2 - 5*x^2)^2*Log[-2 + 5*x^2])/(1250*(2 - 5*x^
2)^2)
```

Maple [A] time = 0.01, size = 39, normalized size = 0.9

$$-\frac{x^2}{250}+\frac{2}{625\left(5x^2-2\right)^2}-\frac{3\ln\left(5x^2-2\right)}{625}+\frac{6}{3125x^2-1250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(-5*x^2+2)^3,x)
```

```
[Out] -1/250*x^2+2/625/(5*x^2-2)^2-3/625*ln(5*x^2-2)+6/625/(5*x^2-2)
```

Maxima [A] time = 0.92504, size = 53, normalized size = 1.15

$$-\frac{1}{250}x^2+\frac{2\left(3x^2-1\right)}{125\left(25x^4-20x^2+4\right)}-\frac{3}{625}\log\left(5x^2-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-5*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -1/250*x^2 + 2/125*(3*x^2 - 1)/(25*x^4 - 20*x^2 + 4) - 3/625*log(5*x^2 - 2)
```

Fricas [A] time = 1.70484, size = 143, normalized size = 3.11

$$-\frac{125x^6-100x^4-40x^2+6\left(25x^4-20x^2+4\right)\log\left(5x^2-2\right)+20}{1250\left(25x^4-20x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-5*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] -1/1250*(125*x^6 - 100*x^4 - 40*x^2 + 6*(25*x^4 - 20*x^2 + 4)*log(5*x^2 - 2
) + 20)/(25*x^4 - 20*x^2 + 4)
```

Sympy [A] time = 0.121358, size = 34, normalized size = 0.74

$$-\frac{x^2}{250}+\frac{6x^2-2}{3125x^4-2500x^2+500}-\frac{3\log\left(5x^2-2\right)}{625}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(-5*x**2+2)**3,x)
```

```
[Out] -x**2/250 + (6*x**2 - 2)/(3125*x**4 - 2500*x**2 + 500) - 3*log(5*x**2 - 2)/
625
```

Giac [A] time = 1.07179, size = 54, normalized size = 1.17

$$-\frac{1}{250}\,x^2+\frac{225\,x^4-120\,x^2+16}{1250\,\left(5\,x^2-2\right)^2}-\frac{3}{625}\,\log\left(\left|5\,x^2-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(-5*x^2+2)^3,x, algorithm="giac")
```

```
[Out] -1/250*x^2 + 1/1250*(225*x^4 - 120*x^2 + 16)/(5*x^2 - 2)^2 - 3/625*log(abs(
5*x^2 - 2))
```

3.186 $\int \frac{1}{(-2+x)^3(1+x)^2} \, dx$

Optimal. Leaf size=44

$$\frac{2}{27(x-2)}+\frac{1}{27(x+1)}-\frac{1}{18(x-2)^2}+\frac{1}{27}\log(x-2)-\frac{1}{27}\log(x+1)$$

```
[Out] -1/(18*(-2 + x)^2) + 2/(27*(-2 + x)) + 1/(27*(1 + x)) + Log[-2 + x]/27 - Lo
g[1 + x]/27
```

Rubi [A] time = 0.0170013, antiderivative size = 50, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{2}{27(2-x)}+\frac{1}{27(x+1)}-\frac{1}{18(2-x)^2}+\frac{1}{27}\log(2-x)-\frac{1}{27}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-2 + x)^3*(1 + x)^2),x]
```

```
[Out] -1/(18*(2 - x)^2) - 2/(27*(2 - x)) + 1/(27*(1 + x)) + Log[2 - x]/27 - Log[1
 + x]/27
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-2+x)^3(1+x)^2} \, dx &= \int \left(\frac{1}{9(-2+x)^3}-\frac{2}{27(-2+x)^2}+\frac{1}{27(-2+x)}-\frac{1}{27(1+x)^2}-\frac{1}{27(1+x)}\right) dx\\ &= -\frac{1}{18(2-x)^2}-\frac{2}{27(2-x)}+\frac{1}{27(1+x)}+\frac{1}{27}\log(2-x)-\frac{1}{27}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0207521, size = 39, normalized size = 0.89

$$\frac{1}{54}\left(\frac{3\left(2x^2-5x-1\right)}{(x-2)^2(x+1)}+2\log(x-2)-2\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-2 + x)^3*(1 + x)^2),x]
```

```
[Out] ((3*(-1 - 5*x + 2*x^2))/((-2 + x)^2*(1 + x)) + 2*Log[-2 + x] - 2*Log[1 + x]
)/54
```

Maple [A] time = 0.009, size = 35, normalized size = 0.8

$$-\frac{1}{18\left(-2+x\right)^{2}}+\frac{2}{-54+27x}+\frac{1}{27+27x}+\frac{\ln\left(-2+x\right)}{27}-\frac{\ln\left(1+x\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2+x)^3/(1+x)^2,x)
```

```
[Out] -1/18/(-2+x)^2+2/27/(-2+x)+1/27/(1+x)+1/27*ln(-2+x)-1/27*ln(1+x)
```

Maxima [A] time = 0.927378, size = 50, normalized size = 1.14

$$\frac{2x^{2}-5x-1}{18\left(x^{3}-3x^{2}+4\right)}-\frac{1}{27}\log\left(x+1\right)+\frac{1}{27}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)^3/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/18*(2*x^2 - 5*x - 1)/(x^3 - 3*x^2 + 4) - 1/27*log(x + 1) + 1/27*log(x - 2
)
```

Fricas [A] time = 1.76286, size = 147, normalized size = 3.34

$$\frac{6x^{2}-2\left(x^{3}-3x^{2}+4\right)\log\left(x+1\right)+2\left(x^{3}-3x^{2}+4\right)\log\left(x-2\right)-15x-3}{54\left(x^{3}-3x^{2}+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)^3/(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/54*(6*x^2 - 2*(x^3 - 3*x^2 + 4)*log(x + 1) + 2*(x^3 - 3*x^2 + 4)*log(x -
2) - 15*x - 3)/(x^3 - 3*x^2 + 4)
```

Sympy [A] time = 0.128278, size = 34, normalized size = 0.77

$$\frac{2x^{2}-5x-1}{18x^{3}-54x^{2}+72}+\frac{\log\left(x-2\right)}{27}-\frac{\log\left(x+1\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)**3/(1+x)**2,x)
```

```
[Out] (2*x**2 - 5*x - 1)/(18*x**3 - 54*x**2 + 72) + log(x - 2)/27 - log(x + 1)/27
```

Giac [A] time = 1.05946, size = 58, normalized size = 1.32

$$\frac{1}{27\left(x+1\right)}-\frac{\frac{18}{x+1}-5}{162\left(\frac{3}{x+1}-1\right)^{2}}+\frac{1}{27}\log\left(\left|-\frac{3}{x+1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2+x)^3/(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/27/(x + 1) - 1/162*(18/(x + 1) - 5)/(3/(x + 1) - 1)^2 + 1/27*log(abs(-3/(
x + 1) + 1))
```

3.187 $\int \frac{1}{(2+x)^3(3+x)^4}\,dx$

Optimal. Leaf size=54

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

```
[Out] -1/(2*(2 + x)^2) + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x
) + 10*Log[2 + x] - 10*Log[3 + x]
```

Rubi [A] time = 0.0182847, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/((2 + x)^3*(3 + x)^4),x]
```

```
[Out] -1/(2*(2 + x)^2) + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x
) + 10*Log[2 + x] - 10*Log[3 + x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2+x)^3(3+x)^4}\,dx &= \int \left(\frac{1}{(2+x)^3}-\frac{4}{(2+x)^2}+\frac{10}{2+x}-\frac{1}{(3+x)^4}-\frac{3}{(3+x)^3}-\frac{6}{(3+x)^2}-\frac{10}{3+x}\right)dx\\ &= -\frac{1}{2(2+x)^2}+\frac{4}{2+x}+\frac{1}{3(3+x)^3}+\frac{3}{2(3+x)^2}+\frac{6}{3+x}+10\log(2+x)-10\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.014764, size = 54, normalized size = 1.

$$\frac{4}{x+2}+\frac{6}{x+3}-\frac{1}{2(x+2)^2}+\frac{3}{2(x+3)^2}+\frac{1}{3(x+3)^3}+10\log(x+2)-10\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((2 + x)^3*(3 + x)^4),x]
```

```
[Out] -1/(2*(2 + x)^2) + 4/(2 + x) + 1/(3*(3 + x)^3) + 3/(2*(3 + x)^2) + 6/(3 + x
) + 10*Log[2 + x] - 10*Log[3 + x]
```

Maple [A] time = 0., size = 49, normalized size = 0.9

$$-\frac{1}{2\,(2+x)^{2}}+4\,(2+x)^{-1}+\frac{1}{3\,(3+x)^{3}}+\frac{3}{2\,(3+x)^{2}}+6\,(3+x)^{-1}+10\,\ln(2+x)-10\,\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+x)^3/(3+x)^4,x)
```

```
[Out] -1/2/(2+x)^2+4/(2+x)+1/3/(3+x)^3+3/2/(3+x)^2+6/(3+x)+10*ln(2+x)-10*ln(3+x)
```

Maxima [A] time = 0.926761, size = 81, normalized size = 1.5

$$\frac{60\,x^{4}+630\,x^{3}+2450\,x^{2}+4175\,x+2627}{6\left(x^{5}+13\,x^{4}+67\,x^{3}+171\,x^{2}+216\,x+108\right)}-10\,\log(x+3)+10\,\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 +
171*x^2 + 216*x + 108) - 10*log(x + 3) + 10*log(x + 2)
```

Fricas [B] time = 1.7558, size = 306, normalized size = 5.67

$$\frac{60\,x^{4}+630\,x^{3}+2450\,x^{2}-60\left(x^{5}+13\,x^{4}+67\,x^{3}+171\,x^{2}+216\,x+108\right)\log(x+3)+60\left(x^{5}+13\,x^{4}+67\,x^{3}+171\,x\right.}{6\left(x^{5}+13\,x^{4}+67\,x^{3}+171\,x^{2}+216\,x+108\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 - 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 21
6*x + 108)*log(x + 3) + 60*(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108)*
log(x + 2) + 4175*x + 2627)/(x^5 + 13*x^4 + 67*x^3 + 171*x^2 + 216*x + 108)
```

Sympy [A] time = 0.160245, size = 58, normalized size = 1.07

$$\frac{60x^{4}+630x^{3}+2450x^{2}+4175x+2627}{6x^{5}+78x^{4}+402x^{3}+1026x^{2}+1296x+648}+10\log(x+2)-10\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)**3/(3+x)**4,x)
```

```
[Out] (60*x**4 + 630*x**3 + 2450*x**2 + 4175*x + 2627)/(6*x**5 + 78*x**4 + 402*x*
*3 + 1026*x**2 + 1296*x + 648) + 10*log(x + 2) - 10*log(x + 3)
```

Giac [A] time = 1.05064, size = 63, normalized size = 1.17

$$\frac{60\,x^4 + 630\,x^3 + 2450\,x^2 + 4175\,x + 2627}{6\,(x+3)^3(x+2)^2} - 10\,\log\left(|x+3|\right) + 10\,\log\left(|x+2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+x)^3/(3+x)^4,x, algorithm="giac")
```

```
[Out] 1/6*(60*x^4 + 630*x^3 + 2450*x^2 + 4175*x + 2627)/((x + 3)^3*(x + 2)^2) - 1
0*log(abs(x + 3)) + 10*log(abs(x + 2))
```

3.188 $\int \frac{x^5}{(3+x)^2} dx$

Optimal. Leaf size=36

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + \frac{243}{x+3} + 405\log(x+3)$$

```
[Out] -108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3 + x) + 405*Log[3 + x]
```

Rubi [A] time = 0.0151688, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + \frac{243}{x+3} + 405\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(3 + x)^2,x]
```

```
[Out] -108*x + (27*x^2)/2 - 2*x^3 + x^4/4 + 243/(3 + x) + 405*Log[3 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(3+x)^2} dx &= \int \left(-108 + 27x - 6x^2 + x^3 - \frac{243}{(3+x)^2} + \frac{405}{3+x}\right) dx \\
&= -108x + \frac{27x^2}{2} - 2x^3 + \frac{x^4}{4} + \frac{243}{3+x} + 405\log(3+x)
\end{aligned}$$

Mathematica [A] time = 0.0136431, size = 36, normalized size = 1.

$$\frac{1}{4}\left(x^4 - 8x^3 + 54x^2 - 432x + \frac{972}{x+3} - 2079\right) + 405\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(3 + x)^2,x]
```

```
[Out] (-2079 - 432*x + 54*x^2 - 8*x^3 + x^4 + 972/(3 + x))/4 + 405*Log[3 + x]
```

Maple [A] time = 0., size = 33, normalized size = 0.9

$$-108\,x + \frac{27\,x^2}{2} - 2\,x^3 + \frac{x^4}{4} + 243\,(3+x)^{-1} + 405\,\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(3+x)^2,x)
```

```
[Out] -108*x+27/2*x^2-2*x^3+1/4*x^4+243/(3+x)+405*ln(3+x)
```

Maxima [A] time = 0.929725, size = 43, normalized size = 1.19

$$\frac{1}{4}\,x^4 - 2\,x^3 + \frac{27}{2}\,x^2 - 108\,x + \frac{243}{x+3} + 405\,\log\left(x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3+x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 - 2*x^3 + 27/2*x^2 - 108*x + 243/(x + 3) + 405*log(x + 3)
```

Fricas [A] time = 1.58525, size = 117, normalized size = 3.25

$$\frac{x^5 - 5\,x^4 + 30\,x^3 - 270\,x^2 + 1620\,(x+3)\log\left(x+3\right) - 1296\,x + 972}{4\,(x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3+x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(x^5 - 5*x^4 + 30*x^3 - 270*x^2 + 1620*(x + 3)*log(x + 3) - 1296*x + 97
2)/(x + 3)
```

Sympy [A] time = 0.080228, size = 31, normalized size = 0.86

$$\frac{x^4}{4} - 2x^3 + \frac{27x^2}{2} - 108x + 405\log\left(x+3\right) + \frac{243}{x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(3+x)**2,x)
```

```
[Out] x**4/4 - 2*x**3 + 27*x**2/2 - 108*x + 405*log(x + 3) + 243/(x + 3)
```

Giac [A] time = 1.05162, size = 61, normalized size = 1.69

$$-\frac{1}{4}\,(x+3)^4\left(\frac{20}{x+3} - \frac{180}{(x+3)^2} + \frac{1080}{(x+3)^3} - 1\right) + \frac{243}{x+3} + 405\,\log\left(|x+3|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(3+x)^2,x, algorithm="giac")
```

```
[Out] -1/4*(x + 3)^4*(20/(x + 3) - 180/(x + 3)^2 + 1080/(x + 3)^3 - 1) + 243/(x +
 3) + 405*log(abs(x + 3))
```

3.189 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right) dx$

Optimal. Leaf size=44

$$\frac{1}{2}x^2(ac1 + 2bb1) + ab1x + \frac{1}{3}x^3(2bc1 + b1c) + \frac{1}{4}cc1x^4$$

```
[Out] a*b1*x + ((2*b*b1 + a*c1)*x^2)/2 + ((b1*c + 2*b*c1)*x^3)/3 + (c*c1*x^4)/4
```

Rubi [A] time = 0.0365853, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {631}

$$\frac{1}{2}x^2(ac1 + 2bb1) + ab1x + \frac{1}{3}x^3(2bc1 + b1c) + \frac{1}{4}cc1x^4$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2),x]
```

```
[Out] a*b1*x + ((2*b*b1 + a*c1)*x^2)/2 + ((b1*c + 2*b*c1)*x^3)/3 + (c*c1*x^4)/4
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int (b1 + c1x)\left(a + 2bx + cx^2\right) dx &= \int \left(ab1 + (2bb1 + ac1)x + (b1c + 2bc1)x^2 + cc1x^3\right) dx\\ &= ab1x + \frac{1}{2}(2bb1 + ac1)x^2 + \frac{1}{3}(b1c + 2bc1)x^3 + \frac{1}{4}cc1x^4\end{aligned}$$

Mathematica [A] time = 0.0116867, size = 41, normalized size = 0.93

$$\frac{1}{12}x(6a(2b1 + c1x) + x(4b(3b1 + 2c1x) + cx(4b1 + 3c1x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2),x]
```

```
[Out] (x*(6*a*(2*b1 + c1*x) + x*(4*b*(3*b1 + 2*c1*x) + c*x*(4*b1 + 3*c1*x))))/12
```

Maple [A] time = 0.003, size = 39, normalized size = 0.9

$$ab1\,x + \frac{(ac1 + 2\,bb1)\,x^2}{2} + \frac{(2\,bc1 + b1\,c)\,x^3}{3} + \frac{cc1\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)*(c*x^2+2*b*x+a),x)
```

```
[Out] a*b1*x+1/2*(a*c1+2*b*b1)*x^2+1/3*(2*b*c1+b1*c)*x^3+1/4*c*c1*x^4
```

Maxima [A] time = 0.925517, size = 51, normalized size = 1.16

$$\frac{1}{4}cc_1x^4 + \frac{1}{3}(b_1c + 2\,bc_1)x^3 + ab_1x + \frac{1}{2}(2\,bb_1 + ac_1)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*c*c1*x^4 + 1/3*(b1*c + 2*b*c1)*x^3 + a*b1*x + 1/2*(2*b*b1 + a*c1)*x^2
```

Fricas [A] time = 1.47087, size = 107, normalized size = 2.43

$$\frac{1}{4}x^4c_1c + \frac{1}{3}x^3cb_1 + \frac{2}{3}x^3c_1b + x^2b_1b + \frac{1}{2}x^2c_1a + xb_1a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*c1*c + 1/3*x^3*c*b1 + 2/3*x^3*c1*b + x^2*b1*b + 1/2*x^2*c1*a + x*b1
*a
```

Sympy [A] time = 0.058321, size = 39, normalized size = 0.89

$$ab_1x + \frac{cc_1x^4}{4} + x^3\left(\frac{2bc_1}{3} + \frac{b_1c}{3}\right) + x^2\left(\frac{ac_1}{2} + bb_1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a),x)
```

```
[Out] a*b1*x + c*c1*x**4/4 + x**3*(2*b*c1/3 + b1*c/3) + x**2*(a*c1/2 + b*b1)
```

Giac [A] time = 1.0673, size = 53, normalized size = 1.2

$$\frac{1}{4}cc_1x^4 + \frac{1}{3}b_1cx^3 + \frac{2}{3}bc_1x^3 + bb_1x^2 + \frac{1}{2}ac_1x^2 + ab_1x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a),x, algorithm="giac")
```

```
[Out] 1/4*c*c1*x^4 + 1/3*b1*c*x^3 + 2/3*b*c1*x^3 + b*b1*x^2 + 1/2*a*c1*x^2 + a*b1
*x
```

3.190 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)^2\,dx$

Optimal. Leaf size=96

$$a^2\mathrm{b1}x + \frac{1}{2}x^4\left(ac\mathrm{c1} + 2b^2\mathrm{c1} + 2b\mathrm{b1}c\right) + \frac{2}{3}x^3\left(2ab\mathrm{c1} + a\mathrm{b1}c + 2b^2\mathrm{b1}\right) + \frac{1}{2}ax^2(a\mathrm{c1} + 4b\mathrm{b1}) + \frac{1}{5}cx^5(4b\mathrm{c1} + \mathrm{b1}c) + \frac{1}{6}c$$

```
[Out] a^2*b1*x + (a*(4*b*b1 + a*c1)*x^2)/2 + (2*(2*b^2*b1 + a*b1*c + 2*a*b*c1)*x^
3)/3 + ((2*b*b1*c + 2*b^2*c1 + a*c*c1)*x^4)/2 + (c*(b1*c + 4*b*c1)*x^5)/5 +
 (c^2*c1*x^6)/6
```

Rubi [A] time = 0.10442, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {631}

$$a^2\mathrm{b1}x + \frac{1}{2}x^4\left(ac\mathrm{c1} + 2b^2\mathrm{c1} + 2b\mathrm{b1}c\right) + \frac{2}{3}x^3\left(2ab\mathrm{c1} + a\mathrm{b1}c + 2b^2\mathrm{b1}\right) + \frac{1}{2}ax^2(a\mathrm{c1} + 4b\mathrm{b1}) + \frac{1}{5}cx^5(4b\mathrm{c1} + \mathrm{b1}c) + \frac{1}{6}c$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] a^2*b1*x + (a*(4*b*b1 + a*c1)*x^2)/2 + (2*(2*b^2*b1 + a*b1*c + 2*a*b*c1)*x^
3)/3 + ((2*b*b1*c + 2*b^2*c1 + a*c*c1)*x^4)/2 + (c*(b1*c + 4*b*c1)*x^5)/5 +
 (c^2*c1*x^6)/6
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\int (\mathrm{b1} + \mathrm{c1}x)\left(a + 2bx + cx^2\right)^2\,dx = \int \left(a^2\mathrm{b1} + a(4b\mathrm{b1} + a\mathrm{c1})x + 2\left(2b^2\mathrm{b1} + a\mathrm{b1}c + 2ab\mathrm{c1}\right)x^2 + 2\left(2b\mathrm{b1}c + 2b^2\mathrm{c1}\right.\right.$$
$$= a^2\mathrm{b1}x + \frac{1}{2}a(4b\mathrm{b1} + a\mathrm{c1})x^2 + \frac{2}{3}\left(2b^2\mathrm{b1} + a\mathrm{b1}c + 2ab\mathrm{c1}\right)x^3 + \frac{1}{2}\left(2b\mathrm{b1}c + 2b^2\mathrm{c1}\right.$$

Mathematica [A] time = 0.028031, size = 91, normalized size = 0.95

$$\frac{1}{30}x\left(15a^2(2\mathrm{b1} + \mathrm{c1}x) + 5ax(4b(3\mathrm{b1} + 2\mathrm{c1}x) + cx(4\mathrm{b1} + 3\mathrm{c1}x)) + x^2\left(10b^2(4\mathrm{b1} + 3\mathrm{c1}x) + 6bcx(5\mathrm{b1} + 4\mathrm{c1}x) + c^2x^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] (x*(15*a^2*(2*b1 + c1*x) + 5*a*x*(4*b*(3*b1 + 2*c1*x) + c*x*(4*b1 + 3*c1*x)
) + x^2*(10*b^2*(4*b1 + 3*c1*x) + 6*b*c*x*(5*b1 + 4*c1*x) + c^2*x^2*(6*b1 +
 5*c1*x))))/30
```

Maple [A] time = 0.001, size = 95, normalized size = 1.

$$\frac{c^2\mathrm{c1}\,x^6}{6}+\frac{\left(4\,\mathrm{c1}\,bc+\mathrm{b1}\,c^2\right)x^5}{5}+\frac{\left(4\,b\mathrm{b1}\,c+\mathrm{c1}\,\left(2\,ac+4\,b^2\right)\right)x^4}{4}+\frac{\left(\mathrm{b1}\,\left(2\,ac+4\,b^2\right)+4\,abc\mathrm{1}\right)x^3}{3}+\frac{\left(\mathrm{c1}\,a^2+4\,\mathrm{b1}\,ab\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)*(c*x^2+2*b*x+a)^2,x)
```

```
[Out] 1/6*c^2*c1*x^6+1/5*(4*b*c*c1+b1*c^2)*x^5+1/4*(4*b*b1*c+c1*(2*a*c+4*b^2))*x^
4+1/3*(b1*(2*a*c+4*b^2)+4*a*b*c1)*x^3+1/2*(a^2*c1+4*a*b*b1)*x^2+a^2*b1*x
```

Maxima [A] time = 0.932681, size = 123, normalized size = 1.28

$$\frac{1}{6}\,c^2c_1x^6+\frac{1}{5}\left(b_1c^2+4\,bcc_1\right)x^5+\frac{1}{2}\left(2\,bb_1c+\left(2\,b^2+ac\right)c_1\right)x^4+a^2b_1x+\frac{2}{3}\left(2\,b^2b_1+ab_1c+2\,abc_1\right)x^3+\frac{1}{2}\left(4\,abb_1+a^2c_1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*c^2*c1*x^6 + 1/5*(b1*c^2 + 4*b*c*c1)*x^5 + 1/2*(2*b*b1*c + (2*b^2 + a*c
)*c1)*x^4 + a^2*b1*x + 2/3*(2*b^2*b1 + a*b1*c + 2*a*b*c1)*x^3 + 1/2*(4*a*b*
b1 + a^2*c1)*x^2
```

Fricas [A] time = 1.56473, size = 252, normalized size = 2.62

$$\frac{1}{6}x^6c_1c^2+\frac{1}{5}x^5c^2b_1+\frac{4}{5}x^5c_1cb+x^4cb_1b+x^4c_1b^2+\frac{1}{2}x^4c_1ca+\frac{4}{3}x^3b_1b^2+\frac{2}{3}x^3cb_1a+\frac{4}{3}x^3c_1ba+2x^2b_1ba+\frac{1}{2}x^2c_1a^2+xb_1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^6*c1*c^2 + 1/5*x^5*c^2*b1 + 4/5*x^5*c1*c*b + x^4*c*b1*b + x^4*c1*b^2
+ 1/2*x^4*c1*c*a + 4/3*x^3*b1*b^2 + 2/3*x^3*c*b1*a + 4/3*x^3*c1*b*a + 2*x^2
*b1*b*a + 1/2*x^2*c1*a^2 + x*b1*a^2
```

Sympy [A] time = 0.075041, size = 100, normalized size = 1.04

$$a^2b_1x+\frac{c^2c_1x^6}{6}+x^5\left(\frac{4bcc_1}{5}+\frac{b_1c^2}{5}\right)+x^4\left(\frac{acc_1}{2}+b^2c_1+bb_1c\right)+x^3\left(\frac{4abc_1}{3}+\frac{2ab_1c}{3}+\frac{4b^2b_1}{3}\right)+x^2\left(\frac{a^2c_1}{2}+2abb_1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a)**2,x)
```

```
[Out] a**2*b1*x + c**2*c1*x**6/6 + x**5*(4*b*c*c1/5 + b1*c**2/5) + x**4*(a*c*c1/2
 + b**2*c1 + b*b1*c) + x**3*(4*a*b*c1/3 + 2*a*b1*c/3 + 4*b**2*b1/3) + x**2*
(a**2*c1/2 + 2*a*b*b1)
```

Giac [A] time = 1.05815, size = 132, normalized size = 1.38

$$\frac{1}{6}c^2c_1x^6 + \frac{1}{5}b_1c^2x^5 + \frac{4}{5}bcc_1x^5 + bb_1cx^4 + b^2c_1x^4 + \frac{1}{2}acc_1x^4 + \frac{4}{3}b^2b_1x^3 + \frac{2}{3}ab_1cx^3 + \frac{4}{3}abc_1x^3 + 2\,abb_1x^2 + \frac{1}{2}a^2c_1x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*c^2*c1*x^6 + 1/5*b1*c^2*x^5 + 4/5*b*c*c1*x^5 + b*b1*c*x^4 + b^2*c1*x^4
+ 1/2*a*c*c1*x^4 + 4/3*b^2*b1*x^3 + 2/3*a*b1*c*x^3 + 4/3*a*b*c1*x^3 + 2*a*b
*b1*x^2 + 1/2*a^2*c1*x^2 + a^2*b1*x
```

3.191 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)^3\,dx$

Optimal. Leaf size=167

$$\frac{1}{4}x^4\left(3a^2c\text{c1} + 12ab^2\text{c1} + 12ab\text{b1}c + 8b^3\text{b1}\right) + \frac{1}{2}a^2x^2(a\text{c1} + 6b\text{b1}) + a^3\text{b1}x + \frac{1}{5}x^5\left(12abc\text{c1} + 3a\text{b1}c^2 + 12b^2\text{b1}c + 8b^3\text{c}\right.$$

```
[Out] a^3*b1*x + (a^2*(6*b*b1 + a*c1)*x^2)/2 + a*(4*b^2*b1 + a*b1*c + 2*a*b*c1)*x
^3 + ((8*b^3*b1 + 12*a*b*b1*c + 12*a*b^2*c1 + 3*a^2*c*c1)*x^4)/4 + ((12*b^2
*b1*c + 3*a*b1*c^2 + 8*b^3*c1 + 12*a*b*c*c1)*x^5)/5 + (c*(2*b*b1*c + 4*b^2*
c1 + a*c*c1)*x^6)/2 + (c^2*(b1*c + 6*b*c1)*x^7)/7 + (c^3*c1*x^8)/8
```

Rubi [A] time = 0.187678, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {631}

$$\frac{1}{4}x^4\left(3a^2c\text{c1} + 12ab^2\text{c1} + 12ab\text{b1}c + 8b^3\text{b1}\right) + \frac{1}{2}a^2x^2(a\text{c1} + 6b\text{b1}) + a^3\text{b1}x + \frac{1}{5}x^5\left(12abc\text{c1} + 3a\text{b1}c^2 + 12b^2\text{b1}c + 8b^3\text{c}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] a^3*b1*x + (a^2*(6*b*b1 + a*c1)*x^2)/2 + a*(4*b^2*b1 + a*b1*c + 2*a*b*c1)*x
^3 + ((8*b^3*b1 + 12*a*b*b1*c + 12*a*b^2*c1 + 3*a^2*c*c1)*x^4)/4 + ((12*b^2
*b1*c + 3*a*b1*c^2 + 8*b^3*c1 + 12*a*b*c*c1)*x^5)/5 + (c*(2*b*b1*c + 4*b^2*
c1 + a*c*c1)*x^6)/2 + (c^2*(b1*c + 6*b*c1)*x^7)/7 + (c^3*c1*x^8)/8
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\int (\text{b1} + \text{c1}x)\left(a + 2bx + cx^2\right)^3\,dx = \int \left(a^3\text{b1} + a^2(6b\text{b1} + a\text{c1})x + 3a\left(4b^2\text{b1} + a\text{b1}c + 2abc\text{1}\right)x^2 + \left(8b^3\text{b1} + 12ab\text{b1}c\right.\right.$$
$$= a^3\text{b1}x + \frac{1}{2}a^2(6b\text{b1} + a\text{c1})x^2 + a\left(4b^2\text{b1} + a\text{b1}c + 2abc\text{1}\right)x^3 + \frac{1}{4}\left(8b^3\text{b1} + 12ab\text{b1}c\right.$$

Mathematica [A] time = 0.0329432, size = 167, normalized size = 1.

$$\frac{1}{4}x^4\left(3a^2c\text{c1} + 12ab^2\text{c1} + 12ab\text{b1}c + 8b^3\text{b1}\right) + \frac{1}{2}a^2x^2(a\text{c1} + 6b\text{b1}) + a^3\text{b1}x + \frac{1}{5}x^5\left(12abc\text{c1} + 3a\text{b1}c^2 + 12b^2\text{b1}c + 8b^3\text{c}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] a^3*b1*x + (a^2*(6*b*b1 + a*c1)*x^2)/2 + a*(4*b^2*b1 + a*b1*c + 2*a*b*c1)*x
^3 + ((8*b^3*b1 + 12*a*b*b1*c + 12*a*b^2*c1 + 3*a^2*c*c1)*x^4)/4 + ((12*b^2
```

```
*b1*c + 3*a*b1*c^2 + 8*b^3*c1 + 12*a*b*c*c1)*x^5)/5 + (c*(2*b*b1*c + 4*b^2*
c1 + a*c*c1)*x^6)/2 + (c^2*(b1*c + 6*b*c1)*x^7)/7 + (c^3*c1*x^8)/8
```

Maple [A] time = 0.001, size = 237, normalized size = 1.4

$$\frac{c^3 c1\, x^8}{8}+\frac{\left(6\, c1\, bc^2+b1\, c^3\right) x^7}{7}+\frac{\left(6\, b1\, bc^2+c1\, \left(ac^2+8\, b^2 c+c\left(2\, ac+4\, b^2\right)\right)\right) x^6}{6}+\frac{\left(b1\, \left(ac^2+8\, b^2 c+c\left(2\, ac+4\, b^2\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)*(c*x^2+2*b*x+a)^3,x)
```

```
[Out] 1/8*c^3*c1*x^8+1/7*(6*b*c^2*c1+b1*c^3)*x^7+1/6*(6*b1*b*c^2+c1*(a*c^2+8*b^2*
c+c*(2*a*c+4*b^2)))*x^6+1/5*(b1*(a*c^2+8*b^2*c+c*(2*a*c+4*b^2))+c1*(8*a*b*c
+2*b*(2*a*c+4*b^2)))*x^5+1/4*(b1*(8*a*b*c+2*b*(2*a*c+4*b^2))+c1*(a*(2*a*c+4
*b^2)+8*b^2*a+c*a^2))*x^4+1/3*(b1*(a*(2*a*c+4*b^2)+8*b^2*a+c*a^2)+6*c1*a^2*
b)*x^3+1/2*(a^3*c1+6*a^2*b*b1)*x^2+a^3*b1*x
```

Maxima [A] time = 0.933209, size = 231, normalized size = 1.38

$$\frac{1}{8}\, c^3 c_1 x^8+\frac{1}{7}\left(b_1 c^3+6\, bc^2 c_1\right) x^7+\frac{1}{2}\left(2\, bb_1 c^2+\left(4\, b^2 c+ac^2\right) c_1\right) x^6+\frac{1}{5}\left(12\, b^2 b_1 c+3\, ab_1 c^2+4\left(2\, b^3+3\, abc\right) c_1\right) x^5+a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*c^3*c1*x^8 + 1/7*(b1*c^3 + 6*b*c^2*c1)*x^7 + 1/2*(2*b*b1*c^2 + (4*b^2*c
 + a*c^2)*c1)*x^6 + 1/5*(12*b^2*b1*c + 3*a*b1*c^2 + 4*(2*b^3 + 3*a*b*c)*c1)
*x^5 + a^3*b1*x + 1/4*(8*b^3*b1 + 12*a*b*b1*c + 3*(4*a*b^2 + a^2*c)*c1)*x^4
 + (4*a*b^2*b1 + a^2*b1*c + 2*a^2*b*c1)*x^3 + 1/2*(6*a^2*b*b1 + a^3*c1)*x^2
```

Fricas [A] time = 1.51364, size = 460, normalized size = 2.75

$$\frac{1}{8} x^8 c_1 c^3+\frac{1}{7} x^7 c^3 b_1+\frac{6}{7} x^7 c_1 c^2 b+x^6 c^2 b_1 b+2 x^6 c_1 c b^2+\frac{1}{2} x^6 c_1 c^2 a+\frac{12}{5} x^5 c b_1 b^2+\frac{8}{5} x^5 c_1 b^3+\frac{3}{5} x^5 c^2 b_1 a+\frac{12}{5} x^5 c_1 cba+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*x^8*c1*c^3 + 1/7*x^7*c^3*b1 + 6/7*x^7*c1*c^2*b + x^6*c^2*b1*b + 2*x^6*c
1*c*b^2 + 1/2*x^6*c1*c^2*a + 12/5*x^5*c*b1*b^2 + 8/5*x^5*c1*b^3 + 3/5*x^5*c
^2*b1*a + 12/5*x^5*c1*c*b*a + 2*x^4*b1*b^3 + 3*x^4*c*b1*b*a + 3*x^4*c1*b^2*
a + 3/4*x^4*c1*c*a^2 + 4*x^3*b1*b^2*a + x^3*c*b1*a^2 + 2*x^3*c1*b*a^2 + 3*x
^2*b1*b*a^2 + 1/2*x^2*c1*a^3 + x*b1*a^3
```

Sympy [A] time = 0.087628, size = 189, normalized size = 1.13

$$a^3 b_1 x+\frac{c^3 c_1 x^8}{8}+x^7\left(\frac{6bc^2 c_1}{7}+\frac{b_1 c^3}{7}\right)+x^6\left(\frac{ac^2 c_1}{2}+2b^2 cc_1+bb_1 c^2\right)+x^5\left(\frac{12abcc_1}{5}+\frac{3ab_1 c^2}{5}+\frac{8b^3 c_1}{5}+\frac{12b^2 b_1 c}{5}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a)**3,x)
```

```
[Out] a**3*b1*x + c**3*c1*x**8/8 + x**7*(6*b*c**2*c1/7 + b1*c**3/7) + x**6*(a*c**
2*c1/2 + 2*b**2*c*c1 + b*b1*c**2) + x**5*(12*a*b*c*c1/5 + 3*a*b1*c**2/5 + 8
*b**3*c1/5 + 12*b**2*b1*c/5) + x**4*(3*a**2*c*c1/4 + 3*a*b**2*c1 + 3*a*b*b1
*c + 2*b**3*b1) + x**3*(2*a**2*b*c1 + a**2*b1*c + 4*a*b**2*b1) + x**2*(a**3
*c1/2 + 3*a**2*b*b1)
```

Giac [A] time = 1.06303, size = 254, normalized size = 1.52

$$\frac{1}{8}c^3c_1x^8 + \frac{1}{7}b_1c^3x^7 + \frac{6}{7}bc^2c_1x^7 + bb_1c^2x^6 + 2\,b^2cc_1x^6 + \frac{1}{2}ac^2c_1x^6 + \frac{12}{5}b^2b_1cx^5 + \frac{3}{5}ab_1c^2x^5 + \frac{8}{5}b^3c_1x^5 + \frac{12}{5}abcc_1x^5 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/8*c^3*c1*x^8 + 1/7*b1*c^3*x^7 + 6/7*b*c^2*c1*x^7 + b*b1*c^2*x^6 + 2*b^2*c
*c1*x^6 + 1/2*a*c^2*c1*x^6 + 12/5*b^2*b1*c*x^5 + 3/5*a*b1*c^2*x^5 + 8/5*b^3
*c1*x^5 + 12/5*a*b*c*c1*x^5 + 2*b^3*b1*x^4 + 3*a*b*b1*c*x^4 + 3*a*b^2*c1*x^
4 + 3/4*a^2*c*c1*x^4 + 4*a*b^2*b1*x^3 + a^2*b1*c*x^3 + 2*a^2*b*c1*x^3 + 3*a
^2*b*b1*x^2 + 1/2*a^3*c1*x^2 + a^3*b1*x
```

3.192 $\int (\mathbf{b1}+\mathbf{c1}x)\left(a+2bx+cx^2\right)^4\,dx$

Optimal. Leaf size=263

$$\frac{1}{3}x^6\left(3a^2c^2\text{c1}+24ab^2c\text{c1}+12ab\text{b1}c^2+16b^3\text{b1}c+8b^4\text{c1}\right)+\frac{2}{5}x^5\left(12a^2bc\text{c1}+3a^2\text{b1}c^2+24ab^2\text{b1}c+16ab^3\text{c1}+8b^4\text{b}\right.$$

```
[Out] a^4*b1*x + (a^3*(8*b*b1 + a*c1)*x^2)/2 + (4*a^2*(6*b^2*b1 + a*b1*c + 2*a*b*
c1)*x^3)/3 + a*(8*b^3*b1 + 6*a*b*b1*c + 6*a*b^2*c1 + a^2*c*c1)*x^4 + (2*(8*
b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 + 16*a*b^3*c1 + 12*a^2*b*c*c1)*x^5)/5
 + ((16*b^3*b1*c + 12*a*b*b1*c^2 + 8*b^4*c1 + 24*a*b^2*c*c1 + 3*a^2*c^2*c1)
*x^6)/3 + (4*c*(6*b^2*b1*c + a*b1*c^2 + 8*b^3*c1 + 6*a*b*c*c1)*x^7)/7 + (c^
2*(2*b*b1*c + 6*b^2*c1 + a*c*c1)*x^8)/2 + (c^3*(b1*c + 8*b*c1)*x^9)/9 + (c^
4*c1*x^10)/10
```

Rubi [A] time = 0.337079, antiderivative size = 263, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {631}

$$\frac{1}{3}x^6\left(3a^2c^2\text{c1}+24ab^2c\text{c1}+12ab\text{b1}c^2+16b^3\text{b1}c+8b^4\text{c1}\right)+\frac{2}{5}x^5\left(12a^2bc\text{c1}+3a^2\text{b1}c^2+24ab^2\text{b1}c+16ab^3\text{c1}+8b^4\text{b}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x]
```

```
[Out] a^4*b1*x + (a^3*(8*b*b1 + a*c1)*x^2)/2 + (4*a^2*(6*b^2*b1 + a*b1*c + 2*a*b*
c1)*x^3)/3 + a*(8*b^3*b1 + 6*a*b*b1*c + 6*a*b^2*c1 + a^2*c*c1)*x^4 + (2*(8*
b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 + 16*a*b^3*c1 + 12*a^2*b*c*c1)*x^5)/5
 + ((16*b^3*b1*c + 12*a*b*b1*c^2 + 8*b^4*c1 + 24*a*b^2*c*c1 + 3*a^2*c^2*c1)
*x^6)/3 + (4*c*(6*b^2*b1*c + a*b1*c^2 + 8*b^3*c1 + 6*a*b*c*c1)*x^7)/7 + (c^
2*(2*b*b1*c + 6*b^2*c1 + a*c*c1)*x^8)/2 + (c^3*(b1*c + 8*b*c1)*x^9)/9 + (c^
4*c1*x^10)/10
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^4\,dx=\int \left(a^4\text{b1}+a^3(8b\text{b1}+a\text{c1})x+4a^2\left(6b^2\text{b1}+a\text{b1}c+2abc\text{c1}\right)x^2+4a\left(8b^3\text{b1}+6a\right.\right.$$

$$=a^4\text{b1}x+\frac{1}{2}a^3(8b\text{b1}+a\text{c1})x^2+\frac{4}{3}a^2\left(6b^2\text{b1}+a\text{b1}c+2abc\text{c1}\right)x^3+a\left(8b^3\text{b1}+6ab\right.$$

Mathematica [A] time = 0.0594917, size = 263, normalized size = 1.

$$\frac{1}{3}x^6\left(3a^2c^2\text{c1}+24ab^2c\text{c1}+12ab\text{b1}c^2+16b^3\text{b1}c+8b^4\text{c1}\right)+\frac{2}{5}x^5\left(12a^2bc\text{c1}+3a^2\text{b1}c^2+24ab^2\text{b1}c+16ab^3\text{c1}+8b^4\text{b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^4,x]
```

```
[Out] a^4*b1*x + (a^3*(8*b*b1 + a*c1)*x^2)/2 + (4*a^2*(6*b^2*b1 + a*b1*c + 2*a*b*
c1)*x^3)/3 + a*(8*b^3*b1 + 6*a*b*b1*c + 6*a*b^2*c1 + a^2*c*c1)*x^4 + (2*(8*
b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 + 16*a*b^3*c1 + 12*a^2*b*c*c1)*x^5)/5
 + ((16*b^3*b1*c + 12*a*b*b1*c^2 + 8*b^4*c1 + 24*a*b^2*c*c1 + 3*a^2*c^2*c1)
*x^6)/3 + (4*c*(6*b^2*b1*c + a*b1*c^2 + 8*b^3*c1 + 6*a*b*c*c1)*x^7)/7 + (c^
2*(2*b*b1*c + 6*b^2*c1 + a*c*c1)*x^8)/2 + (c^3*(b1*c + 8*b*c1)*x^9)/9 + (c^
4*c1*x^10)/10
```

Maple [A] time = 0.001, size = 363, normalized size = 1.4

$$\frac{c^4 \mathrm{c1}\, x^{10}}{10}+\frac{\left(8\,\mathrm{c1}\, b c^3+\mathrm{b1}\, c^4\right) x^9}{9}+\frac{\left(8\,\mathrm{b1}\, b c^3+\mathrm{c1}\, \left(2\left(2\,ac+4\,b^2\right)c^2+16\,b^2c^2\right)\right) x^8}{8}+\frac{\left(\mathrm{b1}\, \left(2\left(2\,ac+4\,b^2\right)c^2+16\,b^2c^2\right)+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)*(c*x^2+2*b*x+a)^4,x)
```

```
[Out] 1/10*c^4*c1*x^10+1/9*(8*b*c^3*c1+b1*c^4)*x^9+1/8*(8*b1*b*c^3+c1*(2*(2*a*c+4
*b^2)*c^2+16*b^2*c^2))*x^8+1/7*(b1*(2*(2*a*c+4*b^2)*c^2+16*b^2*c^2)+c1*(8*a
*b*c^2+8*(2*a*c+4*b^2)*b*c))*x^7+1/6*(b1*(8*a*b*c^2+8*(2*a*c+4*b^2)*b*c)+c1
*(2*a^2*c^2+32*a*b^2*c+(2*a*c+4*b^2)^2))*x^6+1/5*(b1*(2*a^2*c^2+32*a*b^2*c+
(2*a*c+4*b^2)^2)+c1*(8*a^2*b*c+8*a*b*(2*a*c+4*b^2)))*x^5+1/4*(b1*(8*a^2*b*c
+8*a*b*(2*a*c+4*b^2))+c1*(2*a^2*(2*a*c+4*b^2)+16*b^2*a^2))*x^4+1/3*(b1*(2*a
^2*(2*a*c+4*b^2)+16*b^2*a^2)+8*c1*a^3*b)*x^3+1/2*(a^4*c1+8*a^3*b*b1)*x^2+a^
4*b1*x
```

Maxima [A] time = 0.936922, size = 369, normalized size = 1.4

$$\frac{1}{10}\,c^4c_1x^{10}+\frac{1}{9}\left(b_1c^4+8\,bc^3c_1\right)x^9+\frac{1}{2}\left(2\,bb_1c^3+\left(6\,b^2c^2+ac^3\right)c_1\right)x^8+\frac{4}{7}\left(6\,b^2b_1c^2+ab_1c^3+2\left(4\,b^3c+3\,abc^2\right)c_1\right)x^7+\frac{1}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/10*c^4*c1*x^10 + 1/9*(b1*c^4 + 8*b*c^3*c1)*x^9 + 1/2*(2*b*b1*c^3 + (6*b^2
*c^2 + a*c^3)*c1)*x^8 + 4/7*(6*b^2*b1*c^2 + a*b1*c^3 + 2*(4*b^3*c + 3*a*b*c
^2)*c1)*x^7 + 1/3*(16*b^3*b1*c + 12*a*b*b1*c^2 + (8*b^4 + 24*a*b^2*c + 3*a^
2*c^2)*c1)*x^6 + a^4*b1*x + 2/5*(8*b^4*b1 + 24*a*b^2*b1*c + 3*a^2*b1*c^2 +
4*(4*a*b^3 + 3*a^2*b*c)*c1)*x^5 + (8*a*b^3*b1 + 6*a^2*b*b1*c + (6*a^2*b^2 +
 a^3*c)*c1)*x^4 + 4/3*(6*a^2*b^2*b1 + a^3*b1*c + 2*a^3*b*c1)*x^3 + 1/2*(8*a
^3*b*b1 + a^4*c1)*x^2
```

Fricas [A] time = 1.5445, size = 752, normalized size = 2.86

$$\frac{1}{10}x^{10}c_1c^4+\frac{1}{9}x^9c^4b_1+\frac{8}{9}x^9c_1c^3b+x^8c^3b_1b+3x^8c_1c^2b^2+\frac{1}{2}x^8c_1c^3a+\frac{24}{7}x^7c^2b_1b^2+\frac{32}{7}x^7c_1cb^3+\frac{4}{7}x^7c^3b_1a+\frac{24}{7}x^7c_1c^2b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*c1*c^4 + 1/9*x^9*c^4*b1 + 8/9*x^9*c1*c^3*b + x^8*c^3*b1*b + 3*x^8
*c1*c^2*b^2 + 1/2*x^8*c1*c^3*a + 24/7*x^7*c^2*b1*b^2 + 32/7*x^7*c1*c*b^3 +
4/7*x^7*c^3*b1*a + 24/7*x^7*c1*c^2*b*a + 16/3*x^6*c*b1*b^3 + 8/3*x^6*c1*b^4
 + 4*x^6*c^2*b1*b*a + 8*x^6*c1*c*b^2*a + x^6*c1*c^2*a^2 + 16/5*x^5*b1*b^4 +
 48/5*x^5*c*b1*b^2*a + 32/5*x^5*c1*b^3*a + 6/5*x^5*c^2*b1*a^2 + 24/5*x^5*c1
*c*b*a^2 + 8*x^4*b1*b^3*a + 6*x^4*c*b1*b*a^2 + 6*x^4*c1*b^2*a^2 + x^4*c1*c*
a^3 + 8*x^3*b1*b^2*a^2 + 4/3*x^3*c*b1*a^3 + 8/3*x^3*c1*b*a^3 + 4*x^2*b1*b*a
^3 + 1/2*x^2*c1*a^4 + x*b1*a^4
```

Sympy [A] time = 0.108007, size = 313, normalized size = 1.19

$$a^4b_1x + \frac{c^4c_1x^{10}}{10} + x^9\left(\frac{8bc^3c_1}{9} + \frac{b_1c^4}{9}\right) + x^8\left(\frac{ac^3c_1}{2} + 3b^2c^2c_1 + bb_1c^3\right) + x^7\left(\frac{24abc^2c_1}{7} + \frac{4ab_1c^3}{7} + \frac{32b^3cc_1}{7} + \frac{24b^2b_1c}{7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a)**4,x)
```

```
[Out] a**4*b1*x + c**4*c1*x**10/10 + x**9*(8*b*c**3*c1/9 + b1*c**4/9) + x**8*(a*c
**3*c1/2 + 3*b**2*c**2*c1 + b*b1*c**3) + x**7*(24*a*b*c**2*c1/7 + 4*a*b1*c*
*3/7 + 32*b**3*c*c1/7 + 24*b**2*b1*c**2/7) + x**6*(a**2*c**2*c1 + 8*a*b**2*
c*c1 + 4*a*b*b1*c**2 + 8*b**4*c1/3 + 16*b**3*b1*c/3) + x**5*(24*a**2*b*c*c1
/5 + 6*a**2*b1*c**2/5 + 32*a*b**3*c1/5 + 48*a*b**2*b1*c/5 + 16*b**4*b1/5) +
 x**4*(a**3*c*c1 + 6*a**2*b**2*c1 + 6*a**2*b*b1*c + 8*a*b**3*b1) + x**3*(8*
a**3*b*c1/3 + 4*a**3*b1*c/3 + 8*a**2*b**2*b1) + x**2*(a**4*c1/2 + 4*a**3*b*
b1)
```

Giac [A] time = 1.05069, size = 414, normalized size = 1.57

$$\frac{1}{10}c^4c_1x^{10} + \frac{1}{9}b_1c^4x^9 + \frac{8}{9}bc^3c_1x^9 + bb_1c^3x^8 + 3b^2c^2c_1x^8 + \frac{1}{2}ac^3c_1x^8 + \frac{24}{7}b^2b_1c^2x^7 + \frac{4}{7}ab_1c^3x^7 + \frac{32}{7}b^3cc_1x^7 + \frac{24}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/10*c^4*c1*x^10 + 1/9*b1*c^4*x^9 + 8/9*b*c^3*c1*x^9 + b*b1*c^3*x^8 + 3*b^2
*c^2*c1*x^8 + 1/2*a*c^3*c1*x^8 + 24/7*b^2*b1*c^2*x^7 + 4/7*a*b1*c^3*x^7 + 3
2/7*b^3*c*c1*x^7 + 24/7*a*b*c^2*c1*x^7 + 16/3*b^3*b1*c*x^6 + 4*a*b*b1*c^2*x
^6 + 8/3*b^4*c1*x^6 + 8*a*b^2*c*c1*x^6 + a^2*c^2*c1*x^6 + 16/5*b^4*b1*x^5 +
 48/5*a*b^2*b1*c*x^5 + 6/5*a^2*b1*c^2*x^5 + 32/5*a*b^3*c1*x^5 + 24/5*a^2*b*
c*c1*x^5 + 8*a*b^3*b1*x^4 + 6*a^2*b*b1*c*x^4 + 6*a^2*b^2*c1*x^4 + a^3*c*c1*
x^4 + 8*a^2*b^2*b1*x^3 + 4/3*a^3*b1*c*x^3 + 8/3*a^3*b*c1*x^3 + 4*a^3*b*b1*x
^2 + 1/2*a^4*c1*x^2 + a^4*b1*x
```

3.193 $\int (\mathbf{b1}+\mathbf{c1}x)\left(a+2bx+cx^2\right)^n\,dx$

Optimal. Leaf size=159

$$\frac{\text{c1}\left(a+2bx+cx^2\right)^{n+1}}{2c(n+1)}-\frac{2^n(\text{b1}c-b\text{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{-n-1}\left(a+2bx+cx^2\right)^{n+1}{}_2F_1\left(-n,n+1;n+2;\frac{b+cx+\sqrt{b^2-ac}}{2\sqrt{b^2-ac}}\right)}{c(n+1)\sqrt{b^2-ac}}$$

```
[Out] (c1*(a + 2*b*x + c*x^2)^(1 + n))/(2*c*(1 + n)) - (2^n*(b1*c - b*c1)*(-((b -
 Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(-1 - n)*(a + 2*b*x + c*x^2)^(1 +
 n)*Hypergeometric2F1[-n, 1 + n, 2 + n, (b + Sqrt[b^2 - a*c] + c*x)/(2*Sqrt
[b^2 - a*c])])/(c*Sqrt[b^2 - a*c]*(1 + n))
```

Rubi [A] time = 0.0817077, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {640, 624}

$$\frac{\text{c1}\left(a+2bx+cx^2\right)^{n+1}}{2c(n+1)}-\frac{2^n(\text{b1}c-b\text{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{-n-1}\left(a+2bx+cx^2\right)^{n+1}\text{Hypergeometric2F1}\left(-n,n+1,n+2,\right.}{c(n+1)\sqrt{b^2-ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] (c1*(a + 2*b*x + c*x^2)^(1 + n))/(2*c*(1 + n)) - (2^n*(b1*c - b*c1)*(-((b -
 Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(-1 - n)*(a + 2*b*x + c*x^2)^(1 +
 n)*Hypergeometric2F1[-n, 1 + n, 2 + n, (b + Sqrt[b^2 - a*c] + c*x)/(2*Sqrt
[b^2 - a*c])])/(c*Sqrt[b^2 - a*c]*(1 + n))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[4*p]
```

Rubi steps

$$\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^n\,dx=\frac{\text{c1}\left(a+2bx+cx^2\right)^{1+n}}{2c(1+n)}+\frac{(2\text{b1}c-2b\text{c1})\int\left(a+2bx+cx^2\right)^n\,dx}{2c}$$

$$=\frac{\text{c1}\left(a+2bx+cx^2\right)^{1+n}}{2c(1+n)}-\frac{2^n(\text{b1}c-b\text{c1})\left(-\frac{b-\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-1-n}\left(a+2bx+cx^2\right)^{1+n}{}_2F_1\left(\right.}{c\sqrt{b^2-ac}(1+n)}$$

Mathematica [C] time = 0.442925, size = 267, normalized size = 1.68

$$\frac{1}{2}(a+x(2b+cx))^n\left(\frac{\text{b1}2^{n+1}\left(-\sqrt{b^2-ac}+b+cx\right)\left(\frac{\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{-n}\text{Hypergeometric2F1}\left(-n,n+1,n+2,\frac{\sqrt{b^2-ac}-b-cx}{2\sqrt{b^2-ac}}\right.}{c(n+1)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b1 + c1*x)*(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] ((a + x*(2*b + c*x))^n*((c1*x^2*AppellF1[2, -n, -n, 3, -((c*x)/(b + Sqrt[b^
2 - a*c])), (c*x)/(-b + Sqrt[b^2 - a*c])])/(((b - Sqrt[b^2 - a*c] + c*x)/(b
 - Sqrt[b^2 - a*c]))^n*((b + Sqrt[b^2 - a*c] + c*x)/(b + Sqrt[b^2 - a*c]))^
n) + (2^(1 + n)*b1*(b - Sqrt[b^2 - a*c] + c*x)*Hypergeometric2F1[-n, 1 + n,
 2 + n, (-b + Sqrt[b^2 - a*c] - c*x)/(2*Sqrt[b^2 - a*c])])/(c*(1 + n)*((b +
 Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c])^n)))/2
```

Maple [F] time = 0.145, size = 0, normalized size = 0.

$$\int (\text{c1}\,x+\text{b1})\left(cx^2+2\,bx+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)*(c*x^2+2*b*x+a)^n,x)
```

```
[Out] int((c1*x+b1)*(c*x^2+2*b*x+a)^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (c_1x+b_1)\left(cx^2+2\,bx+a\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x, algorithm="maxima")
```

```
[Out] integrate((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((c_1x+b_1)\left(cx^2+2\,bx+a\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x, algorithm="fricas")
```

```
[Out] integral((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (b_1 + c_1 x) \left(a + 2bx + cx^2\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x**2+2*b*x+a)**n,x)
```

```
[Out] Integral((b1 + c1*x)*(a + 2*b*x + c*x**2)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (c_1 x + b_1) \left(cx^2 + 2\,bx + a\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)*(c*x^2+2*b*x+a)^n,x, algorithm="giac")
```

```
[Out] integrate((c1*x + b1)*(c*x^2 + 2*b*x + a)^n, x)
```

3.194 $\int \frac{\mathbf{b1}+\mathbf{c1}x}{a+2bx+cx^2}\,dx$

Optimal. Leaf size=65

$$\frac{\text{c1}\log\left(a+2bx+cx^2\right)}{2c}-\frac{(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{c\sqrt{b^2-ac}}$$

```
[Out] -(((b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(c*Sqrt[b^2 - a*c])) +
 (c1*Log[a + 2*b*x + c*x^2])/(2*c)
```

Rubi [A] time = 0.0415227, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {634, 618, 206, 628}

$$\frac{\text{c1}\log\left(a+2bx+cx^2\right)}{2c}-\frac{(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{c\sqrt{b^2-ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2),x]
```

```
[Out] -(((b1*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(c*Sqrt[b^2 - a*c])) +
 (c1*Log[a + 2*b*x + c*x^2])/(2*c)
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\text{b1} + \text{c1}x}{a + 2bx + cx^2} \, dx = \frac{\text{c1} \int \frac{2b+2cx}{a+2bx+cx^2} \, dx}{2c} + \frac{(2\text{b1}c - 2b\text{c1}) \int \frac{1}{a+2bx+cx^2} \, dx}{2c}$$
$$= \frac{\text{c1} \log\left(a + 2bx + cx^2\right)}{2c} - \frac{(2\text{b1}c - 2b\text{c1}) \, \text{Subst}\left(\int \frac{1}{4(b^2-ac)-x^2} \, dx, x, 2b + 2cx\right)}{c}$$
$$= -\frac{(\text{b1}c - b\text{c1}) \tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{c\sqrt{b^2 - ac}} + \frac{\text{c1} \log\left(a + 2bx + cx^2\right)}{2c}$$

Mathematica [A] time = 0.0431969, size = 66, normalized size = 1.02

$$\frac{(\text{b1}c - b\text{c1}) \tan^{-1}\left(\frac{b+cx}{\sqrt{ac-b^2}}\right)}{c\sqrt{ac - b^2}} + \frac{\text{c1} \log\left(a + 2bx + cx^2\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2),x]
```

```
[Out] ((b1*c - b*c1)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/(c*Sqrt[-b^2 + a*c]) + (
c1*Log[a + 2*b*x + c*x^2])/(2*c)
```

Maple [A] time = 0.004, size = 95, normalized size = 1.5

$$\frac{\text{c1} \ln\left(cx^2 + 2\,bx + a\right)}{2\,c} + \text{b1} \arctan\left(\frac{2\,cx + 2\,b}{2} \frac{1}{\sqrt{ac - b^2}}\right) \frac{1}{\sqrt{ac - b^2}} - \frac{b\text{c1}}{c} \arctan\left(\frac{2\,cx + 2\,b}{2} \frac{1}{\sqrt{ac - b^2}}\right) \frac{1}{\sqrt{ac - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)/(c*x^2+2*b*x+a),x)
```

```
[Out] 1/2*c1*ln(c*x^2+2*b*x+a)/c+1/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^
2)^(1/2))*b1-1/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2))*c1*b
/c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.75041, size = 446, normalized size = 6.86

$$\left[\frac{\left(b^2-ac\right)c_1\log\left(cx^2+2bx+a\right)-\sqrt{b^2-ac}(b_1c-bc_1)\log\left(\frac{c^2x^2+2bcx+2b^2-ac+2\sqrt{b^2-ac}(cx+b)}{cx^2+2bx+a}\right)}{2\left(b^2c-ac^2\right)},\frac{\left(b^2-ac\right)c_1\log\left(cx^2+2bx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a),x, algorithm="fricas")
```

```
[Out] [1/2*((b^2 - a*c)*c1*log(c*x^2 + 2*b*x + a) - sqrt(b^2 - a*c)*(b1*c - b*c1)
*log((c^2*x^2 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2
 + 2*b*x + a)))/(b^2*c - a*c^2), 1/2*((b^2 - a*c)*c1*log(c*x^2 + 2*b*x + a)
 - 2*sqrt(-b^2 + a*c)*(b1*c - b*c1)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(b^2
 - a*c)))/(b^2*c - a*c^2)]
```

Sympy [B] time = 0.609779, size = 246, normalized size = 3.78

$$\left(\frac{c_1}{2c}-\frac{\sqrt{-ac+b^2}\,(bc_1-b_1c)}{2c\left(ac-b^2\right)}\right)\log\left(x+\frac{-2ac\left(\frac{c_1}{2c}-\frac{\sqrt{-ac+b^2}(bc_1-b_1c)}{2c(ac-b^2)}\right)+ac_1+2b^2\left(\frac{c_1}{2c}-\frac{\sqrt{-ac+b^2}(bc_1-b_1c)}{2c(ac-b^2)}\right)-bb_1}{bc_1-b_1c}\right)+\left(\frac{c_1}{2c}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x**2+2*b*x+a),x)
```

```
[Out] (c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2)))*log(x + (-2
*a*c*(c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2))) + a*c1
 + 2*b**2*(c1/(2*c) - sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a*c - b**2))) -
 b*b1)/(b*c1 - b1*c)) + (c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2*c*(a
*c - b**2)))*log(x + (-2*a*c*(c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*c)/(2
*c*(a*c - b**2))) + a*c1 + 2*b**2*(c1/(2*c) + sqrt(-a*c + b**2)*(b*c1 - b1*
c)/(2*c*(a*c - b**2))) - b*b1)/(b*c1 - b1*c))
```

Giac [A] time = 1.06189, size = 81, normalized size = 1.25

$$\frac{c_1\log\left(cx^2+2bx+a\right)}{2c}+\frac{(b_1c-bc_1)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{\sqrt{-b^2+ac}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*c1*log(c*x^2 + 2*b*x + a)/c + (b1*c - b*c1)*arctan((c*x + b)/sqrt(-b^2
+ a*c))/(sqrt(-b^2 + a*c)*c)
```

3.195 $\int \frac{\mathbf{b1}+\mathbf{c1}x}{\left(a+2bx+cx^2\right)^2}\,dx$

Optimal. Leaf size=89

$$\frac{(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{2\left(b^2-ac\right)^{3/2}}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)}$$

```
[Out] -(b*b1 - a*c1 + (b1*c - b*c1)*x)/(2*(b^2 - a*c)*(a + 2*b*x + c*x^2)) + ((b1
*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(2*(b^2 - a*c)^(3/2))
```

Rubi [A] time = 0.0447332, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {638, 618, 206}

$$\frac{(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{2\left(b^2-ac\right)^{3/2}}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] -(b*b1 - a*c1 + (b1*c - b*c1)*x)/(2*(b^2 - a*c)*(a + 2*b*x + c*x^2)) + ((b1
*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(2*(b^2 - a*c)^(3/2))
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\mathrm{b1}+\mathrm{c1}x}{\left(a+2bx+cx^2\right)^2}\,dx = -\frac{b\mathrm{b1}-a\mathrm{c1}+(\mathrm{b1}c-b\mathrm{c1})x}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)} - \frac{(\mathrm{b1}c-b\mathrm{c1})\int \frac{1}{a+2bx+cx^2}\,dx}{2\left(b^2-ac\right)}$$

$$= -\frac{b\mathrm{b1}-a\mathrm{c1}+(\mathrm{b1}c-b\mathrm{c1})x}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)} + \frac{(\mathrm{b1}c-b\mathrm{c1})\,\mathrm{Subst}\left(\int \frac{1}{4\left(b^2-ac\right)-x^2}\,dx, x, 2b+2cx\right)}{b^2-ac}$$

$$= -\frac{b\mathrm{b1}-a\mathrm{c1}+(\mathrm{b1}c-b\mathrm{c1})x}{2\left(b^2-ac\right)\left(a+2bx+cx^2\right)} + \frac{(\mathrm{b1}c-b\mathrm{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{2\left(b^2-ac\right)^{3/2}}$$

Mathematica [A] time = 0.0800663, size = 88, normalized size = 0.99

$$\frac{\frac{(b\mathrm{c1}-\mathrm{b1}c)\tan^{-1}\left(\frac{b+cx}{\sqrt{ac-b^2}}\right)}{\sqrt{ac-b^2}} + \frac{a\mathrm{c1}-b\mathrm{b1}+b\mathrm{c1}x-\mathrm{b1}cx}{a+x(2b+cx)}}{2\left(b^2-ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^2,x]
```

```
[Out] ((-(b*b1) + a*c1 - b1*c*x + b*c1*x)/(a + x*(2*b + c*x)) + ((-(b1*c) + b*c1)
*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(2*(b^2 - a*c))
```

Maple [A] time = 0.003, size = 146, normalized size = 1.6

$$\frac{(-2\,b\mathrm{c1}+2\,\mathrm{b1}\,c)\,x+2\,b\mathrm{b1}-2\,a\mathrm{c1}}{\left(4\,ac-4\,b^2\right)\left(cx^2+2\,bx+a\right)} - 2\,\frac{b\mathrm{c1}}{\left(4\,ac-4\,b^2\right)\sqrt{ac-b^2}}\arctan\left(1/2\,\frac{2\,cx+2\,b}{\sqrt{ac-b^2}}\right) + 2\,\frac{\mathrm{b1}\,c}{\left(4\,ac-4\,b^2\right)\sqrt{ac-b^2}}\,\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)/(c*x^2+2*b*x+a)^2,x)
```

```
[Out] ((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)-2/(4*a*c-4
*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2))*b*c1+2/(4*a*c
-4*b^2)/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2))*b1*c
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.84848, size = 964, normalized size = 10.83

$$\left[-\frac{2\,b^3b_1 - 2\,abb_1c - \left(ab_1c - abc_1 + \left(b_1c^2 - bcc_1\right)x^2 + 2\left(bb_1c - b^2c_1\right)x\right)\sqrt{b^2 - ac}\log\left(\frac{c^2x^2+2\,bcx+2\,b^2-ac+2\,\sqrt{b^2-ac}(cx+b)}{cx^2+2\,bx+a}\right) - 2\,(\cdots}{4\left(ab^4 - 2\,a^2b^2c + a^3c^2 + \left(b^4c - 2\,ab^2c^2 + a^2c^3\right)x^2 + 2\left(b^5 - 2\,ab^3c + a^2bc^2\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*b^3*b1 - 2*a*b*b1*c - (a*b1*c - a*b*c1 + (b1*c^2 - b*c*c1)*x^2 + 2
*(b*b1*c - b^2*c1)*x)*sqrt(b^2 - a*c)*log((c^2*x^2 + 2*b*c*x + 2*b^2 - a*c
+ 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2 + 2*b*x + a)) - 2*(a*b^2 - a^2*c)*c1
+ 2*(b^2*b1*c - a*b1*c^2 - (b^3 - a*b*c)*c1)*x)/(a*b^4 - 2*a^2*b^2*c + a^3*
c^2 + (b^4*c - 2*a*b^2*c^2 + a^2*c^3)*x^2 + 2*(b^5 - 2*a*b^3*c + a^2*b*c^2)
*x), -1/2*(b^3*b1 - a*b*b1*c - (a*b1*c - a*b*c1 + (b1*c^2 - b*c*c1)*x^2 + 2
*(b*b1*c - b^2*c1)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(
b^2 - a*c)) - (a*b^2 - a^2*c)*c1 + (b^2*b1*c - a*b1*c^2 - (b^3 - a*b*c)*c1)
*x)/(a*b^4 - 2*a^2*b^2*c + a^3*c^2 + (b^4*c - 2*a*b^2*c^2 + a^2*c^3)*x^2 +
2*(b^5 - 2*a*b^3*c + a^2*b*c^2)*x)]
```

Sympy [B] time = 1.0452, size = 323, normalized size = 3.63

$$\frac{\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}\,(bc_1 - b_1c)\log\left(x + \frac{-a^2c^2\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(bc_1-b_1c)+2ab^2c\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(bc_1-b_1c)-b^4\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}(bc_1-b_1c)+b^2c_1-bb_1c}{bcc_1-b_1c^2}\right)}{4} - \frac{\sqrt{-\frac{1}{\left(ac-b^2\right)^3}}\,(bc\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x**2+2*b*x+a)**2,x)
```

```
[Out] sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c)*log(x + (-a**2*c**2*sqrt(-1/(a*c - b
**2)**3)*(b*c1 - b1*c) + 2*a*b**2*c*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c)
- b**4*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c) + b**2*c1 - b*b1*c)/(b*c*c1 -
 b1*c**2))/4 - sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c)*log(x + (a**2*c**2*sq
rt(-1/(a*c - b**2)**3)*(b*c1 - b1*c) - 2*a*b**2*c*sqrt(-1/(a*c - b**2)**3)*
(b*c1 - b1*c) + b**4*sqrt(-1/(a*c - b**2)**3)*(b*c1 - b1*c) + b**2*c1 - b*b
1*c)/(b*c*c1 - b1*c**2))/4 - (a*c1 - b*b1 + x*(b*c1 - b1*c))/(2*a**2*c - 2*
a*b**2 + x**2*(2*a*c**2 - 2*b**2*c) + x*(4*a*b*c - 4*b**3))
```

Giac [A] time = 1.0706, size = 124, normalized size = 1.39

$$-\frac{(b_1c - bc_1)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{2\left(b^2 - ac\right)\sqrt{-b^2 + ac}} - \frac{b_1cx - bc_1x + bb_1 - ac_1}{2\left(cx^2 + 2\,bx + a\right)\left(b^2 - ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b1*c - b*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^2 - a*c)*sqrt(-b^
2 + a*c)) - 1/2*(b1*c*x - b*c1*x + b*b1 - a*c1)/((c*x^2 + 2*b*x + a)*(b^2 -
 a*c))
```

3.196 $\int \frac{\text{b1}+\text{c1}x}{\left(a+2bx+cx^2\right)^3}\,dx$

Optimal. Leaf size=130

$$\frac{3(b+cx)(\text{b1}c-b\text{c1})}{8\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{4\left(b^2-ac\right)\left(a+2bx+cx^2\right)^2}-\frac{3c(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{8\left(b^2-ac\right)^{5/2}}$$

```
[Out] -(b*b1 - a*c1 + (b1*c - b*c1)*x)/(4*(b^2 - a*c)*(a + 2*b*x + c*x^2)^2) + (3
*(b1*c - b*c1)*(b + c*x))/(8*(b^2 - a*c)^2*(a + 2*b*x + c*x^2)) - (3*c*(b1*
c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(8*(b^2 - a*c)^(5/2))
```

Rubi [A] time = 0.072158, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {638, 614, 618, 206}

$$\frac{3(b+cx)(\text{b1}c-b\text{c1})}{8\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1})+b\text{b1}}{4\left(b^2-ac\right)\left(a+2bx+cx^2\right)^2}-\frac{3c(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{8\left(b^2-ac\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] -(b*b1 - a*c1 + (b1*c - b*c1)*x)/(4*(b^2 - a*c)*(a + 2*b*x + c*x^2)^2) + (3
*(b1*c - b*c1)*(b + c*x))/(8*(b^2 - a*c)^2*(a + 2*b*x + c*x^2)) - (3*c*(b1*
c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(8*(b^2 - a*c)^(5/2))
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\text{b1}+\text{c1}x}{\left(a+2bx+cx^2\right)^3}\,dx = -\frac{b\text{b1}-a\text{c1}+(\text{b1}c-b\text{c1})x}{4\left(b^2-ac\right)\left(a+2bx+cx^2\right)^2}-\frac{(3(\text{b1}c-b\text{c1}))\int \frac{1}{\left(a+2bx+cx^2\right)^2}\,dx}{4\left(b^2-ac\right)}$$

$$= -\frac{b\text{b1}-a\text{c1}+(\text{b1}c-b\text{c1})x}{4\left(b^2-ac\right)\left(a+2bx+cx^2\right)^2}+\frac{3(\text{b1}c-b\text{c1})(b+cx)}{8\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)}+\frac{(3c(\text{b1}c-b\text{c1}))\int \frac{1}{a+2bx+cx^2}\cdots}{8\left(b^2-ac\right)^2}$$

$$= -\frac{b\text{b1}-a\text{c1}+(\text{b1}c-b\text{c1})x}{4\left(b^2-ac\right)\left(a+2bx+cx^2\right)^2}+\frac{3(\text{b1}c-b\text{c1})(b+cx)}{8\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)}-\frac{(3c(\text{b1}c-b\text{c1}))\,\text{Subst}\left(\int \frac{\cdots}{4(\cdots}\cdots\right.}{4\left(b^2-\cdots\right.}$$

$$= -\frac{b\text{b1}-a\text{c1}+(\text{b1}c-b\text{c1})x}{4\left(b^2-ac\right)\left(a+2bx+cx^2\right)^2}+\frac{3(\text{b1}c-b\text{c1})(b+cx)}{8\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)}-\frac{3c(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-\cdots}}\cdots\right.}{8\left(b^2-ac\right)^{5/2}}$$

Mathematica [A] time = 0.138804, size = 127, normalized size = 0.98

$$\frac{\frac{2\left(b^2-ac\right)(a\text{c1}-b\text{b1}+b\text{c1}x-\text{b1}cx)}{(a+x(2b+cx))^2}+\frac{3c(\text{b1}c-b\text{c1})\tan^{-1}\left(\frac{b+cx}{\sqrt{ac-b^2}}\right)}{\sqrt{ac-b^2}}+\frac{3(b+cx)(\text{b1}c-b\text{c1})}{a+x(2b+cx)}}{8\left(b^2-ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^3,x]
```

```
[Out] ((2*(b^2 - a*c)*(-(b*b1) + a*c1 - b1*c*x + b*c1*x))/(a + x*(2*b + c*x))^2 +
 (3*(b1*c - b*c1)*(b + c*x))/(a + x*(2*b + c*x)) + (3*c*(b1*c - b*c1)*ArcTa
n[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(8*(b^2 - a*c)^2)
```

Maple [B] time = 0.004, size = 274, normalized size = 2.1

$$\frac{(-2\,b\text{c1}+2\,\text{b1}\,c)\,x+2\,b\text{b1}-2\,a\text{c1}}{\left(8\,ac-8\,b^2\right)\left(cx^2+2\,bx+a\right)^2}-6\,\frac{cxb\text{c1}}{\left(4\,ac-4\,b^2\right)^2\left(cx^2+2\,bx+a\right)}+6\,\frac{xc^2\text{b1}}{\left(4\,ac-4\,b^2\right)^2\left(cx^2+2\,bx+a\right)}-6\,\frac{\cdots}{\left(4\,ac-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)/(c*x^2+2*b*x+a)^3,x)
```

```
[Out] 1/2*((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)^2-6/(4
*a*c-4*b^2)^2/(c*x^2+2*b*x+a)*x*c*b*c1+6/(4*a*c-4*b^2)^2/(c*x^2+2*b*x+a)*x*
c^2*b1-6/(4*a*c-4*b^2)^2/(c*x^2+2*b*x+a)*b^2*c1+6/(4*a*c-4*b^2)^2/(c*x^2+2*
b*x+a)*b*b1*c-6/(4*a*c-4*b^2)^2*c/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a
*c-b^2)^(1/2))*b*c1+6/(4*a*c-4*b^2)^2*c^2/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x
+2*b)/(a*c-b^2)^(1/2))*b1
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.92107, size = 2279, normalized size = 17.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*b^5*b1 - 14*a*b^3*b1*c + 10*a^2*b*b1*c^2 - 6*(b^2*b1*c^3 - a*b1*c
^4 - (b^3*c^2 - a*b*c^3)*c1)*x^3 - 18*(b^3*b1*c^2 - a*b*b1*c^3 - (b^4*c - a
*b^2*c^2)*c1)*x^2 + 3*(a^2*b1*c^2 - a^2*b*c*c1 + (b1*c^4 - b*c^3*c1)*x^4 +
4*(b*b1*c^3 - b^2*c^2*c1)*x^3 + 2*(2*b^2*b1*c^2 + a*b1*c^3 - (2*b^3*c + a*b
*c^2)*c1)*x^2 + 4*(a*b*b1*c^2 - a*b^2*c*c1)*x)*sqrt(b^2 - a*c)*log((c^2*x^2
 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2 + 2*b*x + a)
) + 2*(a*b^4 + a^2*b^2*c - 2*a^3*c^2)*c1 - 2*(4*b^4*b1*c + a*b^2*b1*c^2 - 5
*a^2*b1*c^3 - (4*b^5 + a*b^3*c - 5*a^2*b*c^2)*c1)*x)/(a^2*b^6 - 3*a^3*b^4*c
 + 3*a^4*b^2*c^2 - a^5*c^3 + (b^6*c^2 - 3*a*b^4*c^3 + 3*a^2*b^2*c^4 - a^3*c
^5)*x^4 + 4*(b^7*c - 3*a*b^5*c^2 + 3*a^2*b^3*c^3 - a^3*b*c^4)*x^3 + 2*(2*b^
8 - 5*a*b^6*c + 3*a^2*b^4*c^2 + a^3*b^2*c^3 - a^4*c^4)*x^2 + 4*(a*b^7 - 3*a
^2*b^5*c + 3*a^3*b^3*c^2 - a^4*b*c^3)*x), -1/8*(2*b^5*b1 - 7*a*b^3*b1*c + 5
*a^2*b*b1*c^2 - 3*(b^2*b1*c^3 - a*b1*c^4 - (b^3*c^2 - a*b*c^3)*c1)*x^3 - 9*
(b^3*b1*c^2 - a*b*b1*c^3 - (b^4*c - a*b^2*c^2)*c1)*x^2 + 3*(a^2*b1*c^2 - a^
2*b*c*c1 + (b1*c^4 - b*c^3*c1)*x^4 + 4*(b*b1*c^3 - b^2*c^2*c1)*x^3 + 2*(2*b
^2*b1*c^2 + a*b1*c^3 - (2*b^3*c + a*b*c^2)*c1)*x^2 + 4*(a*b*b1*c^2 - a*b^2*
c*c1)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(b^2 - a*c)) +
 (a*b^4 + a^2*b^2*c - 2*a^3*c^2)*c1 - (4*b^4*b1*c + a*b^2*b1*c^2 - 5*a^2*b1
*c^3 - (4*b^5 + a*b^3*c - 5*a^2*b*c^2)*c1)*x)/(a^2*b^6 - 3*a^3*b^4*c + 3*a^
4*b^2*c^2 - a^5*c^3 + (b^6*c^2 - 3*a*b^4*c^3 + 3*a^2*b^2*c^4 - a^3*c^5)*x^4
 + 4*(b^7*c - 3*a*b^5*c^2 + 3*a^2*b^3*c^3 - a^3*b*c^4)*x^3 + 2*(2*b^8 - 5*a
*b^6*c + 3*a^2*b^4*c^2 + a^3*b^2*c^3 - a^4*c^4)*x^2 + 4*(a*b^7 - 3*a^2*b^5*
c + 3*a^3*b^3*c^2 - a^4*b*c^3)*x)]
```

Sympy [B] time = 2.03427, size = 622, normalized size = 4.78

$$\frac{3c\sqrt{-\frac{1}{\left(ac-b^2\right)^5}}\left(bc_1-b_1c\right)\log\left(x+\frac{-3a^3c^4\sqrt{-\frac{1}{\left(ac-b^2\right)^5}}\left(bc_1-b_1c\right)+9a^2b^2c^3\sqrt{-\frac{1}{\left(ac-b^2\right)^5}}\left(bc_1-b_1c\right)-9ab^4c^2\sqrt{-\frac{1}{\left(ac-b^2\right)^5}}\left(bc_1-b_1c\right)+3b^6c\sqrt{-\frac{1}{\left(ac-b^2\right)^5}}\left(bc_1-b_1\right.}{3bc^2c_1-3b_1c^3}\right.}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x**2+2*b*x+a)**3,x)
```

```
[Out] 3*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c)*log(x + (-3*a**3*c**4*sqrt(-1/(a
*c - b**2)**5)*(b*c1 - b1*c) + 9*a**2*b**2*c**3*sqrt(-1/(a*c - b**2)**5)*(b
*c1 - b1*c) - 9*a*b**4*c**2*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 3*b**6
*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 3*b**2*c*c1 - 3*b*b1*c**2)/(3*b
*c**2*c1 - 3*b1*c**3))/16 - 3*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c)*log(
x + (3*a**3*c**4*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) - 9*a**2*b**2*c**3*
sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 9*a*b**4*c**2*sqrt(-1/(a*c - b**2)
**5)*(b*c1 - b1*c) - 3*b**6*c*sqrt(-1/(a*c - b**2)**5)*(b*c1 - b1*c) + 3*b*
*2*c*c1 - 3*b*b1*c**2)/(3*b*c**2*c1 - 3*b1*c**3))/16 - (2*a**2*c*c1 + a*b**
2*c1 - 5*a*b*b1*c + 2*b**3*b1 + x**3*(3*b*c**2*c1 - 3*b1*c**3) + x**2*(9*b*
*2*c*c1 - 9*b*b1*c**2) + x*(5*a*b*c*c1 - 5*a*b1*c**2 + 4*b**3*c1 - 4*b**2*b
1*c))/(8*a**4*c**2 - 16*a**3*b**2*c + 8*a**2*b**4 + x**4*(8*a**2*c**4 - 16*
a*b**2*c**3 + 8*b**4*c**2) + x**3*(32*a**2*b*c**3 - 64*a*b**3*c**2 + 32*b**
5*c) + x**2*(16*a**3*c**3 - 48*a*b**4*c + 32*b**6) + x*(32*a**3*b*c**2 - 64
*a**2*b**3*c + 32*a*b**5))
```

Giac [A] time = 1.07355, size = 262, normalized size = 2.02

$$\frac{3\left(b_1c^2 - bcc_1\right)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{8\left(b^4 - 2\,ab^2c + a^2c^2\right)\sqrt{-b^2 + ac}} + \frac{3\,b_1c^3x^3 - 3\,bc^2c_1x^3 + 9\,bb_1c^2x^2 - 9\,b^2cc_1x^2 + 4\,b^2b_1cx + 5\,ab_1c^2x - 4\,b^3c_1x - 5\,ab}{8\left(b^4 - 2\,ab^2c + a^2c^2\right)\left(cx^2 + 2\,bx + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^3,x, algorithm="giac")
```

```
[Out] 3/8*(b1*c^2 - b*c*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^4 - 2*a*b^2*c
+ a^2*c^2)*sqrt(-b^2 + a*c)) + 1/8*(3*b1*c^3*x^3 - 3*b*c^2*c1*x^3 + 9*b*b1*
c^2*x^2 - 9*b^2*c*c1*x^2 + 4*b^2*b1*c*x + 5*a*b1*c^2*x - 4*b^3*c1*x - 5*a*b
*c*c1*x - 2*b^3*b1 + 5*a*b*b1*c - a*b^2*c1 - 2*a^2*c*c1)/((b^4 - 2*a*b^2*c
+ a^2*c^2)*(c*x^2 + 2*b*x + a)^2)
```

3.197 $\int \frac{\mathbf{b1}+\mathbf{c1}x}{(a+2bx+cx^2)^4}\,dx$

Optimal. Leaf size=173

$$\frac{5c^2(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{16\left(b^2-ac\right)^{7/2}}-\frac{5c(b+cx)(\text{b1}c-b\text{c1})}{16\left(b^2-ac\right)^3\left(a+2bx+cx^2\right)}+\frac{5(b+cx)(\text{b1}c-b\text{c1})}{24\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)^2}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1}\cdots}{6\left(b^2-ac\right)\left(a+2bx\cdots\right.}$$

```
[Out] -(b*b1 - a*c1 + (b1*c - b*c1)*x)/(6*(b^2 - a*c)*(a + 2*b*x + c*x^2)^3) + (5
*(b1*c - b*c1)*(b + c*x))/(24*(b^2 - a*c)^2*(a + 2*b*x + c*x^2)^2) - (5*c*(
b1*c - b*c1)*(b + c*x))/(16*(b^2 - a*c)^3*(a + 2*b*x + c*x^2)) + (5*c^2*(b1
*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(16*(b^2 - a*c)^(7/2))
```

Rubi [A] time = 0.109845, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {638, 614, 618, 206}

$$\frac{5c^2(\text{b1}c-b\text{c1})\tanh^{-1}\left(\frac{b+cx}{\sqrt{b^2-ac}}\right)}{16\left(b^2-ac\right)^{7/2}}-\frac{5c(b+cx)(\text{b1}c-b\text{c1})}{16\left(b^2-ac\right)^3\left(a+2bx+cx^2\right)}+\frac{5(b+cx)(\text{b1}c-b\text{c1})}{24\left(b^2-ac\right)^2\left(a+2bx+cx^2\right)^2}-\frac{-a\text{c1}+x(\text{b1}c-b\text{c1}\cdots}{6\left(b^2-ac\right)\left(a+2bx\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x]
```

```
[Out] -(b*b1 - a*c1 + (b1*c - b*c1)*x)/(6*(b^2 - a*c)*(a + 2*b*x + c*x^2)^3) + (5
*(b1*c - b*c1)*(b + c*x))/(24*(b^2 - a*c)^2*(a + 2*b*x + c*x^2)^2) - (5*c*(
b1*c - b*c1)*(b + c*x))/(16*(b^2 - a*c)^3*(a + 2*b*x + c*x^2)) + (5*c^2*(b1
*c - b*c1)*ArcTanh[(b + c*x)/Sqrt[b^2 - a*c]])/(16*(b^2 - a*c)^(7/2))
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\mathrm{b1} + \mathrm{c1}x}{\left(a + 2bx + cx^2\right)^4} dx = -\frac{b\mathrm{b1} - a\mathrm{c1} + (\mathrm{b1}c - b\mathrm{c1})x}{6\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)^3} - \frac{(5(\mathrm{b1}c - b\mathrm{c1}))\int \frac{1}{\left(a+2bx+cx^2\right)^3} dx}{6\left(b^2 - ac\right)}$$

$$= -\frac{b\mathrm{b1} - a\mathrm{c1} + (\mathrm{b1}c - b\mathrm{c1})x}{6\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)^3} + \frac{5(\mathrm{b1}c - b\mathrm{c1})(b + cx)}{24\left(b^2 - ac\right)^2\left(a + 2bx + cx^2\right)^2} + \frac{(5c(\mathrm{b1}c - b\mathrm{c1}))\int \frac{1}{\left(a+2bx+c\right.}}{8\left(b^2 - ac\right)^2}$$

$$= -\frac{b\mathrm{b1} - a\mathrm{c1} + (\mathrm{b1}c - b\mathrm{c1})x}{6\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)^3} + \frac{5(\mathrm{b1}c - b\mathrm{c1})(b + cx)}{24\left(b^2 - ac\right)^2\left(a + 2bx + cx^2\right)^2} - \frac{5c(\mathrm{b1}c - b\mathrm{c1})(b + cx)}{16\left(b^2 - ac\right)^3\left(a + 2bx + c\right.}$$

$$= -\frac{b\mathrm{b1} - a\mathrm{c1} + (\mathrm{b1}c - b\mathrm{c1})x}{6\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)^3} + \frac{5(\mathrm{b1}c - b\mathrm{c1})(b + cx)}{24\left(b^2 - ac\right)^2\left(a + 2bx + cx^2\right)^2} - \frac{5c(\mathrm{b1}c - b\mathrm{c1})(b + cx)}{16\left(b^2 - ac\right)^3\left(a + 2bx + c\right.}$$

$$= -\frac{b\mathrm{b1} - a\mathrm{c1} + (\mathrm{b1}c - b\mathrm{c1})x}{6\left(b^2 - ac\right)\left(a + 2bx + cx^2\right)^3} + \frac{5(\mathrm{b1}c - b\mathrm{c1})(b + cx)}{24\left(b^2 - ac\right)^2\left(a + 2bx + cx^2\right)^2} - \frac{5c(\mathrm{b1}c - b\mathrm{c1})(b + cx)}{16\left(b^2 - ac\right)^3\left(a + 2bx + c\right.}$$

Mathematica [A] time = 0.21874, size = 168, normalized size = 0.97

$$\frac{\frac{15c^2(b\mathrm{c1}-\mathrm{b1}c)\tan^{-1}\left(\frac{b+cx}{\sqrt{ac-b^2}}\right)}{\sqrt{ac-b^2}} - \frac{10\left(b^2-ac\right)(b+cx)(b\mathrm{c1}-\mathrm{b1}c)}{(a+x(2b+cx))^2} + \frac{8\left(b^2-ac\right)^2(a\mathrm{c1}-b\mathrm{b1}+b\mathrm{c1}x-\mathrm{b1}cx)}{(a+x(2b+cx))^3} + \frac{15c(b+cx)(b\mathrm{c1}-\mathrm{b1}c)}{a+x(2b+cx)}}{48\left(b^2-ac\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^4,x]
```

```
[Out] ((8*(b^2 - a*c)^2*(-(b*b1) + a*c1 - b1*c*x + b*c1*x))/(a + x*(2*b + c*x))^3
 - (10*(b^2 - a*c)*(-(b1*c) + b*c1)*(b + c*x))/(a + x*(2*b + c*x))^2 + (15*
c*(-(b1*c) + b*c1)*(b + c*x))/(a + x*(2*b + c*x)) + (15*c^2*(-(b1*c) + b*c1
)*ArcTan[(b + c*x)/Sqrt[-b^2 + a*c]])/Sqrt[-b^2 + a*c])/(48*(b^2 - a*c)^3)
```

Maple [B] time = 0.006, size = 405, normalized size = 2.3

$$\frac{(-2b\mathrm{c1} + 2\mathrm{b1}c)x + 2b\mathrm{b1} - 2a\mathrm{c1}}{\left(12ac - 12b^2\right)\left(cx^2 + 2bx + a\right)^3} - \frac{10cxb\mathrm{c1}}{3\left(4ac - 4b^2\right)^2\left(cx^2 + 2bx + a\right)^2} + \frac{10xc^2\mathrm{b1}}{3\left(4ac - 4b^2\right)^2\left(cx^2 + 2bx + a\right)^2} - \frac{}{3\left(4ac\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)/(c*x^2+2*b*x+a)^4,x)
```

```
[Out] 1/3*((-2*b*c1+2*b1*c)*x+2*b*b1-2*a*c1)/(4*a*c-4*b^2)/(c*x^2+2*b*x+a)^3-10/3
/(4*a*c-4*b^2)^2/(c*x^2+2*b*x+a)^2*x*c*b*c1+10/3/(4*a*c-4*b^2)^2/(c*x^2+2*b
*x+a)^2*x*c^2*b1-10/3/(4*a*c-4*b^2)^2/(c*x^2+2*b*x+a)^2*b^2*c1+10/3/(4*a*c-
4*b^2)^2/(c*x^2+2*b*x+a)^2*b*b1*c-20/(4*a*c-4*b^2)^3*c^2/(c*x^2+2*b*x+a)*x*
b*c1+20/(4*a*c-4*b^2)^3*c^3/(c*x^2+2*b*x+a)*x*b1-20/(4*a*c-4*b^2)^3*c/(c*x^
```

```
2+2*b*x+a)*b^2*c1+20/(4*a*c-4*b^2)^3*c^2/(c*x^2+2*b*x+a)*b*b1-20/(4*a*c-4*b
^2)^3*c^2/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2))*b*c1+20/(
4*a*c-4*b^2)^3*c^3/(a*c-b^2)^(1/2)*arctan(1/2*(2*c*x+2*b)/(a*c-b^2)^(1/2))*
b1
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.10304, size = 4097, normalized size = 23.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(16*b^7*b1 - 68*a*b^5*b1*c + 118*a^2*b^3*b1*c^2 - 66*a^3*b*b1*c^3 +
30*(b^2*b1*c^5 - a*b1*c^6 - (b^3*c^4 - a*b*c^5)*c1)*x^5 + 150*(b^3*b1*c^4 -
 a*b*b1*c^5 - (b^4*c^3 - a*b^2*c^4)*c1)*x^4 + 20*(11*b^4*b1*c^3 - 7*a*b^2*b
1*c^4 - 4*a^2*b1*c^5 - (11*b^5*c^2 - 7*a*b^3*c^3 - 4*a^2*b*c^4)*c1)*x^3 + 6
0*(b^5*b1*c^2 + 3*a*b^3*b1*c^3 - 4*a^2*b*b1*c^4 - (b^6*c + 3*a*b^4*c^2 - 4*
a^2*b^2*c^3)*c1)*x^2 - 15*(a^3*b1*c^3 - a^3*b*c^2*c1 + (b1*c^6 - b*c^5*c1)*
x^6 + 6*(b*b1*c^5 - b^2*c^4*c1)*x^5 + 3*(4*b^2*b1*c^4 + a*b1*c^5 - (4*b^3*c
^3 + a*b*c^4)*c1)*x^4 + 4*(2*b^3*b1*c^3 + 3*a*b*b1*c^4 - (2*b^4*c^2 + 3*a*b
^2*c^3)*c1)*x^3 + 3*(4*a*b^2*b1*c^3 + a^2*b1*c^4 - (4*a*b^3*c^2 + a^2*b*c^3
)*c1)*x^2 + 6*(a^2*b*b1*c^3 - a^2*b^2*c^2*c1)*x)*sqrt(b^2 - a*c)*log((c^2*x
^2 + 2*b*c*x + 2*b^2 - a*c + 2*sqrt(b^2 - a*c)*(c*x + b))/(c*x^2 + 2*b*x +
a)) + 2*(2*a*b^6 - 11*a^2*b^4*c + a^3*b^2*c^2 + 8*a^4*c^3)*c1 - 6*(4*b^6*b1
*c - 22*a*b^4*b1*c^2 + 7*a^2*b^2*b1*c^3 + 11*a^3*b1*c^4 - (4*b^7 - 22*a*b^5
*c + 7*a^2*b^3*c^2 + 11*a^3*b*c^3)*c1)*x)/(a^3*b^8 - 4*a^4*b^6*c + 6*a^5*b^
4*c^2 - 4*a^6*b^2*c^3 + a^7*c^4 + (b^8*c^3 - 4*a*b^6*c^4 + 6*a^2*b^4*c^5 -
4*a^3*b^2*c^6 + a^4*c^7)*x^6 + 6*(b^9*c^2 - 4*a*b^7*c^3 + 6*a^2*b^5*c^4 - 4
*a^3*b^3*c^5 + a^4*b*c^6)*x^5 + 3*(4*b^10*c - 15*a*b^8*c^2 + 20*a^2*b^6*c^3
 - 10*a^3*b^4*c^4 + a^5*c^6)*x^4 + 4*(2*b^11 - 5*a*b^9*c + 10*a^3*b^5*c^3 -
 10*a^4*b^3*c^4 + 3*a^5*b*c^5)*x^3 + 3*(4*a*b^10 - 15*a^2*b^8*c + 20*a^3*b^
6*c^2 - 10*a^4*b^4*c^3 + a^6*c^5)*x^2 + 6*(a^2*b^9 - 4*a^3*b^7*c + 6*a^4*b^
5*c^2 - 4*a^5*b^3*c^3 + a^6*b*c^4)*x), -1/48*(8*b^7*b1 - 34*a*b^5*b1*c + 59
*a^2*b^3*b1*c^2 - 33*a^3*b*b1*c^3 + 15*(b^2*b1*c^5 - a*b1*c^6 - (b^3*c^4 -
a*b*c^5)*c1)*x^5 + 75*(b^3*b1*c^4 - a*b*b1*c^5 - (b^4*c^3 - a*b^2*c^4)*c1)*
x^4 + 10*(11*b^4*b1*c^3 - 7*a*b^2*b1*c^4 - 4*a^2*b1*c^5 - (11*b^5*c^2 - 7*a
*b^3*c^3 - 4*a^2*b*c^4)*c1)*x^3 + 30*(b^5*b1*c^2 + 3*a*b^3*b1*c^3 - 4*a^2*b
*b1*c^4 - (b^6*c + 3*a*b^4*c^2 - 4*a^2*b^2*c^3)*c1)*x^2 - 15*(a^3*b1*c^3 -
a^3*b*c^2*c1 + (b1*c^6 - b*c^5*c1)*x^6 + 6*(b*b1*c^5 - b^2*c^4*c1)*x^5 + 3*
(4*b^2*b1*c^4 + a*b1*c^5 - (4*b^3*c^3 + a*b*c^4)*c1)*x^4 + 4*(2*b^3*b1*c^3
+ 3*a*b*b1*c^4 - (2*b^4*c^2 + 3*a*b^2*c^3)*c1)*x^3 + 3*(4*a*b^2*b1*c^3 + a^
2*b1*c^4 - (4*a*b^3*c^2 + a^2*b*c^3)*c1)*x^2 + 6*(a^2*b*b1*c^3 - a^2*b^2*c^
2*c1)*x)*sqrt(-b^2 + a*c)*arctan(-sqrt(-b^2 + a*c)*(c*x + b)/(b^2 - a*c)) +
 (2*a*b^6 - 11*a^2*b^4*c + a^3*b^2*c^2 + 8*a^4*c^3)*c1 - 3*(4*b^6*b1*c - 22
```

```
*a*b^4*b1*c^2 + 7*a^2*b^2*b1*c^3 + 11*a^3*b1*c^4 - (4*b^7 - 22*a*b^5*c + 7*
a^2*b^3*c^2 + 11*a^3*b*c^3)*c1)*x)/(a^3*b^8 - 4*a^4*b^6*c + 6*a^5*b^4*c^2 -
 4*a^6*b^2*c^3 + a^7*c^4 + (b^8*c^3 - 4*a*b^6*c^4 + 6*a^2*b^4*c^5 - 4*a^3*b
^2*c^6 + a^4*c^7)*x^6 + 6*(b^9*c^2 - 4*a*b^7*c^3 + 6*a^2*b^5*c^4 - 4*a^3*b^
3*c^5 + a^4*b*c^6)*x^5 + 3*(4*b^10*c - 15*a*b^8*c^2 + 20*a^2*b^6*c^3 - 10*a
^3*b^4*c^4 + a^5*c^6)*x^4 + 4*(2*b^11 - 5*a*b^9*c + 10*a^3*b^5*c^3 - 10*a^4
*b^3*c^4 + 3*a^5*b*c^5)*x^3 + 3*(4*a*b^10 - 15*a^2*b^8*c + 20*a^3*b^6*c^2 -
 10*a^4*b^4*c^3 + a^6*c^5)*x^2 + 6*(a^2*b^9 - 4*a^3*b^7*c + 6*a^4*b^5*c^2 -
 4*a^5*b^3*c^3 + a^6*b*c^4)*x)]
```

Sympy [B] time = 3.87369, size = 1027, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x**2+2*b*x+a)**4,x)
```

```
[Out] 5*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c)*log(x + (-5*a**4*c**6*sqrt(-1
/(a*c - b**2)**7)*(b*c1 - b1*c) + 20*a**3*b**2*c**5*sqrt(-1/(a*c - b**2)**7
)*(b*c1 - b1*c) - 30*a**2*b**4*c**4*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c)
+ 20*a*b**6*c**3*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) - 5*b**8*c**2*sqrt(
-1/(a*c - b**2)**7)*(b*c1 - b1*c) + 5*b**2*c**2*c1 - 5*b*b1*c**3)/(5*b*c**3
*c1 - 5*b1*c**4))/32 - 5*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c)*log(x
+ (5*a**4*c**6*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) - 20*a**3*b**2*c**5*s
qrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) + 30*a**2*b**4*c**4*sqrt(-1/(a*c - b*
*2)**7)*(b*c1 - b1*c) - 20*a*b**6*c**3*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*
c) + 5*b**8*c**2*sqrt(-1/(a*c - b**2)**7)*(b*c1 - b1*c) + 5*b**2*c**2*c1 -
5*b*b1*c**3)/(5*b*c**3*c1 - 5*b1*c**4))/32 - (8*a**3*c**2*c1 + 9*a**2*b**2*
c*c1 - 33*a**2*b*b1*c**2 - 2*a*b**4*c1 + 26*a*b**3*b1*c - 8*b**5*b1 + x**5*
(15*b*c**4*c1 - 15*b1*c**5) + x**4*(75*b**2*c**3*c1 - 75*b*b1*c**4) + x**3*
(40*a*b*c**3*c1 - 40*a*b1*c**4 + 110*b**3*c**2*c1 - 110*b**2*b1*c**3) + x**
2*(120*a*b**2*c**2*c1 - 120*a*b*b1*c**3 + 30*b**4*c*c1 - 30*b**3*b1*c**2) +
 x*(33*a**2*b*c**2*c1 - 33*a**2*b1*c**3 + 54*a*b**3*c*c1 - 54*a*b**2*b1*c**
2 - 12*b**5*c1 + 12*b**4*b1*c))/(48*a**6*c**3 - 144*a**5*b**2*c**2 + 144*a*
*4*b**4*c - 48*a**3*b**6 + x**6*(48*a**3*c**6 - 144*a**2*b**2*c**5 + 144*a*
b**4*c**4 - 48*b**6*c**3) + x**5*(288*a**3*b*c**5 - 864*a**2*b**3*c**4 + 86
4*a*b**5*c**3 - 288*b**7*c**2) + x**4*(144*a**4*c**5 + 144*a**3*b**2*c**4 -
 1296*a**2*b**4*c**3 + 1584*a*b**6*c**2 - 576*b**8*c) + x**3*(576*a**4*b*c*
*4 - 1344*a**3*b**3*c**3 + 576*a**2*b**5*c**2 + 576*a*b**7*c - 384*b**9) +
x**2*(144*a**5*c**4 + 144*a**4*b**2*c**3 - 1296*a**3*b**4*c**2 + 1584*a**2*
b**6*c - 576*a*b**8) + x*(288*a**5*b*c**3 - 864*a**4*b**3*c**2 + 864*a**3*b
**5*c - 288*a**2*b**7))
```

Giac [B] time = 1.07513, size = 490, normalized size = 2.83

$$-\frac{5\left(b_1c^3 - bc^2c_1\right)\arctan\left(\frac{cx+b}{\sqrt{-b^2+ac}}\right)}{16\left(b^6 - 3\,ab^4c + 3\,a^2b^2c^2 - a^3c^3\right)\sqrt{-b^2+ac}} - \frac{15\,b_1c^5x^5 - 15\,bc^4c_1x^5 + 75\,bb_1c^4x^4 - 75\,b^2c^3c_1x^4 + 110\,b^2b_1c^3x^3 + 4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/(c*x^2+2*b*x+a)^4,x, algorithm="giac")
```

```
[Out] -5/16*(b1*c^3 - b*c^2*c1)*arctan((c*x + b)/sqrt(-b^2 + a*c))/((b^6 - 3*a*b^
4*c + 3*a^2*b^2*c^2 - a^3*c^3)*sqrt(-b^2 + a*c)) - 1/48*(15*b1*c^5*x^5 - 15
*b*c^4*c1*x^5 + 75*b*b1*c^4*x^4 - 75*b^2*c^3*c1*x^4 + 110*b^2*b1*c^3*x^3 +
40*a*b1*c^4*x^3 - 110*b^3*c^2*c1*x^3 - 40*a*b*c^3*c1*x^3 + 30*b^3*b1*c^2*x^
2 + 120*a*b*b1*c^3*x^2 - 30*b^4*c*c1*x^2 - 120*a*b^2*c^2*c1*x^2 - 12*b^4*b1
*c*x + 54*a*b^2*b1*c^2*x + 33*a^2*b1*c^3*x + 12*b^5*c1*x - 54*a*b^3*c*c1*x
- 33*a^2*b*c^2*c1*x + 8*b^5*b1 - 26*a*b^3*b1*c + 33*a^2*b*b1*c^2 + 2*a*b^4*
c1 - 9*a^2*b^2*c*c1 - 8*a^3*c^2*c1)/((b^6 - 3*a*b^4*c + 3*a^2*b^2*c^2 - a^3
*c^3)*(c*x^2 + 2*b*x + a)^3)
```

3.198 $\int (\mathbf{b1} + \mathbf{c1}x)\left(a + 2bx + cx^2\right)^{-n} dx$

Optimal. Leaf size=169

$$\frac{\text{c1}\left(a+2bx+cx^2\right)^{1-n}}{2c(1-n)}-\frac{2^{-n}(\text{b1}c-b\text{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{n-1}\left(a+2bx+cx^2\right)^{1-n}{}_2F_1\left(1-n,n;2-n;\frac{b+cx+\sqrt{b^2-ac}}{2\sqrt{b^2-ac}}\right)}{c(1-n)\sqrt{b^2-ac}}$$

```
[Out] (c1*(a + 2*b*x + c*x^2)^(1 - n))/(2*c*(1 - n)) - ((b1*c - b*c1)*(-((b - Sqr
t[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(-1 + n)*(a + 2*b*x + c*x^2)^(1 - n)*
Hypergeometric2F1[1 - n, n, 2 - n, (b + Sqrt[b^2 - a*c] + c*x)/(2*Sqrt[b^2
- a*c])])/(2^n*c*Sqrt[b^2 - a*c]*(1 - n))
```

Rubi [A] time = 0.0793389, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {640, 624}

$$\frac{\text{c1}\left(a+2bx+cx^2\right)^{1-n}}{2c(1-n)}-\frac{2^{-n}(\text{b1}c-b\text{c1})\left(-\frac{-\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}}\right)^{n-1}\left(a+2bx+cx^2\right)^{1-n}\text{Hypergeometric2F1}\left(1-n,n,2-n\right.}{c(1-n)\sqrt{b^2-ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] (c1*(a + 2*b*x + c*x^2)^(1 - n))/(2*c*(1 - n)) - ((b1*c - b*c1)*(-((b - Sqr
t[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c]))^(-1 + n)*(a + 2*b*x + c*x^2)^(1 - n)*
Hypergeometric2F1[1 - n, n, 2 - n, (b + Sqrt[b^2 - a*c] + c*x)/(2*Sqrt[b^2
- a*c])])/(2^n*c*Sqrt[b^2 - a*c]*(1 - n))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[4*p]
```

Rubi steps

$$\begin{aligned}\int (\text{b1}+\text{c1}x)\left(a+2bx+cx^2\right)^{-n} dx &= \frac{\text{c1}\left(a+2bx+cx^2\right)^{1-n}}{2c(1-n)}+\frac{(2\text{b1}c-2b\text{c1})\int\left(a+2bx+cx^2\right)^{-n} dx}{2c}\\ &= \frac{\text{c1}\left(a+2bx+cx^2\right)^{1-n}}{2c(1-n)}-\frac{2^{-n}(\text{b1}c-b\text{c1})\left(-\frac{b-\sqrt{b^2-ac}+cx}{\sqrt{b^2-ac}}\right)^{-1+n}\left(a+2bx+cx^2\right)^{1-n}}{c\sqrt{b^2-ac}(1-n)}\end{aligned}$$

Mathematica [C] time = 0.344348, size = 264, normalized size = 1.56

$$\frac{1}{2}(a+x(2b+cx))^{-n}\left(\text{c1}x^2\left(\frac{-\sqrt{b^2-ac}+b+cx}{b-\sqrt{b^2-ac}}\right)^n\left(\frac{\sqrt{b^2-ac}+b+cx}{\sqrt{b^2-ac}+b}\right)^n F_1\left(2;n,n;3;-\frac{cx}{b+\sqrt{b^2-ac}},\frac{cx}{\sqrt{b^2-ac}-b}\right)-\frac{\text{b1}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(b1 + c1*x)/(a + 2*b*x + c*x^2)^n,x]
```

```
[Out] (c1*x^2*((b - Sqrt[b^2 - a*c] + c*x)/(b - Sqrt[b^2 - a*c]))^n*((b + Sqrt[b^
2 - a*c] + c*x)/(b + Sqrt[b^2 - a*c]))^n*AppellF1[2, n, n, 3, -((c*x)/(b +
Sqrt[b^2 - a*c])), (c*x)/(-b + Sqrt[b^2 - a*c])] - (2^(1 - n)*b1*(b - Sqrt[
b^2 - a*c] + c*x)*((b + Sqrt[b^2 - a*c] + c*x)/Sqrt[b^2 - a*c])^n*Hypergeom
etric2F1[1 - n, n, 2 - n, (-b + Sqrt[b^2 - a*c] - c*x)/(2*Sqrt[b^2 - a*c])]
)/(c*(-1 + n)))/(2*(a + x*(2*b + c*x))^n)
```

Maple [F] time = 0.081, size = 0, normalized size = 0.

$$\int \frac{\text{c1}\,x+\text{b1}}{\left(cx^2+2\,bx+a\right)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c1*x+b1)/((c*x^2+2*b*x+a)^n),x)
```

```
[Out] int((c1*x+b1)/((c*x^2+2*b*x+a)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c_1x+b_1}{\left(cx^2+2\,bx+a\right)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/((c*x^2+2*b*x+a)^n),x, algorithm="maxima")
```

```
[Out] integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{c_1x+b_1}{\left(cx^2+2\,bx+a\right)^n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/((c*x^2+2*b*x+a)^n),x, algorithm="fricas")
```

```
[Out] integral((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/((c*x**2+2*b*x+a)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{c_1x + b_1}{\left(cx^2 + 2\,bx + a\right)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c1*x+b1)/((c*x^2+2*b*x+a)^n),x, algorithm="giac")
```

```
[Out] integrate((c1*x + b1)/(c*x^2 + 2*b*x + a)^n, x)
```

3.199 $\int \frac{x}{3+6x+2x^2}\,dx$

Optimal. Leaf size=49

$$\frac{1}{4}\left(1-\sqrt{3}\right)\log\left(2x-\sqrt{3}+3\right)+\frac{1}{4}\left(1+\sqrt{3}\right)\log\left(2x+\sqrt{3}+3\right)$$

```
[Out] ((1 - Sqrt[3])*Log[3 - Sqrt[3] + 2*x])/4 + ((1 + Sqrt[3])*Log[3 + Sqrt[3] +
 2*x])/4
```

Rubi [A] time = 0.0155641, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {632, 31}

$$\frac{1}{4}\left(1-\sqrt{3}\right)\log\left(2x-\sqrt{3}+3\right)+\frac{1}{4}\left(1+\sqrt{3}\right)\log\left(2x+\sqrt{3}+3\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(3 + 6*x + 2*x^2),x]
```

```
[Out] ((1 - Sqrt[3])*Log[3 - Sqrt[3] + 2*x])/4 + ((1 + Sqrt[3])*Log[3 + Sqrt[3] +
 2*x])/4
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{3+6x+2x^2}\,dx &= \frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{1}{3-\sqrt{3}+2x}\,dx+\frac{1}{2}\left(1+\sqrt{3}\right)\int \frac{1}{3+\sqrt{3}+2x}\,dx\\ &= \frac{1}{4}\left(1-\sqrt{3}\right)\log\left(3-\sqrt{3}+2x\right)+\frac{1}{4}\left(1+\sqrt{3}\right)\log\left(3+\sqrt{3}+2x\right)\end{aligned}$$

Mathematica [A] time = 0.02062, size = 44, normalized size = 0.9

$$\frac{1}{4}\left(\left(1+\sqrt{3}\right)\log\left(2x+\sqrt{3}+3\right)-\left(\sqrt{3}-1\right)\log\left(-2x+\sqrt{3}-3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(3 + 6*x + 2*x^2),x]
```

```
[Out] (-((-1 + Sqrt[3])*Log[-3 + Sqrt[3] - 2*x]) + (1 + Sqrt[3])*Log[3 + Sqrt[3]
+ 2*x])/4
```

Maple [A] time = 0.003, size = 31, normalized size = 0.6

$$\frac{\ln\left(2x^2+6x+3\right)}{4}+\frac{\sqrt{3}}{2}\text{Artanh}\left(\frac{(6+4x)\sqrt{3}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2*x^2+6*x+3),x)
```

```
[Out] 1/4*ln(2*x^2+6*x+3)+1/2*3^(1/2)*arctanh(1/6*(6+4*x)*3^(1/2))
```

Maxima [A] time = 1.40655, size = 55, normalized size = 1.12

$$-\frac{1}{4}\sqrt{3}\log\left(\frac{2x-\sqrt{3}+3}{2x+\sqrt{3}+3}\right)+\frac{1}{4}\log\left(2x^2+6x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x^2+6*x+3),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(3)*log((2*x - sqrt(3) + 3)/(2*x + sqrt(3) + 3)) + 1/4*log(2*x^2 +
 6*x + 3)
```

Fricas [A] time = 1.6719, size = 136, normalized size = 2.78

$$\frac{1}{4}\sqrt{3}\log\left(\frac{2x^2+\sqrt{3}(2x+3)+6x+6}{2x^2+6x+3}\right)+\frac{1}{4}\log\left(2x^2+6x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x^2+6*x+3),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*log((2*x^2 + sqrt(3)*(2*x + 3) + 6*x + 6)/(2*x^2 + 6*x + 3)) +
1/4*log(2*x^2 + 6*x + 3)
```

Sympy [A] time = 0.09926, size = 46, normalized size = 0.94

$$\left(\frac{1}{4}-\frac{\sqrt{3}}{4}\right)\log\left(x-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)+\left(\frac{1}{4}+\frac{\sqrt{3}}{4}\right)\log\left(x+\frac{\sqrt{3}}{2}+\frac{3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x**2+6*x+3),x)
```

```
[Out] (1/4 - sqrt(3)/4)*log(x - sqrt(3)/2 + 3/2) + (1/4 + sqrt(3)/4)*log(x + sqrt
(3)/2 + 3/2)
```

Giac [A] time = 1.05922, size = 62, normalized size = 1.27

$$-\frac{1}{4}\sqrt{3}\log\left(\frac{\left|4x-2\sqrt{3}+6\right|}{\left|4x+2\sqrt{3}+6\right|}\right)+\frac{1}{4}\log\left(\left|2x^2+6x+3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2*x^2+6*x+3),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(3)*log(abs(4*x - 2*sqrt(3) + 6)/abs(4*x + 2*sqrt(3) + 6)) + 1/4*l
og(abs(2*x^2 + 6*x + 3))
```

3.200 $\int \frac{-3+2x}{\left(3+6x+2x^2\right)^3}\,dx$

Optimal. Leaf size=61

$$-\frac{2x+3}{2\left(2x^2+6x+3\right)}+\frac{4x+5}{4\left(2x^2+6x+3\right)^2}+\frac{\tanh^{-1}\left(\frac{2x+3}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (5 + 4*x)/(4*(3 + 6*x + 2*x^2)^2) - (3 + 2*x)/(2*(3 + 6*x + 2*x^2)) + ArcTa
nh[(3 + 2*x)/Sqrt[3]]/Sqrt[3]
```

Rubi [A] time = 0.0221754, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {638, 614, 618, 206}

$$-\frac{2x+3}{2\left(2x^2+6x+3\right)}+\frac{4x+5}{4\left(2x^2+6x+3\right)^2}+\frac{\tanh^{-1}\left(\frac{2x+3}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 2*x)/(3 + 6*x + 2*x^2)^3,x]
```

```
[Out] (5 + 4*x)/(4*(3 + 6*x + 2*x^2)^2) - (3 + 2*x)/(2*(3 + 6*x + 2*x^2)) + ArcTa
nh[(3 + 2*x)/Sqrt[3]]/Sqrt[3]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-3+2x}{\left(3+6x+2x^2\right)^3}\,dx &= \frac{5+4x}{4\left(3+6x+2x^2\right)^2}+3\int \frac{1}{\left(3+6x+2x^2\right)^2}\,dx\\ &= \frac{5+4x}{4\left(3+6x+2x^2\right)^2}-\frac{3+2x}{2\left(3+6x+2x^2\right)}-\int \frac{1}{3+6x+2x^2}\,dx\\ &= \frac{5+4x}{4\left(3+6x+2x^2\right)^2}-\frac{3+2x}{2\left(3+6x+2x^2\right)}+2\,\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,6+4x\right)\\ &= \frac{5+4x}{4\left(3+6x+2x^2\right)^2}-\frac{3+2x}{2\left(3+6x+2x^2\right)}+\frac{\tanh^{-1}\left(\frac{3+2x}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.04588, size = 70, normalized size = 1.15

$$\frac{1}{12}\left(-\frac{3\left(8x^3+36x^2+44x+13\right)}{\left(2x^2+6x+3\right)^2}-2\sqrt{3}\log\left(-2x+\sqrt{3}-3\right)+2\sqrt{3}\log\left(2x+\sqrt{3}+3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 2*x)/(3 + 6*x + 2*x^2)^3,x]
```

```
[Out] ((-3*(13 + 44*x + 36*x^2 + 8*x^3))/(3 + 6*x + 2*x^2)^2 - 2*Sqrt[3]*Log[-3 +
 Sqrt[3] - 2*x] + 2*Sqrt[3]*Log[3 + Sqrt[3] + 2*x])/12
```

Maple [A] time = 0.002, size = 56, normalized size = 0.9

$$-\frac{-24\,x-30}{24\,\left(2\,x^2+6\,x+3\right)^2}-\frac{6+4\,x}{8\,x^2+24\,x+12}+\frac{\sqrt{3}}{3}\text{Artanh}\left(\frac{(6+4\,x)\,\sqrt{3}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+2*x)/(2*x^2+6*x+3)^3,x)
```

```
[Out] -1/24*(-24*x-30)/(2*x^2+6*x+3)^2-1/4*(6+4*x)/(2*x^2+6*x+3)+1/3*3^(1/2)*arct
anh(1/6*(6+4*x)*3^(1/2))
```

Maxima [A] time = 1.41719, size = 90, normalized size = 1.48

$$-\frac{1}{6}\,\sqrt{3}\log\left(\frac{2\,x-\sqrt{3}+3}{2\,x+\sqrt{3}+3}\right)-\frac{8\,x^3+36\,x^2+44\,x+13}{4\,\left(4\,x^4+24\,x^3+48\,x^2+36\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(2*x^2+6*x+3)^3,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log((2*x - sqrt(3) + 3)/(2*x + sqrt(3) + 3)) - 1/4*(8*x^3 + 36
*x^2 + 44*x + 13)/(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)
```

Fricas [A] time = 1.62373, size = 251, normalized size = 4.11

$$-\frac{24\,x^3-2\,\sqrt{3}\left(4\,x^4+24\,x^3+48\,x^2+36\,x+9\right)\log\left(\frac{2\,x^2+\sqrt{3}(2\,x+3)+6\,x+6}{2\,x^2+6\,x+3}\right)+108\,x^2+132\,x+39}{12\left(4\,x^4+24\,x^3+48\,x^2+36\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(2*x^2+6*x+3)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(24*x^3 - 2*sqrt(3)*(4*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)*log((2*x^2 +
 sqrt(3)*(2*x + 3) + 6*x + 6)/(2*x^2 + 6*x + 3)) + 108*x^2 + 132*x + 39)/(4
*x^4 + 24*x^3 + 48*x^2 + 36*x + 9)
```

Sympy [A] time = 0.146863, size = 75, normalized size = 1.23

$$-\frac{8x^3+36x^2+44x+13}{16x^4+96x^3+192x^2+144x+36}-\frac{\sqrt{3}\log\left(x-\frac{\sqrt{3}}{2}+\frac{3}{2}\right)}{6}+\frac{\sqrt{3}\log\left(x+\frac{\sqrt{3}}{2}+\frac{3}{2}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(2*x**2+6*x+3)**3,x)
```

```
[Out] -(8*x**3 + 36*x**2 + 44*x + 13)/(16*x**4 + 96*x**3 + 192*x**2 + 144*x + 36)
 - sqrt(3)*log(x - sqrt(3)/2 + 3/2)/6 + sqrt(3)*log(x + sqrt(3)/2 + 3/2)/6
```

Giac [A] time = 1.06024, size = 82, normalized size = 1.34

$$-\frac{1}{6}\,\sqrt{3}\log\left(\frac{\left|4\,x-2\,\sqrt{3}+6\right|}{\left|4\,x+2\,\sqrt{3}+6\right|}\right)-\frac{8\,x^3+36\,x^2+44\,x+13}{4\left(2\,x^2+6\,x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+2*x)/(2*x^2+6*x+3)^3,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(abs(4*x - 2*sqrt(3) + 6)/abs(4*x + 2*sqrt(3) + 6)) - 1/4*(
8*x^3 + 36*x^2 + 44*x + 13)/(2*x^2 + 6*x + 3)^2
```

3.201 $\int \frac{-1+x}{\left(4+5x+x^2\right)^2} \, dx$

Optimal. Leaf size=36

$$\frac{7x+13}{9\left(x^2+5x+4\right)}+\frac{7}{27}\log(x+1)-\frac{7}{27}\log(x+4)$$

```
[Out] (13 + 7*x)/(9*(4 + 5*x + x^2)) + (7*Log[1 + x])/27 - (7*Log[4 + x])/27
```

Rubi [A] time = 0.0079996, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {638, 616, 31}

$$\frac{7x+13}{9\left(x^2+5x+4\right)}+\frac{7}{27}\log(x+1)-\frac{7}{27}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(4 + 5*x + x^2)^2,x]
```

```
[Out] (13 + 7*x)/(9*(4 + 5*x + x^2)) + (7*Log[1 + x])/27 - (7*Log[4 + x])/27
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x}{\left(4+5x+x^2\right)^2} \, dx &= \frac{13+7x}{9\left(4+5x+x^2\right)}+\frac{7}{9}\int \frac{1}{4+5x+x^2} \, dx \\
&= \frac{13+7x}{9\left(4+5x+x^2\right)}+\frac{7}{27}\int \frac{1}{1+x} \, dx-\frac{7}{27}\int \frac{1}{4+x} \, dx \\
&= \frac{13+7x}{9\left(4+5x+x^2\right)}+\frac{7}{27}\log(1+x)-\frac{7}{27}\log(4+x)
\end{aligned}$$

Mathematica [A] time = 0.0172021, size = 33, normalized size = 0.92

$$\frac{1}{27}\left(\frac{21x+39}{x^2+5x+4}+7\log(x+1)-7\log(x+4)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(4 + 5*x + x^2)^2,x]
```

```
[Out] ((39 + 21*x)/(4 + 5*x + x^2) + 7*Log[1 + x] - 7*Log[4 + x])/27
```

Maple [A] time = 0.008, size = 28, normalized size = 0.8

$$\frac{2}{9+9x}+\frac{7\ln(1+x)}{27}+\frac{5}{36+9x}-\frac{7\ln(4+x)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(x^2+5*x+4)^2,x)
```

```
[Out] 2/9/(1+x)+7/27*ln(1+x)+5/9/(4+x)-7/27*ln(4+x)
```

Maxima [A] time = 0.923678, size = 41, normalized size = 1.14

$$\frac{7x+13}{9\left(x^2+5x+4\right)}-\frac{7}{27}\log(x+4)+\frac{7}{27}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2+5*x+4)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(7*x + 13)/(x^2 + 5*x + 4) - 7/27*log(x + 4) + 7/27*log(x + 1)
```

Fricas [A] time = 1.72885, size = 131, normalized size = 3.64

$$-\frac{7\left(x^2+5x+4\right)\log(x+4)-7\left(x^2+5x+4\right)\log(x+1)-21x-39}{27\left(x^2+5x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2+5*x+4)^2,x, algorithm="fricas")
```

```
[Out] -1/27*(7*(x^2 + 5*x + 4)*log(x + 4) - 7*(x^2 + 5*x + 4)*log(x + 1) - 21*x -
 39)/(x^2 + 5*x + 4)
```

Sympy [A] time = 0.113766, size = 31, normalized size = 0.86

$$\frac{7x+13}{9x^2+45x+36}+\frac{7\log(x+1)}{27}-\frac{7\log(x+4)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**2+5*x+4)**2,x)
```

```
[Out] (7*x + 13)/(9*x**2 + 45*x + 36) + 7*log(x + 1)/27 - 7*log(x + 4)/27
```

Giac [A] time = 1.0542, size = 43, normalized size = 1.19

$$\frac{7x+13}{9\left(x^2+5x+4\right)}-\frac{7}{27}\log\left(|x+4|\right)+\frac{7}{27}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2+5*x+4)^2,x, algorithm="giac")
```

```
[Out] 1/9*(7*x + 13)/(x^2 + 5*x + 4) - 7/27*log(abs(x + 4)) + 7/27*log(abs(x + 1)
)
```

3.202 $\int \frac{1}{\left(2+3x+x^2\right)^5}\,dx$

Optimal. Leaf size=87

$$\frac{35(2x+3)}{x^2+3x+2}-\frac{35(2x+3)}{6\left(x^2+3x+2\right)^2}+\frac{7(2x+3)}{6\left(x^2+3x+2\right)^3}-\frac{2x+3}{4\left(x^2+3x+2\right)^4}+70\log(x+1)-70\log(x+2)$$

```
[Out] -(3 + 2*x)/(4*(2 + 3*x + x^2)^4) + (7*(3 + 2*x))/(6*(2 + 3*x + x^2)^3) - (3
5*(3 + 2*x))/(6*(2 + 3*x + x^2)^2) + (35*(3 + 2*x))/(2 + 3*x + x^2) + 70*Lo
g[1 + x] - 70*Log[2 + x]
```

Rubi [A] time = 0.0232128, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {614, 616, 31}

$$\frac{35(2x+3)}{x^2+3x+2}-\frac{35(2x+3)}{6\left(x^2+3x+2\right)^2}+\frac{7(2x+3)}{6\left(x^2+3x+2\right)^3}-\frac{2x+3}{4\left(x^2+3x+2\right)^4}+70\log(x+1)-70\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + x^2)^(-5),x]
```

```
[Out] -(3 + 2*x)/(4*(2 + 3*x + x^2)^4) + (7*(3 + 2*x))/(6*(2 + 3*x + x^2)^3) - (3
5*(3 + 2*x))/(6*(2 + 3*x + x^2)^2) + (35*(3 + 2*x))/(2 + 3*x + x^2) + 70*Lo
g[1 + x] - 70*Log[2 + x]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(2+3x+x^2\right)^5}\,dx &= -\frac{3+2x}{4\left(2+3x+x^2\right)^4}-\frac{7}{2}\int \frac{1}{\left(2+3x+x^2\right)^4}\,dx\\ &= -\frac{3+2x}{4\left(2+3x+x^2\right)^4}+\frac{7(3+2x)}{6\left(2+3x+x^2\right)^3}+\frac{35}{3}\int \frac{1}{\left(2+3x+x^2\right)^3}\,dx\\ &= -\frac{3+2x}{4\left(2+3x+x^2\right)^4}+\frac{7(3+2x)}{6\left(2+3x+x^2\right)^3}-\frac{35(3+2x)}{6\left(2+3x+x^2\right)^2}-35\int \frac{1}{\left(2+3x+x^2\right)^2}\,dx\\ &= -\frac{3+2x}{4\left(2+3x+x^2\right)^4}+\frac{7(3+2x)}{6\left(2+3x+x^2\right)^3}-\frac{35(3+2x)}{6\left(2+3x+x^2\right)^2}+\frac{35(3+2x)}{2+3x+x^2}+70\int \frac{1}{2+3x+x^2}\,dx\\ &= -\frac{3+2x}{4\left(2+3x+x^2\right)^4}+\frac{7(3+2x)}{6\left(2+3x+x^2\right)^3}-\frac{35(3+2x)}{6\left(2+3x+x^2\right)^2}+\frac{35(3+2x)}{2+3x+x^2}+70\int \frac{1}{1+x}\,dx-70\int\\ &= -\frac{3+2x}{4\left(2+3x+x^2\right)^4}+\frac{7(3+2x)}{6\left(2+3x+x^2\right)^3}-\frac{35(3+2x)}{6\left(2+3x+x^2\right)^2}+\frac{35(3+2x)}{2+3x+x^2}+70\log(1+x)-70\log\end{aligned}$$

Mathematica [A] time = 0.0246254, size = 87, normalized size = 1.

$$\frac{-2x-3}{4\left(x^2+3x+2\right)^4}+\frac{35(2x+3)}{x^2+3x+2}-\frac{35(2x+3)}{6\left(x^2+3x+2\right)^2}+\frac{7(2x+3)}{6\left(x^2+3x+2\right)^3}+70\log(x+1)-70\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + x^2)^(-5),x]
```

```
[Out] (-3 - 2*x)/(4*(2 + 3*x + x^2)^4) + (7*(3 + 2*x))/(6*(2 + 3*x + x^2)^3) - (3
5*(3 + 2*x))/(6*(2 + 3*x + x^2)^2) + (35*(3 + 2*x))/(2 + 3*x + x^2) + 70*Lo
g[1 + x] - 70*Log[2 + x]
```

Maple [A] time = 0.01, size = 70, normalized size = 0.8

$$\frac{1}{4\,(2+x)^4}+\frac{5}{3\,(2+x)^3}+\frac{15}{2\,(2+x)^2}+35\,(2+x)^{-1}-70\,\ln(2+x)-\frac{1}{4\,(1+x)^4}+\frac{5}{3\,(1+x)^3}-\frac{15}{2\,(1+x)^2}+35\,(1+x)^{-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+3*x+2)^5,x)
```

```
[Out] 1/4/(2+x)^4+5/3/(2+x)^3+15/2/(2+x)^2+35/(2+x)-70*ln(2+x)-1/4/(1+x)^4+5/3/(1
+x)^3-15/2/(1+x)^2+35/(1+x)+70*ln(1+x)
```

Maxima [A] time = 0.928019, size = 122, normalized size = 1.4

$$\frac{840\,x^7+8820\,x^6+38920\,x^5+93450\,x^4+131768\,x^3+109116\,x^2+49176\,x+9315}{12\left(x^8+12\,x^7+62\,x^6+180\,x^5+321\,x^4+360\,x^3+248\,x^2+96\,x+16\right)}-70\,\log(x+2)+70\,\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+2)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 109116*x^2
+ 49176*x + 9315)/(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 24
8*x^2 + 96*x + 16) - 70*log(x + 2) + 70*log(x + 1)
```

Fricas [B] time = 1.82358, size = 481, normalized size = 5.53

$$\frac{840\,x^{7}+8820\,x^{6}+38920\,x^{5}+93450\,x^{4}+131768\,x^{3}+109116\,x^{2}-840\left(x^{8}+12\,x^{7}+62\,x^{6}+180\,x^{5}+321\,x^{4}+360\,x\right.}{12\left(x^{8}+12\,x^{7}+62\,x^{6}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+2)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 109116*x^2
- 840*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x
 + 16)*log(x + 2) + 840*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^
3 + 248*x^2 + 96*x + 16)*log(x + 1) + 49176*x + 9315)/(x^8 + 12*x^7 + 62*x^
6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)
```

Sympy [A] time = 0.193934, size = 88, normalized size = 1.01

$$\frac{840x^{7}+8820x^{6}+38920x^{5}+93450x^{4}+131768x^{3}+109116x^{2}+49176x+9315}{12x^{8}+144x^{7}+744x^{6}+2160x^{5}+3852x^{4}+4320x^{3}+2976x^{2}+1152x+192}+70\log\left(x+1\right)-70\log\left(x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+3*x+2)**5,x)
```

```
[Out] (840*x**7 + 8820*x**6 + 38920*x**5 + 93450*x**4 + 131768*x**3 + 109116*x**2
 + 49176*x + 9315)/(12*x**8 + 144*x**7 + 744*x**6 + 2160*x**5 + 3852*x**4 +
 4320*x**3 + 2976*x**2 + 1152*x + 192) + 70*log(x + 1) - 70*log(x + 2)
```

Giac [A] time = 1.05658, size = 84, normalized size = 0.97

$$\frac{840\,x^{7}+8820\,x^{6}+38920\,x^{5}+93450\,x^{4}+131768\,x^{3}+109116\,x^{2}+49176\,x+9315}{12\left(x^{2}+3\,x+2\right)^{4}}-70\,\log\left(|x+2|\right)+70\,\log\left(|x+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+3*x+2)^5,x, algorithm="giac")
```

```
[Out] 1/12*(840*x^7 + 8820*x^6 + 38920*x^5 + 93450*x^4 + 131768*x^3 + 109116*x^2
+ 49176*x + 9315)/(x^2 + 3*x + 2)^4 - 70*log(abs(x + 2)) + 70*log(abs(x + 1
))
```

3.203 $\int \frac{1}{x^3(7-6x+2x^2)^2} dx$

Optimal. Leaf size=81

$$-\frac{2-3x}{35x^2(2x^2-6x+7)}-\frac{1}{490x^2}-\frac{40\log(2x^2-6x+7)}{2401}-\frac{69}{1715x}+\frac{80\log(x)}{2401}-\frac{234\tan^{-1}\left(\frac{3-2x}{\sqrt{5}}\right)}{12005\sqrt{5}}$$

```
[Out] -1/(490*x^2) - 69/(1715*x) - (2 - 3*x)/(35*x^2*(7 - 6*x + 2*x^2)) - (234*Ar
cTan[(3 - 2*x)/Sqrt[5]])/(12005*Sqrt[5]) + (80*Log[x])/2401 - (40*Log[7 - 6
*x + 2*x^2])/2401
```

Rubi [A] time = 0.0561464, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {740, 800, 634, 618, 204, 628}

$$-\frac{2-3x}{35x^2(2x^2-6x+7)}-\frac{1}{490x^2}-\frac{40\log(2x^2-6x+7)}{2401}-\frac{69}{1715x}+\frac{80\log(x)}{2401}-\frac{234\tan^{-1}\left(\frac{3-2x}{\sqrt{5}}\right)}{12005\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(7 - 6*x + 2*x^2)^2),x]
```

```
[Out] -1/(490*x^2) - 69/(1715*x) - (2 - 3*x)/(35*x^2*(7 - 6*x + 2*x^2)) - (234*Ar
cTan[(3 - 2*x)/Sqrt[5]])/(12005*Sqrt[5]) + (80*Log[x])/2401 - (40*Log[7 - 6
*x + 2*x^2])/2401
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(7-6x+2x^2\right)^2}\,dx &= -\frac{2-3x}{35x^2\left(7-6x+2x^2\right)}+\frac{1}{140}\int \frac{4+36x}{x^3\left(7-6x+2x^2\right)}\,dx\\
&= -\frac{2-3x}{35x^2\left(7-6x+2x^2\right)}+\frac{1}{140}\int \left(\frac{4}{7x^3}+\frac{276}{49x^2}+\frac{1600}{343x}-\frac{8(-717+400x)}{343\left(7-6x+2x^2\right)}\right)dx\\
&= -\frac{1}{490x^2}-\frac{69}{1715x}-\frac{2-3x}{35x^2\left(7-6x+2x^2\right)}+\frac{80\log(x)}{2401}-\frac{2\int \frac{-717+400x}{7-6x+2x^2}\,dx}{12005}\\
&= -\frac{1}{490x^2}-\frac{69}{1715x}-\frac{2-3x}{35x^2\left(7-6x+2x^2\right)}+\frac{80\log(x)}{2401}-\frac{40\int \frac{-6+4x}{7-6x+2x^2}\,dx}{2401}+\frac{234\int \frac{1}{7-6x+2x^2}}{12005}\\
&= -\frac{1}{490x^2}-\frac{69}{1715x}-\frac{2-3x}{35x^2\left(7-6x+2x^2\right)}+\frac{80\log(x)}{2401}-\frac{40\log\left(7-6x+2x^2\right)}{2401}-\frac{468\,\mathrm{Subst}}{}\\
&= -\frac{1}{490x^2}-\frac{69}{1715x}-\frac{2-3x}{35x^2\left(7-6x+2x^2\right)}-\frac{234\tan^{-1}\left(\frac{3-2x}{\sqrt{5}}\right)}{12005\sqrt{5}}+\frac{80\log(x)}{2401}-\frac{40\log\left(7-6x\right.}{2401}
\end{aligned}$$

Mathematica [A] time = 0.0348387, size = 70, normalized size = 0.86

$$\frac{-\frac{140(9x-41)}{2x^2-6x+7}-\frac{1225}{x^2}-2000\log\left(2x^2-6x+7\right)-\frac{4200}{x}+4000\log(x)+468\sqrt{5}\tan^{-1}\left(\frac{2x-3}{\sqrt{5}}\right)}{120050}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(7 - 6*x + 2*x^2)^2),x]
```

```
[Out] (-1225/x^2 - 4200/x - (140*(-41 + 9*x))/(7 - 6*x + 2*x^2) + 468*Sqrt[5]*Arc
Tan[(-3 + 2*x)/Sqrt[5]] + 4000*Log[x] - 2000*Log[7 - 6*x + 2*x^2])/120050
```

Maple [A] time = 0.013, size = 62, normalized size = 0.8

$$-\frac{1}{98\,x^2}-\frac{12}{343\,x}+\frac{80\,\ln(x)}{2401}-\frac{4}{2401}\left(\frac{63\,x}{20}-\frac{287}{20}\right)\left(x^2-3\,x+\frac{7}{2}\right)^{-1}-\frac{40\,\ln\left(2\,x^2-6\,x+7\right)}{2401}+\frac{234\sqrt{5}}{60025}\arctan\left(\frac{(4\,x-}{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(2*x^2-6*x+7)^2,x)
```

```
[Out] -1/98/x^2-12/343/x+80/2401*ln(x)-4/2401*(63/20*x-287/20)/(x^2-3*x+7/2)-40/2
401*ln(2*x^2-6*x+7)+234/60025*5^(1/2)*arctan(1/10*(4*x-6)*5^(1/2))
```

Maxima [A] time = 1.41005, size = 93, normalized size = 1.15

$$\frac{234}{60025}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}(2x-3)\right)-\frac{276x^3-814x^2+630x+245}{3430\left(2x^4-6x^3+7x^2\right)}-\frac{40}{2401}\log\left(2x^2-6x+7\right)+\frac{80}{2401}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(2*x^2-6*x+7)^2,x, algorithm="maxima")
```

```
[Out] 234/60025*sqrt(5)*arctan(1/5*sqrt(5)*(2*x - 3)) - 1/3430*(276*x^3 - 814*x^2
 + 630*x + 245)/(2*x^4 - 6*x^3 + 7*x^2) - 40/2401*log(2*x^2 - 6*x + 7) + 80
/2401*log(x)
```

Fricas [A] time = 1.7611, size = 315, normalized size = 3.89

$$-\frac{9660x^3-468\sqrt{5}\left(2x^4-6x^3+7x^2\right)\arctan\left(\frac{1}{5}\sqrt{5}(2x-3)\right)-28490x^2+2000\left(2x^4-6x^3+7x^2\right)\log\left(2x^2-6x+7\right)}{120050\left(2x^4-6x^3+7x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(2*x^2-6*x+7)^2,x, algorithm="fricas")
```

```
[Out] -1/120050*(9660*x^3 - 468*sqrt(5)*(2*x^4 - 6*x^3 + 7*x^2)*arctan(1/5*sqrt(5
)*(2*x - 3)) - 28490*x^2 + 2000*(2*x^4 - 6*x^3 + 7*x^2)*log(2*x^2 - 6*x + 7
) - 4000*(2*x^4 - 6*x^3 + 7*x^2)*log(x) + 22050*x + 8575)/(2*x^4 - 6*x^3 +
7*x^2)
```

Sympy [A] time = 0.202163, size = 80, normalized size = 0.99

$$\frac{80\log(x)}{2401}-\frac{40\log\left(x^2-3x+\frac{7}{2}\right)}{2401}+\frac{234\sqrt{5}\operatorname{atan}\left(\frac{2\sqrt{5}x}{5}-\frac{3\sqrt{5}}{5}\right)}{60025}-\frac{276x^3-814x^2+630x+245}{6860x^4-20580x^3+24010x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(2*x**2-6*x+7)**2,x)
```

```
[Out] 80*log(x)/2401 - 40*log(x**2 - 3*x + 7/2)/2401 + 234*sqrt(5)*atan(2*sqrt(5)
*x/5 - 3*sqrt(5)/5)/60025 - (276*x**3 - 814*x**2 + 630*x + 245)/(6860*x**4
- 20580*x**3 + 24010*x**2)
```

Giac [A] time = 1.07603, size = 90, normalized size = 1.11

$$\frac{234}{60025}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}(2x-3)\right)-\frac{276x^3-814x^2+630x+245}{3430\left(2x^2-6x+7\right)x^2}-\frac{40}{2401}\log\left(2x^2-6x+7\right)+\frac{80}{2401}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(2*x^2-6*x+7)^2,x, algorithm="giac")
```

```
[Out] 234/60025*sqrt(5)*arctan(1/5*sqrt(5)*(2*x - 3)) - 1/3430*(276*x^3 - 814*x^2
 + 630*x + 245)/((2*x^2 - 6*x + 7)*x^2) - 40/2401*log(2*x^2 - 6*x + 7) + 80
/2401*log(abs(x))
```

3.204 $\int \frac{x^9}{(2+3x+x^2)^5}\,dx$

Optimal. Leaf size=104

$$\frac{(3x+4)x^8}{4(x^2+3x+2)^4}-\frac{(81x+110)x^6}{12(x^2+3x+2)^3}+\frac{(135x+184)x^4}{2(x^2+3x+2)^2}-\frac{(1593x+2206)x^2}{2(x^2+3x+2)}+735x-1471\log(x+1)+1472\log(x+2$$

```
[Out] 735*x + (x^8*(4 + 3*x))/(4*(2 + 3*x + x^2)^4) - (x^6*(110 + 81*x))/(12*(2 +
 3*x + x^2)^3) + (x^4*(184 + 135*x))/(2*(2 + 3*x + x^2)^2) - (x^2*(2206 + 1
593*x))/(2*(2 + 3*x + x^2)) - 1471*Log[1 + x] + 1472*Log[2 + x]
```

Rubi [A] time = 0.0737919, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {738, 818, 773, 632, 31}

$$\frac{(3x+4)x^8}{4(x^2+3x+2)^4}-\frac{(81x+110)x^6}{12(x^2+3x+2)^3}+\frac{(135x+184)x^4}{2(x^2+3x+2)^2}-\frac{(1593x+2206)x^2}{2(x^2+3x+2)}+735x-1471\log(x+1)+1472\log(x+2$$

Antiderivative was successfully verified.

```
[In] Int[x^9/(2 + 3*x + x^2)^5,x]
```


```
[Out] 735*x + (x^8*(4 + 3*x))/(4*(2 + 3*x + x^2)^4) - (x^6*(110 + 81*x))/(12*(2 +
 3*x + x^2)^3) + (x^4*(184 + 135*x))/(2*(2 + 3*x + x^2)^2) - (x^2*(2206 + 1
593*x))/(2*(2 + 3*x + x^2)) - 1471*Log[1 + x] + 1472*Log[2 + x]
```


Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c*(
b*e*f + b*d*g + 2*a*e*g))*x))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[1/(c*(p
+ 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp[2
*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*(m
 - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m +
 p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p +
 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m,
2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3,
0])
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{\left(2+3x+x^2\right)^5}\,dx &= \frac{x^8(4+3x)}{4\left(2+3x+x^2\right)^4}-\frac{1}{4}\int \frac{x^7(32+3x)}{\left(2+3x+x^2\right)^4}\,dx \\
&= \frac{x^8(4+3x)}{4\left(2+3x+x^2\right)^4}-\frac{x^6(110+81x)}{12\left(2+3x+x^2\right)^3}-\frac{1}{12}\int \frac{(-660-72x)x^5}{\left(2+3x+x^2\right)^3}\,dx \\
&= \frac{x^8(4+3x)}{4\left(2+3x+x^2\right)^4}-\frac{x^6(110+81x)}{12\left(2+3x+x^2\right)^3}+\frac{x^4(184+135x)}{2\left(2+3x+x^2\right)^2}-\frac{1}{24}\int \frac{x^3(8832+1476x)}{\left(2+3x+x^2\right)^2}\,dx \\
&= \frac{x^8(4+3x)}{4\left(2+3x+x^2\right)^4}-\frac{x^6(110+81x)}{12\left(2+3x+x^2\right)^3}+\frac{x^4(184+135x)}{2\left(2+3x+x^2\right)^2}-\frac{x^2(2206+1593x)}{2\left(2+3x+x^2\right)}-\frac{1}{24}\int \frac{(-5294\cdots}{2\cdots} \\
&= 735x+\frac{x^8(4+3x)}{4\left(2+3x+x^2\right)^4}-\frac{x^6(110+81x)}{12\left(2+3x+x^2\right)^3}+\frac{x^4(184+135x)}{2\left(2+3x+x^2\right)^2}-\frac{x^2(2206+1593x)}{2\left(2+3x+x^2\right)}-\frac{1}{24}\int\cdots \\
&= 735x+\frac{x^8(4+3x)}{4\left(2+3x+x^2\right)^4}-\frac{x^6(110+81x)}{12\left(2+3x+x^2\right)^3}+\frac{x^4(184+135x)}{2\left(2+3x+x^2\right)^2}-\frac{x^2(2206+1593x)}{2\left(2+3x+x^2\right)}-1471\cdots \\
&= 735x+\frac{x^8(4+3x)}{4\left(2+3x+x^2\right)^4}-\frac{x^6(110+81x)}{12\left(2+3x+x^2\right)^3}+\frac{x^4(184+135x)}{2\left(2+3x+x^2\right)^2}-\frac{x^2(2206+1593x)}{2\left(2+3x+x^2\right)}-1471\,\mathrm{l}\cdots
\end{aligned}$$

Mathematica [A] time = 0.0222318, size = 87, normalized size = 0.84

$$\frac{3(456x+451)}{4\left(x^2+3x+2\right)^2}-\frac{2(729x+1114)}{x^2+3x+2}+\frac{1998x+415}{12\left(x^2+3x+2\right)^3}+\frac{513x+514}{4\left(x^2+3x+2\right)^4}-1471\log(x+1)+1472\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/(2 + 3*x + x^2)^5,x]
```

```
[Out] (514 + 513*x)/(4*(2 + 3*x + x^2)^4) + (415 + 1998*x)/(12*(2 + 3*x + x^2)^3)
 + (3*(451 + 456*x))/(4*(2 + 3*x + x^2)^2) - (2*(1114 + 729*x))/(2 + 3*x +
x^2) - 1471*Log[1 + x] + 1472*Log[2 + x]
```

Maple [A] time = 0.008, size = 70, normalized size = 0.7

$$-128\,(2+x)^{-4}-\frac{256}{3\,(2+x)^{3}}-384\,(2+x)^{-2}-1024\,(2+x)^{-1}+1472\,\ln(2+x)+\frac{1}{4\,(1+x)^{4}}-\frac{14}{3\,(1+x)^{3}}+48\,(1+x)^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(x^2+3*x+2)^5,x)
```

```
[Out] -128/(2+x)^4-256/3/(2+x)^3-384/(2+x)^2-1024/(2+x)+1472*ln(2+x)+1/4/(1+x)^4-
14/3/(1+x)^3+48/(1+x)^2-434/(1+x)-1471*ln(1+x)
```

Maxima [A] time = 0.935198, size = 122, normalized size = 1.17

$$-\frac{17496\,x^{7}+184200\,x^{6}+813888\,x^{5}+1955853\,x^{4}+2759400\,x^{3}+2286008\,x^{2}+1030560\,x+195280}{12\left(x^{8}+12\,x^{7}+62\,x^{6}+180\,x^{5}+321\,x^{4}+360\,x^{3}+248\,x^{2}+96\,x+16\right)}+1472\,\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^2+3*x+2)^5,x, algorithm="maxima")
```

```
[Out] -1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 22
86008*x^2 + 1030560*x + 195280)/(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4
+ 360*x^3 + 248*x^2 + 96*x + 16) + 1472*log(x + 2) - 1471*log(x + 1)
```

Fricas [A] time = 1.6261, size = 505, normalized size = 4.86

$$-\frac{17496\,x^{7}+184200\,x^{6}+813888\,x^{5}+1955853\,x^{4}+2759400\,x^{3}+2286008\,x^{2}-17664\left(x^{8}+12\,x^{7}+62\,x^{6}+180\,x^{5}+32\right.}{12\left(x^{8}+12\,x^{7}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^2+3*x+2)^5,x, algorithm="fricas")
```

```
[Out] -1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 22
86008*x^2 - 17664*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 24
8*x^2 + 96*x + 16)*log(x + 2) + 17652*(x^8 + 12*x^7 + 62*x^6 + 180*x^5 + 32
1*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)*log(x + 1) + 1030560*x + 195280)/(x^
8 + 12*x^7 + 62*x^6 + 180*x^5 + 321*x^4 + 360*x^3 + 248*x^2 + 96*x + 16)
```

Sympy [A] time = 0.18975, size = 88, normalized size = 0.85

$$-\frac{17496x^{7}+184200x^{6}+813888x^{5}+1955853x^{4}+2759400x^{3}+2286008x^{2}+1030560x+195280}{12x^{8}+144x^{7}+744x^{6}+2160x^{5}+3852x^{4}+4320x^{3}+2976x^{2}+1152x+192}-1471\log(x+1)+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(x**2+3*x+2)**5,x)
```

```
[Out] -(17496*x**7 + 184200*x**6 + 813888*x**5 + 1955853*x**4 + 2759400*x**3 + 22
86008*x**2 + 1030560*x + 195280)/(12*x**8 + 144*x**7 + 744*x**6 + 2160*x**5
```

```
 + 3852*x**4 + 4320*x**3 + 2976*x**2 + 1152*x + 192) - 1471*log(x + 1) + 14
72*log(x + 2)
```

Giac [A] time = 1.05595, size = 84, normalized size = 0.81

$$-\frac{17496\,x^7 + 184200\,x^6 + 813888\,x^5 + 1955853\,x^4 + 2759400\,x^3 + 2286008\,x^2 + 1030560\,x + 195280}{12\,(x+2)^4(x+1)^4} + 1472\,\log\left(|x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(x^2+3*x+2)^5,x, algorithm="giac")
```

```
[Out] -1/12*(17496*x^7 + 184200*x^6 + 813888*x^5 + 1955853*x^4 + 2759400*x^3 + 22
86008*x^2 + 1030560*x + 195280)/((x + 2)^4*(x + 1)^4) + 1472*log(abs(x + 2)
) - 1471*log(abs(x + 1))
```

3.205 $\int \frac{(1+2x)^2}{\left(3+5x+2x^2\right)^5}\,dx$

Optimal. Leaf size=102

$$\frac{620(4x+5)}{2x^2+5x+3}-\frac{155(4x+5)}{3\left(2x^2+5x+3\right)^2}+\frac{62x+73}{3\left(2x^2+5x+3\right)^3}+\frac{(2x+1)(6x+7)}{4\left(2x^2+5x+3\right)^4}+2480\log(x+1)-2480\log(2x+3)$$

```
[Out] ((1 + 2*x)*(7 + 6*x))/(4*(3 + 5*x + 2*x^2)^4) + (73 + 62*x)/(3*(3 + 5*x + 2
*x^2)^3) - (155*(5 + 4*x))/(3*(3 + 5*x + 2*x^2)^2) + (620*(5 + 4*x))/(3 + 5
*x + 2*x^2) + 2480*Log[1 + x] - 2480*Log[3 + 2*x]
```

Rubi [A] time = 0.0371751, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {738, 638, 614, 616, 31}

$$\frac{620(4x+5)}{2x^2+5x+3}-\frac{155(4x+5)}{3\left(2x^2+5x+3\right)^2}+\frac{62x+73}{3\left(2x^2+5x+3\right)^3}+\frac{(2x+1)(6x+7)}{4\left(2x^2+5x+3\right)^4}+2480\log(x+1)-2480\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)^2/(3 + 5*x + 2*x^2)^5,x]
```

```
[Out] ((1 + 2*x)*(7 + 6*x))/(4*(3 + 5*x + 2*x^2)^4) + (73 + 62*x)/(3*(3 + 5*x + 2
*x^2)^3) - (155*(5 + 4*x))/(3*(3 + 5*x + 2*x^2)^2) + (620*(5 + 4*x))/(3 + 5
*x + 2*x^2) + 2480*Log[1 + x] - 2480*Log[3 + 2*x]
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2x)^2}{\left(3+5x+2x^2\right)^5}\,dx &= \frac{(1+2x)(7+6x)}{4\left(3+5x+2x^2\right)^4}-\frac{1}{4}\int \frac{-28-72x}{\left(3+5x+2x^2\right)^4}\,dx\\
&= \frac{(1+2x)(7+6x)}{4\left(3+5x+2x^2\right)^4}+\frac{73+62x}{3\left(3+5x+2x^2\right)^3}+\frac{310}{3}\int \frac{1}{\left(3+5x+2x^2\right)^3}\,dx\\
&= \frac{(1+2x)(7+6x)}{4\left(3+5x+2x^2\right)^4}+\frac{73+62x}{3\left(3+5x+2x^2\right)^3}-\frac{155(5+4x)}{3\left(3+5x+2x^2\right)^2}-620\int \frac{1}{\left(3+5x+2x^2\right)^2}\,dx\\
&= \frac{(1+2x)(7+6x)}{4\left(3+5x+2x^2\right)^4}+\frac{73+62x}{3\left(3+5x+2x^2\right)^3}-\frac{155(5+4x)}{3\left(3+5x+2x^2\right)^2}+\frac{620(5+4x)}{3+5x+2x^2}+2480\int \frac{}{3+5\cdots}\\
&= \frac{(1+2x)(7+6x)}{4\left(3+5x+2x^2\right)^4}+\frac{73+62x}{3\left(3+5x+2x^2\right)^3}-\frac{155(5+4x)}{3\left(3+5x+2x^2\right)^2}+\frac{620(5+4x)}{3+5x+2x^2}+4960\int \frac{1}{2+2\cdots}\\
&= \frac{(1+2x)(7+6x)}{4\left(3+5x+2x^2\right)^4}+\frac{73+62x}{3\left(3+5x+2x^2\right)^3}-\frac{155(5+4x)}{3\left(3+5x+2x^2\right)^2}+\frac{620(5+4x)}{3+5x+2x^2}+2480\log(1+\cdots\end{aligned}$$

Mathematica [A] time = 0.0513033, size = 99, normalized size = 0.97

$$\frac{620(4x+5)}{2x^2+5x+3}-\frac{155(4x+5)}{3\left(2x^2+5x+3\right)^2}+\frac{31(4x+5)}{6\left(2x^2+5x+3\right)^3}-\frac{10x+11}{4\left(2x^2+5x+3\right)^4}+2480\log(2(x+1))-2480\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)^2/(3 + 5*x + 2*x^2)^5,x]
```

```
[Out] -(11 + 10*x)/(4*(3 + 5*x + 2*x^2)^4) + (31*(5 + 4*x))/(6*(3 + 5*x + 2*x^2)^
3) - (155*(5 + 4*x))/(3*(3 + 5*x + 2*x^2)^2) + (620*(5 + 4*x))/(3 + 5*x + 2
*x^2) + 2480*Log[2*(1 + x)] - 2480*Log[3 + 2*x]
```

Maple [A] time = 0.01, size = 80, normalized size = 0.8

$$16\,(3+2\,x)^{-4}+\frac{256}{3\,(3+2\,x)^3}+328\,(3+2\,x)^{-2}+1360\,(3+2\,x)^{-1}-2480\,\ln(3+2\,x)-\frac{1}{4\,(1+x)^4}+\frac{14}{3\,(1+x)^3}-52$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^2/(2*x^2+5*x+3)^5,x)
```

```
[Out] 16/(3+2*x)^4+256/3/(3+2*x)^3+328/(3+2*x)^2+1360/(3+2*x)-2480*ln(3+2*x)-1/4/
(1+x)^4+14/3/(1+x)^3-52/(1+x)^2+560/(1+x)+2480*ln(1+x)
```

Maxima [A] time = 0.932153, size = 127, normalized size = 1.25

$$\frac{238080\,x^7+2083200\,x^6+7757440\,x^5+15934000\,x^4+19495776\,x^3+14209160\,x^2+5712464\,x+977397}{12\left(16\,x^8+160\,x^7+696\,x^6+1720\,x^5+2641\,x^4+2580\,x^3+1566\,x^2+540\,x+81\right)}-2480\,\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x, algorithm="maxima")
```

```
[Out] 1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3
+ 14209160*x^2 + 5712464*x + 977397)/(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5
 + 2641*x^4 + 2580*x^3 + 1566*x^2 + 540*x + 81) - 2480*log(2*x + 3) + 2480*
log(x + 1)
```

Fricas [A] time = 1.74688, size = 555, normalized size = 5.44

$$\frac{238080\,x^7+2083200\,x^6+7757440\,x^5+15934000\,x^4+19495776\,x^3+14209160\,x^2-29760\left(16\,x^8+160\,x^7+696\,x^6+\right.}{12\left(16\,\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x, algorithm="fricas")
```

```
[Out] 1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3
+ 14209160*x^2 - 29760*(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5 + 2641*x^4 +
2580*x^3 + 1566*x^2 + 540*x + 81)*log(2*x + 3) + 29760*(16*x^8 + 160*x^7 +
696*x^6 + 1720*x^5 + 2641*x^4 + 2580*x^3 + 1566*x^2 + 540*x + 81)*log(x + 1
) + 5712464*x + 977397)/(16*x^8 + 160*x^7 + 696*x^6 + 1720*x^5 + 2641*x^4 +
 2580*x^3 + 1566*x^2 + 540*x + 81)
```

Sympy [A] time = 0.208913, size = 90, normalized size = 0.88

$$\frac{238080x^7+2083200x^6+7757440x^5+15934000x^4+19495776x^3+14209160x^2+5712464x+977397}{192x^8+1920x^7+8352x^6+20640x^5+31692x^4+30960x^3+18792x^2+6480x+972}+2480\log\left(x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**2/(2*x**2+5*x+3)**5,x)
```

```
[Out] (238080*x**7 + 2083200*x**6 + 7757440*x**5 + 15934000*x**4 + 19495776*x**3
+ 14209160*x**2 + 5712464*x + 977397)/(192*x**8 + 1920*x**7 + 8352*x**6 + 2
0640*x**5 + 31692*x**4 + 30960*x**3 + 18792*x**2 + 6480*x + 972) + 2480*log
(x + 1) - 2480*log(x + 3/2)
```

Giac [A] time = 1.06351, size = 89, normalized size = 0.87

$$\frac{238080\,x^7+2083200\,x^6+7757440\,x^5+15934000\,x^4+19495776\,x^3+14209160\,x^2+5712464\,x+977397}{12\left(2\,x^2+5\,x+3\right)^4}-2480\,\log($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^2/(2*x^2+5*x+3)^5,x, algorithm="giac")
```

```
[Out] 1/12*(238080*x^7 + 2083200*x^6 + 7757440*x^5 + 15934000*x^4 + 19495776*x^3
+ 14209160*x^2 + 5712464*x + 977397)/(2*x^2 + 5*x + 3)^4 - 2480*log(abs(2*x
 + 3)) + 2480*log(abs(x + 1))
```

3.206 $\int \frac{(a-bx^2)^3}{x^7} \, dx$

Optimal. Leaf size=40

$$\frac{3a^2b}{4x^4} - \frac{a^3}{6x^6} - \frac{3ab^2}{2x^2} - b^3\log(x)$$

```
[Out] -a^3/(6*x^6) + (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) - b^3*Log[x]
```

Rubi [A] time = 0.0231058, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {266, 43}

$$\frac{3a^2b}{4x^4} - \frac{a^3}{6x^6} - \frac{3ab^2}{2x^2} - b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*x^2)^3/x^7,x]
```

```
[Out] -a^3/(6*x^6) + (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) - b^3*Log[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a-bx^2)^3}{x^7} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(a-bx)^3}{x^4} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{a^3}{x^4} - \frac{3a^2b}{x^3} + \frac{3ab^2}{x^2} - \frac{b^3}{x}\right) dx, x, x^2\right) \\
&= -\frac{a^3}{6x^6} + \frac{3a^2b}{4x^4} - \frac{3ab^2}{2x^2} - b^3\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0054169, size = 40, normalized size = 1.

$$\frac{3a^2b}{4x^4} - \frac{a^3}{6x^6} - \frac{3ab^2}{2x^2} - b^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*x^2)^3/x^7,x]
```

```
[Out] -a^3/(6*x^6) + (3*a^2*b)/(4*x^4) - (3*a*b^2)/(2*x^2) - b^3*Log[x]
```

Maple [A] time = 0.007, size = 35, normalized size = 0.9

$$-\frac{a^3}{6\,x^6}+\frac{3\,a^2b}{4\,x^4}-\frac{3\,b^2a}{2\,x^2}-b^3\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^2+a)^3/x^7,x)
```

```
[Out] -1/6*a^3/x^6+3/4*a^2*b/x^4-3/2*a*b^2/x^2-b^3*ln(x)
```

Maxima [A] time = 0.930659, size = 53, normalized size = 1.32

$$-\frac{1}{2}\,b^3\log\left(x^2\right)-\frac{18\,ab^2x^4-9\,a^2bx^2+2\,a^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^3/x^7,x, algorithm="maxima")
```

```
[Out] -1/2*b^3*log(x^2) - 1/12*(18*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3)/x^6
```

Fricas [A] time = 1.73601, size = 92, normalized size = 2.3

$$-\frac{12\,b^3x^6\log(x)+18\,ab^2x^4-9\,a^2bx^2+2\,a^3}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^3/x^7,x, algorithm="fricas")
```

```
[Out] -1/12*(12*b^3*x^6*log(x) + 18*a*b^2*x^4 - 9*a^2*b*x^2 + 2*a^3)/x^6
```

Sympy [A] time = 0.377801, size = 37, normalized size = 0.92

$$-b^3\log(x)-\frac{2a^3-9a^2bx^2+18ab^2x^4}{12x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**2+a)**3/x**7,x)
```

```
[Out] -b**3*log(x) - (2*a**3 - 9*a**2*b*x**2 + 18*a*b**2*x**4)/(12*x**6)
```

Giac [A] time = 1.05544, size = 63, normalized size = 1.58

$$-\frac{1}{2}\,b^{3}\log\left(x^{2}\right)+\frac{11\,b^{3}x^{6}-18\,ab^{2}x^{4}+9\,a^{2}bx^{2}-2\,a^{3}}{12\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^2+a)^3/x^7,x, algorithm="giac")
```

```
[Out] -1/2*b^3*log(x^2) + 1/12*(11*b^3*x^6 - 18*a*b^2*x^4 + 9*a^2*b*x^2 - 2*a^3)/
x^6
```

3.207 $\int \frac{x^{13}}{\left(a^4+x^4\right)^5} \, dx$

Optimal. Leaf size=83

$$-\frac{x^{10}}{16\left(a^4+x^4\right)^4}-\frac{5x^6}{96\left(a^4+x^4\right)^3}+\frac{5x^2}{256a^4\left(a^4+x^4\right)}-\frac{5x^2}{128\left(a^4+x^4\right)^2}+\frac{5\tan^{-1}\left(\frac{x^2}{a^2}\right)}{256a^6}$$

```
[Out] -x^10/(16*(a^4 + x^4)^4) - (5*x^6)/(96*(a^4 + x^4)^3) - (5*x^2)/(128*(a^4 +
 x^4)^2) + (5*x^2)/(256*a^4*(a^4 + x^4)) + (5*ArcTan[x^2/a^2])/(256*a^6)
```

Rubi [A] time = 0.0391206, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {275, 288, 199, 203}

$$-\frac{x^{10}}{16\left(a^4+x^4\right)^4}-\frac{5x^6}{96\left(a^4+x^4\right)^3}+\frac{5x^2}{256a^4\left(a^4+x^4\right)}-\frac{5x^2}{128\left(a^4+x^4\right)^2}+\frac{5\tan^{-1}\left(\frac{x^2}{a^2}\right)}{256a^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^13/(a^4 + x^4)^5,x]
```

```
[Out] -x^10/(16*(a^4 + x^4)^4) - (5*x^6)/(96*(a^4 + x^4)^3) - (5*x^2)/(128*(a^4 +
 x^4)^2) + (5*x^2)/(256*a^4*(a^4 + x^4)) + (5*ArcTan[x^2/a^2])/(256*a^6)
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^{13}}{\left(a^4+x^4\right)^5}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^6}{\left(a^4+x^2\right)^5}\,dx, x, x^2\right)$$
$$= -\frac{x^{10}}{16\left(a^4+x^4\right)^4} + \frac{5}{16}\,\text{Subst}\left(\int \frac{x^4}{\left(a^4+x^2\right)^4}\,dx, x, x^2\right)$$
$$= -\frac{x^{10}}{16\left(a^4+x^4\right)^4} - \frac{5x^6}{96\left(a^4+x^4\right)^3} + \frac{5}{32}\,\text{Subst}\left(\int \frac{x^2}{\left(a^4+x^2\right)^3}\,dx, x, x^2\right)$$
$$= -\frac{x^{10}}{16\left(a^4+x^4\right)^4} - \frac{5x^6}{96\left(a^4+x^4\right)^3} - \frac{5x^2}{128\left(a^4+x^4\right)^2} + \frac{5}{128}\,\text{Subst}\left(\int \frac{1}{\left(a^4+x^2\right)^2}\,dx, x, x^2\right)$$
$$= -\frac{x^{10}}{16\left(a^4+x^4\right)^4} - \frac{5x^6}{96\left(a^4+x^4\right)^3} - \frac{5x^2}{128\left(a^4+x^4\right)^2} + \frac{5x^2}{256a^4\left(a^4+x^4\right)} + \frac{5\,\text{Subst}\left(\int \frac{1}{a^4+x^2}\,dx, x, x^2\right)}{256a^4}$$
$$= -\frac{x^{10}}{16\left(a^4+x^4\right)^4} - \frac{5x^6}{96\left(a^4+x^4\right)^3} - \frac{5x^2}{128\left(a^4+x^4\right)^2} + \frac{5x^2}{256a^4\left(a^4+x^4\right)} + \frac{5\tan^{-1}\left(\frac{x^2}{a^2}\right)}{256a^6}$$

Mathematica [A] time = 0.0218465, size = 62, normalized size = 0.75

$$\frac{15\tan^{-1}\left(\frac{x^2}{a^2}\right) - \frac{a^2x^2\left(55a^8x^4+73a^4x^8+15a^{12}-15x^{12}\right)}{\left(a^4+x^4\right)^4}}{768a^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^13/(a^4 + x^4)^5,x]
```

```
[Out] (-((a^2*x^2*(15*a^12 + 55*a^8*x^4 + 73*a^4*x^8 - 15*x^12))/(a^4 + x^4)^4) +
 15*ArcTan[x^2/a^2])/(768*a^6)
```

Maple [A] time = 0.011, size = 56, normalized size = 0.7

$$\frac{1}{2\left(a^4+x^4\right)^4}\left(\frac{5\,x^{14}}{128\,a^4} - \frac{73\,x^{10}}{384} - \frac{55\,x^6a^4}{384} - \frac{5\,a^8x^2}{128}\right) + \frac{5}{256\,a^6}\arctan\left(\frac{x^2}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^13/(a^4+x^4)^5,x)
```

```
[Out] 1/2*(5/128/a^4*x^14-73/384*x^10-55/384*x^6*a^4-5/128*a^8*x^2)/(a^4+x^4)^4+5
/256*arctan(x^2/a^2)/a^6
```

Maxima [A] time = 1.54016, size = 112, normalized size = 1.35

$$-\frac{15\,a^{12}x^2 + 55\,a^8x^6 + 73\,a^4x^{10} - 15\,x^{14}}{768\left(a^{20} + 4\,a^{16}x^4 + 6\,a^{12}x^8 + 4\,a^8x^{12} + a^4x^{16}\right)} + \frac{5\arctan\left(\frac{x^2}{a^2}\right)}{256\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(a^4+x^4)^5,x, algorithm="maxima")
```

```
[Out] -1/768*(15*a^12*x^2 + 55*a^8*x^6 + 73*a^4*x^10 - 15*x^14)/(a^20 + 4*a^16*x^
4 + 6*a^12*x^8 + 4*a^8*x^12 + a^4*x^16) + 5/256*arctan(x^2/a^2)/a^6
```

Fricas [A] time = 1.76912, size = 263, normalized size = 3.17

$$-\frac{15\,a^{14}x^{2}+55\,a^{10}x^{6}+73\,a^{6}x^{10}-15\,a^{2}x^{14}-15\left(a^{16}+4\,a^{12}x^{4}+6\,a^{8}x^{8}+4\,a^{4}x^{12}+x^{16}\right)\arctan\left(\frac{x^{2}}{a^{2}}\right)}{768\left(a^{22}+4\,a^{18}x^{4}+6\,a^{14}x^{8}+4\,a^{10}x^{12}+a^{6}x^{16}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(a^4+x^4)^5,x, algorithm="fricas")
```

```
[Out] -1/768*(15*a^14*x^2 + 55*a^10*x^6 + 73*a^6*x^10 - 15*a^2*x^14 - 15*(a^16 +
4*a^12*x^4 + 6*a^8*x^8 + 4*a^4*x^12 + x^16)*arctan(x^2/a^2))/(a^22 + 4*a^18
*x^4 + 6*a^14*x^8 + 4*a^10*x^12 + a^6*x^16)
```

Sympy [C] time = 35.7445, size = 102, normalized size = 1.23

$$\frac{-15a^{12}x^{2}-55a^{8}x^{6}-73a^{4}x^{10}+15x^{14}}{768a^{20}+3072a^{16}x^{4}+4608a^{12}x^{8}+3072a^{8}x^{12}+768a^{4}x^{16}}+\frac{-\frac{5i\log\left(-ia^{2}+x^{2}\right)}{512}+\frac{5i\log\left(ia^{2}+x^{2}\right)}{512}}{a^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**13/(a**4+x**4)**5,x)
```

```
[Out] (-15*a**12*x**2 - 55*a**8*x**6 - 73*a**4*x**10 + 15*x**14)/(768*a**20 + 307
2*a**16*x**4 + 4608*a**12*x**8 + 3072*a**8*x**12 + 768*a**4*x**16) + (-5*I*
log(-I*a**2 + x**2)/512 + 5*I*log(I*a**2 + x**2)/512)/a**6
```

Giac [A] time = 1.06157, size = 78, normalized size = 0.94

$$\frac{5\,\arctan\left(\frac{x^{2}}{a^{2}}\right)}{256\,a^{6}}-\frac{15\,a^{12}x^{2}+55\,a^{8}x^{6}+73\,a^{4}x^{10}-15\,x^{14}}{768\left(a^{4}+x^{4}\right)^{4}a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^13/(a^4+x^4)^5,x, algorithm="giac")
```

```
[Out] 5/256*arctan(x^2/a^2)/a^6 - 1/768*(15*a^12*x^2 + 55*a^8*x^6 + 73*a^4*x^10 -
 15*x^14)/((a^4 + x^4)^4*a^4)
```

3.208 $\int \left(2\sqrt{x}-x\right)^2 x^{3/2} \left(1+x^2\right) dx$

Optimal. Leaf size=49

$$\frac{2x^{13/2}}{13}-\frac{2x^6}{3}+\frac{8x^{11/2}}{11}+\frac{2x^{9/2}}{9}-x^4+\frac{8x^{7/2}}{7}$$

```
[Out] (8*x^(7/2))/7 - x^4 + (2*x^(9/2))/9 + (8*x^(11/2))/11 - (2*x^6)/3 + (2*x^(1
3/2))/13
```

Rubi [A] time = 0.0566495, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1584, 1820, 266, 43}

$$\frac{2x^{13/2}}{13}-\frac{2x^6}{3}+\frac{8x^{11/2}}{11}+\frac{2x^{9/2}}{9}-x^4+\frac{8x^{7/2}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sqrt[x] - x)^2*x^(3/2)*(1 + x^2),x]
```

```
[Out] (8*x^(7/2))/7 - x^4 + (2*x^(9/2))/9 + (8*x^(11/2))/11 - (2*x^6)/3 + (2*x^(1
3/2))/13
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \left(2\sqrt{x}-x\right)^2 x^{3/2}\left(1+x^2\right) dx &= \int \left(2-\sqrt{x}\right)^2 x^{5/2}\left(1+x^2\right) dx \\ &= \int \left(\left(-2+\sqrt{x}\right)^2 x^{5/2}+\left(-2+\sqrt{x}\right)^2 x^{9/2}\right) dx \\ &= \int \left(-2+\sqrt{x}\right)^2 x^{5/2}\, dx+\int \left(-2+\sqrt{x}\right)^2 x^{9/2}\, dx \\ &= 2\,\text{Subst}\left(\int (-2+x)^2x^6\, dx, x, \sqrt{x}\right)+2\,\text{Subst}\left(\int (-2+x)^2x^{10}\, dx, x, \sqrt{x}\right) \\ &= 2\,\text{Subst}\left(\int \left(4x^6-4x^7+x^8\right) dx, x, \sqrt{x}\right)+2\,\text{Subst}\left(\int \left(4x^{10}-4x^{11}+x^{12}\right) dx, x, \sqrt{x}\right) \\ &= \frac{8x^{7/2}}{7}-x^4+\frac{2x^{9/2}}{9}+\frac{8x^{11/2}}{11}-\frac{2x^6}{3}+\frac{2x^{13/2}}{13}\end{aligned}$$

Mathematica [A] time = 0.0218665, size = 49, normalized size = 1.

$$\frac{2x^{13/2}}{13}-\frac{2x^6}{3}+\frac{8x^{11/2}}{11}+\frac{2x^{9/2}}{9}-x^4+\frac{8x^{7/2}}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sqrt[x] - x)^2*x^(3/2)*(1 + x^2),x]
```

```
[Out] (8*x^(7/2))/7 - x^4 + (2*x^(9/2))/9 + (8*x^(11/2))/11 - (2*x^6)/3 + (2*x^(1
3/2))/13
```

Maple [A] time = 0.002, size = 32, normalized size = 0.7

$$\frac{8}{7}x^{\frac{7}{2}}-x^4+\frac{2}{9}x^{\frac{9}{2}}+\frac{8}{11}x^{\frac{11}{2}}-\frac{2\,x^6}{3}+\frac{2}{13}x^{\frac{13}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x)
```

```
[Out] 8/7*x^(7/2)-x^4+2/9*x^(9/2)+8/11*x^(11/2)-2/3*x^6+2/13*x^(13/2)
```

Maxima [A] time = 0.957597, size = 42, normalized size = 0.86

$$\frac{2}{13}x^{\frac{13}{2}}-\frac{2}{3}x^6+\frac{8}{11}x^{\frac{11}{2}}+\frac{2}{9}x^{\frac{9}{2}}-x^4+\frac{8}{7}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 2/13*x^(13/2) - 2/3*x^6 + 8/11*x^(11/2) + 2/9*x^(9/2) - x^4 + 8/7*x^(7/2)
```

Fricas [A] time = 1.68706, size = 103, normalized size = 2.1

$$-\frac{2}{3}x^6-x^4+\frac{2}{9009}\left(693\,x^6+3276\,x^5+1001\,x^4+5148\,x^3\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -2/3*x^6 - x^4 + 2/9009*(693*x^6 + 3276*x^5 + 1001*x^4 + 5148*x^3)*sqrt(x)
```

Sympy [A] time = 2.23406, size = 42, normalized size = 0.86

$$\frac{2x^{\frac{13}{2}}}{13} + \frac{8x^{\frac{11}{2}}}{11} + \frac{2x^{\frac{9}{2}}}{9} + \frac{8x^{\frac{7}{2}}}{7} - \frac{2x^6}{3} - x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2)*(x**2+1)*(-x+2*x**(1/2))**2,x)
```

```
[Out] 2*x**(13/2)/13 + 8*x**(11/2)/11 + 2*x**(9/2)/9 + 8*x**(7/2)/7 - 2*x**6/3 -
x**4
```

Giac [A] time = 1.05136, size = 42, normalized size = 0.86

$$\frac{2}{13}x^{\frac{13}{2}} - \frac{2}{3}x^6 + \frac{8}{11}x^{\frac{11}{2}} + \frac{2}{9}x^{\frac{9}{2}} - x^4 + \frac{8}{7}x^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2)*(x^2+1)*(-x+2*x^(1/2))^2,x, algorithm="giac")
```

```
[Out] 2/13*x^(13/2) - 2/3*x^6 + 8/11*x^(11/2) + 2/9*x^(9/2) - x^4 + 8/7*x^(7/2)
```

3.209 $\int \left(-3x^{3/5}+x^{3/2}\right)^2 \left(-\frac{x^{2/3}}{3}+4x^{3/2}\right) dx$

Optimal. Leaf size=55

$$\frac{8x^{11/2}}{11}-\frac{x^{14/3}}{14}-\frac{120x^{23/5}}{23}+\frac{60x^{113/30}}{113}+\frac{360x^{37/10}}{37}-\frac{45x^{43/15}}{43}$$

```
[Out] (-45*x^(43/15))/43 + (360*x^(37/10))/37 + (60*x^(113/30))/113 - (120*x^(23/
5))/23 - x^(14/3)/14 + (8*x^(11/2))/11
```

Rubi [A] time = 0.237876, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1593, 1584, 1820}

$$\frac{8x^{11/2}}{11}-\frac{x^{14/3}}{14}-\frac{120x^{23/5}}{23}+\frac{60x^{113/30}}{113}+\frac{360x^{37/10}}{37}-\frac{45x^{43/15}}{43}$$

Antiderivative was successfully verified.

```
[In] Int[(-3*x^(3/5) + x^(3/2))^2*(-x^(2/3)/3 + 4*x^(3/2)),x]
```

```
[Out] (-45*x^(43/15))/43 + (360*x^(37/10))/37 + (60*x^(113/30))/113 - (120*x^(23/
5))/23 - x^(14/3)/14 + (8*x^(11/2))/11
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1820

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :>
Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m,
n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])
```

Rubi steps

$$\begin{aligned}
\int \left(-3x^{3/5}+x^{3/2}\right)^2\left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)dx &= \int \left(-3+x^{9/10}\right)^2 x^{6/5}\left(-\frac{x^{2/3}}{3}+4x^{3/2}\right)dx\\
&= \int \left(-\frac{1}{3}+4x^{5/6}\right)\left(-3+x^{9/10}\right)^2 x^{28/15}\,dx\\
&= 30\,\text{Subst}\left(\int x^{85}\left(-\frac{1}{3}+4x^{25}\right)\left(-3+x^{27}\right)^2 dx, x, \sqrt[30]{x}\right)\\
&= 30\,\text{Subst}\left(\int \left(-3x^{85}+36x^{110}+2x^{112}-24x^{137}-\frac{x^{139}}{3}+4x^{164}\right)dx, x, \sqrt[30]{x}\right)\\
&= -\frac{45x^{43/15}}{43}+\frac{360x^{37/10}}{37}+\frac{60x^{113/30}}{113}-\frac{120x^{23/5}}{23}-\frac{x^{14/3}}{14}+\frac{8x^{11/2}}{11}
\end{aligned}$$

Mathematica [A] time = 0.0459141, size = 55, normalized size = 1.

$$\frac{8x^{11/2}}{11}-\frac{x^{14/3}}{14}-\frac{120x^{23/5}}{23}+\frac{60x^{113/30}}{113}+\frac{360x^{37/10}}{37}-\frac{45x^{43/15}}{43}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3*x^(3/5) + x^(3/2))^2*(-x^(2/3)/3 + 4*x^(3/2)),x]
```

```
[Out] (-45*x^(43/15))/43 + (360*x^(37/10))/37 + (60*x^(113/30))/113 - (120*x^(23/
5))/23 - x^(14/3)/14 + (8*x^(11/2))/11
```

Maple [A] time = 0.003, size = 32, normalized size = 0.6

$$-\frac{45}{43}x^{\frac{43}{15}}+\frac{360}{37}x^{\frac{37}{10}}+\frac{60}{113}x^{\frac{113}{30}}-\frac{120}{23}x^{\frac{23}{5}}-\frac{1}{14}x^{\frac{14}{3}}+\frac{8}{11}x^{\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x)
```

```
[Out] -45/43*x^(43/15)+360/37*x^(37/10)+60/113*x^(113/30)-120/23*x^(23/5)-1/14*x^
(14/3)+8/11*x^(11/2)
```

Maxima [A] time = 0.940615, size = 42, normalized size = 0.76

$$\frac{8}{11}x^{\frac{11}{2}}-\frac{1}{14}x^{\frac{14}{3}}-\frac{120}{23}x^{\frac{23}{5}}+\frac{60}{113}x^{\frac{113}{30}}+\frac{360}{37}x^{\frac{37}{10}}-\frac{45}{43}x^{\frac{43}{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x, algorithm="max
ima")
```

```
[Out] 8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/30) + 360/3
7*x^(37/10) - 45/43*x^(43/15)
```

Fricas [A] time = 1.6393, size = 143, normalized size = 2.6

$$\frac{8}{11}x^{\frac{11}{2}}-\frac{1}{14}x^{\frac{14}{3}}-\frac{120}{23}x^{\frac{23}{5}}+\frac{60}{113}x^{\frac{113}{30}}+\frac{360}{37}x^{\frac{37}{10}}-\frac{45}{43}x^{\frac{43}{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x, algorithm="fri
cas")
```

```
[Out] 8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/30) + 360/3
7*x^(37/10) - 45/43*x^(43/15)
```

Sympy [A] time = 2.03727, size = 48, normalized size = 0.87

$$\frac{60x^{\frac{113}{30}}}{113} - \frac{45x^{\frac{43}{15}}}{43} + \frac{360x^{\frac{37}{10}}}{37} - \frac{120x^{\frac{23}{5}}}{23} - \frac{x^{\frac{14}{3}}}{14} + \frac{8x^{\frac{11}{2}}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**(3/5)+x**(3/2))**2*(-1/3*x**(2/3)+4*x**(3/2)),x)
```

```
[Out] 60*x**(113/30)/113 - 45*x**(43/15)/43 + 360*x**(37/10)/37 - 120*x**(23/5)/2
3 - x**(14/3)/14 + 8*x**(11/2)/11
```

Giac [A] time = 1.07293, size = 42, normalized size = 0.76

$$\frac{8}{11}x^{\frac{11}{2}} - \frac{1}{14}x^{\frac{14}{3}} - \frac{120}{23}x^{\frac{23}{5}} + \frac{60}{113}x^{\frac{113}{30}} + \frac{360}{37}x^{\frac{37}{10}} - \frac{45}{43}x^{\frac{43}{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^(3/5)+x^(3/2))^2*(-1/3*x^(2/3)+4*x^(3/2)),x, algorithm="gia
c")
```

```
[Out] 8/11*x^(11/2) - 1/14*x^(14/3) - 120/23*x^(23/5) + 60/113*x^(113/30) + 360/3
7*x^(37/10) - 45/43*x^(43/15)
```

3.210 $\int \frac{1}{1+\sqrt{1+x}}\,dx$

Optimal. Leaf size=22

$$2\sqrt{x+1}-2\log\left(\sqrt{x+1}+1\right)$$

```
[Out] 2*Sqrt[1 + x] - 2*Log[1 + Sqrt[1 + x]]
```

Rubi [A] time = 0.0091146, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {247, 190, 43}

$$2\sqrt{x+1}-2\log\left(\sqrt{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[1 + x])^(-1),x]

[Out] 2*Sqrt[1 + x] - 2*Log[1 + Sqrt[1 + x]]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+\sqrt{1+x}}\,dx &= \text{Subst}\left(\int \frac{1}{1+\sqrt{x}}\,dx,x,1+x\right)\\
&= 2\,\text{Subst}\left(\int \frac{x}{1+x}\,dx,x,\sqrt{1+x}\right)\\
&= 2\,\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx,x,\sqrt{1+x}\right)\\
&= 2\sqrt{1+x}-2\log\left(1+\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0077022, size = 22, normalized size = 1.

$$2\sqrt{x+1}-2\log\left(\sqrt{x+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[1 + x])^(-1),x]
```

```
[Out] 2*Sqrt[1 + x] - 2*Log[1 + Sqrt[1 + x]]
```

Maple [A] time = 0.001, size = 31, normalized size = 1.4

$$2\sqrt{1+x}+\ln\left(-1+\sqrt{1+x}\right)-\ln\left(1+\sqrt{1+x}\right)-\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(1+x)^(1/2)),x)
```

```
[Out] 2*(1+x)^(1/2)+ln(-1+(1+x)^(1/2))-ln(1+(1+x)^(1/2))-ln(x)
```

Maxima [A] time = 0.93172, size = 24, normalized size = 1.09

$$2\sqrt{x+1}-2\log\left(\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

Fricas [A] time = 1.66271, size = 54, normalized size = 2.45

$$2\sqrt{x+1}-2\log\left(\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

Sympy [A] time = 0.111209, size = 19, normalized size = 0.86

$$2\sqrt{x+1}-2\log\left(\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+x)**(1/2)),x)
```

```
[Out] 2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

Giac [A] time = 1.05055, size = 24, normalized size = 1.09

$$2\sqrt{x+1} - 2\log\left(\sqrt{x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x + 1) - 2*log(sqrt(x + 1) + 1)
```

3.211 $\int \frac{x}{1+\sqrt{1+x}} dx$

Optimal. Leaf size=15

$$\frac{2}{3}(x+1)^{3/2}-x$$

```
[Out] -x + (2*(1 + x)^(3/2))/3
```

Rubi [A] time = 0.0063466, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {371}

$$\frac{2}{3}(x+1)^{3/2}-x$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + Sqrt[1 + x]),x]
```

```
[Out] -x + (2*(1 + x)^(3/2))/3
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Sim
plifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+\sqrt{1+x}} dx &= \text{Subst}\left(\int \left(-1+\sqrt{x}\right) dx, x, 1+x\right)\\ &= -x+\frac{2}{3}(1+x)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0150741, size = 19, normalized size = 1.27

$$2\left(\frac{1}{3}(x+1)^{3/2}-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + Sqrt[1 + x]),x]
```

```
[Out] 2*(-x/2 + (1 + x)^(3/2)/3)
```

Maple [A] time = 0.002, size = 13, normalized size = 0.9

$$\frac{2}{3}(1+x)^{\frac{3}{2}}-1-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+(1+x)^(1/2)),x)
```

```
[Out] 2/3*(1+x)^(3/2)-1-x
```

Maxima [A] time = 0.932082, size = 16, normalized size = 1.07

$$\frac{2}{3}(x+1)^{\frac{3}{2}} - x - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*(x + 1)^(3/2) - x - 1
```

Fricas [A] time = 1.74389, size = 31, normalized size = 2.07

$$\frac{2}{3}(x+1)^{\frac{3}{2}} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*(x + 1)^(3/2) - x
```

Sympy [B] time = 0.751358, size = 22, normalized size = 1.47

$$\frac{2x\sqrt{x+1}}{3} - x + \frac{2\sqrt{x+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+(1+x)**(1/2)),x)
```

```
[Out] 2*x*sqrt(x + 1)/3 - x + 2*sqrt(x + 1)/3
```

Giac [A] time = 1.04315, size = 16, normalized size = 1.07

$$\frac{2}{3}(x+1)^{\frac{3}{2}} - x - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2) - x - 1
```

3.212 $\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx$

Optimal. Leaf size=25

$$x+4\sqrt{x+1}+4\log\left(1-\sqrt{x+1}\right)$$

```
[Out] x + 4*Sqrt[1 + x] + 4*Log[1 - Sqrt[1 + x]]
```

Rubi [A] time = 0.0173864, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {431, 376, 77}

$$x+4\sqrt{x+1}+4\log\left(1-\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[1 + x])/(-1 + Sqrt[1 + x]),x]
```

```
[Out] x + 4*Sqrt[1 + x] + 4*Log[1 - Sqrt[1 + x]]
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+\sqrt{1+x}}{-1+\sqrt{1+x}}\,dx &= \text{Subst}\left(\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx,x,1+x\right)\\
&= 2\,\text{Subst}\left(\int \frac{x(1+x)}{-1+x}\,dx,x,\sqrt{1+x}\right)\\
&= 2\,\text{Subst}\left(\int \left(2+\frac{2}{-1+x}+x\right)dx,x,\sqrt{1+x}\right)\\
&= x+4\sqrt{1+x}+4\log\left(1-\sqrt{1+x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0104995, size = 24, normalized size = 0.96

$$x+4\left(\sqrt{x+1}+\log\left(1-\sqrt{x+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[1 + x])/(-1 + Sqrt[1 + x]),x]
```

```
[Out] x + 4*(Sqrt[1 + x] + Log[1 - Sqrt[1 + x]])
```

Maple [A] time = 0.002, size = 21, normalized size = 0.8

$$1+x+4\sqrt{1+x}+4\ln\left(-1+\sqrt{1+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x)
```

```
[Out] 1+x+4*(1+x)^(1/2)+4*ln(-1+(1+x)^(1/2))
```

Maxima [A] time = 0.941341, size = 27, normalized size = 1.08

$$x+4\sqrt{x+1}+4\log\left(\sqrt{x+1}-1\right)+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1) + 1
```

Fricas [A] time = 1.71695, size = 59, normalized size = 2.36

$$x+4\sqrt{x+1}+4\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1)
```

Sympy [A] time = 0.139086, size = 20, normalized size = 0.8

$$x+4\sqrt{x+1}+4\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+x)**(1/2))/(-1+(1+x)**(1/2)),x)
```

```
[Out] x + 4*sqrt(x + 1) + 4*log(sqrt(x + 1) - 1)
```

Giac [A] time = 1.07094, size = 28, normalized size = 1.12

$$x + 4\sqrt{x + 1} + 4\log\left(\left|\sqrt{x + 1} - 1\right|\right) + 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(1+x)^(1/2))/(-1+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] x + 4*sqrt(x + 1) + 4*log(abs(sqrt(x + 1) - 1)) + 1
```

3.213 $\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx$

Optimal. Leaf size=33

$$3\sqrt[3]{x+1}+6\sqrt[6]{x+1}+6\log\left(1-\sqrt[6]{x+1}\right)$$

```
[Out] 6*(1 + x)^(1/6) + 3*(1 + x)^(1/3) + 6*Log[1 - (1 + x)^(1/6)]
```

Rubi [A] time = 0.0190299, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {2012, 1593, 266, 43}

$$3\sqrt[3]{x+1}+6\sqrt[6]{x+1}+6\log\left(1-\sqrt[6]{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[1 + x] + (1 + x)^(2/3))^(-1),x]
```

```
[Out] 6*(1 + x)^(1/6) + 3*(1 + x)^(1/3) + 6*Log[1 - (1 + x)^(1/6)]
```

Rule 2012

```
Int[((a_.)*(u_)^(j_.) + (b_.)*(u_)^(n_.))^(p_), x_Symbol] :> Dist[1/Coeffic
ient[u, x, 1], Subst[Int[(a*x^j + b*x^n)^p, x], x, u], x] /; FreeQ[{a, b, j
, n, p}, x] && LinearQ[u, x] && NeQ[u, x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-\sqrt{1+x}+(1+x)^{2/3}}\,dx &= \text{Subst}\left(\int \frac{1}{-\sqrt{x}+x^{2/3}}\,dx, x, 1+x\right)\\ &= \text{Subst}\left(\int \frac{1}{\left(-1+\sqrt[6]{x}\right)\sqrt{x}}\,dx, x, 1+x\right)\\ &= 6\,\text{Subst}\left(\int \frac{x^2}{-1+x}\,dx, x, \sqrt[6]{1+x}\right)\\ &= 6\,\text{Subst}\left(\int \left(1+\frac{1}{-1+x}+x\right)dx, x, \sqrt[6]{1+x}\right)\\ &= 6\sqrt[6]{1+x}+3\sqrt[3]{1+x}+6\log\left(1-\sqrt[6]{1+x}\right)\end{aligned}$$

Mathematica [A] time = 0.0214325, size = 33, normalized size = 1.

$$3\left(\sqrt[3]{x+1}+2\sqrt[6]{x+1}+2\log\left(1-\sqrt[6]{x+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[1 + x] + (1 + x)^(2/3))^(-1),x]
```

```
[Out] 3*(2*(1 + x)^(1/6) + (1 + x)^(1/3) + 2*Log[1 - (1 + x)^(1/6)])
```

Maple [B] time = 0.031, size = 111, normalized size = 3.4

$$6\sqrt[6]{1+x}+3\sqrt[3]{1+x}+\ln(x)+2\ln\left(-1+\sqrt[6]{1+x}\right)-\ln\left(\sqrt[3]{1+x}+\sqrt[6]{1+x}+1\right)-2\ln\left(1+\sqrt[6]{1+x}\right)+\ln\left(\sqrt[3]{1+x}-\sqrt[6]{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1+x)^(2/3)-(1+x)^(1/2)),x)
```

```
[Out] 6*(1+x)^(1/6)+3*(1+x)^(1/3)+ln(x)+2*ln(-1+(1+x)^(1/6))-ln((1+x)^(1/3)+(1+x)
^(1/6)+1)-2*ln(1+(1+x)^(1/6))+ln((1+x)^(1/3)-(1+x)^(1/6)+1)-ln(1+(1+x)^(1/2
))+ln(-1+(1+x)^(1/2))+2*ln(-1+(1+x)^(1/3))-ln((1+x)^(2/3)+(1+x)^(1/3)+1)
```

Maxima [A] time = 0.927302, size = 34, normalized size = 1.03

$$3\,(x+1)^{\frac{1}{3}}+6\,(x+1)^{\frac{1}{6}}+6\,\log\left((x+1)^{\frac{1}{6}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*log((x + 1)^(1/6) - 1)
```

Fricas [A] time = 1.72372, size = 84, normalized size = 2.55

$$3\,(x+1)^{\frac{1}{3}}+6\,(x+1)^{\frac{1}{6}}+6\,\log\left((x+1)^{\frac{1}{6}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*log((x + 1)^(1/6) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x+1)^{\frac{2}{3}} - \sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)**(2/3)-(1+x)**(1/2)),x)
```

```
[Out] Integral(1/((x + 1)**(2/3) - sqrt(x + 1)), x)
```

Giac [A] time = 1.0838, size = 35, normalized size = 1.06

$$3\,(x+1)^{\frac{1}{3}} + 6\,(x+1)^{\frac{1}{6}} + 6\,\log\left(\left|(x+1)^{\frac{1}{6}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1+x)^(2/3)-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 3*(x + 1)^(1/3) + 6*(x + 1)^(1/6) + 6*log(abs((x + 1)^(1/6) - 1))
```

3.214 $\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx$

Optimal. Leaf size=29

$$\frac{12}{7}\left(\sqrt[4]{x}+1\right)^{7/3}-3\left(\sqrt[4]{x}+1\right)^{4/3}$$

```
[Out] -3*(1 + x^(1/4))^(4/3) + (12*(1 + x^(1/4))^(7/3))/7
```

Rubi [A] time = 0.0089935, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {266, 43}

$$\frac{12}{7}\left(\sqrt[4]{x}+1\right)^{7/3}-3\left(\sqrt[4]{x}+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(1/4))^(1/3)/Sqrt[x],x]
```

```
[Out] -3*(1 + x^(1/4))^(4/3) + (12*(1 + x^(1/4))^(7/3))/7
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{\sqrt{x}}\,dx &= 4\,\text{Subst}\left(\int x\sqrt[3]{1+x}\,dx, x, \sqrt[4]{x}\right)\\
&= 4\,\text{Subst}\left(\int \left(-\sqrt[3]{1+x}+(1+x)^{4/3}\right)dx, x, \sqrt[4]{x}\right)\\
&= -3\left(1+\sqrt[4]{x}\right)^{4/3}+\frac{12}{7}\left(1+\sqrt[4]{x}\right)^{7/3}
\end{aligned}$$

Mathematica [A] time = 0.0082711, size = 24, normalized size = 0.83

$$\frac{3}{7}\left(\sqrt[4]{x}+1\right)^{4/3}\left(4\sqrt[4]{x}-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(1/4))^(1/3)/Sqrt[x],x]
```

```
[Out] (3*(1 + x^(1/4))^(4/3)*(-3 + 4*x^(1/4)))/7
```

Maple [A] time = 0.002, size = 20, normalized size = 0.7

$$-3\left(1+\sqrt[4]{x}\right)^{4/3}+\frac{12}{7}\left(1+\sqrt[4]{x}\right)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/4))^(1/3)/x^(1/2),x)
```

```
[Out] -3*(1+x^(1/4))^(4/3)+12/7*(1+x^(1/4))^(7/3)
```

Maxima [A] time = 0.935212, size = 26, normalized size = 0.9

$$\frac{12}{7}\left(x^{\frac{1}{4}}+1\right)^{\frac{7}{3}}-3\left(x^{\frac{1}{4}}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/4))^(1/3)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 12/7*(x^(1/4) + 1)^(7/3) - 3*(x^(1/4) + 1)^(4/3)
```

Fricas [A] time = 1.78556, size = 69, normalized size = 2.38

$$\frac{3}{7}\left(4\sqrt{x}+x^{\frac{1}{4}}-3\right)\left(x^{\frac{1}{4}}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/4))^(1/3)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 3/7*(4*sqrt(x) + x^(1/4) - 3)*(x^(1/4) + 1)^(1/3)
```

Sympy [B] time = 1.32971, size = 134, normalized size = 4.62

$$\frac{12x^{\frac{7}{4}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x}-\frac{6x^{\frac{5}{4}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x}+\frac{9x^{\frac{5}{4}}}{7x^{\frac{5}{4}}+7x}+\frac{15x^{\frac{3}{2}}\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x}-\frac{9x\sqrt[3]{\sqrt[4]{x}+1}}{7x^{\frac{5}{4}}+7x}+\frac{9x}{7x^{\frac{5}{4}}+7x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/4))**(1/3)/x**(1/2),x)
```

```
[Out] 12*x**(7/4)*(x**(1/4) + 1)**(1/3)/(7*x**(5/4) + 7*x) - 6*x**(5/4)*(x**(1/4)
 + 1)**(1/3)/(7*x**(5/4) + 7*x) + 9*x**(5/4)/(7*x**(5/4) + 7*x) + 15*x**(3/
2)*(x**(1/4) + 1)**(1/3)/(7*x**(5/4) + 7*x) - 9*x*(x**(1/4) + 1)**(1/3)/(7*
x**(5/4) + 7*x) + 9*x/(7*x**(5/4) + 7*x)
```

Giac [A] time = 1.049, size = 26, normalized size = 0.9

$$\frac{12}{7}\left(x^{\frac{1}{4}}+1\right)^{\frac{7}{3}}-3\left(x^{\frac{1}{4}}+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/4))^(1/3)/x^(1/2),x, algorithm="giac")
```

```
[Out] 12/7*(x^(1/4) + 1)^(7/3) - 3*(x^(1/4) + 1)^(4/3)
```

3.215 $\int \frac{1}{x^3(1+x)^{3/2}}\,dx$

Optimal. Leaf size=52

$$-\frac{1}{2x^2\sqrt{x+1}}+\frac{5}{4x\sqrt{x+1}}+\frac{15}{4\sqrt{x+1}}-\frac{15}{4}\tanh^{-1}\left(\sqrt{x+1}\right)$$

```
[Out] 15/(4*Sqrt[1 + x]) - 1/(2*x^2*Sqrt[1 + x]) + 5/(4*x*Sqrt[1 + x]) - (15*ArcT
anh[Sqrt[1 + x]])/4
```

Rubi [A] time = 0.010101, antiderivative size = 53, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {51, 63, 207}

$$-\frac{5\sqrt{x+1}}{2x^2}+\frac{2}{x^2\sqrt{x+1}}+\frac{15\sqrt{x+1}}{4x}-\frac{15}{4}\tanh^{-1}\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x)^(3/2)),x]
```

```
[Out] 2/(x^2*Sqrt[1 + x]) - (5*Sqrt[1 + x])/(2*x^2) + (15*Sqrt[1 + x])/(4*x) - (1
5*ArcTanh[Sqrt[1 + x]])/4
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3(1+x)^{3/2}}\,dx &= \frac{2}{x^2\sqrt{1+x}} + 5\int \frac{1}{x^3\sqrt{1+x}}\,dx \\
&= \frac{2}{x^2\sqrt{1+x}} - \frac{5\sqrt{1+x}}{2x^2} - \frac{15}{4}\int \frac{1}{x^2\sqrt{1+x}}\,dx \\
&= \frac{2}{x^2\sqrt{1+x}} - \frac{5\sqrt{1+x}}{2x^2} + \frac{15\sqrt{1+x}}{4x} + \frac{15}{8}\int \frac{1}{x\sqrt{1+x}}\,dx \\
&= \frac{2}{x^2\sqrt{1+x}} - \frac{5\sqrt{1+x}}{2x^2} + \frac{15\sqrt{1+x}}{4x} + \frac{15}{4}\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x}\right) \\
&= \frac{2}{x^2\sqrt{1+x}} - \frac{5\sqrt{1+x}}{2x^2} + \frac{15\sqrt{1+x}}{4x} - \frac{15}{4}\tanh^{-1}\left(\sqrt{1+x}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0042073, size = 20, normalized size = 0.38

$$\frac{2\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};x+1\right)}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x)^(3/2)),x]
```

```
[Out] (2*Hypergeometric2F1[-1/2, 3, 1/2, 1 + x])/Sqrt[1 + x]
```

Maple [A] time = 0.012, size = 73, normalized size = 1.4

$$2\frac{1}{\sqrt{1+x}} + \frac{1}{8}\left(1+\sqrt{1+x}\right)^{-2} + \frac{7}{8}\left(1+\sqrt{1+x}\right)^{-1} - \frac{15}{8}\ln\left(1+\sqrt{1+x}\right) - \frac{1}{8}\left(-1+\sqrt{1+x}\right)^{-2} + \frac{7}{8}\left(-1+\sqrt{1+x}\right)^{-1} + $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(1+x)^(3/2),x)
```

```
[Out] 2/(1+x)^(1/2)+1/8/(1+(1+x)^(1/2))^2+7/8/(1+(1+x)^(1/2))-15/8*ln(1+(1+x)^(1/
2))-1/8/(-1+(1+x)^(1/2))^2+7/8/(-1+(1+x)^(1/2))+15/8*ln(-1+(1+x)^(1/2))
```

Maxima [A] time = 0.936766, size = 74, normalized size = 1.42

$$\frac{15\,(x+1)^2 - 25\,x - 17}{4\left((x+1)^{\frac{5}{2}} - 2\,(x+1)^{\frac{3}{2}} + \sqrt{x+1}\right)} - \frac{15}{8}\,\log\left(\sqrt{x+1}+1\right) + \frac{15}{8}\,\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(15*(x + 1)^2 - 25*x - 17)/((x + 1)^(5/2) - 2*(x + 1)^(3/2) + sqrt(x +
1)) - 15/8*log(sqrt(x + 1) + 1) + 15/8*log(sqrt(x + 1) - 1)
```

Fricas [A] time = 1.78244, size = 174, normalized size = 3.35

$$-\frac{15\left(x^3+x^2\right)\log\left(\sqrt{x+1}+1\right)-15\left(x^3+x^2\right)\log\left(\sqrt{x+1}-1\right)-2\left(15x^2+5x-2\right)\sqrt{x+1}}{8\left(x^3+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/8*(15*(x^3 + x^2)*log(sqrt(x + 1) + 1) - 15*(x^3 + x^2)*log(sqrt(x + 1)
- 1) - 2*(15*x^2 + 5*x - 2)*sqrt(x + 1))/(x^3 + x^2)
```

Sympy [B] time = 2.69473, size = 3966, normalized size = 76.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(1+x)**(3/2),x)
```

```
[Out] Piecewise((-30*(x + 1)**(17/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(
x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(
9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt
(x + 1)) - 15*I*pi*(x + 1)**(17/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2)
+ 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x +
 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 240*
(x + 1)**(15/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2)
+ 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x +
 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 120*
I*pi*(x + 1)**(15/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)*
*(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 2
24*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 840*(x + 1)**(13/2
)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)*
*(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 2
24*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 420*I*pi*(x + 1)**
(13/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*
(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5
/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 1680*(x + 1)**(11/2)*acoth(sqrt(
x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448
*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(
5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 840*I*pi*(x + 1)**(11/2)/(8*(x
+ 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/
2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x +
 1)**(3/2) + 8*sqrt(x + 1)) - 2100*(x + 1)**(9/2)*acoth(sqrt(x + 1))/(8*(x
+ 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/
2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x +
 1)**(3/2) + 8*sqrt(x + 1)) - 1050*I*pi*(x + 1)**(9/2)/(8*(x + 1)**(17/2) -
 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x +
1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8
*sqrt(x + 1)) + 1680*(x + 1)**(7/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) -
 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x +
1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8
*sqrt(x + 1)) + 840*I*pi*(x + 1)**(7/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(1
5/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448
```

```
*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) -
 840*(x + 1)**(5/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15
/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*
(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) -
420*I*pi*(x + 1)**(5/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x +
1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2)
+ 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 240*(x + 1)**(3
/2)*acoth(sqrt(x + 1))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1
)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) +
 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 120*I*pi*(x + 1)
**(3/2)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448
*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(
5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 30*sqrt(x + 1)*acoth(sqrt(x + 1
))/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x +
 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2)
- 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 15*I*pi*sqrt(x + 1)/(8*(x + 1)**(17/
2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(
x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2)
 + 8*sqrt(x + 1)) + 30*(x + 1)**8/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) +
 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x +
1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 230*(
x + 1)**7/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 4
48*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)*
*(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 766*(x + 1)**6/(8*(x + 1)**(1
7/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560
*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/
2) + 8*sqrt(x + 1)) - 1446*(x + 1)**5/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/
2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(
x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 1
690*(x + 1)**4/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2
) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x
+ 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) - 1250*(x + 1)**3/(8*(x +
1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2)
 + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1
)**(3/2) + 8*sqrt(x + 1)) + 570*(x + 1)**2/(8*(x + 1)**(17/2) - 64*(x + 1)*
*(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) -
448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)
) - 146*(x + 1)/(8*(x + 1)**(17/2) - 64*(x + 1)**(15/2) + 224*(x + 1)**(13/
2) - 448*(x + 1)**(11/2) + 560*(x + 1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x
 + 1)**(5/2) - 64*(x + 1)**(3/2) + 8*sqrt(x + 1)) + 16/(8*(x + 1)**(17/2) -
 64*(x + 1)**(15/2) + 224*(x + 1)**(13/2) - 448*(x + 1)**(11/2) + 560*(x +
1)**(9/2) - 448*(x + 1)**(7/2) + 224*(x + 1)**(5/2) - 64*(x + 1)**(3/2) + 8
*sqrt(x + 1)), Abs(x + 1) > 1), (-15*(x + 1)**(17/2)*atanh(sqrt(x + 1))/(4*
(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(
11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(
x + 1)**(3/2) + 4*sqrt(x + 1)) + 120*(x + 1)**(15/2)*atanh(sqrt(x + 1))/(4*
(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(
11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(
x + 1)**(3/2) + 4*sqrt(x + 1)) - 420*(x + 1)**(13/2)*atanh(sqrt(x + 1))/(4*
(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(
11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(
x + 1)**(3/2) + 4*sqrt(x + 1)) + 840*(x + 1)**(11/2)*atanh(sqrt(x + 1))/(4*
(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(
11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(
x + 1)**(3/2) + 4*sqrt(x + 1)) - 1050*(x + 1)**(9/2)*atanh(sqrt(x + 1))/(4*
(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(
11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(
x + 1)**(3/2) + 4*sqrt(x + 1)) + 840*(x + 1)**(7/2)*atanh(sqrt(x + 1))/(4*(
x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(1
```

```
1/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x
 + 1)**(3/2) + 4*sqrt(x + 1)) - 420*(x + 1)**(5/2)*atanh(sqrt(x + 1))/(4*(x
 + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11
/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x
+ 1)**(3/2) + 4*sqrt(x + 1)) + 120*(x + 1)**(3/2)*atanh(sqrt(x + 1))/(4*(x
+ 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/
2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x +
 1)**(3/2) + 4*sqrt(x + 1)) - 15*sqrt(x + 1)*atanh(sqrt(x + 1))/(4*(x + 1)*
*(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) +
280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**
(3/2) + 4*sqrt(x + 1)) + 15*(x + 1)**8/(4*(x + 1)**(17/2) - 32*(x + 1)**(15
/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*
(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) -
115*(x + 1)**7/(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2
) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x
+ 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) + 383*(x + 1)**6/(4*(x + 1
)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2)
+ 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)
**(3/2) + 4*sqrt(x + 1)) - 723*(x + 1)**5/(4*(x + 1)**(17/2) - 32*(x + 1)**
(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 2
24*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1))
 + 845*(x + 1)**4/(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(1
3/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*
(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) - 625*(x + 1)**3/(4*(x
+ 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/
2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x +
 1)**(3/2) + 4*sqrt(x + 1)) + 285*(x + 1)**2/(4*(x + 1)**(17/2) - 32*(x + 1
)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2)
- 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x +
1)) - 73*(x + 1)/(4*(x + 1)**(17/2) - 32*(x + 1)**(15/2) + 112*(x + 1)**(13
/2) - 224*(x + 1)**(11/2) + 280*(x + 1)**(9/2) - 224*(x + 1)**(7/2) + 112*(
x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4*sqrt(x + 1)) + 8/(4*(x + 1)**(17/2) -
 32*(x + 1)**(15/2) + 112*(x + 1)**(13/2) - 224*(x + 1)**(11/2) + 280*(x +
1)**(9/2) - 224*(x + 1)**(7/2) + 112*(x + 1)**(5/2) - 32*(x + 1)**(3/2) + 4
*sqrt(x + 1)), True))
```

Giac [A] time = 1.07459, size = 66, normalized size = 1.27

$$\frac{2}{\sqrt{x+1}} + \frac{7\,(x+1)^{\frac{3}{2}} - 9\,\sqrt{x+1}}{4\,x^2} - \frac{15}{8}\,\log\left(\sqrt{x+1}+1\right) + \frac{15}{8}\,\log\left(\left|\sqrt{x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(1+x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/sqrt(x + 1) + 1/4*(7*(x + 1)^(3/2) - 9*sqrt(x + 1))/x^2 - 15/8*log(sqrt(x
 + 1) + 1) + 15/8*log(abs(sqrt(x + 1) - 1))
```

3.216 $\int \frac{1}{(1-x)^{7/2}x^5}\,dx$

Optimal. Leaf size=118

$$-\frac{143}{96(1-x)^{5/2}x^2}-\frac{13}{24(1-x)^{5/2}x^3}-\frac{1}{4(1-x)^{5/2}x^4}+\frac{3003}{64\sqrt{1-x}}-\frac{429}{64(1-x)^{5/2}x}+\frac{1001}{64(1-x)^{3/2}}+\frac{3003}{320(1-x)^{5/2}}-\frac{3003}{64}\text{t}$$

```
[Out] 3003/(320*(1 - x)^(5/2)) + 1001/(64*(1 - x)^(3/2)) + 3003/(64*Sqrt[1 - x])
- 1/(4*(1 - x)^(5/2)*x^4) - 13/(24*(1 - x)^(5/2)*x^3) - 143/(96*(1 - x)^(5/
2)*x^2) - 429/(64*(1 - x)^(5/2)*x) - (3003*ArcTanh[Sqrt[1 - x]])/64
```

Rubi [A] time = 0.0375306, antiderivative size = 127, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 206}

$$-\frac{1001\sqrt{1-x}}{32x^2}-\frac{1001\sqrt{1-x}}{40x^3}-\frac{429\sqrt{1-x}}{20x^4}+\frac{286}{15\sqrt{1-x}x^4}+\frac{26}{15(1-x)^{3/2}x^4}+\frac{2}{5(1-x)^{5/2}x^4}-\frac{3003\sqrt{1-x}}{64x}-\frac{3003}{64}\text{tanh}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - x)^(7/2)*x^5),x]
```

```
[Out] 2/(5*(1 - x)^(5/2)*x^4) + 26/(15*(1 - x)^(3/2)*x^4) + 286/(15*Sqrt[1 - x]*x
^4) - (429*Sqrt[1 - x])/(20*x^4) - (1001*Sqrt[1 - x])/(40*x^3) - (1001*Sqrt
[1 - x])/(32*x^2) - (3003*Sqrt[1 - x])/(64*x) - (3003*ArcTanh[Sqrt[1 - x]])
/64
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(1-x)^{7/2}x^5}\,dx &= \frac{2}{5(1-x)^{5/2}x^4} + \frac{13}{5}\int \frac{1}{(1-x)^{5/2}x^5}\,dx \\
&= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{143}{15}\int \frac{1}{(1-x)^{3/2}x^5}\,dx \\
&= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}x^4} + \frac{429}{5}\int \frac{1}{\sqrt{1-x}x^5}\,dx \\
&= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}x^4} - \frac{429\sqrt{1-x}}{20x^4} + \frac{3003}{40}\int \frac{1}{\sqrt{1-x}x^4}\,dx \\
&= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}x^4} - \frac{429\sqrt{1-x}}{20x^4} - \frac{1001\sqrt{1-x}}{40x^3} + \frac{1001}{16}\int \frac{1}{\sqrt{1-x}x^3}\,dx \\
&= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}x^4} - \frac{429\sqrt{1-x}}{20x^4} - \frac{1001\sqrt{1-x}}{40x^3} - \frac{1001\sqrt{1-x}}{32x^2} + \frac{3003}{64}\int \\
&= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}x^4} - \frac{429\sqrt{1-x}}{20x^4} - \frac{1001\sqrt{1-x}}{40x^3} - \frac{1001\sqrt{1-x}}{32x^2} - \frac{3003\sqrt{1-}}{64x} \\
&= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}x^4} - \frac{429\sqrt{1-x}}{20x^4} - \frac{1001\sqrt{1-x}}{40x^3} - \frac{1001\sqrt{1-x}}{32x^2} - \frac{3003\sqrt{1-}}{64x} \\
&= \frac{2}{5(1-x)^{5/2}x^4} + \frac{26}{15(1-x)^{3/2}x^4} + \frac{286}{15\sqrt{1-x}x^4} - \frac{429\sqrt{1-x}}{20x^4} - \frac{1001\sqrt{1-x}}{40x^3} - \frac{1001\sqrt{1-x}}{32x^2} - \frac{3003\sqrt{1-}}{64x}
\end{aligned}
$$

Mathematica [C] time = 0.0060539, size = 26, normalized size = 0.22

$$\frac{2\,{}_2F_1\left(-\frac{5}{2},5;-\frac{3}{2};1-x\right)}{5(1-x)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - x)^(7/2)*x^5),x]
```

```
[Out] (2*Hypergeometric2F1[-5/2, 5, -3/2, 1 - x])/(5*(1 - x)^(5/2))
```

Maple [A] time = 0.015, size = 157, normalized size = 1.3

$$\frac{2}{5}(1-x)^{-\frac{5}{2}} + \frac{10}{3}(1-x)^{-\frac{3}{2}} + 30\frac{1}{\sqrt{1-x}} + \frac{1}{64}\left(1+\sqrt{1-x}\right)^{-4} + \frac{17}{96}\left(1+\sqrt{1-x}\right)^{-3} + \frac{159}{128}\left(1+\sqrt{1-x}\right)^{-2} + \frac{1083}{128}\left(1+\sqrt{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-x)^(7/2)/x^5,x)
```

```
[Out] 2/5/(1-x)^(5/2)+10/3/(1-x)^(3/2)+30/(1-x)^(1/2)+1/64/(1+(1-x)^(1/2))^4+17/9
6/(1+(1-x)^(1/2))^3+159/128/(1+(1-x)^(1/2))^2+1083/128/(1+(1-x)^(1/2))-3003
/128*ln(1+(1-x)^(1/2))-1/64/(-1+(1-x)^(1/2))^4+17/96/(-1+(1-x)^(1/2))^3-159
/128/(-1+(1-x)^(1/2))^2+1083/128/(-1+(1-x)^(1/2))+3003/128*ln(-1+(1-x)^(1/2
))
```

Maxima [A] time = 0.936513, size = 150, normalized size = 1.27

$$\frac{45045\,(x-1)^{6}+165165\,(x-1)^{5}+219219\,(x-1)^{4}+119691\,(x-1)^{3}+18304\,(x-1)^{2}-1664\,x+2048}{960\left((-x+1)^{\frac{13}{2}}-4\,(-x+1)^{\frac{11}{2}}+6\,(-x+1)^{\frac{9}{2}}-4\,(-x+1)^{\frac{7}{2}}+(-x+1)^{\frac{5}{2}}\right)}-\frac{3003}{128}\log\left(\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/960*(45045*(x - 1)^6 + 165165*(x - 1)^5 + 219219*(x - 1)^4 + 119691*(x -
1)^3 + 18304*(x - 1)^2 - 1664*x + 2048)/((-x + 1)^(13/2) - 4*(-x + 1)^(11/2
) + 6*(-x + 1)^(9/2) - 4*(-x + 1)^(7/2) + (-x + 1)^(5/2)) - 3003/128*log(sq
rt(-x + 1) + 1) + 3003/128*log(sqrt(-x + 1) - 1)
```

Fricas [A] time = 1.80789, size = 328, normalized size = 2.78

$$-\frac{45045\left(x^{7}-3\,x^{6}+3\,x^{5}-x^{4}\right)\log\left(\sqrt{-x+1}+1\right)-45045\left(x^{7}-3\,x^{6}+3\,x^{5}-x^{4}\right)\log\left(\sqrt{-x+1}-1\right)+2\left(45045\,x^{6}-1\right.}{1920\left(x^{7}-3\,x^{6}+3\,x^{5}-x^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/x^5,x, algorithm="fricas")
```

```
[Out] -1/1920*(45045*(x^7 - 3*x^6 + 3*x^5 - x^4)*log(sqrt(-x + 1) + 1) - 45045*(x
^7 - 3*x^6 + 3*x^5 - x^4)*log(sqrt(-x + 1) - 1) + 2*(45045*x^6 - 105105*x^5
 + 69069*x^4 - 6435*x^3 - 1430*x^2 - 520*x - 240)*sqrt(-x + 1))/(x^7 - 3*x^
6 + 3*x^5 - x^4)
```

Sympy [C] time = 15.7468, size = 971, normalized size = 8.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)**(7/2)/x**5,x)
```

```
[Out] Piecewise((45045*I*x**7*asin(1/sqrt(x))/(960*x**7 - 2880*x**6 + 2880*x**5 -
 960*x**4) - 45045*I*x**6*sqrt(x - 1)/(960*x**7 - 2880*x**6 + 2880*x**5 - 9
60*x**4) - 135135*I*x**6*asin(1/sqrt(x))/(960*x**7 - 2880*x**6 + 2880*x**5
- 960*x**4) + 105105*I*x**5*sqrt(x - 1)/(960*x**7 - 2880*x**6 + 2880*x**5 -
 960*x**4) + 135135*I*x**5*asin(1/sqrt(x))/(960*x**7 - 2880*x**6 + 2880*x**
5 - 960*x**4) - 69069*I*x**4*sqrt(x - 1)/(960*x**7 - 2880*x**6 + 2880*x**5
- 960*x**4) - 45045*I*x**4*asin(1/sqrt(x))/(960*x**7 - 2880*x**6 + 2880*x**
5 - 960*x**4) + 6435*I*x**3*sqrt(x - 1)/(960*x**7 - 2880*x**6 + 2880*x**5 -
 960*x**4) + 1430*I*x**2*sqrt(x - 1)/(960*x**7 - 2880*x**6 + 2880*x**5 - 96
0*x**4) + 520*I*x*sqrt(x - 1)/(960*x**7 - 2880*x**6 + 2880*x**5 - 960*x**4)
 + 240*I*sqrt(x - 1)/(960*x**7 - 2880*x**6 + 2880*x**5 - 960*x**4), Abs(x)
> 1), (45045*x**7*log(x)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) -
90090*x**7*log(sqrt(1 - x) + 1)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x
**4) + 45045*I*pi*x**7/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) - 90
090*x**6*sqrt(1 - x)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) - 1351
35*x**6*log(x)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) + 270270*x**
```

```
6*log(sqrt(1 - x) + 1)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) - 13
5135*I*pi*x**6/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) + 210210*x**
5*sqrt(1 - x)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) + 135135*x**5
*log(x)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) - 270270*x**5*log(s
qrt(1 - x) + 1)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) + 135135*I*
pi*x**5/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) - 138138*x**4*sqrt(
1 - x)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) - 45045*x**4*log(x)/
(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) + 90090*x**4*log(sqrt(1 - x
) + 1)/(1920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) - 45045*I*pi*x**4/(1
920*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) + 12870*x**3*sqrt(1 - x)/(192
0*x**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) + 2860*x**2*sqrt(1 - x)/(1920*x
**7 - 5760*x**6 + 5760*x**5 - 1920*x**4) + 1040*x*sqrt(1 - x)/(1920*x**7 -
5760*x**6 + 5760*x**5 - 1920*x**4) + 480*sqrt(1 - x)/(1920*x**7 - 5760*x**6
 + 5760*x**5 - 1920*x**4), True))
```

Giac [A] time = 1.06646, size = 140, normalized size = 1.19

$$\frac{2\left(225\,(x-1)^2-25\,x+28\right)}{15\,(x-1)^2\sqrt{-x+1}}-\frac{3249\,(x-1)^3\sqrt{-x+1}+10633\,(x-1)^2\sqrt{-x+1}-11767\,(-x+1)^{\frac{3}{2}}+4431\,\sqrt{-x+1}}{192\,x^4}-\frac{3003}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-x)^(7/2)/x^5,x, algorithm="giac")
```

```
[Out] 2/15*(225*(x - 1)^2 - 25*x + 28)/((x - 1)^2*sqrt(-x + 1)) - 1/192*(3249*(x
- 1)^3*sqrt(-x + 1) + 10633*(x - 1)^2*sqrt(-x + 1) - 11767*(-x + 1)^(3/2) +
 4431*sqrt(-x + 1))/x^4 - 3003/128*log(sqrt(-x + 1) + 1) + 3003/128*log(abs
(sqrt(-x + 1) - 1))
```

3.217 $\int \frac{1}{(-1+x)^{2/3}x^5}\,dx$

Optimal. Leaf size=104

$$\frac{11\sqrt[3]{x-1}}{27x^2}+\frac{11\sqrt[3]{x-1}}{36x^3}+\frac{\sqrt[3]{x-1}}{4x^4}+\frac{55\sqrt[3]{x-1}}{81x}+\frac{55}{81}\log\left(\sqrt[3]{x-1}+1\right)-\frac{55\log(x)}{243}-\frac{110\tan^{-1}\left(\frac{1-2\sqrt[3]{x-1}}{\sqrt{3}}\right)}{81\sqrt{3}}$$

```
[Out] (-1 + x)^(1/3)/(4*x^4) + (11*(-1 + x)^(1/3))/(36*x^3) + (11*(-1 + x)^(1/3))
/(27*x^2) + (55*(-1 + x)^(1/3))/(81*x) - (110*ArcTan[(1 - 2*(-1 + x)^(1/3))
/Sqrt[3]])/(81*Sqrt[3]) + (55*Log[1 + (-1 + x)^(1/3)])/81 - (55*Log[x])/243
```

Rubi [A] time = 0.0400664, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {51, 58, 618, 204, 31}

$$\frac{11\sqrt[3]{x-1}}{27x^2}+\frac{11\sqrt[3]{x-1}}{36x^3}+\frac{\sqrt[3]{x-1}}{4x^4}+\frac{55\sqrt[3]{x-1}}{81x}+\frac{55}{81}\log\left(\sqrt[3]{x-1}+1\right)-\frac{55\log(x)}{243}-\frac{110\tan^{-1}\left(\frac{1-2\sqrt[3]{x-1}}{\sqrt{3}}\right)}{81\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)^(2/3)*x^5),x]
```

```
[Out] (-1 + x)^(1/3)/(4*x^4) + (11*(-1 + x)^(1/3))/(36*x^3) + (11*(-1 + x)^(1/3))
/(27*x^2) + (55*(-1 + x)^(1/3))/(81*x) - (110*ArcTan[(1 - 2*(-1 + x)^(1/3))
/Sqrt[3]])/(81*Sqrt[3]) + (55*Log[1 + (-1 + x)^(1/3)])/81 - (55*Log[x])/243
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-1+x)^{2/3}x^5}\,dx &= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11}{12}\int \frac{1}{(-1+x)^{2/3}x^4}\,dx\\
&= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11\sqrt[3]{-1+x}}{36x^3}+\frac{22}{27}\int \frac{1}{(-1+x)^{2/3}x^3}\,dx\\
&= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11\sqrt[3]{-1+x}}{36x^3}+\frac{11\sqrt[3]{-1+x}}{27x^2}+\frac{55}{81}\int \frac{1}{(-1+x)^{2/3}x^2}\,dx\\
&= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11\sqrt[3]{-1+x}}{36x^3}+\frac{11\sqrt[3]{-1+x}}{27x^2}+\frac{55\sqrt[3]{-1+x}}{81x}+\frac{110}{243}\int \frac{1}{(-1+x)^{2/3}x}\,dx\\
&= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11\sqrt[3]{-1+x}}{36x^3}+\frac{11\sqrt[3]{-1+x}}{27x^2}+\frac{55\sqrt[3]{-1+x}}{81x}-\frac{55\log(x)}{243}+\frac{55}{81}\mathrm{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt[3]{-1
\end{aligned}$$

$$= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11\sqrt[3]{-1+x}}{36x^3}+\frac{11\sqrt[3]{-1+x}}{27x^2}+\frac{55\sqrt[3]{-1+x}}{81x}+\frac{55}{81}\log\left(1+\sqrt[3]{-1+x}\right)-\frac{55\log(x)}{243}-\frac{110}{81}\mathrm{Su}$$

$$= \frac{\sqrt[3]{-1+x}}{4x^4}+\frac{11\sqrt[3]{-1+x}}{36x^3}+\frac{11\sqrt[3]{-1+x}}{27x^2}+\frac{55\sqrt[3]{-1+x}}{81x}-\frac{110\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+x}}{\sqrt{3}}\right)}{81\sqrt{3}}+\frac{55}{81}\log\left(1+\sqrt[3]{-1+\right.$$

Mathematica [C] time = 0.0048085, size = 22, normalized size = 0.21

$$3\sqrt[3]{x-1}\,{}_2F_1\left(\frac{1}{3},5;\frac{4}{3};1-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)^(2/3)*x^5),x]
```

```
[Out] 3*(-1 + x)^(1/3)*Hypergeometric2F1[1/3, 5, 4/3, 1 - x]
```

Maple [B] time = 0.015, size = 158, normalized size = 1.5

$$-\frac{1}{324}\left(1+\sqrt[3]{-1+x}\right)^{-4}-\frac{5}{243}\left(1+\sqrt[3]{-1+x}\right)^{-3}-\frac{20}{243}\left(1+\sqrt[3]{-1+x}\right)^{-2}-\frac{25}{81}\left(1+\sqrt[3]{-1+x}\right)^{-1}+\frac{110}{243}\ln\left(1+\sqrt[3]{-1+x}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)^(2/3)/x^5,x)
```

```
[Out] -1/324/(1+(-1+x)^(1/3))^4-5/243/(1+(-1+x)^(1/3))^3-20/243/(1+(-1+x)^(1/3))^
2-25/81/(1+(-1+x)^(1/3))+110/243*ln(1+(-1+x)^(1/3))-1/243*(-75*(-1+x)^(7/3)
+190*(-1+x)^2-350*(-1+x)^(5/3)+1157/4*(-1+x)^(4/3)+149/4-138*x-116*(-1+x)^(
2/3)+137*(-1+x)^(1/3))/((-1+x)^(2/3)-(-1+x)^(1/3)+1)^4-55/243*ln((-1+x)^(2/
```

```
3)-(-1+x)^(1/3)+1)+110/243*3^(1/2)*arctan(1/3*(2*(-1+x)^(1/3)-1)*3^(1/2))
```

Maxima [A] time = 1.41639, size = 142, normalized size = 1.37

$$\frac{110}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(x-1)^{\frac{1}{3}}-1\right)\right)+\frac{220(x-1)^{\frac{10}{3}}+792(x-1)^{\frac{7}{3}}+1023(x-1)^{\frac{4}{3}}+532(x-1)^{\frac{1}{3}}}{324\left((x-1)^{4}+4(x-1)^{3}+6(x-1)^{2}+4x-3\right)}-\frac{55}{243}\log\left((x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(2/3)/x^5,x, algorithm="maxima")
```

```
[Out] 110/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x - 1)^(1/3) - 1)) + 1/324*(220*(x -
 1)^(10/3) + 792*(x - 1)^(7/3) + 1023*(x - 1)^(4/3) + 532*(x - 1)^(1/3))/((
x - 1)^4 + 4*(x - 1)^3 + 6*(x - 1)^2 + 4*x - 3) - 55/243*log((x - 1)^(2/3)
- (x - 1)^(1/3) + 1) + 110/243*log((x - 1)^(1/3) + 1)
```

Fricas [A] time = 1.69599, size = 282, normalized size = 2.71

$$\frac{440\sqrt{3}x^{4}\arctan\left(\frac{2}{3}\sqrt{3}(x-1)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-220x^{4}\log\left((x-1)^{\frac{2}{3}}-(x-1)^{\frac{1}{3}}+1\right)+440x^{4}\log\left((x-1)^{\frac{1}{3}}+1\right)+3\left(220x^{3}\right.}{972x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(2/3)/x^5,x, algorithm="fricas")
```

```
[Out] 1/972*(440*sqrt(3)*x^4*arctan(2/3*sqrt(3)*(x - 1)^(1/3) - 1/3*sqrt(3)) - 22
0*x^4*log((x - 1)^(2/3) - (x - 1)^(1/3) + 1) + 440*x^4*log((x - 1)^(1/3) +
1) + 3*(220*x^3 + 132*x^2 + 99*x + 81)*(x - 1)^(1/3))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**(2/3)/x**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07167, size = 111, normalized size = 1.07

$$\frac{110}{243}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(x-1)^{\frac{1}{3}}-1\right)\right)+\frac{220(x-1)^{\frac{10}{3}}+792(x-1)^{\frac{7}{3}}+1023(x-1)^{\frac{4}{3}}+532(x-1)^{\frac{1}{3}}}{324x^{4}}-\frac{55}{243}\log\left((x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^(2/3)/x^5,x, algorithm="giac")
```

```
[Out] 110/243*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x - 1)^(1/3) - 1)) + 1/324*(220*(x -
 1)^(10/3) + 792*(x - 1)^(7/3) + 1023*(x - 1)^(4/3) + 532*(x - 1)^(1/3))/x^
4 - 55/243*log((x - 1)^(2/3) - (x - 1)^(1/3) + 1) + 110/243*log((x - 1)^(1/
3) + 1)
```

3.218 $\int \sqrt{\frac{1-x}{1+x}}\,dx$

Optimal. Leaf size=38

$$\sqrt{\frac{1-x}{x+1}}(x+1)-2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

```
[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rubi [A] time = 0.0131231, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1959, 288, 204}

$$\sqrt{\frac{1-x}{x+1}}(x+1)-2\tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x)/(1 + x)],x]

[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{1-x}{1+x}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{x^2}{\left(-1-x^2\right)^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right)\\
&= \sqrt{\frac{1-x}{1+x}}(1+x)+2\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, \sqrt{\frac{1-x}{1+x}}\right)\\
&= \sqrt{\frac{1-x}{1+x}}(1+x)-2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0226805, size = 67, normalized size = 1.76

$$\frac{\sqrt{\frac{1-x}{x+1}}\sqrt{x+1}\left(\sqrt{x+1}(x-1)+2\sqrt{1-x}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)}{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x)/(1 + x)],x]
```

```
[Out] (Sqrt[(1 - x)/(1 + x)]*Sqrt[1 + x]*((-1 + x)*Sqrt[1 + x] + 2*Sqrt[1 - x]*Ar
cSin[Sqrt[1 - x]/Sqrt[2]]))/(-1 + x)
```

Maple [A] time = 0.006, size = 39, normalized size = 1.

$$(1+x)\sqrt{-\frac{-1+x}{1+x}}\left(\sqrt{-x^2+1}+\arcsin(x)\right)\frac{1}{\sqrt{-(1+x)(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)/(1+x))^(1/2),x)
```

```
[Out] (-(-1+x)/(1+x))^(1/2)*(1+x)/(-(1+x)*(-1+x))^(1/2)*((-x^2+1)^(1/2)+arcsin(x)
)
```

Maxima [A] time = 1.40565, size = 58, normalized size = 1.53

$$-\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}-2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*arctan(sqrt(-(x - 1)/(x
 + 1)))
```

Fricas [A] time = 1.78142, size = 90, normalized size = 2.37

$$(x+1)\sqrt{-\frac{x-1}{x+1}}-2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="fricas")
```

```
[Out] (x + 1)*sqrt(-(x - 1)/(x + 1)) - 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{1-x}{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))**(1/2),x)
```

```
[Out] Integral(sqrt((1 - x)/(x + 1)), x)
```

Giac [A] time = 1.05383, size = 39, normalized size = 1.03

$$\frac{1}{2}\,\pi \mathrm{sgn}\left(x+1\right)+\arcsin\left(x\right)\mathrm{sgn}\left(x+1\right)+\sqrt{-x^2+1}\mathrm{sgn}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*pi*sgn(x + 1) + arcsin(x)*sgn(x + 1) + sqrt(-x^2 + 1)*sgn(x + 1)
```

3.219 $\int x\sqrt{\frac{-a+x}{b-x}}\,dx$

Optimal. Leaf size=92

$$\frac{1}{2}(b-x)^2\sqrt{\frac{x-a}{b-x}}+\frac{1}{4}(a-5b)(b-x)\sqrt{\frac{x-a}{b-x}}-\frac{1}{4}(a-b)(a+3b)\tan^{-1}\left(\sqrt{\frac{x-a}{b-x}}\right)$$

```
[Out] ((a - 5*b)*(b - x)*Sqrt[(-a + x)/(b - x)])/4 + ((b - x)^2*Sqrt[(-a + x)/(b
- x)])/2 - ((a - b)*(a + 3*b)*ArcTan[Sqrt[(-a + x)/(b - x)]])/4
```

Rubi [A] time = 0.0589155, antiderivative size = 95, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {1960, 455, 385, 203}

$$\frac{1}{2}(b-x)^2\sqrt{-\frac{a-x}{b-x}}+\frac{1}{4}(a-5b)(b-x)\sqrt{-\frac{a-x}{b-x}}-\frac{1}{4}(a-b)(a+3b)\tan^{-1}\left(\sqrt{-\frac{a-x}{b-x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[(-a + x)/(b - x)],x]
```

```
[Out] ((a - 5*b)*Sqrt[-((a - x)/(b - x))]*(b - x))/4 + (Sqrt[-((a - x)/(b - x))]*
(b - x)^2)/2 - ((a - b)*(a + 3*b)*ArcTan[Sqrt[-((a - x)/(b - x))]])/4
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int x\sqrt{\frac{-a+x}{b-x}}\,dx=-\left((2(a-b))\,\mathrm{Subst}\left(\int\frac{x^2\left(a+bx^2\right)}{\left(1+x^2\right)^3}\,dx,x,\sqrt{\frac{-a+x}{b-x}}\right)\right)$$
$$=\frac{1}{2}\sqrt{-\frac{a-x}{b-x}}(b-x)^2-\frac{1}{2}(-a+b)\,\mathrm{Subst}\left(\int\frac{-a+b-4bx^2}{\left(1+x^2\right)^2}\,dx,x,\sqrt{\frac{-a+x}{b-x}}\right)$$
$$=\frac{1}{4}(a-5b)\sqrt{-\frac{a-x}{b-x}}(b-x)+\frac{1}{2}\sqrt{-\frac{a-x}{b-x}}(b-x)^2-\frac{1}{4}((a-b)(a+3b))\,\mathrm{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{\frac{-a}{b}}\right.$$
$$=\frac{1}{4}(a-5b)\sqrt{-\frac{a-x}{b-x}}(b-x)+\frac{1}{2}\sqrt{-\frac{a-x}{b-x}}(b-x)^2-\frac{1}{4}(a-b)(a+3b)\tan^{-1}\left(\sqrt{-\frac{a-x}{b-x}}\right)$$

Mathematica [A] time = 0.2566, size = 115, normalized size = 1.25

$$\frac{\sqrt{\frac{x-a}{b-x}}\left((b-x)(a-3b-2x)\sqrt{\frac{a-x}{a-b}}-\sqrt{a-b}(a+3b)\sqrt{b-x}\sinh^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{a-b}}\right)\right)}{4\sqrt{\frac{a-x}{a-b}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[(-a + x)/(b - x)],x]
```

```
[Out] (Sqrt[(-a + x)/(b - x)]*((a - 3*b - 2*x)*Sqrt[(a - x)/(a - b)]*(b - x) - Sq
rt[a - b]*(a + 3*b)*Sqrt[b - x]*ArcSinh[Sqrt[b - x]/Sqrt[a - b]]))/(4*Sqrt[
(a - x)/(a - b)])
```

Maple [B] time = 0.019, size = 208, normalized size = 2.3

$$\frac{-b+x}{8}\sqrt{-\frac{-a+x}{-b+x}}\left(\arctan\left(\frac{-a+2\,x-b}{2}\frac{1}{\sqrt{-ab+ax+bx-x^2}}\right)a^2+2\,b\arctan\left(1/2\,\frac{-a+2\,x-b}{\sqrt{-ab+ax+bx-x^2}}\right)a-3\,\mathrm{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((-a+x)/(b-x))^(1/2),x)
```

```
[Out] 1/8*(-(-a+x)/(-b+x))^(1/2)*(-b+x)*(arctan(1/2*(-a+2*x-b)/(-a*b+a*x+b*x-x^2)
^(1/2))*a^2+2*b*arctan(1/2*(-a+2*x-b)/(-a*b+a*x+b*x-x^2)^(1/2))*a-3*arctan(
1/2*(-a+2*x-b)/(-a*b+a*x+b*x-x^2)^(1/2))*b^2+4*(-a*b+a*x+b*x-x^2)^(1/2)*x-2
*(-a*b+a*x+b*x-x^2)^(1/2)*a+6*(-a*b+a*x+b*x-x^2)^(1/2)*b)/(-(-b+x)*(-a+x))^
(1/2)
```

Maxima [A] time = 1.42183, size = 176, normalized size = 1.91

$$-\frac{1}{4}\left(a^2+2\,ab-3\,b^2\right)\arctan\left(\sqrt{-\frac{a-x}{b-x}}\right)-\frac{\left(a^2-6\,ab+5\,b^2\right)\left(-\frac{a-x}{b-x}\right)^{\frac{3}{2}}-\left(a^2+2\,ab-3\,b^2\right)\sqrt{-\frac{a-x}{b-x}}}{4\left(\frac{(a-x)^2}{(b-x)^2}-\frac{2\,(a-x)}{b-x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-a+x)/(b-x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(a^2 + 2*a*b - 3*b^2)*arctan(sqrt(-(a - x)/(b - x))) - 1/4*((a^2 - 6*a
*b + 5*b^2)*(-(a - x)/(b - x))^(3/2) - (a^2 + 2*a*b - 3*b^2)*sqrt(-(a - x)/
(b - x)))/((a - x)^2/(b - x)^2 - 2*(a - x)/(b - x) + 1)
```

Fricas [A] time = 1.79824, size = 165, normalized size = 1.79

$$-\frac{1}{4}\left(a^2+2\,ab-3\,b^2\right)\arctan\left(\sqrt{-\frac{a-x}{b-x}}\right)+\frac{1}{4}\left(ab-3\,b^2-(a-b)x+2\,x^2\right)\sqrt{-\frac{a-x}{b-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-a+x)/(b-x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(a^2 + 2*a*b - 3*b^2)*arctan(sqrt(-(a - x)/(b - x))) + 1/4*(a*b - 3*b^
2 - (a - b)*x + 2*x^2)*sqrt(-(a - x)/(b - x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-a+x)/(b-x))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09141, size = 139, normalized size = 1.51

$$\frac{1}{8}\left(a^2\mathrm{sgn}\left(-b+x\right)+2\,ab\mathrm{sgn}\left(-b+x\right)-3\,b^2\mathrm{sgn}\left(-b+x\right)\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\mathrm{sgn}\left(-a+b\right)-\frac{1}{4}\,\sqrt{-ab+ax+bx-x^2}(a\mathrm{sg}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((-a+x)/(b-x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(a^2*sgn(-b + x) + 2*a*b*sgn(-b + x) - 3*b^2*sgn(-b + x))*arcsin((a + b
 - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqrt(-a*b + a*x + b*x - x^2)*(a*sgn(-b +
 x) - 3*b*sgn(-b + x) - 2*x*sgn(-b + x))
```

3.220 $\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\left(-25+x^2\right)}\,dx$

Optimal. Leaf size=54

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{4}\sqrt{x-5}\sqrt{x+3}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{5}\sqrt{x+3}}{\sqrt{x-5}}\right)}{3\sqrt{5}}$$

```
[Out] ArcTan[(Sqrt[-5 + x]*Sqrt[3 + x])/4]/6 + ArcTanh[(Sqrt[5]*Sqrt[3 + x])/Sqrt
[-5 + x]]/(3*Sqrt[5])
```

Rubi [A] time = 0.0959866, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1586, 178, 92, 203, 93, 206}

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{4}\sqrt{x-5}\sqrt{x+3}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{5}\sqrt{x+3}}{\sqrt{x-5}}\right)}{3\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-5 + x]*Sqrt[3 + x])/((-1 + x)*(-25 + x^2)),x]
```

```
[Out] ArcTan[(Sqrt[-5 + x]*Sqrt[3 + x])/4]/6 + ArcTanh[(Sqrt[5]*Sqrt[3 + x])/Sqrt
[-5 + x]]/(3*Sqrt[5])
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 178

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))^(q_))/(((a_.) + (b_.)*(
x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[
((e + f*x)^(p - 1)*(g + h*x)^q)/(a + b*x), x], x] - Dist[(d*e - c*f)/(b*c -
 a*d), Int[((e + f*x)^(p - 1)*(g + h*x)^q)/(c + d*x), x], x] /; FreeQ[{a, b
, c, d, e, f, g, h, q}, x] && LtQ[0, p, 1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-5+x}\sqrt{3+x}}{(-1+x)\left(-25+x^2\right)}\,dx &= \int \frac{\sqrt{3+x}}{\sqrt{-5+x}(-1+x)(5+x)}\,dx\\ &= \frac{1}{3}\int \frac{1}{\sqrt{-5+x}\sqrt{3+x}(5+x)}\,dx+\frac{2}{3}\int \frac{1}{\sqrt{-5+x}(-1+x)\sqrt{3+x}}\,dx\\ &= \frac{2}{3}\,\text{Subst}\left(\int \frac{1}{2-10x^2}\,dx,x,\frac{\sqrt{3+x}}{\sqrt{-5+x}}\right)+\frac{2}{3}\,\text{Subst}\left(\int \frac{1}{16+x^2}\,dx,x,\sqrt{-5+x}\sqrt{3+x}\right)\\ &= \frac{1}{6}\tan^{-1}\left(\frac{1}{4}\sqrt{-5+x}\sqrt{3+x}\right)+\frac{\tanh^{-1}\left(\frac{\sqrt{5}\sqrt{3+x}}{\sqrt{-5+x}}\right)}{3\sqrt{5}}\end{aligned}$$

Mathematica [A] time = 0.102016, size = 47, normalized size = 0.87

$$\frac{1}{15}\left(5\tan^{-1}\left(\sqrt{\frac{x-5}{x+3}}\right)+\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{\frac{x-5}{x+3}}}{\sqrt{5}}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[-5 + x]*Sqrt[3 + x])/((-1 + x)*(-25 + x^2)),x]
```

```
[Out] (5*ArcTan[Sqrt[(-5 + x)/(3 + x)]] + Sqrt[5]*ArcTanh[Sqrt[(-5 + x)/(3 + x)]/
Sqrt[5]])/15
```

Maple [A] time = 0.023, size = 64, normalized size = 1.2

$$\frac{1}{30}\sqrt{-5+x}\sqrt{3+x}\left(\sqrt{5}\text{Artanh}\left(\frac{(5+3x)\sqrt{5}}{5}\frac{1}{\sqrt{x^2-2x-15}}\right)-5\arctan\left(4\frac{1}{\sqrt{x^2-2x-15}}\right)\right)\frac{1}{\sqrt{x^2-2x-15}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x)
```

```
[Out] 1/30*(-5+x)^(1/2)*(3+x)^(1/2)*(5^(1/2)*arctanh(1/5*(5+3*x)*5^(1/2)/(x^2-2*x
-15)^(1/2))-5*arctan(4/(x^2-2*x-15)^(1/2)))/(x^2-2*x-15)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+3}\sqrt{x-5}}{(x^2-25)(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + 3)*sqrt(x - 5)/((x^2 - 25)*(x - 1)), x)
```

Fricas [A] time = 1.82628, size = 211, normalized size = 3.91

$$\frac{1}{30}\sqrt{5}\log\left(\frac{\sqrt{x+3}\sqrt{x-5}\left(3\sqrt{5}+5\right)+\sqrt{5}(3x+5)+9x+15}{x+5}\right)+\frac{1}{3}\arctan\left(\frac{1}{4}\sqrt{x+3}\sqrt{x-5}-\frac{1}{4}x+\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x, algorithm="fricas")
```

```
[Out] 1/30*sqrt(5)*log((sqrt(x + 3)*sqrt(x - 5)*(3*sqrt(5) + 5) + sqrt(5)*(3*x +
5) + 9*x + 15)/(x + 5)) + 1/3*arctan(1/4*sqrt(x + 3)*sqrt(x - 5) - 1/4*x +
1/4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+3}}{\sqrt{x-5}\,(x-1)\,(x+5)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+x)**(1/2)*(3+x)**(1/2)/(-1+x)/(x**2-25),x)
```

```
[Out] Integral(sqrt(x + 3)/(sqrt(x - 5)*(x - 1)*(x + 5)), x)
```

Giac [B] time = 1.08187, size = 100, normalized size = 1.85

$$-\frac{1}{30}\sqrt{5}\log\left(\frac{\left(\sqrt{x+3}-\sqrt{x-5}\right)^2-4\sqrt{5}+12}{\left(\sqrt{x+3}-\sqrt{x-5}\right)^2+4\sqrt{5}+12}\right)-\frac{1}{3}\arctan\left(\frac{1}{8}\left(\sqrt{x+3}-\sqrt{x-5}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+x)^(1/2)*(3+x)^(1/2)/(-1+x)/(x^2-25),x, algorithm="giac")
```

```
[Out] -1/30*sqrt(5)*log(((sqrt(x + 3) - sqrt(x - 5))^2 - 4*sqrt(5) + 12)/((sqrt(x
 + 3) - sqrt(x - 5))^2 + 4*sqrt(5) + 12)) - 1/3*arctan(1/8*(sqrt(x + 3) - s
qrt(x - 5))^2)
```

3.221 $\int \frac{x^2\sqrt{1+x}\,\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx$

Optimal. Leaf size=304

$$\frac{1}{6}\sqrt{x+1}\left(1-x^2\right)^{5/4}+\frac{x\left(1-x^2\right)^{5/4}}{6\sqrt{1-x}}+\frac{7\left(1-x^2\right)^{5/4}}{24\sqrt{1-x}}+\frac{1}{24}(x+1)^{3/4}(1-x)^{5/4}+\frac{5}{16}\sqrt[4]{x+1}(1-x)^{3/4}-\frac{1}{16}(x+1)^{3/4}\sqrt[4]{1-x}+$$

```
[Out] (5*(1 - x)^(3/4)*(1 + x)^(1/4))/16 - ((1 - x)^(1/4)*(1 + x)^(3/4))/16 + ((1
 - x)^(5/4)*(1 + x)^(3/4))/24 + (7*(1 - x^2)^(5/4))/(24*Sqrt[1 - x]) + (x*(
1 - x^2)^(5/4))/(6*Sqrt[1 - x]) + (Sqrt[1 + x]*(1 - x^2)^(5/4))/6 - (3*ArcT
an[1 - (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)])/(8*Sqrt[2]) + (3*ArcTan[1 +
(Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)])/(8*Sqrt[2]) + Log[1 + Sqrt[1 - x]/S
qrt[1 + x] - (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)]/(8*Sqrt[2]) - Log[1 + S
qrt[1 - x]/Sqrt[1 + x] + (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)]/(8*Sqrt[2])
```

Rubi [A] time = 0.822812, antiderivative size = 319, normalized size of antiderivative = 1.05, number of steps used = 33, number of rules used = 16, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2103, 795, 675, 50, 63, 240, 211, 1165, 628, 1162, 617, 204, 1633, 793, 331, 297}

$$\frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}}+\frac{1}{6}\sqrt{x+1}\left(1-x^2\right)^{5/4}+\frac{1}{6}(1-x)^{7/4}(x+1)^{5/4}+\frac{1}{24}(1-x)^{5/4}(x+1)^{3/4}-\frac{1}{16}\sqrt[4]{1-x}(x+1)^{3/4}+\frac{5}{24}(1-x)^{7/4}\sqrt[4]{x}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 + x]*(1 - x^2)^(1/4))/(Sqrt[1 - x]*(Sqrt[1 - x] - Sqrt[1 +
x])),x]
```

```
[Out] (-5*(1 - x)^(3/4)*(1 + x)^(1/4))/48 + (5*(1 - x)^(7/4)*(1 + x)^(1/4))/24 -
((1 - x)^(1/4)*(1 + x)^(3/4))/16 + ((1 - x)^(5/4)*(1 + x)^(3/4))/24 + ((1 -
 x)^(7/4)*(1 + x)^(5/4))/6 + (Sqrt[1 + x]*(1 - x^2)^(5/4))/6 + (1 - x^2)^(9
/4)/(3*(1 - x)^(3/2)) - (3*ArcTan[1 - (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)
])/(8*Sqrt[2]) + (3*ArcTan[1 + (Sqrt[2]*(1 - x)^(1/4))/(1 + x)^(1/4)])/(8*S
qrt[2]) + Log[1 + Sqrt[1 - x]/Sqrt[1 + x] - (Sqrt[2]*(1 - x)^(1/4))/(1 + x)
^(1/4)]/(8*Sqrt[2]) - Log[1 + Sqrt[1 - x]/Sqrt[1 + x] + (Sqrt[2]*(1 - x)^(1
/4))/(1 + x)^(1/4)]/(8*Sqrt[2])
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
 x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 795

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[(g*(d + e*x)^m*(a + c*x^2)^(p + 1))/(c*(m + 2*p + 2)),
 x] + Dist[(m*(d*g + e*f) + 2*e*f*(p + 1))/(e*(m + 2*p + 2)), Int[(d + e*x)
^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && EqQ[c*d^2
 + a*e^2, 0] && NeQ[m + 2*p + 2, 0] && NeQ[m, 2]
```

Rule 675

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && E
qQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m
, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1633

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Dist[d*e, Int[(d + e*x)^(m - 1)*PolynomialQuotient[Pq, a*e + c*d*x, x]*(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x]
&& EqQ[c*d^2 + a*e^2, 0] && EqQ[PolynomialRemainder[Pq, a*e + c*d*x, x], 0]
```

Rule 793

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
), x_Symbol] :> Simp[((d*g - e*f)*(d + e*x)^m*(a + c*x^2)^(p + 1))/(2*c*d*(
m + p + 1)), x] + Dist[(m*(g*c*d + c*e*f) + 2*e*c*f*(p + 1))/(e*(2*c*d)*(m
+ p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e
, f, g, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p
+ 1, 0]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p
+ 1, 0]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rubi steps

$$\int \frac{x^2\sqrt{1+x}\sqrt[4]{1-x^2}}{\sqrt{1-x}\left(\sqrt{1-x}-\sqrt{1+x}\right)}\,dx = -\left(\frac{1}{2}\int x\sqrt{1+x}\sqrt[4]{1-x^2}\,dx\right)-\frac{1}{2}\int \frac{x(1+x)\sqrt[4]{1-x^2}}{\sqrt{1-x}}\,dx$$

$$= \frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4}-\frac{1}{12}\int \sqrt{1+x}\sqrt[4]{1-x^2}\,dx-\frac{1}{2}\int \frac{x\left(1-x^2\right)^{5/4}}{(1-x)^{3/2}}\,dx$$

$$= \frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4}+\frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}}-\frac{1}{12}\int \sqrt[4]{1-x}(1+x)^{3/4}\,dx-\frac{1}{2}\int \frac{\left(1-x^2\right)^{5/4}}{\sqrt{1-x}}\,dx$$

$$= \frac{1}{24}(1-x)^{5/4}(1+x)^{3/4}+\frac{1}{6}\sqrt{1+x}\left(1-x^2\right)^{5/4}+\frac{\left(1-x^2\right)^{9/4}}{3(1-x)^{3/2}}-\frac{1}{16}\int \frac{\sqrt[4]{1-x}}{\sqrt[4]{1+x}}\,dx-\frac{1}{2}$$

$$= -\frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4}+\frac{1}{24}(1-x)^{5/4}(1+x)^{3/4}+\frac{1}{6}(1-x)^{7/4}(1+x)^{5/4}+\frac{1}{6}\sqrt{1+x}\,(1$$

$$= \frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4}+\frac{1}{24}(1-x)^{5/4}(1+x)^{3/4}+\frac{1}{6}(1-x)^{7/4}(1+$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x}+\frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4}+\frac{1}{24}(1-x)^{5/4}(1+$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x}+\frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4}+\frac{1}{24}(1-x)^{5/4}(1+$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x}+\frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4}+\frac{1}{24}(1-x)^{5/4}(1+$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x}+\frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4}+\frac{1}{24}(1-x)^{5/4}(1+$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x}+\frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4}+\frac{1}{24}(1-x)^{5/4}(1+$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x}+\frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4}+\frac{1}{24}(1-x)^{5/4}(1+$$

$$= -\frac{5}{48}(1-x)^{3/4}\sqrt[4]{1+x}+\frac{5}{24}(1-x)^{7/4}\sqrt[4]{1+x}-\frac{1}{16}\sqrt[4]{1-x}(1+x)^{3/4}+\frac{1}{24}(1-x)^{5/4}(1+$$

Mathematica [C] time = 0.484503, size = 153, normalized size = 0.5

$$\frac{\sqrt[4]{1-x^2}\,{}_2F_1\left(\frac{1}{4},\frac{1}{4};\frac{5}{4};\frac{1-x}{2}\right)}{8\sqrt[4]{2}\sqrt[4]{x+1}}+\frac{5\left(1-x^2\right)^{3/4}{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{1-x}{2}\right)}{24\ 2^{3/4}(x+1)^{3/4}}-\frac{1}{48}\sqrt{x+1}\sqrt[4]{1-x^2}\left(8x^2-\frac{\sqrt{1-x^2}\left(8x^2+22x+29\right)}{x+1}+2x\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 + x]*(1 - x^2)^(1/4))/(Sqrt[1 - x]*(Sqrt[1 - x] - Sqr
t[1 + x])),x]
```

```
[Out] -(Sqrt[1 + x]*(1 - x^2)^(1/4)*(-7 + 2*x + 8*x^2 - (Sqrt[1 - x^2]*(29 + 22*x
 + 8*x^2))/(1 + x)))/48 + ((1 - x^2)^(1/4)*Hypergeometric2F1[1/4, 1/4, 5/4,
 (1 - x)/2])/(8*2^(1/4)*(1 + x)^(1/4)) + (5*(1 - x^2)^(3/4)*Hypergeometric2
```

```
F1[3/4, 3/4, 7/4, (1 - x)/2])/(24*2^(3/4)*(1 + x)^(3/4))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int x^2 \sqrt[4]{-x^2+1}\sqrt{1+x}\frac{1}{\sqrt{1-x}}\left(\sqrt{1-x}-\sqrt{1+x}\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/2)),x)
```

```
[Out] int(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}x^2}{\sqrt{-x+1}\left(\sqrt{x+1}-\sqrt{-x+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/
2)),x, algorithm="maxima")
```

```
[Out] -integrate((-x^2 + 1)^(1/4)*sqrt(x + 1)*x^2/(sqrt(-x + 1)*(sqrt(x + 1) - sq
rt(-x + 1))), x)
```

Fricas [B] time = 2.19136, size = 1611, normalized size = 5.3

$$-\frac{1}{48}\left(8x^2+2x-7\right)\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}+\frac{1}{48}\left(8x^2+22x+29\right)\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{-x+1}-\frac{1}{16}\sqrt{2}\arctan\left(\frac{\sqrt{2}(x+1)\sqrt{\frac{\sqrt{2}\left(-x^2+1\right.}{}}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/
2)),x, algorithm="fricas")
```

```
[Out] -1/48*(8*x^2 + 2*x - 7)*(-x^2 + 1)^(1/4)*sqrt(x + 1) + 1/48*(8*x^2 + 22*x +
 29)*(-x^2 + 1)^(1/4)*sqrt(-x + 1) - 1/16*sqrt(2)*arctan((sqrt(2)*(x + 1)*s
qrt((sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x + 1) + x + sqrt(-x^2 + 1) + 1)/(x + 1)
) - sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x + 1) - x - 1)/(x + 1)) - 1/16*sqrt(2)*a
rctan((sqrt(2)*(x + 1)*sqrt(-(sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x + 1) - x - sq
rt(-x^2 + 1) - 1)/(x + 1)) - sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x + 1) + x + 1)/
(x + 1)) - 5/16*sqrt(2)*arctan((sqrt(2)*(x - 1)*sqrt((sqrt(2)*(-x^2 + 1)^(1
/4)*sqrt(-x + 1) + x - sqrt(-x^2 + 1) - 1)/(x - 1)) - sqrt(2)*(-x^2 + 1)^(1
/4)*sqrt(-x + 1) - x + 1)/(x - 1)) - 5/16*sqrt(2)*arctan((sqrt(2)*(x - 1)*s
qrt(-(sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(-x + 1) - x + sqrt(-x^2 + 1) + 1)/(x -
1)) - sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(-x + 1) + x - 1)/(x - 1)) + 1/64*sqrt(2
)*log(4*(sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x + 1) + x + sqrt(-x^2 + 1) + 1)/(x
```

```
+ 1)) - 1/64*sqrt(2)*log(-4*(sqrt(2)*(-x^2 + 1)^(1/4)*sqrt(x + 1) - x - sqr
t(-x^2 + 1) - 1)/(x + 1)) + 5/64*sqrt(2)*log(4*(sqrt(2)*(-x^2 + 1)^(1/4)*sq
rt(-x + 1) + x - sqrt(-x^2 + 1) - 1)/(x - 1)) - 5/64*sqrt(2)*log(-4*(sqrt(2
)*(-x^2 + 1)^(1/4)*sqrt(-x + 1) - x + sqrt(-x^2 + 1) + 1)/(x - 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+1)**(1/4)*(1+x)**(1/2)/(1-x)**(1/2)/((1-x)**(1/2)-(1+
x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(-x^2+1\right)^{\frac{1}{4}}\sqrt{x+1}x^2}{\sqrt{-x+1}\left(\sqrt{x+1}-\sqrt{-x+1}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/4)*(1+x)^(1/2)/(1-x)^(1/2)/((1-x)^(1/2)-(1+x)^(1/
2)),x, algorithm="giac")
```

```
[Out] integrate(-(-x^2 + 1)^(1/4)*sqrt(x + 1)*x^2/(sqrt(-x + 1)*(sqrt(x + 1) - sq
rt(-x + 1))), x)
```

3.222 $\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}} dx$

Optimal. Leaf size=292

$$-\frac{1}{12}(1-x)^{2/3}\sqrt[3]{x+1}(1-3x)-\frac{1}{4}(1-x)(x+3)+\frac{1}{12}\sqrt[3]{1-x}(x+1)^{2/3}(3x+1)+\frac{1}{12}\sqrt[6]{1-x}(x+1)^{5/6}(3x+2)-\frac{1}{12}(1-x)^{5/6}$$

```
[Out] -((1 - 3*x)*(1 - x)^(2/3)*(1 + x)^(1/3))/12 + (Sqrt[1 - x]*x*Sqrt[1 + x])/4
 - ((1 - x)*(3 + x))/4 + ((1 - x)^(1/3)*(1 + x)^(2/3)*(1 + 3*x))/12 + ((1 -
 x)^(1/6)*(1 + x)^(5/6)*(2 + 3*x))/12 - ((1 - x)^(5/6)*(1 + x)^(1/6)*(10 +
3*x))/12 + ArcTan[(1 + x)^(1/6)/(1 - x)^(1/6)]/6 - (4*ArcTan[((1 - x)^(1/3)
 - 2*(1 + x)^(1/3))/(Sqrt[3]*(1 - x)^(1/3))])/(3*Sqrt[3]) - (5*ArcTan[((1 -
 x)^(1/3) - (1 + x)^(1/3))/((1 - x)^(1/6)*(1 + x)^(1/6))])/6 + ArcTanh[(Sqr
t[3]*(1 - x)^(1/6)*(1 + x)^(1/6))/((1 - x)^(1/3) + (1 + x)^(1/3))]/(6*Sqrt[
3])
```

Rubi [A] time = 1.84631, antiderivative size = 522, normalized size of antiderivative = 1.79, number of steps used = 46, number of rules used = 21, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6688, 6742, 50, 60, 517, 195, 216, 675, 890, 63, 240, 209, 634, 618, 204, 628, 203, 26, 21, 331, 295}

$$\frac{x^2}{4}+\frac{1}{4}\sqrt{1-x^2}x+\frac{x}{2}-\frac{1}{4}(1-x)^{5/6}(x+1)^{7/6}-\frac{1}{4}(1-x)^{7/6}(x+1)^{5/6}+\frac{5}{12}\sqrt[6]{1-x}(x+1)^{5/6}-\frac{1}{4}(1-x)^{4/3}(x+1)^{2/3}+\frac{1}{3}\sqrt[3]{1-}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[1 - x]*x*(1 + x)^(2/3))/(-((1 - x)^(5/6)*(1 + x)^(1/3)) + (1 - x)
^(2/3)*Sqrt[1 + x]),x]
```

```
[Out] x/2 + x^2/4 - (7*(1 - x)^(5/6)*(1 + x)^(1/6))/12 + ((1 - x)^(2/3)*(1 + x)^(
1/3))/6 - ((1 - x)^(5/3)*(1 + x)^(1/3))/4 + ((1 - x)^(1/3)*(1 + x)^(2/3))/3
 - ((1 - x)^(4/3)*(1 + x)^(2/3))/4 + (5*(1 - x)^(1/6)*(1 + x)^(5/6))/12 - (
(1 - x)^(7/6)*(1 + x)^(5/6))/4 - ((1 - x)^(5/6)*(1 + x)^(7/6))/4 + (x*Sqrt[
1 - x^2])/4 + ArcSin[x]/4 - (2*ArcTan[(1 - x)^(1/6)/(1 + x)^(1/6)])/3 + (2*
ArcTan[1/Sqrt[3] - (2*(1 - x)^(1/3))/(Sqrt[3]*(1 + x)^(1/3))])/(3*Sqrt[3])
+ ArcTan[Sqrt[3] - (2*(1 - x)^(1/6))/(1 + x)^(1/6)]/3 - ArcTan[Sqrt[3] + (2
*(1 - x)^(1/6))/(1 + x)^(1/6)]/3 - (2*ArcTan[1/Sqrt[3] - (2*(1 + x)^(1/3))/
(Sqrt[3]*(1 - x)^(1/3))])/(3*Sqrt[3]) - Log[1 - x]/9 + Log[1 + x]/9 + Log[1
 + (1 - x)^(1/3)/(1 + x)^(1/3)]/3 - Log[1 + (1 - x)^(1/3)/(1 + x)^(1/3) - (
Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(12*Sqrt[3]) + Log[1 + (1 - x)^(1/3)/
(1 + x)^(1/3) + (Sqrt[3]*(1 - x)^(1/6))/(1 + x)^(1/6)]/(12*Sqrt[3]) - Log[1
 + (1 + x)^(1/3)/(1 - x)^(1/3)]/3
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 517

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :> Int[u*(a1*a2 + b1*b2
*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && E
qQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && Gt
Q[a2, 0]))
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 675

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[(
d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && E
qQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m
, 0]
```

Rule 890

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p, x
] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 +
 a*e^2, 0] &&  !IntegerQ[p] && GtQ[a, 0] && GtQ[d, 0] &&  !IGtQ[m, 0] &&  !
IGtQ[n, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 26

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_))^(p_.), x
_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b,
c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] &&
 GtQ[a, 0] && LtQ[d, 0]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 295

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[a/b, n]], s = Denominator[Rt[a/b, n]], k, u}, Simp[u = Int[(r*Cos[((2*k
 - 1)*m*Pi)/n] - s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k
- 1)*Pi)/n]*x + s^2*x^2), x] + Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k
 - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]
; (2*(-1)^(m/2)*r^(m + 2)*Int[1/(r^2 + s^2*x^2), x])/(a*n*s^m) + Dist[(2*r^
(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x]
&& IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{1-x}x(1+x)^{2/3}}{-(1-x)^{5/6}\sqrt[3]{1+x}+(1-x)^{2/3}\sqrt{1+x}}\,dx = \int \frac{x\sqrt[3]{1+x}}{-\sqrt[3]{1-x}+\sqrt[6]{1-x^2}}\,dx$$

$$= \int \left(\frac{1}{2}(1-x)^{2/3}\sqrt[3]{1+x}+\frac{1}{2}\sqrt[3]{1-x}\sqrt[3]{1+x}\sqrt[6]{1-x^2}+\frac{1}{2}\sqrt[3]{1+x}\sqrt[3]{1-x^2}+\frac{\sqrt[3]{1}}{2}\right.$$

$$= \frac{1}{2}\int (1-x)^{2/3}\sqrt[3]{1+x}\,dx+\frac{1}{2}\int \sqrt[3]{1-x}\sqrt[3]{1+x}\sqrt[6]{1-x^2}\,dx+\frac{1}{2}\int \sqrt[3]{1+x}\sqrt[3]{1}$$

$$= -\frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}+\frac{1}{6}\int \frac{(1-x)^{2/3}}{(1+x)^{2/3}}\,dx+\frac{1}{2}\int \sqrt[3]{1-x}(1+x)^{2/3}\,dx+\frac{1}{2}\int$$

$$= \frac{x}{2}+\frac{x^2}{4}+\frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x}-\frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}-\frac{1}{4}(1-x)^{4/3}(1+x)^{2/3}-\frac{1}{4}$$

$$= \frac{x}{2}+\frac{x^2}{4}+\frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x}-\frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}+\frac{1}{3}\sqrt[3]{1-x}(1+x)^{2/3}-\frac{1}{4}($$

$$= \frac{x}{2}+\frac{x^2}{4}-\frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x}+\frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x}-\frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}+\frac{1}{3}$$

$$= \frac{x}{2}+\frac{x^2}{4}-\frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x}+\frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x}-\frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}+\frac{1}{3}$$

$$= \frac{x}{2}+\frac{x^2}{4}-\frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x}+\frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x}-\frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}+\frac{1}{3}$$

$$= \frac{x}{2}+\frac{x^2}{4}-\frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x}+\frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x}-\frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}+\frac{1}{3}$$

$$= \frac{x}{2}+\frac{x^2}{4}-\frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x}+\frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x}-\frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}+\frac{1}{3}$$

$$= \frac{x}{2}+\frac{x^2}{4}-\frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x}+\frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x}-\frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}+\frac{1}{3}$$

$$= \frac{x}{2}+\frac{x^2}{4}-\frac{7}{12}(1-x)^{5/6}\sqrt[6]{1+x}+\frac{1}{6}(1-x)^{2/3}\sqrt[3]{1+x}-\frac{1}{4}(1-x)^{5/3}\sqrt[3]{1+x}+\frac{1}{3}$$

Mathematica [C] time = 0.675763, size = 348, normalized size = 1.19

$$-\frac{5\sqrt{1-x^2}\,{}_2F_1\left(\frac{1}{6},\frac{1}{6};\frac{7}{6};\frac{1-x}{2}\right)}{6\sqrt[6]{2}\sqrt[3]{1-x}\sqrt{x+1}}-\frac{2^{2/3}\sqrt[3]{1-x^2}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{1-x}{2}\right)}{3\sqrt[3]{x+1}}-\frac{1}{12}\sqrt[3]{x+1}\left(-4\,2^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{x+1}{2}\right)+\frac{(3x+10)\left(1-x\right.}{x+1}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[1 - x]*x*(1 + x)^(2/3))/(-((1 - x)^(5/6)*(1 + x)^(1/3)) + (
1 - x)^(2/3)*Sqrt[1 + x]),x]
```

```
[Out] ((1 - x)^(5/6)*(1 + x)^(5/6)*ArcSin[x])/(4*(1 - x^2)^(5/6)) - (5*Sqrt[1 - x
^2]*Hypergeometric2F1[1/6, 1/6, 7/6, (1 - x)/2])/(6*2^(1/6)*(1 - x)^(1/3)*S
```

```
qrt[1 + x]) - (2^(2/3)*(1 - x^2)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (1
- x)/2])/(3*(1 + x)^(1/3)) - ((1 + x)^(1/3)*((1 - 3*x)*(1 - x)^(2/3) - (3*(
1 - x)^(1/3)*x*(2 + x))/(1 - x^2)^(1/3) - 3*(1 - x)^(1/3)*x*(1 - x^2)^(1/6)
 - (1 + 3*x)*(1 - x^2)^(1/3) - ((2 + 3*x)*Sqrt[1 - x^2])/(1 - x)^(1/3) + ((
10 + 3*x)*(1 - x^2)^(5/6))/(1 + x) - 4*2^(2/3)*Hypergeometric2F1[1/3, 1/3,
4/3, (1 + x)/2]))/12 - (7*(1 - x^2)^(5/6)*Hypergeometric2F1[5/6, 5/6, 11/6,
 (1 - x)/2])/(30*2^(5/6)*(1 + x)^(5/6))
```

Maple [F] time = 0.039, size = 0, normalized size = 0.

$$\int x\,(1+x)^{\frac{2}{3}}\sqrt{1-x}\left(-(1-x)^{\frac{5}{6}}\sqrt[3]{1+x}+(1-x)^{\frac{2}{3}}\sqrt{1+x}\right)^{-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(1+x)^(
1/2)),x)
```

```
[Out] int(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(1+x)^(
1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^{\frac{2}{3}}x\sqrt{-x+1}}{\sqrt{x+1}(-x+1)^{\frac{2}{3}}-(x+1)^{\frac{1}{3}}(-x+1)^{\frac{5}{6}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(
1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)^(2/3)*x*sqrt(-x + 1)/(sqrt(x + 1)*(-x + 1)^(2/3) - (x + 1
)^(1/3)*(-x + 1)^(5/6)), x)
```

Fricas [B] time = 2.66366, size = 2724, normalized size = 9.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(
1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*x^2 + 1/12*(3*x + 2)*(x + 1)^(5/6)*(-x + 1)^(1/6) + 1/12*(3*x + 1)*(x +
 1)^(2/3)*(-x + 1)^(1/3) + 1/4*sqrt(x + 1)*x*sqrt(-x + 1) + 1/12*(3*x - 1)*
(x + 1)^(1/3)*(-x + 1)^(2/3) - 1/12*(3*x + 10)*(x + 1)^(1/6)*(-x + 1)^(5/6)
 - 2/9*sqrt(3)*arctan(-1/3*(sqrt(3)*(x + 1) - 2*sqrt(3)*(x + 1)^(2/3)*(-x +
 1)^(1/3))/(x + 1)) - 2/9*sqrt(3)*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt(3)*(
x + 1)^(1/3)*(-x + 1)^(2/3))/(x - 1)) - 5/72*sqrt(3)*log(100*(sqrt(3)*(x +
1)^(5/6)*(-x + 1)^(1/6) + x + (x + 1)^(2/3)*(-x + 1)^(1/3) + 1)/(x + 1)) +
5/72*sqrt(3)*log(-100*(sqrt(3)*(x + 1)^(5/6)*(-x + 1)^(1/6) - x - (x + 1)^(
```

```
2/3)*(-x + 1)^(1/3) - 1)/(x + 1)) - 7/72*sqrt(3)*log(196*(sqrt(3)*(x + 1)^(
1/6)*(-x + 1)^(5/6) + x - (x + 1)^(1/3)*(-x + 1)^(2/3) - 1)/(x - 1)) + 7/72
*sqrt(3)*log(-196*(sqrt(3)*(x + 1)^(1/6)*(-x + 1)^(5/6) - x + (x + 1)^(1/3)
*(-x + 1)^(2/3) + 1)/(x - 1)) + 1/2*x + 5/18*arctan(-(sqrt(3)*(x + 1) - 2*(
x + 1)*sqrt((sqrt(3)*(x + 1)^(5/6)*(-x + 1)^(1/6) + x + (x + 1)^(2/3)*(-x +
 1)^(1/3) + 1)/(x + 1)) + 2*(x + 1)^(5/6)*(-x + 1)^(1/6))/(x + 1)) + 5/18*a
rctan((sqrt(3)*(x + 1) + 2*(x + 1)*sqrt(-(sqrt(3)*(x + 1)^(5/6)*(-x + 1)^(1
/6) - x - (x + 1)^(2/3)*(-x + 1)^(1/3) - 1)/(x + 1)) - 2*(x + 1)^(5/6)*(-x
+ 1)^(1/6))/(x + 1)) + 7/18*arctan(-(sqrt(3)*(x - 1) - 2*(x - 1)*sqrt((sqrt
(3)*(x + 1)^(1/6)*(-x + 1)^(5/6) + x - (x + 1)^(1/3)*(-x + 1)^(2/3) - 1)/(x
 - 1)) + 2*(x + 1)^(1/6)*(-x + 1)^(5/6))/(x - 1)) + 7/18*arctan((sqrt(3)*(x
 - 1) + 2*(x - 1)*sqrt(-(sqrt(3)*(x + 1)^(1/6)*(-x + 1)^(5/6) - x + (x + 1)
^(1/3)*(-x + 1)^(2/3) + 1)/(x - 1)) - 2*(x + 1)^(1/6)*(-x + 1)^(5/6))/(x -
1)) - 5/18*arctan((-x + 1)^(1/6)/(x + 1)^(1/6)) - 7/18*arctan((x + 1)^(1/6)
*(-x + 1)^(5/6)/(x - 1)) - 1/2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - 2
/9*log((x + (x + 1)^(2/3)*(-x + 1)^(1/3) + 1)/(x + 1)) + 1/9*log((x - (x +
1)^(2/3)*(-x + 1)^(1/3) + (x + 1)^(1/3)*(-x + 1)^(2/3) + 1)/(x + 1)) - 1/9*
log((x - (x + 1)^(2/3)*(-x + 1)^(1/3) + (x + 1)^(1/3)*(-x + 1)^(2/3) - 1)/(
x - 1)) + 2/9*log(-(x - (x + 1)^(1/3)*(-x + 1)^(2/3) - 1)/(x - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{1-x}\,(x+1)^{\frac{2}{3}}}{-(1-x)^{\frac{5}{6}}\sqrt[3]{x+1}+(1-x)^{\frac{2}{3}}\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(2/3)*(1-x)**(1/2)/(-(1-x)**(5/6)*(1+x)**(1/3)+(1-x)**(2
/3)*(1+x)**(1/2)),x)
```

```
[Out] Integral(x*sqrt(1 - x)*(x + 1)**(2/3)/(-(1 - x)**(5/6)*(x + 1)**(1/3) + (1
- x)**(2/3)*sqrt(x + 1)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(2/3)*(1-x)^(1/2)/(-(1-x)^(5/6)*(1+x)^(1/3)+(1-x)^(2/3)*(
1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.223 $\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}} dx$

Optimal. Leaf size=25

$$-\frac{3(x-1)(x+1)}{2\sqrt[3]{(x-1)^4(x+1)^2}}$$

```
[Out] (-3*(-1 + x)*(1 + x))/(2*((-1 + x)^4*(1 + x)^2)^(1/3))
```

Rubi [A] time = 0.0128641, antiderivative size = 29, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6719, 37}

$$\frac{3(1-x)(x+1)}{2\sqrt[3]{(1-x)^4(x+1)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^4*(1 + x)^2)^(-1/3),x]
```

```
[Out] (3*(1 - x)*(1 + x))/(2*((1 - x)^4*(1 + x)^2)^(1/3))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}} dx &= \frac{\left((-1+x)^{4/3}(1+x)^{2/3}\right)\int \frac{1}{(-1+x)^{4/3}(1+x)^{2/3}} dx}{\sqrt[3]{(-1+x)^4(1+x)^2}}\\ &= \frac{3(1-x)(1+x)}{2\sqrt[3]{(1-x)^4(1+x)^2}}\end{aligned}$$

Mathematica [A] time = 0.0078621, size = 25, normalized size = 1.

$$-\frac{3(x-1)(x+1)}{2\sqrt[3]{(x-1)^4(x+1)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^4*(1 + x)^2)^(-1/3),x]
```

```
[Out] (-3*(-1 + x)*(1 + x))/(2*((-1 + x)^4*(1 + x)^2)^(1/3))
```

Maple [A] time = 0.001, size = 22, normalized size = 0.9

$$-\frac{(-3+3x)(1+x)}{2}\frac{1}{\sqrt[3]{(-1+x)^4(1+x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)^4*(1+x)^2)^(1/3),x)
```

```
[Out] -3/2*(-1+x)*(1+x)/((-1+x)^4*(1+x)^2)^(1/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x+1)^2(x-1)^4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)^2*(x - 1)^4)^(-1/3), x)
```

Fricas [B] time = 1.77818, size = 108, normalized size = 4.32

$$-\frac{3\left(x^6-2x^5-x^4+4x^3-x^2-2x+1\right)^{\frac{2}{3}}}{2\left(x^4-2x^3+2x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/2*(x^6 - 2*x^5 - x^4 + 4*x^3 - x^2 - 2*x + 1)^(2/3)/(x^4 - 2*x^3 + 2*x -
 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{(x-1)^4(x+1)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**4*(1+x)**2)**(1/3),x)
```

```
[Out] Integral(((x - 1)**4*(x + 1)**2)**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x+1)^2(x-1)^4\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^4*(1+x)^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(((x + 1)^2*(x - 1)^4)^(-1/3), x)
```

3.224 $\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx$

Optimal. Leaf size=25

$$\frac{4(x-1)(x+2)}{3\sqrt[4]{(x-1)^3(x+2)^5}}$$

```
[Out] (4*(-1 + x)*(2 + x))/(3*((-1 + x)^3*(2 + x)^5)^(1/4))
```

Rubi [A] time = 0.016036, antiderivative size = 30, normalized size of antiderivative = 1.2, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6719, 37}

$$-\frac{4(1-x)(x+2)}{3\sqrt[4]{-(1-x)^3(x+2)^5}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^3*(2 + x)^5)^(-1/4),x]
```

```
[Out] (-4*(1 - x)*(2 + x))/(3*(-((1 - x)^3*(2 + x)^5))^(1/4))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}\,dx &= \frac{\left((-1+x)^{3/4}(2+x)^{5/4}\right)\int \frac{1}{(-1+x)^{3/4}(2+x)^{5/4}}\,dx}{\sqrt[4]{(-1+x)^3(2+x)^5}}\\ &= -\frac{4(1-x)(2+x)}{3\sqrt[4]{-(1-x)^3(2+x)^5}}\end{aligned}$$

Mathematica [A] time = 0.0125301, size = 25, normalized size = 1.

$$\frac{4(x-1)(x+2)}{3\sqrt[4]{(x-1)^3(x+2)^5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^3*(2 + x)^5)^(-1/4),x]
```

```
[Out] (4*(-1 + x)*(2 + x))/(3*((-1 + x)^3*(2 + x)^5)^(1/4))
```

Maple [A] time = 0.003, size = 22, normalized size = 0.9

$$\frac{(-4+4x)(2+x)}{3}\frac{1}{\sqrt[4]{(-1+x)^3(2+x)^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)^3*(2+x)^5)^(1/4),x)
```

```
[Out] 4/3*(-1+x)*(2+x)/((-1+x)^3*(2+x)^5)^(1/4)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x+2)^5(x-1)^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^3*(2+x)^5)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(((x + 2)^5*(x - 1)^3)^(-1/4), x)
```

Fricas [B] time = 1.74502, size = 169, normalized size = 6.76

$$\frac{4\left(x^8+7x^7+13x^6-11x^5-50x^4-8x^3+64x^2+16x-32\right)^{\frac{3}{4}}}{3\left(x^6+6x^5+9x^4-8x^3-24x^2+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^3*(2+x)^5)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/3*(x^8 + 7*x^7 + 13*x^6 - 11*x^5 - 50*x^4 - 8*x^3 + 64*x^2 + 16*x - 32)^(
3/4)/(x^6 + 6*x^5 + 9*x^4 - 8*x^3 - 24*x^2 + 16)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**3*(2+x)**5)**(1/4),x)
```

```
[Out] Integral(((x - 1)**3*(x + 2)**5)**(-1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x+2)^5(x-1)^3\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^3*(2+x)^5)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(((x + 2)^5*(x - 1)^3)^(-1/4), x)
```

3.225 $\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx$

Optimal. Leaf size=53

$$\frac{9(x-1)^2(x+1)}{16\sqrt[3]{(x-1)^7(x+1)^2}}-\frac{3(x-1)(x+1)}{8\sqrt[3]{(x-1)^7(x+1)^2}}$$

```
[Out] (-3*(-1 + x)*(1 + x))/(8*((-1 + x)^7*(1 + x)^2)^(1/3)) + (9*(-1 + x)^2*(1 +
 x))/(16*((-1 + x)^7*(1 + x)^2)^(1/3))
```

Rubi [A] time = 0.0198843, antiderivative size = 63, normalized size of antiderivative = 1.19, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {6719, 45, 37}

$$\frac{9(x+1)(1-x)^2}{16\sqrt[3]{-(1-x)^7(x+1)^2}}+\frac{3(x+1)(1-x)}{8\sqrt[3]{-(1-x)^7(x+1)^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^7*(1 + x)^2)^(-1/3),x]
```

```
[Out] (3*(1 - x)*(1 + x))/(8*(-((1 - x)^7*(1 + x)^2))^(1/3)) + (9*(1 - x)^2*(1 +
x))/(16*(-((1 - x)^7*(1 + x)^2))^(1/3))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] && !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] || !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}\,dx = \frac{\left((-1+x)^{7/3}(1+x)^{2/3}\right)\int \frac{1}{(-1+x)^{7/3}(1+x)^{2/3}}\,dx}{\sqrt[3]{(-1+x)^7(1+x)^2}}$$
$$= \frac{3(1-x)(1+x)}{8\sqrt[3]{-(1-x)^7(1+x)^2}} - \frac{\left(3(-1+x)^{7/3}(1+x)^{2/3}\right)\int \frac{1}{(-1+x)^{4/3}(1+x)^{2/3}}\,dx}{8\sqrt[3]{(-1+x)^7(1+x)^2}}$$
$$= \frac{3(1-x)(1+x)}{8\sqrt[3]{-(1-x)^7(1+x)^2}} + \frac{9(1-x)^2(1+x)}{16\sqrt[3]{-(1-x)^7(1+x)^2}}$$

Mathematica [A] time = 0.0155586, size = 30, normalized size = 0.57

$$\frac{3(x-1)(x+1)(3x-5)}{16\sqrt[3]{(x-1)^7(x+1)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^7*(1 + x)^2)^(-1/3),x]
```

```
[Out] (3*(-1 + x)*(1 + x)*(-5 + 3*x))/(16*((-1 + x)^7*(1 + x)^2)^(1/3))
```

Maple [A] time = 0.001, size = 27, normalized size = 0.5

$$\frac{(3+3x)(-1+x)(3x-5)}{16}\frac{1}{\sqrt[3]{(-1+x)^7(1+x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)^7*(1+x)^2)^(1/3),x)
```

```
[Out] 3/16*(1+x)*(-1+x)*(3*x-5)/((-1+x)^7*(1+x)^2)^(1/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x+1)^2(x-1)^7\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)^2*(x - 1)^7)^(-1/3), x)
```

Fricas [A] time = 1.67343, size = 184, normalized size = 3.47

$$\frac{3\left(x^9-5x^8+8x^7-14x^5+14x^4-8x^2+5x-1\right)^{\frac{2}{3}}(3x-5)}{16\left(x^7-5x^6+9x^5-5x^4-5x^3+9x^2-5x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/16*(x^9 - 5*x^8 + 8*x^7 - 14*x^5 + 14*x^4 - 8*x^2 + 5*x - 1)^(2/3)*(3*x -
 5)/(x^7 - 5*x^6 + 9*x^5 - 5*x^4 - 5*x^3 + 9*x^2 - 5*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{(x-1)^7 (x+1)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**7*(1+x)**2)**(1/3),x)
```

```
[Out] Integral(((x - 1)**7*(x + 1)**2)**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x+1)^2(x-1)^7\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^7*(1+x)^2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(((x + 1)^2*(x - 1)^7)^(-1/3), x)
```

3.226 $\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{2}\log(x+1)-\frac{3}{2}\log\left(1-\frac{x-1}{\sqrt[3]{(x-1)^2(x+1)}}\right)+\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x-1)}{\sqrt[3]{(x-1)^2(x+1)}}+1}{\sqrt{3}}\right)$$

```
[Out] Sqrt[3]*ArcTan[(1 + (2*(-1 + x))/((-1 + x)^2*(1 + x))^(1/3))/Sqrt[3]] - Log
[1 + x]/2 - (3*Log[1 - (-1 + x)/((-1 + x)^2*(1 + x))^(1/3)])/2
```

Rubi [B] time = 0.120864, antiderivative size = 188, normalized size of antiderivative = 2.81, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2067, 2064, 60}

$$-\frac{(3-3x)^{2/3}\sqrt[3]{x+1}\log\left(-\frac{8}{3}(x-1)\right)}{2\,3^{2/3}\sqrt[3]{x^3-x^2-x+1}}-\frac{\sqrt[3]{3}(3-3x)^{2/3}\sqrt[3]{x+1}\log\left(\frac{\sqrt[3]{3}\sqrt[3]{x+1}}{\sqrt[3]{3-3x}}+1\right)}{2\sqrt[3]{x^3-x^2-x+1}}-\frac{(3-3x)^{2/3}\sqrt[3]{x+1}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x+1}}{\sqrt[6]{3}\sqrt[3]{3-3x}}\right)}{\sqrt[6]{3}\sqrt[3]{x^3-x^2-x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^2*(1 + x))^(-1/3),x]
```

```
[Out] -(((3 - 3*x)^(2/3)*(1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2*(1 + x)^(1/3))/(3^(1
/6)*(3 - 3*x)^(1/3))])/(3^(1/6)*(1 - x - x^2 + x^3)^(1/3))) - ((3 - 3*x)^(2
/3)*(1 + x)^(1/3)*Log[(-8*(-1 + x))/3])/(2*3^(2/3)*(1 - x - x^2 + x^3)^(1/3
)) - (3^(1/3)*(3 - 3*x)^(2/3)*(1 + x)^(1/3)*Log[1 + (3^(1/3)*(1 + x)^(1/3))
/(3 - 3*x)^(1/3)])/(2*(1 - x - x^2 + x^3)^(1/3))
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 2064

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt[3]{\frac{16}{27}-\frac{4x}{3}+x^3}}\,dx, x, -\frac{1}{3}+x\right)$$

$$= \frac{\left(4\ 2^{2/3}(1-x)^{2/3}\sqrt[3]{1+x}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{16}{9}-\frac{8x}{3}\right)^{2/3}\sqrt[3]{\frac{16}{9}+\frac{4x}{3}}}\,dx, x, -\frac{1}{3}+x\right)}{3\sqrt[3]{1-x-x^2+x^3}}$$

$$= -\frac{\sqrt{3}(1-x)^{2/3}\sqrt[3]{1+x}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1+x}}{\sqrt{3}\sqrt[3]{1-x}}\right)}{\sqrt[3]{1-x-x^2+x^3}} - \frac{(1-x)^{2/3}\sqrt[3]{1+x}\log(1-x)}{2\sqrt[3]{1-x-x^2+x^3}} - \frac{3(1-x)^{2/3}\sqrt[3]{1-}}{2\sqrt[3]{1}}$$

Mathematica [C] time = 0.0111695, size = 49, normalized size = 0.73

$$\frac{3(x-1)\sqrt[3]{x+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{1-x}{2}\right)}{\sqrt[3]{2}\sqrt[3]{(x-1)^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x)^2*(1 + x))^(-1/3),x]
```

```
[Out] (3*(-1 + x)*(1 + x)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (1 - x)/2])/(2^(
1/3)*((-1 + x)^2*(1 + x))^(1/3))
```

Maple [F] time = 0.008, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{(-1+x)^2(1+x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((-1+x)^2*(1+x))^(1/3),x)
```

```
[Out] int(1/((-1+x)^2*(1+x))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^2*(1+x))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)*(x - 1)^2)^(-1/3), x)
```

Fricas [B] time = 1.71082, size = 331, normalized size = 4.94

$$-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x-1)+2\sqrt{3}\left(x^3-x^2-x+1\right)^{\frac{1}{3}}}{3\,(x-1)}\right)+\frac{1}{2}\log\left(\frac{x^2+\left(x^3-x^2-x+1\right)^{\frac{1}{3}}(x-1)-2\,x+\left(x^3-x^2-x+1\right)^{\frac{2}{3}}+}{x^2-2\,x+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^2*(1+x))^(1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt(3)*(x^3 - x^2 - x + 1)^(1/3))
/(x - 1)) + 1/2*log((x^2 + (x^3 - x^2 - x + 1)^(1/3)*(x - 1) - 2*x + (x^3 -
 x^2 - x + 1)^(2/3) + 1)/(x^2 - 2*x + 1)) - log(-(x - (x^3 - x^2 - x + 1)^(
1/3) - 1)/(x - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{(x-1)^2\,(x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**2*(1+x))**(1/3),x)
```

```
[Out] Integral(((x - 1)**2*(x + 1))**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)^2*(1+x))^(1/3),x, algorithm="giac")
```

```
[Out] integrate(((x + 1)*(x - 1)^2)^(-1/3), x)
```

3.227 $\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx$

Optimal. Leaf size=122

$$-\frac{4(x-2)(x+1)}{3\sqrt{(x-2)(x+1)^3}}-\frac{\sqrt{2}\sqrt{x-2}(x+1)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x+1}}{\sqrt{x-2}}\right)}{\sqrt{(x-2)(x+1)^3}}+\frac{2\sqrt{x-2}(x+1)^{3/2}\sinh^{-1}\left(\frac{\sqrt{x-2}}{\sqrt{3}}\right)}{\sqrt{(x-2)(x+1)^3}}$$

```
[Out] (-4*(-2 + x)*(1 + x))/(3*Sqrt[(-2 + x)*(1 + x)^3]) + (2*Sqrt[-2 + x]*(1 + x
)^(3/2)*ArcSinh[Sqrt[-2 + x]/Sqrt[3]])/Sqrt[(-2 + x)*(1 + x)^3] - (Sqrt[2]*
Sqrt[-2 + x]*(1 + x)^(3/2)*ArcTan[(Sqrt[2]*Sqrt[1 + x])/Sqrt[-2 + x]])/Sqrt
[(-2 + x)*(1 + x)^3]
```

Rubi [A] time = 0.345962, antiderivative size = 133, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {1593, 6719, 1614, 21, 105, 54, 215, 93, 204}

$$\frac{4(2-x)(x+1)}{3\sqrt{-(2-x)(x+1)^3}}-\frac{\sqrt{2}\sqrt{x-2}(x+1)^{3/2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{x+1}}{\sqrt{x-2}}\right)}{\sqrt{-(2-x)(x+1)^3}}+\frac{2\sqrt{x-2}(x+1)^{3/2}\sinh^{-1}\left(\frac{\sqrt{x-2}}{\sqrt{3}}\right)}{\sqrt{-(2-x)(x+1)^3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1) + x)/Sqrt[(-2 + x)*(1 + x)^3],x]
```

```
[Out] (4*(2 - x)*(1 + x))/(3*Sqrt[-((2 - x)*(1 + x)^3)]) + (2*Sqrt[-2 + x]*(1 + x
)^(3/2)*ArcSinh[Sqrt[-2 + x]/Sqrt[3]])/Sqrt[-((2 - x)*(1 + x)^3)] - (Sqrt[2
]*Sqrt[-2 + x]*(1 + x)^(3/2)*ArcTan[(Sqrt[2]*Sqrt[1 + x])/Sqrt[-2 + x]])/Sq
rt[-((2 - x)*(1 + x)^3)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 1614

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || ( !RationalQ[m] && (SumSimplerQ[m, -1] ||  !SumSimplerQ[n, -1])))
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\frac{1}{x}+x}{\sqrt{(-2+x)(1+x)^3}}\,dx &= \int \frac{1+x^2}{x\sqrt{(-2+x)(1+x)^3}}\,dx \\
&= \frac{\left(\sqrt{-2+x}(1+x)^{3/2}\right)\int \frac{1+x^2}{\sqrt{-2+x}x(1+x)^{3/2}}\,dx}{\sqrt{(-2+x)(1+x)^3}} \\
&= \frac{4(2-x)(1+x)}{3\sqrt{-(2-x)(1+x)^3}} - \frac{\left(2\sqrt{-2+x}(1+x)^{3/2}\right)\int \frac{-\frac{3}{2}-\frac{3x}{2}}{\sqrt{-2+x}x\sqrt{1+x}}\,dx}{3\sqrt{(-2+x)(1+x)^3}} \\
&= \frac{4(2-x)(1+x)}{3\sqrt{-(2-x)(1+x)^3}} + \frac{\left(\sqrt{-2+x}(1+x)^{3/2}\right)\int \frac{\sqrt{1+x}}{\sqrt{-2+x}x}\,dx}{\sqrt{(-2+x)(1+x)^3}} \\
&= \frac{4(2-x)(1+x)}{3\sqrt{-(2-x)(1+x)^3}} + \frac{\left(\sqrt{-2+x}(1+x)^{3/2}\right)\int \frac{1}{\sqrt{-2+x}\sqrt{1+x}}\,dx}{\sqrt{(-2+x)(1+x)^3}} + \frac{\left(\sqrt{-2+x}(1+x)^{3/2}\right)\int \frac{1}{\sqrt{-2+x}\ldots}}{\sqrt{(-2+x)(1+x)^3}} \\
&= \frac{4(2-x)(1+x)}{3\sqrt{-(2-x)(1+x)^3}} + \frac{\left(2\sqrt{-2+x}(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{1}{-1-2x^2}\,dx, x, \frac{\sqrt{1+x}}{\sqrt{-2+x}}\right)}{\sqrt{(-2+x)(1+x)^3}} + \frac{\left(2\sqrt{-2+x}(1\ldots\right.}{\ldots} \\
&= \frac{4(2-x)(1+x)}{3\sqrt{-(2-x)(1+x)^3}} + \frac{2\sqrt{-2+x}(1+x)^{3/2}\sinh^{-1}\left(\frac{\sqrt{-2+x}}{\sqrt{3}}\right)}{\sqrt{-(2-x)(1+x)^3}} - \frac{\sqrt{2}\sqrt{-2+x}(1+x)^{3/2}\tan^{-1}\left(\ldots\right.}{\sqrt{-(2-x)(1+x)^3}}
\end{aligned}
$$

Mathematica [A] time = 0.130333, size = 114, normalized size = 0.93

$$
-\frac{(x+1)\left(-4(2-x)^{3/2}-6(x-2)\sqrt{x+1}\sin^{-1}\left(\frac{\sqrt{2-x}}{\sqrt{3}}\right)-3\sqrt{2}\sqrt{-(x-2)^2}\sqrt{x+1}\tan^{-1}\left(\frac{\sqrt{\frac{x-2}{x+1}}}{\sqrt{2}}\right)\right)}{3\sqrt{2-x}\sqrt{(x-2)(x+1)^3}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^(-1) + x)/Sqrt[(-2 + x)*(1 + x)^3],x]
```

```
[Out] -((1 + x)*(-4*(2 - x)^(3/2) - 6*(-2 + x)*Sqrt[1 + x]*ArcSin[Sqrt[2 - x]/Sqr
t[3]] - 3*Sqrt[2]*Sqrt[-(-2 + x)^2]*Sqrt[1 + x]*ArcTan[Sqrt[(-2 + x)/(1 + x
)]/Sqrt[2]]))/(3*Sqrt[2 - x]*Sqrt[(-2 + x)*(1 + x)^3])
```

Maple [A] time = 0.018, size = 118, normalized size = 1.

$$
\frac{1}{6}\left(-3\sqrt{2}\arctan\left(1/4\,\frac{(4+x)\sqrt{2}}{\sqrt{x^2-x-2}}\right)x+6\,\ln\left(x-1/2+\sqrt{x^2-x-2}\right)x-3\sqrt{2}\arctan\left(1/4\,\frac{(4+x)\sqrt{2}}{\sqrt{x^2-x-2}}\right)+6\,\ln\left(x-1\ldots\right.\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x)
```

```
[Out] 1/6*(-3*2^(1/2)*arctan(1/4*(4+x)*2^(1/2)/(x^2-x-2)^(1/2))*x+6*ln(x-1/2+(x^2
-x-2)^(1/2))*x-3*2^(1/2)*arctan(1/4*(4+x)*2^(1/2)/(x^2-x-2)^(1/2))+6*ln(x-1
/2+(x^2-x-2)^(1/2))-8*(x^2-x-2)^(1/2))*((1+x)*(-2+x))^(1/2)/((-2+x)*(1+x)^3
)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + \frac{1}{x}}{\sqrt{(x+1)^3(x-2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 1/x)/sqrt((x + 1)^3*(x - 2)), x)
```

Fricas [A] time = 1.7969, size = 375, normalized size = 3.07

$$\frac{3\sqrt{2}\left(x^2+2x+1\right)\arctan\left(-\frac{\sqrt{2}\left(x^2+x\right)-\sqrt{2}\sqrt{x^4+x^3-3x^2-5x-2}}{2(x+1)}\right)-4x^2-3\left(x^2+2x+1\right)\log\left(-\frac{2x^2+x-2\sqrt{x^4+x^3-3x^2-5x-2}-1}{x+1}\right)-8x}{3\left(x^2+2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*sqrt(2)*(x^2 + 2*x + 1)*arctan(-1/2*(sqrt(2)*(x^2 + x) - sqrt(2)*sqr
t(x^4 + x^3 - 3*x^2 - 5*x - 2))/(x + 1)) - 4*x^2 - 3*(x^2 + 2*x + 1)*log(-(
2*x^2 + x - 2*sqrt(x^4 + x^3 - 3*x^2 - 5*x - 2) - 1)/(x + 1)) - 8*x - 4*sqr
t(x^4 + x^3 - 3*x^2 - 5*x - 2) - 4)/(x^2 + 2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{x\sqrt{(x-2)(x+1)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)/((-2+x)*(1+x)**3)**(1/2),x)
```

```
[Out] Integral((x**2 + 1)/(x*sqrt((x - 2)*(x + 1)**3)), x)
```

Giac [A] time = 1.19029, size = 230, normalized size = 1.89

$$-\frac{\sqrt{2}\arcsin\left(\frac{4}{3x}+\frac{1}{3}\right)}{2\,\mathrm{sgn}\left(\frac{1}{x^2}+\frac{1}{x^3}\right)}-\frac{\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(2\sqrt{2}\sqrt{-\frac{1}{x}-\frac{2}{x^2}+1}-3\right)}{\frac{4}{x}+1}+6\right|}{\left|4\sqrt{2}+\frac{2\left(2\sqrt{2}\sqrt{-\frac{1}{x}-\frac{2}{x^2}+1}-3\right)}{\frac{4}{x}+1}+6\right|}\right)}{\mathrm{sgn}\left(\frac{1}{x^2}+\frac{1}{x^3}\right)}+\frac{8\sqrt{2}}{3\left(\frac{2\sqrt{2}\sqrt{-\frac{1}{x}-\frac{2}{x^2}+1}-3}{\frac{4}{x}+1}-1\right)\mathrm{sgn}\left(\frac{1}{x^2}+\frac{1}{x^3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)/((-2+x)*(1+x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arcsin(4/3/x + 1/3)/sgn(1/x^2 + 1/x^3) - log(abs(-4*sqrt(2) +
2*(2*sqrt(2)*sqrt(-1/x - 2/x^2 + 1) - 3)/(4/x + 1) + 6)/abs(4*sqrt(2) + 2*(
2*sqrt(2)*sqrt(-1/x - 2/x^2 + 1) - 3)/(4/x + 1) + 6))/sgn(1/x^2 + 1/x^3) +
8/3*sqrt(2)/(((2*sqrt(2)*sqrt(-1/x - 2/x^2 + 1) - 3)/(4/x + 1) - 1)*sgn(1/x
^2 + 1/x^3))
```

3.228 $\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx$

Optimal. Leaf size=150

$$-\frac{\sqrt[3]{(x-1)^2(x+1)}}{x}+\frac{\log(x)}{6}-\frac{2}{3}\log(x+1)-\frac{3}{2}\log\left(1-\frac{x-1}{\sqrt[3]{(x-1)^2(x+1)}}\right)-\frac{1}{2}\log\left(\frac{x-1}{\sqrt[3]{(x-1)^2(x+1)}}+1\right)-\frac{\tan^{-1}\left(\frac{1-\frac{\ldots}{\sqrt[3]{(\ldots}}}{\ldots}\right)}{\sqrt{3}}$$

```
[Out] -(((-1 + x)^2*(1 + x))^(1/3)/x) - ArcTan[(1 - (2*(-1 + x))/((-1 + x)^2*(1 +
 x))^(1/3))/Sqrt[3]]/Sqrt[3] - Sqrt[3]*ArcTan[(1 + (2*(-1 + x))/((-1 + x)^2
*(1 + x))^(1/3))/Sqrt[3]] + Log[x]/6 - (2*Log[1 + x])/3 - (3*Log[1 - (-1 +
x)/((-1 + x)^2*(1 + x))^(1/3)])/2 - Log[1 + (-1 + x)/((-1 + x)^2*(1 + x))^(
1/3)]/2
```

Rubi [B] time = 0.327181, antiderivative size = 404, normalized size of antiderivative = 2.69, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {2081, 2077, 97, 157, 60, 91}

$$-\frac{\sqrt[3]{x^3-x^2-x+1}}{x}+\frac{\sqrt[3]{x^3-x^2-x+1}\log(x)}{2\sqrt[3]{3}(3-3x)^{2/3}\sqrt[3]{x+1}}-\frac{3^{2/3}\sqrt[3]{x^3-x^2-x+1}\log\left(\frac{4(x+1)}{3}\right)}{2(3-3x)^{2/3}\sqrt[3]{x+1}}-\frac{3\,3^{2/3}\sqrt[3]{x^3-x^2-x+1}\log\left(\frac{\sqrt[3]{3-3x}}{\sqrt[3]{3}\sqrt[3]{x+1}}\ldots\right)}{2(3-3x)^{2/3}\sqrt[3]{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x)^2*(1 + x))^(1/3)/x^2,x]
```

```
[Out] -((1 - x - x^2 + x^3)^(1/3)/x) - (3*3^(1/6)*(1 - x - x^2 + x^3)^(1/3)*ArcTa
n[1/Sqrt[3] - (2*(3 - 3*x)^(1/3))/(3^(5/6)*(1 + x)^(1/3))])/((3 - 3*x)^(2/3
)*(1 + x)^(1/3)) - (3^(1/6)*(1 - x - x^2 + x^3)^(1/3)*ArcTan[1/Sqrt[3] + (2
*(3 - 3*x)^(1/3))/(3^(5/6)*(1 + x)^(1/3))])/((3 - 3*x)^(2/3)*(1 + x)^(1/3))
 + ((1 - x - x^2 + x^3)^(1/3)*Log[x])/(2*3^(1/3)*(3 - 3*x)^(2/3)*(1 + x)^(1
/3)) - (3^(2/3)*(1 - x - x^2 + x^3)^(1/3)*Log[(4*(1 + x))/3])/(2*(3 - 3*x)^
(2/3)*(1 + x)^(1/3)) - (3*3^(2/3)*(1 - x - x^2 + x^3)^(1/3)*Log[1 + (3 - 3*
x)^(1/3)/(3^(1/3)*(1 + x)^(1/3))])/(2*(3 - 3*x)^(2/3)*(1 + x)^(1/3)) - (3^(
2/3)*(1 - x - x^2 + x^3)^(1/3)*Log[(2/3)^(2/3)*(3 - 3*x)^(1/3) - (2^(2/3)*(
1 + x)^(1/3))/3^(1/3)])/(2*(3 - 3*x)^(2/3)*(1 + x)^(1/3))
```

Rule 2081

```
Int[(P3_)^(p_.)*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> With[{a = Coeff[P3
, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Su
bst[Int[((3*d*e - c*f)/(3*d) + f*x)^m*Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27
*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c
, 0]] /; FreeQ[{e, f, m, p}, x] && PolyQ[P3, x, 3]
```

Rule 2077

```
Int[((e_.) + (f_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_S
ymbol] :> Dist[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int
[(e + f*x)^m*(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d, e
, f, m, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&  !IntegerQ[p]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rule 91

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)*((e_.) + (f_.)
*(x_))), x_Symbol] :> With[{q = Rt[(d*e - c*f)/(b*e - a*f), 3]}, -Simp[(Sqr
t[3]*q*ArcTan[1/Sqrt[3] + (2*q*(a + b*x)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])
/(d*e - c*f), x] + (Simp[(q*Log[e + f*x])/(2*(d*e - c*f)), x] - Simp[(3*q*L
og[q*(a + b*x)^(1/3) - (c + d*x)^(1/3)])/(2*(d*e - c*f)), x])] /; FreeQ[{a,
 b, c, d, e, f}, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{(-1+x)^2(1+x)}}{x^2}\,dx = \text{Subst}\left(\int \frac{\sqrt[3]{\frac{16}{27}-\frac{4x}{3}+x^3}}{\left(\frac{1}{3}+x\right)^2}\,dx, x, -\frac{1}{3}+x\right)$$

$$= \frac{\left(3\sqrt[3]{1-x-x^2+x^3}\right)\text{Subst}\left(\int \frac{\left(\frac{16}{9}-\frac{8x}{3}\right)^{2/3}\sqrt[3]{\frac{16}{9}+\frac{4x}{3}}}{\left(\frac{1}{3}+x\right)^2}\,dx, x, -\frac{1}{3}+x\right)}{4\ 2^{2/3}(1-x)^{2/3}\sqrt[3]{1+x}}$$

$$= -\frac{\sqrt[3]{1-x-x^2+x^3}}{x} + \frac{\left(3\sqrt[3]{1-x-x^2+x^3}\right)\text{Subst}\left(\int \frac{-\frac{64}{27}-\frac{32x}{9}}{\sqrt[3]{\frac{16}{9}-\frac{8x}{3}}\left(\frac{1}{3}+x\right)\left(\frac{16}{9}+\frac{4x}{3}\right)^{2/3}}\,dx, x, -\frac{1}{3}+x\right)}{4\ 2^{2/3}(1-x)^{2/3}\sqrt[3]{1+x}}$$

$$= -\frac{\sqrt[3]{1-x-x^2+x^3}}{x} - \frac{\left(4\sqrt[3]{2}\sqrt[3]{1-x-x^2+x^3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{\frac{16}{9}-\frac{8x}{3}}\left(\frac{1}{3}+x\right)\left(\frac{16}{9}+\frac{4x}{3}\right)^{2/3}}\,dx, x, -\frac{1}{3}+x\right.}{9(1-x)^{2/3}\sqrt[3]{1+x}}$$

$$= -\frac{\sqrt[3]{1-x-x^2+x^3}}{x} - \frac{\sqrt{3}\sqrt[3]{1-x-x^2+x^3}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1-x}}{\sqrt{3}\sqrt[3]{1+x}}\right)}{(1-x)^{2/3}\sqrt[3]{1+x}} - \frac{\sqrt[3]{1-x-x^2+x^3}\tan^{-1}}{\sqrt{3}(1-x)^{2/3}\sqrt[3]{}}$$

Mathematica [C] time = 0.0668508, size = 112, normalized size = 0.75

$$\frac{\sqrt[3]{(x-1)^2(x+1)}\left(3(x+1)\left(3\ 2^{2/3}\sqrt[3]{1-x}x\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{x+1}{2}\right)-2x+2\right)-2\left(\frac{1}{x}+1\right)^{2/3}\sqrt[3]{\frac{x-1}{x}}xF_1\left(1;\frac{1}{3},\frac{2}{3};2;\frac{1}{x},-\frac{1}{x}\right)\right)}{6x\left(x^2-1\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((-1 + x)^2*(1 + x))^(1/3)/x^2,x]
```

```
[Out] (((-1 + x)^2*(1 + x))^(1/3)*(-2*(1 + x^(-1))^(2/3)*((-1 + x)/x)^(1/3)*x*App
ellF1[1, 1/3, 2/3, 2, x^(-1), -x^(-1)] + 3*(1 + x)*(2 - 2*x + 3*2^(2/3)*(1
- x)^(1/3)*x*Hypergeometric2F1[1/3, 1/3, 4/3, (1 + x)/2])))/(6*x*(-1 + x^2)
)
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{1}{x^2}\sqrt[3]{(-1+x)^2\,(1+x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((-1+x)^2*(1+x))^(1/3)/x^2,x)
```

```
[Out] int(((-1+x)^2*(1+x))^(1/3)/x^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^2*(1+x))^(1/3)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(((x + 1)*(x - 1)^2)^(1/3)/x^2, x)
```

Fricas [B] time = 1.88514, size = 730, normalized size = 4.87

$$6\sqrt{3}x\arctan\left(\frac{\sqrt{3}(x-1)+2\sqrt{3}\left(x^3-x^2-x+1\right)^{\frac{1}{3}}}{3\,(x-1)}\right)-2\sqrt{3}x\arctan\left(-\frac{\sqrt{3}(x-1)-2\sqrt{3}\left(x^3-x^2-x+1\right)^{\frac{1}{3}}}{3\,(x-1)}\right)+3\,x\log\left(\frac{x^2+\left(x^3-x^2-x+1\right)^{\frac{1}{3}}(x-1)-2\,x+\left(x^3-\right.}{x^2-2\,x+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^2*(1+x))^(1/3)/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(6*sqrt(3)*x*arctan(1/3*(sqrt(3)*(x - 1) + 2*sqrt(3)*(x^3 - x^2 - x + 1
)^(1/3))/(x - 1)) - 2*sqrt(3)*x*arctan(-1/3*(sqrt(3)*(x - 1) - 2*sqrt(3)*(x
^3 - x^2 - x + 1)^(1/3))/(x - 1)) + 3*x*log((x^2 + (x^3 - x^2 - x + 1)^(1/3
```

```
)*(x - 1) - 2*x + (x^3 - x^2 - x + 1)^(2/3) + 1)/(x^2 - 2*x + 1)) + x*log((
x^2 - (x^3 - x^2 - x + 1)^(1/3)*(x - 1) - 2*x + (x^3 - x^2 - x + 1)^(2/3) +
 1)/(x^2 - 2*x + 1)) - 2*x*log((x + (x^3 - x^2 - x + 1)^(1/3) - 1)/(x - 1))
 - 6*x*log(-(x - (x^3 - x^2 - x + 1)^(1/3) - 1)/(x - 1)) - 6*(x^3 - x^2 - x
 + 1)^(1/3))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{(x-1)^2\,(x+1)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)**2*(1+x))**(1/3)/x**2,x)
```

```
[Out] Integral(((x - 1)**2*(x + 1))**(1/3)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left((x+1)(x-1)^2\right)^{\frac{1}{3}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((-1+x)^2*(1+x))^(1/3)/x^2,x, algorithm="giac")
```

```
[Out] integrate(((x + 1)*(x - 1)^2)^(1/3)/x^2, x)
```

3.229 $\int \frac{1}{\left(-3-2x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{1-x}{12\left(x^2-2x-3\right)^{3/2}}-\frac{1-x}{24\sqrt{x^2-2x-3}}$$

```
[Out] (1 - x)/(12*(-3 - 2*x + x^2)^(3/2)) - (1 - x)/(24*Sqrt[-3 - 2*x + x^2])
```

Rubi [A] time = 0.006261, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {614, 613}

$$\frac{1-x}{12\left(x^2-2x-3\right)^{3/2}}-\frac{1-x}{24\sqrt{x^2-2x-3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 - 2*x + x^2)^(-5/2),x]
```

```
[Out] (1 - x)/(12*(-3 - 2*x + x^2)^(3/2)) - (1 - x)/(24*Sqrt[-3 - 2*x + x^2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-3-2x+x^2\right)^{5/2}}\,dx &= \frac{1-x}{12\left(-3-2x+x^2\right)^{3/2}}-\frac{1}{6}\int \frac{1}{\left(-3-2x+x^2\right)^{3/2}}\,dx\\ &= \frac{1-x}{12\left(-3-2x+x^2\right)^{3/2}}-\frac{1-x}{24\sqrt{-3-2x+x^2}}\end{aligned}$$

Mathematica [A] time = 0.0108458, size = 27, normalized size = 0.63

$$\frac{(x-1)\left(x^2-2x-5\right)}{24\left(x^2-2x-3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 - 2*x + x^2)^(-5/2),x]
```

```
[Out] ((-1 + x)*(-5 - 2*x + x^2))/(24*(-3 - 2*x + x^2)^(3/2))
```

Maple [A] time = 0.003, size = 32, normalized size = 0.7

$$\frac{(1+x)(-3+x)\left(x^3-3x^2-3x+5\right)}{24}\left(x^2-2x-3\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x-3)^(5/2),x)
```

```
[Out] 1/24*(1+x)*(-3+x)*(x^3-3*x^2-3*x+5)/(x^2-2*x-3)^(5/2)
```

Maxima [A] time = 0.940605, size = 69, normalized size = 1.6

$$\frac{x}{24\sqrt{x^2-2x-3}}-\frac{1}{24\sqrt{x^2-2x-3}}-\frac{x}{12\left(x^2-2x-3\right)^{\frac{3}{2}}}+\frac{1}{12\left(x^2-2x-3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-3)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/24*x/sqrt(x^2 - 2*x - 3) - 1/24/sqrt(x^2 - 2*x - 3) - 1/12*x/(x^2 - 2*x -
 3)^(3/2) + 1/12/(x^2 - 2*x - 3)^(3/2)
```

Fricas [B] time = 1.71169, size = 159, normalized size = 3.7

$$\frac{x^4-4x^3-2x^2+\left(x^3-3x^2-3x+5\right)\sqrt{x^2-2x-3}+12x+9}{24\left(x^4-4x^3-2x^2+12x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/24*(x^4 - 4*x^3 - 2*x^2 + (x^3 - 3*x^2 - 3*x + 5)*sqrt(x^2 - 2*x - 3) + 1
2*x + 9)/(x^4 - 4*x^3 - 2*x^2 + 12*x + 9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2-2x-3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x-3)**(5/2),x)
```

```
[Out] Integral((x**2 - 2*x - 3)**(-5/2), x)
```

Giac [A] time = 1.08948, size = 31, normalized size = 0.72

$$\frac{((x-3)x-3)x+5}{24\left(x^2-2x-3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-3)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*(((x - 3)*x - 3)*x + 5)/(x^2 - 2*x - 3)^(3/2)
```

3.230 $\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx$

Optimal. Leaf size=42

$$\frac{(3-x)\sqrt{x+1}\tanh^{-1}\left(\frac{\sqrt{x+1}}{2}\right)}{\sqrt{x^3-5x^2+3x+9}}$$

```
[Out] ((3 - x)*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/Sqrt[9 + 3*x - 5*x^2 + x^3]
```

Rubi [A] time = 0.0426458, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2067, 2064, 63, 206}

$$\frac{(3-x)\sqrt{x+1}\tanh^{-1}\left(\frac{\sqrt{x+1}}{2}\right)}{\sqrt{x^3-5x^2+3x+9}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 + 3*x - 5*x^2 + x^3],x]
```

```
[Out] ((3 - x)*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/Sqrt[9 + 3*x - 5*x^2 + x^3]
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 2064

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{9+3x-5x^2+x^3}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt{\frac{128}{27}-\frac{16x}{3}+x^3}}\,dx, x, -\frac{5}{3}+x\right)$$
$$= \frac{\left(128(3-x)\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)\sqrt{\frac{128}{9}+\frac{16x}{3}}}\,dx, x, -\frac{5}{3}+x\right)}{3\sqrt{3}\sqrt{9+3x-5x^2+x^3}}$$
$$= \frac{\left(16(3-x)\sqrt{1+x}\right)\text{Subst}\left(\int \frac{1}{\frac{128}{3}-2x^2}\,dx, x, \frac{4\sqrt{1+x}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt{9+3x-5x^2+x^3}}$$
$$= \frac{(3-x)\sqrt{1+x}\tanh^{-1}\left(\frac{\sqrt{1+x}}{2}\right)}{\sqrt{9+3x-5x^2+x^3}}$$

Mathematica [A] time = 0.0082524, size = 37, normalized size = 0.88

$$-\frac{(x-3)\sqrt{x+1}\tanh^{-1}\left(\frac{\sqrt{x+1}}{2}\right)}{\sqrt{(x-3)^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 + 3*x - 5*x^2 + x^3],x]
```

```
[Out] -(((-3 + x)*Sqrt[1 + x]*ArcTanh[Sqrt[1 + x]/2])/Sqrt[(-3 + x)^2*(1 + x)])
```

Maple [A] time = 0.008, size = 45, normalized size = 1.1

$$-\frac{-3+x}{2}\sqrt{1+x}\left(\ln\left(\sqrt{1+x}+2\right)-\ln\left(\sqrt{1+x}-2\right)\right)\frac{1}{\sqrt{x^3-5x^2+3x+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-5*x^2+3*x+9)^(1/2),x)
```

```
[Out] -1/2*(-3+x)*(1+x)^(1/2)*(ln((1+x)^(1/2)+2)-ln((1+x)^(1/2)-2))/(x^3-5*x^2+3*
x+9)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3-5x^2+3x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(x^3 - 5*x^2 + 3*x + 9), x)
```

Fricas [A] time = 1.72879, size = 161, normalized size = 3.83

$$-\frac{1}{2}\log\left(\frac{2x+\sqrt{x^3-5x^2+3x+9}-6}{x-3}\right)+\frac{1}{2}\log\left(-\frac{2x-\sqrt{x^3-5x^2+3x+9}-6}{x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log((2*x + sqrt(x^3 - 5*x^2 + 3*x + 9) - 6)/(x - 3)) + 1/2*log(-(2*x -
 sqrt(x^3 - 5*x^2 + 3*x + 9) - 6)/(x - 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^3-5x^2+3x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-5*x**2+3*x+9)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**3 - 5*x**2 + 3*x + 9), x)
```

Giac [A] time = 1.07318, size = 46, normalized size = 1.1

$$-\frac{\log\left(\sqrt{x+1}+2\right)}{2\,\mathrm{sgn}\left(x-3\right)}+\frac{\log\left(\left|\sqrt{x+1}-2\right|\right)}{2\,\mathrm{sgn}\left(x-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(sqrt(x + 1) + 2)/sgn(x - 3) + 1/2*log(abs(sqrt(x + 1) - 2))/sgn(x
- 3)
```

3.231 $\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{3/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{15(x+1)(3-x)^3}{256\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{5(x+1)(3-x)^2}{64\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{(x+1)(3-x)}{8\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{15(x+1)^{3/2}(3-x)^3\tanh^{-1}\left(\frac{\sqrt{x+1}}{2}\right)}{512\left(x^3-5x^2+3x+9\right)^{3/2}}$$

```
[Out] ((3 - x)*(1 + x))/(8*(9 + 3*x - 5*x^2 + x^3)^(3/2)) + (5*(3 - x)^2*(1 + x))
/(64*(9 + 3*x - 5*x^2 + x^3)^(3/2)) - (15*(3 - x)^3*(1 + x))/(256*(9 + 3*x
- 5*x^2 + x^3)^(3/2)) + (15*(3 - x)^3*(1 + x)^(3/2)*ArcTanh[Sqrt[1 + x]/2])
/(512*(9 + 3*x - 5*x^2 + x^3)^(3/2))
```

Rubi [A] time = 0.126954, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2067, 2064, 51, 63, 206}

$$-\frac{15(x+1)(3-x)^3}{256\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{5(x+1)(3-x)^2}{64\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{(x+1)(3-x)}{8\left(x^3-5x^2+3x+9\right)^{3/2}}+\frac{15(x+1)^{3/2}(3-x)^3\tanh^{-1}\left(\frac{\sqrt{x+1}}{2}\right)}{512\left(x^3-5x^2+3x+9\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 3*x - 5*x^2 + x^3)^(-3/2),x]
```

```
[Out] ((3 - x)*(1 + x))/(8*(9 + 3*x - 5*x^2 + x^3)^(3/2)) + (5*(3 - x)^2*(1 + x))
/(64*(9 + 3*x - 5*x^2 + x^3)^(3/2)) - (15*(3 - x)^3*(1 + x))/(256*(9 + 3*x
- 5*x^2 + x^3)^(3/2)) + (15*(3 - x)^3*(1 + x)^(3/2)*ArcTanh[Sqrt[1 + x]/2])
/(512*(9 + 3*x - 5*x^2 + x^3)^(3/2))
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 2064

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{128}{27}-\frac{16x}{3}+x^3\right)^{3/2}}\,dx, x, -\frac{5}{3}+x\right)$$

$$= \frac{\left(2097152(3-x)^3(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^3\left(\frac{128}{9}+\frac{16x}{3}\right)^{3/2}}\,dx, x, -\frac{5}{3}+x\right)}{81\sqrt{3}\left(9+3x-5x^2+x^3\right)^{3/2}}$$

$$= \frac{(3-x)(1+x)}{8\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{\left(20480(3-x)^3(1+x)^{3/2}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^2\left(\frac{128}{9}+\frac{16x}{3}\right)^{3/2}}\,dx,\right.}{27\sqrt{3}\left(9+3x-5x^2+x^3\right)^{3/2}}$$

$$= \frac{(3-x)(1+x)}{8\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{5(3-x)^2(1+x)}{64\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{\left(80(3-x)^3(1+x)^{3/2}\right)\text{Subst}\left(\right.}{3\sqrt{3}\left(9+3\right.}$$

$$= \frac{(3-x)(1+x)}{8\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{5(3-x)^2(1+x)}{64\left(9+3x-5x^2+x^3\right)^{3/2}} - \frac{15(3-x)^3(1+x)}{256\left(9+3x-5x^2+x^3\right)^{3/2}} + $$

$$= \frac{(3-x)(1+x)}{8\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{5(3-x)^2(1+x)}{64\left(9+3x-5x^2+x^3\right)^{3/2}} - \frac{15(3-x)^3(1+x)}{256\left(9+3x-5x^2+x^3\right)^{3/2}} + $$

$$= \frac{(3-x)(1+x)}{8\left(9+3x-5x^2+x^3\right)^{3/2}} + \frac{5(3-x)^2(1+x)}{64\left(9+3x-5x^2+x^3\right)^{3/2}} - \frac{15(3-x)^3(1+x)}{256\left(9+3x-5x^2+x^3\right)^{3/2}} + $$

Mathematica [C] time = 0.0075373, size = 35, normalized size = 0.25

$$\frac{(x-3)\,{}_2F_1\left(-\frac{1}{2},3;\frac{1}{2};\frac{x+1}{4}\right)}{32\sqrt{(x-3)^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 3*x - 5*x^2 + x^3)^(-3/2),x]
```

```
[Out] ((-3 + x)*Hypergeometric2F1[-1/2, 3, 1/2, (1 + x)/4])/(32*Sqrt[(-3 + x)^2*(
1 + x)])
```

Maple [A] time = 0.016, size = 144, normalized size = 1.

$$-\frac{(-3+x)^3(1+x)}{1024}\left(15\ln\left(\sqrt{1+x}+2\right)(1+x)^{5/2}-15\ln\left(\sqrt{1+x}-2\right)(1+x)^{5/2}-120\ln\left(\sqrt{1+x}+2\right)(1+x)^{3/2}+120\ \mathrm{l}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-5*x^2+3*x+9)^(3/2),x)
```

```
[Out] -1/1024*(-3+x)^3*(1+x)*(15*ln((1+x)^(1/2)+2)*(1+x)^(5/2)-15*ln((1+x)^(1/2)-
2)*(1+x)^(5/2)-120*ln((1+x)^(1/2)+2)*(1+x)^(3/2)+120*ln((1+x)^(1/2)-2)*(1+x
)^(3/2)+240*ln((1+x)^(1/2)+2)*(1+x)^(1/2)-240*ln((1+x)^(1/2)-2)*(1+x)^(1/2)
-60*x^2+280*x-172)/(x^3-5*x^2+3*x+9)^(3/2)/((1+x)^(1/2)+2)^2/((1+x)^(1/2)-2
)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3-5x^2+3x+9\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-3/2), x)
```

Fricas [A] time = 1.74365, size = 355, normalized size = 2.55

$$-\frac{15\left(x^4-8x^3+18x^2-27\right)\log\left(\frac{2x+\sqrt{x^3-5x^2+3x+9}-6}{x-3}\right)-15\left(x^4-8x^3+18x^2-27\right)\log\left(-\frac{2x-\sqrt{x^3-5x^2+3x+9}-6}{x-3}\right)-4\sqrt{x^3-5}}{1024\left(x^4-8x^3+18x^2-27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1024*(15*(x^4 - 8*x^3 + 18*x^2 - 27)*log((2*x + sqrt(x^3 - 5*x^2 + 3*x +
 9) - 6)/(x - 3)) - 15*(x^4 - 8*x^3 + 18*x^2 - 27)*log(-(2*x - sqrt(x^3 - 5
*x^2 + 3*x + 9) - 6)/(x - 3)) - 4*sqrt(x^3 - 5*x^2 + 3*x + 9)*(15*x^2 - 70*
x + 43))/(x^4 - 8*x^3 + 18*x^2 - 27)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3-5x^2+3x+9\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-5*x**2+3*x+9)**(3/2),x)
```

```
[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.232 $\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx$

Optimal. Leaf size=75

$$-\frac{3}{2}\log\left(1-\frac{x-3}{\sqrt[3]{x^3-5x^2+3x+9}}\right)+\sqrt{3}\tan^{-1}\left(\frac{\frac{2(x-3)}{\sqrt[3]{x^3-5x^2+3x+9}}+1}{\sqrt{3}}\right)-\frac{1}{2}\log(x+1)$$

```
[Out] Sqrt[3]*ArcTan[(1 + (2*(-3 + x))/(9 + 3*x - 5*x^2 + x^3)^(1/3))/Sqrt[3]] -
Log[1 + x]/2 - (3*Log[1 - (-3 + x)/(9 + 3*x - 5*x^2 + x^3)^(1/3)])/2
```

Rubi [B] time = 0.118254, antiderivative size = 188, normalized size of antiderivative = 2.51, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2067, 2064, 60}

$$-\frac{(9-3x)^{2/3}\sqrt[3]{x+1}\log\left(-\frac{32}{3}(x-3)\right)}{2\,3^{2/3}\sqrt[3]{x^3-5x^2+3x+9}}-\frac{\sqrt[3]{3}(9-3x)^{2/3}\sqrt[3]{x+1}\log\left(\frac{\sqrt[3]{3}\sqrt[3]{x+1}}{\sqrt[3]{9-3x}}+1\right)}{2\sqrt[3]{x^3-5x^2+3x+9}}-\frac{(9-3x)^{2/3}\sqrt[3]{x+1}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{x+1}}{\sqrt[6]{3}\sqrt[3]{9-3x}}\right)}{\sqrt[6]{3}\sqrt[3]{x^3-5x^2+3x+9}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 3*x - 5*x^2 + x^3)^(-1/3),x]
```

```
[Out] -(((9 - 3*x)^(2/3)*(1 + x)^(1/3)*ArcTan[1/Sqrt[3] - (2*(1 + x)^(1/3))/(3^(1
/6)*(9 - 3*x)^(1/3))])/(3^(1/6)*(9 + 3*x - 5*x^2 + x^3)^(1/3))) - ((9 - 3*x
)^(2/3)*(1 + x)^(1/3)*Log[(-32*(-3 + x))/3])/(2*3^(2/3)*(9 + 3*x - 5*x^2 +
x^3)^(1/3)) - (3^(1/3)*(9 - 3*x)^(2/3)*(1 + x)^(1/3)*Log[1 + (3^(1/3)*(1 +
x)^(1/3))/(9 - 3*x)^(1/3)])/(2*(9 + 3*x - 5*x^2 + x^3)^(1/3))
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 2064

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 60

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[-(d/b), 3]}, Simp[(Sqrt[3]*q*ArcTan[1/Sqrt[3] - (2*q*(a + b*x
)^(1/3))/(Sqrt[3]*(c + d*x)^(1/3))])/d, x] + (Simp[(3*q*Log[(q*(a + b*x)^(1
/3))/(c + d*x)^(1/3) + 1])/(2*d), x] + Simp[(q*Log[c + d*x])/(2*d), x])] /;
 FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && NegQ[d/b]
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{9+3x-5x^2+x^3}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt[3]{\frac{128}{27}-\frac{16x}{3}+x^3}}\,dx, x, -\frac{5}{3}+x\right)$$

$$= \frac{\left(16\ 2^{2/3}(3-x)^{2/3}\sqrt[3]{1+x}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^{2/3}\sqrt[3]{\frac{128}{9}+\frac{16x}{3}}}\,dx, x, -\frac{5}{3}+x\right)}{3\sqrt[3]{9+3x-5x^2+x^3}}$$

$$= -\frac{\sqrt{3}(3-x)^{2/3}\sqrt[3]{1+x}\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{1+x}}{\sqrt{3}\sqrt[3]{3-x}}\right)}{\sqrt[3]{9+3x-5x^2+x^3}} - \frac{(3-x)^{2/3}\sqrt[3]{1+x}\log(3-x)}{2\sqrt[3]{9+3x-5x^2+x^3}} - \frac{3(3-x)^{2/3}\sqrt[3]{}}{2\sqrt[3]{9}}$$

Mathematica [C] time = 0.0112024, size = 49, normalized size = 0.65

$$\frac{3(x-3)\sqrt[3]{x+1}\,{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};\frac{3-x}{4}\right)}{2^{2/3}\sqrt[3]{(x-3)^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 3*x - 5*x^2 + x^3)^(-1/3),x]
```

```
[Out] (3*(-3 + x)*(1 + x)^(1/3)*Hypergeometric2F1[1/3, 1/3, 4/3, (3 - x)/4])/(2^(
2/3)*((-3 + x)^2*(1 + x))^(1/3))
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x^3-5x^2+3x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-5*x^2+3*x+9)^(1/3),x)
```

```
[Out] int(1/(x^3-5*x^2+3*x+9)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3-5x^2+3x+9\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-1/3), x)
```

Fricas [B] time = 1.81094, size = 352, normalized size = 4.69

$$-\sqrt{3}\arctan\left(\frac{\sqrt{3}(x-3)+2\sqrt{3}\left(x^3-5x^2+3x+9\right)^{\frac{1}{3}}}{3(x-3)}\right)+\frac{1}{2}\log\left(\frac{x^2+\left(x^3-5x^2+3x+9\right)^{\frac{1}{3}}(x-3)-6x+\left(x^3-5x^2+3\right.}{x^2-6x+9}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x, algorithm="fricas")
```

```
[Out] -sqrt(3)*arctan(1/3*(sqrt(3)*(x - 3) + 2*sqrt(3)*(x^3 - 5*x^2 + 3*x + 9)^(1
/3))/(x - 3)) + 1/2*log((x^2 + (x^3 - 5*x^2 + 3*x + 9)^(1/3)*(x - 3) - 6*x
+ (x^3 - 5*x^2 + 3*x + 9)^(2/3) + 9)/(x^2 - 6*x + 9)) - log(-(x - (x^3 - 5*
x^2 + 3*x + 9)^(1/3) - 3)/(x - 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x^3-5x^2+3x+9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-5*x**2+3*x+9)**(1/3),x)
```

```
[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3-5x^2+3x+9\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-1/3), x)
```

3.233 $\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{2/3}}\,dx$

Optimal. Leaf size=29

$$\frac{3(3-x)(x+1)}{4\left(x^3-5x^2+3x+9\right)^{2/3}}$$

```
[Out] (3*(3 - x)*(1 + x))/(4*(9 + 3*x - 5*x^2 + x^3)^(2/3))
```

Rubi [A] time = 0.0329589, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2067, 2064, 37}

$$\frac{3(3-x)(x+1)}{4\left(x^3-5x^2+3x+9\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 3*x - 5*x^2 + x^3)^(-2/3),x]

[Out] (3*(3 - x)*(1 + x))/(4*(9 + 3*x - 5*x^2 + x^3)^(2/3))
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 2064

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{2/3}}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{128}{27}-\frac{16x}{3}+x^3\right)^{2/3}}\,dx, x, -\frac{5}{3}+x\right)$$

$$= \frac{\left(512\sqrt[3]{2}(3-x)^{4/3}(1+x)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^{4/3}\left(\frac{128}{9}+\frac{16x}{3}\right)^{2/3}}\,dx, x, -\frac{5}{3}+x\right)}{9\left(9+3x-5x^2+x^3\right)^{2/3}}$$

$$= \frac{3(3-x)(1+x)}{4\left(9+3x-5x^2+x^3\right)^{2/3}}$$

Mathematica [A] time = 0.0077999, size = 23, normalized size = 0.79

$$-\frac{3(x-3)(x+1)}{4\left((x-3)^2(x+1)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 3*x - 5*x^2 + x^3)^(-2/3),x]
```

```
[Out] (-3*(-3 + x)*(1 + x))/(4*((-3 + x)^2*(1 + x))^(2/3))
```

Maple [A] time = 0.002, size = 24, normalized size = 0.8

$$-\frac{(3+3x)(-3+x)}{4}\left(x^3-5x^2+3x+9\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-5*x^2+3*x+9)^(2/3),x)
```

```
[Out] -3/4*(1+x)*(-3+x)/(x^3-5*x^2+3*x+9)^(2/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3-5x^2+3x+9\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-2/3), x)
```

Fricas [A] time = 1.74807, size = 59, normalized size = 2.03

$$-\frac{3\left(x^3-5x^2+3x+9\right)^{\frac{1}{3}}}{4(x-3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x, algorithm="fricas")
```

```
[Out] -3/4*(x^3 - 5*x^2 + 3*x + 9)^(1/3)/(x - 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3-5x^2+3x+9\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-5*x**2+3*x+9)**(2/3),x)
```

```
[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-2/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3-5x^2+3x+9\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-2/3), x)
```

