Computer algebra independent integration tests

0-Independent-test-suites/Stewart-Problems

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [376]. This is test number [9].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (376)	% 0. (0)
Mathematica	% 100. (376)	% 0. (0)
Maple	% 100. (376)	% 0. (0)
Maxima	% 98.67 (371)	% 1.33 (5)
Fricas	% 100. (376)	% 0. (0)
Sympy	% 92.29 (347)	% 7.71 (29)
Giac	% 99.47 (374)	% 0.53 (2)

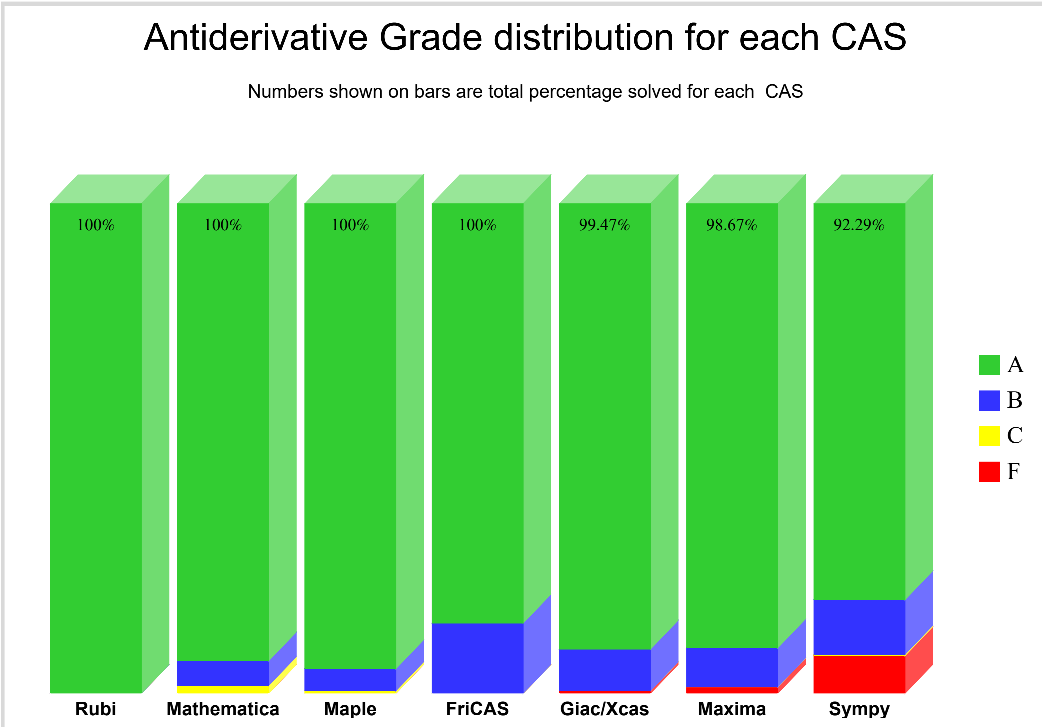
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

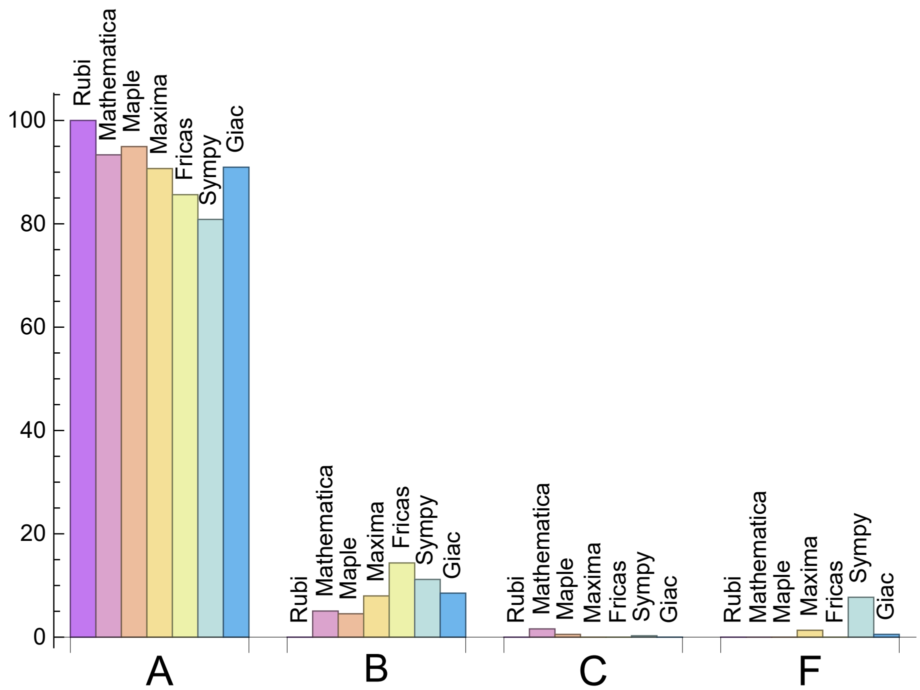
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	93.35	5.05	1.6	0.
Maple	94.95	4.52	0.53	0.
Maxima	90.69	7.98	0.	1.33
Fricas	85.64	14.36	0.	0.
Sympy	80.85	11.17	0.27	7.71
Giac	90.96	8.51	0.	0.53

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.02	22.98	1.	19.	1.
Mathematica	0.02	22.1	1.09	20.	1.
Maple	0.01	21.6	1.01	18.	0.88
Maxima	1.08	29.41	1.4	22.	1.09
Fricas	2.08	76.18	3.44	57.	2.64
Sympy	0.79	34.68	1.72	19.	0.88
Giac	1.06	30.11	1.58	23.	1.1

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{316, 360\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

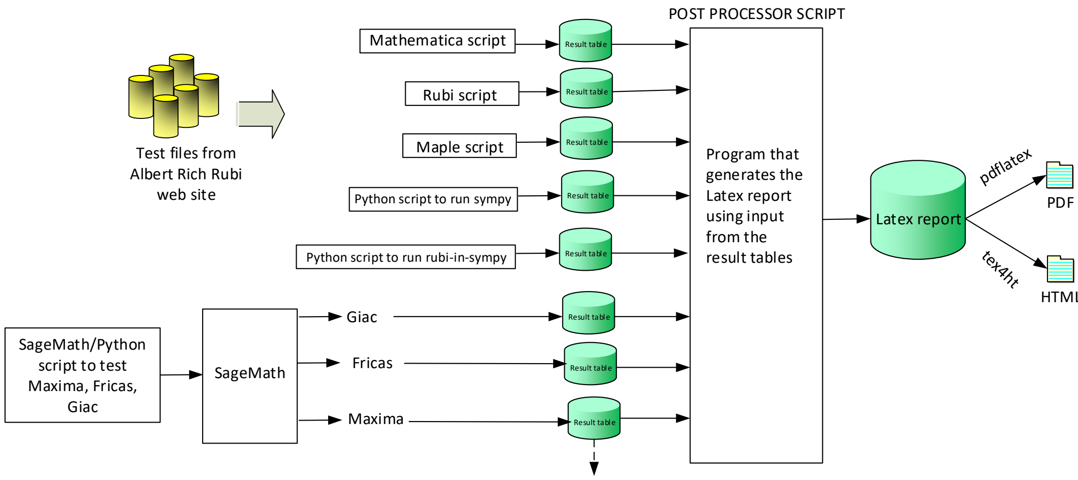
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 213, 214, 215, 216, 217, 218, 219, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 371, 372, 373, 374, 375, 376 }

B grade: { 80, 81, 100, 102, 103, 104, 113, 121, 152, 195, 211, 212, 221, 245, 246, 297, 328, 355, 370 }

C grade: { 220, 235, 236, 244, 316, 334 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 94, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 310, 311, 312, 313, 315, 316, 317, 318, 319, 320, 321, 322, 324, 325, 326, 327, 328, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 371, 372, 373, 374, 375, 376 }

B grade: { 63, 74, 89, 92, 93, 95, 117, 226, 227, 256, 270, 296, 308, 314, 329, 341, 370 }

C grade: { 323, 363 }

F grade: { }

2.1.4 Maxima

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 242, 243, 247, 249, 251, 252, 254, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 272, 273, 274, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376

B grade: { 29, 34, 35, 41, 95, 97, 103, 104, 112, 113, 220, 221, 225, 235, 241, 244, 245, 246, 248, 253, 255, 271, 276, 291, 298, 313, 328, 329, 355, 363 }

C grade: { }

F grade: { 1, 133, 250, 330, 337 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 106, 107, 109, 110, 111, 112, 114, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 236, 237, 238, 239, 240, 242, 243, 245, 247, 252, 253, 254, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 293, 294, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 335, 336, 338, 339, 340, 342, 343, 345, 346, 347, 348, 349, 350, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 371, 372, 373, 374, 375, 376 }

B grade: { 7, 8, 13, 14, 29, 41, 67, 79, 86, 97, 98, 99, 100, 101, 102, 103, 104, 105, 108, 113, 115, 130, 139, 143, 145, 195, 220, 221, 225, 235, 241, 244, 246, 248, 249, 250, 251, 255, 270, 291, 295, 298, 311, 312, 313, 314, 328, 334, 337, 341, 344, 351, 355, 370 }

C grade: { }

F grade: { }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 77, 78, 79, 81, 83, 85, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 103, 105, 107, 109, 110, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 143, 148, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 227, 228, 229, 231, 232, 233, 234, 236, 237, 239, 240, 242, 243, 245, 246, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 267, 268, 269, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 289, 290, 292, 293, 294, 299, 300, 302, 303, 304, 305, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 323, 326, 327, 329, 331, 332, 333, 335, 336, 338, 339, 342, 343, 345, 346, 347, 348, 349, 350, 352, 354, 355, 356, 357, 358, 361, 362, 363, 364, 366, 367, 368, 369, 371, 373, 374, 375, 376 }

B grade: { 7, 8, 42, 67, 75, 76, 80, 82, 84, 86, 87, 100, 102, 104, 106, 108, 111, 113, 127, 139, 142, 181, 195, 226, 230, 235, 241, 266, 270, 283, 291, 296, 297, 298, 306, 320, 330, 334, 341, 344, 370, 372 }

C grade: { 324 }

F grade: { 29, 41, 74, 123, 144, 145, 146, 147, 149, 220, 238, 244, 247, 248, 249, 250, 288, 295, 301, 322, 325, 328, 337, 340, 351, 353, 359, 360, 365 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 112, 114, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 131, 132, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 242, 243, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 256, 257, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 292, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 330, 331, 332, 333, 334, 335, 336, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 370, 371, 372, 373, 374, 375, 376 }

B grade: { 11, 12, 14, 29, 41, 79, 97, 98, 103, 104, 113, 115, 124, 130, 133, 138, 145, 195, 205, 225, 241, 244, 255, 263, 270, 291, 293, 298, 328, 329, 344, 368 }

C grade: { }

F grade: { 269, 337 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	20	12	15
normalized size	1	1.	1.	1.09	0.	1.82	1.09	1.36
time (sec)	N/A	0.002	0.001	0.003	0.	1.896	0.053	1.071

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	3	3	7	2	3
normalized size	1	1.	1.	1.	1.	2.33	0.67	1.
time (sec)	N/A	0.001	0.	0.	0.917	1.798	0.039	1.059

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	11	2	4
normalized size	1	1.	1.	1.5	1.5	5.5	1.	2.
time (sec)	N/A	0.	0.	0.	0.922	1.782	0.05	1.051

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	16	8	11
normalized size	1	1.	1.	1.12	1.38	2.	1.	1.38
time (sec)	N/A	0.002	0.001	0.	0.922	1.9	0.083	1.041

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	12	3	5
normalized size	1	1.	1.	1.25	1.25	3.	0.75	1.25
time (sec)	N/A	0.002	0.001	0.	0.926	2.005	0.053	1.066

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	11	2	3
normalized size	1	1.	1.	1.5	1.5	5.5	1.	1.5
time (sec)	N/A	0.002	0.001	0.001	0.928	1.986	0.053	1.058

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	20	5	3
normalized size	1	1.	1.	1.5	1.5	10.	2.5	1.5
time (sec)	N/A	0.006	0.002	0.003	0.927	1.91	0.059	1.063

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	8	22	7	8
normalized size	1	1.	1.	1.25	2.	5.5	1.75	2.
time (sec)	N/A	0.005	0.002	0.003	0.938	1.847	0.059	1.082

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	5	14	3	5
normalized size	1	1.	1.	1.5	2.5	7.	1.5	2.5
time (sec)	N/A	0.007	0.001	0.006	0.928	1.855	0.061	1.064

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	8	15	5	8
normalized size	1	1.	1.	1.25	2.	3.75	1.25	2.
time (sec)	N/A	0.008	0.002	0.005	0.919	1.866	0.063	1.06

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	12	2	15
normalized size	1	1.	1.	1.5	1.5	6.	1.	7.5
time (sec)	N/A	0.003	0.001	0.	0.919	1.904	0.11	1.059

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	12	2	15
normalized size	1	1.	1.	1.5	1.5	6.	1.	7.5
time (sec)	N/A	0.002	0.001	0.002	0.921	1.891	0.11	1.056

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	4	38	5	8
normalized size	1	1.	1.	1.2	0.8	7.6	1.	1.6
time (sec)	N/A	0.002	0.002	0.	0.929	2.006	0.058	1.06

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	41	3	15
normalized size	1	1.	1.	1.33	1.33	13.67	1.	5.
time (sec)	N/A	0.002	0.002	0.	0.927	2.031	0.058	1.056

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	27	7	11
normalized size	1	1.	1.	1.12	1.38	3.38	0.88	1.38
time (sec)	N/A	0.009	0.002	0.	0.933	1.91	0.167	1.049

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	19	5	11
normalized size	1	1.	1.	1.12	1.38	2.38	0.62	1.38
time (sec)	N/A	0.001	0.001	0.001	0.922	1.828	0.076	1.059

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	12	12	15	28	10	15
normalized size	1	1.	0.63	0.63	0.79	1.47	0.53	0.79
time (sec)	N/A	0.02	0.005	0.002	0.935	1.889	0.078	1.049

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	14	15	46	15	15
normalized size	1	1.	0.74	0.74	0.79	2.42	0.79	0.79
time (sec)	N/A	0.007	0.005	0.	0.928	1.924	0.283	1.047

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	43	12	18
normalized size	1	1.	1.	0.93	1.2	2.87	0.8	1.2
time (sec)	N/A	0.004	0.002	0.001	0.937	2.02	0.184	1.042

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	15	12	15	31	10	15
normalized size	1	1.	0.75	0.6	0.75	1.55	0.5	0.75
time (sec)	N/A	0.008	0.004	0.	0.936	1.831	0.075	1.055

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	26	7	9
normalized size	1	1.	1.	1.14	1.29	3.71	1.	1.29
time (sec)	N/A	0.009	0.002	0.	0.938	1.901	0.164	1.041

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	45	14	19
normalized size	1	1.	1.	0.83	1.06	2.5	0.78	1.06
time (sec)	N/A	0.01	0.011	0.008	0.949	1.947	0.167	1.05

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	35	12	18
normalized size	1	1.	1.	0.82	1.06	2.06	0.71	1.06
time (sec)	N/A	0.003	0.001	0.	0.935	1.873	0.08	1.051

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	28	59	27	28
normalized size	1	1.	0.86	0.83	0.97	2.03	0.93	0.97
time (sec)	N/A	0.025	0.027	0.008	0.941	2.028	0.304	1.04

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	28	59	24	28
normalized size	1	1.	0.86	0.83	0.97	2.03	0.83	0.97
time (sec)	N/A	0.023	0.024	0.006	0.948	1.94	0.292	1.043

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	16	42	15	20
normalized size	1	1.	1.	1.07	1.07	2.8	1.	1.33
time (sec)	N/A	0.003	0.001	0.	0.936	1.751	0.087	1.046

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	41	12	19
normalized size	1	1.	1.	0.94	1.19	2.56	0.75	1.19
time (sec)	N/A	0.004	0.002	0.	1.402	2.025	0.117	1.065

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	18	19	61	24	19
normalized size	1	1.	0.78	0.78	0.83	2.65	1.04	0.83
time (sec)	N/A	0.013	0.003	0.001	0.938	1.956	0.313	1.051

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	100	55	0	139
normalized size	1	1.	1.	1.12	12.5	6.88	0.	17.38
time (sec)	N/A	0.018	0.005	0.004	1.426	2.087	0.	1.102

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	35	12	18
normalized size	1	1.	1.	0.82	1.06	2.06	0.71	1.06
time (sec)	N/A	0.007	0.001	0.001	0.936	1.834	0.083	1.059

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	17	17	22	39	15	22
normalized size	1	1.	0.63	0.63	0.81	1.44	0.56	0.81
time (sec)	N/A	0.031	0.006	0.002	0.934	1.831	0.08	1.05

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	26	65	26	26
normalized size	1	1.	0.81	0.81	0.96	2.41	0.96	0.96
time (sec)	N/A	0.011	0.032	0.004	0.926	1.925	0.293	1.059

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	22	23	62	20	23
normalized size	1	1.	0.74	0.81	0.85	2.3	0.74	0.85
time (sec)	N/A	0.011	0.027	0.007	0.937	1.879	0.458	1.052

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	46	28	7	23
normalized size	1	1.	1.	1.11	5.11	3.11	0.78	2.56
time (sec)	N/A	0.012	0.003	0.001	0.948	1.781	0.171	1.049

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	19	77	45	20	41
normalized size	1	1.	1.	1.	4.05	2.37	1.05	2.16
time (sec)	N/A	0.016	0.013	0.009	0.95	1.771	0.212	1.053

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	11	10	12	23	7	12
normalized size	1	1.	0.69	0.62	0.75	1.44	0.44	0.75
time (sec)	N/A	0.009	0.004	0.001	0.925	1.991	0.073	1.048

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	15	14	18	43	66	18
normalized size	1	1.	0.71	0.67	0.86	2.05	3.14	0.86
time (sec)	N/A	0.008	0.003	0.003	0.934	2.175	2.037	1.049

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	42	14	19
normalized size	1	1.	1.	0.83	1.06	2.33	0.78	1.06
time (sec)	N/A	0.012	0.009	0.005	0.936	2.375	0.162	1.041

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	16	15	19	34	12	19
normalized size	1	1.	0.62	0.58	0.73	1.31	0.46	0.73
time (sec)	N/A	0.018	0.006	0.001	0.938	2.061	0.079	1.054

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	41	12	22
normalized size	1	1.	1.	0.94	1.22	2.28	0.67	1.22
time (sec)	N/A	0.004	0.002	0.001	1.406	2.39	0.118	1.048

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	140	61	0	70
normalized size	1	1.	1.	1.11	15.56	6.78	0.	7.78
time (sec)	N/A	0.016	0.015	0.003	0.945	2.618	0.	1.095

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	76	26	18
normalized size	1	1.	1.	0.82	1.06	4.47	1.53	1.06
time (sec)	N/A	0.009	0.007	0.074	0.934	2.692	0.513	1.059

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	9	18	43	22	18
normalized size	1	1.	1.	0.53	1.06	2.53	1.29	1.06
time (sec)	N/A	0.008	0.006	0.01	0.933	2.415	0.558	1.101

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	39	10	15
normalized size	1	1.	1.	1.09	1.36	3.55	0.91	1.36
time (sec)	N/A	0.01	0.002	0.004	0.926	2.28	0.876	1.058

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	14	12	15	31	10	15
normalized size	1	1.	0.64	0.55	0.68	1.41	0.45	0.68
time (sec)	N/A	0.019	0.002	0.002	0.941	2.326	0.078	1.06

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	9	9	16	20	7	11
normalized size	1	1.	0.6	0.6	1.07	1.33	0.47	0.73
time (sec)	N/A	0.008	0.009	0.	0.944	2.242	0.074	1.055

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	15	19	39	14	19
normalized size	1	1.	0.74	0.79	1.	2.05	0.74	1.
time (sec)	N/A	0.009	0.003	0.004	1.425	2.24	0.091	1.049

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	53	15	18
normalized size	1	1.	1.	0.82	0.82	3.12	0.88	1.06
time (sec)	N/A	0.003	0.003	0.	0.937	2.374	0.376	1.052

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	16	17	15	36	20	15
normalized size	1	1.	0.67	0.71	0.62	1.5	0.83	0.62
time (sec)	N/A	0.008	0.006	0.	0.946	2.218	0.178	1.067

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	12	10	12	30	8	12
normalized size	1	1.	0.86	0.71	0.86	2.14	0.57	0.86
time (sec)	N/A	0.001	0.001	0.	0.937	2.215	0.079	1.05

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	16	54	15	18
normalized size	1	1.	1.	0.82	0.94	3.18	0.88	1.06
time (sec)	N/A	0.003	0.002	0.001	0.938	2.36	0.375	1.048

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	57	20	22
normalized size	1	1.	1.	0.77	1.	2.59	0.91	1.
time (sec)	N/A	0.011	0.012	0.	0.935	2.258	0.294	1.049

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	45	15	22
normalized size	1	1.	1.	0.85	1.1	2.25	0.75	1.1
time (sec)	N/A	0.016	0.009	0.004	0.939	2.307	1.852	1.065

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	19	17	22	42	15	22
normalized size	1	1.	0.68	0.61	0.79	1.5	0.54	0.79
time (sec)	N/A	0.034	0.002	0.001	0.943	2.288	0.085	1.066

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	20	45	15	20
normalized size	1	1.	1.	0.76	0.95	2.14	0.71	0.95
time (sec)	N/A	0.009	0.003	0.	1.42	2.278	0.334	1.056

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	47	15	24
normalized size	1	1.	1.	0.94	1.22	2.61	0.83	1.33
time (sec)	N/A	0.013	0.015	0.005	0.935	2.561	0.217	1.052

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	15	14	18	43	66	18
normalized size	1	1.	0.71	0.67	0.86	2.05	3.14	0.86
time (sec)	N/A	0.006	0.002	0.002	0.94	2.51	2.123	1.061

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	14	15	14	43	14	14
normalized size	1	1.	0.78	0.83	0.78	2.39	0.78	0.78
time (sec)	N/A	0.006	0.006	0.007	0.936	2.583	0.061	1.073

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	36	10	14
normalized size	1	1.	1.	0.79	1.	2.57	0.71	1.
time (sec)	N/A	0.006	0.002	0.005	0.94	2.218	0.058	1.05

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	18	22	59	24	22
normalized size	1	1.	0.92	0.75	0.92	2.46	1.	0.92
time (sec)	N/A	0.013	0.002	0.001	0.953	2.089	0.059	1.058

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	15	31	8	15
normalized size	1	1.	1.15	0.85	1.15	2.38	0.62	1.15
time (sec)	N/A	0.005	0.002	0.	0.928	1.971	0.064	1.077

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	31	18	18	39	12	18
normalized size	1	1.	1.82	1.06	1.06	2.29	0.71	1.06
time (sec)	N/A	0.023	0.01	0.006	0.957	2.219	0.062	1.06

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	31	30	18	73	12	18
normalized size	1	1.	1.82	1.76	1.06	4.29	0.71	1.06
time (sec)	N/A	0.022	0.009	0.004	0.928	2.132	0.063	1.046

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	30	29	24	81	31	30
normalized size	1	1.	0.83	0.81	0.67	2.25	0.86	0.83
time (sec)	N/A	0.045	0.007	0.006	0.937	2.018	0.061	1.065

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	14	19	14	58	14	14
normalized size	1	1.	0.58	0.79	0.58	2.42	0.58	0.58
time (sec)	N/A	0.025	0.004	0.	0.925	2.039	0.066	1.051

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	18	19	19	58	37	19
normalized size	1	1.	0.82	0.86	0.86	2.64	1.68	0.86
time (sec)	N/A	0.009	0.008	0.023	0.936	1.976	0.177	1.056

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	19	105	37	19
normalized size	1	1.	1.	0.75	0.95	5.25	1.85	0.95
time (sec)	N/A	0.014	0.011	0.029	0.933	1.983	0.545	1.06

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	28	26	63	19	26
normalized size	1	1.	1.	1.12	1.04	2.52	0.76	1.04
time (sec)	N/A	0.03	0.01	0.007	0.928	2.003	0.068	1.051

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	24	32	84	36	30
normalized size	1	1.	0.88	0.71	0.94	2.47	1.06	0.88
time (sec)	N/A	0.017	0.002	0.	0.924	1.998	0.061	1.053

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	24	32	82	36	30
normalized size	1	1.	0.88	0.71	0.94	2.41	1.06	0.88
time (sec)	N/A	0.017	0.002	0.029	0.927	2.051	0.057	1.047

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	30	36	24	92	41	30
normalized size	1	1.	0.65	0.78	0.52	2.	0.89	0.65
time (sec)	N/A	0.036	0.014	0.011	0.939	1.995	0.062	1.052

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	23	53	17	23
normalized size	1	1.	1.1	0.81	1.1	2.52	0.81	1.1
time (sec)	N/A	0.007	0.002	0.004	0.929	1.977	0.063	1.045

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	22	36	22	103	31	22
normalized size	1	1.	0.48	0.78	0.48	2.24	0.67	0.48
time (sec)	N/A	0.052	0.007	0.005	0.929	2.105	0.067	1.048

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	34	39	18	58	0	18
normalized size	1	1.	1.62	1.86	0.86	2.76	0.	0.86
time (sec)	N/A	0.022	0.063	0.033	0.93	2.062	0.	1.053

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	18	14	18	49	167	18
normalized size	1	1.	0.86	0.67	0.86	2.33	7.95	0.86
time (sec)	N/A	0.022	0.011	0.02	0.933	2.066	42.137	1.05

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	18	14	16	50	39	16
normalized size	1	1.	0.95	0.74	0.84	2.63	2.05	0.84
time (sec)	N/A	0.02	0.027	0.009	0.927	1.879	0.321	1.048

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	20	42	22	20
normalized size	1	1.	1.	0.79	1.05	2.21	1.16	1.05
time (sec)	N/A	0.014	0.015	0.006	0.931	2.066	0.572	1.056

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	22	39	10	19
normalized size	1	1.	1.	0.93	1.57	2.79	0.71	1.36
time (sec)	N/A	0.013	0.005	0.011	0.93	2.115	0.08	1.071

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	20	29	27	116	20	49
normalized size	1	1.	0.91	1.32	1.23	5.27	0.91	2.23
time (sec)	N/A	0.029	0.023	0.013	0.931	2.069	0.095	1.07

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	36	6	7	23	19	7
normalized size	1	1.	7.2	1.2	1.4	4.6	3.8	1.4
time (sec)	N/A	0.013	0.007	0.021	0.935	2.076	0.318	1.054

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	25	11	20	61	8	14
normalized size	1	1.	2.27	1.	1.82	5.55	0.73	1.27
time (sec)	N/A	0.008	0.012	0.	0.932	1.942	0.381	1.056

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	18	7	8
normalized size	1	1.	1.	1.17	1.33	3.	1.17	1.33
time (sec)	N/A	0.004	0.002	0.002	1.407	1.977	0.062	1.064

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	13	16	36	19	16
normalized size	1	1.	1.29	0.93	1.14	2.57	1.36	1.14
time (sec)	N/A	0.008	0.004	0.001	1.41	1.91	0.066	1.055

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	13	12	51	19	12
normalized size	1	1.	1.55	1.18	1.09	4.64	1.73	1.09
time (sec)	N/A	0.007	0.002	0.027	0.924	1.965	0.061	1.079

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	27	19	20	70	31	20
normalized size	1	1.	1.42	1.	1.05	3.68	1.63	1.05
time (sec)	N/A	0.009	0.003	0.027	0.926	1.948	0.062	1.056

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	11	8	66	29	8
normalized size	1	1.	1.	1.38	1.	8.25	3.62	1.
time (sec)	N/A	0.02	0.002	0.011	0.924	1.977	0.063	1.056

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	27	22	18	69	29	18
normalized size	1	1.	1.59	1.29	1.06	4.06	1.71	1.06
time (sec)	N/A	0.022	0.019	0.012	0.924	1.975	0.065	1.07

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	19	7	8
normalized size	1	1.	1.	0.88	1.	2.38	0.88	1.
time (sec)	N/A	0.012	0.005	0.006	0.927	1.972	0.065	1.054

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	42	19	45	15	19
normalized size	1	1.	1.	2.47	1.12	2.65	0.88	1.12
time (sec)	N/A	0.023	0.023	0.011	0.925	2.237	0.097	1.051

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	20	23	46	77	20	30
normalized size	1	1.	0.91	1.05	2.09	3.5	0.91	1.36
time (sec)	N/A	0.01	0.003	0.002	0.925	2.259	0.104	1.059

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	30	19	24	57	31	24
normalized size	1	1.	1.36	0.86	1.09	2.59	1.41	1.09
time (sec)	N/A	0.013	0.004	0.002	1.411	2.036	0.067	1.058

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	48	27	63	20	27
normalized size	1	1.	1.	2.53	1.42	3.32	1.05	1.42
time (sec)	N/A	0.016	0.01	0.008	0.938	2.028	0.104	1.059

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	58	27	66	20	27
normalized size	1	1.	1.	2.32	1.08	2.64	0.8	1.08
time (sec)	N/A	0.029	0.012	0.012	0.926	2.058	0.115	1.065

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	14	19	7	8
normalized size	1	1.	1.	0.88	1.75	2.38	0.88	1.
time (sec)	N/A	0.013	0.004	0.007	0.925	2.003	0.068	1.058

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	32	49	45	15	19
normalized size	1	1.	1.	1.88	2.88	2.65	0.88	1.12
time (sec)	N/A	0.028	0.013	0.013	0.928	1.945	0.127	1.049

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	14	19	7	8
normalized size	1	1.	1.	0.88	1.75	2.38	0.88	1.
time (sec)	N/A	0.012	0.004	0.01	0.925	2.04	0.067	1.054

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	36	109	27	39
normalized size	1	1.	1.	1.5	2.25	6.81	1.69	2.44
time (sec)	N/A	0.015	0.006	0.008	0.929	2.172	0.109	1.059

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	12	14	53	8	24
normalized size	1	1.	1.	1.5	1.75	6.62	1.	3.
time (sec)	N/A	0.005	0.002	0.003	1.409	2.098	0.059	1.068

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	17	19	90	14	30
normalized size	1	1.	1.	1.21	1.36	6.43	1.	2.14
time (sec)	N/A	0.007	0.003	0.002	0.93	1.972	0.088	1.061

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	37	22	19	112	41	19
normalized size	1	1.	2.18	1.29	1.12	6.59	2.41	1.12
time (sec)	N/A	0.026	0.027	0.013	0.934	1.958	0.067	1.077

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	19	86	14	24
normalized size	1	1.	1.	1.29	1.12	5.06	0.82	1.41
time (sec)	N/A	0.026	0.007	0.013	0.94	2.018	0.103	1.053

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	17	9	11	77	15	8
normalized size	1	1.	3.4	1.8	2.2	15.4	3.	1.6
time (sec)	N/A	0.002	0.003	0.002	0.931	2.151	0.093	1.073

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	47	18	36	150	27	73
normalized size	1	1.	2.94	1.12	2.25	9.38	1.69	4.56
time (sec)	N/A	0.007	0.005	0.031	0.932	2.044	0.112	1.065

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	19	12	23	88	19	26
normalized size	1	1.	2.38	1.5	2.88	11.	2.38	3.25
time (sec)	N/A	0.011	0.004	0.007	0.924	2.153	0.093	1.055

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	17	12	19	73	20	19
normalized size	1	1.	1.31	0.92	1.46	5.62	1.54	1.46
time (sec)	N/A	0.007	0.002	0.026	0.93	1.962	0.057	1.058

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	81	26	18
normalized size	1	1.	1.	0.82	1.06	4.76	1.53	1.06
time (sec)	N/A	0.008	0.007	0.046	0.933	2.082	0.503	1.046

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	35	22	18
normalized size	1	1.	1.	0.82	1.06	2.06	1.29	1.06
time (sec)	N/A	0.008	0.005	0.029	0.93	1.934	0.512	1.072

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	78	26	15
normalized size	1	1.	1.	0.8	1.	5.2	1.73	1.
time (sec)	N/A	0.008	0.006	0.049	0.925	2.13	0.493	1.054

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	9	18	43	24	18
normalized size	1	1.	1.	0.53	1.06	2.53	1.41	1.06
time (sec)	N/A	0.008	0.006	0.01	0.939	2.103	0.533	1.052

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	20	7	8
normalized size	1	1.	1.	0.88	1.	2.5	0.88	1.
time (sec)	N/A	0.013	0.001	0.001	0.928	1.951	0.056	1.053

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	30	81	112	30
normalized size	1	1.	1.	0.77	1.	2.7	3.73	1.
time (sec)	N/A	0.031	0.008	0.	0.931	1.97	14.487	1.067

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	8	6	15	20	7	12
normalized size	1	1.	1.6	1.2	3.	4.	1.4	2.4
time (sec)	N/A	0.022	0.002	0.016	0.93	2.124	0.414	1.063

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	61	20	93	149	32	39
normalized size	1	1.	4.07	1.33	6.2	9.93	2.13	2.6
time (sec)	N/A	0.053	0.009	0.049	1.438	2.096	1.051	1.101

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	22	39	10	24
normalized size	1	1.	1.	0.93	1.57	2.79	0.71	1.71
time (sec)	N/A	0.015	0.005	0.011	0.927	2.167	0.077	1.081

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	20	29	27	116	20	49
normalized size	1	1.	0.91	1.32	1.23	5.27	0.91	2.23
time (sec)	N/A	0.052	0.012	0.013	0.926	2.074	0.095	1.06

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	19	7	8
normalized size	1	1.	1.	0.88	1.	2.38	0.88	1.
time (sec)	N/A	0.015	0.003	0.	0.935	1.996	0.062	1.049

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	42	19	45	15	19
normalized size	1	1.	1.	2.47	1.12	2.65	0.88	1.12
time (sec)	N/A	0.026	0.012	0.	0.935	1.914	0.097	1.056

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	34	28	77	15	53
normalized size	1	1.	1.	1.36	1.12	3.08	0.6	2.12
time (sec)	N/A	0.004	0.007	0.003	1.407	1.795	0.227	1.061

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	38	12	26
normalized size	1	1.	1.	0.81	1.	2.38	0.75	1.62
time (sec)	N/A	0.003	0.003	0.003	1.398	2.046	0.712	1.056

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	20	7	9
normalized size	1	1.	1.	0.89	1.	2.22	0.78	1.
time (sec)	N/A	0.002	0.001	0.003	0.925	1.796	0.125	1.052

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	46	15	24	39	20	26
normalized size	1	1.	2.88	0.94	1.5	2.44	1.25	1.62
time (sec)	N/A	0.002	0.003	0.001	0.974	1.884	1.02	1.057

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	19	35	45	27	31
normalized size	1	1.	0.71	0.61	1.13	1.45	0.87	1.
time (sec)	N/A	0.015	0.008	0.003	1.411	1.823	0.547	1.049

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	28	105	0	31
normalized size	1	1.	1.	0.89	1.04	3.89	0.	1.15
time (sec)	N/A	0.01	0.007	0.005	1.405	2.028	0.	1.068

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	20	19	26	27	45
normalized size	1	1.	1.	1.25	1.19	1.62	1.69	2.81
time (sec)	N/A	0.003	0.003	0.003	1.396	1.848	0.708	1.062

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	25	35	57	39	41
normalized size	1	1.	0.71	0.81	1.13	1.84	1.26	1.32
time (sec)	N/A	0.014	0.009	0.005	1.404	1.898	0.568	1.056

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	15	23	8	15
normalized size	1	1.	1.	1.31	1.15	1.77	0.62	1.15
time (sec)	N/A	0.002	0.	0.003	0.924	1.811	0.128	1.056

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	18	15	41	24	15
normalized size	1	1.	1.	1.2	1.	2.73	1.6	1.
time (sec)	N/A	0.002	0.002	0.003	0.92	2.008	0.18	1.072

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	90	19	26
normalized size	1	1.	1.	0.8	1.04	3.6	0.76	1.04
time (sec)	N/A	0.003	0.007	0.004	1.403	1.98	0.187	1.058

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	18	15	30	39	24	26
normalized size	1	1.	0.72	0.6	1.2	1.56	0.96	1.04
time (sec)	N/A	0.011	0.004	0.005	1.405	1.732	0.342	1.044

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	5	35	3	19
normalized size	1	1.	1.	0.83	0.83	5.83	0.5	3.17
time (sec)	N/A	0.001	0.003	0.001	1.409	1.928	0.125	1.059

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	18	16	20	69	15	34
normalized size	1	1.	0.86	0.76	0.95	3.29	0.71	1.62
time (sec)	N/A	0.003	0.004	0.003	1.397	1.993	0.176	1.054

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	46	26	30	96	66	34
normalized size	1	1.	1.31	0.74	0.86	2.74	1.89	0.97
time (sec)	N/A	0.013	0.02	0.005	1.402	1.879	2.23	1.052

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	28	0	57	78	65
normalized size	1	1.	1.	1.22	0.	2.48	3.39	2.83
time (sec)	N/A	0.004	0.004	0.004	0.	2.059	0.629	1.065

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	26	78	92	32
normalized size	1	1.	1.	0.83	0.87	2.6	3.07	1.07
time (sec)	N/A	0.016	0.006	0.005	1.406	1.975	1.293	1.067

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	25	19	43	37	31
normalized size	1	1.	1.	1.39	1.06	2.39	2.06	1.72
time (sec)	N/A	0.003	0.003	0.003	1.409	1.985	0.744	1.054

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	39	31	32	111	51	32
normalized size	1	1.	1.15	0.91	0.94	3.26	1.5	0.94
time (sec)	N/A	0.006	0.03	0.01	1.404	2.222	1.522	1.076

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	23	30	73	24	30
normalized size	1	1.	1.	0.79	1.03	2.52	0.83	1.03
time (sec)	N/A	0.006	0.009	0.005	1.4	2.271	0.202	1.081

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	19	63	15	50
normalized size	1	1.	1.	0.78	0.83	2.74	0.65	2.17
time (sec)	N/A	0.011	0.004	0.004	1.401	2.29	0.987	1.05

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	53	26	12
normalized size	1	1.	1.	0.77	0.92	4.08	2.	0.92
time (sec)	N/A	0.002	0.002	0.003	0.921	2.368	2.328	1.057

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	22	29	35	66	44	43
normalized size	1	1.	0.71	0.94	1.13	2.13	1.42	1.39
time (sec)	N/A	0.014	0.011	0.004	1.398	2.566	0.593	1.064

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	33	32	42	97	112	35
normalized size	1	1.	0.73	0.71	0.93	2.16	2.49	0.78
time (sec)	N/A	0.01	0.015	0.003	1.406	2.418	2.672	1.062

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	28	26	12
normalized size	1	1.	1.	0.77	0.92	2.15	2.	0.92
time (sec)	N/A	0.002	0.002	0.001	0.926	2.065	0.175	1.046

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	23	16	76	36	16
normalized size	1	1.	1.	1.44	1.	4.75	2.25	1.
time (sec)	N/A	0.001	0.004	0.002	0.921	2.062	0.722	1.064

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	26	49	80	0	31
normalized size	1	1.	0.97	0.79	1.48	2.42	0.	0.94
time (sec)	N/A	0.008	0.039	0.003	1.397	2.169	0.	1.057

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	49	0	24
normalized size	1	1.	1.	0.88	1.	6.12	0.	3.
time (sec)	N/A	0.006	0.005	0.003	1.406	2.107	0.	1.06

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	30	30	59	0	28
normalized size	1	1.	0.96	1.2	1.2	2.36	0.	1.12
time (sec)	N/A	0.006	0.006	0.003	1.402	2.086	0.	1.049

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	47	37	49	85	0	34
normalized size	1	1.	1.07	0.84	1.11	1.93	0.	0.77
time (sec)	N/A	0.017	0.053	0.003	1.406	2.198	0.	1.057

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	23	25	30	82	19	30
normalized size	1	1.	0.88	0.96	1.15	3.15	0.73	1.15
time (sec)	N/A	0.006	0.007	0.001	1.408	2.219	0.113	1.051

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	31	36	80	123	0	49
normalized size	1	1.	0.72	0.84	1.86	2.86	0.	1.14
time (sec)	N/A	0.008	0.009	0.001	0.921	2.306	0.	1.073

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	32	23	30	97	29	30
normalized size	1	1.	0.97	0.7	0.91	2.94	0.88	0.91
time (sec)	N/A	0.025	0.013	0.008	1.407	2.289	1.234	1.058

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	30	72	22	30
normalized size	1	1.	1.	0.77	1.	2.4	0.73	1.
time (sec)	N/A	0.015	0.007	0.004	1.401	2.216	1.115	1.058

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	42	13	11	38	3	22
normalized size	1	1.	3.	0.93	0.79	2.71	0.21	1.57
time (sec)	N/A	0.002	0.003	0.002	0.919	2.029	0.939	1.062

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	38	10	20
normalized size	1	1.	1.	0.93	1.2	2.53	0.67	1.33
time (sec)	N/A	0.004	0.003	0.005	0.922	2.13	0.087	1.052

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	25	21	27	54	19	28
normalized size	1	1.	0.96	0.81	1.04	2.08	0.73	1.08
time (sec)	N/A	0.016	0.004	0.002	0.919	2.041	0.068	1.049

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	68	19	30
normalized size	1	1.	1.	0.8	1.04	2.72	0.76	1.2
time (sec)	N/A	0.042	0.006	0.006	0.918	2.134	0.123	1.048

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	29	25	32	109	20	35
normalized size	1	1.	0.97	0.83	1.07	3.63	0.67	1.17
time (sec)	N/A	0.03	0.016	0.008	0.926	2.073	0.09	1.051

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	65	17	24
normalized size	1	1.	1.	0.78	1.	2.83	0.74	1.04
time (sec)	N/A	0.038	0.005	0.005	1.409	2.052	0.12	1.1

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	32	42	101	34	42
normalized size	1	1.	1.	0.84	1.11	2.66	0.89	1.11
time (sec)	N/A	0.038	0.01	0.004	1.405	2.157	0.106	1.058

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	93	73	104	387	88	100
normalized size	1	1.	0.9	0.71	1.01	3.76	0.85	0.97
time (sec)	N/A	0.537	0.044	0.01	1.413	2.12	0.46	1.058

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	39	130	27	41
normalized size	1	1.	1.	0.85	1.18	3.94	0.82	1.24
time (sec)	N/A	0.047	0.02	0.007	1.404	1.905	0.131	1.063

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	16	20	55	12	20
normalized size	1	1.	0.84	0.84	1.05	2.89	0.63	1.05
time (sec)	N/A	0.003	0.005	0.001	1.404	1.77	0.095	1.051

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	18	46	12	20
normalized size	1	1.	1.	0.74	0.95	2.42	0.63	1.05
time (sec)	N/A	0.003	0.002	0.004	0.925	1.75	0.09	1.073

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	21	16	20	50	17	23
normalized size	1	1.	1.11	0.84	1.05	2.63	0.89	1.21
time (sec)	N/A	0.007	0.003	0.006	0.92	1.882	0.091	1.059

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	27	35	107	26	58
normalized size	1	1.	1.	0.84	1.09	3.34	0.81	1.81
time (sec)	N/A	0.026	0.015	0.008	0.92	1.837	0.123	1.064

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	33	34	46	170	32	42
normalized size	1	1.	0.77	0.79	1.07	3.95	0.74	0.98
time (sec)	N/A	0.042	0.022	0.007	0.928	1.925	0.141	1.076

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	26	72	17	28
normalized size	1	1.	1.	0.86	1.24	3.43	0.81	1.33
time (sec)	N/A	0.011	0.002	0.006	0.923	1.864	0.097	1.055

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	55	17	35
normalized size	1	1.	1.	0.96	1.24	2.2	0.68	1.4
time (sec)	N/A	0.05	0.005	0.007	0.926	1.928	0.087	1.053

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	18	31	8	18
normalized size	1	1.	1.	1.09	1.64	2.82	0.73	1.64
time (sec)	N/A	0.012	0.004	0.004	0.925	1.835	0.087	1.067

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	36	32	42	111	36	42
normalized size	1	1.	1.	0.89	1.17	3.08	1.	1.17
time (sec)	N/A	0.118	0.015	0.005	1.404	1.882	0.169	1.055

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	28	105	22	28
normalized size	1	1.	1.	0.76	0.97	3.62	0.76	0.97
time (sec)	N/A	0.113	0.016	0.009	1.401	1.905	0.157	1.049

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	48	63	236	63	81
normalized size	1	1.	0.9	0.8	1.05	3.93	1.05	1.35
time (sec)	N/A	0.254	0.052	0.009	1.416	2.008	0.228	1.065

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	28	25	41	104	26	34
normalized size	1	1.	0.76	0.68	1.11	2.81	0.7	0.92
time (sec)	N/A	0.008	0.013	0.007	1.414	1.929	0.116	1.056

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	78	68	101	378	88	96
normalized size	1	1.	0.8	0.7	1.04	3.9	0.91	0.99
time (sec)	N/A	0.077	0.041	0.01	1.412	1.97	0.206	1.055

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	36	47	128	46	49
normalized size	1	1.	1.	0.78	1.02	2.78	1.	1.07
time (sec)	N/A	0.059	0.025	0.007	1.411	1.96	0.149	1.064

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	48	38	50	138	48	51
normalized size	1	1.	1.	0.79	1.04	2.88	1.	1.06
time (sec)	N/A	0.024	0.012	0.004	1.43	1.847	0.13	1.056

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	19	14	18	35	10	19
normalized size	1	1.	1.27	0.93	1.2	2.33	0.67	1.27
time (sec)	N/A	0.006	0.003	0.001	0.951	1.806	0.068	1.045

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	8	9	11	24	7	12
normalized size	1	1.	0.8	0.9	1.1	2.4	0.7	1.2
time (sec)	N/A	0.004	0.001	0.002	0.928	1.786	0.068	1.05

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	36	10	18
normalized size	1	1.	1.	0.92	1.15	2.77	0.77	1.38
time (sec)	N/A	0.006	0.004	0.004	0.919	1.918	0.094	1.054

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	35	8	18
normalized size	1	1.	1.	1.09	1.36	3.18	0.73	1.64
time (sec)	N/A	0.001	0.003	0.004	0.926	1.913	0.091	1.063

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	15	13	16	30	10	18
normalized size	1	1.	1.25	1.08	1.33	2.5	0.83	1.5
time (sec)	N/A	0.006	0.003	0.002	0.921	1.81	0.07	1.063

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	19	27	35	49	80	38
normalized size	1	1.	0.73	1.04	1.35	1.88	3.08	1.46
time (sec)	N/A	0.006	0.007	0.006	0.929	1.864	0.197	1.05

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	36	10	19
normalized size	1	1.	1.	0.93	1.14	2.57	0.71	1.36
time (sec)	N/A	0.018	0.003	0.004	0.927	1.68	0.09	1.05

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	19	24	58	17	27
normalized size	1	1.	1.	0.73	0.92	2.23	0.65	1.04
time (sec)	N/A	0.016	0.005	0.005	0.931	1.812	0.088	1.052

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	49	10	20
normalized size	1	1.	1.	1.07	1.36	3.5	0.71	1.43
time (sec)	N/A	0.005	0.004	0.004	0.93	1.87	0.076	1.043

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	59	19	30
normalized size	1	1.	1.	0.87	1.13	2.57	0.83	1.3
time (sec)	N/A	0.008	0.004	0.006	0.935	1.903	0.118	1.055

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	51	15	24
normalized size	1	1.	1.	0.94	1.18	3.	0.88	1.41
time (sec)	N/A	0.04	0.006	0.008	0.933	1.867	0.121	1.07

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	46	8	18
normalized size	1	1.	1.	1.08	1.33	3.83	0.67	1.5
time (sec)	N/A	0.005	0.003	0.006	0.926	1.688	0.072	1.069

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	22	21	27	81	19	28
normalized size	1	1.	0.73	0.7	0.9	2.7	0.63	0.93
time (sec)	N/A	0.01	0.007	0.006	0.929	1.931	0.104	1.052

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	21	27	89	22	35
normalized size	1	1.	0.93	0.75	0.96	3.18	0.79	1.25
time (sec)	N/A	0.01	0.016	0.006	0.926	1.832	0.113	1.048

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	50	14	22
normalized size	1	1.	1.	1.07	1.36	3.57	1.	1.57
time (sec)	N/A	0.023	0.004	0.006	0.927	1.746	0.1	1.057

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	25	18	23	58	17	27
normalized size	1	1.	1.32	0.95	1.21	3.05	0.89	1.42
time (sec)	N/A	0.027	0.007	0.007	0.93	1.781	0.115	1.055

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	35	12	19
normalized size	1	1.	1.	0.93	1.2	2.33	0.8	1.27
time (sec)	N/A	0.009	0.004	0.	0.927	1.849	0.081	1.065

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	36	92	20	41
normalized size	1	1.	1.	0.96	1.44	3.68	0.8	1.64
time (sec)	N/A	0.008	0.01	0.008	0.923	1.742	0.1	1.05

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	30	84	19	24
normalized size	1	1.	1.	0.95	1.43	4.	0.9	1.14
time (sec)	N/A	0.007	0.008	0.005	0.923	1.805	0.086	1.05

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	22	17	22	57	15	24
normalized size	1	1.	2.75	2.12	2.75	7.12	1.88	3.
time (sec)	N/A	0.005	0.003	0.006	0.929	1.772	0.089	1.055

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	32	10	18
normalized size	1	1.	1.	0.93	1.2	2.13	0.67	1.2
time (sec)	N/A	0.009	0.006	0.001	0.925	1.812	0.084	1.065

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	38	12	19
normalized size	1	1.	1.	0.83	1.06	2.11	0.67	1.06
time (sec)	N/A	0.008	0.003	0.001	0.927	1.816	0.069	1.059

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	19	24	58	17	24
normalized size	1	1.	1.	0.95	1.2	2.9	0.85	1.2
time (sec)	N/A	0.009	0.005	0.001	1.406	1.735	0.091	1.061

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	35	90	34	35
normalized size	1	1.	1.	0.87	1.13	2.9	1.1	1.13
time (sec)	N/A	0.014	0.007	0.003	1.406	1.747	0.093	1.059

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	31	22	28	70	22	28
normalized size	1	1.	1.15	0.81	1.04	2.59	0.81	1.04
time (sec)	N/A	0.025	0.005	0.004	1.404	1.935	0.098	1.049

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	28	20	26	65	19	27
normalized size	1	1.	1.22	0.87	1.13	2.83	0.83	1.17
time (sec)	N/A	0.035	0.007	0.004	1.417	1.76	0.119	1.062

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	24	31	85	29	32
normalized size	1	1.	1.	0.86	1.11	3.04	1.04	1.14
time (sec)	N/A	0.022	0.008	0.004	1.413	1.906	0.117	1.056

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	33	43	113	41	45
normalized size	1	1.	1.	0.8	1.05	2.76	1.	1.1
time (sec)	N/A	0.02	0.005	0.001	1.412	1.766	0.117	1.062

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	42	36	47	119	42	49
normalized size	1	1.	1.02	0.88	1.15	2.9	1.02	1.2
time (sec)	N/A	0.024	0.007	0.005	1.412	1.881	0.121	1.057

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	21	27	115	20	63
normalized size	1	1.	0.92	0.88	1.12	4.79	0.83	2.62
time (sec)	N/A	0.034	0.014	0.006	1.403	1.952	0.126	1.068

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	26	19	24	72	19	27
normalized size	1	1.	1.86	1.36	1.71	5.14	1.36	1.93
time (sec)	N/A	0.006	0.004	0.004	1.401	1.903	0.114	1.059

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	25	32	86	29	32
normalized size	1	1.	1.	0.86	1.1	2.97	1.	1.1
time (sec)	N/A	0.114	0.013	0.003	1.408	1.817	0.163	1.063

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	69	19	23
normalized size	1	1.	1.	0.78	1.	3.	0.83	1.
time (sec)	N/A	0.031	0.01	0.003	1.411	1.799	0.155	1.062

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	35	43	123	41	43
normalized size	1	1.	1.	0.9	1.1	3.15	1.05	1.1
time (sec)	N/A	0.014	0.022	0.004	1.421	2.012	0.115	1.06

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	19	46	8	19
normalized size	1	1.	1.	1.1	1.9	4.6	0.8	1.9
time (sec)	N/A	0.017	0.006	0.005	0.926	1.918	0.092	1.064

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	26	12	15	55	12	20
normalized size	1	1.	2.36	1.09	1.36	5.	1.09	1.82
time (sec)	N/A	0.048	0.087	0.026	0.937	2.12	0.215	1.062

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	82	18	23	61	19	23
normalized size	1	1.	4.1	0.9	1.15	3.05	0.95	1.15
time (sec)	N/A	0.052	0.163	0.015	1.408	1.992	0.577	1.059

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	18	46	12	20
normalized size	1	1.	1.	0.74	0.95	2.42	0.63	1.05
time (sec)	N/A	0.004	0.002	0.005	0.926	1.807	0.089	1.06

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	15	39	10	18
normalized size	1	1.	1.	0.71	0.88	2.29	0.59	1.06
time (sec)	N/A	0.002	0.002	0.004	0.924	1.862	0.087	1.054

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	50	17	26
normalized size	1	1.	1.	0.86	1.1	2.38	0.81	1.24
time (sec)	N/A	0.005	0.004	0.005	0.944	1.868	0.122	1.056

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	27	50	127	46	54
normalized size	1	1.	0.9	0.55	1.02	2.59	0.94	1.1
time (sec)	N/A	0.012	0.017	0.002	1.404	1.899	0.106	1.05

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	57	51	68	216	68	72
normalized size	1	1.	0.9	0.81	1.08	3.43	1.08	1.14
time (sec)	N/A	0.087	0.027	0.012	1.418	1.902	0.317	1.06

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	67	54	80	312	63	80
normalized size	1	1.	0.78	0.63	0.93	3.63	0.73	0.93
time (sec)	N/A	0.098	0.044	0.012	1.421	1.981	0.203	1.075

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	29	38	88	44	39
normalized size	1	1.	1.	1.21	1.58	3.67	1.83	1.62
time (sec)	N/A	0.005	0.005	0.007	0.94	1.791	0.716	1.06

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	22	175	367	2130	0	188
normalized size	1	1.	0.11	0.88	1.84	10.65	0.	0.94
time (sec)	N/A	0.378	0.005	0.033	1.44	12.908	0.	1.398

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	41	22	41	109	20	31
normalized size	1	1.	2.28	1.22	2.28	6.06	1.11	1.72
time (sec)	N/A	0.012	0.017	0.031	0.936	1.928	0.251	1.08

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	27	20	43	15	19
normalized size	1	1.	1.	1.5	1.11	2.39	0.83	1.06
time (sec)	N/A	0.006	0.007	0.003	0.929	1.794	0.109	1.045

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	28	21	38	68	26	27
normalized size	1	1.	0.88	0.66	1.19	2.12	0.81	0.84
time (sec)	N/A	0.013	0.011	0.002	0.931	1.909	0.113	1.049

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	42	14	16
normalized size	1	1.	1.	0.81	1.	2.62	0.88	1.
time (sec)	N/A	0.003	0.003	0.003	1.402	1.919	0.149	1.049

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	26	62	26	27
normalized size	1	1.	1.	0.9	2.6	6.2	2.6	2.7
time (sec)	N/A	0.003	0.002	0.003	0.936	1.872	0.482	1.054

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	50	23	30	22	24
normalized size	1	1.	1.	3.57	1.64	2.14	1.57	1.71
time (sec)	N/A	0.004	0.002	0.011	0.927	1.91	0.178	1.054

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	54	28	72	36	30
normalized size	1	1.	1.	1.74	0.9	2.32	1.16	0.97
time (sec)	N/A	0.023	0.01	0.013	0.927	1.899	1.312	1.079

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	21	18	30	49	76	30
normalized size	1	1.	0.66	0.56	0.94	1.53	2.38	0.94
time (sec)	N/A	0.005	0.006	0.003	0.926	1.796	1.076	1.05

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	25	32	81	76	32
normalized size	1	1.	1.	0.81	1.03	2.61	2.45	1.03
time (sec)	N/A	0.008	0.007	0.005	1.406	1.88	1.214	1.05

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	22	20	26	50	117	26
normalized size	1	1.	0.76	0.69	0.9	1.72	4.03	0.9
time (sec)	N/A	0.007	0.007	0.005	0.928	2.055	0.76	1.064

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	26	7	8
normalized size	1	1.	1.	0.88	1.	3.25	0.88	1.
time (sec)	N/A	0.005	0.002	0.003	1.414	2.295	0.291	1.059

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	20	16	20	49	17	22
normalized size	1	1.	0.95	0.76	0.95	2.33	0.81	1.05
time (sec)	N/A	0.011	0.008	0.003	0.951	2.191	0.136	1.051

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	30	66	26	31
normalized size	1	1.	1.	0.77	1.	2.2	0.87	1.03
time (sec)	N/A	0.017	0.013	0.003	0.944	2.129	0.15	1.053

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	17	26	46	26	26
normalized size	1	1.	0.74	0.63	0.96	1.7	0.96	0.96
time (sec)	N/A	0.01	0.007	0.003	0.946	2.099	0.798	1.068

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	29	242	396	1858	561	189
normalized size	1	1.	0.14	1.2	1.97	9.24	2.79	0.94
time (sec)	N/A	0.216	0.007	0.013	1.476	14.307	36.272	1.409

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	24	46	61	167	68	61
normalized size	1	1.	0.39	0.74	0.98	2.69	1.1	0.98
time (sec)	N/A	0.037	0.007	0.007	1.443	2.153	0.701	1.054

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	130	83	111	286	121	111
normalized size	1	1.	1.	0.64	0.85	2.2	0.93	0.85
time (sec)	N/A	0.046	0.037	0.004	0.947	2.129	2.022	1.062