3.234 $\int \frac{1}{(9+3x-5x^2+x^3)^{4/3}}\,dx$

Optimal. Leaf size=92

$$-\frac{27(x+1)(3-x)^3}{320\left(x^3-5x^2+3x+9\right)^{4/3}}+\frac{9(x+1)(3-x)^2}{80\left(x^3-5x^2+3x+9\right)^{4/3}}+\frac{3(x+1)(3-x)}{20\left(x^3-5x^2+3x+9\right)^{4/3}}$$

```
[Out] (3*(3 - x)*(1 + x))/(20*(9 + 3*x - 5*x^2 + x^3)^(4/3)) + (9*(3 - x)^2*(1 +
x))/(80*(9 + 3*x - 5*x^2 + x^3)^(4/3)) - (27*(3 - x)^3*(1 + x))/(320*(9 + 3
*x - 5*x^2 + x^3)^(4/3))
```

Rubi [A] time = 0.0850329, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2067, 2064, 45, 37}

$$-\frac{27(x+1)(3-x)^3}{320\left(x^3-5x^2+3x+9\right)^{4/3}}+\frac{9(x+1)(3-x)^2}{80\left(x^3-5x^2+3x+9\right)^{4/3}}+\frac{3(x+1)(3-x)}{20\left(x^3-5x^2+3x+9\right)^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(9 + 3*x - 5*x^2 + x^3)^(-4/3),x]
```

```
[Out] (3*(3 - x)*(1 + x))/(20*(9 + 3*x - 5*x^2 + x^3)^(4/3)) + (9*(3 - x)^2*(1 +
x))/(80*(9 + 3*x - 5*x^2 + x^3)^(4/3)) - (27*(3 - x)^3*(1 + x))/(320*(9 + 3
*x - 5*x^2 + x^3)^(4/3))
```

Rule 2067

```
Int[(P3_)^(p_), x_Symbol] :> With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1]
, c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*
d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x +
c/(3*d)] /; NeQ[c, 0]] /; FreeQ[p, x] && PolyQ[P3, x, 3]
```

Rule 2064

```
Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] :> Dist[(a + b*x +
d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*
x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] &&
!IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
```

```
1]
```

Rubi steps

$$\int \frac{1}{\left(9+3x-5x^2+x^3\right)^{4/3}}\,dx = \text{Subst}\left(\int \frac{1}{\left(\frac{128}{27}-\frac{16x}{3}+x^3\right)^{4/3}}\,dx, x, -\frac{5}{3}+x\right)$$

$$= \frac{\left(262144\ 2^{2/3}(3-x)^{8/3}(1+x)^{4/3}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^{8/3}\left(\frac{128}{9}+\frac{16x}{3}\right)^{4/3}}\,dx, x, -\frac{5}{3}+x\right)}{81\left(9+3x-5x^2+x^3\right)^{4/3}}$$

$$= \frac{3(3-x)(1+x)}{20\left(9+3x-5x^2+x^3\right)^{4/3}} + \frac{\left(4096\ 2^{2/3}(3-x)^{8/3}(1+x)^{4/3}\right)\text{Subst}\left(\int \frac{1}{\left(\frac{128}{9}-\frac{32x}{3}\right)^{5/3}\left(\frac{128}{9}+\frac{16}{3}\right.}\right.}{45\left(9+3x-5x^2+x^3\right)^{4/3}}$$

$$= \frac{3(3-x)(1+x)}{20\left(9+3x-5x^2+x^3\right)^{4/3}} + \frac{9(3-x)^2(1+x)}{80\left(9+3x-5x^2+x^3\right)^{4/3}} + \frac{\left(16\ 2^{2/3}(3-x)^{8/3}(1+x)^{4/3}\right)\text{S}}{5(9}$$

$$= \frac{3(3-x)(1+x)}{20\left(9+3x-5x^2+x^3\right)^{4/3}} + \frac{9(3-x)^2(1+x)}{80\left(9+3x-5x^2+x^3\right)^{4/3}} - \frac{27(3-x)^3(1+x)}{320\left(9+3x-5x^2+x^3\right)^{4/3}}$$

Mathematica [A] time = 0.0080243, size = 32, normalized size = 0.35

$$\frac{3\left(9x^2-42x+29\right)}{320(x-3)\sqrt[3]{(x-3)^2(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(9 + 3*x - 5*x^2 + x^3)^(-4/3),x]
```

```
[Out] (3*(29 - 42*x + 9*x^2))/(320*(-3 + x)*((-3 + x)^2*(1 + x))^(1/3))
```

Maple [A] time = 0.003, size = 34, normalized size = 0.4

$$\frac{(3+3x)(-3+x)\left(9x^2-42x+29\right)}{320}\left(x^3-5x^2+3x+9\right)^{-\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-5*x^2+3*x+9)^(4/3),x)
```

```
[Out] 3/320*(1+x)*(-3+x)*(9*x^2-42*x+29)/(x^3-5*x^2+3*x+9)^(4/3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3-5x^2+3x+9\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-4/3), x)
```

Fricas [A] time = 1.62835, size = 115, normalized size = 1.25

$$\frac{3 \left(x^3-5 x^2+3 x+9\right)^{\frac{2}{3}} \left(9 x^2-42 x+29\right)}{320 \left(x^4-8 x^3+18 x^2-27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x, algorithm="fricas")
```

```
[Out] 3/320*(x^3 - 5*x^2 + 3*x + 9)^(2/3)*(9*x^2 - 42*x + 29)/(x^4 - 8*x^3 + 18*x
^2 - 27)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3-5x^2+3x+9\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-5*x**2+3*x+9)**(4/3),x)
```

```
[Out] Integral((x**3 - 5*x**2 + 3*x + 9)**(-4/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3-5 x^2+3 x+9\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-5*x^2+3*x+9)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 5*x^2 + 3*x + 9)^(-4/3), x)
```

3.235 $\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sin^{-1}\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcSin[(3 - 4*x)/Sqrt[41]]/Sqrt[2])
```

Rubi [A] time = 0.0137482, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$-\frac{\sin^{-1}\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 + 3*x - 2*x^2],x]

[Out] -(ArcSin[(3 - 4*x)/Sqrt[41]]/Sqrt[2])
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{4+3x-2x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{41}}}\,dx, x, 3-4x\right)}{\sqrt{82}}\\ &= -\frac{\sin^{-1}\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.00997, size = 19, normalized size = 1.

$$-\frac{\sin^{-1}\left(\frac{3-4x}{\sqrt{41}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 + 3*x - 2*x^2],x]
```

```
[Out] -(ArcSin[(3 - 4*x)/Sqrt[41]]/Sqrt[2])
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$\frac{\sqrt{2}}{2}\arcsin\left(\frac{4\sqrt{41}}{41}\left(x-\frac{3}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^2+3*x+4)^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*arcsin(4/41*41^(1/2)*(x-3/4))
```

Maxima [A] time = 1.44799, size = 22, normalized size = 1.16

$$-\frac{1}{2}\sqrt{2}\arcsin\left(-\frac{1}{41}\sqrt{41}(4x-3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+3*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arcsin(-1/41*sqrt(41)*(4*x - 3))
```

Fricas [B] time = 1.75203, size = 93, normalized size = 4.89

$$-\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-2x^2+3x+4}-2\sqrt{2}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+3*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(1/2*(sqrt(2)*sqrt(-2*x^2 + 3*x + 4) - 2*sqrt(2))/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-2x^2+3x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**2+3*x+4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-2*x**2 + 3*x + 4), x)
```

Giac [A] time = 1.08688, size = 22, normalized size = 1.16

$$\frac{1}{2}\,\sqrt{2}\arcsin\left(\frac{1}{41}\,\sqrt{41}(4\,x-3)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+3*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arcsin(1/41*sqrt(41)*(4*x - 3))
```

3.236 $\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx$

Optimal. Leaf size=8

$$-\sin^{-1}(2-x)$$

```
[Out] -ArcSin[2 - x]
```

Rubi [A] time = 0.0052765, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$-\sin^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-3 + 4*x - x^2],x]

[Out] -ArcSin[2 - x]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-3+4x-x^2}}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, 4-2x\right)\right)\\ &= -\sin^{-1}(2-x)\end{aligned}$$

Mathematica [A] time = 0.0059099, size = 8, normalized size = 1.

$$-\sin^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-3 + 4*x - x^2],x]

[Out] -ArcSin[2 - x]
```

Maple [A] time = 0.003, size = 5, normalized size = 0.6

$$\arcsin(-2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+4*x-3)^(1/2),x)
```

```
[Out] arcsin(-2+x)
```

Maxima [B] time = 1.4172, size = 11, normalized size = 1.38

$$-\arcsin(-x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+4*x-3)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-x + 2)
```

Fricas [B] time = 1.79128, size = 74, normalized size = 9.25

$$-\arctan\left(\frac{\sqrt{-x^2+4x-3}(x-2)}{x^2-4x+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+4*x-3)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-x^2 + 4*x - 3)*(x - 2)/(x^2 - 4*x + 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+4x-3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+4*x-3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 + 4*x - 3), x)
```

Giac [A] time = 1.05988, size = 5, normalized size = 0.62

$$\arcsin(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+4*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(x - 2)
```

3.237 $\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx$

Optimal. Leaf size=12

$$\frac{\sin^{-1}(6x+5)}{\sqrt{3}}$$

```
[Out] ArcSin[5 + 6*x]/Sqrt[3]
```

Rubi [A] time = 0.0044271, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$\frac{\sin^{-1}(6x+5)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-2 - 5*x - 3*x^2],x]
```

```
[Out] ArcSin[5 + 6*x]/Sqrt[3]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-2-5x-3x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, -5-6x\right)}{\sqrt{3}}\\ &= \frac{\sin^{-1}(5+6x)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0068594, size = 12, normalized size = 1.

$$\frac{\sin^{-1}(6x+5)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-2 - 5*x - 3*x^2],x]
```

```
[Out] ArcSin[5 + 6*x]/Sqrt[3]
```

Maple [A] time = 0.004, size = 12, normalized size = 1.

$$\frac{\arcsin\left(6x+5\right)\sqrt{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2-5*x-2)^(1/2),x)
```

```
[Out] 1/3*arcsin(6*x+5)*3^(1/2)
```

Maxima [A] time = 1.42477, size = 15, normalized size = 1.25

$$\frac{1}{3}\sqrt{3}\arcsin\left(6x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-5*x-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsin(6*x + 5)
```

Fricas [B] time = 2.03931, size = 115, normalized size = 9.58

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3x^2-5x-2}(6x+5)}{6\left(3x^2+5x+2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-5*x-2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*sqrt(-3*x^2 - 5*x - 2)*(6*x + 5)/(3*x^2 + 5
*x + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^2-5x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2-5*x-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**2 - 5*x - 2), x)
```

Giac [A] time = 1.06858, size = 15, normalized size = 1.25

$$\frac{1}{3}\sqrt{3}\arcsin\left(6x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2-5*x-2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arcsin(6*x + 5)
```

3.238 $\int \frac{1}{\sqrt{1-x^2}(4+x^2)}\,dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}$$

```
[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[1 - x^2])]/(2*Sqrt[5])
```

Rubi [A] time = 0.0075575, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*(4 + x^2)),x]
```

```
[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[1 - x^2])]/(2*Sqrt[5])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-x^2}\,(4+x^2)}\,dx &= \text{Subst}\left(\int \frac{1}{4+5x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}\end{aligned}$$

Mathematica [A] time = 0.0076511, size = 31, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{1-x^2}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*(4 + x^2)),x]

[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[1 - x^2])]/(2*Sqrt[5])
```

Maple [A] time = 0.01, size = 29, normalized size = 0.9

$$-\frac{\sqrt{5}}{10}\arctan\left(\frac{x\sqrt{5}}{2x^2-2}\sqrt{-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+4)/(-x^2+1)^(1/2),x)

[Out] -1/10*5^(1/2)*arctan(1/2*5^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+4)\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)/(-x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((x^2 + 4)*sqrt(-x^2 + 1)), x)
```

Fricas [A] time = 2.07916, size = 70, normalized size = 2.26

$$-\frac{1}{10}\sqrt{5}\arctan\left(\frac{2\sqrt{5}\sqrt{-x^2+1}}{5x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)/(-x^2+1)^(1/2),x, algorithm="fricas")

[Out] -1/10*sqrt(5)*arctan(2/5*sqrt(5)*sqrt(-x^2 + 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(x-1)(x+1)}\,(x^2+4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+4)/(-x**2+1)**(1/2),x)

[Out] Integral(1/(sqrt(-(x - 1)*(x + 1))*(x**2 + 4)), x)
```

Giac [B] time = 1.08879, size = 69, normalized size = 2.23

$$\frac{1}{20}\sqrt{5}\left(\pi \operatorname{sgn}(x) + 2\arctan\left(-\frac{\sqrt{5}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{5\left(\sqrt{-x^2+1}-1\right)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/20*sqrt(5)*(pi*sgn(x) + 2*arctan(-1/5*sqrt(5)*x*((sqrt(-x^2 + 1) - 1)^2/x
^2 - 1)/(sqrt(-x^2 + 1) - 1)))
```

3.239 $\int \frac{1}{(4+x^2)\sqrt{1+4x^2}}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{15}x}{2\sqrt{4x^2+1}}\right)}{2\sqrt{15}}$$

```
[Out] ArcTanh[(Sqrt[15]*x)/(2*Sqrt[1 + 4*x^2])]/(2*Sqrt[15])
```

Rubi [A] time = 0.0099887, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{15}x}{2\sqrt{4x^2+1}}\right)}{2\sqrt{15}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((4 + x^2)*Sqrt[1 + 4*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[15]*x)/(2*Sqrt[1 + 4*x^2])]/(2*Sqrt[15])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(4+x^2\right)\sqrt{1+4x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{4-15x^2}\,dx, x, \frac{x}{\sqrt{1+4x^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{15}x}{2\sqrt{1+4x^2}}\right)}{2\sqrt{15}}\end{aligned}$$

Mathematica [A] time = 0.0108849, size = 31, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{15}x}{2\sqrt{4x^2+1}}\right)}{2\sqrt{15}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((4 + x^2)*Sqrt[1 + 4*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[15]*x)/(2*Sqrt[1 + 4*x^2])]/(2*Sqrt[15])
```

Maple [A] time = 0.008, size = 22, normalized size = 0.7

$$\frac{\sqrt{15}}{30}\mathrm{Artanh}\left(\frac{x\sqrt{15}}{2}\frac{1}{\sqrt{4\,x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+4)/(4*x^2+1)^(1/2),x)
```

```
[Out] 1/30*arctanh(1/2*x*15^(1/2)/(4*x^2+1)^(1/2))*15^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4\,x^2+1}\left(x^2+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(4*x^2 + 1)*(x^2 + 4)), x)
```

Fricas [B] time = 1.97988, size = 157, normalized size = 5.06

$$\frac{1}{60}\,\sqrt{15}\log\left(\frac{961\,x^2+8\,\sqrt{15}\left(31\,x^2+4\right)+4\,\sqrt{4\,x^2+1}\left(31\,\sqrt{15}x+120\,x\right)+124}{x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/60*sqrt(15)*log((961*x^2 + 8*sqrt(15)*(31*x^2 + 4) + 4*sqrt(4*x^2 + 1)*(3
1*sqrt(15)*x + 120*x) + 124)/(x^2 + 4))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+4\right)\sqrt{4x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+4)/(4*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/((x**2 + 4)*sqrt(4*x**2 + 1)), x)
```

Giac [B] time = 1.06585, size = 77, normalized size = 2.48

$$-\frac{1}{60}\sqrt{15}\log\left(\frac{\left(2x-\sqrt{4x^2+1}\right)^2-8\sqrt{15}+31}{\left(2x-\sqrt{4x^2+1}\right)^2+8\sqrt{15}+31}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)/(4*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/60*sqrt(15)*log(((2*x - sqrt(4*x^2 + 1))^2 - 8*sqrt(15) + 31)/((2*x - sq
rt(4*x^2 + 1))^2 + 8*sqrt(15) + 31))
```

3.240 $\int \frac{x}{(3-x^2)\sqrt{5-x^2}} \, dx$

Optimal. Leaf size=24

$$\frac{\tanh^{-1}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[5 - x^2]/Sqrt[2]]/Sqrt[2]
```

Rubi [A] time = 0.0192104, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {444, 63, 207}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((3 - x^2)*Sqrt[5 - x^2]),x]
```

```
[Out] ArcTanh[Sqrt[5 - x^2]/Sqrt[2]]/Sqrt[2]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(3-x^2\right)\sqrt{5-x^2}} \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(3-x)\sqrt{5-x}} \, dx, x, x^2\right) \\
&= -\operatorname{Subst}\left(\int \frac{1}{-2+x^2} \, dx, x, \sqrt{5-x^2}\right) \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.0094691, size = 24, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{5-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((3 - x^2)*Sqrt[5 - x^2]),x]
```

```
[Out] ArcTanh[Sqrt[5 - x^2]/Sqrt[2]]/Sqrt[2]
```

Maple [B] time = 0.025, size = 100, normalized size = 4.2

$$\frac{\sqrt{2}}{4}\mathrm{Artanh}\left(\frac{\left(4+2\sqrt{3}\left(x+\sqrt{3}\right)\right)\sqrt{2}}{4}\frac{1}{\sqrt{-\left(x+\sqrt{3}\right)^2+2\sqrt{3}\left(x+\sqrt{3}\right)+2}}\right)+\frac{\sqrt{2}}{4}\mathrm{Artanh}\left(\frac{\left(4-2\sqrt{3}\left(x-\sqrt{3}\right)\right)\sqrt{2}}{4}\frac{}{\sqrt{-(}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+3)/(-x^2+5)^(1/2),x)
```

```
[Out] 1/4*2^(1/2)*arctanh(1/4*(4+2*3^(1/2)*(x+3^(1/2)))*2^(1/2)/(-(x+3^(1/2))^2+2
*3^(1/2)*(x+3^(1/2))+2)^(1/2))+1/4*2^(1/2)*arctanh(1/4*(4-2*3^(1/2)*(x-3^(1
/2)))*2^(1/2)/(-(x-3^(1/2))^2-2*3^(1/2)*(x-3^(1/2))+2)^(1/2))
```

Maxima [B] time = 1.53592, size = 151, normalized size = 6.29

$$\frac{1}{12}\sqrt{3}\left(\sqrt{3}\sqrt{2}\log\left(\sqrt{3}+\frac{2\sqrt{2}\sqrt{-x^2+5}}{|2x+2\sqrt{3}|}+\frac{4}{|2x+2\sqrt{3}|}\right)+\sqrt{3}\sqrt{2}\log\left(-\sqrt{3}+\frac{2\sqrt{2}\sqrt{-x^2+5}}{|2x-2\sqrt{3}|}+\frac{4}{|2x-2\sqrt{3}|}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*(sqrt(3)*sqrt(2)*log(sqrt(3) + 2*sqrt(2)*sqrt(-x^2 + 5)/abs(2*
x + 2*sqrt(3)) + 4/abs(2*x + 2*sqrt(3))) + sqrt(3)*sqrt(2)*log(-sqrt(3) + 2
*sqrt(2)*sqrt(-x^2 + 5)/abs(2*x - 2*sqrt(3)) + 4/abs(2*x - 2*sqrt(3))))
```

Fricas [B] time = 2.10653, size = 126, normalized size = 5.25

$$\frac{1}{8}\sqrt{2}\log\left(\frac{x^4-4\sqrt{2}\left(x^2-7\right)\sqrt{-x^2+5}-22x^2+89}{x^4-6x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log((x^4 - 4*sqrt(2)*(x^2 - 7)*sqrt(-x^2 + 5) - 22*x^2 + 89)/(x
^4 - 6*x^2 + 9))
```

Sympy [A] time = 3.41892, size = 61, normalized size = 2.54

$$-\begin{cases} -\frac{\sqrt{2}\operatorname{acoth}\left(\frac{\sqrt{2}}{\sqrt{5-x^2}}\right)}{2} & \text{for } \frac{1}{5-x^2} > \frac{1}{2} \\ -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}}{\sqrt{5-x^2}}\right)}{2} & \text{for } \frac{1}{5-x^2} < \frac{1}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+3)/(-x**2+5)**(1/2),x)
```

```
[Out] -Piecewise((-sqrt(2)*acoth(sqrt(2)/sqrt(5 - x**2))/2, 1/(5 - x**2) > 1/2),
(-sqrt(2)*atanh(sqrt(2)/sqrt(5 - x**2))/2, 1/(5 - x**2) < 1/2))
```

Giac [B] time = 1.07389, size = 57, normalized size = 2.38

$$\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}+\sqrt{-x^2+5}\right)-\frac{1}{4}\sqrt{2}\log\left(\left|-\sqrt{2}+\sqrt{-x^2+5}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+3)/(-x^2+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(sqrt(2) + sqrt(-x^2 + 5)) - 1/4*sqrt(2)*log(abs(-sqrt(2) +
sqrt(-x^2 + 5)))
```

3.241 $\int \frac{x}{\sqrt{3-x^2}(5-x^2)} dx$

Optimal. Leaf size=25

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[Sqrt[3 - x^2]/Sqrt[2]]/Sqrt[2])
```

Rubi [A] time = 0.0187771, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {444, 63, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[3 - x^2]*(5 - x^2)),x]

[Out] -(ArcTan[Sqrt[3 - x^2]/Sqrt[2]]/Sqrt[2])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{3-x^2}\left(5-x^2\right)} dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3-x}(5-x)} dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{1}{2+x^2} dx, x, \sqrt{3-x^2}\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.0091463, size = 25, normalized size = 1.

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[3 - x^2]*(5 - x^2)),x]
```

```
[Out] -(ArcTan[Sqrt[3 - x^2]/Sqrt[2]]/Sqrt[2])
```

Maple [B] time = 0.027, size = 100, normalized size = 4.

$$-\frac{\sqrt{2}}{4}\arctan\left(\frac{\left(-4-2\sqrt{5}\left(x-\sqrt{5}\right)\right)\sqrt{2}}{4}\frac{1}{\sqrt{-\left(x-\sqrt{5}\right)^{2}-2\sqrt{5}\left(x-\sqrt{5}\right)-2}}\right)-\frac{\sqrt{2}}{4}\arctan\left(\frac{\left(-4+2\sqrt{5}\left(x+\sqrt{5}\right)\right)\sqrt{2}}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+5)/(-x^2+3)^(1/2),x)
```

```
[Out] -1/4*2^(1/2)*arctan(1/4*(-4-2*5^(1/2)*(x-5^(1/2)))*2^(1/2)/(-(x-5^(1/2))^2-
2*5^(1/2)*(x-5^(1/2))-2)^(1/2))-1/4*2^(1/2)*arctan(1/4*(-4+2*5^(1/2)*(x+5^(
1/2)))*2^(1/2)/(-(x+5^(1/2))^2+2*5^(1/2)*(x+5^(1/2))-2)^(1/2))
```

Maxima [B] time = 1.46323, size = 136, normalized size = 5.44

$$-\frac{1}{20}\sqrt{5}\left(\sqrt{5}\sqrt{2}\arcsin\left(\frac{2\sqrt{5}\sqrt{3}x}{3\left|2x+2\sqrt{5}\right|}+\frac{2\sqrt{3}}{\left|2x+2\sqrt{5}\right|}\right)-\sqrt{5}\sqrt{2}\arcsin\left(\frac{2\sqrt{5}\sqrt{3}x}{3\left|2x-2\sqrt{5}\right|}-\frac{2\sqrt{3}}{\left|2x-2\sqrt{5}\right|}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(5)*(sqrt(5)*sqrt(2)*arcsin(2/3*sqrt(5)*sqrt(3)*x/abs(2*x + 2*sqr
t(5)) + 2*sqrt(3)/abs(2*x + 2*sqrt(5))) - sqrt(5)*sqrt(2)*arcsin(2/3*sqrt(5
)*sqrt(3)*x/abs(2*x - 2*sqrt(5)) - 2*sqrt(3)/abs(2*x - 2*sqrt(5))))
```

Fricas [A] time = 1.99105, size = 93, normalized size = 3.72

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^{2}-1\right)\sqrt{-x^{2}+3}}{4\left(x^{2}-3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/4*sqrt(2)*(x^2 - 1)*sqrt(-x^2 + 3)/(x^2 - 3))
```

Sympy [A] time = 3.30178, size = 24, normalized size = 0.96

$$-\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt{3-x^2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+5)/(-x**2+3)**(1/2),x)
```

```
[Out] -sqrt(2)*atan(sqrt(2)*sqrt(3 - x**2)/2)/2
```

Giac [A] time = 1.05948, size = 27, normalized size = 1.08

$$-\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\sqrt{-x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+5)/(-x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 3))
```

3.242 $\int \frac{1}{\sqrt{2+x^2}(-1+x^4)}\,dx$

Optimal. Leaf size=43

$$-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^2+2}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{x^2+2}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[x/Sqrt[2 + x^2]]/2 - ArcTanh[(Sqrt[3]*x)/Sqrt[2 + x^2]]/(2*Sqrt[3])
```

Rubi [A] time = 0.0197251, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {1175, 377, 206, 203}

$$-\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{x^2+2}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{x^2+2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[2 + x^2]*(-1 + x^4)),x]
```

```
[Out] -ArcTan[x/Sqrt[2 + x^2]]/2 - ArcTanh[(Sqrt[3]*x)/Sqrt[2 + x^2]]/(2*Sqrt[3])
```

Rule 1175

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{r
= Rt[-(a*c), 2]}, -Dist[c/(2*r), Int[(d + e*x^2)^q/(r - c*x^2), x], x] - Di
st[c/(2*r), Int[(d + e*x^2)^q/(r + c*x^2), x], x]] /; FreeQ[{a, c, d, e, q}
, x] && NeQ[c*d^2 + a*e^2, 0] && !IntegerQ[q]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{2+x^2}\left(-1+x^4\right)}\,dx = -\left(\frac{1}{2}\int \frac{1}{\left(1-x^2\right)\sqrt{2+x^2}}\,dx\right) - \frac{1}{2}\int \frac{1}{\left(1+x^2\right)\sqrt{2+x^2}}\,dx$$
$$= -\left(\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{x}{\sqrt{2+x^2}}\right)\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{2+x^2}}\right)$$
$$= -\frac{1}{2}\tan^{-1}\left(\frac{x}{\sqrt{2+x^2}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{2+x^2}}\right)}{2\sqrt{3}}$$

Mathematica [C] time = 0.0454008, size = 96, normalized size = 2.23

$$\frac{1}{12}\left(-3\tan^{-1}\left(\frac{-x+2i}{\sqrt{x^2+2}}\right) + 3\tan^{-1}\left(\frac{x+2i}{\sqrt{x^2+2}}\right) + \sqrt{3}\tanh^{-1}\left(\frac{2-x}{\sqrt{3}\sqrt{x^2+2}}\right) - \sqrt{3}\tanh^{-1}\left(\frac{x+2}{\sqrt{3}\sqrt{x^2+2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[2 + x^2]*(-1 + x^4)),x]
```

```
[Out] (-3*ArcTan[(2*I - x)/Sqrt[2 + x^2]] + 3*ArcTan[(2*I + x)/Sqrt[2 + x^2]] + S
qrt[3]*ArcTanh[(2 - x)/(Sqrt[3]*Sqrt[2 + x^2])] - Sqrt[3]*ArcTanh[(2 + x)/(
Sqrt[3]*Sqrt[2 + x^2])])/12
```

Maple [B] time = 0.018, size = 70, normalized size = 1.6

$$-\frac{1}{2}\arctan\left(x\frac{1}{\sqrt{x^2+2}}\right) + \frac{\sqrt{3}}{12}\text{Artanh}\left(\frac{(4-2x)\sqrt{3}}{6}\frac{1}{\sqrt{(1+x)^2+1-2x}}\right) - \frac{\sqrt{3}}{12}\text{Artanh}\left(\frac{(4+2x)\sqrt{3}}{6}\frac{1}{\sqrt{(-1+x)^2+1+2x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-1)/(x^2+2)^(1/2),x)
```

```
[Out] -1/2*arctan(x/(x^2+2)^(1/2))+1/12*3^(1/2)*arctanh(1/6*(4-2*x)*3^(1/2)/((1+x
)^2+1-2*x)^(1/2))-1/12*3^(1/2)*arctanh(1/6*(4+2*x)*3^(1/2)/((-1+x)^2+1+2*x)
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4-1\right)\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)/(x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 - 1)*sqrt(x^2 + 2)), x)
```

Fricas [B] time = 2.02215, size = 189, normalized size = 4.4

$$\frac{1}{12}\sqrt{3}\log\left(\frac{4x^2-\sqrt{3}\left(2x^2+1\right)-\sqrt{x^2+2}\left(2\sqrt{3}x-3x\right)+2}{x^2-1}\right)-\frac{1}{2}\arctan\left(-x^2+\sqrt{x^2+2}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)/(x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log((4*x^2 - sqrt(3)*(2*x^2 + 1) - sqrt(x^2 + 2)*(2*sqrt(3)*x
- 3*x) + 2)/(x^2 - 1)) - 1/2*arctan(-x^2 + sqrt(x^2 + 2)*x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x-1)(x+1)\left(x^2+1\right)\sqrt{x^2+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-1)/(x**2+2)**(1/2),x)
```

```
[Out] Integral(1/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(x**2 + 2)), x)
```

Giac [B] time = 1.11023, size = 100, normalized size = 2.33

$$-\frac{1}{12}\sqrt{3}\log\left(\frac{\left|2\left(x-\sqrt{x^2+2}\right)^2-4\sqrt{3}-8\right|}{\left|2\left(x-\sqrt{x^2+2}\right)^2+4\sqrt{3}-8\right|}\right)+\frac{1}{2}\arctan\left(\frac{1}{2}\left(x-\sqrt{x^2+2}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1)/(x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*log(abs(2*(x - sqrt(x^2 + 2))^2 - 4*sqrt(3) - 8)/abs(2*(x - s
qrt(x^2 + 2))^2 + 4*sqrt(3) - 8)) + 1/2*arctan(1/2*(x - sqrt(x^2 + 2))^2)
```

3.243 $\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx$

Optimal. Leaf size=62

$$-\frac{\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTanh[(5 + 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])]/(2*Sqrt[7]) - ArcTanh[Sqr
t[4 + 2*x + x^2]/Sqrt[3]]/(2*Sqrt[3])
```

Rubi [A] time = 0.0363772, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1033, 724, 206, 688, 207}

$$-\frac{\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((-1 + x^2)*Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] -ArcTanh[(5 + 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])]/(2*Sqrt[7]) - ArcTanh[Sqr
t[4 + 2*x + x^2]/Sqrt[3]]/(2*Sqrt[3])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(-1+x^2)\sqrt{4+2x+x^2}}\,dx = \frac{1}{2}\int \frac{1}{(-1+x)\sqrt{4+2x+x^2}}\,dx + \frac{1}{2}\int \frac{1}{(1+x)\sqrt{4+2x+x^2}}\,dx$$
$$= 2\,\text{Subst}\left(\int \frac{1}{-12+4x^2}\,dx, x, \sqrt{4+2x+x^2}\right) - \text{Subst}\left(\int \frac{1}{28-x^2}\,dx, x, \frac{10+4x}{\sqrt{4+2x+x^2}}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{10+4x}{2\sqrt{7}\sqrt{4+2x+x^2}}\right)}{2\sqrt{7}} - \frac{\tanh^{-1}\left(\frac{\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Mathematica [A] time = 0.0246336, size = 61, normalized size = 0.98

$$\frac{1}{42}\left(-3\sqrt{7}\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right) - 7\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{(x+1)^2+3}}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-1 + x^2)*Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] (-3*Sqrt[7]*ArcTanh[(5 + 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])] - 7*Sqrt[3]*Ar
cTanh[Sqrt[3 + (1 + x)^2]/Sqrt[3]])/42
```

Maple [A] time = 0.011, size = 49, normalized size = 0.8

$$-\frac{\sqrt{3}}{6}\text{Artanh}\left(\sqrt{3}\frac{1}{\sqrt{(1+x)^2+3}}\right) - \frac{\sqrt{7}}{14}\text{Artanh}\left(\frac{(10+4x)\sqrt{7}}{14}\frac{1}{\sqrt{(-1+x)^2+3+4x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-1)/(x^2+2*x+4)^(1/2),x)
```

```
[Out] -1/6*3^(1/2)*arctanh(3^(1/2)/((1+x)^2+3)^(1/2))-1/14*7^(1/2)*arctanh(1/14*(
10+4*x)*7^(1/2)/((-1+x)^2+3+4*x)^(1/2))
```

Maxima [A] time = 1.46275, size = 73, normalized size = 1.18

$$-\frac{1}{14}\sqrt{7}\,\text{arsinh}\left(\frac{4\sqrt{3}x}{3|2x-2|} + \frac{10\sqrt{3}}{3|2x-2|}\right) - \frac{1}{6}\sqrt{3}\,\text{arsinh}\left(\frac{2\sqrt{3}}{|2x+2|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/14*sqrt(7)*arcsinh(4/3*sqrt(3)*x/abs(2*x - 2) + 10/3*sqrt(3)/abs(2*x - 2
)) - 1/6*sqrt(3)*arcsinh(2*sqrt(3)/abs(2*x + 2))
```

Fricas [A] time = 2.02069, size = 211, normalized size = 3.4

$$\frac{1}{14}\sqrt{7}\log\left(\frac{\sqrt{7}(2x+5)+\sqrt{x^2+2x+4}\left(2\sqrt{7}-7\right)-4x-10}{x-1}\right)+\frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3}-\sqrt{x^2+2x+4}}{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/14*sqrt(7)*log((sqrt(7)*(2*x + 5) + sqrt(x^2 + 2*x + 4)*(2*sqrt(7) - 7) -
 4*x - 10)/(x - 1)) + 1/6*sqrt(3)*log(-(sqrt(3) - sqrt(x^2 + 2*x + 4))/(x +
 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x-1)(x+1)\sqrt{x^2+2x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-1)/(x**2+2*x+4)**(1/2),x)
```

```
[Out] Integral(x/((x - 1)*(x + 1)*sqrt(x**2 + 2*x + 4)), x)
```

Giac [B] time = 1.153, size = 147, normalized size = 2.37

$$\frac{1}{14}\sqrt{7}\log\left(\frac{\left|-2x-2\sqrt{7}+2\sqrt{x^2+2x+4}+2\right|}{\left|-2x+2\sqrt{7}+2\sqrt{x^2+2x+4}+2\right|}\right)+\frac{1}{6}\sqrt{3}\log\left(-\frac{\left|-2x-2\sqrt{3}+2\sqrt{x^2+2x+4}-2\right|}{2\left(x-\sqrt{3}-\sqrt{x^2+2x+4}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1)/(x^2+2*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/14*sqrt(7)*log(abs(-2*x - 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)/abs(-2*x
 + 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)) + 1/6*sqrt(3)*log(-1/2*abs(-2*x
- 2*sqrt(3) + 2*sqrt(x^2 + 2*x + 4) - 2)/(x - sqrt(3) - sqrt(x^2 + 2*x + 4)
 + 1))
```

3.244 $\int \frac{1}{\sqrt{5+2x+x^2}\left(-8+x^3\right)} \, dx$

Optimal. Leaf size=82

$$-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

```
[Out] -ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]/(4*Sqrt[3]) - ArcTanh[(7 + 3
*x)/(Sqrt[13]*Sqrt[5 + 2*x + x^2])]/(12*Sqrt[13]) + ArcTanh[Sqrt[5 + 2*x +
x^2]]/12
```

Rubi [A] time = 0.121105, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {2074, 724, 206, 1025, 982, 204, 1024}

$$-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{x^2+2x+5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[5 + 2*x + x^2]*(-8 + x^3)),x]
```

```
[Out] -ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]/(4*Sqrt[3]) - ArcTanh[(7 + 3
*x)/(Sqrt[13]*Sqrt[5 + 2*x + x^2])]/(12*Sqrt[13]) + ArcTanh[Sqrt[5 + 2*x +
x^2]]/12
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{5+2x+x^2}\left(-8+x^3\right)}\,dx &= \int \left(\frac{1}{12(-2+x)\sqrt{5+2x+x^2}}+\frac{-4-x}{12\left(4+2x+x^2\right)\sqrt{5+2x+x^2}}\right)dx\\
&= \frac{1}{12}\int \frac{1}{(-2+x)\sqrt{5+2x+x^2}}\,dx+\frac{1}{12}\int \frac{-4-x}{\left(4+2x+x^2\right)\sqrt{5+2x+x^2}}\,dx\\
&= -\left(\frac{1}{24}\int \frac{2+2x}{\left(4+2x+x^2\right)\sqrt{5+2x+x^2}}\,dx\right)-\frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{52-x^2}\,dx,x,\frac{14+6x}{\sqrt{5+2x+x^2}}\right)-\\
&= -\frac{\tanh^{-1}\left(\frac{7+3x}{\sqrt{13}\sqrt{5+2x+x^2}}\right)}{12\sqrt{13}}+\frac{1}{6}\,\mathrm{Subst}\left(\int \frac{1}{2-2x^2}\,dx,x,\sqrt{5+2x+x^2}\right)+\mathrm{Subst}\left(\int \frac{1}{-24-\right.\\
&= -\frac{\tan^{-1}\left(\frac{2+2x}{2\sqrt{3}\sqrt{5+2x+x^2}}\right)}{4\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{7+3x}{\sqrt{13}\sqrt{5+2x+x^2}}\right)}{12\sqrt{13}}+\frac{1}{12}\tanh^{-1}\left(\sqrt{5+2x+x^2}\right)
\end{aligned}$$

Mathematica [C] time = 0.31045, size = 159, normalized size = 1.94

$$\frac{1}{312}\left(-2\sqrt{13}\tanh^{-1}\left(\frac{3x+7}{\sqrt{13}\sqrt{x^2+2x+5}}\right)-13\left(\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{2\left(\sqrt[3]{-1}-2\right)x+5i\sqrt{3}+1}{\sqrt{2-2i\sqrt{3}}\sqrt{x^2+2x+5}}\right)+\left(\sqrt{3}-i\right)\tan^{-1}\left(\frac{-2\left(2+(-\right.}{\sqrt{2+2i}}\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[5 + 2*x + x^2]*(-8 + x^3)),x]
```

```
[Out] (-13*((I + Sqrt[3])*ArcTan[(1 + (5*I)*Sqrt[3] + 2*(-2 + (-1)^(1/3))*x)/(Sqr
t[2 - (2*I)*Sqrt[3]]*Sqrt[5 + 2*x + x^2])] + (-I + Sqrt[3])*ArcTan[(1 - (5*
I)*Sqrt[3] - 2*(2 + (-1)^(2/3))*x)/(Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[5 + 2*x +
x^2])]) - 2*Sqrt[13]*ArcTanh[(7 + 3*x)/(Sqrt[13]*Sqrt[5 + 2*x + x^2])])/312
```

Maple [A] time = 0.018, size = 69, normalized size = 0.8

$$\frac{1}{12}\mathrm{Artanh}\left(\sqrt{x^2+2x+5}\right)-\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x+2)\sqrt{3}}{6}\frac{1}{\sqrt{x^2+2x+5}}\right)-\frac{\sqrt{13}}{156}\mathrm{Artanh}\left(\frac{(14+6x)\sqrt{13}}{26}\frac{1}{\sqrt{(-2+x)^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-8)/(x^2+2*x+5)^(1/2),x)
```

```
[Out] 1/12*arctanh((x^2+2*x+5)^(1/2))-1/12*3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)
^(1/2)*(2*x+2))-1/156*13^(1/2)*arctanh(1/26*(14+6*x)*13^(1/2)/((-2+x)^2+6*x
+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3-8)\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 - 8)*sqrt(x^2 + 2*x + 5)), x)
```

Fricas [B] time = 2.14768, size = 479, normalized size = 5.84

$$\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(x+2)+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right)-\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right)+\frac{1}{156}\sqrt{13}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x + 2) + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5))
 - 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) +
1/156*sqrt(13)*log((sqrt(13)*(3*x + 7) + sqrt(x^2 + 2*x + 5)*(3*sqrt(13) -
13) - 9*x - 21)/(x - 2)) - 1/24*log(x^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x
 + 6) + 1/24*log(x^2 - sqrt(x^2 + 2*x + 5)*x + x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x-2)\left(x^2+2x+4\right)\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-8)/(x**2+2*x+5)**(1/2),x)
```

```
[Out] Integral(1/((x - 2)*(x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

Giac [B] time = 1.11625, size = 221, normalized size = 2.7

$$\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}+2\right)\right)-\frac{1}{12}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}\right)\right)+\frac{1}{156}\sqrt{13}\log\left(\frac{|-2x-2}{|-2x+2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-8)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) - 1/12*sqrt
(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5))) + 1/156*sqrt(13)*log(abs
(-2*x - 2*sqrt(13) + 2*sqrt(x^2 + 2*x + 5) + 4)/abs(-2*x + 2*sqrt(13) + 2*s
qrt(x^2 + 2*x + 5) + 4)) - 1/24*log((x - sqrt(x^2 + 2*x + 5))^2 + 4*x - 4*s
qrt(x^2 + 2*x + 5) + 7) + 1/24*log((x - sqrt(x^2 + 2*x + 5))^2 + 3)
```

3.245 $\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx$

Optimal. Leaf size=63

$$\frac{\tan^{-1}\left(\frac{\sqrt{4x^2+4x+5}}{\sqrt{11}}\right)}{\sqrt{11}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{11}{15}}(2x+1)}{\sqrt{4x^2+4x+5}}\right)}{\sqrt{165}}$$

```
[Out] ArcTan[Sqrt[5 + 4*x + 4*x^2]/Sqrt[11]]/Sqrt[11] - ArcTanh[(Sqrt[11/15]*(1 +
 2*x))/Sqrt[5 + 4*x + 4*x^2]]/Sqrt[165]
```

Rubi [A] time = 0.0541548, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1025, 982, 207, 1024, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{4x^2+4x+5}}{\sqrt{11}}\right)}{\sqrt{11}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{11}{15}}(2x+1)}{\sqrt{4x^2+4x+5}}\right)}{\sqrt{165}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((4 + x + x^2)*Sqrt[5 + 4*x + 4*x^2]),x]
```

```
[Out] ArcTan[Sqrt[5 + 4*x + 4*x^2]/Sqrt[11]]/Sqrt[11] - ArcTanh[(Sqrt[11/15]*(1 +
 2*x))/Sqrt[5 + 4*x + 4*x^2]]/Sqrt[165]
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
```

```
EqQ[h*e - 2*g*f, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx = \frac{1}{8}\int \frac{4+8x}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx - \frac{1}{2}\int \frac{1}{(4+x+x^2)\sqrt{5+4x+4x^2}}\,dx$$
$$= 4\,\text{Subst}\left(\int \frac{1}{-240+11x^2}\,dx, x, \frac{4+8x}{\sqrt{5+4x+4x^2}}\right) - \text{Subst}\left(\int \frac{1}{-11-x^2}\,dx, x, \sqrt{5+4}\right.$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{5+4x+4x^2}}{\sqrt{11}}\right)}{\sqrt{11}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\frac{11}{15}}(1+2x)}{\sqrt{5+4x+4x^2}}\right)}{\sqrt{165}}$$

Mathematica [C] time = 0.10611, size = 114, normalized size = 1.81

$$\frac{\left(\sqrt{15}-i\right)\tan^{-1}\left(\frac{-2i\sqrt{15}x-i\sqrt{15}+4}{\sqrt{11}\sqrt{4x^2+4x+5}}\right)+\left(\sqrt{15}+i\right)\tan^{-1}\left(\frac{2i\sqrt{15}x+i\sqrt{15}+4}{\sqrt{11}\sqrt{4x^2+4x+5}}\right)}{2\sqrt{165}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((4 + x + x^2)*Sqrt[5 + 4*x + 4*x^2]),x]
```

```
[Out] ((-I + Sqrt[15])*ArcTan[(4 - I*Sqrt[15] - (2*I)*Sqrt[15]*x)/(Sqrt[11]*Sqrt[
5 + 4*x + 4*x^2])] + (I + Sqrt[15])*ArcTan[(4 + I*Sqrt[15] + (2*I)*Sqrt[15]
*x)/(Sqrt[11]*Sqrt[5 + 4*x + 4*x^2])])/(2*Sqrt[165])
```

Maple [A] time = 0.011, size = 53, normalized size = 0.8

$$\frac{\sqrt{11}}{11}\arctan\left(\frac{\sqrt{11}}{11}\sqrt{4\,x^2+4\,x+5}\right)-\frac{\sqrt{165}}{165}\text{Artanh}\left(\frac{\sqrt{165}\,(8\,x+4)}{60}\frac{1}{\sqrt{4\,x^2+4\,x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x)
```

```
[Out] 1/11*arctan(1/11*(4*x^2+4*x+5)^(1/2)*11^(1/2))*11^(1/2)-1/165*165^(1/2)*arc
tanh(1/60*165^(1/2)*(8*x+4)/(4*x^2+4*x+5)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{4\,x^2+4\,x+5}\left(x^2+x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(4*x^2 + 4*x + 5)*(x^2 + x + 4)), x)
```

Fricas [B] time = 2.23113, size = 1057, normalized size = 16.78

$$\frac{2}{165}\sqrt{165}\sqrt{15}\arctan\left(\frac{1}{60}\sqrt{2}\sqrt{4x^2-\sqrt{4x^2+4x+5}(2x+1)+4x-\sqrt{165}+16}\left(\sqrt{165}\sqrt{15}+15\sqrt{15}\right)+\frac{1}{60}\sqrt{165}\sqrt{1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/165*sqrt(165)*sqrt(15)*arctan(1/60*sqrt(2)*sqrt(4*x^2 - sqrt(4*x^2 + 4*x
+ 5)*(2*x + 1) + 4*x - sqrt(165) + 16)*(sqrt(165)*sqrt(15) + 15*sqrt(15)) +
 1/60*sqrt(165)*sqrt(15)*(2*x + 1) - 1/60*sqrt(4*x^2 + 4*x + 5)*(sqrt(165)*
sqrt(15) + 15*sqrt(15)) + 1/4*sqrt(15)*(2*x + 1)) + 2/165*sqrt(165)*sqrt(15
)*arctan(1/60*sqrt(2)*sqrt(4*x^2 - sqrt(4*x^2 + 4*x + 5)*(2*x + 1) + 4*x +
sqrt(165) + 16)*(sqrt(165)*sqrt(15) - 15*sqrt(15)) + 1/60*sqrt(165)*sqrt(15
)*(2*x + 1) - 1/60*sqrt(4*x^2 + 4*x + 5)*(sqrt(165)*sqrt(15) - 15*sqrt(15))
 - 1/4*sqrt(15)*(2*x + 1)) - 1/330*sqrt(165)*log(460800*x^2 - 115200*sqrt(4
*x^2 + 4*x + 5)*(2*x + 1) + 460800*x + 115200*sqrt(165) + 1843200) + 1/330*
sqrt(165)*log(460800*x^2 - 115200*sqrt(4*x^2 + 4*x + 5)*(2*x + 1) + 460800*
x - 115200*sqrt(165) + 1843200)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(x^2+x+4\right)\sqrt{4x^2+4x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+x+4)/(4*x**2+4*x+5)**(1/2),x)
```

```
[Out] Integral(x/((x**2 + x + 4)*sqrt(4*x**2 + 4*x + 5)), x)
```

Giac [C] time = 1.1623, size = 228, normalized size = 3.62

$$-\frac{1}{330}\sqrt{165}\left(-i\sqrt{15}+1\right)\log\left(-600x+300i\sqrt{15}+300i\sqrt{11}+300\sqrt{4x^2+4x+5}-300\right)+\frac{1}{330}\sqrt{165}\left(-i\sqrt{15}+1\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+4)/(4*x^2+4*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/330*sqrt(165)*(-I*sqrt(15) + 1)*log(-600*x + 300*I*sqrt(15) + 300*I*sqrt
(11) + 300*sqrt(4*x^2 + 4*x + 5) - 300) + 1/330*sqrt(165)*(-I*sqrt(15) + 1)
*log(-600*x + 300*I*sqrt(15) - 300*I*sqrt(11) + 300*sqrt(4*x^2 + 4*x + 5) -
 300) + 1/330*sqrt(165)*(I*sqrt(15) + 1)*log(-600*x - 300*I*sqrt(15) + 300*
I*sqrt(11) + 300*sqrt(4*x^2 + 4*x + 5) - 300) - 1/330*sqrt(165)*(I*sqrt(15)
```

```
+ 1)*log(-600*x - 300*I*sqrt(15) - 300*I*sqrt(11) + 300*sqrt(4*x^2 + 4*x +
5) - 300)
```

3.246 $\int \frac{3+x}{\left(1+x^2\right)\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=56

$$\sqrt{2}\tanh^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+x+1}}\right)-2\sqrt{2}\tan^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+x+1}}\right)$$

```
[Out] -2*Sqrt[2]*ArcTan[(1 - x)/(Sqrt[2]*Sqrt[1 + x + x^2])] + Sqrt[2]*ArcTanh[(1
 + x)/(Sqrt[2]*Sqrt[1 + x + x^2])]
```

Rubi [A] time = 0.0466675, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {1036, 1030, 207, 203}

$$\sqrt{2}\tanh^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+x+1}}\right)-2\sqrt{2}\tan^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + x)/((1 + x^2)*Sqrt[1 + x + x^2]),x]
```

```
[Out] -2*Sqrt[2]*ArcTan[(1 - x)/(Sqrt[2]*Sqrt[1 + x + x^2])] + Sqrt[2]*ArcTanh[(1
 + x)/(Sqrt[2]*Sqrt[1 + x + x^2])]
```

Rule 1036

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist
[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*
e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Si
mp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c
*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] &&
 NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]
```

Rule 1030

```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*
e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[
{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{3+x}{(1+x^2)\sqrt{1+x+x^2}}\,dx = -\left(\frac{1}{2}\int \frac{-4-4x}{(1+x^2)\sqrt{1+x+x^2}}\,dx\right) + \frac{1}{2}\int \frac{2-2x}{(1+x^2)\sqrt{1+x+x^2}}\,dx$$
$$= 4\,\text{Subst}\left(\int \frac{1}{-8+x^2}\,dx, x, \frac{-2-2x}{\sqrt{1+x+x^2}}\right) + 16\,\text{Subst}\left(\int \frac{1}{32+x^2}\,dx, x, \frac{-4+4x}{\sqrt{1+x+x^2}}\right)$$
$$= -2\sqrt{2}\tan^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x+x^2}}\right) + \sqrt{2}\tanh^{-1}\left(\frac{1+x}{\sqrt{2}\sqrt{1+x+x^2}}\right)$$

Mathematica [C] time = 0.0251623, size = 80, normalized size = 1.43

$$\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left((2+i)\tan^{-1}\left(\frac{\sqrt[4]{-1}((2+i)x+(1+2i))}{2\sqrt{x^2+x+1}}\right)+(1+2i)\tanh^{-1}\left(\frac{(-1)^{3/4}((1+2i)x+(2+i))}{2\sqrt{x^2+x+1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + x)/((1 + x^2)*Sqrt[1 + x + x^2]),x]
```

```
[Out] (1/2 + I/2)*(-1)^(3/4)*((2 + I)*ArcTan[((-1)^(1/4)*((1 + 2*I) + (2 + I)*x))
/(2*Sqrt[1 + x + x^2])] + (1 + 2*I)*ArcTanh[((-1)^(3/4)*((2 + I) + (1 + 2*I
)*x))/(2*Sqrt[1 + x + x^2])])
```

Maple [B] time = 0.016, size = 128, normalized size = 2.3

$$\sqrt{2}\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}\left(\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}\right)-2\arctan\left(\frac{\sqrt{2}(-1+x)}{-1-x}\frac{1}{\sqrt{\frac{(-1+x)^2}{(-1-x)^2}+3}}\right)\right)\frac{1}{\sqrt{\left(\frac{(-1+x)^2}{(-1-x)^2}+3\right)\left(1+\frac{-1+x}{-1-x}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x)
```

```
[Out] ((-1+x)^2/(-1-x)^2+3)^(1/2)*2^(1/2)*(arctanh(1/2*((-1+x)^2/(-1-x)^2+3)^(1/2
)*2^(1/2))-2*arctan(2^(1/2)/((-1+x)^2/(-1-x)^2+3)^(1/2)*(-1+x)/(-1-x)))/(((
-1+x)^2/(-1-x)^2+3)/(1+(-1+x)/(-1-x))^2)^(1/2)/(1+(-1+x)/(-1-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+3}{\sqrt{x^2+x+1}(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x + 3)/(sqrt(x^2 + x + 1)*(x^2 + 1)), x)
```

Fricas [B] time = 2.61507, size = 1041, normalized size = 18.59

$$\frac{4}{5}\sqrt{10}\sqrt{5}\arctan\left(\frac{1}{25}\sqrt{5}\sqrt{\sqrt{10}\sqrt{5}(x-1)+10x^2-\sqrt{x^2+x+1}\left(\sqrt{10}\sqrt{5}+10x\right)+5x+15}\left(\sqrt{10}\sqrt{5}+10\right)+\frac{1}{5}\sqrt{10}\sqrt{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/5*sqrt(10)*sqrt(5)*arctan(1/25*sqrt(5)*sqrt(sqrt(10)*sqrt(5)*(x - 1) + 10
*x^2 - sqrt(x^2 + x + 1)*(sqrt(10)*sqrt(5) + 10*x) + 5*x + 15)*(sqrt(10)*sq
rt(5) + 10) + 1/5*sqrt(10)*sqrt(5)*(x + 1) - 1/5*sqrt(x^2 + x + 1)*(sqrt(10
)*sqrt(5) + 10) + 2*x + 1) + 4/5*sqrt(10)*sqrt(5)*arctan(1/5*sqrt(10)*sqrt(
5)*(x + 1) + 1/50*sqrt(-20*sqrt(10)*sqrt(5)*(x - 1) + 200*x^2 + 20*sqrt(x^2
 + x + 1)*(sqrt(10)*sqrt(5) - 10*x) + 100*x + 300)*(sqrt(10)*sqrt(5) - 10)
- 1/5*sqrt(x^2 + x + 1)*(sqrt(10)*sqrt(5) - 10) - 2*x - 1) - 1/10*sqrt(10)*
sqrt(5)*log(20*sqrt(10)*sqrt(5)*(x - 1) + 200*x^2 - 20*sqrt(x^2 + x + 1)*(s
qrt(10)*sqrt(5) + 10*x) + 100*x + 300) + 1/10*sqrt(10)*sqrt(5)*log(-20*sqrt
(10)*sqrt(5)*(x - 1) + 200*x^2 + 20*sqrt(x^2 + x + 1)*(sqrt(10)*sqrt(5) - 1
0*x) + 100*x + 300)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x+3}{\left(x^2+1\right)\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x**2+1)/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral((x + 3)/((x**2 + 1)*sqrt(x**2 + x + 1)), x)
```

Giac [C] time = 1.11181, size = 139, normalized size = 2.48

$$-\left(i-\frac{1}{2}\right)\sqrt{2}\log\left(-(i+1)\,x+\sqrt{2}+(i+1)\,\sqrt{x^2+x+1}-i+1\right)+\left(i+\frac{1}{2}\right)\sqrt{2}\log\left(-(i+1)\,x+i\sqrt{2}+(i+1)\,\sqrt{x^2+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+x)/(x^2+1)/(x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(I - 1/2)*sqrt(2)*log(-(I + 1)*x + sqrt(2) + (I + 1)*sqrt(x^2 + x + 1) - I
 + 1) + (I + 1/2)*sqrt(2)*log(-(I + 1)*x + I*sqrt(2) + (I + 1)*sqrt(x^2 + x
 + 1) + I - 1) - (I + 1/2)*sqrt(2)*log(-(I + 1)*x - I*sqrt(2) + (I + 1)*sqr
t(x^2 + x + 1) + I - 1) + (I - 1/2)*sqrt(2)*log(-(I + 1)*x - sqrt(2) + (I +
 1)*sqrt(x^2 + x + 1) - I + 1)
```

3.247 $\int \frac{1+2x}{\sqrt{-1+6x+x^2}\left(4+4x+3x^2\right)}\,dx$

Optimal. Leaf size=70

$$-\frac{5\tan^{-1}\left(\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{x^2+6x-1}}\right)}{6\sqrt{14}}-\frac{\tanh^{-1}\left(\frac{\sqrt{7}(x+1)}{\sqrt{x^2+6x-1}}\right)}{3\sqrt{7}}$$

```
[Out] (-5*ArcTan[(Sqrt[7/2]*(2 - x))/(2*Sqrt[-1 + 6*x + x^2])])/(6*Sqrt[14]) - Ar
cTanh[(Sqrt[7]*(1 + x))/Sqrt[-1 + 6*x + x^2]]/(3*Sqrt[7])
```

Rubi [A] time = 0.0638688, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1035, 1029, 207, 203}

$$-\frac{5\tan^{-1}\left(\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{x^2+6x-1}}\right)}{6\sqrt{14}}-\frac{\tanh^{-1}\left(\frac{\sqrt{7}(x+1)}{\sqrt{x^2+6x-1}}\right)}{3\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/(Sqrt[-1 + 6*x + x^2]*(4 + 4*x + 3*x^2)),x]
```

```
[Out] (-5*ArcTan[(Sqrt[7/2]*(2 - x))/(2*Sqrt[-1 + 6*x + x^2])])/(6*Sqrt[14]) - Ar
cTanh[(Sqrt[7]*(1 + x))/Sqrt[-1 + 6*x + x^2]]/(3*Sqrt[7])
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1+2x}{\sqrt{-1+6x+x^2}\left(4+4x+3x^2\right)}\,dx = -\left(\frac{1}{42}\int \frac{-70-70x}{\sqrt{-1+6x+x^2}\left(4+4x+3x^2\right)}\,dx\right)+\frac{1}{42}\int \frac{-28+14x}{\sqrt{-1+6x+x^2}\left(4+4x+\right.}$$

$$= -\left(\frac{896}{3}\operatorname{Subst}\left(\int \frac{1}{-200704+28x^2}\,dx, x, \frac{-224-224x}{\sqrt{-1+6x+x^2}}\right)\right)-\frac{2800}{3}\operatorname{Subst}\left(\int\right.$$

$$= -\frac{5\tan^{-1}\left(\frac{\sqrt{\frac{7}{2}}(2-x)}{2\sqrt{-1+6x+x^2}}\right)}{6\sqrt{14}}-\frac{\tanh^{-1}\left(\frac{\sqrt{7}(1+x)}{\sqrt{-1+6x+x^2}}\right)}{3\sqrt{7}}$$

Mathematica [C] time = 0.418217, size = 174, normalized size = 2.49

$$-\frac{\sqrt{7-4i\sqrt{2}}\left(8\sqrt{2}+13i\right)\tan^{-1}\left(\frac{\left(-7-2i\sqrt{2}\right)x-6i\sqrt{2}+9}{\sqrt{7\left(7-4i\sqrt{2}\right)}\sqrt{x^2+6x-1}}\right)+\sqrt{7+4i\sqrt{2}}\left(8\sqrt{2}-13i\right)\tan^{-1}\left(\frac{\left(-7+2i\sqrt{2}\right)x+6i\sqrt{2}+9}{\sqrt{7\left(7+4i\sqrt{2}\right)}\sqrt{x^2+6x-1}}\right)}{108\sqrt{14}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/(Sqrt[-1 + 6*x + x^2]*(4 + 4*x + 3*x^2)),x]
```

```
[Out] -(Sqrt[7 - (4*I)*Sqrt[2]]*(13*I + 8*Sqrt[2])*ArcTan[(9 - (6*I)*Sqrt[2] + (-
7 - (2*I)*Sqrt[2])*x)/(Sqrt[7*(7 - (4*I)*Sqrt[2])]*Sqrt[-1 + 6*x + x^2])] +
 Sqrt[7 + (4*I)*Sqrt[2]]*(-13*I + 8*Sqrt[2])*ArcTan[(9 + (6*I)*Sqrt[2] + (-
7 + (2*I)*Sqrt[2])*x)/(Sqrt[7*(7 + (4*I)*Sqrt[2])]*Sqrt[-1 + 6*x + x^2])])/
(108*Sqrt[14])
```

Maple [B] time = 0.023, size = 158, normalized size = 2.3

$$-\frac{1}{84}\sqrt{-6\frac{(-2+x)^2}{(-1-x)^2}+15}\left(4\sqrt{7}\operatorname{Artanh}\left(1/21\sqrt{-6\frac{(-2+x)^2}{(-1-x)^2}+15}\sqrt{7}\right)-5\sqrt{14}\arctan\left(1/4\frac{\sqrt{14}(-2+x)}{-1-x}\sqrt{-6\frac{(-2+}{(-1-}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x)
```

```
[Out] -1/84*(-6*(-2+x)^2/(-1-x)^2+15)^(1/2)*(4*7^(1/2)*arctanh(1/21*(-6*(-2+x)^2/
(-1-x)^2+15)^(1/2)*7^(1/2))-5*14^(1/2)*arctan(1/4*14^(1/2)*(-6*(-2+x)^2/(-1
-x)^2+15)^(1/2)/(2*(-2+x)^2/(-1-x)^2-5)*(-2+x)/(-1-x)))/(-3*(2*(-2+x)^2/(-1
-x)^2-5)/(1+(-2+x)/(-1-x))^2)^(1/2)/(1+(-2+x)/(-1-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+1}{\left(3x^2+4x+4\right)\sqrt{x^2+6x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 1)/((3*x^2 + 4*x + 4)*sqrt(x^2 + 6*x - 1)), x)
```

Fricas [B] time = 2.84163, size = 1110, normalized size = 15.86

$$\frac{1}{84}\sqrt{14}\sqrt{2}\log\left(13068\sqrt{14}\sqrt{2}(x-2)+78408x^2-13068\sqrt{x^2+6x-1}\left(\sqrt{14}\sqrt{2}+6x+4\right)+287496x+287496\right)-\frac{1}{84}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/84*sqrt(14)*sqrt(2)*log(13068*sqrt(14)*sqrt(2)*(x - 2) + 78408*x^2 - 1306
8*sqrt(x^2 + 6*x - 1)*(sqrt(14)*sqrt(2) + 6*x + 4) + 287496*x + 287496) - 1
/84*sqrt(14)*sqrt(2)*log(-13068*sqrt(14)*sqrt(2)*(x - 2) + 78408*x^2 + 1306
8*sqrt(x^2 + 6*x - 1)*(sqrt(14)*sqrt(2) - 6*x - 4) + 287496*x + 287496) - 5
/42*sqrt(14)*arctan(1/24*sqrt(3)*sqrt(sqrt(14)*sqrt(2)*(x - 2) + 6*x^2 - sq
rt(x^2 + 6*x - 1)*(sqrt(14)*sqrt(2) + 6*x + 4) + 22*x + 22)*(sqrt(14) + sqr
t(2)) + 1/8*sqrt(2)*(x + 3) + 1/8*sqrt(14)*(x + 1) - 1/8*sqrt(x^2 + 6*x - 1
)*(sqrt(14) + sqrt(2))) - 5/42*sqrt(14)*arctan(-1/8*sqrt(2)*(x + 3) + 1/8*s
qrt(14)*(x + 1) + 1/1584*sqrt(-13068*sqrt(14)*sqrt(2)*(x - 2) + 78408*x^2 +
 13068*sqrt(x^2 + 6*x - 1)*(sqrt(14)*sqrt(2) - 6*x - 4) + 287496*x + 287496
)*(sqrt(14) - sqrt(2)) - 1/8*sqrt(x^2 + 6*x - 1)*(sqrt(14) - sqrt(2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+1}{\sqrt{x^2+6x-1}\left(3x^2+4x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(3*x**2+4*x+4)/(x**2+6*x-1)**(1/2),x)
```

```
[Out] Integral((2*x + 1)/(sqrt(x**2 + 6*x - 1)*(3*x**2 + 4*x + 4)), x)
```

Giac [C] time = 1.12576, size = 261, normalized size = 3.73

$$\frac{1}{168}\sqrt{7}\left(-5i\sqrt{2}-4\right)\log\left(-(8i-4)\sqrt{7}\sqrt{2}-(6i+12)x+(2i+4)\sqrt{7}-(8i-4)\sqrt{2}+(6i+12)\sqrt{x^2+6x-1}-4i-8\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(3*x^2+4*x+4)/(x^2+6*x-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/168*sqrt(7)*(-5*I*sqrt(2) - 4)*log(-(8*I - 4)*sqrt(7)*sqrt(2) - (6*I + 12
)*x + (2*I + 4)*sqrt(7) - (8*I - 4)*sqrt(2) + (6*I + 12)*sqrt(x^2 + 6*x - 1
) - 4*I - 8) - 1/168*sqrt(7)*(5*I*sqrt(2) - 4)*log(-(8*I - 4)*sqrt(7)*sqrt(
2) - (6*I + 12)*x - (2*I + 4)*sqrt(7) + (8*I - 4)*sqrt(2) + (6*I + 12)*sqrt
(x^2 + 6*x - 1) - 4*I - 8) + 1/168*sqrt(7)*(5*I*sqrt(2) - 4)*log((4*I - 8)*
sqrt(7)*sqrt(2) - (12*I + 6)*x + (4*I + 2)*sqrt(7) + (4*I - 8)*sqrt(2) + (1
2*I + 6)*sqrt(x^2 + 6*x - 1) - 8*I - 4) - 1/168*sqrt(7)*(-5*I*sqrt(2) - 4)*
log((4*I - 8)*sqrt(7)*sqrt(2) - (12*I + 6)*x - (4*I + 2)*sqrt(7) - (4*I - 8
)*sqrt(2) + (12*I + 6)*sqrt(x^2 + 6*x - 1) - 8*I - 4)
```

3.248 $\int \frac{B+Ax}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$

Optimal. Leaf size=80

$$-\frac{(2A+B)\tan^{-1}\left(\frac{\sqrt{35}(2-x)}{\sqrt{10x^2-22x+13}}\right)}{\sqrt{35}}-\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{10x^2-22x+13}}\right)}{2\sqrt{35}}$$

```
[Out] -(((2*A + B)*ArcTan[(Sqrt[35]*(2 - x))/Sqrt[13 - 22*x + 10*x^2]])/Sqrt[35])
 - ((A + B)*ArcTanh[(Sqrt[35]*(1 - x))/(2*Sqrt[13 - 22*x + 10*x^2])])/(2*Sq
rt[35])
```

Rubi [A] time = 0.123304, antiderivative size = 89, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1035, 1029, 206, 204}

$$-\frac{(2A+B)\tan^{-1}\left(\frac{\sqrt{35}(2-x)}{\sqrt{10x^2-22x+13}}\right)}{\sqrt{35}}-\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{35}(-x(A+B)+A+B)}{2\sqrt{10x^2-22x+13}(A+B)}\right)}{2\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In] Int[(B + A*x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

```
[Out] -(((2*A + B)*ArcTan[(Sqrt[35]*(2 - x))/Sqrt[13 - 22*x + 10*x^2]])/Sqrt[35])
 - ((A + B)*ArcTanh[(Sqrt[35]*(A + B - (A + B)*x))/(2*(A + B)*Sqrt[13 - 22*
x + 10*x^2])])/(2*Sqrt[35])
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{B+Ax}{\left(17-18x+5x^2\right)\sqrt{13-22x+10x^2}}\,dx = \frac{1}{70}\int \frac{140(A+B)-70(A+B)x}{\left(17-18x+5x^2\right)\sqrt{13-22x+10x^2}}\,dx-\frac{1}{70}\int \frac{70(2A+B)-}{\left(17-18x+5x^2\right)}$$

$$= \left(560(A+B)^2\right)\text{Subst}\left(\int \frac{1}{313600(A+B)^2-140x^2}\,dx,x,\frac{-140(A+B)+}{\sqrt{13-22x}}\right.$$

$$= -\frac{(2A+B)\tan^{-1}\left(\frac{\sqrt{35}(2-x)}{\sqrt{13-22x+10x^2}}\right)}{\sqrt{35}}-\frac{(A+B)\tanh^{-1}\left(\frac{\sqrt{35}(A+B-(A+B)x)}{2(A+B)\sqrt{13-22x+10x^2}}\right)}{2\sqrt{35}}$$

Mathematica [C] time = 0.114487, size = 94, normalized size = 1.18

$$\frac{((4-i)A+(2-i)B)\tan^{-1}\left(\frac{(2-18i)-(1-18i)x}{\sqrt{35}\sqrt{10x^2-22x+13}}\right)+((1-4i)A+(1-2i)B)\tanh^{-1}\left(\frac{(18-i)x-(18-2i)}{\sqrt{35}\sqrt{10x^2-22x+13}}\right)}{4\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(B + A*x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

```
[Out] (((4 - I)*A + (2 - I)*B)*ArcTan[((2 - 18*I) - (1 - 18*I)*x)/(Sqrt[35]*Sqrt[
13 - 22*x + 10*x^2])] + ((1 - 4*I)*A + (1 - 2*I)*B)*ArcTanh[((-18 + 2*I) +
(18 - I)*x)/(Sqrt[35]*Sqrt[13 - 22*x + 10*x^2])])/(4*Sqrt[35])
```

Maple [B] time = 0.02, size = 192, normalized size = 2.4

$$\frac{\sqrt{35}}{70}\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\left(\text{Artanh}\left(\frac{2\sqrt{35}}{35}\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\right)A-4\arctan\left(\frac{\sqrt{35}(-2+x)}{1-x}\frac{1}{\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}}\right)A+\text{Artanh}\left(\frac{2}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x)
```

```
[Out] 1/70*((-2+x)^2/(1-x)^2+9)^(1/2)*35^(1/2)*(arctanh(2/35*((-2+x)^2/(1-x)^2+9)
^(1/2)*35^(1/2))*A-4*arctan(35^(1/2)/((-2+x)^2/(1-x)^2+9)^(1/2)*(-2+x)/(1-x
))*A+arctanh(2/35*((-2+x)^2/(1-x)^2+9)^(1/2)*35^(1/2))*B-2*arctan(35^(1/2)/
((-2+x)^2/(1-x)^2+9)^(1/2)*(-2+x)/(1-x))*B)/(((-2+x)^2/(1-x)^2+9)/(1+(-2+x)
/(1-x))^2)^(1/2)/(1+(-2+x)/(1-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Ax+B}{\sqrt{10\,x^2-22\,x+13}\left(5\,x^2-18\,x+17\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] integrate((A*x + B)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{Ax+B}{\left(5x^2-18x+17\right)\sqrt{10x^2-22x+13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(5*x**2-18*x+17)/(10*x**2-22*x+13)**(1/2),x)
```

```
[Out] Integral((A*x + B)/((5*x**2 - 18*x + 17)*sqrt(10*x**2 - 22*x + 13)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A*x+B)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: TypeError
```

3.249 $\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{10x^2-22x+13}}\right)}{2\sqrt{35}}$$

```
[Out] ArcTanh[(Sqrt[35]*(1 - x))/(2*Sqrt[13 - 22*x + 10*x^2])]/(2*Sqrt[35])
```

Rubi [A] time = 0.0238798, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1029, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{10x^2-22x+13}}\right)}{2\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

```
[Out] ArcTanh[(Sqrt[35]*(1 - x))/(2*Sqrt[13 - 22*x + 10*x^2])]/(2*Sqrt[35])
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-2+x}{(17-18x+5x^2)\sqrt{13-22x+10x^2}}\,dx &= 8\,\text{Subst}\left(\int \frac{1}{64-140x^2}\,dx, x, \frac{2-2x}{\sqrt{13-22x+10x^2}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{35}(1-x)}{2\sqrt{13-22x+10x^2}}\right)}{2\sqrt{35}}\end{aligned}$$

Mathematica [C] time = 0.0391635, size = 76, normalized size = 2.

$$\frac{i\left(\tan^{-1}\left(\frac{(2-18i)-(1-18i)x}{\sqrt{35}\sqrt{10x^2-22x+13}}\right)+i\tanh^{-1}\left(\frac{(18-i)x-(18-2i)}{\sqrt{35}\sqrt{10x^2-22x+13}}\right)\right)}{4\sqrt{35}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x)/((17 - 18*x + 5*x^2)*Sqrt[13 - 22*x + 10*x^2]),x]
```

```
[Out] ((I/4)*(ArcTan[((2 - 18*I) - (1 - 18*I)*x)/(Sqrt[35]*Sqrt[13 - 22*x + 10*x^
2])] + I*ArcTanh[((-18 + 2*I) + (18 - I)*x)/(Sqrt[35]*Sqrt[13 - 22*x + 10*x
^2])]))/Sqrt[35]
```

Maple [B] time = 0.01, size = 94, normalized size = 2.5

$$-\frac{\sqrt{35}}{70}\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\,\mathrm{Artanh}\left(\frac{2\sqrt{35}}{35}\sqrt{\frac{(-2+x)^2}{(1-x)^2}+9}\right)\frac{1}{\sqrt{\left(\frac{(-2+x)^2}{(1-x)^2}+9\right)\left(1+\frac{-2+x}{1-x}\right)^{-2}}}\left(1+\frac{-2+x}{1-x}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x)
```

```
[Out] -1/70/(((-2+x)^2/(1-x)^2+9)/(1+(-2+x)/(1-x))^2)^(1/2)/(1+(-2+x)/(1-x))*((-2
+x)^2/(1-x)^2+9)^(1/2)*35^(1/2)*arctanh(2/35*((-2+x)^2/(1-x)^2+9)^(1/2)*35^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-2}{\sqrt{10\,x^2-22\,x+13}\left(5\,x^2-18\,x+17\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((x - 2)/(sqrt(10*x^2 - 22*x + 13)*(5*x^2 - 18*x + 17)), x)
```

Fricas [B] time = 2.10608, size = 254, normalized size = 6.68

$$\frac{1}{280}\sqrt{35}\log\left(\frac{11225\,x^4-47220\,x^3-8\,\sqrt{35}\left(75\,x^3-233\,x^2+245\,x-87\right)\sqrt{10\,x^2-22\,x+13}+75534\,x^2-54372\,x+1484}{25\,x^4-180\,x^3+494\,x^2-612\,x+289}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/280*sqrt(35)*log((11225*x^4 - 47220*x^3 - 8*sqrt(35)*(75*x^3 - 233*x^2 +
245*x - 87)*sqrt(10*x^2 - 22*x + 13) + 75534*x^2 - 54372*x + 14849)/(25*x^4
 - 180*x^3 + 494*x^2 - 612*x + 289))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x-2}{(5x^2-18x+17)\sqrt{10x^2-22x+13}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(5*x**2-18*x+17)/(10*x**2-22*x+13)**(1/2),x)
```

```
[Out] Integral((x - 2)/((5*x**2 - 18*x + 17)*sqrt(10*x**2 - 22*x + 13)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)/(5*x^2-18*x+17)/(10*x^2-22*x+13)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError
```

3.250 $\int x^4\sqrt{5-x^2}\,dx$

Optimal. Leaf size=65

$$\frac{1}{6}\sqrt{5-x^2}x^5-\frac{5}{24}\sqrt{5-x^2}x^3-\frac{25}{16}\sqrt{5-x^2}x+\frac{125}{16}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

```
[Out] (-25*x*Sqrt[5 - x^2])/16 - (5*x^3*Sqrt[5 - x^2])/24 + (x^5*Sqrt[5 - x^2])/6
 + (125*ArcSin[x/Sqrt[5]])/16
```

Rubi [A] time = 0.0173211, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {279, 321, 216}

$$\frac{1}{6}\sqrt{5-x^2}x^5-\frac{5}{24}\sqrt{5-x^2}x^3-\frac{25}{16}\sqrt{5-x^2}x+\frac{125}{16}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[5 - x^2],x]
```

```
[Out] (-25*x*Sqrt[5 - x^2])/16 - (5*x^3*Sqrt[5 - x^2])/24 + (x^5*Sqrt[5 - x^2])/6
 + (125*ArcSin[x/Sqrt[5]])/16
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int x^4\sqrt{5-x^2}\,dx &= \frac{1}{6}x^5\sqrt{5-x^2}+\frac{5}{6}\int\frac{x^4}{\sqrt{5-x^2}}\,dx\\ &= -\frac{5}{24}x^3\sqrt{5-x^2}+\frac{1}{6}x^5\sqrt{5-x^2}+\frac{25}{8}\int\frac{x^2}{\sqrt{5-x^2}}\,dx\\ &= -\frac{25}{16}x\sqrt{5-x^2}-\frac{5}{24}x^3\sqrt{5-x^2}+\frac{1}{6}x^5\sqrt{5-x^2}+\frac{125}{16}\int\frac{1}{\sqrt{5-x^2}}\,dx\\ &= -\frac{25}{16}x\sqrt{5-x^2}-\frac{5}{24}x^3\sqrt{5-x^2}+\frac{1}{6}x^5\sqrt{5-x^2}+\frac{125}{16}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.0218373, size = 40, normalized size = 0.62

$$\frac{1}{48}\left(x\sqrt{5-x^2}\left(8x^4-10x^2-75\right)+375\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Sqrt[5 - x^2],x]
```

```
[Out] (x*Sqrt[5 - x^2]*(-75 - 10*x^2 + 8*x^4) + 375*ArcSin[x/Sqrt[5]])/48
```

Maple [A] time = 0.003, size = 49, normalized size = 0.8

$$-\frac{x^3}{6}\left(-x^2+5\right)^{\frac{3}{2}}-\frac{5\,x}{8}\left(-x^2+5\right)^{\frac{3}{2}}+\frac{25\,x}{16}\sqrt{-x^2+5}+\frac{125}{16}\arcsin\left(\frac{x\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-x^2+5)^(1/2),x)
```

```
[Out] -1/6*x^3*(-x^2+5)^(3/2)-5/8*x*(-x^2+5)^(3/2)+25/16*x*(-x^2+5)^(1/2)+125/16*
arcsin(1/5*x*5^(1/2))
```

Maxima [A] time = 1.54181, size = 65, normalized size = 1.

$$-\frac{1}{6}\left(-x^2+5\right)^{\frac{3}{2}}x^3-\frac{5}{8}\left(-x^2+5\right)^{\frac{3}{2}}x+\frac{25}{16}\sqrt{-x^2+5}x+\frac{125}{16}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+5)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*(-x^2 + 5)^(3/2)*x^3 - 5/8*(-x^2 + 5)^(3/2)*x + 25/16*sqrt(-x^2 + 5)*x
 + 125/16*arcsin(1/5*sqrt(5)*x)
```

Fricas [A] time = 1.7991, size = 107, normalized size = 1.65

$$\frac{1}{48}\left(8\,x^5-10\,x^3-75\,x\right)\sqrt{-x^2+5}-\frac{125}{16}\arctan\left(\frac{\sqrt{-x^2+5}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(8*x^5 - 10*x^3 - 75*x)*sqrt(-x^2 + 5) - 125/16*arctan(sqrt(-x^2 + 5)/
x)
```

Sympy [A] time = 4.53878, size = 155, normalized size = 2.38

$$\begin{cases} \frac{ix^7}{6\sqrt{x^2-5}} - \frac{25ix^5}{24\sqrt{x^2-5}} - \frac{25ix^3}{48\sqrt{x^2-5}} + \frac{125ix}{16\sqrt{x^2-5}} - \frac{125i\operatorname{acosh}\left(\frac{\sqrt{5}x}{5}\right)}{16} & \text{for}\ \frac{|x^2|}{5} > 1 \\ -\frac{x^7}{6\sqrt{5-x^2}} + \frac{25x^5}{24\sqrt{5-x^2}} + \frac{25x^3}{48\sqrt{5-x^2}} - \frac{125x}{16\sqrt{5-x^2}} + \frac{125\operatorname{asin}\left(\frac{\sqrt{5}x}{5}\right)}{16} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-x**2+5)**(1/2),x)
```

```
[Out] Piecewise((I*x**7/(6*sqrt(x**2 - 5)) - 25*I*x**5/(24*sqrt(x**2 - 5)) - 25*I
*x**3/(48*sqrt(x**2 - 5)) + 125*I*x/(16*sqrt(x**2 - 5)) - 125*I*acosh(sqrt(
5)*x/5)/16, Abs(x**2)/5 > 1), (-x**7/(6*sqrt(5 - x**2)) + 25*x**5/(24*sqrt(
5 - x**2)) + 25*x**3/(48*sqrt(5 - x**2)) - 125*x/(16*sqrt(5 - x**2)) + 125*
asin(sqrt(5)*x/5)/16, True))
```

Giac [A] time = 1.06507, size = 49, normalized size = 0.75

$$\frac{1}{48}\left(2\left(4x^2-5\right)x^2-75\right)\sqrt{-x^2+5}x + \frac{125}{16}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*x^2 - 5)*x^2 - 75)*sqrt(-x^2 + 5)*x + 125/16*arcsin(1/5*sqrt(5)*
x)
```

3.251 $\int \frac{1}{x^6\sqrt{2+x^2}}\,dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{x^2+2}}{15x}+\frac{\sqrt{x^2+2}}{15x^3}-\frac{\sqrt{x^2+2}}{10x^5}$$

```
[Out] -Sqrt[2 + x^2]/(10*x^5) + Sqrt[2 + x^2]/(15*x^3) - Sqrt[2 + x^2]/(15*x)
```

Rubi [A] time = 0.0110739, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {271, 264}

$$-\frac{\sqrt{x^2+2}}{15x}+\frac{\sqrt{x^2+2}}{15x^3}-\frac{\sqrt{x^2+2}}{10x^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*Sqrt[2 + x^2]),x]
```

```
[Out] -Sqrt[2 + x^2]/(10*x^5) + Sqrt[2 + x^2]/(15*x^3) - Sqrt[2 + x^2]/(15*x)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^6\sqrt{2+x^2}}\,dx &= -\frac{\sqrt{2+x^2}}{10x^5}-\frac{2}{5}\int \frac{1}{x^4\sqrt{2+x^2}}\,dx\\ &= -\frac{\sqrt{2+x^2}}{10x^5}+\frac{\sqrt{2+x^2}}{15x^3}+\frac{2}{15}\int \frac{1}{x^2\sqrt{2+x^2}}\,dx\\ &= -\frac{\sqrt{2+x^2}}{10x^5}+\frac{\sqrt{2+x^2}}{15x^3}-\frac{\sqrt{2+x^2}}{15x}\end{aligned}$$

Mathematica [A] time = 0.006025, size = 28, normalized size = 0.57

$$-\frac{\sqrt{x^2+2}\left(2x^4-2x^2+3\right)}{30x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*Sqrt[2 + x^2]),x]
```

```
[Out] -(Sqrt[2 + x^2]*(3 - 2*x^2 + 2*x^4))/(30*x^5)
```

Maple [A] time = 0.003, size = 25, normalized size = 0.5

$$-\frac{2x^4-2x^2+3}{30x^5}\sqrt{x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^2+2)^(1/2),x)
```

```
[Out] -1/30*(x^2+2)^(1/2)*(2*x^4-2*x^2+3)/x^5
```

Maxima [A] time = 1.46857, size = 50, normalized size = 1.02

$$-\frac{\sqrt{x^2+2}}{15x}+\frac{\sqrt{x^2+2}}{15x^3}-\frac{\sqrt{x^2+2}}{10x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^2+2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/15*sqrt(x^2 + 2)/x + 1/15*sqrt(x^2 + 2)/x^3 - 1/10*sqrt(x^2 + 2)/x^5
```

Fricas [A] time = 1.8217, size = 74, normalized size = 1.51

$$-\frac{2x^5+\left(2x^4-2x^2+3\right)\sqrt{x^2+2}}{30x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^2+2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/30*(2*x^5 + (2*x^4 - 2*x^2 + 3)*sqrt(x^2 + 2))/x^5
```

Sympy [A] time = 4.00259, size = 41, normalized size = 0.84

$$-\frac{\sqrt{1+\frac{2}{x^2}}}{15}+\frac{\sqrt{1+\frac{2}{x^2}}}{15x^2}-\frac{\sqrt{1+\frac{2}{x^2}}}{10x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**2+2)**(1/2),x)
```

```
[Out] -sqrt(1 + 2/x**2)/15 + sqrt(1 + 2/x**2)/(15*x**2) - sqrt(1 + 2/x**2)/(10*x*
*4)
```

Giac [A] time = 1.07276, size = 69, normalized size = 1.41

$$\frac{32\left(5\left(x-\sqrt{x^2+2}\right)^4-5\left(x-\sqrt{x^2+2}\right)^2+2\right)}{15\left(\left(x-\sqrt{x^2+2}\right)^2-2\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^2+2)^(1/2),x, algorithm="giac")
```

```
[Out] 32/15*(5*(x - sqrt(x^2 + 2))^4 - 5*(x - sqrt(x^2 + 2))^2 + 2)/((x - sqrt(x^
2 + 2))^2 - 2)^5
```

3.252 $\int \frac{1}{\left(3+2x^2\right)^{7/2}} \, dx$

Optimal. Leaf size=49

$$\frac{8x}{405\sqrt{2x^2+3}}+\frac{4x}{135\left(2x^2+3\right)^{3/2}}+\frac{x}{15\left(2x^2+3\right)^{5/2}}$$

```
[Out] x/(15*(3 + 2*x^2)^(5/2)) + (4*x)/(135*(3 + 2*x^2)^(3/2)) + (8*x)/(405*Sqrt[
3 + 2*x^2])
```

Rubi [A] time = 0.0077619, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{8x}{405\sqrt{2x^2+3}}+\frac{4x}{135\left(2x^2+3\right)^{3/2}}+\frac{x}{15\left(2x^2+3\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x^2)^(-7/2),x]
```

```
[Out] x/(15*(3 + 2*x^2)^(5/2)) + (4*x)/(135*(3 + 2*x^2)^(3/2)) + (8*x)/(405*Sqrt[
3 + 2*x^2])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(3+2x^2\right)^{7/2}} \, dx &= \frac{x}{15\left(3+2x^2\right)^{5/2}}+\frac{4}{15}\int \frac{1}{\left(3+2x^2\right)^{5/2}} \, dx\\
&= \frac{x}{15\left(3+2x^2\right)^{5/2}}+\frac{4x}{135\left(3+2x^2\right)^{3/2}}+\frac{8}{135}\int \frac{1}{\left(3+2x^2\right)^{3/2}} \, dx\\
&= \frac{x}{15\left(3+2x^2\right)^{5/2}}+\frac{4x}{135\left(3+2x^2\right)^{3/2}}+\frac{8x}{405\sqrt{3+2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.007732, size = 28, normalized size = 0.57

$$\frac{x\left(32x^4+120x^2+135\right)}{405\left(2x^2+3\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x^2)^(-7/2),x]
```

```
[Out] (x*(135 + 120*x^2 + 32*x^4))/(405*(3 + 2*x^2)^(5/2))
```

Maple [A] time = 0.002, size = 25, normalized size = 0.5

$$\frac{x\left(32\,x^4+120\,x^2+135\right)}{405}\left(2\,x^2+3\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2+3)^(7/2),x)
```

```
[Out] 1/405*x*(32*x^4+120*x^2+135)/(2*x^2+3)^(5/2)
```

Maxima [A] time = 0.959111, size = 50, normalized size = 1.02

$$\frac{8\,x}{405\,\sqrt{2\,x^2+3}}+\frac{4\,x}{135\left(2\,x^2+3\right)^{\frac{3}{2}}}+\frac{x}{15\left(2\,x^2+3\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/405*x/sqrt(2*x^2 + 3) + 4/135*x/(2*x^2 + 3)^(3/2) + 1/15*x/(2*x^2 + 3)^(5
/2)
```

Fricas [A] time = 1.91383, size = 109, normalized size = 2.22

$$\frac{\left(32\,x^5+120\,x^3+135\,x\right)\sqrt{2\,x^2+3}}{405\left(8\,x^6+36\,x^4+54\,x^2+27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/405*(32*x^5 + 120*x^3 + 135*x)*sqrt(2*x^2 + 3)/(8*x^6 + 36*x^4 + 54*x^2 +
 27)
```

Sympy [B] time = 75.9236, size = 139, normalized size = 2.84

$$\frac{32x^5}{1620x^4\sqrt{2x^2+3}+4860x^2\sqrt{2x^2+3}+3645\sqrt{2x^2+3}}+\frac{120x^3}{1620x^4\sqrt{2x^2+3}+4860x^2\sqrt{2x^2+3}+3645\sqrt{2x^2+3}}+\frac{}{1620}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2+3)**(7/2),x)
```

```
[Out] 32*x**5/(1620*x**4*sqrt(2*x**2 + 3) + 4860*x**2*sqrt(2*x**2 + 3) + 3645*sqr
t(2*x**2 + 3)) + 120*x**3/(1620*x**4*sqrt(2*x**2 + 3) + 4860*x**2*sqrt(2*x*
*2 + 3) + 3645*sqrt(2*x**2 + 3)) + 135*x/(1620*x**4*sqrt(2*x**2 + 3) + 4860
*x**2*sqrt(2*x**2 + 3) + 3645*sqrt(2*x**2 + 3))
```

Giac [A] time = 1.09071, size = 35, normalized size = 0.71

$$\frac{\left(8\left(4x^2+15\right)x^2+135\right)x}{405\left(2x^2+3\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)^(7/2),x, algorithm="giac")
```

```
[Out] 1/405*(8*(4*x^2 + 15)*x^2 + 135)*x/(2*x^2 + 3)^(5/2)
```

3.253 $\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=12

$$\log\left(a+\sqrt{x^2+1}\right)$$

```
[Out] Log[a + Sqrt[1 + x^2]]
```

Rubi [A] time = 0.0468336, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2155, 31}

$$\log\left(a+\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^2 + a*Sqrt[1 + x^2]),x]

[Out] Log[a + Sqrt[1 + x^2]]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+x^2+a\sqrt{1+x^2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x+a\sqrt{1+x}}\,dx,x,x^2\right)\\ &= \text{Subst}\left(\int \frac{1}{a+x}\,dx,x,\sqrt{1+x^2}\right)\\ &= \log\left(a+\sqrt{1+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0262214, size = 12, normalized size = 1.

$$\log\left(a+\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^2 + a*Sqrt[1 + x^2]),x]
```

```
[Out] Log[a + Sqrt[1 + x^2]]
```

Maple [B] time = 0.04, size = 328, normalized size = 27.3

$$\frac{1}{a}\sqrt{x^2+1}-\frac{1}{2a}\sqrt{\left(x+\sqrt{(1+a)(a-1)}\right)^2-2\sqrt{(1+a)(a-1)}\left(x+\sqrt{(1+a)(a-1)}\right)+a^2}+\frac{a}{2}\ln\left(\left(2a^2-2\sqrt{(1+a)(a-1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x^2+a*(x^2+1)^(1/2)),x)
```

```
[Out] 1/a*(x^2+1)^(1/2)-1/2/a*((x+((1+a)*(a-1))^(1/2))^2-2*((1+a)*(a-1))^(1/2)*(x
+((1+a)*(a-1))^(1/2))+a^2)^(1/2)+1/2*a/(a^2)^(1/2)*ln((2*a^2-2*((1+a)*(a-1)
)^(1/2)*(x+((1+a)*(a-1))^(1/2))+2*(a^2)^(1/2)*((x+((1+a)*(a-1))^(1/2))^2-2*
((1+a)*(a-1))^(1/2)*(x+((1+a)*(a-1))^(1/2))+a^2)^(1/2))/(x+((1+a)*(a-1))^(1
/2)))-1/2/a*((x-((1+a)*(a-1))^(1/2))^2+2*((1+a)*(a-1))^(1/2)*(x-((1+a)*(a-1
))^(1/2))+a^2)^(1/2)+1/2*a/(a^2)^(1/2)*ln((2*a^2+2*((1+a)*(a-1))^(1/2)*(x-(
(1+a)*(a-1))^(1/2))+2*(a^2)^(1/2)*((x-((1+a)*(a-1))^(1/2))^2+2*((1+a)*(a-1)
)^(1/2)*(x-((1+a)*(a-1))^(1/2))+a^2)^(1/2))/(x-((1+a)*(a-1))^(1/2)))+1/2*ln
(-a^2+x^2+1)
```

Maxima [A] time = 0.945527, size = 14, normalized size = 1.17

$$\log\left(a+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] log(a + sqrt(x^2 + 1))
```

Fricas [B] time = 1.97612, size = 167, normalized size = 13.92

$$\frac{1}{2}\log\left(-a^2+x^2+1\right)-\frac{1}{2}\log\left(ax+x^2-\sqrt{x^2+1}(a+x)+1\right)+\frac{1}{2}\log\left(-ax+x^2+\sqrt{x^2+1}(a-x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-a^2 + x^2 + 1) - 1/2*log(a*x + x^2 - sqrt(x^2 + 1)*(a + x) + 1) +
1/2*log(-a*x + x^2 + sqrt(x^2 + 1)*(a - x) + 1)
```

Sympy [B] time = 1.71292, size = 53, normalized size = 4.42

$$-\frac{a\left(-\frac{\log\left(2a+2\sqrt{x^2+1}\right)}{a}+\frac{\log\left(-2\sqrt{x^2+1}\right)}{a}\right)}{2}+\frac{\log\left(a\sqrt{x^2+1}+x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x**2+a*(x**2+1)**(1/2)),x)
```

```
[Out] -a*(-log(2*a + 2*sqrt(x**2 + 1))/a + log(-2*sqrt(x**2 + 1))/a)/2 + log(a*sq
rt(x**2 + 1) + x**2 + 1)/2
```

Giac [A] time = 1.06548, size = 15, normalized size = 1.25

$$\log\left(\left|a + \sqrt{x^2 + 1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x^2+a*(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] log(abs(a + sqrt(x^2 + 1)))
```

3.254 $\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx$

Optimal. Leaf size=12

$$\frac{1}{\sqrt{x^2+1}}+\sinh^{-1}(x)$$

```
[Out] 1/Sqrt[1 + x^2] + ArcSinh[x]
```

Rubi [A] time = 0.0127955, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1814, 215}

$$\frac{1}{\sqrt{x^2+1}}+\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x + x^2)/(1 + x^2)^(3/2),x]
```

```
[Out] 1/Sqrt[1 + x^2] + ArcSinh[x]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x+x^2}{(1+x^2)^{3/2}}\,dx &= \frac{1}{\sqrt{1+x^2}}+\int \frac{1}{\sqrt{1+x^2}}\,dx\\ &= \frac{1}{\sqrt{1+x^2}}+\sinh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0114939, size = 12, normalized size = 1.

$$\frac{1}{\sqrt{x^2+1}}+\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x + x^2)/(1 + x^2)^(3/2),x]
```

```
[Out] 1/Sqrt[1 + x^2] + ArcSinh[x]
```

Maple [A] time = 0.006, size = 11, normalized size = 0.9

$$\operatorname{Arcsinh}(x)+\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-x+1)/(x^2+1)^(3/2),x)
```

```
[Out] arcsinh(x)+1/(x^2+1)^(1/2)
```

Maxima [A] time = 1.42586, size = 14, normalized size = 1.17

$$\frac{1}{\sqrt{x^2+1}}+\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/sqrt(x^2 + 1) + arcsinh(x)
```

Fricas [B] time = 1.82708, size = 86, normalized size = 7.17

$$-\frac{(x^2+1)\log\left(-x+\sqrt{x^2+1}\right)-\sqrt{x^2+1}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -((x^2 + 1)*log(-x + sqrt(x^2 + 1)) - sqrt(x^2 + 1))/(x^2 + 1)
```

Sympy [B] time = 6.48993, size = 29, normalized size = 2.42

$$\frac{x^2\operatorname{asinh}(x)}{x^2+1}+\frac{\operatorname{asinh}(x)}{x^2+1}+\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-x+1)/(x**2+1)**(3/2),x)
```

```
[Out] x**2*asinh(x)/(x**2 + 1) + asinh(x)/(x**2 + 1) + 1/sqrt(x**2 + 1)
```

Giac [B] time = 1.08002, size = 30, normalized size = 2.5

$$\frac{1}{\sqrt{x^2+1}} - \log\left(-x + \sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)/(x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))
```

3.255 $\int \frac{\sqrt{1+x^2}}{2+x^2} \, dx$

Optimal. Leaf size=27

$$\sinh^{-1}(x)-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{2}}$$

```
[Out] ArcSinh[x] - ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])]/Sqrt[2]
```

Rubi [A] time = 0.0110565, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {402, 215, 377, 206}

$$\sinh^{-1}(x)-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]/(2 + x^2),x]
```

```
[Out] ArcSinh[x] - ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])]/Sqrt[2]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1+x^2}}{2+x^2}\,dx &= \int \frac{1}{\sqrt{1+x^2}}\,dx - \int \frac{1}{\sqrt{1+x^2}\,(2+x^2)}\,dx \\ &= \sinh^{-1}(x) - \text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \frac{x}{\sqrt{1+x^2}}\right) \\ &= \sinh^{-1}(x) - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{1+x^2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0126224, size = 27, normalized size = 1.

$$\sinh^{-1}(x) - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2]/(2 + x^2),x]
```

```
[Out] ArcSinh[x] - ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])]/Sqrt[2]
```

Maple [A] time = 0.01, size = 23, normalized size = 0.9

$$\text{Arcsinh}\,(x) - \frac{\sqrt{2}}{2}\text{Artanh}\left(\frac{x\sqrt{2}}{2}\frac{1}{\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2)/(x^2+2),x)
```

```
[Out] arcsinh(x)-1/2*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\sqrt{x^2+1}x}{x^2+2} + \int \frac{\sqrt{x^2+1}x^4}{x^6+5x^4+8x^2+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^2+2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 1)*x/(x^2 + 2) + integrate(sqrt(x^2 + 1)*x^4/(x^6 + 5*x^4 + 8*x^
2 + 4), x)
```

Fricas [B] time = 1.7168, size = 173, normalized size = 6.41

$$\frac{1}{4}\sqrt{2}\log\left(\frac{9x^2 - 2\sqrt{2}\left(3x^2+2\right) - 2\sqrt{x^2+1}\left(3\sqrt{2}x - 4x\right) + 6}{x^2+2}\right) - \log\left(-x + \sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^2+2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((9*x^2 - 2*sqrt(2)*(3*x^2 + 2) - 2*sqrt(x^2 + 1)*(3*sqrt(2)
*x - 4*x) + 6)/(x^2 + 2)) - log(-x + sqrt(x^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2+1}}{x^2+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2)/(x**2+2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)/(x**2 + 2), x)
```

Giac [B] time = 1.07295, size = 86, normalized size = 3.19

$$\frac{1}{4}\sqrt{2}\log\left(\frac{\left(x-\sqrt{x^2+1}\right)^2-2\sqrt{2}+3}{\left(x-\sqrt{x^2+1}\right)^2+2\sqrt{2}+3}\right)-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2)/(x^2+2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(((x - sqrt(x^2 + 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 + 1)
)^2 + 2*sqrt(2) + 3)) - log(-x + sqrt(x^2 + 1))
```

3.256 $\int \frac{1}{\sqrt{1+x^2}(2+x^2)^2} dx$

Optimal. Leaf size=48

$$\frac{3\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2+1}}{4\left(x^2+2\right)}$$

```
[Out] -(x*Sqrt[1 + x^2])/(4*(2 + x^2)) + (3*ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])])/(
4*Sqrt[2])
```

Rubi [A] time = 0.0135447, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {382, 377, 206}

$$\frac{3\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{x^2+1}}\right)}{4\sqrt{2}}-\frac{x\sqrt{x^2+1}}{4\left(x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 + x^2]*(2 + x^2)^2),x]
```

```
[Out] -(x*Sqrt[1 + x^2])/(4*(2 + x^2)) + (3*ArcTanh[x/(Sqrt[2]*Sqrt[1 + x^2])])/(
4*Sqrt[2])
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}\left(2+x^2\right)^2}\,dx = -\frac{x\sqrt{1+x^2}}{4\left(2+x^2\right)} + \frac{3}{4}\int \frac{1}{\sqrt{1+x^2}\left(2+x^2\right)}\,dx$$
$$= -\frac{x\sqrt{1+x^2}}{4\left(2+x^2\right)} + \frac{3}{4}\,\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \frac{x}{\sqrt{1+x^2}}\right)$$
$$= -\frac{x\sqrt{1+x^2}}{4\left(2+x^2\right)} + \frac{3\tanh^{-1}\left(\frac{x}{\sqrt{2}\sqrt{1+x^2}}\right)}{4\sqrt{2}}$$

Mathematica [A] time = 0.24494, size = 74, normalized size = 1.54

$$\frac{\sqrt{x^2+1}\left(3\sqrt{2}\sqrt{\frac{x^2}{x^2+1}}\left(x^2+2\right)\tanh^{-1}\left(\sqrt{\frac{x^2}{2x^2+2}}\right)-2x^2\right)}{8x\left(x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 + x^2]*(2 + x^2)^2),x]
```

```
[Out] (Sqrt[1 + x^2]*(-2*x^2 + 3*Sqrt[2]*Sqrt[x^2/(1 + x^2)]*(2 + x^2)*ArcTanh[Sq
rt[x^2/(2 + 2*x^2)]]))/(8*x*(2 + x^2))
```

Maple [A] time = 0.014, size = 46, normalized size = 1.

$$\frac{x}{4}\frac{1}{\sqrt{x^2+1}}\left(\frac{x^2}{x^2+1}-2\right)^{-1}+\frac{3\sqrt{2}}{8}\mathrm{Artanh}\left(\frac{x\sqrt{2}}{2}\frac{1}{\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+2)^2/(x^2+1)^(1/2),x)
```

```
[Out] 1/4/(x^2+1)^(1/2)*x/(x^2/(x^2+1)-2)+3/8*arctanh(1/2*x*2^(1/2)/(x^2+1)^(1/2)
)*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+2\right)^2\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^2 + 2)^2*sqrt(x^2 + 1)), x)
```

Fricas [B] time = 1.82398, size = 215, normalized size = 4.48

$$\frac{3\sqrt{2}\left(x^2+2\right)\log\left(\frac{9x^2+2\sqrt{2}\left(3x^2+2\right)+2\sqrt{x^2+1}\left(3\sqrt{2}x+4x\right)+6}{x^2+2}\right)-4x^2-4\sqrt{x^2+1}x-8}{16\left(x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*sqrt(2)*(x^2 + 2)*log((9*x^2 + 2*sqrt(2)*(3*x^2 + 2) + 2*sqrt(x^2 +
 1)*(3*sqrt(2)*x + 4*x) + 6)/(x^2 + 2)) - 4*x^2 - 4*sqrt(x^2 + 1)*x - 8)/(x
^2 + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{x^2+1}\left(x^2+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+2)**2/(x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x**2 + 1)*(x**2 + 2)**2), x)
```

Giac [B] time = 1.07245, size = 136, normalized size = 2.83

$$-\frac{3}{16}\sqrt{2}\log\left(\frac{\left(x-\sqrt{x^2+1}\right)^2-2\sqrt{2}+3}{\left(x-\sqrt{x^2+1}\right)^2+2\sqrt{2}+3}\right)-\frac{3\left(x-\sqrt{x^2+1}\right)^2+1}{2\left(\left(x-\sqrt{x^2+1}\right)^4+6\left(x-\sqrt{x^2+1}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2)^2/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*log(((x - sqrt(x^2 + 1))^2 - 2*sqrt(2) + 3)/((x - sqrt(x^2 +
1))^2 + 2*sqrt(2) + 3)) - 1/2*(3*(x - sqrt(x^2 + 1))^2 + 1)/((x - sqrt(x^2
+ 1))^4 + 6*(x - sqrt(x^2 + 1))^2 + 1)
```

3.257 $\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx$

Optimal. Leaf size=41

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-2}}\right)-\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{x^2-2}}\right)$$

```
[Out] ArcTanh[x/Sqrt[-2 + x^2]] - Sqrt[3/2]*ArcTanh[(Sqrt[2/3]*x)/Sqrt[-2 + x^2]]
```

Rubi [A] time = 0.0218235, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {483, 217, 206, 377, 207}

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-2}}\right)-\sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{x^2-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-6 + x^2)*Sqrt[-2 + x^2]),x]
```

```
[Out] ArcTanh[x/Sqrt[-2 + x^2]] - Sqrt[3/2]*ArcTanh[(Sqrt[2/3]*x)/Sqrt[-2 + x^2]]
```

Rule 483

```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(
n_)), x_Symbol] :> Dist[e^n/b, Int[(e*x)^(m - n)*(c + d*x^n)^q, x], x] - Di
st[(a*e^n)/b, Int[((e*x)^(m - n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; Free
Q[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{(-6+x^2)\sqrt{-2+x^2}}\,dx = 6\int \frac{1}{(-6+x^2)\sqrt{-2+x^2}}\,dx + \int \frac{1}{\sqrt{-2+x^2}}\,dx$$
$$= 6\,\text{Subst}\left(\int \frac{1}{-6+4x^2}\,dx, x, \frac{x}{\sqrt{-2+x^2}}\right) + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-2+x^2}}\right)$$
$$= \tanh^{-1}\left(\frac{x}{\sqrt{-2+x^2}}\right) - \sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{-2+x^2}}\right)$$

Mathematica [A] time = 0.0284174, size = 41, normalized size = 1.

$$\log\left(\sqrt{x^2-2}+x\right) - \sqrt{\frac{3}{2}}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{3}}x}{\sqrt{x^2-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-6 + x^2)*Sqrt[-2 + x^2]),x]
```

```
[Out] -(Sqrt[3/2]*ArcTanh[(Sqrt[2/3]*x)/Sqrt[-2 + x^2]]) + Log[x + Sqrt[-2 + x^2]
]
```

Maple [B] time = 0.036, size = 100, normalized size = 2.4

$$\ln\left(x+\sqrt{x^2-2}\right) - \frac{\sqrt{6}}{4}\text{Artanh}\left(\frac{8+2\left(x-\sqrt{6}\right)\sqrt{6}}{4}\frac{1}{\sqrt{\left(x-\sqrt{6}\right)^2+2\left(x-\sqrt{6}\right)\sqrt{6}+4}}\right) + \frac{\sqrt{6}}{4}\text{Artanh}\left(\frac{8-2\left(x+\sqrt{6}\right)\sqrt{}}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2-6)/(x^2-2)^(1/2),x)
```

```
[Out] ln(x+(x^2-2)^(1/2))-1/4*6^(1/2)*arctanh(1/4*(8+2*(x-6^(1/2))*6^(1/2))/((x-6
^(1/2))^2+2*(x-6^(1/2))*6^(1/2)+4)^(1/2))+1/4*6^(1/2)*arctanh(1/4*(8-2*(x+6
^(1/2))*6^(1/2))/((x+6^(1/2))^2-2*(x+6^(1/2))*6^(1/2)+4)^(1/2))
```

Maxima [B] time = 1.47848, size = 144, normalized size = 3.51

$$\frac{1}{12}\sqrt{6}\left(2\sqrt{6}\log\left(x+\sqrt{x^2-2}\right) - 3\log\left(\sqrt{6}+\frac{4\sqrt{x^2-2}}{\left|2x-2\sqrt{6}\right|}+\frac{8}{\left|2x-2\sqrt{6}\right|}\right) + 3\log\left(-\sqrt{6}+\frac{4\sqrt{x^2-2}}{\left|2x+2\sqrt{6}\right|}+\frac{8}{\left|2x+2\sqrt{6}\right|}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(6)*(2*sqrt(6)*log(x + sqrt(x^2 - 2)) - 3*log(sqrt(6) + 4*sqrt(x^2
 - 2)/abs(2*x - 2*sqrt(6)) + 8/abs(2*x - 2*sqrt(6))) + 3*log(-sqrt(6) + 4*s
```

```
qrt(x^2 - 2)/abs(2*x + 2*sqrt(6)) + 8/abs(2*x + 2*sqrt(6))))
```

Fricas [B] time = 1.8899, size = 211, normalized size = 5.15

$$\frac{1}{4}\sqrt{3}\sqrt{2}\log\left(-\frac{2\sqrt{3}\sqrt{2}\left(5x^2-6\right)-25x^2+2\left(5\sqrt{3}\sqrt{2}x-12x\right)\sqrt{x^2-2}+30}{x^2-6}\right)-\log\left(-x+\sqrt{x^2-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(3)*sqrt(2)*log(-(2*sqrt(3)*sqrt(2)*(5*x^2 - 6) - 25*x^2 + 2*(5*sqr
t(3)*sqrt(2)*x - 12*x)*sqrt(x^2 - 2) + 30)/(x^2 - 6)) - log(-x + sqrt(x^2 -
 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\left(x^2-6\right)\sqrt{x^2-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2-6)/(x**2-2)**(1/2),x)
```

```
[Out] Integral(x**2/((x**2 - 6)*sqrt(x**2 - 2)), x)
```

Giac [B] time = 1.11784, size = 97, normalized size = 2.37

$$-\frac{1}{4}\sqrt{6}\log\left(\frac{\left|2\left(x-\sqrt{x^2-2}\right)^2-8\sqrt{6}-20\right|}{\left|2\left(x-\sqrt{x^2-2}\right)^2+8\sqrt{6}-20\right|}\right)-\frac{1}{2}\log\left(\left(x-\sqrt{x^2-2}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2-6)/(x^2-2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(6)*log(abs(2*(x - sqrt(x^2 - 2))^2 - 8*sqrt(6) - 20)/abs(2*(x - s
qrt(x^2 - 2))^2 + 8*sqrt(6) - 20)) - 1/2*log((x - sqrt(x^2 - 2))^2)
```

3.258 $\int \frac{5+x^2}{\sqrt{1-x^2}(1+x^2)^2}\,dx$

Optimal. Leaf size=47

$$\frac{\sqrt{1-x^2}\,x}{x^2+1}+2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$$

```
[Out] (x*Sqrt[1 - x^2])/(1 + x^2) + 2*Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

Rubi [A] time = 0.020665, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {527, 12, 377, 203}

$$\frac{\sqrt{1-x^2}\,x}{x^2+1}+2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x^2)/(Sqrt[1 - x^2]*(1 + x^2)^2),x]
```

```
[Out] (x*Sqrt[1 - x^2])/(1 + x^2) + 2*Sqrt[2]*ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{5+x^2}{\sqrt{1-x^2}\left(1+x^2\right)^2}\,dx = \frac{x\sqrt{1-x^2}}{1+x^2} - \frac{1}{4}\int -\frac{16}{\sqrt{1-x^2}\left(1+x^2\right)}\,dx$$
$$= \frac{x\sqrt{1-x^2}}{1+x^2} + 4\int \frac{1}{\sqrt{1-x^2}\left(1+x^2\right)}\,dx$$
$$= \frac{x\sqrt{1-x^2}}{1+x^2} + 4\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right)$$
$$= \frac{x\sqrt{1-x^2}}{1+x^2} + 2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)$$

Mathematica [A] time = 0.148822, size = 85, normalized size = 1.81

$$\frac{\sqrt{1-x^2}x}{x^2+1} + \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}}\right)}{\sqrt{2}} + \frac{3x\tanh^{-1}\left(\sqrt{2}\sqrt{\frac{x^2}{x^2-1}}\right)}{\sqrt{2}\sqrt{-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x^2)/(Sqrt[1 - x^2]*(1 + x^2)^2),x]
```

```
[Out] (x*Sqrt[1 - x^2])/(1 + x^2) + ArcTan[(Sqrt[2]*x)/Sqrt[1 - x^2]]/Sqrt[2] + (
3*x*ArcTanh[Sqrt[2]*Sqrt[x^2/(-1 + x^2)]])/(Sqrt[2]*Sqrt[-x^2])
```

Maple [A] time = 0.026, size = 70, normalized size = 1.5

$$-2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}x}{x^2-1}\right) - \frac{x}{2x^2-2}\sqrt{-x^2+1}\left(\frac{\left(-x^2+1\right)x^2}{\left(x^2-1\right)^2}+\frac{1}{2}\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x)
```

```
[Out] -2*2^(1/2)*arctan(2^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)-1/2*(-x^2+1)^(1/2)/(x^2
-1)*x/((-x^2+1)/(x^2-1)^2*x^2+1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+5}{\left(x^2+1\right)^2\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 5)/((x^2 + 1)^2*sqrt(-x^2 + 1)), x)
```

Fricas [A] time = 1.85211, size = 122, normalized size = 2.6

$$-\frac{2\sqrt{2}\left(x^2+1\right)\arctan\left(\frac{\sqrt{2}\sqrt{-x^2+1}}{2x}\right)-\sqrt{-x^2+1}\,x}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(2*sqrt(2)*(x^2 + 1)*arctan(1/2*sqrt(2)*sqrt(-x^2 + 1)/x) - sqrt(-x^2 + 1)
*x)/(x^2 + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5)/(x**2+1)**2/(-x**2+1)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

Giac [B] time = 1.08738, size = 166, normalized size = 3.53

$$\sqrt{2}\left(\pi\operatorname{sgn}(x)+2\arctan\left(-\frac{\sqrt{2}x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{4\left(\sqrt{-x^2+1}-1\right)}\right)\right)-\frac{2\left(\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}\right)}{\left(\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}\right)^2+8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)/(x^2+1)^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(pi*sgn(x) + 2*arctan(-1/4*sqrt(2)*x*((sqrt(-x^2 + 1) - 1)^2/x^2 -
1)/(sqrt(-x^2 + 1) - 1))) - 2*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1
)/x)/((x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)^2 + 8)
```

3.259 $\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=88

$$-4\sqrt{1-x^2}+20\log\left(\sqrt{1-x^2}+5\right)-\frac{25\tan^{-1}\left(\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right)}{2\sqrt{6}}-x+5\sin^{-1}(x)+\frac{25\tan^{-1}\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}$$

```
[Out] -x - 4*Sqrt[1 - x^2] + 5*ArcSin[x] + (25*ArcTan[x/(2*Sqrt[6])])/(2*Sqrt[6])
 - (25*ArcTan[(5*x)/(2*Sqrt[6]*Sqrt[1 - x^2])])/(2*Sqrt[6]) + 20*Log[5 + Sq
rt[1 - x^2]]
```

Rubi [A] time = 0.238756, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6742, 1591, 190, 43, 6740, 203, 402, 216, 377}

$$-4\sqrt{1-x^2}+20\log\left(\sqrt{1-x^2}+5\right)-\frac{25\tan^{-1}\left(\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right)}{2\sqrt{6}}-x+5\sin^{-1}(x)+\frac{25\tan^{-1}\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(4*x - Sqrt[1 - x^2])/(5 + Sqrt[1 - x^2]),x]
```

```
[Out] -x - 4*Sqrt[1 - x^2] + 5*ArcSin[x] + (25*ArcTan[x/(2*Sqrt[6])])/(2*Sqrt[6])
 - (25*ArcTan[(5*x)/(2*Sqrt[6]*Sqrt[1 - x^2])])/(2*Sqrt[6]) + 20*Log[5 + Sq
rt[1 - x^2]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \frac{4x-\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx &= \int \left(\frac{4x}{5+\sqrt{1-x^2}}-\frac{\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\right)dx\\
&= 4\int \frac{x}{5+\sqrt{1-x^2}}\,dx-\int \frac{\sqrt{1-x^2}}{5+\sqrt{1-x^2}}\,dx\\
&= -\left(2\,\mathrm{Subst}\left(\int \frac{1}{5+\sqrt{x}}\,dx,x,1-x^2\right)\right)-\int \left(1-\frac{5}{5+\sqrt{1-x^2}}\right)dx\\
&= -x-4\,\mathrm{Subst}\left(\int \frac{x}{5+x}\,dx,x,\sqrt{1-x^2}\right)+5\int \frac{1}{5+\sqrt{1-x^2}}\,dx\\
&= -x-4\,\mathrm{Subst}\left(\int \left(1-\frac{5}{5+x}\right)dx,x,\sqrt{1-x^2}\right)+5\int \left(\frac{5}{24+x^2}-\frac{\sqrt{1-x^2}}{24+x^2}\right)dx\\
&= -x-4\sqrt{1-x^2}+20\log\left(5+\sqrt{1-x^2}\right)-5\int \frac{\sqrt{1-x^2}}{24+x^2}\,dx+25\int \frac{1}{24+x^2}\,dx\\
&= -x-4\sqrt{1-x^2}+\frac{25\tan^{-1}\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}+20\log\left(5+\sqrt{1-x^2}\right)+5\int \frac{1}{\sqrt{1-x^2}}\,dx-125\int \frac{1}{\sqrt{1-x^2}\,(24+\\
&= -x-4\sqrt{1-x^2}+5\sin^{-1}(x)+\frac{25\tan^{-1}\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}+20\log\left(5+\sqrt{1-x^2}\right)-125\,\mathrm{Subst}\left(\int \frac{1}{24+25x^2}\,d\right.\\
&= -x-4\sqrt{1-x^2}+5\sin^{-1}(x)+\frac{25\tan^{-1}\left(\frac{x}{2\sqrt{6}}\right)}{2\sqrt{6}}-\frac{25\tan^{-1}\left(\frac{5x}{2\sqrt{6}\sqrt{1-x^2}}\right)}{2\sqrt{6}}+20\log\left(5+\sqrt{1-x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.177174, size = 137, normalized size = 1.56

$$-4\sqrt{1-x^2}+10\log\left(x^2+24\right)-10\log\left(\left(x^2+24\right)^2\right)+10\log\left(\left(x^2+24\right)\left(-x^2+10\sqrt{1-x^2}+26\right)\right)+\frac{25\tan^{-1}\left(\frac{4x^2+409}{10\sqrt{6}(}\right.}{2\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*x - Sqrt[1 - x^2])/(5 + Sqrt[1 - x^2]),x]
```

```
[Out] -x - 4*Sqrt[1 - x^2] + 5*ArcSin[x] + (25*ArcTan[x/(2*Sqrt[6])])/(2*Sqrt[6])
 + (25*ArcTan[(96 + 4*x^2 + 409*x*Sqrt[1 - x^2])/(10*Sqrt[6]*(-1 + 17*x^2))
])/(2*Sqrt[6]) + 10*Log[24 + x^2] - 10*Log[(24 + x^2)^2] + 10*Log[(24 + x^2
)*(26 - x^2 + 10*Sqrt[1 - x^2])]
```

Maple [A] time = 0.02, size = 82, normalized size = 0.9

$$\frac{25\sqrt{6}}{12}\arctan\left(\frac{x\sqrt{6}}{12}\right)+10\ln\left(x^2+24\right)-x+5\arcsin(x)+\frac{25\sqrt{6}}{12}\arctan\left(\frac{5x\sqrt{6}}{12x^2-12}\sqrt{-x^2+1}\right)-4\sqrt{-x^2+1}+20$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x)
```

```
[Out] 25/12*arctan(1/12*x*6^(1/2))*6^(1/2)+10*ln(x^2+24)-x+5*arcsin(x)+25/12*6^(1
/2)*arctan(5/12*6^(1/2)*(-x^2+1)^(1/2)/(x^2-1)*x)-4*(-x^2+1)^(1/2)+20*arcta
nh(1/5*(-x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-x-4\sqrt{-x^2+1}+5\int\frac{1}{\sqrt{x+1}\sqrt{-x+1}+5}\,dx+20\log\left(\sqrt{-x^2+1}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] -x - 4*sqrt(-x^2 + 1) + 5*integrate(1/(sqrt(x + 1)*sqrt(-x + 1) + 5), x) +
20*log(sqrt(-x^2 + 1) + 5)
```

Fricas [B] time = 1.98118, size = 450, normalized size = 5.11

$$\frac{25}{12}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}x\right)+\frac{25}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{-x^2+1}-\sqrt{6}}{2x}\right)+\frac{25}{12}\sqrt{6}\arctan\left(\frac{\sqrt{6}\sqrt{-x^2+1}-\sqrt{6}}{3x}\right)-x-4\sqrt{-x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 25/12*sqrt(6)*arctan(1/12*sqrt(6)*x) + 25/12*sqrt(6)*arctan(1/2*(sqrt(6)*sq
rt(-x^2 + 1) - sqrt(6))/x) + 25/12*sqrt(6)*arctan(1/3*(sqrt(6)*sqrt(-x^2 +
```

```
1) - sqrt(6))/x) - x - 4*sqrt(-x^2 + 1) - 10*arctan((sqrt(-x^2 + 1) - 1)/x)
 + 10*log(x^2 + 24) - 10*log(-(x^2 + 6*sqrt(-x^2 + 1) - 6)/x^2) + 10*log((x
^2 - 4*sqrt(-x^2 + 1) + 4)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{4x - \sqrt{1 - x^2}}{\sqrt{1 - x^2} + 5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x-(-x**2+1)**(1/2))/(5+(-x**2+1)**(1/2)),x)
```

```
[Out] Integral((4*x - sqrt(1 - x**2))/(sqrt(1 - x**2) + 5), x)
```

Giac [B] time = 1.13468, size = 182, normalized size = 2.07

$$\frac{25}{12}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}x\right) - \frac{25}{12}\sqrt{6}\arctan\left(-\frac{\sqrt{6}\left(\sqrt{-x^2+1}-1\right)}{3x}\right) - \frac{25}{12}\sqrt{6}\arctan\left(-\frac{\sqrt{6}\left(\sqrt{-x^2+1}-1\right)}{2x}\right) - x - 4\sqrt{-x^2+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x-(-x^2+1)^(1/2))/(5+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 25/12*sqrt(6)*arctan(1/12*sqrt(6)*x) - 25/12*sqrt(6)*arctan(-1/3*sqrt(6)*(s
qrt(-x^2 + 1) - 1)/x) - 25/12*sqrt(6)*arctan(-1/2*sqrt(6)*(sqrt(-x^2 + 1) -
 1)/x) - x - 4*sqrt(-x^2 + 1) + 5*arcsin(x) + 10*log(x^2 + 24) - 10*log(3*(
sqrt(-x^2 + 1) - 1)^2/x^2 + 2) + 10*log(2*(sqrt(-x^2 + 1) - 1)^2/x^2 + 3)
```

3.260 $\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+\left(1+x^2\right)^{3/2}\right)}\,dx$

Optimal. Leaf size=136

$$-\frac{x^2}{6}-\frac{1}{6}\sqrt{x^2+1}x+\frac{8\sqrt{x^2+1}}{9}-\frac{7}{54}\log\left(3x^2+2x+3\right)+\frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)+\frac{7}{27}\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+1}}\right)+\frac{8x}{9}$$

```
[Out] (8*x)/9 - x^2/6 + (8*Sqrt[1 + x^2])/9 - (x*Sqrt[1 + x^2])/6 - (41*ArcSinh[x
])/54 + (4*Sqrt[2]*ArcTan[(1 + 3*x)/(2*Sqrt[2])])/27 + (4*Sqrt[2]*ArcTan[(1
 + x)/(Sqrt[2]*Sqrt[1 + x^2])])/27 + (7*ArcTanh[(1 - x)/(2*Sqrt[1 + x^2])])
/27 - (7*Log[3 + 2*x + 3*x^2])/54
```

Rubi [A] time = 1.49609, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 14, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {6742, 195, 215, 634, 618, 204, 628, 1020, 12, 1081, 1037, 1031, 206, 261}

$$-\frac{x^2}{6}-\frac{1}{6}\sqrt{x^2+1}x+\frac{8\sqrt{x^2+1}}{9}-\frac{7}{54}\log\left(3x^2+2x+3\right)+\frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+1}}\right)+\frac{7}{27}\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+1}}\right)+\frac{8x}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(2 - Sqrt[1 + x^2]))/(Sqrt[1 + x^2]*(1 - x^3 + (1 + x^2)^(3/2))),x
]
```

```
[Out] (8*x)/9 - x^2/6 + (8*Sqrt[1 + x^2])/9 - (x*Sqrt[1 + x^2])/6 - (41*ArcSinh[x
])/54 + (4*Sqrt[2]*ArcTan[(1 + 3*x)/(2*Sqrt[2])])/27 + (4*Sqrt[2]*ArcTan[(1
 + x)/(Sqrt[2]*Sqrt[1 + x^2])])/27 + (7*ArcTanh[(1 - x)/(2*Sqrt[1 + x^2])])
/27 - (7*Log[3 + 2*x + 3*x^2])/54
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1020

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + c*x^2)^p*(d + e*x + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] + Dist[1/(2*f*(p + q + 1)), Int[(a + c*x^2)^(p -
 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p*
(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && N
eQ[p + q + 1, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1081

```
Int[((A_.) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + f*x^2], x], x] + Dis
t[1/c, Int[(A*c - a*C - b*C*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x]
/; FreeQ[{a, b, c, d, f, A, C}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1037

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + b^2*d*f, 2]}, Dist
[1/(2*q), Int[Simp[h*b*d - g*(c*d - a*f - q) + (h*(c*d - a*f + q) + g*b*f)*
x, x]/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[
h*b*d - g*(c*d - a*f + q) + (h*(c*d - a*f - q) + g*b*f)*x, x]/((a + b*x + c
*x^2)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b
^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1031

```
Int[((g_) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[Int[1/Simp[g*(g*b
 - 2*a*h)*(b^2 - 4*a*c) - b*d*x^2, x], x], x, Simp[g*b - 2*a*h - (b*h - 2*g
*c)*x, x]/Sqrt[d + f*x^2]], x] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^
```

```
2 - 4*a*c, 0] && EqQ[b*h^2*d - 2*g*h*(c*d - a*f) - g^2*b*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^2\left(2-\sqrt{1+x^2}\right)}{\sqrt{1+x^2}\left(1-x^3+\left(1+x^2\right)^{3/2}\right)}\,dx = \int\left(-\frac{x^2}{1-x^3+\sqrt{1+x^2}+x^2\sqrt{1+x^2}}-\frac{2x^2}{\sqrt{1+x^2}\left(-1+x^3-\left(1+x^2\right)^{3/2}\right)}\right)dx$$

$$= -\left(2\int\frac{x^2}{\sqrt{1+x^2}\left(-1+x^3-\left(1+x^2\right)^{3/2}\right)}\,dx\right)-\int\frac{x^2}{1-x^3+\sqrt{1+x^2}+x^2\sqrt{1+x^2}}$$

$$= -\left(2\int\left(-\frac{1}{3}+\frac{2}{9\sqrt{1+x^2}}-\frac{x}{3\sqrt{1+x^2}}+\frac{2x}{3\left(3+2x+3x^2\right)}+\frac{3+5x}{9\sqrt{1+x^2}\left(3+2x\right.}\right.\right.$$

$$= \frac{8x}{9}-\frac{x^2}{6}-\frac{1}{9}\int\frac{-3-5x}{3+2x+3x^2}\,dx-\frac{2}{9}\int\frac{3+5x}{\sqrt{1+x^2}\left(3+2x+3x^2\right)}\,dx-\frac{1}{3}\int\sqrt{1}$$

$$= \frac{8x}{9}-\frac{x^2}{6}+\frac{8\sqrt{1+x^2}}{9}-\frac{1}{6}x\sqrt{1+x^2}-\frac{4}{9}\sinh^{-1}(x)+\frac{1}{18}\int\frac{4-4x}{\sqrt{1+x^2}\left(3+2x+3\right.}$$

$$= \frac{8x}{9}-\frac{x^2}{6}+\frac{8\sqrt{1+x^2}}{9}-\frac{1}{6}x\sqrt{1+x^2}-\frac{11}{18}\sinh^{-1}(x)-\frac{7}{54}\log\left(3+2x+3x^2\right)-$$

$$= \frac{8x}{9}-\frac{x^2}{6}+\frac{8\sqrt{1+x^2}}{9}-\frac{1}{6}x\sqrt{1+x^2}-\frac{11}{18}\sinh^{-1}(x)+\frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{1+3x}{2\sqrt{2}}\right)+$$

$$= \frac{8x}{9}-\frac{x^2}{6}+\frac{8\sqrt{1+x^2}}{9}-\frac{1}{6}x\sqrt{1+x^2}-\frac{41}{54}\sinh^{-1}(x)+\frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{1+3x}{2\sqrt{2}}\right)+$$

$$= \frac{8x}{9}-\frac{x^2}{6}+\frac{8\sqrt{1+x^2}}{9}-\frac{1}{6}x\sqrt{1+x^2}-\frac{41}{54}\sinh^{-1}(x)+\frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{1+3x}{2\sqrt{2}}\right)+$$

$$= \frac{8x}{9}-\frac{x^2}{6}+\frac{8\sqrt{1+x^2}}{9}-\frac{1}{6}x\sqrt{1+x^2}-\frac{41}{54}\sinh^{-1}(x)+\frac{4}{27}\sqrt{2}\tan^{-1}\left(\frac{1+3x}{2\sqrt{2}}\right)+$$

Mathematica [C] time = 0.862711, size = 261, normalized size = 1.92

$$\frac{1}{162}\left(-27x^2-27\sqrt{x^2+1}x+144\sqrt{x^2+1}-21\log\left(3x^2+2x+3\right)+\sqrt{1-2i\sqrt{2}}\left(11\sqrt{2}-i\right)\tanh^{-1}\left(\frac{-2i\sqrt{2}x-x+3}{\sqrt{2+4i\sqrt{2}}\sqrt{x^2+1}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(2 - Sqrt[1 + x^2]))/(Sqrt[1 + x^2]*(1 - x^3 + (1 + x^2)^(3/
2))),x]
```

```
[Out] (144*x - 27*x^2 + 144*Sqrt[1 + x^2] - 27*x*Sqrt[1 + x^2] - 123*ArcSinh[x] +
 24*Sqrt[2]*ArcTan[(1 + 3*x)/(2*Sqrt[2])] + Sqrt[1 - (2*I)*Sqrt[2]]*(-I + 1
1*Sqrt[2])*ArcTanh[(3 - x - (2*I)*Sqrt[2]*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[
1 + x^2])] + I*Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(3 - x + (2*I)*Sqrt[2]*x)/(S
qrt[2 - (4*I)*Sqrt[2]]*Sqrt[1 + x^2])] + 11*Sqrt[2 + (4*I)*Sqrt[2]]*ArcTanh
[(3 - x + (2*I)*Sqrt[2]*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[1 + x^2])] - 21*Lo
g[3 + 2*x + 3*x^2])/162
```

Maple [B] time = 0.044, size = 654, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x)
```

```
[Out] 8/9*x-1/6*x^2-7/54*ln(3*x^2+2*x+3)+4/27*2^(1/2)*arctan(1/8*(6*x+2)*2^(1/2))
-41/54*arcsinh(x)-1/12*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(-2^(1/2)*arctan
(1/2*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*(1+x)/(1-x))+5
*arctanh((2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/(1+(1+x)/(1-x))
^2)^(1/2)/(1+(1+x)/(1-x))+3/8*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(-2^(1/2)
*arctan(1/2*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*(1+x)/(
1-x))+arctanh((2*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/(1+(1+x)/(
1-x))^2)^(1/2)/(1+(1+x)/(1-x))-1/6*x*(x^2+1)^(1/2)+8/9*(x^2+1)^(1/2)+1/216*
2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(13*2^(1/2)*arctan(1/2*2^(1/2)*(2*(1+x)
^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*(1+x)/(1-x))+43*arctanh((2*(1+x)^2/
(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/(1+(1+x)/(1-x))^2)^(1/2)/(1+(1+x)/(
1-x))-1/36*2^(1/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)*(-11*2^(1/2)*arctan(1/2*2^(1
/2)*(2*(1+x)^2/(1-x)^2+2)^(1/2)/((1+x)^2/(1-x)^2+1)*(1+x)/(1-x))+arctanh((2
*(1+x)^2/(1-x)^2+2)^(1/2)))/(((1+x)^2/(1-x)^2+1)/(1+(1+x)/(1-x))^2)^(1/2)/(
1+(1+x)/(1-x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)+\int-\frac{3x^{10}-4x^9+5x^8-2x^7}{2\left(2x^{13}+7x^{11}-4x^{10}+11x^9-11x^8+13x^7-13x^6+11x^5-11x^4+4x^3-7x^2-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x, algo
rithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x) + integrate(-1/2*(3*x^10 - 4*x^9 + 5*x^8 -
 2*x^7 + 15*x^6 + 6*x^5 + 9*x^4)/(2*x^13 + 7*x^11 - 4*x^10 + 11*x^9 - 11*x^
8 + 13*x^7 - 13*x^6 + 11*x^5 - 11*x^4 + 4*x^3 - 7*x^2 - 2*(x^12 + 3*x^10 -
2*x^9 + 3*x^8 - 6*x^7 + 2*x^6 - 6*x^5 + 3*x^4 - 2*x^3 + 3*x^2 + 1)*sqrt(x^2
 + 1) - 2), x) + 1/6*log(x^2 + x + 1) + 1/6*log(x - 1)
```

Fricas [A] time = 2.00707, size = 541, normalized size = 3.98

$$-\frac{1}{6}x^2-\frac{1}{18}\sqrt{x^2+1}(3x-16)+\frac{4}{27}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}(3x+1)\right)+\frac{4}{27}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}(3x-1)+\frac{3}{2}\sqrt{2}\sqrt{x^2+1}\right)-\frac{4}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x, algo
rithm="fricas")
```

```
[Out] -1/6*x^2 - 1/18*sqrt(x^2 + 1)*(3*x - 16) + 4/27*sqrt(2)*arctan(1/4*sqrt(2)*
(3*x + 1)) + 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(3*x - 1) + 3/2*sqrt(2)*sqrt(
x^2 + 1)) - 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(x + 1) + 1/2*sqrt(2)*sqrt(x^2
 + 1)) + 8/9*x + 7/54*log(3*x^2 - sqrt(x^2 + 1)*(3*x - 1) - x + 2) - 7/54*l
og(3*x^2 + 2*x + 3) - 7/54*log(x^2 - sqrt(x^2 + 1)*(x + 1) + x + 2) + 41/54
*log(-x + sqrt(x^2 + 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2-(x**2+1)**(1/2))/(1-x**3+(x**2+1)**(3/2))/(x**2+1)**(1/2)
,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10647, size = 238, normalized size = 1.75

$$-\frac{1}{6}x^2 - \frac{1}{18}\sqrt{x^2+1}(3x-16) + \frac{4}{27}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(3x-3\sqrt{x^2+1}-1\right)\right) + \frac{4}{27}\sqrt{2}\arctan\left(\frac{1}{4}\sqrt{2}(3x+1)\right) - \frac{4}{27}\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2-(x^2+1)^(1/2))/(1-x^3+(x^2+1)^(3/2))/(x^2+1)^(1/2),x, algo
rithm="giac")
```

```
[Out] -1/6*x^2 - 1/18*sqrt(x^2 + 1)*(3*x - 16) + 4/27*sqrt(2)*arctan(-1/2*sqrt(2)
*(3*x - 3*sqrt(x^2 + 1) - 1)) + 4/27*sqrt(2)*arctan(1/4*sqrt(2)*(3*x + 1))
- 4/27*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 1) + 1)) + 8/9*x + 7/54*
log(3*(x - sqrt(x^2 + 1))^2 - 2*x + 2*sqrt(x^2 + 1) + 1) - 7/54*log((x - sq
rt(x^2 + 1))^2 + 2*x - 2*sqrt(x^2 + 1) + 3) - 7/54*log(3*x^2 + 2*x + 3) + 4
1/54*log(-x + sqrt(x^2 + 1))
```

3.261 $\int x\sqrt{2rx-x^2}\,dx$

Optimal. Leaf size=64

$$r^3\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}$$

```
[Out] -(r*(r - x)*Sqrt[2*r*x - x^2])/2 - (2*r*x - x^2)^(3/2)/3 + r^3*ArcTan[x/Sqr
t[2*r*x - x^2]]
```

Rubi [A] time = 0.0172441, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {640, 612, 620, 203}

$$r^3\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[2*r*x - x^2],x]
```

```
[Out] -(r*(r - x)*Sqrt[2*r*x - x^2])/2 - (2*r*x - x^2)^(3/2)/3 + r^3*ArcTan[x/Sqr
t[2*r*x - x^2]]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{2rx-x^2}\,dx &= -\frac{1}{3}\left(2rx-x^2\right)^{3/2}+r\int\sqrt{2rx-x^2}\,dx\\ &= -\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}+\frac{1}{2}r^3\int\frac{1}{\sqrt{2rx-x^2}}\,dx\\ &= -\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}+r^3\,\mathrm{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{x}{\sqrt{2rx-x^2}}\right)\\ &= -\frac{1}{2}r(r-x)\sqrt{2rx-x^2}-\frac{1}{3}\left(2rx-x^2\right)^{3/2}+r^3\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.109978, size = 72, normalized size = 1.12

$$\frac{1}{6}\sqrt{-x(x-2r)}\left(\frac{6r^{5/2}\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{2}\sqrt{r}}\right)}{\sqrt{x}\sqrt{2-\frac{x}{r}}}-3r^2-rx+2x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[2*r*x - x^2],x]
```

```
[Out] (Sqrt[-(x*(-2*r + x))]*(-3*r^2 - r*x + 2*x^2 + (6*r^(5/2)*ArcSin[Sqrt[x]/(S
qrt[2]*Sqrt[r])])/(Sqrt[x]*Sqrt[2 - x/r])))/6
```

Maple [A] time = 0.007, size = 73, normalized size = 1.1

$$-\frac{1}{3}\left(2\,rx-x^2\right)^{\frac{3}{2}}+\frac{rx}{2}\sqrt{2\,rx-x^2}-\frac{r^2}{2}\sqrt{2\,rx-x^2}+\frac{r^3}{2}\arctan\left((x-r)\frac{1}{\sqrt{2\,rx-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(2*r*x-x^2)^(1/2),x)
```

```
[Out] -1/3*(2*r*x-x^2)^(3/2)+1/2*r*x*(2*r*x-x^2)^(1/2)-1/2*(2*r*x-x^2)^(1/2)*r^2+
1/2*r^3*arctan((x-r)/(2*r*x-x^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*r*x-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7791, size = 108, normalized size = 1.69

$$-r^3\arctan\left(\frac{\sqrt{2\,rx-x^2}}{x}\right)-\frac{1}{6}\left(3\,r^2+rx-2\,x^2\right)\sqrt{2\,rx-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*r*x-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -r^3*arctan(sqrt(2*r*x - x^2)/x) - 1/6*(3*r^2 + r*x - 2*x^2)*sqrt(2*r*x - x
^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{-x\left(-2r+x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*r*x-x**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-x*(-2*r + x)), x)
```

Giac [A] time = 1.08457, size = 61, normalized size = 0.95

$$-\frac{1}{2}\,r^3\arcsin\left(\frac{r-x}{r}\right)\mathrm{sgn}\left(r\right)-\frac{1}{6}\left(3\,r^2+\left(r-2\,x\right)x\right)\sqrt{2\,rx-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(2*r*x-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*r^3*arcsin((r - x)/r)*sgn(r) - 1/6*(3*r^2 + (r - 2*x)*x)*sqrt(2*r*x -
x^2)
```

3.262 $\int x^2\sqrt{2rx-x^2}\,dx$

Optimal. Leaf size=89

$$-\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2}+\frac{5}{4}r^4\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{5}{12}r\left(2rx-x^2\right)^{3/2}-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}$$

```
[Out] (-5*r^2*(r - x)*Sqrt[2*r*x - x^2])/8 - (5*r*(2*r*x - x^2)^(3/2))/12 - (x*(2
*r*x - x^2)^(3/2))/4 + (5*r^4*ArcTan[x/Sqrt[2*r*x - x^2]])/4
```

Rubi [A] time = 0.0296294, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {670, 640, 612, 620, 203}

$$-\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2}+\frac{5}{4}r^4\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{5}{12}r\left(2rx-x^2\right)^{3/2}-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[2*r*x - x^2],x]
```

```
[Out] (-5*r^2*(r - x)*Sqrt[2*r*x - x^2])/8 - (5*r*(2*r*x - x^2)^(3/2))/12 - (x*(2
*r*x - x^2)^(3/2))/4 + (5*r^4*ArcTan[x/Sqrt[2*r*x - x^2]])/4
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\sqrt{2rx-x^2}\,dx &= -\frac{1}{4}x\left(2rx-x^2\right)^{3/2}+\frac{1}{4}(5r)\int x\sqrt{2rx-x^2}\,dx\\
&= -\frac{5}{12}r\left(2rx-x^2\right)^{3/2}-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}+\frac{1}{4}\left(5r^2\right)\int\sqrt{2rx-x^2}\,dx\\
&= -\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2}-\frac{5}{12}r\left(2rx-x^2\right)^{3/2}-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}+\frac{1}{8}\left(5r^4\right)\int\frac{1}{\sqrt{2rx-x^2}}\,dx\\
&= -\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2}-\frac{5}{12}r\left(2rx-x^2\right)^{3/2}-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}+\frac{1}{4}\left(5r^4\right)\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\frac{}{\sqrt{2r}}\right.\\
&= -\frac{5}{8}r^2(r-x)\sqrt{2rx-x^2}-\frac{5}{12}r\left(2rx-x^2\right)^{3/2}-\frac{1}{4}x\left(2rx-x^2\right)^{3/2}+\frac{5}{4}r^4\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.102408, size = 80, normalized size = 0.9

$$\frac{1}{24}\sqrt{-x(x-2r)}\left(-5r^2x+\frac{30r^{7/2}\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{2}\sqrt{r}}\right)}{\sqrt{x}\sqrt{2-\frac{x}{r}}}-15r^3-2rx^2+6x^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[2*r*x - x^2],x]
```

```
[Out] (Sqrt[-(x*(-2*r + x))]*(-15*r^3 - 5*r^2*x - 2*r*x^2 + 6*x^3 + (30*r^(7/2)*A
rcSin[Sqrt[x]/(Sqrt[2]*Sqrt[r])])/(Sqrt[x]*Sqrt[2 - x/r])))/24
```

Maple [A] time = 0.003, size = 91, normalized size = 1.

$$-\frac{x}{4}\left(2\,rx-x^2\right)^{\frac{3}{2}}-\frac{5\,r}{12}\left(2\,rx-x^2\right)^{\frac{3}{2}}+\frac{5\,r^2x}{8}\sqrt{2\,rx-x^2}-\frac{5\,r^3}{8}\sqrt{2\,rx-x^2}+\frac{5\,r^4}{8}\arctan\left((x-r)\frac{1}{\sqrt{2\,rx-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*r*x-x^2)^(1/2),x)
```

```
[Out] -1/4*x*(2*r*x-x^2)^(3/2)-5/12*r*(2*r*x-x^2)^(3/2)+5/8*r^2*(2*r*x-x^2)^(1/2)
*x-5/8*(2*r*x-x^2)^(1/2)*r^3+5/8*r^4*arctan((x-r)/(2*r*x-x^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*r*x-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.1459, size = 135, normalized size = 1.52

$$-\frac{5}{4}\,r^4\arctan\left(\frac{\sqrt{2\,rx-x^2}}{x}\right)-\frac{1}{24}\left(15\,r^3+5\,r^2x+2\,rx^2-6\,x^3\right)\sqrt{2\,rx-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*r*x-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -5/4*r^4*arctan(sqrt(2*r*x - x^2)/x) - 1/24*(15*r^3 + 5*r^2*x + 2*r*x^2 - 6
*x^3)*sqrt(2*r*x - x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{-x\,(-2r+x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*r*x-x**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-x*(-2*r + x)), x)
```

Giac [A] time = 1.08082, size = 73, normalized size = 0.82

$$-\frac{5}{8}\,r^4\arcsin\left(\frac{r-x}{r}\right)\mathrm{sgn}\left(r\right)-\frac{1}{24}\left(15\,r^3+\left(5\,r^2+2\,(r-3\,x)x\right)x\right)\sqrt{2\,rx-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*r*x-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -5/8*r^4*arcsin((r - x)/r)*sgn(r) - 1/24*(15*r^3 + (5*r^2 + 2*(r - 3*x)*x)*
x)*sqrt(2*r*x - x^2)
```

3.263 $\int x^3\sqrt{2rx-x^2}\,dx$

Optimal. Leaf size=113

$$-\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2}-\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}+\frac{7}{4}r^5\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

```
[Out] (-7*r^3*(r - x)*Sqrt[2*r*x - x^2])/8 - (7*r^2*(2*r*x - x^2)^(3/2))/12 - (7*
r*x*(2*r*x - x^2)^(3/2))/20 - (x^2*(2*r*x - x^2)^(3/2))/5 + (7*r^5*ArcTan[x
/Sqrt[2*r*x - x^2]])/4
```

Rubi [A] time = 0.0427148, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {670, 640, 612, 620, 203}

$$-\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2}-\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}+\frac{7}{4}r^5\tan^{-1}\left(\frac{x}{\sqrt{2rx-x^2}}\right)-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[2*r*x - x^2],x]
```

```
[Out] (-7*r^3*(r - x)*Sqrt[2*r*x - x^2])/8 - (7*r^2*(2*r*x - x^2)^(3/2))/12 - (7*
r*x*(2*r*x - x^2)^(3/2))/20 - (x^2*(2*r*x - x^2)^(3/2))/5 + (7*r^5*ArcTan[x
/Sqrt[2*r*x - x^2]])/4
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{2rx-x^2}\,dx &= -\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{1}{5}(7r)\int x^2\sqrt{2rx-x^2}\,dx\\ &= -\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{1}{4}\left(7r^2\right)\int x\sqrt{2rx-x^2}\,dx\\ &= -\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{1}{4}\left(7r^3\right)\int \sqrt{2rx-x^2}\,dx\\ &= -\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2}-\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{1}{8}\left(7r^5\right)\int\\ &= -\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2}-\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{1}{4}\left(7r^5\right)\mathrm{Su}\\ &= -\frac{7}{8}r^3(r-x)\sqrt{2rx-x^2}-\frac{7}{12}r^2\left(2rx-x^2\right)^{3/2}-\frac{7}{20}rx\left(2rx-x^2\right)^{3/2}-\frac{1}{5}x^2\left(2rx-x^2\right)^{3/2}+\frac{7}{4}r^5\tan^{-1}\end{aligned}$$

Mathematica [A] time = 0.114559, size = 88, normalized size = 0.78

$$\frac{1}{120}\sqrt{-x(x-2r)}\left(-14r^2x^2-35r^3x+\frac{210r^{9/2}\sin^{-1}\left(\frac{\sqrt{x}}{\sqrt{2}\sqrt{r}}\right)}{\sqrt{x}\sqrt{2-\frac{x}{r}}}-105r^4-6rx^3+24x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[2*r*x - x^2],x]
```

```
[Out] (Sqrt[-(x*(-2*r + x))]*(-105*r^4 - 35*r^3*x - 14*r^2*x^2 - 6*r*x^3 + 24*x^4
 + (210*r^(9/2)*ArcSin[Sqrt[x]/(Sqrt[2]*Sqrt[r])])/(Sqrt[x]*Sqrt[2 - x/r]))
)/120
```

Maple [A] time = 0.005, size = 111, normalized size = 1.

$$-\frac{x^2}{5}\left(2\,rx-x^2\right)^{\frac{3}{2}}-\frac{7\,rx}{20}\left(2\,rx-x^2\right)^{\frac{3}{2}}-\frac{7\,r^2}{12}\left(2\,rx-x^2\right)^{\frac{3}{2}}+\frac{7\,r^3x}{8}\sqrt{2\,rx-x^2}-\frac{7\,r^4}{8}\sqrt{2\,rx-x^2}+\frac{7\,r^5}{8}\arctan\left((x-r)\frac{}{\sqrt{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(2*r*x-x^2)^(1/2),x)
```

```
[Out] -1/5*x^2*(2*r*x-x^2)^(3/2)-7/20*r*x*(2*r*x-x^2)^(3/2)-7/12*r^2*(2*r*x-x^2)^
(3/2)+7/8*r^3*(2*r*x-x^2)^(1/2)*x-7/8*(2*r*x-x^2)^(1/2)*r^4+7/8*r^5*arctan(
(x-r)/(2*r*x-x^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*r*x-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03297, size = 158, normalized size = 1.4

$$-\frac{7}{4}\,r^5\arctan\left(\frac{\sqrt{2\,rx-x^2}}{x}\right)-\frac{1}{120}\left(105\,r^4+35\,r^3x+14\,r^2x^2+6\,rx^3-24\,x^4\right)\sqrt{2\,rx-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*r*x-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -7/4*r^5*arctan(sqrt(2*r*x - x^2)/x) - 1/120*(105*r^4 + 35*r^3*x + 14*r^2*x
^2 + 6*r*x^3 - 24*x^4)*sqrt(2*r*x - x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\sqrt{-x\,(-2r+x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(2*r*x-x**2)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-x*(-2*r + x)), x)
```

Giac [A] time = 1.06861, size = 85, normalized size = 0.75

$$-\frac{7}{8}\,r^5\arcsin\left(\frac{r-x}{r}\right)\mathrm{sgn}\left(r\right)-\frac{1}{120}\left(105\,r^4+\left(35\,r^3+2\left(7\,r^2+3\,(r-4\,x)x\right)x\right)x\right)\sqrt{2\,rx-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(2*r*x-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -7/8*r^5*arcsin((r - x)/r)*sgn(r) - 1/120*(105*r^4 + (35*r^3 + 2*(7*r^2 + 3
*(r - 4*x)*x)*x)*x)*sqrt(2*r*x - x^2)
```

3.264 $\int \frac{1}{(-1+x^2)\sqrt{2x+x^2}}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{2}\tan^{-1}\left(\sqrt{x^2+2x}\right)-\frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+2x}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[Sqrt[2*x + x^2]]/2 - ArcTanh[(1 + 2*x)/(Sqrt[3]*Sqrt[2*x + x^2])]/(
2*Sqrt[3])
```

Rubi [A] time = 0.0305797, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {984, 688, 204, 724, 206}

$$-\frac{1}{2}\tan^{-1}\left(\sqrt{x^2+2x}\right)-\frac{\tanh^{-1}\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+2x}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x^2)*Sqrt[2*x + x^2]),x]
```

```
[Out] -ArcTan[Sqrt[2*x + x^2]]/2 - ArcTanh[(1 + 2*x)/(Sqrt[3]*Sqrt[2*x + x^2])]/(
2*Sqrt[3])
```

Rule 984

```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Dist[1/2, Int[1/((a - Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x],
 x] + Dist[1/2, Int[1/((a + Rt[-(a*c), 2]*x)*Sqrt[d + e*x + f*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(-1+x^2\right)\sqrt{2x+x^2}}\,dx &= \frac{1}{2}\int \frac{1}{(-1-x)\sqrt{2x+x^2}}\,dx+\frac{1}{2}\int \frac{1}{(-1+x)\sqrt{2x+x^2}}\,dx\\
&= 2\,\mathrm{Subst}\left(\int \frac{1}{-4-4x^2}\,dx,x,\sqrt{2x+x^2}\right)-\mathrm{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{2+4x}{\sqrt{2x+x^2}}\right)\\
&= -\frac{1}{2}\tan^{-1}\left(\sqrt{2x+x^2}\right)-\frac{\tanh^{-1}\left(\frac{2+4x}{2\sqrt{3}\sqrt{2x+x^2}}\right)}{2\sqrt{3}}
\end{aligned}
$$

Mathematica [A] time = 0.0293119, size = 64, normalized size = 1.31

$$
-\frac{\sqrt{x}\sqrt{x+2}\left(3\tan^{-1}\left(\sqrt{\frac{x}{x+2}}\right)+\sqrt{3}\tanh^{-1}\left(\sqrt{3}\sqrt{\frac{x}{x+2}}\right)\right)}{3\sqrt{x(x+2)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x^2)*Sqrt[2*x + x^2]),x]
```

```
[Out] -(Sqrt[x]*Sqrt[2 + x]*(3*ArcTan[Sqrt[x/(2 + x)]] + Sqrt[3]*ArcTanh[Sqrt[3]*
Sqrt[x/(2 + x)]]))/(3*Sqrt[x*(2 + x)])
```

Maple [A] time = 0.012, size = 42, normalized size = 0.9

$$
\frac{1}{2}\arctan\left(\frac{1}{\sqrt{(1+x)^2-1}}\right)-\frac{\sqrt{3}}{6}\mathrm{Artanh}\left(\frac{(2+4\,x)\sqrt{3}}{6}\frac{1}{\sqrt{(-1+x)^2-1+4\,x}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)/(x^2+2*x)^(1/2),x)
```

```
[Out] 1/2*arctan(1/((1+x)^2-1)^(1/2))-1/6*3^(1/2)*arctanh(1/6*(2+4*x)*3^(1/2)/((-
1+x)^2-1+4*x)^(1/2))
```

Maxima [A] time = 1.46375, size = 73, normalized size = 1.49

$$
-\frac{1}{6}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^2+2\,x}}{|2\,x-2|}+\frac{6}{|2\,x-2|}+2\right)+\frac{1}{2}\arcsin\left(\frac{2}{|2\,x+2|}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*log(2*sqrt(3)*sqrt(x^2 + 2*x)/abs(2*x - 2) + 6/abs(2*x - 2) +
2) + 1/2*arcsin(2/abs(2*x + 2))
```

Fricas [A] time = 2.16267, size = 170, normalized size = 3.47

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3}(2x+1)+\sqrt{x^2+2x}\left(2\sqrt{3}-3\right)-4x-2}{x-1}\right)-\arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(sqrt(3)*(2*x + 1) + sqrt(x^2 + 2*x)*(2*sqrt(3) - 3) - 4*x
 - 2)/(x - 1)) - arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x(x+2)}(x-1)(x+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)/(x**2+2*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(x + 2))*(x - 1)*(x + 1)), x)
```

Giac [A] time = 1.11763, size = 96, normalized size = 1.96

$$\frac{1}{6}\sqrt{3}\log\left(\frac{\left|-2x-2\sqrt{3}+2\sqrt{x^2+2x}+2\right|}{\left|-2x+2\sqrt{3}+2\sqrt{x^2+2x}+2\right|}\right)-\arctan\left(-x+\sqrt{x^2+2x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)/(x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*log(abs(-2*x - 2*sqrt(3) + 2*sqrt(x^2 + 2*x) + 2)/abs(-2*x + 2*
sqrt(3) + 2*sqrt(x^2 + 2*x) + 2)) - arctan(-x + sqrt(x^2 + 2*x) - 1)
```

3.265 $\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx$

Optimal. Leaf size=79

$$-\frac{2\sqrt{2x-x^2}}{3(x+1)}-\frac{5\sqrt{2x-x^2}}{6(x+1)^2}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}\sqrt{2x-x^2}}\right)}{2\sqrt{3}}$$

```
[Out] (-5*Sqrt[2*x - x^2])/(6*(1 + x)^2) - (2*Sqrt[2*x - x^2])/(3*(1 + x)) + ArcT
an[(1 - 2*x)/(Sqrt[3]*Sqrt[2*x - x^2])]/(2*Sqrt[3])
```

Rubi [A] time = 0.044347, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {834, 806, 724, 204}

$$-\frac{2\sqrt{2x-x^2}}{3(x+1)}-\frac{5\sqrt{2x-x^2}}{6(x+1)^2}+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}\sqrt{2x-x^2}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x)/((1 + x)^3*Sqrt[2*x - x^2]),x]
```

```
[Out] (-5*Sqrt[2*x - x^2])/(6*(1 + x)^2) - (2*Sqrt[2*x - x^2])/(3*(1 + x)) + ArcT
an[(1 - 2*x)/(Sqrt[3]*Sqrt[2*x - x^2])]/(2*Sqrt[3])
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-2+3x}{(1+x)^3\sqrt{2x-x^2}}\,dx &= -\frac{5\sqrt{2x-x^2}}{6(1+x)^2}+\frac{1}{6}\int \frac{-7+5x}{(1+x)^2\sqrt{2x-x^2}}\,dx\\ &= -\frac{5\sqrt{2x-x^2}}{6(1+x)^2}-\frac{2\sqrt{2x-x^2}}{3(1+x)}-\frac{1}{2}\int \frac{1}{(1+x)\sqrt{2x-x^2}}\,dx\\ &= -\frac{5\sqrt{2x-x^2}}{6(1+x)^2}-\frac{2\sqrt{2x-x^2}}{3(1+x)}+\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx,x,\frac{-2+4x}{\sqrt{2x-x^2}}\right)\\ &= -\frac{5\sqrt{2x-x^2}}{6(1+x)^2}-\frac{2\sqrt{2x-x^2}}{3(1+x)}-\frac{\tan^{-1}\left(\frac{-2+4x}{2\sqrt{3}\sqrt{2x-x^2}}\right)}{2\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0558168, size = 72, normalized size = 0.91

$$\frac{x\left(4x^2+x-18\right)-2\sqrt{3}\sqrt{x-2}\sqrt{x}(x+1)^2\tanh^{-1}\left(\frac{\sqrt{\frac{x-2}{x}}}{\sqrt{3}}\right)}{6\sqrt{-(x-2)x}(x+1)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(-2 + 3*x)/((1 + x)^3*Sqrt[2*x - x^2]),x]
```

```
[Out] (x*(-18 + x + 4*x^2) - 2*Sqrt[3]*Sqrt[-2 + x]*Sqrt[x]*(1 + x)^2*ArcTanh[Sqr
t[(-2 + x)/x]/Sqrt[3]])/(6*Sqrt[-((-2 + x)*x)]*(1 + x)^2)
```

Maple [A] time = 0.013, size = 74, normalized size = 0.9

$$-\frac{5}{6\,(1+x)^2}\sqrt{-(1+x)^2+1+4\,x}-\frac{2}{3+3\,x}\sqrt{-(1+x)^2+1+4\,x}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(-2+4\,x)\sqrt{3}}{6}\frac{1}{\sqrt{-(1+x)^2+1+4\,x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x)
```

```
[Out] -5/6/(1+x)^2*(-(1+x)^2+1+4*x)^(1/2)-2/3/(1+x)*(-(1+x)^2+1+4*x)^(1/2)-1/6*3^
(1/2)*arctan(1/6*(-2+4*x)*3^(1/2)/(-(1+x)^2+1+4*x)^(1/2))
```

Maxima [A] time = 1.48565, size = 89, normalized size = 1.13

$$-\frac{1}{6}\sqrt{3}\arcsin\left(\frac{2\,x}{|x+1|}-\frac{1}{|x+1|}\right)-\frac{5\sqrt{-x^2+2\,x}}{6\left(x^2+2\,x+1\right)}-\frac{2\sqrt{-x^2+2\,x}}{3\,(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arcsin(2*x/abs(x + 1) - 1/abs(x + 1)) - 5/6*sqrt(-x^2 + 2*x)/(
x^2 + 2*x + 1) - 2/3*sqrt(-x^2 + 2*x)/(x + 1)
```

Fricas [A] time = 2.08249, size = 158, normalized size = 2.

$$\frac{2\sqrt{3}\left(x^2+2x+1\right)\arctan\left(\frac{\sqrt{3}\sqrt{-x^2+2x}}{3x}\right)-\sqrt{-x^2+2x}(4x+9)}{6\left(x^2+2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*sqrt(3)*(x^2 + 2*x + 1)*arctan(1/3*sqrt(3)*sqrt(-x^2 + 2*x)/x) - sqr
t(-x^2 + 2*x)*(4*x + 9))/(x^2 + 2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{3x-2}{\sqrt{-x(x-2)}\,(x+1)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+3*x)/(1+x)**3/(-x**2+2*x)**(1/2),x)
```

```
[Out] Integral((3*x - 2)/(sqrt(-x*(x - 2))*(x + 1)**3), x)
```

Giac [B] time = 1.07121, size = 198, normalized size = 2.51

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{-x^2+2x}-1\right)}{x-1}-1\right)\right)+\frac{\frac{34\left(\sqrt{-x^2+2x}-1\right)}{x-1}-\frac{39\left(\sqrt{-x^2+2x}-1\right)^2}{(x-1)^2}+\frac{18\left(\sqrt{-x^2+2x}-1\right)^3}{(x-1)^3}-26}{24\left(\frac{\sqrt{-x^2+2x}-1}{x-1}-\frac{\left(\sqrt{-x^2+2x}-1\right)^2}{(x-1)^2}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+3*x)/(1+x)^3/(-x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 + 2*x) - 1)/(x - 1) - 1)) + 1/
24*(34*(sqrt(-x^2 + 2*x) - 1)/(x - 1) - 39*(sqrt(-x^2 + 2*x) - 1)^2/(x - 1)
^2 + 18*(sqrt(-x^2 + 2*x) - 1)^3/(x - 1)^3 - 26)/((sqrt(-x^2 + 2*x) - 1)/(x
 - 1) - (sqrt(-x^2 + 2*x) - 1)^2/(x - 1)^2 - 1)^2
```

3.266 $\int \frac{1}{\sqrt{1+x+x^2}} dx$

Optimal. Leaf size=12

$$\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] ArcSinh[(1 + 2*x)/Sqrt[3]]
```

Rubi [A] time = 0.0075392, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {619, 215}

$$\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x + x^2],x]
```

```
[Out] ArcSinh[(1 + 2*x)/Sqrt[3]]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+x+x^2}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}} dx, x, 1+2x\right)}{\sqrt{3}}\\ &= \sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A] time = 0.0055488, size = 12, normalized size = 1.

$$\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x + x^2],x]
```

```
[Out] ArcSinh[(1 + 2*x)/Sqrt[3]]
```

Maple [A] time = 0.003, size = 10, normalized size = 0.8

$$\operatorname{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+x+1)^(1/2),x)
```

```
[Out] arcsinh(2/3*3^(1/2)*(x+1/2))
```

Maxima [A] time = 1.45002, size = 15, normalized size = 1.25

$$\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Fricas [A] time = 2.02633, size = 51, normalized size = 4.25

$$-\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2 + x + 1), x)
```

Giac [A] time = 1.07154, size = 24, normalized size = 2.

$$-\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.267 $\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=53

$$\frac{1}{3}\sqrt{x^2+x+1}x^2-\frac{1}{24}(10x+1)\sqrt{x^2+x+1}+\frac{7}{16}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] (x^2*Sqrt[1 + x + x^2])/3 - ((1 + 10*x)*Sqrt[1 + x + x^2])/24 + (7*ArcSinh[
(1 + 2*x)/Sqrt[3]])/16
```

Rubi [A] time = 0.023292, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {742, 779, 619, 215}

$$\frac{1}{3}\sqrt{x^2+x+1}x^2-\frac{1}{24}(10x+1)\sqrt{x^2+x+1}+\frac{7}{16}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[1 + x + x^2],x]
```

```
[Out] (x^2*Sqrt[1 + x + x^2])/3 - ((1 + 10*x)*Sqrt[1 + x + x^2])/24 + (7*ArcSinh[
(1 + 2*x)/Sqrt[3]])/16
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1+x+x^2}}\,dx = \frac{1}{3}x^2\sqrt{1+x+x^2}+\frac{1}{3}\int \frac{\left(-2-\frac{5x}{2}\right)x}{\sqrt{1+x+x^2}}\,dx$$
$$= \frac{1}{3}x^2\sqrt{1+x+x^2}-\frac{1}{24}(1+10x)\sqrt{1+x+x^2}+\frac{7}{16}\int \frac{1}{\sqrt{1+x+x^2}}\,dx$$
$$= \frac{1}{3}x^2\sqrt{1+x+x^2}-\frac{1}{24}(1+10x)\sqrt{1+x+x^2}+\frac{7\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,1+2x\right)}{16\sqrt{3}}$$
$$= \frac{1}{3}x^2\sqrt{1+x+x^2}-\frac{1}{24}(1+10x)\sqrt{1+x+x^2}+\frac{7}{16}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)$$

Mathematica [A] time = 0.0165027, size = 41, normalized size = 0.77

$$\frac{1}{48}\left(2\sqrt{x^2+x+1}\left(8x^2-10x-1\right)+21\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[1 + x + x^2],x]
```

```
[Out] (2*Sqrt[1 + x + x^2]*(-1 - 10*x + 8*x^2) + 21*ArcSinh[(1 + 2*x)/Sqrt[3]])/4
8
```

Maple [A] time = 0.005, size = 47, normalized size = 0.9

$$\frac{x^2}{3}\sqrt{x^2+x+1}-\frac{5x}{12}\sqrt{x^2+x+1}-\frac{1}{24}\sqrt{x^2+x+1}+\frac{7}{16}\mathrm{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2+x+1)^(1/2),x)
```

```
[Out] 1/3*x^2*(x^2+x+1)^(1/2)-5/12*x*(x^2+x+1)^(1/2)-1/24*(x^2+x+1)^(1/2)+7/16*ar
csinh(2/3*3^(1/2)*(x+1/2))
```

Maxima [A] time = 1.44799, size = 65, normalized size = 1.23

$$\frac{1}{3}\sqrt{x^2+x+1}x^2-\frac{5}{12}\sqrt{x^2+x+1}x-\frac{1}{24}\sqrt{x^2+x+1}+\frac{7}{16}\,\mathrm{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^2 + x + 1)*x^2 - 5/12*sqrt(x^2 + x + 1)*x - 1/24*sqrt(x^2 + x +
1) + 7/16*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Fricas [A] time = 2.12527, size = 116, normalized size = 2.19

$$\frac{1}{24}\left(8x^2-10x-1\right)\sqrt{x^2+x+1}-\frac{7}{16}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(8*x^2 - 10*x - 1)*sqrt(x^2 + x + 1) - 7/16*log(-2*x + 2*sqrt(x^2 + x
+ 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^3}{\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(x**2 + x + 1), x)
```

Giac [A] time = 1.07601, size = 53, normalized size = 1.

$$\frac{1}{24}\left(2\left(4x-5\right)x-1\right)\sqrt{x^2+x+1}-\frac{7}{16}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(2*(4*x - 5)*x - 1)*sqrt(x^2 + x + 1) - 7/16*log(-2*x + 2*sqrt(x^2 + x
 + 1) - 1)
```

3.268 $\int \frac{1}{\left(1+x+x^2\right)^{3/2}} dx$

Optimal. Leaf size=19

$$\frac{2(2x+1)}{3\sqrt{x^2+x+1}}$$

```
[Out] (2*(1 + 2*x))/(3*Sqrt[1 + x + x^2])
```

Rubi [A] time = 0.0019644, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {613}

$$\frac{2(2x+1)}{3\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)^(-3/2),x]
```

```
[Out] (2*(1 + 2*x))/(3*Sqrt[1 + x + x^2])
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(1+x+x^2\right)^{3/2}} dx = \frac{2(1+2x)}{3\sqrt{1+x+x^2}}$$

Mathematica [A] time = 0.0041424, size = 19, normalized size = 1.

$$\frac{2(2x+1)}{3\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)^(-3/2),x]
```

```
[Out] (2*(1 + 2*x))/(3*Sqrt[1 + x + x^2])
```

Maple [A] time = 0.004, size = 16, normalized size = 0.8

$$\frac{2+4x}{3}\frac{1}{\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+x+1)^(3/2),x)
```

```
[Out] 2/3*(1+2*x)/(x^2+x+1)^(1/2)
```

Maxima [A] time = 0.942825, size = 30, normalized size = 1.58

$$\frac{4\,x}{3\,\sqrt{x^2+x+1}}+\frac{2}{3\,\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 4/3*x/sqrt(x^2 + x + 1) + 2/3/sqrt(x^2 + x + 1)
```

Fricas [B] time = 2.24154, size = 90, normalized size = 4.74

$$\frac{2\left(2\,x^2+\sqrt{x^2+x+1}(2\,x+1)+2\,x+2\right)}{3\left(x^2+x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*x^2 + sqrt(x^2 + x + 1)*(2*x + 1) + 2*x + 2)/(x^2 + x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+x+1)**(3/2),x)
```

```
[Out] Integral((x**2 + x + 1)**(-3/2), x)
```

Giac [A] time = 1.0733, size = 20, normalized size = 1.05

$$\frac{2\,(2\,x+1)}{3\,\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*x + 1)/sqrt(x^2 + x + 1)
```

3.269 $\int \frac{x}{(1+x+x^2)^{3/2}} \, dx$

Optimal. Leaf size=17

$$-\frac{2(x+2)}{3\sqrt{x^2+x+1}}$$

[Out] (-2*(2 + x))/(3*Sqrt[1 + x + x^2])

Rubi [A] time = 0.003884, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {636}

$$-\frac{2(x+2)}{3\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

[In] Int[x/(1 + x + x^2)^(3/2),x]

[Out] (-2*(2 + x))/(3*Sqrt[1 + x + x^2])

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{x}{(1+x+x^2)^{3/2}} \, dx = -\frac{2(2+x)}{3\sqrt{1+x+x^2}}$$

Mathematica [A] time = 0.0338795, size = 17, normalized size = 1.

$$-\frac{2(x+2)}{3\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 + x + x^2)^(3/2),x]

[Out] (-2*(2 + x))/(3*Sqrt[1 + x + x^2])

Maple [A] time = 0.002, size = 14, normalized size = 0.8

$$-\frac{4+2x}{3}\frac{1}{\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+x+1)^(3/2),x)
```

```
[Out] -2/3*(2+x)/(x^2+x+1)^(1/2)
```

Maxima [A] time = 0.958795, size = 30, normalized size = 1.76

$$-\frac{2x}{3\sqrt{x^2+x+1}}-\frac{4}{3\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/3*x/sqrt(x^2 + x + 1) - 4/3/sqrt(x^2 + x + 1)
```

Fricas [B] time = 2.05286, size = 84, normalized size = 4.94

$$-\frac{2\left(x^2+\sqrt{x^2+x+1}(x+2)+x+1\right)}{3\left(x^2+x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/3*(x^2 + sqrt(x^2 + x + 1)*(x + 2) + x + 1)/(x^2 + x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\left(x^2+x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+x+1)**(3/2),x)
```

```
[Out] Integral(x/(x**2 + x + 1)**(3/2), x)
```

Giac [A] time = 1.06772, size = 18, normalized size = 1.06

$$-\frac{2(x+2)}{3\sqrt{x^2+x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(x + 2)/sqrt(x^2 + x + 1)
```

3.270 $\int \frac{x^3}{(1+x+x^2)^{3/2}}\,dx$

Optimal. Leaf size=56

$$-\frac{2(x+2)x^2}{3\sqrt{x^2+x+1}}+\frac{1}{3}(2x+5)\sqrt{x^2+x+1}-\frac{3}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] (-2*x^2*(2 + x))/(3*Sqrt[1 + x + x^2]) + ((5 + 2*x)*Sqrt[1 + x + x^2])/3 -
(3*ArcSinh[(1 + 2*x)/Sqrt[3]])/2
```

Rubi [A] time = 0.0240697, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {738, 779, 619, 215}

$$-\frac{2(x+2)x^2}{3\sqrt{x^2+x+1}}+\frac{1}{3}(2x+5)\sqrt{x^2+x+1}-\frac{3}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x + x^2)^(3/2),x]
```

```
[Out] (-2*x^2*(2 + x))/(3*Sqrt[1 + x + x^2]) + ((5 + 2*x)*Sqrt[1 + x + x^2])/3 -
(3*ArcSinh[(1 + 2*x)/Sqrt[3]])/2
```

Rule 738

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*
c)), Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*c
*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p +
 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p, -1] && GtQ[m, 1] &&
IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 779

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x)*(a + b*x + c*x^2)^(p + 1))/(2*c^2*(p + 1)*(2*p + 3)), x
] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p +
3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(1+x+x^2\right)^{3/2}}\,dx &= -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}}+\frac{2}{3}\int \frac{x(4+2x)}{\sqrt{1+x+x^2}}\,dx \\
&= -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}}+\frac{1}{3}(5+2x)\sqrt{1+x+x^2}-\frac{3}{2}\int \frac{1}{\sqrt{1+x+x^2}}\,dx \\
&= -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}}+\frac{1}{3}(5+2x)\sqrt{1+x+x^2}-\frac{1}{2}\sqrt{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,1+2x\right) \\
&= -\frac{2x^2(2+x)}{3\sqrt{1+x+x^2}}+\frac{1}{3}(5+2x)\sqrt{1+x+x^2}-\frac{3}{2}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0118343, size = 48, normalized size = 0.86

$$\frac{6x^2-9\sqrt{x^2+x+1}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+14x+10}{6\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x + x^2)^(3/2),x]
```

```
[Out] (10 + 14*x + 6*x^2 - 9*Sqrt[1 + x + x^2]*ArcSinh[(1 + 2*x)/Sqrt[3]])/(6*Sqr
t[1 + x + x^2])
```

Maple [A] time = 0.004, size = 61, normalized size = 1.1

$$x^2\frac{1}{\sqrt{x^2+x+1}}+\frac{3x}{2}\frac{1}{\sqrt{x^2+x+1}}+\frac{5}{4}\frac{1}{\sqrt{x^2+x+1}}+\frac{5+10x}{12}\frac{1}{\sqrt{x^2+x+1}}-\frac{3}{2}\text{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2+x+1)^(3/2),x)
```

```
[Out] x^2/(x^2+x+1)^(1/2)+3/2*x/(x^2+x+1)^(1/2)+5/4/(x^2+x+1)^(1/2)+5/12*(1+2*x)/
(x^2+x+1)^(1/2)-3/2*arcsinh(2/3*3^(1/2)*(x+1/2))
```

Maxima [A] time = 1.43428, size = 63, normalized size = 1.12

$$\frac{x^2}{\sqrt{x^2+x+1}}+\frac{7x}{3\sqrt{x^2+x+1}}+\frac{5}{3\sqrt{x^2+x+1}}-\frac{3}{2}\,\text{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] x^2/sqrt(x^2 + x + 1) + 7/3*x/sqrt(x^2 + x + 1) + 5/3/sqrt(x^2 + x + 1) - 3
/2*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Fricas [A] time = 2.04525, size = 184, normalized size = 3.29

$$\frac{19\,x^{2} + 18\,\left(x^{2} + x + 1\right)\log\left(-2\,x + 2\,\sqrt{x^{2} + x + 1} - 1\right) + 4\,\left(3\,x^{2} + 7\,x + 5\right)\sqrt{x^{2} + x + 1} + 19\,x + 19}{12\,\left(x^{2} + x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*(19*x^2 + 18*(x^2 + x + 1)*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) + 4*(3*
x^2 + 7*x + 5)*sqrt(x^2 + x + 1) + 19*x + 19)/(x^2 + x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^{3}}{\left(x^{2} + x + 1\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**2+x+1)**(3/2),x)
```

```
[Out] Integral(x**3/(x**2 + x + 1)**(3/2), x)
```

Giac [A] time = 1.07698, size = 51, normalized size = 0.91

$$\frac{(3\,x + 7)x + 5}{3\,\sqrt{x^{2} + x + 1}} + \frac{3}{2}\,\log\left(-2\,x + 2\,\sqrt{x^{2} + x + 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*((3*x + 7)*x + 5)/sqrt(x^2 + x + 1) + 3/2*log(-2*x + 2*sqrt(x^2 + x + 1
) - 1)
```

3.271 $\int x^2\sqrt{1+x+x^2}\,dx$

Optimal. Leaf size=65

$$\frac{1}{4}x\left(x^2+x+1\right)^{3/2}-\frac{5}{24}\left(x^2+x+1\right)^{3/2}+\frac{1}{64}(2x+1)\sqrt{x^2+x+1}+\frac{3}{128}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] ((1 + 2*x)*Sqrt[1 + x + x^2])/64 - (5*(1 + x + x^2)^(3/2))/24 + (x*(1 + x +
 x^2)^(3/2))/4 + (3*ArcSinh[(1 + 2*x)/Sqrt[3]])/128
```

Rubi [A] time = 0.0242333, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {742, 640, 612, 619, 215}

$$\frac{1}{4}x\left(x^2+x+1\right)^{3/2}-\frac{5}{24}\left(x^2+x+1\right)^{3/2}+\frac{1}{64}(2x+1)\sqrt{x^2+x+1}+\frac{3}{128}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[1 + x + x^2],x]
```

```
[Out] ((1 + 2*x)*Sqrt[1 + x + x^2])/64 - (5*(1 + x + x^2)^(3/2))/24 + (x*(1 + x +
 x^2)^(3/2))/4 + (3*ArcSinh[(1 + 2*x)/Sqrt[3]])/128
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{1+x+x^2}\,dx &= \frac{1}{4}x\left(1+x+x^2\right)^{3/2}+\frac{1}{4}\int\left(-1-\frac{5x}{2}\right)\sqrt{1+x+x^2}\,dx\\ &= -\frac{5}{24}\left(1+x+x^2\right)^{3/2}+\frac{1}{4}x\left(1+x+x^2\right)^{3/2}+\frac{1}{16}\int\sqrt{1+x+x^2}\,dx\\ &= \frac{1}{64}(1+2x)\sqrt{1+x+x^2}-\frac{5}{24}\left(1+x+x^2\right)^{3/2}+\frac{1}{4}x\left(1+x+x^2\right)^{3/2}+\frac{3}{128}\int\frac{1}{\sqrt{1+x+x^2}}\,dx\\ &= \frac{1}{64}(1+2x)\sqrt{1+x+x^2}-\frac{5}{24}\left(1+x+x^2\right)^{3/2}+\frac{1}{4}x\left(1+x+x^2\right)^{3/2}+\frac{1}{128}\sqrt{3}\,\text{Subst}\left(\int\frac{1}{\sqrt{1+\frac{x^2}{3}}}\right.\\ &= \frac{1}{64}(1+2x)\sqrt{1+x+x^2}-\frac{5}{24}\left(1+x+x^2\right)^{3/2}+\frac{1}{4}x\left(1+x+x^2\right)^{3/2}+\frac{3}{128}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A] time = 0.0207481, size = 46, normalized size = 0.71

$$\frac{1}{384}\left(2\sqrt{x^2+x+1}\left(48x^3+8x^2+14x-37\right)+9\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[1 + x + x^2],x]
```

```
[Out] (2*Sqrt[1 + x + x^2]*(-37 + 14*x + 8*x^2 + 48*x^3) + 9*ArcSinh[(1 + 2*x)/Sq
rt[3]])/384
```

Maple [A] time = 0.005, size = 49, normalized size = 0.8

$$\frac{x}{4}\left(x^2+x+1\right)^{\frac{3}{2}}-\frac{5}{24}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{1+2x}{64}\sqrt{x^2+x+1}+\frac{3}{128}\text{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2+x+1)^(1/2),x)
```

```
[Out] 1/4*x*(x^2+x+1)^(3/2)-5/24*(x^2+x+1)^(3/2)+1/64*(1+2*x)*(x^2+x+1)^(1/2)+3/1
28*arcsinh(2/3*3^(1/2)*(x+1/2))
```

Maxima [A] time = 1.43718, size = 76, normalized size = 1.17

$$\frac{1}{4}\left(x^2+x+1\right)^{\frac{3}{2}}x-\frac{5}{24}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{1}{32}\sqrt{x^2+x+1}x+\frac{1}{64}\sqrt{x^2+x+1}+\frac{3}{128}\,\text{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(x^2 + x + 1)^(3/2)*x - 5/24*(x^2 + x + 1)^(3/2) + 1/32*sqrt(x^2 + x +
1)*x + 1/64*sqrt(x^2 + x + 1) + 3/128*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Fricas [A] time = 2.02652, size = 132, normalized size = 2.03

$$\frac{1}{192}\left(48\,x^3+8\,x^2+14\,x-37\right)\sqrt{x^2+x+1}-\frac{3}{128}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/192*(48*x^3 + 8*x^2 + 14*x - 37)*sqrt(x^2 + x + 1) - 3/128*log(-2*x + 2*s
qrt(x^2 + x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2\sqrt{x^2+x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(x**2 + x + 1), x)
```

Giac [A] time = 1.06932, size = 59, normalized size = 0.91

$$\frac{1}{192}\left(2\left(4\left(6\,x+1\right)x+7\right)x-37\right)\sqrt{x^2+x+1}-\frac{3}{128}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*(2*(4*(6*x + 1)*x + 7)*x - 37)*sqrt(x^2 + x + 1) - 3/128*log(-2*x + 2
*sqrt(x^2 + x + 1) - 1)
```

3.272 $\int \left(1+x+x^2\right)^{3/2} \, dx$

Optimal. Leaf size=55

$$\frac{1}{8}(2x+1)\left(x^2+x+1\right)^{3/2}+\frac{9}{64}(2x+1)\sqrt{x^2+x+1}+\frac{27}{128}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] (9*(1 + 2*x)*Sqrt[1 + x + x^2])/64 + ((1 + 2*x)*(1 + x + x^2)^(3/2))/8 + (2
7*ArcSinh[(1 + 2*x)/Sqrt[3]])/128
```

Rubi [A] time = 0.0138136, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {612, 619, 215}

$$\frac{1}{8}(2x+1)\left(x^2+x+1\right)^{3/2}+\frac{9}{64}(2x+1)\sqrt{x^2+x+1}+\frac{27}{128}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)^(3/2),x]
```

```
[Out] (9*(1 + 2*x)*Sqrt[1 + x + x^2])/64 + ((1 + 2*x)*(1 + x + x^2)^(3/2))/8 + (2
7*ArcSinh[(1 + 2*x)/Sqrt[3]])/128
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \left(1+x+x^2\right)^{3/2} \, dx &= \frac{1}{8}(1+2x)\left(1+x+x^2\right)^{3/2}+\frac{9}{16}\int \sqrt{1+x+x^2} \, dx\\
&= \frac{9}{64}(1+2x)\sqrt{1+x+x^2}+\frac{1}{8}(1+2x)\left(1+x+x^2\right)^{3/2}+\frac{27}{128}\int \frac{1}{\sqrt{1+x+x^2}} \, dx\\
&= \frac{9}{64}(1+2x)\sqrt{1+x+x^2}+\frac{1}{8}(1+2x)\left(1+x+x^2\right)^{3/2}+\frac{1}{128}\left(9\sqrt{3}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}} \, dx,x,1+\right.\\
&= \frac{9}{64}(1+2x)\sqrt{1+x+x^2}+\frac{1}{8}(1+2x)\left(1+x+x^2\right)^{3/2}+\frac{27}{128}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0151152, size = 46, normalized size = 0.84

$$\frac{1}{128}\left(2\sqrt{x^2+x+1}\left(16x^3+24x^2+42x+17\right)+27\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)^(3/2),x]
```

```
[Out] (2*Sqrt[1 + x + x^2]*(17 + 42*x + 24*x^2 + 16*x^3) + 27*ArcSinh[(1 + 2*x)/S
qrt[3]])/128
```

Maple [A] time = 0.002, size = 43, normalized size = 0.8

$$\frac{1+2x}{8}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{9+18x}{64}\sqrt{x^2+x+1}+\frac{27}{128}\text{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)^(3/2),x)
```

```
[Out] 1/8*(1+2*x)*(x^2+x+1)^(3/2)+9/64*(1+2*x)*(x^2+x+1)^(1/2)+27/128*arcsinh(2/3
*3^(1/2)*(x+1/2))
```

Maxima [A] time = 1.43395, size = 76, normalized size = 1.38

$$\frac{1}{4}\left(x^2+x+1\right)^{\frac{3}{2}}x+\frac{1}{8}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{9}{32}\sqrt{x^2+x+1}x+\frac{9}{64}\sqrt{x^2+x+1}+\frac{27}{128}\,\text{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(x^2 + x + 1)^(3/2)*x + 1/8*(x^2 + x + 1)^(3/2) + 9/32*sqrt(x^2 + x + 1
)*x + 9/64*sqrt(x^2 + x + 1) + 27/128*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Fricas [A] time = 2.10879, size = 134, normalized size = 2.44

$$\frac{1}{64}\left(16x^3+24x^2+42x+17\right)\sqrt{x^2+x+1}-\frac{27}{128}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/64*(16*x^3 + 24*x^2 + 42*x + 17)*sqrt(x^2 + x + 1) - 27/128*log(-2*x + 2*
sqrt(x^2 + x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2+x+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)**(3/2),x)
```

```
[Out] Integral((x**2 + x + 1)**(3/2), x)
```

Giac [A] time = 1.0686, size = 59, normalized size = 1.07

$$\frac{1}{64}\,(2\,(4\,(2\,x+3)x+21)x+17)\sqrt{x^2+x+1}-\frac{27}{128}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(2*(4*(2*x + 3)*x + 21)*x + 17)*sqrt(x^2 + x + 1) - 27/128*log(-2*x +
2*sqrt(x^2 + x + 1) - 1)
```

3.273 $\int \left(1+x+x^2\right)^{5/2} dx$

Optimal. Leaf size=74

$$\frac{1}{12}(2x+1)\left(x^2+x+1\right)^{5/2}+\frac{5}{64}(2x+1)\left(x^2+x+1\right)^{3/2}+\frac{45}{512}(2x+1)\sqrt{x^2+x+1}+\frac{135\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{1024}$$

```
[Out] (45*(1 + 2*x)*Sqrt[1 + x + x^2])/512 + (5*(1 + 2*x)*(1 + x + x^2)^(3/2))/64
 + ((1 + 2*x)*(1 + x + x^2)^(5/2))/12 + (135*ArcSinh[(1 + 2*x)/Sqrt[3]])/10
24
```

Rubi [A] time = 0.0190233, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {612, 619, 215}

$$\frac{1}{12}(2x+1)\left(x^2+x+1\right)^{5/2}+\frac{5}{64}(2x+1)\left(x^2+x+1\right)^{3/2}+\frac{45}{512}(2x+1)\sqrt{x^2+x+1}+\frac{135\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{1024}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2)^(5/2),x]
```

```
[Out] (45*(1 + 2*x)*Sqrt[1 + x + x^2])/512 + (5*(1 + 2*x)*(1 + x + x^2)^(3/2))/64
 + ((1 + 2*x)*(1 + x + x^2)^(5/2))/12 + (135*ArcSinh[(1 + 2*x)/Sqrt[3]])/10
24
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(1+x+x^2\right)^{5/2} dx = \frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2} + \frac{5}{8}\int \left(1+x+x^2\right)^{3/2} dx$$
$$= \frac{5}{64}(1+2x)\left(1+x+x^2\right)^{3/2} + \frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2} + \frac{45}{128}\int \sqrt{1+x+x^2}\,dx$$
$$= \frac{45}{512}(1+2x)\sqrt{1+x+x^2} + \frac{5}{64}(1+2x)\left(1+x+x^2\right)^{3/2} + \frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2} + \frac{135\int \frac{}{\sqrt{1}}}{102}$$
$$= \frac{45}{512}(1+2x)\sqrt{1+x+x^2} + \frac{5}{64}(1+2x)\left(1+x+x^2\right)^{3/2} + \frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2} + \frac{\left(45\sqrt{3}\right) S}{}$$
$$= \frac{45}{512}(1+2x)\sqrt{1+x+x^2} + \frac{5}{64}(1+2x)\left(1+x+x^2\right)^{3/2} + \frac{1}{12}(1+2x)\left(1+x+x^2\right)^{5/2} + \frac{135\sinh}{10}$$

Mathematica [A] time = 0.0206426, size = 56, normalized size = 0.76

$$\frac{2\sqrt{x^2+x+1}\left(256x^5+640x^4+1264x^3+1256x^2+1142x+383\right)+405\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{3072}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2)^(5/2),x]
```

```
[Out] (2*Sqrt[1 + x + x^2]*(383 + 1142*x + 1256*x^2 + 1264*x^3 + 640*x^4 + 256*x^
5) + 405*ArcSinh[(1 + 2*x)/Sqrt[3]])/3072
```

Maple [A] time = 0.003, size = 58, normalized size = 0.8

$$\frac{1+2x}{12}\left(x^2+x+1\right)^{\frac{5}{2}} + \frac{5+10x}{64}\left(x^2+x+1\right)^{\frac{3}{2}} + \frac{45+90x}{512}\sqrt{x^2+x+1} + \frac{135}{1024}\mathrm{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x+1)^(5/2),x)
```

```
[Out] 1/12*(1+2*x)*(x^2+x+1)^(5/2)+5/64*(1+2*x)*(x^2+x+1)^(3/2)+45/512*(1+2*x)*(x
^2+x+1)^(1/2)+135/1024*arcsinh(2/3*3^(1/2)*(x+1/2))
```

Maxima [A] time = 1.44686, size = 104, normalized size = 1.41

$$\frac{1}{6}\left(x^2+x+1\right)^{\frac{5}{2}}x + \frac{1}{12}\left(x^2+x+1\right)^{\frac{5}{2}} + \frac{5}{32}\left(x^2+x+1\right)^{\frac{3}{2}}x + \frac{5}{64}\left(x^2+x+1\right)^{\frac{3}{2}} + \frac{45}{256}\sqrt{x^2+x+1}x + \frac{45}{512}\sqrt{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(x^2 + x + 1)^(5/2)*x + 1/12*(x^2 + x + 1)^(5/2) + 5/32*(x^2 + x + 1)^(
3/2)*x + 5/64*(x^2 + x + 1)^(3/2) + 45/256*sqrt(x^2 + x + 1)*x + 45/512*sqr
```

```
t(x^2 + x + 1) + 135/1024*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Fricas [A] time = 2.18232, size = 176, normalized size = 2.38

$$\frac{1}{1536}\left(256\,x^{5}+640\,x^{4}+1264\,x^{3}+1256\,x^{2}+1142\,x+383\right)\sqrt{x^{2}+x+1}-\frac{135}{1024}\,\log\left(-2\,x+2\,\sqrt{x^{2}+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(256*x^5 + 640*x^4 + 1264*x^3 + 1256*x^2 + 1142*x + 383)*sqrt(x^2 +
x + 1) - 135/1024*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(x^{2}+x+1\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x+1)**(5/2),x)
```

```
[Out] Integral((x**2 + x + 1)**(5/2), x)
```

Giac [A] time = 1.06681, size = 73, normalized size = 0.99

$$\frac{1}{1536}\,(2\,(4\,(2\,(8\,(2\,x+5)x+79)x+157)x+571)x+383)\sqrt{x^{2}+x+1}-\frac{135}{1024}\,\log\left(-2\,x+2\,\sqrt{x^{2}+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/1536*(2*(4*(2*(8*(2*x + 5)*x + 79)*x + 157)*x + 571)*x + 383)*sqrt(x^2 +
x + 1) - 135/1024*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.274 $\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=38

$$\frac{1}{2}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)-\frac{\sqrt{x^2+x+1}}{x}$$

```
[Out] -(Sqrt[1 + x + x^2]/x) + ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])]/2
```

Rubi [A] time = 0.0131642, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {730, 724, 206}

$$\frac{1}{2}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)-\frac{\sqrt{x^2+x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[1 + x + x^2]),x]

[Out] -(Sqrt[1 + x + x^2]/x) + ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])]/2
```

Rule 730

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{1+x+x^2}}\,dx &= -\frac{\sqrt{1+x+x^2}}{x}-\frac{1}{2}\int \frac{1}{x\sqrt{1+x+x^2}}\,dx\\
&= -\frac{\sqrt{1+x+x^2}}{x}+\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{2+x}{\sqrt{1+x+x^2}}\right)\\
&= -\frac{\sqrt{1+x+x^2}}{x}+\frac{1}{2}\tanh^{-1}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.009296, size = 38, normalized size = 1.

$$\frac{1}{2}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)-\frac{\sqrt{x^2+x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[1 + x + x^2]),x]
```

```
[Out] -(Sqrt[1 + x + x^2]/x) + ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])]/2
```

Maple [A] time = 0.005, size = 31, normalized size = 0.8

$$\frac{1}{2}\text{Artanh}\left(\frac{2+x}{2}\frac{1}{\sqrt{x^2+x+1}}\right)-\frac{1}{x}\sqrt{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^2+x+1)^(1/2),x)
```

```
[Out] 1/2*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))-(x^2+x+1)^(1/2)/x
```

Maxima [A] time = 1.45017, size = 50, normalized size = 1.32

$$-\frac{\sqrt{x^2+x+1}}{x}+\frac{1}{2}\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3|x|}+\frac{2\sqrt{3}}{3|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x^2 + x + 1)/x + 1/2*arcsinh(1/3*sqrt(3)*x/abs(x) + 2/3*sqrt(3)/abs(x
))
```

Fricas [A] time = 2.15017, size = 144, normalized size = 3.79

$$\frac{x\log\left(-x+\sqrt{x^2+x+1}+1\right)-x\log\left(-x+\sqrt{x^2+x+1}-1\right)-2x-2\sqrt{x^2+x+1}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x*log(-x + sqrt(x^2 + x + 1) + 1) - x*log(-x + sqrt(x^2 + x + 1) - 1)
- 2*x - 2*sqrt(x^2 + x + 1))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(x**2 + x + 1)), x)
```

Giac [B] time = 1.08955, size = 90, normalized size = 2.37

$$\frac{x-\sqrt{x^2+x+1}+2}{\left(x-\sqrt{x^2+x+1}\right)^2-1}+\frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right)-\frac{1}{2}\,\log\left(\left|-x+\sqrt{x^2+x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] (x - sqrt(x^2 + x + 1) + 2)/((x - sqrt(x^2 + x + 1))^2 - 1) + 1/2*log(abs(-
x + sqrt(x^2 + x + 1) + 1)) - 1/2*log(abs(-x + sqrt(x^2 + x + 1) - 1))
```

3.275 $\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=57

$$\frac{3\sqrt{x^2+x+1}}{4x}-\frac{\sqrt{x^2+x+1}}{2x^2}+\frac{1}{8}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

```
[Out] -Sqrt[1 + x + x^2]/(2*x^2) + (3*Sqrt[1 + x + x^2])/(4*x) + ArcTanh[(2 + x)/
(2*Sqrt[1 + x + x^2])]/8
```

Rubi [A] time = 0.0239868, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {744, 806, 724, 206}

$$\frac{3\sqrt{x^2+x+1}}{4x}-\frac{\sqrt{x^2+x+1}}{2x^2}+\frac{1}{8}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[1 + x + x^2]),x]
```

```
[Out] -Sqrt[1 + x + x^2]/(2*x^2) + (3*Sqrt[1 + x + x^2])/(4*x) + ArcTanh[(2 + x)/
(2*Sqrt[1 + x + x^2])]/8
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{1+x+x^2}}\,dx &= -\frac{\sqrt{1+x+x^2}}{2x^2}-\frac{1}{2}\int \frac{\frac{3}{2}+x}{x^2\sqrt{1+x+x^2}}\,dx\\
&= -\frac{\sqrt{1+x+x^2}}{2x^2}+\frac{3\sqrt{1+x+x^2}}{4x}-\frac{1}{8}\int \frac{1}{x\sqrt{1+x+x^2}}\,dx\\
&= -\frac{\sqrt{1+x+x^2}}{2x^2}+\frac{3\sqrt{1+x+x^2}}{4x}+\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{2+x}{\sqrt{1+x+x^2}}\right)\\
&= -\frac{\sqrt{1+x+x^2}}{2x^2}+\frac{3\sqrt{1+x+x^2}}{4x}+\frac{1}{8}\tanh^{-1}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0142266, size = 43, normalized size = 0.75

$$\frac{1}{8}\left(\frac{2\sqrt{x^2+x+1}(3x-2)}{x^2}+\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[1 + x + x^2]),x]
```

```
[Out] ((2*(-2 + 3*x)*Sqrt[1 + x + x^2])/x^2 + ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2
])])/8
```

Maple [A] time = 0.003, size = 44, normalized size = 0.8

$$\frac{1}{8}\mathrm{Artanh}\left(\frac{2+x}{2}\frac{1}{\sqrt{x^2+x+1}}\right)-\frac{1}{2x^2}\sqrt{x^2+x+1}+\frac{3}{4x}\sqrt{x^2+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^2+x+1)^(1/2),x)
```

```
[Out] 1/8*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))-1/2*(x^2+x+1)^(1/2)/x^2+3/4*(x^2+x+1
)^(1/2)/x
```

Maxima [A] time = 1.43785, size = 68, normalized size = 1.19

$$\frac{3\sqrt{x^2+x+1}}{4x}-\frac{\sqrt{x^2+x+1}}{2x^2}+\frac{1}{8}\,\mathrm{arsinh}\left(\frac{\sqrt{3}x}{3|x|}+\frac{2\sqrt{3}}{3|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(x^2 + x + 1)/x - 1/2*sqrt(x^2 + x + 1)/x^2 + 1/8*arcsinh(1/3*sqrt(
3)*x/abs(x) + 2/3*sqrt(3)/abs(x))
```

Fricas [A] time = 2.06931, size = 169, normalized size = 2.96

$$\frac{x^2 \log\left(-x + \sqrt{x^2 + x + 1} + 1\right) - x^2 \log\left(-x + \sqrt{x^2 + x + 1} - 1\right) + 6\,x^2 + 2\,\sqrt{x^2 + x + 1}(3\,x - 2)}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(x^2*log(-x + sqrt(x^2 + x + 1) + 1) - x^2*log(-x + sqrt(x^2 + x + 1) -
 1) + 6*x^2 + 2*sqrt(x^2 + x + 1)*(3*x - 2))/x^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{x^2 + x + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(1/(x**3*sqrt(x**2 + x + 1)), x)
```

Giac [A] time = 1.08011, size = 113, normalized size = 1.98

$$\frac{\left(x - \sqrt{x^2 + x + 1}\right)^3 + 9\,x - 9\,\sqrt{x^2 + x + 1} + 8}{4\left(\left(x - \sqrt{x^2 + x + 1}\right)^2 - 1\right)^2} + \frac{1}{8}\log\left(\left|-x + \sqrt{x^2 + x + 1} + 1\right|\right) - \frac{1}{8}\log\left(\left|-x + \sqrt{x^2 + x + 1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*((x - sqrt(x^2 + x + 1))^3 + 9*x - 9*sqrt(x^2 + x + 1) + 8)/((x - sqrt(
x^2 + x + 1))^2 - 1)^2 + 1/8*log(abs(-x + sqrt(x^2 + x + 1) + 1)) - 1/8*log
(abs(-x + sqrt(x^2 + x + 1) - 1))
```

3.276 $\int \frac{1}{x^2(1+x+x^2)^{3/2}}\,dx$

Optimal. Leaf size=62

$$\frac{2(1-x)}{3x\sqrt{x^2+x+1}}-\frac{5\sqrt{x^2+x+1}}{3x}+\frac{3}{2}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

```
[Out] (2*(1 - x))/(3*x*Sqrt[1 + x + x^2]) - (5*Sqrt[1 + x + x^2])/(3*x) + (3*ArcT
anh[(2 + x)/(2*Sqrt[1 + x + x^2])])/2
```

Rubi [A] time = 0.0266347, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {740, 806, 724, 206}

$$\frac{2(1-x)}{3x\sqrt{x^2+x+1}}-\frac{5\sqrt{x^2+x+1}}{3x}+\frac{3}{2}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(1 + x + x^2)^(3/2)),x]
```

```
[Out] (2*(1 - x))/(3*x*Sqrt[1 + x + x^2]) - (5*Sqrt[1 + x + x^2])/(3*x) + (3*ArcT
anh[(2 + x)/(2*Sqrt[1 + x + x^2])])/2
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^2\left(1+x+x^2\right)^{3/2}}\,dx &= \frac{2(1-x)}{3x\sqrt{1+x+x^2}}+\frac{2}{3}\int \frac{\frac{5}{2}-x}{x^2\sqrt{1+x+x^2}}\,dx\\
&= \frac{2(1-x)}{3x\sqrt{1+x+x^2}}-\frac{5\sqrt{1+x+x^2}}{3x}-\frac{3}{2}\int \frac{1}{x\sqrt{1+x+x^2}}\,dx\\
&= \frac{2(1-x)}{3x\sqrt{1+x+x^2}}-\frac{5\sqrt{1+x+x^2}}{3x}+3\,\mathrm{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{2+x}{\sqrt{1+x+x^2}}\right)\\
&= \frac{2(1-x)}{3x\sqrt{1+x+x^2}}-\frac{5\sqrt{1+x+x^2}}{3x}+\frac{3}{2}\tanh^{-1}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0163208, size = 50, normalized size = 0.81

$$\frac{3}{2}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)-\frac{5x^2+7x+3}{3x\sqrt{x^2+x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(1 + x + x^2)^(3/2)),x]
```

```
[Out] -(3 + 7*x + 5*x^2)/(3*x*Sqrt[1 + x + x^2]) + (3*ArcTanh[(2 + x)/(2*Sqrt[1 +
 x + x^2])])/2
```

Maple [A] time = 0.004, size = 56, normalized size = 0.9

$$-\frac{1}{x}\frac{1}{\sqrt{x^2+x+1}}-\frac{3}{2}\frac{1}{\sqrt{x^2+x+1}}-\frac{5+10\,x}{6}\frac{1}{\sqrt{x^2+x+1}}+\frac{3}{2}\mathrm{Artanh}\left(\frac{2+x}{2}\frac{1}{\sqrt{x^2+x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^2+x+1)^(3/2),x)
```

```
[Out] -1/x/(x^2+x+1)^(1/2)-3/2/(x^2+x+1)^(1/2)-5/6*(1+2*x)/(x^2+x+1)^(1/2)+3/2*ar
ctanh(1/2*(2+x)/(x^2+x+1)^(1/2))
```

Maxima [A] time = 1.46165, size = 78, normalized size = 1.26

$$-\frac{5\,x}{3\,\sqrt{x^2+x+1}}-\frac{7}{3\,\sqrt{x^2+x+1}}-\frac{1}{\sqrt{x^2+x+1}x}+\frac{3}{2}\,\mathrm{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|}+\frac{2\,\sqrt{3}}{3\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -5/3*x/sqrt(x^2 + x + 1) - 7/3/sqrt(x^2 + x + 1) - 1/(sqrt(x^2 + x + 1)*x)
+ 3/2*arcsinh(1/3*sqrt(3)*x/abs(x) + 2/3*sqrt(3)/abs(x))
```

Fricas [B] time = 2.21108, size = 258, normalized size = 4.16

$$-\frac{10\,x^{3}+10\,x^{2}-9\left(x^{3}+x^{2}+x\right)\log\left(-x+\sqrt{x^{2}+x+1}+1\right)+9\left(x^{3}+x^{2}+x\right)\log\left(-x+\sqrt{x^{2}+x+1}-1\right)+2\left(5\,x^{2}+\right.}{6\left(x^{3}+x^{2}+x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/6*(10*x^3 + 10*x^2 - 9*(x^3 + x^2 + x)*log(-x + sqrt(x^2 + x + 1) + 1) +
 9*(x^3 + x^2 + x)*log(-x + sqrt(x^2 + x + 1) - 1) + 2*(5*x^2 + 7*x + 3)*sq
rt(x^2 + x + 1) + 10*x)/(x^3 + x^2 + x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^{2}\left(x^{2}+x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**2+x+1)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(x**2 + x + 1)**(3/2)), x)
```

Giac [A] time = 1.08326, size = 108, normalized size = 1.74

$$-\frac{2\,(x+2)}{3\,\sqrt{x^{2}+x+1}}+\frac{x-\sqrt{x^{2}+x+1}+2}{\left(x-\sqrt{x^{2}+x+1}\right)^{2}-1}+\frac{3}{2}\,\log\left(\left|-x+\sqrt{x^{2}+x+1}+1\right|\right)-\frac{3}{2}\,\log\left(\left|-x+\sqrt{x^{2}+x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^2+x+1)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(x + 2)/sqrt(x^2 + x + 1) + (x - sqrt(x^2 + x + 1) + 2)/((x - sqrt(x^2
 + x + 1))^2 - 1) + 3/2*log(abs(-x + sqrt(x^2 + x + 1) + 1)) - 3/2*log(abs(
-x + sqrt(x^2 + x + 1) - 1))
```

3.277 $\int \frac{1}{x^3(1+x+x^2)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}+\frac{37\sqrt{x^2+x+1}}{12x}-\frac{7\sqrt{x^2+x+1}}{6x^2}-\frac{3}{8}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

```
[Out] (2*(1 - x))/(3*x^2*Sqrt[1 + x + x^2]) - (7*Sqrt[1 + x + x^2])/(6*x^2) + (37
*Sqrt[1 + x + x^2])/(12*x) - (3*ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])])/8
```

Rubi [A] time = 0.0371738, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {740, 834, 806, 724, 206}

$$\frac{2(1-x)}{3x^2\sqrt{x^2+x+1}}+\frac{37\sqrt{x^2+x+1}}{12x}-\frac{7\sqrt{x^2+x+1}}{6x^2}-\frac{3}{8}\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(1 + x + x^2)^(3/2)),x]
```

```
[Out] (2*(1 - x))/(3*x^2*Sqrt[1 + x + x^2]) - (7*Sqrt[1 + x + x^2])/(6*x^2) + (37
*Sqrt[1 + x + x^2])/(12*x) - (3*ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])])/8
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b*
x + c*x^2)^(p + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
```

```
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\left(1+x+x^2\right)^{3/2}}\,dx &= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}}+\frac{2}{3}\int \frac{\frac{7}{2}-2x}{x^3\sqrt{1+x+x^2}}\,dx\\
&= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}}-\frac{7\sqrt{1+x+x^2}}{6x^2}-\frac{1}{3}\int \frac{\frac{37}{4}+\frac{7x}{2}}{x^2\sqrt{1+x+x^2}}\,dx\\
&= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}}-\frac{7\sqrt{1+x+x^2}}{6x^2}+\frac{37\sqrt{1+x+x^2}}{12x}+\frac{3}{8}\int \frac{1}{x\sqrt{1+x+x^2}}\,dx\\
&= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}}-\frac{7\sqrt{1+x+x^2}}{6x^2}+\frac{37\sqrt{1+x+x^2}}{12x}-\frac{3}{4}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{2+x}{\sqrt{1+x+x^2}}\right)\\
&= \frac{2(1-x)}{3x^2\sqrt{1+x+x^2}}-\frac{7\sqrt{1+x+x^2}}{6x^2}+\frac{37\sqrt{1+x+x^2}}{12x}-\frac{3}{8}\tanh^{-1}\left(\frac{2+x}{2\sqrt{1+x+x^2}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0130602, size = 65, normalized size = 0.82

$$
\frac{74x^3+46x^2-9\sqrt{x^2+x+1}x^2\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)+30x-12}{24x^2\sqrt{x^2+x+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(1 + x + x^2)^(3/2)),x]
```

```
[Out] (-12 + 30*x + 46*x^2 + 74*x^3 - 9*x^2*Sqrt[1 + x + x^2]*ArcTanh[(2 + x)/(2*
Sqrt[1 + x + x^2])])/(24*x^2*Sqrt[1 + x + x^2])
```

Maple [A] time = 0.005, size = 69, normalized size = 0.9

$$
-\frac{1}{2x^2}\frac{1}{\sqrt{x^2+x+1}}+\frac{5}{4x}\frac{1}{\sqrt{x^2+x+1}}+\frac{3}{8}\frac{1}{\sqrt{x^2+x+1}}+\frac{37+74x}{24}\frac{1}{\sqrt{x^2+x+1}}-\frac{3}{8}\text{Artanh}\left(\frac{2+x}{2}\frac{1}{\sqrt{x^2+x+1}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^2+x+1)^(3/2),x)
```

```
[Out] -1/2/x^2/(x^2+x+1)^(1/2)+5/4/x/(x^2+x+1)^(1/2)+3/8/(x^2+x+1)^(1/2)+37/24*(1
+2*x)/(x^2+x+1)^(1/2)-3/8*arctanh(1/2*(2+x)/(x^2+x+1)^(1/2))
```

Maxima [A] time = 1.66985, size = 96, normalized size = 1.22

$$\frac{37\,x}{12\,\sqrt{x^2+x+1}}+\frac{23}{12\,\sqrt{x^2+x+1}}+\frac{5}{4\,\sqrt{x^2+x+1}x}-\frac{1}{2\,\sqrt{x^2+x+1}x^2}-\frac{3}{8}\,\operatorname{arsinh}\left(\frac{\sqrt{3}x}{3\,|x|}+\frac{2\,\sqrt{3}}{3\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+x+1)^(3/2),x, algorithm="maxima")
```

```
[Out] 37/12*x/sqrt(x^2 + x + 1) + 23/12/sqrt(x^2 + x + 1) + 5/4/(sqrt(x^2 + x + 1
)*x) - 1/2/(sqrt(x^2 + x + 1)*x^2) - 3/8*arcsinh(1/3*sqrt(3)*x/abs(x) + 2/3
*sqrt(3)/abs(x))
```

Fricas [A] time = 2.11412, size = 284, normalized size = 3.59

$$\frac{74\,x^4+74\,x^3+74\,x^2-9\left(x^4+x^3+x^2\right)\log\left(-x+\sqrt{x^2+x+1}+1\right)+9\left(x^4+x^3+x^2\right)\log\left(-x+\sqrt{x^2+x+1}-1\right)+2\,(3}{24\left(x^4+x^3+x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+x+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(74*x^4 + 74*x^3 + 74*x^2 - 9*(x^4 + x^3 + x^2)*log(-x + sqrt(x^2 + x
+ 1) + 1) + 9*(x^4 + x^3 + x^2)*log(-x + sqrt(x^2 + x + 1) - 1) + 2*(37*x^3
 + 23*x^2 + 15*x - 6)*sqrt(x^2 + x + 1))/(x^4 + x^3 + x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\left(x^2+x+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**2+x+1)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(x**2 + x + 1)**(3/2)), x)
```

Giac [A] time = 1.09846, size = 158, normalized size = 2.

$$\frac{2\,(2\,x+1)}{3\,\sqrt{x^2+x+1}}-\frac{3\left(x-\sqrt{x^2+x+1}\right)^3+8\left(x-\sqrt{x^2+x+1}\right)^2-13\,x+13\,\sqrt{x^2+x+1}-16}{4\left(\left(x-\sqrt{x^2+x+1}\right)^2-1\right)^2}-\frac{3}{8}\,\log\left(\left|-x+\sqrt{x^2+x+1}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2+x+1)^(3/2),x, algorithm="giac")
```

```
[Out] 2/3*(2*x + 1)/sqrt(x^2 + x + 1) - 1/4*(3*(x - sqrt(x^2 + x + 1))^3 + 8*(x -
 sqrt(x^2 + x + 1))^2 - 13*x + 13*sqrt(x^2 + x + 1) - 16)/((x - sqrt(x^2 +
x + 1))^2 - 1)^2 - 3/8*log(abs(-x + sqrt(x^2 + x + 1) + 1)) + 3/8*log(abs(-
x + sqrt(x^2 + x + 1) - 1))
```

3.278 $\int \frac{1}{(1+x)\sqrt{1+x+x^2}} dx$

Optimal. Leaf size=22

$$-\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)$$

```
[Out] -ArcTanh[(1 - x)/(2*Sqrt[1 + x + x^2])]
```

Rubi [A] time = 0.0092097, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {724, 206}

$$-\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*Sqrt[1 + x + x^2]),x]
```

```
[Out] -ArcTanh[(1 - x)/(2*Sqrt[1 + x + x^2])]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)\sqrt{1+x+x^2}} dx &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{4-x^2} dx, x, \frac{1-x}{\sqrt{1+x+x^2}}\right)\right)\\ &= -\tanh^{-1}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0031113, size = 22, normalized size = 1.

$$-\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*Sqrt[1 + x + x^2]),x]
```

```
[Out] -ArcTanh[(1 - x)/(2*Sqrt[1 + x + x^2])]
```

Maple [A] time = 0.003, size = 22, normalized size = 1.

$$-\mathrm{Artanh}\left(\frac{1-x}{2}\frac{1}{\sqrt{(1+x)^2-x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(x^2+x+1)^(1/2),x)
```

```
[Out] -arctanh(1/2*(1-x)/((1+x)^2-x)^(1/2))
```

Maxima [A] time = 1.68206, size = 34, normalized size = 1.55

$$\mathrm{arsinh}\left(\frac{\sqrt{3}x}{3\,|x+1|}-\frac{\sqrt{3}}{3\,|x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(1/3*sqrt(3)*x/abs(x + 1) - 1/3*sqrt(3)/abs(x + 1))
```

Fricas [A] time = 2.02714, size = 86, normalized size = 3.91

$$-\log\left(-x+\sqrt{x^2+x+1}\right)+\log\left(-x+\sqrt{x^2+x+1}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + x + 1)) + log(-x + sqrt(x^2 + x + 1) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(x+1)\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x**2+x+1)**(1/2),x)
```

```
[Out] Integral(1/((x + 1)*sqrt(x**2 + x + 1)), x)
```

Giac [A] time = 1.09693, size = 43, normalized size = 1.95

$$-\log\left(\left|-x+\sqrt{x^2+x+1}\right|\right)+\log\left(\left|-x+\sqrt{x^2+x+1}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x + 1) - 2))
```

3.279 $\int \frac{1}{\sqrt{4+2x+x^2}(-x+x^3)}\,dx$

Optimal. Leaf size=86

$$\frac{1}{2}\tanh^{-1}\left(\frac{x+4}{2\sqrt{x^2+2x+4}}\right)-\frac{\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] ArcTanh[(4 + x)/(2*Sqrt[4 + 2*x + x^2])]/2 - ArcTanh[(5 + 2*x)/(Sqrt[7]*Sqr
t[4 + 2*x + x^2])]/(2*Sqrt[7]) - ArcTanh[Sqrt[4 + 2*x + x^2]/Sqrt[3]]/(2*Sq
rt[3])
```

Rubi [A] time = 0.278194, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1593, 6725, 724, 206, 1033, 688, 207}

$$\frac{1}{2}\tanh^{-1}\left(\frac{x+4}{2\sqrt{x^2+2x+4}}\right)-\frac{\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{2\sqrt{7}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+2x+4}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[4 + 2*x + x^2]*(-x + x^3)),x]
```

```
[Out] ArcTanh[(4 + x)/(2*Sqrt[4 + 2*x + x^2])]/2 - ArcTanh[(5 + 2*x)/(Sqrt[7]*Sqr
t[4 + 2*x + x^2])]/(2*Sqrt[7]) - ArcTanh[Sqrt[4 + 2*x + x^2]/Sqrt[3]]/(2*Sq
rt[3])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 688

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a +
 b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && E
qQ[2*c*d - b*e, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{4+2x+x^2}\left(-x+x^3\right)}\,dx = \int \frac{1}{x\left(-1+x^2\right)\sqrt{4+2x+x^2}}\,dx$$

$$= \int \left(-\frac{1}{x\sqrt{4+2x+x^2}} + \frac{x}{\left(-1+x^2\right)\sqrt{4+2x+x^2}}\right)dx$$

$$= -\int \frac{1}{x\sqrt{4+2x+x^2}}\,dx + \int \frac{x}{\left(-1+x^2\right)\sqrt{4+2x+x^2}}\,dx$$

$$= \frac{1}{2}\int \frac{1}{(-1+x)\sqrt{4+2x+x^2}}\,dx + \frac{1}{2}\int \frac{1}{(1+x)\sqrt{4+2x+x^2}}\,dx + 2\,\text{Subst}\left(\int \frac{1}{16-x^2}\,dx\right.$$

$$= \frac{1}{2}\tanh^{-1}\left(\frac{4+x}{2\sqrt{4+2x+x^2}}\right) + 2\,\text{Subst}\left(\int \frac{1}{-12+4x^2}\,dx, x, \sqrt{4+2x+x^2}\right) - \text{Subst}\left(\int\right.$$

$$= \frac{1}{2}\tanh^{-1}\left(\frac{4+x}{2\sqrt{4+2x+x^2}}\right) - \frac{\tanh^{-1}\left(\frac{10+4x}{2\sqrt{7}\sqrt{4+2x+x^2}}\right)}{2\sqrt{7}} - \frac{\tanh^{-1}\left(\frac{\sqrt{4+2x+x^2}}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Mathematica [A] time = 0.0602414, size = 83, normalized size = 0.97

$$\frac{1}{42}\left(21\tanh^{-1}\left(\frac{x+4}{2\sqrt{x^2+2x+4}}\right) - 3\sqrt{7}\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right) - 7\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{(x+1)^2+3}}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[4 + 2*x + x^2]*(-x + x^3)),x]
```

```
[Out] (21*ArcTanh[(4 + x)/(2*Sqrt[4 + 2*x + x^2])] - 3*Sqrt[7]*ArcTanh[(5 + 2*x)/
(Sqrt[7]*Sqrt[4 + 2*x + x^2])] - 7*Sqrt[3]*ArcTanh[Sqrt[3 + (1 + x)^2]/Sqrt
[3]])/42
```

Maple [A] time = 0.01, size = 69, normalized size = 0.8

$$\frac{1}{2}\mathrm{Artanh}\left(\frac{8+2x}{4}\frac{1}{\sqrt{x^2+2x+4}}\right)-\frac{\sqrt{3}}{6}\mathrm{Artanh}\left(\sqrt{3}\frac{1}{\sqrt{(1+x)^2+3}}\right)-\frac{\sqrt{7}}{14}\mathrm{Artanh}\left(\frac{(10+4x)\sqrt{7}}{14}\frac{1}{\sqrt{(-1+x)^2+3+\cdots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-x)/(x^2+2*x+4)^(1/2),x)
```

```
[Out] 1/2*arctanh(1/4*(8+2*x)/(x^2+2*x+4)^(1/2))-1/6*3^(1/2)*arctanh(3^(1/2)/((1+
x)^2+3)^(1/2))-1/14*7^(1/2)*arctanh(1/14*(10+4*x)*7^(1/2)/((-1+x)^2+3+4*x)^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^3-x)\sqrt{x^2+2x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 - x)*sqrt(x^2 + 2*x + 4)), x)
```

Fricas [A] time = 2.16378, size = 319, normalized size = 3.71

$$\frac{1}{14}\sqrt{7}\log\left(\frac{\sqrt{7}(2x+5)+\sqrt{x^2+2x+4}(2\sqrt{7}-7)-4x-10}{x-1}\right)+\frac{1}{6}\sqrt{3}\log\left(-\frac{\sqrt{3}-\sqrt{x^2+2x+4}}{x+1}\right)+\frac{1}{2}\log\left(-x+\sqrt{x\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/14*sqrt(7)*log((sqrt(7)*(2*x + 5) + sqrt(x^2 + 2*x + 4)*(2*sqrt(7) - 7) -
 4*x - 10)/(x - 1)) + 1/6*sqrt(3)*log(-(sqrt(3) - sqrt(x^2 + 2*x + 4))/(x +
 1)) + 1/2*log(-x + sqrt(x^2 + 2*x + 4) + 2) - 1/2*log(-x + sqrt(x^2 + 2*x
+ 4) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x(x-1)(x+1)\sqrt{x^2+2x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-x)/(x**2+2*x+4)**(1/2),x)
```

```
[Out] Integral(1/(x*(x - 1)*(x + 1)*sqrt(x**2 + 2*x + 4)), x)
```

Giac [B] time = 1.15977, size = 198, normalized size = 2.3

$$\frac{1}{14}\sqrt{7}\log\left(\frac{\left|-2x-2\sqrt{7}+2\sqrt{x^2+2x+4}+2\right|}{\left|-2x+2\sqrt{7}+2\sqrt{x^2+2x+4}+2\right|}\right)+\frac{1}{6}\sqrt{3}\log\left(-\frac{\left|-2x-2\sqrt{3}+2\sqrt{x^2+2x+4}-2\right|}{2\left(x-\sqrt{3}-\sqrt{x^2+2x+4}+1\right)}\right)+\frac{1}{2}\log\left(\left|-x+\sqrt{x^2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x)/(x^2+2*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/14*sqrt(7)*log(abs(-2*x - 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)/abs(-2*x
 + 2*sqrt(7) + 2*sqrt(x^2 + 2*x + 4) + 2)) + 1/6*sqrt(3)*log(-1/2*abs(-2*x
- 2*sqrt(3) + 2*sqrt(x^2 + 2*x + 4) - 2)/(x - sqrt(3) - sqrt(x^2 + 2*x + 4)
 + 1)) + 1/2*log(abs(-x + sqrt(x^2 + 2*x + 4) + 2)) - 1/2*log(abs(-x + sqrt
(x^2 + 2*x + 4) - 2))
```

3.280 $\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2} \, dx$

Optimal. Leaf size=62

$$\frac{\sqrt{x^2+2x+4}}{1-x}-\frac{2\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{\sqrt{7}}+\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$$

```
[Out] Sqrt[4 + 2*x + x^2]/(1 - x) + ArcSinh[(1 + x)/Sqrt[3]] - (2*ArcTanh[(5 + 2*
x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])])/Sqrt[7]
```

Rubi [A] time = 0.0465202, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {732, 843, 619, 215, 724, 206}

$$\frac{\sqrt{x^2+2x+4}}{1-x}-\frac{2\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{\sqrt{7}}+\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + 2*x + x^2]/(-1 + x)^2,x]
```

```
[Out] Sqrt[4 + 2*x + x^2]/(1 - x) + ArcSinh[(1 + x)/Sqrt[3]] - (2*ArcTanh[(5 + 2*
x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])])/Sqrt[7]
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{4+2x+x^2}}{(-1+x)^2}\,dx &= \frac{\sqrt{4+2x+x^2}}{1-x}+\frac{1}{2}\int \frac{2+2x}{(-1+x)\sqrt{4+2x+x^2}}\,dx\\
&= \frac{\sqrt{4+2x+x^2}}{1-x}+2\int \frac{1}{(-1+x)\sqrt{4+2x+x^2}}\,dx+\int \frac{1}{\sqrt{4+2x+x^2}}\,dx\\
&= \frac{\sqrt{4+2x+x^2}}{1-x}-4\,\text{Subst}\left(\int \frac{1}{28-x^2}\,dx,x,\frac{10+4x}{\sqrt{4+2x+x^2}}\right)+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{12}}}\,dx,x,2+2x\right)}{2\sqrt{3}}\\
&= \frac{\sqrt{4+2x+x^2}}{1-x}+\sinh^{-1}\left(\frac{1+x}{\sqrt{3}}\right)-\frac{2\tanh^{-1}\left(\frac{5+2x}{\sqrt{7}\sqrt{4+2x+x^2}}\right)}{\sqrt{7}}
\end{aligned}$$

Mathematica [A] time = 0.0382395, size = 61, normalized size = 0.98

$$-\frac{\sqrt{x^2+2x+4}}{x-1}-\frac{2\tanh^{-1}\left(\frac{2x+5}{\sqrt{7}\sqrt{x^2+2x+4}}\right)}{\sqrt{7}}+\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + 2*x + x^2]/(-1 + x)^2,x]
```

```
[Out] -(Sqrt[4 + 2*x + x^2]/(-1 + x)) + ArcSinh[(1 + x)/Sqrt[3]] - (2*ArcTanh[(5
+ 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])])/Sqrt[7]
```

Maple [A] time = 0.007, size = 91, normalized size = 1.5

$$-\frac{1}{-7+7x}\left((-1+x)^2+3+4x\right)^{\frac{3}{2}}+\frac{2}{7}\sqrt{(-1+x)^2+3+4x}+\text{Arcsinh}\left(\frac{(1+x)\sqrt{3}}{3}\right)-\frac{2\sqrt{7}}{7}\text{Artanh}\left(\frac{(10+4x)\sqrt{7}}{14}\frac{}{\sqrt{(-1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x+4)^(1/2)/(-1+x)^2,x)
```

```
[Out] -1/7/(-1+x)*((-1+x)^2+3+4*x)^(3/2)+2/7*((-1+x)^2+3+4*x)^(1/2)+arcsinh(1/3*(
1+x)*3^(1/2))-2/7*7^(1/2)*arctanh(1/14*(10+4*x)*7^(1/2)/((-1+x)^2+3+4*x)^(1
```

```
/2))+1/14*(2*x+2)*((-1+x)^2+3+4*x)^(1/2)
```

Maxima [A] time = 1.45849, size = 82, normalized size = 1.32

$$-\frac{2}{7}\sqrt{7}\operatorname{arsinh}\left(\frac{2\sqrt{3}x}{3|x-1|}+\frac{5\sqrt{3}}{3|x-1|}\right)-\frac{\sqrt{x^2+2x+4}}{x-1}+\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x, algorithm="maxima")
```

```
[Out] -2/7*sqrt(7)*arcsinh(2/3*sqrt(3)*x/abs(x - 1) + 5/3*sqrt(3)/abs(x - 1)) - s
qrt(x^2 + 2*x + 4)/(x - 1) + arcsinh(1/3*sqrt(3)*x + 1/3*sqrt(3))
```

Fricas [A] time = 1.9008, size = 263, normalized size = 4.24

$$\frac{2\sqrt{7}(x-1)\log\left(\frac{\sqrt{7}(2x+5)+\sqrt{x^2+2x+4}(2\sqrt{7}-7)-4x-10}{x-1}\right)-7(x-1)\log\left(-x+\sqrt{x^2+2x+4}-1\right)-7x-7\sqrt{x^2+2x+4}+7}{7(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/7*(2*sqrt(7)*(x - 1)*log((sqrt(7)*(2*x + 5) + sqrt(x^2 + 2*x + 4)*(2*sqrt
(7) - 7) - 4*x - 10)/(x - 1)) - 7*(x - 1)*log(-x + sqrt(x^2 + 2*x + 4) - 1)
 - 7*x - 7*sqrt(x^2 + 2*x + 4) + 7)/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x^2+2x+4}}{(x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x+4)**(1/2)/(-1+x)**2,x)
```

```
[Out] Integral(sqrt(x**2 + 2*x + 4)/(x - 1)**2, x)
```

Giac [B] time = 1.15964, size = 204, normalized size = 3.29

$$-\frac{2}{7}\sqrt{7}\log\left(2\sqrt{7}+7\sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}+\frac{7\sqrt{7}}{x-1}\right)\operatorname{sgn}\left(\frac{1}{x-1}\right)+\log\left(\sqrt{\frac{4}{x-1}+\frac{7}{(x-1)^2}+1}+\frac{\sqrt{7}}{x-1}+1\right)\operatorname{sgn}\left(\frac{1}{x-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+4)^(1/2)/(-1+x)^2,x, algorithm="giac")
```

```
[Out] -2/7*sqrt(7)*log(2*sqrt(7) + 7*sqrt(4/(x - 1) + 7/(x - 1)^2 + 1) + 7*sqrt(7
)/(x - 1))*sgn(1/(x - 1)) + log(sqrt(4/(x - 1) + 7/(x - 1)^2 + 1) + sqrt(7)
/(x - 1) + 1)*sgn(1/(x - 1)) - log(abs(sqrt(4/(x - 1) + 7/(x - 1)^2 + 1) +
sqrt(7)/(x - 1) - 1))*sgn(1/(x - 1)) - sqrt(4/(x - 1) + 7/(x - 1)^2 + 1)*sg
n(1/(x - 1))
```

3.281 $\int \frac{3+2x}{(3+2x+x^2)^2\sqrt{4+2x+x^2}}\,dx$

Optimal. Leaf size=76

$$-\frac{\sqrt{x^2+2x+4}(3-x)}{4\left(x^2+2x+3\right)}-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+2x+4}}\right)}{4\sqrt{2}}+\tanh^{-1}\left(\sqrt{x^2+2x+4}\right)$$

```
[Out] -((3 - x)*Sqrt[4 + 2*x + x^2])/(4*(3 + 2*x + x^2)) - ArcTan[(1 + x)/(Sqrt[2
]*Sqrt[4 + 2*x + x^2])]/(4*Sqrt[2]) + ArcTanh[Sqrt[4 + 2*x + x^2]]
```

Rubi [A] time = 0.0677463, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1016, 1025, 982, 204, 1024, 206}

$$-\frac{\sqrt{x^2+2x+4}(3-x)}{4\left(x^2+2x+3\right)}-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{2}\sqrt{x^2+2x+4}}\right)}{4\sqrt{2}}+\tanh^{-1}\left(\sqrt{x^2+2x+4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/((3 + 2*x + x^2)^2*Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] -((3 - x)*Sqrt[4 + 2*x + x^2])/(4*(3 + 2*x + x^2)) - ArcTan[(1 + x)/(Sqrt[2
]*Sqrt[4 + 2*x + x^2])]/(4*Sqrt[2]) + ArcTanh[Sqrt[4 + 2*x + x^2]]
```

Rule 1016

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x + c*x^2)^(p + 1)
*(d + e*x + f*x^2)^(q + 1)*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*
c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))
 - h*(b*c*d - 2*a*c*e + a*b*f))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d -
a*e)*(c*e - b*f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*
x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)
 + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*(-(h*c*e
))))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g*
b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2*
a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(
p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*
(-(h*c*e))))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*c
*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x}{\left(3+2x+x^2\right)^2\sqrt{4+2x+x^2}}\,dx &= -\frac{(3-x)\sqrt{4+2x+x^2}}{4\left(3+2x+x^2\right)}+\frac{1}{8}\int \frac{-10-8x}{\left(3+2x+x^2\right)\sqrt{4+2x+x^2}}\,dx\\ &= -\frac{(3-x)\sqrt{4+2x+x^2}}{4\left(3+2x+x^2\right)}-\frac{1}{4}\int \frac{1}{\left(3+2x+x^2\right)\sqrt{4+2x+x^2}}\,dx-\frac{1}{2}\int \frac{2+}{\left(3+2x+x^2\right)}\\ &= -\frac{(3-x)\sqrt{4+2x+x^2}}{4\left(3+2x+x^2\right)}+2\,\text{Subst}\left(\int \frac{1}{2-2x^2}\,dx,x,\sqrt{4+2x+x^2}\right)+\text{Subst}\left(\int \frac{}{-1}\right.\\ &= -\frac{(3-x)\sqrt{4+2x+x^2}}{4\left(3+2x+x^2\right)}-\frac{\tan^{-1}\left(\frac{2+2x}{2\sqrt{2}\sqrt{4+2x+x^2}}\right)}{4\sqrt{2}}+\tanh^{-1}\left(\sqrt{4+2x+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.368974, size = 146, normalized size = 1.92

$$\frac{1}{32}\left(8\left(\frac{\sqrt{x^2+2x+4}(x-3)}{x^2+2x+3}-2\log\left(\left(x^2+2x+3\right)^2\right)+2\log\left(\left(x^2+2x+3\right)\left(x^2+2\sqrt{x^2+2x+4}+2x+5\right)\right)\right)-4\sqrt{2}\tan\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/((3 + 2*x + x^2)^2*Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] (-4*Sqrt[2]*ArcTan[(Sqrt[2]*(4 + 10*x + 5*x^2))/(12 + 4*x^2 + 11*Sqrt[4 + 2
*x + x^2] + x*(8 + 11*Sqrt[4 + 2*x + x^2]))] + 8*(((-3 + x)*Sqrt[4 + 2*x +
x^2])/(3 + 2*x + x^2) - 2*Log[(3 + 2*x + x^2)^2] + 2*Log[(3 + 2*x + x^2)*(5
```

```
+ 2*x + x^2 + 2*Sqrt[4 + 2*x + x^2])]))/32
```

Maple [A] time = 0.024, size = 123, normalized size = 1.6

$$-\frac{1}{2}\left(1+\sqrt{x^2+2x+4}\right)^{-1}+\frac{1}{2}\ln\left(1+\sqrt{x^2+2x+4}\right)-\frac{1}{2}\left(-1+\sqrt{x^2+2x+4}\right)^{-1}-\frac{1}{2}\ln\left(-1+\sqrt{x^2+2x+4}\right)+\frac{3+}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x)
```

```
[Out] -1/2/(1+(x^2+2*x+4)^(1/2))+1/2*ln(1+(x^2+2*x+4)^(1/2))-1/2/(-1+(x^2+2*x+4)^
(1/2))-1/2*ln(-1+(x^2+2*x+4)^(1/2))+3/4*(1+x)/(x^2+2*x+4)^(1/2)/((1+x)^2/(x
^2+2*x+4)+2)-1/8*arctan(1/2*(1+x)*2^(1/2)/(x^2+2*x+4)^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+3}{\sqrt{x^2+2x+4}\left(x^2+2x+3\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x + 3)/(sqrt(x^2 + 2*x + 4)*(x^2 + 2*x + 3)^2), x)
```

Fricas [B] time = 1.88254, size = 504, normalized size = 6.63

$$\sqrt{2}\left(x^2+2x+3\right)\arctan\left(-\frac{1}{2}\sqrt{2}(x+2)+\frac{1}{2}\sqrt{2}\sqrt{x^2+2x+4}\right)-\sqrt{2}\left(x^2+2x+3\right)\arctan\left(-\frac{1}{2}\sqrt{2}x+\frac{1}{2}\sqrt{2}\sqrt{x^2+2x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*(x^2 + 2*x + 3)*arctan(-1/2*sqrt(2)*(x + 2) + 1/2*sqrt(2)*sqrt
(x^2 + 2*x + 4)) - sqrt(2)*(x^2 + 2*x + 3)*arctan(-1/2*sqrt(2)*x + 1/2*sqrt
(2)*sqrt(x^2 + 2*x + 4)) + 2*x^2 - 4*(x^2 + 2*x + 3)*log(x^2 - sqrt(x^2 + 2
*x + 4)*(x + 2) + 3*x + 5) + 4*(x^2 + 2*x + 3)*log(x^2 - sqrt(x^2 + 2*x + 4
)*x + x + 3) + 2*sqrt(x^2 + 2*x + 4)*(x - 3) + 4*x + 6)/(x^2 + 2*x + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x+3}{\left(x^2+2x+3\right)^2\sqrt{x^2+2x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x**2+2*x+3)**2/(x**2+2*x+4)**(1/2),x)
```

```
[Out] Integral((2*x + 3)/((x**2 + 2*x + 3)**2*sqrt(x**2 + 2*x + 4)), x)
```

Giac [B] time = 1.09552, size = 317, normalized size = 4.17

$$\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2+2x+4}+2\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(x-\sqrt{x^2+2x+4}\right)\right)+\frac{4\left(x-\sqrt{x^2+2\right.}{2\left(\left(x-\sqrt{x^2+2x+4}\right)^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x^2+2*x+3)^2/(x^2+2*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 2*x + 4) + 2)) - 1/8*sqrt(2
)*arctan(-1/2*sqrt(2)*(x - sqrt(x^2 + 2*x + 4))) + 1/2*(4*(x - sqrt(x^2 + 2
*x + 4))^3 + 13*(x - sqrt(x^2 + 2*x + 4))^2 + 26*x - 26*sqrt(x^2 + 2*x + 4)
 + 26)/((x - sqrt(x^2 + 2*x + 4))^4 + 4*(x - sqrt(x^2 + 2*x + 4))^3 + 8*(x
- sqrt(x^2 + 2*x + 4))^2 + 8*x - 8*sqrt(x^2 + 2*x + 4) + 12) - 1/2*log((x -
 sqrt(x^2 + 2*x + 4))^2 + 4*x - 4*sqrt(x^2 + 2*x + 4) + 6) + 1/2*log((x - s
qrt(x^2 + 2*x + 4))^2 + 2)
```

3.282 $\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\left(-3+x+2x^2\right)}\,dx$

Optimal. Leaf size=36

$$\frac{\sqrt{x^2+2x-3}}{2(1-x)}+\sqrt{x^2+2x-3}$$

```
[Out] Sqrt[-3 + 2*x + x^2] + Sqrt[-3 + 2*x + x^2]/(2*(1 - x))
```

Rubi [A] time = 0.127551, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 1586, 1638, 650}

$$\frac{\sqrt{x^2+2x-3}}{2(1-x)}+\sqrt{x^2+2x-3}$$

Antiderivative was successfully verified.

```
[In] Int[(3*x^2 + 2*x^3)/(Sqrt[-3 + 2*x + x^2]*(-3 + x + 2*x^2)),x]
```

```
[Out] Sqrt[-3 + 2*x + x^2] + Sqrt[-3 + 2*x + x^2]/(2*(1 - x))
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 1638

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[(f*(d + e*x)^(m + q - 1)*(a + b*x + c*x^2)^(p + 1))/(c*e^(q - 1)*(m + q
 + 2*p + 1)), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q + e*f*(m + p + q)*(d + e*x)^(q - 2)*(b*d - 2*a*e + (2*c*d - b
*e)*x), x], x], x] /; NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m,
p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2,
 0]
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d -
b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \frac{3x^2+2x^3}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx = \int \frac{x^2(3+2x)}{\sqrt{-3+2x+x^2}\,(-3+x+2x^2)}\,dx$$
$$= \int \frac{x^2}{(-1+x)\sqrt{-3+2x+x^2}}\,dx$$
$$= \sqrt{-3+2x+x^2} + \int \frac{1}{(-1+x)\sqrt{-3+2x+x^2}}\,dx$$
$$= \sqrt{-3+2x+x^2} + \frac{\sqrt{-3+2x+x^2}}{2(1-x)}$$

Mathematica [A] time = 0.0111896, size = 26, normalized size = 0.72

$$\frac{2x^2+3x-9}{2\sqrt{x^2+2x-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x^2 + 2*x^3)/(Sqrt[-3 + 2*x + x^2]*(-3 + x + 2*x^2)),x]
```

```
[Out] (-9 + 3*x + 2*x^2)/(2*Sqrt[-3 + 2*x + x^2])
```

Maple [A] time = 0.005, size = 21, normalized size = 0.6

$$\frac{(-3+2x)(3+x)}{2}\frac{1}{\sqrt{x^2+2x-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x)
```

```
[Out] 1/2*(-3+2*x)*(3+x)/(x^2+2*x-3)^(1/2)
```

Maxima [A] time = 1.46946, size = 38, normalized size = 1.06

$$\sqrt{x^2+2x-3} - \frac{\sqrt{x^2+2x-3}}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x, algorithm="maxima"
)
```

```
[Out] sqrt(x^2 + 2*x - 3) - 1/2*sqrt(x^2 + 2*x - 3)/(x - 1)
```

Fricas [A] time = 1.88616, size = 58, normalized size = 1.61

$$\frac{\sqrt{x^2+2x-3}(2x-3)}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/2*sqrt(x^2 + 2*x - 3)*(2*x - 3)/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{(x-1)(x+3)}(x-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+3*x**2)/(2*x**2+x-3)/(x**2+2*x-3)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt((x - 1)*(x + 3))*(x - 1)), x)
```

Giac [A] time = 1.07088, size = 41, normalized size = 1.14

$$\sqrt{x^2+2x-3}+\frac{2}{x-\sqrt{x^2+2x-3}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+3*x^2)/(2*x^2+x-3)/(x^2+2*x-3)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 2*x - 3) + 2/(x - sqrt(x^2 + 2*x - 3) - 1)
```

3.283 $\int \frac{1+x^4}{(1+x+x^2)\sqrt{2+x+x^2}}\,dx$

Optimal. Leaf size=87

$$\frac{1}{2}\sqrt{x^2+x+2}\,x-\frac{7}{4}\sqrt{x^2+x+2}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+x+2}}\right)}{\sqrt{3}}-\tanh^{-1}\left(\sqrt{x^2+x+2}\right)-\frac{1}{8}\sinh^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)$$

```
[Out] (-7*Sqrt[2 + x + x^2])/4 + (x*Sqrt[2 + x + x^2])/2 - ArcSinh[(1 + 2*x)/Sqrt
[7]]/8 + ArcTan[(1 + 2*x)/(Sqrt[3]*Sqrt[2 + x + x^2])]/Sqrt[3] - ArcTanh[Sq
rt[2 + x + x^2]]
```

Rubi [A] time = 0.209347, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6728, 640, 619, 215, 742, 1025, 982, 204, 1024, 206}

$$\frac{1}{2}\sqrt{x^2+x+2}\,x-\frac{7}{4}\sqrt{x^2+x+2}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}\sqrt{x^2+x+2}}\right)}{\sqrt{3}}-\tanh^{-1}\left(\sqrt{x^2+x+2}\right)-\frac{1}{8}\sinh^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/((1 + x + x^2)*Sqrt[2 + x + x^2]),x]
```

```
[Out] (-7*Sqrt[2 + x + x^2])/4 + (x*Sqrt[2 + x + x^2])/2 - ArcSinh[(1 + 2*x)/Sqrt
[7]]/8 + ArcTan[(1 + 2*x)/(Sqrt[3]*Sqrt[2 + x + x^2])]/Sqrt[3] - ArcTanh[Sq
rt[2 + x + x^2]]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
```

```
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 1025

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(h*e - 2*g*f)/(2*f), Int[1/(
(a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2
*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e
 - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1024

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g, Subst[Int[1/(b*d - a*e -
 b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &&
 EqQ[h*e - 2*g*f, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1+x^4}{\left(1+x+x^2\right)\sqrt{2+x+x^2}}\,dx = \int \left(-\frac{x}{\sqrt{2+x+x^2}}+\frac{x^2}{\sqrt{2+x+x^2}}+\frac{1+x}{\left(1+x+x^2\right)\sqrt{2+x+x^2}}\right)dx$$
$$= -\int \frac{x}{\sqrt{2+x+x^2}}\,dx+\int \frac{x^2}{\sqrt{2+x+x^2}}\,dx+\int \frac{1+x}{\left(1+x+x^2\right)\sqrt{2+x+x^2}}\,dx$$
$$= -\sqrt{2+x+x^2}+\frac{1}{2}x\sqrt{2+x+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{2+x+x^2}}\,dx+\frac{1}{2}\int \frac{-2-\frac{3x}{2}}{\sqrt{2+x+x^2}}\,dx+\frac{1}{2}\int$$
$$= -\frac{7}{4}\sqrt{2+x+x^2}+\frac{1}{2}x\sqrt{2+x+x^2}-\frac{5}{8}\int \frac{1}{\sqrt{2+x+x^2}}\,dx+\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{7}}}\,dx,x,1+\right.}{2\sqrt{7}}$$
$$= -\frac{7}{4}\sqrt{2+x+x^2}+\frac{1}{2}x\sqrt{2+x+x^2}+\frac{1}{2}\sinh^{-1}\left(\frac{1+2x}{\sqrt{7}}\right)+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}\sqrt{2+x+x^2}}\right)}{\sqrt{3}}-\tanh^{-1}$$
$$= -\frac{7}{4}\sqrt{2+x+x^2}+\frac{1}{2}x\sqrt{2+x+x^2}-\frac{1}{8}\sinh^{-1}\left(\frac{1+2x}{\sqrt{7}}\right)+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}\sqrt{2+x+x^2}}\right)}{\sqrt{3}}-\tanh^{-1}$$

Mathematica [C] time = 0.146555, size = 134, normalized size = 1.54

$$\frac{1}{24}\left(6\sqrt{x^2+x+2}(2x-7)-4i\left(\sqrt{3}-3i\right)\tanh^{-1}\left(\frac{-2i\sqrt{3}x-i\sqrt{3}+7}{4\sqrt{x^2+x+2}}\right)+4i\left(\sqrt{3}+3i\right)\tanh^{-1}\left(\frac{2i\sqrt{3}x+i\sqrt{3}+7}{4\sqrt{x^2+x+2}}\right)-3\sinh\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/((1 + x + x^2)*Sqrt[2 + x + x^2]),x]
```

```
[Out] (6*(-7 + 2*x)*Sqrt[2 + x + x^2] - 3*ArcSinh[(1 + 2*x)/Sqrt[7]] - (4*I)*(-3*
I + Sqrt[3])*ArcTanh[(7 - I*Sqrt[3] - (2*I)*Sqrt[3]*x)/(4*Sqrt[2 + x + x^2]
)] + (4*I)*(3*I + Sqrt[3])*ArcTanh[(7 + I*Sqrt[3] + (2*I)*Sqrt[3]*x)/(4*Sqr
t[2 + x + x^2])])/24
```

Maple [A] time = 0.017, size = 69, normalized size = 0.8

$$\frac{x}{2}\sqrt{x^2+x+2}-\frac{7}{4}\sqrt{x^2+x+2}-\frac{1}{8}\text{Arcsinh}\left(\frac{2\sqrt{7}}{7}\left(x+\frac{1}{2}\right)\right)-\text{Artanh}\left(\sqrt{x^2+x+2}\right)+\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\frac{1}{\sqrt{x^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x)
```

```
[Out] 1/2*x*(x^2+x+2)^(1/2)-7/4*(x^2+x+2)^(1/2)-1/8*arcsinh(2/7*7^(1/2)*(x+1/2))-
arctanh((x^2+x+2)^(1/2))+1/3*arctan(1/3*(1+2*x)*3^(1/2)/(x^2+x+2)^(1/2))*3^
(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4+1}{\sqrt{x^2+x+2}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)/(sqrt(x^2 + x + 2)*(x^2 + x + 1)), x)
```

Fricas [B] time = 1.86978, size = 455, normalized size = 5.23

$$\frac{1}{4}\sqrt{x^2+x+2}(2x-7)-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(2x+3)+\frac{2}{3}\sqrt{3}\sqrt{x^2+x+2}\right)+\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(2x-1)+\frac{2}{3}\sqrt{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^2 + x + 2)*(2*x - 7) - 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x + 3)
 + 2/3*sqrt(3)*sqrt(x^2 + x + 2)) + 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x -
1) + 2/3*sqrt(3)*sqrt(x^2 + x + 2)) + 1/2*log(2*x^2 - sqrt(x^2 + x + 2)*(2*
x + 3) + 4*x + 5) - 1/2*log(2*x^2 - sqrt(x^2 + x + 2)*(2*x - 1) + 3) + 1/8*
log(-2*x + 2*sqrt(x^2 + x + 2) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4+1}{\left(x^2+x+1\right)\sqrt{x^2+x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/(x**2+x+1)/(x**2+x+2)**(1/2),x)
```

```
[Out] Integral((x**4 + 1)/((x**2 + x + 1)*sqrt(x**2 + x + 2)), x)
```

Giac [B] time = 1.08002, size = 200, normalized size = 2.3

$$\frac{1}{4}\sqrt{x^2+x+2}(2x-7)-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(2x-2\sqrt{x^2+x+2}+3\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(2x-2\sqrt{x^2+x+2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/(x^2+x+1)/(x^2+x+2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^2 + x + 2)*(2*x - 7) - 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - 2*
sqrt(x^2 + x + 2) + 3)) + 1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(2*x - 2*sqrt(x^2
 + x + 2) - 1)) + 1/2*log((x - sqrt(x^2 + x + 2))^2 + 3*x - 3*sqrt(x^2 + x
+ 2) + 3) - 1/2*log((x - sqrt(x^2 + x + 2))^2 - x + sqrt(x^2 + x + 2) + 1)
+ 1/8*log(-2*x + 2*sqrt(x^2 + x + 2) - 1)
```

3.284 $\int \frac{1}{(4+2x+x^2)^{7/2}}\,dx$

Optimal. Leaf size=58

$$\frac{8(x+1)}{405\sqrt{x^2+2x+4}}+\frac{4(x+1)}{135\left(x^2+2x+4\right)^{3/2}}+\frac{x+1}{15\left(x^2+2x+4\right)^{5/2}}$$

```
[Out] (1 + x)/(15*(4 + 2*x + x^2)^(5/2)) + (4*(1 + x))/(135*(4 + 2*x + x^2)^(3/2)
) + (8*(1 + x))/(405*Sqrt[4 + 2*x + x^2])
```

Rubi [A] time = 0.0107579, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {614, 613}

$$\frac{8(x+1)}{405\sqrt{x^2+2x+4}}+\frac{4(x+1)}{135\left(x^2+2x+4\right)^{3/2}}+\frac{x+1}{15\left(x^2+2x+4\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 2*x + x^2)^(-7/2),x]
```

```
[Out] (1 + x)/(15*(4 + 2*x + x^2)^(5/2)) + (4*(1 + x))/(135*(4 + 2*x + x^2)^(3/2)
) + (8*(1 + x))/(405*Sqrt[4 + 2*x + x^2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(4+2x+x^2\right)^{7/2}}\,dx &= \frac{1+x}{15\left(4+2x+x^2\right)^{5/2}}+\frac{4}{15}\int \frac{1}{\left(4+2x+x^2\right)^{5/2}}\,dx\\
&= \frac{1+x}{15\left(4+2x+x^2\right)^{5/2}}+\frac{4(1+x)}{135\left(4+2x+x^2\right)^{3/2}}+\frac{8}{135}\int \frac{1}{\left(4+2x+x^2\right)^{3/2}}\,dx\\
&= \frac{1+x}{15\left(4+2x+x^2\right)^{5/2}}+\frac{4(1+x)}{135\left(4+2x+x^2\right)^{3/2}}+\frac{8(1+x)}{405\sqrt{4+2x+x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0111127, size = 39, normalized size = 0.67

$$\frac{(x+1)\left(8x^4+32x^3+108x^2+152x+203\right)}{405\left(x^2+2x+4\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 2*x + x^2)^(-7/2),x]
```

```
[Out] ((1 + x)*(203 + 152*x + 108*x^2 + 32*x^3 + 8*x^4))/(405*(4 + 2*x + x^2)^(5/
2))
```

Maple [A] time = 0.003, size = 38, normalized size = 0.7

$$\frac{8\,x^5+40\,x^4+140\,x^3+260\,x^2+355\,x+203}{405}\left(x^2+2\,x+4\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+2*x+4)^(7/2),x)
```

```
[Out] 1/405*(8*x^5+40*x^4+140*x^3+260*x^2+355*x+203)/(x^2+2*x+4)^(5/2)
```

Maxima [A] time = 0.963592, size = 103, normalized size = 1.78

$$\frac{8\,x}{405\,\sqrt{x^2+2\,x+4}}+\frac{8}{405\,\sqrt{x^2+2\,x+4}}+\frac{4\,x}{135\left(x^2+2\,x+4\right)^{\frac{3}{2}}}+\frac{4}{135\left(x^2+2\,x+4\right)^{\frac{3}{2}}}+\frac{x}{15\left(x^2+2\,x+4\right)^{\frac{5}{2}}}+\frac{}{15\left(x^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/405*x/sqrt(x^2 + 2*x + 4) + 8/405/sqrt(x^2 + 2*x + 4) + 4/135*x/(x^2 + 2*
x + 4)^(3/2) + 4/135/(x^2 + 2*x + 4)^(3/2) + 1/15*x/(x^2 + 2*x + 4)^(5/2) +
 1/15/(x^2 + 2*x + 4)^(5/2)
```

Fricas [B] time = 1.78644, size = 262, normalized size = 4.52

$$\frac{8\,x^6+48\,x^5+192\,x^4+448\,x^3+768\,x^2+\left(8\,x^5+40\,x^4+140\,x^3+260\,x^2+355\,x+203\right)\sqrt{x^2+2\,x+4}+768\,x+512}{405\left(x^6+6\,x^5+24\,x^4+56\,x^3+96\,x^2+96\,x+64\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/405*(8*x^6 + 48*x^5 + 192*x^4 + 448*x^3 + 768*x^2 + (8*x^5 + 40*x^4 + 140
*x^3 + 260*x^2 + 355*x + 203)*sqrt(x^2 + 2*x + 4) + 768*x + 512)/(x^6 + 6*x
^5 + 24*x^4 + 56*x^3 + 96*x^2 + 96*x + 64)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2+2x+4)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+2*x+4)**(7/2),x)
```

```
[Out] Integral((x**2 + 2*x + 4)**(-7/2), x)
```

Giac [A] time = 1.09836, size = 45, normalized size = 0.78

$$\frac{(4\,((2\,(x+5)x+35)x+65)x+355)x+203}{405\,(x^2+2\,x+4)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)^(7/2),x, algorithm="giac")
```

```
[Out] 1/405*((4*((2*(x + 5)*x + 35)*x + 65)*x + 355)*x + 203)/(x^2 + 2*x + 4)^(5/
2)
```

3.285 $\int \frac{1}{\left(1+8x+3x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=47

$$\frac{2(3x+4)}{169\sqrt{3x^2+8x+1}}-\frac{3x+4}{39\left(3x^2+8x+1\right)^{3/2}}$$

```
[Out] -(4 + 3*x)/(39*(1 + 8*x + 3*x^2)^(3/2)) + (2*(4 + 3*x))/(169*Sqrt[1 + 8*x +
 3*x^2])
```

Rubi [A] time = 0.0072238, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {614, 613}

$$\frac{2(3x+4)}{169\sqrt{3x^2+8x+1}}-\frac{3x+4}{39\left(3x^2+8x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 8*x + 3*x^2)^(-5/2),x]
```

```
[Out] -(4 + 3*x)/(39*(1 + 8*x + 3*x^2)^(3/2)) + (2*(4 + 3*x))/(169*Sqrt[1 + 8*x +
 3*x^2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+8x+3x^2\right)^{5/2}}\,dx &= -\frac{4+3x}{39\left(1+8x+3x^2\right)^{3/2}}-\frac{2}{13}\int \frac{1}{\left(1+8x+3x^2\right)^{3/2}}\,dx\\ &= -\frac{4+3x}{39\left(1+8x+3x^2\right)^{3/2}}+\frac{2(4+3x)}{169\sqrt{1+8x+3x^2}}\end{aligned}$$

Mathematica [A] time = 0.0163809, size = 33, normalized size = 0.7

$$\frac{(3x+4)\left(18x^2+48x-7\right)}{507\left(3x^2+8x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 8*x + 3*x^2)^(-5/2),x]
```

```
[Out] ((4 + 3*x)*(-7 + 48*x + 18*x^2))/(507*(1 + 8*x + 3*x^2)^(3/2))
```

Maple [A] time = 0.002, size = 30, normalized size = 0.6

$$\frac{54 x^3 + 216 x^2 + 171 x - 28}{507} \left(3 x^2 + 8 x + 1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^2+8*x+1)^(5/2),x)
```

```
[Out] 1/507*(54*x^3+216*x^2+171*x-28)/(3*x^2+8*x+1)^(3/2)
```

Maxima [A] time = 0.961582, size = 80, normalized size = 1.7

$$\frac{6 x}{169 \sqrt{3 x^2 + 8 x + 1}} + \frac{8}{169 \sqrt{3 x^2 + 8 x + 1}} - \frac{x}{13 \left(3 x^2 + 8 x + 1\right)^{\frac{3}{2}}} - \frac{4}{39 \left(3 x^2 + 8 x + 1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+8*x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 6/169*x/sqrt(3*x^2 + 8*x + 1) + 8/169/sqrt(3*x^2 + 8*x + 1) - 1/13*x/(3*x^2
 + 8*x + 1)^(3/2) - 4/39/(3*x^2 + 8*x + 1)^(3/2)
```

Fricas [A] time = 1.64053, size = 197, normalized size = 4.19

$$-\frac{252 x^4 + 1344 x^3 + 1960 x^2 - \left(54 x^3 + 216 x^2 + 171 x - 28\right) \sqrt{3 x^2 + 8 x + 1} + 448 x + 28}{507 \left(9 x^4 + 48 x^3 + 70 x^2 + 16 x + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+8*x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/507*(252*x^4 + 1344*x^3 + 1960*x^2 - (54*x^3 + 216*x^2 + 171*x - 28)*sqr
t(3*x^2 + 8*x + 1) + 448*x + 28)/(9*x^4 + 48*x^3 + 70*x^2 + 16*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(3x^2 + 8x + 1\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**2+8*x+1)**(5/2),x)

[Out] Integral((3*x**2 + 8*x + 1)**(-5/2), x)
```

Giac [A] time = 1.07882, size = 36, normalized size = 0.77

$$\frac{9\,(6\,(x+4)x+19)x-28}{507\,\left(3\,x^2+8\,x+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^2+8*x+1)^(5/2),x, algorithm="giac")

[Out] 1/507*(9*(6*(x + 4)*x + 19)*x - 28)/(3*x^2 + 8*x + 1)^(3/2)
```

3.286 $\int \frac{1}{(5+4x-3x^2)^{5/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{2(2-3x)}{361\sqrt{-3x^2+4x+5}}-\frac{2-3x}{57\left(-3x^2+4x+5\right)^{3/2}}$$

```
[Out] -(2 - 3*x)/(57*(5 + 4*x - 3*x^2)^(3/2)) - (2*(2 - 3*x))/(361*Sqrt[5 + 4*x -
 3*x^2])
```

Rubi [A] time = 0.0076841, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {614, 613}

$$-\frac{2(2-3x)}{361\sqrt{-3x^2+4x+5}}-\frac{2-3x}{57\left(-3x^2+4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 4*x - 3*x^2)^(-5/2),x]
```

```
[Out] -(2 - 3*x)/(57*(5 + 4*x - 3*x^2)^(3/2)) - (2*(2 - 3*x))/(361*Sqrt[5 + 4*x -
 3*x^2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(5+4x-3x^2\right)^{5/2}}\,dx &= -\frac{2-3x}{57\left(5+4x-3x^2\right)^{3/2}}+\frac{2}{19}\int \frac{1}{\left(5+4x-3x^2\right)^{3/2}}\,dx\\ &= -\frac{2-3x}{57\left(5+4x-3x^2\right)^{3/2}}-\frac{2(2-3x)}{361\sqrt{5+4x-3x^2}}\end{aligned}$$

Mathematica [A] time = 0.011454, size = 33, normalized size = 0.7

$$-\frac{(3x-2)\left(18x^2-24x-49\right)}{1083\left(-3x^2+4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 4*x - 3*x^2)^(-5/2),x]
```

```
[Out] -((-2 + 3*x)*(-49 - 24*x + 18*x^2))/(1083*(5 + 4*x - 3*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 30, normalized size = 0.6

$$-\frac{54\,x^3-108\,x^2-99\,x+98}{1083}\left(-3\,x^2+4\,x+5\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3*x^2+4*x+5)^(5/2),x)
```

```
[Out] -1/1083*(54*x^3-108*x^2-99*x+98)/(-3*x^2+4*x+5)^(3/2)
```

Maxima [A] time = 0.95672, size = 80, normalized size = 1.7

$$\frac{6\,x}{361\,\sqrt{-3\,x^2+4\,x+5}}-\frac{4}{361\,\sqrt{-3\,x^2+4\,x+5}}+\frac{x}{19\left(-3\,x^2+4\,x+5\right)^{\frac{3}{2}}}-\frac{2}{57\left(-3\,x^2+4\,x+5\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+5)^(5/2),x, algorithm="maxima")
```

```
[Out] 6/361*x/sqrt(-3*x^2 + 4*x + 5) - 4/361/sqrt(-3*x^2 + 4*x + 5) + 1/19*x/(-3*
x^2 + 4*x + 5)^(3/2) - 2/57/(-3*x^2 + 4*x + 5)^(3/2)
```

Fricas [A] time = 1.84248, size = 136, normalized size = 2.89

$$-\frac{\left(54\,x^3-108\,x^2-99\,x+98\right)\sqrt{-3\,x^2+4\,x+5}}{1083\left(9\,x^4-24\,x^3-14\,x^2+40\,x+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+5)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/1083*(54*x^3 - 108*x^2 - 99*x + 98)*sqrt(-3*x^2 + 4*x + 5)/(9*x^4 - 24*x
^3 - 14*x^2 + 40*x + 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(-3x^2+4x+5\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x**2+4*x+5)**(5/2),x)
```

```
[Out] Integral((-3*x**2 + 4*x + 5)**(-5/2), x)
```

Giac [A] time = 1.0955, size = 53, normalized size = 1.13

$$-\frac{(9(6(x-2)x-11)x+98)\sqrt{-3x^2+4x+5}}{1083\left(3x^2-4x-5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3*x^2+4*x+5)^(5/2),x, algorithm="giac")
```

```
[Out] -1/1083*(9*(6*(x - 2)*x - 11)*x + 98)*sqrt(-3*x^2 + 4*x + 5)/(3*x^2 - 4*x -
 5)^2
```

3.287 $\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{\sqrt{x^2+2x+2}}{x+1}+\frac{1}{x+1}+\sinh^{-1}(x+1)$$

```
[Out] (1 + x)^(-1) - Sqrt[2 + 2*x + x^2]/(1 + x) + ArcSinh[1 + x]
```

Rubi [A] time = 0.0385381, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6742, 684, 619, 215}

$$-\frac{\sqrt{x^2+2x+2}}{x+1}+\frac{1}{x+1}+\sinh^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[2 + 2*x + x^2])^(-1),x]
```

```
[Out] (1 + x)^(-1) - Sqrt[2 + 2*x + x^2]/(1 + x) + ArcSinh[1 + x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 684

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[(b*p)/(d*e*(m + 1)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p - 1), x]
, x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e
, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !(IntegerQ[m/2]
&& LtQ[m + 2*p + 3, 0]) && IntegerQ[2*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1+\sqrt{2+2x+x^2}}\,dx &= \int \left(-\frac{1}{(1+x)^2}+\frac{\sqrt{2+2x+x^2}}{(1+x)^2}\right)dx \\
&= \frac{1}{1+x}+\int \frac{\sqrt{2+2x+x^2}}{(1+x)^2}\,dx \\
&= \frac{1}{1+x}-\frac{\sqrt{2+2x+x^2}}{1+x}+\int \frac{1}{\sqrt{2+2x+x^2}}\,dx \\
&= \frac{1}{1+x}-\frac{\sqrt{2+2x+x^2}}{1+x}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{4}}}\,dx,x,2+2x\right) \\
&= \frac{1}{1+x}-\frac{\sqrt{2+2x+x^2}}{1+x}+\sinh^{-1}(1+x)
\end{aligned}
$$

Mathematica [A] time = 0.0202459, size = 30, normalized size = 1.03

$$\frac{-\sqrt{x^2+2x+2}+(x+1)\sinh^{-1}(x+1)+1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[2 + 2*x + x^2])^(-1),x]
```

```
[Out] (1 - Sqrt[2 + 2*x + x^2] + (1 + x)*ArcSinh[1 + x])/(1 + x)
```

Maple [A] time = 0.007, size = 40, normalized size = 1.4

$$-\frac{1}{1+x}\left((1+x)^2+1\right)^{\frac{3}{2}}+(1+x)\sqrt{(1+x)^2+1}+\text{Arcsinh}\,(1+x)+(1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(x^2+2*x+2)^(1/2)),x)
```

```
[Out] -1/(1+x)*((1+x)^2+1)^(3/2)+(1+x)*((1+x)^2+1)^(1/2)+arcsinh(1+x)+1/(1+x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2\,x+2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^2+2*x+2)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 2*x + 2) + 1), x)
```

Fricas [A] time = 1.83724, size = 108, normalized size = 3.72

$$-\frac{(x+1)\log\left(-x+\sqrt{x^2+2x+2}-1\right)+x+\sqrt{x^2+2x+2}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^2+2*x+2)^(1/2)),x, algorithm="fricas")
```

```
[Out] -((x + 1)*log(-x + sqrt(x^2 + 2*x + 2) - 1) + x + sqrt(x^2 + 2*x + 2))/(x +
 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2x+2}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x**2+2*x+2)**(1/2)),x)
```

```
[Out] Integral(1/(sqrt(x**2 + 2*x + 2) + 1), x)
```

Giac [B] time = 1.08477, size = 81, normalized size = 2.79

$$\frac{2}{\left(x-\sqrt{x^2+2x+2}\right)^2+2x-2\sqrt{x^2+2x+2}}+\frac{1}{x+1}-\log\left(-x+\sqrt{x^2+2x+2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x^2+2*x+2)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/((x - sqrt(x^2 + 2*x + 2))^2 + 2*x - 2*sqrt(x^2 + 2*x + 2)) + 1/(x + 1) -
 log(-x + sqrt(x^2 + 2*x + 2) - 1)
```

3.288 $\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=45

$$\sqrt{x^2+x+1}+2\log\left(\sqrt{x^2+x+1}+x\right)-x-\frac{3}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] -x + Sqrt[1 + x + x^2] - (3*ArcSinh[(1 + 2*x)/Sqrt[3]])/2 + 2*Log[x + Sqrt[
1 + x + x^2]]
```

Rubi [A] time = 0.0324864, antiderivative size = 59, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2116, 893}

$$\frac{3}{2\left(2\left(\sqrt{x^2+x+1}+x\right)+1\right)}+2\log\left(\sqrt{x^2+x+1}+x\right)-\frac{3}{2}\log\left(2\left(\sqrt{x^2+x+1}+x\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sqrt[1 + x + x^2])^(-1),x]
```

```
[Out] 3/(2*(1 + 2*(x + Sqrt[1 + x + x^2]))) + 2*Log[x + Sqrt[1 + x + x^2]] - (3*L
og[1 + 2*(x + Sqrt[1 + x + x^2])])/2
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x+\sqrt{1+x+x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1+x+x^2}{x(1+2x)^2}\,dx, x, x+\sqrt{1+x+x^2}\right)\\
&= 2\,\text{Subst}\left(\int \left(\frac{1}{x}-\frac{3}{2(1+2x)^2}-\frac{3}{2(1+2x)}\right)dx, x, x+\sqrt{1+x+x^2}\right)\\
&= \frac{3}{2\left(1+2\left(x+\sqrt{1+x+x^2}\right)\right)}+2\log\left(x+\sqrt{1+x+x^2}\right)-\frac{3}{2}\log\left(1+2\left(x+\sqrt{1+x+x^2}\right)\right)
\end{aligned}$$

Mathematica [A] time = 0.0296762, size = 59, normalized size = 1.31

$$\frac{3}{2\left(2\left(\sqrt{x^2+x+1}+x\right)+1\right)}+2\log\left(\sqrt{x^2+x+1}+x\right)-\frac{3}{2}\log\left(2\left(\sqrt{x^2+x+1}+x\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[1 + x + x^2])^(-1),x]
```

```
[Out] 3/(2*(1 + 2*(x + Sqrt[1 + x + x^2]))) + 2*Log[x + Sqrt[1 + x + x^2]] - (3*L
og[1 + 2*(x + Sqrt[1 + x + x^2])])/2
```

Maple [A] time = 0.005, size = 52, normalized size = 1.2

$$\sqrt{(1+x)^2-x}-\frac{1}{2}\mathrm{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)-\mathrm{Artanh}\left(\frac{1-x}{2}\frac{1}{\sqrt{(1+x)^2-x}}\right)-x+\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(x^2+x+1)^(1/2)),x)
```

```
[Out] ((1+x)^2-x)^(1/2)-1/2*arcsinh(2/3*3^(1/2)*(x+1/2))-arctanh(1/2*(1-x)/((1+x)
^2-x)^(1/2))-x+ln(1+x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x+\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2+x+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(x + sqrt(x^2 + x + 1)), x)
```

Fricas [A] time = 1.72745, size = 193, normalized size = 4.29

$$-x+\sqrt{x^2+x+1}+\log(x+1)-\log\left(-x+\sqrt{x^2+x+1}\right)+\log\left(-x+\sqrt{x^2+x+1}-2\right)+\frac{1}{2}\log\left(-2x+2\sqrt{x^2+x+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2+x+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -x + sqrt(x^2 + x + 1) + log(x + 1) - log(-x + sqrt(x^2 + x + 1)) + log(-x
+ sqrt(x^2 + x + 1) - 2) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x+\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x**2+x+1)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(x**2 + x + 1)), x)
```

Giac [A] time = 1.09049, size = 89, normalized size = 1.98

$$-x+\sqrt{x^2+x+1}+\frac{1}{2}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)+\log\left(|x+1|\right)-\log\left(\left|-x+\sqrt{x^2+x+1}\right|\right)+\log\left(\left|-x+\sqrt{x^2+x+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(x^2+x+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -x + sqrt(x^2 + x + 1) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) + log(abs(
x + 1)) - log(abs(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x + 1)
 - 2))
```

3.289 $\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx$

Optimal. Leaf size=79

$$-\frac{x^4}{6}-\frac{x^3}{9}+\frac{1}{6}\left(x^2+x+1\right)^{3/2}x-\frac{5}{36}\left(x^2+x+1\right)^{3/2}+\frac{1}{96}(2x+1)\sqrt{x^2+x+1}+\frac{1}{64}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] -x^3/9 - x^4/6 + ((1 + 2*x)*Sqrt[1 + x + x^2])/96 - (5*(1 + x + x^2)^(3/2))
/36 + (x*(1 + x + x^2)^(3/2))/6 + ArcSinh[(1 + 2*x)/Sqrt[3]]/64
```

Rubi [A] time = 0.135887, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {6742, 742, 640, 612, 619, 215}

$$-\frac{x^4}{6}-\frac{x^3}{9}+\frac{1}{6}\left(x^2+x+1\right)^{3/2}x-\frac{5}{36}\left(x^2+x+1\right)^{3/2}+\frac{1}{96}(2x+1)\sqrt{x^2+x+1}+\frac{1}{64}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + 2*x + 2*Sqrt[1 + x + x^2]),x]
```

```
[Out] -x^3/9 - x^4/6 + ((1 + 2*x)*Sqrt[1 + x + x^2])/96 - (5*(1 + x + x^2)^(3/2))
/36 + (x*(1 + x + x^2)^(3/2))/6 + ArcSinh[(1 + 2*x)/Sqrt[3]]/64
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 742

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m +
 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(
a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[Rat
ionalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuad
raticQ[a, b, c, d, e, m, p, x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{1+2x+2\sqrt{1+x+x^2}}\,dx &= \int \left(-\frac{x^2}{3}-\frac{2x^3}{3}+\frac{2}{3}x^2\sqrt{1+x+x^2}\right)dx \\
&= -\frac{x^3}{9}-\frac{x^4}{6}+\frac{2}{3}\int x^2\sqrt{1+x+x^2}\,dx \\
&= -\frac{x^3}{9}-\frac{x^4}{6}+\frac{1}{6}x\left(1+x+x^2\right)^{3/2}+\frac{1}{6}\int\left(-1-\frac{5x}{2}\right)\sqrt{1+x+x^2}\,dx \\
&= -\frac{x^3}{9}-\frac{x^4}{6}-\frac{5}{36}\left(1+x+x^2\right)^{3/2}+\frac{1}{6}x\left(1+x+x^2\right)^{3/2}+\frac{1}{24}\int\sqrt{1+x+x^2}\,dx \\
&= -\frac{x^3}{9}-\frac{x^4}{6}+\frac{1}{96}(1+2x)\sqrt{1+x+x^2}-\frac{5}{36}\left(1+x+x^2\right)^{3/2}+\frac{1}{6}x\left(1+x+x^2\right)^{3/2}+\frac{1}{64}\int \frac{}{\sqrt{1}} \\
&= -\frac{x^3}{9}-\frac{x^4}{6}+\frac{1}{96}(1+2x)\sqrt{1+x+x^2}-\frac{5}{36}\left(1+x+x^2\right)^{3/2}+\frac{1}{6}x\left(1+x+x^2\right)^{3/2}+\frac{\text{Subst}\left(\int\right.}{} \\
&= -\frac{x^3}{9}-\frac{x^4}{6}+\frac{1}{96}(1+2x)\sqrt{1+x+x^2}-\frac{5}{36}\left(1+x+x^2\right)^{3/2}+\frac{1}{6}x\left(1+x+x^2\right)^{3/2}+\frac{1}{64}\sinh^{-}
\end{aligned}$$

Mathematica [A] time = 0.0742065, size = 71, normalized size = 0.9

$$\frac{1}{576}\left(-96x^4-64x^3+96\left(x^2+x+1\right)^{3/2}x-80\left(x^2+x+1\right)^{3/2}+6(2x+1)\sqrt{x^2+x+1}+9\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + 2*x + 2*Sqrt[1 + x + x^2]),x]
```

```
[Out] (-64*x^3 - 96*x^4 + 6*(1 + 2*x)*Sqrt[1 + x + x^2] - 80*(1 + x + x^2)^(3/2)
+ 96*x*(1 + x + x^2)^(3/2) + 9*ArcSinh[(1 + 2*x)/Sqrt[3]])/576
```

Maple [A] time = 0.003, size = 59, normalized size = 0.8

$$-\frac{x^3}{9}-\frac{x^4}{6}+\frac{x}{6}\left(x^2+x+1\right)^{\frac{3}{2}}-\frac{5}{36}\left(x^2+x+1\right)^{\frac{3}{2}}+\frac{1+2x}{96}\sqrt{x^2+x+1}+\frac{1}{64}\text{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x)
```

```
[Out] -1/9*x^3-1/6*x^4+1/6*x*(x^2+x+1)^(3/2)-5/36*(x^2+x+1)^(3/2)+1/96*(1+2*x)*(x
^2+x+1)^(1/2)+1/64*arcsinh(2/3*3^(1/2)*(x+1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{2\,x+2\,\sqrt{x^2+x+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(2*x + 2*sqrt(x^2 + x + 1) + 1), x)
```

Fricas [A] time = 1.86522, size = 159, normalized size = 2.01

$$-\frac{1}{6}\,x^4-\frac{1}{9}\,x^3+\frac{1}{288}\,\left(48\,x^3+8\,x^2+14\,x-37\right)\sqrt{x^2+x+1}-\frac{1}{64}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/6*x^4 - 1/9*x^3 + 1/288*(48*x^3 + 8*x^2 + 14*x - 37)*sqrt(x^2 + x + 1) -
 1/64*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{2x+2\sqrt{x^2+x+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+2*x+2*(x**2+x+1)**(1/2)),x)
```

```
[Out] Integral(x**2/(2*x + 2*sqrt(x**2 + x + 1) + 1), x)
```

Giac [A] time = 1.06957, size = 73, normalized size = 0.92

$$-\frac{1}{6}\,x^4-\frac{1}{9}\,x^3+\frac{1}{288}\,(2\,(4\,(6\,x+1)x+7)x-37)\sqrt{x^2+x+1}-\frac{1}{64}\,\log\left(-2\,x+2\,\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+2*x+2*(x^2+x+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -1/6*x^4 - 1/9*x^3 + 1/288*(2*(4*(6*x + 1)*x + 7)*x - 37)*sqrt(x^2 + x + 1)
 - 1/64*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.290 $\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}} \, dx$

Optimal. Leaf size=80

$$-3\sqrt{x^2+x+1}+4\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)-\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)+x+\log(x)-4\log(x+1)+\frac{5}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] x - 3*Sqrt[1 + x + x^2] + (5*ArcSinh[(1 + 2*x)/Sqrt[3]])/2 + 4*ArcTanh[(1 -
 x)/(2*Sqrt[1 + x + x^2])] - ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])] + Log[x
] - 4*Log[1 + x]
```

Rubi [A] time = 0.343471, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 26, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {6742, 734, 843, 619, 215, 724, 206, 6740}

$$-3\sqrt{x^2+x+1}+4\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)-\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)+x+\log(x)-4\log(x+1)+\frac{5}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-3*x + Sqrt[1 + x + x^2])/(-1 + Sqrt[1 + x + x^2]),x]
```


```
[Out] x - 3*Sqrt[1 + x + x^2] + (5*ArcSinh[(1 + 2*x)/Sqrt[3]])/2 + 4*ArcTanh[(1 -
 x)/(2*Sqrt[1 + x + x^2])] - ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])] + Log[x
] - 4*Log[1 + x]
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 6740

```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Rubi steps

$$\int \frac{-3x+\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx = \int \left(-\frac{3x}{-1+\sqrt{1+x+x^2}}+\frac{\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\right)dx$$

$$= -\left(3\int \frac{x}{-1+\sqrt{1+x+x^2}}\,dx\right)+\int \frac{\sqrt{1+x+x^2}}{-1+\sqrt{1+x+x^2}}\,dx$$

$$= -\left(3\int \left(\frac{1}{1+x}+\frac{\sqrt{1+x+x^2}}{1+x}\right)dx\right)+\int \left(1+\frac{1}{-1+\sqrt{1+x+x^2}}\right)dx$$

$$= x-3\log(1+x)-3\int \frac{\sqrt{1+x+x^2}}{1+x}\,dx+\int \frac{1}{-1+\sqrt{1+x+x^2}}\,dx$$

$$= x-3\sqrt{1+x+x^2}-3\log(1+x)+\frac{3}{2}\int \frac{-1+x}{(1+x)\sqrt{1+x+x^2}}\,dx+\int \left(\frac{1}{-1-x}+\frac{1}{x}+\frac{\sqrt{1+x}}{x}\right.$$

$$= x-3\sqrt{1+x+x^2}+\log(x)-4\log(1+x)+\frac{3}{2}\int \frac{1}{\sqrt{1+x+x^2}}\,dx-3\int \frac{1}{(1+x)\sqrt{1+x+x}}$$

$$= x-3\sqrt{1+x+x^2}+\log(x)-4\log(1+x)-\frac{1}{2}\int \frac{-2-x}{x\sqrt{1+x+x^2}}\,dx+\frac{1}{2}\int \frac{-1+x}{(1+x)\sqrt{1+x+}}$$

$$= x-3\sqrt{1+x+x^2}+\frac{3}{2}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+3\tanh^{-1}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right)+\log(x)-4\log(1+x$$

$$= x-3\sqrt{1+x+x^2}+\frac{3}{2}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+3\tanh^{-1}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right)+\log(x)-4\log(1+x$$

$$= x-3\sqrt{1+x+x^2}+\frac{5}{2}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)+4\tanh^{-1}\left(\frac{1-x}{2\sqrt{1+x+x^2}}\right)-\tanh^{-1}\left(\frac{2+x}{2\sqrt{1+x+}}\right.$$

Mathematica [A] time = 0.110335, size = 80, normalized size = 1.

$$-3\sqrt{x^2+x+1}+4\tanh^{-1}\left(\frac{1-x}{2\sqrt{x^2+x+1}}\right)-\tanh^{-1}\left(\frac{x+2}{2\sqrt{x^2+x+1}}\right)+x+\log(x)-4\log(x+1)+\frac{5}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3*x + Sqrt[1 + x + x^2])/(-1 + Sqrt[1 + x + x^2]),x]
```

```
[Out] x - 3*Sqrt[1 + x + x^2] + (5*ArcSinh[(1 + 2*x)/Sqrt[3]])/2 + 4*ArcTanh[(1 -
 x)/(2*Sqrt[1 + x + x^2])] - ArcTanh[(2 + x)/(2*Sqrt[1 + x + x^2])] + Log[x
] - 4*Log[1 + x]
```

Maple [A] time = 0.01, size = 80, normalized size = 1.

$$x-4\ln(1+x)+\ln(x)+\sqrt{x^2+x+1}+\frac{5}{2}\mathrm{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)-\mathrm{Artanh}\left(\frac{2+x}{2}\frac{1}{\sqrt{x^2+x+1}}\right)-4\sqrt{(1+x)^2-x+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x)
```

```
[Out] x-4*ln(1+x)+ln(x)+(x^2+x+1)^(1/2)+5/2*arcsinh(2/3*3^(1/2)*(x+1/2))-arctanh(
1/2*(2+x)/(x^2+x+1)^(1/2))-4*((1+x)^2-x)^(1/2)+4*arctanh(1/2*(1-x)/((1+x)^2
-x)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3}{4}x^2+\frac{1}{2}x+\int-\frac{3x^3+2x^2-x}{2\left(x^2+x-2\sqrt{x^2+x+1}+2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x, algorithm="maxima"
)
```

```
[Out] 3/4*x^2 + 1/2*x + integrate(-1/2*(3*x^3 + 2*x^2 - x)/(x^2 + x - 2*sqrt(x^2
+ x + 1) + 2), x)
```

Fricas [A] time = 1.93481, size = 306, normalized size = 3.82

$$x-3\sqrt{x^2+x+1}-4\log(x+1)+\log(x)-\log\left(-x+\sqrt{x^2+x+1}+1\right)+4\log\left(-x+\sqrt{x^2+x+1}\right)+\log\left(-x+\sqrt{x^2+\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x, algorithm="fricas"
)
```

```
[Out] x - 3*sqrt(x^2 + x + 1) - 4*log(x + 1) + log(x) - log(-x + sqrt(x^2 + x + 1
) + 1) + 4*log(-x + sqrt(x^2 + x + 1)) + log(-x + sqrt(x^2 + x + 1) - 1) -
4*log(-x + sqrt(x^2 + x + 1) - 2) - 5/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{3x}{\sqrt{x^2+x+1}-1}\,dx-\int -\frac{\sqrt{x^2+x+1}}{\sqrt{x^2+x+1}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x+(x**2+x+1)**(1/2))/(-1+(x**2+x+1)**(1/2)),x)
```

```
[Out] -Integral(3*x/(sqrt(x**2 + x + 1) - 1), x) - Integral(-sqrt(x**2 + x + 1)/(
sqrt(x**2 + x + 1) - 1), x)
```

Giac [A] time = 1.14201, size = 142, normalized size = 1.78

$$x-3\sqrt{x^2+x+1}-\frac{5}{2}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)-4\log\left(|x+1|\right)+\log\left(|x|\right)-\log\left(\left|-x+\sqrt{x^2+x+1}+1\right|\right)+4\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x+(x^2+x+1)^(1/2))/(-1+(x^2+x+1)^(1/2)),x, algorithm="giac")
```

```
[Out] x - 3*sqrt(x^2 + x + 1) - 5/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1) - 4*log(a
bs(x + 1)) + log(abs(x)) - log(abs(-x + sqrt(x^2 + x + 1) + 1)) + 4*log(abs
(-x + sqrt(x^2 + x + 1))) + log(abs(-x + sqrt(x^2 + x + 1) - 1)) - 4*log(ab
s(-x + sqrt(x^2 + x + 1) - 2))
```

3.291 $\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx$

Optimal. Leaf size=158

$$\frac{1}{2}\sqrt{x^2+2x+4}(x+1)+\frac{1}{4}(2x+1)\sqrt{x^2+x+1}-2\sqrt{x^2+x+1}-2\sqrt{x^2+2x+4}-2\sqrt{7}\tanh^{-1}\left(\frac{5x+1}{2\sqrt{7}\sqrt{x^2+x+1}}\right)+2\sqrt{7}$$

```
[Out] -2*Sqrt[1 + x + x^2] + ((1 + 2*x)*Sqrt[1 + x + x^2])/4 - 2*Sqrt[4 + 2*x + x
^2] + ((1 + x)*Sqrt[4 + 2*x + x^2])/2 + (11*ArcSinh[(1 + x)/Sqrt[3]])/2 + (
43*ArcSinh[(1 + 2*x)/Sqrt[3]])/8 - 2*Sqrt[7]*ArcTanh[(1 + 5*x)/(2*Sqrt[7]*S
qrt[1 + x + x^2])] + 2*Sqrt[7]*ArcTanh[(1 - 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^
2])]
```

Rubi [A] time = 0.525733, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 36, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {6742, 734, 843, 619, 215, 724, 206, 612}

$$\frac{1}{2}\sqrt{x^2+2x+4}(x+1)+\frac{1}{4}(2x+1)\sqrt{x^2+x+1}-2\sqrt{x^2+x+1}-2\sqrt{x^2+2x+4}-2\sqrt{7}\tanh^{-1}\left(\frac{5x+1}{2\sqrt{7}\sqrt{x^2+x+1}}\right)+2\sqrt{7}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(-Sqrt[1 + x + x^2] + Sqrt[4 + 2*x + x^2]),x]
```


```
[Out] -2*Sqrt[1 + x + x^2] + ((1 + 2*x)*Sqrt[1 + x + x^2])/4 - 2*Sqrt[4 + 2*x + x
^2] + ((1 + x)*Sqrt[4 + 2*x + x^2])/2 + (11*ArcSinh[(1 + x)/Sqrt[3]])/2 + (
43*ArcSinh[(1 + 2*x)/Sqrt[3]])/8 - 2*Sqrt[7]*ArcTanh[(1 + 5*x)/(2*Sqrt[7]*S
qrt[1 + x + x^2])] + 2*Sqrt[7]*ArcTanh[(1 - 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^
2])]
```


Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rubi steps

$$\int \frac{1+x}{-\sqrt{1+x+x^2}+\sqrt{4+2x+x^2}}\,dx = \int \left(-\frac{1}{\sqrt{1+x+x^2}-\sqrt{4+2x+x^2}}-\frac{x}{\sqrt{1+x+x^2}-\sqrt{4+2x+x^2}}\right) dx$$

$$= -\int \frac{1}{\sqrt{1+x+x^2}-\sqrt{4+2x+x^2}}\,dx - \int \frac{x}{\sqrt{1+x+x^2}-\sqrt{4+2x+x^2}}\,dx$$

$$= -\int \left(-\frac{\sqrt{1+x+x^2}}{3+x}-\frac{\sqrt{4+2x+x^2}}{3+x}\right) dx - \int \left(-\sqrt{1+x+x^2}+\frac{3\sqrt{1+x+x^2}}{3+x}\right.$$

$$= -\left(3\int \frac{\sqrt{1+x+x^2}}{3+x}\,dx\right) - 3\int \frac{\sqrt{4+2x+x^2}}{3+x}\,dx + \int \sqrt{1+x+x^2}\,dx + \int \frac{\sqrt{1}}{}$$

$$= -2\sqrt{1+x+x^2}+\frac{1}{4}(1+2x)\sqrt{1+x+x^2}-2\sqrt{4+2x+x^2}+\frac{1}{2}(1+x)\sqrt{4+2x+}$$

$$= -2\sqrt{1+x+x^2}+\frac{1}{4}(1+2x)\sqrt{1+x+x^2}-2\sqrt{4+2x+x^2}+\frac{1}{2}(1+x)\sqrt{4+2x+}$$

$$= -2\sqrt{1+x+x^2}+\frac{1}{4}(1+2x)\sqrt{1+x+x^2}-2\sqrt{4+2x+x^2}+\frac{1}{2}(1+x)\sqrt{4+2x+}$$

$$= -2\sqrt{1+x+x^2}+\frac{1}{4}(1+2x)\sqrt{1+x+x^2}-2\sqrt{4+2x+x^2}+\frac{1}{2}(1+x)\sqrt{4+2x+}$$

Mathematica [A] time = 0.277175, size = 151, normalized size = 0.96

$$\frac{1}{8}\left(2\left(2\sqrt{x^2+x+1}x+2\sqrt{x^2+2x+4}x-7\sqrt{x^2+x+1}-6\sqrt{x^2+2x+4}-8\sqrt{7}\tanh^{-1}\left(\frac{5x+1}{2\sqrt{7}\sqrt{x^2+x+1}}\right)+8\sqrt{7}\tan\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(-Sqrt[1 + x + x^2] + Sqrt[4 + 2*x + x^2]),x]
```

```
[Out] (44*ArcSinh[(1 + x)/Sqrt[3]] + 43*ArcSinh[(1 + 2*x)/Sqrt[3]] + 2*(-7*Sqrt[1
 + x + x^2] + 2*x*Sqrt[1 + x + x^2] - 6*Sqrt[4 + 2*x + x^2] + 2*x*Sqrt[4 +
2*x + x^2] - 8*Sqrt[7]*ArcTanh[(1 + 5*x)/(2*Sqrt[7]*Sqrt[1 + x + x^2])] + 8
*Sqrt[7]*ArcTanh[(1 - 2*x)/(Sqrt[7]*Sqrt[4 + 2*x + x^2])]))/8
```

Maple [A] time = 0.012, size = 140, normalized size = 0.9

$$-2\sqrt{(3+x)^2-5x-8}+\frac{43}{8}\mathrm{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)+2\sqrt{7}\mathrm{Artanh}\left(1/14\,\frac{(-1-5x)\sqrt{7}}{\sqrt{(3+x)^2-5x-8}}\right)-2\sqrt{(3+x)^2-4x-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x)
```

```
[Out] -2*((3+x)^2-5*x-8)^(1/2)+43/8*arcsinh(2/3*3^(1/2)*(x+1/2))+2*7^(1/2)*arctan
h(1/14*(-1-5*x)*7^(1/2)/((3+x)^2-5*x-8)^(1/2))-2*((3+x)^2-4*x-5)^(1/2)+11/2
*arcsinh(1/3*(1+x)*3^(1/2))+2*7^(1/2)*arctanh(1/14*(2-4*x)*7^(1/2)/((3+x)^2
-4*x-5)^(1/2))+1/4*(1+2*x)*(x^2+x+1)^(1/2)+1/4*(2*x+2)*(x^2+2*x+4)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^2+2x+4}-\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate((x + 1)/(sqrt(x^2 + 2*x + 4) - sqrt(x^2 + x + 1)), x)
```

Fricas [A] time = 1.88251, size = 460, normalized size = 2.91

$$\frac{1}{4}\sqrt{x^2+x+1}(2x-7)+\frac{1}{2}\sqrt{x^2+2x+4}(x-3)+2\sqrt{7}\log\left(\frac{2\sqrt{7}(5x+1)+2\sqrt{x^2+x+1}\left(5\sqrt{7}-14\right)-25x-5}{x+3}\right)+2\sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^2 + x + 1)*(2*x - 7) + 1/2*sqrt(x^2 + 2*x + 4)*(x - 3) + 2*sqrt(
7)*log((2*sqrt(7)*(5*x + 1) + 2*sqrt(x^2 + x + 1)*(5*sqrt(7) - 14) - 25*x -
 5)/(x + 3)) + 2*sqrt(7)*log((sqrt(7)*(2*x - 1) + sqrt(x^2 + 2*x + 4)*(2*sq
rt(7) - 7) - 4*x + 2)/(x + 3)) - 11/2*log(-x + sqrt(x^2 + 2*x + 4) - 1) - 4
3/8*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{-\sqrt{x^2+x+1}+\sqrt{x^2+2x+4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-(x**2+x+1)**(1/2)+(x**2+2*x+4)**(1/2)),x)
```

```
[Out] Integral((x + 1)/(-sqrt(x**2 + x + 1) + sqrt(x**2 + 2*x + 4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^2+2x+4}-\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-(x^2+x+1)^(1/2)+(x^2+2*x+4)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^2 + 2*x + 4) - sqrt(x^2 + x + 1)), x)
```

3.292 $\int \frac{1}{\sqrt{-1+x}x^3} dx$

Optimal. Leaf size=41

$$\frac{\sqrt{x-1}}{2x^2}+\frac{3\sqrt{x-1}}{4x}+\frac{3}{4}\tan^{-1}\left(\sqrt{x-1}\right)$$

```
[Out] Sqrt[-1 + x]/(2*x^2) + (3*Sqrt[-1 + x])/(4*x) + (3*ArcTan[Sqrt[-1 + x]])/4
```

Rubi [A] time = 0.0085171, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {51, 63, 203}

$$\frac{\sqrt{x-1}}{2x^2}+\frac{3\sqrt{x-1}}{4x}+\frac{3}{4}\tan^{-1}\left(\sqrt{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-1 + x]*x^3),x]
```

```
[Out] Sqrt[-1 + x]/(2*x^2) + (3*Sqrt[-1 + x])/(4*x) + (3*ArcTan[Sqrt[-1 + x]])/4
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{-1+x}x^3}\,dx &= \frac{\sqrt{-1+x}}{2x^2} + \frac{3}{4}\int \frac{1}{\sqrt{-1+x}x^2}\,dx \\
&= \frac{\sqrt{-1+x}}{2x^2} + \frac{3\sqrt{-1+x}}{4x} + \frac{3}{8}\int \frac{1}{\sqrt{-1+x}x}\,dx \\
&= \frac{\sqrt{-1+x}}{2x^2} + \frac{3\sqrt{-1+x}}{4x} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\right) \\
&= \frac{\sqrt{-1+x}}{2x^2} + \frac{3\sqrt{-1+x}}{4x} + \frac{3}{4}\tan^{-1}\left(\sqrt{-1+x}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0039461, size = 22, normalized size = 0.54

$$2\sqrt{x-1}\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-1 + x]*x^3),x]
```

```
[Out] 2*Sqrt[-1 + x]*Hypergeometric2F1[1/2, 3, 3/2, 1 - x]
```

Maple [A] time = 0.007, size = 30, normalized size = 0.7

$$\frac{3}{4}\arctan\left(\sqrt{-1+x}\right) + \frac{1}{2x^2}\sqrt{-1+x} + \frac{3}{4x}\sqrt{-1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(-1+x)^(1/2),x)
```

```
[Out] 3/4*arctan((-1+x)^(1/2))+1/2*(-1+x)^(1/2)/x^2+3/4*(-1+x)^(1/2)/x
```

Maxima [A] time = 1.44212, size = 51, normalized size = 1.24

$$\frac{3\,(x-1)^{\frac{3}{2}} + 5\sqrt{x-1}}{4\left((x-1)^2 + 2x - 1\right)} + \frac{3}{4}\arctan\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*(x - 1)^(3/2) + 5*sqrt(x - 1))/((x - 1)^2 + 2*x - 1) + 3/4*arctan(sq
rt(x - 1))
```

Fricas [A] time = 1.89877, size = 82, normalized size = 2.

$$\frac{3x^2\arctan\left(\sqrt{x-1}\right) + (3x+2)\sqrt{x-1}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*x^2*arctan(sqrt(x - 1)) + (3*x + 2)*sqrt(x - 1))/x^2
```

Sympy [A] time = 2.67197, size = 131, normalized size = 3.2

$$\begin{cases} \frac{3i \operatorname{acosh}\left(\frac{1}{\sqrt{x}}\right)}{4} - \frac{3i}{4\sqrt{x}\sqrt{-1+\frac{1}{x}}} + \frac{i}{4x^{\frac{3}{2}}\sqrt{-1+\frac{1}{x}}} + \frac{i}{2x^{\frac{5}{2}}\sqrt{-1+\frac{1}{x}}} & \text{for}\ \frac{1}{|x|} > 1 \\ -\frac{3\operatorname{asin}\left(\frac{1}{\sqrt{x}}\right)}{4} + \frac{3}{4\sqrt{x}\sqrt{1-\frac{1}{x}}} - \frac{1}{4x^{\frac{3}{2}}\sqrt{1-\frac{1}{x}}} - \frac{1}{2x^{\frac{5}{2}}\sqrt{1-\frac{1}{x}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(-1+x)**(1/2),x)
```

```
[Out] Piecewise((3*I*acosh(1/sqrt(x))/4 - 3*I/(4*sqrt(x)*sqrt(-1 + 1/x)) + I/(4*x
**(3/2)*sqrt(-1 + 1/x)) + I/(2*x**(5/2)*sqrt(-1 + 1/x)), 1/Abs(x) > 1), (-3
*asin(1/sqrt(x))/4 + 3/(4*sqrt(x)*sqrt(1 - 1/x)) - 1/(4*x**(3/2)*sqrt(1 - 1
/x)) - 1/(2*x**(5/2)*sqrt(1 - 1/x)), True))
```

Giac [A] time = 1.07476, size = 39, normalized size = 0.95

$$\frac{3\,(x-1)^{\frac{3}{2}} + 5\,\sqrt{x-1}}{4\,x^2} + \frac{3}{4}\,\arctan\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(-1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*(x - 1)^(3/2) + 5*sqrt(x - 1))/x^2 + 3/4*arctan(sqrt(x - 1))
```

3.293 $\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx$

Optimal. Leaf size=13

$$-\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

```
[Out] -(1 - 3/x)^(-1/3)
```

Rubi [A] time = 0.0037591, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {261}

$$-\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 - 3/x)^(4/3)*x^2),x]
```

```
[Out] -(1 - 3/x)^(-1/3)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(1-\frac{3}{x}\right)^{4/3} x^2} \, dx = -\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

Mathematica [A] time = 0.0054422, size = 13, normalized size = 1.

$$-\frac{1}{\sqrt[3]{\frac{x-3}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 - 3/x)^(4/3)*x^2),x]
```

```
[Out] -((-3 + x)/x)^(-1/3)
```

Maple [A] time = 0.001, size = 18, normalized size = 1.4

$$-\frac{-3+x}{x}\left(\frac{-3+x}{x}\right)^{-\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-3/x)^(4/3)/x^2,x)
```

```
[Out] -(-3+x)/x/((-3+x)/x)^(4/3)
```

Maxima [A] time = 0.953112, size = 15, normalized size = 1.15

$$-\frac{1}{\left(-\frac{3}{x}+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3/x)^(4/3)/x^2,x, algorithm="maxima")
```

```
[Out] -1/(-3/x + 1)^(1/3)
```

Fricas [A] time = 1.78042, size = 41, normalized size = 3.15

$$-\frac{x\left(\frac{x-3}{x}\right)^{\frac{2}{3}}}{x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3/x)^(4/3)/x^2,x, algorithm="fricas")
```

```
[Out] -x*((x - 3)/x)^(2/3)/(x - 3)
```

Sympy [A] time = 0.916459, size = 10, normalized size = 0.77

$$-\frac{1}{\sqrt[3]{1-\frac{3}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3/x)**(4/3)/x**2,x)
```

```
[Out] -1/(1 - 3/x)**(1/3)
```

Giac [A] time = 1.06431, size = 15, normalized size = 1.15

$$-\frac{1}{\left(-\frac{3}{x}+1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-3/x)^(4/3)/x^2,x, algorithm="giac")
```

```
[Out] -1/(-3/x + 1)^(1/3)
```

3.294 $\int \frac{(-1+3x)^{4/3}}{x^2} \, dx$

Optimal. Leaf size=71

$$-\frac{(3x-1)^{4/3}}{x}+12\sqrt[3]{3x-1}+2\log(x)-6\log\left(\sqrt[3]{3x-1}+1\right)+4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{3x-1}}{\sqrt{3}}\right)$$

```
[Out] 12*(-1 + 3*x)^(1/3) - (-1 + 3*x)^(4/3)/x + 4*Sqrt[3]*ArcTan[(1 - 2*(-1 + 3*
x)^(1/3))/Sqrt[3]] + 2*Log[x] - 6*Log[1 + (-1 + 3*x)^(1/3)]
```

Rubi [A] time = 0.0275319, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {47, 50, 58, 618, 204, 31}

$$-\frac{(3x-1)^{4/3}}{x}+12\sqrt[3]{3x-1}+2\log(x)-6\log\left(\sqrt[3]{3x-1}+1\right)+4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{3x-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x)^(4/3)/x^2,x]
```

```
[Out] 12*(-1 + 3*x)^(1/3) - (-1 + 3*x)^(4/3)/x + 4*Sqrt[3]*ArcTan[(1 - 2*(-1 + 3*
x)^(1/3))/Sqrt[3]] + 2*Log[x] - 6*Log[1 + (-1 + 3*x)^(1/3)]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(-1+3x)^{4/3}}{x^2}\,dx &= -\frac{(-1+3x)^{4/3}}{x}+4\int \frac{\sqrt[3]{-1+3x}}{x}\,dx\\ &= 12\sqrt[3]{-1+3x}-\frac{(-1+3x)^{4/3}}{x}-4\int \frac{1}{x(-1+3x)^{2/3}}\,dx\\ &= 12\sqrt[3]{-1+3x}-\frac{(-1+3x)^{4/3}}{x}+2\log(x)-6\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,\sqrt[3]{-1+3x}\right)-6\,\text{Subst}\left(\int \frac{1}{1-x}\right.\\ &= 12\sqrt[3]{-1+3x}-\frac{(-1+3x)^{4/3}}{x}+2\log(x)-6\log\left(1+\sqrt[3]{-1+3x}\right)+12\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1\right.\\ &= 12\sqrt[3]{-1+3x}-\frac{(-1+3x)^{4/3}}{x}+4\sqrt{3}\tan^{-1}\left(\frac{1-2\sqrt[3]{-1+3x}}{\sqrt{3}}\right)+2\log(x)-6\log\left(1+\sqrt[3]{-1+3x}\right)\end{aligned}$$

Mathematica [C] time = 0.005079, size = 26, normalized size = 0.37

$$\frac{9}{7}(3x-1)^{7/3}\,{}_2F_1\left(2,\frac{7}{3};\frac{10}{3};1-3x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x)^(4/3)/x^2,x]
```

```
[Out] (9*(-1 + 3*x)^(7/3)*Hypergeometric2F1[2, 7/3, 10/3, 1 - 3*x])/7
```

Maple [A] time = 0.013, size = 109, normalized size = 1.5

$$9\sqrt[3]{3x-1}-\left(1+\sqrt[3]{3x-1}\right)^{-1}-4\ln\left(1+\sqrt[3]{3x-1}\right)+\left(1+\sqrt[3]{3x-1}\right)\left((3x-1)^{\frac{2}{3}}-\sqrt[3]{3x-1}+1\right)^{-1}+2\ln\left((3x-1)^{2/3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x-1)^(4/3)/x^2,x)
```

```
[Out] 9*(3*x-1)^(1/3)-1/(1+(3*x-1)^(1/3))-4*ln(1+(3*x-1)^(1/3))+(1+(3*x-1)^(1/3))
/((3*x-1)^(2/3)-(3*x-1)^(1/3)+1)+2*ln((3*x-1)^(2/3)-(3*x-1)^(1/3)+1)-4*3^(1
/2)*arctan(1/3*(2*(3*x-1)^(1/3)-1)*3^(1/2))
```

Maxima [A] time = 1.43274, size = 103, normalized size = 1.45

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(3x-1)^{\frac{1}{3}}-1\right)\right)+9(3x-1)^{\frac{1}{3}}+\frac{(3x-1)^{\frac{1}{3}}}{x}+2\log\left((3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1\right)-4\log\left((3x-1)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+3*x)^(4/3)/x^2,x, algorithm="maxima")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(3*x - 1)^(1/3) - 1)) + 9*(3*x - 1)^(1/3)
+ (3*x - 1)^(1/3)/x + 2*log((3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) - 4*log(
(3*x - 1)^(1/3) + 1)
```

Fricas [A] time = 1.77593, size = 238, normalized size = 3.35

$$-\frac{4\sqrt{3}x\arctan\left(\frac{2}{3}\sqrt{3}(3x-1)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-2x\log\left((3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1\right)+4x\log\left((3x-1)^{\frac{1}{3}}+1\right)-(9x+1)(3x-}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+3*x)^(4/3)/x^2,x, algorithm="fricas")
```

```
[Out] -(4*sqrt(3)*x*arctan(2/3*sqrt(3)*(3*x - 1)^(1/3) - 1/3*sqrt(3)) - 2*x*log((
3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) + 4*x*log((3*x - 1)^(1/3) + 1) - (9*x
 + 1)*(3*x - 1)^(1/3))/x
```

Sympy [C] time = 2.12979, size = 541, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+3*x)**(4/3)/x**2,x)
```

```
[Out] 189*3**(1/3)*(x - 1/3)**(4/3)*exp(I*pi/3)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/
3)*gamma(10/3) + 3*exp(I*pi/3)*gamma(10/3)) + 84*3**(1/3)*(x - 1/3)**(1/3)*
exp(I*pi/3)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)
*gamma(10/3)) + 84*(x - 1/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(I*pi/
3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*gam
ma(10/3)) - 84*(x - 1/3)*exp(I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_pol
ar(I*pi) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/
3)*gamma(10/3)) + 84*(x - 1/3)*exp(2*I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)
*exp_polar(5*I*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) +
 3*exp(I*pi/3)*gamma(10/3)) + 28*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(I
*pi/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)
*gamma(10/3)) - 28*exp(I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(I*p
i) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*gam
ma(10/3)) + 28*exp(2*I*pi/3)*log(-3**(1/3)*(x - 1/3)**(1/3)*exp_polar(5*I*p
i/3) + 1)*gamma(7/3)/(9*(x - 1/3)*exp(I*pi/3)*gamma(10/3) + 3*exp(I*pi/3)*g
amma(10/3))
```

Giac [A] time = 1.08422, size = 103, normalized size = 1.45

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2(3x-1)^{\frac{1}{3}}-1\right)\right)+9(3x-1)^{\frac{1}{3}}+\frac{(3x-1)^{\frac{1}{3}}}{x}+2\log\left((3x-1)^{\frac{2}{3}}-(3x-1)^{\frac{1}{3}}+1\right)-4\log\left((3x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+3*x)^(4/3)/x^2,x, algorithm="giac")
```

```
[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(3*x - 1)^(1/3) - 1)) + 9*(3*x - 1)^(1/3)
+ (3*x - 1)^(1/3)/x + 2*log((3*x - 1)^(2/3) - (3*x - 1)^(1/3) + 1) - 4*log(
(3*x - 1)^(1/3) + 1)
```

3.295 $\int (4-3x)^{4/3} x^2 \, dx$

Optimal. Leaf size=40

$$-\frac{1}{117}(4-3x)^{13/3}+\frac{4}{45}(4-3x)^{10/3}-\frac{16}{63}(4-3x)^{7/3}$$

```
[Out] (-16*(4 - 3*x)^(7/3))/63 + (4*(4 - 3*x)^(10/3))/45 - (4 - 3*x)^(13/3)/117
```

Rubi [A] time = 0.0072651, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$-\frac{1}{117}(4-3x)^{13/3}+\frac{4}{45}(4-3x)^{10/3}-\frac{16}{63}(4-3x)^{7/3}$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 3*x)^(4/3)*x^2,x]
```

```
[Out] (-16*(4 - 3*x)^(7/3))/63 + (4*(4 - 3*x)^(10/3))/45 - (4 - 3*x)^(13/3)/117
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (4-3x)^{4/3}x^2\,dx &= \int \left(\frac{16}{9}(4-3x)^{4/3}-\frac{8}{9}(4-3x)^{7/3}+\frac{1}{9}(4-3x)^{10/3}\right)dx\\ &= -\frac{16}{63}(4-3x)^{7/3}+\frac{4}{45}(4-3x)^{10/3}-\frac{1}{117}(4-3x)^{13/3}\end{aligned}$$

Mathematica [A] time = 0.012353, size = 23, normalized size = 0.57

$$-\frac{1}{455}(4-3x)^{7/3}\left(35x^2+28x+16\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 3*x)^(4/3)*x^2,x]
```

```
[Out] -((4 - 3*x)^(7/3)*(16 + 28*x + 35*x^2))/455
```

Maple [A] time = 0.003, size = 20, normalized size = 0.5

$$-\frac{35\,x^2+28\,x+16}{455}(4-3\,x)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4-3*x)^(4/3)*x^2,x)
```

```
[Out] -1/455*(35*x^2+28*x+16)*(4-3*x)^(7/3)
```

Maxima [A] time = 0.959427, size = 38, normalized size = 0.95

$$-\frac{1}{117}\,(-3\,x+4)^{\frac{13}{3}}+\frac{4}{45}\,(-3\,x+4)^{\frac{10}{3}}-\frac{16}{63}\,(-3\,x+4)^{\frac{7}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-3*x)^(4/3)*x^2,x, algorithm="maxima")
```

```
[Out] -1/117*(-3*x + 4)^(13/3) + 4/45*(-3*x + 4)^(10/3) - 16/63*(-3*x + 4)^(7/3)
```

Fricas [A] time = 1.76462, size = 90, normalized size = 2.25

$$-\frac{1}{455}\left(315\,x^4-588\,x^3+32\,x^2+64\,x+256\right)(-3\,x+4)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-3*x)^(4/3)*x^2,x, algorithm="fricas")
```

```
[Out] -1/455*(315*x^4 - 588*x^3 + 32*x^2 + 64*x + 256)*(-3*x + 4)^(1/3)
```

Sympy [B] time = 1.71752, size = 180, normalized size = 4.5

$$\begin{cases} -\frac{9x^4\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{13}+\frac{84x^3\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{65}-\frac{32x^2\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455}-\frac{64x\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455}-\frac{256\sqrt[3]{3x-4}e^{\frac{i\pi}{3}}}{455} & \text{for}\ \frac{3|x|}{4}>1 \\ -\frac{9x^4\sqrt[3]{4-3x}}{13}+\frac{84x^3\sqrt[3]{4-3x}}{65}-\frac{32x^2\sqrt[3]{4-3x}}{455}-\frac{64x\sqrt[3]{4-3x}}{455}-\frac{256\sqrt[3]{4-3x}}{455} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-3*x)**(4/3)*x**2,x)
```

```
[Out] Piecewise((-9*x**4*(3*x - 4)**(1/3)*exp(I*pi/3)/13 + 84*x**3*(3*x - 4)**(1/
3)*exp(I*pi/3)/65 - 32*x**2*(3*x - 4)**(1/3)*exp(I*pi/3)/455 - 64*x*(3*x -
4)**(1/3)*exp(I*pi/3)/455 - 256*(3*x - 4)**(1/3)*exp(I*pi/3)/455, 3*Abs(x)/
4 > 1), (-9*x**4*(4 - 3*x)**(1/3)/13 + 84*x**3*(4 - 3*x)**(1/3)/65 - 32*x**
2*(4 - 3*x)**(1/3)/455 - 64*x*(4 - 3*x)**(1/3)/455 - 256*(4 - 3*x)**(1/3)/4
55, True))
```

Giac [A] time = 1.07843, size = 66, normalized size = 1.65

$$-\frac{1}{117}\,(3\,x-4)^4(-3\,x+4)^{\frac{1}{3}}-\frac{4}{45}\,(3\,x-4)^3(-3\,x+4)^{\frac{1}{3}}-\frac{16}{63}\,(3\,x-4)^2(-3\,x+4)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-3*x)^(4/3)*x^2,x, algorithm="giac")
```

```
[Out] -1/117*(3*x - 4)^4*(-3*x + 4)^(1/3) - 4/45*(3*x - 4)^3*(-3*x + 4)^(1/3) - 1
6/63*(3*x - 4)^2*(-3*x + 4)^(1/3)
```

3.296 $\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx$

Optimal. Leaf size=48

$$4\left(1-2\sqrt[3]{x}\right)^{3/4}+6\tan^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)-6\tanh^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)$$

```
[Out] 4*(1 - 2*x^(1/3))^(3/4) + 6*ArcTan[(1 - 2*x^(1/3))^(1/4)] - 6*ArcTanh[(1 -
2*x^(1/3))^(1/4)]
```

Rubi [A] time = 0.0176575, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {266, 50, 63, 298, 203, 206}

$$4\left(1-2\sqrt[3]{x}\right)^{3/4}+6\tan^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)-6\tanh^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^(1/3))^(3/4)/x,x]
```

```
[Out] 4*(1 - 2*x^(1/3))^(3/4) + 6*ArcTan[(1 - 2*x^(1/3))^(1/4)] - 6*ArcTanh[(1 -
2*x^(1/3))^(1/4)]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1-2\sqrt[3]{x}\right)^{3/4}}{x}\,dx &= 3\,\text{Subst}\left(\int \frac{(1-2x)^{3/4}}{x}\,dx, x, \sqrt[3]{x}\right)\\
&= 4\left(1-2\sqrt[3]{x}\right)^{3/4} + 3\,\text{Subst}\left(\int \frac{1}{\sqrt[4]{1-2x}x}\,dx, x, \sqrt[3]{x}\right)\\
&= 4\left(1-2\sqrt[3]{x}\right)^{3/4} - 6\,\text{Subst}\left(\int \frac{x^2}{\frac{1}{2}-\frac{x^4}{2}}\,dx, x, \sqrt[4]{1-2\sqrt[3]{x}}\right)\\
&= 4\left(1-2\sqrt[3]{x}\right)^{3/4} - 6\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1-2\sqrt[3]{x}}\right) + 6\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{1-2\sqrt[3]{x}}\right)\\
&= 4\left(1-2\sqrt[3]{x}\right)^{3/4} + 6\tan^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right) - 6\tanh^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0123142, size = 48, normalized size = 1.

$$4\left(1-2\sqrt[3]{x}\right)^{3/4} + 6\tan^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right) - 6\tanh^{-1}\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^(1/3))^(3/4)/x,x]
```

```
[Out] 4*(1 - 2*x^(1/3))^(3/4) + 6*ArcTan[(1 - 2*x^(1/3))^(1/4)] - 6*ArcTanh[(1 -
2*x^(1/3))^(1/4)]
```

Maple [A] time = 0.007, size = 53, normalized size = 1.1

$$4\left(1-2\sqrt[3]{x}\right)^{3/4} + 3\ln\left(-1+\sqrt[4]{1-2\sqrt[3]{x}}\right) - 3\ln\left(1+\sqrt[4]{1-2\sqrt[3]{x}}\right) + 6\arctan\left(\sqrt[4]{1-2\sqrt[3]{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2*x^(1/3))^(3/4)/x,x)
```

```
[Out] 4*(1-2*x^(1/3))^(3/4)+3*ln(-1+(1-2*x^(1/3))^(1/4))-3*ln(1+(1-2*x^(1/3))^(1/
4))+6*arctan((1-2*x^(1/3))^(1/4))
```

Maxima [A] time = 1.46042, size = 70, normalized size = 1.46

$$4\left(-2x^{\frac{1}{3}}+1\right)^{\frac{3}{4}}+6\arctan\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}\right)-3\log\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}+1\right)+3\log\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x^(1/3))^(3/4)/x,x, algorithm="maxima")
```

```
[Out] 4*(-2*x^(1/3) + 1)^(3/4) + 6*arctan((-2*x^(1/3) + 1)^(1/4)) - 3*log((-2*x^(
1/3) + 1)^(1/4) + 1) + 3*log((-2*x^(1/3) + 1)^(1/4) - 1)
```

Fricas [A] time = 1.83267, size = 180, normalized size = 3.75

$$4\left(-2x^{\frac{1}{3}}+1\right)^{\frac{3}{4}}+6\arctan\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}\right)-3\log\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}+1\right)+3\log\left(\left(-2x^{\frac{1}{3}}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x^(1/3))^(3/4)/x,x, algorithm="fricas")
```

```
[Out] 4*(-2*x^(1/3) + 1)^(3/4) + 6*arctan((-2*x^(1/3) + 1)^(1/4)) - 3*log((-2*x^(
1/3) + 1)^(1/4) + 1) + 3*log((-2*x^(1/3) + 1)^(1/4) - 1)
```

Sympy [C] time = 3.48925, size = 51, normalized size = 1.06

$$-\frac{3\cdot 2^{\frac{3}{4}}\sqrt[4]{x}e^{\frac{3i\pi}{4}}\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{3}{4}\\ \frac{1}{4}\end{matrix}\middle|\frac{1}{2\sqrt[3]{x}}\right)}{\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x**(1/3))**(3/4)/x,x)
```

```
[Out] -3*2**(3/4)*x**(1/4)*exp(3*I*pi/4)*gamma(-3/4)*hyper((-3/4, -3/4), (1/4,),
1/(2*x**(1/3)))/gamma(1/4)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*x^(1/3))^(3/4)/x,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.297 $\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx$

Optimal. Leaf size=69

$$\frac{1}{26}\left(3-2\sqrt{x}\right)^{13/4}-\frac{1}{2}\left(3-2\sqrt{x}\right)^{9/4}+\frac{27}{10}\left(3-2\sqrt{x}\right)^{5/4}-\frac{27}{2}\sqrt[4]{3-2\sqrt{x}}$$

```
[Out] (-27*(3 - 2*Sqrt[x])^(1/4))/2 + (27*(3 - 2*Sqrt[x])^(5/4))/10 - (3 - 2*Sqrt
[x])^(9/4)/2 + (3 - 2*Sqrt[x])^(13/4)/26
```

Rubi [A] time = 0.0195488, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{26}\left(3-2\sqrt{x}\right)^{13/4}-\frac{1}{2}\left(3-2\sqrt{x}\right)^{9/4}+\frac{27}{10}\left(3-2\sqrt{x}\right)^{5/4}-\frac{27}{2}\sqrt[4]{3-2\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(3 - 2*Sqrt[x])^(3/4),x]
```

```
[Out] (-27*(3 - 2*Sqrt[x])^(1/4))/2 + (27*(3 - 2*Sqrt[x])^(5/4))/10 - (3 - 2*Sqrt
[x])^(9/4)/2 + (3 - 2*Sqrt[x])^(13/4)/26
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(3-2\sqrt{x}\right)^{3/4}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^3}{(3-2x)^{3/4}}\,dx,x,\sqrt{x}\right)\\
&= 2\,\text{Subst}\left(\int \left(\frac{27}{8(3-2x)^{3/4}}-\frac{27}{8}\sqrt[4]{3-2x}+\frac{9}{8}(3-2x)^{5/4}-\frac{1}{8}(3-2x)^{9/4}\right)dx,x,\sqrt{x}\right)\\
&= -\frac{27}{2}\sqrt[4]{3-2\sqrt{x}}+\frac{27}{10}\left(3-2\sqrt{x}\right)^{5/4}-\frac{1}{2}\left(3-2\sqrt{x}\right)^{9/4}+\frac{1}{26}\left(3-2\sqrt{x}\right)^{13/4}
\end{aligned}$$

Mathematica [A] time = 0.0127291, size = 36, normalized size = 0.52

$$-\frac{4}{65}\sqrt[4]{3-2\sqrt{x}}\left(5x^{3/2}+10x+24\sqrt{x}+144\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(3 - 2*Sqrt[x])^(3/4),x]
```

```
[Out] (-4*(3 - 2*Sqrt[x])^(1/4)*(144 + 24*Sqrt[x] + 10*x + 5*x^(3/2)))/65
```

Maple [A] time = 0.003, size = 46, normalized size = 0.7

$$-\frac{27}{2}\sqrt[4]{3-2\sqrt{x}}+\frac{27}{10}\left(3-2\sqrt{x}\right)^{\frac{5}{4}}-\frac{1}{2}\left(3-2\sqrt{x}\right)^{\frac{9}{4}}+\frac{1}{26}\left(3-2\sqrt{x}\right)^{\frac{13}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(3-2*x^(1/2))^(3/4),x)
```

```
[Out] -27/2*(3-2*x^(1/2))^(1/4)+27/10*(3-2*x^(1/2))^(5/4)-1/2*(3-2*x^(1/2))^(9/4)
+1/26*(3-2*x^(1/2))^(13/4)
```

Maxima [A] time = 0.954979, size = 61, normalized size = 0.88

$$\frac{1}{26}\left(-2\sqrt{x}+3\right)^{\frac{13}{4}}-\frac{1}{2}\left(-2\sqrt{x}+3\right)^{\frac{9}{4}}+\frac{27}{10}\left(-2\sqrt{x}+3\right)^{\frac{5}{4}}-\frac{27}{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3-2*x^(1/2))^(3/4),x, algorithm="maxima")
```

```
[Out] 1/26*(-2*sqrt(x) + 3)^(13/4) - 1/2*(-2*sqrt(x) + 3)^(9/4) + 27/10*(-2*sqrt(
x) + 3)^(5/4) - 27/2*(-2*sqrt(x) + 3)^(1/4)
```

Fricas [A] time = 1.84317, size = 86, normalized size = 1.25

$$-\frac{4}{65}\left((5x+24)\sqrt{x}+10x+144\right)\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3-2*x^(1/2))^(3/4),x, algorithm="fricas")
```

```
[Out] -4/65*((5*x + 24)*sqrt(x) + 10*x + 144)*(-2*sqrt(x) + 3)^(1/4)
```

Sympy [B] time = 2.05704, size = 3305, normalized size = 47.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3-2*x**(1/2))**(3/4),x)
```

```
[Out] Piecewise((1280*3**(1/4)*x**(25/2)*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-
37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(
```

```
17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9
+ 47385*3**(1/4)*x**8) + 26304*3**(1/4)*x**(23/2)*(2*sqrt(x) - 3)**(1/4)*ex
p(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 18954
0*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900
*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 200016*3**(1/4)*x**(21/2)*(2*sqrt(x
) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x
**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4
)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 331776*sqrt(3)*x**(
21/2)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1
/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/
4)*x**9 + 47385*3**(1/4)*x**8) - 2123820*3**(1/4)*x**(19/2)*(2*sqrt(x) - 3)
**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/
2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**1
0 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 2488320*sqrt(3)*x**(19/2)
/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x
**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x*
*9 + 47385*3**(1/4)*x**8) - 1609632*3**(1/4)*x**(17/2)*(2*sqrt(x) - 3)**(1/
4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) -
189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 3
15900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 1679616*sqrt(3)*x**(17/2)/(-37
440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17
/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 +
47385*3**(1/4)*x**8) - 8960*3**(1/4)*x**12*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*
pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1
/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/
4)*x**9 + 47385*3**(1/4)*x**8) - 18432*3**(1/4)*x**11*(2*sqrt(x) - 3)**(1/4
)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 1
89540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 31
5900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 36864*sqrt(3)*x**11/(-37440*3**
(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4
160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3
**(1/4)*x**8) + 965520*3**(1/4)*x**10*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)
/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x
**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x*
*9 + 47385*3**(1/4)*x**8) + 1244160*sqrt(3)*x**10/(-37440*3**(1/4)*x**(21/2
) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x
**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8)
+ 2548584*3**(1/4)*x**9*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1
/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 416
0*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**
(1/4)*x**8) + 2799360*sqrt(3)*x**9/(-37440*3**(1/4)*x**(21/2) - 280800*3**(
1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3
**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 419904*3**(1/
4)*x**8*(2*sqrt(x) - 3)**(1/4)*exp(-3*I*pi/4)/(-37440*3**(1/4)*x**(21/2) -
280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11
 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 41
9904*sqrt(3)*x**8/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) -
189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 3
15900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8), 2*Abs(sqrt(x))/3 > 1), (-1280*3
**(1/4)*x**(25/2)*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 28080
0*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 14
0400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 26304*3
**(1/4)*x**(23/2)*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 28080
0*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 14
0400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 200016*
3**(1/4)*x**(21/2)*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 2808
00*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 1
40400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 331776
*sqrt(3)*x**(21/2)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) -
```

```
 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 +
315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 2123820*3**(1/4)*x**(19/2)*(3
 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2)
 - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10
+ 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 2488320*sqrt(3)*x**(19/2)/(
-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**
(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9
 + 47385*3**(1/4)*x**8) + 1609632*3**(1/4)*x**(17/2)*(3 - 2*sqrt(x))**(1/4)
/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x
**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x*
*9 + 47385*3**(1/4)*x**8) - 1679616*sqrt(3)*x**(17/2)/(-37440*3**(1/4)*x**(
21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/
4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x*
*8) + 8960*3**(1/4)*x**12*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2)
 - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x*
*11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) +
 18432*3**(1/4)*x**11*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 2
80800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11
+ 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 368
64*sqrt(3)*x**11/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 1
89540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 31
5900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 965520*3**(1/4)*x**10*(3 - 2*sq
rt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 1895
40*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 31590
0*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 1244160*sqrt(3)*x**10/(-37440*3**(
1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 41
60*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3*
*(1/4)*x**8) - 2548584*3**(1/4)*x**9*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4
)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*
3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1
/4)*x**8) + 2799360*sqrt(3)*x**9/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/
4)*x**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**
(1/4)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) - 419904*3**(1/4)
*x**8*(3 - 2*sqrt(x))**(1/4)/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x
**(19/2) - 189540*3**(1/4)*x**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4
)*x**10 + 315900*3**(1/4)*x**9 + 47385*3**(1/4)*x**8) + 419904*sqrt(3)*x**8
/(-37440*3**(1/4)*x**(21/2) - 280800*3**(1/4)*x**(19/2) - 189540*3**(1/4)*x
**(17/2) + 4160*3**(1/4)*x**11 + 140400*3**(1/4)*x**10 + 315900*3**(1/4)*x*
*9 + 47385*3**(1/4)*x**8), True))
```

Giac [A] time = 1.07862, size = 85, normalized size = 1.23

$$-\frac{1}{26}\left(2\sqrt{x}-3\right)^{3}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}-\frac{1}{2}\left(2\sqrt{x}-3\right)^{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}+\frac{27}{10}\left(-2\sqrt{x}+3\right)^{\frac{5}{4}}-\frac{27}{2}\left(-2\sqrt{x}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(3-2*x^(1/2))^(3/4),x, algorithm="giac")
```

```
[Out] -1/26*(2*sqrt(x) - 3)^3*(-2*sqrt(x) + 3)^(1/4) - 1/2*(2*sqrt(x) - 3)^2*(-2*
sqrt(x) + 3)^(1/4) + 27/10*(-2*sqrt(x) + 3)^(5/4) - 27/2*(-2*sqrt(x) + 3)^(
1/4)
```

3.298 $\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx$

Optimal. Leaf size=193

$$-\frac{\left(2\sqrt{x}-1\right)^{5/4}}{x}-\frac{5\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{x}}-\frac{5\log\left(\sqrt{2\sqrt{x}-1}-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{4\sqrt{2}}+\frac{5\log\left(\sqrt{2\sqrt{x}-1}+\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{4\sqrt{2}}-\frac{5\tan^{-}}{}$$

```
[Out] -((-1 + 2*Sqrt[x])^(5/4)/x) - (5*(-1 + 2*Sqrt[x])^(1/4))/(2*Sqrt[x]) - (5*A
rcTan[1 - Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4)])/(2*Sqrt[2]) + (5*ArcTan[1 + Sqrt
[2]*(-1 + 2*Sqrt[x])^(1/4)])/(2*Sqrt[2]) - (5*Log[1 - Sqrt[2]*(-1 + 2*Sqrt[
x])^(1/4) + Sqrt[-1 + 2*Sqrt[x]]])/(4*Sqrt[2]) + (5*Log[1 + Sqrt[2]*(-1 + 2
*Sqrt[x])^(1/4) + Sqrt[-1 + 2*Sqrt[x]]])/(4*Sqrt[2])
```

Rubi [A] time = 0.113249, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {266, 47, 63, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\left(2\sqrt{x}-1\right)^{5/4}}{x}-\frac{5\sqrt[4]{2\sqrt{x}-1}}{2\sqrt{x}}-\frac{5\log\left(\sqrt{2\sqrt{x}-1}-\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{4\sqrt{2}}+\frac{5\log\left(\sqrt{2\sqrt{x}-1}+\sqrt{2}\sqrt[4]{2\sqrt{x}-1}+1\right)}{4\sqrt{2}}-\frac{5\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*Sqrt[x])^(5/4)/x^2,x]
```

```
[Out] -((-1 + 2*Sqrt[x])^(5/4)/x) - (5*(-1 + 2*Sqrt[x])^(1/4))/(2*Sqrt[x]) - (5*A
rcTan[1 - Sqrt[2]*(-1 + 2*Sqrt[x])^(1/4)])/(2*Sqrt[2]) + (5*ArcTan[1 + Sqrt
[2]*(-1 + 2*Sqrt[x])^(1/4)])/(2*Sqrt[2]) - (5*Log[1 - Sqrt[2]*(-1 + 2*Sqrt[
x])^(1/4) + Sqrt[-1 + 2*Sqrt[x]]])/(4*Sqrt[2]) + (5*Log[1 + Sqrt[2]*(-1 + 2
*Sqrt[x])^(1/4) + Sqrt[-1 + 2*Sqrt[x]]])/(4*Sqrt[2])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x^2}\,dx = 2\,\text{Subst}\left(\int \frac{(-1+2x)^{5/4}}{x^3}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x}+\frac{5}{2}\,\text{Subst}\left(\int \frac{\sqrt[4]{-1+2x}}{x^2}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x}-\frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}}+\frac{5}{4}\,\text{Subst}\left(\int \frac{1}{x(-1+2x)^{3/4}}\,dx,x,\sqrt{x}\right)$$
$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x}-\frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}}+\frac{5}{2}\,\text{Subst}\left(\int \frac{1}{\frac{1}{2}+\frac{x^4}{2}}\,dx,x,\sqrt[4]{-1+2\sqrt{x}}\right)$$
$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x}-\frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}}+\frac{5}{4}\,\text{Subst}\left(\int \frac{1-x^2}{\frac{1}{2}+\frac{x^4}{2}}\,dx,x,\sqrt[4]{-1+2\sqrt{x}}\right)+\frac{5}{4}\,\text{Subst}\left(\int \frac{1+x^2}{\frac{1}{2}+\frac{x^4}{2}}\right.$$
$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x}-\frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}}+\frac{5}{4}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx,x,\sqrt[4]{-1+2\sqrt{x}}\right)+\frac{5}{4}\,\text{Subst}\left(\int\right.$$
$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x}-\frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}}-\frac{5\log\left(1-\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}+\sqrt{-1+2\sqrt{x}}\right)}{4\sqrt{2}}+\frac{5\log\left(1+\sqrt{2}\sqrt[4]{-\right.}{}$$
$$= -\frac{\left(-1+2\sqrt{x}\right)^{5/4}}{x}-\frac{5\sqrt[4]{-1+2\sqrt{x}}}{2\sqrt{x}}-\frac{5\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}\right)}{2\sqrt{2}}+\frac{5\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}\right)}{2\sqrt{2}}$$

Mathematica [C] time = 0.0077126, size = 34, normalized size = 0.18

$$\frac{32}{9}\left(2\sqrt{x}-1\right)^{9/4}\,{}_2F_1\left(\frac{9}{4},3;\frac{13}{4};1-2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*Sqrt[x])^(5/4)/x^2,x]
```

```
[Out] (32*(-1 + 2*Sqrt[x])^(9/4)*Hypergeometric2F1[9/4, 3, 13/4, 1 - 2*Sqrt[x]])/
9
```

Maple [A] time = 0.013, size = 130, normalized size = 0.7

$$8\,\frac{1}{x}\left(-\frac{9\left(-1+2\sqrt{x}\right)^{5/4}}{32}-\frac{5\sqrt[4]{-1+2\sqrt{x}}}{32}\right)+\frac{5\sqrt{2}}{4}\arctan\left(1+\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}\right)+\frac{5\sqrt{2}}{4}\arctan\left(-1+\sqrt{2}\sqrt[4]{-1+2\sqrt{x}}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+2*x^(1/2))^(5/4)/x^2,x)
```

```
[Out] 8*(-9/32*(-1+2*x^(1/2))^(5/4)-5/32*(-1+2*x^(1/2))^(1/4))/x+5/4*arctan(1+2^(
1/2)*(-1+2*x^(1/2))^(1/4))*2^(1/2)+5/4*arctan(-1+2^(1/2)*(-1+2*x^(1/2))^(1/
4))*2^(1/2)+5/8*2^(1/2)*ln((1+2^(1/2)*(-1+2*x^(1/2))^(1/4)+(-1+2*x^(1/2))^(
```

```
1/2))/(1-2^(1/2)*(-1+2*x^(1/2))^(1/4)+(-1+2*x^(1/2))^(1/2)))
```

Maxima [A] time = 1.46602, size = 212, normalized size = 1.1

$$\frac{5}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right)+\frac{5}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right)+\frac{5}{8}\sqrt{2}\log\left(\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x^(1/2))^(5/4)/x^2,x, algorithm="maxima")
```

```
[Out] 5/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4))) + 5/4*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(2*sqrt(x) - 1)^(1/4))) + 5/8*sqrt(
2)*log(sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - 5/8*sqrt(
2)*log(-sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - (9*(2*sq
rt(x) - 1)^(5/4) + 5*(2*sqrt(x) - 1)^(1/4))/((2*sqrt(x) - 1)^2 + 4*sqrt(x)
- 1)
```

Fricas [A] time = 1.89518, size = 643, normalized size = 3.33

$$-\frac{20\sqrt{2}x\arctan\left(\sqrt{2}\sqrt{\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}+\sqrt{2\sqrt{x}-1}+1}-\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}-1\right)+20\sqrt{2}x\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{-4\sqrt{2}\left(2\sqrt{x}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x^(1/2))^(5/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(20*sqrt(2)*x*arctan(sqrt(2)*sqrt(sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt
(2*sqrt(x) - 1) + 1) - sqrt(2)*(2*sqrt(x) - 1)^(1/4) - 1) + 20*sqrt(2)*x*ar
ctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*(2*sqrt(x) - 1)^(1/4) + 4*sqrt(2*sqrt(x) -
 1) + 4) - sqrt(2)*(2*sqrt(x) - 1)^(1/4) + 1) - 5*sqrt(2)*x*log(4*sqrt(2)*(
2*sqrt(x) - 1)^(1/4) + 4*sqrt(2*sqrt(x) - 1) + 4) + 5*sqrt(2)*x*log(-4*sqrt
(2)*(2*sqrt(x) - 1)^(1/4) + 4*sqrt(2*sqrt(x) - 1) + 4) + 4*(9*sqrt(x) - 2)*
(2*sqrt(x) - 1)^(1/4))/x
```

Sympy [C] time = 19.9278, size = 44, normalized size = 0.23

$$-\frac{4\sqrt[4]{2}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{5}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{e^{2i\pi}}{2\sqrt{x}}\right)}{x^{\frac{3}{8}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x**(1/2))**(5/4)/x**2,x)
```

```
[Out] -4*2**(1/4)*gamma(3/4)*hyper((-5/4, 3/4), (7/4,), exp_polar(2*I*pi)/(2*sqrt
(x)))/(x**(3/8)*gamma(7/4))
```

Giac [A] time = 1.07839, size = 192, normalized size = 0.99

$$\frac{5}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right)+\frac{5}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\left(2\sqrt{x}-1\right)^{\frac{1}{4}}\right)\right)+\frac{5}{8}\sqrt{2}\log\left(\sqrt{2}\left(2\sqrt{x}-1\right)^{\frac{1}{4}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+2*x^(1/2))^(5/4)/x^2,x, algorithm="giac")
```

```
[Out] 5/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*(2*sqrt(x) - 1)^(1/4))) + 5/4*s
qrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*(2*sqrt(x) - 1)^(1/4))) + 5/8*sqrt(
2)*log(sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - 5/8*sqrt(
2)*log(-sqrt(2)*(2*sqrt(x) - 1)^(1/4) + sqrt(2*sqrt(x) - 1) + 1) - 1/4*(9*(
2*sqrt(x) - 1)^(5/4) + 5*(2*sqrt(x) - 1)^(1/4))/x
```

3.299 $\int x^6 \sqrt[3]{1+x^7}\,dx$

Optimal. Leaf size=13

$$\frac{3}{28}\left(x^7+1\right)^{4/3}$$

```
[Out] (3*(1 + x^7)^(4/3))/28
```

Rubi [A] time = 0.0027798, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{3}{28}\left(x^7+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(1 + x^7)^(1/3),x]
```

```
[Out] (3*(1 + x^7)^(4/3))/28
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^6 \sqrt[3]{1+x^7}\,dx=\frac{3}{28}\left(1+x^7\right)^{4/3}$$

Mathematica [A] time = 0.0029953, size = 13, normalized size = 1.

$$\frac{3}{28}\left(x^7+1\right)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(1 + x^7)^(1/3),x]
```

```
[Out] (3*(1 + x^7)^(4/3))/28
```

Maple [B] time = 0.006, size = 37, normalized size = 2.9

$$\frac{(3+3x)\left(x^6-x^5+x^4-x^3+x^2-x+1\right)}{28}\sqrt[3]{x^7+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(x^7+1)^(1/3),x)
```

```
[Out] 3/28*(1+x)*(x^6-x^5+x^4-x^3+x^2-x+1)*(x^7+1)^(1/3)
```

Maxima [A] time = 0.937423, size = 12, normalized size = 0.92

$$\frac{3}{28}\left(x^7+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^7+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/28*(x^7 + 1)^(4/3)
```

Fricas [A] time = 1.73788, size = 30, normalized size = 2.31

$$\frac{3}{28}\left(x^7+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^7+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/28*(x^7 + 1)^(4/3)
```

Sympy [B] time = 0.560205, size = 26, normalized size = 2.

$$\frac{3x^7\sqrt[3]{x^7+1}}{28}+\frac{3\sqrt[3]{x^7+1}}{28}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(x**7+1)**(1/3),x)
```

```
[Out] 3*x**7*(x**7 + 1)**(1/3)/28 + 3*(x**7 + 1)**(1/3)/28
```

Giac [A] time = 1.06003, size = 12, normalized size = 0.92

$$\frac{3}{28}\left(x^7+1\right)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(x^7+1)^(1/3),x, algorithm="giac")
```

```
[Out] 3/28*(x^7 + 1)^(4/3)
```

3.300 $\int \frac{x^6}{\left(1+x^7\right)^{5/3}}\,dx$

Optimal. Leaf size=13

$$-\frac{3}{14\left(x^7+1\right)^{2/3}}$$

```
[Out] -3/(14*(1 + x^7)^(2/3))
```

Rubi [A] time = 0.0029647, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{3}{14\left(x^7+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(1 + x^7)^(5/3),x]

[Out] -3/(14*(1 + x^7)^(2/3))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^6}{\left(1+x^7\right)^{5/3}}\,dx=-\frac{3}{14\left(1+x^7\right)^{2/3}}$$

Mathematica [A] time = 0.0026527, size = 13, normalized size = 1.

$$-\frac{3}{14\left(x^7+1\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(1 + x^7)^(5/3),x]