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	40	50	63	0	38
normalized size	1	1.	1.	1.67	2.08	2.62	0.	1.58
time (sec)	N/A	0.009	0.008	0.007	1.425	2.215	0.	1.067

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	47	10	16
normalized size	1	1.	1.	1.09	1.36	4.27	0.91	1.45
time (sec)	N/A	0.021	0.007	0.031	0.938	2.335	0.196	1.056

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	42	14	20
normalized size	1	1.	1.	0.94	1.18	2.47	0.82	1.18
time (sec)	N/A	0.032	0.013	0.007	0.95	2.255	0.112	1.066

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	10	28	68	26	28
normalized size	1	1.	1.	0.83	2.33	5.67	2.17	2.33
time (sec)	N/A	0.008	0.003	0.005	0.935	2.063	1.041	1.05

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	36	47	95	32	50
normalized size	1	1.	1.	1.29	1.68	3.39	1.14	1.79
time (sec)	N/A	0.013	0.009	0.002	0.951	2.028	1.243	1.068

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	22	41	95	20	31
normalized size	1	1.	1.	0.51	0.95	2.21	0.47	0.72
time (sec)	N/A	0.019	0.011	0.011	0.928	2.306	0.209	1.083

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	24	19	53	126	0	50
normalized size	1	1.	1.14	0.9	2.52	6.	0.	2.38
time (sec)	N/A	0.01	0.02	0.019	1.408	2.226	0.	1.126

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	24	16	34	68	14	23
normalized size	1	1.	2.18	1.45	3.09	6.18	1.27	2.09
time (sec)	N/A	0.011	0.014	0.027	0.944	2.252	0.237	1.099

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	43	22	41	109	20	31
normalized size	1	1.	2.39	1.22	2.28	6.06	1.11	1.72
time (sec)	N/A	0.012	0.016	0.031	0.943	2.222	0.249	1.081

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	24	34	132	0	38
normalized size	1	1.	1.46	1.	1.42	5.5	0.	1.58
time (sec)	N/A	0.052	0.02	0.036	0.954	2.318	0.	1.067

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	39	24	297	132	0	38
normalized size	1	1.	1.62	1.	12.38	5.5	0.	1.58
time (sec)	N/A	0.027	0.029	0.052	0.964	2.343	0.	1.061

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	24	31	115	0	34
normalized size	1	1.	1.	1.33	1.72	6.39	0.	1.89
time (sec)	N/A	0.03	0.017	0.03	0.93	2.358	0.	1.057

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	0	242	0	82
normalized size	1	1.	1.06	0.97	0.	6.72	0.	2.28
time (sec)	N/A	0.022	0.043	0.046	0.	2.25	0.	1.116

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	99	2866	35
normalized size	1	1.	1.	1.07	1.33	6.6	191.07	2.33
time (sec)	N/A	0.025	0.042	0.	1.447	2.523	40.291	1.066

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	10	14	11	24	7	12
normalized size	1	1.	0.83	1.17	0.92	2.	0.58	1.
time (sec)	N/A	0.002	0.001	0.001	0.917	2.011	0.072	1.059

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	35	31	12	15
normalized size	1	1.	1.	0.71	2.06	1.82	0.71	0.88
time (sec)	N/A	0.003	0.002	0.001	0.926	2.095	0.121	1.041

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	14	30	7	11
normalized size	1	1.	0.67	0.75	1.17	2.5	0.58	0.92
time (sec)	N/A	0.009	0.008	0.006	0.921	2.082	0.349	1.068

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	24	36	109	27	39
normalized size	1	1.	1.	1.5	2.25	6.81	1.69	2.44
time (sec)	N/A	0.014	0.007	0.	0.936	1.973	0.111	1.067

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	42	19	45	15	19
normalized size	1	1.	1.	2.47	1.12	2.65	0.88	1.12
time (sec)	N/A	0.025	0.016	0.	0.945	1.85	0.096	1.064

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	16	17	15	36	20	15
normalized size	1	1.	0.67	0.71	0.62	1.5	0.83	0.62
time (sec)	N/A	0.009	0.006	0.	0.942	1.867	0.176	1.063

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	31	41	108	36	45
normalized size	1	1.	1.	0.74	0.98	2.57	0.86	1.07
time (sec)	N/A	0.043	0.007	0.007	0.971	1.709	0.13	1.064

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	22	7	8
normalized size	1	1.	1.	0.88	1.	2.75	0.88	1.
time (sec)	N/A	0.014	0.001	0.003	0.956	1.871	0.41	1.057

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	28	8	15
normalized size	1	1.	1.	0.92	1.17	2.33	0.67	1.25
time (sec)	N/A	0.006	0.003	0.003	0.958	1.797	0.069	1.046

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	4	4	12	3	4
normalized size	1	1.	1.	0.8	0.8	2.4	0.6	0.8
time (sec)	N/A	0.006	0.005	0.002	0.971	1.941	0.624	1.055

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	14	19	14	58	14	14
normalized size	1	1.	0.58	0.79	0.58	2.42	0.58	0.58
time (sec)	N/A	0.028	0.004	0.	0.969	1.996	0.062	1.051

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	42	8	24
normalized size	1	1.	1.	1.12	1.38	5.25	1.	3.
time (sec)	N/A	0.023	0.023	0.029	0.954	2.058	0.129	1.085

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	17	15	23	8	15
normalized size	1	1.	1.	1.31	1.15	1.77	0.62	1.15
time (sec)	N/A	0.002	0.001	0.	0.957	1.878	0.125	1.058

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	36	12	18
normalized size	1	1.	1.	0.82	1.06	2.12	0.71	1.06
time (sec)	N/A	0.007	0.001	0.001	0.973	1.889	0.086	1.058

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	19	24	58	107	24
normalized size	1	1.	1.	0.79	1.	2.42	4.46	1.
time (sec)	N/A	0.006	0.006	0.006	1.422	1.902	1.249	1.059

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	46	8	18
normalized size	1	1.	1.	1.08	1.33	3.83	0.67	1.5
time (sec)	N/A	0.005	0.003	0.004	0.935	1.818	0.078	1.051

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	22	49	15	22
normalized size	1	1.	1.	1.06	1.38	3.06	0.94	1.38
time (sec)	N/A	0.005	0.002	0.002	1.431	1.935	0.117	1.055

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	30	39	103	32	0
normalized size	1	1.	1.	1.36	1.77	4.68	1.45	0.
time (sec)	N/A	0.056	0.021	0.002	0.95	1.972	1.791	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	43	26	122	54	57
normalized size	1	1.	0.81	1.59	0.96	4.52	2.	2.11
time (sec)	N/A	0.006	0.017	0.001	0.93	1.816	0.808	1.098

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	22	41	45	15	19
normalized size	1	1.	1.	1.29	2.41	2.65	0.88	1.12
time (sec)	N/A	0.025	0.011	0.012	0.943	2.046	0.105	1.072

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	54	15	22
normalized size	1	1.	1.	0.77	1.	2.45	0.68	1.
time (sec)	N/A	0.008	0.003	0.003	1.414	1.769	0.091	1.072

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	32	68	24	35
normalized size	1	1.	0.88	0.78	1.	2.12	0.75	1.09
time (sec)	N/A	0.01	0.009	0.003	1.448	1.984	0.182	1.071

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	47	65	73	54
normalized size	1	1.	1.	0.83	1.57	2.17	2.43	1.8
time (sec)	N/A	0.015	0.005	0.004	1.425	1.878	1.3	1.06

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	36	10	20
normalized size	1	1.	1.	1.08	1.38	2.77	0.77	1.54
time (sec)	N/A	0.003	0.003	0.001	0.948	1.706	0.089	1.087

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	14	17	59	46	17	36
normalized size	1	1.	0.88	1.06	3.69	2.88	1.06	2.25
time (sec)	N/A	0.027	0.013	0.004	0.953	1.85	0.326	1.092

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	20	14	20	38	14	24
normalized size	1	1.	1.18	0.82	1.18	2.24	0.82	1.41
time (sec)	N/A	0.009	0.005	0.003	0.937	1.889	0.092	1.068

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	22	3	4
normalized size	1	1.	1.	1.33	1.33	7.33	1.	1.33
time (sec)	N/A	0.018	0.004	0.01	1.422	1.94	0.224	1.068

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	55	20	22
normalized size	1	1.	1.	0.77	1.	2.5	0.91	1.
time (sec)	N/A	0.011	0.013	0.	0.941	2.038	0.295	1.068

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	20	7	12
normalized size	1	1.	1.	1.11	1.33	2.22	0.78	1.33
time (sec)	N/A	0.004	0.004	0.003	0.968	2.028	0.06	1.048

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	27	8	14
normalized size	1	1.	1.	0.92	1.17	2.25	0.67	1.17
time (sec)	N/A	0.02	0.007	0.	0.955	2.002	0.082	1.073

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	26	63	26	26
normalized size	1	1.	0.81	0.81	0.96	2.33	0.96	0.96
time (sec)	N/A	0.01	0.027	0.007	0.954	1.97	0.297	1.059

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	65	26	18
normalized size	1	1.	1.	0.82	1.06	3.82	1.53	1.06
time (sec)	N/A	0.008	0.007	0.041	0.948	2.001	0.532	1.057

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	69	19	27
normalized size	1	1.	1.	0.8	1.04	2.76	0.76	1.08
time (sec)	N/A	0.014	0.005	0.004	1.447	1.815	0.112	1.067

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	28	46	39	74	29	66
normalized size	1	1.	0.72	1.18	1.	1.9	0.74	1.69
time (sec)	N/A	0.015	0.009	0.002	0.953	1.915	0.107	1.045

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	16	14	18	34	12	18
normalized size	1	1.	0.62	0.54	0.69	1.31	0.46	0.69
time (sec)	N/A	0.027	0.003	0.002	0.969	1.921	0.085	1.046

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	11	14	26	12	14
normalized size	1	1.	1.5	0.92	1.17	2.17	1.	1.17
time (sec)	N/A	0.005	0.008	0.003	1.466	2.016	0.063	1.064

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	30	30	61	0	28
normalized size	1	1.	0.96	1.2	1.2	2.44	0.	1.12
time (sec)	N/A	0.005	0.006	0.004	1.418	1.984	0.	1.071

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	22	28	65	20	28
normalized size	1	1.	0.85	0.81	1.04	2.41	0.74	1.04
time (sec)	N/A	0.015	0.004	0.002	0.939	1.979	0.34	1.061

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	12	15	38	15	15
normalized size	1	1.	1.	0.63	0.79	2.	0.79	0.79
time (sec)	N/A	0.003	0.003	0.002	0.94	1.901	1.855	1.051

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	26	51	19	22
normalized size	1	1.	1.	1.	4.33	8.5	3.17	3.67
time (sec)	N/A	0.01	0.003	0.002	0.936	1.907	0.101	1.051

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	36	10	14
normalized size	1	1.	1.	0.79	1.	2.57	0.71	1.
time (sec)	N/A	0.013	0.004	0.002	1.418	1.839	0.096	1.052

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	30	10	43
normalized size	1	1.	1.	0.75	0.92	2.5	0.83	3.58
time (sec)	N/A	0.003	0.002	0.003	1.415	2.152	0.218	1.055

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	26	24	81	31	30
normalized size	1	1.	0.88	0.76	0.71	2.38	0.91	0.88
time (sec)	N/A	0.033	0.008	0.001	0.938	2.305	0.06	1.064

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	7	11	74	0	8
normalized size	1	1.	1.	0.58	0.92	6.17	0.	0.67
time (sec)	N/A	0.007	0.006	0.003	1.404	2.105	0.	1.064

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	59	19	53	44	19
normalized size	1	1.	1.	3.69	1.19	3.31	2.75	1.19
time (sec)	N/A	0.046	0.03	0.016	0.928	2.02	2.747	1.052

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	20	6	7	32	12	9
normalized size	1	1.	2.86	0.86	1.	4.57	1.71	1.29
time (sec)	N/A	0.013	0.005	0.014	0.929	2.149	2.022	1.054

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	20	51	15	22
normalized size	1	1.	1.	1.	3.33	8.5	2.5	3.67
time (sec)	N/A	0.017	0.002	0.003	0.949	1.902	0.096	1.05

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	35	43	117	41	45
normalized size	1	1.	1.	0.81	1.	2.72	0.95	1.05
time (sec)	N/A	0.02	0.008	0.005	1.408	1.987	0.122	1.044

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	29	38	100	88	42	77
normalized size	1	1.	0.78	1.03	2.7	2.38	1.14	2.08
time (sec)	N/A	0.076	0.016	0.005	0.969	1.822	2.097	1.046

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	35	0	9
normalized size	1	1.	1.	0.89	1.	3.89	0.	1.
time (sec)	N/A	0.023	0.005	0.02	0.932	2.032	0.	1.056

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	34	12	22
normalized size	1	1.	1.	0.85	1.1	1.7	0.6	1.1
time (sec)	N/A	0.002	0.	0.	0.937	1.598	0.049	1.044

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	12	12	27	8	14
normalized size	1	1.	1.	1.	1.	2.25	0.67	1.17
time (sec)	N/A	0.021	0.008	0.005	0.942	1.918	0.079	1.052

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	51	15	23
normalized size	1	1.	1.	0.86	1.1	2.43	0.71	1.1
time (sec)	N/A	0.011	0.005	0.004	0.928	1.814	0.096	1.05

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	22	41	105	20	31
normalized size	1	1.	1.	0.51	0.95	2.44	0.47	0.72
time (sec)	N/A	0.018	0.012	0.012	0.937	2.29	0.223	1.08

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	18	15	22	58	144	22
normalized size	1	1.	0.75	0.62	0.92	2.42	6.	0.92
time (sec)	N/A	0.004	0.006	0.003	0.937	1.99	0.926	1.052

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	24	26	22	55	34	22
normalized size	1	1.	0.63	0.68	0.58	1.45	0.89	0.58
time (sec)	N/A	0.019	0.008	0.002	0.937	2.198	0.318	1.056

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	93	41	126	39	41
normalized size	1	1.	1.	2.51	1.11	3.41	1.05	1.11
time (sec)	N/A	0.036	0.013	0.012	1.412	2.082	1.264	1.052

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	16	20	55	14	23
normalized size	1	1.	1.	0.94	1.18	3.24	0.82	1.35
time (sec)	N/A	0.029	0.004	0.007	0.936	2.143	0.097	1.057

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	29	35	51	76	39	35
normalized size	1	1.	0.72	0.88	1.27	1.9	0.98	0.88
time (sec)	N/A	0.067	0.037	0.007	0.952	2.268	0.315	1.048

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	7	59	5	7
normalized size	1	1.	1.	1.2	1.4	11.8	1.	1.4
time (sec)	N/A	0.009	2.479	0.007	0.926	2.284	0.506	1.052

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	5	49	3	5
normalized size	1	1.	1.	0.83	0.83	8.17	0.5	0.83
time (sec)	N/A	0.001	0.004	0.003	1.401	2.208	0.131	1.071

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	24	30	84	157	61	30
normalized size	1	1.	0.65	0.81	2.27	4.24	1.65	0.81
time (sec)	N/A	0.011	0.007	0.005	0.944	2.038	0.14	1.062

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	58	24	93	17	24
normalized size	1	1.	1.	2.76	1.14	4.43	0.81	1.14
time (sec)	N/A	0.029	0.023	0.028	0.932	2.325	0.099	1.085

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	32	76	41	32
normalized size	1	1.	1.	0.83	1.07	2.53	1.37	1.07
time (sec)	N/A	0.034	0.058	0.008	0.936	2.18	0.192	1.054

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	4	4	27	4	4	18	3	4
normalized size	1	1.	6.75	1.	1.	4.5	0.75	1.
time (sec)	N/A	0.021	0.004	0.003	0.925	2.185	0.866	1.048

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	20	38	10	20
normalized size	1	1.	1.	0.92	1.54	2.92	0.77	1.54
time (sec)	N/A	0.005	0.003	0.004	0.929	2.021	0.09	1.048

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	24	24	28	55	22	28
normalized size	1	1.	0.55	0.55	0.64	1.25	0.5	0.64
time (sec)	N/A	0.039	0.009	0.001	0.937	2.107	0.083	1.05

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	28	36	90	37	36
normalized size	1	1.	1.	0.68	0.88	2.2	0.9	0.88
time (sec)	N/A	0.012	0.01	0.005	1.407	2.128	2.931	1.053

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	54	114	18
normalized size	1	1.	1.	0.8	1.04	2.16	4.56	0.72
time (sec)	N/A	0.03	0.01	0.046	0.928	2.309	15.042	1.051

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	24	7	14
normalized size	1	1.	1.	0.92	1.17	2.	0.58	1.17
time (sec)	N/A	0.001	0.001	0.001	0.937	2.351	0.081	1.049

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	62	41	58	90	0	41
normalized size	1	1.	1.51	1.	1.41	2.2	0.	1.
time (sec)	N/A	0.013	0.023	0.008	1.411	2.276	0.	1.072

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	27	119	36	80	29	38
normalized size	1	1.	0.79	3.5	1.06	2.35	0.85	1.12
time (sec)	N/A	0.037	0.02	0.	1.417	2.355	2.89	1.068

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	23	46	42	23
normalized size	1	1.	1.	0.95	1.21	2.42	2.21	1.21
time (sec)	N/A	0.006	0.004	0.001	1.42	2.092	0.102	1.053

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	39	30	53	101	0	42
normalized size	1	1.	1.03	0.79	1.39	2.66	0.	1.11
time (sec)	N/A	0.012	0.015	0.003	1.415	2.129	0.	1.058

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	30	7	11
normalized size	1	1.	1.	0.75	0.92	2.5	0.58	0.92
time (sec)	N/A	0.005	0.004	0.001	1.407	2.144	0.112	1.069

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	27	35	89	36	35
normalized size	1	1.	1.	0.87	1.13	2.87	1.16	1.13
time (sec)	N/A	0.015	0.01	0.001	1.405	2.113	0.1	1.054

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	37	16	163	95	0	203
normalized size	1	1.	3.7	1.6	16.3	9.5	0.	20.3
time (sec)	N/A	0.01	0.01	0.004	1.422	2.116	0.	1.16

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	30	39	61	22	41
normalized size	1	1.	1.	2.	2.6	4.07	1.47	2.73
time (sec)	N/A	0.007	0.004	0.006	0.926	1.842	0.132	1.071

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	14	0	45	63	18
normalized size	1	1.	1.	0.67	0.	2.14	3.	0.86
time (sec)	N/A	0.006	0.016	0.002	0.	1.841	0.352	1.05

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	26	53	19	24
normalized size	1	1.	0.7	0.78	1.13	2.3	0.83	1.04
time (sec)	N/A	0.017	0.011	0.005	0.933	1.933	0.108	1.05

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	54	17	22
normalized size	1	1.	1.	0.85	1.1	2.7	0.85	1.1
time (sec)	N/A	0.008	0.007	0.004	0.927	2.013	0.481	1.049

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	24	49	14	27
normalized size	1	1.	1.	0.89	1.33	2.72	0.78	1.5
time (sec)	N/A	0.008	0.003	0.006	0.924	1.818	0.121	1.053

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	19	20	26	74	20	27
normalized size	1	1.	1.58	1.67	2.17	6.17	1.67	2.25
time (sec)	N/A	0.012	0.005	0.005	0.935	1.985	0.109	1.059

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	11	14	42	12	22
normalized size	1	1.	1.	1.38	1.75	5.25	1.5	2.75
time (sec)	N/A	0.037	0.008	0.032	1.41	2.158	1.311	1.066

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	16	43	44	16
normalized size	1	1.	1.	0.72	0.89	2.39	2.44	0.89
time (sec)	N/A	0.003	0.002	0.006	1.406	1.855	0.979	1.059

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	0	72	0	0
normalized size	1	1.	1.	0.91	0.	6.55	0.	0.
time (sec)	N/A	0.05	0.016	0.03	0.	2.553	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	30	85	27	30
normalized size	1	1.	1.	0.77	1.	2.83	0.9	1.
time (sec)	N/A	0.006	0.008	0.004	1.446	1.995	0.207	1.072

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	20	25	28	57	26	28
normalized size	1	1.	0.83	1.04	1.17	2.38	1.08	1.17
time (sec)	N/A	0.037	0.015	0.004	0.952	1.965	0.555	1.083

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	38	42	66	109	0	54
normalized size	1	1.	0.76	0.84	1.32	2.18	0.	1.08
time (sec)	N/A	0.015	0.018	0.004	1.423	1.949	0.	1.059

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	47	12	153	51	12
normalized size	1	1.	1.	4.27	1.09	13.91	4.64	1.09
time (sec)	N/A	0.001	0.002	0.	0.931	1.575	0.055	1.057

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	31	28	26	61	20	26
normalized size	1	1.	1.24	1.12	1.04	2.44	0.8	1.04
time (sec)	N/A	0.027	0.01	0.006	0.949	2.075	0.062	1.058

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	26	68	26	26
normalized size	1	1.	0.81	0.81	0.96	2.52	0.96	0.96
time (sec)	N/A	0.01	0.027	0.006	0.937	1.872	0.459	1.052

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	41	24	46	182	36	95
normalized size	1	1.	1.71	1.	1.92	7.58	1.5	3.96
time (sec)	N/A	0.011	0.009	0.009	0.98	2.024	0.116	1.066

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	35	47	45	84	17	59
normalized size	1	1.	1.03	1.38	1.32	2.47	0.5	1.74
time (sec)	N/A	0.008	0.015	0.005	1.424	1.891	0.225	1.073

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	47	70	75	51
normalized size	1	1.	1.	0.83	1.57	2.33	2.5	1.7
time (sec)	N/A	0.017	0.005	0.004	1.426	1.967	1.485	1.05

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	19	17	22	43	15	22
normalized size	1	1.	0.59	0.53	0.69	1.34	0.47	0.69
time (sec)	N/A	0.019	0.006	0.001	0.942	1.733	0.079	1.066

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	31	18	23	47	26	23
normalized size	1	1.	1.35	0.78	1.	2.04	1.13	1.
time (sec)	N/A	0.037	0.022	0.006	0.958	2.046	0.322	1.059

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	24	22	28	57	19	28
normalized size	1	1.	0.89	0.81	1.04	2.11	0.7	1.04
time (sec)	N/A	0.015	0.004	0.002	1.416	2.059	0.175	1.055

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	32	31	72	73	29	43
normalized size	1	1.	0.84	0.82	1.89	1.92	0.76	1.13
time (sec)	N/A	0.023	0.011	0.002	1.438	2.185	0.562	1.08

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	5	5	82	0	8
normalized size	1	1.	1.	1.	1.	16.4	0.	1.6
time (sec)	N/A	0.01	0.005	0.002	0.953	1.923	0.	1.067

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	28	59	27	28
normalized size	1	1.	0.86	0.83	0.97	2.03	0.93	0.97
time (sec)	N/A	0.028	0.024	0.	0.941	1.922	0.316	1.062

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	33	29	49	126	0	35
normalized size	1	1.	0.92	0.81	1.36	3.5	0.	0.97
time (sec)	N/A	0.01	0.016	0.003	1.431	2.054	0.	1.061

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	31	23	30	65	24	30
normalized size	1	1.	1.11	0.82	1.07	2.32	0.86	1.07
time (sec)	N/A	0.019	0.011	0.007	1.443	1.88	0.115	1.058

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	58	25	57	138	46	51
normalized size	1	1.	2.23	0.96	2.19	5.31	1.77	1.96
time (sec)	N/A	0.014	0.113	0.032	0.946	2.161	0.135	1.082

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	30	32	32	95	46	30
normalized size	1	1.	0.65	0.7	0.7	2.07	1.	0.65
time (sec)	N/A	0.02	0.006	0.007	0.949	1.927	0.059	1.057

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	15	17	22	92	17	22
normalized size	1	1.	0.75	0.85	1.1	4.6	0.85	1.1
time (sec)	N/A	0.035	0.015	0.006	0.946	2.091	6.153	1.051

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	22	22	62	14	26
normalized size	1	1.	1.	1.05	1.05	2.95	0.67	1.24
time (sec)	N/A	0.026	0.016	0.007	0.926	2.025	0.09	1.052

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	31	63	182	0	68
normalized size	1	1.	0.89	0.84	1.7	4.92	0.	1.84
time (sec)	N/A	0.041	0.019	0.016	1.406	3.008	0.	1.069

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	28	28	40	26	39	69	0	38
normalized size	1	1.	1.43	0.93	1.39	2.46	0.	1.36
time (sec)	N/A	0.008	0.029	0.003	0.93	1.967	0.	1.064

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	23	17	20	58	17	20
normalized size	1	1.	1.21	0.89	1.05	3.05	0.89	1.05
time (sec)	N/A	0.009	0.002	0.027	0.942	1.963	0.058	1.078

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	26	25	32	59	22	32
normalized size	1	1.	0.57	0.54	0.7	1.28	0.48	0.7
time (sec)	N/A	0.04	0.007	0.002	0.925	1.965	0.082	1.055

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	34	45	45	34	23
normalized size	1	1.	1.	1.89	2.5	2.5	1.89	1.28
time (sec)	N/A	0.007	0.003	0.043	0.932	1.811	1.012	1.074

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	22	22	26	63	24	26
normalized size	1	1.	0.81	0.81	0.96	2.33	0.89	0.96
time (sec)	N/A	0.009	0.042	0.007	0.943	1.919	0.288	1.053

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	22	18	17	22	41	0	22
normalized size	1	1.22	1.	0.94	1.22	2.28	0.	1.22
time (sec)	N/A	0.03	0.006	0.002	0.931	1.928	0.	1.048

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	23	22	28	65	20	28
normalized size	1	1.	0.85	0.81	1.04	2.41	0.74	1.04
time (sec)	N/A	0.016	0.004	0.	0.92	2.031	0.325	1.065

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	27	63	19	27
normalized size	1	1.	1.	0.81	1.04	2.42	0.73	1.04
time (sec)	N/A	0.012	0.006	0.004	1.412	1.863	1.077	1.069

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	11	14	12	34	15	721
normalized size	1	1.	0.73	0.93	0.8	2.27	1.	48.07
time (sec)	N/A	0.017	0.015	0.009	0.944	1.971	2.25	1.094

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	35	35	46	89	122	39
normalized size	1	1.	0.74	0.74	0.98	1.89	2.6	0.83
time (sec)	N/A	0.01	0.018	0.005	1.409	2.022	2.636	1.068

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	36	27	12	70	27	12
normalized size	1	1.	3.27	2.45	1.09	6.36	2.45	1.09
time (sec)	N/A	0.002	0.001	0.001	0.93	1.569	0.051	1.059

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	18	39	12	18
normalized size	1	1.	1.	1.06	1.06	2.29	0.71	1.06
time (sec)	N/A	0.022	0.006	0.006	0.923	2.005	0.059	1.055

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	27	22	18	69	29	18
normalized size	1	1.	1.59	1.29	1.06	4.06	1.71	1.06
time (sec)	N/A	0.023	0.016	0.	0.924	2.228	0.063	1.061

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	26	49	36	26
normalized size	1	1.	0.67	0.56	0.96	1.81	1.33	0.96
time (sec)	N/A	0.005	0.006	0.002	0.925	2.095	0.844	1.057

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	18	22	59	24	22
normalized size	1	1.	0.92	0.75	0.92	2.46	1.	0.92
time (sec)	N/A	0.009	0.002	0.004	0.92	1.921	0.056	1.063

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	27	57	12	22
normalized size	1	1.	1.	1.42	2.25	4.75	1.	1.83
time (sec)	N/A	0.006	0.003	0.001	0.924	2.076	0.086	1.077

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	25	22	46	68	53	38
normalized size	1	1.	0.62	0.55	1.15	1.7	1.32	0.95
time (sec)	N/A	0.014	0.007	0.005	1.404	1.82	2.029	1.054

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [19] had the largest ratio of [1.]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	3	0.333
2	A	1	1	1.	3	0.333
3	A	1	1	1.	3	0.333
4	A	1	1	1.	3	0.333
5	A	1	1	1.	2	0.5
6	A	1	1	1.	2	0.5
7	A	2	2	1.	4	0.5
8	A	2	2	1.	4	0.5
9	A	2	2	1.	5	0.4
10	A	2	2	1.	5	0.4
11	A	1	1	1.	2	0.5
12	A	1	1	1.	2	0.5
13	A	1	1	1.	2	0.5
14	A	1	1	1.	2	0.5
15	A	2	2	1.	4	0.5
16	A	1	1	1.	2	0.5
17	A	3	2	1.	7	0.286
18	A	1	1	1.	6	0.167
19	A	2	2	1.	2	1.
20	A	2	2	1.	7	0.286
21	A	2	2	1.	4	0.5
22	A	2	2	1.	6	0.333
23	A	1	1	1.	4	0.25
24	A	3	2	1.	8	0.25
25	A	3	2	1.	8	0.25
26	A	2	2	1.	4	0.5
27	A	2	2	1.	2	1.
28	A	3	3	1.	6	0.5
29	A	2	2	1.	6	0.333
30	A	1	1	1.	6	0.167
31	A	4	2	1.	7	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
32	A	1	1	1.	10	0.1
33	A	1	1	1.	10	0.1
34	A	2	2	1.	4	0.5
35	A	2	2	1.	6	0.333
36	A	2	2	1.	7	0.286
37	A	1	1	1.	8	0.125
38	A	2	2	1.	6	0.333
39	A	3	2	1.	9	0.222
40	A	2	2	1.	2	1.
41	A	2	2	1.	6	0.333
42	A	1	1	1.	9	0.111
43	A	1	1	1.	9	0.111
44	A	2	2	1.	6	0.333
45	A	2	2	1.	9	0.222
46	A	2	2	1.	9	0.222
47	A	2	2	1.	5	0.4
48	A	1	1	1.	3	0.333
49	A	3	3	1.	7	0.429
50	A	1	1	1.	6	0.167
51	A	1	1	1.	3	0.333
52	A	3	3	1.	6	0.5
53	A	3	3	1.	8	0.375
54	A	3	2	1.	9	0.222
55	A	3	3	1.	4	0.75
56	A	2	2	1.	6	0.333
57	A	1	1	1.	8	0.125
58	A	2	2	1.	6	0.333
59	A	2	2	1.	4	0.5
60	A	3	2	1.	4	0.5
61	A	2	1	1.	4	0.25
62	A	3	2	1.	9	0.222
63	A	3	2	1.	9	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	4	3	1.	9	0.333
65	A	3	3	1.	9	0.333
66	A	1	1	1.	10	0.1
67	A	3	2	1.	11	0.182
68	A	4	3	1.	9	0.333
69	A	4	2	1.	4	0.5
70	A	4	2	1.	4	0.5
71	A	4	3	1.	13	0.231
72	A	2	1	1.	4	0.25
73	A	5	3	1.	9	0.333
74	A	3	2	1.	11	0.182
75	A	3	2	1.	11	0.182
76	A	3	3	1.	14	0.214
77	A	3	2	1.	8	0.25
78	A	3	2	1.	7	0.286
79	A	4	3	1.	9	0.333
80	A	2	2	1.	9	0.222
81	A	1	1	1.	8	0.125
82	A	2	2	1.	4	0.5
83	A	3	2	1.	4	0.5
84	A	2	1	1.	4	0.25
85	A	2	1	1.	4	0.25
86	A	2	2	1.	9	0.222
87	A	3	2	1.	9	0.222
88	A	2	2	1.	7	0.286
89	A	3	2	1.	9	0.222
90	A	3	2	1.	4	0.5
91	A	4	2	1.	4	0.5
92	A	3	2	1.	7	0.286
93	A	3	2	1.	9	0.222
94	A	2	2	1.	7	0.286
95	A	3	2	1.	9	0.222

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	2	2	1.	7	0.286
97	A	2	2	1.	7	0.286
98	A	2	2	1.	4	0.5
99	A	2	2	1.	4	0.5
100	A	3	2	1.	9	0.222
101	A	3	2	1.	9	0.222
102	A	1	1	1.	2	0.5
103	A	2	2	1.	4	0.5
104	A	3	3	1.	5	0.6
105	A	2	1	1.	4	0.25
106	A	1	1	1.	9	0.111
107	A	1	1	1.	7	0.143
108	A	1	1	1.	9	0.111
109	A	1	1	1.	9	0.111
110	A	2	2	1.	7	0.286
111	A	5	2	1.	11	0.182
112	A	2	2	1.	13	0.154
113	A	6	4	1.	10	0.4
114	A	3	2	1.	7	0.286
115	A	4	3	1.	9	0.333
116	A	2	2	1.	7	0.286
117	A	3	2	1.	9	0.222
118	A	2	2	1.	15	0.133
119	A	1	1	1.	13	0.077
120	A	1	1	1.	11	0.091
121	A	2	2	1.	13	0.154
122	A	3	2	1.	15	0.133
123	A	3	3	1.	16	0.188
124	A	1	1	1.	15	0.067
125	A	3	2	1.	15	0.133
126	A	1	1	1.	13	0.077
127	A	1	1	1.	13	0.077

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
128	A	2	2	1.	11	0.182
129	A	3	2	1.	13	0.154
130	A	1	1	1.	9	0.111
131	A	2	2	1.	9	0.222
132	A	4	4	1.	13	0.308
133	A	1	1	1.	17	0.059
134	A	4	4	1.	15	0.267
135	A	1	1	1.	15	0.067
136	A	3	3	1.	17	0.176
137	A	2	2	1.	15	0.133
138	A	3	3	1.	13	0.231
139	A	1	1	1.	11	0.091
140	A	3	2	1.	15	0.133
141	A	3	3	1.	15	0.2
142	A	2	2	1.	12	0.167
143	A	1	1	1.	11	0.091
144	A	3	3	1.	13	0.231
145	A	2	2	1.	12	0.167
146	A	2	2	1.	14	0.143
147	A	4	4	1.	17	0.235
148	A	3	3	1.	10	0.3
149	A	2	2	1.	14	0.143
150	A	3	3	1.	17	0.176
151	A	4	4	1.	11	0.364
152	A	2	2	1.	11	0.182
153	A	3	2	1.	12	0.167
154	A	3	2	1.	11	0.182
155	A	3	2	1.	25	0.08
156	A	2	1	1.	29	0.034
157	A	6	5	1.	20	0.25
158	A	6	5	1.	23	0.217
159	A	14	10	1.	32	0.312

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
160	A	6	5	1.	26	0.192
161	A	2	2	1.	7	0.286
162	A	3	2	1.	11	0.182
163	A	4	3	1.	14	0.214
164	A	2	1	1.	23	0.043
165	A	3	2	1.	22	0.091
166	A	3	2	1.	11	0.182
167	A	4	3	1.	26	0.115
168	A	3	2	1.	16	0.125
169	A	8	4	1.	25	0.16
170	A	6	3	1.	23	0.13
171	A	6	5	1.	26	0.192
172	A	3	2	1.	11	0.182
173	A	8	7	1.	20	0.35
174	A	7	6	1.	23	0.261
175	A	8	8	1.	11	0.727
176	A	2	1	1.	9	0.111
177	A	2	1	1.	7	0.143
178	A	2	1	1.	16	0.062
179	A	3	2	1.	11	0.182
180	A	2	1	1.	13	0.077
181	A	3	2	1.	11	0.182
182	A	3	2	1.	15	0.133
183	A	5	3	1.	20	0.15
184	A	2	1	1.	11	0.091
185	A	2	1	1.	16	0.062
186	A	3	2	1.	25	0.08
187	A	3	2	1.	12	0.167
188	A	2	1	1.	11	0.091
189	A	2	1	1.	14	0.071
190	A	3	2	1.	22	0.091
191	A	2	1	1.	24	0.042

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
192	A	1	1	1.	20	0.05
193	A	2	1	1.	9	0.111
194	A	2	1	1.	9	0.111
195	A	3	3	1.	11	0.273
196	A	1	1	1.	22	0.045
197	A	3	2	1.	11	0.182
198	A	4	4	1.	14	0.286
199	A	4	4	1.	10	0.4
200	A	6	5	1.	23	0.217
201	A	5	4	1.	23	0.174
202	A	6	5	1.	15	0.333
203	A	6	6	1.	7	0.857
204	A	7	7	1.	11	0.636
205	A	5	4	1.	21	0.19
206	A	4	4	1.	11	0.364
207	A	6	4	1.	30	0.133
208	A	7	6	1.	24	0.25
209	A	3	3	1.	14	0.214
210	A	3	2	1.	16	0.125
211	A	2	2	1.	21	0.095
212	A	3	3	1.	15	0.2
213	A	3	2	1.	10	0.2
214	A	1	1	1.	9	0.111
215	A	3	2	1.	18	0.111
216	A	3	2	1.	10	0.2
217	A	6	5	1.	43	0.116
218	A	7	5	1.	50	0.1
219	A	3	3	1.	11	0.273
220	A	9	9	1.	15	0.6
221	A	2	2	1.	11	0.182
222	A	3	2	1.	9	0.222
223	A	3	2	1.	9	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
224	A	3	3	1.	11	0.273
225	A	2	2	1.	11	0.182
226	A	2	2	1.	11	0.182
227	A	4	2	1.	13	0.154
228	A	2	1	1.	11	0.091
229	A	3	3	1.	13	0.231
230	A	3	2	1.	11	0.182
231	A	3	3	1.	13	0.231
232	A	3	2	1.	17	0.118
233	A	4	3	1.	17	0.176
234	A	3	2	1.	13	0.154
235	A	10	9	1.	21	0.429
236	A	9	9	1.	13	0.692
237	A	4	3	1.	13	0.231
238	A	5	5	1.	13	0.385
239	A	2	2	1.	12	0.167
240	A	4	3	1.	20	0.15
241	A	3	3	1.	9	0.333
242	A	4	4	1.	11	0.364
243	A	4	3	1.	8	0.375
244	A	2	2	1.	7	0.286
245	A	2	2	1.	10	0.2
246	A	2	2	1.	11	0.182
247	A	6	6	1.	7	0.857
248	A	4	2	1.	11	0.182
249	A	4	3	1.	9	0.333
250	A	2	2	1.	11	0.182
251	A	2	1	1.	19	0.053
252	A	1	1	1.	9	0.111
253	A	2	1	1.	13	0.077
254	A	1	1	1.	8	0.125
255	A	2	2	1.	7	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
256	A	3	2	1.	9	0.222
257	A	3	3	1.	7	0.429
258	A	3	2	1.	20	0.1
259	A	2	2	1.	10	0.2
260	A	2	1	1.	11	0.091
261	A	2	2	1.	7	0.286
262	A	3	3	1.	9	0.333
263	A	1	1	1.	15	0.067
264	A	1	1	1.	13	0.077
265	A	1	1	1.	6	0.167
266	A	3	3	1.	13	0.231
267	A	2	1	1.	7	0.143
268	A	3	3	1.	6	0.5
269	A	4	4	1.	16	0.25
270	A	3	2	1.	9	0.222
271	A	3	2	1.	9	0.222
272	A	4	4	1.	12	0.333
273	A	3	3	1.	4	0.75
274	A	4	4	1.	15	0.267
275	A	3	2	1.	12	0.167
276	A	3	2	1.	6	0.333
277	A	1	1	1.	21	0.048
278	A	2	2	1.	11	0.182
279	A	3	3	1.	6	0.5
280	A	1	1	1.	4	0.25
281	A	3	2	1.	13	0.154
282	A	1	1	1.	10	0.1
283	A	1	1	1.	9	0.111
284	A	5	4	1.	11	0.364
285	A	3	2	1.	8	0.25
286	A	2	2	1.	11	0.182
287	A	2	2	1.	6	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
288	A	2	2	1.	14	0.143
289	A	4	3	1.	6	0.5
290	A	2	1	1.	15	0.067
291	A	2	2	1.	13	0.154
292	A	3	3	1.	14	0.214
293	A	2	2	1.	9	0.222
294	A	4	3	1.	9	0.333
295	A	2	2	1.	14	0.143
296	A	3	2	1.	22	0.091
297	A	2	2	1.	7	0.286
298	A	2	2	1.	13	0.154
299	A	6	6	1.	7	0.857
300	A	6	2	1.	6	0.333
301	A	1	3	1.	8	0.375
302	A	1	0	1.	10	0.
303	A	3	2	1.	15	0.133
304	A	4	3	1.	16	0.188
305	A	4	3	1.	8	0.375
306	A	2	1	1.	9	0.111
307	A	4	3	1.	7	0.429
308	A	5	4	1.	12	0.333
309	A	3	2	1.	17	0.118
310	A	8	4	1.	13	0.308
311	A	2	2	1.	6	0.333
312	A	1	1	1.	11	0.091
313	A	2	1	1.	9	0.111
314	A	3	2	1.	13	0.154
315	A	6	5	1.	6	0.833
316	A	1	1	1.	12	0.083
317	A	4	4	1.	11	0.364
318	A	4	2	1.	9	0.222
319	A	7	4	1.	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
320	A	5	2	1.	11	0.182
321	A	1	1	1.	6	0.167
322	A	3	3	1.	15	0.2
323	A	5	5	1.	13	0.385
324	A	3	3	1.	13	0.231
325	A	3	3	1.	12	0.25
326	A	2	2	1.	11	0.182
327	A	4	4	1.	12	0.333
328	A	2	2	1.	6	0.333
329	A	2	2	1.	13	0.154
330	A	3	3	1.	15	0.2
331	A	4	3	1.	16	0.188
332	A	2	2	1.	12	0.167
333	A	4	4	1.	8	0.5
334	A	3	3	1.	13	0.231
335	A	4	4	1.	17	0.235
336	A	2	2	1.	13	0.154
337	A	5	4	1.	17	0.235
338	A	2	2	1.	15	0.133
339	A	4	2	1.	6	0.333
340	A	4	4	1.	14	0.286
341	A	1	1	1.	9	0.111
342	A	3	2	1.	9	0.222
343	A	1	1	1.	10	0.1
344	A	2	2	1.	8	0.25
345	A	3	3	1.	15	0.2
346	A	4	4	1.	15	0.267
347	A	3	2	1.	9	0.222
348	A	3	2	1.	13	0.154
349	A	3	3	1.	6	0.5
350	A	5	5	1.	8	0.625
351	A	2	2	1.	8	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
352	A	3	2	1.	8	0.25
353	A	3	3	1.	14	0.214
354	A	3	2	1.	15	0.133
355	A	3	2	1.	4	0.5
356	A	4	2	1.	6	0.333
357	A	4	4	1.	10	0.4
358	A	3	2	1.	15	0.133
359	A	4	4	1.	13	0.308
360	A	3	3	1.	13	0.231
361	A	2	1	1.	4	0.25
362	A	5	2	1.	9	0.222
363	A	3	3	1.	13	0.231
364	A	1	1	1.	10	0.1
365	A	4	4	1.22	10	0.4
366	A	4	3	1.	6	0.5
367	A	4	4	1.	11	0.364
368	A	4	3	1.	9	0.333
369	A	3	3	1.	15	0.2
370	A	1	1	1.	11	0.091
371	A	3	2	1.	9	0.222
372	A	3	2	1.	9	0.222
373	A	2	1	1.	11	0.091
374	A	3	2	1.	4	0.5
375	A	2	2	1.	4	0.5
376	A	3	2	1.	13	0.154

Chapter 3

Listing of integrals

3.1 $\int x^n \, dx$

Optimal. Leaf size=11

$$\frac{x^{n+1}}{n+1}$$

```
[Out] x^(1 + n)/(1 + n)
```

Rubi [A] time = 0.0023506, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {30}

$$\frac{x^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^n,x]
```

```
[Out] x^(1 + n)/(1 + n)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^n\,dx=\frac{x^{1+n}}{1+n}$$

Mathematica [A] time = 0.0014014, size = 11, normalized size = 1.

$$\frac{x^{n+1}}{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n,x]

[Out] x^(1 + n)/(1 + n)
```

Maple [A] time = 0.003, size = 12, normalized size = 1.1

$$\frac{x^{1+n}}{1+n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n,x)

[Out] x^(1+n)/(1+n)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89629, size = 20, normalized size = 1.82

$$\frac{xx^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="fricas")
```

```
[Out] x*x^n/(n + 1)
```

Sympy [A] time = 0.053293, size = 12, normalized size = 1.09

$$\begin{cases} \frac{x^{n+1}}{n+1} & \text{for}\: n \neq -1 \\ \log{\left(x \right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n,x)
```

```
[Out] Piecewise((x**(n + 1)/(n + 1), Ne(n, -1)), (log(x), True))
```

Giac [A] time = 1.0705, size = 15, normalized size = 1.36

$$\frac{x^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n,x, algorithm="giac")
```

```
[Out] x^(n + 1)/(n + 1)
```

3.2 $\int e^x \, dx$

Optimal. Leaf size=3

$$e^x$$

```
[Out] E^x
```

Rubi [A] time = 0.0009641, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2194}

$$e^x$$

Antiderivative was successfully verified.

```
[In] Int[E^x,x]
```

```
[Out] E^x
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^x \, dx = e^x$$

Mathematica [A] time = 0.0000985, size = 3, normalized size = 1.

$$e^x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x,x]
```

```
[Out] E^x
```

Maple [A] time = 0., size = 3, normalized size = 1.

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x),x)
```

```
[Out] exp(x)
```

Maxima [A] time = 0.91739, size = 3, normalized size = 1.

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x, algorithm="maxima")
```

```
[Out] e^x
```

Fricas [A] time = 1.79761, size = 7, normalized size = 2.33

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x, algorithm="fricas")
```

```
[Out] e^x
```

Sympy [A] time = 0.039106, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x)
```

```
[Out] exp(x)
```

Giac [A] time = 1.05884, size = 3, normalized size = 1.

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x, algorithm="giac")
```

```
[Out] e^x
```

3.3 $\int \frac{1}{x} \, dx$

Optimal. Leaf size=2

$$\log(x)$$

```
[Out] Log[x]
```

Rubi [A] time = 0.0001821, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {29}

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1),x]
```

```
[Out] Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x} \, dx = \log(x)$$

Mathematica [A] time = 0.0000746, size = 2, normalized size = 1.

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1),x]
```

```
[Out] Log[x]
```

Maple [A] time = 0., size = 3, normalized size = 1.5

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
```

```
[Out] ln(x)
```

Maxima [A] time = 0.922497, size = 3, normalized size = 1.5

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="maxima")
```

```
[Out] log(x)
```

Fricas [A] time = 1.7818, size = 11, normalized size = 5.5

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="fricas")
```

```
[Out] log(x)
```

Sympy [A] time = 0.04991, size = 2, normalized size = 1.

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x)
```

```
[Out] log(x)
```

Giac [A] time = 1.05146, size = 4, normalized size = 2.

$$\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="giac")
```

```
[Out] log(abs(x))
```

3.4 $\int a^x\,dx$

Optimal. Leaf size=8

$$\frac{a^x}{\log(a)}$$

```
[Out] a^x/Log[a]
```

Rubi [A] time = 0.002335, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2194}

$$\frac{a^x}{\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[a^x,x]
```

```
[Out] a^x/Log[a]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int a^x\,dx = \frac{a^x}{\log(a)}$$

Mathematica [A] time = 0.0005207, size = 8, normalized size = 1.

$$\frac{a^x}{\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^x,x]
```

```
[Out] a^x/Log[a]
```

Maple [A] time = 0., size = 9, normalized size = 1.1

$$\frac{a^x}{\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x,x)
```

```
[Out] a^x/ln(a)
```

Maxima [A] time = 0.9221, size = 11, normalized size = 1.38

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x,x, algorithm="maxima")
```

```
[Out] a^x/log(a)
```

Fricas [A] time = 1.89986, size = 16, normalized size = 2.

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x,x, algorithm="fricas")
```

```
[Out] a^x/log(a)
```

Sympy [A] time = 0.082901, size = 8, normalized size = 1.

$$\begin{cases} \frac{a^x}{\log(a)} & \text{for } \log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x,x)
```

```
[Out] Piecewise((a**x/log(a), Ne(log(a), 0)), (x, True))
```

Giac [A] time = 1.04125, size = 11, normalized size = 1.38

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x,x, algorithm="giac")
```

```
[Out] a^x/log(a)
```

3.5 $\int \sin(x)\,dx$

Optimal. Leaf size=4

$$-\cos(x)$$

```
[Out] -Cos[x]
```

Rubi [A] time = 0.0022196, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2638}

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x],x]

[Out] -Cos[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sin(x)\,dx = -\cos(x)$$

Mathematica [A] time = 0.000953, size = 4, normalized size = 1.

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x],x]
```

```
[Out] -Cos[x]
```

Maple [A] time = 0., size = 5, normalized size = 1.3

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x),x)
```

```
[Out] -cos(x)
```

Maxima [A] time = 0.926118, size = 5, normalized size = 1.25

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x, algorithm="maxima")
```

```
[Out] -cos(x)
```

Fricas [A] time = 2.00473, size = 12, normalized size = 3.

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x, algorithm="fricas")
```

```
[Out] -cos(x)
```

Sympy [A] time = 0.053336, size = 3, normalized size = 0.75

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x)
```

```
[Out] -cos(x)
```

Giac [A] time = 1.06566, size = 5, normalized size = 1.25

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x, algorithm="giac")
```

```
[Out] -cos(x)
```

3.6 $\int \cos(x) \, dx$

Optimal. Leaf size=2

$$\sin(x)$$

```
[Out] Sin[x]
```

Rubi [A] time = 0.0018374, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2637}

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x],x]
```

```
[Out] Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(x) \, dx = \sin(x)$$

Mathematica [A] time = 0.0008872, size = 2, normalized size = 1.

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x],x]
```

```
[Out] Sin[x]
```

Maple [A] time = 0.001, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x),x)
```

```
[Out] sin(x)
```

Maxima [A] time = 0.928332, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="maxima")
```

```
[Out] sin(x)
```

Fricas [A] time = 1.98609, size = 11, normalized size = 5.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="fricas")
```

```
[Out] sin(x)
```

Sympy [A] time = 0.052728, size = 2, normalized size = 1.

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x)
```

```
[Out] sin(x)
```

Giac [A] time = 1.05768, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="giac")
```

```
[Out] sin(x)
```

3.7 $\int \sec^2(x)\,dx$

Optimal. Leaf size=2

$$\tan(x)$$

```
[Out] Tan[x]
```

Rubi [A] time = 0.0055703, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3767, 8}

$$\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2,x]
```

```
[Out] Tan[x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\,dx &= -\operatorname{Subst}\left(\int 1\,dx, x, -\tan(x)\right)\\ &= \tan(x)\end{aligned}$$

Mathematica [A] time = 0.0016016, size = 2, normalized size = 1.

$$\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2,x]
```

```
[Out] Tan[x]
```

Maple [A] time = 0.003, size = 3, normalized size = 1.5

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2,x)
```

```
[Out] tan(x)
```

Maxima [A] time = 0.926789, size = 3, normalized size = 1.5

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2,x, algorithm="maxima")
```

```
[Out] tan(x)
```

Fricas [B] time = 1.90967, size = 20, normalized size = 10.

$$\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2,x, algorithm="fricas")
```

```
[Out] sin(x)/cos(x)
```

Sympy [B] time = 0.058736, size = 5, normalized size = 2.5

$$\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2,x)
```

```
[Out] sin(x)/cos(x)
```

Giac [A] time = 1.06331, size = 3, normalized size = 1.5

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2,x, algorithm="giac")
```

```
[Out] tan(x)
```

3.8 $\int \csc^2(x)\,dx$

Optimal. Leaf size=4

$$-\cot(x)$$

```
[Out] -Cot[x]
```

Rubi [A] time = 0.005372, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3767, 8}

$$-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2,x]
```

```
[Out] -Cot[x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \csc^2(x)\,dx &= -\text{Subst}\left(\int 1\,dx, x, \cot(x)\right)\\ &= -\cot(x)\end{aligned}$$

Mathematica [A] time = 0.0016446, size = 4, normalized size = 1.

$$-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2,x]
```

```
[Out] -Cot[x]
```

Maple [A] time = 0.003, size = 5, normalized size = 1.3

$$-\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2,x)
```

```
[Out] -cot(x)
```

Maxima [A] time = 0.937823, size = 8, normalized size = 2.

$$-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2,x, algorithm="maxima")
```

```
[Out] -1/tan(x)
```

Fricas [B] time = 1.84732, size = 22, normalized size = 5.5

$$-\frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2,x, algorithm="fricas")
```

```
[Out] -cos(x)/sin(x)
```

Sympy [B] time = 0.05926, size = 7, normalized size = 1.75

$$-\frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2,x)
```

```
[Out] -cos(x)/sin(x)
```

Giac [A] time = 1.08226, size = 8, normalized size = 2.

$$-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2,x, algorithm="giac")
```

```
[Out] -1/tan(x)
```

3.9 $\int \sec(x) \tan(x) \, dx$

Optimal. Leaf size=2

$$\sec(x)$$

```
[Out] Sec[x]
```

Rubi [A] time = 0.0065987, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2606, 8}

$$\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Tan[x],x]

[Out] Sec[x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \sec(x) \tan(x) \, dx &= \text{Subst}\left(\int 1 \, dx, x, \sec(x)\right) \\ &= \sec(x) \end{aligned}$$

Mathematica [A] time = 0.0012405, size = 2, normalized size = 1.

$$\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Tan[x],x]
```

```
[Out] Sec[x]
```

Maple [A] time = 0.006, size = 3, normalized size = 1.5

$$\sec(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x),x)
```

```
[Out] sec(x)
```

Maxima [A] time = 0.927725, size = 5, normalized size = 2.5

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x),x, algorithm="maxima")
```

```
[Out] 1/cos(x)
```

Fricas [A] time = 1.85484, size = 14, normalized size = 7.

$$\frac{1}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x),x, algorithm="fricas")
```

```
[Out] 1/cos(x)
```

Sympy [A] time = 0.061392, size = 3, normalized size = 1.5

$$\frac{1}{\cos{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x),x)
```

```
[Out] 1/cos(x)
```

Giac [A] time = 1.06429, size = 5, normalized size = 2.5

$$\frac{1}{\cos{(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x),x, algorithm="giac")
```

```
[Out] 1/cos(x)
```

3.10 $\int \cot(x) \csc(x) \, dx$

Optimal. Leaf size=4

$$-\csc(x)$$

[Out] -Csc[x]

Rubi [A] time = 0.0078946, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2606, 8}

$$-\csc(x)$$

Antiderivative was successfully verified.

[In] Int[Cot[x]*Csc[x],x]

[Out] -Csc[x]

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \cot(x) \csc(x) \, dx &= -\text{Subst}\left(\int 1 \, dx, x, \csc(x)\right) \\ &= -\csc(x) \end{aligned}$$

Mathematica [A] time = 0.0015108, size = 4, normalized size = 1.

$$-\csc(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Csc[x],x]
```

```
[Out] -Csc[x]
```

Maple [A] time = 0.005, size = 5, normalized size = 1.3

$$-\csc(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*csc(x),x)
```

```
[Out] -csc(x)
```

Maxima [A] time = 0.91894, size = 8, normalized size = 2.

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x),x, algorithm="maxima")
```

```
[Out] -1/sin(x)
```

Fricas [A] time = 1.8661, size = 15, normalized size = 3.75

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x),x, algorithm="fricas")
```

```
[Out] -1/sin(x)
```

Sympy [A] time = 0.062908, size = 5, normalized size = 1.25

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x),x)
```

```
[Out] -1/sin(x)
```

Giac [A] time = 1.0601, size = 8, normalized size = 2.

$$-\frac{1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*csc(x),x, algorithm="giac")
```

```
[Out] -1/sin(x)
```

3.11 $\int \sinh(x)\,dx$

Optimal. Leaf size=2

$$\cosh(x)$$

```
[Out] Cosh[x]
```

Rubi [A] time = 0.003047, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {2638}

$$\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[x],x]

[Out] Cosh[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sinh(x)\,dx = \cosh(x)$$

Mathematica [A] time = 0.0010163, size = 2, normalized size = 1.

$$\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[x],x]
```

```
[Out] Cosh[x]
```

Maple [A] time = 0., size = 3, normalized size = 1.5

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(x),x)
```

```
[Out] cosh(x)
```

Maxima [A] time = 0.918757, size = 3, normalized size = 1.5

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x, algorithm="maxima")
```

```
[Out] cosh(x)
```

Fricas [A] time = 1.90352, size = 12, normalized size = 6.

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x, algorithm="fricas")
```

```
[Out] cosh(x)
```

Sympy [A] time = 0.109566, size = 2, normalized size = 1.

$$\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x)
```

```
[Out] cosh(x)
```

Giac [B] time = 1.05852, size = 15, normalized size = 7.5

$$\frac{1}{2}e^{(-x)} + \frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(x),x, algorithm="giac")
```

```
[Out] 1/2*e^(-x) + 1/2*e^x
```

3.12 $\int \cosh(x)\,dx$

Optimal. Leaf size=2

$$\sinh(x)$$

```
[Out] Sinh[x]
```

Rubi [A] time = 0.0020568, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2637}

$$\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x],x]

[Out] Sinh[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cosh(x)\,dx = \sinh(x)$$

Mathematica [A] time = 0.0011039, size = 2, normalized size = 1.

$$\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x],x]
```

```
[Out] Sinh[x]
```

Maple [A] time = 0.002, size = 3, normalized size = 1.5

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x),x)
```

```
[Out] sinh(x)
```

Maxima [A] time = 0.92125, size = 3, normalized size = 1.5

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x, algorithm="maxima")
```

```
[Out] sinh(x)
```

Fricas [A] time = 1.89097, size = 12, normalized size = 6.