[Out] -3/(14*(1 + x^7)^(2/3))
```

Maple [B] time = 0.004, size = 37, normalized size = 2.9

$$-\frac{(3+3x)\left(x^6-x^5+x^4-x^3+x^2-x+1\right)}{14}\left(x^7+1\right)^{-\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(x^7+1)^(5/3),x)
```

```
[Out] -3/14*(1+x)*(x^6-x^5+x^4-x^3+x^2-x+1)/(x^7+1)^(5/3)
```

Maxima [A] time = 0.942658, size = 12, normalized size = 0.92

$$-\frac{3}{14\left(x^7+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^7+1)^(5/3),x, algorithm="maxima")
```

```
[Out] -3/14/(x^7 + 1)^(2/3)
```

Fricas [A] time = 1.77936, size = 31, normalized size = 2.38

$$-\frac{3}{14\left(x^7+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^7+1)^(5/3),x, algorithm="fricas")
```

```
[Out] -3/14/(x^7 + 1)^(2/3)
```

Sympy [A] time = 0.572686, size = 12, normalized size = 0.92

$$-\frac{3}{14\left(x^7+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(x**7+1)**(5/3),x)
```

```
[Out] -3/(14*(x**7 + 1)**(2/3))
```

Giac [A] time = 1.06213, size = 12, normalized size = 0.92

$$-\frac{3}{14\left(x^7+1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(x^7+1)^(5/3),x, algorithm="giac")
```

```
[Out] -3/14/(x^7 + 1)^(2/3)
```

3.301 $\int \frac{1}{x(-27+2x^7)^{2/3}}\,dx$

Optimal. Leaf size=59

$$\frac{1}{42}\log\left(\sqrt[3]{2x^7-27}+3\right)-\frac{\tan^{-1}\left(\frac{3-2\sqrt[3]{2x^7-27}}{3\sqrt{3}}\right)}{21\sqrt{3}}-\frac{\log(x)}{18}$$

```
[Out] -ArcTan[(3 - 2*(-27 + 2*x^7)^(1/3))/(3*Sqrt[3])]/(21*Sqrt[3]) - Log[x]/18 +
 Log[3 + (-27 + 2*x^7)^(1/3)]/42
```

Rubi [A] time = 0.0371693, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {266, 58, 618, 204, 31}

$$\frac{1}{42}\log\left(\sqrt[3]{2x^7-27}+3\right)-\frac{\tan^{-1}\left(\frac{3-2\sqrt[3]{2x^7-27}}{3\sqrt{3}}\right)}{21\sqrt{3}}-\frac{\log(x)}{18}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-27 + 2*x^7)^(2/3)),x]
```

```
[Out] -ArcTan[(3 - 2*(-27 + 2*x^7)^(1/3))/(3*Sqrt[3])]/(21*Sqrt[3]) - Log[x]/18 +
 Log[3 + (-27 + 2*x^7)^(1/3)]/42
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
 x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x\left(-27+2x^7\right)^{2/3}}\,dx = \frac{1}{7}\operatorname{Subst}\left(\int \frac{1}{x(-27+2x)^{2/3}}\,dx, x, x^7\right)$$
$$= -\frac{\log(x)}{18} + \frac{1}{42}\operatorname{Subst}\left(\int \frac{1}{3+x}\,dx, x, \sqrt[3]{-27+2x^7}\right) + \frac{1}{14}\operatorname{Subst}\left(\int \frac{1}{9-3x+x^2}\,dx, x, \sqrt[3]{-27+2\right.$$
$$= -\frac{\log(x)}{18} + \frac{1}{42}\log\left(3+\sqrt[3]{-27+2x^7}\right) - \frac{1}{7}\operatorname{Subst}\left(\int \frac{1}{-27-x^2}\,dx, x, -3+2\sqrt[3]{-27+2x^7}\right)$$
$$= -\frac{\tan^{-1}\left(\frac{3-2\sqrt[3]{-27+2x^7}}{3\sqrt{3}}\right)}{21\sqrt{3}} - \frac{\log(x)}{18} + \frac{1}{42}\log\left(3+\sqrt[3]{-27+2x^7}\right)$$

Mathematica [A] time = 0.0221199, size = 84, normalized size = 1.42

$$\frac{1}{63}\log\left(\sqrt[3]{2x^7-27}+3\right) - \frac{1}{126}\log\left(\left(2x^7-27\right)^{2/3} - 3\sqrt[3]{2x^7-27}+9\right) + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{2x^7-27}-3}{3\sqrt{3}}\right)}{21\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-27 + 2*x^7)^(2/3)),x]
```

```
[Out] ArcTan[(-3 + 2*(-27 + 2*x^7)^(1/3))/(3*Sqrt[3])]/(21*Sqrt[3]) + Log[3 + (-2
7 + 2*x^7)^(1/3)]/63 - Log[9 - 3*(-27 + 2*x^7)^(1/3) + (-27 + 2*x^7)^(2/3)]
/126
```

Maple [C] time = 0.058, size = 74, normalized size = 1.3

$$\frac{1}{63\,\Gamma(2/3)}\left(-\operatorname{signum}\left(-1+\frac{2x^7}{27}\right)\right)^{\frac{2}{3}}\left(\left(\frac{\pi\sqrt{3}}{6}-\frac{9\ln(3)}{2}+7\ln(x)+\ln(2)+i\pi\right)\Gamma\left(\frac{2}{3}\right)+\frac{4\,\Gamma(2/3)\,x^7}{81}{}_3F_2(1,1,\frac{5}{3};2,2;\frac{2x^7}{27}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(2*x^7-27)^(2/3),x)
```

```
[Out] 1/63/signum(-1+2/27*x^7)^(2/3)*(-signum(-1+2/27*x^7))^(2/3)*((1/6*Pi*3^(1/2
)-9/2*ln(3)+7*ln(x)+ln(2)+I*Pi)*GAMMA(2/3)+4/81*GAMMA(2/3)*x^7*hypergeom([1
,1,5/3],[2,2],2/27*x^7))/GAMMA(2/3)
```

Maxima [A] time = 1.4436, size = 86, normalized size = 1.46

$$\frac{1}{63}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(2\left(2x^7-27\right)^{\frac{1}{3}}-3\right)\right) - \frac{1}{126}\log\left(\left(2x^7-27\right)^{\frac{2}{3}} - 3\left(2x^7-27\right)^{\frac{1}{3}}+9\right) + \frac{1}{63}\log\left(\left(2x^7-27\right)^{\frac{1}{3}}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x^7-27)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/63*sqrt(3)*arctan(1/9*sqrt(3)*(2*(2*x^7 - 27)^(1/3) - 3)) - 1/126*log((2*
x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(1/3) + 9) + 1/63*log((2*x^7 - 27)^(1/3) +
 3)
```

Fricas [A] time = 1.94792, size = 217, normalized size = 3.68

$$\frac{1}{63}\sqrt{3}\arctan\left(\frac{2}{9}\sqrt{3}\left(2x^{7}-27\right)^{\frac{1}{3}}-\frac{1}{3}\sqrt{3}\right)-\frac{1}{126}\log\left(\left(2x^{7}-27\right)^{\frac{2}{3}}-3\left(2x^{7}-27\right)^{\frac{1}{3}}+9\right)+\frac{1}{63}\log\left(\left(2x^{7}-27\right)^{\frac{1}{3}}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x^7-27)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/63*sqrt(3)*arctan(2/9*sqrt(3)*(2*x^7 - 27)^(1/3) - 1/3*sqrt(3)) - 1/126*l
og((2*x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(1/3) + 9) + 1/63*log((2*x^7 - 27)^(
1/3) + 3)
```

Sympy [C] time = 1.03546, size = 42, normalized size = 0.71

$$-\frac{\sqrt[3]{2}\Gamma\left(\frac{2}{3}\right){}_{2}F_{1}\left(\begin{matrix}\frac{2}{3},\frac{2}{3}\\\frac{5}{3}\end{matrix}\middle|\frac{27e^{2i\pi}}{2x^{7}}\right)}{14x^{\frac{14}{3}}\Gamma\left(\frac{5}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x**7-27)**(2/3),x)
```

```
[Out] -2**(1/3)*gamma(2/3)*hyper((2/3, 2/3), (5/3,), 27*exp_polar(2*I*pi)/(2*x**7
))/(14*x**(14/3)*gamma(5/3))
```

Giac [A] time = 1.09344, size = 86, normalized size = 1.46

$$\frac{1}{63}\sqrt{3}\arctan\left(\frac{1}{9}\sqrt{3}\left(2\left(2x^{7}-27\right)^{\frac{1}{3}}-3\right)\right)-\frac{1}{126}\log\left(\left(2x^{7}-27\right)^{\frac{2}{3}}-3\left(2x^{7}-27\right)^{\frac{1}{3}}+9\right)+\frac{1}{63}\log\left(\left(2x^{7}-27\right)^{\frac{1}{3}}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(2*x^7-27)^(2/3),x, algorithm="giac")
```

```
[Out] 1/63*sqrt(3)*arctan(1/9*sqrt(3)*(2*(2*x^7 - 27)^(1/3) - 3)) - 1/126*log((2*
x^7 - 27)^(2/3) - 3*(2*x^7 - 27)^(1/3) + 9) + 1/63*log((2*x^7 - 27)^(1/3) +
 3)
```

3.302 $\int \frac{(1+x^7)^{2/3}}{x^8} dx$

Optimal. Leaf size=70

$$-\frac{(x^7+1)^{2/3}}{7x^7}+\frac{1}{7}\log\left(1-\sqrt[3]{x^7+1}\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^7+1}+1}{\sqrt{3}}\right)}{7\sqrt{3}}-\frac{\log(x)}{3}$$

```
[Out] -(1 + x^7)^(2/3)/(7*x^7) + (2*ArcTan[(1 + 2*(1 + x^7)^(1/3))/Sqrt[3]])/(7*S
qrt[3]) - Log[x]/3 + Log[1 - (1 + x^7)^(1/3)]/7
```

Rubi [A] time = 0.0378973, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {266, 47, 55, 618, 204, 31}

$$-\frac{(x^7+1)^{2/3}}{7x^7}+\frac{1}{7}\log\left(1-\sqrt[3]{x^7+1}\right)+\frac{2\tan^{-1}\left(\frac{2\sqrt[3]{x^7+1}+1}{\sqrt{3}}\right)}{7\sqrt{3}}-\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^7)^(2/3)/x^8,x]
```

```
[Out] -(1 + x^7)^(2/3)/(7*x^7) + (2*ArcTan[(1 + 2*(1 + x^7)^(1/3))/Sqrt[3]])/(7*S
qrt[3]) - Log[x]/3 + Log[1 - (1 + x^7)^(1/3)]/7
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1+x^7\right)^{2/3}}{x^8}\,dx &= \frac{1}{7}\operatorname{Subst}\left(\int \frac{(1+x)^{2/3}}{x^2}\,dx, x, x^7\right)\\
&= -\frac{\left(1+x^7\right)^{2/3}}{7x^7}+\frac{2}{21}\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{1+x}}\,dx, x, x^7\right)\\
&= -\frac{\left(1+x^7\right)^{2/3}}{7x^7}-\frac{\log(x)}{3}-\frac{1}{7}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[3]{1+x^7}\right)+\frac{1}{7}\operatorname{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, \sqrt[3]{1+\right.\\
&= -\frac{\left(1+x^7\right)^{2/3}}{7x^7}-\frac{\log(x)}{3}+\frac{1}{7}\log\left(1-\sqrt[3]{1+x^7}\right)-\frac{2}{7}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2\sqrt[3]{1+x^7}\right)\\
&= -\frac{\left(1+x^7\right)^{2/3}}{7x^7}+\frac{2\tan^{-1}\left(\frac{1+2\sqrt[3]{1+x^7}}{\sqrt{3}}\right)}{7\sqrt{3}}-\frac{\log(x)}{3}+\frac{1}{7}\log\left(1-\sqrt[3]{1+x^7}\right)
\end{aligned}$$

Mathematica [C] time = 0.0057785, size = 26, normalized size = 0.37

$$\frac{3}{35}\left(x^7+1\right)^{5/3}{}_2F_1\left(\frac{5}{3},2;\frac{8}{3};x^7+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^7)^(2/3)/x^8,x]
```

```
[Out] (3*(1 + x^7)^(5/3)*Hypergeometric2F1[5/3, 2, 8/3, 1 + x^7])/35
```

Maple [C] time = 0.039, size = 76, normalized size = 1.1

$$-\frac{1}{7x^7}\left(x^7+1\right)^{\frac{2}{3}}+\frac{\sqrt{3}\Gamma\left(\frac{2}{3}\right)}{21\pi}\left(\frac{2\pi\sqrt{3}}{3\Gamma(2/3)}\left(-\frac{\pi\sqrt{3}}{6}-\frac{3\ln(3)}{2}+7\ln(x)\right)-\frac{2\pi\sqrt{3}x^7}{9\Gamma(2/3)}{}_3F_2(1,1,\frac{4}{3};2,2;-x^7)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7+1)^(2/3)/x^8,x)
```

```
[Out] -1/7*(x^7+1)^(2/3)/x^7+1/21/Pi*3^(1/2)*GAMMA(2/3)*(2/3*(-1/6*Pi*3^(1/2)-3/2
*ln(3)+7*ln(x))*Pi*3^(1/2)/GAMMA(2/3)-2/9*Pi*3^(1/2)/GAMMA(2/3)*x^7*hyperge
om([1,1,4/3],[2,2],-x^7))
```

Maxima [A] time = 1.45476, size = 89, normalized size = 1.27

$$\frac{2}{21}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^7+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^7+1\right)^{\frac{2}{3}}}{7x^7}-\frac{1}{21}\log\left(\left(x^7+1\right)^{\frac{2}{3}}+\left(x^7+1\right)^{\frac{1}{3}}+1\right)+\frac{2}{21}\log\left(\left(x^7+1\right)^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^7+1)^(2/3)/x^8,x, algorithm="maxima")
```

```
[Out] 2/21*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^7 + 1)^(1/3) + 1)) - 1/7*(x^7 + 1)^(2
/3)/x^7 - 1/21*log((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3) + 1) + 2/21*log((x^7 +
 1)^(1/3) - 1)
```

Fricas [A] time = 1.81303, size = 240, normalized size = 3.43

$$\frac{2\sqrt{3}x^7\arctan\left(\frac{2}{3}\sqrt{3}\left(x^7+1\right)^{\frac{1}{3}}+\frac{1}{3}\sqrt{3}\right)-x^7\log\left(\left(x^7+1\right)^{\frac{2}{3}}+\left(x^7+1\right)^{\frac{1}{3}}+1\right)+2x^7\log\left(\left(x^7+1\right)^{\frac{1}{3}}-1\right)-3\left(x^7+1\right)^{\frac{2}{3}}}{21x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^7+1)^(2/3)/x^8,x, algorithm="fricas")
```

```
[Out] 1/21*(2*sqrt(3)*x^7*arctan(2/3*sqrt(3)*(x^7 + 1)^(1/3) + 1/3*sqrt(3)) - x^7
*log((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3) + 1) + 2*x^7*log((x^7 + 1)^(1/3) - 1
) - 3*(x^7 + 1)^(2/3))/x^7
```

Sympy [C] time = 1.89561, size = 34, normalized size = 0.49

$$-\frac{\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}-\frac{2}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{e^{i\pi}}{x^7}\right)}{7x^{\frac{7}{3}}\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**7+1)**(2/3)/x**8,x)
```

```
[Out] -gamma(1/3)*hyper((-2/3, 1/3), (4/3,), exp_polar(I*pi)/x**7)/(7*x**(7/3)*ga
mma(4/3))
```

Giac [A] time = 1.09018, size = 90, normalized size = 1.29

$$\frac{2}{21}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(x^7+1\right)^{\frac{1}{3}}+1\right)\right)-\frac{\left(x^7+1\right)^{\frac{2}{3}}}{7x^7}-\frac{1}{21}\log\left(\left(x^7+1\right)^{\frac{2}{3}}+\left(x^7+1\right)^{\frac{1}{3}}+1\right)+\frac{2}{21}\log\left(\left|\left(x^7+1\right)^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^7+1)^(2/3)/x^8,x, algorithm="giac")
```

```
[Out] 2/21*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^7 + 1)^(1/3) + 1)) - 1/7*(x^7 + 1)^(2
/3)/x^7 - 1/21*log((x^7 + 1)^(2/3) + (x^7 + 1)^(1/3) + 1) + 2/21*log(abs((x
^7 + 1)^(1/3) - 1))
```

3.303 $\int \frac{\sqrt[4]{3+4x^4}}{x^2} dx$

Optimal. Leaf size=68

$$-\frac{\sqrt[4]{4x^4+3}}{x}-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{\sqrt{2}}$$

```
[Out] -((3 + 4*x^4)^(1/4)/x) - ArcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2] + Ar
cTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2]
```

Rubi [A] time = 0.0208885, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {277, 331, 298, 203, 206}

$$-\frac{\sqrt[4]{4x^4+3}}{x}-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*x^4)^(1/4)/x^2,x]
```

```
[Out] -((3 + 4*x^4)^(1/4)/x) - ArcTan[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2] + Ar
cTanh[(Sqrt[2]*x)/(3 + 4*x^4)^(1/4)]/Sqrt[2]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[4]{3+4x^4}}{x^2}\,dx &= -\frac{\sqrt[4]{3+4x^4}}{x}+4\int \frac{x^2}{\left(3+4x^4\right)^{3/4}}\,dx\\ &= -\frac{\sqrt[4]{3+4x^4}}{x}+4\,\text{Subst}\left(\int \frac{x^2}{1-4x^4}\,dx,x,\frac{x}{\sqrt[4]{3+4x^4}}\right)\\ &= -\frac{\sqrt[4]{3+4x^4}}{x}+\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\frac{x}{\sqrt[4]{3+4x^4}}\right)-\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\frac{x}{\sqrt[4]{3+4x^4}}\right)\\ &= -\frac{\sqrt[4]{3+4x^4}}{x}-\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.0040984, size = 27, normalized size = 0.4

$$-\frac{\sqrt[4]{3}\,{}_2F_1\left(-\frac{1}{4},-\frac{1}{4};\frac{3}{4};-\frac{4x^4}{3}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*x^4)^(1/4)/x^2,x]
```

```
[Out] -((3^(1/4)*Hypergeometric2F1[-1/4, -1/4, 3/4, (-4*x^4)/3])/x)
```

Maple [C] time = 0.04, size = 35, normalized size = 0.5

$$-\frac{1}{x}\sqrt[4]{4\,x^4+3}+\frac{4\,\sqrt[4]{3}x^3}{9}{}_2F_1(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{4\,x^4}{3})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^4+3)^(1/4)/x^2,x)
```

```
[Out] -(4*x^4+3)^(1/4)/x+4/9*3^(1/4)*x^3*hypergeom([3/4,3/4],[7/4],-4/3*x^4)
```

Maxima [A] time = 1.46827, size = 112, normalized size = 1.65

$$\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4\,x^4+3\right)^{\frac{1}{4}}}{2\,x}\right)-\frac{1}{4}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}}\right)-\frac{\left(4\,x^4+3\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+3)^(1/4)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 1/4*sqrt(2)*log(-(sqr
t(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) - (4*x^4 + 3)^
(1/4)/x
```

Fricas [B] time = 26.1558, size = 390, normalized size = 5.74

$$-\frac{2\sqrt{2}x\arctan\left(\frac{4}{3}\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}x^3+\frac{2}{3}\sqrt{2}\left(4x^4+3\right)^{\frac{3}{4}}x\right)-\sqrt{2}x\log\left(-256x^8-192x^4-4\sqrt{2}\left(16x^5+3x\right)\left(4x^4+3\right)^{\frac{3}{4}}-8\right.}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+3)^(1/4)/x^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*sqrt(2)*x*arctan(4/3*sqrt(2)*(4*x^4 + 3)^(1/4)*x^3 + 2/3*sqrt(2)*(4
*x^4 + 3)^(3/4)*x) - sqrt(2)*x*log(-256*x^8 - 192*x^4 - 4*sqrt(2)*(16*x^5 +
 3*x)*(4*x^4 + 3)^(3/4) - 8*sqrt(2)*(16*x^7 + 9*x^3)*(4*x^4 + 3)^(1/4) - 16
*(8*x^6 + 3*x^2)*sqrt(4*x^4 + 3) - 9) + 8*(4*x^4 + 3)^(1/4))/x
```

Sympy [C] time = 1.05336, size = 41, normalized size = 0.6

$$\frac{\sqrt[4]{3}\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{4},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\right|\frac{4x^4e^{i\pi}}{3}\right)}{4x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**4+3)**(1/4)/x**2,x)
```

```
[Out] 3**(1/4)*gamma(-1/4)*hyper((-1/4, -1/4), (3/4,), 4*x**4*exp_polar(I*pi)/3)/
(4*x*gamma(3/4))
```

Giac [A] time = 1.08321, size = 112, normalized size = 1.65

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}}{2x}\right)-\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}}\right)-\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^4+3)^(1/4)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 1/4*sqrt(2)*log(-(sqr
t(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) - (4*x^4 + 3)^
(1/4)/x
```

3.304 $\int x^2 \left(3+4x^4\right)^{5/4} dx$

Optimal. Leaf size=93

$$\frac{1}{8}\left(4x^4+3\right)^{5/4}x^3+\frac{15}{32}\sqrt[4]{4x^4+3}x^3-\frac{45\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}}+\frac{45\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}}$$

```
[Out] (15*x^3*(3 + 4*x^4)^(1/4))/32 + (x^3*(3 + 4*x^4)^(5/4))/8 - (45*ArcTan[(Sqr
t[2]*x)/(3 + 4*x^4)^(1/4)])/(128*Sqrt[2]) + (45*ArcTanh[(Sqrt[2]*x)/(3 + 4*
x^4)^(1/4)])/(128*Sqrt[2])
```

Rubi [A] time = 0.0293518, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {279, 331, 298, 203, 206}

$$\frac{1}{8}\left(4x^4+3\right)^{5/4}x^3+\frac{15}{32}\sqrt[4]{4x^4+3}x^3-\frac{45\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}}+\frac{45\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(3 + 4*x^4)^(5/4),x]
```

```
[Out] (15*x^3*(3 + 4*x^4)^(1/4))/32 + (x^3*(3 + 4*x^4)^(5/4))/8 - (45*ArcTan[(Sqr
t[2]*x)/(3 + 4*x^4)^(1/4)])/(128*Sqrt[2]) + (45*ArcTanh[(Sqrt[2]*x)/(3 + 4*
x^4)^(1/4)])/(128*Sqrt[2])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(3+4x^4\right)^{5/4}dx &= \frac{1}{8}x^3\left(3+4x^4\right)^{5/4}+\frac{15}{8}\int x^2\sqrt[4]{3+4x^4}\,dx\\ &= \frac{15}{32}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^3\left(3+4x^4\right)^{5/4}+\frac{45}{32}\int\frac{x^2}{\left(3+4x^4\right)^{3/4}}dx\\ &= \frac{15}{32}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^3\left(3+4x^4\right)^{5/4}+\frac{45}{32}\text{Subst}\left(\int\frac{x^2}{1-4x^4}dx,x,\frac{x}{\sqrt[4]{3+4x^4}}\right)\\ &= \frac{15}{32}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^3\left(3+4x^4\right)^{5/4}+\frac{45}{128}\text{Subst}\left(\int\frac{1}{1-2x^2}dx,x,\frac{x}{\sqrt[4]{3+4x^4}}\right)-\frac{45}{128}\text{Subst}\left(\int\frac{}{1}\right.\\ &= \frac{15}{32}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^3\left(3+4x^4\right)^{5/4}-\frac{45\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}+\frac{45\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{128\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.0046128, size = 26, normalized size = 0.28

$$\sqrt[4]{3}x^3\,{}_2F_1\left(-\frac{5}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^4}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(3 + 4*x^4)^(5/4),x]
```

```
[Out] 3^(1/4)*x^3*Hypergeometric2F1[-5/4, 3/4, 7/4, (-4*x^4)/3]
```

Maple [C] time = 0.021, size = 42, normalized size = 0.5

$$\frac{x^3\left(16x^4+27\right)}{32}\sqrt[4]{4x^4+3}+\frac{5\sqrt[4]{3}x^3}{32}\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(4*x^4+3)^(5/4),x)
```

```
[Out] 1/32*x^3*(16*x^4+27)*(4*x^4+3)^(1/4)+5/32*3^(1/4)*x^3*hypergeom([3/4,3/4],[
7/4],-4/3*x^4)
```

Maxima [A] time = 1.46926, size = 176, normalized size = 1.89

$$\frac{45}{256}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}}{2x}\right)-\frac{45}{512}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}}\right)+\frac{9\left(\frac{20\left(4x^4+3\right)^{\frac{1}{4}}}{x}-\frac{9\left(4x^4+3\right)^{\frac{5}{4}}}{x^5}\right)}{32\left(\frac{8\left(4x^4+3\right)}{x^4}-\frac{\left(4x^4+3\right)^2}{x^8}-16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^4+3)^(5/4),x, algorithm="maxima")
```

```
[Out] 45/256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 45/512*sqrt(2)*log
(-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) + 9/32*(
20*(4*x^4 + 3)^(1/4)/x - 9*(4*x^4 + 3)^(5/4)/x^5)/(8*(4*x^4 + 3)/x^4 - (4*x
^4 + 3)^2/x^8 - 16)
```

Fricas [A] time = 1.86701, size = 298, normalized size = 3.2

$$\frac{45}{256}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}}{2x}\right)+\frac{45}{512}\sqrt{2}\log\left(8x^4+4\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}x^3+4\sqrt{4x^4+3}x^2+2\sqrt{2}\left(4x^4+3\right)^{\frac{3}{4}}x+3\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^4+3)^(5/4),x, algorithm="fricas")
```

```
[Out] 45/256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 45/512*sqrt(2)*log
(8*x^4 + 4*sqrt(2)*(4*x^4 + 3)^(1/4)*x^3 + 4*sqrt(4*x^4 + 3)*x^2 + 2*sqrt(2
)*(4*x^4 + 3)^(3/4)*x + 3) + 1/32*(16*x^7 + 27*x^3)*(4*x^4 + 3)^(1/4)
```

Sympy [C] time = 2.09364, size = 41, normalized size = 0.44

$$\frac{3\sqrt[4]{3}x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{5}{4},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{4x^4e^{i\pi}}{3}\right)}{4\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(4*x**4+3)**(5/4),x)
```

```
[Out] 3*3**(1/4)*x**3*gamma(3/4)*hyper((-5/4, 3/4), (7/4,), 4*x**4*exp_polar(I*pi
)/3)/(4*gamma(7/4))
```

Giac [A] time = 1.08375, size = 149, normalized size = 1.6

$$\frac{1}{32}x^8\left(\frac{9\left(4x^4+3\right)^{\frac{1}{4}}\left(\frac{3}{x^4}+4\right)}{x}-\frac{20\left(4x^4+3\right)^{\frac{1}{4}}}{x}\right)+\frac{45}{256}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}}{2x}\right)-\frac{45}{512}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(4*x^4+3)^(5/4),x, algorithm="giac")
```

```
[Out] 1/32*x^8*(9*(4*x^4 + 3)^(1/4)*(3/x^4 + 4)/x - 20*(4*x^4 + 3)^(1/4)/x) + 45/
256*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) - 45/512*sqrt(2)*log(-(
sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x))
```

3.305 $\int x^6 \sqrt[4]{3+4x^4}\,dx$

Optimal. Leaf size=93

$$\frac{1}{8}\sqrt[4]{4x^4+3}x^7+\frac{3}{128}\sqrt[4]{4x^4+3}x^3+\frac{27\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}}-\frac{27\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}}$$

```
[Out] (3*x^3*(3 + 4*x^4)^(1/4))/128 + (x^7*(3 + 4*x^4)^(1/4))/8 + (27*ArcTan[(Sqr
t[2]*x)/(3 + 4*x^4)^(1/4)])/(512*Sqrt[2]) - (27*ArcTanh[(Sqrt[2]*x)/(3 + 4*
x^4)^(1/4)])/(512*Sqrt[2])
```

Rubi [A] time = 0.0285984, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {279, 321, 331, 298, 203, 206}

$$\frac{1}{8}\sqrt[4]{4x^4+3}x^7+\frac{3}{128}\sqrt[4]{4x^4+3}x^3+\frac{27\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}}-\frac{27\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{4x^4+3}}\right)}{512\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(3 + 4*x^4)^(1/4),x]
```

```
[Out] (3*x^3*(3 + 4*x^4)^(1/4))/128 + (x^7*(3 + 4*x^4)^(1/4))/8 + (27*ArcTan[(Sqr
t[2]*x)/(3 + 4*x^4)^(1/4)])/(512*Sqrt[2]) - (27*ArcTanh[(Sqrt[2]*x)/(3 + 4*
x^4)^(1/4)])/(512*Sqrt[2])
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
```

```
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x^6\sqrt[4]{3+4x^4}\,dx &= \frac{1}{8}x^7\sqrt[4]{3+4x^4}+\frac{3}{8}\int\frac{x^6}{\left(3+4x^4\right)^{3/4}}\,dx\\
&= \frac{3}{128}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^7\sqrt[4]{3+4x^4}-\frac{27}{128}\int\frac{x^2}{\left(3+4x^4\right)^{3/4}}\,dx\\
&= \frac{3}{128}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^7\sqrt[4]{3+4x^4}-\frac{27}{128}\text{Subst}\left(\int\frac{x^2}{1-4x^4}\,dx,x,\frac{x}{\sqrt[4]{3+4x^4}}\right)\\
&= \frac{3}{128}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^7\sqrt[4]{3+4x^4}-\frac{27}{512}\text{Subst}\left(\int\frac{1}{1-2x^2}\,dx,x,\frac{x}{\sqrt[4]{3+4x^4}}\right)+\frac{27}{512}\text{Subst}\left(\int\frac{1}{1+
\right.\\
&= \frac{3}{128}x^3\sqrt[4]{3+4x^4}+\frac{1}{8}x^7\sqrt[4]{3+4x^4}+\frac{27\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}}-\frac{27\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{3+4x^4}}\right)}{512\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.0136698, size = 43, normalized size = 0.46

$$\frac{1}{32}x^3\left(\left(4x^4+3\right)^{5/4}-3\sqrt[4]{3}\,{}_2F_1\left(-\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^4}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(3 + 4*x^4)^(1/4),x]
```

```
[Out] (x^3*((3 + 4*x^4)^(5/4) - 3*3^(1/4)*Hypergeometric2F1[-1/4, 3/4, 7/4, (-4*x
^4)/3]))/32
```

Maple [C] time = 0.02, size = 42, normalized size = 0.5

$$\frac{x^3\left(16x^4+3\right)}{128}\sqrt[4]{4x^4+3}-\frac{3\sqrt[4]{3}x^3}{128}{}_2\text{F}_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};-\frac{4x^4}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(4*x^4+3)^(1/4),x)
```

```
[Out] 1/128*x^3*(16*x^4+3)*(4*x^4+3)^(1/4)-3/128*3^(1/4)*x^3*hypergeom([3/4,3/4],
[7/4],-4/3*x^4)
```

Maxima [A] time = 1.43927, size = 174, normalized size = 1.87

$$-\frac{27}{1024}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}}{2x}\right)+\frac{27}{2048}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{\left(4x^4+3\right)^{\frac{1}{4}}}{x}}\right)-\frac{9\left(\frac{12\left(4x^4+3\right)^{\frac{1}{4}}}{x}+\frac{\left(4x^4+3\right)^{\frac{5}{4}}}{x^5}\right)}{128\left(\frac{8\left(4x^4+3\right)}{x^4}-\frac{\left(4x^4+3\right)^2}{x^8}-16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(4*x^4+3)^(1/4),x, algorithm="maxima")
```

```
[Out] -27/1024*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 27/2048*sqrt(2)*
log(-(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x)) - 9/1
28*(12*(4*x^4 + 3)^(1/4)/x + (4*x^4 + 3)^(5/4)/x^5)/(8*(4*x^4 + 3)/x^4 - (4
*x^4 + 3)^2/x^8 - 16)
```

Fricas [A] time = 1.89777, size = 302, normalized size = 3.25

$$-\frac{27}{1024}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}}{2x}\right)+\frac{27}{2048}\sqrt{2}\log\left(8x^4-4\sqrt{2}\left(4x^4+3\right)^{\frac{1}{4}}x^3+4\sqrt{4x^4+3}x^2-2\sqrt{2}\left(4x^4+3\right)^{\frac{3}{4}}x+3\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(4*x^4+3)^(1/4),x, algorithm="fricas")
```

```
[Out] -27/1024*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 27/2048*sqrt(2)*
log(8*x^4 - 4*sqrt(2)*(4*x^4 + 3)^(1/4)*x^3 + 4*sqrt(4*x^4 + 3)*x^2 - 2*sqr
t(2)*(4*x^4 + 3)^(3/4)*x + 3) + 1/128*(16*x^7 + 3*x^3)*(4*x^4 + 3)^(1/4)
```

Sympy [C] time = 1.59965, size = 39, normalized size = 0.42

$$\frac{\sqrt[4]{3}x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\frac{4x^4e^{i\pi}}{3}\right)}{4\Gamma\left(\frac{11}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(4*x**4+3)**(1/4),x)
```

```
[Out] 3**(1/4)*x**7*gamma(7/4)*hyper((-1/4, 7/4), (11/4,), 4*x**4*exp_polar(I*pi)
/3)/(4*gamma(11/4))
```

Giac [A] time = 1.10721, size = 147, normalized size = 1.58

$$\frac{1}{128}\,x^{8}\left(\frac{\left(4\,x^{4}+3\right)^{\frac{1}{4}}\left(\frac{3}{x^{4}}+4\right)}{x}+\frac{12\left(4\,x^{4}+3\right)^{\frac{1}{4}}}{x}\right)-\frac{27}{1024}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(4\,x^{4}+3\right)^{\frac{1}{4}}}{2\,x}\right)+\frac{27}{2048}\,\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\left(4\,x^{4}+3\right)^{\frac{1}{4}}}{x}}{\sqrt{2}+\frac{\left(4\,x^{4}+3\right)^{\frac{1}{4}}}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(4*x^4+3)^(1/4),x, algorithm="giac")
```

```
[Out] 1/128*x^8*((4*x^4 + 3)^(1/4)*(3/x^4 + 4)/x + 12*(4*x^4 + 3)^(1/4)/x) - 27/1
024*sqrt(2)*arctan(1/2*sqrt(2)*(4*x^4 + 3)^(1/4)/x) + 27/2048*sqrt(2)*log(-
(sqrt(2) - (4*x^4 + 3)^(1/4)/x)/(sqrt(2) + (4*x^4 + 3)^(1/4)/x))
```

3.306 $\int \sqrt[3]{x\left(1-x^2\right)}\,dx$

Optimal. Leaf size=93

$$\frac{1}{2}\sqrt[3]{x\left(1-x^2\right)}x-\frac{1}{4}\log\left(\sqrt[3]{x\left(1-x^2\right)}+x\right)+\frac{\tan^{-1}\left(\frac{2x-\sqrt[3]{x\left(1-x^2\right)}}{\sqrt{3}\sqrt[3]{x\left(1-x^2\right)}}\right)}{2\sqrt{3}}+\frac{\log(x)}{12}$$

```
[Out] (x*(x*(1 - x^2))^(1/3))/2 + ArcTan[(2*x - (x*(1 - x^2))^(1/3))/(Sqrt[3]*(x*
(1 - x^2))^(1/3))]/(2*Sqrt[3]) + Log[x]/12 - Log[x + (x*(1 - x^2))^(1/3)]/4
```

Rubi [B] time = 0.143999, antiderivative size = 200, normalized size of antiderivative = 2.15, number of steps used = 12, number of rules used = 12, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {1979, 2004, 2032, 329, 275, 331, 292, 31, 634, 618, 204, 628}

$$\frac{1}{2}\sqrt[3]{x-x^3}x+\frac{\left(1-x^2\right)^{2/3}x^{2/3}\log\left(\frac{x^{4/3}}{\left(1-x^2\right)^{2/3}}-\frac{x^{2/3}}{\sqrt[3]{1-x^2}}+1\right)}{12\left(x-x^3\right)^{2/3}}-\frac{\left(1-x^2\right)^{2/3}x^{2/3}\log\left(\frac{x^{2/3}}{\sqrt[3]{1-x^2}}+1\right)}{6\left(x-x^3\right)^{2/3}}-\frac{\left(1-x^2\right)^{2/3}x^{2/3}\tan^{-1}\left(\frac{1-\frac{2x^{2/3}}{\sqrt[3]{1-x^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\left(x-x^3\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - x^2))^(1/3),x]
```

```
[Out] (x*(x - x^3)^(1/3))/2 - (x^(2/3)*(1 - x^2)^(2/3)*ArcTan[(1 - (2*x^(2/3))/(1
 - x^2)^(1/3))/Sqrt[3]])/(2*Sqrt[3]*(x - x^3)^(2/3)) + (x^(2/3)*(1 - x^2)^(
2/3)*Log[1 + x^(4/3)/(1 - x^2)^(2/3) - x^(2/3)/(1 - x^2)^(1/3)])/(12*(x - x
^3)^(2/3)) - (x^(2/3)*(1 - x^2)^(2/3)*Log[1 + x^(2/3)/(1 - x^2)^(1/3)])/(6*
(x - x^3)^(2/3))
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 331

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^(p + (m +
 1)/n), Subst[Int[x^m/(1 - b*x^n)^(p + (m + 1)/n + 1), x], x, x/(a + b*x^n)
^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2
^(-1)] && IntegersQ[m, p + (m + 1)/n]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt[3]{x\left(1-x^2\right)}\,dx = \int \sqrt[3]{x-x^3}\,dx$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{1}{3}\int \frac{x}{\left(x-x^3\right)^{2/3}}\,dx$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{\left(x^{2/3}\left(1-x^2\right)^{2/3}\right)\int \frac{\sqrt[3]{x}}{\left(1-x^2\right)^{2/3}}\,dx}{3\left(x-x^3\right)^{2/3}}$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{\left(x^{2/3}\left(1-x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x^3}{\left(1-x^6\right)^{2/3}}\,dx, x, \sqrt[3]{x}\right)}{\left(x-x^3\right)^{2/3}}$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{\left(x^{2/3}\left(1-x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{\left(1-x^3\right)^{2/3}}\,dx, x, x^{2/3}\right)}{2\left(x-x^3\right)^{2/3}}$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{\left(x^{2/3}\left(1-x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{2\left(x-x^3\right)^{2/3}}$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} - \frac{\left(x^{2/3}\left(1-x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{6\left(x-x^3\right)^{2/3}} + \frac{\left(x^{2/3}\left(1-x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \ldots\right)}{6\left(x-x^3\right)^{2/3}}$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} - \frac{x^{2/3}\left(1-x^2\right)^{2/3}\log\left(1+\frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{6\left(x-x^3\right)^{2/3}} + \frac{\left(x^{2/3}\left(1-x^2\right)^{2/3}\right)\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{12\left(x-x^3\right)^{2/3}} + \ldots$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} + \frac{x^{2/3}\left(1-x^2\right)^{2/3}\log\left(1+\frac{x^{4/3}}{\left(1-x^2\right)^{2/3}}-\frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{12\left(x-x^3\right)^{2/3}} - \frac{x^{2/3}\left(1-x^2\right)^{2/3}\log\left(1+\frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{6\left(x-x^3\right)^{2/3}} - \ldots$$
$$= \frac{1}{2}x\sqrt[3]{x-x^3} - \frac{x^{2/3}\left(1-x^2\right)^{2/3}\tan^{-1}\left(\frac{1-\frac{2x^{2/3}}{\sqrt[3]{1-x^2}}}{\sqrt{3}}\right)}{2\sqrt{3}\left(x-x^3\right)^{2/3}} + \frac{x^{2/3}\left(1-x^2\right)^{2/3}\log\left(1+\frac{x^{4/3}}{\left(1-x^2\right)^{2/3}}-\frac{x^{2/3}}{\sqrt[3]{1-x^2}}\right)}{12\left(x-x^3\right)^{2/3}} - \ldots$$

Mathematica [C] time = 0.0088829, size = 40, normalized size = 0.43

$$\frac{3x\sqrt[3]{x-x^3}\,{}_2F_1\left(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^2\right)}{4\sqrt[3]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 - x^2))^(1/3),x]
```

```
[Out] (3*x*(x - x^3)^(1/3)*Hypergeometric2F1[-1/3, 2/3, 5/3, x^2])/(4*(1 - x^2)^(
1/3))
```

Maple [C] time = 0.026, size = 15, normalized size = 0.2

$$\frac{3}{4}x^{\frac{4}{3}}{}_2\text{F}_1(-\frac{1}{3},\frac{2}{3};\frac{5}{3};x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(-x^2+1))^(1/3),x)
```

```
[Out] 3/4*x^(4/3)*hypergeom([-1/3,2/3],[5/3],x^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^2-1\right)x\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(-x^2+1))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((-(x^2 - 1)*x)^(1/3), x)
```

Fricas [A] time = 3.46897, size = 347, normalized size = 3.73

$$-\frac{1}{6}\,\sqrt{3}\arctan\left(\frac{44032959556\,\sqrt{3}\left(-x^3+x\right)^{\frac{1}{3}}x-\sqrt{3}\left(16754327161\,x^2-2707204793\right)+10524305234\,\sqrt{3}\left(-x^3+x\right)^{\frac{2}{3}}}{81835897185\,x^2-1102302937}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(-x^2+1))^(1/3),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan((44032959556*sqrt(3)*(-x^3 + x)^(1/3)*x - sqrt(3)*(1675
4327161*x^2 - 2707204793) + 10524305234*sqrt(3)*(-x^3 + x)^(2/3))/(81835897
185*x^2 - 1102302937)) + 1/2*(-x^3 + x)^(1/3)*x - 1/12*log(3*(-x^3 + x)^(1/
3)*x + 3*(-x^3 + x)^(2/3) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{x\left(1-x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(-x**2+1))**(1/3),x)
```

```
[Out] Integral((x*(1 - x**2))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\left(x^2-1\right)x\right)^{\frac{1}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(-x^2+1))^(1/3),x, algorithm="giac")

[Out] integrate((-(x^2 - 1)*x)^(1/3), x)
```

3.307 $\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx$

Optimal. Leaf size=126

$$\frac{3}{40}\sqrt{\left(\sqrt[3]{x}+1\right)x}x^{2/3}+\frac{21}{128}\tanh^{-1}\left(\frac{x^{2/3}}{\sqrt{\left(\sqrt[3]{x}+1\right)x}}\right)+\frac{3}{5}\sqrt{\left(\sqrt[3]{x}+1\right)x}x-\frac{7}{80}\sqrt{\left(\sqrt[3]{x}+1\right)x}\sqrt[3]{x}+\frac{7}{64}\sqrt{\left(\sqrt[3]{x}+1\right)x}-\frac{21\sqrt{(}}{1}$$

```
[Out] (7*Sqrt[(1 + x^(1/3))*x])/64 - (21*Sqrt[(1 + x^(1/3))*x])/(128*x^(1/3)) - (
7*x^(1/3)*Sqrt[(1 + x^(1/3))*x])/80 + (3*x^(2/3)*Sqrt[(1 + x^(1/3))*x])/40
+ (3*x*Sqrt[(1 + x^(1/3))*x])/5 + (21*ArcTanh[x^(2/3)/Sqrt[(1 + x^(1/3))*x]
])/128
```

Rubi [A] time = 0.115334, antiderivative size = 114, normalized size of antiderivative = 0.9, number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1979, 2004, 2024, 2010, 2029, 206}

$$\frac{3}{5}\sqrt{x^{4/3}+x}x+\frac{3}{40}\sqrt{x^{4/3}+x}x^{2/3}-\frac{7}{80}\sqrt{x^{4/3}+x}\sqrt[3]{x}+\frac{7}{64}\sqrt{x^{4/3}+x}-\frac{21\sqrt{x^{4/3}+x}}{128\sqrt[3]{x}}+\frac{21}{128}\tanh^{-1}\left(\frac{x^{2/3}}{\sqrt{x^{4/3}+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x^(1/3))*x],x]
```

```
[Out] (7*Sqrt[x + x^(4/3)])/64 - (21*Sqrt[x + x^(4/3)])/(128*x^(1/3)) - (7*x^(1/3
)*Sqrt[x + x^(4/3)])/80 + (3*x^(2/3)*Sqrt[x + x^(4/3)])/40 + (3*x*Sqrt[x +
x^(4/3)])/5 + (21*ArcTanh[x^(2/3)/Sqrt[x + x^(4/3)]])/128
```

Rule 1979

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && G
eneralizedBinomialQ[u, x] &&  !GeneralizedBinomialMatchQ[u, x]
```

Rule 2004

```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(x*(a*x^j
 + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j +
 b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j, n
] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

Rule 2024

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p
 + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), In
t[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x
] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && GtQ
[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]
```

Rule 2010

```
Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[(-2*Sqrt
[a*x^j + b*x^n])/(b*(n - 2)*x^(n - 1)), x] - Dist[(a*(2*n - j - 2))/(b*(n -
 2)), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] &&
LtQ[2*(n - 1), j, n]
```

Rule 2029

```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist
[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\left(1+\sqrt[3]{x}\right)x}\,dx &= \int \sqrt{x+x^{4/3}}\,dx\\
&= \frac{3}{5}x\sqrt{x+x^{4/3}}+\frac{1}{10}\int \frac{x}{\sqrt{x+x^{4/3}}}\,dx\\
&= \frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}}+\frac{3}{5}x\sqrt{x+x^{4/3}}-\frac{7}{80}\int \frac{x^{2/3}}{\sqrt{x+x^{4/3}}}\,dx\\
&= -\frac{7}{80}\sqrt[3]{x}\sqrt{x+x^{4/3}}+\frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}}+\frac{3}{5}x\sqrt{x+x^{4/3}}+\frac{7}{96}\int \frac{\sqrt[3]{x}}{\sqrt{x+x^{4/3}}}\,dx\\
&= \frac{7}{64}\sqrt{x+x^{4/3}}-\frac{7}{80}\sqrt[3]{x}\sqrt{x+x^{4/3}}+\frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}}+\frac{3}{5}x\sqrt{x+x^{4/3}}-\frac{7}{128}\int \frac{1}{\sqrt{x+x^{4/3}}}\,dx\\
&= \frac{7}{64}\sqrt{x+x^{4/3}}-\frac{21\sqrt{x+x^{4/3}}}{128\sqrt[3]{x}}-\frac{7}{80}\sqrt[3]{x}\sqrt{x+x^{4/3}}+\frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}}+\frac{3}{5}x\sqrt{x+x^{4/3}}+\frac{7}{256}\int \frac{1}{\sqrt[3]{x}\sqrt{x+\ldots}}\\
&= \frac{7}{64}\sqrt{x+x^{4/3}}-\frac{21\sqrt{x+x^{4/3}}}{128\sqrt[3]{x}}-\frac{7}{80}\sqrt[3]{x}\sqrt{x+x^{4/3}}+\frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}}+\frac{3}{5}x\sqrt{x+x^{4/3}}+\frac{21}{128}\,\mathrm{Subst}\left(\int \ldots\right.\\
&= \frac{7}{64}\sqrt{x+x^{4/3}}-\frac{21\sqrt{x+x^{4/3}}}{128\sqrt[3]{x}}-\frac{7}{80}\sqrt[3]{x}\sqrt{x+x^{4/3}}+\frac{3}{40}x^{2/3}\sqrt{x+x^{4/3}}+\frac{3}{5}x\sqrt{x+x^{4/3}}+\frac{21}{128}\tanh^{-1}\left(\frac{\ldots}{\sqrt{\ldots}}\right.
\end{aligned}$$

Mathematica [A] time = 0.0488544, size = 83, normalized size = 0.66

$$\frac{\sqrt{x^{4/3}+x}\left(\sqrt{\sqrt[3]{x}+1}\sqrt[6]{x}\left(384x^{4/3}-56x^{2/3}+48x+70\sqrt[3]{x}-105\right)+105\sinh^{-1}\left(\sqrt[6]{x}\right)\right)}{640\sqrt{\sqrt[3]{x}+1}\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x^(1/3))*x],x]
```

```
[Out] (Sqrt[x + x^(4/3)]*(Sqrt[1 + x^(1/3)]*x^(1/6)*(-105 + 70*x^(1/3) - 56*x^(2/
3) + 48*x + 384*x^(4/3)) + 105*ArcSinh[x^(1/6)]))/(640*Sqrt[1 + x^(1/3)]*Sq
rt[x])
```

Maple [A] time = 0.01, size = 108, normalized size = 0.9

$$\frac{1}{1280}\sqrt{\left(\sqrt[3]{x}+1\right)x}\left(768\,x^{2/3}\left(x^{2/3}+\sqrt[3]{x}\right)^{3/2}-672\,\sqrt[3]{x}\left(x^{2/3}+\sqrt[3]{x}\right)^{3/2}+560\,\left(x^{2/3}+\sqrt[3]{x}\right)^{3/2}-420\,\sqrt{x^{2/3}+\sqrt[3]{x}}\sqrt[3]{x}-210\,\sqrt{x^{2\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^(1/3)+1)*x)^(1/2),x)
```

```
[Out] 1/1280*((x^(1/3)+1)*x)^(1/2)*(768*x^(2/3)*(x^(2/3)+x^(1/3))^(3/2)-672*x^(1/
3)*(x^(2/3)+x^(1/3))^(3/2)+560*(x^(2/3)+x^(1/3))^(3/2)-420*(x^(2/3)+x^(1/3)
)^(1/2)*x^(1/3)-210*(x^(2/3)+x^(1/3))^(1/2)+105*ln(1/2+x^(1/3)+(x^(2/3)+x^(
1/3))^(1/2)))/x^(1/3)/((x^(1/3)+1)*x^(1/3))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x\left(x^{\frac{1}{3}}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x^(1/3))*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x*(x^(1/3) + 1)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x^(1/3))*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x\left(\sqrt[3]{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x**(1/3))*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(x**(1/3) + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x^(1/3))*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.308 $\int \frac{x^3}{(-1+x^4)\sqrt{1+2x^8}}\,dx$

Optimal. Leaf size=34

$$-\frac{\tanh^{-1}\left(\frac{2x^4+1}{\sqrt{3}\sqrt{2x^8+1}}\right)}{4\sqrt{3}}$$

```
[Out] -ArcTanh[(1 + 2*x^4)/(Sqrt[3]*Sqrt[1 + 2*x^8])]/(4*Sqrt[3])
```

Rubi [A] time = 0.0317954, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1469, 725, 206}

$$-\frac{\tanh^{-1}\left(\frac{2x^4+1}{\sqrt{3}\sqrt{2x^8+1}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((-1 + x^4)*Sqrt[1 + 2*x^8]),x]
```

```
[Out] -ArcTanh[(1 + 2*x^4)/(Sqrt[3]*Sqrt[1 + 2*x^8])]/(4*Sqrt[3])
```

Rule 1469

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(-1+x^4\right)\sqrt{1+2x^8}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{(-1+x)\sqrt{1+2x^2}}\,dx, x, x^4\right)\\
&= -\left(\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{3-x^2}\,dx, x, \frac{1+2x^4}{\sqrt{1+2x^8}}\right)\right)\\
&= -\frac{\tanh^{-1}\left(\frac{1+2x^4}{\sqrt{3}\sqrt{1+2x^8}}\right)}{4\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.0134825, size = 29, normalized size = 0.85

$$-\frac{\tanh^{-1}\left(\frac{2x^4+1}{\sqrt{6x^8+3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((-1 + x^4)*Sqrt[1 + 2*x^8]),x]
```

```
[Out] -ArcTanh[(1 + 2*x^4)/Sqrt[3 + 6*x^8]]/(4*Sqrt[3])
```

Maple [F] time = 0.053, size = 0, normalized size = 0.

$$\int \frac{x^3}{x^4-1}\frac{1}{\sqrt{2x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^4-1)/(2*x^8+1)^(1/2),x)
```

```
[Out] int(x^3/(x^4-1)/(2*x^8+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{2x^8+1}\,(x^4-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(2*x^8 + 1)*(x^4 - 1)), x)
```

Fricas [A] time = 2.40605, size = 128, normalized size = 3.76

$$\frac{1}{12}\sqrt{3}\log\left(\frac{2x^4-\sqrt{3}(2x^4+1)-\sqrt{2x^8+1}(\sqrt{3}-3)+1}{x^4-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*log((2*x^4 - sqrt(3)*(2*x^4 + 1) - sqrt(2*x^8 + 1)*(sqrt(3) -
3) + 1)/(x^4 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(x-1)(x+1)\left(x^2+1\right)\sqrt{2x^8+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**4-1)/(2*x**8+1)**(1/2),x)
```

```
[Out] Integral(x**3/((x - 1)*(x + 1)*(x**2 + 1)*sqrt(2*x**8 + 1)), x)
```

Giac [B] time = 1.13792, size = 95, normalized size = 2.79

$$\frac{1}{12}\sqrt{3}\log\left(-\frac{\left|-2\sqrt{2}x^4-2\sqrt{3}+2\sqrt{2}+2\sqrt{2x^8+1}\right|}{2\left(\sqrt{2}x^4-\sqrt{3}-\sqrt{2}-\sqrt{2x^8+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^4-1)/(2*x^8+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*log(-1/2*abs(-2*sqrt(2)*x^4 - 2*sqrt(3) + 2*sqrt(2) + 2*sqrt(2
*x^8 + 1))/(sqrt(2)*x^4 - sqrt(3) - sqrt(2) - sqrt(2*x^8 + 1)))
```

3.309 $\int x^9\sqrt{1+x^5+x^{10}}\,dx$

Optimal. Leaf size=58

$$\frac{1}{15}\left(x^{10}+x^5+1\right)^{3/2}-\frac{1}{40}\left(2x^5+1\right)\sqrt{x^{10}+x^5+1}-\frac{3}{80}\sinh^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)$$

```
[Out] -((1 + 2*x^5)*Sqrt[1 + x^5 + x^10])/40 + (1 + x^5 + x^10)^(3/2)/15 - (3*Arc
Sinh[(1 + 2*x^5)/Sqrt[3]])/80
```

Rubi [A] time = 0.0370013, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {1357, 640, 612, 619, 215}

$$\frac{1}{15}\left(x^{10}+x^5+1\right)^{3/2}-\frac{1}{40}\left(2x^5+1\right)\sqrt{x^{10}+x^5+1}-\frac{3}{80}\sinh^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^9*Sqrt[1 + x^5 + x^10],x]
```

```
[Out] -((1 + 2*x^5)*Sqrt[1 + x^5 + x^10])/40 + (1 + x^5 + x^10)^(3/2)/15 - (3*Arc
Sinh[(1 + 2*x^5)/Sqrt[3]])/80
```

Rule 1357

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^9\sqrt{1+x^5+x^{10}}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int x\sqrt{1+x+x^2}\,dx,x,x^5\right)\\
&= \frac{1}{15}\left(1+x^5+x^{10}\right)^{3/2}-\frac{1}{10}\,\text{Subst}\left(\int \sqrt{1+x+x^2}\,dx,x,x^5\right)\\
&= -\frac{1}{40}\left(1+2x^5\right)\sqrt{1+x^5+x^{10}}+\frac{1}{15}\left(1+x^5+x^{10}\right)^{3/2}-\frac{3}{80}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}}\,dx,x,x^5\right)\\
&= -\frac{1}{40}\left(1+2x^5\right)\sqrt{1+x^5+x^{10}}+\frac{1}{15}\left(1+x^5+x^{10}\right)^{3/2}-\frac{1}{80}\sqrt{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,1+2x^5\right)\\
&= -\frac{1}{40}\left(1+2x^5\right)\sqrt{1+x^5+x^{10}}+\frac{1}{15}\left(1+x^5+x^{10}\right)^{3/2}-\frac{3}{80}\sinh^{-1}\left(\frac{1+2x^5}{\sqrt{3}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0168163, size = 47, normalized size = 0.81

$$\frac{1}{240}\left(2\sqrt{x^{10}+x^5+1}\left(8x^{10}+2x^5+5\right)-9\sinh^{-1}\left(\frac{2x^5+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*Sqrt[1 + x^5 + x^10],x]

[Out] (2*Sqrt[1 + x^5 + x^10]*(5 + 2*x^5 + 8*x^10) - 9*ArcSinh[(1 + 2*x^5)/Sqrt[3
]])/240
```

Maple [F] time = 0.025, size = 0, normalized size = 0.

$$\int x^9\sqrt{x^{10}+x^5+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(x^10+x^5+1)^(1/2),x)

[Out] int(x^9*(x^10+x^5+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^{10}+x^5+1}x^9\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(x^10+x^5+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x^10 + x^5 + 1)*x^9, x)
```

Fricas [A] time = 2.37243, size = 131, normalized size = 2.26

$$\frac{1}{120}\left(8x^{10}+2x^{5}+5\right)\sqrt{x^{10}+x^{5}+1}+\frac{3}{80}\log\left(-2x^{5}+2\sqrt{x^{10}+x^{5}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(x^10+x^5+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/120*(8*x^10 + 2*x^5 + 5)*sqrt(x^10 + x^5 + 1) + 3/80*log(-2*x^5 + 2*sqrt(
x^10 + x^5 + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^{9}\sqrt{\left(x^{2}+x+1\right)\left(x^{8}-x^{7}+x^{5}-x^{4}+x^{3}-x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(x**10+x**5+1)**(1/2),x)
```

```
[Out] Integral(x**9*sqrt((x**2 + x + 1)*(x**8 - x**7 + x**5 - x**4 + x**3 - x + 1
)), x)
```

Giac [A] time = 1.07607, size = 66, normalized size = 1.14

$$\frac{1}{120}\sqrt{x^{10}+x^{5}+1}\left(2\left(4x^{5}+1\right)x^{5}+5\right)+\frac{3}{80}\log\left(-2x^{5}+2\sqrt{x^{10}+x^{5}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(x^10+x^5+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/120*sqrt(x^10 + x^5 + 1)*(2*(4*x^5 + 1)*x^5 + 5) + 3/80*log(-2*x^5 + 2*sq
rt(x^10 + x^5 + 1) - 1)
```

3.310 $\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx$

Optimal. Leaf size=71

$$\frac{3\sqrt{x^4+2x^2+4}}{64x^2}-\frac{\sqrt{x^4+2x^2+4}}{16x^4}+\frac{1}{128}\tanh^{-1}\left(\frac{x^2+4}{2\sqrt{x^4+2x^2+4}}\right)$$

```
[Out] -Sqrt[4 + 2*x^2 + x^4]/(16*x^4) + (3*Sqrt[4 + 2*x^2 + x^4])/(64*x^2) + ArcT
anh[(4 + x^2)/(2*Sqrt[4 + 2*x^2 + x^4])]/128
```

Rubi [A] time = 0.0518186, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1114, 744, 806, 724, 206}

$$\frac{3\sqrt{x^4+2x^2+4}}{64x^2}-\frac{\sqrt{x^4+2x^2+4}}{16x^4}+\frac{1}{128}\tanh^{-1}\left(\frac{x^2+4}{2\sqrt{x^4+2x^2+4}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*Sqrt[4 + 2*x^2 + x^4]),x]
```

```
[Out] -Sqrt[4 + 2*x^2 + x^4]/(16*x^4) + (3*Sqrt[4 + 2*x^2 + x^4])/(64*x^2) + ArcT
anh[(4 + x^2)/(2*Sqrt[4 + 2*x^2 + x^4])]/128
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 - b*d*e + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 - b*d*e + a*e^2)), Int[(
d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && Ne
Q[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumS
implerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

Rule 806

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + b
*x + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2)), x] - Dist[(b*(e*f
 + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m
+ 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\sqrt{4+2x^2+x^4}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x^3\sqrt{4+2x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{4+2x^2+x^4}}{16x^4} - \frac{1}{16}\,\text{Subst}\left(\int \frac{3+x}{x^2\sqrt{4+2x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{4+2x^2+x^4}}{16x^4} + \frac{3\sqrt{4+2x^2+x^4}}{64x^2} - \frac{1}{64}\,\text{Subst}\left(\int \frac{1}{x\sqrt{4+2x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{4+2x^2+x^4}}{16x^4} + \frac{3\sqrt{4+2x^2+x^4}}{64x^2} + \frac{1}{32}\,\text{Subst}\left(\int \frac{1}{16-x^2}\,dx, x, \frac{2\left(4+x^2\right)}{\sqrt{4+2x^2+x^4}}\right)\\
&= -\frac{\sqrt{4+2x^2+x^4}}{16x^4} + \frac{3\sqrt{4+2x^2+x^4}}{64x^2} + \frac{1}{128}\tanh^{-1}\left(\frac{4+x^2}{2\sqrt{4+2x^2+x^4}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0278847, size = 55, normalized size = 0.77

$$\frac{1}{128}\left(\frac{2\sqrt{x^4+2x^2+4}\left(3x^2-4\right)}{x^4}+\tanh^{-1}\left(\frac{x^2+4}{2\sqrt{x^4+2x^2+4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*Sqrt[4 + 2*x^2 + x^4]),x]
```

```
[Out] ((2*(-4 + 3*x^2)*Sqrt[4 + 2*x^2 + x^4])/x^4 + ArcTanh[(4 + x^2)/(2*Sqrt[4 +
 2*x^2 + x^4])])/128
```

Maple [A] time = 0.01, size = 60, normalized size = 0.9

$$-\frac{1}{16\,x^4}\sqrt{x^4+2\,x^2+4}+\frac{3}{64\,x^2}\sqrt{x^4+2\,x^2+4}+\frac{1}{128}\text{Artanh}\left(\frac{2\,x^2+8}{4}\frac{1}{\sqrt{x^4+2\,x^2+4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^4+2*x^2+4)^(1/2),x)
```

```
[Out] -1/16*(x^4+2*x^2+4)^(1/2)/x^4+3/64*(x^4+2*x^2+4)^(1/2)/x^2+1/128*arctanh(1/
4*(2*x^2+8)/(x^4+2*x^2+4)^(1/2))
```

Maxima [A] time = 1.46907, size = 70, normalized size = 0.99

$$\frac{3\,\sqrt{x^4+2\,x^2+4}}{64\,x^2}-\frac{\sqrt{x^4+2\,x^2+4}}{16\,x^4}+\frac{1}{128}\,\text{arsinh}\left(\frac{1}{3}\,\sqrt{3}+\frac{4\,\sqrt{3}}{3\,x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/64*sqrt(x^4 + 2*x^2 + 4)/x^2 - 1/16*sqrt(x^4 + 2*x^2 + 4)/x^4 + 1/128*arc
sinh(1/3*sqrt(3) + 4/3*sqrt(3)/x^2)
```

Fricas [A] time = 2.36044, size = 196, normalized size = 2.76

$$\frac{x^4 \log\left(-x^2 + \sqrt{x^4 + 2\,x^2 + 4} + 2\right) - x^4 \log\left(-x^2 + \sqrt{x^4 + 2\,x^2 + 4} - 2\right) + 6\,x^4 + 2\,\sqrt{x^4 + 2\,x^2 + 4}\left(3\,x^2 - 4\right)}{128\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/128*(x^4*log(-x^2 + sqrt(x^4 + 2*x^2 + 4) + 2) - x^4*log(-x^2 + sqrt(x^4
+ 2*x^2 + 4) - 2) + 6*x^4 + 2*sqrt(x^4 + 2*x^2 + 4)*(3*x^2 - 4))/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^5\sqrt{x^4 + 2x^2 + 4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**4+2*x**2+4)**(1/2),x)
```

```
[Out] Integral(1/(x**5*sqrt(x**4 + 2*x**2 + 4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^4 + 2\,x^2 + 4}x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^4+2*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^4 + 2*x^2 + 4)*x^5), x)
```

3.311 $\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}} \, dx$

Optimal. Leaf size=21

$$\tanh^{-1}\left(\frac{x^2+1}{\sqrt{x^4+3x^2+1}}\right)$$

```
[Out] ArcTanh[(1 + x^2)/Sqrt[1 + 3*x^2 + x^4]]
```

Rubi [A] time = 0.0291543, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1251, 838, 206}

$$\tanh^{-1}\left(\frac{x^2+1}{\sqrt{x^4+3x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(x*Sqrt[1 + 3*x^2 + x^4]),x]
```

```
[Out] ArcTanh[(1 + x^2)/Sqrt[1 + 3*x^2 + x^4]]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 838

```
Int[((f_) + (g_.)*(x_))/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_
.)*(x_)^2]), x_Symbol] :> Dist[(4*f*(a - d))/(b*d - a*e), Subst[Int[1/(4*(a
 - d) - x^2), x], x, (2*(a - d) + (b - e)*x)/Sqrt[a + b*x + c*x^2]], x] /;
FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[4*c*(a - d) - (b - e)^2, 0] && EqQ[e
*f*(b - e) - 2*g*(b*d - a*e), 0] && NeQ[b*d - a*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^2}{x\sqrt{1+3x^2+x^4}} \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{-1+x}{x\sqrt{1+3x+x^2}} \, dx, x, x^2\right)\\ &= 2\operatorname{Subst}\left(\int \frac{1}{4-x^2} \, dx, x, \frac{2\left(1+x^2\right)}{\sqrt{1+3x^2+x^4}}\right)\\ &= \tanh^{-1}\left(\frac{1+x^2}{\sqrt{1+3x^2+x^4}}\right)\end{aligned}$$

Mathematica [B] time = 0.0153564, size = 57, normalized size = 2.71

$$\frac{1}{2}\left(\tanh^{-1}\left(\frac{2x^2+3}{2\sqrt{x^4+3x^2+1}}\right)+\tanh^{-1}\left(\frac{3x^2+2}{2\sqrt{x^4+3x^2+1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(x*Sqrt[1 + 3*x^2 + x^4]),x]
```

```
[Out] (ArcTanh[(3 + 2*x^2)/(2*Sqrt[1 + 3*x^2 + x^4])] + ArcTanh[(2 + 3*x^2)/(2*Sq
rt[1 + 3*x^2 + x^4])])/2
```

Maple [B] time = 0.017, size = 46, normalized size = 2.2

$$\frac{1}{2}\text{Artanh}\left(\frac{3x^2+2}{2}\frac{1}{\sqrt{x^4+3x^2+1}}\right)+\frac{1}{2}\ln\left(x^2+\frac{3}{2}+\sqrt{x^4+3x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x)
```

```
[Out] 1/2*arctanh(1/2*(3*x^2+2)/(x^4+3*x^2+1)^(1/2))+1/2*ln(x^2+3/2+(x^4+3*x^2+1)
^(1/2))
```

Maxima [B] time = 0.978983, size = 70, normalized size = 3.33

$$\frac{1}{2}\log\left(2x^2+2\sqrt{x^4+3x^2+1}+3\right)+\frac{1}{2}\log\left(\frac{2\sqrt{x^4+3x^2+1}}{x^2}+\frac{2}{x^2}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*log(2*x^2 + 2*sqrt(x^4 + 3*x^2 + 1) + 3) + 1/2*log(2*sqrt(x^4 + 3*x^2 +
 1)/x^2 + 2/x^2 + 3)
```

Fricas [B] time = 2.13552, size = 149, normalized size = 7.1

$$-\frac{1}{2}\log\left(4x^4+11x^2-\sqrt{x^4+3x^2+1}\left(4x^2+5\right)+5\right)+\frac{1}{2}\log\left(-x^2+\sqrt{x^4+3x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(4*x^4 + 11*x^2 - sqrt(x^4 + 3*x^2 + 1)*(4*x^2 + 5) + 5) + 1/2*log(
-x^2 + sqrt(x^4 + 3*x^2 + 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x-1)(x+1)}{x\sqrt{x^4+3x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/x/(x**4+3*x**2+1)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(x*sqrt(x**4 + 3*x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2-1}{\sqrt{x^4+3\,x^2+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^4+3*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^4 + 3*x^2 + 1)*x), x)
```

3.312 $\int \left(-3x+2x^3\right)\left(-3x^2+x^4\right)^{3/5}\,dx$

Optimal. Leaf size=17

$$\frac{5}{16}\left(x^4-3x^2\right)^{8/5}$$

```
[Out] (5*(-3*x^2 + x^4)^(8/5))/16
```

Rubi [A] time = 0.0102869, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1588}

$$\frac{5}{16}\left(x^4-3x^2\right)^{8/5}$$

Antiderivative was successfully verified.

```
[In] Int[(-3*x + 2*x^3)*(-3*x^2 + x^4)^(3/5),x]
```

```
[Out] (5*(-3*x^2 + x^4)^(8/5))/16
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \left(-3x+2x^3\right)\left(-3x^2+x^4\right)^{3/5}\,dx=\frac{5}{16}\left(-3x^2+x^4\right)^{8/5}$$

Mathematica [C] time = 0.0545341, size = 75, normalized size = 4.41

$$\frac{5\left(x^2\left(x^2-3\right)\right)^{3/5}\left(16x^4\,{}_2F_1\left(-\frac{3}{5},\frac{13}{5};\frac{18}{5};\frac{x^2}{3}\right)-39x^2\,{}_2F_1\left(-\frac{3}{5},\frac{8}{5};\frac{13}{5};\frac{x^2}{3}\right)\right)}{208\left(1-\frac{x^2}{3}\right)^{3/5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3*x + 2*x^3)*(-3*x^2 + x^4)^(3/5),x]
```

```
[Out] (5*(x^2*(-3 + x^2))^(3/5)*(-39*x^2*Hypergeometric2F1[-3/5, 8/5, 13/5, x^2/3
] + 16*x^4*Hypergeometric2F1[-3/5, 13/5, 18/5, x^2/3]))/(208*(1 - x^2/3)^(3
/5))
```

Maple [A] time = 0.005, size = 22, normalized size = 1.3

$$\frac{5x^2(x^2-3)}{16}(x^4-3x^2)^{\frac{3}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x)
```

```
[Out] 5/16*(x^4-3*x^2)^(3/5)*x^2*(x^2-3)
```

Maxima [A] time = 0.964498, size = 18, normalized size = 1.06

$$\frac{5}{16}(x^4-3x^2)^{\frac{8}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x, algorithm="maxima")
```

```
[Out] 5/16*(x^4 - 3*x^2)^(8/5)
```

Fricas [A] time = 1.9955, size = 35, normalized size = 2.06

$$\frac{5}{16}(x^4-3x^2)^{\frac{8}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x, algorithm="fricas")
```

```
[Out] 5/16*(x^4 - 3*x^2)^(8/5)
```

Sympy [B] time = 1.24886, size = 36, normalized size = 2.12

$$\frac{5x^4(x^4-3x^2)^{\frac{3}{5}}}{16}-\frac{15x^2(x^4-3x^2)^{\frac{3}{5}}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-3*x)*(x**4-3*x**2)**(3/5),x)
```

```
[Out] 5*x**4*(x**4 - 3*x**2)**(3/5)/16 - 15*x**2*(x**4 - 3*x**2)**(3/5)/16
```

Giac [A] time = 1.08591, size = 18, normalized size = 1.06

$$\frac{5}{16}(x^4-3x^2)^{\frac{8}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-3*x)*(x^4-3*x^2)^(3/5),x, algorithm="giac")
```

```
[Out] 5/16*(x^4 - 3*x^2)^(8/5)
```

3.313 $\int \frac{-2x^5+3x^8-x^2\left(-1+3x^3\right)^{2/3}}{\left(-1+3x^3\right)^{3/4}}\,dx$

Optimal. Leaf size=46

$$\frac{4}{243}\left(3x^3-1\right)^{9/4}-\frac{4}{33}\left(3x^3-1\right)^{11/12}-\frac{4}{27}\sqrt[4]{3x^3-1}$$

```
[Out] (-4*(-1 + 3*x^3)^(1/4))/27 - (4*(-1 + 3*x^3)^(11/12))/33 + (4*(-1 + 3*x^3)^
(9/4))/243
```

Rubi [A] time = 0.193269, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {6742, 266, 43, 261}

$$\frac{4}{243}\left(3x^3-1\right)^{9/4}-\frac{4}{33}\left(3x^3-1\right)^{11/12}-\frac{4}{27}\sqrt[4]{3x^3-1}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*x^5 + 3*x^8 - x^2*(-1 + 3*x^3)^(2/3))/(-1 + 3*x^3)^(3/4),x]
```

```
[Out] (-4*(-1 + 3*x^3)^(1/4))/27 - (4*(-1 + 3*x^3)^(11/12))/33 + (4*(-1 + 3*x^3)^
(9/4))/243
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{-2x^5+3x^8-x^2\left(-1+3x^3\right)^{2/3}}{\left(-1+3x^3\right)^{3/4}}\,dx = \int \left(-\frac{2x^5}{\left(-1+3x^3\right)^{3/4}}+\frac{3x^8}{\left(-1+3x^3\right)^{3/4}}-\frac{x^2}{\sqrt[12]{-1+3x^3}}\right)dx$$
$$= -\left(2\int \frac{x^5}{\left(-1+3x^3\right)^{3/4}}\,dx\right)+3\int \frac{x^8}{\left(-1+3x^3\right)^{3/4}}\,dx-\int \frac{x^2}{\sqrt[12]{-1+3x^3}}\,dx$$
$$= -\frac{4}{33}\left(-1+3x^3\right)^{11/12}-\frac{2}{3}\text{Subst}\left(\int \frac{x}{(-1+3x)^{3/4}}\,dx,x,x^3\right)+\text{Subst}\left(\int \frac{x^2}{(-1+3x)^{3/4}}\right.$$
$$= -\frac{4}{33}\left(-1+3x^3\right)^{11/12}-\frac{2}{3}\text{Subst}\left(\int \left(\frac{1}{3(-1+3x)^{3/4}}+\frac{1}{3}\sqrt[4]{-1+3x}\right)dx,x,x^3\right)+\text{Su}$$
$$= -\frac{4}{27}\sqrt[4]{-1+3x^3}-\frac{4}{33}\left(-1+3x^3\right)^{11/12}+\frac{4}{243}\left(-1+3x^3\right)^{9/4}$$

Mathematica [A] time = 0.0805153, size = 40, normalized size = 0.87

$$-\frac{4\sqrt[4]{3x^3-1}\left(-99x^6+66x^3+81\left(3x^3-1\right)^{2/3}+88\right)}{2673}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*x^5 + 3*x^8 - x^2*(-1 + 3*x^3)^(2/3))/(-1 + 3*x^3)^(3/4),x]
```

```
[Out] (-4*(-1 + 3*x^3)^(1/4)*(88 + 66*x^3 - 99*x^6 + 81*(-1 + 3*x^3)^(2/3)))/2673
```

Maple [C] time = 0.067, size = 116, normalized size = 2.5

$$-\frac{x^6}{3}\left(-\text{signum}\left(3x^3-1\right)\right)^{\frac{3}{4}}{}_2F_1(\frac{3}{4},2;3;3x^3)\left(\text{signum}\left(3x^3-1\right)\right)^{-\frac{3}{4}}+\frac{x^9}{3}\left(-\text{signum}\left(3x^3-1\right)\right)^{\frac{3}{4}}{}_2F_1(\frac{3}{4},3;4;3x^3)\left(\text{sign}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x)
```

```
[Out] -1/3/signum(3*x^3-1)^(3/4)*(-signum(3*x^3-1))^(3/4)*x^6*hypergeom([3/4,2],[
3],3*x^3)+1/3/signum(3*x^3-1)^(3/4)*(-signum(3*x^3-1))^(3/4)*x^9*hypergeom(
[3/4,3],[4],3*x^3)-1/3/signum(3*x^3-1)^(1/12)*(-signum(3*x^3-1))^(1/12)*x^3
*hypergeom([1/12,1],[2],3*x^3)
```

Maxima [A] time = 0.972361, size = 46, normalized size = 1.

$$\frac{4}{243}\left(3x^3-1\right)^{\frac{9}{4}}-\frac{4}{33}\left(3x^3-1\right)^{\frac{11}{12}}-\frac{4}{27}\left(3x^3-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x, algorithm="
maxima")
```

```
[Out] 4/243*(3*x^3 - 1)^(9/4) - 4/33*(3*x^3 - 1)^(11/12) - 4/27*(3*x^3 - 1)^(1/4)
```

Fricas [A] time = 2.03497, size = 97, normalized size = 2.11

$$\frac{4}{243}\left(9x^6-6x^3-8\right)\left(3x^3-1\right)^{\frac{1}{4}}-\frac{4}{33}\left(3x^3-1\right)^{\frac{11}{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x, algorithm="
fricas")
```

```
[Out] 4/243*(9*x^6 - 6*x^3 - 8)*(3*x^3 - 1)^(1/4) - 4/33*(3*x^3 - 1)^(11/12)
```

Sympy [C] time = 3.08764, size = 221, normalized size = 4.8

$$-\frac{4\left(3x^3-1\right)^{\frac{11}{12}}}{33}-2\left(\begin{cases}\frac{4x^3\sqrt[4]{3x^3-1}}{45}+\frac{16\sqrt[4]{3x^3-1}}{135} & \text{for } 3\left|x^3\right|>1\\ -\frac{4x^3\sqrt[4]{1-3x^3}e^{-\frac{3i\pi}{4}}}{45}-\frac{16\sqrt[4]{1-3x^3}e^{-\frac{3i\pi}{4}}}{135} & \text{otherwise}\end{cases}\right)+3\left(\begin{cases}\frac{4x^6\sqrt[4]{3x^3-1}}{81}+\frac{32x^3\sqrt[4]{3x^3-1}}{1215}+\frac{128\sqrt[4]{3x^3-1}}{3645}\\ \frac{4x^6\sqrt[4]{1-3x^3}e^{\frac{i\pi}{4}}}{81}+\frac{32x^3\sqrt[4]{1-3x^3}e^{\frac{i\pi}{4}}}{1215}+\frac{128\sqrt[4]{1-3x^3}e}{3645}\end{cases}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**5+3*x**8-x**2*(3*x**3-1)**(2/3))/(3*x**3-1)**(3/4),x)
```

```
[Out] -4*(3*x**3 - 1)**(11/12)/33 - 2*Piecewise((4*x**3*(3*x**3 - 1)**(1/4)/45 +
16*(3*x**3 - 1)**(1/4)/135, 3*Abs(x**3) > 1), (-4*x**3*(1 - 3*x**3)**(1/4)*
exp(-3*I*pi/4)/45 - 16*(1 - 3*x**3)**(1/4)*exp(-3*I*pi/4)/135, True)) + 3*P
iecewise((4*x**6*(3*x**3 - 1)**(1/4)/81 + 32*x**3*(3*x**3 - 1)**(1/4)/1215
+ 128*(3*x**3 - 1)**(1/4)/3645, 3*Abs(x**3) > 1), (4*x**6*(1 - 3*x**3)**(1/
4)*exp(I*pi/4)/81 + 32*x**3*(1 - 3*x**3)**(1/4)*exp(I*pi/4)/1215 + 128*(1 -
 3*x**3)**(1/4)*exp(I*pi/4)/3645, True))
```

Giac [A] time = 1.07351, size = 46, normalized size = 1.

$$\frac{4}{243}\left(3x^3-1\right)^{\frac{9}{4}}-\frac{4}{33}\left(3x^3-1\right)^{\frac{11}{12}}-\frac{4}{27}\left(3x^3-1\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^5+3*x^8-x^2*(3*x^3-1)^(2/3))/(3*x^3-1)^(3/4),x, algorithm="
giac")
```

```
[Out] 4/243*(3*x^3 - 1)^(9/4) - 4/33*(3*x^3 - 1)^(11/12) - 4/27*(3*x^3 - 1)^(1/4)
```

3.314 $\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}}\,dx$

Optimal. Leaf size=78

$$-\frac{\log\left(x^3-1\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}x-\sqrt[3]{x^3+2}\right)}{2\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}$$

```
[Out] -(ArcTan[(1 + (2*3^(1/3)*x)/(2 + x^3)^(1/3))/Sqrt[3]]/3^(5/6)) - Log[-1 + x
^3]/(6*3^(1/3)) + Log[3^(1/3)*x - (2 + x^3)^(1/3)]/(2*3^(1/3))
```

Rubi [A] time = 0.0708598, antiderivative size = 107, normalized size of antiderivative = 1.37, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {377, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(1-\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(\frac{3^{2/3}x^2}{\left(x^3+2\right)^{2/3}}+\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+1\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{2x}{\sqrt[6]{3}\sqrt[3]{x^3+2}}+\frac{1}{\sqrt{3}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x^3)*(2 + x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2*x)/(3^(1/6)*(2 + x^3)^(1/3))]/3^(5/6)) + Log[1 - (3
^(1/3)*x)/(2 + x^3)^(1/3)]/(3*3^(1/3)) - Log[1 + (3^(2/3)*x^2)/(2 + x^3)^(2
/3) + (3^(1/3)*x)/(2 + x^3)^(1/3)]/(6*3^(1/3))
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(-1+x^3\right)\sqrt[3]{2+x^3}}\,dx &= \text{Subst}\left(\int \frac{1}{-1+3x^3}\,dx, x, \frac{x}{\sqrt[3]{2+x^3}}\right)\\
&= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+\sqrt[3]{3}x}\,dx, x, \frac{x}{\sqrt[3]{2+x^3}}\right)+\frac{1}{3}\text{Subst}\left(\int \frac{-2-\sqrt[3]{3}x}{1+\sqrt[3]{3}x+3^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{2+x^3}}\right)\\
&= \frac{\log\left(1-\frac{\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}\right)}{3\sqrt[3]{3}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{3}x+3^{2/3}x^2}\,dx, x, \frac{x}{\sqrt[3]{2+x^3}}\right)-\frac{\text{Subst}\left(\int \frac{\sqrt[3]{3}+2\ 3^{2/3}x}{1+\sqrt[3]{3}x+3^{2/3}x^2}\right.}{6\sqrt[3]{3}}\\
&= \frac{\log\left(1-\frac{\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(1+\frac{3^{2/3}x^2}{\left(2+x^3\right)^{2/3}}+\frac{\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}\right)}{6\sqrt[3]{3}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}\right)}{\sqrt[3]{3}}\\
&= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3^{5/6}}+\frac{\log\left(1-\frac{\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(1+\frac{3^{2/3}x^2}{\left(2+x^3\right)^{2/3}}+\frac{\sqrt[3]{3}x}{\sqrt[3]{2+x^3}}\right)}{6\sqrt[3]{3}}
\end{aligned}$$

Mathematica [A] time = 0.0766959, size = 104, normalized size = 1.33

$$\frac{\sqrt{3}\left(2\log\left(1-\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}\right)-\log\left(\frac{3^{2/3}x^2}{\left(x^3+2\right)^{2/3}}+\frac{\sqrt[3]{3}x}{\sqrt[3]{x^3+2}}+1\right)\right)-6\tan^{-1}\left(\frac{2x}{\sqrt[6]{3}\sqrt[3]{x^3+2}}+\frac{1}{\sqrt{3}}\right)}{6\ 3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x^3)*(2 + x^3)^(1/3)),x]
```

```
[Out] (-6*ArcTan[1/Sqrt[3] + (2*x)/(3^(1/6)*(2 + x^3)^(1/3))] + Sqrt[3]*(2*Log[1
- (3^(1/3)*x)/(2 + x^3)^(1/3)] - Log[1 + (3^(2/3)*x^2)/(2 + x^3)^(2/3) + (3
^(1/3)*x)/(2 + x^3)^(1/3)]))/(6*3^(5/6))
```

Maple [F] time = 0.042, size = 0, normalized size = 0.

$$\int \frac{1}{x^3-1}\frac{1}{\sqrt[3]{x^3+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-1)/(x^3+2)^(1/3),x)
```

```
[Out] int(1/(x^3-1)/(x^3+2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+2\right)^{\frac{1}{3}}\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)/(x^3+2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 2)^(1/3)*(x^3 - 1)), x)
```

Fricas [B] time = 27.4718, size = 624, normalized size = 8.

$$\frac{1}{27}\cdot 3^{\frac{2}{3}}\log\left(\frac{9\cdot 3^{\frac{1}{3}}\left(x^3+2\right)^{\frac{1}{3}}x^2-2\cdot 3^{\frac{2}{3}}\left(x^3-1\right)-9\left(x^3+2\right)^{\frac{2}{3}}x}{x^3-1}\right)-\frac{1}{54}\cdot 3^{\frac{2}{3}}\log\left(\frac{3\cdot 3^{\frac{2}{3}}\left(7x^4+2x\right)\left(x^3+2\right)^{\frac{2}{3}}+3^{\frac{1}{3}}\left(31x^6+4\right.}{x^6-2x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/27*3^(2/3)*log((9*3^(1/3)*(x^3 + 2)^(1/3)*x^2 - 2*3^(2/3)*(x^3 - 1) - 9*(
x^3 + 2)^(2/3)*x)/(x^3 - 1)) - 1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 2*x)*(x
^3 + 2)^(2/3) + 3^(1/3)*(31*x^6 + 46*x^3 + 4) + 9*(5*x^5 + 4*x^2)*(x^3 + 2)
^(1/3))/(x^6 - 2*x^3 + 1)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)*(7*
x^7 - 5*x^4 - 2*x)*(x^3 + 2)^(2/3) - 3^(1/3)*(127*x^9 + 402*x^6 + 192*x^3 +
 8) - 18*(31*x^8 + 46*x^5 + 4*x^2)*(x^3 + 2)^(1/3))/(251*x^9 + 462*x^6 + 24
*x^3 - 8))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x-1)\sqrt[3]{x^3+2}\left(x^2+x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-1)/(x**3+2)**(1/3),x)
```

```
[Out] Integral(1/((x - 1)*(x**3 + 2)**(1/3)*(x**2 + x + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+2\right)^{\frac{1}{3}}\left(x^3-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1)/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 2)^(1/3)*(x^3 - 1)), x)
```

3.315 $\int \frac{1}{(1+x^4)\sqrt[4]{2+x^4}}\,dx$

Optimal. Leaf size=141

$$-\frac{\log\left(\frac{x^2}{\sqrt{x^4+2}}-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{4\sqrt{2}}+\frac{\log\left(\frac{x^2}{\sqrt{x^4+2}}+\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{2\sqrt{2}}$$

```
[Out] -ArcTan[1 - (Sqrt[2]*x)/(2 + x^4)^(1/4)]/(2*Sqrt[2]) + ArcTan[1 + (Sqrt[2]*
x)/(2 + x^4)^(1/4)]/(2*Sqrt[2]) - Log[1 + x^2/Sqrt[2 + x^4] - (Sqrt[2]*x)/(
2 + x^4)^(1/4)]/(4*Sqrt[2]) + Log[1 + x^2/Sqrt[2 + x^4] + (Sqrt[2]*x)/(2 +
x^4)^(1/4)]/(4*Sqrt[2])
```

Rubi [A] time = 0.0602254, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {377, 211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(\frac{x^2}{\sqrt{x^4+2}}-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{4\sqrt{2}}+\frac{\log\left(\frac{x^2}{\sqrt{x^4+2}}+\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^4)*(2 + x^4)^(1/4)),x]
```

```
[Out] -ArcTan[1 - (Sqrt[2]*x)/(2 + x^4)^(1/4)]/(2*Sqrt[2]) + ArcTan[1 + (Sqrt[2]*
x)/(2 + x^4)^(1/4)]/(2*Sqrt[2]) - Log[1 + x^2/Sqrt[2 + x^4] - (Sqrt[2]*x)/(
2 + x^4)^(1/4)]/(4*Sqrt[2]) + Log[1 + x^2/Sqrt[2 + x^4] + (Sqrt[2]*x)/(2 +
x^4)^(1/4)]/(4*Sqrt[2])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(1+x^4\right)\sqrt[4]{2+x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^4}\,dx, x, \frac{x}{\sqrt[4]{2+x^4}}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \frac{x}{\sqrt[4]{2+x^4}}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \frac{x}{\sqrt[4]{2+x^4}}\right)\\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \frac{x}{\sqrt[4]{2+x^4}}\right)+\frac{1}{4}\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \frac{x}{\sqrt[4]{2+x^4}}\right)-\\
&= -\frac{\log\left(1+\frac{x^2}{\sqrt{2+x^4}}-\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}}+\frac{\log\left(1+\frac{x^2}{\sqrt{2+x^4}}+\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{2\sqrt{2}}-\\
&= -\frac{\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{2\sqrt{2}}-\frac{\log\left(1+\frac{x^2}{\sqrt{2+x^4}}-\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}}+\frac{\log\left(1+\frac{x^2}{\sqrt{2+x^4}}+\frac{\sqrt{2}x}{\sqrt[4]{2+x^4}}\right)}{4\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0484045, size = 120, normalized size = 0.85

$$
\frac{-\log\left(\frac{x^2}{\sqrt{x^4+2}}-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)+\log\left(\frac{x^2}{\sqrt{x^4+2}}+\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)-2\tan^{-1}\left(1-\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}\right)+2\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt[4]{x^4+2}}+1\right)}{4\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^4)*(2 + x^4)^(1/4)),x]
```

```
[Out] (-2*ArcTan[1 - (Sqrt[2]*x)/(2 + x^4)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*x)/(2 +
 x^4)^(1/4)] - Log[1 + x^2/Sqrt[2 + x^4] - (Sqrt[2]*x)/(2 + x^4)^(1/4)] + L
og[1 + x^2/Sqrt[2 + x^4] + (Sqrt[2]*x)/(2 + x^4)^(1/4)])/(4*Sqrt[2])
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \frac{1}{x^4+1}\frac{1}{\sqrt[4]{x^4+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1)/(x^4+2)^(1/4),x)
```

```
[Out] int(1/(x^4+1)/(x^4+2)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+2\right)^{\frac{1}{4}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(x^4+2)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 2)^(1/4)*(x^4 + 1)), x)
```

Fricas [B] time = 49.4755, size = 1062, normalized size = 7.53

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(x^4+2\right)^{\frac{3}{4}}x^2-\sqrt{2}\left(x^4+2\right)^{\frac{5}{4}}-\left(2\,x^5-\sqrt{2}\left(x^4+2\right)^{\frac{3}{4}}x^2-\sqrt{2}\left(x^4+2\right)^{\frac{5}{4}}+4\,x\right)\sqrt{\frac{x^4+\sqrt{2}\left(x^4+2\right)^{\frac{1}{4}}x^3+2\sqrt{x^4+2}x^2+\sqrt{}}{x^4+1}}}{2\left(x^5+2\,x\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(x^4+2)^(1/4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*(sqrt(2)*(x^4 + 2)^(3/4)*x^2 - sqrt(2)*(x^4 + 2)^(5/
4) - (2*x^5 - sqrt(2)*(x^4 + 2)^(3/4)*x^2 - sqrt(2)*(x^4 + 2)^(5/4) + 4*x)*
sqrt((x^4 + sqrt(2)*(x^4 + 2)^(1/4)*x^3 + 2*sqrt(x^4 + 2)*x^2 + sqrt(2)*(x^
4 + 2)^(3/4)*x + 1)/(x^4 + 1)))/(x^5 + 2*x)) + 1/4*sqrt(2)*arctan(1/2*(sqrt
(2)*(x^4 + 2)^(3/4)*x^2 - sqrt(2)*(x^4 + 2)^(5/4) + (2*x^5 + sqrt(2)*(x^4 +
 2)^(3/4)*x^2 + sqrt(2)*(x^4 + 2)^(5/4) + 4*x)*sqrt((x^4 - sqrt(2)*(x^4 + 2
)^(1/4)*x^3 + 2*sqrt(x^4 + 2)*x^2 - sqrt(2)*(x^4 + 2)^(3/4)*x + 1)/(x^4 + 1
)))/(x^5 + 2*x)) + 1/16*sqrt(2)*log(4*(x^4 + sqrt(2)*(x^4 + 2)^(1/4)*x^3 +
2*sqrt(x^4 + 2)*x^2 + sqrt(2)*(x^4 + 2)^(3/4)*x + 1)/(x^4 + 1)) - 1/16*sqrt
(2)*log(4*(x^4 - sqrt(2)*(x^4 + 2)^(1/4)*x^3 + 2*sqrt(x^4 + 2)*x^2 - sqrt(2
)*(x^4 + 2)^(3/4)*x + 1)/(x^4 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+1\right)\sqrt[4]{x^4+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)/(x**4+2)**(1/4),x)
```

```
[Out] Integral(1/((x**4 + 1)*(x**4 + 2)**(1/4)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+2\right)^{\frac{1}{4}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(x^4+2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 2)^(1/4)*(x^4 + 1)), x)
```

3.316 $\int \frac{-1+x^3}{\sqrt[3]{2+x^3}} \, dx$

Optimal. Leaf size=63

$$\frac{1}{3}\left(x^3+2\right)^{2/3}x+\frac{5}{6}\log\left(\sqrt[3]{x^3+2}-x\right)-\frac{5\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (x*(2 + x^3)^(2/3))/3 - (5*ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]])/(3*
Sqrt[3]) + (5*Log[-x + (2 + x^3)^(1/3)])/6
```

Rubi [A] time = 0.0112845, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {388, 239}

$$\frac{1}{3}\left(x^3+2\right)^{2/3}x+\frac{5}{6}\log\left(\sqrt[3]{x^3+2}-x\right)-\frac{5\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)/(2 + x^3)^(1/3),x]
```

```
[Out] (x*(2 + x^3)^(2/3))/3 - (5*ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3]])/(3*
Sqrt[3]) + (5*Log[-x + (2 + x^3)^(1/3)])/6
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]
*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^
3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x^3}{\sqrt[3]{2+x^3}} \, dx &= \frac{1}{3}x\left(2+x^3\right)^{2/3}-\frac{5}{3}\int \frac{1}{\sqrt[3]{2+x^3}} \, dx\\ &= \frac{1}{3}x\left(2+x^3\right)^{2/3}-\frac{5\tan^{-1}\left(\frac{1+\frac{2x}{\sqrt[3]{2+x^3}}}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{5}{6}\log\left(-x+\sqrt[3]{2+x^3}\right)\end{aligned}$$

Mathematica [A] time = 0.0338833, size = 91, normalized size = 1.44

$$\frac{1}{18}\left(6\left(x^3+2\right)^{2/3}x+10\log\left(1-\frac{x}{\sqrt[3]{x^3+2}}\right)-5\log\left(\frac{x^2}{\left(x^3+2\right)^{2/3}}+\frac{x}{\sqrt[3]{x^3+2}}+1\right)-10\sqrt{3}\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+2}}+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)/(2 + x^3)^(1/3),x]
```

```
[Out] (6*x*(2 + x^3)^(2/3) - 10*Sqrt[3]*ArcTan[(1 + (2*x)/(2 + x^3)^(1/3))/Sqrt[3
]] + 10*Log[1 - x/(2 + x^3)^(1/3)] - 5*Log[1 + x^2/(2 + x^3)^(2/3) + x/(2 +
 x^3)^(1/3)])/18
```

Maple [C] time = 0.027, size = 29, normalized size = 0.5

$$\frac{x}{3}\left(x^3+2\right)^{\frac{2}{3}}-\frac{5\,x2^{2/3}}{6}{}_2F_1\left(\frac{1}{3},\frac{1}{3};\frac{4}{3};-\frac{x^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-1)/(x^3+2)^(1/3),x)
```

```
[Out] 1/3*x*(x^3+2)^(2/3)-5/6*2^(2/3)*x*hypergeom([1/3,1/3],[4/3],-1/2*x^3)
```

Maxima [A] time = 1.471, size = 127, normalized size = 2.02

$$\frac{5}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(x^3+2\right)^{\frac{1}{3}}}{x}+1\right)\right)+\frac{2\left(x^3+2\right)^{\frac{2}{3}}}{3x^2\left(\frac{x^3+2}{x^3}-1\right)}-\frac{5}{18}\log\left(\frac{\left(x^3+2\right)^{\frac{1}{3}}}{x}+\frac{\left(x^3+2\right)^{\frac{2}{3}}}{x^2}+1\right)+\frac{5}{9}\log\left(\frac{\left(x^3+2\right)^{\frac{1}{3}}}{x}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="maxima")
```

```
[Out] 5/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(x^3 + 2)^(1/3)/x + 1)) + 2/3*(x^3 + 2)^(
2/3)/(x^2*((x^3 + 2)/x^3 - 1)) - 5/18*log((x^3 + 2)^(1/3)/x + (x^3 + 2)^(2/
3)/x^2 + 1) + 5/9*log((x^3 + 2)^(1/3)/x - 1)
```

Fricas [A] time = 1.91578, size = 252, normalized size = 4.

$$\frac{1}{3}\left(x^3+2\right)^{\frac{2}{3}}x+\frac{5}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}x+2\sqrt{3}\left(x^3+2\right)^{\frac{1}{3}}}{3x}\right)+\frac{5}{9}\log\left(-\frac{x-\left(x^3+2\right)^{\frac{1}{3}}}{x}\right)-\frac{5}{18}\log\left(\frac{x^2+\left(x^3+2\right)^{\frac{1}{3}}x+\left(x^3+2\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + 2)^(2/3)*x + 5/9*sqrt(3)*arctan(1/3*(sqrt(3)*x + 2*sqrt(3)*(x^3
+ 2)^(1/3))/x) + 5/9*log(-(x - (x^3 + 2)^(1/3))/x) - 5/18*log((x^2 + (x^3 +
```

```
2)^(1/3)*x + (x^3 + 2)^(2/3))/x^2)
```

Sympy [C] time = 1.82774, size = 71, normalized size = 1.13

$$\frac{2^{\frac{2}{3}}x^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{4}{3}\\\frac{7}{3}\end{matrix}\middle|\frac{x^3e^{i\pi}}{2}\right)}{6\Gamma\left(\frac{7}{3}\right)}-\frac{2^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{3}\\\frac{4}{3}\end{matrix}\middle|\frac{x^3e^{i\pi}}{2}\right)}{6\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-1)/(x**3+2)**(1/3),x)
```

```
[Out] 2**(2/3)*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), x**3*exp_polar(I*pi)/2)/
(6*gamma(7/3)) - 2**(2/3)*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), x**3*exp_p
olar(I*pi)/2)/(6*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3-1}{\left(x^3+2\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-1)/(x^3+2)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((x^3 - 1)/(x^3 + 2)^(1/3), x)
```

3.317 $\int \frac{(1+x^4)^{3/4}}{(2+x^4)^2}\,dx$

Optimal. Leaf size=74

$$\frac{(x^4+1)^{3/4}x}{8(x^4+2)}+\frac{3\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{16\ 2^{3/4}}+\frac{3\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{16\ 2^{3/4}}$$

```
[Out] (x*(1 + x^4)^(3/4))/(8*(2 + x^4)) + (3*ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))])
/(16*2^(3/4)) + (3*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))])/(16*2^(3/4))
```

Rubi [A] time = 0.0243437, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {378, 377, 212, 206, 203}

$$\frac{(x^4+1)^{3/4}x}{8(x^4+2)}+\frac{3\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{16\ 2^{3/4}}+\frac{3\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{16\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)^(3/4)/(2 + x^4)^2,x]
```

```
[Out] (x*(1 + x^4)^(3/4))/(8*(2 + x^4)) + (3*ArcTan[x/(2^(1/4)*(1 + x^4)^(1/4))])
/(16*2^(3/4)) + (3*ArcTanh[x/(2^(1/4)*(1 + x^4)^(1/4))])/(16*2^(3/4))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1+x^4\right)^{3/4}}{\left(2+x^4\right)^2}\,dx &= \frac{x\left(1+x^4\right)^{3/4}}{8\left(2+x^4\right)}+\frac{3}{8}\int \frac{1}{\sqrt[4]{1+x^4}\left(2+x^4\right)}\,dx\\ &= \frac{x\left(1+x^4\right)^{3/4}}{8\left(2+x^4\right)}+\frac{3}{8}\,\text{Subst}\left(\int \frac{1}{2-x^4}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)\\ &= \frac{x\left(1+x^4\right)^{3/4}}{8\left(2+x^4\right)}+\frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{2}-x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)}{16\sqrt{2}}+\frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{2}+x^2}\,dx,x,\frac{x}{\sqrt[4]{1+x^4}}\right)}{16\sqrt{2}}\\ &= \frac{x\left(1+x^4\right)^{3/4}}{8\left(2+x^4\right)}+\frac{3\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\ 2^{3/4}}+\frac{3\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{1+x^4}}\right)}{16\ 2^{3/4}}\end{aligned}$$

Mathematica [C] time = 0.0097326, size = 41, normalized size = 0.55

$$\frac{x\,{}_2F_1\left(-\frac{3}{4},\frac{1}{4};\frac{5}{4};-\frac{x^4}{x^4+2}\right)}{2\ 2^{3/4}\sqrt[4]{x^4+2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^4)^(3/4)/(2 + x^4)^2,x]
```

```
[Out] (x*Hypergeometric2F1[-3/4, 1/4, 5/4, -(x^4/(2 + x^4))])/(2*2^(3/4)*(2 + x^4
)^(1/4))
```

Maple [F] time = 0.033, size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+2\right)^2}\left(x^4+1\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)^(3/4)/(x^4+2)^2,x)
```

```
[Out] int((x^4+1)^(3/4)/(x^4+2)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+1\right)^{\frac{3}{4}}}{\left(x^4+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(3/4)/(x^4+2)^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)
```

Fricas [B] time = 52.7589, size = 668, normalized size = 9.03

$$-\frac{12 \cdot 8^{\frac{3}{4}} \left(x^4+2\right) \arctan\left(-\frac{8^{\frac{3}{4}} \left(x^4+1\right)^{\frac{1}{4}} x^3+4 \cdot 8^{\frac{1}{4}} \left(x^4+1\right)^{\frac{3}{4}} x-2^{\frac{1}{4}} \left(8^{\frac{3}{4}} \sqrt{x^4+1} x^2+8^{\frac{1}{4}} \left(3 x^4+2\right)\right)}{2 \left(x^4+2\right)}\right)-3 \cdot 8^{\frac{3}{4}} \left(x^4+2\right) \log\left(\frac{8 \sqrt{2} \left(x^4+1\right)^{\frac{1}{4}} x^3+8 \cdot 8^{\frac{1}{4}} \sqrt{x^4 \cdots}}{\cdots}\right)\cdots}{512 \left(x^4+\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(3/4)/(x^4+2)^2,x, algorithm="fricas")
```

```
[Out] -1/512*(12*8^(3/4)*(x^4 + 2)*arctan(-1/2*(8^(3/4)*(x^4 + 1)^(1/4)*x^3 + 4*8
^(1/4)*(x^4 + 1)^(3/4)*x - 2^(1/4)*(8^(3/4)*sqrt(x^4 + 1)*x^2 + 8^(1/4)*(3*
x^4 + 2)))/(x^4 + 2)) - 3*8^(3/4)*(x^4 + 2)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*
x^3 + 8*8^(1/4)*sqrt(x^4 + 1)*x^2 + 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4
)*x)/(x^4 + 2)) + 3*8^(3/4)*(x^4 + 2)*log((8*sqrt(2)*(x^4 + 1)^(1/4)*x^3 -
8*8^(1/4)*sqrt(x^4 + 1)*x^2 - 8^(3/4)*(3*x^4 + 2) + 16*(x^4 + 1)^(3/4)*x)/(
x^4 + 2)) - 64*(x^4 + 1)^(3/4)*x)/(x^4 + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+1\right)^{\frac{3}{4}}}{\left(x^4+2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)**(3/4)/(x**4+2)**2,x)
```

```
[Out] Integral((x**4 + 1)**(3/4)/(x**4 + 2)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^4+1\right)^{\frac{3}{4}}}{\left(x^4+2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)^(3/4)/(x^4+2)^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 1)^(3/4)/(x^4 + 2)^2, x)
```

3.318 $\int \frac{(-2+x^5)^2}{(3+x^5)^{16/5}}\,dx$

Optimal. Leaf size=48

$$\frac{97x}{891\sqrt[5]{x^5+3}}+\frac{5x}{297\left(x^5+3\right)^{6/5}}-\frac{5\left(x^5-2\right)x}{33\left(x^5+3\right)^{11/5}}$$

```
[Out] (-5*x*(-2 + x^5))/(33*(3 + x^5)^(11/5)) + (5*x)/(297*(3 + x^5)^(6/5)) + (97
*x)/(891*(3 + x^5)^(1/5))
```

Rubi [A] time = 0.0117009, antiderivative size = 59, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {378, 191}

$$\frac{x\left(2-x^5\right)^2}{33\left(x^5+3\right)^{11/5}}+\frac{10x\left(2-x^5\right)}{297\left(x^5+3\right)^{6/5}}+\frac{100x}{891\sqrt[5]{x^5+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^5)^2/(3 + x^5)^(16/5),x]
```

```
[Out] (x*(2 - x^5)^2)/(33*(3 + x^5)^(11/5)) + (10*x*(2 - x^5))/(297*(3 + x^5)^(6/
5)) + (100*x)/(891*(3 + x^5)^(1/5))
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(-2+x^5\right)^2}{\left(3+x^5\right)^{16/5}}\,dx &= \frac{x\left(2-x^5\right)^2}{33\left(3+x^5\right)^{11/5}}-\frac{20}{33}\int \frac{-2+x^5}{\left(3+x^5\right)^{11/5}}\,dx\\
&= \frac{x\left(2-x^5\right)^2}{33\left(3+x^5\right)^{11/5}}+\frac{10x\left(2-x^5\right)}{297\left(3+x^5\right)^{6/5}}+\frac{100}{297}\int \frac{1}{\left(3+x^5\right)^{6/5}}\,dx\\
&= \frac{x\left(2-x^5\right)^2}{33\left(3+x^5\right)^{11/5}}+\frac{10x\left(2-x^5\right)}{297\left(3+x^5\right)^{6/5}}+\frac{100x}{891\sqrt[5]{3+x^5}}
\end{aligned}$$

Mathematica [A] time = 0.0087394, size = 26, normalized size = 0.54

$$\frac{x\left(97x^{10}+462x^5+1188\right)}{891\left(x^5+3\right)^{11/5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^5)^2/(3 + x^5)^(16/5),x]
```

```
[Out] (x*(1188 + 462*x^5 + 97*x^10))/(891*(3 + x^5)^(11/5))
```

Maple [A] time = 0.006, size = 23, normalized size = 0.5

$$\frac{x\left(97\,x^{10}+462\,x^5+1188\right)}{891}\left(x^5+3\right)^{-\frac{11}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5-2)^2/(x^5+3)^(16/5),x)
```

```
[Out] 1/891*x*(97*x^10+462*x^5+1188)/(x^5+3)^(11/5)
```

Maxima [B] time = 0.95903, size = 99, normalized size = 2.06

$$-\frac{4\,x^{11}\left(\frac{11\,\left(x^5+3\right)}{x^5}-\frac{33\,\left(x^5+3\right)^2}{x^{10}}-3\right)}{891\,\left(x^5+3\right)^{\frac{11}{5}}}-\frac{2\,x^{11}\left(\frac{11\,\left(x^5+3\right)}{x^5}-6\right)}{297\,\left(x^5+3\right)^{\frac{11}{5}}}+\frac{x^{11}}{33\,\left(x^5+3\right)^{\frac{11}{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-2)^2/(x^5+3)^(16/5),x, algorithm="maxima")
```

```
[Out] -4/891*x^11*(11*(x^5 + 3)/x^5 - 33*(x^5 + 3)^2/x^10 - 3)/(x^5 + 3)^(11/5) -
 2/297*x^11*(11*(x^5 + 3)/x^5 - 6)/(x^5 + 3)^(11/5) + 1/33*x^11/(x^5 + 3)^(
11/5)
```

Fricas [A] time = 1.50801, size = 111, normalized size = 2.31

$$\frac{\left(97\,x^{11}+462\,x^6+1188\,x\right)\left(x^5+3\right)^{\frac{4}{5}}}{891\,\left(x^{15}+9\,x^{10}+27\,x^5+27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-2)^2/(x^5+3)^(16/5),x, algorithm="fricas")
```

```
[Out] 1/891*(97*x^11 + 462*x^6 + 1188*x)*(x^5 + 3)^(4/5)/(x^15 + 9*x^10 + 27*x^5
+ 27)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5-2)**2/(x**5+3)**(16/5),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^5-2\right)^2}{\left(x^5+3\right)^{\frac{16}{5}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5-2)^2/(x^5+3)^(16/5),x, algorithm="giac")
```

```
[Out] integrate((x^5 - 2)^2/(x^5 + 3)^(16/5), x)
```

3.319 $\int \frac{1}{(3x+3x^2+x^3)\sqrt[3]{3+3x+3x^2+x^3}}\,dx$

Optimal. Leaf size=90

$$-\frac{\log\left(1-(x+1)^3\right)}{6\sqrt[3]{3}}+\frac{\log\left(\sqrt[3]{3}(x+1)-\sqrt[3]{(x+1)^3+2}\right)}{2\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}$$

```
[Out] -(ArcTan[(1 + (2*3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3))/Sqrt[3]]/3^(5/6))
- Log[1 - (1 + x)^3]/(6*3^(1/3)) + Log[3^(1/3)*(1 + x) - (2 + (1 + x)^3)^(1
/3)]/(2*3^(1/3))
```

Rubi [A] time = 0.0824654, antiderivative size = 123, normalized size of antiderivative = 1.37, number of steps used = 9, number of rules used = 9, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {432, 431, 377, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(1-\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}\right)}{3\sqrt[3]{3}}-\frac{\log\left(\frac{3^{2/3}(x+1)^2}{\left((x+1)^3+2\right)^{2/3}}+\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)}{6\sqrt[3]{3}}-\frac{\tan^{-1}\left(\frac{2(x+1)}{\sqrt[6]{3}\sqrt[3]{(x+1)^3+2}}+\frac{1}{\sqrt{3}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3*x + 3*x^2 + x^3)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] -(ArcTan[1/Sqrt[3] + (2*(1 + x))/(3^(1/6)*(2 + (1 + x)^3)^(1/3))]/3^(5/6))
+ Log[1 - (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)]/(3*3^(1/3)) - Log[1 + (3
^(2/3)*(1 + x)^2)/(2 + (1 + x)^3)^(2/3) + (3^(1/3)*(1 + x))/(2 + (1 + x)^3)
^(1/3)]/(6*3^(1/3))
```

Rule 432

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[NormalizePseudoBinomial[u, x]^p
*NormalizePseudoBinomial[v, x]^q, x] /; FreeQ[{p, q}, x] && PseudoBinomialP
airQ[u, v, x]
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symb
ol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q,
x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u,
 x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
```

```
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(3x+3x^2+x^3\right)\sqrt[3]{3+3x+3x^2+x^3}}\,dx = \int \frac{1}{\left(-1+(1+x)^3\right)\sqrt[3]{2+(1+x)^3}}\,dx$$

$$= \text{Subst}\left(\int \frac{1}{\left(-1+x^3\right)\sqrt[3]{2+x^3}}\,dx, x, 1+x\right)$$

$$= \text{Subst}\left(\int \frac{1}{-1+3x^3}\,dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right)$$

$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{-1+\sqrt[3]{3}x}\,dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{-2-\sqrt[3]{3}x}{1+\sqrt[3]{3}x+3^{2/3}\cdots}\right.$$

$$= \frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+\sqrt[3]{3}x+3^{2/3}x^2}\,dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right) \cdots$$

$$= \frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}} - \frac{\log\left(1+\frac{3^{2/3}(1+x)^2}{\left(2+(1+x)^3\right)^{2/3}}+\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{6\sqrt[3]{3}} + \frac{\text{Subst}\left(\int \frac{1}{-3-x^2} \cdots\right.}{\cdots}$$

$$= -\frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}}{\sqrt{3}}\right)}{3^{5/6}} + \frac{\log\left(1-\frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}} - \frac{\log\left(1+\frac{3^{2/3}(1+x)^2}{\left(2+(1+x)^3\right)^{2/3}}+\frac{\sqrt[3]{3}(1+\cdots}{\sqrt[3]{2+(1+\cdots}}\right)}{6\sqrt[3]{3}}$$

Mathematica [A] time = 0.143695, size = 120, normalized size = 1.33

$$\frac{\sqrt{3}\left(2\log\left(1-\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}\right)-\log\left(\frac{3^{2/3}(x+1)^2}{\left((x+1)^3+2\right)^{2/3}}+\frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1\right)\right)-6\tan^{-1}\left(\frac{2(x+1)}{\sqrt[6]{3}\sqrt[3]{(x+1)^3+2}}+\frac{1}{\sqrt{3}}\right)}{6\ 3^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3*x + 3*x^2 + x^3)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]
```

```
[Out] (-6*ArcTan[1/Sqrt[3] + (2*(1 + x))/(3^(1/6)*(2 + (1 + x)^3)^(1/3))] + Sqrt[
3]*(2*Log[1 - (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)] - Log[1 + (3^(2/3)*(
1 + x)^2)/(2 + (1 + x)^3)^(2/3) + (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)])
)/(6*3^(5/6))
```

Maple [F] time = 0.022, size = 0, normalized size = 0.

$$\int \frac{1}{x^3+3x^2+3x}\frac{1}{\sqrt[3]{x^3+3x^2+3x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x)
```

```
[Out] int(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+3x^2+3x+3\right)^{\frac{1}{3}}\left(x^3+3x^2+3x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^3 + 3*x^2 + 3*x)), x)
```

Fricas [B] time = 71.2991, size = 1281, normalized size = 14.23

$$-\frac{1}{54}\cdot 3^{\frac{2}{3}}\log\left(\frac{3\cdot 3^{\frac{2}{3}}\left(7x^4+28x^3+42x^2+30x+9\right)\left(x^3+3x^2+3x+3\right)^{\frac{2}{3}}+3^{\frac{1}{3}}\left(31x^6+186x^5+465x^4+666x^3+6\right.}{x^6+6x^5+15x^4+18x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="fricas")
```

```
[Out] -1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 28*x^3 + 42*x^2 + 30*x + 9)*(x^3 + 3*
x^2 + 3*x + 3)^(2/3) + 3^(1/3)*(31*x^6 + 186*x^5 + 465*x^4 + 666*x^3 + 603*
```

```
x^2 + 324*x + 81) + 9*(5*x^5 + 25*x^4 + 50*x^3 + 54*x^2 + 33*x + 9)*(x^3 +
3*x^2 + 3*x + 3)^(1/3))/(x^6 + 6*x^5 + 15*x^4 + 18*x^3 + 9*x^2)) + 1/27*3^(
2/3)*log((2*3^(2/3)*(x^3 + 3*x^2 + 3*x) - 9*3^(1/3)*(x^3 + 3*x^2 + 3*x + 3)
^(1/3)*(x^2 + 2*x + 1) + 9*(x^3 + 3*x^2 + 3*x + 3)^(2/3)*(x + 1))/(x^3 + 3*
x^2 + 3*x)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)*(7*x^7 + 49*x^6 +
147*x^5 + 240*x^4 + 225*x^3 + 117*x^2 + 27*x)*(x^3 + 3*x^2 + 3*x + 3)^(2/3)
 - 3^(1/3)*(127*x^9 + 1143*x^8 + 4572*x^7 + 11070*x^6 + 18414*x^5 + 22032*x
^4 + 18900*x^3 + 11178*x^2 + 4131*x + 729) - 18*(31*x^8 + 248*x^7 + 868*x^6
 + 1782*x^5 + 2400*x^4 + 2196*x^3 + 1332*x^2 + 486*x + 81)*(x^3 + 3*x^2 + 3
*x + 3)^(1/3))/(251*x^9 + 2259*x^8 + 9036*x^7 + 21546*x^6 + 34398*x^5 + 385
56*x^4 + 30348*x^3 + 16038*x^2 + 5103*x + 729))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\left(x^2+3x+3\right)\sqrt[3]{x^3+3x^2+3x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+3*x**2+3*x)/(x**3+3*x**2+3*x+3)**(1/3),x)
```

```
[Out] Integral(1/(x*(x**2 + 3*x + 3)*(x**3 + 3*x**2 + 3*x + 3)**(1/3)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^3+3\,x^2+3\,x+3\right)^{\frac{1}{3}}\left(x^3+3\,x^2+3\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+3*x^2+3*x)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="giac")
```

```
[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^3 + 3*x^2 + 3*x)), x)
```

3.320 $\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Rubi [A] time = 0.033677, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1699, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{\left(1+x^2\right)\sqrt{1+x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.096177, size = 40, normalized size = 1.74

$$\sqrt[4]{-1}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right),-1\right)-2\Pi\left(-i;i\sinh^{-1}\left(\sqrt[4]{-1}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] (-1)^(1/4)*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - 2*EllipticPi[-I, I*Arc
Sinh[(-1)^(1/4)*x], -1])
```

Maple [C] time = 0.007, size = 112, normalized size = 4.9

$$-\frac{\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right),i\right)}{\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}}\sqrt{1-ix^2}\sqrt{1+ix^2}\frac{1}{\sqrt{x^4+1}}-2\frac{(-1)^{3/4}\sqrt{1-ix^2}\sqrt{1+ix^2}\mathrm{EllipticPi}\left(\sqrt[4]{-1}x,i,\sqrt{-i}-(-1)^{3/4}\right)}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x)
```

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-2*(-1)^(3/4)*(1-I*x^2)^(1/2)*(
1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4
))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2-1}{\sqrt{x^4+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

Fricas [A] time = 1.76918, size = 61, normalized size = 2.65

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{x^2\sqrt{x^4+1}+\sqrt{x^4+1}}\,dx-\int -\frac{1}{x^2\sqrt{x^4+1}+\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**2+1)/(x**4+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x) - Integral(-1/(x*
*2*sqrt(x**4 + 1) + sqrt(x**4 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-1}{\sqrt{x^4+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

3.321 $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=23

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Rubi [A] time = 0.0359511, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1699, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{x^4+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2]
```

Rule 1699

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{\left(1-x^2\right)\sqrt{1+x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1+x^4}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.0899468, size = 36, normalized size = 1.57

$$\sqrt[4]{-1}\left(\text{EllipticF}\left(i\sinh^{-1}\left(\sqrt[4]{-1}x\right),-1\right)-2\Pi\left(i;\sin^{-1}\left((-1)^{3/4}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] (-1)^(1/4)*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - 2*EllipticPi[I, ArcSin
[(-1)^(3/4)*x], -1])
```

Maple [C] time = 0.006, size = 112, normalized size = 4.9

$$-\frac{\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}\right),i\right)}{\frac{\sqrt{2}}{2}+\frac{i}{2}\sqrt{2}}\sqrt{1-ix^2}\sqrt{1+ix^2}\frac{1}{\sqrt{x^4+1}}-2\frac{(-1)^{3/4}\sqrt{1-ix^2}\sqrt{1+ix^2}\mathrm{EllipticPi}\left(\sqrt[4]{-1}x,-i,\sqrt{-i}-(-1}{\sqrt{x^4+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x)
```

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-2*(-1)^(3/4)*(1-I*x^2)^(1/2)*(
1+I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/
4))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2+1}{\sqrt{x^4+1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

Fricas [B] time = 1.77016, size = 111, normalized size = 4.83

$$\frac{1}{4}\sqrt{2}\log\left(\frac{x^4+2\sqrt{2}\sqrt{x^4+1}x+2x^2+1}{x^4-2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((x^4 + 2*sqrt(2)*sqrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2
+ 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx-\int \frac{1}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(-x**2+1)/(x**4+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x) - Integral(1/(x**
2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2+1}{\sqrt{x^4+1}\,(x^2-1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 1)/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

3.322 $\int \frac{1+x^2}{x\sqrt{1+x^4}} \, dx$

Optimal. Leaf size=16

$$\tanh^{-1}\left(\frac{x^2-1}{\sqrt{x^4+1}}\right)$$

```
[Out] ArcTanh[(-1 + x^2)/Sqrt[1 + x^4]]
```

Rubi [A] time = 0.0241054, antiderivative size = 23, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1252, 844, 215, 266, 63, 207}

$$\frac{1}{2}\sinh^{-1}\left(x^2\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(x*Sqrt[1 + x^4]),x]

[Out] ArcSinh[x^2]/2 - ArcTanh[Sqrt[1 + x^4]]/2
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{x\sqrt{1+x^4}}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1+x}{x\sqrt{1+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^2\right)+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\sinh^{-1}\left(x^2\right)+\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^4\right)\\ &= \frac{1}{2}\sinh^{-1}\left(x^2\right)+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^4}\right)\\ &= \frac{1}{2}\sinh^{-1}\left(x^2\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{1+x^4}\right)\end{aligned}$$

Mathematica [A] time = 0.0169876, size = 21, normalized size = 1.31

$$\frac{1}{2}\left(\sinh^{-1}\left(x^2\right)-\tanh^{-1}\left(\sqrt{x^4+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(x*Sqrt[1 + x^4]),x]
```

```
[Out] (ArcSinh[x^2] - ArcTanh[Sqrt[1 + x^4]])/2
```

Maple [A] time = 0.009, size = 18, normalized size = 1.1

$$-\frac{1}{2}\mathrm{Artanh}\left(\frac{1}{\sqrt{x^4+1}}\right)+\frac{\mathrm{Arcsinh}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/x/(x^4+1)^(1/2),x)
```

```
[Out] -1/2*arctanh(1/(x^4+1)^(1/2))+1/2*arcsinh(x^2)
```

Maxima [B] time = 1.51227, size = 77, normalized size = 4.81

$$-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)+\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right)-\frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1) + 1/4*log(sqrt(x^4
 + 1)/x^2 + 1) - 1/4*log(sqrt(x^4 + 1)/x^2 - 1)
```

Fricas [B] time = 1.55958, size = 123, normalized size = 7.69

$$-\frac{1}{2}\log\left(2x^4 - x^2 - \sqrt{x^4+1}\left(2x^2-1\right)+1\right) + \frac{1}{2}\log\left(-x^2 + \sqrt{x^4+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*x^4 - x^2 - sqrt(x^4 + 1)*(2*x^2 - 1) + 1) + 1/2*log(-x^2 + sqrt
(x^4 + 1) - 1)
```

Sympy [A] time = 3.95985, size = 14, normalized size = 0.88

$$-\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2} + \frac{\operatorname{asinh}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/x/(x**4+1)**(1/2),x)
```

```
[Out] -asinh(x**(-2))/2 + asinh(x**2)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2+1}{\sqrt{x^4+1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)/(sqrt(x^4 + 1)*x), x)
```

3.323 $\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=16

$$\tanh^{-1}\left(\frac{x^2+1}{\sqrt{x^4+1}}\right)$$

```
[Out] ArcTanh[(1 + x^2)/Sqrt[1 + x^4]]
```

Rubi [A] time = 0.024671, antiderivative size = 23, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1252, 844, 215, 266, 63, 207}

$$\frac{1}{2}\sinh^{-1}\left(x^2\right)+\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)/(x*Sqrt[1 + x^4]),x]

[Out] ArcSinh[x^2]/2 + ArcTanh[Sqrt[1 + x^4]]/2
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{-1+x^2}{x\sqrt{1+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{-1+x}{x\sqrt{1+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx, x, x^2\right) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}\sinh^{-1}\left(x^2\right) - \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^4\right)\\
&= \frac{1}{2}\sinh^{-1}\left(x^2\right) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^4}\right)\\
&= \frac{1}{2}\sinh^{-1}\left(x^2\right) + \frac{1}{2}\tanh^{-1}\left(\sqrt{1+x^4}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0143672, size = 19, normalized size = 1.19

$$\frac{1}{2}\left(\sinh^{-1}\left(x^2\right) + \tanh^{-1}\left(\sqrt{x^4+1}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)/(x*Sqrt[1 + x^4]),x]
```

```
[Out] (ArcSinh[x^2] + ArcTanh[Sqrt[1 + x^4]])/2
```

Maple [A] time = 0.005, size = 18, normalized size = 1.1

$$\frac{1}{2}\operatorname{Artanh}\left(\frac{1}{\sqrt{x^4+1}}\right) + \frac{\operatorname{Arcsinh}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)/x/(x^4+1)^(1/2),x)
```

```
[Out] 1/2*arctanh(1/(x^4+1)^(1/2))+1/2*arcsinh(x^2)
```

Maxima [B] time = 1.45403, size = 77, normalized size = 4.81

$$\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right) - \frac{1}{4}\log\left(\sqrt{x^4+1}-1\right) + \frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}+1\right) - \frac{1}{4}\log\left(\frac{\sqrt{x^4+1}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*log(sqrt(x^4 + 1) + 1) - 1/4*log(sqrt(x^4 + 1) - 1) + 1/4*log(sqrt(x^4
+ 1)/x^2 + 1) - 1/4*log(sqrt(x^4 + 1)/x^2 - 1)
```

Fricas [B] time = 1.4965, size = 123, normalized size = 7.69

$$-\frac{1}{2}\log\left(2x^4 + x^2 - \sqrt{x^4+1}\left(2x^2+1\right) + 1\right) + \frac{1}{2}\log\left(-x^2 + \sqrt{x^4+1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*x^4 + x^2 - sqrt(x^4 + 1)*(2*x^2 + 1) + 1) + 1/2*log(-x^2 + sqrt
(x^4 + 1) + 1)
```

Sympy [A] time = 4.09624, size = 14, normalized size = 0.88

$$\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2} + \frac{\operatorname{asinh}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/x/(x**4+1)**(1/2),x)
```

```
[Out] asinh(x**(-2))/2 + asinh(x**2)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 - 1}{\sqrt{x^4 + 1}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)/x/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)/(sqrt(x^4 + 1)*x), x)
```

3.324 $\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{x^4+x^2+1}}\right)}{\sqrt{3}}$$

```
[Out] ArcTanh[(Sqrt[3]*x)/Sqrt[1 + x^2 + x^4]]/Sqrt[3]
```

Rubi [A] time = 0.0432189, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1698, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{x^4+x^2+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ArcTanh[(Sqrt[3]*x)/Sqrt[1 + x^2 + x^4]]/Sqrt[3]
```

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2), x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^2+x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1-3x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{3}x}{\sqrt{1+x^2+x^4}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [C] time = 1.23863, size = 522, normalized size = 20.08

$$(-1)^{2/3}\left(-\sqrt[3]{-1}\left(\sqrt[3]{-1}-1\right)^2\left(1+\sqrt[3]{-1}\right)\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)-2i\sqrt{3}\sqrt{\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2)/((1 - x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] ((-1)^(2/3)*(-((-1)^(1/3)*(-1 + (-1)^(1/3))^2*(1 + (-1)^(1/3))*Sqrt[1 + (-1
)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*EllipticF[I*ArcSinh[(-1)^(5/6)*x], (-
1)^(2/3)]) - (2*I)*Sqrt[3]*((-1)^(1/3) - x)^2*Sqrt[((-1)^(2/3) - x)/((1 + (
-1)^(1/3))*((-1)^(1/3) - x))]*Sqrt[((-1)^(2/3) + x)/(-1 + x - (-1)^(1/3)*x)
]*Sqrt[(1 + x - (-1)^(1/3)*x)/((1 + (-1)^(1/3))*((-1)^(1/3) - x))]*((1 + (-
1)^(1/3))*EllipticF[ArcSin[Sqrt[(1 + x - (-1)^(1/3)*x)/((1 + (-1)^(1/3))*((
-1)^(1/3) - x))]], -3] - 2*(-1)^(1/3)*EllipticPi[-1, ArcSin[Sqrt[(1 + x - (
-1)^(1/3)*x)/((1 + (-1)^(1/3))*((-1)^(1/3) - x))]], -3]) + (2*I)*Sqrt[3]*((
-1)^(1/3) - x)^2*Sqrt[((-1)^(2/3) - x)/((1 + (-1)^(1/3))*((-1)^(1/3) - x))]
*Sqrt[((-1)^(2/3) + x)/(-1 + x - (-1)^(1/3)*x)]*Sqrt[(1 + x - (-1)^(1/3)*x)
/((1 + (-1)^(1/3))*((-1)^(1/3) - x))]*((-1 + (-1)^(1/3))*EllipticF[ArcSin[S
qrt[(1 + x - (-1)^(1/3)*x)/((1 + (-1)^(1/3))*((-1)^(1/3) - x))]], -3] - 2*(
-1)^(1/3)*EllipticPi[3, ArcSin[Sqrt[(1 + x - (-1)^(1/3)*x)/((1 + (-1)^(1/3)
)*((-1)^(1/3) - x))]], -3])))/((1 - (-1)^(2/3))*Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.157, size = 184, normalized size = 7.1

$$-2\,\frac{\sqrt{1-\left(-1/2+i/2\sqrt{3}\right)x^2}\sqrt{1-\left(-1/2-i/2\sqrt{3}\right)x^2}\,\mathrm{EllipticF}\left(1/2\,x\sqrt{-2+2\,i\sqrt{3}},1/2\,\sqrt{-2+2\,i\sqrt{3}}\right)}{\sqrt{-2+2\,i\sqrt{3}}\sqrt{x^4+x^2+1}}+2\,\frac{\sqrt{1-\left(-1/2+i/2\right.}}{\sqrt{-1/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x)
```

```
[Out] -2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I
*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/
2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*
3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*El
lipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I*3^
(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2+1}{\sqrt{x^4+x^2+1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 + 1)/(sqrt(x^4 + x^2 + 1)*(x^2 - 1)), x)
```

Fricas [B] time = 1.73039, size = 119, normalized size = 4.58

$$\frac{1}{6}\,\sqrt{3}\log\left(\frac{x^4+2\,\sqrt{3}\sqrt{x^4+x^2+1}x+4\,x^2+1}{x^4-2\,x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log((x^4 + 2*sqrt(3)*sqrt(x^4 + x^2 + 1)*x + 4*x^2 + 1)/(x^4 -
2*x^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{x^2\sqrt{x^4+x^2+1}-\sqrt{x^4+x^2+1}}\,dx-\int \frac{1}{x^2\sqrt{x^4+x^2+1}-\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(-x**2+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + x**2 + 1) - sqrt(x**4 + x**2 + 1)), x) - I
ntegral(1/(x**2*sqrt(x**4 + x**2 + 1) - sqrt(x**4 + x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2+1}{\sqrt{x^4+x^2+1}\left(x^2-1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(-x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 1)/(sqrt(x^4 + x^2 + 1)*(x^2 - 1)), x)
```

3.325 $\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=15

$$\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]

Rubi [A] time = 0.0390042, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1698, 203}

$$\tan^{-1}\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]

[Out] ArcTan[x/Sqrt[1 + x^2 + x^4]]

Rule 1698

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> Dist[A, Subst[Int[1/(d - (b*d - 2*a*e)*x^2),
x], x, x/Sqrt[a + b*x^2 + c*x^4]], x] /; FreeQ[{a, b, c, d, e, A, B}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[c*d^2 - a*e^2,
 0] && EqQ[B*d + A*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^2+x^4}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{1+x^2+x^4}}\right)\\ &= \tan^{-1}\left(\frac{x}{\sqrt{1+x^2+x^4}}\right)\end{aligned}$$

Mathematica [C] time = 0.129821, size = 94, normalized size = 6.27

$$-\frac{(-1)^{2/3}\sqrt{\sqrt[3]{-1}x^2+1}\sqrt{1-(-1)^{2/3}x^2}\left(\text{EllipticF}\left(i\sinh^{-1}\left((-1)^{5/6}x\right),(-1)^{2/3}\right)+2\Pi\left(\sqrt[3]{-1};-i\sinh^{-1}\left((-1)^{5/6}x\right)|(-1)^{2/3}\right)\right)}{\sqrt{x^4+x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)/((1 + x^2)*Sqrt[1 + x^2 + x^4]),x]
```

```
[Out] -(((-1)^(2/3)*Sqrt[1 + (-1)^(1/3)*x^2]*Sqrt[1 - (-1)^(2/3)*x^2]*(EllipticF[
I*ArcSinh[(-1)^(5/6)*x], (-1)^(2/3)] + 2*EllipticPi[(-1)^(1/3), (-I)*ArcSin
h[(-1)^(5/6)*x], (-1)^(2/3)]))/Sqrt[1 + x^2 + x^4])
```

Maple [C] time = 0.023, size = 188, normalized size = 12.5

$$-2\,\frac{\sqrt{1-\left(-1/2+i/2\sqrt{3}\right)x^2}\sqrt{1-\left(-1/2-i/2\sqrt{3}\right)x^2}\,\mathrm{EllipticF}\left(1/2\,x\sqrt{-2+2\,i\sqrt{3}},1/2\,\sqrt{-2+2\,i\sqrt{3}}\right)}{\sqrt{-2+2\,i\sqrt{3}}\sqrt{x^4+x^2+1}}+2\,\frac{\sqrt{1+1/2\,x^2-}}{\sqrt{-1/}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x)
```

```
[Out] -2/(-2+2*I*3^(1/2))^(1/2)*(1-(-1/2+1/2*I*3^(1/2))*x^2)^(1/2)*(1-(-1/2-1/2*I
*3^(1/2))*x^2)^(1/2)/(x^4+x^2+1)^(1/2)*EllipticF(1/2*x*(-2+2*I*3^(1/2))^(1/
2),1/2*(-2+2*I*3^(1/2))^(1/2))+2/(-1/2+1/2*I*3^(1/2))^(1/2)*(1+1/2*x^2-1/2*
I*x^2*3^(1/2))^(1/2)*(1+1/2*x^2+1/2*I*x^2*3^(1/2))^(1/2)/(x^4+x^2+1)^(1/2)*
EllipticPi((-1/2+1/2*I*3^(1/2))^(1/2)*x,-1/(-1/2+1/2*I*3^(1/2)),(-1/2-1/2*I
*3^(1/2))^(1/2)/(-1/2+1/2*I*3^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2-1}{\sqrt{x^4+x^2+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

Fricas [A] time = 1.7684, size = 42, normalized size = 2.8

$$\arctan\left(\frac{x}{\sqrt{x^4+x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(x/sqrt(x^4 + x^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{x^2}{x^2\sqrt{x^4+x^2+1}+\sqrt{x^4+x^2+1}}\,dx-\int-\frac{1}{x^2\sqrt{x^4+x^2+1}+\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(x**2+1)/(x**4+x**2+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + x**2 + 1) + sqrt(x**4 + x**2 + 1)), x) - I
ntegral(-1/(x**2*sqrt(x**4 + x**2 + 1) + sqrt(x**4 + x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2-1}{\sqrt{x^4+x^2+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(x^2+1)/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/(sqrt(x^4 + x^2 + 1)*(x^2 + 1)), x)
```

3.326 $\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=16

$$\frac{\sqrt{x^4+x^2+1}}{x}$$

```
[Out] Sqrt[1 + x^2 + x^4]/x
```

Rubi [A] time = 0.0180566, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1590}

$$\frac{\sqrt{x^4+x^2+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)/(x^2*Sqrt[1 + x^2 + x^4]),x]

[Out] Sqrt[1 + x^2 + x^4]/x
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] :> With[{p = Expon[Pp, x], q =
 Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int \frac{-1+x^4}{x^2\sqrt{1+x^2+x^4}}\,dx = \frac{\sqrt{1+x^2+x^4}}{x}$$

Mathematica [A] time = 0.0360015, size = 16, normalized size = 1.

$$\frac{\sqrt{x^4+x^2+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)/(x^2*Sqrt[1 + x^2 + x^4]),x]

[Out] Sqrt[1 + x^2 + x^4]/x
```

Maple [A] time = 0.006, size = 29, normalized size = 1.8

$$\frac{(x^2+x+1)(x^2-x+1)}{x}\frac{1}{\sqrt{x^4+x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x)
```

```
[Out] (x^2+x+1)*(x^2-x+1)/(x^4+x^2+1)^(1/2)/x
```

Maxima [A] time = 1.15252, size = 30, normalized size = 1.88

$$\frac{\sqrt{x^2+x+1}\sqrt{x^2-x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + x + 1)*sqrt(x^2 - x + 1)/x
```

Fricas [A] time = 1.49778, size = 31, normalized size = 1.94

$$\frac{\sqrt{x^4+x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^4 + x^2 + 1)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(x-1)(x+1)(x^2+1)}{x^2\sqrt{(x^2-x+1)(x^2+x+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-1)/x**2/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)*(x**2 + 1)/(x**2*sqrt((x**2 - x + 1)*(x**2 + x + 1
))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x^4-1}{\sqrt{x^4+x^2+1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-1)/x^2/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 - 1)/(sqrt(x^4 + x^2 + 1)*x^2), x)
```

3.327 $\int \frac{1-x^2}{\left(1+2ax+x^2\right)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx$

Optimal. Leaf size=74

$$\frac{\tan^{-1}\left(\frac{2x\left(a^2-b+1\right)+ax^2+a}{\sqrt{2}\sqrt{1-b}\sqrt{2ax^3+2ax+2bx^2+x^4+1}}\right)}{\sqrt{2}\sqrt{1-b}}$$

```
[Out] ArcTan[(a + 2*(1 + a^2 - b)*x + a*x^2)/(Sqrt[2]*Sqrt[1 - b]*Sqrt[1 + 2*a*x
+ 2*b*x^2 + 2*a*x^3 + x^4])]/(Sqrt[2]*Sqrt[1 - b])
```

Rubi [A] time = 0.204305, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {2084}

$$\frac{\tan^{-1}\left(\frac{2x\left(a^2-b+1\right)+ax^2+a}{\sqrt{2}\sqrt{1-b}\sqrt{2ax^3+2ax+2bx^2+x^4+1}}\right)}{\sqrt{2}\sqrt{1-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)/((1 + 2*a*x + x^2)*Sqrt[1 + 2*a*x + 2*b*x^2 + 2*a*x^3 + x^4])
,x]
```

```
[Out] ArcTan[(a + 2*(1 + a^2 - b)*x + a*x^2)/(Sqrt[2]*Sqrt[1 - b]*Sqrt[1 + 2*a*x
+ 2*b*x^2 + 2*a*x^3 + x^4])]/(Sqrt[2]*Sqrt[1 - b])
```

Rule 2084

```
Int[((f_) + (g_.)*(x_)^2)/(((d_) + (e_.)*(x_) + (d_.)*(x_)^2)*Sqrt[(a_) + (
b_.)*(x_) + (c_.)*(x_)^2 + (b_.)*(x_)^3 + (a_.)*(x_)^4]), x_Symbol] :> Simp
[(a*f*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*Rt[a^2*(2*a - c),
 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(d*Rt[a^2*(2*a - c), 2]), x] /
; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g, 0] &&
PosQ[a^2*(2*a - c)]
```

Rubi steps

$$\int \frac{1-x^2}{\left(1+2ax+x^2\right)\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\,dx = \frac{\tan^{-1}\left(\frac{a+2\left(1+a^2-b\right)x+ax^2}{\sqrt{2}\sqrt{1-b}\sqrt{1+2ax+2bx^2+2ax^3+x^4}}\right)}{\sqrt{2}\sqrt{1-b}}$$

Mathematica [C] time = 6.45998, size = 17955, normalized size = 242.64

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - x^2)/((1 + 2*a*x + x^2)*Sqrt[1 + 2*a*x + 2*b*x^2 + 2*a*x^3 +
 x^4]),x]
```

```
[Out] Result too large to show
```

Maple [C] time = 0.114, size = 247419, normalized size = 3343.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2 - 1}{\sqrt{2\,ax^3 + x^4 + 2\,bx^2 + 2\,ax + 1}\left(2\,ax + x^2 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x, alg
orithm="maxima")
```

```
[Out] -integrate((x^2 - 1)/(sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(2*a*x + x^
2 + 1)), x)
```

Fricas [A] time = 3.10011, size = 633, normalized size = 8.55

$$\left[\frac{\sqrt{2}\log\left(\frac{4\,a^3x^3+\left(a^2+2\,b-2\right)x^4+4\,a^3x+2\left(2\,a^4+5\,a^2-2\left(2\,a^2+3\right)b+4\,b^2+2\right)x^2+a^2-\frac{2\sqrt{2}\sqrt{2\,ax^3+x^4+2\,bx^2+2\,ax+1}\left((ab-a)x^2+ab-2\left(a^2-\left(a^2+2\right)b+b^2+1\right)x-a\right)}{\sqrt{b-1}}+2\,b-2}{4\,ax^3+x^4+2\left(2\,a^2+1\right)x^2+4\,ax+1}\right)}{4\sqrt{b-1}},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x, alg
orithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log((4*a^3*x^3 + (a^2 + 2*b - 2)*x^4 + 4*a^3*x + 2*(2*a^4 + 5*
a^2 - 2*(2*a^2 + 3)*b + 4*b^2 + 2)*x^2 + a^2 - 2*sqrt(2)*sqrt(2*a*x^3 + x^4
 + 2*b*x^2 + 2*a*x + 1)*((a*b - a)*x^2 + a*b - 2*(a^2 - (a^2 + 2)*b + b^2 +
 1)*x - a)/sqrt(b - 1) + 2*b - 2)/(4*a*x^3 + x^4 + 2*(2*a^2 + 1)*x^2 + 4*a*
x + 1))/sqrt(b - 1), 1/2*sqrt(2)*sqrt(-1/(b - 1))*arctan(sqrt(2)*sqrt(2*a*x
^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(b - 1)*sqrt(-1/(b - 1))/(a*x^2 + 2*(a^2 -
b + 1)*x + a))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2}{2ax\sqrt{2ax^3 + 2ax + 2bx^2 + x^4 + 1} + x^2\sqrt{2ax^3 + 2ax + 2bx^2 + x^4 + 1} + \sqrt{2ax^3 + 2ax + 2bx^2 + x^4 + 1}}\,dx - \int -\frac{}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)/(2*a*x+x**2+1)/(2*a*x**3+x**4+2*b*x**2+2*a*x+1)**(1/2),
x)
```

```
[Out] -Integral(x**2/(2*a*x*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + x**2*s
qrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + sqrt(2*a*x**3 + 2*a*x + 2*b*x
**2 + x**4 + 1)), x) - Integral(-1/(2*a*x*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2
+ x**4 + 1) + x**2*sqrt(2*a*x**3 + 2*a*x + 2*b*x**2 + x**4 + 1) + sqrt(2*a*
x**3 + 2*a*x + 2*b*x**2 + x**4 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{x^2 - 1}{\sqrt{2\,ax^3 + x^4 + 2\,bx^2 + 2\,ax + 1}\left(2\,ax + x^2 + 1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)/(2*a*x+x^2+1)/(2*a*x^3+x^4+2*b*x^2+2*a*x+1)^(1/2),x, alg
orithm="giac")
```

```
[Out] integrate(-(x^2 - 1)/(sqrt(2*a*x^3 + x^4 + 2*b*x^2 + 2*a*x + 1)*(2*a*x + x^
2 + 1)), x)
```

3.328 $\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx$

Optimal. Leaf size=22

$$\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

```
[Out] ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

Rubi [A] time = 0.064396, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2128, 203}

$$\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

Rule 2128

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+x^4\right)\sqrt{-x^2+\sqrt{1+x^4}}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)\\ &= \tan^{-1}\left(\frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right)\end{aligned}$$

Mathematica [A] time = 1.00667, size = 24, normalized size = 1.09

$$\cot^{-1}\left(\frac{\sqrt{\sqrt{x^4+1}-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcCot[Sqrt[-x^2 + Sqrt[1 + x^4]]/x]
```

Maple [F] time = 0.024, size = 0, normalized size = 0.

$$\int \frac{1}{x^4+1}\frac{1}{\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

```
[Out] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+1\right)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

Fricas [B] time = 6.07889, size = 149, normalized size = 6.77

$$-\frac{1}{4}\arctan\left(\frac{4\left(10\,x^7-6\,x^3+\left(7\,x^5-x\right)\sqrt{x^4+1}\right)\sqrt{-x^2+\sqrt{x^4+1}}}{17\,x^8-46\,x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*arctan(4*(10*x^7 - 6*x^3 + (7*x^5 - x)*sqrt(x^4 + 1))*sqrt(-x^2 + sqrt
(x^4 + 1))/(17*x^8 - 46*x^4 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+\sqrt{x^4+1}}\left(x^4+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1)/(-x**2+(x**4+1)**(1/2))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 + sqrt(x**4 + 1))*(x**4 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^4+1\right)\sqrt{-x^2+\sqrt{x^4+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)
```

3.329 $\int \frac{1}{\left(1+x^{2n}\right)\sqrt{-x^2+\left(1+x^{2n}\right)^{\frac{1}{n}}}}\,dx$

Optimal. Leaf size=24

$$\tan^{-1}\left(\frac{x}{\sqrt{\left(x^{2n}+1\right)^{\frac{1}{n}}-x^2}}\right)$$

```
[Out] ArcTan[x/Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]]
```

Rubi [A] time = 0.0678271, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {2128, 203}

$$\tan^{-1}\left(\frac{x}{\sqrt{\left(x^{2n}+1\right)^{\frac{1}{n}}-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^(2*n))*Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]),x]
```

```
[Out] ArcTan[x/Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]]
```

Rule 2128

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x
_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x
/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ
[p, 2/n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+x^{2n}\right)\sqrt{-x^2+\left(1+x^{2n}\right)^{\frac{1}{n}}}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{-x^2+\left(1+x^{2n}\right)^{\frac{1}{n}}}}\right)\\ &= \tan^{-1}\left(\frac{x}{\sqrt{-x^2+\left(1+x^{2n}\right)^{\frac{1}{n}}}}\right)\end{aligned}$$

Mathematica [A] time = 0.0813074, size = 26, normalized size = 1.08

$$\cot^{-1}\left(\frac{\sqrt{\left(x^{2n}+1\right)^{\frac{1}{n}}-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^(2*n))*Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]),x]
```

```
[Out] ArcCot[Sqrt[-x^2 + (1 + x^(2*n))^n^(-1)]/x]
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{1}{1+x^{2n}}\frac{1}{\sqrt{-x^2+\sqrt[n]{1+x^{2n}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x)
```

```
[Out] int(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+\left(x^{2n}+1\right)^{\left(\frac{1}{n}\right)}}\left(x^{2n}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x, algorithm="maxima
")
```

```
[Out] integrate(1/(sqrt(-x^2 + (x^(2*n) + 1)^(1/n))*(x^(2*n) + 1)), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x, algorithm="fricas
")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2 + (x^{2n}+1)^{\frac{1}{n}}}\,(x^{2n}+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**(2*n))/(-x**2+(1+x**(2*n))**(1/n))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-x**2 + (x**(2*n) + 1)**(1/n))*(x**(2*n) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2 + (x^{2n}+1)^{\left(\frac{1}{n}\right)}}\,(x^{2n}+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(2*n))/(-x^2+(1+x^(2*n))^(1/n))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-x^2 + (x^(2*n) + 1)^(1/n))*(x^(2*n) + 1)), x)
```

3.330 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0066023, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\,dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0016566, size = 14, normalized size = 1.

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2,x]
```

```
[Out] x/2 + Sin[2*x]/4
```

Maple [A] time = 0., size = 11, normalized size = 0.8

$$\frac{x}{2}+\frac{\cos\left(x\right)\sin\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
```

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A] time = 1.02639, size = 14, normalized size = 1.

$$\frac{1}{2}\,x+\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A] time = 1.54087, size = 36, normalized size = 2.57

$$\frac{1}{2}\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.056366, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

Giac [A] time = 1.05331, size = 14, normalized size = 1.

$$\frac{1}{2}\,x+\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

3.331 $\int \cos^3(x)\,dx$

Optimal. Leaf size=11

$$\sin(x) - \frac{\sin^3(x)}{3}$$

```
[Out] Sin[x] - Sin[x]^3/3
```

Rubi [A] time = 0.0065874, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2633}

$$\sin(x) - \frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3,x]
```

```
[Out] Sin[x] - Sin[x]^3/3
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\,dx &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(x)\right)\\ &= \sin(x) - \frac{\sin^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.0017568, size = 15, normalized size = 1.36

$$\frac{3\sin(x)}{4} + \frac{1}{12}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3,x]
```

```
[Out] (3*Sin[x])/4 + Sin[3*x]/12
```

Maple [A] time = 0.003, size = 11, normalized size = 1.

$$\frac{\left(2+(\cos(x))^2\right)\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3,x)
```

```
[Out] 1/3*(2+cos(x)^2)*sin(x)
```

Maxima [A] time = 0.927702, size = 12, normalized size = 1.09

$$-\frac{1}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="maxima")
```

```
[Out] -1/3*sin(x)^3 + sin(x)
```

Fricas [A] time = 1.48112, size = 36, normalized size = 3.27

$$\frac{1}{3}\left(\cos(x)^2 + 2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + 2)*sin(x)
```

Sympy [A] time = 0.058163, size = 8, normalized size = 0.73

$$-\frac{\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3,x)
```

```
[Out] -sin(x)**3/3 + sin(x)
```

Giac [A] time = 1.0437, size = 12, normalized size = 1.09

$$-\frac{1}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="giac")
```

```
[Out] -1/3*sin(x)^3 + sin(x)
```

3.332 $\int \sin^4(x)\,dx$

Optimal. Leaf size=24

$$\frac{3x}{8}-\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{8}\sin(x)\cos(x)$$

```
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rubi [A] time = 0.0099481, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{3x}{8}-\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^4(x)\,dx &= -\frac{1}{4}\cos(x)\sin^3(x)+\frac{3}{4}\int \sin^2(x)\,dx\\ &= -\frac{3}{8}\cos(x)\sin(x)-\frac{1}{4}\cos(x)\sin^3(x)+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}-\frac{3}{8}\cos(x)\sin(x)-\frac{1}{4}\cos(x)\sin^3(x)\end{aligned}$$

Mathematica [A] time = 0.0017592, size = 22, normalized size = 0.92

$$\frac{3x}{8}-\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32
```

Maple [A] time = 0., size = 18, normalized size = 0.8

$$-\frac{\cos(x)}{4}\left((\sin(x))^3+\frac{3\sin(x)}{2}\right)+\frac{3x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4,x)
```

```
[Out] -1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x
```

Maxima [A] time = 0.932326, size = 22, normalized size = 0.92

$$\frac{3}{8}x+\frac{1}{32}\sin(4x)-\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

Fricas [A] time = 1.60014, size = 59, normalized size = 2.46

$$\frac{1}{8}\left(2\cos(x)^3-5\cos(x)\right)\sin(x)+\frac{3}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(2*cos(x)^3 - 5*cos(x))*sin(x) + 3/8*x
```

Sympy [A] time = 0.057036, size = 24, normalized size = 1.

$$\frac{3x}{8}-\frac{\sin^3(x)\cos(x)}{4}-\frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4,x)
```

```
[Out] 3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

Giac [A] time = 1.04816, size = 22, normalized size = 0.92

$$\frac{3}{8}x+\frac{1}{32}\sin(4x)-\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

3.333 $\int \cos^6(x)\,dx$

Optimal. Leaf size=34

$$\frac{5x}{16}+\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{24}\sin(x)\cos^3(x)+\frac{5}{16}\sin(x)\cos(x)$$

```
[Out] (5*x)/16 + (5*Cos[x]*Sin[x])/16 + (5*Cos[x]^3*Sin[x])/24 + (Cos[x]^5*Sin[x]
)/6
```

Rubi [A] time = 0.0192104, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{5x}{16}+\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{24}\sin(x)\cos^3(x)+\frac{5}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6,x]
```

```
[Out] (5*x)/16 + (5*Cos[x]*Sin[x])/16 + (5*Cos[x]^3*Sin[x])/24 + (Cos[x]^5*Sin[x]
)/6
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^6(x)\,dx &= \frac{1}{6}\cos^5(x)\sin(x)+\frac{5}{6}\int \cos^4(x)\,dx\\
&= \frac{5}{24}\cos^3(x)\sin(x)+\frac{1}{6}\cos^5(x)\sin(x)+\frac{5}{8}\int \cos^2(x)\,dx\\
&= \frac{5}{16}\cos(x)\sin(x)+\frac{5}{24}\cos^3(x)\sin(x)+\frac{1}{6}\cos^5(x)\sin(x)+\frac{5\int 1\,dx}{16}\\
&= \frac{5x}{16}+\frac{5}{16}\cos(x)\sin(x)+\frac{5}{24}\cos^3(x)\sin(x)+\frac{1}{6}\cos^5(x)\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.0020545, size = 30, normalized size = 0.88

$$\frac{5x}{16}+\frac{15}{64}\sin(2x)+\frac{3}{64}\sin(4x)+\frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6,x]
```

```
[Out] (5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192
```

Maple [A] time = 0.005, size = 24, normalized size = 0.7

$$\frac{\sin(x)}{6}\left((\cos(x))^5+\frac{5(\cos(x))^3}{4}+\frac{15\cos(x)}{8}\right)+\frac{5x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6,x)
```

```
[Out] 1/6*(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)+5/16*x
```

Maxima [A] time = 0.934904, size = 32, normalized size = 0.94

$$-\frac{1}{48}\sin(2x)^3+\frac{5}{16}x+\frac{3}{64}\sin(4x)+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6,x, algorithm="maxima")
```

```
[Out] -1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) + 1/4*sin(2*x)
```

Fricas [A] time = 1.59037, size = 82, normalized size = 2.41

$$\frac{1}{48}\left(8\cos(x)^5+10\cos(x)^3+15\cos(x)\right)\sin(x)+\frac{5}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(8*cos(x)^5 + 10*cos(x)^3 + 15*cos(x))*sin(x) + 5/16*x
```

Sympy [A] time = 0.057266, size = 36, normalized size = 1.06

$$\frac{5x}{16}+\frac{\sin(x)\cos^5(x)}{6}+\frac{5\sin(x)\cos^3(x)}{24}+\frac{5\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6,x)
```

```
[Out] 5*x/16 + sin(x)*cos(x)**5/6 + 5*sin(x)*cos(x)**3/24 + 5*sin(x)*cos(x)/16
```

Giac [A] time = 1.05931, size = 30, normalized size = 0.88

$$\frac{5}{16}x+\frac{1}{192}\sin(6x)+\frac{3}{64}\sin(4x)+\frac{15}{64}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6,x, algorithm="giac")
```

```
[Out] 5/16*x + 1/192*sin(6*x) + 3/64*sin(4*x) + 15/64*sin(2*x)
```

3.334 $\int \sin^8(x)\,dx$

Optimal. Leaf size=44

$$\frac{35x}{128}-\frac{1}{8}\sin^7(x)\cos(x)-\frac{7}{48}\sin^5(x)\cos(x)-\frac{35}{192}\sin^3(x)\cos(x)-\frac{35}{128}\sin(x)\cos(x)$$

```
[Out] (35*x)/128 - (35*Cos[x]*Sin[x])/128 - (35*Cos[x]*Sin[x]^3)/192 - (7*Cos[x]*
Sin[x]^5)/48 - (Cos[x]*Sin[x]^7)/8
```

Rubi [A] time = 0.0214529, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{35x}{128}-\frac{1}{8}\sin^7(x)\cos(x)-\frac{7}{48}\sin^5(x)\cos(x)-\frac{35}{192}\sin^3(x)\cos(x)-\frac{35}{128}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^8,x]
```

```
[Out] (35*x)/128 - (35*Cos[x]*Sin[x])/128 - (35*Cos[x]*Sin[x]^3)/192 - (7*Cos[x]*
Sin[x]^5)/48 - (Cos[x]*Sin[x]^7)/8
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \sin^8(x)\,dx &= -\frac{1}{8}\cos(x)\sin^7(x)+\frac{7}{8}\int \sin^6(x)\,dx\\
&= -\frac{7}{48}\cos(x)\sin^5(x)-\frac{1}{8}\cos(x)\sin^7(x)+\frac{35}{48}\int \sin^4(x)\,dx\\
&= -\frac{35}{192}\cos(x)\sin^3(x)-\frac{7}{48}\cos(x)\sin^5(x)-\frac{1}{8}\cos(x)\sin^7(x)+\frac{35}{64}\int \sin^2(x)\,dx\\
&= -\frac{35}{128}\cos(x)\sin(x)-\frac{35}{192}\cos(x)\sin^3(x)-\frac{7}{48}\cos(x)\sin^5(x)-\frac{1}{8}\cos(x)\sin^7(x)+\frac{35\int 1\,dx}{128}\\
&= \frac{35x}{128}-\frac{35}{128}\cos(x)\sin(x)-\frac{35}{192}\cos(x)\sin^3(x)-\frac{7}{48}\cos(x)\sin^5(x)-\frac{1}{8}\cos(x)\sin^7(x)
\end{aligned}$$

Mathematica [A] time = 0.0025816, size = 38, normalized size = 0.86

$$\frac{35x}{128}-\frac{7}{32}\sin(2x)+\frac{7}{128}\sin(4x)-\frac{1}{96}\sin(6x)+\frac{\sin(8x)}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^8,x]
```

```
[Out] (35*x)/128 - (7*Sin[2*x])/32 + (7*Sin[4*x])/128 - Sin[6*x]/96 + Sin[8*x]/10
24
```

Maple [A] time = 0.034, size = 30, normalized size = 0.7

$$-\frac{\cos(x)}{8}\left((\sin(x))^{7}+\frac{7(\sin(x))^{5}}{6}+\frac{35(\sin(x))^{3}}{24}+\frac{35\sin(x)}{16}\right)+\frac{35x}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^8,x)
```

```
[Out] -1/8*(sin(x)^7+7/6*sin(x)^5+35/24*sin(x)^3+35/16*sin(x))*cos(x)+35/128*x
```

Maxima [A] time = 0.940387, size = 41, normalized size = 0.93

$$\frac{1}{24}\sin(2x)^{3}+\frac{35}{128}x+\frac{1}{1024}\sin(8x)+\frac{7}{128}\sin(4x)-\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^8,x, algorithm="maxima")
```

```
[Out] 1/24*sin(2*x)^3 + 35/128*x + 1/1024*sin(8*x) + 7/128*sin(4*x) - 1/4*sin(2*x
)
```

Fricas [A] time = 1.69731, size = 111, normalized size = 2.52

$$\frac{1}{384}\left(48\cos(x)^{7}-200\cos(x)^{5}+326\cos(x)^{3}-279\cos(x)\right)\sin(x)+\frac{35}{128}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^8,x, algorithm="fricas")
```

```
[Out] 1/384*(48*cos(x)^7 - 200*cos(x)^5 + 326*cos(x)^3 - 279*cos(x))*sin(x) + 35/
128*x
```

Sympy [A] time = 0.059871, size = 48, normalized size = 1.09

$$\frac{35x}{128}-\frac{\sin^{7}(x)\cos(x)}{8}-\frac{7\sin^{5}(x)\cos(x)}{48}-\frac{35\sin^{3}(x)\cos(x)}{192}-\frac{35\sin(x)\cos(x)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**8,x)
```

```
[Out] 35*x/128 - sin(x)**7*cos(x)/8 - 7*sin(x)**5*cos(x)/48 - 35*sin(x)**3*cos(x)
/192 - 35*sin(x)*cos(x)/128
```

Giac [A] time = 1.07199, size = 38, normalized size = 0.86

$$\frac{35}{128}\,x + \frac{1}{1024}\,\sin\left(8\,x\right) - \frac{1}{96}\,\sin\left(6\,x\right) + \frac{7}{128}\,\sin\left(4\,x\right) - \frac{7}{32}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^8,x, algorithm="giac")
```

```
[Out] 35/128*x + 1/1024*sin(8*x) - 1/96*sin(6*x) + 7/128*sin(4*x) - 7/32*sin(2*x)
```

3.335 $\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$

Optimal. Leaf size=20

$$\frac{3x}{8}+\frac{\cos(x)}{2}-\frac{1}{8}\sin(x)\cos(x)$$

```
[Out] (3*x)/8 + Cos[x]/2 - (Cos[x]*Sin[x])/8
```

Rubi [B] time = 0.0160949, antiderivative size = 64, normalized size of antiderivative = 3.2, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2635, 8}

$$\frac{3x}{8}+\frac{1}{2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^3\left(\frac{x}{2}+\frac{\pi}{4}\right)+\frac{3}{4}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Pi/4 + x/2]^4,x]
```

```
[Out] (3*x)/8 + (3*Cos[Pi/4 + x/2]*Sin[Pi/4 + x/2])/4 + (Cos[Pi/4 + x/2]^3*Sin[Pi
/4 + x/2])/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx &= \frac{1}{2}\cos^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{3}{4}\int \sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)\,dx\\ &= \frac{3}{4}\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{1}{2}\cos^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}+\frac{3}{4}\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{1}{2}\cos^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.017465, size = 21, normalized size = 1.05

$$\frac{1}{16}(6x+8\cos(x)-2\sin(x)\cos(x)+3\pi)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Pi/4 + x/2]^4,x]
```

```
[Out] (3*Pi + 6*x + 8*Cos[x] - 2*Cos[x]*Sin[x])/16
```

Maple [B] time = 0.01, size = 39, normalized size = 2.

$$\frac{1}{2}\left(\left(\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)^{3}+\frac{3}{2}\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{3\pi}{16}+\frac{3x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/4*Pi+1/2*x)^4,x)
```

```
[Out] 1/2*(cos(1/4*Pi+1/2*x)^3+3/2*cos(1/4*Pi+1/2*x))*sin(1/4*Pi+1/2*x)+3/16*Pi+3
/8*x
```

Maxima [A] time = 0.933846, size = 31, normalized size = 1.55

$$\frac{3}{16}\pi+\frac{3}{8}x+\frac{1}{16}\sin\left(\pi+2x\right)+\frac{1}{2}\sin\left(\frac{1}{2}\pi+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/4*pi+1/2*x)^4,x, algorithm="maxima")
```

```
[Out] 3/16*pi + 3/8*x + 1/16*sin(pi + 2*x) + 1/2*sin(1/2*pi + x)
```

Fricas [B] time = 1.67131, size = 112, normalized size = 5.6

$$\frac{1}{4}\left(2\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^{3}+3\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)\right)\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+\frac{3}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/4*pi+1/2*x)^4,x, algorithm="fricas")
```

```
[Out] 1/4*(2*cos(1/4*pi + 1/2*x)^3 + 3*cos(1/4*pi + 1/2*x))*sin(1/4*pi + 1/2*x) +
 3/8*x
```

Sympy [B] time = 0.615847, size = 99, normalized size = 4.95

$$\frac{3x\sin^{4}\left(\frac{x}{2}+\frac{\pi}{4}\right)}{8}+\frac{3x\sin^{2}\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^{2}\left(\frac{x}{2}+\frac{\pi}{4}\right)}{4}+\frac{3x\cos^{4}\left(\frac{x}{2}+\frac{\pi}{4}\right)}{8}+\frac{3\sin^{3}\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos\left(\frac{x}{2}+\frac{\pi}{4}\right)}{4}+\frac{5\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\cos^{3}\left(\frac{x}{2}\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/4*pi+1/2*x)**4,x)
```

```
[Out] 3*x*sin(x/2 + pi/4)**4/8 + 3*x*sin(x/2 + pi/4)**2*cos(x/2 + pi/4)**2/4 + 3*
x*cos(x/2 + pi/4)**4/8 + 3*sin(x/2 + pi/4)**3*cos(x/2 + pi/4)/4 + 5*sin(x/2
 + pi/4)*cos(x/2 + pi/4)**3/4
```

Giac [A] time = 1.05228, size = 19, normalized size = 0.95

$$\frac{3}{8}x + \frac{1}{2}\cos(x) - \frac{1}{16}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/4*pi+1/2*x)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/2*cos(x) - 1/16*sin(2*x)
```

3.336 $\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx$

Optimal. Leaf size=31

$$\frac{1}{9}\cos^3\left(\frac{\pi}{12}-3x\right)-\frac{1}{3}\cos\left(\frac{\pi}{12}-3x\right)$$

```
[Out] -Cos[Pi/12 - 3*x]/3 + Cos[Pi/12 - 3*x]^3/9
```

Rubi [A] time = 0.0107794, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2633}

$$\frac{1}{9}\cos^3\left(\frac{\pi}{12}-3x\right)-\frac{1}{3}\cos\left(\frac{\pi}{12}-3x\right)$$

Antiderivative was successfully verified.

```
[In] Int[-Sin[Pi/12 - 3*x]^3,x]
```

```
[Out] -Cos[Pi/12 - 3*x]/3 + Cos[Pi/12 - 3*x]^3/9
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int -\sin^3\left(\frac{\pi}{12}-3x\right)\,dx &= -\left(\frac{1}{3}\operatorname{Subst}\left(\int\left(1-x^2\right)\,dx,x,\cos\left(\frac{\pi}{12}-3x\right)\right)\right)\\ &= -\frac{1}{3}\cos\left(\frac{\pi}{12}-3x\right)+\frac{1}{9}\cos^3\left(\frac{\pi}{12}-3x\right)\end{aligned}$$

Mathematica [A] time = 0.0163376, size = 31, normalized size = 1.

$$\frac{1}{36}\cos\left(3\left(\frac{\pi}{12}-3x\right)\right)-\frac{1}{4}\cos\left(\frac{\pi}{12}-3x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-Sin[Pi/12 - 3*x]^3,x]
```

```
[Out] -Cos[Pi/12 - 3*x]/4 + Cos[3*(Pi/12 - 3*x)]/36
```

Maple [A] time = 0.007, size = 23, normalized size = 0.7

$$-\frac{1}{9}\left(2+\left(\cos\left(\frac{5\pi}{12}+3x\right)\right)^2\right)\sin\left(\frac{5\pi}{12}+3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(5/12*Pi+3*x)^3,x)
```

```
[Out] -1/9*(2+cos(5/12*Pi+3*x)^2)*sin(5/12*Pi+3*x)
```

Maxima [A] time = 0.936318, size = 31, normalized size = 1.

$$\frac{1}{9}\sin\left(\frac{5}{12}\pi + 3x\right)^3 - \frac{1}{3}\sin\left(\frac{5}{12}\pi + 3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(5/12*pi+3*x)^3,x, algorithm="maxima")
```

```
[Out] 1/9*sin(5/12*pi + 3*x)^3 - 1/3*sin(5/12*pi + 3*x)
```

Fricas [A] time = 1.5673, size = 70, normalized size = 2.26

$$-\frac{1}{9}\left(\cos\left(\frac{5}{12}\pi + 3x\right)^2 + 2\right)\sin\left(\frac{5}{12}\pi + 3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(5/12*pi+3*x)^3,x, algorithm="fricas")
```

```
[Out] -1/9*(cos(5/12*pi + 3*x)^2 + 2)*sin(5/12*pi + 3*x)
```

Sympy [A] time = 0.309147, size = 39, normalized size = 1.26

$$-\frac{2\sin^3\left(3x + \frac{5\pi}{12}\right)}{9} - \frac{\sin\left(3x + \frac{5\pi}{12}\right)\cos^2\left(3x + \frac{5\pi}{12}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(5/12*pi+3*x)**3,x)
```

```
[Out] -2*sin(3*x + 5*pi/12)**3/9 - sin(3*x + 5*pi/12)*cos(3*x + 5*pi/12)**2/3
```

Giac [A] time = 1.06113, size = 31, normalized size = 1.

$$\frac{1}{9}\sin\left(\frac{5}{12}\pi + 3x\right)^3 - \frac{1}{3}\sin\left(\frac{5}{12}\pi + 3x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-cos(5/12*pi+3*x)^3,x, algorithm="giac")
```

```
[Out] 1/9*sin(5/12*pi + 3*x)^3 - 1/3*sin(5/12*pi + 3*x)
```

3.337 $\int \csc^6(x)\,dx$

Optimal. Leaf size=21

$$-\frac{1}{5}\cot^5(x)-\frac{2\cot^3(x)}{3}-\cot(x)$$

```
[Out] -Cot[x] - (2*Cot[x]^3)/3 - Cot[x]^5/5
```

Rubi [A] time = 0.0089103, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3767}

$$-\frac{1}{5}\cot^5(x)-\frac{2\cot^3(x)}{3}-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^6,x]
```

```
[Out] -Cot[x] - (2*Cot[x]^3)/3 - Cot[x]^5/5
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^6(x)\,dx &= -\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, \cot(x)\right)\\ &= -\cot(x)-\frac{2\cot^3(x)}{3}-\frac{\cot^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.0030074, size = 27, normalized size = 1.29

$$-\frac{8\cot(x)}{15}-\frac{1}{5}\cot(x)\csc^4(x)-\frac{4}{15}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^6,x]
```

```
[Out] (-8*Cot[x])/15 - (4*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5
```

Maple [A] time = 0.028, size = 18, normalized size = 0.9

$$\left(-\frac{8}{15}-\frac{(\csc(x))^4}{5}-\frac{4(\csc(x))^2}{15}\right)\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^6,x)
```

```
[Out] (-8/15-1/5*csc(x)^4-4/15*csc(x)^2)*cot(x)
```

Maxima [A] time = 0.936538, size = 27, normalized size = 1.29

$$-\frac{15 \tan(x)^4 + 10 \tan(x)^2 + 3}{15 \tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

Fricas [B] time = 1.58287, size = 112, normalized size = 5.33

$$-\frac{8 \cos(x)^5 - 20 \cos(x)^3 + 15 \cos(x)}{15 \left(\cos(x)^4 - 2 \cos(x)^2 + 1\right) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/15*(8*cos(x)^5 - 20*cos(x)^3 + 15*cos(x))/((cos(x)^4 - 2*cos(x)^2 + 1)*s
in(x))
```

Sympy [A] time = 0.058264, size = 32, normalized size = 1.52

$$-\frac{8 \cos(x)}{15 \sin(x)} - \frac{4 \cos(x)}{15 \sin^3(x)} - \frac{\cos(x)}{5 \sin^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)**6,x)
```

```
[Out] -8*cos(x)/(15*sin(x)) - 4*cos(x)/(15*sin(x)**3) - cos(x)/(5*sin(x)**5)
```

Giac [A] time = 1.06628, size = 27, normalized size = 1.29

$$-\frac{15 \tan(x)^4 + 10 \tan(x)^2 + 3}{15 \tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/tan(x)^5
```

3.338 $\int \csc^7(x)\,dx$

Optimal. Leaf size=36

$$-\frac{5}{16}\tanh^{-1}(\cos(x))-\frac{1}{6}\cot(x)\csc^5(x)-\frac{5}{24}\cot(x)\csc^3(x)-\frac{5}{16}\cot(x)\csc(x)$$

```
[Out] (-5*ArcTanh[Cos[x]])/16 - (5*Cot[x]*Csc[x])/16 - (5*Cot[x]*Csc[x]^3)/24 - (
Cot[x]*Csc[x]^5)/6
```

Rubi [A] time = 0.0195031, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3768, 3770}

$$-\frac{5}{16}\tanh^{-1}(\cos(x))-\frac{1}{6}\cot(x)\csc^5(x)-\frac{5}{24}\cot(x)\csc^3(x)-\frac{5}{16}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^7,x]
```

```
[Out] (-5*ArcTanh[Cos[x]])/16 - (5*Cot[x]*Csc[x])/16 - (5*Cot[x]*Csc[x]^3)/24 - (
Cot[x]*Csc[x]^5)/6
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^7(x)\,dx &= -\frac{1}{6}\cot(x)\csc^5(x)+\frac{5}{6}\int \csc^5(x)\,dx\\ &= -\frac{5}{24}\cot(x)\csc^3(x)-\frac{1}{6}\cot(x)\csc^5(x)+\frac{5}{8}\int \csc^3(x)\,dx\\ &= -\frac{5}{16}\cot(x)\csc(x)-\frac{5}{24}\cot(x)\csc^3(x)-\frac{1}{6}\cot(x)\csc^5(x)+\frac{5}{16}\int \csc(x)\,dx\\ &= -\frac{5}{16}\tanh^{-1}(\cos(x))-\frac{5}{16}\cot(x)\csc(x)-\frac{5}{24}\cot(x)\csc^3(x)-\frac{1}{6}\cot(x)\csc^5(x)\end{aligned}$$

Mathematica [B] time = 0.0081332, size = 95, normalized size = 2.64

$$-\frac{1}{384}\csc^6\left(\frac{x}{2}\right)-\frac{1}{64}\csc^4\left(\frac{x}{2}\right)-\frac{5}{64}\csc^2\left(\frac{x}{2}\right)+\frac{1}{384}\sec^6\left(\frac{x}{2}\right)+\frac{1}{64}\sec^4\left(\frac{x}{2}\right)+\frac{5}{64}\sec^2\left(\frac{x}{2}\right)+\frac{5}{16}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{5}{16}\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^7,x]
```

```
[Out] (-5*Csc[x/2]^2)/64 - Csc[x/2]^4/64 - Csc[x/2]^6/384 - (5*Log[Cos[x/2]])/16
+ (5*Log[Sin[x/2]])/16 + (5*Sec[x/2]^2)/64 + Sec[x/2]^4/64 + Sec[x/2]^6/384
```

Maple [A] time = 0.034, size = 32, normalized size = 0.9

$$\left(-\frac{(\csc(x))^5}{6} - \frac{5\,(\csc(x))^3}{24} - \frac{5\csc(x)}{16}\right)\cot(x) + \frac{5\ln(\csc(x) - \cot(x))}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^7,x)
```

```
[Out] (-1/6*csc(x)^5-5/24*csc(x)^3-5/16*csc(x))*cot(x)+5/16*ln(csc(x)-cot(x))
```

Maxima [A] time = 0.923332, size = 73, normalized size = 2.03

$$\frac{15\cos(x)^5 - 40\cos(x)^3 + 33\cos(x)}{48\left(\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1\right)} - \frac{5}{32}\log(\cos(x) + 1) + \frac{5}{32}\log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^7,x, algorithm="maxima")
```

```
[Out] 1/48*(15*cos(x)^5 - 40*cos(x)^3 + 33*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos
(x)^2 - 1) - 5/32*log(cos(x) + 1) + 5/32*log(cos(x) - 1)
```

Fricas [B] time = 1.77784, size = 302, normalized size = 8.39

$$\frac{30\cos(x)^5 - 80\cos(x)^3 - 15\left(\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1\right)\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + 15\left(\cos(x)^6 - 3\cos(x)^4 + \right.}{96\left(\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^7,x, algorithm="fricas")
```

```
[Out] 1/96*(30*cos(x)^5 - 80*cos(x)^3 - 15*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 -
1)*log(1/2*cos(x) + 1/2) + 15*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*log(
-1/2*cos(x) + 1/2) + 66*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

Sympy [A] time = 0.158818, size = 60, normalized size = 1.67

$$\frac{15\cos^5(x) - 40\cos^3(x) + 33\cos(x)}{48\cos^6(x) - 144\cos^4(x) + 144\cos^2(x) - 48} + \frac{5\log(\cos(x) - 1)}{32} - \frac{5\log(\cos(x) + 1)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**7,x)
```

```
[Out] (15*cos(x)**5 - 40*cos(x)**3 + 33*cos(x))/(48*cos(x)**6 - 144*cos(x)**4 + 1
44*cos(x)**2 - 48) + 5*log(cos(x) - 1)/32 - 5*log(cos(x) + 1)/32
```

Giac [B] time = 1.05699, size = 151, normalized size = 4.19

$$-\frac{\left(\frac{9(\cos(x)-1)}{\cos(x)+1}-\frac{45(\cos(x)-1)^2}{(\cos(x)+1)^2}+\frac{110(\cos(x)-1)^3}{(\cos(x)+1)^3}-1\right)(\cos(x)+1)^3}{384(\cos(x)-1)^3}-\frac{15(\cos(x)-1)}{128(\cos(x)+1)}+\frac{3(\cos(x)-1)^2}{128(\cos(x)+1)^2}-\frac{(\cos(x)-1)^3}{384(\cos(x)+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^7,x, algorithm="giac")
```

```
[Out] -1/384*(9*(cos(x) - 1)/(cos(x) + 1) - 45*(cos(x) - 1)^2/(cos(x) + 1)^2 + 11
0*(cos(x) - 1)^3/(cos(x) + 1)^3 - 1)*(cos(x) + 1)^3/(cos(x) - 1)^3 - 15/128
*(cos(x) - 1)/(cos(x) + 1) + 3/128*(cos(x) - 1)^2/(cos(x) + 1)^2 - 1/384*(c
os(x) - 1)^3/(cos(x) + 1)^3 + 5/32*log(-(cos(x) - 1)/(cos(x) + 1))
```

3.339 $\int \sec^{12}(x)\,dx$

Optimal. Leaf size=41

$$\frac{\tan^{11}(x)}{11}+\frac{5\tan^9(x)}{9}+\frac{10\tan^7(x)}{7}+2\tan^5(x)+\frac{5\tan^3(x)}{3}+\tan(x)$$

```
[Out] Tan[x] + (5*Tan[x]^3)/3 + 2*Tan[x]^5 + (10*Tan[x]^7)/7 + (5*Tan[x]^9)/9 + T
an[x]^11/11
```

Rubi [A] time = 0.0165278, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3767}

$$\frac{\tan^{11}(x)}{11}+\frac{5\tan^9(x)}{9}+\frac{10\tan^7(x)}{7}+2\tan^5(x)+\frac{5\tan^3(x)}{3}+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^12,x]
```

```
[Out] Tan[x] + (5*Tan[x]^3)/3 + 2*Tan[x]^5 + (10*Tan[x]^7)/7 + (5*Tan[x]^9)/9 + T
an[x]^11/11
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^{12}(x)\,dx &= -\text{Subst}\left(\int \left(1+5x^2+10x^4+10x^6+5x^8+x^{10}\right)dx, x, -\tan(x)\right)\\ &= \tan(x)+\frac{5\tan^3(x)}{3}+2\tan^5(x)+\frac{10\tan^7(x)}{7}+\frac{5\tan^9(x)}{9}+\frac{\tan^{11}(x)}{11}\end{aligned}$$

Mathematica [A] time = 0.0038647, size = 57, normalized size = 1.39

$$\frac{256\tan(x)}{693}+\frac{1}{11}\tan(x)\sec^{10}(x)+\frac{10}{99}\tan(x)\sec^8(x)+\frac{80}{693}\tan(x)\sec^6(x)+\frac{32}{231}\tan(x)\sec^4(x)+\frac{128}{693}\tan(x)\sec^2(x$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^12,x]
```

```
[Out] (256*Tan[x])/693 + (128*Sec[x]^2*Tan[x])/693 + (32*Sec[x]^4*Tan[x])/231 + (
80*Sec[x]^6*Tan[x])/693 + (10*Sec[x]^8*Tan[x])/99 + (Sec[x]^10*Tan[x])/11
```

Maple [A] time = 0.033, size = 37, normalized size = 0.9

$$-\left(-\frac{256}{693}-\frac{(\sec(x))^{10}}{11}-\frac{10(\sec(x))^8}{99}-\frac{80(\sec(x))^6}{693}-\frac{32(\sec(x))^4}{231}-\frac{128(\sec(x))^2}{693}\right)\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^12,x)
```

```
[Out] -(-256/693-1/11*sec(x)^10-10/99*sec(x)^8-80/693*sec(x)^6-32/231*sec(x)^4-12
8/693*sec(x)^2)*tan(x)
```

Maxima [A] time = 0.932061, size = 45, normalized size = 1.1

$$\frac{1}{11}\tan(x)^{11}+\frac{5}{9}\tan(x)^{9}+\frac{10}{7}\tan(x)^{7}+2\tan(x)^{5}+\frac{5}{3}\tan(x)^{3}+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^12,x, algorithm="maxima")
```

```
[Out] 1/11*tan(x)^11 + 5/9*tan(x)^9 + 10/7*tan(x)^7 + 2*tan(x)^5 + 5/3*tan(x)^3 +
 tan(x)
```

Fricas [A] time = 1.609, size = 138, normalized size = 3.37

$$\frac{\left(256\cos(x)^{10}+128\cos(x)^{8}+96\cos(x)^{6}+80\cos(x)^{4}+70\cos(x)^{2}+63\right)\sin(x)}{693\cos(x)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^12,x, algorithm="fricas")
```

```
[Out] 1/693*(256*cos(x)^10 + 128*cos(x)^8 + 96*cos(x)^6 + 80*cos(x)^4 + 70*cos(x)
^2 + 63)*sin(x)/cos(x)^11
```

Sympy [A] time = 0.059934, size = 66, normalized size = 1.61

$$\frac{256\sin(x)}{693\cos(x)}+\frac{128\sin(x)}{693\cos^{3}(x)}+\frac{32\sin(x)}{231\cos^{5}(x)}+\frac{80\sin(x)}{693\cos^{7}(x)}+\frac{10\sin(x)}{99\cos^{9}(x)}+\frac{\sin(x)}{11\cos^{11}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**12,x)
```

```
[Out] 256*sin(x)/(693*cos(x)) + 128*sin(x)/(693*cos(x)**3) + 32*sin(x)/(231*cos(x
)**5) + 80*sin(x)/(693*cos(x)**7) + 10*sin(x)/(99*cos(x)**9) + sin(x)/(11*c
os(x)**11)
```

Giac [A] time = 1.04604, size = 45, normalized size = 1.1

$$\frac{1}{11}\tan(x)^{11}+\frac{5}{9}\tan(x)^{9}+\frac{10}{7}\tan(x)^{7}+2\tan(x)^{5}+\frac{5}{3}\tan(x)^{3}+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^12,x, algorithm="giac")
```

```
[Out] 1/11*tan(x)^11 + 5/9*tan(x)^9 + 10/7*tan(x)^7 + 2*tan(x)^5 + 5/3*tan(x)^3 +
 tan(x)
```

3.340 $\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx$

Optimal. Leaf size=40

$$\frac{1}{6}\tanh^{-1}\left(\sin\left(3x+\frac{\pi}{4}\right)\right)+\frac{1}{6}\tan\left(3x+\frac{\pi}{4}\right)\sec\left(3x+\frac{\pi}{4}\right)$$

```
[Out] ArcTanh[Sin[Pi/4 + 3*x]]/6 + (Sec[Pi/4 + 3*x]*Tan[Pi/4 + 3*x])/6
```

Rubi [A] time = 0.0123502, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3768, 3770}

$$\frac{1}{6}\tanh^{-1}\left(\sin\left(3x+\frac{\pi}{4}\right)\right)+\frac{1}{6}\tan\left(3x+\frac{\pi}{4}\right)\sec\left(3x+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[Pi/4 + 3*x]^3,x]
```

```
[Out] ArcTanh[Sin[Pi/4 + 3*x]]/6 + (Sec[Pi/4 + 3*x]*Tan[Pi/4 + 3*x])/6
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^3\left(\frac{\pi}{4}+3x\right)\,dx&=\frac{1}{6}\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)+\frac{1}{2}\int \csc\left(\frac{\pi}{4}-3x\right)\,dx\\&=\frac{1}{6}\tanh^{-1}\left(\sin\left(\frac{\pi}{4}+3x\right)\right)+\frac{1}{6}\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)\end{aligned}$$

Mathematica [A] time = 0.0119868, size = 40, normalized size = 1.

$$\frac{1}{6}\tanh^{-1}\left(\sin\left(3x+\frac{\pi}{4}\right)\right)+\frac{1}{6}\tan\left(3x+\frac{\pi}{4}\right)\sec\left(3x+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[Pi/4 + 3*x]^3,x]
```

```
[Out] ArcTanh[Sin[Pi/4 + 3*x]]/6 + (Sec[Pi/4 + 3*x]*Tan[Pi/4 + 3*x])/6
```

Maple [A] time = 0.036, size = 40, normalized size = 1.

$$\frac{1}{6}\sec\left(\frac{\pi}{4}+3x\right)\tan\left(\frac{\pi}{4}+3x\right)+\frac{1}{6}\ln\left(\sec\left(\frac{\pi}{4}+3x\right)+\tan\left(\frac{\pi}{4}+3x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(1/4*Pi+3*x)^3,x)
```

```
[Out] 1/6*sec(1/4*Pi+3*x)*tan(1/4*Pi+3*x)+1/6*ln(sec(1/4*Pi+3*x)+tan(1/4*Pi+3*x))
```

Maxima [A] time = 0.92955, size = 69, normalized size = 1.72

$$-\frac{\sin\left(\frac{1}{4}\pi+3x\right)}{6\left(\sin\left(\frac{1}{4}\pi+3x\right)^2-1\right)}+\frac{1}{12}\log\left(\sin\left(\frac{1}{4}\pi+3x\right)+1\right)-\frac{1}{12}\log\left(\sin\left(\frac{1}{4}\pi+3x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(1/4*pi+3*x)^3,x, algorithm="maxima")
```

```
[Out] -1/6*sin(1/4*pi + 3*x)/(sin(1/4*pi + 3*x)^2 - 1) + 1/12*log(sin(1/4*pi + 3*
x) + 1) - 1/12*log(sin(1/4*pi + 3*x) - 1)
```

Fricas [B] time = 1.60991, size = 198, normalized size = 4.95

$$\frac{\cos\left(\frac{1}{4}\pi+3x\right)^2\log\left(\sin\left(\frac{1}{4}\pi+3x\right)+1\right)-\cos\left(\frac{1}{4}\pi+3x\right)^2\log\left(-\sin\left(\frac{1}{4}\pi+3x\right)+1\right)+2\sin\left(\frac{1}{4}\pi+3x\right)}{12\cos\left(\frac{1}{4}\pi+3x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(1/4*pi+3*x)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(cos(1/4*pi + 3*x)^2*log(sin(1/4*pi + 3*x) + 1) - cos(1/4*pi + 3*x)^2*
log(-sin(1/4*pi + 3*x) + 1) + 2*sin(1/4*pi + 3*x))/cos(1/4*pi + 3*x)^2
```

Sympy [B] time = 1.29094, size = 388, normalized size = 9.7

$$-\frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right)\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}+\frac{2\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right)\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}+\frac{\pi}{8}\right)+6}-\frac{\log\left(\tan\left(\frac{3x}{2}+\frac{\pi}{8}\right)-1\right.}{6\tan^4\left(\frac{3x}{2}+\frac{\pi}{8}\right)-12\tan^2\left(\frac{3x}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(1/4*pi+3*x)**3,x)
```

```
[Out] -log(tan(3*x/2 + pi/8) - 1)*tan(3*x/2 + pi/8)**4/(6*tan(3*x/2 + pi/8)**4 -
12*tan(3*x/2 + pi/8)**2 + 6) + 2*log(tan(3*x/2 + pi/8) - 1)*tan(3*x/2 + pi/
8)**2/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) - log(tan(3*x/
```

```
2 + pi/8) - 1)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + log
(tan(3*x/2 + pi/8) + 1)*tan(3*x/2 + pi/8)**4/(6*tan(3*x/2 + pi/8)**4 - 12*t
an(3*x/2 + pi/8)**2 + 6) - 2*log(tan(3*x/2 + pi/8) + 1)*tan(3*x/2 + pi/8)**
2/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + log(tan(3*x/2 +
pi/8) + 1)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + 2*tan(3
*x/2 + pi/8)**3/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6) + 2*
tan(3*x/2 + pi/8)/(6*tan(3*x/2 + pi/8)**4 - 12*tan(3*x/2 + pi/8)**2 + 6)
```

Giac [A] time = 1.06233, size = 72, normalized size = 1.8

$$-\frac{\sin\left(\frac{1}{4}\,\pi+3\,x\right)}{6\left(\sin\left(\frac{1}{4}\,\pi+3\,x\right)^{2}-1\right)}+\frac{1}{12}\,\log\left(\sin\left(\frac{1}{4}\,\pi+3\,x\right)+1\right)-\frac{1}{12}\,\log\left(-\sin\left(\frac{1}{4}\,\pi+3\,x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(1/4*pi+3*x)^3,x, algorithm="giac")
```

```
[Out] -1/6*sin(1/4*pi + 3*x)/(sin(1/4*pi + 3*x)^2 - 1) + 1/12*log(sin(1/4*pi + 3*
x) + 1) - 1/12*log(-sin(1/4*pi + 3*x) + 1)
```

3.341 $\int \tan^6(x)\,dx$

Optimal. Leaf size=22

$$-x+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}+\tan(x)$$

```
[Out] -x + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

Rubi [A] time = 0.0137087, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 8}

$$-x+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^6,x]
```

```
[Out] -x + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tan^6(x)\,dx &= \frac{\tan^5(x)}{5}-\int \tan^4(x)\,dx\\ &= -\frac{1}{3}\tan^3(x)+\frac{\tan^5(x)}{5}+\int \tan^2(x)\,dx\\ &= \tan(x)-\frac{\tan^3(x)}{3}+\frac{\tan^5(x)}{5}-\int 1\,dx\\ &= -x+\tan(x)-\frac{\tan^3(x)}{3}+\frac{\tan^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.0043884, size = 30, normalized size = 1.36

$$-x+\frac{23\tan(x)}{15}+\frac{1}{5}\tan(x)\sec^4(x)-\frac{11}{15}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^6,x]
```

```
[Out] -x + (23*Tan[x])/15 - (11*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

Maple [A] time = 0.002, size = 19, normalized size = 0.9

$$-x+\tan(x)-\frac{(\tan(x))^3}{3}+\frac{(\tan(x))^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^6,x)
```

```
[Out] -x+tan(x)-1/3*tan(x)^3+1/5*tan(x)^5
```

Maxima [A] time = 1.41193, size = 24, normalized size = 1.09

$$\frac{1}{5}\tan(x)^5-\frac{1}{3}\tan(x)^3-x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^6,x, algorithm="maxima")
```

```
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

Fricas [A] time = 1.52648, size = 57, normalized size = 2.59

$$\frac{1}{5}\tan(x)^5-\frac{1}{3}\tan(x)^3-x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^6,x, algorithm="fricas")
```

```
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

Sympy [A] time = 0.068596, size = 31, normalized size = 1.41

$$-x+\frac{\sin^5(x)}{5\cos^5(x)}-\frac{\sin^3(x)}{3\cos^3(x)}+\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**6,x)
```

```
[Out] -x + sin(x)**5/(5*cos(x)**5) - sin(x)**3/(3*cos(x)**3) + sin(x)/cos(x)
```

Giac [A] time = 1.04852, size = 24, normalized size = 1.09

$$\frac{1}{5}\tan(x)^5-\frac{1}{3}\tan(x)^3-x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^6,x, algorithm="giac")
```

```
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

3.342 $\int \cot^5(x)\,dx$

Optimal. Leaf size=20

$$-\frac{1}{4}\cot^4(x)+\frac{\cot^2(x)}{2}+\log(\sin(x))$$

```
[Out] Cot[x]^2/2 - Cot[x]^4/4 + Log[Sin[x]]
```

Rubi [A] time = 0.0150477, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 3475}

$$-\frac{1}{4}\cot^4(x)+\frac{\cot^2(x)}{2}+\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^5,x]
```

```
[Out] Cot[x]^2/2 - Cot[x]^4/4 + Log[Sin[x]]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^5(x)\,dx &= -\frac{1}{4}\cot^4(x)-\int \cot^3(x)\,dx\\ &= \frac{\cot^2(x)}{2}-\frac{\cot^4(x)}{4}+\int \cot(x)\,dx\\ &= \frac{\cot^2(x)}{2}-\frac{\cot^4(x)}{4}+\log(\sin(x))\end{aligned}$$

Mathematica [A] time = 0.0031104, size = 16, normalized size = 0.8

$$-\frac{1}{4}\csc^4(x)+\csc^2(x)+\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^5,x]
```

```
[Out] Csc[x]^2 - Csc[x]^4/4 + Log[Sin[x]]
```

Maple [A] time = 0.007, size = 26, normalized size = 1.3

$$-\frac{\ln\left((\tan(x))^2+1\right)}{2}-\frac{1}{4\,(\tan(x))^4}+\ln(\tan(x))+\frac{1}{2\,(\tan(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(x)^5,x)
```

```
[Out] -1/2*ln(tan(x)^2+1)-1/4/tan(x)^4+ln(tan(x))+1/2/tan(x)^2
```

Maxima [A] time = 0.932898, size = 30, normalized size = 1.5

$$\frac{4\,\sin(x)^2-1}{4\,\sin(x)^4}+\frac{1}{2}\,\log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(4*sin(x)^2 - 1)/sin(x)^4 + 1/2*log(sin(x)^2)
```

Fricas [B] time = 1.72129, size = 116, normalized size = 5.8

$$\frac{2\,\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)\tan(x)^4+3\,\tan(x)^4+2\,\tan(x)^2-1}{4\,\tan(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*log(tan(x)^2/(tan(x)^2 + 1))*tan(x)^4 + 3*tan(x)^4 + 2*tan(x)^2 - 1)
/tan(x)^4
```

Sympy [A] time = 0.1013, size = 19, normalized size = 0.95

$$\frac{4\sin^2(x)-1}{4\sin^4(x)}+\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x)**5,x)
```

```
[Out] (4*sin(x)**2 - 1)/(4*sin(x)**4) + log(sin(x))
```

Giac [B] time = 1.06858, size = 50, normalized size = 2.5

$$-\frac{3\,\tan\left(x\right)^{4}-2\,\tan\left(x\right)^{2}+1}{4\,\tan\left(x\right)^{4}}-\frac{1}{2}\,\log\left(\tan\left(x\right)^{2}+1\right)+\frac{1}{2}\,\log\left(\tan\left(x\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x)^5,x, algorithm="giac")
```

```
[Out] -1/4*(3*tan(x)^4 - 2*tan(x)^2 + 1)/tan(x)^4 - 1/2*log(tan(x)^2 + 1) + 1/2*l
og(tan(x)^2)
```

3.343 $\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx$

Optimal. Leaf size=32

$$x-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)+3\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

```
[Out] x + 3*Cot[Pi/4 + x/3] - Cot[Pi/4 + x/3]^3
```

Rubi [A] time = 0.0139363, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3473, 8}

$$x-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)+3\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[Pi/4 + x/3]^4,x]
```

```
[Out] x + 3*Cot[Pi/4 + x/3] - Cot[Pi/4 + x/3]^3
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cot^4\left(\frac{\pi}{4}+\frac{x}{3}\right)\,dx &= -\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)-\int \tan^2\left(\frac{\pi}{4}-\frac{x}{3}\right)\,dx\\ &= 3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)+\int 1\,dx\\ &= x+3\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)-\cot^3\left(\frac{\pi}{4}+\frac{x}{3}\right)\end{aligned}$$

Mathematica [C] time = 0.0315336, size = 40, normalized size = 1.25

$$-\cot^3\left(\frac{x}{3}+\frac{\pi}{4}\right)\,_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-\tan^2\left(\frac{x}{3}+\frac{\pi}{4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[Pi/4 + x/3]^4,x]
```

```
[Out] -(Cot[Pi/4 + x/3]^3*Hypergeometric2F1[-3/2, 1, -1/2, -Tan[Pi/4 + x/3]^2])
```

Maple [A] time = 0.003, size = 28, normalized size = 0.9

$$-\left(\cot\left(\frac{\pi}{4}+\frac{x}{3}\right)\right)^{3}+3\cot\left(\pi/4+x/3\right)-\frac{3\pi}{4}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(1/4*Pi+1/3*x)^4,x)
```

```
[Out] -cot(1/4*Pi+1/3*x)^3+3*cot(1/4*Pi+1/3*x)-3/4*Pi+x
```

Maxima [A] time = 1.42652, size = 41, normalized size = 1.28

$$\frac{3}{4}\pi+x+\frac{3\tan\left(\frac{1}{4}\pi+\frac{1}{3}x\right)^{2}-1}{\tan\left(\frac{1}{4}\pi+\frac{1}{3}x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(1/4*pi+1/3*x)^4,x, algorithm="maxima")
```

```
[Out] 3/4*pi + x + (3*tan(1/4*pi + 1/3*x)^2 - 1)/tan(1/4*pi + 1/3*x)^3
```

Fricas [B] time = 1.68143, size = 207, normalized size = 6.47

$$\frac{4\cos\left(\frac{1}{2}\pi+\frac{2}{3}x\right)^{2}+\left(x\cos\left(\frac{1}{2}\pi+\frac{2}{3}x\right)-x\right)\sin\left(\frac{1}{2}\pi+\frac{2}{3}x\right)+2\cos\left(\frac{1}{2}\pi+\frac{2}{3}x\right)-2}{\left(\cos\left(\frac{1}{2}\pi+\frac{2}{3}x\right)-1\right)\sin\left(\frac{1}{2}\pi+\frac{2}{3}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(1/4*pi+1/3*x)^4,x, algorithm="fricas")
```

```
[Out] (4*cos(1/2*pi + 2/3*x)^2 + (x*cos(1/2*pi + 2/3*x) - x)*sin(1/2*pi + 2/3*x)
+ 2*cos(1/2*pi + 2/3*x) - 2)/((cos(1/2*pi + 2/3*x) - 1)*sin(1/2*pi + 2/3*x)
)
```

Sympy [A] time = 0.194499, size = 20, normalized size = 0.62

$$x-\cot^{3}\left(\frac{x}{3}+\frac{\pi}{4}\right)+3\cot\left(\frac{x}{3}+\frac{\pi}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(1/4*pi+1/3*x)**4,x)
```

```
[Out] x - cot(x/3 + pi/4)**3 + 3*cot(x/3 + pi/4)
```

Giac [B] time = 1.09137, size = 72, normalized size = 2.25

$$\frac{3}{4}\pi + \frac{1}{8}\tan\left(\frac{1}{8}\pi + \frac{1}{6}x\right)^3 + x + \frac{15\tan\left(\frac{1}{8}\pi + \frac{1}{6}x\right)^2 - 1}{8\tan\left(\frac{1}{8}\pi + \frac{1}{6}x\right)^3} - \frac{15}{8}\tan\left(\frac{1}{8}\pi + \frac{1}{6}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(1/4*pi+1/3*x)^4,x, algorithm="giac")
```

```
[Out] 3/4*pi + 1/8*tan(1/8*pi + 1/6*x)^3 + x + 1/8*(15*tan(1/8*pi + 1/6*x)^2 - 1)
/tan(1/8*pi + 1/6*x)^3 - 15/8*tan(1/8*pi + 1/6*x)
```

3.344 $\int \cos^6(x)\sin^4(x)\,dx$

Optimal. Leaf size=56

$$\frac{3x}{256}-\frac{1}{10}\sin^3(x)\cos^7(x)-\frac{3}{80}\sin(x)\cos^7(x)+\frac{1}{160}\sin(x)\cos^5(x)+\frac{1}{128}\sin(x)\cos^3(x)+\frac{3}{256}\sin(x)\cos(x)$$

```
[Out] (3*x)/256 + (3*Cos[x]*Sin[x])/256 + (Cos[x]^3*Sin[x])/128 + (Cos[x]^5*Sin[x
])/160 - (3*Cos[x]^7*Sin[x])/80 - (Cos[x]^7*Sin[x]^3)/10
```

Rubi [A] time = 0.0623641, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{3x}{256}-\frac{1}{10}\sin^3(x)\cos^7(x)-\frac{3}{80}\sin(x)\cos^7(x)+\frac{1}{160}\sin(x)\cos^5(x)+\frac{1}{128}\sin(x)\cos^3(x)+\frac{3}{256}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6*Sin[x]^4,x]
```

```
[Out] (3*x)/256 + (3*Cos[x]*Sin[x])/256 + (Cos[x]^3*Sin[x])/128 + (Cos[x]^5*Sin[x
])/160 - (3*Cos[x]^7*Sin[x])/80 - (Cos[x]^7*Sin[x]^3)/10
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^6(x)\sin^4(x)\,dx &= -\frac{1}{10}\cos^7(x)\sin^3(x)+\frac{3}{10}\int \cos^6(x)\sin^2(x)\,dx\\
&= -\frac{3}{80}\cos^7(x)\sin(x)-\frac{1}{10}\cos^7(x)\sin^3(x)+\frac{3}{80}\int \cos^6(x)\,dx\\
&= \frac{1}{160}\cos^5(x)\sin(x)-\frac{3}{80}\cos^7(x)\sin(x)-\frac{1}{10}\cos^7(x)\sin^3(x)+\frac{1}{32}\int \cos^4(x)\,dx\\
&= \frac{1}{128}\cos^3(x)\sin(x)+\frac{1}{160}\cos^5(x)\sin(x)-\frac{3}{80}\cos^7(x)\sin(x)-\frac{1}{10}\cos^7(x)\sin^3(x)+\frac{3}{128}\int \cos^2(x
\\
&= \frac{3}{256}\cos(x)\sin(x)+\frac{1}{128}\cos^3(x)\sin(x)+\frac{1}{160}\cos^5(x)\sin(x)-\frac{3}{80}\cos^7(x)\sin(x)-\frac{1}{10}\cos^7(x)\sin\\
&= \frac{3x}{256}+\frac{3}{256}\cos(x)\sin(x)+\frac{1}{128}\cos^3(x)\sin(x)+\frac{1}{160}\cos^5(x)\sin(x)-\frac{3}{80}\cos^7(x)\sin(x)-\frac{1}{10}\cos
\end{aligned}$$

Mathematica [A] time = 0.0154152, size = 46, normalized size = 0.82

$$\frac{3x}{256}+\frac{1}{512}\sin(2x)-\frac{1}{256}\sin(4x)-\frac{\sin(6x)}{1024}+\frac{\sin(8x)}{2048}+\frac{\sin(10x)}{5120}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6*Sin[x]^4,x]
```

```
[Out] (3*x)/256 + Sin[2*x]/512 - Sin[4*x]/256 - Sin[6*x]/1024 + Sin[8*x]/2048 + S
in[10*x]/5120
```

Maple [A] time = 0.008, size = 42, normalized size = 0.8

$$-\frac{(\cos(x))^7(\sin(x))^3}{10}-\frac{3(\cos(x))^7\sin(x)}{80}+\frac{\sin(x)}{160}\left((\cos(x))^5+\frac{5(\cos(x))^3}{4}+\frac{15\cos(x)}{8}\right)+\frac{3x}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6*sin(x)^4,x)
```

```
[Out] -1/10*cos(x)^7*sin(x)^3-3/80*cos(x)^7*sin(x)+1/160*(cos(x)^5+5/4*cos(x)^3+1
5/8*cos(x))*sin(x)+3/256*x
```

Maxima [A] time = 0.930434, size = 32, normalized size = 0.57

$$\frac{1}{320}\sin(2x)^5+\frac{3}{256}x+\frac{1}{2048}\sin(8x)-\frac{1}{256}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^4,x, algorithm="maxima")
```

```
[Out] 1/320*sin(2*x)^5 + 3/256*x + 1/2048*sin(8*x) - 1/256*sin(4*x)
```

Fricas [A] time = 1.98124, size = 127, normalized size = 2.27

$$\frac{1}{1280}\left(128\cos(x)^9-176\cos(x)^7+8\cos(x)^5+10\cos(x)^3+15\cos(x)\right)\sin(x)+\frac{3}{256}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/1280*(128*cos(x)^9 - 176*cos(x)^7 + 8*cos(x)^5 + 10*cos(x)^3 + 15*cos(x))
*sin(x) + 3/256*x
```

Sympy [A] time = 0.06141, size = 56, normalized size = 1.

$$\frac{3x}{256} + \frac{\sin(x)\cos^9(x)}{10} - \frac{11\sin(x)\cos^7(x)}{80} + \frac{\sin(x)\cos^5(x)}{160} + \frac{\sin(x)\cos^3(x)}{128} + \frac{3\sin(x)\cos(x)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6*sin(x)**4,x)
```

```
[Out] 3*x/256 + sin(x)*cos(x)**9/10 - 11*sin(x)*cos(x)**7/80 + sin(x)*cos(x)**5/1
60 + sin(x)*cos(x)**3/128 + 3*sin(x)*cos(x)/256
```

Giac [A] time = 1.05848, size = 46, normalized size = 0.82

$$\frac{3}{256}x + \frac{1}{5120}\sin(10x) + \frac{1}{2048}\sin(8x) - \frac{1}{1024}\sin(6x) - \frac{1}{256}\sin(4x) + \frac{1}{512}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/256*x + 1/5120*sin(10*x) + 1/2048*sin(8*x) - 1/1024*sin(6*x) - 1/256*sin(
4*x) + 1/512*sin(2*x)
```

3.345 $\int \cos^6(x)\sin^7(x)\,dx$

Optimal. Leaf size=33

$$\frac{\cos^{13}(x)}{13}-\frac{3\cos^{11}(x)}{11}+\frac{\cos^9(x)}{3}-\frac{\cos^7(x)}{7}$$

```
[Out] -Cos[x]^7/7 + Cos[x]^9/3 - (3*Cos[x]^11)/11 + Cos[x]^13/13
```

Rubi [A] time = 0.0300866, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2565, 270}

$$\frac{\cos^{13}(x)}{13}-\frac{3\cos^{11}(x)}{11}+\frac{\cos^9(x)}{3}-\frac{\cos^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6*Sin[x]^7,x]
```

```
[Out] -Cos[x]^7/7 + Cos[x]^9/3 - (3*Cos[x]^11)/11 + Cos[x]^13/13
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^6(x)\sin^7(x)\,dx &= -\operatorname{Subst}\left(\int x^6\left(1-x^2\right)^3\,dx,x,\cos(x)\right)\\
&= -\operatorname{Subst}\left(\int \left(x^6-3x^8+3x^{10}-x^{12}\right)dx,x,\cos(x)\right)\\
&= -\frac{1}{7}\cos^7(x)+\frac{\cos^9(x)}{3}-\frac{3\cos^{11}(x)}{11}+\frac{\cos^{13}(x)}{13}
\end{aligned}$$

Mathematica [A] time = 0.0272894, size = 55, normalized size = 1.67

$$-\frac{5\cos(x)}{1024}-\frac{5\cos(3x)}{4096}+\frac{3\cos(5x)}{4096}+\frac{3\cos(7x)}{14336}-\frac{\cos(9x)}{6144}-\frac{\cos(11x)}{45056}+\frac{\cos(13x)}{53248}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6*Sin[x]^7,x]
```

```
[Out] (-5*Cos[x])/1024 - (5*Cos[3*x])/4096 + (3*Cos[5*x])/4096 + (3*Cos[7*x])/143
36 - Cos[9*x]/6144 - Cos[11*x]/45056 + Cos[13*x]/53248
```

Maple [A] time = 0.008, size = 38, normalized size = 1.2

$$-\frac{(\cos(x))^7(\sin(x))^6}{13}-\frac{6(\sin(x))^4(\cos(x))^7}{143}-\frac{8(\sin(x))^2(\cos(x))^7}{429}-\frac{16(\cos(x))^7}{3003}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6*sin(x)^7,x)
```

```
[Out] -1/13*cos(x)^7*sin(x)^6-6/143*sin(x)^4*cos(x)^7-8/429*sin(x)^2*cos(x)^7-16/
3003*cos(x)^7
```

Maxima [A] time = 0.934259, size = 34, normalized size = 1.03

$$\frac{1}{13}\cos(x)^{13}-\frac{3}{11}\cos(x)^{11}+\frac{1}{3}\cos(x)^9-\frac{1}{7}\cos(x)^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^7,x, algorithm="maxima")
```

```
[Out] 1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

Fricas [A] time = 2.05043, size = 85, normalized size = 2.58

$$\frac{1}{13}\cos(x)^{13}-\frac{3}{11}\cos(x)^{11}+\frac{1}{3}\cos(x)^9-\frac{1}{7}\cos(x)^7$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^7,x, algorithm="fricas")
```

```
[Out] 1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

Sympy [A] time = 0.064692, size = 27, normalized size = 0.82

$$\frac{\cos^{13}(x)}{13}-\frac{3\cos^{11}(x)}{11}+\frac{\cos^9(x)}{3}-\frac{\cos^7(x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6*sin(x)**7,x)
```

```
[Out] cos(x)**13/13 - 3*cos(x)**11/11 + cos(x)**9/3 - cos(x)**7/7
```

Giac [A] time = 1.04935, size = 34, normalized size = 1.03

$$\frac{1}{13}\cos(x)^{13} - \frac{3}{11}\cos(x)^{11} + \frac{1}{3}\cos(x)^{9} - \frac{1}{7}\cos(x)^{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^7,x, algorithm="giac")
```

```
[Out] 1/13*cos(x)^13 - 3/11*cos(x)^11 + 1/3*cos(x)^9 - 1/7*cos(x)^7
```

3.346 $\int \sin^{10}(x)\tan(x)\,dx$

Optimal. Leaf size=46

$$\frac{\cos^{10}(x)}{10}-\frac{5\cos^8(x)}{8}+\frac{5\cos^6(x)}{3}-\frac{5\cos^4(x)}{2}+\frac{5\cos^2(x)}{2}-\log(\cos(x))$$

```
[Out] (5*Cos[x]^2)/2 - (5*Cos[x]^4)/2 + (5*Cos[x]^6)/3 - (5*Cos[x]^8)/8 + Cos[x]^
10/10 - Log[Cos[x]]
```

Rubi [A] time = 0.0280991, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2590, 266, 43}

$$\frac{\cos^{10}(x)}{10}-\frac{5\cos^8(x)}{8}+\frac{5\cos^6(x)}{3}-\frac{5\cos^4(x)}{2}+\frac{5\cos^2(x)}{2}-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^10*Tan[x],x]
```

```
[Out] (5*Cos[x]^2)/2 - (5*Cos[x]^4)/2 + (5*Cos[x]^6)/3 - (5*Cos[x]^8)/8 + Cos[x]^
10/10 - Log[Cos[x]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sin^{10}(x)\tan(x)\,dx &= -\text{Subst}\left(\int \frac{\left(1-x^2\right)^5}{x}\,dx,x,\cos(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^5}{x}\,dx,x,\cos^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-5+\frac{1}{x}+10x-10x^2+5x^3-x^4\right)dx,x,\cos^2(x)\right)\right)\\
&= \frac{5\cos^2(x)}{2}-\frac{5\cos^4(x)}{2}+\frac{5\cos^6(x)}{3}-\frac{5\cos^8(x)}{8}+\frac{\cos^{10}(x)}{10}-\log(\cos(x))
\end{aligned}$$

Mathematica [A] time = 0.0149302, size = 46, normalized size = 1.

$$\frac{\cos^{10}(x)}{10}-\frac{5\cos^8(x)}{8}+\frac{5\cos^6(x)}{3}-\frac{5\cos^4(x)}{2}+\frac{5\cos^2(x)}{2}-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^10*Tan[x],x]
```

```
[Out] (5*Cos[x]^2)/2 - (5*Cos[x]^4)/2 + (5*Cos[x]^6)/3 - (5*Cos[x]^8)/8 + Cos[x]^
10/10 - Log[Cos[x]]
```

Maple [A] time = 0.01, size = 37, normalized size = 0.8

$$-\frac{(\sin(x))^{10}}{10}-\frac{(\sin(x))^8}{8}-\frac{(\sin(x))^6}{6}-\frac{(\sin(x))^4}{4}-\frac{(\sin(x))^2}{2}-\ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^11/cos(x),x)
```

```
[Out] -1/10*sin(x)^10-1/8*sin(x)^8-1/6*sin(x)^6-1/4*sin(x)^4-1/2*sin(x)^2-ln(cos(
x))
```

Maxima [A] time = 0.935469, size = 54, normalized size = 1.17

$$-\frac{1}{10}\sin(x)^{10}-\frac{1}{8}\sin(x)^8-\frac{1}{6}\sin(x)^6-\frac{1}{4}\sin(x)^4-\frac{1}{2}\sin(x)^2-\frac{1}{2}\log\left(\sin(x)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^11/cos(x),x, algorithm="maxima")
```

```
[Out] -1/10*sin(x)^10 - 1/8*sin(x)^8 - 1/6*sin(x)^6 - 1/4*sin(x)^4 - 1/2*sin(x)^2
 - 1/2*log(sin(x)^2 - 1)
```

Fricas [A] time = 2.12827, size = 123, normalized size = 2.67

$$\frac{1}{10}\cos(x)^{10}-\frac{5}{8}\cos(x)^8+\frac{5}{3}\cos(x)^6-\frac{5}{2}\cos(x)^4+\frac{5}{2}\cos(x)^2-\log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^11/cos(x),x, algorithm="fricas")
```

```
[Out] 1/10*cos(x)^10 - 5/8*cos(x)^8 + 5/3*cos(x)^6 - 5/2*cos(x)^4 + 5/2*cos(x)^2
- log(-cos(x))
```

Sympy [A] time = 0.090494, size = 44, normalized size = 0.96

$$-\log(\cos(x)) + \frac{\cos^{10}(x)}{10} - \frac{5\cos^8(x)}{8} + \frac{5\cos^6(x)}{3} - \frac{5\cos^4(x)}{2} + \frac{5\cos^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**11/cos(x),x)
```

```
[Out] -log(cos(x)) + cos(x)**10/10 - 5*cos(x)**8/8 + 5*cos(x)**6/3 - 5*cos(x)**4/
2 + 5*cos(x)**2/2
```

Giac [A] time = 1.07015, size = 51, normalized size = 1.11

$$\frac{1}{10}\cos(x)^{10} - \frac{5}{8}\cos(x)^8 + \frac{5}{3}\cos(x)^6 - \frac{5}{2}\cos(x)^4 + \frac{5}{2}\cos(x)^2 - \frac{1}{2}\log\left(\cos(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^11/cos(x),x, algorithm="giac")
```

```
[Out] 1/10*cos(x)^10 - 5/8*cos(x)^8 + 5/3*cos(x)^6 - 5/2*cos(x)^4 + 5/2*cos(x)^2
- 1/2*log(cos(x)^2)
```

3.347 $\int \csc^6(x)\sec^6(x)\,dx$

Optimal. Leaf size=41

$$\frac{\tan^5(x)}{5}+\frac{5\tan^3(x)}{3}+10\tan(x)-\frac{1}{5}\cot^5(x)-\frac{5\cot^3(x)}{3}-10\cot(x)$$

```
[Out] -10*Cot[x] - (5*Cot[x]^3)/3 - Cot[x]^5/5 + 10*Tan[x] + (5*Tan[x]^3)/3 + Tan
[x]^5/5
```

Rubi [A] time = 0.0338857, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2620, 270}

$$\frac{\tan^5(x)}{5}+\frac{5\tan^3(x)}{3}+10\tan(x)-\frac{1}{5}\cot^5(x)-\frac{5\cot^3(x)}{3}-10\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^6*Sec[x]^6,x]
```

```
[Out] -10*Cot[x] - (5*Cot[x]^3)/3 - Cot[x]^5/5 + 10*Tan[x] + (5*Tan[x]^3)/3 + Tan
[x]^5/5
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc^6(x)\sec^6(x)\,dx &= \text{Subst}\left(\int \frac{\left(1+x^2\right)^5}{x^6}\,dx,x,\tan(x)\right)\\
&= \text{Subst}\left(\int \left(10+\frac{1}{x^6}+\frac{5}{x^4}+\frac{10}{x^2}+5x^2+x^4\right)dx,x,\tan(x)\right)\\
&= -10\cot(x)-\frac{5\cot^3(x)}{3}-\frac{\cot^5(x)}{5}+10\tan(x)+\frac{5\tan^3(x)}{3}+\frac{\tan^5(x)}{5}
\end{aligned}$$

Mathematica [A] time = 0.0363721, size = 53, normalized size = 1.29

$$\frac{128\tan(x)}{15}-\frac{128\cot(x)}{15}-\frac{1}{5}\cot(x)\csc^4(x)-\frac{19}{15}\cot(x)\csc^2(x)+\frac{1}{5}\tan(x)\sec^4(x)+\frac{19}{15}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^6*Sec[x]^6,x]
```

```
[Out] (-128*Cot[x])/15 - (19*Cot[x]*Csc[x]^2)/15 - (Cot[x]*Csc[x]^4)/5 + (128*Tan
[x])/15 + (19*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

Maple [A] time = 0.011, size = 56, normalized size = 1.4

$$\frac{1}{5\left(\sin\left(x\right)\right)^{5}\left(\cos\left(x\right)\right)^{5}}-\frac{2}{5\left(\sin\left(x\right)\right)^{5}\left(\cos\left(x\right)\right)^{3}}+\frac{16}{15\left(\cos\left(x\right)\right)^{3}\left(\sin\left(x\right)\right)^{3}}-\frac{32}{15\cos\left(x\right)\left(\sin\left(x\right)\right)^{3}}+\frac{128}{15\cos\left(x\right)\sin\left(x\right)}-256$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^6/sin(x)^6,x)
```

```
[Out] 1/5/sin(x)^5/cos(x)^5-2/5/sin(x)^5/cos(x)^3+16/15/sin(x)^3/cos(x)^3-32/15/s
in(x)^3/cos(x)+128/15/cos(x)/sin(x)-256/15*cot(x)
```

Maxima [A] time = 0.943402, size = 50, normalized size = 1.22

$$\frac{1}{5}\tan\left(x\right)^{5}+\frac{5}{3}\tan\left(x\right)^{3}-\frac{150\tan\left(x\right)^{4}+25\tan\left(x\right)^{2}+3}{15\tan\left(x\right)^{5}}+10\tan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^6/sin(x)^6,x, algorithm="maxima")
```

```
[Out] 1/5*tan(x)^5 + 5/3*tan(x)^3 - 1/15*(150*tan(x)^4 + 25*tan(x)^2 + 3)/tan(x)^
5 + 10*tan(x)
```

Fricas [A] time = 1.93181, size = 174, normalized size = 4.24

$$-\frac{256\cos\left(x\right)^{10}-640\cos\left(x\right)^{8}+480\cos\left(x\right)^{6}-80\cos\left(x\right)^{4}-10\cos\left(x\right)^{2}-3}{15\left(\cos\left(x\right)^{9}-2\cos\left(x\right)^{7}+\cos\left(x\right)^{5}\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^6/sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/15*(256*cos(x)^10 - 640*cos(x)^8 + 480*cos(x)^6 - 80*cos(x)^4 - 10*cos(x
)^2 - 3)/((cos(x)^9 - 2*cos(x)^7 + cos(x)^5)*sin(x))
```

Sympy [A] time = 0.067319, size = 44, normalized size = 1.07

$$-\frac{256\cos\left(2x\right)}{15\sin\left(2x\right)}-\frac{128\cos\left(2x\right)}{15\sin^{3}\left(2x\right)}-\frac{32\cos\left(2x\right)}{5\sin^{5}\left(2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**6/sin(x)**6,x)
```

```
[Out] -256*cos(2*x)/(15*sin(2*x)) - 128*cos(2*x)/(15*sin(2*x)**3) - 32*cos(2*x)/(
5*sin(2*x)**5)
```

Giac [A] time = 1.06296, size = 35, normalized size = 0.85

$$-\frac{32\left(15\tan\left(2x\right)^{4}+10\tan\left(2x\right)^{2}+3\right)}{15\tan\left(2x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^6/sin(x)^6,x, algorithm="giac")
```

```
[Out] -32/15*(15*tan(2*x)^4 + 10*tan(2*x)^2 + 3)/tan(2*x)^5
```

3.348 $\int \cos^2(x)\sin^2(x)\,dx$

Optimal. Leaf size=24

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rubi [A] time = 0.0272315, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\sin^2(x)\,dx &= -\frac{1}{4}\cos^3(x)\sin(x)+\frac{1}{4}\int \cos^2(x)\,dx\\ &= \frac{1}{8}\cos(x)\sin(x)-\frac{1}{4}\cos^3(x)\sin(x)+\frac{\int 1\,dx}{8}\\ &= \frac{x}{8}+\frac{1}{8}\cos(x)\sin(x)-\frac{1}{4}\cos^3(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0042527, size = 14, normalized size = 0.58

$$\frac{x}{8}-\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 - Sin[4*x]/32
```

Maple [A] time = 0.004, size = 19, normalized size = 0.8

$$\frac{x}{8} + \frac{\cos(x)\sin(x)}{8} - \frac{(\cos(x))^3 \sin(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2,x)
```

```
[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

Maxima [A] time = 0.931042, size = 14, normalized size = 0.58

$$\frac{1}{8}x - \frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

Fricas [A] time = 1.82253, size = 58, normalized size = 2.42

$$-\frac{1}{8}\left(2\cos(x)^3 - \cos(x)\right)\sin(x) + \frac{1}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

Sympy [A] time = 0.062906, size = 14, normalized size = 0.58

$$\frac{x}{8} - \frac{\sin(2x)\cos(2x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**2,x)
```

```
[Out] x/8 - sin(2*x)*cos(2*x)/16
```

Giac [A] time = 1.06056, size = 14, normalized size = 0.58

$$\frac{1}{8}x - \frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

3.349 $\int \cos^4(x)\sin^4(x)\,dx$

Optimal. Leaf size=46

$$\frac{3x}{128}-\frac{1}{8}\sin^3(x)\cos^5(x)-\frac{1}{16}\sin(x)\cos^5(x)+\frac{1}{64}\sin(x)\cos^3(x)+\frac{3}{128}\sin(x)\cos(x)$$

```
[Out] (3*x)/128 + (3*Cos[x]*Sin[x])/128 + (Cos[x]^3*Sin[x])/64 - (Cos[x]^5*Sin[x]
)/16 - (Cos[x]^5*Sin[x]^3)/8
```

Rubi [A] time = 0.0569754, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{3x}{128}-\frac{1}{8}\sin^3(x)\cos^5(x)-\frac{1}{16}\sin(x)\cos^5(x)+\frac{1}{64}\sin(x)\cos^3(x)+\frac{3}{128}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4*Sin[x]^4,x]
```

```
[Out] (3*x)/128 + (3*Cos[x]*Sin[x])/128 + (Cos[x]^3*Sin[x])/64 - (Cos[x]^5*Sin[x]
)/16 - (Cos[x]^5*Sin[x]^3)/8
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(x)\sin^4(x)\,dx &= -\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{3}{8}\int \cos^4(x)\sin^2(x)\,dx\\
&= -\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{1}{16}\int \cos^4(x)\,dx\\
&= \frac{1}{64}\cos^3(x)\sin(x)-\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{3}{64}\int \cos^2(x)\,dx\\
&= \frac{3}{128}\cos(x)\sin(x)+\frac{1}{64}\cos^3(x)\sin(x)-\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{3\int 1\,dx}{128}\\
&= \frac{3x}{128}+\frac{3}{128}\cos(x)\sin(x)+\frac{1}{64}\cos^3(x)\sin(x)-\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)
\end{aligned}$$

Mathematica [A] time = 0.0067803, size = 22, normalized size = 0.48

$$\frac{3x}{128}-\frac{1}{128}\sin(4x)+\frac{\sin(8x)}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4*Sin[x]^4,x]
```

```
[Out] (3*x)/128 - Sin[4*x]/128 + Sin[8*x]/1024
```

Maple [A] time = 0., size = 36, normalized size = 0.8

$$-\frac{(\cos(x))^5(\sin(x))^3}{8}-\frac{(\cos(x))^5\sin(x)}{16}+\frac{\sin(x)}{64}\left((\cos(x))^3+\frac{3\cos(x)}{2}\right)+\frac{3x}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*sin(x)^4,x)
```

```
[Out] -1/8*cos(x)^5*sin(x)^3-1/16*cos(x)^5*sin(x)+1/64*(cos(x)^3+3/2*cos(x))*sin(
x)+3/128*x
```

Maxima [A] time = 0.93297, size = 22, normalized size = 0.48

$$\frac{3}{128}x+\frac{1}{1024}\sin(8x)-\frac{1}{128}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="maxima")
```

```
[Out] 3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

Fricas [A] time = 1.76376, size = 103, normalized size = 2.24

$$\frac{1}{128}\left(16\cos(x)^7-24\cos(x)^5+2\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{3}{128}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(16*cos(x)^7 - 24*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 3/128*x
```

Sympy [A] time = 0.066674, size = 31, normalized size = 0.67

$$\frac{3x}{128}-\frac{\sin^3(2x)\cos(2x)}{128}-\frac{3\sin(2x)\cos(2x)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4*sin(x)**4,x)
```

```
[Out] 3*x/128 - sin(2*x)**3*cos(2*x)/128 - 3*sin(2*x)*cos(2*x)/256
```

Giac [A] time = 1.0656, size = 22, normalized size = 0.48

$$\frac{3}{128}\,x + \frac{1}{1024}\,\sin\left(8\,x\right) - \frac{1}{128}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

3.350 $\int \cos^6(x) \sin^6(x) \, dx$

Optimal. Leaf size=68

$$\frac{5x}{1024} - \frac{1}{12}\sin^5(x)\cos^7(x) - \frac{1}{24}\sin^3(x)\cos^7(x) - \frac{1}{64}\sin(x)\cos^7(x) + \frac{1}{384}\sin(x)\cos^5(x) + \frac{5\sin(x)\cos^3(x)}{1536} + \frac{5\sin(x)\cos(x)}{1024}$$

```
[Out] (5*x)/1024 + (5*Cos[x]*Sin[x])/1024 + (5*Cos[x]^3*Sin[x])/1536 + (Cos[x]^5*
Sin[x])/384 - (Cos[x]^7*Sin[x])/64 - (Cos[x]^7*Sin[x]^3)/24 - (Cos[x]^7*Sin
[x]^5)/12
```

Rubi [A] time = 0.084813, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{5x}{1024} - \frac{1}{12}\sin^5(x)\cos^7(x) - \frac{1}{24}\sin^3(x)\cos^7(x) - \frac{1}{64}\sin(x)\cos^7(x) + \frac{1}{384}\sin(x)\cos^5(x) + \frac{5\sin(x)\cos^3(x)}{1536} + \frac{5\sin(x)\cos(x)}{1024}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6*Sin[x]^6,x]
```

```
[Out] (5*x)/1024 + (5*Cos[x]*Sin[x])/1024 + (5*Cos[x]^3*Sin[x])/1536 + (Cos[x]^5*
Sin[x])/384 - (Cos[x]^7*Sin[x])/64 - (Cos[x]^7*Sin[x]^3)/24 - (Cos[x]^7*Sin
[x]^5)/12
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^6(x)\sin^6(x)\,dx &= -\frac{1}{12}\cos^7(x)\sin^5(x) + \frac{5}{12}\int \cos^6(x)\sin^4(x)\,dx \\ &= -\frac{1}{24}\cos^7(x)\sin^3(x) - \frac{1}{12}\cos^7(x)\sin^5(x) + \frac{1}{8}\int \cos^6(x)\sin^2(x)\,dx \\ &= -\frac{1}{64}\cos^7(x)\sin(x) - \frac{1}{24}\cos^7(x)\sin^3(x) - \frac{1}{12}\cos^7(x)\sin^5(x) + \frac{1}{64}\int \cos^6(x)\,dx \\ &= \frac{1}{384}\cos^5(x)\sin(x) - \frac{1}{64}\cos^7(x)\sin(x) - \frac{1}{24}\cos^7(x)\sin^3(x) - \frac{1}{12}\cos^7(x)\sin^5(x) + \frac{5}{384}\int \cos \\ &= \frac{5\cos^3(x)\sin(x)}{1536} + \frac{1}{384}\cos^5(x)\sin(x) - \frac{1}{64}\cos^7(x)\sin(x) - \frac{1}{24}\cos^7(x)\sin^3(x) - \frac{1}{12}\cos^7(x)\sin \\ &= \frac{5\cos(x)\sin(x)}{1024} + \frac{5\cos^3(x)\sin(x)}{1536} + \frac{1}{384}\cos^5(x)\sin(x) - \frac{1}{64}\cos^7(x)\sin(x) - \frac{1}{24}\cos^7(x)\sin^3 \\ &= \frac{5x}{1024} + \frac{5\cos(x)\sin(x)}{1024} + \frac{5\cos^3(x)\sin(x)}{1536} + \frac{1}{384}\cos^5(x)\sin(x) - \frac{1}{64}\cos^7(x)\sin(x) - \frac{1}{24}\cos\end{aligned}$$

Mathematica [A] time = 0.0152357, size = 30, normalized size = 0.44

$$\frac{5x}{1024} - \frac{15\sin(4x)}{8192} + \frac{3\sin(8x)}{8192} - \frac{\sin(12x)}{24576}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6*Sin[x]^6,x]
```

```
[Out] (5*x)/1024 - (15*Sin[4*x])/8192 + (3*Sin[8*x])/8192 - Sin[12*x]/24576
```

Maple [A] time = 0.008, size = 52, normalized size = 0.8

$$-\frac{(\cos(x))^7(\sin(x))^5}{12} - \frac{(\cos(x))^7(\sin(x))^3}{24} - \frac{(\cos(x))^7\sin(x)}{64} + \frac{\sin(x)}{384}\left((\cos(x))^5 + \frac{5(\cos(x))^3}{4} + \frac{15\cos(x)}{8}\right) + \frac{5}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6*sin(x)^6,x)
```

```
[Out] -1/12*cos(x)^7*sin(x)^5-1/24*cos(x)^7*sin(x)^3-1/64*cos(x)^7*sin(x)+1/384*(
cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)+5/1024*x
```

Maxima [A] time = 0.938378, size = 32, normalized size = 0.47

$$\frac{1}{6144}\sin(4x)^3 + \frac{5}{1024}x + \frac{3}{8192}\sin(8x) - \frac{1}{512}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^6,x, algorithm="maxima")
```

```
[Out] 1/6144*sin(4*x)^3 + 5/1024*x + 3/8192*sin(8*x) - 1/512*sin(4*x)
```

Fricas [A] time = 2.01354, size = 151, normalized size = 2.22

$$-\frac{1}{3072}\left(256\cos(x)^{11}-640\cos(x)^{9}+432\cos(x)^{7}-8\cos(x)^{5}-10\cos(x)^{3}-15\cos(x)\right)\sin(x)+\frac{5}{1024}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/3072*(256*cos(x)^11 - 640*cos(x)^9 + 432*cos(x)^7 - 8*cos(x)^5 - 10*cos(
x)^3 - 15*cos(x))*sin(x) + 5/1024*x
```

Sympy [A] time = 0.06577, size = 46, normalized size = 0.68

$$\frac{5x}{1024}-\frac{\sin^{5}(2x)\cos(2x)}{768}-\frac{5\sin^{3}(2x)\cos(2x)}{3072}-\frac{5\sin(2x)\cos(2x)}{2048}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6*sin(x)**6,x)
```

```
[Out] 5*x/1024 - sin(2*x)**5*cos(2*x)/768 - 5*sin(2*x)**3*cos(2*x)/3072 - 5*sin(2
*x)*cos(2*x)/2048
```

Giac [A] time = 1.05155, size = 30, normalized size = 0.44

$$\frac{5}{1024}x-\frac{1}{24576}\sin(12x)+\frac{3}{8192}\sin(8x)-\frac{15}{8192}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6*sin(x)^6,x, algorithm="giac")
```

```
[Out] 5/1024*x - 1/24576*sin(12*x) + 3/8192*sin(8*x) - 15/8192*sin(4*x)
```

3.351 $\int \cos^8(x)\sin^8(x)\,dx$

Optimal. Leaf size=90

$$\frac{35x}{32768}-\frac{1}{16}\sin^7(x)\cos^9(x)-\frac{1}{32}\sin^5(x)\cos^9(x)-\frac{5}{384}\sin^3(x)\cos^9(x)-\frac{1}{256}\sin(x)\cos^9(x)+\frac{\sin(x)\cos^7(x)}{2048}+\frac{7\sin}{}$$

```
[Out] (35*x)/32768 + (35*Cos[x]*Sin[x])/32768 + (35*Cos[x]^3*Sin[x])/49152 + (7*C
os[x]^5*Sin[x])/12288 + (Cos[x]^7*Sin[x])/2048 - (Cos[x]^9*Sin[x])/256 - (5
*Cos[x]^9*Sin[x]^3)/384 - (Cos[x]^9*Sin[x]^5)/32 - (Cos[x]^9*Sin[x]^7)/16
```

Rubi [A] time = 0.136664, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{35x}{32768}-\frac{1}{16}\sin^7(x)\cos^9(x)-\frac{1}{32}\sin^5(x)\cos^9(x)-\frac{5}{384}\sin^3(x)\cos^9(x)-\frac{1}{256}\sin(x)\cos^9(x)+\frac{\sin(x)\cos^7(x)}{2048}+\frac{7\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^8*Sin[x]^8,x]
```

```
[Out] (35*x)/32768 + (35*Cos[x]*Sin[x])/32768 + (35*Cos[x]^3*Sin[x])/49152 + (7*C
os[x]^5*Sin[x])/12288 + (Cos[x]^7*Sin[x])/2048 - (Cos[x]^9*Sin[x])/256 - (5
*Cos[x]^9*Sin[x]^3)/384 - (Cos[x]^9*Sin[x]^5)/32 - (Cos[x]^9*Sin[x]^7)/16
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^8(x)\sin^8(x)\,dx &= -\frac{1}{16}\cos^9(x)\sin^7(x) + \frac{7}{16}\int \cos^8(x)\sin^6(x)\,dx \\
&= -\frac{1}{32}\cos^9(x)\sin^5(x) - \frac{1}{16}\cos^9(x)\sin^7(x) + \frac{5}{32}\int \cos^8(x)\sin^4(x)\,dx \\
&= -\frac{5}{384}\cos^9(x)\sin^3(x) - \frac{1}{32}\cos^9(x)\sin^5(x) - \frac{1}{16}\cos^9(x)\sin^7(x) + \frac{5}{128}\int \cos^8(x)\sin^2(x)\,dx \\
&= -\frac{1}{256}\cos^9(x)\sin(x) - \frac{5}{384}\cos^9(x)\sin^3(x) - \frac{1}{32}\cos^9(x)\sin^5(x) - \frac{1}{16}\cos^9(x)\sin^7(x) + \frac{1}{256}\int \mathrm{co} \\
&= \frac{\cos^7(x)\sin(x)}{2048} - \frac{1}{256}\cos^9(x)\sin(x) - \frac{5}{384}\cos^9(x)\sin^3(x) - \frac{1}{32}\cos^9(x)\sin^5(x) - \frac{1}{16}\cos^9(x)\sin^7 \\
&= \frac{7\cos^5(x)\sin(x)}{12288} + \frac{\cos^7(x)\sin(x)}{2048} - \frac{1}{256}\cos^9(x)\sin(x) - \frac{5}{384}\cos^9(x)\sin^3(x) - \frac{1}{32}\cos^9(x)\sin^5(x \\
&= \frac{35\cos^3(x)\sin(x)}{49152} + \frac{7\cos^5(x)\sin(x)}{12288} + \frac{\cos^7(x)\sin(x)}{2048} - \frac{1}{256}\cos^9(x)\sin(x) - \frac{5}{384}\cos^9(x)\sin^3(x) \\
&= \frac{35\cos(x)\sin(x)}{32768} + \frac{35\cos^3(x)\sin(x)}{49152} + \frac{7\cos^5(x)\sin(x)}{12288} + \frac{\cos^7(x)\sin(x)}{2048} - \frac{1}{256}\cos^9(x)\sin(x) - \\
&= \frac{35x}{32768} + \frac{35\cos(x)\sin(x)}{32768} + \frac{35\cos^3(x)\sin(x)}{49152} + \frac{7\cos^5(x)\sin(x)}{12288} + \frac{\cos^7(x)\sin(x)}{2048} - \frac{1}{256}\cos^9(x)
\end{aligned}$$

Mathematica [A] time = 0.0148192, size = 38, normalized size = 0.42

$$\frac{35x}{32768} - \frac{7\sin(4x)}{16384} + \frac{7\sin(8x)}{65536} - \frac{\sin(12x)}{49152} + \frac{\sin(16x)}{524288}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^8*Sin[x]^8,x]
```

```
[Out] (35*x)/32768 - (7*Sin[4*x])/16384 + (7*Sin[8*x])/65536 - Sin[12*x]/49152 +
Sin[16*x]/524288
```

Maple [A] time = 0.033, size = 68, normalized size = 0.8

$$-\frac{(\cos(x))^9(\sin(x))^7}{16} - \frac{(\cos(x))^9(\sin(x))^5}{32} - \frac{5(\cos(x))^9(\sin(x))^3}{384} - \frac{(\cos(x))^9\sin(x)}{256} + \frac{\sin(x)}{2048}\left((\cos(x))^7 + \frac{7(\cos(x)}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^8*sin(x)^8,x)
```

```
[Out] -1/16*cos(x)^9*sin(x)^7-1/32*cos(x)^9*sin(x)^5-5/384*cos(x)^9*sin(x)^3-1/25
6*cos(x)^9*sin(x)+1/2048*(cos(x)^7+7/6*cos(x)^5+35/24*cos(x)^3+35/16*cos(x)
)*sin(x)+35/32768*x
```

Maxima [A] time = 0.944744, size = 41, normalized size = 0.46

$$\frac{1}{12288}\sin(4x)^3 + \frac{35}{32768}x + \frac{1}{524288}\sin(16x) + \frac{7}{65536}\sin(8x) - \frac{1}{2048}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^8*sin(x)^8,x, algorithm="maxima")
```

```
[Out] 1/12288*sin(4*x)^3 + 35/32768*x + 1/524288*sin(16*x) + 7/65536*sin(8*x) - 1
/2048*sin(4*x)
```

Fricas [A] time = 2.00414, size = 208, normalized size = 2.31

$$\frac{1}{98304}\left(6144\cos(x)^{15} - 21504\cos(x)^{13} + 25856\cos(x)^{11} - 10880\cos(x)^{9} + 48\cos(x)^{7} + 56\cos(x)^{5} + 70\cos(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^8*sin(x)^8,x, algorithm="fricas")
```

```
[Out] 1/98304*(6144*cos(x)^15 - 21504*cos(x)^13 + 25856*cos(x)^11 - 10880*cos(x)^
9 + 48*cos(x)^7 + 56*cos(x)^5 + 70*cos(x)^3 + 105*cos(x))*sin(x) + 35/32768
*x
```

Sympy [A] time = 0.069368, size = 61, normalized size = 0.68

$$\frac{35x}{32768} - \frac{\sin^{7}(2x)\cos(2x)}{4096} - \frac{7\sin^{5}(2x)\cos(2x)}{24576} - \frac{35\sin^{3}(2x)\cos(2x)}{98304} - \frac{35\sin(2x)\cos(2x)}{65536}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**8*sin(x)**8,x)
```

```
[Out] 35*x/32768 - sin(2*x)**7*cos(2*x)/4096 - 7*sin(2*x)**5*cos(2*x)/24576 - 35*
sin(2*x)**3*cos(2*x)/98304 - 35*sin(2*x)*cos(2*x)/65536
```

Giac [A] time = 1.05709, size = 38, normalized size = 0.42

$$\frac{35}{32768}x + \frac{1}{524288}\sin(16x) - \frac{1}{49152}\sin(12x) + \frac{7}{65536}\sin(8x) - \frac{7}{16384}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^8*sin(x)^8,x, algorithm="giac")
```

```
[Out] 35/32768*x + 1/524288*sin(16*x) - 1/49152*sin(12*x) + 7/65536*sin(8*x) - 7/
16384*sin(4*x)
```

3.352 $\int \cos^{2m}(x)\sin^{2m}(x)\,dx$

Optimal. Leaf size=68

$$\frac{\sin^{2m+1}(x)\cos^{2m-1}(x)\cos^2(x)^{\frac{1}{2}-m}\,{}_2F_1\left(\frac{1}{2}(1-2m),\frac{1}{2}(2m+1);\frac{1}{2}(2m+3);\sin^2(x)\right)}{2m+1}$$

```
[Out] (Cos[x]^(-1 + 2*m)*(Cos[x]^2)^(1/2 - m)*Hypergeometric2F1[(1 - 2*m)/2, (1 +
 2*m)/2, (3 + 2*m)/2, Sin[x]^2]*Sin[x]^(1 + 2*m))/(1 + 2*m)
```

Rubi [A] time = 0.0374064, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2577}

$$\frac{\sin^{2m+1}(x)\cos^{2m-1}(x)\cos^2(x)^{\frac{1}{2}-m}\text{Hypergeometric2F1}\left(\frac{1}{2}(1-2m),\frac{1}{2}(2m+1),\frac{1}{2}(2m+3),\sin^2(x)\right)}{2m+1}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^(2*m)*Sin[x]^(2*m),x]
```

```
[Out] (Cos[x]^(-1 + 2*m)*(Cos[x]^2)^(1/2 - m)*Hypergeometric2F1[(1 - 2*m)/2, (1 +
 2*m)/2, (3 + 2*m)/2, Sin[x]^2]*Sin[x]^(1 + 2*m))/(1 + 2*m)
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \cos^{2m}(x)\sin^{2m}(x)\,dx = \frac{\cos^{-1+2m}(x)\cos^2(x)^{\frac{1}{2}-m}\,{}_2F_1\left(\frac{1}{2}(1-2m),\frac{1}{2}(1+2m);\frac{1}{2}(3+2m);\sin^2(x)\right)\sin^{1+2m}(x)}{1+2m}$$

Mathematica [A] time = 0.0662521, size = 58, normalized size = 0.85

$$\frac{\sin^{2m+1}(x)\cos^{2m-1}(x)\cos^2(x)^{\frac{1}{2}-m}\,{}_2F_1\left(\frac{1}{2}-m,m+\frac{1}{2};m+\frac{3}{2};\sin^2(x)\right)}{2m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^(2*m)*Sin[x]^(2*m),x]
```

```
[Out] (Cos[x]^(-1 + 2*m)*(Cos[x]^2)^(1/2 - m)*Hypergeometric2F1[1/2 - m, 1/2 + m,
 3/2 + m, Sin[x]^2]*Sin[x]^(1 + 2*m))/(1 + 2*m)
```

Maple [F] time = 0.298, size = 0, normalized size = 0.

$$\int (\cos (x))^{2m} (\sin (x))^{2m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^(2*m)*sin(x)^(2*m),x)
```

```
[Out] int(cos(x)^(2*m)*sin(x)^(2*m),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos (x)^{2m} \sin (x)^{2m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)^(2*m)*sin(x)^(2*m), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\cos (x)^{2m} \sin (x)^{2m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="fricas")
```

```
[Out] integral(cos(x)^(2*m)*sin(x)^(2*m), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^{2m} (x) \cos^{2m} (x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**(2*m)*sin(x)**(2*m),x)
```

```
[Out] Integral(sin(x)**(2*m)*cos(x)**(2*m), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cos (x)^{2m} \sin (x)^{2m} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^(2*m)*sin(x)^(2*m),x, algorithm="giac")
```

```
[Out] integrate(cos(x)^(2*m)*sin(x)^(2*m), x)
```

3.353 $\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx$

Optimal. Leaf size=32

$$\frac{1}{2}\log\left(\tan\left(2x+\frac{\pi}{4}\right)\right)-\frac{1}{4}\cot^2\left(2x+\frac{\pi}{4}\right)$$

```
[Out] -Cot[Pi/4 + 2*x]^2/4 + Log[Tan[Pi/4 + 2*x]]/2
```

Rubi [A] time = 0.0212909, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2620, 14}

$$\frac{1}{2}\log\left(\tan\left(2x+\frac{\pi}{4}\right)\right)-\frac{1}{4}\cot^2\left(2x+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[Pi/4 + 2*x]^3*Sec[Pi/4 + 2*x],x]
```

```
[Out] -Cot[Pi/4 + 2*x]^2/4 + Log[Tan[Pi/4 + 2*x]]/2
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \csc^3\left(\frac{\pi}{4}+2x\right)\sec\left(\frac{\pi}{4}+2x\right)\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1+x^2}{x^3}\,dx,x,\tan\left(\frac{\pi}{4}+2x\right)\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int\left(\frac{1}{x^3}+\frac{1}{x}\right)dx,x,\tan\left(\frac{\pi}{4}+2x\right)\right)\\ &= -\frac{1}{4}\cot^2\left(\frac{\pi}{4}+2x\right)+\frac{1}{2}\log\left(\tan\left(\frac{\pi}{4}+2x\right)\right)\end{aligned}$$

Mathematica [A] time = 0.0668821, size = 45, normalized size = 1.41

$$\frac{1}{4}\left(-\csc^2\left(2x+\frac{\pi}{4}\right)+2\log\left(\sin\left(2x+\frac{\pi}{4}\right)\right)-2\log\left(\cos\left(\frac{1}{4}(8x+\pi)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[Pi/4 + 2*x]^3*Sec[Pi/4 + 2*x],x]
```

```
[Out] (-Csc[Pi/4 + 2*x]^2 - 2*Log[Cos[(Pi + 8*x)/4]] + 2*Log[Sin[Pi/4 + 2*x]])/4
```

Maple [A] time = 0.019, size = 25, normalized size = 0.8

$$-\frac{1}{4}\left(\sin\left(\frac{\pi}{4}+2x\right)\right)^{-2}+\frac{1}{2}\ln\left(\tan\left(\frac{\pi}{4}+2x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(1/4*Pi+2*x)/sin(1/4*Pi+2*x)^3,x)
```

```
[Out] -1/4/sin(1/4*Pi+2*x)^2+1/2*ln(tan(1/4*Pi+2*x))
```

Maxima [A] time = 0.93955, size = 55, normalized size = 1.72

$$-\frac{1}{4\sin\left(\frac{1}{4}\pi+2x\right)^{2}}-\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)^{2}-1\right)+\frac{1}{4}\log\left(\sin\left(\frac{1}{4}\pi+2x\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x, algorithm="maxima")
```

```
[Out] -1/4/sin(1/4*pi + 2*x)^2 - 1/4*log(sin(1/4*pi + 2*x)^2 - 1) + 1/4*log(sin(1
/4*pi + 2*x)^2)
```

Fricas [B] time = 1.81058, size = 207, normalized size = 6.47

$$-\frac{\left(\cos\left(\frac{1}{4}\pi+2x\right)^{2}-1\right)\log\left(\cos\left(\frac{1}{4}\pi+2x\right)^{2}\right)-\left(\cos\left(\frac{1}{4}\pi+2x\right)^{2}-1\right)\log\left(-\frac{1}{4}\cos\left(\frac{1}{4}\pi+2x\right)^{2}+\frac{1}{4}\right)-1}{4\left(\cos\left(\frac{1}{4}\pi+2x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((cos(1/4*pi + 2*x)^2 - 1)*log(cos(1/4*pi + 2*x)^2) - (cos(1/4*pi + 2*
x)^2 - 1)*log(-1/4*cos(1/4*pi + 2*x)^2 + 1/4) - 1)/(cos(1/4*pi + 2*x)^2 - 1
)
```

Sympy [B] time = 1.7433, size = 54, normalized size = 1.69

$$-\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)-1\right)}{2}-\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)+1\right)}{2}+\frac{\log\left(\tan\left(x+\frac{\pi}{8}\right)\right)}{2}-\frac{\tan^{2}\left(x+\frac{\pi}{8}\right)}{16}-\frac{1}{16\tan^{2}\left(x+\frac{\pi}{8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)**3,x)
```

```
[Out] -log(tan(x + pi/8) - 1)/2 - log(tan(x + pi/8) + 1)/2 + log(tan(x + pi/8))/2
 - tan(x + pi/8)**2/16 - 1/(16*tan(x + pi/8)**2)
```

Giac [B] time = 1.10798, size = 178, normalized size = 5.56

$$-\frac{\left(\frac{4\left(\cos\left(\frac{1}{4}\pi+2x\right)-1\right)}{\cos\left(\frac{1}{4}\pi+2x\right)+1}-1\right)\left(\cos\left(\frac{1}{4}\pi+2x\right)+1\right)}{16\left(\cos\left(\frac{1}{4}\pi+2x\right)-1\right)}+\frac{\cos\left(\frac{1}{4}\pi+2x\right)-1}{16\left(\cos\left(\frac{1}{4}\pi+2x\right)+1\right)}+\frac{1}{4}\log\left(-\frac{\cos\left(\frac{1}{4}\pi+2x\right)-1}{\cos\left(\frac{1}{4}\pi+2x\right)+1}\right)-\frac{1}{2}\log\left(\left|-\frac{\cos}{\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(1/4*pi+2*x)/sin(1/4*pi+2*x)^3,x, algorithm="giac")
```

```
[Out] -1/16*(4*(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1) - 1)*(cos(1/4*pi +
 2*x) + 1)/(cos(1/4*pi + 2*x) - 1) + 1/16*(cos(1/4*pi + 2*x) - 1)/(cos(1/4*
pi + 2*x) + 1) + 1/4*log(-(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1))
- 1/2*log(abs(-(cos(1/4*pi + 2*x) - 1)/(cos(1/4*pi + 2*x) + 1) - 1))
```

3.354 $\int \sec^2(x)\tan^2(x)\,dx$

Optimal. Leaf size=8

$$\frac{\tan^3(x)}{3}$$

```
[Out] Tan[x]^3/3
```

Rubi [A] time = 0.021766, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 30}

$$\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Tan[x]^2,x]
```

```
[Out] Tan[x]^3/3
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\tan^2(x)\,dx &= \text{Subst}\left(\int x^2\,dx, x, \tan(x)\right)\\ &= \frac{\tan^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.0027176, size = 8, normalized size = 1.

$$\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Tan[x]^2,x]
```

```
[Out] Tan[x]^3/3
```

Maple [A] time = 0.01, size = 11, normalized size = 1.4

$$\frac{(\sin(x))^3}{3(\cos(x))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*tan(x)^2,x)
```

```
[Out] 1/3*sin(x)^3/cos(x)^3
```

Maxima [A] time = 0.932575, size = 8, normalized size = 1.

$$\frac{1}{3}\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*tan(x)^3
```

Fricas [B] time = 1.74863, size = 50, normalized size = 6.25

$$-\frac{\left(\cos(x)^2-1\right)\sin(x)}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(x)^2 - 1)*sin(x)/cos(x)^3
```

Sympy [B] time = 0.062806, size = 17, normalized size = 2.12

$$-\frac{\sin(x)}{3\cos(x)}+\frac{\sin(x)}{3\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*tan(x)**2,x)
```

```
[Out] -sin(x)/(3*cos(x)) + sin(x)/(3*cos(x)**3)
```

Giac [A] time = 1.07314, size = 8, normalized size = 1.

$$\frac{1}{3}\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^2,x, algorithm="giac")
```

```
[Out] 1/3*tan(x)^3
```

3.355 $\int \cot^3(x)\csc(x)\,dx$

Optimal. Leaf size=11

$$\csc(x) - \frac{\csc^3(x)}{3}$$

```
[Out] Csc[x] - Csc[x]^3/3
```

Rubi [A] time = 0.0156247, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2606}

$$\csc(x) - \frac{\csc^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Csc[x],x]
```

```
[Out] Csc[x] - Csc[x]^3/3
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}\int \cot^3(x)\csc(x)\,dx &= -\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \csc(x)\right)\\ &= \csc(x) - \frac{\csc^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.00721, size = 11, normalized size = 1.

$$\csc(x) - \frac{\csc^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Csc[x],x]
```

```
[Out] Csc[x] - Csc[x]^3/3
```

Maple [B] time = 0.009, size = 32, normalized size = 2.9

$$-\frac{(\cos(x))^4}{3(\sin(x))^3} + \frac{(\cos(x))^4}{3\sin(x)} + \frac{\left(2+(\cos(x))^2\right)\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*csc(x),x)
```

```
[Out] -1/3/sin(x)^3*cos(x)^4+1/3/sin(x)*cos(x)^4+1/3*(2+cos(x)^2)*sin(x)
```

Maxima [A] time = 0.943858, size = 19, normalized size = 1.73

$$\frac{3\,\sin\left(x\right)^{2}-1}{3\,\sin\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x),x, algorithm="maxima")
```

```
[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

Fricas [B] time = 1.86833, size = 62, normalized size = 5.64

$$\frac{3\,\cos\left(x\right)^{2}-2}{3\left(\cos\left(x\right)^{2}-1\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x),x, algorithm="fricas")
```

```
[Out] 1/3*(3*cos(x)^2 - 2)/((cos(x)^2 - 1)*sin(x))
```

Sympy [A] time = 0.089148, size = 14, normalized size = 1.27

$$\frac{3\sin^{2}\left(x\right)-1}{3\sin^{3}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3*csc(x),x)
```

```
[Out] (3*sin(x)**2 - 1)/(3*sin(x)**3)
```

Giac [A] time = 1.08197, size = 19, normalized size = 1.73

$$\frac{3\,\sin\left(x\right)^{2}-1}{3\,\sin\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x),x, algorithm="giac")
```

```
[Out] 1/3*(3*sin(x)^2 - 1)/sin(x)^3
```

3.356 $\int \sec^3(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^3(x)}{3}$$

```
[Out] Sec[x]^3/3
```

Rubi [A] time = 0.0125706, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 30}

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3*Tan[x],x]
```

```
[Out] Sec[x]^3/3
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^3(x)\tan(x)\,dx &= \text{Subst}\left(\int x^2\,dx, x, \sec(x)\right)\\ &= \frac{\sec^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.005183, size = 8, normalized size = 1.

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3*Tan[x],x]
```

```
[Out] Sec[x]^3/3
```

Maple [A] time = 0.008, size = 7, normalized size = 0.9

$$\frac{(\sec(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3*tan(x),x)
```

```
[Out] 1/3*sec(x)^3
```

Maxima [A] time = 0.930542, size = 8, normalized size = 1.

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="maxima")
```

```
[Out] 1/3/cos(x)^3
```

Fricas [A] time = 1.8274, size = 19, normalized size = 2.38

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="fricas")
```

```
[Out] 1/3/cos(x)^3
```

Sympy [A] time = 0.063587, size = 7, normalized size = 0.88

$$\frac{1}{3\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3*tan(x),x)
```

```
[Out] 1/(3*cos(x)**3)
```

Giac [A] time = 1.06664, size = 8, normalized size = 1.

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="giac")
```

```
[Out] 1/3/cos(x)^3
```

3.357 $\int \cot^2(x)\csc^3(x)\,dx$

Optimal. Leaf size=26

$$\frac{1}{8}\tanh^{-1}(\cos(x))-\frac{1}{4}\cot(x)\csc^3(x)+\frac{1}{8}\cot(x)\csc(x)$$

```
[Out] ArcTanh[Cos[x]]/8 + (Cot[x]*Csc[x])/8 - (Cot[x]*Csc[x]^3)/4
```

Rubi [A] time = 0.0313486, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2611, 3768, 3770}

$$\frac{1}{8}\tanh^{-1}(\cos(x))-\frac{1}{4}\cot(x)\csc^3(x)+\frac{1}{8}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2*Csc[x]^3,x]
```

```
[Out] ArcTanh[Cos[x]]/8 + (Cot[x]*Csc[x])/8 - (Cot[x]*Csc[x]^3)/4
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(x)\csc^3(x)\,dx &= -\frac{1}{4}\cot(x)\csc^3(x)-\frac{1}{4}\int \csc^3(x)\,dx\\ &= \frac{1}{8}\cot(x)\csc(x)-\frac{1}{4}\cot(x)\csc^3(x)-\frac{1}{8}\int \csc(x)\,dx\\ &= \frac{1}{8}\tanh^{-1}(\cos(x))+\frac{1}{8}\cot(x)\csc(x)-\frac{1}{4}\cot(x)\csc^3(x)\end{aligned}$$

Mathematica [B] time = 0.018436, size = 71, normalized size = 2.73

$$-\frac{1}{64}\csc^4\left(\frac{x}{2}\right)+\frac{1}{32}\csc^2\left(\frac{x}{2}\right)+\frac{1}{64}\sec^4\left(\frac{x}{2}\right)-\frac{1}{32}\sec^2\left(\frac{x}{2}\right)-\frac{1}{8}\log\left(\sin\left(\frac{x}{2}\right)\right)+\frac{1}{8}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2*Csc[x]^3,x]
```

```
[Out] Csc[x/2]^2/32 - Csc[x/2]^4/64 + Log[Cos[x/2]]/8 - Log[Sin[x/2]]/8 - Sec[x/2
]^2/32 + Sec[x/2]^4/64
```

Maple [A] time = 0.012, size = 36, normalized size = 1.4

$$-\frac{(\cos(x))^3}{4(\sin(x))^4}-\frac{(\cos(x))^3}{8(\sin(x))^2}-\frac{\cos(x)}{8}-\frac{\ln(\csc(x)-\cot(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2*csc(x)^3,x)
```

```
[Out] -1/4*cos(x)^3/sin(x)^4-1/8/sin(x)^2*cos(x)^3-1/8*cos(x)-1/8*ln(csc(x)-cot(x
))
```

Maxima [A] time = 0.933742, size = 51, normalized size = 1.96

$$-\frac{\cos(x)^3+\cos(x)}{8\left(\cos(x)^4-2\cos(x)^2+1\right)}+\frac{1}{16}\log(\cos(x)+1)-\frac{1}{16}\log(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*csc(x)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(cos(x)^3 + cos(x))/(cos(x)^4 - 2*cos(x)^2 + 1) + 1/16*log(cos(x) + 1)
 - 1/16*log(cos(x) - 1)
```

Fricas [B] time = 1.87957, size = 221, normalized size = 8.5

$$-\frac{2\cos(x)^3-\left(\cos(x)^4-2\cos(x)^2+1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\left(\cos(x)^4-2\cos(x)^2+1\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2\cos(x}{16\left(\cos(x)^4-2\cos(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*csc(x)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*cos(x)^3 - (cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*cos(x) + 1/2) + (co
s(x)^4 - 2*cos(x)^2 + 1)*log(-1/2*cos(x) + 1/2) + 2*cos(x))/(cos(x)^4 - 2*c
os(x)^2 + 1)
```

Sympy [A] time = 0.132958, size = 39, normalized size = 1.5

$$-\frac{\cos^3(x)+\cos(x)}{8\cos^4(x)-16\cos^2(x)+8}-\frac{\log(\cos(x)-1)}{16}+\frac{\log(\cos(x)+1)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2*csc(x)**3,x)
```

```
[Out] -(cos(x)**3 + cos(x))/(8*cos(x)**4 - 16*cos(x)**2 + 8) - log(cos(x) - 1)/16
 + log(cos(x) + 1)/16
```

Giac [B] time = 1.08961, size = 63, normalized size = 2.42

$$-\frac{\frac{1}{\cos(x)} + \cos(x)}{8\left(\left(\frac{1}{\cos(x)} + \cos(x)\right)^2 - 4\right)} + \frac{1}{32}\log\left(\left|\frac{1}{\cos(x)} + \cos(x) + 2\right|\right) - \frac{1}{32}\log\left(\left|\frac{1}{\cos(x)} + \cos(x) - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2*csc(x)^3,x, algorithm="giac")
```

```
[Out] -1/8*(1/cos(x) + cos(x))/((1/cos(x) + cos(x))^2 - 4) + 1/32*log(abs(1/cos(x
) + cos(x) + 2)) - 1/32*log(abs(1/cos(x) + cos(x) - 2))
```

3.358 $\int \cot^3(x)\csc^4(x)\,dx$

Optimal. Leaf size=17

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Rubi [A] time = 0.0260194, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Csc[x]^4,x]
```

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \cot^3(x)\csc^4(x)\,dx &= -\operatorname{Subst}\left(\int x^3\left(-1+x^2\right)\,dx,x,\csc(x)\right)\\
&= -\operatorname{Subst}\left(\int \left(-x^3+x^5\right)\,dx,x,\csc(x)\right)\\
&= \frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}
\end{aligned}$$

Mathematica [A] time = 0.0079304, size = 17, normalized size = 1.

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Csc[x]^4,x]
```

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Maple [A] time = 0.008, size = 22, normalized size = 1.3

$$-\frac{(\cos(x))^4}{6(\sin(x))^6}-\frac{(\cos(x))^4}{12(\sin(x))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/sin(x)^7,x)
```

```
[Out] -1/6/sin(x)^6*cos(x)^4-1/12/sin(x)^4*cos(x)^4
```

Maxima [A] time = 0.929369, size = 19, normalized size = 1.12

$$\frac{3\sin(x)^2-2}{12\sin(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="maxima")
```

```
[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

Fricas [B] time = 1.78264, size = 86, normalized size = 5.06

$$\frac{3\cos(x)^2-1}{12\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="fricas")
```

```
[Out] 1/12*(3*cos(x)^2 - 1)/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

Sympy [A] time = 0.095965, size = 14, normalized size = 0.82

$$\frac{3\sin^2(x)-2}{12\sin^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/sin(x)**7,x)
```

```
[Out] (3*sin(x)**2 - 2)/(12*sin(x)**6)
```

Giac [A] time = 1.1006, size = 24, normalized size = 1.41

$$\frac{3\,\cos\left(x\right)^{2}-1}{12\,{\left(\cos\left(x\right)^{2}-1\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/sin(x)^7,x, algorithm="giac")
```

```
[Out] 1/12*(3*cos(x)^2 - 1)/(cos(x)^2 - 1)^3
```

3.359 $\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx$

Optimal. Leaf size=31

$$\frac{2}{11}\sec^{\frac{11}{2}}(x)-\frac{4}{7}\sec^{\frac{7}{2}}(x)+\frac{2}{3}\sec^{\frac{3}{2}}(x)$$

```
[Out] (2*Sec[x]^(3/2))/3 - (4*Sec[x]^(7/2))/7 + (2*Sec[x]^(11/2))/11
```

Rubi [A] time = 0.0258051, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2622, 270}

$$\frac{2}{11}\sec^{\frac{11}{2}}(x)-\frac{4}{7}\sec^{\frac{7}{2}}(x)+\frac{2}{3}\sec^{\frac{3}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^(13/2)*Sin[x]^5,x]
```

```
[Out] (2*Sec[x]^(3/2))/3 - (4*Sec[x]^(7/2))/7 + (2*Sec[x]^(11/2))/11
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{13}{2}}(x)\sin^5(x)\,dx &= \text{Subst}\left(\int \sqrt{x}\left(-1+x^2\right)^2\,dx,x,\sec(x)\right)\\ &= \text{Subst}\left(\int \left(\sqrt{x}-2x^{5/2}+x^{9/2}\right)dx,x,\sec(x)\right)\\ &= \frac{2}{3}\sec^{\frac{3}{2}}(x)-\frac{4}{7}\sec^{\frac{7}{2}}(x)+\frac{2}{11}\sec^{\frac{11}{2}}(x)\end{aligned}$$

Mathematica [A] time = 0.0501065, size = 24, normalized size = 0.77

$$\frac{1}{924}(44\cos(2x)+77\cos(4x)+135)\sec^{\frac{11}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^(13/2)*Sin[x]^5,x]
```

```
[Out] ((135 + 44*Cos[2*x] + 77*Cos[4*x])*Sec[x]^(11/2))/924
```

Maple [B] time = 0.092, size = 49, normalized size = 1.6

$$\frac{32}{231}\left(77\left(\sin\left(x/2\right)\right)^{8}-154\left(\sin\left(x/2\right)\right)^{6}+99\left(\sin\left(x/2\right)\right)^{4}-22\left(\sin\left(x/2\right)\right)^{2}+2\right)\left(-2\left(\sin\left(x/2\right)\right)^{2}+1\right)^{-\frac{11}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^(3/2)*tan(x)^5,x)
```

```
[Out] 32/231/(-2*sin(1/2*x)^2+1)^(11/2)*(77*sin(1/2*x)^8-154*sin(1/2*x)^6+99*sin(
1/2*x)^4-22*sin(1/2*x)^2+2)
```

Maxima [A] time = 0.938604, size = 26, normalized size = 0.84

$$\frac{2}{3\cos\left(x\right)^{\frac{3}{2}}}-\frac{4}{7\cos\left(x\right)^{\frac{7}{2}}}+\frac{2}{11\cos\left(x\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="maxima")
```

```
[Out] 2/3/cos(x)^(3/2) - 4/7/cos(x)^(7/2) + 2/11/cos(x)^(11/2)
```

Fricas [A] time = 1.87227, size = 73, normalized size = 2.35

$$\frac{2\left(77\cos\left(x\right)^{4}-66\cos\left(x\right)^{2}+21\right)}{231\cos\left(x\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="fricas")
```

```
[Out] 2/231*(77*cos(x)^4 - 66*cos(x)^2 + 21)/cos(x)^(11/2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**(3/2)*tan(x)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10041, size = 31, normalized size = 1.

$$\frac{2 \left(77 \cos \left(x\right)^{4} - 66 \cos \left(x\right)^{2} + 21\right) \mathrm{sgn}\left(\cos \left(x\right)\right)}{231 \cos \left(x\right)^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^(3/2)*tan(x)^5,x, algorithm="giac")
```

```
[Out] 2/231*(77*cos(x)^4 - 66*cos(x)^2 + 21)*sgn(cos(x))/cos(x)^(11/2)
```

3.360 $\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx$

Optimal. Leaf size=21

$$\frac{2}{9}\tan^{\frac{9}{2}}(x)+\frac{2}{5}\tan^{\frac{5}{2}}(x)$$

```
[Out] (2*Tan[x]^(5/2))/5 + (2*Tan[x]^(9/2))/9
```

Rubi [A] time = 0.0241418, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2607, 14}

$$\frac{2}{9}\tan^{\frac{9}{2}}(x)+\frac{2}{5}\tan^{\frac{5}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4*Tan[x]^(3/2),x]
```

```
[Out] (2*Tan[x]^(5/2))/5 + (2*Tan[x]^(9/2))/9
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sec^4(x)\tan^{\frac{3}{2}}(x)\,dx &= \text{Subst}\left(\int x^{3/2}\left(1+x^2\right)\,dx,x,\tan(x)\right)\\ &= \text{Subst}\left(\int \left(x^{3/2}+x^{7/2}\right)\,dx,x,\tan(x)\right)\\ &= \frac{2}{5}\tan^{\frac{5}{2}}(x)+\frac{2}{9}\tan^{\frac{9}{2}}(x)\end{aligned}$$

Mathematica [A] time = 0.0365843, size = 22, normalized size = 1.05

$$\frac{2}{45}(2\cos(2x)+7)\tan^{\frac{5}{2}}(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4*Tan[x]^(3/2),x]
```

```
[Out] (2*(7 + 2*Cos[2*x])*Sec[x]^2*Tan[x]^(5/2))/45
```

Maple [A] time = 0.259, size = 26, normalized size = 1.2

$$\frac{\left(8\left(\cos\left(x\right)\right)^{2}+10\right)\sin\left(x\right)}{45\left(\cos\left(x\right)\right)^{3}}\left(\frac{\sin\left(x\right)}{\cos\left(x\right)}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4*tan(x)^(3/2),x)
```

```
[Out] 2/45*(4*cos(x)^2+5)*sin(x)*(sin(x)/cos(x))^(3/2)/cos(x)^3
```

Maxima [A] time = 0.935958, size = 18, normalized size = 0.86

$$\frac{2}{9}\tan\left(x\right)^{\frac{9}{2}}+\frac{2}{5}\tan\left(x\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/9*tan(x)^(9/2) + 2/5*tan(x)^(5/2)
```

Fricas [A] time = 2.46133, size = 86, normalized size = 4.1

$$-\frac{2\left(4\cos\left(x\right)^{4}+\cos\left(x\right)^{2}-5\right)\sqrt{\frac{\sin(x)}{\cos(x)}}}{45\cos\left(x\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="fricas")
```

```
[Out] -2/45*(4*cos(x)^4 + cos(x)^2 - 5)*sqrt(sin(x)/cos(x))/cos(x)^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4*tan(x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.0933, size = 18, normalized size = 0.86

$$\frac{2}{9}\tan(x)^{\frac{9}{2}} + \frac{2}{5}\tan(x)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^(3/2),x, algorithm="giac")
```

```
[Out] 2/9*tan(x)^(9/2) + 2/5*tan(x)^(5/2)
```

3.361 $\int \cot^4(x) \csc^3(x) \, dx$

Optimal. Leaf size=38

$$-\frac{1}{16}\tanh^{-1}(\cos(x))-\frac{1}{6}\cot^3(x)\csc^3(x)+\frac{1}{8}\cot(x)\csc^3(x)-\frac{1}{16}\cot(x)\csc(x)$$

```
[Out] -ArcTanh[Cos[x]]/16 - (Cot[x]*Csc[x])/16 + (Cot[x]*Csc[x]^3)/8 - (Cot[x]^3*
Csc[x]^3)/6
```

Rubi [A] time = 0.0530602, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2611, 3768, 3770}

$$-\frac{1}{16}\tanh^{-1}(\cos(x))-\frac{1}{6}\cot^3(x)\csc^3(x)+\frac{1}{8}\cot(x)\csc^3(x)-\frac{1}{16}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4*Csc[x]^3,x]
```

```
[Out] -ArcTanh[Cos[x]]/16 - (Cot[x]*Csc[x])/16 + (Cot[x]*Csc[x]^3)/8 - (Cot[x]^3*
Csc[x]^3)/6
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^4(x)\csc^3(x)\,dx &= -\frac{1}{6}\cot^3(x)\csc^3(x)-\frac{1}{2}\int \cot^2(x)\csc^3(x)\,dx\\
&= \frac{1}{8}\cot(x)\csc^3(x)-\frac{1}{6}\cot^3(x)\csc^3(x)+\frac{1}{8}\int \csc^3(x)\,dx\\
&= -\frac{1}{16}\cot(x)\csc(x)+\frac{1}{8}\cot(x)\csc^3(x)-\frac{1}{6}\cot^3(x)\csc^3(x)+\frac{1}{16}\int \csc(x)\,dx\\
&= -\frac{1}{16}\tanh^{-1}(\cos(x))-\frac{1}{16}\cot(x)\csc(x)+\frac{1}{8}\cot(x)\csc^3(x)-\frac{1}{6}\cot^3(x)\csc^3(x)
\end{aligned}$$

Mathematica [B] time = 0.0192547, size = 95, normalized size = 2.5

$$-\frac{1}{384}\csc^6\left(\frac{x}{2}\right)+\frac{1}{64}\csc^4\left(\frac{x}{2}\right)-\frac{1}{64}\csc^2\left(\frac{x}{2}\right)+\frac{1}{384}\sec^6\left(\frac{x}{2}\right)-\frac{1}{64}\sec^4\left(\frac{x}{2}\right)+\frac{1}{64}\sec^2\left(\frac{x}{2}\right)+\frac{1}{16}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{16}\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4*Csc[x]^3,x]
```

```
[Out] -Csc[x/2]^2/64 + Csc[x/2]^4/64 - Csc[x/2]^6/384 - Log[Cos[x/2]]/16 + Log[Si
n[x/2]]/16 + Sec[x/2]^2/64 - Sec[x/2]^4/64 + Sec[x/2]^6/384
```

Maple [A] time = 0.016, size = 52, normalized size = 1.4

$$-\frac{(\cos(x))^5}{6(\sin(x))^6}-\frac{(\cos(x))^5}{24(\sin(x))^4}+\frac{(\cos(x))^5}{48(\sin(x))^2}+\frac{(\cos(x))^3}{48}+\frac{\cos(x)}{16}+\frac{\ln(\csc(x)-\cot(x))}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4*csc(x)^3,x)
```

```
[Out] -1/6/sin(x)^6*cos(x)^5-1/24/sin(x)^4*cos(x)^5+1/48/sin(x)^2*cos(x)^5+1/48*c
os(x)^3+1/16*cos(x)+1/16*ln(csc(x)-cot(x))
```

Maxima [A] time = 0.932084, size = 73, normalized size = 1.92

$$\frac{3\cos(x)^5+8\cos(x)^3-3\cos(x)}{48\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)}-\frac{1}{32}\log(\cos(x)+1)+\frac{1}{32}\log(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*csc(x)^3,x, algorithm="maxima")
```

```
[Out] 1/48*(3*cos(x)^5 + 8*cos(x)^3 - 3*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)
^2 - 1) - 1/32*log(cos(x) + 1) + 1/32*log(cos(x) - 1)
```

Fricas [B] time = 2.55986, size = 297, normalized size = 7.82

$$\frac{6\cos(x)^5+16\cos(x)^3-3\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+3\left(\cos(x)^6-3\cos(x)^4+3\cos\right.}{96\left(\cos(x)^6-3\cos(x)^4+3\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*csc(x)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(6*cos(x)^5 + 16*cos(x)^3 - 3*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
*log(1/2*cos(x) + 1/2) + 3*(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*log(-1/
2*cos(x) + 1/2) - 6*cos(x))/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

Sympy [A] time = 0.159786, size = 56, normalized size = 1.47

$$\frac{3\cos^5(x) + 8\cos^3(x) - 3\cos(x)}{48\cos^6(x) - 144\cos^4(x) + 144\cos^2(x) - 48} + \frac{\log(\cos(x) - 1)}{32} - \frac{\log(\cos(x) + 1)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4*csc(x)**3,x)
```

```
[Out] (3*cos(x)**5 + 8*cos(x)**3 - 3*cos(x))/(48*cos(x)**6 - 144*cos(x)**4 + 144*
cos(x)**2 - 48) + log(cos(x) - 1)/32 - log(cos(x) + 1)/32
```

Giac [A] time = 1.0869, size = 59, normalized size = 1.55

$$\frac{3\cos(x)^5 + 8\cos(x)^3 - 3\cos(x)}{48\left(\cos(x)^2 - 1\right)^3} - \frac{1}{32}\log(\cos(x) + 1) + \frac{1}{32}\log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*csc(x)^3,x, algorithm="giac")
```

```
[Out] 1/48*(3*cos(x)^5 + 8*cos(x)^3 - 3*cos(x))/(cos(x)^2 - 1)^3 - 1/32*log(cos(x
) + 1) + 1/32*log(-cos(x) + 1)
```

3.362 $\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$

Optimal. Leaf size=76

$$-\frac{1}{4}\tanh^{-1}\left(\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)+\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)-\frac{1}{4}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

```
[Out] -ArcTanh[Sin[Pi/4 + x/2]]/4 - (Sec[Pi/4 + x/2]*Tan[Pi/4 + x/2])/4 + (Sec[Pi
/4 + x/2]^3*Tan[Pi/4 + x/2])/2
```

Rubi [A] time = 0.039172, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {2611, 3768, 3770}

$$-\frac{1}{4}\tanh^{-1}\left(\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)+\frac{1}{2}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)-\frac{1}{4}\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec\left(\frac{x}{2}+\frac{\pi}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2]^2,x]
```

```
[Out] -ArcTanh[Sin[Pi/4 + x/2]]/4 - (Sec[Pi/4 + x/2]*Tan[Pi/4 + x/2])/4 + (Sec[Pi
/4 + x/2]^3*Tan[Pi/4 + x/2])/2
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx=\frac{1}{2}\sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)-\frac{1}{4}\int \sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\,dx$$
$$=-\frac{1}{4}\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{1}{2}\sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)-\frac{1}{8}\int \csc\left(\frac{\pi}{4}-\frac{x}{2}\right)\,d$$
$$=-\frac{1}{4}\tanh^{-1}\left(\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)-\frac{1}{4}\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)+\frac{1}{2}\sec^3\left(\frac{\pi}{4}+\frac{x}{2}\right)\tan\left(\frac{\pi}{4}\right.$$

Mathematica [A] time = 0.0735997, size = 74, normalized size = 0.97

$$-\frac{1}{4}\tanh^{-1}\left(\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\right)+\frac{1}{2}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^4\left(\frac{1}{4}(2x+\pi)\right)-\frac{1}{4}\sin\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^2\left(\frac{1}{4}(2x+\pi)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[Pi/4 + x/2]^3*Tan[Pi/4 + x/2]^2,x]
```

```
[Out] -ArcTanh[Sin[Pi/4 + x/2]]/4 - (Sec[(Pi + 2*x)/4]^2*Sin[Pi/4 + x/2])/4 + (Se
c[(Pi + 2*x)/4]^4*Sin[Pi/4 + x/2])/2
```

Maple [A] time = 0.023, size = 76, normalized size = 1.

$$\frac{1}{2}\left(\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)^3\left(\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)^{-4}+\frac{1}{4}\left(\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)^3\left(\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)^{-2}+\frac{1}{4}\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)-\frac{1}{4}\ln\left(\sec\left(\frac{\pi}{4}+\frac{x}{2}\right)+\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(1/4*Pi+1/2*x)^3*tan(1/4*Pi+1/2*x)^2,x)
```

```
[Out] 1/2*sin(1/4*Pi+1/2*x)^3/cos(1/4*Pi+1/2*x)^4+1/4*sin(1/4*Pi+1/2*x)^3/cos(1/4
*Pi+1/2*x)^2+1/4*sin(1/4*Pi+1/2*x)-1/4*ln(sec(1/4*Pi+1/2*x)+tan(1/4*Pi+1/2*
x))
```

Maxima [A] time = 0.947992, size = 100, normalized size = 1.32

$$\frac{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^3+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}{4\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^4-2\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^2+1\right)}-\frac{1}{8}\log\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+1\right)+\frac{1}{8}\log\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(sin(1/4*pi + 1/2*x)^3 + sin(1/4*pi + 1/2*x))/(sin(1/4*pi + 1/2*x)^4 -
2*sin(1/4*pi + 1/2*x)^2 + 1) - 1/8*log(sin(1/4*pi + 1/2*x) + 1) + 1/8*log(s
in(1/4*pi + 1/2*x) - 1)
```

Fricas [A] time = 2.6527, size = 252, normalized size = 3.32

$$-\frac{\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^4\log\left(\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+1\right)-\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^4\log\left(-\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+1\right)+2\left(\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^2-2\right)}{8\cos\left(\frac{1}{4}\pi+\frac{1}{2}x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(cos(1/4*pi + 1/2*x)^4*log(sin(1/4*pi + 1/2*x) + 1) - cos(1/4*pi + 1/2
*x)^4*log(-sin(1/4*pi + 1/2*x) + 1) + 2*(cos(1/4*pi + 1/2*x)^2 - 2)*sin(1/4
*pi + 1/2*x))/cos(1/4*pi + 1/2*x)^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \tan^2\left(\frac{x}{2}+\frac{\pi}{4}\right)\sec^3\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/4*pi+1/2*x)**3*tan(1/4*pi+1/2*x)**2,x)
```

```
[Out] Integral(tan(x/2 + pi/4)**2*sec(x/2 + pi/4)**3, x)
```

Giac [A] time = 1.18049, size = 128, normalized size = 1.68

$$\frac{\frac{1}{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}{4\left(\left(\frac{1}{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)\right)^2-4\right)}-\frac{1}{16}\log\left(\left|\frac{1}{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}+\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)+2\right|\right)+\frac{1}{16}\log\left(\left|\frac{1}{\sin\left(\frac{1}{4}\pi+\frac{1}{2}x\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(1/4*pi+1/2*x)^3*tan(1/4*pi+1/2*x)^2,x, algorithm="giac")
```

```
[Out] 1/4*(1/sin(1/4*pi + 1/2*x) + sin(1/4*pi + 1/2*x))/((1/sin(1/4*pi + 1/2*x) +
 sin(1/4*pi + 1/2*x))^2 - 4) - 1/16*log(abs(1/sin(1/4*pi + 1/2*x) + sin(1/4
*pi + 1/2*x) + 2)) + 1/16*log(abs(1/sin(1/4*pi + 1/2*x) + sin(1/4*pi + 1/2*
x) - 2))
```

3.363 $\int \left(1+\cot^3(x)\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx$

Optimal. Leaf size=88

$$\frac{1}{3}a^2\tan^3(x)+a^2\tan(x)-\frac{1}{2}a^2\cot^2(x)+\left(a^2+4\right)\log(\sin(x))+4ax+4a\cot(x)+(4-a)a\log(\cos(x))+\frac{x}{2}+\cos^4(x)$$

```
[Out] x/2 + 4*a*x + 2*Cos[x]^2 + Cos[x]^4 + 4*a*Cot[x] - (a^2*Cot[x]^2)/2 + (4 -
a)*a*Log[Cos[x]] + (4 + a^2)*Log[Sin[x]] + (Cos[x]*Sin[x])/2 - Cos[x]^3*Sin
[x] + a^2*Tan[x] + (a^2*Tan[x]^3)/3
```

Rubi [A] time = 0.550653, antiderivative size = 84, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1805, 1802, 635, 203, 260}

$$\frac{1}{3}a^2\tan^3(x)+a^2\tan(x)-\frac{1}{2}a^2\cot^2(x)+\left(a^2+4\right)\log(\tan(x))+\frac{1}{2}(8a+1)x+4a\cot(x)+4(a+1)\log(\cos(x))+\cos^4$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cot[x]^3)*(a*Sec[x]^2 - Sin[2*x])^2,x]
```

```
[Out] ((1 + 8*a)*x)/2 + 4*a*Cot[x] - (a^2*Cot[x]^2)/2 + 4*(1 + a)*Log[Cos[x]] + (
4 + a^2)*Log[Tan[x]] + Cos[x]^4*(1 - Tan[x]) + a^2*Tan[x] + (a^2*Tan[x]^3)/
3 + (Cos[x]^2*(4 + Tan[x]))/2
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \left(1+\cot^3(x)\right)\left(a\sec^2(x)-\sin(2x)\right)^2\,dx=\text{Subst}\left(\int\frac{\left(1+x^3\right)\left(a-2x+2ax^2+ax^4\right)^2}{x^3\left(1+x^2\right)^3}\,dx,x,\tan(x)\right)$$

$$=\cos^4(x)(1-\tan(x))-\frac{1}{4}\,\text{Subst}\left(\int\frac{-4a^2+16ax-4\left(4+3a^2\right)x^2-4\left(1-4a\right.}{}\right.$$

$$=\cos^4(x)(1-\tan(x))+\frac{1}{2}\cos^2(x)(4+\tan(x))+\frac{1}{8}\,\text{Subst}\left(\int\frac{8a^2-32ax+16}{}\right.$$

$$=\cos^4(x)(1-\tan(x))+\frac{1}{2}\cos^2(x)(4+\tan(x))+\frac{1}{8}\,\text{Subst}\left(\int\left(8a^2+\frac{8a^2}{x^3}-\frac{32}{x}\right.\right.$$

$$=4a\cot(x)-\frac{1}{2}a^2\cot^2(x)+\left(4+a^2\right)\log(\tan(x))+\cos^4(x)(1-\tan(x))+a^2\,\text{ta}$$

$$=4a\cot(x)-\frac{1}{2}a^2\cot^2(x)+\left(4+a^2\right)\log(\tan(x))+\cos^4(x)(1-\tan(x))+a^2\,\text{ta}$$

$$=\frac{1}{2}(1+8a)x+4a\cot(x)-\frac{1}{2}a^2\cot^2(x)+4(1+a)\log(\cos(x))+\left(4+a^2\right)\log(\text{t}$$

Mathematica [A] time = 1.78038, size = 127, normalized size = 1.44

$$-\frac{2\sin(x)\cos^3(x)\left(\sin(2x)-a\sec^2(x)\right)^2\left(-8a^2(\cos(2x)+2)\sec^2(x)-3\cot(x)\left(-4a^2\csc^2(x)+8a^2\log(\sin(x))-8a^2\log(\text{c}\right.\right.}{3(-4a+2\sin(2x)+\text{si}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cot[x]^3)*(a*Sec[x]^2 - Sin[2*x])^2,x]
```

```
[Out] (-2*Cos[x]^3*Sin[x]*(-(a*Sec[x]^2) + Sin[2*x])^2*(-96*a*Cot[x]^2 - 8*a^2*(2
 + Cos[2*x])*Sec[x]^2 - 3*Cot[x]*(4*x + 32*a*x + 12*Cos[2*x] + Cos[4*x] - 4
*a^2*Csc[x]^2 + 32*a*Log[Cos[x]] - 8*a^2*Log[Cos[x]] + 32*Log[Sin[x]] + 8*a
^2*Log[Sin[x]] - Sin[4*x])))/(3*(-4*a + 2*Sin[2*x] + Sin[4*x])^2)
```

Maple [B] time = 0.196, size = 186, normalized size = 2.1

$$-4\left((\cos(x))^5+5/4\,(\cos(x))^3+\frac{15\cos(x)}{8}\right)\sin(x)-4\cot(x)+(\cos(x))^4+4\,ax+8\,\left((\cos(x))^3+3/2\,\cos(x)\right)\sin(x)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x)
```

```
[Out] -4*(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)-4*cot(x)+cos(x)^4+4*a*x+8*(co
s(x)^3+3/2*cos(x))*sin(x)+2*cos(x)^2+1/2*x+4*ln(sin(x))+2*a*cot(x)^2+4*a*ln
(sin(x))-2/3*a^2*cot(x)-1/2*a^2/sin(x)^2+a^2*ln(tan(x))-4*a*ln(tan(x))-2*a/
sin(x)^2+4*a*cot(x)+2/sin(x)^2*cos(x)^8+8/sin(x)*cos(x)^5-2/sin(x)^2*cos(x)
^6-4/sin(x)*cos(x)^7+2*cos(x)^6+1/3*a^2/sin(x)/cos(x)+1/3*a^2/sin(x)/cos(x)
```

^3

Maxima [A] time = 1.44376, size = 155, normalized size = 1.76

$$\frac{1}{3}\left(\tan(x)^3 + 3\tan(x)\right)a^2 - \frac{1}{2}a^2\left(\frac{1}{\sin(x)^2} + \log\left(\sin(x)^2 - 1\right) - \log\left(\sin(x)^2\right)\right) + 4a\left(x + \frac{1}{\tan(x)}\right) + 2a\log\left(-\sin(x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(x)^3 + 3*tan(x))*a^2 - 1/2*a^2*(1/sin(x)^2 + log(sin(x)^2 - 1) - l
og(sin(x)^2)) + 4*a*(x + 1/tan(x)) + 2*a*log(-sin(x)^2 + 1) + 1/2*x + 1/8*c
os(4*x) + 3/2*cos(2*x) + 2*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 2*log(
cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 1/8*sin(4*x)
```

Fricas [B] time = 2.78211, size = 487, normalized size = 5.53

$$\frac{24\cos(x)^9 + 24\cos(x)^7 + 3(4(8a+1)x - 27)\cos(x)^5 + 3\left(4a^2 - 4(8a+1)x + 11\right)\cos(x)^3 - 12\left(\left(a^2 - 4a\right)\cos(x)^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(24*cos(x)^9 + 24*cos(x)^7 + 3*(4*(8*a + 1)*x - 27)*cos(x)^5 + 3*(4*a^
2 - 4*(8*a + 1)*x + 11)*cos(x)^3 - 12*((a^2 - 4*a)*cos(x)^5 - (a^2 - 4*a)*c
os(x)^3)*log(cos(x)^2) + 12*((a^2 + 4)*cos(x)^5 - (a^2 + 4)*cos(x)^3)*log(-
1/4*cos(x)^2 + 1/4) - 4*(6*cos(x)^8 - 9*cos(x)^6 - (4*a^2 - 24*a - 3)*cos(x
)^4 + 2*a^2*cos(x)^2 + 2*a^2)*sin(x))/(cos(x)^5 - cos(x)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)**3)*(a*sec(x)**2-sin(2*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12373, size = 201, normalized size = 2.28

$$\frac{1}{3}a^2\tan(x)^3 + a^2\tan(x) + \frac{1}{2}(8a+1)x - 2(a+1)\log\left(\tan(x)^2 + 1\right) + \left(a^2 + 4\right)\log\left(|\tan(x)|\right) - \frac{a^2\tan(x)^6 - 4a\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cot(x)^3)*(a*sec(x)^2-sin(2*x))^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*tan(x)^3 + a^2*tan(x) + 1/2*(8*a + 1)*x - 2*(a + 1)*log(tan(x)^2 +
1) + (a^2 + 4)*log(abs(tan(x))) - 1/2*(a^2*tan(x)^6 - 4*a*tan(x)^6 + 3*a^2*
tan(x)^4 - 8*a*tan(x)^5 - 8*a*tan(x)^4 - tan(x)^5 + 3*a^2*tan(x)^2 - 16*a*t
an(x)^3 - 4*tan(x)^4 - 4*a*tan(x)^2 + tan(x)^3 + a^2 - 8*a*tan(x) - 6*tan(x
)^2)/(tan(x)^3 + tan(x))^2
```

3.364 $\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4\,dx$

Optimal. Leaf size=70

$$\frac{227x}{32}-\frac{3}{80}\sin^5(x)+\frac{3\sin^4(x)}{8}-\frac{3\sin^3(x)}{2}-3\sin(x)-\frac{2\cos^3(x)}{3}-3\cos^2(x)+10\cos(x)-\frac{1}{16}\sin^3(x)\cos(x)-\frac{99}{32}\sin$$

```
[Out] (227*x)/32 + 10*Cos[x] - 3*Cos[x]^2 - (2*Cos[x]^3)/3 - 3*Sin[x] - (99*Cos[x
]*Sin[x])/32 - (3*Sin[x]^3)/2 - (Cos[x]*Sin[x]^3)/16 + (3*Sin[x]^4)/8 - (3*
Sin[x]^5)/80
```

Rubi [A] time = 0.153064, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4401, 2637, 2668, 2669, 2635, 8, 641}

$$\frac{227x}{32}-\frac{3}{80}\sin^5(x)+\frac{3\sin^4(x)}{8}-\frac{3\sin^3(x)}{2}-3\sin(x)-\frac{2\cos^3(x)}{3}-3\cos^2(x)+10\cos(x)-\frac{1}{16}\sin^3(x)\cos(x)-\frac{99}{32}\sin$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 3*Cos[x])*(1 - Sin[x]/2)^4,x]
```

```
[Out] (227*x)/32 + 10*Cos[x] - 3*Cos[x]^2 - (2*Cos[x]^3)/3 - 3*Sin[x] - (99*Cos[x
]*Sin[x])/32 - (3*Sin[x]^3)/2 - (Cos[x]*Sin[x]^3)/16 + (3*Sin[x]^4)/8 - (3*
Sin[x]^5)/80
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rubi steps

$$\int (4-3\cos(x))\left(1-\frac{\sin(x)}{2}\right)^4 dx = \int \left(4-3\cos(x)+2(-4+3\cos(x))\sin(x)-\frac{3}{2}(-4+3\cos(x))\sin^2(x)+\frac{1}{2}(-4+3\cos$$
$$= 4x-\frac{1}{16}\int(-4+3\cos(x))\sin^4(x)\,dx+\frac{1}{2}\int(-4+3\cos(x))\sin^3(x)\,dx-\frac{3}{2}\int(-4+3$$
$$= 4x-3\sin(x)-\frac{3\sin^3(x)}{2}-\frac{3\sin^5(x)}{80}-\frac{1}{54}\text{Subst}\left(\int(-4+x)\left(9-x^2\right)dx,x,3\cos(x\right.$$
$$= 4x+8\cos(x)-3\cos^2(x)-3\sin(x)-3\cos(x)\sin(x)-\frac{3\sin^3(x)}{2}-\frac{1}{16}\cos(x)\sin^3(x)$$
$$= 7x+10\cos(x)-3\cos^2(x)-\frac{2\cos^3(x)}{3}-3\sin(x)-\frac{99}{32}\cos(x)\sin(x)-\frac{3\sin^3(x)}{2}-\frac{1}{16}$$
$$= \frac{227x}{32}+10\cos(x)-3\cos^2(x)-\frac{2\cos^3(x)}{3}-3\sin(x)-\frac{99}{32}\cos(x)\sin(x)-\frac{3\sin^3(x)}{2}-$$

Mathematica [A] time = 0.0805028, size = 74, normalized size = 1.06

$$\frac{227x}{32}-\frac{531\sin(x)}{128}-\frac{25}{16}\sin(2x)+\frac{99}{256}\sin(3x)+\frac{1}{128}\sin(4x)-\frac{3\sin(5x)}{1280}+\frac{19\cos(x)}{2}-\frac{27}{16}\cos(2x)-\frac{1}{6}\cos(3x)+\frac{3}{64}\text{co}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 3*Cos[x])*(1 - Sin[x]/2)^4,x]
```

```
[Out] (227*x)/32 + (19*Cos[x])/2 - (27*Cos[2*x])/16 - Cos[3*x]/6 + (3*Cos[4*x])/6
4 - (531*Sin[x])/128 - (25*Sin[2*x])/16 + (99*Sin[3*x])/256 + Sin[4*x]/128
- (3*Sin[5*x])/1280
```

Maple [A] time = 0.039, size = 66, normalized size = 0.9

$$\frac{227\,x}{32}+8\,\cos(x)-3\,\cos(x)\sin(x)+\frac{\left(4+2\,(\sin(x))^2\right)\cos(x)}{3}-\frac{\cos(x)}{16}\left((\sin(x))^3+\frac{3\,\sin(x)}{2}\right)-3\,\sin(x)-3\,(\cos(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4-3*cos(x))*(1-1/2*sin(x))^4,x)
```

```
[Out] 227/32*x+8*cos(x)-3*cos(x)*sin(x)+2/3*(2+sin(x)^2)*cos(x)-1/16*(sin(x)^3+3/
2*sin(x))*cos(x)-3*sin(x)-3*cos(x)^2-3/2*sin(x)^3+3/8*sin(x)^4-3/80*sin(x)^
```

5

Maxima [A] time = 0.942261, size = 73, normalized size = 1.04

$$-\frac{3}{80}\sin(x)^5+\frac{3}{8}\sin(x)^4-\frac{2}{3}\cos(x)^3-\frac{3}{2}\sin(x)^3-3\cos(x)^2+\frac{227}{32}x+10\cos(x)+\frac{1}{128}\sin(4x)-\frac{25}{16}\sin(2x)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x, algorithm="maxima")
```

```
[Out] -3/80*sin(x)^5 + 3/8*sin(x)^4 - 2/3*cos(x)^3 - 3/2*sin(x)^3 - 3*cos(x)^2 +
227/32*x + 10*cos(x) + 1/128*sin(4*x) - 25/16*sin(2*x) - 3*sin(x)
```

Fricas [A] time = 2.1721, size = 194, normalized size = 2.77

$$\frac{3}{8}\cos(x)^4-\frac{2}{3}\cos(x)^3-\frac{15}{4}\cos(x)^2-\frac{1}{160}\left(6\cos(x)^4-10\cos(x)^3-252\cos(x)^2+505\cos(x)+726\right)\sin(x)+\frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x, algorithm="fricas")
```

```
[Out] 3/8*cos(x)^4 - 2/3*cos(x)^3 - 15/4*cos(x)^2 - 1/160*(6*cos(x)^4 - 10*cos(x)
^3 - 252*cos(x)^2 + 505*cos(x) + 726)*sin(x) + 227/32*x + 10*cos(x)
```

Sympy [B] time = 1.67097, size = 162, normalized size = 2.31

$$\frac{3x\sin^4(x)}{32}+\frac{3x\sin^2(x)\cos^2(x)}{16}+3x\sin^2(x)+\frac{3x\cos^4(x)}{32}+3x\cos^2(x)+4x-\frac{3\sin^5(x)}{80}-\frac{5\sin^3(x)\cos(x)}{32}-\frac{3\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-3*cos(x))*(1-1/2*sin(x))**4,x)
```

```
[Out] 3*x*sin(x)**4/32 + 3*x*sin(x)**2*cos(x)**2/16 + 3*x*sin(x)**2 + 3*x*cos(x)*
*4/32 + 3*x*cos(x)**2 + 4*x - 3*sin(x)**5/80 - 5*sin(x)**3*cos(x)/32 - 3*si
n(x)**3/2 - 3*sin(x)**2*cos(x)**2/4 + 2*sin(x)**2*cos(x) + 3*sin(x)**2 - 3*
sin(x)*cos(x)**3/32 - 3*sin(x)*cos(x) - 3*sin(x) - 3*cos(x)**4/8 + 4*cos(x)
**3/3 + 8*cos(x)
```

Giac [A] time = 1.09989, size = 73, normalized size = 1.04

$$\frac{227}{32}x+\frac{3}{64}\cos(4x)-\frac{1}{6}\cos(3x)-\frac{27}{16}\cos(2x)+\frac{19}{2}\cos(x)-\frac{3}{1280}\sin(5x)+\frac{1}{128}\sin(4x)+\frac{99}{256}\sin(3x)-\frac{25}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-3*cos(x))*(1-1/2*sin(x))^4,x, algorithm="giac")
```

```
[Out] 227/32*x + 3/64*cos(4*x) - 1/6*cos(3*x) - 27/16*cos(2*x) + 19/2*cos(x) - 3/
1280*sin(5*x) + 1/128*sin(4*x) + 99/256*sin(3*x) - 25/16*sin(2*x) - 531/128
*sin(x)
```

3.365 $\int \left(\frac{1}{2}-3 \cot(x)\right) (3-2 \cot(x))^3 \, dx$

Optimal. Leaf size=33

$$-\frac{285x}{2}+(3-2\cot(x))^3+5(3-2\cot(x))^2-42\cot(x)+4\log(\sin(x))$$

```
[Out] (-285*x)/2 + 5*(3 - 2*Cot[x])^2 + (3 - 2*Cot[x])^3 - 42*Cot[x] + 4*Log[Sin[
x]]
```

Rubi [A] time = 0.0624847, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3528, 3525, 3475}

$$-\frac{285x}{2}+(3-2\cot(x))^3+5(3-2\cot(x))^2-42\cot(x)+4\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(1/2 - 3*Cot[x])*(3 - 2*Cot[x])^3,x]
```

```
[Out] (-285*x)/2 + 5*(3 - 2*Cot[x])^2 + (3 - 2*Cot[x])^3 - 42*Cot[x] + 4*Log[Sin[
x]]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{1}{2}-3\cot(x)\right)(3-2\cot(x))^3\,dx &= (3-2\cot(x))^3+\int \left(-\frac{9}{2}-10\cot(x)\right)(3-2\cot(x))^2\,dx\\
&= 5(3-2\cot(x))^2+(3-2\cot(x))^3+\int \left(-\frac{67}{2}-21\cot(x)\right)(3-2\cot(x))\,dx\\
&= -\frac{285x}{2}+5(3-2\cot(x))^2+(3-2\cot(x))^3-42\cot(x)+4\int \cot(x)\,dx\\
&= -\frac{285x}{2}+5(3-2\cot(x))^2+(3-2\cot(x))^3-42\cot(x)+4\log(\sin(x))
\end{aligned}$$

Mathematica [A] time = 0.023241, size = 29, normalized size = 0.88

$$-\frac{285x}{2}-148\cot(x)+56\csc^2(x)+4\log(\sin(x))-8\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1/2 - 3*Cot[x])*(3 - 2*Cot[x])^3,x]
```

```
[Out] (-285*x)/2 - 148*Cot[x] + 56*Csc[x]^2 - 8*Cot[x]*Csc[x]^2 + 4*Log[Sin[x]]
```

Maple [A] time = 0.004, size = 33, normalized size = 1.

$$-8\,(\cot(x))^3+56\,(\cot(x))^2-156\,\cot(x)-2\,\ln\left((\cot(x))^2+1\right)+\frac{285\,\pi}{4}-\frac{285\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/2-3*cot(x))*(3-2*cot(x))^3,x)
```

```
[Out] -8*cot(x)^3+56*cot(x)^2-156*cot(x)-2*ln(cot(x)^2+1)+285/4*Pi-285/2*x
```

Maxima [A] time = 1.41367, size = 49, normalized size = 1.48

$$-\frac{285}{2}\,x-\frac{4\left(39\,\tan(x)^2-14\,\tan(x)+2\right)}{\tan(x)^3}-2\,\log\left(\tan(x)^2+1\right)+4\,\log(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x, algorithm="maxima")
```

```
[Out] -285/2*x - 4*(39*tan(x)^2 - 14*tan(x) + 2)/tan(x)^3 - 2*log(tan(x)^2 + 1) +
 4*log(tan(x))
```

Fricas [B] time = 2.1512, size = 220, normalized size = 6.67

$$\frac{4\,(\cos(2\,x)-1)\log\left(-\frac{1}{2}\,\cos(2\,x)+\frac{1}{2}\right)\sin(2\,x)-296\,\cos(2\,x)^2-(285\,x\cos(2\,x)-285\,x+224)\sin(2\,x)+32\,\cos(2}{2\,(\cos(2\,x)-1)\sin(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*(cos(2*x) - 1)*log(-1/2*cos(2*x) + 1/2)*sin(2*x) - 296*cos(2*x)^2 -
(285*x*cos(2*x) - 285*x + 224)*sin(2*x) + 32*cos(2*x) + 328)/((cos(2*x) - 1
)*sin(2*x))
```

Sympy [A] time = 0.612355, size = 39, normalized size = 1.18

$$-\frac{285x}{2}-2\log\left(\tan^{2}(x)+1\right)+4\log(\tan(x))-\frac{156}{\tan(x)}+\frac{56}{\tan^{2}(x)}-\frac{8}{\tan^{3}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2-3*cot(x))*(3-2*cot(x))**3,x)
```

```
[Out] -285*x/2 - 2*log(tan(x)**2 + 1) + 4*log(tan(x)) - 156/tan(x) + 56/tan(x)**2
 - 8/tan(x)**3
```

Giac [B] time = 1.14233, size = 101, normalized size = 3.06

$$\tan\left(\frac{1}{2}x\right)^{3}+14\tan\left(\frac{1}{2}x\right)^{2}-\frac{285}{2}x-\frac{22\tan\left(\frac{1}{2}x\right)^{3}+225\tan\left(\frac{1}{2}x\right)^{2}-42\tan\left(\frac{1}{2}x\right)+3}{3\tan\left(\frac{1}{2}x\right)^{3}}-4\log\left(\tan\left(\frac{1}{2}x\right)^{2}+1\right)+4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/2-3*cot(x))*(3-2*cot(x))^3,x, algorithm="giac")
```

```
[Out] tan(1/2*x)^3 + 14*tan(1/2*x)^2 - 285/2*x - 1/3*(22*tan(1/2*x)^3 + 225*tan(1
/2*x)^2 - 42*tan(1/2*x) + 3)/tan(1/2*x)^3 - 4*log(tan(1/2*x)^2 + 1) + 4*log
(abs(tan(1/2*x))) + 75*tan(1/2*x)
```

3.366 $\int \cos(5x)\sec^5(x)\,dx$

Optimal. Leaf size=16

$$16x+\frac{5\tan^3(x)}{3}-15\tan(x)$$

```
[Out] 16*x - 15*Tan[x] + (5*Tan[x]^3)/3
```

Rubi [A] time = 0.0428634, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1153, 203}

$$16x+\frac{5\tan^3(x)}{3}-15\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[5*x]*Sec[x]^5,x]
```

```
[Out] 16*x - 15*Tan[x] + (5*Tan[x]^3)/3
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos(5x)\sec^5(x)\,dx &= \text{Subst}\left(\int \frac{1-10x^2+5x^4}{1+x^2}\,dx,x,\tan(x)\right)\\
&= \text{Subst}\left(\int \left(-15+5x^2+\frac{16}{1+x^2}\right)dx,x,\tan(x)\right)\\
&= -15\tan(x)+\frac{5\tan^3(x)}{3}+16\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)\\
&= 16x-15\tan(x)+\frac{5\tan^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.0144063, size = 20, normalized size = 1.25

$$16x-\frac{50\tan(x)}{3}+\frac{5}{3}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[5*x]*Sec[x]^5,x]
```

```
[Out] 16*x - (50*Tan[x])/3 + (5*Sec[x]^2*Tan[x])/3
```

Maple [A] time = 0.059, size = 21, normalized size = 1.3

$$16\,x-5\,\left(-2/3-1/3\,(\sec(x))^{2}\right)\tan(x)-20\,\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(5*x)/cos(x)^5,x)
```

```
[Out] 16*x-5*(-2/3-1/3*sec(x)^2)*tan(x)-20*tan(x)
```

Maxima [A] time = 1.43112, size = 19, normalized size = 1.19

$$\frac{5}{3}\,\tan(x)^{3}+16\,x-15\,\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/cos(x)^5,x, algorithm="maxima")
```

```
[Out] 5/3*tan(x)^3 + 16*x - 15*tan(x)
```

Fricas [A] time = 2.18106, size = 80, normalized size = 5.

$$\frac{48\,x\cos(x)^{3}-5\left(10\,\cos(x)^{2}-1\right)\sin(x)}{3\,\cos(x)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/cos(x)^5,x, algorithm="fricas")
```

```
[Out] 1/3*(48*x*cos(x)^3 - 5*(10*cos(x)^2 - 1)*sin(x))/cos(x)^3
```

Sympy [A] time = 110.271, size = 24, normalized size = 1.5

$$16x-\frac{20\sin(x)}{\cos(x)}+\frac{5\tan^{3}(x)}{3}+5\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/cos(x)**5,x)
```

```
[Out] 16*x - 20*sin(x)/cos(x) + 5*tan(x)**3/3 + 5*tan(x)
```

Giac [A] time = 1.06622, size = 19, normalized size = 1.19

$$\frac{5}{3}\tan(x)^3 + 16x - 15\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/cos(x)^5,x, algorithm="giac")
```

```
[Out] 5/3*tan(x)^3 + 16*x - 15*tan(x)
```

3.367 $\int \cos(4x)\sec(x)\,dx$

Optimal. Leaf size=12

$$\tanh^{-1}(\sin(x)) - \frac{8\sin^3(x)}{3}$$

```
[Out] ArcTanh[Sin[x]] - (8*Sin[x]^3)/3
```

Rubi [A] time = 0.0231566, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4364, 1153, 206}

$$\tanh^{-1}(\sin(x)) - \frac{8\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[4*x]*Sec[x],x]
```

```
[Out] ArcTanh[Sin[x]] - (8*Sin[x]^3)/3
```

Rule 4364

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos(4x)\sec(x)\,dx &= \text{Subst}\left(\int \frac{1-8x^2+8x^4}{1-x^2}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \left(-8x^2+\frac{1}{1-x^2}\right)dx, x, \sin(x)\right)\\
&= -\frac{8}{3}\sin^3(x) + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)\\
&= \tanh^{-1}(\sin(x)) - \frac{8\sin^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.0103242, size = 12, normalized size = 1.

$$\tanh^{-1}(\sin(x)) - \frac{8\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[4*x]*Sec[x],x]
```

```
[Out] ArcTanh[Sin[x]] - (8*Sin[x]^3)/3
```

Maple [B] time = 0.021, size = 22, normalized size = 1.8

$$\ln(\sec(x) + \tan(x)) + \frac{\left(16 + 8\,(\cos(x))^2\right)\sin(x)}{3} - 8\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)/cos(x),x)
```

```
[Out] ln(sec(x)+tan(x))+8/3*(2+cos(x)^2)*sin(x)-8*sin(x)
```

Maxima [A] time = 0.930704, size = 28, normalized size = 2.33

$$-\frac{8}{3}\sin(x)^3 + \frac{1}{2}\log(\sin(x) + 1) - \frac{1}{2}\log(\sin(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/cos(x),x, algorithm="maxima")
```

```
[Out] -8/3*sin(x)^3 + 1/2*log(sin(x) + 1) - 1/2*log(sin(x) - 1)
```

Fricas [B] time = 2.30134, size = 97, normalized size = 8.08

$$\frac{8}{3}\left(\cos(x)^2 - 1\right)\sin(x) + \frac{1}{2}\log(\sin(x) + 1) - \frac{1}{2}\log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/cos(x),x, algorithm="fricas")
```

```
[Out] 8/3*(cos(x)^2 - 1)*sin(x) + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Sympy [A] time = 1.3245, size = 24, normalized size = 2.

$$-\frac{\log(\sin(x) - 1)}{2} + \frac{\log(\sin(x) + 1)}{2} - \frac{8\sin^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/cos(x),x)
```

```
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2 - 8*sin(x)**3/3
```

Giac [B] time = 1.06832, size = 31, normalized size = 2.58

$$-\frac{8}{3}\sin(x)^3+\frac{1}{2}\log(\sin(x)+1)-\frac{1}{2}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/cos(x),x, algorithm="giac")
```

```
[Out] -8/3*sin(x)^3 + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

3.368 $\int \cos(x)\cos(4x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

```
[Out] Sin[3*x]/6 + Sin[5*x]/10
```

Rubi [A] time = 0.0082524, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4283}

$$\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[4*x],x]

[Out] Sin[3*x]/6 + Sin[5*x]/10
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\cos(4x)\,dx=\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

Mathematica [A] time = 0.0058287, size = 17, normalized size = 1.

$$\frac{1}{6}\sin(3x)+\frac{1}{10}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[4*x],x]

[Out] Sin[3*x]/6 + Sin[5*x]/10
```

Maple [A] time = 0.046, size = 14, normalized size = 0.8

$$\frac{\sin(3x)}{6}+\frac{\sin(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(4*x),x)
```

```
[Out] 1/6*sin(3*x)+1/10*sin(5*x)
```

Maxima [A] time = 0.924469, size = 18, normalized size = 1.06

$$\frac{1}{10} \sin(5x) + \frac{1}{6} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="maxima")
```

```
[Out] 1/10*sin(5*x) + 1/6*sin(3*x)
```

Fricas [A] time = 2.20529, size = 59, normalized size = 3.47

$$\frac{1}{15} \left(24 \cos(x)^4 - 8 \cos(x)^2 - 1\right) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="fricas")
```

```
[Out] 1/15*(24*cos(x)^4 - 8*cos(x)^2 - 1)*sin(x)
```

Sympy [A] time = 0.51626, size = 20, normalized size = 1.18

$$-\frac{\sin(x)\cos(4x)}{15} + \frac{4\sin(4x)\cos(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x)
```

```
[Out] -sin(x)*cos(4*x)/15 + 4*sin(4*x)*cos(x)/15
```

Giac [A] time = 1.11297, size = 18, normalized size = 1.06

$$\frac{1}{10} \sin(5x) + \frac{1}{6} \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(4*x),x, algorithm="giac")
```

```
[Out] 1/10*sin(5*x) + 1/6*sin(3*x)
```

3.369 $\int \cos(4x) \sec^5(x) \, dx$

Optimal. Leaf size=26

$$\frac{35}{8}\tanh^{-1}(\sin(x)) + \frac{1}{4}\tan(x)\sec^3(x) - \frac{29}{8}\tan(x)\sec(x)$$

```
[Out] (35*ArcTanh[Sin[x]])/8 - (29*Sec[x]*Tan[x])/8 + (Sec[x]^3*Tan[x])/4
```

Rubi [A] time = 0.0300102, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {4364, 1157, 385, 206}

$$\frac{35}{8}\tanh^{-1}(\sin(x)) + \frac{1}{4}\tan(x)\sec^3(x) - \frac{29}{8}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[4*x]*Sec[x]^5,x]
```

```
[Out] (35*ArcTanh[Sin[x]])/8 - (29*Sec[x]*Tan[x])/8 + (Sec[x]^3*Tan[x])/4
```

Rule 4364

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos(4x)\sec^5(x)\,dx &= \text{Subst}\left(\int \frac{1-8x^2+8x^4}{\left(1-x^2\right)^3}\,dx, x, \sin(x)\right)\\ &= \frac{1}{4}\sec^3(x)\tan(x) - \frac{1}{4}\text{Subst}\left(\int \frac{-3+32x^2}{\left(1-x^2\right)^2}\,dx, x, \sin(x)\right)\\ &= -\frac{29}{8}\sec(x)\tan(x) + \frac{1}{4}\sec^3(x)\tan(x) + \frac{35}{8}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)\\ &= \frac{35}{8}\tanh^{-1}(\sin(x)) - \frac{29}{8}\sec(x)\tan(x) + \frac{1}{4}\sec^3(x)\tan(x)\end{aligned}$$

Mathematica [A] time = 0.0497023, size = 26, normalized size = 1.

$$\frac{1}{8}\left(35\tanh^{-1}(\sin(x)) - 27\tan(x)\sec^3(x) + 29\tan^3(x)\sec(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[4*x]*Sec[x]^5,x]
```

```
[Out] (35*ArcTanh[Sin[x]] - 27*Sec[x]^3*Tan[x] + 29*Sec[x]*Tan[x]^3)/8
```

Maple [A] time = 0.023, size = 31, normalized size = 1.2

$$-\left(-\frac{(\sec(x))^3}{4} - \frac{3\sec(x)}{8}\right)\tan(x) + \frac{35\ln(\sec(x)+\tan(x))}{8} - 4\sec(x)\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)/cos(x)^5,x)
```

```
[Out] -(-1/4*sec(x)^3-3/8*sec(x))*tan(x)+35/8*ln(sec(x)+tan(x))-4*sec(x)*tan(x)
```

Maxima [B] time = 0.938973, size = 73, normalized size = 2.81

$$\frac{5\sin(x)^3 - 3\sin(x)}{8\left(\sin(x)^4 - 2\sin(x)^2 + 1\right)} + \frac{3\sin(x)}{\sin(x)^2 - 1} + \frac{35}{16}\log(\sin(x)+1) - \frac{35}{16}\log(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/cos(x)^5,x, algorithm="maxima")
```

```
[Out] 1/8*(5*sin(x)^3 - 3*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 3*sin(x)/(sin(x)^
2 - 1) + 35/16*log(sin(x) + 1) - 35/16*log(sin(x) - 1)
```

Fricas [B] time = 2.45168, size = 142, normalized size = 5.46

$$\frac{35\cos(x)^4\log(\sin(x)+1) - 35\cos(x)^4\log(-\sin(x)+1) - 2\left(29\cos(x)^2 - 2\right)\sin(x)}{16\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/cos(x)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(35*cos(x)^4*log(sin(x) + 1) - 35*cos(x)^4*log(-sin(x) + 1) - 2*(29*co
s(x)^2 - 2)*sin(x))/cos(x)^4
```

Sympy [B] time = 108.838, size = 75, normalized size = 2.88

$$-\frac{35\log(\sin(x)-1)}{16}+\frac{35\log(\sin(x)+1)}{16}-\frac{3\sin^3(x)}{8\sin^4(x)-16\sin^2(x)+8}+\frac{5\sin(x)}{8\sin^4(x)-16\sin^2(x)+8}+\frac{8\sin(x)}{2\sin^2(x)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/cos(x)**5,x)
```

```
[Out] -35*log(sin(x) - 1)/16 + 35*log(sin(x) + 1)/16 - 3*sin(x)**3/(8*sin(x)**4 -
 16*sin(x)**2 + 8) + 5*sin(x)/(8*sin(x)**4 - 16*sin(x)**2 + 8) + 8*sin(x)/(
2*sin(x)**2 - 2)
```

Giac [A] time = 1.07727, size = 51, normalized size = 1.96

$$\frac{29\sin(x)^3-27\sin(x)}{8\left(\sin(x)^2-1\right)^2}+\frac{35}{16}\log(\sin(x)+1)-\frac{35}{16}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/cos(x)^5,x, algorithm="giac")
```

```
[Out] 1/8*(29*sin(x)^3 - 27*sin(x))/(sin(x)^2 - 1)^2 + 35/16*log(sin(x) + 1) - 35
/16*log(-sin(x) + 1)
```

3.370 $\int \cos^4(x)\cos(4x)\,dx$

Optimal. Leaf size=38

$$\frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)$$

```
[Out] x/16 + Sin[2*x]/8 + (3*Sin[4*x])/32 + Sin[6*x]/24 + Sin[8*x]/128
```

Rubi [A] time = 0.0302545, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4354, 2637}

$$\frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4*Cos[4*x],x]

[Out] x/16 + Sin[2*x]/8 + (3*Sin[4*x])/32 + Sin[6*x]/24 + Sin[8*x]/128
```

Rule 4354

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.), x_Symbol
] :> Int[ExpandTrigReduce[ActivateTrig[F[a + b*x]^p*G[c + d*x]^q], x], x] /
; FreeQ[{a, b, c, d}, x] && (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] ||
 EqQ[G, cos]) && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(x)\cos(4x)\,dx &= \int \left(\frac{1}{16}+\frac{1}{4}\cos(2x)+\frac{3}{8}\cos(4x)+\frac{1}{4}\cos(6x)+\frac{1}{16}\cos(8x)\right)dx\\ &= \frac{x}{16}+\frac{1}{16}\int \cos(8x)\,dx+\frac{1}{4}\int \cos(2x)\,dx+\frac{1}{4}\int \cos(6x)\,dx+\frac{3}{8}\int \cos(4x)\,dx\\ &= \frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)\end{aligned}$$

Mathematica [A] time = 0.0103867, size = 38, normalized size = 1.

$$\frac{x}{16}+\frac{1}{8}\sin(2x)+\frac{3}{32}\sin(4x)+\frac{1}{24}\sin(6x)+\frac{1}{128}\sin(8x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4*Cos[4*x],x]

[Out] x/16 + Sin[2*x]/8 + (3*Sin[4*x])/32 + Sin[6*x]/24 + Sin[8*x]/128
```

Maple [A] time = 0.062, size = 29, normalized size = 0.8

$$\frac{x}{16}+\frac{\sin\left(2x\right)}{8}+\frac{3\sin\left(4x\right)}{32}+\frac{\sin\left(6x\right)}{24}+\frac{\sin\left(8x\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*cos(4*x),x)
```

```
[Out] 1/16*x+1/8*sin(2*x)+3/32*sin(4*x)+1/24*sin(6*x)+1/128*sin(8*x)
```

Maxima [A] time = 0.958005, size = 41, normalized size = 1.08

$$-\frac{1}{6}\sin\left(2x\right)^{3}+\frac{1}{16}x+\frac{1}{128}\sin\left(8x\right)+\frac{3}{32}\sin\left(4x\right)+\frac{1}{4}\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cos(4*x),x, algorithm="maxima")
```

```
[Out] -1/6*sin(2*x)^3 + 1/16*x + 1/128*sin(8*x) + 3/32*sin(4*x) + 1/4*sin(2*x)
```

Fricas [A] time = 2.47478, size = 99, normalized size = 2.61

$$\frac{1}{48}\left(48\cos\left(x\right)^{7}-8\cos\left(x\right)^{5}+2\cos\left(x\right)^{3}+3\cos\left(x\right)\right)\sin\left(x\right)+\frac{1}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cos(4*x),x, algorithm="fricas")
```

```
[Out] 1/48*(48*cos(x)^7 - 8*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 1/16*x
```

Sympy [B] time = 27.5657, size = 144, normalized size = 3.79

$$\frac{x\sin^{4}(x)\cos(4x)}{16}-\frac{x\sin^{3}(x)\sin(4x)\cos(x)}{4}-\frac{3x\sin^{2}(x)\cos^{2}(x)\cos(4x)}{8}+\frac{x\sin(x)\sin(4x)\cos^{3}(x)}{4}+\frac{x\cos^{4}(x)\cos(4x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4*cos(4*x),x)
```

```
[Out] x*sin(x)**4*cos(4*x)/16 - x*sin(x)**3*sin(4*x)*cos(x)/4 - 3*x*sin(x)**2*cos
(x)**2*cos(4*x)/8 + x*sin(x)*sin(4*x)*cos(x)**3/4 + x*cos(x)**4*cos(4*x)/16
 - sin(x)**4*sin(4*x)/3 - 61*sin(x)**3*cos(x)*cos(4*x)/48 + 7*sin(x)**2*sin
(4*x)*cos(x)**2/4 + 15*sin(x)*cos(x)**3*cos(4*x)/16
```

Giac [A] time = 1.08375, size = 38, normalized size = 1.

$$\frac{1}{16}x+\frac{1}{128}\sin\left(8x\right)+\frac{1}{24}\sin\left(6x\right)+\frac{3}{32}\sin\left(4x\right)+\frac{1}{8}\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cos(4*x),x, algorithm="giac")
```

```
[Out] 1/16*x + 1/128*sin(8*x) + 1/24*sin(6*x) + 3/32*sin(4*x) + 1/8*sin(2*x)
```

3.371 $\int \cos(5x)\csc^5(x)\,dx$

Optimal. Leaf size=20

$$-\frac{1}{4}\csc^4(x)+6\csc^2(x)+16\log(\sin(x))$$

```
[Out] 6*Csc[x]^2 - Csc[x]^4/4 + 16*Log[Sin[x]]
```

Rubi [A] time = 0.0461352, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4366, 1247, 698}

$$-\frac{1}{4}\csc^4(x)+6\csc^2(x)+16\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[5*x]*Csc[x]^5,x]
```

```
[Out] 6*Csc[x]^2 - Csc[x]^4/4 + 16*Log[Sin[x]]
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\begin{aligned}
\int \cos(5x)\csc^5(x)\,dx &= -\text{Subst}\left(\int \frac{x\left(5-20x^2+16x^4\right)}{\left(1-x^2\right)^3}\,dx,x,\cos(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{5-20x+16x^2}{(1-x)^3}\,dx,x,\cos^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^3}-\frac{12}{(-1+x)^2}-\frac{16}{-1+x}\right)dx,x,\cos^2(x)\right)\right)\\
&= 6\csc^2(x)-\frac{\csc^4(x)}{4}+16\log(\sin(x))
\end{aligned}$$

Mathematica [A] time = 0.01147, size = 20, normalized size = 1.

$$-\frac{1}{4}\csc^4(x)+6\csc^2(x)+16\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[5*x]*Csc[x]^5,x]
```

```
[Out] 6*Csc[x]^2 - Csc[x]^4/4 + 16*Log[Sin[x]]
```

Maple [A] time = 0.048, size = 35, normalized size = 1.8

$$-\frac{5}{4\,(\sin(x))^4}+5\,\frac{(\cos(x))^4}{(\sin(x))^4}-4\,(\cot(x))^4+8\,(\cot(x))^2+16\,\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(5*x)/sin(x)^5,x)
```

```
[Out] -5/4/sin(x)^4+5/sin(x)^4*cos(x)^4-4*cot(x)^4+8*cot(x)^2+16*ln(sin(x))
```

Maxima [A] time = 0.942205, size = 45, normalized size = 2.25

$$\frac{5}{\sin(x)^2}+\frac{4\,\sin(x)^2-1}{4\,\sin(x)^4}+\frac{11}{2}\,\log\left(\sin(x)^2\right)+5\,\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/sin(x)^5,x, algorithm="maxima")
```

```
[Out] 5/sin(x)^2 + 1/4*(4*sin(x)^2 - 1)/sin(x)^4 + 11/2*log(sin(x)^2) + 5*log(sin
(x))
```

Fricas [B] time = 2.36719, size = 138, normalized size = 6.9

$$-\frac{24\,\cos(x)^2-64\left(\cos(x)^4-2\,\cos(x)^2+1\right)\log\left(\frac{1}{2}\,\sin(x)\right)-23}{4\left(\cos(x)^4-2\,\cos(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/sin(x)^5,x, algorithm="fricas")
```

```
[Out] -1/4*(24*cos(x)^2 - 64*(cos(x)^4 - 2*cos(x)^2 + 1)*log(1/2*sin(x)) - 23)/(c
os(x)^4 - 2*cos(x)^2 + 1)
```

Sympy [A] time = 105.548, size = 22, normalized size = 1.1

$$8\log\left(\sin^2(x)\right)+\frac{6}{\sin^2(x)}-\frac{1}{4\sin^4(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/sin(x)**5,x)
```

```
[Out] 8*log(sin(x)**2) + 6/sin(x)**2 - 1/(4*sin(x)**4)
```

Giac [B] time = 1.10203, size = 136, normalized size = 6.8

$$-\frac{\left(\frac{92(\cos(x)-1)}{\cos(x)+1}+\frac{768(\cos(x)-1)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)^2}{64(\cos(x)-1)^2}-\frac{23(\cos(x)-1)}{16(\cos(x)+1)}-\frac{(\cos(x)-1)^2}{64(\cos(x)+1)^2}-16\log\left(-\frac{\cos(x)-1}{\cos(x)+1}+1\right)+8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)/sin(x)^5,x, algorithm="giac")
```

```
[Out] -1/64*(92*(cos(x) - 1)/(cos(x) + 1) + 768*(cos(x) - 1)^2/(cos(x) + 1)^2 + 1
)*(cos(x) + 1)^2/(cos(x) - 1)^2 - 23/16*(cos(x) - 1)/(cos(x) + 1) - 1/64*(c
os(x) - 1)^2/(cos(x) + 1)^2 - 16*log(-(cos(x) - 1)/(cos(x) + 1) + 1) + 8*lo
g(-(cos(x) - 1)/(cos(x) + 1))
```

3.372 $\int \csc^4(x)\sin(4x)\,dx$

Optimal. Leaf size=12

$$-2\csc^2(x)-8\log(\sin(x))$$

```
[Out] -2*Csc[x]^2 - 8*Log[Sin[x]]
```

Rubi [A] time = 0.0214947, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {14}

$$-2\csc^2(x)-8\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4*Sin[4*x],x]
```

```
[Out] -2*Csc[x]^2 - 8*Log[Sin[x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \csc^4(x)\sin(4x)\,dx &= \text{Subst}\left(\int \frac{4-8x^2}{x^3}\,dx,x,\sin(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{4}{x^3}-\frac{8}{x}\right)dx,x,\sin(x)\right)\\ &= -2\csc^2(x)-8\log(\sin(x))\end{aligned}$$

Mathematica [A] time = 0.0091205, size = 12, normalized size = 1.

$$-2\csc^2(x)-8\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4*Sin[4*x],x]
```

```
[Out] -2*Csc[x]^2 - 8*Log[Sin[x]]
```

Maple [A] time = 0.05, size = 19, normalized size = 1.6

$$2\,(\sin(x))^{-2}-4\,(\cot(x))^2-8\,\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(4*x)/sin(x)^4,x)
```

```
[Out] 2/sin(x)^2-4*cot(x)^2-8*ln(sin(x))
```

Maxima [A] time = 0.936607, size = 26, normalized size = 2.17

$$-\frac{2}{\sin(x)^2} - 2\log\left(\sin(x)^2\right) - 4\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*x)/sin(x)^4,x, algorithm="maxima")
```

```
[Out] -2/sin(x)^2 - 2*log(sin(x)^2) - 4*log(sin(x))
```

Fricas [B] time = 2.24487, size = 78, normalized size = 6.5

$$-\frac{2\left(4\left(\cos(x)^2-1\right)\log\left(\frac{1}{2}\sin(x)\right)-1\right)}{\cos(x)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*x)/sin(x)^4,x, algorithm="fricas")
```

```
[Out] -2*(4*(cos(x)^2 - 1)*log(1/2*sin(x)) - 1)/(cos(x)^2 - 1)
```

Sympy [A] time = 15.3909, size = 14, normalized size = 1.17

$$-8\log(\sin(x)) - \frac{2}{\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*x)/sin(x)**4,x)
```

```
[Out] -8*log(sin(x)) - 2/sin(x)**2
```

Giac [B] time = 1.08126, size = 96, normalized size = 8.

$$\frac{\left(\frac{8(\cos(x)-1)}{\cos(x)+1}+1\right)(\cos(x)+1)}{2(\cos(x)-1)} + \frac{\cos(x)-1}{2(\cos(x)+1)} + 8\log\left(-\frac{\cos(x)-1}{\cos(x)+1}+1\right) - 4\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(4*x)/sin(x)^4,x, algorithm="giac")
```

```
[Out] 1/2*(8*(cos(x) - 1)/(cos(x) + 1) + 1)*(cos(x) + 1)/(cos(x) - 1) + 1/2*(cos(
x) - 1)/(cos(x) + 1) + 8*log(-(cos(x) - 1)/(cos(x) + 1) + 1) - 4*log(-(cos(
x) - 1)/(cos(x) + 1))
```

3.373 $\int \frac{\cot(x)}{2+\sin(2x)}\,dx$

Optimal. Leaf size=64

$$-\frac{x}{2\sqrt{3}}+\frac{1}{2}\log(\sin(x))+\frac{\tan^{-1}\left(\frac{1-2\cos^2(x)}{2\sin(x)\cos(x)+\sqrt{3}+2}\right)}{2\sqrt{3}}-\frac{1}{4}\log(\sin(x)\cos(x)+1)$$

```
[Out] -x/(2*Sqrt[3]) + ArcTan[(1 - 2*Cos[x]^2)/(2 + Sqrt[3] + 2*Cos[x]*Sin[x])]/(
2*Sqrt[3]) + Log[Sin[x]]/2 - Log[1 + Cos[x]*Sin[x]]/4
```

Rubi [A] time = 0.0784877, antiderivative size = 65, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {705, 29, 634, 618, 204, 628}

$$-\frac{x}{2\sqrt{3}}-\frac{1}{4}\log\left(\tan^2(x)+\tan(x)+1\right)+\frac{1}{2}\log(\tan(x))+\frac{\tan^{-1}\left(\frac{1-2\cos^2(x)}{2\sin(x)\cos(x)+\sqrt{3}+2}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(2 + Sin[2*x]),x]
```

```
[Out] -x/(2*Sqrt[3]) + ArcTan[(1 - 2*Cos[x]^2)/(2 + Sqrt[3] + 2*Cos[x]*Sin[x])]/(
2*Sqrt[3]) + Log[Tan[x]]/2 - Log[1 + Tan[x] + Tan[x]^2]/4
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{2+\sin(2x)}\,dx &= \text{Subst}\left(\int \frac{1}{x\left(2+2x+2x^2\right)}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tan(x)\right) + \frac{1}{2}\text{Subst}\left(\int \frac{-2-2x}{2+2x+2x^2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\log(\tan(x)) - \frac{1}{4}\text{Subst}\left(\int \frac{2+4x}{2+2x+2x^2}\,dx, x, \tan(x)\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{2+2x+2x^2}\,dx, x, \tan(
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2}\log(\tan(x)) - \frac{1}{4}\log\left(1+\tan(x)+\tan^2(x)\right) + \text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, 2+4\tan(x)\right)\\
&= -\frac{x}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{1-2\cos^2(x)}{2+\sqrt{3}+2\cos(x)\sin(x)}\right)}{2\sqrt{3}} + \frac{1}{2}\log(\tan(x)) - \frac{1}{4}\log\left(1+\tan(x)+\tan^2(x)\right)
\end{aligned}$$

Mathematica [A] time = 0.0362993, size = 39, normalized size = 0.61

$$\frac{1}{12}\left(-2\sqrt{3}\tan^{-1}\left(\frac{2\tan(x)+1}{\sqrt{3}}\right)+6\log(\sin(x))-3\log(\sin(2x)+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(2 + Sin[2*x]),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 + 2*Tan[x])/Sqrt[3]] + 6*Log[Sin[x]] - 3*Log[2 + Sin[
2*x]])/12
```

Maple [A] time = 0.062, size = 35, normalized size = 0.6

$$\frac{\ln(\tan(x))}{2} - \frac{\ln\left(1+\tan(x)+(\tan(x))^2\right)}{4} - \frac{\sqrt{3}}{6}\arctan\left(\frac{(2\tan(x)+1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(x)/(2+sin(2*x)),x)
```

```
[Out] 1/2*ln(tan(x))-1/4*ln(1+tan(x)+tan(x)^2)-1/6*3^(1/2)*arctan(1/3*(2*tan(x)+1
)*3^(1/2))
```

Maxima [B] time = 1.50802, size = 281, normalized size = 4.39

$$-\frac{1}{24}\sqrt{3}\left(\sqrt{3}\log\left(-2(4\sin(2x)+1)\cos(4x)+\cos(4x)^2+16\cos(2x)^2+8\cos(2x)\sin(4x)+\sin(4x)^2+16\sin(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/(2+sin(2*x)),x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(3)*(sqrt(3)*log(-2*(4*sin(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 16*c
os(2*x)^2 + 8*cos(2*x)*sin(4*x) + sin(4*x)^2 + 16*sin(2*x)^2 + 8*sin(2*x) +
 1) - 2*sqrt(3)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) - 2*sqrt(3)*log(cos
(x)^2 + sin(x)^2 - 2*cos(x) + 1) - 2*arctan2(2*sqrt(3)*cos(2*x)/(cos(2*x)^2
 - 2*(sqrt(3) - 2)*sin(2*x) + sin(2*x)^2 - 4*sqrt(3) + 7), (cos(2*x)^2 + si
n(2*x)^2 + 4*sin(2*x) + 1)/(cos(2*x)^2 - 2*(sqrt(3) - 2)*sin(2*x) + sin(2*x
)^2 - 4*sqrt(3) + 7)))
```

Fricas [A] time = 2.46668, size = 223, normalized size = 3.48

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)+\sqrt{3}}{3\left(2\cos(x)^2-1\right)}\right)-\frac{1}{8}\log\left(-\cos(x)^4+\cos(x)^2+2\cos(x)\sin(x)+1\right)+\frac{1}{4}\log\left(-\frac{1}{4}\cos(x)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/(2+sin(2*x)),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) + sqrt(3))/(2*cos(x)^2 -
1)) - 1/8*log(-cos(x)^4 + cos(x)^2 + 2*cos(x)*sin(x) + 1) + 1/4*log(-1/4*co
s(x)^2 + 1/4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos(x)}{(\sin(2x)+2)\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/(2+sin(2*x)),x)
```

```
[Out] Integral(cos(x)/((sin(2*x) + 2)*sin(x)), x)
```

Giac [A] time = 1.14812, size = 101, normalized size = 1.58

$$-\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)-\cos(2x)-2\sin(2x)-1}{\sqrt{3}\cos(2x)+\sqrt{3}-2\cos(2x)+\sin(2x)+2}\right)\right)-\frac{1}{4}\log\left(\tan(x)^2+\tan(x)+1\right)+\frac{1}{2}\log(|\tan(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/(2+sin(2*x)),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) - cos(2*x) - 2*sin(2*x) - 1)/(s
qrt(3)*cos(2*x) + sqrt(3) - 2*cos(2*x) + sin(2*x) + 2))) - 1/4*log(tan(x)^2
 + tan(x) + 1) + 1/2*log(abs(tan(x)))
```

3.374 $\int \cos(x)\cot(x)\sec(3x)\,dx$

Optimal. Leaf size=11

$$-\frac{1}{2}\log\left(\csc^2(x)-4\right)$$

```
[Out] -Log[-4 + Csc[x]^2]/2
```

Rubi [A] time = 0.0332537, antiderivative size = 17, normalized size of antiderivative = 1.55, number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4356, 266, 36, 31, 29}

$$\log(\sin(x))-\frac{1}{2}\log\left(1-4\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[x]*Sec[3*x],x]
```

```
[Out] Log[Sin[x]] - Log[1 - 4*Sin[x]^2]/2
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \cos(x)\cot(x)\sec(3x)\,dx = \text{Subst}\left(\int \frac{1}{x(1-4x^2)}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1-4x)x}\,dx, x, \sin^2(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^2(x)\right) + 2\,\text{Subst}\left(\int \frac{1}{1-4x}\,dx, x, \sin^2(x)\right)$$
$$= \log(\sin(x)) - \frac{1}{2}\log\left(1-4\sin^2(x)\right)$$

Mathematica [A] time = 0.0130305, size = 17, normalized size = 1.55

$$\log(\sin(x)) - \frac{1}{2}\log\left(1-4\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[x]*Sec[3*x],x]
```

```
[Out] Log[Sin[x]] - Log[1 - 4*Sin[x]^2]/2
```

Maple [B] time = 0.044, size = 27, normalized size = 2.5

$$\frac{\ln(\cos(x)+1)}{2} + \frac{\ln(\cos(x)-1)}{2} - \frac{\ln\left(4(\cos(x))^2-3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/cos(3*x)/sin(x),x)
```

```
[Out] 1/2*ln(cos(x)+1)+1/2*ln(cos(x)-1)-1/2*ln(4*cos(x)^2-3)
```

Maxima [B] time = 0.955627, size = 124, normalized size = 11.27

$$-\frac{1}{4}\log\left(-2(\cos(2x)-1)\cos(4x) + \cos(4x)^2 + \cos(2x)^2 + \sin(4x)^2 - 2\sin(4x)\sin(2x) + \sin(2x)^2 - 2\cos(2x) +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/cos(3*x)/sin(x),x, algorithm="maxima")
```

```
[Out] -1/4*log(-2*(cos(2*x) - 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + sin(4*x)^2
- 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*cos(2*x) + 1) + 1/2*log(cos(x)^2 + s
in(x)^2 + 2*cos(x) + 1) + 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [A] time = 2.21157, size = 59, normalized size = 5.36

$$-\frac{1}{2}\log\left(4\cos(x)^2-3\right) + \log\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/cos(3*x)/sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(4*cos(x)^2 - 3) + log(1/2*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(x)}{\sin(x)\cos(3x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/cos(3*x)/sin(x),x)
```