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x, algorithm="fricas")
```

```
[Out] sinh(x)
```

Sympy [A] time = 0.109905, size = 2, normalized size = 1.

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x)
```

```
[Out] sinh(x)
```

Giac [B] time = 1.05604, size = 15, normalized size = 7.5

$$-\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x),x, algorithm="giac")
```

```
[Out] -1/2*e^(-x) + 1/2*e^x
```

3.13 $\int \tan(x)\,dx$

Optimal. Leaf size=5

$$-\log(\cos(x))$$

```
[Out] -Log[Cos[x]]
```

Rubi [A] time = 0.0020498, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3475}

$$-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x],x]

[Out] -Log[Cos[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tan(x)\,dx = -\log(\cos(x))$$

Mathematica [A] time = 0.0022787, size = 5, normalized size = 1.

$$-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x],x]
```

```
[Out] -Log[Cos[x]]
```

Maple [A] time = 0., size = 6, normalized size = 1.2

$$-\ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x),x)
```

```
[Out] -ln(cos(x))
```

Maxima [A] time = 0.928878, size = 4, normalized size = 0.8

$$\log(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x, algorithm="maxima")
```

```
[Out] log(sec(x))
```

Fricas [B] time = 2.00598, size = 38, normalized size = 7.6

$$-\frac{1}{2}\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] time = 0.05756, size = 5, normalized size = 1.

$$-\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x)
```

```
[Out] -log(cos(x))
```

Giac [A] time = 1.05968, size = 8, normalized size = 1.6

$$-\log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x, algorithm="giac")
```

```
[Out] -log(abs(cos(x)))
```

3.14 $\int \cot(x)\,dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

```
[Out] Log[Sin[x]]
```

Rubi [A] time = 0.001929, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3475}

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x],x]

[Out] Log[Sin[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(x)\,dx = \log(\sin(x))$$

Mathematica [A] time = 0.0020149, size = 3, normalized size = 1.

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Maple [A] time = 0., size = 4, normalized size = 1.3

$$\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x),x)
```

```
[Out] ln(sin(x))
```

Maxima [A] time = 0.926513, size = 4, normalized size = 1.33

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="maxima")
```

```
[Out] log(sin(x))
```

Fricas [B] time = 2.03097, size = 41, normalized size = 13.67

$$\frac{1}{2}\log\left(-\frac{1}{2}\cos(2x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(-1/2*cos(2*x) + 1/2)
```

Sympy [A] time = 0.058282, size = 3, normalized size = 1.

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x)
```

```
[Out] log(sin(x))
```

Giac [B] time = 1.05605, size = 15, normalized size = 5.

$$\frac{1}{2} \log \left(-\cos (x)^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="giac")
```

```
[Out] 1/2*log(-cos(x)^2 + 1)
```

3.15 $\int x\sin(x)\,dx$

Optimal. Leaf size=8

$$\sin(x) - x\cos(x)$$

```
[Out] -(x*Cos[x]) + Sin[x]
```

Rubi [A] time = 0.0085667, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3296, 2637}

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x],x]
```

```
[Out] -(x*Cos[x]) + Sin[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sin(x)\,dx &= -x\cos(x) + \int \cos(x)\,dx\\ &= -x\cos(x) + \sin(x)\end{aligned}$$

Mathematica [A] time = 0.0020667, size = 8, normalized size = 1.

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x],x]
```

```
[Out] -(x*Cos[x]) + Sin[x]
```

Maple [A] time = 0., size = 9, normalized size = 1.1

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x),x)
```

```
[Out] -x*cos(x)+sin(x)
```

Maxima [A] time = 0.933074, size = 11, normalized size = 1.38

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="maxima")
```

```
[Out] -x*cos(x) + sin(x)
```

Fricas [A] time = 1.90965, size = 27, normalized size = 3.38

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="fricas")
```

```
[Out] -x*cos(x) + sin(x)
```

Sympy [A] time = 0.166954, size = 7, normalized size = 0.88

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x)
```

```
[Out] -x*cos(x) + sin(x)
```

Giac [A] time = 1.04858, size = 11, normalized size = 1.38

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="giac")
```

```
[Out] -x*cos(x) + sin(x)
```

3.16 $\int \log(x)\,dx$

Optimal. Leaf size=8

$$x\log(x)-x$$

```
[Out] -x + x*Log[x]
```

Rubi [A] time = 0.0009977, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2295}

$$x\log(x)-x$$

Antiderivative was successfully verified.

```
[In] Int[Log[x],x]
```

```
[Out] -x + x*Log[x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int \log(x)\,dx=-x+x\log(x)$$

Mathematica [A] time = 0.0005906, size = 8, normalized size = 1.

$$x\log(x)-x$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x],x]
```

```
[Out] -x + x*Log[x]
```

Maple [A] time = 0.001, size = 9, normalized size = 1.1

$$-x + x\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x),x)
```

```
[Out] -x+x*ln(x)
```

Maxima [A] time = 0.921812, size = 11, normalized size = 1.38

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="maxima")
```

```
[Out] x*log(x) - x
```

Fricas [A] time = 1.82787, size = 19, normalized size = 2.38

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="fricas")
```

```
[Out] x*log(x) - x
```

Sympy [A] time = 0.076374, size = 5, normalized size = 0.62

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x),x)
```

```
[Out] x*log(x) - x
```

Giac [A] time = 1.05865, size = 11, normalized size = 1.38

$$x \log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="giac")
```

```
[Out] x*log(x) - x
```

3.17 $\int e^x x^2 \, dx$

Optimal. Leaf size=19

$$e^x x^2 - 2e^x x + 2e^x$$

```
[Out] 2*E^x - 2*E^x*x + E^x*x^2
```

Rubi [A] time = 0.0201952, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2176, 2194}

$$e^x x^2 - 2e^x x + 2e^x$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x^2,x]
```

```
[Out] 2*E^x - 2*E^x*x + E^x*x^2
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^x x^2 \, dx &= e^x x^2 - 2 \int e^x x \, dx \\
&= -2e^x x + e^x x^2 + 2 \int e^x \, dx \\
&= 2e^x - 2e^x x + e^x x^2
\end{aligned}$$

Mathematica [A] time = 0.0047733, size = 12, normalized size = 0.63

$$e^x\left(x^2-2x+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x^2,x]
```

```
[Out] E^x*(2 - 2*x + x^2)
```

Maple [A] time = 0.002, size = 12, normalized size = 0.6

$$\left(x^2-2\,x+2\right)\mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x^2,x)
```

```
[Out] (x^2-2*x+2)*exp(x)
```

Maxima [A] time = 0.934845, size = 15, normalized size = 0.79

$$\left(x^2-2\,x+2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2,x, algorithm="maxima")
```

```
[Out] (x^2 - 2*x + 2)*e^x
```

Fricas [A] time = 1.88894, size = 28, normalized size = 1.47

$$\left(x^2-2\,x+2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2,x, algorithm="fricas")
```

```
[Out] (x^2 - 2*x + 2)*e^x
```

Sympy [A] time = 0.078241, size = 10, normalized size = 0.53

$$\left(x^2 - 2x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x**2,x)
```

```
[Out] (x**2 - 2*x + 2)*exp(x)
```

Giac [A] time = 1.04944, size = 15, normalized size = 0.79

$$\left(x^2 - 2\,x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2,x, algorithm="giac")
```

```
[Out] (x^2 - 2*x + 2)*e^x
```

3.18 $\int e^x \sin(x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^x\cos(x)$$

```
[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rubi [A] time = 0.0074345, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4432}

$$\frac{1}{2}e^x\sin(x)-\frac{1}{2}e^x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[x],x]
```

```
[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\sin(x)\,dx=-\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)$$

Mathematica [A] time = 0.005285, size = 14, normalized size = 0.74

$$\frac{1}{2}e^x(\sin(x)-\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[x],x]
```

```
[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

Maple [A] time = 0., size = 14, normalized size = 0.7

$$-\frac{e^x \cos(x)}{2} + \frac{e^x \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x)
```

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Maxima [A] time = 0.928081, size = 15, normalized size = 0.79

$$-\frac{1}{2}(\cos(x) - \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Fricas [A] time = 1.9241, size = 46, normalized size = 2.42

$$-\frac{1}{2} \cos(x) e^x + \frac{1}{2} e^x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A] time = 0.282974, size = 15, normalized size = 0.79

$$\frac{e^{x}\sin{(x)}}{2}-\frac{e^{x}\cos{(x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x)
```

```
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

Giac [A] time = 1.0473, size = 15, normalized size = 0.79

$$-\frac{1}{2}\left(\cos{(x)}-\sin{(x)}\right)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*(cos(x) - sin(x))*e^x
```

3.19 $\int \tan^{-1}(x)\,dx$

Optimal. Leaf size=15

$$x\tan^{-1}(x)-\frac{1}{2}\log\left(x^2+1\right)$$

```
[Out] x*ArcTan[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0038938, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {4846, 260}

$$x\tan^{-1}(x)-\frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[x],x]
```

```
[Out] x*ArcTan[x] - Log[1 + x^2]/2
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \tan^{-1}(x)\,dx &= x\tan^{-1}(x)-\int \frac{x}{1+x^2}\,dx\\ &= x\tan^{-1}(x)-\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0016189, size = 15, normalized size = 1.

$$x\tan^{-1}(x)-\frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[x],x]
```

```
[Out] x*ArcTan[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.001, size = 14, normalized size = 0.9

$$x\arctan(x)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x),x)
```

```
[Out] x*arctan(x)-1/2*ln(x^2+1)
```

Maxima [A] time = 0.936874, size = 18, normalized size = 1.2

$$x\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x),x, algorithm="maxima")
```

```
[Out] x*arctan(x) - 1/2*log(x^2 + 1)
```

Fricas [A] time = 2.0195, size = 43, normalized size = 2.87

$$x\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x),x, algorithm="fricas")
```

```
[Out] x*arctan(x) - 1/2*log(x^2 + 1)
```

Sympy [A] time = 0.184029, size = 12, normalized size = 0.8

$$x\,\mathrm{atan}\,(x) - \frac{\log\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x),x)
```

```
[Out] x*atan(x) - log(x**2 + 1)/2
```

Giac [A] time = 1.04172, size = 18, normalized size = 1.2

$$x\arctan\,(x) - \frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x),x, algorithm="giac")
```

```
[Out] x*arctan(x) - 1/2*log(x^2 + 1)
```

3.20 $\int e^{2x} x \, dx$

Optimal. Leaf size=20

$$\frac{1}{2}e^{2x}x - \frac{e^{2x}}{4}$$

```
[Out] -E^(2*x)/4 + (E^(2*x)*x)/2
```

Rubi [A] time = 0.0084638, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2176, 2194}

$$\frac{1}{2}e^{2x}x - \frac{e^{2x}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*x,x]

[Out] -E^(2*x)/4 + (E^(2*x)*x)/2
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{2x} x \, dx &= \frac{1}{2}e^{2x}x - \frac{1}{2}\int e^{2x} \, dx\\ &= -\frac{e^{2x}}{4} + \frac{1}{2}e^{2x}x\end{aligned}$$

Mathematica [A] time = 0.0042397, size = 15, normalized size = 0.75

$$e^{2x}\left(\frac{x}{2}-\frac{1}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*x,x]
```

```
[Out] E^(2*x)*(-1/4 + x/2)
```

Maple [A] time = 0., size = 12, normalized size = 0.6

$$\frac{(2x-1)\,e^{2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*x,x)
```

```
[Out] 1/4*(2*x-1)*exp(2*x)
```

Maxima [A] time = 0.936104, size = 15, normalized size = 0.75

$$\frac{1}{4}\,(2x-1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*x,x, algorithm="maxima")
```

```
[Out] 1/4*(2*x - 1)*e^(2*x)
```

Fricas [A] time = 1.8314, size = 31, normalized size = 1.55

$$\frac{1}{4}\,(2x-1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*x,x, algorithm="fricas")
```

```
[Out] 1/4*(2*x - 1)*e^(2*x)
```

Sympy [A] time = 0.075245, size = 10, normalized size = 0.5

$$\frac{(2x-1)\,e^{2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*x,x)
```

```
[Out] (2*x - 1)*exp(2*x)/4
```

Giac [A] time = 1.05513, size = 15, normalized size = 0.75

$$\frac{1}{4}\,(2\,x-1)e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*x,x, algorithm="giac")
```

```
[Out] 1/4*(2*x - 1)*e^(2*x)
```

3.21 $\int x \cos(x)\, dx$

Optimal. Leaf size=7

$$x \sin(x) + \cos(x)$$

```
[Out] Cos[x] + x*Sin[x]
```

Rubi [A] time = 0.008778, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3296, 2638}

$$x \sin(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int x \cos(x)\, dx &= x \sin(x) - \int \sin(x)\, dx \\ &= \cos(x) + x \sin(x) \end{aligned}$$

Mathematica [A] time = 0.0017807, size = 7, normalized size = 1.

$$x\sin(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Maple [A] time = 0., size = 8, normalized size = 1.1

$$\cos(x) + x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)
```

```
[Out] cos(x)+x*sin(x)
```

Maxima [A] time = 0.938032, size = 9, normalized size = 1.29

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="maxima")
```

```
[Out] x*sin(x) + cos(x)
```

Fricas [A] time = 1.90055, size = 26, normalized size = 3.71

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="fricas")
```

```
[Out] x*sin(x) + cos(x)
```

Sympy [A] time = 0.164299, size = 7, normalized size = 1.

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x)
```

```
[Out] x*sin(x) + cos(x)
```

Giac [A] time = 1.04116, size = 9, normalized size = 1.29

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="giac")
```

```
[Out] x*sin(x) + cos(x)
```

3.22 $\int x\sin(4x)\,dx$

Optimal. Leaf size=18

$$\frac{1}{16}\sin(4x)-\frac{1}{4}x\cos(4x)$$

```
[Out] -(x*Cos[4*x])/4 + Sin[4*x]/16
```

Rubi [A] time = 0.0096751, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2637}

$$\frac{1}{16}\sin(4x)-\frac{1}{4}x\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[4*x],x]

[Out] -(x*Cos[4*x])/4 + Sin[4*x]/16
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sin(4x)\,dx &= -\frac{1}{4}x\cos(4x)+\frac{1}{4}\int\cos(4x)\,dx\\ &= -\frac{1}{4}x\cos(4x)+\frac{1}{16}\sin(4x)\end{aligned}$$

Mathematica [A] time = 0.010573, size = 18, normalized size = 1.

$$\frac{1}{16}\sin(4x)-\frac{1}{4}x\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[4*x],x]
```

```
[Out] -(x*Cos[4*x])/4 + Sin[4*x]/16
```

Maple [A] time = 0.008, size = 15, normalized size = 0.8

$$-\frac{x\cos(4x)}{4}+\frac{\sin(4x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(4*x),x)
```

```
[Out] -1/4*x*cos(4*x)+1/16*sin(4*x)
```

Maxima [A] time = 0.948765, size = 19, normalized size = 1.06

$$-\frac{1}{4}x\cos(4x)+\frac{1}{16}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(4*x),x, algorithm="maxima")
```

```
[Out] -1/4*x*cos(4*x) + 1/16*sin(4*x)
```

Fricas [A] time = 1.94667, size = 45, normalized size = 2.5

$$-\frac{1}{4}x\cos(4x)+\frac{1}{16}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(4*x),x, algorithm="fricas")
```

```
[Out] -1/4*x*x*cos(4*x) + 1/16*sin(4*x)
```

Sympy [A] time = 0.167363, size = 14, normalized size = 0.78

$$-\frac{x\cos(4x)}{4}+\frac{\sin(4x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(4*x),x)
```

```
[Out] -x*cos(4*x)/4 + sin(4*x)/16
```

Giac [A] time = 1.0504, size = 19, normalized size = 1.06

$$-\frac{1}{4}x\cos(4x)+\frac{1}{16}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(4*x),x, algorithm="giac")
```

```
[Out] -1/4*x*x*cos(4*x) + 1/16*sin(4*x)
```

3.23 $\int x \log(x) \, dx$

Optimal. Leaf size=17

$$\frac{1}{2}x^2\log(x) - \frac{x^2}{4}$$

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rubi [A] time = 0.0034969, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2304}

$$\frac{1}{2}x^2\log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x],x]
```

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x \log(x) \, dx = -\frac{x^2}{4} + \frac{1}{2}x^2\log(x)$$

Mathematica [A] time = 0.000605, size = 17, normalized size = 1.

$$\frac{1}{2}x^2\log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x],x]
```

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Maple [A] time = 0., size = 14, normalized size = 0.8

$$-\frac{x^2}{4}+\frac{x^2 \ln \left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x),x)
```

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Maxima [A] time = 0.935179, size = 18, normalized size = 1.06

$$\frac{1}{2} \, x^{2} \log \left(x\right) - \frac{1}{4} \, x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Fricas [A] time = 1.87314, size = 35, normalized size = 2.06

$$\frac{1}{2} \, x^{2} \log \left(x\right) - \frac{1}{4} \, x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A] time = 0.080408, size = 12, normalized size = 0.71

$$\frac{x^2\log(x)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x),x)
```

```
[Out] x**2*log(x)/2 - x**2/4
```

Giac [A] time = 1.05056, size = 18, normalized size = 1.06

$$\frac{1}{2}x^2\log(x) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

3.24 $\int x^2 \cos(3x)\,dx$

Optimal. Leaf size=29

$$\frac{1}{3}x^2\sin(3x) - \frac{2}{27}\sin(3x) + \frac{2}{9}x\cos(3x)$$

```
[Out] (2*x*Cos[3*x])/9 - (2*Sin[3*x])/27 + (x^2*Sin[3*x])/3
```

Rubi [A] time = 0.025313, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3296, 2637}

$$\frac{1}{3}x^2\sin(3x) - \frac{2}{27}\sin(3x) + \frac{2}{9}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[3*x],x]
```

```
[Out] (2*x*Cos[3*x])/9 - (2*Sin[3*x])/27 + (x^2*Sin[3*x])/3
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^2\cos(3x)\,dx &= \frac{1}{3}x^2\sin(3x) - \frac{2}{3}\int x\sin(3x)\,dx \\
&= \frac{2}{9}x\cos(3x) + \frac{1}{3}x^2\sin(3x) - \frac{2}{9}\int \cos(3x)\,dx \\
&= \frac{2}{9}x\cos(3x) - \frac{2}{27}\sin(3x) + \frac{1}{3}x^2\sin(3x)
\end{aligned}$$

Mathematica [A] time = 0.0267297, size = 25, normalized size = 0.86

$$\frac{1}{27}\left(9x^2-2\right)\sin(3x)+\frac{2}{9}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[3*x],x]
```

```
[Out] (2*x*Cos[3*x])/9 + ((-2 + 9*x^2)*Sin[3*x])/27
```

Maple [A] time = 0.008, size = 24, normalized size = 0.8

$$\frac{2\,x\cos\left(3\,x\right)}{9}-\frac{2\,\sin\left(3\,x\right)}{27}+\frac{x^2\sin\left(3\,x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(3*x),x)
```

```
[Out] 2/9*x*cos(3*x)-2/27*sin(3*x)+1/3*x^2*sin(3*x)
```

Maxima [A] time = 0.941103, size = 28, normalized size = 0.97

$$\frac{2}{9}\,x\cos\left(3\,x\right)+\frac{1}{27}\left(9\,x^2-2\right)\sin\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x),x, algorithm="maxima")
```

```
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

Fricas [A] time = 2.02833, size = 59, normalized size = 2.03

$$\frac{2}{9}\,x\cos\left(3\,x\right)+\frac{1}{27}\left(9\,x^2-2\right)\sin\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x),x, algorithm="fricas")
```

```
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

Sympy [A] time = 0.303979, size = 27, normalized size = 0.93

$$\frac{x^2\sin(3x)}{3} + \frac{2x\cos(3x)}{9} - \frac{2\sin(3x)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(3*x),x)
```

```
[Out] x**2*sin(3*x)/3 + 2*x*cos(3*x)/9 - 2*sin(3*x)/27
```

Giac [A] time = 1.04041, size = 28, normalized size = 0.97

$$\frac{2}{9}\,x\cos(3\,x) + \frac{1}{27}\left(9\,x^2 - 2\right)\sin(3\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x),x, algorithm="giac")
```

```
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

3.25 $\int x^2 \sin(2x)\, dx$

Optimal. Leaf size=29

$$-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)$$

```
[Out] Cos[2*x]/4 - (x^2*Cos[2*x])/2 + (x*Sin[2*x])/2
```

Rubi [A] time = 0.0226745, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3296, 2638}

$$-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[2*x],x]
```

```
[Out] Cos[2*x]/4 - (x^2*Cos[2*x])/2 + (x*Sin[2*x])/2
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sin(2x)\,dx &= -\frac{1}{2}x^2\cos(2x)+\int x\cos(2x)\,dx\\ &= -\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)-\frac{1}{2}\int\sin(2x)\,dx\\ &= \frac{1}{4}\cos(2x)-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)\end{aligned}$$

Mathematica [A] time = 0.0240711, size = 25, normalized size = 0.86

$$\frac{1}{2}x\sin(2x)-\frac{1}{4}\left(2x^2-1\right)\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[2*x],x]
```

```
[Out] -((-1 + 2*x^2)*Cos[2*x])/4 + (x*Sin[2*x])/2
```

Maple [A] time = 0.006, size = 24, normalized size = 0.8

$$\frac{\cos(2x)}{4}-\frac{x^2\cos(2x)}{2}+\frac{x\sin(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(2*x),x)
```

```
[Out] 1/4*cos(2*x)-1/2*x^2*cos(2*x)+1/2*x*sin(2*x)
```

Maxima [A] time = 0.947915, size = 28, normalized size = 0.97

$$-\frac{1}{4}\left(2x^2-1\right)\cos(2x)+\frac{1}{2}x\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^2 - 1)*cos(2*x) + 1/2*x*sin(2*x)
```

Fricas [A] time = 1.93983, size = 59, normalized size = 2.03

$$-\frac{1}{4}\left(2x^2-1\right)\cos(2x)+\frac{1}{2}x\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(2*x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^2 - 1)*cos(2*x) + 1/2*x*sin(2*x)
```

Sympy [A] time = 0.292341, size = 24, normalized size = 0.83

$$-\frac{x^2\cos(2x)}{2}+\frac{x\sin(2x)}{2}+\frac{\cos(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(2*x),x)
```

```
[Out] -x**2*cos(2*x)/2 + x*sin(2*x)/2 + cos(2*x)/4
```

Giac [A] time = 1.04342, size = 28, normalized size = 0.97

$$-\frac{1}{4}\left(2x^2-1\right)\cos(2x)+\frac{1}{2}x\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(2*x),x, algorithm="giac")
```

```
[Out] -1/4*(2*x^2 - 1)*cos(2*x) + 1/2*x*sin(2*x)
```

3.26 $\int \log^2(x)\,dx$

Optimal. Leaf size=15

$$2x + x\log^2(x) - 2x\log(x)$$

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Rubi [A] time = 0.0033866, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2296, 2295}

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^2,x]
```

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \log^2(x)\,dx &= x\log^2(x) - 2\int \log(x)\,dx\\ &= 2x - 2x\log(x) + x\log^2(x)\end{aligned}$$

Mathematica [A] time = 0.0009109, size = 15, normalized size = 1.

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^2,x]
```

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Maple [A] time = 0., size = 16, normalized size = 1.1

$$2\,x - 2\,x\ln(x) + x(\ln(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2,x)
```

```
[Out] 2*x-2*x*ln(x)+x*ln(x)^2
```

Maxima [A] time = 0.936374, size = 16, normalized size = 1.07

$$\left(\log(x)^2 - 2\log(x) + 2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="maxima")
```

```
[Out] (log(x)^2 - 2*log(x) + 2)*x
```

Fricas [A] time = 1.75099, size = 42, normalized size = 2.8

$$x\log(x)^2 - 2\,x\log(x) + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="fricas")
```

```
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

Sympy [A] time = 0.087332, size = 15, normalized size = 1.

$$x \log (x)^2 - 2x \log (x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2,x)
```

```
[Out] x*log(x)**2 - 2*x*log(x) + 2*x
```

Giac [A] time = 1.04614, size = 20, normalized size = 1.33

$$x \log (x)^2 - 2 \, x \log (x) + 2 \, x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="giac")
```

```
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

3.27 $\int \sin^{-1}(x)\,dx$

Optimal. Leaf size=16

$$\sqrt{1-x^2}+x\sin^{-1}(x)$$

[Out] Sqrt[1 - x^2] + x*ArcSin[x]

Rubi [A] time = 0.0038397, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {4619, 261}

$$\sqrt{1-x^2}+x\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[ArcSin[x],x]

[Out] Sqrt[1 - x^2] + x*ArcSin[x]

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sin^{-1}(x)\,dx &= x\sin^{-1}(x)-\int \frac{x}{\sqrt{1-x^2}}\,dx\\ &= \sqrt{1-x^2}+x\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0020695, size = 16, normalized size = 1.

$$\sqrt{1-x^2}+x\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x],x]
```

```
[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

Maple [A] time = 0., size = 15, normalized size = 0.9

$$x\arcsin(x)+\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x),x)
```

```
[Out] x*arcsin(x)+(-x^2+1)^(1/2)
```

Maxima [A] time = 1.40238, size = 19, normalized size = 1.19

$$x\arcsin(x)+\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x),x, algorithm="maxima")
```

```
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

Fricas [A] time = 2.02459, size = 41, normalized size = 2.56

$$x\arcsin(x)+\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x),x, algorithm="fricas")
```

```
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

Sympy [A] time = 0.11681, size = 12, normalized size = 0.75

$$x \operatorname{asin}(x) + \sqrt{1 - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x),x)
```

```
[Out] x*asin(x) + sqrt(1 - x**2)
```

Giac [A] time = 1.06511, size = 19, normalized size = 1.19

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x),x, algorithm="giac")
```

```
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

3.28 $\int t \cos(t) \sin(t) \, dt$

Optimal. Leaf size=23

$$-\frac{t}{4}+\frac{1}{2} t \sin^2(t)+\frac{1}{4} \sin(t) \cos(t)$$

```
[Out] -t/4 + (Cos[t]*Sin[t])/4 + (t*Sin[t]^2)/2
```

Rubi [A] time = 0.0127517, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3443, 2635, 8}

$$-\frac{t}{4}+\frac{1}{2} t \sin^2(t)+\frac{1}{4} \sin(t) \cos(t)$$

Antiderivative was successfully verified.

```
[In] Int[t*Cos[t]*Sin[t],t]
```

```
[Out] -t/4 + (Cos[t]*Sin[t])/4 + (t*Sin[t]^2)/2
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int t\cos(t)\sin(t)\,dt &= \frac{1}{2}t\sin^2(t) - \frac{1}{2}\int \sin^2(t)\,dt\\ &= \frac{1}{4}\cos(t)\sin(t) + \frac{1}{2}t\sin^2(t) - \frac{\int 1\,dt}{4}\\ &= -\frac{t}{4} + \frac{1}{4}\cos(t)\sin(t) + \frac{1}{2}t\sin^2(t)\end{aligned}$$

Mathematica [A] time = 0.0028449, size = 18, normalized size = 0.78

$$\frac{1}{8}\sin(2t) - \frac{1}{4}t\cos(2t)$$

Antiderivative was successfully verified.

```
[In] Integrate[t*Cos[t]*Sin[t],t]
```

```
[Out] -(t*Cos[2*t])/4 + Sin[2*t]/8
```

Maple [A] time = 0.001, size = 18, normalized size = 0.8

$$-\frac{t(\cos(t))^2}{2} + \frac{\cos(t)\sin(t)}{4} + \frac{t}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t*cos(t)*sin(t),t)
```

```
[Out] -1/2*t*cos(t)^2+1/4*cos(t)*sin(t)+1/4*t
```

Maxima [A] time = 0.938485, size = 19, normalized size = 0.83

$$-\frac{1}{4}t\cos(2t) + \frac{1}{8}\sin(2t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*cos(t)*sin(t),t, algorithm="maxima")
```

```
[Out] -1/4*t*cos(2*t) + 1/8*sin(2*t)
```

Fricas [A] time = 1.95604, size = 61, normalized size = 2.65

$$-\frac{1}{2}\,t\cos\left(t\right)^{2}+\frac{1}{4}\,\cos\left(t\right)\sin\left(t\right)+\frac{1}{4}\,t$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*cos(t)*sin(t),t, algorithm="fricas")
```

```
[Out] -1/2*t*cos(t)^2 + 1/4*cos(t)*sin(t) + 1/4*t
```

Sympy [A] time = 0.313081, size = 24, normalized size = 1.04

$$\frac{t\sin^{2}\left(t\right)}{4}-\frac{t\cos^{2}\left(t\right)}{4}+\frac{\sin\left(t\right)\cos\left(t\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*cos(t)*sin(t),t)
```

```
[Out] t*sin(t)**2/4 - t*cos(t)**2/4 + sin(t)*cos(t)/4
```

Giac [A] time = 1.05105, size = 19, normalized size = 0.83

$$-\frac{1}{4}\,t\cos\left(2\,t\right)+\frac{1}{8}\,\sin\left(2\,t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*cos(t)*sin(t),t, algorithm="giac")
```

```
[Out] -1/4*t*cos(2*t) + 1/8*sin(2*t)
```

3.29 $\int t \sec^2(t)\, dt$

Optimal. Leaf size=8

$$t\tan(t) + \log(\cos(t))$$

```
[Out] Log[Cos[t]] + t*Tan[t]
```

Rubi [A] time = 0.0180556, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4184, 3475}

$$t\tan(t) + \log(\cos(t))$$

Antiderivative was successfully verified.

```
[In] Int[t*Sec[t]^2,t]
```

```
[Out] Log[Cos[t]] + t*Tan[t]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int t\sec^2(t)\, dt &= t\tan(t) - \int \tan(t)\, dt\\ &= \log(\cos(t)) + t\tan(t)\end{aligned}$$

Mathematica [A] time = 0.004618, size = 8, normalized size = 1.

$$t\tan(t)+\log(\cos(t))$$

Antiderivative was successfully verified.