```
[Out] Integral(cos(x)**2/(sin(x)*cos(3*x)), x)
```

Giac [B] time = 1.11134, size = 32, normalized size = 2.91

$$\frac{1}{2}\log\left(-\cos(x)^2+1\right)-\frac{1}{2}\log\left(\left|4\cos(x)^2-3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/cos(3*x)/sin(x),x, algorithm="giac")
```

```
[Out] 1/2*log(-cos(x)^2 + 1) - 1/2*log(abs(4*cos(x)^2 - 3))
```

3.375 $\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx$

Optimal. Leaf size=7

$$-\tan^{-1}(\cos(2x))$$

```
[Out] -ArcTan[Cos[2*x]]
```

Rubi [A] time = 0.0391762, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {12, 1107, 617, 204}

$$-\tan^{-1}(\cos(2x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/(Cos[x]^4 + Sin[x]^4),x]

[Out] -ArcTan[Cos[2*x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(2x)}{\cos^4(x)+\sin^4(x)}\,dx &= \text{Subst}\left(\int \frac{2x}{1-2x^2+2x^4}\,dx, x, \sin(x)\right)\\
&= 2\,\text{Subst}\left(\int \frac{x}{1-2x^2+2x^4}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{1-2x+2x^2}\,dx, x, \sin^2(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-2\sin^2(x)\right)\\
&= -\tan^{-1}\left(1-2\sin^2(x)\right)
\end{aligned}$$

Mathematica [A] time = 0.0310197, size = 7, normalized size = 1.

$$-\tan^{-1}(\cos(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/(Cos[x]^4 + Sin[x]^4),x]
```

```
[Out] -ArcTan[Cos[2*x]]
```

Maple [A] time = 0.035, size = 12, normalized size = 1.7

$$-\arctan\left(2\left(\cos\left(x\right)\right)^{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(cos(x)^4+sin(x)^4),x)
```

```
[Out] -arctan(2*cos(x)^2-1)
```

Maxima [A] time = 1.42329, size = 12, normalized size = 1.71

$$\arctan\left(2\sin\left(x\right)^{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x, algorithm="maxima")
```

```
[Out] arctan(2*sin(x)^2 - 1)
```

Fricas [A] time = 2.27802, size = 34, normalized size = 4.86

$$-\arctan\left(2\cos\left(x\right)^{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x, algorithm="fricas")
```

```
[Out] -arctan(2*cos(x)^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(cos(x)**4+sin(x)**4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07174, size = 12, normalized size = 1.71

$$\arctan\left(2\,\sin\left(x\right)^{2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(cos(x)^4+sin(x)^4),x, algorithm="giac")
```

```
[Out] arctan(2*sin(x)^2 - 1)
```

3.376 $\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=53

$$\frac{x}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{\cos(x)-\sqrt{3}\sin(x)}{\sin(x)+\sqrt{3}\cos(x)+2(2+\sqrt{3})}\right)}{\sqrt{3}}$$

```
[Out] x/(2*Sqrt[3]) + ArcTan[(Cos[x] - Sqrt[3]*Sin[x])/(2*(2 + Sqrt[3]) + Sqrt[3]
*Cos[x] + Sin[x])]/Sqrt[3]
```

Rubi [A] time = 0.0981272, antiderivative size = 83, normalized size of antiderivative = 1.57, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3124, 617, 204}

$$\frac{x}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{(3-4\sqrt{3})\sin(x)+(4-\sqrt{3})\cos(x)}{(4-\sqrt{3})\sin(x)-(3-4\sqrt{3})\cos(x)+2(5+2\sqrt{3})}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + Sqrt[3]*Cos[x] + Sin[x])^(-1),x]
```

```
[Out] x/(2*Sqrt[3]) + ArcTan[((4 - Sqrt[3])*Cos[x] + (3 - 4*Sqrt[3])*Sin[x])/(2*(
5 + 2*Sqrt[3]) - (3 - 4*Sqrt[3])*Cos[x] + (4 - Sqrt[3])*Sin[x])]/Sqrt[3]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{4+\sqrt{3}\cos(x)+\sin(x)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{4+\sqrt{3}+2x+\left(4-\sqrt{3}\right)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= -\left(2\,\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx, x, 1+\left(4-\sqrt{3}\right)\tan\left(\frac{x}{2}\right)\right)\right)$$
$$= \frac{x}{2\sqrt{3}} + \frac{\tan^{-1}\left(\frac{\left(4-\sqrt{3}\right)\cos(x)+\left(3-4\sqrt{3}\right)\sin(x)}{2\left(5+2\sqrt{3}\right)-\left(3-4\sqrt{3}\right)\cos(x)+\left(4-\sqrt{3}\right)\sin(x)}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.0586049, size = 33, normalized size = 0.62

$$-\frac{\tan^{-1}\left(\frac{\left(\sqrt{3}-4\right)\tan\left(\frac{x}{2}\right)-1}{2\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + Sqrt[3]*Cos[x] + Sin[x])^(-1),x]
```

```
[Out] -(ArcTan[(-1 + (-4 + Sqrt[3])*Tan[x/2])/(2*Sqrt[3])]/Sqrt[3])
```

Maple [A] time = 0.046, size = 43, normalized size = 0.8

$$-52\,\frac{1}{\left(\sqrt{3}-4\right)\left(16\,\sqrt{3}+12\right)}\arctan\left(\frac{26\,\tan(x/2)+2\,\sqrt{3}+8}{16\,\sqrt{3}+12}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+sin(x)+cos(x)*3^(1/2)),x)
```

```
[Out] -52/(3^(1/2)-4)/(16*3^(1/2)+12)*arctan((26*tan(1/2*x)+2*3^(1/2)+8)/(16*3^(1
/2)+12))
```

Maxima [A] time = 1.41039, size = 36, normalized size = 0.68

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{6}\sqrt{3}\left(\frac{\left(\sqrt{3}-4\right)\sin(x)}{\cos(x)+1}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/6*sqrt(3)*((sqrt(3) - 4)*sin(x)/(cos(x) + 1) - 1))
```

Fricas [A] time = 2.36854, size = 128, normalized size = 2.42

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{2\left(\left(4\sqrt{3}\cos(x)+3\right)\sin(x)+\sqrt{3}\cos(x)+3\right)}{3\left(4\cos(x)^2-3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(2/3*((4*sqrt(3)*cos(x) + 3)*sin(x) + sqrt(3)*cos(x) + 3)
/(4*cos(x)^2 - 3))
```

Sympy [B] time = 8.79338, size = 107, normalized size = 2.02

$$-\frac{3274042252104260754602221214805345211931850862603092}{-9822126756312782263806663644416035635795552587809278287269344468680621585946752563395790411185}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+sin(x)+cos(x)*3**(1/2)),x)
```

```
[Out] -32740422521042607546022212148053452119318508626030927624231148228935405286
489175211319301372839079350465571750721183129*sqrt(3)*(atan(-tan(x/2)/2 + 2
*sqrt(3)*tan(x/2)/3 + sqrt(3)/6) + pi*floor((x/2 - pi/2)/pi))/(-98221267563
127822638066636444160356357955525878092782872693444686806215859467525633957
904118517238051396715252163549387 + 567080752677181058341878323874840680777
336021498392871277469379330093857007343756148068212677826348337704415266270
45716*sqrt(3)) + 5670807526771810583418783238748406807773360214983928712774
6937933009385700734375614806821267782634833770441526627045716*(atan(-tan(x/
2)/2 + 2*sqrt(3)*tan(x/2)/3 + sqrt(3)/6) + pi*floor((x/2 - pi/2)/pi))/(-982
212675631278226380666364441603563579555258780927828726934446868062158594675
25633957904118517238051396715252163549387 + 5670807526771810583418783238748
406807773360214983928712774693793300938570073437561480682126778263483377044
1526627045716*sqrt(3))
```

Giac [A] time = 1.06747, size = 105, normalized size = 1.98

$$\frac{\left(x + 2 \arctan\left(\frac{\sqrt{3}\cos(x) - 8\sqrt{3}\sin(x) + \sqrt{3} + 4\cos(x) + 7\sin(x) + 4}{8\sqrt{3}\cos(x) + \sqrt{3}\sin(x) + 8\sqrt{3} - 7\cos(x) + 4\sin(x) + 19}\right)\right)\left(\sqrt{3} + 4\right)}{2\left(4\sqrt{3} + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+sin(x)+cos(x)*3^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*(x + 2*arctan((sqrt(3)*cos(x) - 8*sqrt(3)*sin(x) + sqrt(3) + 4*cos(x) +
 7*sin(x) + 4)/(8*sqrt(3)*cos(x) + sqrt(3)*sin(x) + 8*sqrt(3) - 7*cos(x) +
4*sin(x) + 19)))*(sqrt(3) + 4)/(4*sqrt(3) + 3)
```

3.377 $\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx$

Optimal. Leaf size=33

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{23}(\cos(x)-\sin(x))}{3\sin(x)+3\cos(x)+8}\right)}{\sqrt{23}}$$

```
[Out] -(ArcTanh[(Sqrt[23]*(Cos[x] - Sin[x]))/(8 + 3*Cos[x] + 3*Sin[x])]/Sqrt[23])
```

Rubi [B] time = 0.0735646, antiderivative size = 94, normalized size of antiderivative = 2.85, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3124, 618, 206}

$$\frac{\log\left(\sqrt{23}\sin(x)-4\sin(x)-4\sqrt{23}\cos(x)+19\cos(x)+4\left(5-\sqrt{23}\right)\right)}{2\sqrt{23}}-\frac{\log\left(-\sqrt{23}\sin(x)-4\sin(x)+4\sqrt{23}\cos(x)+19\cos(x)\right.}{2\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 4*Cos[x] + 4*Sin[x])^(-1),x]
```

```
[Out] -Log[4*(5 + Sqrt[23]) + 19*Cos[x] + 4*Sqrt[23]*Cos[x] - 4*Sin[x] - Sqrt[23]
*Sin[x]]/(2*Sqrt[23]) + Log[4*(5 - Sqrt[23]) + 19*Cos[x] - 4*Sqrt[23]*Cos[x
] - 4*Sin[x] + Sqrt[23]*Sin[x]]/(2*Sqrt[23])
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{3+4\cos(x)+4\sin(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{7+8x-x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)\\ &= -\left(4\,\text{Subst}\left(\int \frac{1}{92-x^2}\,dx,x,8-2\tan\left(\frac{x}{2}\right)\right)\right)\\ &= -\frac{\log\left(4\left(5+\sqrt{23}\right)+19\cos(x)+4\sqrt{23}\cos(x)-4\sin(x)-\sqrt{23}\sin(x)\right)}{2\sqrt{23}}+\frac{\log\left(4\left(5-\sqrt{2\right.}}{}\end{aligned}$$

Mathematica [A] time = 0.0429494, size = 22, normalized size = 0.67

$$\frac{2\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-4}{\sqrt{23}}\right)}{\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 4*Cos[x] + 4*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTanh[(-4 + Tan[x/2])/Sqrt[23]])/Sqrt[23]
```

Maple [A] time = 0.031, size = 20, normalized size = 0.6

$$\frac{2\sqrt{23}}{23}\mathrm{Artanh}\left(\frac{\sqrt{23}}{46}\left(-8+2\tan(x/2)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3+4*cos(x)+4*sin(x)),x)
```

```
[Out] 2/23*23^(1/2)*arctanh(1/46*(-8+2*tan(1/2*x))*23^(1/2))
```

Maxima [A] time = 1.4106, size = 53, normalized size = 1.61

$$-\frac{1}{23}\sqrt{23}\log\left(-\frac{\sqrt{23}-\frac{\sin(x)}{\cos(x)+1}+4}{\sqrt{23}+\frac{\sin(x)}{\cos(x)+1}-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+4*cos(x)+4*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/23*sqrt(23)*log(-(sqrt(23) - sin(x)/(cos(x) + 1) + 4)/(sqrt(23) + sin(x)
/(cos(x) + 1) - 4))
```

Fricas [B] time = 2.22366, size = 230, normalized size = 6.97

$$\frac{1}{46}\sqrt{23}\log\left(-\frac{6\sqrt{23}\cos(x)^2+8\left(\sqrt{23}-3\right)\cos(x)-2\left(4\sqrt{23}-7\cos(x)+12\right)\sin(x)-3\sqrt{23}-48}{8\left(4\cos(x)+3\right)\sin(x)+24\cos(x)+25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+4*cos(x)+4*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/46*sqrt(23)*log(-(6*sqrt(23)*cos(x)^2 + 8*(sqrt(23) - 3)*cos(x) - 2*(4*sq
rt(23) - 7*cos(x) + 12)*sin(x) - 3*sqrt(23) - 48)/(8*(4*cos(x) + 3)*sin(x)
+ 24*cos(x) + 25))
```

Sympy [A] time = 1.71092, size = 39, normalized size = 1.18

$$\frac{\sqrt{23}\log\left(\tan\left(\frac{x}{2}\right)-4+\sqrt{23}\right)}{23}-\frac{\sqrt{23}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{23}-4\right)}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+4*cos(x)+4*sin(x)),x)
```

```
[Out] sqrt(23)*log(tan(x/2) - 4 + sqrt(23))/23 - sqrt(23)*log(tan(x/2) - sqrt(23)
 - 4)/23
```

Giac [A] time = 1.15066, size = 50, normalized size = 1.52

$$-\frac{1}{23}\sqrt{23}\log\left(\frac{\left|-2\sqrt{23}+2\tan\left(\frac{1}{2}x\right)-8\right|}{\left|2\sqrt{23}+2\tan\left(\frac{1}{2}x\right)-8\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3+4*cos(x)+4*sin(x)),x, algorithm="giac")
```

```
[Out] -1/23*sqrt(23)*log(abs(-2*sqrt(23) + 2*tan(1/2*x) - 8)/abs(2*sqrt(23) + 2*t
an(1/2*x) - 8))
```

3.378 $\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx$

Optimal. Leaf size=27

$$\frac{x}{3}+\frac{1}{3}\tan^{-1}\left(\frac{2\sin(x)\cos(x)}{2\sin^2(x)+1}\right)$$

```
[Out] x/3 + ArcTan[(2*Cos[x]*Sin[x])/(1 + 2*Sin[x]^2)]/3
```

Rubi [A] time = 0.021039, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {203}

$$\frac{x}{3}+\frac{1}{3}\tan^{-1}\left(\frac{2\sin(x)\cos(x)}{2\sin^2(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 3*Cos[x]^2 + 5*Sin[x]^2)^(-1),x]
```

```
[Out] x/3 + ArcTan[(2*Cos[x]*Sin[x])/(1 + 2*Sin[x]^2)]/3
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4-3\cos^2(x)+5\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1+9x^2}\,dx,x,\tan(x)\right)\\ &= \frac{x}{3}+\frac{1}{3}\tan^{-1}\left(\frac{2\cos(x)\sin(x)}{1+2\sin^2(x)}\right)\end{aligned}$$

Mathematica [A] time = 0.0214716, size = 9, normalized size = 0.33

$$\frac{1}{3}\tan^{-1}(3\tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 3*Cos[x]^2 + 5*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[3*Tan[x]]/3
```

Maple [A] time = 0.026, size = 8, normalized size = 0.3

$$\frac{\arctan(3\tan(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-3*cos(x)^2+5*sin(x)^2),x)
```

```
[Out] 1/3*arctan(3*tan(x))
```

Maxima [A] time = 1.41616, size = 9, normalized size = 0.33

$$\frac{1}{3}\arctan\left(3\tan\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/3*arctan(3*tan(x))
```

Fricas [A] time = 2.14946, size = 70, normalized size = 2.59

$$-\frac{1}{6}\arctan\left(\frac{10\cos\left(x\right)^{2}-9}{6\cos\left(x\right)\sin\left(x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/6*arctan(1/6*(10*cos(x)^2 - 9)/(cos(x)*sin(x)))
```

Sympy [B] time = 14.4745, size = 148, normalized size = 5.48

$$\frac{\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{17-12\sqrt{2}}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor}{6\sqrt{2}\sqrt{17-12\sqrt{2}}+9\sqrt{17-12\sqrt{2}}}+\frac{\sqrt{17-12\sqrt{2}}\sqrt{12\sqrt{2}+17}\left(\operatorname{atan}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{12\sqrt{2}+17}}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{6\sqrt{2}\sqrt{17-12\sqrt{2}}+9\sqrt{17-12\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-3*cos(x)**2+5*sin(x)**2),x)
```

```
[Out] (atan(tan(x/2)/sqrt(17 - 12*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(6*sqrt(
2)*sqrt(17 - 12*sqrt(2)) + 9*sqrt(17 - 12*sqrt(2))) + sqrt(17 - 12*sqrt(2))
*sqrt(12*sqrt(2) + 17)*(atan(tan(x/2)/sqrt(12*sqrt(2) + 17)) + pi*floor((x/
2 - pi/2)/pi))/(6*sqrt(2)*sqrt(17 - 12*sqrt(2)) + 9*sqrt(17 - 12*sqrt(2)))
```

Giac [A] time = 1.1132, size = 27, normalized size = 1.

$$\frac{1}{3}x-\frac{1}{3}\arctan\left(\frac{\sin\left(2x\right)}{\cos\left(2x\right)-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-3*cos(x)^2+5*sin(x)^2),x, algorithm="giac")

[Out] 1/3*x - 1/3*arctan(sin(2*x)/(cos(2*x) - 2))
```

3.379 $\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx$

Optimal. Leaf size=28

$$\frac{4x}{25}+\frac{2}{5(\tan(x)+2)}-\frac{3}{25}\log(\sin(x)+2\cos(x))$$

```
[Out] (4*x)/25 - (3*Log[2*Cos[x] + Sin[x]])/25 + 2/(5*(2 + Tan[x]))
```

Rubi [A] time = 0.0407054, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {801, 635, 203, 260}

$$\frac{4x}{25}+\frac{2}{5(\tan(x)+2)}-\frac{3}{25}\log(\sin(x)+2\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 4*Cot[x] + Tan[x])^(-1),x]
```

```
[Out] (4*x)/25 - (3*Log[2*Cos[x] + Sin[x]])/25 + 2/(5*(2 + Tan[x]))
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{4+4\cot(x)+\tan(x)}\,dx = \text{Subst}\left(\int \frac{x}{(2+x)^2\left(1+x^2\right)}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{2}{5(2+x)^2} - \frac{3}{25(2+x)} + \frac{4+3x}{25\left(1+x^2\right)}\right)dx, x, \tan(x)\right)$$
$$= -\frac{3}{25}\log(2+\tan(x)) + \frac{2}{5(2+\tan(x))} + \frac{1}{25}\text{Subst}\left(\int \frac{4+3x}{1+x^2}\,dx, x, \tan(x)\right)$$
$$= -\frac{3}{25}\log(2+\tan(x)) + \frac{2}{5(2+\tan(x))} + \frac{3}{25}\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \tan(x)\right) + \frac{4}{25}\text{Subs}$$
$$= \frac{4x}{25} - \frac{3}{25}\log(\cos(x)) - \frac{3}{25}\log(2+\tan(x)) + \frac{2}{5(2+\tan(x))}$$

Mathematica [A] time = 0.0404914, size = 41, normalized size = 1.46

$$\frac{4x-3\log(\sin(x)+2\cos(x))+\cot(x)(8x-6\log(\sin(x)+2\cos(x)))-5}{50\cot(x)+25}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 4*Cot[x] + Tan[x])^(-1),x]
```

```
[Out] (-5 + 4*x + Cot[x]*(8*x - 6*Log[2*Cos[x] + Sin[x]]) - 3*Log[2*Cos[x] + Sin[
x]])/(25 + 50*Cot[x])
```

Maple [A] time = 0.1, size = 29, normalized size = 1.

$$\frac{2}{10+5\tan(x)} - \frac{3\ln(2+\tan(x))}{25} + \frac{3\ln\left((\tan(x))^2+1\right)}{50} + \frac{4x}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+4*cot(x)+tan(x)),x)
```

```
[Out] 2/5/(2+tan(x))-3/25*ln(2+tan(x))+3/50*ln(tan(x)^2+1)+4/25*x
```

Maxima [A] time = 1.42053, size = 38, normalized size = 1.36

$$\frac{4}{25}x + \frac{2}{5(\tan(x)+2)} + \frac{3}{50}\log\left(\tan(x)^2+1\right) - \frac{3}{25}\log(\tan(x)+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+4*cot(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] 4/25*x + 2/5/(tan(x) + 2) + 3/50*log(tan(x)^2 + 1) - 3/25*log(tan(x) + 2)
```

Fricas [B] time = 2.16238, size = 153, normalized size = 5.46

$$-\frac{3\left(\tan\left(x\right)+2\right)\log\left(\frac{\tan\left(x\right)^{2}+4\,\tan\left(x\right)+4}{\tan\left(x\right)^{2}+1}\right)-8\left(x-1\right)\tan\left(x\right)-16\,x-4}{50\left(\tan\left(x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+4*cot(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] -1/50*(3*(tan(x) + 2)*log((tan(x)^2 + 4*tan(x) + 4)/(tan(x)^2 + 1)) - 8*(x
- 1)*tan(x) - 16*x - 4)/(tan(x) + 2)
```

Sympy [B] time = 0.510224, size = 105, normalized size = 3.75

$$\frac{8x\tan\left(x\right)}{50\tan\left(x\right)+100}+\frac{16x}{50\tan\left(x\right)+100}-\frac{6\log\left(\tan\left(x\right)+2\right)\tan\left(x\right)}{50\tan\left(x\right)+100}-\frac{12\log\left(\tan\left(x\right)+2\right)}{50\tan\left(x\right)+100}+\frac{3\log\left(\tan^{2}\left(x\right)+1\right)\tan\left(x\right)}{50\tan\left(x\right)+100}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+4*cot(x)+tan(x)),x)
```

```
[Out] 8*x*tan(x)/(50*tan(x) + 100) + 16*x/(50*tan(x) + 100) - 6*log(tan(x) + 2)*t
an(x)/(50*tan(x) + 100) - 12*log(tan(x) + 2)/(50*tan(x) + 100) + 3*log(tan(
x)**2 + 1)*tan(x)/(50*tan(x) + 100) + 6*log(tan(x)**2 + 1)/(50*tan(x) + 100
) - 10*tan(x)/(50*tan(x) + 100)
```

Giac [A] time = 1.11721, size = 39, normalized size = 1.39

$$\frac{4}{25}\,x+\frac{2}{5\left(\tan\left(x\right)+2\right)}+\frac{3}{50}\,\log\left(\tan\left(x\right)^{2}+1\right)-\frac{3}{25}\,\log\left(\left|\tan\left(x\right)+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+4*cot(x)+tan(x)),x, algorithm="giac")
```

```
[Out] 4/25*x + 2/5/(tan(x) + 2) + 3/50*log(tan(x)^2 + 1) - 3/25*log(abs(tan(x) +
2))
```

3.380 $\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx$

Optimal. Leaf size=67

$$\frac{8x}{15\sqrt{15}}+\frac{4\tan(x)+1}{15\left(2\tan^2(x)+\tan(x)+2\right)}-\frac{8\tan^{-1}\left(\frac{1-2\cos^2(x)}{2\sin(x)\cos(x)+\sqrt{15}+4}\right)}{15\sqrt{15}}$$

```
[Out] (8*x)/(15*Sqrt[15]) - (8*ArcTan[(1 - 2*Cos[x]^2)/(4 + Sqrt[15] + 2*Cos[x]*S
in[x])])/(15*Sqrt[15]) + (1 + 4*Tan[x])/(15*(2 + Tan[x] + 2*Tan[x]^2))
```

Rubi [A] time = 0.0456749, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {614, 618, 204}

$$\frac{8x}{15\sqrt{15}}+\frac{4\tan(x)+1}{15\left(2\tan^2(x)+\tan(x)+2\right)}-\frac{8\tan^{-1}\left(\frac{1-2\cos^2(x)}{2\sin(x)\cos(x)+\sqrt{15}+4}\right)}{15\sqrt{15}}$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sec[x] + Sin[x])^(-2),x]
```

```
[Out] (8*x)/(15*Sqrt[15]) - (8*ArcTan[(1 - 2*Cos[x]^2)/(4 + Sqrt[15] + 2*Cos[x]*S
in[x])])/(15*Sqrt[15]) + (1 + 4*Tan[x])/(15*(2 + Tan[x] + 2*Tan[x]^2))
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(2\sec(x)+\sin(x))^2}\,dx &= \text{Subst}\left(\int \frac{1}{\left(2+x+2x^2\right)^2}\,dx, x, \tan(x)\right)\\ &= \frac{1+4\tan(x)}{15\left(2+\tan(x)+2\tan^2(x)\right)}+\frac{4}{15}\text{Subst}\left(\int \frac{1}{2+x+2x^2}\,dx, x, \tan(x)\right)\\ &= \frac{1+4\tan(x)}{15\left(2+\tan(x)+2\tan^2(x)\right)}-\frac{8}{15}\text{Subst}\left(\int \frac{1}{-15-x^2}\,dx, x, 1+4\tan(x)\right)\\ &= \frac{8x}{15\sqrt{15}}-\frac{8\tan^{-1}\left(\frac{1-2\cos^2(x)}{4+\sqrt{15}+2\cos(x)\sin(x)}\right)}{15\sqrt{15}}+\frac{1+4\tan(x)}{15\left(2+\tan(x)+2\tan^2(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.114695, size = 58, normalized size = 0.87

$$\frac{(\sin(2x)+4)\sec^2(x)\left(15(\cos(2x)-15)+8\sqrt{15}(\sin(2x)+4)\tan^{-1}\left(\frac{4\tan(x)+1}{\sqrt{15}}\right)\right)}{900(\sin(x)+2\sec(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sec[x] + Sin[x])^(-2),x]
```

```
[Out] (Sec[x]^2*(4 + Sin[2*x])*(15*(-15 + Cos[2*x]) + 8*Sqrt[15]*ArcTan[(1 + 4*Ta
n[x])/Sqrt[15]]*(4 + Sin[2*x])))/(900*(2*Sec[x] + Sin[x])^2)
```

Maple [A] time = 0.063, size = 39, normalized size = 0.6

$$\frac{1+4\tan(x)}{30+15\tan(x)+30(\tan(x))^2}+\frac{8\sqrt{15}}{225}\arctan\left(\frac{(1+4\tan(x))\sqrt{15}}{15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*sec(x)+sin(x))^2,x)
```

```
[Out] 1/15*(1+4*tan(x))/(2+tan(x)+2*tan(x)^2)+8/225*15^(1/2)*arctan(1/15*(1+4*tan
(x))*15^(1/2))
```

Maxima [A] time = 1.42277, size = 51, normalized size = 0.76

$$\frac{8}{225}\sqrt{15}\arctan\left(\frac{1}{15}\sqrt{15}(4\tan(x)+1)\right)+\frac{4\tan(x)+1}{15\left(2\tan(x)^2+\tan(x)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sec(x)+sin(x))^2,x, algorithm="maxima")
```

```
[Out] 8/225*sqrt(15)*arctan(1/15*sqrt(15)*(4*tan(x) + 1)) + 1/15*(4*tan(x) + 1)/(
2*tan(x)^2 + tan(x) + 2)
```

Fricas [A] time = 2.22084, size = 212, normalized size = 3.16

$$\frac{4\left(\sqrt{15}\cos\left(x\right)\sin\left(x\right)+2\sqrt{15}\right)\arctan\left(\frac{8\sqrt{15}\cos(x)\sin(x)+\sqrt{15}}{15\left(2\cos(x)^{2}-1\right)}\right)+15\cos\left(x\right)^{2}-120}{225\left(\cos\left(x\right)\sin\left(x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sec(x)+sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/225*(4*(sqrt(15)*cos(x)*sin(x) + 2*sqrt(15))*arctan(1/15*(8*sqrt(15)*cos(
x)*sin(x) + sqrt(15))/(2*cos(x)^2 - 1)) + 15*cos(x)^2 - 120)/(cos(x)*sin(x)
 + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(\sin\left(x\right)+2\sec\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sec(x)+sin(x))**2,x)
```

```
[Out] Integral((sin(x) + 2*sec(x))**(-2), x)
```

Giac [A] time = 1.10296, size = 105, normalized size = 1.57

$$\frac{8}{225}\sqrt{15}\left(x+\arctan\left(-\frac{\sqrt{15}\sin\left(2x\right)-\cos\left(2x\right)-4\sin\left(2x\right)-1}{\sqrt{15}\cos\left(2x\right)+\sqrt{15}-4\cos\left(2x\right)+\sin\left(2x\right)+4}\right)\right)+\frac{4\tan\left(x\right)+1}{15\left(2\tan\left(x\right)^{2}+\tan\left(x\right)+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sec(x)+sin(x))^2,x, algorithm="giac")
```

```
[Out] 8/225*sqrt(15)*(x + arctan(-(sqrt(15)*sin(2*x) - cos(2*x) - 4*sin(2*x) - 1)
/(sqrt(15)*cos(2*x) + sqrt(15) - 4*cos(2*x) + sin(2*x) + 4))) + 1/15*(4*tan
(x) + 1)/(2*tan(x)^2 + tan(x) + 2)
```

3.381 $\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx$

Optimal. Leaf size=55

$$\frac{x}{6\sqrt{6}}+\frac{\tan(x)}{6\left(2\tan^2(x)+3\right)}-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{6}+2}\right)}{6\sqrt{6}}$$

```
[Out] x/(6*Sqrt[6]) - ArcTan[(Cos[x]*Sin[x])/(2 + Sqrt[6] + Cos[x]^2)]/(6*Sqrt[6]
) + Tan[x]/(6*(3 + 2*Tan[x]^2))
```

Rubi [A] time = 0.0315169, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {199, 203}

$$\frac{x}{6\sqrt{6}}+\frac{\tan(x)}{6\left(2\tan^2(x)+3\right)}-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\cos^2(x)+\sqrt{6}+2}\right)}{6\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + 2*Sec[x])^(-2),x]
```

```
[Out] x/(6*Sqrt[6]) - ArcTan[(Cos[x]*Sin[x])/(2 + Sqrt[6] + Cos[x]^2)]/(6*Sqrt[6]
) + Tan[x]/(6*(3 + 2*Tan[x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(\cos(x)+2\sec(x))^2}\,dx &= \text{Subst}\left(\int \frac{1}{\left(3+2x^2\right)^2}\,dx,x,\tan(x)\right)\\ &= \frac{\tan(x)}{6\left(3+2\tan^2(x)\right)}+\frac{1}{6}\text{Subst}\left(\int \frac{1}{3+2x^2}\,dx,x,\tan(x)\right)\\ &= \frac{x}{6\sqrt{6}}-\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{2+\sqrt{6}+\cos^2(x)}\right)}{6\sqrt{6}}+\frac{\tan(x)}{6\left(3+2\tan^2(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.0841971, size = 54, normalized size = 0.98

$$\frac{(\cos(2x)+5)\sec^4(x)\left(6\sin(2x)+\sqrt{6}(\cos(2x)+5)\tan^{-1}\left(\sqrt{\frac{2}{3}}\tan(x)\right)\right)}{144\left(2\sec^2(x)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + 2*Sec[x])^(-2),x]
```

```
[Out] ((5 + Cos[2*x])*Sec[x]^4*(Sqrt[6]*ArcTan[Sqrt[2/3]*Tan[x]]*(5 + Cos[2*x]) +
 6*Sin[2*x]))/(144*(1 + 2*Sec[x]^2)^2)
```

Maple [A] time = 0.038, size = 29, normalized size = 0.5

$$\frac{\tan(x)}{18+12(\tan(x))^2}+\frac{\sqrt{6}}{36}\arctan\left(\frac{\tan(x)\sqrt{6}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)+2*sec(x))^2,x)
```

```
[Out] 1/6*tan(x)/(3+2*tan(x)^2)+1/36*6^(1/2)*arctan(1/3*tan(x)*6^(1/2))
```

Maxima [A] time = 1.41602, size = 38, normalized size = 0.69

$$\frac{1}{36}\sqrt{6}\arctan\left(\frac{1}{3}\sqrt{6}\tan(x)\right)+\frac{\tan(x)}{6\left(2\tan(x)^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+2*sec(x))^2,x, algorithm="maxima")
```

```
[Out] 1/36*sqrt(6)*arctan(1/3*sqrt(6)*tan(x)) + 1/6*tan(x)/(2*tan(x)^2 + 3)
```

Fricas [A] time = 2.12823, size = 184, normalized size = 3.35

$$-\frac{\left(\sqrt{6}\cos(x)^2+2\sqrt{6}\right)\arctan\left(\frac{5\sqrt{6}\cos(x)^2-2\sqrt{6}}{12\cos(x)\sin(x)}\right)-12\cos(x)\sin(x)}{72\left(\cos(x)^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+2*sec(x))^2,x, algorithm="fricas")
```

```
[Out] -1/72*((sqrt(6)*cos(x)^2 + 2*sqrt(6))*arctan(1/12*(5*sqrt(6)*cos(x)^2 - 2*s
qrt(6))/(cos(x)*sin(x))) - 12*cos(x)*sin(x))/(cos(x)^2 + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\cos(x) + 2\sec(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+2*sec(x))**2,x)
```

```
[Out] Integral((cos(x) + 2*sec(x))**(-2), x)
```

Giac [A] time = 1.08689, size = 82, normalized size = 1.49

$$\frac{1}{36}\,\sqrt{6}\left(x + \arctan\left(-\frac{\sqrt{6}\sin(2\,x) - 2\,\sin(2\,x)}{\sqrt{6}\cos(2\,x) + \sqrt{6} - 2\,\cos(2\,x) + 2}\right)\right) + \frac{\tan(x)}{6\left(2\,\tan(x)^2 + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+2*sec(x))^2,x, algorithm="giac")
```

```
[Out] 1/36*sqrt(6)*(x + arctan(-(sqrt(6)*sin(2*x) - 2*sin(2*x))/(sqrt(6)*cos(2*x)
 + sqrt(6) - 2*cos(2*x) + 2))) + 1/6*tan(x)/(2*tan(x)^2 + 3)
```

3.382 $\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx$

Optimal. Leaf size=42

$$-\frac{67x}{250}-\frac{29}{50(3\tan(x)+1)}-\frac{7}{10(3\tan(x)+1)^2}-\frac{28}{125}\log(3\sin(x)+\cos(x))$$

```
[Out] (-67*x)/250 - (28*Log[Cos[x] + 3*Sin[x]])/125 - 7/(10*(1 + 3*Tan[x])^2) - 2
9/(50*(1 + 3*Tan[x]))
```

Rubi [A] time = 0.0958575, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3628, 3529, 3531, 3530}

$$-\frac{67x}{250}-\frac{29}{50(3\tan(x)+1)}-\frac{7}{10(3\tan(x)+1)^2}-\frac{28}{125}\log(3\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(5 - Tan[x] - 6*Tan[x]^2)/(1 + 3*Tan[x])^3,x]
```

```
[Out] (-67*x)/250 - (28*Log[Cos[x] + 3*Sin[x]])/125 - 7/(10*(1 + 3*Tan[x])^2) - 2
9/(50*(1 + 3*Tan[x]))
```

Rule 3628

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*tan[(e_.) +
 (f_.)*(x_)] + (C_.)*tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b^2
- a*b*B + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x
] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[b*B + a*(A -
C) - (A*b - a*B - b*C)*Tan[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
 C}, x] && NeQ[A*b^2 - a*b*B + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{5-\tan(x)-6\tan^2(x)}{(1+3\tan(x))^3}\,dx &= -\frac{7}{10(1+3\tan(x))^2}+\frac{1}{10}\int \frac{8-34\tan(x)}{(1+3\tan(x))^2}\,dx\\ &= -\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}+\frac{1}{100}\int \frac{-94-58\tan(x)}{1+3\tan(x)}\,dx\\ &= -\frac{67x}{250}-\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}-\frac{28}{125}\int \frac{3-\tan(x)}{1+3\tan(x)}\,dx\\ &= -\frac{67x}{250}-\frac{28}{125}\log(\cos(x)+3\sin(x))-\frac{7}{10(1+3\tan(x))^2}-\frac{29}{50(1+3\tan(x))}\end{aligned}$$

Mathematica [A] time = 0.21688, size = 70, normalized size = 1.67

$$-\frac{670x+560\log(3\sin(x)+\cos(x))-4\cos(2x)(134x+112\log(3\sin(x)+\cos(x))-405)+6\sin(2x)(67x+56\log(3\sin(}{500(3\sin(x)+\cos(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - Tan[x] - 6*Tan[x]^2)/(1 + 3*Tan[x])^3,x]
```

```
[Out] -(-1305 + 670*x + 560*Log[Cos[x] + 3*Sin[x]] - 4*Cos[2*x]*(-405 + 134*x + 1
12*Log[Cos[x] + 3*Sin[x]]) + 6*(-90 + 67*x + 56*Log[Cos[x] + 3*Sin[x]])*Sin
[2*x])/(500*(Cos[x] + 3*Sin[x])^2)
```

Maple [A] time = 0.025, size = 43, normalized size = 1.

$$\frac{14\ln\left((\tan(x))^2+1\right)}{125}-\frac{7}{10\,(1+3\,\tan(x))^2}-\frac{29}{50+150\,\tan(x)}-\frac{28\ln(1+3\,\tan(x))}{125}-\frac{67\,x}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x)
```

```
[Out] 14/125*ln(tan(x)^2+1)-7/10/(1+3*tan(x))^2-29/50/(1+3*tan(x))-28/125*ln(1+3*
tan(x))-67/250*x
```

Maxima [A] time = 1.41712, size = 59, normalized size = 1.4

$$-\frac{67}{250}\,x-\frac{87\,\tan(x)+64}{50\left(9\,\tan(x)^2+6\,\tan(x)+1\right)}+\frac{14}{125}\,\log\left(\tan(x)^2+1\right)-\frac{28}{125}\,\log(3\,\tan(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x, algorithm="maxima")
```

```
[Out] -67/250*x - 1/50*(87*tan(x) + 64)/(9*tan(x)^2 + 6*tan(x) + 1) + 14/125*log(
tan(x)^2 + 1) - 28/125*log(3*tan(x) + 1)
```

Fricas [B] time = 2.24171, size = 243, normalized size = 5.79

$$-\frac{9\,(134\,x-1)\tan\left(x\right)^{2}+56\left(9\,\tan\left(x\right)^{2}+6\,\tan\left(x\right)+1\right)\log\left(\frac{9\,\tan\left(x\right)^{2}+6\,\tan\left(x\right)+1}{\tan\left(x\right)^{2}+1}\right)+12\,(67\,x+72)\tan\left(x\right)+134\,x+63}{500\left(9\,\tan\left(x\right)^{2}+6\,\tan\left(x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x, algorithm="fricas")
```

```
[Out] -1/500*(9*(134*x - 1)*tan(x)^2 + 56*(9*tan(x)^2 + 6*tan(x) + 1)*log((9*tan(
x)^2 + 6*tan(x) + 1)/(tan(x)^2 + 1)) + 12*(67*x + 72)*tan(x) + 134*x + 639)
/(9*tan(x)^2 + 6*tan(x) + 1)
```

Sympy [B] time = 0.539959, size = 253, normalized size = 6.02

$$-\frac{1206x\tan^{2}\left(x\right)}{4500\tan^{2}\left(x\right)+3000\tan\left(x\right)+500}-\frac{804x\tan\left(x\right)}{4500\tan^{2}\left(x\right)+3000\tan\left(x\right)+500}-\frac{134x}{4500\tan^{2}\left(x\right)+3000\tan\left(x\right)+500}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5-tan(x)-6*tan(x)**2)/(1+3*tan(x))**3,x)
```

```
[Out] -1206*x*tan(x)**2/(4500*tan(x)**2 + 3000*tan(x) + 500) - 804*x*tan(x)/(4500
*tan(x)**2 + 3000*tan(x) + 500) - 134*x/(4500*tan(x)**2 + 3000*tan(x) + 500
) - 1008*log(tan(x) + 1/3)*tan(x)**2/(4500*tan(x)**2 + 3000*tan(x) + 500) -
 672*log(tan(x) + 1/3)*tan(x)/(4500*tan(x)**2 + 3000*tan(x) + 500) - 112*lo
g(tan(x) + 1/3)/(4500*tan(x)**2 + 3000*tan(x) + 500) + 504*log(tan(x)**2 +
1)*tan(x)**2/(4500*tan(x)**2 + 3000*tan(x) + 500) + 336*log(tan(x)**2 + 1)*
tan(x)/(4500*tan(x)**2 + 3000*tan(x) + 500) + 56*log(tan(x)**2 + 1)/(4500*t
an(x)**2 + 3000*tan(x) + 500) + 1305*tan(x)**2/(4500*tan(x)**2 + 3000*tan(x
) + 500) - 495/(4500*tan(x)**2 + 3000*tan(x) + 500)
```

Giac [A] time = 1.12964, size = 53, normalized size = 1.26

$$-\frac{67}{250}\,x-\frac{87\,\tan\left(x\right)+64}{50\,(3\,\tan\left(x\right)+1)^{2}}+\frac{14}{125}\,\log\left(\tan\left(x\right)^{2}+1\right)-\frac{28}{125}\,\log\left(|3\,\tan\left(x\right)+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5-tan(x)-6*tan(x)^2)/(1+3*tan(x))^3,x, algorithm="giac")
```

```
[Out] -67/250*x - 1/50*(87*tan(x) + 64)/(3*tan(x) + 1)^2 + 14/125*log(tan(x)^2 +
1) - 28/125*log(abs(3*tan(x) + 1))
```

3.383 $\int \cos^2(x)\sec(3x)\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\tanh^{-1}(2\sin(x))$$

[Out] ArcTanh[2*Sin[x]]/2

Rubi [A] time = 0.0224052, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {206}

$$\frac{1}{2}\tanh^{-1}(2\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2*Sec[3*x],x]

[Out] ArcTanh[2*Sin[x]]/2

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\sec(3x)\,dx &= \text{Subst}\left(\int \frac{1}{1-4x^2}\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}\tanh^{-1}(2\sin(x))\end{aligned}$$

Mathematica [A] time = 0.0066682, size = 9, normalized size = 1.

$$\frac{1}{2}\tanh^{-1}(2\sin(x))$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^2*Sec[3*x],x]

[Out] ArcTanh[2*Sin[x]]/2

Maple [B] time = 0.04, size = 20, normalized size = 2.2

$$-\frac{\ln(2\sin(x)-1)}{4}+\frac{\ln(1+2\sin(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/cos(3*x),x)
```

```
[Out] -1/4*ln(2*sin(x)-1)+1/4*ln(1+2*sin(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)^2}{\cos(3x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/cos(3*x),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)^2/cos(3*x), x)
```

Fricas [B] time = 2.19179, size = 65, normalized size = 7.22

$$\frac{1}{4}\log(2\sin(x)+1) - \frac{1}{4}\log(-2\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/cos(3*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(2*sin(x) + 1) - 1/4*log(-2*sin(x) + 1)
```

Sympy [B] time = 6.42523, size = 76, normalized size = 8.44

$$-\frac{\log(\sin(3x)-1)}{12} + \frac{\log(\sin(3x)+1)}{12} - \frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{6} + \frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{6} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right)-4\tan\left(\frac{x}{2}\right)+1\right)}{12} + \frac{\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/cos(3*x),x)
```

```
[Out] -log(sin(3*x) - 1)/12 + log(sin(3*x) + 1)/12 - log(tan(x/2) - 1)/6 + log(ta
n(x/2) + 1)/6 - log(tan(x/2)**2 - 4*tan(x/2) + 1)/12 + log(tan(x/2)**2 + 4*
tan(x/2) + 1)/12
```

Giac [B] time = 1.10587, size = 28, normalized size = 3.11

$$\frac{1}{4}\log(|2\sin(x)+1|) - \frac{1}{4}\log(|2\sin(x)-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/cos(3*x),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(2*sin(x) + 1)) - 1/4*log(abs(2*sin(x) - 1))
```

3.384 $\int \sec(2x)\sin(x)\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2]
```

Rubi [A] time = 0.0144944, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4357, 207}

$$\frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*x]*Sin[x],x]
```

```
[Out] ArcTanh[Sqrt[2]*Cos[x]]/Sqrt[2]
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sec(2x)\sin(x)\,dx &= -\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \cos(x)\right)\\
&= \frac{\tanh^{-1}\left(\sqrt{2}\cos(x)\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.371911, size = 174, normalized size = 11.6

$$\frac{4\tanh^{-1}\left(\tan\left(\frac{x}{2}\right)+\sqrt{2}\right)-\log\left(-\sqrt{2}\sin(x)-\sqrt{2}\cos(x)+2\right)+\log\left(-\sqrt{2}\sin(x)+\sqrt{2}\cos(x)+2\right)+2i\tan^{-1}\left(\frac{\cos\left(\frac{x}{2}\right)-\left(\sqrt{2}-\right.}{\left(1+\sqrt{2}\right)\cos\left(\frac{}{}\right.}\right.}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*x]*Sin[x],x]
```

```
[Out] ((2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])*Cos[x/2]
- Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*Sin[x/2])/((-1 + Sqrt
[2])*Cos[x/2] - Sin[x/2])] + 4*ArcTanh[Sqrt[2] + Tan[x/2]] - Log[2 - Sqrt[2
]*Cos[x] - Sqrt[2]*Sin[x]] + Log[2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])/(4*S
qrt[2])
```

Maple [A] time = 0.02, size = 13, normalized size = 0.9

$$\frac{\operatorname{Artanh}\left(\cos\left(x\right)\sqrt{2}\right)\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/cos(2*x),x)
```

```
[Out] 1/2*arctanh(cos(x)*2^(1/2))*2^(1/2)
```

Maxima [B] time = 1.44863, size = 174, normalized size = 11.6

$$\frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}\sin\left(2x\right)\sin\left(x\right)+2\left(\sqrt{2}\cos\left(x\right)+1\right)\cos\left(2x\right)+\cos\left(2x\right)^{2}+2\cos\left(x\right)^{2}+\sin\left(2x\right)^{2}+2\sin\left(x\right)^{2}+2\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/cos(2*x),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(2*sqrt(2)*sin(2*x)*sin(x) + 2*(sqrt(2)*cos(x) + 1)*cos(2*x)
 + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 1
) - 1/8*sqrt(2)*log(-2*sqrt(2)*sin(2*x)*sin(x) - 2*(sqrt(2)*cos(x) - 1)*cos
(2*x) + cos(2*x)^2 + 2*cos(x)^2 + sin(2*x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x
) + 1)
```

Fricas [B] time = 2.16725, size = 97, normalized size = 6.47

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{2\cos\left(x\right)^{2}+2\sqrt{2}\cos\left(x\right)+1}{2\cos\left(x\right)^{2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/cos(2*x),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(2*cos(x)^2 + 2*sqrt(2)*cos(x) + 1)/(2*cos(x)^2 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(x\right)}{\cos\left(2x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/cos(2*x),x)
```

```
[Out] Integral(sin(x)/cos(2*x), x)
```

Giac [B] time = 1.14186, size = 66, normalized size = 4.4

$$\frac{1}{4}\,\sqrt{2}\log\left(\frac{\left|-4\,\sqrt{2}-\frac{2\,(\cos(x)-1)}{\cos(x)+1}-6\right|}{\left|4\,\sqrt{2}-\frac{2\,(\cos(x)-1)}{\cos(x)+1}-6\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/cos(2*x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6)/abs(4*sqr
t(2) - 2*(cos(x) - 1)/(cos(x) + 1) - 6))
```

3.385 $\int \sec(2x)\sin^2(x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\tanh^{-1}(2\sin(x)\cos(x))-\frac{x}{2}$$

```
[Out] -x/2 + ArcTanh[2*Cos[x]*Sin[x]]/4
```

Rubi [A] time = 0.0417367, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {298, 203, 206}

$$\frac{1}{4}\tanh^{-1}(2\sin(x)\cos(x))-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*x]*Sin[x]^2,x]
```

```
[Out] -x/2 + ArcTanh[2*Cos[x]*Sin[x]]/4
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sec(2x)\sin^2(x)\,dx &= \text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\tan(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\tan(x)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)\\ &= -\frac{x}{2}+\frac{1}{4}\tanh^{-1}(2\cos(x)\sin(x))\end{aligned}$$

Mathematica [A] time = 0.0145743, size = 28, normalized size = 1.65

$$-\frac{x}{2}-\frac{1}{4}\log(\cos(x)-\sin(x))+\frac{1}{4}\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*x]*Sin[x]^2,x]
```

```
[Out] -x/2 - Log[Cos[x] - Sin[x]]/4 + Log[Cos[x] + Sin[x]]/4
```

Maple [A] time = 0.037, size = 19, normalized size = 1.1

$$\frac{\ln(1+\tan(x))}{4}-\frac{\ln(-1+\tan(x))}{4}-\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/cos(2*x),x)
```

```
[Out] 1/4*ln(1+tan(x))-1/4*ln(-1+tan(x))-1/2*x
```

Maxima [B] time = 1.53131, size = 173, normalized size = 10.18

$$-\frac{1}{2}x-\frac{1}{8}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)+\frac{1}{8}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/cos(2*x),x, algorithm="maxima")
```

```
[Out] -1/2*x - 1/8*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) + 1/8*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*sqrt(2)*s
in(x) + 2) + 1/8*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) + 2*sqrt(2)
*sin(x) + 2) - 1/8*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*cos(x) - 2*sqrt(
2)*sin(x) + 2)
```

Fricas [A] time = 2.20918, size = 96, normalized size = 5.65

$$-\frac{1}{2}x+\frac{1}{8}\log(2\cos(x)\sin(x)+1)-\frac{1}{8}\log(-2\cos(x)\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/cos(2*x),x, algorithm="fricas")
```

```
[Out] -1/2*x + 1/8*log(2*cos(x)*sin(x) + 1) - 1/8*log(-2*cos(x)*sin(x) + 1)
```

Sympy [A] time = 1.23341, size = 22, normalized size = 1.29

$$-\frac{x}{2}-\frac{\log(\sin(2x)-1)}{8}+\frac{\log(\sin(2x)+1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/cos(2*x),x)
```

```
[Out] -x/2 - log(sin(2*x) - 1)/8 + log(sin(2*x) + 1)/8
```

Giac [A] time = 1.11681, size = 27, normalized size = 1.59

$$-\frac{1}{2}x + \frac{1}{4}\log\left(|\tan\left(x\right) + 1|\right) - \frac{1}{4}\log\left(|\tan\left(x\right) - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/cos(2*x),x, algorithm="giac")
```

```
[Out] -1/2*x + 1/4*log(abs(tan(x) + 1)) - 1/4*log(abs(tan(x) - 1))
```

3.386 $\int \sec(3x)\sin^3(x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{3}\log(\cos(x))-\frac{1}{24}\log\left(3-4\cos^2(x)\right)$$

```
[Out] Log[Cos[x]]/3 - Log[3 - 4*Cos[x]^2]/24
```

Rubi [A] time = 0.041225, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4366, 446, 72}

$$\frac{1}{3}\log(\cos(x))-\frac{1}{24}\log\left(3-4\cos^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[3*x]*Sin[x]^3,x]
```

```
[Out] Log[Cos[x]]/3 - Log[3 - 4*Cos[x]^2]/24
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sec(3x)\sin^3(x)\,dx &= -\text{Subst}\left(\int \frac{-1+x^2}{x\left(3-4x^2\right)}\,dx,x,\cos(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{-1+x}{(3-4x)x}\,dx,x,\cos^2(x)\right)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{3x}+\frac{1}{3(-3+4x)}\right)dx,x,\cos^2(x)\right)\right)\\
&= \frac{1}{3}\log(\cos(x))-\frac{1}{24}\log\left(3-4\cos^2(x)\right)
\end{aligned}$$

Mathematica [A] time = 0.0153123, size = 21, normalized size = 1.

$$\frac{1}{3}\log(\cos(x)) - \frac{1}{24}\log\left(1 - 4\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[3*x]*Sin[x]^3,x]
```

```
[Out] Log[Cos[x]]/3 - Log[1 - 4*Sin[x]^2]/24
```

Maple [A] time = 0.053, size = 18, normalized size = 0.9

$$\frac{\ln(\cos(x))}{3} - \frac{\ln\left(4\,(\cos(x))^2 - 3\right)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/cos(3*x),x)
```

```
[Out] 1/3*ln(cos(x))-1/24*ln(4*cos(x)^2-3)
```

Maxima [B] time = 1.4324, size = 109, normalized size = 5.19

$$-\frac{1}{48}\log\left(-2\left(\cos(2x) - 1\right)\cos(4x) + \cos(4x)^2 + \cos(2x)^2 + \sin(4x)^2 - 2\sin(4x)\sin(2x) + \sin(2x)^2 - 2\cos(2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/cos(3*x),x, algorithm="maxima")
```

```
[Out] -1/48*log(-2*(cos(2*x) - 1)*cos(4*x) + cos(4*x)^2 + cos(2*x)^2 + sin(4*x)^2
 - 2*sin(4*x)*sin(2*x) + sin(2*x)^2 - 2*cos(2*x) + 1) + 1/6*log(cos(2*x)^2
+ sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A] time = 2.32023, size = 62, normalized size = 2.95

$$-\frac{1}{24}\log\left(4\cos(x)^2 - 3\right) + \frac{1}{3}\log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/cos(3*x),x, algorithm="fricas")
```

```
[Out] -1/24*log(4*cos(x)^2 - 3) + 1/3*log(-cos(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^3(x)}{\cos(3x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/cos(3*x),x)
```

```
[Out] Integral(sin(x)**3/cos(3*x), x)
```

Giac [B] time = 1.1141, size = 120, normalized size = 5.71

$$-\frac{1}{8}\log\left(-\frac{\cos(x)+1}{\cos(x)-1}-\frac{\cos(x)-1}{\cos(x)+1}+2\right)+\frac{1}{6}\log\left(-\frac{\cos(x)+1}{\cos(x)-1}-\frac{\cos(x)-1}{\cos(x)+1}-2\right)-\frac{1}{24}\log\left(\left|-\frac{\cos(x)+1}{\cos(x)-1}-\frac{\cos(x)-1}{\cos(x)+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/cos(3*x),x, algorithm="giac")
```

```
[Out] -1/8*log(-(cos(x) + 1)/(cos(x) - 1) - (cos(x) - 1)/(cos(x) + 1) + 2) + 1/6*
log(-(cos(x) + 1)/(cos(x) - 1) - (cos(x) - 1)/(cos(x) + 1) - 2) - 1/24*log(
abs(-(cos(x) + 1)/(cos(x) - 1) - (cos(x) - 1)/(cos(x) + 1) - 14))
```

3.387 $\int \cos(x)\csc(3x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3 - 4\sin^2(x)\right)$$

```
[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

Rubi [A] time = 0.0233493, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4356, 266, 36, 31, 29}

$$\frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3 - 4\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Csc[3*x],x]

[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\csc(3x)\,dx &= \text{Subst}\left(\int \frac{1}{x\left(3-4x^2\right)}\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(3-4x)x}\,dx, x, \sin^2(x)\right)\\ &= \frac{1}{6}\text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin^2(x)\right) + \frac{2}{3}\text{Subst}\left(\int \frac{1}{3-4x}\,dx, x, \sin^2(x)\right)\\ &= \frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3-4\sin^2(x)\right)\end{aligned}$$

Mathematica [A] time = 0.0074523, size = 21, normalized size = 1.

$$\frac{1}{3}\log(\sin(x)) - \frac{1}{6}\log\left(3-4\sin^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Csc[3*x],x]
```

```
[Out] Log[Sin[x]]/3 - Log[3 - 4*Sin[x]^2]/6
```

Maple [A] time = 0.051, size = 34, normalized size = 1.6

$$\frac{\ln(\cos(x)+1)}{6} - \frac{\ln(2\cos(x)-1)}{6} + \frac{\ln(\cos(x)-1)}{6} - \frac{\ln(2\cos(x)+1)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(3*x),x)
```

```
[Out] 1/6*ln(cos(x)+1)-1/6*ln(2*cos(x)-1)+1/6*ln(cos(x)-1)-1/6*ln(2*cos(x)+1)
```

Maxima [B] time = 1.41161, size = 174, normalized size = 8.29

$$-\frac{1}{12}\log\left(2(\cos(x)+1)\cos(2x) + \cos(2x)^2 + \cos(x)^2 + \sin(2x)^2 + 2\sin(2x)\sin(x) + \sin(x)^2 + 2\cos(x) + 1\right) - \frac{1}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/12*log(2*(cos(x) + 1)*cos(2*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 + 2*
sin(2*x)*sin(x) + sin(x)^2 + 2*cos(x) + 1) - 1/12*log(-2*(cos(x) - 1)*cos(2
*x) + cos(2*x)^2 + cos(x)^2 + sin(2*x)^2 - 2*sin(2*x)*sin(x) + sin(x)^2 - 2
*cos(x) + 1) + 1/6*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/6*log(cos(x)
^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [A] time = 2.22208, size = 65, normalized size = 3.1

$$-\frac{1}{6}\log\left(4\cos(x)^2-1\right) + \frac{1}{3}\log\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(3*x),x, algorithm="fricas")
```

```
[Out] -1/6*log(4*cos(x)^2 - 1) + 1/3*log(1/2*sin(x))
```

Sympy [A] time = 1.1751, size = 17, normalized size = 0.81

$$-\frac{\log\left(4\sin^2(x)-3\right)}{6}+\frac{\log(\sin(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(3*x),x)
```

```
[Out] -log(4*sin(x)**2 - 3)/6 + log(sin(x))/3
```

Giac [A] time = 1.08619, size = 41, normalized size = 1.95

$$-\frac{1}{6}\log\left(\left|-\frac{3(\cos(x)+1)}{\cos(x)-1}-\frac{3(\cos(x)-1)}{\cos(x)+1}-10\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(3*x),x, algorithm="giac")
```

```
[Out] -1/6*log(abs(-3*(cos(x) + 1)/(cos(x) - 1) - 3*(cos(x) - 1)/(cos(x) + 1) - 1
0))
```

3.388 $\int \csc(4x)\sin(x)\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

```
[Out] -ArcTanh[Sin[x]]/4 + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

Rubi [A] time = 0.0244572, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1093, 207}

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[4*x]*Sin[x],x]
```

```
[Out] -ArcTanh[Sin[x]]/4 + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \csc(4x)\sin(x)\,dx &= \text{Subst}\left(\int \frac{1}{4-12x^2+8x^4}\,dx, x, \sin(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx, x, \sin(x)\right)-2\,\text{Subst}\left(\int \frac{1}{-4+8x^2}\,dx, x, \sin(x)\right)\\ &= -\frac{1}{4}\tanh^{-1}(\sin(x))+\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.022535, size = 26, normalized size = 1.

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[4*x]*Sin[x],x]
```

```
[Out] -ArcTanh[Sin[x]]/4 + ArcTanh[Sqrt[2]*Sin[x]]/(2*Sqrt[2])
```

Maple [A] time = 0.056, size = 28, normalized size = 1.1

$$\frac{\operatorname{Artanh}\left(\sin\left(x\right)\sqrt{2}\right)\sqrt{2}}{4}-\frac{\ln\left(1+\sin\left(x\right)\right)}{8}+\frac{\ln\left(-1+\sin\left(x\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(4*x),x)
```

```
[Out] 1/4*arctanh(sin(x)*2^(1/2))*2^(1/2)-1/8*ln(1+sin(x))+1/8*ln(-1+sin(x))
```

Maxima [B] time = 1.48325, size = 231, normalized size = 8.88

$$\frac{1}{16}\sqrt{2}\log\left(2\cos\left(x\right)^{2}+2\sin\left(x\right)^{2}+2\sqrt{2}\cos\left(x\right)+2\sqrt{2}\sin\left(x\right)+2\right)-\frac{1}{16}\sqrt{2}\log\left(2\cos\left(x\right)^{2}+2\sin\left(x\right)^{2}+2\sqrt{2}\cos\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(4*x),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*
sqrt(2)*sin(x) + 2) + 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/16*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 -
 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^2 + 2
*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [B] time = 2.0654, size = 158, normalized size = 6.08

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{2\cos\left(x\right)^{2}-2\sqrt{2}\sin\left(x\right)-3}{2\cos\left(x\right)^{2}-1}\right)-\frac{1}{8}\log\left(\sin\left(x\right)+1\right)+\frac{1}{8}\log\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(4*x),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1/
8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(4*x),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.11601, size = 65, normalized size = 2.5

$$-\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin(x)\right|}{\left|2\sqrt{2}+4\sin(x)\right|}\right)-\frac{1}{8}\log(\sin(x)+1)+\frac{1}{8}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(4*x),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1/
8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

3.389 $\int \csc(4x)\sin^3(x)\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{4\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

```
[Out] -ArcTanh[Sin[x]]/4 + ArcTanh[Sqrt[2]*Sin[x]]/(4*Sqrt[2])
```

Rubi [A] time = 0.0346002, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1130, 207}

$$\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{4\sqrt{2}}-\frac{1}{4}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[4*x]*Sin[x]^3,x]
```

```
[Out] -ArcTanh[Sin[x]]/4 + ArcTanh[Sqrt[2]*Sin[x]]/(4*Sqrt[2])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \csc(4x)\sin^3(x)\,dx &= \text{Subst}\left(\int \frac{x^2}{4-12x^2+8x^4}\,dx,x,\sin(x)\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{-8+8x^2}\,dx,x,\sin(x)\right)-\text{Subst}\left(\int \frac{1}{-4+8x^2}\,dx,x,\sin(x)\right)\\ &= -\frac{1}{4}\tanh^{-1}(\sin(x))+\frac{\tanh^{-1}\left(\sqrt{2}\sin(x)\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.315892, size = 218, normalized size = 8.38

$$\frac{2\log\left(2\sin(x)+\sqrt{2}\right)+4\sqrt{2}\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-4\sqrt{2}\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(-\sqrt{2}\sin(x)-\sqrt{2}\cos(x)+2\right.}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[4*x]*Sin[x]^3,x]
```

```
[Out] ((-2*I)*ArcTan[(Cos[x/2] - (-1 + Sqrt[2])*Sin[x/2])/((1 + Sqrt[2])*Cos[x/2]
 - Sin[x/2])] - (2*I)*ArcTan[(Cos[x/2] - (1 + Sqrt[2])*Sin[x/2])/((-1 + Sqr
t[2])*Cos[x/2] - Sin[x/2])] + 4*Sqrt[2]*Log[Cos[x/2] - Sin[x/2]] - 4*Sqrt[2
]*Log[Cos[x/2] + Sin[x/2]] + 2*Log[Sqrt[2] + 2*Sin[x]] - Log[2 - Sqrt[2]*Co
s[x] - Sqrt[2]*Sin[x]] - Log[2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])/(16*Sqrt
[2])
```

Maple [A] time = 0.066, size = 28, normalized size = 1.1

$$\frac{\mathrm{Artanh}\left(\sin(x)\sqrt{2}\right)\sqrt{2}}{8}-\frac{\ln(1+\sin(x))}{8}+\frac{\ln(-1+\sin(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/sin(4*x),x)
```

```
[Out] 1/8*arctanh(sin(x)*2^(1/2))*2^(1/2)-1/8*ln(1+sin(x))+1/8*ln(-1+sin(x))
```

Maxima [B] time = 1.47293, size = 231, normalized size = 8.88

$$\frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)+2\sqrt{2}\sin(x)+2\right)-\frac{1}{32}\sqrt{2}\log\left(2\cos(x)^2+2\sin(x)^2+2\sqrt{2}\cos(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/sin(4*x),x, algorithm="maxima")
```

```
[Out] 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) + 2*sqrt(2)*sin
(x) + 2) - 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 + 2*sqrt(2)*cos(x) - 2*
sqrt(2)*sin(x) + 2) + 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 - 2*sqrt(2)*
cos(x) + 2*sqrt(2)*sin(x) + 2) - 1/32*sqrt(2)*log(2*cos(x)^2 + 2*sin(x)^2 -
 2*sqrt(2)*cos(x) - 2*sqrt(2)*sin(x) + 2) - 1/8*log(cos(x)^2 + sin(x)^2 + 2
*sin(x) + 1) + 1/8*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [B] time = 2.03274, size = 159, normalized size = 6.12

$$\frac{1}{16}\sqrt{2}\log\left(-\frac{2\cos(x)^2-2\sqrt{2}\sin(x)-3}{2\cos(x)^2-1}\right)-\frac{1}{8}\log(\sin(x)+1)+\frac{1}{8}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/sin(4*x),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2)*log(-(2*cos(x)^2 - 2*sqrt(2)*sin(x) - 3)/(2*cos(x)^2 - 1)) - 1
/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/sin(4*x),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.12643, size = 65, normalized size = 2.5

$$-\frac{1}{16}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(x\right)\right|}{\left|2\sqrt{2}+4\sin\left(x\right)\right|}\right)-\frac{1}{8}\log\left(\sin\left(x\right)+1\right)+\frac{1}{8}\log\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/sin(4*x),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))) - 1
/8*log(sin(x) + 1) + 1/8*log(-sin(x) + 1)
```

3.390 $\int \sqrt{1+\sin(2x)}\,dx$

Optimal. Leaf size=16

$$-\frac{\cos(2x)}{\sqrt{\sin(2x)+1}}$$

```
[Out] -(Cos[2*x]/Sqrt[1 + Sin[2*x]])
```

Rubi [A] time = 0.0086676, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2646}

$$-\frac{\cos(2x)}{\sqrt{\sin(2x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sin[2*x]],x]

[Out] -(Cos[2*x]/Sqrt[1 + Sin[2*x]])
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{1+\sin(2x)}\,dx = -\frac{\cos(2x)}{\sqrt{1+\sin(2x)}}$$

Mathematica [A] time = 0.0138993, size = 25, normalized size = 1.56

$$\frac{\sqrt{\sin(2x)+1}(\sin(x)-\cos(x))}{\sin(x)+\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sin[2*x]],x]

[Out] ((-Cos[x] + Sin[x])*Sqrt[1 + Sin[2*x]])/(Cos[x] + Sin[x])
```

Maple [A] time = 0.033, size = 22, normalized size = 1.4

$$\frac{-1+\sin(2x)}{\cos(2x)}\sqrt{1+\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(2*x))^(1/2),x)
```

```
[Out] (-1+sin(2*x))*(1+sin(2*x))^(1/2)/cos(2*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(2x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(2*x) + 1), x)
```

Fricas [B] time = 1.80544, size = 99, normalized size = 6.19

$$-\frac{(\cos(2x) - \sin(2x) + 1)\sqrt{\sin(2x) + 1}}{\cos(2x) + \sin(2x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] -(cos(2*x) - sin(2*x) + 1)*sqrt(sin(2*x) + 1)/(cos(2*x) + sin(2*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(2x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(2*x))**(1/2),x)
```

```
[Out] Integral(sqrt(sin(2*x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(2x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(2*x) + 1), x)
```

3.391 $\int \sqrt{1-\sin(2x)}\,dx$

Optimal. Leaf size=17

$$\frac{\cos(2x)}{\sqrt{1-\sin(2x)}}$$

```
[Out] Cos[2*x]/Sqrt[1 - Sin[2*x]]
```

Rubi [A] time = 0.0119692, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2646}

$$\frac{\cos(2x)}{\sqrt{1-\sin(2x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sin[2*x]],x]

[Out] Cos[2*x]/Sqrt[1 - Sin[2*x]]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{1-\sin(2x)}\,dx = \frac{\cos(2x)}{\sqrt{1-\sin(2x)}}$$

Mathematica [A] time = 0.0135429, size = 27, normalized size = 1.59

$$\frac{\sqrt{1-\sin(2x)}(\sin(x)+\cos(x))}{\cos(x)-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sin[2*x]],x]

[Out] ((Cos[x] + Sin[x])*Sqrt[1 - Sin[2*x]])/(Cos[x] - Sin[x])
```

Maple [A] time = 0.033, size = 31, normalized size = 1.8

$$-\frac{(-1+\sin(2x))(1+\sin(2x))}{\cos(2x)}\frac{1}{\sqrt{1-\sin(2x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(2*x))^(1/2),x)
```

```
[Out] -(-1+sin(2*x))*(1+sin(2*x))/cos(2*x)/(1-sin(2*x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\sin(2x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-sin(2*x) + 1), x)
```

Fricas [B] time = 1.81164, size = 99, normalized size = 5.82

$$\frac{(\cos(2x)+\sin(2x)+1)\sqrt{-\sin(2x)+1}}{\cos(2x)-\sin(2x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] (cos(2*x) + sin(2*x) + 1)*sqrt(-sin(2*x) + 1)/(cos(2*x) - sin(2*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{1-\sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2*x))**(1/2),x)
```

```
[Out] Integral(sqrt(1 - sin(2*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\sin(2x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-sin(2*x) + 1), x)
```

3.392 $\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx$

Optimal. Leaf size=27

$$\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{\cos(2x)+1}}\right)}{\sqrt{2}}$$

```
[Out] ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 + Cos[2*x]])]/Sqrt[2]
```

Rubi [A] time = 0.0125423, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2649, 206}

$$\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{\cos(2x)+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Cos[2*x]],x]
```

```
[Out] ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 + Cos[2*x]])]/Sqrt[2]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+\cos(2x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, -\frac{\sin(2x)}{\sqrt{1+\cos(2x)}}\right)\\ &= \frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1+\cos(2x)}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0143964, size = 47, normalized size = 1.74

$$-\frac{\cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)}{\sqrt{\cos(2x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Cos[2*x]],x]
```

```
[Out] -((Cos[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]))/Sqrt[1 + C
os[2*x]])
```

Maple [C] time = 0.015, size = 9, normalized size = 0.3

$$\frac{\sqrt{2}\,\mathrm{InverseJacobiAM}\left(x,1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(2*x))^(1/2),x)
```

```
[Out] 1/2*2^(1/2)*InverseJacobiAM(x,1)
```

Maxima [A] time = 1.58802, size = 55, normalized size = 2.04

$$\frac{1}{4}\sqrt{2}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\sin\left(x\right)+1\right)-\frac{1}{4}\sqrt{2}\log\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}-2\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*sqrt(2)*log(cos(x
)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [B] time = 1.85242, size = 161, normalized size = 5.96

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{\cos\left(2x\right)^{2}-2\sqrt{2}\sqrt{\cos\left(2x\right)+1}\sin\left(2x\right)-2\cos\left(2x\right)-3}{\cos\left(2x\right)^{2}+2\cos\left(2x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(cos(2*x)^2 - 2*sqrt(2)*sqrt(cos(2*x) + 1)*sin(2*x) - 2*co
s(2*x) - 3)/(cos(2*x)^2 + 2*cos(2*x) + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{\cos\left(2x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(cos(2*x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\cos(2x) + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(cos(2*x) + 1), x)
```

3.393 $\int \frac{1}{\sqrt{1-\cos(2x)}} \, dx$

Optimal. Leaf size=30

$$-\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 - Cos[2*x]])]/Sqrt[2])
```

Rubi [A] time = 0.0131854, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2649, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - Cos[2*x]],x]
```

```
[Out] -(ArcTanh[Sin[2*x]/(Sqrt[2]*Sqrt[1 - Cos[2*x]])]/Sqrt[2])
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-\cos(2x)}} \, dx &= -\text{Subst}\left(\int \frac{1}{2-x^2} \, dx, x, \frac{\sin(2x)}{\sqrt{1-\cos(2x)}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sin(2x)}{\sqrt{2}\sqrt{1-\cos(2x)}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0154276, size = 33, normalized size = 1.1

$$-\frac{\sin(x)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{\sqrt{1-\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - Cos[2*x]],x]
```

```
[Out] -(((Log[Cos[x/2]] - Log[Sin[x/2]])*Sin[x])/Sqrt[1 - Cos[2*x]])
```

Maple [A] time = 0.034, size = 17, normalized size = 0.6

$$-\frac{\sin(x)\operatorname{Artanh}(\cos(x))\sqrt{2}}{2}\frac{1}{\sqrt{(\sin(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(2*x))^(1/2),x)
```

```
[Out] -1/2*sin(x)*arctanh(cos(x))*2^(1/2)/(sin(x)^2)^(1/2)
```

Maxima [B] time = 1.61261, size = 136, normalized size = 4.53

$$-\frac{1}{4}\sqrt{2}\log\left(\cos\left(\frac{1}{2}\arctan\left(\sin\left(2x\right),\cos\left(2x\right)\right)\right)^2+\sin\left(\frac{1}{2}\arctan\left(\sin\left(2x\right),\cos\left(2x\right)\right)\right)^2+2\cos\left(\frac{1}{2}\arctan\left(\sin\left(2x\right),c\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(2)*log(cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(1/2*arctan2(s
in(2*x), cos(2*x)))^2 + 2*cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1) + 1/4*s
qrt(2)*log(cos(1/2*arctan2(sin(2*x), cos(2*x)))^2 + sin(1/2*arctan2(sin(2*x
), cos(2*x)))^2 - 2*cos(1/2*arctan2(sin(2*x), cos(2*x))) + 1)
```

Fricas [B] time = 1.81252, size = 167, normalized size = 5.57

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{(\cos(2x)+3)\sin(2x)-2\left(\sqrt{2}\cos(2x)+\sqrt{2}\right)\sqrt{-\cos(2x)+1}}{(\cos(2x)-1)\sin(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-((cos(2*x) + 3)*sin(2*x) - 2*(sqrt(2)*cos(2*x) + sqrt(2))*
sqrt(-cos(2*x) + 1))/((cos(2*x) - 1)*sin(2*x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{1-\cos(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(2*x))**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - cos(2*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\cos(2x) + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-cos(2*x) + 1), x)
```

3.394 $\int \frac{1}{(1-\cos(3x))^{3/2}} dx$

Optimal. Leaf size=53

$$-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sin(3x)}{\sqrt{2}\sqrt{1-\cos(3x)}}\right)}{6\sqrt{2}}$$

```
[Out] -ArcTanh[Sin[3*x]/(Sqrt[2]*Sqrt[1 - Cos[3*x]])]/(6*Sqrt[2]) - Sin[3*x]/(6*(
1 - Cos[3*x])^(3/2))
```

Rubi [A] time = 0.0282741, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2650, 2649, 206}

$$-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sin(3x)}{\sqrt{2}\sqrt{1-\cos(3x)}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[3*x])^(-3/2),x]
```

```
[Out] -ArcTanh[Sin[3*x]/(Sqrt[2]*Sqrt[1 - Cos[3*x]])]/(6*Sqrt[2]) - Sin[3*x]/(6*(
1 - Cos[3*x])^(3/2))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2649

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[-2/d, S
ubst[Int[1/(2*a - x^2), x], x, (b*Cos[c + d*x])/Sqrt[a + b*Sin[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1-\cos(3x))^{3/2}} dx &= -\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}+\frac{1}{4}\int \frac{1}{\sqrt{1-\cos(3x)}} dx\\
&= -\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}-\frac{1}{6}\text{Subst}\left(\int \frac{1}{2-x^2} dx, x, \frac{\sin(3x)}{\sqrt{1-\cos(3x)}}\right)\\
&= -\frac{\tanh^{-1}\left(\frac{\sin(3x)}{\sqrt{2}\sqrt{1-\cos(3x)}}\right)}{6\sqrt{2}}-\frac{\sin(3x)}{6(1-\cos(3x))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.118476, size = 61, normalized size = 1.15

$$-\frac{\sin^3\left(\frac{3x}{2}\right)\left(\csc^2\left(\frac{3x}{4}\right)-\sec^2\left(\frac{3x}{4}\right)-4\log\left(\sin\left(\frac{3x}{4}\right)\right)+4\log\left(\cos\left(\frac{3x}{4}\right)\right)\right)}{12(1-\cos(3x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[3*x])^(-3/2),x]
```

```
[Out] -((Csc[(3*x)/4]^2 + 4*Log[Cos[(3*x)/4]] - 4*Log[Sin[(3*x)/4]] - Sec[(3*x)/4
]^2)*Sin[(3*x)/2]^3)/(12*(1 - Cos[3*x])^(3/2))
```

Maple [A] time = 0.047, size = 52, normalized size = 1.

$$-\frac{\sqrt{2}}{6}\left(\frac{1}{2}\cos\left(\frac{3x}{2}\right)+\frac{1}{4}\left(\ln\left(\cos\left(\frac{3x}{2}\right)+1\right)-\ln\left(\cos\left(\frac{3x}{2}\right)-1\right)\right)\left(\sin\left(\frac{3x}{2}\right)\right)^2\right)\left(\sin\left(\frac{3x}{2}\right)\right)^{-1}\frac{1}{\sqrt{\left(\sin\left(\frac{3x}{2}\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(3*x))^(3/2),x)
```

```
[Out] -1/6*(1/2*cos(3/2*x)+1/4*(ln(cos(3/2*x)+1)-ln(cos(3/2*x)-1))*sin(3/2*x)^2)/
sin(3/2*x)*2^(1/2)/(sin(3/2*x)^2)^(1/2)
```

Maxima [B] time = 1.67138, size = 585, normalized size = 11.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(3*x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(sin(6*x) - 2*sin(3*x))*cos(3/2*pi + 3/2*arctan2(sin(3*x), cos(3*x)
)) - 4*(sin(6*x) - 2*sin(3*x))*cos(1/2*pi + 1/2*arctan2(sin(3*x), cos(3*x))
) + (2*(2*cos(3*x) - 1)*cos(6*x) - cos(6*x)^2 - 4*cos(3*x)^2 - sin(6*x)^2 +
 4*sin(6*x)*sin(3*x) - 4*sin(3*x)^2 + 4*cos(3*x) - 1)*log(cos(1/2*arctan2(s
in(3*x), cos(3*x)))^2 + sin(1/2*arctan2(sin(3*x), cos(3*x)))^2 + 2*cos(1/2*
arctan2(sin(3*x), cos(3*x))) + 1) - (2*(2*cos(3*x) - 1)*cos(6*x) - cos(6*x)
^2 - 4*cos(3*x)^2 - sin(6*x)^2 + 4*sin(6*x)*sin(3*x) - 4*sin(3*x)^2 + 4*cos
(3*x) - 1)*log(cos(1/2*arctan2(sin(3*x), cos(3*x)))^2 + sin(1/2*arctan2(sin
(3*x), cos(3*x)))^2 - 2*cos(1/2*arctan2(sin(3*x), cos(3*x))) + 1) - 4*(cos(
6*x) - 2*cos(3*x) + 1)*sin(3/2*pi + 3/2*arctan2(sin(3*x), cos(3*x))) + 4*(c
os(6*x) - 2*cos(3*x) + 1)*sin(1/2*pi + 1/2*arctan2(sin(3*x), cos(3*x))))/(s
qrt(2)*cos(6*x)^2 + 4*sqrt(2)*cos(3*x)^2 + sqrt(2)*sin(6*x)^2 - 4*sqrt(2)*s
in(6*x)*sin(3*x) + 4*sqrt(2)*sin(3*x)^2 - 2*(2*sqrt(2)*cos(3*x) - sqrt(2))*
cos(6*x) - 4*sqrt(2)*cos(3*x) + sqrt(2))
```

Fricas [B] time = 1.9398, size = 300, normalized size = 5.66

$$\frac{\left(\sqrt{2}\cos\left(3\,x\right)-\sqrt{2}\right)\log\left(-\frac{\left(\cos(3\,x)+3\right)\sin(3\,x)-2\left(\sqrt{2}\cos(3\,x)+\sqrt{2}\right)\sqrt{-\cos(3\,x)+1}}{\left(\cos(3\,x)-1\right)\sin(3\,x)}\right)\sin\left(3\,x\right)+4\left(\cos\left(3\,x\right)+1\right)\sqrt{-\cos\left(3\,x\right)+1}}{24\left(\cos\left(3\,x\right)-1\right)\sin\left(3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(3*x))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*((sqrt(2)*cos(3*x) - sqrt(2))*log(-((cos(3*x) + 3)*sin(3*x) - 2*(sqrt(
2)*cos(3*x) + sqrt(2))*sqrt(-cos(3*x) + 1))/((cos(3*x) - 1)*sin(3*x)))*sin(
3*x) + 4*(cos(3*x) + 1)*sqrt(-cos(3*x) + 1))/((cos(3*x) - 1)*sin(3*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(1-\cos\left(3x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(3*x))**(3/2),x)
```

```
[Out] Integral((1 - cos(3*x))**(-3/2), x)
```

Giac [A] time = 1.20578, size = 80, normalized size = 1.51

$$-\frac{\sqrt{2}\left(\frac{2\sqrt{\tan\left(\frac{3}{2}\,x\right)^{2}+1}}{\tan\left(\frac{3}{2}\,x\right)^{2}}+\log\left(\sqrt{\tan\left(\frac{3}{2}\,x\right)^{2}+1}+1\right)-\log\left(\sqrt{\tan\left(\frac{3}{2}\,x\right)^{2}+1}-1\right)\right)}{24\,\mathrm{sgn}\left(\tan\left(\frac{3}{2}\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(3*x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*sqrt(2)*(2*sqrt(tan(3/2*x)^2 + 1)/tan(3/2*x)^2 + log(sqrt(tan(3/2*x)^
2 + 1) + 1) - log(sqrt(tan(3/2*x)^2 + 1) - 1))/sgn(tan(3/2*x))
```

3.395 $\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2} dx$

Optimal. Leaf size=73

$$\frac{3}{5}\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}\cos\left(\frac{2x}{3}\right)+\frac{8}{5}\sqrt{1-\sin\left(\frac{2x}{3}\right)}\cos\left(\frac{2x}{3}\right)+\frac{32\cos\left(\frac{2x}{3}\right)}{5\sqrt{1-\sin\left(\frac{2x}{3}\right)}}$$

```
[Out] (32*Cos[(2*x)/3])/(5*Sqrt[1 - Sin[(2*x)/3]]) + (8*Cos[(2*x)/3]*Sqrt[1 - Sin
[(2*x)/3]])/5 + (3*Cos[(2*x)/3]*(1 - Sin[(2*x)/3])^(3/2))/5
```

Rubi [A] time = 0.0325273, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2647, 2646}

$$\frac{3}{5}\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}\cos\left(\frac{2x}{3}\right)+\frac{8}{5}\sqrt{1-\sin\left(\frac{2x}{3}\right)}\cos\left(\frac{2x}{3}\right)+\frac{32\cos\left(\frac{2x}{3}\right)}{5\sqrt{1-\sin\left(\frac{2x}{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[(2*x)/3])^(5/2),x]
```

```
[Out] (32*Cos[(2*x)/3])/(5*Sqrt[1 - Sin[(2*x)/3]]) + (8*Cos[(2*x)/3]*Sqrt[1 - Sin
[(2*x)/3]])/5 + (3*Cos[(2*x)/3]*(1 - Sin[(2*x)/3])^(3/2))/5
```

Rule 2647

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(a*(2*n - 1))/n, In
t[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2
 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2} dx &= \frac{3}{5}\cos\left(\frac{2x}{3}\right)\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}+\frac{8}{5}\int \left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2} dx\\
&= \frac{8}{5}\cos\left(\frac{2x}{3}\right)\sqrt{1-\sin\left(\frac{2x}{3}\right)}+\frac{3}{5}\cos\left(\frac{2x}{3}\right)\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}+\frac{32}{15}\int \sqrt{1-\sin\left(\frac{2x}{3}\right)}\,dx\\
&= \frac{32\cos\left(\frac{2x}{3}\right)}{5\sqrt{1-\sin\left(\frac{2x}{3}\right)}}+\frac{8}{5}\cos\left(\frac{2x}{3}\right)\sqrt{1-\sin\left(\frac{2x}{3}\right)}+\frac{3}{5}\cos\left(\frac{2x}{3}\right)\left(1-\sin\left(\frac{2x}{3}\right)\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.166133, size = 76, normalized size = 1.04

$$\frac{\left(1-\sin\left(\frac{2x}{3}\right)\right)^{5/2}\left(150\sin\left(\frac{x}{3}\right)-25\sin(x)-3\sin\left(\frac{5x}{3}\right)+150\cos\left(\frac{x}{3}\right)+25\cos(x)-3\cos\left(\frac{5x}{3}\right)\right)}{20\left(\cos\left(\frac{x}{3}\right)-\sin\left(\frac{x}{3}\right)\right)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[(2*x)/3])^(5/2),x]
```

```
[Out] ((1 - Sin[(2*x)/3])^(5/2)*(150*Cos[x/3] + 25*Cos[x] - 3*Cos[(5*x)/3] + 150*
Sin[x/3] - 25*Sin[x] - 3*Sin[(5*x)/3]))/(20*(Cos[x/3] - Sin[x/3])^5)
```

Maple [A] time = 0.037, size = 47, normalized size = 0.6

$$-\frac{1}{5}\left(-1+\sin\left(\frac{2x}{3}\right)\right)\left(1+\sin\left(\frac{2x}{3}\right)\right)\left(3\left(\sin\left(2/3x\right)\right)^2-14\sin\left(2/3x\right)+43\right)\left(\cos\left(\frac{2x}{3}\right)\right)^{-1}\frac{1}{\sqrt{1-\sin\left(\frac{2x}{3}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(2/3*x))^(5/2),x)
```

```
[Out] -1/5*(-1+sin(2/3*x))*(1+sin(2/3*x))*(3*sin(2/3*x)^2-14*sin(2/3*x)+43)/cos(2
/3*x)/(1-sin(2/3*x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-\sin\left(\frac{2}{3}x\right)+1\right)^{\frac{5}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2/3*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-sin(2/3*x) + 1)^(5/2), x)
```

Fricas [A] time = 1.88725, size = 223, normalized size = 3.05

$$-\frac{\left(3\cos\left(\frac{2}{3}x\right)^3-11\cos\left(\frac{2}{3}x\right)^2+\left(3\cos\left(\frac{2}{3}x\right)^2+14\cos\left(\frac{2}{3}x\right)-32\right)\sin\left(\frac{2}{3}x\right)-46\cos\left(\frac{2}{3}x\right)-32\right)\sqrt{-\sin\left(\frac{2}{3}x\right)+1}}{5\left(\cos\left(\frac{2}{3}x\right)-\sin\left(\frac{2}{3}x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2/3*x))^(5/2),x, algorithm="fricas")
```

```
[Out] -1/5*(3*cos(2/3*x)^3 - 11*cos(2/3*x)^2 + (3*cos(2/3*x)^2 + 14*cos(2/3*x) -
32)*sin(2/3*x) - 46*cos(2/3*x) - 32)*sqrt(-sin(2/3*x) + 1)/(cos(2/3*x) - si
```

```
n(2/3*x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2/3*x))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2/3*x))^(5/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.396 $\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx$

Optimal. Leaf size=55

$$\frac{1}{12}(2\sin(x)+1)^{3/2}-\frac{1}{2}\sqrt{2\sin(x)+1}-\frac{4}{\sqrt[4]{2\sin(x)+1}}+\frac{3}{4\sqrt{2\sin(x)+1}}$$

```
[Out] 3/(4*Sqrt[1 + 2*Sin[x]]) - 4/(1 + 2*Sin[x])^(1/4) - Sqrt[1 + 2*Sin[x]]/2 +
(1 + 2*Sin[x])^(3/2)/12
```

Rubi [A] time = 0.150597, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4356, 14}

$$\frac{1}{12}(2\sin(x)+1)^{3/2}-\frac{1}{2}\sqrt{2\sin(x)+1}-\frac{4}{\sqrt[4]{2\sin(x)+1}}+\frac{3}{4\sqrt{2\sin(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*(-Cos[x]^2 + 2*(1 + 2*Sin[x])^(1/4)))/(1 + 2*Sin[x])^(3/2),x]
```

```
[Out] 3/(4*Sqrt[1 + 2*Sin[x]]) - 4/(1 + 2*Sin[x])^(1/4) - Sqrt[1 + 2*Sin[x]]/2 +
(1 + 2*Sin[x])^(3/2)/12
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)\left(-\cos^2(x)+2\sqrt[4]{1+2\sin(x)}\right)}{(1+2\sin(x))^{3/2}}\,dx &= \text{Subst}\left(\int \frac{-1+x^2+2\sqrt[4]{1+2x}}{(1+2x)^{3/2}}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{-3+8x-2x^4+x^8}{x^3}\,dx, x, \sqrt[4]{1+2\sin(x)}\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{3}{x^3}+\frac{8}{x^2}-2x+x^5\right)dx, x, \sqrt[4]{1+2\sin(x)}\right)\\
&= \frac{3}{4\sqrt{1+2\sin(x)}}-\frac{4}{\sqrt[4]{1+2\sin(x)}}-\frac{1}{2}\sqrt{1+2\sin(x)}+\frac{1}{12}(1+2\sin(x))^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.0771201, size = 36, normalized size = 0.65

$$-\frac{4\sin(x)+24\sqrt[4]{2\sin(x)+1}+\cos(2x)-3}{6\sqrt{2\sin(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*(-Cos[x]^2 + 2*(1 + 2*Sin[x])^(1/4)))/(1 + 2*Sin[x])^(3/2
),x]
```

```
[Out] -(-3 + Cos[2*x] + 4*Sin[x] + 24*(1 + 2*Sin[x])^(1/4))/(6*Sqrt[1 + 2*Sin[x]]
)
```

Maple [A] time = 0.436, size = 42, normalized size = 0.8

$$-4\frac{1}{\sqrt[4]{1+2\sin(x)}}+\frac{1}{12}(1+2\sin(x))^{\frac{3}{2}}+\frac{3}{4}\frac{1}{\sqrt{1+2\sin(x)}}-\frac{1}{2}\sqrt{1+2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x)
```

```
[Out] -4/(1+2*sin(x))^(1/4)+1/12*(1+2*sin(x))^(3/2)+3/4/(1+2*sin(x))^(1/2)-1/2*(1
+2*sin(x))^(1/2)
```

Maxima [A] time = 0.929482, size = 58, normalized size = 1.05

$$\frac{1}{12}(2\sin(x)+1)^{\frac{3}{2}}-\frac{16(2\sin(x)+1)^{\frac{1}{4}}-3}{4\sqrt{2\sin(x)+1}}-\frac{1}{2}\sqrt{2\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x, alg
orithm="maxima")
```

```
[Out] 1/12*(2*sin(x) + 1)^(3/2) - 1/4*(16*(2*sin(x) + 1)^(1/4) - 3)/sqrt(2*sin(x)
 + 1) - 1/2*sqrt(2*sin(x) + 1)
```

Fricas [A] time = 2.08143, size = 127, normalized size = 2.31

$$-\frac{\left(\cos(x)^2+2\sin(x)-2\right)\sqrt{2\sin(x)+1}+12(2\sin(x)+1)^{\frac{3}{4}}}{3(2\sin(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x, alg
orithm="fricas")
```

```
[Out] -1/3*((cos(x)^2 + 2*sin(x) - 2)*sqrt(2*sin(x) + 1) + 12*(2*sin(x) + 1)^(3/4
))/(2*sin(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-cos(x)**2+2*(1+2*sin(x))**(1/4))/(1+2*sin(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(\cos\left(x\right)^{2} - 2\left(2\sin\left(x\right) + 1\right)^{\frac{1}{4}}\right)\cos\left(x\right)}{\left(2\sin\left(x\right) + 1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-cos(x)^2+2*(1+2*sin(x))^(1/4))/(1+2*sin(x))^(3/2),x, alg
orithm="giac")
```

```
[Out] integrate(-(cos(x)^2 - 2*(2*sin(x) + 1)^(1/4))*cos(x)/(2*sin(x) + 1)^(3/2),
 x)
```

3.397 $\int \sqrt{\tan(x)}\,dx$

Optimal. Leaf size=98

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}+\frac{\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+\right.}{2\sqrt{2}}$$

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[x
]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 +
Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2])
```

Rubi [A] time = 0.0687999, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.333, Rules used = {3476, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}+\frac{\log\left(\tan(x)-\sqrt{2}\sqrt{\tan(x)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\tan(x)+\sqrt{2}\sqrt{\tan(x)}+\right.}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[x]],x]
```

```
[Out] -(ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*Sqrt[Tan[x
]]]/Sqrt[2] + Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]) - Log[1 +
Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/(2*Sqrt[2])
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\tan(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(x)\right)\\
&= 2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)\\
&= -\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right) + \text{Subst}\left(\int \frac{1+x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)\\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(x)}\right) + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(x)}\right) + \frac{\text{Subst}\left(\int \frac{}{-1}\right.}{}\\
&= \frac{\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt{2}\sqrt{\tan(}\right.}{\sqrt{2}}\\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}} + \frac{\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}} - \frac{\log\left(1+\sqrt{2}\sqrt{\right.}{}
\end{aligned}
$$

Mathematica [C] time = 0.0122845, size = 24, normalized size = 0.24

$$\frac{2}{3}\tan^{\frac{3}{2}}(x)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[x]],x]
```

```
[Out] (2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[x]^2]*Tan[x]^(3/2))/3
```

Maple [A] time = 0.001, size = 49, normalized size = 0.5

$$\frac{\cos(x)\sqrt{2}\arccos(\cos(x)-\sin(x))}{2}\sqrt{\tan(x)}\frac{1}{\sqrt{\cos(x)\sin(x)}}-\frac{\sqrt{2}}{2}\ln\left(\cos(x)+\sqrt{2}\sqrt{\tan(x)}\cos(x)+\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^(1/2),x)
```

```
[Out] 1/2*tan(x)^(1/2)/(cos(x)*sin(x))^(1/2)*cos(x)*2^(1/2)*arccos(cos(x)-sin(x))
-1/2*2^(1/2)*ln(cos(x)+2^(1/2)*tan(x)^(1/2)*cos(x)+sin(x))
```

Maxima [A] time = 1.41856, size = 108, normalized size = 1.1

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 1/2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - 1/4*sqrt(2)*log(sqrt(2)*sqr
t(tan(x)) + tan(x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) +
1)
```

Fricas [B] time = 1.88925, size = 572, normalized size = 5.84

$$-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{\frac{\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}\cos(x)+\cos(x)+\sin(x)}{\cos(x)}}-\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}-1\right)-\sqrt{2}\arctan\left(\sqrt{2}\sqrt{-\frac{\sqrt{2}\sqrt{\frac{\sin(x)}{\cos(x)}}\cos(x)}{\cos}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*arctan(sqrt(2)*sqrt((sqrt(2)*sqrt(sin(x)/cos(x))*cos(x) + cos(x) +
 sin(x))/cos(x)) - sqrt(2)*sqrt(sin(x)/cos(x)) - 1) - sqrt(2)*arctan(sqrt(2
)*sqrt(-(sqrt(2)*sqrt(sin(x)/cos(x))*cos(x) - cos(x) - sin(x))/cos(x)) - sq
rt(2)*sqrt(sin(x)/cos(x)) + 1) - 1/4*sqrt(2)*log(4*(sqrt(2)*sqrt(sin(x)/cos
(x))*cos(x) + cos(x) + sin(x))/cos(x)) + 1/4*sqrt(2)*log(-4*(sqrt(2)*sqrt(s
in(x)/cos(x))*cos(x) - cos(x) - sin(x))/cos(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\tan(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**(1/2),x)
```

```
[Out] Integral(sqrt(tan(x)), x)
```

Giac [A] time = 1.1425, size = 108, normalized size = 1.1

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)+\frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan(x)}+\tan(x)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) + 1/2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) - 1/4*sqrt(2)*log(sqrt(2)*sqr
t(tan(x)) + tan(x) + 1) + 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) +
1)
```

3.398 $\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{10}\sqrt{3}\tan^{-1}\left(\frac{1-2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right)+\frac{3}{20}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)-\frac{1}{20}\log\left(\tan^{2}(5x)+1\right)$$

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Tan[5*x]^(2/3))/Sqrt[3]])/10 + (3*Log[1 + Tan[5*x]^
(2/3)])/20 - Log[1 + Tan[5*x]^2]/20
```

Rubi [A] time = 0.0584444, antiderivative size = 69, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {3476, 329, 275, 200, 31, 634, 618, 204, 628}

$$-\frac{1}{10}\sqrt{3}\tan^{-1}\left(\frac{1-2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right)+\frac{1}{10}\log\left(\tan^{\frac{2}{3}}(5x)+1\right)-\frac{1}{20}\log\left(\tan^{\frac{4}{3}}(5x)-\tan^{\frac{2}{3}}(5x)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[5*x]^(-1/3),x]
```

```
[Out] -(Sqrt[3]*ArcTan[(1 - 2*Tan[5*x]^(2/3))/Sqrt[3]])/10 + Log[1 + Tan[5*x]^(2/
3)]/10 - Log[1 - Tan[5*x]^(2/3) + Tan[5*x]^(4/3)]/20
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx &= \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{x}\left(1+x^2\right)}\,dx, x, \tan(5x)\right)\\
&= \frac{3}{5}\,\text{Subst}\left(\int \frac{x}{1+x^6}\,dx, x, \sqrt[3]{\tan(5x)}\right)\\
&= \frac{3}{10}\,\text{Subst}\left(\int \frac{1}{1+x^3}\,dx, x, \tan^{\frac{2}{3}}(5x)\right)\\
&= \frac{1}{10}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \tan^{\frac{2}{3}}(5x)\right) + \frac{1}{10}\,\text{Subst}\left(\int \frac{2-x}{1-x+x^2}\,dx, x, \tan^{\frac{2}{3}}(5x)\right)\\
&= \frac{1}{10}\log\left(1+\tan^{\frac{2}{3}}(5x)\right) - \frac{1}{20}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \tan^{\frac{2}{3}}(5x)\right) + \frac{3}{20}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \text{t}\right.\\
&= \frac{1}{10}\log\left(1+\tan^{\frac{2}{3}}(5x)\right) - \frac{1}{20}\log\left(1-\tan^{\frac{2}{3}}(5x)+\tan^{\frac{4}{3}}(5x)\right) - \frac{3}{10}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tan\right.\\
&= -\frac{1}{10}\sqrt{3}\tan^{-1}\left(\frac{1-2\tan^{\frac{2}{3}}(5x)}{\sqrt{3}}\right) + \frac{1}{10}\log\left(1+\tan^{\frac{2}{3}}(5x)\right) - \frac{1}{20}\log\left(1-\tan^{\frac{2}{3}}(5x)+\tan^{\frac{4}{3}}(5x)\right)
\end{aligned}
$$

Mathematica [A] time = 0.050121, size = 69, normalized size = 1.21

$$
\frac{1}{10}\sqrt{3}\tan^{-1}\left(\frac{2\tan^{\frac{2}{3}}(5x)-1}{\sqrt{3}}\right) + \frac{1}{10}\log\left(\tan^{\frac{2}{3}}(5x)+1\right) - \frac{1}{20}\log\left(\tan^{\frac{4}{3}}(5x)-\tan^{\frac{2}{3}}(5x)+1\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[5*x]^(-1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(-1 + 2*Tan[5*x]^(2/3))/Sqrt[3]])/10 + Log[1 + Tan[5*x]^(2/
3)]/10 - Log[1 - Tan[5*x]^(2/3) + Tan[5*x]^(4/3)]/20
```

Maple [A] time = 0.012, size = 53, normalized size = 0.9

$$\frac{1}{10}\ln\left(1+(\tan(5x))^{\frac{2}{3}}\right)-\frac{1}{20}\ln\left(1-(\tan(5x))^{\frac{2}{3}}+(\tan(5x))^{\frac{4}{3}}\right)+\frac{\sqrt{3}}{10}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,(\tan(5x))^{2/3}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(5*x)^(1/3),x)
```

```
[Out] 1/10*ln(1+tan(5*x)^(2/3))-1/20*ln(1-tan(5*x)^(2/3)+tan(5*x)^(4/3))+1/10*3^(
1/2)*arctan(1/3*(2*tan(5*x)^(2/3)-1)*3^(1/2))
```

Maxima [A] time = 1.4146, size = 70, normalized size = 1.23

$$\frac{1}{10}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\tan(5x)^{\frac{2}{3}}-1\right)\right)-\frac{1}{20}\log\left(\tan(5x)^{\frac{4}{3}}-\tan(5x)^{\frac{2}{3}}+1\right)+\frac{1}{10}\log\left(\tan(5x)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(5*x)^(1/3),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(5*x)^(2/3) - 1)) - 1/20*log(tan(5*x)
^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*log(tan(5*x)^(2/3) + 1)
```

Fricas [A] time = 1.86973, size = 192, normalized size = 3.37

$$\frac{1}{10}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\tan(5x)^{\frac{2}{3}}-\frac{1}{3}\sqrt{3}\right)-\frac{1}{20}\log\left(\tan(5x)^{\frac{4}{3}}-\tan(5x)^{\frac{2}{3}}+1\right)+\frac{1}{10}\log\left(\tan(5x)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(5*x)^(1/3),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(3)*arctan(2/3*sqrt(3)*tan(5*x)^(2/3) - 1/3*sqrt(3)) - 1/20*log(ta
n(5*x)^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*log(tan(5*x)^(2/3) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{\tan(5x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(5*x)**(1/3),x)
```

```
[Out] Integral(tan(5*x)**(-1/3), x)
```

Giac [A] time = 1.16806, size = 70, normalized size = 1.23

$$\frac{1}{10}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\tan\left(5x\right)^{\frac{2}{3}}-1\right)\right)-\frac{1}{20}\log\left(\tan\left(5x\right)^{\frac{4}{3}}-\tan\left(5x\right)^{\frac{2}{3}}+1\right)+\frac{1}{10}\log\left(\tan\left(5x\right)^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(5*x)^(1/3),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(5*x)^(2/3) - 1)) - 1/20*log(tan(5*x)
^(4/3) - tan(5*x)^(2/3) + 1) + 1/10*log(tan(5*x)^(2/3) + 1)
```

3.399 $\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx$

Optimal. Leaf size=87

$$-\frac{9\tan^{-1}\left(\frac{1-3\tan(2x)}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{250\sqrt{2}}-\frac{3}{25\sqrt{3\tan(2x)+4}}+\frac{13\tanh^{-1}\left(\frac{\tan(2x)+3}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{250\sqrt{2}}$$

```
[Out] (-9*ArcTan[(1 - 3*Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*x]])])/(250*Sqrt[2])
+ (13*ArcTanh[(3 + Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*x]])])/(250*Sqrt[2])
 - 3/(25*Sqrt[4 + 3*Tan[2*x]])
```

Rubi [A] time = 0.113088, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3483, 3536, 3535, 203, 207}

$$-\frac{9\tan^{-1}\left(\frac{1-3\tan(2x)}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{250\sqrt{2}}-\frac{3}{25\sqrt{3\tan(2x)+4}}+\frac{13\tanh^{-1}\left(\frac{\tan(2x)+3}{\sqrt{2}\sqrt{3\tan(2x)+4}}\right)}{250\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 3*Tan[2*x])^(-3/2),x]
```

```
[Out] (-9*ArcTan[(1 - 3*Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*x]])])/(250*Sqrt[2])
+ (13*ArcTanh[(3 + Tan[2*x])/(Sqrt[2]*Sqrt[4 + 3*Tan[2*x]])])/(250*Sqrt[2])
 - 3/(25*Sqrt[4 + 3*Tan[2*x]])
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3536

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> With[{q = Rt[a^2 + b^2, 2]}, Dist[1/(2*q), Int[(a
*c + b*d + c*q + (b*c - a*d + d*q)*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]],
x], x] - Dist[1/(2*q), Int[(a*c + b*d - c*q + (b*c - a*d - d*q)*Tan[e + f*x
])/Sqrt[a + b*Tan[e + f*x]], x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ
[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && NeQ[c^2 + d^2, 0] && NeQ[2*a*c*d - b
*(c^2 - d^2), 0] && (PerfectSquareQ[a^2 + b^2] || RationalQ[a, b, c, d])
```

Rule 3535

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*tan[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Dist[(-2*d^2)/f, Subst[Int[1/(2*b*c*d - 4*a*d^2 +
 x^2), x], x, (b*c - 2*a*d - b*d*Tan[e + f*x])/Sqrt[a + b*Tan[e + f*x]]], x
] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0]
 && NeQ[c^2 + d^2, 0] && EqQ[2*a*c*d - b*(c^2 - d^2), 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(4+3\tan(2x))^{3/2}}\,dx &= -\frac{3}{25\sqrt{4+3\tan(2x)}}+\frac{1}{25}\int \frac{4-3\tan(2x)}{\sqrt{4+3\tan(2x)}}\,dx\\ &= -\frac{3}{25\sqrt{4+3\tan(2x)}}+\frac{1}{250}\int \frac{27+9\tan(2x)}{\sqrt{4+3\tan(2x)}}\,dx-\frac{1}{250}\int \frac{-13+39\tan(2x)}{\sqrt{4+3\tan(2x)}}\,dx\\ &= -\frac{3}{25\sqrt{4+3\tan(2x)}}-\frac{81}{250}\text{Subst}\left(\int \frac{1}{162+x^2}\,dx,x,\frac{9-27\tan(2x)}{\sqrt{4+3\tan(2x)}}\right)+\frac{1521}{250}\text{Subst}\left(\int \frac{}{-27}\right.\\ &= -\frac{9\tan^{-1}\left(\frac{1-3\tan(2x)}{\sqrt{2}\sqrt{4+3\tan(2x)}}\right)}{250\sqrt{2}}+\frac{13\tanh^{-1}\left(\frac{3+\tan(2x)}{\sqrt{2}\sqrt{4+3\tan(2x)}}\right)}{250\sqrt{2}}-\frac{3}{25\sqrt{4+3\tan(2x)}}\end{aligned}$$

Mathematica [C] time = 0.0925458, size = 73, normalized size = 0.84

$$-\frac{(3+4i)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\left(\frac{4}{25}-\frac{3i}{25}\right)(3\tan(2x)+4)\right)+(3-4i)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\left(\frac{4}{25}+\frac{3i}{25}\right)(3\tan(2x)+4)\right)}{50\sqrt{3\tan(2x)+4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 3*Tan[2*x])^(-3/2),x]
```

```
[Out] -((3 + 4*I)*Hypergeometric2F1[-1/2, 1, 1/2, (4/25 - (3*I)/25)*(4 + 3*Tan[2*
x])] + (3 - 4*I)*Hypergeometric2F1[-1/2, 1, 1/2, (4/25 + (3*I)/25)*(4 + 3*T
an[2*x])])/(50*Sqrt[4 + 3*Tan[2*x]])
```

Maple [A] time = 0.049, size = 130, normalized size = 1.5

$$-\frac{13\sqrt{2}}{1000}\ln\left(3\tan(2x)+9-3\sqrt{4+3\tan(2x)}\sqrt{2}\right)+\frac{9\sqrt{2}}{500}\arctan\left(\frac{\sqrt{2}}{2}\left(2\sqrt{4+3\tan(2x)}-3\sqrt{2}\right)\right)+\frac{13\sqrt{2}}{1000}\ln\left(3\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+3*tan(2*x))^(3/2),x)
```

```
[Out] -13/1000*2^(1/2)*ln(3*tan(2*x)+9-3*(4+3*tan(2*x))^(1/2)*2^(1/2))+9/500*2^(1
/2)*arctan(1/2*(2*(4+3*tan(2*x))^(1/2)-3*2^(1/2))*2^(1/2))+13/1000*2^(1/2)*
ln(3*tan(2*x)+9+3*(4+3*tan(2*x))^(1/2)*2^(1/2))+9/500*2^(1/2)*arctan(1/2*(2
*(4+3*tan(2*x))^(1/2)+3*2^(1/2))*2^(1/2))-3/25/(4+3*tan(2*x))^(1/2)
```

Maxima [B] time = 2.49861, size = 4338, normalized size = 49.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+3*tan(2*x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/18000*(2000*(3*cos(4*x) + sin(4*x))*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(
8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^3 + 2000*
(3*cos(4*x) + sin(4*x))*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*
x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))*sin(1/2*arctan2(-3*cos(8
*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4
))^2 - 2000*(cos(4*x) - 3*sin(4*x) - 3)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin
(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^3 - 80*(
48*cos(4*x) + 25*sin(4*x) - 27)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) +
8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4)) - 80*(25*(cos(4*
x) - 3*sin(4*x) - 3)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x)
+ 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 - 25*cos(4*x) + 48*sin(4*
x) + 75)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8
*x) + 8*cos(4*x) + 3*sin(8*x) + 4)) + 9*(18*(sqrt(2)*cos(1/2*arctan2(-3*cos
(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) +
 4))^2 + sqrt(2)*sin(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3,
 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2)*arctan2(1/3*25^(1/4)*(25*cos
(4*x)^4 + 25*sin(4*x)^4 + 64*cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x) +
25)*sin(4*x)^2 + 48*sin(4*x)^3 + 78*cos(4*x)^2 + 48*(cos(4*x)^2 + 2*cos(4*x
) + 1)*sin(4*x) + 64*cos(4*x) + 25)^(1/4)*sin(1/2*arctan2(-8/3*cos(4*x)^2 +
 2/9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*cos(4*
x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*cos(4*x) + 25/
9)) + cos(4*x) - 4/3*sin(4*x), 1/3*25^(1/4)*(25*cos(4*x)^4 + 25*sin(4*x)^4
+ 64*cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(
4*x)^3 + 78*cos(4*x)^2 + 48*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos
(4*x) + 25)^(1/4)*cos(1/2*arctan2(-8/3*cos(4*x)^2 + 2/9*(7*cos(4*x) + 16)*s
in(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*cos(4*x)^2 + 8/3*(2*cos(4*x) +
 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*cos(4*x) + 25/9)) - 4/3*cos(4*x) - sin
(4*x) - 4/3) + 18*(sqrt(2)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin
(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 + sqrt(2)*sin(1/2*a
rctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) +
 3*sin(8*x) + 4))^2)*arctan2(2/3*4^(1/4)*(4*cos(4*x)^4 + 4*sin(4*x)^4 + 16*
cos(4*x)^3 + (8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3 +
 33*cos(4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*x) +
13)^(1/4)*sin(1/2*arctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*cos(4*x) + 8/3
, 16/9*cos(4*x)^2 - 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9))
 + 4/3*sin(4*x) + 1, 2/3*4^(1/4)*(4*cos(4*x)^4 + 4*sin(4*x)^4 + 16*cos(4*x)
^3 + (8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3 + 33*cos(
4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*x) + 13)^(1/4
)*cos(1/2*arctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*cos(4*x) + 8/3, 16/9*c
os(4*x)^2 - 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9)) + 4/3*c
os(4*x) + 4/3) + 13*(sqrt(2)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*s
in(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 + sqrt(2)*sin(1/2
*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x)
 + 3*sin(8*x) + 4))^2)*log(-2/9*25^(1/4)*(25*cos(4*x)^4 + 25*sin(4*x)^4 + 6
4*cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x
)^3 + 78*cos(4*x)^2 + 48*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos(4*
x) + 25)^(1/4)*(4*cos(4*x) + 3*sin(4*x) + 4)*cos(1/2*arctan2(-8/3*cos(4*x)^
2 + 2/9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*cos
(4*x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*cos(4*x) +
25/9)) + 5/9*sqrt(25*cos(4*x)^4 + 25*sin(4*x)^4 + 64*cos(4*x)^3 + 2*(25*cos
(4*x)^2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x)^3 + 78*cos(4*x)^2 + 48
*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos(4*x) + 25)*cos(1/2*arctan2
(-8/3*cos(4*x)^2 + 2/9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*co
s(4*x), 7/9*cos(4*x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 3
2/9*cos(4*x) + 25/9))^2 + 2/9*25^(1/4)*(25*cos(4*x)^4 + 25*sin(4*x)^4 + 64*
```

```
cos(4*x)^3 + 2*(25*cos(4*x)^2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x)^
3 + 78*cos(4*x)^2 + 48*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos(4*x)
 + 25)^(1/4)*(3*cos(4*x) - 4*sin(4*x))*sin(1/2*arctan2(-8/3*cos(4*x)^2 + 2/
9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x), 7/9*cos(4*x)^
2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*cos(4*x) + 25/9))
 + 5/9*sqrt(25*cos(4*x)^4 + 25*sin(4*x)^4 + 64*cos(4*x)^3 + 2*(25*cos(4*x)^
2 + 32*cos(4*x) + 25)*sin(4*x)^2 + 48*sin(4*x)^3 + 78*cos(4*x)^2 + 48*(cos(
4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 64*cos(4*x) + 25)*sin(1/2*arctan2(-8/3*
cos(4*x)^2 + 2/9*(7*cos(4*x) + 16)*sin(4*x) + 8/3*sin(4*x)^2 - 8/3*cos(4*x)
, 7/9*cos(4*x)^2 + 8/3*(2*cos(4*x) + 1)*sin(4*x) - 7/9*sin(4*x)^2 + 32/9*co
s(4*x) + 25/9))^2 + 25/9*cos(4*x)^2 + 25/9*sin(4*x)^2 + 32/9*cos(4*x) + 8/3
*sin(4*x) + 16/9) - 13*(sqrt(2)*cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) +
8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 + sqrt(2)*sin(
1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4
*x) + 3*sin(8*x) + 4))^2)*log(16/9*4^(1/4)*(4*cos(4*x)^4 + 4*sin(4*x)^4 + 1
6*cos(4*x)^3 + (8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3
 + 33*cos(4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*x)
+ 13)^(1/4)*(cos(4*x) + 1)*cos(1/2*arctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8
/3*cos(4*x) + 8/3, 16/9*cos(4*x)^2 - 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*
sin(4*x) + 16/9)) + 8/9*sqrt(4*cos(4*x)^4 + 4*sin(4*x)^4 + 16*cos(4*x)^3 +
(8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^2 + 12*sin(4*x)^3 + 33*cos(4*x)^
2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*x) + 34*cos(4*x) + 13)*cos(1/2*a
rctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*cos(4*x) + 8/3, 16/9*cos(4*x)^2 -
 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9))^2 + 4/9*4^(1/4)*(4
*cos(4*x)^4 + 4*sin(4*x)^4 + 16*cos(4*x)^3 + (8*cos(4*x)^2 + 16*cos(4*x) +
17)*sin(4*x)^2 + 12*sin(4*x)^3 + 33*cos(4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x
) + 1)*sin(4*x) + 34*cos(4*x) + 13)^(1/4)*(4*sin(4*x) + 3)*sin(1/2*arctan2(
32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*cos(4*x) + 8/3, 16/9*cos(4*x)^2 - 16/9*s
in(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(4*x) + 16/9)) + 8/9*sqrt(4*cos(4*x)^4 +
 4*sin(4*x)^4 + 16*cos(4*x)^3 + (8*cos(4*x)^2 + 16*cos(4*x) + 17)*sin(4*x)^
2 + 12*sin(4*x)^3 + 33*cos(4*x)^2 + 12*(cos(4*x)^2 + 2*cos(4*x) + 1)*sin(4*
x) + 34*cos(4*x) + 13)*sin(1/2*arctan2(32/9*(cos(4*x) + 1)*sin(4*x) + 8/3*c
os(4*x) + 8/3, 16/9*cos(4*x)^2 - 16/9*sin(4*x)^2 + 32/9*cos(4*x) - 8/3*sin(
4*x) + 16/9))^2 + 16/9*cos(4*x)^2 + 16/9*sin(4*x)^2 + 32/9*cos(4*x) + 8/3*s
in(4*x) + 25/9))*(2*(32*cos(4*x) - 24*sin(4*x) + 7)*cos(8*x) + 25*cos(8*x)^
2 + 64*cos(4*x)^2 + 16*(3*cos(4*x) + 4*sin(4*x) + 3)*sin(8*x) + 25*sin(8*x)
^2 + 64*sin(4*x)^2 + 64*cos(4*x) + 48*sin(4*x) + 25)^(1/4))/((2*(32*cos(4*x
) - 24*sin(4*x) + 7)*cos(8*x) + 25*cos(8*x)^2 + 64*cos(4*x)^2 + 16*(3*cos(4
*x) + 4*sin(4*x) + 3)*sin(8*x) + 25*sin(8*x)^2 + 64*sin(4*x)^2 + 64*cos(4*x
) + 48*sin(4*x) + 25)^(1/4)*(cos(1/2*arctan2(-3*cos(8*x) + 4*sin(8*x) + 8*s
in(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin(8*x) + 4))^2 + sin(1/2*arctan2
(-3*cos(8*x) + 4*sin(8*x) + 8*sin(4*x) + 3, 4*cos(8*x) + 8*cos(4*x) + 3*sin
(8*x) + 4))^2))
```

Fricas [B] time = 2.0897, size = 1613, normalized size = 18.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+3*tan(2*x))^(3/2),x, algorithm="fricas")
```

```
[Out] -1/5000*(36*(7*sqrt(10)*sqrt(5)*cos(2*x)^2 + 24*sqrt(10)*sqrt(5)*cos(2*x)*s
in(2*x) + 9*sqrt(10)*sqrt(5))*arctan(1/25*sqrt(15)*sqrt(10)*sqrt(5)*sqrt((s
qrt(10)*sqrt(5)*sqrt((4*cos(2*x) + 3*sin(2*x))/cos(2*x))*cos(2*x) + 15*cos(
2*x) + 5*sin(2*x))/cos(2*x)) - 1/5*sqrt(10)*sqrt(5)*sqrt((4*cos(2*x) + 3*si
n(2*x))/cos(2*x)) - 3) + 36*(7*sqrt(10)*sqrt(5)*cos(2*x)^2 + 24*sqrt(10)*sq
```

```
rt(5)*cos(2*x)*sin(2*x) + 9*sqrt(10)*sqrt(5))*arctan(1/25*sqrt(15)*sqrt(10)
*sqrt(5)*sqrt(-(sqrt(10)*sqrt(5)*sqrt((4*cos(2*x) + 3*sin(2*x))/cos(2*x))*c
os(2*x) - 15*cos(2*x) - 5*sin(2*x))/cos(2*x)) - 1/5*sqrt(10)*sqrt(5)*sqrt((
4*cos(2*x) + 3*sin(2*x))/cos(2*x)) + 3) - 13*(7*sqrt(10)*sqrt(5)*cos(2*x)^2
 + 24*sqrt(10)*sqrt(5)*cos(2*x)*sin(2*x) + 9*sqrt(10)*sqrt(5))*log(9375*(sq
rt(10)*sqrt(5)*sqrt((4*cos(2*x) + 3*sin(2*x))/cos(2*x))*cos(2*x) + 15*cos(2
*x) + 5*sin(2*x))/cos(2*x)) + 13*(7*sqrt(10)*sqrt(5)*cos(2*x)^2 + 24*sqrt(1
0)*sqrt(5)*cos(2*x)*sin(2*x) + 9*sqrt(10)*sqrt(5))*log(-9375*(sqrt(10)*sqrt
(5)*sqrt((4*cos(2*x) + 3*sin(2*x))/cos(2*x))*cos(2*x) - 15*cos(2*x) - 5*sin
(2*x))/cos(2*x)) + 600*(4*cos(2*x)^2 + 3*cos(2*x)*sin(2*x))*sqrt((4*cos(2*x
) + 3*sin(2*x))/cos(2*x)))/(7*cos(2*x)^2 + 24*cos(2*x)*sin(2*x) + 9)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\tan(2x)+4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+3*tan(2*x))**(3/2),x)
```

```
[Out] Integral((3*tan(2*x) + 4)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(3\tan(2x)+4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+3*tan(2*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*tan(2*x) + 4)^(-3/2), x)
```

3.400 $\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{2}{3}\sqrt{4-3\tan(x)}+\frac{8}{3\sqrt{4-3\tan(x)}}+\frac{1}{3}\log(4-3\tan(x))$$

```
[Out] Log[4 - 3*Tan[x]]/3 + 8/(3*Sqrt[4 - 3*Tan[x]]) + (2*Sqrt[4 - 3*Tan[x]])/3
```

Rubi [A] time = 0.146582, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {4342, 43}

$$\frac{2}{3}\sqrt{4-3\tan(x)}+\frac{8}{3\sqrt{4-3\tan(x)}}+\frac{1}{3}\log(4-3\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(-Sqrt[4 - 3*Tan[x]] + 3*Tan[x]))/(4 - 3*Tan[x])^(3/2),x]
```

```
[Out] Log[4 - 3*Tan[x]]/3 + 8/(3*Sqrt[4 - 3*Tan[x]]) + (2*Sqrt[4 - 3*Tan[x]])/3
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x)\left(-\sqrt{4-3\tan(x)}+3\tan(x)\right)}{(4-3\tan(x))^{3/2}}\,dx &= \text{Subst}\left(\int \left(\frac{3x}{(4-3x)^{3/2}}+\frac{1}{-4+3x}\right)dx, x, \tan(x)\right)\\
&= \frac{1}{3}\log(4-3\tan(x))+3\,\text{Subst}\left(\int \frac{x}{(4-3x)^{3/2}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{3}\log(4-3\tan(x))+3\,\text{Subst}\left(\int \left(\frac{4}{3(4-3x)^{3/2}}-\frac{1}{3\sqrt{4-3x}}\right)dx, x, \tan(x)\right)\\
&= \frac{1}{3}\log(4-3\tan(x))+\frac{8}{3\sqrt{4-3\tan(x)}}+\frac{2}{3}\sqrt{4-3\tan(x)}
\end{aligned}$$

Mathematica [A] time = 1.20141, size = 38, normalized size = 0.95

$$\frac{-6\tan(x)+\sqrt{4-3\tan(x)}\log(4-3\tan(x))+16}{3\sqrt{4-3\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(-Sqrt[4 - 3*Tan[x]] + 3*Tan[x]))/(4 - 3*Tan[x])^(3/2),
x]
```

```
[Out] (16 + Log[4 - 3*Tan[x]]*Sqrt[4 - 3*Tan[x]] - 6*Tan[x])/(3*Sqrt[4 - 3*Tan[x]
])
```

Maple [B] time = 0.461, size = 219, normalized size = 5.5

$$\frac{(\cos(x)-1)^2(\cos(x)+1)^2}{(12\cos(x)-9\sin(x))(\sin(x))^4}\left(16\sqrt{\frac{4\cos(x)-3\sin(x)}{\cos(x)}}\cos(x)+4\cos(x)\ln\left(-\frac{\cos(x)-2\sin(x)-1}{\sin(x)}\right)-4\cos(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x)
```

```
[Out] 1/3*(cos(x)-1)^2*(cos(x)+1)^2*(16*((4*cos(x)-3*sin(x))/cos(x))^(1/2)*cos(x)
+4*cos(x)*ln(-(cos(x)-2*sin(x)-1)/sin(x))-4*cos(x)*ln(-(-sin(x)-1+cos(x))/s
in(x))+4*cos(x)*ln(-(2*cos(x)+sin(x)-2)/sin(x))-4*cos(x)*ln(-(sin(x)-1+cos(
x))/sin(x))-6*((4*cos(x)-3*sin(x))/cos(x))^(1/2)*sin(x)-3*sin(x)*ln(-(cos(x
)-2*sin(x)-1)/sin(x))+3*sin(x)*ln(-(-sin(x)-1+cos(x))/sin(x))-3*sin(x)*ln(-
(2*cos(x)+sin(x)-2)/sin(x))+3*sin(x)*ln(-(sin(x)-1+cos(x))/sin(x)))/(4*cos(
x)-3*sin(x))/sin(x)^4
```

Maxima [A] time = 0.934669, size = 41, normalized size = 1.02

$$\frac{2}{3}\sqrt{-3\tan(x)+4}+\frac{8}{3\sqrt{-3\tan(x)+4}}+\frac{1}{3}\log(-3\tan(x)+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x, alg
orithm="maxima")
```

```
[Out] 2/3*sqrt(-3*tan(x) + 4) + 8/3/sqrt(-3*tan(x) + 4) + 1/3*log(-3*tan(x) + 4)
```

Fricas [B] time = 2.07839, size = 259, normalized size = 6.48

$$\frac{(4\cos(x)-3\sin(x))\log\left(\frac{7}{4}\cos(x)^2-6\cos(x)\sin(x)+\frac{9}{4}\right)-(4\cos(x)-3\sin(x))\log\left(\cos(x)^2\right)+4\sqrt{\frac{4\cos(x)-3\sin}{\cos(x)}}}{6(4\cos(x)-3\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x, alg
orithm="fricas")
```

```
[Out] 1/6*((4*cos(x) - 3*sin(x))*log(7/4*cos(x)^2 - 6*cos(x)*sin(x) + 9/4) - (4*c
os(x) - 3*sin(x))*log(cos(x)^2) + 4*sqrt((4*cos(x) - 3*sin(x))/cos(x))*(8*c
```

```
os(x) - 3*sin(x)))/(4*cos(x) - 3*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(4-3*tan(x))**(1/2)+3*tan(x))/cos(x)**2/(4-3*tan(x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20298, size = 42, normalized size = 1.05

$$\frac{2}{3}\sqrt{-3\tan(x)+4}+\frac{8}{3\sqrt{-3\tan(x)+4}}+\frac{1}{3}\log(|-3\tan(x)+4|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-(4-3*tan(x))^(1/2)+3*tan(x))/cos(x)^2/(4-3*tan(x))^(3/2),x, alg
orithm="giac")
```

```
[Out] 2/3*sqrt(-3*tan(x) + 4) + 8/3/sqrt(-3*tan(x) + 4) + 1/3*log(abs(-3*tan(x) +
 4))
```

3.401 $\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx$

Optimal. Leaf size=84

$$-\frac{x}{2}+\frac{\tan^{-1}\left(\frac{1-\tan(x)}{\sqrt{2}\sqrt{\tan(x)}}\right)}{\sqrt{2}}+\frac{1}{1-\sqrt{\tan(x)}}+\log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{2}\log(\cos(x))+\frac{\tanh^{-1}\left(\frac{\tan(x)+1}{\sqrt{2}\sqrt{\tan(x)}}\right)}{\sqrt{2}}$$

```
[Out] -x/2 + ArcTan[(1 - Tan[x])/(Sqrt[2]*Sqrt[Tan[x]])]/Sqrt[2] + ArcTanh[(1 + T
an[x])/(Sqrt[2]*Sqrt[Tan[x]])]/Sqrt[2] + Log[Cos[x]]/2 + Log[1 - Sqrt[Tan[x
]]] + (1 - Sqrt[Tan[x]])^(-1)
```

Rubi [A] time = 0.371099, antiderivative size = 133, normalized size of antiderivative = 1.58, number of steps used = 19, number of rules used = 13, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {3670, 6725, 1831, 297, 1162, 617, 204, 1165, 628, 1248, 635, 203, 260}

$$-\frac{x}{2}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)}{\sqrt{2}}+\frac{1}{1-\sqrt{\tan(x)}}+\log\left(1-\sqrt{\tan(x)}\right)-\frac{\log\left(\tan(x)-\sqrt{2}\sqrt{\text{ta}}\right.}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(-1 + Sqrt[Tan[x]])^2,x]
```

```
[Out] -x/2 + ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Sqrt[T
an[x]]]/Sqrt[2] + Log[Cos[x]]/2 + Log[1 - Sqrt[Tan[x]]] - Log[1 - Sqrt[2]*S
qrt[Tan[x]] + Tan[x]]/(2*Sqrt[2]) + Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]/
(2*Sqrt[2]) + (1 - Sqrt[Tan[x]])^(-1)
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1831

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[
{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)
))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{
a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
```

```
b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\tan(x)}{\left(-1+\sqrt{\tan(x)}\right)^2}\,dx = \text{Subst}\left(\int \frac{x}{\left(-1+\sqrt{x}\right)^2(1+x^2)}\,dx, x, \tan(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{x^3}{(-1+x)^2(1+x^4)}\,dx, x, \sqrt{\tan(x)}\right)$$
$$= 2\,\text{Subst}\left(\int \left(\frac{1}{2(-1+x)^2}+\frac{1}{2(-1+x)}-\frac{x(1+x)^2}{2(1+x^4)}\right)dx, x, \sqrt{\tan(x)}\right)$$
$$= \log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}-\text{Subst}\left(\int \frac{x(1+x)^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)$$
$$= \log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}-\text{Subst}\left(\int \left(\frac{2x^2}{1+x^4}+\frac{x(1+x^2)}{1+x^4}\right)dx, x, \sqrt{\tan(x)}\right)$$
$$= \log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}-2\,\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)-\text{Subst}\left(\int \frac{x(1+$$
$$= \log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}-\frac{1}{2}\text{Subst}\left(\int \frac{1+x}{1+x^2}\,dx, x, \tan(x)\right)+\text{Subst}\left(\int \frac{1-x^2}{1+x^4}$$
$$= \log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{1-\sqrt{\tan(x)}}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(x)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{x}{1+x}$$
$$= -\frac{x}{2}+\frac{1}{2}\log(\cos(x))+\log\left(1-\sqrt{\tan(x)}\right)-\frac{\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)}{2\sqrt{2}}+\frac{\log\left(1+\sqrt{2}\sqrt{\right.}}{2}$$
$$= -\frac{x}{2}+\frac{\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}-\frac{\tan^{-1}\left(1+\sqrt{2}\sqrt{\tan(x)}\right)}{\sqrt{2}}+\frac{1}{2}\log(\cos(x))+\log\left(1-\sqrt{\tan(\right.}$$

Mathematica [C] time = 0.292647, size = 62, normalized size = 0.74

$$-\frac{2}{3}\tan^{\frac{3}{2}}(x)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\tan^2(x)\right)-\frac{1}{2}\tan^{-1}(\tan(x))+\frac{1}{1-\sqrt{\tan(x)}}+\log\left(1-\sqrt{\tan(x)}\right)+\frac{1}{2}\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(-1 + Sqrt[Tan[x]])^2,x]
```

```
[Out] -ArcTan[Tan[x]]/2 + Log[Cos[x]]/2 + Log[1 - Sqrt[Tan[x]]] + (1 - Sqrt[Tan[x
]])^(-1) - (2*Hypergeometric2F1[3/4, 1, 7/4, -Tan[x]^2]*Tan[x]^(3/2))/3
```

Maple [A] time = 0.024, size = 97, normalized size = 1.2

$$-\left(-1+\sqrt{\tan(x)}\right)^{-1}+\ln\left(-1+\sqrt{\tan(x)}\right)-\frac{\sqrt{2}}{2}\arctan\left(1+\sqrt{2}\sqrt{\tan(x)}\right)-\frac{\sqrt{2}}{2}\arctan\left(-1+\sqrt{2}\sqrt{\tan(x)}\right)-\frac{\sqrt{2}}{4}\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(-1+tan(x)^(1/2))^2,x)
```

```
[Out] -1/(-1+tan(x)^(1/2))+ln(-1+tan(x)^(1/2))-1/2*arctan(1+2^(1/2)*tan(x)^(1/2))
*2^(1/2)-1/2*arctan(-1+2^(1/2)*tan(x)^(1/2))*2^(1/2)-1/4*2^(1/2)*ln((1-2^(1
```

```
/2)*tan(x)^(1/2)+tan(x))/(1+2^(1/2)*tan(x)^(1/2)+tan(x)))-1/4*ln(tan(x)^2+1
)-1/2*x
```

Maxima [A] time = 1.42359, size = 158, normalized size = 1.88

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2}-2\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)-\frac{1}{4}\sqrt{2}\left(\sqrt{2}+2\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)-\frac{1}{8}\sqrt{2}\left(\sqrt{2}-2\right)\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-1+tan(x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2) - 2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) -
1/4*sqrt(2)*(sqrt(2) + 2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) -
 1/8*sqrt(2)*(sqrt(2) - 2)*log(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/8*sqr
t(2)*(sqrt(2) + 2)*log(-sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/(sqrt(tan(x)
) - 1) + log(sqrt(tan(x)) - 1)
```

Fricas [C] time = 11.7255, size = 2503, normalized size = 29.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-1+tan(x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(2*sqrt(-I) - I + 1)*(tan(x) - 1)*log(-1/2*(2*sqrt(-I) - I + 1)^2*(
4*(-1)^(1/4) + 2*I + 1) - (2*(-1)^(1/4) + I + 1)^3 - ((2*(-1)^(1/4) + I + 1
)^2 - 8*(-1)^(1/4) - 4*I - 3)*(2*sqrt(-I) - I + 1) + 4*(2*(-1)^(1/4) + I +
1)^2 + 6*sqrt(tan(x)) - 16*(-1)^(1/4) - 8*I - 9) + 2*(2*(-1)^(1/4) + I + 1)
*(tan(x) - 1)*log((2*(-1)^(1/4) + I + 1)^3 - 7/2*(2*(-1)^(1/4) + I + 1)^2 +
 6*sqrt(tan(x)) + 14*(-1)^(1/4) + 7*I + 14) - ((2*sqrt(-I) - I + 1)*(tan(x)
 - 1) + (2*(-1)^(1/4) + I + 1)*(tan(x) - 1) - 4*sqrt(-3/16*(2*sqrt(-I) - I
+ 1)^2 - 3/16*(2*(-1)^(1/4) + I + 1)^2 - 1/8*(2*sqrt(-I) - I + 1)*(2*(-1)^(
1/4) + I - 3) + (-1)^(1/4) + 1/2*I - 1/2)*(tan(x) - 1) - 4*tan(x) + 4)*log(
1/4*(2*sqrt(-I) - I + 1)^2*(4*(-1)^(1/4) + 2*I + 1) + 1/2*((2*(-1)^(1/4) +
I + 1)^2 - 8*(-1)^(1/4) - 4*I - 3)*(2*sqrt(-I) - I + 1) - 1/4*(2*(-1)^(1/4)
 + I + 1)^2 + sqrt(-3/16*(2*sqrt(-I) - I + 1)^2 - 3/16*(2*(-1)^(1/4) + I +
1)^2 - 1/8*(2*sqrt(-I) - I + 1)*(2*(-1)^(1/4) + I - 3) + (-1)^(1/4) + 1/2*I
 - 1/2)*((2*sqrt(-I) - I + 1)*(4*(-1)^(1/4) + 2*I + 1) - 2*(-1)^(1/4) - I +
 1) + 6*sqrt(tan(x)) + (-1)^(1/4) + 1/2*I - 5/2) - ((2*sqrt(-I) - I + 1)*(t
an(x) - 1) + (2*(-1)^(1/4) + I + 1)*(tan(x) - 1) + 4*sqrt(-3/16*(2*sqrt(-I)
 - I + 1)^2 - 3/16*(2*(-1)^(1/4) + I + 1)^2 - 1/8*(2*sqrt(-I) - I + 1)*(2*(
-1)^(1/4) + I - 3) + (-1)^(1/4) + 1/2*I - 1/2)*(tan(x) - 1) - 4*tan(x) + 4)
*log(1/4*(2*sqrt(-I) - I + 1)^2*(4*(-1)^(1/4) + 2*I + 1) + 1/2*((2*(-1)^(1/
4) + I + 1)^2 - 8*(-1)^(1/4) - 4*I - 3)*(2*sqrt(-I) - I + 1) - 1/4*(2*(-1)^
(1/4) + I + 1)^2 - sqrt(-3/16*(2*sqrt(-I) - I + 1)^2 - 3/16*(2*(-1)^(1/4) +
 I + 1)^2 - 1/8*(2*sqrt(-I) - I + 1)*(2*(-1)^(1/4) + I - 3) + (-1)^(1/4) +
1/2*I - 1/2)*((2*sqrt(-I) - I + 1)*(4*(-1)^(1/4) + 2*I + 1) - 2*(-1)^(1/4)
- I + 1) + 6*sqrt(tan(x)) + (-1)^(1/4) + 1/2*I - 5/2) - 8*(tan(x) - 1)*log(
sqrt(tan(x)) - 1) + 8*sqrt(tan(x)) + 8)/(tan(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\left(\sqrt{\tan(x)}-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-1+tan(x)**(1/2))**2,x)
```

```
[Out] Integral(tan(x)/(sqrt(tan(x)) - 1)**2, x)
```

Giac [A] time = 1.21259, size = 150, normalized size = 1.79

$$-\frac{1}{2}\left(\sqrt{2}-1\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2\sqrt{\tan(x)}\right)\right)-\frac{1}{2}\left(\sqrt{2}+1\right)\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2\sqrt{\tan(x)}\right)\right)+\frac{1}{4}\sqrt{2}\log\left(\sqrt{2}\sqrt{\tan}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-1+tan(x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(2) - 1)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(tan(x)))) - 1/2*(sq
rt(2) + 1)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(tan(x)))) + 1/4*sqrt(2)*lo
g(sqrt(2)*sqrt(tan(x)) + tan(x) + 1) - 1/4*sqrt(2)*log(-sqrt(2)*sqrt(tan(x)
) + tan(x) + 1) - 1/(sqrt(tan(x)) - 1) - 1/4*log(tan(x)^2 + 1) + log(abs(sq
rt(tan(x)) - 1))
```

3.402 $\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))-\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

```
[Out] -ArcSin[Cos[x] - Sin[x]]/2 - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

Rubi [A] time = 0.015072, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4306}

$$-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))-\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/Sqrt[Sin[2*x]],x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]]/2 - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx=-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))-\frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

Mathematica [A] time = 0.0376623, size = 31, normalized size = 1.

$$\frac{1}{2}\left(-\sin^{-1}(\cos(x)-\sin(x))-\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/Sqrt[Sin[2*x]],x]
```

```
[Out] (-ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/2
```

Maple [C] time = 0.041, size = 266, normalized size = 8.6

$$-\frac{1}{2}\sqrt{-\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)\left(2\sqrt{1+\tan(x/2)}\sqrt{-2\tan(x/2)+2}\sqrt{-\tan(x/2)}\text{EllipticE}\left(\sqrt{1+\text{ta}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/sin(2*x)^(1/2),x)
```

```
[Out] -1/2*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(2*(1+tan(1/2*x)
)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x)
)^(1/2),1/2*2^(1/2))*tan(1/2*x)^2-(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1
/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2
*x)^2+2*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*El
lipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))-(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*
x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))
+2*tan(1/2*x)^4-2*tan(1/2*x)^2)/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(tan(1/
2*x)^2+1)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)/sqrt(sin(2*x)), x)
```

Fricas [B] time = 2.01286, size = 455, normalized size = 14.68

$$\frac{1}{4}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)-\frac{1}{4}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\mathrm{si}}{\cos(x)-\sin(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))
/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/4*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin
(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) + 1/8*log(-32*cos(x)^4 + 4*sqrt(
2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) +
32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/sin(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(x)/sqrt(sin(2*x)), x)
```

3.403 $\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))$$

```
[Out] -ArcSin[Cos[x] - Sin[x]]/2 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

Rubi [A] time = 0.0146254, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4305}

$$\frac{1}{2}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/Sqrt[Sin[2*x]],x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]]/2 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx=-\frac{1}{2}\sin^{-1}(\cos(x)-\sin(x))+\frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)$$

Mathematica [A] time = 0.0290116, size = 29, normalized size = 0.94

$$\frac{1}{2}\left(\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\sin^{-1}(\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/Sqrt[Sin[2*x]],x]
```

```
[Out] (-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/2
```

Maple [C] time = 0.048, size = 98, normalized size = 3.2

$$\sqrt{-\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)\sqrt{1+\tan\left(\frac{x}{2}\right)}\sqrt{-2\tan(x/2)+2}\sqrt{-\tan\left(\frac{x}{2}\right)}\text{EllipticF}\left(\sqrt{1+\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(2*x)^(1/2),x)
```

```
[Out] (-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/(tan(1/2*x)*(tan(1/2*
x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(
1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)/sqrt(sin(2*x)), x)
```

Fricas [B] time = 1.98261, size = 455, normalized size = 14.68

$$\frac{1}{4}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)-\frac{1}{4}\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))
/(cos(x)^2 + 2*cos(x)*sin(x) - 1)) - 1/4*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin
(x)) - cos(x) - sin(x))/(cos(x) - sin(x))) - 1/8*log(-32*cos(x)^4 + 4*sqrt(
2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) +
32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(x)/sqrt(sin(2*x)), x)
```

3.404 $\int \sin(x)\sqrt{\sin(2x)}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{4}\sin^{-1}(\cos(x)-\sin(x))-\frac{1}{2}\sqrt{\sin(2x)}\cos(x)+\frac{1}{4}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

```
[Out] -ArcSin[Cos[x] - Sin[x]]/4 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/4 - (Cos
[x]*Sqrt[Sin[2*x]])/2
```

Rubi [A] time = 0.0297949, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4302, 4305}

$$-\frac{1}{4}\sin^{-1}(\cos(x)-\sin(x))-\frac{1}{2}\sqrt{\sin(2x)}\cos(x)+\frac{1}{4}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sqrt[Sin[2*x]],x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]]/4 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/4 - (Cos
[x]*Sqrt[Sin[2*x]])/2
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \sin(x)\sqrt{\sin(2x)}\,dx &= -\frac{1}{2}\cos(x)\sqrt{\sin(2x)}+\frac{1}{2}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx\\ &= -\frac{1}{4}\sin^{-1}(\cos(x)-\sin(x))+\frac{1}{4}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)-\frac{1}{2}\cos(x)\sqrt{\sin(2x)}\end{aligned}$$

Mathematica [A] time = 0.0352929, size = 41, normalized size = 0.91

$$\frac{1}{4}\left(-\sin^{-1}(\cos(x)-\sin(x))-2\sqrt{\sin(2x)}\cos(x)+\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sqrt[Sin[2*x]],x]
```

```
[Out] (-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]] - 2*Cos[x
]*Sqrt[Sin[2*x]])/4
```

Maple [C] time = 0.053, size = 171, normalized size = 3.8

$$\sqrt{-\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)\left(\sqrt{1+\tan\left(\frac{x}{2}\right)}\sqrt{-2\tan(x/2)+2}\sqrt{-\tan\left(\frac{x}{2}\right)}\mathrm{EllipticF}\left(\sqrt{1+\tan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(2*x)^(1/2),x)
```

```
[Out] (-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*((1+tan(1/2*x))^(1/2)
*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2)
,1/2*2^(1/2))*tan(1/2*x)^2+(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-t
an(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))+2*tan(1/2*x)^3
-2*tan(1/2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x)
)^(1/2)/(tan(1/2*x)^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(2x)}\sin(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(2*x))*sin(x), x)
```

Fricas [B] time = 1.97212, size = 513, normalized size = 11.4

$$-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}\cos(x)+\frac{1}{8}\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)-\frac{1}{8}\arctan\left(-\frac{2}{\ }\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*sqrt(cos(x)*sin(x))*cos(x) + 1/8*arctan(-(sqrt(2)*sqrt(cos(x)*
sin(x))*(cos(x) - sin(x)) + cos(x)*sin(x))/(cos(x)^2 + 2*cos(x)*sin(x) - 1)
) - 1/8*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin(x)) - cos(x) - sin(x))/(cos(x) -
 sin(x))) - 1/16*log(-32*cos(x)^4 + 4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1
)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x)^2 + 16*cos(x)*sin(x) +
 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(2x)}\sin(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(2*x))*sin(x), x)
```

3.405 $\int (\cos(x) - \sin(x))\sqrt{\sin(2x)}\, dx$

Optimal. Leaf size=47

$$\frac{1}{2}\sin(x)\sqrt{\sin(2x)} + \frac{1}{2}\sqrt{\sin(2x)}\cos(x) - \frac{1}{2}\log\left(\sin(x) + \sqrt{\sin(2x)} + \cos(x)\right)$$

```
[Out] -Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2 + (Cos[x]*Sqrt[Sin[2*x]])/2 + (Sin
[x]*Sqrt[Sin[2*x]])/2
```

Rubi [A] time = 0.09618, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {4401, 4301, 4306, 4302, 4305}

$$\frac{1}{2}\sin(x)\sqrt{\sin(2x)} + \frac{1}{2}\sqrt{\sin(2x)}\cos(x) - \frac{1}{2}\log\left(\sin(x) + \sqrt{\sin(2x)} + \cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] - Sin[x])*Sqrt[Sin[2*x]],x]
```

```
[Out] -Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/2 + (Cos[x]*Sqrt[Sin[2*x]])/2 + (Sin
[x]*Sqrt[Sin[2*x]])/2
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 4301

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(2*Sin[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p*
g)/(2*p + 1), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && G
tQ[p, 0] && IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rule 4302

```
Int[sin[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(-2*Cos[a + b*x]*(g*Sin[c + d*x])^p)/(d*(2*p + 1)), x] + Dist[(2*p
*g)/(2*p + 1), Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{
a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &&
GtQ[p, 0] && IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
```

```
a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}
\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx &= \int \left(\cos(x)\sqrt{\sin(2x)}-\sin(x)\sqrt{\sin(2x)}\right)\,dx\\
&= \int \cos(x)\sqrt{\sin(2x)}\,dx - \int \sin(x)\sqrt{\sin(2x)}\,dx\\
&= \frac{1}{2}\cos(x)\sqrt{\sin(2x)}+\frac{1}{2}\sin(x)\sqrt{\sin(2x)}-\frac{1}{2}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx+\frac{1}{2}\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx\\
&= -\frac{1}{2}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right)+\frac{1}{2}\cos(x)\sqrt{\sin(2x)}+\frac{1}{2}\sin(x)\sqrt{\sin(2x)}
\end{aligned}$$

Mathematica [A] time = 0.0618678, size = 43, normalized size = 0.91

$$\frac{1}{2}\left(\sin(x)\sqrt{\sin(2x)}+\sqrt{\sin(2x)}\cos(x)-\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] - Sin[x])*Sqrt[Sin[2*x]],x]
```

```
[Out] (-Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]] + Cos[x]*Sqrt[Sin[2*x]] + Sin[x]*Sq
rt[Sin[2*x]])/2
```

Maple [C] time = 0.08, size = 443, normalized size = 9.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)-sin(x))*sin(2*x)^(1/2),x)
```

```
[Out] -(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(3*(1+tan(1/2*x))^(1
/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1
/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x
)^2-4*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Elli
pticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1
/2*x)))^(1/2)*tan(1/2*x)^2+3*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(
-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*
(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)-4*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+
2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(t
an(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)-4*(tan(1/2*x)^3-tan(1/2*x))^
(1/2)*tan(1/2*x)^4+2*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1
/2*x)^3-4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^2-2*(tan(1/2*x)*(tan(1
/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1
/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/(tan(1/2*x)^2+1)/(tan(1/2*x)*(tan(1/2*x
)-1)*(1+tan(1/2*x)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((cos(x) - sin(x))*sqrt(sin(2*x)), x)
```

Fricas [B] time = 1.89208, size = 262, normalized size = 5.57

$$\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)+\sin(x))+\frac{1}{8}\log\left(-32\cos(x)^4+4\sqrt{2}\left(4\cos(x)^3-\left(4\cos(x)^2+1\right)\sin(x)-5\cos(x)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) + sin(x)) + 1/8*log(-32*cos(x)^4 +
4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin
(x)) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))*sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int(\cos(x)-\sin(x))\sqrt{\sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)-sin(x))*sin(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((cos(x) - sin(x))*sqrt(sin(2*x)), x)
```

3.406 $\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)} dx$

Optimal. Leaf size=61

$$\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4\sqrt{\sin(2x)}} - \frac{1}{16}\sin^{-1}(\cos(x) - \sin(x)) + \frac{1}{16}\log\left(\sin(x) + \sqrt{\sin(2x)} + \cos(x)\right)$$

```
[Out] -ArcSin[Cos[x] - Sin[x]]/16 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/16 + Si
n[x]^5/(5*Sin[2*x]^(5/2)) - Sin[x]/(4*Sqrt[Sin[2*x]])
```

Rubi [A] time = 0.0801925, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4294, 4308, 4305}

$$\frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4\sqrt{\sin(2x)}} - \frac{1}{16}\sin^{-1}(\cos(x) - \sin(x)) + \frac{1}{16}\log\left(\sin(x) + \sqrt{\sin(2x)} + \cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^7/Sin[2*x]^(7/2),x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]]/16 + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/16 + Si
n[x]^5/(5*Sin[2*x]^(5/2)) - Sin[x]/(4*Sqrt[Sin[2*x]])
```

Rule 4294

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[(e^2*(e*Sin[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p +
1))/(2*b*g*(p + 1)), x] + Dist[(e^4*(m + p - 1))/(4*g^2*(p + 1)), Int[(e*Si
n[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d,
e, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 2]
 && LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4305

```
Int[cos[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] + Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx &= \frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{1}{4}\int \frac{\sin^3(x)}{\sin^{\frac{3}{2}}(2x)}\,dx\\ &= \frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4\sqrt{\sin(2x)}} + \frac{1}{16}\int \csc(x)\sqrt{\sin(2x)}\,dx\\ &= \frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4\sqrt{\sin(2x)}} + \frac{1}{8}\int \frac{\cos(x)}{\sqrt{\sin(2x)}}\,dx\\ &= -\frac{1}{16}\sin^{-1}(\cos(x)-\sin(x)) + \frac{1}{16}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) + \frac{\sin^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{\sin(x)}{4\sqrt{\sin(2x)}}\end{aligned}$$

Mathematica [A] time = 0.0879294, size = 50, normalized size = 0.82

$$\frac{1}{80}\left(2\sqrt{\sin(2x)}\sec(x)\left(\sec^2(x)-6\right)+5\left(\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)-\sin^{-1}(\cos(x)-\sin(x))\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^7/Sin[2*x]^(7/2),x]
```

```
[Out] (5*(-ArcSin[Cos[x] - Sin[x]] + Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]) + 2*S
ec[x]*(-6 + Sec[x]^2)*Sqrt[Sin[2*x]])/80
```

Maple [C] time = 0.105, size = 510, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^7/sin(2*x)^(7/2),x)
```

```
[Out] 1/2688*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(5*(1+tan(1/2*
x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*
x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^14+35*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)
+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*t
an(1/2*x)^12+10*tan(1/2*x)^15+105*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1
/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2
*x)^10+66*tan(1/2*x)^13+175*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-
tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^8-
1014*tan(1/2*x)^11+175*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1
/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^6+2002*
tan(1/2*x)^9+105*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))
^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^4-2002*tan(1/
2*x)^7+35*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*
EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^2+1014*tan(1/2*x)^5+
5*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Elliptic
F((1+tan(1/2*x))^(1/2),1/2*2^(1/2))-66*tan(1/2*x)^3-10*tan(1/2*x))/(tan(1/2
*x)*(tan(1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/(tan(1/2*x)^2+1
)^7
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)^7/sin(2*x)^(7/2), x)
```

Fricas [B] time = 2.07311, size = 594, normalized size = 9.74

$$10\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)\cos(x)^3-10\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cos(x)-\sin(x)}{\cos(x)-\sin(x)}\right)\cos(x)^3-5\cos($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/320*(10*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin(x)) + cos(x)*s
in(x))/(cos(x)^2 + 2*cos(x)*sin(x) - 1))*cos(x)^3 - 10*arctan(-(2*sqrt(2)*s
qrt(cos(x)*sin(x)) - cos(x) - sin(x))/(cos(x) - sin(x)))*cos(x)^3 - 5*cos(x
)^3*log(-32*cos(x)^4 + 4*sqrt(2)*(4*cos(x)^3 - (4*cos(x)^2 + 1)*sin(x) - 5*
cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x)^2 + 16*cos(x)*sin(x) + 1) - 48*cos(
x)^3 - 8*sqrt(2)*(6*cos(x)^2 - 1)*sqrt(cos(x)*sin(x)))/cos(x)^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**7/sin(2*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^7/sin(2*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(sin(x)^7/sin(2*x)^(7/2), x)
```

3.407 $\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx$

Optimal. Leaf size=61

$$-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{1}{16}\sin^{-1}(\cos(x)-\sin(x))+\frac{\cos(x)}{4\sqrt{\sin(2x)}}-\frac{1}{16}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

```
[Out] -ArcSin[Cos[x] - Sin[x]]/16 - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/16 - Co
s[x]^5/(5*Sin[2*x]^(5/2)) + Cos[x]/(4*Sqrt[Sin[2*x]])
```

Rubi [A] time = 0.082215, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4293, 4307, 4306}

$$-\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{1}{16}\sin^{-1}(\cos(x)-\sin(x))+\frac{\cos(x)}{4\sqrt{\sin(2x)}}-\frac{1}{16}\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^7/Sin[2*x]^(7/2),x]
```

```
[Out] -ArcSin[Cos[x] - Sin[x]]/16 - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]]/16 - Co
s[x]^5/(5*Sin[2*x]^(5/2)) + Cos[x]/(4*Sqrt[Sin[2*x]])
```

Rule 4293

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> Simp[(e^2*(e*Cos[a + b*x])^(m - 2)*(g*Sin[c + d*x])^(p + 1
))/(2*b*g*(p + 1)), x] + Dist[(e^4*(m + p - 1))/(4*g^2*(p + 1)), Int[(e*Cos
[a + b*x])^(m - 4)*(g*Sin[c + d*x])^(p + 2), x], x] /; FreeQ[{a, b, c, d, e
, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && GtQ[m, 2]
&& LtQ[p, -1] && (GtQ[m, 3] || EqQ[p, -3/2]) && IntegersQ[2*m, 2*p]
```

Rule 4307

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/cos[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4306

```
Int[sin[(a_.) + (b_.)*(x_)]/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> -Si
mp[ArcSin[Cos[a + b*x] - Sin[a + b*x]]/d, x] - Simp[Log[Cos[a + b*x] + Sin[
a + b*x] + Sqrt[Sin[c + d*x]]]/d, x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c -
 a*d, 0] && EqQ[d/b, 2]
```

Rubi steps

$$\int \frac{\cos^7(x)}{\sin^{\frac{7}{2}}(2x)}\,dx = -\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{1}{4}\int \frac{\cos^3(x)}{\sin^{\frac{3}{2}}(2x)}\,dx$$
$$= -\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} + \frac{\cos(x)}{4\sqrt{\sin(2x)}} + \frac{1}{16}\int \sec(x)\sqrt{\sin(2x)}\,dx$$
$$= -\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} + \frac{\cos(x)}{4\sqrt{\sin(2x)}} + \frac{1}{8}\int \frac{\sin(x)}{\sqrt{\sin(2x)}}\,dx$$
$$= -\frac{1}{16}\sin^{-1}(\cos(x)-\sin(x)) - \frac{1}{16}\log\left(\cos(x)+\sin(x)+\sqrt{\sin(2x)}\right) - \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} + \frac{\cos(x)}{4\sqrt{\sin(2x)}}$$

Mathematica [A] time = 0.0739581, size = 56, normalized size = 0.92

$$\sqrt{\sin(2x)}\left(\frac{3\csc(x)}{20} - \frac{\csc^3(x)}{40}\right) + \frac{1}{16}\left(-\sin^{-1}(\cos(x)-\sin(x)) - \log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^7/Sin[2*x]^(7/2),x]
```

```
[Out] (-ArcSin[Cos[x] - Sin[x]] - Log[Cos[x] + Sin[x] + Sqrt[Sin[2*x]]])/16 + ((3
*Csc[x])/20 - Csc[x]^3/40)*Sqrt[Sin[2*x]]
```

Maple [C] time = 0.058, size = 1108, normalized size = 18.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/sin(2*x)^(7/2),x)
```

```
[Out] 1/160*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(192*(tan(1/2*x)*(t
an(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/
2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/
2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^6-96*(tan(1/2*x)*(tan(1/2*x)^2-1))
^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Ell
ipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(
1/2*x)))^(1/2)*tan(1/2*x)^6-(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)
*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^10-384*(tan(1/2*x)*(tan(1/
2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))
^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-
1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^4+192*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/
2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Ellipti
cF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*
x)))^(1/2)*tan(1/2*x)^4+96*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^
3-tan(1/2*x))^(1/2)*tan(1/2*x)^8+3*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan
(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^8+48*(tan(1/2*x)^3-
tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*
x)^8+192*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1
/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/
2))*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^2-96*(tan(1
/2*x)*(tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*
```

```
(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)
*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^2-192*(tan(1/2*x)*(tan(1/2
*x)^2-1))^(1/2)*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*tan(1/2*x)^6+14*(tan(1/2*x)
*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*t
an(1/2*x)^6-144*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*
(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^6+96*tan(1/2*x)^4*(tan(1/2*x)^3-tan(1/2*x)
)^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)+14*tan(1/2*x)^4*(tan(1/2*x)*(ta
n(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)+144*(
tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(
1/2)*tan(1/2*x)^4+3*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*(tan(1/
2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^2-48*(tan(1/2*x)^3-tan(1/2*x))^(1/
2)*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^2-(tan(1/2*x
)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2))
/(tan(1/2*x)^2-1)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/(tan(1/2*x)*(tan(1/2*x)-1
)*(1+tan(1/2*x)))^(1/2)/(tan(1/2*x)-1)/(1+tan(1/2*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(x)^7/sin(2*x)^(7/2), x)
```

Fricas [B] time = 2.13289, size = 684, normalized size = 11.21

$$10\left(\cos(x)^2-1\right)\arctan\left(-\frac{\sqrt{2}\sqrt{\cos(x)\sin(x)}(\cos(x)-\sin(x))+\cos(x)\sin(x)}{\cos(x)^2+2\cos(x)\sin(x)-1}\right)\sin(x)-10\left(\cos(x)^2-1\right)\arctan\left(-\frac{2\sqrt{2}\sqrt{\cos(x)\sin(x)}-\cdots}{\cos(x)-\sin\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/320*(10*(cos(x)^2 - 1)*arctan(-(sqrt(2)*sqrt(cos(x)*sin(x))*(cos(x) - sin
(x)) + cos(x)*sin(x))/(cos(x)^2 + 2*cos(x)*sin(x) - 1))*sin(x) - 10*(cos(x)
^2 - 1)*arctan(-(2*sqrt(2)*sqrt(cos(x)*sin(x)) - cos(x) - sin(x))/(cos(x) -
 sin(x)))*sin(x) + 5*(cos(x)^2 - 1)*log(-32*cos(x)^4 + 4*sqrt(2)*(4*cos(x)^
3 - (4*cos(x)^2 + 1)*sin(x) - 5*cos(x))*sqrt(cos(x)*sin(x)) + 32*cos(x)^2 +
 16*cos(x)*sin(x) + 1)*sin(x) + 8*sqrt(2)*(6*cos(x)^2 - 5)*sqrt(cos(x)*sin(
x)) + 48*(cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**7/sin(2*x)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)^7}{\sin(2x)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/sin(2*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cos(x)^7/sin(2*x)^(7/2), x)
```

3.408 $\int \csc^5(x) \sin^{\frac{3}{2}}(2x) \, dx$

Optimal. Leaf size=16

$$-\frac{1}{5} \sin^{\frac{5}{2}}(2x) \csc^5(x)$$

[Out] -(Csc[x]^5*Sin[2*x]^(5/2))/5

Rubi [A] time = 0.0224134, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {4292}

$$-\frac{1}{5} \sin^{\frac{5}{2}}(2x) \csc^5(x)$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^5*Sin[2*x]^(3/2),x]

[Out] -(Csc[x]^5*Sin[2*x]^(5/2))/5

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\int \csc^5(x) \sin^{\frac{3}{2}}(2x) \, dx = -\frac{1}{5} \csc^5(x) \sin^{\frac{5}{2}}(2x)$$

Mathematica [A] time = 0.0322744, size = 16, normalized size = 1.

$$-\frac{1}{5} \sin^{\frac{5}{2}}(2x) \csc^5(x)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^5*Sin[2*x]^(3/2),x]

[Out] -(Csc[x]^5*Sin[2*x]^(5/2))/5

Maple [C] time = 0.049, size = 508, normalized size = 31.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)^(3/2)/sin(x)^5,x)
```

```
[Out] 1/5*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(96*(tan(1/2*x)*(tan(
1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x
))^(1/2)*EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x
)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^2-48*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1
/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*Ellipt
icF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2
*x)))^(1/2)*tan(1/2*x)^2-(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*(t
an(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^6+28*(tan(1/2*x)^3-tan(1/2*x)
)^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^4+40*ta
n(1/2*x)^4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1
/2)+tan(1/2*x)^4*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*(tan(1/2*
x)*(tan(1/2*x)^2-1))^(1/2)-28*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(
tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^2+(tan(1/2*x)*(tan(1/2*x)^2-
1))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^2-(ta
n(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))
^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1
/2*x)))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(2x)^{\frac{3}{2}}}{\sin(x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="maxima")
```

```
[Out] integrate(sin(2*x)^(3/2)/sin(x)^5, x)
```

Fricas [B] time = 1.93462, size = 124, normalized size = 7.75

$$\frac{4\left(\sqrt{2}\sqrt{\cos(x)\sin(x)}\cos(x)^2+\left(\cos(x)^2-1\right)\sin(x)\right)}{5\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="fricas")
```

```
[Out] 4/5*(sqrt(2)*sqrt(cos(x)*sin(x))*cos(x)^2 + (cos(x)^2 - 1)*sin(x))/((cos(x)
^2 - 1)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)**(3/2)/sin(x)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(2x)^{\frac{3}{2}}}{\sin(x)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)^(3/2)/sin(x)^5,x, algorithm="giac")
```

```
[Out] integrate(sin(2*x)^(3/2)/sin(x)^5, x)
```

3.409 $\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx$

Optimal. Leaf size=31

$$\frac{1}{5}\sqrt{\sin(2x)}\sec^3(x)+\frac{4}{5}\sqrt{\sin(2x)}\sec(x)$$

```
[Out] (4*Sec[x]*Sqrt[Sin[2*x]])/5 + (Sec[x]^3*Sqrt[Sin[2*x]])/5
```

Rubi [A] time = 0.0413526, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4299, 4291}

$$\frac{1}{5}\sqrt{\sin(2x)}\sec^3(x)+\frac{4}{5}\sqrt{\sin(2x)}\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3/Sqrt[Sin[2*x]],x]
```

```
[Out] (4*Sec[x]*Sqrt[Sin[2*x]])/5 + (Sec[x]^3*Sqrt[Sin[2*x]])/5
```

Rule 4299

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p
_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(2*b*g
*(m + p + 1)), x] + Dist[(m + 2*p + 2)/(e^2*(m + p + 1)), Int[(e*Cos[a + b*
x])^(m + 2)*(g*Sin[c + d*x])^p, x], x] /; FreeQ[{a, b, c, d, e, g, p}, x] &
& EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p] && LtQ[m, -1] && NeQ[m
+ 2*p + 2, 0] && NeQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Rule 4291

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> -Simp[((e*Cos[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*
m), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/
b, 2] &&  !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(x)}{\sqrt{\sin(2x)}}\,dx &= \frac{1}{5}\sec^3(x)\sqrt{\sin(2x)}+\frac{4}{5}\int \frac{\sec(x)}{\sqrt{\sin(2x)}}\,dx\\ &= \frac{4}{5}\sec(x)\sqrt{\sin(2x)}+\frac{1}{5}\sec^3(x)\sqrt{\sin(2x)}\end{aligned}$$

Mathematica [A] time = 0.028768, size = 20, normalized size = 0.65

$$\frac{1}{5}\sqrt{\sin(2x)}\sec(x)\left(\sec^2(x)+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3/Sqrt[Sin[2*x]],x]
```

```
[Out] (Sec[x]*(4 + Sec[x]^2)*Sqrt[Sin[2*x]])/5
```

Maple [C] time = 0.05, size = 286, normalized size = 9.2

$$\frac{1}{12}\sqrt{-\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)\left(5\sqrt{1+\tan(x/2)}\sqrt{-2\tan(x/2)+2}\sqrt{-\tan(x/2)}\text{EllipticF}\left(\sqrt{1+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^3/sin(2*x)^(1/2),x)
```

```
[Out] 1/12*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)*(5*(1+tan(1/2*x)
)^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x)
)^(1/2),1/2*2^(1/2))*tan(1/2*x)^6+15*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)
^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(
1/2*x)^4-14*tan(1/2*x)^7+15*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-
tan(1/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)^2+
2*tan(1/2*x)^5+5*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))
^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))-2*tan(1/2*x)^3+14*tan(1/
2*x))/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)/(
tan(1/2*x)^2+1)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos(x)^3\sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^3/sin(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(cos(x)^3*sqrt(sin(2*x))), x)
```

Fricas [A] time = 1.95203, size = 100, normalized size = 3.23

$$\frac{4\cos(x)^3+\sqrt{2}\left(4\cos(x)^2+1\right)\sqrt{\cos(x)\sin(x)}}{5\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^3/sin(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(4*cos(x)^3 + sqrt(2)*(4*cos(x)^2 + 1)*sqrt(cos(x)*sin(x)))/cos(x)^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**3/sin(2*x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\cos(x)^3 \sqrt{\sin(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^3/sin(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(cos(x)^3*sqrt(sin(2*x))), x)
```

3.410 $\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)} \, dx$

Optimal. Leaf size=29

$$\frac{4\sin(x)}{3\sqrt{\sin(2x)}} - \frac{2\cos(x)}{3\sin^{\frac{3}{2}}(2x)}$$

```
[Out] (-2*Cos[x])/(3*Sin[2*x]^(3/2)) + (4*Sin[x])/(3*Sqrt[Sin[2*x]])
```

Rubi [A] time = 0.0476002, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4308, 4303, 4292}

$$\frac{4\sin(x)}{3\sqrt{\sin(2x)}} - \frac{2\cos(x)}{3\sin^{\frac{3}{2}}(2x)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/Sin[2*x]^(3/2),x]
```

```
[Out] (-2*Cos[x])/(3*Sin[2*x]^(3/2)) + (4*Sin[x])/(3*Sqrt[Sin[2*x]])
```

Rule 4308

```
Int[((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_)/sin[(a_.) + (b_.)*(x_)], x_Symbol]
 :> Dist[2*g, Int[Cos[a + b*x]*(g*Sin[c + d*x])^(p - 1), x], x] /; FreeQ[{a
, b, c, d, g, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !IntegerQ[p] &
& IntegerQ[2*p]
```

Rule 4303

```
Int[cos[(a_.) + (b_.)*(x_)]*((g_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol]
 :> Simp[(Cos[a + b*x]*(g*Sin[c + d*x])^(p + 1))/(2*b*g*(p + 1)), x] + Dist
[(2*p + 3)/(2*g*(p + 1)), Int[Sin[a + b*x]*(g*Sin[c + d*x])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, g}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && !Inte
gerQ[p] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 4292

```
Int[((e_.)*sin[(a_.) + (b_.)*(x_)])^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Simp[((e*Sin[a + b*x])^m*(g*Sin[c + d*x])^(p + 1))/(b*g*m
), x] /; FreeQ[{a, b, c, d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, 2] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(x)}{\sin^{\frac{3}{2}}(2x)} \, dx &= 2\int \frac{\cos(x)}{\sin^{\frac{5}{2}}(2x)} \, dx \\
&= -\frac{2\cos(x)}{3\sin^{\frac{3}{2}}(2x)} + \frac{4}{3}\int \frac{\sin(x)}{\sin^{\frac{3}{2}}(2x)} \, dx \\
&= -\frac{2\cos(x)}{3\sin^{\frac{3}{2}}(2x)} + \frac{4\sin(x)}{3\sqrt{\sin(2x)}}
\end{aligned}$$

Mathematica [A] time = 0.0300172, size = 24, normalized size = 0.83

$$\sqrt{\sin(2x)}\left(\frac{\sec(x)}{2}-\frac{1}{6}\cot(x)\csc(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/Sin[2*x]^(3/2),x]
```

```
[Out] (-(Cot[x]*Csc[x])/6 + Sec[x]/2)*Sqrt[Sin[2*x]]
```

Maple [C] time = 0.039, size = 121, normalized size = 4.2

$$-\frac{1}{12}\sqrt{-\tan\left(\frac{x}{2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)^{-1}}\left(\left(\tan\left(\frac{x}{2}\right)\right)^2-1\right)\left(2\sqrt{1+\tan(x/2)}\sqrt{-2\tan(x/2)+2}\sqrt{-\tan(x/2)}\mathrm{EllipticF}\left(\sqrt{1+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)/sin(2*x)^(3/2),x)
```

```
[Out] -1/12*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)*(tan(1/2*x)^2-1)/tan(1/2*x)*(2*(
1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*EllipticF((
1+tan(1/2*x))^(1/2),1/2*2^(1/2))*tan(1/2*x)-tan(1/2*x)^4+1)/(tan(1/2*x)*(ta
n(1/2*x)^2-1))^(1/2)/(tan(1/2*x)^3-tan(1/2*x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(2x)^{\frac{3}{2}}\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)/sin(2*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sin(2*x)^(3/2)*sin(x)), x)
```

Fricas [B] time = 1.88893, size = 130, normalized size = 4.48

$$\frac{4\cos(x)^3+\sqrt{2}\left(4\cos(x)^2-3\right)\sqrt{\cos(x)\sin(x)}-4\cos(x)}{6\left(\cos(x)^3-\cos(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)/sin(2*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*cos(x)^3 + sqrt(2)*(4*cos(x)^2 - 3)*sqrt(cos(x)*sin(x)) - 4*cos(x))/
(cos(x)^3 - cos(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)/sin(2*x)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(2x)^{\frac{3}{2}} \sin(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)/sin(2*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sin(2*x)^(3/2)*sin(x)), x)
```

3.411 $\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{\left(\sin^2(x)-\sin(2x)\right)\sin^{\frac{5}{2}}(2x)}\,dx$

Optimal. Leaf size=68

$$-\frac{9\cos(x)}{16\sqrt{\sin(2x)}}+\frac{\cos(x)\cot^2(x)}{20\sqrt{\sin(2x)}}-\frac{5\cos(x)\cot(x)}{24\sqrt{\sin(2x)}}+\frac{33}{32}\tanh^{-1}\left(\frac{1}{2}\sqrt{\sin(2x)}\sec(x)\right)$$

```
[Out] (33*ArcTanh[(Sec[x]*Sqrt[Sin[2*x]])/2])/32 - (9*Cos[x])/(16*Sqrt[Sin[2*x]])
 - (5*Cos[x]*Cot[x])/(24*Sqrt[Sin[2*x]]) + (Cos[x]*Cot[x]^2)/(20*Sqrt[Sin[2
*x]])
```

Rubi [A] time = 0.855936, antiderivative size = 95, normalized size of antiderivative = 1.4, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4390, 1619, 63, 207}

$$\frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)}-\frac{5\sin(x)\cos^4(x)}{6\sin^{\frac{5}{2}}(2x)}-\frac{9\sin^2(x)\cos^3(x)}{4\sin^{\frac{5}{2}}(2x)}+\frac{33\sin^5(x)\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*(Cos[2*x] - 3*Tan[x]))/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^(5/2))
,x]
```

```
[Out] Cos[x]^5/(5*Sin[2*x]^(5/2)) - (5*Cos[x]^4*Sin[x])/(6*Sin[2*x]^(5/2)) - (9*C
os[x]^3*Sin[x]^2)/(4*Sin[2*x]^(5/2)) + (33*ArcTanh[Sqrt[Tan[x]]/Sqrt[2]]*Si
n[x]^5)/(4*Sqrt[2]*Sin[2*x]^(5/2)*Tan[x]^(5/2))
```

Rule 4390

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[(u*Sin
[v/2]^(2*m))/(c*Tan[v/2])^m, x]}, Dist[((c*Sin[v])^m*(c*Tan[v/2])^m)/Sin[v/
2]^(2*m), Int[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], (u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rule 1619

```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_)), x_Symbol] :> I
nt[ExpandIntegrand[1/Sqrt[c + d*x], (Px*(c + d*x)^(n + 1/2))/(a + b*x), x],
 x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0] && GtQ
[Expon[Px, x], 2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(x)(\cos(2x)-3\tan(x))}{\left(\sin^2(x)-\sin(2x)\right)\sin^{\frac{5}{2}}(2x)}\,dx = \frac{\sin^5(x)\int \frac{\csc^2(x)(\cos(2x)-3\tan(x))}{\left(\sin^2(x)-\sin(2x)\right)\sqrt{\tan(x)}}\,dx}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$= \frac{\sin^5(x)\,\text{Subst}\left(\int \frac{-1+3x+x^2+3x^3}{(2-x)x^{7/2}}\,dx, x, \tan(x)\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$= \frac{\sin^5(x)\,\text{Subst}\left(\int \left(-\frac{1}{2x^{7/2}}+\frac{5}{4x^{5/2}}+\frac{9}{8x^{3/2}}-\frac{33}{8(-2+x)\sqrt{x}}\right)dx, x, \tan(x)\right)}{\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

$$= \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{5\cos^4(x)\sin(x)}{6\sin^{\frac{5}{2}}(2x)} - \frac{9\cos^3(x)\sin^2(x)}{4\sin^{\frac{5}{2}}(2x)} - \frac{\left(33\sin^5(x)\right)\text{Subst}\left(\int \frac{1}{(-2+x}\right.}{8\sin^{\frac{5}{2}}(2x)\tan}$$

$$= \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{5\cos^4(x)\sin(x)}{6\sin^{\frac{5}{2}}(2x)} - \frac{9\cos^3(x)\sin^2(x)}{4\sin^{\frac{5}{2}}(2x)} - \frac{\left(33\sin^5(x)\right)\text{Subst}\left(\int \frac{1}{-2+x^2}\right.}{4\sin^{\frac{5}{2}}(2x)\tan}$$

$$= \frac{\cos^5(x)}{5\sin^{\frac{5}{2}}(2x)} - \frac{5\cos^4(x)\sin(x)}{6\sin^{\frac{5}{2}}(2x)} - \frac{9\cos^3(x)\sin^2(x)}{4\sin^{\frac{5}{2}}(2x)} + \frac{33\tanh^{-1}\left(\frac{\sqrt{\tan(x)}}{\sqrt{2}}\right)\sin^5(x)}{4\sqrt{2}\sin^{\frac{5}{2}}(2x)\tan^{\frac{5}{2}}(x)}$$

Mathematica [C] time = 6.0357, size = 150, normalized size = 2.21

$$\frac{\sqrt{\sin(2x)}\cos(x)(\cos(2x)-3\tan(x))\left(\frac{1}{15}\csc(x)\left(-50\cot(x)+12\csc^2(x)-147\right)-33\sqrt{\frac{\cos(x)}{2\cos(x)-2}}\sqrt{\tan\left(\frac{x}{2}\right)}\sec(x)\left(\text{Elli}\right.\right.}{16(-6\sin(x)+\cos(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*(Cos[2*x] - 3*Tan[x]))/((Sin[x]^2 - Sin[2*x])*Sin[2*x]^
(5/2)),x]
```

```
[Out] (Cos[x]*Sqrt[Sin[2*x]]*((Csc[x]*(-147 - 50*Cot[x] + 12*Csc[x]^2))/15 - 33*S
qrt[Cos[x]/(-2 + 2*Cos[x])]*(EllipticF[ArcSin[1/Sqrt[Tan[x/2]]], -1] + Elli
pticPi[-2/(-1 + Sqrt[5]), -ArcSin[1/Sqrt[Tan[x/2]]], -1] + EllipticPi[(-1 +
 Sqrt[5])/2, -ArcSin[1/Sqrt[Tan[x/2]]], -1])*Sec[x]*Sqrt[Tan[x/2]])*(Cos[2*
x] - 3*Tan[x]))/(16*(Cos[x] + Cos[3*x] - 6*Sin[x]))
```

Maple [C] time = 0.215, size = 761, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x)
```

```
[Out] 1/3840*(-tan(1/2*x)/(tan(1/2*x)^2-1))^(1/2)/tan(1/2*x)^3*(932*(tan(1/2*x)*(
tan(1/2*x)^2-1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1
/2*x))^(1/2)*EllipticF((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1
/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^2-3024*(tan(1/2*x)*(tan(1/2*x)^2-
1))^(1/2)*(1+tan(1/2*x))^(1/2)*(-2*tan(1/2*x)+2)^(1/2)*(-tan(1/2*x))^(1/2)*
EllipticE((1+tan(1/2*x))^(1/2),1/2*2^(1/2))*(tan(1/2*x)*(tan(1/2*x)-1)*(1+t
an(1/2*x)))^(1/2)*tan(1/2*x)^2+24*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*(tan(
1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^6+3*(tan(1/2*x)^3-ta
n(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*sum((34*_a
lpha^3+13*_alpha^2+34*_alpha-21)*(_alpha^3+2*_alpha-3)*(1+tan(1/2*x))^(1/2)
*(1-tan(1/2*x))^(1/2)*(-tan(1/2*x))^(1/2)/(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/
2)*EllipticPi((1+tan(1/2*x))^(1/2),-1/4*_alpha^3-1/2*_alpha+3/4,1/2*2^(1/2)
),_alpha=RootOf(_Z^4+_Z^3+2*_Z^2-_Z+1))*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)
*2^(1/2)*tan(1/2*x)^2+200*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*
(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2)*tan(1/2*x)^5-552*(tan(1/2*x)^3-tan(1/2*
x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^4-192
0*tan(1/2*x)^4*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1)
)^(1/2)-24*tan(1/2*x)^4*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*(t
an(1/2*x)*(tan(1/2*x)^2-1))^(1/2)+552*(tan(1/2*x)^3-tan(1/2*x))^(1/2)*(tan(
1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1/2*x)^2-24*(tan(1/2*x)*(ta
n(1/2*x)^2-1))^(1/2)*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*tan(1
/2*x)^2-200*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)*(tan(1/2*x)*(t
an(1/2*x)^2-1))^(1/2)*tan(1/2*x)+24*(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x
)))^(1/2)*(tan(1/2*x)*(tan(1/2*x)^2-1))^(1/2))/(tan(1/2*x)^3-tan(1/2*x))^(1
/2)/(tan(1/2*x)*(tan(1/2*x)-1)*(1+tan(1/2*x)))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x
, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.11844, size = 506, normalized size = 7.44

$$-\frac{495\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\sqrt{2}\sqrt{\cos(x)\sin(x)}(4\cos(x)+3\sin(x))+\frac{1}{2}\cos(x)^2+\frac{7}{2}\cos(x)\sin(x)+\frac{1}{2}\right)\sin(x)-495\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x
, algorithm="fricas")
```

```
[Out] -1/1920*(495*(cos(x)^2 - 1)*log(-1/2*sqrt(2)*sqrt(cos(x)*sin(x))*(4*cos(x)
+ 3*sin(x)) + 1/2*cos(x)^2 + 7/2*cos(x)*sin(x) + 1/2)*sin(x) - 495*(cos(x)^
2 - 1)*log(1/2*cos(x)^2 + 1/2*sqrt(2)*sqrt(cos(x)*sin(x))*sin(x) - 1/2*cos(
x)*sin(x) + 1/2)*sin(x) + 4*sqrt(2)*(147*cos(x)^2 - 50*cos(x)*sin(x) - 135)
*sqrt(cos(x)*sin(x)) + 388*(cos(x)^2 - 1)*sin(x))/((cos(x)^2 - 1)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*(cos(2*x)-3*tan(x))/(sin(x)**2-sin(2*x))/sin(2*x)**(5/2
),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(\cos(2x) - 3\tan(x))\cos(x)^3}{\left(\sin(x)^2 - \sin(2x)\right)\sin(2x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*(cos(2*x)-3*tan(x))/(sin(x)^2-sin(2*x))/sin(2*x)^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((cos(2*x) - 3*tan(x))*cos(x)^3/((sin(x)^2 - sin(2*x))*sin(2*x)^(5
/2)), x)
```

3.412 $\int \sqrt{\sec^4(x)\tan(x)}\,dx$

Optimal. Leaf size=19

$$\frac{2}{3}\sin(x)\cos(x)\sqrt{\tan(x)\sec^4(x)}$$

```
[Out] (2*Cos[x]*Sin[x]*Sqrt[Sec[x]^4*Tan[x]])/3
```

Rubi [A] time = 0.123945, antiderivative size = 29, normalized size of antiderivative = 1.53, number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1999, 1954, 1250, 30}

$$\frac{2\tan^2(x)\sec^2(x)}{3\sqrt{\tan^5(x)+2\tan^3(x)+\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[x]^4*Tan[x]],x]
```

```
[Out] (2*Sec[x]^2*Tan[x]^2)/(3*Sqrt[Tan[x] + 2*Tan[x]^3 + Tan[x]^5])
```

Rule 1999

```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_), x_Symbol] :> Int[(f*x)^m*ExpandToSu
m[z, x]*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p}, x] && BinomialQ[z, x] &
& GeneralizedTrinomialQ[u, x] && EqQ[BinomialDegree[z, x] - GeneralizedTrin
omialDegree[u, x], 0] && !(BinomialMatchQ[z, x] && GeneralizedTrinomialMat
chQ[u, x])
```

Rule 1954

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.))^(p_
)*((A_) + (B_.)*(x_)^(q_)), x_Symbol] :> Dist[(a*x^j + b*x^k + c*x^n)^p/(x^
(j*p)*(a + b*x^(k - j) + c*x^(2*(k - j)))^p), Int[x^(m + j*p)*(A + B*x^(k -
 j))*(a + b*x^(k - j) + c*x^(2*(k - j)))^p, x], x] /; FreeQ[{a, b, c, A, B,
 j, k, m, p}, x] && EqQ[q, k - j] && EqQ[n, 2*k - j] && !IntegerQ[p] && Po
sQ[k - j]
```

Rule 1250

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] && !IntegerQ[p]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sqrt{\sec^4(x)\tan(x)}\,dx = \text{Subst}\left(\int \frac{x\left(1+x^2\right)}{\sqrt{x\left(1+x^2\right)^2}}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \frac{x\left(1+x^2\right)}{\sqrt{x+2x^3+x^5}}\,dx, x, \tan(x)\right)$$
$$= \frac{\left(\sqrt{\tan(x)}\sqrt{1+2\tan^2(x)+\tan^4(x)}\right)\text{Subst}\left(\int \frac{\sqrt{x}\left(1+x^2\right)}{\sqrt{1+2x^2+x^4}}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)+2\tan^3(x)+\tan^5(x)}}$$
$$= \frac{\left(\sec^2(x)\sqrt{\tan(x)}\right)\text{Subst}\left(\int \sqrt{x}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)+2\tan^3(x)+\tan^5(x)}}$$
$$= \frac{2\sec^2(x)\tan^2(x)}{3\sqrt{\tan(x)+2\tan^3(x)+\tan^5(x)}}$$

Mathematica [A] time = 0.0074824, size = 19, normalized size = 1.

$$\frac{2}{3}\sin(x)\cos(x)\sqrt{\tan(x)\sec^4(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[x]^4*Tan[x]],x]
```

```
[Out] (2*Cos[x]*Sin[x]*Sqrt[Sec[x]^4*Tan[x]])/3
```

Maple [A] time = 0.126, size = 16, normalized size = 0.8

$$\frac{2\cos(x)\sin(x)}{3}\sqrt{\frac{\sin(x)}{(\cos(x))^5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)/cos(x)^5)^(1/2),x)
```

```
[Out] 2/3*cos(x)*sin(x)*(sin(x)/cos(x)^5)^(1/2)
```

Maxima [A] time = 1.49072, size = 8, normalized size = 0.42

$$\frac{2}{3}\tan(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)/cos(x)^5)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*tan(x)^(3/2)
```

Fricas [A] time = 2.12696, size = 55, normalized size = 2.89

$$\frac{2}{3}\sqrt{\frac{\sin(x)}{\cos(x)^5}}\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)/cos(x)^5)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(sin(x)/cos(x)^5)*cos(x)*sin(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)/cos(x)**5)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{\sin(x)}{\cos(x)^5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)/cos(x)^5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(x)/cos(x)^5), x)
```

3.413 $\int \sqrt{\sin^4(x)\tan(x)}\,dx$

Optimal. Leaf size=92

$$-\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)}+\frac{3\tan^{-1}\left(\frac{(1-\cot(x))\csc^2(x)\sqrt{\sin^4(x)\tan(x)}}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{3\log\left(\sin(x)+\cos(x)-\sqrt{2}\cot(x)\csc(x)\sqrt{\sin^4(\right.}{4\sqrt{2}}$$

```
[Out] (3*ArcTan[((1 - Cot[x])*Csc[x]^2*Sqrt[Sin[x]^4*Tan[x]])/Sqrt[2]])/(4*Sqrt[2
]) + (3*Log[Cos[x] + Sin[x] - Sqrt[2]*Cot[x]*Csc[x]*Sqrt[Sin[x]^4*Tan[x]]])
/(4*Sqrt[2]) - (Cot[x]*Sqrt[Sin[x]^4*Tan[x]])/2
```

Rubi [B] time = 0.239033, antiderivative size = 204, normalized size of antiderivative = 2.22, number of steps used = 13, number of rules used = 9, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {6719, 288, 329, 297, 1162, 617, 204, 1165, 628}

$$-\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)}-\frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)\sec^2(x)\sqrt{\sin^4(x)\tan(x)}}{4\sqrt{2}\tan^{\frac{5}{2}}(x)}+\frac{3\tan^{-1}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)\sec^2(x)\sqrt{}}{4\sqrt{2}\tan^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sin[x]^4*Tan[x]],x]
```

```
[Out] -(Cot[x]*Sqrt[Sin[x]^4*Tan[x]])/2 - (3*ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]*Sec
[x]^2*Sqrt[Sin[x]^4*Tan[x]])/(4*Sqrt[2]*Tan[x]^(5/2)) + (3*ArcTan[1 + Sqrt[
2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Sin[x]^4*Tan[x]])/(4*Sqrt[2]*Tan[x]^(5/2)) +
 (3*Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Sin[x]^4*Tan[x]])/
(8*Sqrt[2]*Tan[x]^(5/2)) - (3*Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sec[x]
^2*Sqrt[Sin[x]^4*Tan[x]])/(8*Sqrt[2]*Tan[x]^(5/2))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \sqrt{\sin^4(x)\tan(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{\frac{x^5}{(1+x^2)^2}}}{1+x^2}\,dx, x, \tan(x)\right)$$

$$= \frac{\left(\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{x^{5/2}}{(1+x^2)^2}\,dx, x, \tan(x)\right)}{\tan^{\frac{5}{2}}(x)}$$

$$= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} + \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{\sqrt{x}}{1+x^2}\,dx, x, \tan(x)\right)}{4\tan^{\frac{5}{2}}(x)}$$

$$= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} + \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)}{2\tan^{\frac{5}{2}}(x)}$$

$$= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} - \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{1-x^2}{1+x^4}\,dx, x, \sqrt{\tan(x)}\right)}{4\tan^{\frac{5}{2}}(x)} + \frac{(3}{}$$

$$= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} + \frac{\left(3\sec^2(x)\sqrt{\sin^4(x)\tan(x)}\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{2}x+x^2}\,dx, x, \sqrt{\tan(x)}\right)}{8\tan^{\frac{5}{2}}(x)}$$

$$= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} + \frac{3\log\left(1-\sqrt{2}\sqrt{\tan(x)}+\tan(x)\right)\sec^2(x)\sqrt{\sin^4(x)\tan(x)}}{8\sqrt{2}\tan^{\frac{5}{2}}(x)} - \frac{3\,\mathrm{lo}}{}$$

$$= -\frac{1}{2}\cot(x)\sqrt{\sin^4(x)\tan(x)} - \frac{3\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)\sec^2(x)\sqrt{\sin^4(x)\tan(x)}}{4\sqrt{2}\tan^{\frac{5}{2}}(x)} + \frac{3\tan^{-1}\left(1\right.}{}$$

Mathematica [A] time = 0.0742365, size = 66, normalized size = 0.72

$$-\frac{1}{8}\sqrt{\sin(2x)}\csc^3(x)\sqrt{\sin^4(x)\tan(x)}\left(2\sin(x)\sqrt{\sin(2x)}+3\sin^{-1}(\cos(x)-\sin(x))+3\log\left(\sin(x)+\sqrt{\sin(2x)}+\cos(x\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sin[x]^4*Tan[x]],x]
```

```
[Out] -(Csc[x]^3*(3*ArcSin[Cos[x] - Sin[x]] + 3*Log[Cos[x] + Sin[x] + Sqrt[Sin[2*
x]]] + 2*Sin[x]*Sqrt[Sin[2*x]])*Sqrt[Sin[2*x]]*Sqrt[Sin[x]^4*Tan[x]])/8
```

Maple [C] time = 0.141, size = 310, normalized size = 3.4

$$-\frac{\sqrt{32}(\cos(x)-1)(\cos(x)+1)^2}{32(\sin(x))^5}\left(-3i\sqrt{\frac{\cos(x)-1}{\sin(x)}}\sqrt{\frac{\sin(x)-1+\cos(x)}{\sin(x)}}\sqrt{\frac{1-\cos(x)+\sin(x)}{\sin(x)}}\text{EllipticPi}\left(\sqrt{\frac{1-\mathrm{co}}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)^5/cos(x))^(1/2),x)
```

```
[Out] -1/32*32^(1/2)*(cos(x)-1)*(-3*I*((cos(x)-1)/sin(x))^(1/2)*((sin(x)-1+cos(x)
)/sin(x))^(1/2)*((1-cos(x)+sin(x))/sin(x))^(1/2)*EllipticPi(((1-cos(x)+sin(
x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+3*I*((cos(x)-1)/sin(x))^(1/2)*((si
n(x)-1+cos(x))/sin(x))^(1/2)*((1-cos(x)+sin(x))/sin(x))^(1/2)*EllipticPi(((
1-cos(x)+sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-3*((cos(x)-1)/sin(x))
^(1/2)*((sin(x)-1+cos(x))/sin(x))^(1/2)*((1-cos(x)+sin(x))/sin(x))^(1/2)*El
lipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))-3*((cos(x)
-1)/sin(x))^(1/2)*((sin(x)-1+cos(x))/sin(x))^(1/2)*((1-cos(x)+sin(x))/sin(x
))^(1/2)*EllipticPi(((1-cos(x)+sin(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))
+2*cos(x)^2*2^(1/2)-2*cos(x)*2^(1/2))*(cos(x)+1)^2*(sin(x)^5/cos(x))^(1/2)/
sin(x)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{\sin(x)^5}{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^5/cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(x)^5/cos(x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^5/cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{\sin^5(x)}{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)**5/cos(x))**(1/2),x)
```

```
[Out] Integral(sqrt(sin(x)**5/cos(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{\sin(x)^5}{\cos(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^5/cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(x)^5/cos(x)), x)
```

3.414 $\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx$

Optimal. Leaf size=47

$$\frac{3}{5}\sin(x)\cos^3(x)\sqrt[3]{\tan^2(x)\sec^{12}(x)}+\frac{3}{11}\sin^3(x)\cos(x)\sqrt[3]{\tan^2(x)\sec^{12}(x)}$$

```
[Out] (3*Cos[x]^3*Sin[x]*(Sec[x]^12*Tan[x]^2)^(1/3))/5 + (3*Cos[x]*Sin[x]^3*(Sec[
x]^12*Tan[x]^2)^(1/3))/11
```

Rubi [A] time = 0.144341, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6719, 14}

$$\frac{3}{5}\sin(x)\cos^3(x)\sqrt[3]{\tan^2(x)\sec^{12}(x)}+\frac{3}{11}\sin^3(x)\cos(x)\sqrt[3]{\tan^2(x)\sec^{12}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^12*Tan[x]^2)^(1/3),x]
```

```
[Out] (3*Cos[x]^3*Sin[x]*(Sec[x]^12*Tan[x]^2)^(1/3))/5 + (3*Cos[x]*Sin[x]^3*(Sec[
x]^12*Tan[x]^2)^(1/3))/11
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{\sec^{12}(x)\tan^2(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt[3]{x^2\left(1+x^2\right)^6}}{1+x^2}\,dx,x,\tan(x)\right)\\ &= \frac{\left(\cos^4(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}\right)\text{Subst}\left(\int x^{2/3}\left(1+x^2\right)\,dx,x,\tan(x)\right)}{\tan^{\frac{2}{3}}(x)}\\ &= \frac{\left(\cos^4(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}\right)\text{Subst}\left(\int \left(x^{2/3}+x^{8/3}\right)\,dx,x,\tan(x)\right)}{\tan^{\frac{2}{3}}(x)}\\ &= \frac{3}{5}\cos^3(x)\sin(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}+\frac{3}{11}\cos(x)\sin^3(x)\sqrt[3]{\sec^{12}(x)\tan^2(x)}\end{aligned}$$

Mathematica [A] time = 0.146176, size = 63, normalized size = 1.34

$$\frac{3\sin(x)\cos(x)\left(8\left(-\tan^2(x)\right)^{5/6}+3\cos(2x)\left(\left(-\tan^2(x)\right)^{5/6}-1\right)-3\right)\sqrt[3]{\tan^2(x)\sec^{12}(x)}}{55\left(-\tan^2(x)\right)^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^12*Tan[x]^2)^(1/3),x]
```

```
[Out] (3*Cos[x]*Sin[x]*(Sec[x]^12*Tan[x]^2)^(1/3)*(-3 + 8*(-Tan[x]^2)^(5/6) + 3*C
os[2*x]*(-1 + (-Tan[x]^2)^(5/6))))/(55*(-Tan[x]^2)^(5/6))
```

Maple [F] time = 0.348, size = 0, normalized size = 0.

$$\int \sqrt[3]{\frac{(\sin(x))^2}{(\cos(x))^{14}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(x)^2/cos(x)^14)^(1/3),x)
```

```
[Out] int((sin(x)^2/cos(x)^14)^(1/3),x)
```

Maxima [A] time = 1.52159, size = 18, normalized size = 0.38

$$\frac{3}{11}\tan(x)^{\frac{11}{3}}+\frac{3}{5}\tan(x)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/11*tan(x)^(11/3) + 3/5*tan(x)^(5/3)
```

Fricas [A] time = 2.87204, size = 96, normalized size = 2.04

$$\frac{3}{55}\left(6\cos(x)^3+5\cos(x)\right)\left(-\frac{\cos(x)^2-1}{\cos(x)^{14}}\right)^{\frac{1}{3}}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/55*(6*cos(x)^3 + 5*cos(x))*(-(cos(x)^2 - 1)/cos(x)^14)^(1/3)*sin(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)**2/cos(x)**14)**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\frac{\sin(x)^2}{\cos(x)^{14}}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sin(x)^2/cos(x)^14)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((sin(x)^2/cos(x)^14)^(1/3), x)
```

3.415 $\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx$

Optimal. Leaf size=70

$$-\frac{4\sin(x)\cos^5(x)}{9\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}-\frac{8\sin^3(x)\cos^3(x)}{\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}+\frac{4\sin^5(x)\cos(x)}{7\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}$$

```
[Out] (-4*Cos[x]^5*Sin[x])/(9*(Cos[x]^11*Sin[x]^13)^(1/4)) - (8*Cos[x]^3*Sin[x]^3
)/(Cos[x]^11*Sin[x]^13)^(1/4) + (4*Cos[x]*Sin[x]^5)/(7*(Cos[x]^11*Sin[x]^13
)^(1/4))
```

Rubi [A] time = 0.195545, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6719, 270}

$$-\frac{4\sin(x)\cos^5(x)}{9\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}-\frac{8\sin^3(x)\cos^3(x)}{\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}+\frac{4\sin^5(x)\cos(x)}{7\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^11*Sin[x]^13)^(-1/4),x]
```

```
[Out] (-4*Cos[x]^5*Sin[x])/(9*(Cos[x]^11*Sin[x]^13)^(1/4)) - (8*Cos[x]^3*Sin[x]^3
)/(Cos[x]^11*Sin[x]^13)^(1/4) + (4*Cos[x]*Sin[x]^5)/(7*(Cos[x]^11*Sin[x]^13
)^(1/4))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}\,dx = \text{Subst}\left(\int \frac{1}{\sqrt[4]{\frac{x^{13}}{(1+x^2)^{12}}}\,(1+x^2)}\,dx, x, \tan(x)\right)$$
$$= \frac{\left(\cos^6(x)\tan^{\frac{13}{4}}(x)\right)\text{Subst}\left(\int \frac{(1+x^2)^2}{x^{13/4}}\,dx, x, \tan(x)\right)}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}$$
$$= \frac{\left(\cos^6(x)\tan^{\frac{13}{4}}(x)\right)\text{Subst}\left(\int \left(\frac{1}{x^{13/4}}+\frac{2}{x^{5/4}}+x^{3/4}\right)dx, x, \tan(x)\right)}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}$$
$$= -\frac{4\cos^5(x)\sin(x)}{9\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}} - \frac{8\cos^3(x)\sin^3(x)}{\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}} + \frac{4\cos(x)\sin^5(x)}{7\sqrt[4]{\cos^{11}(x)\sin^{13}(x)}}$$

Mathematica [A] time = 0.0543274, size = 35, normalized size = 0.5

$$-\frac{4\sin(x)\cos(x)(8\cos(2x)-16\cos(4x)+15)}{63\sqrt[4]{\sin^{13}(x)\cos^{11}(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^11*Sin[x]^13)^(-1/4),x]
```

```
[Out] (-4*Cos[x]*(15 + 8*Cos[2*x] - 16*Cos[4*x])*Sin[x])/(63*(Cos[x]^11*Sin[x]^13
)^(1/4))
```

Maple [F] time = 0.29, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{(\cos(x))^{11}(\sin(x))^{13}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^11*sin(x)^13)^(1/4),x)
```

```
[Out] int(1/(cos(x)^11*sin(x)^13)^(1/4),x)
```

Maxima [A] time = 1.51458, size = 104, normalized size = 1.49

$$\frac{4}{23}\tan(x)^{\frac{23}{4}} + \frac{8}{15}\tan(x)^{\frac{15}{4}} + \frac{4}{7}\tan(x)^{\frac{7}{4}} - \frac{4\left(35\tan(x)^7 + 161\tan(x)^5 + 345\tan(x)^3 - 805\tan(x)\right)}{805\tan(x)^{\frac{5}{4}}} + \frac{4\left(21\tan(x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x, algorithm="maxima")
```

```
[Out] 4/23*tan(x)^(23/4) + 8/15*tan(x)^(15/4) + 4/7*tan(x)^(7/4) - 4/805*(35*tan(
x)^7 + 161*tan(x)^5 + 345*tan(x)^3 - 805*tan(x))/tan(x)^(5/4) + 4/315*(21*t
an(x)^7 + 135*tan(x)^5 - 945*tan(x)^3 - 35*tan(x))/tan(x)^(13/4)
```

Fricas [A] time = 3.10193, size = 336, normalized size = 4.8

$$\frac{4\left(128\,\cos\left(x\right)^{4}-144\,\cos\left(x\right)^{2}+9\right)\left(\left(\cos\left(x\right)^{23}-6\,\cos\left(x\right)^{21}+15\,\cos\left(x\right)^{19}-20\,\cos\left(x\right)^{17}+15\,\cos\left(x\right)^{15}-6\,\cos\left(x\right)^{13}\right.\right.}{63\left(\cos\left(x\right)^{22}-6\,\cos\left(x\right)^{20}+15\,\cos\left(x\right)^{18}-20\,\cos\left(x\right)^{16}+15\,\cos\left(x\right)^{14}-6\,\cos\left(x\right)^{12}+\cos\left(x\right)^{10}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x, algorithm="fricas")
```

```
[Out] 4/63*(128*cos(x)^4 - 144*cos(x)^2 + 9)*((cos(x)^23 - 6*cos(x)^21 + 15*cos(x
)^19 - 20*cos(x)^17 + 15*cos(x)^15 - 6*cos(x)^13 + cos(x)^11)*sin(x))^(3/4)
/(cos(x)^22 - 6*cos(x)^20 + 15*cos(x)^18 - 20*cos(x)^16 + 15*cos(x)^14 - 6*
cos(x)^12 + cos(x)^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)**11*sin(x)**13)**(1/4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\cos\left(x\right)^{11}\sin\left(x\right)^{13}\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)^11*sin(x)^13)^(1/4),x, algorithm="giac")
```

```
[Out] integrate((cos(x)^11*sin(x)^13)^(-1/4), x)
```

3.416 $\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx$

Optimal. Leaf size=108

$$-\frac{\sqrt{\sin(2x)}\cos(x)\sin^{-1}(\cos(x)-\sin(x))}{\sqrt{\sin(x)\cos^3(x)}}-\frac{\sin(2x)}{\sqrt{\sin(x)\cos^3(x)}}-\frac{\sqrt{\sin(2x)}\cos(x)\tanh^{-1}(\sin(x))}{\sqrt{\sin(x)\cos^3(x)}}-\sqrt{2}\log\left(\sin(x)+\cos(x)\right.$$

```
[Out] -(Sqrt[2]*Log[Cos[x] + Sin[x] - Sqrt[2]*Sec[x]*Sqrt[Cos[x]^3*Sin[x]]]) - (A
rcSin[Cos[x] - Sin[x]]*Cos[x]*Sqrt[Sin[2*x]])/Sqrt[Cos[x]^3*Sin[x]] - (ArcT
anh[Sin[x]]*Cos[x]*Sqrt[Sin[2*x]])/Sqrt[Cos[x]^3*Sin[x]] - Sin[2*x]/Sqrt[Co
s[x]^3*Sin[x]]
```

Rubi [B] time = 1.53986, antiderivative size = 234, normalized size of antiderivative = 2.17, number of steps used = 27, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {6719, 6725, 215, 329, 211, 1165, 628, 1162, 617, 204, 321}

$$-2\sec^2(x)\sqrt{\sin(x)\cos^3(x)}-\frac{\sqrt{2}\tan^{-1}\left(1-\sqrt{2}\sqrt{\tan(x)}\right)\sec^2(x)\sqrt{\sin(x)\cos^3(x)}}{\sqrt{\tan(x)}}+\frac{\sqrt{2}\tan^{-1}\left(\sqrt{2}\sqrt{\tan(x)}+1\right)\sec^2(x)\sqrt{}}{\sqrt{\tan(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[2*x] - Sqrt[Sin[2*x]])/Sqrt[Cos[x]^3*Sin[x]],x]
```

```
[Out] -2*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]] - Sqrt[2]*ArcSinh[Tan[x]]*Cot[x]*(Sec[x]^
2)^(3/2)*Sqrt[Cos[x]*Sin[x]]*Sqrt[Cos[x]^3*Sin[x]] - (Sqrt[2]*ArcTan[1 - Sq
rt[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] + (Sqrt[2]
*ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[
x]] - (Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]
])/(Sqrt[2]*Sqrt[Tan[x]]) + (Log[1 + Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^
2*Sqrt[Cos[x]^3*Sin[x]])/(Sqrt[2]*Sqrt[Tan[x]])
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \frac{\cos(2x)-\sqrt{\sin(2x)}}{\sqrt{\cos^3(x)\sin(x)}}\,dx = \text{Subst}\left(\int \frac{\sqrt{\frac{x}{(1+x^2)^2}}\left(1-x^2-\frac{x}{\sqrt{\frac{x}{2+2x^2}}}\right)}{x}\,dx, x, \tan(x)\right)$$

$$= \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int \frac{1-x^2-\frac{x}{\sqrt{\frac{x}{2+2x^2}}}}{\sqrt{x}(1+x^2)}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)}}$$

$$= \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\left(-\frac{\sqrt{2}\sqrt{\frac{x}{1+x^2}}}{\sqrt{x}}+\frac{1}{\sqrt{x}(1+x^2)}-\frac{x^{3/2}}{1+x^2}\right)dx, x, \tan(x)\right)}{\sqrt{\tan(x)}}$$

$$= \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(1+x^2)}\,dx, x, \tan(x)\right)}{\sqrt{\tan(x)}} - \frac{\left(\sec^2(x)\sqrt{\cos^3(x)\sin(x)}\right)\text{Subs}}{\sqrt{\tan(x}}$$

$$= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)} - \left(\sqrt{2}\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}\sqrt{\cos^3(x)\sin(x)}\right)\text{Subst}\left(\int\right.$$

$$= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)} - \sqrt{2}\sinh^{-1}(\tan(x))\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}\sqrt{\cos^3(x)\sin}$$

$$= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)} - \sqrt{2}\sinh^{-1}(\tan(x))\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}\sqrt{\cos^3(x)\sin}$$

$$= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)} - \sqrt{2}\sinh^{-1}(\tan(x))\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}\sqrt{\cos^3(x)\sin}$$

$$= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)} - \sqrt{2}\sinh^{-1}(\tan(x))\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}\sqrt{\cos^3(x)\sin}$$

$$= -2\sec^2(x)\sqrt{\cos^3(x)\sin(x)} - \sqrt{2}\sinh^{-1}(\tan(x))\cot(x)\sec^2(x)^{3/2}\sqrt{\cos(x)\sin(x)}\sqrt{\cos^3(x)\sin}$$

Mathematica [C] time = 0.261313, size = 105, normalized size = 0.97

$$\frac{-4\sin(x)\cos^3(x)\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\cos^2(x)\right)-3\sqrt[4]{\sin^2(x)}\cos(x)\left(2\sin(x)+\sqrt{\sin(2x)}\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\right.\right.\right.}{3\sqrt[4]{\sin^2(x)}\sqrt{\sin(x)\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[2*x] - Sqrt[Sin[2*x]])/Sqrt[Cos[x]^3*Sin[x]],x]
```

```
[Out] (-4*Cos[x]^3*Hypergeometric2F1[3/4, 3/4, 7/4, Cos[x]^2]*Sin[x] - 3*Cos[x]*(
Sin[x]^2)^(1/4)*(2*Sin[x] + (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin
[x/2]])*Sqrt[Sin[2*x]]))/(3*Sqrt[Cos[x]^3*Sin[x]]*(Sin[x]^2)^(1/4))
```

Maple [C] time = 0.317, size = 247, normalized size = 2.3

$$-2\frac{\cos(x)\sin(x)}{\sqrt{(\cos(x))^3\sin(x)}}-\frac{\cos(x)\sqrt{2}(\sin(x))^2}{\cos(x)-1}\left(i\mathrm{EllipticPi}\left(\sqrt{-\frac{-\sin(x)-1+\cos(x)}{\sin(x)}},\frac{1}{2}-\frac{i}{2},\frac{\sqrt{2}}{2}\right)-i\mathrm{EllipticPi}\left(\sqrt{-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x)
```

```
[Out] -2*cos(x)*sin(x)/(cos(x)^3*sin(x))^(1/2)-2^(1/2)*(I*EllipticPi((-(-sin(x)-1
+cos(x))/sin(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*EllipticPi((-(-sin(x)-1+cos
(x))/sin(x))^(1/2),1/2+1/2*I,1/2*2^(1/2))+EllipticPi((-(-sin(x)-1+cos(x))/s
in(x))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi((-(-sin(x)-1+cos(x))/sin(x))
^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*EllipticF((-(-sin(x)-1+cos(x))/sin(x))^(1/2
),1/2*2^(1/2)))*cos(x)*sin(x)^2*(-(-sin(x)-1+cos(x))/sin(x))^(1/2)*((sin(x)
-1+cos(x))/sin(x))^(1/2)*((cos(x)-1)/sin(x))^(1/2)/(cos(x)-1)/(cos(x)^3*sin
(x))^(1/2)+2*2^(1/2)*arctanh((cos(x)-1)/sin(x))*cos(x)*(cos(x)*sin(x))^(1/2
)/(cos(x)^3*sin(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x, algorithm="m
axima")
```

```
[Out] 1/2*sqrt(2)*integrate(2*((((cos(4*x) + 1)*cos(1/2*arctan2(sin(x), -cos(x) +
 1)) - sin(4*x)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin(
x), cos(x) + 1)) + (cos(1/2*arctan2(sin(x), -cos(x) + 1))*sin(4*x) + (cos(4
*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(1/2*arctan2(sin(x), cos
(x) + 1)))*cos(3/2*arctan2(sin(2*x), cos(2*x) + 1)) + ((cos(1/2*arctan2(sin
(x), -cos(x) + 1))*sin(4*x) + (cos(4*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x
) + 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) - ((cos(4*x) + 1)*cos(1/2*arc
tan2(sin(x), -cos(x) + 1)) - sin(4*x)*sin(1/2*arctan2(sin(x), -cos(x) + 1))
)*sin(1/2*arctan2(sin(x), cos(x) + 1)))*sin(3/2*arctan2(sin(2*x), cos(2*x)
+ 1)))/((cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(3/4)*(cos(x)^2 + sin(x)
^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4)), x) -
1/2*sqrt(2)*integrate(-2*(((cos(1/2*arctan2(sin(x), -cos(x) + 1))*sin(4*x)
+ (cos(4*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin
(x), cos(x) + 1)) - ((cos(4*x) + 1)*cos(1/2*arctan2(sin(x), -cos(x) + 1)) -
 sin(4*x)*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(1/2*arctan2(sin(x), co
s(x) + 1)))*cos(3/2*arctan2(sin(2*x), cos(2*x) + 1)) - (((cos(4*x) + 1)*cos
(1/2*arctan2(sin(x), -cos(x) + 1)) - sin(4*x)*sin(1/2*arctan2(sin(x), -cos(
x) + 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) + (cos(1/2*arctan2(sin(x), -
cos(x) + 1))*sin(4*x) + (cos(4*x) + 1)*sin(1/2*arctan2(sin(x), -cos(x) + 1)
))*sin(1/2*arctan2(sin(x), cos(x) + 1)))*sin(3/2*arctan2(sin(2*x), cos(2*x)
 + 1)))/((cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(3/4)*(cos(x)^2 + sin(x
)^2 + 2*cos(x) + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4)), x) -
 1/2*sqrt(2)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + 1/2*sqrt(2)*log(cos(
x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [B] time = 76.0529, size = 2007, normalized size = 18.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x, algorithm="f
ricas")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(-1/2*(2*cos(x)^4 - 2*cos(x)^3*sin(x) - 2*cos(x)^2 -
sqrt(2)*sqrt(cos(x)^3*sin(x))*sqrt((4*cos(x)^2*sin(x) + 2*sqrt(2)*sqrt(cos(
x)^3*sin(x))*(cos(x) + sin(x)) + cos(x))/cos(x)) - sqrt(2)*sqrt(cos(x)^3*si
n(x)))/(cos(x)^4 + cos(x)^3*sin(x) - cos(x)^2))*cos(x)^2 + 2*sqrt(2)*arctan
(1/2*(2*cos(x)^4 - 2*cos(x)^3*sin(x) - 2*cos(x)^2 + sqrt(2)*sqrt(cos(x)^3*s
in(x))*sqrt((4*cos(x)^2*sin(x) - 2*sqrt(2)*sqrt(cos(x)^3*sin(x))*(cos(x) +
sin(x)) + cos(x))/cos(x)) + sqrt(2)*sqrt(cos(x)^3*sin(x)))/(cos(x)^4 + cos(
x)^3*sin(x) - cos(x)^2))*cos(x)^2 - 2*sqrt(2)*arctan(-1/2*(sqrt(2)*sqrt(cos
(x)^3*sin(x))*(cos(x) - sin(x)) + (2*cos(x)^2*sin(x) - sqrt(2)*sqrt(cos(x)^
3*sin(x))*(cos(x) + sin(x)))*sqrt((4*cos(x)^2*sin(x) + 2*sqrt(2)*sqrt(cos(x
)^3*sin(x))*(cos(x) + sin(x)) + cos(x))/cos(x)))/(cos(x)^2*sin(x)))*cos(x)^
2 - 2*sqrt(2)*arctan(-1/2*(sqrt(2)*sqrt(cos(x)^3*sin(x))*(cos(x) - sin(x))
- (2*cos(x)^2*sin(x) + sqrt(2)*sqrt(cos(x)^3*sin(x))*(cos(x) + sin(x)))*sqr
t((4*cos(x)^2*sin(x) - 2*sqrt(2)*sqrt(cos(x)^3*sin(x))*(cos(x) + sin(x)) +
cos(x))/cos(x)))/(cos(x)^2*sin(x)))*cos(x)^2 - sqrt(2)*cos(x)^2*log((4*cos(
x)^2*sin(x) + 2*sqrt(2)*sqrt(cos(x)^3*sin(x))*(cos(x) + sin(x)) + cos(x))/c
os(x)) + sqrt(2)*cos(x)^2*log((4*cos(x)^2*sin(x) - 2*sqrt(2)*sqrt(cos(x)^3*
sin(x))*(cos(x) + sin(x)) + cos(x))/cos(x)) - sqrt(2)*cos(x)^2*log((cos(x)^
6 - 8*cos(x)^4 + 4*sqrt(cos(x)^3*sin(x))*(cos(x)^2 - 2)*sqrt(cos(x)*sin(x))
 + 8*cos(x)^2)/cos(x)^6) + 8*sqrt(cos(x)^3*sin(x)))/cos(x)^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(2*x)-sin(2*x)**(1/2))/(cos(x)**3*sin(x))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{\sin(2x)} - \cos(2x)}{\sqrt{\cos(x)^3 \sin(x)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(2*x)-sin(2*x)^(1/2))/(cos(x)^3*sin(x))^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(-(sqrt(sin(2*x)) - cos(2*x))/sqrt(cos(x)^3*sin(x)), x)
```

3.417 $\int \frac{\sqrt{\cos(x)\sin^3(x)}-2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)}+\sqrt{\tan(x)}}\,dx$

Optimal. Leaf size=364

$$-\sqrt[4]{2}\tan^{-1}\left(\frac{\sqrt{2}-\tan(x)}{2^{3/4}\sqrt{\tan(x)}}\right)+\frac{4}{\sqrt{\tan(x)}}-\sqrt[4]{2}\coth^{-1}\left(\frac{\tan(x)+\sqrt{2}}{2^{3/4}\sqrt{\tan(x)}}\right)-2\sqrt{2}\tan^{-1}\left(\frac{\cos(x)(\cos(x)-\sin(x))}{\sqrt{2}\sqrt{\sin(x)\cos^3(x)}}\right)+\sqrt[4]{2}\tan$$

```
[Out] -2*Sqrt[2]*ArcCoth[(Cos[x]*(Cos[x] + Sin[x]))/(Sqrt[2]*Sqrt[Cos[x]^3*Sin[x]
])] + 2^(1/4)*ArcCoth[(Cos[x]*(Sqrt[2]*Cos[x] + Sin[x]))/(2^(3/4)*Sqrt[Cos[
x]^3*Sin[x]])] - 2^(1/4)*ArcCoth[(Sqrt[2] + Tan[x])/(2^(3/4)*Sqrt[Tan[x]])]
 - 2*Sqrt[2]*ArcTan[(Cos[x]*(Cos[x] - Sin[x]))/(Sqrt[2]*Sqrt[Cos[x]^3*Sin[x
]])] + 2^(1/4)*ArcTan[(Cos[x]*(Sqrt[2]*Cos[x] - Sin[x]))/(2^(3/4)*Sqrt[Cos[
x]^3*Sin[x]])] - 2^(1/4)*ArcTan[(Sqrt[2] - Tan[x])/(2^(3/4)*Sqrt[Tan[x]])]
+ 4*Csc[x]*Sec[x]*Sqrt[Cos[x]^3*Sin[x]] + (Csc[x]^2*Log[1 + Cos[x]^2]*Sec[x
]^2*Sqrt[Cos[x]^3*Sin[x]]*Sqrt[Cos[x]*Sin[x]^3])/4 + (Csc[x]^2*Log[Sin[x]]*
Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]]*Sqrt[Cos[x]*Sin[x]^3])/2 + 4/Sqrt[Tan[x]] -
(Csc[x]^2*Log[1 + Cos[x]^2]*Sqrt[Cos[x]*Sin[x]^3]*Sqrt[Tan[x]])/4 + (Csc[x]
^2*Log[Sin[x]]*Sqrt[Cos[x]*Sin[x]^3]*Sqrt[Tan[x]])/2
```

Rubi [A] time = 4.73026, antiderivative size = 665, normalized size of antiderivative = 1.83, number of steps used = 66, number of rules used = 21, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.512, Rules used = {6725, 6742, 6719, 325, 329, 297, 1162, 617, 204, 1165, 628, 466, 482, 6733, 15, 29, 266, 36, 31, 260, 444}

$$-\sqrt[4]{2}\tan^{-1}\left(1-\sqrt[4]{2}\sqrt{\tan(x)}\right)+\sqrt[4]{2}\tan^{-1}\left(\sqrt[4]{2}\sqrt{\tan(x)}+1\right)+\frac{4}{\sqrt{\tan(x)}}+\frac{\log\left(\tan(x)-2^{3/4}\sqrt{\tan(x)}+\sqrt{2}\right)}{2^{3/4}}-\frac{\log\left(\text{ta}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[Cos[x]*Sin[x]^3] - 2*Sin[2*x])/(-Sqrt[Cos[x]^3*Sin[x]] + Sqrt[Tan
[x]]),x]
```

```
[Out] -(2^(1/4)*ArcTan[1 - 2^(1/4)*Sqrt[Tan[x]]]) + 2^(1/4)*ArcTan[1 + 2^(1/4)*Sq
rt[Tan[x]]] + Log[Sqrt[2] - 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]/2^(3/4) - Log[Sq
rt[2] + 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]/2^(3/4) + 4*Csc[x]*Sec[x]*Sqrt[Cos[x
]^3*Sin[x]] - (Csc[x]^2*Log[Sec[x]^2]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]]*Sqrt[C
os[x]*Sin[x]^3])/2 + Csc[x]^2*Log[Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[
x]]*Sqrt[Cos[x]*Sin[x]^3] + (Csc[x]^2*Log[2 + Tan[x]^2]*Sec[x]^2*Sqrt[Cos[x
]^3*Sin[x]]*Sqrt[Cos[x]*Sin[x]^3])/4 + (Log[Tan[x]]*Sec[x]^2*Sqrt[Cos[x]*Si
n[x]^3])/(2*Tan[x]^(3/2)) - (Log[2 + Tan[x]^2]*Sec[x]^2*Sqrt[Cos[x]*Sin[x]^
3])/(4*Tan[x]^(3/2)) + 4/Sqrt[Tan[x]] + (2^(1/4)*ArcTan[1 - 2^(1/4)*Sqrt[Ta
n[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] - (2^(1/4)*ArcTan[1 + 2
^(1/4)*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] - (2*Sqrt
[2]*ArcTan[1 - Sqrt[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[T
an[x]] + (2*Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[Tan[x]]]*Sec[x]^2*Sqrt[Cos[x]^3
*Sin[x]])/Sqrt[Tan[x]] + (Sqrt[2]*Log[1 - Sqrt[2]*Sqrt[Tan[x]] + Tan[x]]*Se
c[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] - (Sqrt[2]*Log[1 + Sqrt[2]*Sqrt[
Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/Sqrt[Tan[x]] - (Log[Sqrt[
2] - 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sqrt[Cos[x]^3*Sin[x]])/(2^(3/4
)*Sqrt[Tan[x]]) + (Log[Sqrt[2] + 2^(3/4)*Sqrt[Tan[x]] + Tan[x]]*Sec[x]^2*Sq
rt[Cos[x]^3*Sin[x]])/(2^(3/4)*Sqrt[Tan[x]])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 466

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/e, Subst[Int[x^(k*(m
 + 1) - 1)*(a + (b*x^(k*n))/e^n)^p*(c + (d*x^(k*n))/e^n)^q, x], x, (e*x)^(1
/k)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n
, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[b/(b*c - a*d), Int[(e*x)^m/(a + b*x^n), x], x] - Dist[d/
(b*c - a*d), Int[(e*x)^m/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 &&  !IntegerQ[m]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(x)\sin^3(x)} - 2\sin(2x)}{-\sqrt{\cos^3(x)\sin(x)} + \sqrt{\tan(x)}}\,dx = \text{Subst}\left(\int \frac{\sqrt{\frac{x^3}{(1+x^2)^2}} - \frac{4x}{1+x^2}}{(1+x^2)\left(\sqrt{x} - \sqrt{\frac{x}{(1+x^2)^2}}\right)}\,dx, x, \tan(x)\right)$$

$$= \text{Subst}\left(\int \left(\frac{4x}{(1+x^2)^2\left(-\sqrt{x} + \sqrt{\frac{x}{(1+x^2)^2}}\right)} - \frac{\sqrt{\frac{x^3}{(1+x^2)^2}}}{(1+x^2)\left(-\sqrt{x} + \sqrt{\frac{x}{(1+x^2)^2}}\right)}\right) dx, x, \text{t}$$

$$= 4\,\text{Subst}\left(\int \frac{x}{(1+x^2)^2\left(-\sqrt{x} + \sqrt{\frac{x}{(1+x^2)^2}}\right)}\,dx, x, \tan(x)\right) - \text{Subst}\left(\int \frac{\sqrt{}}{(1+x^2)\left(-\right.}\right.$$

$$= 4\,\text{Subst}\left(\int \left(-\frac{1}{2x^{3/2}} - \frac{\sqrt{\frac{x}{(1+x^2)^2}}}{2x^2} + \frac{\sqrt{x}}{2(2+x^2)} + \frac{\sqrt{\frac{x}{(1+x^2)^2}}}{2(2+x^2)}\right) dx, x, \tan(x)\right) - \frac{(\text{s}}{}$$

$$= \frac{4}{\sqrt{\tan(x)}} - 2\,\text{Subst}\left(\int \frac{\sqrt{\frac{x}{(1+x^2)^2}}}{x^2}\,dx, x, \tan(x)\right) + 2\,\text{Subst}\left(\int \frac{\sqrt{x}}{2+x^2}\,dx, x, \text{tan}\right.$$

$$= \frac{4}{\sqrt{\tan(x)}} + 4\,\text{Subst}\left(\int \frac{x^2}{2+x^4}\,dx, x, \sqrt{\tan(x)}\right) - \frac{\left(2\sec^2(x)\sqrt{\cos(x)\sin^3(x)}\right)\text{S}}{}$$

$$= 4\csc(x)\sec(x)\sqrt{\cos^3(x)\sin(x)} + \frac{4}{\sqrt{\tan(x)}} - 2\,\text{Subst}\left(\int \frac{\sqrt{2} - x^2}{2+x^4}\,dx, x, \sqrt{\tan(x}\right.$$

$$= 4\csc(x)\sec(x)\sqrt{\cos^3(x)\sin(x)} + \frac{4}{\sqrt{\tan(x)}} + \frac{\text{Subst}\left(\int \frac{2^{3/4}+2x}{-\sqrt{2}-2^{3/4}x-x^2}\,dx, x, \sqrt{\tan(}\right.}{2^{3/4}}$$

$$= \frac{\log\left(\sqrt{2} - 2^{3/4}\sqrt{\tan(x)} + \tan(x)\right)}{2^{3/4}} - \frac{\log\left(\sqrt{2} + 2^{3/4}\sqrt{\tan(x)} + \tan(x)\right)}{2^{3/4}} + 4\csc($$

$$= -\sqrt[4]{2}\tan^{-1}\left(1 - \sqrt[4]{2}\sqrt{\tan(x)}\right) + \sqrt[4]{2}\tan^{-1}\left(1 + \sqrt[4]{2}\sqrt{\tan(x)}\right) + \frac{\log\left(\sqrt{2} - 2^{3/4}\sqrt{\text{t}}\right.}{2^{3}}$$

$$= -\sqrt[4]{2}\tan^{-1}\left(1 - \sqrt[4]{2}\sqrt{\tan(x)}\right) + \sqrt[4]{2}\tan^{-1}\left(1 + \sqrt[4]{2}\sqrt{\tan(x)}\right) + \frac{\log\left(\sqrt{2} - 2^{3/4}\sqrt{\text{t}}\right.}{2^{3}}$$

$$= -\sqrt[4]{2}\tan^{-1}\left(1 - \sqrt[4]{2}\sqrt{\tan(x)}\right) + \sqrt[4]{2}\tan^{-1}\left(1 + \sqrt[4]{2}\sqrt{\tan(x)}\right) + \frac{\log\left(\sqrt{2} - 2^{3/4}\sqrt{\text{t}}\right.}{2^{3}}$$

Mathematica [C] time = 14.5279, size = 475, normalized size = 1.3

$$-\frac{2\left(2-\sin^2(x)\right)\cos^2(x)^{3/4}\tan^{\frac{3}{2}}(x)\,{}_2F_1\left(\frac{3}{4},\frac{3}{4};\frac{7}{4};\frac{\sin^2(x)}{2\left(1-\frac{\sin^2(x)}{2}\right)}\right)}{3\left(1-\frac{\sin^2(x)}{2}\right)^{3/4}\left(\sin^2(x)-2\right)}+\frac{4}{\sqrt{\tan(x)}}+\sqrt{2\sin(2x)+\sin(4x)}\left(\sqrt{2}\tan(x)+\sqrt{2}\cot(x)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[x]*Sin[x]^3] - 2*Sin[2*x])/(-Sqrt[Cos[x]^3*Sin[x]] + Sq
rt[Tan[x]]),x]
```

```
[Out] -(Cos[x]*Csc[x/2]*(4*Log[Sec[x/2]^2] - 2*Log[Tan[x/2]] - Log[1 + Tan[x/2]^4
])*Sec[x/2]*Sqrt[Cos[x]*Sin[x]^3])/(8*Sqrt[Cos[x]^3*Sin[x]]) + ((1 + I)*((4
 + 4*I)*EllipticPi[-I, -ArcSin[Sqrt[Tan[x/2]]], -1] - (4 + 4*I)*EllipticPi[
I, -ArcSin[Sqrt[Tan[x/2]]], -1] + (-1)^(1/4)*(-EllipticPi[-(-1)^(1/4), -Arc
Sin[Sqrt[Tan[x/2]]], -1] + EllipticPi[(-1)^(1/4), -ArcSin[Sqrt[Tan[x/2]]],
-1] - EllipticPi[-(-1)^(3/4), -ArcSin[Sqrt[Tan[x/2]]], -1] + EllipticPi[(-1
)^(3/4), -ArcSin[Sqrt[Tan[x/2]]], -1]))*Sec[x/2]^4*Sqrt[Cos[x]^3*Sin[x]])/(
Sqrt[Cos[x]*Sec[x/2]^2]*Sqrt[Tan[x/2]]*(-1 + Tan[x/2]^2)) + 4/Sqrt[Tan[x]]
- (2*(Cos[x]^2)^(3/4)*Hypergeometric2F1[3/4, 3/4, 7/4, Sin[x]^2/(2*(1 - Sin
[x]^2/2))]*(2 - Sin[x]^2)*Tan[x]^(3/2))/(3*(1 - Sin[x]^2/2)^(3/4)*(-2 + Sin
[x]^2)) + Sqrt[2*Sin[2*x] + Sin[4*x]]*(Sqrt[2]*Cot[x] + Sqrt[2]*Tan[x]) + (
Csc[x]^2*(4*Log[Sqrt[Tan[x]]] - Log[2 + Tan[x]^2])*Sec[x]^2*Sqrt[2*Sin[2*x]
 - Sin[4*x]]*Sqrt[Tan[x]]*(2 + Tan[x]^2))/(4*Sqrt[2]*(3 + Cos[2*x])*(1 + Ta
n[x]^2)^2)
```

Maple [C] time = 3.754, size = 23475, normalized size = 64.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+tan(x)^
(1/2)),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+t
an(x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*integrate(-1/4*(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(1/4)*(((((sqr
t(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*
sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(4*x) - (sqrt(2)*cos(3*x) + 2*sqrt(2)
*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2
```

```
)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x)
 - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(1/2*arctan2(
sin(x), -cos(x) + 1)) - ((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*c
os(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) +
(sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x)
+ 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) - 2*sqrt
(2)*cos(2*x) - sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqr
t(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin(x),
 cos(x) + 1)) + (((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) -
 sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2
)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqr
t(2)*sin(2*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) - 2*sqrt(2)*cos
(2*x) - sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*si
n(x))*cos(1/2*arctan2(sin(x), -cos(x) + 1)) + ((sqrt(2)*cos(3*x) + 2*sqrt(2
)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(
2)*sin(x))*cos(4*x) - (sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(
x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqr
t(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*
sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*s
in(1/2*arctan2(sin(x), cos(x) + 1)))*cos(1/2*arctan2(sin(2*x), cos(2*x) + 1
)) - ((((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*s
in(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2)*cos(3*x)
 + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2
*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) - sq
rt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(
1/2*arctan2(sin(x), -cos(x) + 1)) + ((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x)
 + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))
*cos(4*x) - (sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(
2)*sin(3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3
*x) - 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*si
n(2*x) - sqrt(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arc
tan2(sin(x), cos(x) + 1)) - (((sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt
(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(4*
x) - (sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(
3*x) - 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) - 2
*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x)
- sqrt(2)*sin(x))*cos(1/2*arctan2(sin(x), -cos(x) + 1)) - ((sqrt(2)*cos(3*x
) + 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(
2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) +
sqrt(2)*cos(x) + sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*si
n(4*x) - sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) + sqrt(2)*s
in(3*x) + 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos
(x) + 1)))*sin(1/2*arctan2(sin(x), cos(x) + 1)))*sin(1/2*arctan2(sin(2*x),
cos(2*x) + 1)))/((((2*(2*cos(2*x) + cos(x))*cos(3*x) + cos(3*x)^2 + 4*cos(2
*x)^2 + 4*cos(2*x)*cos(x) + cos(x)^2 + 2*(2*sin(2*x) + sin(x))*sin(3*x) + s
in(3*x)^2 + 4*sin(2*x)^2 + 4*sin(2*x)*sin(x) + sin(x)^2)*cos(1/2*arctan2(si
n(x), -cos(x) + 1))^2 + (2*(2*cos(2*x) + cos(x))*cos(3*x) + cos(3*x)^2 + 4*
cos(2*x)^2 + 4*cos(2*x)*cos(x) + cos(x)^2 + 2*(2*sin(2*x) + sin(x))*sin(3*x
) + sin(3*x)^2 + 4*sin(2*x)^2 + 4*sin(2*x)*sin(x) + sin(x)^2)*sin(1/2*arcta
n2(sin(x), -cos(x) + 1))^2)*cos(1/2*arctan2(sin(x), cos(x) + 1))^2 + ((2*(2
*cos(2*x) + cos(x))*cos(3*x) + cos(3*x)^2 + 4*cos(2*x)^2 + 4*cos(2*x)*cos(x
) + cos(x)^2 + 2*(2*sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 4*sin(2*x)^2
 + 4*sin(2*x)*sin(x) + sin(x)^2)*cos(1/2*arctan2(sin(x), -cos(x) + 1))^2 +
(2*(2*cos(2*x) + cos(x))*cos(3*x) + cos(3*x)^2 + 4*cos(2*x)^2 + 4*cos(2*x)*
cos(x) + cos(x)^2 + 2*(2*sin(2*x) + sin(x))*sin(3*x) + sin(3*x)^2 + 4*sin(2
*x)^2 + 4*sin(2*x)*sin(x) + sin(x)^2)*sin(1/2*arctan2(sin(x), -cos(x) + 1))
^2)*sin(1/2*arctan2(sin(x), cos(x) + 1))^2)*(cos(x)^2 + sin(x)^2 + 2*cos(x)
 + 1)^(1/4)*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4)), x) + 2*integrate(1
/4*(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)^(1/4)*((((sqrt(2)*cos(3*x) -
```

```
 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x
) + sqrt(2)*sin(x))*cos(4*x) - (sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqr
t(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4
*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(
3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(1/2*arctan2(sin(x), -cos(x)
 + 1)) - ((sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)
*sin(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2)*cos(3*
x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin
(2*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) -
sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*si
n(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin(x), cos(x) + 1)) +
 (((sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*
x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2)*cos(3*x) - 2*
sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) +
 sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) - sqrt(2)
*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(1/2*a
rctan2(sin(x), -cos(x) + 1)) + ((sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sq
rt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(
4*x) - (sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*si
n(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) +
 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x
) - sqrt(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(1/2*arctan2(
sin(x), cos(x) + 1)))*cos(1/2*arctan2(sin(2*x), cos(2*x) + 1)) - ((((sqrt(2
)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqr
t(2)*sin(2*x) - sqrt(2)*sin(x))*cos(4*x) + (sqrt(2)*cos(3*x) - 2*sqrt(2)*co
s(2*x) + sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*s
in(x))*sin(4*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) +
sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(1/2*arctan2(sin
(x), -cos(x) + 1)) + ((sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(
x) + sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(4*x) - (sq
rt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2
*sqrt(2)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)
*cos(2*x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2
)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*cos(1/2*arctan2(sin(x), co
s(x) + 1)) - (((sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) + sq
rt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*cos(4*x) - (sqrt(2)*c
os(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2
)*sin(2*x) - sqrt(2)*sin(x))*sin(4*x) - sqrt(2)*cos(3*x) + 2*sqrt(2)*cos(2*
x) - sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*sin(x
))*cos(1/2*arctan2(sin(x), -cos(x) + 1)) - ((sqrt(2)*cos(3*x) - 2*sqrt(2)*c
os(2*x) + sqrt(2)*cos(x) - sqrt(2)*sin(3*x) + 2*sqrt(2)*sin(2*x) - sqrt(2)*
sin(x))*cos(4*x) + (sqrt(2)*cos(3*x) - 2*sqrt(2)*cos(2*x) + sqrt(2)*cos(x)
+ sqrt(2)*sin(3*x) - 2*sqrt(2)*sin(2*x) + sqrt(2)*sin(x))*sin(4*x) - sqrt(2
)*cos(3*x) + 2*sqrt(2)*cos(2*x) - sqrt(2)*cos(x) + sqrt(2)*sin(3*x) - 2*sqr
t(2)*sin(2*x) + sqrt(2)*sin(x))*sin(1/2*arctan2(sin(x), -cos(x) + 1)))*sin(
1/2*arctan2(sin(x), cos(x) + 1)))*sin(1/2*arctan2(sin(2*x), cos(2*x) + 1)))
/((((2*(2*cos(2*x) - cos(x))*cos(3*x) - cos(3*x)^2 - 4*cos(2*x)^2 + 4*cos(2
*x)*cos(x) - cos(x)^2 + 2*(2*sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - 4*s
in(2*x)^2 + 4*sin(2*x)*sin(x) - sin(x)^2)*cos(1/2*arctan2(sin(x), -cos(x) +
 1))^2 + (2*(2*cos(2*x) - cos(x))*cos(3*x) - cos(3*x)^2 - 4*cos(2*x)^2 + 4*
cos(2*x)*cos(x) - cos(x)^2 + 2*(2*sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2
- 4*sin(2*x)^2 + 4*sin(2*x)*sin(x) - sin(x)^2)*sin(1/2*arctan2(sin(x), -cos
(x) + 1))^2)*cos(1/2*arctan2(sin(x), cos(x) + 1))^2 + ((2*(2*cos(2*x) - cos
(x))*cos(3*x) - cos(3*x)^2 - 4*cos(2*x)^2 + 4*cos(2*x)*cos(x) - cos(x)^2 +
2*(2*sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - 4*sin(2*x)^2 + 4*sin(2*x)*s
in(x) - sin(x)^2)*cos(1/2*arctan2(sin(x), -cos(x) + 1))^2 + (2*(2*cos(2*x)
- cos(x))*cos(3*x) - cos(3*x)^2 - 4*cos(2*x)^2 + 4*cos(2*x)*cos(x) - cos(x)
^2 + 2*(2*sin(2*x) - sin(x))*sin(3*x) - sin(3*x)^2 - 4*sin(2*x)^2 + 4*sin(2
*x)*sin(x) - sin(x)^2)*sin(1/2*arctan2(sin(x), -cos(x) + 1))^2)*sin(1/2*arc
```

```
tan2(sin(x), cos(x) + 1))^2)*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)^(1/4)*(co
s(x)^2 + sin(x)^2 - 2*cos(x) + 1)^(1/4)), x) + 1/2*log(cos(x)^2 + sin(x)^2
+ 2*cos(x) + 1) + 1/2*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+t
an(x)^(1/2)),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*sin(2*x)+(cos(x)*sin(x)**3)**(1/2))/(-(cos(x)**3*sin(x))**(1/
2)+tan(x)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{\cos(x)\sin(x)^3} - 2\sin(2x)}{\sqrt{\cos(x)^3\sin(x)} - \sqrt{\tan(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*sin(2*x)+(cos(x)*sin(x)^3)^(1/2))/(-(cos(x)^3*sin(x))^(1/2)+t
an(x)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(-(sqrt(cos(x)*sin(x)^3) - 2*sin(2*x))/(sqrt(cos(x)^3*sin(x)) - sq
rt(tan(x))), x)
```

3.418 $\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{\left(\cos^5(x)\sin(x)\right)^{2/3}}\,dx$

Optimal. Leaf size=125

$$-\frac{9\sin^4(x)}{10\left(\sin(x)\cos^5(x)\right)^{2/3}}-\frac{9}{4}\sec^8(x)\left(\sin(x)\cos^5(x)\right)^{4/3}+\frac{3}{14}\tan^4(x)\sqrt[3]{\sin(x)\cos^5(x)}\sqrt[3]{\tan(x)\sec^6(x)}+\frac{3}{4}\tan^2(x)\sqrt[3]{\sin(x)}$$

```
[Out] (-9*Sin[x]^4)/(10*(Cos[x]^5*Sin[x])^(2/3)) - (9*Sec[x]^8*(Cos[x]^5*Sin[x])^
(4/3))/4 + (3*(Cos[x]^5*Sin[x])^(1/3)*(Sec[x]^6*Tan[x])^(1/3))/2 + (3*(Cos[
x]^5*Sin[x])^(1/3)*Tan[x]^2*(Sec[x]^6*Tan[x])^(1/3))/4 + (3*(Cos[x]^5*Sin[x
])^(1/3)*Tan[x]^4*(Sec[x]^6*Tan[x])^(1/3))/14
```

Rubi [A] time = 1.01891, antiderivative size = 141, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6719, 6733, 6742, 14}

$$-\frac{9\sin^2(x)\cos^2(x)}{4\left(\sin(x)\cos^5(x)\right)^{2/3}}-\frac{9\sin^4(x)}{10\left(\sin(x)\cos^5(x)\right)^{2/3}}+\frac{3\sin^3(x)\cos^3(x)\sqrt[3]{\tan(x)\sec^6(x)}}{4\left(\sin(x)\cos^5(x)\right)^{2/3}}+\frac{3\sin^5(x)\cos(x)\sqrt[3]{\tan(x)\sec^6(x)}}{14\left(\sin(x)\cos^5(x)\right)^{2/3}}+$$

Antiderivative was successfully verified.

```
[In] Int[(-3*Tan[x] + (Sec[x]^6*Tan[x])^(1/3))/(Cos[x]^5*Sin[x])^(2/3),x]
```

```
[Out] (-9*Cos[x]^2*Sin[x]^2)/(4*(Cos[x]^5*Sin[x])^(2/3)) - (9*Sin[x]^4)/(10*(Cos[
x]^5*Sin[x])^(2/3)) + (3*Cos[x]^5*Sin[x]*(Sec[x]^6*Tan[x])^(1/3))/(2*(Cos[x
]^5*Sin[x])^(2/3)) + (3*Cos[x]^3*Sin[x]^3*(Sec[x]^6*Tan[x])^(1/3))/(4*(Cos[
x]^5*Sin[x])^(2/3)) + (3*Cos[x]*Sin[x]^5*(Sec[x]^6*Tan[x])^(1/3))/(14*(Cos[
x]^5*Sin[x])^(2/3))
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ
[v, x] && !FreeQ[w, x]
```

Rule 6733

```
Int[(u_)*(x_)^(m_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k, Subst[I
nt[x^(k*(m + 1) - 1)*(u /. x -> x^k), x], x, x^(1/k)], x]] /; FractionQ[m]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{-3\tan(x)+\sqrt[3]{\sec^6(x)\tan(x)}}{\left(\cos^5(x)\sin(x)\right)^{2/3}}\,dx = \text{Subst}\left(\int \frac{-3x+\sqrt[3]{x\left(1+x^2\right)^3}}{\left(\frac{x}{\left(1+x^2\right)^3}\right)^{2/3}\left(1+x^2\right)}\,dx, x, \tan(x)\right)$$

$$= \frac{\left(\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int \frac{\left(1+x^2\right)\left(-3x+\sqrt[3]{x\left(1+x^2\right)^3}\right)}{x^{2/3}}\,dx, x, \tan(x)\right)}{\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

$$= \frac{\left(3\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int \left(1+x^6\right)\left(-3x^3+\sqrt[3]{x^3\left(1+x^6\right)^3}\right)dx, x, \sqrt[3]{\tan(x)}\right)}{\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

$$= \frac{\left(3\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int \left(-3x^3+\sqrt[3]{x^3\left(1+x^6\right)^3}-x^6\left(3x^3-\sqrt[3]{\left(x+x^7\right)^3}\right)\right)\right.}{\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

$$= -\frac{9\cos^2(x)\sin^2(x)}{4\left(\cos^5(x)\sin(x)\right)^{2/3}} + \frac{\left(3\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int \sqrt[3]{x^3\left(1+x^6\right)^3}\,dx, x, \sqrt[3]{\text{ta}}\right.}{\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

$$= -\frac{9\cos^2(x)\sin^2(x)}{4\left(\cos^5(x)\sin(x)\right)^{2/3}} - \frac{\left(3\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int \left(3x^9-x^6\sqrt[3]{x^3\left(1+x^6\right)^3}\right.\right.}{\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

$$= -\frac{9\cos^2(x)\sin^2(x)}{4\left(\cos^5(x)\sin(x)\right)^{2/3}} - \frac{9\sin^4(x)}{10\left(\cos^5(x)\sin(x)\right)^{2/3}} + \frac{\left(3\cos^4(x)\tan^{\frac{2}{3}}(x)\right)\text{Subst}\left(\int\right.}{\left(\cos^5(x\right.}$$

$$= -\frac{9\cos^2(x)\sin^2(x)}{4\left(\cos^5(x)\sin(x)\right)^{2/3}} - \frac{9\sin^4(x)}{10\left(\cos^5(x)\sin(x)\right)^{2/3}} + \frac{3\cos^5(x)\sin(x)\sqrt[3]{\sec^6(x)\tan(}}{2\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

$$= -\frac{9\cos^2(x)\sin^2(x)}{4\left(\cos^5(x)\sin(x)\right)^{2/3}} - \frac{9\sin^4(x)}{10\left(\cos^5(x)\sin(x)\right)^{2/3}} + \frac{3\cos^5(x)\sin(x)\sqrt[3]{\sec^6(x)\tan(}}{2\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

$$= -\frac{9\cos^2(x)\sin^2(x)}{4\left(\cos^5(x)\sin(x)\right)^{2/3}} - \frac{9\sin^4(x)}{10\left(\cos^5(x)\sin(x)\right)^{2/3}} + \frac{3\cos^5(x)\sin(x)\sqrt[3]{\sec^6(x)\tan(}}{2\left(\cos^5(x)\sin(x)\right)^{2/3}}$$

Mathematica [A] time = 0.393317, size = 58, normalized size = 0.46

$$-\frac{3\sin(x)\left(924\sin(x)+252\sin(3x)-5(158\cos(x)+57\cos(3x)+9\cos(5x))\sqrt[3]{\tan(x)\sec^6(x)}\right)}{2240\left(\sin(x)\cos^5(x)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3*Tan[x] + (Sec[x]^6*Tan[x])^(1/3))/(Cos[x]^5*Sin[x])^(2/3),x]
```

```
[Out] (-3*Sin[x]*(924*Sin[x] + 252*Sin[3*x] - 5*(158*Cos[x] + 57*Cos[3*x] + 9*Cos
[5*x])*(Sec[x]^6*Tan[x])^(1/3)))/(2240*(Cos[x]^5*Sin[x])^(2/3))
```

Maple [F] time = 0.75, size = 0, normalized size = 0.

$$\int \left(\sqrt[3]{\frac{\sin \left(x \right)}{\left(\cos \left(x \right)\right)^{7}}}-3 \tan \left(x \right)\right) \left(\left(\cos \left(x \right)\right)^{5} \sin \left(x \right)\right)^{-\frac{2}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x)
```

```
[Out] int(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x)
```

Maxima [A] time = 1.73825, size = 81, normalized size = 0.65

$$-\frac{3}{20} \, \tan \left(x\right)^{\frac{20}{3}} - \frac{3}{7} \, \tan \left(x\right)^{\frac{14}{3}} - \frac{9}{10} \, \tan \left(x\right)^{\frac{10}{3}} - \frac{3}{8} \, \tan \left(x\right)^{\frac{8}{3}} - \frac{9}{4} \, \tan \left(x\right)^{\frac{4}{3}} + \frac{3 \, {\left(14 \, \tan \left(x\right)^{7} + 60 \, \tan \left(x\right)^{5} + 105 \, \tan \left(x\right)^{3} + \right.}}{280 \, \tan \left(x\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, alg
orithm="maxima")
```

```
[Out] -3/20*tan(x)^(20/3) - 3/7*tan(x)^(14/3) - 9/10*tan(x)^(10/3) - 3/8*tan(x)^(
8/3) - 9/4*tan(x)^(4/3) + 3/280*(14*tan(x)^7 + 60*tan(x)^5 + 105*tan(x)^3 +
 140*tan(x))/tan(x)^(1/3)
```

Fricas [A] time = 2.9936, size = 182, normalized size = 1.46

$$-\frac{3 \, {\left(\cos \left(x\right)^{5} \sin \left(x\right)\right)}^{\frac{1}{3}} {\left(21 \, {\left(3 \, \cos \left(x\right)^{2} + 2\right)} \sin \left(x\right) - 5 \, {\left(9 \, \cos \left(x\right)^{5} + 3 \, \cos \left(x\right)^{3} + 2 \, \cos \left(x\right)\right)} \left(\frac{\sin\left(x\right)}{\cos\left(x\right)^{7}}\right)^{\frac{1}{3}}\right)}}{140 \, \cos \left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, alg
orithm="fricas")
```

```
[Out] -3/140*(cos(x)^5*sin(x))^(1/3)*(21*(3*cos(x)^2 + 2)*sin(x) - 5*(9*cos(x)^5
+ 3*cos(x)^3 + 2*cos(x))*(sin(x)/cos(x)^7)^(1/3))/cos(x)^5
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((sin(x)/cos(x)**7)**(1/3)-3*tan(x))/(cos(x)**5*sin(x))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{\sin(x)}{\cos(x)^7}\right)^{\frac{1}{3}} - 3\tan(x)}{\left(\cos(x)^5\sin(x)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((sin(x)/cos(x)^7)^(1/3)-3*tan(x))/(cos(x)^5*sin(x))^(2/3),x, alg
orithm="giac")
```

```
[Out] integrate(((sin(x)/cos(x)^7)^(1/3) - 3*tan(x))/(cos(x)^5*sin(x))^(2/3), x)
```

3.419 $\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx$

Optimal. Leaf size=73

$$-\frac{1}{6}\cos(x)\left(2\cos^2(x)+1\right)^{5/2}-\frac{5}{24}\cos(x)\left(2\cos^2(x)+1\right)^{3/2}-\frac{5}{16}\cos(x)\sqrt{2\cos^2(x)+1}-\frac{5\sinh^{-1}\left(\sqrt{2}\cos(x)\right)}{16\sqrt{2}}$$

```
[Out] (-5*ArcSinh[Sqrt[2]*Cos[x]])/(16*Sqrt[2]) - (5*Cos[x]*Sqrt[1 + 2*Cos[x]^2])
/16 - (5*Cos[x]*(1 + 2*Cos[x]^2)^(3/2))/24 - (Cos[x]*(1 + 2*Cos[x]^2)^(5/2)
)/6
```

Rubi [A] time = 0.0465499, antiderivative size = 67, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 195, 215}

$$-\frac{1}{6}\cos(x)(\cos(2x)+2)^{5/2}-\frac{5}{24}\cos(x)(\cos(2x)+2)^{3/2}-\frac{5}{16}\cos(x)\sqrt{\cos(2x)+2}-\frac{5\sinh^{-1}\left(\sqrt{2}\cos(x)\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*Cos[x]^2)^(5/2)*Sin[x],x]
```

```
[Out] (-5*ArcSinh[Sqrt[2]*Cos[x]])/(16*Sqrt[2]) - (5*Cos[x]*Sqrt[2 + Cos[2*x]])/1
6 - (5*Cos[x]*(2 + Cos[2*x])^(3/2))/24 - (Cos[x]*(2 + Cos[2*x])^(5/2))/6
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \left(1+2\cos^2(x)\right)^{5/2}\sin(x)\,dx &= -\operatorname{Subst}\left(\int \left(1+2x^2\right)^{5/2}\,dx,x,\cos(x)\right)\\ &= -\frac{1}{6}\cos(x)(2+\cos(2x))^{5/2}-\frac{5}{6}\operatorname{Subst}\left(\int \left(1+2x^2\right)^{3/2}\,dx,x,\cos(x)\right)\\ &= -\frac{5}{24}\cos(x)(2+\cos(2x))^{3/2}-\frac{1}{6}\cos(x)(2+\cos(2x))^{5/2}-\frac{5}{8}\operatorname{Subst}\left(\int \sqrt{1+2x^2}\,dx,x,\right.\\ &= -\frac{5}{16}\cos(x)\sqrt{2+\cos(2x)}-\frac{5}{24}\cos(x)(2+\cos(2x))^{3/2}-\frac{1}{6}\cos(x)(2+\cos(2x))^{5/2}-\frac{5}{16}\\ &= -\frac{5\sinh^{-1}\left(\sqrt{2}\cos(x)\right)}{16\sqrt{2}}-\frac{5}{16}\cos(x)\sqrt{2+\cos(2x)}-\frac{5}{24}\cos(x)(2+\cos(2x))^{3/2}-\frac{1}{6}\cos\end{aligned}$$

Mathematica [A] time = 0.138242, size = 61, normalized size = 0.84

$$\frac{1}{96}\left(-2\sqrt{\cos(2x)+2}(92\cos(x)+23\cos(3x)+2\cos(5x))-15\sqrt{2}\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)+2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*Cos[x]^2)^(5/2)*Sin[x],x]
```

```
[Out] (-2*Sqrt[2 + Cos[2*x]]*(92*Cos[x] + 23*Cos[3*x] + 2*Cos[5*x]) - 15*Sqrt[2]*
Log[Sqrt[2]*Cos[x] + Sqrt[2 + Cos[2*x]]])/96
```

Maple [A] time = 0.012, size = 56, normalized size = 0.8

$$-\frac{5\cos(x)}{24}\left(1+2(\cos(x))^2\right)^{\frac{3}{2}}-\frac{\cos(x)}{6}\left(1+2(\cos(x))^2\right)^{\frac{5}{2}}-\frac{5\operatorname{Arcsinh}\left(\cos(x)\sqrt{2}\right)\sqrt{2}}{32}-\frac{5\cos(x)}{16}\sqrt{1+2(\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*cos(x)^2)^(5/2)*sin(x),x)
```

```
[Out] -5/24*cos(x)*(1+2*cos(x)^2)^(3/2)-1/6*cos(x)*(1+2*cos(x)^2)^(5/2)-5/32*arcs
inh(cos(x)*2^(1/2))*2^(1/2)-5/16*cos(x)*(1+2*cos(x)^2)^(1/2)
```

Maxima [A] time = 1.46665, size = 74, normalized size = 1.01

$$-\frac{1}{6}\left(2\cos(x)^2+1\right)^{\frac{5}{2}}\cos(x)-\frac{5}{24}\left(2\cos(x)^2+1\right)^{\frac{3}{2}}\cos(x)-\frac{5}{32}\sqrt{2}\operatorname{arsinh}\left(\sqrt{2}\cos(x)\right)-\frac{5}{16}\sqrt{2\cos(x)^2+1}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*cos(x)^2)^(5/2)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/6*(2*cos(x)^2 + 1)^(5/2)*cos(x) - 5/24*(2*cos(x)^2 + 1)^(3/2)*cos(x) - 5
/32*sqrt(2)*arcsinh(sqrt(2)*cos(x)) - 5/16*sqrt(2*cos(x)^2 + 1)*cos(x)
```

Fricas [A] time = 3.39366, size = 352, normalized size = 4.82

$$-\frac{1}{48}\left(32\cos(x)^5+52\cos(x)^3+33\cos(x)\right)\sqrt{2\cos(x)^2+1}+\frac{5}{256}\sqrt{2}\log\left(2048\cos(x)^8+2048\cos(x)^6+640\cos(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*cos(x)^2)^(5/2)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/48*(32*cos(x)^5 + 52*cos(x)^3 + 33*cos(x))*sqrt(2*cos(x)^2 + 1) + 5/256*
sqrt(2)*log(2048*cos(x)^8 + 2048*cos(x)^6 + 640*cos(x)^4 + 64*cos(x)^2 - 8*
(128*sqrt(2)*cos(x)^7 + 96*sqrt(2)*cos(x)^5 + 20*sqrt(2)*cos(x)^3 + sqrt(2)
*cos(x))*sqrt(2*cos(x)^2 + 1) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*cos(x)**2)**(5/2)*sin(x),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08495, size = 74, normalized size = 1.01

$$-\frac{1}{48}\left(4\left(8\cos(x)^2+13\right)\cos(x)^2+33\right)\sqrt{2\cos(x)^2+1}\cos(x)+\frac{5}{32}\sqrt{2}\log\left(-\sqrt{2}\cos(x)+\sqrt{2\cos(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*cos(x)^2)^(5/2)*sin(x),x, algorithm="giac")
```

```
[Out] -1/48*(4*(8*cos(x)^2 + 13)*cos(x)^2 + 33)*sqrt(2*cos(x)^2 + 1)*cos(x) + 5/3
2*sqrt(2)*log(-sqrt(2)*cos(x) + sqrt(2*cos(x)^2 + 1))
```

3.420 $\int \cos(x)\left(5\cos^2(x)+\sin^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=69

$$\frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}+\frac{25}{24}\sin(x)\left(5-4\sin^2(x)\right)^{3/2}+\frac{125}{16}\sin(x)\sqrt{5-4\sin^2(x)}+\frac{625}{32}\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{5}}\right)$$

```
[Out] (625*ArcSin[(2*Sin[x])/Sqrt[5]])/32 + (125*Sin[x]*Sqrt[5 - 4*Sin[x]^2])/16
+ (25*Sin[x]*(5 - 4*Sin[x]^2)^(3/2))/24 + (Sin[x]*(5 - 4*Sin[x]^2)^(5/2))/6
```

Rubi [A] time = 0.0533013, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4356, 195, 216}

$$\frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2}+\frac{25}{24}\sin(x)\left(5-4\sin^2(x)\right)^{3/2}+\frac{125}{16}\sin(x)\sqrt{5-4\sin^2(x)}+\frac{625}{32}\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(5*Cos[x]^2 + Sin[x]^2)^(5/2),x]
```

```
[Out] (625*ArcSin[(2*Sin[x])/Sqrt[5]])/32 + (125*Sin[x]*Sqrt[5 - 4*Sin[x]^2])/16
+ (25*Sin[x]*(5 - 4*Sin[x]^2)^(3/2))/24 + (Sin[x]*(5 - 4*Sin[x]^2)^(5/2))/6
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \cos(x)\left(5\cos^2(x)+\sin^2(x)\right)^{5/2}dx = \text{Subst}\left(\int \left(5-4x^2\right)^{5/2}dx, x, \sin(x)\right)$$
$$= \frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2} + \frac{25}{6}\text{Subst}\left(\int \left(5-4x^2\right)^{3/2}dx, x, \sin(x)\right)$$
$$= \frac{25}{24}\sin(x)\left(5-4\sin^2(x)\right)^{3/2} + \frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)^{5/2} + \frac{125}{8}\text{Subst}\left(\int \sqrt{5-4x}\right.$$
$$= \frac{125}{16}\sin(x)\sqrt{5-4\sin^2(x)} + \frac{25}{24}\sin(x)\left(5-4\sin^2(x)\right)^{3/2} + \frac{1}{6}\sin(x)\left(5-4\sin^2(x)\right)$$
$$= \frac{625}{32}\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{5}}\right) + \frac{125}{16}\sin(x)\sqrt{5-4\sin^2(x)} + \frac{25}{24}\sin(x)\left(5-4\sin^2(x)\right)^{3/2} +$$

Mathematica [A] time = 0.0734648, size = 48, normalized size = 0.7

$$\frac{1}{96}\left(1875\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{5}}\right) + 2(515\sin(x)+90\sin(3x)+8\sin(5x))\sqrt{2\cos(2x)+3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(5*Cos[x]^2 + Sin[x]^2)^(5/2),x]
```

```
[Out] (1875*ArcSin[(2*Sin[x])/Sqrt[5]] + 2*Sqrt[3 + 2*Cos[2*x]]*(515*Sin[x] + 90*
Sin[3*x] + 8*Sin[5*x]))/96
```

Maple [A] time = 0.092, size = 103, normalized size = 1.5

$$\frac{1}{192\sin(x)}\sqrt{\left(4(\cos(x))^2+1\right)(\sin(x))^2}\left(512\sqrt{-4(\sin(x))^4+5(\sin(x))^2}(\sin(x))^4 - 2080\sqrt{-4(\sin(x))^4+5(\sin(x))}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x)
```

```
[Out] 1/192*((4*cos(x)^2+1)*sin(x)^2)^(1/2)*(512*(-4*sin(x)^4+5*sin(x)^2)^(1/2)*s
in(x)^4-2080*(-4*sin(x)^4+5*sin(x)^2)^(1/2)*sin(x)^2+3300*(-4*sin(x)^4+5*si
n(x)^2)^(1/2)+1875*arcsin(-1+8/5*sin(x)^2))/sin(x)/(4*cos(x)^2+1)^(1/2)
```

Maxima [A] time = 1.43121, size = 72, normalized size = 1.04

$$\frac{1}{6}\left(-4\sin(x)^2+5\right)^{\frac{5}{2}}\sin(x) + \frac{25}{24}\left(-4\sin(x)^2+5\right)^{\frac{3}{2}}\sin(x) + \frac{125}{16}\sqrt{-4\sin(x)^2+5}\sin(x) + \frac{625}{32}\arcsin\left(\frac{2}{5}\sqrt{5}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-4*sin(x)^2 + 5)^(5/2)*sin(x) + 25/24*(-4*sin(x)^2 + 5)^(3/2)*sin(x) +
 125/16*sqrt(-4*sin(x)^2 + 5)*sin(x) + 625/32*arcsin(2/5*sqrt(5)*sin(x))
```

Fricas [A] time = 3.0801, size = 296, normalized size = 4.29

$$\frac{1}{48}\left(128\cos(x)^4+264\cos(x)^2+433\right)\sqrt{4\cos(x)^2+1}\sin(x)+\frac{625}{64}\arctan\left(\frac{4\left(8\cos(x)^2-3\right)\sqrt{4\cos(x)^2+1}\sin(x)}{64\cos(x)^4-23\cos(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/48*(128*cos(x)^4 + 264*cos(x)^2 + 433)*sqrt(4*cos(x)^2 + 1)*sin(x) + 625/
64*arctan((4*(8*cos(x)^2 - 3)*sqrt(4*cos(x)^2 + 1)*sin(x) - 25*cos(x)*sin(x
))/(64*cos(x)^4 - 23*cos(x)^2 - 16)) + 625/64*arctan(sin(x)/cos(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(5*cos(x)**2+sin(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08751, size = 55, normalized size = 0.8

$$\frac{1}{48}\left(8\left(16\sin(x)^2-65\right)\sin(x)^2+825\right)\sqrt{-4\sin(x)^2+5}\sin(x)+\frac{625}{32}\arcsin\left(\frac{2}{5}\sqrt{5}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(5*cos(x)^2+sin(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/48*(8*(16*sin(x)^2 - 65)*sin(x)^2 + 825)*sqrt(-4*sin(x)^2 + 5)*sin(x) + 6
25/32*arcsin(2/5*sqrt(5)*sin(x))
```

3.421 $\int \cos(x)\left(-\cos^2(x)-5\sin^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=58

$$\frac{1}{4}\sin(x)\left(-4\sin^2(x)-1\right)^{3/2}-\frac{3}{8}\sin(x)\sqrt{-4\sin^2(x)-1}+\frac{3}{16}\tan^{-1}\left(\frac{2\sin(x)}{\sqrt{-4\sin^2(x)-1}}\right)$$

```
[Out] (3*ArcTan[(2*Sin[x])/Sqrt[-1 - 4*Sin[x]^2]])/16 - (3*Sin[x]*Sqrt[-1 - 4*Sin
[x]^2])/8 + (Sin[x]*(-1 - 4*Sin[x]^2)^(3/2))/4
```

Rubi [A] time = 0.051131, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4356, 195, 217, 203}

$$\frac{1}{4}\sin(x)\left(-4\sin^2(x)-1\right)^{3/2}-\frac{3}{8}\sin(x)\sqrt{-4\sin^2(x)-1}+\frac{3}{16}\tan^{-1}\left(\frac{2\sin(x)}{\sqrt{-4\sin^2(x)-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(-Cos[x]^2 - 5*Sin[x]^2)^(3/2),x]
```

```
[Out] (3*ArcTan[(2*Sin[x])/Sqrt[-1 - 4*Sin[x]^2]])/16 - (3*Sin[x]*Sqrt[-1 - 4*Sin
[x]^2])/8 + (Sin[x]*(-1 - 4*Sin[x]^2)^(3/2))/4
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos(x)\left(-\cos^2(x)-5\sin^2(x)\right)^{3/2} dx = \text{Subst}\left(\int \left(-1-4x^2\right)^{3/2} dx, x, \sin(x)\right)$$
$$= \frac{1}{4}\sin(x)\left(-1-4\sin^2(x)\right)^{3/2} - \frac{3}{4}\text{Subst}\left(\int \sqrt{-1-4x^2}\, dx, x, \sin(x)\right)$$
$$= -\frac{3}{8}\sin(x)\sqrt{-1-4\sin^2(x)} + \frac{1}{4}\sin(x)\left(-1-4\sin^2(x)\right)^{3/2} + \frac{3}{8}\text{Subst}\left(\int \frac{}{\sqrt{-1}}\right.$$
$$= -\frac{3}{8}\sin(x)\sqrt{-1-4\sin^2(x)} + \frac{1}{4}\sin(x)\left(-1-4\sin^2(x)\right)^{3/2} + \frac{3}{8}\text{Subst}\left(\int \frac{1}{1+}\right.$$
$$= \frac{3}{16}\tan^{-1}\left(\frac{2\sin(x)}{\sqrt{-1-4\sin^2(x)}}\right) - \frac{3}{8}\sin(x)\sqrt{-1-4\sin^2(x)} + \frac{1}{4}\sin(x)\left(-1-4\,\text{si}\right.$$

Mathematica [A] time = 0.0750157, size = 61, normalized size = 1.05

$$\frac{\sqrt{2\cos(2x)-3}\left(2(2\sin(3x)-11\sin(x))\sqrt{3-2\cos(2x)}-3\sinh^{-1}(2\sin(x))\right)}{16\sqrt{4\sin^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(-Cos[x]^2 - 5*Sin[x]^2)^(3/2),x]
```

```
[Out] (Sqrt[-3 + 2*Cos[2*x]]*(-3*ArcSinh[2*Sin[x]] + 2*Sqrt[3 - 2*Cos[2*x]]*(-11*
Sin[x] + 2*Sin[3*x])))/(16*Sqrt[1 + 4*Sin[x]^2])
```

Maple [A] time = 0.082, size = 82, normalized size = 1.4

$$\frac{1}{32\sin(x)}\sqrt{\left(4(\cos(x))^2-5\right)(\sin(x))^2}\left(-32\sqrt{-4(\sin(x))^4-(\sin(x))^2}(\sin(x))^2-20\sqrt{-4(\sin(x))^4-(\sin(x))^2}+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x)
```

```
[Out] 1/32*((4*cos(x)^2-5)*sin(x)^2)^(1/2)*(-32*(-4*sin(x)^4-sin(x)^2)^(1/2)*sin(
x)^2-20*(-4*sin(x)^4-sin(x)^2)^(1/2)+3*arcsin(8*sin(x)^2+1))/sin(x)/(4*cos(
x)^2-5)^(1/2)
```

Maxima [C] time = 1.48356, size = 49, normalized size = 0.84

$$\frac{1}{4}\left(-4\sin(x)^2-1\right)^{\frac{3}{2}}\sin(x) - \frac{3}{8}\sqrt{-4\sin(x)^2-1}\sin(x) - \frac{3}{16}i\,\text{arsinh}(2\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x, algorithm="maxima")
```

[Out] 1/4*(-4*sin(x)^2 - 1)^(3/2)*sin(x) - 3/8*sqrt(-4*sin(x)^2 - 1)*sin(x) - 3/16*I*arcsinh(2*sin(x))

Fricas [C] time = 2.78899, size = 431, normalized size = 7.43

$$\frac{1}{128}\left(12i\,e^{(4ix)}\log\left(-\frac{1}{2}\sqrt{e^{(4ix)}-3\,e^{(2ix)}+1}\left(4\,e^{(2ix)}-5\right)+2\,e^{(4ix)}-\frac{11}{2}\,e^{(2ix)}+\frac{5}{2}\right)-12i\,e^{(4ix)}\log\left(\sqrt{e^{(4ix)}-3\,e^{(2ix)}+1}-e\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x, algorithm="fricas")

[Out] 1/128*(12*I*e^(4*I*x)*log(-1/2*sqrt(e^(4*I*x) - 3*e^(2*I*x) + 1)*(4*e^(2*I*x) - 5) + 2*e^(4*I*x) - 11/2*e^(2*I*x) + 5/2) - 12*I*e^(4*I*x)*log(sqrt(e^(4*I*x) - 3*e^(2*I*x) + 1) - e^(2*I*x) - 1) + (-16*I*e^(6*I*x) + 88*I*e^(4*I*x) - 88*I*e^(2*I*x) + 16*I)*sqrt(e^(4*I*x) - 3*e^(2*I*x) + 1) - 145*I*e^(4*I*x))*e^(-4*I*x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(-cos(x)**2-5*sin(x)**2)**(3/2),x)

[Out] Timed out

Giac [C] time = 1.11235, size = 41, normalized size = 0.71

$$-\frac{1}{8}i\left(8\,\sin\left(x\right)^{2}+5\right)\sqrt{4\,\sin\left(x\right)^{2}+1}\,\sin\left(x\right)-\frac{3}{16}i\,\arcsin\left(2i\,\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*(-cos(x)^2-5*sin(x)^2)^(3/2),x, algorithm="giac")

[Out] -1/8*I*(8*sin(x)^2 + 5)*sqrt(4*sin(x)^2 + 1)*sin(x) - 3/16*I*arcsin(2*I*sin(x))

3.422 $\int \frac{\sin(x)}{\left(5\cos^2(x)-2\sin^2(x)\right)^{7/2}}\,dx$

Optimal. Leaf size=55

$$\frac{\cos(x)}{15\sqrt{7\cos^2(x)-2}}-\frac{\cos(x)}{15\left(7\cos^2(x)-2\right)^{3/2}}+\frac{\cos(x)}{10\left(7\cos^2(x)-2\right)^{5/2}}$$

```
[Out] Cos[x]/(10*(-2 + 7*Cos[x]^2)^(5/2)) - Cos[x]/(15*(-2 + 7*Cos[x]^2)^(3/2)) +
 Cos[x]/(15*Sqrt[-2 + 7*Cos[x]^2])
```

Rubi [A] time = 0.0535616, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {4357, 192, 191}

$$\frac{\cos(x)}{15\sqrt{7\cos^2(x)-2}}-\frac{\cos(x)}{15\left(7\cos^2(x)-2\right)^{3/2}}+\frac{\cos(x)}{10\left(7\cos^2(x)-2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(5*Cos[x]^2 - 2*Sin[x]^2)^(7/2),x]
```

```
[Out] Cos[x]/(10*(-2 + 7*Cos[x]^2)^(5/2)) - Cos[x]/(15*(-2 + 7*Cos[x]^2)^(3/2)) +
 Cos[x]/(15*Sqrt[-2 + 7*Cos[x]^2])
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sin(x)}{\left(5\cos^2(x)-2\sin^2(x)\right)^{7/2}}\,dx = -\text{Subst}\left(\int \frac{1}{\left(-2+7x^2\right)^{7/2}}\,dx, x, \cos(x)\right)$$
$$= \frac{\cos(x)}{10\left(-2+7\cos^2(x)\right)^{5/2}} + \frac{2}{5}\text{Subst}\left(\int \frac{1}{\left(-2+7x^2\right)^{5/2}}\,dx, x, \cos(x)\right)$$
$$= \frac{\cos(x)}{10\left(-2+7\cos^2(x)\right)^{5/2}} - \frac{\cos(x)}{15\left(-2+7\cos^2(x)\right)^{3/2}} - \frac{2}{15}\text{Subst}\left(\int \frac{1}{\left(-2+7x^2\right)^{3/2}}\,dx, x, \text{co}\right.$$
$$= \frac{\cos(x)}{10\left(-2+7\cos^2(x)\right)^{5/2}} - \frac{\cos(x)}{15\left(-2+7\cos^2(x)\right)^{3/2}} + \frac{\cos(x)}{15\sqrt{-2+7\cos^2(x)}}$$

Mathematica [A] time = 0.0923111, size = 37, normalized size = 0.67

$$\frac{\cos(x)(56\cos(2x)+49\cos(4x)+67)}{15\sqrt{2}(7\cos(2x)+3)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(5*Cos[x]^2 - 2*Sin[x]^2)^(7/2),x]
```

```
[Out] (Cos[x]*(67 + 56*Cos[2*x] + 49*Cos[4*x]))/(15*Sqrt[2]*(3 + 7*Cos[2*x])^(5/2
))
```

Maple [A] time = 0.03, size = 44, normalized size = 0.8

$$\frac{\cos(x)}{10}\left(-2+7(\cos(x))^2\right)^{-\frac{5}{2}} - \frac{\cos(x)}{15}\left(-2+7(\cos(x))^2\right)^{-\frac{3}{2}} + \frac{\cos(x)}{15}\frac{1}{\sqrt{-2+7(\cos(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x)
```

```
[Out] 1/10*cos(x)/(-2+7*cos(x)^2)^(5/2)-1/15*cos(x)/(-2+7*cos(x)^2)^(3/2)+1/15*co
s(x)/(-2+7*cos(x)^2)^(1/2)
```

Maxima [A] time = 0.960126, size = 58, normalized size = 1.05

$$\frac{\cos(x)}{15\sqrt{7\cos(x)^2-2}} - \frac{\cos(x)}{15\left(7\cos(x)^2-2\right)^{\frac{3}{2}}} + \frac{\cos(x)}{10\left(7\cos(x)^2-2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*cos(x)/sqrt(7*cos(x)^2 - 2) - 1/15*cos(x)/(7*cos(x)^2 - 2)^(3/2) + 1/1
0*cos(x)/(7*cos(x)^2 - 2)^(5/2)
```

Fricas [A] time = 3.54351, size = 155, normalized size = 2.82

$$\frac{\left(98\,\cos\left(x\right)^{5}-70\,\cos\left(x\right)^{3}+15\,\cos\left(x\right)\right)\sqrt{7\,\cos\left(x\right)^{2}-2}}{30\left(343\,\cos\left(x\right)^{6}-294\,\cos\left(x\right)^{4}+84\,\cos\left(x\right)^{2}-8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/30*(98*cos(x)^5 - 70*cos(x)^3 + 15*cos(x))*sqrt(7*cos(x)^2 - 2)/(343*cos(
x)^6 - 294*cos(x)^4 + 84*cos(x)^2 - 8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(5*cos(x)**2-2*sin(x)**2)**(7/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11863, size = 41, normalized size = 0.75

$$\frac{\left(14\left(7\,\cos\left(x\right)^{2}-5\right)\cos\left(x\right)^{2}+15\right)\cos\left(x\right)}{30\left(7\,\cos\left(x\right)^{2}-2\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(5*cos(x)^2-2*sin(x)^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/30*(14*(7*cos(x)^2 - 5)*cos(x)^2 + 15)*cos(x)/(7*cos(x)^2 - 2)^(5/2)
```

3.423 $\int \frac{\cos(x)\cos(2x)}{\left(2-5\sin^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2\sin^{-1}\left(\sqrt{\frac{5}{2}}\sin(x)\right)}{5\sqrt{5}}+\frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}$$

```
[Out] (2*ArcSin[Sqrt[5/2]*Sin[x]])/(5*Sqrt[5]) + Sin[x]/(10*Sqrt[2 - 5*Sin[x]^2])
```

Rubi [A] time = 0.0668324, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4356, 385, 216}

$$\frac{2\sin^{-1}\left(\sqrt{\frac{5}{2}}\sin(x)\right)}{5\sqrt{5}}+\frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Cos[2*x])/(2 - 5*Sin[x]^2)^(3/2),x]
```

```
[Out] (2*ArcSin[Sqrt[5/2]*Sin[x]])/(5*Sqrt[5]) + Sin[x]/(10*Sqrt[2 - 5*Sin[x]^2])
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\cos(x)\cos(2x)}{\left(2-5\sin^2(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1-2x^2}{\left(2-5x^2\right)^{3/2}}\,dx, x, \sin(x)\right)$$
$$= \frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}} + \frac{2}{5}\text{Subst}\left(\int \frac{1}{\sqrt{2-5x^2}}\,dx, x, \sin(x)\right)$$
$$= \frac{2\sin^{-1}\left(\sqrt{\frac{5}{2}}\sin(x)\right)}{5\sqrt{5}} + \frac{\sin(x)}{10\sqrt{2-5\sin^2(x)}}$$

Mathematica [A] time = 0.0941733, size = 39, normalized size = 1.

$$\frac{1}{50}\left(4\sqrt{5}\sin^{-1}\left(\sqrt{\frac{5}{2}}\sin(x)\right)+\frac{5\sin(x)}{\sqrt{2-5\sin^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Cos[2*x])/(2 - 5*Sin[x]^2)^(3/2),x]
```

```
[Out] (4*Sqrt[5]*ArcSin[Sqrt[5/2]*Sin[x]] + (5*Sin[x])/Sqrt[2 - 5*Sin[x]^2])/50
```

Maple [B] time = 0.113, size = 58, normalized size = 1.5

$$\frac{1}{250\,(\cos(x))^2-150}\left(20\,\sqrt{5}\arcsin\left(1/2\,\sin(x)\,\sqrt{10}\right)(\cos(x))^2+5\,\sin(x)\,\sqrt{5\,(\cos(x))^2-3}-12\,\arcsin\left(1/2\,\sin(x)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x)
```

```
[Out] 1/50/(5*cos(x)^2-3)*(20*5^(1/2)*arcsin(1/2*sin(x)*10^(1/2))*cos(x)^2+5*sin(
x)*(5*cos(x)^2-3)^(1/2)-12*arcsin(1/2*sin(x)*10^(1/2))*5^(1/2))
```

Maxima [B] time = 1.73337, size = 967, normalized size = 24.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/50*(5*cos(1/2*arctan2(5*sin(4*x) - 2*sin(2*x), 5*cos(4*x) - 2*cos(2*x) +
5))*sin(2*x) - 5*(cos(2*x) - 1)*sin(1/2*arctan2(5*sin(4*x) - 2*sin(2*x), 5*
cos(4*x) - 2*cos(2*x) + 5)) + 2*(-10*(2*cos(2*x) - 5)*cos(4*x) + 25*cos(4*x
)^2 + 4*cos(2*x)^2 + 25*sin(4*x)^2 - 20*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 -
20*cos(2*x) + 25)^(1/4)*(sqrt(5)*arctan2(1/12*sqrt(6)*(sqrt(6)*(25/36)^(1/4
)*(25*cos(2*x)^4 + 25*sin(2*x)^4 - 20*cos(2*x)^3 + 2*(25*cos(2*x)^2 - 10*co
s(2*x) - 23)*sin(2*x)^2 + 54*cos(2*x)^2 - 20*cos(2*x) + 25)^(1/4)*sin(1/2*a
```

```
rctan2(5/12*(5*cos(2*x) - 1)*sin(2*x), 25/24*cos(2*x)^2 - 25/24*sin(2*x)^2
- 5/12*cos(2*x) + 25/24)) + 5*sin(2*x)), 5/12*sqrt(6)*cos(2*x) + 1/2*(25/36
)^(1/4)*(25*cos(2*x)^4 + 25*sin(2*x)^4 - 20*cos(2*x)^3 + 2*(25*cos(2*x)^2 -
 10*cos(2*x) - 23)*sin(2*x)^2 + 54*cos(2*x)^2 - 20*cos(2*x) + 25)^(1/4)*cos
(1/2*arctan2(5/12*(5*cos(2*x) - 1)*sin(2*x), 25/24*cos(2*x)^2 - 25/24*sin(2
*x)^2 - 5/12*cos(2*x) + 25/24)) - 1/12*sqrt(6)) + sqrt(5)*arctan2(1/12*sqrt
(6)*(sqrt(6)*(1/36)^(1/4)*(cos(2*x)^4 + sin(2*x)^4 - 20*cos(2*x)^3 + 2*(cos
(2*x)^2 - 10*cos(2*x) + 1)*sin(2*x)^2 + 198*cos(2*x)^2 - 980*cos(2*x) + 240
1)^(1/4)*sin(1/2*arctan2(1/12*(cos(2*x) - 5)*sin(2*x), 1/24*cos(2*x)^2 - 1/
24*sin(2*x)^2 - 5/12*cos(2*x) + 49/24)) + sin(2*x)), 1/12*sqrt(6)*cos(2*x)
+ 1/2*(1/36)^(1/4)*(cos(2*x)^4 + sin(2*x)^4 - 20*cos(2*x)^3 + 2*(cos(2*x)^2
 - 10*cos(2*x) + 1)*sin(2*x)^2 + 198*cos(2*x)^2 - 980*cos(2*x) + 2401)^(1/4
)*cos(1/2*arctan2(1/12*(cos(2*x) - 5)*sin(2*x), 1/24*cos(2*x)^2 - 1/24*sin(
2*x)^2 - 5/12*cos(2*x) + 49/24)) - 5/12*sqrt(6))))/(-10*(2*cos(2*x) - 5)*co
s(4*x) + 25*cos(4*x)^2 + 4*cos(2*x)^2 + 25*sin(4*x)^2 - 20*sin(4*x)*sin(2*x
) + 4*sin(2*x)^2 - 20*cos(2*x) + 25)^(1/4)
```

Fricas [B] time = 3.21994, size = 302, normalized size = 7.74

$$-\frac{\left(5\sqrt{5}\cos\left(x\right)^{2}-3\sqrt{5}\right)\arctan\left(\frac{\left(50\sqrt{5}\cos\left(x\right)^{4}-80\sqrt{5}\cos\left(x\right)^{2}+31\sqrt{5}\right)\sqrt{5\cos\left(x\right)^{2}-3}}{10\left(25\cos\left(x\right)^{4}-35\cos\left(x\right)^{2}+12\right)\sin\left(x\right)}\right)-5\sqrt{5\cos\left(x\right)^{2}-3}\sin\left(x\right)}{50\left(5\cos\left(x\right)^{2}-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/50*((5*sqrt(5)*cos(x)^2 - 3*sqrt(5))*arctan(1/10*(50*sqrt(5)*cos(x)^4 -
80*sqrt(5)*cos(x)^2 + 31*sqrt(5))*sqrt(5*cos(x)^2 - 3)/((25*cos(x)^4 - 35*c
os(x)^2 + 12)*sin(x))) - 5*sqrt(5*cos(x)^2 - 3)*sin(x))/(5*cos(x)^2 - 3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)/(2-5*sin(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10228, size = 51, normalized size = 1.31

$$\frac{2}{25}\sqrt{5}\arcsin\left(\frac{1}{2}\sqrt{10}\sin\left(x\right)\right)-\frac{\sqrt{-5\sin\left(x\right)^{2}+2}\sin\left(x\right)}{10\left(5\sin\left(x\right)^{2}-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)/(2-5*sin(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 2/25*sqrt(5)*arcsin(1/2*sqrt(10)*sin(x)) - 1/10*sqrt(-5*sin(x)^2 + 2)*sin(x
)/(5*sin(x)^2 - 2)
```

3.424 $\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=48

$$\frac{295\cos(x)}{243\sqrt{9-4\cos^2(x)}}-\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}}-\frac{1}{2}\sin^{-1}\left(\frac{2\cos(x)}{3}\right)$$

```
[Out] -ArcSin[(2*Cos[x])/3]/2 - (55*Cos[x])/(27*(9 - 4*Cos[x]^2)^(3/2)) + (295*Co
s[x])/(243*Sqrt[9 - 4*Cos[x]^2])
```

Rubi [A] time = 0.0722855, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1157, 385, 216}

$$\frac{295\cos(x)}{243\sqrt{9-4\cos^2(x)}}-\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}}-\frac{1}{2}\sin^{-1}\left(\frac{2\cos(x)}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[5*x]/(5*Cos[x]^2 + 9*Sin[x]^2)^(5/2),x]
```

```
[Out] -ArcSin[(2*Cos[x])/3]/2 - (55*Cos[x])/(27*(9 - 4*Cos[x]^2)^(3/2)) + (295*Co
s[x])/(243*Sqrt[9 - 4*Cos[x]^2])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sin(5x)}{\left(5\cos^2(x)+9\sin^2(x)\right)^{5/2}}\,dx = -\text{Subst}\left(\int \frac{1-12x^2+16x^4}{\left(9-4x^2\right)^{5/2}}\,dx,x,\cos(x)\right)$$
$$= -\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}} + \frac{1}{27}\text{Subst}\left(\int \frac{52+108x^2}{\left(9-4x^2\right)^{3/2}}\,dx,x,\cos(x)\right)$$
$$= -\frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}} + \frac{295\cos(x)}{243\sqrt{9-4\cos^2(x)}} - \text{Subst}\left(\int \frac{1}{\sqrt{9-4x^2}}\,dx,x,\cos(x)\right)$$
$$= -\frac{1}{2}\sin^{-1}\left(\frac{2\cos(x)}{3}\right) - \frac{55\cos(x)}{27\left(9-4\cos^2(x)\right)^{3/2}} + \frac{295\cos(x)}{243\sqrt{9-4\cos^2(x)}}$$

Mathematica [C] time = 0.303501, size = 63, normalized size = 1.31

$$\frac{2550\cos(x)-590\cos(3x)+243i(7-2\cos(2x))^{3/2}\log\left(\sqrt{7-2\cos(2x)}+2i\cos(x)\right)}{486(7-2\cos(2x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[5*x]/(5*Cos[x]^2 + 9*Sin[x]^2)^(5/2),x]
```

```
[Out] (2550*Cos[x] - 590*Cos[3*x] + (243*I)*(7 - 2*Cos[2*x])^(3/2)*Log[(2*I)*Cos[
x] + Sqrt[7 - 2*Cos[2*x]]])/(486*(7 - 2*Cos[2*x])^(3/2))
```

Maple [A] time = 0.074, size = 53, normalized size = 1.1

$$-\frac{4\left(\cos(x)\right)^3}{3}\left(9-4\left(\cos(x)\right)^2\right)^{-\frac{3}{2}} + \frac{214\cos(x)}{243}\frac{1}{\sqrt{9-4\left(\cos(x)\right)^2}} - \frac{1}{2}\arcsin\left(\frac{2\cos(x)}{3}\right) + \frac{26\cos(x)}{27}\left(9-4\left(\cos(x)\right)^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x)
```

```
[Out] -4/3*cos(x)^3/(9-4*cos(x)^2)^(3/2)+214/243*cos(x)/(9-4*cos(x)^2)^(1/2)-1/2*
arcsin(2/3*cos(x))+26/27*cos(x)/(9-4*cos(x)^2)^(3/2)
```

Maxima [A] time = 1.48914, size = 93, normalized size = 1.94

$$-2\left(\frac{2\cos(x)^2}{\left(-4\cos(x)^2+9\right)^{\frac{3}{2}}} - \frac{3}{\left(-4\cos(x)^2+9\right)^{\frac{3}{2}}}\right)\cos(x) + \frac{52\cos(x)}{243\sqrt{-4\cos(x)^2+9}} + \frac{26\cos(x)}{27\left(-4\cos(x)^2+9\right)^{\frac{3}{2}}} - \frac{1}{2}\arcsin\left(\frac{2}{3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -2*(2*cos(x)^2/(-4*cos(x)^2 + 9)^(3/2) - 3/(-4*cos(x)^2 + 9)^(3/2))*cos(x)
+ 52/243*cos(x)/sqrt(-4*cos(x)^2 + 9) + 26/27*cos(x)/(-4*cos(x)^2 + 9)^(3/2
```

```
) - 1/2*arcsin(2/3*cos(x))
```

Fricas [B] time = 3.00686, size = 413, normalized size = 8.6

$$\frac{243\left(16\cos(x)^4-72\cos(x)^2+81\right)\arctan\left(-\frac{81\cos(x)\sin(x)-4\left(8\cos(x)^3-9\cos(x)\right)\sqrt{-4\cos(x)^2+9}}{64\cos(x)^4-225\cos(x)^2+81}\right)-243\left(16\cos(x)^4-72\cos(x)^2 \dots\right.}{972\left(16\cos(x)^4-72\cos(x)^2+81\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/972*(243*(16*cos(x)^4 - 72*cos(x)^2 + 81)*arctan(-(81*cos(x)*sin(x) - 4*(
8*cos(x)^3 - 9*cos(x))*sqrt(-4*cos(x)^2 + 9))/(64*cos(x)^4 - 225*cos(x)^2 +
 81)) - 243*(16*cos(x)^4 - 72*cos(x)^2 + 81)*arctan(sin(x)/cos(x)) - 80*(59
*cos(x)^3 - 108*cos(x))*sqrt(-4*cos(x)^2 + 9))/(16*cos(x)^4 - 72*cos(x)^2 +
 81)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)/(5*cos(x)**2+9*sin(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11154, size = 54, normalized size = 1.12

$$-\frac{20\left(59\cos(x)^2-108\right)\sqrt{-4\cos(x)^2+9}\cos(x)}{243\left(4\cos(x)^2-9\right)^2}-\frac{1}{2}\arcsin\left(\frac{2}{3}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(5*x)/(5*cos(x)^2+9*sin(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -20/243*(59*cos(x)^2 - 108)*sqrt(-4*cos(x)^2 + 9)*cos(x)/(4*cos(x)^2 - 9)^2
 - 1/2*arcsin(2/3*cos(x))
```

3.425 $\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=49

$$\frac{1}{8}\sqrt{4\sin^2(x)-5}-\frac{5}{8\sqrt{4\sin^2(x)-5}}-\frac{1}{4\left(4\sin^2(x)-5\right)^{3/2}}$$

```
[Out] -1/(4*(-5 + 4*Sin[x]^2)^(3/2)) - 5/(8*Sqrt[-5 + 4*Sin[x]^2]) + Sqrt[-5 + 4*
Sin[x]^2]/8
```

Rubi [A] time = 0.115832, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {4356, 1247, 698}

$$\frac{1}{8}\sqrt{4\sin^2(x)-5}-\frac{5}{8\sqrt{4\sin^2(x)-5}}-\frac{1}{4\left(4\sin^2(x)-5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Cos[2*x]*Sin[3*x])/(-5 + 4*Sin[x]^2)^(5/2),x]
```

```
[Out] -1/(4*(-5 + 4*Sin[x]^2)^(3/2)) - 5/(8*Sqrt[-5 + 4*Sin[x]^2]) + Sqrt[-5 + 4*
Sin[x]^2]/8
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rubi steps

$$\int \frac{\cos(x)\cos(2x)\sin(3x)}{\left(-5+4\sin^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{x\left(3-10x^2+8x^4\right)}{\left(-5+4x^2\right)^{5/2}}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{3-10x+8x^2}{(-5+4x)^{5/2}}\,dx, x, \sin^2(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{3}{(-5+4x)^{5/2}}+\frac{5}{2(-5+4x)^{3/2}}+\frac{1}{2\sqrt{-5+4x}}\right)dx, x, \sin^2(x)\right)$$
$$= -\frac{1}{4\left(-5+4\sin^2(x)\right)^{3/2}} - \frac{5}{8\sqrt{-5+4\sin^2(x)}} + \frac{1}{8}\sqrt{-5+4\sin^2(x)}$$

Mathematica [A] time = 0.0834154, size = 28, normalized size = 0.57

$$\frac{11\cos(2x)+\cos(4x)+12}{4\left(4\sin^2(x)-5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Cos[2*x]*Sin[3*x])/(-5 + 4*Sin[x]^2)^(5/2),x]
```

```
[Out] (12 + 11*Cos[2*x] + Cos[4*x])/(4*(-5 + 4*Sin[x]^2)^(3/2))
```

Maple [A] time = 0.046, size = 46, normalized size = 0.9

$$2\frac{(\cos(x))^4}{\left(-4(\cos(x))^2-1\right)^{3/2}}+\frac{7(\cos(x))^2}{2}\left(-4(\cos(x))^2-1\right)^{-\frac{3}{2}}+\frac{1}{2}\left(-4(\cos(x))^2-1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x)
```

```
[Out] 2*cos(x)^4/(-4*cos(x)^2-1)^(3/2)+7/2*cos(x)^2/(-4*cos(x)^2-1)^(3/2)+1/2/(-4
*cos(x)^2-1)^(3/2)
```

Maxima [B] time = 1.02905, size = 259, normalized size = 5.29

$$-\frac{(\cos(11x)+14\cos(9x)+58\cos(7x)+94\cos(5x)+58\cos(3x)+15\cos(x))\cos\left(\frac{5}{2}\arctan(\sin(4x)+3\sin(\right.}{8\left(2(3\cos(2x)+1)\cos(4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x, algorithm="maxi
ma")
```

```
[Out] -1/8*((cos(11*x) + 14*cos(9*x) + 58*cos(7*x) + 94*cos(5*x) + 58*cos(3*x) +
15*cos(x))*cos(5/2*arctan2(sin(4*x) + 3*sin(2*x), -cos(4*x) - 3*cos(2*x) -
1)) - (sin(11*x) + 14*sin(9*x) + 58*sin(7*x) + 94*sin(5*x) + 58*sin(3*x) +
```

```
13*sin(x))*sin(5/2*arctan2(sin(4*x) + 3*sin(2*x), -cos(4*x) - 3*cos(2*x) -
1)))/(2*(3*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 9*cos(2*x)^2 + sin(4*x)^2
+ 6*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*cos(2*x) + 1)^(5/4)
```

Fricas [A] time = 3.17695, size = 4, normalized size = 0.08

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x, algorithm="fric
as")

[Out] 0
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)**2)**(5/2),x)

[Out] Timed out
```

Giac [A] time = 1.09792, size = 45, normalized size = 0.92

$$\frac{1}{8}\sqrt{4\sin\left(x\right)^{2}-5}-\frac{20\sin\left(x\right)^{2}-23}{8\left(4\sin\left(x\right)^{2}-5\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*sin(3*x)/(-5+4*sin(x)^2)^(5/2),x, algorithm="giac
")

[Out] 1/8*sqrt(4*sin(x)^2 - 5) - 1/8*(20*sin(x)^2 - 23)/(4*sin(x)^2 - 5)^(3/2)
```

3.426 $\int \frac{\csc^2(x)(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x))}{\sqrt{-5+\sin^2(x)}}\,dx$

Optimal. Leaf size=111

$$2\sqrt{\sin^2(x)-5}-\frac{2\tan^{-1}\left(\frac{\sqrt{\sin^2(x)-5}}{\sqrt{5}}\right)}{\sqrt{5}}-2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{\sin^2(x)-5}}\right)+\frac{2}{5}\sqrt{\sin^2(x)-5}\csc(x)+2\tan^{-1}\left(\frac{\cos(x)}{\sqrt{\sin^2(x)-5}}\right)-$$

```
[Out] 2*ArcTan[Cos[x]/Sqrt[-5 + Sin[x]^2]] - ArcTan[(Sqrt[5]*Cos[x])/Sqrt[-5 + Si
n[x]^2]]/Sqrt[5] - (2*ArcTan[Sqrt[-5 + Sin[x]^2]/Sqrt[5]])/Sqrt[5] - 2*ArcT
anh[Sin[x]/Sqrt[-5 + Sin[x]^2]] + 2*Sqrt[-5 + Sin[x]^2] + (2*Csc[x]*Sqrt[-5
 + Sin[x]^2])/5
```

Rubi [A] time = 0.574979, antiderivative size = 119, normalized size of antiderivative = 1.07, number of steps used = 18, number of rules used = 13, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.394, Rules used = {4401, 4356, 451, 217, 206, 4366, 6725, 203, 261, 1010, 377, 444, 63}

$$2\sqrt{-\cos^2(x)-4}-2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{\sin^2(x)-5}}\right)+2\tan^{-1}\left(\frac{\cos(x)}{\sqrt{-\cos^2(x)-4}}\right)-\frac{\tan^{-1}\left(\frac{\sqrt{5}\cos(x)}{\sqrt{-\cos^2(x)-4}}\right)}{\sqrt{5}}-\frac{2\tan^{-1}\left(\frac{\sqrt{-\cos^2(x)-4}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]^2*(-2*Cos[x]^3*(-1 + Sin[x]) + Cos[2*x]*Sin[x]))/Sqrt[-5 + Sin[
x]^2],x]
```

```
[Out] 2*ArcTan[Cos[x]/Sqrt[-4 - Cos[x]^2]] - ArcTan[(Sqrt[5]*Cos[x])/Sqrt[-4 - Co
s[x]^2]]/Sqrt[5] - (2*ArcTan[Sqrt[-4 - Cos[x]^2]/Sqrt[5]])/Sqrt[5] - 2*ArcT
anh[Sin[x]/Sqrt[-5 + Sin[x]^2]] + 2*Sqrt[-4 - Cos[x]^2] + (2*Csc[x]*Sqrt[-5
 + Sin[x]^2])/5
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 1010

```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,
Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p, q}
, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rubi steps

$$\int \frac{\csc^2(x)\left(-2\cos^3(x)(-1+\sin(x))+\cos(2x)\sin(x)\right)}{\sqrt{-5+\sin^2(x)}}\,dx = \int\left(\frac{2\cos(x)\cot^2(x)}{\sqrt{-5+\sin^2(x)}}+\frac{\left(-2\cos^3(x)+\cos(2x)\right)\csc(x)}{\sqrt{-5+\sin^2(x)}}\right)dx$$

$$= 2\int \frac{\cos(x)\cot^2(x)}{\sqrt{-5+\sin^2(x)}}\,dx + \int \frac{\left(-2\cos^3(x)+\cos(2x)\right)\csc(x)}{\sqrt{-5+\sin^2(x)}}$$

$$= 2\,\mathrm{Subst}\left(\int \frac{1-x^2}{x^2\sqrt{-5+x^2}}\,dx, x, \sin(x)\right) - \mathrm{Subst}\left(\int \frac{-1+}{\sqrt{-4-}}\right.$$

$$= \frac{2}{5}\csc(x)\sqrt{-5+\sin^2(x)} - 2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-5+x^2}}\,dx, x, \sin\right.$$

$$= \frac{2}{5}\csc(x)\sqrt{-5+\sin^2(x)} + 2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-4-x^2}}\,dx, x, \cos\right.$$

$$= -2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{-5+\sin^2(x)}}\right) + 2\sqrt{-4-\cos^2(x)} + \frac{2}{5}\csc(x$$

$$= 2\tan^{-1}\left(\frac{\cos(x)}{\sqrt{-4-\cos^2(x)}}\right) - 2\tanh^{-1}\left(\frac{\sin(x)}{\sqrt{-5+\sin^2(x)}}\right) +$$

$$= 2\tan^{-1}\left(\frac{\cos(x)}{\sqrt{-4-\cos^2(x)}}\right) - \frac{\tan^{-1}\left(\frac{\sqrt{5}\cos(x)}{\sqrt{-4-\cos^2(x)}}\right)}{\sqrt{5}} - 2\tanh$$

$$= 2\tan^{-1}\left(\frac{\cos(x)}{\sqrt{-4-\cos^2(x)}}\right) - \frac{\tan^{-1}\left(\frac{\sqrt{5}\cos(x)}{\sqrt{-4-\cos^2(x)}}\right)}{\sqrt{5}} - \frac{2\tan^{-1}}{}$$

Mathematica [C] time = 2.32738, size = 338, normalized size = 3.05

$$\frac{(16-32i)\sqrt{5}\sqrt{\frac{(1+2i)(\cos(x)-2i)}{\cos(x)+1}}\sqrt{\frac{(1-2i)(\cos(x)+2i)}{\cos(x)+1}}\cos^2\left(\frac{x}{2}\right)\mathrm{EllipticF}\left(\sin^{-1}\left(\frac{(1+2i)\tan\left(\frac{x}{2}\right)}{\sqrt{5}}\right), -\frac{7}{25}+\frac{24i}{25}\right) - (32-64i)\sqrt{5}\sqrt{\frac{(1}{}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]^2*(-2*Cos[x]^3*(-1 + Sin[x]) + Cos[2*x]*Sin[x]))/Sqrt[-5
+ Sin[x]^2],x]
```

```
[Out] ((16 - 32*I)*Sqrt[5]*Cos[x/2]^2*Sqrt[((1 + 2*I)*(-2*I + Cos[x]))/(1 + Cos[x
])]*Sqrt[((1 - 2*I)*(2*I + Cos[x]))/(1 + Cos[x])]*EllipticF[ArcSin[((1 + 2*
I)*Tan[x/2])/Sqrt[5]], -7/25 + (24*I)/25] - (32 - 64*I)*Sqrt[5]*Cos[x/2]^2*
Sqrt[((1 + 2*I)*(-2*I + Cos[x]))/(1 + Cos[x])]*Sqrt[((1 - 2*I)*(2*I + Cos[x
]))/(1 + Cos[x])]*EllipticPi[3/5 + (4*I)/5, ArcSin[((1 + 2*I)*Tan[x/2])/Sqr
t[5]], -7/25 + (24*I)/25] - 5*(85 + Sqrt[10]*ArcTan[(Sqrt[10]*Cos[x])/Sqrt[
-9 - Cos[2*x]]]*Sqrt[-9 - Cos[2*x]] + 2*Sqrt[10]*ArcTan[Sqrt[-9 - Cos[2*x]]
```

```
/Sqrt[10]]*Sqrt[-9 - Cos[2*x]] + 18*Csc[x] + 2*Cos[2*x]*Csc[x] + (10*I)*Sqr
t[2]*Sqrt[-9 - Cos[2*x]]*Log[I*Sqrt[2]*Cos[x] + Sqrt[-9 - Cos[2*x]]] + 5*Cs
c[x]*Sin[3*x]))/(25*Sqrt[2]*Sqrt[-9 - Cos[2*x]])
```

Maple [A] time = 0.184, size = 130, normalized size = 1.2

$$-2\ln\left(\sin(x)+\sqrt{-5+(\sin(x))^2}\right)+2\sqrt{-5+(\sin(x))^2}+\frac{2\sqrt{5}}{5}\arctan\left(\sqrt{5}\frac{1}{\sqrt{-5+(\sin(x))^2}}\right)+\frac{2}{5\sin(x)}\sqrt{-5+(\sin(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)^(1/2),
x)
```

```
[Out] -2*ln(sin(x)+(-5+sin(x)^2)^(1/2))+2*(-5+sin(x)^2)^(1/2)+2/5*5^(1/2)*arctan(
5^(1/2)/(-5+sin(x)^2)^(1/2))+2/5*(-5+sin(x)^2)^(1/2)/sin(x)+1/10*((-5+sin(x
)^2)*cos(x)^2)^(1/2)*(5^(1/2)*arctan(1/5*(3*sin(x)^2-5)*5^(1/2)/(-cos(x)^4-
4*cos(x)^2)^(1/2))+10*arcsin(1+1/2*cos(x)^2))/cos(x)/(-5+sin(x)^2)^(1/2)
```

Maxima [C] time = 1.51108, size = 155, normalized size = 1.4

$$\frac{2}{5}\sqrt{5}\arcsin\left(\frac{\sqrt{5}}{|\sin(x)|}\right)-\frac{1}{10}i\sqrt{5}\operatorname{arsinh}\left(\frac{\cos(x)}{2(\cos(x)+1)}-\frac{2}{\cos(x)+1}\right)-\frac{1}{10}i\sqrt{5}\operatorname{arsinh}\left(-\frac{\cos(x)}{2(\cos(x)-1)}-\frac{2}{\cos(x)-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)^
(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*sqrt(5)*arcsin(sqrt(5)/abs(sin(x))) - 1/10*I*sqrt(5)*arcsinh(1/2*cos(x)
/(cos(x) + 1) - 2/(cos(x) + 1)) - 1/10*I*sqrt(5)*arcsinh(-1/2*cos(x)/(cos(x
) - 1) - 2/(cos(x) - 1)) + 2*sqrt(sin(x)^2 - 5) + 2/5*sqrt(sin(x)^2 - 5)/si
n(x) - 2*I*arcsinh(1/2*cos(x)) - 2*log(2*sqrt(sin(x)^2 - 5) + 2*sin(x))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)^
(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cos(x)**3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)**2/(-5+sin(x)**
2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.78554, size = 356, normalized size = 3.21

$$\pi \operatorname{sgn}\left(-2i\sqrt{\cos(x)^2+4}-4i\right)\operatorname{sgn}(\cos(x))-\frac{1}{5}\sqrt{5}\left(\pi \operatorname{sgn}\left(-2i\sqrt{\cos(x)^2+4}-4i\right)\operatorname{sgn}(\cos(x))+2\arctan\left(\frac{\sqrt{5}\left(\frac{\left(i\sqrt{\right.}\right.}{}\right.}{5\left(-2\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cos(x)^3*(-1+sin(x))+cos(2*x)*sin(x))/sin(x)^2/(-5+sin(x)^2)^
(1/2),x, algorithm="giac")
```

```
[Out] pi*sgn(-2*I*sqrt(cos(x)^2 + 4) - 4*I)*sgn(cos(x)) - 1/5*sqrt(5)*(pi*sgn(-2*
I*sqrt(cos(x)^2 + 4) - 4*I)*sgn(cos(x)) + 2*arctan(1/5*sqrt(5)*((I*sqrt(cos
(x)^2 + 4) + 2*I)^2/cos(x)^2 - 1)*cos(x)/(-2*I*sqrt(cos(x)^2 + 4) - 4*I)))
+ 1/10*sqrt(5)*(pi*sgn(-2*I*sqrt(cos(x)^2 + 4) - 4*I)*sgn(cos(x)) + 2*arcta
n(1/5*sqrt(5)*((-I*sqrt(cos(x)^2 + 4) - 2*I)^2/cos(x)^2 - 1)*cos(x)/(-2*I*s
qrt(cos(x)^2 + 4) - 4*I))) - 2/5*sqrt(5)*arctan(1/5*sqrt(5)*sqrt(sin(x)^2 -
 5)) + 2*sqrt(sin(x)^2 - 5) + 4/((sqrt(sin(x)^2 - 5) - sin(x))^2 + 5) + 2*a
rctan(((I*sqrt(cos(x)^2 + 4) + 2*I)^2/cos(x)^2 - 1)*cos(x)/(-2*I*sqrt(cos(x
)^2 + 4) - 4*I)) + log((sqrt(sin(x)^2 - 5) - sin(x))^2)
```

3.427 $\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx$

Optimal. Leaf size=112

$$\frac{5\sin^{-1}\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}}+\frac{3}{4}\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)-\frac{1}{2}\sin(x)\sqrt{4\cos^2(x)-1}-\frac{1}{2}\sin(x)\sqrt{8\cos^2(x)-1}-\frac{3}{4}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{4\cos^2(x)-1}}\right)$$

```
[Out] (5*ArcSin[2*Sqrt[2/7]*Sin[x]])/(4*Sqrt[2]) + (3*ArcSin[(2*Sin[x])/Sqrt[3]])
/4 - (3*ArcTan[Sin[x]/Sqrt[-1 + 4*Cos[x]^2]])/4 - (3*ArcTan[Sin[x]/Sqrt[-1
+ 8*Cos[x]^2]])/4 - (Sqrt[-1 + 4*Cos[x]^2]*Sin[x])/2 - (Sqrt[-1 + 8*Cos[x]^
2]*Sin[x])/2
```

Rubi [A] time = 0.444286, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 6, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6742, 402, 216, 377, 204, 195}

$$\frac{5\sin^{-1}\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}}+\frac{3}{4}\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)-\frac{1}{2}\sin(x)\sqrt{7-8\sin^2(x)}-\frac{1}{2}\sin(x)\sqrt{3-4\sin^2(x)}-\frac{3}{4}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{7-8\sin^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]/(-Sqrt[-1 + 8*Cos[x]^2] + Sqrt[3*Cos[x]^2 - Sin[x]^2]),x]
```

```
[Out] (5*ArcSin[2*Sqrt[2/7]*Sin[x]])/(4*Sqrt[2]) + (3*ArcSin[(2*Sin[x])/Sqrt[3]])
/4 - (3*ArcTan[Sin[x]/Sqrt[7 - 8*Sin[x]^2]])/4 - (3*ArcTan[Sin[x]/Sqrt[3 -
4*Sin[x]^2]])/4 - (Sin[x]*Sqrt[7 - 8*Sin[x]^2])/2 - (Sin[x]*Sqrt[3 - 4*Sin[
x]^2])/2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int \frac{\cos(3x)}{-\sqrt{-1+8\cos^2(x)}+\sqrt{3\cos^2(x)-\sin^2(x)}}\,dx = \text{Subst}\left(\int \frac{-1+4x^2}{\sqrt{7-8x^2}-\sqrt{3-4x^2}}\,dx, x, \sin(x)\right)$$

$$= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{7-8x^2}-\sqrt{3-4x^2}}+\frac{4x^2}{\sqrt{7-8x^2}-\sqrt{3-4x^2}}\right) dx, x,$$

$$= 4\,\text{Subst}\left(\int \frac{x^2}{\sqrt{7-8x^2}-\sqrt{3-4x^2}}\,dx, x, \sin(x)\right) - \text{Subst}\left(\int \frac{}{\sqrt{7-}}\right.$$

$$= 4\,\text{Subst}\left(\int \left(-\frac{1}{4}\sqrt{7-8x^2}-\frac{1}{4}\sqrt{3-4x^2}-\frac{\sqrt{7-8x^2}}{4\left(-1+x^2\right)}-\frac{\sqrt{3-4x^2}}{4\left(-1+x^2\right.}\right.\right.$$

$$= \frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt{7-8x^2}}{-1+x^2}\,dx, x, \sin(x)\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{\sqrt{3-4x^2}}{-1+x^2}\,dx, x\right.$$

$$= -\frac{1}{2}\sin(x)\sqrt{7-8\sin^2(x)} - \frac{1}{2}\sin(x)\sqrt{3-4\sin^2(x)} - \frac{1}{4}\,\text{Subst}\left(\int\right.$$

$$= -\frac{11\sin^{-1}\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}} + 2\sqrt{2}\sin^{-1}\left(2\sqrt{\frac{2}{7}}\sin(x)\right) + \frac{3}{4}\sin^{-1}\left(\frac{2\,\text{si}}{}\right.$$

$$= -\frac{11\sin^{-1}\left(2\sqrt{\frac{2}{7}}\sin(x)\right)}{4\sqrt{2}} + 2\sqrt{2}\sin^{-1}\left(2\sqrt{\frac{2}{7}}\sin(x)\right) + \frac{3}{4}\sin^{-1}\left(\frac{2\,\text{si}}{}\right.$$

Mathematica [A] time = 0.344358, size = 156, normalized size = 1.39

$$\frac{1}{8}\left(5\sqrt{2}\sin^{-1}\left(2\sqrt{\frac{2}{7}}\sin(x)\right)+6\sin^{-1}\left(\frac{2\sin(x)}{\sqrt{3}}\right)-4\sin(x)\sqrt{2\cos(2x)+1}-4\sin(x)\sqrt{4\cos(2x)+3}+3\tan^{-1}\left(\frac{7-}{\sqrt{4\,\text{co}}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]/(-Sqrt[-1 + 8*Cos[x]^2] + Sqrt[3*Cos[x]^2 - Sin[x]^2]),x
]
```

```
[Out] (5*Sqrt[2]*ArcSin[2*Sqrt[2/7]*Sin[x]] + 6*ArcSin[(2*Sin[x])/Sqrt[3]] + 3*Ar
cTan[(7 - 8*Sin[x])/Sqrt[3 + 4*Cos[2*x]]] + 3*ArcTan[(3 - 4*Sin[x])/Sqrt[1
+ 2*Cos[2*x]]] - 3*ArcTan[(3 + 4*Sin[x])/Sqrt[1 + 2*Cos[2*x]]] - 3*ArcTan[(
7 + 8*Sin[x])/Sqrt[3 + 4*Cos[2*x]]] - 4*Sqrt[1 + 2*Cos[2*x]]*Sin[x] - 4*Sqr
t[3 + 4*Cos[2*x]]*Sin[x])/8
```

Maple [C] time = 4.062, size = 97512, normalized size = 870.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x)
```

```
[Out] result too large to display
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x,
algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 4.09137, size = 632, normalized size = 5.64

$$-\frac{5}{32}\sqrt{2}\arctan\left(\frac{\left(512\sqrt{2}\cos\left(x\right)^{4}-576\sqrt{2}\cos\left(x\right)^{2}+113\sqrt{2}\right)\sqrt{8\cos\left(x\right)^{2}-1}}{16\left(128\cos\left(x\right)^{4}-88\cos\left(x\right)^{2}+9\right)\sin\left(x\right)}\right)-\frac{1}{2}\sqrt{8\cos\left(x\right)^{2}-1}\sin\left(x\right)-\frac{1}{2}\sqrt{4\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x,
algorithm="fricas")
```

```
[Out] -5/32*sqrt(2)*arctan(1/16*(512*sqrt(2)*cos(x)^4 - 576*sqrt(2)*cos(x)^2 + 11
3*sqrt(2))*sqrt(8*cos(x)^2 - 1)/((128*cos(x)^4 - 88*cos(x)^2 + 9)*sin(x)))
- 1/2*sqrt(8*cos(x)^2 - 1)*sin(x) - 1/2*sqrt(4*cos(x)^2 - 1)*sin(x) + 3/8*a
rctan((4*(8*cos(x)^2 - 5)*sqrt(4*cos(x)^2 - 1)*sin(x) - 9*cos(x)*sin(x))/(6
4*cos(x)^4 - 71*cos(x)^2 + 16)) + 3/8*arctan(sin(x)/cos(x)) + 3/8*arctan(1/
2*(9*cos(x)^2 - 2)/(sqrt(8*cos(x)^2 - 1)*sin(x))) + 3/4*arctan(sqrt(4*cos(x
)^2 - 1)/sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(3x\right)}{\sqrt{-\sin^{2}\left(x\right)+3\cos^{2}\left(x\right)}-\sqrt{8\cos^{2}\left(x\right)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(-(-1+8*cos(x)**2)**(1/2)+(3*cos(x)**2-sin(x)**2)**(1/2)
),x)
```

```
[Out] Integral(cos(3*x)/(sqrt(-sin(x)**2 + 3*cos(x)**2) - sqrt(8*cos(x)**2 - 1)),
 x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\cos(3x)}{\sqrt{8 \cos(x)^2 - 1} - \sqrt{3 \cos(x)^2 - \sin(x)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)/(-(-1+8*cos(x)^2)^(1/2)+(3*cos(x)^2-sin(x)^2)^(1/2)),x,
algorithm="giac")
```

```
[Out] integrate(-cos(3*x)/(sqrt(8*cos(x)^2 - 1) - sqrt(3*cos(x)^2 - sin(x)^2)), x
)
```

3.428 $\int \left(2-3 \sin^2(x)\right)^{3/5} \sin(4x)\,dx$

Optimal. Leaf size=33

$$\frac{5}{36}\left(2-3\sin^2(x)\right)^{8/5}-\frac{20}{117}\left(2-3\sin^2(x)\right)^{13/5}$$

```
[Out] (5*(2 - 3*Sin[x]^2)^(8/5))/36 - (20*(2 - 3*Sin[x]^2)^(13/5))/117
```

Rubi [A] time = 0.0608089, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {12, 444, 43}

$$\frac{5}{36}\left(2-3\sin^2(x)\right)^{8/5}-\frac{20}{117}\left(2-3\sin^2(x)\right)^{13/5}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*Sin[x]^2)^(3/5)*Sin[4*x],x]
```

```
[Out] (5*(2 - 3*Sin[x]^2)^(8/5))/36 - (20*(2 - 3*Sin[x]^2)^(13/5))/117
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(2-3\sin^2(x)\right)^{3/5}\sin(4x)\,dx &= \text{Subst}\left(\int 4x\left(2-3x^2\right)^{3/5}\left(1-2x^2\right)\,dx,x,\sin(x)\right)\\
&= 4\,\text{Subst}\left(\int x\left(2-3x^2\right)^{3/5}\left(1-2x^2\right)\,dx,x,\sin(x)\right)\\
&= 2\,\text{Subst}\left(\int (2-3x)^{3/5}(1-2x)\,dx,x,\sin^2(x)\right)\\
&= 2\,\text{Subst}\left(\int \left(-\frac{1}{3}(2-3x)^{3/5}+\frac{2}{3}(2-3x)^{8/5}\right)\,dx,x,\sin^2(x)\right)\\
&= \frac{5}{36}\left(2-3\sin^2(x)\right)^{8/5}-\frac{20}{117}\left(2-3\sin^2(x)\right)^{13/5}
\end{aligned}$$

Mathematica [A] time = 0.0413974, size = 29, normalized size = 0.88

$$-\frac{5(3\cos(2x)+1)^{8/5}(24\cos(2x)-5)}{936\ 2^{3/5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*Sin[x]^2)^(3/5)*Sin[4*x],x]
```

```
[Out] (-5*(1 + 3*Cos[2*x])^(8/5)*(-5 + 24*Cos[2*x]))/(936*2^(3/5))
```

Maple [A] time = 0.101, size = 38, normalized size = 1.2

$$\frac{5}{12}\left(3\left(\cos\left(x\right)\right)^{2}-1\right)^{\frac{8}{5}}-\frac{20}{117}\left(\frac{1}{2}+\frac{3\cos\left(2x\right)}{2}\right)^{\frac{13}{5}}-\frac{5}{18}\left(\frac{1}{2}+\frac{3\cos\left(2x\right)}{2}\right)^{\frac{8}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2-3*sin(x)^2)^(3/5)*sin(4*x),x)
```

```
[Out] 5/12*(3*cos(x)^2-1)^(8/5)-20/117*(1/2+3/2*cos(2*x))^(13/5)-5/18*(1/2+3/2*co
s(2*x))^(8/5)
```

Maxima [A] time = 0.969511, size = 34, normalized size = 1.03

$$-\frac{20}{117}\left(-3\sin\left(x\right)^{2}+2\right)^{\frac{13}{5}}+\frac{5}{36}\left(-3\sin\left(x\right)^{2}+2\right)^{\frac{8}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x, algorithm="maxima")
```

```
[Out] -20/117*(-3*sin(x)^2 + 2)^(13/5) + 5/36*(-3*sin(x)^2 + 2)^(8/5)
```

Fricas [A] time = 3.46549, size = 89, normalized size = 2.7

$$-\frac{5}{468}\left(144\cos\left(x\right)^{4}-135\cos\left(x\right)^{2}+29\right)\left(3\cos\left(x\right)^{2}-1\right)^{\frac{3}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x, algorithm="fricas")
```

```
[Out] -5/468*(144*cos(x)^4 - 135*cos(x)^2 + 29)*(3*cos(x)^2 - 1)^(3/5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*sin(x)**2)**(3/5)*sin(4*x),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-3 \sin(x)^2 + 2\right)^{\frac{3}{5}} \sin(4x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2-3*sin(x)^2)^(3/5)*sin(4*x),x, algorithm="giac")
```

```
[Out] integrate((-3*sin(x)^2 + 2)^(3/5)*sin(4*x), x)
```

3.429 $\int \cos(x)\sqrt{\cos(2x)}\,dx$

Optimal. Leaf size=33

$$\frac{\sin^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}+\frac{1}{2}\sin(x)\sqrt{\cos(2x)}$$

```
[Out] ArcSin[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + (Sqrt[Cos[2*x]]*Sin[x])/2
```

Rubi [A] time = 0.0220386, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4356, 195, 216}

$$\frac{\sin^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}+\frac{1}{2}\sin(x)\sqrt{\cos(2x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sqrt[Cos[2*x]],x]
```

```
[Out] ArcSin[Sqrt[2]*Sin[x]]/(2*Sqrt[2]) + (Sqrt[Cos[2*x]]*Sin[x])/2
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sqrt{\cos(2x)}\,dx &= \text{Subst}\left(\int \sqrt{1-2x^2}\,dx,x,\sin(x)\right)\\ &= \frac{1}{2}\sqrt{\cos(2x)}\sin(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-2x^2}}\,dx,x,\sin(x)\right)\\ &= \frac{\sin^{-1}\left(\sqrt{2}\sin(x)\right)}{2\sqrt{2}}+\frac{1}{2}\sqrt{\cos(2x)}\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0183587, size = 32, normalized size = 0.97

$$\frac{1}{4}\left(\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right)+2\sin(x)\sqrt{\cos(2x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sqrt[Cos[2*x]],x]
```

```
[Out] (Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] + 2*Sqrt[Cos[2*x]]*Sin[x])/4
```

Maple [B] time = 0.056, size = 61, normalized size = 1.9

$$\frac{1}{8\sin(x)}\sqrt{\left(2(\cos(x))^2-1\right)(\sin(x))^2}\left(\sqrt{2}\arcsin\left(4(\sin(x))^2-1\right)+4\sqrt{-2(\sin(x))^4+(\sin(x))^2}\right)\frac{1}{\sqrt{2(\cos(x))^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x)^(1/2),x)
```

```
[Out] 1/8*((2*cos(x)^2-1)*sin(x)^2)^(1/2)*(2^(1/2)*arcsin(4*sin(x)^2-1)+4*(-2*sin
(x)^4+sin(x)^2)^(1/2))/sin(x)/(2*cos(x)^2-1)^(1/2)
```

Maxima [B] time = 1.60023, size = 659, normalized size = 19.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*sqrt(2)*(2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*a
rctan2(sin(4*x), cos(4*x) + 1))*sin(2*x) - (cos(2*x) - 1)*sin(1/2*arctan2(s
in(4*x), cos(4*x) + 1))) + arctan2(-(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) +
 1)^(1/4)*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(2*x) - cos(2*x)*sin
(1/2*arctan2(sin(4*x), cos(4*x) + 1))), (cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*
x) + 1)^(1/4)*(cos(2*x)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x)
*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))) + 1) - arctan2(-(cos(4*x)^2 + si
n(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))*
sin(2*x) - cos(2*x)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))), (cos(4*x)^2
+ sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(2*x)*cos(1/2*arctan2(sin(4*x), co
s(4*x) + 1)) + sin(2*x)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))) - 1) - ar
ctan2((cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*sin(1/2*arctan2(sin(
4*x), cos(4*x) + 1)), (cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(
1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) + arctan2((cos(4*x)^2 + sin(4*x)^
2 + 2*cos(4*x) + 1)^(1/4)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1)), (cos(4*
x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x
) + 1)) - 1))
```

Fricas [B] time = 3.21268, size = 242, normalized size = 7.33

$$-\frac{1}{16}\sqrt{2}\arctan\left(\frac{\left(32\sqrt{2}\cos(x)^4-48\sqrt{2}\cos(x)^2+17\sqrt{2}\right)\sqrt{2\cos(x)^2-1}}{8\left(8\cos(x)^4-10\cos(x)^2+3\right)\sin(x)}\right)+\frac{1}{2}\sqrt{2\cos(x)^2-1}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*sqrt(2)*arctan(1/8*(32*sqrt(2)*cos(x)^4 - 48*sqrt(2)*cos(x)^2 + 17*sq
rt(2))*sqrt(2*cos(x)^2 - 1)/((8*cos(x)^4 - 10*cos(x)^2 + 3)*sin(x))) + 1/2*
sqrt(2*cos(x)^2 - 1)*sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(x)\sqrt{\cos(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)**(1/2),x)
```

```
[Out] Integral(cos(x)*sqrt(cos(2*x)), x)
```

Giac [A] time = 1.11515, size = 36, normalized size = 1.09

$$\frac{1}{4}\sqrt{2}\arcsin\left(\sqrt{2}\sin(x)\right) + \frac{1}{2}\sqrt{-2\sin(x)^2 + 1}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arcsin(sqrt(2)*sin(x)) + 1/2*sqrt(-2*sin(x)^2 + 1)*sin(x)
```

3.430 $\int \cos^{\frac{3}{2}}(2x) \sin(x) \, dx$

Optimal. Leaf size=55

$$-\frac{1}{4} \cos(x) \cos^{\frac{3}{2}}(2x) + \frac{3}{8} \cos(x) \sqrt{\cos(2x)} - \frac{3 \tanh^{-1}\left(\frac{\sqrt{2} \cos(x)}{\sqrt{\cos(2x)}}\right)}{8\sqrt{2}}$$

```
[Out] (-3*ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]])/(8*Sqrt[2]) + (3*Cos[x]*Sqrt[
Cos[2*x]])/8 - (Cos[x]*Cos[2*x]^(3/2))/4
```

Rubi [A] time = 0.0326769, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4357, 195, 217, 206}

$$-\frac{1}{4} \cos(x) \cos^{\frac{3}{2}}(2x) + \frac{3}{8} \cos(x) \sqrt{\cos(2x)} - \frac{3 \tanh^{-1}\left(\frac{\sqrt{2} \cos(x)}{\sqrt{\cos(2x)}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]^(3/2)*Sin[x],x]
```

```
[Out] (-3*ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]])/(8*Sqrt[2]) + (3*Cos[x]*Sqrt[
Cos[2*x]])/8 - (Cos[x]*Cos[2*x]^(3/2))/4
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(2x)\sin(x)\,dx &= -\operatorname{Subst}\left(\int \left(-1+2x^2\right)^{3/2}\,dx,x,\cos(x)\right)\\ &= -\frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)+\frac{3}{4}\operatorname{Subst}\left(\int \sqrt{-1+2x^2}\,dx,x,\cos(x)\right)\\ &= \frac{3}{8}\cos(x)\sqrt{\cos(2x)}-\frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)-\frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+2x^2}}\,dx,x,\cos(x)\right)\\ &= \frac{3}{8}\cos(x)\sqrt{\cos(2x)}-\frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)-\frac{3}{8}\operatorname{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\frac{\cos(x)}{\sqrt{\cos(2x)}}\right)\\ &= -\frac{3\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{8\sqrt{2}}+\frac{3}{8}\cos(x)\sqrt{\cos(2x)}-\frac{1}{4}\cos(x)\cos^{\frac{3}{2}}(2x)\end{aligned}$$

Mathematica [A] time = 0.095451, size = 49, normalized size = 0.89

$$-\frac{1}{8}\sqrt{\cos(2x)}(\cos(3x)-2\cos(x))-\frac{3\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]^(3/2)*Sin[x],x]
```

```
[Out] -(Sqrt[Cos[2*x]]*(-2*Cos[x] + Cos[3*x]))/8 - (3*Log[Sqrt[2]*Cos[x] + Sqrt[C
os[2*x]]])/(8*Sqrt[2])
```

Maple [A] time = 0.038, size = 55, normalized size = 1.

$$-\frac{(\cos(x))^3}{2}\sqrt{2(\cos(x))^2-1}+\frac{5\cos(x)}{8}\sqrt{2(\cos(x))^2-1}-\frac{3\sqrt{2}}{16}\ln\left(\cos(x)\sqrt{2}+\sqrt{2(\cos(x))^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^(3/2)*sin(x),x)
```

```
[Out] -1/2*cos(x)^3*(2*cos(x)^2-1)^(1/2)+5/8*cos(x)*(2*cos(x)^2-1)^(1/2)-3/16*ln(
cos(x)*2^(1/2)+(2*cos(x)^2-1)^(1/2))*2^(1/2)
```

Maxima [B] time = 1.63463, size = 1067, normalized size = 19.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/128*sqrt(2)*(4*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(((cos(4
*x) - 2)*cos(1/2*arctan2(sin(4*x), cos(4*x))) + sin(4*x)*sin(1/2*arctan2(si
n(4*x), cos(4*x))) + cos(4*x) - 2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))
 - (cos(1/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x) - (cos(4*x) - 2)*sin(1/2*
arctan2(sin(4*x), cos(4*x))) - sin(4*x))*sin(1/2*arctan2(sin(4*x), cos(4*x)
```

```
 + 1))) + 3*log(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arct
an2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x)
+ 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + 2*(cos(4*x)^2 + sin(4*x)^
2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) - 3
*log(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x
), cos(4*x) + 1))^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*sin(1/
2*arctan2(sin(4*x), cos(4*x) + 1))^2 - 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4
*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 1) + 3*log(((cos(
1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2
)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(1/2*arctan2(sin(4*x), c
os(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2)*sin(1/2*arctan2(sin(4
*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1) + 2*(
cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x),
cos(4*x) + 1))*cos(1/2*arctan2(sin(4*x), cos(4*x))) + sin(1/2*arctan2(sin(4
*x), cos(4*x) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*x)))) + 1) - 3*log(((co
s(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))
^2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(1/2*arctan2(sin(4*x),
 cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2)*sin(1/2*arctan2(sin
(4*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1) - 2
*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1/2*arctan2(sin(4*x)
, cos(4*x) + 1))*cos(1/2*arctan2(sin(4*x), cos(4*x))) + sin(1/2*arctan2(sin
(4*x), cos(4*x) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*x)))) + 1))
```

Fricas [B] time = 3.43034, size = 329, normalized size = 5.98

$$-\frac{1}{8}\left(4\cos(x)^3-5\cos(x)\right)\sqrt{2\cos(x)^2-1}+\frac{3}{128}\sqrt{2}\log\left(2048\cos(x)^8-2048\cos(x)^6+640\cos(x)^4-64\cos(x)^2-8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/8*(4*cos(x)^3 - 5*cos(x))*sqrt(2*cos(x)^2 - 1) + 3/128*sqrt(2)*log(2048*
cos(x)^8 - 2048*cos(x)^6 + 640*cos(x)^4 - 64*cos(x)^2 - 8*(128*sqrt(2)*cos(
x)^7 - 96*sqrt(2)*cos(x)^5 + 20*sqrt(2)*cos(x)^3 - sqrt(2)*cos(x))*sqrt(2*c
os(x)^2 - 1) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)**(3/2)*sin(x),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11543, size = 65, normalized size = 1.18

$$-\frac{1}{8}\left(4\cos(x)^2-5\right)\sqrt{2\cos(x)^2-1}\cos(x)+\frac{3}{16}\sqrt{2}\log\left(\left|-\sqrt{2}\cos(x)+\sqrt{2\cos(x)^2-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^(3/2)*sin(x),x, algorithm="giac")
```

```
[Out] -1/8*(4*cos(x)^2 - 5)*sqrt(2*cos(x)^2 - 1)*cos(x) + 3/16*sqrt(2)*log(abs(-s
qrt(2)*cos(x) + sqrt(2*cos(x)^2 - 1)))
```

3.431 $\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx$

Optimal. Leaf size=16

$$-\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

```
[Out] -Cos[3*x]/(3*Cos[2*x]^(3/2))
```

Rubi [A] time = 0.0134062, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4331}

$$-\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/Cos[2*x]^(5/2),x]
```

```
[Out] -Cos[3*x]/(3*Cos[2*x]^(3/2))
```

Rule 4331

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)], x_Symbol
] :> -Simp[((m + 2)*(e*Cos[a + b*x])^(m + 1)*Cos[(m + 1)*(a + b*x)])/(d*e*(
m + 1)), x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b
, Abs[m + 2]]
```

Rubi steps

$$\int \frac{\sin(x)}{\cos^{\frac{5}{2}}(2x)}\,dx = -\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

Mathematica [A] time = 0.0357061, size = 16, normalized size = 1.

$$-\frac{\cos(3x)}{3\cos^{\frac{3}{2}}(2x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/Cos[2*x]^(5/2),x]
```

```
[Out] -Cos[3*x]/(3*Cos[2*x]^(3/2))
```

Maple [B] time = 0.075, size = 39, normalized size = 2.4

$$\frac{\cos(x)\left(4(\sin(x))^2-1\right)}{12(\sin(x))^4-12(\sin(x))^2+3}\sqrt{-2(\sin(x))^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/cos(2*x)^(5/2),x)
```

```
[Out] 1/3/(4*sin(x)^4-4*sin(x)^2+1)*(-2*sin(x)^2+1)^(1/2)*cos(x)*(4*sin(x)^2-1)
```

Maxima [B] time = 1.51037, size = 122, normalized size = 7.62

$$-\frac{\sqrt{2}\sin\left(\frac{3}{2}\arctan\left(\sin\left(4x\right),\cos\left(4x\right)+1\right)\right)\sin\left(\frac{3}{2}\arctan\left(\sin\left(4x\right),\cos\left(4x\right)\right)\right)+\left(\sqrt{2}\cos\left(\frac{3}{2}\arctan\left(\sin\left(4x\right),\cos\right.\right.\right.}{3\left(\cos\left(4x\right)^2+\sin\left(4x\right)^2+2\cos\left(4x\right)+1\right)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(sqrt(2)*sin(3/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(3/2*arctan2(sin(
4*x), cos(4*x))) + (sqrt(2)*cos(3/2*arctan2(sin(4*x), cos(4*x))) + sqrt(2))
*cos(3/2*arctan2(sin(4*x), cos(4*x) + 1)))/(cos(4*x)^2 + sin(4*x)^2 + 2*cos
(4*x) + 1)^(3/4)
```

Fricas [B] time = 3.198, size = 109, normalized size = 6.81

$$-\frac{\left(4\cos\left(x\right)^3-3\cos\left(x\right)\right)\sqrt{2\cos\left(x\right)^2-1}}{3\left(4\cos\left(x\right)^4-4\cos\left(x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(4*cos(x)^3 - 3*cos(x))*sqrt(2*cos(x)^2 - 1)/(4*cos(x)^4 - 4*cos(x)^2
+ 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/cos(2*x)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27446, size = 62, normalized size = 3.88

$$\frac{\left(\left(\tan\left(\frac{1}{2}x\right)^2-15\right)\tan\left(\frac{1}{2}x\right)^2+15\right)\tan\left(\frac{1}{2}x\right)^2-1}{3\left(\tan\left(\frac{1}{2}x\right)^4-6\tan\left(\frac{1}{2}x\right)^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/cos(2*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(((tan(1/2*x)^2 - 15)*tan(1/2*x)^2 + 15)*tan(1/2*x)^2 - 1)/(tan(1/2*x)^
4 - 6*tan(1/2*x)^2 + 1)^(3/2)
```

3.432 $\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx$

Optimal. Leaf size=49

$$2\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right)-\frac{5}{2}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right)-\frac{1}{2}\sqrt{\cos(2x)}\tan(x)\sec(x)$$

```
[Out] 2*Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - (5*ArcTan[Sin[x]/Sqrt[Cos[2*x]]])/2 - (S
qrt[Cos[2*x]]*Sec[x]*Tan[x])/2
```

Rubi [A] time = 0.0574821, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {4364, 413, 523, 216, 377, 203}

$$2\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right)-\frac{5}{2}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right)-\frac{1}{2}\sqrt{\cos(2x)}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]^(3/2)*Sec[x]^3,x]
```

```
[Out] 2*Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - (5*ArcTan[Sin[x]/Sqrt[Cos[2*x]]])/2 - (S
qrt[Cos[2*x]]*Sec[x]*Tan[x])/2
```

Rule 4364

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x^
2)^((n - 1)/2), Sin[c*(a + b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /
; FunctionOfQ[Sin[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && IntegerQ
[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(2x)\sec^3(x)\,dx &= \text{Subst}\left(\int \frac{\left(1-2x^2\right)^{3/2}}{\left(1-x^2\right)^2}\,dx, x, \sin(x)\right)\\
&= -\frac{1}{2}\sqrt{\cos(2x)}\sec(x)\tan(x) - \frac{1}{2}\text{Subst}\left(\int \frac{-3+8x^2}{\sqrt{1-2x^2}\left(1-x^2\right)}\,dx, x, \sin(x)\right)\\
&= -\frac{1}{2}\sqrt{\cos(2x)}\sec(x)\tan(x) - \frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-2x^2}\left(1-x^2\right)}\,dx, x, \sin(x)\right) + 4\,\text{Subst}\left(\int \frac{1}{\sqrt{1-2}}\right.\\
&= 2\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right) - \frac{1}{2}\sqrt{\cos(2x)}\sec(x)\tan(x) - \frac{5}{2}\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\sin(x)}{\sqrt{\cos(2x)}}\right)\\
&= 2\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right) - \frac{5}{2}\tan^{-1}\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right) - \frac{1}{2}\sqrt{\cos(2x)}\sec(x)\tan(x)
\end{aligned}$$

Mathematica [A] time = 0.092552, size = 49, normalized size = 1.

$$\frac{1}{2}\left(4\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right) - 5\tan^{-1}\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right) - \sqrt{\cos(2x)}\tan(x)\sec(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]^(3/2)*Sec[x]^3,x]
```

```
[Out] (4*Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - 5*ArcTan[Sin[x]/Sqrt[Cos[2*x]]] - Sqrt[
Cos[2*x]]*Sec[x]*Tan[x])/2
```

Maple [B] time = 0.067, size = 100, normalized size = 2.

$$-\frac{1}{4\left(\cos(x)\right)^2\sin(x)}\sqrt{\left(2\left(\cos(x)\right)^2-1\right)\left(\sin(x)\right)^2}\left(4\sqrt{2}\arcsin\left(4\left(\cos(x)\right)^2-3\right)\left(\cos(x)\right)^2 - 5\arctan\left(1/2\frac{3\left(\cos\right.}{\sqrt{-2\left(\sin(x)\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^(3/2)/cos(x)^3,x)
```

```
[Out] -1/4*((2*cos(x)^2-1)*sin(x)^2)^(1/2)*(4*2^(1/2)*arcsin(4*cos(x)^2-3)*cos(x)
^2-5*arctan(1/2*(3*cos(x)^2-2)/(-2*sin(x)^4+sin(x)^2)^(1/2))*cos(x)^2+2*(-2
*sin(x)^4+sin(x)^2)^(1/2))/cos(x)^2/sin(x)/(2*cos(x)^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(2x)^{\frac{3}{2}}}{\cos(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="maxima")
```

```
[Out] integrate(cos(2*x)^(3/2)/cos(x)^3, x)
```

Fricas [B] time = 3.84434, size = 365, normalized size = 7.45

$$-\frac{2\sqrt{2}\arctan\left(\frac{\left(32\sqrt{2}\cos(x)^4-48\sqrt{2}\cos(x)^2+17\sqrt{2}\right)\sqrt{2\cos(x)^2-1}}{8\left(8\cos(x)^4-10\cos(x)^2+3\right)\sin(x)}\right)\cos(x)^2-5\arctan\left(\frac{3\cos(x)^2-2}{2\sqrt{2\cos(x)^2-1}\sin(x)}\right)\cos(x)^2+2\sqrt{2\cos}}{4\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(2)*arctan(1/8*(32*sqrt(2)*cos(x)^4 - 48*sqrt(2)*cos(x)^2 + 17*
sqrt(2))*sqrt(2*cos(x)^2 - 1)/((8*cos(x)^4 - 10*cos(x)^2 + 3)*sin(x)))*cos(
x)^2 - 5*arctan(1/2*(3*cos(x)^2 - 2)/(sqrt(2*cos(x)^2 - 1)*sin(x)))*cos(x)^
2 + 2*sqrt(2*cos(x)^2 - 1)*sin(x))/cos(x)^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)**(3/2)/cos(x)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(2x)^{\frac{3}{2}}}{\cos(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^(3/2)/cos(x)^3,x, algorithm="giac")
```

```
[Out] integrate(cos(2*x)^(3/2)/cos(x)^3, x)
```