```
[In] Integrate[t*Sec[t]^2,t]
```

```
[Out] Log[Cos[t]] + t*Tan[t]
```

Maple [A] time = 0.004, size = 9, normalized size = 1.1

$$\ln(\cos(t))+t\tan(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t*sec(t)^2,t)
```

```
[Out] ln(cos(t))+t*tan(t)
```

Maxima [B] time = 1.42639, size = 100, normalized size = 12.5

$$\frac{\left(\cos(2t)^2+\sin(2t)^2+2\cos(2t)+1\right)\log\left(\cos(2t)^2+\sin(2t)^2+2\cos(2t)+1\right)+4t\sin(2t)}{2\left(\cos(2t)^2+\sin(2t)^2+2\cos(2t)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*sec(t)^2,t, algorithm="maxima")
```

```
[Out] 1/2*((cos(2*t)^2 + sin(2*t)^2 + 2*cos(2*t) + 1)*log(cos(2*t)^2 + sin(2*t)^2
 + 2*cos(2*t) + 1) + 4*t*sin(2*t))/(cos(2*t)^2 + sin(2*t)^2 + 2*cos(2*t) +
1)
```

Fricas [B] time = 2.08662, size = 55, normalized size = 6.88

$$\frac{\cos(t)\log(-\cos(t))+t\sin(t)}{\cos(t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*sec(t)^2,t, algorithm="fricas")
```

```
[Out] (cos(t)*log(-cos(t)) + t*sin(t))/cos(t)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int t\sec^2(t)\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*sec(t)**2,t)
```

```
[Out] Integral(t*sec(t)**2, t)
```

Giac [B] time = 1.10245, size = 139, normalized size = 17.38

$$\frac{\log\left(\frac{4\left(\tan\left(\frac{1}{2}t\right)^4-2\tan\left(\frac{1}{2}t\right)^2+1\right)}{\tan\left(\frac{1}{2}t\right)^4+2\tan\left(\frac{1}{2}t\right)^2+1}\right)\tan\left(\frac{1}{2}t\right)^2-4t\tan\left(\frac{1}{2}t\right)-\log\left(\frac{4\left(\tan\left(\frac{1}{2}t\right)^4-2\tan\left(\frac{1}{2}t\right)^2+1\right)}{\tan\left(\frac{1}{2}t\right)^4+2\tan\left(\frac{1}{2}t\right)^2+1}\right)}{2\left(\tan\left(\frac{1}{2}t\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*sec(t)^2,t, algorithm="giac")
```

```
[Out] 1/2*(log(4*(tan(1/2*t)^4 - 2*tan(1/2*t)^2 + 1)/(tan(1/2*t)^4 + 2*tan(1/2*t)
^2 + 1))*tan(1/2*t)^2 - 4*t*tan(1/2*t) - log(4*(tan(1/2*t)^4 - 2*tan(1/2*t)
^2 + 1)/(tan(1/2*t)^4 + 2*tan(1/2*t)^2 + 1)))/(tan(1/2*t)^2 - 1)
```

3.30 $\int t^2 \log(t)\, dt$

Optimal. Leaf size=17

$$\frac{1}{3}t^3 \log(t) - \frac{t^3}{9}$$

```
[Out] -t^3/9 + (t^3*Log[t])/3
```

Rubi [A] time = 0.0073478, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2304}

$$\frac{1}{3}t^3 \log(t) - \frac{t^3}{9}$$

Antiderivative was successfully verified.

```
[In] Int[t^2*Log[t],t]
```

```
[Out] -t^3/9 + (t^3*Log[t])/3
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int t^2 \log(t)\, dt = -\frac{t^3}{9} + \frac{1}{3}t^3 \log(t)$$

Mathematica [A] time = 0.0009988, size = 17, normalized size = 1.

$$\frac{1}{3}t^3 \log(t) - \frac{t^3}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[t^2*Log[t],t]
```

```
[Out] -t^3/9 + (t^3*Log[t])/3
```

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$-\frac{t^3}{9} + \frac{t^3 \ln(t)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^2*ln(t),t)
```

```
[Out] -1/9*t^3+1/3*t^3*ln(t)
```

Maxima [A] time = 0.935756, size = 18, normalized size = 1.06

$$\frac{1}{3}\,t^3 \log(t) - \frac{1}{9}\,t^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^2*log(t),t, algorithm="maxima")
```

```
[Out] 1/3*t^3*log(t) - 1/9*t^3
```

Fricas [A] time = 1.83429, size = 35, normalized size = 2.06

$$\frac{1}{3}\,t^3 \log(t) - \frac{1}{9}\,t^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^2*log(t),t, algorithm="fricas")
```

```
[Out] 1/3*t^3*log(t) - 1/9*t^3
```

Sympy [A] time = 0.082899, size = 12, normalized size = 0.71

$$\frac{t^3 \log(t)}{3} - \frac{t^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t**2*ln(t),t)
```

```
[Out] t**3*log(t)/3 - t**3/9
```

Giac [A] time = 1.05941, size = 18, normalized size = 1.06

$$\frac{1}{3} t^3 \log(t) - \frac{1}{9} t^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^2*log(t),t, algorithm="giac")
```

```
[Out] 1/3*t^3*log(t) - 1/9*t^3
```

3.31 $\int e^t t^3 \, dt$

Optimal. Leaf size=27

$$e^t t^3 - 3e^t t^2 + 6e^t t - 6e^t$$

```
[Out] -6*E^t + 6*E^t*t - 3*E^t*t^2 + E^t*t^3
```

Rubi [A] time = 0.0307144, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2176, 2194}

$$e^t t^3 - 3e^t t^2 + 6e^t t - 6e^t$$

Antiderivative was successfully verified.

```
[In] Int[E^t*t^3,t]
```

```
[Out] -6*E^t + 6*E^t*t - 3*E^t*t^2 + E^t*t^3
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int e^t t^3 \, dt &= e^t t^3 - 3 \int e^t t^2 \, dt \\
&= -3e^t t^2 + e^t t^3 + 6 \int e^t t \, dt \\
&= 6e^t t - 3e^t t^2 + e^t t^3 - 6 \int e^t \, dt \\
&= -6e^t + 6e^t t - 3e^t t^2 + e^t t^3
\end{aligned}
$$

Mathematica [A] time = 0.0060535, size = 17, normalized size = 0.63

$$e^t \left(t^3 - 3t^2 + 6t - 6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t*t^3,t]
```

```
[Out] E^t*(-6 + 6*t - 3*t^2 + t^3)
```

Maple [A] time = 0.002, size = 17, normalized size = 0.6

$$\left(t^3 - 3\,t^2 + 6\,t - 6\right) \mathrm{e}^t$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)*t^3,t)
```

```
[Out] (t^3-3*t^2+6*t-6)*exp(t)
```

Maxima [A] time = 0.93371, size = 22, normalized size = 0.81

$$\left(t^3 - 3\,t^2 + 6\,t - 6\right) e^t$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*t^3,t, algorithm="maxima")
```

```
[Out] (t^3 - 3*t^2 + 6*t - 6)*e^t
```

Fricas [A] time = 1.83083, size = 39, normalized size = 1.44

$$\left(t^{3}-3\,t^{2}+6\,t-6\right)e^{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*t^3,t, algorithm="fricas")
```

```
[Out] (t^3 - 3*t^2 + 6*t - 6)*e^t
```

Sympy [A] time = 0.079841, size = 15, normalized size = 0.56

$$\left(t^{3}-3t^{2}+6t-6\right)e^{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*t**3,t)
```

```
[Out] (t**3 - 3*t**2 + 6*t - 6)*exp(t)
```

Giac [A] time = 1.05015, size = 22, normalized size = 0.81

$$\left(t^{3}-3\,t^{2}+6\,t-6\right)e^{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*t^3,t, algorithm="giac")
```

```
[Out] (t^3 - 3*t^2 + 6*t - 6)*e^t
```

3.32 $\int e^{2t} \sin(3t)\, dt$

Optimal. Leaf size=27

$$\frac{2}{13}e^{2t}\sin(3t) - \frac{3}{13}e^{2t}\cos(3t)$$

```
[Out] (-3*E^(2*t)*Cos[3*t])/13 + (2*E^(2*t)*Sin[3*t])/13
```

Rubi [A] time = 0.011464, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4432}

$$\frac{2}{13}e^{2t}\sin(3t) - \frac{3}{13}e^{2t}\cos(3t)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*t)*Sin[3*t],t]
```

```
[Out] (-3*E^(2*t)*Cos[3*t])/13 + (2*E^(2*t)*Sin[3*t])/13
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{2t}\sin(3t)\,dt = -\frac{3}{13}e^{2t}\cos(3t) + \frac{2}{13}e^{2t}\sin(3t)$$

Mathematica [A] time = 0.0324587, size = 22, normalized size = 0.81

$$\frac{1}{13}e^{2t}(2\sin(3t) - 3\cos(3t))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*t)*Sin[3*t],t]
```

```
[Out] (E^(2*t)*(-3*Cos[3*t] + 2*Sin[3*t]))/13
```

Maple [A] time = 0.004, size = 22, normalized size = 0.8

$$-\frac{3\,e^{2\,t}\cos\left(3\,t\right)}{13}+\frac{2\,e^{2\,t}\sin\left(3\,t\right)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*t)*sin(3*t),t)
```

```
[Out] -3/13*exp(2*t)*cos(3*t)+2/13*exp(2*t)*sin(3*t)
```

Maxima [A] time = 0.925916, size = 26, normalized size = 0.96

$$-\frac{1}{13}\,\left(3\,\cos\left(3\,t\right)-2\,\sin\left(3\,t\right)\right)e^{\left(2\,t\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)*sin(3*t),t, algorithm="maxima")
```

```
[Out] -1/13*(3*cos(3*t) - 2*sin(3*t))*e^(2*t)
```

Fricas [A] time = 1.92505, size = 65, normalized size = 2.41

$$-\frac{3}{13}\,\cos\left(3\,t\right)e^{\left(2\,t\right)}+\frac{2}{13}\,e^{\left(2\,t\right)}\sin\left(3\,t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)*sin(3*t),t, algorithm="fricas")
```

```
[Out] -3/13*cos(3*t)*e^(2*t) + 2/13*e^(2*t)*sin(3*t)
```

Sympy [A] time = 0.293081, size = 26, normalized size = 0.96

$$\frac{2e^{2t}\sin(3t)}{13} - \frac{3e^{2t}\cos(3t)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)*sin(3*t),t)
```

```
[Out] 2*exp(2*t)*sin(3*t)/13 - 3*exp(2*t)*cos(3*t)/13
```

Giac [A] time = 1.0594, size = 26, normalized size = 0.96

$$-\frac{1}{13}\left(3\cos(3t) - 2\sin(3t)\right)e^{(2t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)*sin(3*t),t, algorithm="giac")
```

```
[Out] -1/13*(3*cos(3*t) - 2*sin(3*t))*e^(2*t)
```

3.33 $\int e^{-t}\cos(3t)\,dt$

Optimal. Leaf size=27

$$\frac{3}{10}e^{-t}\sin(3t)-\frac{1}{10}e^{-t}\cos(3t)$$

```
[Out] -Cos[3*t]/(10*E^t) + (3*Sin[3*t])/(10*E^t)
```

Rubi [A] time = 0.0111672, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4433}

$$\frac{3}{10}e^{-t}\sin(3t)-\frac{1}{10}e^{-t}\cos(3t)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*t]/E^t,t]
```

```
[Out] -Cos[3*t]/(10*E^t) + (3*Sin[3*t])/(10*E^t)
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-t}\cos(3t)\,dt=-\frac{1}{10}e^{-t}\cos(3t)+\frac{3}{10}e^{-t}\sin(3t)$$

Mathematica [A] time = 0.0269654, size = 20, normalized size = 0.74

$$-\frac{1}{10}e^{-t}(\cos(3t)-3\sin(3t))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*t]/E^t,t]
```

```
[Out] -(Cos[3*t] - 3*Sin[3*t])/(10*E^t)
```

Maple [A] time = 0.007, size = 22, normalized size = 0.8

$$-\frac{e^{-t}\cos(3t)}{10}+\frac{3e^{-t}\sin(3t)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*t)/exp(t),t)
```

```
[Out] -1/10*exp(-t)*cos(3*t)+3/10*exp(-t)*sin(3*t)
```

Maxima [A] time = 0.936622, size = 23, normalized size = 0.85

$$-\frac{1}{10}(\cos(3t)-3\sin(3t))e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*t)/exp(t),t, algorithm="maxima")
```

```
[Out] -1/10*(cos(3*t) - 3*sin(3*t))*e^(-t)
```

Fricas [A] time = 1.87896, size = 62, normalized size = 2.3

$$-\frac{1}{10}\cos(3t)e^{(-t)}+\frac{3}{10}e^{(-t)}\sin(3t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*t)/exp(t),t, algorithm="fricas")
```

```
[Out] -1/10*cos(3*t)*e^(-t) + 3/10*e^(-t)*sin(3*t)
```

Sympy [A] time = 0.458049, size = 20, normalized size = 0.74

$$\frac{3e^{-t}\sin(3t)}{10} - \frac{e^{-t}\cos(3t)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*t)/exp(t),t)
```

```
[Out] 3*exp(-t)*sin(3*t)/10 - exp(-t)*cos(3*t)/10
```

Giac [A] time = 1.05177, size = 23, normalized size = 0.85

$$-\frac{1}{10}\left(\cos(3\,t) - 3\,\sin(3\,t)\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*t)/exp(t),t, algorithm="giac")
```

```
[Out] -1/10*(cos(3*t) - 3*sin(3*t))*e^(-t)
```

3.34 $\int y\sinh(y)\,dy$

Optimal. Leaf size=9

$$y\cosh(y)-\sinh(y)$$

```
[Out] y*Cosh[y] - Sinh[y]
```

Rubi [A] time = 0.0115834, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3296, 2637}

$$y\cosh(y)-\sinh(y)$$

Antiderivative was successfully verified.

```
[In] Int[y*Sinh[y],y]
```

```
[Out] y*Cosh[y] - Sinh[y]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int y\sinh(y)\,dy &= y\cosh(y)-\int \cosh(y)\,dy\\ &= y\cosh(y)-\sinh(y)\end{aligned}$$

Mathematica [A] time = 0.0031957, size = 9, normalized size = 1.

$$y\cosh(y) - \sinh(y)$$

Antiderivative was successfully verified.

```
[In] Integrate[y*Sinh[y],y]
```

```
[Out] y*Cosh[y] - Sinh[y]
```

Maple [A] time = 0.001, size = 10, normalized size = 1.1

$$y\cosh\left(y\right) - \sinh\left(y\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(y*sinh(y),y)
```

```
[Out] y*cosh(y)-sinh(y)
```

Maxima [B] time = 0.948028, size = 46, normalized size = 5.11

$$\frac{1}{2}\,y^2\sinh\left(y\right) + \frac{1}{4}\left(y^2 + 2\,y + 2\right)e^{(-y)} - \frac{1}{4}\left(y^2 - 2\,y + 2\right)e^y$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(y*sinh(y),y, algorithm="maxima")
```

```
[Out] 1/2*y^2*sinh(y) + 1/4*(y^2 + 2*y + 2)*e^(-y) - 1/4*(y^2 - 2*y + 2)*e^y
```

Fricas [A] time = 1.78134, size = 28, normalized size = 3.11

$$y\cosh\left(y\right) - \sinh\left(y\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(y*sinh(y),y, algorithm="fricas")
```

```
[Out] y*cosh(y) - sinh(y)
```

Sympy [A] time = 0.171068, size = 7, normalized size = 0.78

$$y\cosh\left(y\right) - \sinh\left(y\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(y*sinh(y),y)
```

```
[Out] y*cosh(y) - sinh(y)
```

Giac [A] time = 1.04867, size = 23, normalized size = 2.56

$$\frac{1}{2}\left(y+1\right)e^{\left(-y\right)} + \frac{1}{2}\left(y-1\right)e^{y}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(y*sinh(y),y, algorithm="giac")
```

```
[Out] 1/2*(y + 1)*e^(-y) + 1/2*(y - 1)*e^y
```

3.35 $\int y \cosh(ay) \, dy$

Optimal. Leaf size=19

$$\frac{y \sinh(ay)}{a} - \frac{\cosh(ay)}{a^2}$$

```
[Out] -(Cosh[a*y]/a^2) + (y*Sinh[a*y])/a
```

Rubi [A] time = 0.0162827, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2638}

$$\frac{y \sinh(ay)}{a} - \frac{\cosh(ay)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[y*Cosh[a*y],y]
```

```
[Out] -(Cosh[a*y]/a^2) + (y*Sinh[a*y])/a
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int y \cosh(ay) \, dy &= \frac{y \sinh(ay)}{a} - \frac{\int \sinh(ay) \, dy}{a} \\ &= -\frac{\cosh(ay)}{a^2} + \frac{y \sinh(ay)}{a}\end{aligned}$$

Mathematica [A] time = 0.0129749, size = 19, normalized size = 1.

$$\frac{y\sinh(ay)}{a}-\frac{\cosh(ay)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[y*Cosh[a*y],y]
```

```
[Out] -(Cosh[a*y]/a^2) + (y*Sinh[a*y])/a
```

Maple [A] time = 0.009, size = 19, normalized size = 1.

$$\frac{ya\sinh\left(ay\right)-\cosh\left(ay\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(y*cosh(a*y),y)
```

```
[Out] 1/a^2*(y*a*sinh(a*y)-cosh(a*y))
```

Maxima [B] time = 0.950136, size = 77, normalized size = 4.05

$$\frac{1}{2}\,y^2\cosh\left(ay\right)-\frac{1}{4}\,a\left(\frac{\left(a^2y^2-2\,ay+2\right)e^{(ay)}}{a^3}+\frac{\left(a^2y^2+2\,ay+2\right)e^{(-ay)}}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(y*cosh(a*y),y, algorithm="maxima")
```

```
[Out] 1/2*y^2*cosh(a*y) - 1/4*a*((a^2*y^2 - 2*a*y + 2)*e^(a*y)/a^3 + (a^2*y^2 + 2
*a*y + 2)*e^(-a*y)/a^3)
```

Fricas [A] time = 1.77083, size = 45, normalized size = 2.37

$$\frac{ay\sinh\left(ay\right)-\cosh\left(ay\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(y*cosh(a*y),y, algorithm="fricas")
```

```
[Out] (a*y*sinh(a*y) - cosh(a*y))/a^2
```

Sympy [A] time = 0.21223, size = 20, normalized size = 1.05

$$\begin{cases} \frac{y\sinh\left(ay\right)}{a}-\frac{\cosh\left(ay\right)}{a^2} & \text{for } a\neq 0 \\ \frac{y^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(y*cosh(a*y),y)
```

```
[Out] Piecewise((y*sinh(a*y)/a - cosh(a*y)/a**2, Ne(a, 0)), (y**2/2, True))
```

Giac [A] time = 1.05336, size = 41, normalized size = 2.16

$$\frac{\left(ay-1\right)e^{\left(ay\right)}}{2\,a^2}-\frac{\left(ay+1\right)e^{\left(-ay\right)}}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(y*cosh(a*y),y, algorithm="giac")
```

```
[Out] 1/2*(a*y - 1)*e^(a*y)/a^2 - 1/2*(a*y + 1)*e^(-a*y)/a^2
```

3.36 $\int e^{-t} t \, dt$

Optimal. Leaf size=16

$$-e^{-t}t - e^{-t}$$

```
[Out] -E^(-t) - t/E^t
```

Rubi [A] time = 0.0086793, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2176, 2194}

$$-e^{-t}t - e^{-t}$$

Antiderivative was successfully verified.

```
[In] Int[t/E^t,t]
```

```
[Out] -E^(-t) - t/E^t
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int e^{-t} t \, dt &= -e^{-t} t + \int e^{-t} \, dt \\ &= -e^{-t} - e^{-t} t \end{aligned}$$

Mathematica [A] time = 0.0043066, size = 11, normalized size = 0.69

$$e^{-t}(-t-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[t/E^t,t]
```

```
[Out] (-1 - t)/E^t
```

Maple [A] time = 0.001, size = 10, normalized size = 0.6

$$-\frac{1+t}{e^t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t/exp(t),t)
```

```
[Out] -(1+t)/exp(t)
```

Maxima [A] time = 0.924738, size = 12, normalized size = 0.75

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t, algorithm="maxima")
```

```
[Out] -(t + 1)*e^(-t)
```

Fricas [A] time = 1.9907, size = 23, normalized size = 1.44

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t, algorithm="fricas")
```

```
[Out] -(t + 1)*e^(-t)
```

Sympy [A] time = 0.073225, size = 7, normalized size = 0.44

$$(-t-1)\,e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t)
```

```
[Out] (-t - 1)*exp(-t)
```

Giac [A] time = 1.04802, size = 12, normalized size = 0.75

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t, algorithm="giac")
```

```
[Out] -(t + 1)*e^(-t)
```

3.37 $\int \sqrt{t} \log(t)\, dt$

Optimal. Leaf size=21

$$\frac{2}{3}t^{3/2}\log(t) - \frac{4t^{3/2}}{9}$$

```
[Out] (-4*t^(3/2))/9 + (2*t^(3/2)*Log[t])/3
```

Rubi [A] time = 0.0078074, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2304}

$$\frac{2}{3}t^{3/2}\log(t) - \frac{4t^{3/2}}{9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[t]*Log[t],t]
```

```
[Out] (-4*t^(3/2))/9 + (2*t^(3/2)*Log[t])/3
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{t}\log(t)\, dt = -\frac{4t^{3/2}}{9} + \frac{2}{3}t^{3/2}\log(t)$$

Mathematica [A] time = 0.0028059, size = 15, normalized size = 0.71

$$\frac{2}{9}t^{3/2}(3\log(t) - 2)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[t]*Log[t],t]
```

```
[Out] (2*t^(3/2)*(-2 + 3*Log[t]))/9
```

Maple [A] time = 0.003, size = 14, normalized size = 0.7

$$-\frac{4}{9}t^{\frac{3}{2}}+\frac{2\ln\left(t\right)}{3}t^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(t)*t^(1/2),t)
```

```
[Out] -4/9*t^(3/2)+2/3*t^(3/2)*ln(t)
```

Maxima [A] time = 0.934268, size = 18, normalized size = 0.86

$$\frac{2}{3}\,t^{\frac{3}{2}}\log\left(t\right)-\frac{4}{9}\,t^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)*t^(1/2),t, algorithm="maxima")
```

```
[Out] 2/3*t^(3/2)*log(t) - 4/9*t^(3/2)
```

Fricas [A] time = 2.17516, size = 43, normalized size = 2.05

$$\frac{2}{9}\left(3\,t\log\left(t\right)-2\,t\right)\sqrt{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)*t^(1/2),t, algorithm="fricas")
```

```
[Out] 2/9*(3*t*log(t) - 2*t)*sqrt(t)
```

Sympy [A] time = 2.037, size = 66, normalized size = 3.14

$$\begin{cases} \frac{2t^{\frac{3}{2}}\log(t)}{3} - \frac{4t^{\frac{3}{2}}}{9} & \text{for } |t| < 1 \\ -\frac{2t^{\frac{3}{2}}\log\left(\frac{1}{t}\right)}{3} - \frac{4t^{\frac{3}{2}}}{9} & \text{for } \frac{1}{|t|} < 1 \\ -G_{3,3}^{2,1}\left(\begin{matrix} 1 & \frac{5}{2}, \frac{5}{2} \\ \frac{3}{2}, \frac{3}{2} & 0 \end{matrix} \middle| t\right) + G_{3,3}^{0,3}\left(\begin{matrix} \frac{5}{2}, \frac{5}{2}, 1 & \\ & \frac{3}{2}, \frac{3}{2}, 0 \end{matrix} \middle| t\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(t)*t**(1/2),t)
```

```
[Out] Piecewise((2*t**(3/2)*log(t)/3 - 4*t**(3/2)/9, Abs(t) < 1), (-2*t**(3/2)*lo
g(1/t)/3 - 4*t**(3/2)/9, 1/Abs(t) < 1), (-meijerg(((1,), (5/2, 5/2)), ((3/2
, 3/2), (0,)), t) + meijerg(((5/2, 5/2, 1), ()), ((), (3/2, 3/2, 0)), t), T
rue))
```

Giac [A] time = 1.04944, size = 18, normalized size = 0.86

$$\frac{2}{3}\,t^{\frac{3}{2}}\log(t) - \frac{4}{9}\,t^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)*t^(1/2),t, algorithm="giac")
```

```
[Out] 2/3*t^(3/2)*log(t) - 4/9*t^(3/2)
```

3.38 $\int x\cos(2x)\,dx$

Optimal. Leaf size=18

$$\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)$$

```
[Out] Cos[2*x]/4 + (x*Sin[2*x])/2
```

Rubi [A] time = 0.0122093, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2638}

$$\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[2*x],x]
```

```
[Out] Cos[2*x]/4 + (x*Sin[2*x])/2
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cos(2x)\,dx &= \frac{1}{2}x\sin(2x)-\frac{1}{2}\int\sin(2x)\,dx\\ &= \frac{1}{4}\cos(2x)+\frac{1}{2}x\sin(2x)\end{aligned}$$

Mathematica [A] time = 0.0090039, size = 18, normalized size = 1.

$$\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x],x]
```

```
[Out] Cos[2*x]/4 + (x*Sin[2*x])/2
```

Maple [A] time = 0.005, size = 15, normalized size = 0.8

$$\frac{\cos(2x)}{4}+\frac{x\sin(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x),x)
```

```
[Out] 1/4*cos(2*x)+1/2*x*sin(2*x)
```

Maxima [A] time = 0.935605, size = 19, normalized size = 1.06

$$\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x),x, algorithm="maxima")
```

```
[Out] 1/2*x*sin(2*x) + 1/4*cos(2*x)
```

Fricas [A] time = 2.37485, size = 42, normalized size = 2.33

$$\frac{1}{2}x\sin(2x)+\frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x),x, algorithm="fricas")
```

```
[Out] 1/2*x*sin(2*x) + 1/4*cos(2*x)
```

Sympy [A] time = 0.162415, size = 14, normalized size = 0.78

$$\frac{x\sin(2x)}{2}+\frac{\cos(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x),x)
```

```
[Out] x*sin(2*x)/2 + cos(2*x)/4
```

Giac [A] time = 1.04112, size = 19, normalized size = 1.06

$$\frac{1}{2}\,x\sin(2\,x)+\frac{1}{4}\,\cos(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x),x, algorithm="giac")
```

```
[Out] 1/2*x*sin(2*x) + 1/4*cos(2*x)
```

3.39 $\int e^{-x}x^2\,dx$

Optimal. Leaf size=26

$$-e^{-x}x^2-2e^{-x}x-2e^{-x}$$

```
[Out] -2/E^x - (2*x)/E^x - x^2/E^x
```

Rubi [A] time = 0.0179771, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$-e^{-x}x^2-2e^{-x}x-2e^{-x}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^x,x]
```

```
[Out] -2/E^x - (2*x)/E^x - x^2/E^x
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int e^{-x}x^2\,dx &= -e^{-x}x^2+2\int e^{-x}x\,dx\\
&= -2e^{-x}x-e^{-x}x^2+2\int e^{-x}\,dx\\
&= -2e^{-x}-2e^{-x}x-e^{-x}x^2
\end{aligned}$$

Mathematica [A] time = 0.0059813, size = 16, normalized size = 0.62

$$e^{-x}\left(-x^2-2x-2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^x,x]
```

```
[Out] (-2 - 2*x - x^2)/E^x
```

Maple [A] time = 0.001, size = 15, normalized size = 0.6

$$-\frac{x^2+2\,x+2}{\mathrm{e}^{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/exp(x),x)
```

```
[Out] -(x^2+2*x+2)/exp(x)
```

Maxima [A] time = 0.938155, size = 19, normalized size = 0.73

$$-\left(x^2+2\,x+2\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/exp(x),x, algorithm="maxima")
```

```
[Out] -(x^2 + 2*x + 2)*e^(-x)
```

Fricas [A] time = 2.06137, size = 34, normalized size = 1.31

$$-\left(x^2+2\,x+2\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/exp(x),x, algorithm="fricas")
```

```
[Out] -(x^2 + 2*x + 2)*e^(-x)
```