3.433 $\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx$

Optimal. Leaf size=87

$$-\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)}-\frac{11\cos(x)}{20\cos^{\frac{3}{2}}(2x)}+\frac{63\cos(x)}{20\sqrt{\cos(2x)}}+\frac{3\sin^2(x)\cos(x)}{10\cos^{\frac{5}{2}}(2x)}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]]/Sqrt[2]) - (11*Cos[x])/(20*Cos[2
*x]^(3/2)) - (2*Cos[x]^3)/(3*Cos[2*x]^(3/2)) + (63*Cos[x])/(20*Sqrt[Cos[2*x
]]) + (3*Cos[x]*Sin[x]^2)/(10*Cos[2*x]^(5/2))
```

Rubi [A] time = 0.207827, antiderivative size = 91, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {4377, 12, 452, 288, 217, 206, 4366, 378, 191}

$$-\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)}+\frac{13\cos(x)}{5\sqrt{\cos(2x)}}+\frac{3\sin^4(x)\cos(x)}{5\cos^{\frac{5}{2}}(2x)}-\frac{4\sin^2(x)\cos(x)}{5\cos^{\frac{3}{2}}(2x)}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x]^2*(3*Sin[x]^3 - Cos[x]*Sin[4*x]))/Cos[2*x]^(7/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]]/Sqrt[2]) - (2*Cos[x]^3)/(3*Cos[2
*x]^(3/2)) + (13*Cos[x])/(5*Sqrt[Cos[2*x]]) - (4*Cos[x]*Sin[x]^2)/(5*Cos[2*
x]^(3/2)) + (3*Cos[x]*Sin[x]^4)/(5*Cos[2*x]^(5/2))
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 452

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b
*e*(m + 1)), x] + Dist[d/b, Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) +
1, 0] && NeQ[m, -1]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(x)\left(3\sin^3(x)-\cos(x)\sin(4x)\right)}{\cos^{\frac{7}{2}}(2x)}\,dx &= 3\int \frac{\sin^5(x)}{\cos^{\frac{7}{2}}(2x)}\,dx-\int \frac{\cos(x)\sin^2(x)\sin(4x)}{\cos^{\frac{7}{2}}(2x)}\,dx\\
&=-\left(3\,\mathrm{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(-1+2x^2\right)^{7/2}}\,dx,x,\cos(x)\right)\right)+\mathrm{Subst}\left(\int \frac{4x^2\left(1-x^2\right)}{\left(-1+2x^2\right)^{5/2}}\,d\right.\\
&=\frac{3\cos(x)\sin^4(x)}{5\cos^{\frac{5}{2}}(2x)}+\frac{12}{5}\,\mathrm{Subst}\left(\int \frac{1-x^2}{\left(-1+2x^2\right)^{5/2}}\,dx,x,\cos(x)\right)+4\,\mathrm{Subst}\left(\right.\\
&=-\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)}-\frac{4\cos(x)\sin^2(x)}{5\cos^{\frac{3}{2}}(2x)}+\frac{3\cos(x)\sin^4(x)}{5\cos^{\frac{5}{2}}(2x)}-\frac{8}{5}\,\mathrm{Subst}\left(\int \frac{1}{\left(-1+2\right.}\right.\\
&=-\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)}+\frac{13\cos(x)}{5\sqrt{\cos(2x)}}-\frac{4\cos(x)\sin^2(x)}{5\cos^{\frac{3}{2}}(2x)}+\frac{3\cos(x)\sin^4(x)}{5\cos^{\frac{5}{2}}(2x)}-\mathrm{Subst}\\
&=-\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)}+\frac{13\cos(x)}{5\sqrt{\cos(2x)}}-\frac{4\cos(x)\sin^2(x)}{5\cos^{\frac{3}{2}}(2x)}+\frac{3\cos(x)\sin^4(x)}{5\cos^{\frac{5}{2}}(2x)}-\mathrm{Subst}\\
&=-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}}-\frac{2\cos^3(x)}{3\cos^{\frac{3}{2}}(2x)}+\frac{13\cos(x)}{5\sqrt{\cos(2x)}}-\frac{4\cos(x)\sin^2(x)}{5\cos^{\frac{3}{2}}(2x)}+\frac{3\,\mathrm{c}}{5}
\end{aligned}$$

Mathematica [A] time = 0.200984, size = 62, normalized size = 0.71

$$\frac{250\cos(x)+45\cos(3x)+169\cos(5x)-120\sqrt{2}\cos^{\frac{5}{2}}(2x)\log\left(\sqrt{2}\cos(x)+\sqrt{\cos(2x)}\right)}{240\cos^{\frac{5}{2}}(2x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x]^2*(3*Sin[x]^3 - Cos[x]*Sin[4*x]))/Cos[2*x]^(7/2),x]
```

```
[Out] (250*Cos[x] + 45*Cos[3*x] + 169*Cos[5*x] - 120*Sqrt[2]*Cos[2*x]^(5/2)*Log[S
qrt[2]*Cos[x] + Sqrt[Cos[2*x]]])/(240*Cos[2*x]^(5/2))
```

Maple [B] time = 0.168, size = 180, normalized size = 2.1

$$-\frac{1}{240\,(\sin(x))^6-360\,(\sin(x))^4+180\,(\sin(x))^2-30}\left(120\ln\left(\cos(x)\sqrt{2}+\sqrt{-2\,(\sin(x))^2+1}\right)\sqrt{2}(\sin(x))^6+338\sqrt{-2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x)
```

```
[Out] -1/30/(8*sin(x)^6-12*sin(x)^4+6*sin(x)^2-1)*(120*ln(cos(x)*2^(1/2)+(-2*sin(
x)^2+1)^(1/2))*2^(1/2)*sin(x)^6+338*(-2*sin(x)^2+1)^(1/2)*cos(x)*sin(x)^4-1
80*ln(cos(x)*2^(1/2)+(-2*sin(x)^2+1)^(1/2))*2^(1/2)*sin(x)^4-276*(-2*sin(x)
^2+1)^(1/2)*sin(x)^2*cos(x)+90*ln(cos(x)*2^(1/2)+(-2*sin(x)^2+1)^(1/2))*2^(
1/2)*sin(x)^2+58*(-2*sin(x)^2+1)^(1/2)*cos(x)-15*ln(cos(x)*2^(1/2)+(-2*sin(
x)^2+1)^(1/2))*2^(1/2))
```

Maxima [B] time = 2.03273, size = 1835, normalized size = 21.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x, algorithm
="maxima")
```

```
[Out] 1/48*(4*(4*sqrt(2)*sin(4*x)*sin(5/2*arctan2(sin(4*x), cos(4*x))) + 4*(sqrt(
2)*cos(4*x) + sqrt(2))*cos(5/2*arctan2(sin(4*x), cos(4*x))) + 3*sqrt(2)*cos
(8*x) + 7*sqrt(2)*cos(4*x) + 4*sqrt(2))*cos(5/2*arctan2(sin(4*x), cos(4*x)
+ 1)) + 12*sqrt(2)*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(3/2*a
rctan2(sin(4*x), cos(4*x) + 1)) - 12*(sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)
^2 + 2*sqrt(2)*cos(4*x) + sqrt(2))*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))
 - 4*(4*sqrt(2)*cos(5/2*arctan2(sin(4*x), cos(4*x)))*sin(4*x) - 4*(sqrt(2)*
cos(4*x) + sqrt(2))*sin(5/2*arctan2(sin(4*x), cos(4*x))) - 3*sqrt(2)*sin(8*
x) - 7*sqrt(2)*sin(4*x))*sin(5/2*arctan2(sin(4*x), cos(4*x) + 1)) - 3*(cos(
4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*((sqrt(2)*cos(4*x)^2 + sqrt(2)*
sin(4*x)^2 + 2*sqrt(2)*cos(4*x) + sqrt(2))*log(sqrt(cos(4*x)^2 + sin(4*x)^2
 + 2*cos(4*x) + 1)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos(4*
x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1)
)^2 + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(si
```

```
n(4*x), cos(4*x) + 1)) + 1) - (sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)^2 + 2*
sqrt(2)*cos(4*x) + sqrt(2))*log(sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) +
 1)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + sqrt(cos(4*x)^2 + sin(4*x)
^2 + 2*cos(4*x) + 1)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 - 2*(cos(4*
x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x
) + 1)) + 1) + (sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)^2 + 2*sqrt(2)*cos(4*x
) + sqrt(2))*log(((cos(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2
(sin(4*x), cos(4*x)))^2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(
1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2
)*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + sin(4*x)^2
+ 2*cos(4*x) + 1) + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos
(1/2*arctan2(sin(4*x), cos(4*x) + 1))*cos(1/2*arctan2(sin(4*x), cos(4*x)))
+ sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*
x)))) + 1) - (sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)^2 + 2*sqrt(2)*cos(4*x)
+ sqrt(2))*log(((cos(1/2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(s
in(4*x), cos(4*x)))^2)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + (cos(1/
2*arctan2(sin(4*x), cos(4*x)))^2 + sin(1/2*arctan2(sin(4*x), cos(4*x)))^2)*
sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2)*sqrt(cos(4*x)^2 + sin(4*x)^2 +
2*cos(4*x) + 1) - 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(1
/2*arctan2(sin(4*x), cos(4*x) + 1))*cos(1/2*arctan2(sin(4*x), cos(4*x))) +
sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))*sin(1/2*arctan2(sin(4*x), cos(4*x)
))) + 1)))/(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(5/4) + 1/20*(((15*co
s(8*x) + 70*cos(4*x) + 43)*cos(5/2*arctan2(sin(4*x), cos(4*x))) + 5*(3*sin(
8*x) + 14*sin(4*x))*sin(5/2*arctan2(sin(4*x), cos(4*x))) - 12)*cos(5/2*arct
an2(sin(4*x), cos(4*x) + 1)) + 15*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1
)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) - (5*(3*sin(8*x) + 14*sin(4*x))*
cos(5/2*arctan2(sin(4*x), cos(4*x))) - (15*cos(8*x) + 70*cos(4*x) + 43)*sin
(5/2*arctan2(sin(4*x), cos(4*x))))*sin(5/2*arctan2(sin(4*x), cos(4*x) + 1))
 + 40*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(3/2*arctan2(sin(4*
x), cos(4*x) + 1)))/((sqrt(2)*cos(4*x)^2 + sqrt(2)*sin(4*x)^2 + 2*sqrt(2)*c
os(4*x) + sqrt(2))*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4))
```

Fricas [B] time = 3.79861, size = 506, normalized size = 5.82

$$\frac{15\left(8\sqrt{2}\cos(x)^6 - 12\sqrt{2}\cos(x)^4 + 6\sqrt{2}\cos(x)^2 - \sqrt{2}\right)\log\left(2048\cos(x)^8 - 2048\cos(x)^6 + 640\cos(x)^4 - 64\cos(\right.}{240}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x, algorithm
="fricas")
```

```
[Out] 1/240*(15*(8*sqrt(2)*cos(x)^6 - 12*sqrt(2)*cos(x)^4 + 6*sqrt(2)*cos(x)^2 -
sqrt(2))*log(2048*cos(x)^8 - 2048*cos(x)^6 + 640*cos(x)^4 - 64*cos(x)^2 - 8
*(128*sqrt(2)*cos(x)^7 - 96*sqrt(2)*cos(x)^5 + 20*sqrt(2)*cos(x)^3 - sqrt(2
)*cos(x))*sqrt(2*cos(x)^2 - 1) + 1) + 16*(169*cos(x)^5 - 200*cos(x)^3 + 60*
cos(x))*sqrt(2*cos(x)^2 - 1))/(8*cos(x)^6 - 12*cos(x)^4 + 6*cos(x)^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*sin(x)**3-cos(x)*sin(4*x))/cos(2*x)**(7/2)/csc(x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13867, size = 74, normalized size = 0.85

$$\frac{1}{2}\sqrt{2}\log\left(\left|-\sqrt{2}\cos\left(x\right)+\sqrt{2\cos\left(x\right)^{2}-1}\right|\right)+\frac{\left(\left(169\cos\left(x\right)^{2}-200\right)\cos\left(x\right)^{2}+60\right)\cos\left(x\right)}{15\left(2\cos\left(x\right)^{2}-1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*sin(x)^3-cos(x)*sin(4*x))/cos(2*x)^(7/2)/csc(x)^2,x, algorithm
="giac")
```

```
[Out] 1/2*sqrt(2)*log(abs(-sqrt(2)*cos(x) + sqrt(2*cos(x)^2 - 1))) + 1/15*((169*c
os(x)^2 - 200)*cos(x)^2 + 60)*cos(x)/(2*cos(x)^2 - 1)^(5/2)
```

3.434 $\int \left(4-5 \sec^2(x)\right)^{3/2} dx$

Optimal. Leaf size=68

$$8 \tan^{-1}\left(\frac{2 \tan(x)}{\sqrt{-5 \tan^2(x)-1}}\right)-\frac{7}{2} \sqrt{5} \tan^{-1}\left(\frac{\sqrt{5} \tan(x)}{\sqrt{-5 \tan^2(x)-1}}\right)-\frac{5}{2} \tan(x) \sqrt{-5 \tan^2(x)-1}$$

```
[Out] 8*ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]] - (7*Sqrt[5]*ArcTan[(Sqrt[5]*Tan
[x])/Sqrt[-1 - 5*Tan[x]^2]])/2 - (5*Tan[x]*Sqrt[-1 - 5*Tan[x]^2])/2
```

Rubi [A] time = 0.0554331, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4128, 416, 523, 217, 203, 377}

$$8 \tan^{-1}\left(\frac{2 \tan(x)}{\sqrt{-5 \tan^2(x)-1}}\right)-\frac{7}{2} \sqrt{5} \tan^{-1}\left(\frac{\sqrt{5} \tan(x)}{\sqrt{-5 \tan^2(x)-1}}\right)-\frac{5}{2} \tan(x) \sqrt{-5 \tan^2(x)-1}$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 5*Sec[x]^2)^(3/2),x]
```

```
[Out] 8*ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]] - (7*Sqrt[5]*ArcTan[(Sqrt[5]*Tan
[x])/Sqrt[-1 - 5*Tan[x]^2]])/2 - (5*Tan[x]*Sqrt[-1 - 5*Tan[x]^2])/2
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \left(4-5\sec^2(x)\right)^{3/2} dx = \text{Subst}\left(\int \frac{\left(-1-5x^2\right)^{3/2}}{1+x^2} dx, x, \tan(x)\right)$$

$$= -\frac{5}{2}\tan(x)\sqrt{-1-5\tan^2(x)} + \frac{1}{2}\text{Subst}\left(\int \frac{-3-35x^2}{\sqrt{-1-5x^2}\left(1+x^2\right)} dx, x, \tan(x)\right)$$

$$= -\frac{5}{2}\tan(x)\sqrt{-1-5\tan^2(x)} + 16\,\text{Subst}\left(\int \frac{1}{\sqrt{-1-5x^2}\left(1+x^2\right)} dx, x, \tan(x)\right) - \frac{35}{2}\text{Subst}\left(\int$$

$$= -\frac{5}{2}\tan(x)\sqrt{-1-5\tan^2(x)} + 16\,\text{Subst}\left(\int \frac{1}{1+4x^2} dx, x, \frac{\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{35}{2}\text{Subst}\left(\int \frac{}{1}\right.$$

$$= 8\tan^{-1}\left(\frac{2\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{7}{2}\sqrt{5}\tan^{-1}\left(\frac{\sqrt{5}\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{5}{2}\tan(x)\sqrt{-1-5\tan^2(x)}$$

Mathematica [C] time = 0.206649, size = 115, normalized size = 1.69

$$-\frac{\left(4\cos^2(x)-5\right)\sec(x)\sqrt{4-5\sec^2(x)}\left(5\sin(x)\sqrt{2\cos(2x)-3}+16i\cos^2(x)\log\left(\sqrt{2\cos(2x)-3}+2i\sin(x)\right)+7\sqrt{5}\cos^2\right.}{2(2\cos(2x)-3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 5*Sec[x]^2)^(3/2),x]
```

```
[Out] -((-5 + 4*Cos[x]^2)*Sec[x]*Sqrt[4 - 5*Sec[x]^2]*(7*Sqrt[5]*ArcTan[(Sqrt[5]*
Sin[x])/Sqrt[-3 + 2*Cos[2*x]]]*Cos[x]^2 + (16*I)*Cos[x]^2*Log[Sqrt[-3 + 2*C
os[2*x]] + (2*I)*Sin[x]] + 5*Sqrt[-3 + 2*Cos[2*x]]*Sin[x]))/(2*(-3 + 2*Cos[
2*x])^(3/2))
```

Maple [C] time = 0.264, size = 754, normalized size = 11.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4-5*sec(x)^2)^(3/2),x)
```

```
[Out] 1/2*I/(-9-4*5^(1/2))^(1/2)/(2+5^(1/2))*(70*I*(-2*(2*cos(x)*5^(1/2)-2*5^(1/2
)+4*cos(x)-5)/(cos(x)+1))^(1/2)*((2*cos(x)*5^(1/2)-2*5^(1/2)-4*cos(x)+5)/(c
```

```
os(x)+1))^(1/2)*EllipticPi((-9-4*5^(1/2))^(1/2)*(cos(x)-1)/sin(x),-1/(9+4*5
^(1/2)),(-9+4*5^(1/2))^(1/2)/(-9-4*5^(1/2))^(1/2))*2^(1/2)*sin(x)*cos(x)^2*
5^(1/2)-64*I*(-2*(2*cos(x)*5^(1/2)-2*5^(1/2)+4*cos(x)-5)/(cos(x)+1))^(1/2)*
((2*cos(x)*5^(1/2)-2*5^(1/2)-4*cos(x)+5)/(cos(x)+1))^(1/2)*EllipticPi((-9-4
*5^(1/2))^(1/2)*(cos(x)-1)/sin(x),1/(9+4*5^(1/2)),(-9+4*5^(1/2))^(1/2)/(-9-
4*5^(1/2))^(1/2))*2^(1/2)*sin(x)*cos(x)^2*5^(1/2)-3*I*(-2*(2*cos(x)*5^(1/2)
-2*5^(1/2)+4*cos(x)-5)/(cos(x)+1))^(1/2)*((2*cos(x)*5^(1/2)-2*5^(1/2)-4*cos
(x)+5)/(cos(x)+1))^(1/2)*EllipticF(I*(cos(x)-1)*(2+5^(1/2))/sin(x),9-4*5^(1
/2))*2^(1/2)*sin(x)*cos(x)^2*5^(1/2)+140*I*(-2*(2*cos(x)*5^(1/2)-2*5^(1/2)+
4*cos(x)-5)/(cos(x)+1))^(1/2)*((2*cos(x)*5^(1/2)-2*5^(1/2)-4*cos(x)+5)/(cos
(x)+1))^(1/2)*EllipticPi((-9-4*5^(1/2))^(1/2)*(cos(x)-1)/sin(x),-1/(9+4*5^(
1/2)),(-9+4*5^(1/2))^(1/2)/(-9-4*5^(1/2))^(1/2))*2^(1/2)*sin(x)*cos(x)^2-12
8*I*(-2*(2*cos(x)*5^(1/2)-2*5^(1/2)+4*cos(x)-5)/(cos(x)+1))^(1/2)*((2*cos(x
)*5^(1/2)-2*5^(1/2)-4*cos(x)+5)/(cos(x)+1))^(1/2)*EllipticPi((-9-4*5^(1/2))
^(1/2)*(cos(x)-1)/sin(x),1/(9+4*5^(1/2)),(-9+4*5^(1/2))^(1/2)/(-9-4*5^(1/2)
)^(1/2))*2^(1/2)*sin(x)*cos(x)^2-6*I*(-2*(2*cos(x)*5^(1/2)-2*5^(1/2)+4*cos(
x)-5)/(cos(x)+1))^(1/2)*((2*cos(x)*5^(1/2)-2*5^(1/2)-4*cos(x)+5)/(cos(x)+1)
)^(1/2)*EllipticF(I*(cos(x)-1)*(2+5^(1/2))/sin(x),9-4*5^(1/2))*2^(1/2)*sin(
x)*cos(x)^2-80*cos(x)^3*5^(1/2)-180*cos(x)^3+80*cos(x)^2*5^(1/2)+180*cos(x)
^2+100*cos(x)*5^(1/2)+225*cos(x)-100*5^(1/2)-225)*((4*cos(x)^2-5)/cos(x)^2)
^(3/2)*sin(x)*cos(x)/(cos(x)-1)/(4*cos(x)^2-5)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-5 \sec(x)^2 + 4\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-5*sec(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-5*sec(x)^2 + 4)^(3/2), x)
```

Fricas [B] time = 3.72658, size = 417, normalized size = 6.13

$$\frac{7\sqrt{5}\arctan\left(\frac{\sqrt{5}\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\cos(x)}{5\sin(x)}\right)\cos(x) + 8\arctan\left(\frac{4\left(8\cos(x)^3-9\cos(x)\right)\sqrt{\frac{4\cos(x)^2-5}{\cos(x)^2}}\sin(x)+\cos(x)\sin(x)}{64\cos(x)^4-143\cos(x)^2+80}\right)\cos(x) - 8\arctan}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-5*sec(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(7*sqrt(5)*arctan(1/5*sqrt(5)*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*cos(x)/si
n(x))*cos(x) + 8*arctan((4*(8*cos(x)^3 - 9*cos(x))*sqrt((4*cos(x)^2 - 5)/co
s(x)^2)*sin(x) + cos(x)*sin(x))/(64*cos(x)^4 - 143*cos(x)^2 + 80))*cos(x) -
 8*arctan(sin(x)/cos(x))*cos(x) - 5*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*sin(x))
/cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4 - 5\sec^2(x)\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-5*sec(x)**2)**(3/2),x)
```

```
[Out] Integral((4 - 5*sec(x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-5 \sec (x)^2+4\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4-5*sec(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-5*sec(x)^2 + 4)^(3/2), x)
```

3.435 $\int \frac{1}{\left(4-5\sec^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=40

$$\frac{1}{8}\tan^{-1}\left(\frac{2\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)-\frac{5\tan(x)}{4\sqrt{-5\tan^2(x)-1}}$$

```
[Out] ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]]/8 - (5*Tan[x])/(4*Sqrt[-1 - 5*Tan[
x]^2])
```

Rubi [A] time = 0.0301136, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4128, 382, 377, 203}

$$\frac{1}{8}\tan^{-1}\left(\frac{2\tan(x)}{\sqrt{-5\tan^2(x)-1}}\right)-\frac{5\tan(x)}{4\sqrt{-5\tan^2(x)-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 5*Sec[x]^2)^(-3/2),x]
```

```
[Out] ArcTan[(2*Tan[x])/Sqrt[-1 - 5*Tan[x]^2]]/8 - (5*Tan[x])/(4*Sqrt[-1 - 5*Tan[
x]^2])
```

Rule 4128

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + b + b*ff^2*x^2)^p/
(1 + ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] &
& NeQ[a + b, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(4-5\sec^2(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(-1-5x^2\right)^{3/2}\left(1+x^2\right)}\,dx, x, \tan(x)\right)$$
$$= -\frac{5\tan(x)}{4\sqrt{-1-5\tan^2(x)}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{-1-5x^2}\left(1+x^2\right)}\,dx, x, \tan(x)\right)$$
$$= -\frac{5\tan(x)}{4\sqrt{-1-5\tan^2(x)}} + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+4x^2}\,dx, x, \frac{\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right)$$
$$= \frac{1}{8}\tan^{-1}\left(\frac{2\tan(x)}{\sqrt{-1-5\tan^2(x)}}\right) - \frac{5\tan(x)}{4\sqrt{-1-5\tan^2(x)}}$$

Mathematica [A] time = 0.124404, size = 79, normalized size = 1.98

$$-\frac{(2\cos(2x)-3)^{3/2}\sec^3(x)\left(10\sin(x)\sqrt{3-2\cos(2x)}+(2\cos(2x)-3)\sinh^{-1}(2\sin(x))\right)}{8\sqrt{-\left(4\sin^2(x)+1\right)^2}\left(4-5\sec^2(x)\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(4 - 5*Sec[x]^2)^(-3/2),x]
```

```
[Out] -((-3 + 2*Cos[2*x])^(3/2)*Sec[x]^3*(ArcSinh[2*Sin[x]]*(-3 + 2*Cos[2*x]) + 1
0*Sqrt[3 - 2*Cos[2*x]]*Sin[x]))/(8*(4 - 5*Sec[x]^2)^(3/2)*Sqrt[-(1 + 4*Sin[
x]^2)^2])
```

Maple [C] time = 0.211, size = 473, normalized size = 11.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-5*sec(x)^2)^(3/2),x)
```

```
[Out] -1/4*I/(-9-4*5^(1/2))^(1/2)/(2+5^(1/2))*(4*cos(x)^2-5)*(2*I*sin(x)*2^(1/2)*
5^(1/2)*EllipticPi((-9-4*5^(1/2))^(1/2)*(cos(x)-1)/sin(x),1/(9+4*5^(1/2)),(
-9+4*5^(1/2))^(1/2)/(-9-4*5^(1/2))^(1/2))*(-2*(2*cos(x)*5^(1/2)-2*5^(1/2)+4
*cos(x)-5)/(cos(x)+1))^(1/2)*((2*cos(x)*5^(1/2)-2*5^(1/2)-4*cos(x)+5)/(cos(
x)+1))^(1/2)-I*sin(x)*2^(1/2)*5^(1/2)*EllipticF(I*(cos(x)-1)*(2+5^(1/2))/si
n(x),9-4*5^(1/2))*(-2*(2*cos(x)*5^(1/2)-2*5^(1/2)+4*cos(x)-5)/(cos(x)+1))^(
1/2)*((2*cos(x)*5^(1/2)-2*5^(1/2)-4*cos(x)+5)/(cos(x)+1))^(1/2)+4*I*sin(x)*
2^(1/2)*EllipticPi((-9-4*5^(1/2))^(1/2)*(cos(x)-1)/sin(x),1/(9+4*5^(1/2)),(
-9+4*5^(1/2))^(1/2)/(-9-4*5^(1/2))^(1/2))*(-2*(2*cos(x)*5^(1/2)-2*5^(1/2)+4
*cos(x)-5)/(cos(x)+1))^(1/2)*((2*cos(x)*5^(1/2)-2*5^(1/2)-4*cos(x)+5)/(cos(
x)+1))^(1/2)-2*I*sin(x)*2^(1/2)*EllipticF(I*(cos(x)-1)*(2+5^(1/2))/sin(x),9
-4*5^(1/2))*(-2*(2*cos(x)*5^(1/2)-2*5^(1/2)+4*cos(x)-5)/(cos(x)+1))^(1/2)*(
(2*cos(x)*5^(1/2)-2*5^(1/2)-4*cos(x)+5)/(cos(x)+1))^(1/2)+20*cos(x)*5^(1/2)
+45*cos(x)-20*5^(1/2)-45)*sin(x)/(cos(x)-1)/cos(x)^3/((4*cos(x)^2-5)/cos(x)
^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-5 \sec\left(x\right)^{2}+4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-5*sec(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-5*sec(x)^2 + 4)^(-3/2), x)
```

Fricas [B] time = 3.38956, size = 351, normalized size = 8.78

$$-\frac{20 \sqrt{\frac{4 \cos\left(x\right)^{2}-5}{\cos\left(x\right)^{2}}} \cos\left(x\right) \sin\left(x\right) - \left(4 \cos\left(x\right)^{2}-5\right) \arctan\left(\frac{4 \left(8 \cos\left(x\right)^{3}-9 \cos\left(x\right)\right) \sqrt{\frac{4 \cos\left(x\right)^{2}-5}{\cos\left(x\right)^{2}}} \sin\left(x\right)+\cos\left(x\right) \sin\left(x\right)}{64 \cos\left(x\right)^{4}-143 \cos\left(x\right)^{2}+80}\right) + \left(4 \cos\left(x\right)^{2}\right.}{16 \left(4 \cos\left(x\right)^{2}-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-5*sec(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/16*(20*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*cos(x)*sin(x) - (4*cos(x)^2 - 5)*
arctan((4*(8*cos(x)^3 - 9*cos(x))*sqrt((4*cos(x)^2 - 5)/cos(x)^2)*sin(x) +
cos(x)*sin(x))/(64*cos(x)^4 - 143*cos(x)^2 + 80)) + (4*cos(x)^2 - 5)*arctan
(sin(x)/cos(x)))/(4*cos(x)^2 - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(4-5 \sec^{2}\left(x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-5*sec(x)**2)**(3/2),x)
```

```
[Out] Integral((4 - 5*sec(x)**2)**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-5 \sec\left(x\right)^{2}+4\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-5*sec(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-5*sec(x)^2 + 4)^(-3/2), x)
```

3.436 $\int \frac{-2\cot^2(x)+\sin(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=94

$$-\frac{1}{8}\cos(x)\sqrt{5\tan^2(x)+1}-\frac{\cos(x)}{4\sqrt{5\tan^2(x)+1}}-\frac{1}{4}\tanh^{-1}\left(\frac{2\tan(x)}{\sqrt{5\tan^2(x)+1}}\right)+\frac{9}{2}\sqrt{5\tan^2(x)+1}\cot(x)-\frac{5\cot(x)}{2\sqrt{5\tan^2(x)+1}}$$

```
[Out] -ArcTanh[(2*Tan[x])/Sqrt[1 + 5*Tan[x]^2]]/4 - Cos[x]/(4*Sqrt[1 + 5*Tan[x]^2
]) - (5*Cot[x])/(2*Sqrt[1 + 5*Tan[x]^2]) - (Cos[x]*Sqrt[1 + 5*Tan[x]^2])/8
+ (9*Cot[x]*Sqrt[1 + 5*Tan[x]^2])/2
```

Rubi [A] time = 0.18858, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4377, 12, 3670, 472, 583, 377, 206, 3664, 271, 191}

$$-\frac{5\sec(x)}{8\sqrt{5\sec^2(x)-4}}+\frac{\cos(x)}{4\sqrt{5\sec^2(x)-4}}-\frac{1}{4}\tanh^{-1}\left(\frac{2\tan(x)}{\sqrt{5\tan^2(x)+1}}\right)+\frac{9}{2}\sqrt{5\tan^2(x)+1}\cot(x)-\frac{5\cot(x)}{2\sqrt{5\tan^2(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*Cot[x]^2 + Sin[x])/(1 + 5*Tan[x]^2)^(3/2),x]
```

```
[Out] -ArcTanh[(2*Tan[x])/Sqrt[1 + 5*Tan[x]^2]]/4 + Cos[x]/(4*Sqrt[-4 + 5*Sec[x]^
2]) - (5*Sec[x])/(8*Sqrt[-4 + 5*Sec[x]^2]) - (5*Cot[x])/(2*Sqrt[1 + 5*Tan[x
]^2]) + (9*Cot[x]*Sqrt[1 + 5*Tan[x]^2])/2
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
```

```
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^
m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1
), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m
 - 1)/2]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{-2\cot^2(x)+\sin(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx = \int -\frac{2\cot^2(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx + \int \frac{\sin(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx$$

$$= -\left(2\int \frac{\cot^2(x)}{\left(1+5\tan^2(x)\right)^{3/2}}\,dx\right) + \text{Subst}\left(\int \frac{1}{x^2\left(-4+5x^2\right)^{3/2}}\,dx, x, \sec(x)\right)$$

$$= \frac{\cos(x)}{4\sqrt{-4+5\sec^2(x)}} - 2\,\text{Subst}\left(\int \frac{1}{x^2\left(1+x^2\right)\left(1+5x^2\right)^{3/2}}\,dx, x, \tan(x)\right) + \frac{5}{2}\,\text{Subst}\left(\int \frac{}{\left(-4+\right.}\right.$$

$$= \frac{\cos(x)}{4\sqrt{-4+5\sec^2(x)}} - \frac{5\sec(x)}{8\sqrt{-4+5\sec^2(x)}} - \frac{5\cot(x)}{2\sqrt{1+5\tan^2(x)}} + \frac{1}{2}\,\text{Subst}\left(\int \frac{-9-10x^2}{x^2\left(1+x^2\right)\sqrt{1+}}\right.$$

$$= \frac{\cos(x)}{4\sqrt{-4+5\sec^2(x)}} - \frac{5\sec(x)}{8\sqrt{-4+5\sec^2(x)}} - \frac{5\cot(x)}{2\sqrt{1+5\tan^2(x)}} + \frac{9}{2}\cot(x)\sqrt{1+5\tan^2(x)} - \frac{1}{2}\,\text{S}$$

$$= \frac{\cos(x)}{4\sqrt{-4+5\sec^2(x)}} - \frac{5\sec(x)}{8\sqrt{-4+5\sec^2(x)}} - \frac{5\cot(x)}{2\sqrt{1+5\tan^2(x)}} + \frac{9}{2}\cot(x)\sqrt{1+5\tan^2(x)} - \frac{1}{2}\,\text{S}$$

$$= -\frac{1}{4}\tanh^{-1}\left(\frac{2\tan(x)}{\sqrt{1+5\tan^2(x)}}\right) + \frac{\cos(x)}{4\sqrt{-4+5\sec^2(x)}} - \frac{5\sec(x)}{8\sqrt{-4+5\sec^2(x)}} - \frac{5\cot(x)}{2\sqrt{1+5\tan^2(x)}}$$

Mathematica [A] time = 0.512624, size = 131, normalized size = 1.39

$$-\frac{\sin^2(x)(2\cos(2x)-3)^{3/2}\tan(x)\left(2\cot^2(x)\csc(x)-1\right)\left(\sqrt{4\sin^2(x)+1}\left(16\csc^3(x)-3\csc^2(x)+164\csc(x)-2\right)-2\left(\csc\right.\right.}{2\sqrt{-(3-2\cos(2x))^2}\sqrt{5\tan^2(x)+1}\left(\cot^2(x)+5\right)(-3\sin(x)+\sin(3x)+4\cos(2x)+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*Cot[x]^2 + Sin[x])/(1 + 5*Tan[x]^2)^(3/2),x]
```

```
[Out] -((-3 + 2*Cos[2*x])^(3/2)*(-1 + 2*Cot[x]^2*Csc[x])*Sin[x]^2*(-2*ArcSinh[2*S
in[x]]*(4 + Csc[x]^2) + (-2 + 164*Csc[x] - 3*Csc[x]^2 + 16*Csc[x]^3)*Sqrt[1
 + 4*Sin[x]^2])*Tan[x])/(2*Sqrt[-(3 - 2*Cos[2*x])^2]*(5 + Cot[x]^2)*(4 + 4*
Cos[2*x] - 3*Sin[x] + Sin[3*x])*Sqrt[1 + 5*Tan[x]^2])
```

Maple [C] time = 0.412, size = 975, normalized size = 10.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x)
```

```
[Out] -1/8*I/(-9+4*5^(1/2))^(1/2)/(2+5^(1/2))^2/(-2+5^(1/2))^2/(4*cos(x)^2-5)^2*(
-4*I*2^(1/2)*cos(x)*sin(x)*EllipticF(I*(-2+5^(1/2))*(cos(x)-1)/sin(x),9+4*5
^(1/2))*((2*cos(x)*5^(1/2)-2*5^(1/2)-4*cos(x)+5)/(cos(x)+1))^(1/2)*(-2*(2*c
os(x)*5^(1/2)-2*5^(1/2)+4*cos(x)-5)/(cos(x)+1))^(1/2)+3*I*arctanh(1/2*(-16)
^(1/2)*cos(x)*(cos(x)-1)/sin(x)^2/(-(4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2))*cos
(x)*sin(x)*5^(1/2)*(-(4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2)-4*I*EllipticF(I*(-2
```

```
+5^(1/2))*(cos(x)-1)/sin(x),9+4*5^(1/2))*sin(x)*2^(1/2)*((2*cos(x)*5^(1/2)-
2*5^(1/2)-4*cos(x)+5)/(cos(x)+1))^(1/2)*(-2*(2*cos(x)*5^(1/2)-2*5^(1/2)+4*c
os(x)-5)/(cos(x)+1))^(1/2)+8*I*EllipticPi((-9+4*5^(1/2))^(1/2)*(cos(x)-1)/s
in(x),-1/(-9+4*5^(1/2)),(-9-4*5^(1/2))^(1/2)/(-9+4*5^(1/2))^(1/2))*sin(x)*2
^(1/2)*((2*cos(x)*5^(1/2)-2*5^(1/2)-4*cos(x)+5)/(cos(x)+1))^(1/2)*(-2*(2*co
s(x)*5^(1/2)-2*5^(1/2)+4*cos(x)-5)/(cos(x)+1))^(1/2)+8*I*EllipticPi((-9+4*5
^(1/2))^(1/2)*(cos(x)-1)/sin(x),-1/(-9+4*5^(1/2)),(-9-4*5^(1/2))^(1/2)/(-9+
4*5^(1/2))^(1/2))*cos(x)*sin(x)*2^(1/2)*((2*cos(x)*5^(1/2)-2*5^(1/2)-4*cos(
x)+5)/(cos(x)+1))^(1/2)*(-2*(2*cos(x)*5^(1/2)-2*5^(1/2)+4*cos(x)-5)/(cos(x)
+1))^(1/2)-6*I*arctanh(1/2*(-16)^(1/2)*cos(x)*(cos(x)-1)/sin(x)^2/(-(4*cos(
x)^2-5)/(cos(x)+1)^2)^(1/2))*sin(x)*(-(4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2)+3*
I*sin(x)*5^(1/2)*arctanh(1/2*(-16)^(1/2)*cos(x)*(cos(x)-1)/sin(x)^2/(-(4*co
s(x)^2-5)/(cos(x)+1)^2)^(1/2))*(-(4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2)+3*arcta
n(2*cos(x)*(cos(x)-1)/sin(x)^2/(-(4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2))*cos(x)
*sin(x)*5^(1/2)*(-(4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2)-6*I*cos(x)*sin(x)*arct
anh(1/2*(-16)^(1/2)*cos(x)*(cos(x)-1)/sin(x)^2/(-(4*cos(x)^2-5)/(cos(x)+1)^
2)^(1/2))*(-(4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2)-6*arctan(2*cos(x)*(cos(x)-1)
/sin(x)^2/(-(4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2))*(-(4*cos(x)^2-5)/(cos(x)+1)
^2)^(1/2)*sin(x)*cos(x)+3*sin(x)*5^(1/2)*arctan(2*cos(x)*(cos(x)-1)/sin(x)^
2/(-(4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2))*(-(4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2
)-2*cos(x)^2*sin(x)*5^(1/2)-6*arctan(2*cos(x)*(cos(x)-1)/sin(x)^2/(-(4*cos(
x)^2-5)/(cos(x)+1)^2)^(1/2))*sin(x)*(-(4*cos(x)^2-5)/(cos(x)+1)^2)^(1/2)+4*
cos(x)^2*sin(x)+164*cos(x)^2*5^(1/2)-328*cos(x)^2+5*sin(x)*5^(1/2)-10*sin(x
)-180*5^(1/2)+360)*cos(x)^3*(-(4*cos(x)^2-5)/cos(x)^2)^(3/2)/sin(x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [A] time = 4.15037, size = 285, normalized size = 3.03

$$\frac{2\left(4\,\cos\left(x\right)^{2}-5\right)\log\left(\sqrt{-\frac{4\,\cos\left(x\right)^{2}-5}{\cos\left(x\right)^{2}}}\cos\left(x\right)-2\,\sin\left(x\right)\right)\sin\left(x\right)+\left(164\,\cos\left(x\right)^{3}-\left(2\,\cos\left(x\right)^{3}-5\,\cos\left(x\right)\right)\sin\left(x\right)-18\right.}{8\left(4\,\cos\left(x\right)^{2}-5\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(4*cos(x)^2 - 5)*log(sqrt(-(4*cos(x)^2 - 5)/cos(x)^2)*cos(x) - 2*sin
(x))*sin(x) + (164*cos(x)^3 - (2*cos(x)^3 - 5*cos(x))*sin(x) - 180*cos(x))*
sqrt(-(4*cos(x)^2 - 5)/cos(x)^2))/((4*cos(x)^2 - 5)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{\sin\left(x\right)}{5\sqrt{5\tan^{2}\left(x\right)+1}\tan^{2}\left(x\right)+\sqrt{5\tan^{2}\left(x\right)+1}}\,dx-\int \frac{2\cot^{2}\left(x\right)}{5\sqrt{5\tan^{2}\left(x\right)+1}\tan^{2}\left(x\right)+\sqrt{5\tan^{2}\left(x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cot(x)**2+sin(x))/(1+5*tan(x)**2)**(3/2),x)
```

```
[Out] -Integral(-sin(x)/(5*sqrt(5*tan(x)**2 + 1)*tan(x)**2 + sqrt(5*tan(x)**2 + 1
)), x) - Integral(2*cot(x)**2/(5*sqrt(5*tan(x)**2 + 1)*tan(x)**2 + sqrt(5*t
an(x)**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{2 \cot(x)^2 - \sin(x)}{\left(5 \tan(x)^2 + 1\right)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*cot(x)^2+sin(x))/(1+5*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-(2*cot(x)^2 - sin(x))/(5*tan(x)^2 + 1)^(3/2), x)
```

3.437 $\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{3}\tan^3(x)\sqrt{4-\cot^2(x)}-\frac{2}{3}\tan(x)\sqrt{4-\cot^2(x)}$$

```
[Out] (-2*Sqrt[4 - Cot[x]^2]*Tan[x])/3 - (Sqrt[4 - Cot[x]^2]*Tan[x]^3)/3
```

Rubi [A] time = 0.149407, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {12, 434, 453, 191}

$$-\frac{1}{3}\tan^3(x)\sqrt{4-\cot^2(x)}-\frac{2}{3}\tan(x)\sqrt{4-\cot^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[((-3 + Cos[2*x])*Sec[x]^4)/Sqrt[4 - Cot[x]^2],x]
```

```
[Out] (-2*Sqrt[4 - Cot[x]^2]*Tan[x])/3 - (Sqrt[4 - Cot[x]^2]*Tan[x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 434

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{(-3+\cos(2x))\sec^4(x)}{\sqrt{4-\cot^2(x)}}\,dx = \text{Subst}\left(\int \frac{2\left(-1-2x^2\right)}{\sqrt{4-\frac{1}{x^2}}}\,dx, x, \tan(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{-1-2x^2}{\sqrt{4-\frac{1}{x^2}}}\,dx, x, \tan(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{\left(-2-\frac{1}{x^2}\right)x^2}{\sqrt{4-\frac{1}{x^2}}}\,dx, x, \tan(x)\right)$$
$$= -\frac{1}{3}\sqrt{4-\cot^2(x)}\tan^3(x) - \frac{8}{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{4-\frac{1}{x^2}}}\,dx, x, \tan(x)\right)$$
$$= -\frac{2}{3}\sqrt{4-\cot^2(x)}\tan(x) - \frac{1}{3}\sqrt{4-\cot^2(x)}\tan^3(x)$$

Mathematica [A] time = 0.0784498, size = 36, normalized size = 0.92

$$\frac{(\cos(2x)+3)(5\cos(2x)-3)\csc(x)\sec^3(x)}{12\sqrt{4-\cot^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-3 + Cos[2*x])*Sec[x]^4)/Sqrt[4 - Cot[x]^2],x]
```

```
[Out] ((3 + Cos[2*x])*(-3 + 5*Cos[2*x])*Csc[x]*Sec[x]^3)/(12*Sqrt[4 - Cot[x]^2])
```

Maple [A] time = 0.457, size = 61, normalized size = 1.6

$$-\frac{\left(5\,(\cos(x))^2+2\right)\sin(x)\sqrt{4}}{12\,(\cos(x))^3}\sqrt{-\frac{-4+5\,(\cos(x))^2}{(\sin(x))^2}}+\frac{\sin(x)}{2\cos(x)}\sqrt{-\frac{-4+5\,(\cos(x))^2}{(\sin(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x)
```

```
[Out] -1/12*(5*cos(x)^2+2)*sin(x)*(-(-4+5*cos(x)^2)/sin(x)^2)^(1/2)*4^(1/2)/cos(x
)^3+1/2*(-(-4+5*cos(x)^2)/sin(x)^2)^(1/2)*sin(x)/cos(x)
```

Maxima [B] time = 0.975043, size = 85, normalized size = 2.18

$$-\frac{1}{48}\left(-\frac{1}{\tan(x)^2}+4\right)^{\frac{3}{2}}\tan(x)^3+\frac{3}{16}\sqrt{-\frac{1}{\tan(x)^2}+4}\tan(x)-\frac{8\tan(x)^4+26\tan(x)^2-7}{8\sqrt{2\tan(x)+1}\sqrt{2\tan(x)-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/48*(-1/tan(x)^2 + 4)^(3/2)*tan(x)^3 + 3/16*sqrt(-1/tan(x)^2 + 4)*tan(x)
- 1/8*(8*tan(x)^4 + 26*tan(x)^2 - 7)/(sqrt(2*tan(x) + 1)*sqrt(2*tan(x) - 1)
)
```

Fricas [A] time = 3.44099, size = 101, normalized size = 2.59

$$-\frac{\left(\cos\left(x\right)^{2}+1\right)\sqrt{\frac{5\,\cos(x)^{2}-4}{\cos(x)^{2}-1}}\,\sin\left(x\right)}{3\,\cos\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(cos(x)^2 + 1)*sqrt((5*cos(x)^2 - 4)/(cos(x)^2 - 1))*sin(x)/cos(x)^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+cos(2*x))/cos(x)**4/(4-cot(x)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(2\,x\right)-3}{\sqrt{-\cot\left(x\right)^{2}+4}\cos\left(x\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+cos(2*x))/cos(x)^4/(4-cot(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((cos(2*x) - 3)/(sqrt(-cot(x)^2 + 4)*cos(x)^4), x)
```

3.438 $\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))(5-4\sec^2(x))^{3/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2}{15\sqrt{5-4\sec^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

```
[Out] -ArcTanh[Sqrt[5 - 4*Sec[x]^2]/Sqrt[3]]/(6*Sqrt[3]) - ArcTanh[Sqrt[5 - 4*Sec
[x]^2]/Sqrt[5]]/(5*Sqrt[5]) - 2/(15*Sqrt[5 - 4*Sec[x]^2])
```

Rubi [A] time = 1.22989, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.387, Rules used = {4373, 6725, 261, 266, 51, 63, 206, 514, 446, 85, 156, 207}

$$-\frac{2}{15\sqrt{5-4\sec^2(x)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{3}}\right)}{6\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((3 + Sin[x]^2)*Tan[x]^3)/((-2 + Cos[x]^2)*(5 - 4*Sec[x]^2)^(3/2)),x]
```

```
[Out] -ArcTanh[Sqrt[5 - 4*Sec[x]^2]/Sqrt[3]]/(6*Sqrt[3]) - ArcTanh[Sqrt[5 - 4*Sec
[x]^2]/Sqrt[5]]/(5*Sqrt[5]) - 2/(15*Sqrt[5 - 4*Sec[x]^2])
```

Rule 4373

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[(b*c*d^(n - 1))^(-1), Subst[Int[SubstF
or[(1 - d^2*x^2)^((n - 1)/2)/x^n, Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(
a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b,
c}, x] && IntegerQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
```

```
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 85

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)),
x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*
x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && LtQ[p, -1]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(3+\sin^2(x))\tan^3(x)}{(-2+\cos^2(x))(5-4\sec^2(x))^{3/2}}\,dx = -\text{Subst}\left(\int \frac{(1-x^2)(4-x^2)}{\left(5-\frac{4}{x^2}\right)^{3/2}x^3(-2+x^2)}\,dx, x, \cos(x)\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{2}{\left(5-\frac{4}{x^2}\right)^{3/2}x^3} + \frac{3}{2\left(5-\frac{4}{x^2}\right)^{3/2}x} - \frac{x}{2\left(5-\frac{4}{x^2}\right)^{3/2}(-2+x^2)}\right)dx, x$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x}{\left(5-\frac{4}{x^2}\right)^{3/2}(-2+x^2)}\,dx, x, \cos(x)\right) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{\left(5-\frac{4}{x^2}\right)^{3/2}x}\,dx$$

$$= -\frac{1}{2\sqrt{5-4\sec^2(x)}} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(5-\frac{4}{x^2}\right)^{3/2}\left(1-\frac{2}{x^2}\right)x}\,dx, x, \cos(x)\right) + \frac{3}{4}\text{Subs}$$

$$= -\frac{1}{5\sqrt{5-4\sec^2(x)}} + \frac{3}{20}\text{Subst}\left(\int \frac{1}{\sqrt{5-4x}x}\,dx, x, \sec^2(x)\right) - \frac{1}{4}\text{Subst}\left(\int \frac{}{(5-}\right.$$

$$= -\frac{2}{15\sqrt{5-4\sec^2(x)}} + \frac{1}{120}\text{Subst}\left(\int \frac{-6-8x}{\sqrt{5-4x}(1-2x)x}\,dx, x, \sec^2(x)\right) - \frac{3}{40}\text{Su}$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{10\sqrt{5}} - \frac{2}{15\sqrt{5-4\sec^2(x)}} - \frac{1}{20}\text{Subst}\left(\int \frac{1}{\sqrt{5-4x}x}\,dx, x, \text{s}\right.$$

$$= -\frac{3\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{10\sqrt{5}} - \frac{2}{15\sqrt{5-4\sec^2(x)}} + \frac{1}{40}\text{Subst}\left(\int \frac{1}{\frac{5}{4}-\frac{x^2}{4}}\,dx, x, \sqrt{5-}\right.$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{3}}\right)}{6\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{\sqrt{5-4\sec^2(x)}}{\sqrt{5}}\right)}{5\sqrt{5}} - \frac{2}{15\sqrt{5-4\sec^2(x)}}$$

Mathematica [B] time = 2.61882, size = 255, normalized size = 3.49

$$\frac{15\sin^2(x)\sqrt{15\cos(2x)-9}\tanh^{-1}\left(\frac{\sqrt{5\cos(2x)-3}}{\sqrt{6}\sqrt{\cos^2(x)}}\right) - 2\left(15\sqrt{2}\sqrt{\sin^2(x)}\sqrt{\sin^2(2x)} + 9\sqrt{5}\sin^2(x)\sqrt{5\cos(2x)-3}\left(\log\left(10\sin^2\right.\right.\right.}{22}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((3 + Sin[x]^2)*Tan[x]^3)/((-2 + Cos[x]^2)*(5 - 4*Sec[x]^2)^(3/2)
),x]
```

```
[Out] (15*ArcTanh[Sqrt[-3 + 5*Cos[2*x]]/(Sqrt[6]*Sqrt[Cos[x]^2])]*Sqrt[-9 + 15*Co
s[2*x]]*Sin[x]^2 - 2*(9*Sqrt[5]*Sqrt[-3 + 5*Cos[2*x]]*(Log[10*Sin[x]^2] - L
og[5*(-Sqrt[-3 + 5*Cos[2*x]] + Cos[2*x]*Sqrt[-3 + 5*Cos[2*x]] + Sqrt[10]*Sq
rt[Sin[x]^2]*Sqrt[Sin[2*x]^2])])*Sin[x]^2 + 15*Sqrt[2]*Sqrt[Sin[x]^2]*Sqrt[
Sin[2*x]^2] + 10*ArcTanh[(Sqrt[6]*Cos[x])/Sqrt[-3 + 5*Cos[2*x]]]*Sqrt[-9 +
15*Cos[2*x]]*Sec[x]*Sqrt[Sin[x]^2]*Sqrt[Sin[2*x]^2]))/(225*Sqrt[10 - 8*Sec[
x]^2]*Sqrt[Sin[x]^2]*Sqrt[Sin[2*x]^2])
```

Maple [B] time = 0.276, size = 1615, normalized size = 22.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+sin(x)^2)*tan(x)^3/(cos(x)^2-2)/(5-4*sec(x)^2)^(3/2),x)
```

```
[Out] -3/5/(6+2*5^(1/2)-2^(1/2))/(6+2*5^(1/2)+2^(1/2))/(-6+2*5^(1/2)-2^(1/2))/(2*
3^(1/2)+6^(1/2))/(-6+2*5^(1/2)+2^(1/2))/(2*3^(1/2)-6^(1/2))/(5+2*5^(1/2))/(
-5+2*5^(1/2))*(-4+5*cos(x)^2)*(50*3^(1/2)*cos(x)*2^(1/2)*((-4+5*cos(x)^2)/(
cos(x)+1)^2)^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2)*(cos(x)-1)*(5*co
s(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((-4+5*cos(x)^2)/(cos(x)+1)^2)
^(1/2))+50*3^(1/2)*cos(x)*2^(1/2)*((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2)*arct
anh(1/2/(2*3^(1/2)-6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x)+
4*2^(1/2)-4)/sin(x)^2/((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2))-25*cos(x)*2^(1/
2)*((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^(
1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((-4+5*co
s(x)^2)/(cos(x)+1)^2)^(1/2))*6^(1/2)+25*cos(x)*2^(1/2)*((-4+5*cos(x)^2)/(co
s(x)+1)^2)^(1/2)*arctanh(1/2/(2*3^(1/2)-6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(
x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin(x)^2/((-4+5*cos(x)^2)/(cos(x)+1)^2)^(
1/2))*6^(1/2)+100*3^(1/2)*((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2)*cos(x)*arcta
nh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)+10*cos(x)+4
*2^(1/2)+4)/sin(x)^2/((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2))-100*3^(1/2)*((-4
+5*cos(x)^2)/(cos(x)+1)^2)^(1/2)*cos(x)*arctanh(1/2/(2*3^(1/2)-6^(1/2))*4^(
1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin(x)^2/((-4+5*co
s(x)^2)/(cos(x)+1)^2)^(1/2))+50*3^(1/2)*((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2
)*2^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1
/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2))+5
0*3^(1/2)*((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2)*2^(1/2)*arctanh(1/2/(2*3^(1/
2)-6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/sin
(x)^2/((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2))+72*((-4+5*cos(x)^2)/(cos(x)+1)^
2)^(1/2)*cos(x)*5^(1/2)*arctanh(1/2*5^(1/2)*cos(x)*4^(1/2)*(cos(x)-1)/sin(x
)^2/((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2))-50*((-4+5*cos(x)^2)/(cos(x)+1)^2)
^(1/2)*cos(x)*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*
2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2
))*6^(1/2)-50*((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2)*cos(x)*arctanh(1/2/(2*3^
(1/2)-6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/2)-4)/
sin(x)^2/((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2))*6^(1/2)-25*((-4+5*cos(x)^2)/
(cos(x)+1)^2)^(1/2)*2^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2)*(cos(x)
-1)*(5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((-4+5*cos(x)^2)/(cos
(x)+1)^2)^(1/2))*6^(1/2)+25*((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2)*2^(1/2)*ar
ctanh(1/2/(2*3^(1/2)-6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x
)+4*2^(1/2)-4)/sin(x)^2/((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2))*6^(1/2)+100*3
^(1/2)*((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2))
*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((-4+
5*cos(x)^2)/(cos(x)+1)^2)^(1/2))-100*3^(1/2)*((-4+5*cos(x)^2)/(cos(x)+1)^2)
^(1/2)*arctanh(1/2/(2*3^(1/2)-6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)
-10*cos(x)+4*2^(1/2)-4)/sin(x)^2/((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2))+72*(
(-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2)*5^(1/2)*arctanh(1/2*5^(1/2)*cos(x)*4^(1
/2)*(cos(x)-1)/sin(x)^2/((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2))-50*((-4+5*cos
(x)^2)/(cos(x)+1)^2)^(1/2)*arctanh(1/2/(2*3^(1/2)+6^(1/2))*4^(1/2)*(cos(x)-
1)*(5*cos(x)*2^(1/2)+10*cos(x)+4*2^(1/2)+4)/sin(x)^2/((-4+5*cos(x)^2)/(cos(
x)+1)^2)^(1/2))*6^(1/2)-50*((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2)*arctanh(1/2
/(2*3^(1/2)-6^(1/2))*4^(1/2)*(cos(x)-1)*(5*cos(x)*2^(1/2)-10*cos(x)+4*2^(1/
2)-4)/sin(x)^2/((-4+5*cos(x)^2)/(cos(x)+1)^2)^(1/2))*6^(1/2)-240*cos(x))/co
s(x)^3/((-4+5*cos(x)^2)/cos(x)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sin\left(x\right)^2+3\right)\tan\left(x\right)^3}{\left(\cos\left(x\right)^2-2\right)\left(-4\sec\left(x\right)^2+5\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((sin(x)^2 + 3)*tan(x)^3/((cos(x)^2 - 2)*(-4*sec(x)^2 + 5)^(3/2)),
 x)
```

Fricas [B] time = 4.45508, size = 799, normalized size = 10.95

$$-\frac{480\sqrt{\frac{5\cos(x)^2-4}{\cos(x)^2}}\cos\left(x\right)^2-18\left(5\sqrt{5}\cos\left(x\right)^2-4\sqrt{5}\right)\log\left(625\cos\left(x\right)^8-1000\cos\left(x\right)^6+500\cos\left(x\right)^4-80\cos\left(x\right)^2-(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/3600*(480*sqrt((5*cos(x)^2 - 4)/cos(x)^2)*cos(x)^2 - 18*(5*sqrt(5)*cos(x
)^2 - 4*sqrt(5))*log(625*cos(x)^8 - 1000*cos(x)^6 + 500*cos(x)^4 - 80*cos(x
)^2 - (125*sqrt(5)*cos(x)^8 - 150*sqrt(5)*cos(x)^6 + 50*sqrt(5)*cos(x)^4 -
4*sqrt(5)*cos(x)^2)*sqrt((5*cos(x)^2 - 4)/cos(x)^2) + 2) - 25*(5*sqrt(3)*co
s(x)^2 - 4*sqrt(3))*log((1921*cos(x)^8 - 3464*cos(x)^6 + 2040*cos(x)^4 - 41
6*cos(x)^2 - 8*(62*sqrt(3)*cos(x)^8 - 87*sqrt(3)*cos(x)^6 + 36*sqrt(3)*cos(
x)^4 - 4*sqrt(3)*cos(x)^2)*sqrt((5*cos(x)^2 - 4)/cos(x)^2) + 16)/(cos(x)^8
- 8*cos(x)^6 + 24*cos(x)^4 - 32*cos(x)^2 + 16)))/(5*cos(x)^2 - 4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(x)**2)*tan(x)**3/(-2+cos(x)**2)/(5-4*sec(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.35068, size = 231, normalized size = 3.16

$$-\frac{1}{4500}\sqrt{15}\sqrt{5}\left(6i\sqrt{15}\pi+12\sqrt{15}\log\left(2\right)-25\log\left(-\frac{\sqrt{15}+5}{\sqrt{15}-5}\right)\right)\mathrm{sgn}\left(\cos\left(x\right)\right)-\frac{\sqrt{15}\sqrt{5}\log\left(-\frac{2\left(\left(\sqrt{5}\cos(x)-\sqrt{5\cos(x)^2-4}\right)^2-4\cdots\right.}{\left|2\left(\sqrt{5}\cos(x)-\sqrt{5\cos(x)^2-4}\right)^2+8\cdots\right|}\right.}{180\,\mathrm{sgn}\left(\cos\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sin(x)^2)*tan(x)^3/(-2+cos(x)^2)/(5-4*sec(x)^2)^(3/2),x, algor
ithm="giac")
```

```
[Out] -1/4500*sqrt(15)*sqrt(5)*(6*I*sqrt(15)*pi + 12*sqrt(15)*log(2) - 25*log(-(s
qrt(15) + 5)/(sqrt(15) - 5)))*sgn(cos(x)) - 1/180*sqrt(15)*sqrt(5)*log(-2*(
(sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))^2 - 4*sqrt(15) - 16)/abs(2*(sqrt(5)
*cos(x) - sqrt(5*cos(x)^2 - 4))^2 + 8*sqrt(15) - 32))/sgn(cos(x)) + 1/50*sq
rt(5)*log((sqrt(5)*cos(x) - sqrt(5*cos(x)^2 - 4))^2)/sgn(cos(x)) - 2/15*cos
(x)/(sqrt(5*cos(x)^2 - 4)*sgn(cos(x)))
```

3.439 $\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$-\frac{7\tan(x)}{8\sqrt{9\tan^2(x)+4}}+\frac{3}{8}\log\left(9\tan^2(x)+4\right)-\frac{3}{4}\log(\tan(x))-\frac{\cot(x)}{4\sqrt{9\tan^2(x)+4}}$$

```
[Out] (-3*Log[Tan[x]])/4 + (3*Log[4 + 9*Tan[x]^2])/8 - Cot[x]/(4*Sqrt[4 + 9*Tan[x
]^2]) - (7*Tan[x])/(8*Sqrt[4 + 9*Tan[x]^2])
```

Rubi [A] time = 0.849005, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 48, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.146, Rules used = {6742, 191, 271, 266, 36, 29, 31}

$$-\frac{7\tan(x)}{8\sqrt{9\tan^2(x)+4}}+\frac{3}{8}\log\left(9\tan^2(x)+4\right)-\frac{3}{4}\log(\tan(x))-\frac{\cot(x)}{4\sqrt{9\tan^2(x)+4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]^2*(Sec[x]^2 - 3*Tan[x]*Sqrt[4*Sec[x]^2 + 5*Tan[x]^2]))/(4*Sec[x
]^2 + 5*Tan[x]^2)^(3/2),x]
```

```
[Out] (-3*Log[Tan[x]])/4 + (3*Log[4 + 9*Tan[x]^2])/8 - Cot[x]/(4*Sqrt[4 + 9*Tan[x
]^2]) - (7*Tan[x])/(8*Sqrt[4 + 9*Tan[x]^2])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\csc^2(x)\left(\sec^2(x)-3\tan(x)\sqrt{4\sec^2(x)+5\tan^2(x)}\right)}{\left(4\sec^2(x)+5\tan^2(x)\right)^{3/2}}\,dx = \text{Subst}\left(\int \frac{1+x^2-3x\sqrt{4+9x^2}}{x^2\left(4+9x^2\right)^{3/2}}\,dx, x, \tan(x)\right)$$

$$= \text{Subst}\left(\int \left(\frac{1}{\left(4+9x^2\right)^{3/2}}+\frac{1}{x^2\left(4+9x^2\right)^{3/2}}-\frac{3}{x\left(4+9x^2\right)}\right.\right.$$

$$= -\left(3\,\text{Subst}\left(\int \frac{1}{x\left(4+9x^2\right)}\,dx, x, \tan(x)\right)\right)+\text{Subst}\left(\int \right.$$

$$= -\frac{\cot(x)}{4\sqrt{4+9\tan^2(x)}}+\frac{\tan(x)}{4\sqrt{4+9\tan^2(x)}}-\frac{3}{2}\,\text{Subst}\left(\int \right.$$

$$= -\frac{\cot(x)}{4\sqrt{4+9\tan^2(x)}}-\frac{7\tan(x)}{8\sqrt{4+9\tan^2(x)}}-\frac{3}{8}\,\text{Subst}\left(\int \frac{1}{x}\right.$$

$$= -\frac{3}{4}\log(\tan(x))+\frac{3}{8}\log\left(4+9\tan^2(x)\right)-\frac{\cot(x)}{4\sqrt{4+9\tan^2(}}$$

Mathematica [B] time = 1.81516, size = 116, normalized size = 2.04

$$\frac{-5\tan(x)+5\cot(x)-9\csc(x)\sec(x)-6\sqrt{2}\log\left(\tan\left(\frac{x}{2}\right)\right)\sqrt{5\tan^2(x)+13\sec^2(x)-5}+6\sqrt{\frac{13-5\cos(2x)}{\cos(2x)+1}}\log\left(\tan^4\left(\frac{x}{2}\right.\right.}{16\sqrt{\frac{13-5\cos(2x)}{\cos(2x)+1}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]^2*(Sec[x]^2 - 3*Tan[x]*Sqrt[4*Sec[x]^2 + 5*Tan[x]^2]))/(4
*Sec[x]^2 + 5*Tan[x]^2)^(3/2),x]
```

```
[Out] (5*Cot[x] + 6*Sqrt[(13 - 5*Cos[2*x])/(1 + Cos[2*x])]*Log[1 + 7*Tan[x/2]^2 +
 Tan[x/2]^4] - 9*Csc[x]*Sec[x] - 5*Tan[x] - 6*Sqrt[2]*Log[Tan[x/2]]*Sqrt[-5
 + 13*Sec[x]^2 + 5*Tan[x]^2])/(16*Sqrt[(13 - 5*Cos[2*x])/(1 + Cos[2*x])])
```

Maple [B] time = 0.337, size = 117, normalized size = 2.1

$$-\frac{1}{8\left(\cos(x)\right)^3\sin(x)}\left(6\left(-\frac{5\left(\cos(x)\right)^2-9}{\left(\cos(x)\right)^2}\right)^{3/2}\ln\left(-\frac{\cos(x)-1}{\sin(x)}\right)\left(\cos(x)\right)^3\sin(x)-3\left(-\frac{5\left(\cos(x)\right)^2-9}{\left(\cos(x)\right)^2}\right)^{3/2}\ln\left(-\frac{5\,(c}{(co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec(x)^2+
5*tan(x)^2)^(3/2),x)
```

```
[Out] -1/8*(6*(-(5*cos(x)^2-9)/cos(x)^2)^(3/2)*ln(-(cos(x)-1)/sin(x))*cos(x)^3*si
n(x)-3*(-(5*cos(x)^2-9)/cos(x)^2)^(3/2)*ln(-(5*cos(x)^2-9)/(cos(x)+1)^2)*co
s(x)^3*sin(x)+25*cos(x)^4-80*cos(x)^2+63)/(-(5*cos(x)^2-9)/cos(x)^2)^(3/2)/
cos(x)^3/sin(x)
```

Maxima [A] time = 1.428, size = 63, normalized size = 1.11

$$-\frac{7\tan(x)}{8\sqrt{9\tan(x)^2+4}}-\frac{1}{4\sqrt{9\tan(x)^2+4}\tan(x)}+\frac{3}{8}\log\left(9\tan(x)^2+4\right)-\frac{3}{4}\log(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec
(x)^2+5*tan(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -7/8*tan(x)/sqrt(9*tan(x)^2 + 4) - 1/4/(sqrt(9*tan(x)^2 + 4)*tan(x)) + 3/8*
log(9*tan(x)^2 + 4) - 3/4*log(tan(x))
```

Fricas [A] time = 3.26581, size = 252, normalized size = 4.42

$$\frac{3\left(5\cos(x)^2-9\right)\log\left(-\frac{5}{4}\cos(x)^2+\frac{9}{4}\right)\sin(x)-6\left(5\cos(x)^2-9\right)\log\left(\frac{1}{2}\sin(x)\right)\sin(x)-\left(5\cos(x)^3-7\cos(x)\right)\sqrt{-\frac{5}{}}}{8\left(5\cos(x)^2-9\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec
(x)^2+5*tan(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(3*(5*cos(x)^2 - 9)*log(-5/4*cos(x)^2 + 9/4)*sin(x) - 6*(5*cos(x)^2 - 9
)*log(1/2*sin(x))*sin(x) - (5*cos(x)^3 - 7*cos(x))*sqrt(-(5*cos(x)^2 - 9)/c
os(x)^2))/((5*cos(x)^2 - 9)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)**2-3*(4*sec(x)**2+5*tan(x)**2)**(1/2)*tan(x))/sin(x)**2/(
4*sec(x)**2+5*tan(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(x)^2 - 3\sqrt{4\sec(x)^2 + 5\tan(x)^2}\tan(x)}{\left(4\sec(x)^2 + 5\tan(x)^2\right)^{\frac{3}{2}}\sin(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sec(x)^2-3*(4*sec(x)^2+5*tan(x)^2)^(1/2)*tan(x))/sin(x)^2/(4*sec
(x)^2+5*tan(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((sec(x)^2 - 3*sqrt(4*sec(x)^2 + 5*tan(x)^2)*tan(x))/((4*sec(x)^2
+ 5*tan(x)^2)^(3/2)*sin(x)^2), x)
```

3.440 $\int \tan(x)\left(1+5\tan^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=66

$$\frac{1}{5}\left(5\tan^2(x)+1\right)^{5/2}-\frac{4}{3}\left(5\tan^2(x)+1\right)^{3/2}+16\sqrt{5\tan^2(x)+1}-32\tan^{-1}\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right)$$

```
[Out] -32*ArcTan[Sqrt[1 + 5*Tan[x]^2]/2] + 16*Sqrt[1 + 5*Tan[x]^2] - (4*(1 + 5*Ta
n[x]^2)^(3/2))/3 + (1 + 5*Tan[x]^2)^(5/2)/5
```

Rubi [A] time = 0.0706004, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3670, 444, 50, 63, 203}

$$\frac{1}{5}\left(5\tan^2(x)+1\right)^{5/2}-\frac{4}{3}\left(5\tan^2(x)+1\right)^{3/2}+16\sqrt{5\tan^2(x)+1}-32\tan^{-1}\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]*(1 + 5*Tan[x]^2)^(5/2),x]
```

```
[Out] -32*ArcTan[Sqrt[1 + 5*Tan[x]^2]/2] + 16*Sqrt[1 + 5*Tan[x]^2] - (4*(1 + 5*Ta
n[x]^2)^(3/2))/3 + (1 + 5*Tan[x]^2)^(5/2)/5
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \tan(x)\left(1+5\tan^2(x)\right)^{5/2} dx = \text{Subst}\left(\int \frac{x\left(1+5x^2\right)^{5/2}}{1+x^2} dx, x, \tan(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(1+5x)^{5/2}}{1+x} dx, x, \tan^2(x)\right)$$
$$= \frac{1}{5}\left(1+5\tan^2(x)\right)^{5/2} - 2\,\text{Subst}\left(\int \frac{(1+5x)^{3/2}}{1+x} dx, x, \tan^2(x)\right)$$
$$= -\frac{4}{3}\left(1+5\tan^2(x)\right)^{3/2} + \frac{1}{5}\left(1+5\tan^2(x)\right)^{5/2} + 8\,\text{Subst}\left(\int \frac{\sqrt{1+5x}}{1+x} dx, x, \tan^2(x)\right)$$
$$= 16\sqrt{1+5\tan^2(x)} - \frac{4}{3}\left(1+5\tan^2(x)\right)^{3/2} + \frac{1}{5}\left(1+5\tan^2(x)\right)^{5/2} - 32\,\text{Subst}\left(\int \frac{}{(1+} \right.$$
$$= 16\sqrt{1+5\tan^2(x)} - \frac{4}{3}\left(1+5\tan^2(x)\right)^{3/2} + \frac{1}{5}\left(1+5\tan^2(x)\right)^{5/2} - \frac{64}{5}\text{Subst}\left(\int \frac{1}{\frac{4}{5}+} \right.$$
$$= -32\tan^{-1}\left(\frac{1}{2}\sqrt{1+5\tan^2(x)}\right) + 16\sqrt{1+5\tan^2(x)} - \frac{4}{3}\left(1+5\tan^2(x)\right)^{3/2} + \frac{1}{5}\left(1+5\right.$$

Mathematica [C] time = 0.225238, size = 49, normalized size = 0.74

$$\frac{5\sqrt{5}\left(5\tan^2(x)+1\right)^{5/2}\,{}_2F_1\left(-\frac{5}{2},-\frac{5}{2};-\frac{3}{2};\frac{4\cos^2(x)}{5}\right)}{(3-2\cos(2x))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*(1 + 5*Tan[x]^2)^(5/2),x]
```

```
[Out] (5*Sqrt[5]*Hypergeometric2F1[-5/2, -5/2, -3/2, (4*Cos[x]^2)/5]*(1 + 5*Tan[x
]^2)^(5/2))/(3 - 2*Cos[2*x])^(5/2)
```

Maple [A] time = 0.025, size = 61, normalized size = 0.9

$$5\,(\tan(x))^4\sqrt{1+5\,(\tan(x))^2} - \frac{14\,(\tan(x))^2}{3}\sqrt{1+5\,(\tan(x))^2} + \frac{223}{15}\sqrt{1+5\,(\tan(x))^2} - 32\,\arctan\left(1/2\sqrt{1+5\,(\text{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(1+5*tan(x)^2)^(5/2),x)
```

```
[Out] 5*tan(x)^4*(1+5*tan(x)^2)^(1/2)-14/3*tan(x)^2*(1+5*tan(x)^2)^(1/2)+223/15*(
1+5*tan(x)^2)^(1/2)-32*arctan(1/2*(1+5*tan(x)^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(5\,\tan\left(x\right)^{2}+1\right)^{\frac{5}{2}}\tan\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((5*tan(x)^2 + 1)^(5/2)*tan(x), x)
```

Fricas [A] time = 3.06739, size = 157, normalized size = 2.38

$$\frac{1}{15}\left(75\,\tan\left(x\right)^{4}-70\,\tan\left(x\right)^{2}+223\right)\sqrt{5\,\tan\left(x\right)^{2}+1}-16\,\arctan\left(\frac{5\,\tan\left(x\right)^{2}-3}{4\,\sqrt{5\,\tan\left(x\right)^{2}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(75*tan(x)^4 - 70*tan(x)^2 + 223)*sqrt(5*tan(x)^2 + 1) - 16*arctan(1/4
*(5*tan(x)^2 - 3)/sqrt(5*tan(x)^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(5\tan^{2}\left(x\right)+1\right)^{\frac{5}{2}}\tan\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+5*tan(x)**2)**(5/2),x)
```

```
[Out] Integral((5*tan(x)**2 + 1)**(5/2)*tan(x), x)
```

Giac [A] time = 1.08715, size = 70, normalized size = 1.06

$$\frac{1}{5}\left(5\,\tan\left(x\right)^{2}+1\right)^{\frac{5}{2}}-\frac{4}{3}\left(5\,\tan\left(x\right)^{2}+1\right)^{\frac{3}{2}}+16\,\sqrt{5\,\tan\left(x\right)^{2}+1}-32\,\arctan\left(\frac{1}{2}\,\sqrt{5\,\tan\left(x\right)^{2}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+5*tan(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/5*(5*tan(x)^2 + 1)^(5/2) - 4/3*(5*tan(x)^2 + 1)^(3/2) + 16*sqrt(5*tan(x)^
2 + 1) - 32*arctan(1/2*sqrt(5*tan(x)^2 + 1))
```

3.441 $\int \frac{\tan(x)}{\left(1+5\tan^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=54

$$\frac{1}{32}\tan^{-1}\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right)+\frac{1}{16\sqrt{5\tan^2(x)+1}}-\frac{1}{12\left(5\tan^2(x)+1\right)^{3/2}}$$

```
[Out] ArcTan[Sqrt[1 + 5*Tan[x]^2]/2]/32 - 1/(12*(1 + 5*Tan[x]^2)^(3/2)) + 1/(16*S
qrt[1 + 5*Tan[x]^2])
```

Rubi [A] time = 0.0601066, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3670, 444, 51, 63, 203}

$$\frac{1}{32}\tan^{-1}\left(\frac{1}{2}\sqrt{5\tan^2(x)+1}\right)+\frac{1}{16\sqrt{5\tan^2(x)+1}}-\frac{1}{12\left(5\tan^2(x)+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(1 + 5*Tan[x]^2)^(5/2),x]
```

```
[Out] ArcTan[Sqrt[1 + 5*Tan[x]^2]/2]/32 - 1/(12*(1 + 5*Tan[x]^2)^(3/2)) + 1/(16*S
qrt[1 + 5*Tan[x]^2])
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
```

```
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan(x)}{\left(1+5\tan^2(x)\right)^{5/2}}\,dx &= \text{Subst}\left(\int \frac{x}{\left(1+x^2\right)\left(1+5x^2\right)^{5/2}}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)(1+5x)^{5/2}}\,dx, x, \tan^2(x)\right)\\
&= -\frac{1}{12\left(1+5\tan^2(x)\right)^{3/2}} - \frac{1}{8}\text{Subst}\left(\int \frac{1}{(1+x)(1+5x)^{3/2}}\,dx, x, \tan^2(x)\right)\\
&= -\frac{1}{12\left(1+5\tan^2(x)\right)^{3/2}} + \frac{1}{16\sqrt{1+5\tan^2(x)}} + \frac{1}{32}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{1+5x}}\,dx, x, \tan^2(x)\right)\\
&= -\frac{1}{12\left(1+5\tan^2(x)\right)^{3/2}} + \frac{1}{16\sqrt{1+5\tan^2(x)}} + \frac{1}{80}\text{Subst}\left(\int \frac{1}{\frac{4}{5}+\frac{x^2}{5}}\,dx, x, \sqrt{1+5\tan^2(x)}\right)\\
&= \frac{1}{32}\tan^{-1}\left(\frac{1}{2}\sqrt{1+5\tan^2(x)}\right) - \frac{1}{12\left(1+5\tan^2(x)\right)^{3/2}} + \frac{1}{16\sqrt{1+5\tan^2(x)}}
\end{aligned}
$$

Mathematica [A] time = 0.681154, size = 71, normalized size = 1.31

$$
\frac{(2\cos(2x)-3)\sec^5(x)\left(-6\cos(x)+8\cos(3x)-3(2\cos(2x)-3)^{3/2}\log\left(2\cos(x)+\sqrt{2\cos(2x)-3}\right)\right)}{96\left(5\tan^2(x)+1\right)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(1 + 5*Tan[x]^2)^(5/2),x]
```

```
[Out] ((-3 + 2*Cos[2*x])*(-6*Cos[x] + 8*Cos[3*x] - 3*(-3 + 2*Cos[2*x])^(3/2)*Log[
2*Cos[x] + Sqrt[-3 + 2*Cos[2*x]]])*Sec[x]^5)/(96*(1 + 5*Tan[x]^2)^(5/2))
```

Maple [A] time = 0.02, size = 41, normalized size = 0.8

$$
\frac{1}{32}\arctan\left(\frac{1}{2}\sqrt{1+5\left(\tan(x)\right)^2}\right)+\frac{1}{16}\frac{1}{\sqrt{1+5\left(\tan(x)\right)^2}}-\frac{1}{12}\left(1+5\left(\tan(x)\right)^2\right)^{-\frac{3}{2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(1+5*tan(x)^2)^(5/2),x)
```

```
[Out] 1/32*arctan(1/2*(1+5*tan(x)^2)^(1/2))+1/16/(1+5*tan(x)^2)^(1/2)-1/12/(1+5*t
an(x)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\left(5\tan(x)^2+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/(5*tan(x)^2 + 1)^(5/2), x)
```

Fricas [A] time = 3.07106, size = 227, normalized size = 4.2

$$\frac{3\left(25\tan(x)^4+10\tan(x)^2+1\right)\arctan\left(\frac{5\tan(x)^2-3}{4\sqrt{5\tan(x)^2+1}}\right)+4\left(15\tan(x)^2-1\right)\sqrt{5\tan(x)^2+1}}{192\left(25\tan(x)^4+10\tan(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(3*(25*tan(x)^4 + 10*tan(x)^2 + 1)*arctan(1/4*(5*tan(x)^2 - 3)/sqrt(5
*tan(x)^2 + 1)) + 4*(15*tan(x)^2 - 1)*sqrt(5*tan(x)^2 + 1))/(25*tan(x)^4 +
10*tan(x)^2 + 1)
```

Sympy [A] time = 5.40261, size = 46, normalized size = 0.85

$$\frac{\operatorname{atan}\left(\frac{\sqrt{5\tan^2(x)+1}}{2}\right)}{32}+\frac{1}{16\sqrt{5\tan^2(x)+1}}-\frac{1}{12\left(5\tan^2(x)+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+5*tan(x)**2)**(5/2),x)
```

```
[Out] atan(sqrt(5*tan(x)**2 + 1)/2)/32 + 1/(16*sqrt(5*tan(x)**2 + 1)) - 1/(12*(5*
tan(x)**2 + 1)**(3/2))
```

Giac [A] time = 1.07443, size = 49, normalized size = 0.91

$$\frac{15\tan(x)^2-1}{48\left(5\tan(x)^2+1\right)^{\frac{3}{2}}}+\frac{1}{32}\arctan\left(\frac{1}{2}\sqrt{5\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(1+5*tan(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/48*(15*tan(x)^2 - 1)/(5*tan(x)^2 + 1)^(3/2) + 1/32*arctan(1/2*sqrt(5*tan(
x)^2 + 1))
```

3.442 $\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx$

Optimal. Leaf size=133

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{a^3-b^3}}+\frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}}+\frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}}$$

```
[Out] (Sqrt[3]*ArcTan[(1 + (2*(a^3 + b^3*Tan[x]^2)^(1/3))/(a^3 - b^3)^(1/3))/Sqrt
[3]])/(2*(a^3 - b^3)^(1/3)) + Log[Cos[x]]/(2*(a^3 - b^3)^(1/3)) + (3*Log[(a
^3 - b^3)^(1/3) - (a^3 + b^3*Tan[x]^2)^(1/3)])/(4*(a^3 - b^3)^(1/3))
```

Rubi [A] time = 0.155863, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3670, 444, 55, 617, 204, 31}

$$\frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}+1}{\sqrt{3}}\right)}{2\sqrt[3]{a^3-b^3}}+\frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}}+\frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a^3 + b^3*Tan[x]^2)^(1/3),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + (2*(a^3 + b^3*Tan[x]^2)^(1/3))/(a^3 - b^3)^(1/3))/Sqrt
[3]])/(2*(a^3 - b^3)^(1/3)) + Log[Cos[x]]/(2*(a^3 - b^3)^(1/3)) + (3*Log[(a
^3 - b^3)^(1/3) - (a^3 + b^3*Tan[x]^2)^(1/3)])/(4*(a^3 - b^3)^(1/3))
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\tan(x)}{\sqrt[3]{a^3+b^3\tan^2(x)}}\,dx = \text{Subst}\left(\int \frac{x}{\left(1+x^2\right)\sqrt[3]{a^3+b^3x^2}}\,dx, x, \tan(x)\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(1+x)\sqrt[3]{a^3+b^3x}}\,dx, x, \tan^2(x)\right)$$

$$= \frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{\left(a^3-b^3\right)^{2/3}+\sqrt[3]{a^3-b^3}x+x^2}\,dx, x, \sqrt[3]{a^3+b^3\tan^2(x)}\right) - \frac{3\,\text{Subst}}{}$$

$$= \frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}} + \frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}} - \frac{3\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+\frac{2\sqrt[3]{a^3+b^3\tan^2(}}{\sqrt[3]{a^3-b^3}}\right.}{2\sqrt[3]{a^3-b^3}}$$

$$= \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}}{\sqrt{3}}\right)}{2\sqrt[3]{a^3-b^3}} + \frac{\log(\cos(x))}{2\sqrt[3]{a^3-b^3}} + \frac{3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)}{4\sqrt[3]{a^3-b^3}}$$

Mathematica [A] time = 0.220266, size = 105, normalized size = 0.79

$$\frac{2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a^3+b^3\tan^2(x)}}{\sqrt[3]{a^3-b^3}}+1}{\sqrt{3}}\right)+3\log\left(\sqrt[3]{a^3-b^3}-\sqrt[3]{a^3+b^3\tan^2(x)}\right)+2\log(\cos(x))}{4\sqrt[3]{a^3-b^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a^3 + b^3*Tan[x]^2)^(1/3),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + (2*(a^3 + b^3*Tan[x]^2)^(1/3))/(a^3 - b^3)^(1/3))/Sq
rt[3]] + 2*Log[Cos[x]] + 3*Log[(a^3 - b^3)^(1/3) - (a^3 + b^3*Tan[x]^2)^(1/
3)])/(4*(a^3 - b^3)^(1/3))
```

Maple [F] time = 0.106, size = 0, normalized size = 0.

$$\int \tan(x) \frac{1}{\sqrt[3]{a^3 + b^3 (\tan(x))^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x)
```

```
[Out] int(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\left(b^3 \tan(x)^2 + a^3\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate(tan(x)/(b^3*tan(x)^2 + a^3)^(1/3), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\sqrt[3]{a^3 + b^3 \tan^2(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a**3+b**3*tan(x)**2)**(1/3),x)
```

```
[Out] Integral(tan(x)/(a**3 + b**3*tan(x)**2)**(1/3), x)
```

Giac [A] time = 1.2334, size = 251, normalized size = 1.89

$$\frac{3\left(a^3 - b^3\right)^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2\left(b^3 \tan(x)^2 + a^3\right)^{\frac{1}{3}} + \left(a^3 - b^3\right)^{\frac{1}{3}}\right)}{3\left(a^3 - b^3\right)^{\frac{1}{3}}}\right)}{2\left(\sqrt{3}a^3 - \sqrt{3}b^3\right)} - \frac{\log\left(\left(b^3 \tan(x)^2 + a^3\right)^{\frac{2}{3}} + \left(b^3 \tan(x)^2 + a^3\right)^{\frac{1}{3}}\left(a^3 - b^3\right)^{\frac{1}{3}} + \left(a^3 - \right.\right.}{4\left(a^3 - b^3\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a^3+b^3*tan(x)^2)^(1/3),x, algorithm="giac")
```

```
[Out] 3/2*(a^3 - b^3)^(2/3)*arctan(1/3*sqrt(3)*(2*(b^3*tan(x)^2 + a^3)^(1/3) + (a
^3 - b^3)^(1/3))/(a^3 - b^3)^(1/3))/(sqrt(3)*a^3 - sqrt(3)*b^3) - 1/4*log((
b^3*tan(x)^2 + a^3)^(2/3) + (b^3*tan(x)^2 + a^3)^(1/3)*(a^3 - b^3)^(1/3) +
(a^3 - b^3)^(2/3))/(a^3 - b^3)^(1/3) + 1/2*log(abs((b^3*tan(x)^2 + a^3)^(1/
3) - (a^3 - b^3)^(1/3)))/(a^3 - b^3)^(1/3)
```

3.443 $\int \tan(x)\left(1-7\tan^2(x)\right)^{2/3}\,dx$

Optimal. Leaf size=69

$$2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{1-7\tan^2(x)}+1}{\sqrt{3}}\right)+\frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}+3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right)+2\log(\cos(x))$$

```
[Out] 2*Sqrt[3]*ArcTan[(1 + (1 - 7*Tan[x]^2)^(1/3))/Sqrt[3]] + 2*Log[Cos[x]] + 3*
Log[2 - (1 - 7*Tan[x]^2)^(1/3)] + (3*(1 - 7*Tan[x]^2)^(2/3))/4
```

Rubi [A] time = 0.0871591, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3670, 444, 50, 55, 618, 204, 31}

$$2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{1-7\tan^2(x)}+1}{\sqrt{3}}\right)+\frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}+3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right)+2\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]*(1 - 7*Tan[x]^2)^(2/3),x]
```

```
[Out] 2*Sqrt[3]*ArcTan[(1 + (1 - 7*Tan[x]^2)^(1/3))/Sqrt[3]] + 2*Log[Cos[x]] + 3*
Log[2 - (1 - 7*Tan[x]^2)^(1/3)] + (3*(1 - 7*Tan[x]^2)^(2/3))/4
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
```

```
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \tan(x)\left(1-7\tan^2(x)\right)^{2/3}\,dx &= \text{Subst}\left(\int \frac{x\left(1-7x^2\right)^{2/3}}{1+x^2}\,dx, x, \tan(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1-7x)^{2/3}}{1+x}\,dx, x, \tan^2(x)\right)\\
&= \frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3} + 4\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{1-7x}(1+x)}\,dx, x, \tan^2(x)\right)\\
&= 2\log(\cos(x)) + \frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3} - 3\,\text{Subst}\left(\int \frac{1}{2-x}\,dx, x, \sqrt[3]{1-7\tan^2(x)}\right) + 6\,\text{Sul}\\
&= 2\log(\cos(x)) + 3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right) + \frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3} - 12\,\text{Subst}\left(\int \frac{1}{-12-}\right.\\
&= 2\sqrt{3}\tan^{-1}\left(\frac{1+\sqrt[3]{1-7\tan^2(x)}}{\sqrt{3}}\right) + 2\log(\cos(x)) + 3\log\left(2-\sqrt[3]{1-7\tan^2(x)}\right) + \frac{3}{4}\left(1-\right.
\end{aligned}$$

Mathematica [C] time = 0.138839, size = 42, normalized size = 0.61

$$-\frac{3}{4}\left(1-7\tan^2(x)\right)^{2/3}\left({}_2F_1\left(\frac{2}{3},1;\frac{5}{3};\frac{1}{8}(4\cos(2x)-3)\sec^2(x)\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]*(1 - 7*Tan[x]^2)^(2/3),x]
```

```
[Out] (-3*(-1 + Hypergeometric2F1[2/3, 1, 5/3, ((-3 + 4*Cos[2*x])*Sec[x]^2)/8])*(
1 - 7*Tan[x]^2)^(2/3))/4
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \tan(x)\left(1-7(\tan(x))^2\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(1-7*tan(x)^2)^(2/3),x)
```

```
[Out] int(tan(x)*(1-7*tan(x)^2)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-7 \tan(x)^2 + 1\right)^{\frac{2}{3}} \tan(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((-7*tan(x)^2 + 1)^(2/3)*tan(x), x)
```

Fricas [B] time = 11.5915, size = 360, normalized size = 5.22

$$2\sqrt{3}\arctan\left(\frac{7\sqrt{3}\tan(x)^2 + 4\sqrt{3}\left(-7\tan(x)^2 + 1\right)^{\frac{2}{3}} - 16\sqrt{3}\left(-7\tan(x)^2 + 1\right)^{\frac{1}{3}} - \sqrt{3}}{7\tan(x)^2 - 65}\right) + \frac{3}{4}\left(-7\tan(x)^2 + 1\right)^{\frac{2}{3}} + \text{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x, algorithm="fricas")
```

```
[Out] 2*sqrt(3)*arctan((7*sqrt(3)*tan(x)^2 + 4*sqrt(3)*(-7*tan(x)^2 + 1)^(2/3) -
16*sqrt(3)*(-7*tan(x)^2 + 1)^(1/3) - sqrt(3))/(7*tan(x)^2 - 65)) + 3/4*(-7*
tan(x)^2 + 1)^(2/3) + log((7*tan(x)^2 + 6*(-7*tan(x)^2 + 1)^(2/3) - 12*(-7*
tan(x)^2 + 1)^(1/3) + 7)/(tan(x)^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(1 - 7\tan^2(x)\right)^{\frac{2}{3}} \tan(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1-7*tan(x)**2)**(2/3),x)
```

```
[Out] Integral((1 - 7*tan(x)**2)**(2/3)*tan(x), x)
```

Giac [A] time = 1.10444, size = 107, normalized size = 1.55

$$2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\left(-7\tan(x)^2 + 1\right)^{\frac{1}{3}} + 1\right)\right) + \frac{3}{4}\left(-7\tan(x)^2 + 1\right)^{\frac{2}{3}} - \log\left(\left(-7\tan(x)^2 + 1\right)^{\frac{2}{3}} + 2\left(-7\tan(x)^2 + 1\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1-7*tan(x)^2)^(2/3),x, algorithm="giac")
```

```
[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*((-7*tan(x)^2 + 1)^(1/3) + 1)) + 3/4*(-7*tan(x
)^2 + 1)^(2/3) - log((-7*tan(x)^2 + 1)^(2/3) + 2*(-7*tan(x)^2 + 1)^(1/3) +
4) + 2*log(abs((-7*tan(x)^2 + 1)^(1/3) - 2))
```

3.444 $\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx$

Optimal. Leaf size=52

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}$$

```
[Out] -(ArcTan[(a^4 + b^4*Csc[x]^2)^(1/4)/a]/a) + ArcTanh[(a^4 + b^4*Csc[x]^2)^(1
/4)/a]/a
```

Rubi [A] time = 0.0893266, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {4139, 266, 63, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a^4 + b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] -(ArcTan[(a^4 + b^4*Csc[x]^2)^(1/4)/a]/a) + ArcTanh[(a^4 + b^4*Csc[x]^2)^(1
/4)/a]/a
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a^4+b^4x^2}}\,dx, x, \csc(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a^4+b^4x}}\,dx, x, \csc^2(x)\right)\right)\\
&= -\frac{2\,\text{Subst}\left(\int \frac{x^2}{-\frac{a^4}{b^4}+\frac{x^4}{b^4}}\,dx, x, \sqrt[4]{a^4+b^4\csc^2(x)}\right)}{b^4}\\
&= \text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, \sqrt[4]{a^4+b^4\csc^2(x)}\right) - \text{Subst}\left(\int \frac{1}{a^2+x^2}\,dx, x, \sqrt[4]{a^4+b^4\csc^2(x)}\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4+b^4\csc^2(x)}}{a}\right)}{a}
\end{aligned}$$

Mathematica [B] time = 0.407236, size = 256, normalized size = 4.92

$$\frac{\sqrt[4]{a^4\cos(2x)-a^4-2b^4}\left(-\log\left(\frac{a^2\sin(x)}{\sqrt{a^4\left(-\sin^2(x)\right)-b^4}}-\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{a^4\left(-\sin^2(x)\right)-b^4}}+1\right)+\log\left(\frac{a^2\sin(x)}{\sqrt{a^4\left(-\sin^2(x)\right)-b^4}}+\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{a^4\left(-\sin^2(x)\right)-b^4}}+1\right)-2\,\text{ta}\right.}{2\,2^{3/4}a\sqrt{\sin(x)}\sqrt[4]{a^4+b^4\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a^4 + b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] ((-a^4 - 2*b^4 + a^4*Cos[2*x])^(1/4)*(-2*ArcTan[1 - (Sqrt[2]*a*Sqrt[Sin[x]]
)/(-b^4 - a^4*Sin[x]^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*a*Sqrt[Sin[x]])/(-b^
4 - a^4*Sin[x]^2)^(1/4)] - Log[1 + (a^2*Sin[x])/Sqrt[-b^4 - a^4*Sin[x]^2] -
 (Sqrt[2]*a*Sqrt[Sin[x]])/(-b^4 - a^4*Sin[x]^2)^(1/4)] + Log[1 + (a^2*Sin[x
])/Sqrt[-b^4 - a^4*Sin[x]^2] + (Sqrt[2]*a*Sqrt[Sin[x]])/(-b^4 - a^4*Sin[x]^
2)^(1/4)]))/(2*2^(3/4)*a*(a^4 + b^4*Csc[x]^2)^(1/4)*Sqrt[Sin[x]])
```

Maple [F] time = 0.112, size = 0, normalized size = 0.

$$\int \cot(x)\frac{1}{\sqrt[4]{a^4+b^4(\csc(x))^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x)

[Out] int(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x)
```

Maxima [A] time = 1.42158, size = 96, normalized size = 1.85

$$-\frac{\arctan\left(\frac{\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a}+\frac{\log\left(a+\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}-\frac{\log\left(-a+\left(a^4+\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x, algorithm="maxima")

[Out] -arctan((a^4 + b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(a + (a^4 + b^4/sin(x)^2)^
(1/4))/a - 1/2*log(-a + (a^4 + b^4/sin(x)^2)^(1/4))/a
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x, algorithm="fricas")

[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\sqrt[4]{a^4+b^4\csc^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a**4+b**4*csc(x)**2)**(1/4),x)

[Out] Integral(cot(x)/(a**4 + b**4*csc(x)**2)**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\left(b^4\csc(x)^2+a^4\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a^4+b^4*csc(x)^2)^(1/4),x, algorithm="giac")

[Out] integrate(cot(x)/(b^4*csc(x)^2 + a^4)^(1/4), x)
```

3.445 $\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx$

Optimal. Leaf size=54

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}$$

```
[Out] -(ArcTan[(a^4 - b^4*Csc[x]^2)^(1/4)/a]/a) + ArcTanh[(a^4 - b^4*Csc[x]^2)^(1
/4)/a]/a
```

Rubi [A] time = 0.0874378, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4139, 266, 63, 298, 203, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(a^4 - b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] -(ArcTan[(a^4 - b^4*Csc[x]^2)^(1/4)/a]/a) + ArcTanh[(a^4 - b^4*Csc[x]^2)^(1
/4)/a]/a
```

Rule 4139

```
Int[((a_) + (b_.)*((c_.)*sec[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Di
st[1/f, Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p)/x, x],
 x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m
 - 1)/2] && (GtQ[m, 0] || EqQ[n, 2] || EqQ[n, 4] || IGtQ[p, 0] || IntegersQ
[2*n, p])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{\sqrt[4]{a^4-b^4\csc^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a^4-b^4x^2}}\,dx, x, \csc(x)\right)\\
&= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt[4]{a^4-b^4x}}\,dx, x, \csc^2(x)\right)\right)\\
&= \frac{2\,\text{Subst}\left(\int \frac{x^2}{\frac{a^4}{b^4}-\frac{x^4}{b^4}}\,dx, x, \sqrt[4]{a^4-b^4\csc^2(x)}\right)}{b^4}\\
&= \text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, \sqrt[4]{a^4-b^4\csc^2(x)}\right) - \text{Subst}\left(\int \frac{1}{a^2+x^2}\,dx, x, \sqrt[4]{a^4-b^4\csc^2(x)}\right)\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a^4-b^4\csc^2(x)}}{a}\right)}{a}
\end{aligned}$$

Mathematica [B] time = 0.445366, size = 245, normalized size = 4.54

$$\frac{\sqrt[4]{a^4\cos(2x)-a^4+2b^4}\left(-\log\left(\frac{a^2\sin(x)}{\sqrt{b^4-a^4\sin^2(x)}}-\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{b^4-a^4\sin^2(x)}}+1\right)+\log\left(\frac{a^2\sin(x)}{\sqrt{b^4-a^4\sin^2(x)}}+\frac{\sqrt{2}a\sqrt{\sin(x)}}{\sqrt[4]{b^4-a^4\sin^2(x)}}+1\right)-2\tan^{-1}\left(1\right.\right.}{2\,2^{3/4}a\sqrt{\sin(x)}\sqrt[4]{a^4-b^4\csc^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(a^4 - b^4*Csc[x]^2)^(1/4),x]
```

```
[Out] ((-a^4 + 2*b^4 + a^4*Cos[2*x])^(1/4)*(-2*ArcTan[1 - (Sqrt[2]*a*Sqrt[Sin[x]]
)/(b^4 - a^4*Sin[x]^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*a*Sqrt[Sin[x]])/(b^4
- a^4*Sin[x]^2)^(1/4)] - Log[1 + (a^2*Sin[x])/Sqrt[b^4 - a^4*Sin[x]^2] - (S
qrt[2]*a*Sqrt[Sin[x]])/(b^4 - a^4*Sin[x]^2)^(1/4)] + Log[1 + (a^2*Sin[x])/S
qrt[b^4 - a^4*Sin[x]^2] + (Sqrt[2]*a*Sqrt[Sin[x]])/(b^4 - a^4*Sin[x]^2)^(1/
4)]))/(2*2^(3/4)*a*(a^4 - b^4*Csc[x]^2)^(1/4)*Sqrt[Sin[x]])
```

Maple [F] time = 0.103, size = 0, normalized size = 0.

$$\int \cot(x)\frac{1}{\sqrt[4]{a^4-b^4(\csc(x))^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x)
```

```
[Out] int(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x)
```

Maxima [A] time = 1.42083, size = 100, normalized size = 1.85

$$-\frac{\arctan\left(\frac{\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}}{a}\right)}{a}+\frac{\log\left(a+\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}-\frac{\log\left(-a+\left(a^4-\frac{b^4}{\sin(x)^2}\right)^{\frac{1}{4}}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x, algorithm="maxima")
```

```
[Out] -arctan((a^4 - b^4/sin(x)^2)^(1/4)/a)/a + 1/2*log(a + (a^4 - b^4/sin(x)^2)^
(1/4))/a - 1/2*log(-a + (a^4 - b^4/sin(x)^2)^(1/4))/a
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\sqrt[4]{\left(a^2-b^2\csc(x)\right)\left(a^2+b^2\csc(x)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a**4-b**4*csc(x)**2)**(1/4),x)
```

```
[Out] Integral(cot(x)/((a**2 - b**2*csc(x))*(a**2 + b**2*csc(x)))**(1/4), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\left(-b^4\csc(x)^2+a^4\right)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a^4-b^4*csc(x)^2)^(1/4),x, algorithm="giac")
```

```
[Out] integrate(cot(x)/(-b^4*csc(x)^2 + a^4)^(1/4), x)
```

3.446 $\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{\left(1-3\sec^2(x)\right)^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}\,dx$

Optimal. Leaf size=133

$$-\frac{1}{4}\left(1-3\sec^2(x)\right)^{2/3}-\sqrt[6]{1-3\sec^2(x)}+\frac{1}{2\left(1-\sqrt{1-3\sec^2(x)}\right)}+\frac{1}{4}\log\left(\sec^2(x)\right)-\frac{3}{2}\log\left(1-\sqrt[6]{1-3\sec^2(x)}\right)+\frac{1}{3}\log$$

```
[Out] Sqrt[3]*ArcTan[(1 + 2*(1 - 3*Sec[x]^2)^(1/6))/Sqrt[3]] + Log[Sec[x]^2]/4 -
(3*Log[1 - (1 - 3*Sec[x]^2)^(1/6)])/2 + Log[1 - Sqrt[1 - 3*Sec[x]^2]]/3 - (
1 - 3*Sec[x]^2)^(1/6) - (1 - 3*Sec[x]^2)^(2/3)/4 + 1/(2*(1 - Sqrt[1 - 3*Sec
[x]^2]))
```

Rubi [A] time = 5.10991, antiderivative size = 174, normalized size of antiderivative = 1.31, number of steps used = 29, number of rules used = 16, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.262, Rules used = {4361, 6742, 6684, 261, 6697, 341, 57, 618, 204, 31, 6688, 266, 47, 63, 206, 25}

$$\frac{\cos^2(x)}{6}-\frac{1}{4}\left(1-3\sec^2(x)\right)^{2/3}-\sqrt[6]{1-3\sec^2(x)}-\frac{3}{2}\log\left(1-\sqrt[6]{1-3\sec^2(x)}\right)+\frac{1}{2}\log\left(1-\sqrt{1-3\sec^2(x)}\right)+\frac{1}{6}\cos^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*Tan[x]*((1 - 3*Sec[x]^2)^(1/3)*Sin[x]^2 + 3*Tan[x]^2))/((1 -
3*Sec[x]^2)^(5/6)*(1 - Sqrt[1 - 3*Sec[x]^2])),x]
```

```
[Out] Sqrt[3]*ArcTan[(1 + 2*(1 - 3*Sec[x]^2)^(1/6))/Sqrt[3]] + ArcTanh[Sqrt[1 - 3
*Sec[x]^2]]/2 + Cos[x]^2/6 + Log[1 - Sqrt[-((3 - Cos[x]^2)*Sec[x]^2)]]/3 -
(3*Log[1 - (1 - 3*Sec[x]^2)^(1/6)])/2 + Log[1 - Sqrt[1 - 3*Sec[x]^2]]/2 - (
1 - 3*Sec[x]^2)^(1/6) + (Cos[x]^2*Sqrt[1 - 3*Sec[x]^2])/6 - (1 - 3*Sec[x]^2
)^(2/3)/4
```

Rule 4361

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6684

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6697

```
Int[(u_.)*(v_)^(m_.)*((a_.) + (b_.)*(y_)^(n_))^(p_.), x_Symbol] :> Module[{
q, r}, Dist[q*r, Subst[Int[x^m*(a + b*x^n)^p, x], x, y], x] /;  !FalseQ[r =
 Divides[y^m, v^m, x]] &&  !FalseQ[q = DerivativeDivides[y, u, x]]] /; Free
Q[{a, b, m, n, p}, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 25

```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.),
x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; F
reeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d,
0] &&  !(IntegerQ[m] && NegQ[n])
```

Rubi steps

$$\int \frac{\sec^2(x)\tan(x)\left(\sqrt[3]{1-3\sec^2(x)}\sin^2(x)+3\tan^2(x)\right)}{\left(1-3\sec^2(x)\right)^{5/6}\left(1-\sqrt{1-3\sec^2(x)}\right)}dx = -\text{Subst}\left(\int \frac{(1-x^2)\left(3+\sqrt[3]{1-\frac{3}{x^2}}x^2\right)}{\left(1-\sqrt{1-\frac{3}{x^2}}\right)\left(1-\frac{3}{x^2}\right)^{5/6}x^5}dx, x, \cos(x)\right)$$

$$= -\text{Subst}\left(\int\left(\frac{-3-x^2\sqrt[3]{\frac{-3+x^2}{x^2}}}{\left(1-\frac{3}{x^2}\right)^{5/6}x^5\left(-1+\sqrt{\frac{-3+x^2}{x^2}}\right)}+\frac{3+x^2}{\left(1-\frac{3}{x^2}\right)^{5/6}x^3\left(\right.}\right.\right.$$

$$= -\text{Subst}\left(\int \frac{-3-x^2\sqrt[3]{\frac{-3+x^2}{x^2}}}{\left(1-\frac{3}{x^2}\right)^{5/6}x^5\left(-1+\sqrt{\frac{-3+x^2}{x^2}}\right)}dx, x, \cos(x)\right) - \text{Su}$$

$$= -\text{Subst}\left(\int\left(\frac{1}{\sqrt{1-\frac{3}{x^2}}x^3\left(1-\sqrt{\frac{-3+x^2}{x^2}}\right)}-\frac{3}{\left(1-\frac{3}{x^2}\right)^{5/6}x^5\left(-1+\right.}\right.\right.$$

$$= 3\,\text{Subst}\left(\int \frac{1}{\left(1-\frac{3}{x^2}\right)^{5/6}x^5\left(-1+\sqrt{\frac{-3+x^2}{x^2}}\right)}dx, x, \cos(x)\right) - 3\,\text{S}$$

$$= \frac{1}{3}\log\left(1-\sqrt{-\left(3-\cos^2(x)\right)\sec^2(x)}\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{\left(-1+\sqrt{\right.}}\right.$$

$$= \frac{1}{3}\log\left(1-\sqrt{-\left(3-\cos^2(x)\right)\sec^2(x)}\right) - \text{Subst}\left(\int \frac{1}{\left(1-\frac{3}{x^2}\right)^{5/}}\right.$$

$$= \frac{\cos^2(x)}{6} + \frac{1}{3}\log\left(1-\sqrt{-\left(3-\cos^2(x)\right)\sec^2(x)}\right) + \frac{1}{2}\log\left(1-\right.$$

$$= \frac{\cos^2(x)}{6} - \frac{3}{2}\log\left(1-\sqrt[6]{-\left(3-\cos^2(x)\right)\sec^2(x)}\right) + \frac{1}{3}\log\left(1-\right.$$

$$= \sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[6]{-\left(3-\cos^2(x)\right)\sec^2(x)}}{\sqrt{3}}\right) + \frac{\cos^2(x)}{6} - \frac{3}{2}\text{lo}$$

$$= \sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[6]{-\left(3-\cos^2(x)\right)\sec^2(x)}}{\sqrt{3}}\right) + \frac{\cos^2(x)}{6} - \frac{3}{2}\text{lo}$$

$$= \sqrt{3}\tan^{-1}\left(\frac{1+2\sqrt[6]{-\left(3-\cos^2(x)\right)\sec^2(x)}}{\sqrt{3}}\right) + \frac{1}{2}\tanh^{-1}\left(\sqrt{1-}\right.$$

Mathematica [C] time = 56.1542, size = 4397, normalized size = 33.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[x]^2*Tan[x]*((1 - 3*Sec[x]^2)^(1/3)*Sin[x]^2 + 3*Tan[x]^2))/
((1 - 3*Sec[x]^2)^(5/6)*(1 - Sqrt[1 - 3*Sec[x]^2])),x]
```

```
[Out] (-3*(6 + ((-5 + Cos[2*x])/(1 + Cos[2*x]))^(1/3) + Cos[2*x]*((-5 + Cos[2*x])
/(1 + Cos[2*x]))^(1/3))*(3*Sec[x]^2 + (1 - 3*Sec[x]^2)^(1/3))*Sin[x]^2*Tan[
x]*(-2 - 3*Tan[x]^2)^(5/6)*(1 + Tan[x]^2)*(2 + 3*Tan[x]^2)*(-8*AppellF1[1,
1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2] + 4*AppellF1[2, 1/2, 2, 3, (-3*Tan[x
]^2)/2, -Tan[x]^2]*Tan[x]^2 + 3*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Ta
n[x]^2]*Tan[x]^2)^2*((4*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]
+ 3*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2])*Tan[x]^2*(30*3^(2/3
)*Hypergeometric2F1[1/3, 1/3, 4/3, (3 + 3*Tan[x]^2)^(-1)]*Sqrt[-2 - 3*Tan[x
]^2]*(1 + Tan[x]^2)*((2 + 3*Tan[x]^2)/(1 + Tan[x]^2))^(1/3) + 12*3^(1/6)*Hy
pergeometric2F1[5/6, 5/6, 11/6, (3 + 3*Tan[x]^2)^(-1)]*(1 + Tan[x]^2)*((2 +
 3*Tan[x]^2)/(1 + Tan[x]^2))^(5/6) + 5*(2*Log[1 + Tan[x]^2]*(-2 - 3*Tan[x]^
2)^(5/6)*(1 + Tan[x]^2) + 9*Tan[x]^4*(4 + Sqrt[-2 - 3*Tan[x]^2]) + 3*Tan[x]
^2*(20 - 2*(-2 - 3*Tan[x]^2)^(1/3) + 5*Sqrt[-2 - 3*Tan[x]^2]) + 2*(12 - 2*(
-2 - 3*Tan[x]^2)^(1/3) + 3*Sqrt[-2 - 3*Tan[x]^2] + (-2 - 3*Tan[x]^2)^(5/6))
)) - 8*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*(30*3^(2/3)*Hyper
geometric2F1[1/3, 1/3, 4/3, (3 + 3*Tan[x]^2)^(-1)]*Sqrt[-2 - 3*Tan[x]^2]*(1
 + Tan[x]^2)*((2 + 3*Tan[x]^2)/(1 + Tan[x]^2))^(1/3) + 12*3^(1/6)*Hypergeom
etric2F1[5/6, 5/6, 11/6, (3 + 3*Tan[x]^2)^(-1)]*(1 + Tan[x]^2)*((2 + 3*Tan[
x]^2)/(1 + Tan[x]^2))^(5/6) + 5*(2*Log[1 + Tan[x]^2]*(-2 - 3*Tan[x]^2)^(5/6
)*(1 + Tan[x]^2) + 9*Tan[x]^4*(4 + Sqrt[-2 - 3*Tan[x]^2]) + Tan[x]^2*(60 -
7*(-2 - 3*Tan[x]^2)^(1/3) + 15*Sqrt[-2 - 3*Tan[x]^2]) + 2*(12 - 2*(-2 - 3*T
an[x]^2)^(1/3) + 3*Sqrt[-2 - 3*Tan[x]^2] + (-2 - 3*Tan[x]^2)^(5/6))))))/(10
*2^(1/6)*(-1 + Sqrt[(-5 + Cos[2*x])/(1 + Cos[2*x])])*(1 - 3*Sec[x]^2)^(5/6)
*(6 + (1 - 3*Sec[x]^2)^(1/3) + Cos[2*x]*(1 - 3*Sec[x]^2)^(1/3))*(-4 - 6*Tan
[x]^2)^(5/6)*(-8*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2] + (4*Ap
pellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2] + 3*AppellF1[2, 3/2, 1, 3,
 (-3*Tan[x]^2)/2, -Tan[x]^2])*Tan[x]^2)*(1152*AppellF1[1, 1/2, 1, 2, (-3*Ta
n[x]^2)/2, -Tan[x]^2]^2*Tan[x]^3 + 2880*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2
)/2, -Tan[x]^2]^2*Tan[x]^5 - 1152*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -
Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5 - 864
*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3,
(-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5 + 1728*AppellF1[1, 1/2, 1, 2, (-3*Tan[
x]^2)/2, -Tan[x]^2]^2*Tan[x]^7 - 2880*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/
2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7 +
 288*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7 - 2160*A
ppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-
3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7 + 432*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^
2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^
7 + 162*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7 - 172
8*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3,
 (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9 + 720*AppellF1[2, 1/2, 2, 3, (-3*Tan[
x]^2)/2, -Tan[x]^2]^2*Tan[x]^9 - 1296*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/
2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9 +
 1080*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1
, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9 + 405*AppellF1[2, 3/2, 1, 3, (-3*
Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^9 + 432*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^
2)/2, -Tan[x]^2]^2*Tan[x]^11 + 648*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2,
-Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^11 + 2
43*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^11 + 720*App
ellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^3*(-2 - 3*Tan[x]^2
)^(1/3) - 192*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2
, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^3*(-2 - 3*Tan[x]^2)^(1/3) -
 144*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1,
 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^3*(-2 - 3*Tan[x]^2)^(1/3) + 1008*App
ellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^5*(-2 - 3*Tan[x]^2
)^(1/3) - 1032*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[
2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*(-2 - 3*Tan[x]^2)^(1/3)
- 774*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1
, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*(-2 - 3*Tan[x]^2)^(1/3) + 128*App
```

ellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[3, 1/2, 3, 4, (-3*
Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*(-2 - 3*Tan[x]^2)^(1/3) + 96*AppellF1[1, 1
/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[3, 3/2, 2, 4, (-3*Tan[x]^2)/
2, -Tan[x]^2]*Tan[x]^5*(-2 - 3*Tan[x]^2)^(1/3) + 108*AppellF1[1, 1/2, 1, 2,
 (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[3, 5/2, 1, 4, (-3*Tan[x]^2)/2, -Tan[x
]^2]*Tan[x]^5*(-2 - 3*Tan[x]^2)^(1/3) - 1080*AppellF1[1, 1/2, 1, 2, (-3*Tan
[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan
[x]^7*(-2 - 3*Tan[x]^2)^(1/3) + 96*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2,
-Tan[x]^2]^2*Tan[x]^7*(-2 - 3*Tan[x]^2)^(1/3) - 810*AppellF1[1, 1/2, 1, 2,
(-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]
^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(1/3) + 144*AppellF1[2, 1/2, 2, 3, (-3*Tan[x
]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x
]^7*(-2 - 3*Tan[x]^2)^(1/3) + 54*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -T
an[x]^2]^2*Tan[x]^7*(-2 - 3*Tan[x]^2)^(1/3) + 320*AppellF1[1, 1/2, 1, 2, (-
3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[3, 1/2, 3, 4, (-3*Tan[x]^2)/2, -Tan[x]^2
]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(1/3) + 240*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^
2)/2, -Tan[x]^2]*AppellF1[3, 3/2, 2, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^
7*(-2 - 3*Tan[x]^2)^(1/3) + 270*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Ta
n[x]^2]*AppellF1[3, 5/2, 1, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3
*Tan[x]^2)^(1/3) + 144*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2
Tan[x]^9(-2 - 3*Tan[x]^2)^(1/3) + 216*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2
)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9
*(-2 - 3*Tan[x]^2)^(1/3) + 81*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[
x]^2]^2*Tan[x]^9*(-2 - 3*Tan[x]^2)^(1/3) + 192*AppellF1[1, 1/2, 1, 2, (-3*T
an[x]^2)/2, -Tan[x]^2]*AppellF1[3, 1/2, 3, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*T
an[x]^9*(-2 - 3*Tan[x]^2)^(1/3) + 144*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/
2, -Tan[x]^2]*AppellF1[3, 3/2, 2, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9*(
-2 - 3*Tan[x]^2)^(1/3) + 162*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x
]^2]*AppellF1[3, 5/2, 1, 4, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9*(-2 - 3*Ta
n[x]^2)^(1/3) + 1152*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*T
an[x]^3*Sqrt[-2 - 3*Tan[x]^2] + 2880*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2
, -Tan[x]^2]^2*Tan[x]^5*Sqrt[-2 - 3*Tan[x]^2] - 1152*AppellF1[1, 1/2, 1, 2,
 (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x
]^2]*Tan[x]^5*Sqrt[-2 - 3*Tan[x]^2] - 864*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]
^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]
^5*Sqrt[-2 - 3*Tan[x]^2] + 1728*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Ta
n[x]^2]^2*Tan[x]^7*Sqrt[-2 - 3*Tan[x]^2] - 2880*AppellF1[1, 1/2, 1, 2, (-3*
Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*
Tan[x]^7*Sqrt[-2 - 3*Tan[x]^2] + 288*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2
, -Tan[x]^2]^2*Tan[x]^7*Sqrt[-2 - 3*Tan[x]^2] - 2160*AppellF1[1, 1/2, 1, 2,
 (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x
]^2]*Tan[x]^7*Sqrt[-2 - 3*Tan[x]^2] + 432*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]
^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]
^7*Sqrt[-2 - 3*Tan[x]^2] + 162*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan
[x]^2]^2*Tan[x]^7*Sqrt[-2 - 3*Tan[x]^2] - 1728*AppellF1[1, 1/2, 1, 2, (-3*T
an[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*T
an[x]^9*Sqrt[-2 - 3*Tan[x]^2] + 720*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2,
 -Tan[x]^2]^2*Tan[x]^9*Sqrt[-2 - 3*Tan[x]^2] - 1296*AppellF1[1, 1/2, 1, 2,
(-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]
^2]*Tan[x]^9*Sqrt[-2 - 3*Tan[x]^2] + 1080*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]
^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]
^9*Sqrt[-2 - 3*Tan[x]^2] + 405*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan
[x]^2]^2*Tan[x]^9*Sqrt[-2 - 3*Tan[x]^2] + 432*AppellF1[2, 1/2, 2, 3, (-3*Ta
n[x]^2)/2, -Tan[x]^2]^2*Tan[x]^11*Sqrt[-2 - 3*Tan[x]^2] + 648*AppellF1[2, 1
/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/
2, -Tan[x]^2]*Tan[x]^11*Sqrt[-2 - 3*Tan[x]^2] + 243*AppellF1[2, 3/2, 1, 3,
(-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^11*Sqrt[-2 - 3*Tan[x]^2] + 384*AppellF
1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^3*(-2 - 3*Tan[x]^2)^(5
/6) + 576*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^5*(-2

```
 - 3*Tan[x]^2)^(5/6) - 384*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^
2]*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*(-2 - 3*Tan[
x]^2)^(5/6) - 288*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*Appell
F1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^5*(-2 - 3*Tan[x]^2)^(5/
6) - 576*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 1/2
, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(5/6) + 96*A
ppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7*(-2 - 3*Tan[x]
^2)^(5/6) - 432*AppellF1[1, 1/2, 1, 2, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1
[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(5/6)
 + 144*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*AppellF1[2, 3/2,
1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^7*(-2 - 3*Tan[x]^2)^(5/6) + 54*App
ellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^7*(-2 - 3*Tan[x]^2
)^(5/6) + 144*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*Tan[x]^9
*(-2 - 3*Tan[x]^2)^(5/6) + 216*AppellF1[2, 1/2, 2, 3, (-3*Tan[x]^2)/2, -Tan
[x]^2]*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]*Tan[x]^9*(-2 - 3*
Tan[x]^2)^(5/6) + 81*AppellF1[2, 3/2, 1, 3, (-3*Tan[x]^2)/2, -Tan[x]^2]^2*T
an[x]^9*(-2 - 3*Tan[x]^2)^(5/6)))
```

Maple [F] time = 180., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{(\cos(x))^2}\left(\sqrt[3]{1-3(\sec(x))^2}(\sin(x))^2+3(\tan(x))^2\right)\left(1-3(\sec(x))^2\right)^{-\frac{5}{6}}\left(1-\sqrt{1-3(\sec(x))^2}\right)^{-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*sec(x)^
2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x)
```

```
[Out] int(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*sec(x)^
2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*s
ec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*s
ec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*((1-3*sec(x)**2)**(1/3)*sin(x)**2+3*tan(x)**2)/cos(x)**2/(
1-3*sec(x)**2)**(5/6)/(1-(1-3*sec(x)**2)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(\left(-3\sec\left(x\right)^{2}+1\right)^{\frac{1}{3}}\sin\left(x\right)^{2}+3\tan\left(x\right)^{2}\right)\tan\left(x\right)}{\left(-3\sec\left(x\right)^{2}+1\right)^{\frac{5}{6}}\left(\sqrt{-3\sec\left(x\right)^{2}+1}-1\right)\cos\left(x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*((1-3*sec(x)^2)^(1/3)*sin(x)^2+3*tan(x)^2)/cos(x)^2/(1-3*s
ec(x)^2)^(5/6)/(1-(1-3*sec(x)^2)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(-((-3*sec(x)^2 + 1)^(1/3)*sin(x)^2 + 3*tan(x)^2)*tan(x)/((-3*sec(
x)^2 + 1)^(5/6)*(sqrt(-3*sec(x)^2 + 1) - 1)*cos(x)^2), x)
```

3.447 $\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx$

Optimal. Leaf size=100

$$\frac{2\tan^3(x)}{3(\tan(x)\tan(2x))^{3/2}}+\frac{3\tan(x)}{4\sqrt{\tan(x)\tan(2x)}}+\frac{\tan(x)}{2(\tan(x)\tan(2x))^{3/2}}+2\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right)-\frac{11\tanh^{-1}\left(\frac{\sqrt{2}\ldots}{\sqrt{\tan(\ldots}}\right)}{4\sqrt{2}}$$

```
[Out] 2*ArcTanh[Tan[x]/Sqrt[Tan[x]*Tan[2*x]]] - (11*ArcTanh[(Sqrt[2]*Tan[x])/Sqrt
[Tan[x]*Tan[2*x]]])/(4*Sqrt[2]) + Tan[x]/(2*(Tan[x]*Tan[2*x])^(3/2)) + (2*T
an[x]^3)/(3*(Tan[x]*Tan[2*x])^(3/2)) + (3*Tan[x])/(4*Sqrt[Tan[x]*Tan[2*x]])
```

Rubi [B] time = 1.22598, antiderivative size = 208, normalized size of antiderivative = 2.08, number of steps used = 21, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {4397, 12, 6719, 6725, 266, 47, 50, 63, 203, 444}

$$\frac{\left(1-\tan^2(x)\right)\tan(x)}{3\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}}+\frac{3\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}}-\frac{11\tan^{-1}\left(\sqrt{\tan^2(x)-1}\right)\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{\tan^2(x)-1}}+\frac{2\tan^{-1}\left(\frac{\sqrt{\tan^2(x)-1}}{\sqrt{2}}\right)\tan(x)}{\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{\tan^2(x)-1}}+\frac{\left(1-\tan^2\ldots\right.}{4\sqrt{2}\sqrt{\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*(-Cos[2*x] + 2*Tan[x]^2))/(Tan[x]*Tan[2*x])^(3/2),x]
```

```
[Out] (3*Tan[x])/(4*Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]) + (Cot[x]*(1 - Tan[x]^
2))/(4*Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]) + (Tan[x]*(1 - Tan[x]^2))/(3*
Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]) - (11*ArcTan[Sqrt[-1 + Tan[x]^2]]*Ta
n[x])/(4*Sqrt[2]*Sqrt[Tan[x]^2/(1 - Tan[x]^2)]*Sqrt[-1 + Tan[x]^2]) + (2*Ar
cTan[Sqrt[-1 + Tan[x]^2]/Sqrt[2]]*Tan[x])/(Sqrt[Tan[x]^2/(1 - Tan[x]^2)]*Sq
rt[-1 + Tan[x]^2])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6719

```
Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[
p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^
(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] &&  !IntegerQ[p] &&  !FreeQ
[v, x] &&  !FreeQ[w, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$\int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(\tan(x)\tan(2x))^{3/2}}\,dx = \int \frac{\sec^2(x)\left(-\cos(2x)+2\tan^2(x)\right)}{(-1+\sec(2x))^{3/2}}\,dx$$

$$= \text{Subst}\left(\int \frac{(1-x^2)(-1+3x^2+2x^4)}{2\sqrt{2}x^2\sqrt{\frac{x^2}{1-x^2}}(1+x^2)}\,dx, x, \tan(x)\right)$$

$$= \frac{\text{Subst}\left(\int \frac{(1-x^2)(-1+3x^2+2x^4)}{x^2\sqrt{\frac{x^2}{1-x^2}}(1+x^2)}\,dx, x, \tan(x)\right)}{2\sqrt{2}}$$

$$= \frac{\tan(x)\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}(-1+3x^2+2x^4)}{x^3(1+x^2)}\,dx, x, \tan(x)\right)}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$= \frac{\tan(x)\,\text{Subst}\left(\int \left(-\frac{(1-x^2)^{3/2}}{x^3}+\frac{4(1-x^2)^{3/2}}{x}-\frac{2x(1-x^2)^{3/2}}{1+x^2}\right)dx, x, \tan(x)\right)}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}}$$

$$= -\frac{\tan(x)\,\text{Subst}\left(\int \frac{(1-x^2)^{3/2}}{x^3}\,dx, x, \tan(x)\right)}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} - \frac{\tan(x)\,\text{Subst}\left(\int \frac{x(1-x^2)^{3/2}}{1+x^2}\,dx, x, \text{t}\right.}{\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x}}$$

$$= -\frac{\tan(x)\,\text{Subst}\left(\int \frac{(1-x)^{3/2}}{x^2}\,dx, x, \tan^2(x)\right)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} - \frac{\tan(x)\,\text{Subst}\left(\int \frac{(1-x)^{3/2}}{1+x}\,dx, x, \text{tar}\right.}{2\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x}}$$

$$= \frac{\cot(x)\left(1-\tan^2(x)\right)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{\tan(x)\left(1-\tan^2(x)\right)}{3\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\sqrt{2}\tan(x)\left(1-\tan^2(x)\right)}{3\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{(3}{}$$

$$= \frac{3\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\cot(x)\left(1-\tan^2(x)\right)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{\tan(x)\left(1-\tan^2(x)\right)}{3\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\sqrt{2}\tan(x)}{3\sqrt{1}}$$

$$= \frac{3\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\cot(x)\left(1-\tan^2(x)\right)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{\tan(x)\left(1-\tan^2(x)\right)}{3\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} + \frac{\sqrt{2}\tan(x)}{3\sqrt{1}}$$

$$= \frac{3\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}} - \frac{3\tanh^{-1}\left(\sqrt{1-\tan^2(x)}\right)\tan(x)}{4\sqrt{2}\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\tan^2(x)}} - \frac{\sqrt{2}\tanh^{-1}\left(\sqrt{1-\tan^2}\right.}{\sqrt{\frac{\tan^2(x)}{1-\tan^2(x)}}\sqrt{1-\text{t}}}$$

Mathematica [A] time = 4.59417, size = 168, normalized size = 1.68

$$\frac{\tan^2(2x)\left(2\tan^2(x)-\cos(2x)\right)\left(-3\sin(x)\cos(x)\tan^{-1}\left(\sqrt{\tan^2(x)-1}\right)\sqrt{\tan^2(x)-1}+\frac{4\sqrt{2}\cos(2x)\tan(x)\left(\sqrt{2}\tanh^{-1}\left(\sqrt{1-\text{ta}}\right.\right.}{\sqrt{1-\text{ta}}}\right.}{2(6\cos(2x)+\cos(4x)-3)(\tan(x)\,\text{tan}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*(-Cos[2*x] + 2*Tan[x]^2))/(Tan[x]*Tan[2*x])^(3/2),x]
```

```
[Out] ((-Cos[2*x] + 2*Tan[x]^2)*((4*Sqrt[2]*(-2*ArcTanh[Sqrt[2 - 2*Tan[x]^2]/2] +
 Sqrt[2]*ArcTanh[Sqrt[1 - Tan[x]^2]])*Cos[2*x]*Tan[x])/Sqrt[1 - Tan[x]^2] -
 3*ArcTan[Sqrt[-1 + Tan[x]^2]]*Cos[x]*Sin[x]*Sqrt[-1 + Tan[x]^2] + (-3*Cot[
x] - 4*Cos[x]*Sin[x] + (5 + 9*Cos[2*x])*Tan[x]^3)/3)*Tan[2*x]^2)/(2*(-3 + 6
*Cos[2*x] + Cos[4*x])*(Tan[x]*Tan[2*x])^(3/2))
```

Maple [B] time = 0.482, size = 559, normalized size = 5.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x)
```

```
[Out] 1/96*2^(1/2)*4^(1/2)*(cos(x)-1)^2*(48*cos(x)^4*arctanh(1/2*2^(1/2)*cos(x)*4
^(1/2)*(cos(x)-1)/sin(x)^2/((2*cos(x)^2-1)/(cos(x)+1)^2)^(1/2))*2^(1/2)-201
*cos(x)^4*ln(-2*(cos(x)^2*((2*cos(x)^2-1)/(cos(x)+1)^2)^(1/2)-2*cos(x)^2+co
s(x)-((2*cos(x)^2-1)/(cos(x)+1)^2)^(1/2)+1)/sin(x)^2)+168*cos(x)^4*ln(-4*(c
os(x)^2*((2*cos(x)^2-1)/(cos(x)+1)^2)^(1/2)-2*cos(x)^2+cos(x)-((2*cos(x)^2-
1)/(cos(x)+1)^2)^(1/2)+1)/sin(x)^2)-33*cos(x)^4*arctanh(1/2*4^(1/2)*(2*cos(
x)^2-3*cos(x)+1)/sin(x)^2/((2*cos(x)^2-1)/(cos(x)+1)^2)^(1/2))-22*cos(x)^4*
((2*cos(x)^2-1)/(cos(x)+1)^2)^(1/2)-48*cos(x)^3*arctanh(1/2*2^(1/2)*cos(x)*
4^(1/2)*(cos(x)-1)/sin(x)^2/((2*cos(x)^2-1)/(cos(x)+1)^2)^(1/2))*2^(1/2)+20
1*cos(x)^3*ln(-2*(cos(x)^2*((2*cos(x)^2-1)/(cos(x)+1)^2)^(1/2)-2*cos(x)^2+c
os(x)-((2*cos(x)^2-1)/(cos(x)+1)^2)^(1/2)+1)/sin(x)^2)-168*cos(x)^3*ln(-4*(
cos(x)^2*((2*cos(x)^2-1)/(cos(x)+1)^2)^(1/2)-2*cos(x)^2+cos(x)-((2*cos(x)^2
-1)/(cos(x)+1)^2)^(1/2)+1)/sin(x)^2)+33*cos(x)^3*arctanh(1/2*4^(1/2)*(2*cos
(x)^2-3*cos(x)+1)/sin(x)^2/((2*cos(x)^2-1)/(cos(x)+1)^2)^(1/2))+36*cos(x)^2
*((2*cos(x)^2-1)/(cos(x)+1)^2)^(1/2)-8*((2*cos(x)^2-1)/(cos(x)+1)^2)^(1/2))
/sin(x)^3/cos(x)^3/(sin(x)^2/(2*cos(x)^2-1))^(3/2)/((2*cos(x)^2-1)/(cos(x)+
1)^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2 \tan (x)^2 - \cos (2 x)}{(\tan (2 x) \tan (x))^{\frac{3}{2}} \cos (x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x, algori
thm="maxima")
```

```
[Out] integrate((2*tan(x)^2 - cos(2*x))/((tan(2*x)*tan(x))^(3/2)*cos(x)^2), x)
```

Fricas [B] time = 3.30021, size = 784, normalized size = 7.84

$$-\frac{24 \left(\cos (x)^5 - \cos (x)^3\right) \log \left(-\frac{4 \sqrt{2} \left(8 \cos(x)^5 - 6 \cos(x)^3 + \cos(x)\right) \sqrt{-\frac{\cos(x)^2-1}{2 \cos(x)^2-1}} - \left(32 \cos(x)^4 - 16 \cos(x)^2 + 1\right) \sin(x)}{\sin(x)}\right) \sin (x) - 33 \left(\sqrt{2} \cos (x \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x, algori
thm="fricas")
```

```
[Out] -1/48*(24*(cos(x)^5 - cos(x)^3)*log(-(4*sqrt(2)*(8*cos(x)^5 - 6*cos(x)^3 +
cos(x))*sqrt(-(cos(x)^2 - 1)/(2*cos(x)^2 - 1)) - (32*cos(x)^4 - 16*cos(x)^2
 + 1)*sin(x))/sin(x))*sin(x) - 33*(sqrt(2)*cos(x)^5 - sqrt(2)*cos(x)^3)*log
(4*(sqrt(2)*(2*(3*sqrt(2) - 4)*cos(x)^3 - (3*sqrt(2) - 4)*cos(x))*sqrt(-(co
s(x)^2 - 1)/(2*cos(x)^2 - 1)) + (3*(2*sqrt(2) - 3)*cos(x)^2 - 2*sqrt(2) + 3
)*sin(x))/((cos(x)^2 - 1)*sin(x)))*sin(x) - 2*sqrt(2)*(22*cos(x)^6 - 47*cos
(x)^4 + 26*cos(x)^2 - 4)*sqrt(-(cos(x)^2 - 1)/(2*cos(x)^2 - 1)) - 44*(cos(x
)^5 - cos(x)^3)*sin(x))/((cos(x)^5 - cos(x)^3)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(2*x)+2*tan(x)**2)/cos(x)**2/(tan(x)*tan(2*x))**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34416, size = 265, normalized size = 2.65

$$\frac{11\sqrt{2}\log\left(\sqrt{-\tan(x)^2+1}+1\right)}{16\,\mathrm{sgn}\left(\tan(x)^2-1\right)\mathrm{sgn}(\tan(x))}-\frac{11\sqrt{2}\log\left(-\sqrt{-\tan(x)^2+1}+1\right)}{16\,\mathrm{sgn}\left(\tan(x)^2-1\right)\mathrm{sgn}(\tan(x))}-\frac{2\sqrt{2}\left(-\tan(x)^2+1\right)^{\frac{3}{2}}+3\sqrt{2}\sqrt{-\tan(x)^2}}{12\,\mathrm{sgn}\left(\tan(x)^2-1\right)\mathrm{sgn}(\tan(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(2*x)+2*tan(x)^2)/cos(x)^2/(tan(x)*tan(2*x))^(3/2),x, algori
thm="giac")
```

```
[Out] 11/16*sqrt(2)*log(sqrt(-tan(x)^2 + 1) + 1)/(sgn(tan(x)^2 - 1)*sgn(tan(x)))
- 11/16*sqrt(2)*log(-sqrt(-tan(x)^2 + 1) + 1)/(sgn(tan(x)^2 - 1)*sgn(tan(x)
)) - 1/12*(2*sqrt(2)*(-tan(x)^2 + 1)^(3/2) + 3*sqrt(2)*sqrt(-tan(x)^2 + 1))
/(sgn(tan(x)^2 - 1)*sgn(tan(x))) + log((sqrt(2) - sqrt(-tan(x)^2 + 1))/(sqr
t(2) + sqrt(-tan(x)^2 + 1)))/(sgn(tan(x)^2 - 1)*sgn(tan(x))) - 1/8*sqrt(2)*
sqrt(-tan(x)^2 + 1)/(sgn(tan(x)^2 - 1)*sgn(tan(x))*tan(x)^2)
```

3.448 $\int \frac{\tan(x)}{\left(a^3-b^3\cos^n(x)\right)^{4/3}}\,dx$

Optimal. Leaf size=112

$$-\frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}-\frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^4n}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3\cos^n(x)}+a}{\sqrt{3}a}\right)}{a^4n}+\frac{\log(\cos(x))}{2a^4}$$

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*Cos[x]^n)^(1/3))/(Sqrt[3]*a)])/(a^4*n))
 - 3/(a^3*n*(a^3 - b^3*Cos[x]^n)^(1/3)) + Log[Cos[x]]/(2*a^4) - (3*Log[a -
(a^3 - b^3*Cos[x]^n)^(1/3)])/(2*a^4*n)
```

Rubi [A] time = 0.168556, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {3230, 266, 51, 55, 617, 204, 31}

$$-\frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}-\frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^4n}-\frac{\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{a^3-b^3\cos^n(x)}+a}{\sqrt{3}a}\right)}{a^4n}+\frac{\log(\cos(x))}{2a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(a^3 - b^3*Cos[x]^n)^(4/3),x]
```

```
[Out] -((Sqrt[3]*ArcTan[(a + 2*(a^3 - b^3*Cos[x]^n)^(1/3))/(Sqrt[3]*a)])/(a^4*n))
 - 3/(a^3*n*(a^3 - b^3*Cos[x]^n)^(1/3)) + Log[Cos[x]]/(2*a^4) - (3*Log[a -
(a^3 - b^3*Cos[x]^n)^(1/3)])/(2*a^4*n)
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
```

```
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(x)}{\left(a^3-b^3\cos^n(x)\right)^{4/3}}\,dx &= -\operatorname{Subst}\left(\int \frac{1}{x\left(a^3-b^3x^n\right)^{4/3}}\,dx, x, \cos(x)\right)\\
&= -\frac{\operatorname{Subst}\left(\int \frac{1}{x\left(a^3-b^3x\right)^{4/3}}\,dx, x, \cos^n(x)\right)}{n}\\
&= -\frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}-\frac{\operatorname{Subst}\left(\int \frac{1}{x\sqrt[3]{a^3-b^3x}}\,dx, x, \cos^n(x)\right)}{a^3n}\\
&= -\frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}+\frac{\log(\cos(x))}{2a^4}+\frac{3\operatorname{Subst}\left(\int \frac{1}{a-x}\,dx, x, \sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^4n}-\frac{3\operatorname{Sub}}{}\\
&= -\frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}+\frac{\log(\cos(x))}{2a^4}-\frac{3\log\left(a-\sqrt[3]{a^3-b^3\cos^n(x)}\right)}{2a^4n}+\frac{3\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\right.}{}\\
&= -\frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{a^3-b^3\cos^n(x)}}{a}}{\sqrt{3}}\right)}{a^4n}-\frac{3}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}+\frac{\log(\cos(x))}{2a^4}-\frac{3\log\left(a-\sqrt[3]{a^3-b^3}\right.}{2a^4n}
\end{aligned}$$

Mathematica [C] time = 0.0420222, size = 47, normalized size = 0.42

$$-\frac{3\,{}_2F_1\left(-\frac{1}{3},1;\frac{2}{3};1-\frac{b^3\cos^n(x)}{a^3}\right)}{a^3n\sqrt[3]{a^3-b^3\cos^n(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(a^3 - b^3*Cos[x]^n)^(4/3),x]
```

```
[Out] (-3*Hypergeometric2F1[-1/3, 1, 2/3, 1 - (b^3*Cos[x]^n)/a^3])/(a^3*n*(a^3 -
b^3*Cos[x]^n)^(1/3))
```

Maple [A] time = 0.027, size = 137, normalized size = 1.2

$$\frac{1}{2\,na^4}\ln\left(\left(a^3-b^3\left(\cos\left(x\right)\right)^n\right)^{\frac{2}{3}}+a\sqrt[3]{a^3-b^3\left(\cos\left(x\right)\right)^n}+a^2\right)-\frac{\sqrt{3}}{na^4}\arctan\left(\frac{\sqrt{3}}{3\,a}\left(a+2\,\sqrt[3]{a^3-b^3\left(\cos\left(x\right)\right)^n}\right)\right)-\frac{1}{na^4}\ln\left(-a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x)
```

```
[Out] 1/2/n/a^4*ln((a^3-b^3*cos(x)^n)^(2/3)+a*(a^3-b^3*cos(x)^n)^(1/3)+a^2)-arcta
n(1/3*(a+2*(a^3-b^3*cos(x)^n)^(1/3))/a*3^(1/2))*3^(1/2)/a^4/n-1/n/a^4*ln(-a
+(a^3-b^3*cos(x)^n)^(1/3))-3/a^3/n/(a^3-b^3*cos(x)^n)^(1/3)
```

Maxima [A] time = 1.42737, size = 184, normalized size = 1.64

$$-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(a+2\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}\right)}{3\,a}\right)}{a^4n}+\frac{\log\left(a^2+\left(-b^3\cos\left(x\right)^n+a^3\right)^{\frac{1}{3}}a+\left(-b^3\cos\left(x\right)^n+a^3\right)^{\frac{2}{3}}\right)}{2\,a^4n}-\frac{\log\left(-a+\left(-b^3\cos\left(x\right.\right.\right.}{a^4n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x, algorithm="maxima")
```

```
[Out] -sqrt(3)*arctan(1/3*sqrt(3)*(a + 2*(-b^3*cos(x)^n + a^3)^(1/3))/a)/(a^4*n)
+ 1/2*log(a^2 + (-b^3*cos(x)^n + a^3)^(1/3)*a + (-b^3*cos(x)^n + a^3)^(2/3)
)/(a^4*n) - log(-a + (-b^3*cos(x)^n + a^3)^(1/3))/(a^4*n) - 3/((-b^3*cos(x)
^n + a^3)^(1/3)*a^3*n)
```

Fricas [A] time = 2.95077, size = 450, normalized size = 4.02

$$-\frac{2\left(\sqrt{3}b^3\cos\left(x\right)^n-\sqrt{3}a^3\right)\arctan\left(\frac{\sqrt{3}a+2\,\sqrt{3}\left(-b^3\cos(x)^n+a^3\right)^{\frac{1}{3}}}{3\,a}\right)-\left(b^3\cos\left(x\right)^n-a^3\right)\log\left(a^2+\left(-b^3\cos\left(x\right)^n+a^3\right)^{\frac{1}{3}}a+\left(-b^3\right.\right.}{2\left(a^4b^3n\cos\left(x\right)^n-a^7n\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x, algorithm="fricas")
```

```
[Out] -1/2*(2*(sqrt(3)*b^3*cos(x)^n - sqrt(3)*a^3)*arctan(1/3*(sqrt(3)*a + 2*sqrt
(3)*(-b^3*cos(x)^n + a^3)^(1/3))/a) - (b^3*cos(x)^n - a^3)*log(a^2 + (-b^3*
cos(x)^n + a^3)^(1/3)*a + (-b^3*cos(x)^n + a^3)^(2/3)) + 2*(b^3*cos(x)^n -
a^3)*log(-a + (-b^3*cos(x)^n + a^3)^(1/3)) - 6*(-b^3*cos(x)^n + a^3)^(2/3)*
a)/(a^4*b^3*n*cos(x)^n - a^7*n)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a**3-b**3*cos(x)**n)**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\left(-b^3 \cos(x)^n + a^3\right)^{\frac{4}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(a^3-b^3*cos(x)^n)^(4/3),x, algorithm="giac")
```

```
[Out] integrate(tan(x)/(-b^3*cos(x)^n + a^3)^(4/3), x)
```

3.449 $\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx$

Optimal. Leaf size=95

$$-\frac{2}{15}\left(2\cos^9(x)+1\right)^{5/6}+\frac{\tan^{-1}\left(\frac{1-\sqrt[3]{2\cos^9(x)+1}}{\sqrt{3}\sqrt[6]{2\cos^9(x)+1}}\right)}{3\sqrt{3}}+\frac{1}{3}\tanh^{-1}\left(\sqrt[6]{2\cos^9(x)+1}\right)-\frac{1}{9}\tanh^{-1}\left(\sqrt{2\cos^9(x)+1}\right)$$

```
[Out] ArcTan[(1 - (1 + 2*Cos[x]^9)^(1/3))/(Sqrt[3]*(1 + 2*Cos[x]^9)^(1/6))]/(3*Sq
rt[3]) + ArcTanh[(1 + 2*Cos[x]^9)^(1/6)]/3 - ArcTanh[Sqrt[1 + 2*Cos[x]^9]]/
9 - (2*(1 + 2*Cos[x]^9)^(5/6))/15
```

Rubi [A] time = 0.259556, antiderivative size = 162, normalized size of antiderivative = 1.71, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3230, 266, 50, 63, 296, 634, 618, 204, 628, 206}

$$-\frac{2}{15}\left(2\cos^9(x)+1\right)^{5/6}-\frac{1}{18}\log\left(\sqrt[3]{2\cos^9(x)+1}-\sqrt[6]{2\cos^9(x)+1}+1\right)+\frac{1}{18}\log\left(\sqrt[3]{2\cos^9(x)+1}+\sqrt[6]{2\cos^9(x)+1}+1\right)+$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*Cos[x]^9)^(5/6)*Tan[x],x]
```

```
[Out] ArcTan[(1 - 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]]/(3*Sqrt[3]) - ArcTan[(1 + 2*
(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]]/(3*Sqrt[3]) + (2*ArcTanh[(1 + 2*Cos[x]^9)^
(1/6)])/9 - (2*(1 + 2*Cos[x]^9)^(5/6))/15 - Log[1 - (1 + 2*Cos[x]^9)^(1/6)
+ (1 + 2*Cos[x]^9)^(1/3)]/18 + Log[1 + (1 + 2*Cos[x]^9)^(1/6) + (1 + 2*Cos[
x]^9)^(1/3)]/18
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 296

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[(2*k*m*Pi)/n] - s*Cos[(2*k*(m + 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*
x + s^2*x^2), x] + Int[(r*Cos[(2*k*m*Pi)/n] + s*Cos[(2*k*(m + 1)*Pi)/n]*x)/
(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^(m + 2)*Int[1/(r^2 - s^
2*x^2), x])/(a*n*s^m) + Dist[(2*r^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 2)
/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && Lt
Q[m, n - 1] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(1+2\cos^9(x)\right)^{5/6}\tan(x)\,dx = -\text{Subst}\left(\int \frac{\left(1+2x^9\right)^{5/6}}{x}\,dx, x, \cos(x)\right)$$
$$= -\left(\frac{1}{9}\text{Subst}\left(\int \frac{(1+2x)^{5/6}}{x}\,dx, x, \cos^9(x)\right)\right)$$
$$= -\frac{2}{15}\left(1+2\cos^9(x)\right)^{5/6} - \frac{1}{9}\text{Subst}\left(\int \frac{1}{x\sqrt[6]{1+2x}}\,dx, x, \cos^9(x)\right)$$
$$= -\frac{2}{15}\left(1+2\cos^9(x)\right)^{5/6} - \frac{1}{3}\text{Subst}\left(\int \frac{x^4}{-\frac{1}{2}+\frac{x^6}{2}}\,dx, x, \sqrt[6]{1+2\cos^9(x)}\right)$$
$$= -\frac{2}{15}\left(1+2\cos^9(x)\right)^{5/6} + \frac{2}{9}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[6]{1+2\cos^9(x)}\right) + \frac{2}{9}\text{Subst}\left(\int \frac{-\frac{1}{2}}{1-x}\right.$$
$$= \frac{2}{9}\tanh^{-1}\left(\sqrt[6]{1+2\cos^9(x)}\right) - \frac{2}{15}\left(1+2\cos^9(x)\right)^{5/6} - \frac{1}{18}\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[6]{1}\right.$$
$$= \frac{2}{9}\tanh^{-1}\left(\sqrt[6]{1+2\cos^9(x)}\right) - \frac{2}{15}\left(1+2\cos^9(x)\right)^{5/6} - \frac{1}{18}\log\left(1-\sqrt[6]{1+2\cos^9(x)}+\sqrt[3]{1+}\right.$$
$$= \frac{\tan^{-1}\left(\frac{1-2\sqrt[6]{1+2\cos^9(x)}}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2\sqrt[6]{1+2\cos^9(x)}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{2}{9}\tanh^{-1}\left(\sqrt[6]{1+2\cos^9(x)}\right) - \frac{2}{15}\left(1\right.$$

Mathematica [A] time = 0.130941, size = 154, normalized size = 1.62

$$\frac{1}{90}\left(-12\left(2\cos^9(x)+1\right)^{5/6} - 5\log\left(\sqrt[3]{2\cos^9(x)+1} - \sqrt[6]{2\cos^9(x)+1}+1\right) + 5\log\left(\sqrt[3]{2\cos^9(x)+1} + \sqrt[6]{2\cos^9(x)+1}+1\right) -\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*Cos[x]^9)^(5/6)*Tan[x],x]
```

```
[Out] (10*Sqrt[3]*ArcTan[(1 - 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]] - 10*Sqrt[3]*Arc
Tan[(1 + 2*(1 + 2*Cos[x]^9)^(1/6))/Sqrt[3]] + 20*ArcTanh[(1 + 2*Cos[x]^9)^(
1/6)] - 12*(1 + 2*Cos[x]^9)^(5/6) - 5*Log[1 - (1 + 2*Cos[x]^9)^(1/6) + (1 +
 2*Cos[x]^9)^(1/3)] + 5*Log[1 + (1 + 2*Cos[x]^9)^(1/6) + (1 + 2*Cos[x]^9)^(
1/3)])/90
```

Maple [F] time = 0.11, size = 0, normalized size = 0.

$$\int \left(1+2\left(\cos(x)\right)^9\right)^{\frac{5}{6}}\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*cos(x)^9)^(5/6)*tan(x),x)
```

```
[Out] int((1+2*cos(x)^9)^(5/6)*tan(x),x)
```

Maxima [B] time = 1.41716, size = 196, normalized size = 2.06

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}+1\right)\right) - \frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos(x)^9+1\right)^{\frac{1}{6}}-1\right)\right) - \frac{2}{15}\left(2\cos(x)^9+1\right)^{\frac{5}{6}} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*cos(x)^9)^(5/6)*tan(x),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) + 1)) - 1/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) - 1)) - 2/15*(2*cos(x)^9 +
1)^(5/6) + 1/18*log((2*cos(x)^9 + 1)^(1/3) + (2*cos(x)^9 + 1)^(1/6) + 1) -
1/18*log((2*cos(x)^9 + 1)^(1/3) - (2*cos(x)^9 + 1)^(1/6) + 1) + 1/9*log((2*
cos(x)^9 + 1)^(1/6) + 1) - 1/9*log((2*cos(x)^9 + 1)^(1/6) - 1)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*cos(x)^9)^(5/6)*tan(x),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*cos(x)**9)**(5/6)*tan(x),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14937, size = 197, normalized size = 2.07

$$-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos\left(x\right)^{9}+1\right)^{\frac{1}{6}}+1\right)\right)-\frac{1}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(2\cos\left(x\right)^{9}+1\right)^{\frac{1}{6}}-1\right)\right)-\frac{2}{15}\left(2\cos\left(x\right)^{9}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*cos(x)^9)^(5/6)*tan(x),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) + 1)) - 1/9*sqrt(
3)*arctan(1/3*sqrt(3)*(2*(2*cos(x)^9 + 1)^(1/6) - 1)) - 2/15*(2*cos(x)^9 +
1)^(5/6) + 1/18*log((2*cos(x)^9 + 1)^(1/3) + (2*cos(x)^9 + 1)^(1/6) + 1) -
1/18*log((2*cos(x)^9 + 1)^(1/3) - (2*cos(x)^9 + 1)^(1/6) + 1) + 1/9*log((2*
cos(x)^9 + 1)^(1/6) + 1) - 1/9*log(abs((2*cos(x)^9 + 1)^(1/6) - 1))
```

3.450 $\int \frac{\cos(x)\sin^8(x)}{\left(2-5\sin^3(x)\right)^{4/3}}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{625}\left(2-5\sin^3(x)\right)^{5/3}+\frac{2}{125}\left(2-5\sin^3(x)\right)^{2/3}+\frac{4}{125\sqrt[3]{2-5\sin^3(x)}}$$

```
[Out] 4/(125*(2 - 5*Sin[x]^3)^(1/3)) + (2*(2 - 5*Sin[x]^3)^(2/3))/125 - (2 - 5*Si
n[x]^3)^(5/3)/625
```

Rubi [A] time = 0.108198, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4334, 266, 43}

$$-\frac{1}{625}\left(2-5\sin^3(x)\right)^{5/3}+\frac{2}{125}\left(2-5\sin^3(x)\right)^{2/3}+\frac{4}{125\sqrt[3]{2-5\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x]^8)/(2 - 5*Sin[x]^3)^(4/3),x]
```

```
[Out] 4/(125*(2 - 5*Sin[x]^3)^(1/3)) + (2*(2 - 5*Sin[x]^3)^(2/3))/125 - (2 - 5*Si
n[x]^3)^(5/3)/625
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(x)\sin^8(x)}{\left(2-5\sin^3(x)\right)^{4/3}}\,dx = \text{Subst}\left(\int \frac{x^8}{\left(2-5x^3\right)^{4/3}}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \frac{x^2}{(2-5x)^{4/3}}\,dx, x, \sin^3(x)\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \left(\frac{4}{25(2-5x)^{4/3}} - \frac{4}{25\sqrt[3]{2-5x}} + \frac{1}{25}(2-5x)^{2/3}\right)dx, x, \sin^3(x)\right)$$
$$= \frac{4}{125\sqrt[3]{2-5\sin^3(x)}} + \frac{2}{125}\left(2-5\sin^3(x)\right)^{2/3} - \frac{1}{625}\left(2-5\sin^3(x)\right)^{5/3}$$

Mathematica [A] time = 0.461911, size = 30, normalized size = 0.61

$$\frac{-25\sin^6(x)-30\sin^3(x)+36}{625\sqrt[3]{2-5\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x]^8)/(2 - 5*Sin[x]^3)^(4/3),x]
```

```
[Out] (36 - 30*Sin[x]^3 - 25*Sin[x]^6)/(625*(2 - 5*Sin[x]^3)^(1/3))
```

Maple [F] time = 0.61, size = 0, normalized size = 0.

$$\int \cot(x)(\sin(x))^9\left(2-5(\sin(x))^3\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x)
```

```
[Out] int(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x)
```

Maxima [A] time = 0.929972, size = 50, normalized size = 1.02

$$-\frac{1}{625}\left(-5\sin(x)^3+2\right)^{\frac{5}{3}} + \frac{2}{125}\left(-5\sin(x)^3+2\right)^{\frac{2}{3}} + \frac{4}{125\left(-5\sin(x)^3+2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x, algorithm="maxima")
```

```
[Out] -1/625*(-5*sin(x)^3 + 2)^(5/3) + 2/125*(-5*sin(x)^3 + 2)^(2/3) + 4/125/(-5*
sin(x)^3 + 2)^(1/3)
```

Fricas [A] time = 3.06118, size = 158, normalized size = 3.22

$$\frac{25 \cos(x)^6 - 75 \cos(x)^4 + 75 \cos(x)^2 + 30\left(\cos(x)^2 - 1\right)\sin(x) + 11}{625\left(5\left(\cos(x)^2 - 1\right)\sin(x) + 2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x, algorithm="fricas")
```

```
[Out] 1/625*(25*cos(x)^6 - 75*cos(x)^4 + 75*cos(x)^2 + 30*(cos(x)^2 - 1)*sin(x) +
 11)/(5*(cos(x)^2 - 1)*sin(x) + 2)^(1/3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*sin(x)**9/(2-5*sin(x)**3)**(4/3),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.0856, size = 50, normalized size = 1.02

$$-\frac{1}{625}\left(-5 \sin(x)^3 + 2\right)^{\frac{5}{3}} + \frac{2}{125}\left(-5 \sin(x)^3 + 2\right)^{\frac{2}{3}} + \frac{4}{125\left(-5 \sin(x)^3 + 2\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*sin(x)^9/(2-5*sin(x)^3)^(4/3),x, algorithm="giac")
```

```
[Out] -1/625*(-5*sin(x)^3 + 2)^(5/3) + 2/125*(-5*sin(x)^3 + 2)^(2/3) + 4/125/(-5*
sin(x)^3 + 2)^(1/3)
```

3.451 $\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx$

Optimal. Leaf size=20

$$-\frac{3}{32}\left(\sqrt[3]{1-8\tan^2(x)}+1\right)^2$$

```
[Out] (-3*(1 + (1 - 8*Tan[x]^2)^(1/3))^2)/32
```

Rubi [A] time = 0.225151, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {4342, 6686}

$$-\frac{3}{32}\left(\sqrt[3]{1-8\tan^2(x)}+1\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x]^2*Tan[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3),x
]
```

```
[Out] (-3*(1 + (1 - 8*Tan[x]^2)^(1/3))^2)/32
```

Rule 4342

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^2, x_Symbol] :> With[{d = FreeFac
tors[Tan[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Tan[c*(a +
b*x)]/d, u, x], x], x, Tan[c*(a + b*x)]/d], x] /; FunctionOfQ[Tan[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && NonsumQ[u] && (EqQ[F, Sec] ||
 EqQ[F, sec])
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)\tan(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx &= \text{Subst}\left(\int \frac{x\left(1+\sqrt[3]{1-8x^2}\right)}{\left(1-8x^2\right)^{2/3}}\,dx, x, \tan(x)\right)\\ &= -\frac{3}{32}\left(1+\sqrt[3]{1-8\tan^2(x)}\right)^2\end{aligned}$$

Mathematica [B] time = 0.19986, size = 42, normalized size = 2.1

$$-\frac{3(9\cos(2x)-7)\left(\sqrt[3]{1-8\tan^2(x)}+2\right)\sec^2(x)}{64\left(1-8\tan^2(x)\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x]^2*Tan[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(
2/3),x]
```

```
[Out] (-3*(-7 + 9*Cos[2*x])*Sec[x]^2*(2 + (1 - 8*Tan[x]^2)^(1/3)))/(64*(1 - 8*Tan
[x]^2)^(2/3))
```

Maple [A] time = 0.031, size = 26, normalized size = 1.3

$$-\frac{3}{16}\sqrt[3]{1-8\left(\tan\left(x\right)\right)^{2}}-\frac{3}{32}\left(1-8\left(\tan\left(x\right)\right)^{2}\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x)
```

```
[Out] -3/16*(1-8*tan(x)^2)^(1/3)-3/32*(1-8*tan(x)^2)^(2/3)
```

Maxima [B] time = 1.17046, size = 116, normalized size = 5.8

$$-\frac{3\left(\frac{\left(9\sin(x)^{2}-1\right)(3\sin(x)-1)^{\frac{1}{3}}(\sin(x)+1)^{\frac{1}{3}}(\sin(x)-1)^{\frac{1}{3}}}{(3\sin(x)+1)^{\frac{1}{3}}}+\frac{2\left(9\sin(x)^{2}-1\right)(\sin(x)+1)^{\frac{2}{3}}(\sin(x)-1)^{\frac{2}{3}}}{(3\sin(x)+1)^{\frac{2}{3}}}\right)}{32\left(\sin\left(x\right)^{2}-1\right)\left(3\sin\left(x\right)-1\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="maxima")
```

```
[Out] -3/32*((9*sin(x)^2 - 1)*(3*sin(x) - 1)^(1/3)*(sin(x) + 1)^(1/3)*(sin(x) - 1
)^(1/3)/(3*sin(x) + 1)^(1/3) + 2*(9*sin(x)^2 - 1)*(sin(x) + 1)^(2/3)*(sin(x
) - 1)^(2/3)/(3*sin(x) + 1)^(2/3))/((sin(x)^2 - 1)*(3*sin(x) - 1)^(2/3))
```

Fricas [B] time = 2.29881, size = 111, normalized size = 5.55

$$-\frac{3}{32}\left(\frac{9\cos\left(x\right)^{2}-8}{\cos\left(x\right)^{2}}\right)^{\frac{2}{3}}-\frac{3}{16}\left(\frac{9\cos\left(x\right)^{2}-8}{\cos\left(x\right)^{2}}\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="fricas")
```

```
[Out] -3/32*((9*cos(x)^2 - 8)/cos(x)^2)^(2/3) - 3/16*((9*cos(x)^2 - 8)/cos(x)^2)^
(1/3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+(1-8*tan(x)**2)**(1/3))/cos(x)**2/(1-8*tan(x)**2)**(2/3
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12657, size = 34, normalized size = 1.7

$$-\frac{3}{32}\left(-8\,\tan\left(x\right)^{2}+1\right)^{\frac{2}{3}}-\frac{3}{16}\left(-8\,\tan\left(x\right)^{2}+1\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="giac")
```

```
[Out] -3/32*(-8*tan(x)^2 + 1)^(2/3) - 3/16*(-8*tan(x)^2 + 1)^(1/3)
```

3.452 $\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx$

Optimal. Leaf size=27

$$\frac{3}{2}\log\left(1-\sqrt[3]{1-8\tan^2(x)}\right)-\log(\tan(x))$$

```
[Out] -Log[Tan[x]] + (3*Log[1 - (1 - 8*Tan[x]^2)^(1/3)])/2
```

Rubi [A] time = 0.967354, antiderivative size = 35, normalized size of antiderivative = 1.3, number of steps used = 15, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {4366, 6725, 514, 444, 57, 618, 204, 31, 55}

$$\frac{3}{2}\log\left(1-\sqrt[3]{9-8\sec^2(x)}\right)-\frac{1}{2}\log\left(1-\sec^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]*Sec[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/3),x]

[Out] -Log[1 - Sec[x]^2]/2 + (3*Log[1 - (9 - 8*Sec[x]^2)^(1/3)])/2
```

Rule 4366

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_), x_Symbol] :> With[{d = Free
Factors[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[(1 - d^2*x
^2)^((n - 1)/2), Cos[c*(a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x]
/; FunctionOfQ[Cos[c*(a + b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 57

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2),
```

```
 x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/
3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x
])] /; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 55

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(1/3)), x_Symbol] :> With[
{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q), x
] + (Dist[3/(2*b), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)],
x] - Dist[3/(2*b*q), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])] /;
FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]
```

Rubi steps

$$\int \frac{\csc(x)\sec(x)\left(1+\sqrt[3]{1-8\tan^2(x)}\right)}{\left(1-8\tan^2(x)\right)^{2/3}}\,dx = -\text{Subst}\left(\int \frac{1+\sqrt[3]{9-\frac{8}{x^2}}}{\left(9-\frac{8}{x^2}\right)^{2/3}x\left(1-x^2\right)}\,dx,x,\cos(x)\right)$$

$$= -\text{Subst}\left(\int\left(-\frac{1}{\left(9-\frac{8}{x^2}\right)^{2/3}x\left(-1+x^2\right)}-\frac{1}{\sqrt[3]{9-\frac{8}{x^2}}x\left(-1+x^2\right)}\right)dx,x,\cos(x\right.$$

$$= \text{Subst}\left(\int \frac{1}{\left(9-\frac{8}{x^2}\right)^{2/3}x\left(-1+x^2\right)}\,dx,x,\cos(x)\right)+\text{Subst}\left(\int \frac{1}{\sqrt[3]{9-\frac{8}{x^2}}x\left(-\right.}\right.$$

$$= \text{Subst}\left(\int \frac{1}{\left(9-\frac{8}{x^2}\right)^{2/3}\left(1-\frac{1}{x^2}\right)x^3}\,dx,x,\cos(x)\right)+\text{Subst}\left(\int \frac{1}{\sqrt[3]{9-\frac{8}{x^2}}\left(1-\right.}\right.$$

$$= -\left(\frac{1}{2}\text{Subst}\left(\int \frac{1}{(9-8x)^{2/3}(1-x)}\,dx,x,\sec^2(x)\right)\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt[3]{9-8x}}\right.$$

$$= -\log(\tan(x))-2\left(\frac{3}{4}\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[3]{9-8\sec^2(x)}\right)\right)$$

$$= \frac{3}{2}\log\left(1-\sqrt[3]{9-8\sec^2(x)}\right)-\log(\tan(x))$$

Mathematica [B] time = 4.10294, size = 58, normalized size = 2.15

$$\frac{1}{4}\left(5\log\left(1-\sqrt[3]{1-8\tan^2(x)}\right)-\log\left(\left(1-8\tan^2(x)\right)^{2/3}+\sqrt[3]{1-8\tan^2(x)}+1\right)-2\log(\tan(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]*Sec[x]*(1 + (1 - 8*Tan[x]^2)^(1/3)))/(1 - 8*Tan[x]^2)^(2/
3),x]
```

```
[Out] (-2*Log[Tan[x]] + 5*Log[1 - (1 - 8*Tan[x]^2)^(1/3)] - Log[1 + (1 - 8*Tan[x]
^2)^(1/3) + (1 - 8*Tan[x]^2)^(2/3)])/4
```

Maple [F] time = 0.485, size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{(\cos(x))^2}\left(1+\sqrt[3]{1-8(\tan(x))^2}\right)\left(1-8(\tan(x))^2\right)^{-\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x)
```

```
[Out] int(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\left(-8\tan(x)^2+1\right)^{\frac{1}{3}}+1\right)\cot(x)}{\left(-8\tan(x)^2+1\right)^{\frac{2}{3}}\cos(x)^2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="maxima")
```

```
[Out] integrate(((-8*tan(x)^2 + 1)^(1/3) + 1)*cot(x)/((-8*tan(x)^2 + 1)^(2/3)*cos
(x)^2), x)
```

Fricas [B] time = 16.9191, size = 281, normalized size = 10.41

$$-\frac{1}{2}\log\left(\frac{16\left(145\cos(x)^4-200\cos(x)^2+3\left(11\cos(x)^4-8\cos(x)^2\right)\left(\frac{9\cos(x)^2-8}{\cos(x)^2}\right)^{\frac{2}{3}}+3\left(19\cos(x)^4-16\cos(x)^2\right)\left(\frac{9\cos}{\cos}\right.\right.}{\cos(x)^4-2\cos(x)^2+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="fricas")
```

```
[Out] -1/2*log(16*(145*cos(x)^4 - 200*cos(x)^2 + 3*(11*cos(x)^4 - 8*cos(x)^2)*((9
*cos(x)^2 - 8)/cos(x)^2)^(2/3) + 3*(19*cos(x)^4 - 16*cos(x)^2)*((9*cos(x)^2
 - 8)/cos(x)^2)^(1/3) + 64)/(cos(x)^4 - 2*cos(x)^2 + 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1+(1-8*tan(x)**2)**(1/3))/cos(x)**2/(1-8*tan(x)**2)**(2/3
),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13704, size = 55, normalized size = 2.04

$$-\frac{1}{2}\log\left(\left(-8\tan\left(x\right)^{2}+1\right)^{\frac{2}{3}}+\left(-8\tan\left(x\right)^{2}+1\right)^{\frac{1}{3}}+1\right)+\log\left(-\left(-8\tan\left(x\right)^{2}+1\right)^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(1+(1-8*tan(x)^2)^(1/3))/cos(x)^2/(1-8*tan(x)^2)^(2/3),x,
algorithm="giac")
```

```
[Out] -1/2*log((-8*tan(x)^2 + 1)^(2/3) + (-8*tan(x)^2 + 1)^(1/3) + 1) + log(-(-8*
tan(x)^2 + 1)^(1/3) + 1)
```

3.453 $\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx$

Optimal. Leaf size=101

$$2\sqrt[4]{5\sin^2(x)-1}-\frac{\sqrt[4]{5\sin^2(x)-1}}{2\left(\sqrt{5\sin^2(x)-1}+2\right)}-\frac{3\tan^{-1}\left(\frac{\sqrt[4]{5\sin^2(x)-1}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{5\sin^2(x)-1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] (-3*ArcTan[(-1 + 5*Sin[x]^2)^(1/4)/Sqrt[2]])/Sqrt[2] - ArcTanh[(-1 + 5*Sin[
x]^2)^(1/4)/Sqrt[2]]/(2*Sqrt[2]) + 2*(-1 + 5*Sin[x]^2)^(1/4) - (-1 + 5*Sin[
x]^2)^(1/4)/(2*(2 + Sqrt[-1 + 5*Sin[x]^2]))
```

Rubi [A] time = 1.40508, antiderivative size = 126, normalized size of antiderivative = 1.25, number of steps used = 14, number of rules used = 10, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4361, 6742, 6697, 341, 50, 63, 203, 470, 522, 207}

$$2\sqrt[4]{4-5\cos^2(x)}-\frac{\sqrt[4]{4-5\cos^2(x)}}{2\left(\sqrt{4-5\cos^2(x)}+2\right)}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)+\frac{\tan^{-1}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[((5*Cos[x]^2 - Sqrt[-1 + 5*Sin[x]^2])*Tan[x])/((-1 + 5*Sin[x]^2)^(1/4)*
(2 + Sqrt[-1 + 5*Sin[x]^2])),x]
```

```
[Out] ArcTan[(4 - 5*Cos[x]^2)^(1/4)/Sqrt[2]]/Sqrt[2] - 2*Sqrt[2]*ArcTan[(4 - 5*Co
s[x]^2)^(1/4)/Sqrt[2]] - ArcTanh[(4 - 5*Cos[x]^2)^(1/4)/Sqrt[2]]/(2*Sqrt[2]
) + 2*(4 - 5*Cos[x]^2)^(1/4) - (4 - 5*Cos[x]^2)^(1/4)/(2*(2 + Sqrt[4 - 5*Co
s[x]^2]))
```

Rule 4361

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 6697

```
Int[(u_.)*(v_)^(m_.)*((a_.) + (b_.)*(y_)^(n_))^(p_.), x_Symbol] :> Module[{
q, r}, Dist[q*r, Subst[Int[x^m*(a + b*x^n)^p, x], x, y], x] /;  !FalseQ[r =
 Divides[y^m, v^m, x]] &&  !FalseQ[q = DerivativeDivides[y, u, x]]] /; Free
Q[{a, b, m, n, p}, x]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(5\cos^2(x)-\sqrt{-1+5\sin^2(x)}\right)\tan(x)}{\sqrt[4]{-1+5\sin^2(x)}\left(2+\sqrt{-1+5\sin^2(x)}\right)}\,dx = -\operatorname{Subst}\left(\int \frac{5x^2-\sqrt{4-5x^2}}{\sqrt[4]{4-5x^2}\left(2x+x\sqrt{4-5x^2}\right)}\,dx,x,\cos(x)\right)$$

$$= -\operatorname{Subst}\left(\int\left(\frac{5x}{\sqrt[4]{4-5x^2}\left(2+\sqrt{4-5x^2}\right)}-\frac{\sqrt[4]{4-5x^2}}{x\left(2+\sqrt{4-5x^2}\right)}\right)dx,x,\cos(x\ldots$$

$$= -\left(5\operatorname{Subst}\left(\int \frac{x}{\sqrt[4]{4-5x^2}\left(2+\sqrt{4-5x^2}\right)}\,dx,x,\cos(x)\right)\right)+\operatorname{Subst}\left(\int \frac{\ldots}{x(\ldots}\right.$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt[4]{4-5x}}{\left(2+\sqrt{4-5x}\right)x}\,dx,x,\cos^2(x)\right)+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\left(2+\sqrt{x}\right)\sqrt[4]{\ldots}}\right.$$

$$= 2\operatorname{Subst}\left(\int \frac{x^4}{\left(-2+x^2\right)\left(2+x^2\right)^2}\,dx,x,\sqrt[4]{4-5\cos^2(x)}\right)+\operatorname{Subst}\left(\int \frac{\sqrt{\ldots}}{2+\ldots}\right.$$

$$= 2\sqrt[4]{4-5\cos^2(x)}-\frac{\sqrt[4]{4-5\cos^2(x)}}{2\left(2+\sqrt{4-5\cos^2(x)}\right)}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{-4+6x^2}{\left(-2+x^2\right)\left(2+\ldots\right.}\right.$$

$$= 2\sqrt[4]{4-5\cos^2(x)}-\frac{\sqrt[4]{4-5\cos^2(x)}}{2\left(2+\sqrt{4-5\cos^2(x)}\right)}+\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-2+x^2}\,dx,x,\ldots\right.$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)}{\sqrt{2}}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{4-5\cos^2(x)}}{\sqrt{2}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{4-5\cos^2(\ldots}}{\sqrt{2}}\right)}{2\sqrt{2}}\ldots$$

Mathematica [A] time = 0.410263, size = 89, normalized size = 0.88

$$\frac{1}{4}\left(-2\sqrt[4]{4-5\cos^2(x)}\left(\frac{1}{\sqrt{4-5\cos^2(x)}+2}-4\right)-6\sqrt{2}\tan^{-1}\left(\frac{\sqrt[4]{3-5\cos(2x)}}{2^{3/4}}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt[4]{3-5\cos(2x)}}{2^{3/4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((5*Cos[x]^2 - Sqrt[-1 + 5*Sin[x]^2])*Tan[x])/((-1 + 5*Sin[x]^2)^
(1/4)*(2 + Sqrt[-1 + 5*Sin[x]^2])),x]
```

```
[Out] (-6*Sqrt[2]*ArcTan[(3 - 5*Cos[2*x])^(1/4)/2^(3/4)] - Sqrt[2]*ArcTanh[(3 - 5
*Cos[2*x])^(1/4)/2^(3/4)] - 2*(4 - 5*Cos[x]^2)^(1/4)*(-4 + (2 + Sqrt[4 - 5*
Cos[x]^2])^(-1)))/4
```

Maple [F] time = 0.742, size = 0, normalized size = 0.

$$\int \tan(x)\left(5\,(\cos(x))^2-\sqrt{-1+5\,(\sin(x))^2}\right)\frac{1}{\sqrt[4]{-1+5\,(\sin(x))^2}}\left(2+\sqrt{-1+5\,(\sin(x))^2}\right)^{-1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(2+(-1+
5*sin(x)^2)^(1/2)),x)
```

```
[Out] int((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(2+(-1+
5*sin(x)^2)^(1/2)),x)
```

Maxima [A] time = 1.47469, size = 135, normalized size = 1.34

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(5\sin(x)^2-1\right)^{\frac{1}{4}}\right)+\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-\left(5\sin(x)^2-1\right)^{\frac{1}{4}}}{\sqrt{2}+\left(5\sin(x)^2-1\right)^{\frac{1}{4}}}\right)+2\left(5\sin(x)^2-1\right)^{\frac{1}{4}}-\frac{\left(5\sin(x)^2-\right.}{2\left(\sqrt{5\sin(x)^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(
2+(-1+5*sin(x)^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(5*sin(x)^2 - 1)^(1/4)) + 1/8*sqrt(2)*log(-
(sqrt(2) - (5*sin(x)^2 - 1)^(1/4))/(sqrt(2) + (5*sin(x)^2 - 1)^(1/4))) + 2*
(5*sin(x)^2 - 1)^(1/4) - 1/2*(5*sin(x)^2 - 1)^(1/4)/(sqrt(5*sin(x)^2 - 1) +
 2)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(
2+(-1+5*sin(x)^2)^(1/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*cos(x)**2-(-1+5*sin(x)**2)**(1/2))*tan(x)/(-1+5*sin(x)**2)**(1
/4)/(2+(-1+5*sin(x)**2)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*cos(x)^2-(-1+5*sin(x)^2)^(1/2))*tan(x)/(-1+5*sin(x)^2)^(1/4)/(
2+(-1+5*sin(x)^2)^(1/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.454 $\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx$

Optimal. Leaf size=25

$$-\frac{3}{64} \cos^{\frac{8}{3}}(2x) - \frac{3}{40} \cos^{\frac{5}{3}}(2x)$$

```
[Out] (-3*Cos[2*x]^(5/3))/40 - (3*Cos[2*x]^(8/3))/64
```

Rubi [A] time = 0.0606313, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4357, 266, 43}

$$-\frac{3}{64} \cos^{\frac{8}{3}}(2x) - \frac{3}{40} \cos^{\frac{5}{3}}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Cos[2*x]^(2/3)*Sin[x],x]
```

```
[Out] (-3*Cos[2*x]^(5/3))/40 - (3*Cos[2*x]^(8/3))/64
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \cos^3(x) \cos^{\frac{2}{3}}(2x) \sin(x) \, dx &= -\text{Subst}\left(\int x^3 \left(-1+2x^2\right)^{2/3} \, dx, x, \cos(x)\right)\\
&= -\left(\frac{1}{2} \text{Subst}\left(\int x(-1+2x)^{2/3} \, dx, x, \cos^2(x)\right)\right)\\
&= -\left(\frac{1}{2} \text{Subst}\left(\int \left(\frac{1}{2}(-1+2x)^{2/3}+\frac{1}{2}(-1+2x)^{5/3}\right) dx, x, \cos^2(x)\right)\right)\\
&= -\frac{3}{40} \left(-1+2\cos^2(x)\right)^{5/3}-\frac{3}{64} \left(-1+2\cos^2(x)\right)^{8/3}
\end{aligned}$$

Mathematica [C] time = 0.303941, size = 140, normalized size = 5.6

$$-\frac{3}{40}\cos^{\frac{5}{3}}(2x)-\frac{3e^{-6ix}\sqrt[3]{1+e^{4ix}}\left(2e^{4ix}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{3};\frac{2}{3};-e^{4ix}\right)+e^{8ix}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-e^{4ix}\right)+\left(1+e^{4ix}\right)^{2/3}\left(1+e^{8ix}\right)\right)}{256\;2^{2/3}\sqrt[3]{e^{-2ix}+e^{2ix}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Cos[2*x]^(2/3)*Sin[x],x]
```

```
[Out] (-3*Cos[2*x]^(5/3))/40 - (3*(1 + E^((4*I)*x))^(1/3)*((1 + E^((4*I)*x))^(2/3
)*(1 + E^((8*I)*x)) + 2*E^((4*I)*x)*Hypergeometric2F1[-1/3, 1/3, 2/3, -E^((
4*I)*x)] + E^((8*I)*x)*Hypergeometric2F1[1/3, 2/3, 5/3, -E^((4*I)*x)]))/(25
6*2^(2/3)*E^((6*I)*x)*(E^((-2*I)*x) + E^((2*I)*x))^(1/3))
```

Maple [F] time = 0.17, size = 0, normalized size = 0.

$$\int (\cos(x))^4 (\cos(2x))^{\frac{2}{3}} \tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*cos(2*x)^(2/3)*tan(x),x)
```

```
[Out] int(cos(x)^4*cos(2*x)^(2/3)*tan(x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \cos(2x)^{\frac{2}{3}} \cos(x)^4 \tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x, algorithm="maxima")
```

```
[Out] integrate(cos(2*x)^(2/3)*cos(x)^4*tan(x), x)
```

Fricas [A] time = 2.42886, size = 84, normalized size = 3.36

$$-\frac{3}{320}\left(20\cos(x)^4-4\cos(x)^2-3\right)\left(2\cos(x)^2-1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x, algorithm="fricas")
```

```
[Out] -3/320*(20*cos(x)^4 - 4*cos(x)^2 - 3)*(2*cos(x)^2 - 1)^(2/3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4*cos(2*x)**(2/3)*tan(x),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18457, size = 34, normalized size = 1.36

$$-\frac{3}{64}\left(2\cos\left(x\right)^{2}-1\right)^{\frac{8}{3}}-\frac{3}{40}\left(2\cos\left(x\right)^{2}-1\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*cos(2*x)^(2/3)*tan(x),x, algorithm="giac")
```

```
[Out] -3/64*(2*cos(x)^2 - 1)^(8/3) - 3/40*(2*cos(x)^2 - 1)^(5/3)
```

3.455 $\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx$

Optimal. Leaf size=102

$$\frac{1}{36}\cos^{\frac{9}{4}}(2x)-\frac{1}{5}\cos^{\frac{5}{4}}(2x)+\frac{7}{4}\sqrt[4]{\cos(2x)}+\frac{\tan^{-1}\left(\frac{1-\sqrt{\cos(2x)}}{\sqrt{2}\sqrt[4]{\cos(2x)}}\right)}{\sqrt{2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\cos(2x)}+1}{\sqrt{2}\sqrt[4]{\cos(2x)}}\right)}{\sqrt{2}}$$

```
[Out] ArcTan[(1 - Sqrt[Cos[2*x]])/(Sqrt[2]*Cos[2*x]^(1/4))]/Sqrt[2] - ArcTanh[(1
+ Sqrt[Cos[2*x]])/(Sqrt[2]*Cos[2*x]^(1/4))]/Sqrt[2] + (7*Cos[2*x]^(1/4))/4
- Cos[2*x]^(5/4)/5 + Cos[2*x]^(9/4)/36
```

Rubi [A] time = 0.192432, antiderivative size = 154, normalized size of antiderivative = 1.51, number of steps used = 14, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4361, 446, 88, 63, 211, 1165, 628, 1162, 617, 204}

$$\frac{1}{36}\cos^{\frac{9}{4}}(2x)-\frac{1}{5}\cos^{\frac{5}{4}}(2x)+\frac{7}{4}\sqrt[4]{\cos(2x)}+\frac{\log\left(\sqrt{\cos(2x)}-\sqrt{2}\sqrt[4]{\cos(2x)}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{\cos(2x)}+\sqrt{2}\sqrt[4]{\cos(2x)}+1\right)}{2\sqrt{2}}+$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x]^6*Tan[x])/Cos[2*x]^(3/4),x]
```

```
[Out] ArcTan[1 - Sqrt[2]*Cos[2*x]^(1/4)]/Sqrt[2] - ArcTan[1 + Sqrt[2]*Cos[2*x]^(1
/4)]/Sqrt[2] + (7*Cos[2*x]^(1/4))/4 - Cos[2*x]^(5/4)/5 + Cos[2*x]^(9/4)/36
+ Log[1 - Sqrt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2*x]]]/(2*Sqrt[2]) - Log[1 + Sq
rt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2*x]]]/(2*Sqrt[2])
```

Rule 4361

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[(b*c)^(-1), Subst[Int[SubstFor[1/x, Cos[c*(
a + b*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a
+ b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Tan] || EqQ[F, tan])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin^6(x)\tan(x)}{\cos^{\frac{3}{4}}(2x)}\,dx = -\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x\left(-1+2x^2\right)^{3/4}}\,dx, x, \cos(x)\right)$$
$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \frac{(1-x)^3}{x(-1+2x)^{3/4}}\,dx, x, \cos^2(x)\right)\right)$$
$$= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{7}{4(-1+2x)^{3/4}} + \frac{1}{x(-1+2x)^{3/4}} + \sqrt[4]{-1+2x} - \frac{1}{4}(-1+2x)^{5/4}\right)dx, x, \cos^2(x)\right)\right.$$
$$= \frac{7}{4}\sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5}\left(-1+2\cos^2(x)\right)^{5/4} + \frac{1}{36}\left(-1+2\cos^2(x)\right)^{9/4} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(-1+2x)\right.$$
$$= \frac{7}{4}\sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5}\left(-1+2\cos^2(x)\right)^{5/4} + \frac{1}{36}\left(-1+2\cos^2(x)\right)^{9/4} - \operatorname{Subst}\left(\int \frac{1}{\frac{1}{2}+\frac{x^4}{2}}\,dx, x,\right.$$
$$= \frac{7}{4}\sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5}\left(-1+2\cos^2(x)\right)^{5/4} + \frac{1}{36}\left(-1+2\cos^2(x)\right)^{9/4} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1-x^2}{\frac{1}{2}+\frac{x^4}{2}}\,dx,\right.$$
$$= \frac{7}{4}\sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5}\left(-1+2\cos^2(x)\right)^{5/4} + \frac{1}{36}\left(-1+2\cos^2(x)\right)^{9/4} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-\sqrt{2}x+\right.$$
$$= \frac{7}{4}\sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5}\left(-1+2\cos^2(x)\right)^{5/4} + \frac{1}{36}\left(-1+2\cos^2(x)\right)^{9/4} + \frac{\log\left(1-\sqrt{2}\sqrt[4]{-1+2\cos\right.}{2}$$
$$= \frac{\tan^{-1}\left(1-\sqrt{2}\sqrt[4]{-1+2\cos^2(x)}\right)}{\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}\sqrt[4]{-1+2\cos^2(x)}\right)}{\sqrt{2}} + \frac{7}{4}\sqrt[4]{-1+2\cos^2(x)} - \frac{1}{5}\left(\right.$$

Mathematica [A] time = 0.116848, size = 153, normalized size = 1.5

$$\frac{1}{360}\left(-72\cos^{\frac{5}{4}}(2x) + 5\cos(4x)\sqrt[4]{\cos(2x)} + 635\sqrt[4]{\cos(2x)} + 90\sqrt{2}\log\left(\sqrt{\cos(2x)} - \sqrt{2}\sqrt[4]{\cos(2x)} + 1\right) - 90\sqrt{2}\log\left(\sqrt{\cos}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x]^6*Tan[x])/Cos[2*x]^(3/4),x]
```

```
[Out] (180*Sqrt[2]*ArcTan[1 - Sqrt[2]*Cos[2*x]^(1/4)] - 180*Sqrt[2]*ArcTan[1 + Sq
rt[2]*Cos[2*x]^(1/4)] + 635*Cos[2*x]^(1/4) - 72*Cos[2*x]^(5/4) + 5*Cos[2*x]
^(1/4)*Cos[4*x] + 90*Sqrt[2]*Log[1 - Sqrt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2*x]
]] - 90*Sqrt[2]*Log[1 + Sqrt[2]*Cos[2*x]^(1/4) + Sqrt[Cos[2*x]]])/360
```

Maple [F] time = 0.295, size = 0, normalized size = 0.

$$\int (\sin(x))^6 \tan(x) (\cos(2x))^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6*tan(x)/cos(2*x)^(3/4),x)
```

```
[Out] int(sin(x)^6*tan(x)/cos(2*x)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(x)^6 \tan(x)}{\cos(2x)^{\frac{3}{4}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**6*tan(x)/cos(2*x)**(3/4),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6*tan(x)/cos(2*x)^(3/4),x, algorithm="giac")
```

```
[Out] Timed out
```

3.456 $\int \sqrt{\tan(x)\tan(2x)}\,dx$

Optimal. Leaf size=17

$$-\tanh^{-1}\left(\frac{\tan(x)}{\sqrt{\tan(x)\tan(2x)}}\right)$$

```
[Out] -ArcTanh[Tan[x]/Sqrt[Tan[x]*Tan[2*x]]]
```

Rubi [A] time = 0.0190383, antiderivative size = 18, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4397, 3774, 207}

$$-\tanh^{-1}\left(\frac{\tan(2x)}{\sqrt{\sec(2x)-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Tan[x]*Tan[2*x]],x]

[Out] -ArcTanh[Tan[2*x]/Sqrt[-1 + Sec[2*x]]]
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3774

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\tan(x)\tan(2x)}\,dx &= \int \sqrt{-1+\sec(2x)}\,dx\\ &= -\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -\frac{\tan(2x)}{\sqrt{-1+\sec(2x)}}\right)\\ &= -\tanh^{-1}\left(\frac{\tan(2x)}{\sqrt{-1+\sec(2x)}}\right)\end{aligned}$$

Mathematica [B] time = 0.0692801, size = 45, normalized size = 2.65

$$-\frac{\sqrt{\cos(2x)}\sqrt{\tan(x)\tan(2x)}\csc(x)\tanh^{-1}\left(\frac{\sqrt{2}\cos(x)}{\sqrt{\cos(2x)}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Tan[x]*Tan[2*x]],x]
```

```
[Out] -((ArcTanh[(Sqrt[2]*Cos[x])/Sqrt[Cos[2*x]]]*Sqrt[Cos[2*x]]*Csc[x]*Sqrt[Tan[
x]*Tan[2*x]])/Sqrt[2])
```

Maple [B] time = 0.144, size = 88, normalized size = 5.2

$$-\frac{\sqrt{4}\sin(x)}{2\cos(x)-2}\sqrt{\frac{-(\cos(x))^2+1}{2(\cos(x))^2-1}}\sqrt{\frac{2(\cos(x))^2-1}{(\cos(x)+1)^2}}\mathrm{Artanh}\left(\frac{\cos(x)\sqrt{2}\sqrt{4}(\cos(x)-1)}{2(\sin(x))^2}\frac{1}{\sqrt{\frac{2(\cos(x))^2-1}{(\cos(x)+1)^2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(x)*tan(2*x))^(1/2),x)
```

```
[Out] -1/2*4^(1/2)*((-cos(x)^2+1)/(2*cos(x)^2-1))^(1/2)*sin(x)*((2*cos(x)^2-1)/(c
os(x)+1)^2)^(1/2)*arctanh(1/2*2^(1/2)*cos(x)*4^(1/2)*(cos(x)-1)/sin(x)^2/((
2*cos(x)^2-1)/(cos(x)+1)^2)^(1/2))/(cos(x)-1)
```

Maxima [B] time = 1.48664, size = 350, normalized size = 20.59

$$\frac{1}{4}\log\left(4\sqrt{\cos(4x)^2+\sin(4x)^2+2\cos(4x)+1}\cos\left(\frac{1}{2}\arctan(\sin(4x),\cos(4x)+1)\right)^2+4\sqrt{\cos(4x)^2+\sin(4x)^2-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(x)*tan(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*log(4*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)*cos(1/2*arctan2(si
n(4*x), cos(4*x) + 1))^2 + 4*sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)
*sin(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + 8*(cos(4*x)^2 + sin(4*x)^2 +
2*cos(4*x) + 1)^(1/4)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + 4) - 1/4*l
og(cos(2*x)^2 + sin(2*x)^2 + sqrt(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)
*(cos(1/2*arctan2(sin(4*x), cos(4*x) + 1))^2 + sin(1/2*arctan2(sin(4*x), co
s(4*x) + 1))^2) + 2*(cos(4*x)^2 + sin(4*x)^2 + 2*cos(4*x) + 1)^(1/4)*(cos(2
*x)*cos(1/2*arctan2(sin(4*x), cos(4*x) + 1)) + sin(2*x)*sin(1/2*arctan2(sin
(4*x), cos(4*x) + 1))))
```

Fricas [B] time = 2.87459, size = 150, normalized size = 8.82

$$\frac{1}{2}\log\left(-\frac{\tan(x)^3-2\sqrt{2}\left(\tan(x)^2-1\right)\sqrt{-\frac{\tan(x)^2}{\tan(x)^2-1}}-3\tan(x)}{\tan(x)^3+\tan(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(x)*tan(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(tan(x)^3 - 2*sqrt(2)*(tan(x)^2 - 1)*sqrt(-tan(x)^2/(tan(x)^2 - 1)
) - 3*tan(x))/(tan(x)^3 + tan(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\tan(x)\tan(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(x)*tan(2*x))**(1/2),x)
```

```
[Out] Integral(sqrt(tan(x)*tan(2*x)), x)
```

Giac [B] time = 1.21842, size = 115, normalized size = 6.76

$$\frac{1}{4}\sqrt{2}\left(\left(\sqrt{2}\log\left(\sqrt{2}+\sqrt{-\tan(x)^2+1}\right)-\sqrt{2}\log\left(\sqrt{2}-\sqrt{-\tan(x)^2+1}\right)\right)\operatorname{sgn}\left(\tan(x)^2-1\right)\operatorname{sgn}(\tan(x))+\left(\sqrt{2}\log\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((tan(x)*tan(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*((sqrt(2)*log(sqrt(2) + sqrt(-tan(x)^2 + 1)) - sqrt(2)*log(sqrt
(2) - sqrt(-tan(x)^2 + 1)))*sgn(tan(x)^2 - 1)*sgn(tan(x)) + (sqrt(2)*log(sq
rt(2) + 1) - sqrt(2)*log(sqrt(2) - 1))*sgn(tan(x)))
```

3.457 $\int \sqrt{\cot(2x)\tan(x)}\,dx$

Optimal. Leaf size=32

$$\tan^{-1}\left(\frac{\sqrt{2}\tan(x)}{\sqrt{1-\tan^2(x)}}\right)-\frac{\sin^{-1}(\tan(x))}{\sqrt{2}}$$

```
[Out] -(ArcSin[Tan[x]]/Sqrt[2]) + ArcTan[(Sqrt[2]*Tan[x])/Sqrt[1 - Tan[x]^2]]
```

Rubi [A] time = 0.0385622, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {12, 402, 216, 377, 203}

$$\tan^{-1}\left(\frac{\sqrt{2}\tan(x)}{\sqrt{1-\tan^2(x)}}\right)-\frac{\sin^{-1}(\tan(x))}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cot[2*x]*Tan[x]],x]
```

```
[Out] -(ArcSin[Tan[x]]/Sqrt[2]) + ArcTan[(Sqrt[2]*Tan[x])/Sqrt[1 - Tan[x]^2]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{\cot(2x)\tan(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{2}\left(1+x^2\right)}\,dx, x, \tan(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{1+x^2}\,dx, x, \tan(x)\right)}{\sqrt{2}}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, \tan(x)\right)}{\sqrt{2}} + \sqrt{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\left(1+x^2\right)}\,dx, x, \tan(x)\right)$$
$$= -\frac{\sin^{-1}(\tan(x))}{\sqrt{2}} + \sqrt{2}\,\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{\tan(x)}{\sqrt{1-\tan^2(x)}}\right)$$
$$= -\frac{\sin^{-1}(\tan(x))}{\sqrt{2}} + \tan^{-1}\left(\frac{\sqrt{2}\tan(x)}{\sqrt{1-\tan^2(x)}}\right)$$

Mathematica [A] time = 0.067457, size = 52, normalized size = 1.62

$$\frac{\cos(x)\sqrt{\tan(x)\cot(2x)}\left(\sqrt{2}\sin^{-1}\left(\sqrt{2}\sin(x)\right) - \tan^{-1}\left(\frac{\sin(x)}{\sqrt{\cos(2x)}}\right)\right)}{\sqrt{\cos(2x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cot[2*x]*Tan[x]],x]
```

```
[Out] ((Sqrt[2]*ArcSin[Sqrt[2]*Sin[x]] - ArcTan[Sin[x]/Sqrt[Cos[2*x]]])*Cos[x]*Sq
rt[Cot[2*x]*Tan[x]])/Sqrt[Cos[2*x]]
```

Maple [C] time = 0.327, size = 242, normalized size = 7.6

$$\frac{\sqrt{2}\left(2+\sqrt{2}\right)\cos(x)\,(\sin(x))^2}{2\sqrt{3+2\sqrt{2}}\left(1+\sqrt{2}\right)(\cos(x)-1)\left(2\,(\cos(x))^2-1\right)}\left(4\,\text{EllipticPi}\left(\frac{\sqrt{3+2\sqrt{2}}\,(\cos(x)-1)}{\sin(x)}, -\left(3+2\sqrt{2}\right)^{-1}, \frac{\sqrt{3-2}}{\sqrt{3+2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(2*x)/cot(x))^(1/2),x)
```

```
[Out] 1/2*2^(1/2)/(3+2*2^(1/2))^(1/2)/(1+2^(1/2))*(4*EllipticPi((3+2*2^(1/2))^(1/
2)*(cos(x)-1)/sin(x),-1/(3+2*2^(1/2)),(3-2*2^(1/2))^(1/2)/(3+2*2^(1/2))^(1/
2))-2*EllipticPi((3+2*2^(1/2))^(1/2)*(cos(x)-1)/sin(x),1/(3+2*2^(1/2)),(3-2
*2^(1/2))^(1/2)/(3+2*2^(1/2))^(1/2))-EllipticF((cos(x)-1)*(1+2^(1/2))/sin(x
),3-2*2^(1/2)))*(2+2^(1/2))*cos(x)*(-2*(cos(x)*2^(1/2)-2*cos(x)-2^(1/2)+1)/
(cos(x)+1))^(1/2)*((cos(x)*2^(1/2)+2*cos(x)-2^(1/2)-1)/(cos(x)+1))^(1/2)*si
n(x)^2*((2*cos(x)^2-1)/cos(x)^2)^(1/2)/(cos(x)-1)/(2*cos(x)^2-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(2*x)/cot(x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [B] time = 2.67048, size = 328, normalized size = 10.25

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(3\cos(2x)^2+2\cos(2x)-1\right)\sqrt{\frac{\cos(2x)}{\cos(2x)+1}}}{4\cos(2x)\sin(2x)}\right)-\frac{1}{2}\arctan\left(\frac{\sqrt{2}\left(2\sqrt{2}\cos(2x)^2+\sqrt{2}\cos(2x)-\sqrt{2}\right)\sqrt{\frac{\cos(2x)}{\cos(2x)+1}}}{4\cos(2x)\sin(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(2*x)/cot(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*(3*cos(2*x)^2 + 2*cos(2*x) - 1)*sqrt(cos(2*x
)/(cos(2*x) + 1))/(cos(2*x)*sin(2*x))) - 1/2*arctan(1/4*sqrt(2)*(2*sqrt(2)*
cos(2*x)^2 + sqrt(2)*cos(2*x) - sqrt(2))*sqrt(cos(2*x)/(cos(2*x) + 1))/(cos
(2*x)*sin(2*x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{\cot(2x)}{\cot(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(2*x)/cot(x))**(1/2),x)
```

```
[Out] Integral(sqrt(cot(2*x)/cot(x)), x)
```

Giac [C] time = 1.23145, size = 188, normalized size = 5.88

$$\frac{1}{2}\left(\pi-\sqrt{2}\arctan(-i)-\sqrt{2}\arctan\left(\sqrt{2}\right)-i\log\left(2\sqrt{2}+3\right)\right)\operatorname{sgn}(\sin(2x))-\frac{\sqrt{2}\left(\left(\left(\sqrt{2}\arcsin\left(4\cos(x)^2-3\right)+2\arcta\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cot(2*x)/cot(x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(pi - sqrt(2)*arctan(-I) - sqrt(2)*arctan(sqrt(2)) - I*log(2*sqrt(2) +
3))*sgn(sin(2*x)) - 1/4*sqrt(2)*((sqrt(2)*arcsin(4*cos(x)^2 - 3) + 2*arctan
(1/4*sqrt(2)*(3*(2*sqrt(2)*sqrt(-2*cos(x)^4 + 3*cos(x)^2 - 1) - 1)/(4*cos(x
)^2 - 3) - 1)))*sgn(cos(x)) + (-I*sqrt(2)*log(2*I*sqrt(2) + 3*I) - 2*arctan
(-I))*sgn(cos(x)))/(sgn(cos(x))*sgn(sin(2*x)))
```

3.458 $\int \frac{1}{x^5(5+x^2)} \, dx$

Optimal. Leaf size=31

$$\frac{1}{50x^2} - \frac{1}{20x^4} - \frac{1}{250}\log\left(x^2+5\right) + \frac{\log(x)}{125}$$

```
[Out] -1/(20*x^4) + 1/(50*x^2) + Log[x]/125 - Log[5 + x^2]/250
```

Rubi [A] time = 0.0148877, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {266, 44}

$$\frac{1}{50x^2} - \frac{1}{20x^4} - \frac{1}{250}\log\left(x^2+5\right) + \frac{\log(x)}{125}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(5 + x^2)),x]
```

```
[Out] -1/(20*x^4) + 1/(50*x^2) + Log[x]/125 - Log[5 + x^2]/250
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\left(5+x^2\right)} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(5+x)} \, dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{5x^3} - \frac{1}{25x^2} + \frac{1}{125x} - \frac{1}{125(5+x)}\right) dx, x, x^2\right)\\
&= -\frac{1}{20x^4} + \frac{1}{50x^2} + \frac{\log(x)}{125} - \frac{1}{250}\log\left(5+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0036833, size = 31, normalized size = 1.

$$\frac{1}{50x^2} - \frac{1}{20x^4} - \frac{1}{250}\log\left(x^2+5\right) + \frac{\log(x)}{125}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(5 + x^2)),x]
```

```
[Out] -1/(20*x^4) + 1/(50*x^2) + Log[x]/125 - Log[5 + x^2]/250
```

Maple [A] time = 0.006, size = 24, normalized size = 0.8

$$-\frac{1}{20\,x^4}+\frac{1}{50\,x^2}+\frac{\ln\left(x\right)}{125}-\frac{\ln\left(x^2+5\right)}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(x^2+5),x)
```

```
[Out] -1/20/x^4+1/50/x^2+1/125*ln(x)-1/250*ln(x^2+5)
```

Maxima [A] time = 0.931927, size = 36, normalized size = 1.16

$$\frac{2\,x^2-5}{100\,x^4}-\frac{1}{250}\,\log\left(x^2+5\right)+\frac{1}{250}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^2+5),x, algorithm="maxima")
```

```
[Out] 1/100*(2*x^2 - 5)/x^4 - 1/250*log(x^2 + 5) + 1/250*log(x^2)
```

Fricas [A] time = 2.29247, size = 84, normalized size = 2.71

$$-\frac{2\,x^4\log\left(x^2+5\right)-4\,x^4\log\left(x\right)-10\,x^2+25}{500\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^2+5),x, algorithm="fricas")
```

```
[Out] -1/500*(2*x^4*log(x^2 + 5) - 4*x^4*log(x) - 10*x^2 + 25)/x^4
```

Sympy [A] time = 0.112281, size = 24, normalized size = 0.77

$$\frac{\log\left(x\right)}{125}-\frac{\log\left(x^2+5\right)}{250}+\frac{2x^2-5}{100x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(x**2+5),x)
```

```
[Out] log(x)/125 - log(x**2 + 5)/250 + (2*x**2 - 5)/(100*x**4)
```

Giac [A] time = 1.14959, size = 43, normalized size = 1.39

$$-\frac{3x^4 - 10x^2 + 25}{500x^4} - \frac{1}{250}\log\left(x^2 + 5\right) + \frac{1}{250}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(x^2+5),x, algorithm="giac")
```

```
[Out] -1/500*(3*x^4 - 10*x^2 + 25)/x^4 - 1/250*log(x^2 + 5) + 1/250*log(x^2)
```

3.459 $\int \frac{1}{x^6(5+x^2)}\,dx$

Optimal. Leaf size=39

$$\frac{1}{75x^3}-\frac{1}{25x^5}-\frac{1}{125x}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

```
[Out] -1/(25*x^5) + 1/(75*x^3) - 1/(125*x) - ArcTan[x/Sqrt[5]]/(125*Sqrt[5])
```

Rubi [A] time = 0.0139152, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {325, 203}

$$\frac{1}{75x^3}-\frac{1}{25x^5}-\frac{1}{125x}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*(5 + x^2)),x]
```

```
[Out] -1/(25*x^5) + 1/(75*x^3) - 1/(125*x) - ArcTan[x/Sqrt[5]]/(125*Sqrt[5])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^6\left(5+x^2\right)}\,dx &= -\frac{1}{25x^5}-\frac{1}{5}\int \frac{1}{x^4\left(5+x^2\right)}\,dx\\
&= -\frac{1}{25x^5}+\frac{1}{75x^3}+\frac{1}{25}\int \frac{1}{x^2\left(5+x^2\right)}\,dx\\
&= -\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{1}{125}\int \frac{1}{5+x^2}\,dx\\
&= -\frac{1}{25x^5}+\frac{1}{75x^3}-\frac{1}{125x}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.0137162, size = 39, normalized size = 1.

$$\frac{1}{75x^3}-\frac{1}{25x^5}-\frac{1}{125x}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)}{125\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*(5 + x^2)),x]
```

```
[Out] -1/(25*x^5) + 1/(75*x^3) - 1/(125*x) - ArcTan[x/Sqrt[5]]/(125*Sqrt[5])
```

Maple [A] time = 0.006, size = 29, normalized size = 0.7

$$-\frac{1}{25\,x^5}+\frac{1}{75\,x^3}-\frac{1}{125\,x}-\frac{\sqrt{5}}{625}\arctan\left(\frac{x\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(x^2+5),x)
```

```
[Out] -1/25/x^5+1/75/x^3-1/125/x-1/625*arctan(1/5*x*5^(1/2))*5^(1/2)
```

Maxima [A] time = 1.44339, size = 41, normalized size = 1.05

$$-\frac{1}{625}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}x\right)-\frac{3\,x^4-5\,x^2+15}{375\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^2+5),x, algorithm="maxima")
```

```
[Out] -1/625*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/375*(3*x^4 - 5*x^2 + 15)/x^5
```

Fricas [A] time = 2.08892, size = 100, normalized size = 2.56

$$-\frac{3\sqrt{5}x^5\arctan\left(\frac{1}{5}\sqrt{5}x\right)+15\,x^4-25\,x^2+75}{1875\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^2+5),x, algorithm="fricas")
```

```
[Out] -1/1875*(3*sqrt(5)*x^5*arctan(1/5*sqrt(5)*x) + 15*x^4 - 25*x^2 + 75)/x^5
```

Sympy [A] time = 0.125101, size = 34, normalized size = 0.87

$$-\frac{\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}x}{5}\right)}{625}-\frac{3x^4-5x^2+15}{375x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(x**2+5),x)
```

```
[Out] -sqrt(5)*atan(sqrt(5)*x/5)/625 - (3*x**4 - 5*x**2 + 15)/(375*x**5)
```

Giac [A] time = 1.11461, size = 41, normalized size = 1.05

$$-\frac{1}{625}\,\sqrt{5}\arctan\left(\frac{1}{5}\,\sqrt{5}x\right)-\frac{3\,x^4-5\,x^2+15}{375\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(x^2+5),x, algorithm="giac")
```

```
[Out] -1/625*sqrt(5)*arctan(1/5*sqrt(5)*x) - 1/375*(3*x^4 - 5*x^2 + 15)/x^5
```

3.460 $\int \frac{1}{x\left(-4+x^2\right)^4} \, dx$

Optimal. Leaf size=58

$$\frac{1}{128\left(4-x^2\right)}+\frac{1}{64\left(4-x^2\right)^2}+\frac{1}{24\left(4-x^2\right)^3}-\frac{1}{512}\log\left(4-x^2\right)+\frac{\log(x)}{256}$$

```
[Out] 1/(24*(4 - x^2)^3) + 1/(64*(4 - x^2)^2) + 1/(128*(4 - x^2)) + Log[x]/256 -
Log[4 - x^2]/512
```

Rubi [A] time = 0.0319356, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {266, 44}

$$\frac{1}{128\left(4-x^2\right)}+\frac{1}{64\left(4-x^2\right)^2}+\frac{1}{24\left(4-x^2\right)^3}-\frac{1}{512}\log\left(4-x^2\right)+\frac{\log(x)}{256}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-4 + x^2)^4),x]
```

```
[Out] 1/(24*(4 - x^2)^3) + 1/(64*(4 - x^2)^2) + 1/(128*(4 - x^2)) + Log[x]/256 -
Log[4 - x^2]/512
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(-4+x^2\right)^4} \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(-4+x)^4 x} \, dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{4(-4+x)^4}-\frac{1}{16(-4+x)^3}+\frac{1}{64(-4+x)^2}-\frac{1}{256(-4+x)}+\frac{1}{256x}\right) dx, x, x^2\right)\\
&= \frac{1}{24\left(4-x^2\right)^3}+\frac{1}{64\left(4-x^2\right)^2}+\frac{1}{128\left(4-x^2\right)}+\frac{\log(x)}{256}-\frac{1}{512}\log\left(4-x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0159794, size = 40, normalized size = 0.69

$$\frac{-\frac{4\left(3x^4-30x^2+88\right)}{\left(x^2-4\right)^3}-3\log\left(4-x^2\right)+6\log(x)}{1536}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-4 + x^2)^4),x]
```

```
[Out] ((-4*(88 - 30*x^2 + 3*x^4))/(-4 + x^2)^3 + 6*Log[x] - 3*Log[4 - x^2])/1536
```

Maple [A] time = 0.012, size = 60, normalized size = 1.

$$\frac{1}{1536\left(2+x\right)^{3}}+\frac{3}{2048\left(2+x\right)^{2}}+\frac{11}{8192+4096x}-\frac{\ln\left(2+x\right)}{512}+\frac{\ln\left(x\right)}{256}-\frac{1}{1536\left(-2+x\right)^{3}}+\frac{3}{2048\left(-2+x\right)^{2}}-\frac{11}{-8192+40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2-4)^4,x)
```

```
[Out] 1/1536/(2+x)^3+3/2048/(2+x)^2+11/4096/(2+x)-1/512*ln(2+x)+1/256*ln(x)-1/153
6/(-2+x)^3+3/2048/(-2+x)^2-11/4096/(-2+x)-1/512*ln(-2+x)
```

Maxima [A] time = 0.92482, size = 62, normalized size = 1.07

$$-\frac{3\,x^{4}-30\,x^{2}+88}{384\,\left(x^{6}-12\,x^{4}+48\,x^{2}-64\right)}-\frac{1}{512}\,\log\left(x^{2}-4\right)+\frac{1}{512}\,\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-4)^4,x, algorithm="maxima")
```

```
[Out] -1/384*(3*x^4 - 30*x^2 + 88)/(x^6 - 12*x^4 + 48*x^2 - 64) - 1/512*log(x^2 -
 4) + 1/512*log(x^2)
```

Fricas [A] time = 2.00235, size = 201, normalized size = 3.47

$$-\frac{12\,x^{4}-120\,x^{2}+3\,\left(x^{6}-12\,x^{4}+48\,x^{2}-64\right)\log\left(x^{2}-4\right)-6\,\left(x^{6}-12\,x^{4}+48\,x^{2}-64\right)\log\left(x\right)+352}{1536\,\left(x^{6}-12\,x^{4}+48\,x^{2}-64\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-4)^4,x, algorithm="fricas")
```

```
[Out] -1/1536*(12*x^4 - 120*x^2 + 3*(x^6 - 12*x^4 + 48*x^2 - 64)*log(x^2 - 4) - 6
*(x^6 - 12*x^4 + 48*x^2 - 64)*log(x) + 352)/(x^6 - 12*x^4 + 48*x^2 - 64)
```

Sympy [A] time = 0.145878, size = 41, normalized size = 0.71

$$-\frac{3x^{4}-30x^{2}+88}{384x^{6}-4608x^{4}+18432x^{2}-24576}+\frac{\log\left(x\right)}{256}-\frac{\log\left(x^{2}-4\right)}{512}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2-4)**4,x)
```

```
[Out] -(3*x**4 - 30*x**2 + 88)/(384*x**6 - 4608*x**4 + 18432*x**2 - 24576) + log(
x)/256 - log(x**2 - 4)/512
```

Giac [A] time = 1.13846, size = 57, normalized size = 0.98

$$\frac{11\,x^6 - 156\,x^4 + 768\,x^2 - 1408}{3072\,\left(x^2 - 4\right)^3} + \frac{1}{512}\,\log\left(x^2\right) - \frac{1}{512}\,\log\left(\left|x^2 - 4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-4)^4,x, algorithm="giac")
```

```
[Out] 1/3072*(11*x^6 - 156*x^4 + 768*x^2 - 1408)/(x^2 - 4)^3 + 1/512*log(x^2) - 1
/512*log(abs(x^2 - 4))
```

3.461 $\int \frac{1}{x\left(-2+x^2\right)^{5/2}} dx$

Optimal. Leaf size=52

$$\frac{1}{4\sqrt{x^2-2}}-\frac{1}{6\left(x^2-2\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{x^2-2}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -1/(6*(-2 + x^2)^(3/2)) + 1/(4*Sqrt[-2 + x^2]) + ArcTan[Sqrt[-2 + x^2]/Sqrt
[2]]/(4*Sqrt[2])
```

Rubi [A] time = 0.023171, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 51, 63, 203}

$$\frac{1}{4\sqrt{x^2-2}}-\frac{1}{6\left(x^2-2\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{x^2-2}}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(-2 + x^2)^(5/2)),x]
```

```
[Out] -1/(6*(-2 + x^2)^(3/2)) + 1/(4*Sqrt[-2 + x^2]) + ArcTan[Sqrt[-2 + x^2]/Sqrt
[2]]/(4*Sqrt[2])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(-2+x^2\right)^{5/2}}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{(-2+x)^{5/2}x}\,dx, x, x^2\right)\\
&= -\frac{1}{6\left(-2+x^2\right)^{3/2}} - \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{(-2+x)^{3/2}x}\,dx, x, x^2\right)\\
&= -\frac{1}{6\left(-2+x^2\right)^{3/2}} + \frac{1}{4\sqrt{-2+x^2}} + \frac{1}{8}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{-2+x}x}\,dx, x, x^2\right)\\
&= -\frac{1}{6\left(-2+x^2\right)^{3/2}} + \frac{1}{4\sqrt{-2+x^2}} + \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \sqrt{-2+x^2}\right)\\
&= -\frac{1}{6\left(-2+x^2\right)^{3/2}} + \frac{1}{4\sqrt{-2+x^2}} + \frac{\tan^{-1}\left(\frac{\sqrt{-2+x^2}}{\sqrt{2}}\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.0062441, size = 30, normalized size = 0.58

$$-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{x^2}{2}\right)}{6\left(x^2-2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(-2 + x^2)^(5/2)),x]
```

```
[Out] -Hypergeometric2F1[-3/2, 1, -1/2, 1 - x^2/2]/(6*(-2 + x^2)^(3/2))
```

Maple [A] time = 0.005, size = 37, normalized size = 0.7

$$-\frac{1}{6}\left(x^2-2\right)^{-\frac{3}{2}}+\frac{1}{4}\frac{1}{\sqrt{x^2-2}}-\frac{\sqrt{2}}{8}\arctan\left(\sqrt{2}\frac{1}{\sqrt{x^2-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2-2)^(5/2),x)
```

```
[Out] -1/6/(x^2-2)^(3/2)+1/4/(x^2-2)^(1/2)-1/8*2^(1/2)*arctan(2^(1/2)/(x^2-2)^(1/
2))
```

Maxima [A] time = 1.40477, size = 45, normalized size = 0.87

$$-\frac{1}{8}\sqrt{2}\arcsin\left(\frac{\sqrt{2}}{|x|}\right)+\frac{1}{4\sqrt{x^2-2}}-\frac{1}{6\left(x^2-2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*arcsin(sqrt(2)/abs(x)) + 1/4/sqrt(x^2 - 2) - 1/6/(x^2 - 2)^(3/
2)
```

Fricas [A] time = 2.06344, size = 180, normalized size = 3.46

$$\frac{3\sqrt{2}\left(x^4-4x^2+4\right)\arctan\left(-\frac{1}{2}\sqrt{2}x+\frac{1}{2}\sqrt{2}\sqrt{x^2-2}\right)+\left(3x^2-8\right)\sqrt{x^2-2}}{12\left(x^4-4x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(3*sqrt(2)*(x^4 - 4*x^2 + 4)*arctan(-1/2*sqrt(2)*x + 1/2*sqrt(2)*sqrt(
x^2 - 2)) + (3*x^2 - 8)*sqrt(x^2 - 2))/(x^4 - 4*x^2 + 4)
```

Sympy [C] time = 5.23291, size = 986, normalized size = 18.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2-2)**(5/2),x)
```

```
[Out] Piecewise((6*I*x**4*log(x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2))
 - 3*I*x**4*log(x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 6*
x**4*asin(sqrt(2)/x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 6*s
qrt(2)*x**2*sqrt(x**2 - 2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2))
 - 24*I*x**2*log(x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 12*I
*x**2*log(x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 24*x**2*
asin(sqrt(2)/x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 16*sqrt(
2)*sqrt(x**2 - 2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 24*I*l
og(x)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*I*log(x**2)/(24
*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 24*asin(sqrt(2)/x)/(24*sqrt
(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)), Abs(x**2)/2 > 1), (-3*I*x**4*log(
x**2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 6*I*x**4*log(sqrt(
1 - x**2/2) + 1)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 3*pi*x*
*4/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 3*I*x**4*log(2)/(24*s
qrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 6*sqrt(2)*I*x**2*sqrt(2 - x**
2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 12*I*x**2*log(x**2)/(
24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 24*I*x**2*log(sqrt(1 - x*
*2/2) + 1)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 12*pi*x**2/(2
4*sqrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*I*x**2*log(2)/(24*sqrt(
2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 16*sqrt(2)*I*sqrt(2 - x**2)/(24*s
qrt(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*I*log(x**2)/(24*sqrt(2)*x*
*4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) + 24*I*log(sqrt(1 - x**2/2) + 1)/(24*sqr
t(2)*x**4 - 96*sqrt(2)*x**2 + 96*sqrt(2)) - 12*pi/(24*sqrt(2)*x**4 - 96*sqr
t(2)*x**2 + 96*sqrt(2)) + 12*I*log(2)/(24*sqrt(2)*x**4 - 96*sqrt(2)*x**2 +
96*sqrt(2)), True))
```

Giac [A] time = 1.0771, size = 47, normalized size = 0.9

$$\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-2}\right)+\frac{3x^2-8}{12\left(x^2-2\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2-2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 2)) + 1/12*(3*x^2 - 8)/(x^2 - 2)^
(3/2)
```

3.462 $\int \frac{\left(-10+x^2\right)^{5/2}}{x} dx$

Optimal. Leaf size=61

$$\frac{1}{5}\left(x^2-10\right)^{5/2}-\frac{10}{3}\left(x^2-10\right)^{3/2}+100\sqrt{x^2-10}-100\sqrt{10}\tan^{-1}\left(\frac{\sqrt{x^2-10}}{\sqrt{10}}\right)$$

```
[Out] 100*Sqrt[-10 + x^2] - (10*(-10 + x^2)^(3/2))/3 + (-10 + x^2)^(5/2)/5 - 100*
Sqrt[10]*ArcTan[Sqrt[-10 + x^2]/Sqrt[10]]
```

Rubi [A] time = 0.0318791, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 50, 63, 203}

$$\frac{1}{5}\left(x^2-10\right)^{5/2}-\frac{10}{3}\left(x^2-10\right)^{3/2}+100\sqrt{x^2-10}-100\sqrt{10}\tan^{-1}\left(\frac{\sqrt{x^2-10}}{\sqrt{10}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-10 + x^2)^(5/2)/x,x]
```

```
[Out] 100*Sqrt[-10 + x^2] - (10*(-10 + x^2)^(3/2))/3 + (-10 + x^2)^(5/2)/5 - 100*
Sqrt[10]*ArcTan[Sqrt[-10 + x^2]/Sqrt[10]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{(-10+x^2)^{5/2}}{x}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{(-10+x)^{5/2}}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{5}\left(-10+x^2\right)^{5/2} - 5\,\text{Subst}\left(\int \frac{(-10+x)^{3/2}}{x}\,dx, x, x^2\right)$$
$$= -\frac{10}{3}\left(-10+x^2\right)^{3/2} + \frac{1}{5}\left(-10+x^2\right)^{5/2} + 50\,\text{Subst}\left(\int \frac{\sqrt{-10+x}}{x}\,dx, x, x^2\right)$$
$$= 100\sqrt{-10+x^2} - \frac{10}{3}\left(-10+x^2\right)^{3/2} + \frac{1}{5}\left(-10+x^2\right)^{5/2} - 500\,\text{Subst}\left(\int \frac{1}{\sqrt{-10+x}x}\,dx, x, x^2\right)$$
$$= 100\sqrt{-10+x^2} - \frac{10}{3}\left(-10+x^2\right)^{3/2} + \frac{1}{5}\left(-10+x^2\right)^{5/2} - 1000\,\text{Subst}\left(\int \frac{1}{10+x^2}\,dx, x, \sqrt{-10+x^2}\right)$$
$$= 100\sqrt{-10+x^2} - \frac{10}{3}\left(-10+x^2\right)^{3/2} + \frac{1}{5}\left(-10+x^2\right)^{5/2} - 100\sqrt{10}\tan^{-1}\left(\frac{\sqrt{-10+x^2}}{\sqrt{10}}\right)$$

Mathematica [A] time = 0.0177892, size = 47, normalized size = 0.77

$$\frac{1}{15}\sqrt{x^2-10}\left(3x^4-110x^2+2300\right) - 100\sqrt{10}\tan^{-1}\left(\sqrt{\frac{x^2}{10}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-10 + x^2)^(5/2)/x,x]
```

```
[Out] (Sqrt[-10 + x^2]*(2300 - 110*x^2 + 3*x^4))/15 - 100*Sqrt[10]*ArcTan[Sqrt[-1
 + x^2/10]]
```

Maple [A] time = 0.005, size = 46, normalized size = 0.8

$$\frac{1}{5}\left(x^2-10\right)^{\frac{5}{2}} - \frac{10}{3}\left(x^2-10\right)^{\frac{3}{2}} + 100\sqrt{x^2-10} + 100\sqrt{10}\arctan\left(\frac{\sqrt{10}}{\sqrt{x^2-10}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-10)^(5/2)/x,x)
```

```
[Out] 1/5*(x^2-10)^(5/2)-10/3*(x^2-10)^(3/2)+100*(x^2-10)^(1/2)+100*10^(1/2)*arct
an(10^(1/2)/(x^2-10)^(1/2))
```

Maxima [A] time = 1.45614, size = 57, normalized size = 0.93

$$\frac{1}{5}\left(x^2-10\right)^{\frac{5}{2}} - \frac{10}{3}\left(x^2-10\right)^{\frac{3}{2}} + 100\sqrt{10}\arcsin\left(\frac{\sqrt{10}}{|x|}\right) + 100\sqrt{x^2-10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] 1/5*(x^2 - 10)^(5/2) - 10/3*(x^2 - 10)^(3/2) + 100*sqrt(10)*arcsin(sqrt(10)
/abs(x)) + 100*sqrt(x^2 - 10)
```

Fricas [A] time = 2.05554, size = 158, normalized size = 2.59

$$\frac{1}{15}\left(3x^4-110x^2+2300\right)\sqrt{x^2-10}-200\sqrt{10}\arctan\left(-\frac{1}{10}\sqrt{10}x+\frac{1}{10}\sqrt{10}\sqrt{x^2-10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)^(5/2)/x,x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^4 - 110*x^2 + 2300)*sqrt(x^2 - 10) - 200*sqrt(10)*arctan(-1/10*sq
rt(10)*x + 1/10*sqrt(10)*sqrt(x^2 - 10))
```

Sympy [C] time = 9.04472, size = 167, normalized size = 2.74

$$\begin{cases}\frac{x^4\sqrt{x^2-10}}{5}-\frac{22x^2\sqrt{x^2-10}}{3}+\frac{460\sqrt{x^2-10}}{3}-100\sqrt{10}i\log(x)+50\sqrt{10}i\log\left(x^2\right)+100\sqrt{10}\operatorname{asin}\left(\frac{\sqrt{10}}{x}\right) & \text{for}\ \frac{|x^2|}{10}>1\\ \frac{ix^4\sqrt{10-x^2}}{5}-\frac{22ix^2\sqrt{10-x^2}}{3}+\frac{460i\sqrt{10-x^2}}{3}+50\sqrt{10}i\log\left(x^2\right)-100\sqrt{10}i\log\left(\sqrt{1-\frac{x^2}{10}}+1\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-10)**(5/2)/x,x)
```

```
[Out] Piecewise((x**4*sqrt(x**2 - 10)/5 - 22*x**2*sqrt(x**2 - 10)/3 + 460*sqrt(x*
*2 - 10)/3 - 100*sqrt(10)*I*log(x) + 50*sqrt(10)*I*log(x**2) + 100*sqrt(10)
*asin(sqrt(10)/x), Abs(x**2)/10 > 1), (I*x**4*sqrt(10 - x**2)/5 - 22*I*x**2
*sqrt(10 - x**2)/3 + 460*I*sqrt(10 - x**2)/3 + 50*sqrt(10)*I*log(x**2) - 10
0*sqrt(10)*I*log(sqrt(1 - x**2/10) + 1), True))
```

Giac [A] time = 1.06717, size = 62, normalized size = 1.02

$$\frac{1}{5}\left(x^2-10\right)^{\frac{5}{2}}-\frac{10}{3}\left(x^2-10\right)^{\frac{3}{2}}-100\sqrt{10}\arctan\left(\frac{1}{10}\sqrt{10}\sqrt{x^2-10}\right)+100\sqrt{x^2-10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-10)^(5/2)/x,x, algorithm="giac")
```

```
[Out] 1/5*(x^2 - 10)^(5/2) - 10/3*(x^2 - 10)^(3/2) - 100*sqrt(10)*arctan(1/10*sqr
t(10)*sqrt(x^2 - 10)) + 100*sqrt(x^2 - 10)
```

3.463 $\int x^{1+2n}\,dx$

Optimal. Leaf size=16

$$\frac{x^{2(n+1)}}{2(n+1)}$$

```
[Out] x^(2*(1 + n))/(2*(1 + n))
```

Rubi [A] time = 0.0025427, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {30}

$$\frac{x^{2(n+1)}}{2(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + 2*n),x]

[Out] x^(2*(1 + n))/(2*(1 + n))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{1+2n}\,dx = \frac{x^{2(1+n)}}{2(1+n)}$$

Mathematica [A] time = 0.0022625, size = 15, normalized size = 0.94

$$\frac{x^{2n+2}}{2n+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + 2*n),x]

[Out] x^(2 + 2*n)/(2 + 2*n)
```

Maple [A] time = 0.002, size = 15, normalized size = 0.9

$$\frac{x^{2+2n}}{2+2n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+2*n),x)
```

```
[Out] 1/2*x^(2+2*n)/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08276, size = 36, normalized size = 2.25

$$\frac{xx^{2\,n+1}}{2\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*n),x, algorithm="fricas")
```

```
[Out] 1/2*x*x^(2*n + 1)/(n + 1)
```

Sympy [A] time = 0.054665, size = 19, normalized size = 1.19

$$\begin{cases} \frac{x^{2n+2}}{2n+2} & \text{for}\ 2n+1 \neq -1 \\ \log{(x)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+2*n),x)
```

```
[Out] Piecewise((x**(2*n + 2)/(2*n + 2), Ne(2*n + 1, -1)), (log(x), True))
```

Giac [A] time = 1.07528, size = 19, normalized size = 1.19

$$\frac{x^{2\,n+2}}{2\,(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+2*n),x, algorithm="giac")
```

```
[Out] 1/2*x^(2*n + 2)/(n + 1)
```

3.464 $\int \frac{x^7}{(-5+x^2)^3}\,dx$

Optimal. Leaf size=46

$$\frac{x^2}{2}+\frac{75}{2(5-x^2)}-\frac{125}{4(5-x^2)^2}+\frac{15}{2}\log(5-x^2)$$

```
[Out] x^2/2 - 125/(4*(5 - x^2)^2) + 75/(2*(5 - x^2)) + (15*Log[5 - x^2])/2
```

Rubi [A] time = 0.0237345, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {266, 43}

$$\frac{x^2}{2}+\frac{75}{2(5-x^2)}-\frac{125}{4(5-x^2)^2}+\frac{15}{2}\log(5-x^2)$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(-5 + x^2)^3,x]
```

```
[Out] x^2/2 - 125/(4*(5 - x^2)^2) + 75/(2*(5 - x^2)) + (15*Log[5 - x^2])/2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(-5+x^2)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^3}{(-5+x)^3}\,dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(1+\frac{125}{(-5+x)^3}+\frac{75}{(-5+x)^2}+\frac{15}{-5+x}\right)dx, x, x^2\right)\\
&= \frac{x^2}{2}-\frac{125}{4(5-x^2)^2}+\frac{75}{2(5-x^2)}+\frac{15}{2}\log(5-x^2)
\end{aligned}$$

Mathematica [A] time = 0.0127481, size = 36, normalized size = 0.78

$$\frac{1}{4}\left(2x^2-\frac{150}{x^2-5}-\frac{125}{(x^2-5)^2}+30\log(x^2-5)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(-5 + x^2)^3,x]
```

```
[Out] (2*x^2 - 125/(-5 + x^2)^2 - 150/(-5 + x^2) + 30*Log[-5 + x^2])/4
```

Maple [A] time = 0.009, size = 33, normalized size = 0.7

$$\frac{x^2}{2} - \frac{75}{2x^2 - 10} + \frac{15\ln\left(x^2-5\right)}{2} - \frac{125}{4\left(x^2-5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^2-5)^3,x)
```

```
[Out] 1/2*x^2-75/2/(x^2-5)+15/2*ln(x^2-5)-125/4/(x^2-5)^2
```

Maxima [A] time = 0.947302, size = 47, normalized size = 1.02

$$\frac{1}{2}x^2 - \frac{25\left(6x^2-25\right)}{4\left(x^4-10x^2+25\right)} + \frac{15}{2}\log\left(x^2-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^2-5)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 25/4*(6*x^2 - 25)/(x^4 - 10*x^2 + 25) + 15/2*log(x^2 - 5)
```

Fricas [A] time = 1.86473, size = 130, normalized size = 2.83

$$\frac{2x^6 - 20x^4 - 100x^2 + 30\left(x^4-10x^2+25\right)\log\left(x^2-5\right) + 625}{4\left(x^4-10x^2+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^2-5)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*x^6 - 20*x^4 - 100*x^2 + 30*(x^4 - 10*x^2 + 25)*log(x^2 - 5) + 625)/
(x^4 - 10*x^2 + 25)
```

Sympy [A] time = 0.111764, size = 32, normalized size = 0.7

$$\frac{x^2}{2} - \frac{150x^2 - 625}{4x^4 - 40x^2 + 100} + \frac{15\log\left(x^2-5\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**2-5)**3,x)
```

```
[Out] x**2/2 - (150*x**2 - 625)/(4*x**4 - 40*x**2 + 100) + 15*log(x**2 - 5)/2
```

Giac [A] time = 1.05348, size = 49, normalized size = 1.07

$$\frac{1}{2}x^2 - \frac{5\left(9x^4 - 60x^2 + 100\right)}{4\left(x^2-5\right)^2} + \frac{15}{2}\log\left(\left|x^2-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^2-5)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 5/4*(9*x^4 - 60*x^2 + 100)/(x^2 - 5)^2 + 15/2*log(abs(x^2 - 5))
```

3.465 $\int \frac{-4x^3+3x^5}{\left(-1+x^2\right)^5}\,dx$

Optimal. Leaf size=40

$$-\frac{3}{4\left(1-x^2\right)^2}+\frac{1}{3\left(1-x^2\right)^3}+\frac{1}{8\left(1-x^2\right)^4}$$

```
[Out] 1/(8*(1 - x^2)^4) + 1/(3*(1 - x^2)^3) - 3/(4*(1 - x^2)^2)
```

Rubi [A] time = 0.0353437, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1593, 446, 77}

$$-\frac{3}{4\left(1-x^2\right)^2}+\frac{1}{3\left(1-x^2\right)^3}+\frac{1}{8\left(1-x^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(-4*x^3 + 3*x^5)/(-1 + x^2)^5,x]
```

```
[Out] 1/(8*(1 - x^2)^4) + 1/(3*(1 - x^2)^3) - 3/(4*(1 - x^2)^2)
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int \frac{-4x^3+3x^5}{\left(-1+x^2\right)^5}\,dx &= \int \frac{x^3\left(-4+3x^2\right)}{\left(-1+x^2\right)^5}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{x(-4+3x)}{(-1+x)^5}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^5}+\frac{2}{(-1+x)^4}+\frac{3}{(-1+x)^3}\right)dx, x, x^2\right) \\
&= \frac{1}{8\left(1-x^2\right)^4}+\frac{1}{3\left(1-x^2\right)^3}-\frac{3}{4\left(1-x^2\right)^2}
\end{aligned}
$$

Mathematica [A] time = 0.0081096, size = 23, normalized size = 0.57

$$\frac{-18x^4+28x^2-7}{24\left(x^2-1\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4*x^3 + 3*x^5)/(-1 + x^2)^5,x]
```

```
[Out] (-7 + 28*x^2 - 18*x^4)/(24*(-1 + x^2)^4)
```

Maple [A] time = 0.01, size = 58, normalized size = 1.5

$$\frac{1}{128\,(1+x)^4}+\frac{11}{192\,(1+x)^3}-\frac{27}{256\,(1+x)^2}-\frac{27}{256+256\,x}+\frac{1}{128\,(-1+x)^4}-\frac{11}{192\,(-1+x)^3}-\frac{27}{256\,(-1+x)^2}+\frac{}{-256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^5-4*x^3)/(x^2-1)^5,x)
```

```
[Out] 1/128/(1+x)^4+11/192/(1+x)^3-27/256/(1+x)^2-27/256/(1+x)+1/128/(-1+x)^4-11/
192/(-1+x)^3-27/256/(-1+x)^2+27/256/(-1+x)
```

Maxima [A] time = 0.933901, size = 49, normalized size = 1.22

$$-\frac{18\,x^4-28\,x^2+7}{24\left(x^8-4\,x^6+6\,x^4-4\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-4*x^3)/(x^2-1)^5,x, algorithm="maxima")
```

```
[Out] -1/24*(18*x^4 - 28*x^2 + 7)/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

Fricas [A] time = 1.94571, size = 85, normalized size = 2.12

$$-\frac{18\,x^4-28\,x^2+7}{24\left(x^8-4\,x^6+6\,x^4-4\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-4*x^3)/(x^2-1)^5,x, algorithm="fricas")
```

```
[Out] -1/24*(18*x^4 - 28*x^2 + 7)/(x^8 - 4*x^6 + 6*x^4 - 4*x^2 + 1)
```

Sympy [A] time = 0.129208, size = 34, normalized size = 0.85

$$-\frac{18x^4 - 28x^2 + 7}{24x^8 - 96x^6 + 144x^4 - 96x^2 + 24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**5-4*x**3)/(x**2-1)**5,x)
```

```
[Out] -(18*x**4 - 28*x**2 + 7)/(24*x**8 - 96*x**6 + 144*x**4 - 96*x**2 + 24)
```

Giac [A] time = 1.05319, size = 28, normalized size = 0.7

$$-\frac{18\,x^4 - 28\,x^2 + 7}{24\,{\left(x^2 - 1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^5-4*x^3)/(x^2-1)^5,x, algorithm="giac")
```

```
[Out] -1/24*(18*x^4 - 28*x^2 + 7)/(x^2 - 1)^4
```

3.466 $\int x^3 \left(1+x^2\right)^{9/14} \, dx$

Optimal. Leaf size=27

$$\frac{7}{37} \left(x^2+1\right)^{37/14}-\frac{7}{23} \left(x^2+1\right)^{23/14}$$

```
[Out] (-7*(1 + x^2)^(23/14))/23 + (7*(1 + x^2)^(37/14))/37
```

Rubi [A] time = 0.0114618, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{7}{37} \left(x^2+1\right)^{37/14}-\frac{7}{23} \left(x^2+1\right)^{23/14}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x^2)^(9/14),x]
```

```
[Out] (-7*(1 + x^2)^(23/14))/23 + (7*(1 + x^2)^(37/14))/37
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(1+x^2\right)^{9/14} \, dx &= \frac{1}{2} \text{Subst}\left(\int x(1+x)^{9/14} \, dx, x, x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(-(1+x)^{9/14}+(1+x)^{23/14}\right) \, dx, x, x^2\right)\\
&= -\frac{7}{23} \left(1+x^2\right)^{23/14}+\frac{7}{37} \left(1+x^2\right)^{37/14}
\end{aligned}$$

Mathematica [A] time = 0.0063217, size = 20, normalized size = 0.74

$$\frac{7}{851} \left(x^2+1\right)^{23/14} \left(23 x^2-14\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x^2)^(9/14),x]
```

```
[Out] (7*(1 + x^2)^(23/14)*(-14 + 23*x^2))/851
```

Maple [A] time = 0.004, size = 17, normalized size = 0.6

$$\frac{161 x^2 - 98}{851}\left(x^2+1\right)^{\frac{23}{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+1)^(9/14),x)
```

```
[Out] 7/851*(x^2+1)^(23/14)*(23*x^2-14)
```

Maxima [A] time = 0.945455, size = 26, normalized size = 0.96

$$\frac{7}{37}\left(x^2+1\right)^{\frac{37}{14}} - \frac{7}{23}\left(x^2+1\right)^{\frac{23}{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(9/14),x, algorithm="maxima")
```

```
[Out] 7/37*(x^2 + 1)^(37/14) - 7/23*(x^2 + 1)^(23/14)
```

Fricas [A] time = 1.95499, size = 62, normalized size = 2.3

$$\frac{7}{851}\left(23 x^4 + 9 x^2 - 14\right)\left(x^2+1\right)^{\frac{9}{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(9/14),x, algorithm="fricas")
```

```
[Out] 7/851*(23*x^4 + 9*x^2 - 14)*(x^2 + 1)^(9/14)
```

Sympy [A] time = 18.7864, size = 41, normalized size = 1.52

$$\frac{7x^4\left(x^2+1\right)^{\frac{9}{14}}}{37} + \frac{63x^2\left(x^2+1\right)^{\frac{9}{14}}}{851} - \frac{98\left(x^2+1\right)^{\frac{9}{14}}}{851}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+1)**(9/14),x)
```

```
[Out] 7*x**4*(x**2 + 1)**(9/14)/37 + 63*x**2*(x**2 + 1)**(9/14)/851 - 98*(x**2 +
1)**(9/14)/851
```

Giac [A] time = 1.07927, size = 26, normalized size = 0.96

$$\frac{7}{37}\left(x^2+1\right)^{\frac{37}{14}}-\frac{7}{23}\left(x^2+1\right)^{\frac{23}{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)^(9/14),x, algorithm="giac")
```

```
[Out] 7/37*(x^2 + 1)^(37/14) - 7/23*(x^2 + 1)^(23/14)
```

3.467 $\int \frac{x^5}{\left(-4+x^2\right)^{13/6}}\,dx$

Optimal. Leaf size=38

$$\frac{3}{5}\left(x^2-4\right)^{5/6}-\frac{24}{\sqrt[6]{x^2-4}}-\frac{48}{7\left(x^2-4\right)^{7/6}}$$

```
[Out] -48/(7*(-4 + x^2)^(7/6)) - 24/(-4 + x^2)^(1/6) + (3*(-4 + x^2)^(5/6))/5
```

Rubi [A] time = 0.0154403, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{5}\left(x^2-4\right)^{5/6}-\frac{24}{\sqrt[6]{x^2-4}}-\frac{48}{7\left(x^2-4\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(-4 + x^2)^(13/6),x]
```

```
[Out] -48/(7*(-4 + x^2)^(7/6)) - 24/(-4 + x^2)^(1/6) + (3*(-4 + x^2)^(5/6))/5
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(-4+x^2\right)^{13/6}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(-4+x)^{13/6}}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{16}{(-4+x)^{13/6}}+\frac{8}{(-4+x)^{7/6}}+\frac{1}{\sqrt[6]{-4+x}}\right)dx,x,x^2\right)\\
&= -\frac{48}{7\left(-4+x^2\right)^{7/6}}-\frac{24}{\sqrt[6]{-4+x^2}}+\frac{3}{5}\left(-4+x^2\right)^{5/6}
\end{aligned}$$

Mathematica [A] time = 0.0090649, size = 25, normalized size = 0.66

$$\frac{3\left(7x^4-336x^2+1152\right)}{35\left(x^2-4\right)^{7/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(-4 + x^2)^(13/6),x]
```

```
[Out] (3*(1152 - 336*x^2 + 7*x^4))/(35*(-4 + x^2)^(7/6))
```

Maple [A] time = 0.004, size = 28, normalized size = 0.7

$$\frac{(-6+3x)(2+x)\left(7x^4-336x^2+1152\right)}{35}\left(x^2-4\right)^{-\frac{13}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^2-4)^(13/6),x)
```

```
[Out] 3/35*(-2+x)*(2+x)*(7*x^4-336*x^2+1152)/(x^2-4)^(13/6)
```

Maxima [A] time = 0.976044, size = 38, normalized size = 1.

$$\frac{3}{5}\left(x^2-4\right)^{\frac{5}{6}}-\frac{24}{\left(x^2-4\right)^{\frac{1}{6}}}-\frac{48}{7\left(x^2-4\right)^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2-4)^(13/6),x, algorithm="maxima")
```

```
[Out] 3/5*(x^2 - 4)^(5/6) - 24/(x^2 - 4)^(1/6) - 48/7/(x^2 - 4)^(7/6)
```

Fricas [A] time = 2.02865, size = 89, normalized size = 2.34

$$\frac{3\left(7x^4-336x^2+1152\right)\left(x^2-4\right)^{\frac{5}{6}}}{35\left(x^4-8x^2+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2-4)^(13/6),x, algorithm="fricas")
```

```
[Out] 3/35*(7*x^4 - 336*x^2 + 1152)*(x^2 - 4)^(5/6)/(x^4 - 8*x^2 + 16)
```

Sympy [B] time = 6.20819, size = 82, normalized size = 2.16

$$\frac{21x^4}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}}-\frac{1008x^2}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}}+\frac{3456}{35x^2\sqrt[6]{x^2-4}-140\sqrt[6]{x^2-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**2-4)**(13/6),x)
```

```
[Out] 21*x**4/(35*x**2*(x**2 - 4)**(1/6) - 140*(x**2 - 4)**(1/6)) - 1008*x**2/(35
*x**2*(x**2 - 4)**(1/6) - 140*(x**2 - 4)**(1/6)) + 3456/(35*x**2*(x**2 - 4)
**(1/6) - 140*(x**2 - 4)**(1/6))
```

Giac [A] time = 1.06423, size = 35, normalized size = 0.92

$$\frac{3}{5}\left(x^2-4\right)^{\frac{5}{6}}-\frac{24\left(7x^2-26\right)}{7\left(x^2-4\right)^{\frac{7}{6}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2-4)^(13/6),x, algorithm="giac")
```

```
[Out] 3/5*(x^2 - 4)^(5/6) - 24/7*(7*x^2 - 26)/(x^2 - 4)^(7/6)
```

3.468 $\int \frac{1}{(1+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=33

$$\frac{2x}{3\sqrt{2x^2+1}}+\frac{x}{3\left(2x^2+1\right)^{3/2}}$$

```
[Out] x/(3*(1 + 2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 + 2*x^2])
```

Rubi [A] time = 0.0042804, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{2x}{3\sqrt{2x^2+1}}+\frac{x}{3\left(2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2)^(-5/2),x]
```

```
[Out] x/(3*(1 + 2*x^2)^(3/2)) + (2*x)/(3*Sqrt[1 + 2*x^2])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+2x^2\right)^{5/2}}\,dx &= \frac{x}{3\left(1+2x^2\right)^{3/2}}+\frac{2}{3}\int \frac{1}{\left(1+2x^2\right)^{3/2}}\,dx\\ &= \frac{x}{3\left(1+2x^2\right)^{3/2}}+\frac{2x}{3\sqrt{1+2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0053973, size = 23, normalized size = 0.7

$$\frac{x\left(4x^2+3\right)}{3\left(2x^2+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2)^(-5/2),x]
```

```
[Out] (x*(3 + 4*x^2))/(3*(1 + 2*x^2)^(3/2))
```

Maple [A] time = 0.001, size = 20, normalized size = 0.6

$$\frac{x\left(4x^2+3\right)}{3}\left(2x^2+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2+1)^(5/2),x)
```

```
[Out] 1/3*x*(4*x^2+3)/(2*x^2+1)^(3/2)
```

Maxima [A] time = 0.937497, size = 34, normalized size = 1.03

$$\frac{2x}{3\sqrt{2x^2+1}}+\frac{x}{3\left(2x^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/sqrt(2*x^2 + 1) + 1/3*x/(2*x^2 + 1)^(3/2)
```

Fricas [A] time = 2.07408, size = 74, normalized size = 2.24

$$\frac{\left(4x^3+3x\right)\sqrt{2x^2+1}}{3\left(4x^4+4x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(4*x^3 + 3*x)*sqrt(2*x^2 + 1)/(4*x^4 + 4*x^2 + 1)
```

Sympy [B] time = 6.52519, size = 61, normalized size = 1.85

$$\frac{4x^3}{6x^2\sqrt{2x^2+1}+3\sqrt{2x^2+1}}+\frac{3x}{6x^2\sqrt{2x^2+1}+3\sqrt{2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2+1)**(5/2),x)
```

```
[Out] 4*x**3/(6*x**2*sqrt(2*x**2 + 1) + 3*sqrt(2*x**2 + 1)) + 3*x/(6*x**2*sqrt(2*
x**2 + 1) + 3*sqrt(2*x**2 + 1))
```

Giac [A] time = 1.07411, size = 26, normalized size = 0.79

$$\frac{(4x^2+3)x}{3(2x^2+1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 + 3)*x/(2*x^2 + 1)^(3/2)
```

3.469 $\int \frac{1}{\left(-1-2x+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{1-x}{6\left(x^2-2x-1\right)^{3/2}}-\frac{1-x}{6\sqrt{x^2-2x-1}}$$

```
[Out] (1 - x)/(6*(-1 - 2*x + x^2)^(3/2)) - (1 - x)/(6*Sqrt[-1 - 2*x + x^2])
```

Rubi [A] time = 0.0061111, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {614, 613}

$$\frac{1-x}{6\left(x^2-2x-1\right)^{3/2}}-\frac{1-x}{6\sqrt{x^2-2x-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 2*x + x^2)^(-5/2),x]
```

```
[Out] (1 - x)/(6*(-1 - 2*x + x^2)^(3/2)) - (1 - x)/(6*Sqrt[-1 - 2*x + x^2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-1-2x+x^2\right)^{5/2}}\,dx &= \frac{1-x}{6\left(-1-2x+x^2\right)^{3/2}}-\frac{1}{3}\int \frac{1}{\left(-1-2x+x^2\right)^{3/2}}\,dx\\ &= \frac{1-x}{6\left(-1-2x+x^2\right)^{3/2}}-\frac{1-x}{6\sqrt{-1-2x+x^2}}\end{aligned}$$

Mathematica [A] time = 0.0138116, size = 26, normalized size = 0.6

$$\frac{x^3-3x^2+2}{6\left(x^2-2x-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x + x^2)^(-5/2),x]
```

```
[Out] (2 - 3*x^2 + x^3)/(6*(-1 - 2*x + x^2)^(3/2))
```

Maple [A] time = 0.003, size = 23, normalized size = 0.5

$$\frac{x^3-3x^2+2}{6}\left(x^2-2x-1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x-1)^(5/2),x)
```

```
[Out] 1/6*(x^3-3*x^2+2)/(x^2-2*x-1)^(3/2)
```

Maxima [A] time = 0.934023, size = 69, normalized size = 1.6

$$\frac{x}{6\sqrt{x^2-2x-1}}-\frac{1}{6\sqrt{x^2-2x-1}}-\frac{x}{6\left(x^2-2x-1\right)^{\frac{3}{2}}}+\frac{1}{6\left(x^2-2x-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*x/sqrt(x^2 - 2*x - 1) - 1/6/sqrt(x^2 - 2*x - 1) - 1/6*x/(x^2 - 2*x - 1)
^(3/2) + 1/6/(x^2 - 2*x - 1)^(3/2)
```

Fricas [A] time = 2.11103, size = 147, normalized size = 3.42

$$\frac{x^4-4x^3+2x^2+\left(x^3-3x^2+2\right)\sqrt{x^2-2x-1}+4x+1}{6\left(x^4-4x^3+2x^2+4x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-1)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(x^4 - 4*x^3 + 2*x^2 + (x^3 - 3*x^2 + 2)*sqrt(x^2 - 2*x - 1) + 4*x + 1)
/(x^4 - 4*x^3 + 2*x^2 + 4*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(x^2-2x-1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x-1)**(5/2),x)
```

```
[Out] Integral((x**2 - 2*x - 1)**(-5/2), x)
```

Giac [A] time = 1.06268, size = 28, normalized size = 0.65

$$\frac{(x-3)x^2+2}{6\left(x^2-2x-1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x-1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/6*((x - 3)*x^2 + 2)/(x^2 - 2*x - 1)^(3/2)
```

3.470 $\int \frac{1}{x^4(-8+x^2)^{3/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{x}{192\sqrt{x^2-8}}+\frac{1}{48\sqrt{x^2-8}x}+\frac{1}{24\sqrt{x^2-8}x^3}$$

```
[Out] 1/(24*x^3*Sqrt[-8 + x^2]) + 1/(48*x*Sqrt[-8 + x^2]) - x/(192*Sqrt[-8 + x^2]
)
```

Rubi [A] time = 0.0094631, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {271, 191}

$$-\frac{x}{192\sqrt{x^2-8}}+\frac{1}{48\sqrt{x^2-8}x}+\frac{1}{24\sqrt{x^2-8}x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(-8 + x^2)^(3/2)),x]
```

```
[Out] 1/(24*x^3*Sqrt[-8 + x^2]) + 1/(48*x*Sqrt[-8 + x^2]) - x/(192*Sqrt[-8 + x^2]
)
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4\left(-8+x^2\right)^{3/2}}\,dx &= \frac{1}{24x^3\sqrt{-8+x^2}}+\frac{1}{6}\int \frac{1}{x^2\left(-8+x^2\right)^{3/2}}\,dx\\
&= \frac{1}{24x^3\sqrt{-8+x^2}}+\frac{1}{48x\sqrt{-8+x^2}}+\frac{1}{24}\int \frac{1}{\left(-8+x^2\right)^{3/2}}\,dx\\
&= \frac{1}{24x^3\sqrt{-8+x^2}}+\frac{1}{48x\sqrt{-8+x^2}}-\frac{x}{192\sqrt{-8+x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0052976, size = 28, normalized size = 0.6

$$\frac{-x^4+4x^2+8}{192x^3\sqrt{x^2-8}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(-8 + x^2)^(3/2)),x]
```

```
[Out] (8 + 4*x^2 - x^4)/(192*x^3*Sqrt[-8 + x^2])
```

Maple [A] time = 0.002, size = 23, normalized size = 0.5

$$-\frac{x^4-4x^2-8}{192x^3}\frac{1}{\sqrt{x^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^2-8)^(3/2),x)
```

```
[Out] -1/192*(x^4-4*x^2-8)/x^3/(x^2-8)^(1/2)
```

Maxima [A] time = 1.40514, size = 47, normalized size = 1.

$$-\frac{x}{192\sqrt{x^2-8}}+\frac{1}{48\sqrt{x^2-8}x}+\frac{1}{24\sqrt{x^2-8}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^2-8)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/192*x/sqrt(x^2 - 8) + 1/48/(sqrt(x^2 - 8)*x) + 1/24/(sqrt(x^2 - 8)*x^3)
```

Fricas [A] time = 2.04825, size = 95, normalized size = 2.02

$$-\frac{x^5-8x^3+\left(x^4-4x^2-8\right)\sqrt{x^2-8}}{192\left(x^5-8x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^2-8)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/192*(x^5 - 8*x^3 + (x^4 - 4*x^2 - 8)*sqrt(x^2 - 8))/(x^5 - 8*x^3)
```

Sympy [A] time = 4.75237, size = 151, normalized size = 3.21

$$\begin{cases} -\frac{ix^4\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2}+\frac{4ix^2\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2}+\frac{8i\sqrt{-1+\frac{8}{x^2}}}{192x^4-1536x^2} & \text{for } \frac{8}{|x^2|}>1 \\ -\frac{x^4\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2}+\frac{4x^2\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2}+\frac{8\sqrt{1-\frac{8}{x^2}}}{192x^4-1536x^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**2-8)**(3/2),x)
```

```
[Out] Piecewise((-I*x**4*sqrt(-1 + 8/x**2)/(192*x**4 - 1536*x**2) + 4*I*x**2*sqrt
(-1 + 8/x**2)/(192*x**4 - 1536*x**2) + 8*I*sqrt(-1 + 8/x**2)/(192*x**4 - 15
36*x**2), 8/Abs(x**2) > 1), (-x**4*sqrt(1 - 8/x**2)/(192*x**4 - 1536*x**2)
+ 4*x**2*sqrt(1 - 8/x**2)/(192*x**4 - 1536*x**2) + 8*sqrt(1 - 8/x**2)/(192*
x**4 - 1536*x**2), True))
```

Giac [A] time = 1.07795, size = 84, normalized size = 1.79

$$-\frac{x}{512\sqrt{x^2-8}}-\frac{3\left(x-\sqrt{x^2-8}\right)^4+96\left(x-\sqrt{x^2-8}\right)^2+320}{96\left(\left(x-\sqrt{x^2-8}\right)^2+8\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^2-8)^(3/2),x, algorithm="giac")
```

```
[Out] -1/512*x/sqrt(x^2 - 8) - 1/96*(3*(x - sqrt(x^2 - 8))^4 + 96*(x - sqrt(x^2 -
 8))^2 + 320)/((x - sqrt(x^2 - 8))^2 + 8)^3
```

3.471 $\int \frac{(5+x^2)^2}{x^{13/3}} \, dx$

Optimal. Leaf size=28

$$\frac{3x^{2/3}}{2} - \frac{15}{2x^{4/3}} - \frac{15}{2x^{10/3}}$$

```
[Out] -15/(2*x^(10/3)) - 15/(2*x^(4/3)) + (3*x^(2/3))/2
```

Rubi [A] time = 0.0055334, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {270}

$$\frac{3x^{2/3}}{2} - \frac{15}{2x^{4/3}} - \frac{15}{2x^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x^2)^2/x^(13/3),x]
```

```
[Out] -15/(2*x^(10/3)) - 15/(2*x^(4/3)) + (3*x^(2/3))/2
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(5+x^2)^2}{x^{13/3}} \, dx &= \int \left(\frac{25}{x^{13/3}} + \frac{10}{x^{7/3}} + \frac{1}{\sqrt[3]{x}} \right) dx \\ &= -\frac{15}{2x^{10/3}} - \frac{15}{2x^{4/3}} + \frac{3x^{2/3}}{2} \end{aligned}$$

Mathematica [A] time = 0.0051637, size = 19, normalized size = 0.68

$$\frac{3\left(x^4 - 5x^2 - 5\right)}{2x^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x^2)^2/x^(13/3),x]
```

```
[Out] (3*(-5 - 5*x^2 + x^4))/(2*x^(10/3))
```

Maple [A] time = 0.005, size = 16, normalized size = 0.6

$$\frac{3x^4 - 15x^2 - 15}{2} x^{-\frac{10}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5)^2/x^(13/3),x)
```

```
[Out] 3/2*(x^4-5*x^2-5)/x^(10/3)
```

Maxima [A] time = 0.932026, size = 22, normalized size = 0.79

$$\frac{3}{2}x^{\frac{2}{3}} - \frac{15\left(x^2+1\right)}{2x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)^2/x^(13/3),x, algorithm="maxima")
```

```
[Out] 3/2*x^(2/3) - 15/2*(x^2 + 1)/x^(10/3)
```

Fricas [A] time = 2.0601, size = 43, normalized size = 1.54

$$\frac{3\left(x^4-5x^2-5\right)}{2x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)^2/x^(13/3),x, algorithm="fricas")
```

```
[Out] 3/2*(x^4 - 5*x^2 - 5)/x^(10/3)
```

Sympy [A] time = 8.78522, size = 24, normalized size = 0.86

$$\frac{3x^{\frac{2}{3}}}{2} - \frac{15}{2x^{\frac{4}{3}}} - \frac{15}{2x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5)**2/x**(13/3),x)
```

```
[Out] 3*x**(2/3)/2 - 15/(2*x**(4/3)) - 15/(2*x**(10/3))
```

Giac [A] time = 1.06838, size = 22, normalized size = 0.79

$$\frac{3}{2}x^{\frac{2}{3}} - \frac{15\left(x^2+1\right)}{2x^{\frac{10}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)^2/x^(13/3),x, algorithm="giac")
```

```
[Out] 3/2*x^(2/3) - 15/2*(x^2 + 1)/x^(10/3)
```

3.472 $\int \frac{1}{x^7(1+x^2)^3}\,dx$

Optimal. Leaf size=52

$$-\frac{2}{x^2+1}-\frac{3}{x^2}-\frac{1}{4\left(x^2+1\right)^2}+\frac{3}{4x^4}-\frac{1}{6x^6}+5\log\left(x^2+1\right)-10\log(x)$$

```
[Out] -1/(6*x^6) + 3/(4*x^4) - 3/x^2 - 1/(4*(1 + x^2)^2) - 2/(1 + x^2) - 10*Log[x
] + 5*Log[1 + x^2]
```

Rubi [A] time = 0.0280003, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {266, 44}

$$-\frac{2}{x^2+1}-\frac{3}{x^2}-\frac{1}{4\left(x^2+1\right)^2}+\frac{3}{4x^4}-\frac{1}{6x^6}+5\log\left(x^2+1\right)-10\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^7*(1 + x^2)^3),x]
```

```
[Out] -1/(6*x^6) + 3/(4*x^4) - 3/x^2 - 1/(4*(1 + x^2)^2) - 2/(1 + x^2) - 10*Log[x
] + 5*Log[1 + x^2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^7\left(1+x^2\right)^3}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x^4(1+x)^3}\,dx,x,x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{1}{x^4}-\frac{3}{x^3}+\frac{6}{x^2}-\frac{10}{x}+\frac{1}{(1+x)^3}+\frac{4}{(1+x)^2}+\frac{10}{1+x}\right)dx,x,x^2\right)\\ &= -\frac{1}{6x^6}+\frac{3}{4x^4}-\frac{3}{x^2}-\frac{1}{4\left(1+x^2\right)^2}-\frac{2}{1+x^2}-10\log(x)+5\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0262952, size = 49, normalized size = 0.94

$$-\frac{60x^8+90x^6+20x^4-5x^2+2}{12x^6\left(x^2+1\right)^2}+5\log\left(x^2+1\right)-10\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^7*(1 + x^2)^3),x]
```

```
[Out] -(2 - 5*x^2 + 20*x^4 + 90*x^6 + 60*x^8)/(12*x^6*(1 + x^2)^2) - 10*Log[x] +
5*Log[1 + x^2]
```

Maple [A] time = 0.011, size = 47, normalized size = 0.9

$$-\frac{1}{6x^{6}}+\frac{3}{4x^{4}}-3x^{-2}-\frac{1}{4\left(x^{2}+1\right)^{2}}-2\left(x^{2}+1\right)^{-1}-10\ln\left(x\right)+5\ln\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^7/(x^2+1)^3,x)
```

```
[Out] -1/6/x^6+3/4/x^4-3/x^2-1/4/(x^2+1)^2-2/(x^2+1)-10*ln(x)+5*ln(x^2+1)
```

Maxima [A] time = 0.942041, size = 72, normalized size = 1.38

$$-\frac{60x^{8}+90x^{6}+20x^{4}-5x^{2}+2}{12\left(x^{10}+2x^{8}+x^{6}\right)}+5\log\left(x^{2}+1\right)-5\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(60*x^8 + 90*x^6 + 20*x^4 - 5*x^2 + 2)/(x^10 + 2*x^8 + x^6) + 5*log(x
^2 + 1) - 5*log(x^2)
```

Fricas [A] time = 2.2616, size = 189, normalized size = 3.63

$$-\frac{60x^{8}+90x^{6}+20x^{4}-5x^{2}-60\left(x^{10}+2x^{8}+x^{6}\right)\log\left(x^{2}+1\right)+120\left(x^{10}+2x^{8}+x^{6}\right)\log\left(x\right)+2}{12\left(x^{10}+2x^{8}+x^{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(60*x^8 + 90*x^6 + 20*x^4 - 5*x^2 - 60*(x^10 + 2*x^8 + x^6)*log(x^2 +
 1) + 120*(x^10 + 2*x^8 + x^6)*log(x) + 2)/(x^10 + 2*x^8 + x^6)
```

Sympy [A] time = 0.165768, size = 49, normalized size = 0.94

$$-10\log\left(x\right)+5\log\left(x^{2}+1\right)-\frac{60x^{8}+90x^{6}+20x^{4}-5x^{2}+2}{12x^{10}+24x^{8}+12x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**7/(x**2+1)**3,x)
```

```
[Out] -10*log(x) + 5*log(x**2 + 1) - (60*x**8 + 90*x**6 + 20*x**4 - 5*x**2 + 2)/(
12*x**10 + 24*x**8 + 12*x**6)
```

Giac [A] time = 1.07521, size = 78, normalized size = 1.5

$$-\frac{30\,x^4+68\,x^2+39}{4\left(x^2+1\right)^2}+\frac{110\,x^6-36\,x^4+9\,x^2-2}{12\,x^6}+5\,\log\left(x^2+1\right)-5\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^7/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] -1/4*(30*x^4 + 68*x^2 + 39)/(x^2 + 1)^2 + 1/12*(110*x^6 - 36*x^4 + 9*x^2 -
2)/x^6 + 5*log(x^2 + 1) - 5*log(x^2)
```

3.473 $\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{(2+x^2)^{3/2}}\,dx$

Optimal. Leaf size=25

$$-\frac{9\left(\frac{2}{x^2}+1\right)^{7/9}x}{10\sqrt{x^2+2}}$$

```
[Out] (-9*(1 + 2/x^2)^(7/9)*x)/(10*Sqrt[2 + x^2])
```

Rubi [A] time = 0.0569161, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {1991, 435, 264}

$$-\frac{9\left(\frac{2}{x^2}+1\right)^{7/9}x}{10\sqrt{x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + x^2)/x^2)^(7/9)/(2 + x^2)^(3/2),x]
```

```
[Out] (-9*(1 + 2/x^2)^(7/9)*x)/(10*Sqrt[2 + x^2])
```

Rule 1991

```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && BinomialQ[v, x] &&
!(BinomialMatchQ[u, x] && BinomialMatchQ[v, x])
```

Rule 435

```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbo
l] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart
[q], Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x], x] /; FreeQ[{a, b, c, d
, n, p, q}, x] && EqQ[mn, -n] &&  !IntegerQ[q] &&  !IntegerQ[p]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\left(\frac{2+x^2}{x^2}\right)^{7/9}}{\left(2+x^2\right)^{3/2}}\,dx = \int \frac{\left(1+\frac{2}{x^2}\right)^{7/9}}{\left(2+x^2\right)^{3/2}}\,dx$$
$$= \frac{\left(\left(1+\frac{2}{x^2}\right)^{7/9} x^{14/9}\right)\int \frac{1}{x^{14/9}\left(2+x^2\right)^{13/18}}\,dx}{\left(2+x^2\right)^{7/9}}$$
$$= -\frac{9\left(1+\frac{2}{x^2}\right)^{7/9} x}{10\sqrt{2+x^2}}$$

Mathematica [A] time = 0.0085718, size = 25, normalized size = 1.

$$-\frac{9\left(\frac{2}{x^2}+1\right)^{7/9} x}{10\sqrt{x^2+2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + x^2)/x^2)^(7/9)/(2 + x^2)^(3/2),x]
```

```
[Out] (-9*(1 + 2/x^2)^(7/9)*x)/(10*Sqrt[2 + x^2])
```

Maple [A] time = 0.004, size = 22, normalized size = 0.9

$$-\frac{9x}{10}\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}\frac{1}{\sqrt{x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x)
```

```
[Out] -9/10*x/(x^2+2)^(1/2)*((x^2+2)/x^2)^(7/9)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{\left(x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2), x)
```

Fricas [A] time = 4.55282, size = 61, normalized size = 2.44

$$-\frac{9\,x\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{10\,\sqrt{x^2+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x, algorithm="fricas")
```

```
[Out] -9/10*x*((x^2 + 2)/x^2)^(7/9)/sqrt(x^2 + 2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2+2)/x**2)**(7/9)/(x**2+2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\frac{x^2+2}{x^2}\right)^{\frac{7}{9}}}{\left(x^2+2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x^2+2)/x^2)^(7/9)/(x^2+2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(((x^2 + 2)/x^2)^(7/9)/(x^2 + 2)^(3/2), x)
```

3.474 $\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx$

Optimal. Leaf size=50

$$\frac{x^5}{175\left(\sqrt{10}-x^2\right)^{5/2}}+\frac{x^5}{7\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}$$

```
[Out] x^5/(7*Sqrt[10]*(Sqrt[10] - x^2)^(7/2)) + x^5/(175*(Sqrt[10] - x^2)^(5/2))
```

Rubi [A] time = 0.0151225, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {271, 264}

$$\frac{x^5}{5\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}-\frac{x^7}{175\left(\sqrt{10}-x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[10] - x^2)^(9/2),x]
```

```
[Out] x^5/(5*Sqrt[10]*(Sqrt[10] - x^2)^(7/2)) - x^7/(175*(Sqrt[10] - x^2)^(7/2))
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx &= \frac{x^5}{5\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}-\frac{1}{5}\sqrt{\frac{2}{5}}\int \frac{x^6}{\left(\sqrt{10}-x^2\right)^{9/2}}\,dx\\ &= \frac{x^5}{5\sqrt{10}\left(\sqrt{10}-x^2\right)^{7/2}}-\frac{x^7}{175\left(\sqrt{10}-x^2\right)^{7/2}}\end{aligned}$$

Mathematica [A] time = 0.0232837, size = 35, normalized size = 0.7

$$\frac{7\sqrt{10}x^5-2x^7}{350\left(\sqrt{10}-x^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[10] - x^2)^(9/2),x]
```

```
[Out] (7*Sqrt[10]*x^5 - 2*x^7)/(350*(Sqrt[10] - x^2)^(7/2))
```

Maple [A] time = 0.032, size = 28, normalized size = 0.6

$$\frac{x^5\left(-2x^2+7\sqrt{10}\right)}{350}\left(-x^2+\sqrt{10}\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+10^(1/2))^(9/2),x)
```

```
[Out] 1/350*x^5*(-2*x^2+7*10^(1/2))/(-x^2+10^(1/2))^(7/2)
```

Maxima [B] time = 1.41785, size = 107, normalized size = 2.14

$$\frac{x}{175\sqrt{-x^2+\sqrt{10}}}+\frac{\sqrt{10}x}{350\left(-x^2+\sqrt{10}\right)^{\frac{3}{2}}}+\frac{x^3}{4\left(-x^2+\sqrt{10}\right)^{\frac{7}{2}}}+\frac{3x}{140\left(-x^2+\sqrt{10}\right)^{\frac{5}{2}}}-\frac{3\sqrt{10}x}{28\left(-x^2+\sqrt{10}\right)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+10^(1/2))^(9/2),x, algorithm="maxima")
```

```
[Out] 1/175*x/sqrt(-x^2 + sqrt(10)) + 1/350*sqrt(10)*x/(-x^2 + sqrt(10))^(3/2) +
1/4*x^3/(-x^2 + sqrt(10))^(7/2) + 3/140*x/(-x^2 + sqrt(10))^(5/2) - 3/28*sq
rt(10)*x/(-x^2 + sqrt(10))^(7/2)
```

Fricas [A] time = 2.2025, size = 196, normalized size = 3.92

$$-\frac{\left(2x^{15}-160x^{11}-2600x^7+\sqrt{10}\left(x^{13}-340x^9-700x^5\right)\right)\sqrt{-x^2+\sqrt{10}}}{350\left(x^{16}-40x^{12}+600x^8-4000x^4+10000\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+10^(1/2))^(9/2),x, algorithm="fricas")
```

```
[Out] -1/350*(2*x^15 - 160*x^11 - 2600*x^7 + sqrt(10)*(x^13 - 340*x^9 - 700*x^5))
*sqrt(-x^2 + sqrt(10))/(x^16 - 40*x^12 + 600*x^8 - 4000*x^4 + 10000)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+10**(1/2))**(9/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16634, size = 132, normalized size = 2.64

$$-\frac{16\left(7\left(\frac{x}{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}-\frac{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}{x}\right)^2+20\right)}{175\left(\frac{x}{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}-\frac{\sqrt{-x^2+\sqrt{10}}-10^{\frac{1}{4}}}{x}\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+10^(1/2))^(9/2),x, algorithm="giac")
```

```
[Out] -16/175*(7*(x/(sqrt(-x^2 + sqrt(10)) - 10^(1/4)) - (sqrt(-x^2 + sqrt(10)) -
 10^(1/4))/x)^2 + 20)/(x/(sqrt(-x^2 + sqrt(10)) - 10^(1/4)) - (sqrt(-x^2 +
sqrt(10)) - 10^(1/4))/x)^7
```

3.475 $\int \frac{x^2}{(3-x^2)^{3/2}} \, dx$

Optimal. Leaf size=24

$$\frac{x}{\sqrt{3-x^2}} - \sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

```
[Out] x/Sqrt[3 - x^2] - ArcSin[x/Sqrt[3]]
```

Rubi [A] time = 0.0053872, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {288, 216}

$$\frac{x}{\sqrt{3-x^2}} - \sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(3 - x^2)^(3/2),x]
```

```
[Out] x/Sqrt[3 - x^2] - ArcSin[x/Sqrt[3]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\left(3-x^2\right)^{3/2}} \, dx &= \frac{x}{\sqrt{3-x^2}} - \int \frac{1}{\sqrt{3-x^2}} \, dx \\
&= \frac{x}{\sqrt{3-x^2}} - \sin^{-1}\left(\frac{x}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0156706, size = 24, normalized size = 1.

$$\frac{x}{\sqrt{3-x^2}} - \sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(3 - x^2)^(3/2),x]
```

```
[Out] x/Sqrt[3 - x^2] - ArcSin[x/Sqrt[3]]
```

Maple [A] time = 0.005, size = 22, normalized size = 0.9

$$-\arcsin\left(\frac{x\sqrt{3}}{3}\right)+x\frac{1}{\sqrt{-x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+3)^(3/2),x)
```

```
[Out] -arcsin(1/3*x*3^(1/2))+x/(-x^2+3)^(1/2)
```

Maxima [A] time = 1.40437, size = 28, normalized size = 1.17

$$\frac{x}{\sqrt{-x^2+3}}-\arcsin\left(\frac{1}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(-x^2 + 3) - arcsin(1/3*sqrt(3)*x)
```

Fricas [A] time = 2.14949, size = 90, normalized size = 3.75

$$\frac{\left(x^2-3\right)\arctan\left(\frac{\sqrt{-x^2+3}}{x}\right)-\sqrt{-x^2+3}x}{x^2-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] ((x^2 - 3)*arctan(sqrt(-x^2 + 3)/x) - sqrt(-x^2 + 3)*x)/(x^2 - 3)
```

Sympy [B] time = 0.516362, size = 49, normalized size = 2.04

$$-\frac{x^2\operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)}{x^2-3}-\frac{x\sqrt{3-x^2}}{x^2-3}+\frac{3\operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)}{x^2-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+3)**(3/2),x)
```

```
[Out] -x**2*asin(sqrt(3)*x/3)/(x**2 - 3) - x*sqrt(3 - x**2)/(x**2 - 3) + 3*asin(s
qrt(3)*x/3)/(x**2 - 3)
```

Giac [A] time = 1.08977, size = 39, normalized size = 1.62

$$-\frac{\sqrt{-x^2+3}x}{x^2-3} - \arcsin\left(\frac{1}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 3)*x/(x^2 - 3) - arcsin(1/3*sqrt(3)*x)
```

3.476 $\int \frac{(25-x^2)^{3/2}}{x^4} \, dx$

Optimal. Leaf size=40

$$-\frac{(25-x^2)^{3/2}}{3x^3}+\frac{\sqrt{25-x^2}}{x}+\sin^{-1}\left(\frac{x}{5}\right)$$

```
[Out] Sqrt[25 - x^2]/x - (25 - x^2)^(3/2)/(3*x^3) + ArcSin[x/5]
```

Rubi [A] time = 0.0085835, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {277, 216}

$$-\frac{(25-x^2)^{3/2}}{3x^3}+\frac{\sqrt{25-x^2}}{x}+\sin^{-1}\left(\frac{x}{5}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(25 - x^2)^(3/2)/x^4,x]

[Out] Sqrt[25 - x^2]/x - (25 - x^2)^(3/2)/(3*x^3) + ArcSin[x/5]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{(25-x^2)^{3/2}}{x^4} \, dx &= -\frac{(25-x^2)^{3/2}}{3x^3}-\int \frac{\sqrt{25-x^2}}{x^2} \, dx\\ &= \frac{\sqrt{25-x^2}}{x}-\frac{(25-x^2)^{3/2}}{3x^3}+\int \frac{1}{\sqrt{25-x^2}} \, dx\\ &= \frac{\sqrt{25-x^2}}{x}-\frac{(25-x^2)^{3/2}}{3x^3}+\sin^{-1}\left(\frac{x}{5}\right)\end{aligned}$$

Mathematica [C] time = 0.0031677, size = 24, normalized size = 0.6

$$-\frac{125\,{}_2F_1\left(-\frac{3}{2},-\frac{3}{2};-\frac{1}{2};\frac{x^2}{25}\right)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(25 - x^2)^(3/2)/x^4,x]
```

```
[Out] (-125*Hypergeometric2F1[-3/2, -3/2, -1/2, x^2/25])/(3*x^3)
```

Maple [A] time = 0.005, size = 58, normalized size = 1.5

$$-\frac{1}{75\,x^3}\left(-x^2+25\right)^{\frac{5}{2}}+\frac{2}{1875\,x}\left(-x^2+25\right)^{\frac{5}{2}}+\frac{2\,x}{1875}\left(-x^2+25\right)^{\frac{3}{2}}+\frac{x}{25}\sqrt{-x^2+25}+\arcsin\left(\frac{x}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+25)^(3/2)/x^4,x)
```

```
[Out] -1/75/x^3*(-x^2+25)^(5/2)+2/1875/x*(-x^2+25)^(5/2)+2/1875*x*(-x^2+25)^(3/2)
+1/25*x*(-x^2+25)^(1/2)+arcsin(1/5*x)
```

Maxima [A] time = 1.42977, size = 61, normalized size = 1.52

$$\frac{1}{25}\sqrt{-x^2+25}x+\frac{2\left(-x^2+25\right)^{\frac{3}{2}}}{75\,x}-\frac{\left(-x^2+25\right)^{\frac{5}{2}}}{75\,x^3}+\arcsin\left(\frac{1}{5}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+25)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/25*sqrt(-x^2 + 25)*x + 2/75*(-x^2 + 25)^(3/2)/x - 1/75*(-x^2 + 25)^(5/2)/
x^3 + arcsin(1/5*x)
```

Fricas [A] time = 2.01717, size = 109, normalized size = 2.72

$$-\frac{6\,x^3\arctan\left(\frac{\sqrt{-x^2+25}-5}{x}\right)-\left(4\,x^2-25\right)\sqrt{-x^2+25}}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+25)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] -1/3*(6*x^3*arctan((sqrt(-x^2 + 25) - 5)/x) - (4*x^2 - 25)*sqrt(-x^2 + 25))
/x^3
```

Sympy [A] time = 2.56946, size = 32, normalized size = 0.8

$$\operatorname{asin}\left(\frac{x}{5}\right)+\frac{4\sqrt{25-x^2}}{3x}-\frac{25\sqrt{25-x^2}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+25)**(3/2)/x**4,x)
```

```
[Out] asin(x/5) + 4*sqrt(25 - x**2)/(3*x) - 25*sqrt(25 - x**2)/(3*x**3)
```

Giac [B] time = 1.0877, size = 104, normalized size = 2.6

$$-\frac{x^{3}\left(\frac{15\left(\sqrt{-x^{2}+25}-5\right)^{2}}{x^{2}}-1\right)}{24\left(\sqrt{-x^{2}+25}-5\right)^{3}}+\frac{5\left(\sqrt{-x^{2}+25}-5\right)}{8x}-\frac{\left(\sqrt{-x^{2}+25}-5\right)^{3}}{24x^{3}}+\arcsin\left(\frac{1}{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+25)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] -1/24*x^3*(15*(sqrt(-x^2 + 25) - 5)^2/x^2 - 1)/(sqrt(-x^2 + 25) - 5)^3 + 5/
8*(sqrt(-x^2 + 25) - 5)/x - 1/24*(sqrt(-x^2 + 25) - 5)^3/x^3 + arcsin(1/5*x
)
```

3.477 $\int \frac{1}{\left(1-2x^2\right)^{7/2}}\,dx$

Optimal. Leaf size=49

$$\frac{8x}{15\sqrt{1-2x^2}}+\frac{4x}{15\left(1-2x^2\right)^{3/2}}+\frac{x}{5\left(1-2x^2\right)^{5/2}}$$

```
[Out] x/(5*(1 - 2*x^2)^(5/2)) + (4*x)/(15*(1 - 2*x^2)^(3/2)) + (8*x)/(15*Sqrt[1 -
 2*x^2])
```

Rubi [A] time = 0.0074578, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {192, 191}

$$\frac{8x}{15\sqrt{1-2x^2}}+\frac{4x}{15\left(1-2x^2\right)^{3/2}}+\frac{x}{5\left(1-2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x^2)^(-7/2),x]
```

```
[Out] x/(5*(1 - 2*x^2)^(5/2)) + (4*x)/(15*(1 - 2*x^2)^(3/2)) + (8*x)/(15*Sqrt[1 -
 2*x^2])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1-2x^2\right)^{7/2}}\,dx &= \frac{x}{5\left(1-2x^2\right)^{5/2}}+\frac{4}{5}\int \frac{1}{\left(1-2x^2\right)^{5/2}}\,dx\\
&= \frac{x}{5\left(1-2x^2\right)^{5/2}}+\frac{4x}{15\left(1-2x^2\right)^{3/2}}+\frac{8}{15}\int \frac{1}{\left(1-2x^2\right)^{3/2}}\,dx\\
&= \frac{x}{5\left(1-2x^2\right)^{5/2}}+\frac{4x}{15\left(1-2x^2\right)^{3/2}}+\frac{8x}{15\sqrt{1-2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0081629, size = 28, normalized size = 0.57

$$\frac{x\left(32x^4-40x^2+15\right)}{15\left(1-2x^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x^2)^(-7/2),x]
```

```
[Out] (x*(15 - 40*x^2 + 32*x^4))/(15*(1 - 2*x^2)^(5/2))
```

Maple [A] time = 0.002, size = 25, normalized size = 0.5

$$\frac{x\left(32\,x^{4}-40\,x^{2}+15\right)}{15}\left(-2\,x^{2}+1\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-2*x^2+1)^(7/2),x)
```

```
[Out] 1/15*x*(32*x^4-40*x^2+15)/(-2*x^2+1)^(5/2)
```

Maxima [A] time = 0.924798, size = 50, normalized size = 1.02

$$\frac{8\,x}{15\,\sqrt{-2\,x^{2}+1}}+\frac{4\,x}{15\left(-2\,x^{2}+1\right)^{\frac{3}{2}}}+\frac{x}{5\left(-2\,x^{2}+1\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+1)^(7/2),x, algorithm="maxima")
```

```
[Out] 8/15*x/sqrt(-2*x^2 + 1) + 4/15*x/(-2*x^2 + 1)^(3/2) + 1/5*x/(-2*x^2 + 1)^(5
/2)
```

Fricas [A] time = 2.10818, size = 105, normalized size = 2.14

$$-\frac{\left(32\,x^{5}-40\,x^{3}+15\,x\right)\sqrt{-2\,x^{2}+1}}{15\left(8\,x^{6}-12\,x^{4}+6\,x^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+1)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(32*x^5 - 40*x^3 + 15*x)*sqrt(-2*x^2 + 1)/(8*x^6 - 12*x^4 + 6*x^2 - 1
)
```

Sympy [B] time = 85.6648, size = 291, normalized size = 5.94

$$\begin{cases} -\frac{32ix^{5}}{60x^{4}\sqrt{2x^{2}-1}-60x^{2}\sqrt{2x^{2}-1}+15\sqrt{2x^{2}-1}}+\frac{40ix^{3}}{60x^{4}\sqrt{2x^{2}-1}-60x^{2}\sqrt{2x^{2}-1}+15\sqrt{2x^{2}-1}}-\frac{15ix}{60x^{4}\sqrt{2x^{2}-1}-60x^{2}\sqrt{2x^{2}-1}+15\sqrt{2x^{2}-1}} & \text{for}\ 2\left|x^{2}\right|>1 \\ \frac{32x^{5}}{60x^{4}\sqrt{1-2x^{2}}-60x^{2}\sqrt{1-2x^{2}}+15\sqrt{1-2x^{2}}}-\frac{40x^{3}}{60x^{4}\sqrt{1-2x^{2}}-60x^{2}\sqrt{1-2x^{2}}+15\sqrt{1-2x^{2}}}+\frac{15x}{60x^{4}\sqrt{1-2x^{2}}-60x^{2}\sqrt{1-2x^{2}}+15\sqrt{1-2x^{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x**2+1)**(7/2),x)
```

```
[Out] Piecewise((-32*I*x**5/(60*x**4*sqrt(2*x**2 - 1) - 60*x**2*sqrt(2*x**2 - 1)
+ 15*sqrt(2*x**2 - 1)) + 40*I*x**3/(60*x**4*sqrt(2*x**2 - 1) - 60*x**2*sqrt
(2*x**2 - 1) + 15*sqrt(2*x**2 - 1)) - 15*I*x/(60*x**4*sqrt(2*x**2 - 1) - 60
*x**2*sqrt(2*x**2 - 1) + 15*sqrt(2*x**2 - 1)), 2*Abs(x**2) > 1), (32*x**5/(
60*x**4*sqrt(1 - 2*x**2) - 60*x**2*sqrt(1 - 2*x**2) + 15*sqrt(1 - 2*x**2))
- 40*x**3/(60*x**4*sqrt(1 - 2*x**2) - 60*x**2*sqrt(1 - 2*x**2) + 15*sqrt(1
- 2*x**2)) + 15*x/(60*x**4*sqrt(1 - 2*x**2) - 60*x**2*sqrt(1 - 2*x**2) + 15
*sqrt(1 - 2*x**2)), True))
```

Giac [A] time = 1.08415, size = 47, normalized size = 0.96

$$-\frac{\left(8\left(4x^2-5\right)x^2+15\right)\sqrt{-2x^2+1}\,x}{15\left(2x^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-2*x^2+1)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*(8*(4*x^2 - 5)*x^2 + 15)*sqrt(-2*x^2 + 1)*x/(2*x^2 - 1)^3
```

3.478 $\int \frac{1}{\left(-7+6x-x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=47

$$-\frac{3-x}{6\sqrt{-x^2+6x-7}}-\frac{3-x}{6\left(-x^2+6x-7\right)^{3/2}}$$

```
[Out] -(3 - x)/(6*(-7 + 6*x - x^2)^(3/2)) - (3 - x)/(6*Sqrt[-7 + 6*x - x^2])
```

Rubi [A] time = 0.0074342, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {614, 613}

$$-\frac{3-x}{6\sqrt{-x^2+6x-7}}-\frac{3-x}{6\left(-x^2+6x-7\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(-7 + 6*x - x^2)^(-5/2),x]
```

```
[Out] -(3 - x)/(6*(-7 + 6*x - x^2)^(3/2)) - (3 - x)/(6*Sqrt[-7 + 6*x - x^2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-7+6x-x^2\right)^{5/2}}\,dx &= -\frac{3-x}{6\left(-7+6x-x^2\right)^{3/2}}+\frac{1}{3}\int \frac{1}{\left(-7+6x-x^2\right)^{3/2}}\,dx\\ &= -\frac{3-x}{6\left(-7+6x-x^2\right)^{3/2}}-\frac{3-x}{6\sqrt{-7+6x-x^2}}\end{aligned}$$

Mathematica [A] time = 0.0110864, size = 29, normalized size = 0.62

$$-\frac{(x-3)\left(x^2-6x+6\right)}{6\left(-x^2+6x-7\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-7 + 6*x - x^2)^(-5/2),x]
```

```
[Out] -((-3 + x)*(6 - 6*x + x^2))/(6*(-7 + 6*x - x^2)^(3/2))
```

Maple [A] time = 0.003, size = 28, normalized size = 0.6

$$-\frac{x^3-9x^2+24x-18}{6}\left(-x^2+6x-7\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+6*x-7)^(5/2),x)
```

```
[Out] -1/6*(x^3-9*x^2+24*x-18)/(-x^2+6*x-7)^(3/2)
```

Maxima [A] time = 0.932898, size = 80, normalized size = 1.7

$$\frac{x}{6\sqrt{-x^2+6x-7}}-\frac{1}{2\sqrt{-x^2+6x-7}}+\frac{x}{6\left(-x^2+6x-7\right)^{\frac{3}{2}}}-\frac{1}{2\left(-x^2+6x-7\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+6*x-7)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*x/sqrt(-x^2 + 6*x - 7) - 1/2/sqrt(-x^2 + 6*x - 7) + 1/6*x/(-x^2 + 6*x -
 7)^(3/2) - 1/2/(-x^2 + 6*x - 7)^(3/2)
```

Fricas [A] time = 2.11454, size = 120, normalized size = 2.55

$$-\frac{\left(x^3-9x^2+24x-18\right)\sqrt{-x^2+6x-7}}{6\left(x^4-12x^3+50x^2-84x+49\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+6*x-7)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(x^3 - 9*x^2 + 24*x - 18)*sqrt(-x^2 + 6*x - 7)/(x^4 - 12*x^3 + 50*x^2
- 84*x + 49)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^2+6x-7\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+6*x-7)**(5/2),x)
```

```
[Out] Integral((-x**2 + 6*x - 7)**(-5/2), x)
```

Giac [A] time = 1.08344, size = 47, normalized size = 1.

$$-\frac{(((x-9)x+24)x-18)\sqrt{-x^2+6x-7}}{6(x^2-6x+7)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+6*x-7)^(5/2),x, algorithm="giac")
```

```
[Out] -1/6*(((x - 9)*x + 24)*x - 18)*sqrt(-x^2 + 6*x - 7)/(x^2 - 6*x + 7)^2
```

3.479 $\int \left(1-2x-2x^2\right)^3 \, dx$

Optimal. Leaf size=36

$$-\frac{8x^7}{7}-4x^6-\frac{12x^5}{5}+4x^4+2x^3-3x^2+x$$

```
[Out] x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

Rubi [A] time = 0.0116741, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {611}

$$-\frac{8x^7}{7}-4x^6-\frac{12x^5}{5}+4x^4+2x^3-3x^2+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x - 2*x^2)^3,x]

[Out] x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

Rule 611

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegr
and[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && IGtQ[p, 0] && (EqQ[a, 0] ||  !PerfectSquareQ[b^2 - 4*a*c])
```

Rubi steps

$$\begin{aligned}\int \left(1-2x-2x^2\right)^3 \, dx &= \int \left(1-6x+6x^2+16x^3-12x^4-24x^5-8x^6\right) dx\\ &= x-3x^2+2x^3+4x^4-\frac{12x^5}{5}-4x^6-\frac{8x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0012721, size = 36, normalized size = 1.

$$-\frac{8x^7}{7}-4x^6-\frac{12x^5}{5}+4x^4+2x^3-3x^2+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x - 2*x^2)^3,x]

[Out] x - 3*x^2 + 2*x^3 + 4*x^4 - (12*x^5)/5 - 4*x^6 - (8*x^7)/7
```

Maple [A] time = 0.001, size = 33, normalized size = 0.9

$$x-3\,x^2+2\,x^3+4\,x^4-\frac{12\,x^5}{5}-4\,x^6-\frac{8\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2-2*x+1)^3,x)
```

```
[Out] x-3*x^2+2*x^3+4*x^4-12/5*x^5-4*x^6-8/7*x^7
```

Maxima [A] time = 0.945616, size = 43, normalized size = 1.19

$$-\frac{8}{7}x^7 - 4x^6 - \frac{12}{5}x^5 + 4x^4 + 2x^3 - 3x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2-2*x+1)^3,x, algorithm="maxima")
```

```
[Out] -8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

Fricas [A] time = 1.82526, size = 77, normalized size = 2.14

$$-\frac{8}{7}x^7 - 4x^6 - \frac{12}{5}x^5 + 4x^4 + 2x^3 - 3x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2-2*x+1)^3,x, algorithm="fricas")
```

```
[Out] -8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

Sympy [A] time = 0.055361, size = 34, normalized size = 0.94

$$-\frac{8x^7}{7} - 4x^6 - \frac{12x^5}{5} + 4x^4 + 2x^3 - 3x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x**2-2*x+1)**3,x)
```

```
[Out] -8*x**7/7 - 4*x**6 - 12*x**5/5 + 4*x**4 + 2*x**3 - 3*x**2 + x
```

Giac [A] time = 1.06285, size = 43, normalized size = 1.19

$$-\frac{8}{7}x^7 - 4x^6 - \frac{12}{5}x^5 + 4x^4 + 2x^3 - 3x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*x^2-2*x+1)^3,x, algorithm="giac")
```

```
[Out] -8/7*x^7 - 4*x^6 - 12/5*x^5 + 4*x^4 + 2*x^3 - 3*x^2 + x
```

3.480 $\int(-1+5x)\left(-1-x+x^2\right)^2\,dx$

Optimal. Leaf size=39

$$\frac{5x^6}{6}-\frac{11x^5}{5}-\frac{3x^4}{4}+\frac{11x^3}{3}+\frac{3x^2}{2}-x$$

```
[Out] -x + (3*x^2)/2 + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

Rubi [A] time = 0.0141987, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {631}

$$\frac{5x^6}{6}-\frac{11x^5}{5}-\frac{3x^4}{4}+\frac{11x^3}{3}+\frac{3x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 5*x)*(-1 - x + x^2)^2,x]
```

```
[Out] -x + (3*x^2)/2 + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}\int(-1+5x)\left(-1-x+x^2\right)^2\,dx&=\int\left(-1+3x+11x^2-3x^3-11x^4+5x^5\right)dx\\&=-x+\frac{3x^2}{2}+\frac{11x^3}{3}-\frac{3x^4}{4}-\frac{11x^5}{5}+\frac{5x^6}{6}\end{aligned}$$

Mathematica [A] time = 0.0013804, size = 39, normalized size = 1.

$$\frac{5x^6}{6}-\frac{11x^5}{5}-\frac{3x^4}{4}+\frac{11x^3}{3}+\frac{3x^2}{2}-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 5*x)*(-1 - x + x^2)^2,x]
```

```
[Out] -x + (3*x^2)/2 + (11*x^3)/3 - (3*x^4)/4 - (11*x^5)/5 + (5*x^6)/6
```

Maple [A] time = 0.001, size = 30, normalized size = 0.8

$$-x+\frac{3\,x^2}{2}+\frac{11\,x^3}{3}-\frac{3\,x^4}{4}-\frac{11\,x^5}{5}+\frac{5\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+5*x)*(x^2-x-1)^2,x)
```

```
[Out] -x+3/2*x^2+11/3*x^3-3/4*x^4-11/5*x^5+5/6*x^6
```

Maxima [A] time = 0.929819, size = 39, normalized size = 1.

$$\frac{5}{6}\,x^{6} - \frac{11}{5}\,x^{5} - \frac{3}{4}\,x^{4} + \frac{11}{3}\,x^{3} + \frac{3}{2}\,x^{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+5*x)*(x^2-x-1)^2,x, algorithm="maxima")
```

```
[Out] 5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

Fricas [A] time = 1.75093, size = 74, normalized size = 1.9

$$\frac{5}{6}x^{6} - \frac{11}{5}x^{5} - \frac{3}{4}x^{4} + \frac{11}{3}x^{3} + \frac{3}{2}x^{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+5*x)*(x^2-x-1)^2,x, algorithm="fricas")
```

```
[Out] 5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

Sympy [A] time = 0.057382, size = 34, normalized size = 0.87

$$\frac{5x^{6}}{6} - \frac{11x^{5}}{5} - \frac{3x^{4}}{4} + \frac{11x^{3}}{3} + \frac{3x^{2}}{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+5*x)*(x**2-x-1)**2,x)
```

```
[Out] 5*x**6/6 - 11*x**5/5 - 3*x**4/4 + 11*x**3/3 + 3*x**2/2 - x
```

Giac [A] time = 1.06274, size = 39, normalized size = 1.

$$\frac{5}{6}\,x^{6} - \frac{11}{5}\,x^{5} - \frac{3}{4}\,x^{4} + \frac{11}{3}\,x^{3} + \frac{3}{2}\,x^{2} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+5*x)*(x^2-x-1)^2,x, algorithm="giac")
```

```
[Out] 5/6*x^6 - 11/5*x^5 - 3/4*x^4 + 11/3*x^3 + 3/2*x^2 - x
```

3.481 $\int \frac{1+3x}{(1-8x+2x^2)^{5/2}}\,dx$

Optimal. Leaf size=47

$$\frac{1-2x}{6\left(2x^2-8x+1\right)^{3/2}}-\frac{2(2-x)}{21\sqrt{2x^2-8x+1}}$$

```
[Out] (1 - 2*x)/(6*(1 - 8*x + 2*x^2)^(3/2)) - (2*(2 - x))/(21*Sqrt[1 - 8*x + 2*x^
2])
```

Rubi [A] time = 0.0089818, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {638, 613}

$$\frac{1-2x}{6\left(2x^2-8x+1\right)^{3/2}}-\frac{2(2-x)}{21\sqrt{2x^2-8x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 3*x)/(1 - 8*x + 2*x^2)^(5/2),x]
```

```
[Out] (1 - 2*x)/(6*(1 - 8*x + 2*x^2)^(3/2)) - (2*(2 - x))/(21*Sqrt[1 - 8*x + 2*x^
2])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+3x}{\left(1-8x+2x^2\right)^{5/2}}\,dx &= \frac{1-2x}{6\left(1-8x+2x^2\right)^{3/2}}-\frac{2}{3}\int \frac{1}{\left(1-8x+2x^2\right)^{3/2}}\,dx\\ &= \frac{1-2x}{6\left(1-8x+2x^2\right)^{3/2}}-\frac{2(2-x)}{21\sqrt{1-8x+2x^2}}\end{aligned}$$

Mathematica [A] time = 0.0468753, size = 33, normalized size = 0.7

$$\frac{8x^3-48x^2+54x-1}{42\left(2x^2-8x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 3*x)/(1 - 8*x + 2*x^2)^(5/2),x]
```

```
[Out] (-1 + 54*x - 48*x^2 + 8*x^3)/(42*(1 - 8*x + 2*x^2)^(3/2))
```

Maple [A] time = 0.003, size = 30, normalized size = 0.6

$$\frac{8x^3-48x^2+54x-1}{42}\left(2x^2-8x+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+3*x)/(2*x^2-8*x+1)^(5/2),x)
```

```
[Out] 1/42*(8*x^3-48*x^2+54*x-1)/(2*x^2-8*x+1)^(3/2)
```

Maxima [A] time = 0.937834, size = 80, normalized size = 1.7

$$\frac{2x}{21\sqrt{2x^2-8x+1}}-\frac{4}{21\sqrt{2x^2-8x+1}}-\frac{x}{3\left(2x^2-8x+1\right)^{\frac{3}{2}}}+\frac{1}{6\left(2x^2-8x+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/21*x/sqrt(2*x^2 - 8*x + 1) - 4/21/sqrt(2*x^2 - 8*x + 1) - 1/3*x/(2*x^2 -
8*x + 1)^(3/2) + 1/6/(2*x^2 - 8*x + 1)^(3/2)
```

Fricas [A] time = 2.01195, size = 180, normalized size = 3.83

$$-\frac{4x^4-32x^3+68x^2-\left(8x^3-48x^2+54x-1\right)\sqrt{2x^2-8x+1}-16x+1}{42\left(4x^4-32x^3+68x^2-16x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/42*(4*x^4 - 32*x^3 + 68*x^2 - (8*x^3 - 48*x^2 + 54*x - 1)*sqrt(2*x^2 - 8
*x + 1) - 16*x + 1)/(4*x^4 - 32*x^3 + 68*x^2 - 16*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{3x+1}{\left(2x^2-8x+1\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+3*x)/(2*x**2-8*x+1)**(5/2),x)
```

```
[Out] Integral((3*x + 1)/(2*x**2 - 8*x + 1)**(5/2), x)
```

Giac [A] time = 1.08805, size = 36, normalized size = 0.77

$$\frac{2\,(4\,(x-6)x+27)x-1}{42\,\left(2\,x^2-8\,x+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+3*x)/(2*x^2-8*x+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/42*(2*(4*(x - 6)*x + 27)*x - 1)/(2*x^2 - 8*x + 1)^(3/2)
```

3.482 $\int \frac{-1-8x+8x^3}{\left(1+2x-4x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{4(x+1)}{15\left(-4x^2+2x+1\right)^{3/2}}-\frac{122x+7}{75\sqrt{-4x^2+2x+1}}$$

```
[Out] (-4*(1 + x))/(15*(1 + 2*x - 4*x^2)^(3/2)) - (7 + 122*x)/(75*Sqrt[1 + 2*x -
4*x^2])
```

Rubi [A] time = 0.0216626, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1660, 636}

$$-\frac{4(x+1)}{15\left(-4x^2+2x+1\right)^{3/2}}-\frac{122x+7}{75\sqrt{-4x^2+2x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 8*x + 8*x^3)/(1 + 2*x - 4*x^2)^(5/2),x]
```

```
[Out] (-4*(1 + x))/(15*(1 + 2*x - 4*x^2)^(3/2)) - (7 + 122*x)/(75*Sqrt[1 + 2*x -
4*x^2])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 636

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x
 + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1-8x+8x^3}{\left(1+2x-4x^2\right)^{5/2}}\,dx &= -\frac{4(1+x)}{15\left(1+2x-4x^2\right)^{3/2}}-\frac{1}{30}\int \frac{46+60x}{\left(1+2x-4x^2\right)^{3/2}}\,dx\\ &= -\frac{4(1+x)}{15\left(1+2x-4x^2\right)^{3/2}}-\frac{7+122x}{75\sqrt{1+2x-4x^2}}\end{aligned}$$

Mathematica [A] time = 0.112204, size = 33, normalized size = 0.73

$$-\frac{-488x^3+216x^2+156x+27}{75\left(-4x^2+2x+1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 8*x + 8*x^3)/(1 + 2*x - 4*x^2)^(5/2),x]
```

```
[Out] -(27 + 156*x + 216*x^2 - 488*x^3)/(75*(1 + 2*x - 4*x^2)^(3/2))
```

Maple [A] time = 0.005, size = 30, normalized size = 0.7

$$\frac{488\,x^3-216\,x^2-156\,x-27}{75}\left(-4\,x^2+2\,x+1\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x)
```

```
[Out] 1/75*(488*x^3-216*x^2-156*x-27)/(-4*x^2+2*x+1)^(3/2)
```

Maxima [B] time = 0.955753, size = 103, normalized size = 2.29

$$-\frac{122\,x}{75\,\sqrt{-4\,x^2+2\,x+1}}+\frac{2\,x^2}{\left(-4\,x^2+2\,x+1\right)^{\frac{3}{2}}}+\frac{61}{150\,\sqrt{-4\,x^2+2\,x+1}}-\frac{19\,x}{15\left(-4\,x^2+2\,x+1\right)^{\frac{3}{2}}}-\frac{23}{30\left(-4\,x^2+2\,x+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x, algorithm="maxima")
```

```
[Out] -122/75*x/sqrt(-4*x^2 + 2*x + 1) + 2*x^2/(-4*x^2 + 2*x + 1)^(3/2) + 61/150/
sqrt(-4*x^2 + 2*x + 1) - 19/15*x/(-4*x^2 + 2*x + 1)^(3/2) - 23/30/(-4*x^2 +
 2*x + 1)^(3/2)
```

Fricas [A] time = 2.02849, size = 194, normalized size = 4.31

$$-\frac{432\,x^4-432\,x^3-108\,x^2-\left(488\,x^3-216\,x^2-156\,x-27\right)\sqrt{-4\,x^2+2\,x+1}+108\,x+27}{75\left(16\,x^4-16\,x^3-4\,x^2+4\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/75*(432*x^4 - 432*x^3 - 108*x^2 - (488*x^3 - 216*x^2 - 156*x - 27)*sqrt(
-4*x^2 + 2*x + 1) + 108*x + 27)/(16*x^4 - 16*x^3 - 4*x^2 + 4*x + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**3-8*x-1)/(-4*x**2+2*x+1)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09662, size = 55, normalized size = 1.22

$$\frac{(4(2(61x-27)x-39)x-27)\sqrt{-4x^2+2x+1}}{75\left(4x^2-2x-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3-8*x-1)/(-4*x^2+2*x+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/75*(4*(2*(61*x - 27)*x - 39)*x - 27)*sqrt(-4*x^2 + 2*x + 1)/(4*x^2 - 2*x
- 1)^2
```

3.483 $\int x^2 \cos^5(x)\,dx$

Optimal. Leaf size=83

$$\frac{8}{15}x^2\sin(x)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{4}{15}x^2\sin(x)\cos^2(x)-\frac{2\sin^5(x)}{125}+\frac{76\sin^3(x)}{675}-\frac{298\sin(x)}{225}+\frac{2}{25}x\cos^5(x)+\frac{8}{45}x\,\mathrm{c}$$

```
[Out] (16*x*Cos[x])/15 + (8*x*Cos[x]^3)/45 + (2*x*Cos[x]^5)/25 - (298*Sin[x])/225
 + (8*x^2*Sin[x])/15 + (4*x^2*Cos[x]^2*Sin[x])/15 + (x^2*Cos[x]^4*Sin[x])/5
 + (76*Sin[x]^3)/675 - (2*Sin[x]^5)/125
```

Rubi [A] time = 0.0937162, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3311, 3296, 2637, 2633}

$$\frac{8}{15}x^2\sin(x)+\frac{1}{5}x^2\sin(x)\cos^4(x)+\frac{4}{15}x^2\sin(x)\cos^2(x)-\frac{2\sin^5(x)}{125}+\frac{76\sin^3(x)}{675}-\frac{298\sin(x)}{225}+\frac{2}{25}x\cos^5(x)+\frac{8}{45}x\,\mathrm{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x]^5,x]
```

```
[Out] (16*x*Cos[x])/15 + (8*x*Cos[x]^3)/45 + (2*x*Cos[x]^5)/25 - (298*Sin[x])/225
 + (8*x^2*Sin[x])/15 + (4*x^2*Cos[x]^2*Sin[x])/15 + (x^2*Cos[x]^4*Sin[x])/5
 + (76*Sin[x]^3)/675 - (2*Sin[x]^5)/125
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int x^2\cos^5(x)\,dx = \frac{2}{25}x\cos^5(x)+\frac{1}{5}x^2\cos^4(x)\sin(x)-\frac{2}{25}\int\cos^5(x)\,dx+\frac{4}{5}\int x^2\cos^3(x)\,dx$$
$$=\frac{8}{45}x\cos^3(x)+\frac{2}{25}x\cos^5(x)+\frac{4}{15}x^2\cos^2(x)\sin(x)+\frac{1}{5}x^2\cos^4(x)\sin(x)+\frac{2}{25}\text{Subst}\left(\int\left(1-2x^2+x^4\right)\right.$$
$$=\frac{8}{45}x\cos^3(x)+\frac{2}{25}x\cos^5(x)-\frac{2\sin(x)}{25}+\frac{8}{15}x^2\sin(x)+\frac{4}{15}x^2\cos^2(x)\sin(x)+\frac{1}{5}x^2\cos^4(x)\sin(x)+\frac{4\,\mathrm{si}}{}$$
$$=\frac{16}{15}x\cos(x)+\frac{8}{45}x\cos^3(x)+\frac{2}{25}x\cos^5(x)-\frac{58\sin(x)}{225}+\frac{8}{15}x^2\sin(x)+\frac{4}{15}x^2\cos^2(x)\sin(x)+\frac{1}{5}x^2\cos^4($$
$$=\frac{16}{15}x\cos(x)+\frac{8}{45}x\cos^3(x)+\frac{2}{25}x\cos^5(x)-\frac{298\sin(x)}{225}+\frac{8}{15}x^2\sin(x)+\frac{4}{15}x^2\cos^2(x)\sin(x)+\frac{1}{5}x^2\cos$$

Mathematica [A] time = 0.0539509, size = 67, normalized size = 0.81

$$\frac{5}{8}\left(x^2-2\right)\sin(x)+\frac{5}{432}\left(9x^2-2\right)\sin(3x)+\frac{\left(25x^2-2\right)\sin(5x)}{2000}+\frac{5}{4}x\cos(x)+\frac{5}{72}x\cos(3x)+\frac{1}{200}x\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x]^5,x]
```

```
[Out] (5*x*Cos[x])/4 + (5*x*Cos[3*x])/72 + (x*Cos[5*x])/200 + (5*(-2 + x^2)*Sin[x
])/8 + (5*(-2 + 9*x^2)*Sin[3*x])/432 + ((-2 + 25*x^2)*Sin[5*x])/2000
```

Maple [A] time = 0.012, size = 70, normalized size = 0.8

$$\frac{x^2\sin(x)}{5}\left(\frac{8}{3}+(\cos(x))^4+\frac{4\,(\cos(x))^2}{3}\right)-\frac{16\,\sin(x)}{15}+\frac{16\,x\cos(x)}{15}+\frac{2\,x\,(\cos(x))^5}{25}-\frac{2\,\sin(x)}{125}\left(\frac{8}{3}+(\cos(x))^4+\frac{4\,(\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^5,x)
```

```
[Out] 1/5*x^2*(8/3+cos(x)^4+4/3*cos(x)^2)*sin(x)-16/15*sin(x)+16/15*x*cos(x)+2/25
*x*cos(x)^5-2/125*(8/3+cos(x)^4+4/3*cos(x)^2)*sin(x)+8/45*x*cos(x)^3-8/135*
(2+cos(x)^2)*sin(x)
```

Maxima [A] time = 0.960549, size = 74, normalized size = 0.89

$$\frac{1}{200}x\cos(5x)+\frac{5}{72}x\cos(3x)+\frac{5}{4}x\cos(x)+\frac{1}{2000}\left(25x^2-2\right)\sin(5x)+\frac{5}{432}\left(9x^2-2\right)\sin(3x)+\frac{5}{8}\left(x^2-2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^5,x, algorithm="maxima")
```

```
[Out] 1/200*x*cos(5*x) + 5/72*x*cos(3*x) + 5/4*x*cos(x) + 1/2000*(25*x^2 - 2)*sin
(5*x) + 5/432*(9*x^2 - 2)*sin(3*x) + 5/8*(x^2 - 2)*sin(x)
```

Fricas [A] time = 2.31121, size = 190, normalized size = 2.29

$$\frac{2}{25}\,x\cos\left(x\right)^{5}+\frac{8}{45}\,x\cos\left(x\right)^{3}+\frac{16}{15}\,x\cos\left(x\right)+\frac{1}{3375}\left(27\left(25\,x^{2}-2\right)\cos\left(x\right)^{4}+4\left(225\,x^{2}-68\right)\cos\left(x\right)^{2}+1800\,x^{2}-414$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^5,x, algorithm="fricas")
```

```
[Out] 2/25*x*cos(x)^5 + 8/45*x*cos(x)^3 + 16/15*x*cos(x) + 1/3375*(27*(25*x^2 - 2
)*cos(x)^4 + 4*(225*x^2 - 68)*cos(x)^2 + 1800*x^2 - 4144)*sin(x)
```

Sympy [A] time = 3.46872, size = 112, normalized size = 1.35

$$\frac{8x^{2}\sin^{5}\left(x\right)}{15}+\frac{4x^{2}\sin^{3}\left(x\right)\cos^{2}\left(x\right)}{3}+x^{2}\sin\left(x\right)\cos^{4}\left(x\right)+\frac{16x\sin^{4}\left(x\right)\cos\left(x\right)}{15}+\frac{104x\sin^{2}\left(x\right)\cos^{3}\left(x\right)}{45}+\frac{298x\cos^{5}\left(x\right)}{225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x)**5,x)
```

```
[Out] 8*x**2*sin(x)**5/15 + 4*x**2*sin(x)**3*cos(x)**2/3 + x**2*sin(x)*cos(x)**4
+ 16*x*sin(x)**4*cos(x)/15 + 104*x*sin(x)**2*cos(x)**3/45 + 298*x*cos(x)**5
/225 - 4144*sin(x)**5/3375 - 1712*sin(x)**3*cos(x)**2/675 - 298*sin(x)*cos(
x)**4/225
```

Giac [A] time = 1.07878, size = 74, normalized size = 0.89

$$\frac{1}{200}\,x\cos\left(5\,x\right)+\frac{5}{72}\,x\cos\left(3\,x\right)+\frac{5}{4}\,x\cos\left(x\right)+\frac{1}{2000}\left(25\,x^{2}-2\right)\sin\left(5\,x\right)+\frac{5}{432}\left(9\,x^{2}-2\right)\sin\left(3\,x\right)+\frac{5}{8}\left(x^{2}-2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^5,x, algorithm="giac")
```

```
[Out] 1/200*x*cos(5*x) + 5/72*x*cos(3*x) + 5/4*x*cos(x) + 1/2000*(25*x^2 - 2)*sin
(5*x) + 5/432*(9*x^2 - 2)*sin(3*x) + 5/8*(x^2 - 2)*sin(x)
```

3.484 $\int x^3 \sin^3(x) \, dx$

Optimal. Leaf size=73

$$\frac{1}{3}x^2 \sin^3(x) + 2x^2 \sin(x) - \frac{2}{3}x^3 \cos(x) - \frac{1}{3}x^3 \sin^2(x) \cos(x) - \frac{2\sin^3(x)}{27} - \frac{40\sin(x)}{9} + \frac{40}{9}x\cos(x) + \frac{2}{9}x\sin^2(x)\cos(x)$$

```
[Out] (40*x*Cos[x])/9 - (2*x^3*Cos[x])/3 - (40*Sin[x])/9 + 2*x^2*Sin[x] + (2*x*Co
s[x]*Sin[x]^2)/9 - (x^3*Cos[x]*Sin[x]^2)/3 - (2*Sin[x]^3)/27 + (x^2*Sin[x]^
3)/3
```

Rubi [A] time = 0.0842397, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3311, 3296, 2637, 3310}

$$\frac{1}{3}x^2 \sin^3(x) + 2x^2 \sin(x) - \frac{2}{3}x^3 \cos(x) - \frac{1}{3}x^3 \sin^2(x) \cos(x) - \frac{2\sin^3(x)}{27} - \frac{40\sin(x)}{9} + \frac{40}{9}x\cos(x) + \frac{2}{9}x\sin^2(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[x]^3,x]
```

```
[Out] (40*x*Cos[x])/9 - (2*x^3*Cos[x])/3 - (40*Sin[x])/9 + 2*x^2*Sin[x] + (2*x*Co
s[x]*Sin[x]^2)/9 - (x^3*Cos[x]*Sin[x]^2)/3 - (2*Sin[x]^3)/27 + (x^2*Sin[x]^
3)/3
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int x^3\sin^3(x)\,dx &= -\frac{1}{3}x^3\cos(x)\sin^2(x)+\frac{1}{3}x^2\sin^3(x)+\frac{2}{3}\int x^3\sin(x)\,dx-\frac{2}{3}\int x\sin^3(x)\,dx\\ &= -\frac{2}{3}x^3\cos(x)+\frac{2}{9}x\cos(x)\sin^2(x)-\frac{1}{3}x^3\cos(x)\sin^2(x)-\frac{2\sin^3(x)}{27}+\frac{1}{3}x^2\sin^3(x)-\frac{4}{9}\int x\sin(x)\,dx\\ &= \frac{4}{9}x\cos(x)-\frac{2}{3}x^3\cos(x)+2x^2\sin(x)+\frac{2}{9}x\cos(x)\sin^2(x)-\frac{1}{3}x^3\cos(x)\sin^2(x)-\frac{2\sin^3(x)}{27}+\frac{1}{3}x^2\sin\\ &= \frac{40}{9}x\cos(x)-\frac{2}{3}x^3\cos(x)-\frac{4\sin(x)}{9}+2x^2\sin(x)+\frac{2}{9}x\cos(x)\sin^2(x)-\frac{1}{3}x^3\cos(x)\sin^2(x)-\frac{2\sin^3(}{27}\\ &= \frac{40}{9}x\cos(x)-\frac{2}{3}x^3\cos(x)-\frac{40\sin(x)}{9}+2x^2\sin(x)+\frac{2}{9}x\cos(x)\sin^2(x)-\frac{1}{3}x^3\cos(x)\sin^2(x)-\frac{2\sin}{27}\end{aligned}$$

Mathematica [A] time = 0.0958343, size = 51, normalized size = 0.7

$$\frac{1}{108}\left(243\left(x^2-2\right)\sin(x)-\left(9x^2-2\right)\sin(3x)-81x\left(x^2-6\right)\cos(x)+3x\left(3x^2-2\right)\cos(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x]^3,x]
```

```
[Out] (-81*x*(-6 + x^2)*Cos[x] + 3*x*(-2 + 3*x^2)*Cos[3*x] + 243*(-2 + x^2)*Sin[x
] - (-2 + 9*x^2)*Sin[3*x])/108
```

Maple [A] time = 0.023, size = 57, normalized size = 0.8

$$-\frac{x^3\left(2+(\sin(x))^2\right)\cos(x)}{3}+2x^2\sin(x)-\frac{40\sin(x)}{9}+4x\cos(x)+\frac{x^2(\sin(x))^3}{3}+\frac{2x\left(2+(\sin(x))^2\right)\cos(x)}{9}-\frac{2\,(\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x)^3,x)
```

```
[Out] -1/3*x^3*(2+sin(x)^2)*cos(x)+2*x^2*sin(x)-40/9*sin(x)+4*x*cos(x)+1/3*x^2*si
n(x)^3+2/9*x*(2+sin(x)^2)*cos(x)-2/27*sin(x)^3
```

Maxima [A] time = 0.976939, size = 66, normalized size = 0.9

$$\frac{1}{36}\left(3x^3-2x\right)\cos(3x)-\frac{3}{4}\left(x^3-6x\right)\cos(x)-\frac{1}{108}\left(9x^2-2\right)\sin(3x)+\frac{9}{4}\left(x^2-2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/36*(3*x^3 - 2*x)*cos(3*x) - 3/4*(x^3 - 6*x)*cos(x) - 1/108*(9*x^2 - 2)*si
n(3*x) + 9/4*(x^2 - 2)*sin(x)
```

Fricas [A] time = 2.24924, size = 146, normalized size = 2.

$$\frac{1}{9}\left(3x^3-2x\right)\cos\left(x\right)^3-\frac{1}{3}\left(3x^3-14x\right)\cos\left(x\right)-\frac{1}{27}\left(\left(9x^2-2\right)\cos\left(x\right)^2-63x^2+122\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/9*(3*x^3 - 2*x)*cos(x)^3 - 1/3*(3*x^3 - 14*x)*cos(x) - 1/27*((9*x^2 - 2)*
cos(x)^2 - 63*x^2 + 122)*sin(x)
```

Sympy [A] time = 1.95342, size = 92, normalized size = 1.26

$$-x^3\sin^2\left(x\right)\cos\left(x\right)-\frac{2x^3\cos^3\left(x\right)}{3}+\frac{7x^2\sin^3\left(x\right)}{3}+2x^2\sin\left(x\right)\cos^2\left(x\right)+\frac{14x\sin^2\left(x\right)\cos\left(x\right)}{3}+\frac{40x\cos^3\left(x\right)}{9}-\frac{122\sin^3\left(x\right)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(x)**3,x)
```

```
[Out] -x**3*sin(x)**2*cos(x) - 2*x**3*cos(x)**3/3 + 7*x**2*sin(x)**3/3 + 2*x**2*s
in(x)*cos(x)**2 + 14*x*sin(x)**2*cos(x)/3 + 40*x*cos(x)**3/9 - 122*sin(x)**
3/27 - 40*sin(x)*cos(x)**2/9
```

Giac [A] time = 1.07071, size = 66, normalized size = 0.9

$$\frac{1}{36}\left(3x^3-2x\right)\cos\left(3x\right)-\frac{3}{4}\left(x^3-6x\right)\cos\left(x\right)-\frac{1}{108}\left(9x^2-2\right)\sin\left(3x\right)+\frac{9}{4}\left(x^2-2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/36*(3*x^3 - 2*x)*cos(3*x) - 3/4*(x^3 - 6*x)*cos(x) - 1/108*(9*x^2 - 2)*si
n(3*x) + 9/4*(x^2 - 2)*sin(x)
```

3.485 $\int x^2 \sin^6(x)\,dx$

Optimal. Leaf size=105

$$\frac{5x^3}{48}-\frac{1}{6}x^2\sin^5(x)\cos(x)-\frac{5}{24}x^2\sin^3(x)\cos(x)-\frac{5}{16}x^2\sin(x)\cos(x)-\frac{245x}{1152}+\frac{1}{18}x\sin^6(x)+\frac{5}{48}x\sin^4(x)+\frac{5}{16}x\sin^2($$

```
[Out] (-245*x)/1152 + (5*x^3)/48 + (245*Cos[x]*Sin[x])/1152 - (5*x^2*Cos[x]*Sin[x
])/16 + (5*x*Sin[x]^2)/16 + (65*Cos[x]*Sin[x]^3)/1728 - (5*x^2*Cos[x]*Sin[x
]^3)/24 + (5*x*Sin[x]^4)/48 + (Cos[x]*Sin[x]^5)/108 - (x^2*Cos[x]*Sin[x]^5)
/6 + (x*Sin[x]^6)/18
```

Rubi [A] time = 0.109744, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3311, 30, 2635, 8}

$$\frac{5x^3}{48}-\frac{1}{6}x^2\sin^5(x)\cos(x)-\frac{5}{24}x^2\sin^3(x)\cos(x)-\frac{5}{16}x^2\sin(x)\cos(x)-\frac{245x}{1152}+\frac{1}{18}x\sin^6(x)+\frac{5}{48}x\sin^4(x)+\frac{5}{16}x\sin^2($$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[x]^6,x]
```


```
[Out] (-245*x)/1152 + (5*x^3)/48 + (245*Cos[x]*Sin[x])/1152 - (5*x^2*Cos[x]*Sin[x
])/16 + (5*x*Sin[x]^2)/16 + (65*Cos[x]*Sin[x]^3)/1728 - (5*x^2*Cos[x]*Sin[x
]^3)/24 + (5*x*Sin[x]^4)/48 + (Cos[x]*Sin[x]^5)/108 - (x^2*Cos[x]*Sin[x]^5)
/6 + (x*Sin[x]^6)/18
```


Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int x^2\sin^6(x)\,dx = -\frac{1}{6}x^2\cos(x)\sin^5(x)+\frac{1}{18}x\sin^6(x)-\frac{1}{18}\int\sin^6(x)\,dx+\frac{5}{6}\int x^2\sin^4(x)\,dx$$
$$= -\frac{5}{24}x^2\cos(x)\sin^3(x)+\frac{5}{48}x\sin^4(x)+\frac{1}{108}\cos(x)\sin^5(x)-\frac{1}{6}x^2\cos(x)\sin^5(x)+\frac{1}{18}x\sin^6(x)-\frac{5}{108}\int$$
$$= -\frac{5}{16}x^2\cos(x)\sin(x)+\frac{5}{16}x\sin^2(x)+\frac{65\cos(x)\sin^3(x)}{1728}-\frac{5}{24}x^2\cos(x)\sin^3(x)+\frac{5}{48}x\sin^4(x)+\frac{1}{108}\cos$$
$$= \frac{5x^3}{48}+\frac{245\cos(x)\sin(x)}{1152}-\frac{5}{16}x^2\cos(x)\sin(x)+\frac{5}{16}x\sin^2(x)+\frac{65\cos(x)\sin^3(x)}{1728}-\frac{5}{24}x^2\cos(x)\sin^3(x)$$
$$= -\frac{245x}{1152}+\frac{5x^3}{48}+\frac{245\cos(x)\sin(x)}{1152}-\frac{5}{16}x^2\cos(x)\sin(x)+\frac{5}{16}x\sin^2(x)+\frac{65\cos(x)\sin^3(x)}{1728}-\frac{5}{24}x^2\cos$$

Mathematica [A] time = 0.086196, size = 70, normalized size = 0.67

$$\frac{1440x^3-1620\left(2x^2-1\right)\sin(2x)+81\left(8x^2-1\right)\sin(4x)-4\left(18x^2-1\right)\sin(6x)-3240x\cos(2x)+324x\cos(4x)-24x\cos(}{13824}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[x]^6,x]
```

```
[Out] (1440*x^3 - 3240*x*Cos[2*x] + 324*x*Cos[4*x] - 24*x*Cos[6*x] - 1620*(-1 + 2
*x^2)*Sin[2*x] + 81*(-1 + 8*x^2)*Sin[4*x] - 4*(-1 + 18*x^2)*Sin[6*x])/13824
```

Maple [A] time = 0.03, size = 96, normalized size = 0.9

$$x^2\left(-\frac{\cos(x)}{6}\left((\sin(x))^5+\frac{5(\sin(x))^3}{4}+\frac{15\sin(x)}{8}\right)+\frac{5x}{16}\right)+\frac{x(\sin(x))^6}{18}+\frac{\cos(x)}{108}\left((\sin(x))^5+\frac{5(\sin(x))^3}{4}+\frac{15\sin(x}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x)^6,x)
```

```
[Out] x^2*(-1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x)+1/18*x*sin(x)^
6+1/108*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+115/1152*x+5/48*x*sin(x)
^4+5/192*(sin(x)^3+3/2*sin(x))*cos(x)-5/16*x*cos(x)^2+5/32*cos(x)*sin(x)-5/
24*x^3
```

Maxima [A] time = 0.942559, size = 89, normalized size = 0.85

$$\frac{5}{48}x^3-\frac{1}{576}x\cos(6x)+\frac{3}{128}x\cos(4x)-\frac{15}{64}x\cos(2x)-\frac{1}{3456}\left(18x^2-1\right)\sin(6x)+\frac{3}{512}\left(8x^2-1\right)\sin(4x)-\frac{15}{128}\left(2x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^6,x, algorithm="maxima")
```

```
[Out] 5/48*x^3 - 1/576*x*cos(6*x) + 3/128*x*cos(4*x) - 15/64*x*cos(2*x) - 1/3456*
(18*x^2 - 1)*sin(6*x) + 3/512*(8*x^2 - 1)*sin(4*x) - 15/128*(2*x^2 - 1)*sin
(2*x)
```

Fricas [A] time = 2.31521, size = 240, normalized size = 2.29

$$-\frac{1}{18}\,x\cos\left(x\right)^{6}+\frac{13}{48}\,x\cos\left(x\right)^{4}+\frac{5}{48}\,x^{3}-\frac{11}{16}\,x\cos\left(x\right)^{2}-\frac{1}{3456}\left(32\left(18\,x^{2}-1\right)\cos\left(x\right)^{5}-2\left(936\,x^{2}-97\right)\cos\left(x\right)^{3}+3\left(7\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/18*x*cos(x)^6 + 13/48*x*cos(x)^4 + 5/48*x^3 - 11/16*x*cos(x)^2 - 1/3456*
(32*(18*x^2 - 1)*cos(x)^5 - 2*(936*x^2 - 97)*cos(x)^3 + 3*(792*x^2 - 299)*c
os(x))*sin(x) + 299/1152*x
```

Sympy [A] time = 5.74307, size = 192, normalized size = 1.83

$$\frac{5x^{3}\sin^{6}(x)}{48}+\frac{5x^{3}\sin^{4}(x)\cos^{2}(x)}{16}+\frac{5x^{3}\sin^{2}(x)\cos^{4}(x)}{16}+\frac{5x^{3}\cos^{6}(x)}{48}-\frac{11x^{2}\sin^{5}(x)\cos(x)}{16}-\frac{5x^{2}\sin^{3}(x)\cos^{3}(x}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x)**6,x)
```

```
[Out] 5*x**3*sin(x)**6/48 + 5*x**3*sin(x)**4*cos(x)**2/16 + 5*x**3*sin(x)**2*cos(
x)**4/16 + 5*x**3*cos(x)**6/48 - 11*x**2*sin(x)**5*cos(x)/16 - 5*x**2*sin(x
)**3*cos(x)**3/6 - 5*x**2*sin(x)*cos(x)**5/16 + 299*x*sin(x)**6/1152 + 35*x
*sin(x)**4*cos(x)**2/384 - 125*x*sin(x)**2*cos(x)**4/384 - 245*x*cos(x)**6/
1152 + 299*sin(x)**5*cos(x)/1152 + 25*sin(x)**3*cos(x)**3/54 + 245*sin(x)*c
os(x)**5/1152
```

Giac [A] time = 1.06228, size = 89, normalized size = 0.85

$$\frac{5}{48}\,x^{3}-\frac{1}{576}\,x\cos\left(6\,x\right)+\frac{3}{128}\,x\cos\left(4\,x\right)-\frac{15}{64}\,x\cos\left(2\,x\right)-\frac{1}{3456}\left(18\,x^{2}-1\right)\sin\left(6\,x\right)+\frac{3}{512}\left(8\,x^{2}-1\right)\sin\left(4\,x\right)-\frac{15}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^6,x, algorithm="giac")
```

```
[Out] 5/48*x^3 - 1/576*x*cos(6*x) + 3/128*x*cos(4*x) - 15/64*x*cos(2*x) - 1/3456*
(18*x^2 - 1)*sin(6*x) + 3/512*(8*x^2 - 1)*sin(4*x) - 15/128*(2*x^2 - 1)*sin
(2*x)
```

3.486 $\int x^2 \cos(x) \sin^2(x) \, dx$

Optimal. Leaf size=44

$$\frac{1}{3}x^2 \sin^3(x) - \frac{2\sin^3(x)}{27} - \frac{4\sin(x)}{9} + \frac{4}{9}x\cos(x) + \frac{2}{9}x\sin^2(x)\cos(x)$$

```
[Out] (4*x*Cos[x])/9 - (4*Sin[x])/9 + (2*x*Cos[x]*Sin[x]^2)/9 - (2*Sin[x]^3)/27 +
 (x^2*Sin[x]^3)/3
```

Rubi [A] time = 0.0420206, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3443, 3310, 3296, 2637}

$$\frac{1}{3}x^2 \sin^3(x) - \frac{2\sin^3(x)}{27} - \frac{4\sin(x)}{9} + \frac{4}{9}x\cos(x) + \frac{2}{9}x\sin^2(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x]*Sin[x]^2,x]
```


```
[Out] (4*x*Cos[x])/9 - (4*Sin[x])/9 + (2*x*Cos[x]*Sin[x]^2)/9 - (2*Sin[x]^3)/27 +
 (x^2*Sin[x]^3)/3
```


Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^2\cos(x)\sin^2(x)\,dx = \frac{1}{3}x^2\sin^3(x) - \frac{2}{3}\int x\sin^3(x)\,dx$$
$$= \frac{2}{9}x\cos(x)\sin^2(x) - \frac{2\sin^3(x)}{27} + \frac{1}{3}x^2\sin^3(x) - \frac{4}{9}\int x\sin(x)\,dx$$
$$= \frac{4}{9}x\cos(x) + \frac{2}{9}x\cos(x)\sin^2(x) - \frac{2\sin^3(x)}{27} + \frac{1}{3}x^2\sin^3(x) - \frac{4}{9}\int\cos(x)\,dx$$
$$= \frac{4}{9}x\cos(x) - \frac{4\sin(x)}{9} + \frac{2}{9}x\cos(x)\sin^2(x) - \frac{2\sin^3(x)}{27} + \frac{1}{3}x^2\sin^3(x)$$

Mathematica [A] time = 0.121809, size = 39, normalized size = 0.89

$$\frac{1}{54}\left(\sin(x)\left(9x^2+\left(2-9x^2\right)\cos(2x)-26\right)+27x\cos(x)-3x\cos(3x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x]*Sin[x]^2,x]
```

```
[Out] (27*x*Cos[x] - 3*x*Cos[3*x] + (-26 + 9*x^2 + (2 - 9*x^2)*Cos[2*x])*Sin[x])/
54
```

Maple [A] time = 0.004, size = 32, normalized size = 0.7

$$\frac{x^2(\sin(x))^3}{3}+\frac{2x\left(2+(\sin(x))^2\right)\cos(x)}{9}-\frac{2(\sin(x))^3}{27}-\frac{4\sin(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)*sin(x)^2,x)
```

```
[Out] 1/3*x^2*sin(x)^3+2/9*x*(2+sin(x)^2)*cos(x)-2/27*sin(x)^3-4/9*sin(x)
```

Maxima [A] time = 0.95696, size = 47, normalized size = 1.07

$$-\frac{1}{18}x\cos(3x)+\frac{1}{2}x\cos(x)-\frac{1}{108}\left(9x^2-2\right)\sin(3x)+\frac{1}{4}\left(x^2-2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)*sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/18*x*cos(3*x) + 1/2*x*cos(x) - 1/108*(9*x^2 - 2)*sin(3*x) + 1/4*(x^2 - 2
)*sin(x)
```

Fricas [A] time = 2.25054, size = 111, normalized size = 2.52

$$-\frac{2}{9}x\cos(x)^3+\frac{2}{3}x\cos(x)-\frac{1}{27}\left(\left(9x^2-2\right)\cos(x)^2-9x^2+14\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -2/9*x*cos(x)^3 + 2/3*x*cos(x) - 1/27*((9*x^2 - 2)*cos(x)^2 - 9*x^2 + 14)*s
in(x)
```

Sympy [A] time = 1.20993, size = 53, normalized size = 1.2

$$\frac{x^2 \sin^3(x)}{3} + \frac{2x \sin^2(x) \cos(x)}{3} + \frac{4x \cos^3(x)}{9} - \frac{14 \sin^3(x)}{27} - \frac{4 \sin(x) \cos^2(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x)*sin(x)**2,x)
```

```
[Out] x**2*sin(x)**3/3 + 2*x*sin(x)**2*cos(x)/3 + 4*x*cos(x)**3/9 - 14*sin(x)**3/
27 - 4*sin(x)*cos(x)**2/9
```

Giac [A] time = 1.08224, size = 47, normalized size = 1.07

$$-\frac{1}{18}\,x\cos(3\,x) + \frac{1}{2}\,x\cos(x) - \frac{1}{108}\left(9\,x^2 - 2\right)\sin(3\,x) + \frac{1}{4}\left(x^2 - 2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)*sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/18*x*cos(3*x) + 1/2*x*cos(x) - 1/108*(9*x^2 - 2)*sin(3*x) + 1/4*(x^2 - 2
)*sin(x)
```

3.487 $\int x \cos^2(x) \cot^2(x) \, dx$

Optimal. Leaf size=33

$$-\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x\cot(x) + \log(\sin(x)) - \frac{1}{2}x\sin(x)\cos(x)$$

```
[Out] (-3*x^2)/4 - Cos[x]^2/4 - x*Cot[x] + Log[Sin[x]] - (x*Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0544898, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4408, 3310, 30, 3720, 3475}

$$-\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x\cot(x) + \log(\sin(x)) - \frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] (-3*x^2)/4 - Cos[x]^2/4 - x*Cot[x] + Log[Sin[x]] - (x*Cos[x]*Sin[x])/2
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int x\cos^2(x)\cot^2(x)\,dx = -\int x\cos^2(x)\,dx + \int x\cot^2(x)\,dx$$
$$= -\frac{1}{4}\cos^2(x) - x\cot(x) - \frac{1}{2}x\cos(x)\sin(x) - \frac{\int x\,dx}{2} - \int x\,dx + \int \cot(x)\,dx$$
$$= -\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x\cot(x) + \log(\sin(x)) - \frac{1}{2}x\cos(x)\sin(x)$$

Mathematica [A] time = 0.0286489, size = 33, normalized size = 1.

$$-\frac{3x^2}{4} - \frac{1}{4}x\sin(2x) - \frac{1}{8}\cos(2x) - x\cot(x) + \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] (-3*x^2)/4 - Cos[2*x]/8 - x*Cot[x] + Log[Sin[x]] - (x*Sin[2*x])/4
```

Maple [B] time = 0.193, size = 125, normalized size = 3.8

$$-\frac{x}{4\tan(x)} + \ln(\tan(x)) - \frac{\ln\left((\tan(x))^2+1\right)}{2} - \frac{1}{4\tan(x)\left((\tan(x))^2+1\right)^2}\left(x + x^2\tan(x) + x^2(\tan(x))^5 - \frac{(\tan(x))^5}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^4/sin(x)^2,x)
```

```
[Out] -1/4*x/tan(x)+ln(tan(x))-1/2*ln(tan(x)^2+1)-1/4*(x+x^2*tan(x)+x^2*tan(x)^5-
1/2*tan(x)^5+1/2*tan(x)+4*x*x*tan(x)^2+3*x*x*tan(x)^4+2*x^2*tan(x)^3)/(tan(x)^2
+1)^2/tan(x)-(1/2*x+1/2*x*x*tan(x)^2+1/2*x^2*tan(x)+1/2*x^2*tan(x)^3)/tan(x)/
(tan(x)^2+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^4/sin(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 2.0248, size = 138, normalized size = 4.18

$$\frac{4\,x\cos(x)^3 - 12\,x\cos(x) - \left(6\,x^2 + 2\,\cos(x)^2 - 1\right)\sin(x) + 8\,\log\left(\frac{1}{2}\,\sin(x)\right)\sin(x)}{8\,\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^4/sin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*x*cos(x)^3 - 12*x*cos(x) - (6*x^2 + 2*cos(x)^2 - 1)*sin(x) + 8*log(1
/2*sin(x))*sin(x))/sin(x)
```

Sympy [B] time = 2.54221, size = 507, normalized size = 15.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)**4/sin(x)**2,x)
```

```
[Out] -3*x**2*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 6*x**2*t
an(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 3*x**2*tan(x/2)/(
4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 2*x*tan(x/2)**6/(4*tan(x/2)**
5 + 8*tan(x/2)**3 + 4*tan(x/2)) + 6*x*tan(x/2)**4/(4*tan(x/2)**5 + 8*tan(x/
2)**3 + 4*tan(x/2)) - 6*x*tan(x/2)**2/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*ta
n(x/2)) - 2*x/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 4*log(tan(x/2)
**2 + 1)*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) - 8*log(t
an(x/2)**2 + 1)*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) -
4*log(tan(x/2)**2 + 1)*tan(x/2)/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)
) + 4*log(tan(x/2))*tan(x/2)**5/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)
) + 8*log(tan(x/2))*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)
) + 4*log(tan(x/2))*tan(x/2)/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2)) +
 4*tan(x/2)**3/(4*tan(x/2)**5 + 8*tan(x/2)**3 + 4*tan(x/2))
```

Giac [B] time = 1.12583, size = 278, normalized size = 8.42

$$-\frac{6\,x^2\tan\left(\frac{1}{2}\,x\right)^5-4\,x\tan\left(\frac{1}{2}\,x\right)^6-4\,\log\left(\frac{16\,\tan\left(\frac{1}{2}\,x\right)^2}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^5+12\,x^2\tan\left(\frac{1}{2}\,x\right)^3-12\,x\tan\left(\frac{1}{2}\,x\right)^4+\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^4/sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/8*(6*x^2*tan(1/2*x)^5 - 4*x*tan(1/2*x)^6 - 4*log(16*tan(1/2*x)^2/(tan(1/
2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^5 + 12*x^2*tan(1/2*x)^3 - 12*x*tan
(1/2*x)^4 + tan(1/2*x)^5 - 8*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*
x)^2 + 1))*tan(1/2*x)^3 + 6*x^2*tan(1/2*x) + 12*x*tan(1/2*x)^2 - 6*tan(1/2*
x)^3 - 4*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x
) + 4*x + tan(1/2*x))/(tan(1/2*x)^5 + 2*tan(1/2*x)^3 + tan(1/2*x))
```

3.488 $\int x \sec(x) \tan^3(x)\, dx$

Optimal. Leaf size=30

$$\frac{1}{3}x\sec^3(x) - x\sec(x) + \frac{5}{6}\tanh^{-1}(\sin(x)) - \frac{1}{6}\tan(x)\sec(x)$$

```
[Out] (5*ArcTanh[Sin[x]])/6 - x*Sec[x] + (x*Sec[x]^3)/3 - (Sec[x]*Tan[x])/6
```

Rubi [A] time = 0.0407061, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2606, 4417, 3770, 3768}

$$\frac{1}{3}x\sec^3(x) - x\sec(x) + \frac{5}{6}\tanh^{-1}(\sin(x)) - \frac{1}{6}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[x]*Tan[x]^3,x]
```

```
[Out] (5*ArcTanh[Sin[x]])/6 - x*Sec[x] + (x*Sec[x]^3)/3 - (Sec[x]*Tan[x])/6
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4417

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Sec[a + b*x]^n*Tan[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d, n, p}, x] && IGtQ[m, 0] && (IntegerQ[n/2] || Int
egerQ[(p - 1)/2])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$
\begin{aligned}
\int x\sec(x)\tan^3(x)\,dx &= -x\sec(x)+\frac{1}{3}x\sec^3(x)-\int\left(-\sec(x)+\frac{\sec^3(x)}{3}\right)dx\\
&= -x\sec(x)+\frac{1}{3}x\sec^3(x)-\frac{1}{3}\int\sec^3(x)\,dx+\int\sec(x)\,dx\\
&= \tanh^{-1}(\sin(x))-x\sec(x)+\frac{1}{3}x\sec^3(x)-\frac{1}{6}\sec(x)\tan(x)-\frac{1}{6}\int\sec(x)\,dx\\
&= \frac{5}{6}\tanh^{-1}(\sin(x))-x\sec(x)+\frac{1}{3}x\sec^3(x)-\frac{1}{6}\sec(x)\tan(x)
\end{aligned}
$$

Mathematica [B] time = 0.11333, size = 104, normalized size = 3.47

$$
-\frac{1}{24}\sec^3(x)\left(4x+2\sin(2x)+12x\cos(2x)+5\cos(3x)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+15\cos(x)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\right.\right.
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x]*Tan[x]^3,x]
```

```
[Out] -(Sec[x]^3*(4*x + 12*x*Cos[2*x] + 5*Cos[3*x]*Log[Cos[x/2] - Sin[x/2]] + 15*
Cos[x]*(Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]) - 5*Cos[3*x]*L
og[Cos[x/2] + Sin[x/2]] + 2*Sin[2*x]))/24
```

Maple [A] time = 0.122, size = 30, normalized size = 1.

$$
-\frac{x}{\cos(x)}+\frac{5\ln(\sec(x)+\tan(x))}{6}+\frac{x}{3(\cos(x))^3}-\frac{\sec(x)\tan(x)}{6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^3/cos(x)^4,x)
```

```
[Out] -x/cos(x)+5/6*ln(sec(x)+tan(x))+1/3*x/cos(x)^3-1/6*sec(x)*tan(x)
```

Maxima [B] time = 1.51156, size = 836, normalized size = 27.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x)^4,x, algorithm="maxima")
```

```
[Out] -1/12*(48*x*sin(3*x)*sin(2*x) + 4*(6*x*cos(5*x) + 4*x*cos(3*x) + 6*x*cos(x)
 + sin(5*x) - sin(x))*cos(6*x) + 12*(6*x*cos(4*x) + 6*x*cos(2*x) + 2*x - si
n(4*x) - sin(2*x))*cos(5*x) + 12*(4*x*cos(3*x) + 6*x*cos(x) - sin(x))*cos(4
*x) + 16*(3*x*cos(2*x) + x)*cos(3*x) + 12*(6*x*cos(x) - sin(x))*cos(2*x) +
24*x*cos(x) - 5*(2*(3*cos(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*
(3*cos(2*x) + 1)*cos(4*x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin
(2*x))*sin(6*x) + sin(6*x)^2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9*sin(
2*x)^2 + 6*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + 5*(2*(3*
cos(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*(3*cos(2*x) + 1)*cos(4
```

```
*x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin(2*x))*sin(6*x) + sin(
6*x)^2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*cos(2*x) +
1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) + 4*(6*x*sin(5*x) + 4*x*sin(3*x)
 + 6*x*sin(x) - cos(5*x) + cos(x))*sin(6*x) + 4*(18*x*sin(4*x) + 18*x*sin(2
*x) + 3*cos(4*x) + 3*cos(2*x) + 1)*sin(5*x) + 12*(4*x*sin(3*x) + 6*x*sin(x)
 + cos(x))*sin(4*x) + 12*(6*x*sin(x) + cos(x))*sin(2*x) - 4*sin(x))/(2*(3*c
os(4*x) + 3*cos(2*x) + 1)*cos(6*x) + cos(6*x)^2 + 6*(3*cos(2*x) + 1)*cos(4*
x) + 9*cos(4*x)^2 + 9*cos(2*x)^2 + 6*(sin(4*x) + sin(2*x))*sin(6*x) + sin(6
*x)^2 + 9*sin(4*x)^2 + 18*sin(4*x)*sin(2*x) + 9*sin(2*x)^2 + 6*cos(2*x) + 1
)
```

Fricas [A] time = 1.9149, size = 154, normalized size = 5.13

$$\frac{5 \cos(x)^3 \log(\sin(x)+1) - 5 \cos(x)^3 \log(-\sin(x)+1) - 12 x \cos(x)^2 - 2 \cos(x) \sin(x) + 4 x}{12 \cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(5*cos(x)^3*log(sin(x) + 1) - 5*cos(x)^3*log(-sin(x) + 1) - 12*x*cos(x
)^2 - 2*cos(x)*sin(x) + 4*x)/cos(x)^3
```

Sympy [B] time = 1.81592, size = 551, normalized size = 18.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**3/cos(x)**4,x)
```

```
[Out] 4*x*tan(x/2)**6/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 12*
x*tan(x/2)**4/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 12*x*
tan(x/2)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 4*x/(6*
tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 5*log(tan(x/2) - 1)*ta
n(x/2)**6/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 15*log(ta
n(x/2) - 1)*tan(x/2)**4/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 -
6) - 15*log(tan(x/2) - 1)*tan(x/2)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*
tan(x/2)**2 - 6) + 5*log(tan(x/2) - 1)/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18
*tan(x/2)**2 - 6) + 5*log(tan(x/2) + 1)*tan(x/2)**6/(6*tan(x/2)**6 - 18*tan
(x/2)**4 + 18*tan(x/2)**2 - 6) - 15*log(tan(x/2) + 1)*tan(x/2)**4/(6*tan(x/
2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 15*log(tan(x/2) + 1)*tan(x/2
)**2/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 5*log(tan(x/2)
 + 1)/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) - 2*tan(x/2)**5
/(6*tan(x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6) + 2*tan(x/2)/(6*tan(
x/2)**6 - 18*tan(x/2)**4 + 18*tan(x/2)**2 - 6)
```

Giac [B] time = 1.52935, size = 460, normalized size = 15.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x)^4,x, algorithm="giac")
```

```
[Out] 1/12*(8*x*tan(1/2*x)^6 + 5*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2
*x)^2 + 1))*tan(1/2*x)^6 - 5*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1
/2*x)^2 + 1))*tan(1/2*x)^6 - 24*x*tan(1/2*x)^4 - 15*log(2*(tan(1/2*x)^2 + 2
*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4 + 15*log(2*(tan(1/2*x)^2
- 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4 - 4*tan(1/2*x)^5 - 24*
x*tan(1/2*x)^2 + 15*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 +
 1))*tan(1/2*x)^2 - 15*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^
2 + 1))*tan(1/2*x)^2 + 8*x - 5*log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan
(1/2*x)^2 + 1)) + 5*log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 +
 1)) + 4*tan(1/2*x))/(tan(1/2*x)^6 - 3*tan(1/2*x)^4 + 3*tan(1/2*x)^2 - 1)
```

3.489 $\int x\sec^2(x)\tan(x)\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}x\sec^2(x)-\frac{\tan(x)}{2}$$

```
[Out] (x*Sec[x]^2)/2 - Tan[x]/2
```

Rubi [A] time = 0.0177302, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3757, 3767, 8}

$$\frac{1}{2}x\sec^2(x)-\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[x]^2*Tan[x],x]

[Out] (x*Sec[x]^2)/2 - Tan[x]/2
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int x\sec^2(x)\tan(x)\,dx&=\frac{1}{2}x\sec^2(x)-\frac{1}{2}\int\sec^2(x)\,dx\\&=\frac{1}{2}x\sec^2(x)+\frac{1}{2}\text{Subst}\left(\int 1\,dx,x,-\tan(x)\right)\\&=\frac{1}{2}x\sec^2(x)-\frac{\tan(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0155963, size = 16, normalized size = 1.

$$\frac{1}{2}x\sec^2(x)-\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x]^2*Tan[x],x]
```

```
[Out] (x*Sec[x]^2)/2 - Tan[x]/2
```

Maple [A] time = 0.005, size = 13, normalized size = 0.8

$$\frac{x}{2\left(\cos\left(x\right)\right)^{2}}-\frac{\tan\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)/cos(x)^3,x)
```

```
[Out] 1/2*x/cos(x)^2-1/2*tan(x)
```

Maxima [B] time = 0.944647, size = 178, normalized size = 11.12

$$\frac{4\,x\cos\left(2\,x\right)^{2}+4\,x\sin\left(2\,x\right)^{2}+\left(2\,x\cos\left(2\,x\right)+\sin\left(2\,x\right)\right)\cos\left(4\,x\right)+2\,x\cos\left(2\,x\right)+\left(2\,x\sin\left(2\,x\right)-\cos\left(2\,x\right)-1\right)\sin\left(4\,x\right.}{2\left(2\,\cos\left(2\,x\right)+1\right)\cos\left(4\,x\right)+\cos\left(4\,x\right)^{2}+4\,\cos\left(2\,x\right)^{2}+\sin\left(4\,x\right)^{2}+4\,\sin\left(4\,x\right)\sin\left(2\,x\right)+4\,\sin\left(2\,x\right)^{2}+4\,\cos\left(2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/cos(x)^3,x, algorithm="maxima")
```

```
[Out] (4*x*cos(2*x)^2 + 4*x*sin(2*x)^2 + (2*x*cos(2*x) + sin(2*x))*cos(4*x) + 2*x
*cos(2*x) + (2*x*sin(2*x) - cos(2*x) - 1)*sin(4*x) - sin(2*x))/(2*(2*cos(2*
x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(
2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

Fricas [A] time = 1.79746, size = 47, normalized size = 2.94

$$-\frac{\cos\left(x\right)\sin\left(x\right)-x}{2\,\cos\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/cos(x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(cos(x)*sin(x) - x)/cos(x)^2
```

Sympy [B] time = 1.05299, size = 128, normalized size = 8.

$$\frac{x\tan^{4}\left(\frac{x}{2}\right)}{2\tan^{4}\left(\frac{x}{2}\right)-4\tan^{2}\left(\frac{x}{2}\right)+2}+\frac{2x\tan^{2}\left(\frac{x}{2}\right)}{2\tan^{4}\left(\frac{x}{2}\right)-4\tan^{2}\left(\frac{x}{2}\right)+2}+\frac{x}{2\tan^{4}\left(\frac{x}{2}\right)-4\tan^{2}\left(\frac{x}{2}\right)+2}+\frac{2\tan^{3}\left(\frac{x}{2}\right)}{2\tan^{4}\left(\frac{x}{2}\right)-4\tan^{2}\left(\frac{x}{2}\right)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/cos(x)**3,x)
```

```
[Out] x*tan(x/2)**4/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) + 2*x*tan(x/2)**2/(2*tan(
x/2)**4 - 4*tan(x/2)**2 + 2) + x/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) + 2*ta
n(x/2)**3/(2*tan(x/2)**4 - 4*tan(x/2)**2 + 2) - 2*tan(x/2)/(2*tan(x/2)**4 -
 4*tan(x/2)**2 + 2)
```

Giac [B] time = 1.07766, size = 72, normalized size = 4.5

$$\frac{x\tan\left(\frac{1}{2}x\right)^4+2\,x\tan\left(\frac{1}{2}x\right)^2+2\,\tan\left(\frac{1}{2}x\right)^3+x-2\,\tan\left(\frac{1}{2}x\right)}{2\left(\tan\left(\frac{1}{2}x\right)^4-2\,\tan\left(\frac{1}{2}x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)/cos(x)^3,x, algorithm="giac")
```

```
[Out] 1/2*(x*tan(1/2*x)^4 + 2*x*tan(1/2*x)^2 + 2*tan(1/2*x)^3 + x - 2*tan(1/2*x))
/(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)
```

3.490 $\int x\sin^2(x)\tan(x)\,dx$

Optimal. Leaf size=62

$$\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)+\frac{ix^2}{2}+\frac{x}{4}-x\log\left(1+e^{2ix}\right)-\frac{1}{2}x\sin^2(x)-\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] x/4 + (I/2)*x^2 - x*Log[1 + E^((2*I)*x)] + (I/2)*PolyLog[2, -E^((2*I)*x)] -
 (Cos[x]*Sin[x])/4 - (x*Sin[x]^2)/2
```

Rubi [A] time = 0.071817, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {4407, 3443, 2635, 8, 3719, 2190, 2279, 2391}

$$\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)+\frac{ix^2}{2}+\frac{x}{4}-x\log\left(1+e^{2ix}\right)-\frac{1}{2}x\sin^2(x)-\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x]^2*Tan[x],x]
```

```
[Out] x/4 + (I/2)*x^2 - x*Log[1 + E^((2*I)*x)] + (I/2)*PolyLog[2, -E^((2*I)*x)] -
 (Cos[x]*Sin[x])/4 - (x*Sin[x]^2)/2
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int x\sin^2(x)\tan(x)\,dx &= -\int x\cos(x)\sin(x)\,dx+\int x\tan(x)\,dx\\
&= \frac{ix^2}{2}-\frac{1}{2}x\sin^2(x)-2i\int\frac{e^{2ix}x}{1+e^{2ix}}\,dx+\frac{1}{2}\int\sin^2(x)\,dx\\
&= \frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)-\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x\sin^2(x)+\frac{\int 1\,dx}{4}+\int\log\left(1+e^{2ix}\right)dx\\
&= \frac{x}{4}+\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)-\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x\sin^2(x)-\frac{1}{2}i\,\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx,x,e^{2ix}\right)\\
&= \frac{x}{4}+\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)+\frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right)-\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x\sin^2(x)
\end{aligned}$$

Mathematica [A] time = 0.0146834, size = 57, normalized size = 0.92

$$\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)+\frac{ix^2}{2}-x\log\left(1+e^{2ix}\right)-\frac{1}{8}\sin(2x)+\frac{1}{4}x\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x]^2*Tan[x],x]
```

```
[Out] (I/2)*x^2 + (x*Cos[2*x])/4 - x*Log[1 + E^((2*I)*x)] + (I/2)*PolyLog[2, -E^(
(2*I)*x)] - Sin[2*x]/8
```

Maple [A] time = 0.06, size = 57, normalized size = 0.9

$$\frac{i}{2}x^2+\frac{(i+2x)\,e^{2ix}}{16}+\frac{(-i+2x)\,e^{-2ix}}{16}-x\ln\left(1+e^{2ix}\right)+\frac{i}{2}\text{polylog}\left(2,-e^{2ix}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^3/cos(x),x)
```

```
[Out] 1/2*I*x^2+1/16*(I+2*x)*exp(2*I*x)+1/16*(-I+2*x)*exp(-2*I*x)-x*ln(1+exp(2*I*
x))+1/2*I*polylog(2,-exp(2*I*x))
```

Maxima [A] time = 1.44558, size = 89, normalized size = 1.44

$$\frac{1}{2}ix^2 - ix\arctan\left(\sin\left(2x\right),\cos\left(2x\right)+1\right) + \frac{1}{4}x\cos\left(2x\right) - \frac{1}{2}x\log\left(\cos\left(2x\right)^2 + \sin\left(2x\right)^2 + 2\cos\left(2x\right)+1\right) + \frac{1}{2}i\,\mathrm{Li}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x),x, algorithm="maxima")
```

```
[Out] 1/2*I*x^2 - I*x*arctan2(sin(2*x), cos(2*x) + 1) + 1/4*x*cos(2*x) - 1/2*x*lo
g(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + 1/2*I*dilog(-e^(2*I*x)) - 1/8
*sin(2*x)
```

Fricas [B] time = 2.0748, size = 432, normalized size = 6.97

$$\frac{1}{2}x\cos\left(x\right)^2 - \frac{1}{2}x\log\left(i\cos\left(x\right)+\sin\left(x\right)+1\right) - \frac{1}{2}x\log\left(i\cos\left(x\right)-\sin\left(x\right)+1\right) - \frac{1}{2}x\log\left(-i\cos\left(x\right)+\sin\left(x\right)+1\right) - \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(x)^2 - 1/2*x*log(I*cos(x) + sin(x) + 1) - 1/2*x*log(I*cos(x) - si
n(x) + 1) - 1/2*x*log(-I*cos(x) + sin(x) + 1) - 1/2*x*log(-I*cos(x) - sin(x
) + 1) - 1/4*cos(x)*sin(x) - 1/4*x - 1/2*I*dilog(I*cos(x) + sin(x)) + 1/2*I
*dilog(I*cos(x) - sin(x)) + 1/2*I*dilog(-I*cos(x) + sin(x)) - 1/2*I*dilog(-
I*cos(x) - sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin^3\left(x\right)}{\cos\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**3/cos(x),x)
```

```
[Out] Integral(x*sin(x)**3/cos(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sin\left(x\right)^3}{\cos\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x),x, algorithm="giac")
```

```
[Out] integrate(x*sin(x)^3/cos(x), x)
```

3.491 $\int x\tan^3(x)\,dx$

Optimal. Leaf size=59

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{ix^2}{2}+\frac{x}{2}+x\log\left(1+e^{2ix}\right)+\frac{1}{2}x\tan^2(x)-\frac{\tan(x)}{2}$$

```
[Out] x/2 - (I/2)*x^2 + x*Log[1 + E^((2*I)*x)] - (I/2)*PolyLog[2, -E^((2*I)*x)] -
 Tan[x]/2 + (x*Tan[x]^2)/2
```

Rubi [A] time = 0.0610901, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.167, Rules used = {3720, 3473, 8, 3719, 2190, 2279, 2391}

$$-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right)-\frac{ix^2}{2}+\frac{x}{2}+x\log\left(1+e^{2ix}\right)+\frac{1}{2}x\tan^2(x)-\frac{\tan(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Tan[x]^3,x]
```

```
[Out] x/2 - (I/2)*x^2 + x*Log[1 + E^((2*I)*x)] - (I/2)*PolyLog[2, -E^((2*I)*x)] -
 Tan[x]/2 + (x*Tan[x]^2)/2
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int x\tan^3(x)\,dx &= \frac{1}{2}x\tan^2(x) - \frac{1}{2}\int \tan^2(x)\,dx - \int x\tan(x)\,dx \\
&= -\frac{ix^2}{2} - \frac{\tan(x)}{2} + \frac{1}{2}x\tan^2(x) + 2i\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx + \frac{\int 1\,dx}{2} \\
&= \frac{x}{2} - \frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) - \frac{\tan(x)}{2} + \frac{1}{2}x\tan^2(x) - \int \log\left(1+e^{2ix}\right)\,dx \\
&= \frac{x}{2} - \frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) - \frac{\tan(x)}{2} + \frac{1}{2}x\tan^2(x) + \frac{1}{2}i\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2ix}\right) \\
&= \frac{x}{2} - \frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) - \frac{1}{2}i\text{Li}_2\left(-e^{2ix}\right) - \frac{\tan(x)}{2} + \frac{1}{2}x\tan^2(x)
\end{aligned}
$$

Mathematica [A] time = 0.0133528, size = 54, normalized size = 0.92

$$
-\frac{1}{2}i\text{PolyLog}\left(2,-e^{2ix}\right) - \frac{ix^2}{2} + x\log\left(1+e^{2ix}\right) - \frac{\tan(x)}{2} + \frac{1}{2}x\sec^2(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tan[x]^3,x]
```

```
[Out] (-I/2)*x^2 + x*Log[1 + E^((2*I)*x)] - (I/2)*PolyLog[2, -E^((2*I)*x)] + (x*S
ec[x]^2)/2 - Tan[x]/2
```

Maple [A] time = 0.197, size = 59, normalized size = 1.

$$
-\frac{i}{2}x^2 + \frac{-i\mathrm{e}^{2\,ix} + 2\,x\mathrm{e}^{2\,ix} - i}{\left(1+\mathrm{e}^{2\,ix}\right)^2} + x\ln\left(1+\mathrm{e}^{2\,ix}\right) - \frac{i}{2}\text{polylog}\left(2,-\mathrm{e}^{2\,ix}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^3/cos(x)^3,x)
```

```
[Out] -1/2*I*x^2+(-I*exp(2*I*x)+2*x*exp(2*I*x)-I)/(1+exp(2*I*x))^2+x*ln(1+exp(2*I
*x))-1/2*I*polylog(2,-exp(2*I*x))
```

Maxima [B] time = 1.56025, size = 288, normalized size = 4.88

$$
-\frac{x^2\cos(4\,x) + i\,x^2\sin(4\,x) + x^2 - (2\,x\cos(4\,x) + 4\,x\cos(2\,x) + 2i\,x\sin(4\,x) + 4i\,x\sin(2\,x) + 2\,x)\arctan(\sin(2\,x),}{}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x)^3,x, algorithm="maxima")
```

```
[Out] -(x^2*cos(4*x) + I*x^2*sin(4*x) + x^2 - (2*x*cos(4*x) + 4*x*cos(2*x) + 2*I*
x*sin(4*x) + 4*I*x*sin(2*x) + 2*x)*arctan2(sin(2*x), cos(2*x) + 1) + 2*(x^2
 + 2*I*x + 1)*cos(2*x) + (cos(4*x) + 2*cos(2*x) + I*sin(4*x) + 2*I*sin(2*x)
 + 1)*dilog(-e^(2*I*x)) - (-I*x*cos(4*x) - 2*I*x*cos(2*x) + x*sin(4*x) + 2*
x*sin(2*x) - I*x)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) - (-2*I*x^2
 + 4*x - 2*I)*sin(2*x) + 2)/(-2*I*cos(4*x) - 4*I*cos(2*x) + 2*sin(4*x) + 4*
sin(2*x) - 2*I)
```

Fricas [B] time = 2.11879, size = 473, normalized size = 8.02

$$x \cos(x)^2 \log(i \cos(x) + \sin(x) + 1) + x \cos(x)^2 \log(i \cos(x) - \sin(x) + 1) + x \cos(x)^2 \log(-i \cos(x) + \sin(x) + 1) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos(x)^2*log(I*cos(x) + sin(x) + 1) + x*cos(x)^2*log(I*cos(x) - sin(
x) + 1) + x*cos(x)^2*log(-I*cos(x) + sin(x) + 1) + x*cos(x)^2*log(-I*cos(x)
 - sin(x) + 1) + I*cos(x)^2*dilog(I*cos(x) + sin(x)) - I*cos(x)^2*dilog(I*c
os(x) - sin(x)) - I*cos(x)^2*dilog(-I*cos(x) + sin(x)) + I*cos(x)^2*dilog(-
I*cos(x) - sin(x)) - cos(x)*sin(x) + x)/cos(x)^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin^3(x)}{\cos^3(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**3/cos(x)**3,x)
```

```
[Out] Integral(x*sin(x)**3/cos(x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sin(x)^3}{\cos(x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3/cos(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*sin(x)^3/cos(x)^3, x)
```

3.492 $\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx$

Optimal. Leaf size=12

$$\frac{2}{\frac{\cot(x)}{x}+1}$$

```
[Out] 2/(1 + Cot[x]/x)
```

Rubi [A] time = 0.0988092, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6711, 32}

$$\frac{2}{\frac{\cot(x)}{x}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + Sin[2*x])/(Cos[x] + x*Sin[x])^2,x]
```

```
[Out] 2/(1 + Cot[x]/x)
```

Rule 6711

```
Int[(u_)*((a_.)*(v_)^(p_.) + (b_.)*(w_)^(q_.))^(m_.), x_Symbol] :> With[{c
= Simplify[u/(p*w*D[v, x] - q*v*D[w, x])]}, Dist[c*p, Subst[Int[(b + a*x^p)
^m, x], x, v*w^(m*q + 1)], x] /; FreeQ[c, x]] /; FreeQ[{a, b, m, p, q}, x]
&& EqQ[p + q*(m*p + 1), 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{2x+\sin(2x)}{(\cos(x)+x\sin(x))^2}\,dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{(1+x)^2}\,dx, x, \frac{\cot(x)}{x}\right)\right)\\
&= \frac{2}{1+\frac{\cot(x)}{x}}
\end{aligned}$$

Mathematica [A] time = 0.213904, size = 14, normalized size = 1.17

$$\frac{2x\sin(x)}{x\sin(x)+\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + Sin[2*x])/(Cos[x] + x*Sin[x])^2,x]
```

```
[Out] (2*x*Sin[x])/(Cos[x] + x*Sin[x])
```

Maple [C] time = 0.711, size = 44, normalized size = 3.7

$$\frac{-2i}{x+i} - \frac{4ix}{(x+i)\left(x\mathrm{e}^{2ix} - x + i\mathrm{e}^{2ix} + i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x)
```

```
[Out] -2*I/(x+I)-4*I*x/(x+I)/(x*exp(2*I*x)-x+I*exp(2*I*x)+I)
```

Maxima [B] time = 1.35291, size = 105, normalized size = 8.75

$$-\frac{2\left(\cos(2x)^2 + 2x\sin(2x) + \sin(2x)^2 + 2\cos(2x) + 1\right)}{\left(x^2+1\right)\cos(2x)^2 + \left(x^2+1\right)\sin(2x)^2 + x^2 - 2\left(x^2-1\right)\cos(2x) + 4x\sin(2x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -2*(cos(2*x)^2 + 2*x*sin(2*x) + sin(2*x)^2 + 2*cos(2*x) + 1)/((x^2 + 1)*cos
(2*x)^2 + (x^2 + 1)*sin(2*x)^2 + x^2 - 2*(x^2 - 1)*cos(2*x) + 4*x*sin(2*x)
+ 1)
```

Fricas [A] time = 1.95124, size = 42, normalized size = 3.5

$$-\frac{2\cos(x)}{x\sin(x) + \cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -2*cos(x)/(x*sin(x) + cos(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x + \sin(2x)}{(x\sin(x) + \cos(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))**2,x)
```

```
[Out] Integral((2*x + sin(2*x))/(x*sin(x) + cos(x))**2, x)
```

Giac [A] time = 1.06736, size = 14, normalized size = 1.17

$$-\frac{2}{x\tan(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x+sin(2*x))/(cos(x)+x*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2/(x*tan(x) + 1)
```

3.493 $\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx$

Optimal. Leaf size=20

$$\frac{x\csc(x)}{x\cos(x)-\sin(x)}-\cot(x)$$

```
[Out] -Cot[x] + (x*Csc[x])/(x*Cos[x] - Sin[x])
```

Rubi [A] time = 0.0341266, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4594, 3767, 8}

$$\frac{x\csc(x)}{x\cos(x)-\sin(x)}-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(x*Cos[x] - Sin[x])^2,x]

[Out] -Cot[x] + (x*Csc[x])/(x*Cos[x] - Sin[x])
```

Rule 4594

```
Int[(x_)^2/(Cos[(a_.)*(x_)]*(d_.)*(x_) + (c_.)*Sin[(a_.)*(x_)])^2, x_Symbol
] :> Simp[x/(a*d*Sin[a*x]*(c*Sin[a*x] + d*x*Cos[a*x])), x] + Dist[1/d^2, In
t[1/Sin[a*x]^2, x], x] /; FreeQ[{a, c, d}, x] && EqQ[a*c + d, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(x\cos(x)-\sin(x))^2}\,dx &= \frac{x\csc(x)}{x\cos(x)-\sin(x)}+\int \csc^2(x)\,dx\\ &= \frac{x\csc(x)}{x\cos(x)-\sin(x)}-\text{Subst}\left(\int 1\,dx,x,\cot(x)\right)\\ &= -\cot(x)+\frac{x\csc(x)}{x\cos(x)-\sin(x)}\end{aligned}$$

Mathematica [A] time = 0.207802, size = 19, normalized size = 0.95

$$\frac{x\sin(x)+\cos(x)}{x\cos(x)-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(x*Cos[x] - Sin[x])^2,x]
```

```
[Out] (Cos[x] + x*Sin[x])/(x*Cos[x] - Sin[x])
```

Maple [A] time = 0.214, size = 37, normalized size = 1.9

$$\left(-1+\left(\tan\left(\frac{x}{2}\right)\right)^2-2\,x\tan\left(x/2\right)\right)\left(x\left(\tan\left(\frac{x}{2}\right)\right)^2-x+2\,\tan\left(x/2\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x*cos(x)-sin(x))^2,x)
```

```
[Out] (-1+tan(1/2*x)^2-2*x*tan(1/2*x))/(x*tan(1/2*x)^2-x+2*tan(1/2*x))
```

Maxima [B] time = 0.954805, size = 93, normalized size = 4.65

$$\frac{2\left(2\,x\cos\left(2\,x\right)+\left(x^2-1\right)\sin\left(2\,x\right)\right)}{\left(x^2+1\right)\cos\left(2\,x\right)^2+\left(x^2+1\right)\sin\left(2\,x\right)^2+x^2+2\left(x^2-1\right)\cos\left(2\,x\right)-4\,x\sin\left(2\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x*cos(x)-sin(x))^2,x, algorithm="maxima")
```

```
[Out] 2*(2*x*cos(2*x) + (x^2 - 1)*sin(2*x))/((x^2 + 1)*cos(2*x)^2 + (x^2 + 1)*sin
(2*x)^2 + x^2 + 2*(x^2 - 1)*cos(2*x) - 4*x*sin(2*x) + 1)
```

Fricas [A] time = 1.82124, size = 55, normalized size = 2.75

$$\frac{x\sin\left(x\right)+\cos\left(x\right)}{x\cos\left(x\right)-\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x*cos(x)-sin(x))^2,x, algorithm="fricas")
```

```
[Out] (x*sin(x) + cos(x))/(x*cos(x) - sin(x))
```

Sympy [B] time = 2.35619, size = 66, normalized size = 3.3

$$-\frac{2x\tan\left(\frac{x}{2}\right)}{x\tan^2\left(\frac{x}{2}\right)-x+2\tan\left(\frac{x}{2}\right)}+\frac{\tan^2\left(\frac{x}{2}\right)}{x\tan^2\left(\frac{x}{2}\right)-x+2\tan\left(\frac{x}{2}\right)}-\frac{1}{x\tan^2\left(\frac{x}{2}\right)-x+2\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x*cos(x)-sin(x))**2,x)
```

```
[Out] -2*x*tan(x/2)/(x*tan(x/2)**2 - x + 2*tan(x/2)) + tan(x/2)**2/(x*tan(x/2)**2
 - x + 2*tan(x/2)) - 1/(x*tan(x/2)**2 - x + 2*tan(x/2))
```

Giac [A] time = 1.06728, size = 53, normalized size = 2.65

$$-\frac{2\,x\tan\left(\frac{1}{2}\,x\right)-\tan\left(\frac{1}{2}\,x\right)^{2}+1}{x\tan\left(\frac{1}{2}\,x\right)^{2}-x+2\,\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x*cos(x)-sin(x))^2,x, algorithm="giac")
```

```
[Out] -(2*x*tan(1/2*x) - tan(1/2*x)^2 + 1)/(x*tan(1/2*x)^2 - x + 2*tan(1/2*x))
```

3.494 $\int a^{mx} b^{nx}\,dx$

Optimal. Leaf size=22

$$\frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}$$

```
[Out] (a^(m*x)*b^(n*x))/(m*Log[a] + n*Log[b])
```

Rubi [A] time = 0.0263098, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2287, 2194}

$$\frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}$$

Antiderivative was successfully verified.