Sympy [A] time = 0.078733, size = 12, normalized size = 0.46

$$\left(-x^2-2x-2\right)e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/exp(x),x)
```

```
[Out] (-x**2 - 2*x - 2)*exp(-x)
```

Giac [A] time = 1.05395, size = 19, normalized size = 0.73

$$-\left(x^2+2\,x+2\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/exp(x),x, algorithm="giac")
```

```
[Out] -(x^2 + 2*x + 2)*e^(-x)
```

3.40 $\int \cos^{-1}(x)\,dx$

Optimal. Leaf size=18

$$x\cos^{-1}(x)-\sqrt{1-x^2}$$

```
[Out] -Sqrt[1 - x^2] + x*ArcCos[x]
```

Rubi [A] time = 0.0041468, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {4620, 261}

$$x\cos^{-1}(x)-\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCos[x],x]
```

```
[Out] -Sqrt[1 - x^2] + x*ArcCos[x]
```

Rule 4620

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*ArcCos[c*x])^n, x] + Dist[b*c*n, Int[(x*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^{-1}(x)\,dx &= x\cos^{-1}(x)+\int \frac{x}{\sqrt{1-x^2}}\,dx\\ &= -\sqrt{1-x^2}+x\cos^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0024326, size = 18, normalized size = 1.

$$x\cos^{-1}(x) - \sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCos[x],x]
```

```
[Out] -Sqrt[1 - x^2] + x*ArcCos[x]
```

Maple [A] time = 0.001, size = 17, normalized size = 0.9

$$x\arccos(x) - \sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccos(x),x)
```

```
[Out] x*arccos(x)-(-x^2+1)^(1/2)
```

Maxima [A] time = 1.40595, size = 22, normalized size = 1.22

$$x\arccos(x) - \sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x),x, algorithm="maxima")
```

```
[Out] x*arccos(x) - sqrt(-x^2 + 1)
```

Fricas [A] time = 2.39014, size = 41, normalized size = 2.28

$$x\arccos(x) - \sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x),x, algorithm="fricas")
```

```
[Out] x*arccos(x) - sqrt(-x^2 + 1)
```

Sympy [A] time = 0.117869, size = 12, normalized size = 0.67

$$x \operatorname{acos}(x) - \sqrt{1 - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acos(x),x)
```

```
[Out] x*acos(x) - sqrt(1 - x**2)
```

Giac [A] time = 1.04753, size = 22, normalized size = 1.22

$$x \arccos(x) - \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccos(x),x, algorithm="giac")
```

```
[Out] x*arccos(x) - sqrt(-x^2 + 1)
```

3.41 $\int x \csc^2(x)\,dx$

Optimal. Leaf size=9

$$\log(\sin(x)) - x\cot(x)$$

```
[Out] -(x*Cot[x]) + Log[Sin[x]]
```

Rubi [A] time = 0.0156671, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4184, 3475}

$$\log(\sin(x)) - x\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[x]^2,x]
```

```
[Out] -(x*Cot[x]) + Log[Sin[x]]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\csc^2(x)\,dx &= -x\cot(x) + \int \cot(x)\,dx\\ &= -x\cot(x) + \log(\sin(x))\end{aligned}$$

Mathematica [A] time = 0.0152348, size = 9, normalized size = 1.

$$\log(\sin(x)) - x\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[x]^2,x]
```

```
[Out] -(x*Cot[x]) + Log[Sin[x]]
```

Maple [A] time = 0.003, size = 10, normalized size = 1.1

$$-x\cot(x) + \ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)^2,x)
```

```
[Out] -x*cot(x)+ln(sin(x))
```

Maxima [B] time = 0.944753, size = 140, normalized size = 15.56

$$\frac{\left(\cos(2x)^2 + \sin(2x)^2 - 2\cos(2x) + 1\right)\log\left(\cos(x)^2 + \sin(x)^2 + 2\cos(x) + 1\right) + \left(\cos(2x)^2 + \sin(2x)^2 - 2\cos(2x) + \right.}{2\left(\cos(2x)^2 + \sin(2x)^2 - 2\cos(2x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2
*cos(x) + 1) + (cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)*log(cos(x)^2 + si
n(x)^2 - 2*cos(x) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x
) + 1)
```

Fricas [B] time = 2.61758, size = 61, normalized size = 6.78

$$-\frac{x\cos(x) - \log\left(\frac{1}{2}\sin(x)\right)\sin(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2,x, algorithm="fricas")
```

```
[Out] -(x*cos(x) - log(1/2*sin(x))*sin(x))/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\csc^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)**2,x)
```

```
[Out] Integral(x*csc(x)**2, x)
```

Giac [B] time = 1.0947, size = 70, normalized size = 7.78

$$\frac{x\tan\left(\frac{1}{2}\,x\right)^2 + \log\left(\frac{16\,\tan\left(\frac{1}{2}\,x\right)^2}{\tan\left(\frac{1}{2}\,x\right)^4 + 2\,\tan\left(\frac{1}{2}\,x\right)^2 + 1}\right)\tan\left(\frac{1}{2}\,x\right) - x}{2\,\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x*tan(1/2*x)^2 + log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 +
1))*tan(1/2*x) - x)/tan(1/2*x)
```

3.42 $\int \cos(5x)\sin(3x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\cos(2x)-\frac{1}{16}\cos(8x)$$

```
[Out] Cos[2*x]/4 - Cos[8*x]/16
```

Rubi [A] time = 0.0085067, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4284}

$$\frac{1}{4}\cos(2x)-\frac{1}{16}\cos(8x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[5*x]*Sin[3*x],x]
```

```
[Out] Cos[2*x]/4 - Cos[8*x]/16
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(5x)\sin(3x)\,dx=\frac{1}{4}\cos(2x)-\frac{1}{16}\cos(8x)$$

Mathematica [A] time = 0.0074548, size = 17, normalized size = 1.

$$\frac{\cos^2(x)}{2}-\frac{1}{16}\cos(8x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[5*x]*Sin[3*x],x]
```

```
[Out] Cos[x]^2/2 - Cos[8*x]/16
```

Maple [A] time = 0.074, size = 14, normalized size = 0.8

$$\frac{\cos(2x)}{4} - \frac{\cos(8x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(5*x)*sin(3*x),x)
```

```
[Out] 1/4*cos(2*x)-1/16*cos(8*x)
```

Maxima [A] time = 0.933849, size = 18, normalized size = 1.06

$$-\frac{1}{16}\cos(8x) + \frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/16*cos(8*x) + 1/4*cos(2*x)
```

Fricas [A] time = 2.69187, size = 76, normalized size = 4.47

$$-8\cos(x)^8 + 16\cos(x)^6 - 10\cos(x)^4 + \frac{5}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] -8*cos(x)^8 + 16*cos(x)^6 - 10*cos(x)^4 + 5/2*cos(x)^2
```

Sympy [B] time = 0.512751, size = 26, normalized size = 1.53

$$\frac{5\sin(3x)\sin(5x)}{16}+\frac{3\cos(3x)\cos(5x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)*sin(3*x),x)
```

```
[Out] 5*sin(3*x)*sin(5*x)/16 + 3*cos(3*x)*cos(5*x)/16
```

Giac [A] time = 1.05937, size = 18, normalized size = 1.06

$$-\frac{1}{16}\cos(8x)+\frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(5*x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -1/16*cos(8*x) + 1/4*cos(2*x)
```

3.43 $\int \sin(2x)\sin(4x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\sin(2x) - \frac{1}{12}\sin(6x)$$

```
[Out] Sin[2*x]/4 - Sin[6*x]/12
```

Rubi [A] time = 0.0080002, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4282}

$$\frac{1}{4}\sin(2x) - \frac{1}{12}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]*Sin[4*x],x]

[Out] Sin[2*x]/4 - Sin[6*x]/12
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(2x)\sin(4x)\,dx = \frac{1}{4}\sin(2x) - \frac{1}{12}\sin(6x)$$

Mathematica [A] time = 0.0061412, size = 17, normalized size = 1.

$$\frac{1}{4}\sin(2x) - \frac{1}{12}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]*Sin[4*x],x]
```

```
[Out] Sin[2*x]/4 - Sin[6*x]/12
```

Maple [A] time = 0.01, size = 9, normalized size = 0.5

$$\frac{(\sin(2x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(4*x),x)
```

```
[Out] 1/3*sin(2*x)^3
```

Maxima [A] time = 0.932666, size = 18, normalized size = 1.06

$$-\frac{1}{12}\sin(6x)+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*sin(4*x),x, algorithm="maxima")
```

```
[Out] -1/12*sin(6*x) + 1/4*sin(2*x)
```

Fricas [A] time = 2.41495, size = 43, normalized size = 2.53

$$-\frac{1}{3}\left(\cos(2x)^2-1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*sin(4*x),x, algorithm="fricas")
```

```
[Out] -1/3*(cos(2*x)^2 - 1)*sin(2*x)
```

Sympy [A] time = 0.558186, size = 22, normalized size = 1.29

$$-\frac{\sin(2x)\cos(4x)}{3}+\frac{\sin(4x)\cos(2x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*sin(4*x),x)
```

```
[Out] -sin(2*x)*cos(4*x)/3 + sin(4*x)*cos(2*x)/6
```

Giac [A] time = 1.10083, size = 18, normalized size = 1.06

$$-\frac{1}{12}\sin(6x)+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*sin(4*x),x, algorithm="giac")
```

```
[Out] -1/12*sin(6*x) + 1/4*sin(2*x)
```

3.44 $\int \cos(x)\log(\sin(x))\,dx$

Optimal. Leaf size=11

$$\sin(x)\log(\sin(x)) - \sin(x)$$

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

Rubi [A] time = 0.0098451, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2637, 2554}

$$\sin(x)\log(\sin(x)) - \sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Log[Sin[x]],x]
```

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\log(\sin(x))\,dx &= \log(\sin(x))\sin(x) - \int \cos(x)\,dx\\ &= -\sin(x) + \log(\sin(x))\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0023458, size = 11, normalized size = 1.

$$\sin(x)\log(\sin(x)) - \sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[Sin[x]],x]
```

```
[Out] -Sin[x] + Log[Sin[x]]*Sin[x]
```

Maple [A] time = 0.004, size = 12, normalized size = 1.1

$$-\sin(x) + \ln(\sin(x))\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(sin(x)),x)
```

```
[Out] -sin(x)+ln(sin(x))*sin(x)
```

Maxima [A] time = 0.926163, size = 15, normalized size = 1.36

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="maxima")
```

```
[Out] log(sin(x))*sin(x) - sin(x)
```

Fricas [A] time = 2.27957, size = 39, normalized size = 3.55

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="fricas")
```

```
[Out] log(sin(x))*sin(x) - sin(x)
```

Sympy [A] time = 0.876365, size = 10, normalized size = 0.91

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(sin(x)),x)
```

```
[Out] log(sin(x))*sin(x) - sin(x)
```

Giac [A] time = 1.05834, size = 15, normalized size = 1.36

$$\log(\sin(x))\sin(x) - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x)),x, algorithm="giac")
```

```
[Out] log(sin(x))*sin(x) - sin(x)
```

3.45 $\int e^{x^2} x^3 \, dx$

Optimal. Leaf size=22

$$\frac{1}{2}e^{x^2}x^2 - \frac{e^{x^2}}{2}$$

```
[Out] -E^x^2/2 + (E^x^2*x^2)/2
```

Rubi [A] time = 0.019292, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2212, 2209}

$$\frac{1}{2}e^{x^2}x^2 - \frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*x^3,x]
```

```
[Out] -E^x^2/2 + (E^x^2*x^2)/2
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int e^{x^2} x^3 \, dx = \frac{1}{2} e^{x^2} x^2 - \int e^{x^2} x \, dx$$
$$= -\frac{e^{x^2}}{2} + \frac{1}{2} e^{x^2} x^2$$

<u>Mathematica</u> [A] time = 0.0016353, size = 14, normalized size = 0.64

$$\frac{1}{2} e^{x^2} \left(x^2 - 1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*x^3,x]
```

```
[Out] (E^x^2*(-1 + x^2))/2
```

<u>Maple</u> [A] time = 0.002, size = 12, normalized size = 0.6

$$\frac{\left(x^2 - 1\right) e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*x^3,x)
```

```
[Out] 1/2*(x^2-1)*exp(x^2)
```

<u>Maxima</u> [A] time = 0.941293, size = 15, normalized size = 0.68

$$\frac{1}{2} \left(x^2 - 1\right) e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

Fricas [A] time = 2.32601, size = 31, normalized size = 1.41

$$\frac{1}{2}\left(x^2-1\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

Sympy [A] time = 0.077963, size = 10, normalized size = 0.45

$$\frac{\left(x^2-1\right)e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x**3,x)
```

```
[Out] (x**2 - 1)*exp(x**2)/2
```

Giac [A] time = 1.06031, size = 15, normalized size = 0.68

$$\frac{1}{2}\left(x^2-1\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x^3,x, algorithm="giac")
```

```
[Out] 1/2*(x^2 - 1)*e^(x^2)
```

3.46 $\int e^x(3+2x)\,dx$

Optimal. Leaf size=15

$$e^x(2x+3)-2e^x$$

```
[Out] -2*E^x + E^x*(3 + 2*x)
```

Rubi [A] time = 0.0083929, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$e^x(2x+3)-2e^x$$

Antiderivative was successfully verified.

```
[In] Int[E^x*(3 + 2*x),x]
```

```
[Out] -2*E^x + E^x*(3 + 2*x)
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^x(3+2x)\,dx &= e^x(3+2x)-2\int e^x\,dx\\ &= -2e^x+e^x(3+2x)\end{aligned}$$

Mathematica [A] time = 0.009235, size = 9, normalized size = 0.6

$$e^x(2x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*(3 + 2*x),x]
```

```
[Out] E^x*(1 + 2*x)
```

Maple [A] time = 0., size = 9, normalized size = 0.6

$$(1+2x)\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*(3+2*x),x)
```

```
[Out] (1+2*x)*exp(x)
```

Maxima [A] time = 0.944416, size = 16, normalized size = 1.07

$$2\,(x-1)e^x+3\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(3+2*x),x, algorithm="maxima")
```

```
[Out] 2*(x - 1)*e^x + 3*e^x
```

Fricas [A] time = 2.2422, size = 20, normalized size = 1.33

$$(2\,x+1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(3+2*x),x, algorithm="fricas")
```

```
[Out] (2*x + 1)*e^x
```

Sympy [A] time = 0.074453, size = 7, normalized size = 0.47

$$(2x+1)\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(3+2*x),x)
```

```
[Out] (2*x + 1)*exp(x)
```

Giac [A] time = 1.05474, size = 11, normalized size = 0.73

$$(2\,x+1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*(3+2*x),x, algorithm="giac")
```

```
[Out] (2*x + 1)*e^x
```

3.47 $\int 5^x x\,dx$

Optimal. Leaf size=19

$$\frac{5^x x}{\log(5)}-\frac{5^x}{\log^2(5)}$$

```
[Out] -(5^x/Log[5]^2) + (5^x*x)/Log[5]
```

Rubi [A] time = 0.0085244, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2176, 2194}

$$\frac{5^x x}{\log(5)}-\frac{5^x}{\log^2(5)}$$

Antiderivative was successfully verified.

```
[In] Int[5^x*x,x]

[Out] -(5^x/Log[5]^2) + (5^x*x)/Log[5]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int 5^x x\,dx = \frac{5^x x}{\log(5)} - \frac{\int 5^x\,dx}{\log(5)}$$
$$= -\frac{5^x}{\log^2(5)} + \frac{5^x x}{\log(5)}$$

Mathematica [A] time = 0.0028747, size = 14, normalized size = 0.74

$$\frac{5^x(x\log(5)-1)}{\log^2(5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[5^x*x,x]
```

```
[Out] (5^x*(-1 + x*Log[5]))/Log[5]^2
```

Maple [A] time = 0.004, size = 15, normalized size = 0.8

$$\frac{(\ln(5)\,x-1)\,5^x}{(\ln(5))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5^x*x,x)
```

```
[Out] (ln(5)*x-1)*5^x/ln(5)^2
```

Maxima [A] time = 1.42486, size = 19, normalized size = 1.

$$\frac{(x\log(5)-1)5^x}{\log(5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5^x*x,x, algorithm="maxima")
```

```
[Out] (x*log(5) - 1)*5^x/log(5)^2
```

Fricas [A] time = 2.23962, size = 39, normalized size = 2.05

$$\frac{(x\log(5)-1)5^x}{\log(5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5^x*x,x, algorithm="fricas")
```

```
[Out] (x*log(5) - 1)*5^x/log(5)^2
```

Sympy [A] time = 0.091439, size = 14, normalized size = 0.74

$$\frac{5^x(x\log(5)-1)}{\log(5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5**x*x,x)
```

```
[Out] 5**x*(x*log(5) - 1)/log(5)**2
```

Giac [A] time = 1.04894, size = 19, normalized size = 1.

$$\frac{(x\log(5)-1)5^x}{\log(5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5^x*x,x, algorithm="giac")
```

```
[Out] (x*log(5) - 1)*5^x/log(5)^2
```

3.48 $\int \cos(\log(x))\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x\sin(\log(x)) + \frac{1}{2}x\cos(\log(x))$$

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rubi [A] time = 0.0029784, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4476}

$$\frac{1}{2}x\sin(\log(x)) + \frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Log[x]],x]
```

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rule 4476

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(x*
Cos[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] + Simp[(b*d*n*x*Sin[d*(a +
 b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\int \cos(\log(x))\,dx = \frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

Mathematica [A] time = 0.0026487, size = 17, normalized size = 1.

$$\frac{1}{2}x\sin(\log(x)) + \frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Log[x]],x]
```

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Maple [A] time = 0., size = 14, normalized size = 0.8

$$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(ln(x)),x)
```

```
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Maxima [A] time = 0.936863, size = 14, normalized size = 0.82

$$\frac{1}{2}\,x(\cos(\log(x))+\sin(\log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x*(cos(log(x)) + sin(log(x)))
```

Fricas [A] time = 2.37387, size = 53, normalized size = 3.12

$$\frac{1}{2}\,x\cos(\log(x))+\frac{1}{2}\,x\sin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A] time = 0.375787, size = 15, normalized size = 0.88

$$\frac{x\sin(\log(x))}{2}+\frac{x\cos(\log(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(ln(x)),x)
```

```
[Out] x*sin(log(x))/2 + x*cos(log(x))/2
```

Giac [A] time = 1.05204, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x\cos(\log(x))+\frac{1}{2}\,x\sin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="giac")
```

```
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

3.49 $\int e^{\sqrt{x}}\,dx$

Optimal. Leaf size=24

$$2e^{\sqrt{x}}\sqrt{x}-2e^{\sqrt{x}}$$

```
[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rubi [A] time = 0.0077433, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2207, 2176, 2194}

$$2e^{\sqrt{x}}\sqrt{x}-2e^{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[E^Sqrt[x],x]
```

```
[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rule 2207

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Dist[k/d, Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (c
 + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !Inte
gerQ[n]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int e^x x\,dx, x, \sqrt{x}\right)$$
$$= 2e^{\sqrt{x}}\sqrt{x} - 2\,\text{Subst}\left(\int e^x\,dx, x, \sqrt{x}\right)$$
$$= -2e^{\sqrt{x}} + 2e^{\sqrt{x}}\sqrt{x}$$

Mathematica [A] time = 0.005531, size = 16, normalized size = 0.67

$$2e^{\sqrt{x}}\left(\sqrt{x}-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sqrt[x],x]
```

```
[Out] 2*E^Sqrt[x]*(-1 + Sqrt[x])
```

Maple [A] time = 0., size = 17, normalized size = 0.7

$$-2\,\mathrm{e}^{\sqrt{x}} + 2\,\mathrm{e}^{\sqrt{x}}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^(1/2)),x)
```

```
[Out] -2*exp(x^(1/2))+2*exp(x^(1/2))*x^(1/2)
```

Maxima [A] time = 0.946347, size = 15, normalized size = 0.62

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Fricas [A] time = 2.21775, size = 36, normalized size = 1.5

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Sympy [A] time = 0.177795, size = 20, normalized size = 0.83

$$2\sqrt{x}e^{\sqrt{x}}-2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*exp(sqrt(x)) - 2*exp(sqrt(x))
```

Giac [A] time = 1.06747, size = 15, normalized size = 0.62

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

3.50 $\int \log\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=14

$$x\log\left(\sqrt{x}\right)-\frac{x}{2}$$

```
[Out] -x/2 + x*Log[Sqrt[x]]
```

Rubi [A] time = 0.0011754, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2295}

$$x\log\left(\sqrt{x}\right)-\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[Sqrt[x]],x]
```

```
[Out] -x/2 + x*Log[Sqrt[x]]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int \log\left(\sqrt{x}\right)\,dx=-\frac{x}{2}+x\log\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.0005846, size = 12, normalized size = 0.86

$$\frac{1}{2}(x\log(x)-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[Sqrt[x]],x]
```

```
[Out] (-x + x*Log[x])/2
```

Maple [A] time = 0., size = 10, normalized size = 0.7

$$-\frac{x}{2}+\frac{x\ln\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*ln(x),x)
```

```
[Out] -1/2*x+1/2*x*ln(x)
```

Maxima [A] time = 0.937011, size = 12, normalized size = 0.86

$$\frac{1}{2}\,x\log\left(x\right)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(x),x, algorithm="maxima")
```

```
[Out] 1/2*x*log(x) - 1/2*x
```

Fricas [A] time = 2.215, size = 30, normalized size = 2.14

$$\frac{1}{2}\,x\log\left(x\right)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*x*log(x) - 1/2*x
```

Sympy [A] time = 0.079031, size = 8, normalized size = 0.57

$$\frac{x\log(x)}{2} - \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*ln(x),x)
```

```
[Out] x*log(x)/2 - x/2
```

Giac [A] time = 1.0501, size = 12, normalized size = 0.86

$$\frac{1}{2}\,x\log(x) - \frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x*log(x) - 1/2*x
```

3.51 $\int \sin(\log(x))\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x\sin(\log(x)) - \frac{1}{2}x\cos(\log(x))$$

```
[Out] -(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rubi [A] time = 0.0030924, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4475}

$$\frac{1}{2}x\sin(\log(x)) - \frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Log[x]],x]
```

```
[Out] -(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rule 4475

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(x*
Sin[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] - Simp[(b*d*n*x*Cos[d*(a +
 b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\int \sin(\log(x))\,dx = -\frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

Mathematica [A] time = 0.0019087, size = 17, normalized size = 1.

$$\frac{1}{2}x\sin(\log(x)) - \frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Log[x]],x]
```

```
[Out] -(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$-\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(ln(x)),x)
```

```
[Out] -1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Maxima [A] time = 0.93754, size = 16, normalized size = 0.94

$$-\frac{1}{2}\,x(\cos(\log(x))-\sin(\log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x)),x, algorithm="maxima")
```

```
[Out] -1/2*x*(cos(log(x)) - sin(log(x)))
```

Fricas [A] time = 2.35962, size = 54, normalized size = 3.18

$$-\frac{1}{2}\,x\cos(\log(x))+\frac{1}{2}\,x\sin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x)),x, algorithm="fricas")
```

```
[Out] -1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A] time = 0.374718, size = 15, normalized size = 0.88

$$\frac{x\sin(\log(x))}{2} - \frac{x\cos(\log(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(ln(x)),x)
```

```
[Out] x*sin(log(x))/2 - x*cos(log(x))/2
```

Giac [A] time = 1.04808, size = 18, normalized size = 1.06

$$-\frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x)),x, algorithm="giac")
```

```
[Out] -1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

3.52 $\int \sin\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=22

$$2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rubi [A] time = 0.0108838, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3361, 3296, 2637}

$$2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[x]],x]
```

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sin\left(\sqrt{x}\right) dx &= 2\,\text{Subst}\left(\int x\sin(x)\,dx, x, \sqrt{x}\right) \\
&= -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\,\text{Subst}\left(\int \cos(x)\,dx, x, \sqrt{x}\right) \\
&= -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0122321, size = 22, normalized size = 1.

$$2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[x]],x]
```

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Maple [A] time = 0., size = 17, normalized size = 0.8

$$2\,\sin\left(\sqrt{x}\right) - 2\,\cos\left(\sqrt{x}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2)),x)
```

```
[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

Maxima [A] time = 0.934698, size = 22, normalized size = 1.

$$-2\,\sqrt{x}\cos\left(\sqrt{x}\right) + 2\,\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Fricas [A] time = 2.25847, size = 57, normalized size = 2.59

$$-2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Sympy [A] time = 0.293694, size = 20, normalized size = 0.91

$$-2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/2)),x)
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Giac [A] time = 1.04925, size = 22, normalized size = 1.

$$-2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="giac")
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

3.53 $\int x^5 \cos(x^3)\, dx$

Optimal. Leaf size=20

$$\frac{1}{3}x^3\sin\left(x^3\right)+\frac{\cos\left(x^3\right)}{3}$$

```
[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

Rubi [A] time = 0.0160509, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3380, 3296, 2638}

$$\frac{1}{3}x^3\sin\left(x^3\right)+\frac{\cos\left(x^3\right)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Cos[x^3],x]
```

```
[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^5 \cos\left(x^3\right)\,dx &= \frac{1}{3}\,\text{Subst}\left(\int x\cos(x)\,dx, x, x^3\right)\\
&= \frac{1}{3}x^3\sin\left(x^3\right) - \frac{1}{3}\,\text{Subst}\left(\int \sin(x)\,dx, x, x^3\right)\\
&= \frac{\cos\left(x^3\right)}{3} + \frac{1}{3}x^3\sin\left(x^3\right)
\end{aligned}
$$

Mathematica [A] time = 0.0087507, size = 20, normalized size = 1.

$$\frac{1}{3}x^3\sin\left(x^3\right) + \frac{\cos\left(x^3\right)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cos[x^3],x]
```

```
[Out] Cos[x^3]/3 + (x^3*Sin[x^3])/3
```

Maple [A] time = 0.004, size = 17, normalized size = 0.9

$$\frac{\cos\left(x^3\right)}{3} + \frac{x^3\sin\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cos(x^3),x)
```

```
[Out] 1/3*cos(x^3)+1/3*x^3*sin(x^3)
```

Maxima [A] time = 0.939193, size = 22, normalized size = 1.1

$$\frac{1}{3}\,x^3\sin\left(x^3\right) + \frac{1}{3}\,\cos\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(x^3),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

Fricas [A] time = 2.30663, size = 45, normalized size = 2.25

$$\frac{1}{3}\,x^3\sin\left(x^3\right)+\frac{1}{3}\,\cos\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(x^3),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

Sympy [A] time = 1.85221, size = 15, normalized size = 0.75

$$\frac{x^3\sin\left(x^3\right)}{3}+\frac{\cos\left(x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cos(x**3),x)
```

```
[Out] x**3*sin(x**3)/3 + cos(x**3)/3
```

Giac [A] time = 1.06456, size = 22, normalized size = 1.1

$$\frac{1}{3}\,x^3\sin\left(x^3\right)+\frac{1}{3}\,\cos\left(x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cos(x^3),x, algorithm="giac")
```

```
[Out] 1/3*x^3*sin(x^3) + 1/3*cos(x^3)
```

3.54 $\int e^{x^2} x^5 \, dx$

Optimal. Leaf size=28

$$\frac{1}{2}e^{x^2}x^4 - e^{x^2}x^2 + e^{x^2}$$

```
[Out] E^x^2 - E^x^2*x^2 + (E^x^2*x^4)/2
```

Rubi [A] time = 0.0337833, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2212, 2209}

$$\frac{1}{2}e^{x^2}x^4 - e^{x^2}x^2 + e^{x^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*x^5,x]
```

```
[Out] E^x^2 - E^x^2*x^2 + (E^x^2*x^4)/2
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int e^{x^2}x^5\,dx = \frac{1}{2}e^{x^2}x^4 - 2\int e^{x^2}x^3\,dx$$
$$= -e^{x^2}x^2 + \frac{1}{2}e^{x^2}x^4 + 2\int e^{x^2}x\,dx$$
$$= e^{x^2} - e^{x^2}x^2 + \frac{1}{2}e^{x^2}x^4$$

Mathematica [A] time = 0.0021054, size = 19, normalized size = 0.68

$$\frac{1}{2}e^{x^2}\left(x^4 - 2x^2 + 2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*x^5,x]
```

```
[Out] (E^x^2*(2 - 2*x^2 + x^4))/2
```

Maple [A] time = 0.001, size = 17, normalized size = 0.6

$$\frac{\left(x^4 - 2\,x^2 + 2\right)\mathrm{e}^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*x^5,x)
```

```
[Out] 1/2*(x^4-2*x^2+2)*exp(x^2)
```

Maxima [A] time = 0.943267, size = 22, normalized size = 0.79

$$\frac{1}{2}\left(x^4 - 2\,x^2 + 2\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x^5,x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 - 2*x^2 + 2)*e^(x^2)
```

Fricas [A] time = 2.28847, size = 42, normalized size = 1.5

$$\frac{1}{2}\left(x^4 - 2x^2 + 2\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x^5,x, algorithm="fricas")
```