```
[In] Int[a^(m*x)*b^(n*x),x]
```

```
[Out] (a^(m*x)*b^(n*x))/(m*Log[a] + n*Log[b])
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int a^{mx}b^{nx}\,dx &= \int e^{x(m\log(a)+n\log(b))}\,dx\\ &= \frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}\end{aligned}$$

Mathematica [A] time = 0.0135011, size = 22, normalized size = 1.

$$\frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^(m*x)*b^(n*x),x]
```

```
[Out] (a^(m*x)*b^(n*x))/(m*Log[a] + n*Log[b])
```

Maple [A] time = 0.004, size = 23, normalized size = 1.1

$$\frac{a^{mx}b^{nx}}{m\ln(a)+n\ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^(m*x)*b^(n*x),x)
```

```
[Out] a^(m*x)*b^(n*x)/(m*ln(a)+n*ln(b))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(m*x)*b^(n*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91508, size = 53, normalized size = 2.41

$$\frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(m*x)*b^(n*x),x, algorithm="fricas")
```

```
[Out] a^(m*x)*b^(n*x)/(m*log(a) + n*log(b))
```

Sympy [A] time = 0.654854, size = 42, normalized size = 1.91

$$\begin{cases} \frac{a^{mx}b^{nx}}{m\log(a)+n\log(b)} & \text{for}\ m\neq -\frac{n\log(b)}{\log(a)} \\ b^{nx}xe^{-nx\log(b)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**(m*x)*b**(n*x),x)
```

```
[Out] Piecewise((a**(m*x)*b**(n*x)/(m*log(a) + n*log(b)), Ne(m, -n*log(b)/log(a))
), (b**(n*x)*x*exp(-n*x*log(b)), True))
```

Giac [C] time = 1.14874, size = 439, normalized size = 19.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(m*x)*b^(n*x),x, algorithm="giac")
```

```
[Out] 2*(2*(m*log(abs(a)) + n*log(abs(b)))*cos(-1/2*pi*m*x*sgn(a) - 1/2*pi*n*x*sg
n(b) + 1/2*pi*m*x + 1/2*pi*n*x)/((pi*m*sgn(a) + pi*n*sgn(b) - pi*m - pi*n)^
2 + 4*(m*log(abs(a)) + n*log(abs(b)))^2) - (pi*m*sgn(a) + pi*n*sgn(b) - pi*
m - pi*n)*sin(-1/2*pi*m*x*sgn(a) - 1/2*pi*n*x*sgn(b) + 1/2*pi*m*x + 1/2*pi*
n*x)/((pi*m*sgn(a) + pi*n*sgn(b) - pi*m - pi*n)^2 + 4*(m*log(abs(a)) + n*lo
g(abs(b)))^2))*e^((m*log(abs(a)) + n*log(abs(b)))*x) - 1/2*I*(-2*I*e^(1/2*I
*pi*m*x*sgn(a) + 1/2*I*pi*n*x*sgn(b) - 1/2*I*pi*m*x - 1/2*I*pi*n*x)/(I*pi*m
*sgn(a) + I*pi*n*sgn(b) - I*pi*m - I*pi*n + 2*m*log(abs(a)) + 2*n*log(abs(b
))) + 2*I*e^(-1/2*I*pi*m*x*sgn(a) - 1/2*I*pi*n*x*sgn(b) + 1/2*I*pi*m*x + 1/
2*I*pi*n*x)/(-I*pi*m*sgn(a) - I*pi*n*sgn(b) + I*pi*m + I*pi*n + 2*m*log(abs
(a)) + 2*n*log(abs(b))))*e^((m*log(abs(a)) + n*log(abs(b)))*x)
```

3.495 $\int a^{-x} b^{-x} \left(a^x-b^x\right)^2 \, dx$

Optimal. Leaf size=34

$$\frac{a^x b^{-x}-a^{-x} b^x}{\log (a)-\log (b)}-2 x$$

```
[Out] -2*x + (a^x/b^x - b^x/a^x)/(Log[a] - Log[b])
```

Rubi [A] time = 0.207676, antiderivative size = 41, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2287, 6742, 2194, 8}

$$-\frac{a^{-x} b^x}{\log (a)-\log (b)}+\frac{a^x b^{-x}}{\log (a)-\log (b)}-2 x$$

Antiderivative was successfully verified.

```
[In] Int[(a^x - b^x)^2/(a^x*b^x),x]
```

```
[Out] -2*x + a^x/(b^x*(Log[a] - Log[b])) - b^x/(a^x*(Log[a] - Log[b]))
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int a^{-x} b^{-x} \left(a^x-b^x\right)^2 \, dx &= \int \left(a^x-b^x\right)^2 e^{-x (\log (a)+\log (b))} \, dx\\
&= \int \left(a^{2 x} e^{-x (\log (a)+\log (b))}-2 a^x b^x e^{-x (\log (a)+\log (b))}+b^{2 x} e^{-x (\log (a)+\log (b))}\right) \, dx\\
&= -\left(2 \int a^x b^x e^{-x (\log (a)+\log (b))} \, dx\right)+\int a^{2 x} e^{-x (\log (a)+\log (b))} \, dx+\int b^{2 x} e^{-x (\log (a)+\log (b))} \, dx\\
&= -(2 \int 1 \, dx)+\int e^{-x (\log (a)-\log (b))} \, dx+\int e^{x (\log (a)-\log (b))} \, dx\\
&= -2 x+\frac{a^x b^{-x}}{\log (a)-\log (b)}-\frac{a^{-x} b^x}{\log (a)-\log (b)}
\end{aligned}$$

Mathematica [A] time = 0.0499614, size = 46, normalized size = 1.35

$$\frac{e^{x(\log(a)-\log(b))}}{\log(a)-\log(b)}+\frac{e^{x(\log(b)-\log(a))}}{\log(b)-\log(a)}-2x$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^x - b^x)^2/(a^x*b^x),x]
```

```
[Out] -2*x + E^(x*(Log[a] - Log[b]))/(Log[a] - Log[b]) + E^(x*(-Log[a] + Log[b]))
/(-Log[a] + Log[b])
```

Maple [A] time = 0.017, size = 65, normalized size = 1.9

$$\frac{1}{e^{x\ln(a)}e^{x\ln(b)}}\left(\frac{\left(e^{x\ln(a)}\right)^2}{\ln(a)-\ln(b)}-\frac{\left(e^{x\ln(b)}\right)^2}{\ln(a)-\ln(b)}-2\,xe^{x\ln(a)}e^{x\ln(b)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^x-b^x)^2/(a^x)/(b^x),x)
```

```
[Out] (1/(ln(a)-ln(b))*exp(x*ln(a))^2-1/(ln(a)-ln(b))*exp(x*ln(b))^2-2*x*exp(x*ln
(a))*exp(x*ln(b)))/exp(x*ln(a))/exp(x*ln(b))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^x-b^x)^2/(a^x)/(b^x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.82862, size = 113, normalized size = 3.32

$$-\frac{2\,(x\log(a)-x\log(b))a^x b^x-a^{2\,x}+b^{2\,x}}{a^x b^x(\log(a)-\log(b))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^x-b^x)^2/(a^x)/(b^x),x, algorithm="fricas")
```

```
[Out] -(2*(x*log(a) - x*log(b))*a^x*b^x - a^(2*x) + b^(2*x))/(a^x*b^x*(log(a) - l
og(b)))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**x-b**x)**2/(a**x)/(b**x),x)
```

```
[Out] Exception raised: TypeError
```

Giac [C] time = 1.30936, size = 589, normalized size = 17.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^x-b^x)^2/(a^x)/(b^x),x, algorithm="giac")
```

```
[Out] 2*(2*(log(abs(a)) - log(abs(b)))*cos(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((
pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2) - (pi*sgn(a) -
pi*sgn(b))*sin(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(b))
^2 + 4*(log(abs(a)) - log(abs(b)))^2))*e^(x*(log(abs(a)) - log(abs(b)))) -
1/2*I*(-2*I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b))/(I*pi*sgn(a) - I*pi*s
gn(b) + 2*log(abs(a)) - 2*log(abs(b))) + 2*I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*
pi*x*sgn(b))/(-I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a)) - 2*log(abs(b))))*
e^(x*(log(abs(a)) - log(abs(b)))) - 2*(2*(log(abs(a)) - log(abs(b)))*cos(1/
2*pi*x*sgn(a) - 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)
) - log(abs(b)))^2) - (pi*sgn(a) - pi*sgn(b))*sin(1/2*pi*x*sgn(a) - 1/2*pi*
x*sgn(b))/((pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2))*e^
(-x*(log(abs(a)) - log(abs(b)))) - 1/2*I*(2*I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*
pi*x*sgn(b))/(I*pi*sgn(a) - I*pi*sgn(b) - 2*log(abs(a)) + 2*log(abs(b))) -
2*I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*pi*x*sgn(b))/(-I*pi*sgn(a) + I*pi*sgn(b)
- 2*log(abs(a)) + 2*log(abs(b))))*e^(-x*(log(abs(a)) - log(abs(b)))) - 2*x
```

3.496 $\int (-e^{-x} + e^x)\,dx$

Optimal. Leaf size=9

$$e^{-x} + e^x$$

```
[Out] E^(-x) + E^x
```

Rubi [A] time = 0.0034646, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2194}

$$e^{-x} + e^x$$

Antiderivative was successfully verified.

```
[In] Int[-E^(-x) + E^x,x]
```

```
[Out] E^(-x) + E^x
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int (-e^{-x} + e^x)\,dx &= -\int e^{-x}\,dx + \int e^x\,dx\\ &= e^{-x} + e^x\end{aligned}$$

Mathematica [A] time = 0.0030865, size = 9, normalized size = 1.

$$e^{-x} + e^x$$

Antiderivative was successfully verified.

```
[In] Integrate[-E^(-x) + E^x,x]
```

```
[Out] E^(-x) + E^x
```

Maple [A] time = 0.002, size = 8, normalized size = 0.9

$$(\mathrm{e}^x)^{-1} + \mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/exp(x)+exp(x),x)
```

```
[Out] 1/exp(x)+exp(x)
```

Maxima [A] time = 0.927173, size = 9, normalized size = 1.

$$e^{(-x)} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/exp(x)+exp(x),x, algorithm="maxima")
```

```
[Out] e^(-x) + e^x
```

Fricas [A] time = 1.68799, size = 30, normalized size = 3.33

$$\left(e^{(2x)} + 1\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/exp(x)+exp(x),x, algorithm="fricas")
```

```
[Out] (e^(2*x) + 1)*e^(-x)
```

Sympy [A] time = 0.08325, size = 7, normalized size = 0.78

$$e^x + e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/exp(x)+exp(x),x)
```

```
[Out] exp(x) + exp(-x)
```

Giac [A] time = 1.08643, size = 9, normalized size = 1.

$$e^{(-x)} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/exp(x)+exp(x),x, algorithm="giac")
```

```
[Out] e^(-x) + e^x
```

3.497 $\int (-e^{-x}+e^x)^2\,dx$

Optimal. Leaf size=22

$$-2x-\frac{e^{-2x}}{2}+\frac{e^{2x}}{2}$$

```
[Out] -1/(2*E^(2*x)) + E^(2*x)/2 - 2*x
```

Rubi [A] time = 0.0177986, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2282, 266, 43}

$$-2x-\frac{e^{-2x}}{2}+\frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-E^(-x) + E^x)^2,x]
```

```
[Out] -1/(2*E^(2*x)) + E^(2*x)/2 - 2*x
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (-e^{-x}+e^x)^2\,dx &= \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^3}\,dx,x,e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx,x,e^{2x}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)\,dx,x,e^{2x}\right)\\ &= -\frac{1}{2}e^{-2x}+\frac{e^{2x}}{2}-2x\end{aligned}$$

Mathematica [A] time = 0.0058902, size = 22, normalized size = 1.

$$-2x-\frac{e^{-2x}}{2}+\frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-E^(-x) + E^x)^2,x]
```

```
[Out] -1/(2*E^(2*x)) + E^(2*x)/2 - 2*x
```

Maple [A] time = 0.006, size = 19, normalized size = 0.9

$$\frac{(e^x)^2}{2}-2\ln(e^x)-\frac{1}{2(e^x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/exp(x)+exp(x))^2,x)
```

```
[Out] 1/2*exp(x)^2-2*ln(exp(x))-1/2/exp(x)^2
```

Maxima [A] time = 0.926443, size = 22, normalized size = 1.

$$-2x+\frac{1}{2}e^{(2x)}-\frac{1}{2}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^2,x, algorithm="maxima")
```

```
[Out] -2*x + 1/2*e^(2*x) - 1/2*e^(-2*x)
```

Fricas [A] time = 1.86786, size = 58, normalized size = 2.64

$$-\frac{1}{2}\left(4xe^{(2x)}-e^{(4x)}+1\right)e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*x*e^(2*x) - e^(4*x) + 1)*e^(-2*x)
```

Sympy [A] time = 0.101106, size = 17, normalized size = 0.77

$$-2x+\frac{e^{2x}}{2}-\frac{e^{-2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))**2,x)
```

```
[Out] -2*x + exp(2*x)/2 - exp(-2*x)/2
```

Giac [A] time = 1.05804, size = 32, normalized size = 1.45

$$\frac{1}{2}\left(2e^{(2x)} - 1\right)e^{(-2x)} - 2x + \frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*e^(2*x) - 1)*e^(-2*x) - 2*x + 1/2*e^(2*x)
```

3.498 $\int (-e^{-x} + e^x)^3 \, dx$

Optimal. Leaf size=31

$$\frac{e^{-3x}}{3} - 3e^{-x} - 3e^x + \frac{e^{3x}}{3}$$

```
[Out] 1/(3*E^(3*x)) - 3/E^x - 3*E^x + E^(3*x)/3
```

Rubi [A] time = 0.019334, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2282, 270}

$$\frac{e^{-3x}}{3} - 3e^{-x} - 3e^x + \frac{e^{3x}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(-E^(-x) + E^x)^3,x]
```

```
[Out] 1/(3*E^(3*x)) - 3/E^x - 3*E^x + E^(3*x)/3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (-e^{-x} + e^x)^3 \, dx &= \text{Subst}\left(\int \frac{\left(-1+x^2\right)^3}{x^4} \, dx, x, e^x\right) \\
&= \text{Subst}\left(\int \left(-3 - \frac{1}{x^4} + \frac{3}{x^2} + x^2\right) \, dx, x, e^x\right) \\
&= \frac{e^{-3x}}{3} - 3e^{-x} - 3e^x + \frac{e^{3x}}{3}
\end{aligned}$$

Mathematica [A] time = 0.0131351, size = 30, normalized size = 0.97

$$\frac{1}{3} e^{-3x} \left(-9e^{2x} - 9e^{4x} + e^{6x} + 1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-E^(-x) + E^x)^3,x]
```

```
[Out] (1 - 9*E^(2*x) - 9*E^(4*x) + E^(6*x))/(3*E^(3*x))
```

Maple [A] time = 0.005, size = 24, normalized size = 0.8

$$\frac{(e^x)^3}{3} - 3\,e^x + \frac{1}{3\,(e^x)^3} - 3\,(e^x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/exp(x)+exp(x))^3,x)
```

```
[Out] 1/3*exp(x)^3-3*exp(x)+1/3/exp(x)^3-3/exp(x)
```

Maxima [A] time = 0.922947, size = 31, normalized size = 1.

$$\frac{1}{3}\,e^{(3\,x)} - 3\,e^{(-x)} + \frac{1}{3}\,e^{(-3\,x)} - 3\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*e^(3*x) - 3*e^(-x) + 1/3*e^(-3*x) - 3*e^x
```

Fricas [A] time = 1.73138, size = 70, normalized size = 2.26

$$\frac{1}{3}\left(e^{(6\,x)} - 9\,e^{(4\,x)} - 9\,e^{(2\,x)} + 1\right)e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(e^(6*x) - 9*e^(4*x) - 9*e^(2*x) + 1)*e^(-3*x)
```

Sympy [A] time = 0.126128, size = 24, normalized size = 0.77

$$\frac{e^{3x}}{3} - 3e^{x} - 3e^{-x} + \frac{e^{-3x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))**3,x)
```

```
[Out] exp(3*x)/3 - 3*exp(x) - 3*exp(-x) + exp(-3*x)/3
```

Giac [A] time = 1.06519, size = 34, normalized size = 1.1

$$-\frac{1}{3}\left(9\,e^{(2\,x)}-1\right)e^{(-3\,x)}+\frac{1}{3}\,e^{(3\,x)}-3\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^3,x, algorithm="giac")
```

```
[Out] -1/3*(9*e^(2*x) - 1)*e^(-3*x) + 1/3*e^(3*x) - 3*e^x
```

3.499 $\int (-e^{-x}+e^x)^4 \, dx$

Optimal. Leaf size=36

$$6x-\frac{e^{-4x}}{4}+2e^{-2x}-2e^{2x}+\frac{e^{4x}}{4}$$

```
[Out] -1/(4*E^(4*x)) + 2/E^(2*x) - 2*E^(2*x) + E^(4*x)/4 + 6*x
```

Rubi [A] time = 0.0255405, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2282, 266, 43}

$$6x-\frac{e^{-4x}}{4}+2e^{-2x}-2e^{2x}+\frac{e^{4x}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(-E^(-x) + E^x)^4,x]
```

```
[Out] -1/(4*E^(4*x)) + 2/E^(2*x) - 2*E^(2*x) + E^(4*x)/4 + 6*x
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (-e^{-x}+e^x)^4 \, dx &= \text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{x^5} \, dx, x, e^x\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^4}{x^3} \, dx, x, e^{2x}\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(-4+\frac{1}{x^3}-\frac{4}{x^2}+\frac{6}{x}+x\right) dx, x, e^{2x}\right)\\ &= -\frac{1}{4}e^{-4x}+2e^{-2x}-2e^{2x}+\frac{e^{4x}}{4}+6x\end{aligned}$$

Mathematica [A] time = 0.0209597, size = 34, normalized size = 0.94

$$\frac{1}{4}\left(24x-e^{-4x}+8e^{-2x}-8e^{2x}+e^{4x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-E^(-x) + E^x)^4,x]
```

```
[Out] (-E^(-4*x) + 8/E^(2*x) - 8*E^(2*x) + E^(4*x) + 24*x)/4
```

Maple [A] time = 0.007, size = 31, normalized size = 0.9

$$\frac{\left(e^{x}\right)^{4}}{4}-2\left(e^{x}\right)^{2}+6\ln\left(e^{x}\right)-\frac{1}{4\left(e^{x}\right)^{4}}+2\left(e^{x}\right)^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/exp(x)+exp(x))^4,x)
```

```
[Out] 1/4*exp(x)^4-2*exp(x)^2+6*ln(exp(x))-1/4/exp(x)^4+2/exp(x)^2
```

Maxima [A] time = 0.929387, size = 38, normalized size = 1.06

$$6\,x+\frac{1}{4}\,e^{(4\,x)}-2\,e^{(2\,x)}+2\,e^{(-2\,x)}-\frac{1}{4}\,e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^4,x, algorithm="maxima")
```

```
[Out] 6*x + 1/4*e^(4*x) - 2*e^(2*x) + 2*e^(-2*x) - 1/4*e^(-4*x)
```

Fricas [A] time = 1.80387, size = 90, normalized size = 2.5

$$\frac{1}{4}\left(24\,xe^{(4\,x)}+e^{(8\,x)}-8\,e^{(6\,x)}+8\,e^{(2\,x)}-1\right)e^{(-4\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(24*x*e^(4*x) + e^(8*x) - 8*e^(6*x) + 8*e^(2*x) - 1)*e^(-4*x)
```

Sympy [A] time = 0.137358, size = 31, normalized size = 0.86

$$6x+\frac{e^{4x}}{4}-2e^{2x}+2e^{-2x}-\frac{e^{-4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))**4,x)
```

```
[Out] 6*x + exp(4*x)/4 - 2*exp(2*x) + 2*exp(-2*x) - exp(-4*x)/4
```

Giac [A] time = 1.06391, size = 49, normalized size = 1.36

$$-\frac{1}{4}\left(18\,e^{(4\,x)}-8\,e^{(2\,x)}+1\right)e^{(-4\,x)}+6\,x+\frac{1}{4}\,e^{(4\,x)}-2\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^4,x, algorithm="giac")
```

```
[Out] -1/4*(18*e^(4*x) - 8*e^(2*x) + 1)*e^(-4*x) + 6*x + 1/4*e^(4*x) - 2*e^(2*x)
```

3.500 $\int \left(-e^{-x}+e^x\right)^n\,dx$

Optimal. Leaf size=48

$$-\frac{\left(1-e^{2x}\right)\left(e^x-e^{-x}\right)^n\,{}_2F_1\left(1,\frac{n+2}{2};1-\frac{n}{2};e^{2x}\right)}{n}$$

```
[Out] -(((-E^(-x) + E^x)^n*(1 - E^(2*x))*Hypergeometric2F1[1, (2 + n)/2, 1 - n/2,
 E^(2*x)])/n)
```

Rubi [A] time = 0.0470604, antiderivative size = 52, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2282, 2032, 365, 364}

$$-\frac{\left(e^x-e^{-x}\right)^n\left(1-e^{2x}\right)^{-n}\text{Hypergeometric2F1}\left(-n,-\frac{n}{2},1-\frac{n}{2},e^{2x}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[(-E^(-x) + E^x)^n,x]
```

```
[Out] -(((-E^(-x) + E^x)^n*Hypergeometric2F1[-n, -n/2, 1 - n/2, E^(2*x)])/((1 - E
^(2*x))^n*n))
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2032

```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Dist[(c^IntPart[m]*(c*x)^FracPart[m]*(a*x^j + b*x^n)^FracPart[p])/(x^(
FracPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]), Int[x^(m + j*p
)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int (-e^{-x}+e^x)^n\,dx = \text{Subst}\left(\int \frac{\left(-\frac{1}{x}+x\right)^n}{x}\,dx,x,e^x\right)$$
$$= \left((e^x)^n(-e^{-x}+e^x)^n\left(-1+e^{2x}\right)^{-n}\right)\text{Subst}\left(\int x^{-1-n}\left(-1+x^2\right)^n\,dx,x,e^x\right)$$
$$= \left((e^x)^n(-e^{-x}+e^x)^n\left(1-e^{2x}\right)^{-n}\right)\text{Subst}\left(\int x^{-1-n}\left(1-x^2\right)^n\,dx,x,e^x\right)$$
$$= -\frac{(-e^{-x}+e^x)^n\left(1-e^{2x}\right)^{-n}\,{}_2F_1\left(-n,-\frac{n}{2};1-\frac{n}{2};e^{2x}\right)}{n}$$

Mathematica [A] time = 0.0116902, size = 45, normalized size = 0.94

$$\frac{\left(e^{2x}-1\right)(e^x-e^{-x})^n\,{}_2F_1\left(1,\frac{n}{2}+1;1-\frac{n}{2};e^{2x}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-E^(-x) + E^x)^n,x]
```

```
[Out] ((-E^(-x) + E^x)^n*(-1 + E^(2*x))*Hypergeometric2F1[1, 1 + n/2, 1 - n/2, E^
(2*x)])/n
```

Maple [F] time = 0.127, size = 0, normalized size = 0.

$$\int \left(-\left(e^x\right)^{-1}+e^x\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/exp(x)+exp(x))^n,x)
```

```
[Out] int((-1/exp(x)+exp(x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^{(-x)}+e^x\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^n,x, algorithm="maxima")
```

```
[Out] integrate((-e^(-x) + e^x)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-e^{(-x)}+e^x\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^n,x, algorithm="fricas")
```

```
[Out] integral((-e^(-x) + e^x)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e^x - e^{-x})^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))**n,x)
```

```
[Out] Integral((exp(x) - exp(-x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-e^{(-x)} + e^x\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))^n,x, algorithm="giac")
```

```
[Out] integrate((-e^(-x) + e^x)^n, x)
```

3.501 $\int \left(a^{-4x}-a^{2x}\right)^3 \, dx$

Optimal. Leaf size=43

$$-\frac{a^{-12x}}{12\log(a)}+\frac{a^{-6x}}{2\log(a)}-\frac{a^{6x}}{6\log(a)}+3x$$

```
[Out] 3*x - 1/(12*a^(12*x)*Log[a]) + 1/(2*a^(6*x)*Log[a]) - a^(6*x)/(6*Log[a])
```

Rubi [A] time = 0.0272945, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2282, 266, 43}

$$-\frac{a^{-12x}}{12\log(a)}+\frac{a^{-6x}}{2\log(a)}-\frac{a^{6x}}{6\log(a)}+3x$$

Antiderivative was successfully verified.

```
[In] Int[(a^(-4*x) - a^(2*x))^3,x]
```

```
[Out] 3*x - 1/(12*a^(12*x)*Log[a]) + 1/(2*a^(6*x)*Log[a]) - a^(6*x)/(6*Log[a])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(a^{-4x}-a^{2x}\right)^3 dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^3\right)^3}{x^7} dx, x, a^{2x}\right)}{2\log(a)}$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x)^3}{x^3} dx, x, a^{6x}\right)}{6\log(a)}$$
$$= \frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^3}-\frac{3}{x^2}+\frac{3}{x}\right) dx, x, a^{6x}\right)}{6\log(a)}$$
$$= 3x - \frac{a^{-12x}}{12\log(a)} + \frac{a^{-6x}}{2\log(a)} - \frac{a^{6x}}{6\log(a)}$$

Mathematica [A] time = 0.0315887, size = 33, normalized size = 0.77

$$-\frac{a^{-12x}-6a^{-6x}+2a^{6x}-36x\log(a)}{12\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(-4*x) - a^(2*x))^3,x]
```

```
[Out] -(a^(-12*x) - 6/a^(6*x) + 2*a^(6*x) - 36*x*Log[a])/(12*Log[a])
```

Maple [A] time = 0.019, size = 56, normalized size = 1.3

$$\frac{1}{\left(e^{2x\ln(a)}\right)^6}\left(-\frac{1}{12\ln(a)}+3x\left(e^{2x\ln(a)}\right)^6+\frac{\left(e^{2x\ln(a)}\right)^3}{2\ln(a)}-\frac{\left(e^{2x\ln(a)}\right)^9}{6\ln(a)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(a^(4*x))-a^(2*x))^3,x)
```

```
[Out] (-1/12/ln(a)+3*x*exp(2*x*ln(a))^6+1/2/ln(a)*exp(2*x*ln(a))^3-1/6/ln(a)*exp(
2*x*ln(a))^9)/exp(2*x*ln(a))^6
```

Maxima [A] time = 0.925228, size = 55, normalized size = 1.28

$$3x-\frac{a^{6x}}{6\log(a)}-\frac{1}{12a^{12x}\log(a)}+\frac{1}{2a^{6x}\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(a^(4*x))-a^(2*x))^3,x, algorithm="maxima")
```

```
[Out] 3*x - 1/6*a^(6*x)/log(a) - 1/12/(a^(12*x)*log(a)) + 1/2/(a^(6*x)*log(a))
```

Fricas [A] time = 1.91736, size = 103, normalized size = 2.4

$$\frac{36\,a^{12\,x}x\log\left(a\right) - 2\,a^{18\,x} + 6\,a^{6\,x} - 1}{12\,a^{12\,x}\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(a^(4*x))-a^(2*x))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(36*a^(12*x)*x*log(a) - 2*a^(18*x) + 6*a^(6*x) - 1)/(a^(12*x)*log(a))
```

Sympy [A] time = 0.21758, size = 54, normalized size = 1.26

$$3x + \begin{cases} \frac{-24a^{6x}\log\left(a\right)^{2} + 72a^{-6x}\log\left(a\right)^{2} - 12a^{-12x}\log\left(a\right)^{2}}{144\log\left(a\right)^{3}} & \text{for}\: 144\log\left(a\right)^{3} \neq 0 \\ -3x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(a**(4*x))-a**(2*x))**3,x)
```

```
[Out] 3*x + Piecewise(((-24*a**(6*x)*log(a)**2 + 72*a**(-6*x)*log(a)**2 - 12*a**(
-12*x)*log(a)**2)/(144*log(a)**3), Ne(144*log(a)**3, 0)), (-3*x, True))
```

Giac [A] time = 1.06233, size = 62, normalized size = 1.44

$$-\frac{2\,a^{6\,x} + \frac{9\,a^{12\,x} - 6\,a^{6\,x} + 1}{a^{12\,x}} - 6\,\log\left(a^{6\,x}\right)}{12\,\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(a^(4*x))-a^(2*x))^3,x, algorithm="giac")
```

```
[Out] -1/12*(2*a^(6*x) + (9*a^(12*x) - 6*a^(6*x) + 1)/a^(12*x) - 6*log(a^(6*x)))/
log(a)
```

3.502 $\int \left(a^{kx}+a^{lx}\right) dx$

Optimal. Leaf size=27

$$\frac{a^{kx}}{k\log(a)}+\frac{a^{lx}}{l\log(a)}$$

```
[Out] a^(k*x)/(k*Log[a]) + a^(l*x)/(l*Log[a])
```

Rubi [A] time = 0.010812, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2194}

$$\frac{a^{kx}}{k\log(a)}+\frac{a^{lx}}{l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[a^(k*x) + a^(l*x),x]
```

```
[Out] a^(k*x)/(k*Log[a]) + a^(l*x)/(l*Log[a])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a^{kx}+a^{lx}\right) dx &= \int a^{kx}\, dx+\int a^{lx}\, dx\\ &= \frac{a^{kx}}{k\log(a)}+\frac{a^{lx}}{l\log(a)}\end{aligned}$$

Mathematica [A] time = 0.0058437, size = 27, normalized size = 1.

$$\frac{a^{kx}}{k\log(a)}+\frac{a^{lx}}{l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^(k*x) + a^(l*x),x]
```

```
[Out] a^(k*x)/(k*Log[a]) + a^(l*x)/(l*Log[a])
```

Maple [A] time = 0.003, size = 28, normalized size = 1.

$$\frac{a^{kx}}{k\ln(a)}+\frac{a^{lx}}{l\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^(k*x)+a^(l*x),x)
```

```
[Out] a^(k*x)/k/ln(a)+a^(l*x)/l/ln(a)
```

Maxima [A] time = 0.948154, size = 36, normalized size = 1.33

$$\frac{a^{kx}}{k\log\left(a\right)}+\frac{a^{lx}}{l\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(k*x)+a^(l*x),x, algorithm="maxima")
```

```
[Out] a^(k*x)/(k*log(a)) + a^(l*x)/(l*log(a))
```

Fricas [A] time = 1.76971, size = 51, normalized size = 1.89

$$\frac{a^{lx}k+a^{kx}l}{kl\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(k*x)+a^(l*x),x, algorithm="fricas")
```

```
[Out] (a^(l*x)*k + a^(k*x)*l)/(k*l*log(a))
```

Sympy [A] time = 0.286201, size = 29, normalized size = 1.07

$$\begin{cases}\frac{a^{kx}}{k\log\left(a\right)} & \text{for}\: k\log\left(a\right)\neq 0\\x & \text{otherwise}\end{cases}+\begin{cases}\frac{a^{lx}}{l\log\left(a\right)} & \text{for}\: l\log\left(a\right)\neq 0\\x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**(k*x)+a**(l*x),x)
```

```
[Out] Piecewise((a**(k*x)/(k*log(a)), Ne(k*log(a), 0)), (x, True)) + Piecewise((a
**(l*x)/(l*log(a)), Ne(l*log(a), 0)), (x, True))
```

Giac [A] time = 1.06494, size = 36, normalized size = 1.33

$$\frac{a^{kx}}{k\log\left(a\right)}+\frac{a^{lx}}{l\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(k*x)+a^(l*x),x, algorithm="giac")
```

```
[Out] a^(k*x)/(k*log(a)) + a^(l*x)/(l*log(a))
```

3.503 $\int \left(a^{kx}+a^{lx}\right)^2 \, dx$

Optimal. Leaf size=53

$$\frac{2a^{x(k+l)}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) + (2*a^((k + l)*x))/((k + l
)*Log[a])
```

Rubi [A] time = 0.0784878, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6742, 2194}

$$\frac{2a^{x(k+l)}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(k*x) + a^(l*x))^2,x]
```

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) + (2*a^((k + l)*x))/((k + l
)*Log[a])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a^{kx}+a^{lx}\right)^2 \, dx &= \frac{\text{Subst}\left(\int \left(e^{kx}+e^{lx}\right)^2 \, dx, x, x\log(a)\right)}{\log(a)}\\
&= \frac{\text{Subst}\left(\int \left(e^{2kx}+e^{2lx}+2e^{(k+l)x}\right) dx, x, x\log(a)\right)}{\log(a)}\\
&= \frac{\text{Subst}\left(\int e^{2kx}\, dx, x, x\log(a)\right)}{\log(a)}+\frac{\text{Subst}\left(\int e^{2lx}\, dx, x, x\log(a)\right)}{\log(a)}+\frac{2\,\text{Subst}\left(\int e^{(k+l)x}\, dx, x, x\log(a)\right)}{\log(a)}\\
&= \frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}+\frac{2a^{(k+l)x}}{(k+l)\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.0422056, size = 53, normalized size = 1.

$$\frac{2a^{x(k+l)}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(k*x) + a^(l*x))^2,x]
```

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) + (2*a^((k + l)*x))/((k + l
)*Log[a])
```

Maple [A] time = 0.019, size = 59, normalized size = 1.1

$$\frac{\left(\mathrm{e}^{kx\ln(a)}\right)^2}{2\,k\ln(a)}+\frac{\left(\mathrm{e}^{lx\ln(a)}\right)^2}{2\,l\ln(a)}+2\,\frac{\mathrm{e}^{kx\ln(a)}\mathrm{e}^{lx\ln(a)}}{\ln(a)\,(k+l)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x)+a^(l*x))^2,x)
```

```
[Out] 1/2/k/ln(a)*exp(k*x*ln(a))^2+1/2/l/ln(a)*exp(l*x*ln(a))^2+2/ln(a)/(k+l)*exp
(k*x*ln(a))*exp(l*x*ln(a))
```

Maxima [A] time = 0.922317, size = 69, normalized size = 1.3

$$\frac{2\,a^{kx+lx}}{(k+l)\log(a)}+\frac{a^{2\,kx}}{2\,k\log(a)}+\frac{a^{2\,lx}}{2\,l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^2,x, algorithm="maxima")
```

```
[Out] 2*a^(k*x + l*x)/((k + l)*log(a)) + 1/2*a^(2*k*x)/(k*log(a)) + 1/2*a^(2*l*x)
/(l*log(a))
```

Fricas [A] time = 1.92044, size = 138, normalized size = 2.6

$$\frac{4\,a^{kx}a^{lx}kl+\left(kl+l^2\right)a^{2\,kx}+\left(k^2+kl\right)a^{2\,lx}}{2\left(k^2l+kl^2\right)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^(k*x)*a^(l*x)*k*l + (k*l + l^2)*a^(2*k*x) + (k^2 + k*l)*a^(2*l*x))
/((k^2*l + k*l^2)*log(a))
```

Sympy [A] time = 2.0471, size = 250, normalized size = 4.72

$$\begin{cases} 4x & \text{for } a = 1 \\ \frac{a^{2lx}}{2l\log(a)}+\frac{2a^{lx}}{l\log(a)}+x & \text{for } k = 0 \\ \frac{a^{2lx}}{2l\log(a)}+2x-\frac{a^{-2lx}}{2l\log(a)} & \text{for } k = -l \\ \frac{a^{2kx}}{2k\log(a)}+\frac{2a^{kx}}{k\log(a)}+x & \text{for } l = 0 \\ \frac{a^{2kx}kl}{2k^2l\log(a)+2kl^2\log(a)}+\frac{a^{2kx}l^2}{2k^2l\log(a)+2kl^2\log(a)}+\frac{4a^{kx}a^{lx}kl}{2k^2l\log(a)+2kl^2\log(a)}+\frac{a^{2lx}k^2}{2k^2l\log(a)+2kl^2\log(a)}+\frac{a^{2lx}kl}{2k^2l\log(a)+2kl^2\log(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(k*x)+a**(l*x))**2,x)
```

```
[Out] Piecewise((4*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))),
(a**(2*l*x)/(2*l*log(a)) + 2*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(2*l*x
)/(2*l*log(a)) + 2*x - a**(-2*l*x)/(2*l*log(a)), Eq(k, -l)), (a**(2*k*x)/(2
*k*log(a)) + 2*a**(k*x)/(k*log(a)) + x, Eq(l, 0)), (a**(2*k*x)*k*l/(2*k**2*
l*log(a) + 2*k*l**2*log(a)) + a**(2*k*x)*l**2/(2*k**2*l*log(a) + 2*k*l**2*l
og(a)) + 4*a**(k*x)*a**(l*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(
2*l*x)*k**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x)*k*l/(2*k**2*l*
log(a) + 2*k*l**2*log(a)), True))
```

Giac [C] time = 1.15727, size = 933, normalized size = 17.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^2,x, algorithm="giac")
```

```
[Out] (2*k*cos(-pi*k*x*sgn(a) + pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2 + (pi*k*
sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-pi*k*x*sgn(a) + pi*k*x)/(4*k^
2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(2*k*x) + (2*l*cos(-pi*l*
x*sgn(a) + pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^
2) - (pi*l*sgn(a) - pi*l)*sin(-pi*l*x*sgn(a) + pi*l*x)/(4*l^2*log(abs(a))^2
 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(2*l*x) - 1/2*I*abs(a)^(2*k*x)*(-I*e^(I*
pi*k*x*sgn(a) - I*pi*k*x)/(I*pi*k*sgn(a) - I*pi*k + 2*k*log(abs(a))) + I*e^
(-I*pi*k*x*sgn(a) + I*pi*k*x)/(-I*pi*k*sgn(a) + I*pi*k + 2*k*log(abs(a))))
- 1/2*I*abs(a)^(2*l*x)*(-I*e^(I*pi*l*x*sgn(a) - I*pi*l*x)/(I*pi*l*sgn(a) -
I*pi*l + 2*l*log(abs(a))) + I*e^(-I*pi*l*x*sgn(a) + I*pi*l*x)/(-I*pi*l*sgn(
a) + I*pi*l + 2*l*log(abs(a)))) + 4*(2*(k*log(abs(a)) + l*log(abs(a)))*cos(
-1/2*pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*x)/((pi*k*sg
n(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2)
- (pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)*sin(-1/2*pi*k*x*sgn(a) - 1/2*pi
*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*x)/((pi*k*sgn(a) + pi*l*sgn(a) - pi*k -
 pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2))*e^((k*log(abs(a)) + l*log(
abs(a)))*x) - 1/2*I*(-4*I*e^(1/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l*x*sgn(a) - 1/
2*I*pi*k*x - 1/2*I*pi*l*x)/(I*pi*k*sgn(a) + I*pi*l*sgn(a) - I*pi*k - I*pi*l
 + 2*k*log(abs(a)) + 2*l*log(abs(a))) + 4*I*e^(-1/2*I*pi*k*x*sgn(a) - 1/2*I
*pi*l*x*sgn(a) + 1/2*I*pi*k*x + 1/2*I*pi*l*x)/(-I*pi*k*sgn(a) - I*pi*l*sgn(
a) + I*pi*k + I*pi*l + 2*k*log(abs(a)) + 2*l*log(abs(a))))*e^((k*log(abs(a)
) + l*log(abs(a)))*x)
```

3.504 $\int \left(a^{kx}+a^{lx}\right)^3\,dx$

Optimal. Leaf size=79

$$\frac{3a^{x(2k+l)}}{\log(a)(2k+l)}+\frac{3a^{x(k+2l)}}{\log(a)(k+2l)}+\frac{a^{3kx}}{3k\log(a)}+\frac{a^{3lx}}{3l\log(a)}$$

```
[Out] a^(3*k*x)/(3*k*Log[a]) + a^(3*l*x)/(3*l*Log[a]) + (3*a^((2*k + l)*x))/((2*k
 + l)*Log[a]) + (3*a^((k + 2*l)*x))/((k + 2*l)*Log[a])
```

Rubi [A] time = 0.100182, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6742, 2194}

$$\frac{3a^{x(2k+l)}}{\log(a)(2k+l)}+\frac{3a^{x(k+2l)}}{\log(a)(k+2l)}+\frac{a^{3kx}}{3k\log(a)}+\frac{a^{3lx}}{3l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(k*x) + a^(l*x))^3,x]
```

```
[Out] a^(3*k*x)/(3*k*Log[a]) + a^(3*l*x)/(3*l*Log[a]) + (3*a^((2*k + l)*x))/((2*k
 + l)*Log[a]) + (3*a^((k + 2*l)*x))/((k + 2*l)*Log[a])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a^{kx}+a^{lx}\right)^3\,dx &= \frac{\text{Subst}\left(\int \left(e^{kx}+e^{lx}\right)^3\,dx,x,x\log(a)\right)}{\log(a)}\\ &= \frac{\text{Subst}\left(\int \left(e^{3kx}+e^{3lx}+3e^{(2k+l)x}+3e^{(k+2l)x}\right)dx,x,x\log(a)\right)}{\log(a)}\\ &= \frac{\text{Subst}\left(\int e^{3kx}\,dx,x,x\log(a)\right)}{\log(a)}+\frac{\text{Subst}\left(\int e^{3lx}\,dx,x,x\log(a)\right)}{\log(a)}+\frac{3\,\text{Subst}\left(\int e^{(2k+l)x}\,dx,x,x\log(a)\right)}{\log(a)}\\ &= \frac{a^{3kx}}{3k\log(a)}+\frac{a^{3lx}}{3l\log(a)}+\frac{3a^{(2k+l)x}}{(2k+l)\log(a)}+\frac{3a^{(k+2l)x}}{(k+2l)\log(a)}\end{aligned}$$

Mathematica [A] time = 0.087738, size = 65, normalized size = 0.82

$$\frac{\frac{9a^{x(2k+l)}}{2k+l}+\frac{9a^{x(k+2l)}}{k+2l}+\frac{a^{3kx}}{k}+\frac{a^{3lx}}{l}}{3\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(k*x) + a^(l*x))^3,x]
```

```
[Out] (a^(3*k*x)/k + a^(3*l*x)/l + (9*a^((2*k + l)*x))/(2*k + l) + (9*a^((k + 2*l
)*x))/(k + 2*l))/(3*Log[a])
```

Maple [A] time = 0.03, size = 90, normalized size = 1.1

$$\frac{\left(\mathrm{e}^{kx\ln(a)}\right)^{3}}{3k\ln(a)}+\frac{\left(\mathrm{e}^{lx\ln(a)}\right)^{3}}{3l\ln(a)}+3\,\frac{\mathrm{e}^{kx\ln(a)}\left(\mathrm{e}^{lx\ln(a)}\right)^{2}}{\ln(a)(k+2l)}+3\,\frac{\left(\mathrm{e}^{kx\ln(a)}\right)^{2}\mathrm{e}^{lx\ln(a)}}{\ln(a)(2k+l)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x)+a^(l*x))^3,x)
```

```
[Out] 1/3/k/ln(a)*exp(k*x*ln(a))^3+1/3/l/ln(a)*exp(l*x*ln(a))^3+3/ln(a)/(k+2*l)*e
xp(k*x*ln(a))*exp(l*x*ln(a))^2+3/ln(a)/(2*k+l)*exp(k*x*ln(a))^2*exp(l*x*ln(
a))
```

Maxima [A] time = 0.942141, size = 104, normalized size = 1.32

$$\frac{3\,a^{2kx+lx}}{(2k+l)\log(a)}+\frac{3\,a^{kx+2lx}}{(k+2l)\log(a)}+\frac{a^{3kx}}{3k\log(a)}+\frac{a^{3lx}}{3l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^3,x, algorithm="maxima")
```

```
[Out] 3*a^(2*k*x + l*x)/((2*k + l)*log(a)) + 3*a^(k*x + 2*l*x)/((k + 2*l)*log(a))
 + 1/3*a^(3*k*x)/(k*log(a)) + 1/3*a^(3*l*x)/(l*log(a))
```

Fricas [A] time = 1.7881, size = 278, normalized size = 3.52

$$\frac{9\left(2k^{2}l+kl^{2}\right)a^{kx}a^{2lx}+9\left(k^{2}l+2kl^{2}\right)a^{2kx}a^{lx}+\left(2k^{2}l+5kl^{2}+2l^{3}\right)a^{3kx}+\left(2k^{3}+5k^{2}l+2kl^{2}\right)a^{3lx}}{3\left(2k^{3}l+5k^{2}l^{2}+2kl^{3}\right)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(9*(2*k^2*l + k*l^2)*a^(k*x)*a^(2*l*x) + 9*(k^2*l + 2*k*l^2)*a^(2*k*x)*
a^(l*x) + (2*k^2*l + 5*k*l^2 + 2*l^3)*a^(3*k*x) + (2*k^3 + 5*k^2*l + 2*k*l^
2)*a^(3*l*x))/((2*k^3*l + 5*k^2*l^2 + 2*k*l^3)*log(a))
```

Sympy [A] time = 30.8855, size = 665, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(k*x)+a**(l*x))**3,x)
```

```
[Out] Piecewise((8*x, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))),
(a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(2*l*log(a)) + 3*a**(l*x)/(l*log(a)
) + x, Eq(k, 0)), (a**(3*l*x)/(3*l*log(a)) + 3*x - a**(-3*l*x)/(l*log(a)) -
 a**(-6*l*x)/(6*l*log(a)), Eq(k, -2*l)), (2*a**(3*l*x/2)/(l*log(a)) + a**(3
*l*x)/(3*l*log(a)) + 3*x - 2*a**(-3*l*x/2)/(3*l*log(a)), Eq(k, -l/2)), (a**
(3*k*x)/(3*k*log(a)) + 3*a**(2*k*x)/(2*k*log(a)) + 3*a**(k*x)/(k*log(a)) +
x, Eq(l, 0)), (2*a**(3*k*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) +
 6*k*l**3*log(a)) + 5*a**(3*k*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log
(a) + 6*k*l**3*log(a)) + 2*a**(3*k*x)*l**3/(6*k**3*l*log(a) + 15*k**2*l**2*
log(a) + 6*k*l**3*log(a)) + 9*a**(2*k*x)*a**(l*x)*k**2*l/(6*k**3*l*log(a) +
 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a**(2*k*x)*a**(l*x)*k*l**2/(6*
k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a**(k*x)*a**(2*
l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 9*a
**(k*x)*a**(2*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3
*log(a)) + 2*a**(3*l*x)*k**3/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l
**3*log(a)) + 5*a**(3*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) +
6*k*l**3*log(a)) + 2*a**(3*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(
a) + 6*k*l**3*log(a)), True))
```

Giac [C] time = 1.20619, size = 1395, normalized size = 17.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^3,x, algorithm="giac")
```

```
[Out] 2/3*(2*k*cos(-3/2*pi*k*x*sgn(a) + 3/2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a)
)^2 + (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-3/2*pi*k*x*sgn(a)
 + 3/2*pi*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(3*k*
x) + 2/3*(2*l*cos(-3/2*pi*l*x*sgn(a) + 3/2*pi*l*x)*log(abs(a))/(4*l^2*log(a
bs(a))^2 + (pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-3/2*pi*l*x*s
gn(a) + 3/2*pi*l*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^
(3*l*x) - 1/2*I*abs(a)^(3*k*x)*(-2*I*e^(3/2*I*pi*k*x*sgn(a) - 3/2*I*pi*k*x)
/(3*I*pi*k*sgn(a) - 3*I*pi*k + 6*k*log(abs(a))) + 2*I*e^(-3/2*I*pi*k*x*sgn(
a) + 3/2*I*pi*k*x)/(-3*I*pi*k*sgn(a) + 3*I*pi*k + 6*k*log(abs(a)))) - 1/2*I
*abs(a)^(3*l*x)*(-2*I*e^(3/2*I*pi*l*x*sgn(a) - 3/2*I*pi*l*x)/(3*I*pi*l*sgn(
a) - 3*I*pi*l + 6*l*log(abs(a))) + 2*I*e^(-3/2*I*pi*l*x*sgn(a) + 3/2*I*pi*l
*x)/(-3*I*pi*l*sgn(a) + 3*I*pi*l + 6*l*log(abs(a)))) + 6*(2*(2*k*log(abs(a)
) + l*log(abs(a)))*cos(-pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi
*l*x)/((2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a))
 + l*log(abs(a)))^2) - (2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)*sin(-p
i*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*l*x)/((2*pi*k*sgn(a) + p
i*l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a)) + l*log(abs(a)))^2))*e^(
(2*k*log(abs(a)) + l*log(abs(a)))*x) - 1/2*I*(-6*I*e^(I*pi*k*x*sgn(a) + 1/2
*I*pi*l*x*sgn(a) - I*pi*k*x - 1/2*I*pi*l*x)/(2*I*pi*k*sgn(a) + I*pi*l*sgn(a
) - 2*I*pi*k - I*pi*l + 4*k*log(abs(a)) + 2*l*log(abs(a))) + 6*I*e^(-I*pi*k
*x*sgn(a) - 1/2*I*pi*l*x*sgn(a) + I*pi*k*x + 1/2*I*pi*l*x)/(-2*I*pi*k*sgn(a
) - I*pi*l*sgn(a) + 2*I*pi*k + I*pi*l + 4*k*log(abs(a)) + 2*l*log(abs(a))))
*e^((2*k*log(abs(a)) + l*log(abs(a)))*x) + 6*(2*(k*log(abs(a)) + 2*l*log(ab
s(a)))*cos(-1/2*pi*k*x*sgn(a) - pi*l*x*sgn(a) + 1/2*pi*k*x + pi*l*x)/((pi*k
*sgn(a) + 2*pi*l*sgn(a) - pi*k - 2*pi*l)^2 + 4*(k*log(abs(a)) + 2*l*log(abs
(a)))^2) - (pi*k*sgn(a) + 2*pi*l*sgn(a) - pi*k - 2*pi*l)*sin(-1/2*pi*k*x*sg
```

```
n(a) - pi*l*x*sgn(a) + 1/2*pi*k*x + pi*l*x)/((pi*k*sgn(a) + 2*pi*l*sgn(a) -
 pi*k - 2*pi*l)^2 + 4*(k*log(abs(a)) + 2*l*log(abs(a)))^2))*e^((k*log(abs(a
)) + 2*l*log(abs(a)))*x) - 1/2*I*(-6*I*e^(1/2*I*pi*k*x*sgn(a) + I*pi*l*x*sg
n(a) - 1/2*I*pi*k*x - I*pi*l*x)/(I*pi*k*sgn(a) + 2*I*pi*l*sgn(a) - I*pi*k -
 2*I*pi*l + 2*k*log(abs(a)) + 4*l*log(abs(a))) + 6*I*e^(-1/2*I*pi*k*x*sgn(a
) - I*pi*l*x*sgn(a) + 1/2*I*pi*k*x + I*pi*l*x)/(-I*pi*k*sgn(a) - 2*I*pi*l*s
gn(a) + I*pi*k + 2*I*pi*l + 2*k*log(abs(a)) + 4*l*log(abs(a))))*e^((k*log(a
bs(a)) + 2*l*log(abs(a)))*x)
```

3.505 $\int \left(a^{kx}+a^{lx}\right)^4\,dx$

Optimal. Leaf size=98

$$\frac{3a^{2x(k+l)}}{\log(a)(k+l)}+\frac{4a^{x(3k+l)}}{\log(a)(3k+l)}+\frac{4a^{x(k+3l)}}{\log(a)(k+3l)}+\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}$$

```
[Out] a^(4*k*x)/(4*k*Log[a]) + a^(4*l*x)/(4*l*Log[a]) + (3*a^(2*(k + l)*x))/((k +
 l)*Log[a]) + (4*a^((3*k + l)*x))/((3*k + l)*Log[a]) + (4*a^((k + 3*l)*x))/
((k + 3*l)*Log[a])
```

Rubi [A] time = 0.124742, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {6742, 2194}

$$\frac{3a^{2x(k+l)}}{\log(a)(k+l)}+\frac{4a^{x(3k+l)}}{\log(a)(3k+l)}+\frac{4a^{x(k+3l)}}{\log(a)(k+3l)}+\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(k*x) + a^(l*x))^4,x]
```

```
[Out] a^(4*k*x)/(4*k*Log[a]) + a^(4*l*x)/(4*l*Log[a]) + (3*a^(2*(k + l)*x))/((k +
 l)*Log[a]) + (4*a^((3*k + l)*x))/((3*k + l)*Log[a]) + (4*a^((k + 3*l)*x))/
((k + 3*l)*Log[a])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a^{kx}+a^{lx}\right)^4\,dx &= \frac{\operatorname{Subst}\left(\int \left(e^{kx}+e^{lx}\right)^4\,dx,x,x\log(a)\right)}{\log(a)}\\
&= \frac{\operatorname{Subst}\left(\int \left(e^{4kx}+e^{4lx}+6e^{2(k+l)x}+4e^{(3k+l)x}+4e^{(k+3l)x}\right)dx,x,x\log(a)\right)}{\log(a)}\\
&= \frac{\operatorname{Subst}\left(\int e^{4kx}\,dx,x,x\log(a)\right)}{\log(a)}+\frac{\operatorname{Subst}\left(\int e^{4lx}\,dx,x,x\log(a)\right)}{\log(a)}+\frac{4\operatorname{Subst}\left(\int e^{(3k+l)x}\,dx,x,x\log(a)\right)}{\log(a)}\\
&= \frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}+\frac{3a^{2(k+l)x}}{(k+l)\log(a)}+\frac{4a^{(3k+l)x}}{(3k+l)\log(a)}+\frac{4a^{(k+3l)x}}{(k+3l)\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.0910069, size = 80, normalized size = 0.82

$$\frac{\frac{12a^{2x(k+l)}}{k+l}+\frac{16a^{x(3k+l)}}{3k+l}+\frac{16a^{x(k+3l)}}{k+3l}+\frac{a^{4kx}}{k}+\frac{a^{4lx}}{l}}{4\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(k*x) + a^(l*x))^4,x]
```

```
[Out] (a^(4*k*x)/k + a^(4*l*x)/l + (12*a^(2*(k + l)*x))/(k + l) + (16*a^((3*k + l
)*x))/(3*k + l) + (16*a^((k + 3*l)*x))/(k + 3*l))/(4*Log[a])
```

Maple [A] time = 0.016, size = 109, normalized size = 1.1

$$\frac{\left(a^{kx}\right)^{4}}{4k\ln\left(a\right)}+4\frac{\left(a^{kx}\right)^{3}a^{lx}}{\ln\left(a\right)\left(3k+l\right)}+3\frac{\left(a^{kx}\right)^{2}\left(a^{lx}\right)^{2}}{\ln\left(a\right)\left(k+l\right)}+4\frac{a^{kx}\left(a^{lx}\right)^{3}}{\ln\left(a\right)\left(k+3l\right)}+\frac{\left(a^{lx}\right)^{4}}{4l\ln\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x)+a^(l*x))^4,x)
```

```
[Out] 1/4/ln(a)/k*(a^(k*x))^4+4*(a^(k*x))^3/ln(a)/(3*k+l)*a^(l*x)+3*(a^(k*x))^2/l
n(a)/(k+l)*(a^(l*x))^2+4*a^(k*x)/ln(a)/(k+3*l)*(a^(l*x))^3+1/4/ln(a)/l*(a^(
l*x))^4
```

Maxima [A] time = 0.932314, size = 134, normalized size = 1.37

$$\frac{4\,a^{3\,kx+lx}}{(3\,k+l)\log\left(a\right)}+\frac{4\,a^{kx+3\,lx}}{(k+3\,l)\log\left(a\right)}+\frac{3\,a^{2\,kx+2\,lx}}{(k+l)\log\left(a\right)}+\frac{a^{4\,kx}}{4\,k\log\left(a\right)}+\frac{a^{4\,lx}}{4\,l\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^4,x, algorithm="maxima")
```

```
[Out] 4*a^(3*k*x + l*x)/((3*k + l)*log(a)) + 4*a^(k*x + 3*l*x)/((k + 3*l)*log(a))
 + 3*a^(2*k*x + 2*l*x)/((k + l)*log(a)) + 1/4*a^(4*k*x)/(k*log(a)) + 1/4*a^
(4*l*x)/(l*log(a))
```

Fricas [B] time = 1.90731, size = 448, normalized size = 4.57

$$\frac{16\left(3\,k^{3}l+4\,k^{2}l^{2}+kl^{3}\right)a^{kx}a^{3\,lx}+12\left(3\,k^{3}l+10\,k^{2}l^{2}+3\,kl^{3}\right)a^{2\,kx}a^{2\,lx}+16\left(k^{3}l+4\,k^{2}l^{2}+3\,kl^{3}\right)a^{3\,kx}a^{lx}+\left(3\,k^{3}l+13\,k^{2}l^{2}+\right.}{4\left(3\,k^{4}l+13\,k^{3}l^{2}+13\,k^{2}l^{3}+3\,kl^{4}\right)\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(16*(3*k^3*l + 4*k^2*l^2 + k*l^3)*a^(k*x)*a^(3*l*x) + 12*(3*k^3*l + 10*
k^2*l^2 + 3*k*l^3)*a^(2*k*x)*a^(2*l*x) + 16*(k^3*l + 4*k^2*l^2 + 3*k*l^3)*a
^(3*k*x)*a^(l*x) + (3*k^3*l + 13*k^2*l^2 + 13*k*l^3 + 3*l^4)*a^(4*k*x) + (3
*k^4 + 13*k^3*l + 13*k^2*l^2 + 3*k*l^3)*a^(4*l*x))/((3*k^4*l + 13*k^3*l^2 +
 13*k^2*l^3 + 3*k*l^4)*log(a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(k*x)+a**(l*x))**4,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.24725, size = 1835, normalized size = 18.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*k*cos(-2*pi*k*x*sgn(a) + 2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2
+ (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-2*pi*k*x*sgn(a) + 2*p
i*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(4*k*x) + 1/2
*(2*l*cos(-2*pi*l*x*sgn(a) + 2*pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2 + (
pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-2*pi*l*x*sgn(a) + 2*pi*l
*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(4*l*x) - 1/2*I*
abs(a)^(4*k*x)*(-I*e^(2*I*pi*k*x*sgn(a) - 2*I*pi*k*x)/(2*I*pi*k*sgn(a) - 2*
I*pi*k + 4*k*log(abs(a))) + I*e^(-2*I*pi*k*x*sgn(a) + 2*I*pi*k*x)/(-2*I*pi*
k*sgn(a) + 2*I*pi*k + 4*k*log(abs(a)))) - 1/2*I*abs(a)^(4*l*x)*(-I*e^(2*I*p
i*l*x*sgn(a) - 2*I*pi*l*x)/(2*I*pi*l*sgn(a) - 2*I*pi*l + 4*l*log(abs(a))) +
 I*e^(-2*I*pi*l*x*sgn(a) + 2*I*pi*l*x)/(-2*I*pi*l*sgn(a) + 2*I*pi*l + 4*l*l
og(abs(a)))) + 8*(2*(3*k*log(abs(a)) + l*log(abs(a)))*cos(-3/2*pi*k*x*sgn(a
) - 1/2*pi*l*x*sgn(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(a) + pi*l*sgn
(a) - 3*pi*k - pi*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^2) - (3*pi*k*s
gn(a) + pi*l*sgn(a) - 3*pi*k - pi*l)*sin(-3/2*pi*k*x*sgn(a) - 1/2*pi*l*x*sg
n(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(a) + pi*l*sgn(a) - 3*pi*k - pi
*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^2))*e^((3*k*log(abs(a)) + l*log
(abs(a)))*x) - 1/2*I*(-8*I*e^(3/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l*x*sgn(a) - 3
/2*I*pi*k*x - 1/2*I*pi*l*x)/(3*I*pi*k*sgn(a) + I*pi*l*sgn(a) - 3*I*pi*k - I
*pi*l + 6*k*log(abs(a)) + 2*l*log(abs(a))) + 8*I*e^(-3/2*I*pi*k*x*sgn(a) -
1/2*I*pi*l*x*sgn(a) + 3/2*I*pi*k*x + 1/2*I*pi*l*x)/(-3*I*pi*k*sgn(a) - I*pi
*l*sgn(a) + 3*I*pi*k + I*pi*l + 6*k*log(abs(a)) + 2*l*log(abs(a))))*e^((3*k
*log(abs(a)) + l*log(abs(a)))*x) + 8*(2*(k*log(abs(a)) + 3*l*log(abs(a)))*c
os(-1/2*pi*k*x*sgn(a) - 3/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 3/2*pi*l*x)/((pi*k
*sgn(a) + 3*pi*l*sgn(a) - pi*k - 3*pi*l)^2 + 4*(k*log(abs(a)) + 3*l*log(abs
(a)))^2) - (pi*k*sgn(a) + 3*pi*l*sgn(a) - pi*k - 3*pi*l)*sin(-1/2*pi*k*x*sg
n(a) - 3/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 3/2*pi*l*x)/((pi*k*sgn(a) + 3*pi*l*
sgn(a) - pi*k - 3*pi*l)^2 + 4*(k*log(abs(a)) + 3*l*log(abs(a)))^2))*e^((k*l
og(abs(a)) + 3*l*log(abs(a)))*x) - 1/2*I*(-8*I*e^(1/2*I*pi*k*x*sgn(a) + 3/2
*I*pi*l*x*sgn(a) - 1/2*I*pi*k*x - 3/2*I*pi*l*x)/(I*pi*k*sgn(a) + 3*I*pi*l*s
gn(a) - I*pi*k - 3*I*pi*l + 2*k*log(abs(a)) + 6*l*log(abs(a))) + 8*I*e^(-1/
2*I*pi*k*x*sgn(a) - 3/2*I*pi*l*x*sgn(a) + 1/2*I*pi*k*x + 3/2*I*pi*l*x)/(-I*
pi*k*sgn(a) - 3*I*pi*l*sgn(a) + I*pi*k + 3*I*pi*l + 2*k*log(abs(a)) + 6*l*l
og(abs(a))))*e^((k*log(abs(a)) + 3*l*log(abs(a)))*x) + 6*(2*(k*log(abs(a))
+ l*log(abs(a)))*cos(-pi*k*x*sgn(a) - pi*l*x*sgn(a) + pi*k*x + pi*l*x)/((pi
*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a))
)^2) - (pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)*sin(-pi*k*x*sgn(a) - pi*l*
x*sgn(a) + pi*k*x + pi*l*x)/((pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)^2 +
```

```
4*(k*log(abs(a)) + l*log(abs(a)))^2))*e^(2*(k*log(abs(a)) + l*log(abs(a)))*
x) - 1/2*I*(-6*I*e^(I*pi*k*x*sgn(a) + I*pi*l*x*sgn(a) - I*pi*k*x - I*pi*l*x
)/(I*pi*k*sgn(a) + I*pi*l*sgn(a) - I*pi*k - I*pi*l + 2*k*log(abs(a)) + 2*l*
log(abs(a))) + 6*I*e^(-I*pi*k*x*sgn(a) - I*pi*l*x*sgn(a) + I*pi*k*x + I*pi*
l*x)/(-I*pi*k*sgn(a) - I*pi*l*sgn(a) + I*pi*k + I*pi*l + 2*k*log(abs(a)) +
2*l*log(abs(a))))*e^(2*(k*log(abs(a)) + l*log(abs(a)))*x)
```

3.506 $\int \left(a^{kx}+a^{lx}\right)^n \, dx$

Optimal. Leaf size=72

$$\frac{\left(a^{x(k-l)}+1\right)\left(a^{kx}+a^{lx}\right)^n {}_2F_1\left(1,\frac{kn}{k-l}+1;\frac{ln}{k-l}+1;-a^{(k-l)x}\right)}{ln\log(a)}$$

```
[Out] ((1 + a^((k - l)*x))*(a^(k*x) + a^(l*x))^n*Hypergeometric2F1[1, 1 + (k*n)/(
k - l), 1 + (l*n)/(k - l), -a^((k - l)*x)])/(l*n*Log[a])
```

Rubi [A] time = 0.0771031, antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2285, 2251}

$$\frac{\left(a^{x(-(k-l))}+1\right)^{-n}\left(a^{kx}+a^{lx}\right)^n \text{Hypergeometric2F1}\left(-n,-\frac{kn}{k-l},1-\frac{kn}{k-l},-a^{x(-(k-l))}\right)}{kn\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(k*x) + a^(l*x))^n,x]
```

```
[Out] ((a^(k*x) + a^(l*x))^n*Hypergeometric2F1[-n, -((k*n)/(k - l)), 1 - (k*n)/(k
 - l), -a^(-((k - l)*x))])/((1 + a^(-((k - l)*x)))^n*k*n*Log[a])
```

Rule 2285

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a^{kx}+a^{lx}\right)^n \, dx &= \left(a^{-knx}\left(1+a^{-(k-l)x}\right)^{-n}\left(a^{kx}+a^{lx}\right)^n\right)\int a^{knx}\left(1+a^{-(k-l)x}\right)^n \, dx\\ &= \frac{\left(1+a^{-(k-l)x}\right)^{-n}\left(a^{kx}+a^{lx}\right)^n {}_2F_1\left(-n,-\frac{kn}{k-l};1-\frac{kn}{k-l};-a^{-(k-l)x}\right)}{kn\log(a)}\end{aligned}$$

Mathematica [A] time = 0.0286806, size = 73, normalized size = 1.01

$$\frac{\left(a^{x(l-k)}+1\right)\left(a^{kx}+a^{lx}\right)^n {}_2F_1\left(1,\frac{kn}{l-k}+n+1;\frac{kn}{l-k}+1;-a^{(l-k)x}\right)}{kn\log(a)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^(k*x) + a^(l*x))^n,x]
```

```
[Out] ((a^(k*x) + a^(l*x))^n*(1 + a^((-k + l)*x))*Hypergeometric2F1[1, 1 + n + (k
*n)/(-k + l), 1 + (k*n)/(-k + l), -a^((-k + l)*x)])/(k*n*Log[a])
```

Maple [F] time = 0.05, size = 0, normalized size = 0.

$$\int \left(a^{kx} + a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x)+a^(l*x))^n,x)
```

```
[Out] int((a^(k*x)+a^(l*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx} + a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((a^(k*x) + a^(l*x))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^{kx} + a^{lx}\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^n,x, algorithm="fricas")
```

```
[Out] integral((a^(k*x) + a^(l*x))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx} + a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(k*x)+a**(l*x))**n,x)
```

```
[Out] Integral((a**(k*x) + a**(l*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx} + a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)+a^(l*x))^n,x, algorithm="giac")
```

```
[Out] integrate((a^(k*x) + a^(l*x))^n, x)
```

3.507 $\int \left(a^{kx}-a^{lx}\right) dx$

Optimal. Leaf size=28

$$\frac{a^{kx}}{k\log(a)}-\frac{a^{lx}}{l\log(a)}$$

```
[Out] a^(k*x)/(k*Log[a]) - a^(l*x)/(l*Log[a])
```

Rubi [A] time = 0.0083801, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2194}

$$\frac{a^{kx}}{k\log(a)}-\frac{a^{lx}}{l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[a^(k*x) - a^(l*x),x]

[Out] a^(k*x)/(k*Log[a]) - a^(l*x)/(l*Log[a])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a^{kx}-a^{lx}\right) dx &= \int a^{kx}\,dx-\int a^{lx}\,dx\\ &= \frac{a^{kx}}{k\log(a)}-\frac{a^{lx}}{l\log(a)}\end{aligned}$$

Mathematica [A] time = 0.0061269, size = 28, normalized size = 1.

$$\frac{a^{kx}}{k\log(a)}-\frac{a^{lx}}{l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^(k*x) - a^(l*x),x]

[Out] a^(k*x)/(k*Log[a]) - a^(l*x)/(l*Log[a])
```

Maple [A] time = 0.001, size = 29, normalized size = 1.

$$\frac{a^{kx}}{k\ln(a)}-\frac{a^{lx}}{l\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^(k*x)-a^(l*x),x)
```

```
[Out] a^(k*x)/k/ln(a)-a^(l*x)/l/ln(a)
```

Maxima [A] time = 0.934184, size = 38, normalized size = 1.36

$$\frac{a^{kx}}{k\log(a)} - \frac{a^{lx}}{l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(k*x)-a^(l*x),x, algorithm="maxima")
```

```
[Out] a^(k*x)/(k*log(a)) - a^(l*x)/(l*log(a))
```

Fricas [A] time = 1.87292, size = 53, normalized size = 1.89

$$-\frac{a^{lx}k - a^{kx}l}{kl\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(k*x)-a^(l*x),x, algorithm="fricas")
```

```
[Out] -(a^(l*x)*k - a^(k*x)*l)/(k*l*log(a))
```

Sympy [A] time = 0.287002, size = 29, normalized size = 1.04

$$\begin{cases} \frac{a^{kx}}{k\log(a)} & \text{for}\: k\log(a) \neq 0 \\ x & \text{otherwise} \end{cases} - \begin{cases} \frac{a^{lx}}{l\log(a)} & \text{for}\: l\log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**(k*x)-a**(l*x),x)
```

```
[Out] Piecewise((a**(k*x)/(k*log(a)), Ne(k*log(a), 0)), (x, True)) - Piecewise((a
**(l*x)/(l*log(a)), Ne(l*log(a), 0)), (x, True))
```

Giac [A] time = 1.12463, size = 38, normalized size = 1.36

$$\frac{a^{kx}}{k\log(a)} - \frac{a^{lx}}{l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(k*x)-a^(l*x),x, algorithm="giac")
```

```
[Out] a^(k*x)/(k*log(a)) - a^(l*x)/(l*log(a))
```

3.508 $\int \left(a^{kx}-a^{lx}\right)^2 \, dx$

Optimal. Leaf size=53

$$-\frac{2a^{x(k+l)}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) - (2*a^((k + l)*x))/((k + l
)*Log[a])
```

Rubi [A] time = 0.071417, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6742, 2194}

$$-\frac{2a^{x(k+l)}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(k*x) - a^(l*x))^2,x]
```

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) - (2*a^((k + l)*x))/((k + l
)*Log[a])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a^{kx}-a^{lx}\right)^2 \, dx &= \frac{\text{Subst}\left(\int \left(e^{kx}-e^{lx}\right)^2 \, dx,x,x\log(a)\right)}{\log(a)}\\
&= \frac{\text{Subst}\left(\int \left(e^{2kx}+e^{2lx}-2e^{(k+l)x}\right) \, dx,x,x\log(a)\right)}{\log(a)}\\
&= \frac{\text{Subst}\left(\int e^{2kx} \, dx,x,x\log(a)\right)}{\log(a)}+\frac{\text{Subst}\left(\int e^{2lx} \, dx,x,x\log(a)\right)}{\log(a)}-\frac{2\,\text{Subst}\left(\int e^{(k+l)x} \, dx,x,x\log(a)\right)}{\log(a)}\\
&= \frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}-\frac{2a^{(k+l)x}}{(k+l)\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.0336438, size = 53, normalized size = 1.

$$-\frac{2a^{x(k+l)}}{\log(a)(k+l)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(k*x) - a^(l*x))^2,x]
```

```
[Out] a^(2*k*x)/(2*k*Log[a]) + a^(2*l*x)/(2*l*Log[a]) - (2*a^((k + l)*x))/((k + l
)*Log[a])
```

Maple [A] time = 0.012, size = 59, normalized size = 1.1

$$\frac{\left(e^{kx\ln(a)}\right)^2}{2k\ln(a)}+\frac{\left(e^{lx\ln(a)}\right)^2}{2l\ln(a)}-2\frac{e^{kx\ln(a)}e^{lx\ln(a)}}{\ln(a)(k+l)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x)-a^(l*x))^2,x)
```

```
[Out] 1/2/k/ln(a)*exp(k*x*ln(a))^2+1/2/l/ln(a)*exp(l*x*ln(a))^2-2/ln(a)/(k+l)*exp
(k*x*ln(a))*exp(l*x*ln(a))
```

Maxima [A] time = 0.926778, size = 69, normalized size = 1.3

$$-\frac{2a^{kx+lx}}{(k+l)\log(a)}+\frac{a^{2kx}}{2k\log(a)}+\frac{a^{2lx}}{2l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^2,x, algorithm="maxima")
```

```
[Out] -2*a^(k*x + l*x)/((k + l)*log(a)) + 1/2*a^(2*k*x)/(k*log(a)) + 1/2*a^(2*l*x
)/(l*log(a))
```

Fricas [A] time = 1.86213, size = 139, normalized size = 2.62

$$-\frac{4a^{kx}a^{lx}kl-\left(kl+l^2\right)a^{2kx}-\left(k^2+kl\right)a^{2lx}}{2\left(k^2l+kl^2\right)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^(k*x)*a^(l*x)*k*l - (k*l + l^2)*a^(2*k*x) - (k^2 + k*l)*a^(2*l*x)
)/((k^2*l + k*l^2)*log(a))
```

Sympy [A] time = 2.03262, size = 248, normalized size = 4.68

$$\begin{cases} 0 & \text{for } a = 1 \wedge \\ \frac{a^{2lx}}{2l\log(a)}-\frac{2a^{lx}}{l\log(a)}+x & \text{for } k = 0 \\ \frac{a^{2lx}}{2l\log(a)}-2x-\frac{a^{-2lx}}{2l\log(a)} & \text{for } k = -l \\ \frac{a^{2kx}}{2k\log(a)}-\frac{2a^{kx}}{k\log(a)}+x & \text{for } l = 0 \\ \frac{a^{2kx}kl}{2k^2l\log(a)+2kl^2\log(a)}+\frac{a^{2kx}l^2}{2k^2l\log(a)+2kl^2\log(a)}-\frac{4a^{kx}a^{lx}kl}{2k^2l\log(a)+2kl^2\log(a)}+\frac{a^{2lx}k^2}{2k^2l\log(a)+2kl^2\log(a)}+\frac{a^{2lx}kl}{2k^2l\log(a)+2kl^2\log(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(k*x)-a**(l*x))**2,x)
```

```
[Out] Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))), (a
**(2*l*x)/(2*l*log(a)) - 2*a**(l*x)/(l*log(a)) + x, Eq(k, 0)), (a**(2*l*x)/
(2*l*log(a)) - 2*x - a**(-2*l*x)/(2*l*log(a)), Eq(k, -l)), (a**(2*k*x)/(2*k
*log(a)) - 2*a**(k*x)/(k*log(a)) + x, Eq(l, 0)), (a**(2*k*x)*k*l/(2*k**2*l*
log(a) + 2*k*l**2*log(a)) + a**(2*k*x)*l**2/(2*k**2*l*log(a) + 2*k*l**2*log
(a)) - 4*a**(k*x)*a**(l*x)*k*l/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*
l*x)*k**2/(2*k**2*l*log(a) + 2*k*l**2*log(a)) + a**(2*l*x)*k*l/(2*k**2*l*lo
g(a) + 2*k*l**2*log(a)), True))
```

Giac [C] time = 1.20967, size = 933, normalized size = 17.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^2,x, algorithm="giac")
```

```
[Out] (2*k*cos(-pi*k*x*sgn(a) + pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2 + (pi*k*
sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-pi*k*x*sgn(a) + pi*k*x)/(4*k^
2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(2*k*x) + (2*l*cos(-pi*l*
x*sgn(a) + pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^
2) - (pi*l*sgn(a) - pi*l)*sin(-pi*l*x*sgn(a) + pi*l*x)/(4*l^2*log(abs(a))^2
 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(2*l*x) - 1/2*I*abs(a)^(2*k*x)*(-I*e^(I*
pi*k*x*sgn(a) - I*pi*k*x)/(I*pi*k*sgn(a) - I*pi*k + 2*k*log(abs(a))) + I*e^
(-I*pi*k*x*sgn(a) + I*pi*k*x)/(-I*pi*k*sgn(a) + I*pi*k + 2*k*log(abs(a))))
- 1/2*I*abs(a)^(2*l*x)*(-I*e^(I*pi*l*x*sgn(a) - I*pi*l*x)/(I*pi*l*sgn(a) -
I*pi*l + 2*l*log(abs(a))) + I*e^(-I*pi*l*x*sgn(a) + I*pi*l*x)/(-I*pi*l*sgn(
a) + I*pi*l + 2*l*log(abs(a)))) - 4*(2*(k*log(abs(a)) + l*log(abs(a)))*cos(
-1/2*pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*x)/((pi*k*sg
n(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2)
- (pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)*sin(-1/2*pi*k*x*sgn(a) - 1/2*pi
*l*x*sgn(a) + 1/2*pi*k*x + 1/2*pi*l*x)/((pi*k*sgn(a) + pi*l*sgn(a) - pi*k -
 pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^2))*e^((k*log(abs(a)) + l*log(
abs(a)))*x) - 1/2*I*(4*I*e^(1/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l*x*sgn(a) - 1/2
*I*pi*k*x - 1/2*I*pi*l*x)/(I*pi*k*sgn(a) + I*pi*l*sgn(a) - I*pi*k - I*pi*l
+ 2*k*log(abs(a)) + 2*l*log(abs(a))) - 4*I*e^(-1/2*I*pi*k*x*sgn(a) - 1/2*I*
pi*l*x*sgn(a) + 1/2*I*pi*k*x + 1/2*I*pi*l*x)/(-I*pi*k*sgn(a) - I*pi*l*sgn(a
) + I*pi*k + I*pi*l + 2*k*log(abs(a)) + 2*l*log(abs(a))))*e^((k*log(abs(a))
 + l*log(abs(a)))*x)
```

3.509 $\int \left(a^{kx}-a^{lx}\right)^3 \, dx$

Optimal. Leaf size=79

$$-\frac{3a^{x(2k+l)}}{\log(a)(2k+l)}+\frac{3a^{x(k+2l)}}{\log(a)(k+2l)}+\frac{a^{3kx}}{3k\log(a)}-\frac{a^{3lx}}{3l\log(a)}$$

```
[Out] a^(3*k*x)/(3*k*Log[a]) - a^(3*l*x)/(3*l*Log[a]) - (3*a^((2*k + l)*x))/((2*k
 + l)*Log[a]) + (3*a^((k + 2*l)*x))/((k + 2*l)*Log[a])
```

Rubi [A] time = 0.0831905, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6742, 2194}

$$-\frac{3a^{x(2k+l)}}{\log(a)(2k+l)}+\frac{3a^{x(k+2l)}}{\log(a)(k+2l)}+\frac{a^{3kx}}{3k\log(a)}-\frac{a^{3lx}}{3l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(k*x) - a^(l*x))^3,x]
```

```
[Out] a^(3*k*x)/(3*k*Log[a]) - a^(3*l*x)/(3*l*Log[a]) - (3*a^((2*k + l)*x))/((2*k
 + l)*Log[a]) + (3*a^((k + 2*l)*x))/((k + 2*l)*Log[a])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a^{kx}-a^{lx}\right)^3 \, dx &= \frac{\text{Subst}\left(\int \left(e^{kx}-e^{lx}\right)^3 \, dx,x,x\log(a)\right)}{\log(a)}\\
&= \frac{\text{Subst}\left(\int \left(e^{3kx}-e^{3lx}-3e^{(2k+l)x}+3e^{(k+2l)x}\right) dx,x,x\log(a)\right)}{\log(a)}\\
&= \frac{\text{Subst}\left(\int e^{3kx}\,dx,x,x\log(a)\right)}{\log(a)}-\frac{\text{Subst}\left(\int e^{3lx}\,dx,x,x\log(a)\right)}{\log(a)}-\frac{3\,\text{Subst}\left(\int e^{(2k+l)x}\,dx,x,x\log(a)\right)}{\log(a)}\\
&= \frac{a^{3kx}}{3k\log(a)}-\frac{a^{3lx}}{3l\log(a)}-\frac{3a^{(2k+l)x}}{(2k+l)\log(a)}+\frac{3a^{(k+2l)x}}{(k+2l)\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.0678, size = 66, normalized size = 0.84

$$\frac{-\frac{9a^{x(2k+l)}}{2k+l}+\frac{9a^{x(k+2l)}}{k+2l}+\frac{a^{3kx}}{k}-\frac{a^{3lx}}{l}}{3\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(k*x) - a^(l*x))^3,x]
```

```
[Out] (a^(3*k*x)/k - a^(3*l*x)/l - (9*a^((2*k + l)*x))/(2*k + l) + (9*a^((k + 2*l
)*x))/(k + 2*l))/(3*Log[a])
```

Maple [A] time = 0.026, size = 90, normalized size = 1.1

$$\frac{\left(e^{kx\ln(a)}\right)^{3}}{3k\ln(a)}-\frac{\left(e^{lx\ln(a)}\right)^{3}}{3l\ln(a)}+3\frac{e^{kx\ln(a)}\left(e^{lx\ln(a)}\right)^{2}}{\ln(a)(k+2l)}-3\frac{\left(e^{kx\ln(a)}\right)^{2}e^{lx\ln(a)}}{\ln(a)(2k+l)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x)-a^(l*x))^3,x)
```

```
[Out] 1/3/k/ln(a)*exp(k*x*ln(a))^3-1/3/l/ln(a)*exp(l*x*ln(a))^3+3/ln(a)/(k+2*l)*e
xp(k*x*ln(a))*exp(l*x*ln(a))^2-3/ln(a)/(2*k+l)*exp(k*x*ln(a))^2*exp(l*x*ln(
a))
```

Maxima [A] time = 0.936595, size = 104, normalized size = 1.32

$$-\frac{3\,a^{2\,kx+lx}}{(2\,k+l)\log(a)}+\frac{3\,a^{kx+2\,lx}}{(k+2\,l)\log(a)}+\frac{a^{3\,kx}}{3\,k\log(a)}-\frac{a^{3\,lx}}{3\,l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^3,x, algorithm="maxima")
```

```
[Out] -3*a^(2*k*x + l*x)/((2*k + l)*log(a)) + 3*a^(k*x + 2*l*x)/((k + 2*l)*log(a)
) + 1/3*a^(3*k*x)/(k*log(a)) - 1/3*a^(3*l*x)/(l*log(a))
```

Fricas [A] time = 1.88177, size = 278, normalized size = 3.52

$$\frac{9\left(2\,k^{2}l+kl^{2}\right)a^{kx}a^{2\,lx}-9\left(k^{2}l+2\,kl^{2}\right)a^{2\,kx}a^{lx}+\left(2\,k^{2}l+5\,kl^{2}+2\,l^{3}\right)a^{3\,kx}-\left(2\,k^{3}+5\,k^{2}l+2\,kl^{2}\right)a^{3\,lx}}{3\left(2\,k^{3}l+5\,k^{2}l^{2}+2\,kl^{3}\right)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^3,x, algorithm="fricas")
```

```
[Out] 1/3*(9*(2*k^2*l + k*l^2)*a^(k*x)*a^(2*l*x) - 9*(k^2*l + 2*k*l^2)*a^(2*k*x)*
a^(l*x) + (2*k^2*l + 5*k*l^2 + 2*l^3)*a^(3*k*x) - (2*k^3 + 5*k^2*l + 2*k*l^
2)*a^(3*l*x))/((2*k^3*l + 5*k^2*l^2 + 2*k*l^3)*log(a))
```

Sympy [A] time = 31.378, size = 663, normalized size = 8.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(k*x)-a**(l*x))**3,x)
```

```
[Out] Piecewise((0, Eq(a, 1) & (Eq(a, 1) | Eq(k, 0)) & (Eq(a, 1) | Eq(l, 0))), (-
a**(3*l*x)/(3*l*log(a)) + 3*a**(2*l*x)/(2*l*log(a)) - 3*a**(l*x)/(l*log(a))
 + x, Eq(k, 0)), (-a**(3*l*x)/(3*l*log(a)) + 3*x + a**(-3*l*x)/(l*log(a)) -
 a**(-6*l*x)/(6*l*log(a)), Eq(k, -2*l)), (2*a**(3*l*x/2)/(l*log(a)) - a**(3
*l*x)/(3*l*log(a)) - 3*x - 2*a**(-3*l*x/2)/(3*l*log(a)), Eq(k, -l/2)), (a**
(3*k*x)/(3*k*log(a)) - 3*a**(2*k*x)/(2*k*log(a)) + 3*a**(k*x)/(k*log(a)) -
x, Eq(l, 0)), (2*a**(3*k*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) +
 6*k*l**3*log(a)) + 5*a**(3*k*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log
(a) + 6*k*l**3*log(a)) + 2*a**(3*k*x)*l**3/(6*k**3*l*log(a) + 15*k**2*l**2*
log(a) + 6*k*l**3*log(a)) - 9*a**(2*k*x)*a**(l*x)*k**2*l/(6*k**3*l*log(a) +
 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) - 18*a**(2*k*x)*a**(l*x)*k*l**2/(6*
k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 18*a**(k*x)*a**(2*
l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3*log(a)) + 9*a
**(k*x)*a**(2*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l**3
*log(a)) - 2*a**(3*l*x)*k**3/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) + 6*k*l
**3*log(a)) - 5*a**(3*l*x)*k**2*l/(6*k**3*l*log(a) + 15*k**2*l**2*log(a) +
6*k*l**3*log(a)) - 2*a**(3*l*x)*k*l**2/(6*k**3*l*log(a) + 15*k**2*l**2*log(
a) + 6*k*l**3*log(a)), True))
```

Giac [C] time = 1.24789, size = 1395, normalized size = 17.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^3,x, algorithm="giac")
```

```
[Out] 2/3*(2*k*cos(-3/2*pi*k*x*sgn(a) + 3/2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a)
)^2 + (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-3/2*pi*k*x*sgn(a)
 + 3/2*pi*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(3*k*
x) - 2/3*(2*l*cos(-3/2*pi*l*x*sgn(a) + 3/2*pi*l*x)*log(abs(a))/(4*l^2*log(a
bs(a))^2 + (pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-3/2*pi*l*x*s
gn(a) + 3/2*pi*l*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^
(3*l*x) - 1/2*I*abs(a)^(3*k*x)*(-2*I*e^(3/2*I*pi*k*x*sgn(a) - 3/2*I*pi*k*x)
/(3*I*pi*k*sgn(a) - 3*I*pi*k + 6*k*log(abs(a))) + 2*I*e^(-3/2*I*pi*k*x*sgn(
a) + 3/2*I*pi*k*x)/(-3*I*pi*k*sgn(a) + 3*I*pi*k + 6*k*log(abs(a)))) - 1/2*I
*abs(a)^(3*l*x)*(2*I*e^(3/2*I*pi*l*x*sgn(a) - 3/2*I*pi*l*x)/(3*I*pi*l*sgn(a
) - 3*I*pi*l + 6*l*log(abs(a))) - 2*I*e^(-3/2*I*pi*l*x*sgn(a) + 3/2*I*pi*l*
x)/(-3*I*pi*l*sgn(a) + 3*I*pi*l + 6*l*log(abs(a)))) - 6*(2*(2*k*log(abs(a))
 + l*log(abs(a)))*cos(-pi*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*
l*x)/((2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a))
+ l*log(abs(a)))^2) - (2*pi*k*sgn(a) + pi*l*sgn(a) - 2*pi*k - pi*l)*sin(-pi
*k*x*sgn(a) - 1/2*pi*l*x*sgn(a) + pi*k*x + 1/2*pi*l*x)/((2*pi*k*sgn(a) + pi
*l*sgn(a) - 2*pi*k - pi*l)^2 + 4*(2*k*log(abs(a)) + l*log(abs(a)))^2))*e^((
2*k*log(abs(a)) + l*log(abs(a)))*x) - 1/2*I*(6*I*e^(I*pi*k*x*sgn(a) + 1/2*I
*pi*l*x*sgn(a) - I*pi*k*x - 1/2*I*pi*l*x)/(2*I*pi*k*sgn(a) + I*pi*l*sgn(a)
- 2*I*pi*k - I*pi*l + 4*k*log(abs(a)) + 2*l*log(abs(a))) - 6*I*e^(-I*pi*k*x
*sgn(a) - 1/2*I*pi*l*x*sgn(a) + I*pi*k*x + 1/2*I*pi*l*x)/(-2*I*pi*k*sgn(a)
- I*pi*l*sgn(a) + 2*I*pi*k + I*pi*l + 4*k*log(abs(a)) + 2*l*log(abs(a))))*e
^((2*k*log(abs(a)) + l*log(abs(a)))*x) + 6*(2*(k*log(abs(a)) + 2*l*log(abs(
a)))*cos(-1/2*pi*k*x*sgn(a) - pi*l*x*sgn(a) + 1/2*pi*k*x + pi*l*x)/((pi*k*s
gn(a) + 2*pi*l*sgn(a) - pi*k - 2*pi*l)^2 + 4*(k*log(abs(a)) + 2*l*log(abs(a
)))^2) - (pi*k*sgn(a) + 2*pi*l*sgn(a) - pi*k - 2*pi*l)*sin(-1/2*pi*k*x*sgn(
```

```
a) - pi*l*x*sgn(a) + 1/2*pi*k*x + pi*l*x)/((pi*k*sgn(a) + 2*pi*l*sgn(a) - p
i*k - 2*pi*l)^2 + 4*(k*log(abs(a)) + 2*l*log(abs(a)))^2))*e^((k*log(abs(a))
 + 2*l*log(abs(a)))*x) - 1/2*I*(-6*I*e^(1/2*I*pi*k*x*sgn(a) + I*pi*l*x*sgn(
a) - 1/2*I*pi*k*x - I*pi*l*x)/(I*pi*k*sgn(a) + 2*I*pi*l*sgn(a) - I*pi*k - 2
*I*pi*l + 2*k*log(abs(a)) + 4*l*log(abs(a))) + 6*I*e^(-1/2*I*pi*k*x*sgn(a)
- I*pi*l*x*sgn(a) + 1/2*I*pi*k*x + I*pi*l*x)/(-I*pi*k*sgn(a) - 2*I*pi*l*sgn
(a) + I*pi*k + 2*I*pi*l + 2*k*log(abs(a)) + 4*l*log(abs(a))))*e^((k*log(abs
(a)) + 2*l*log(abs(a)))*x)
```

3.510 $\int \left(a^{kx}-a^{lx}\right)^4 dx$

Optimal. Leaf size=98

$$\frac{3a^{2x(k+l)}}{\log(a)(k+l)}-\frac{4a^{x(3k+l)}}{\log(a)(3k+l)}-\frac{4a^{x(k+3l)}}{\log(a)(k+3l)}+\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}$$

```
[Out] a^(4*k*x)/(4*k*Log[a]) + a^(4*l*x)/(4*l*Log[a]) + (3*a^(2*(k + l)*x))/((k +
 l)*Log[a]) - (4*a^((3*k + l)*x))/((3*k + l)*Log[a]) - (4*a^((k + 3*l)*x))/
((k + 3*l)*Log[a])
```

Rubi [A] time = 0.0984358, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6742, 2194}

$$\frac{3a^{2x(k+l)}}{\log(a)(k+l)}-\frac{4a^{x(3k+l)}}{\log(a)(3k+l)}-\frac{4a^{x(k+3l)}}{\log(a)(k+3l)}+\frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(k*x) - a^(l*x))^4,x]
```

```
[Out] a^(4*k*x)/(4*k*Log[a]) + a^(4*l*x)/(4*l*Log[a]) + (3*a^(2*(k + l)*x))/((k +
 l)*Log[a]) - (4*a^((3*k + l)*x))/((3*k + l)*Log[a]) - (4*a^((k + 3*l)*x))/
((k + 3*l)*Log[a])
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \left(a^{kx}-a^{lx}\right)^4 dx &= \frac{\text{Subst}\left(\int \left(e^{kx}-e^{lx}\right)^4 dx, x, x\log(a)\right)}{\log(a)}\\ &= \frac{\text{Subst}\left(\int \left(e^{4kx}+e^{4lx}+6e^{2(k+l)x}-4e^{(3k+l)x}-4e^{(k+3l)x}\right) dx, x, x\log(a)\right)}{\log(a)}\\ &= \frac{\text{Subst}\left(\int e^{4kx} dx, x, x\log(a)\right)}{\log(a)}+\frac{\text{Subst}\left(\int e^{4lx} dx, x, x\log(a)\right)}{\log(a)}-\frac{4\,\text{Subst}\left(\int e^{(3k+l)x} dx, x, x\log(a)\right)}{\log(a)}\\ &= \frac{a^{4kx}}{4k\log(a)}+\frac{a^{4lx}}{4l\log(a)}+\frac{3a^{2(k+l)x}}{(k+l)\log(a)}-\frac{4a^{(3k+l)x}}{(3k+l)\log(a)}-\frac{4a^{(k+3l)x}}{(k+3l)\log(a)}\end{aligned}$$

Mathematica [A] time = 0.0700836, size = 80, normalized size = 0.82

$$\frac{\frac{12a^{2x(k+l)}}{k+l}-\frac{16a^{x(3k+l)}}{3k+l}-\frac{16a^{x(k+3l)}}{k+3l}+\frac{a^{4kx}}{k}+\frac{a^{4lx}}{l}}{4\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^(k*x) - a^(l*x))^4,x]
```

```
[Out] (a^(4*k*x)/k + a^(4*l*x)/l + (12*a^(2*(k + l)*x))/(k + l) - (16*a^((3*k + l
)*x))/(3*k + l) - (16*a^((k + 3*l)*x))/(k + 3*l))/(4*Log[a])
```

Maple [A] time = 0.015, size = 109, normalized size = 1.1

$$\frac{(a^{kx})^4}{4k\ln(a)} - 4\frac{(a^{kx})^3 a^{lx}}{\ln(a)(3k+l)} + 3\frac{(a^{kx})^2 (a^{lx})^2}{\ln(a)(k+l)} - 4\frac{a^{kx}(a^{lx})^3}{\ln(a)(k+3l)} + \frac{(a^{lx})^4}{4l\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x)-a^(l*x))^4,x)
```

```
[Out] 1/4/ln(a)/k*(a^(k*x))^4-4*(a^(k*x))^3/ln(a)/(3*k+l)*a^(l*x)+3*(a^(k*x))^2/l
n(a)/(k+l)*(a^(l*x))^2-4*a^(k*x)/ln(a)/(k+3*l)*(a^(l*x))^3+1/4/ln(a)/l*(a^(
l*x))^4
```

Maxima [A] time = 0.931511, size = 134, normalized size = 1.37

$$-\frac{4a^{3kx+lx}}{(3k+l)\log(a)} - \frac{4a^{kx+3lx}}{(k+3l)\log(a)} + \frac{3a^{2kx+2lx}}{(k+l)\log(a)} + \frac{a^{4kx}}{4k\log(a)} + \frac{a^{4lx}}{4l\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^4,x, algorithm="maxima")
```

```
[Out] -4*a^(3*k*x + l*x)/((3*k + l)*log(a)) - 4*a^(k*x + 3*l*x)/((k + 3*l)*log(a)
) + 3*a^(2*k*x + 2*l*x)/((k + l)*log(a)) + 1/4*a^(4*k*x)/(k*log(a)) + 1/4*a
^(4*l*x)/(l*log(a))
```

Fricas [B] time = 1.94092, size = 450, normalized size = 4.59

$$-\frac{16\left(3k^3l + 4k^2l^2 + kl^3\right)a^{kx}a^{3lx} - 12\left(3k^3l + 10k^2l^2 + 3kl^3\right)a^{2kx}a^{2lx} + 16\left(k^3l + 4k^2l^2 + 3kl^3\right)a^{3kx}a^{lx} - \left(3k^3l + 13k^2l^2 + \right.}{4\left(3k^4l + 13k^3l^2 + 13k^2l^3 + 3kl^4\right)\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^4,x, algorithm="fricas")
```

```
[Out] -1/4*(16*(3*k^3*l + 4*k^2*l^2 + k*l^3)*a^(k*x)*a^(3*l*x) - 12*(3*k^3*l + 10
*k^2*l^2 + 3*k*l^3)*a^(2*k*x)*a^(2*l*x) + 16*(k^3*l + 4*k^2*l^2 + 3*k*l^3)*
a^(3*k*x)*a^(l*x) - (3*k^3*l + 13*k^2*l^2 + 13*k*l^3 + 3*l^4)*a^(4*k*x) - (
3*k^4 + 13*k^3*l + 13*k^2*l^2 + 3*k*l^3)*a^(4*l*x))/((3*k^4*l + 13*k^3*l^2
+ 13*k^2*l^3 + 3*k*l^4)*log(a))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(k*x)-a**(l*x))**4,x)
```

```
[Out] Timed out
```

Giac [C] time = 1.35651, size = 1835, normalized size = 18.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^4,x, algorithm="giac")
```

```
[Out] 1/2*(2*k*cos(-2*pi*k*x*sgn(a) + 2*pi*k*x)*log(abs(a))/(4*k^2*log(abs(a))^2
+ (pi*k*sgn(a) - pi*k)^2) - (pi*k*sgn(a) - pi*k)*sin(-2*pi*k*x*sgn(a) + 2*p
i*k*x)/(4*k^2*log(abs(a))^2 + (pi*k*sgn(a) - pi*k)^2))*abs(a)^(4*k*x) + 1/2
*(2*l*cos(-2*pi*l*x*sgn(a) + 2*pi*l*x)*log(abs(a))/(4*l^2*log(abs(a))^2 + (
pi*l*sgn(a) - pi*l)^2) - (pi*l*sgn(a) - pi*l)*sin(-2*pi*l*x*sgn(a) + 2*pi*l
*x)/(4*l^2*log(abs(a))^2 + (pi*l*sgn(a) - pi*l)^2))*abs(a)^(4*l*x) - 1/2*I*
abs(a)^(4*k*x)*(-I*e^(2*I*pi*k*x*sgn(a) - 2*I*pi*k*x)/(2*I*pi*k*sgn(a) - 2*
I*pi*k + 4*k*log(abs(a))) + I*e^(-2*I*pi*k*x*sgn(a) + 2*I*pi*k*x)/(-2*I*pi*
k*sgn(a) + 2*I*pi*k + 4*k*log(abs(a)))) - 1/2*I*abs(a)^(4*l*x)*(-I*e^(2*I*p
i*l*x*sgn(a) - 2*I*pi*l*x)/(2*I*pi*l*sgn(a) - 2*I*pi*l + 4*l*log(abs(a))) +
 I*e^(-2*I*pi*l*x*sgn(a) + 2*I*pi*l*x)/(-2*I*pi*l*sgn(a) + 2*I*pi*l + 4*l*l
og(abs(a)))) - 8*(2*(3*k*log(abs(a)) + l*log(abs(a)))*cos(-3/2*pi*k*x*sgn(a
) - 1/2*pi*l*x*sgn(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(a) + pi*l*sgn
(a) - 3*pi*k - pi*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^2) - (3*pi*k*s
gn(a) + pi*l*sgn(a) - 3*pi*k - pi*l)*sin(-3/2*pi*k*x*sgn(a) - 1/2*pi*l*x*sg
n(a) + 3/2*pi*k*x + 1/2*pi*l*x)/((3*pi*k*sgn(a) + pi*l*sgn(a) - 3*pi*k - pi
*l)^2 + 4*(3*k*log(abs(a)) + l*log(abs(a)))^2))*e^((3*k*log(abs(a)) + l*log
(abs(a)))*x) - 1/2*I*(8*I*e^(3/2*I*pi*k*x*sgn(a) + 1/2*I*pi*l*x*sgn(a) - 3/
2*I*pi*k*x - 1/2*I*pi*l*x)/(3*I*pi*k*sgn(a) + I*pi*l*sgn(a) - 3*I*pi*k - I*
pi*l + 6*k*log(abs(a)) + 2*l*log(abs(a))) - 8*I*e^(-3/2*I*pi*k*x*sgn(a) - 1
/2*I*pi*l*x*sgn(a) + 3/2*I*pi*k*x + 1/2*I*pi*l*x)/(-3*I*pi*k*sgn(a) - I*pi*
l*sgn(a) + 3*I*pi*k + I*pi*l + 6*k*log(abs(a)) + 2*l*log(abs(a))))*e^((3*k*
log(abs(a)) + l*log(abs(a)))*x) - 8*(2*(k*log(abs(a)) + 3*l*log(abs(a)))*co
s(-1/2*pi*k*x*sgn(a) - 3/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 3/2*pi*l*x)/((pi*k*
sgn(a) + 3*pi*l*sgn(a) - pi*k - 3*pi*l)^2 + 4*(k*log(abs(a)) + 3*l*log(abs(
a)))^2) - (pi*k*sgn(a) + 3*pi*l*sgn(a) - pi*k - 3*pi*l)*sin(-1/2*pi*k*x*sgn
(a) - 3/2*pi*l*x*sgn(a) + 1/2*pi*k*x + 3/2*pi*l*x)/((pi*k*sgn(a) + 3*pi*l*s
gn(a) - pi*k - 3*pi*l)^2 + 4*(k*log(abs(a)) + 3*l*log(abs(a)))^2))*e^((k*lo
g(abs(a)) + 3*l*log(abs(a)))*x) - 1/2*I*(8*I*e^(1/2*I*pi*k*x*sgn(a) + 3/2*I
*pi*l*x*sgn(a) - 1/2*I*pi*k*x - 3/2*I*pi*l*x)/(I*pi*k*sgn(a) + 3*I*pi*l*sgn
(a) - I*pi*k - 3*I*pi*l + 2*k*log(abs(a)) + 6*l*log(abs(a))) - 8*I*e^(-1/2*
I*pi*k*x*sgn(a) - 3/2*I*pi*l*x*sgn(a) + 1/2*I*pi*k*x + 3/2*I*pi*l*x)/(-I*pi
*k*sgn(a) - 3*I*pi*l*sgn(a) + I*pi*k + 3*I*pi*l + 2*k*log(abs(a)) + 6*l*log
(abs(a))))*e^((k*log(abs(a)) + 3*l*log(abs(a)))*x) + 6*(2*(k*log(abs(a)) +
l*log(abs(a)))*cos(-pi*k*x*sgn(a) - pi*l*x*sgn(a) + pi*k*x + pi*l*x)/((pi*k
*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*(k*log(abs(a)) + l*log(abs(a)))^
2) - (pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)*sin(-pi*k*x*sgn(a) - pi*l*x*
sgn(a) + pi*k*x + pi*l*x)/((pi*k*sgn(a) + pi*l*sgn(a) - pi*k - pi*l)^2 + 4*
```

```
(k*log(abs(a)) + l*log(abs(a)))^2))*e^(2*(k*log(abs(a)) + l*log(abs(a)))*x)
 - 1/2*I*(-6*I*e^(I*pi*k*x*sgn(a) + I*pi*l*x*sgn(a) - I*pi*k*x - I*pi*l*x)/
(I*pi*k*sgn(a) + I*pi*l*sgn(a) - I*pi*k - I*pi*l + 2*k*log(abs(a)) + 2*l*lo
g(abs(a))) + 6*I*e^(-I*pi*k*x*sgn(a) - I*pi*l*x*sgn(a) + I*pi*k*x + I*pi*l*
x)/(-I*pi*k*sgn(a) - I*pi*l*sgn(a) + I*pi*k + I*pi*l + 2*k*log(abs(a)) + 2*
l*log(abs(a))))*e^(2*(k*log(abs(a)) + l*log(abs(a)))*x)
```

3.511 $\int \left(a^{kx}-a^{lx}\right)^n\,dx$

Optimal. Leaf size=74

$$\frac{\left(1-a^{x(k-l)}\right)\left(a^{kx}-a^{lx}\right)^n {}_2F_1\left(1,\frac{kn}{k-l}+1;\frac{ln}{k-l}+1;a^{(k-l)x}\right)}{ln\log(a)}$$

```
[Out] ((1 - a^((k - l)*x))*(a^(k*x) - a^(l*x))^n*Hypergeometric2F1[1, 1 + (k*n)/(
k - l), 1 + (l*n)/(k - l), a^((k - l)*x)])/(l*n*Log[a])
```

Rubi [A] time = 0.0623792, antiderivative size = 82, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2285, 2251}

$$\frac{\left(1-a^{x(-(k-l))}\right)^{-n}\left(a^{kx}-a^{lx}\right)^n \text{Hypergeometric2F1}\left(-n,-\frac{kn}{k-l},1-\frac{kn}{k-l},a^{x(-(k-l))}\right)}{kn\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^(k*x) - a^(l*x))^n,x]
```

```
[Out] ((a^(k*x) - a^(l*x))^n*Hypergeometric2F1[-n, -((k*n)/(k - l)), 1 - (k*n)/(k
 - l), a^(-((k - l)*x))])/((1 - a^(-((k - l)*x)))^n*k*n*Log[a])
```

Rule 2285

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Dist[(a*F^
v + b*F^w)^n/(F^(n*v)*(a + b*F^ExpandToSum[w - v, x])^n), Int[u*F^(n*v)*(a
+ b*F^ExpandToSum[w - v, x])^n, x], x] /; FreeQ[{F, a, b, n}, x] &&  !Integ
erQ[n] && LinearQ[{v, w}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \left(a^{kx}-a^{lx}\right)^n\,dx &= \left(a^{-knx}\left(1-a^{-(k-l)x}\right)^{-n}\left(a^{kx}-a^{lx}\right)^n\right)\int a^{knx}\left(1-a^{-(k-l)x}\right)^n\,dx\\ &= \frac{\left(1-a^{-(k-l)x}\right)^{-n}\left(a^{kx}-a^{lx}\right)^n {}_2F_1\left(-n,-\frac{kn}{k-l};1-\frac{kn}{k-l};a^{-(k-l)x}\right)}{kn\log(a)}\end{aligned}$$

Mathematica [A] time = 0.0260134, size = 75, normalized size = 1.01

$$\frac{\left(1-a^{x(l-k)}\right)\left(a^{kx}-a^{lx}\right)^n {}_2F_1\left(1,\frac{kn}{l-k}+n+1;\frac{kn}{l-k}+1;a^{(l-k)x}\right)}{kn\log(a)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^(k*x) - a^(l*x))^n,x]
```

```
[Out] ((a^(k*x) - a^(l*x))^n*(1 - a^((-k + l)*x))*Hypergeometric2F1[1, 1 + n + (k
*n)/(-k + l), 1 + (k*n)/(-k + l), a^((-k + l)*x)])/(k*n*Log[a])
```

Maple [F] time = 0.048, size = 0, normalized size = 0.

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^(k*x)-a^(l*x))^n,x)
```

```
[Out] int((a^(k*x)-a^(l*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((a^(k*x) - a^(l*x))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^{kx} - a^{lx}\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^n,x, algorithm="fricas")
```

```
[Out] integral((a^(k*x) - a^(l*x))^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx} - a^{lx}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**(k*x)-a**(l*x))**n,x)
```

```
[Out] Integral((a**(k*x) - a**(l*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a^{kx} - a^{lx}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^(k*x)-a^(l*x))^n,x, algorithm="giac")
```

```
[Out] integrate((a^(k*x) - a^(l*x))^n, x)
```

3.512 $\int (1 + a^{mx}) \, dx$

Optimal. Leaf size=15

$$\frac{a^{mx}}{m \log(a)} + x$$

```
[Out] x + a^(m*x)/(m*Log[a])
```

Rubi [A] time = 0.0055083, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2194}

$$\frac{a^{mx}}{m \log(a)} + x$$

Antiderivative was successfully verified.

```
[In] Int[1 + a^(m*x),x]
```

```
[Out] x + a^(m*x)/(m*Log[a])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int (1 + a^{mx}) \, dx &= x + \int a^{mx} \, dx \\ &= x + \frac{a^{mx}}{m \log(a)}\end{aligned}$$

Mathematica [A] time = 0.0038371, size = 15, normalized size = 1.

$$\frac{a^{mx}}{m \log(a)} + x$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + a^(m*x),x]
```

```
[Out] x + a^(m*x)/(m*Log[a])
```

Maple [A] time = 0., size = 16, normalized size = 1.1

$$x + \frac{a^{mx}}{m \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1+a^(m*x),x)
```

```
[Out] x+a^(m*x)/m/ln(a)
```

Maxima [A] time = 0.923701, size = 20, normalized size = 1.33

$$x + \frac{a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+a^(m*x),x, algorithm="maxima")
```

```
[Out] x + a^(m*x)/(m*log(a))
```

Fricas [A] time = 1.82672, size = 47, normalized size = 3.13

$$\frac{mx \log(a) + a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+a^(m*x),x, algorithm="fricas")
```

```
[Out] (m*x*log(a) + a^(m*x))/(m*log(a))
```

Sympy [A] time = 0.087906, size = 15, normalized size = 1.

$$x + \begin{cases} \frac{a^{mx}}{m \log(a)} & \text{for } m \log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+a**(m*x),x)
```

```
[Out] x + Piecewise((a**(m*x)/(m*log(a)), Ne(m*log(a), 0)), (x, True))
```

Giac [A] time = 1.12403, size = 20, normalized size = 1.33

$$x + \frac{a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1+a^(m*x),x, algorithm="giac")
```

```
[Out] x + a^(m*x)/(m*log(a))
```

3.513 $\int (1 + a^{mx})^2 \, dx$

Optimal. Leaf size=33

$$\frac{2a^{mx}}{m\log(a)} + \frac{a^{2mx}}{2m\log(a)} + x$$

```
[Out] x + (2*a^(m*x))/(m*Log[a]) + a^(2*m*x)/(2*m*Log[a])
```

Rubi [A] time = 0.0134742, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2282, 43}

$$\frac{2a^{mx}}{m\log(a)} + \frac{a^{2mx}}{2m\log(a)} + x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^(m*x))^2,x]
```

```
[Out] x + (2*a^(m*x))/(m*Log[a]) + a^(2*m*x)/(2*m*Log[a])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (1 + a^{mx})^2 \, dx &= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x} \, dx, x, a^{mx}\right)}{m\log(a)} \\
&= \frac{\text{Subst}\left(\int \left(2 + \frac{1}{x} + x\right) dx, x, a^{mx}\right)}{m\log(a)} \\
&= x + \frac{2a^{mx}}{m\log(a)} + \frac{a^{2mx}}{2m\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.0102533, size = 31, normalized size = 0.94

$$\frac{2a^{mx} + \frac{1}{2}a^{2mx} + mx\log(a)}{m\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^(m*x))^2,x]
```

```
[Out] (2*a^(m*x) + a^(2*m*x)/2 + m*x*Log[a])/(m*Log[a])
```

Maple [A] time = 0.008, size = 46, normalized size = 1.4

$$\frac{(a^{mx})^2}{2\,m\ln(a)}+2\,\frac{a^{mx}}{m\ln(a)}+\frac{\ln(a^{mx})}{m\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+a^(m*x))^2,x)
```

```
[Out] 1/2/m/ln(a)*(a^(m*x))^2+2*a^(m*x)/m/ln(a)+1/m/ln(a)*ln(a^(m*x))
```

Maxima [A] time = 0.928311, size = 42, normalized size = 1.27

$$x+\frac{a^{2\,mx}}{2\,m\log(a)}+\frac{2\,a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^2,x, algorithm="maxima")
```

```
[Out] x + 1/2*a^(2*m*x)/(m*log(a)) + 2*a^(m*x)/(m*log(a))
```

Fricas [A] time = 1.81448, size = 74, normalized size = 2.24

$$\frac{2\,mx\log(a)+a^{2\,mx}+4\,a^{mx}}{2\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*m*x*log(a) + a^(2*m*x) + 4*a^(m*x))/(m*log(a))
```

Sympy [A] time = 0.110519, size = 46, normalized size = 1.39

$$x+\begin{cases}\frac{a^{2mx}m\log(a)+4a^{mx}m\log(a)}{2m^2\log(a)^2} & \text{for } 2m^2\log(a)^2\neq 0\\ 3x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a**(m*x))**2,x)
```

```
[Out] x + Piecewise(((a**(2*m*x)*m*log(a) + 4*a**(m*x)*m*log(a))/(2*m**2*log(a)**
2), Ne(2*m**2*log(a)**2, 0)), (3*x, True))
```

Giac [A] time = 1.0866, size = 41, normalized size = 1.24

$$\frac{2\,mx\log\left(|a|\right) + a^{2\,mx} + 4\,a^{mx}}{2\,m\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*m*x*log(abs(a)) + a^(2*m*x) + 4*a^(m*x))/(m*log(a))
```

3.514 $\int (1 + a^{mx})^3 \, dx$

Optimal. Leaf size=50

$$\frac{3a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{2m\log(a)} + \frac{a^{3mx}}{3m\log(a)} + x$$

```
[Out] x + (3*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(2*m*Log[a]) + a^(3*m*x)/(3*m*Lo
g[a])
```

Rubi [A] time = 0.0157746, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2282, 43}

$$\frac{3a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{2m\log(a)} + \frac{a^{3mx}}{3m\log(a)} + x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^(m*x))^3,x]
```

```
[Out] x + (3*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(2*m*Log[a]) + a^(3*m*x)/(3*m*Lo
g[a])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (1 + a^{mx})^3 \, dx &= \frac{\text{Subst}\left(\int \frac{(1+x)^3}{x} \, dx, x, a^{mx}\right)}{m\log(a)} \\
&= \frac{\text{Subst}\left(\int \left(3 + \frac{1}{x} + 3x + x^2\right) dx, x, a^{mx}\right)}{m\log(a)} \\
&= x + \frac{3a^{mx}}{m\log(a)} + \frac{3a^{2mx}}{2m\log(a)} + \frac{a^{3mx}}{3m\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.0257468, size = 35, normalized size = 0.7

$$\frac{\left(9a^{mx} + 2a^{2mx} + 18\right)a^{mx}}{6m\log(a)} + x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^(m*x))^3,x]
```

```
[Out] x + (a^(m*x)*(18 + 9*a^(m*x) + 2*a^(2*m*x)))/(6*m*Log[a])
```

Maple [A] time = 0.002, size = 62, normalized size = 1.2

$$\frac{(a^{mx})^3}{3\,m\ln(a)}+\frac{3\,(a^{mx})^2}{2\,m\ln(a)}+3\,\frac{a^{mx}}{m\ln(a)}+\frac{\ln(a^{mx})}{m\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+a^(m*x))^3,x)
```

```
[Out] 1/3/m/ln(a)*(a^(m*x))^3+3/2/m/ln(a)*(a^(m*x))^2+3*a^(m*x)/m/ln(a)+1/m/ln(a)
*ln(a^(m*x))
```

Maxima [A] time = 0.934788, size = 62, normalized size = 1.24

$$x+\frac{a^{3\,mx}}{3\,m\log(a)}+\frac{3\,a^{2\,mx}}{2\,m\log(a)}+\frac{3\,a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^3,x, algorithm="maxima")
```

```
[Out] x + 1/3*a^(3*m*x)/(m*log(a)) + 3/2*a^(2*m*x)/(m*log(a)) + 3*a^(m*x)/(m*log(
a))
```

Fricas [A] time = 1.80834, size = 97, normalized size = 1.94

$$\frac{6\,mx\log(a)+2\,a^{3\,mx}+9\,a^{2\,mx}+18\,a^{mx}}{6\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*m*x*log(a) + 2*a^(3*m*x) + 9*a^(2*m*x) + 18*a^(m*x))/(m*log(a))
```

Sympy [A] time = 0.133762, size = 71, normalized size = 1.42

$$x+\begin{cases}\frac{2a^{3mx}m^2\log(a)^2+9a^{2mx}m^2\log(a)^2+18a^{mx}m^2\log(a)^2}{6m^3\log(a)^3} & \text{for } 6m^3\log(a)^3\neq 0\\ 7x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a**(m*x))**3,x)
```

```
[Out] x + Piecewise(((2*a**(3*m*x)*m**2*log(a)**2 + 9*a**(2*m*x)*m**2*log(a)**2 +
 18*a**(m*x)*m**2*log(a)**2)/(6*m**3*log(a)**3), Ne(6*m**3*log(a)**3, 0)),
(7*x, True))
```

Giac [A] time = 1.08985, size = 54, normalized size = 1.08

$$\frac{6\,mx\log\left(|a|\right) + 2\,a^{3\,mx} + 9\,a^{2\,mx} + 18\,a^{mx}}{6\,m\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^3,x, algorithm="giac")
```

```
[Out] 1/6*(6*m*x*log(abs(a)) + 2*a^(3*m*x) + 9*a^(2*m*x) + 18*a^(m*x))/(m*log(a))
```

3.515 $\int (1+a^{mx})^4 \, dx$

Optimal. Leaf size=65

$$\frac{4a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{m\log(a)}+\frac{4a^{3mx}}{3m\log(a)}+\frac{a^{4mx}}{4m\log(a)}+x$$

```
[Out] x + (4*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(m*Log[a]) + (4*a^(3*m*x))/(3*m*
Log[a]) + a^(4*m*x)/(4*m*Log[a])
```

Rubi [A] time = 0.0191191, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2282, 43}

$$\frac{4a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{m\log(a)}+\frac{4a^{3mx}}{3m\log(a)}+\frac{a^{4mx}}{4m\log(a)}+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^(m*x))^4,x]
```

```
[Out] x + (4*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(m*Log[a]) + (4*a^(3*m*x))/(3*m*
Log[a]) + a^(4*m*x)/(4*m*Log[a])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (1+a^{mx})^4 \, dx &= \frac{\text{Subst}\left(\int \frac{(1+x)^4}{x} \, dx, x, a^{mx}\right)}{m\log(a)}\\
&= \frac{\text{Subst}\left(\int \left(4+\frac{1}{x}+6x+4x^2+x^3\right) dx, x, a^{mx}\right)}{m\log(a)}\\
&= x+\frac{4a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{m\log(a)}+\frac{4a^{3mx}}{3m\log(a)}+\frac{a^{4mx}}{4m\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.0197996, size = 49, normalized size = 0.75

$$\frac{4a^{mx}+3a^{2mx}+\frac{4}{3}a^{3mx}+\frac{1}{4}a^{4mx}+mx\log(a)}{m\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^(m*x))^4,x]
```

```
[Out] (4*a^(m*x) + 3*a^(2*m*x) + (4*a^(3*m*x))/3 + a^(4*m*x)/4 + m*x*Log[a])/(m*L
og[a])
```

Maple [A] time = 0.002, size = 78, normalized size = 1.2

$$\frac{(a^{mx})^4}{4\,m\ln(a)}+\frac{4\,(a^{mx})^3}{3\,m\ln(a)}+3\,\frac{(a^{mx})^2}{m\ln(a)}+4\,\frac{a^{mx}}{m\ln(a)}+\frac{\ln(a^{mx})}{m\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+a^(m*x))^4,x)
```

```
[Out] 1/4/m/ln(a)*(a^(m*x))^4+4/3/m/ln(a)*(a^(m*x))^3+3/m/ln(a)*(a^(m*x))^2+4*a^(
m*x)/m/ln(a)+1/m/ln(a)*ln(a^(m*x))
```

Maxima [A] time = 0.921888, size = 82, normalized size = 1.26

$$x+\frac{a^{4\,mx}}{4\,m\log(a)}+\frac{4\,a^{3\,mx}}{3\,m\log(a)}+\frac{3\,a^{2\,mx}}{m\log(a)}+\frac{4\,a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^4,x, algorithm="maxima")
```

```
[Out] x + 1/4*a^(4*m*x)/(m*log(a)) + 4/3*a^(3*m*x)/(m*log(a)) + 3*a^(2*m*x)/(m*lo
g(a)) + 4*a^(m*x)/(m*log(a))
```

Fricas [A] time = 1.67255, size = 122, normalized size = 1.88

$$\frac{12\,mx\log(a)+3\,a^{4\,mx}+16\,a^{3\,mx}+36\,a^{2\,mx}+48\,a^{mx}}{12\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(12*m*x*log(a) + 3*a^(4*m*x) + 16*a^(3*m*x) + 36*a^(2*m*x) + 48*a^(m*x
))/(m*log(a))
```

Sympy [A] time = 0.155902, size = 88, normalized size = 1.35

$$x+\begin{cases}\frac{3a^{4mx}m^3\log(a)^3+16a^{3mx}m^3\log(a)^3+36a^{2mx}m^3\log(a)^3+48a^{mx}m^3\log(a)^3}{12m^4\log(a)^4} & \text{for } 12m^4\log(a)^4\neq 0\\ 15x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a**(m*x))**4,x)
```

```
[Out] x + Piecewise(((3*a**(4*m*x)*m**3*log(a)**3 + 16*a**(3*m*x)*m**3*log(a)**3
+ 36*a**(2*m*x)*m**3*log(a)**3 + 48*a**(m*x)*m**3*log(a)**3)/(12*m**4*log(a
)**4), Ne(12*m**4*log(a)**4, 0)), (15*x, True))
```

Giac [A] time = 1.13746, size = 65, normalized size = 1.

$$\frac{12\,mx\log\left(|a|\right) + 3\,a^{4\,mx} + 16\,a^{3\,mx} + 36\,a^{2\,mx} + 48\,a^{mx}}{12\,m\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^4,x, algorithm="giac")
```

```
[Out] 1/12*(12*m*x*log(abs(a)) + 3*a^(4*m*x) + 16*a^(3*m*x) + 36*a^(2*m*x) + 48*a
^(m*x))/(m*log(a))
```

3.516 $\int (1 + a^{mx})^n \, dx$

Optimal. Leaf size=40

$$-\frac{(a^{mx}+1)^{n+1} \, {}_2F_1(1,n+1;n+2;a^{mx}+1)}{m(n+1)\log(a)}$$

```
[Out] -(((1 + a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + a^(m*x)])/(
m*(1 + n)*Log[a]))
```

Rubi [A] time = 0.0210817, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2282, 65}

$$-\frac{(a^{mx}+1)^{n+1} \text{Hypergeometric2F1}(1,n+1,n+2,a^{mx}+1)}{m(n+1)\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a^(m*x))^n,x]
```

```
[Out] -(((1 + a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + a^(m*x)])/(
m*(1 + n)*Log[a]))
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int (1+a^{mx})^n \, dx &= \frac{\text{Subst}\left(\int \frac{(1+x)^n}{x} \, dx, x, a^{mx}\right)}{m\log(a)}\\ &= -\frac{(1+a^{mx})^{1+n} \, {}_2F_1(1,1+n;2+n;1+a^{mx})}{m(1+n)\log(a)}\end{aligned}$$

Mathematica [A] time = 0.0142504, size = 40, normalized size = 1.

$$-\frac{(a^{mx}+1)^{n+1} \, {}_2F_1(1,n+1;n+2;a^{mx}+1)}{m(n+1)\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a^(m*x))^n,x]
```

```
[Out] -(((1 + a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + a^(m*x)])/(
m*(1 + n)*Log[a]))
```

Maple [F] time = 0.029, size = 0, normalized size = 0.

$$\int (1 + a^{mx})^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+a^(m*x))^n,x)
```

```
[Out] int((1+a^(m*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a^{mx} + 1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((a^(m*x) + 1)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a^{mx} + 1)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^n,x, algorithm="fricas")
```

```
[Out] integral((a^(m*x) + 1)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a^{mx} + 1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a**(m*x))**n,x)
```

```
[Out] Integral((a**(m*x) + 1)**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a^{mx} + 1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+a^(m*x))^n,x, algorithm="giac")
```

```
[Out] integrate((a^(m*x) + 1)^n, x)
```

3.517 $\int (1 - a^{mx}) \, dx$

Optimal. Leaf size=16

$$x - \frac{a^{mx}}{m \log(a)}$$

```
[Out] x - a^(m*x)/(m*Log[a])
```

Rubi [A] time = 0.0050012, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2194}

$$x - \frac{a^{mx}}{m \log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[1 - a^(m*x),x]

[Out] x - a^(m*x)/(m*Log[a])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int (1 - a^{mx}) \, dx &= x - \int a^{mx} \, dx \\ &= x - \frac{a^{mx}}{m \log(a)} \end{aligned}$$

Mathematica [A] time = 0.0042407, size = 16, normalized size = 1.

$$x - \frac{a^{mx}}{m \log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1 - a^(m*x),x]

[Out] x - a^(m*x)/(m*Log[a])
```

Maple [A] time = 0.001, size = 17, normalized size = 1.1

$$x - \frac{a^{mx}}{m \ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1-a^(m*x),x)
```

```
[Out] x-a^(m*x)/m/ln(a)
```

Maxima [A] time = 0.926793, size = 22, normalized size = 1.38

$$x - \frac{a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-a^(m*x),x, algorithm="maxima")
```

```
[Out] x - a^(m*x)/(m*log(a))
```

Fricas [A] time = 1.83819, size = 47, normalized size = 2.94

$$\frac{mx \log(a) - a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-a^(m*x),x, algorithm="fricas")
```

```
[Out] (m*x*log(a) - a^(m*x))/(m*log(a))
```

Sympy [A] time = 0.090836, size = 19, normalized size = 1.19

$$x + \begin{cases} -\frac{a^{mx}}{m \log(a)} & \text{for } m \log(a) \neq 0 \\ -x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-a**(m*x),x)
```

```
[Out] x + Piecewise((-a**(m*x)/(m*log(a)), Ne(m*log(a), 0)), (-x, True))
```

Giac [A] time = 1.16075, size = 22, normalized size = 1.38

$$x - \frac{a^{mx}}{m \log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1-a^(m*x),x, algorithm="giac")
```

```
[Out] x - a^(m*x)/(m*log(a))
```

3.518 $\int (1-a^{mx})^2\,dx$

Optimal. Leaf size=33

$$-\frac{2a^{mx}}{m\log(a)}+\frac{a^{2mx}}{2m\log(a)}+x$$

```
[Out] x - (2*a^(m*x))/(m*Log[a]) + a^(2*m*x)/(2*m*Log[a])
```

Rubi [A] time = 0.0134689, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 43}

$$-\frac{2a^{mx}}{m\log(a)}+\frac{a^{2mx}}{2m\log(a)}+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^(m*x))^2,x]
```

```
[Out] x - (2*a^(m*x))/(m*Log[a]) + a^(2*m*x)/(2*m*Log[a])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (1-a^{mx})^2\,dx &= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx, x, a^{mx}\right)}{m\log(a)}\\ &= \frac{\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)dx, x, a^{mx}\right)}{m\log(a)}\\ &= x-\frac{2a^{mx}}{m\log(a)}+\frac{a^{2mx}}{2m\log(a)}\end{aligned}$$

Mathematica [A] time = 0.015045, size = 25, normalized size = 0.76

$$\frac{(a^{mx}-4)\,a^{mx}}{2m\log(a)}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^(m*x))^2,x]
```

```
[Out] x + (a^(m*x)*(-4 + a^(m*x)))/(2*m*Log[a])
```

Maple [A] time = 0.002, size = 46, normalized size = 1.4

$$\frac{(a^{mx})^2}{2\,m\ln(a)} - 2\,\frac{a^{mx}}{m\ln(a)} + \frac{\ln(a^{mx})}{m\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-a^(m*x))^2,x)
```

```
[Out] 1/2/m/ln(a)*(a^(m*x))^2-2*a^(m*x)/m/ln(a)+1/m/ln(a)*ln(a^(m*x))
```

Maxima [A] time = 0.931394, size = 42, normalized size = 1.27

$$x + \frac{a^{2\,mx}}{2\,m\log(a)} - \frac{2\,a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^2,x, algorithm="maxima")
```

```
[Out] x + 1/2*a^(2*m*x)/(m*log(a)) - 2*a^(m*x)/(m*log(a))
```

Fricas [A] time = 1.65989, size = 74, normalized size = 2.24

$$\frac{2\,mx\log(a) + a^{2\,mx} - 4\,a^{mx}}{2\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*m*x*log(a) + a^(2*m*x) - 4*a^(m*x))/(m*log(a))
```

Sympy [A] time = 0.11048, size = 46, normalized size = 1.39

$$x + \begin{cases} \frac{a^{2mx} m\log(a) - 4a^{mx} m\log(a)}{2m^2\log(a)^2} & \text{for}\ 2m^2\log(a)^2 \neq 0 \\ -x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a**(m*x))**2,x)
```

```
[Out] x + Piecewise(((a**(2*m*x)*m*log(a) - 4*a**(m*x)*m*log(a))/(2*m**2*log(a)**
2), Ne(2*m**2*log(a)**2, 0)), (-x, True))
```

Giac [A] time = 1.07327, size = 41, normalized size = 1.24

$$\frac{2\,mx\log\left(|a|\right)+a^{2\,mx}-4\,a^{mx}}{2\,m\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*m*x*log(abs(a)) + a^(2*m*x) - 4*a^(m*x))/(m*log(a))
```

3.519 $\int (1-a^{mx})^3 \, dx$

Optimal. Leaf size=50

$$-\frac{3a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{2m\log(a)}-\frac{a^{3mx}}{3m\log(a)}+x$$

```
[Out] x - (3*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(2*m*Log[a]) - a^(3*m*x)/(3*m*Lo
g[a])
```

Rubi [A] time = 0.0167983, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 43}

$$-\frac{3a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{2m\log(a)}-\frac{a^{3mx}}{3m\log(a)}+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^(m*x))^3,x]
```

```
[Out] x - (3*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(2*m*Log[a]) - a^(3*m*x)/(3*m*Lo
g[a])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (1-a^{mx})^3 \, dx &= \frac{\text{Subst}\left(\int \frac{(1-x)^3}{x} \, dx, x, a^{mx}\right)}{m\log(a)}\\ &= \frac{\text{Subst}\left(\int \left(-3+\frac{1}{x}+3x-x^2\right) \, dx, x, a^{mx}\right)}{m\log(a)}\\ &= x-\frac{3a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{2m\log(a)}-\frac{a^{3mx}}{3m\log(a)}\end{aligned}$$

Mathematica [A] time = 0.025566, size = 35, normalized size = 0.7

$$x-\frac{a^{mx}\left(-9a^{mx}+2a^{2mx}+18\right)}{6m\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^(m*x))^3,x]
```

```
[Out] x - (a^(m*x)*(18 - 9*a^(m*x) + 2*a^(2*m*x)))/(6*m*Log[a])
```

Maple [A] time = 0.002, size = 62, normalized size = 1.2

$$-\frac{(a^{mx})^3}{3m\ln(a)}+\frac{3(a^{mx})^2}{2m\ln(a)}-3\frac{a^{mx}}{m\ln(a)}+\frac{\ln(a^{mx})}{m\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-a^(m*x))^3,x)
```

```
[Out] -1/3/m/ln(a)*(a^(m*x))^3+3/2/m/ln(a)*(a^(m*x))^2-3*a^(m*x)/m/ln(a)+1/m/ln(a
)*ln(a^(m*x))
```

Maxima [A] time = 0.930163, size = 62, normalized size = 1.24

$$x-\frac{a^{3mx}}{3m\log(a)}+\frac{3a^{2mx}}{2m\log(a)}-\frac{3a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^3,x, algorithm="maxima")
```

```
[Out] x - 1/3*a^(3*m*x)/(m*log(a)) + 3/2*a^(2*m*x)/(m*log(a)) - 3*a^(m*x)/(m*log(
a))
```

Fricas [A] time = 1.93649, size = 97, normalized size = 1.94

$$\frac{6mx\log(a)-2a^{3mx}+9a^{2mx}-18a^{mx}}{6m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^3,x, algorithm="fricas")
```

```
[Out] 1/6*(6*m*x*log(a) - 2*a^(3*m*x) + 9*a^(2*m*x) - 18*a^(m*x))/(m*log(a))
```

Sympy [A] time = 0.137024, size = 71, normalized size = 1.42

$$x+\begin{cases}\frac{-2a^{3mx}m^2\log(a)^2+9a^{2mx}m^2\log(a)^2-18a^{mx}m^2\log(a)^2}{6m^3\log(a)^3} & \text{for } 6m^3\log(a)^3\neq 0\\ -x & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a**(m*x))**3,x)
```

```
[Out] x + Piecewise(((-2*a**(3*m*x)*m**2*log(a)**2 + 9*a**(2*m*x)*m**2*log(a)**2
- 18*a**(m*x)*m**2*log(a)**2)/(6*m**3*log(a)**3), Ne(6*m**3*log(a)**3, 0)),
 (-x, True))
```

Giac [A] time = 1.08117, size = 54, normalized size = 1.08

$$\frac{6\,mx\log\left(|a|\right)-2\,a^{3\,mx}+9\,a^{2\,mx}-18\,a^{mx}}{6\,m\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^3,x, algorithm="giac")
```

```
[Out] 1/6*(6*m*x*log(abs(a)) - 2*a^(3*m*x) + 9*a^(2*m*x) - 18*a^(m*x))/(m*log(a))
```

3.520 $\int (1-a^{mx})^4 \, dx$

Optimal. Leaf size=65

$$-\frac{4a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{m\log(a)}-\frac{4a^{3mx}}{3m\log(a)}+\frac{a^{4mx}}{4m\log(a)}+x$$

```
[Out] x - (4*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(m*Log[a]) - (4*a^(3*m*x))/(3*m*
Log[a]) + a^(4*m*x)/(4*m*Log[a])
```

Rubi [A] time = 0.0191933, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 43}

$$-\frac{4a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{m\log(a)}-\frac{4a^{3mx}}{3m\log(a)}+\frac{a^{4mx}}{4m\log(a)}+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^(m*x))^4,x]
```

```
[Out] x - (4*a^(m*x))/(m*Log[a]) + (3*a^(2*m*x))/(m*Log[a]) - (4*a^(3*m*x))/(3*m*
Log[a]) + a^(4*m*x)/(4*m*Log[a])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (1-a^{mx})^4 \, dx &= \frac{\text{Subst}\left(\int \frac{(1-x)^4}{x} \, dx, x, a^{mx}\right)}{m\log(a)}\\ &= \frac{\text{Subst}\left(\int \left(-4+\frac{1}{x}+6x-4x^2+x^3\right) dx, x, a^{mx}\right)}{m\log(a)}\\ &= x-\frac{4a^{mx}}{m\log(a)}+\frac{3a^{2mx}}{m\log(a)}-\frac{4a^{3mx}}{3m\log(a)}+\frac{a^{4mx}}{4m\log(a)}\end{aligned}$$

Mathematica [A] time = 0.0161545, size = 49, normalized size = 0.75

$$\frac{-4a^{mx}+3a^{2mx}-\frac{4}{3}a^{3mx}+\frac{1}{4}a^{4mx}+mx\log(a)}{m\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^(m*x))^4,x]
```

```
[Out] (-4*a^(m*x) + 3*a^(2*m*x) - (4*a^(3*m*x))/3 + a^(4*m*x)/4 + m*x*Log[a])/(m*
Log[a])
```

Maple [A] time = 0.002, size = 78, normalized size = 1.2

$$\frac{(a^{mx})^4}{4m\ln(a)} - \frac{4\,(a^{mx})^3}{3m\ln(a)} + 3\,\frac{(a^{mx})^2}{m\ln(a)} - 4\,\frac{a^{mx}}{m\ln(a)} + \frac{\ln(a^{mx})}{m\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-a^(m*x))^4,x)
```

```
[Out] 1/4/m/ln(a)*(a^(m*x))^4-4/3/m/ln(a)*(a^(m*x))^3+3/m/ln(a)*(a^(m*x))^2-4*a^(
m*x)/m/ln(a)+1/m/ln(a)*ln(a^(m*x))
```

Maxima [A] time = 0.929881, size = 82, normalized size = 1.26

$$x + \frac{a^{4\,mx}}{4\,m\log(a)} - \frac{4\,a^{3\,mx}}{3\,m\log(a)} + \frac{3\,a^{2\,mx}}{m\log(a)} - \frac{4\,a^{mx}}{m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^4,x, algorithm="maxima")
```

```
[Out] x + 1/4*a^(4*m*x)/(m*log(a)) - 4/3*a^(3*m*x)/(m*log(a)) + 3*a^(2*m*x)/(m*lo
g(a)) - 4*a^(m*x)/(m*log(a))
```

Fricas [A] time = 1.84396, size = 122, normalized size = 1.88

$$\frac{12\,mx\log(a) + 3\,a^{4\,mx} - 16\,a^{3\,mx} + 36\,a^{2\,mx} - 48\,a^{mx}}{12\,m\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(12*m*x*log(a) + 3*a^(4*m*x) - 16*a^(3*m*x) + 36*a^(2*m*x) - 48*a^(m*x
))/(m*log(a))
```

Sympy [A] time = 0.155208, size = 88, normalized size = 1.35

$$x + \begin{cases} \frac{3a^{4mx}m^3\log(a)^3 - 16a^{3mx}m^3\log(a)^3 + 36a^{2mx}m^3\log(a)^3 - 48a^{mx}m^3\log(a)^3}{12m^4\log(a)^4} & \text{for}\ 12m^4\log(a)^4 \neq 0 \\ -x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a**(m*x))**4,x)
```

```
[Out] x + Piecewise(((3*a**(4*m*x)*m**3*log(a)**3 - 16*a**(3*m*x)*m**3*log(a)**3
+ 36*a**(2*m*x)*m**3*log(a)**3 - 48*a**(m*x)*m**3*log(a)**3)/(12*m**4*log(a
)**4), Ne(12*m**4*log(a)**4, 0)), (-x, True))
```

Giac [A] time = 1.10778, size = 65, normalized size = 1.

$$\frac{12\,mx\log\left(|a|\right) + 3\,a^{4\,mx} - 16\,a^{3\,mx} + 36\,a^{2\,mx} - 48\,a^{mx}}{12\,m\log\left(a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^4,x, algorithm="giac")
```

```
[Out] 1/12*(12*m*x*log(abs(a)) + 3*a^(4*m*x) - 16*a^(3*m*x) + 36*a^(2*m*x) - 48*a
^(m*x))/(m*log(a))
```

3.521 $\int (1 - a^{mx})^n \, dx$

Optimal. Leaf size=44

$$-\frac{(1-a^{mx})^{n+1} \, {}_2F_1(1,n+1;n+2;1-a^{mx})}{m(n+1)\log(a)}$$

```
[Out] -(((1 - a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 - a^(m*x)])/(
m*(1 + n)*Log[a]))
```

Rubi [A] time = 0.0187026, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 65}

$$-\frac{(1-a^{mx})^{n+1} \, \text{Hypergeometric2F1}(1,n+1,n+2,1-a^{mx})}{m(n+1)\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - a^(m*x))^n,x]
```

```
[Out] -(((1 - a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 - a^(m*x)])/(
m*(1 + n)*Log[a]))
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}
\int (1-a^{mx})^n \, dx &= \frac{\text{Subst}\left(\int \frac{(1-x)^n}{x} \, dx, x, a^{mx}\right)}{m\log(a)} \\
&= -\frac{(1-a^{mx})^{1+n} \, {}_2F_1(1,1+n;2+n;1-a^{mx})}{m(1+n)\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.0138956, size = 44, normalized size = 1.

$$-\frac{(1-a^{mx})^{n+1} \, {}_2F_1(1,n+1;n+2;1-a^{mx})}{m(n+1)\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - a^(m*x))^n,x]
```

```
[Out] -(((1 - a^(m*x))^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, 1 - a^(m*x)])/(
m*(1 + n)*Log[a]))
```

Maple [F] time = 0.032, size = 0, normalized size = 0.

$$\int (1 - a^{mx})^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-a^(m*x))^n,x)
```

```
[Out] int((1-a^(m*x))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-a^{mx} + 1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((-a^(m*x) + 1)^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-a^{mx} + 1)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^n,x, algorithm="fricas")
```

```
[Out] integral((-a^(m*x) + 1)^n, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (1 - a^{mx})^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a**(m*x))**n,x)
```

```
[Out] Integral((1 - a**(m*x))**n, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-a^{mx} + 1)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-a^(m*x))^n,x, algorithm="giac")
```

```
[Out] integrate((-a^(m*x) + 1)^n, x)
```

3.522 $\int \frac{1}{b+ae^{nx}}\,dx$

Optimal. Leaf size=24

$$\frac{x}{b}-\frac{\log\left(ae^{nx}+b\right)}{bn}$$

```
[Out] x/b - Log[b + a*E^(n*x)]/(b*n)
```

Rubi [A] time = 0.0161057, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2282, 36, 29, 31}

$$\frac{x}{b}-\frac{\log\left(ae^{nx}+b\right)}{bn}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*E^(n*x))^(-1),x]
```

```
[Out] x/b - Log[b + a*E^(n*x)]/(b*n)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{b+ae^{nx}}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x(b+ax)}\,dx,x,e^{nx}\right)}{n}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,e^{nx}\right)}{bn}-\frac{a\operatorname{Subst}\left(\int \frac{1}{b+ax}\,dx,x,e^{nx}\right)}{bn}\\
&= \frac{x}{b}-\frac{\log\left(b+ae^{nx}\right)}{bn}
\end{aligned}$$

Mathematica [A] time = 0.0055943, size = 24, normalized size = 1.

$$\frac{x}{b} - \frac{\log(ae^{nx} + b)}{bn}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*E^(n*x))^(-1),x]
```

```
[Out] x/b - Log[b + a*E^(n*x)]/(b*n)
```

Maple [A] time = 0.007, size = 31, normalized size = 1.3

$$\frac{\ln(e^{nx})}{nb} - \frac{\ln(b + ae^{nx})}{nb}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b+a*exp(n*x)),x)
```

```
[Out] 1/n/b*ln(exp(n*x))-ln(b+a*exp(n*x))/b/n
```

Maxima [A] time = 0.935134, size = 31, normalized size = 1.29

$$\frac{x}{b} - \frac{\log\left(ae^{(nx)} + b\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b+a*exp(n*x)),x, algorithm="maxima")
```

```
[Out] x/b - log(a*e^(n*x) + b)/(b*n)
```

Fricas [A] time = 1.84173, size = 46, normalized size = 1.92

$$\frac{nx - \log\left(ae^{(nx)} + b\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b+a*exp(n*x)),x, algorithm="fricas")
```

```
[Out] (n*x - log(a*e^(n*x) + b))/(b*n)
```

Sympy [A] time = 0.112088, size = 15, normalized size = 0.62

$$\frac{x}{b} - \frac{\log\left(e^{nx} + \frac{b}{a}\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b+a*exp(n*x)),x)
```

```
[Out] x/b - log(exp(n*x) + b/a)/(b*n)
```

Giac [A] time = 1.09529, size = 32, normalized size = 1.33

$$\frac{x}{b} - \frac{\log\left(\left|ae^{(nx)} + b\right|\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b+a*exp(n*x)),x, algorithm="giac")
```

```
[Out] x/b - log(abs(a*e^(n*x) + b))/(b*n)
```

3.523 $\int \frac{e^x}{b+ae^{3x}}\,dx$

Optimal. Leaf size=100

$$\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{2\sqrt[3]{a}b^{2/3}}-\frac{\log\left(ae^{3x}+b\right)}{6\sqrt[3]{a}b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}e^x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}$$

```
[Out] -(ArcTan[(b^(1/3) - 2*a^(1/3)*E^x)/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^(1/3)*b^(2
/3))) + Log[b^(1/3) + a^(1/3)*E^x]/(2*a^(1/3)*b^(2/3)) - Log[b + a*E^(3*x)]
/(6*a^(1/3)*b^(2/3))
```

Rubi [A] time = 0.0943053, antiderivative size = 123, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {2249, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}e^{2x}-\sqrt[3]{a}\sqrt[3]{b}e^x+b^{2/3}\right)}{6\sqrt[3]{a}b^{2/3}}+\frac{\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}-2\sqrt[3]{a}e^x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(b + a*E^(3*x)),x]
```

```
[Out] -(ArcTan[(b^(1/3) - 2*a^(1/3)*E^x)/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^(1/3)*b^(2
/3))) + Log[b^(1/3) + a^(1/3)*E^x]/(3*a^(1/3)*b^(2/3)) - Log[b^(2/3) - a^(1
/3)*b^(1/3)*E^x + a^(2/3)*E^(2*x)]/(6*a^(1/3)*b^(2/3))
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^x}{b+ae^{3x}}\,dx &= \text{Subst}\left(\int \frac{1}{b+ax^3}\,dx,x,e^x\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{b}+\sqrt[3]{a}x}\,dx,x,e^x\right)}{3b^{2/3}}+\frac{\text{Subst}\left(\int \frac{2\sqrt[3]{b}-\sqrt[3]{a}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx,x,e^x\right)}{3b^{2/3}}\\
&= \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}e^x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\text{Subst}\left(\int \frac{-\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx,x,e^x\right)}{6\sqrt[3]{a}b^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx,x,e^x\right)}{2\sqrt[3]{b}}\\
&= \frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}e^x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}e^x+a^{2/3}e^{2x}\right)}{6\sqrt[3]{a}b^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2\sqrt[3]{a}e^x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}b^{2/3}}\\
&= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}e^x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}+\frac{\log\left(\sqrt[3]{b}+\sqrt[3]{a}e^x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\log\left(b^{2/3}-\sqrt[3]{a}\sqrt[3]{b}e^x+a^{2/3}e^{2x}\right)}{6\sqrt[3]{a}b^{2/3}}
\end{aligned}
$$

Mathematica [A] time = 0.0551443, size = 97, normalized size = 0.97

$$
-\frac{\log\left(a^{2/3}e^{2x}-\sqrt[3]{a}\sqrt[3]{b}e^x+b^{2/3}\right)-2\log\left(\sqrt[3]{a}e^x+\sqrt[3]{b}\right)+2\sqrt{3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{a}e^x}{\sqrt[3]{b}}}{\sqrt{3}}\right)}{6\sqrt[3]{a}b^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(b + a*E^(3*x)),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(1 - (2*a^(1/3)*E^x)/b^(1/3))/Sqrt[3]] - 2*Log[b^(1/3) +
 a^(1/3)*E^x] + Log[b^(2/3) - a^(1/3)*b^(1/3)*E^x + a^(2/3)*E^(2*x)])/(6*a^
(1/3)*b^(2/3))
```

Maple [A] time = 0.007, size = 95, normalized size = 1.

$$\frac{1}{3a}\ln\left(\mathrm{e}^{x}+\sqrt[3]{\frac{b}{a}}\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}}-\frac{1}{6a}\ln\left(\left(\mathrm{e}^{x}\right)^{2}-\sqrt[3]{\frac{b}{a}}\mathrm{e}^{x}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}}+\frac{\sqrt{3}}{3a}\arctan\left(\frac{\sqrt{3}}{3}\left(2\,\mathrm{e}^{x}\frac{1}{\sqrt[3]{\frac{b}{a}}}-1\right)\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(b+a*exp(3*x)),x)
```

```
[Out] 1/3/a/(b/a)^(2/3)*ln(exp(x)+(b/a)^(1/3))-1/6/a/(b/a)^(2/3)*ln(exp(x)^2-(b/a
)^(1/3)*exp(x)+(b/a)^(2/3))+1/3/a/(b/a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2
/(b/a)^(1/3)*exp(x)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(b+a*exp(3*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.87355, size = 795, normalized size = 7.95

$$\left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{\left(ab^{2}\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,abe^{(3\,x)}-3\left(ab^{2}\right)^{\frac{1}{3}}be^{x}-b^{2}+3\sqrt{\frac{1}{3}}\left(2\,abe^{(2\,x)}+\left(ab^{2}\right)^{\frac{2}{3}}e^{x}-\left(ab^{2}\right)^{\frac{1}{3}}b\right)\sqrt{-\frac{\left(ab^{2}\right)^{\frac{1}{3}}}{a}}}{ae^{(3\,x)}+b}\right)-\left(ab^{2}\right)^{\frac{2}{3}}\log\left(abe^{(2\,x)}-\left(ab^{2}\right)^{\frac{2}{3}}e^{x}+\right.}{6\,ab^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(b+a*exp(3*x)),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a*b^2)^(1/3)/a)*log((2*a*b*e^(3*x) - 3*(a*b^2)
^(1/3)*b*e^x - b^2 + 3*sqrt(1/3)*(2*a*b*e^(2*x) + (a*b^2)^(2/3)*e^x - (a*b^
2)^(1/3)*b)*sqrt(-(a*b^2)^(1/3)/a))/(a*e^(3*x) + b)) - (a*b^2)^(2/3)*log(a*
b*e^(2*x) - (a*b^2)^(2/3)*e^x + (a*b^2)^(1/3)*b) + 2*(a*b^2)^(2/3)*log(a*b*
e^x + (a*b^2)^(2/3)))/(a*b^2), 1/6*(6*sqrt(1/3)*a*b*sqrt((a*b^2)^(1/3)/a)*a
rctan(sqrt(1/3)*(2*(a*b^2)^(2/3)*e^x - (a*b^2)^(1/3)*b)*sqrt((a*b^2)^(1/3)/
a)/b^2) - (a*b^2)^(2/3)*log(a*b*e^(2*x) - (a*b^2)^(2/3)*e^x + (a*b^2)^(1/3)
*b) + 2*(a*b^2)^(2/3)*log(a*b*e^x + (a*b^2)^(2/3)))/(a*b^2)]
```

Sympy [A] time = 0.160282, size = 22, normalized size = 0.22

$$\mathrm{RootSum}\left(27z^{3}ab^{2}-1,\left(i\mapsto i\log\left(3ib+e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(b+a*exp(3*x)),x)
```

```
[Out] RootSum(27*_z**3*a*b**2 - 1, Lambda(_i, _i*log(3*_i*b + exp(x))))
```

Giac [A] time = 1.12688, size = 157, normalized size = 1.57

$$-\frac{\left(-\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{b}{a}\right)^{\frac{1}{3}}+e^{x}\right|\right)}{3\,b}+\frac{\sqrt{3}\left(-a^{2}b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{b}{a}\right)^{\frac{1}{3}}+2\,e^{x}\right)}{3\left(-\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,ab}+\frac{\left(-a^{2}b\right)^{\frac{1}{3}}\log\left(\left(-\frac{b}{a}\right)^{\frac{1}{3}}e^{x}+\left(-\frac{b}{a}\right)^{\frac{2}{3}}+e^{(2\,x)}\right)}{6\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(b+a*exp(3*x)),x, algorithm="giac")
```

```
[Out] -1/3*(-b/a)^(1/3)*log(abs(-(-b/a)^(1/3) + e^x))/b + 1/3*sqrt(3)*(-a^2*b)^(1
/3)*arctan(1/3*sqrt(3)*((-b/a)^(1/3) + 2*e^x)/(-b/a)^(1/3))/(a*b) + 1/6*(-a
^2*b)^(1/3)*log((-b/a)^(1/3)*e^x + (-b/a)^(2/3) + e^(2*x))/(a*b)
```

3.524 $\int \frac{-1+e^x}{1+e^x} dx$

Optimal. Leaf size=12

$$2\log(e^x+1)-x$$

```
[Out] -x + 2*Log[1 + E^x]
```

Rubi [A] time = 0.0218481, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2282, 72}

$$2\log(e^x+1)-x$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + E^x)/(1 + E^x),x]
```

```
[Out] -x + 2*Log[1 + E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+e^x}{1+e^x}\,dx &= \text{Subst}\left(\int \frac{-1+x}{x(1+x)}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{x}+\frac{2}{1+x}\right)dx, x, e^x\right)\\
&= -x+2\log\left(1+e^x\right)
\end{aligned}$$

Mathematica [A] time = 0.0081372, size = 12, normalized size = 1.

$$2\log(e^x+1)-x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + E^x)/(1 + E^x),x]
```

```
[Out] -x + 2*Log[1 + E^x]
```

Maple [A] time = 0.005, size = 14, normalized size = 1.2

$$-\ln\left(e^{x}\right)+2\ln\left(1+e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+exp(x))/(1+exp(x)),x)
```

```
[Out] -ln(exp(x))+2*ln(1+exp(x))
```

Maxima [A] time = 0.927006, size = 15, normalized size = 1.25

$$-x+2\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(x))/(1+exp(x)),x, algorithm="maxima")
```

```
[Out] -x + 2*log(e^x + 1)
```

Fricas [A] time = 1.86974, size = 28, normalized size = 2.33

$$-x+2\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(x))/(1+exp(x)),x, algorithm="fricas")
```

```
[Out] -x + 2*log(e^x + 1)
```

Sympy [A] time = 0.077769, size = 8, normalized size = 0.67

$$-x+2\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(x))/(1+exp(x)),x)
```

```
[Out] -x + 2*log(exp(x) + 1)
```

Giac [A] time = 1.11951, size = 15, normalized size = 1.25

$$-x+2\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(x))/(1+exp(x)),x, algorithm="giac")
```

```
[Out] -x + 2*log(e^x + 1)
```

3.525 $\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx$

Optimal. Leaf size=47

$$\frac{1}{12}\log\left(-2e^{2x}+3e^{4x}+1\right)-\frac{\tan^{-1}\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)}{6\sqrt{2}}$$

```
[Out] -ArcTan[(1 - 3*E^(2*x))/Sqrt[2]]/(6*Sqrt[2]) + Log[1 - 2*E^(2*x) + 3*E^(4*x
)]/12
```

Rubi [A] time = 0.0591705, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2282, 634, 618, 204, 628}

$$\frac{1}{12}\log\left(-2e^{2x}+3e^{4x}+1\right)-\frac{\tan^{-1}\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)}{6\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(4*x)/(1 - 2*E^(2*x) + 3*E^(4*x)),x]
```

```
[Out] -ArcTan[(1 - 3*E^(2*x))/Sqrt[2]]/(6*Sqrt[2]) + Log[1 - 2*E^(2*x) + 3*E^(4*x
)]/12
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{4x}}{1-2e^{2x}+3e^{4x}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{1-2x+3x^2}\,dx,x,e^{2x}\right)\\ &= \frac{1}{12}\text{Subst}\left(\int \frac{-2+6x}{1-2x+3x^2}\,dx,x,e^{2x}\right)+\frac{1}{6}\text{Subst}\left(\int \frac{1}{1-2x+3x^2}\,dx,x,e^{2x}\right)\\ &= \frac{1}{12}\log\left(1-2e^{2x}+3e^{4x}\right)-\frac{1}{3}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,x,-2+6e^{2x}\right)\\ &= -\frac{\tan^{-1}\left(\frac{1-3e^{2x}}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{1}{12}\log\left(1-2e^{2x}+3e^{4x}\right)\end{aligned}$$

Mathematica [A] time = 0.020503, size = 44, normalized size = 0.94

$$\frac{1}{12}\left(\log\left(-2e^{2x}+3e^{4x}+1\right)+\sqrt{2}\tan^{-1}\left(\frac{3e^{2x}-1}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(4*x)/(1 - 2*E^(2*x) + 3*E^(4*x)),x]
```

```
[Out] (Sqrt[2]*ArcTan[(-1 + 3*E^(2*x))/Sqrt[2]] + Log[1 - 2*E^(2*x) + 3*E^(4*x)])
/12
```

Maple [A] time = 0.007, size = 38, normalized size = 0.8

$$\frac{\ln\left(1-2\left(e^{x}\right)^{2}+3\left(e^{x}\right)^{4}\right)}{12}+\frac{\sqrt{2}}{12}\arctan\left(\frac{\left(6\left(e^{x}\right)^{2}-2\right)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x)
```

```
[Out] 1/12*ln(1-2*exp(x)^2+3*exp(x)^4)+1/12*2^(1/2)*arctan(1/4*(6*exp(x)^2-2)*2^(
1/2))
```

Maxima [A] time = 1.41344, size = 50, normalized size = 1.06

$$\frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(3e^{(2x)}-1\right)\right)+\frac{1}{12}\log\left(3e^{(4x)}-2e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*e^(2*x) - 1)) + 1/12*log(3*e^(4*x) - 2*e
^(2*x) + 1)
```

Fricas [A] time = 1.78919, size = 127, normalized size = 2.7

$$\frac{1}{12}\sqrt{2}\arctan\left(\frac{3}{2}\sqrt{2}e^{(2x)}-\frac{1}{2}\sqrt{2}\right)+\frac{1}{12}\log\left(3e^{(4x)}-2e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(2)*arctan(3/2*sqrt(2)*e^(2*x) - 1/2*sqrt(2)) + 1/12*log(3*e^(4*x)
 - 2*e^(2*x) + 1)
```

Sympy [A] time = 0.134202, size = 22, normalized size = 0.47

$$\operatorname{RootSum}\left(96z^2-16z+1,\left(i\mapsto i\log\left(8i+e^{2x}-1\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x)
```

```
[Out] RootSum(96*_z**2 - 16*_z + 1, Lambda(_i, _i*log(8*_i + exp(2*x) - 1)))
```

Giac [A] time = 1.13426, size = 50, normalized size = 1.06

$$\frac{1}{12}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(3e^{(2x)}-1\right)\right)+\frac{1}{12}\log\left(3e^{(4x)}-2e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(4*x)/(1-2*exp(2*x)+3*exp(4*x)),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*arctan(1/2*sqrt(2)*(3*e^(2*x) - 1)) + 1/12*log(3*e^(4*x) - 2*e
^(2*x) + 1)
```

3.526 $\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx$

Optimal. Leaf size=39

$$e^x+\frac{e^{2x}}{2}+\log(1-e^x)-\frac{1}{2}\log\left(e^{2x}+1\right)-\tan^{-1}(e^x)$$

```
[Out] E^x + E^(2*x)/2 - ArcTan[E^x] + Log[1 - E^x] - Log[1 + E^(2*x)]/2
```

Rubi [A] time = 0.0615397, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {2282, 2074, 635, 203, 260}

$$e^x+\frac{e^{2x}}{2}+\log(1-e^x)-\frac{1}{2}\log\left(e^{2x}+1\right)-\tan^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[(E^x + E^(5*x))/(-1 + E^x - E^(2*x) + E^(3*x)),x]

[Out] E^x + E^(2*x)/2 - ArcTan[E^x] + Log[1 - E^x] - Log[1 + E^(2*x)]/2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^x+e^{5x}}{-1+e^x-e^{2x}+e^{3x}}\,dx &= \text{Subst}\left(\int \frac{-1-x^4}{1-x+x^2-x^3}\,dx,x,e^x\right)\\
&= \text{Subst}\left(\int \left(1+\frac{1}{-1+x}+x+\frac{-1-x}{1+x^2}\right)dx,x,e^x\right)\\
&= e^x+\frac{e^{2x}}{2}+\log\left(1-e^x\right)+\text{Subst}\left(\int \frac{-1-x}{1+x^2}\,dx,x,e^x\right)\\
&= e^x+\frac{e^{2x}}{2}+\log\left(1-e^x\right)-\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,e^x\right)-\text{Subst}\left(\int \frac{x}{1+x^2}\,dx,x,e^x\right)\\
&= e^x+\frac{e^{2x}}{2}-\tan^{-1}\left(e^x\right)+\log\left(1-e^x\right)-\frac{1}{2}\log\left(1+e^{2x}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0423789, size = 51, normalized size = 1.31

$$
\frac{1}{2}\left(2e^x+e^{2x}+(-1+i)\log\left(-e^x+i\right)+2\log\left(1-e^x\right)-(1+i)\log\left(e^x+i\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x + E^(5*x))/(-1 + E^x - E^(2*x) + E^(3*x)),x]
```

```
[Out] (2*E^x + E^(2*x) - (1 - I)*Log[I - E^x] + 2*Log[1 - E^x] - (1 + I)*Log[I +
E^x])/2
```

Maple [A] time = 0.012, size = 29, normalized size = 0.7

$$
-\frac{\ln\left(\left(e^x\right)^2+1\right)}{2}-\arctan\left(e^x\right)+\ln\left(-1+e^x\right)+e^x+\frac{\left(e^x\right)^2}{2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x)
```

```
[Out] -1/2*ln(exp(x)^2+1)-arctan(exp(x))+ln(-1+exp(x))+exp(x)+1/2*exp(x)^2
```

Maxima [A] time = 1.41349, size = 38, normalized size = 0.97

$$
-\arctan\left(e^x\right)+\frac{1}{2}e^{(2x)}+e^x-\frac{1}{2}\log\left(e^{(2x)}+1\right)+\log\left(e^x-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x, algorithm="max
ima")
```

```
[Out] -arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(e^x - 1)
```

Fricas [A] time = 1.94914, size = 97, normalized size = 2.49

$$
-\arctan\left(e^x\right)+\frac{1}{2}e^{(2x)}+e^x-\frac{1}{2}\log\left(e^{(2x)}+1\right)+\log\left(e^x-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x, algorithm="fri
cas")
```

```
[Out] -arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(e^x - 1)
```

Sympy [A] time = 0.186641, size = 48, normalized size = 1.23

$$\frac{e^{2x}}{2} + e^{x} + \log\left(e^{x} - 1\right) + \operatorname{RootSum}\left(2z^{2} + 2z + 1, \left(i \mapsto i \log\left(\frac{4i^{2}}{5} - \frac{6i}{5} + e^{x} - \frac{3}{5}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x)
```

```
[Out] exp(2*x)/2 + exp(x) + log(exp(x) - 1) + RootSum(2*_z**2 + 2*_z + 1, Lambda(
_i, _i*log(4*_i**2/5 - 6*_i/5 + exp(x) - 3/5)))
```

Giac [A] time = 1.13056, size = 39, normalized size = 1.

$$-\arctan\left(e^{x}\right) + \frac{1}{2}\, e^{(2\,x)} + e^{x} - \frac{1}{2}\, \log\left(e^{(2\,x)} + 1\right) + \log\left(\left|e^{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((exp(x)+exp(5*x))/(-1+exp(x)-exp(2*x)+exp(3*x)),x, algorithm="gia
c")
```

```
[Out] -arctan(e^x) + 1/2*e^(2*x) + e^x - 1/2*log(e^(2*x) + 1) + log(abs(e^x - 1))
```

3.527 $\int e^{nx} (a + be^{nx})^{r/s} \, dx$

Optimal. Leaf size=30

$$\frac{s\,(a+be^{nx})^{\frac{r+s}{s}}}{bn(r+s)}$$

```
[Out] ((a + b*E^(n*x))^((r + s)/s)*s)/(b*n*(r + s))
```

Rubi [A] time = 0.0384192, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2246, 32}

$$\frac{s\,(a+be^{nx})^{\frac{r+s}{s}}}{bn(r+s)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(n*x)*(a + b*E^(n*x))^(r/s),x]
```

```
[Out] ((a + b*E^(n*x))^((r + s)/s)*s)/(b*n*(r + s))
```

Rule 2246

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
 e, n, p}, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{nx}\,(a+be^{nx})^{r/s}\,dx &= \frac{\text{Subst}\left(\int (a+bx)^{r/s}\,dx, x, e^{nx}\right)}{n}\\ &= \frac{(a+be^{nx})^{\frac{r+s}{s}}\,s}{bn(r+s)}\end{aligned}$$

Mathematica [A] time = 0.0381094, size = 30, normalized size = 1.

$$\frac{s\,(a+be^{nx})^{\frac{r}{s}+1}}{bnr+bns}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(n*x)*(a + b*E^(n*x))^(r/s),x]
```

```
[Out] ((a + b*E^(n*x))^(1 + r/s)*s)/(b*n*r + b*n*s)
```

Maple [A] time = 0.003, size = 33, normalized size = 1.1

$$\frac{1}{nb}(a+b\mathrm{e}^{nx})^{\frac{r}{s}+1}\left(\frac{r}{s}+1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(n*x)*(a+b*exp(n*x))^(r/s),x)
```

```
[Out] 1/n*(a+b*exp(n*x))^(r/s+1)/b/(r/s+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.81068, size = 80, normalized size = 2.67

$$\frac{\left(bse^{(nx)}+as\right)\left(be^{(nx)}+a\right)^{\frac{r}{s}}}{bnr+bns}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x, algorithm="fricas")
```

```
[Out] (b*s*e^(n*x) + a*s)*(b*e^(n*x) + a)^(r/s)/(b*n*r + b*n*s)
```

Sympy [A] time = 1.20665, size = 94, normalized size = 3.13

$$\begin{cases} \frac{x}{a} & \text{for}\: b=0\wedge n=0\wedge r=-s \\ \frac{a^{\frac{r}{s}}e^{nx}}{n} & \text{for}\: b=0 \\ x(a+b)^{\frac{r}{s}} & \text{for}\: n=0 \\ \frac{\log\left(\frac{a}{b}+e^{nx}\right)}{bn} & \text{for}\: r=-s \\ \frac{as(a+be^{nx})^{\frac{r}{s}}}{bnr+bns}+\frac{bs(a+be^{nx})^{\frac{r}{s}}e^{nx}}{bnr+bns} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*x)*(a+b*exp(n*x))**(r/s),x)
```

```
[Out] Piecewise((x/a, Eq(b, 0) & Eq(n, 0) & Eq(r, -s)), (a**(r/s)*exp(n*x)/n, Eq(
b, 0)), (x*(a + b)**(r/s), Eq(n, 0)), (log(a/b + exp(n*x))/(b*n), Eq(r, -s)
), (a*s*(a + b*exp(n*x))**(r/s)/(b*n*r + b*n*s) + b*s*(a + b*exp(n*x))**(r/
s)*exp(n*x)/(b*n*r + b*n*s), True))
```

Giac [A] time = 1.16899, size = 43, normalized size = 1.43

$$\frac{\left(be^{(nx)} + a\right)^{\frac{r}{s}+1}}{bn\left(\frac{r}{s} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(n*x)*(a+b*exp(n*x))^(r/s),x, algorithm="giac")
```

```
[Out] (b*e^(n*x) + a)^(r/s + 1)/(b*n*(r/s + 1))
```

3.528 $\int \sqrt[4]{1-2e^{x/3}}\,dx$

Optimal. Leaf size=54

$$12\sqrt[4]{1-2e^{x/3}}-6\tan^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right)-6\tanh^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right)$$

```
[Out] 12*(1 - 2*E^(x/3))^(1/4) - 6*ArcTan[(1 - 2*E^(x/3))^(1/4)] - 6*ArcTanh[(1 -
 2*E^(x/3))^(1/4)]
```

Rubi [A] time = 0.021602, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2282, 50, 63, 212, 206, 203}

$$12\sqrt[4]{1-2e^{x/3}}-6\tan^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right)-6\tanh^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*E^(x/3))^(1/4),x]
```

```
[Out] 12*(1 - 2*E^(x/3))^(1/4) - 6*ArcTan[(1 - 2*E^(x/3))^(1/4)] - 6*ArcTanh[(1 -
 2*E^(x/3))^(1/4)]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt[4]{1-2e^{x/3}}\,dx &= 3\,\text{Subst}\left(\int \frac{\sqrt[4]{1-2x}}{x}\,dx, x, e^{x/3}\right)\\
&= 12\sqrt[4]{1-2e^{x/3}} + 3\,\text{Subst}\left(\int \frac{1}{(1-2x)^{3/4}x}\,dx, x, e^{x/3}\right)\\
&= 12\sqrt[4]{1-2e^{x/3}} - 6\,\text{Subst}\left(\int \frac{1}{\frac{1}{2}-\frac{x^4}{2}}\,dx, x, \sqrt[4]{1-2e^{x/3}}\right)\\
&= 12\sqrt[4]{1-2e^{x/3}} - 6\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt[4]{1-2e^{x/3}}\right) - 6\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt[4]{1-2e^{x/3}}\right)\\
&= 12\sqrt[4]{1-2e^{x/3}} - 6\tan^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right) - 6\tanh^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0141379, size = 54, normalized size = 1.

$$12\sqrt[4]{1-2e^{x/3}} - 6\tan^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right) - 6\tanh^{-1}\left(\sqrt[4]{1-2e^{x/3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*E^(x/3))^(1/4),x]
```

```
[Out] 12*(1 - 2*E^(x/3))^(1/4) - 6*ArcTan[(1 - 2*E^(x/3))^(1/4)] - 6*ArcTanh[(1 -
 2*E^(x/3))^(1/4)]
```

Maple [A] time = 0.006, size = 57, normalized size = 1.1

$$12\sqrt[4]{1-2\,e^{x/3}} + 3\ln\left(-1+\sqrt[4]{1-2\,e^{x/3}}\right) - 3\ln\left(1+\sqrt[4]{1-2\,e^{x/3}}\right) - 6\arctan\left(\sqrt[4]{1-2\,e^{x/3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-2*exp(1/3*x))^(1/4),x)
```

```
[Out] 12*(1-2*exp(1/3*x))^(1/4)+3*ln(-1+(1-2*exp(1/3*x))^(1/4))-3*ln(1+(1-2*exp(1
/3*x))^(1/4))-6*arctan((1-2*exp(1/3*x))^(1/4))
```

Maxima [A] time = 1.42446, size = 76, normalized size = 1.41

$$12\left(-2\,e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}} - 6\arctan\left(\left(-2\,e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}\right) - 3\log\left(\left(-2\,e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}+1\right) + 3\log\left(\left(-2\,e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*exp(1/3*x))^(1/4),x, algorithm="maxima")
```

```
[Out] 12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4)) - 3*log((-
2*e^(1/3*x) + 1)^(1/4) + 1) + 3*log((-2*e^(1/3*x) + 1)^(1/4) - 1)
```

Fricas [A] time = 1.69538, size = 192, normalized size = 3.56

$$12\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}-6\arctan\left(\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}\right)-3\log\left(\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}+1\right)+3\log\left(\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*exp(1/3*x))^(1/4),x, algorithm="fricas")
```

```
[Out] 12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4)) - 3*log((-
2*e^(1/3*x) + 1)^(1/4) + 1) + 3*log((-2*e^(1/3*x) + 1)^(1/4) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[4]{1-2e^{\frac{x}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*exp(1/3*x))**(1/4),x)
```

```
[Out] Integral((1 - 2*exp(x/3))**(1/4), x)
```

Giac [A] time = 1.14576, size = 77, normalized size = 1.43

$$12\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}-6\arctan\left(\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}\right)-3\log\left(\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}+1\right)+3\log\left(\left|\left(-2e^{\left(\frac{1}{3}x\right)}+1\right)^{\frac{1}{4}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-2*exp(1/3*x))^(1/4),x, algorithm="giac")
```

```
[Out] 12*(-2*e^(1/3*x) + 1)^(1/4) - 6*arctan((-2*e^(1/3*x) + 1)^(1/4)) - 3*log((-
2*e^(1/3*x) + 1)^(1/4) + 1) + 3*log(abs((-2*e^(1/3*x) + 1)^(1/4) - 1))
```

3.529 $\int (a+be^{nx})^{r/s}\,dx$

Optimal. Leaf size=59

$$-\frac{s\left(a+be^{nx}\right)^{\frac{r+s}{s}}\,{}_2F_1\left(1,\frac{r+s}{s};\frac{r}{s}+2;\frac{e^{nx}b}{a}+1\right)}{an(r+s)}$$

```
[Out] -(((a + b*E^(n*x))^((r + s)/s)*s*Hypergeometric2F1[1, (r + s)/s, 2 + r/s, 1
 + (b*E^(n*x))/a])/(a*n*(r + s)))
```

Rubi [A] time = 0.0305085, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2282, 65}

$$-\frac{s\left(a+be^{nx}\right)^{\frac{r+s}{s}}\text{Hypergeometric2F1}\left(1,\frac{r+s}{s},\frac{r}{s}+2,\frac{be^{nx}}{a}+1\right)}{an(r+s)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*E^(n*x))^(r/s),x]
```

```
[Out] -(((a + b*E^(n*x))^((r + s)/s)*s*Hypergeometric2F1[1, (r + s)/s, 2 + r/s, 1
 + (b*E^(n*x))/a])/(a*n*(r + s)))
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int (a+be^{nx})^{r/s}\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^{r/s}}{x}\,dx,x,e^{nx}\right)}{n}\\ &= -\frac{\left(a+be^{nx}\right)^{\frac{r+s}{s}} s\,{}_2F_1\left(1,\frac{r+s}{s};2+\frac{r}{s};1+\frac{be^{nx}}{a}\right)}{an(r+s)}\end{aligned}$$

Mathematica [A] time = 0.0213791, size = 59, normalized size = 1.

$$-\frac{s\left(a+be^{nx}\right)^{\frac{r+s}{s}}\,{}_2F_1\left(1,\frac{r+s}{s};\frac{r}{s}+2;\frac{e^{nx}b}{a}+1\right)}{an(r+s)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*E^(n*x))^(r/s),x]
```

```
[Out] -(((a + b*E^(n*x))^((r + s)/s)*s*Hypergeometric2F1[1, (r + s)/s, 2 + r/s, 1
 + (b*E^(n*x))/a])/(a*n*(r + s)))
```

Maple [F] time = 0.027, size = 0, normalized size = 0.

$$\int (a + b e^{nx})^{\frac{r}{s}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*exp(n*x))^(r/s),x)
```

```
[Out] int((a+b*exp(n*x))^(r/s),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b e^{(nx)} + a\right)^{\frac{r}{s}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*exp(n*x))^(r/s),x, algorithm="maxima")
```

```
[Out] integrate((b*e^(n*x) + a)^(r/s), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b e^{(nx)} + a\right)^{\frac{r}{s}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*exp(n*x))^(r/s),x, algorithm="fricas")
```

```
[Out] integral((b*e^(n*x) + a)^(r/s), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b e^{nx})^{\frac{r}{s}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*exp(n*x))**(r/s),x)
```

```
[Out] Integral((a + b*exp(n*x))**(r/s), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(be^{(nx)} + a\right)^{\frac{r}{s}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*exp(n*x))^(r/s),x, algorithm="giac")
```

```
[Out] integrate((b*e^(n*x) + a)^(r/s), x)
```

3.530 $\int \frac{e^x}{\sqrt{a^2+e^{2x}}} dx$

Optimal. Leaf size=18

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)$$

[Out] ArcTanh[E^x/Sqrt[a^2 + E^(2*x)]]

Rubi [A] time = 0.0282844, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2249, 217, 206}

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)$$

Antiderivative was successfully verified.

[In] Int[E^x/Sqrt[a^2 + E^(2*x)],x]

[Out] ArcTanh[E^x/Sqrt[a^2 + E^(2*x)]]

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^x}{\sqrt{a^2+e^{2x}}} dx &= \text{Subst}\left(\int \frac{1}{\sqrt{a^2+x^2}} dx, x, e^x\right)\\
&= \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{e^x}{\sqrt{a^2+e^{2x}}}\right)\\
&= \tanh^{-1}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0053018, size = 18, normalized size = 1.

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{a^2+e^{2x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/Sqrt[a^2 + E^(2*x)],x]
```

```
[Out] ArcTanh[E^x/Sqrt[a^2 + E^(2*x)]]
```

Maple [A] time = 0.01, size = 15, normalized size = 0.8

$$\ln\left(e^x+\sqrt{a^2+(e^x)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(a^2+exp(2*x))^(1/2),x)
```

```
[Out] ln(exp(x)+(a^2+exp(x)^2)^(1/2))
```

Maxima [A] time = 0.930315, size = 12, normalized size = 0.67

$$\operatorname{arsinh}\left(\frac{e^x}{\sqrt{a^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(a^2+exp(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(e^x/sqrt(a^2))
```

Fricas [A] time = 1.88472, size = 45, normalized size = 2.5

$$-\log\left(\sqrt{a^2+e^{(2x)}}-e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(a^2+exp(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] -log(sqrt(a^2 + e^(2*x)) - e^x)
```

Sympy [A] time = 0.629821, size = 31, normalized size = 1.72

$$\begin{cases} \operatorname{asinh}\left(\sqrt{\frac{1}{a^2}}e^x\right) & \text{for}\ a^2>0 \\ \operatorname{acosh}\left(\sqrt{-\frac{1}{a^2}}e^x\right) & \text{for}\ a^2<0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(a**2+exp(2*x))**(1/2),x)
```

```
[Out] Piecewise((asinh(sqrt(a**(-2))*exp(x)), a**2 > 0), (acosh(sqrt(-1/a**2)*exp
(x)), a**2 < 0))
```

Giac [A] time = 1.13445, size = 24, normalized size = 1.33

$$-\log\left(\sqrt{a^2 + e^{(2x)}} - e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(a^2+exp(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] -log(sqrt(a^2 + e^(2*x)) - e^x)
```

3.531 $\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx$

Optimal. Leaf size=20

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{e^{2x}-a^2}}\right)$$

```
[Out] ArcTanh[E^x/Sqrt[-a^2 + E^(2*x)]]
```

Rubi [A] time = 0.02868, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2249, 217, 206}

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{e^{2x}-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/Sqrt[-a^2 + E^(2*x)],x]
```

```
[Out] ArcTanh[E^x/Sqrt[-a^2 + E^(2*x)]]
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Log[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^x}{\sqrt{-a^2+e^{2x}}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{-a^2+x^2}}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{e^x}{\sqrt{-a^2+e^{2x}}}\right)\\
&= \tanh^{-1}\left(\frac{e^x}{\sqrt{-a^2+e^{2x}}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0052604, size = 20, normalized size = 1.

$$\tanh^{-1}\left(\frac{e^x}{\sqrt{e^{2x}-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/Sqrt[-a^2 + E^(2*x)],x]
```

```
[Out] ArcTanh[E^x/Sqrt[-a^2 + E^(2*x)]]
```

Maple [A] time = 0.009, size = 17, normalized size = 0.9

$$\ln\left(e^x+\sqrt{-a^2+(e^x)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(-a^2+exp(2*x))^(1/2),x)
```

```
[Out] ln(exp(x)+(-a^2+exp(x)^2)^(1/2))
```

Maxima [A] time = 0.931632, size = 27, normalized size = 1.35

$$\log\left(2\sqrt{-a^2+e^{(2x)}}+2e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*sqrt(-a^2 + e^(2*x)) + 2*e^x)
```

Fricas [A] time = 1.88553, size = 46, normalized size = 2.3

$$-\log\left(\sqrt{-a^2+e^{(2x)}}-e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] -log(sqrt(-a^2 + e^(2*x)) - e^x)
```

Sympy [A] time = 0.647886, size = 31, normalized size = 1.55

$$\begin{cases} \operatorname{asinh}\left(\sqrt{-\frac{1}{a^2}}e^x\right) & \text{for } a^2<0 \\ \operatorname{acosh}\left(\sqrt{\frac{1}{a^2}}e^x\right) & \text{for } a^2>0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-a**2+exp(2*x))**(1/2),x)
```

```
[Out] Piecewise((asinh(sqrt(-1/a**2)*exp(x)), a**2 < 0), (acosh(sqrt(a**(-2))*exp
(x)), a**2 > 0))
```

Giac [A] time = 1.14326, size = 27, normalized size = 1.35

$$-\log\left(-\sqrt{-a^2+e^{(2x)}}+e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-a^2+exp(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] -log(-sqrt(-a^2 + e^(2*x)) + e^x)
```

3.532 $\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx$

Optimal. Leaf size=40

$$\frac{2}{3}\tanh^{-1}\left(\frac{2-5e^{3x/4}}{4\sqrt{e^{3x/4}+e^{3x/2}-2}}\right)$$

```
[Out] (2*ArcTanh[(2 - 5*E^((3*x)/4))/(4*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)])])/3
```

Rubi [A] time = 0.102506, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2282, 724, 206}

$$\frac{2}{3}\tanh^{-1}\left(\frac{2-5e^{3x/4}}{4\sqrt{e^{3x/4}+e^{3x/2}-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*x)/4)/((-2 + E^((3*x)/4))*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)]),x
]
```


```
[Out] (2*ArcTanh[(2 - 5*E^((3*x)/4))/(4*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)])])/3
```


Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{3x/4}}{\left(-2+e^{3x/4}\right)\sqrt{-2+e^{3x/4}+e^{3x/2}}}\,dx &= \frac{4}{3}\,\text{Subst}\left(\int \frac{1}{(-2+x)\sqrt{-2+x+x^2}}\,dx, x, e^{3x/4}\right)\\
&= -\left(\frac{8}{3}\,\text{Subst}\left(\int \frac{1}{16-x^2}\,dx, x, \frac{-2+5e^{3x/4}}{\sqrt{-2+e^{3x/4}+e^{3x/2}}}\right)\right)\\
&= \frac{2}{3}\tanh^{-1}\left(\frac{2-5e^{3x/4}}{4\sqrt{-2+e^{3x/4}+e^{3x/2}}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0196298, size = 40, normalized size = 1.

$$-\frac{2}{3}\tanh^{-1}\left(\frac{5e^{3x/4}-2}{4\sqrt{e^{3x/4}+e^{3x/2}-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*x)/4)/((-2 + E^((3*x)/4))*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/
2)]),x]
```

```
[Out] (-2*ArcTanh[(-2 + 5*E^((3*x)/4))/(4*Sqrt[-2 + E^((3*x)/4) + E^((3*x)/2)])])
/3
```

Maple [F] time = 0.019, size = 0, normalized size = 0.

$$\int \mathrm{e}^{\frac{3x}{4}}\left(-2+\mathrm{e}^{\frac{3x}{4}}\right)^{-1}\frac{1}{\sqrt{-2+\mathrm{e}^{\frac{3x}{4}}+\mathrm{e}^{\frac{3x}{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x)
```

```
[Out] int(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x)
```

Maxima [A] time = 1.41629, size = 53, normalized size = 1.32

$$-\frac{2}{3}\log\left(\frac{4\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}}{\left|e^{\left(\frac{3}{4}x\right)}-2\right|}+\frac{8}{\left|e^{\left(\frac{3}{4}x\right)}-2\right|}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, al
gorithm="maxima")
```

```
[Out] -2/3*log(4*sqrt(e^(3/2*x) + e^(3/4*x) - 2)/abs(e^(3/4*x) - 2) + 8/abs(e^(3/
4*x) - 2) + 5)
```

Fricas [A] time = 1.9534, size = 154, normalized size = 3.85

$$-\frac{2}{3}\log\left(\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}-e^{\left(\frac{3}{4}x\right)}+4\right)+\frac{2}{3}\log\left(\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}-e^{\left(\frac{3}{4}x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, al
gorithm="fricas")
```

```
[Out] -2/3*log(sqrt(e^(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x) + 4) + 2/3*log(sqrt(e^
(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\frac{3x}{4}}}{\left(e^{\frac{3x}{4}}-2\right)\sqrt{e^{\frac{3x}{4}}+e^{\frac{3x}{2}}-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))**(1/2),x)
```

```
[Out] Integral(exp(3*x/4)/((exp(3*x/4) - 2)*sqrt(exp(3*x/4) + exp(3*x/2) - 2)), x
)
```

Giac [A] time = 1.20086, size = 65, normalized size = 1.62

$$-\frac{2}{3}\log\left(\left|\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}-e^{\left(\frac{3}{4}x\right)}+4\right|\right)+\frac{2}{3}\log\left(\left|\sqrt{e^{\left(\frac{3}{2}x\right)}+e^{\left(\frac{3}{4}x\right)}-2}-e^{\left(\frac{3}{4}x\right)}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3/4*x)/(-2+exp(3/4*x))/(-2+exp(3/4*x)+exp(3/2*x))^(1/2),x, al
gorithm="giac")
```

```
[Out] -2/3*log(abs(sqrt(e^(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x) + 4)) + 2/3*log(ab
s(sqrt(e^(3/2*x) + e^(3/4*x) - 2) - e^(3/4*x)))
```

3.533 $\int e^{-2x} \left(-3+e^{7x}\right)^{2/3} dx$

Optimal. Leaf size=37

$$\frac{1}{6} e^{-2x} \left(e^{7x}-3\right)^{5/3} {}_2F_1\left(1,\frac{29}{21};\frac{5}{7};\frac{e^{7x}}{3}\right)$$

```
[Out] ((-3 + E^(7*x))^(5/3)*Hypergeometric2F1[1, 29/21, 5/7, E^(7*x)/3])/(6*E^(2*
x))
```

Rubi [A] time = 0.0517065, antiderivative size = 57, normalized size of antiderivative = 1.54, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2249, 335, 365, 364}

$$-\frac{3^{2/3} e^{-2x} \left(e^{7x}-3\right)^{2/3} \text{Hypergeometric2F1}\left(-\frac{2}{3},-\frac{2}{7},\frac{5}{7},\frac{e^{7x}}{3}\right)}{2 \left(3-e^{7x}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + E^(7*x))^(2/3)/E^(2*x),x]
```

```
[Out] -(3^(2/3)*(-3 + E^(7*x))^(2/3)*Hypergeometric2F1[-2/3, -2/7, 5/7, E^(7*x)/3
])/(2*E^(2*x)*(3 - E^(7*x))^(2/3))
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{-2x}\left(-3+e^{7x}\right)^{2/3}dx = -\text{Subst}\left(\int\left(-3+\frac{1}{x^7}\right)^{2/3}x\,dx,x,e^{-x}\right)$$
$$= \text{Subst}\left(\int\frac{\left(-3+x^7\right)^{2/3}}{x^3}dx,x,e^x\right)$$
$$= \frac{\left(-3+e^{7x}\right)^{2/3}\text{Subst}\left(\int\frac{\left(1-\frac{x^7}{3}\right)^{2/3}}{x^3}dx,x,e^x\right)}{\left(1-\frac{e^{7x}}{3}\right)^{2/3}}$$
$$= -\frac{3^{2/3}e^{-2x}\left(-3+e^{7x}\right)^{2/3}{}_2F_1\left(-\frac{2}{3},-\frac{2}{7};\frac{5}{7};\frac{e^{7x}}{3}\right)}{2\left(3-e^{7x}\right)^{2/3}}$$

Mathematica [A] time = 0.0163485, size = 54, normalized size = 1.46

$$-\frac{e^{-2x}\left(e^{7x}-3\right)^{2/3}{}_2F_1\left(-\frac{2}{3},-\frac{2}{7};\frac{5}{7};\frac{e^{7x}}{3}\right)}{2\left(1-\frac{e^{7x}}{3}\right)^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + E^(7*x))^(2/3)/E^(2*x),x]
```

```
[Out] -((-3 + E^(7*x))^(2/3)*Hypergeometric2F1[-2/3, -2/7, 5/7, E^(7*x)/3])/(2*E^
(2*x)*(1 - E^(7*x)/3)^(2/3))
```

Maple [F] time = 0.021, size = 0, normalized size = 0.

$$\int\frac{1}{e^{2x}}\left(-3+e^{7x}\right)^{\frac{2}{3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+exp(7*x))^(2/3)/exp(2*x),x)
```

```
[Out] int((-3+exp(7*x))^(2/3)/exp(2*x),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e^{(7x)}-3\right)^{\frac{2}{3}}e^{(-2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+exp(7*x))^(2/3)/exp(2*x),x, algorithm="maxima")
```

```
[Out] integrate((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^{(7x)} - 3\right)^{\frac{2}{3}} e^{(-2x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+exp(7*x))^(2/3)/exp(2*x),x, algorithm="fricas")
```

```
[Out] integral((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e^{7x} - 3\right)^{\frac{2}{3}} e^{-2x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+exp(7*x))**(2/3)/exp(2*x),x)
```

```
[Out] Integral((exp(7*x) - 3)**(2/3)*exp(-2*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(e^{(7x)} - 3\right)^{\frac{2}{3}} e^{(-2x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+exp(7*x))^(2/3)/exp(2*x),x, algorithm="giac")
```

```
[Out] integrate((e^(7*x) - 3)^(2/3)*e^(-2*x), x)
```

3.534 $\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}} \, dx$

Optimal. Leaf size=73

$$\frac{8}{13}\left(3-e^{x/2}\right)^{13/4}-8\left(3-e^{x/2}\right)^{9/4}+\frac{216}{5}\left(3-e^{x/2}\right)^{5/4}-216\sqrt[4]{3-e^{x/2}}$$

```
[Out] -216*(3 - E^(x/2))^(1/4) + (216*(3 - E^(x/2))^(5/4))/5 - 8*(3 - E^(x/2))^(9
/4) + (8*(3 - E^(x/2))^(13/4))/13
```

Rubi [A] time = 0.0474549, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2248, 43}

$$\frac{8}{13}\left(3-e^{x/2}\right)^{13/4}-8\left(3-e^{x/2}\right)^{9/4}+\frac{216}{5}\left(3-e^{x/2}\right)^{5/4}-216\sqrt[4]{3-e^{x/2}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(3 - E^(x/2))^(3/4),x]
```

```
[Out] -216*(3 - E^(x/2))^(1/4) + (216*(3 - E^(x/2))^(5/4))/5 - 8*(3 - E^(x/2))^(9
/4) + (8*(3 - E^(x/2))^(13/4))/13
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2x}}{\left(3-e^{x/2}\right)^{3/4}} \, dx &= 2\,\text{Subst}\left(\int \frac{x^3}{(3-x)^{3/4}} \, dx, x, e^{x/2}\right) \\
&= 2\,\text{Subst}\left(\int \left(\frac{27}{(3-x)^{3/4}}-27\sqrt[4]{3-x}+9(3-x)^{5/4}-(3-x)^{9/4}\right) dx, x, e^{x/2}\right) \\
&= -216\sqrt[4]{3-e^{x/2}}+\frac{216}{5}\left(3-e^{x/2}\right)^{5/4}-8\left(3-e^{x/2}\right)^{9/4}+\frac{8}{13}\left(3-e^{x/2}\right)^{13/4}
\end{aligned}$$

Mathematica [A] time = 0.0195386, size = 44, normalized size = 0.6

$$-\frac{8}{65}\sqrt[4]{3-e^{x/2}}\left(96e^{x/2}+20e^x+5e^{3x/2}+1152\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(3 - E^(x/2))^(3/4),x]
```

```
[Out] (-8*(3 - E^(x/2))^(1/4)*(1152 + 96*E^(x/2) + 20*E^x + 5*E^((3*x)/2)))/65
```

Maple [A] time = 0.019, size = 37, normalized size = 0.5

$$\frac{8}{65}\left(5\,\mathrm{e}^{3/2\,x}+20\,\mathrm{e}^{x}+96\,\mathrm{e}^{x/2}+1152\right)\left(-3+\mathrm{e}^{\frac{x}{2}}\right)\left(3-\mathrm{e}^{\frac{x}{2}}\right)^{-\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(3-exp(1/2*x))^(3/4),x)
```

```
[Out] 8/65/(3-exp(1/2*x))^(3/4)*(5*exp(3/2*x)+20*exp(x)+96*exp(1/2*x)+1152)*(-3+e
xp(1/2*x))
```

Maxima [A] time = 0.931849, size = 66, normalized size = 0.9

$$\frac{8}{13}\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{13}{4}}-8\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{9}{4}}+\frac{216}{5}\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{5}{4}}-216\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x, algorithm="maxima")
```

```
[Out] 8/13*(-e^(1/2*x) + 3)^(13/4) - 8*(-e^(1/2*x) + 3)^(9/4) + 216/5*(-e^(1/2*x)
 + 3)^(5/4) - 216*(-e^(1/2*x) + 3)^(1/4)
```

Fricas [A] time = 1.83639, size = 101, normalized size = 1.38

$$-\frac{8}{65}\left(5\,e^{\left(\frac{3}{2}\,x\right)}+96\,e^{\left(\frac{1}{2}\,x\right)}+20\,e^{x}+1152\right)\left(-e^{\left(\frac{1}{2}\,x\right)}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x, algorithm="fricas")
```

```
[Out] -8/65*(5*e^(3/2*x) + 96*e^(1/2*x) + 20*e^x + 1152)*(-e^(1/2*x) + 3)^(1/4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{e^{2x}}{\left(3-e^{\frac{x}{2}}\right)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(3-exp(1/2*x))**(3/4),x)
```

```
[Out] Integral(exp(2*x)/(3 - exp(x/2))**(3/4), x)
```

Giac [A] time = 1.14824, size = 88, normalized size = 1.21

$$-\frac{8}{13}\left(e^{\left(\frac{1}{2}x\right)}-3\right)^{3}\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{1}{4}}-8\left(e^{\left(\frac{1}{2}x\right)}-3\right)^{2}\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{1}{4}}+\frac{216}{5}\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{5}{4}}-216\left(-e^{\left(\frac{1}{2}x\right)}+3\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(3-exp(1/2*x))^(3/4),x, algorithm="giac")
```

```
[Out] -8/13*(e^(1/2*x) - 3)^3*(-e^(1/2*x) + 3)^(1/4) - 8*(e^(1/2*x) - 3)^2*(-e^(1
/2*x) + 3)^(1/4) + 216/5*(-e^(1/2*x) + 3)^(5/4) - 216*(-e^(1/2*x) + 3)^(1/4
)
```

3.535 $\int e^{-x/2}x^3\,dx$

Optimal. Leaf size=44

$$-2e^{-x/2}x^3-12e^{-x/2}x^2-48e^{-x/2}x-96e^{-x/2}$$

```
[Out] -96/E^(x/2) - (48*x)/E^(x/2) - (12*x^2)/E^(x/2) - (2*x^3)/E^(x/2)
```

Rubi [A] time = 0.0317714, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2176, 2194}

$$-2e^{-x/2}x^3-12e^{-x/2}x^2-48e^{-x/2}x-96e^{-x/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^(x/2),x]
```

```
[Out] -96/E^(x/2) - (48*x)/E^(x/2) - (12*x^2)/E^(x/2) - (2*x^3)/E^(x/2)
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-x/2}x^3\,dx&=-2e^{-x/2}x^3+6\int e^{-x/2}x^2\,dx\\&=-12e^{-x/2}x^2-2e^{-x/2}x^3+24\int e^{-x/2}x\,dx\\&=-48e^{-x/2}x-12e^{-x/2}x^2-2e^{-x/2}x^3+48\int e^{-x/2}\,dx\\&=-96e^{-x/2}-48e^{-x/2}x-12e^{-x/2}x^2-2e^{-x/2}x^3\end{aligned}$$

Mathematica [A] time = 0.0071218, size = 23, normalized size = 0.52

$$e^{-x/2}\left(-2x^3-12x^2-48x-96\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^(x/2),x]
```

```
[Out] (-96 - 48*x - 12*x^2 - 2*x^3)/E^(x/2)
```

Maple [A] time = 0.002, size = 22, normalized size = 0.5

$$-2\,\frac{x^{3}+6\,x^{2}+24\,x+48}{e^{x/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/exp(1/2*x),x)
```

```
[Out] -2*(x^3+6*x^2+24*x+48)/exp(1/2*x)
```

Maxima [A] time = 0.937732, size = 26, normalized size = 0.59

$$-2\left(x^{3}+6\,x^{2}+24\,x+48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/exp(1/2*x),x, algorithm="maxima")
```

```
[Out] -2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

Fricas [A] time = 1.75255, size = 55, normalized size = 1.25

$$-2\left(x^{3}+6\,x^{2}+24\,x+48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/exp(1/2*x),x, algorithm="fricas")
```

```
[Out] -2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

Sympy [A] time = 0.084769, size = 20, normalized size = 0.45

$$\left(-2x^{3}-12x^{2}-48x-96\right)e^{-\frac{x}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/exp(1/2*x),x)
```

```
[Out] (-2*x**3 - 12*x**2 - 48*x - 96)*exp(-x/2)
```

Giac [A] time = 1.08907, size = 26, normalized size = 0.59

$$-2\left(x^{3}+6\,x^{2}+24\,x+48\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/exp(1/2*x),x, algorithm="giac")

[Out] -2*(x^3 + 6*x^2 + 24*x + 48)*e^(-1/2*x)
```

3.536 $\int \frac{e^{-x/2}}{x^3} \, dx$

Optimal. Leaf size=39

$$\frac{1}{8}\text{ExpIntegralEi}\left(-\frac{x}{2}\right)-\frac{e^{-x/2}}{2x^2}+\frac{e^{-x/2}}{4x}$$

```
[Out] -1/(2*E^(x/2)*x^2) + 1/(4*E^(x/2)*x) + ExpIntegralEi[-x/2]/8
```

Rubi [A] time = 0.0330314, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2177, 2178}

$$\frac{1}{8}\text{ExpIntegralEi}\left(-\frac{x}{2}\right)-\frac{e^{-x/2}}{2x^2}+\frac{e^{-x/2}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^(x/2)*x^3),x]
```

```
[Out] -1/(2*E^(x/2)*x^2) + 1/(4*E^(x/2)*x) + ExpIntegralEi[-x/2]/8
```

Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !$UseGamma === True
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{-x/2}}{x^3} \, dx &= -\frac{e^{-x/2}}{2x^2}-\frac{1}{4}\int \frac{e^{-x/2}}{x^2} \, dx \\
&= -\frac{e^{-x/2}}{2x^2}+\frac{e^{-x/2}}{4x}+\frac{1}{8}\int \frac{e^{-x/2}}{x} \, dx \\
&= -\frac{e^{-x/2}}{2x^2}+\frac{e^{-x/2}}{4x}+\frac{\text{Ei}\left(-\frac{x}{2}\right)}{8}
\end{aligned}$$

Mathematica [A] time = 0.0222686, size = 26, normalized size = 0.67

$$\frac{1}{8}\left(\text{ExpIntegralEi}\left(-\frac{x}{2}\right)+\frac{2e^{-x/2}(x-2)}{x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^(x/2)*x^3),x]
```

```
[Out] ((2*(-2 + x))/(E^(x/2)*x^2) + ExpIntegralEi[-x/2])/8
```

Maple [A] time = 0.001, size = 31, normalized size = 0.8

$$-\frac{1}{2x^2}\left(e^{\frac{x}{2}}\right)^{-1}+\frac{1}{4x}\left(e^{\frac{x}{2}}\right)^{-1}-\frac{1}{8}\mathrm{Ei}\left(1,\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(1/2*x)/x^3,x)
```

```
[Out] -1/2/exp(1/2*x)/x^2+1/4/exp(1/2*x)/x-1/8*Ei(1,1/2*x)
```

Maxima [A] time = 1.02505, size = 9, normalized size = 0.23

$$-\frac{1}{4}\Gamma\left(-2,\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(1/2*x)/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*gamma(-2, 1/2*x)
```

Fricas [A] time = 1.77915, size = 66, normalized size = 1.69

$$\frac{x^2\mathrm{Ei}\left(-\frac{1}{2}x\right)+2(x-2)e^{\left(-\frac{1}{2}x\right)}}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(1/2*x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/8*(x^2*Ei(-1/2*x) + 2*(x - 2)*e^(-1/2*x))/x^2
```

Sympy [C] time = 1.44202, size = 32, normalized size = 0.82

$$\frac{\mathrm{Ei}\left(\frac{xe^{i\pi}}{2}\right)}{8}+\frac{e^{-\frac{x}{2}}}{4x}-\frac{e^{-\frac{x}{2}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(1/2*x)/x**3,x)
```

```
[Out] Ei(x*exp_polar(I*pi)/2)/8 + exp(-x/2)/(4*x) - exp(-x/2)/(2*x**2)
```

Giac [A] time = 1.11147, size = 36, normalized size = 0.92

$$\frac{x^2 \text{Ei}\left(-\frac{1}{2}x\right) + 2xe^{\left(-\frac{1}{2}x\right)} - 4e^{\left(-\frac{1}{2}x\right)}}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(1/2*x)/x^3,x, algorithm="giac")
```

```
[Out] 1/8*(x^2*Ei(-1/2*x) + 2*x*e^(-1/2*x) - 4*e^(-1/2*x))/x^2
```

3.537 $\int a^{3x}x^2\,dx$

Optimal. Leaf size=44

$$\frac{x^2a^{3x}}{3\log(a)}-\frac{2xa^{3x}}{9\log^2(a)}+\frac{2a^{3x}}{27\log^3(a)}$$

```
[Out] (2*a^(3*x))/(27*Log[a]^3) - (2*a^(3*x)*x)/(9*Log[a]^2) + (a^(3*x)*x^2)/(3*L
og[a])
```

Rubi [A] time = 0.0215737, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$\frac{x^2a^{3x}}{3\log(a)}-\frac{2xa^{3x}}{9\log^2(a)}+\frac{2a^{3x}}{27\log^3(a)}$$

Antiderivative was successfully verified.

```
[In] Int[a^(3*x)*x^2,x]
```

```
[Out] (2*a^(3*x))/(27*Log[a]^3) - (2*a^(3*x)*x)/(9*Log[a]^2) + (a^(3*x)*x^2)/(3*L
og[a])
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int a^{3x}x^2\,dx &= \frac{a^{3x}x^2}{3\log(a)}-\frac{2\int a^{3x}x\,dx}{3\log(a)}\\
&= -\frac{2a^{3x}x}{9\log^2(a)}+\frac{a^{3x}x^2}{3\log(a)}+\frac{2\int a^{3x}\,dx}{9\log^2(a)}\\
&= \frac{2a^{3x}}{27\log^3(a)}-\frac{2a^{3x}x}{9\log^2(a)}+\frac{a^{3x}x^2}{3\log(a)}
\end{aligned}$$

Mathematica [A] time = 0.010054, size = 29, normalized size = 0.66

$$\frac{a^{3x}\left(9x^2\log^2(a)-6x\log(a)+2\right)}{27\log^3(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^(3*x)*x^2,x]
```

```
[Out] (a^(3*x)*(2 - 6*x*Log[a] + 9*x^2*Log[a]^2))/(27*Log[a]^3)
```

Maple [A] time = 0.006, size = 28, normalized size = 0.6

$$\frac{\left(9\,x^2\,(\ln(a))^2-6\,x\ln(a)+2\right)a^{3\,x}}{27\,(\ln(a))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^(3*x)*x^2,x)
```

```
[Out] 1/27*(9*x^2*ln(a)^2-6*x*ln(a)+2)*a^(3*x)/ln(a)^3
```

Maxima [A] time = 0.94159, size = 36, normalized size = 0.82

$$\frac{\left(9\,x^2\log(a)^2-6\,x\log(a)+2\right)a^{3\,x}}{27\,\log(a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(3*x)*x^2,x, algorithm="maxima")
```

```
[Out] 1/27*(9*x^2*log(a)^2 - 6*x*log(a) + 2)*a^(3*x)/log(a)^3
```

Fricas [A] time = 1.90109, size = 77, normalized size = 1.75

$$\frac{\left(9\,x^2\log(a)^2-6\,x\log(a)+2\right)a^{3\,x}}{27\,\log(a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(3*x)*x^2,x, algorithm="fricas")
```

```
[Out] 1/27*(9*x^2*log(a)^2 - 6*x*log(a) + 2)*a^(3*x)/log(a)^3
```

Sympy [A] time = 0.108499, size = 39, normalized size = 0.89

$$\begin{cases} \frac{a^{3x}\left(9x^2\log(a)^2-6x\log(a)+2\right)}{27\log(a)^3} & \text{for } 27\log(a)^3 \neq 0 \\ \frac{x^3}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**(3*x)*x**2,x)
```

```
[Out] Piecewise((a**(3*x)*(9*x**2*log(a)**2 - 6*x*log(a) + 2)/(27*log(a)**3), Ne(
27*log(a)**3, 0)), (x**3/3, True))
```

Giac [C] time = 1.18254, size = 1115, normalized size = 25.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^(3*x)*x^2,x, algorithm="giac")
```

```
[Out] -1/27*((6*(3*pi*x^2*log(abs(a))*sgn(a) - 3*pi*x^2*log(abs(a)) - pi*x*sgn(a)
 + pi*x)*(pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a))
^2)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a))^2)^
2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^3)^2) -
 (9*pi^2*x^2*sgn(a) - 9*pi^2*x^2 + 18*x^2*log(abs(a))^2 - 12*x*log(abs(a))
+ 4)*(3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^3)/((p
i^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*log(abs(a))^2)^2 + (3*
pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(abs(a))^3)^2))*cos(-3/
2*pi*x*sgn(a) + 3/2*pi*x) - ((9*pi^2*x^2*sgn(a) - 9*pi^2*x^2 + 18*x^2*log(a
bs(a))^2 - 12*x*log(abs(a)) + 4)*(pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) -
 pi^3 + 3*pi*log(abs(a))^2)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^
3 + 3*pi*log(abs(a))^2)^2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a))
 + 2*log(abs(a))^3)^2) + 6*(3*pi*x^2*log(abs(a))*sgn(a) - 3*pi*x^2*log(abs(
a)) - pi*x*sgn(a) + pi*x)*(3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) +
 2*log(abs(a))^3)/((pi^3*sgn(a) - 3*pi*log(abs(a))^2*sgn(a) - pi^3 + 3*pi*l
og(abs(a))^2)^2 + (3*pi^2*log(abs(a))*sgn(a) - 3*pi^2*log(abs(a)) + 2*log(a
bs(a))^3)^2))*sin(-3/2*pi*x*sgn(a) + 3/2*pi*x))*abs(a)^(3*x) + 1/2*I*abs(a)
^(3*x)*((36*I*pi^2*x^2*sgn(a) - 72*pi*x^2*log(abs(a))*sgn(a) - 36*I*pi^2*x^
2 + 72*pi*x^2*log(abs(a)) + 72*I*x^2*log(abs(a))^2 + 24*pi*x*sgn(a) - 24*pi
*x - 48*I*x*log(abs(a)) + 16*I)*e^(3/2*I*pi*x*sgn(a) - 3/2*I*pi*x)/(-108*I*
pi^3*sgn(a) + 324*pi^2*log(abs(a))*sgn(a) + 324*I*pi*log(abs(a))^2*sgn(a) +
 108*I*pi^3 - 324*pi^2*log(abs(a)) - 324*I*pi*log(abs(a))^2 + 216*log(abs(a
))^3) - (36*I*pi^2*x^2*sgn(a) + 72*pi*x^2*log(abs(a))*sgn(a) - 36*I*pi^2*x^
2 - 72*pi*x^2*log(abs(a)) + 72*I*x^2*log(abs(a))^2 - 24*pi*x*sgn(a) + 24*pi
*x - 48*I*x*log(abs(a)) + 16*I)*e^(-3/2*I*pi*x*sgn(a) + 3/2*I*pi*x)/(108*I*
pi^3*sgn(a) + 324*pi^2*log(abs(a))*sgn(a) - 324*I*pi*log(abs(a))^2*sgn(a) -
 108*I*pi^3 - 324*pi^2*log(abs(a)) + 324*I*pi*log(abs(a))^2 + 216*log(abs(a
))^3))
```

3.538 $\int e^{x^2} x \left(1+x^2\right) dx$

Optimal. Leaf size=12

$$\frac{1}{2}e^{x^2}x^2$$

[Out] (E^x^2*x^2)/2

Rubi [A] time = 0.0487273, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2226, 2209, 2212}

$$\frac{1}{2}e^{x^2}x^2$$

Antiderivative was successfully verified.

[In] Int[E^x^2*x*(1 + x^2),x]

[Out] (E^x^2*x^2)/2

Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
, c, d, n}, x] && PolynomialQ[u, x]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}
\int e^{x^2} x \left(1+x^2\right) dx &= \int \left(e^{x^2} x + e^{x^2} x^3\right) dx \\
&= \int e^{x^2} x \, dx + \int e^{x^2} x^3 \, dx \\
&= \frac{e^{x^2}}{2} + \frac{1}{2}e^{x^2}x^2 - \int e^{x^2} x \, dx \\
&= \frac{1}{2}e^{x^2}x^2
\end{aligned}$$

Mathematica [A] time = 0.0048944, size = 12, normalized size = 1.

$$\frac{1}{2}e^{x^2}x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*x*(1 + x^2),x]
```

```
[Out] (E^x^2*x^2)/2
```

Maple [A] time = 0.002, size = 10, normalized size = 0.8

$$\frac{e^{x^2}x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*x*(x^2+1),x)
```

```
[Out] 1/2*exp(x^2)*x^2
```

Maxima [A] time = 0.928401, size = 24, normalized size = 2.

$$\frac{1}{2}\left(x^2-1\right)e^{\left(x^2\right)}+\frac{1}{2}e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x*(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*(x^2 - 1)*e^(x^2) + 1/2*e^(x^2)
```

Fricas [A] time = 1.78307, size = 23, normalized size = 1.92

$$\frac{1}{2}x^2e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x*(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*e^(x^2)
```

Sympy [A] time = 0.086483, size = 8, normalized size = 0.67

$$\frac{x^2e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x*(x**2+1),x)
```

```
[Out] x**2*exp(x**2)/2
```

Giac [A] time = 1.13533, size = 24, normalized size = 2.

$$\frac{1}{2}\left(x^2-1\right)e^{\left(x^2\right)}+\frac{1}{2}e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x*(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*(x^2 - 1)*e^(x^2) + 1/2*e^(x^2)
```

3.539 $\int \frac{x}{(e^{-x}+e^x)^2} dx$

Optimal. Leaf size=32

$$-\frac{x}{2\left(e^{2x}+1\right)}+\frac{x}{2}-\frac{1}{4}\log\left(e^{2x}+1\right)$$

```
[Out] x/2 - x/(2*(1 + E^(2*x))) - Log[1 + E^(2*x)]/4
```

Rubi [A] time = 0.0550637, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2283, 2191, 2282, 36, 29, 31}

$$-\frac{x}{2\left(e^{2x}+1\right)}+\frac{x}{2}-\frac{1}{4}\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(E^(-x) + E^x)^2,x]
```

```
[Out] x/2 - x/(2*(1 + E^(2*x))) - Log[1 + E^(2*x)]/4
```

Rule 2283

```
Int[(u_.)*((a_.)*(F_)^(v_) + (b_.)*(F_)^(w_))^(n_), x_Symbol] :> Int[u*F^(n
*v)*(a + b*F^ExpandToSum[w - v, x])^n, x] /; FreeQ[{F, a, b, n}, x] && ILtQ
[n, 0] && LinearQ[{v, w}, x]
```

Rule 2191

```
Int[((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((a_.) + (b_.)*((F_)^((g_.)*(
(e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :>
 Simp[((c + d*x)^m*(a + b*(F^(g*(e + f*x)))^n)^(p + 1))/(b*f*g*n*(p + 1)*Lo
g[F]), x] - Dist[(d*m)/(b*f*g*n*(p + 1)*Log[F]), Int[(c + d*x)^(m - 1)*(a +
 b*(F^(g*(e + f*x)))^n)^(p + 1), x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m
, n, p}, x] && NeQ[p, -1]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(e^{-x}+e^x)^2}\,dx &= \int \frac{e^{2x}x}{\left(1+e^{2x}\right)^2}\,dx\\
&= -\frac{x}{2\left(1+e^{2x}\right)}+\frac{1}{2}\int \frac{1}{1+e^{2x}}\,dx\\
&= -\frac{x}{2\left(1+e^{2x}\right)}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,e^{2x}\right)\\
&= -\frac{x}{2\left(1+e^{2x}\right)}+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,e^{2x}\right)-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,e^{2x}\right)\\
&= \frac{x}{2}-\frac{x}{2\left(1+e^{2x}\right)}-\frac{1}{4}\log\left(1+e^{2x}\right)
\end{aligned}$$

Mathematica [A] time = 0.0350698, size = 31, normalized size = 0.97

$$\frac{e^{2x}x}{2e^{2x}+2}-\frac{1}{4}\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(E^(-x) + E^x)^2,x]
```

```
[Out] (E^(2*x)*x)/(2 + 2*E^(2*x)) - Log[1 + E^(2*x)]/4
```

Maple [A] time = 0.01, size = 26, normalized size = 0.8

$$-\frac{\ln\left(\left(e^{x}\right)^{2}+1\right)}{4}+\frac{\left(e^{x}\right)^{2}x}{2\left(e^{x}\right)^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(exp(-x)+exp(x))^2,x)
```

```
[Out] -1/4*ln(exp(x)^2+1)+1/2*x*exp(x)^2/(exp(x)^2+1)
```

Maxima [A] time = 1.43541, size = 34, normalized size = 1.06

$$\frac{xe^{(2x)}}{2\left(e^{(2x)}+1\right)}-\frac{1}{4}\log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(exp(-x)+exp(x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*x*e^(2*x)/(e^(2*x) + 1) - 1/4*log(e^(2*x) + 1)
```

Fricas [A] time = 1.80123, size = 89, normalized size = 2.78

$$\frac{2\,xe^{(2\,x)} - \left(e^{(2\,x)} + 1\right)\log\left(e^{(2\,x)} + 1\right)}{4\left(e^{(2\,x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(exp(-x)+exp(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*x*e^(2*x) - (e^(2*x) + 1)*log(e^(2*x) + 1))/(e^(2*x) + 1)
```

Sympy [A] time = 0.098359, size = 24, normalized size = 0.75

$$-\frac{x}{2} + \frac{x}{2 + 2e^{-2x}} - \frac{\log\left(1 + e^{-2x}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(exp(-x)+exp(x))**2,x)
```

```
[Out] -x/2 + x/(2 + 2*exp(-2*x)) - log(1 + exp(-2*x))/4
```

Giac [A] time = 1.1543, size = 54, normalized size = 1.69

$$\frac{2\,xe^{(2\,x)} - e^{(2\,x)}\log\left(e^{(2\,x)} + 1\right) - \log\left(e^{(2\,x)} + 1\right)}{4\left(e^{(2\,x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(exp(-x)+exp(x))^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*x*e^(2*x) - e^(2*x)*log(e^(2*x) + 1) - log(e^(2*x) + 1))/(e^(2*x) +
1)
```

3.540 $\int \frac{e^x(1-x-x^2)}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=15

$$e^x\sqrt{1-x^2}$$

```
[Out] E^x*Sqrt[1 - x^2]
```

Rubi [A] time = 0.0569821, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {2288}

$$e^x\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 - x - x^2))/Sqrt[1 - x^2],x]
```

```
[Out] E^x*Sqrt[1 - x^2]
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\int \frac{e^x\left(1-x-x^2\right)}{\sqrt{1-x^2}}\,dx = e^x\sqrt{1-x^2}$$

Mathematica [A] time = 0.0257325, size = 15, normalized size = 1.

$$e^x\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 - x - x^2))/Sqrt[1 - x^2],x]
```

```
[Out] E^x*Sqrt[1 - x^2]
```

Maple [A] time = 0.005, size = 20, normalized size = 1.3

$$-e^x(1+x)(-1+x)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x)
```

```
[Out] -exp(x)*(1+x)*(-1+x)/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.09179, size = 28, normalized size = 1.87

$$-\frac{(x^2-1)e^x}{\sqrt{x+1}\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(x^2 - 1)*e^x/(sqrt(x + 1)*sqrt(-x + 1))
```

Fricas [A] time = 2.07677, size = 27, normalized size = 1.8

$$\sqrt{-x^2+1}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(-x^2 + 1)*e^x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{e^x}{\sqrt{1-x^2}}\,dx - \int \frac{xe^x}{\sqrt{1-x^2}}\,dx - \int \frac{x^2e^x}{\sqrt{1-x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(-x**2-x+1)/(-x**2+1)**(1/2),x)
```

```
[Out] -Integral(-exp(x)/sqrt(1 - x**2), x) - Integral(x*exp(x)/sqrt(1 - x**2), x)
 - Integral(x**2*exp(x)/sqrt(1 - x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(x^2+x-1)e^x}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(-x^2-x+1)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + x - 1)*e^x/sqrt(-x^2 + 1), x)
```

3.541 $\int e^{-3x}\cos(2x)\,dx$

Optimal. Leaf size=27

$$\frac{2}{13}e^{-3x}\sin(2x)-\frac{3}{13}e^{-3x}\cos(2x)$$

```
[Out] (-3*Cos[2*x])/(13*E^(3*x)) + (2*Sin[2*x])/(13*E^(3*x))
```

Rubi [A] time = 0.0105735, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4433}

$$\frac{2}{13}e^{-3x}\sin(2x)-\frac{3}{13}e^{-3x}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]/E^(3*x),x]
```

```
[Out] (-3*Cos[2*x])/(13*E^(3*x)) + (2*Sin[2*x])/(13*E^(3*x))
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-3x}\cos(2x)\,dx=-\frac{3}{13}e^{-3x}\cos(2x)+\frac{2}{13}e^{-3x}\sin(2x)$$

Mathematica [A] time = 0.0263124, size = 22, normalized size = 0.81

$$\frac{1}{13}e^{-3x}(2\sin(2x)-3\cos(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]/E^(3*x),x]
```

```
[Out] (-3*Cos[2*x] + 2*Sin[2*x])/(13*E^(3*x))
```

Maple [A] time = 0.007, size = 22, normalized size = 0.8

$$-\frac{3\,\mathrm{e}^{-3\,x}\cos(2\,x)}{13}+\frac{2\,\mathrm{e}^{-3\,x}\sin(2\,x)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)/exp(3*x),x)
```

```
[Out] -3/13*exp(-3*x)*cos(2*x)+2/13*exp(-3*x)*sin(2*x)
```

Maxima [A] time = 0.946883, size = 26, normalized size = 0.96

$$-\frac{1}{13}\left(3\cos\left(2x\right)-2\sin\left(2x\right)\right)e^{\left(-3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/exp(3*x),x, algorithm="maxima")
```

```
[Out] -1/13*(3*cos(2*x) - 2*sin(2*x))*e^(-3*x)
```

Fricas [A] time = 2.19539, size = 68, normalized size = 2.52

$$-\frac{3}{13}\cos\left(2x\right)e^{\left(-3x\right)}+\frac{2}{13}e^{\left(-3x\right)}\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/exp(3*x),x, algorithm="fricas")
```

```
[Out] -3/13*cos(2*x)*e^(-3*x) + 2/13*e^(-3*x)*sin(2*x)
```

Sympy [A] time = 0.477693, size = 26, normalized size = 0.96

$$\frac{2e^{-3x}\sin\left(2x\right)}{13}-\frac{3e^{-3x}\cos\left(2x\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/exp(3*x),x)
```

```
[Out] 2*exp(-3*x)*sin(2*x)/13 - 3*exp(-3*x)*cos(2*x)/13
```

Giac [A] time = 1.10919, size = 26, normalized size = 0.96

$$-\frac{1}{13}\left(3\cos\left(2x\right)-2\sin\left(2x\right)\right)e^{\left(-3x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)/exp(3*x),x, algorithm="giac")
```

```
[Out] -1/13*(3*cos(2*x) - 2*sin(2*x))*e^(-3*x)
```

3.542 $\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}} \, dx$

Optimal. Leaf size=35

$$\frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}} - \frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

```
[Out] (-30*Cos[x/2])/(13*(E^x)^(1/3)) + (6*Sin[x/2])/(13*(E^x)^(1/3))
```

Rubi [A] time = 0.111145, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {2281, 6742, 4433, 4432}

$$\frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}} - \frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x/2] + Sin[x/2])/(E^x)^(1/3),x]
```

```
[Out] (-30*Cos[x/2])/(13*(E^x)^(1/3)) + (6*Sin[x/2])/(13*(E^x)^(1/3))
```

Rule 2281

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] && !IntegerQ[n]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int \frac{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}{\sqrt[3]{e^x}}\,dx = \frac{e^{x/3}\int e^{-x/3}\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)dx}{\sqrt[3]{e^x}}$$
$$= \frac{\left(6e^{x/3}\right)\text{Subst}\left(\int e^{-2x}(\cos(3x)+\sin(3x))\,dx,x,\frac{x}{6}\right)}{\sqrt[3]{e^x}}$$
$$= \frac{\left(6e^{x/3}\right)\text{Subst}\left(\int \left(e^{-2x}\cos(3x)+e^{-2x}\sin(3x)\right)dx,x,\frac{x}{6}\right)}{\sqrt[3]{e^x}}$$
$$= \frac{\left(6e^{x/3}\right)\text{Subst}\left(\int e^{-2x}\cos(3x)\,dx,x,\frac{x}{6}\right)}{\sqrt[3]{e^x}} + \frac{\left(6e^{x/3}\right)\text{Subst}\left(\int e^{-2x}\sin(3x)\,dx,x,\frac{x}{6}\right)}{\sqrt[3]{e^x}}$$
$$= -\frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}} + \frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

Mathematica [A] time = 0.0585615, size = 26, normalized size = 0.74

$$\frac{6\left(\sin\left(\frac{x}{2}\right)-5\cos\left(\frac{x}{2}\right)\right)}{13\sqrt[3]{e^x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x/2] + Sin[x/2])/(E^x)^(1/3),x]
```

```
[Out] (6*(-5*Cos[x/2] + Sin[x/2]))/(13*(E^x)^(1/3))
```

Maple [A] time = 0.014, size = 22, normalized size = 0.6

$$-\frac{30}{13}e^{-\frac{x}{3}}\cos\left(\frac{x}{2}\right)+\frac{6}{13}e^{-\frac{x}{3}}\sin\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x)
```

```
[Out] -30/13*exp(-1/3*x)*cos(1/2*x)+6/13*exp(-1/3*x)*sin(1/2*x)
```

Maxima [A] time = 0.929995, size = 53, normalized size = 1.51

$$-\frac{6}{13}\left(3\cos\left(\frac{1}{2}x\right)+2\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)}-\frac{6}{13}\left(2\cos\left(\frac{1}{2}x\right)-3\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x, algorithm="maxima")
```

```
[Out] -6/13*(3*cos(1/2*x) + 2*sin(1/2*x))*e^(-1/3*x) - 6/13*(2*cos(1/2*x) - 3*sin
(1/2*x))*e^(-1/3*x)
```

Fricas [A] time = 2.17471, size = 80, normalized size = 2.29

$$-\frac{30}{13}\cos\left(\frac{1}{2}x\right)e^{\left(-\frac{1}{3}x\right)}+\frac{6}{13}e^{\left(-\frac{1}{3}x\right)}\sin\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x, algorithm="fricas")
```

```
[Out] -30/13*cos(1/2*x)*e^(-1/3*x) + 6/13*e^(-1/3*x)*sin(1/2*x)
```

Sympy [A] time = 0.829124, size = 29, normalized size = 0.83

$$\frac{6\sin\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}-\frac{30\cos\left(\frac{x}{2}\right)}{13\sqrt[3]{e^x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(1/2*x)+sin(1/2*x))/exp(x)**(1/3),x)
```

```
[Out] 6*sin(x/2)/(13*exp(x)**(1/3)) - 30*cos(x/2)/(13*exp(x)**(1/3))
```

Giac [A] time = 1.0828, size = 53, normalized size = 1.51

$$-\frac{6}{13}\left(3\cos\left(\frac{1}{2}x\right)+2\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)}-\frac{6}{13}\left(2\cos\left(\frac{1}{2}x\right)-3\sin\left(\frac{1}{2}x\right)\right)e^{\left(-\frac{1}{3}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(1/2*x)+sin(1/2*x))/exp(x)^(1/3),x, algorithm="giac")
```

```
[Out] -6/13*(3*cos(1/2*x) + 2*sin(1/2*x))*e^(-1/3*x) - 6/13*(2*cos(1/2*x) - 3*sin
(1/2*x))*e^(-1/3*x)
```

3.543 $\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx$

Optimal. Leaf size=57

$$\frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}-\frac{4\log(3)\cos\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}$$

```
[Out] (-4*Cos[(3*x)/2]*Log[3])/(3*(3^(3*x))^(1/4)*(4 + Log[3]^2)) + (8*Sin[(3*x)/
2])/(3*(3^(3*x))^(1/4)*(4 + Log[3]^2))
```

Rubi [A] time = 0.0284353, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2281, 4433}

$$\frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}-\frac{4\log(3)\cos\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[(3*x)/2]/(3^(3*x))^(1/4),x]
```

```
[Out] (-4*Cos[(3*x)/2]*Log[3])/(3*(3^(3*x))^(1/4)*(4 + Log[3]^2)) + (8*Sin[(3*x)/
2])/(3*(3^(3*x))^(1/4)*(4 + Log[3]^2))
```

Rule 2281

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos\left(\frac{3x}{2}\right)}{\sqrt[4]{3^{3x}}}\,dx &= \frac{3^{3x/4}\int 3^{-3x/4}\cos\left(\frac{3x}{2}\right)\,dx}{\sqrt[4]{3^{3x}}}\\ &= -\frac{4\cos\left(\frac{3x}{2}\right)\log(3)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}+\frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\left(4+\log^2(3)\right)}\end{aligned}$$

Mathematica [A] time = 0.066293, size = 37, normalized size = 0.65

$$-\frac{4\left(\log(3)\cos\left(\frac{3x}{2}\right)-2\sin\left(\frac{3x}{2}\right)\right)}{3\sqrt[4]{27^x}\left(4+\log^2(3)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[(3*x)/2]/(3^(3*x))^(1/4),x]
```

```
[Out] (-4*(Cos[(3*x)/2]*Log[3] - 2*Sin[(3*x)/2]))/(3*(27^x)^(1/4)*(4 + Log[3]^2))
```

Maple [C] time = 0.05, size = 55, normalized size = 1.

$$-\frac{-4\,i\mathrm{e}^{-\frac{3i}{2}x}+2\,\mathrm{e}^{-3/2\,ix}\ln(3)+4\,i\mathrm{e}^{\frac{3i}{2}x}+2\,\ln(3)\,\mathrm{e}^{3/2\,ix}}{(6\,i+3\,\ln(3))\,(-2\,i+\ln(3))}\frac{1}{\sqrt[4]{27^{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3/2*x)/(3^(3*x))^(1/4),x)
```

```
[Out] -2/3/(2*I+ln(3))/(-2*I+ln(3))/(27^x)^(1/4)*(-2*I*exp(-3/2*I*x)+exp(-3/2*I*x
)*ln(3)+2*I*exp(3/2*I*x)+ln(3)*exp(3/2*I*x))
```

Maxima [A] time = 1.41806, size = 42, normalized size = 0.74

$$-\frac{4\left(\cos\left(\frac{3}{2}\,x\right)\log(3)-2\,\sin\left(\frac{3}{2}\,x\right)\right)}{3\left(\log(3)^{2}+4\right)3^{\frac{3}{4}\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="maxima")
```

```
[Out] -4/3*(cos(3/2*x)*log(3) - 2*sin(3/2*x))/((log(3)^2 + 4)*3^(3/4*x))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 3.06312, size = 70, normalized size = 1.23

$$\frac{8\sin\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\log(3)^{2}+12\sqrt[4]{3^{3x}}}-\frac{4\log(3)\cos\left(\frac{3x}{2}\right)}{3\sqrt[4]{3^{3x}}\log(3)^{2}+12\sqrt[4]{3^{3x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3/2*x)/(3**(3*x))**(1/4),x)
```

```
[Out] 8*sin(3*x/2)/(3*(3**(3*x))**(1/4)*log(3)**2 + 12*(3**(3*x))**(1/4)) - 4*log
(3)*cos(3*x/2)/(3*(3**(3*x))**(1/4)*log(3)**2 + 12*(3**(3*x))**(1/4))
```

Giac [A] time = 1.13985, size = 53, normalized size = 0.93

$$-\frac{4\left(\frac{\cos\left(\frac{3}{2}x\right)\log(3)}{\log(3)^2+4}-\frac{2\sin\left(\frac{3}{2}x\right)}{\log(3)^2+4}\right)}{3\cdot 3^{\frac{3}{4}x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3/2*x)/(3^(3*x))^(1/4),x, algorithm="giac")
```

```
[Out] -4/3*(cos(3/2*x)*log(3)/(log(3)^2 + 4) - 2*sin(3/2*x)/(log(3)^2 + 4))/3^(3/
4*x)
```

3.544 $\int e^{mx} \cos^2(x)\, dx$

Optimal. Leaf size=54

$$\frac{2e^{mx}}{m\left(m^2+4\right)}+\frac{me^{mx}\cos^2(x)}{m^2+4}+\frac{2e^{mx}\sin(x)\cos(x)}{m^2+4}$$

```
[Out] (2*E^(m*x))/(m*(4 + m^2)) + (E^(m*x)*m*Cos[x]^2)/(4 + m^2) + (2*E^(m*x)*Cos
[x]*Sin[x])/(4 + m^2)
```

Rubi [A] time = 0.0230822, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4435, 2194}

$$\frac{2e^{mx}}{m\left(m^2+4\right)}+\frac{me^{mx}\cos^2(x)}{m^2+4}+\frac{2e^{mx}\sin(x)\cos(x)}{m^2+4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(m*x)*Cos[x]^2,x]
```

```
[Out] (2*E^(m*x))/(m*(4 + m^2)) + (E^(m*x)*m*Cos[x]^2)/(4 + m^2) + (2*E^(m*x)*Cos
[x]*Sin[x])/(4 + m^2)
```

Rule 4435

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x]^m)/(e^2*m^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[(e*m*F^(c*(a + b*x))*Sin[d
+ e*x]*Cos[d + e*x]^(m - 1))/(e^2*m^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{mx}\cos^2(x)\,dx &= \frac{e^{mx}m\cos^2(x)}{4+m^2}+\frac{2e^{mx}\cos(x)\sin(x)}{4+m^2}+\frac{2\int e^{mx}\,dx}{4+m^2}\\ &= \frac{2e^{mx}}{m\left(4+m^2\right)}+\frac{e^{mx}m\cos^2(x)}{4+m^2}+\frac{2e^{mx}\cos(x)\sin(x)}{4+m^2}\end{aligned}$$

Mathematica [A] time = 0.0351539, size = 39, normalized size = 0.72

$$\frac{e^{mx}\left(m^2\cos(2x)+m^2+2m\sin(2x)+4\right)}{2m\left(m^2+4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(m*x)*Cos[x]^2,x]

[Out] (E^(m*x)*(4 + m^2 + m^2*Cos[2*x] + 2*m*Sin[2*x]))/(2*m*(4 + m^2))
```

Maple [A] time = 0.02, size = 45, normalized size = 0.8

$$\frac{e^{mx}}{2m}+\frac{m e^{mx}\cos(2x)}{2m^2+8}+\frac{e^{mx}\sin(2x)}{m^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)*cos(x)^2,x)

[Out] 1/2*exp(m*x)/m+1/2*m/(m^2+4)*exp(m*x)*cos(2*x)+1/(m^2+4)*exp(m*x)*sin(2*x)
```

Maxima [A] time = 0.942968, size = 61, normalized size = 1.13

$$\frac{m^2\cos(2x)e^{(mx)}+2me^{(mx)}\sin(2x)+\left(m^2+4\right)e^{(mx)}}{2\left(m^3+4m\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*cos(x)^2,x, algorithm="maxima")

[Out] 1/2*(m^2*cos(2*x)*e^(m*x) + 2*m*e^(m*x)*sin(2*x) + (m^2 + 4)*e^(m*x))/(m^3
+ 4*m)
```

Fricas [A] time = 2.08634, size = 95, normalized size = 1.76

$$\frac{2m\cos(x)e^{(mx)}\sin(x)+\left(m^2\cos(x)^2+2\right)e^{(mx)}}{m^3+4m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*cos(x)^2,x, algorithm="fricas")

[Out] (2*m*cos(x)*e^(m*x)*sin(x) + (m^2*cos(x)^2 + 2)*e^(m*x))/(m^3 + 4*m)
```

Sympy [A] time = 3.42764, size = 265, normalized size = 4.91

$$\begin{cases} \frac{x\sin^2(x)}{2}+\frac{x\cos^2(x)}{2}+\frac{\sin(x)\cos(x)}{2} & \text{for } m=0 \\ -\frac{xe^{-2ix}\sin^2(x)}{4}+\frac{ixe^{-2ix}\sin(x)\cos(x)}{2}+\frac{xe^{-2ix}\cos^2(x)}{4}-\frac{e^{-2ix}\sin(x)\cos(x)}{4}+\frac{ie^{-2ix}\cos^2(x)}{2} & \text{for } m=-2i \\ -\frac{xe^{2ix}\sin^2(x)}{4}-\frac{ixe^{2ix}\sin(x)\cos(x)}{2}+\frac{xe^{2ix}\cos^2(x)}{4}-\frac{e^{2ix}\sin(x)\cos(x)}{4}-\frac{ie^{2ix}\cos^2(x)}{2} & \text{for } m=2i \\ \frac{m^2e^{mx}\cos^2(x)}{m^3+4m}+\frac{2me^{mx}\sin(x)\cos(x)}{m^3+4m}+\frac{2e^{mx}\sin^2(x)}{m^3+4m}+\frac{2e^{mx}\cos^2(x)}{m^3+4m} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*cos(x)**2,x)
```

```
[Out] Piecewise((x*sin(x)**2/2 + x*cos(x)**2/2 + sin(x)*cos(x)/2, Eq(m, 0)), (-x*
exp(-2*I*x)*sin(x)**2/4 + I*x*exp(-2*I*x)*sin(x)*cos(x)/2 + x*exp(-2*I*x)*c
os(x)**2/4 - exp(-2*I*x)*sin(x)*cos(x)/4 + I*exp(-2*I*x)*cos(x)**2/2, Eq(m,
 -2*I)), (-x*exp(2*I*x)*sin(x)**2/4 - I*x*exp(2*I*x)*sin(x)*cos(x)/2 + x*ex
p(2*I*x)*cos(x)**2/4 - exp(2*I*x)*sin(x)*cos(x)/4 - I*exp(2*I*x)*cos(x)**2/
2, Eq(m, 2*I)), (m**2*exp(m*x)*cos(x)**2/(m**3 + 4*m) + 2*m*exp(m*x)*sin(x)
*cos(x)/(m**3 + 4*m) + 2*exp(m*x)*sin(x)**2/(m**3 + 4*m) + 2*exp(m*x)*cos(x
)**2/(m**3 + 4*m), True))
```

Giac [A] time = 1.09468, size = 58, normalized size = 1.07

$$\frac{1}{2}\left(\frac{m\cos(2x)}{m^2+4}+\frac{2\sin(2x)}{m^2+4}\right)e^{(mx)}+\frac{e^{(mx)}}{2m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(m*cos(2*x)/(m^2 + 4) + 2*sin(2*x)/(m^2 + 4))*e^(m*x) + 1/2*e^(m*x)/m
```

3.545 $\int e^{mx} \sin^3(x)\,dx$

Optimal. Leaf size=82

$$\frac{me^{mx}\sin^3(x)}{m^2+9}+\frac{6me^{mx}\sin(x)}{m^4+10m^2+9}-\frac{6e^{mx}\cos(x)}{m^4+10m^2+9}-\frac{3e^{mx}\sin^2(x)\cos(x)}{m^2+9}$$

```
[Out] (-6*E^(m*x)*Cos[x])/(9 + 10*m^2 + m^4) + (6*E^(m*x)*m*Sin[x])/(9 + 10*m^2 +
 m^4) - (3*E^(m*x)*Cos[x]*Sin[x]^2)/(9 + m^2) + (E^(m*x)*m*Sin[x]^3)/(9 + m
^2)
```

Rubi [A] time = 0.0353972, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4434, 4432}

$$\frac{me^{mx}\sin^3(x)}{m^2+9}+\frac{6me^{mx}\sin(x)}{m^4+10m^2+9}-\frac{6e^{mx}\cos(x)}{m^4+10m^2+9}-\frac{3e^{mx}\sin^2(x)\cos(x)}{m^2+9}$$

Antiderivative was successfully verified.

```
[In] Int[E^(m*x)*Sin[x]^3,x]
```


```
[Out] (-6*E^(m*x)*Cos[x])/(9 + 10*m^2 + m^4) + (6*E^(m*x)*m*Sin[x])/(9 + 10*m^2 +
 m^4) - (3*E^(m*x)*Cos[x]*Sin[x]^2)/(9 + m^2) + (E^(m*x)*m*Sin[x]^3)/(9 + m
^2)
```


Rule 4434

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x]^n)/(e^2*n^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[(e*n*F^(c*(a + b*x))*Cos[d
+ e*x]*Sin[d + e*x]^(n - 1))/(e^2*n^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int e^{mx}\sin^3(x)\,dx &= -\frac{3e^{mx}\cos(x)\sin^2(x)}{9+m^2}+\frac{e^{mx}m\sin^3(x)}{9+m^2}+\frac{6\int e^{mx}\sin(x)\,dx}{9+m^2}\\ &= -\frac{6e^{mx}\cos(x)}{9+10m^2+m^4}+\frac{6e^{mx}m\sin(x)}{9+10m^2+m^4}-\frac{3e^{mx}\cos(x)\sin^2(x)}{9+m^2}+\frac{e^{mx}m\sin^3(x)}{9+m^2}\end{aligned}$$

Mathematica [A] time = 0.166801, size = 64, normalized size = 0.78

$$\frac{e^{mx}\left(-3\left(m^2+9\right)\cos(x)+3\left(m^2+1\right)\cos(3x)-2m\sin(x)\left(\left(m^2+1\right)\cos(2x)-m^2-13\right)\right)}{4\left(m^4+10m^2+9\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(m*x)*Sin[x]^3,x]
```

```
[Out] (E^(m*x)*(-3*(9 + m^2)*Cos[x] + 3*(1 + m^2)*Cos[3*x] - 2*m*(-13 - m^2 + (1
+ m^2)*Cos[2*x])*Sin[x]))/(4*(9 + 10*m^2 + m^4))
```

Maple [A] time = 0.023, size = 68, normalized size = 0.8

$$-\frac{3\,\mathrm{e}^{mx}\cos(x)}{4\,m^2+4}+\frac{3\,m\mathrm{e}^{mx}\sin(x)}{4\,m^2+4}+\frac{3\,\mathrm{e}^{mx}\cos(3\,x)}{4\,m^2+36}-\frac{m\mathrm{e}^{mx}\sin(3\,x)}{4\,m^2+36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)*sin(x)^3,x)
```

```
[Out] -3/4/(m^2+1)*exp(m*x)*cos(x)+3/4*m/(m^2+1)*exp(m*x)*sin(x)+3/4/(m^2+9)*exp(
m*x)*cos(3*x)-1/4*m/(m^2+9)*exp(m*x)*sin(3*x)
```

Maxima [A] time = 0.976098, size = 99, normalized size = 1.21

$$\frac{3\left(m^2+1\right)\cos(3\,x)\,e^{(mx)}-3\left(m^2+9\right)\cos(x)\,e^{(mx)}-\left(m^3+m\right)e^{(mx)}\sin(3\,x)+3\left(m^3+9\,m\right)e^{(mx)}\sin(x)}{4\left(m^4+10\,m^2+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(3*(m^2 + 1)*cos(3*x)*e^(m*x) - 3*(m^2 + 9)*cos(x)*e^(m*x) - (m^3 + m)*
e^(m*x)*sin(3*x) + 3*(m^3 + 9*m)*e^(m*x)*sin(x))/(m^4 + 10*m^2 + 9)
```

Fricas [A] time = 1.89389, size = 165, normalized size = 2.01

$$\frac{\left(m^3-\left(m^3+m\right)\cos(x)^2+7\,m\right)e^{(mx)}\sin(x)+3\left(\left(m^2+1\right)\cos(x)^3-\left(m^2+3\right)\cos(x)\right)e^{(mx)}}{m^4+10\,m^2+9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*sin(x)^3,x, algorithm="fricas")
```

```
[Out] ((m^3 - (m^3 + m)*cos(x)^2 + 7*m)*e^(m*x)*sin(x) + 3*((m^2 + 1)*cos(x)^3 -
(m^2 + 3)*cos(x))*e^(m*x))/(m^4 + 10*m^2 + 9)
```

Sympy [A] time = 14.6775, size = 644, normalized size = 7.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*sin(x)**3,x)
```

```
[Out] Piecewise((x*exp(-3*I*x)*sin(x)**3/8 - 3*I*x*exp(-3*I*x)*sin(x)**2*cos(x)/8
 - 3*x*exp(-3*I*x)*sin(x)*cos(x)**2/8 + I*x*exp(-3*I*x)*cos(x)**3/8 - 7*exp
(-3*I*x)*sin(x)**2*cos(x)/8 + 9*I*exp(-3*I*x)*sin(x)*cos(x)**2/8 + 5*exp(-3
*I*x)*cos(x)**3/12, Eq(m, -3*I)), (3*x*exp(-I*x)*sin(x)**3/8 - 3*I*x*exp(-I
*x)*sin(x)**2*cos(x)/8 + 3*x*exp(-I*x)*sin(x)*cos(x)**2/8 - 3*I*x*exp(-I*x)
*cos(x)**3/8 - 5*exp(-I*x)*sin(x)**2*cos(x)/8 + I*exp(-I*x)*sin(x)*cos(x)**
2/8 - exp(-I*x)*cos(x)**3/4, Eq(m, -I)), (3*x*exp(I*x)*sin(x)**3/8 + 3*I*x*
exp(I*x)*sin(x)**2*cos(x)/8 + 3*x*exp(I*x)*sin(x)*cos(x)**2/8 + 3*I*x*exp(I
*x)*cos(x)**3/8 - 5*exp(I*x)*sin(x)**2*cos(x)/8 - I*exp(I*x)*sin(x)*cos(x)*
*2/8 - exp(I*x)*cos(x)**3/4, Eq(m, I)), (x*exp(3*I*x)*sin(x)**3/8 + 3*I*x*e
xp(3*I*x)*sin(x)**2*cos(x)/8 - 3*x*exp(3*I*x)*sin(x)*cos(x)**2/8 - I*x*exp(
3*I*x)*cos(x)**3/8 - 7*exp(3*I*x)*sin(x)**2*cos(x)/8 - 9*I*exp(3*I*x)*sin(x
)*cos(x)**2/8 + 5*exp(3*I*x)*cos(x)**3/12, Eq(m, 3*I)), (m**3*exp(m*x)*sin(
x)**3/(m**4 + 10*m**2 + 9) - 3*m**2*exp(m*x)*sin(x)**2*cos(x)/(m**4 + 10*m*
*2 + 9) + 7*m*exp(m*x)*sin(x)**3/(m**4 + 10*m**2 + 9) + 6*m*exp(m*x)*sin(x)
*cos(x)**2/(m**4 + 10*m**2 + 9) - 9*exp(m*x)*sin(x)**2*cos(x)/(m**4 + 10*m*
*2 + 9) - 6*exp(m*x)*cos(x)**3/(m**4 + 10*m**2 + 9), True))
```

Giac [A] time = 1.10856, size = 85, normalized size = 1.04

$$-\frac{1}{4}\left(\frac{m\sin(3x)}{m^2+9}-\frac{3\cos(3x)}{m^2+9}\right)e^{(mx)}+\frac{3}{4}\left(\frac{m\sin(x)}{m^2+1}-\frac{\cos(x)}{m^2+1}\right)e^{(mx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*sin(x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(m*sin(3*x)/(m^2 + 9) - 3*cos(3*x)/(m^2 + 9))*e^(m*x) + 3/4*(m*sin(x)/
(m^2 + 1) - cos(x)/(m^2 + 1))*e^(m*x)
```

3.546 $\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx$

Optimal. Leaf size=79

$$\frac{32\sin\left(\frac{x}{3}\right)}{65\sqrt{e^x}}-\frac{2\cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}}-\frac{48\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}}+\frac{4\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)}{5\sqrt{e^x}}$$

```
[Out] (-48*Cos[x/3])/(65*Sqrt[E^x]) - (2*Cos[x/3]^3)/(5*Sqrt[E^x]) + (32*Sin[x/3]
)/(65*Sqrt[E^x]) + (4*Cos[x/3]^2*Sin[x/3])/(5*Sqrt[E^x])
```

Rubi [A] time = 0.0449323, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2281, 4435, 4433}

$$\frac{32\sin\left(\frac{x}{3}\right)}{65\sqrt{e^x}}-\frac{2\cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}}-\frac{48\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}}+\frac{4\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)}{5\sqrt{e^x}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x/3]^3/Sqrt[E^x],x]
```

```
[Out] (-48*Cos[x/3])/(65*Sqrt[E^x]) - (2*Cos[x/3]^3)/(5*Sqrt[E^x]) + (32*Sin[x/3]
)/(65*Sqrt[E^x]) + (4*Cos[x/3]^2*Sin[x/3])/(5*Sqrt[E^x])
```

Rule 2281

```
Int[(u_.)*((a_.)*(F_)^(v_))^(n_), x_Symbol] :> Dist[(a*F^v)^n/F^(n*v), Int[
u*F^(n*v), x], x] /; FreeQ[{F, a, n}, x] &&  !IntegerQ[n]
```

Rule 4435

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x]^m)/(e^2*m^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[(e*m*F^(c*(a + b*x))*Sin[d
+ e*x]*Cos[d + e*x]^(m - 1))/(e^2*m^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int \frac{\cos^3\left(\frac{x}{3}\right)}{\sqrt{e^x}}\,dx = \frac{e^{x/2}\int e^{-x/2}\cos^3\left(\frac{x}{3}\right)\,dx}{\sqrt{e^x}}$$
$$= -\frac{2\cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}} + \frac{4\cos^2\left(\frac{x}{3}\right)\sin\left(\frac{x}{3}\right)}{5\sqrt{e^x}} + \frac{\left(8e^{x/2}\right)\int e^{-x/2}\cos\left(\frac{x}{3}\right)\,dx}{15\sqrt{e^x}}$$
$$= -\frac{48\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}} - \frac{2\cos^3\left(\frac{x}{3}\right)}{5\sqrt{e^x}} + \frac{32\sin\left(\frac{x}{3}\right)}{65\sqrt{e^x}} + \frac{4\cos^2\left(\frac{x}{3}\right)\sin\left(\frac{x}{3}\right)}{5\sqrt{e^x}}$$

Mathematica [A] time = 0.0380714, size = 36, normalized size = 0.46

$$\frac{90\sin\left(\frac{x}{3}\right) + 26\sin(x) - 135\cos\left(\frac{x}{3}\right) - 13\cos(x)}{130\sqrt{e^x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x/3]^3/Sqrt[E^x],x]
```

```
[Out] (-135*Cos[x/3] - 13*Cos[x] + 90*Sin[x/3] + 26*Sin[x])/(130*Sqrt[E^x])
```

Maple [A] time = 0.039, size = 38, normalized size = 0.5

$$-\frac{\cos(x)}{10}e^{-\frac{x}{2}} + \frac{\sin(x)}{5}e^{-\frac{x}{2}} - \frac{27}{26}e^{-\frac{x}{2}}\cos\left(\frac{x}{3}\right) + \frac{9}{13}e^{-\frac{x}{2}}\sin\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(1/3*x)^3/exp(x)^(1/2),x)
```

```
[Out] -1/10*exp(-1/2*x)*cos(x)+1/5*exp(-1/2*x)*sin(x)-27/26*exp(-1/2*x)*cos(1/3*x
)+9/13*exp(-1/2*x)*sin(1/3*x)
```

Maxima [A] time = 0.950952, size = 36, normalized size = 0.46

$$-\frac{1}{130}\left(135\cos\left(\frac{1}{3}x\right) + 13\cos(x) - 90\sin\left(\frac{1}{3}x\right) - 26\sin(x)\right)e^{\left(-\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/3*x)^3/exp(x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/130*(135*cos(1/3*x) + 13*cos(x) - 90*sin(1/3*x) - 26*sin(x))*e^(-1/2*x)
```

Fricas [A] time = 1.90172, size = 138, normalized size = 1.75

$$\frac{4}{65}\left(13\cos\left(\frac{1}{3}x\right)^2 + 8\right)e^{\left(-\frac{1}{2}x\right)}\sin\left(\frac{1}{3}x\right) - \frac{2}{65}\left(13\cos\left(\frac{1}{3}x\right)^3 + 24\cos\left(\frac{1}{3}x\right)\right)e^{\left(-\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/3*x)^3/exp(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/65*(13*cos(1/3*x)^2 + 8)*e^(-1/2*x)*sin(1/3*x) - 2/65*(13*cos(1/3*x)^3 +
24*cos(1/3*x))*e^(-1/2*x)
```

Sympy [A] time = 2.64452, size = 76, normalized size = 0.96

$$\frac{32\sin^3\left(\frac{x}{3}\right)}{65\sqrt{e^x}} - \frac{48\sin^2\left(\frac{x}{3}\right)\cos\left(\frac{x}{3}\right)}{65\sqrt{e^x}} + \frac{84\sin\left(\frac{x}{3}\right)\cos^2\left(\frac{x}{3}\right)}{65\sqrt{e^x}} - \frac{74\cos^3\left(\frac{x}{3}\right)}{65\sqrt{e^x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/3*x)**3/exp(x)**(1/2),x)
```

```
[Out] 32*sin(x/3)**3/(65*sqrt(exp(x))) - 48*sin(x/3)**2*cos(x/3)/(65*sqrt(exp(x))
) + 84*sin(x/3)*cos(x/3)**2/(65*sqrt(exp(x))) - 74*cos(x/3)**3/(65*sqrt(exp
(x)))
```

Giac [A] time = 1.11083, size = 45, normalized size = 0.57

$$-\frac{9}{26}\left(3\cos\left(\frac{1}{3}x\right) - 2\sin\left(\frac{1}{3}x\right)\right)e^{\left(-\frac{1}{2}x\right)} - \frac{1}{10}\left(\cos\left(x\right) - 2\sin\left(x\right)\right)e^{\left(-\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(1/3*x)^3/exp(x)^(1/2),x, algorithm="giac")
```

```
[Out] -9/26*(3*cos(1/3*x) - 2*sin(1/3*x))*e^(-1/2*x) - 1/10*(cos(x) - 2*sin(x))*e
^(-1/2*x)
```

3.547 $\int e^{2x}\cos^2(x)\sin^2(x)\,dx$

Optimal. Leaf size=36

$$\frac{e^{2x}}{16}-\frac{1}{40}e^{2x}\sin(4x)-\frac{1}{80}e^{2x}\cos(4x)$$

```
[Out] E^(2*x)/16 - (E^(2*x)*Cos[4*x])/80 - (E^(2*x)*Sin[4*x])/40
```

Rubi [A] time = 0.0437049, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4469, 2194, 4433}

$$\frac{e^{2x}}{16}-\frac{1}{40}e^{2x}\sin(4x)-\frac{1}{80}e^{2x}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*Cos[x]^2*Sin[x]^2,x]
```

```
[Out] E^(2*x)/16 - (E^(2*x)*Cos[4*x])/80 - (E^(2*x)*Sin[4*x])/40
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int e^{2x}\cos^2(x)\sin^2(x)\,dx&=\int\left(\frac{e^{2x}}{8}-\frac{1}{8}e^{2x}\cos(4x)\right)dx\\&=\frac{1}{8}\int e^{2x}\,dx-\frac{1}{8}\int e^{2x}\cos(4x)\,dx\\&=\frac{e^{2x}}{16}-\frac{1}{80}e^{2x}\cos(4x)-\frac{1}{40}e^{2x}\sin(4x)\end{aligned}$$

Mathematica [A] time = 0.0324941, size = 21, normalized size = 0.58

$$-\frac{1}{80}e^{2x}(2\sin(4x)+\cos(4x)-5)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*Cos[x]^2*Sin[x]^2,x]
```

```
[Out] -(E^(2*x)*(-5 + Cos[4*x] + 2*Sin[4*x]))/80
```

Maple [A] time = 0.007, size = 28, normalized size = 0.8

$$-\frac{e^{2x}\cos(4x)}{80}-\frac{e^{2x}\sin(4x)}{40}+\frac{(e^{x})^{2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*cos(x)^2*sin(x)^2,x)
```

```
[Out] -1/80*exp(2*x)*cos(4*x)-1/40*exp(2*x)*sin(4*x)+1/16*exp(x)^2
```

Maxima [A] time = 0.961003, size = 36, normalized size = 1.

$$-\frac{1}{80}\cos(4x)e^{(2x)}-\frac{1}{40}e^{(2x)}\sin(4x)+\frac{1}{16}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/80*cos(4*x)*e^(2*x) - 1/40*e^(2*x)*sin(4*x) + 1/16*e^(2*x)
```

Fricas [A] time = 1.86622, size = 120, normalized size = 3.33

$$-\frac{1}{10}\left(2\cos(x)^{3}-\cos(x)\right)e^{(2x)}\sin(x)-\frac{1}{20}\left(2\cos(x)^{4}-2\cos(x)^{2}-1\right)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/10*(2*cos(x)^3 - cos(x))*e^(2*x)*sin(x) - 1/20*(2*cos(x)^4 - 2*cos(x)^2
 - 1)*e^(2*x)
```

Sympy [B] time = 5.17987, size = 70, normalized size = 1.94

$$\frac{e^{2x}\sin^{4}(x)}{20}+\frac{e^{2x}\sin^{3}(x)\cos(x)}{10}+\frac{e^{2x}\sin^{2}(x)\cos^{2}(x)}{5}-\frac{e^{2x}\sin(x)\cos^{3}(x)}{10}+\frac{e^{2x}\cos^{4}(x)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(x)**2*sin(x)**2,x)
```

```
[Out] exp(2*x)*sin(x)**4/20 + exp(2*x)*sin(x)**3*cos(x)/10 + exp(2*x)*sin(x)**2*c
os(x)**2/5 - exp(2*x)*sin(x)*cos(x)**3/10 + exp(2*x)*cos(x)**4/20
```

Giac [A] time = 1.12767, size = 32, normalized size = 0.89

$$-\frac{1}{80}\left(\cos\left(4x\right)+2\sin\left(4x\right)\right)e^{\left(2x\right)}+\frac{1}{16}e^{\left(2x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*cos(x)^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/80*(cos(4*x) + 2*sin(4*x))*e^(2*x) + 1/16*e^(2*x)
```

3.548 $\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx$

Optimal. Leaf size=36

$$\frac{e^{3x}}{24}-\frac{1}{60}e^{3x}\sin(6x)-\frac{1}{120}e^{3x}\cos(6x)$$

```
[Out] E^(3*x)/24 - (E^(3*x)*Cos[6*x])/120 - (E^(3*x)*Sin[6*x])/60
```

Rubi [A] time = 0.0433467, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4469, 2194, 4433}

$$\frac{e^{3x}}{24}-\frac{1}{60}e^{3x}\sin(6x)-\frac{1}{120}e^{3x}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*x)*Cos[(3*x)/2]^2*Sin[(3*x)/2]^2,x]
```

```
[Out] E^(3*x)/24 - (E^(3*x)*Cos[6*x])/120 - (E^(3*x)*Sin[6*x])/60
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int e^{3x}\cos^2\left(\frac{3x}{2}\right)\sin^2\left(\frac{3x}{2}\right)\,dx &= \int\left(\frac{e^{3x}}{8}-\frac{1}{8}e^{3x}\cos(6x)\right)dx\\ &=\frac{1}{8}\int e^{3x}\,dx-\frac{1}{8}\int e^{3x}\cos(6x)\,dx\\ &=\frac{e^{3x}}{24}-\frac{1}{120}e^{3x}\cos(6x)-\frac{1}{60}e^{3x}\sin(6x)\end{aligned}$$

Mathematica [A] time = 0.0378083, size = 21, normalized size = 0.58

$$-\frac{1}{120}e^{3x}(2\sin(6x)+\cos(6x)-5)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*x)*Cos[(3*x)/2]^2*Sin[(3*x)/2]^2,x]
```

```
[Out] -(E^(3*x)*(-5 + Cos[6*x] + 2*Sin[6*x]))/120
```

Maple [B] time = 0.017, size = 63, normalized size = 1.8

$$-\frac{(12\cos(x)+24\sin(x))\,e^{3x}(\cos(x))^5}{45}+\frac{(6\cos(x)+8\sin(x))\,e^{3x}(\cos(x))^3}{15}-\frac{(3\cos(x)+2\sin(x))\,e^{3x}\cos(x)}{20}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x)
```

```
[Out] -4/45*(3*cos(x)+6*sin(x))*exp(3*x)*cos(x)^5+2/15*(3*cos(x)+4*sin(x))*exp(3*
x)*cos(x)^3-1/20*(3*cos(x)+2*sin(x))*exp(3*x)*cos(x)+1/20*exp(x)^3
```

Maxima [A] time = 0.946844, size = 36, normalized size = 1.

$$-\frac{1}{120}\cos(6x)\,e^{(3x)}-\frac{1}{60}\,e^{(3x)}\sin(6x)+\frac{1}{24}\,e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x, algorithm="maxima")
```

```
[Out] -1/120*cos(6*x)*e^(3*x) - 1/60*e^(3*x)*sin(6*x) + 1/24*e^(3*x)
```

Fricas [A] time = 1.919, size = 147, normalized size = 4.08

$$-\frac{1}{15}\left(2\cos\left(\frac{3}{2}x\right)^3-\cos\left(\frac{3}{2}x\right)\right)e^{(3x)}\sin\left(\frac{3}{2}x\right)-\frac{1}{30}\left(2\cos\left(\frac{3}{2}x\right)^4-2\cos\left(\frac{3}{2}x\right)^2-1\right)e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(3/2*x)^3 - cos(3/2*x))*e^(3*x)*sin(3/2*x) - 1/30*(2*cos(3/2*x)
^4 - 2*cos(3/2*x)^2 - 1)*e^(3*x)
```

Sympy [B] time = 5.62001, size = 99, normalized size = 2.75

$$\frac{e^{3x}\sin^4\left(\frac{3x}{2}\right)}{30}+\frac{e^{3x}\sin^3\left(\frac{3x}{2}\right)\cos\left(\frac{3x}{2}\right)}{15}+\frac{2e^{3x}\sin^2\left(\frac{3x}{2}\right)\cos^2\left(\frac{3x}{2}\right)}{15}-\frac{e^{3x}\sin\left(\frac{3x}{2}\right)\cos^3\left(\frac{3x}{2}\right)}{15}+\frac{e^{3x}\cos^4\left(\frac{3x}{2}\right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*cos(3/2*x)**2*sin(3/2*x)**2,x)
```

[Out] exp(3*x)*sin(3*x/2)**4/30 + exp(3*x)*sin(3*x/2)**3*cos(3*x/2)/15 + 2*exp(3*x)*sin(3*x/2)**2*cos(3*x/2)**2/15 - exp(3*x)*sin(3*x/2)*cos(3*x/2)**3/15 + exp(3*x)*cos(3*x/2)**4/30

Giac [A] time = 1.12675, size = 32, normalized size = 0.89

$$-\frac{1}{120}\left(\cos\left(6\,x\right)+2\,\sin\left(6\,x\right)\right)e^{\left(3\,x\right)}+\frac{1}{24}\,e^{\left(3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(3*x)*cos(3/2*x)^2*sin(3/2*x)^2,x, algorithm="giac")

[Out] -1/120*(cos(6*x) + 2*sin(6*x))*e^(3*x) + 1/24*e^(3*x)

3.549 $\int e^{mx} \tan^2(x)\,dx$

Optimal. Leaf size=58

$$-\frac{e^{mx}}{m}+\frac{4e^{(m+2i)x}\,{}_2F_1\left(2,1-\frac{im}{2};2-\frac{im}{2};-e^{2ix}\right)}{m+2i}$$

```
[Out] -(E^(m*x)/m) + (4*E^((2*I + m)*x)*Hypergeometric2F1[2, 1 - (I/2)*m, 2 - (I/
2)*m, -E^((2*I)*x)])/(2*I + m)
```

Rubi [A] time = 0.0774187, antiderivative size = 85, normalized size of antiderivative = 1.47, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4442, 2194, 2251}

$$\frac{4e^{mx}\text{Hypergeometric2F1}\left(1,-\frac{im}{2},1-\frac{im}{2},-e^{2ix}\right)}{m}-\frac{4e^{mx}\text{Hypergeometric2F1}\left(2,-\frac{im}{2},1-\frac{im}{2},-e^{2ix}\right)}{m}-\frac{e^{mx}}{m}$$

Antiderivative was successfully verified.

```
[In] Int[E^(m*x)*Tan[x]^2,x]
```

```
[Out] -(E^(m*x)/m) + (4*E^(m*x)*Hypergeometric2F1[1, (-I/2)*m, 1 - (I/2)*m, -E^((
2*I)*x)])/m - (4*E^(m*x)*Hypergeometric2F1[2, (-I/2)*m, 1 - (I/2)*m, -E^((2
*I)*x)])/m
```

Rule 4442

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 - E^(2*I*(d + e*x)
))^n)/(1 + E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int e^{mx}\tan^2(x)\,dx = -\int \left(e^{mx} + \frac{4e^{mx}}{\left(1+e^{2ix}\right)^2} - \frac{4e^{mx}}{1+e^{2ix}}\right)dx$$
$$= -\left(4\int \frac{e^{mx}}{\left(1+e^{2ix}\right)^2}\,dx\right) + 4\int \frac{e^{mx}}{1+e^{2ix}}\,dx - \int e^{mx}\,dx$$
$$= -\frac{e^{mx}}{m} + \frac{4e^{mx}\,{}_2F_1\left(1,-\frac{im}{2};1-\frac{im}{2};-e^{2ix}\right)}{m} - \frac{4e^{mx}\,{}_2F_1\left(2,-\frac{im}{2};1-\frac{im}{2};-e^{2ix}\right)}{m}$$

Mathematica [A] time = 0.236516, size = 97, normalized size = 1.67

$$\frac{e^{mx}\left(\frac{im^2e^{2ix}\,{}_2F_1\left(1,1-\frac{im}{2};2-\frac{im}{2};-e^{2ix}\right)}{m+2i} - im\,{}_2F_1\left(1,-\frac{im}{2};1-\frac{im}{2};-e^{2ix}\right) + m\tan(x) - 1\right)}{m}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(m*x)*Tan[x]^2,x]
```

```
[Out] (E^(m*x)*(-1 + (I*E^((2*I)*x)*m^2*Hypergeometric2F1[1, 1 - (I/2)*m, 2 - (I/
2)*m, -E^((2*I)*x)])/(2*I + m) - I*m*Hypergeometric2F1[1, (-I/2)*m, 1 - (I/
2)*m, -E^((2*I)*x)] + m*Tan[x]))/m
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int e^{mx}\left(\tan(x)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)*tan(x)^2,x)
```

```
[Out] int(exp(m*x)*tan(x)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(e^{(mx)}\tan(x)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] integral(e^(m*x)*tan(x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{mx} \tan^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*tan(x)**2,x)
```

```
[Out] Integral(exp(m*x)*tan(x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{(mx)} \tan(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)*tan(x)^2,x, algorithm="giac")
```

```
[Out] integrate(e^(m*x)*tan(x)^2, x)
```

3.550 $\int e^{mx}\csc^2(x)\,dx$

Optimal. Leaf size=45

$$-\frac{4e^{(m+2i)x}\,{}_2F_1\left(2,1-\frac{im}{2};2-\frac{im}{2};e^{2ix}\right)}{m+2i}$$

```
[Out] (-4*E^((2*I + m)*x)*Hypergeometric2F1[2, 1 - (I/2)*m, 2 - (I/2)*m, E^((2*I)
*x)])/(2*I + m)
```

Rubi [A] time = 0.0180852, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4453}

$$-\frac{4e^{(m+2i)x}\text{Hypergeometric2F1}\left(2,1-\frac{im}{2},2-\frac{im}{2},e^{2ix}\right)}{m+2i}$$

Antiderivative was successfully verified.

```
[In] Int[E^(m*x)*Csc[x]^2,x]
```

```
[Out] (-4*E^((2*I + m)*x)*Hypergeometric2F1[2, 1 - (I/2)*m, 2 - (I/2)*m, E^((2*I)
*x)])/(2*I + m)
```

Rule 4453

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F]
)/(2*e), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rubi steps

$$\int e^{mx}\csc^2(x)\,dx=-\frac{4e^{(2i+m)x}\,{}_2F_1\left(2,1-\frac{im}{2};2-\frac{im}{2};e^{2ix}\right)}{2i+m}$$

Mathematica [A] time = 0.174203, size = 90, normalized size = 2.

$$\frac{e^{mx}\left(me^{2ix}\,{}_2F_1\left(1,1-\frac{im}{2};2-\frac{im}{2};e^{2ix}\right)+(m+2i)\left({}_2F_1\left(1,-\frac{im}{2};1-\frac{im}{2};e^{2ix}\right)-i\cot(x)\right)\right)}{-2+im}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(m*x)*Csc[x]^2,x]
```

```
[Out] (E^(m*x)*(E^((2*I)*x)*m*Hypergeometric2F1[1, 1 - (I/2)*m, 2 - (I/2)*m, E^((
2*I)*x)] + (2*I + m)*((-I)*Cot[x] + Hypergeometric2F1[1, (-I/2)*m, 1 - (I/2
)*m, E^((2*I)*x)])))/(-2 + I*m)
```

Maple [F] time = 0.083, size = 0, normalized size = 0.

$$\int \frac{\mathrm{e}^{mx}}{\left(\sin \left(x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)/sin(x)^2,x)
```

```
[Out] int(exp(m*x)/sin(x)^2,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/sin(x)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{e^{(mx)}}{\cos\left(x\right)^{2}-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/sin(x)^2,x, algorithm="fricas")
```

```
[Out] integral(-e^(m*x)/(cos(x)^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{mx}}{\sin^{2}\left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/sin(x)**2,x)
```

```
[Out] Integral(exp(m*x)/sin(x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(mx)}}{\sin\left(x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/sin(x)^2,x, algorithm="giac")
```

```
[Out] integrate(e^(m*x)/sin(x)^2, x)
```

3.551 $\int e^{mx} \sec^3(x) \, dx$

Optimal. Leaf size=51

$$\frac{8e^{(m+3i)x} {}_2F_1\left(3, \frac{1}{2}(3-im); \frac{1}{2}(5-im); -e^{2ix}\right)}{m+3i}$$

```
[Out] (8*E^((3*I + m)*x)*Hypergeometric2F1[3, (3 - I*m)/2, (5 - I*m)/2, -E^((2*I)
*x)])/(3*I + m)
```

Rubi [A] time = 0.0363634, antiderivative size = 77, normalized size of antiderivative = 1.51, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4448, 4451}

$$(-m+i)\left(-e^{(m+i)x}\right) \text{Hypergeometric2F1}\left(1, \frac{1}{2}(1-im), \frac{1}{2}(3-im), -e^{2ix}\right) - \frac{1}{2}me^{mx}\sec(x) + \frac{1}{2}e^{mx}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(m*x)*Sec[x]^3,x]
```

```
[Out] -(E^((I + m)*x)*(I - m)*Hypergeometric2F1[1, (1 - I*m)/2, (3 - I*m)/2, -E^(
(2*I)*x)]) - (E^(m*x)*m*Sec[x])/2 + (E^(m*x)*Sec[x]*Tan[x])/2
```

Rule 4448

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> -Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sec[d + e*x]^(n - 2))/(e^2*(n - 1)*
(n - 2)), x] + (Dist[(e^2*(n - 2)^2 + b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n - 2
)), Int[F^(c*(a + b*x))*Sec[d + e*x]^(n - 2), x], x] + Simp[(F^(c*(a + b*x)
)*Sec[d + e*x]^(n - 1)*Sin[d + e*x])/(e*(n - 1)), x]) /; FreeQ[{F, a, b, c,
 d, e}, x] && NeQ[b^2*c^2*Log[F]^2 + e^2*(n - 2)^2, 0] && GtQ[n, 1] && NeQ[
n, 2]
```

Rule 4451

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[(2^n*E^(I*n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F])/(2*e), -E^(2*I*(d + e*x))
])/(I*e*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int e^{mx}\sec^3(x)\,dx &= -\frac{1}{2}e^{mx}m\sec(x) + \frac{1}{2}e^{mx}\sec(x)\tan(x) + \frac{1}{2}\left(1+m^2\right)\int e^{mx}\sec(x)\,dx\\ &= -e^{(i+m)x}(i-m)\,{}_2F_1\left(1, \frac{1}{2}(1-im); \frac{1}{2}(3-im); -e^{2ix}\right) - \frac{1}{2}e^{mx}m\sec(x) + \frac{1}{2}e^{mx}\sec(x)\tan(x)\end{aligned}$$

Mathematica [A] time = 0.0470558, size = 66, normalized size = 1.29

$$\frac{1}{2}e^{mx}\left(\sec(x)(\tan(x)-m) + 2(m-i)e^{ix}\,{}_2F_1\left(1, \frac{1}{2}-\frac{im}{2}; \frac{3}{2}-\frac{im}{2}; -e^{2ix}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(m*x)*Sec[x]^3,x]
```

```
[Out] (E^(m*x)*(2*E^(I*x)*(-I + m)*Hypergeometric2F1[1, 1/2 - (I/2)*m, 3/2 - (I/2
)*m, -E^((2*I)*x)] + Sec[x]*(-m + Tan[x])))/2
```

Maple [F] time = 0.113, size = 0, normalized size = 0.

$$\int \frac{e^{mx}}{(\cos(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)/cos(x)^3,x)
```

```
[Out] int(exp(m*x)/cos(x)^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/cos(x)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^{(mx)}}{\cos(x)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/cos(x)^3,x, algorithm="fricas")
```

```
[Out] integral(e^(m*x)/cos(x)^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{mx}}{\cos^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/cos(x)**3,x)
```

```
[Out] Integral(exp(m*x)/cos(x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(mx)}}{\cos(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/cos(x)^3,x, algorithm="giac")
```

```
[Out] integrate(e^(m*x)/cos(x)^3, x)
```

3.552 $\int \frac{e^x}{1+\cos(x)}\,dx$

Optimal. Leaf size=28

$$(1-i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;-e^{ix}\right)$$

```
[Out] (1 - I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(I*x)]
```

Rubi [A] time = 0.0312807, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4457, 4451}

$$(1-i)e^{(1+i)x}\text{Hypergeometric2F1}\left(1-i,2,2-i,-e^{ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 + Cos[x]),x]
```

```
[Out] (1 - I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(I*x)]
```

Rule 4457

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 + (e*x)/2]^
(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f - g, 0] && ILt
Q[n, 0]
```

Rule 4451

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Simp[(2^n*E^(I*n*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[n, n/2
 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F])/(2*e), -E^(2*I*(d + e*x))
])/(I*e*n + b*c*Log[F]), x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{1+\cos(x)}\,dx &= \frac{1}{2}\int e^x \sec^2\left(\frac{x}{2}\right)\,dx\\ &= (1-i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;-e^{ix}\right)\end{aligned}$$

Mathematica [A] time = 0.0097364, size = 28, normalized size = 1.

$$(1-i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;-e^{ix}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 + Cos[x]),x]
```

```
[Out] (1 - I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(I*x)]
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \frac{e^{x}}{\cos (x)+1}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(cos(x)+1),x)
```

```
[Out] int(exp(x)/(cos(x)+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(\left(\cos (x)^{2}+\sin (x)^{2}+2 \cos (x)+1\right)\int \frac{e^{x}\sin(x)}{\cos(x)^{2}+\sin(x)^{2}+2\cos(x)+1}dx-e^{x}\sin (x)\right)}{\cos (x)^{2}+\sin (x)^{2}+2 \cos (x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+cos(x)),x, algorithm="maxima")
```

```
[Out] -2*((cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 + s
in(x)^2 + 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x) +
 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^{x}}{\cos (x)+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] integral(e^x/(cos(x) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{x}}{\cos (x)+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+cos(x)),x)
```

```
[Out] Integral(exp(x)/(cos(x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{x}}{\cos (x)+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+cos(x)),x, algorithm="giac")
```

```
[Out] integrate(e^x/(cos(x) + 1), x)
```

3.553 $\int \frac{e^x}{1-\cos(x)}\,dx$

Optimal. Leaf size=26

$$(-1+i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;e^{ix}\right)$$

```
[Out] (-1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, E^(I*x)]
```

Rubi [A] time = 0.0321949, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4458, 4453}

$$(-1+i)e^{(1+i)x}\text{Hypergeometric2F1}\left(1-i,2,2-i,e^{ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 - Cos[x]),x]
```

```
[Out] (-1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, E^(I*x)]
```

Rule 4458

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_))), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Sin[d/2 + (e*x)/2]^
(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f + g, 0] && ILt
Q[n, 0]
```

Rule 4453

```
Int[Csc[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-2*I)^n*E^(I*n*(d + e*x))*(F^(c*(a + b*x))/(I*e*n + b*c*Log[F]
))*Hypergeometric2F1[n, n/2 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F]
)/(2*e), E^(2*I*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ
[n]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{1-\cos(x)}\,dx &= \frac{1}{2}\int e^x\csc^2\left(\frac{x}{2}\right)dx\\ &= (-1+i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;e^{ix}\right)\end{aligned}$$

Mathematica [B] time = 0.07335, size = 84, normalized size = 3.23

$$\frac{(1+i)e^x\sin\left(\frac{x}{2}\right)\left((1+i)\,{}_2F_1\left(-i,1;1-i;e^{ix}\right)\sin\left(\frac{x}{2}\right)+e^{ix}\,{}_2F_1\left(1,1-i;2-i;e^{ix}\right)\sin\left(\frac{x}{2}\right)+(1-i)\cos\left(\frac{x}{2}\right)\right)}{\cos(x)-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 - Cos[x]),x]
```

```
[Out] ((1 + I)*E^x*Sin[x/2]*((1 - I)*Cos[x/2] + (1 + I)*Hypergeometric2F1[-I, 1,
1 - I, E^(I*x)]*Sin[x/2] + E^(I*x)*Hypergeometric2F1[1, 1 - I, 2 - I, E^(I*
x)]*Sin[x/2]))/(-1 + Cos[x])
```

Maple [F] time = 0.035, size = 0, normalized size = 0.

$$\int \frac{e^x}{1-\cos (x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(1-cos(x)),x)
```

```
[Out] int(exp(x)/(1-cos(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(\left(\cos (x)^2+\sin (x)^2-2\,\cos (x)+1\right)\int \frac{e^x\sin (x)}{\cos (x)^2+\sin (x)^2-2\,\cos (x)+1}\,dx-e^x\sin (x)\right)}{\cos (x)^2+\sin (x)^2-2\,\cos (x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-cos(x)),x, algorithm="maxima")
```

```
[Out] 2*((cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 - 2*cos(x) +
1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{e^x}{\cos (x)-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] integral(-e^x/(cos(x) - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e^x}{\cos (x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-cos(x)),x)
```

```
[Out] -Integral(exp(x)/(cos(x) - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{e^x}{\cos(x) - 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-cos(x)),x, algorithm="giac")
```

```
[Out] integrate(-e^x/(cos(x) - 1), x)
```

3.554 $\int \frac{e^x}{1+\sin(x)}\,dx$

Optimal. Leaf size=30

$$(-1+i)e^{(1-i)x}\,{}_2F_1\left(1+i,2;2+i;-ie^{-ix}\right)$$

```
[Out] (-1 + I)*E^((1 - I)*x)*Hypergeometric2F1[1 + I, 2, 2 + I, (-I)/E^(I*x)]
```

Rubi [A] time = 0.0342418, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4456, 4450}

$$(-1+i)e^{(1-i)x}\text{Hypergeometric2F1}\left(1+i,2,2+i,-ie^{-ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 + Sin[x]),x]
```

```
[Out] (-1 + I)*E^((1 - I)*x)*Hypergeometric2F1[1 + I, 2, 2 + I, (-I)/E^(I*x)]
```

Rule 4456

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 + (e*x)/2 -
 (f*Pi)/(4*g)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f
^2 - g^2, 0] && ILtQ[n, 0]
```

Rule 4450

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n
_.), x_Symbol] :> Simp[(2^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*F^(c*(a + b*x))*
Hypergeometric2F1[n, n/2 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F])/(
2*e), -(E^(2*I*k*Pi)*E^(2*I*(d + e*x)))])/(I*e*n + b*c*Log[F]), x] /; FreeQ
[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{1+\sin(x)}\,dx &= \frac{1}{2}\int e^x\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)dx\\ &= (-1+i)e^{(1-i)x}\,{}_2F_1\left(1+i,2;2+i;-ie^{-ix}\right)\end{aligned}$$

Mathematica [B] time = 0.572315, size = 61, normalized size = 2.03

$$\frac{2e^x\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}-(1-i)(\sinh(x)+\cosh(x))(1-(1-i)\,{}_2F_1(-i,1;1-i;i\cos(x)-\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 + Sin[x]),x]
```

```
[Out] (2*E^x*Sin[x/2])/(Cos[x/2] + Sin[x/2]) - (1 - I)*(1 - (1 - I)*Hypergeometri
c2F1[-I, 1, 1 - I, I*Cos[x] - Sin[x]])*(Cosh[x] + Sinh[x])
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \frac{e^{x}}{1+\sin\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(1+sin(x)),x)
```

```
[Out] int(exp(x)/(1+sin(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(\cos\left(x\right)e^{x}-\left(\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\,\sin\left(x\right)+1\right)\int\frac{\cos(x)e^{x}}{\cos(x)^{2}+\sin(x)^{2}+2\,\sin(x)+1}\,dx\right)}{\cos\left(x\right)^{2}+\sin\left(x\right)^{2}+2\,\sin\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+sin(x)),x, algorithm="maxima")
```

```
[Out] -2*(cos(x)*e^x - (cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate(cos(x)*e^x/
(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 + 2*sin(x) +
 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{e^{x}}{\sin\left(x\right)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+sin(x)),x, algorithm="fricas")
```

```
[Out] integral(e^x/(sin(x) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{x}}{\sin\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+sin(x)),x)
```

```
[Out] Integral(exp(x)/(sin(x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^x}{\sin(x) + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+sin(x)),x, algorithm="giac")
```

```
[Out] integrate(e^x/(sin(x) + 1), x)
```

3.555 $\int \frac{e^x}{1-\sin(x)}\,dx$

Optimal. Leaf size=30

$$(1+i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;-ie^{ix}\right)$$

```
[Out] (1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, (-I)*E^(I*x)]
```

Rubi [A] time = 0.0340965, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4456, 4450}

$$(1+i)e^{(1+i)x}\text{Hypergeometric2F1}\left(1-i,2,2-i,-ie^{ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 - Sin[x]),x]
```

```
[Out] (1 + I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, (-I)*E^(I*x)]
```

Rule 4456

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]
)^(n_.), x_Symbol] :> Dist[2^n*f^n, Int[F^(c*(a + b*x))*Cos[d/2 + (e*x)/2 -
 (f*Pi)/(4*g)]^(2*n), x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] && EqQ[f
^2 - g^2, 0] && ILtQ[n, 0]
```

Rule 4450

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sec[(d_.) + Pi*(k_.) + (e_.)*(x_)]^(n
_.), x_Symbol] :> Simp[(2^n*E^(I*k*n*Pi)*E^(I*n*(d + e*x))*F^(c*(a + b*x))*
Hypergeometric2F1[n, n/2 - (I*b*c*Log[F])/(2*e), 1 + n/2 - (I*b*c*Log[F])/(
2*e), -(E^(2*I*k*Pi)*E^(2*I*(d + e*x)))])/(I*e*n + b*c*Log[F]), x] /; FreeQ
[{F, a, b, c, d, e}, x] && IntegerQ[4*k] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{1-\sin(x)}\,dx &= \frac{1}{2}\int e^x\sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)dx\\ &= (1+i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;-ie^{ix}\right)\end{aligned}$$

Mathematica [B] time = 0.62303, size = 61, normalized size = 2.03

$$\frac{2e^x\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}+(1+i)(\sinh(x)+\cosh(x))(1-(1+i)\,{}_2F_1(-i,1;1-i;\sin(x)-i\cos(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 - Sin[x]),x]
```

```
[Out] (2*E^x*Sin[x/2])/(Cos[x/2] - Sin[x/2]) + (1 + I)*(1 - (1 + I)*Hypergeometri
c2F1[-I, 1, 1 - I, (-I)*Cos[x] + Sin[x]])*(Cosh[x] + Sinh[x])
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \frac{e^{x}}{1-\sin \left(x \right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(1-sin(x)),x)
```

```
[Out] int(exp(x)/(1-sin(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2 \left(\cos \left(x\right) e^{x} - \left(\cos \left(x\right)^{2} + \sin \left(x\right)^{2} - 2 \, \sin \left(x\right) + 1\right) \int \frac{\cos\left(x\right) e^{x}}{\cos\left(x\right)^{2} + \sin\left(x\right)^{2} - 2 \, \sin\left(x\right) + 1}\,{d x}\right)}{\cos \left(x\right)^{2} + \sin \left(x\right)^{2} - 2 \, \sin \left(x\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-sin(x)),x, algorithm="maxima")
```

```
[Out] 2*(cos(x)*e^x - (cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)*integrate(cos(x)*e^x/(
cos(x)^2 + sin(x)^2 - 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 - 2*sin(x) +
1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{e^{x}}{\sin \left(x\right) - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-sin(x)),x, algorithm="fricas")
```

```
[Out] integral(-e^x/(sin(x) - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e^{x}}{\sin \left(x \right)} - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-sin(x)),x)
```

```
[Out] -Integral(exp(x)/(sin(x) - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{e^x}{\sin(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-sin(x)),x, algorithm="giac")
```

```
[Out] integrate(-e^x/(sin(x) - 1), x)
```

3.556 $\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx$

Optimal. Leaf size=15

$$-\frac{e^x\sin(x)}{1-\cos(x)}$$

```
[Out] -((E^x*Sin[x])/(1 - Cos[x]))
```

Rubi [A] time = 0.0299952, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2288}

$$-\frac{e^x\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 - Sin[x]))/(1 - Cos[x]),x]

[Out] -((E^x*Sin[x])/(1 - Cos[x]))
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx=-\frac{e^x\sin(x)}{1-\cos(x)}$$

Mathematica [A] time = 0.21783, size = 11, normalized size = 0.73

$$-e^x\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 - Sin[x]))/(1 - Cos[x]),x]

[Out] -(E^x*Cot[x/2])
```

Maple [B] time = 0.047, size = 33, normalized size = 2.2

$$\left(-e^x\left(\tan\left(\frac{x}{2}\right)\right)^2-e^x\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-1}\left(\tan\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1-sin(x))/(1-cos(x)),x)
```

```
[Out] (-exp(x)*tan(1/2*x)^2-exp(x))/(tan(1/2*x)^2+1)/tan(1/2*x)
```

Maxima [A] time = 1.27011, size = 30, normalized size = 2.

$$-\frac{2 e^{x} \sin\left(x\right)}{\cos\left(x\right)^{2} + \sin\left(x\right)^{2} - 2 \cos\left(x\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1-cos(x)),x, algorithm="maxima")
```

```
[Out] -2*e^x*sin(x)/(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)
```

Fricas [A] time = 2.16965, size = 35, normalized size = 2.33

$$-\frac{(\cos\left(x\right) + 1)e^{x}}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)*e^x/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(\sin\left(x\right) - 1) e^{x}}{\cos\left(x\right) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1-cos(x)),x)
```

```
[Out] Integral((sin(x) - 1)*exp(x)/(cos(x) - 1), x)
```

Giac [A] time = 1.1411, size = 14, normalized size = 0.93

$$-\frac{e^{x}}{\tan\left(\frac{1}{2}\, x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1-cos(x)),x, algorithm="giac")
```

```
[Out] -e^x/tan(1/2*x)
```

3.557 $\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx$

Optimal. Leaf size=41

$$\frac{e^x\sin(x)}{1-\cos(x)}-(2-2i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;e^{ix}\right)$$

```
[Out] (-2 + 2*I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, E^(I*x)] + (E^x
*Sin[x])/(1 - Cos[x])
```

Rubi [A] time = 0.113198, antiderivative size = 45, normalized size of antiderivative = 1.1, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4463, 4461, 4443, 2194, 2251, 2288}

$$-4ie^x\text{Hypergeometric2F1}\left(-i,1,1-i,e^{ix}\right)+2ie^x-\frac{e^x\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 + Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] (2*I)*E^x - (4*I)*E^x*Hypergeometric2F1[-I, 1, 1 - I, E^(I*x)] - (E^x*Sin[x
])/(1 - Cos[x])
```

Rule 4463

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*((h_) + (i_.)*Sin[(d_.) + (e_.)*(x_)
]))/(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_)), x_Symbol] :> Dist[2*i, Int[F^(c
*(a + b*x))*(Sin[d + e*x]/(f + g*Cos[d + e*x])), x], x] + Int[F^(c*(a + b*x
))*((h - i*Sin[d + e*x])/(f + g*Cos[d + e*x])), x] /; FreeQ[{F, a, b, c, d,
 e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h +
f*i, 0]
```

Rule 4461

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Dist[f^n, Int[F^(c*(a +
 b*x))*Cot[d/2 + (e*x)/2]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] &
& EqQ[f + g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rule 4443

```
Int[Cot[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Dist[(-I)^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 + E^(2*I*(d + e
*x)))^n)/(1 - E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e},
 x] && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
```

```
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{e^x(1+\sin(x))}{1-\cos(x)}\,dx &= 2\int \frac{e^x\sin(x)}{1-\cos(x)}\,dx+\int \frac{e^x(1-\sin(x))}{1-\cos(x)}\,dx\\
&= -\frac{e^x\sin(x)}{1-\cos(x)}+2\int e^x\cot\left(\frac{x}{2}\right)dx\\
&= -\frac{e^x\sin(x)}{1-\cos(x)}-2i\int\left(-e^x-\frac{2e^x}{-1+e^{ix}}\right)dx\\
&= -\frac{e^x\sin(x)}{1-\cos(x)}+2i\int e^x\,dx+4i\int\frac{e^x}{-1+e^{ix}}\,dx\\
&= 2ie^x-4ie^x\,{}_2F_1\left(-i,1;1-i;e^{ix}\right)-\frac{e^x\sin(x)}{1-\cos(x)}
\end{aligned}
$$

Mathematica [B] time = 0.208887, size = 100, normalized size = 2.44

$$
\frac{2e^x\sin\left(\frac{x}{2}\right)(\sin(x)+1)\left(2i\,{}_2F_1\left(-i,1;1-i;e^{ix}\right)\sin\left(\frac{x}{2}\right)+(1+i)e^{ix}\,{}_2F_1\left(1,1-i;2-i;e^{ix}\right)\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{(\cos(x)-1)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 + Sin[x]))/(1 - Cos[x]),x]
```

```
[Out] (2*E^x*Sin[x/2]*(Cos[x/2] + (2*I)*Hypergeometric2F1[-I, 1, 1 - I, E^(I*x)]*
Sin[x/2] + (1 + I)*E^(I*x)*Hypergeometric2F1[1, 1 - I, 2 - I, E^(I*x)]*Sin[
x/2])*(1 + Sin[x]))/((-1 + Cos[x])*(Cos[x/2] + Sin[x/2])^2)
```

Maple [F] time = 0.064, size = 0, normalized size = 0.

$$
\int \frac{e^x\left(1+\sin(x)\right)}{1-\cos(x)}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1+sin(x))/(1-cos(x)),x)
```

```
[Out] int(exp(x)*(1+sin(x))/(1-cos(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$
\frac{2\left(2\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)\int\frac{e^x\sin(x)}{\cos(x)^2+\sin(x)^2-2\cos(x)+1}\,dx-e^x\sin(x)\right)}{\cos(x)^2+\sin(x)^2-2\cos(x)+1}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1-cos(x)),x, algorithm="maxima")
```

```
[Out] 2*(2*(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 +
sin(x)^2 - 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 - 2*cos(x)
+ 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{e^x \sin(x) + e^x}{\cos(x) - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] integral(-(e^x*sin(x) + e^x)/(cos(x) - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{e^x}{\cos(x) - 1}\, dx - \int \frac{e^x \sin(x)}{\cos(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1-cos(x)),x)
```

```
[Out] -Integral(exp(x)/(cos(x) - 1), x) - Integral(exp(x)*sin(x)/(cos(x) - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(\sin(x) + 1)e^x}{\cos(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1-cos(x)),x, algorithm="giac")
```

```
[Out] integrate(-(sin(x) + 1)*e^x/(cos(x) - 1), x)
```

3.558 $\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx$

Optimal. Leaf size=12

$$\frac{e^x \sin(x)}{\cos(x)+1}$$

```
[Out] (E^x*Sin[x])/(1 + Cos[x])
```

Rubi [A] time = 0.0277275, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2288}

$$\frac{e^x \sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 + Sin[x]))/(1 + Cos[x]),x]
```

```
[Out] (E^x*Sin[x])/(1 + Cos[x])
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx = \frac{e^x \sin(x)}{1+\cos(x)}$$

Mathematica [A] time = 0.180259, size = 10, normalized size = 0.83

$$e^x \tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 + Sin[x]))/(1 + Cos[x]),x]
```

```
[Out] E^x*Tan[x/2]
```

Maple [A] time = 0.034, size = 8, normalized size = 0.7

$$e^x \tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1+sin(x))/(cos(x)+1),x)
```

```
[Out] exp(x)*tan(1/2*x)
```

Maxima [A] time = 1.20976, size = 30, normalized size = 2.5

$$\frac{2 e^{x} \sin(x)}{\cos(x)^{2} + \sin(x)^{2} + 2 \cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1+cos(x)),x, algorithm="maxima")
```

```
[Out] 2*e^x*sin(x)/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
```

Fricas [A] time = 1.95081, size = 34, normalized size = 2.83

$$\frac{e^{x} \sin(x)}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] e^x*sin(x)/(cos(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(\sin(x) + 1) e^{x}}{\cos(x) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1+cos(x)),x)
```

```
[Out] Integral((sin(x) + 1)*exp(x)/(cos(x) + 1), x)
```

Giac [A] time = 1.15052, size = 9, normalized size = 0.75

$$e^{x} \tan\left(\frac{1}{2} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sin(x))/(1+cos(x)),x, algorithm="giac")
```

```
[Out] e^x*tan(1/2*x)
```

3.559 $\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx$

Optimal. Leaf size=42

$$-\frac{e^x\sin(x)}{\cos(x)+1}+(2-2i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;-e^{ix}\right)$$

```
[Out] (2 - 2*I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, -E^(I*x)] - (E^x
*Sin[x])/(1 + Cos[x])
```

Rubi [A] time = 0.111279, antiderivative size = 44, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4463, 4460, 4442, 2194, 2251, 2288}

$$-4ie^x\text{Hypergeometric2F1}\left(-i,1,1-i,-e^{ix}\right)+2ie^x+\frac{e^x\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 - Sin[x]))/(1 + Cos[x]),x]
```

```
[Out] (2*I)*E^x - (4*I)*E^x*Hypergeometric2F1[-I, 1, 1 - I, -E^(I*x)] + (E^x*Sin[
x])/(1 + Cos[x])
```

Rule 4463

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*((h_) + (i_.)*Sin[(d_.) + (e_.)*(x_)
]))/(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_)), x_Symbol] :> Dist[2*i, Int[F^(c
*(a + b*x))*(Sin[d + e*x]/(f + g*Cos[d + e*x])), x], x] + Int[F^(c*(a + b*x
))*((h - i*Sin[d + e*x])/(f + g*Cos[d + e*x])), x] /; FreeQ[{F, a, b, c, d,
 e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h +
f*i, 0]
```

Rule 4460

```
Int[(Cos[(d_.) + (e_.)*(x_)]*(g_.) + (f_))^(n_.)*(F_)^((c_.)*((a_.) + (b_.)
*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Dist[f^n, Int[F^(c*(a +
 b*x))*Tan[d/2 + (e*x)/2]^m, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x] &
& EqQ[f - g, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rule 4442

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 - E^(2*I*(d + e*x)
))^n)/(1 + E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
```

```
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x(1-\sin(x))}{1+\cos(x)}\,dx &= -\left(2\int \frac{e^x\sin(x)}{1+\cos(x)}\,dx\right)+\int \frac{e^x(1+\sin(x))}{1+\cos(x)}\,dx\\ &= \frac{e^x\sin(x)}{1+\cos(x)}-2\int e^x\tan\left(\frac{x}{2}\right)dx\\ &= \frac{e^x\sin(x)}{1+\cos(x)}-2i\int\left(-e^x+\frac{2e^x}{1+e^{ix}}\right)dx\\ &= \frac{e^x\sin(x)}{1+\cos(x)}+2i\int e^x\,dx-4i\int\frac{e^x}{1+e^{ix}}\,dx\\ &= 2ie^x-4ie^x\,{}_2F_1\left(-i,1;1-i;-e^{ix}\right)+\frac{e^x\sin(x)}{1+\cos(x)}\end{aligned}$$

Mathematica [B] time = 0.213225, size = 87, normalized size = 2.07

$$-\frac{2e^x\cos\left(\frac{x}{2}\right)\left(2i\,{}_2F_1\left(-i,1;1-i;-e^{ix}\right)\cos\left(\frac{x}{2}\right)-(1+i)e^{ix}\,{}_2F_1\left(1,1-i;2-i;-e^{ix}\right)\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 - Sin[x]))/(1 + Cos[x]),x]
```

```
[Out] (-2*E^x*Cos[x/2]*((2*I)*Cos[x/2]*Hypergeometric2F1[-I, 1, 1 - I, -E^(I*x)]
 - (1 + I)*E^(I*x)*Cos[x/2]*Hypergeometric2F1[1, 1 - I, 2 - I, -E^(I*x)] - S
in[x/2]))/(1 + Cos[x])
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \frac{e^x(1-\sin(x))}{\cos(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1-sin(x))/(cos(x)+1),x)
```

```
[Out] int(exp(x)*(1-sin(x))/(cos(x)+1),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(2\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)\int\frac{e^x\sin(x)}{\cos(x)^2+\sin(x)^2+2\cos(x)+1}\,dx-e^x\sin(x)\right)}{\cos(x)^2+\sin(x)^2+2\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1+cos(x)),x, algorithm="maxima")
```

```
[Out] -2*(2*(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)*integrate(e^x*sin(x)/(cos(x)^2 +
 sin(x)^2 + 2*cos(x) + 1), x) - e^x*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x)
 + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{e^{x}\sin\left(x\right)-e^{x}}{\cos\left(x\right)+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] integral(-(e^x*sin(x) - e^x)/(cos(x) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{e^{x}}{\cos\left(x\right)+1}\,dx-\int \frac{e^{x}\sin\left(x\right)}{\cos\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1+cos(x)),x)
```

```
[Out] -Integral(-exp(x)/(cos(x) + 1), x) - Integral(exp(x)*sin(x)/(cos(x) + 1), x
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(\sin\left(x\right)-1\right)e^{x}}{\cos\left(x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sin(x))/(1+cos(x)),x, algorithm="giac")
```

```
[Out] integrate(-(sin(x) - 1)*e^x/(cos(x) + 1), x)
```

3.560 $\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx$

Optimal. Leaf size=46

$$-\frac{e^x\cos(x)}{1-\sin(x)}+(2+2i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;-ie^{ix}\right)$$

```
[Out] (2 + 2*I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, (-I)*E^(I*x)] -
(E^x*Cos[x])/(1 - Sin[x])
```

Rubi [A] time = 0.128552, antiderivative size = 48, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4462, 4459, 4442, 2194, 2251, 2288}

$$-4ie^x\text{Hypergeometric2F1}\left(-i,1,1-i,-ie^{ix}\right)+2ie^x+\frac{e^x\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 - Cos[x]))/(1 - Sin[x]),x]
```

```
[Out] (2*I)*E^x - (4*I)*E^x*Hypergeometric2F1[-I, 1, 1 - I, (-I)*E^(I*x)] + (E^x*
Cos[x])/(1 - Sin[x])
```

Rule 4462

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*(Cos[(d_.) + (e_.)*(x_)]*(i_.) + (h_
)))/((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Dist[2*i, Int[F^(c
*(a + b*x))*(Cos[d + e*x]/(f + g*Sin[d + e*x])), x], x] + Int[F^(c*(a + b*x
))*((h - i*Cos[d + e*x])/(f + g*Sin[d + e*x])), x] /; FreeQ[{F, a, b, c, d,
 e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h -
f*i, 0]
```

Rule 4459

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Dist[g^n, Int[F^(c*(a +
 b*x))*Tan[(f*Pi)/(4*g) - d/2 - (e*x)/2]^m, x], x] /; FreeQ[{F, a, b, c, d,
 e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rule 4442

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 - E^(2*I*(d + e*x)
))^n)/(1 + E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
```

```
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx &= -\left(2\int \frac{e^x\cos(x)}{1-\sin(x)}\,dx\right)+\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx\\
&= \frac{e^x\cos(x)}{1-\sin(x)}-2\int e^x\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)dx\\
&= \frac{e^x\cos(x)}{1-\sin(x)}-2i\int\left(-e^x+\frac{2e^x}{1+e^{2i\left(\frac{\pi}{4}+\frac{x}{2}\right)}}\right)dx\\
&= \frac{e^x\cos(x)}{1-\sin(x)}+2i\int e^x\,dx-4i\int\frac{e^x}{1+e^{2i\left(\frac{\pi}{4}+\frac{x}{2}\right)}}\,dx\\
&= 2ie^x-4ie^x\,{}_2F_1\left(-i,1;1-i;-ie^{ix}\right)+\frac{e^x\cos(x)}{1-\sin(x)}
\end{aligned}$$

Mathematica [A] time = 0.700616, size = 72, normalized size = 1.57

$$\frac{1}{2}(\cos(x)-1)\csc^2\left(\frac{x}{2}\right)\left(4i(\sinh(x)+\cosh(x))\,{}_2F_1(-i,1;1-i;\sin(x)-i\cos(x))-\frac{e^x\left((1+2i)\cot\left(\frac{x}{2}\right)+(1-2i)\right)}{\cot\left(\frac{x}{2}\right)-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 - Cos[x]))/(1 - Sin[x]),x]
```

```
[Out] ((-1 + Cos[x])*Csc[x/2]^2*(-((E^x*((1 - 2*I) + (1 + 2*I)*Cot[x/2]))/(-1 + C
ot[x/2])) + (4*I)*Hypergeometric2F1[-I, 1, 1 - I, (-I)*Cos[x] + Sin[x]]*(Co
sh[x] + Sinh[x])))/2
```

Maple [F] time = 0.079, size = 0, normalized size = 0.

$$\int \frac{e^x(1-\cos(x))}{1-\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1-cos(x))/(1-sin(x)),x)
```

```
[Out] int(exp(x)*(1-cos(x))/(1-sin(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\left(\cos(x)e^x-2\left(\cos(x)^2+\sin(x)^2-2\sin(x)+1\right)\int\frac{\cos(x)e^x}{\cos(x)^2+\sin(x)^2-2\sin(x)+1}\,dx\right)}{\cos(x)^2+\sin(x)^2-2\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-cos(x))/(1-sin(x)),x, algorithm="maxima")
```

```
[Out] 2*(cos(x)*e^x - 2*(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)*integrate(cos(x)*e^x
/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 - 2*sin(x)
+ 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(\cos(x)-1)e^x}{\sin(x)-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-cos(x))/(1-sin(x)),x, algorithm="fricas")
```

```
[Out] integral((cos(x) - 1)*e^x/(sin(x) - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(\cos(x)-1)\,e^x}{\sin(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-cos(x))/(1-sin(x)),x)
```

```
[Out] Integral((cos(x) - 1)*exp(x)/(sin(x) - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(\cos(x)-1)e^x}{\sin(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-cos(x))/(1-sin(x)),x, algorithm="giac")
```

```
[Out] integrate((cos(x) - 1)*e^x/(sin(x) - 1), x)
```

3.561 $\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx$

Optimal. Leaf size=14

$$\frac{e^x\cos(x)}{1-\sin(x)}$$

```
[Out] (E^x*Cos[x])/(1 - Sin[x])
```

Rubi [A] time = 0.0240413, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2288}

$$\frac{e^x\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 + Cos[x]))/(1 - Sin[x]),x]

[Out] (E^x*Cos[x])/(1 - Sin[x])
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\int \frac{e^x(1+\cos(x))}{1-\sin(x)}\,dx=\frac{e^x\cos(x)}{1-\sin(x)}$$

Mathematica [A] time = 0.0748232, size = 23, normalized size = 1.64

$$-\frac{e^x\left(\tan\left(\frac{x}{2}\right)+1\right)}{\tan\left(\frac{x}{2}\right)-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 + Cos[x]))/(1 - Sin[x]),x]

[Out] -((E^x*(1 + Tan[x/2]))/(-1 + Tan[x/2]))
```

Maple [B] time = 0.069, size = 53, normalized size = 3.8

$$\left(-e^x\tan\left(\frac{x}{2}\right)-e^x\left(\tan\left(\frac{x}{2}\right)\right)^2-e^x\left(\tan\left(\frac{x}{2}\right)\right)^3-e^x\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-1}\left(\tan\left(\frac{x}{2}\right)-1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(cos(x)+1)/(1-sin(x)),x)
```

```
[Out] (-exp(x)*tan(1/2*x)-exp(x)*tan(1/2*x)^2-exp(x)*tan(1/2*x)^3-exp(x))/(tan(1/
2*x)^2+1)/(tan(1/2*x)-1)
```

Maxima [A] time = 1.21475, size = 30, normalized size = 2.14

$$\frac{2\cos(x)e^x}{\cos(x)^2+\sin(x)^2-2\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1-sin(x)),x, algorithm="maxima")
```

```
[Out] 2*cos(x)*e^x/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [A] time = 1.81892, size = 74, normalized size = 5.29

$$\frac{(\cos(x)+1)e^x+e^x\sin(x)}{\cos(x)-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1-sin(x)),x, algorithm="fricas")
```

```
[Out] ((cos(x) + 1)*e^x + e^x*sin(x))/(cos(x) - sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{e^x}{\sin(x)-1}\,dx-\int\frac{e^x\cos(x)}{\sin(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1-sin(x)),x)
```

```
[Out] -Integral(exp(x)/(sin(x) - 1), x) - Integral(exp(x)*cos(x)/(sin(x) - 1), x)
```

Giac [A] time = 1.14211, size = 27, normalized size = 1.93

$$-\frac{e^x\tan\left(\frac{1}{2}x\right)+e^x}{\tan\left(\frac{1}{2}x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1-sin(x)),x, algorithm="giac")
```

```
[Out] -(e^x*tan(1/2*x) + e^x)/(tan(1/2*x) - 1)
```

3.562 $\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx$

Optimal. Leaf size=43

$$\frac{e^x\cos(x)}{\sin(x)+1}-(2+2i)e^{(1+i)x}\,{}_2F_1\left(1-i,2;2-i;ie^{ix}\right)$$

```
[Out] (-2 - 2*I)*E^((1 + I)*x)*Hypergeometric2F1[1 - I, 2, 2 - I, I*E^(I*x)] + (E
^x*Cos[x])/(1 + Sin[x])
```

Rubi [A] time = 0.127377, antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4462, 4459, 4442, 2194, 2251, 2288}

$$4ie^x\text{Hypergeometric2F1}\left(i,1,1+i,-ie^{-ix}\right)-2ie^x-\frac{e^x\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 + Cos[x]))/(1 + Sin[x]),x]
```

```
[Out] (-2*I)*E^x + (4*I)*E^x*Hypergeometric2F1[I, 1, 1 + I, (-I)/E^(I*x)] - (E^x*
Cos[x])/(1 + Sin[x])
```

Rule 4462

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_)))*(Cos[(d_.) + (e_.)*(x_)]*(i_.) + (h_
)))/((f_) + (g_.)*Sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Dist[2*i, Int[F^(c
*(a + b*x))*(Cos[d + e*x]/(f + g*Sin[d + e*x])), x], x] + Int[F^(c*(a + b*x
))*((h - i*Cos[d + e*x])/(f + g*Sin[d + e*x])), x] /; FreeQ[{F, a, b, c, d,
 e, f, g, h, i}, x] && EqQ[f^2 - g^2, 0] && EqQ[h^2 - i^2, 0] && EqQ[g*h -
f*i, 0]
```

Rule 4459

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_) +
 (g_.)*Sin[(d_.) + (e_.)*(x_)])^(n_.), x_Symbol] :> Dist[g^n, Int[F^(c*(a +
 b*x))*Tan[(f*Pi)/(4*g) - d/2 - (e*x)/2]^m, x], x] /; FreeQ[{F, a, b, c, d,
 e, f, g}, x] && EqQ[f^2 - g^2, 0] && IntegersQ[m, n] && EqQ[m + n, 0]
```

Rule 4442

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Tan[(d_.) + (e_.)*(x_)]^(n_.), x_Symb
ol] :> Dist[I^n, Int[ExpandIntegrand[(F^(c*(a + b*x))*(1 - E^(2*I*(d + e*x)
))^n)/(1 + E^(2*I*(d + e*x)))^n, x], x], x] /; FreeQ[{F, a, b, c, d, e}, x]
 && IntegerQ[n]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2251

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_)*(G_)^((h_.)*((f_.
) + (g_.)*(x_))), x_Symbol] :> Simp[(a^p*G^(h*(f + g*x))*Hypergeometric2F1[
```

```
-p, (g*h*Log[G])/(d*e*Log[F]), (g*h*Log[G])/(d*e*Log[F]) + 1, Simplify[-((b
*F^(e*(c + d*x)))/a)]])/(g*h*Log[G]), x] /; FreeQ[{F, G, a, b, c, d, e, f,
g, h, p}, x] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^x(1+\cos(x))}{1+\sin(x)}\,dx &= 2\int \frac{e^x\cos(x)}{1+\sin(x)}\,dx+\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx\\ &= -\frac{e^x\cos(x)}{1+\sin(x)}+2\int e^x\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)dx\\ &= -\frac{e^x\cos(x)}{1+\sin(x)}+2i\int \left(-e^x+\frac{2e^x}{1+e^{2i\left(\frac{\pi}{4}-\frac{x}{2}\right)}}\right)dx\\ &= -\frac{e^x\cos(x)}{1+\sin(x)}-2i\int e^x\,dx+4i\int \frac{e^x}{1+e^{2i\left(\frac{\pi}{4}-\frac{x}{2}\right)}}\,dx\\ &= -2ie^x+4ie^x\,{}_2F_1\left(i,1;1+i;-ie^{-ix}\right)-\frac{e^x\cos(x)}{1+\sin(x)}\end{aligned}$$

Mathematica [A] time = 0.186466, size = 73, normalized size = 1.7

$$\frac{1}{2}(\cos(x)+1)\sec^2\left(\frac{x}{2}\right)\left(\frac{e^x\left((1+2i)\tan\left(\frac{x}{2}\right)-(1-2i)\right)}{\tan\left(\frac{x}{2}\right)+1}-4i(\sinh(x)+\cosh(x))\,{}_2F_1(-i,1;1-i;i\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 + Cos[x]))/(1 + Sin[x]),x]
```

```
[Out] ((1 + Cos[x])*Sec[x/2]^2*((-4*I)*Hypergeometric2F1[-I, 1, 1 - I, I*Cos[x] -
 Sin[x]]*(Cosh[x] + Sinh[x]) + (E^x*((-1 + 2*I) + (1 + 2*I)*Tan[x/2]))/(1 +
 Tan[x/2])))/2
```

Maple [F] time = 0.069, size = 0, normalized size = 0.

$$\int \frac{e^x(\cos(x)+1)}{1+\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(cos(x)+1)/(1+sin(x)),x)
```

```
[Out] int(exp(x)*(cos(x)+1)/(1+sin(x)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(\cos(x)e^x-2\left(\cos(x)^2+\sin(x)^2+2\sin(x)+1\right)\int \frac{\cos(x)e^x}{\cos(x)^2+\sin(x)^2+2\sin(x)+1}\,dx\right)}{\cos(x)^2+\sin(x)^2+2\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1+sin(x)),x, algorithm="maxima")
```

```
[Out] -2*(cos(x)*e^x - 2*(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate(cos(x)*e^
x/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1), x))/(cos(x)^2 + sin(x)^2 + 2*sin(x)
 + 1)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(\cos(x)+1)e^{x}}{\sin(x)+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1+sin(x)),x, algorithm="fricas")
```

```
[Out] integral((cos(x) + 1)*e^x/(sin(x) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(\cos(x)+1)\, e^{x}}{\sin(x)+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1+sin(x)),x)
```

```
[Out] Integral((cos(x) + 1)*exp(x)/(sin(x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(\cos(x)+1)e^{x}}{\sin(x)+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+cos(x))/(1+sin(x)),x, algorithm="giac")
```

```
[Out] integrate((cos(x) + 1)*e^x/(sin(x) + 1), x)
```

3.563 $\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx$

Optimal. Leaf size=13

$$-\frac{e^x\cos(x)}{\sin(x)+1}$$

```
[Out] -((E^x*Cos[x])/(1 + Sin[x]))
```

Rubi [A] time = 0.0230758, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {2288}

$$-\frac{e^x\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 - Cos[x]))/(1 + Sin[x]),x]

[Out] -((E^x*Cos[x])/(1 + Sin[x]))
```

Rule 2288

```
Int[(y_.)*(F_)^(u_)*((v_) + (w_)), x_Symbol] :> With[{z = (v*y)/(Log[F]*D[u
, x])}, Simp[F^u*z, x] /; EqQ[D[z, x], w*y]] /; FreeQ[F, x]
```

Rubi steps

$$\int \frac{e^x(1-\cos(x))}{1+\sin(x)}\,dx = -\frac{e^x\cos(x)}{1+\sin(x)}$$

Mathematica [A] time = 0.0630134, size = 23, normalized size = 1.77

$$-\frac{e^x\left(\cot\left(\frac{x}{2}\right)-1\right)}{\cot\left(\frac{x}{2}\right)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 - Cos[x]))/(1 + Sin[x]),x]

[Out] -((E^x*(-1 + Cot[x/2]))/(1 + Cot[x/2]))
```

Maple [B] time = 0.056, size = 51, normalized size = 3.9

$$\left(e^x\tan\left(\frac{x}{2}\right)+e^x\left(\tan\left(\frac{x}{2}\right)\right)^3-e^x\left(\tan\left(\frac{x}{2}\right)\right)^2-e^x\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-1}\left(1+\tan\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1-cos(x))/(1+sin(x)),x)
```

```
[Out] (exp(x)*tan(1/2*x)+exp(x)*tan(1/2*x)^3-exp(x)*tan(1/2*x)^2-exp(x))/(tan(1/2
*x)^2+1)/(1+tan(1/2*x))
```

Maxima [A] time = 1.21953, size = 30, normalized size = 2.31

$$-\frac{2 \cos(x) e^{x}}{\cos(x)^{2}+\sin(x)^{2}+2 \sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-cos(x))/(1+sin(x)),x, algorithm="maxima")
```

```
[Out] -2*cos(x)*e^x/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)
```

Fricas [A] time = 1.85127, size = 76, normalized size = 5.85

$$-\frac{(\cos(x)+1)e^{x}-e^{x} \sin(x)}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-cos(x))/(1+sin(x)),x, algorithm="fricas")
```

```
[Out] -((cos(x) + 1)*e^x - e^x*sin(x))/(cos(x) + sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int -\frac{e^{x}}{\sin(x)+1}\, dx-\int \frac{e^{x} \cos(x)}{\sin(x)+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-cos(x))/(1+sin(x)),x)
```

```
[Out] -Integral(-exp(x)/(sin(x) + 1), x) - Integral(exp(x)*cos(x)/(sin(x) + 1), x
)
```

Giac [A] time = 1.10581, size = 28, normalized size = 2.15

$$\frac{e^{x} \tan\left(\frac{1}{2} x\right)-e^{x}}{\tan\left(\frac{1}{2} x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-cos(x))/(1+sin(x)),x, algorithm="giac")
```

```
[Out] (e^x*tan(1/2*x) - e^x)/(tan(1/2*x) + 1)
```

3.564 $\int e^x x \cos(x)\, dx$

Optimal. Leaf size=30

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

```
[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

Rubi [A] time = 0.0387418, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4433, 4466, 4432}

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x*Cos[x],x]
```

```
[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}
\int e^x x \cos(x)\, dx &= \frac{1}{2}e^x x \cos(x) + \frac{1}{2}e^x x \sin(x) - \int \left(\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)\right) dx \\
&= \frac{1}{2}e^x x \cos(x) + \frac{1}{2}e^x x \sin(x) - \frac{1}{2}\int e^x \cos(x)\, dx - \frac{1}{2}\int e^x \sin(x)\, dx \\
&= \frac{1}{2}e^x x \cos(x) - \frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x)
\end{aligned}$$

Mathematica [A] time = 0.0262369, size = 18, normalized size = 0.6

$$\frac{1}{2}e^x((x-1)\sin(x)+x\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x*Cos[x],x]
```

```
[Out] (E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2
```

Maple [A] time = 0.004, size = 20, normalized size = 0.7

$$\frac{e^x x\cos(x)}{2}-\left(-\frac{x}{2}+\frac{1}{2}\right)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x*cos(x),x)
```

```
[Out] 1/2*exp(x)*x*cos(x)-(-1/2*x+1/2)*exp(x)*sin(x)
```

Maxima [A] time = 0.940203, size = 23, normalized size = 0.77

$$\frac{1}{2}x\cos(x)e^x+\frac{1}{2}(x-1)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="maxima")
```

```
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Fricas [A] time = 1.92316, size = 58, normalized size = 1.93

$$\frac{1}{2}x\cos(x)e^x+\frac{1}{2}(x-1)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Sympy [A] time = 0.793402, size = 27, normalized size = 0.9

$$\frac{xe^x\sin(x)}{2}+\frac{xe^x\cos(x)}{2}-\frac{e^x\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x)
```

```
[Out] x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2
```

Giac [A] time = 1.11615, size = 20, normalized size = 0.67

$$\frac{1}{2}\left(x\cos\left(x\right)+\left(x-1\right)\sin\left(x\right)\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="giac")
```

```
[Out] 1/2*(x*cos(x) + (x - 1)*sin(x))*e^x
```

3.565 $\int e^x x^2 \sin(x)\,dx$

Optimal. Leaf size=50

$$\frac{1}{2}e^x x^2 \sin(x) - \frac{1}{2}e^x x^2 \cos(x) - \frac{1}{2}e^x \sin(x) + e^x x \cos(x) - \frac{1}{2}e^x \cos(x)$$

```
[Out] -(E^x*Cos[x])/2 + E^x*x*Cos[x] - (E^x*x^2*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x
*x^2*Sin[x])/2
```

Rubi [A] time = 0.107696, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4432, 4465, 14, 4433, 4466}

$$\frac{1}{2}e^x x^2 \sin(x) - \frac{1}{2}e^x x^2 \cos(x) - \frac{1}{2}e^x \sin(x) + e^x x \cos(x) - \frac{1}{2}e^x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x^2*Sin[x],x]
```

```
[Out] -(E^x*Cos[x])/2 + E^x*x*Cos[x] - (E^x*x^2*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x
*x^2*Sin[x])/2
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int e^x x^2 \sin(x)\,dx &= -\frac{1}{2}e^x x^2\cos(x) + \frac{1}{2}e^x x^2\sin(x) - 2\int x\left(-\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x\sin(x)\right)dx \\
&= -\frac{1}{2}e^x x^2\cos(x) + \frac{1}{2}e^x x^2\sin(x) - 2\int\left(-\frac{1}{2}e^x x\cos(x) + \frac{1}{2}e^x x\sin(x)\right)dx \\
&= -\frac{1}{2}e^x x^2\cos(x) + \frac{1}{2}e^x x^2\sin(x) + \int e^x x\cos(x)\,dx - \int e^x x\sin(x)\,dx \\
&= e^x x\cos(x) - \frac{1}{2}e^x x^2\cos(x) + \frac{1}{2}e^x x^2\sin(x) + \int\left(-\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x\sin(x)\right)dx - \int\left(\frac{1}{2}e^x\cos(x) + \frac{1}{2}e^x \right. \\
&= e^x x\cos(x) - \frac{1}{2}e^x x^2\cos(x) + \frac{1}{2}e^x x^2\sin(x) - 2\left(\frac{1}{2}\int e^x\cos(x)\,dx\right) \\
&= e^x x\cos(x) - \frac{1}{2}e^x x^2\cos(x) + \frac{1}{2}e^x x^2\sin(x) - 2\left(\frac{1}{4}e^x\cos(x) + \frac{1}{4}e^x\sin(x)\right)
\end{aligned}$$

Mathematica [A] time = 0.0357919, size = 25, normalized size = 0.5

$$\frac{1}{2}e^x\left(\left(x^2-1\right)\sin(x) - (x-1)^2\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x^2*Sin[x],x]

[Out] (E^x*(-((-1 + x)^2*Cos[x]) + (-1 + x^2)*Sin[x]))/2
```

Maple [A] time = 0.003, size = 27, normalized size = 0.5

$$\left(-\frac{x^2}{2} + x - \frac{1}{2}\right)e^x\cos(x) + \left(\frac{x^2}{2} - \frac{1}{2}\right)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x^2*sin(x),x)

[Out] (-1/2*x^2+x-1/2)*exp(x)*cos(x)+(1/2*x^2-1/2)*exp(x)*sin(x)
```

Maxima [A] time = 0.979202, size = 35, normalized size = 0.7

$$-\frac{1}{2}\left(x^2 - 2x + 1\right)\cos(x)e^x + \frac{1}{2}\left(x^2-1\right)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="maxima")

[Out] -1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

Fricas [A] time = 1.74131, size = 81, normalized size = 1.62

$$-\frac{1}{2}\left(x^2-2x+1\right)\cos(x)e^x+\frac{1}{2}\left(x^2-1\right)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2 - 2*x + 1)*cos(x)*e^x + 1/2*(x^2 - 1)*e^x*sin(x)
```

Sympy [A] time = 2.10396, size = 48, normalized size = 0.96

$$\frac{x^2e^x\sin(x)}{2}-\frac{x^2e^x\cos(x)}{2}+xe^x\cos(x)-\frac{e^x\sin(x)}{2}-\frac{e^x\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x**2*sin(x),x)
```

```
[Out] x**2*exp(x)*sin(x)/2 - x**2*exp(x)*cos(x)/2 + x*exp(x)*cos(x) - exp(x)*sin(
x)/2 - exp(x)*cos(x)/2
```

Giac [A] time = 1.09025, size = 34, normalized size = 0.68

$$-\frac{1}{2}\left(\left(x^2-2x+1\right)\cos(x)-\left(x^2-1\right)\sin(x)\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*((x^2 - 2*x + 1)*cos(x) - (x^2 - 1)*sin(x))*e^x
```

3.566 $\int e^{-3x}x^2\sin(x)\,dx$

Optimal. Leaf size=75

$$-\frac{3}{10}e^{-3x}x^2\sin(x)-\frac{1}{10}e^{-3x}x^2\cos(x)-\frac{4}{25}e^{-3x}x\sin(x)-\frac{9}{250}e^{-3x}\sin(x)-\frac{3}{25}e^{-3x}x\cos(x)-\frac{13}{250}e^{-3x}\cos(x)$$

```
[Out] (-13*Cos[x])/(250*E^(3*x)) - (3*x*Cos[x])/(25*E^(3*x)) - (x^2*Cos[x])/(10*E
^(3*x)) - (9*Sin[x])/(250*E^(3*x)) - (4*x*Sin[x])/(25*E^(3*x)) - (3*x^2*Sin
[x])/(10*E^(3*x))
```

Rubi [A] time = 0.140277, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4432, 4465, 14, 4433, 4466}

$$-\frac{3}{10}e^{-3x}x^2\sin(x)-\frac{1}{10}e^{-3x}x^2\cos(x)-\frac{4}{25}e^{-3x}x\sin(x)-\frac{9}{250}e^{-3x}\sin(x)-\frac{3}{25}e^{-3x}x\cos(x)-\frac{13}{250}e^{-3x}\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sin[x])/E^(3*x),x]
```

```
[Out] (-13*Cos[x])/(250*E^(3*x)) - (3*x*Cos[x])/(25*E^(3*x)) - (x^2*Cos[x])/(10*E
^(3*x)) - (9*Sin[x])/(250*E^(3*x)) - (4*x*Sin[x])/(25*E^(3*x)) - (3*x^2*Sin
[x])/(10*E^(3*x))
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
```

```
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\int e^{-3x}x^2\sin(x)\,dx = -\frac{1}{10}e^{-3x}x^2\cos(x) - \frac{3}{10}e^{-3x}x^2\sin(x) - 2\int x\left(-\frac{1}{10}e^{-3x}\cos(x) - \frac{3}{10}e^{-3x}\sin(x)\right)dx$$
$$= -\frac{1}{10}e^{-3x}x^2\cos(x) - \frac{3}{10}e^{-3x}x^2\sin(x) - 2\int\left(-\frac{1}{10}e^{-3x}x\cos(x) - \frac{3}{10}e^{-3x}x\sin(x)\right)dx$$
$$= -\frac{1}{10}e^{-3x}x^2\cos(x) - \frac{3}{10}e^{-3x}x^2\sin(x) + \frac{1}{5}\int e^{-3x}x\cos(x)\,dx + \frac{3}{5}\int e^{-3x}x\sin(x)\,dx$$
$$= -\frac{3}{25}e^{-3x}x\cos(x) - \frac{1}{10}e^{-3x}x^2\cos(x) - \frac{4}{25}e^{-3x}x\sin(x) - \frac{3}{10}e^{-3x}x^2\sin(x) - \frac{1}{5}\int\left(-\frac{3}{10}e^{-3x}\cos(x)\right.$$
$$= -\frac{3}{25}e^{-3x}x\cos(x) - \frac{1}{10}e^{-3x}x^2\cos(x) - \frac{4}{25}e^{-3x}x\sin(x) - \frac{3}{10}e^{-3x}x^2\sin(x) - \frac{1}{50}\int e^{-3x}\sin(x)\,dx +$$
$$= -\frac{2}{125}e^{-3x}\cos(x) - \frac{3}{25}e^{-3x}x\cos(x) - \frac{1}{10}e^{-3x}x^2\cos(x) - \frac{6}{125}e^{-3x}\sin(x) - \frac{4}{25}e^{-3x}x\sin(x) - \frac{3}{10}e^{-3}$$

Mathematica [A] time = 0.0347447, size = 38, normalized size = 0.51

$$\frac{1}{250}e^{-3x}\left(-\left(75x^2+40x+9\right)\sin(x)-\left(25x^2+30x+13\right)\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sin[x])/E^(3*x),x]
```

```
[Out] (-((13 + 30*x + 25*x^2)*Cos[x]) - (9 + 40*x + 75*x^2)*Sin[x])/(250*E^(3*x))
```

Maple [A] time = 0.006, size = 36, normalized size = 0.5

$$\left(-\frac{x^2}{10}-\frac{3x}{25}-\frac{13}{250}\right)e^{-3x}\cos(x)+\left(-\frac{3x^2}{10}-\frac{4x}{25}-\frac{9}{250}\right)e^{-3x}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x)/exp(3*x),x)
```

```
[Out] (-1/10*x^2-3/25*x-13/250)*exp(-3*x)*cos(x)+(-3/10*x^2-4/25*x-9/250)*exp(-3*
x)*sin(x)
```

Maxima [A] time = 0.969134, size = 45, normalized size = 0.6

$$-\frac{1}{250}\left(\left(25x^2+30x+13\right)\cos(x)+\left(75x^2+40x+9\right)\sin(x)\right)e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)/exp(3*x),x, algorithm="maxima")
```

```
[Out] -1/250*((25*x^2 + 30*x + 13)*cos(x) + (75*x^2 + 40*x + 9)*sin(x))*e^(-3*x)
```

Fricas [A] time = 1.90528, size = 120, normalized size = 1.6

$$-\frac{1}{250}\left(25\,x^2+30\,x+13\right)\cos\left(x\right)e^{(-3\,x)}-\frac{1}{250}\left(75\,x^2+40\,x+9\right)e^{(-3\,x)}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)/exp(3*x),x, algorithm="fricas")
```

```
[Out] -1/250*(25*x^2 + 30*x + 13)*cos(x)*e^(-3*x) - 1/250*(75*x^2 + 40*x + 9)*e^(
-3*x)*sin(x)
```

Sympy [A] time = 2.22796, size = 80, normalized size = 1.07

$$-\frac{3x^2e^{-3x}\sin\left(x\right)}{10}-\frac{x^2e^{-3x}\cos\left(x\right)}{10}-\frac{4xe^{-3x}\sin\left(x\right)}{25}-\frac{3xe^{-3x}\cos\left(x\right)}{25}-\frac{9e^{-3x}\sin\left(x\right)}{250}-\frac{13e^{-3x}\cos\left(x\right)}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x)/exp(3*x),x)
```

```
[Out] -3*x**2*exp(-3*x)*sin(x)/10 - x**2*exp(-3*x)*cos(x)/10 - 4*x*exp(-3*x)*sin(
x)/25 - 3*x*exp(-3*x)*cos(x)/25 - 9*exp(-3*x)*sin(x)/250 - 13*exp(-3*x)*cos
(x)/250
```

Giac [A] time = 1.15373, size = 45, normalized size = 0.6

$$-\frac{1}{250}\left(\left(25\,x^2+30\,x+13\right)\cos\left(x\right)+\left(75\,x^2+40\,x+9\right)\sin\left(x\right)\right)e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)/exp(3*x),x, algorithm="giac")
```

```
[Out] -1/250*((25*x^2 + 30*x + 13)*cos(x) + (75*x^2 + 40*x + 9)*sin(x))*e^(-3*x)
```

3.567 $\int e^{x/2}x^2\cos^3(x)\,dx$

Optimal. Leaf size=187

$$\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{12}{37}e^{x/2}x^2\sin(x)\cos^2(x)-\frac{24}{125}e^{x/2}\sin(x)-\frac{24}{25}e^{x/2}x\sin(x)-$$

```
[Out] (-132*E^(x/2)*Cos[x])/125 + (18*E^(x/2)*x*Cos[x])/25 + (48*E^(x/2)*x^2*Cos[
x])/185 + (2*E^(x/2)*x^2*Cos[x]^3)/37 - (428*E^(x/2)*Cos[3*x])/50653 + (70*
E^(x/2)*x*Cos[3*x])/1369 - (24*E^(x/2)*Sin[x])/125 - (24*E^(x/2)*x*Sin[x])/
25 + (96*E^(x/2)*x^2*Sin[x])/185 + (12*E^(x/2)*x^2*Cos[x]^2*Sin[x])/37 - (7
92*E^(x/2)*Sin[3*x])/50653 - (24*E^(x/2)*x*Sin[3*x])/1369
```

Rubi [A] time = 0.478478, antiderivative size = 253, normalized size of antiderivative = 1.35, number of steps used = 31, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {4435, 4433, 4466, 14, 4432, 4469, 4465, 4470}

$$\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{12}{37}e^{x/2}x^2\sin(x)\cos^2(x)-\frac{1218672e^{x/2}\sin(x)}{6331625}-\frac{32556e^{x/2}x\,\mathrm{s}}{34225}$$

Antiderivative was successfully verified.

```
[In] Int[E^(x/2)*x^2*Cos[x]^3,x]
```

```
[Out] (-6687696*E^(x/2)*Cos[x])/6331625 + (24792*E^(x/2)*x*Cos[x])/34225 + (48*E^
(x/2)*x^2*Cos[x])/185 + (16*E^(x/2)*Cos[x]^3)/50653 - (8*E^(x/2)*x*Cos[x]^3
)/1369 + (2*E^(x/2)*x^2*Cos[x]^3)/37 - (432*E^(x/2)*Cos[3*x])/50653 + (72*E
^(x/2)*x*Cos[3*x])/1369 - (1218672*E^(x/2)*Sin[x])/6331625 - (32556*E^(x/2)
*x*Sin[x])/34225 + (96*E^(x/2)*x^2*Sin[x])/185 + (96*E^(x/2)*Cos[x]^2*Sin[x
])/50653 - (48*E^(x/2)*x*Cos[x]^2*Sin[x])/1369 + (12*E^(x/2)*x^2*Cos[x]^2*S
in[x])/37 - (816*E^(x/2)*Sin[3*x])/50653 - (12*E^(x/2)*x*Sin[3*x])/1369
```

Rule 4435

```
Int[Cos[(d_.) + (e_.)*(x_)]^(m_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x]^m)/(e^2*m^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(m*(m - 1)*e^2)/(e^2*m^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Cos[d + e*x]^(m - 2), x], x] + Simp[(e*m*F^(c*(a + b*x))*Sin[d
+ e*x]*Cos[d + e*x]^(m - 1))/(e^2*m^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*m^2 + b^2*c^2*Log[F]^2, 0] && GtQ[m, 1]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4470

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(x_)^(p
_.)*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[x^p*F^
(c*(a + b*x)), Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c,
d, e, f, g}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int e^{x/2}x^2\cos^3(x)\,dx = \frac{48}{185}e^{x/2}x^2\cos(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{12}{37}e^{x/2}x^2\cos^2(x)\sin(x)-2\int x\left(\frac{48}{185}e^{x/2}\right.$$

$$=\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{12}{37}e^{x/2}x^2\cos^2(x)\sin(x)-2\int\left(\frac{48}{185}e^{x/2}x\right.$$

$$=\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{2}{37}e^{x/2}x^2\cos^3(x)+\frac{96}{185}e^{x/2}x^2\sin(x)+\frac{12}{37}e^{x/2}x^2\cos^2(x)\sin(x)-\frac{4}{37}\int e^{x/2}x\cos$$

$$=\frac{20352e^{x/2}x\cos(x)}{34225}+\frac{48}{185}e^{x/2}x^2\cos(x)-\frac{8e^{x/2}x\cos^3(x)}{1369}+\frac{2}{37}e^{x/2}x^2\cos^3(x)-\frac{30336e^{x/2}x\sin(x)}{34225}+$$

$$=\frac{20352e^{x/2}x\cos(x)}{34225}+\frac{48}{185}e^{x/2}x^2\cos(x)-\frac{8e^{x/2}x\cos^3(x)}{1369}+\frac{2}{37}e^{x/2}x^2\cos^3(x)-\frac{30336e^{x/2}x\sin(x)}{34225}+$$

$$=-\frac{48384e^{x/2}\cos(x)}{171125}+\frac{24792e^{x/2}x\cos(x)}{34225}+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{16e^{x/2}\cos^3(x)}{50653}-\frac{8e^{x/2}x\cos^3(x)}{1369}+\frac{2}{37}$$

$$=-\frac{1780608e^{x/2}\cos(x)}{6331625}+\frac{24792e^{x/2}x\cos(x)}{34225}+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{16e^{x/2}\cos^3(x)}{50653}-\frac{8e^{x/2}x\cos^3(x)}{1369}+$$

$$=-\frac{2482128e^{x/2}\cos(x)}{6331625}+\frac{24792e^{x/2}x\cos(x)}{34225}+\frac{48}{185}e^{x/2}x^2\cos(x)+\frac{16e^{x/2}\cos^3(x)}{50653}-\frac{8e^{x/2}x\cos^3(x)}{1369}+$$

Mathematica [A] time = 0.148817, size = 72, normalized size = 0.39

$$\frac{e^{x/2}\left(303918\left(25x^2-40x-8\right)\sin(x)+750\left(1369x^2-296x-264\right)\sin(3x)+151959\left(25x^2+60x-88\right)\cos(x)+125\left(\right.\right.}{12663250}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(x/2)*x^2*Cos[x]^3,x]
```

```
[Out] (E^(x/2)*(151959*(-88 + 60*x + 25*x^2)*Cos[x] + 125*(-856 + 5180*x + 1369*x
^2)*Cos[3*x] + 303918*(-8 - 40*x + 25*x^2)*Sin[x] + 750*(-264 - 296*x + 136
9*x^2)*Sin[3*x]))/12663250
```

Maple [A] time = 0.039, size = 78, normalized size = 0.4

$$\frac{\cos(3x)}{4}\left(\frac{2x^2}{37}+\frac{280x}{1369}-\frac{1712}{50653}\right)e^{\frac{x}{2}}-\frac{\sin(3x)}{4}\left(-\frac{12x^2}{37}+\frac{96x}{1369}+\frac{3168}{50653}\right)e^{\frac{x}{2}}+\frac{3\cos(x)}{4}\left(\frac{2x^2}{5}+\frac{24x}{25}-\frac{176}{125}\right)e^{\frac{x}{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1/2*x)*x^2*cos(x)^3,x)
```

```
[Out] 1/4*(2/37*x^2+280/1369*x-1712/50653)*exp(1/2*x)*cos(3*x)-1/4*(-12/37*x^2+96
/1369*x+3168/50653)*exp(1/2*x)*sin(3*x)+3/4*(2/5*x^2+24/25*x-176/125)*exp(1
/2*x)*cos(x)-3/4*(-4/5*x^2+32/25*x+32/125)*exp(1/2*x)*sin(x)
```

Maxima [A] time = 0.990361, size = 104, normalized size = 0.56

$$\frac{1}{101306}\left(1369x^2+5180x-856\right)\cos(3x)e^{\left(\frac{1}{2}x\right)}+\frac{3}{250}\left(25x^2+60x-88\right)\cos(x)e^{\left(\frac{1}{2}x\right)}+\frac{3}{50653}\left(1369x^2-296x-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)*x^2*cos(x)^3,x, algorithm="maxima")
```

```
[Out] 1/101306*(1369*x^2 + 5180*x - 856)*cos(3*x)*e^(1/2*x) + 3/250*(25*x^2 + 60*
x - 88)*cos(x)*e^(1/2*x) + 3/50653*(1369*x^2 - 296*x - 264)*e^(1/2*x)*sin(3
*x) + 3/125*(25*x^2 - 40*x - 8)*e^(1/2*x)*sin(x)
```

Fricas [A] time = 2.01906, size = 279, normalized size = 1.49

$$\frac{12}{6331625}\left(125\left(1369x^2-296x-264\right)\cos(x)^2+273800x^2-497280x-93056\right)e^{\left(\frac{1}{2}x\right)}\sin(x)+\frac{2}{6331625}\left(125\left(1369x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)*x^2*cos(x)^3,x, algorithm="fricas")
```

```
[Out] 12/6331625*(125*(1369*x^2 - 296*x - 264)*cos(x)^2 + 273800*x^2 - 497280*x -
 93056)*e^(1/2*x)*sin(x) + 2/6331625*(125*(1369*x^2 + 5180*x - 856)*cos(x)^
3 + 24*(34225*x^2 + 74740*x - 135952)*cos(x))*e^(1/2*x)
```

Sympy [A] time = 12.812, size = 202, normalized size = 1.08

$$\frac{96x^2e^{\frac{x}{2}}\sin^3(x)}{185}+\frac{48x^2e^{\frac{x}{2}}\sin^2(x)\cos(x)}{185}+\frac{156x^2e^{\frac{x}{2}}\sin(x)\cos^2(x)}{185}+\frac{58x^2e^{\frac{x}{2}}\cos^3(x)}{185}-\frac{32256xe^{\frac{x}{2}}\sin^3(x)}{34225}+\frac{19392xe^{\frac{x}{2}}\ldots}{3\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)*x**2*cos(x)**3,x)
```

```
[Out] 96*x**2*exp(x/2)*sin(x)**3/185 + 48*x**2*exp(x/2)*sin(x)**2*cos(x)/185 + 15
6*x**2*exp(x/2)*sin(x)*cos(x)**2/185 + 58*x**2*exp(x/2)*cos(x)**3/185 - 322
56*x*exp(x/2)*sin(x)**3/34225 + 19392*x*exp(x/2)*sin(x)**2*cos(x)/34225 - 3
4656*x*exp(x/2)*sin(x)*cos(x)**2/34225 + 26392*x*exp(x/2)*cos(x)**3/34225 -
 1116672*exp(x/2)*sin(x)**3/6331625 - 6525696*exp(x/2)*sin(x)**2*cos(x)/633
1625 - 1512672*exp(x/2)*sin(x)*cos(x)**2/6331625 - 6739696*exp(x/2)*cos(x)*
*3/6331625
```

Giac [A] time = 1.08749, size = 99, normalized size = 0.53

$$\frac{1}{101306}\left(\left(1369\,x^2+5180\,x-856\right)\cos(3\,x)+6\left(1369\,x^2-296\,x-264\right)\sin(3\,x)\right)e^{\left(\frac{1}{2}\,x\right)}+\frac{3}{250}\left(\left(25\,x^2+60\,x-88\right)\cos(x)\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)*x^2*cos(x)^3,x, algorithm="giac")
```

```
[Out] 1/101306*((1369*x^2 + 5180*x - 856)*cos(3*x) + 6*(1369*x^2 - 296*x - 264)*s
in(3*x))*e^(1/2*x) + 3/250*((25*x^2 + 60*x - 88)*cos(x) + 2*(25*x^2 - 40*x
- 8)*sin(x))*e^(1/2*x)
```

3.568 $\int e^{2x}x^2\sin(4x)\,dx$

Optimal. Leaf size=87

$$\frac{1}{10}e^{2x}x^2\sin(4x)-\frac{1}{5}e^{2x}x^2\cos(4x)+\frac{3}{50}e^{2x}x\sin(4x)-\frac{11}{500}e^{2x}\sin(4x)+\frac{2}{25}e^{2x}x\cos(4x)+\frac{1}{250}e^{2x}\cos(4x)$$

```
[Out] (E^(2*x)*Cos[4*x])/250 + (2*E^(2*x)*x*Cos[4*x])/25 - (E^(2*x)*x^2*Cos[4*x])
/5 - (11*E^(2*x)*Sin[4*x])/500 + (3*E^(2*x)*x*Sin[4*x])/50 + (E^(2*x)*x^2*S
in[4*x])/10
```

Rubi [A] time = 0.156602, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {4432, 4465, 14, 4433, 4466}

$$\frac{1}{10}e^{2x}x^2\sin(4x)-\frac{1}{5}e^{2x}x^2\cos(4x)+\frac{3}{50}e^{2x}x\sin(4x)-\frac{11}{500}e^{2x}\sin(4x)+\frac{2}{25}e^{2x}x\cos(4x)+\frac{1}{250}e^{2x}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*x^2*Sin[4*x],x]
```

```
[Out] (E^(2*x)*Cos[4*x])/250 + (2*E^(2*x)*x*Cos[4*x])/25 - (E^(2*x)*x^2*Cos[4*x])
/5 - (11*E^(2*x)*Sin[4*x])/500 + (3*E^(2*x)*x*Sin[4*x])/50 + (E^(2*x)*x^2*S
in[4*x])/10
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
```

```
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int e^{2x}x^2\sin(4x)\,dx &= -\frac{1}{5}e^{2x}x^2\cos(4x)+\frac{1}{10}e^{2x}x^2\sin(4x)-2\int x\left(-\frac{1}{5}e^{2x}\cos(4x)+\frac{1}{10}e^{2x}\sin(4x)\right)dx\\
&= -\frac{1}{5}e^{2x}x^2\cos(4x)+\frac{1}{10}e^{2x}x^2\sin(4x)-2\int\left(-\frac{1}{5}e^{2x}x\cos(4x)+\frac{1}{10}e^{2x}x\sin(4x)\right)dx\\
&= -\frac{1}{5}e^{2x}x^2\cos(4x)+\frac{1}{10}e^{2x}x^2\sin(4x)-\frac{1}{5}\int e^{2x}x\sin(4x)\,dx+\frac{2}{5}\int e^{2x}x\cos(4x)\,dx\\
&= \frac{2}{25}e^{2x}x\cos(4x)-\frac{1}{5}e^{2x}x^2\cos(4x)+\frac{3}{50}e^{2x}x\sin(4x)+\frac{1}{10}e^{2x}x^2\sin(4x)+\frac{1}{5}\int\left(-\frac{1}{5}e^{2x}\cos(4x)+\frac{1}{10}e\right.\\
&= \frac{2}{25}e^{2x}x\cos(4x)-\frac{1}{5}e^{2x}x^2\cos(4x)+\frac{3}{50}e^{2x}x\sin(4x)+\frac{1}{10}e^{2x}x^2\sin(4x)+\frac{1}{50}\int e^{2x}\sin(4x)\,dx-2\left(\frac{1}{2}\right.\\
&= \frac{3}{250}e^{2x}\cos(4x)+\frac{2}{25}e^{2x}x\cos(4x)-\frac{1}{5}e^{2x}x^2\cos(4x)-\frac{3}{500}e^{2x}\sin(4x)+\frac{3}{50}e^{2x}x\sin(4x)+\frac{1}{10}e^{2x}x^2\,\mathrm{si}
\end{aligned}$$

Mathematica [A] time = 0.077555, size = 40, normalized size = 0.46

$$\frac{1}{500}e^{2x}\left(\left(50x^2+30x-11\right)\sin(4x)+\left(-100x^2+40x+2\right)\cos(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*x^2*Sin[4*x],x]
```

```
[Out] (E^(2*x)*((2 + 40*x - 100*x^2)*Cos[4*x] + (-11 + 30*x + 50*x^2)*Sin[4*x]))/
500
```

Maple [A] time = 0.004, size = 40, normalized size = 0.5

$$\left(-\frac{x^2}{5}+\frac{2x}{25}+\frac{1}{250}\right)e^{2x}\cos(4x)+\left(\frac{x^2}{10}+\frac{3x}{50}-\frac{11}{500}\right)e^{2x}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*x^2*sin(4*x),x)
```

```
[Out] (-1/5*x^2+2/25*x+1/250)*exp(2*x)*cos(4*x)+(1/10*x^2+3/50*x-11/500)*exp(2*x)
*sin(4*x)
```

Maxima [A] time = 0.956134, size = 55, normalized size = 0.63

$$-\frac{1}{250}\left(50x^2-20x-1\right)\cos(4x)e^{(2x)}+\frac{1}{500}\left(50x^2+30x-11\right)e^{(2x)}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*x^2*sin(4*x),x, algorithm="maxima")
```

```
[Out] -1/250*(50*x^2 - 20*x - 1)*cos(4*x)*e^(2*x) + 1/500*(50*x^2 + 30*x - 11)*e^
(2*x)*sin(4*x)
```

Fricas [A] time = 1.75816, size = 123, normalized size = 1.41

$$-\frac{1}{250}\left(50\,x^2-20\,x-1\right)\cos\left(4\,x\right)e^{(2\,x)}+\frac{1}{500}\left(50\,x^2+30\,x-11\right)e^{(2\,x)}\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*x^2*sin(4*x),x, algorithm="fricas")
```

```
[Out] -1/250*(50*x^2 - 20*x - 1)*cos(4*x)*e^(2*x) + 1/500*(50*x^2 + 30*x - 11)*e^
(2*x)*sin(4*x)
```

Sympy [A] time = 2.20705, size = 85, normalized size = 0.98

$$\frac{x^2e^{2x}\sin\left(4x\right)}{10}-\frac{x^2e^{2x}\cos\left(4x\right)}{5}+\frac{3xe^{2x}\sin\left(4x\right)}{50}+\frac{2xe^{2x}\cos\left(4x\right)}{25}-\frac{11e^{2x}\sin\left(4x\right)}{500}+\frac{e^{2x}\cos\left(4x\right)}{250}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*x**2*sin(4*x),x)
```

```
[Out] x**2*exp(2*x)*sin(4*x)/10 - x**2*exp(2*x)*cos(4*x)/5 + 3*x*exp(2*x)*sin(4*x
)/50 + 2*x*exp(2*x)*cos(4*x)/25 - 11*exp(2*x)*sin(4*x)/500 + exp(2*x)*cos(4
*x)/250
```

Giac [A] time = 1.12231, size = 53, normalized size = 0.61

$$-\frac{1}{500}\left(2\left(50\,x^2-20\,x-1\right)\cos\left(4\,x\right)-\left(50\,x^2+30\,x-11\right)\sin\left(4\,x\right)\right)e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*x^2*sin(4*x),x, algorithm="giac")
```

```
[Out] -1/500*(2*(50*x^2 - 20*x - 1)*cos(4*x) - (50*x^2 + 30*x - 11)*sin(4*x))*e^(
2*x)
```

3.569 $\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx$

Optimal. Leaf size=185

$$\frac{1}{5}e^{x/2}x^2\sin(x)-\frac{3}{37}e^{x/2}x^2\sin(3x)+\frac{1}{10}e^{x/2}x^2\cos(x)-\frac{1}{74}e^{x/2}x^2\cos(3x)-\frac{8}{25}e^{x/2}x\sin(x)+\frac{24e^{x/2}x\sin(3x)}{1369}-\frac{8}{125}e^{x/2}\sin(x$$

```
[Out] (-44*E^(x/2)*Cos[x])/125 + (6*E^(x/2)*x*Cos[x])/25 + (E^(x/2)*x^2*Cos[x])/1
0 + (428*E^(x/2)*Cos[3*x])/50653 - (70*E^(x/2)*x*Cos[3*x])/1369 - (E^(x/2)*
x^2*Cos[3*x])/74 - (8*E^(x/2)*Sin[x])/125 - (8*E^(x/2)*x*Sin[x])/25 + (E^(x
/2)*x^2*Sin[x])/5 + (792*E^(x/2)*Sin[3*x])/50653 + (24*E^(x/2)*x*Sin[3*x])/
1369 - (3*E^(x/2)*x^2*Sin[3*x])/37
```

Rubi [A] time = 0.356214, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4470, 4433, 4466, 14, 4432, 4465}

$$\frac{1}{5}e^{x/2}x^2\sin(x)-\frac{3}{37}e^{x/2}x^2\sin(3x)+\frac{1}{10}e^{x/2}x^2\cos(x)-\frac{1}{74}e^{x/2}x^2\cos(3x)-\frac{8}{25}e^{x/2}x\sin(x)+\frac{24e^{x/2}x\sin(3x)}{1369}-\frac{8}{125}e^{x/2}\sin(x$$

Antiderivative was successfully verified.

```
[In] Int[E^(x/2)*x^2*Cos[x]*Sin[x]^2,x]
```


```
[Out] (-44*E^(x/2)*Cos[x])/125 + (6*E^(x/2)*x*Cos[x])/25 + (E^(x/2)*x^2*Cos[x])/1
0 + (428*E^(x/2)*Cos[3*x])/50653 - (70*E^(x/2)*x*Cos[3*x])/1369 - (E^(x/2)*
x^2*Cos[3*x])/74 - (8*E^(x/2)*Sin[x])/125 - (8*E^(x/2)*x*Sin[x])/25 + (E^(x
/2)*x^2*Sin[x])/5 + (792*E^(x/2)*Sin[3*x])/50653 + (24*E^(x/2)*x*Sin[3*x])/
1369 - (3*E^(x/2)*x^2*Sin[3*x])/37
```


Rule 4470

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*(x_)^(p
_.)*Sin[(d_.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[x^p*F^
(c*(a + b*x)), Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c,
d, e, f, g}, x] && IGtQ[m, 0] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\int e^{x/2}x^2\cos(x)\sin^2(x)\,dx = \int\left(\frac{1}{4}e^{x/2}x^2\cos(x)-\frac{1}{4}e^{x/2}x^2\cos(3x)\right)dx$$
$$= \frac{1}{4}\int e^{x/2}x^2\cos(x)\,dx-\frac{1}{4}\int e^{x/2}x^2\cos(3x)\,dx$$
$$= \frac{1}{10}e^{x/2}x^2\cos(x)-\frac{1}{74}e^{x/2}x^2\cos(3x)+\frac{1}{5}e^{x/2}x^2\sin(x)-\frac{3}{37}e^{x/2}x^2\sin(3x)-\frac{1}{2}\int x\left(\frac{2}{5}e^{x/2}\text{co}\right.$$
$$= \frac{1}{10}e^{x/2}x^2\cos(x)-\frac{1}{74}e^{x/2}x^2\cos(3x)+\frac{1}{5}e^{x/2}x^2\sin(x)-\frac{3}{37}e^{x/2}x^2\sin(3x)-\frac{1}{2}\int\left(\frac{2}{5}e^{x/2}x\,\text{co}\right.$$
$$= \frac{1}{10}e^{x/2}x^2\cos(x)-\frac{1}{74}e^{x/2}x^2\cos(3x)+\frac{1}{5}e^{x/2}x^2\sin(x)-\frac{3}{37}e^{x/2}x^2\sin(3x)+\frac{1}{37}\int e^{x/2}x\cos($$
$$= \frac{6}{25}e^{x/2}x\cos(x)+\frac{1}{10}e^{x/2}x^2\cos(x)-\frac{70e^{x/2}x\cos(3x)}{1369}-\frac{1}{74}e^{x/2}x^2\cos(3x)-\frac{8}{25}e^{x/2}x\sin(x)+$$
$$= \frac{6}{25}e^{x/2}x\cos(x)+\frac{1}{10}e^{x/2}x^2\cos(x)-\frac{70e^{x/2}x\cos(3x)}{1369}-\frac{1}{74}e^{x/2}x^2\cos(3x)-\frac{8}{25}e^{x/2}x\sin(x)+$$
$$= -\frac{12}{125}e^{x/2}\cos(x)+\frac{6}{25}e^{x/2}x\cos(x)+\frac{1}{10}e^{x/2}x^2\cos(x)+\frac{140e^{x/2}\cos(3x)}{50653}-\frac{70e^{x/2}x\cos(3x)}{1369}$$

Mathematica [A] time = 0.210563, size = 76, normalized size = 0.41

$$\frac{e^{x/2}\left(50653\left(2\left(25x^2-40x-8\right)\sin(x)+\left(25x^2+60x-88\right)\cos(x)\right)-125\left(6\left(1369x^2-296x-264\right)\sin(3x)+\left(1369x^2\right.\right.\right.}{12663250}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(x/2)*x^2*Cos[x]*Sin[x]^2,x]
```

```
[Out] (E^(x/2)*(50653*((-88 + 60*x + 25*x^2)*Cos[x] + 2*(-8 - 40*x + 25*x^2)*Sin[
x]) - 125*((-856 + 5180*x + 1369*x^2)*Cos[3*x] + 6*(-264 - 296*x + 1369*x^2
)*Sin[3*x])))/12663250
```

Maple [A] time = 0.024, size = 78, normalized size = 0.4

$$\frac{\cos(x)}{4}\left(\frac{2x^2}{5}+\frac{24x}{25}-\frac{176}{125}\right)e^{\frac{x}{2}}-\frac{\sin(x)}{4}\left(-\frac{4x^2}{5}+\frac{32x}{25}+\frac{32}{125}\right)e^{\frac{x}{2}}-\frac{\cos(3x)}{4}\left(\frac{2x^2}{37}+\frac{280x}{1369}-\frac{1712}{50653}\right)e^{\frac{x}{2}}+\frac{\sin(3x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x)
```

```
[Out] 1/4*(2/5*x^2+24/25*x-176/125)*exp(1/2*x)*cos(x)-1/4*(-4/5*x^2+32/25*x+32/12
5)*exp(1/2*x)*sin(x)-1/4*(2/37*x^2+280/1369*x-1712/50653)*exp(1/2*x)*cos(3*
x)+1/4*(-12/37*x^2+96/1369*x+3168/50653)*exp(1/2*x)*sin(3*x)
```

Maxima [A] time = 0.98504, size = 104, normalized size = 0.56

$$-\frac{1}{101306}\left(1369\,x^2+5180\,x-856\right)\cos\left(3\,x\right)e^{\left(\frac{1}{2}\,x\right)}+\frac{1}{250}\left(25\,x^2+60\,x-88\right)\cos\left(x\right)e^{\left(\frac{1}{2}\,x\right)}-\frac{3}{50653}\left(1369\,x^2-296\,x-264\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/101306*(1369*x^2 + 5180*x - 856)*cos(3*x)*e^(1/2*x) + 1/250*(25*x^2 + 60
*x - 88)*cos(x)*e^(1/2*x) - 3/50653*(1369*x^2 - 296*x - 264)*e^(1/2*x)*sin(
3*x) + 1/125*(25*x^2 - 40*x - 8)*e^(1/2*x)*sin(x)
```

Fricas [A] time = 2.25934, size = 282, normalized size = 1.52

$$-\frac{4}{6331625}\left(375\left(1369\,x^2-296\,x-264\right)\cos\left(x\right)^2-444925\,x^2+534280\,x+126056\right)e^{\left(\frac{1}{2}\,x\right)}\sin\left(x\right)-\frac{2}{6331625}\left(125\left(1369\,x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -4/6331625*(375*(1369*x^2 - 296*x - 264)*cos(x)^2 - 444925*x^2 + 534280*x +
 126056)*e^(1/2*x)*sin(x) - 2/6331625*(125*(1369*x^2 + 5180*x - 856)*cos(x)
^3 - (444925*x^2 + 1245420*x - 1194616)*cos(x))*e^(1/2*x)
```

Sympy [A] time = 12.7763, size = 202, normalized size = 1.09

$$\frac{52x^2e^{\frac{x}{2}}\sin^3(x)}{185}+\frac{26x^2e^{\frac{x}{2}}\sin^2(x)\cos(x)}{185}-\frac{8x^2e^{\frac{x}{2}}\sin(x)\cos^2(x)}{185}+\frac{16x^2e^{\frac{x}{2}}\cos^3(x)}{185}-\frac{11552xe^{\frac{x}{2}}\sin^3(x)}{34225}+\frac{13464xe^{\frac{x}{2}}\sin}{342}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)*x**2*cos(x)*sin(x)**2,x)
```

```
[Out] 52*x**2*exp(x/2)*sin(x)**3/185 + 26*x**2*exp(x/2)*sin(x)**2*cos(x)/185 - 8*
x**2*exp(x/2)*sin(x)*cos(x)**2/185 + 16*x**2*exp(x/2)*cos(x)**3/185 - 11552
*x*exp(x/2)*sin(x)**3/34225 + 13464*x*exp(x/2)*sin(x)**2*cos(x)/34225 - 915
2*x*exp(x/2)*sin(x)*cos(x)**2/34225 + 6464*x*exp(x/2)*cos(x)**3/34225 - 504
224*exp(x/2)*sin(x)**3/6331625 - 2389232*exp(x/2)*sin(x)**2*cos(x)/6331625
- 108224*exp(x/2)*sin(x)*cos(x)**2/6331625 - 2175232*exp(x/2)*cos(x)**3/633
1625
```

Giac [A] time = 1.14673, size = 99, normalized size = 0.54

$$-\frac{1}{101306}\left(\left(1369\,x^2+5180\,x-856\right)\cos\left(3\,x\right)+6\left(1369\,x^2-296\,x-264\right)\sin\left(3\,x\right)\right)e^{\left(\frac{1}{2}\,x\right)}+\frac{1}{250}\left(\left(25\,x^2+60\,x-88\right)\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/2*x)*x^2*cos(x)*sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/101306*((1369*x^2 + 5180*x - 856)*cos(3*x) + 6*(1369*x^2 - 296*x - 264)*
sin(3*x))*e^(1/2*x) + 1/250*((25*x^2 + 60*x - 88)*cos(x) + 2*(25*x^2 - 40*x
 - 8)*sin(x))*e^(1/2*x)
```

3.570 $\int \cosh(x)\,dx$

Optimal. Leaf size=2

$$\sinh(x)$$

[Out] Sinh[x]

Rubi [A] time = 0.0026886, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2637}

$$\sinh(x)$$

Antiderivative was successfully verified.

[In] Int[Cosh[x],x]

[Out] Sinh[x]

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh(x)\,dx = \sinh(x)$$

Mathematica [A] time = 0.0022323, size = 2, normalized size = 1.

$$\sinh(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x],x]

[Out] Sinh[x]

Maple [A] time = 0., size = 3, normalized size = 1.5

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x),x)

[Out] sinh(x)

Maxima [A] time = 0.932734, size = 3, normalized size = 1.5

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x, algorithm="maxima")
```

```
[Out] sinh(x)
```

Fricas [A] time = 2.0578, size = 12, normalized size = 6.

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x, algorithm="fricas")
```

```
[Out] sinh(x)
```

Sympy [A] time = 0.1112, size = 2, normalized size = 1.

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x)
```

```
[Out] sinh(x)
```

Giac [B] time = 1.12708, size = 15, normalized size = 7.5

$$-\frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x, algorithm="giac")
```

```
[Out] -1/2*e^(-x) + 1/2*e^x
```

3.571 $\int \sinh(x)\, dx$

Optimal. Leaf size=2

$$\cosh(x)$$

[Out] Cosh[x]

Rubi [A] time = 0.0029761, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2638}

$$\cosh(x)$$

Antiderivative was successfully verified.

[In] Int[Sinh[x],x]

[Out] Cosh[x]

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sinh(x)\, dx = \cosh(x)$$

Mathematica [A] time = 0.0010051, size = 2, normalized size = 1.

$$\cosh(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[x],x]

[Out] Cosh[x]

Maple [A] time = 0.002, size = 3, normalized size = 1.5

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(x),x)

[Out] cosh(x)

Maxima [A] time = 0.926615, size = 3, normalized size = 1.5

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x, algorithm="maxima")
```

```
[Out] cosh(x)
```

Fricas [A] time = 1.99209, size = 12, normalized size = 6.

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x, algorithm="fricas")
```

```
[Out] cosh(x)
```

Sympy [A] time = 0.110663, size = 2, normalized size = 1.

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x)
```

```
[Out] cosh(x)
```

Giac [B] time = 1.10054, size = 15, normalized size = 7.5

$$\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x, algorithm="giac")
```

```
[Out] 1/2*e^(-x) + 1/2*e^x
```

3.572 $\int \tanh(x)\,dx$

Optimal. Leaf size=3

$$\log(\cosh(x))$$

[Out] Log[Cosh[x]]

Rubi [A] time = 0.0034148, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3475}

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

[In] Int[Tanh[x],x]

[Out] Log[Cosh[x]]

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

Mathematica [A] time = 0.0019616, size = 3, normalized size = 1.

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x],x]

[Out] Log[Cosh[x]]

Maple [A] time = 0.001, size = 4, normalized size = 1.3

$$\ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x),x)

[Out] ln(cosh(x))

Maxima [A] time = 0.924554, size = 4, normalized size = 1.33

$$\log(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="maxima")
```

```
[Out] log(cosh(x))
```

Fricas [B] time = 2.0849, size = 55, normalized size = 18.33

$$-x + \log\left(\frac{2\cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="fricas")
```

```
[Out] -x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [B] time = 0.118635, size = 7, normalized size = 2.33

$$x - \log(\tanh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x)
```

```
[Out] x - log(tanh(x) + 1)
```

Giac [B] time = 1.13741, size = 15, normalized size = 5.

$$-x + \log\left(e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="giac")
```

```
[Out] -x + log(e^(2*x) + 1)
```

3.573 $\int \coth(x)\,dx$

Optimal. Leaf size=3

$$\log(\sinh(x))$$

```
[Out] Log[Sinh[x]]
```

Rubi [A] time = 0.0035209, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3475}

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x],x]
```

```
[Out] Log[Sinh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \coth(x)\,dx = \log(\sinh(x))$$

Mathematica [A] time = 0.0035221, size = 3, normalized size = 1.

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x],x]
```

```
[Out] Log[Sinh[x]]
```

Maple [A] time = 0.001, size = 4, normalized size = 1.3

$$\ln(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x),x)
```

```
[Out] ln(sinh(x))
```

Maxima [A] time = 0.92252, size = 4, normalized size = 1.33

$$\log(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x, algorithm="maxima")
```

```
[Out] log(sinh(x))
```

Fricas [B] time = 2.16589, size = 55, normalized size = 18.33

$$-x + \log\left(\frac{2\sinh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x, algorithm="fricas")
```

```
[Out] -x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [B] time = 0.290359, size = 12, normalized size = 4.

$$x - \log(\tanh(x) + 1) + \log(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x)
```

```
[Out] x - log(tanh(x) + 1) + log(tanh(x))
```

Giac [B] time = 1.08425, size = 16, normalized size = 5.33

$$-x + \log\left(\left|e^{(2x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x, algorithm="giac")
```

```
[Out] -x + log(abs(e^(2*x) - 1))
```

3.574 $\int \operatorname{sech}(x)\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\sinh(x))$$

```
[Out] ArcTan[Sinh[x]]
```

Rubi [A] time = 0.0032907, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3770}

$$\tan^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x],x]
```

```
[Out] ArcTan[Sinh[x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{sech}(x)\,dx = \tan^{-1}(\sinh(x))$$

Mathematica [B] time = 0.0039811, size = 9, normalized size = 3.

$$2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x],x]
```

```
[Out] 2*ArcTan[Tanh[x/2]]
```

Maple [A] time = 0., size = 4, normalized size = 1.3

$$\arctan(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x),x)
```

```
[Out] arctan(sinh(x))
```

Maxima [A] time = 0.927777, size = 4, normalized size = 1.33

$$\arctan(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x, algorithm="maxima")
```

```
[Out] arctan(sinh(x))
```

Fricas [B] time = 2.08164, size = 39, normalized size = 13.

$$2\arctan(\cosh(x) + \sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x)
```

```
[Out] Integral(sech(x), x)
```

Giac [A] time = 1.09022, size = 7, normalized size = 2.33

$$2\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x)
```

3.575 $\int \operatorname{csch}(x)\,dx$

Optimal. Leaf size=5

$$-\tanh^{-1}(\cosh(x))$$

```
[Out] -ArcTanh[Cosh[x]]
```

Rubi [A] time = 0.0036311, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3770}

$$-\tanh^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x],x]

[Out] -ArcTanh[Cosh[x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}(x)\,dx = -\tanh^{-1}(\cosh(x))$$

Mathematica [A] time = 0.0041317, size = 7, normalized size = 1.4

$$\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x],x]

[Out] Log[Tanh[x/2]]
```

Maple [A] time = 0.001, size = 6, normalized size = 1.2

$$\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x),x)
```

```
[Out] ln(tanh(1/2*x))
```

Maxima [A] time = 0.936344, size = 7, normalized size = 1.4

$$\log\left(\tanh\left(\frac{1}{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x),x, algorithm="maxima")
```

```
[Out] log(tanh(1/2*x))
```

Fricas [B] time = 2.12289, size = 78, normalized size = 15.6

$$-\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x),x, algorithm="fricas")
```

```
[Out] -log(cosh(x) + sinh(x) + 1) + log(cosh(x) + sinh(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\operatorname{csch}\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x),x)
```

```
[Out] Integral(csch(x), x)
```

Giac [B] time = 1.13312, size = 19, normalized size = 3.8

$$-\log\left(e^{x}+1\right)+\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x),x, algorithm="giac")
```

```
[Out] -log(e^x + 1) + log(abs(e^x - 1))
```

3.576 $\int \cosh^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)$$

```
[Out] x/2 + (Cosh[x]*Sinh[x])/2
```

Rubi [A] time = 0.0071077, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2,x]
```

```
[Out] x/2 + (Cosh[x]*Sinh[x])/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cosh^2(x)\,dx &= \frac{1}{2}\cosh(x)\sinh(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cosh(x)\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.0018328, size = 14, normalized size = 1.

$$\frac{x}{2}+\frac{1}{4}\sinh(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2,x]
```

```
[Out] x/2 + Sinh[2*x]/4
```

Maple [A] time = 0.006, size = 11, normalized size = 0.8

$$\frac{x}{2}+\frac{\cosh\left(x\right)\sinh\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2,x)
```

```
[Out] 1/2*x+1/2*cosh(x)*sinh(x)
```

Maxima [A] time = 0.926173, size = 22, normalized size = 1.57

$$\frac{1}{2}\,x+\frac{1}{8}\,e^{\left(2\,x\right)}-\frac{1}{8}\,e^{\left(-2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x)
```

Fricas [A] time = 2.03191, size = 39, normalized size = 2.79

$$\frac{1}{2}\,\cosh\left(x\right)\sinh\left(x\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cosh(x)*sinh(x) + 1/2*x
```

Sympy [B] time = 0.184481, size = 24, normalized size = 1.71

$$-\frac{x\sinh^{2}\left(x\right)}{2}+\frac{x\cosh^{2}\left(x\right)}{2}+\frac{\sinh\left(x\right)\cosh\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2,x)
```

```
[Out] -x*sinh(x)**2/2 + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2
```

Giac [B] time = 1.09885, size = 32, normalized size = 2.29

$$-\frac{1}{8}\,{\left(2\,e^{\left(2\,x\right)}+1\right)}e^{\left(-2\,x\right)}+\frac{1}{2}\,x+\frac{1}{8}\,e^{\left(2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2,x, algorithm="giac")
```

```
[Out] -1/8*(2*e^(2*x) + 1)*e^(-2*x) + 1/2*x + 1/8*e^(2*x)
```

3.577 $\int \sinh^5(x)\,dx$

Optimal. Leaf size=19

$$\frac{\cosh^5(x)}{5}-\frac{2\cosh^3(x)}{3}+\cosh(x)$$

```
[Out] Cosh[x] - (2*Cosh[x]^3)/3 + Cosh[x]^5/5
```

Rubi [A] time = 0.01108, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2633}

$$\frac{\cosh^5(x)}{5}-\frac{2\cosh^3(x)}{3}+\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^5,x]
```

```
[Out] Cosh[x] - (2*Cosh[x]^3)/3 + Cosh[x]^5/5
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh^5(x)\,dx &= \text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\cosh(x)\right)\\ &= \cosh(x)-\frac{2\cosh^3(x)}{3}+\frac{\cosh^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.0019877, size = 23, normalized size = 1.21

$$\frac{5\cosh(x)}{8}-\frac{5}{48}\cosh(3x)+\frac{1}{80}\cosh(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^5,x]
```

```
[Out] (5*Cosh[x])/8 - (5*Cosh[3*x])/48 + Cosh[5*x]/80
```

Maple [A] time = 0.033, size = 18, normalized size = 1.

$$\left(\frac{8}{15}+\frac{(\sinh(x))^4}{5}-\frac{4(\sinh(x))^2}{15}\right)\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^5,x)
```

```
[Out] (8/15+1/5*sinh(x)^4-4/15*sinh(x)^2)*cosh(x)
```

Maxima [B] time = 0.93214, size = 47, normalized size = 2.47

$$\frac{1}{160}e^{(5x)} - \frac{5}{96}e^{(3x)} + \frac{5}{16}e^{(-x)} - \frac{5}{96}e^{(-3x)} + \frac{1}{160}e^{(-5x)} + \frac{5}{16}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5,x, algorithm="maxima")
```

```
[Out] 1/160*e^(5*x) - 5/96*e^(3*x) + 5/16*e^(-x) - 5/96*e^(-3*x) + 1/160*e^(-5*x)
 + 5/16*e^x
```

Fricas [B] time = 2.10196, size = 155, normalized size = 8.16

$$\frac{1}{80}\cosh(x)^{5} + \frac{1}{16}\cosh(x)\sinh(x)^{4} - \frac{5}{48}\cosh(x)^{3} + \frac{1}{16}\left(2\cosh(x)^{3} - 5\cosh(x)\right)\sinh(x)^{2} + \frac{5}{8}\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5,x, algorithm="fricas")
```

```
[Out] 1/80*cosh(x)^5 + 1/16*cosh(x)*sinh(x)^4 - 5/48*cosh(x)^3 + 1/16*(2*cosh(x)^
3 - 5*cosh(x))*sinh(x)^2 + 5/8*cosh(x)
```

Sympy [A] time = 1.12174, size = 29, normalized size = 1.53

$$\sinh^{4}(x)\cosh(x) - \frac{4\sinh^{2}(x)\cosh^{3}(x)}{3} + \frac{8\cosh^{5}(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**5,x)
```

```
[Out] sinh(x)**4*cosh(x) - 4*sinh(x)**2*cosh(x)**3/3 + 8*cosh(x)**5/15
```

Giac [B] time = 1.10703, size = 50, normalized size = 2.63

$$\frac{1}{480}\left(150\,e^{(4x)} - 25\,e^{(2x)} + 3\right)e^{(-5x)} + \frac{1}{160}e^{(5x)} - \frac{5}{96}e^{(3x)} + \frac{5}{16}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^5,x, algorithm="giac")
```

```
[Out] 1/480*(150*e^(4*x) - 25*e^(2*x) + 3)*e^(-5*x) + 1/160*e^(5*x) - 5/96*e^(3*x
) + 5/16*e^x
```

3.578 $\int \tanh^4(x)\,dx$

Optimal. Leaf size=14

$$x-\frac{1}{3}\tanh^3(x)-\tanh(x)$$

```
[Out] x - Tanh[x] - Tanh[x]^3/3
```

Rubi [A] time = 0.0111177, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 8}

$$x-\frac{1}{3}\tanh^3(x)-\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]^4,x]
```

```
[Out] x - Tanh[x] - Tanh[x]^3/3
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \tanh^4(x)\,dx &= -\frac{1}{3}\tanh^3(x)+\int \tanh^2(x)\,dx\\
&= -\tanh(x)-\frac{\tanh^3(x)}{3}+\int 1\,dx\\
&= x-\tanh(x)-\frac{\tanh^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.0030748, size = 18, normalized size = 1.29

$$x-\frac{4\tanh(x)}{3}+\frac{1}{3}\tanh(x)\text{sech}^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4,x]
```

```
[Out] x - (4*Tanh[x])/3 + (Sech[x]^2*Tanh[x])/3
```

Maple [B] time = 0.002, size = 26, normalized size = 1.9

$$-\frac{(\tanh(x))^3}{3}-\tanh(x)-\frac{\ln(-1+\tanh(x))}{2}+\frac{\ln(1+\tanh(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^4,x)
```

```
[Out] -1/3*tanh(x)^3-tanh(x)-1/2*ln(-1+tanh(x))+1/2*ln(1+tanh(x))
```

Maxima [B] time = 0.948404, size = 51, normalized size = 3.64

$$x-\frac{4\left(3e^{(-2x)}+3e^{(-4x)}+2\right)}{3\left(3e^{(-2x)}+3e^{(-4x)}+e^{(-6x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4,x, algorithm="maxima")
```

```
[Out] x - 4/3*(3*e^(-2*x) + 3*e^(-4*x) + 2)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) +
 1)
```

Fricas [B] time = 2.10164, size = 221, normalized size = 15.79

$$\frac{(3x+4)\cosh(x)^3+3(3x+4)\cosh(x)\sinh(x)^2-12\cosh(x)^2\sinh(x)-4\sinh(x)^3+3(3x+4)\cosh(x)}{3\left(\cosh(x)^3+3\cosh(x)\sinh(x)^2+3\cosh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*((3*x + 4)*cosh(x)^3 + 3*(3*x + 4)*cosh(x)*sinh(x)^2 - 12*cosh(x)^2*sin
h(x) - 4*sinh(x)^3 + 3*(3*x + 4)*cosh(x))/(cosh(x)^3 + 3*cosh(x)*sinh(x)^2
+ 3*cosh(x))
```

Sympy [A] time = 0.216398, size = 10, normalized size = 0.71

$$x-\frac{\tanh^3(x)}{3}-\tanh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**4,x)
```

```
[Out] x - tanh(x)**3/3 - tanh(x)
```

Giac [B] time = 1.13156, size = 35, normalized size = 2.5

$$x + \frac{4\left(3e^{(4x)} + 3e^{(2x)} + 2\right)}{3\left(e^{(2x)} + 1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^4,x, algorithm="giac")
```

```
[Out] x + 4/3*(3*e^(4*x) + 3*e^(2*x) + 2)/(e^(2*x) + 1)^3
```

3.579 $\int \mathrm{csch}^3(x)\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\mathrm{csch}(x)$$

```
[Out] ArcTanh[Cosh[x]]/2 - (Coth[x]*Csch[x])/2
```

Rubi [A] time = 0.0117557, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3768, 3770}

$$\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\mathrm{csch}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[x]^3,x]
```

```
[Out] ArcTanh[Cosh[x]]/2 - (Coth[x]*Csch[x])/2
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \mathrm{csch}^3(x)\,dx&=-\frac{1}{2}\coth(x)\mathrm{csch}(x)-\frac{1}{2}\int \mathrm{csch}(x)\,dx\\&=\frac{1}{2}\tanh^{-1}(\cosh(x))-\frac{1}{2}\coth(x)\mathrm{csch}(x)\end{aligned}$$

Mathematica [B] time = 0.0037539, size = 36, normalized size = 2.25

$$-\frac{1}{8}\mathrm{csch}^2\left(\frac{x}{2}\right)-\frac{1}{8}\mathrm{sech}^2\left(\frac{x}{2}\right)-\frac{1}{2}\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[x]^3,x]
```

```
[Out] -Csch[x/2]^2/8 - Log[Tanh[x/2]]/2 - Sech[x/2]^2/8
```

Maple [A] time = 0.036, size = 11, normalized size = 0.7

$$-\frac{\coth(x)\operatorname{csch}(x)}{2}+\operatorname{Artanh}(e^{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(x)^3,x)
```

```
[Out] -1/2*coth(x)*csch(x)+arctanh(exp(x))
```

Maxima [B] time = 0.938495, size = 61, normalized size = 3.81

$$\frac{e^{(-x)}+e^{(-3x)}}{2e^{(-2x)}-e^{(-4x)}-1}+\frac{1}{2}\log\left(e^{(-x)}+1\right)-\frac{1}{2}\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3,x, algorithm="maxima")
```

```
[Out] (e^(-x) + e^(-3*x))/(2*e^(-2*x) - e^(-4*x) - 1) + 1/2*log(e^(-x) + 1) - 1/2
*log(e^(-x) - 1)
```

Fricas [B] time = 2.11272, size = 737, normalized size = 46.06

$$-\frac{2\cosh(x)^{3}+6\cosh(x)\sinh(x)^{2}+2\sinh(x)^{3}-\left(\cosh(x)^{4}+4\cosh(x)\sinh(x)^{3}+\sinh(x)^{4}+2\left(3\cosh(x)^{2}-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cosh(x)^3 + 6*cosh(x)*sinh(x)^2 + 2*sinh(x)^3 - (cosh(x)^4 + 4*cosh
(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4
*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) + 1) + (cosh(x)^4
 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh
(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2
*(3*cosh(x)^2 + 1)*sinh(x) + 2*cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 +
sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - co
sh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\operatorname{csch}^{3}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)**3,x)
```

```
[Out] Integral(csch(x)**3, x)
```

Giac [B] time = 1.09089, size = 61, normalized size = 3.81

$$-\frac{e^{(-x)} + e^{x}}{\left(e^{(-x)} + e^{x}\right)^{2} - 4} + \frac{1}{4} \log\left(e^{(-x)} + e^{x} + 2\right) - \frac{1}{4} \log\left(e^{(-x)} + e^{x} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(x)^3,x, algorithm="giac")
```

```
[Out] -(e^(-x) + e^x)/((e^(-x) + e^x)^2 - 4) + 1/4*log(e^(-x) + e^x + 2) - 1/4*lo
g(e^(-x) + e^x - 2)
```

3.580 $\int \mathbf{sech}^5(x)\,dx$

Optimal. Leaf size=26

$$\frac{3}{8}\tan^{-1}(\sinh(x))+\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{3}{8}\tanh(x)\text{sech}(x)$$

```
[Out] (3*ArcTan[Sinh[x]])/8 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x])/4
```

Rubi [A] time = 0.0167193, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3768, 3770}

$$\frac{3}{8}\tan^{-1}(\sinh(x))+\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{3}{8}\tanh(x)\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^5,x]
```

```
[Out] (3*ArcTan[Sinh[x]])/8 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x])/4
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \text{sech}^5(x)\,dx &= \frac{1}{4}\text{sech}^3(x)\tanh(x)+\frac{3}{4}\int \text{sech}^3(x)\,dx\\ &= \frac{3}{8}\text{sech}(x)\tanh(x)+\frac{1}{4}\text{sech}^3(x)\tanh(x)+\frac{3}{8}\int \text{sech}(x)\,dx\\ &= \frac{3}{8}\tan^{-1}(\sinh(x))+\frac{3}{8}\text{sech}(x)\tanh(x)+\frac{1}{4}\text{sech}^3(x)\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.0037519, size = 30, normalized size = 1.15

$$\frac{3}{4}\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{4}\tanh(x)\text{sech}^3(x)+\frac{3}{8}\tanh(x)\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^5,x]
```

```
[Out] (3*ArcTan[Tanh[x/2]])/4 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x])/4
```

Maple [A] time = 0.035, size = 21, normalized size = 0.8

$$\left(\frac{(\operatorname{sech}(x))^3}{4}+\frac{3\operatorname{sech}(x)}{8}\right)\tanh(x)+\frac{3\arctan(e^x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cosh(x)^5,x)
```

```
[Out] (1/4*sech(x)^3+3/8*sech(x))*tanh(x)+3/4*arctan(exp(x))
```

Maxima [B] time = 1.41523, size = 82, normalized size = 3.15

$$\frac{3\,e^{(-x)}+11\,e^{(-3\,x)}-11\,e^{(-5\,x)}-3\,e^{(-7\,x)}}{4\left(4\,e^{(-2\,x)}+6\,e^{(-4\,x)}+4\,e^{(-6\,x)}+e^{(-8\,x)}+1\right)}-\frac{3}{4}\arctan\left(e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(x)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(3*e^(-x) + 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*e^(-2*x) + 6*e^(
-4*x) + 4*e^(-6*x) + e^(-8*x) + 1) - 3/4*arctan(e^(-x))
```

Fricas [B] time = 2.09317, size = 1547, normalized size = 59.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(x)^5,x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(x)^7 + 21*cosh(x)*sinh(x)^6 + 3*sinh(x)^7 + (63*cosh(x)^2 + 11)
*sinh(x)^5 + 11*cosh(x)^5 + 5*(21*cosh(x)^3 + 11*cosh(x))*sinh(x)^4 + (105*
cosh(x)^4 + 110*cosh(x)^2 - 11)*sinh(x)^3 - 11*cosh(x)^3 + (63*cosh(x)^5 +
110*cosh(x)^3 - 33*cosh(x))*sinh(x)^2 + 3*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7
+ sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3
+ 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*
cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cos
h(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(
x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + s
inh(x)) + (21*cosh(x)^6 + 55*cosh(x)^4 - 33*cosh(x)^2 - 3)*sinh(x) - 3*cosh
(x))/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sin
h(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x
)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(
x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 +
 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + co
sh(x))*sinh(x) + 1)
```

Sympy [B] time = 3.98338, size = 422, normalized size = 16.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(x)**5,x)
```

```
[Out] 3*tanh(x/2)**8*atan(tanh(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(
x/2)**4 + 16*tanh(x/2)**2 + 4) - 5*tanh(x/2)**7/(4*tanh(x/2)**8 + 16*tanh(x
/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 12*tanh(x/2)**6*atan(tanh
(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**
2 + 4) + 3*tanh(x/2)**5/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4
 + 16*tanh(x/2)**2 + 4) + 18*tanh(x/2)**4*atan(tanh(x/2))/(4*tanh(x/2)**8 +
 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) - 3*tanh(x/2)**3/
(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4)
+ 12*tanh(x/2)**2*atan(tanh(x/2))/(4*tanh(x/2)**8 + 16*tanh(x/2)**6 + 24*ta
nh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 5*tanh(x/2)/(4*tanh(x/2)**8 + 16*tanh(x
/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4) + 3*atan(tanh(x/2))/(4*tanh
(x/2)**8 + 16*tanh(x/2)**6 + 24*tanh(x/2)**4 + 16*tanh(x/2)**2 + 4)
```

Giac [B] time = 1.16145, size = 81, normalized size = 3.12

$$\frac{3}{16}\pi - \frac{3\left(e^{(-x)} - e^{x}\right)^{3} + 20\,e^{(-x)} - 20\,e^{x}}{4\left(\left(e^{(-x)} - e^{x}\right)^{2} + 4\right)^{2}} + \frac{3}{8}\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cosh(x)^5,x, algorithm="giac")
```

```
[Out] 3/16*pi - 1/4*(3*(e^(-x) - e^x)^3 + 20*e^(-x) - 20*e^x)/((e^(-x) - e^x)^2 +
 4)^2 + 3/8*arctan(1/2*(e^(2*x) - 1)*e^(-x))
```

3.581 $\int \sinh^4(x)\tanh(x)\,dx$

Optimal. Leaf size=18

$$\frac{\cosh^4(x)}{4}-\cosh^2(x)+\log(\cosh(x))$$

```
[Out] -Cosh[x]^2 + Cosh[x]^4/4 + Log[Cosh[x]]
```

Rubi [A] time = 0.0225872, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2590, 266, 43}

$$\frac{\cosh^4(x)}{4}-\cosh^2(x)+\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]^4*Tanh[x],x]

[Out] -Cosh[x]^2 + Cosh[x]^4/4 + Log[Cosh[x]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \sinh^4(x)\tanh(x)\,dx &= \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x}\,dx,x,\cosh(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1-x)^2}{x}\,dx,x,\cosh^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-2+\frac{1}{x}+x\right)dx,x,\cosh^2(x)\right)\\
&= -\cosh^2(x)+\frac{\cosh^4(x)}{4}+\log(\cosh(x))
\end{aligned}$$

Mathematica [A] time = 0.0060303, size = 18, normalized size = 1.

$$\frac{\cosh^4(x)}{4}-\cosh^2(x)+\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]^4*Tanh[x],x]
```

```
[Out] -Cosh[x]^2 + Cosh[x]^4/4 + Log[Cosh[x]]
```

Maple [A] time = 0.013, size = 17, normalized size = 0.9

$$\frac{(\sinh(x))^4}{4}-\frac{(\sinh(x))^2}{2}+\ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/sech(x)^4,x)
```

```
[Out] 1/4*sinh(x)^4-1/2*sinh(x)^2+ln(cosh(x))
```

Maxima [B] time = 1.40772, size = 47, normalized size = 2.61

$$-\frac{1}{64}\left(12\,e^{(-2\,x)}-1\right)e^{(4\,x)}+x-\frac{3}{16}\,e^{(-2\,x)}+\frac{1}{64}\,e^{(-4\,x)}+\log\left(e^{(-2\,x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/sech(x)^4,x, algorithm="maxima")
```

```
[Out] -1/64*(12*e^(-2*x) - 1)*e^(4*x) + x - 3/16*e^(-2*x) + 1/64*e^(-4*x) + log(e
^(-2*x) + 1)
```

Fricas [B] time = 2.1511, size = 863, normalized size = 47.94

$$\frac{\cosh(x)^8+8\cosh(x)\sinh(x)^7+\sinh(x)^8+4\left(7\cosh(x)^2-3\right)\sinh(x)^6-12\cosh(x)^6+8\left(7\cosh(x)^3-9\cosh(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/sech(x)^4,x, algorithm="fricas")
```

```
[Out] 1/64*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 3)*sin
h(x)^6 - 12*cosh(x)^6 + 8*(7*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 - 64*x*cosh(x
)^4 + 2*(35*cosh(x)^4 - 90*cosh(x)^2 - 32*x)*sinh(x)^4 + 8*(7*cosh(x)^5 - 3
0*cosh(x)^3 - 32*x*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 - 45*cosh(x)^4 - 96*
x*cosh(x)^2 - 3)*sinh(x)^2 - 12*cosh(x)^2 + 64*(cosh(x)^4 + 4*cosh(x)^3*sin
h(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)*log(2*cosh(
x)/(cosh(x) - sinh(x))) + 8*(cosh(x)^7 - 9*cosh(x)^5 - 32*x*cosh(x)^3 - 3*c
osh(x))*sinh(x) + 1)/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)
```

```
^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(x)}{\operatorname{sech}^4(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**5/sech(x)**4,x)
```

```
[Out] Integral(tanh(x)**5/sech(x)**4, x)
```

Giac [B] time = 1.1268, size = 58, normalized size = 3.22

$$\frac{1}{64}\left(48\,e^{(4\,x)} - 12\,e^{(2\,x)} + 1\right)e^{(-4\,x)} - x + \frac{1}{64}\,e^{(4\,x)} - \frac{3}{16}\,e^{(2\,x)} + \log\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/sech(x)^4,x, algorithm="giac")
```

```
[Out] 1/64*(48*e^(4*x) - 12*e^(2*x) + 1)*e^(-4*x) - x + 1/64*e^(4*x) - 3/16*e^(2*
x) + log(e^(2*x) + 1)
```

3.582 $\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx$

Optimal. Leaf size=31

$$-\frac{4}{19}\operatorname{sech}^{\frac{19}{4}}(x)+\frac{8}{11}\operatorname{sech}^{\frac{11}{4}}(x)-\frac{4}{3}\operatorname{sech}^{\frac{3}{4}}(x)$$

```
[Out] (-4*Sech[x]^(3/4))/3 + (8*Sech[x]^(11/4))/11 - (4*Sech[x]^(19/4))/19
```

Rubi [A] time = 0.0311796, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2622, 270}

$$-\frac{4}{19}\operatorname{sech}^{\frac{19}{4}}(x)+\frac{8}{11}\operatorname{sech}^{\frac{11}{4}}(x)-\frac{4}{3}\operatorname{sech}^{\frac{3}{4}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^(23/4)*Sinh[x]^5,x]
```

```
[Out] (-4*Sech[x]^(3/4))/3 + (8*Sech[x]^(11/4))/11 - (4*Sech[x]^(19/4))/19
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^{\frac{23}{4}}(x)\sinh^5(x)\,dx&=-\operatorname{Subst}\left(\int\frac{(-1+x^2)^2}{\sqrt[4]{x}}\,dx,x,\operatorname{sech}(x)\right)\\&=-\operatorname{Subst}\left(\int\left(\frac{1}{\sqrt[4]{x}}-2x^{7/4}+x^{15/4}\right)dx,x,\operatorname{sech}(x)\right)\\&=-\frac{4}{3}\operatorname{sech}^{\frac{3}{4}}(x)+\frac{8}{11}\operatorname{sech}^{\frac{11}{4}}(x)-\frac{4}{19}\operatorname{sech}^{\frac{19}{4}}(x)\end{aligned}$$

Mathematica [A] time = 0.0530409, size = 27, normalized size = 0.87

$$\operatorname{sech}^{\frac{3}{4}}(x)\left(-\frac{4}{19}\operatorname{sech}^4(x)+\frac{8\operatorname{sech}^2(x)}{11}-\frac{4}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^(23/4)*Sinh[x]^5,x]
```

```
[Out] Sech[x]^(3/4)*(-4/3 + (8*Sech[x]^2)/11 - (4*Sech[x]^4)/19)
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int (\operatorname{sech}(x))^{\frac{3}{4}} (\tanh(x))^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^(3/4)*tanh(x)^5,x)
```

```
[Out] int(sech(x)^(3/4)*tanh(x)^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}(x)^{\frac{3}{4}} \tanh(x)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="maxima")
```

```
[Out] integrate(sech(x)^(3/4)*tanh(x)^5, x)
```

Fricas [B] time = 2.11994, size = 1253, normalized size = 40.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="fricas")
```

```
[Out] -4/627*2^(3/4)*(209*cosh(x)^8 + 1672*cosh(x)*sinh(x)^7 + 209*sinh(x)^8 + 76
*(77*cosh(x)^2 + 5)*sinh(x)^6 + 380*cosh(x)^6 + 152*(77*cosh(x)^3 + 15*cosh
(x))*sinh(x)^5 + 10*(1463*cosh(x)^4 + 570*cosh(x)^2 + 87)*sinh(x)^4 + 870*c
osh(x)^4 + 8*(1463*cosh(x)^5 + 950*cosh(x)^3 + 435*cosh(x))*sinh(x)^3 + 4*(
1463*cosh(x)^6 + 1425*cosh(x)^4 + 1305*cosh(x)^2 + 95)*sinh(x)^2 + 380*cosh
(x)^2 + 8*(209*cosh(x)^7 + 285*cosh(x)^5 + 435*cosh(x)^3 + 95*cosh(x))*sinh
(x) + 209)*((cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2
+ 1))^(3/4)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 +
 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35
*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 1
0*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh
(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)
^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**(3/4)*tanh(x)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{sech}(x)^{\frac{3}{4}} \tanh(x)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^(3/4)*tanh(x)^5,x, algorithm="giac")
```

```
[Out] integrate(sech(x)^(3/4)*tanh(x)^5, x)
```

3.583 $\int \frac{1}{a+b\cosh(x)}\,dx$

Optimal. Leaf size=41

$$\frac{2\tanh^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] (2*ArcTanh[((a - b)*Tanh[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Rubi [A] time = 0.0491715, antiderivative size = 42, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2659, 208}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[x])^(-1),x]
```

```
[Out] (2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b])
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\cosh(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{a+b-(a-b)x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\\ &= \frac{2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}\end{aligned}$$

Mathematica [A] time = 0.0316551, size = 41, normalized size = 1.

$$-\frac{2\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[x])^(-1),x]

[Out] (-2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]
```

Maple [A] time = 0.01, size = 36, normalized size = 0.9

$$2\frac{1}{\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tanh(x/2)}{\sqrt{(a+b)(a-b)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*cosh(x)),x)

[Out] 2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 2.10459, size = 460, normalized size = 11.22

$$\left[\frac{\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\left(b^2\cosh(x)+ab\right)\sinh(x)-2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)}{\sqrt{a^2-b^2}}, -\frac{2\sqrt{-a^2+b^2}\arctan\left(-\right.}{a^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)),x, algorithm="fricas")

[Out] [log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*
cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*
cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b))/sqr
t(a^2 - b^2), -2*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*s
inh(x) + a)/(a^2 - b^2))/(a^2 - b^2)]
```

Sympy [A] time = 10.6283, size = 126, normalized size = 3.07

$$\begin{cases} \tilde{\infty}\,\mathrm{atan}\left(\tanh\left(\frac{x}{2}\right)\right) & \text{for } a = 0 \wedge b = 0 \\ -\frac{1}{b\tanh\left(\frac{x}{2}\right)} & \text{for } a = -b \\ \frac{\tanh\left(\frac{x}{2}\right)}{b} & \text{for } a = b \\ -\frac{\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)),x)
```

```
[Out] Piecewise((zoo*atan(tanh(x/2)), Eq(a, 0) & Eq(b, 0)), (-1/(b*tanh(x/2)), Eq
(a, -b)), (tanh(x/2)/b, Eq(a, b)), (-log(-sqrt(a/(a - b) + b/(a - b)) + tan
h(x/2))/(a*sqrt(a/(a - b) + b/(a - b)) - b*sqrt(a/(a - b) + b/(a - b))) + l
og(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*sqrt(a/(a - b) + b/(a - b))
- b*sqrt(a/(a - b) + b/(a - b))), True))
```

Giac [A] time = 1.1686, size = 43, normalized size = 1.05

$$\frac{2\arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*cosh(x)),x, algorithm="giac")
```

```
[Out] 2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

3.584 $\int \frac{1}{(1+\cosh(x))^2} dx$

Optimal. Leaf size=25

$$\frac{\sinh(x)}{3(\cosh(x)+1)} + \frac{\sinh(x)}{3(\cosh(x)+1)^2}$$

```
[Out] Sinh[x]/(3*(1 + Cosh[x])^2) + Sinh[x]/(3*(1 + Cosh[x]))
```

Rubi [A] time = 0.0166621, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2650, 2648}

$$\frac{\sinh(x)}{3(\cosh(x)+1)} + \frac{\sinh(x)}{3(\cosh(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cosh[x])^(-2),x]
```

```
[Out] Sinh[x]/(3*(1 + Cosh[x])^2) + Sinh[x]/(3*(1 + Cosh[x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+\cosh(x))^2} dx &= \frac{\sinh(x)}{3(1+\cosh(x))^2} + \frac{1}{3}\int \frac{1}{1+\cosh(x)} dx \\ &= \frac{\sinh(x)}{3(1+\cosh(x))^2} + \frac{\sinh(x)}{3(1+\cosh(x))} \end{aligned}$$

Mathematica [A] time = 0.0114978, size = 16, normalized size = 0.64

$$\frac{\sinh(x)(\cosh(x)+2)}{3(\cosh(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cosh[x])^(-2),x]
```

```
[Out] ((2 + Cosh[x])*Sinh[x])/(3*(1 + Cosh[x])^2)
```

Maple [A] time = 0.005, size = 16, normalized size = 0.6

$$-\frac{1}{6}\left(\tanh\left(\frac{x}{2}\right)\right)^{3}+\frac{1}{2}\tanh\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cosh(x))^2,x)
```

```
[Out] -1/6*tanh(1/2*x)^3+1/2*tanh(1/2*x)
```

Maxima [B] time = 0.938846, size = 66, normalized size = 2.64

$$\frac{2\,e^{(-x)}}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1}+\frac{2}{3\left(3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x))^2,x, algorithm="maxima")
```

```
[Out] 2*e^(-x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 2/3/(3*e^(-x) + 3*e^(-2*x
) + e^(-3*x) + 1)
```

Fricas [B] time = 2.00102, size = 211, normalized size = 8.44

$$-\frac{2\,(3\,\cosh\left(x\right)+3\,\sinh\left(x\right)+1)}{3\left(\cosh\left(x\right)^{3}+3\,(\cosh\left(x\right)+1)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}+3\,\cosh\left(x\right)^{2}+3\left(\cosh\left(x\right)^{2}+2\,\cosh\left(x\right)+1\right)\sinh\left(x\right)+3\,\cosh\left(x\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(3*cosh(x) + 3*sinh(x) + 1)/(cosh(x)^3 + 3*(cosh(x) + 1)*sinh(x)^2 + s
inh(x)^3 + 3*cosh(x)^2 + 3*(cosh(x)^2 + 2*cosh(x) + 1)*sinh(x) + 3*cosh(x)
+ 1)
```

Sympy [A] time = 0.409911, size = 14, normalized size = 0.56

$$-\frac{\tanh^{3}\left(\frac{x}{2}\right)}{6}+\frac{\tanh\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x))**2,x)
```

```
[Out] -tanh(x/2)**3/6 + tanh(x/2)/2
```

Giac [A] time = 1.1271, size = 19, normalized size = 0.76

$$-\frac{2\,(3\,e^{x}+1)}{3\,(e^{x}+1)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cosh(x))^2,x, algorithm="giac")
```

```
[Out] -2/3*(3*e^x + 1)/(e^x + 1)^3
```

3.585 $\int \frac{1}{a+b\tanh(x)}\,dx$

Optimal. Leaf size=39

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rubi [A] time = 0.0499977, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3484, 3530}

$$\frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x)/(a^2 - b^2) - (b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Rule 3484

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[(a*x)/(a
^2 + b^2), x] + Dist[b/(a^2 + b^2), Int[(b - a*Tan[c + d*x])/(a + b*Tan[c +
 d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\tanh(x)}\,dx &= \frac{ax}{a^2-b^2}-\frac{(ib)\int \frac{-ib-ia\tanh(x)}{a+b\tanh(x)}\,dx}{a^2-b^2}\\ &= \frac{ax}{a^2-b^2}-\frac{b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}\end{aligned}$$

Mathematica [A] time = 0.0569922, size = 29, normalized size = 0.74

$$\frac{ax-b\log(a\cosh(x)+b\sinh(x))}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[x])^(-1),x]
```

```
[Out] (a*x - b*Log[a*Cosh[x] + b*Sinh[x]])/(a^2 - b^2)
```

Maple [A] time = 0.018, size = 55, normalized size = 1.4

$$\frac{\ln\left(1+\tanh\left(x\right)\right)}{2a-2b}-\frac{\ln\left(-1+\tanh\left(x\right)\right)}{2a+2b}-\frac{b\ln\left(a+b\tanh\left(x\right)\right)}{\left(a+b\right)\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*tanh(x)),x)
```

```
[Out] 1/(2*a-2*b)*ln(1+tanh(x))-1/(2*a+2*b)*ln(-1+tanh(x))-b/(a+b)/(a-b)*ln(a+b*t
anh(x))
```

Maxima [A] time = 0.942096, size = 55, normalized size = 1.41

$$-\frac{b\log\left(-(a-b)e^{(-2x)}-a-b\right)}{a^2-b^2}+\frac{x}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="maxima")
```

```
[Out] -b*log(-(a - b)*e^(-2*x) - a - b)/(a^2 - b^2) + x/(a + b)
```

Fricas [A] time = 2.23332, size = 108, normalized size = 2.77

$$\frac{(a+b)x-b\log\left(\frac{2\left(a\cosh(x)+b\sinh(x)\right)}{\cosh(x)-\sinh(x)}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="fricas")
```

```
[Out] ((a + b)*x - b*log(2*(a*cosh(x) + b*sinh(x))/(cosh(x) - sinh(x))))/(a^2 - b
^2)
```

Sympy [A] time = 0.662489, size = 146, normalized size = 3.74

$$\begin{cases} \tilde{\infty}\left(x-\log\left(\tanh\left(x\right)+1\right)+\log\left(\tanh\left(x\right)\right)\right) & \text{for } a=0\wedge b=0 \\ \frac{x}{a} & \text{for } b=0 \\ -\frac{x\tanh\left(x\right)}{2b\tanh\left(x\right)-2b}+\frac{x}{2b\tanh\left(x\right)-2b}+\frac{1}{2b\tanh\left(x\right)-2b} & \text{for } a=-b \\ \frac{x\tanh\left(x\right)}{2b\tanh\left(x\right)+2b}+\frac{x}{2b\tanh\left(x\right)+2b}-\frac{1}{2b\tanh\left(x\right)+2b} & \text{for } a=b \\ \frac{ax}{a^2-b^2}-\frac{bx}{a^2-b^2}-\frac{b\log\left(\frac{a}{b}+\tanh\left(x\right)\right)}{a^2-b^2}+\frac{b\log\left(\tanh\left(x\right)+1\right)}{a^2-b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)),x)
```

```
[Out] Piecewise((zoo*(x - log(tanh(x) + 1) + log(tanh(x))), Eq(a, 0) & Eq(b, 0)),
 (x/a, Eq(b, 0)), (-x*tanh(x)/(2*b*tanh(x) - 2*b) + x/(2*b*tanh(x) - 2*b) +
 1/(2*b*tanh(x) - 2*b), Eq(a, -b)), (x*tanh(x)/(2*b*tanh(x) + 2*b) + x/(2*b
*tanh(x) + 2*b) - 1/(2*b*tanh(x) + 2*b), Eq(a, b)), (a*x/(a**2 - b**2) - b*
x/(a**2 - b**2) - b*log(a/b + tanh(x))/(a**2 - b**2) + b*log(tanh(x) + 1)/(
a**2 - b**2), True))
```

Giac [A] time = 1.12581, size = 58, normalized size = 1.49

$$-\frac{b\log\left(\left|ae^{(2x)} + be^{(2x)} + a - b\right|\right)}{a^2 - b^2} + \frac{x}{a - b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)),x, algorithm="giac")
```

```
[Out] -b*log(abs(a*e^(2*x) + b*e^(2*x) + a - b))/(a^2 - b^2) + x/(a - b)
```

3.586 $\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 + b^2]]/(a*Sqrt[a^2 + b^2])
```

Rubi [A] time = 0.0353816, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3181, 208}

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + b^2*Cosh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 + b^2]]/(a*Sqrt[a^2 + b^2])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a^2+b^2\cosh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a^2-\left(a^2+b^2\right)x^2}\,dx, x, \coth(x)\right)\\ &= \frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A] time = 0.0638357, size = 31, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + b^2*Cosh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 + b^2]]/(a*Sqrt[a^2 + b^2])
```

Maple [B] time = 0.047, size = 98, normalized size = 3.2

$$\frac{1}{2a}\ln\left(\sqrt{a^2+b^2}\left(\tanh\left(\frac{x}{2}\right)\right)^2+2\,a\tanh(x/2)+\sqrt{a^2+b^2}\right)\frac{1}{\sqrt{a^2+b^2}}-\frac{1}{2a}\ln\left(\sqrt{a^2+b^2}\left(\tanh\left(\frac{x}{2}\right)\right)^2-2\,a\tanh(x/2)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+b^2*cosh(x)^2),x)
```

```
[Out] 1/2/a/(a^2+b^2)^(1/2)*ln((a^2+b^2)^(1/2)*tanh(1/2*x)^2+2*a*tanh(1/2*x)+(a^2
+b^2)^(1/2))-1/2/a/(a^2+b^2)^(1/2)*ln((a^2+b^2)^(1/2)*tanh(1/2*x)^2-2*a*tan
h(1/2*x)+(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.14807, size = 743, normalized size = 23.97

$$\frac{\sqrt{a^2+b^2}\log\left(\frac{b^4\cosh(x)^4+4\,b^4\cosh(x)\sinh(x)^3+b^4\sinh(x)^4+8\,a^4+8\,a^2b^2+b^4+2\left(2\,a^2b^2+b^4\right)\cosh(x)^2+2\left(3\,b^4\cosh(x)^2+2\,a^2b^2+b^4\right)\sinh(x)^2+4\left(b^4\cosh(\right.}{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4+2\left(2\,a^2+b^2\right)\cosh(x)^2+2\left(3\,b^2\cosh(x)^2+2\,a^2+\right.}\right.}{2\left(a^3+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a^2 + b^2)*log((b^4*cosh(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sinh
(x)^4 + 8*a^4 + 8*a^2*b^2 + b^4 + 2*(2*a^2*b^2 + b^4)*cosh(x)^2 + 2*(3*b^4*
cosh(x)^2 + 2*a^2*b^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 + (2*a^2*b^2 + b^
4)*cosh(x))*sinh(x) - 4*(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) + a*b^2*
sinh(x)^2 + 2*a^3 + a*b^2)*sqrt(a^2 + b^2))/(b^2*cosh(x)^4 + 4*b^2*cosh(x)*
sinh(x)^3 + b^2*sinh(x)^4 + 2*(2*a^2 + b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2
+ 2*a^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + (2*a^2 + b^2)*cosh(x))*
sinh(x)))/(a^3 + a*b^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+b**2*cosh(x)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.10479, size = 107, normalized size = 3.45

$$\frac{\log\left(\frac{b^2e^{(2x)}+2a^2+b^2-2\sqrt{a^2+b^2}|a|}{b^2e^{(2x)}+2a^2+b^2+2\sqrt{a^2+b^2}|a|}\right)}{2\sqrt{a^2+b^2}|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+b^2*cosh(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*log((b^2*e^(2*x) + 2*a^2 + b^2 - 2*sqrt(a^2 + b^2)*abs(a))/(b^2*e^(2*x)
 + 2*a^2 + b^2 + 2*sqrt(a^2 + b^2)*abs(a)))/(sqrt(a^2 + b^2)*abs(a))
```

3.587 $\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 - b^2]]/(a*Sqrt[a^2 - b^2])
```

Rubi [A] time = 0.0369354, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3181, 208}

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - b^2*Cosh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 - b^2]]/(a*Sqrt[a^2 - b^2])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a^2-b^2\cosh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a^2-\left(a^2-b^2\right)x^2}\,dx, x, \coth(x)\right)\\ &= \frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}\end{aligned}$$

Mathematica [A] time = 0.0447526, size = 35, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{a\tanh(x)}{\sqrt{a^2-b^2}}\right)}{a\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - b^2*Cosh[x]^2)^(-1),x]
```

```
[Out] ArcTanh[(a*Tanh[x])/Sqrt[a^2 - b^2]]/(a*Sqrt[a^2 - b^2])
```

Maple [B] time = 0.024, size = 74, normalized size = 2.1

$$\frac{1}{a}\mathrm{Artanh}\left((a+b)\tanh\left(\frac{x}{2}\right)\frac{1}{\sqrt{(a+b)(a-b)}}\right)\frac{1}{\sqrt{(a+b)(a-b)}}+\frac{1}{a}\mathrm{Artanh}\left((a-b)\tanh\left(\frac{x}{2}\right)\frac{1}{\sqrt{(a+b)(a-b)}}\right)\frac{1}{\sqrt{(a+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2-b^2*cosh(x)^2),x)
```

```
[Out] 1/a/((a+b)*(a-b))^(1/2)*arctanh((a+b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))+1/a/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-b^2*cosh(x)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.24971, size = 945, normalized size = 27.

$$\left[\frac{\sqrt{a^2-b^2}\log\left(\frac{b^4\cosh(x)^4+4\,b^4\cosh(x)\sinh(x)^3+b^4\sinh(x)^4+8\,a^4-8\,a^2b^2+b^4-2\left(2\,a^2b^2-b^4\right)\cosh(x)^2+2\left(3\,b^4\cosh(x)^2-2\,a^2b^2+b^4\right)\sinh(x)^2+4\left(b^4\,\mathrm{co}\right.}{b^2\cosh(x)^4+4\,b^2\cosh(x)\sinh(x)^3+b^2\sinh(x)^4-2\left(2\,a^2-b^2\right)\cosh(x)^2+2\left(3\,b^2\cosh(x)^2-2\right.}\right.}{2\left(a^3-ab^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-b^2*cosh(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(a^2 - b^2)*log((b^4*cosh(x)^4 + 4*b^4*cosh(x)*sinh(x)^3 + b^4*sin
h(x)^4 + 8*a^4 - 8*a^2*b^2 + b^4 - 2*(2*a^2*b^2 - b^4)*cosh(x)^2 + 2*(3*b^4
*cosh(x)^2 - 2*a^2*b^2 + b^4)*sinh(x)^2 + 4*(b^4*cosh(x)^3 - (2*a^2*b^2 - b
^4)*cosh(x))*sinh(x) + 4*(a*b^2*cosh(x)^2 + 2*a*b^2*cosh(x)*sinh(x) + a*b^2
*sinh(x)^2 - 2*a^3 + a*b^2)*sqrt(a^2 - b^2))/(b^2*cosh(x)^4 + 4*b^2*cosh(x)
*sinh(x)^3 + b^2*sinh(x)^4 - 2*(2*a^2 - b^2)*cosh(x)^2 + 2*(3*b^2*cosh(x)^2
 - 2*a^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 - (2*a^2 - b^2)*cosh(x))
*sinh(x)))/(a^3 - a*b^2), sqrt(-a^2 + b^2)*arctan(-1/2*(b^2*cosh(x)^2 + 2*b
^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 - 2*a^2 + b^2)*sqrt(-a^2 + b^2)/(a^3 - a
*b^2))/(a^3 - a*b^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2-b**2*cosh(x)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12546, size = 68, normalized size = 1.94

$$-\frac{\arctan\left(\frac{b^2e^{(2x)}-2a^2+b^2}{2\sqrt{-a^2+b^2}a}\right)}{\sqrt{-a^2+b^2}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-b^2*cosh(x)^2),x, algorithm="giac")
```

```
[Out] -arctan(1/2*(b^2*e^(2*x) - 2*a^2 + b^2)/(sqrt(-a^2 + b^2)*a))/(sqrt(-a^2 +
b^2)*a)
```

3.588 $\int \frac{1}{1-\sinh^4(x)}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}$$

```
[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(2*Sqrt[2]) + Tanh[x]/2
```

Rubi [A] time = 0.0174735, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3209, 388, 206}

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^4)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(2*Sqrt[2]) + Tanh[x]/2
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\sinh^4(x)}\,dx &= \text{Subst}\left(\int \frac{1-x^2}{1-2x^2}\,dx,x,\tanh(x)\right)\\ &= \frac{\tanh(x)}{2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\tanh(x)\right)\\ &= \frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0939069, size = 24, normalized size = 0.96

$$\frac{1}{4}\left(\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)+2\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^4)^(-1),x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + 2*Tanh[x])/4
```

Maple [B] time = 0.021, size = 55, normalized size = 2.2

$$\tanh\left(\frac{x}{2}\right)\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-1}+\frac{\sqrt{2}}{4}\mathrm{Artanh}\left(\frac{\sqrt{2}}{4}(2\tanh(x/2)-2)\right)+\frac{\sqrt{2}}{4}\mathrm{Artanh}\left(\frac{\sqrt{2}}{4}(2\tanh(x/2)+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^4),x)
```

```
[Out] tanh(1/2*x)/(tanh(1/2*x)^2+1)+1/4*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(
1/2))+1/4*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))
```

Maxima [B] time = 1.44069, size = 93, normalized size = 3.72

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)+\frac{1}{e^{(-2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/8*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) + 1/(e^(-2*x) + 1)
```

Fricas [B] time = 2.15747, size = 382, normalized size = 15.28

$$\frac{\left(\sqrt{2}\cosh(x)^2+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^2+\sqrt{2}\right)\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)+3\left(2\sqrt{2}-3\right)\sinh(x)^2}{\cosh(x)^2+\sinh(x)^2-3}\right.}{8\left(\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="fricas")
```

```
[Out] 1/8*((sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + s
qrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(
x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 -
3)) - 8)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**4),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16645, size = 65, normalized size = 2.6

$$-\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)-\frac{1}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6)) - 1/(e^(2*x) + 1)
```

3.589 $\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{3(\tanh(x)+1)}-\frac{4\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] (-4*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(3*(1 + Tanh[x]))
```

Rubi [A] time = 0.136227, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2074, 618, 204}

$$-\frac{1}{3(\tanh(x)+1)}-\frac{4\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]^3 - Sinh[x]^3)/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] (-4*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(3*(1 + Tanh[x]))
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^3(x)-\sinh^3(x)}{\cosh^3(x)+\sinh^3(x)}\,dx = \text{Subst}\left(\int \frac{1+x+x^2}{1+x+x^3+x^4}\,dx, x, \tanh(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{3(1+x)^2}+\frac{2}{3\left(1-x+x^2\right)}\right)dx, x, \tanh(x)\right)$$
$$= -\frac{1}{3(1+\tanh(x))}+\frac{2}{3}\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \tanh(x)\right)$$
$$= -\frac{1}{3(1+\tanh(x))}-\frac{4}{3}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\tanh(x)\right)$$
$$= -\frac{4\tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}-\frac{1}{3(1+\tanh(x))}$$

Mathematica [A] time = 0.128968, size = 37, normalized size = 1.12

$$\frac{1}{18}\left(3\sinh(2x)-3\cosh(2x)+8\sqrt{3}\tan^{-1}\left(\frac{2\tanh(x)-1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cosh[x]^3 - Sinh[x]^3)/(Cosh[x]^3 + Sinh[x]^3),x]
```

```
[Out] (8*Sqrt[3]*ArcTan[(-1 + 2*Tanh[x])/Sqrt[3]] - 3*Cosh[2*x] + 3*Sinh[2*x])/18
```

Maple [C] time = 0.084, size = 78, normalized size = 2.4

$$-\frac{2}{3}\left(1+\tanh\left(\frac{x}{2}\right)\right)^{-2}+\frac{2}{3}\left(1+\tanh\left(\frac{x}{2}\right)\right)^{-1}+\frac{2i}{9}\sqrt{3}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+\left(-1-i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)+1\right)-\frac{2i}{9}\sqrt{3}\ln\left(\left(\tanh\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x)
```

```
[Out] -2/3/(1+tanh(1/2*x))^2+2/3/(1+tanh(1/2*x))+2/9*I*3^(1/2)*ln(tanh(1/2*x)^2+(
-1-I*3^(1/2))*tanh(1/2*x)+1)-2/9*I*3^(1/2)*ln(tanh(1/2*x)^2+(-1+I*3^(1/2))*
tanh(1/2*x)+1)
```

Maxima [B] time = 1.46238, size = 95, normalized size = 2.88

$$\frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)-\frac{4}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)-\frac{1}{6}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x, algorithm="maxima"
)
```

```
[Out] 4/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2))
) - 4/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sqrt
```

```
(2))) - 1/6*e^(-2*x)
```

Fricas [B] time = 2.22686, size = 265, normalized size = 8.03

$$-\frac{8 \left(\sqrt{3} \cosh\left(x\right)^{2} + 2 \sqrt{3} \cosh\left(x\right) \sinh\left(x\right) + \sqrt{3} \sinh\left(x\right)^{2}\right) \arctan\left(-\frac{\sqrt{3} \cosh(x)+\sqrt{3} \sinh(x)}{3 (\cosh(x)-\sinh(x))}\right) + 3}{18 \left(\cosh\left(x\right)^{2} + 2 \cosh\left(x\right) \sinh\left(x\right) + \sinh\left(x\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x, algorithm="fricas"
)
```

```
[Out] -1/18*(8*(sqrt(3)*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2
)*arctan(-1/3*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) + 3)
/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] time = 1.99149, size = 102, normalized size = 3.09

$$\frac{4 \sqrt{3} \sinh\left(x\right) \operatorname{atan}\left(\frac{2 \sqrt{3} \sinh\left(x\right)}{3 \cosh\left(x\right)} - \frac{\sqrt{3}}{3}\right)}{9 \sinh\left(x\right) + 9 \cosh\left(x\right)} + \frac{4 \sqrt{3} \cosh\left(x\right) \operatorname{atan}\left(\frac{2 \sqrt{3} \sinh\left(x\right)}{3 \cosh\left(x\right)} - \frac{\sqrt{3}}{3}\right)}{9 \sinh\left(x\right) + 9 \cosh\left(x\right)} - \frac{3 \cosh\left(x\right)}{9 \sinh\left(x\right) + 9 \cosh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)**3-sinh(x)**3)/(cosh(x)**3+sinh(x)**3),x)
```

```
[Out] 4*sqrt(3)*sinh(x)*atan(2*sqrt(3)*sinh(x)/(3*cosh(x)) - sqrt(3)/3)/(9*sinh(x
) + 9*cosh(x)) + 4*sqrt(3)*cosh(x)*atan(2*sqrt(3)*sinh(x)/(3*cosh(x)) - sqr
t(3)/3)/(9*sinh(x) + 9*cosh(x)) - 3*cosh(x)/(9*sinh(x) + 9*cosh(x))
```

Giac [A] time = 1.08848, size = 30, normalized size = 0.91

$$\frac{4}{9} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} e^{\left(2 x\right)}\right) - \frac{1}{6} e^{\left(-2 x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cosh(x)^3-sinh(x)^3)/(cosh(x)^3+sinh(x)^3),x, algorithm="giac")
```

```
[Out] 4/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) - 1/6*e^(-2*x)
```

3.590 $\int \cosh(x)\cosh(2x)\cosh(3x)\,dx$

Optimal. Leaf size=30

$$\frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{1}{16}\sinh(4x)+\frac{1}{24}\sinh(6x)$$

```
[Out] x/4 + Sinh[2*x]/8 + Sinh[4*x]/16 + Sinh[6*x]/24
```

Rubi [A] time = 0.0345891, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2637}

$$\frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{1}{16}\sinh(4x)+\frac{1}{24}\sinh(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Cosh[2*x]*Cosh[3*x],x]
```

```
[Out] x/4 + Sinh[2*x]/8 + Sinh[4*x]/16 + Sinh[6*x]/24
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cosh(x)\cosh(2x)\cosh(3x)\,dx &= \int \left(\frac{1}{4}+\frac{1}{4}\cosh(2x)+\frac{1}{4}\cosh(4x)+\frac{1}{4}\cosh(6x)\right)dx\\ &= \frac{x}{4}+\frac{1}{4}\int \cosh(2x)\,dx+\frac{1}{4}\int \cosh(4x)\,dx+\frac{1}{4}\int \cosh(6x)\,dx\\ &= \frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{1}{16}\sinh(4x)+\frac{1}{24}\sinh(6x)\end{aligned}$$

Mathematica [A] time = 0.0099438, size = 30, normalized size = 1.

$$\frac{x}{4}+\frac{1}{8}\sinh(2x)+\frac{1}{16}\sinh(4x)+\frac{1}{24}\sinh(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Cosh[2*x]*Cosh[3*x],x]
```

```
[Out] x/4 + Sinh[2*x]/8 + Sinh[4*x]/16 + Sinh[6*x]/24
```

Maple [A] time = 0.039, size = 23, normalized size = 0.8

$$\frac{x}{4}+\frac{\sinh(2x)}{8}+\frac{\sinh(4x)}{16}+\frac{\sinh(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*cosh(2*x)*cosh(3*x),x)
```

```
[Out] 1/4*x+1/8*sinh(2*x)+1/16*sinh(4*x)+1/24*sinh(6*x)
```

Maxima [A] time = 0.951413, size = 57, normalized size = 1.9

$$\frac{1}{96}\left(3e^{(-2x)}+6e^{(-4x)}+2\right)e^{(6x)}+\frac{1}{4}x-\frac{1}{16}e^{(-2x)}-\frac{1}{32}e^{(-4x)}-\frac{1}{48}e^{(-6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(2*x)*cosh(3*x),x, algorithm="maxima")
```

```
[Out] 1/96*(3*e^(-2*x) + 6*e^(-4*x) + 2)*e^(6*x) + 1/4*x - 1/16*e^(-2*x) - 1/32*e
^(-4*x) - 1/48*e^(-6*x)
```

Fricas [A] time = 2.08525, size = 166, normalized size = 5.53

$$\frac{1}{4}\cosh(x)\sinh(x)^5+\frac{1}{12}\left(10\cosh(x)^3+3\cosh(x)\right)\sinh(x)^3+\frac{1}{4}\left(\cosh(x)^5+\cosh(x)^3+\cosh(x)\right)\sinh(x)+\frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(2*x)*cosh(3*x),x, algorithm="fricas")
```

```
[Out] 1/4*cosh(x)*sinh(x)^5 + 1/12*(10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 1/4*(co
sh(x)^5 + cosh(x)^3 + cosh(x))*sinh(x) + 1/4*x
```

Sympy [B] time = 13.0053, size = 116, normalized size = 3.87

$$\frac{x\sinh(x)\sinh(2x)\cosh(3x)}{4}-\frac{x\sinh(x)\sinh(3x)\cosh(2x)}{4}-\frac{x\sinh(2x)\sinh(3x)\cosh(x)}{4}+\frac{x\cosh(x)\cosh(2x)\cos}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(2*x)*cosh(3*x),x)
```

```
[Out] x*sinh(x)*sinh(2*x)*cosh(3*x)/4 - x*sinh(x)*sinh(3*x)*cosh(2*x)/4 - x*sinh(
2*x)*sinh(3*x)*cosh(x)/4 + x*cosh(x)*cosh(2*x)*cosh(3*x)/4 - 3*sinh(x)*sinh
(2*x)*sinh(3*x)/8 + sinh(x)*cosh(2*x)*cosh(3*x)/3 + 5*sinh(2*x)*cosh(x)*cos
h(3*x)/24
```

Giac [B] time = 1.09158, size = 65, normalized size = 2.17

$$-\frac{1}{96}\left(22\,e^{(6\,x)}+6\,e^{(4\,x)}+3\,e^{(2\,x)}+2\right)e^{(-6\,x)}+\frac{1}{4}\,x+\frac{1}{48}\,e^{(6\,x)}+\frac{1}{32}\,e^{(4\,x)}+\frac{1}{16}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*cosh(2*x)*cosh(3*x),x, algorithm="giac")
```

```
[Out] -1/96*(22*e^(6*x) + 6*e^(4*x) + 3*e^(2*x) + 2)*e^(-6*x) + 1/4*x + 1/48*e^(6
*x) + 1/32*e^(4*x) + 1/16*e^(2*x)
```

3.591 $\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)\,dx$

Optimal. Leaf size=30

$$-\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)$$

```
[Out] -x/4 + Sinh[2*x]/8 - Sinh[3*x]/12 + Sinh[5*x]/20
```

Rubi [A] time = 0.0343048, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4355, 2637}

$$-\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[(3*x)/2]*Sinh[x]*Sinh[(5*x)/2],x]
```

```
[Out] -x/4 + Sinh[2*x]/8 - Sinh[3*x]/12 + Sinh[5*x]/20
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cosh\left(\frac{3x}{2}\right)\sinh(x)\sinh\left(\frac{5x}{2}\right)dx &= -\int\left(\frac{1}{4}-\frac{1}{4}\cosh(2x)+\frac{1}{4}\cosh(3x)-\frac{1}{4}\cosh(5x)\right)dx\\ &= -\frac{x}{4}+\frac{1}{4}\int\cosh(2x)\,dx-\frac{1}{4}\int\cosh(3x)\,dx+\frac{1}{4}\int\cosh(5x)\,dx\\ &= -\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)\end{aligned}$$

Mathematica [A] time = 0.0098559, size = 30, normalized size = 1.

$$-\frac{x}{4}+\frac{1}{8}\sinh(2x)-\frac{1}{12}\sinh(3x)+\frac{1}{20}\sinh(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[(3*x)/2]*Sinh[x]*Sinh[(5*x)/2],x]
```

```
[Out] -x/4 + Sinh[2*x]/8 - Sinh[3*x]/12 + Sinh[5*x]/20
```

Maple [A] time = 0.099, size = 23, normalized size = 0.8

$$-\frac{x}{4}+\frac{\sinh(2x)}{8}-\frac{\sinh(3x)}{12}+\frac{\sinh(5x)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x)
```

```
[Out] -1/4*x+1/8*sinh(2*x)-1/12*sinh(3*x)+1/20*sinh(5*x)
```

Maxima [A] time = 0.951566, size = 57, normalized size = 1.9

$$-\frac{1}{240}\left(10\,e^{(-2x)}-15\,e^{(-3x)}-6\right)e^{(5x)}-\frac{1}{4}\,x-\frac{1}{16}\,e^{(-2x)}+\frac{1}{24}\,e^{(-3x)}-\frac{1}{40}\,e^{(-5x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x, algorithm="maxima")
```

```
[Out] -1/240*(10*e^(-2*x) - 15*e^(-3*x) - 6)*e^(5*x) - 1/4*x - 1/16*e^(-2*x) + 1/
24*e^(-3*x) - 1/40*e^(-5*x)
```

Fricas [B] time = 2.20653, size = 362, normalized size = 12.07

$$6\cosh\left(\frac{1}{2}x\right)^{3}\sinh\left(\frac{1}{2}x\right)^{7}+\frac{1}{2}\cosh\left(\frac{1}{2}x\right)\sinh\left(\frac{1}{2}x\right)^{9}+\frac{1}{10}\left(126\cosh\left(\frac{1}{2}x\right)^{5}-5\cosh\left(\frac{1}{2}x\right)\right)\sinh\left(\frac{1}{2}x\right)^{5}+\frac{1}{6}\left(36\text{ c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x, algorithm="fricas")
```

```
[Out] 6*cosh(1/2*x)^3*sinh(1/2*x)^7 + 1/2*cosh(1/2*x)*sinh(1/2*x)^9 + 1/10*(126*c
osh(1/2*x)^5 - 5*cosh(1/2*x))*sinh(1/2*x)^5 + 1/6*(36*cosh(1/2*x)^7 - 10*co
sh(1/2*x)^3 + 3*cosh(1/2*x))*sinh(1/2*x)^3 + 1/2*(cosh(1/2*x)^9 - cosh(1/2*
x)^5 + cosh(1/2*x)^3)*sinh(1/2*x) - 1/4*x
```

Sympy [B] time = 12.0042, size = 138, normalized size = 4.6

$$-\frac{x\sinh(x)\sinh\left(\frac{3x}{2}\right)\cosh\left(\frac{5x}{2}\right)}{4}+\frac{x\sinh(x)\sinh\left(\frac{5x}{2}\right)\cosh\left(\frac{3x}{2}\right)}{4}+\frac{x\sinh\left(\frac{3x}{2}\right)\sinh\left(\frac{5x}{2}\right)\cosh(x)}{4}-\frac{x\cosh(x)\cosh\left(\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x)
```

```
[Out] -x*sinh(x)*sinh(3*x/2)*cosh(5*x/2)/4 + x*sinh(x)*sinh(5*x/2)*cosh(3*x/2)/4
+ x*sinh(3*x/2)*sinh(5*x/2)*cosh(x)/4 - x*cosh(x)*cosh(3*x/2)*cosh(5*x/2)/4
```

```
 - sinh(x)*sinh(3*x/2)*sinh(5*x/2)/12 + 7*sinh(x)*cosh(3*x/2)*cosh(5*x/2)/2
0 - sinh(3*x/2)*cosh(x)*cosh(5*x/2)/15
```

Giac [B] time = 1.08488, size = 65, normalized size = 2.17

$$\frac{1}{240}\left(137\,e^{(5\,x)}-15\,e^{(3\,x)}+10\,e^{(2\,x)}-6\right)e^{(-5\,x)}-\frac{1}{4}\,x+\frac{1}{40}\,e^{(5\,x)}-\frac{1}{24}\,e^{(3\,x)}+\frac{1}{16}\,e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(3/2*x)*sinh(x)*sinh(5/2*x),x, algorithm="giac")
```

```
[Out] 1/240*(137*e^(5*x) - 15*e^(3*x) + 10*e^(2*x) - 6)*e^(-5*x) - 1/4*x + 1/40*e
^(5*x) - 1/24*e^(3*x) + 1/16*e^(2*x)
```

3.592 $\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\left(\sinh^2(x)+\sinh(2x)\right)}\,dx$

Optimal. Leaf size=69

$$\frac{1}{6}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{\sinh(2x)}}\right)+\frac{\cosh(x)}{\sqrt{\sinh(2x)}}+\sqrt{2}\tan^{-1}\left(\text{sech}(x)\sqrt{\sinh(x)\cosh(x)}\right)-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\text{sech}(x)\sqrt{\sinh(x)\cosh(x)}\right)$$

```
[Out] Sqrt[2]*ArcTan[Sech[x]*Sqrt[Cosh[x]*Sinh[x]]] + ArcTan[Sinh[x]/Sqrt[Sinh[2*
x]]]/6 - (Sqrt[2]*ArcTanh[Sech[x]*Sqrt[Cosh[x]*Sinh[x]]])/3 + Cosh[x]/Sqrt[
Sinh[2*x]]
```

Rubi [A] time = 0.973021, antiderivative size = 102, normalized size of antiderivative = 1.48, number of steps used = 8, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {4390, 6725, 207, 203}

$$-\frac{2\sinh(x)\tanh^{-1}\left(\sqrt{\tanh(x)}\right)}{3\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}+\frac{\cosh(x)}{\sqrt{\sinh(2x)}}+\frac{2\sinh(x)\tan^{-1}\left(\sqrt{\tanh(x)}\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}+\frac{\sinh(x)\tan^{-1}\left(\frac{\sqrt{\tanh(x)}}{\sqrt{2}}\right)}{3\sqrt{2}\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cosh[x]*(-Cosh[2*x] + Tanh[x]))/(Sqrt[Sinh[2*x]]*(Sinh[x]^2 + Sinh[2*x
])),x]
```

```
[Out] Cosh[x]/Sqrt[Sinh[2*x]] + (2*ArcTan[Sqrt[Tanh[x]]]*Sinh[x])/(Sqrt[Sinh[2*x]
]*Sqrt[Tanh[x]]) + (ArcTan[Sqrt[Tanh[x]]/Sqrt[2]]*Sinh[x])/(3*Sqrt[2]*Sqrt[
Sinh[2*x]]*Sqrt[Tanh[x]]) - (2*ArcTanh[Sqrt[Tanh[x]]]*Sinh[x])/(3*Sqrt[Sinh
[2*x]]*Sqrt[Tanh[x]])
```

Rule 4390

```
Int[(u_)*((c_.)*sin[v_])^(m_), x_Symbol] :> With[{w = FunctionOfTrig[(u*Sin
[v/2]^(2*m))/(c*Tan[v/2])^m, x]}, Dist[((c*Sin[v])^m*(c*Tan[v/2])^m)/Sin[v/
2]^(2*m), Int[(u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x], x] /;  !FalseQ[w] && F
unctionOfQ[NonfreeFactors[Tan[w], x], (u*Sin[v/2]^(2*m))/(c*Tan[v/2])^m, x]
] /; FreeQ[c, x] && LinearQ[v, x] && IntegerQ[m + 1/2] &&  !SumQ[u] && Inve
rseFunctionFreeQ[u, x]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh(x)(-\cosh(2x)+\tanh(x))}{\sqrt{\sinh(2x)}\left(\sinh^2(x)+\sinh(2x)\right)}\,dx = \frac{\sinh(x)\int \frac{-\cosh(2x)+\tanh(x)}{\left(\sinh^2(x)+\sinh(2x)\right)\sqrt{\tanh(x)}}\,dx}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$= \frac{\sinh(x)\,\text{Subst}\left(\int \frac{-1+x-x^2-x^3}{x^{3/2}(2+x)(1-x^2)}\,dx, x, \tanh(x)\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$= \frac{(2\sinh(x))\,\text{Subst}\left(\int \frac{1-x^2+x^4+x^6}{x^2(2+x^2)(-1+x^4)}\,dx, x, \sqrt{\tanh(x)}\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$= \frac{(2\sinh(x))\,\text{Subst}\left(\int \left(-\frac{1}{2x^2}+\frac{1}{3(-1+x^2)}+\frac{1}{1+x^2}+\frac{1}{6(2+x^2)}\right)dx, x, \sqrt{\tanh(x)}\right)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}}$$

$$= \frac{\cosh(x)}{\sqrt{\sinh(2x)}} + \frac{\sinh(x)\,\text{Subst}\left(\int \frac{1}{2+x^2}\,dx, x, \sqrt{\tanh(x)}\right)}{3\sqrt{\sinh(2x)}\sqrt{\tanh(x)}} + \frac{(2\sinh(x))\,\text{Subst}\left(\int\right.}{3\sqrt{\sinh(2}}$$

$$= \frac{\cosh(x)}{\sqrt{\sinh(2x)}} + \frac{2\tan^{-1}\left(\sqrt{\tanh(x)}\right)\sinh(x)}{\sqrt{\sinh(2x)}\sqrt{\tanh(x)}} + \frac{\tan^{-1}\left(\frac{\sqrt{\tanh(x)}}{\sqrt{2}}\right)\sinh(x)}{3\sqrt{2}\sqrt{\sinh(2x)}\sqrt{\tanh(x)}} - \frac{2\tan}{3}$$

Mathematica [C] time = 29.6156, size = 392, normalized size = 5.68

$$\sqrt{\sinh(2x)}(\tanh(x)-\cosh(2x))\left(-3\coth(x)+\frac{\sqrt[4]{-1}\cosh(x)\sqrt{\tanh^3\left(\frac{x}{2}\right)+\tanh\left(\frac{x}{2}\right)}\left(\frac{8\sqrt[6]{-1}\left(2\left(\sqrt[3]{-1}-1\right)\Pi\left(i;\sin^{-1}\left((-1)^{3/4}\sqrt{\tanh\left(\frac{x}{2}\right)}\right)\middle|-1\right)+\left(3-3i\sqrt{3}\right)\Pi\left(-i\right.\right.}{}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cosh[x]*(-Cosh[2*x] + Tanh[x]))/(Sqrt[Sinh[2*x]]*(Sinh[x]^2 + Si
nh[2*x])),x]
```

```
[Out] (Sqrt[Sinh[2*x]]*(-3*Coth[x] + ((-1)^(1/4)*Cosh[x]*Sqrt[Tanh[x/2] + Tanh[x/
2]^3]*((-9*Coth[x/2]*(EllipticF[I*ArcSinh[(-1)^(1/4)/Sqrt[Tanh[x/2]]], -1]
- EllipticPi[-(-1)^(1/6), I*ArcSinh[(-1)^(1/4)/Sqrt[Tanh[x/2]]], -1] - Elli
pticPi[-(-1)^(5/6), I*ArcSinh[(-1)^(1/4)/Sqrt[Tanh[x/2]]], -1]))/Sqrt[1 + C
oth[x/2]^2] + (8*(-1)^(1/6)*((3 - (3*I)*Sqrt[3])*EllipticPi[-I, I*ArcSinh[(
-1)^(1/4)*Sqrt[Tanh[x/2]]], -1] + 2*(-1 + (-1)^(1/3))*EllipticPi[I, ArcSin[
(-1)^(3/4)*Sqrt[Tanh[x/2]]], -1] + I*(I + Sqrt[3])*EllipticPi[-(-1)^(1/6),
I*ArcSinh[(-1)^(1/4)*Sqrt[Tanh[x/2]]], -1] + 2*(-1 + (-1)^(1/3))*EllipticPi
[-(-1)^(5/6), I*ArcSinh[(-1)^(1/4)*Sqrt[Tanh[x/2]]], -1]))/((-I + Sqrt[3])*
Sqrt[1 + Tanh[x/2]^2])))/((1 + Cosh[x])*Sqrt[Sinh[2*x]/(1 + Cosh[x])^2]*Sqr
t[Tanh[x/2]]))*(-Cosh[2*x] + Tanh[x]))/(3*(Cosh[x] + Cosh[3*x] - 2*Sinh[x])
)
```

Maple [C] time = 0.217, size = 609, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2),x)
```

```
[Out] 1/12*((tanh(1/2*x)^2+1)*tanh(1/2*x)/(tanh(1/2*x)^2-1)^2)^(1/2)*(tanh(1/2*x)
^2-1)*(-12*I*(-I*(tanh(1/2*x)+I))^(1/2)*2^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)
*(I*tanh(1/2*x))^(1/2)*EllipticPi((-I*(tanh(1/2*x)+I))^(1/2),1/2-1/2*I,1/2*
2^(1/2))*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)+9*I*(-I*(tanh(1/2*x)+I))^(1/
2)*2^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)*EllipticF((-I*
(tanh(1/2*x)+I))^(1/2),1/2*2^(1/2))*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)+4
*I*(-I*(tanh(1/2*x)+I))^(1/2)*2^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1
/2*x))^(1/2)*EllipticPi((-I*(tanh(1/2*x)+I))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(
(tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)+12*(-I*(tanh(1/2*x)+I))^(1/2)*2^(1/2)*
(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)*EllipticPi((-I*(tanh(1/2*
x)+I))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)+4
*(-I*(tanh(1/2*x)+I))^(1/2)*2^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2
*x))^(1/2)*EllipticPi((-I*(tanh(1/2*x)+I))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((t
anh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)+I*2^(1/2)*sum(_alpha*(I*_alpha+1+I)*(-I*
(tanh(1/2*x)+I))^(1/2)*(-I*(-tanh(1/2*x)+I))^(1/2)*(I*tanh(1/2*x))^(1/2)/((
tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)*EllipticPi((-I*(tanh(1/2*x)+I))^(1/2),_
alpha+1-I,1/2*2^(1/2)),_alpha=RootOf(_Z^2+_Z+1))*(tanh(1/2*x)^3+tanh(1/2*x)
)^(1/2)*((tanh(1/2*x)^2+1)*tanh(1/2*x))^(1/2)-6*(tanh(1/2*x)^3+tanh(1/2*x))
^(1/2)*tanh(1/2*x)^2-6*(tanh(1/2*x)^3+tanh(1/2*x))^(1/2))/(tanh(1/2*x)^2+1)
/tanh(1/2*x)/(tanh(1/2*x)^3+tanh(1/2*x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{(\cosh(2x) - \tanh(x))\cosh(x)}{\left(\sinh(x)^2 + \sinh(2x)\right)\sqrt{\sinh(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2
),x, algorithm="maxima")
```

```
[Out] -integrate((cosh(2*x) - tanh(x))*cosh(x)/((sinh(x)^2 + sinh(2*x))*sqrt(sinh
(2*x))), x)
```

Fricas [B] time = 2.47632, size = 1337, normalized size = 19.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2
),x, algorithm="fricas")
```

```
[Out] -1/12*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*arctan(1/2*(sqrt(2)*
cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + 3*sqrt(2))*sqrt
(cosh(x)*sinh(x)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(cosh(x)^4 +
```

```
4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)
^4 - 1)) + 6*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(
x)^2 - sqrt(2))*arctan(2*sqrt(cosh(x)*sinh(x)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 +
4*cosh(x)*sinh(x)^3 + sinh(x)^4 - 1)) - (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh
(x)*sinh(x) + sqrt(2)*sinh(x)^2 - sqrt(2))*log(2*cosh(x)^4 + 8*cosh(x)^3*si
nh(x) + 12*cosh(x)^2*sinh(x)^2 + 8*cosh(x)*sinh(x)^3 + 2*sinh(x)^4 - 4*(cos
h(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(cosh(x)*sinh(x)/(cosh(x)^2 - 2
*cosh(x)*sinh(x) + sinh(x)^2)) - 1) - 12*sqrt(2)*sqrt(cosh(x)*sinh(x)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2 - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)**2+sinh(2*x))/sinh(2*x)**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{(\cosh(2x) - \tanh(x))\cosh(x)}{\left(\sinh(x)^2 + \sinh(2x)\right)\sqrt{\sinh(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*(-cosh(2*x)+tanh(x))/(sinh(x)^2+sinh(2*x))/sinh(2*x)^(1/2
),x, algorithm="giac")
```

```
[Out] integrate(-(cosh(2*x) - tanh(x))*cosh(x)/((sinh(x)^2 + sinh(2*x))*sqrt(sinh
(2*x))), x)
```

3.593 $\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=37

$$\frac{2\cosh(x)}{243\sqrt{4\cosh^2(x)-9}}-\frac{\cosh(x)}{27\left(4\cosh^2(x)-9\right)^{3/2}}$$

```
[Out] -Cosh[x]/(27*(-9 + 4*Cosh[x]^2)^(3/2)) + (2*Cosh[x])/(243*Sqrt[-9 + 4*Cosh[
x]^2])
```

Rubi [A] time = 0.0452293, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 192, 191}

$$\frac{2\cosh(x)}{243\sqrt{4\cosh^2(x)-9}}-\frac{\cosh(x)}{27\left(4\cosh^2(x)-9\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x]/(-9 + 4*Cosh[x]^2)^(5/2),x]
```

```
[Out] -Cosh[x]/(27*(-9 + 4*Cosh[x]^2)^(3/2)) + (2*Cosh[x])/(243*Sqrt[-9 + 4*Cosh[
x]^2])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sinh(x)}{\left(-9+4\cosh^2(x)\right)^{5/2}}\,dx = \text{Subst}\left(\int \frac{1}{\left(-9+4x^2\right)^{5/2}}\,dx, x, \cosh(x)\right)$$
$$= -\frac{\cosh(x)}{27\left(-9+4\cosh^2(x)\right)^{3/2}} - \frac{2}{27}\text{Subst}\left(\int \frac{1}{\left(-9+4x^2\right)^{3/2}}\,dx, x, \cosh(x)\right)$$
$$= -\frac{\cosh(x)}{27\left(-9+4\cosh^2(x)\right)^{3/2}} + \frac{2\cosh(x)}{243\sqrt{-9+4\cosh^2(x)}}$$

Mathematica [A] time = 0.0709346, size = 26, normalized size = 0.7

$$\frac{\cosh(x)(4\cosh(2x)-23)}{243(2\cosh(2x)-7)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x]/(-9 + 4*Cosh[x]^2)^(5/2),x]
```

```
[Out] (Cosh[x]*(-23 + 4*Cosh[2*x]))/(243*(-7 + 2*Cosh[2*x])^(3/2))
```

Maple [A] time = 0.012, size = 30, normalized size = 0.8

$$-\frac{\cosh(x)}{27}\left(-9+4\,(\cosh(x))^2\right)^{-\frac{3}{2}} + \frac{2\,\cosh(x)}{243}\frac{1}{\sqrt{-9+4\,(\cosh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x)
```

```
[Out] -1/27*cosh(x)/(-9+4*cosh(x)^2)^(3/2)+2/243*cosh(x)/(-9+4*cosh(x)^2)^(1/2)
```

Maple [B] time = 1.03513, size = 169, normalized size = 4.57

$$-\frac{1855\,e^{(-2\,x)} - 8485\,e^{(-4\,x)} + 5285\,e^{(-6\,x)} - 980\,e^{(-8\,x)} + 56\,e^{(-10\,x)} - 106}{12150\left(3\,e^{(-x)} + e^{(-2\,x)} + 1\right)^{\frac{5}{2}}\left(-3\,e^{(-x)} + e^{(-2\,x)} + 1\right)^{\frac{5}{2}}} + \frac{980\,e^{(-2\,x)} - 5285\,e^{(-4\,x)} + 8485\,e^{(-6\,x)} - 1855\,e^{(-}}{12150\left(3\,e^{(-x)} + e^{(-2\,x)} + 1\right)^{\frac{5}{2}}\left(-3\,e^{(-x)} +\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12150*(1855*e^(-2*x) - 8485*e^(-4*x) + 5285*e^(-6*x) - 980*e^(-8*x) + 56
*e^(-10*x) - 106)/((3*e^(-x) + e^(-2*x) + 1)^(5/2)*(-3*e^(-x) + e^(-2*x) +
1)^(5/2)) + 1/12150*(980*e^(-2*x) - 5285*e^(-4*x) + 8485*e^(-6*x) - 1855*e^
(-8*x) + 106*e^(-10*x) - 56)/((3*e^(-x) + e^(-2*x) + 1)^(5/2)*(-3*e^(-x) +
e^(-2*x) + 1)^(5/2))
```

Fricas [B] time = 2.05894, size = 1585, normalized size = 42.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/486*(2*cosh(x)^8 + 16*cosh(x)*sinh(x)^7 + 2*sinh(x)^8 + 28*(2*cosh(x)^2 -
 1)*sinh(x)^6 - 28*cosh(x)^6 + 56*(2*cosh(x)^3 - 3*cosh(x))*sinh(x)^5 + 2*(
70*cosh(x)^4 - 210*cosh(x)^2 + 51)*sinh(x)^4 + 102*cosh(x)^4 + 8*(14*cosh(x
)^5 - 70*cosh(x)^3 + 51*cosh(x))*sinh(x)^3 + 4*(14*cosh(x)^6 - 105*cosh(x)^
4 + 153*cosh(x)^2 - 7)*sinh(x)^2 - 28*cosh(x)^2 + 8*(2*cosh(x)^7 - 21*cosh(
x)^5 + 51*cosh(x)^3 - 7*cosh(x))*sinh(x) + (2*cosh(x)^6 + 12*cosh(x)*sinh(x
)^5 + 2*sinh(x)^6 + 3*(10*cosh(x)^2 - 7)*sinh(x)^4 - 21*cosh(x)^4 + 4*(10*c
osh(x)^3 - 21*cosh(x))*sinh(x)^3 + 3*(10*cosh(x)^4 - 42*cosh(x)^2 - 7)*sinh
(x)^2 - 21*cosh(x)^2 + 6*(2*cosh(x)^5 - 14*cosh(x)^3 - 7*cosh(x))*sinh(x) +
 2)*sqrt((2*cosh(x)^2 + 2*sinh(x)^2 - 7)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s
inh(x)^2)) + 2)/(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 14*(2*cosh(x
)^2 - 1)*sinh(x)^6 - 14*cosh(x)^6 + 28*(2*cosh(x)^3 - 3*cosh(x))*sinh(x)^5
+ (70*cosh(x)^4 - 210*cosh(x)^2 + 51)*sinh(x)^4 + 51*cosh(x)^4 + 4*(14*cosh
(x)^5 - 70*cosh(x)^3 + 51*cosh(x))*sinh(x)^3 + 2*(14*cosh(x)^6 - 105*cosh(x
)^4 + 153*cosh(x)^2 - 7)*sinh(x)^2 - 14*cosh(x)^2 + 4*(2*cosh(x)^7 - 21*cos
h(x)^5 + 51*cosh(x)^3 - 7*cosh(x))*sinh(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(-9+4*cosh(x)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14142, size = 54, normalized size = 1.46

$$\frac{\left(\left(2\,e^{(2\,x)} - 21\right)e^{(2\,x)} - 21\right)e^{(2\,x)} + 2}{486\,\left(e^{(4\,x)} - 7\,e^{(2\,x)} + 1\right)^{\frac{3}{2}}} - \frac{1}{243}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)/(-9+4*cosh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/486*(((2*e^(2*x) - 21)*e^(2*x) - 21)*e^(2*x) + 2)/(e^(4*x) - 7*e^(2*x) +
1)^(3/2) - 1/243
```

3.594 $\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$2\sqrt{1-\sinh^2(x)}+\frac{2}{\sqrt{1-\sinh^2(x)}}$$

```
[Out] 2/Sqrt[1 - Sinh[x]^2] + 2*Sqrt[1 - Sinh[x]^2]
```

Rubi [A] time = 0.107906, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 266, 43}

$$2\sqrt{1-\sinh^2(x)}+\frac{2}{\sqrt{1-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sinh[x]^2*Sinh[2*x])/(1 - Sinh[x]^2)^(3/2),x]
```

```
[Out] 2/Sqrt[1 - Sinh[x]^2] + 2*Sqrt[1 - Sinh[x]^2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(1-\sinh^2(x)\right)^{3/2}}\,dx = i\,\text{Subst}\left(\int -\frac{2ix^3}{\left(1-x^2\right)^{3/2}}\,dx, x, \sinh(x)\right)$$
$$= 2\,\text{Subst}\left(\int \frac{x^3}{\left(1-x^2\right)^{3/2}}\,dx, x, \sinh(x)\right)$$
$$= \text{Subst}\left(\int \frac{x}{(1-x)^{3/2}}\,dx, x, \sinh^2(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{(1-x)^{3/2}} - \frac{1}{\sqrt{1-x}}\right)dx, x, \sinh^2(x)\right)$$
$$= \frac{2}{\sqrt{1-\sinh^2(x)}} + 2\sqrt{1-\sinh^2(x)}$$

Mathematica [A] time = 0.0539855, size = 21, normalized size = 0.72

$$\frac{5-\cosh(2x)}{\sqrt{1-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sinh[x]^2*Sinh[2*x])/(1 - Sinh[x]^2)^(3/2),x]
```

```
[Out] (5 - Cosh[2*x])/Sqrt[1 - Sinh[x]^2]
```

Maple [C] time = 0.044, size = 28, normalized size = 1.

$$\text{` int/indef0`}\left(-2\,\frac{(\sinh(x))^3}{\left(-1+(\sinh(x))^2\right)\sqrt{1-(\sinh(x))^2}}, \sinh(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x)
```

```
[Out] `int/indef0`(-2*sinh(x)^3/(-1+sinh(x)^2)/(1-sinh(x)^2)^(1/2),sinh(x))
```

Maxima [B] time = 1.65, size = 239, normalized size = 8.24

$$-\frac{16\,e^{(-x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}} + \frac{62\,e^{(-3\,x)}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\,e^{(-x)}-e^{(-2\,x)}+1\right)^{\frac{3}{2}}} - \frac{16\,e^{(-5}}{\left(2\,e^{(-x)}+e^{(-2\,x)}-1\right)^{\frac{3}{2}}\left(2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -16*e^(-x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x) + 1)^(3/2)
) + 62*e^(-3*x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x) + 1)^
```

```
(3/2)) - 16*e^(-5*x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x)
+ 1)^(3/2)) + e^(-7*x)/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x
) + 1)^(3/2)) + e^x/((2*e^(-x) + e^(-2*x) - 1)^(3/2)*(2*e^(-x) - e^(-2*x) +
 1)^(3/2))
```

Fricas [B] time = 2.16068, size = 548, normalized size = 18.9

$$\frac{\sqrt{2}\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2 - 5\right)\sinh(x)^2 - 10\cosh(x)^2 + 4\left(\cosh(x)^3 - 5\right.\right.}{\cosh(x)^5 + 5\cosh(x)\sinh(x)^4 + \sinh(x)^5 + 2\left(5\cosh(x)^2 - 3\right)\sinh(x)^3 - 6\cosh(x)^3 + 2\left(5\cosh(x)^3 - 9\cosh(x)\right)\mathrm{s}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 5)*
sinh(x)^2 - 10*cosh(x)^2 + 4*(cosh(x)^3 - 5*cosh(x))*sinh(x) + 1)*sqrt(-(co
sh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(cosh
(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5 + 2*(5*cosh(x)^2 - 3)*sinh(x)^3 - 6
*cosh(x)^3 + 2*(5*cosh(x)^3 - 9*cosh(x))*sinh(x)^2 + (5*cosh(x)^4 - 18*cosh
(x)^2 + 1)*sinh(x) + cosh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(x)\sinh(2x)}{\left(-\left(\sinh(x)-1\right)\left(\sinh(x)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)**2*sinh(2*x)/(1-sinh(x)**2)**(3/2),x)
```

```
[Out] Integral(sinh(x)**2*sinh(2*x)/(-(sinh(x) - 1)*(sinh(x) + 1))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(2x)\sinh(x)^2}{\left(-\sinh(x)^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x)^2*sinh(2*x)/(1-sinh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(2*x)*sinh(x)^2/(-sinh(x)^2 + 1)^(3/2), x)
```

3.595 $\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}} \, dx$

Optimal. Leaf size=15

$$\frac{\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

```
[Out] ArcSinh[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

Rubi [A] time = 0.0185004, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4356, 215}

$$\frac{\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/Sqrt[Cosh[2*x]],x]
```

```
[Out] ArcSinh[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh(x)}{\sqrt{\cosh(2x)}} \, dx &= \text{Subst}\left(\int \frac{1}{\sqrt{1+2x^2}} \, dx, x, \sinh(x)\right) \\ &= \frac{\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.0107721, size = 15, normalized size = 1.

$$\frac{\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/Sqrt[Cosh[2*x]],x]
```

```
[Out] ArcSinh[Sqrt[2]*Sinh[x]]/Sqrt[2]
```

Maple [B] time = 0.048, size = 63, normalized size = 4.2

$$\frac{\sqrt{2}}{4\sinh(x)}\sqrt{\left(2(\cosh(x))^2-1\right)(\sinh(x))^2}\ln\left(\sqrt{2}(\sinh(x))^2+\sqrt{2(\sinh(x))^4+(\sinh(x))^2}+\frac{\sqrt{2}}{4}\right)\frac{1}{\sqrt{2(\cosh(x))^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/cosh(2*x)^(1/2),x)
```

```
[Out] 1/4*((2*cosh(x)^2-1)*sinh(x)^2)^(1/2)*ln(2^(1/2)*sinh(x)^2+(2*sinh(x)^4+sin
h(x)^2)^(1/2)+1/4*2^(1/2))*2^(1/2)/sinh(x)/(2*cosh(x)^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/cosh(2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(x)/sqrt(cosh(2*x)), x)
```

Fricas [B] time = 2.30835, size = 1636, normalized size = 109.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/cosh(2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)
^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 +
5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(x)^
5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 + 30
*cosh(x)^2 - 4)*sinh(x)^2 + sqrt(2)*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh
(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 + 4*(5*cosh(x)^3 - 3*co
sh(x))*sinh(x)^3 + (15*cosh(x)^4 - 18*cosh(x)^2 + 4)*sinh(x)^2 + 4*cosh(x)^
2 + 2*(3*cosh(x)^5 - 6*cosh(x)^3 + 4*cosh(x))*sinh(x) - 4)*sqrt((cosh(x)^2
+ sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*cosh(x)^2 + 2
*(4*cosh(x)^7 - 9*cosh(x)^5 + 10*cosh(x)^3 - 4*cosh(x))*sinh(x) + 4)/(cosh(
x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^
3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/8*sqrt(2
)*log((cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh
(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt((cosh(
x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + cosh(x)^2
+ 2*(2*cosh(x)^3 + cosh(x))*sinh(x) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + s
inh(x)^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/cosh(2*x)**(1/2),x)
```

```
[Out] Integral(cosh(x)/sqrt(cosh(2*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(x)}{\sqrt{\cosh(2\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/cosh(2*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(x)/sqrt(cosh(2*x)), x)
```

3.596 $\int x \tanh^2(x) \, dx$

Optimal. Leaf size=16

$$\frac{x^2}{2} - x \tanh(x) + \log(\cosh(x))$$

```
[Out] x^2/2 + Log[Cosh[x]] - x*Tanh[x]
```

Rubi [A] time = 0.0194905, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3720, 3475, 30}

$$\frac{x^2}{2} - x \tanh(x) + \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Tanh[x]^2,x]

[Out] x^2/2 + Log[Cosh[x]] - x*Tanh[x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int x \tanh^2(x) \, dx &= -x \tanh(x) + \int x \, dx + \int \tanh(x) \, dx \\ &= \frac{x^2}{2} + \log(\cosh(x)) - x \tanh(x) \end{aligned}$$

Mathematica [A] time = 0.020748, size = 16, normalized size = 1.

$$\frac{x^2}{2} - x \tanh(x) + \log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tanh[x]^2,x]
```

```
[Out] x^2/2 + Log[Cosh[x]] - x*Tanh[x]
```

Maple [A] time = 0.013, size = 28, normalized size = 1.8

$$\frac{x^2}{2} - 2\,x + 2\,\frac{x}{1 + \mathrm{e}^{2\,x}} + \ln\left(1 + \mathrm{e}^{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tanh(x)^2,x)
```

```
[Out] 1/2*x^2-2*x+2*x/(1+exp(2*x))+ln(1+exp(2*x))
```

Maxima [B] time = 1.56092, size = 66, normalized size = 4.12

$$-\frac{xe^{(2\,x)}}{e^{(2\,x)} + 1} + \frac{x^2 + \left(x^2 - 2\,x\right)e^{(2\,x)}}{2\left(e^{(2\,x)} + 1\right)} + \log\left(e^{(2\,x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(x)^2,x, algorithm="maxima")
```

```
[Out] -x*e^(2*x)/(e^(2*x) + 1) + 1/2*(x^2 + (x^2 - 2*x)*e^(2*x))/(e^(2*x) + 1) +
log(e^(2*x) + 1)
```

Fricas [B] time = 2.18759, size = 305, normalized size = 19.06

$$\frac{\left(x^2 - 4\,x\right)\cosh\left(x\right)^2 + 2\left(x^2 - 4\,x\right)\cosh\left(x\right)\sinh\left(x\right) + \left(x^2 - 4\,x\right)\sinh\left(x\right)^2 + x^2 + 2\left(\cosh\left(x\right)^2 + 2\,\cosh\left(x\right)\sinh\left(x\right) + s\right.}{2\left(\cosh\left(x\right)^2 + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 4*x)*cosh(x)^2 + 2*(x^2 - 4*x)*cosh(x)*sinh(x) + (x^2 - 4*x)*si
nh(x)^2 + x^2 + 2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(2*cos
h(x)/(cosh(x) - sinh(x))))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [A] time = 0.171118, size = 17, normalized size = 1.06

$$\frac{x^2}{2} - x\tanh\left(x\right) + x - \log\left(\tanh\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(x)**2,x)
```

```
[Out] x**2/2 - x*tanh(x) + x - log(tanh(x) + 1)
```

Giac [B] time = 1.16759, size = 69, normalized size = 4.31

$$\frac{x^2e^{(2x)} + x^2 - 4xe^{(2x)} + 2e^{(2x)}\log\left(e^{(2x)} + 1\right) + 2\log\left(e^{(2x)} + 1\right)}{2\left(e^{(2x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tanh(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e^(2*x) + x^2 - 4*x*e^(2*x) + 2*e^(2*x)*log(e^(2*x) + 1) + 2*log(e
^(2*x) + 1))/(e^(2*x) + 1)
```

3.597 $\int x \coth^2(x) \, dx$

Optimal. Leaf size=16

$$\frac{x^2}{2} - x \coth(x) + \log(\sinh(x))$$

```
[Out] x^2/2 - x*Coth[x] + Log[Sinh[x]]
```

Rubi [A] time = 0.0207453, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3720, 3475, 30}

$$\frac{x^2}{2} - x \coth(x) + \log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Coth[x]^2,x]

[Out] x^2/2 - x*Coth[x] + Log[Sinh[x]]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x \coth^2(x) \, dx &= -x \coth(x) + \int x \, dx + \int \coth(x) \, dx \\ &= \frac{x^2}{2} - x \coth(x) + \log(\sinh(x))\end{aligned}$$

Mathematica [A] time = 0.0190905, size = 16, normalized size = 1.

$$\frac{x^2}{2} - x \coth(x) + \log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Coth[x]^2,x]
```

```
[Out] x^2/2 - x*Coth[x] + Log[Sinh[x]]
```

Maple [A] time = 0.013, size = 28, normalized size = 1.8

$$\frac{x^2}{2}-2\,x-2\,\frac{x}{-1+\mathrm{e}^{2\,x}}+\ln\left(-1+\mathrm{e}^{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*coth(x)^2,x)
```

```
[Out] 1/2*x^2-2*x-2*x/(-1+exp(2*x))+ln(-1+exp(2*x))
```

Maxima [B] time = 1.1065, size = 72, normalized size = 4.5

$$-\frac{xe^{(2\,x)}}{e^{(2\,x)}-1}-\frac{x^2-\left(x^2-2\,x\right)e^{(2\,x)}}{2\left(e^{(2\,x)}-1\right)}+\log\left(e^x+1\right)+\log\left(e^x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(x)^2,x, algorithm="maxima")
```

```
[Out] -x*e^(2*x)/(e^(2*x) - 1) - 1/2*(x^2 - (x^2 - 2*x)*e^(2*x))/(e^(2*x) - 1) +
log(e^x + 1) + log(e^x - 1)
```

Fricas [B] time = 2.24242, size = 305, normalized size = 19.06

$$\frac{\left(x^2-4\,x\right)\cosh\left(x\right)^2+2\left(x^2-4\,x\right)\cosh\left(x\right)\sinh\left(x\right)+\left(x^2-4\,x\right)\sinh\left(x\right)^2-x^2+2\left(\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sinh\left(x\right)+\mathrm{sinl}\right.}{2\left(\cosh\left(x\right)^2+2\,\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 4*x)*cosh(x)^2 + 2*(x^2 - 4*x)*cosh(x)*sinh(x) + (x^2 - 4*x)*si
nh(x)^2 - x^2 + 2*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*log(2*sin
h(x)/(cosh(x) - sinh(x))))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)
```

Sympy [A] time = 0.907406, size = 22, normalized size = 1.38

$$\frac{x^2}{2}+x-\frac{x}{\tanh\left(x\right)}-\log\left(\tanh\left(x\right)+1\right)+\log\left(\tanh\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(x)**2,x)
```

```
[Out] x**2/2 + x - x/tanh(x) - log(tanh(x) + 1) + log(tanh(x))
```

Giac [B] time = 1.146, size = 72, normalized size = 4.5

$$\frac{x^2e^{(2\,x)} - x^2 - 4\,xe^{(2\,x)} + 2\,e^{(2\,x)}\log\left(e^{(2\,x)} - 1\right) - 2\,\log\left(e^{(2\,x)} - 1\right)}{2\left(e^{(2\,x)} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*coth(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x^2*e^(2*x) - x^2 - 4*x*e^(2*x) + 2*e^(2*x)*log(e^(2*x) - 1) - 2*log(e
^(2*x) - 1))/(e^(2*x) - 1)
```

3.598 $\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx$

Optimal. Leaf size=20

$$e^x x - e^x + \frac{e^{2x}}{2}$$

```
[Out] -E^x + E^(2*x)/2 + E^x*x
```

Rubi [A] time = 0.0670704, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {5648, 6742, 2176, 2194, 2282, 12, 14}

$$e^x x - e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x + Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
```

```
[Out] -E^x + E^(2*x)/2 + E^x*x
```

Rule 5648

```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a*v)/b))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x+\cosh(x)+\sinh(x)}{\cosh(x)-\sinh(x)}\,dx &= \int e^x(x+\cosh(x)+\sinh(x))\,dx \\
&= \int \left(e^x x+e^x\cosh(x)+e^x\sinh(x)\right)\,dx \\
&= \int e^x x\,dx+\int e^x\cosh(x)\,dx+\int e^x\sinh(x)\,dx \\
&= e^x x-\int e^x\,dx+\text{Subst}\left(\int \frac{-1+x^2}{2x}\,dx,x,e^x\right)+\text{Subst}\left(\int \frac{1+x^2}{2x}\,dx,x,e^x\right) \\
&= -e^x+e^x x+\frac{1}{2}\text{Subst}\left(\int \frac{-1+x^2}{x}\,dx,x,e^x\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1+x^2}{x}\,dx,x,e^x\right) \\
&= -e^x+e^x x+\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x}+x\right)\,dx,x,e^x\right)+\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{x}+x\right)\,dx,x,e^x\right) \\
&= -e^x+\frac{e^{2x}}{2}+e^x x
\end{aligned}
$$

Mathematica [A] time = 0.0909661, size = 23, normalized size = 1.15

$$(x-1)\sinh(x)+\frac{1}{2}\cosh(2x)+(x+\sinh(x)-1)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Cosh[x] + Sinh[x])/(Cosh[x] - Sinh[x]),x]
```

```
[Out] Cosh[2*x]/2 + (-1 + x)*Sinh[x] + Cosh[x]*(-1 + x + Sinh[x])
```

Maple [B] time = 0.101, size = 38, normalized size = 1.9

$$x\cosh(x)-\sinh(x)+\sinh(x)x-\cosh(x)+2\left(-1+\tanh(x/2)\right)^{-2}+2\left(-1+\tanh(x/2)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)
```

```
[Out] x*cosh(x)-sinh(x)+sinh(x)*x-cosh(x)+2/(-1+tanh(1/2*x))^2+2/(-1+tanh(1/2*x))
```

Maxima [A] time = 0.971821, size = 18, normalized size = 0.9

$$(x-1)e^x+\frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="maxima")
```

```
[Out] (x - 1)*e^x + 1/2*e^(2*x)
```

Fricas [A] time = 2.0234, size = 74, normalized size = 3.7

$$\frac{2\,x + \cosh\left(x\right) + \sinh\left(x\right) - 2}{2\,\left(\cosh\left(x\right) - \sinh\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x + cosh(x) + sinh(x) - 2)/(cosh(x) - sinh(x))
```

Sympy [A] time = 0.343859, size = 26, normalized size = 1.3

$$\frac{x}{-\sinh\left(x\right) + \cosh\left(x\right)} + \frac{\cosh\left(x\right)}{-\sinh\left(x\right) + \cosh\left(x\right)} - \frac{1}{-\sinh\left(x\right) + \cosh\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x)
```

```
[Out] x/(-sinh(x) + cosh(x)) + cosh(x)/(-sinh(x) + cosh(x)) - 1/(-sinh(x) + cosh(
x))
```

Giac [A] time = 1.09971, size = 15, normalized size = 0.75

$$\frac{1}{2}\,(2\,x + e^{x} - 2)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x)+sinh(x))/(cosh(x)-sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2*(2*x + e^x - 2)*e^x
```

3.599 $\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)} \, dx$

Optimal. Leaf size=15

$$x-(1-x)\tanh\left(\frac{x}{2}\right)$$

```
[Out] x - (1 - x)*Tanh[x/2]
```

Rubi [A] time = 0.130112, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6742, 3318, 4184, 3475}

$$x-(1-x)\tanh\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Cosh[x] + Sinh[x])/(1 + Cosh[x]),x]
```

```
[Out] x - (1 - x)*Tanh[x/2]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3318

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1*(e + (Pi*a)/(2*b)))/2 +
 (f*x)/2]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x+\cosh(x)+\sinh(x)}{1+\cosh(x)}\,dx &= \int \left(\frac{x+\cosh(x)}{1+\cosh(x)}+\tanh\left(\frac{x}{2}\right)\right)dx\\ &= \int \frac{x+\cosh(x)}{1+\cosh(x)}\,dx+\int \tanh\left(\frac{x}{2}\right)dx\\ &= 2\log\left(\cosh\left(\frac{x}{2}\right)\right)+\int \left(1+\frac{-1+x}{1+\cosh(x)}\right)dx\\ &= x+2\log\left(\cosh\left(\frac{x}{2}\right)\right)+\int \frac{-1+x}{1+\cosh(x)}\,dx\\ &= x+2\log\left(\cosh\left(\frac{x}{2}\right)\right)+\frac{1}{2}\int (-1+x)\mathrm{sech}^2\left(\frac{x}{2}\right)dx\\ &= x+2\log\left(\cosh\left(\frac{x}{2}\right)\right)-(1-x)\tanh\left(\frac{x}{2}\right)-\int \tanh\left(\frac{x}{2}\right)dx\\ &= x-(1-x)\tanh\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.074841, size = 20, normalized size = 1.33

$$\frac{\sinh(x)\left(x+x\coth\left(\frac{x}{2}\right)-1\right)}{\cosh(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Cosh[x] + Sinh[x])/(1 + Cosh[x]),x]
```

```
[Out] ((-1 + x + x*Coth[x/2])*Sinh[x])/(1 + Cosh[x])
```

Maple [A] time = 0.024, size = 16, normalized size = 1.1

$$2x-2\frac{-1+x}{1+e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+cosh(x)+sinh(x))/(1+cosh(x)),x)
```

```
[Out] 2*x-2*(-1+x)/(1+exp(x))
```

Maxima [B] time = 0.963221, size = 47, normalized size = 3.13

$$x+\frac{2\,xe^x}{e^x+1}-\frac{2}{e^{(-x)}+1}+\log\left(\cosh\left(x\right)+1\right)-2\,\log\left(e^x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x, algorithm="maxima")
```

```
[Out] x + 2*x*e^x/(e^x + 1) - 2/(e^(-x) + 1) + log(cosh(x) + 1) - 2*log(e^x + 1)
```

Fricas [A] time = 2.12291, size = 74, normalized size = 4.93

$$\frac{2\left(x\cosh\left(x\right)+x\sinh\left(x\right)+1\right)}{\cosh\left(x\right)+\sinh\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x, algorithm="fricas")
```

```
[Out] 2*(x*cosh(x) + x*sinh(x) + 1)/(cosh(x) + sinh(x) + 1)
```

Sympy [A] time = 0.428544, size = 12, normalized size = 0.8

$$x\tanh\left(\frac{x}{2}\right)+x-\tanh\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x)
```

```
[Out] x*tanh(x/2) + x - tanh(x/2)
```

Giac [A] time = 1.10866, size = 19, normalized size = 1.27

$$\frac{2\left(xe^{x}+1\right)}{e^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+cosh(x)+sinh(x))/(1+cosh(x)),x, algorithm="giac")
```

```
[Out] 2*(x*e^x + 1)/(e^x + 1)
```

3.600 $\int e^{2x}\operatorname{csch}^4(x)\,dx$

Optimal. Leaf size=20

$$\frac{8e^{6x}}{3\left(1-e^{2x}\right)^3}$$

```
[Out] (8*E^(6*x))/(3*(1 - E^(2*x))^3)
```

Rubi [A] time = 0.0183473, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2282, 12, 264}

$$\frac{8e^{6x}}{3\left(1-e^{2x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*Csch[x]^4,x]
```

```
[Out] (8*E^(6*x))/(3*(1 - E^(2*x))^3)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int e^{2x}\operatorname{csch}^4(x)\,dx &= \operatorname{Subst}\left(\int \frac{16x^5}{\left(1-x^2\right)^4}\,dx,x,e^x\right)\\ &= 16\operatorname{Subst}\left(\int \frac{x^5}{\left(1-x^2\right)^4}\,dx,x,e^x\right)\\ &= \frac{8e^{6x}}{3\left(1-e^{2x}\right)^3}\end{aligned}$$

Mathematica [A] time = 0.0145626, size = 20, normalized size = 1.

$$\frac{8e^{6x}}{3\left(1-e^{2x}\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*Csch[x]^4,x]
```

```
[Out] (8*E^(6*x))/(3*(1 - E^(2*x))^3)
```

Maple [A] time = 0.079, size = 20, normalized size = 1.

$$-\frac{1}{3\left(\tanh\left(x\right)\right)^{3}}-\left(\tanh\left(x\right)\right)^{-2}-\left(\tanh\left(x\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/sinh(x)^4,x)
```

```
[Out] -1/3/tanh(x)^3-1/tanh(x)^2-1/tanh(x)
```

Maxima [A] time = 0.938714, size = 30, normalized size = 1.5

$$\frac{8}{3\left(3\,e^{(-2\,x)}-3\,e^{(-4\,x)}+e^{(-6\,x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/sinh(x)^4,x, algorithm="maxima")
```

```
[Out] 8/3/(3*e^(-2*x) - 3*e^(-4*x) + e^(-6*x) - 1)
```

Fricas [B] time = 2.14174, size = 254, normalized size = 12.7

$$-\frac{8\left(4\,\cosh\left(x\right)^{2}+4\,\cosh\left(x\right)\sinh\left(x\right)+4\,\sinh\left(x\right)^{2}-3\right)}{3\left(\cosh\left(x\right)^{4}+4\,\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+2\left(3\,\cosh\left(x\right)^{2}-2\right)\sinh\left(x\right)^{2}-4\,\cosh\left(x\right)^{2}+4\left(\cosh\left(x\right)^{3}-\cosh\left(x\right)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/sinh(x)^4,x, algorithm="fricas")
```

```
[Out] -8/3*(4*cosh(x)^2 + 4*cosh(x)*sinh(x) + 4*sinh(x)^2 - 3)/(cosh(x)^4 + 4*cos
h(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 2)*sinh(x)^2 - 4*cosh(x)^2 +
4*(cosh(x)^3 - cosh(x))*sinh(x) + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{2x}}{\sinh^4\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/sinh(x)**4,x)
```

```
[Out] Integral(exp(2*x)/sinh(x)**4, x)
```

Giac [A] time = 1.14871, size = 32, normalized size = 1.6

$$-\frac{8\left(3\,e^{(4\,x)}-3\,e^{(2\,x)}+1\right)}{3\left(e^{(2\,x)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/sinh(x)^4,x, algorithm="giac")
```

```
[Out] -8/3*(3*e^(4*x) - 3*e^(2*x) + 1)/(e^(2*x) - 1)^3
```

3.601 $\int e^{-2x}\mathbf{sech}^4(x)\,dx$

Optimal. Leaf size=13

$$-\frac{8}{3\left(e^{2x}+1\right)^3}$$

```
[Out] -8/(3*(1 + E^(2*x))^3)
```

Rubi [A] time = 0.01701, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2282, 12, 261}

$$-\frac{8}{3\left(e^{2x}+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^4/E^(2*x),x]
```

```
[Out] -8/(3*(1 + E^(2*x))^3)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int e^{-2x}\text{sech}^4(x)\,dx &= \text{Subst}\left(\int \frac{16x}{\left(1+x^2\right)^4}\,dx, x, e^x\right)\\ &= 16\,\text{Subst}\left(\int \frac{x}{\left(1+x^2\right)^4}\,dx, x, e^x\right)\\ &= -\frac{8}{3\left(1+e^{2x}\right)^3}\end{aligned}$$

Mathematica [A] time = 0.0095893, size = 13, normalized size = 1.

$$-\frac{8}{3\left(e^{2x}+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^4/E^(2*x),x]
```

```
[Out] -8/(3*(1 + E^(2*x))^3)
```

Maple [B] time = 0.043, size = 52, normalized size = 4.

$$-2\,\frac{-\left(\tanh\left(x/2\right)\right)^5+2\,\left(\tanh\left(x/2\right)\right)^4-10/3\,\left(\tanh\left(x/2\right)\right)^3+2\,\left(\tanh\left(x/2\right)\right)^2-\tanh\left(x/2\right)}{\left(\left(\tanh\left(x/2\right)\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(2*x)/cosh(x)^4,x)
```

```
[Out] -2*(-tanh(1/2*x)^5+2*tanh(1/2*x)^4-10/3*tanh(1/2*x)^3+2*tanh(1/2*x)^2-tanh(
1/2*x))/(tanh(1/2*x)^2+1)^3
```

Maxima [B] time = 0.976321, size = 101, normalized size = 7.77

$$\frac{8\,e^{(-2\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1}+\frac{8\,e^{(-4\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1}+\frac{8}{3\left(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*x)/cosh(x)^4,x, algorithm="maxima")
```

```
[Out] 8*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 8*e^(-4*x)/(3*e^(-2*x
) + 3*e^(-4*x) + e^(-6*x) + 1) + 8/3/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) +
1)
```

Fricas [B] time = 2.11093, size = 338, normalized size = 26.

$$-\frac{}{3\left(\cosh\left(x\right)^6+6\,\cosh\left(x\right)\sinh\left(x\right)^5+\sinh\left(x\right)^6+3\left(5\,\cosh\left(x\right)^2+1\right)\sinh\left(x\right)^4+3\,\cosh\left(x\right)^4+4\left(5\,\cosh\left(x\right)^3+3\,\cosh\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*x)/cosh(x)^4,x, algorithm="fricas")
```

```
[Out] -8/3/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sin
h(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)
^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3
+ cosh(x))*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{-2x}}{\cosh^4(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*x)/cosh(x)**4,x)
```

```
[Out] Integral(exp(-2*x)/cosh(x)**4, x)
```

Giac [A] time = 1.08337, size = 14, normalized size = 1.08

$$-\frac{8}{3\left(e^{(2x)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(2*x)/cosh(x)^4,x, algorithm="giac")
```

```
[Out] -8/3/(e^(2*x) + 1)^3
```

3.602 $\int \frac{e^x}{\cosh(x)-\sinh(x)} \, dx$

Optimal. Leaf size=9

$$\frac{e^{2x}}{2}$$

```
[Out] E^(2*x)/2
```

Rubi [A] time = 0.016691, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2282, 30}

$$\frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(Cosh[x] - Sinh[x]),x]
```

```
[Out] E^(2*x)/2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{e^x}{\cosh(x)-\sinh(x)} \, dx &= \text{Subst}\left(\int x \, dx, x, e^x\right) \\ &= \frac{e^{2x}}{2} \end{aligned}$$

Mathematica [A] time = 0.0030923, size = 9, normalized size = 1.

$$\frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(Cosh[x] - Sinh[x]),x]
```

```
[Out] E^(2*x)/2
```

Maple [B] time = 0.002, size = 14, normalized size = 1.6

$$-\frac{e^{x}}{-2\cosh(x)+2\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(cosh(x)-sinh(x)),x)
```

```
[Out] -1/2*exp(x)/(-cosh(x)+sinh(x))
```

Maxima [A] time = 0.943977, size = 8, normalized size = 0.89

$$\frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)-sinh(x)),x, algorithm="maxima")
```

```
[Out] 1/2*e^(2*x)
```

Fricas [B] time = 2.02849, size = 61, normalized size = 6.78

$$\frac{\cosh(x)+\sinh(x)}{2(\cosh(x)-\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)-sinh(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(x) + sinh(x))/(cosh(x) - sinh(x))
```

Sympy [B] time = 0.380086, size = 12, normalized size = 1.33

$$\frac{e^{x}}{-2\sinh(x)+2\cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)-sinh(x)),x)
```

```
[Out] exp(x)/(-2*sinh(x) + 2*cosh(x))
```

Giac [A] time = 1.09636, size = 8, normalized size = 0.89

$$\frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)-sinh(x)),x, algorithm="giac")
```

```
[Out] 1/2*e^(2*x)
```

3.603 $\int \frac{e^{mx}}{\cosh(x)+\sinh(x)} \, dx$

Optimal. Leaf size=13

$$\frac{e^{(m-1)x}}{m-1}$$

```
[Out] E^((-1 + m)*x)/(-1 + m)
```

Rubi [A] time = 0.0291286, antiderivative size = 19, normalized size of antiderivative = 1.46, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5648, 2227, 2194}

$$-\frac{e^{-(1-m)x}}{1-m}$$

Antiderivative was successfully verified.

```
[In] Int[E^(m*x)/(Cosh[x] + Sinh[x]),x]
```

```
[Out] -(1/(E^((1 - m)*x)*(1 - m)))
```

Rule 5648

```
Int[(u_.)*(Cosh[v_]*(a_.) + (b_.)*Sinh[v_])^(n_.), x_Symbol] :> Int[u*(a*E^
((a*v)/b))^n, x] /; FreeQ[{a, b, n}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2227

```
Int[(u_.)*(F_)^((a_.) + (b_.)*(v_)), x_Symbol] :> Int[u*F^(a + b*NormalizeP
owerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && Powe
rOfLinearQ[v, x] &&  !PowerOfLinearMatchQ[v, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{mx}}{\cosh(x)+\sinh(x)} \, dx &= \int e^{-x+mx} \, dx\\ &= \int e^{-(1-m)x} \, dx\\ &= -\frac{e^{-(1-m)x}}{1-m}\end{aligned}$$

Mathematica [A] time = 0.0302059, size = 18, normalized size = 1.38

$$\frac{e^{mx}(\cosh(x)-\sinh(x))}{m-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(m*x)/(Cosh[x] + Sinh[x]),x]
```

```
[Out] (E^(m*x)*(Cosh[x] - Sinh[x]))/(-1 + m)
```

Maple [A] time = 0.002, size = 18, normalized size = 1.4

$$\frac{e^{mx}}{(-1+m)(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(m*x)/(cosh(x)+sinh(x)),x)
```

```
[Out] 1/(-1+m)*exp(m*x)/(cosh(x)+sinh(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/(cosh(x)+sinh(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.11526, size = 82, normalized size = 6.31

$$\frac{\cosh(mx)+\sinh(mx)}{(m-1)\cosh(x)+(m-1)\sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/(cosh(x)+sinh(x)),x, algorithm="fricas")
```

```
[Out] (cosh(m*x) + sinh(m*x))/((m - 1)*cosh(x) + (m - 1)*sinh(x))
```

Sympy [A] time = 0.54703, size = 32, normalized size = 2.46

$$\begin{cases} \frac{e^{mx}}{m\sinh(x)+m\cosh(x)-\sinh(x)-\cosh(x)} & \text{for}\ m\neq 1 \\ \frac{xe^{x}}{\sinh(x)+\cosh(x)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/(cosh(x)+sinh(x)),x)
```

```
[Out] Piecewise((exp(m*x)/(m*sinh(x) + m*cosh(x) - sinh(x) - cosh(x)), Ne(m, 1)),
 (x*exp(x)/(sinh(x) + cosh(x)), True))
```

Giac [A] time = 1.11001, size = 22, normalized size = 1.69

$$\frac{e^{(mx)}}{me^x - e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(m*x)/(cosh(x)+sinh(x)),x, algorithm="giac")
```

```
[Out] e^(m*x)/(m*e^x - e^x)
```

3.604 $\int \frac{e^x}{\cosh(x)+\sinh(x)} \, dx$

Optimal. Leaf size=1

$$x$$

```
[Out] x
```

Rubi [A] time = 0.0149549, antiderivative size = 1, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2282, 29}

$$x$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(Cosh[x] + Sinh[x]),x]
```

```
[Out] x
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned} \int \frac{e^x}{\cosh(x)+\sinh(x)} \, dx &= \text{Subst}\left(\int \frac{1}{x} \, dx, x, e^x\right) \\ &= x \end{aligned}$$

Mathematica [A] time = 0.0012358, size = 1, normalized size = 1.

$$x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(Cosh[x] + Sinh[x]),x]
```

```
[Out] x
```

Maple [A] time = 0.018, size = 2, normalized size = 2.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(cosh(x)+sinh(x)),x)
```

```
[Out] x
```

Maxima [A] time = 0.936195, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)+sinh(x)),x, algorithm="maxima")
```

```
[Out] x
```

Fricas [A] time = 1.91013, size = 4, normalized size = 4.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)+sinh(x)),x, algorithm="fricas")
```

```
[Out] x
```

Sympy [B] time = 0.353225, size = 10, normalized size = 10.

$$\frac{xe^{x}}{\sinh(x) + \cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)+sinh(x)),x)
```

```
[Out] x*exp(x)/(sinh(x) + cosh(x))
```

Giac [A] time = 1.08285, size = 1, normalized size = 1.

$$x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(cosh(x)+sinh(x)),x, algorithm="giac")
```

```
[Out] x
```

3.605 $\int \frac{e^x}{1-\cosh(x)} \, dx$

Optimal. Leaf size=22

$$-\frac{2}{1-e^x}-2\log\left(1-e^x\right)$$

[Out] -2/(1 - E^x) - 2*Log[1 - E^x]

Rubi [A] time = 0.0242695, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2282, 12, 43}

$$-\frac{2}{1-e^x}-2\log\left(1-e^x\right)$$

Antiderivative was successfully verified.

[In] Int[E^x/(1 - Cosh[x]),x]

[Out] -2/(1 - E^x) - 2*Log[1 - E^x]

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{1-\cosh(x)}\,dx &= \text{Subst}\left(\int -\frac{2x}{(1-x)^2}\,dx,x,e^x\right)\\ &= -\left(2\,\text{Subst}\left(\int \frac{x}{(1-x)^2}\,dx,x,e^x\right)\right)\\ &= -\left(2\,\text{Subst}\left(\int \left(\frac{1}{(-1+x)^2}+\frac{1}{-1+x}\right)dx,x,e^x\right)\right)\\ &= -\frac{2}{1-e^x}-2\log\left(1-e^x\right)\end{aligned}$$

Mathematica [A] time = 0.0318872, size = 36, normalized size = 1.64

$$\frac{4\left(\frac{1}{1-e^x}+\log\left(1-e^x\right)\right)\sinh^2\left(\frac{x}{2}\right)}{1-\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 - Cosh[x]),x]
```

```
[Out] (4*((1 - E^x)^(-1) + Log[1 - E^x])*Sinh[x/2]^2)/(1 - Cosh[x])
```

Maple [A] time = 0.01, size = 24, normalized size = 1.1

$$\left(\tanh\left(\frac{x}{2}\right)\right)^{-1}-2\ln\left(\tanh\left(x/2\right)\right)+2\ln\left(-1+\tanh\left(x/2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(1-cosh(x)),x)
```

```
[Out] 1/tanh(1/2*x)-2*ln(tanh(1/2*x))+2*ln(-1+tanh(1/2*x))
```

Maxima [A] time = 0.948572, size = 22, normalized size = 1.

$$\frac{2}{e^x-1}-2\log\left(e^x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-cosh(x)),x, algorithm="maxima")
```

```
[Out] 2/(e^x - 1) - 2*log(e^x - 1)
```

Fricas [A] time = 1.98946, size = 115, normalized size = 5.23

$$-\frac{2\left(\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)-1\right)}{\cosh\left(x\right)+\sinh\left(x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-cosh(x)),x, algorithm="fricas")
```

```
[Out] -2*((cosh(x) + sinh(x) - 1)*log(cosh(x) + sinh(x) - 1) - 1)/(cosh(x) + sinh
(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{e^x}{\cosh\left(x\right)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-cosh(x)),x)
```

```
[Out] -Integral(exp(x)/(cosh(x) - 1), x)
```

Giac [A] time = 1.11136, size = 23, normalized size = 1.05

$$\frac{2}{e^x - 1} - 2\,\log\left(|e^x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1-cosh(x)),x, algorithm="giac")
```

```
[Out] 2/(e^x - 1) - 2*log(abs(e^x - 1))
```

3.606 $\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx$

Optimal. Leaf size=13

$$e^x+\frac{2}{e^x+1}$$

```
[Out] E^x + 2/(1 + E^x)
```

Rubi [A] time = 0.030643, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2282, 683}

$$e^x+\frac{2}{e^x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 + Sinh[x]))/(1 + Cosh[x]),x]

[Out] E^x + 2/(1 + E^x)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] &&  !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{e^x(1+\sinh(x))}{1+\cosh(x)}\,dx &= \text{Subst}\left(\int \frac{-1+2x+x^2}{(1+x)^2}\,dx,x,e^x\right)\\ &= \text{Subst}\left(\int \left(1-\frac{2}{(1+x)^2}\right)dx,x,e^x\right)\\ &= e^x+\frac{2}{1+e^x}\end{aligned}$$

Mathematica [A] time = 0.0258615, size = 18, normalized size = 1.38

$$\frac{e^x+e^{2x}+2}{e^x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 + Sinh[x]))/(1 + Cosh[x]),x]

[Out] (2 + E^x + E^(2*x))/(1 + E^x)
```

Maple [A] time = 0.012, size = 18, normalized size = 1.4

$$-\tanh\left(\frac{x}{2}\right) - 2\left(-1 + \tanh\left(x/2\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1+sinh(x))/(1+cosh(x)),x)

[Out] -tanh(1/2*x)-2/(-1+tanh(1/2*x))
```

Maxima [A] time = 0.980585, size = 15, normalized size = 1.15

$$\frac{2}{e^x + 1} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x, algorithm="maxima")

[Out] 2/(e^x + 1) + e^x
```

Fricas [A] time = 2.03628, size = 69, normalized size = 5.31

$$\frac{3\cosh\left(x\right) - \sinh\left(x\right) + 1}{\cosh\left(x\right) - \sinh\left(x\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x, algorithm="fricas")

[Out] (3*cosh(x) - sinh(x) + 1)/(cosh(x) - sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(\sinh\left(x\right) + 1\right)e^{x}}{\cosh\left(x\right) + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x)

[Out] Integral((sinh(x) + 1)*exp(x)/(cosh(x) + 1), x)
```

Giac [A] time = 1.12289, size = 15, normalized size = 1.15

$$\frac{2}{e^x + 1} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1+sinh(x))/(1+cosh(x)),x, algorithm="giac")
```

```
[Out] 2/(e^x + 1) + e^x
```

3.607 $\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx$

Optimal. Leaf size=15

$$e^x - \frac{2}{1-e^x}$$

```
[Out] E^x - 2/(1 - E^x)
```

Rubi [A] time = 0.0332623, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2282, 683}

$$e^x - \frac{2}{1-e^x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*(1 - Sinh[x]))/(1 - Cosh[x]),x]

[Out] E^x - 2/(1 - E^x)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 683

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] &
& IGtQ[p, 0] && !(EqQ[m, 3] && NeQ[p, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{e^x(1-\sinh(x))}{1-\cosh(x)}\,dx &= \text{Subst}\left(\int \frac{-1-2x+x^2}{(1-x)^2}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \left(1-\frac{2}{(-1+x)^2}\right)dx, x, e^x\right)\\
&= e^x - \frac{2}{1-e^x}
\end{aligned}$$

Mathematica [A] time = 0.0288623, size = 20, normalized size = 1.33

$$\frac{-e^x+e^{2x}+2}{e^x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*(1 - Sinh[x]))/(1 - Cosh[x]),x]
```

```
[Out] (2 - E^x + E^(2*x))/(-1 + E^x)
```

Maple [A] time = 0.017, size = 18, normalized size = 1.2

$$\left(\tanh\left(\frac{x}{2}\right)\right)^{-1} - 2\left(-1 + \tanh(x/2)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(1-sinh(x))/(1-cosh(x)),x)
```

```
[Out] 1/tanh(1/2*x)-2/(-1+tanh(1/2*x))
```

Maxima [A] time = 0.973092, size = 15, normalized size = 1.

$$\frac{2}{e^x - 1} + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x, algorithm="maxima")
```

```
[Out] 2/(e^x - 1) + e^x
```

Fricas [A] time = 1.98821, size = 70, normalized size = 4.67

$$-\frac{3\cosh(x) - \sinh(x) - 1}{\cosh(x) - \sinh(x) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x, algorithm="fricas")
```

```
[Out] -(3*cosh(x) - sinh(x) - 1)/(cosh(x) - sinh(x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(\sinh(x) - 1)e^x}{\cosh(x) - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x)
```

```
[Out] Integral((sinh(x) - 1)*exp(x)/(cosh(x) - 1), x)
```

<u>Giac</u> [A] time = 1.09202, size = 15, normalized size = 1.

$$\frac{2}{e^{x}-1}+e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(1-sinh(x))/(1-cosh(x)),x, algorithm="giac")
```

```
[Out] 2/(e^x - 1) + e^x
```

3.608 $\int x^m \log(x)\, dx$

Optimal. Leaf size=26

$$\frac{x^{m+1}\log(x)}{m+1} - \frac{x^{m+1}}{(m+1)^2}$$

```
[Out] -(x^(1 + m)/(1 + m)^2) + (x^(1 + m)*Log[x])/(1 + m)
```

Rubi [A] time = 0.0101041, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2304}

$$\frac{x^{m+1}\log(x)}{m+1} - \frac{x^{m+1}}{(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Log[x],x]

[Out] -(x^(1 + m)/(1 + m)^2) + (x^(1 + m)*Log[x])/(1 + m)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^m \log(x)\, dx = -\frac{x^{1+m}}{(1+m)^2} + \frac{x^{1+m}\log(x)}{1+m}$$

Mathematica [A] time = 0.0080626, size = 19, normalized size = 0.73

$$\frac{x^{m+1}((m+1)\log(x)-1)}{(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Log[x],x]

[Out] (x^(1 + m)*(-1 + (1 + m)*Log[x]))/(1 + m)^2
```

Maple [A] time = 0.007, size = 34, normalized size = 1.3

$$\frac{x\ln(x)\,e^{m\ln(x)}}{1+m} - \frac{xe^{m\ln(x)}}{m^2+2m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*ln(x),x)
```

```
[Out] 1/(1+m)*x*ln(x)*exp(m*ln(x))-1/(m^2+2*m+1)*x*exp(m*ln(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08615, size = 59, normalized size = 2.27

$$\frac{((m+1)x\log(x)-x)x^m}{m^2+2m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(x),x, algorithm="fricas")
```

```
[Out] ((m + 1)*x*log(x) - x)*x^m/(m^2 + 2*m + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*ln(x),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^m \log(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(x),x, algorithm="giac")
```

```
[Out] integrate(x^m*log(x), x)
```

3.609 $\int x^m \log^2(x)\, dx$

Optimal. Leaf size=42

$$\frac{2x^{m+1}}{(m+1)^3} + \frac{x^{m+1}\log^2(x)}{m+1} - \frac{2x^{m+1}\log(x)}{(m+1)^2}$$

```
[Out] (2*x^(1 + m))/(1 + m)^3 - (2*x^(1 + m)*Log[x])/(1 + m)^2 + (x^(1 + m)*Log[x
]^2)/(1 + m)
```

Rubi [A] time = 0.0238764, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2305, 2304}

$$\frac{2x^{m+1}}{(m+1)^3} + \frac{x^{m+1}\log^2(x)}{m+1} - \frac{2x^{m+1}\log(x)}{(m+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Log[x]^2,x]
```

```
[Out] (2*x^(1 + m))/(1 + m)^3 - (2*x^(1 + m)*Log[x])/(1 + m)^2 + (x^(1 + m)*Log[x
]^2)/(1 + m)
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^m \log^2(x)\, dx &= \frac{x^{1+m}\log^2(x)}{1+m} - \frac{2\int x^m \log(x)\, dx}{1+m}\\ &= \frac{2x^{1+m}}{(1+m)^3} - \frac{2x^{1+m}\log(x)}{(1+m)^2} + \frac{x^{1+m}\log^2(x)}{1+m}\end{aligned}$$

Mathematica [A] time = 0.010345, size = 30, normalized size = 0.71

$$\frac{x^{m+1}\left((m+1)^2\log^2(x) - 2(m+1)\log(x) + 2\right)}{(m+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Log[x]^2,x]
```

```
[Out] (x^(1 + m)*(2 - 2*(1 + m)*Log[x] + (1 + m)^2*Log[x]^2))/(1 + m)^3
```

Maple [A] time = 0.008, size = 61, normalized size = 1.5

$$\frac{x\left(\ln\left(x\right)\right)^{2}\mathrm{e}^{m\ln\left(x\right)}}{1+m}+2\,\frac{x\mathrm{e}^{m\ln\left(x\right)}}{m^{3}+3\,m^{2}+3\,m+1}-2\,\frac{x\ln\left(x\right)\mathrm{e}^{m\ln\left(x\right)}}{m^{2}+2\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*ln(x)^2,x)
```

```
[Out] 1/(1+m)*x*ln(x)^2*exp(m*ln(x))+2/(m^3+3*m^2+3*m+1)*x*exp(m*ln(x))-2/(m^2+2*
m+1)*x*ln(x)*exp(m*ln(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.13438, size = 115, normalized size = 2.74

$$\frac{\left(\left(m^{2}+2\,m+1\right)x\log\left(x\right)^{2}-2\,(m+1)x\log\left(x\right)+2\,x\right)x^{m}}{m^{3}+3\,m^{2}+3\,m+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(x)^2,x, algorithm="fricas")
```

```
[Out] ((m^2 + 2*m + 1)*x*log(x)^2 - 2*(m + 1)*x*log(x) + 2*x)*x^m/(m^3 + 3*m^2 +
3*m + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*ln(x)**2,x)
```

```
[Out] Exception raised: TypeError
```

Giac [A] time = 1.09837, size = 113, normalized size = 2.69

$$-\frac{2\,mxx^{m}\log\left(x\right)}{\left(m^{2}+2\,m+1\right)\left(m+1\right)}+\frac{x^{m+1}\log\left(x\right)^{2}}{m+1}-\frac{2\,xx^{m}\log\left(x\right)}{\left(m^{2}+2\,m+1\right)\left(m+1\right)}+\frac{2\,xx^{m}}{\left(m^{2}+2\,m+1\right)\left(m+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*log(x)^2,x, algorithm="giac")
```

```
[Out] -2*m*x*x^m*log(x)/((m^2 + 2*m + 1)*(m + 1)) + x^(m + 1)*log(x)^2/(m + 1) -
2*x*x^m*log(x)/((m^2 + 2*m + 1)*(m + 1)) + 2*x*x^m/((m^2 + 2*m + 1)*(m + 1)
)
```

3.610 $\int \frac{\log^2(x)}{x^{5/2}} dx$

Optimal. Leaf size=34

$$-\frac{16}{27x^{3/2}}-\frac{2\log^2(x)}{3x^{3/2}}-\frac{8\log(x)}{9x^{3/2}}$$

```
[Out] -16/(27*x^(3/2)) - (8*Log[x])/(9*x^(3/2)) - (2*Log[x]^2)/(3*x^(3/2))
```

Rubi [A] time = 0.0202339, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2305, 2304}

$$-\frac{16}{27x^{3/2}}-\frac{2\log^2(x)}{3x^{3/2}}-\frac{8\log(x)}{9x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^2/x^(5/2),x]
```

```
[Out] -16/(27*x^(3/2)) - (8*Log[x])/(9*x^(3/2)) - (2*Log[x]^2)/(3*x^(3/2))
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^2(x)}{x^{5/2}} dx &= -\frac{2\log^2(x)}{3x^{3/2}}+\frac{4}{3}\int \frac{\log(x)}{x^{5/2}} dx\\ &= -\frac{16}{27x^{3/2}}-\frac{8\log(x)}{9x^{3/2}}-\frac{2\log^2(x)}{3x^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.0068995, size = 21, normalized size = 0.62

$$-\frac{2\left(9\log^2(x)+12\log(x)+8\right)}{27x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^2/x^(5/2),x]
```

```
[Out] (-2*(8 + 12*Log[x] + 9*Log[x]^2))/(27*x^(3/2))
```

Maple [A] time = 0.016, size = 23, normalized size = 0.7

$$-\frac{16}{27}x^{-\frac{3}{2}}-\frac{8\ln(x)}{9}x^{-\frac{3}{2}}-\frac{2(\ln(x))^2}{3}x^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2/x^(5/2),x)
```

```
[Out] -16/27/x^(3/2)-8/9*ln(x)/x^(3/2)-2/3*ln(x)^2/x^(3/2)
```

Maxima [A] time = 0.955466, size = 30, normalized size = 0.88

$$-\frac{2\log(x)^2}{3x^{\frac{3}{2}}}-\frac{8\log(x)}{9x^{\frac{3}{2}}}-\frac{16}{27x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2/x^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*log(x)^2/x^(3/2) - 8/9*log(x)/x^(3/2) - 16/27/x^(3/2)
```

Fricas [A] time = 2.15775, size = 59, normalized size = 1.74

$$-\frac{2\left(9\log(x)^2+12\log(x)+8\right)}{27x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2/x^(5/2),x, algorithm="fricas")
```

```
[Out] -2/27*(9*log(x)^2 + 12*log(x) + 8)/x^(3/2)
```

Sympy [A] time = 6.13746, size = 34, normalized size = 1.

$$-\frac{2\log(x)^2}{3x^{\frac{3}{2}}}-\frac{8\log(x)}{9x^{\frac{3}{2}}}-\frac{16}{27x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2/x**(5/2),x)
```

```
[Out] -2*log(x)**2/(3*x**(3/2)) - 8*log(x)/(9*x**(3/2)) - 16/(27*x**(3/2))
```

Giac [A] time = 1.16687, size = 30, normalized size = 0.88

$$-\frac{2\,\log\left(x\right)^{2}}{3\,x^{\frac{3}{2}}}-\frac{8\,\log\left(x\right)}{9\,x^{\frac{3}{2}}}-\frac{16}{27\,x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2/x^(5/2),x, algorithm="giac")
```

```
[Out] -2/3*log(x)^2/x^(3/2) - 8/9*log(x)/x^(3/2) - 16/27/x^(3/2)
```

3.611 $\int (a+bx)\log(x)\,dx$

Optimal. Leaf size=28

$$-ax+ax\log(x)-\frac{bx^2}{4}+\frac{1}{2}bx^2\log(x)$$

```
[Out] -(a*x) - (b*x^2)/4 + a*x*Log[x] + (b*x^2*Log[x])/2
```

Rubi [A] time = 0.0112043, antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2313}

$$\frac{1}{2}\log(x)\left(2ax+bx^2\right)-ax-\frac{bx^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*Log[x],x]
```

```
[Out] -(a*x) - (b*x^2)/4 + ((2*a*x + b*x^2)*Log[x])/2
```

Rule 2313

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]
), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
 d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (a+bx)\log(x)\,dx &= \frac{1}{2}\left(2ax+bx^2\right)\log(x)-\int \left(a+\frac{bx}{2}\right)dx\\ &= -ax-\frac{bx^2}{4}+\frac{1}{2}\left(2ax+bx^2\right)\log(x)\end{aligned}$$

Mathematica [A] time = 0.001755, size = 28, normalized size = 1.

$$-ax+ax\log(x)-\frac{bx^2}{4}+\frac{1}{2}bx^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*Log[x],x]
```

```
[Out] -(a*x) - (b*x^2)/4 + a*x*Log[x] + (b*x^2*Log[x])/2
```

Maple [A] time = 0.002, size = 25, normalized size = 0.9

$$-ax-\frac{bx^2}{4}+ax\ln(x)+\frac{bx^2\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*ln(x),x)
```

```
[Out] -a*x-1/4*b*x^2+a*x*ln(x)+1/2*b*x^2*ln(x)
```

Maxima [A] time = 0.989507, size = 34, normalized size = 1.21

$$-\frac{1}{4}bx^2 - ax + \frac{1}{2}\left(bx^2 + 2\,ax\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(x),x, algorithm="maxima")
```

```
[Out] -1/4*b*x^2 - a*x + 1/2*(b*x^2 + 2*a*x)*log(x)
```

Fricas [A] time = 2.04165, size = 63, normalized size = 2.25

$$-\frac{1}{4}bx^2 - ax + \frac{1}{2}\left(bx^2 + 2\,ax\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(x),x, algorithm="fricas")
```

```
[Out] -1/4*b*x^2 - a*x + 1/2*(b*x^2 + 2*a*x)*log(x)
```

Sympy [A] time = 0.10186, size = 22, normalized size = 0.79

$$-ax - \frac{bx^2}{4} + \left(ax + \frac{bx^2}{2}\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*ln(x),x)
```

```
[Out] -a*x - b*x**2/4 + (a*x + b*x**2/2)*log(x)
```

Giac [A] time = 1.09703, size = 32, normalized size = 1.14

$$\frac{1}{2}bx^2\log\left(x\right) - \frac{1}{4}bx^2 + ax\log\left(x\right) - ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*log(x),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*log(x) - 1/4*b*x^2 + a*x*log(x) - a*x
```

3.612 $\int (a+bx)^3 \log(x)\, dx$

Optimal. Leaf size=67

$$-\frac{3}{4}a^2bx^2-\frac{a^4\log(x)}{4b}-a^3x-\frac{1}{3}ab^2x^3+\frac{\log(x)(a+bx)^4}{4b}-\frac{b^3x^4}{16}$$

```
[Out] -(a^3*x) - (3*a^2*b*x^2)/4 - (a*b^2*x^3)/3 - (b^3*x^4)/16 - (a^4*Log[x])/(4
*b) + ((a + b*x)^4*Log[x])/(4*b)
```

Rubi [A] time = 0.0333638, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {32, 2313, 12, 43}

$$-\frac{3}{4}a^2bx^2-\frac{a^4\log(x)}{4b}-a^3x-\frac{1}{3}ab^2x^3+\frac{\log(x)(a+bx)^4}{4b}-\frac{b^3x^4}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*Log[x],x]
```

```
[Out] -(a^3*x) - (3*a^2*b*x^2)/4 - (a*b^2*x^3)/3 - (b^3*x^4)/16 - (a^4*Log[x])/(4
*b) + ((a + b*x)^4*Log[x])/(4*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2313

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]
), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
 d, e, n, r}, x] && IGtQ[q, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+bx)^3\log(x)\,dx = \frac{(a+bx)^4\log(x)}{4b} - \int \frac{(a+bx)^4}{4bx}\,dx$$
$$= \frac{(a+bx)^4\log(x)}{4b} - \frac{\int \frac{(a+bx)^4}{x}\,dx}{4b}$$
$$= \frac{(a+bx)^4\log(x)}{4b} - \frac{\int \left(4a^3b + \frac{a^4}{x} + 6a^2b^2x + 4ab^3x^2 + b^4x^3\right)dx}{4b}$$
$$= -a^3x - \frac{3}{4}a^2bx^2 - \frac{1}{3}ab^2x^3 - \frac{b^3x^4}{16} - \frac{a^4\log(x)}{4b} + \frac{(a+bx)^4\log(x)}{4b}$$

Mathematica [A] time = 0.0219597, size = 81, normalized size = 1.21

$$-\frac{3}{4}a^2bx^2 + \frac{3}{2}a^2bx^2\log(x) - a^3x + a^3x\log(x) - \frac{1}{3}ab^2x^3 + ab^2x^3\log(x) - \frac{1}{16}b^3x^4 + \frac{1}{4}b^3x^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^3*Log[x],x]
```

```
[Out] -(a^3*x) - (3*a^2*b*x^2)/4 - (a*b^2*x^3)/3 - (b^3*x^4)/16 + a^3*x*Log[x] +
(3*a^2*b*x^2*Log[x])/2 + a*b^2*x^3*Log[x] + (b^3*x^4*Log[x])/4
```

Maple [A] time = 0.001, size = 72, normalized size = 1.1

$$\frac{b^3x^4\ln(x)}{4} - \frac{b^3x^4}{16} + b^2ax^3\ln(x) - \frac{ab^2x^3}{3} + \frac{3a^2bx^2\ln(x)}{2} - \frac{3a^2bx^2}{4} + \ln(x)xa^3 - a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*ln(x),x)
```

```
[Out] 1/4*b^3*x^4*ln(x)-1/16*b^3*x^4+b^2*a*x^3*ln(x)-1/3*a*b^2*x^3+3/2*a^2*b*x^2*
ln(x)-3/4*a^2*b*x^2+ln(x)*x*a^3-a^3*x
```

Maxima [A] time = 0.938009, size = 93, normalized size = 1.39

$$-\frac{1}{16}b^3x^4 - \frac{1}{3}ab^2x^3 - \frac{3}{4}a^2bx^2 - a^3x + \frac{1}{4}\left(b^3x^4 + 4ab^2x^3 + 6a^2bx^2 + 4a^3x\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(x),x, algorithm="maxima")
```

```
[Out] -1/16*b^3*x^4 - 1/3*a*b^2*x^3 - 3/4*a^2*b*x^2 - a^3*x + 1/4*(b^3*x^4 + 4*a*
b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(x)
```

Fricas [A] time = 2.0462, size = 157, normalized size = 2.34

$$-\frac{1}{16}b^3x^4 - \frac{1}{3}ab^2x^3 - \frac{3}{4}a^2bx^2 - a^3x + \frac{1}{4}\left(b^3x^4 + 4ab^2x^3 + 6a^2bx^2 + 4a^3x\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(x),x, algorithm="fricas")
```

```
[Out] -1/16*b^3*x^4 - 1/3*a*b^2*x^3 - 3/4*a^2*b*x^2 - a^3*x + 1/4*(b^3*x^4 + 4*a*
b^2*x^3 + 6*a^2*b*x^2 + 4*a^3*x)*log(x)
```

Sympy [A] time = 0.133159, size = 71, normalized size = 1.06

$$-a^3x - \frac{3a^2bx^2}{4} - \frac{ab^2x^3}{3} - \frac{b^3x^4}{16} + \left(a^3x + \frac{3a^2bx^2}{2} + ab^2x^3 + \frac{b^3x^4}{4}\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*ln(x),x)
```

```
[Out] -a**3*x - 3*a**2*b*x**2/4 - a*b**2*x**3/3 - b**3*x**4/16 + (a**3*x + 3*a**2
*b*x**2/2 + a*b**2*x**3 + b**3*x**4/4)*log(x)
```

Giac [A] time = 1.1419, size = 96, normalized size = 1.43

$$\frac{1}{4}b^3x^4\log(x) - \frac{1}{16}b^3x^4 + ab^2x^3\log(x) - \frac{1}{3}ab^2x^3 + \frac{3}{2}a^2bx^2\log(x) - \frac{3}{4}a^2bx^2 + a^3x\log(x) - a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*log(x),x, algorithm="giac")
```

```
[Out] 1/4*b^3*x^4*log(x) - 1/16*b^3*x^4 + a*b^2*x^3*log(x) - 1/3*a*b^2*x^3 + 3/2*
a^2*b*x^2*log(x) - 3/4*a^2*b*x^2 + a^3*x*log(x) - a^3*x
```

3.613 $\int \left(-1-8 \log^2(x)+3 \log^3(x)\right) \, dx$

Optimal. Leaf size=23

$$-35 x+3 x \log^3(x)-17 x \log^2(x)+34 x \log(x)$$

```
[Out] -35*x + 34*x*Log[x] - 17*x*Log[x]^2 + 3*x*Log[x]^3
```

Rubi [A] time = 0.0138408, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2296, 2295}

$$-35 x+3 x \log^3(x)-17 x \log^2(x)+34 x \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[-1 - 8*Log[x]^2 + 3*Log[x]^3,x]
```

```
[Out] -35*x + 34*x*Log[x] - 17*x*Log[x]^2 + 3*x*Log[x]^3
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \left(-1-8 \log^2(x)+3 \log^3(x)\right) \, dx &= -x+3 \int \log^3(x) \, dx-8 \int \log^2(x) \, dx\\
&= -x-8 x \log^2(x)+3 x \log^3(x)-9 \int \log^2(x) \, dx+16 \int \log(x) \, dx\\
&= -17 x+16 x \log(x)-17 x \log^2(x)+3 x \log^3(x)+18 \int \log(x) \, dx\\
&= -35 x+34 x \log(x)-17 x \log^2(x)+3 x \log^3(x)
\end{aligned}$$

Mathematica [A] time = 0.002495, size = 23, normalized size = 1.

$$-35 x+3 x \log^3(x)-17 x \log^2(x)+34 x \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[-1 - 8*Log[x]^2 + 3*Log[x]^3,x]
```

```
[Out] -35*x + 34*x*Log[x] - 17*x*Log[x]^2 + 3*x*Log[x]^3
```

Maple [A] time = 0.002, size = 24, normalized size = 1.

$$-35\,x+34\,x\ln\left(x\right)-17\,x\left(\ln\left(x\right)\right)^{2}+3\,x\left(\ln\left(x\right)\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1-8*ln(x)^2+3*ln(x)^3,x)
```

```
[Out] -35*x+34*x*ln(x)-17*x*ln(x)^2+3*x*ln(x)^3
```

Maxima [A] time = 0.946304, size = 49, normalized size = 2.13

$$3\left(\log\left(x\right)^{3}-3\,\log\left(x\right)^{2}+6\,\log\left(x\right)-6\right)x-8\left(\log\left(x\right)^{2}-2\,\log\left(x\right)+2\right)x-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1-8*log(x)^2+3*log(x)^3,x, algorithm="maxima")
```

```
[Out] 3*(log(x)^3 - 3*log(x)^2 + 6*log(x) - 6)*x - 8*(log(x)^2 - 2*log(x) + 2)*x
- x
```

Fricas [A] time = 2.11497, size = 69, normalized size = 3.

$$3\,x\log\left(x\right)^{3}-17\,x\log\left(x\right)^{2}+34\,x\log\left(x\right)-35\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1-8*log(x)^2+3*log(x)^3,x, algorithm="fricas")
```

```
[Out] 3*x*log(x)^3 - 17*x*log(x)^2 + 34*x*log(x) - 35*x
```

Sympy [A] time = 0.1018, size = 26, normalized size = 1.13

$$3x\log\left(x\right)^{3}-17x\log\left(x\right)^{2}+34x\log\left(x\right)-35x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1-8*ln(x)**2+3*ln(x)**3,x)
```

```
[Out] 3*x*log(x)**3 - 17*x*log(x)**2 + 34*x*log(x) - 35*x
```

Giac [A] time = 1.13521, size = 31, normalized size = 1.35

$$3\,x\log\left(x\right)^{3}-17\,x\log\left(x\right)^{2}+34\,x\log\left(x\right)-35\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1-8*log(x)^2+3*log(x)^3,x, algorithm="giac")
```

```
[Out] 3*x*log(x)^3 - 17*x*log(x)^2 + 34*x*log(x) - 35*x
```

3.614 $\int (1+x^4) (1-2\log(x)+\log^3(x)) \, dx$

Optimal. Leaf size=60

$$\frac{169x^5}{625}+\frac{1}{5}x^5\log^3(x)-\frac{3}{25}x^5\log^2(x)-\frac{44}{125}x^5\log(x)-3x+x\log^3(x)-3x\log^2(x)+4x\log(x)$$

```
[Out] -3*x + (169*x^5)/625 + 4*x*Log[x] - (44*x^5*Log[x])/125 - 3*x*Log[x]^2 - (3
*x^5*Log[x]^2)/25 + x*Log[x]^3 + (x^5*Log[x]^3)/5
```

Rubi [A] time = 0.0906383, antiderivative size = 73, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6742, 2313, 12, 2330, 2296, 2295, 2305, 2304}

$$\frac{169x^5}{625}+\frac{1}{5}x^5\log^3(x)-\frac{3}{25}x^5\log^2(x)+\frac{6}{125}x^5\log(x)-\frac{2}{5}\left(x^5+5x\right)\log(x)-3x+x\log^3(x)-3x\log^2(x)+6x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)*(1 - 2*Log[x] + Log[x]^3),x]
```

```
[Out] -3*x + (169*x^5)/625 + 6*x*Log[x] + (6*x^5*Log[x])/125 - (2*(5*x + x^5)*Log
[x])/5 - 3*x*Log[x]^2 - (3*x^5*Log[x]^2)/25 + x*Log[x]^3 + (x^5*Log[x]^3)/5
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2313

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]
), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
 d, e, n, r}, x] && IGtQ[q, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2330

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(1+x^4\right)\left(1-2\log(x)+\log^3(x)\right)\,dx &= \int \left(1+x^4-2\left(1+x^4\right)\log(x)+\left(1+x^4\right)\log^3(x)\right)\,dx \\
&= x+\frac{x^5}{5}-2\int \left(1+x^4\right)\log(x)\,dx+\int \left(1+x^4\right)\log^3(x)\,dx \\
&= x+\frac{x^5}{5}-\frac{2}{5}\left(5x+x^5\right)\log(x)+2\int \frac{1}{5}\left(5+x^4\right)\,dx+\int \left(\log^3(x)+x^4\log^3(x)\right) \\
&= x+\frac{x^5}{5}-\frac{2}{5}\left(5x+x^5\right)\log(x)+\frac{2}{5}\int \left(5+x^4\right)\,dx+\int \log^3(x)\,dx+\int x^4\log^3 \\
&= 3x+\frac{7x^5}{25}-\frac{2}{5}\left(5x+x^5\right)\log(x)+x\log^3(x)+\frac{1}{5}x^5\log^3(x)-\frac{3}{5}\int x^4\log^2(x)\,d \\
&= 3x+\frac{7x^5}{25}-\frac{2}{5}\left(5x+x^5\right)\log(x)-3x\log^2(x)-\frac{3}{25}x^5\log^2(x)+x\log^3(x)+\frac{1}{5}x \\
&= -3x+\frac{169x^5}{625}+6x\log(x)+\frac{6}{125}x^5\log(x)-\frac{2}{5}\left(5x+x^5\right)\log(x)-3x\log^2(x)-
\end{aligned}$$

Mathematica [A] time = 0.0036325, size = 60, normalized size = 1.

$$\frac{169x^5}{625}+\frac{1}{5}x^5\log^3(x)-\frac{3}{25}x^5\log^2(x)-\frac{44}{125}x^5\log(x)-3x+x\log^3(x)-3x\log^2(x)+4x\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)*(1 - 2*Log[x] + Log[x]^3),x]
```

```
[Out] -3*x + (169*x^5)/625 + 4*x*Log[x] - (44*x^5*Log[x])/125 - 3*x*Log[x]^2 - (3
*x^5*Log[x]^2)/25 + x*Log[x]^3 + (x^5*Log[x]^3)/5
```

Maple [A] time = 0.002, size = 53, normalized size = 0.9

$$-3\,x+\frac{169\,x^5}{625}+4\,x\ln(x)-\frac{44\,x^5\ln(x)}{125}-3\,x\left(\ln(x)\right)^2-\frac{3\,x^5\left(\ln(x)\right)^2}{25}+x\left(\ln(x)\right)^3+\frac{x^5\left(\ln(x)\right)^3}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)*(1-2*ln(x)+ln(x)^3),x)
```

```
[Out] -3*x+169/625*x^5+4*x*ln(x)-44/125*x^5*ln(x)-3*x*ln(x)^2-3/25*x^5*ln(x)^2+x*
ln(x)^3+1/5*x^5*ln(x)^3
```

Maxima [A] time = 0.95742, size = 89, normalized size = 1.48

$$\frac{1}{625}\left(125\log(x)^3-75\log(x)^2+30\log(x)-6\right)x^5-\frac{2}{25}x^5(5\log(x)-1)+\frac{1}{5}x^5+\left(\log(x)^3-3\log(x)^2+6\log(x)-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(1-2*log(x)+log(x)^3),x, algorithm="maxima")
```

```
[Out] 1/625*(125*log(x)^3 - 75*log(x)^2 + 30*log(x) - 6)*x^5 - 2/25*x^5*(5*log(x)
 - 1) + 1/5*x^5 + (log(x)^3 - 3*log(x)^2 + 6*log(x) - 6)*x - 2*x*(log(x) -
1) + x
```

Fricas [A] time = 2.06399, size = 144, normalized size = 2.4

$$\frac{169}{625}x^5+\frac{1}{5}\left(x^5+5x\right)\log(x)^3-\frac{3}{25}\left(x^5+25x\right)\log(x)^2-\frac{4}{125}\left(11x^5-125x\right)\log(x)-3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(1-2*log(x)+log(x)^3),x, algorithm="fricas")
```

```
[Out] 169/625*x^5 + 1/5*(x^5 + 5*x)*log(x)^3 - 3/25*(x^5 + 25*x)*log(x)^2 - 4/125
*(11*x^5 - 125*x)*log(x) - 3*x
```

Sympy [A] time = 0.130412, size = 51, normalized size = 0.85

$$\frac{169x^5}{625}-3x+\left(-\frac{44x^5}{125}+4x\right)\log(x)+\left(-\frac{3x^5}{25}-3x\right)\log(x)^2+\left(\frac{x^5}{5}+x\right)\log(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)*(1-2*ln(x)+ln(x)**3),x)
```

```
[Out] 169*x**5/625 - 3*x + (-44*x**5/125 + 4*x)*log(x) + (-3*x**5/25 - 3*x)*log(x
)**2 + (x**5/5 + x)*log(x)**3
```

Giac [A] time = 1.13155, size = 70, normalized size = 1.17

$$\frac{1}{5}x^5\log(x)^3-\frac{3}{25}x^5\log(x)^2-\frac{44}{125}x^5\log(x)+\frac{169}{625}x^5+x\log(x)^3-3x\log(x)^2+4x\log(x)-3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)*(1-2*log(x)+log(x)^3),x, algorithm="giac")
```

```
[Out] 1/5*x^5*log(x)^3 - 3/25*x^5*log(x)^2 - 44/125*x^5*log(x) + 169/625*x^5 + x*
log(x)^3 - 3*x*log(x)^2 + 4*x*log(x) - 3*x
```

3.615 $\int \frac{1}{x^3 \log^4(x)}\,dx$

Optimal. Leaf size=43

$$-\frac{4}{3}\text{ExpIntegralEi}(-2\log(x))+\frac{1}{3x^2\log^2(x)}-\frac{1}{3x^2\log^3(x)}-\frac{2}{3x^2\log(x)}$$

```
[Out] (-4*ExpIntegralEi[-2*Log[x]])/3 - 1/(3*x^2*Log[x]^3) + 1/(3*x^2*Log[x]^2) -
 2/(3*x^2*Log[x])
```

Rubi [A] time = 0.0592722, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2306, 2309, 2178}

$$-\frac{4}{3}\text{ExpIntegralEi}(-2\log(x))+\frac{1}{3x^2\log^2(x)}-\frac{1}{3x^2\log^3(x)}-\frac{2}{3x^2\log(x)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Log[x]^4),x]
```


```
[Out] (-4*ExpIntegralEi[-2*Log[x]])/3 - 1/(3*x^2*Log[x]^3) + 1/(3*x^2*Log[x]^2) -
 2/(3*x^2*Log[x])
```


Rule 2306

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^3\log^4(x)}\,dx &= -\frac{1}{3x^2\log^3(x)} - \frac{2}{3}\int \frac{1}{x^3\log^3(x)}\,dx \\
&= -\frac{1}{3x^2\log^3(x)} + \frac{1}{3x^2\log^2(x)} + \frac{2}{3}\int \frac{1}{x^3\log^2(x)}\,dx \\
&= -\frac{1}{3x^2\log^3(x)} + \frac{1}{3x^2\log^2(x)} - \frac{2}{3x^2\log(x)} - \frac{4}{3}\int \frac{1}{x^3\log(x)}\,dx \\
&= -\frac{1}{3x^2\log^3(x)} + \frac{1}{3x^2\log^2(x)} - \frac{2}{3x^2\log(x)} - \frac{4}{3}\,\text{Subst}\left(\int \frac{e^{-2x}}{x}\,dx, x, \log(x)\right) \\
&= -\frac{4}{3}\text{Ei}(-2\log(x)) - \frac{1}{3x^2\log^3(x)} + \frac{1}{3x^2\log^2(x)} - \frac{2}{3x^2\log(x)}
\end{aligned}
$$

Mathematica [A] time = 0.0163129, size = 43, normalized size = 1.

$$-\frac{4}{3}\text{ExpIntegralEi}(-2\log(x)) + \frac{1}{3x^2\log^2(x)} - \frac{1}{3x^2\log^3(x)} - \frac{2}{3x^2\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Log[x]^4),x]
```

```
[Out] (-4*ExpIntegralEi[-2*Log[x]])/3 - 1/(3*x^2*Log[x]^3) + 1/(3*x^2*Log[x]^2) -
 2/(3*x^2*Log[x])
```

Maple [A] time = 0.005, size = 37, normalized size = 0.9

$$-\frac{1}{3\,x^2\,(\ln(x))^3} + \frac{1}{3\,x^2\,(\ln(x))^2} - \frac{2}{3\,x^2\ln(x)} + \frac{4\,\text{Ei}(1,2\,\ln(x))}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/ln(x)^4,x)
```

```
[Out] -1/3/x^2/ln(x)^3+1/3/x^2/ln(x)^2-2/3/x^2/ln(x)+4/3*Ei(1,2*ln(x))
```

Maxima [A] time = 1.05405, size = 11, normalized size = 0.26

$$-8\,\Gamma(-3, 2\,\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(x)^4,x, algorithm="maxima")
```

```
[Out] -8*gamma(-3, 2*log(x))
```

Fricas [A] time = 2.04585, size = 115, normalized size = 2.67

$$-\frac{4\,x^2\log(x)^3\,\text{log\_integral}\left(\frac{1}{x^2}\right) + 2\,\log(x)^2 - \log(x) + 1}{3\,x^2\log(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(x)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(4*x^2*log(x)^3*log_integral(x^(-2)) + 2*log(x)^2 - log(x) + 1)/(x^2*l
og(x)^3)
```

Sympy [A] time = 0.675919, size = 32, normalized size = 0.74

$$-\frac{4\operatorname{Ei}(-2\log(x))}{3}+\frac{-2\log(x)^2+\log(x)-1}{3x^2\log(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/ln(x)**4,x)
```

```
[Out] -4*Ei(-2*log(x))/3 + (-2*log(x)**2 + log(x) - 1)/(3*x**2*log(x)**3)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x^3\log(x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/log(x)^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x^3*log(x)^4), x)
```

3.616 $\int \frac{\log(x)}{a+bx}\,dx$

Optimal. Leaf size=29

$$\frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}+\frac{\log(x)\log\left(\frac{bx}{a}+1\right)}{b}$$

```
[Out] (Log[x]*Log[1 + (b*x)/a])/b + PolyLog[2, -((b*x)/a)]/b
```

Rubi [A] time = 0.0202085, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2317, 2391}

$$\frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}+\frac{\log(x)\log\left(\frac{bx}{a}+1\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(a + b*x),x]
```

```
[Out] (Log[x]*Log[1 + (b*x)/a])/b + PolyLog[2, -((b*x)/a)]/b
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{a+bx}\,dx &= \frac{\log(x)\log\left(1+\frac{bx}{a}\right)}{b}-\frac{\int \frac{\log\left(1+\frac{bx}{a}\right)}{x}\,dx}{b}\\ &= \frac{\log(x)\log\left(1+\frac{bx}{a}\right)}{b}+\frac{\text{Li}_2\left(-\frac{bx}{a}\right)}{b}\end{aligned}$$

Mathematica [A] time = 0.0024513, size = 30, normalized size = 1.03

$$\frac{\text{PolyLog}\left(2,-\frac{bx}{a}\right)}{b}+\frac{\log(x)\log\left(\frac{a+bx}{a}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(a + b*x),x]
```

```
[Out] (Log[x]*Log[(a + b*x)/a])/b + PolyLog[2, -((b*x)/a)]/b
```

Maple [A] time = 0.007, size = 32, normalized size = 1.1

$$\frac{1}{b}\mathrm{dilog}\left(\frac{bx+a}{a}\right)+\frac{\ln(x)}{b}\ln\left(\frac{bx+a}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(b*x+a),x)
```

```
[Out] dilog((b*x+a)/a)/b+ln(x)*ln((b*x+a)/a)/b
```

Maxima [A] time = 0.958278, size = 34, normalized size = 1.17

$$\frac{\log\left(\frac{bx}{a}+1\right)\log(x)+\mathrm{Li}_2\left(-\frac{bx}{a}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(b*x+a),x, algorithm="maxima")
```

```
[Out] (log(b*x/a + 1)*log(x) + dilog(-b*x/a))/b
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log(x)}{bx+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral(log(x)/(b*x + a), x)
```

Sympy [C] time = 7.30554, size = 151, normalized size = 5.21

$$\begin{cases} \frac{\log\left(\frac{a}{b}\right)\log\left(\frac{a}{b}+x\right)}{b}+\frac{i\pi\log\left(\frac{a}{b}+x\right)}{b}-\frac{\mathrm{Li}_2\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} & \text{for} \\ -\frac{\log\left(\frac{a}{b}\right)\log\left(\frac{1}{\frac{a}{b}+x}\right)}{b}-\frac{i\pi\log\left(\frac{1}{\frac{a}{b}+x}\right)}{b}-\frac{\mathrm{Li}_2\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} & \text{for} \\ -\frac{G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle|\frac{a}{b}+x\right)\log\left(\frac{a}{b}\right)}{b}-\frac{i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle|\frac{a}{b}+x\right)}{b}+\frac{G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|\frac{a}{b}+x\right)\log\left(\frac{a}{b}\right)}{b}+\frac{i\pi G_{2,2}^{0,2}\left(\begin{matrix}1,1 & \\ & 0,0\end{matrix}\middle|\frac{a}{b}+x\right)}{b}-\frac{\mathrm{Li}_2\left(\frac{b\left(\frac{a}{b}+x\right)}{a}\right)}{b} & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(b*x+a),x)
```

```
[Out] Piecewise((log(a/b)*log(a/b + x)/b + I*pi*log(a/b + x)/b - polylog(2, b*(a/
b + x)/a)/b, Abs(a/b + x) < 1), (-log(a/b)*log(1/(a/b + x))/b - I*pi*log(1/
(a/b + x))/b - polylog(2, b*(a/b + x)/a)/b, 1/Abs(a/b + x) < 1), (-meijerg(
((), (1, 1)), ((0, 0), ()), a/b + x)*log(a/b)/b - I*pi*meijerg(((), (1, 1))
, ((0, 0), ()), a/b + x)/b + meijerg(((1, 1), ()), ((), (0, 0)), a/b + x)*l
og(a/b)/b + I*pi*meijerg(((1, 1), ()), ((), (0, 0)), a/b + x)/b - polylog(2
, b*(a/b + x)/a)/b, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x)}{bx + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(x)/(b*x + a), x)
```

3.617 $\int \frac{\log(x)}{(a+bx)^2} dx$

Optimal. Leaf size=29

$$\frac{x\log(x)}{a(a+bx)} - \frac{\log(a+bx)}{ab}$$

```
[Out] (x*Log[x])/(a*(a + b*x)) - Log[a + b*x]/(a*b)
```

Rubi [A] time = 0.013455, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2314, 31}

$$\frac{x\log(x)}{a(a+bx)} - \frac{\log(a+bx)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(a + b*x)^2,x]

[Out] (x*Log[x])/(a*(a + b*x)) - Log[a + b*x]/(a*b)
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\log(x)}{(a+bx)^2} dx &= \frac{x\log(x)}{a(a+bx)} - \frac{\int \frac{1}{a+bx} dx}{a} \\ &= \frac{x\log(x)}{a(a+bx)} - \frac{\log(a+bx)}{ab} \end{aligned}$$

Mathematica [A] time = 0.0164849, size = 27, normalized size = 0.93

$$\frac{\frac{x\log(x)}{a+bx} - \frac{\log(a+bx)}{b}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(a + b*x)^2,x]

[Out] ((x*Log[x])/(a + b*x) - Log[a + b*x]/b)/a
```

Maple [A] time = 0.005, size = 30, normalized size = 1.

$$\frac{x\ln(x)}{a(bx+a)}-\frac{\ln(bx+a)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(b*x+a)^2,x)
```

```
[Out] x*ln(x)/a/(b*x+a)-ln(b*x+a)/a/b
```

Maxima [A] time = 0.93178, size = 51, normalized size = 1.76

$$-\frac{\frac{\log(bx+a)}{a}-\frac{\log(x)}{a}}{b}-\frac{\log(x)}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(log(b*x + a)/a - log(x)/a)/b - log(x)/((b*x + a)*b)
```

Fricas [A] time = 2.22414, size = 77, normalized size = 2.66

$$\frac{bx\log(x)-(bx+a)\log(bx+a)}{ab^2x+a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b*x*log(x) - (b*x + a)*log(b*x + a))/(a*b^2*x + a^2*b)
```

Sympy [A] time = 0.358122, size = 24, normalized size = 0.83

$$-\frac{\log(x)}{ab+b^2x}+\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(b*x+a)**2,x)
```

```
[Out] -log(x)/(a*b + b**2*x) + (log(x) - log(a/b + x))/(a*b)
```

Giac [A] time = 1.09642, size = 49, normalized size = 1.69

$$-\frac{\log(x)}{(bx+a)b}+\frac{\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -log(x)/((b*x + a)*b) + log(abs(-a/(b*x + a) + 1))/(a*b)
```

3.618 $\int \frac{\log^n(x)}{x}\,dx$

Optimal. Leaf size=12

$$\frac{\log^{n+1}(x)}{n+1}$$

```
[Out] Log[x]^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0174533, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2302, 30}

$$\frac{\log^{n+1}(x)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^n/x,x]
```

```
[Out] Log[x]^(1 + n)/(1 + n)
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^n(x)}{x}\,dx &= \text{Subst}\left(\int x^n\,dx, x, \log(x)\right)\\ &= \frac{\log^{1+n}(x)}{1+n}\end{aligned}$$

Mathematica [A] time = 0.0033826, size = 12, normalized size = 1.

$$\frac{\log^{n+1}(x)}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^n/x,x]
```

```
[Out] Log[x]^(1 + n)/(1 + n)
```

Maple [A] time = 0.002, size = 13, normalized size = 1.1

$$\frac{(\ln(x))^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^n/x,x)
```

```
[Out] ln(x)^(1+n)/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^n/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.40761, size = 34, normalized size = 2.83

$$\frac{\log(x)^{n}\log(x)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^n/x,x, algorithm="fricas")
```

```
[Out] log(x)^n*log(x)/(n + 1)
```

Sympy [A] time = 0.865165, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\log(x)^{n+1}}{n+1} & \text{for}\ n \neq -1 \\ \log(\log(x)) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**n/x,x)
```

```
[Out] Piecewise((log(x)**(n + 1)/(n + 1), Ne(n, -1)), (log(log(x)), True))
```

Giac [A] time = 1.13295, size = 16, normalized size = 1.33

$$\frac{\log(x)^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^n/x,x, algorithm="giac")

[Out] log(x)^(n + 1)/(n + 1)
```

3.619 $\int \frac{(a+b\log(x))^n}{x}\,dx$

Optimal. Leaf size=19

$$\frac{(a+b\log(x))^{n+1}}{b(n+1)}$$

```
[Out] (a + b*Log[x])^(1 + n)/(b*(1 + n))
```

Rubi [A] time = 0.0282467, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2302, 30}

$$\frac{(a+b\log(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Log[x])^n/x,x]
```

```
[Out] (a + b*Log[x])^(1 + n)/(b*(1 + n))
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\log(x))^n}{x}\,dx &= \frac{\text{Subst}\left(\int x^n\,dx, x, a+b\log(x)\right)}{b}\\ &= \frac{(a+b\log(x))^{1+n}}{b(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0116802, size = 19, normalized size = 1.

$$\frac{(a+b\log(x))^{n+1}}{b(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Log[x])^n/x,x]
```

```
[Out] (a + b*Log[x])^(1 + n)/(b*(1 + n))
```

Maple [A] time = 0.001, size = 20, normalized size = 1.1

$$\frac{(a+b\ln(x))^{1+n}}{b(1+n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*ln(x))^n/x,x)
```

```
[Out] (a+b*ln(x))^(1+n)/b/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(x))^n/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.36099, size = 58, normalized size = 3.05

$$\frac{(b\log(x)+a)(b\log(x)+a)^n}{bn+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(x))^n/x,x, algorithm="fricas")
```

```
[Out] (b*log(x) + a)*(b*log(x) + a)^n/(b*n + b)
```

Sympy [A] time = 1.25934, size = 36, normalized size = 1.89

$$-\begin{cases} -a^n\log(x) & \text{for } b=0 \\ -\frac{\begin{cases} \frac{(a+b\log(x))^{n+1}}{n+1} & \text{for } n\neq -1 \\ \log(a+b\log(x)) & \text{otherwise} \end{cases}}{b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*ln(x))**n/x,x)
```

```
[Out] -Piecewise((-a**n*log(x), Eq(b, 0)), (-Piecewise(((a + b*log(x))**(n + 1)/(
n + 1), Ne(n, -1)), (log(a + b*log(x)), True))/b, True))
```

Giac [A] time = 1.14566, size = 26, normalized size = 1.37

$$\frac{(b\log(x)+a)^{n+1}}{b(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*log(x))^n/x,x, algorithm="giac")
```

```
[Out] (b*log(x) + a)^(n + 1)/(b*(n + 1))
```

3.620 $\int \frac{1}{x(a+b\log(x))}\,dx$

Optimal. Leaf size=11

$$\frac{\log(a+b\log(x))}{b}$$

```
[Out] Log[a + b*Log[x]]/b
```

Rubi [A] time = 0.0237006, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2302, 29}

$$\frac{\log(a+b\log(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*Log[x])),x]

[Out] Log[a + b*Log[x]]/b
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(a+b\log(x))}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, a+b\log(x)\right)}{b}\\ &= \frac{\log(a+b\log(x))}{b}\end{aligned}$$

Mathematica [A] time = 0.0147001, size = 11, normalized size = 1.

$$\frac{\log(a+b\log(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*Log[x])),x]

[Out] Log[a + b*Log[x]]/b
```

Maple [A] time = 0.001, size = 12, normalized size = 1.1

$$\frac{\ln \left(a+b\ln \left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*ln(x)),x)
```

```
[Out] ln(a+b*ln(x))/b
```

Maxima [A] time = 0.934022, size = 15, normalized size = 1.36

$$\frac{\log \left(b\log \left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*log(x)),x, algorithm="maxima")
```

```
[Out] log(b*log(x) + a)/b
```

Fricas [A] time = 2.39609, size = 28, normalized size = 2.55

$$\frac{\log \left(b\log \left(x\right)+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*log(x)),x, algorithm="fricas")
```

```
[Out] log(b*log(x) + a)/b
```

Sympy [A] time = 0.112607, size = 8, normalized size = 0.73

$$\frac{\log \left(\frac{a}{b}+\log \left(x\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*ln(x)),x)
```

```
[Out] log(a/b + log(x))/b
```

Giac [B] time = 1.08645, size = 41, normalized size = 3.73

$$\frac{\log \left(\frac{1}{4}\,\pi^2 b^2 (\operatorname{sgn}\left(x\right)-1)^2+(b\log \left(|x|\right)+a)^2\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*log(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(1/4*pi^2*b^2*(sgn(x) - 1)^2 + (b*log(abs(x)) + a)^2)/b
```

3.621 $\int \frac{(a+b\log(x))^{-n}}{x}\,dx$

Optimal. Leaf size=23

$$\frac{(a+b\log(x))^{1-n}}{b(1-n)}$$

```
[Out] (a + b*Log[x])^(1 - n)/(b*(1 - n))
```

Rubi [A] time = 0.0324734, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2302, 30}

$$\frac{(a+b\log(x))^{1-n}}{b(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*Log[x])^n),x]

[Out] (a + b*Log[x])^(1 - n)/(b*(1 - n))
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\log(x))^{-n}}{x}\,dx &= \frac{\text{Subst}\left(\int x^{-n}\,dx, x, a+b\log(x)\right)}{b}\\ &= \frac{(a+b\log(x))^{1-n}}{b(1-n)}\end{aligned}$$

Mathematica [A] time = 0.011729, size = 23, normalized size = 1.

$$\frac{(a+b\log(x))^{1-n}}{b(1-n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*Log[x])^n),x]

[Out] (a + b*Log[x])^(1 - n)/(b*(1 - n))
```

Maple [A] time = 0., size = 24, normalized size = 1.

$$\frac{(a + b\ln(x))^{1-n}}{b(1-n)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/((a+b*ln(x))^n),x)
```

```
[Out] (a+b*ln(x))^(1-n)/b/(1-n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((a+b*log(x))^n),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.44858, size = 62, normalized size = 2.7

$$-\frac{b\log(x) + a}{(bn - b)(b\log(x) + a)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((a+b*log(x))^n),x, algorithm="fricas")
```

```
[Out] -(b*log(x) + a)/((b*n - b)*(b*log(x) + a)^n)
```

Sympy [A] time = 37.2145, size = 71, normalized size = 3.09

$$\begin{cases} \frac{\log(x)}{a} & \text{for } b = 0 \wedge n = 1 \\ a^{-n}\log(x) & \text{for } b = 0 \\ \frac{\log\left(\frac{a}{b} + \log(x)\right)}{b} & \text{for } n = 1 \\ -\frac{a}{bn(a+b\log(x))^n - b(a+b\log(x))^n} - \frac{b\log(x)}{bn(a+b\log(x))^n - b(a+b\log(x))^n} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((a+b*ln(x))**n),x)
```

```
[Out] Piecewise((log(x)/a, Eq(b, 0) & Eq(n, 1)), (a**(-n)*log(x), Eq(b, 0)), (log
(a/b + log(x))/b, Eq(n, 1)), (-a/(b*n*(a + b*log(x))**n - b*(a + b*log(x))*
*n) - b*log(x)/(b*n*(a + b*log(x))**n - b*(a + b*log(x))**n), True))
```

Giac [A] time = 1.0971, size = 30, normalized size = 1.3

$$-\frac{(b\log(x)+a)^{-n+1}}{b(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((a+b*log(x))^n),x, algorithm="giac")
```

```
[Out] -(b*log(x) + a)^(-n + 1)/(b*(n - 1))
```

3.622 $\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx$

Optimal. Leaf size=16

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)$$

```
[Out] ArcTanh[Log[x]/Sqrt[a^2 + Log[x]^2]]
```

Rubi [A] time = 0.0376583, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {217, 206}

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 + Log[x]^2]),x]
```

```
[Out] ArcTanh[Log[x]/Sqrt[a^2 + Log[x]^2]]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a^2+\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{a^2+x^2}}\,dx, x, \log(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)\\
&= \tanh^{-1}\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)
\end{aligned}$$

Mathematica [B] time = 0.021873, size = 46, normalized size = 2.88

$$\frac{1}{2}\log\left(\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}+1\right)-\frac{1}{2}\log\left(1-\frac{\log(x)}{\sqrt{a^2+\log^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 + Log[x]^2]),x]
```

```
[Out] -Log[1 - Log[x]/Sqrt[a^2 + Log[x]^2]]/2 + Log[1 + Log[x]/Sqrt[a^2 + Log[x]^
2]]/2
```

Maple [A] time = 0.004, size = 15, normalized size = 0.9

$$\ln\left(\ln(x)+\sqrt{a^2+(\ln(x))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2+ln(x)^2)^(1/2),x)
```

```
[Out] ln(ln(x)+(a^2+ln(x)^2)^(1/2))
```

Maxima [A] time = 0.948843, size = 12, normalized size = 0.75

$$\operatorname{arsinh}\left(\frac{\log(x)}{\sqrt{a^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+log(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(log(x)/sqrt(a^2))
```

Fricas [A] time = 2.4954, size = 50, normalized size = 3.12

$$-\log\left(\sqrt{a^2+\log(x)^2}-\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+log(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(sqrt(a^2 + log(x)^2) - log(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\sqrt{a^2+\log(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2+ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a**2 + log(x)**2)), x)
```

Giac [A] time = 1.11912, size = 24, normalized size = 1.5

$$-\log\left(\sqrt{a^2 + \log(x)^2} - \log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2+log(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(sqrt(a^2 + log(x)^2) - log(x))
```

3.623 $\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx$

Optimal. Leaf size=18

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{\log^2(x)-a^2}}\right)$$

```
[Out] ArcTanh[Log[x]/Sqrt[-a^2 + Log[x]^2]]
```

Rubi [A] time = 0.042227, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {217, 206}

$$\tanh^{-1}\left(\frac{\log(x)}{\sqrt{\log^2(x)-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-a^2 + Log[x]^2]),x]
```

```
[Out] ArcTanh[Log[x]/Sqrt[-a^2 + Log[x]^2]]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{-a^2+\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{-a^2+x^2}}\,dx, x, \log(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\log(x)}{\sqrt{-a^2+\log^2(x)}}\right)\\
&= \tanh^{-1}\left(\frac{\log(x)}{\sqrt{-a^2+\log^2(x)}}\right)
\end{aligned}$$

Mathematica [B] time = 0.0241887, size = 50, normalized size = 2.78

$$\frac{1}{2}\log\left(\frac{\log(x)}{\sqrt{\log^2(x)-a^2}}+1\right)-\frac{1}{2}\log\left(1-\frac{\log(x)}{\sqrt{\log^2(x)-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-a^2 + Log[x]^2]),x]
```

```
[Out] -Log[1 - Log[x]/Sqrt[-a^2 + Log[x]^2]]/2 + Log[1 + Log[x]/Sqrt[-a^2 + Log[x
]^2]]/2
```

Maple [A] time = 0.007, size = 17, normalized size = 0.9

$$\ln\left(\ln(x)+\sqrt{-a^2+(\ln(x))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-a^2+ln(x)^2)^(1/2),x)
```

```
[Out] ln(ln(x)+(-a^2+ln(x)^2)^(1/2))
```

Maxima [A] time = 0.935055, size = 27, normalized size = 1.5

$$\log\left(2\sqrt{-a^2+\log(x)^2}+2\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2+log(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*sqrt(-a^2 + log(x)^2) + 2*log(x))
```

Fricas [A] time = 2.41528, size = 51, normalized size = 2.83

$$-\log\left(\sqrt{-a^2+\log(x)^2}-\log(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2+log(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(sqrt(-a^2 + log(x)^2) - log(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-(a-\log(x))(a+\log(x))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a**2+ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(a - log(x))*(a + log(x)))), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a^2+log(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.624 $\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx$

Optimal. Leaf size=18

$$\tan^{-1}\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

```
[Out] ArcTan[Log[x]/Sqrt[a^2 - Log[x]^2]]
```

Rubi [A] time = 0.041188, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {217, 203}

$$\tan^{-1}\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a^2 - Log[x]^2]),x]
```

```
[Out] ArcTan[Log[x]/Sqrt[a^2 - Log[x]^2]]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{a^2-\log^2(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{a^2-x^2}}\,dx, x, \log(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)\\ &= \tan^{-1}\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)\end{aligned}$$

Mathematica [A] time = 0.0241556, size = 18, normalized size = 1.

$$\tan^{-1}\left(\frac{\log(x)}{\sqrt{a^2-\log^2(x)}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a^2 - Log[x]^2]),x]
```

```
[Out] ArcTan[Log[x]/Sqrt[a^2 - Log[x]^2]]
```

Maple [A] time = 0.006, size = 17, normalized size = 0.9

$$\arctan\left(\ln(x)\frac{1}{\sqrt{a^2-(\ln(x))^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a^2-ln(x)^2)^(1/2),x)
```

```
[Out] arctan(ln(x)/(a^2-ln(x)^2)^(1/2))
```

Maxima [A] time = 1.4427, size = 12, normalized size = 0.67

$$\arcsin\left(\frac{\log(x)}{\sqrt{a^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2-log(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(log(x)/sqrt(a^2))
```

Fricas [A] time = 2.47214, size = 63, normalized size = 3.5

$$-2\arctan\left(-\frac{a-\sqrt{a^2-\log(x)^2}}{\log(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2-log(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(-(a - sqrt(a^2 - log(x)^2))/log(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\sqrt{(a-\log(x))(a+\log(x))}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a**2-ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((a - log(x))*(a + log(x)))), x)
```

Giac [A] time = 1.38456, size = 14, normalized size = 0.78

$$\arcsin\left(\frac{\log(x)}{a}\right)\operatorname{sgn}(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a^2-log(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(log(x)/a)*sgn(a)
```

3.625 $\int \frac{1}{x \log(x)\sqrt{a^2+\log^2(x)}} \, dx$

Optimal. Leaf size=22

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

```
[Out] -(ArcTanh[Sqrt[a^2 + Log[x]^2]/a]/a)
```

Rubi [A] time = 0.0879265, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {266, 63, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Log[x]*Sqrt[a^2 + Log[x]^2]),x]

[Out] -(ArcTanh[Sqrt[a^2 + Log[x]^2]/a]/a)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\log(x)\sqrt{a^2+\log^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt{a^2+x^2}}\,dx, x, \log(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{a^2+x}}\,dx, x, \log^2(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{-a^2+x^2}\,dx, x, \sqrt{a^2+\log^2(x)}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

Mathematica [A] time = 0.0171483, size = 22, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2+\log^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Log[x]*Sqrt[a^2 + Log[x]^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[a^2 + Log[x]^2]/a]/a)
```

Maple [A] time = 0.005, size = 37, normalized size = 1.7

$$-\ln\left(\frac{1}{\ln(x)}\left(2a^2+2\sqrt{a^2}\sqrt{a^2+(\ln(x))^2}\right)\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x)/(a^2+ln(x)^2)^(1/2),x)
```

```
[Out] -1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2+ln(x)^2)^(1/2))/ln(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.3981, size = 117, normalized size = 5.32

$$-\frac{\log\left(a+\sqrt{a^2+\log(x)^2}-\log(x)\right)-\log\left(-a+\sqrt{a^2+\log(x)^2}-\log(x)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(log(a + sqrt(a^2 + log(x)^2) - log(x)) - log(-a + sqrt(a^2 + log(x)^2) -
log(x)))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a^2 + \log(x)^2}\log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x)/(a**2+ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a**2 + log(x)**2)*log(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(a^2+log(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.626 $\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx$

Optimal. Leaf size=24

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}$$

```
[Out] -(ArcTanh[Sqrt[a^2 - Log[x]^2]/a]/a)
```

Rubi [A] time = 0.0981423, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {266, 63, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Log[x]*Sqrt[a^2 - Log[x]^2]),x]

[Out] -(ArcTanh[Sqrt[a^2 - Log[x]^2]/a]/a)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\log(x)\sqrt{a^2-\log^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt{a^2-x^2}}\,dx, x, \log(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{a^2-xx}}\,dx, x, \log^2(x)\right)$$
$$= -\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, \sqrt{a^2-\log^2(x)}\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}$$

Mathematica [A] time = 0.0156425, size = 24, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a^2-\log^2(x)}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Log[x]*Sqrt[a^2 - Log[x]^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[a^2 - Log[x]^2]/a]/a)
```

Maple [A] time = 0.007, size = 39, normalized size = 1.6

$$-\ln\left(\frac{1}{\ln(x)}\left(2a^2+2\sqrt{a^2}\sqrt{a^2-(\ln(x))^2}\right)\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x)/(a^2-ln(x)^2)^(1/2),x)
```

```
[Out] -1/(a^2)^(1/2)*ln((2*a^2+2*(a^2)^(1/2)*(a^2-ln(x)^2)^(1/2))/ln(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.37175, size = 58, normalized size = 2.42

$$\frac{\log\left(-\frac{a-\sqrt{a^2-\log(x)^2}}{\log(x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] log(-(a - sqrt(a^2 - log(x)^2))/log(x))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{(a-\log(x))(a+\log(x))}\log(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x)/(a**2-ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt((a - log(x))*(a + log(x)))*log(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(a^2-log(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.627 $\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx$

Optimal. Leaf size=23

$$\frac{\tan^{-1}\left(\frac{\sqrt{\log^2(x)-a^2}}{a}\right)}{a}$$

```
[Out] ArcTan[Sqrt[-a^2 + Log[x]^2]/a]/a
```

Rubi [A] time = 0.098703, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {266, 63, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\log^2(x)-a^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Log[x]*Sqrt[-a^2 + Log[x]^2]),x]
```

```
[Out] ArcTan[Sqrt[-a^2 + Log[x]^2]/a]/a
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\log(x)\sqrt{-a^2+\log^2(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt{-a^2+x^2}}\,dx, x, \log(x)\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{-a^2+x}}\,dx, x, \log^2(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{a^2+x^2}\,dx, x, \sqrt{-a^2+\log^2(x)}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{-a^2+\log^2(x)}}{a}\right)}{a}$$

Mathematica [A] time = 0.0156958, size = 23, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{\log^2(x)-a^2}}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Log[x]*Sqrt[-a^2 + Log[x]^2]),x]
```

```
[Out] ArcTan[Sqrt[-a^2 + Log[x]^2]/a]/a
```

Maple [A] time = 0.005, size = 43, normalized size = 1.9

$$-\ln\left(\frac{1}{\ln(x)}\left(-2a^2+2\sqrt{-a^2}\sqrt{-a^2+(\ln(x))^2}\right)\right)\frac{1}{\sqrt{-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x)/(-a^2+ln(x)^2)^(1/2),x)
```

```
[Out] -1/(-a^2)^(1/2)*ln((-2*a^2+2*(-a^2)^(1/2)*(-a^2+ln(x)^2)^(1/2))/ln(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.33496, size = 65, normalized size = 2.83

$$\frac{2\arctan\left(\frac{\sqrt{-a^2+\log(x)^2}-\log(x)}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan((sqrt(-a^2 + log(x)^2) - log(x))/a)/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-\left(a-\log\left(x\right)\right)\left(a+\log\left(x\right)\right)}\log\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x)/(-a**2+ln(x)**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(a - log(x))*(a + log(x)))*log(x)), x)
```

Giac [A] time = 1.06522, size = 28, normalized size = 1.22

$$\frac{\arctan\left(\frac{\sqrt{-a^2+\log(x)^2}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)/(-a^2+log(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(-a^2 + log(x)^2)/a)/a
```

3.628 $\int \frac{\log(\log(x))}{x} \, dx$

Optimal. Leaf size=11

$$\log(x)\log(\log(x)) - \log(x)$$

```
[Out] -Log[x] + Log[x]*Log[Log[x]]
```

Rubi [A] time = 0.0072318, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2521}

$$\log(x)\log(\log(x)) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Log[x]]/x,x]
```

```
[Out] -Log[x] + Log[x]*Log[Log[x]]
```

Rule 2521

```
Int[((a_.) + Log[Log[(d_.)*(x_)^(n_.)]^(p_.)*(c_.)]*(b_.))/(x_), x_Symbol]
:> Simp[(Log[d*x^n]*(a + b*Log[c*Log[d*x^n]^p]))/n, x] - Simp[b*p*Log[x], x
] /; FreeQ[{a, b, c, d, n, p}, x]
```

Rubi steps

$$\int \frac{\log(\log(x))}{x} \, dx = -\log(x) + \log(x)\log(\log(x))$$

Mathematica [A] time = 0.0041991, size = 11, normalized size = 1.

$$\log(x)\log(\log(x)) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[x]]/x,x]
```

```
[Out] -Log[x] + Log[x]*Log[Log[x]]
```

Maple [A] time = 0.002, size = 12, normalized size = 1.1

$$-\ln(x) + \ln(x)\ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x))/x,x)
```

```
[Out] -ln(x)+ln(x)*ln(ln(x))
```

Maxima [A] time = 0.950311, size = 15, normalized size = 1.36

$$\log(x)\log(\log(x)) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))/x,x, algorithm="maxima")
```

```
[Out] log(x)*log(log(x)) - log(x)
```

Fricas [A] time = 2.45788, size = 39, normalized size = 3.55

$$\log(x)\log(\log(x)) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))/x,x, algorithm="fricas")
```

```
[Out] log(x)*log(log(x)) - log(x)
```

Sympy [A] time = 0.274747, size = 10, normalized size = 0.91

$$\log(x)\log(\log(x)) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x))/x,x)
```

```
[Out] log(x)*log(log(x)) - log(x)
```

Giac [A] time = 1.06199, size = 15, normalized size = 1.36

$$\log(x)\log(\log(x)) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))/x,x, algorithm="giac")
```

```
[Out] log(x)*log(log(x)) - log(x)
```

3.629 $\int \frac{\log^2(\log(x))}{x} \, dx$

Optimal. Leaf size=20

$$\log(x)\log^2(\log(x)) - 2\log(x)\log(\log(x)) + 2\log(x)$$

```
[Out] 2*Log[x] - 2*Log[x]*Log[Log[x]] + Log[x]*Log[Log[x]]^2
```

Rubi [A] time = 0.0177781, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2296, 2295}

$$\log(x)\log^2(\log(x)) - 2\log(x)\log(\log(x)) + 2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Log[x]]^2/x,x]
```

```
[Out] 2*Log[x] - 2*Log[x]*Log[Log[x]] + Log[x]*Log[Log[x]]^2
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^2(\log(x))}{x} \, dx &= \text{Subst}\left(\int \log^2(x) \, dx, x, \log(x)\right) \\
&= \log(x)\log^2(\log(x)) - 2\,\text{Subst}\left(\int \log(x) \, dx, x, \log(x)\right) \\
&= 2\log(x) - 2\log(x)\log(\log(x)) + \log(x)\log^2(\log(x))
\end{aligned}$$

Mathematica [A] time = 0.0059768, size = 20, normalized size = 1.

$$\log(x)\log^2(\log(x)) - 2\log(x)\log(\log(x)) + 2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[x]]^2/x,x]
```

```
[Out] 2*Log[x] - 2*Log[x]*Log[Log[x]] + Log[x]*Log[Log[x]]^2
```

Maple [A] time = 0.003, size = 21, normalized size = 1.1

$$2\ln(x) - 2\ln(x)\ln(\ln(x)) + \ln(x)(\ln(\ln(x)))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x))^2/x,x)
```

```
[Out] 2*ln(x)-2*ln(x)*ln(ln(x))+ln(x)*ln(ln(x))^2
```

Maxima [A] time = 0.958777, size = 20, normalized size = 1.

$$\left(\log(\log(x))^2 - 2\log(\log(x)) + 2\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^2/x,x, algorithm="maxima")
```

```
[Out] (log(log(x))^2 - 2*log(log(x)) + 2)*log(x)
```

Fricas [A] time = 2.41174, size = 76, normalized size = 3.8

$$\log(x)\log(\log(x))^2 - 2\log(x)\log(\log(x)) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^2/x,x, algorithm="fricas")
```

```
[Out] log(x)*log(log(x))^2 - 2*log(x)*log(log(x)) + 2*log(x)
```

Sympy [A] time = 0.316487, size = 24, normalized size = 1.2

$$\log(x)\log(\log(x))^2 - 2\log(x)\log(\log(x)) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x))**2/x,x)
```

```
[Out] log(x)*log(log(x))**2 - 2*log(x)*log(log(x)) + 2*log(x)
```

Giac [A] time = 1.06349, size = 27, normalized size = 1.35

$$\log(x)\log(\log(x))^2 - 2\log(x)\log(\log(x)) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^2/x,x, algorithm="giac")
```

```
[Out] log(x)*log(log(x))^2 - 2*log(x)*log(log(x)) + 2*log(x)
```

3.630 $\int \frac{\log^3(\log(x))}{x}\,dx$

Optimal. Leaf size=29

$$\log(x)\log^3(\log(x))-3\log(x)\log^2(\log(x))+6\log(x)\log(\log(x))-6\log(x)$$

```
[Out] -6*Log[x] + 6*Log[x]*Log[Log[x]] - 3*Log[x]*Log[Log[x]]^2 + Log[x]*Log[Log[
x]]^3
```

Rubi [A] time = 0.0215285, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2296, 2295}

$$\log(x)\log^3(\log(x))-3\log(x)\log^2(\log(x))+6\log(x)\log(\log(x))-6\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Log[x]]^3/x,x]
```

```
[Out] -6*Log[x] + 6*Log[x]*Log[Log[x]] - 3*Log[x]*Log[Log[x]]^2 + Log[x]*Log[Log[
x]]^3
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^3(\log(x))}{x}\,dx &= \text{Subst}\left(\int \log^3(x)\,dx,x,\log(x)\right)\\
&= \log(x)\log^3(\log(x))-3\,\text{Subst}\left(\int \log^2(x)\,dx,x,\log(x)\right)\\
&= -3\log(x)\log^2(\log(x))+\log(x)\log^3(\log(x))+6\,\text{Subst}\left(\int \log(x)\,dx,x,\log(x)\right)\\
&= -6\log(x)+6\log(x)\log(\log(x))-3\log(x)\log^2(\log(x))+\log(x)\log^3(\log(x))
\end{aligned}$$

Mathematica [A] time = 0.0088646, size = 29, normalized size = 1.

$$\log(x)\log^3(\log(x))-3\log(x)\log^2(\log(x))+6\log(x)\log(\log(x))-6\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[x]]^3/x,x]
```

```
[Out] -6*Log[x] + 6*Log[x]*Log[Log[x]] - 3*Log[x]*Log[Log[x]]^2 + Log[x]*Log[Log[
x]]^3
```

Maple [A] time = 0.002, size = 30, normalized size = 1.

$$-6\ln(x)+6\ln(x)\ln(\ln(x))-3\ln(x)(\ln(\ln(x)))^2+\ln(x)(\ln(\ln(x)))^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x))^3/x,x)
```

```
[Out] -6*ln(x)+6*ln(x)*ln(ln(x))-3*ln(x)*ln(ln(x))^2+ln(x)*ln(ln(x))^3
```

Maxima [A] time = 1.04443, size = 30, normalized size = 1.03

$$\left(\log(\log(x))^3-3\log(\log(x))^2+6\log(\log(x))-6\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^3/x,x, algorithm="maxima")
```

```
[Out] (log(log(x))^3 - 3*log(log(x))^2 + 6*log(log(x)) - 6)*log(x)
```

Fricas [A] time = 2.32822, size = 109, normalized size = 3.76

$$\log(x)\log(\log(x))^3-3\log(x)\log(\log(x))^2+6\log(x)\log(\log(x))-6\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^3/x,x, algorithm="fricas")
```

```
[Out] log(x)*log(log(x))^3 - 3*log(x)*log(log(x))^2 + 6*log(x)*log(log(x)) - 6*lo
g(x)
```

Sympy [A] time = 0.356041, size = 36, normalized size = 1.24

$$\log(x)\log(\log(x))^3-3\log(x)\log(\log(x))^2+6\log(x)\log(\log(x))-6\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x))**3/x,x)
```

```
[Out] log(x)*log(log(x))**3 - 3*log(x)*log(log(x))**2 + 6*log(x)*log(log(x)) - 6*
log(x)
```

Giac [A] time = 1.06163, size = 39, normalized size = 1.34

$$\log(x)\log(\log(x))^3-3\log(x)\log(\log(x))^2+6\log(x)\log(\log(x))-6\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^3/x,x, algorithm="giac")
```

```
[Out] log(x)*log(log(x))^3 - 3*log(x)*log(log(x))^2 + 6*log(x)*log(log(x)) - 6*lo
g(x)
```

3.631 $\int \frac{\log^4(\log(x))}{x} dx$

Optimal. Leaf size=38

$$\log(x)\log^4(\log(x)) - 4\log(x)\log^3(\log(x)) + 12\log(x)\log^2(\log(x)) - 24\log(x)\log(\log(x)) + 24\log(x)$$

```
[Out] 24*Log[x] - 24*Log[x]*Log[Log[x]] + 12*Log[x]*Log[Log[x]]^2 - 4*Log[x]*Log[
Log[x]]^3 + Log[x]*Log[Log[x]]^4
```

Rubi [A] time = 0.0256756, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2296, 2295}

$$\log(x)\log^4(\log(x)) - 4\log(x)\log^3(\log(x)) + 12\log(x)\log^2(\log(x)) - 24\log(x)\log(\log(x)) + 24\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Log[x]]^4/x,x]
```

```
[Out] 24*Log[x] - 24*Log[x]*Log[Log[x]] + 12*Log[x]*Log[Log[x]]^2 - 4*Log[x]*Log[
Log[x]]^3 + Log[x]*Log[Log[x]]^4
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log^4(\log(x))}{x} dx &= \text{Subst}\left(\int \log^4(x)\,dx, x, \log(x)\right)\\
&= \log(x)\log^4(\log(x)) - 4\,\text{Subst}\left(\int \log^3(x)\,dx, x, \log(x)\right)\\
&= -4\log(x)\log^3(\log(x)) + \log(x)\log^4(\log(x)) + 12\,\text{Subst}\left(\int \log^2(x)\,dx, x, \log(x)\right)\\
&= 12\log(x)\log^2(\log(x)) - 4\log(x)\log^3(\log(x)) + \log(x)\log^4(\log(x)) - 24\,\text{Subst}(\int \log(x)\,dx, x, \text{l}\\
&= 24\log(x) - 24\log(x)\log(\log(x)) + 12\log(x)\log^2(\log(x)) - 4\log(x)\log^3(\log(x)) + \log(x)\log^4(\text{l}
\end{aligned}$$

Mathematica [A] time = 0.0060759, size = 38, normalized size = 1.

$$\log(x)\log^4(\log(x)) - 4\log(x)\log^3(\log(x)) + 12\log(x)\log^2(\log(x)) - 24\log(x)\log(\log(x)) + 24\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[x]]^4/x,x]
```

```
[Out] 24*Log[x] - 24*Log[x]*Log[Log[x]] + 12*Log[x]*Log[Log[x]]^2 - 4*Log[x]*Log[
Log[x]]^3 + Log[x]*Log[Log[x]]^4
```

Maple [A] time = 0.002, size = 39, normalized size = 1.

$$24 \ln(x) - 24 \ln(x) \ln(\ln(x)) + 12 \ln(x) (\ln(\ln(x)))^2 - 4 \ln(x) (\ln(\ln(x)))^3 + \ln(x) (\ln(\ln(x)))^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x))^4/x,x)
```

```
[Out] 24*ln(x)-24*ln(x)*ln(ln(x))+12*ln(x)*ln(ln(x))^2-4*ln(x)*ln(ln(x))^3+ln(x)*
ln(ln(x))^4
```

Maxima [A] time = 0.981408, size = 39, normalized size = 1.03

$$\left(\log(\log(x))^4 - 4 \log(\log(x))^3 + 12 \log(\log(x))^2 - 24 \log(\log(x)) + 24\right) \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^4/x,x, algorithm="maxima")
```

```
[Out] (log(log(x))^4 - 4*log(log(x))^3 + 12*log(log(x))^2 - 24*log(log(x)) + 24)*
log(x)
```

Fricas [A] time = 2.75544, size = 147, normalized size = 3.87

$$\log(x) \log(\log(x))^4 - 4 \log(x) \log(\log(x))^3 + 12 \log(x) \log(\log(x))^2 - 24 \log(x) \log(\log(x)) + 24 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^4/x,x, algorithm="fricas")
```

```
[Out] log(x)*log(log(x))^4 - 4*log(x)*log(log(x))^3 + 12*log(x)*log(log(x))^2 - 2
4*log(x)*log(log(x)) + 24*log(x)
```

Sympy [A] time = 0.396785, size = 48, normalized size = 1.26

$$\log(x) \log(\log(x))^4 - 4 \log(x) \log(\log(x))^3 + 12 \log(x) \log(\log(x))^2 - 24 \log(x) \log(\log(x)) + 24 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x))**4/x,x)
```

```
[Out] log(x)*log(log(x))**4 - 4*log(x)*log(log(x))**3 + 12*log(x)*log(log(x))**2
- 24*log(x)*log(log(x)) + 24*log(x)
```

Giac [A] time = 1.06818, size = 51, normalized size = 1.34

$$\log(x)\log(\log(x))^4 - 4\log(x)\log(\log(x))^3 + 12\log(x)\log(\log(x))^2 - 24\log(x)\log(\log(x)) + 24\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^4/x,x, algorithm="giac")
```

```
[Out] log(x)*log(log(x))^4 - 4*log(x)*log(log(x))^3 + 12*log(x)*log(log(x))^2 - 2
4*log(x)*log(log(x)) + 24*log(x)
```

3.632 $\int \frac{\log^n(\log(x))}{x}\,dx$

Optimal. Leaf size=24

$$(-\log(\log(x)))^{-n}\log^n(\log(x))\text{Gamma}(n+1,-\log(\log(x)))$$

```
[Out] (Gamma[1 + n, -Log[Log[x]]]*Log[Log[x]]^n)/(-Log[Log[x]])^n
```

Rubi [A] time = 0.032174, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2299, 2181}

$$(-\log(\log(x)))^{-n}\log^n(\log(x))\text{Gamma}(n+1,-\log(\log(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Log[x]]^n/x,x]
```

```
[Out] (Gamma[1 + n, -Log[Log[x]]]*Log[Log[x]]^n)/(-Log[Log[x]])^n
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^n(\log(x))}{x}\,dx &= \text{Subst}\left(\int \log^n(x)\,dx, x, \log(x)\right)\\ &= \text{Subst}\left(\int e^x x^n\,dx, x, \log(\log(x))\right)\\ &= \Gamma(1+n,-\log(\log(x)))(-\log(\log(x)))^{-n}\log^n(\log(x))\end{aligned}$$

Mathematica [A] time = 0.0171608, size = 24, normalized size = 1.

$$(-\log(\log(x)))^{-n}\log^n(\log(x))\text{Gamma}(n+1,-\log(\log(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Log[x]]^n/x,x]
```

```
[Out] (Gamma[1 + n, -Log[Log[x]]]*Log[Log[x]]^n)/(-Log[Log[x]])^n
```

Maple [F] time = 0.023, size = 0, normalized size = 0.

$$\int \frac{(\ln (\ln (x)))^{n}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(x))^n/x,x)
```

```
[Out] int(ln(ln(x))^n/x,x)
```

Maxima [A] time = 1.05007, size = 39, normalized size = 1.62

$$-\left(-\log\left(\log\left(x\right)\right)\right)^{-n-1}\log\left(\log\left(x\right)\right)^{n+1}\Gamma\left(n+1,-\log\left(\log\left(x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^n/x,x, algorithm="maxima")
```

```
[Out] -(-log(log(x)))^(-n - 1)*log(log(x))^(n + 1)*gamma(n + 1, -log(log(x)))
```

Fricas [A] time = 2.77683, size = 51, normalized size = 2.12

$$\cos\left(\pi n\right)\Gamma\left(n+1,-\log\left(\log\left(x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^n/x,x, algorithm="fricas")
```

```
[Out] cos(pi*n)*gamma(n + 1, -log(log(x)))
```

Sympy [A] time = 2.76667, size = 24, normalized size = 1.

$$\left(-\log\left(\log\left(x\right)\right)\right)^{-n}\log\left(\log\left(x\right)\right)^{n}\Gamma\left(n+1,-\log\left(\log\left(x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(x))**n/x,x)
```

```
[Out] (-log(log(x)))**(-n)*log(log(x))**n*uppergamma(n + 1, -log(log(x)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\log\left(x\right)\right)^{n}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(x))^n/x,x, algorithm="giac")
```

```
[Out] integrate(log(log(x))^n/x, x)
```

3.633 $\int \frac{\cot(x)}{\log(\sin(x))} \, dx$

Optimal. Leaf size=4

$$\log(\log(\sin(x)))$$

```
[Out] Log[Log[Sin[x]]]
```

Rubi [A] time = 0.0200642, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4338, 2302, 29}

$$\log(\log(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Log[Sin[x]],x]

[Out] Log[Log[Sin[x]]]
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned} \int \frac{\cot(x)}{\log(\sin(x))} \, dx &= \text{Subst}\left(\int \frac{1}{x \log(x)} \, dx, x, \sin(x)\right) \\ &= \text{Subst}\left(\int \frac{1}{x} \, dx, x, \log(\sin(x))\right) \\ &= \log(\log(\sin(x))) \end{aligned}$$

Mathematica [A] time = 0.0105987, size = 4, normalized size = 1.

$$\log(\log(\sin(x)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Log[Sin[x]],x]
```

```
[Out] Log[Log[Sin[x]]]
```

Maple [A] time = 0.008, size = 5, normalized size = 1.3

$$\ln(\ln(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/ln(sin(x)),x)
```

```
[Out] ln(ln(sin(x)))
```

Maxima [A] time = 0.937304, size = 5, normalized size = 1.25

$$\log(\log(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(sin(x)),x, algorithm="maxima")
```

```
[Out] log(log(sin(x)))
```

Fricas [A] time = 2.3799, size = 24, normalized size = 6.

$$\log(\log(\sin(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(sin(x)),x, algorithm="fricas")
```

```
[Out] log(log(sin(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\log(\sin(x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/ln(sin(x)),x)
```

```
[Out] Integral(cot(x)/log(sin(x)), x)
```

Giac [A] time = 1.06849, size = 7, normalized size = 1.75

$$\log(|\log(\sin(x))|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/log(sin(x)),x, algorithm="giac")
```

```
[Out] log(abs(log(sin(x))))
```

3.634 $\int (\cos(x) + \sec(x)) \tan(x) \, dx$

Optimal. Leaf size=7

$$\sec(x) - \cos(x)$$

```
[Out] -Cos[x] + Sec[x]
```

Rubi [A] time = 0.0465842, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4236}

$$\sec(x) - \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Sec[x])*Tan[x],x]

[Out] -Cos[x] + Sec[x]
```

Rule 4236

```
Int[(u_)*((A_.) + cos[(a_.) + (b_.)*(x_)]*(B_.) + (C_.)*sec[(a_.) + (b_.)*(
x_)]), x_Symbol] :> Int[(ActivateTrig[u]*(C + A*Cos[a + b*x] + B*Cos[a + b*
x]^2))/Cos[a + b*x], x] /; FreeQ[{a, b, A, B, C}, x]
```

Rubi steps

$$\begin{aligned}
\int (\cos(x) + \sec(x)) \tan(x) \, dx &= \int \left(1 + \cos^2(x)\right) \sec(x) \tan(x) \, dx \\
&= -\text{Subst}\left(\int \left(1 + \frac{1}{x^2}\right) dx, x, \cos(x)\right) \\
&= -\cos(x) + \sec(x)
\end{aligned}$$

Mathematica [A] time = 0.0040395, size = 7, normalized size = 1.

$$\sec(x) - \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sec[x])*Tan[x],x]

[Out] -Cos[x] + Sec[x]
```

Maple [A] time = 0.013, size = 10, normalized size = 1.4

$$(\cos(x))^{-1} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/cos(x)+cos(x))*tan(x),x)
```

```
[Out] 1/cos(x)-cos(x)
```

Maxima [A] time = 0.937552, size = 12, normalized size = 1.71

$$\frac{1}{\cos(x)} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/cos(x)+cos(x))*tan(x),x, algorithm="maxima")
```

```
[Out] 1/cos(x) - cos(x)
```

Fricas [A] time = 2.44796, size = 32, normalized size = 4.57

$$-\frac{\cos(x)^2 - 1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/cos(x)+cos(x))*tan(x),x, algorithm="fricas")
```

```
[Out] -(cos(x)^2 - 1)/cos(x)
```

Sympy [A] time = 2.42084, size = 7, normalized size = 1.

$$-\cos(x) + \frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/cos(x)+cos(x))*tan(x),x)
```

```
[Out] -cos(x) + 1/cos(x)
```

Giac [A] time = 1.06459, size = 12, normalized size = 1.71

$$\frac{1}{\cos(x)} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/cos(x)+cos(x))*tan(x),x, algorithm="giac")
```

```
[Out] 1/cos(x) - cos(x)
```

3.635 $\int \log(\cosh(x))\sinh(x)\,dx$

Optimal. Leaf size=11

$$\cosh(x)\log(\cosh(x)) - \cosh(x)$$

```
[Out] -Cosh[x] + Cosh[x]*Log[Cosh[x]]
```

Rubi [A] time = 0.0112569, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2638, 2554}

$$\cosh(x)\log(\cosh(x)) - \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cosh[x]]*Sinh[x],x]
```

```
[Out] -Cosh[x] + Cosh[x]*Log[Cosh[x]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \log(\cosh(x))\sinh(x)\,dx &= \cosh(x)\log(\cosh(x)) - \int \sinh(x)\,dx\\ &= -\cosh(x) + \cosh(x)\log(\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.0065869, size = 11, normalized size = 1.

$$\cosh(x)\log(\cosh(x)) - \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cosh[x]]*Sinh[x],x]
```

```
[Out] -Cosh[x] + Cosh[x]*Log[Cosh[x]]
```

Maple [A] time = 0.004, size = 12, normalized size = 1.1

$$-\cosh(x) + \cosh(x)\ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cosh(x))*sinh(x),x)
```

```
[Out] -cosh(x)+cosh(x)*ln(cosh(x))
```

Maxima [A] time = 0.947536, size = 15, normalized size = 1.36

$$\cosh(x)\log(\cosh(x)) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x))*sinh(x),x, algorithm="maxima")
```

```
[Out] cosh(x)*log(cosh(x)) - cosh(x)
```

Fricas [B] time = 2.20455, size = 185, normalized size = 16.82

$$-\frac{\cosh(x)^2 - \left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)\log(\cosh(x)) + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1}{2\left(\cosh(x) + \sinh(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x))*sinh(x),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh(x)^2 - (cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log(cosh(
x)) + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

Sympy [A] time = 0.939075, size = 10, normalized size = 0.91

$$\log(\cosh(x))\cosh(x) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cosh(x))*sinh(x),x)
```

```
[Out] log(cosh(x))*cosh(x) - cosh(x)
```

Giac [B] time = 1.06502, size = 43, normalized size = 3.91

$$\frac{1}{2}\left(e^{(-x)} + e^x\right)\log\left(\frac{1}{2}e^{(-x)} + \frac{1}{2}e^x\right) - \frac{1}{2}e^{(-x)} - \frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x))*sinh(x),x, algorithm="giac")
```

```
[Out] 1/2*(e^(-x) + e^x)*log(1/2*e^(-x) + 1/2*e^x) - 1/2*e^(-x) - 1/2*e^x
```

3.636 $\int \log(\cosh(x)) \tanh(x) \, dx$

Optimal. Leaf size=9

$$\frac{1}{2} \log^2(\cosh(x))$$

```
[Out] Log[Cosh[x]]^2/2
```

Rubi [A] time = 0.0154075, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3475, 4341, 2301}

$$\frac{1}{2} \log^2(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cosh[x]]*Tanh[x],x]

[Out] Log[Cosh[x]]^2/2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4341

```
Int[(u_)*Tanh[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cosh[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Cosh[c*(a
+ b*x)]/d, u, x], x], x, Cosh[c*(a + b*x)]/d], x] /; FunctionOfQ[Cosh[c*(a
+ b*x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \log(\cosh(x)) \tanh(x) \, dx &= \text{Subst}\left(\int \frac{\log(x)}{x} \, dx, x, \cosh(x)\right)\\ &= \frac{1}{2} \log^2(\cosh(x))\end{aligned}$$

Mathematica [A] time = 0.003558, size = 9, normalized size = 1.

$$\frac{1}{2} \log^2(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cosh[x]]*Tanh[x],x]
```

```
[Out] Log[Cosh[x]]^2/2
```

Maple [A] time = 0.008, size = 8, normalized size = 0.9

$$\frac{\left(\ln \left(\cosh \left(x\right)\right)\right)^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cosh(x))*tanh(x),x)
```

```
[Out] 1/2*ln(cosh(x))^2
```

Maxima [A] time = 0.941918, size = 9, normalized size = 1.

$$\frac{1}{2} \log \left(\cosh \left(x\right)\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x))*tanh(x),x, algorithm="maxima")
```

```
[Out] 1/2*log(cosh(x))^2
```

Fricas [A] time = 2.39197, size = 27, normalized size = 3.

$$\frac{1}{2} \log \left(\cosh \left(x\right)\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x))*tanh(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(cosh(x))^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(\cosh \left(x\right)\right) \tanh \left(x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cosh(x))*tanh(x),x)
```

```
[Out] Integral(log(cosh(x))*tanh(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log \left(\cosh \left(x\right)\right) \tanh \left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cosh(x))*tanh(x),x, algorithm="giac")
```

```
[Out] integrate(log(cosh(x))*tanh(x), x)
```

3.637 $\int \log\left(x-\sqrt{1+x^2}\right)\,dx$

Optimal. Leaf size=26

$$\sqrt{x^2+1}+x\log\left(x-\sqrt{x^2+1}\right)$$

```
[Out] Sqrt[1 + x^2] + x*Log[x - Sqrt[1 + x^2]]
```

Rubi [A] time = 0.0053162, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2534, 261}

$$\sqrt{x^2+1}+x\log\left(x-\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x - Sqrt[1 + x^2]],x]
```

```
[Out] Sqrt[1 + x^2] + x*Log[x - Sqrt[1 + x^2]]
```

Rule 2534

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]], x_Symbol] :
> Simp[x*Log[d + e*x + f*Sqrt[a + c*x^2]], x] - Dist[a*c*f^2, Int[x/(d*e*(a
 + c*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e,
f}, x] && EqQ[e^2 - c*f^2, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \log\left(x-\sqrt{1+x^2}\right)\,dx &= x\log\left(x-\sqrt{1+x^2}\right)+\int \frac{x}{\sqrt{1+x^2}}\,dx\\ &= \sqrt{1+x^2}+x\log\left(x-\sqrt{1+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0202813, size = 26, normalized size = 1.

$$\sqrt{x^2+1}+x\log\left(x-\sqrt{x^2+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x - Sqrt[1 + x^2]],x]
```

```
[Out] Sqrt[1 + x^2] + x*Log[x - Sqrt[1 + x^2]]
```

Maple [A] time = 0.002, size = 23, normalized size = 0.9

$$x\ln\left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x-(x^2+1)^(1/2)),x)
```

```
[Out] x*ln(x-(x^2+1)^(1/2))+(x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x\log\left(x-\sqrt{x^2+1}\right)-x+\arctan(x)+\int -\frac{x}{x^3-\left(x^2+1\right)^{\frac{3}{2}}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x-(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(x - sqrt(x^2 + 1)) - x + arctan(x) + integrate(-x/(x^3 - (x^2 + 1)^(3
/2) + x), x)
```

Fricas [A] time = 2.36471, size = 57, normalized size = 2.19

$$x\log\left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x-(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] x*log(x - sqrt(x^2 + 1)) + sqrt(x^2 + 1)
```

Sympy [A] time = 9.29586, size = 20, normalized size = 0.77

$$x\log\left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x-(x**2+1)**(1/2)),x)
```

```
[Out] x*log(x - sqrt(x**2 + 1)) + sqrt(x**2 + 1)
```

Giac [A] time = 1.05946, size = 30, normalized size = 1.15

$$x\log\left(x-\sqrt{x^2+1}\right)+\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x-(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(x - sqrt(x^2 + 1)) + sqrt(x^2 + 1)
```

3.638 $\int \frac{\log(-1+x)}{x^3}\,dx$

Optimal. Leaf size=35

$$-\frac{\log(x-1)}{2x^2}+\frac{1}{2x}+\frac{1}{2}\log(1-x)-\frac{\log(x)}{2}$$

```
[Out] 1/(2*x) + Log[1 - x]/2 - Log[-1 + x]/(2*x^2) - Log[x]/2
```

Rubi [A] time = 0.0145775, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2395, 44}

$$-\frac{\log(x-1)}{2x^2}+\frac{1}{2x}+\frac{1}{2}\log(1-x)-\frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[-1 + x]/x^3,x]
```

```
[Out] 1/(2*x) + Log[1 - x]/2 - Log[-1 + x]/(2*x^2) - Log[x]/2
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(-1+x)}{x^3}\,dx &= -\frac{\log(-1+x)}{2x^2}+\frac{1}{2}\int \frac{1}{(-1+x)x^2}\,dx\\
&= -\frac{\log(-1+x)}{2x^2}+\frac{1}{2}\int \left(\frac{1}{-1+x}-\frac{1}{x^2}-\frac{1}{x}\right)dx\\
&= \frac{1}{2x}+\frac{1}{2}\log(1-x)-\frac{\log(-1+x)}{2x^2}-\frac{\log(x)}{2}
\end{aligned}$$

Mathematica [A] time = 0.0102805, size = 27, normalized size = 0.77

$$\frac{1}{2}\left(-\frac{\log(x-1)}{x^2}+\frac{1}{x}+\log(1-x)-\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-1 + x]/x^3,x]
```

```
[Out] (x^(-1) + Log[1 - x] - Log[-1 + x]/x^2 - Log[x])/2
```

Maple [A] time = 0.011, size = 26, normalized size = 0.7

$$-\frac{\ln(x)}{2}+\frac{1}{2x}+\frac{\ln(-1+x)(-1+x)(1+x)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-1+x)/x^3,x)
```

```
[Out] -1/2*ln(x)+1/2/x+1/2*ln(-1+x)*(-1+x)*(1+x)/x^2
```

Maxima [A] time = 0.944461, size = 34, normalized size = 0.97

$$\frac{1}{2x}-\frac{\log(x-1)}{2x^2}+\frac{1}{2}\log(x-1)-\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2/x - 1/2*log(x - 1)/x^2 + 1/2*log(x - 1) - 1/2*log(x)
```

Fricas [A] time = 2.37088, size = 68, normalized size = 1.94

$$-\frac{x^2\log(x)-\left(x^2-1\right)\log(x-1)-x}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(x^2*log(x) - (x^2 - 1)*log(x - 1) - x)/x^2
```

Sympy [A] time = 0.127624, size = 26, normalized size = 0.74

$$-\frac{\log(x)}{2}+\frac{\log(x-1)}{2}+\frac{1}{2x}-\frac{\log(x-1)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-1+x)/x**3,x)
```

```
[Out] -log(x)/2 + log(x - 1)/2 + 1/(2*x) - log(x - 1)/(2*x**2)
```

Giac [A] time = 1.0585, size = 36, normalized size = 1.03

$$\frac{1}{2x} - \frac{\log(x-1)}{2x^2} + \frac{1}{2}\log(|x-1|) - \frac{1}{2}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-1+x)/x^3,x, algorithm="giac")
```

```
[Out] 1/2/x - 1/2*log(x - 1)/x^2 + 1/2*log(abs(x - 1)) - 1/2*log(abs(x))
```

3.639 $\int (-e^{-x}+e^x) \log \left(1+e^{2x}\right) \, dx$

Optimal. Leaf size=32

$$-2e^x+e^{-x}\log\left(e^{2x}+1\right)+e^x\log\left(e^{2x}+1\right)$$

```
[Out] -2*E^x + Log[1 + E^(2*x)]/E^x + E^x*Log[1 + E^(2*x)]
```

Rubi [A] time = 0.0538533, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2282, 2476, 2448, 321, 203, 2455}

$$-2e^x+e^{-x}\log\left(e^{2x}+1\right)+e^x\log\left(e^{2x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-E^(-x) + E^x)*Log[1 + E^(2*x)],x]
```

```
[Out] -2*E^x + Log[1 + E^(2*x)]/E^x + E^x*Log[1 + E^(2*x)]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (-e^{-x}+e^x)\log\left(1+e^{2x}\right)dx = \text{Subst}\left(\int \frac{\left(-1+x^2\right)\log\left(1+x^2\right)}{x^2}dx, x, e^x\right)$$
$$= \text{Subst}\left(\int \left(\log\left(1+x^2\right)-\frac{\log\left(1+x^2\right)}{x^2}\right)dx, x, e^x\right)$$
$$= \text{Subst}\left(\int \log\left(1+x^2\right)dx, x, e^x\right) - \text{Subst}\left(\int \frac{\log\left(1+x^2\right)}{x^2}dx, x, e^x\right)$$
$$= e^{-x}\log\left(1+e^{2x}\right)+e^x\log\left(1+e^{2x}\right)-2\,\text{Subst}\left(\int \frac{1}{1+x^2}dx, x, e^x\right)-2\,\text{Subst}\left(\int \frac{x^2}{1+ \right.$$
$$= -2e^x-2\tan^{-1}\left(e^x\right)+e^{-x}\log\left(1+e^{2x}\right)+e^x\log\left(1+e^{2x}\right)+2\,\text{Subst}\left(\int \frac{1}{1+x^2}dx, x, e\right.$$
$$= -2e^x+e^{-x}\log\left(1+e^{2x}\right)+e^x\log\left(1+e^{2x}\right)$$

Mathematica [A] time = 0.0313868, size = 24, normalized size = 0.75

$$(e^{-x}+e^x)\log\left(e^{2x}+1\right)-2e^x$$

Antiderivative was successfully verified.

```
[In] Integrate[(-E^(-x) + E^x)*Log[1 + E^(2*x)],x]
```

```
[Out] -2*E^x + (E^(-x) + E^x)*Log[1 + E^(2*x)]
```

Maple [A] time = 0.016, size = 32, normalized size = 1.

$$\frac{\left(e^x\right)^2\ln\left(\left(e^x\right)^2+1\right)-2\left(e^x\right)^2+\ln\left(\left(e^x\right)^2+1\right)}{e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/exp(x)+exp(x))*ln(1+exp(2*x)),x)
```

```
[Out] (exp(x)^2*ln(exp(x)^2+1)-2*exp(x)^2+ln(exp(x)^2+1))/exp(x)
```

Maxima [A] time = 0.952153, size = 27, normalized size = 0.84

$$\left(e^{(-x)}+e^x\right)\log\left(e^{(2x)}+1\right)-2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x, algorithm="maxima")
```

```
[Out] (e^(-x) + e^x)*log(e^(2*x) + 1) - 2*e^x
```

Fricas [A] time = 2.39275, size = 72, normalized size = 2.25

$$\left(\left(e^{(2x)}+1\right)\log\left(e^{(2x)}+1\right)-2e^{(2x)}\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x, algorithm="fricas")
```

```
[Out] ((e^(2*x) + 1)*log(e^(2*x) + 1) - 2*e^(2*x))*e^(-x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ShapeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))*ln(1+exp(2*x)),x)
```

```
[Out] Exception raised: ShapeError
```

Giac [A] time = 1.061, size = 27, normalized size = 0.84

$$\left(e^{(-x)}+e^{x}\right)\log\left(e^{(2x)}+1\right)-2e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1/exp(x)+exp(x))*log(1+exp(2*x)),x, algorithm="giac")
```

```
[Out] (e^(-x) + e^x)*log(e^(2*x) + 1) - 2*e^x
```

3.640 $\int e^{3x/2} \log(-1+e^x)\,dx$

Optimal. Leaf size=52

$$-\frac{4e^{x/2}}{3}-\frac{4}{9}e^{3x/2}+\frac{2}{3}e^{3x/2}\log\left(e^x-1\right)+\frac{4}{3}\tanh^{-1}\left(e^{x/2}\right)$$

```
[Out] (-4*E^(x/2))/3 - (4*E^((3*x)/2))/9 + (4*ArcTanh[E^(x/2)])/3 + (2*E^((3*x)/2
)*Log[-1 + E^x])/3
```

Rubi [A] time = 0.0436347, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2194, 2554, 12, 2248, 302, 207}

$$-\frac{4e^{x/2}}{3}-\frac{4}{9}e^{3x/2}+\frac{2}{3}e^{3x/2}\log\left(e^x-1\right)+\frac{4}{3}\tanh^{-1}\left(e^{x/2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^((3*x)/2)*Log[-1 + E^x],x]
```

```
[Out] (-4*E^(x/2))/3 - (4*E^((3*x)/2))/9 + (4*ArcTanh[E^(x/2)])/3 + (2*E^((3*x)/2
)*Log[-1 + E^x])/3
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int e^{3x/2}\log(-1+e^x)\,dx &= \frac{2}{3}e^{3x/2}\log(-1+e^x)-\int\frac{2e^{5x/2}}{3(-1+e^x)}\,dx\\
&= \frac{2}{3}e^{3x/2}\log(-1+e^x)-\frac{2}{3}\int\frac{e^{5x/2}}{-1+e^x}\,dx\\
&= \frac{2}{3}e^{3x/2}\log(-1+e^x)-\frac{4}{3}\,\text{Subst}\left(\int\frac{x^4}{-1+x^2}\,dx,x,e^{x/2}\right)\\
&= \frac{2}{3}e^{3x/2}\log(-1+e^x)-\frac{4}{3}\,\text{Subst}\left(\int\left(1+x^2+\frac{1}{-1+x^2}\right)dx,x,e^{x/2}\right)\\
&= -\frac{4e^{x/2}}{3}-\frac{4}{9}e^{3x/2}+\frac{2}{3}e^{3x/2}\log(-1+e^x)-\frac{4}{3}\,\text{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,e^{x/2}\right)\\
&= -\frac{4e^{x/2}}{3}-\frac{4}{9}e^{3x/2}+\frac{4}{3}\tanh^{-1}\left(e^{x/2}\right)+\frac{2}{3}e^{3x/2}\log(-1+e^x)
\end{aligned}$$

Mathematica [A] time = 0.0333664, size = 42, normalized size = 0.81

$$\frac{2}{9}\left(e^{x/2}\left(3e^x\log\left(e^x-1\right)-2\left(e^x+3\right)\right)+6\tanh^{-1}\left(e^{x/2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^((3*x)/2)*Log[-1 + E^x],x]

[Out] (2*(6*ArcTanh[E^(x/2)] + E^(x/2)*(-2*(3 + E^x) + 3*E^x*Log[-1 + E^x])))/9
```

Maple [A] time = 0.023, size = 43, normalized size = 0.8

$$\frac{2\ln\left(-1+e^x\right)}{3}e^{\frac{3x}{2}}-\frac{4}{9}e^{\frac{3x}{2}}-\frac{4}{3}e^{\frac{x}{2}}+\frac{2}{3}\ln\left(e^{\frac{x}{2}}+1\right)-\frac{2}{3}\ln\left(-1+e^{\frac{x}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(3/2*x)*ln(-1+exp(x)),x)

[Out] 2/3*exp(3/2*x)*ln(-1+exp(x))-4/9*exp(3/2*x)-4/3*exp(1/2*x)+2/3*ln(exp(1/2*x
)+1)-2/3*ln(-1+exp(1/2*x))
```

Maxima [A] time = 0.950337, size = 57, normalized size = 1.1

$$\frac{2}{3}e^{\left(\frac{3}{2}x\right)}\log\left(e^x-1\right)-\frac{4}{9}e^{\left(\frac{3}{2}x\right)}-\frac{4}{3}e^{\left(\frac{1}{2}x\right)}+\frac{2}{3}\log\left(e^{\left(\frac{1}{2}x\right)}+1\right)-\frac{2}{3}\log\left(e^{\left(\frac{1}{2}x\right)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3/2*x)*log(-1+exp(x)),x, algorithm="maxima")
```

```
[Out] 2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*log(e^(1/2
*x) + 1) - 2/3*log(e^(1/2*x) - 1)
```

Fricas [A] time = 2.37824, size = 149, normalized size = 2.87

$$\frac{2}{3}e^{\left(\frac{3}{2}x\right)}\log\left(e^{x}-1\right)-\frac{4}{9}e^{\left(\frac{3}{2}x\right)}-\frac{4}{3}e^{\left(\frac{1}{2}x\right)}+\frac{2}{3}\log\left(e^{\left(\frac{1}{2}x\right)}+1\right)-\frac{2}{3}\log\left(e^{\left(\frac{1}{2}x\right)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3/2*x)*log(-1+exp(x)),x, algorithm="fricas")
```

```
[Out] 2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*log(e^(1/2
*x) + 1) - 2/3*log(e^(1/2*x) - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3/2*x)*ln(-1+exp(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.07113, size = 58, normalized size = 1.12

$$\frac{2}{3}e^{\left(\frac{3}{2}x\right)}\log\left(e^{x}-1\right)-\frac{4}{9}e^{\left(\frac{3}{2}x\right)}-\frac{4}{3}e^{\left(\frac{1}{2}x\right)}+\frac{2}{3}\log\left(e^{\left(\frac{1}{2}x\right)}+1\right)-\frac{2}{3}\log\left(\left|e^{\left(\frac{1}{2}x\right)}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3/2*x)*log(-1+exp(x)),x, algorithm="giac")
```

```
[Out] 2/3*e^(3/2*x)*log(e^x - 1) - 4/9*e^(3/2*x) - 4/3*e^(1/2*x) + 2/3*log(e^(1/2
*x) + 1) - 2/3*log(abs(e^(1/2*x) - 1))
```

3.641 $\int \cos^3(x)\log(\sin(x))\,dx$

Optimal. Leaf size=30

$$\frac{\sin^3(x)}{9}-\sin(x)-\frac{1}{3}\sin^3(x)\log(\sin(x))+\sin(x)\log(\sin(x))$$

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x] + Sin[x]^3/9 - (Log[Sin[x]]*Sin[x]^3)/3
```

Rubi [A] time = 0.0348104, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2633, 2554, 12, 4356}

$$\frac{\sin^3(x)}{9}-\sin(x)-\frac{1}{3}\sin^3(x)\log(\sin(x))+\sin(x)\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Log[Sin[x]],x]
```

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x] + Sin[x]^3/9 - (Log[Sin[x]]*Sin[x]^3)/3
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4356

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\log(\sin(x))\,dx &= \log(\sin(x))\sin(x)-\frac{1}{3}\log(\sin(x))\sin^3(x)-\int \frac{1}{6}\cos(x)(5+\cos(2x))\,dx\\ &= \log(\sin(x))\sin(x)-\frac{1}{3}\log(\sin(x))\sin^3(x)-\frac{1}{6}\int \cos(x)(5+\cos(2x))\,dx\\ &= \log(\sin(x))\sin(x)-\frac{1}{3}\log(\sin(x))\sin^3(x)-\frac{1}{6}\operatorname{Subst}\left(\int \left(6-2x^2\right)\,dx,x,\sin(x)\right)\\ &= -\sin(x)+\log(\sin(x))\sin(x)+\frac{\sin^3(x)}{9}-\frac{1}{3}\log(\sin(x))\sin^3(x)\end{aligned}$$

Mathematica [A] time = 0.0077247, size = 30, normalized size = 1.

$$\frac{\sin^3(x)}{9} - \sin(x) - \frac{1}{3}\sin^3(x)\log(\sin(x)) + \sin(x)\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Log[Sin[x]],x]
```

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x] + Sin[x]^3/9 - (Log[Sin[x]]*Sin[x]^3)/3
```

Maple [C] time = 0.035, size = 126, normalized size = 4.2

$$-\frac{i}{24}\mathrm{e}^{3ix}\ln(2\sin(x)) + \frac{i}{72}\mathrm{e}^{3ix} + \frac{11i}{24}\mathrm{e}^{ix} - \frac{3i}{8}\mathrm{e}^{ix}\ln(2\sin(x)) + \frac{3i}{8}\mathrm{e}^{-ix}\ln(2\sin(x)) - \frac{11i}{24}\mathrm{e}^{-ix} + \frac{i}{24}\mathrm{e}^{-3ix}\ln(2\sin(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*ln(sin(x)),x)
```

```
[Out] -1/24*I*exp(3*I*x)*ln(2*sin(x))+1/72*I*exp(3*I*x)+11/24*I*exp(I*x)-3/8*I*ex
p(I*x)*ln(2*sin(x))+3/8*I*exp(-I*x)*ln(2*sin(x))-11/24*I*exp(-I*x)+1/24*I*e
xp(-3*I*x)*ln(2*sin(x))-1/72*I*exp(-3*I*x)+1/24*I*ln(2)*exp(3*I*x)+3/8*I*ln
(2)*exp(I*x)-1/24*I*ln(2)*exp(-3*I*x)-3/8*I*ln(2)*exp(-I*x)
```

Maxima [A] time = 0.958534, size = 34, normalized size = 1.13

$$\frac{1}{9}\sin(x)^3 - \frac{1}{3}\left(\sin(x)^3 - 3\sin(x)\right)\log(\sin(x)) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*log(sin(x)),x, algorithm="maxima")
```

```
[Out] 1/9*sin(x)^3 - 1/3*(sin(x)^3 - 3*sin(x))*log(sin(x)) - sin(x)
```

Fricas [A] time = 2.61872, size = 90, normalized size = 3.

$$\frac{1}{3}\left(\cos(x)^2 + 2\right)\log(\sin(x))\sin(x) - \frac{1}{9}\left(\cos(x)^2 + 8\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*log(sin(x)),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + 2)*log(sin(x))*sin(x) - 1/9*(cos(x)^2 + 8)*sin(x)
```

Sympy [A] time = 5.09148, size = 42, normalized size = 1.4

$$\frac{2\log(\sin(x))\sin^3(x)}{3} + \log(\sin(x))\sin(x)\cos^2(x) - \frac{8\sin^3(x)}{9} - \sin(x)\cos^2(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*ln(sin(x)),x)
```

```
[Out] 2*log(sin(x))*sin(x)**3/3 + log(sin(x))*sin(x)*cos(x)**2 - 8*sin(x)**3/9 -
sin(x)*cos(x)**2
```

Giac [A] time = 1.07313, size = 35, normalized size = 1.17

$$-\frac{1}{3}\log\left(\sin\left(x\right)\right)\sin\left(x\right)^{3}+\frac{1}{9}\sin\left(x\right)^{3}+\log\left(\sin\left(x\right)\right)\sin\left(x\right)-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*log(sin(x)),x, algorithm="giac")
```

```
[Out] -1/3*log(sin(x))*sin(x)^3 + 1/9*sin(x)^3 + log(sin(x))*sin(x) - sin(x)
```

3.642 $\int \log(\tan(x))\sec^4(x)\,dx$

Optimal. Leaf size=30

$$-\frac{\tan^3(x)}{9}-\tan(x)+\frac{1}{3}\tan^3(x)\log(\tan(x))+\tan(x)\log(\tan(x))$$

```
[Out] -Tan[x] + Log[Tan[x]]*Tan[x] - Tan[x]^3/9 + (Log[Tan[x]]*Tan[x]^3)/3
```

Rubi [A] time = 0.0582128, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3767, 2554, 12}

$$-\frac{\tan^3(x)}{9}-\tan(x)+\frac{1}{3}\tan^3(x)\log(\tan(x))+\tan(x)\log(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Log[Tan[x]]*Sec[x]^4,x]
```

```
[Out] -Tan[x] + Log[Tan[x]]*Tan[x] - Tan[x]^3/9 + (Log[Tan[x]]*Tan[x]^3)/3
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\begin{aligned}
\int \log(\tan(x))\sec^4(x)\,dx &= \log(\tan(x))\tan(x)+\frac{1}{3}\log(\tan(x))\tan^3(x)-\int \frac{1}{3}(2+\cos(2x))\sec^4(x)\,dx\\
&= \log(\tan(x))\tan(x)+\frac{1}{3}\log(\tan(x))\tan^3(x)-\frac{1}{3}\int (2+\cos(2x))\sec^4(x)\,dx\\
&= \log(\tan(x))\tan(x)+\frac{1}{3}\log(\tan(x))\tan^3(x)-\frac{1}{3}\operatorname{Subst}\left(\int \left(3+x^2\right)\,dx,x,\tan(x)\right)\\
&= -\tan(x)+\log(\tan(x))\tan(x)-\frac{\tan^3(x)}{9}+\frac{1}{3}\log(\tan(x))\tan^3(x)
\end{aligned}$$

Mathematica [A] time = 0.0709439, size = 29, normalized size = 0.97

$$\frac{1}{9}\tan(x)\left(\sec^2(x)(6\log(\tan(x))+3\cos(2x)\log(\tan(x))-1)-8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Tan[x]]*Sec[x]^4,x]
```

```
[Out] ((-8 + (-1 + 6*Log[Tan[x]] + 3*Cos[2*x]*Log[Tan[x]])*Sec[x]^2)*Tan[x])/9
```

Maple [B] time = 0.153, size = 55, normalized size = 1.8

$$\frac{\sin(x)}{9(\cos(x))^3}\left(6(\cos(x))^2\ln\left(1/2\frac{\sin(x)}{\cos(x)}\right)+6(\cos(x))^2\ln(2)-8(\cos(x))^2+3\ln\left(1/2\frac{\sin(x)}{\cos(x)}\right)+3\ln(2)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(tan(x))/cos(x)^4,x)
```

```
[Out] 1/9*(6*cos(x)^2*ln(1/2*sin(x)/cos(x))+6*cos(x)^2*ln(2)-8*cos(x)^2+3*ln(1/2*
sin(x)/cos(x))+3*ln(2)-1)*sin(x)/cos(x)^3
```

Maxima [A] time = 0.944442, size = 34, normalized size = 1.13

$$-\frac{1}{9}\tan(x)^3+\frac{1}{3}\left(\tan(x)^3+3\tan(x)\right)\log(\tan(x))-\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tan(x))/cos(x)^4,x, algorithm="maxima")
```

```
[Out] -1/9*tan(x)^3 + 1/3*(tan(x)^3 + 3*tan(x))*log(tan(x)) - tan(x)
```

Fricas [A] time = 2.61341, size = 117, normalized size = 3.9

$$\frac{3\left(2\cos(x)^2+1\right)\log\left(\frac{\sin(x)}{\cos(x)}\right)\sin(x)-\left(8\cos(x)^2+1\right)\sin(x)}{9\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tan(x))/cos(x)^4,x, algorithm="fricas")
```

```
[Out] 1/9*(3*(2*cos(x)^2 + 1)*log(sin(x)/cos(x))*sin(x) - (8*cos(x)^2 + 1)*sin(x)
)/cos(x)^3
```

Sympy [A] time = 160.155, size = 46, normalized size = 1.53

$$\frac{\log(\tan(x))\tan^3(x)}{3}+\log(\tan(x))\tan(x)-\frac{\sin^3(x)}{9\cos^3(x)}+\frac{\sin(x)}{3\cos(x)}-\frac{4\tan(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(tan(x))/cos(x)**4,x)
```

```
[Out] log(tan(x))*tan(x)**3/3 + log(tan(x))*tan(x) - sin(x)**3/(9*cos(x)**3) + si
n(x)/(3*cos(x)) - 4*tan(x)/3
```

Giac [A] time = 1.06601, size = 35, normalized size = 1.17

$$\frac{1}{3}\log(\tan(x))\tan(x)^3 - \frac{1}{9}\tan(x)^3 + \log(\tan(x))\tan(x) - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tan(x))/cos(x)^4,x, algorithm="giac")
```

```
[Out] 1/3*log(tan(x))*tan(x)^3 - 1/9*tan(x)^3 + log(tan(x))*tan(x) - tan(x)
```

3.643 $\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx$

Optimal. Leaf size=28

$$-\frac{x}{2}+\tan\left(\frac{x}{2}\right)+\frac{\sin(x)\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x)+1}$$

```
[Out] -x/2 + (Log[Cos[x/2]]*Sin[x])/(1 + Cos[x]) + Tan[x/2]
```

Rubi [A] time = 0.0347725, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {2648, 2554, 12, 3473, 8}

$$-\frac{x}{2}+\tan\left(\frac{x}{2}\right)+\frac{\sin(x)\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[Cos[x/2]]/(1 + Cos[x]),x]
```

```
[Out] -x/2 + (Log[Cos[x/2]]*Sin[x])/(1 + Cos[x]) + Tan[x/2]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{1+\cos(x)}\,dx &= \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)} - \int -\frac{1}{2}\tan^2\left(\frac{x}{2}\right)dx \\ &= \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)} + \frac{1}{2}\int \tan^2\left(\frac{x}{2}\right)dx \\ &= \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)} + \tan\left(\frac{x}{2}\right) - \frac{\int 1\,dx}{2} \\ &= -\frac{x}{2} + \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)\sin(x)}{1+\cos(x)} + \tan\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0845528, size = 32, normalized size = 1.14

$$-\frac{\sin(x)\left(x\cot\left(\frac{x}{2}\right)-2\left(\log\left(\cos\left(\frac{x}{2}\right)\right)+1\right)\right)}{2(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Cos[x/2]]/(1 + Cos[x]),x]
```

```
[Out] -((x*Cot[x/2] - 2*(1 + Log[Cos[x/2]]))*Sin[x])/(2*(1 + Cos[x]))
```

Maple [C] time = 0.163, size = 164, normalized size = 5.9

$$\frac{-2\,i\ln\left(e^{\frac{i}{2}x}\right)}{e^{ix}+1} + \frac{1}{e^{ix}+1}\left(-i\ln\left(e^{ix}+1\right)e^{ix} + \pi\,\mathrm{csgn}\left(i\left(e^{ix}+1\right)\right)\mathrm{csgn}\left(ie^{-\frac{i}{2}x}\right)\mathrm{csgn}\left(i\cos\left(\frac{x}{2}\right)\right) - \pi\,\mathrm{csgn}\left(i\left(e^{ix}+1\right)\right)\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(cos(1/2*x))/(cos(x)+1),x)
```

```
[Out] -2*I/(exp(I*x)+1)*ln(exp(1/2*I*x))+(-I*ln(exp(I*x)+1)*exp(I*x)+Pi*csgn(I*(e
xp(I*x)+1))*csgn(I*exp(-1/2*I*x))*csgn(I*cos(1/2*x))-Pi*csgn(I*(exp(I*x)+1)
)*csgn(I*cos(1/2*x))^2-Pi*csgn(I*exp(-1/2*I*x))*csgn(I*cos(1/2*x))^2+Pi*csg
n(I*cos(1/2*x))^3-x*exp(I*x)+I*ln(exp(I*x)+1)-2*I*ln(2)+2*I-x)/(exp(I*x)+1)
```

Maxima [B] time = 0.955665, size = 76, normalized size = 2.71

$$\frac{\log\left(\cos\left(\frac{1}{2}\,x\right)\right)\sin\left(x\right)}{\cos\left(x\right)+1} - \frac{x\cos\left(x\right)^2 + x\sin\left(x\right)^2 + 2\,x\cos\left(x\right) + x - 4\,\sin\left(x\right)}{2\left(\cos\left(x\right)^2 + \sin\left(x\right)^2 + 2\,\cos\left(x\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(1/2*x))/(1+cos(x)),x, algorithm="maxima")
```

```
[Out] log(cos(1/2*x))*sin(x)/(cos(x) + 1) - 1/2*(x*cos(x)^2 + x*sin(x)^2 + 2*x*co
s(x) + x - 4*sin(x))/(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1)
```

Fricas [A] time = 2.61238, size = 105, normalized size = 3.75

$$-\frac{x\cos\left(\frac{1}{2}x\right) - 2\log\left(\cos\left(\frac{1}{2}x\right)\right)\sin\left(\frac{1}{2}x\right) - 2\sin\left(\frac{1}{2}x\right)}{2\cos\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(1/2*x))/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(x*cos(1/2*x) - 2*log(cos(1/2*x))*sin(1/2*x) - 2*sin(1/2*x))/cos(1/2*x
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\cos\left(\frac{x}{2}\right)\right)}{\cos(x) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(cos(1/2*x))/(1+cos(x)),x)
```

```
[Out] Integral(log(cos(x/2))/(cos(x) + 1), x)
```

Giac [A] time = 1.08722, size = 58, normalized size = 2.07

$$-\frac{1}{2}x - \frac{2\log\left(\cos\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)}{\left(x^2+1\right)\left(\frac{x^2-1}{x^2+1} - 1\right)} + \tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(cos(1/2*x))/(1+cos(x)),x, algorithm="giac")
```

```
[Out] -1/2*x - 2*log(cos(1/2*x))*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
 + tan(1/2*x)
```

3.644 $\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2} \, dx$

Optimal. Leaf size=60

$$-\frac{2x}{3}+\frac{8\sin(x)}{9(\cos(x)+1)}-\frac{\sin(x)}{9(\cos(x)+1)^2}+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

```
[Out] (-2*x)/3 - Sin[x]/(9*(1 + Cos[x])^2) + (8*Sin[x])/(9*(1 + Cos[x])) - (Log[S
in[x]]*Sin[x])/(3*(1 + Cos[x])^2) + (2*Log[Sin[x]]*Sin[x])/(3*(1 + Cos[x]))
```

Rubi [A] time = 0.132164, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {2750, 2648, 2554, 12, 2968, 3019, 2735}

$$-\frac{2x}{3}+\frac{8\sin(x)}{9(\cos(x)+1)}-\frac{\sin(x)}{9(\cos(x)+1)^2}+\frac{2\sin(x)\log(\sin(x))}{3(\cos(x)+1)}-\frac{\sin(x)\log(\sin(x))}{3(\cos(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Log[Sin[x]])/(1 + Cos[x])^2,x]
```


```
[Out] (-2*x)/3 - Sin[x]/(9*(1 + Cos[x])^2) + (8*Sin[x])/(9*(1 + Cos[x])) - (Log[S
in[x]]*Sin[x])/(3*(1 + Cos[x])^2) + (2*Log[Sin[x]]*Sin[x])/(3*(1 + Cos[x]))
```


Rule 2750

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), In
t[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && N
eQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2968

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3019

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((A*b - a
*B + b*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[1
/(a^2*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[a*A*(m + 1) + m*(b*
B - a*C) + b*C*(2*m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A,
B, C}, x] && LtQ[m, -1] && EqQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)\log(\sin(x))}{(1+\cos(x))^2}\,dx &= -\frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2}+\frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))}-\int \frac{\cos(x)(1+2\cos(x))}{3(1+\cos(x))^2}\,dx \\
&= -\frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2}+\frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))}-\frac{1}{3}\int \frac{\cos(x)(1+2\cos(x))}{(1+\cos(x))^2}\,dx \\
&= -\frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2}+\frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))}-\frac{1}{3}\int \frac{\cos(x)+2\cos^2(x)}{(1+\cos(x))^2}\,dx \\
&= -\frac{\sin(x)}{9(1+\cos(x))^2}-\frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2}+\frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))}+\frac{1}{9}\int \frac{2-6\cos(x)}{1+\cos(x)}\,dx \\
&= -\frac{2x}{3}-\frac{\sin(x)}{9(1+\cos(x))^2}-\frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2}+\frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))}+\frac{8}{9}\int \frac{1}{1+\cos(x)}\,dx \\
&= -\frac{2x}{3}-\frac{\sin(x)}{9(1+\cos(x))^2}+\frac{8\sin(x)}{9(1+\cos(x))}-\frac{\log(\sin(x))\sin(x)}{3(1+\cos(x))^2}+\frac{2\log(\sin(x))\sin(x)}{3(1+\cos(x))}
\end{aligned}$$

Mathematica [A] time = 0.147594, size = 56, normalized size = 0.93

$$-\frac{1}{18}\sec^3\left(\frac{x}{2}\right)\left(9x\cos\left(\frac{x}{2}\right)+3x\cos\left(\frac{3x}{2}\right)-\sin\left(\frac{x}{2}\right)(3\log(\sin(x))+\cos(x)(6\log(\sin(x))+8)+7)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Log[Sin[x]])/(1 + Cos[x])^2,x]
```

```
[Out] -(Sec[x/2]^3*(9*x*Cos[x/2] + 3*x*Cos[(3*x)/2] - (7 + 3*Log[Sin[x]] + Cos[x]
*(8 + 6*Log[Sin[x]])*Sin[x/2]))/18
```

Maple [B] time = 0.086, size = 106, normalized size = 1.8

$$-\frac{1}{9(\sin(x))^3}\left(12\sin(x)(\cos(x))^2\arctan\left(\frac{\cos(x)-1}{\sin(x)}\right)-6(\cos(x))^3\ln(2)-6(\cos(x))^3\ln(1/2\sin(x))-8(\cos(x))^3+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(sin(x))/(cos(x)+1)^2,x)
```

```
[Out] -1/9*(12*sin(x)*cos(x)^2*arctan((cos(x)-1)/sin(x))-6*cos(x)^3*ln(2)-6*cos(x
)^3*ln(1/2*sin(x))-8*cos(x)^3+9*cos(x)^2*ln(2)+9*cos(x)^2*ln(1/2*sin(x))-12
```

```
*arctan((cos(x)-1)/sin(x))*sin(x)+9*cos(x)^2+6*cos(x)-3*ln(2)-3*ln(1/2*sin(
x))-7)/sin(x)^3
```

Maxima [A] time = 1.43172, size = 116, normalized size = 1.93

$$\frac{1}{6}\left(\frac{3\sin(x)}{\cos(x)+1} - \frac{\sin(x)^3}{(\cos(x)+1)^3}\right)\log\left(\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}\right) + \frac{5\sin(x)}{6(\cos(x)+1)} - \frac{\sin(x)^3}{18(\cos(x)+1)^3} - \frac{4}{3}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*sin(x)/(cos(x) + 1) - sin(x)^3/(cos(x) + 1)^3)*log(2*sin(x)/((sin(x)
^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1))) + 5/6*sin(x)/(cos(x) + 1) - 1/18*sin(
x)^3/(cos(x) + 1)^3 - 4/3*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 2.49149, size = 174, normalized size = 2.9

$$-\frac{6x\cos(x)^2 - 3(2\cos(x)+1)\log(\sin(x))\sin(x) + 12x\cos(x) - (8\cos(x)+7)\sin(x) + 6x}{9\left(\cos(x)^2 + 2\cos(x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x, algorithm="fricas")
```

```
[Out] -1/9*(6*x*cos(x)^2 - 3*(2*cos(x) + 1)*log(sin(x))*sin(x) + 12*x*cos(x) - (8
*cos(x) + 7)*sin(x) + 6*x)/(cos(x)^2 + 2*cos(x) + 1)
```

Sympy [A] time = 11.4039, size = 107, normalized size = 1.78

$$-\frac{2x}{3} + \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan^3\left(\frac{x}{2}\right)}{6} - \frac{\log\left(\tan^2\left(\frac{x}{2}\right)+1\right)\tan\left(\frac{x}{2}\right)}{2} - \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)\tan^3\left(\frac{x}{2}\right)}{6} + \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)\tan\left(\frac{x}{2}\right)}{2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(sin(x))/(1+cos(x))**2,x)
```

```
[Out] -2*x/3 + log(tan(x/2)**2 + 1)*tan(x/2)**3/6 - log(tan(x/2)**2 + 1)*tan(x/2)
/2 - log(tan(x/2))*tan(x/2)**3/6 + log(tan(x/2))*tan(x/2)/2 - log(2)*tan(x/
2)**3/6 - tan(x/2)**3/18 + log(2)*tan(x/2)/2 + 5*tan(x/2)/6
```

Giac [A] time = 1.21404, size = 49, normalized size = 0.82

$$-\frac{1}{18}\tan\left(\frac{1}{2}x\right)^3 - \frac{1}{6}\left(\tan\left(\frac{1}{2}x\right)^3 - 3\tan\left(\frac{1}{2}x\right)\right)\log(\sin(x)) - \frac{2}{3}x + \frac{5}{6}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))/(1+cos(x))^2,x, algorithm="giac")
```

```
[Out] -1/18*tan(1/2*x)^3 - 1/6*(tan(1/2*x)^3 - 3*tan(1/2*x))*log(sin(x)) - 2/3*x
+ 5/6*tan(1/2*x)
```

3.645 $\int \frac{\cos^{-1}(x)^2}{x^5} dx$

Optimal. Leaf size=65

$$-\frac{1}{12x^2}-\frac{\cos^{-1}(x)^2}{4x^4}+\frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x}+\frac{\sqrt{1-x^2}\cos^{-1}(x)}{6x^3}+\frac{\log(x)}{3}$$

```
[Out] -1/(12*x^2) + (Sqrt[1 - x^2]*ArcCos[x])/(6*x^3) + (Sqrt[1 - x^2]*ArcCos[x])
/(3*x) - ArcCos[x]^2/(4*x^4) + Log[x]/3
```

Rubi [A] time = 0.109141, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4628, 4702, 4682, 29, 30}

$$-\frac{1}{12x^2}-\frac{\cos^{-1}(x)^2}{4x^4}+\frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x}+\frac{\sqrt{1-x^2}\cos^{-1}(x)}{6x^3}+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[x]^2/x^5,x]
```

```
[Out] -1/(12*x^2) + (Sqrt[1 - x^2]*ArcCos[x])/(6*x^3) + (Sqrt[1 - x^2]*ArcCos[x])
/(3*x) - ArcCos[x]^2/(4*x^4) + Log[x]/3
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4702

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart
[p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 4682

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^F
racPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 - c
^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &
& NeQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{-1}(x)^2}{x^5}\,dx &= -\frac{\cos^{-1}(x)^2}{4x^4}-\frac{1}{2}\int \frac{\cos^{-1}(x)}{x^4\sqrt{1-x^2}}\,dx\\
&= \frac{\sqrt{1-x^2}\cos^{-1}(x)}{6x^3}-\frac{\cos^{-1}(x)^2}{4x^4}+\frac{1}{6}\int \frac{1}{x^3}\,dx-\frac{1}{3}\int \frac{\cos^{-1}(x)}{x^2\sqrt{1-x^2}}\,dx\\
&= -\frac{1}{12x^2}+\frac{\sqrt{1-x^2}\cos^{-1}(x)}{6x^3}+\frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x}-\frac{\cos^{-1}(x)^2}{4x^4}+\frac{1}{3}\int \frac{1}{x}\,dx\\
&= -\frac{1}{12x^2}+\frac{\sqrt{1-x^2}\cos^{-1}(x)}{6x^3}+\frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x}-\frac{\cos^{-1}(x)^2}{4x^4}+\frac{\log(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.0321626, size = 52, normalized size = 0.8

$$-\frac{1}{12x^2}-\frac{\cos^{-1}(x)^2}{4x^4}+\frac{\sqrt{1-x^2}\left(2x^2+1\right)\cos^{-1}(x)}{6x^3}+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[x]^2/x^5,x]
```

```
[Out] -1/(12*x^2) + (Sqrt[1 - x^2]*(1 + 2*x^2)*ArcCos[x])/(6*x^3) - ArcCos[x]^2/(
4*x^4) + Log[x]/3
```

Maple [A] time = 0.033, size = 52, normalized size = 0.8

$$-\frac{1}{12\,x^2}-\frac{(\arccos(x))^2}{4\,x^4}+\frac{\ln(x)}{3}+\frac{\arccos(x)}{6\,x^3}\sqrt{-x^2+1}+\frac{\arccos(x)}{3\,x}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x)^2/x^5,x)
```

```
[Out] -1/12/x^2-1/4*arccos(x)^2/x^4+1/3*ln(x)+1/6*arccos(x)*(-x^2+1)^(1/2)/x^3+1/
3*arccos(x)*(-x^2+1)^(1/2)/x
```

Maxima [A] time = 1.42872, size = 69, normalized size = 1.06

$$\frac{1}{6}\left(\frac{2\sqrt{-x^2+1}}{x}+\frac{\sqrt{-x^2+1}}{x^3}\right)\arccos(x)-\frac{1}{12\,x^2}-\frac{\arccos(x)^2}{4\,x^4}+\frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*(2*sqrt(-x^2 + 1)/x + sqrt(-x^2 + 1)/x^3)*arccos(x) - 1/12/x^2 - 1/4*ar
ccos(x)^2/x^4 + 1/3*log(x)
```

Fricas [A] time = 2.81129, size = 119, normalized size = 1.83

$$\frac{4 x^4 \log(x) + 2 \left(2 x^3 + x\right) \sqrt{-x^2 + 1} \arccos(x) - x^2 - 3 \arccos(x)^2}{12 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^4*log(x) + 2*(2*x^3 + x)*sqrt(-x^2 + 1)*arccos(x) - x^2 - 3*arcco
s(x)^2)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{acos}^2(x)}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x)**2/x**5,x)
```

```
[Out] Integral(acos(x)**2/x**5, x)
```

Giac [B] time = 1.11555, size = 140, normalized size = 2.15

$$-\frac{1}{48} \left(\frac{x^3 \left(\frac{9 \left(\sqrt{-x^2+1}-1\right)^2}{x^2} + 1 \right)}{\left(\sqrt{-x^2+1} - 1\right)^3} - \frac{9 \left(\sqrt{-x^2+1} - 1\right)}{x} - \frac{\left(\sqrt{-x^2+1} - 1\right)^3}{x^3} \right) \arccos(x) - \frac{2 x^2 + 1}{12 x^2} - \frac{\arccos(x)^2}{4 x^4} + \frac{1}{6} \log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)^2/x^5,x, algorithm="giac")
```

```
[Out] -1/48*(x^3*(9*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1)^3 - 9*(s
qrt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^3/x^3)*arccos(x) - 1/12*(2*x^2
+ 1)/x^2 - 1/4*arccos(x)^2/x^4 + 1/6*log(x^2)
```

3.646 $\int x^2 \sin^{-1}(x)^2 \, dx$

Optimal. Leaf size=61

$$-\frac{2x^3}{27}+\frac{1}{3}x^3\sin^{-1}(x)^2+\frac{2}{9}\sqrt{1-x^2}x^2\sin^{-1}(x)+\frac{4}{9}\sqrt{1-x^2}\sin^{-1}(x)-\frac{4x}{9}$$

```
[Out] (-4*x)/9 - (2*x^3)/27 + (4*Sqrt[1 - x^2]*ArcSin[x])/9 + (2*x^2*Sqrt[1 - x^2
]*ArcSin[x])/9 + (x^3*ArcSin[x]^2)/3
```

Rubi [A] time = 0.0938961, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4627, 4707, 4677, 8, 30}

$$-\frac{2x^3}{27}+\frac{1}{3}x^3\sin^{-1}(x)^2+\frac{2}{9}\sqrt{1-x^2}x^2\sin^{-1}(x)+\frac{4}{9}\sqrt{1-x^2}\sin^{-1}(x)-\frac{4x}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSin[x]^2,x]
```

```
[Out] (-4*x)/9 - (2*x^3)/27 + (4*Sqrt[1 - x^2]*ArcSin[x])/9 + (2*x^2*Sqrt[1 - x^2
]*ArcSin[x])/9 + (x^3*ArcSin[x]^2)/3
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4707

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcSin[c*x])^n)/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4677

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p +
 1)), x] + Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sin^{-1}(x)^2 \, dx &= \frac{1}{3}x^3 \sin^{-1}(x)^2 - \frac{2}{3}\int \frac{x^3 \sin^{-1}(x)}{\sqrt{1-x^2}} \, dx \\
&= \frac{2}{9}x^2\sqrt{1-x^2}\sin^{-1}(x) + \frac{1}{3}x^3 \sin^{-1}(x)^2 - \frac{2\int x^2 \, dx}{9} - \frac{4}{9}\int \frac{x \sin^{-1}(x)}{\sqrt{1-x^2}} \, dx \\
&= -\frac{2x^3}{27} + \frac{4}{9}\sqrt{1-x^2}\sin^{-1}(x) + \frac{2}{9}x^2\sqrt{1-x^2}\sin^{-1}(x) + \frac{1}{3}x^3 \sin^{-1}(x)^2 - \frac{4\int 1 \, dx}{9} \\
&= -\frac{4x}{9} - \frac{2x^3}{27} + \frac{4}{9}\sqrt{1-x^2}\sin^{-1}(x) + \frac{2}{9}x^2\sqrt{1-x^2}\sin^{-1}(x) + \frac{1}{3}x^3 \sin^{-1}(x)^2
\end{aligned}$$

Mathematica [A] time = 0.0222281, size = 42, normalized size = 0.69

$$\frac{1}{27}\left(-2\left(x^2+6\right)x + 9x^3\sin^{-1}(x)^2 + 6\sqrt{1-x^2}\left(x^2+2\right)\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSin[x]^2,x]
```

```
[Out] (-2*x*(6 + x^2) + 6*Sqrt[1 - x^2]*(2 + x^2)*ArcSin[x] + 9*x^3*ArcSin[x]^2)/
27
```

Maple [A] time = 0.023, size = 37, normalized size = 0.6

$$\frac{x^3 (\arcsin(x))^2}{3} + \frac{2\arcsin(x)\left(x^2+2\right)}{9}\sqrt{-x^2+1} - \frac{2x^3}{27} - \frac{4x}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsin(x)^2,x)
```

```
[Out] 1/3*x^3*arcsin(x)^2+2/9*arcsin(x)*(x^2+2)*(-x^2+1)^(1/2)-2/27*x^3-4/9*x
```

Maxima [A] time = 1.44083, size = 63, normalized size = 1.03

$$\frac{1}{3}x^3 \arcsin(x)^2 - \frac{2}{27}x^3 + \frac{2}{9}\left(\sqrt{-x^2+1}x^2 + 2\sqrt{-x^2+1}\right)\arcsin(x) - \frac{4}{9}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsin(x)^2 - 2/27*x^3 + 2/9*(sqrt(-x^2 + 1)*x^2 + 2*sqrt(-x^2 + 1)
)*arcsin(x) - 4/9*x
```

Fricas [A] time = 2.44157, size = 109, normalized size = 1.79

$$\frac{1}{3}\,x^3\arcsin\left(x\right)^2 - \frac{2}{27}\,x^3 + \frac{2}{9}\,\left(x^2+2\right)\sqrt{-x^2+1}\arcsin\left(x\right) - \frac{4}{9}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arcsin(x)^2 - 2/27*x^3 + 2/9*(x^2 + 2)*sqrt(-x^2 + 1)*arcsin(x) - 4
/9*x
```

Sympy [A] time = 0.597459, size = 54, normalized size = 0.89

$$\frac{x^3\operatorname{asin}^2\left(x\right)}{3} - \frac{2x^3}{27} + \frac{2x^2\sqrt{1-x^2}\operatorname{asin}\left(x\right)}{9} - \frac{4x}{9} + \frac{4\sqrt{1-x^2}\operatorname{asin}\left(x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asin(x)**2,x)
```

```
[Out] x**3*asin(x)**2/3 - 2*x**3/27 + 2*x**2*sqrt(1 - x**2)*asin(x)/9 - 4*x/9 + 4
*sqrt(1 - x**2)*asin(x)/9
```

Giac [A] time = 1.07623, size = 77, normalized size = 1.26

$$\frac{1}{3}\,\left(x^2-1\right)x\arcsin\left(x\right)^2 + \frac{1}{3}\,x\arcsin\left(x\right)^2 - \frac{2}{9}\,\left(-x^2+1\right)^{\frac{3}{2}}\arcsin\left(x\right) - \frac{2}{27}\,\left(x^2-1\right)x + \frac{2}{3}\,\sqrt{-x^2+1}\arcsin\left(x\right) - \frac{14}{27}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)^2,x, algorithm="giac")
```

```
[Out] 1/3*(x^2 - 1)*x*arcsin(x)^2 + 1/3*x*arcsin(x)^2 - 2/9*(-x^2 + 1)^(3/2)*arcs
in(x) - 2/27*(x^2 - 1)*x + 2/3*sqrt(-x^2 + 1)*arcsin(x) - 14/27*x
```

3.647 $\int x^3 \tan^{-1}(x)^2 \, dx$

Optimal. Leaf size=53

$$\frac{x^2}{12} - \frac{1}{3}\log\left(x^2+1\right) + \frac{1}{4}x^4\tan^{-1}(x)^2 - \frac{1}{6}x^3\tan^{-1}(x) + \frac{1}{2}x\tan^{-1}(x) - \frac{1}{4}\tan^{-1}(x)^2$$

```
[Out] x^2/12 + (x*ArcTan[x])/2 - (x^3*ArcTan[x])/6 - ArcTan[x]^2/4 + (x^4*ArcTan[
x]^2)/4 - Log[1 + x^2]/3
```

Rubi [A] time = 0.116431, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {4852, 4916, 266, 43, 4846, 260, 4884}

$$\frac{x^2}{12} - \frac{1}{3}\log\left(x^2+1\right) + \frac{1}{4}x^4\tan^{-1}(x)^2 - \frac{1}{6}x^3\tan^{-1}(x) + \frac{1}{2}x\tan^{-1}(x) - \frac{1}{4}\tan^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcTan[x]^2,x]
```

```
[Out] x^2/12 + (x*ArcTan[x])/2 - (x^3*ArcTan[x])/6 - ArcTan[x]^2/4 + (x^4*ArcTan[
x]^2)/4 - Log[1 + x^2]/3
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
```

```
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\tan^{-1}(x)^2\,dx &= \frac{1}{4}x^4\tan^{-1}(x)^2-\frac{1}{2}\int\frac{x^4\tan^{-1}(x)}{1+x^2}\,dx\\ &= \frac{1}{4}x^4\tan^{-1}(x)^2-\frac{1}{2}\int x^2\tan^{-1}(x)\,dx+\frac{1}{2}\int\frac{x^2\tan^{-1}(x)}{1+x^2}\,dx\\ &= -\frac{1}{6}x^3\tan^{-1}(x)+\frac{1}{4}x^4\tan^{-1}(x)^2+\frac{1}{6}\int\frac{x^3}{1+x^2}\,dx+\frac{1}{2}\int\tan^{-1}(x)\,dx-\frac{1}{2}\int\frac{\tan^{-1}(x)}{1+x^2}\,dx\\ &= \frac{1}{2}x\tan^{-1}(x)-\frac{1}{6}x^3\tan^{-1}(x)-\frac{1}{4}\tan^{-1}(x)^2+\frac{1}{4}x^4\tan^{-1}(x)^2+\frac{1}{12}\text{Subst}\left(\int\frac{x}{1+x}\,dx,x,x^2\right)-\frac{1}{2}\int\\ &= \frac{1}{2}x\tan^{-1}(x)-\frac{1}{6}x^3\tan^{-1}(x)-\frac{1}{4}\tan^{-1}(x)^2+\frac{1}{4}x^4\tan^{-1}(x)^2-\frac{1}{4}\log\left(1+x^2\right)+\frac{1}{12}\text{Subst}\left(\int\left(1+\right.\right.\\ &= \frac{x^2}{12}+\frac{1}{2}x\tan^{-1}(x)-\frac{1}{6}x^3\tan^{-1}(x)-\frac{1}{4}\tan^{-1}(x)^2+\frac{1}{4}x^4\tan^{-1}(x)^2-\frac{1}{3}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0149791, size = 37, normalized size = 0.7

$$\frac{1}{12}\left(x^2-4\log\left(x^2+1\right)-2\left(x^2-3\right)x\tan^{-1}(x)+3\left(x^4-1\right)\tan^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcTan[x]^2,x]
```

```
[Out] (x^2 - 2*x*(-3 + x^2)*ArcTan[x] + 3*(-1 + x^4)*ArcTan[x]^2 - 4*Log[1 + x^2]
)/12
```

Maple [A] time = 0.01, size = 42, normalized size = 0.8

$$\frac{x^2}{12}+\frac{x\arctan(x)}{2}-\frac{x^3\arctan(x)}{6}-\frac{(\arctan(x))^2}{4}+\frac{x^4(\arctan(x))^2}{4}-\frac{\ln\left(x^2+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(x)^2,x)
```

```
[Out] 1/12*x^2+1/2*x*arctan(x)-1/6*x^3*arctan(x)-1/4*arctan(x)^2+1/4*x^4*arctan(x
)^2-1/3*ln(x^2+1)
```

Maxima [A] time = 1.44738, size = 59, normalized size = 1.11

$$\frac{1}{4}x^4\arctan(x)^2+\frac{1}{12}x^2-\frac{1}{6}\left(x^3-3x+3\arctan(x)\right)\arctan(x)+\frac{1}{4}\arctan(x)^2-\frac{1}{3}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan(x)^2 + 1/12*x^2 - 1/6*(x^3 - 3*x + 3*arctan(x))*arctan(x) +
1/4*arctan(x)^2 - 1/3*log(x^2 + 1)
```

Fricas [A] time = 2.54543, size = 115, normalized size = 2.17

$$\frac{1}{4}\left(x^4-1\right)\arctan(x)^2+\frac{1}{12}x^2-\frac{1}{6}\left(x^3-3x\right)\arctan(x)-\frac{1}{3}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(x^4 - 1)*arctan(x)^2 + 1/12*x^2 - 1/6*(x^3 - 3*x)*arctan(x) - 1/3*log(
x^2 + 1)
```

Sympy [A] time = 0.61409, size = 44, normalized size = 0.83

$$\frac{x^4\operatorname{atan}^2(x)}{4}-\frac{x^3\operatorname{atan}(x)}{6}+\frac{x^2}{12}+\frac{x\operatorname{atan}(x)}{2}-\frac{\log\left(x^2+1\right)}{3}-\frac{\operatorname{atan}^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(x)**2,x)
```

```
[Out] x**4*atan(x)**2/4 - x**3*atan(x)/6 + x**2/12 + x*atan(x)/2 - log(x**2 + 1)/
3 - atan(x)**2/4
```

Giac [A] time = 1.08217, size = 55, normalized size = 1.04

$$\frac{1}{4}x^4\arctan(x)^2-\frac{1}{6}x^3\arctan(x)+\frac{1}{12}x^2+\frac{1}{2}x\arctan(x)-\frac{1}{4}\arctan(x)^2-\frac{1}{3}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^4*arctan(x)^2 - 1/6*x^3*arctan(x) + 1/12*x^2 + 1/2*x*arctan(x) - 1/4*
arctan(x)^2 - 1/3*log(x^2 + 1)
```

3.648 $\int \frac{\tan^{-1}(x)^2}{x^5} dx$

Optimal. Leaf size=61

$$-\frac{1}{12x^2}+\frac{1}{3}\log\left(x^2+1\right)-\frac{\tan^{-1}(x)^2}{4x^4}-\frac{\tan^{-1}(x)}{6x^3}-\frac{2\log(x)}{3}+\frac{1}{4}\tan^{-1}(x)^2+\frac{\tan^{-1}(x)}{2x}$$

```
[Out] -1/(12*x^2) - ArcTan[x]/(6*x^3) + ArcTan[x]/(2*x) + ArcTan[x]^2/4 - ArcTan[
x]^2/(4*x^4) - (2*Log[x])/3 + Log[1 + x^2]/3
```

Rubi [A] time = 0.126128, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {4852, 4918, 266, 44, 36, 29, 31, 4884}

$$-\frac{1}{12x^2}+\frac{1}{3}\log\left(x^2+1\right)-\frac{\tan^{-1}(x)^2}{4x^4}-\frac{\tan^{-1}(x)}{6x^3}-\frac{2\log(x)}{3}+\frac{1}{4}\tan^{-1}(x)^2+\frac{\tan^{-1}(x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x]^2/x^5,x]
```

```
[Out] -1/(12*x^2) - ArcTan[x]/(6*x^3) + ArcTan[x]/(2*x) + ArcTan[x]^2/4 - ArcTan[
x]^2/(4*x^4) - (2*Log[x])/3 + Log[1 + x^2]/3
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
```

```
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(x)^2}{x^5}\,dx &= -\frac{\tan^{-1}(x)^2}{4x^4}+\frac{1}{2}\int \frac{\tan^{-1}(x)}{x^4\left(1+x^2\right)}\,dx\\
&= -\frac{\tan^{-1}(x)^2}{4x^4}+\frac{1}{2}\int \frac{\tan^{-1}(x)}{x^4}\,dx-\frac{1}{2}\int \frac{\tan^{-1}(x)}{x^2\left(1+x^2\right)}\,dx\\
&= -\frac{\tan^{-1}(x)}{6x^3}-\frac{\tan^{-1}(x)^2}{4x^4}+\frac{1}{6}\int \frac{1}{x^3\left(1+x^2\right)}\,dx-\frac{1}{2}\int \frac{\tan^{-1}(x)}{x^2}\,dx+\frac{1}{2}\int \frac{\tan^{-1}(x)}{1+x^2}\,dx\\
&= -\frac{\tan^{-1}(x)}{6x^3}+\frac{\tan^{-1}(x)}{2x}+\frac{1}{4}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)^2}{4x^4}+\frac{1}{12}\operatorname{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx,x,x^2\right)-\frac{1}{2}\int \frac{}{x(1}\\
&= -\frac{\tan^{-1}(x)}{6x^3}+\frac{\tan^{-1}(x)}{2x}+\frac{1}{4}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)^2}{4x^4}+\frac{1}{12}\operatorname{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx,x,x^2\right)-\\
&= -\frac{1}{12x^2}-\frac{\tan^{-1}(x)}{6x^3}+\frac{\tan^{-1}(x)}{2x}+\frac{1}{4}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)^2}{4x^4}-\frac{\log(x)}{6}+\frac{1}{12}\log\left(1+x^2\right)-\frac{1}{4}\operatorname{Subst}\left(\right.\\
&= -\frac{1}{12x^2}-\frac{\tan^{-1}(x)}{6x^3}+\frac{\tan^{-1}(x)}{2x}+\frac{1}{4}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)^2}{4x^4}-\frac{2\log(x)}{3}+\frac{1}{3}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0205726, size = 56, normalized size = 0.92

$$-\frac{1}{12x^2}+\frac{1}{3}\log\left(x^2+1\right)+\frac{\left(x^4-1\right)\tan^{-1}(x)^2}{4x^4}+\frac{\left(3x^2-1\right)\tan^{-1}(x)}{6x^3}-\frac{2\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]^2/x^5,x]
```

```
[Out] -1/(12*x^2) + ((-1 + 3*x^2)*ArcTan[x])/(6*x^3) + ((-1 + x^4)*ArcTan[x]^2)/(
4*x^4) - (2*Log[x])/3 + Log[1 + x^2]/3
```

Maple [A] time = 0.017, size = 48, normalized size = 0.8

$$-\frac{1}{12\,x^2}-\frac{\arctan(x)}{6\,x^3}+\frac{\arctan(x)}{2\,x}+\frac{(\arctan(x))^2}{4}-\frac{(\arctan(x))^2}{4\,x^4}-\frac{2\,\ln(x)}{3}+\frac{\ln\left(x^2+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)^2/x^5,x)
```

```
[Out] -1/12/x^2-1/6*arctan(x)/x^3+1/2*arctan(x)/x+1/4*arctan(x)^2-1/4*arctan(x)^2
/x^4-2/3*ln(x)+1/3*ln(x^2+1)
```

Maxima [A] time = 1.43238, size = 86, normalized size = 1.41

$$\frac{1}{6}\left(\frac{3x^2-1}{x^3}+3\arctan(x)\right)\arctan(x)-\frac{3x^2\arctan(x)^2-4x^2\log\left(x^2+1\right)+8x^2\log(x)+1}{12x^2}-\frac{\arctan(x)^2}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*((3*x^2 - 1)/x^3 + 3*arctan(x))*arctan(x) - 1/12*(3*x^2*arctan(x)^2 - 4
*x^2*log(x^2 + 1) + 8*x^2*log(x) + 1)/x^2 - 1/4*arctan(x)^2/x^4
```

Fricas [A] time = 2.51856, size = 140, normalized size = 2.3

$$\frac{4x^4\log\left(x^2+1\right)-8x^4\log(x)+3\left(x^4-1\right)\arctan(x)^2-x^2+2\left(3x^3-x\right)\arctan(x)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*x^4*log(x^2 + 1) - 8*x^4*log(x) + 3*(x^4 - 1)*arctan(x)^2 - x^2 + 2
*(3*x^3 - x)*arctan(x))/x^4
```

Sympy [A] time = 0.901114, size = 53, normalized size = 0.87

$$-\frac{2\log(x)}{3}+\frac{\log\left(x^2+1\right)}{3}+\frac{\operatorname{atan}^2(x)}{4}+\frac{\operatorname{atan}(x)}{2x}-\frac{1}{12x^2}-\frac{\operatorname{atan}(x)}{6x^3}-\frac{\operatorname{atan}^2(x)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)**2/x**5,x)
```

```
[Out] -2*log(x)/3 + log(x**2 + 1)/3 + atan(x)**2/4 + atan(x)/(2*x) - 1/(12*x**2)
- atan(x)/(6*x**3) - atan(x)**2/(4*x**4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\arctan(x)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)^2/x^5, x)
```

3.649 $\int x^3 \csc^{-1}(x)^2\,dx$

Optimal. Leaf size=63

$$\frac{x^2}{12}+\frac{1}{4}x^4\csc^{-1}(x)^2+\frac{1}{6}\sqrt{1-\frac{1}{x^2}}\,x^3\csc^{-1}(x)+\frac{1}{3}\sqrt{1-\frac{1}{x^2}}\,x\csc^{-1}(x)+\frac{\log(x)}{3}$$

```
[Out] x^2/12 + (Sqrt[1 - x^(-2)]*x*ArcCsc[x])/3 + (Sqrt[1 - x^(-2)]*x^3*ArcCsc[x]
)/6 + (x^4*ArcCsc[x]^2)/4 + Log[x]/3
```

Rubi [A] time = 0.0671453, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5223, 3758, 4185, 4184, 3475}

$$\frac{x^2}{12}+\frac{1}{4}x^4\csc^{-1}(x)^2+\frac{1}{6}\sqrt{1-\frac{1}{x^2}}\,x^3\csc^{-1}(x)+\frac{1}{3}\sqrt{1-\frac{1}{x^2}}\,x\csc^{-1}(x)+\frac{\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcCsc[x]^2,x]
```

```
[Out] x^2/12 + (Sqrt[1 - x^(-2)]*x*ArcCsc[x])/3 + (Sqrt[1 - x^(-2)]*x^3*ArcCsc[x]
)/6 + (x^4*ArcCsc[x]^2)/4 + Log[x]/3
```

Rule 5223

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> -Dist[
(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csc[x]^(m + 1)*Cot[x], x], x, ArcCs
c[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n,
 0] || LtQ[m, -1])
```

Rule 3758

```
Int[Cot[(a_.) + (b_.)*(x_)^(n_.)]^(q_.)*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)
*(x_)^(m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^p)/(b*n*p), x
] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Csc[a + b*x^n]^p, x], x] /; Fre
eQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \csc^{-1}(x)^2\,dx &= -\operatorname{Subst}\left(\int x^2 \cot(x)\csc^4(x)\,dx, x, \csc^{-1}(x)\right)\\
&= \frac{1}{4}x^4\csc^{-1}(x)^2 - \frac{1}{2}\operatorname{Subst}\left(\int x\csc^4(x)\,dx, x, \csc^{-1}(x)\right)\\
&= \frac{x^2}{12} + \frac{1}{6}\sqrt{1-\frac{1}{x^2}}x^3\csc^{-1}(x) + \frac{1}{4}x^4\csc^{-1}(x)^2 - \frac{1}{3}\operatorname{Subst}\left(\int x\csc^2(x)\,dx, x, \csc^{-1}(x)\right)\\
&= \frac{x^2}{12} + \frac{1}{3}\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x) + \frac{1}{6}\sqrt{1-\frac{1}{x^2}}x^3\csc^{-1}(x) + \frac{1}{4}x^4\csc^{-1}(x)^2 - \frac{1}{3}\operatorname{Subst}\left(\int \cot(x)\,dx, x, c\right.\\
&= \frac{x^2}{12} + \frac{1}{3}\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x) + \frac{1}{6}\sqrt{1-\frac{1}{x^2}}x^3\csc^{-1}(x) + \frac{1}{4}x^4\csc^{-1}(x)^2 + \frac{\log(x)}{3}
\end{aligned}
$$

Mathematica [A] time = 0.0475762, size = 42, normalized size = 0.67

$$
\frac{1}{12}\left(x^2 + 3x^4\csc^{-1}(x)^2 + 2\sqrt{1-\frac{1}{x^2}}\left(x^2+2\right)x\csc^{-1}(x) + 4\log(x)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcCsc[x]^2,x]
```

```
[Out] (x^2 + 2*Sqrt[1 - x^(-2)]*x*(2 + x^2)*ArcCsc[x] + 3*x^4*ArcCsc[x]^2 + 4*Log
[x])/12
```

Maple [A] time = 0.034, size = 56, normalized size = 0.9

$$
\frac{x^4\left(\operatorname{arccsc}(x)\right)^2}{4} + \frac{\operatorname{arccsc}(x)\,x^3}{6}\sqrt{\frac{x^2-1}{x^2}} + \frac{x^2}{12} + \frac{\operatorname{arccsc}(x)\,x}{3}\sqrt{\frac{x^2-1}{x^2}} - \frac{\ln\left(x^{-1}\right)}{3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccsc(x)^2,x)
```

```
[Out] 1/4*x^4*arccsc(x)^2+1/6*arccsc(x)*x^3*((x^2-1)/x^2)^(1/2)+1/12*x^2+1/3*((x^
2-1)/x^2)^(1/2)*arccsc(x)*x-1/3*ln(1/x)
```

Maxima [A] time = 1.66807, size = 128, normalized size = 2.03

$$
\frac{1}{4}x^4\operatorname{arccsc}(x)^2 + \frac{2x^4\arctan\left(1, \sqrt{x+1}\sqrt{x-1}\right) + 2x^2\arctan\left(1, \sqrt{x+1}\sqrt{x-1}\right) + \left(x^2 + 2\log\left(x^2\right)\right)\sqrt{x+1}\sqrt{x-1} -}{12\sqrt{x+1}\sqrt{x-1}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccsc(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arccsc(x)^2 + 1/12*(2*x^4*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + 2*x
^2*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + (x^2 + 2*log(x^2))*sqrt(x + 1)*sqr
t(x - 1) - 4*arctan2(1, sqrt(x + 1)*sqrt(x - 1)))/(sqrt(x + 1)*sqrt(x - 1))
```

Fricas [A] time = 2.62521, size = 115, normalized size = 1.83

$$\frac{1}{4}x^4\operatorname{arccsc}(x)^2+\frac{1}{6}\left(x^2+2\right)\sqrt{x^2-1}\operatorname{arccsc}(x)+\frac{1}{12}x^2+\frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccsc(x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*arccsc(x)^2 + 1/6*(x^2 + 2)*sqrt(x^2 - 1)*arccsc(x) + 1/12*x^2 + 1/
3*log(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\operatorname{acsc}^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acsc(x)**2,x)
```

```
[Out] Integral(x**3*acsc(x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccsc(x)^2,x, algorithm="giac")
```

```
[Out] sage2
```

3.650 $\int \frac{\sec^{-1}(x)^4}{x^5} \, dx$

Optimal. Leaf size=148

$$-\frac{45}{128x^2}-\frac{3}{128x^4}-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{8x}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}+\frac{9\sec^{-1}(x)^2}{16x^2}+\frac{3\sec^{-1}(x)^2}{16x^4}-\frac{45\sqrt{1-\frac{1}{x^2}}\,\mathrm{se}}{64x}$$

```
[Out] -3/(128*x^4) - 45/(128*x^2) - (3*Sqrt[1 - x^(-2)]*ArcSec[x])/(32*x^3) - (45
*Sqrt[1 - x^(-2)]*ArcSec[x])/(64*x) - (45*ArcSec[x]^2)/128 + (3*ArcSec[x]^2
)/(16*x^4) + (9*ArcSec[x]^2)/(16*x^2) + (Sqrt[1 - x^(-2)]*ArcSec[x]^3)/(4*x
^3) + (3*Sqrt[1 - x^(-2)]*ArcSec[x]^3)/(8*x) + (3*ArcSec[x]^4)/32 - ArcSec[
x]^4/(4*x^4)
```

Rubi [A] time = 0.147337, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5222, 3444, 3311, 30, 3310}

$$-\frac{45}{128x^2}-\frac{3}{128x^4}-\frac{\sec^{-1}(x)^4}{4x^4}+\frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{8x}+\frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3}+\frac{9\sec^{-1}(x)^2}{16x^2}+\frac{3\sec^{-1}(x)^2}{16x^4}-\frac{45\sqrt{1-\frac{1}{x^2}}\,\mathrm{se}}{64x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[x]^4/x^5,x]
```

```
[Out] -3/(128*x^4) - 45/(128*x^2) - (3*Sqrt[1 - x^(-2)]*ArcSec[x])/(32*x^3) - (45
*Sqrt[1 - x^(-2)]*ArcSec[x])/(64*x) - (45*ArcSec[x]^2)/128 + (3*ArcSec[x]^2
)/(16*x^4) + (9*ArcSec[x]^2)/(16*x^2) + (Sqrt[1 - x^(-2)]*ArcSec[x]^3)/(4*x
^3) + (3*Sqrt[1 - x^(-2)]*ArcSec[x]^3)/(8*x) + (3*ArcSec[x]^4)/32 - ArcSec[
x]^4/(4*x^4)
```

Rule 5222

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[1
/c^(m + 1), Subst[Int[(a + b*x)^n*Sec[x]^(m + 1)*Tan[x], x], x, ArcSec[c*x]
], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] ||
 LtQ[m, -1])
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int \frac{\sec^{-1}(x)^4}{x^5}\,dx = \text{Subst}\left(\int x^4\cos^3(x)\sin(x)\,dx, x, \sec^{-1}(x)\right)$$
$$= -\frac{\sec^{-1}(x)^4}{4x^4} + \text{Subst}\left(\int x^3\cos^4(x)\,dx, x, \sec^{-1}(x)\right)$$
$$= \frac{3\sec^{-1}(x)^2}{16x^4} + \frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3} - \frac{\sec^{-1}(x)^4}{4x^4} - \frac{3}{8}\text{Subst}\left(\int x\cos^4(x)\,dx, x, \sec^{-1}(x)\right) + \frac{3}{4}\text{Subst}\left(\int x\right.$$
$$= -\frac{3}{128x^4} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{32x^3} + \frac{3\sec^{-1}(x)^2}{16x^4} + \frac{9\sec^{-1}(x)^2}{16x^2} + \frac{\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{4x^3} + \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^3}{8x}$$
$$= -\frac{3}{128x^4} - \frac{45}{128x^2} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{32x^3} - \frac{45\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{64x} + \frac{3\sec^{-1}(x)^2}{16x^4} + \frac{9\sec^{-1}(x)^2}{16x^2} + \frac{\sqrt{1-\frac{1}{x^2}}\,\text{s}}{4x}$$
$$= -\frac{3}{128x^4} - \frac{45}{128x^2} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{32x^3} - \frac{45\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)}{64x} - \frac{45}{128}\sec^{-1}(x)^2 + \frac{3\sec^{-1}(x)^2}{16x^4} + \frac{9\sec^{-1}}{16x^2}$$

Mathematica [A] time = 0.0738931, size = 92, normalized size = 0.62

$$\frac{-45x^2+4\left(3x^4-8\right)\sec^{-1}(x)^4+16\sqrt{1-\frac{1}{x^2}}x\left(3x^2+2\right)\sec^{-1}(x)^3+\left(-45x^4+72x^2+24\right)\sec^{-1}(x)^2-6\sqrt{1-\frac{1}{x^2}}x\left(15x^2+2\right.}{128x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[x]^4/x^5,x]
```

```
[Out] (-3 - 45*x^2 - 6*Sqrt[1 - x^(-2)]*x*(2 + 15*x^2)*ArcSec[x] + (24 + 72*x^2 -
 45*x^4)*ArcSec[x]^2 + 16*Sqrt[1 - x^(-2)]*x*(2 + 3*x^2)*ArcSec[x]^3 + 4*(-
8 + 3*x^4)*ArcSec[x]^4)/(128*x^4)
```

Maple [A] time = 0.062, size = 165, normalized size = 1.1

$$-\frac{(\text{arcsec}(x))^4}{4x^4} + \frac{(\text{arcsec}(x))^3}{8x^3}\left(3\,\text{arcsec}(x)\,x^3 + 3x^2\sqrt{\frac{x^2-1}{x^2}} + 2\sqrt{\frac{x^2-1}{x^2}}\right) + \frac{3(\text{arcsec}(x))^2}{16x^4} - \frac{3\,\text{arcsec}(x)}{64x^3}\left(3\,\text{arcsec}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x)^4/x^5,x)
```

```
[Out] -1/4*arcsec(x)^4/x^4+1/8*arcsec(x)^3*(3*arcsec(x)*x^3+3*x^2*((x^2-1)/x^2)^(
1/2)+2*((x^2-1)/x^2)^(1/2))/x^3+3/16*arcsec(x)^2/x^4-3/64*arcsec(x)*(3*arcs
```

```
ec(x)*x^3+3*x^2*((x^2-1)/x^2)^(1/2)+2*((x^2-1)/x^2)^(1/2))/x^3+45/128*arcse
c(x)^2-3/128/x^4-45/128/x^2+9/16*arcsec(x)^2/x^2-9/16*arcsec(x)*(arcsec(x)*
x+((x^2-1)/x^2)^(1/2))/x+9/32-9/32*arcsec(x)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$8x^4\int \frac{12\left(x^2-1\right)\log\left(x^2\right)^2\log(x)^2-16\left(x^2-1\right)\log\left(x^2\right)\log(x)^3+8\left(x^2-1\right)\log(x)^4+\left(x^2-4\left(x^2-1\right)\log(x)-1\right)\log\left(x^2\right)^3-12\left(4\left(x^2-1\right)\log(x)^2+\left(x^2-4\left(x^2-1\right)\log(\right.\right.}{x^7-x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)^4/x^5,x, algorithm="maxima")
```

```
[Out] 1/64*(64*x^4*integrate(1/8*(12*(x^2 - 1)*log(x^2)^2*log(x)^2 - 16*(x^2 - 1)
*log(x^2)*log(x)^3 + 8*(x^2 - 1)*log(x)^4 + (x^2 - 4*(x^2 - 1)*log(x) - 1)*
log(x^2)^3 - 12*(4*(x^2 - 1)*log(x)^2 + (x^2 - 4*(x^2 - 1)*log(x) - 1)*log(
x^2))*arctan(sqrt(x + 1)*sqrt(x - 1))^2 + 2*(4*arctan(sqrt(x + 1)*sqrt(x -
1))^3 - 3*arctan(sqrt(x + 1)*sqrt(x - 1))*log(x^2)^2)*sqrt(x + 1)*sqrt(x -
1))/(x^7 - x^5), x) - 16*arctan(sqrt(x + 1)*sqrt(x - 1))^4 + 24*arctan(sqrt
(x + 1)*sqrt(x - 1))^2*log(x^2)^2 - log(x^2)^4)/x^4
```

Fricas [A] time = 2.7025, size = 220, normalized size = 1.49

$$\frac{4\left(3x^4-8\right)\operatorname{arcsec}(x)^4-3\left(15x^4-24x^2-8\right)\operatorname{arcsec}(x)^2-45x^2+2\left(8\left(3x^2+2\right)\operatorname{arcsec}(x)^3-3\left(15x^2+2\right)\operatorname{arcsec}\right.}{128x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)^4/x^5,x, algorithm="fricas")
```

```
[Out] 1/128*(4*(3*x^4 - 8)*arcsec(x)^4 - 3*(15*x^4 - 24*x^2 - 8)*arcsec(x)^2 - 45
*x^2 + 2*(8*(3*x^2 + 2)*arcsec(x)^3 - 3*(15*x^2 + 2)*arcsec(x))*sqrt(x^2 -
1) - 3)/x^4
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asec}^4(x)}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x)**4/x**5,x)
```

```
[Out] Integral(asec(x)**4/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arcsec}(x)^4}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)^4/x^5,x, algorithm="giac")
```

```
[Out] integrate(arcsec(x)^4/x^5, x)
```

3.651 $\int \sqrt{1-x^2} \sin^{-1}(x) \, dx$

Optimal. Leaf size=34

$$-\frac{x^2}{4}+\frac{1}{2}\sqrt{1-x^2}x\sin^{-1}(x)+\frac{1}{4}\sin^{-1}(x)^2$$

```
[Out] -x^2/4 + (x*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4
```

Rubi [A] time = 0.0292565, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4647, 4641, 30}

$$-\frac{x^2}{4}+\frac{1}{2}\sqrt{1-x^2}x\sin^{-1}(x)+\frac{1}{4}\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]*ArcSin[x],x]
```

```
[Out] -x^2/4 + (x*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4
```

Rule 4647

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSin[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x^2} \sin^{-1}(x) \, dx &= \frac{1}{2}x\sqrt{1-x^2}\sin^{-1}(x)-\frac{\int x \, dx}{2}+\frac{1}{2}\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} \, dx\\ &= -\frac{x^2}{4}+\frac{1}{2}x\sqrt{1-x^2}\sin^{-1}(x)+\frac{1}{4}\sin^{-1}(x)^2\end{aligned}$$

Mathematica [A] time = 0.009158, size = 30, normalized size = 0.88

$$\frac{1}{4}\left(-x^2+2\sqrt{1-x^2}x\sin^{-1}(x)+\sin^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]*ArcSin[x],x]
```

```
[Out] (-x^2 + 2*x*Sqrt[1 - x^2]*ArcSin[x] + ArcSin[x]^2)/4
```

Maple [A] time = 0.038, size = 31, normalized size = 0.9

$$\frac{\arcsin(x)}{2}\left(x\sqrt{-x^2+1}+\arcsin(x)\right)-\frac{(\arcsin(x))^2}{4}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)*(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsin(x)*(x*(-x^2+1)^(1/2)+arcsin(x))-1/4*arcsin(x)^2-1/4*x^2
```

Maxima [A] time = 1.43341, size = 41, normalized size = 1.21

$$-\frac{1}{4}x^2+\frac{1}{2}\left(\sqrt{-x^2+1}x+\arcsin(x)\right)\arcsin(x)-\frac{1}{4}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arcsin(x) - 1/4*arcsin(x)^2
```

Fricas [A] time = 2.40354, size = 81, normalized size = 2.38

$$\frac{1}{2}\sqrt{-x^2+1}x\arcsin(x)-\frac{1}{4}x^2+\frac{1}{4}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*arcsin(x) - 1/4*x^2 + 1/4*arcsin(x)^2
```

Sympy [A] time = 18.9193, size = 48, normalized size = 1.41

$$\left(\begin{cases}\frac{x\sqrt{1-x^2}}{2}+\frac{\operatorname{asin}(x)}{2} & \text{for } x>-1\wedge x<1\end{cases}\right)\operatorname{asin}(x)-\begin{cases}\text{NaN} & \text{for } x<-1\\ \frac{x^2}{4}+\frac{\operatorname{asin}^2(x)}{4}-\frac{\pi^2}{16}-\frac{1}{4} & \text{for } x<1\\ \text{NaN} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)*(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((x*sqrt(1 - x**2)/2 + asin(x)/2, (x > -1) & (x < 1)))*asin(x) - P
iecewise((nan, x < -1), (x**2/4 + asin(x)**2/4 - pi**2/16 - 1/4, x < 1), (n
an, True))
```

Giac [A] time = 1.08232, size = 36, normalized size = 1.06

$$\frac{1}{2}\sqrt{-x^2+1}x\arcsin(x) - \frac{1}{4}x^2 + \frac{1}{4}\arcsin(x)^2 + \frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*arcsin(x) - 1/4*x^2 + 1/4*arcsin(x)^2 + 1/8
```

3.652 $\int \sqrt{1-x^2} \cos^{-1}(x) \, dx$

Optimal. Leaf size=34

$$\frac{x^2}{4}+\frac{1}{2}\sqrt{1-x^2}x\cos^{-1}(x)-\frac{1}{4}\cos^{-1}(x)^2$$

```
[Out] x^2/4 + (x*Sqrt[1 - x^2]*ArcCos[x])/2 - ArcCos[x]^2/4
```

Rubi [A] time = 0.0299284, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4648, 4642, 30}

$$\frac{x^2}{4}+\frac{1}{2}\sqrt{1-x^2}x\cos^{-1}(x)-\frac{1}{4}\cos^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]*ArcCos[x],x]
```

```
[Out] x^2/4 + (x*Sqrt[1 - x^2]*ArcCos[x])/2 - ArcCos[x]^2/4
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x^2}\cos^{-1}(x)\,dx &= \frac{1}{2}x\sqrt{1-x^2}\cos^{-1}(x)+\frac{\int x\,dx}{2}+\frac{1}{2}\int \frac{\cos^{-1}(x)}{\sqrt{1-x^2}}\,dx\\ &= \frac{x^2}{4}+\frac{1}{2}x\sqrt{1-x^2}\cos^{-1}(x)-\frac{1}{4}\cos^{-1}(x)^2\end{aligned}$$

Mathematica [A] time = 0.0167872, size = 30, normalized size = 0.88

$$\frac{1}{4}\left(x^2+2\sqrt{1-x^2}x\cos^{-1}(x)-\cos^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]*ArcCos[x],x]
```

```
[Out] (x^2 + 2*x*Sqrt[1 - x^2]*ArcCos[x] - ArcCos[x]^2)/4
```

Maple [A] time = 0.044, size = 33, normalized size = 1.

$$-\frac{\arccos(x)}{2}\left(-x\sqrt{-x^2+1}+\arccos(x)\right)+\frac{(\arccos(x))^2}{4}+\frac{x^2}{4}-\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x)*(-x^2+1)^(1/2),x)
```

```
[Out] -1/2*arccos(x)*(-x*(-x^2+1)^(1/2)+arccos(x))+1/4*arccos(x)^2+1/4*x^2-1/4
```

Maxima [A] time = 1.42414, size = 41, normalized size = 1.21

$$\frac{1}{4}x^2+\frac{1}{2}\left(\sqrt{-x^2+1}x+\arcsin(x)\right)\arccos(x)+\frac{1}{4}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*x^2 + 1/2*(sqrt(-x^2 + 1)*x + arcsin(x))*arccos(x) + 1/4*arcsin(x)^2
```

Fricas [A] time = 2.55854, size = 81, normalized size = 2.38

$$\frac{1}{2}\sqrt{-x^2+1}x\arccos(x)+\frac{1}{4}x^2-\frac{1}{4}\arccos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2
```

Sympy [A] time = 19.1584, size = 48, normalized size = 1.41

$$\left(\begin{cases}\frac{x\sqrt{1-x^2}}{2}+\frac{\operatorname{asin}(x)}{2} & \text{for } x>-1\wedge x<1\end{cases}\right)\operatorname{acos}(x)+\begin{cases}\text{NaN} & \text{for } x<-1\\ \frac{x^2}{4}+\frac{\operatorname{asin}^2(x)}{4}-\frac{\pi^2}{16}-\frac{1}{4} & \text{for } x<1\\ \text{NaN} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x)*(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((x*sqrt(1 - x**2)/2 + asin(x)/2, (x > -1) & (x < 1)))*acos(x) + P
iecewise((nan, x < -1), (x**2/4 + asin(x)**2/4 - pi**2/16 - 1/4, x < 1), (n
an, True))
```

Giac [A] time = 1.06837, size = 36, normalized size = 1.06

$$\frac{1}{2}\sqrt{-x^2+1}x\arccos(x)+\frac{1}{4}x^2-\frac{1}{4}\arccos(x)^2-\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x*arccos(x) + 1/4*x^2 - 1/4*arccos(x)^2 - 1/8
```

3.653 $\int x\sqrt{1-x^2}\cos^{-1}(x)\,dx$

Optimal. Leaf size=30

$$\frac{x^3}{9}-\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)-\frac{x}{3}$$

```
[Out] -x/3 + x^3/9 - ((1 - x^2)^(3/2)*ArcCos[x])/3
```

Rubi [A] time = 0.0313971, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {4678}

$$\frac{x^3}{9}-\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)-\frac{x}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 - x^2]*ArcCos[x],x]
```

```
[Out] -x/3 + x^3/9 - ((1 - x^2)^(3/2)*ArcCos[x])/3
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{1-x^2}\cos^{-1}(x)\,dx&=-\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)-\frac{1}{3}\int\left(1-x^2\right)\,dx\\&=-\frac{x}{3}+\frac{x^3}{9}-\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0276791, size = 26, normalized size = 0.87

$$\frac{1}{9}\left(x^3-3\left(1-x^2\right)^{3/2}\cos^{-1}(x)-3x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 - x^2]*ArcCos[x],x]
```

```
[Out] (-3*x + x^3 - 3*(1 - x^2)^(3/2)*ArcCos[x])/9
```

Maple [C] time = 0.15, size = 134, normalized size = 4.5

$$-\frac{i+3\arccos(x)}{72}\left(4ix^3-4\sqrt{-x^2+1}x^2-3ix+\sqrt{-x^2+1}\right)+\frac{\arccos(x)+i}{8}\left(ix-\sqrt{-x^2+1}\right)-\frac{\arccos(x)-i}{8}\left(ix+\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(x)*(-x^2+1)^(1/2),x)
```

```
[Out] -1/72*(I+3*arccos(x))*(4*I*x^3-4*(-x^2+1)^(1/2)*x^2-3*I*x+(-x^2+1)^(1/2))+1
/8*(arccos(x)+I)*(I*x-(-x^2+1)^(1/2))-1/8*(arccos(x)-I)*(I*x+(-x^2+1)^(1/2)
)+1/72*(-I+3*arccos(x))*(4*I*x^3+4*(-x^2+1)^(1/2)*x^2-3*I*x-(-x^2+1)^(1/2))
```

Maxima [A] time = 1.41756, size = 30, normalized size = 1.

$$\frac{1}{9}x^3 - \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos\left(x\right) - \frac{1}{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*x^3 - 1/3*(-x^2 + 1)^(3/2)*arccos(x) - 1/3*x
```

Fricas [A] time = 2.68538, size = 78, normalized size = 2.6

$$\frac{1}{9}x^3 + \frac{1}{3}\left(x^2-1\right)\sqrt{-x^2+1}\arccos\left(x\right) - \frac{1}{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*x^3 + 1/3*(x^2 - 1)*sqrt(-x^2 + 1)*arccos(x) - 1/3*x
```

Sympy [A] time = 1.00505, size = 37, normalized size = 1.23

$$\frac{x^3}{9} + \frac{x^2\sqrt{1-x^2}\operatorname{acos}\left(x\right)}{3} - \frac{x}{3} - \frac{\sqrt{1-x^2}\operatorname{acos}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(x)*(-x**2+1)**(1/2),x)
```

```
[Out] x**3/9 + x**2*sqrt(1 - x**2)*acos(x)/3 - x/3 - sqrt(1 - x**2)*acos(x)/3
```

Giac [A] time = 1.08687, size = 30, normalized size = 1.

$$\frac{1}{9}x^3 - \frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos\left(x\right) - \frac{1}{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/9*x^3 - 1/3*(-x^2 + 1)^(3/2)*arccos(x) - 1/3*x
```

3.654 $\int \left(1-x^2\right)^{3/2} \sin^{-1}(x) \, dx$

Optimal. Leaf size=59

$$\frac{x^4}{16}-\frac{5x^2}{16}+\frac{1}{4}\left(1-x^2\right)^{3/2} x \sin^{-1}(x)+\frac{3}{8}\sqrt{1-x^2} x \sin^{-1}(x)+\frac{3}{16}\sin^{-1}(x)^2$$

```
[Out] (-5*x^2)/16 + x^4/16 + (3*x*Sqrt[1 - x^2]*ArcSin[x])/8 + (x*(1 - x^2)^(3/2)
*ArcSin[x])/4 + (3*ArcSin[x]^2)/16
```

Rubi [A] time = 0.0497776, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4649, 4647, 4641, 30, 14}

$$\frac{x^4}{16}-\frac{5x^2}{16}+\frac{1}{4}\left(1-x^2\right)^{3/2} x \sin^{-1}(x)+\frac{3}{8}\sqrt{1-x^2} x \sin^{-1}(x)+\frac{3}{16}\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^2)^(3/2)*ArcSin[x],x]
```

```
[Out] (-5*x^2)/16 + x^4/16 + (3*x*Sqrt[1 - x^2]*ArcSin[x])/8 + (x*(1 - x^2)^(3/2)
*ArcSin[x])/4 + (3*ArcSin[x]^2)/16
```

Rule 4649

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcSin[c*x])^n)/(2*p + 1), x] + (D
ist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x
] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 - c^2*x
^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 4647

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSin[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(1-x^2\right)^{3/2}\sin^{-1}(x)\,dx &= \frac{1}{4}x\left(1-x^2\right)^{3/2}\sin^{-1}(x)-\frac{1}{4}\int x\left(1-x^2\right)\,dx+\frac{3}{4}\int \sqrt{1-x^2}\sin^{-1}(x)\,dx\\ &= \frac{3}{8}x\sqrt{1-x^2}\sin^{-1}(x)+\frac{1}{4}x\left(1-x^2\right)^{3/2}\sin^{-1}(x)-\frac{1}{4}\int \left(x-x^3\right)\,dx-\frac{3\int x\,dx}{8}+\frac{3}{8}\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}\,d\\ &= -\frac{5x^2}{16}+\frac{x^4}{16}+\frac{3}{8}x\sqrt{1-x^2}\sin^{-1}(x)+\frac{1}{4}x\left(1-x^2\right)^{3/2}\sin^{-1}(x)+\frac{3}{16}\sin^{-1}(x)^2\end{aligned}$$

Mathematica [A] time = 0.0336593, size = 42, normalized size = 0.71

$$\frac{1}{16}\left(x^4-5x^2-2\sqrt{1-x^2}\left(2x^2-5\right)x\sin^{-1}(x)+3\sin^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^2)^(3/2)*ArcSin[x],x]
```

```
[Out] (-5*x^2 + x^4 - 2*x*Sqrt[1 - x^2]*(-5 + 2*x^2)*ArcSin[x] + 3*ArcSin[x]^2)/1
6
```

Maple [A] time = 0.038, size = 58, normalized size = 1.

$$\frac{\arcsin(x)}{8}\left(-2\sqrt{-x^2+1}x^3+5x\sqrt{-x^2+1}+3\arcsin(x)\right)-\frac{3(\arcsin(x))^2}{16}+\frac{\left(x^2-1\right)^2}{16}-\frac{3x^2}{16}+\frac{3}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(3/2)*arcsin(x),x)
```

```
[Out] 1/8*arcsin(x)*(-2*(-x^2+1)^(1/2)*x^3+5*x*(-x^2+1)^(1/2)+3*arcsin(x))-3/16*a
rcsin(x)^2+1/16*(x^2-1)^2-3/16*x^2+3/16
```

Maxima [A] time = 1.40081, size = 68, normalized size = 1.15

$$\frac{1}{16}x^4-\frac{5}{16}x^2+\frac{1}{8}\left(2\left(-x^2+1\right)^{\frac{3}{2}}x+3\sqrt{-x^2+1}x+3\arcsin(x)\right)\arcsin(x)-\frac{3}{16}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x),x, algorithm="maxima")
```

```
[Out] 1/16*x^4 - 5/16*x^2 + 1/8*(2*(-x^2 + 1)^(3/2)*x + 3*sqrt(-x^2 + 1)*x + 3*ar
csin(x))*arcsin(x) - 3/16*arcsin(x)^2
```

Fricas [A] time = 2.56213, size = 115, normalized size = 1.95

$$\frac{1}{16}x^4 - \frac{1}{8}\left(2x^3 - 5x\right)\sqrt{-x^2+1}\arcsin(x) - \frac{5}{16}x^2 + \frac{3}{16}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x),x, algorithm="fricas")
```

```
[Out] 1/16*x^4 - 1/8*(2*x^3 - 5*x)*sqrt(-x^2 + 1)*arcsin(x) - 5/16*x^2 + 3/16*arc
sin(x)^2
```

Sympy [A] time = 2.11774, size = 53, normalized size = 0.9

$$\frac{x^4}{16} - \frac{x^3\sqrt{1-x^2}\operatorname{asin}(x)}{4} - \frac{5x^2}{16} + \frac{5x\sqrt{1-x^2}\operatorname{asin}(x)}{8} + \frac{3\operatorname{asin}^2(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(3/2)*asin(x),x)
```

```
[Out] x**4/16 - x**3*sqrt(1 - x**2)*asin(x)/4 - 5*x**2/16 + 5*x*sqrt(1 - x**2)*as
in(x)/8 + 3*asin(x)**2/16
```

Giac [A] time = 1.09751, size = 68, normalized size = 1.15

$$\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x\arcsin(x) + \frac{3}{8}\sqrt{-x^2+1}x\arcsin(x) + \frac{1}{16}\left(x^2-1\right)^2 - \frac{3}{16}x^2 + \frac{3}{16}\arcsin(x)^2 + \frac{9}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x),x, algorithm="giac")
```

```
[Out] 1/4*(-x^2 + 1)^(3/2)*x*arcsin(x) + 3/8*sqrt(-x^2 + 1)*x*arcsin(x) + 1/16*(x
^2 - 1)^2 - 3/16*x^2 + 3/16*arcsin(x)^2 + 9/128
```

3.655 $\int x\left(1-x^2\right)^{3/2}\sin^{-1}(x)\,dx$

Optimal. Leaf size=37

$$\frac{x^5}{25}-\frac{2x^3}{15}-\frac{1}{5}\left(1-x^2\right)^{5/2}\sin^{-1}(x)+\frac{x}{5}$$

```
[Out] x/5 - (2*x^3)/15 + x^5/25 - ((1 - x^2)^(5/2)*ArcSin[x])/5
```

Rubi [A] time = 0.0373196, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4677, 194}

$$\frac{x^5}{25}-\frac{2x^3}{15}-\frac{1}{5}\left(1-x^2\right)^{5/2}\sin^{-1}(x)+\frac{x}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 - x^2)^(3/2)*ArcSin[x],x]
```

```
[Out] x/5 - (2*x^3)/15 + x^5/25 - ((1 - x^2)^(5/2)*ArcSin[x])/5
```

Rule 4677

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p +
 1)), x] + Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\left(1-x^2\right)^{3/2}\sin^{-1}(x)\,dx &= -\frac{1}{5}\left(1-x^2\right)^{5/2}\sin^{-1}(x)+\frac{1}{5}\int\left(1-x^2\right)^2\,dx\\ &= -\frac{1}{5}\left(1-x^2\right)^{5/2}\sin^{-1}(x)+\frac{1}{5}\int\left(1-2x^2+x^4\right)\,dx\\ &= \frac{x}{5}-\frac{2x^3}{15}+\frac{x^5}{25}-\frac{1}{5}\left(1-x^2\right)^{5/2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0181415, size = 35, normalized size = 0.95

$$\frac{1}{5}\left(\frac{x^5}{5}-\frac{2x^3}{3}-\left(1-x^2\right)^{5/2}\sin^{-1}(x)+x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 - x^2)^(3/2)*ArcSin[x],x]
```

```
[Out] (x - (2*x^3)/3 + x^5/5 - (1 - x^2)^(5/2)*ArcSin[x])/5
```

Maple [A] time = 0.037, size = 37, normalized size = 1.

$$-\frac{\arcsin\left(x\right)\left(x^{2}-1\right)^{2}\sqrt{-x^{2}+1}}{5}+\frac{\left(3\,x^{4}-10\,x^{2}+15\right)x}{75}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+1)^(3/2)*arcsin(x),x)
```

```
[Out] -1/5*arcsin(x)*(x^2-1)^2*(-x^2+1)^(1/2)+1/75*(3*x^4-10*x^2+15)*x
```

Maxima [A] time = 1.42637, size = 36, normalized size = 0.97

$$\frac{1}{25}\,x^{5}-\frac{1}{5}\left(-x^{2}+1\right)^{\frac{5}{2}}\arcsin\left(x\right)-\frac{2}{15}\,x^{3}+\frac{1}{5}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(3/2)*arcsin(x),x, algorithm="maxima")
```

```
[Out] 1/25*x^5 - 1/5*(-x^2 + 1)^(5/2)*arcsin(x) - 2/15*x^3 + 1/5*x
```

Fricas [A] time = 2.58846, size = 105, normalized size = 2.84

$$\frac{1}{25}\,x^{5}-\frac{2}{15}\,x^{3}-\frac{1}{5}\left(x^{4}-2\,x^{2}+1\right)\sqrt{-x^{2}+1}\arcsin\left(x\right)+\frac{1}{5}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(3/2)*arcsin(x),x, algorithm="fricas")
```

```
[Out] 1/25*x^5 - 2/15*x^3 - 1/5*(x^4 - 2*x^2 + 1)*sqrt(-x^2 + 1)*arcsin(x) + 1/5*
x
```

Sympy [B] time = 3.54366, size = 63, normalized size = 1.7

$$\frac{x^{5}}{25}-\frac{x^{4}\sqrt{1-x^{2}}\operatorname{asin}\left(x\right)}{5}-\frac{2x^{3}}{15}+\frac{2x^{2}\sqrt{1-x^{2}}\operatorname{asin}\left(x\right)}{5}+\frac{x}{5}-\frac{\sqrt{1-x^{2}}\operatorname{asin}\left(x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+1)**(3/2)*asin(x),x)
```

```
[Out] x**5/25 - x**4*sqrt(1 - x**2)*asin(x)/5 - 2*x**3/15 + 2*x**2*sqrt(1 - x**2)
*asin(x)/5 + x/5 - sqrt(1 - x**2)*asin(x)/5
```

Giac [A] time = 1.08351, size = 46, normalized size = 1.24

$$\frac{1}{25}\,x^5 - \frac{1}{5}\left(x^2 - 1\right)^2 \sqrt{-x^2 + 1}\,\arcsin\left(x\right) - \frac{2}{15}\,x^3 + \frac{1}{5}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(3/2)*arcsin(x),x, algorithm="giac")
```

```
[Out] 1/25*x^5 - 1/5*(x^2 - 1)^2*sqrt(-x^2 + 1)*arcsin(x) - 2/15*x^3 + 1/5*x
```

3.656 $\int x^3 \left(1-x^2\right)^{3/2} \cos^{-1}(x)\,dx$

Optimal. Leaf size=61

$$-\frac{x^7}{49}+\frac{8x^5}{175}-\frac{x^3}{105}+\frac{1}{7}\left(1-x^2\right)^{7/2}\cos^{-1}(x)-\frac{1}{5}\left(1-x^2\right)^{5/2}\cos^{-1}(x)-\frac{2x}{35}$$

```
[Out] (-2*x)/35 - x^3/105 + (8*x^5)/175 - x^7/49 - ((1 - x^2)^(5/2)*ArcCos[x])/5
+ ((1 - x^2)^(7/2)*ArcCos[x])/7
```

Rubi [A] time = 0.0750953, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {266, 43, 4690, 12, 373}

$$-\frac{x^7}{49}+\frac{8x^5}{175}-\frac{x^3}{105}+\frac{1}{7}\left(1-x^2\right)^{7/2}\cos^{-1}(x)-\frac{1}{5}\left(1-x^2\right)^{5/2}\cos^{-1}(x)-\frac{2x}{35}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 - x^2)^(3/2)*ArcCos[x],x]
```

```
[Out] (-2*x)/35 - x^3/105 + (8*x^5)/175 - x^7/49 - ((1 - x^2)^(5/2)*ArcCos[x])/5
+ ((1 - x^2)^(7/2)*ArcCos[x])/7
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4690

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(1 - c^2*x^2)^p, x]}, Dist[d^p*(a + b*
ArcCos[c*x]), u, x] + Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^
2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Intege
rQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -
2^(-1)] && GtQ[d, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^3\left(1-x^2\right)^{3/2}\cos^{-1}(x)\,dx &= -\frac{1}{5}\left(1-x^2\right)^{5/2}\cos^{-1}(x)+\frac{1}{7}\left(1-x^2\right)^{7/2}\cos^{-1}(x)+\int\frac{1}{35}\left(-2-5x^2\right)\left(1-x^2\right)^2\,dx\\ &= -\frac{1}{5}\left(1-x^2\right)^{5/2}\cos^{-1}(x)+\frac{1}{7}\left(1-x^2\right)^{7/2}\cos^{-1}(x)+\frac{1}{35}\int\left(-2-5x^2\right)\left(1-x^2\right)^2\,dx\\ &= -\frac{1}{5}\left(1-x^2\right)^{5/2}\cos^{-1}(x)+\frac{1}{7}\left(1-x^2\right)^{7/2}\cos^{-1}(x)+\frac{1}{35}\int\left(-2-x^2+8x^4-5x^6\right)dx\\ &= -\frac{2x}{35}-\frac{x^3}{105}+\frac{8x^5}{175}-\frac{x^7}{49}-\frac{1}{5}\left(1-x^2\right)^{5/2}\cos^{-1}(x)+\frac{1}{7}\left(1-x^2\right)^{7/2}\cos^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.057314, size = 47, normalized size = 0.77

$$-\frac{x\left(75x^6-168x^4+35x^2+210\right)}{3675}-\frac{1}{35}\left(5x^2+2\right)\left(1-x^2\right)^{5/2}\cos^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 - x^2)^(3/2)*ArcCos[x],x]
```

```
[Out] -(x*(210 + 35*x^2 - 168*x^4 + 75*x^6))/3675 - ((1 - x^2)^(5/2)*(2 + 5*x^2)*
ArcCos[x])/35
```

Maple [C] time = 0.28, size = 430, normalized size = 7.1

$$\frac{i+7\arccos(x)}{6272}\left(64ix^7-64\sqrt{-x^2+1}x^6-112ix^5+80\sqrt{-x^2+1}x^4+56ix^3-24\sqrt{-x^2+1}x^2-7ix+\sqrt{-x^2+1}\right)-\frac{i+5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-x^2+1)^(3/2)*arccos(x),x)
```

```
[Out] 1/6272*(I+7*arccos(x))*(64*I*x^7-64*(-x^2+1)^(1/2)*x^6-112*I*x^5+80*(-x^2+1
)^(1/2)*x^4+56*I*x^3-24*(-x^2+1)^(1/2)*x^2-7*I*x+(-x^2+1)^(1/2))-1/3200*(I+
5*arccos(x))*(16*I*x^5-16*(-x^2+1)^(1/2)*x^4-20*I*x^3+12*(-x^2+1)^(1/2)*x^2
+5*I*x-(-x^2+1)^(1/2))-1/384*(I+3*arccos(x))*(4*I*x^3-4*(-x^2+1)^(1/2)*x^2-
3*I*x+(-x^2+1)^(1/2))+3/128*(arccos(x)+I)*(I*x-(-x^2+1)^(1/2))-3/128*(arcco
s(x)-I)*(I*x+(-x^2+1)^(1/2))+1/384*(-I+3*arccos(x))*(4*I*x^3+4*(-x^2+1)^(1/
2)*x^2-3*I*x-(-x^2+1)^(1/2))+1/3200*(-I+5*arccos(x))*(16*I*x^5+16*(-x^2+1)^
(1/2)*x^4-20*I*x^3-12*(-x^2+1)^(1/2)*x^2+5*I*x+(-x^2+1)^(1/2))-1/6272*(-I+7
*arccos(x))*(64*I*x^7+64*(-x^2+1)^(1/2)*x^6-112*I*x^5-80*(-x^2+1)^(1/2)*x^4
+56*I*x^3+24*(-x^2+1)^(1/2)*x^2-7*I*x-(-x^2+1)^(1/2))
```

Maxima [A] time = 1.40885, size = 66, normalized size = 1.08

$$-\frac{1}{49}x^7+\frac{8}{175}x^5-\frac{1}{105}x^3-\frac{1}{35}\left(5\left(-x^2+1\right)^{\frac{5}{2}}x^2+2\left(-x^2+1\right)^{\frac{5}{2}}\right)\arccos(x)-\frac{2}{35}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x, algorithm="maxima")
```

```
[Out] -1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*(-x^2 + 1)^(5/2)*x^2 + 2*(-x^2
+ 1)^(5/2))*arccos(x) - 2/35*x
```

Fricas [A] time = 2.45917, size = 138, normalized size = 2.26

$$-\frac{1}{49}x^7 + \frac{8}{175}x^5 - \frac{1}{105}x^3 - \frac{1}{35}\left(5x^6 - 8x^4 + x^2 + 2\right)\sqrt{-x^2 + 1}\arccos(x) - \frac{2}{35}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x, algorithm="fricas")
```

```
[Out] -1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*x^6 - 8*x^4 + x^2 + 2)*sqrt(-x^
2 + 1)*arccos(x) - 2/35*x
```

Sympy [A] time = 153.986, size = 88, normalized size = 1.44

$$-\frac{x^7}{49} - \frac{x^6\sqrt{1-x^2}\operatorname{acos}(x)}{7} + \frac{8x^5}{175} + \frac{8x^4\sqrt{1-x^2}\operatorname{acos}(x)}{35} - \frac{x^3}{105} - \frac{x^2\sqrt{1-x^2}\operatorname{acos}(x)}{35} - \frac{2x}{35} - \frac{2\sqrt{1-x^2}\operatorname{acos}(x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-x**2+1)**(3/2)*acos(x),x)
```

```
[Out] -x**7/49 - x**6*sqrt(1 - x**2)*acos(x)/7 + 8*x**5/175 + 8*x**4*sqrt(1 - x**
2)*acos(x)/35 - x**3/105 - x**2*sqrt(1 - x**2)*acos(x)/35 - 2*x/35 - 2*sqrt
(1 - x**2)*acos(x)/35
```

Giac [A] time = 1.07375, size = 81, normalized size = 1.33

$$-\frac{1}{49}x^7 + \frac{8}{175}x^5 - \frac{1}{105}x^3 - \frac{1}{35}\left(5\left(x^2-1\right)^3\sqrt{-x^2+1} + 7\left(x^2-1\right)^2\sqrt{-x^2+1}\right)\arccos(x) - \frac{2}{35}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(3/2)*arccos(x),x, algorithm="giac")
```

```
[Out] -1/49*x^7 + 8/175*x^5 - 1/105*x^3 - 1/35*(5*(x^2 - 1)^3*sqrt(-x^2 + 1) + 7*
(x^2 - 1)^2*sqrt(-x^2 + 1))*arccos(x) - 2/35*x
```

3.657 $\int \frac{(1-x^2)^{3/2} \cos^{-1}(x)}{x} \, dx$

Optimal. Leaf size=95

$$-i\text{PolyLog}\left(2,-ie^{i\cos^{-1}(x)}\right)+i\text{PolyLog}\left(2,ie^{i\cos^{-1}(x)}\right)-\frac{x^3}{9}+\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)+\sqrt{1-x^2}\cos^{-1}(x)+\frac{4x}{3}+2i\cos^{-1}(x$$

```
[Out] (4*x)/3 - x^3/9 + Sqrt[1 - x^2]*ArcCos[x] + ((1 - x^2)^(3/2)*ArcCos[x])/3 +
 (2*I)*ArcCos[x]*ArcTan[E^(I*ArcCos[x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[x]
)] + I*PolyLog[2, I*E^(I*ArcCos[x])]
```

Rubi [A] time = 0.158151, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {4700, 4698, 4710, 4181, 2279, 2391, 8}

$$-i\text{PolyLog}\left(2,-ie^{i\cos^{-1}(x)}\right)+i\text{PolyLog}\left(2,ie^{i\cos^{-1}(x)}\right)-\frac{x^3}{9}+\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)+\sqrt{1-x^2}\cos^{-1}(x)+\frac{4x}{3}+2i\cos^{-1}(x$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x^2)^(3/2)*ArcCos[x])/x,x]
```

```
[Out] (4*x)/3 - x^3/9 + Sqrt[1 - x^2]*ArcCos[x] + ((1 - x^2)^(3/2)*ArcCos[x])/3 +
 (2*I)*ArcCos[x]*ArcTan[E^(I*ArcCos[x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[x]
)] + I*PolyLog[2, I*E^(I*ArcCos[x])]
```

Rule 4700

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcC
os[c*x])^n)/(f*(m + 2*p + 1)), x] + (Dist[(2*d*p)/(m + 2*p + 1), Int[(f*x)^
m*(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart
[p]*(d + e*x^2)^FracPart[p])/(f*(m + 2*p + 1)*(1 - c^2*x^2)^FracPart[p]), I
nt[(f*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x
]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
GtQ[p, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4698

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcC
os[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x
^2]), Int[((f*x)^m*(a + b*ArcCos[c*x])^n)/Sqrt[1 - c^2*x^2], x], x] + Dist[
(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a
 + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && Eq
Q[c^2*d + e, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4710

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n
*Cos[x]^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1-x^2)^{3/2}\cos^{-1}(x)}{x}\,dx &= \frac{1}{3}(1-x^2)^{3/2}\cos^{-1}(x)+\frac{1}{3}\int (1-x^2)\,dx+\int \frac{\sqrt{1-x^2}\cos^{-1}(x)}{x}\,dx\\
&= \frac{x}{3}-\frac{x^3}{9}+\sqrt{1-x^2}\cos^{-1}(x)+\frac{1}{3}(1-x^2)^{3/2}\cos^{-1}(x)+\int 1\,dx+\int \frac{\cos^{-1}(x)}{x\sqrt{1-x^2}}\,dx\\
&= \frac{4x}{3}-\frac{x^3}{9}+\sqrt{1-x^2}\cos^{-1}(x)+\frac{1}{3}(1-x^2)^{3/2}\cos^{-1}(x)-\text{Subst}\left(\int x\sec(x)\,dx,x,\cos^{-1}(x)\right.\\
&= \frac{4x}{3}-\frac{x^3}{9}+\sqrt{1-x^2}\cos^{-1}(x)+\frac{1}{3}(1-x^2)^{3/2}\cos^{-1}(x)+2i\cos^{-1}(x)\tan^{-1}\left(e^{i\cos^{-1}(x)}\right)+\text{Su}\\
&= \frac{4x}{3}-\frac{x^3}{9}+\sqrt{1-x^2}\cos^{-1}(x)+\frac{1}{3}(1-x^2)^{3/2}\cos^{-1}(x)+2i\cos^{-1}(x)\tan^{-1}\left(e^{i\cos^{-1}(x)}\right)-i\,\text{Su}\\
&= \frac{4x}{3}-\frac{x^3}{9}+\sqrt{1-x^2}\cos^{-1}(x)+\frac{1}{3}(1-x^2)^{3/2}\cos^{-1}(x)+2i\cos^{-1}(x)\tan^{-1}\left(e^{i\cos^{-1}(x)}\right)-i\text{Li}
\end{aligned}$$

Mathematica [A] time = 0.255596, size = 119, normalized size = 1.25

$$-i\text{PolyLog}\left(2,-ie^{i\cos^{-1}(x)}\right)+i\text{PolyLog}\left(2,ie^{i\cos^{-1}(x)}\right)+\sqrt{1-x^2}\cos^{-1}(x)+\frac{1}{36}\left(12(1-x^2)^{3/2}\cos^{-1}(x)+9x-\cos(3\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 - x^2)^(3/2)*ArcCos[x])/x,x]
```

```
[Out] x + Sqrt[1 - x^2]*ArcCos[x] + (9*x + 12*(1 - x^2)^(3/2)*ArcCos[x] - Cos[3*A
rcCos[x]])/36 - ArcCos[x]*Log[1 - I*E^(I*ArcCos[x])] + ArcCos[x]*Log[1 + I*
E^(I*ArcCos[x])] - I*PolyLog[2, (-I)*E^(I*ArcCos[x])] + I*PolyLog[2, I*E^(I
*ArcCos[x])]
```

Maple [B] time = 0.191, size = 227, normalized size = 2.4

$$\frac{i+3\arccos(x)}{72}\left(4ix^3-4\sqrt{-x^2+1}x^2-3ix+\sqrt{-x^2+1}\right)-\frac{5\arccos(x)+5i}{8}\left(ix-\sqrt{-x^2+1}\right)+\frac{5\arccos(x)-5i}{8}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(3/2)*arccos(x)/x,x)
```

```
[Out] 1/72*(I+3*arccos(x))*(4*I*x^3-4*(-x^2+1)^(1/2)*x^2-3*I*x+(-x^2+1)^(1/2))-5/
8*(arccos(x)+I)*(I*x-(-x^2+1)^(1/2))+5/8*(arccos(x)-I)*(I*x+(-x^2+1)^(1/2))
-1/72*(-I+3*arccos(x))*(4*I*x^3+4*(-x^2+1)^(1/2)*x^2-3*I*x-(-x^2+1)^(1/2))+
arccos(x)*ln(1+I*(x+I*(-x^2+1)^(1/2)))-arccos(x)*ln(1-I*(x+I*(-x^2+1)^(1/2)
))-I*dilog(1+I*(x+I*(-x^2+1)^(1/2)))+I*dilog(1-I*(x+I*(-x^2+1)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^2+1\right)^{\frac{3}{2}} \arccos\left(x\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arccos(x)/x,x, algorithm="maxima")
```

```
[Out] integrate((-x^2 + 1)^(3/2)*arccos(x)/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(x^2-1\right)\sqrt{-x^2+1}\arccos\left(x\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arccos(x)/x,x, algorithm="fricas")
```

```
[Out] integral(-(x^2 - 1)*sqrt(-x^2 + 1)*arccos(x)/x, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(3/2)*acos(x)/x,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(-x^2+1\right)^{\frac{3}{2}} \arccos\left(x\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arccos(x)/x,x, algorithm="giac")
```

```
[Out] integrate((-x^2 + 1)^(3/2)*arccos(x)/x, x)
```

3.658 $\int \frac{\left(1-x^2\right)^{3/2}\sin^{-1}(x)}{x^6}\,dx$

Optimal. Leaf size=41

$$\frac{1}{5x^2}-\frac{1}{20x^4}-\frac{\left(1-x^2\right)^{5/2}\sin^{-1}(x)}{5x^5}+\frac{\log(x)}{5}$$

```
[Out] -1/(20*x^4) + 1/(5*x^2) - ((1 - x^2)^(5/2)*ArcSin[x])/(5*x^5) + Log[x]/5
```

Rubi [A] time = 0.0591848, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4681, 266, 43}

$$\frac{1}{5x^2}-\frac{1}{20x^4}-\frac{\left(1-x^2\right)^{5/2}\sin^{-1}(x)}{5x^5}+\frac{\log(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[((1 - x^2)^(3/2)*ArcSin[x])/x^6,x]
```

```
[Out] -1/(20*x^4) + 1/(5*x^2) - ((1 - x^2)^(5/2)*ArcSin[x])/(5*x^5) + Log[x]/5
```

Rule 4681

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcSin[c*x])^n)/(d*f*(m + 1)), x] - Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^F
racPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 - c
^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &
& NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{(1-x^2)^{3/2}\sin^{-1}(x)}{x^6}\,dx = -\frac{(1-x^2)^{5/2}\sin^{-1}(x)}{5x^5} + \frac{1}{5}\int \frac{(1-x^2)^2}{x^5}\,dx$$
$$= -\frac{(1-x^2)^{5/2}\sin^{-1}(x)}{5x^5} + \frac{1}{10}\,\text{Subst}\left(\int \frac{(1-x)^2}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{(1-x^2)^{5/2}\sin^{-1}(x)}{5x^5} + \frac{1}{10}\,\text{Subst}\left(\int \left(\frac{1}{x^3}-\frac{2}{x^2}+\frac{1}{x}\right)dx, x, x^2\right)$$
$$= -\frac{1}{20x^4} + \frac{1}{5x^2} - \frac{(1-x^2)^{5/2}\sin^{-1}(x)}{5x^5} + \frac{\log(x)}{5}$$

Mathematica [A] time = 0.0436426, size = 36, normalized size = 0.88

$$-\frac{-4x^3-4x^5\log(x)+4(1-x^2)^{5/2}\sin^{-1}(x)+x}{20x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 - x^2)^(3/2)*ArcSin[x])/x^6,x]
```

```
[Out] -(x - 4*x^3 + 4*(1 - x^2)^(5/2)*ArcSin[x] - 4*x^5*Log[x])/(20*x^5)
```

Maple [C] time = 0.532, size = 201, normalized size = 4.9

$$-\frac{2i}{5}\arcsin(x) + \frac{1}{(100x^8-200x^6+200x^4-100x^2+20)x^5}\left(-\sqrt{-x^2+1}x^4+ix^5+2\sqrt{-x^2+1}x^2-\sqrt{-x^2+1}\right)(20\text{ a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(3/2)*arcsin(x)/x^6,x)
```

```
[Out] -2/5*I*arcsin(x)+1/20*(-(-x^2+1)^(1/2)*x^4+I*x^5+2*(-x^2+1)^(1/2)*x^2-(-x^2
+1)^(1/2))*(20*arcsin(x)*x^8-4*I*x^8-4*(-x^2+1)^(1/2)*x^7-40*arcsin(x)*x^6+
I*x^6+9*(-x^2+1)^(1/2)*x^5+40*arcsin(x)*x^4-6*(-x^2+1)^(1/2)*x^3-20*x^2*arc
sin(x)+x*(-x^2+1)^(1/2)+4*arcsin(x))/(5*x^8-10*x^6+10*x^4-5*x^2+1)/x^5+1/5*
ln((I*x+(-x^2+1)^(1/2))^2-1)
```

Maxima [A] time = 1.45164, size = 47, normalized size = 1.15

$$-\frac{(-x^2+1)^{\frac{5}{2}}\arcsin(x)}{5x^5} + \frac{4x^2-1}{20x^4} + \frac{1}{10}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*(-x^2 + 1)^(5/2)*arcsin(x)/x^5 + 1/20*(4*x^2 - 1)/x^4 + 1/10*log(x^2)
```

Fricas [A] time = 2.78533, size = 113, normalized size = 2.76

$$\frac{4x^5\log(x) + 4x^3 - 4(x^4 - 2x^2 + 1)\sqrt{-x^2+1}\arcsin(x) - x}{20x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/20*(4*x^5*log(x) + 4*x^3 - 4*(x^4 - 2*x^2 + 1)*sqrt(-x^2 + 1)*arcsin(x) -
 x)/x^5
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(x-1)(x+1))^{\frac{3}{2}}\operatorname{asin}(x)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(3/2)*asin(x)/x**6,x)
```

```
[Out] Integral((-(x - 1)*(x + 1))**(3/2)*asin(x)/x**6, x)
```

Giac [B] time = 1.09303, size = 182, normalized size = 4.44

$$-\frac{1}{160}\left(\frac{x^5\left(\frac{5\left(\sqrt{-x^2+1}-1\right)^2}{x^2} - \frac{10\left(\sqrt{-x^2+1}-1\right)^4}{x^4} - 1\right)}{\left(\sqrt{-x^2+1}-1\right)^5} + \frac{10\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{5\left(\sqrt{-x^2+1}-1\right)^3}{x^3} + \frac{\left(\sqrt{-x^2+1}-1\right)^5}{x^5}\right)\arcsin(x) -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(3/2)*arcsin(x)/x^6,x, algorithm="giac")
```

```
[Out] -1/160*(x^5*(5*(sqrt(-x^2 + 1) - 1)^2/x^2 - 10*(sqrt(-x^2 + 1) - 1)^4/x^4 -
 1)/(sqrt(-x^2 + 1) - 1)^5 + 10*(sqrt(-x^2 + 1) - 1)/x - 5*(sqrt(-x^2 + 1)
- 1)^3/x^3 + (sqrt(-x^2 + 1) - 1)^5/x^5)*arcsin(x) - 1/20*(3*x^4 - 4*x^2 +
1)/x^4 + 1/10*log(x^2)
```

3.659 $\int \frac{x^2 \sin^{-1}(x)}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=34

$$\frac{x^2}{4} - \frac{1}{2}\sqrt{1-x^2}x\sin^{-1}(x) + \frac{1}{4}\sin^{-1}(x)^2$$

```
[Out] x^2/4 - (x*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4
```

Rubi [A] time = 0.0597781, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4707, 4641, 30}

$$\frac{x^2}{4} - \frac{1}{2}\sqrt{1-x^2}x\sin^{-1}(x) + \frac{1}{4}\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] x^2/4 - (x*Sqrt[1 - x^2]*ArcSin[x])/2 + ArcSin[x]^2/4
```

Rule 4707

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcSin[c*x])^n)/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2 \sin^{-1}(x)}{\sqrt{1-x^2}} \, dx &= -\frac{1}{2}x\sqrt{1-x^2}\sin^{-1}(x) + \frac{\int x \, dx}{2} + \frac{1}{2}\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} \, dx \\ &= \frac{x^2}{4} - \frac{1}{2}x\sqrt{1-x^2}\sin^{-1}(x) + \frac{1}{4}\sin^{-1}(x)^2\end{aligned}$$

Mathematica [A] time = 0.0112583, size = 28, normalized size = 0.82

$$\frac{1}{4}\left(x^2 - 2\sqrt{1-x^2}x\sin^{-1}(x) + \sin^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] (x^2 - 2*x*Sqrt[1 - x^2]*ArcSin[x] + ArcSin[x]^2)/4
```

Maple [A] time = 0.035, size = 32, normalized size = 0.9

$$\frac{\arcsin(x)}{2}\left(-x\sqrt{-x^2+1}+\arcsin(x)\right)-\frac{(\arcsin(x))^2}{4}+\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsin(x)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsin(x)*(-x*(-x^2+1)^(1/2)+arcsin(x))-1/4*arcsin(x)^2+1/4*x^2
```

Maxima [A] time = 1.40747, size = 43, normalized size = 1.26

$$\frac{1}{4}x^2-\frac{1}{2}\left(\sqrt{-x^2+1}x-\arcsin(x)\right)\arcsin(x)-\frac{1}{4}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 1/2*(sqrt(-x^2 + 1)*x - arcsin(x))*arcsin(x) - 1/4*arcsin(x)^2
```

Fricas [A] time = 2.59129, size = 82, normalized size = 2.41

$$-\frac{1}{2}\sqrt{-x^2+1}x\arcsin(x)+\frac{1}{4}x^2+\frac{1}{4}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x*arcsin(x) + 1/4*x^2 + 1/4*arcsin(x)^2
```

Sympy [A] time = 0.332589, size = 26, normalized size = 0.76

$$\frac{x^2}{4}-\frac{x\sqrt{1-x^2}\operatorname{asin}(x)}{2}+\frac{\operatorname{asin}^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asin(x)/(-x**2+1)**(1/2),x)
```

```
[Out] x**2/4 - x*sqrt(1 - x**2)*asin(x)/2 + asin(x)**2/4
```

Giac [A] time = 1.07644, size = 36, normalized size = 1.06

$$-\frac{1}{2}\sqrt{-x^2+1}x\arcsin(x)+\frac{1}{4}x^2+\frac{1}{4}\arcsin(x)^2-\frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x*arcsin(x) + 1/4*x^2 + 1/4*arcsin(x)^2 - 1/8
```

3.660 $\int \frac{x^4 \sin^{-1}(x)}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=61

$$\frac{x^4}{16}+\frac{3x^2}{16}-\frac{1}{4}\sqrt{1-x^2}x^3\sin^{-1}(x)-\frac{3}{8}\sqrt{1-x^2}x\sin^{-1}(x)+\frac{3}{16}\sin^{-1}(x)^2$$

```
[Out] (3*x^2)/16 + x^4/16 - (3*x*Sqrt[1 - x^2]*ArcSin[x])/8 - (x^3*Sqrt[1 - x^2]*
ArcSin[x])/4 + (3*ArcSin[x]^2)/16
```

Rubi [A] time = 0.106788, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4707, 4641, 30}

$$\frac{x^4}{16}+\frac{3x^2}{16}-\frac{1}{4}\sqrt{1-x^2}x^3\sin^{-1}(x)-\frac{3}{8}\sqrt{1-x^2}x\sin^{-1}(x)+\frac{3}{16}\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] (3*x^2)/16 + x^4/16 - (3*x*Sqrt[1 - x^2]*ArcSin[x])/8 - (x^3*Sqrt[1 - x^2]*
ArcSin[x])/4 + (3*ArcSin[x]^2)/16
```

Rule 4707

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcSin[c*x])^n)/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx &= -\frac{1}{4}x^3\sqrt{1-x^2}\sin^{-1}(x)+\frac{\int x^3\,dx}{4}+\frac{3}{4}\int \frac{x^2\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx\\
&= \frac{x^4}{16}-\frac{3}{8}x\sqrt{1-x^2}\sin^{-1}(x)-\frac{1}{4}x^3\sqrt{1-x^2}\sin^{-1}(x)+\frac{3\int x\,dx}{8}+\frac{3}{8}\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx\\
&= \frac{3x^2}{16}+\frac{x^4}{16}-\frac{3}{8}x\sqrt{1-x^2}\sin^{-1}(x)-\frac{1}{4}x^3\sqrt{1-x^2}\sin^{-1}(x)+\frac{3}{16}\sin^{-1}(x)^2
\end{aligned}$$

Mathematica [A] time = 0.0250347, size = 43, normalized size = 0.7

$$\frac{1}{16}\left(\left(x^2+3\right)x^2-2\sqrt{1-x^2}\left(2x^2+3\right)x\sin^{-1}(x)+3\sin^{-1}(x)^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcSin[x])/Sqrt[1 - x^2],x]
```

```
[Out] (x^2*(3 + x^2) - 2*x*Sqrt[1 - x^2]*(3 + 2*x^2)*ArcSin[x] + 3*ArcSin[x]^2)/1
6
```

Maple [A] time = 0.044, size = 53, normalized size = 0.9

$$\frac{\arcsin(x)}{8}\left(-2\sqrt{-x^2+1}x^3-3x\sqrt{-x^2+1}+3\arcsin(x)\right)-\frac{3(\arcsin(x))^2}{16}+\frac{x^4}{16}+\frac{3x^2}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arcsin(x)/(-x^2+1)^(1/2),x)
```

```
[Out] 1/8*arcsin(x)*(-2*(-x^2+1)^(1/2)*x^3-3*x*(-x^2+1)^(1/2)+3*arcsin(x))-3/16*a
rcsin(x)^2+1/16*x^4+3/16*x^2
```

Maxima [A] time = 1.42568, size = 70, normalized size = 1.15

$$\frac{1}{16}x^4+\frac{3}{16}x^2-\frac{1}{8}\left(2\sqrt{-x^2+1}x^3+3\sqrt{-x^2+1}x-3\arcsin(x)\right)\arcsin(x)-\frac{3}{16}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*x^4 + 3/16*x^2 - 1/8*(2*sqrt(-x^2 + 1)*x^3 + 3*sqrt(-x^2 + 1)*x - 3*ar
csin(x))*arcsin(x) - 3/16*arcsin(x)^2
```

Fricas [A] time = 2.44136, size = 115, normalized size = 1.89

$$\frac{1}{16}x^4-\frac{1}{8}\left(2x^3+3x\right)\sqrt{-x^2+1}\arcsin(x)+\frac{3}{16}x^2+\frac{3}{16}\arcsin(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*x^4 - 1/8*(2*x^3 + 3*x)*sqrt(-x^2 + 1)*arcsin(x) + 3/16*x^2 + 3/16*arc
sin(x)^2
```

Sympy [A] time = 1.29486, size = 53, normalized size = 0.87

$$\frac{x^4}{16} - \frac{x^3\sqrt{1-x^2}\,\mathrm{asin}\,(x)}{4} + \frac{3x^2}{16} - \frac{3x\sqrt{1-x^2}\,\mathrm{asin}\,(x)}{8} + \frac{3\,\mathrm{asin}^2\,(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asin(x)/(-x**2+1)**(1/2),x)
```

```
[Out] x**4/16 - x**3*sqrt(1 - x**2)*asin(x)/4 + 3*x**2/16 - 3*x*sqrt(1 - x**2)*as
in(x)/8 + 3*asin(x)**2/16
```

Giac [A] time = 1.09011, size = 68, normalized size = 1.11

$$\frac{1}{4}\left(-x^2+1\right)^{\frac{3}{2}}x\arcsin\left(x\right) - \frac{5}{8}\sqrt{-x^2+1}x\arcsin\left(x\right) + \frac{1}{16}\left(x^2-1\right)^2 + \frac{5}{16}x^2 + \frac{3}{16}\arcsin\left(x\right)^2 - \frac{23}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsin(x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(-x^2 + 1)^(3/2)*x*arcsin(x) - 5/8*sqrt(-x^2 + 1)*x*arcsin(x) + 1/16*(x
^2 - 1)^2 + 5/16*x^2 + 3/16*arcsin(x)^2 - 23/128
```

3.661 $\int \frac{x \sin^{-1}(x)}{(1-x^2)^{3/2}} dx$

Optimal. Leaf size=19

$$\frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - \tanh^{-1}(x)$$

```
[Out] ArcSin[x]/Sqrt[1 - x^2] - ArcTanh[x]
```

Rubi [A] time = 0.0375043, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4677, 206}

$$\frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - \tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcSin[x])/(1 - x^2)^(3/2),x]

[Out] ArcSin[x]/Sqrt[1 - x^2] - ArcTanh[x]
```

Rule 4677

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p +
 1)), x] + Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x \sin^{-1}(x)}{(1-x^2)^{3/2}} dx &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - \int \frac{1}{1-x^2} dx \\ &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - \tanh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.010899, size = 19, normalized size = 1.

$$\frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - \tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcSin[x])/(1 - x^2)^(3/2),x]
```

```
[Out] ArcSin[x]/Sqrt[1 - x^2] - ArcTanh[x]
```

Maple [B] time = 0.035, size = 46, normalized size = 2.4

$$-\frac{\arcsin(x)}{x^2-1}\sqrt{-x^2+1}-\ln\left(\frac{1}{\sqrt{-x^2+1}}+x\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsin(x)/(-x^2+1)^(3/2),x)
```

```
[Out] -(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)-ln(1/(-x^2+1)^(1/2)+x/(-x^2+1)^(1/2))
```

Maxima [A] time = 1.40527, size = 34, normalized size = 1.79

$$\frac{\arcsin(x)}{\sqrt{-x^2+1}}-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] arcsin(x)/sqrt(-x^2 + 1) - 1/2*log(x + 1) + 1/2*log(x - 1)
```

Fricas [B] time = 2.53159, size = 123, normalized size = 6.47

$$-\frac{(x^2-1)\log(x+1)-(x^2-1)\log(x-1)+2\sqrt{-x^2+1}\arcsin(x)}{2(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) + 2*sqrt(-x^2 + 1)*arcsin
(x))/(x^2 - 1)
```

Sympy [A] time = 4.89581, size = 20, normalized size = 1.05

$$-\begin{cases}\operatorname{acoth}(x) & \text{for } x^2>1\\ \operatorname{atanh}(x) & \text{for } x^2<1\end{cases}+\frac{\operatorname{asin}(x)}{\sqrt{1-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asin(x)/(-x**2+1)**(3/2),x)
```

```
[Out] -Piecewise((acoth(x), x**2 > 1), (atanh(x), x**2 < 1)) + asin(x)/sqrt(1 - x
**2)
```

Giac [A] time = 1.07836, size = 36, normalized size = 1.89

$$\frac{\arcsin(x)}{\sqrt{-x^2+1}} - \frac{1}{2}\log(|x+1|) + \frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] arcsin(x)/sqrt(-x^2 + 1) - 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1))
```

3.662 $\int \frac{x\cos^{-1}(x)}{\left(1-x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=17

$$\frac{\cos^{-1}(x)}{\sqrt{1-x^2}}+\tanh^{-1}(x)$$

```
[Out] ArcCos[x]/Sqrt[1 - x^2] + ArcTanh[x]
```

Rubi [A] time = 0.0348905, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {4678, 206}

$$\frac{\cos^{-1}(x)}{\sqrt{1-x^2}}+\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCos[x])/(1 - x^2)^(3/2),x]

[Out] ArcCos[x]/Sqrt[1 - x^2] + ArcTanh[x]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos^{-1}(x)}{\left(1-x^2\right)^{3/2}}\,dx &= \frac{\cos^{-1}(x)}{\sqrt{1-x^2}}+\int \frac{1}{1-x^2}\,dx\\ &= \frac{\cos^{-1}(x)}{\sqrt{1-x^2}}+\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0539185, size = 32, normalized size = 1.88

$$\frac{1}{2}\left(\frac{2\cos^{-1}(x)}{\sqrt{1-x^2}}-\log(1-x)+\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCos[x])/(1 - x^2)^(3/2),x]
```

```
[Out] ((2*ArcCos[x])/Sqrt[1 - x^2] - Log[1 - x] + Log[1 + x])/2
```

Maple [B] time = 0.038, size = 47, normalized size = 2.8

$$-\frac{\arccos(x)}{x^2-1}\sqrt{-x^2+1}-\ln\left(\frac{1}{\sqrt{-x^2+1}}-x\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(x)/(-x^2+1)^(3/2),x)
```

```
[Out] -(-x^2+1)^(1/2)/(x^2-1)*arccos(x)-ln(1/(-x^2+1)^(1/2)-x/(-x^2+1)^(1/2))
```

Maxima [A] time = 1.40954, size = 34, normalized size = 2.

$$\frac{\arccos(x)}{\sqrt{-x^2+1}}+\frac{1}{2}\log(x+1)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)/(-x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] arccos(x)/sqrt(-x^2 + 1) + 1/2*log(x + 1) - 1/2*log(x - 1)
```

Fricas [B] time = 2.40786, size = 122, normalized size = 7.18

$$\frac{(x^2-1)\log(x+1)-(x^2-1)\log(x-1)-2\sqrt{-x^2+1}\arccos(x)}{2(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)/(-x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*sqrt(-x^2 + 1)*arccos(
x))/(x^2 - 1)
```

Sympy [A] time = 6.04356, size = 20, normalized size = 1.18

$$\begin{cases}\operatorname{acoth}(x) & \text{for } x^2>1\\ \operatorname{atanh}(x) & \text{for } x^2<1\end{cases}+\frac{\operatorname{acos}(x)}{\sqrt{1-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(x)/(-x**2+1)**(3/2),x)
```

```
[Out] Piecewise((acoth(x), x**2 > 1), (atanh(x), x**2 < 1)) + acos(x)/sqrt(1 - x*
*2)
```

Giac [A] time = 1.08046, size = 36, normalized size = 2.12

$$\frac{\arccos(x)}{\sqrt{-x^2+1}} + \frac{1}{2}\log(|x+1|) - \frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)/(-x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] arccos(x)/sqrt(-x^2 + 1) + 1/2*log(abs(x + 1)) - 1/2*log(abs(x - 1))
```

3.663 $\int \frac{\sin^{-1}(x)}{(1-x^2)^{5/2}}\,dx$

Optimal. Leaf size=62

$$-\frac{1}{6\left(1-x^2\right)}+\frac{1}{3}\log\left(1-x^2\right)+\frac{2x\sin^{-1}(x)}{3\sqrt{1-x^2}}+\frac{x\sin^{-1}(x)}{3\left(1-x^2\right)^{3/2}}$$

```
[Out] -1/(6*(1 - x^2)) + (x*ArcSin[x])/(3*(1 - x^2)^(3/2)) + (2*x*ArcSin[x])/(3*S
qrt[1 - x^2]) + Log[1 - x^2]/3
```

Rubi [A] time = 0.0391695, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4655, 4651, 260, 261}

$$-\frac{1}{6\left(1-x^2\right)}+\frac{1}{3}\log\left(1-x^2\right)+\frac{2x\sin^{-1}(x)}{3\sqrt{1-x^2}}+\frac{x\sin^{-1}(x)}{3\left(1-x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]/(1 - x^2)^(5/2),x]
```

```
[Out] -1/(6*(1 - x^2)) + (x*ArcSin[x])/(3*(1 - x^2)^(3/2)) + (2*x*ArcSin[x])/(3*S
qrt[1 - x^2]) + Log[1 - x^2]/3
```

Rule 4655

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x_
Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*d*(p + 1)
), x] + (Dist[(2*p + 3)/(2*d*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcSi
n[c*x])^n, x], x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*(p
 + 1)*(1 - c^2*x^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcS
in[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0]
 && GtQ[n, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 4651

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[(x*(a + b*ArcSin[c*x])^n)/(d*Sqrt[d + e*x^2]), x] - Dist[(
b*c*n)/Sqrt[d], Int[(x*(a + b*ArcSin[c*x])^(n - 1))/(d + e*x^2), x], x] /;
FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[d, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\sin^{-1}(x)}{\left(1-x^2\right)^{5/2}}\,dx = \frac{x\sin^{-1}(x)}{3\left(1-x^2\right)^{3/2}} - \frac{1}{3}\int \frac{x}{\left(1-x^2\right)^2}\,dx + \frac{2}{3}\int \frac{\sin^{-1}(x)}{\left(1-x^2\right)^{3/2}}\,dx$$
$$= -\frac{1}{6\left(1-x^2\right)} + \frac{x\sin^{-1}(x)}{3\left(1-x^2\right)^{3/2}} + \frac{2x\sin^{-1}(x)}{3\sqrt{1-x^2}} - \frac{2}{3}\int \frac{x}{1-x^2}\,dx$$
$$= -\frac{1}{6\left(1-x^2\right)} + \frac{x\sin^{-1}(x)}{3\left(1-x^2\right)^{3/2}} + \frac{2x\sin^{-1}(x)}{3\sqrt{1-x^2}} + \frac{1}{3}\log\left(1-x^2\right)$$

Mathematica [A] time = 0.0879138, size = 45, normalized size = 0.73

$$\frac{1}{6}\left(\frac{1}{x^2-1} + 2\log\left(1-x^2\right) - \frac{2x\left(2x^2-3\right)\sin^{-1}(x)}{\left(1-x^2\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]/(1 - x^2)^(5/2),x]
```

```
[Out] ((-1 + x^2)^(-1) - (2*x*(-3 + 2*x^2)*ArcSin[x])/(1 - x^2)^(3/2) + 2*Log[1 -
 x^2])/6
```

Maple [A] time = 0.037, size = 63, normalized size = 1.

$$\frac{\arcsin(x)\,x}{3\left(x^2-1\right)^2}\sqrt{-x^2+1} + \frac{1}{6x^2-6} - \frac{2\arcsin(x)\,x}{3x^2-3}\sqrt{-x^2+1} + \frac{\ln\left(-x^2+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)/(-x^2+1)^(5/2),x)
```

```
[Out] 1/3*x*arcsin(x)*(-x^2+1)^(1/2)/(x^2-1)^2+1/6/(x^2-1)-2/3*(-x^2+1)^(1/2)/(x^
2-1)*arcsin(x)*x+1/3*ln(-x^2+1)
```

Maxima [A] time = 1.41139, size = 65, normalized size = 1.05

$$\frac{1}{3}\left(\frac{2x}{\sqrt{-x^2+1}} + \frac{x}{\left(-x^2+1\right)^{\frac{3}{2}}}\right)\arcsin(x) + \frac{1}{6\left(x^2-1\right)} + \frac{1}{3}\log\left(-3x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(-x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x/sqrt(-x^2 + 1) + x/(-x^2 + 1)^(3/2))*arcsin(x) + 1/6/(x^2 - 1) + 1
/3*log(-3*x^2 + 3)
```

Fricas [A] time = 2.53875, size = 151, normalized size = 2.44

$$-\frac{2\left(2x^3-3x\right)\sqrt{-x^2+1}\arcsin\left(x\right)-x^2-2\left(x^4-2x^2+1\right)\log\left(x^2-1\right)+1}{6\left(x^4-2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(2*x^3 - 3*x)*sqrt(-x^2 + 1)*arcsin(x) - x^2 - 2*(x^4 - 2*x^2 + 1)*
log(x^2 - 1) + 1)/(x^4 - 2*x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)/(-x**2+1)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.08977, size = 73, normalized size = 1.18

$$-\frac{\left(2x^2-3\right)\sqrt{-x^2+1}x\arcsin\left(x\right)}{3\left(x^2-1\right)^2}-\frac{2x^2-3}{6\left(x^2-1\right)}+\frac{1}{3}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(2*x^2 - 3)*sqrt(-x^2 + 1)*x*arcsin(x)/(x^2 - 1)^2 - 1/6*(2*x^2 - 3)/(
x^2 - 1) + 1/3*log(abs(x^2 - 1))
```

3.664 $\int \frac{x^3 \sin^{-1}(x)}{\left(1-x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=36

$$\sqrt{1-x^2}\sin^{-1}(x) + \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - x - \tanh^{-1}(x)$$

```
[Out] -x + ArcSin[x]/Sqrt[1 - x^2] + Sqrt[1 - x^2]*ArcSin[x] - ArcTanh[x]
```

Rubi [A] time = 0.0692451, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {266, 43, 4689, 388, 206}

$$\sqrt{1-x^2}\sin^{-1}(x) + \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} - x - \tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcSin[x])/(1 - x^2)^(3/2),x]
```

```
[Out] -x + ArcSin[x]/Sqrt[1 - x^2] + Sqrt[1 - x^2]*ArcSin[x] - ArcTanh[x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 4689

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(p_)
, x_Symbol] :> With[{u = IntHide[x^m*(1 - c^2*x^2)^p, x]}, Dist[d^p*(a + b*
ArcSin[c*x]), u, x] - Dist[b*c*d^p, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^
2], x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && Intege
rQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*p + 3)/2, 0]) && NeQ[p, -
2^(-1)] && GtQ[d, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3 \sin^{-1}(x)}{\left(1-x^2\right)^{3/2}}\,dx &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}+\sqrt{1-x^2}\sin^{-1}(x)-\int \frac{2-x^2}{1-x^2}\,dx\\ &= -x+\frac{\sin^{-1}(x)}{\sqrt{1-x^2}}+\sqrt{1-x^2}\sin^{-1}(x)-\int \frac{1}{1-x^2}\,dx\\ &= -x+\frac{\sin^{-1}(x)}{\sqrt{1-x^2}}+\sqrt{1-x^2}\sin^{-1}(x)-\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.064117, size = 40, normalized size = 1.11

$$\frac{1}{2}\left(-\frac{2\left(x^2-2\right)\sin^{-1}(x)}{\sqrt{1-x^2}}-2x+\log(1-x)-\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSin[x])/(1 - x^2)^(3/2),x]

[Out] (-2*x - (2*(-2 + x^2)*ArcSin[x])/Sqrt[1 - x^2] + Log[1 - x] - Log[1 + x])/2
```

Maple [A] time = 0.219, size = 61, normalized size = 1.7

$$-x+\arcsin(x)\sqrt{-x^2+1}-\frac{\arcsin(x)}{x^2-1}\sqrt{-x^2+1}-\ln\left(\frac{1}{\sqrt{-x^2+1}}+x\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsin(x)/(-x^2+1)^(3/2),x)

[Out] -x+arcsin(x)*(-x^2+1)^(1/2)-(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)-ln(1/(-x^2+1)^
(1/2)+x/(-x^2+1)^(1/2))
```

Maxima [A] time = 1.44542, size = 61, normalized size = 1.69

$$-\left(\frac{x^2}{\sqrt{-x^2+1}}-\frac{2}{\sqrt{-x^2+1}}\right)\arcsin(x)-x-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="maxima")

[Out] -(x^2/sqrt(-x^2 + 1) - 2/sqrt(-x^2 + 1))*arcsin(x) - x - 1/2*log(x + 1) + 1
/2*log(x - 1)
```

Fricas [A] time = 2.50162, size = 155, normalized size = 4.31

$$-\frac{2\,x^3-2\,\left(x^2-2\right)\sqrt{-x^2+1}\arcsin\left(x\right)+\left(x^2-1\right)\log\left(x+1\right)-\left(x^2-1\right)\log\left(x-1\right)-2\,x}{2\,\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^3 - 2*(x^2 - 2)*sqrt(-x^2 + 1)*arcsin(x) + (x^2 - 1)*log(x + 1) -
 (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

Sympy [A] time = 15.3001, size = 37, normalized size = 1.03

$$-x-\left(-\sqrt{1-x^2}-\frac{1}{\sqrt{1-x^2}}\right)\operatorname{asin}\left(x\right)+\frac{\log\left(x-1\right)}{2}-\frac{\log\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asin(x)/(-x**2+1)**(3/2),x)
```

```
[Out] -x - (-sqrt(1 - x**2) - 1/sqrt(1 - x**2))*asin(x) + log(x - 1)/2 - log(x +
1)/2
```

Giac [A] time = 1.08882, size = 54, normalized size = 1.5

$$\left(\sqrt{-x^2+1}+\frac{1}{\sqrt{-x^2+1}}\right)\arcsin\left(x\right)-x-\frac{1}{2}\log\left(|x+1|\right)+\frac{1}{2}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsin(x)/(-x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] (sqrt(-x^2 + 1) + 1/sqrt(-x^2 + 1))*arcsin(x) - x - 1/2*log(abs(x + 1)) + 1
/2*log(abs(x - 1))
```

3.665 $\int \frac{\sin^{-1}(x)}{x(1-x^2)^{3/2}} \, dx$

Optimal. Leaf size=62

$$i\text{PolyLog}\left(2,-e^{i\sin^{-1}(x)}\right)-i\text{PolyLog}\left(2,e^{i\sin^{-1}(x)}\right)+\frac{\sin^{-1}(x)}{\sqrt{1-x^2}}-\tanh^{-1}(x)-2\sin^{-1}(x)\tanh^{-1}\left(e^{i\sin^{-1}(x)}\right)$$

```
[Out] ArcSin[x]/Sqrt[1 - x^2] - 2*ArcSin[x]*ArcTanh[E^(I*ArcSin[x])] - ArcTanh[x]
 + I*PolyLog[2, -E^(I*ArcSin[x])] - I*PolyLog[2, E^(I*ArcSin[x])]
```

Rubi [A] time = 0.11596, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {4705, 4709, 4183, 2279, 2391, 206}

$$i\text{PolyLog}\left(2,-e^{i\sin^{-1}(x)}\right)-i\text{PolyLog}\left(2,e^{i\sin^{-1}(x)}\right)+\frac{\sin^{-1}(x)}{\sqrt{1-x^2}}-\tanh^{-1}(x)-2\sin^{-1}(x)\tanh^{-1}\left(e^{i\sin^{-1}(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]/(x*(1 - x^2)^(3/2)),x]
```

```
[Out] ArcSin[x]/Sqrt[1 - x^2] - 2*ArcSin[x]*ArcTanh[E^(I*ArcSin[x])] - ArcTanh[x]
 + I*PolyLog[2, -E^(I*ArcSin[x])] - I*PolyLog[2, E^(I*ArcSin[x])]
```

Rule 4705

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcSin[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1)),
Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n, x], x] + Dist[(b*c*n
*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 - c^2*x^2)^FracPart[
p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 4709

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[d]), Subst[Int[(a + b*x)^n*Sin
[x]^m, x], x, ArcSin[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d +
e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^{-1}(x)}{x\left(1-x^2\right)^{3/2}}\,dx &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}-\int \frac{1}{1-x^2}\,dx+\int \frac{\sin^{-1}(x)}{x\sqrt{1-x^2}}\,dx\\ &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}-\tanh^{-1}(x)+\text{Subst}\left(\int x\csc(x)\,dx,x,\sin^{-1}(x)\right)\\ &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}-2\sin^{-1}(x)\tanh^{-1}\left(e^{i\sin^{-1}(x)}\right)-\tanh^{-1}(x)-\text{Subst}\left(\int \log\left(1-e^{ix}\right)\,dx,x,\sin^{-1}(x)\right)+\text{Su}\\ &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}-2\sin^{-1}(x)\tanh^{-1}\left(e^{i\sin^{-1}(x)}\right)-\tanh^{-1}(x)+i\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{i\sin^{-1}(x)}\right)-i\,\text{Su}\\ &= \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}-2\sin^{-1}(x)\tanh^{-1}\left(e^{i\sin^{-1}(x)}\right)-\tanh^{-1}(x)+i\text{Li}_2\left(-e^{i\sin^{-1}(x)}\right)-i\text{Li}_2\left(e^{i\sin^{-1}(x)}\right)\end{aligned}$$

Mathematica [A] time = 0.180873, size = 112, normalized size = 1.81

$$i\text{PolyLog}\left(2,-e^{i\sin^{-1}(x)}\right)-i\text{PolyLog}\left(2,e^{i\sin^{-1}(x)}\right)+\frac{\sin^{-1}(x)}{\sqrt{1-x^2}}+\sin^{-1}(x)\log\left(1-e^{i\sin^{-1}(x)}\right)-\sin^{-1}(x)\log\left(1+e^{i\sin^{-1}(x)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSin[x]/(x*(1 - x^2)^(3/2)),x]
```

```
[Out] ArcSin[x]/Sqrt[1 - x^2] + ArcSin[x]*Log[1 - E^(I*ArcSin[x])] - ArcSin[x]*Lo
g[1 + E^(I*ArcSin[x])] + Log[Cos[ArcSin[x]/2] - Sin[ArcSin[x]/2]] - Log[Cos
[ArcSin[x]/2] + Sin[ArcSin[x]/2]] + I*PolyLog[2, -E^(I*ArcSin[x])] - I*Poly
Log[2, E^(I*ArcSin[x])]
```

Maple [A] time = 0.159, size = 97, normalized size = 1.6

$$-\frac{\arcsin(x)}{x^2-1}\sqrt{-x^2+1}+2\,i\arctan\left(ix+\sqrt{-x^2+1}\right)+i\text{dilog}\left(ix+\sqrt{-x^2+1}+1\right)-\arcsin(x)\ln\left(ix+\sqrt{-x^2+1}+1\right)+i\text{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)/x/(-x^2+1)^(3/2),x)
```

```
[Out] -(-x^2+1)^(1/2)/(x^2-1)*arcsin(x)+2*I*arctan(I*x+(-x^2+1)^(1/2))+I*dilog(I*
x+(-x^2+1)^(1/2)+1)-arcsin(x)*ln(I*x+(-x^2+1)^(1/2)+1)+I*dilog(I*x+(-x^2+1)
```

```
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arcsin(x)}{\left(-x^2+1\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x/(-x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsin(x)/((-x^2 + 1)^(3/2)*x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{-x^2+1}\arcsin(x)}{x^5-2x^3+x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x/(-x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)*arcsin(x)/(x^5 - 2*x^3 + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asin}(x)}{x\left(-(x-1)(x+1)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)/x/(-x**2+1)**(3/2),x)
```

```
[Out] Integral(asin(x)/(x*(-(x - 1)*(x + 1))**(3/2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arcsin(x)}{\left(-x^2+1\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x/(-x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arcsin(x)/((-x^2 + 1)^(3/2)*x), x)
```

3.666 $\int \frac{\cos^{-1}(x)}{x^4\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=54

$$\frac{1}{6x^2}-\frac{2\sqrt{1-x^2}\cos^{-1}(x)}{3x}-\frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x^3}-\frac{2\log(x)}{3}$$

```
[Out] 1/(6*x^2) - (Sqrt[1 - x^2]*ArcCos[x])/(3*x^3) - (2*Sqrt[1 - x^2]*ArcCos[x])
/(3*x) - (2*Log[x])/3
```

Rubi [A] time = 0.0909131, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4702, 4682, 29, 30}

$$\frac{1}{6x^2}-\frac{2\sqrt{1-x^2}\cos^{-1}(x)}{3x}-\frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x^3}-\frac{2\log(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[x]/(x^4*Sqrt[1 - x^2]),x]
```

```
[Out] 1/(6*x^2) - (Sqrt[1 - x^2]*ArcCos[x])/(3*x^3) - (2*Sqrt[1 - x^2]*ArcCos[x])
/(3*x) - (2*Log[x])/3
```

Rule 4702

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart
[p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 4682

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^F
racPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(f*x)^(m + 1)*(1 - c
^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
 e, f, m, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] &
& NeQ[m, -1]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\cos^{-1}(x)}{x^4\sqrt{1-x^2}}\,dx = -\frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x^3} - \frac{1}{3}\int \frac{1}{x^3}\,dx + \frac{2}{3}\int \frac{\cos^{-1}(x)}{x^2\sqrt{1-x^2}}\,dx$$
$$= \frac{1}{6x^2} - \frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x^3} - \frac{2\sqrt{1-x^2}\cos^{-1}(x)}{3x} - \frac{2}{3}\int \frac{1}{x}\,dx$$
$$= \frac{1}{6x^2} - \frac{\sqrt{1-x^2}\cos^{-1}(x)}{3x^3} - \frac{2\sqrt{1-x^2}\cos^{-1}(x)}{3x} - \frac{2\log(x)}{3}$$

Mathematica [A] time = 0.0430401, size = 38, normalized size = 0.7

$$\frac{-4x^3\log(x) - 2\sqrt{1-x^2}\left(2x^2+1\right)\cos^{-1}(x) + x}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[x]/(x^4*Sqrt[1 - x^2]),x]
```

```
[Out] (x - 2*Sqrt[1 - x^2]*(1 + 2*x^2)*ArcCos[x] - 4*x^3*Log[x])/(6*x^3)
```

Maple [A] time = 0.043, size = 43, normalized size = 0.8

$$\frac{1}{6\,x^2} - \frac{2\,\ln(x)}{3} - \frac{\arccos(x)}{3\,x^3}\sqrt{-x^2+1} - \frac{2\,\arccos(x)}{3\,x}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x)/x^4/(-x^2+1)^(1/2),x)
```

```
[Out] 1/6/x^2-2/3*ln(x)-1/3*arccos(x)*(-x^2+1)^(1/2)/x^3-2/3*arccos(x)*(-x^2+1)^(
1/2)/x
```

Maxima [A] time = 1.4098, size = 57, normalized size = 1.06

$$-\frac{1}{3}\left(\frac{2\sqrt{-x^2+1}}{x} + \frac{\sqrt{-x^2+1}}{x^3}\right)\arccos(x) + \frac{1}{6\,x^2} - \frac{2}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(2*sqrt(-x^2 + 1)/x + sqrt(-x^2 + 1)/x^3)*arccos(x) + 1/6/x^2 - 2/3*lo
g(x)
```

Fricas [A] time = 2.69828, size = 95, normalized size = 1.76

$$-\frac{4\,x^3\log(x) + 2\left(2\,x^2+1\right)\sqrt{-x^2+1}\arccos(x) - x}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/6*(4*x^3*log(x) + 2*(2*x^2 + 1)*sqrt(-x^2 + 1)*arccos(x) - x)/x^3
```

Sympy [A] time = 114.909, size = 60, normalized size = 1.11

$$\left(\begin{cases} -\frac{\sqrt{1-x^2}}{x} - \frac{\left(1-x^2\right)^{\frac{3}{2}}}{3x^3} & \text{for}\ x > -1 \wedge x < 1 \end{cases}\right)\operatorname{acos}(x) + \begin{cases} \text{NaN} & \text{for}\ x < -1 \\ -\frac{2\log(x)}{3} - \frac{1}{6} + \frac{2i\pi}{3} + \frac{1}{6x^2} & \text{for}\ x < 1 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x)/x**4/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-sqrt(1 - x**2)/x - (1 - x**2)**(3/2)/(3*x**3), (x > -1) & (x <
1)))*acos(x) + Piecewise((nan, x < -1), (-2*log(x)/3 - 1/6 + 2*I*pi/3 + 1/(
6*x**2), x < 1), (nan, True))
```

Giac [B] time = 1.10374, size = 128, normalized size = 2.37

$$\frac{1}{24}\left(\frac{x^3\left(\frac{9\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+1\right)}{\left(\sqrt{-x^2+1}-1\right)^3} - \frac{9\left(\sqrt{-x^2+1}-1\right)}{x} - \frac{\left(\sqrt{-x^2+1}-1\right)^3}{x^3}\right)\arccos(x) + \frac{2x^2+1}{6x^2} - \frac{1}{3}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x)/x^4/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/24*(x^3*(9*(sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1)^3 - 9*(sq
rt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^3/x^3)*arccos(x) + 1/6*(2*x^2 +
1)/x^2 - 1/3*log(x^2)
```

3.667 $\int x\sqrt{1-x^2}\cos^{-1}(x)^2\,dx$

Optimal. Leaf size=66

$$\frac{2}{27}\left(1-x^2\right)^{3/2}+\frac{4\sqrt{1-x^2}}{9}+\frac{2}{9}x^3\cos^{-1}(x)-\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2-\frac{2}{3}x\cos^{-1}(x)$$

```
[Out] (4*Sqrt[1 - x^2])/9 + (2*(1 - x^2)^(3/2))/27 - (2*x*ArcCos[x])/3 + (2*x^3*A
rcCos[x])/9 - ((1 - x^2)^(3/2)*ArcCos[x]^2)/3
```

Rubi [A] time = 0.0717082, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {4678, 4646, 444, 43}

$$\frac{2}{27}\left(1-x^2\right)^{3/2}+\frac{4\sqrt{1-x^2}}{9}+\frac{2}{9}x^3\cos^{-1}(x)-\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2-\frac{2}{3}x\cos^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 - x^2]*ArcCos[x]^2,x]
```

```
[Out] (4*Sqrt[1 - x^2])/9 + (2*(1 - x^2)^(3/2))/27 - (2*x*ArcCos[x])/3 + (2*x^3*A
rcCos[x])/9 - ((1 - x^2)^(3/2)*ArcCos[x]^2)/3
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4646

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbo
l] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCos[c*x], u, x] +
 Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 - c^2*x^2], x], x], x]] /; FreeQ[
{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\sqrt{1-x^2}\cos^{-1}(x)^2\,dx = -\frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2 - \frac{2}{3}\int\left(1-x^2\right)\cos^{-1}(x)\,dx$$

$$= -\frac{2}{3}x\cos^{-1}(x) + \frac{2}{9}x^3\cos^{-1}(x) - \frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2 - \frac{2}{3}\int\frac{x\left(1-\frac{x^2}{3}\right)}{\sqrt{1-x^2}}dx$$

$$= -\frac{2}{3}x\cos^{-1}(x) + \frac{2}{9}x^3\cos^{-1}(x) - \frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2 - \frac{1}{3}\text{Subst}\left(\int\frac{1-\frac{x}{3}}{\sqrt{1-x}}dx, x, x^2\right)$$

$$= -\frac{2}{3}x\cos^{-1}(x) + \frac{2}{9}x^3\cos^{-1}(x) - \frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2 - \frac{1}{3}\text{Subst}\left(\int\left(\frac{2}{3\sqrt{1-x}} + \frac{\sqrt{1-x}}{3}\right) d\right.$$

$$= \frac{4\sqrt{1-x^2}}{9} + \frac{2}{27}\left(1-x^2\right)^{3/2} - \frac{2}{3}x\cos^{-1}(x) + \frac{2}{9}x^3\cos^{-1}(x) - \frac{1}{3}\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2$$

Mathematica [A] time = 0.0531804, size = 50, normalized size = 0.76

$$\frac{1}{27}\left(-2\sqrt{1-x^2}\left(x^2-7\right) - 9\left(1-x^2\right)^{3/2}\cos^{-1}(x)^2 + 6x\left(x^2-3\right)\cos^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 - x^2]*ArcCos[x]^2,x]
```

```
[Out] (-2*Sqrt[1 - x^2]*(-7 + x^2) + 6*x*(-3 + x^2)*ArcCos[x] - 9*(1 - x^2)^(3/2)
*ArcCos[x]^2)/27
```

Maple [C] time = 0.14, size = 158, normalized size = 2.4

$$-\frac{6\,i\arccos(x) + 9\,(\arccos(x))^2 - 2}{216}\left(4\,ix^3 - 4\sqrt{-x^2+1}x^2 - 3\,ix + \sqrt{-x^2+1}\right) + \frac{(\arccos(x))^2 - 2 + 2\,i\arccos(x)}{8}\left(ix -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccos(x)^2*(-x^2+1)^(1/2),x)
```

```
[Out] -1/216*(6*I*arccos(x)+9*arccos(x)^2-2)*(4*I*x^3-4*(-x^2+1)^(1/2)*x^2-3*I*x+
(-x^2+1)^(1/2))+1/8*(arccos(x)^2-2+2*I*arccos(x))*(I*x-(-x^2+1)^(1/2))-1/8*
(arccos(x)^2-2-2*I*arccos(x))*(I*x+(-x^2+1)^(1/2))+1/216*(-6*I*arccos(x)+9*
arccos(x)^2-2)*(4*I*x^3+4*(-x^2+1)^(1/2)*x^2-3*I*x-(-x^2+1)^(1/2))
```

Maxima [A] time = 1.45235, size = 70, normalized size = 1.06

$$-\frac{1}{3}\left(-x^2+1\right)^{\frac{3}{2}}\arccos(x)^2 - \frac{2}{27}\sqrt{-x^2+1}x^2 + \frac{2}{9}\left(x^3 - 3\,x\right)\arccos(x) + \frac{14}{27}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 1)^(3/2)*arccos(x)^2 - 2/27*sqrt(-x^2 + 1)*x^2 + 2/9*(x^3 - 3*
x)*arccos(x) + 14/27*sqrt(-x^2 + 1)
```

Fricas [A] time = 2.52164, size = 119, normalized size = 1.8

$$\frac{2}{9}\left(x^{3}-3\,x\right)\arccos\left(x\right)+\frac{1}{27}\left(9\left(x^{2}-1\right)\arccos\left(x\right)^{2}-2\,x^{2}+14\right)\sqrt{-x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(x^3 - 3*x)*arccos(x) + 1/27*(9*(x^2 - 1)*arccos(x)^2 - 2*x^2 + 14)*sqr
t(-x^2 + 1)
```

Sympy [A] time = 2.94496, size = 78, normalized size = 1.18

$$\frac{2x^{3}\operatorname{acos}\left(x\right)}{9}+\frac{x^{2}\sqrt{1-x^{2}}\operatorname{acos}^{2}\left(x\right)}{3}-\frac{2x^{2}\sqrt{1-x^{2}}}{27}-\frac{2x\operatorname{acos}\left(x\right)}{3}-\frac{\sqrt{1-x^{2}}\operatorname{acos}^{2}\left(x\right)}{3}+\frac{14\sqrt{1-x^{2}}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acos(x)**2*(-x**2+1)**(1/2),x)
```

```
[Out] 2*x**3*acos(x)/9 + x**2*sqrt(1 - x**2)*acos(x)**2/3 - 2*x**2*sqrt(1 - x**2)
/27 - 2*x*acos(x)/3 - sqrt(1 - x**2)*acos(x)**2/3 + 14*sqrt(1 - x**2)/27
```

Giac [A] time = 1.11153, size = 72, normalized size = 1.09

$$\frac{2}{9}\,x^{3}\arccos\left(x\right)-\frac{1}{3}\left(-x^{2}+1\right)^{\frac{3}{2}}\arccos\left(x\right)^{2}-\frac{2}{27}\,\sqrt{-x^{2}+1}x^{2}-\frac{2}{3}\,x\arccos\left(x\right)+\frac{14}{27}\,\sqrt{-x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccos(x)^2*(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*x^3*arccos(x) - 1/3*(-x^2 + 1)^(3/2)*arccos(x)^2 - 2/27*sqrt(-x^2 + 1)*
x^2 - 2/3*x*arccos(x) + 14/27*sqrt(-x^2 + 1)
```

3.668 $\int \frac{x^2 \sin^{-1}(x)^3}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=73

$$-\frac{3x^2}{8} - \frac{1}{2}x\sqrt{1-x^2}\sin^{-1}(x)^3 + \frac{3}{4}x^2\sin^{-1}(x)^2 + \frac{3}{4}x\sqrt{1-x^2}\sin^{-1}(x) + \frac{1}{8}\sin^{-1}(x)^4 - \frac{3}{8}\sin^{-1}(x)^2$$

```
[Out] (-3*x^2)/8 + (3*x*Sqrt[1 - x^2]*ArcSin[x])/4 - (3*ArcSin[x]^2)/8 + (3*x^2*A
rcSin[x]^2)/4 - (x*Sqrt[1 - x^2]*ArcSin[x]^3)/2 + ArcSin[x]^4/8
```

Rubi [A] time = 0.154527, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {4707, 4641, 4627, 30}

$$-\frac{3x^2}{8} - \frac{1}{2}x\sqrt{1-x^2}\sin^{-1}(x)^3 + \frac{3}{4}x^2\sin^{-1}(x)^2 + \frac{3}{4}x\sqrt{1-x^2}\sin^{-1}(x) + \frac{1}{8}\sin^{-1}(x)^4 - \frac{3}{8}\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcSin[x]^3)/Sqrt[1 - x^2],x]
```

```
[Out] (-3*x^2)/8 + (3*x*Sqrt[1 - x^2]*ArcSin[x])/4 - (3*ArcSin[x]^2)/8 + (3*x^2*A
rcSin[x]^2)/4 - (x*Sqrt[1 - x^2]*ArcSin[x]^3)/2 + ArcSin[x]^4/8
```

Rule 4707

```
Int[(((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*(a + b*
ArcSin[c*x])^n)/(e*m), x] + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)
*(a + b*ArcSin[c*x])^n)/Sqrt[d + e*x^2], x], x] + Dist[(b*f*n*Sqrt[1 - c^2*
x^2])/(c*m*Sqrt[d + e*x^2]), Int[(f*x)^(m - 1)*(a + b*ArcSin[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0]
&& GtQ[m, 1] && IntegerQ[m]
```

Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{x^2 \sin^{-1}(x)^3}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}x\sqrt{1-x^2}\sin^{-1}(x)^3 + \frac{1}{2}\int \frac{\sin^{-1}(x)^3}{\sqrt{1-x^2}}\,dx + \frac{3}{2}\int x\sin^{-1}(x)^2\,dx$$
$$= \frac{3}{4}x^2\sin^{-1}(x)^2 - \frac{1}{2}x\sqrt{1-x^2}\sin^{-1}(x)^3 + \frac{1}{8}\sin^{-1}(x)^4 - \frac{3}{2}\int \frac{x^2\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx$$
$$= \frac{3}{4}x\sqrt{1-x^2}\sin^{-1}(x) + \frac{3}{4}x^2\sin^{-1}(x)^2 - \frac{1}{2}x\sqrt{1-x^2}\sin^{-1}(x)^3 + \frac{1}{8}\sin^{-1}(x)^4 - \frac{3\int x\,dx}{4} - \frac{3}{4}\int \frac{\sin^{-1}}{\sqrt{1}}$$
$$= -\frac{3x^2}{8} + \frac{3}{4}x\sqrt{1-x^2}\sin^{-1}(x) - \frac{3}{8}\sin^{-1}(x)^2 + \frac{3}{4}x^2\sin^{-1}(x)^2 - \frac{1}{2}x\sqrt{1-x^2}\sin^{-1}(x)^3 + \frac{1}{8}\sin^{-1}(x)^4$$

Mathematica [A] time = 0.032461, size = 60, normalized size = 0.82

$$\frac{1}{8}\left(-3x^2 - 4x\sqrt{1-x^2}\sin^{-1}(x)^3 + \left(6x^2-3\right)\sin^{-1}(x)^2 + 6x\sqrt{1-x^2}\sin^{-1}(x) + \sin^{-1}(x)^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSin[x]^3)/Sqrt[1 - x^2],x]
```

```
[Out] (-3*x^2 + 6*x*Sqrt[1 - x^2]*ArcSin[x] + (-3 + 6*x^2)*ArcSin[x]^2 - 4*x*Sqrt
[1 - x^2]*ArcSin[x]^3 + ArcSin[x]^4)/8
```

Maple [A] time = 0.053, size = 69, normalized size = 1.

$$\frac{(\arcsin(x))^3}{2}\left(-x\sqrt{-x^2+1} + \arcsin(x)\right) + \frac{3\,(\arcsin(x))^2\left(x^2-1\right)}{4} + \frac{3\arcsin(x)}{4}\left(x\sqrt{-x^2+1} + \arcsin(x)\right) - \frac{3\,(\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsin(x)^3*(-x*(-x^2+1)^(1/2)+arcsin(x))+3/4*arcsin(x)^2*(x^2-1)+3/4*a
rcsin(x)*(x*(-x^2+1)^(1/2)+arcsin(x))-3/8*arcsin(x)^2-3/8*x^2-3/8*arcsin(x)
^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\arcsin(x)^3}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*arcsin(x)^3/sqrt(-x^2 + 1), x)
```

Fricas [A] time = 2.45903, size = 151, normalized size = 2.07

$$\frac{1}{8}\arcsin(x)^4 + \frac{3}{8}\left(2x^2-1\right)\arcsin(x)^2 - \frac{3}{8}x^2 - \frac{1}{4}\left(2x\arcsin(x)^3 - 3x\arcsin(x)\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*arcsin(x)^4 + 3/8*(2*x^2 - 1)*arcsin(x)^2 - 3/8*x^2 - 1/4*(2*x*arcsin(x
)^3 - 3*x*arcsin(x))*sqrt(-x^2 + 1)
```

Sympy [A] time = 1.27156, size = 66, normalized size = 0.9

$$\frac{3x^2\operatorname{asin}^2(x)}{4}-\frac{3x^2}{8}-\frac{x\sqrt{1-x^2}\operatorname{asin}^3(x)}{2}+\frac{3x\sqrt{1-x^2}\operatorname{asin}(x)}{4}+\frac{\operatorname{asin}^4(x)}{8}-\frac{3\operatorname{asin}^2(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asin(x)**3/(-x**2+1)**(1/2),x)
```

```
[Out] 3*x**2*asin(x)**2/4 - 3*x**2/8 - x*sqrt(1 - x**2)*asin(x)**3/2 + 3*x*sqrt(1
 - x**2)*asin(x)/4 + asin(x)**4/8 - 3*asin(x)**2/8
```

Giac [A] time = 1.11686, size = 81, normalized size = 1.11

$$-\frac{1}{2}\sqrt{-x^2+1}x\arcsin(x)^3+\frac{1}{8}\arcsin(x)^4+\frac{3}{4}\left(x^2-1\right)\arcsin(x)^2+\frac{3}{4}\sqrt{-x^2+1}x\arcsin(x)-\frac{3}{8}x^2+\frac{3}{8}\arcsin(x)^2+\frac{3}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x)^3/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 1)*x*arcsin(x)^3 + 1/8*arcsin(x)^4 + 3/4*(x^2 - 1)*arcsin(
x)^2 + 3/4*sqrt(-x^2 + 1)*x*arcsin(x) - 3/8*x^2 + 3/8*arcsin(x)^2 + 3/16
```

3.669 $\int \frac{x \tan^{-1}(x)}{(1+x^2)^2} \, dx$

Optimal. Leaf size=32

$$\frac{x}{4(x^2+1)} - \frac{\tan^{-1}(x)}{2(x^2+1)} + \frac{1}{4}\tan^{-1}(x)$$

```
[Out] x/(4*(1 + x^2)) + ArcTan[x]/4 - ArcTan[x]/(2*(1 + x^2))
```

Rubi [A] time = 0.0259001, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4930, 199, 203}

$$\frac{x}{4(x^2+1)} - \frac{\tan^{-1}(x)}{2(x^2+1)} + \frac{1}{4}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[x])/(1 + x^2)^2,x]

[Out] x/(4*(1 + x^2)) + ArcTan[x]/4 - ArcTan[x]/(2*(1 + x^2))
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x \tan^{-1}(x)}{(1+x^2)^2} \, dx &= -\frac{\tan^{-1}(x)}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{(1+x^2)^2} \, dx \\
&= \frac{x}{4(1+x^2)} - \frac{\tan^{-1}(x)}{2(1+x^2)} + \frac{1}{4}\int \frac{1}{1+x^2} \, dx \\
&= \frac{x}{4(1+x^2)} + \frac{1}{4}\tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(1+x^2)}
\end{aligned}$$

Mathematica [A] time = 0.0199502, size = 21, normalized size = 0.66

$$\frac{(x^2-1)\tan^{-1}(x)+x}{4(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] (x + (-1 + x^2)*ArcTan[x])/(4*(1 + x^2))
```

Maple [A] time = 0.004, size = 27, normalized size = 0.8

$$\frac{x}{4x^2+4}+\frac{\arctan(x)}{4}-\frac{\arctan(x)}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x)/(x^2+1)^2,x)
```

```
[Out] 1/4*x/(x^2+1)+1/4*arctan(x)-1/2*arctan(x)/(x^2+1)
```

Maxima [A] time = 1.40762, size = 35, normalized size = 1.09

$$\frac{x}{4(x^2+1)}-\frac{\arctan(x)}{2(x^2+1)}+\frac{1}{4}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x/(x^2 + 1) - 1/2*arctan(x)/(x^2 + 1) + 1/4*arctan(x)
```

Fricas [A] time = 2.36349, size = 55, normalized size = 1.72

$$\frac{(x^2-1)\arctan(x)+x}{4(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 - 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A] time = 0.687117, size = 31, normalized size = 0.97

$$\frac{x^2\,\mathrm{atan}(x)}{4x^2+4}+\frac{x}{4x^2+4}-\frac{\mathrm{atan}(x)}{4x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x)/(x**2+1)**2,x)
```

```
[Out] x**2*atan(x)/(4*x**2 + 4) + x/(4*x**2 + 4) - atan(x)/(4*x**2 + 4)
```

Giac [A] time = 1.06447, size = 35, normalized size = 1.09

$$\frac{x}{4\left(x^2+1\right)}-\frac{\arctan\left(x\right)}{2\left(x^2+1\right)}+\frac{1}{4}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/4*x/(x^2 + 1) - 1/2*arctan(x)/(x^2 + 1) + 1/4*arctan(x)
```

3.670 $\int \frac{x\tan^{-1}(x)}{\left(1+x^2\right)^3}\,dx$

Optimal. Leaf size=44

$$\frac{3x}{32\left(x^2+1\right)}+\frac{x}{16\left(x^2+1\right)^2}-\frac{\tan^{-1}(x)}{4\left(x^2+1\right)^2}+\frac{3}{32}\tan^{-1}(x)$$

```
[Out] x/(16*(1 + x^2)^2) + (3*x)/(32*(1 + x^2)) + (3*ArcTan[x])/32 - ArcTan[x]/(4
*(1 + x^2)^2)
```

Rubi [A] time = 0.0289532, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4930, 199, 203}

$$\frac{3x}{32\left(x^2+1\right)}+\frac{x}{16\left(x^2+1\right)^2}-\frac{\tan^{-1}(x)}{4\left(x^2+1\right)^2}+\frac{3}{32}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[x])/(1 + x^2)^3,x]
```

```
[Out] x/(16*(1 + x^2)^2) + (3*x)/(32*(1 + x^2)) + (3*ArcTan[x])/32 - ArcTan[x]/(4
*(1 + x^2)^2)
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\tan^{-1}(x)}{\left(1+x^2\right)^3}\,dx &= -\frac{\tan^{-1}(x)}{4\left(1+x^2\right)^2}+\frac{1}{4}\int \frac{1}{\left(1+x^2\right)^3}\,dx \\
&= \frac{x}{16\left(1+x^2\right)^2}-\frac{\tan^{-1}(x)}{4\left(1+x^2\right)^2}+\frac{3}{16}\int \frac{1}{\left(1+x^2\right)^2}\,dx \\
&= \frac{x}{16\left(1+x^2\right)^2}+\frac{3x}{32\left(1+x^2\right)}-\frac{\tan^{-1}(x)}{4\left(1+x^2\right)^2}+\frac{3}{32}\int \frac{1}{1+x^2}\,dx \\
&= \frac{x}{16\left(1+x^2\right)^2}+\frac{3x}{32\left(1+x^2\right)}+\frac{3}{32}\tan^{-1}(x)-\frac{\tan^{-1}(x)}{4\left(1+x^2\right)^2}
\end{aligned}
$$

Mathematica [A] time = 0.0204355, size = 36, normalized size = 0.82

$$\frac{x\left(3x^2+5\right)+\left(3x^4+6x^2-5\right)\tan^{-1}(x)}{32\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[x])/(1 + x^2)^3,x]
```

```
[Out] (x*(5 + 3*x^2) + (-5 + 6*x^2 + 3*x^4)*ArcTan[x])/(32*(1 + x^2)^2)
```

Maple [A] time = 0.005, size = 37, normalized size = 0.8

$$\frac{x}{16\left(x^2+1\right)^2}+\frac{3x}{32x^2+32}+\frac{3\arctan(x)}{32}-\frac{\arctan(x)}{4\left(x^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x)/(x^2+1)^3,x)
```

```
[Out] 1/16*x/(x^2+1)^2+3/32*x/(x^2+1)+3/32*arctan(x)-1/4*arctan(x)/(x^2+1)^2
```

Maxima [A] time = 1.3996, size = 53, normalized size = 1.2

$$\frac{3x^3+5x}{32\left(x^4+2x^2+1\right)}-\frac{\arctan(x)}{4\left(x^2+1\right)^2}+\frac{3}{32}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(3*x^3 + 5*x)/(x^4 + 2*x^2 + 1) - 1/4*arctan(x)/(x^2 + 1)^2 + 3/32*arc
tan(x)
```

Fricas [A] time = 2.43621, size = 95, normalized size = 2.16

$$\frac{3\,x^3+\left(3\,x^4+6\,x^2-5\right)\arctan\left(x\right)+5\,x}{32\,\left(x^4+2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(3*x^3 + (3*x^4 + 6*x^2 - 5)*arctan(x) + 5*x)/(x^4 + 2*x^2 + 1)
```

Sympy [B] time = 1.14173, size = 88, normalized size = 2.

$$\frac{3x^4\operatorname{atan}\left(x\right)}{32x^4+64x^2+32}+\frac{3x^3}{32x^4+64x^2+32}+\frac{6x^2\operatorname{atan}\left(x\right)}{32x^4+64x^2+32}+\frac{5x}{32x^4+64x^2+32}-\frac{5\operatorname{atan}\left(x\right)}{32x^4+64x^2+32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x)/(x**2+1)**3,x)
```

```
[Out] 3*x**4*atan(x)/(32*x**4 + 64*x**2 + 32) + 3*x**3/(32*x**4 + 64*x**2 + 32) +
 6*x**2*atan(x)/(32*x**4 + 64*x**2 + 32) + 5*x/(32*x**4 + 64*x**2 + 32) - 5
*atan(x)/(32*x**4 + 64*x**2 + 32)
```

Giac [A] time = 1.06281, size = 46, normalized size = 1.05

$$\frac{3\,x^3+5\,x}{32\,\left(x^2+1\right)^2}-\frac{\arctan\left(x\right)}{4\,\left(x^2+1\right)^2}+\frac{3}{32}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] 1/32*(3*x^3 + 5*x)/(x^2 + 1)^2 - 1/4*arctan(x)/(x^2 + 1)^2 + 3/32*arctan(x)
```

3.671 $\int \frac{x^2 \tan^{-1}(x)}{1+x^2} \, dx$

Optimal. Leaf size=23

$$-\frac{1}{2} \log \left(x^2+1\right)-\frac{1}{2} \tan^{-1}(x)^2+x \tan^{-1}(x)$$

```
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Rubi [A] time = 0.0484445, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {4916, 4846, 260, 4884}

$$-\frac{1}{2} \log \left(x^2+1\right)-\frac{1}{2} \tan^{-1}(x)^2+x \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[x])/(1 + x^2),x]
```

```
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 \tan^{-1}(x)}{1+x^2} \, dx &= \int \tan^{-1}(x) \, dx-\int \frac{\tan^{-1}(x)}{1+x^2} \, dx \\
&= x \tan^{-1}(x)-\frac{1}{2} \tan^{-1}(x)^2-\int \frac{x}{1+x^2} \, dx \\
&= x \tan^{-1}(x)-\frac{1}{2} \tan^{-1}(x)^2-\frac{1}{2} \log \left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.015335, size = 23, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{1}{2}\tan^{-1}(x)^2+x\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[x])/(1 + x^2),x]
```

```
[Out] x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2
```

Maple [A] time = 0.009, size = 20, normalized size = 0.9

$$x\arctan(x)-\frac{(\arctan(x))^2}{2}-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x)/(x^2+1),x)
```

```
[Out] x*arctan(x)-1/2*arctan(x)^2-1/2*ln(x^2+1)
```

Maxima [A] time = 1.41145, size = 32, normalized size = 1.39

$$(x-\arctan(x))\arctan(x)+\frac{1}{2}\arctan(x)^2-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] (x - arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

Fricas [A] time = 2.40857, size = 68, normalized size = 2.96

$$x\arctan(x)-\frac{1}{2}\arctan(x)^2-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

Sympy [A] time = 0.385271, size = 19, normalized size = 0.83

$$x\operatorname{atan}(x)-\frac{\log\left(x^2+1\right)}{2}-\frac{\operatorname{atan}^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x)/(x**2+1),x)
```

```
[Out] x*atan(x) - log(x**2 + 1)/2 - atan(x)**2/2
```

Giac [A] time = 1.08159, size = 26, normalized size = 1.13

$$x \arctan(x) - \frac{1}{2} \arctan(x)^2 - \frac{1}{2} \log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1),x, algorithm="giac")
```

```
[Out] x*arctan(x) - 1/2*arctan(x)^2 - 1/2*log(x^2 + 1)
```

3.672 $\int \frac{x^3 \tan^{-1}(x)}{1+x^2} dx$

Optimal. Leaf size=67

$$\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)+\frac{1}{2}x^2\tan^{-1}(x)-\frac{x}{2}+\frac{1}{2}i\tan^{-1}(x)^2+\frac{1}{2}\tan^{-1}(x)+\log\left(\frac{2}{1+ix}\right)\tan^{-1}(x)$$

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2 + (I/2)*ArcTan[x]^2 + ArcTan[x]*Log[
2/(1 + I*x)] + (I/2)*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rubi [A] time = 0.0937433, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {4916, 4852, 321, 203, 4920, 4854, 2402, 2315}

$$\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)+\frac{1}{2}x^2\tan^{-1}(x)-\frac{x}{2}+\frac{1}{2}i\tan^{-1}(x)^2+\frac{1}{2}\tan^{-1}(x)+\log\left(\frac{2}{1+ix}\right)\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[x])/(1 + x^2),x]
```

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2 + (I/2)*ArcTan[x]^2 + ArcTan[x]*Log[
2/(1 + I*x)] + (I/2)*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
```

```
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 \tan^{-1}(x)}{1+x^2} \, dx &= \int x \tan^{-1}(x) \, dx - \int \frac{x \tan^{-1}(x)}{1+x^2} \, dx \\
&= \frac{1}{2} x^2 \tan^{-1}(x) + \frac{1}{2} i \tan^{-1}(x)^2 - \frac{1}{2} \int \frac{x^2}{1+x^2} \, dx + \int \frac{\tan^{-1}(x)}{i-x} \, dx \\
&= -\frac{x}{2} + \frac{1}{2} x^2 \tan^{-1}(x) + \frac{1}{2} i \tan^{-1}(x)^2 + \tan^{-1}(x) \log\left(\frac{2}{1+ix}\right) + \frac{1}{2} \int \frac{1}{1+x^2} \, dx - \int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2} \, dx \\
&= -\frac{x}{2} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} x^2 \tan^{-1}(x) + \frac{1}{2} i \tan^{-1}(x)^2 + \tan^{-1}(x) \log\left(\frac{2}{1+ix}\right) + i \operatorname{Subst}\left(\int \frac{\log(2x)}{1-2x} \, d\right. \\
&= -\frac{x}{2} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} x^2 \tan^{-1}(x) + \frac{1}{2} i \tan^{-1}(x)^2 + \tan^{-1}(x) \log\left(\frac{2}{1+ix}\right) + \frac{1}{2} i \operatorname{Li}_2\left(1 - \frac{2}{1+ix}\right)
\end{aligned}$$

Mathematica [A] time = 0.0323023, size = 57, normalized size = 0.85

$$\frac{1}{2} \left(i \text{PolyLog}\left(2, \frac{x+i}{x-i}\right) + \left(x^2 + 2 \log\left(-\frac{2i}{x-i}\right) + 1\right) \tan^{-1}(x) - x + i \tan^{-1}(x)^2 \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[x])/(1 + x^2),x]
```

```
[Out] (-x + I*ArcTan[x]^2 + ArcTan[x]*(1 + x^2 + 2*Log[(-2*I)/(-I + x)]) + I*Poly
Log[2, (I + x)/(-I + x)])/2
```

Maple [B] time = 0.031, size = 128, normalized size = 1.9

$$\frac{x^2 \arctan(x)}{2} - \frac{\arctan(x) \ln\left(x^2+1\right)}{2} - \frac{x}{2} + \frac{\arctan(x)}{2} + \frac{i}{4} \ln\left(-\frac{i}{2}(x+i)\right) \ln(x-i) + \frac{i}{8} (\ln(x-i))^2 - \frac{i}{4} \ln(x-i) \ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(x)/(x^2+1),x)
```

```
[Out] 1/2*x^2*arctan(x)-1/2*arctan(x)*ln(x^2+1)-1/2*x+1/2*arctan(x)+1/4*I*ln(-1/2
*I*(x+I))*ln(x-I)+1/8*I*ln(x-I)^2-1/4*I*ln(x-I)*ln(x^2+1)+1/4*I*dilog(-1/2*
I*(x+I))-1/4*I*ln(1/2*I*(x-I))*ln(x+I)-1/8*I*ln(x+I)^2+1/4*I*ln(x+I)*ln(x^2
+1)-1/4*I*dilog(1/2*I*(x-I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(x)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(x)/(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^3 \arctan(x)}{x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(x)/(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{atan}(x)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(x)/(x**2+1),x)
```

```
[Out] Integral(x**3*atan(x)/(x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(x)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(x)/(x^2 + 1), x)
```

3.673 $\int \frac{x^2 \tan^{-1}(x)}{(1+x^2)^2} dx$

Optimal. Leaf size=34

$$-\frac{1}{4(x^2+1)}-\frac{x\tan^{-1}(x)}{2(x^2+1)}+\frac{1}{4}\tan^{-1}(x)^2$$

```
[Out] -1/(4*(1 + x^2)) - (x*ArcTan[x])/(2*(1 + x^2)) + ArcTan[x]^2/4
```

Rubi [A] time = 0.0470394, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4934, 4884}

$$-\frac{1}{4(x^2+1)}-\frac{x\tan^{-1}(x)}{2(x^2+1)}+\frac{1}{4}\tan^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcTan[x])/(1 + x^2)^2,x]

[Out] -1/(4*(1 + x^2)) - (x*ArcTan[x])/(2*(1 + x^2)) + ArcTan[x]^2/4
```

Rule 4934

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (-Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] +
Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*c^2*d*(q + 1)), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2\tan^{-1}(x)}{(1+x^2)^2}\,dx &= -\frac{1}{4(1+x^2)}-\frac{x\tan^{-1}(x)}{2(1+x^2)}+\frac{1}{2}\int \frac{\tan^{-1}(x)}{1+x^2}\,dx\\ &= -\frac{1}{4(1+x^2)}-\frac{x\tan^{-1}(x)}{2(1+x^2)}+\frac{1}{4}\tan^{-1}(x)^2\end{aligned}$$

Mathematica [A] time = 0.0262093, size = 28, normalized size = 0.82

$$\frac{(x^2+1)\tan^{-1}(x)^2-2x\tan^{-1}(x)-1}{4(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] (-1 - 2*x*ArcTan[x] + (1 + x^2)*ArcTan[x]^2)/(4*(1 + x^2))
```

Maple [A] time = 0.012, size = 29, normalized size = 0.9

$$-\frac{1}{4x^2+4}-\frac{x\arctan(x)}{2x^2+2}+\frac{(\arctan(x))^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x)/(x^2+1)^2,x)
```

```
[Out] -1/4/(x^2+1)-1/2*x*arctan(x)/(x^2+1)+1/4*arctan(x)^2
```

Maxima [A] time = 1.41796, size = 54, normalized size = 1.59

$$-\frac{1}{2}\left(\frac{x}{x^2+1}-\arctan(x)\right)\arctan(x)-\frac{\left(x^2+1\right)\arctan(x)^2+1}{4\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x/(x^2 + 1) - arctan(x))*arctan(x) - 1/4*((x^2 + 1)*arctan(x)^2 + 1)/
(x^2 + 1)
```

Fricas [A] time = 2.52562, size = 80, normalized size = 2.35

$$\frac{\left(x^2+1\right)\arctan(x)^2-2x\arctan(x)-1}{4\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((x^2 + 1)*arctan(x)^2 - 2*x*arctan(x) - 1)/(x^2 + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x)/(x**2+1)**2,x)
```

```
[Out] Exception raised: RecursionError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \arctan(x)}{(x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*arctan(x)/(x^2 + 1)^2, x)
```

3.674 $\int \frac{x^3 \tan^{-1}(x)}{(1+x^2)^2} \, dx$

Optimal. Leaf size=79

$$-\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)-\frac{x}{4\left(x^2+1\right)}+\frac{\tan^{-1}(x)}{2\left(x^2+1\right)}-\frac{1}{2}i\tan^{-1}(x)^2-\frac{1}{4}\tan^{-1}(x)-\log\left(\frac{2}{1+ix}\right)\tan^{-1}(x)$$

```
[Out] -x/(4*(1 + x^2)) - ArcTan[x]/4 + ArcTan[x]/(2*(1 + x^2)) - (I/2)*ArcTan[x]^
2 - ArcTan[x]*Log[2/(1 + I*x)] - (I/2)*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rubi [A] time = 0.11268, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {4964, 4920, 4854, 2402, 2315, 4930, 199, 203}

$$-\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)-\frac{x}{4\left(x^2+1\right)}+\frac{\tan^{-1}(x)}{2\left(x^2+1\right)}-\frac{1}{2}i\tan^{-1}(x)^2-\frac{1}{4}\tan^{-1}(x)-\log\left(\frac{2}{1+ix}\right)\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] -x/(4*(1 + x^2)) - ArcTan[x]/4 + ArcTan[x]/(2*(1 + x^2)) - (I/2)*ArcTan[x]^
2 - ArcTan[x]*Log[2/(1 + I*x)] - (I/2)*PolyLog[2, 1 - 2/(1 + I*x)]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3 \tan^{-1}(x)}{\left(1+x^2\right)^2}\,dx &= -\int \frac{x \tan^{-1}(x)}{\left(1+x^2\right)^2}\,dx + \int \frac{x \tan^{-1}(x)}{1+x^2}\,dx \\
&= \frac{\tan^{-1}(x)}{2\left(1+x^2\right)} - \frac{1}{2} i \tan^{-1}(x)^2 - \frac{1}{2}\int \frac{1}{\left(1+x^2\right)^2}\,dx - \int \frac{\tan^{-1}(x)}{i-x}\,dx \\
&= -\frac{x}{4\left(1+x^2\right)} + \frac{\tan^{-1}(x)}{2\left(1+x^2\right)} - \frac{1}{2} i \tan^{-1}(x)^2 - \tan^{-1}(x)\log\left(\frac{2}{1+ix}\right) - \frac{1}{4}\int \frac{1}{1+x^2}\,dx + \int \frac{\log\left(\frac{2}{1+ix}\right)}{1+x^2} \\
&= -\frac{x}{4\left(1+x^2\right)} - \frac{1}{4}\tan^{-1}(x) + \frac{\tan^{-1}(x)}{2\left(1+x^2\right)} - \frac{1}{2} i \tan^{-1}(x)^2 - \tan^{-1}(x)\log\left(\frac{2}{1+ix}\right) - i\,\text{Subst}\left(\int \frac{\log(}{1-}\right. \\
&= -\frac{x}{4\left(1+x^2\right)} - \frac{1}{4}\tan^{-1}(x) + \frac{\tan^{-1}(x)}{2\left(1+x^2\right)} - \frac{1}{2} i \tan^{-1}(x)^2 - \tan^{-1}(x)\log\left(\frac{2}{1+ix}\right) - \frac{1}{2} i \text{Li}_2\left(1-\frac{2}{1+i}\right.
\end{aligned}
$$

Mathematica [A] time = 0.0673323, size = 64, normalized size = 0.81

$$\frac{1}{2} i \text{PolyLog}\left(2,-e^{2 i \tan^{-1}(x)}\right)+\frac{1}{2} i \tan^{-1}(x)^2-\tan^{-1}(x)\log\left(1+e^{2 i \tan^{-1}(x)}\right)-\frac{1}{8}\sin\left(2\tan^{-1}(x)\right)+\frac{1}{4}\tan^{-1}(x)\cos\left(2\,\text{ta}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] (I/2)*ArcTan[x]^2 + (ArcTan[x]*Cos[2*ArcTan[x]])/4 - ArcTan[x]*Log[1 + E^((
2*I)*ArcTan[x])] + (I/2)*PolyLog[2, -E^((2*I)*ArcTan[x])] - Sin[2*ArcTan[x]
]/8
```

Maple [B] time = 0.013, size = 139, normalized size = 1.8

$$\frac{\arctan(x)\ln\left(x^2+1\right)}{2}+\frac{\arctan(x)}{2x^2+2}-\frac{x}{4x^2+4}-\frac{\arctan(x)}{4}-\frac{i}{4}\ln\left(-\frac{i}{2}(x+i)\right)\ln(x-i)-\frac{i}{8}(\ln(x-i))^2+\frac{i}{4}\ln(x-i)\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(x)/(x^2+1)^2,x)
```

```
[Out] 1/2*arctan(x)*ln(x^2+1)+1/2*arctan(x)/(x^2+1)-1/4*x/(x^2+1)-1/4*arctan(x)-1
/4*I*ln(-1/2*I*(x+I))*ln(x-I)-1/8*I*ln(x-I)^2+1/4*I*ln(x-I)*ln(x^2+1)-1/4*I
*dilog(-1/2*I*(x+I))+1/4*I*ln(1/2*I*(x-I))*ln(x+I)+1/8*I*ln(x+I)^2-1/4*I*ln
(x+I)*ln(x^2+1)+1/4*I*dilog(1/2*I*(x-I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\arctan(x)}{\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^3*arctan(x)/(x^2 + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{x^3\arctan(x)}{x^4+2x^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(x^3*arctan(x)/(x^4 + 2*x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(x)/(x**2+1)**2,x)
```

```
[Out] Exception raised: RecursionError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan(x)}{(x^2+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(x)/(x^2 + 1)^2, x)
```

3.675 $\int \frac{x^5 \tan^{-1}(x)}{(1+x^2)^2} dx$

Optimal. Leaf size=89

$$i\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)+\frac{x}{4(x^2+1)}+\frac{1}{2}x^2\tan^{-1}(x)-\frac{\tan^{-1}(x)}{2(x^2+1)}-\frac{x}{2}+i\tan^{-1}(x)^2+\frac{3}{4}\tan^{-1}(x)+2\log\left(\frac{2}{1+ix}\right)\tan^{-}$$

```
[Out] -x/2 + x/(4*(1 + x^2)) + (3*ArcTan[x])/4 + (x^2*ArcTan[x])/2 - ArcTan[x]/(2
*(1 + x^2)) + I*ArcTan[x]^2 + 2*ArcTan[x]*Log[2/(1 + I*x)] + I*PolyLog[2, 1
 - 2/(1 + I*x)]
```

Rubi [A] time = 0.234194, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 11, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {4964, 4916, 4852, 321, 203, 4920, 4854, 2402, 2315, 4930, 199}

$$i\text{PolyLog}\left(2,1-\frac{2}{1+ix}\right)+\frac{x}{4(x^2+1)}+\frac{1}{2}x^2\tan^{-1}(x)-\frac{\tan^{-1}(x)}{2(x^2+1)}-\frac{x}{2}+i\tan^{-1}(x)^2+\frac{3}{4}\tan^{-1}(x)+2\log\left(\frac{2}{1+ix}\right)\tan^{-}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] -x/2 + x/(4*(1 + x^2)) + (3*ArcTan[x])/4 + (x^2*ArcTan[x])/2 - ArcTan[x]/(2
*(1 + x^2)) + I*ArcTan[x]^2 + 2*ArcTan[x]*Log[2/(1 + I*x)] + I*PolyLog[2, 1
 - 2/(1 + I*x)]
```

Rule 4964

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Dist[1/e, Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Arc
Tan[c*x])^p, x], x] - Dist[d/e, Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan[c
*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p
, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 4930

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[((d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p)/(2*e*(q +
 1)), x] - Dist[(b*p)/(2*c*(q + 1)), Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])^
(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
0] && NeQ[q, -1]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rubi steps

$$\int \frac{x^5 \tan^{-1}(x)}{(1+x^2)^2}\,dx = -\int \frac{x^3 \tan^{-1}(x)}{(1+x^2)^2}\,dx + \int \frac{x^3 \tan^{-1}(x)}{1+x^2}\,dx$$
$$= \int x\tan^{-1}(x)\,dx + \int \frac{x\tan^{-1}(x)}{(1+x^2)^2}\,dx - 2\int \frac{x\tan^{-1}(x)}{1+x^2}\,dx$$
$$= \frac{1}{2}x^2\tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{(1+x^2)^2}\,dx - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx - 2\left(-\frac{1}{2}i\tan^{-1}(x)^2 - \int \frac{\tan^{-1}(x)}{i-x}\,d\right.$$
$$= -\frac{x}{2} + \frac{x}{4(1+x^2)} + \frac{1}{2}x^2\tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(1+x^2)} + \frac{1}{4}\int \frac{1}{1+x^2}\,dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx - 2\left(-\frac{1}{2}i\tan^{-1}(x)^2\right.$$
$$= -\frac{x}{2} + \frac{x}{4(1+x^2)} + \frac{3}{4}\tan^{-1}(x) + \frac{1}{2}x^2\tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(1+x^2)} - 2\left(-\frac{1}{2}i\tan^{-1}(x)^2 - \tan^{-1}(x)\log\left(\frac{2}{1+i}\right.\right.$$
$$= -\frac{x}{2} + \frac{x}{4(1+x^2)} + \frac{3}{4}\tan^{-1}(x) + \frac{1}{2}x^2\tan^{-1}(x) - \frac{\tan^{-1}(x)}{2(1+x^2)} - 2\left(-\frac{1}{2}i\tan^{-1}(x)^2 - \tan^{-1}(x)\log\left(\frac{2}{1+i}\right.\right.$$

Mathematica [A] time = 0.201496, size = 70, normalized size = 0.79

$$\frac{1}{8}\left(-8i\text{PolyLog}\left(2,-e^{2i\tan^{-1}(x)}\right) + 4\left(x^2+1\right)\tan^{-1}(x) - 4x - 8i\tan^{-1}(x)^2 + 16\tan^{-1}(x)\log\left(1+e^{2i\tan^{-1}(x)}\right) + \sin\left(2\tan\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*ArcTan[x])/(1 + x^2)^2,x]
```

```
[Out] (-4*x + 4*(1 + x^2)*ArcTan[x] - (8*I)*ArcTan[x]^2 - 2*ArcTan[x]*Cos[2*ArcTa
n[x]] + 16*ArcTan[x]*Log[1 + E^((2*I)*ArcTan[x])] - (8*I)*PolyLog[2, -E^((2
*I)*ArcTan[x])] + Sin[2*ArcTan[x]])/8
```

Maple [A] time = 0.018, size = 149, normalized size = 1.7

$$\frac{x^2\arctan(x)}{2} - \arctan(x)\ln\left(x^2+1\right) - \frac{\arctan(x)}{2x^2+2} - \frac{x}{2} + \frac{x}{4x^2+4} + \frac{3\arctan(x)}{4} + \frac{i}{4}(\ln(x-i))^2 + \frac{i}{2}\ln\left(-\frac{i}{2}(x+i)\right)\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*arctan(x)/(x^2+1)^2,x)
```

```
[Out] 1/2*x^2*arctan(x)-arctan(x)*ln(x^2+1)-1/2*arctan(x)/(x^2+1)-1/2*x+1/4*x/(x^
2+1)+3/4*arctan(x)+1/4*I*ln(x-I)^2+1/2*I*ln(-1/2*I*(x+I))*ln(x-I)-1/2*I*ln(
x-I)*ln(x^2+1)+1/2*I*dilog(-1/2*I*(x+I))-1/2*I*ln(1/2*I*(x-I))*ln(x+I)-1/4*
I*ln(x+I)^2+1/2*I*ln(x+I)*ln(x^2+1)-1/2*I*dilog(1/2*I*(x-I))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5\arctan(x)}{\left(x^2+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(x)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] integrate(x^5*arctan(x)/(x^2 + 1)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^5 \arctan(x)}{x^4 + 2x^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(x)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] integral(x^5*arctan(x)/(x^4 + 2*x^2 + 1), x)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RecursionError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*atan(x)/(x**2+1)**2,x)
```

```
[Out] Exception raised: RecursionError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \arctan(x)}{(x^2 + 1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*arctan(x)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] integrate(x^5*arctan(x)/(x^2 + 1)^2, x)
```

3.676 $\int \frac{(1+x^2)\tan^{-1}(x)}{x^2}\,dx$

Optimal. Leaf size=22

$$-\log\left(x^2+1\right)+\log(x)+x\tan^{-1}(x)-\frac{\tan^{-1}(x)}{x}$$

```
[Out] -(ArcTan[x]/x) + x*ArcTan[x] + Log[x] - Log[1 + x^2]
```

Rubi [A] time = 0.0341581, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {4950, 4852, 266, 36, 29, 31, 4846, 260}

$$-\log\left(x^2+1\right)+\log(x)+x\tan^{-1}(x)-\frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*ArcTan[x])/x^2,x]

[Out] -(ArcTan[x]/x) + x*ArcTan[x] + Log[x] - Log[1 + x^2]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x^2)\tan^{-1}(x)}{x^2}\,dx &= \int \tan^{-1}(x)\,dx + \int \frac{\tan^{-1}(x)}{x^2}\,dx\\ &= -\frac{\tan^{-1}(x)}{x} + x\tan^{-1}(x) + \int \frac{1}{x(1+x^2)}\,dx - \int \frac{x}{1+x^2}\,dx\\ &= -\frac{\tan^{-1}(x)}{x} + x\tan^{-1}(x) - \frac{1}{2}\log(1+x^2) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, x^2\right)\\ &= -\frac{\tan^{-1}(x)}{x} + x\tan^{-1}(x) - \frac{1}{2}\log(1+x^2) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x}\,d\right.\\ &= -\frac{\tan^{-1}(x)}{x} + x\tan^{-1}(x) + \log(x) - \log(1+x^2)\end{aligned}$$

Mathematica [A] time = 0.0064361, size = 22, normalized size = 1.

$$-\log(x^2+1) + \log(x) + x\tan^{-1}(x) - \frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*ArcTan[x])/x^2,x]
```

```
[Out] -(ArcTan[x]/x) + x*ArcTan[x] + Log[x] - Log[1 + x^2]
```

Maple [A] time = 0.008, size = 23, normalized size = 1.1

$$-\frac{\arctan(x)}{x} + x\arctan(x) + \ln(x) - \ln(x^2+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*arctan(x)/x^2,x)
```

```
[Out] -arctan(x)/x+x*arctan(x)+ln(x)-ln(x^2+1)
```

Maxima [A] time = 1.41546, size = 28, normalized size = 1.27

$$\left(x - \frac{1}{x}\right)\arctan(x) - \log\left(x^2 + 1\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)/x^2,x, algorithm="maxima")
```

```
[Out] (x - 1/x)*arctan(x) - log(x^2 + 1) + log(x)
```

Fricas [A] time = 2.31242, size = 72, normalized size = 3.27

$$\frac{\left(x^2 - 1\right)\arctan(x) - x\log\left(x^2 + 1\right) + x\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)/x^2,x, algorithm="fricas")
```

```
[Out] ((x^2 - 1)*arctan(x) - x*log(x^2 + 1) + x*log(x))/x
```

Sympy [A] time = 0.384592, size = 19, normalized size = 0.86

$$x\operatorname{atan}(x) + \log(x) - \log\left(x^2 + 1\right) - \frac{\operatorname{atan}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*atan(x)/x**2,x)
```

```
[Out] x*atan(x) + log(x) - log(x**2 + 1) - atan(x)/x
```

Giac [A] time = 1.0688, size = 34, normalized size = 1.55

$$\left(x - \frac{1}{x}\right)\arctan(x) - \log\left(x^2 + 1\right) + \frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)/x^2,x, algorithm="giac")
```

```
[Out] (x - 1/x)*arctan(x) - log(x^2 + 1) + 1/2*log(x^2)
```

3.677 $\int \frac{(1+x^2)\tan^{-1}(x)}{x^5}\,dx$

Optimal. Leaf size=31

$$-\frac{1}{12x^3}-\frac{(x^2+1)^2\tan^{-1}(x)}{4x^4}-\frac{1}{4x}$$

```
[Out] -1/(12*x^3) - 1/(4*x) - ((1 + x^2)^2*ArcTan[x])/(4*x^4)
```

Rubi [A] time = 0.0231771, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4944, 14}

$$-\frac{1}{12x^3}-\frac{(x^2+1)^2\tan^{-1}(x)}{4x^4}-\frac{1}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*ArcTan[x])/x^5,x]

[Out] -1/(12*x^3) - 1/(4*x) - ((1 + x^2)^2*ArcTan[x])/(4*x^4)
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x^2)\tan^{-1}(x)}{x^5}\,dx &= -\frac{(1+x^2)^2\tan^{-1}(x)}{4x^4}+\frac{1}{4}\int \frac{1+x^2}{x^4}\,dx\\ &= -\frac{(1+x^2)^2\tan^{-1}(x)}{4x^4}+\frac{1}{4}\int \left(\frac{1}{x^4}+\frac{1}{x^2}\right)dx\\ &= -\frac{1}{12x^3}-\frac{1}{4x}-\frac{(1+x^2)^2\tan^{-1}(x)}{4x^4}\end{aligned}$$

Mathematica [C] time = 0.0075305, size = 59, normalized size = 1.9

$$-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-x^2\right)}{12x^3}-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-x^2\right)}{2x}-\frac{\tan^{-1}(x)}{2x^2}-\frac{\tan^{-1}(x)}{4x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*ArcTan[x])/x^5,x]
```

```
[Out] -ArcTan[x]/(4*x^4) - ArcTan[x]/(2*x^2) - Hypergeometric2F1[-3/2, 1, -1/2, -
x^2]/(12*x^3) - Hypergeometric2F1[-1/2, 1, 1/2, -x^2]/(2*x)
```

Maple [A] time = 0.008, size = 30, normalized size = 1.

$$-\frac{\arctan(x)}{4x^4}-\frac{\arctan(x)}{2x^2}-\frac{\arctan(x)}{4}-\frac{1}{12x^3}-\frac{1}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*arctan(x)/x^5,x)
```

```
[Out] -1/4*arctan(x)/x^4-1/2*arctan(x)/x^2-1/4*arctan(x)-1/12/x^3-1/4/x
```

Maxima [A] time = 1.41116, size = 42, normalized size = 1.35

$$-\frac{3x^2+1}{12x^3}-\frac{(2x^2+1)\arctan(x)}{4x^4}-\frac{1}{4}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*x^2 + 1)/x^3 - 1/4*(2*x^2 + 1)*arctan(x)/x^4 - 1/4*arctan(x)
```

Fricas [A] time = 2.43455, size = 74, normalized size = 2.39

$$-\frac{3x^3+3(x^4+2x^2+1)\arctan(x)+x}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(3*x^3 + 3*(x^4 + 2*x^2 + 1)*arctan(x) + x)/x^4
```

Sympy [A] time = 0.859894, size = 34, normalized size = 1.1

$$-\frac{\operatorname{atan}(x)}{4}-\frac{1}{4x}-\frac{\operatorname{atan}(x)}{2x^2}-\frac{1}{12x^3}-\frac{\operatorname{atan}(x)}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*atan(x)/x**5,x)
```

```
[Out] -atan(x)/4 - 1/(4*x) - atan(x)/(2*x**2) - 1/(12*x**3) - atan(x)/(4*x**4)
```

Giac [A] time = 1.07741, size = 42, normalized size = 1.35

$$-\frac{3\,x^2+1}{12\,x^3}-\frac{\left(2\,x^2+1\right)\arctan\left(x\right)}{4\,x^4}-\frac{1}{4}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)/x^5,x, algorithm="giac")
```

```
[Out] -1/12*(3*x^2 + 1)/x^3 - 1/4*(2*x^2 + 1)*arctan(x)/x^4 - 1/4*arctan(x)
```

3.678 $\int \frac{(1+x^2)^2 \tan^{-1}(x)}{x^5} \, dx$

Optimal. Leaf size=63

$$\frac{1}{2}i\text{PolyLog}(2,-ix)-\frac{1}{2}i\text{PolyLog}(2,ix)-\frac{1}{12x^3}-\frac{\tan^{-1}(x)}{x^2}-\frac{\tan^{-1}(x)}{4x^4}-\frac{3}{4x}-\frac{3}{4}\tan^{-1}(x)$$

```
[Out] -1/(12*x^3) - 3/(4*x) - (3*ArcTan[x])/4 - ArcTan[x]/(4*x^4) - ArcTan[x]/x^2
 + (I/2)*PolyLog[2, (-I)*x] - (I/2)*PolyLog[2, I*x]
```

Rubi [A] time = 0.0836894, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {4948, 4852, 325, 203, 4848, 2391}

$$\frac{1}{2}i\text{PolyLog}(2,-ix)-\frac{1}{2}i\text{PolyLog}(2,ix)-\frac{1}{12x^3}-\frac{\tan^{-1}(x)}{x^2}-\frac{\tan^{-1}(x)}{4x^4}-\frac{3}{4x}-\frac{3}{4}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)^2*ArcTan[x])/x^5,x]
```

```
[Out] -1/(12*x^3) - 3/(4*x) - (3*ArcTan[x])/4 - ArcTan[x]/(4*x^4) - ArcTan[x]/x^2
 + (I/2)*PolyLog[2, (-I)*x] - (I/2)*PolyLog[2, I*x]
```

Rule 4948

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(1+x^2\right)^2 \tan^{-1}(x)}{x^5}\,dx &= \int \left(\frac{\tan^{-1}(x)}{x^5}+\frac{2\tan^{-1}(x)}{x^3}+\frac{\tan^{-1}(x)}{x}\right) dx \\
&= 2\int \frac{\tan^{-1}(x)}{x^3}\,dx+\int \frac{\tan^{-1}(x)}{x^5}\,dx+\int \frac{\tan^{-1}(x)}{x}\,dx \\
&= -\frac{\tan^{-1}(x)}{4x^4}-\frac{\tan^{-1}(x)}{x^2}+\frac{1}{2}i\int \frac{\log(1-ix)}{x}\,dx-\frac{1}{2}i\int \frac{\log(1+ix)}{x}\,dx+\frac{1}{4}\int \frac{1}{x^4\left(1+x^2\right)}\,dx \\
&= -\frac{1}{12x^3}-\frac{1}{x}-\frac{\tan^{-1}(x)}{4x^4}-\frac{\tan^{-1}(x)}{x^2}+\frac{1}{2}i\text{Li}_2(-ix)-\frac{1}{2}i\text{Li}_2(ix)-\frac{1}{4}\int \frac{1}{x^2\left(1+x^2\right)}\,dx-\int \frac{}{1} \\
&= -\frac{1}{12x^3}-\frac{3}{4x}-\tan^{-1}(x)-\frac{\tan^{-1}(x)}{4x^4}-\frac{\tan^{-1}(x)}{x^2}+\frac{1}{2}i\text{Li}_2(-ix)-\frac{1}{2}i\text{Li}_2(ix)+\frac{1}{4}\int \frac{1}{1+x^2}\,d \\
&= -\frac{1}{12x^3}-\frac{3}{4x}-\frac{3}{4}\tan^{-1}(x)-\frac{\tan^{-1}(x)}{4x^4}-\frac{\tan^{-1}(x)}{x^2}+\frac{1}{2}i\text{Li}_2(-ix)-\frac{1}{2}i\text{Li}_2(ix)
\end{aligned}
$$

Mathematica [C] time = 0.0070159, size = 81, normalized size = 1.29

$$
\frac{1}{2}i\text{PolyLog}(2,-ix)-\frac{1}{2}i\text{PolyLog}(2,ix)-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};-x^2\right)}{12x^3}-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-x^2\right)}{x}-\frac{\tan^{-1}(x)}{x^2}-\frac{\tan^{-1}(x)}{4x^4}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + x^2)^2*ArcTan[x])/x^5,x]
```

```
[Out] -ArcTan[x]/(4*x^4) - ArcTan[x]/x^2 - Hypergeometric2F1[-3/2, 1, -1/2, -x^2]
/(12*x^3) - Hypergeometric2F1[-1/2, 1, 1/2, -x^2]/x + (I/2)*PolyLog[2, (-I)
*x] - (I/2)*PolyLog[2, I*x]
```

Maple [A] time = 0.019, size = 79, normalized size = 1.3

$$
\arctan(x)\ln(x)-\frac{\arctan(x)}{4x^4}-\frac{\arctan(x)}{x^2}+\frac{i}{2}\ln(x)\ln(1+ix)-\frac{i}{2}\ln(x)\ln(1-ix)+\frac{i}{2}\text{dilog}(1+ix)-\frac{i}{2}\text{dilog}(1-
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^2*arctan(x)/x^5,x)
```

```
[Out] arctan(x)*ln(x)-1/4*arctan(x)/x^4-arctan(x)/x^2+1/2*I*ln(x)*ln(1+I*x)-1/2*I
*ln(x)*ln(1-I*x)+1/2*I*dilog(1+I*x)-1/2*I*dilog(1-I*x)-3/4*arctan(x)-1/12/x
^3-3/4/x
```

Maxima [A] time = 1.57451, size = 96, normalized size = 1.52

$$-\frac{3\,\pi x^4 \log\left(x^2+1\right) - 12\,x^4 \arctan\left(x\right) \log\left(x\right) + 6i\,x^4 \mathrm{Li}_2\left(i\,x+1\right) - 6i\,x^4 \mathrm{Li}_2\left(-i\,x+1\right) + 9\,x^3 + 3\left(3\,x^4+4\,x^2+1\right) \arctan\left(x\right) + x}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2*arctan(x)/x^5,x, algorithm="maxima")
```

```
[Out] -1/12*(3*pi*x^4*log(x^2 + 1) - 12*x^4*arctan(x)*log(x) + 6*I*x^4*dilog(I*x
+ 1) - 6*I*x^4*dilog(-I*x + 1) + 9*x^3 + 3*(3*x^4 + 4*x^2 + 1)*arctan(x) +
x)/x^4
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(x^4+2\,x^2+1\right) \arctan\left(x\right)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2*arctan(x)/x^5,x, algorithm="fricas")
```

```
[Out] integral((x^4 + 2*x^2 + 1)*arctan(x)/x^5, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^2 \operatorname{atan}\left(x\right)}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**2*atan(x)/x**5,x)
```

```
[Out] Integral((x**2 + 1)**2*atan(x)/x**5, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2+1\right)^2 \arctan\left(x\right)}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^2*arctan(x)/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)^2*arctan(x)/x^5, x)
```

3.679 $\int \frac{\tan^{-1}(x)}{x^2(1+x^2)}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)}{x}$$

```
[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 + Log[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0574401, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {4918, 4852, 266, 36, 29, 31, 4884}

$$-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x]/(x^2*(1 + x^2)),x]
```

```
[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 + Log[x] - Log[1 + x^2]/2
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(x)}{x^2\left(1+x^2\right)}\,dx &= \int \frac{\tan^{-1}(x)}{x^2}\,dx - \int \frac{\tan^{-1}(x)}{1+x^2}\,dx \\
&= -\frac{\tan^{-1}(x)}{x} - \frac{1}{2}\tan^{-1}(x)^2 + \int \frac{1}{x\left(1+x^2\right)}\,dx \\
&= -\frac{\tan^{-1}(x)}{x} - \frac{1}{2}\tan^{-1}(x)^2 + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, x^2\right) \\
&= -\frac{\tan^{-1}(x)}{x} - \frac{1}{2}\tan^{-1}(x)^2 + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right) \\
&= -\frac{\tan^{-1}(x)}{x} - \frac{1}{2}\tan^{-1}(x)^2 + \log(x) - \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0064808, size = 28, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]/(x^2*(1 + x^2)),x]
```

```
[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 + Log[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.012, size = 25, normalized size = 0.9

$$-\frac{\arctan(x)}{x}-\frac{(\arctan(x))^2}{2}+\ln(x)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)/x^2/(x^2+1),x)
```

```
[Out] -arctan(x)/x-1/2*arctan(x)^2+ln(x)-1/2*ln(x^2+1)
```

Maxima [A] time = 1.43856, size = 36, normalized size = 1.29

$$-\left(\frac{1}{x}+\arctan(x)\right)\arctan(x)+\frac{1}{2}\arctan(x)^2-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/x^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] -(1/x + arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*log(x^2 + 1) + log(x)
```

Fricas [A] time = 2.50508, size = 92, normalized size = 3.29

$$-\frac{x\arctan\left(x\right)^{2}+x\log\left(x^{2}+1\right)-2\,x\log\left(x\right)+2\,\arctan\left(x\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/x^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/2*(x*arctan(x)^2 + x*log(x^2 + 1) - 2*x*log(x) + 2*arctan(x))/x
```

Sympy [A] time = 0.595404, size = 22, normalized size = 0.79

$$\log\left(x\right)-\frac{\log\left(x^{2}+1\right)}{2}-\frac{\operatorname{atan}^{2}\left(x\right)}{2}-\frac{\operatorname{atan}\left(x\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)/x**2/(x**2+1),x)
```

```
[Out] log(x) - log(x**2 + 1)/2 - atan(x)**2/2 - atan(x)/x
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\arctan\left(x\right)}{\left(x^{2}+1\right)x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/x^2/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(arctan(x)/((x^2 + 1)*x^2), x)
```

3.680 $\int \frac{\tan^{-1}(x)^2}{x^3} dx$

Optimal. Leaf size=39

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{\tan^{-1}(x)^2}{2x^2}+\log(x)-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)}{x}$$

```
[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 - ArcTan[x]^2/(2*x^2) + Log[x] - Log[1 + x^2
]/2
```

Rubi [A] time = 0.069604, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {4852, 4918, 266, 36, 29, 31, 4884}

$$-\frac{1}{2}\log\left(x^2+1\right)-\frac{\tan^{-1}(x)^2}{2x^2}+\log(x)-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x]^2/x^3,x]
```

```
[Out] -(ArcTan[x]/x) - ArcTan[x]^2/2 - ArcTan[x]^2/(2*x^2) + Log[x] - Log[1 + x^2
]/2
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4918

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[1/d, Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
x] - Dist[e/(d*f^2), Int[((f*x)^(m + 2)*(a + b*ArcTan[c*x])^p)/(d + e*x^2),
 x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^{-1}(x)^2}{x^3}\,dx &= -\frac{\tan^{-1}(x)^2}{2x^2}+\int \frac{\tan^{-1}(x)}{x^2\left(1+x^2\right)}\,dx\\
&= -\frac{\tan^{-1}(x)^2}{2x^2}+\int \frac{\tan^{-1}(x)}{x^2}\,dx-\int \frac{\tan^{-1}(x)}{1+x^2}\,dx\\
&= -\frac{\tan^{-1}(x)}{x}-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)^2}{2x^2}+\int \frac{1}{x\left(1+x^2\right)}\,dx\\
&= -\frac{\tan^{-1}(x)}{x}-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)^2}{2x^2}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,x^2\right)\\
&= -\frac{\tan^{-1}(x)}{x}-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)^2}{2x^2}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x}\,dx,x,x^2\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right)\\
&= -\frac{\tan^{-1}(x)}{x}-\frac{1}{2}\tan^{-1}(x)^2-\frac{\tan^{-1}(x)^2}{2x^2}+\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0194049, size = 38, normalized size = 0.97

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{\left(-x^2-1\right)\tan^{-1}(x)^2}{2x^2}+\log(x)-\frac{\tan^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]^2/x^3,x]
```

```
[Out] -(ArcTan[x]/x) + ((-1 - x^2)*ArcTan[x]^2)/(2*x^2) + Log[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.001, size = 34, normalized size = 0.9

$$-\frac{\arctan(x)}{x}-\frac{(\arctan(x))^2}{2}-\frac{(\arctan(x))^2}{2\,x^2}+\ln(x)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)^2/x^3,x)
```

```
[Out] -arctan(x)/x-1/2*arctan(x)^2-1/2*arctan(x)^2/x^2+ln(x)-1/2*ln(x^2+1)
```

Maxima [A] time = 1.42613, size = 49, normalized size = 1.26

$$-\left(\frac{1}{x}+\arctan(x)\right)\arctan(x)+\frac{1}{2}\arctan(x)^2-\frac{\arctan(x)^2}{2\,x^2}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2/x^3,x, algorithm="maxima")
```

```
[Out] -(1/x + arctan(x))*arctan(x) + 1/2*arctan(x)^2 - 1/2*arctan(x)^2/x^2 - 1/2*
log(x^2 + 1) + log(x)
```

Fricas [A] time = 2.50901, size = 113, normalized size = 2.9

$$-\frac{(x^2+1)\arctan(x)^2 + x^2\log(x^2+1) - 2x^2\log(x) + 2x\arctan(x)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*((x^2 + 1)*arctan(x)^2 + x^2*log(x^2 + 1) - 2*x^2*log(x) + 2*x*arctan(
x))/x^2
```

Sympy [A] time = 0.547456, size = 32, normalized size = 0.82

$$\log(x) - \frac{\log(x^2+1)}{2} - \frac{\operatorname{atan}^2(x)}{2} - \frac{\operatorname{atan}(x)}{x} - \frac{\operatorname{atan}^2(x)}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)**2/x**3,x)
```

```
[Out] log(x) - log(x**2 + 1)/2 - atan(x)**2/2 - atan(x)/x - atan(x)**2/(2*x**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(x)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2/x^3,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)^2/x^3, x)
```

3.681 $\int \frac{(1+x^2)\tan^{-1}(x)^2}{x^5}\,dx$

Optimal. Leaf size=60

$$-\frac{1}{12x^2}-\frac{1}{6}\log\left(x^2+1\right)-\frac{\left(x^2+1\right)^2\tan^{-1}(x)^2}{4x^4}-\frac{\tan^{-1}(x)}{6x^3}+\frac{\log(x)}{3}-\frac{\tan^{-1}(x)}{2x}$$

```
[Out] -1/(12*x^2) - ArcTan[x]/(6*x^3) - ArcTan[x]/(2*x) - ((1 + x^2)^2*ArcTan[x]^
2)/(4*x^4) + Log[x]/3 - Log[1 + x^2]/6
```

Rubi [A] time = 0.0779685, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {4944, 4950, 4852, 266, 44, 36, 29, 31}

$$-\frac{1}{12x^2}-\frac{1}{6}\log\left(x^2+1\right)-\frac{\left(x^2+1\right)^2\tan^{-1}(x)^2}{4x^4}-\frac{\tan^{-1}(x)}{6x^3}+\frac{\log(x)}{3}-\frac{\tan^{-1}(x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*ArcTan[x]^2)/x^5,x]
```

```
[Out] -1/(12*x^2) - ArcTan[x]/(6*x^3) - ArcTan[x]/(2*x) - ((1 + x^2)^2*ArcTan[x]^
2)/(4*x^4) + Log[x]/3 - Log[1 + x^2]/6
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4950

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Dist[d, Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
b*ArcTan[c*x])^p, x], x] + Dist[(c^2*d)/f^2, Int[(f*x)^(m + 2)*(d + e*x^2)^
(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1] &&
 IntegerQ[q]))
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(1+x^2)\tan^{-1}(x)^2}{x^5}\,dx &= -\frac{(1+x^2)^2\tan^{-1}(x)^2}{4x^4}+\frac{1}{2}\int \frac{(1+x^2)\tan^{-1}(x)}{x^4}\,dx\\
&= -\frac{(1+x^2)^2\tan^{-1}(x)^2}{4x^4}+\frac{1}{2}\int \frac{\tan^{-1}(x)}{x^4}\,dx+\frac{1}{2}\int \frac{\tan^{-1}(x)}{x^2}\,dx\\
&= -\frac{\tan^{-1}(x)}{6x^3}-\frac{\tan^{-1}(x)}{2x}-\frac{(1+x^2)^2\tan^{-1}(x)^2}{4x^4}+\frac{1}{6}\int \frac{1}{x^3(1+x^2)}\,dx+\frac{1}{2}\int \frac{1}{x(1+x^2)}\,dx\\
&= -\frac{\tan^{-1}(x)}{6x^3}-\frac{\tan^{-1}(x)}{2x}-\frac{(1+x^2)^2\tan^{-1}(x)^2}{4x^4}+\frac{1}{12}\text{Subst}\left(\int \frac{1}{x^2(1+x)}\,dx,x,x^2\right)+\frac{1}{4}\text{Subst}\\
&= -\frac{\tan^{-1}(x)}{6x^3}-\frac{\tan^{-1}(x)}{2x}-\frac{(1+x^2)^2\tan^{-1}(x)^2}{4x^4}+\frac{1}{12}\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx,x,x^2\right)+\\
&= -\frac{1}{12x^2}-\frac{\tan^{-1}(x)}{6x^3}-\frac{\tan^{-1}(x)}{2x}-\frac{(1+x^2)^2\tan^{-1}(x)^2}{4x^4}+\frac{\log(x)}{3}-\frac{1}{6}\log(1+x^2)
\end{aligned}$$

Mathematica [A] time = 0.0318336, size = 56, normalized size = 0.93

$$\frac{x^2\left(4x^2\log(x)-2x^2\log\left(x^2+1\right)-1\right)-3\left(x^2+1\right)^2\tan^{-1}(x)^2-2\left(3x^3+x\right)\tan^{-1}(x)}{12x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*ArcTan[x]^2)/x^5,x]
```

```
[Out] (-2*(x + 3*x^3)*ArcTan[x] - 3*(1 + x^2)^2*ArcTan[x]^2 + x^2*(-1 + 4*x^2*Log
[x] - 2*x^2*Log[1 + x^2]))/(12*x^4)
```

Maple [A] time = 0.019, size = 57, normalized size = 1.

$$-\frac{(\arctan (x))^{2}}{4 x^{4}}-\frac{(\arctan (x))^{2}}{2 x^{2}}-\frac{(\arctan (x))^{2}}{4}-\frac{\arctan (x)}{6 x^{3}}-\frac{\arctan (x)}{2 x}-\frac{\ln \left(x^{2}+1\right)}{6}-\frac{1}{12 x^{2}}+\frac{\ln (x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*arctan(x)^2/x^5,x)
```

```
[Out] -1/4*arctan(x)^2/x^4-1/2*arctan(x)^2/x^2-1/4*arctan(x)^2-1/6*arctan(x)/x^3-
1/2*arctan(x)/x-1/6*ln(x^2+1)-1/12/x^2+1/3*ln(x)
```

Maxima [A] time = 1.43972, size = 96, normalized size = 1.6

$$-\frac{1}{6}\left(\frac{3 x^{2}+1}{x^{3}}+3 \arctan (x)\right) \arctan (x)+\frac{3 x^{2} \arctan (x)^{2}-2 x^{2} \log \left(x^{2}+1\right)+4 x^{2} \log (x)-1}{12 x^{2}}-\frac{\left(2 x^{2}+1\right) \arctan (}{4 x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)^2/x^5,x, algorithm="maxima")
```

```
[Out] -1/6*((3*x^2 + 1)/x^3 + 3*arctan(x))*arctan(x) + 1/12*(3*x^2*arctan(x)^2 -
2*x^2*log(x^2 + 1) + 4*x^2*log(x) - 1)/x^2 - 1/4*(2*x^2 + 1)*arctan(x)^2/x^
4
```

Fricas [A] time = 2.42932, size = 153, normalized size = 2.55

$$-\frac{2 x^{4} \log \left(x^{2}+1\right)-4 x^{4} \log (x)+3\left(x^{4}+2 x^{2}+1\right) \arctan (x)^{2}+x^{2}+2\left(3 x^{3}+x\right) \arctan (x)}{12 x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)^2/x^5,x, algorithm="fricas")
```

```
[Out] -1/12*(2*x^4*log(x^2 + 1) - 4*x^4*log(x) + 3*(x^4 + 2*x^2 + 1)*arctan(x)^2
+ x^2 + 2*(3*x^3 + x)*arctan(x))/x^4
```

Sympy [A] time = 0.907907, size = 61, normalized size = 1.02

$$\frac{\log (x)}{3}-\frac{\log \left(x^{2}+1\right)}{6}-\frac{\operatorname{atan}^{2}(x)}{4}-\frac{\operatorname{atan}(x)}{2x}-\frac{\operatorname{atan}^{2}(x)}{2x^{2}}-\frac{1}{12x^{2}}-\frac{\operatorname{atan}(x)}{6x^{3}}-\frac{\operatorname{atan}^{2}(x)}{4x^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*atan(x)**2/x**5,x)
```

```
[Out] log(x)/3 - log(x**2 + 1)/6 - atan(x)**2/4 - atan(x)/(2*x) - atan(x)**2/(2*x
**2) - 1/(12*x**2) - atan(x)/(6*x**3) - atan(x)**2/(4*x**4)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2+1)\arctan(x)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*arctan(x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^2 + 1)*arctan(x)^2/x^5, x)
```

3.682 $\int \frac{x^3 \tan^{-1}(x)^2}{(1+x^2)^3} dx$

Optimal. Leaf size=79

$$\frac{5}{32(x^2+1)} - \frac{1}{32(x^2+1)^2} + \frac{x^4 \tan^{-1}(x)^2}{4(x^2+1)^2} + \frac{x^3 \tan^{-1}(x)}{8(x^2+1)^2} + \frac{3x \tan^{-1}(x)}{16(x^2+1)} - \frac{3}{32}\tan^{-1}(x)^2$$

```
[Out] -1/(32*(1 + x^2)^2) + 5/(32*(1 + x^2)) + (x^3*ArcTan[x])/(8*(1 + x^2)^2) +
(3*x*ArcTan[x])/(16*(1 + x^2)) - (3*ArcTan[x]^2)/32 + (x^4*ArcTan[x]^2)/(4*
(1 + x^2)^2)
```

Rubi [A] time = 0.133031, antiderivative size = 82, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {4944, 4938, 4934, 4884}

$$-\frac{x^4}{32(x^2+1)^2} + \frac{3}{32(x^2+1)} + \frac{x^4 \tan^{-1}(x)^2}{4(x^2+1)^2} + \frac{x^3 \tan^{-1}(x)}{8(x^2+1)^2} + \frac{3x \tan^{-1}(x)}{16(x^2+1)} - \frac{3}{32}\tan^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcTan[x]^2)/(1 + x^2)^3,x]
```

```
[Out] -x^4/(32*(1 + x^2)^2) + 3/(32*(1 + x^2)) + (x^3*ArcTan[x])/(8*(1 + x^2)^2)
+ (3*x*ArcTan[x])/(16*(1 + x^2)) - (3*ArcTan[x]^2)/32 + (x^4*ArcTan[x]^2)/(
4*(1 + x^2)^2)
```

Rule 4944

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p)/(d*f*(m + 1)), x] - Dist[(b*c*p)/(f*(m + 1)), Int[(f*x)^
(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b, c
, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0] &
& NeQ[m, -1]
```

Rule 4938

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[(b*(f*x)^m*(d + e*x^2)^(q + 1))/(c*d*m^2), x] +
 (Dist[(f^2*(m - 1))/(c^2*d*m), Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a +
b*ArcTan[c*x]), x], x] - Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*(a + b*A
rcTan[c*x]))/(c^2*d*m), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d
] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

Rule 4934

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> -Simp[(b*(d + e*x^2)^(q + 1))/(4*c^3*d*(q + 1)^2), x] + (-Dist[
1/(2*c^2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x] +
Simp[(x*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]))/(2*c^2*d*(q + 1)), x]) /;
FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\tan^{-1}(x)^2}{\left(1+x^2\right)^3}\,dx &= \frac{x^4\tan^{-1}(x)^2}{4\left(1+x^2\right)^2}-\frac{1}{2}\int \frac{x^4\tan^{-1}(x)}{\left(1+x^2\right)^3}\,dx\\
&= -\frac{x^4}{32\left(1+x^2\right)^2}+\frac{x^3\tan^{-1}(x)}{8\left(1+x^2\right)^2}+\frac{x^4\tan^{-1}(x)^2}{4\left(1+x^2\right)^2}-\frac{3}{8}\int \frac{x^2\tan^{-1}(x)}{\left(1+x^2\right)^2}\,dx\\
&= -\frac{x^4}{32\left(1+x^2\right)^2}+\frac{3}{32\left(1+x^2\right)}+\frac{x^3\tan^{-1}(x)}{8\left(1+x^2\right)^2}+\frac{3x\tan^{-1}(x)}{16\left(1+x^2\right)}+\frac{x^4\tan^{-1}(x)^2}{4\left(1+x^2\right)^2}-\frac{3}{16}\int \frac{\tan^{-1}(x)}{1+x^2}\,dx\\
&= -\frac{x^4}{32\left(1+x^2\right)^2}+\frac{3}{32\left(1+x^2\right)}+\frac{x^3\tan^{-1}(x)}{8\left(1+x^2\right)^2}+\frac{3x\tan^{-1}(x)}{16\left(1+x^2\right)}-\frac{3}{32}\tan^{-1}(x)^2+\frac{x^4\tan^{-1}(x)^2}{4\left(1+x^2\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.0561438, size = 47, normalized size = 0.59

$$\frac{5x^2+2\left(5x^2+3\right)x\tan^{-1}(x)+\left(5x^4-6x^2-3\right)\tan^{-1}(x)^2+4}{32\left(x^2+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcTan[x]^2)/(1 + x^2)^3,x]
```

```
[Out] (4 + 5*x^2 + 2*x*(3 + 5*x^2)*ArcTan[x] + (-3 - 6*x^2 + 5*x^4)*ArcTan[x]^2)/
(32*(1 + x^2)^2)
```

Maple [A] time = 0.025, size = 78, normalized size = 1.

$$\frac{(\arctan(x))^2}{4\left(x^2+1\right)^2}-\frac{(\arctan(x))^2}{2x^2+2}+\frac{5x^3\arctan(x)}{16\left(x^2+1\right)^2}+\frac{3x\arctan(x)}{16\left(x^2+1\right)^2}+\frac{5(\arctan(x))^2}{32}-\frac{1}{32\left(x^2+1\right)^2}+\frac{5}{32x^2+32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arctan(x)^2/(x^2+1)^3,x)
```

```
[Out] 1/4*arctan(x)^2/(x^2+1)^2-1/2*arctan(x)^2/(x^2+1)+5/16*x^3*arctan(x)/(x^2+1
)^2+3/16*x*arctan(x)/(x^2+1)^2+5/32*arctan(x)^2-1/32/(x^2+1)^2+5/32/(x^2+1)
```

Maxima [A] time = 1.48649, size = 127, normalized size = 1.61

$$\frac{1}{16}\left(\frac{5x^3+3x}{x^4+2x^2+1}+5\arctan(x)\right)\arctan(x)-\frac{\left(2x^2+1\right)\arctan(x)^2}{4\left(x^4+2x^2+1\right)}-\frac{5\left(x^4+2x^2+1\right)\arctan(x)^2-5x^2-4}{32\left(x^4+2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)^2/(x^2+1)^3,x, algorithm="maxima")
```

```
[Out] 1/16*((5*x^3 + 3*x)/(x^4 + 2*x^2 + 1) + 5*arctan(x))*arctan(x) - 1/4*(2*x^2
 + 1)*arctan(x)^2/(x^4 + 2*x^2 + 1) - 1/32*(5*(x^4 + 2*x^2 + 1)*arctan(x)^2
 - 5*x^2 - 4)/(x^4 + 2*x^2 + 1)
```

Fricas [A] time = 2.53098, size = 132, normalized size = 1.67

$$\frac{\left(5 x^4-6 x^2-3\right) \arctan (x)^2+5 x^2+2 \left(5 x^3+3 x\right) \arctan (x)+4}{32 \left(x^4+2 x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)^2/(x^2+1)^3,x, algorithm="fricas")
```

```
[Out] 1/32*((5*x^4 - 6*x^2 - 3)*arctan(x)^2 + 5*x^2 + 2*(5*x^3 + 3*x)*arctan(x) +
 4)/(x^4 + 2*x^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{atan}^2(x)}{\left(x^2+1\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*atan(x)**2/(x**2+1)**3,x)
```

```
[Out] Integral(x**3*atan(x)**2/(x**2 + 1)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \arctan (x)^2}{\left(x^2+1\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arctan(x)^2/(x^2+1)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*arctan(x)^2/(x^2 + 1)^3, x)
```

3.683 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx$

Optimal. Leaf size=107

$$\frac{i\sqrt{x^2}\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{x}-\frac{i\sqrt{x^2}\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{x}-\frac{1}{\sqrt{x^2}}-\frac{\sqrt{x^2-1}\sec^{-1}(x)}{x}-\frac{2i\sqrt{x^2}\sec^{-1}(x)\tan^{-1}\left(e^{i\sec^{-1}(x)}\right)}{x}$$

```
[Out] -(1/Sqrt[x^2]) - (Sqrt[-1 + x^2]*ArcSec[x])/x - ((2*I)*Sqrt[x^2]*ArcSec[x]*
ArcTan[E^(I*ArcSec[x])])/x + (I*Sqrt[x^2]*PolyLog[2, (-I)*E^(I*ArcSec[x])])
/x - (I*Sqrt[x^2]*PolyLog[2, I*E^(I*ArcSec[x])])/x
```

Rubi [A] time = 0.153779, antiderivative size = 116, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {5242, 4698, 4710, 4181, 2279, 2391, 8}

$$\frac{i\sqrt{x^2}\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{x}-\frac{i\sqrt{x^2}\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{x}-\frac{1}{\sqrt{x^2}}-\frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x}-\frac{2i\sqrt{x^2}\sec^{-1}(x)\tan^{-1}\left(e^{i\sec^{-1}(x)}\right)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^2]*ArcSec[x])/x^2,x]
```

```
[Out] -(1/Sqrt[x^2]) - (Sqrt[1 - x^(-2)]*Sqrt[x^2]*ArcSec[x])/x - ((2*I)*Sqrt[x^2
]*ArcSec[x]*ArcTan[E^(I*ArcSec[x])])/x + (I*Sqrt[x^2]*PolyLog[2, (-I)*E^(I*
ArcSec[x])])/x - (I*Sqrt[x^2]*PolyLog[2, I*E^(I*ArcSec[x])])/x
```

Rule 5242

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> -Dist[Sqrt[x^2]/x, Subst[Int[((e + d*x^2)^p*(a + b*A
rcCos[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rule 4698

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d_) +
(e_.)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)*Sqrt[d + e*x^2]*(a + b*ArcC
os[c*x])^n)/(f*(m + 2)), x] + (Dist[Sqrt[d + e*x^2]/((m + 2)*Sqrt[1 - c^2*x
^2]), Int[((f*x)^m*(a + b*ArcCos[c*x])^n)/Sqrt[1 - c^2*x^2], x], x] + Dist[
(b*c*n*Sqrt[d + e*x^2])/(f*(m + 2)*Sqrt[1 - c^2*x^2]), Int[(f*x)^(m + 1)*(a
 + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && Eq
Q[c^2*d + e, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 4710

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n
*Cos[x]^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
```

```
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^2}\,dx = -\frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{\sqrt{1-x^2}\cos^{-1}(x)}{x}\,dx, x, \frac{1}{x}\right)}{x}$$
$$= -\frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x} - \frac{\sqrt{x^2}\,\text{Subst}\left(\int 1\,dx, x, \frac{1}{x}\right)}{x} - \frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{\cos^{-1}(x)}{x\sqrt{1-x^2}}\,dx, x, \frac{1}{x}\right)}{x}$$
$$= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x} + \frac{\sqrt{x^2}\,\text{Subst}\left(\int x\sec(x)\,dx, x, \sec^{-1}(x)\right)}{x}$$
$$= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x} - \frac{2i\sqrt{x^2}\sec^{-1}(x)\tan^{-1}\left(e^{i\sec^{-1}(x)}\right)}{x} - \frac{\sqrt{x^2}\,\text{Subst}\left(\int \log\left(1 -\right.\right.}{x}$$
$$= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x} - \frac{2i\sqrt{x^2}\sec^{-1}(x)\tan^{-1}\left(e^{i\sec^{-1}(x)}\right)}{x} + \frac{\left(i\sqrt{x^2}\right)\text{Subst}\left(\int \frac{\log(1}{x}\right.}{x}$$
$$= -\frac{1}{\sqrt{x^2}} - \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{x} - \frac{2i\sqrt{x^2}\sec^{-1}(x)\tan^{-1}\left(e^{i\sec^{-1}(x)}\right)}{x} + \frac{i\sqrt{x^2}\text{Li}_2\left(-ie^{i\sec^{-1}(x)}\right)}{x}$$

Mathematica [A] time = 0.172135, size = 116, normalized size = 1.08

$$-\frac{\sqrt{1-\frac{1}{x^2}}\left(-ix\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)+ix\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)+\sqrt{1-\frac{1}{x^2}}x\sec^{-1}(x)-x\sec^{-1}(x)\log\left(1-ie^{i\sec^{-1}(x}\right.\right.}{\sqrt{x^2-1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[-1 + x^2]*ArcSec[x])/x^2,x]
```

```
[Out] -((Sqrt[1 - x^(-2)]*(1 + Sqrt[1 - x^(-2)]*x*ArcSec[x] - x*ArcSec[x]*Log[1 -
 I*E^(I*ArcSec[x])] + x*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])] - I*x*PolyLog[
2, (-I)*E^(I*ArcSec[x])] + I*x*PolyLog[2, I*E^(I*ArcSec[x])]))/Sqrt[-1 + x^
2])
```

Maple [B] time = 0.391, size = 708, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x)*(x^2-1)^(1/2)/x^2,x)
```

```
[Out] 1/4*(x^2-1)^(1/2)*(((x^2-1)/x^2)^(1/2)*x^3-3*I*x^2-4*((x^2-1)/x^2)^(1/2)*x+
4*I)/(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)/x-1/2*(x^2-1)^(1/2)*(((x^2-1)/x^2)^(1
/2)*x-I)*x/(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)-1/4/(x^2-1)^(1/2)/x*(I*((x^2-1)
/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/2)*x+2*x^2-2)*arcsec(x)+1/4/(x^2-1)^(1
/2)*(I*((x^2-1)/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/2)*x-2*x^2+2)*arcsec(x)
/x+I*(x^2-1)^(1/2)*x^3/(4*I*((x^2-1)/x^2)^(1/2)*x^3-8*I*((x^2-1)/x^2)^(1/2)
*x+8*x^2-8)-1/2/(x^2-1)^(1/2)*(I*x^2+((x^2-1)/x^2)^(1/2)*x-I)*arcsec(x)*ln(
1+I*(1/x+I*(1-1/x^2)^(1/2)))+1/2/(x^2-1)^(1/2)*(I*x^2+((x^2-1)/x^2)^(1/2)*x
-I)*arcsec(x)*ln(1-I*(1/x+I*(1-1/x^2)^(1/2)))-1/2/(x^2-1)^(1/2)*(-I*((x^2-1
)/x^2)^(1/2)*x+x^2-1)*dilog(1+I*(1/x+I*(1-1/x^2)^(1/2)))+1/2/(x^2-1)^(1/2)*
(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)*dilog(1-I*(1/x+I*(1-1/x^2)^(1/2)))+1/2/(x^
2-1)^(1/2)*(I*x^2-((x^2-1)/x^2)^(1/2)*x-I)*arcsec(x)*ln(1+I*(1/x+I*(1-1/x^2
)^(1/2)))-1/2/(x^2-1)^(1/2)*(I*x^2-((x^2-1)/x^2)^(1/2)*x-I)*arcsec(x)*ln(1-
I*(1/x+I*(1-1/x^2)^(1/2)))+1/2/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x+x^2-1
)*dilog(1+I*(1/x+I*(1-1/x^2)^(1/2)))-1/2/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/
2)*x+x^2-1)*dilog(1-I*(1/x+I*(1-1/x^2)^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - 1)*arcsec(x)/x^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - 1)*arcsec(x)/x^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x-1)\,(x+1)}\,\mathrm{asec}\,(x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x)*(x**2-1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt((x - 1)*(x + 1))*asec(x)/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - 1)*arcsec(x)/x^2, x)
```

3.684 $\int \frac{\left(-1+x^2\right)^{5/2} \csc^{-1}(x)}{x^3} \, dx$

Optimal. Leaf size=106

$$\frac{2x^4+3}{12x\sqrt{x^2}}-\frac{7x\log(x)}{3\sqrt{x^2}}+\frac{\left(x^2-1\right)^{5/2}\csc^{-1}(x)}{3x^2}-\frac{5\left(x^2-1\right)^{3/2}\csc^{-1}(x)}{3x^2}-\frac{5\sqrt{x^2-1}\csc^{-1}(x)}{2x^2}-\frac{5x\csc^{-1}(x)^2}{4\sqrt{x^2}}$$

```
[Out] (3 + 2*x^4)/(12*x*Sqrt[x^2]) - (5*Sqrt[-1 + x^2]*ArcCsc[x])/(2*x^2) - (5*(-
1 + x^2)^(3/2)*ArcCsc[x])/(3*x^2) + ((-1 + x^2)^(5/2)*ArcCsc[x])/(3*x^2) -
(5*x*ArcCsc[x]^2)/(4*Sqrt[x^2]) - (7*x*Log[x])/(3*Sqrt[x^2])
```

Rubi [A] time = 0.19574, antiderivative size = 133, normalized size of antiderivative = 1.25, number of steps used = 11, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {5243, 4695, 4647, 4641, 30, 14, 266, 43}

$$\frac{x\sqrt{x^2}}{6}+\frac{\sqrt{x^2}}{4x^3}-\frac{7\sqrt{x^2}\log(x)}{3x}+\frac{1}{3}\left(x^2\right)^{3/2}\left(1-\frac{1}{x^2}\right)^{5/2}\csc^{-1}(x)-\frac{5}{3}\sqrt{x^2}\left(1-\frac{1}{x^2}\right)^{3/2}\csc^{-1}(x)-\frac{5\sqrt{1-\frac{1}{x^2}}\csc^{-1}(x)}{2\sqrt{x^2}}-\frac{5\sqrt{x^2}\,\mathrm{c}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^2)^(5/2)*ArcCsc[x])/x^3,x]
```

```
[Out] Sqrt[x^2]/(4*x^3) + (x*Sqrt[x^2])/6 - (5*Sqrt[1 - x^(-2)]*ArcCsc[x])/(2*Sqr
t[x^2]) - (5*(1 - x^(-2))^(3/2)*Sqrt[x^2]*ArcCsc[x])/3 + ((1 - x^(-2))^(5/2
)*(x^2)^(3/2)*ArcCsc[x])/3 - (5*Sqrt[x^2]*ArcCsc[x]^2)/(4*x) - (7*Sqrt[x^2]
*Log[x])/(3*x)
```

Rule 5243

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> -Dist[Sqrt[x^2]/x, Subst[Int[((e + d*x^2)^p*(a + b*A
rcSin[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rule 4695

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcS
in[c*x])^n)/(f*(m + 1)), x] + (-Dist[(2*e*p)/(f^2*(m + 1)), Int[(f*x)^(m +
2)*(d + e*x^2)^(p - 1)*(a + b*ArcSin[c*x])^n, x], x] - Dist[(b*c*n*d^IntPar
t[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(f
*x)^(m + 1)*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x]) /;
 FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0
] && LtQ[m, -1]
```

Rule 4647

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcSin[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcSin[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] - Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcSin[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
 + e, 0] && GtQ[n, 0]
```

Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\left(-1+x^2\right)^{5/2}\csc^{-1}(x)}{x^3}\,dx = -\frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^{5/2}\sin^{-1}(x)}{x^4}\,dx,x,\frac{1}{x}\right)}{x}$$
$$=\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}\left(x^2\right)^{3/2}\csc^{-1}(x)-\frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^3}\,dx,x,\frac{1}{x}\right)}{3x}+\frac{\left(5\sqrt{x^2}\right)\text{Subst}\left(\int \frac{(1-x}{}\right.}{3x}$$
$$=-\frac{5}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\csc^{-1}(x)+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}\left(x^2\right)^{3/2}\csc^{-1}(x)-\frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx,\right.}{6x}$$
$$=-\frac{5\sqrt{1-\frac{1}{x^2}}\csc^{-1}(x)}{2\sqrt{x^2}}-\frac{5}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\csc^{-1}(x)+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}\left(x^2\right)^{3/2}\csc^{-1}(x)-\frac{\sqrt{x}}{}$$
$$=\frac{\sqrt{x^2}}{4x^3}+\frac{x\sqrt{x^2}}{6}-\frac{5\sqrt{1-\frac{1}{x^2}}\csc^{-1}(x)}{2\sqrt{x^2}}-\frac{5}{3}\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\csc^{-1}(x)+\frac{1}{3}\left(1-\frac{1}{x^2}\right)^{5/2}\left(x^2\right)^{3/2}$$

Mathematica [A] time = 0.294714, size = 86, normalized size = 0.81

$$\frac{\sqrt{x^2-1}\left(4x^2+\csc^{-1}(x)\left(8\sqrt{1-\frac{1}{x^2}}x\left(x^2-7\right)-6\sin\left(2\csc^{-1}(x)\right)\right)+48\log\left(\frac{1}{x}\right)-8\log(x)-30\csc^{-1}(x)^2-3\cos\left(2\csc\right.\right.}{24\sqrt{1-\frac{1}{x^2}}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)^(5/2)*ArcCsc[x])/x^3,x]
```

```
[Out] (Sqrt[-1 + x^2]*(4*x^2 - 30*ArcCsc[x]^2 - 3*Cos[2*ArcCsc[x]] + 48*Log[x^(-1
)] - 8*Log[x] + ArcCsc[x]*(8*Sqrt[1 - x^(-2)]*x*(-7 + x^2) - 6*Sin[2*ArcCsc
[x]])))/(24*Sqrt[1 - x^(-2)]*x)
```

Maple [C] time = 0.322, size = 305, normalized size = 2.9

$$-\frac{5x\left(\operatorname{arccsc}(x)\right)^2}{4}\sqrt{\frac{x^2-1}{x^2}}\frac{1}{\sqrt{x^2-1}}+\frac{2\operatorname{arccsc}(x)+i}{16x^2}\left(i\sqrt{\frac{x^2-1}{x^2}}x^3-2i\sqrt{\frac{x^2-1}{x^2}}x-2x^2+2\right)\frac{1}{\sqrt{x^2-1}}-\frac{-i+2\operatorname{arccsc}(}{16x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(5/2)*arccsc(x)/x^3,x)
```

```
[Out] -5/4/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*arccsc(x)^2+1/16/(x^2-1)^(1/2)/x^2
*(I*((x^2-1)/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/2)*x-2*x^2+2)*(2*arccsc(x)
+I)-1/16/(x^2-1)^(1/2)/x^2*(I*((x^2-1)/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/
2)*x+2*x^2-2)*(-I+2*arccsc(x))-14/3*I/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*a
rccsc(x)+1/6/(x^2-1)^(1/2)*(x^4+7*I*((x^2-1)/x^2)^(1/2)*x-8*x^2+7)*(2*arccs
c(x)*x^4+((x^2-1)/x^2)^(1/2)*x^3-30*arccsc(x)*x^2-7*((x^2-1)/x^2)^(1/2)*x+1
26*arccsc(x)-7*I)/(x^4-15*x^2+63)+7/3/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*l
n((I/x+(1-1/x^2)^(1/2))^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(x^2-1\right)^{\frac{5}{2}}\operatorname{arccsc}(x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)^(5/2)*arccsc(x)/x^3, x)
```

Fricas [A] time = 2.75193, size = 146, normalized size = 1.38

$$\frac{2x^4-15x^2\operatorname{arccsc}(x)^2-28x^2\log(x)+2\left(2x^4-14x^2-3\right)\sqrt{x^2-1}\operatorname{arccsc}(x)+3}{12x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(2*x^4 - 15*x^2*arccsc(x)^2 - 28*x^2*log(x) + 2*(2*x^4 - 14*x^2 - 3)*s
qrt(x^2 - 1)*arccsc(x) + 3)/x^2
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(5/2)*acsc(x)/x**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2-1)^{\frac{5}{2}} \operatorname{arccsc}(x)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(5/2)*arccsc(x)/x^3,x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)^(5/2)*arccsc(x)/x^3, x)
```

3.685 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4} dx$

Optimal. Leaf size=41

$$\frac{1}{3\sqrt{x^2}}-\frac{1}{9(x^2)^{3/2}}+\frac{(x^2-1)^{3/2}\sec^{-1}(x)}{3x^3}$$

```
[Out] -1/(9*(x^2)^(3/2)) + 1/(3*Sqrt[x^2]) + ((-1 + x^2)^(3/2)*ArcSec[x])/(3*x^3)
```

Rubi [A] time = 0.0519343, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {264, 5238, 12, 14}

$$\frac{1}{3\sqrt{x^2}}-\frac{1}{9(x^2)^{3/2}}+\frac{(x^2-1)^{3/2}\sec^{-1}(x)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^2]*ArcSec[x])/x^4,x]
```

```
[Out] -1/(9*(x^2)^(3/2)) + 1/(3*Sqrt[x^2]) + ((-1 + x^2)^(3/2)*ArcSec[x])/(3*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x^4}\,dx = \frac{(-1+x^2)^{3/2}\sec^{-1}(x)}{3x^3} - \frac{x\int \frac{-1+x^2}{3x^4}\,dx}{\sqrt{x^2}}$$
$$= \frac{(-1+x^2)^{3/2}\sec^{-1}(x)}{3x^3} - \frac{x\int \frac{-1+x^2}{x^4}\,dx}{3\sqrt{x^2}}$$
$$= \frac{(-1+x^2)^{3/2}\sec^{-1}(x)}{3x^3} - \frac{x\int \left(-\frac{1}{x^4}+\frac{1}{x^2}\right)dx}{3\sqrt{x^2}}$$
$$= -\frac{1}{9(x^2)^{3/2}} + \frac{1}{3\sqrt{x^2}} + \frac{(-1+x^2)^{3/2}\sec^{-1}(x)}{3x^3}$$

Mathematica [A] time = 0.0526947, size = 48, normalized size = 1.17

$$\frac{\sqrt{1-\frac{1}{x^2}}x\left(3x^2-1\right)+3\left(x^2-1\right)^2\sec^{-1}(x)}{9x^3\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^2]*ArcSec[x])/x^4,x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x*(-1 + 3*x^2) + 3*(-1 + x^2)^2*ArcSec[x])/(9*x^3*Sqrt[-1
 + x^2])
```

Maple [C] time = 0.383, size = 602, normalized size = 14.7

$$-\frac{1}{144x^3}\sqrt{x^2-1}\left(\sqrt{\frac{x^2-1}{x^2}}x^5-5\,ix^4-12\sqrt{\frac{x^2-1}{x^2}}x^3+20\,ix^2+16\sqrt{\frac{x^2-1}{x^2}}x-16\,i\right)\left(-i\sqrt{\frac{x^2-1}{x^2}}x+x^2-1\right)^{-1}-\frac{1}{18x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x)*(x^2-1)^(1/2)/x^4,x)
```

```
[Out] -1/144*(x^2-1)^(1/2)*(((x^2-1)/x^2)^(1/2)*x^5-5*I*x^4-12*((x^2-1)/x^2)^(1/2
)*x^3+20*I*x^2+16*((x^2-1)/x^2)^(1/2)*x-16*I)/(-I*((x^2-1)/x^2)^(1/2)*x+x^2
-1)/x^3-1/18*(x^2-1)^(1/2)*(((x^2-1)/x^2)^(1/2)*x^3-3*I*x^2-4*((x^2-1)/x^2)
^(1/2)*x+4*I)/(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)/x+1/48/(x^2-1)^(1/2)/x^3*(I*
((x^2-1)/x^2)^(1/2)*x^5-8*I*((x^2-1)/x^2)^(1/2)*x^3+4*x^4+8*I*((x^2-1)/x^2)
^(1/2)*x-12*x^2+8)*arcsec(x)+1/24/(x^2-1)^(1/2)/x*(I*((x^2-1)/x^2)^(1/2)*x^
3-2*I*((x^2-1)/x^2)^(1/2)*x+2*x^2-2)*arcsec(x)+1/8*(x^2-1)^(1/2)*(((x^2-1)/
x^2)^(1/2)*x-I)*x/(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)-1/24/(x^2-1)^(1/2)*(I*((
x^2-1)/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/2)*x-2*x^2+2)*arcsec(x)/x-I*(x^2
-1)^(1/2)*x^3/(18*I*((x^2-1)/x^2)^(1/2)*x^3-36*I*((x^2-1)/x^2)^(1/2)*x+36*x
^2-36)-I*(x^2-1)^(1/2)*x^5/(144*I*((x^2-1)/x^2)^(1/2)*x^5-1152*I*((x^2-1)/x
^2)^(1/2)*x^3+576*x^4+1152*I*((x^2-1)/x^2)^(1/2)*x-1728*x^2+1152)-1/48/(x^2
-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x^5-8*I*((x^2-1)/x^2)^(1/2)*x^3-4*x^4+8*I*
((x^2-1)/x^2)^(1/2)*x+12*x^2-8)*arcsec(x)/x^3
```

Maxima [A] time = 1.4774, size = 36, normalized size = 0.88

$$\frac{(x^2-1)^{\frac{3}{2}}\operatorname{arcsec}(x)}{3x^3}+\frac{3x^2-1}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 - 1)^(3/2)*arcsec(x)/x^3 + 1/9*(3*x^2 - 1)/x^3
```

Fricas [A] time = 2.3381, size = 69, normalized size = 1.68

$$\frac{3(x^2-1)^{\frac{3}{2}}\operatorname{arcsec}(x)+3x^2-1}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/9*(3*(x^2 - 1)^(3/2)*arcsec(x) + 3*x^2 - 1)/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x)*(x**2-1)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.10548, size = 101, normalized size = 2.46

$$-\frac{2\arctan\left(-x+\sqrt{x^2-1}\right)}{3\operatorname{sgn}(x)}+\frac{2\left(3\left(x-\sqrt{x^2-1}\right)^4+1\right)\arccos\left(\frac{1}{x}\right)}{3\left(\left(x-\sqrt{x^2-1}\right)^2+1\right)^3}+\frac{3x^2-1}{9x^3\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)*(x^2-1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] -2/3*arctan(-x + sqrt(x^2 - 1))/sgn(x) + 2/3*(3*(x - sqrt(x^2 - 1))^4 + 1)*
arccos(1/x)/((x - sqrt(x^2 - 1))^2 + 1)^3 + 1/9*(3*x^2 - 1)/(x^3*sgn(x))
```

3.686 $\int \frac{\sec^{-1}(x)}{(-1+x^2)^{5/2}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{x^2}}{6(1-x^2)}+\frac{2x\sec^{-1}(x)}{3\sqrt{x^2-1}}-\frac{x\sec^{-1}(x)}{3(x^2-1)^{3/2}}+\frac{5}{6}\coth^{-1}\left(\sqrt{x^2}\right)$$

```
[Out] Sqrt[x^2]/(6*(1 - x^2)) + (5*ArcCoth[Sqrt[x^2]])/6 - (x*ArcSec[x])/(3*(-1 +
 x^2)^(3/2)) + (2*x*ArcSec[x])/(3*Sqrt[-1 + x^2])
```

Rubi [A] time = 0.0306868, antiderivative size = 67, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {192, 191, 5228, 12, 385, 206}

$$\frac{\sqrt{x^2}}{6(1-x^2)}+\frac{2x\sec^{-1}(x)}{3\sqrt{x^2-1}}-\frac{x\sec^{-1}(x)}{3(x^2-1)^{3/2}}+\frac{5x\tanh^{-1}(x)}{6\sqrt{x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[ArcSec[x]/(-1 + x^2)^(5/2),x]
```

```
[Out] Sqrt[x^2]/(6*(1 - x^2)) - (x*ArcSec[x])/(3*(-1 + x^2)^(3/2)) + (2*x*ArcSec[
x])/(3*Sqrt[-1 + x^2]) + (5*x*ArcTanh[x])/(6*Sqrt[x^2])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5228

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSec[c*x], u, x]
- Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegrand[u/(x*Sqrt[c^2*x^2 - 1])
, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2,
0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
```

```
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{-1}(x)}{\left(-1+x^2\right)^{5/2}}\,dx &= -\frac{x\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{2x\sec^{-1}(x)}{3\sqrt{-1+x^2}}-\frac{x\int \frac{-3+2x^2}{3\left(1-x^2\right)^2}\,dx}{\sqrt{x^2}}\\ &= -\frac{x\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{2x\sec^{-1}(x)}{3\sqrt{-1+x^2}}-\frac{x\int \frac{-3+2x^2}{\left(1-x^2\right)^2}\,dx}{3\sqrt{x^2}}\\ &= \frac{\sqrt{x^2}}{6\left(1-x^2\right)}-\frac{x\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{2x\sec^{-1}(x)}{3\sqrt{-1+x^2}}+\frac{(5x)\int \frac{1}{1-x^2}\,dx}{6\sqrt{x^2}}\\ &= \frac{\sqrt{x^2}}{6\left(1-x^2\right)}-\frac{x\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{2x\sec^{-1}(x)}{3\sqrt{-1+x^2}}+\frac{5x\tanh^{-1}(x)}{6\sqrt{x^2}}\end{aligned}$$

Mathematica [A] time = 0.130161, size = 67, normalized size = 1.03

$$\frac{\sqrt{1-\frac{1}{x^2}}x\left(-5\left(x^2-1\right)\log(1-x)+5\left(x^2-1\right)\log(x+1)-2x\right)+4x\left(2x^2-3\right)\sec^{-1}(x)}{12\left(x^2-1\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[x]/(-1 + x^2)^(5/2),x]
```

```
[Out] (4*x*(-3 + 2*x^2)*ArcSec[x] + Sqrt[1 - x^(-2)]*x*(-2*x - 5*(-1 + x^2)*Log[1
 - x] + 5*(-1 + x^2)*Log[1 + x]))/(12*(-1 + x^2)^(3/2))
```

Maple [C] time = 0.24, size = 128, normalized size = 2.

$$\frac{x}{6x^4-12x^2+6}\sqrt{x^2-1}\left(4\,\mathrm{arcsec}\,(x)\,x^2-\sqrt{\frac{x^2-1}{x^2}}x-6\,\mathrm{arcsec}\,(x)\right)-\frac{5x}{6}\sqrt{\frac{x^2-1}{x^2}}\ln\left(x^{-1}+i\sqrt{1-x^{-2}}-1\right)\frac{1}{\sqrt{x^2-1}}+\frac{5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x)/(x^2-1)^(5/2),x)
```

```
[Out] 1/6*(x^2-1)^(1/2)*x/(x^4-2*x^2+1)*(4*arcsec(x)*x^2-((x^2-1)/x^2)^(1/2)*x-6*
arcsec(x))-5/6/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*ln(1/x+I*(1-1/x^2)^(1/2)
-1)+5/6/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*ln(1/x+I*(1-1/x^2)^(1/2)+1)
```

Maxima [A] time = 1.18249, size = 65, normalized size = 1.

$$\frac{1}{3}\left(\frac{2x}{\sqrt{x^2-1}}-\frac{x}{(x^2-1)^{\frac{3}{2}}}\right)\mathrm{arcsec}\,(x)-\frac{x}{6\,(x^2-1)}+\frac{5}{12}\,\log\left(x+1\right)-\frac{5}{12}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x/sqrt(x^2 - 1) - x/(x^2 - 1)^(3/2))*arcsec(x) - 1/6*x/(x^2 - 1) + 5
/12*log(x + 1) - 5/12*log(x - 1)
```

Fricas [A] time = 2.35359, size = 198, normalized size = 3.05

$$-\frac{2\,x^3-4\,(2\,x^3-3\,x)\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)-5\,(x^4-2\,x^2+1)\log\left(x+1\right)+5\,(x^4-2\,x^2+1)\log\left(x-1\right)-2\,x}{12\,(x^4-2\,x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(2*x^3 - 4*(2*x^3 - 3*x)*sqrt(x^2 - 1)*arcsec(x) - 5*(x^4 - 2*x^2 + 1
)*log(x + 1) + 5*(x^4 - 2*x^2 + 1)*log(x - 1) - 2*x)/(x^4 - 2*x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x)/(x**2-1)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11606, size = 78, normalized size = 1.2

$$\frac{(2\,x^2-3)x\arccos\left(\frac{1}{x}\right)}{3\,(x^2-1)^{\frac{3}{2}}}+\frac{5\,\log\left(|x+1|\right)}{12\,\mathrm{sgn}\,(x)}-\frac{5\,\log\left(|x-1|\right)}{12\,\mathrm{sgn}\,(x)}-\frac{x}{6\,(x^2-1)\mathrm{sgn}\,(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*x^2 - 3)*x*arccos(1/x)/(x^2 - 1)^(3/2) + 5/12*log(abs(x + 1))/sgn(x)
 - 5/12*log(abs(x - 1))/sgn(x) - 1/6*x/((x^2 - 1)*sgn(x))
```

3.687 $\int \frac{x^2 \sec^{-1}(x)}{\left(-1+x^2\right)^{5/2}} dx$

Optimal. Leaf size=51

$$\frac{\sqrt{x^2}}{6\left(1-x^2\right)}-\frac{x^3 \sec^{-1}(x)}{3\left(x^2-1\right)^{3/2}}-\frac{1}{6}\coth^{-1}\left(\sqrt{x^2}\right)$$

```
[Out] Sqrt[x^2]/(6*(1 - x^2)) - ArcCoth[Sqrt[x^2]]/6 - (x^3*ArcSec[x])/(3*(-1 + x
^2)^(3/2))
```

Rubi [A] time = 0.063826, antiderivative size = 53, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {264, 5238, 12, 288, 207}

$$\frac{\sqrt{x^2}}{6\left(1-x^2\right)}-\frac{x^3 \sec^{-1}(x)}{3\left(x^2-1\right)^{3/2}}-\frac{x \tanh^{-1}(x)}{6\sqrt{x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^2*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] Sqrt[x^2]/(6*(1 - x^2)) - (x^3*ArcSec[x])/(3*(-1 + x^2)^(3/2)) - (x*ArcTanh
[x])/(6*Sqrt[x^2])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2 \sec^{-1}(x)}{\left(-1+x^2\right)^{5/2}} \, dx &= -\frac{x^3 \sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}-\frac{x \int -\frac{x^2}{3\left(-1+x^2\right)^2} \, dx}{\sqrt{x^2}} \\
&= -\frac{x^3 \sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{x \int \frac{x^2}{\left(-1+x^2\right)^2} \, dx}{3\sqrt{x^2}} \\
&= \frac{\sqrt{x^2}}{6\left(1-x^2\right)}-\frac{x^3 \sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{x \int \frac{1}{-1+x^2} \, dx}{6\sqrt{x^2}} \\
&= \frac{\sqrt{x^2}}{6\left(1-x^2\right)}-\frac{x^3 \sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}-\frac{x \tanh^{-1}(x)}{6\sqrt{x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.116783, size = 61, normalized size = 1.2

$$
\frac{\sqrt{1-\frac{1}{x^2}}x\left(\left(x^2-1\right)\log(1-x)-\left(x^2-1\right)\log(x+1)-2x\right)-4x^3\sec^{-1}(x)}{12\left(x^2-1\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] (-4*x^3*ArcSec[x] + Sqrt[1 - x^(-2)]*x*(-2*x + (-1 + x^2)*Log[1 - x] - (-1
+ x^2)*Log[1 + x]))/(12*(-1 + x^2)^(3/2))
```

Maple [C] time = 0.248, size = 121, normalized size = 2.4

$$
-\frac{x^2}{6x^4-12x^2+6}\sqrt{x^2-1}\left(2\,\mathrm{arcsec}\,(x)\,x+\sqrt{\frac{x^2-1}{x^2}}\right)+\frac{x}{6}\sqrt{\frac{x^2-1}{x^2}}\ln\left(x^{-1}+i\sqrt{1-x^{-2}}-1\right)\frac{1}{\sqrt{x^2-1}}-\frac{x}{6}\sqrt{\frac{x^2-1}{x^2}}\ln\Bigg(
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsec(x)/(x^2-1)^(5/2),x)
```

```
[Out] -1/6*(x^2-1)^(1/2)*x^2/(x^4-2*x^2+1)*(2*arcsec(x)*x+((x^2-1)/x^2)^(1/2))+1/
6/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*ln(1/x+I*(1-1/x^2)^(1/2)-1)-1/6/(x^2-
1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*ln(1/x+I*(1-1/x^2)^(1/2)+1)
```

Maxima [A] time = 1.15586, size = 62, normalized size = 1.22

$$
-\frac{1}{3}\left(\frac{x}{\sqrt{x^2-1}}+\frac{x}{\left(x^2-1\right)^{\frac{3}{2}}}\right)\mathrm{arcsec}\,(x)-\frac{x}{6\left(x^2-1\right)}-\frac{1}{12}\,\log\left(x+1\right)+\frac{1}{12}\,\log\left(x-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(x/sqrt(x^2 - 1) + x/(x^2 - 1)^(3/2))*arcsec(x) - 1/6*x/(x^2 - 1) - 1/
12*log(x + 1) + 1/12*log(x - 1)
```

Fricas [A] time = 2.64376, size = 180, normalized size = 3.53

$$-\frac{4\sqrt{x^2-1}x^3\operatorname{arcsec}(x)+2x^3+\left(x^4-2x^2+1\right)\log(x+1)-\left(x^4-2x^2+1\right)\log(x-1)-2x}{12\left(x^4-2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(4*sqrt(x^2 - 1)*x^3*arcsec(x) + 2*x^3 + (x^4 - 2*x^2 + 1)*log(x + 1)
 - (x^4 - 2*x^2 + 1)*log(x - 1) - 2*x)/(x^4 - 2*x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asec(x)/(x**2-1)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09206, size = 72, normalized size = 1.41

$$-\frac{x^3\arccos\left(\frac{1}{x}\right)}{3\left(x^2-1\right)^{\frac{3}{2}}}-\frac{\log\left(|x+1|\right)}{12\operatorname{sgn}(x)}+\frac{\log\left(|x-1|\right)}{12\operatorname{sgn}(x)}-\frac{x}{6\left(x^2-1\right)\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*x^3*arccos(1/x)/(x^2 - 1)^(3/2) - 1/12*log(abs(x + 1))/sgn(x) + 1/12*l
og(abs(x - 1))/sgn(x) - 1/6*x/((x^2 - 1)*sgn(x))
```

3.688 $\int \frac{x^3 \sec^{-1}(x)}{\left(-1+x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=82

$$\frac{x}{6\sqrt{x^2}\left(1-x^2\right)}-\frac{2x\log(x)}{3\sqrt{x^2}}+\frac{x\log\left(x^2-1\right)}{3\sqrt{x^2}}-\frac{\sec^{-1}(x)}{\sqrt{x^2-1}}-\frac{\sec^{-1}(x)}{3\left(x^2-1\right)^{3/2}}$$

```
[Out] x/(6*Sqrt[x^2]*(1 - x^2)) - ArcSec[x]/(3*(-1 + x^2)^(3/2)) - ArcSec[x]/Sqrt
[-1 + x^2] - (2*x*Log[x])/(3*Sqrt[x^2]) + (x*Log[-1 + x^2])/(3*Sqrt[x^2])
```

Rubi [A] time = 0.0897121, antiderivative size = 84, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {266, 43, 5238, 12, 446, 77}

$$\frac{x}{6\sqrt{x^2}\left(1-x^2\right)}-\frac{2x\log(x)}{3\sqrt{x^2}}+\frac{x\log\left(1-x^2\right)}{3\sqrt{x^2}}-\frac{\sec^{-1}(x)}{\sqrt{x^2-1}}-\frac{\sec^{-1}(x)}{3\left(x^2-1\right)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^3*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] x/(6*Sqrt[x^2]*(1 - x^2)) - ArcSec[x]/(3*(-1 + x^2)^(3/2)) - ArcSec[x]/Sqrt
[-1 + x^2] - (2*x*Log[x])/(3*Sqrt[x^2]) + (x*Log[1 - x^2])/(3*Sqrt[x^2])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3 \sec^{-1}(x)}{\left(-1+x^2\right)^{5/2}}\,dx &= -\frac{\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}-\frac{\sec^{-1}(x)}{\sqrt{-1+x^2}}-\frac{x\int \frac{2-3x^2}{3x\left(1-x^2\right)^2}\,dx}{\sqrt{x^2}}\\
&= -\frac{\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}-\frac{\sec^{-1}(x)}{\sqrt{-1+x^2}}-\frac{x\int \frac{2-3x^2}{x\left(1-x^2\right)^2}\,dx}{3\sqrt{x^2}}\\
&= -\frac{\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}-\frac{\sec^{-1}(x)}{\sqrt{-1+x^2}}-\frac{x\,\text{Subst}\left(\int \frac{2-3x}{(1-x)^2x}\,dx,x,x^2\right)}{6\sqrt{x^2}}\\
&= -\frac{\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}-\frac{\sec^{-1}(x)}{\sqrt{-1+x^2}}-\frac{x\,\text{Subst}\left(\int \left(-\frac{1}{(-1+x)^2}-\frac{2}{-1+x}+\frac{2}{x}\right)dx,x,x^2\right)}{6\sqrt{x^2}}\\
&= \frac{x}{6\sqrt{x^2}\left(1-x^2\right)}-\frac{\sec^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}-\frac{\sec^{-1}(x)}{\sqrt{-1+x^2}}-\frac{2x\log(x)}{3\sqrt{x^2}}+\frac{x\log\left(1-x^2\right)}{3\sqrt{x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.174818, size = 72, normalized size = 0.88

$$
\frac{-\frac{\left(x^2-1\right)\left(4\left(x^2-1\right)\log(x)-2\left(x^2-1\right)\log\left(1-x^2\right)+1\right)}{\sqrt{1-\frac{1}{x^2}}x}-2\left(3x^2-2\right)\sec^{-1}(x)}{6\left(x^2-1\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] (-2*(-2 + 3*x^2)*ArcSec[x] - ((-1 + x^2)*(1 + 4*(-1 + x^2)*Log[x] - 2*(-1 +
 x^2)*Log[1 - x^2]))/(Sqrt[1 - x^(-2)]*x))/(6*(-1 + x^2)^(3/2))
```

Maple [C] time = 0.403, size = 197, normalized size = 2.4

$$
-\frac{4i}{3}x\mathrm{arcsec}(x)\sqrt{\frac{x^2-1}{x^2}}\frac{1}{\sqrt{x^2-1}}+\frac{1}{6x^2\left(4x^6-11x^4+10x^2-3\right)}\sqrt{x^2-1}\left(2i\sqrt{\frac{x^2-1}{x^2}}x^3-2i\sqrt{\frac{x^2-1}{x^2}}x-3x^2+2\right)\Big(8\,\mathrm{a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsec(x)/(x^2-1)^(5/2),x)
```

```
[Out] -4/3*I/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*arcsec(x)+1/6*(x^2-1)^(1/2)/x^2*
(2*I*((x^2-1)/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/2)*x-3*x^2+2)*(8*arcsec(x
)*x^4+2*I*x^4+3*((x^2-1)/x^2)^(1/2)*x^3-6*arcsec(x)*x^2-4*I*x^2-2*((x^2-1)/
x^2)^(1/2)*x+2*I)/(4*x^6-11*x^4+10*x^2-3)+2/3/(x^2-1)^(1/2)*((x^2-1)/x^2)^(
1/2)*x*ln((1/x+I*(1-1/x^2)^(1/2))^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \operatorname{arcsec}(x)}{(x^2-1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3*arcsec(x)/(x^2 - 1)^(5/2), x)
```

Fricas [A] time = 2.62936, size = 186, normalized size = 2.27

$$-\frac{2\left(3x^2-2\right)\sqrt{x^2-1}\operatorname{arcsec}(x)+x^2-2\left(x^4-2x^2+1\right)\log\left(x^2-1\right)+4\left(x^4-2x^2+1\right)\log(x)-1}{6\left(x^4-2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/6*(2*(3*x^2 - 2)*sqrt(x^2 - 1)*arcsec(x) + x^2 - 2*(x^4 - 2*x^2 + 1)*log
(x^2 - 1) + 4*(x^4 - 2*x^2 + 1)*log(x) - 1)/(x^4 - 2*x^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asec(x)/(x**2-1)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11892, size = 86, normalized size = 1.05

$$-\frac{\left(3x^2-2\right)\arccos\left(\frac{1}{x}\right)}{3\left(x^2-1\right)^{\frac{3}{2}}}-\frac{\log\left(x^2\right)}{3\operatorname{sgn}(x)}+\frac{\log\left(\left|x^2-1\right|\right)}{3\operatorname{sgn}(x)}-\frac{2x^2-1}{6\left(x^2-1\right)\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*x^2 - 2)*arccos(1/x)/(x^2 - 1)^(3/2) - 1/3*log(x^2)/sgn(x) + 1/3*lo
g(abs(x^2 - 1))/sgn(x) - 1/6*(2*x^2 - 1)/((x^2 - 1)*sgn(x))
```

3.689 $\int \frac{x^6 \sec^{-1}(x)}{\left(-1+x^2\right)^{5/2}} dx$

Optimal. Leaf size=175

$$\frac{5i\sqrt{x^2}\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{2x}-\frac{5i\sqrt{x^2}\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{2x}+\frac{\sqrt{x^2}\left(2-3x^2\right)}{6\left(x^2-1\right)}+\frac{x^5\sec^{-1}(x)}{2\left(x^2-1\right)^{3/2}}-\frac{5x^3\sec^{-1}(x)}{6\left(x^2-1\right)^{3/2}}-\frac{5x\,\text{s}}{2\text{}}$$

```
[Out] (Sqrt[x^2]*(2 - 3*x^2))/(6*(-1 + x^2)) - (13*ArcCoth[Sqrt[x^2]])/6 - (5*x^3
*ArcSec[x])/(6*(-1 + x^2)^(3/2)) + (x^5*ArcSec[x])/(2*(-1 + x^2)^(3/2)) - (
5*x*ArcSec[x])/(2*Sqrt[-1 + x^2]) - ((5*I)*Sqrt[x^2]*ArcSec[x]*ArcTan[E^(I*
ArcSec[x])])/x + (((5*I)/2)*Sqrt[x^2]*PolyLog[2, (-I)*E^(I*ArcSec[x])])/x -
 (((5*I)/2)*Sqrt[x^2]*PolyLog[2, I*E^(I*ArcSec[x])])/x
```

Rubi [A] time = 0.317106, antiderivative size = 232, normalized size of antiderivative = 1.33, number of steps used = 16, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {5242, 4702, 4706, 4710, 4181, 2279, 2391, 206, 199, 290, 325}

$$\frac{5i\sqrt{x^2}\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)}{2x}-\frac{5i\sqrt{x^2}\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)}{2x}+\frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)}-\frac{3\sqrt{x^2}}{4}-\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}}+\frac{x\sqrt{x^2}\sec^{-1}}{2\left(1-\frac{1}{x^2}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^6*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] -5/(12*(1 - x^(-2))*Sqrt[x^2]) - (3*Sqrt[x^2])/4 + Sqrt[x^2]/(4*(1 - x^(-2)
)) - (13*Sqrt[x^2]*ArcCoth[x])/(6*x) - (5*Sqrt[x^2]*ArcSec[x])/(6*(1 - x^(-
2))^(3/2)*x) - (5*Sqrt[x^2]*ArcSec[x])/(2*Sqrt[1 - x^(-2)]*x) + (x*Sqrt[x^2
]*ArcSec[x])/(2*(1 - x^(-2))^(3/2)) - ((5*I)*Sqrt[x^2]*ArcSec[x]*ArcTan[E^(
I*ArcSec[x])])/x + (((5*I)/2)*Sqrt[x^2]*PolyLog[2, (-I)*E^(I*ArcSec[x])])/x
 - (((5*I)/2)*Sqrt[x^2]*PolyLog[2, I*E^(I*ArcSec[x])])/x
```

Rule 5242

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> -Dist[Sqrt[x^2]/x, Subst[Int[((e + d*x^2)^p*(a + b*A
rcCos[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rule 4702

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a + b
*ArcCos[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[(b*
c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(f*(m + 1)*(1 - c^2*x^2)^FracPart
[p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
 0] && LtQ[m, -1] && IntegerQ[m]
```

Rule 4706

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
```

```
b*ArcCos[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1)),
Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n, x], x] - Dist[(b*c*n
*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*f*(p + 1)*(1 - c^2*x^2)^FracPart[
p]), Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1),
 x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n,
0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || IntegerQ[p] || EqQ[n, 1]
)
```

Rule 4710

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> -Dist[(c^(m + 1)*Sqrt[d])^(-1), Subst[Int[(a + b*x)^n
*Cos[x]^m, x], x, ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*
d + e, 0] && GtQ[d, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
```

```
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rubi steps

$$\int \frac{x^6 \sec^{-1}(x)}{\left(-1+x^2\right)^{5/2}}\,dx = -\frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{\cos^{-1}(x)}{x^3\left(1-x^2\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{x}$$

$$= \frac{x\sqrt{x^2}\sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right)^{3/2}} + \frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)^2}\,dx, x, \frac{1}{x}\right)}{2x} - \frac{\left(5\sqrt{x^2}\right)\text{Subst}\left(\int \frac{\cos^{-1}(x)}{x\left(1-x^2\right)^{5/2}}\,dx, x, \frac{1}{x}\right)}{2x}$$

$$= \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} + \frac{x\sqrt{x^2}\sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right)^{3/2}} + \frac{\left(3\sqrt{x^2}\right)\text{Subst}\left(\int \frac{1}{x^2\left(1-x^2\right)}\,dx, x, \frac{1}{x}\right)}{4x} - \frac{\left(5\sqrt{x^2}\right)\text{Su}}{}$$

$$= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}\,x} + \frac{x\sqrt{x^2}\sec^{-1}(x)}{2\left(1-\frac{1}{x^2}\right)^{3/2}} - \frac{(5}{}$$

$$= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{13\sqrt{x^2}\coth^{-1}(x)}{6x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}\,x} +$$

$$= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{13\sqrt{x^2}\coth^{-1}(x)}{6x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}\,x} +$$

$$= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{13\sqrt{x^2}\coth^{-1}(x)}{6x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}\,x} +$$

$$= -\frac{5}{12\left(1-\frac{1}{x^2}\right)\sqrt{x^2}} - \frac{3\sqrt{x^2}}{4} + \frac{\sqrt{x^2}}{4\left(1-\frac{1}{x^2}\right)} - \frac{13\sqrt{x^2}\coth^{-1}(x)}{6x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{6\left(1-\frac{1}{x^2}\right)^{3/2}x} - \frac{5\sqrt{x^2}\sec^{-1}(x)}{2\sqrt{1-\frac{1}{x^2}}\,x} +$$

Mathematica [B] time = 1.86951, size = 383, normalized size = 2.19

$$-\frac{x^5\left(-60i\sqrt{1-\frac{1}{x^2}}\sin^2\left(2\sec^{-1}(x)\right)\text{PolyLog}\left(2,-ie^{i\sec^{-1}(x)}\right)+60i\sqrt{1-\frac{1}{x^2}}\sin^2\left(2\sec^{-1}(x)\right)\text{PolyLog}\left(2,ie^{i\sec^{-1}(x)}\right)-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^6*ArcSec[x])/(-1 + x^2)^(5/2),x]
```

```
[Out] -(x^5*(22*ArcSec[x] + 40*ArcSec[x]*Cos[2*ArcSec[x]] - 30*ArcSec[x]*Cos[4*Ar
cSec[x]] - 30*Sqrt[1 - x^(-2)]*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])] + 30*Sq
rt[1 - x^(-2)]*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])] + 26*Sqrt[1 - x^(-2)]*L
og[Cos[ArcSec[x]/2]] - 26*Sqrt[1 - x^(-2)]*Log[Sin[ArcSec[x]/2]] + 16*Sin[2
*ArcSec[x]] - (60*I)*Sqrt[1 - x^(-2)]*PolyLog[2, (-I)*E^(I*ArcSec[x])]*Sin[
```

```
2*ArcSec[x]]^2 + (60*I)*Sqrt[1 - x^(-2)]*PolyLog[2, I*E^(I*ArcSec[x])]*Sin[
2*ArcSec[x]]^2 - 15*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])]*Sin[3*ArcSec[x]] +
 15*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])]*Sin[3*ArcSec[x]] + 13*Log[Cos[ArcS
ec[x]/2]]*Sin[3*ArcSec[x]] - 13*Log[Sin[ArcSec[x]/2]]*Sin[3*ArcSec[x]] - 4*
Sin[4*ArcSec[x]] + 15*ArcSec[x]*Log[1 - I*E^(I*ArcSec[x])]*Sin[5*ArcSec[x]]
 - 15*ArcSec[x]*Log[1 + I*E^(I*ArcSec[x])]*Sin[5*ArcSec[x]] - 13*Log[Cos[Ar
cSec[x]/2]]*Sin[5*ArcSec[x]] + 13*Log[Sin[ArcSec[x]/2]]*Sin[5*ArcSec[x]]))/
(96*(-1 + x^2)^(3/2))
```

Maple [A] time = 0.372, size = 240, normalized size = 1.4

$$\frac{x}{6x^4-12x^2+6}\sqrt{x^2-1}\left(3\operatorname{arcsec}(x)x^4-3\sqrt{\frac{x^2-1}{x^2}}x^3-20\operatorname{arcsec}(x)x^2+2\sqrt{\frac{x^2-1}{x^2}}x+15\operatorname{arcsec}(x)\right)-\frac{i}{6}x\sqrt{\frac{x^2-1}{x^2}}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*arcsec(x)/(x^2-1)^(5/2),x)
```

```
[Out] 1/6*(x^2-1)^(1/2)*x/(x^4-2*x^2+1)*(3*arcsec(x)*x^4-3*((x^2-1)/x^2)^(1/2)*x^
3-20*arcsec(x)*x^2+2*((x^2-1)/x^2)^(1/2)*x+15*arcsec(x))-1/6*I*((x^2-1)/x^2
)^(1/2)*x*(15*I*arcsec(x)*ln(1-I*(1/x+I*(1-1/x^2)^(1/2)))-15*I*arcsec(x)*ln
(1+I*(1/x+I*(1-1/x^2)^(1/2)))-13*I*ln(1/x+I*(1-1/x^2)^(1/2)+1)+13*I*ln(1/x+
I*(1-1/x^2)^(1/2)-1)-15*dilog(1+I*(1/x+I*(1-1/x^2)^(1/2)))+15*dilog(1-I*(1/
x+I*(1-1/x^2)^(1/2))))/(x^2-1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6 \operatorname{arcsec}(x)}{(x^2-1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6*arcsec(x)/(x^2 - 1)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2-1}x^6\operatorname{arcsec}(x)}{x^6-3x^4+3x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - 1)*x^6*arcsec(x)/(x^6 - 3*x^4 + 3*x^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*asec(x)/(x**2-1)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6 \operatorname{arcsec}(x)}{(x^2-1)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*arcsec(x)/(x^2-1)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^6*arcsec(x)/(x^2 - 1)^(5/2), x)
```

3.690 $\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=23

$$\frac{1}{\sqrt{x^2}}+\frac{\sqrt{x^2-1}\sec^{-1}(x)}{x}$$

```
[Out] 1/Sqrt[x^2] + (Sqrt[-1 + x^2]*ArcSec[x])/x
```

Rubi [A] time = 0.0478767, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {264, 5238, 30}

$$\frac{1}{\sqrt{x^2}}+\frac{\sqrt{x^2-1}\sec^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[x]/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] 1/Sqrt[x^2] + (Sqrt[-1 + x^2]*ArcSec[x])/x
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 5238

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSec[c*x], u, x] - Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^{-1}(x)}{x^2\sqrt{-1+x^2}}\,dx &= \frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x}-\frac{x\int \frac{1}{x^2}\,dx}{\sqrt{x^2}}\\ &= \frac{1}{\sqrt{x^2}}+\frac{\sqrt{-1+x^2}\sec^{-1}(x)}{x}\end{aligned}$$

Mathematica [A] time = 0.0344072, size = 35, normalized size = 1.52

$$\frac{\sqrt{1-\frac{1}{x^2}}x+\left(x^2-1\right)\sec^{-1}(x)}{x\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[x]/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (Sqrt[1 - x^(-2)]*x + (-1 + x^2)*ArcSec[x])/(x*Sqrt[-1 + x^2])
```

Maple [C] time = 0.223, size = 186, normalized size = 8.1

$$-\frac{1}{4x}\left(\sqrt{\frac{x^2-1}{x^2}}x^3-3ix^2-4\sqrt{\frac{x^2-1}{x^2}}x+4i\right)\frac{1}{\sqrt{x^2-1}}\left(i\sqrt{\frac{x^2-1}{x^2}}x+1\right)^{-1}+\frac{\operatorname{arcsec}(x)}{4x}\left(x^2-2-2i\sqrt{\frac{x^2-1}{x^2}}x\right)\frac{1}{\sqrt{x^2-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x)/x^2/(x^2-1)^(1/2),x)
```

```
[Out] -1/4*(((x^2-1)/x^2)^(1/2)*x^3-3*I*x^2-4*((x^2-1)/x^2)^(1/2)*x+4*I)/(x^2-1)^
(1/2)/(I*((x^2-1)/x^2)^(1/2)*x+1)/x+1/4/(x^2-1)^(1/2)/x*(x^2-2-2*I*((x^2-1)
/x^2)^(1/2)*x)*arcsec(x)+1/2*x*arcsec(x)/(x^2-1)^(1/2)-1/4*I*x^3/(x^2-1)^(1
/2)/(x^2-2-2*I*((x^2-1)/x^2)^(1/2)*x)+1/4/(x^2-1)^(1/2)*(x^2-2+2*I*((x^2-1)
/x^2)^(1/2)*x)*arcsec(x)/x
```

Maxima [A] time = 1.47155, size = 23, normalized size = 1.

$$\frac{\sqrt{x^2-1}\operatorname{arcsec}(x)}{x}+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 1)*arcsec(x)/x + 1/x
```

Fricas [A] time = 2.57726, size = 45, normalized size = 1.96

$$\frac{\sqrt{x^2-1}\operatorname{arcsec}(x)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(x^2 - 1)*arcsec(x) + 1)/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x)/x**2/(x**2-1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.10502, size = 68, normalized size = 2.96

$$\frac{2 \arccos\left(\frac{1}{x}\right)}{\left(x - \sqrt{x^2 - 1}\right)^2 + 1} - \frac{2 \arctan\left(-x + \sqrt{x^2 - 1}\right)}{\operatorname{sgn}(x)} + \frac{1}{x\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)/x^2/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arccos(1/x)/((x - sqrt(x^2 - 1))^2 + 1) - 2*arctan(-x + sqrt(x^2 - 1))/sg
n(x) + 1/(x*sgn(x))
```

3.691 $\int \frac{\csc^{-1}(x)}{x^2(-1+x^2)^{5/2}}\,dx$

Optimal. Leaf size=70

$$-\frac{1}{\sqrt{x^2}}+\frac{\sqrt{x^2}}{6\left(x^2-1\right)}+\frac{\left(8x^4-12x^2+3\right)\csc^{-1}(x)}{3x\left(x^2-1\right)^{3/2}}-\frac{11}{6}\coth^{-1}\left(\sqrt{x^2}\right)$$

```
[Out] -(1/Sqrt[x^2]) + Sqrt[x^2]/(6*(-1 + x^2)) - (11*ArcCoth[Sqrt[x^2]])/6 + ((3
 - 12*x^2 + 8*x^4)*ArcCsc[x])/(3*x*(-1 + x^2)^(3/2))
```

Rubi [A] time = 0.085914, antiderivative size = 91, normalized size of antiderivative = 1.3, number of steps used = 5, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {271, 192, 191, 5239, 12, 1259, 453, 206}

$$-\frac{1}{\sqrt{x^2}}-\frac{\sqrt{x^2}}{6\left(1-x^2\right)}+\frac{8x\csc^{-1}(x)}{3\sqrt{x^2-1}}-\frac{4x\csc^{-1}(x)}{3\left(x^2-1\right)^{3/2}}+\frac{\csc^{-1}(x)}{x\left(x^2-1\right)^{3/2}}-\frac{11x\tanh^{-1}(x)}{6\sqrt{x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[ArcCsc[x]/(x^2*(-1 + x^2)^(5/2)),x]
```

```
[Out] -(1/Sqrt[x^2]) - Sqrt[x^2]/(6*(1 - x^2)) + ArcCsc[x]/(x*(-1 + x^2)^(3/2)) -
 (4*x*ArcCsc[x])/(3*(-1 + x^2)^(3/2)) + (8*x*ArcCsc[x])/(3*Sqrt[-1 + x^2])
- (11*x*ArcTanh[x])/(6*Sqrt[x^2])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 5239

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCsc[c*x], u, x] + Dist[(b*c*x)/Sqrt[c^2*x^2], Int[SimplifyIntegr
and[u/(x*Sqrt[c^2*x^2 - 1]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, p},
 x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (I
GtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2
*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^{-1}(x)}{x^2\left(-1+x^2\right)^{5/2}}\,dx &= \frac{\csc^{-1}(x)}{x\left(-1+x^2\right)^{3/2}}-\frac{4x\csc^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{-1+x^2}}+\frac{x\int\frac{3-12x^2+8x^4}{3x^2\left(1-x^2\right)^2}\,dx}{\sqrt{x^2}}\\
&= \frac{\csc^{-1}(x)}{x\left(-1+x^2\right)^{3/2}}-\frac{4x\csc^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{-1+x^2}}+\frac{x\int\frac{3-12x^2+8x^4}{x^2\left(1-x^2\right)^2}\,dx}{3\sqrt{x^2}}\\
&= -\frac{\sqrt{x^2}}{6\left(1-x^2\right)}+\frac{\csc^{-1}(x)}{x\left(-1+x^2\right)^{3/2}}-\frac{4x\csc^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{-1+x^2}}-\frac{x\int\frac{-6+17x^2}{x^2\left(1-x^2\right)}\,dx}{6\sqrt{x^2}}\\
&= -\frac{1}{\sqrt{x^2}}-\frac{\sqrt{x^2}}{6\left(1-x^2\right)}+\frac{\csc^{-1}(x)}{x\left(-1+x^2\right)^{3/2}}-\frac{4x\csc^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{-1+x^2}}-\frac{(11x)\int\frac{1}{1-x^2}\,dx}{6\sqrt{x^2}}\\
&= -\frac{1}{\sqrt{x^2}}-\frac{\sqrt{x^2}}{6\left(1-x^2\right)}+\frac{\csc^{-1}(x)}{x\left(-1+x^2\right)^{3/2}}-\frac{4x\csc^{-1}(x)}{3\left(-1+x^2\right)^{3/2}}+\frac{8x\csc^{-1}(x)}{3\sqrt{-1+x^2}}-\frac{11x\tanh^{-1}(x)}{6\sqrt{x^2}}
\end{aligned}
$$

Mathematica [A] time = 0.128443, size = 79, normalized size = 1.13

$$
\frac{\sqrt{1-\frac{1}{x^2}}x\left(-10x^2+11\left(x^2-1\right)x\log(1-x)-11\left(x^2-1\right)x\log(x+1)+12\right)+4\left(8x^4-12x^2+3\right)\csc^{-1}(x)}{12x\left(x^2-1\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCsc[x]/(x^2*(-1 + x^2)^(5/2)),x]
```

```
[Out] (4*(3 - 12*x^2 + 8*x^4)*ArcCsc[x] + Sqrt[1 - x^(-2)]*x*(12 - 10*x^2 + 11*x*
(-1 + x^2)*Log[1 - x] - 11*x*(-1 + x^2)*Log[1 + x]))/(12*x*(-1 + x^2)^(3/2)
)
```

Maple [C] time = 0.475, size = 702, normalized size = 10.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsc(x)/x^2/(x^2-1)^(5/2),x)
```

```
[Out] -1/4/(I*((x^2-1)/x^2)^(1/2)*x-1)/x/(x^2-1)^(1/2)*(3*I*x^2-4*I-4*((x^2-1)/x^
2)^(1/2)*x+((x^2-1)/x^2)^(1/2)*x^3)+1/4/(x^2-1)^(1/2)/x*(x^2-2+2*I*((x^2-1)
/x^2)^(1/2)*x)*arccsc(x)+1/2*x*arccsc(x)/(x^2-1)^(1/2)+1/4/(x^2-1)^(1/2)*(x
^2-2-2*I*((x^2-1)/x^2)^(1/2)*x)*arccsc(x)/x+1/4*x^3/(x^2-1)^(1/2)/(I*x^2-2*
((x^2-1)/x^2)^(1/2)*x-2*I)-1/24*x^5*(((x^2-1)/x^2)^(1/2)*x+I)/(x^2-1)^(1/2)
/(I*((x^2-1)/x^2)^(1/2)*x^5-5*I*((x^2-1)/x^2)^(1/2)*x^3-3*x^4+4*I*((x^2-1)/
x^2)^(1/2)*x+7*x^2-4)+1/24*x*(5*I*x^4-20*I*x^2-12*((x^2-1)/x^2)^(1/2)*x^3+(
(x^2-1)/x^2)^(1/2)*x^5+16*I+16*((x^2-1)/x^2)^(1/2)*x)/(x^2-1)^(1/2)/(I*((x^
2-1)/x^2)^(1/2)*x^5-5*I*((x^2-1)/x^2)^(1/2)*x^3-3*x^4+4*I*((x^2-1)/x^2)^(1/
2)*x+7*x^2-4)+2/3*(x^2-1)^(1/2)*x^3/(x^4-2*x^2+1)*arccsc(x)+1/2*(x^2-1)^(1/
2)*x*(x^2-2-2*I*((x^2-1)/x^2)^(1/2)*x)*arccsc(x)/(x^4-2*x^2+1)+1/2*(x^2-1)^
(1/2)*x*(x^2-2+2*I*((x^2-1)/x^2)^(1/2)*x)*arccsc(x)/(x^4-2*x^2+1)-11/12/(x^
2-1)^(1/2)*(((x^2-1)/x^2)^(1/2)*x+I)*ln(I/x+(1-1/x^2)^(1/2)+I)-11/12/(x^2-1
)^(1/2)*(((x^2-1)/x^2)^(1/2)*x-I)*ln(I/x+(1-1/x^2)^(1/2)+I)+11/12/(x^2-1)^(
1/2)*(((x^2-1)/x^2)^(1/2)*x+I)*ln(I/x+(1-1/x^2)^(1/2)-I)+11/12/(x^2-1)^(1/2
)*(((x^2-1)/x^2)^(1/2)*x-I)*ln(I/x+(1-1/x^2)^(1/2)-I)
```

Maxima [B] time = 2.57892, size = 166, normalized size = 2.37

$$\frac{32\,x^4\arctan\bigl(1,\sqrt{x+1}\sqrt{x-1}\bigr)-\bigl(x^3-x\bigr)\sqrt{x+1}\sqrt{x-1}\Bigl(\frac{2\,(5\,x^2-6)}{x^3-x}+11\,\log(x+1)-11\,\log(x-1)\Bigr)-48\,x^2\arctan\bigl(1,}{12\,\bigl(x^3-x\bigr)\sqrt{x+1}\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(32*x^4*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) - (x^3 - x)*sqrt(x + 1)*sq
rt(x - 1)*(2*(5*x^2 - 6)/(x^3 - x) + 11*log(x + 1) - 11*log(x - 1)) - 48*x^
2*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + 12*arctan2(1, sqrt(x + 1)*sqrt(x -
1)))/((x^3 - x)*sqrt(x + 1)*sqrt(x - 1))
```

Fricas [A] time = 2.53592, size = 223, normalized size = 3.19

$$-\frac{10\,x^4-4\,\bigl(8\,x^4-12\,x^2+3\bigr)\sqrt{x^2-1}\,\operatorname{arccsc}(x)-22\,x^2+11\,\bigl(x^5-2\,x^3+x\bigr)\log(x+1)-11\,\bigl(x^5-2\,x^3+x\bigr)\log(x-1)}{12\,\bigl(x^5-2\,x^3+x\bigr)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/12*(10*x^4 - 4*(8*x^4 - 12*x^2 + 3)*sqrt(x^2 - 1)*arccsc(x) - 22*x^2 + 1
1*(x^5 - 2*x^3 + x)*log(x + 1) - 11*(x^5 - 2*x^3 + x)*log(x - 1) + 12)/(x^5
 - 2*x^3 + x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsc(x)/x**2/(x**2-1)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13022, size = 142, normalized size = 2.03

$$\frac{1}{3}\left(\frac{(5x^2-6)x}{(x^2-1)^{\frac{3}{2}}}+\frac{6}{\left(x-\sqrt{x^2-1}\right)^2+1}\right)\arcsin\left(\frac{1}{x}\right)+\frac{2\arctan\left(-x+\sqrt{x^2-1}\right)}{\mathrm{sgn}(x)}-\frac{11\log(|x+1|)}{12\,\mathrm{sgn}(x)}+\frac{11\log(|x-1|)}{12\,\mathrm{sgn}(x)}-\frac{}{6\,(x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x)/x^2/(x^2-1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((5*x^2 - 6)*x/(x^2 - 1)^(3/2) + 6/((x - sqrt(x^2 - 1))^2 + 1))*arcsin(
1/x) + 2*arctan(-x + sqrt(x^2 - 1))/sgn(x) - 11/12*log(abs(x + 1))/sgn(x) +
 11/12*log(abs(x - 1))/sgn(x) - 1/6*(5*x^2 - 6)/((x^3 - x)*sgn(x))
```

3.692 $\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=74

$$\frac{24\sqrt{x^2-1}}{x}+\frac{\sqrt{x^2-1}\csc^{-1}(x)^4}{x}-\frac{4\csc^{-1}(x)^3}{\sqrt{x^2}}-\frac{12\sqrt{x^2-1}\csc^{-1}(x)^2}{x}+\frac{24\csc^{-1}(x)}{\sqrt{x^2}}$$

```
[Out] (24*Sqrt[-1 + x^2])/x + (24*ArcCsc[x])/Sqrt[x^2] - (12*Sqrt[-1 + x^2]*ArcCs
c[x]^2)/x - (4*ArcCsc[x]^3)/Sqrt[x^2] + (Sqrt[-1 + x^2]*ArcCsc[x]^4)/x
```

Rubi [A] time = 0.178698, antiderivative size = 101, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5243, 4677, 4619, 261}

$$\frac{24\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}}{x}+\frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x}-\frac{4\csc^{-1}(x)^3}{\sqrt{x^2}}-\frac{12\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^2}{x}+\frac{24\csc^{-1}(x)}{\sqrt{x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsc[x]^4/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (24*Sqrt[1 - x^(-2)]*Sqrt[x^2])/x + (24*ArcCsc[x])/Sqrt[x^2] - (12*Sqrt[1 -
 x^(-2)]*Sqrt[x^2]*ArcCsc[x]^2)/x - (4*ArcCsc[x]^3)/Sqrt[x^2] + (Sqrt[1 - x
^(-2)]*Sqrt[x^2]*ArcCsc[x]^4)/x
```

Rule 5243

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> -Dist[Sqrt[x^2]/x, Subst[Int[((e + d*x^2)^p*(a + b*A
rcSin[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rule 4677

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p +
 1)), x] + Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{\csc^{-1}(x)^4}{x^2\sqrt{-1+x^2}}\,dx = -\frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{x\sin^{-1}(x)^4}{\sqrt{1-x^2}}\,dx,x,\frac{1}{x}\right)}{x}$$
$$= \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x} - \frac{\left(4\sqrt{x^2}\right)\text{Subst}\left(\int \sin^{-1}(x)^3\,dx,x,\frac{1}{x}\right)}{x}$$
$$= -\frac{4\csc^{-1}(x)^3}{\sqrt{x^2}} + \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x} + \frac{\left(12\sqrt{x^2}\right)\text{Subst}\left(\int \frac{x\sin^{-1}(x)^2}{\sqrt{1-x^2}}\,dx,x,\frac{1}{x}\right)}{x}$$
$$= -\frac{12\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^2}{x} - \frac{4\csc^{-1}(x)^3}{\sqrt{x^2}} + \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x} + \frac{\left(24\sqrt{x^2}\right)\text{Subst}\left(\int \sin^{-1}(x)\,dx\right.}{x}$$
$$= \frac{24\csc^{-1}(x)}{\sqrt{x^2}} - \frac{12\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^2}{x} - \frac{4\csc^{-1}(x)^3}{\sqrt{x^2}} + \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x} - \frac{\left(24\sqrt{x^2}\right)\text{Subst}\left(\right.}{x}$$
$$= \frac{24\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}}{x} + \frac{24\csc^{-1}(x)}{\sqrt{x^2}} - \frac{12\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^2}{x} - \frac{4\csc^{-1}(x)^3}{\sqrt{x^2}} + \frac{\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\csc^{-1}(x)^4}{x}$$

Mathematica [A] time = 0.0588532, size = 76, normalized size = 1.03

$$\frac{24\left(x^2-1\right)+\left(x^2-1\right)\csc^{-1}(x)^4-4\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x)^3-12\left(x^2-1\right)\csc^{-1}(x)^2+24\sqrt{1-\frac{1}{x^2}}x\csc^{-1}(x)}{x\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCsc[x]^4/(x^2*Sqrt[-1 + x^2]),x]
```

```
[Out] (24*(-1 + x^2) + 24*Sqrt[1 - x^(-2)]*x*ArcCsc[x] - 12*(-1 + x^2)*ArcCsc[x]^
2 - 4*Sqrt[1 - x^(-2)]*x*ArcCsc[x]^3 + (-1 + x^2)*ArcCsc[x]^4)/(x*Sqrt[-1 +
 x^2])
```

Maple [C] time = 0.405, size = 443, normalized size = 6.

$$\frac{(\text{arccsc}(x))^3}{x}\left(ix^2-2\sqrt{\frac{x^2-1}{x^2}}x-2i\right)\frac{1}{\sqrt{x^2-1}}+\frac{(\text{arccsc}(x))^4}{4x}\left(x^2-2+2i\sqrt{\frac{x^2-1}{x^2}}x\right)\frac{1}{\sqrt{x^2-1}}+\frac{x(\text{arccsc}(x))^4}{2}\frac{1}{\sqrt{x^2-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsc(x)^4/x^2/(x^2-1)^(1/2),x)
```

```
[Out] 1/(x^2-1)^(1/2)/x*(I*x^2-2*((x^2-1)/x^2)^(1/2)*x-2*I)*arccsc(x)^3+1/4/(x^2-
1)^(1/2)/x*(x^2-2+2*I*((x^2-1)/x^2)^(1/2)*x)*arccsc(x)^4+1/2/(x^2-1)^(1/2)*
x*arccsc(x)^4-6/(x^2-1)^(1/2)/x*(I*x^2-2*((x^2-1)/x^2)^(1/2)*x-2*I)*arccsc(
x)-3/(x^2-1)^(1/2)/x*(x^2-2+2*I*((x^2-1)/x^2)^(1/2)*x)*arccsc(x)^2-6/(x^2-1
)^(1/2)*x*arccsc(x)^2+6*(I*((x^2-1)/x^2)^(1/2)*x^3-4*I*((x^2-1)/x^2)^(1/2)*
x-3*x^2+4)/(x^2-1)^(1/2)/(I*((x^2-1)/x^2)^(1/2)*x-1)/x+12*x/(x^2-1)^(1/2)-1
/(x^2-1)^(1/2)*(I*x^2+2*((x^2-1)/x^2)^(1/2)*x-2*I)*arccsc(x)^3/x+1/4/(x^2-1
)^(1/2)*(x^2-2-2*I*((x^2-1)/x^2)^(1/2)*x)*arccsc(x)^4/x+6/(x^2-1)^(1/2)*(I*
x^2+2*((x^2-1)/x^2)^(1/2)*x-2*I)*arccsc(x)/x-3/(x^2-1)^(1/2)*(x^2-2-2*I*((x
^2-1)/x^2)^(1/2)*x)*arccsc(x)^2/x+6*I*x^3/(x^2-1)^(1/2)/(I*x^2-2*((x^2-1)/x
```

```
^2)^(1/2)*x-2*I)
```

Maxima [A] time = 1.5069, size = 78, normalized size = 1.05

$$\frac{\sqrt{x^2-1}\operatorname{arccsc}(x)^4}{x}-12\sqrt{-\frac{1}{x^2}+1}\operatorname{arccsc}(x)^2-\frac{4\operatorname{arccsc}(x)^3}{x}+24\sqrt{-\frac{1}{x^2}+1}+\frac{24\operatorname{arccsc}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 1)*arccsc(x)^4/x - 12*sqrt(-1/x^2 + 1)*arccsc(x)^2 - 4*arccsc(x)
^3/x + 24*sqrt(-1/x^2 + 1) + 24*arccsc(x)/x
```

Fricas [A] time = 2.58062, size = 117, normalized size = 1.58

$$-\frac{4\operatorname{arccsc}(x)^3-\left(\operatorname{arccsc}(x)^4-12\operatorname{arccsc}(x)^2+24\right)\sqrt{x^2-1}-24\operatorname{arccsc}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(4*arccsc(x)^3 - (arccsc(x)^4 - 12*arccsc(x)^2 + 24)*sqrt(x^2 - 1) - 24*ar
ccsc(x))/x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsc(x)**4/x**2/(x**2-1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arccsc}(x)^4}{\sqrt{x^2-1}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x)^4/x^2/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccsc(x)^4/(sqrt(x^2 - 1)*x^2), x)
```

3.693 $\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx$

Optimal. Leaf size=133

$$\frac{\sqrt{x^2-1}\,(17x^2-2)}{64x^4}+\frac{x\sec^{-1}(x)^3}{8\sqrt{x^2}}-\frac{(x^2-1)^{3/2}\sec^{-1}(x)^2}{4x^4}-\frac{3\sqrt{x^2-1}\sec^{-1}(x)^2}{8x^2}+\frac{(x^2-1)^2\sec^{-1}(x)}{8x^3\sqrt{x^2}}+\frac{9x\sec^{-1}(x)}{64\sqrt{x^2}}-\frac{3\,\mathrm{se}}{8.}$$

```
[Out] (Sqrt[-1 + x^2]*(-2 + 17*x^2))/(64*x^4) - (3*ArcSec[x])/(8*x*Sqrt[x^2]) + (
9*x*ArcSec[x])/(64*Sqrt[x^2]) + ((-1 + x^2)^2*ArcSec[x])/(8*x^3*Sqrt[x^2])
- (3*Sqrt[-1 + x^2]*ArcSec[x]^2)/(8*x^2) - ((-1 + x^2)^(3/2)*ArcSec[x]^2)/(
4*x^4) + (x*ArcSec[x]^3)/(8*Sqrt[x^2])
```

Rubi [A] time = 0.201347, antiderivative size = 172, normalized size of antiderivative = 1.29, number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.529, Rules used = {5242, 4650, 4648, 4642, 4628, 321, 216, 4678, 195}

$$\frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{32\sqrt{x^2}}+\frac{15\sqrt{1-\frac{1}{x^2}}}{64\sqrt{x^2}}-\frac{9\sqrt{x^2}\csc^{-1}(x)}{64x}+\frac{\sqrt{x^2}\sec^{-1}(x)^3}{8x}-\frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sec^{-1}(x)^2}{4\sqrt{x^2}}-\frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^2}{8\sqrt{x^2}}+\frac{\left(1-\frac{1}{x^2}\right)^2\sqrt{x}}{8x}$$

Antiderivative was successfully verified.

```
[In] Int[((-1 + x^2)^(3/2)*ArcSec[x]^2)/x^5,x]
```

```
[Out] (15*Sqrt[1 - x^(-2)])/(64*Sqrt[x^2]) + (1 - x^(-2))^(3/2)/(32*Sqrt[x^2]) -
(9*Sqrt[x^2]*ArcCsc[x])/(64*x) - (3*Sqrt[x^2]*ArcSec[x])/(8*x^3) + ((1 - x^
(-2))^2*Sqrt[x^2]*ArcSec[x])/(8*x) - (3*Sqrt[1 - x^(-2)]*ArcSec[x]^2)/(8*Sq
rt[x^2]) - ((1 - x^(-2))^(3/2)*ArcSec[x]^2)/(4*Sqrt[x^2]) + (Sqrt[x^2]*ArcS
ec[x]^3)/(8*x)
```

Rule 5242

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> -Dist[Sqrt[x^2]/x, Subst[Int[((e + d*x^2)^p*(a + b*A
rcCos[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rule 4650

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n)/(2*p + 1), x] + (D
ist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 - c^2*x
^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
 GtQ[p, 0]
```

Rule 4648

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(x*Sqrt[d + e*x^2]*(a + b*ArcCos[c*x])^n)/2, x] + (Dist[Sqrt
[d + e*x^2]/(2*Sqrt[1 - c^2*x^2]), Int[(a + b*ArcCos[c*x])^n/Sqrt[1 - c^2*x
^2], x], x] + Dist[(b*c*n*Sqrt[d + e*x^2])/(2*Sqrt[1 - c^2*x^2]), Int[x*(a
+ b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d
```

```
 + e, 0] && GtQ[n, 0]
```

Rule 4642

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> -Simp[(a + b*ArcCos[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rubi steps

$$\int \frac{(-1+x^2)^{3/2}\sec^{-1}(x)^2}{x^5}\,dx = -\frac{\sqrt{x^2}\,\text{Subst}\left(\int (1-x^2)^{3/2}\cos^{-1}(x)^2\,dx, x, \frac{1}{x}\right)}{x}$$

$$= -\frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sec^{-1}(x)^2}{4\sqrt{x^2}} - \frac{\sqrt{x^2}\,\text{Subst}\left(\int x(1-x^2)\cos^{-1}(x)\,dx, x, \frac{1}{x}\right)}{2x} - \frac{\left(3\sqrt{x^2}\right)\text{Subst}\left(\int \sqrt{\right.}$$

$$= \frac{\left(1-\frac{1}{x^2}\right)^2\sqrt{x^2}\sec^{-1}(x)}{8x} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^2}{8\sqrt{x^2}} - \frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sec^{-1}(x)^2}{4\sqrt{x^2}} + \frac{\sqrt{x^2}\,\text{Subst}\left(\int (1\right.}{8x}$$

$$= \frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{32\sqrt{x^2}} - \frac{3\sqrt{x^2}\sec^{-1}(x)}{8x^3} + \frac{\left(1-\frac{1}{x^2}\right)^2\sqrt{x^2}\sec^{-1}(x)}{8x} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^2}{8\sqrt{x^2}} - \frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{4\sqrt{\right.}}$$

$$= \frac{15\sqrt{1-\frac{1}{x^2}}}{64\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{32\sqrt{x^2}} - \frac{3\sqrt{x^2}\sec^{-1}(x)}{8x^3} + \frac{\left(1-\frac{1}{x^2}\right)^2\sqrt{x^2}\sec^{-1}(x)}{8x} - \frac{3\sqrt{1-\frac{1}{x^2}}\sec^{-1}(x)^2}{8\sqrt{x^2}}$$

$$= \frac{15\sqrt{1-\frac{1}{x^2}}}{64\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)^{3/2}}{32\sqrt{x^2}} - \frac{9\sqrt{x^2}\csc^{-1}(x)}{64x} - \frac{3\sqrt{x^2}\sec^{-1}(x)}{8x^3} + \frac{\left(1-\frac{1}{x^2}\right)^2\sqrt{x^2}\sec^{-1}(x)}{8x} - \frac{3}{}$$

Mathematica [A] time = 0.240602, size = 84, normalized size = 0.63

$$\frac{\sqrt{x^2-1}\left(32\sec^{-1}(x)^3 + 4\sec^{-1}(x)\left(\cos\left(4\sec^{-1}(x)\right) - 16\cos\left(2\sec^{-1}(x)\right)\right) + 8\sec^{-1}(x)^2\left(\sin\left(4\sec^{-1}(x)\right) - 8\sin\left(2\sec^{-1}\right.\right.\right.}{256\sqrt{1-\frac{1}{x^2}}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((-1 + x^2)^(3/2)*ArcSec[x]^2)/x^5,x]
```

```
[Out] (Sqrt[-1 + x^2]*(32*ArcSec[x]^3 + 4*ArcSec[x]*(-16*Cos[2*ArcSec[x]] + Cos[4
*ArcSec[x]]) + 32*Sin[2*ArcSec[x]] - Sin[4*ArcSec[x]] + 8*ArcSec[x]^2*(-8*S
in[2*ArcSec[x]] + Sin[4*ArcSec[x]])))/(256*Sqrt[1 - x^(-2)]*x)
```

Maple [C] time = 0.355, size = 327, normalized size = 2.5

$$\frac{x\,(\text{arcsec}\,(x))^3}{8}\sqrt{\frac{x^2-1}{x^2}}\frac{1}{\sqrt{x^2-1}} - \frac{4\,i\text{arcsec}\,(x) + 8\,(\text{arcsec}\,(x))^2 - 1}{512\,x^4}\left(i\sqrt{\frac{x^2-1}{x^2}}x^5 - 8\,i\sqrt{\frac{x^2-1}{x^2}}x^3 + 4\,x^4 + 8\,i\sqrt{\frac{x^2-1}{x^2}}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-1)^(3/2)*arcsec(x)^2/x^5,x)
```

```
[Out] 1/8/(x^2-1)^(1/2)*((x^2-1)/x^2)^(1/2)*x*arcsec(x)^3-1/512/(x^2-1)^(1/2)/x^4
*(I*((x^2-1)/x^2)^(1/2)*x^5-8*I*((x^2-1)/x^2)^(1/2)*x^3+4*x^4+8*I*((x^2-1)/
x^2)^(1/2)*x-12*x^2+8)*(4*I*arcsec(x)+8*arcsec(x)^2-1)-1/16/(x^2-1)^(1/2)/x
^2*(I*((x^2-1)/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/2)*x+2*x^2-2)*(2*arcsec(
x)^2-1+2*I*arcsec(x))+1/16/(x^2-1)^(1/2)/x^2*(I*((x^2-1)/x^2)^(1/2)*x^3-2*I
*((x^2-1)/x^2)^(1/2)*x-2*x^2+2)*(2*arcsec(x)^2-1-2*I*arcsec(x))+1/512/(x^2-
1)^(1/2)/x^4*(I*((x^2-1)/x^2)^(1/2)*x^5-8*I*((x^2-1)/x^2)^(1/2)*x^3-4*x^4+8
*I*((x^2-1)/x^2)^(1/2)*x+12*x^2-8)*(-4*I*arcsec(x)+8*arcsec(x)^2-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2-1)^{\frac{3}{2}} \operatorname{arcsec}(x)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - 1)^(3/2)*arcsec(x)^2/x^5, x)
```

Fricas [A] time = 2.39838, size = 163, normalized size = 1.23

$$\frac{8\,x^4 \operatorname{arcsec}(x)^3 + \left(17\,x^4 - 40\,x^2 + 8\right) \operatorname{arcsec}(x) - \left(8\left(5\,x^2 - 2\right) \operatorname{arcsec}(x)^2 - 17\,x^2 + 2\right)\sqrt{x^2-1}}{64\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/64*(8*x^4*arcsec(x)^3 + (17*x^4 - 40*x^2 + 8)*arcsec(x) - (8*(5*x^2 - 2)*
arcsec(x)^2 - 17*x^2 + 2)*sqrt(x^2 - 1))/x^4
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**(3/2)*asec(x)**2/x**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2-1)^{\frac{3}{2}} \operatorname{arcsec}(x)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-1)^(3/2)*arcsec(x)^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((x^2 - 1)^(3/2)*arcsec(x)^2/x^5, x)
```

3.694 $\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx$

Optimal. Leaf size=110

$$\frac{2(1-21x^2)}{27(x^2)^{3/2}}+\frac{(x^2-1)^{3/2}\sec^{-1}(x)^3}{3x^3}+\frac{(x^2-1)\sec^{-1}(x)^2}{3(x^2)^{3/2}}+\frac{2\sec^{-1}(x)^2}{3\sqrt{x^2}}-\frac{2(x^2-1)^{3/2}\sec^{-1}(x)}{9x^3}-\frac{4\sqrt{x^2-1}\sec^{-1}(x)}{3x}$$

```
[Out] (2*(1 - 21*x^2))/(27*(x^2)^(3/2)) - (4*Sqrt[-1 + x^2]*ArcSec[x])/(3*x) - (2
*(-1 + x^2)^(3/2)*ArcSec[x])/(9*x^3) + (2*ArcSec[x]^2)/(3*Sqrt[x^2]) + ((-1
 + x^2)*ArcSec[x]^2)/(3*(x^2)^(3/2)) + ((-1 + x^2)^(3/2)*ArcSec[x]^3)/(3*x^
3)
```

Rubi [A] time = 0.199997, antiderivative size = 146, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {5242, 4678, 4650, 4620, 8}

$$\frac{2\sqrt{x^2}}{27x^4}-\frac{14}{9\sqrt{x^2}}+\frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)^3}{3x}+\frac{\left(1-\frac{1}{x^2}\right)\sec^{-1}(x)^2}{3\sqrt{x^2}}+\frac{2\sec^{-1}(x)^2}{3\sqrt{x^2}}-\frac{2\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)}{9x}-\frac{4\sqrt{1-\frac{1}{x^2}}\sqrt{}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[-1 + x^2]*ArcSec[x]^3)/x^4,x]
```

```
[Out] -14/(9*Sqrt[x^2]) + (2*Sqrt[x^2])/(27*x^4) - (4*Sqrt[1 - x^(-2)]*Sqrt[x^2]*
ArcSec[x])/(3*x) - (2*(1 - x^(-2))^(3/2)*Sqrt[x^2]*ArcSec[x])/(9*x) + (2*Ar
cSec[x]^2)/(3*Sqrt[x^2]) + ((1 - x^(-2))*ArcSec[x]^2)/(3*Sqrt[x^2]) + ((1 -
 x^(-2))^(3/2)*Sqrt[x^2]*ArcSec[x]^3)/(3*x)
```

Rule 5242

```
Int[((a_.) + ArcSec[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)
^2)^(p_), x_Symbol] :> -Dist[Sqrt[x^2]/x, Subst[Int[((e + d*x^2)^p*(a + b*A
rcCos[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x], x] /; FreeQ[{a, b, c, d, e,
n}, x] && IGtQ[n, 0] && EqQ[c^2*d + e, 0] && IntegerQ[m] && IntegerQ[p + 1/
2] && GtQ[e, 0] && LtQ[d, 0]
```

Rule 4678

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCos[c*x])^n)/(2*e*(p +
 1)), x] - Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1
- c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n
 - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n
, 0] && NeQ[p, -1]
```

Rule 4650

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n)/(2*p + 1), x] + (D
ist[(2*d*p)/(2*p + 1), Int[(d + e*x^2)^(p - 1)*(a + b*ArcCos[c*x])^n, x], x
] + Dist[(b*c*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/((2*p + 1)*(1 - c^2*x
^2)^FracPart[p]), Int[x*(1 - c^2*x^2)^(p - 1/2)*(a + b*ArcCos[c*x])^(n - 1)
, x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] &&
```

```
GtQ[p, 0]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sqrt{-1+x^2}\sec^{-1}(x)^3}{x^4}\,dx = -\frac{\sqrt{x^2}\,\mathrm{Subst}\left(\int x\sqrt{1-x^2}\cos^{-1}(x)^3\,dx, x, \frac{1}{x}\right)}{x}$$

$$= \frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)^3}{3x} + \frac{\sqrt{x^2}\,\mathrm{Subst}\left(\int \left(1-x^2\right)\cos^{-1}(x)^2\,dx, x, \frac{1}{x}\right)}{x}$$

$$= \frac{\left(1-\frac{1}{x^2}\right)\sec^{-1}(x)^2}{3\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)^3}{3x} + \frac{\left(2\sqrt{x^2}\right)\mathrm{Subst}\left(\int x\sqrt{1-x^2}\cos^{-1}(x)\,dx\right.}{3x}$$

$$= -\frac{2\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)}{9x} + \frac{2\sec^{-1}(x)^2}{3\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)\sec^{-1}(x)^2}{3\sqrt{x^2}} + \frac{\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x}{3x}$$

$$= -\frac{2}{9\sqrt{x^2}} + \frac{2\sqrt{x^2}}{27x^4} - \frac{4\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{3x} - \frac{2\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)}{9x} + \frac{2\sec^{-1}(x)^2}{3\sqrt{x^2}} + \ldots$$

$$= -\frac{14}{9\sqrt{x^2}} + \frac{2\sqrt{x^2}}{27x^4} - \frac{4\sqrt{1-\frac{1}{x^2}}\sqrt{x^2}\sec^{-1}(x)}{3x} - \frac{2\left(1-\frac{1}{x^2}\right)^{3/2}\sqrt{x^2}\sec^{-1}(x)}{9x} + \frac{2\sec^{-1}(x)^2}{3\sqrt{x^2}} + \ldots$$

Mathematica [A] time = 0.0896488, size = 92, normalized size = 0.84

$$\frac{2\sqrt{1-\frac{1}{x^2}}x\left(1-21x^2\right)+9\left(x^2-1\right)^2\sec^{-1}(x)^3+9\sqrt{1-\frac{1}{x^2}}x\left(3x^2-1\right)\sec^{-1}(x)^2-6\left(7x^4-8x^2+1\right)\sec^{-1}(x)}{27x^3\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^2]*ArcSec[x]^3)/x^4,x]
```

```
[Out] (2*Sqrt[1 - x^(-2)]*x*(1 - 21*x^2) - 6*(1 - 8*x^2 + 7*x^4)*ArcSec[x] + 9*Sq
rt[1 - x^(-2)]*x*(-1 + 3*x^2)*ArcSec[x]^2 + 9*(-1 + x^2)^2*ArcSec[x]^3)/(27
*x^3*Sqrt[-1 + x^2])
```

Maple [C] time = 0.588, size = 1153, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x)
```

```
[Out] 1/48/(x^2-1)^(1/2)/x^3*(I*((x^2-1)/x^2)^(1/2)*x^5-8*I*((x^2-1)/x^2)^(1/2)*x
^3+4*x^4+8*I*((x^2-1)/x^2)^(1/2)*x-12*x^2+8)*arcsec(x)^3+1/24/(x^2-1)^(1/2)
/x*(I*((x^2-1)/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/2)*x+2*x^2-2)*arcsec(x)^
3+10/27*(x^2-1)^(1/2)*(((x^2-1)/x^2)^(1/2)*x^3-3*I*x^2-4*((x^2-1)/x^2)^(1/2
)*x+4*I)/(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)/x-3/4*(x^2-1)^(1/2)*(((x^2-1)/x^2
)^(1/2)*x-I)*x/(-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)-1/72/(x^2-1)^(1/2)/x^3*(I*(
(x^2-1)/x^2)^(1/2)*x^5-8*I*((x^2-1)/x^2)^(1/2)*x^3+4*x^4+8*I*((x^2-1)/x^2)^
(1/2)*x-12*x^2+8)*arcsec(x)-13/36/(x^2-1)^(1/2)/x*(I*((x^2-1)/x^2)^(1/2)*x^
3-2*I*((x^2-1)/x^2)^(1/2)*x+2*x^2-2)*arcsec(x)+I*(x^2-1)^(1/2)*x^5/(216*I*(
(x^2-1)/x^2)^(1/2)*x^5-1728*I*((x^2-1)/x^2)^(1/2)*x^3+864*x^4+1728*I*((x^2-
1)/x^2)^(1/2)*x-2592*x^2+1728)-1/24/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x^
3-2*I*((x^2-1)/x^2)^(1/2)*x-2*x^2+2)*arcsec(x)^3/x+10*I*(x^2-1)^(1/2)*x^3/(
27*I*((x^2-1)/x^2)^(1/2)*x^3-54*I*((x^2-1)/x^2)^(1/2)*x+54*x^2-54)+13/36/(x
^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x^3-2*I*((x^2-1)/x^2)^(1/2)*x-2*x^2+2)*a
rcsec(x)/x-1/48/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x^5-8*I*((x^2-1)/x^2)^
(1/2)*x^3-4*x^4+8*I*((x^2-1)/x^2)^(1/2)*x+12*x^2-8)*arcsec(x)^3/x^3-1/6/(x^
2-1)^(1/2)*(((x^2-1)/x^2)^(1/2)*x^3+2*I*x^2-2*((x^2-1)/x^2)^(1/2)*x-2*I)*ar
csec(x)^2/x+1/72/(x^2-1)^(1/2)*(I*((x^2-1)/x^2)^(1/2)*x^5-8*I*((x^2-1)/x^2)
^(1/2)*x^3-4*x^4+8*I*((x^2-1)/x^2)^(1/2)*x+12*x^2-8)*arcsec(x)/x^3-1/48/(x^
2-1)^(1/2)*(((x^2-1)/x^2)^(1/2)*x^5+4*I*x^4-8*((x^2-1)/x^2)^(1/2)*x^3-12*I*
x^2+8*((x^2-1)/x^2)^(1/2)*x+8*I)*arcsec(x)^2/x^3-1/48/(x^2-1)^(1/2)/x^3*(((
x^2-1)/x^2)^(1/2)*x^5-4*I*x^4-8*((x^2-1)/x^2)^(1/2)*x^3+12*I*x^2+8*((x^2-1)
/x^2)^(1/2)*x-8*I)*arcsec(x)^2-1/6/(x^2-1)^(1/2)/x*(((x^2-1)/x^2)^(1/2)*x^3
-2*I*x^2-2*((x^2-1)/x^2)^(1/2)*x+2*I)*arcsec(x)^2+3/8/(x^2-1)^(1/2)*((x^2-1
)/x^2)^(1/2)*x^2*arcsec(x)^2+1/216*(x^2-1)^(1/2)*(((x^2-1)/x^2)^(1/2)*x^5-5
*I*x^4-12*((x^2-1)/x^2)^(1/2)*x^3+20*I*x^2+16*((x^2-1)/x^2)^(1/2)*x-16*I)/(
-I*((x^2-1)/x^2)^(1/2)*x+x^2-1)/x^3
```

Maxima [A] time = 2.53848, size = 126, normalized size = 1.15

$$\frac{\left(x^2-1\right)^{\frac{3}{2}}\operatorname{arcsec}\left(x\right)^3}{3\,x^3}+\frac{\left(3\,x^2-1\right)\operatorname{arcsec}\left(x\right)^2}{3\,x^3}-\frac{2\left(\left(21\,x^2-1\right)\sqrt{x+1}\sqrt{x-1}+3\left(7\,x^4-8\,x^2+1\right)\arctan\left(\sqrt{x+1}\sqrt{x-1}\right)\right)}{27\,\sqrt{x+1}\sqrt{x-1}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 - 1)^(3/2)*arcsec(x)^3/x^3 + 1/3*(3*x^2 - 1)*arcsec(x)^2/x^3 - 2/2
7*((21*x^2 - 1)*sqrt(x + 1)*sqrt(x - 1) + 3*(7*x^4 - 8*x^2 + 1)*arctan(sqrt
(x + 1)*sqrt(x - 1)))/(sqrt(x + 1)*sqrt(x - 1)*x^3)
```

Fricas [A] time = 2.43214, size = 163, normalized size = 1.48

$$\frac{9\left(3\,x^2-1\right)\operatorname{arcsec}\left(x\right)^2-42\,x^2+3\left(3\left(x^2-1\right)\operatorname{arcsec}\left(x\right)^3-2\left(7\,x^2-1\right)\operatorname{arcsec}\left(x\right)\right)\sqrt{x^2-1}+2}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(9*(3*x^2 - 1)*arcsec(x)^2 - 42*x^2 + 3*(3*(x^2 - 1)*arcsec(x)^3 - 2*(
7*x^2 - 1)*arcsec(x))*sqrt(x^2 - 1) + 2)/x^3
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x)**3*(x**2-1)**(1/2)/x**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-1}\,\mathrm{arcsec}\,(x)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x)^3*(x^2-1)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - 1)*arcsec(x)^3/x^4, x)
```

3.695 $\int \sin^{-1}\left(\sqrt{\frac{-a+x}{a+x}}\right) dx$

Optimal. Leaf size=55

$$(a+x)\sin^{-1}\left(\sqrt{\frac{x-a}{a+x}}\right)-\frac{\sqrt{2}a\sqrt{\frac{x-a}{a+x}}}{\sqrt{\frac{a}{a+x}}}$$

```
[Out] -((Sqrt[2]*a*Sqrt[(-a + x)/(a + x)])/Sqrt[a/(a + x)]) + (a + x)*ArcSin[Sqrt
[(-a + x)/(a + x)]]
```

Rubi [B] time = 0.842875, antiderivative size = 118, normalized size of antiderivative = 2.15, number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4840, 12, 6677, 6720, 385, 217, 206}

$$-\sqrt{2}\sqrt{\frac{a}{a+x}}\sqrt{-\frac{a-x}{a+x}}(a+x)+x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)-\frac{a\sqrt{\frac{a}{a+x}}\tanh^{-1}\left(\frac{\sqrt{-\frac{a-x}{a+x}}}{\sqrt{2}\sqrt{-\frac{a}{a+x}}}\right)}{\sqrt{-\frac{a}{a+x}}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[Sqrt[(-a + x)/(a + x)]],x]
```

```
[Out] -(Sqrt[2]*Sqrt[a/(a + x)]*Sqrt[-((a - x)/(a + x))]*(a + x)) + x*ArcSin[Sqrt
[-((a - x)/(a + x))]] - (a*Sqrt[a/(a + x)]*ArcTanh[Sqrt[-((a - x)/(a + x))]
/(Sqrt[2]*Sqrt[-(a/(a + x))])])/Sqrt[-(a/(a + x))]
```

Rule 4840

```
Int[ArcSin[u_], x_Symbol] :> Simp[x*ArcSin[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/Sqrt[1 - u^2], x], x] /; InverseFunctionFreeQ[u, x] && !Functio
nOfExponentialQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6677

```
Int[(u_)*((c_.)*((a_.) + (b_.)*(x_))^(n_))^(p_), x_Symbol] :> Dist[(c^IntPa
rt[p]*(c*(a + b*x)^n)^FracPart[p])/(a + b*x)^(n*FracPart[p]), Int[u*(a + b*
x)^(n*p), x], x] /; FreeQ[{a, b, c, n, p}, x] && !IntegerQ[p] && !MatchQ[
u, x^(n1_.)*(v_.) /; EqQ[n, n1 + 1]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sin^{-1}\left(\sqrt{\frac{-a+x}{a+x}}\right) dx = x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \int \frac{x\left(\frac{a}{a+x}\right)^{3/2}}{\sqrt{2}a\sqrt{\frac{-a+x}{a+x}}}\,dx$$

$$= x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{\int \frac{x\left(\frac{a}{a+x}\right)^{3/2}}{\sqrt{\frac{-a+x}{a+x}}}\,dx}{\sqrt{2}a}$$

$$= x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{\left(\sqrt{\frac{a}{a+x}}\sqrt{a+x}\right)\int \frac{x}{\sqrt{\frac{-a+x}{a+x}}(a+x)^{3/2}}\,dx}{\sqrt{2}}$$

$$= x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \left(a\sqrt{\frac{a}{a+x}}\sqrt{a+x}\right)\text{Subst}\left(\int \frac{1+x^2}{\sqrt{-\frac{a}{-1+x^2}}\left(-1+x^2\right)^2}\,dx, x, \sqrt{\frac{-a+x}{a+x}}\right)$$

$$= x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{\left(a\sqrt{\frac{a}{a+x}}\right)\text{Subst}\left(\int \frac{1+x^2}{\left(-1+x^2\right)^{3/2}}\,dx, x, \sqrt{\frac{-a+x}{a+x}}\right)}{\sqrt{-\frac{a}{a+x}}}$$

$$= -\sqrt{2}\sqrt{\frac{a}{a+x}}\sqrt{-\frac{a-x}{a+x}}(a+x) + x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{\left(a\sqrt{\frac{a}{a+x}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}}\,dx, x, \sqrt{\cdots}\right.}{\sqrt{-\frac{a}{a+x}}}$$

$$= -\sqrt{2}\sqrt{\frac{a}{a+x}}\sqrt{-\frac{a-x}{a+x}}(a+x) + x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{\left(a\sqrt{\frac{a}{a+x}}\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{\sqrt{\cdots}}{\sqrt{2}\sqrt{-\cdots}}\right.}{\sqrt{-\frac{a}{a+x}}}$$

$$= -\sqrt{2}\sqrt{\frac{a}{a+x}}\sqrt{-\frac{a-x}{a+x}}(a+x) + x\sin^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{a\sqrt{\frac{a}{a+x}}\tanh^{-1}\left(\frac{\sqrt{-\frac{a-x}{a+x}}}{\sqrt{2}\sqrt{-\frac{a}{a+x}}}\right)}{\sqrt{-\frac{a}{a+x}}}$$

Mathematica [A] time = 0.143833, size = 99, normalized size = 1.8

$$x\sin^{-1}\left(\sqrt{\frac{x-a}{a+x}}\right) + \frac{\sqrt{\frac{a}{a+x}}\left(\sqrt{2}\sqrt{a}\sqrt{x-a}\tan^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{2}\sqrt{a}}\right) + 2a - 2x\right)}{\sqrt{2}\sqrt{\frac{x-a}{a+x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[Sqrt[(-a + x)/(a + x)]],x]
```

```
[Out] x*ArcSin[Sqrt[(-a + x)/(a + x)]] + (Sqrt[a/(a + x)]*(2*a - 2*x + Sqrt[2]*Sq
rt[a]*Sqrt[-a + x]*ArcTan[Sqrt[-a + x]/(Sqrt[2]*Sqrt[a])]))/(Sqrt[2]*Sqrt[(
-a + x)/(a + x)])
```

Maple [A] time = 0.025, size = 85, normalized size = 1.6

$$x\arcsin\left(\sqrt{\frac{-a+x}{a+x}}\right)+\frac{\sqrt{2}}{2}\sqrt{-a+x}\sqrt{\frac{a}{a+x}}\left(\sqrt{a}\sqrt{2}\arctan\left(\frac{\sqrt{2}}{2}\sqrt{-a+x}\frac{1}{\sqrt{a}}\right)-2\sqrt{-a+x}\right)\frac{1}{\sqrt{\frac{-a+x}{a+x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(((-a+x)/(a+x))^(1/2)),x)
```

```
[Out] x*arcsin(((-a+x)/(a+x))^(1/2))+1/2/((-a+x)/(a+x))^(1/2)*(-a+x)^(1/2)*2^(1/2
)*(a/(a+x))^(1/2)*(a^(1/2)*2^(1/2)*arctan(1/2*(-a+x)^(1/2)*2^(1/2)/a^(1/2))
-2*(-a+x)^(1/2))
```

Maxima [B] time = 1.4494, size = 139, normalized size = 2.53

$$a\left(\frac{2\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)}{\frac{a-x}{a+x}+1}+\frac{\sqrt{\frac{a-x}{a+x}+1}}{\sqrt{-\frac{a-x}{a+x}}+1}+\frac{\sqrt{\frac{a-x}{a+x}+1}}{\sqrt{-\frac{a-x}{a+x}}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(((-a+x)/(a+x))^(1/2)),x, algorithm="maxima")
```

```
[Out] a*(2*arcsin(sqrt(-(a - x)/(a + x)))/((a - x)/(a + x) + 1) + sqrt((a - x)/(a
 + x) + 1)/(sqrt(-(a - x)/(a + x)) + 1) + sqrt((a - x)/(a + x) + 1)/(sqrt(-
(a - x)/(a + x)) - 1))
```

Fricas [A] time = 2.53532, size = 132, normalized size = 2.4

$$-\sqrt{2}(a+x)\sqrt{-\frac{a-x}{a+x}}\sqrt{\frac{a}{a+x}}+(a+x)\arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(((-a+x)/(a+x))^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*(a + x)*sqrt(-(a - x)/(a + x))*sqrt(a/(a + x)) + (a + x)*arcsin(sq
rt(-(a - x)/(a + x)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(((-a+x)/(a+x))**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arcsin\left(\sqrt{-\frac{a-x}{a+x}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(((-a+x)/(a+x))^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(arcsin(sqrt(-(a - x)/(a + x))), x)
```

3.696 $\int \tan^{-1}\left(\sqrt{\frac{-a+x}{a+x}}\right) dx$

Optimal. Leaf size=40

$$x\tan^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)-a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

```
[Out] x*ArcTan[Sqrt[-((a - x)/(a + x))]] - a*ArcTanh[Sqrt[-((a - x)/(a + x))]]
```

Rubi [A] time = 0.0425593, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {5203, 12, 1961, 208}

$$x\tan^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)-a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[(-a + x)/(a + x)]],x]
```

```
[Out] x*ArcTan[Sqrt[-((a - x)/(a + x))]] - a*ArcTanh[Sqrt[-((a - x)/(a + x))]]
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1961

```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(
u /. x -> (-(a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n
 + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b,
 c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Inte
gerQ[r]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \tan^{-1}\left(\sqrt{\frac{-a+x}{a+x}}\right) dx = x\tan^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \int \frac{a}{2\sqrt{\frac{-a+x}{a+x}}(a+x)} dx$$
$$= x\tan^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{1}{2}a\int \frac{1}{\sqrt{\frac{-a+x}{a+x}}(a+x)} dx$$
$$= x\tan^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - (2a^2)\operatorname{Subst}\left(\int \frac{1}{2a-2ax^2} dx, x, \sqrt{\frac{-a+x}{a+x}}\right)$$
$$= x\tan^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right) - a\tanh^{-1}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

Mathematica [A] time = 0.0488343, size = 71, normalized size = 1.78

$$x\tan^{-1}\left(\sqrt{\frac{x-a}{a+x}}\right) - \frac{a\sqrt{x-a}\tanh^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{a+x}}\right)}{\sqrt{\frac{x-a}{a+x}}\sqrt{a+x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[(-a + x)/(a + x)]],x]
```

```
[Out] x*ArcTan[Sqrt[(-a + x)/(a + x)]] - (a*Sqrt[-a + x]*ArcTanh[Sqrt[-a + x]/Sqr
t[a + x]])/(Sqrt[(-a + x)/(a + x)]*Sqrt[a + x])
```

Maple [A] time = 0.013, size = 64, normalized size = 1.6

$$x\arctan\left(\sqrt{\frac{-a+x}{a+x}}\right) - \frac{(-a+x)a}{2}\ln\left(x+\sqrt{-a^2+x^2}\right)\frac{1}{\sqrt{\frac{-a+x}{a+x}}}\frac{1}{\sqrt{(a+x)(-a+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(((-a+x)/(a+x))^(1/2)),x)
```

```
[Out] x*arctan(((-a+x)/(a+x))^(1/2))-1/2*(-a+x)*a*ln(x+(-a^2+x^2)^(1/2))/((-a+x)/
(a+x))^(1/2)/((a+x)*(-a+x))^(1/2)
```

Maxima [B] time = 1.41736, size = 120, normalized size = 3.

$$\frac{1}{2}a\left(\frac{4\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right)}{\frac{a-x}{a+x}+1} - 2\arctan\left(\sqrt{-\frac{a-x}{a+x}}\right) - \log\left(\sqrt{-\frac{a-x}{a+x}}+1\right) + \log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(((-a+x)/(a+x))^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*a*(4*arctan(sqrt(-(a - x)/(a + x)))/((a - x)/(a + x) + 1) - 2*arctan(sq
rt(-(a - x)/(a + x))) - log(sqrt(-(a - x)/(a + x)) + 1) + log(sqrt(-(a - x)
```

```
/(a + x)) - 1))
```

Fricas [A] time = 2.88018, size = 154, normalized size = 3.85

$$x \arctan\left(\sqrt{-\frac{a-x}{a+x}}\right) - \frac{1}{2}\, a \log\left(\sqrt{-\frac{a-x}{a+x}} + 1\right) + \frac{1}{2}\, a \log\left(\sqrt{-\frac{a-x}{a+x}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(((-a+x)/(a+x))^(1/2)),x, algorithm="fricas")
```

```
[Out] x*arctan(sqrt(-(a - x)/(a + x))) - 1/2*a*log(sqrt(-(a - x)/(a + x)) + 1) +
1/2*a*log(sqrt(-(a - x)/(a + x)) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{atan}\left(\sqrt{\frac{-a+x}{a+x}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(((-a+x)/(a+x))**(1/2)),x)
```

```
[Out] Integral(atan(sqrt((-a + x)/(a + x))), x)
```

Giac [A] time = 1.11585, size = 66, normalized size = 1.65

$$\frac{1}{2}\, a \log\left(\left|-x + \sqrt{-a^2 + x^2}\right|\right) \operatorname{sgn}(a+x) + x \arctan\left(\frac{\sqrt{-a^2 + x^2} \operatorname{sgn}(a+x)}{a+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(((-a+x)/(a+x))^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*a*log(abs(-x + sqrt(-a^2 + x^2)))*sgn(a + x) + x*arctan(sqrt(-a^2 + x^2
)*sgn(a + x)/(a + x))
```

3.697 $\int \frac{\tan^{-1}(x)}{(1+x)^3} dx$

Optimal. Leaf size=39

$$-\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{4(x+1)}+\frac{1}{4}\log(x+1)-\frac{\tan^{-1}(x)}{2(x+1)^2}$$

```
[Out] -1/(4*(1 + x)) - ArcTan[x]/(2*(1 + x)^2) + Log[1 + x]/4 - Log[1 + x^2]/8
```

Rubi [A] time = 0.0293697, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4862, 710, 801, 260}

$$-\frac{1}{8}\log\left(x^2+1\right)-\frac{1}{4(x+1)}+\frac{1}{4}\log(x+1)-\frac{\tan^{-1}(x)}{2(x+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x]/(1 + x)^3,x]
```

```
[Out] -1/(4*(1 + x)) - ArcTan[x]/(2*(1 + x)^2) + Log[1 + x]/4 - Log[1 + x^2]/8
```

Rule 4862

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[((d + e*x)^(q + 1)*(a + b*ArcTan[c*x]))/(e*(q + 1)), x] - Dist[(b*
c)/(e*(q + 1)), Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b,
 c, d, e, q}, x] && NeQ[q, -1]
```

Rule 710

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d
 + e*x)^(m + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/(c*d^2 + a*e^2), In
t[((d + e*x)^(m + 1)*(d - e*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m
}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\tan^{-1}(x)}{(1+x)^3} dx = -\frac{\tan^{-1}(x)}{2(1+x)^2} + \frac{1}{2}\int \frac{1}{(1+x)^2\left(1+x^2\right)} dx$$
$$= -\frac{1}{4(1+x)} - \frac{\tan^{-1}(x)}{2(1+x)^2} + \frac{1}{4}\int \frac{1-x}{(1+x)\left(1+x^2\right)} dx$$
$$= -\frac{1}{4(1+x)} - \frac{\tan^{-1}(x)}{2(1+x)^2} + \frac{1}{4}\int \left(\frac{1}{1+x} - \frac{x}{1+x^2}\right) dx$$
$$= -\frac{1}{4(1+x)} - \frac{\tan^{-1}(x)}{2(1+x)^2} + \frac{1}{4}\log(1+x) - \frac{1}{4}\int \frac{x}{1+x^2} dx$$
$$= -\frac{1}{4(1+x)} - \frac{\tan^{-1}(x)}{2(1+x)^2} + \frac{1}{4}\log(1+x) - \frac{1}{8}\log\left(1+x^2\right)$$

Mathematica [A] time = 0.0277444, size = 35, normalized size = 0.9

$$\frac{1}{8}\left(-\log\left(x^2+1\right) - \frac{2}{x+1} + 2\log(x+1) - \frac{4\tan^{-1}(x)}{(x+1)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x]/(1 + x)^3,x]
```

```
[Out] (-2/(1 + x) - (4*ArcTan[x])/(1 + x)^2 + 2*Log[1 + x] - Log[1 + x^2])/8
```

Maple [A] time = 0.009, size = 32, normalized size = 0.8

$$-\frac{1}{4+4x} - \frac{\arctan(x)}{2(1+x)^2} + \frac{\ln(1+x)}{4} - \frac{\ln\left(x^2+1\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)/(1+x)^3,x)
```

```
[Out] -1/4/(1+x)-1/2*arctan(x)/(1+x)^2+1/4*ln(1+x)-1/8*ln(x^2+1)
```

Maxima [A] time = 1.41133, size = 42, normalized size = 1.08

$$-\frac{1}{4(x+1)} - \frac{\arctan(x)}{2(x+1)^2} - \frac{1}{8}\log\left(x^2+1\right) + \frac{1}{4}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/(1+x)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(x + 1) - 1/2*arctan(x)/(x + 1)^2 - 1/8*log(x^2 + 1) + 1/4*log(x + 1)
```

Fricas [A] time = 2.79862, size = 146, normalized size = 3.74

$$-\frac{(x^2+2x+1)\log(x^2+1)-2(x^2+2x+1)\log(x+1)+2x+4\arctan(x)+2}{8(x^2+2x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/(1+x)^3,x, algorithm="fricas")
```

```
[Out] -1/8*((x^2 + 2*x + 1)*log(x^2 + 1) - 2*(x^2 + 2*x + 1)*log(x + 1) + 2*x + 4
*arctan(x) + 2)/(x^2 + 2*x + 1)
```

Sympy [B] time = 0.662382, size = 153, normalized size = 3.92

$$\frac{2x^2\log(x+1)}{8x^2+16x+8}-\frac{x^2\log(x^2+1)}{8x^2+16x+8}+\frac{x^2}{8x^2+16x+8}+\frac{4x\log(x+1)}{8x^2+16x+8}-\frac{2x\log(x^2+1)}{8x^2+16x+8}+\frac{2\log(x+1)}{8x^2+16x+8}-\frac{\log(x^2+1)}{8x^2+16x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)/(1+x)**3,x)
```

```
[Out] 2*x**2*log(x + 1)/(8*x**2 + 16*x + 8) - x**2*log(x**2 + 1)/(8*x**2 + 16*x +
 8) + x**2/(8*x**2 + 16*x + 8) + 4*x*log(x + 1)/(8*x**2 + 16*x + 8) - 2*x*l
og(x**2 + 1)/(8*x**2 + 16*x + 8) + 2*log(x + 1)/(8*x**2 + 16*x + 8) - log(x
**2 + 1)/(8*x**2 + 16*x + 8) - 4*atan(x)/(8*x**2 + 16*x + 8) - 1/(8*x**2 +
16*x + 8)
```

Giac [A] time = 1.07559, size = 43, normalized size = 1.1

$$-\frac{1}{4(x+1)}-\frac{\arctan(x)}{2(x+1)^2}-\frac{1}{8}\log(x^2+1)+\frac{1}{4}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)/(1+x)^3,x, algorithm="giac")
```

```
[Out] -1/4/(x + 1) - 1/2*arctan(x)/(x + 1)^2 - 1/8*log(x^2 + 1) + 1/4*log(abs(x +
 1))
```

3.698 $\int -\frac{\tan^{-1}(a-x)}{a+x}\,dx$

Optimal. Leaf size=122

$$-\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2}{1-i(a-x)}\right)+\frac{1}{2}i\text{PolyLog}\left(2,1+\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right)+\log\left(\frac{2}{1-i(a-x)}\right)\tan^{-1}(a-x)-\log\left(\right.$$

```
[Out] ArcTan[a - x]*Log[2/(1 - I*(a - x))] - ArcTan[a - x]*Log[(-2*(a + x))/((I -
 2*a)*(1 - I*(a - x)))] - (I/2)*PolyLog[2, 1 - 2/(1 - I*(a - x))] + (I/2)*P
olyLog[2, 1 + (2*(a + x))/((I - 2*a)*(1 - I*(a - x)))]
```

Rubi [A] time = 0.0914153, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5047, 4856, 2402, 2315, 2447}

$$-\frac{1}{2}i\text{PolyLog}\left(2,1-\frac{2}{1-i(a-x)}\right)+\frac{1}{2}i\text{PolyLog}\left(2,1+\frac{2(a+x)}{(-2a+i)(1-i(a-x))}\right)+\log\left(\frac{2}{1-i(a-x)}\right)\tan^{-1}(a-x)-\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[-(ArcTan[a - x]/(a + x)),x]
```

```
[Out] ArcTan[a - x]*Log[2/(1 - I*(a - x))] - ArcTan[a - x]*Log[(-2*(a + x))/((I -
 2*a)*(1 - I*(a - x)))] - (I/2)*PolyLog[2, 1 - 2/(1 - I*(a - x))] + (I/2)*P
olyLog[2, 1 + (2*(a + x))/((I - 2*a)*(1 - I*(a - x)))]
```

Rule 5047

```
Int[((a_.) + ArcTan[(c_) + (d_.)*(x_)]*(b_.))^(p_.)*((e_.) + (f_.)*(x_))^(m
_.), x_Symbol] :> Dist[1/d, Subst[Int[((d*e - c*f)/d + (f*x)/d)^m*(a + b*Ar
cTan[x])^p, x], x, c + d*x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && IG
tQ[p, 0]
```

Rule 4856

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> -S
imp[((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e, x] + (Dist[(b*c)/e, Int[Log
[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Dist[(b*c)/e, Int[Log[(2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))]/(1 + c^2*x^2), x], x] + Simp[((a + b*ArcTan[c
*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e, x]) /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
```

```
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

Rubi steps

$$\int -\frac{\tan^{-1}(a-x)}{a+x}\,dx = \text{Subst}\left(\int \frac{\tan^{-1}(x)}{2a-x}\,dx, x, a-x\right)$$

$$= \tan^{-1}(a-x)\log\left(\frac{2}{1-i(a-x)}\right) - \tan^{-1}(a-x)\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right) - \text{Subst}\left(\int \frac{\log\left(\frac{}{1}\right.}{1+}\right.$$

$$= \tan^{-1}(a-x)\log\left(\frac{2}{1-i(a-x)}\right) - \tan^{-1}(a-x)\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right) + \frac{1}{2}i\text{Li}_2\left(1+\frac{}{(i-2}\right.$$

$$= \tan^{-1}(a-x)\log\left(\frac{2}{1-i(a-x)}\right) - \tan^{-1}(a-x)\log\left(-\frac{2(a+x)}{(i-2a)(1-i(a-x))}\right) - \frac{1}{2}i\text{Li}_2\left(1-\frac{2}{1-i(}\right.$$

Mathematica [A] time = 0.0304267, size = 105, normalized size = 0.86

$$-\frac{1}{2}i\left(\text{PolyLog}\left(2,\frac{a-x+i}{2a+i}\right) - \text{PolyLog}\left(2,\frac{-a+x+i}{-2a+i}\right) - \log(1+i(a-x))\log\left(\frac{a+x}{2a-i}\right) + \log(-ia+ix+1)\log\left(\frac{a+}{2a+}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[-(ArcTan[a - x]/(a + x)),x]
```

```
[Out] (-I/2)*(-(Log[1 + I*(a - x)]*Log[(a + x)/(-I + 2*a)]) + Log[1 - I*a + I*x]*
Log[(a + x)/(I + 2*a)] + PolyLog[2, (I + a - x)/(I + 2*a)] - PolyLog[2, (I
- a + x)/(I - 2*a)])
```

Maple [A] time = 0.013, size = 102, normalized size = 0.8

$$-\ln(a+x)\arctan(a-x) + \frac{i}{2}\ln(a+x)\ln\left(\frac{a-x+i}{2a+i}\right) - \frac{i}{2}\ln(a+x)\ln\left(\frac{-a+x+i}{i-2a}\right) + \frac{i}{2}\text{dilog}\left(\frac{a-x+i}{2a+i}\right) - \frac{i}{2}\text{dilog}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-arctan(a-x)/(a+x),x)
```

```
[Out] -ln(a+x)*arctan(a-x)+1/2*I*ln(a+x)*ln((a-x+I)/(2*a+I))-1/2*I*ln(a+x)*ln((-a
+x+I)/(I-2*a))+1/2*I*dilog((a-x+I)/(2*a+I))-1/2*I*dilog((-a+x+I)/(I-2*a))
```

Maxima [A] time = 1.6248, size = 159, normalized size = 1.3

$$-\frac{1}{2}\arctan\left(\frac{a+x}{4a^2+1}, \frac{2(a^2+ax)}{4a^2+1}\right)\log\left(a^2-2ax+x^2+1\right) + \frac{1}{2}\arctan(-a+x)\log\left(\frac{a^2+2ax+x^2}{4a^2+1}\right) - \frac{1}{2}i\,\text{Li}_2\left(-\frac{-ia}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(a-x)/(a+x),x, algorithm="maxima")
```

```
[Out] -1/2*arctan2((a + x)/(4*a^2 + 1), 2*(a^2 + a*x)/(4*a^2 + 1))*log(a^2 - 2*a*
x + x^2 + 1) + 1/2*arctan(-a + x)*log((a^2 + 2*a*x + x^2)/(4*a^2 + 1)) - 1/
2*I*dilog(-(-I*a + I*x + 1)/(2*I*a - 1)) + 1/2*I*dilog(-(-I*a + I*x - 1)/(2
*I*a + 1))
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan\left(-a+x\right)}{a+x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(a-x)/(a+x),x, algorithm="fricas")
```

```
[Out] integral(arctan(-a + x)/(a + x), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\operatorname{atan}\left(a-x\right)}{a+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-atan(a-x)/(a+x),x)
```

```
[Out] -Integral(atan(a - x)/(a + x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\arctan\left(a-x\right)}{a+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-arctan(a-x)/(a+x),x, algorithm="giac")
```

```
[Out] integrate(-arctan(a - x)/(a + x), x)
```

3.699 $\int \frac{\sin^{-1}\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}} \, dx$

Optimal. Leaf size=28

$$-\frac{\sqrt{x^2}\sin^{-1}\left(\sqrt{1-x^2}\right)^2}{2x}$$

```
[Out] -(Sqrt[x^2]*ArcSin[Sqrt[1 - x^2]]^2)/(2*x)
```

Rubi [A] time = 0.0332714, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4834, 4641}

$$-\frac{\sqrt{x^2}\sin^{-1}\left(\sqrt{1-x^2}\right)^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[Sqrt[1 - x^2]]/Sqrt[1 - x^2],x]

[Out] -(Sqrt[x^2]*ArcSin[Sqrt[1 - x^2]]^2)/(2*x)
```

Rule 4834

```
Int[ArcSin[Sqrt[1 + (b_.)*(x_)^2]]^(n_.)/Sqrt[1 + (b_.)*(x_)^2], x_Symbol]
:> Dist[Sqrt[-(b*x^2)]/(b*x), Subst[Int[ArcSin[x]^n/Sqrt[1 - x^2], x], x, S
qrt[1 + b*x^2]], x] /; FreeQ[{b, n}, x]
```

Rule 4641

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(a + b*ArcSin[c*x])^(n + 1)/(b*c*Sqrt[d]*(n + 1)), x] /; Fre
eQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] && GtQ[d, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^{-1}\left(\sqrt{1-x^2}\right)}{\sqrt{1-x^2}} \, dx &= -\frac{\sqrt{x^2}\,\text{Subst}\left(\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}} \, dx, x, \sqrt{1-x^2}\right)}{x}\\
&= -\frac{\sqrt{x^2}\sin^{-1}\left(\sqrt{1-x^2}\right)^2}{2x}
\end{aligned}$$

Mathematica [A] time = 0.0166199, size = 28, normalized size = 1.

$$-\frac{\sqrt{x^2}\sin^{-1}\left(\sqrt{1-x^2}\right)^2}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[Sqrt[1 - x^2]]/Sqrt[1 - x^2],x]
```

```
[Out] -(Sqrt[x^2]*ArcSin[Sqrt[1 - x^2]]^2)/(2*x)
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \arcsin\left(\sqrt{-x^2+1}\right)\frac{1}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)
```

```
[Out] int(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arcsin\left(\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arcsin(sqrt(-x^2 + 1))/sqrt(-x^2 + 1), x)
```

Fricas [A] time = 2.55128, size = 42, normalized size = 1.5

$$-\frac{1}{2}\arcsin\left(\sqrt{-x^2+1}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*arcsin(sqrt(-x^2 + 1))^2
```

Sympy [A] time = 2.92055, size = 27, normalized size = 0.96

$$\frac{x\operatorname{asin}^2(x)}{2\sqrt{x^2}} + \operatorname{asin}(x)\operatorname{asin}\left(\sqrt{1-x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin((-x**2+1)**(1/2))/(-x**2+1)**(1/2),x)
```

```
[Out] x*asin(x)**2/(2*sqrt(x**2)) + asin(x)*asin(sqrt(1 - x**2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arcsin\left(\sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin((-x^2+1)^(1/2))/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arcsin(sqrt(-x^2 + 1))/sqrt(-x^2 + 1), x)
```

3.700 $\int \frac{x\tan^{-1}\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=31

$$\sqrt{x^2+1}\tan^{-1}\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

```
[Out] Sqrt[1 + x^2]*ArcTan[Sqrt[1 + x^2]] - Log[2 + x^2]/2
```

Rubi [A] time = 0.0380929, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {261, 5207, 260}

$$\sqrt{x^2+1}\tan^{-1}\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcTan[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

```
[Out] Sqrt[1 + x^2]*ArcTan[Sqrt[1 + x^2]] - Log[2 + x^2]/2
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5207

```
Int[((a_.) + ArcTan[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcTan[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcTan[u]), x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\tan^{-1}\left(\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\,dx &= \sqrt{1+x^2}\tan^{-1}\left(\sqrt{1+x^2}\right)-\int \frac{x}{2+x^2}\,dx\\ &= \sqrt{1+x^2}\tan^{-1}\left(\sqrt{1+x^2}\right)-\frac{1}{2}\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0272257, size = 31, normalized size = 1.

$$\sqrt{x^2+1}\tan^{-1}\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcTan[Sqrt[1 + x^2]])/Sqrt[1 + x^2],x]
```

```
[Out] Sqrt[1 + x^2]*ArcTan[Sqrt[1 + x^2]] - Log[2 + x^2]/2
```

Maple [A] time = 0.007, size = 26, normalized size = 0.8

$$-\frac{\ln\left(x^2+2\right)}{2}+\arctan\left(\sqrt{x^2+1}\right)\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x)
```

```
[Out] -1/2*ln(x^2+2)+arctan((x^2+1)^(1/2))*(x^2+1)^(1/2)
```

Maxima [A] time = 0.942101, size = 34, normalized size = 1.1

$$\sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)
```

Fricas [A] time = 2.52711, size = 76, normalized size = 2.45

$$\sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)
```

Sympy [A] time = 3.32846, size = 26, normalized size = 0.84

$$\sqrt{x^2+1}\operatorname{atan}\left(\sqrt{x^2+1}\right)-\frac{\log\left(x^2+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan((x**2+1)**(1/2))/(x**2+1)**(1/2),x)
```

```
[Out] sqrt(x**2 + 1)*atan(sqrt(x**2 + 1)) - log(x**2 + 2)/2
```

Giac [A] time = 1.07154, size = 34, normalized size = 1.1

$$\sqrt{x^2+1}\arctan\left(\sqrt{x^2+1}\right)-\frac{1}{2}\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan((x^2+1)^(1/2))/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 1)*arctan(sqrt(x^2 + 1)) - 1/2*log(x^2 + 2)
```

3.701 $\int \frac{\sin^{-1}(x)}{(1-x)^{5/2}}\,dx$

Optimal. Leaf size=57

$$-\frac{\sqrt{x+1}}{3(1-x)}+\frac{2\sin^{-1}(x)}{3(1-x)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{3\sqrt{2}}$$

```
[Out] -Sqrt[1 + x]/(3*(1 - x)) + (2*ArcSin[x])/(3*(1 - x)^(3/2)) - ArcTanh[Sqrt[1
 + x]/Sqrt[2]]/(3*Sqrt[2])
```

Rubi [A] time = 0.0316633, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4743, 627, 51, 63, 206}

$$-\frac{\sqrt{x+1}}{3(1-x)}+\frac{2\sin^{-1}(x)}{3(1-x)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]/(1 - x)^(5/2),x]
```

```
[Out] -Sqrt[1 + x]/(3*(1 - x)) + (2*ArcSin[x])/(3*(1 - x)^(3/2)) - ArcTanh[Sqrt[1
 + x]/Sqrt[2]]/(3*Sqrt[2])
```

Rule 4743

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_))^(m_.), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(e*(m + 1)), x] -
Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1
))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0]
&& NeQ[m, -1]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^{-1}(x)}{(1-x)^{5/2}}\,dx &= \frac{2\sin^{-1}(x)}{3(1-x)^{3/2}}-\frac{2}{3}\int \frac{1}{(1-x)^{3/2}\sqrt{1-x^2}}\,dx \\
&= \frac{2\sin^{-1}(x)}{3(1-x)^{3/2}}-\frac{2}{3}\int \frac{1}{(1-x)^2\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{3(1-x)}+\frac{2\sin^{-1}(x)}{3(1-x)^{3/2}}-\frac{1}{6}\int \frac{1}{(1-x)\sqrt{1+x}}\,dx \\
&= -\frac{\sqrt{1+x}}{3(1-x)}+\frac{2\sin^{-1}(x)}{3(1-x)^{3/2}}-\frac{1}{3}\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+x}\right) \\
&= -\frac{\sqrt{1+x}}{3(1-x)}+\frac{2\sin^{-1}(x)}{3(1-x)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{1+x}}{\sqrt{2}}\right)}{3\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.111364, size = 61, normalized size = 1.07

$$\frac{1}{6}\left(-\frac{2\left(\sqrt{1-x^2}-2\sin^{-1}(x)\right)}{(1-x)^{3/2}}-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-x^2}}{\sqrt{2-2x}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]/(1 - x)^(5/2),x]
```

```
[Out] ((-2*(Sqrt[1 - x^2] - 2*ArcSin[x]))/(1 - x)^(3/2) - Sqrt[2]*ArcTanh[Sqrt[1
- x^2]/Sqrt[2 - 2*x]])/6
```

Maple [A] time = 0.006, size = 70, normalized size = 1.2

$$\frac{2\arcsin(x)}{3}(1-x)^{-\frac{3}{2}}-\frac{1}{6}\sqrt{1+x}\left(\sqrt{2}\mathrm{Artanh}\left(\sqrt{2}\frac{1}{\sqrt{1+x}}\right)(1-x)+2\sqrt{1+x}\right)\frac{1}{\sqrt{1-x}}\frac{1}{\sqrt{-(1-x)^2+2-2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)/(1-x)^(5/2),x)
```

```
[Out] 2/3*arcsin(x)/(1-x)^(3/2)-1/6/(1-x)^(1/2)*(1+x)^(1/2)*(2^(1/2)*arctanh(2^(1
/2)/(1+x)^(1/2))*(1-x)+2*(1+x)^(1/2))/(-(1-x)^2+2-2*x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2\left(\frac{1}{8}\left(7\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{x+1}}{\sqrt{2}+\sqrt{x+1}}\right)+16\sqrt{x+1}-\frac{4\sqrt{x+1}}{x-1}\right)(x-1)\sqrt{-x+1}+\arctan\left(x,\sqrt{x+1}\sqrt{-x+1}\right)\right)}{3(x-1)\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(1-x)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*(3*(x - 1)*sqrt(-x + 1)*integrate(1/3*sqrt(x + 1)*x^2/(x^5 - x^4 - x^3
 + x^2 + (x^3 - x^2 - x + 1)*e^(log(x + 1) + log(-x + 1))), x) + arctan2(x,
 sqrt(x + 1)*sqrt(-x + 1)))/((x - 1)*sqrt(-x + 1))
```

Fricas [B] time = 2.71643, size = 235, normalized size = 4.12

$$\frac{\sqrt{2}\left(x^2-2x+1\right)\log\left(-\frac{x^2+2\sqrt{2}\sqrt{-x^2+1}\sqrt{-x+1}+2x-3}{x^2-2x+1}\right)-4\sqrt{-x+1}\left(\sqrt{-x^2+1}-2\arcsin(x)\right)}{12\left(x^2-2x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(1-x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(sqrt(2)*(x^2 - 2*x + 1)*log(-(x^2 + 2*sqrt(2)*sqrt(-x^2 + 1)*sqrt(-x
+ 1) + 2*x - 3)/(x^2 - 2*x + 1)) - 4*sqrt(-x + 1)*(sqrt(-x^2 + 1) - 2*arcsi
n(x)))/(x^2 - 2*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\operatorname{asin}(x)}{(1-x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)/(1-x)**(5/2),x)
```

```
[Out] Integral(asin(x)/(1 - x)**(5/2), x)
```

Giac [A] time = 1.0983, size = 78, normalized size = 1.37

$$\frac{1}{12}\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{x+1}}{\sqrt{2}+\sqrt{x+1}}\right)+\frac{\sqrt{x+1}}{3(x-1)}-\frac{2\arcsin(x)}{3(x-1)\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/(1-x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(2)*log((sqrt(2) - sqrt(x + 1))/(sqrt(2) + sqrt(x + 1))) + 1/3*sqr
t(x + 1)/(x - 1) - 2/3*arcsin(x)/((x - 1)*sqrt(-x + 1))
```

3.702 $\int (-1+x)^{5/2} \csc^{-1}(x) \, dx$

Optimal. Leaf size=82

$$\frac{4x\sqrt{x^2-1}\left(3x^2-19x+83\right)}{105\sqrt{x^2}\sqrt{x-1}}+\frac{4x\tanh^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{x-1}}\right)}{7\sqrt{x^2}}+\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)$$

```
[Out] (4*x*Sqrt[-1 + x^2]*(83 - 19*x + 3*x^2))/(105*Sqrt[-1 + x]*Sqrt[x^2]) + (2*
(-1 + x)^(7/2)*ArcCsc[x])/7 + (4*x*ArcTanh[Sqrt[-1 + x^2]/Sqrt[-1 + x]])/(7
*Sqrt[x^2])
```

Rubi [A] time = 0.0784756, antiderivative size = 140, normalized size of antiderivative = 1.71, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {5227, 1574, 892, 88, 63, 207}

$$\frac{4(x+1)^3\sqrt{x-1}}{35\sqrt{1-\frac{1}{x^2}}x}-\frac{20(x+1)^2\sqrt{x-1}}{21\sqrt{1-\frac{1}{x^2}}x}+\frac{4(x+1)\sqrt{x-1}}{\sqrt{1-\frac{1}{x^2}}x}+\frac{4\sqrt{x+1}\sqrt{x-1}\tanh^{-1}\left(\sqrt{x+1}\right)}{7\sqrt{1-\frac{1}{x^2}}x}+\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)^(5/2)*ArcCsc[x],x]
```

```
[Out] (4*Sqrt[-1 + x]*(1 + x))/(Sqrt[1 - x^(-2)]*x) - (20*Sqrt[-1 + x]*(1 + x)^2)
/(21*Sqrt[1 - x^(-2)]*x) + (4*Sqrt[-1 + x]*(1 + x)^3)/(35*Sqrt[1 - x^(-2)]*
x) + (2*(-1 + x)^(7/2)*ArcCsc[x])/7 + (4*Sqrt[-1 + x]*Sqrt[1 + x]*ArcTanh[S
qrt[1 + x]])/(7*Sqrt[1 - x^(-2)]*x)
```

Rule 5227

```
Int[((a_.) + ArcCsc[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCsc[c*x]))/(e*(m + 1)), x] + Dist[b/
(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 - 1/(c^2*x^2)]), x], x] /;
 FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 1574

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[(x^(2*n*FracPart[p])*(a + c/x^(2*n))^FracPart[p])/
(c + a*x^(2*n))^FracPart[p], Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 892

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[(a + c*x^2)^FracPart[p]/((d + e*x)^FracPart[p]*(a
/d + (c*x)/e)^FracPart[p]), Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/
e)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] &&
EqQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[p] &&  !IGtQ[m, 0] &&  !IGtQ[n, 0]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
```

```
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int (-1+x)^{5/2}\csc^{-1}(x)\,dx &= \frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)+\frac{2}{7}\int \frac{(-1+x)^{7/2}}{\sqrt{1-\frac{1}{x^2}}x^2}\,dx\\
&= \frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)+\frac{\left(2\sqrt{-1+x^2}\right)\int \frac{(-1+x)^{7/2}}{x\sqrt{-1+x^2}}\,dx}{7\sqrt{1-\frac{1}{x^2}}x}\\
&= \frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)+\frac{\left(2\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{(-1+x)^3}{x\sqrt{1+x}}\,dx}{7\sqrt{1-\frac{1}{x^2}}x}\\
&= \frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)+\frac{\left(2\sqrt{-1+x}\sqrt{1+x}\right)\int \left(\frac{7}{\sqrt{1+x}}-\frac{1}{x\sqrt{1+x}}-5\sqrt{1+x}+(1+x)^{3/2}\right)dx}{7\sqrt{1-\frac{1}{x^2}}x}\\
&= \frac{4\sqrt{-1+x}(1+x)}{\sqrt{1-\frac{1}{x^2}}x}-\frac{20\sqrt{-1+x}(1+x)^2}{21\sqrt{1-\frac{1}{x^2}}x}+\frac{4\sqrt{-1+x}(1+x)^3}{35\sqrt{1-\frac{1}{x^2}}x}+\frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)-\frac{(2\cdots}{\cdots}\\
&= \frac{4\sqrt{-1+x}(1+x)}{\sqrt{1-\frac{1}{x^2}}x}-\frac{20\sqrt{-1+x}(1+x)^2}{21\sqrt{1-\frac{1}{x^2}}x}+\frac{4\sqrt{-1+x}(1+x)^3}{35\sqrt{1-\frac{1}{x^2}}x}+\frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)-\frac{(4\cdots}{\cdots}\\
&= \frac{4\sqrt{-1+x}(1+x)}{\sqrt{1-\frac{1}{x^2}}x}-\frac{20\sqrt{-1+x}(1+x)^2}{21\sqrt{1-\frac{1}{x^2}}x}+\frac{4\sqrt{-1+x}(1+x)^3}{35\sqrt{1-\frac{1}{x^2}}x}+\frac{2}{7}(-1+x)^{7/2}\csc^{-1}(x)+\frac{4\sqrt{\cdots}}{\cdots}\end{aligned}$$

Mathematica [A] time = 0.0811706, size = 72, normalized size = 0.88

$$\frac{4\sqrt{1-\frac{1}{x^2}}x\left(3x^2-19x+83\right)}{105\sqrt{x-1}}+\frac{4}{7}\tanh^{-1}\left(\frac{\sqrt{1-\frac{1}{x^2}}x}{\sqrt{x-1}}\right)+\frac{2}{7}(x-1)^{7/2}\csc^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)^(5/2)*ArcCsc[x],x]
```

```
[Out] (4*Sqrt[1 - x^(-2)]*x*(83 - 19*x + 3*x^2))/(105*Sqrt[-1 + x]) + (2*(-1 + x)
^(7/2)*ArcCsc[x])/7 + (4*ArcTanh[(Sqrt[1 - x^(-2)]*x)/Sqrt[-1 + x]])/7
```

Maple [A] time = 0.015, size = 76, normalized size = 0.9

$$\frac{2\,\mathrm{arccsc}\,(x)}{7}(-1+x)^{\frac{7}{2}}+\frac{4}{105x}\sqrt{-1+x}\,\sqrt{1+x}\left(3\,(-1+x)^{2}\sqrt{1+x}-13\,(-1+x)\sqrt{1+x}+15\,\mathrm{Artanh}\left(\sqrt{1+x}\right)+67\sqrt{1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^(5/2)*arccsc(x),x)
```

```
[Out] 2/7*(-1+x)^(7/2)*arccsc(x)+4/105*(-1+x)^(1/2)*(1+x)^(1/2)*(3*(-1+x)^2*(1+x)
^(1/2)-13*(-1+x)*(1+x)^(1/2)+15*arctanh((1+x)^(1/2))+67*(1+x)^(1/2))/((-1+x
)*(1+x)/x^2)^(1/2)/x
```

Maxima [A] time = 3.69916, size = 157, normalized size = 1.91

$$\frac{4}{35}(x+1)^{\frac{5}{2}}-\frac{20}{21}(x+1)^{\frac{3}{2}}+\frac{2}{7}\left(x^{3}\arctan\left(1,\sqrt{x+1}\sqrt{x-1}\right)-3\,x^{2}\arctan\left(1,\sqrt{x+1}\sqrt{x-1}\right)+3\,x\arctan\left(1,\sqrt{x+1}\sqrt{x}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(5/2)*arccsc(x),x, algorithm="maxima")
```

```
[Out] 4/35*(x + 1)^(5/2) - 20/21*(x + 1)^(3/2) + 2/7*(x^3*arctan2(1, sqrt(x + 1)*
sqrt(x - 1)) - 3*x^2*arctan2(1, sqrt(x + 1)*sqrt(x - 1)) + 3*x*arctan2(1, s
qrt(x + 1)*sqrt(x - 1)) - arctan2(1, sqrt(x + 1)*sqrt(x - 1)))*sqrt(x - 1)
+ 4*sqrt(x + 1) + 2/7*log(sqrt(x + 1) + 1) - 2/7*log(sqrt(x + 1) - 1)
```

Fricas [B] time = 2.47565, size = 347, normalized size = 4.23

$$\frac{2\left(15\left(x^{4}-4\,x^{3}+6\,x^{2}-4\,x+1\right)\sqrt{x-1}\,\mathrm{arccsc}\,(x)+2\left(3\,x^{2}-19\,x+83\right)\sqrt{x^{2}-1}\sqrt{x-1}+15\,(x-1)\log\left(\frac{x^{2}+\sqrt{x^{2}-1}\sqrt{x-1}-1}{x^{2}-1}\right)\right.}{105\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(5/2)*arccsc(x),x, algorithm="fricas")
```

```
[Out] 2/105*(15*(x^4 - 4*x^3 + 6*x^2 - 4*x + 1)*sqrt(x - 1)*arccsc(x) + 2*(3*x^2
- 19*x + 83)*sqrt(x^2 - 1)*sqrt(x - 1) + 15*(x - 1)*log((x^2 + sqrt(x^2 - 1
)*sqrt(x - 1) - 1)/(x^2 - 1)) - 15*(x - 1)*log(-(x^2 - sqrt(x^2 - 1)*sqrt(x
 - 1) - 1)/(x^2 - 1)))/(x - 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(5/2)*acsc(x),x)
```

[Out] Timed out

Giac [B] time = 1.24695, size = 266, normalized size = 3.24

$$\frac{2}{3}(x-1)^{\frac{3}{2}}\arcsin\left(\frac{1}{x}\right)+\frac{2}{105}\left(15(x-1)^{\frac{7}{2}}+42(x-1)^{\frac{5}{2}}+35(x-1)^{\frac{3}{2}}\right)\arcsin\left(\frac{1}{x}\right)-\frac{4}{15}\left(3(x-1)^{\frac{5}{2}}+5(x-1)^{\frac{3}{2}}\right)\arcsin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(5/2)*arccsc(x),x, algorithm="giac")
```

```
[Out] 2/3*(x - 1)^(3/2)*arcsin(1/x) + 2/105*(15*(x - 1)^(7/2) + 42*(x - 1)^(5/2)
+ 35*(x - 1)^(3/2))*arcsin(1/x) - 4/15*(3*(x - 1)^(5/2) + 5*(x - 1)^(3/2))*
arcsin(1/x) + 4/105*(3*(x + 1)^(5/2) - 11*(x + 1)^(3/2) + 14*sqrt(x + 1))/s
gn((x - 1)^(3/2) + sqrt(x - 1)) - 8/15*((x + 1)^(3/2) - 4*sqrt(x + 1))/sgn(
(x - 1)^(3/2) + sqrt(x - 1)) + 2/7*log(sqrt(x + 1) + 1)/sgn((x - 1)^(3/2) +
 sqrt(x - 1)) - 2/7*log(sqrt(x + 1) - 1)/sgn((x - 1)^(3/2) + sqrt(x - 1)) +
 4/3*sqrt(x + 1)/sgn((x - 1)^(3/2) + sqrt(x - 1))
```

3.703 $\int \sin^{-1}(\sinh(x))\mathbf{sech}^4(x)\,dx$

Optimal. Leaf size=49

$$-\frac{2}{3}\sin^{-1}\left(\frac{\cosh(x)}{\sqrt{2}}\right)+\frac{1}{6}\sqrt{1-\sinh^2(x)}\mathrm{sech}(x)-\frac{1}{3}\tanh^3(x)\sin^{-1}(\sinh(x))+\tanh(x)\sin^{-1}(\sinh(x))$$

```
[Out] (-2*ArcSin[Cosh[x]/Sqrt[2]])/3 + (Sech[x]*Sqrt[1 - Sinh[x]^2])/6 + ArcSin[S
inh[x]]*Tanh[x] - (ArcSin[Sinh[x]]*Tanh[x]^3)/3
```

Rubi [A] time = 0.140496, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3767, 4844, 12, 4357, 451, 216}

$$\frac{1}{6}\sqrt{2-\cosh^2(x)}\mathrm{sech}(x)-\frac{2}{3}\sin^{-1}\left(\frac{\cosh(x)}{\sqrt{2}}\right)-\frac{1}{3}\tanh^3(x)\sin^{-1}(\sinh(x))+\tanh(x)\sin^{-1}(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[Sinh[x]]*Sech[x]^4,x]
```

```
[Out] (-2*ArcSin[Cosh[x]/Sqrt[2]])/3 + (Sqrt[2 - Cosh[x]^2]*Sech[x])/6 + ArcSin[S
inh[x]]*Tanh[x] - (ArcSin[Sinh[x]]*Tanh[x]^3)/3
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4844

```
Int[((a_.) + ArcSin[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcSin[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/Sq
rt[1 - u^2], x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x]
&& InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; Fre
eQ[{c, d, m}, x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4357

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
```

```
Q[m + n, -1]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sin^{-1}(\sinh(x))\mathbf{sech}^4(x)\,dx = \sin^{-1}(\sinh(x))\tanh(x) - \frac{1}{3}\sin^{-1}(\sinh(x))\tanh^3(x) - \int \frac{(2+\cosh(2x))\mathbf{sech}(x)\tanh}{3\sqrt{1-\sinh^2(x)}}$$
$$= \sin^{-1}(\sinh(x))\tanh(x) - \frac{1}{3}\sin^{-1}(\sinh(x))\tanh^3(x) - \frac{1}{3}\int \frac{(2+\cosh(2x))\mathbf{sech}(x)\,\mathrm{ta}}{\sqrt{1-\sinh^2(x)}}$$
$$= \sin^{-1}(\sinh(x))\tanh(x) - \frac{1}{3}\sin^{-1}(\sinh(x))\tanh^3(x) - \frac{1}{3}\,\mathrm{Subst}\left(\int \frac{1+2x^2}{x^2\sqrt{2-x^2}}\,dx, x, c\right.$$
$$= \frac{1}{6}\sqrt{2-\cosh^2(x)}\mathbf{sech}(x) + \sin^{-1}(\sinh(x))\tanh(x) - \frac{1}{3}\sin^{-1}(\sinh(x))\tanh^3(x) - \frac{2}{3}\,\mathrm{S}$$
$$= -\frac{2}{3}\sin^{-1}\left(\frac{\cosh(x)}{\sqrt{2}}\right) + \frac{1}{6}\sqrt{2-\cosh^2(x)}\mathbf{sech}(x) + \sin^{-1}(\sinh(x))\tanh(x) - \frac{1}{3}\sin^{-1}(\mathrm{s}$$

Mathematica [C] time = 0.228016, size = 66, normalized size = 1.35

$$\frac{1}{12}\left(8i\log\left(\sqrt{3-\cosh(2x)} + i\sqrt{2}\cosh(x)\right) + \sqrt{6-2\cosh(2x)}\mathbf{sech}(x) + 4(\cosh(2x)+2)\tanh(x)\mathbf{sech}^2(x)\sin^{-1}(\sinh($$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[Sinh[x]]*Sech[x]^4,x]
```

```
[Out] ((8*I)*Log[I*Sqrt[2]*Cosh[x] + Sqrt[3 - Cosh[2*x]]] + Sqrt[6 - 2*Cosh[2*x]]
*Sech[x] + 4*ArcSin[Sinh[x]]*(2 + Cosh[2*x])*Sech[x]^2*Tanh[x])/12
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int \arcsin\left(\sinh\left(x\right)\right)\left(\mathrm{sech}\left(x\right)\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(sinh(x))*sech(x)^4,x)
```

```
[Out] int(arcsin(sinh(x))*sech(x)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\left(3e^{(2x)}+1\right)\arctan\left(e^{(2x)}-1, \sqrt{e^{(2x)}+2e^{x}-1}\sqrt{-e^{(2x)}+2e^{x}+1}\right) + 16\left(e^{(6x)}+3e^{(4x)}+3e^{(2x)}+1\right)\int -\frac{}{\left(e^{(8x)}-4e^{(6x)}\right.}}{3\left(e^{(6x)}+3e^{(4x)}+3e^{(2x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(4*(3*e^(2*x) + 1)*arctan2(e^(2*x) - 1, sqrt(e^(2*x) + 2*e^x - 1)*sqrt
(-e^(2*x) + 2*e^x + 1)) + 3*(e^(6*x) + 3*e^(4*x) + 3*e^(2*x) + 1)*integrate
(16/3*(3*e^(4*x) + e^(2*x))*e^(1/2*log(e^(2*x) + 2*e^x - 1) + 1/2*log(-e^(2
*x) + 2*e^x + 1))/((e^(8*x) - 4*e^(6*x) - 10*e^(4*x) - 4*e^(2*x) + 1)*e^(lo
g(e^(2*x) + 2*e^x - 1) + log(-e^(2*x) + 2*e^x + 1)) + e^(12*x) - 6*e^(10*x)
 - e^(8*x) + 12*e^(6*x) - e^(4*x) - 6*e^(2*x) + 1), x))/(e^(6*x) + 3*e^(4*x
) + 3*e^(2*x) + 1)
```

Fricas [B] time = 2.55435, size = 1785, normalized size = 36.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(sqrt(2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
+ 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(-(
cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4
*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 + 1)*sinh(x)
^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 +
 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + co
sh(x))*sinh(x) + 1)*arctan(sqrt(2)*(3*cosh(x)^2 + 6*cosh(x)*sinh(x) + 3*sin
h(x)^2 - 1)*sqrt(-(cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 6*(cosh(x)^2
 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x))*sinh(x) + 1)) + 8
*(3*cosh(x)^2 + 6*cosh(x)*sinh(x) + 3*sinh(x)^2 + 1)*arctan(sqrt(2)*(cosh(x
)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-(cosh(x)^2 + sinh(x)^2 - 3)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(cosh(x)^4 + 4*cosh(x)*sinh(x)
^3 + sinh(x)^4 + 6*(cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 4*(cosh(x)^3 -
 3*cosh(x))*sinh(x) + 1)))/(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3
*(5*cosh(x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*si
nh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(co
sh(x)^5 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(sinh(x))*sech(x)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arcsin(\sinh(x))\operatorname{sech}(x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(sinh(x))*sech(x)^4,x, algorithm="giac")
```

```
[Out] integrate(arcsin(sinh(x))*sech(x)^4, x)
```

3.704 $\int \cot^{-1}(\cosh(x))\coth(x)\mathbf{csch}^3(x)\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{\coth(x)}{6}-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

```
[Out] ArcTanh[Tanh[x]/Sqrt[2]]/(6*Sqrt[2]) + Coth[x]/6 - (ArcCot[Cosh[x]]*Csch[x]
^3)/3
```

Rubi [A] time = 0.119881, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {2606, 30, 5208, 12, 453, 206}

$$\frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{\coth(x)}{6}-\frac{1}{3}\text{csch}^3(x)\cot^{-1}(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[Cosh[x]]*Coth[x]*Csch[x]^3,x]
```

```
[Out] ArcTanh[Tanh[x]/Sqrt[2]]/(6*Sqrt[2]) + Coth[x]/6 - (ArcCot[Cosh[x]]*Csch[x]
^3)/3
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5208

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] && !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
```

```
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cot^{-1}(\cosh(x))\coth(x)\operatorname{csch}^3(x)\,dx &= -\frac{1}{3}\cot^{-1}(\cosh(x))\operatorname{csch}^3(x)+\int \frac{2\operatorname{csch}^2(x)}{3(-3-\cosh(2x))}\,dx\\
&= -\frac{1}{3}\cot^{-1}(\cosh(x))\operatorname{csch}^3(x)+\frac{2}{3}\int \frac{\operatorname{csch}^2(x)}{-3-\cosh(2x)}\,dx\\
&= -\frac{1}{3}\cot^{-1}(\cosh(x))\operatorname{csch}^3(x)-\frac{2}{3}\operatorname{Subst}\left(\int \frac{1-x^2}{2x^2\left(2-x^2\right)}\,dx,x,\tanh(x)\right)\\
&= -\frac{1}{3}\cot^{-1}(\cosh(x))\operatorname{csch}^3(x)-\frac{1}{3}\operatorname{Subst}\left(\int \frac{1-x^2}{x^2\left(2-x^2\right)}\,dx,x,\tanh(x)\right)\\
&= \frac{\coth(x)}{6}-\frac{1}{3}\cot^{-1}(\cosh(x))\operatorname{csch}^3(x)+\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,\tanh(x)\right)\\
&= \frac{\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)}{6\sqrt{2}}+\frac{\coth(x)}{6}-\frac{1}{3}\cot^{-1}(\cosh(x))\operatorname{csch}^3(x)
\end{aligned}
$$

Mathematica [A] time = 0.167389, size = 40, normalized size = 1.11

$$
\frac{1}{24}\left(2\sqrt{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{2}}\right)+\operatorname{csch}^3(x)\left(-\cosh(x)+\cosh(3x)-8\cot^{-1}(\cosh(x))\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[Cosh[x]]*Coth[x]*Csch[x]^3,x]
```

```
[Out] (2*Sqrt[2]*ArcTanh[Tanh[x]/Sqrt[2]] + (-8*ArcCot[Cosh[x]] - Cosh[x] + Cosh[
3*x])*Csch[x]^3)/24
```

Maple [C] time = 0.462, size = 854, normalized size = 23.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(cosh(x))*cosh(x)/sinh(x)^4,x)
```

```
[Out] 4/3*I*exp(3*x)/(-1+exp(2*x))^3*ln(exp(2*x)+1+2*I*exp(x))-1/24*(-8-2^(1/2)*l
n(exp(2*x)+(1+2^(1/2))^2)+2^(1/2)*ln(exp(2*x)+(2^(1/2)-1)^2)+16*exp(2*x)+16
*Pi*csgn(I*exp(-x)*(-exp(2*x)-1+2*I*exp(x)))*csgn(exp(-x)*(-exp(2*x)-1+2*I*
exp(x)))*exp(3*x)-16*Pi*csgn(I*exp(-x))*csgn(I*(exp(2*x)+1+2*I*exp(x)))*csg
n(I*exp(-x)*(exp(2*x)+1+2*I*exp(x)))*exp(3*x)+16*Pi*csgn(I*exp(-x)*(-exp(2*
```

```
x)-1+2*I*exp(x)))*csgn(exp(-x)*(-exp(2*x)-1+2*I*exp(x)))^2*exp(3*x)-8*exp(4
*x)-3*2^(1/2)*ln(exp(2*x)+(1+2^(1/2))^2)*exp(4*x)-3*2^(1/2)*ln(exp(2*x)+(2^
(1/2)-1)^2)*exp(2*x)+3*2^(1/2)*ln(exp(2*x)+(1+2^(1/2))^2)*exp(2*x)-2^(1/2)*
ln(exp(2*x)+(2^(1/2)-1)^2)*exp(6*x)+2^(1/2)*ln(exp(2*x)+(1+2^(1/2))^2)*exp(
6*x)+3*2^(1/2)*ln(exp(2*x)+(2^(1/2)-1)^2)*exp(4*x)+32*I*exp(3*x)*ln(exp(2*x
)+1-2*I*exp(x))-16*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(-exp(2*x)-1+2*I*exp(x
)))^2*exp(3*x)+16*Pi*csgn(I*exp(-x))*csgn(I*exp(-x)*(exp(2*x)+1+2*I*exp(x))
)^2*exp(3*x)+16*Pi*csgn(I*(-exp(2*x)-1+2*I*exp(x)))*csgn(I*exp(-x)*(-exp(2*
x)-1+2*I*exp(x)))^2*exp(3*x)+16*Pi*csgn(I*(exp(2*x)+1+2*I*exp(x)))*csgn(I*e
xp(-x)*(exp(2*x)+1+2*I*exp(x)))^2*exp(3*x)+16*Pi*csgn(I*exp(-x)*(exp(2*x)+1
+2*I*exp(x)))*csgn(exp(-x)*(exp(2*x)+1+2*I*exp(x)))^2*exp(3*x)-16*Pi*csgn(I
*exp(-x)*(exp(2*x)+1+2*I*exp(x)))^3*exp(3*x)-16*Pi*csgn(I*exp(-x)*(-exp(2*x
)-1+2*I*exp(x)))^3*exp(3*x)+16*Pi*csgn(exp(-x)*(exp(2*x)+1+2*I*exp(x)))^2*e
xp(3*x)+16*Pi*csgn(exp(-x)*(-exp(2*x)-1+2*I*exp(x)))^3*exp(3*x)+16*Pi*csgn(
exp(-x)*(-exp(2*x)-1+2*I*exp(x)))^2*exp(3*x)-16*Pi*csgn(exp(-x)*(exp(2*x)+1
+2*I*exp(x)))^3*exp(3*x)+16*Pi*csgn(I*exp(-x))*csgn(I*(-exp(2*x)-1+2*I*exp(
x)))*csgn(I*exp(-x)*(-exp(2*x)-1+2*I*exp(x)))*exp(3*x)-16*Pi*csgn(I*exp(-x)
*(exp(2*x)+1+2*I*exp(x)))*csgn(exp(-x)*(exp(2*x)+1+2*I*exp(x)))*exp(3*x))/(
-1+exp(2*x))^3
```

Maxima [B] time = 1.44779, size = 73, normalized size = 2.03

$$-\frac{1}{24}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(-2x)}-3}{2\sqrt{2}+e^{(-2x)}+3}\right)-\frac{1}{3\left(e^{(-2x)}-1\right)}-\frac{\operatorname{arccot}\left(\cosh\left(x\right)\right)}{3\sinh\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="maxima")
```

```
[Out] -1/24*sqrt(2)*log(-(2*sqrt(2) - e^(-2*x) - 3)/(2*sqrt(2) + e^(-2*x) + 3)) -
 1/3/(e^(-2*x) - 1) - 1/3*arccot(cosh(x))/sinh(x)^3
```

Fricas [B] time = 2.30467, size = 1422, normalized size = 39.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(8*cosh(x)^4 + 32*cosh(x)*sinh(x)^3 + 8*sinh(x)^4 + 16*(3*cosh(x)^2 -
1)*sinh(x)^2 - 64*(cosh(x)^3 + 3*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 +
sinh(x)^3)*arctan(2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + si
nh(x)^2 + 1)) - 16*cosh(x)^2 + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*sinh(
x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4 - 3*
sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x)^3 +
 3*(5*sqrt(2)*cosh(x)^4 - 6*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 3*sqrt
(2)*cosh(x)^2 + 6*(sqrt(2)*cosh(x)^5 - 2*sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x
))*sinh(x) - sqrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)
*cosh(x)*sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 + 2*sqrt(2) - 3)/(cosh(x)^2
+ sinh(x)^2 + 3)) + 32*(cosh(x)^3 - cosh(x))*sinh(x) + 8)/(cosh(x)^6 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^2 - 1)*sinh(x)^4 - 3*cosh(x)^4 +
 4*(5*cosh(x)^3 - 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 - 6*cosh(x)^2 + 1)*
sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 - 2*cosh(x)^3 + cosh(x))*sinh(x) - 1
```

)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(cosh(x))*cosh(x)/sinh(x)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.09246, size = 95, normalized size = 2.64

$$\frac{1}{24}\sqrt{2}\log\left(-\frac{2\sqrt{2}-e^{(2x)}-3}{2\sqrt{2}+e^{(2x)}+3}\right)+\frac{1}{3\left(e^{(2x)}-1\right)}+\frac{8\arctan\left(\frac{2}{e^{(-x)}+e^{x}}\right)}{3\left(e^{(-x)}-e^{x}\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(cosh(x))*cosh(x)/sinh(x)^4,x, algorithm="giac")
```

```
[Out] 1/24*sqrt(2)*log(-(2*sqrt(2) - e^(2*x) - 3)/(2*sqrt(2) + e^(2*x) + 3)) + 1/
3/(e^(2*x) - 1) + 8/3*arctan(2/(e^(-x) + e^x))/(e^(-x) - e^x)^3
```

3.705 $\int e^x \sin^{-1}(\tanh(x))\,dx$

Optimal. Leaf size=28

$$e^x \sin^{-1}(\tanh(x)) - \log\left(e^{2x}+1\right)\cosh(x)\sqrt{\operatorname{sech}^2(x)}$$

```
[Out] E^x*ArcSin[Tanh[x]] - Cosh[x]*Log[1 + E^(2*x)]*Sqrt[Sech[x]^2]
```

Rubi [A] time = 0.0769321, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {2194, 4844, 6720, 2282, 12, 260}

$$e^x \sin^{-1}(\tanh(x)) - \log\left(e^{2x}+1\right)\cosh(x)\sqrt{\operatorname{sech}^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[E^x*ArcSin[Tanh[x]],x]
```

```
[Out] E^x*ArcSin[Tanh[x]] - Cosh[x]*Log[1 + E^(2*x)]*Sqrt[Sech[x]^2]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4844

```
Int[((a_.) + ArcSin[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcSin[u], w, x] - Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/Sq
rt[1 - u^2], x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x]
&& InverseFunctionFreeQ[u, x] &&  !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; Fre
eQ[{c, d, m}, x]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(EqQ
[v, x] && EqQ[m, 1])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int e^x \sin^{-1}(\tanh(x))\,dx &= e^x \sin^{-1}(\tanh(x)) - \int e^x \sqrt{\operatorname{sech}^2(x)}\,dx \\
&= e^x \sin^{-1}(\tanh(x)) - \left(\cosh(x)\sqrt{\operatorname{sech}^2(x)}\right)\int e^x \operatorname{sech}(x)\,dx \\
&= e^x \sin^{-1}(\tanh(x)) - \left(\cosh(x)\sqrt{\operatorname{sech}^2(x)}\right)\text{Subst}\left(\int \frac{2x}{1+x^2}\,dx, x, e^x\right) \\
&= e^x \sin^{-1}(\tanh(x)) - \left(2\cosh(x)\sqrt{\operatorname{sech}^2(x)}\right)\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, e^x\right) \\
&= e^x \sin^{-1}(\tanh(x)) - \cosh(x)\log\left(1+e^{2x}\right)\sqrt{\operatorname{sech}^2(x)}
\end{aligned}$$

Mathematica [B] time = 0.837499, size = 64, normalized size = 2.29

$$e^x \sin^{-1}\left(\frac{e^{2x}-1}{e^{2x}+1}\right) - e^{-x}\sqrt{\frac{e^{2x}}{\left(e^{2x}+1\right)^2}}\left(e^{2x}+1\right)\log\left(e^{2x}+1\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^x*ArcSin[Tanh[x]],x]
```

```
[Out] E^x*ArcSin[(-1 + E^(2*x))/(1 + E^(2*x))] - (Sqrt[E^(2*x)/(1 + E^(2*x))^2]*(
1 + E^(2*x))*Log[1 + E^(2*x)])/E^x
```

Maple [F] time = 0.046, size = 0, normalized size = 0.

$$\int e^x \arcsin(\tanh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*arcsin(tanh(x)),x)
```

```
[Out] int(exp(x)*arcsin(tanh(x)),x)
```

Maxima [A] time = 1.64726, size = 22, normalized size = 0.79

$$\arcsin(\tanh(x))\,e^x - \log\left(e^{(2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*arcsin(tanh(x)),x, algorithm="maxima")
```

```
[Out] arcsin(tanh(x))*e^x - log(e^(2*x) + 1)
```

Fricas [A] time = 2.21292, size = 100, normalized size = 3.57

$$(\cosh(x) + \sinh(x))\arctan(\sinh(x)) - \log\left(\frac{2\cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*arcsin(tanh(x)),x, algorithm="fricas")
```

```
[Out] (cosh(x) + sinh(x))*arctan(sinh(x)) - log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \operatorname{asin}(\tanh(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*asin(tanh(x)),x)
```

```
[Out] Integral(exp(x)*asin(tanh(x)), x)
```

Giac [A] time = 1.07862, size = 51, normalized size = 1.82

$$\arcsin\left(\frac{e^{(2x)}}{e^{(2x)} + 1} - \frac{1}{e^{(2x)} + 1}\right)e^x - \log\left(e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*arcsin(tanh(x)),x, algorithm="giac")
```

```
[Out] arcsin(e^(2*x)/(e^(2*x) + 1) - 1/(e^(2*x) + 1))*e^x - log(e^(2*x) + 1)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
    )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
    type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                        #since this estimate of leaf count is bit lower than
```

```
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```