```
[Out] 1/2*(x^4 - 2*x^2 + 2)*e^(x^2)
```

Sympy [A] time = 0.084731, size = 15, normalized size = 0.54

$$\frac{\left(x^4 - 2x^2 + 2\right)e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x**5,x)
```

```
[Out] (x**4 - 2*x**2 + 2)*exp(x**2)/2
```

Giac [A] time = 1.066, size = 22, normalized size = 0.79

$$\frac{1}{2}\left(x^4 - 2x^2 + 2\right)e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x^5,x, algorithm="giac")
```

```
[Out] 1/2*(x^4 - 2*x^2 + 2)*e^(x^2)
```

3.55 $\int x \tan^{-1}(x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}x^2\tan^{-1}(x)-\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rubi [A] time = 0.0085168, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {4852, 321, 203}

$$\frac{1}{2}x^2\tan^{-1}(x)-\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x],x]

[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x \tan^{-1}(x)\,dx &= \frac{1}{2}x^2\tan^{-1}(x) - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx\\
&= -\frac{x}{2} + \frac{1}{2}x^2\tan^{-1}(x) + \frac{1}{2}\int \frac{1}{1+x^2}\,dx\\
&= -\frac{x}{2} + \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}x^2\tan^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.0025206, size = 21, normalized size = 1.

$$\frac{1}{2}x^2\tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x],x]
```

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Maple [A] time = 0., size = 16, normalized size = 0.8

$$-\frac{x}{2} + \frac{\arctan(x)}{2} + \frac{x^2\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x),x)
```

```
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Maxima [A] time = 1.41976, size = 20, normalized size = 0.95

$$\frac{1}{2}x^2\arctan(x) - \frac{1}{2}x + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Fricas [A] time = 2.2782, size = 45, normalized size = 2.14

$$\frac{1}{2}\left(x^2+1\right)\arctan\left(x\right)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

Sympy [A] time = 0.333704, size = 15, normalized size = 0.71

$$\frac{x^2\operatorname{atan}\left(x\right)}{2}-\frac{x}{2}+\frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x),x)
```

```
[Out] x**2*atan(x)/2 - x/2 + atan(x)/2
```

Giac [A] time = 1.05583, size = 20, normalized size = 0.95

$$\frac{1}{2}\,x^2\arctan\left(x\right)-\frac{1}{2}\,x+\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

3.56 $\int x \cos(\pi x) \, dx$

Optimal. Leaf size=18

$$\frac{x \sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2}$$

```
[Out] Cos[Pi*x]/Pi^2 + (x*Sin[Pi*x])/Pi
```

Rubi [A] time = 0.0125108, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2638}

$$\frac{x \sin(\pi x)}{\pi} + \frac{\cos(\pi x)}{\pi^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[Pi*x],x]
```

```
[Out] Cos[Pi*x]/Pi^2 + (x*Sin[Pi*x])/Pi
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int x \cos(\pi x) \, dx &= \frac{x \sin(\pi x)}{\pi} - \frac{\int \sin(\pi x) \, dx}{\pi} \\ &= \frac{\cos(\pi x)}{\pi^2} + \frac{x \sin(\pi x)}{\pi} \end{aligned}$$

Mathematica [A] time = 0.0146212, size = 18, normalized size = 1.

$$\frac{x\sin(\pi x)}{\pi}+\frac{\cos(\pi x)}{\pi^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[Pi*x],x]
```

```
[Out] Cos[Pi*x]/Pi^2 + (x*Sin[Pi*x])/Pi
```

Maple [A] time = 0.005, size = 17, normalized size = 0.9

$$\frac{\cos(\pi x)+x\pi\sin(\pi x)}{\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(Pi*x),x)
```

```
[Out] 1/Pi^2*(cos(Pi*x)+x*Pi*sin(Pi*x))
```

Maxima [A] time = 0.935002, size = 22, normalized size = 1.22

$$\frac{\pi x\sin(\pi x)+\cos(\pi x)}{\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(pi*x),x, algorithm="maxima")
```

```
[Out] (pi*x*sin(pi*x) + cos(pi*x))/pi^2
```

Fricas [A] time = 2.56099, size = 47, normalized size = 2.61

$$\frac{\pi x\sin(\pi x)+\cos(\pi x)}{\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(pi*x),x, algorithm="fricas")
```

```
[Out] (pi*x*sin(pi*x) + cos(pi*x))/pi^2
```

Sympy [A] time = 0.216689, size = 15, normalized size = 0.83

$$\frac{x\sin(\pi x)}{\pi}+\frac{\cos(\pi x)}{\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(pi*x),x)
```

```
[Out] x*sin(pi*x)/pi + cos(pi*x)/pi**2
```

Giac [A] time = 1.05167, size = 24, normalized size = 1.33

$$\frac{x\sin(\pi x)}{\pi}+\frac{\cos(\pi x)}{\pi^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(pi*x),x, algorithm="giac")
```

```
[Out] x*sin(pi*x)/pi + cos(pi*x)/pi^2
```

3.57 $\int \sqrt{x}\log(x)\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}x^{3/2}\log(x)-\frac{4x^{3/2}}{9}$$

```
[Out] (-4*x^(3/2))/9 + (2*x^(3/2)*Log[x])/3
```

Rubi [A] time = 0.0058836, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2304}

$$\frac{2}{3}x^{3/2}\log(x)-\frac{4x^{3/2}}{9}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Log[x],x]
```

```
[Out] (-4*x^(3/2))/9 + (2*x^(3/2)*Log[x])/3
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{x}\log(x)\,dx=-\frac{4x^{3/2}}{9}+\frac{2}{3}x^{3/2}\log(x)$$

Mathematica [A] time = 0.0021311, size = 15, normalized size = 0.71

$$\frac{2}{9}x^{3/2}(3\log(x)-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Log[x],x]
```

```
[Out] (2*x^(3/2)*(-2 + 3*Log[x]))/9
```

Maple [A] time = 0.002, size = 14, normalized size = 0.7

$$-\frac{4}{9}x^{\frac{3}{2}}+\frac{2\ln(x)}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)*x^(1/2),x)
```

```
[Out] -4/9*x^(3/2)+2/3*x^(3/2)*ln(x)
```

Maxima [A] time = 0.940038, size = 18, normalized size = 0.86

$$\frac{2}{3}x^{\frac{3}{2}}\log(x)-\frac{4}{9}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*x^(3/2)*log(x) - 4/9*x^(3/2)
```

Fricas [A] time = 2.50986, size = 43, normalized size = 2.05

$$\frac{2}{9}(3x\log(x)-2x)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/9*(3*x*log(x) - 2*x)*sqrt(x)
```

Sympy [A] time = 2.12344, size = 66, normalized size = 3.14

$$\begin{cases} \frac{2x^{\frac{3}{2}}\log(x)}{3} - \frac{4x^{\frac{3}{2}}}{9} & \text{for } |x| < 1 \\ -\frac{2x^{\frac{3}{2}}\log\left(\frac{1}{x}\right)}{3} - \frac{4x^{\frac{3}{2}}}{9} & \text{for } \frac{1}{|x|} < 1 \\ -G_{3,3}^{2,1}\left(\begin{matrix} 1 & \frac{5}{2}, \frac{5}{2} \\ \frac{3}{2}, \frac{3}{2} & 0 \end{matrix} \middle| x\right) + G_{3,3}^{0,3}\left(\begin{matrix} \frac{5}{2}, \frac{5}{2}, 1 & \\ & \frac{3}{2}, \frac{3}{2}, 0 \end{matrix} \middle| x\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)*x**(1/2),x)
```

```
[Out] Piecewise((2*x**(3/2)*log(x)/3 - 4*x**(3/2)/9, Abs(x) < 1), (-2*x**(3/2)*lo
g(1/x)/3 - 4*x**(3/2)/9, 1/Abs(x) < 1), (-meijerg(((1,), (5/2, 5/2)), ((3/2
, 3/2), (0,)), x) + meijerg(((5/2, 5/2, 1), ()), ((), (3/2, 3/2, 0)), x), T
rue))
```

Giac [A] time = 1.06099, size = 18, normalized size = 0.86

$$\frac{2}{3}\,x^{\frac{3}{2}}\log(x) - \frac{4}{9}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)*x^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*x^(3/2)*log(x) - 4/9*x^(3/2)
```

3.58 $\int \sin^2(3x)\,dx$

Optimal. Leaf size=18

$$\frac{x}{2}-\frac{1}{6}\sin(3x)\cos(3x)$$

```
[Out] x/2 - (Cos[3*x]*Sin[3*x])/6
```

Rubi [A] time = 0.0060824, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2635, 8}

$$\frac{x}{2}-\frac{1}{6}\sin(3x)\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[3*x]^2,x]
```

```
[Out] x/2 - (Cos[3*x]*Sin[3*x])/6
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^2(3x)\,dx &= -\frac{1}{6}\cos(3x)\sin(3x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{6}\cos(3x)\sin(3x)\end{aligned}$$

Mathematica [A] time = 0.0062876, size = 14, normalized size = 0.78

$$\frac{x}{2}-\frac{1}{12}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[3*x]^2,x]
```

```
[Out] x/2 - Sin[6*x]/12
```

Maple [A] time = 0.007, size = 15, normalized size = 0.8

$$\frac{x}{2}-\frac{\cos(3x)\sin(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)^2,x)
```

```
[Out] 1/2*x-1/6*cos(3*x)*sin(3*x)
```

Maxima [A] time = 0.936092, size = 14, normalized size = 0.78

$$\frac{1}{2}x-\frac{1}{12}\sin(6x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/12*sin(6*x)
```

Fricas [A] time = 2.58258, size = 43, normalized size = 2.39

$$-\frac{1}{6}\cos(3x)\sin(3x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)^2,x, algorithm="fricas")
```

```
[Out] -1/6*cos(3*x)*sin(3*x) + 1/2*x
```

Sympy [A] time = 0.060721, size = 14, normalized size = 0.78

$$\frac{x}{2} - \frac{\sin(3x)\cos(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)**2,x)
```

```
[Out] x/2 - sin(3*x)*cos(3*x)/6
```

Giac [A] time = 1.07291, size = 14, normalized size = 0.78

$$\frac{1}{2}\,x - \frac{1}{12}\,\sin(6\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/12*sin(6*x)
```

3.59 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0055584, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\,dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.001569, size = 14, normalized size = 1.

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2,x]
```

```
[Out] x/2 + Sin[2*x]/4
```

Maple [A] time = 0.005, size = 11, normalized size = 0.8

$$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
```

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A] time = 0.939797, size = 14, normalized size = 1.

$$\frac{1}{2}x+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A] time = 2.21765, size = 36, normalized size = 2.57

$$\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.057686, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

Giac [A] time = 1.04969, size = 14, normalized size = 1.

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

3.60 $\int \cos^4(x)\,dx$

Optimal. Leaf size=24

$$\frac{3x}{8}+\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{8}\sin(x)\cos(x)$$

```
[Out] (3*x)/8 + (3*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
```

Rubi [A] time = 0.0129631, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{3x}{8}+\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4,x]
```

```
[Out] (3*x)/8 + (3*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(x)\,dx &= \frac{1}{4}\cos^3(x)\sin(x)+\frac{3}{4}\int \cos^2(x)\,dx\\ &= \frac{3}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}+\frac{3}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0018096, size = 22, normalized size = 0.92

$$\frac{3x}{8}+\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4,x]
```

```
[Out] (3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32
```

Maple [A] time = 0.001, size = 18, normalized size = 0.8

$$\frac{\sin(x)}{4}\left((\cos(x))^3+\frac{3\cos(x)}{2}\right)+\frac{3x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4,x)
```

```
[Out] 1/4*(cos(x)^3+3/2*cos(x))*sin(x)+3/8*x
```

Maxima [A] time = 0.952741, size = 22, normalized size = 0.92

$$\frac{3}{8}x+\frac{1}{32}\sin(4x)+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

Fricas [A] time = 2.08891, size = 59, normalized size = 2.46

$$\frac{1}{8}\left(2\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{3}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(2*cos(x)^3 + 3*cos(x))*sin(x) + 3/8*x
```

Sympy [A] time = 0.058508, size = 24, normalized size = 1.

$$\frac{3x}{8} + \frac{\sin(x)\cos^3(x)}{4} + \frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4,x)
```

```
[Out] 3*x/8 + sin(x)*cos(x)**3/4 + 3*sin(x)*cos(x)/8
```

Giac [A] time = 1.05795, size = 22, normalized size = 0.92

$$\frac{3}{8}x + \frac{1}{32}\sin(4x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

3.61 $\int \sin^3(x)\,dx$

Optimal. Leaf size=13

$$\frac{\cos^3(x)}{3}-\cos(x)$$

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rubi [A] time = 0.0054989, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2633}

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3,x]
```

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(x)\,dx &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(x)\right)\\ &= -\cos(x)+\frac{\cos^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.0017633, size = 15, normalized size = 1.15

$$\frac{1}{12}\cos(3x)-\frac{3\cos(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3,x]
```

```
[Out] (-3*Cos[x])/4 + Cos[3*x]/12
```

Maple [A] time = 0., size = 11, normalized size = 0.9

$$-\frac{\left(2+(\sin(x))^2\right)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x)
```

```
[Out] -1/3*(2+sin(x)^2)*cos(x)
```

Maxima [A] time = 0.928092, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

Fricas [A] time = 1.97143, size = 31, normalized size = 2.38

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

Sympy [A] time = 0.06428, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3,x)
```

```
[Out] cos(x)**3/3 - cos(x)
```

Giac [A] time = 1.07697, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

3.62 $\int \cos^4(x)\sin^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\cos^7(x)}{7}-\frac{\cos^5(x)}{5}$$

```
[Out] -Cos[x]^5/5 + Cos[x]^7/7
```

Rubi [A] time = 0.022827, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2565, 14}

$$\frac{\cos^7(x)}{7}-\frac{\cos^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4*Sin[x]^3,x]
```

```
[Out] -Cos[x]^5/5 + Cos[x]^7/7
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^4(x)\sin^3(x)\,dx = -\operatorname{Subst}\left(\int x^4\left(1-x^2\right)\,dx, x, \cos(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(x^4-x^6\right)\,dx, x, \cos(x)\right)$$
$$= -\frac{1}{5}\cos^5(x)+\frac{\cos^7(x)}{7}$$

Mathematica [A] time = 0.009907, size = 31, normalized size = 1.82

$$-\frac{3\cos(x)}{64}-\frac{1}{64}\cos(3x)+\frac{1}{320}\cos(5x)+\frac{1}{448}\cos(7x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4*Sin[x]^3,x]
```

```
[Out] (-3*Cos[x])/64 - Cos[3*x]/64 + Cos[5*x]/320 + Cos[7*x]/448
```

Maple [A] time = 0.006, size = 18, normalized size = 1.1

$$-\frac{(\cos(x))^5(\sin(x))^2}{7}-\frac{2(\cos(x))^5}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*sin(x)^3,x)
```

```
[Out] -1/7*cos(x)^5*sin(x)^2-2/35*cos(x)^5
```

Maxima [A] time = 0.95728, size = 18, normalized size = 1.06

$$\frac{1}{7}\cos(x)^7-\frac{1}{5}\cos(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/7*cos(x)^7 - 1/5*cos(x)^5
```

Fricas [A] time = 2.21902, size = 39, normalized size = 2.29

$$\frac{1}{7}\cos(x)^7-\frac{1}{5}\cos(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/7*cos(x)^7 - 1/5*cos(x)^5
```

Sympy [A] time = 0.061924, size = 12, normalized size = 0.71

$$\frac{\cos^7(x)}{7}-\frac{\cos^5(x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4*sin(x)**3,x)
```

```
[Out] cos(x)**7/7 - cos(x)**5/5
```

Giac [A] time = 1.0605, size = 18, normalized size = 1.06

$$\frac{1}{7}\cos(x)^7-\frac{1}{5}\cos(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/7*cos(x)^7 - 1/5*cos(x)^5
```

3.63 $\int \cos^3(x)\sin^4(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sin^5(x)}{5}-\frac{\sin^7(x)}{7}$$

```
[Out] Sin[x]^5/5 - Sin[x]^7/7
```

Rubi [A] time = 0.0222875, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2564, 14}

$$\frac{\sin^5(x)}{5}-\frac{\sin^7(x)}{7}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Sin[x]^4,x]
```

```
[Out] Sin[x]^5/5 - Sin[x]^7/7
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\sin^4(x)\,dx &= \text{Subst}\left(\int x^4\left(1-x^2\right)\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \left(x^4-x^6\right)\,dx, x, \sin(x)\right)\\ &= \frac{\sin^5(x)}{5}-\frac{\sin^7(x)}{7}\end{aligned}$$

Mathematica [A] time = 0.0093503, size = 31, normalized size = 1.82

$$\frac{3\sin(x)}{64}-\frac{1}{64}\sin(3x)-\frac{1}{320}\sin(5x)+\frac{1}{448}\sin(7x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Sin[x]^4,x]
```

```
[Out] (3*Sin[x])/64 - Sin[3*x]/64 - Sin[5*x]/320 + Sin[7*x]/448
```

Maple [B] time = 0.004, size = 30, normalized size = 1.8

$$-\frac{(\cos(x))^4(\sin(x))^3}{7}-\frac{3\sin(x)(\cos(x))^4}{35}+\frac{\left(2+(\cos(x))^2\right)\sin(x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^4,x)
```

```
[Out] -1/7*cos(x)^4*sin(x)^3-3/35*sin(x)*cos(x)^4+1/35*(2+cos(x)^2)*sin(x)
```

Maxima [A] time = 0.928444, size = 18, normalized size = 1.06

$$-\frac{1}{7}\sin(x)^7+\frac{1}{5}\sin(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^4,x, algorithm="maxima")
```

```
[Out] -1/7*sin(x)^7 + 1/5*sin(x)^5
```

Fricas [A] time = 2.13228, size = 73, normalized size = 4.29

$$\frac{1}{35}\left(5\cos\left(x\right)^{6}-8\cos\left(x\right)^{4}+\cos\left(x\right)^{2}+2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/35*(5*cos(x)^6 - 8*cos(x)^4 + cos(x)^2 + 2)*sin(x)
```

Sympy [A] time = 0.062694, size = 12, normalized size = 0.71

$$-\frac{\sin^{7}\left(x\right)}{7}+\frac{\sin^{5}\left(x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**4,x)
```

```
[Out] -sin(x)**7/7 + sin(x)**5/5
```

Giac [A] time = 1.04648, size = 18, normalized size = 1.06

$$-\frac{1}{7}\sin\left(x\right)^{7}+\frac{1}{5}\sin\left(x\right)^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^4,x, algorithm="giac")
```

```
[Out] -1/7*sin(x)^7 + 1/5*sin(x)^5
```

3.64 $\int \cos^2(x)\sin^4(x)\,dx$

Optimal. Leaf size=36

$$\frac{x}{16}-\frac{1}{6}\sin^3(x)\cos^3(x)-\frac{1}{8}\sin(x)\cos^3(x)+\frac{1}{16}\sin(x)\cos(x)$$

```
[Out] x/16 + (Cos[x]*Sin[x])/16 - (Cos[x]^3*Sin[x])/8 - (Cos[x]^3*Sin[x]^3)/6
```

Rubi [A] time = 0.0448538, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{x}{16}-\frac{1}{6}\sin^3(x)\cos^3(x)-\frac{1}{8}\sin(x)\cos^3(x)+\frac{1}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sin[x]^4,x]
```

```
[Out] x/16 + (Cos[x]*Sin[x])/16 - (Cos[x]^3*Sin[x])/8 - (Cos[x]^3*Sin[x]^3)/6
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\sin^4(x)\,dx &= -\frac{1}{6}\cos^3(x)\sin^3(x) + \frac{1}{2}\int \cos^2(x)\sin^2(x)\,dx\\ &= -\frac{1}{8}\cos^3(x)\sin(x) - \frac{1}{6}\cos^3(x)\sin^3(x) + \frac{1}{8}\int \cos^2(x)\,dx\\ &= \frac{1}{16}\cos(x)\sin(x) - \frac{1}{8}\cos^3(x)\sin(x) - \frac{1}{6}\cos^3(x)\sin^3(x) + \frac{\int 1\,dx}{16}\\ &= \frac{x}{16} + \frac{1}{16}\cos(x)\sin(x) - \frac{1}{8}\cos^3(x)\sin(x) - \frac{1}{6}\cos^3(x)\sin^3(x)\end{aligned}$$

Mathematica [A] time = 0.0074607, size = 30, normalized size = 0.83

$$\frac{x}{16} - \frac{1}{64}\sin(2x) - \frac{1}{64}\sin(4x) + \frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sin[x]^4,x]
```

```
[Out] x/16 - Sin[2*x]/64 - Sin[4*x]/64 + Sin[6*x]/192
```

Maple [A] time = 0.006, size = 29, normalized size = 0.8

$$\frac{x}{16} + \frac{\cos(x)\sin(x)}{16} - \frac{(\cos(x))^3\sin(x)}{8} - \frac{(\cos(x))^3(\sin(x))^3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^4,x)
```

```
[Out] 1/16*x+1/16*cos(x)*sin(x)-1/8*cos(x)^3*sin(x)-1/6*cos(x)^3*sin(x)^3
```

Maxima [A] time = 0.937497, size = 24, normalized size = 0.67

$$-\frac{1}{48}\sin(2x)^3 + \frac{1}{16}x - \frac{1}{64}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^4,x, algorithm="maxima")
```

```
[Out] -1/48*sin(2*x)^3 + 1/16*x - 1/64*sin(4*x)
```

Fricas [A] time = 2.01846, size = 81, normalized size = 2.25

$$\frac{1}{48}\left(8\,\cos\left(x\right)^{5}-14\,\cos\left(x\right)^{3}+3\,\cos\left(x\right)\right)\sin\left(x\right)+\frac{1}{16}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(8*cos(x)^5 - 14*cos(x)^3 + 3*cos(x))*sin(x) + 1/16*x
```

Sympy [A] time = 0.061066, size = 31, normalized size = 0.86

$$\frac{x}{16}+\frac{\sin^{5}\left(x\right)\cos\left(x\right)}{6}-\frac{\sin^{3}\left(x\right)\cos\left(x\right)}{24}-\frac{\sin\left(x\right)\cos\left(x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**4,x)
```

```
[Out] x/16 + sin(x)**5*cos(x)/6 - sin(x)**3*cos(x)/24 - sin(x)*cos(x)/16
```

Giac [A] time = 1.06472, size = 30, normalized size = 0.83

$$\frac{1}{16}\,x+\frac{1}{192}\,\sin\left(6\,x\right)-\frac{1}{64}\,\sin\left(4\,x\right)-\frac{1}{64}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^4,x, algorithm="giac")
```

```
[Out] 1/16*x + 1/192*sin(6*x) - 1/64*sin(4*x) - 1/64*sin(2*x)
```

3.65 $\int \cos^2(x)\sin^2(x)\,dx$

Optimal. Leaf size=24

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rubi [A] time = 0.0248675, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\sin^2(x)\,dx &= -\frac{1}{4}\cos^3(x)\sin(x) + \frac{1}{4}\int \cos^2(x)\,dx\\ &= \frac{1}{8}\cos(x)\sin(x) - \frac{1}{4}\cos^3(x)\sin(x) + \frac{\int 1\,dx}{8}\\ &= \frac{x}{8} + \frac{1}{8}\cos(x)\sin(x) - \frac{1}{4}\cos^3(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0037463, size = 14, normalized size = 0.58

$$\frac{x}{8} - \frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 - Sin[4*x]/32
```

Maple [A] time = 0., size = 19, normalized size = 0.8

$$\frac{x}{8} + \frac{\cos(x)\sin(x)}{8} - \frac{(\cos(x))^3\sin(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2,x)
```

```
[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

Maxima [A] time = 0.925287, size = 14, normalized size = 0.58

$$\frac{1}{8}x - \frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

Fricas [A] time = 2.03853, size = 58, normalized size = 2.42

$$-\frac{1}{8}\left(2\,\cos\left(x\right)^{3}-\cos\left(x\right)\right)\sin\left(x\right)+\frac{1}{8}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

Sympy [A] time = 0.065533, size = 14, normalized size = 0.58

$$\frac{x}{8}-\frac{\sin\left(2x\right)\cos\left(2x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**2,x)
```

```
[Out] x/8 - sin(2*x)*cos(2*x)/16
```

Giac [A] time = 1.05108, size = 14, normalized size = 0.58

$$\frac{1}{8}\,x-\frac{1}{32}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

3.66 $\int (1-\sin(2x))^2\,dx$

Optimal. Leaf size=22

$$\frac{3x}{2}+\cos(2x)-\frac{1}{4}\sin(2x)\cos(2x)$$

```
[Out] (3*x)/2 + Cos[2*x] - (Cos[2*x]*Sin[2*x])/4
```

Rubi [A] time = 0.0090706, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2644}

$$\frac{3x}{2}+\cos(2x)-\frac{1}{4}\sin(2x)\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[2*x])^2,x]
```

```
[Out] (3*x)/2 + Cos[2*x] - (Cos[2*x]*Sin[2*x])/4
```

Rule 2644

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[((2*a^2 + b
^2)*x)/2, x] + (-Simp[(2*a*b*Cos[c + d*x])/d, x] - Simp[(b^2*Cos[c + d*x]*S
in[c + d*x])/(2*d), x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\int (1-\sin(2x))^2\,dx=\frac{3x}{2}+\cos(2x)-\frac{1}{4}\cos(2x)\sin(2x)$$

Mathematica [A] time = 0.0077577, size = 18, normalized size = 0.82

$$\frac{3x}{2}-\frac{1}{8}\sin(4x)+\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[2*x])^2,x]
```

```
[Out] (3*x)/2 + Cos[2*x] - Sin[4*x]/8
```

Maple [A] time = 0.023, size = 19, normalized size = 0.9

$$\frac{3x}{2} + \cos(2x) - \frac{\cos(2x)\sin(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(2*x))^2,x)
```

```
[Out] 3/2*x+cos(2*x)-1/4*cos(2*x)*sin(2*x)
```

Maxima [A] time = 0.935611, size = 19, normalized size = 0.86

$$\frac{3}{2}x + \cos(2x) - \frac{1}{8}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2*x))^2,x, algorithm="maxima")
```

```
[Out] 3/2*x + cos(2*x) - 1/8*sin(4*x)
```

Fricas [A] time = 1.97602, size = 58, normalized size = 2.64

$$-\frac{1}{4}\cos(2x)\sin(2x) + \frac{3}{2}x + \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2*x))^2,x, algorithm="fricas")
```

```
[Out] -1/4*cos(2*x)*sin(2*x) + 3/2*x + cos(2*x)
```

Sympy [A] time = 0.17712, size = 37, normalized size = 1.68

$$\frac{x\sin^2(2x)}{2} + \frac{x\cos^2(2x)}{2} + x - \frac{\sin(2x)\cos(2x)}{4} + \cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2*x))**2,x)
```

```
[Out] x*sin(2*x)**2/2 + x*cos(2*x)**2/2 + x - sin(2*x)*cos(2*x)/4 + cos(2*x)
```

Giac [A] time = 1.0555, size = 19, normalized size = 0.86

$$\frac{3}{2}\,x + \cos(2\,x) - \frac{1}{8}\,\sin(4\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(2*x))^2,x, algorithm="giac")
```

```
[Out] 3/2*x + cos(2*x) - 1/8*sin(4*x)
```

3.67 $\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx$

Optimal. Leaf size=20

$$\frac{x}{4}-\frac{1}{4}\cos\left(2x+\frac{\pi}{6}\right)$$

```
[Out] x/4 - Cos[Pi/6 + 2*x]/4
```

Rubi [A] time = 0.0144671, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4574, 2638}

$$\frac{x}{4}-\frac{1}{4}\cos\left(2x+\frac{\pi}{6}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[Pi/6 + x],x]
```

```
[Out] x/4 - Cos[Pi/6 + 2*x]/4
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin\left(\frac{\pi}{6}+x\right)\,dx &= \int \left(\frac{1}{4}+\frac{1}{2}\sin\left(\frac{\pi}{6}+2x\right)\right)\,dx\\ &= \frac{x}{4}+\frac{1}{2}\int \sin\left(\frac{\pi}{6}+2x\right)\,dx\\ &= \frac{x}{4}-\frac{1}{4}\cos\left(\frac{\pi}{6}+2x\right)\end{aligned}$$

Mathematica [A] time = 0.0108803, size = 20, normalized size = 1.

$$\frac{x}{4}-\frac{1}{4}\cos\left(2x+\frac{\pi}{6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[Pi/6 + x],x]
```

```
[Out] x/4 - Cos[Pi/6 + 2*x]/4
```

Maple [A] time = 0.029, size = 15, normalized size = 0.8

$$\frac{x}{4}-\frac{1}{4}\cos\left(\frac{\pi}{6}+2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(1/6*Pi+x),x)
```

```
[Out] 1/4*x-1/4*cos(1/6*Pi+2*x)
```

Maxima [A] time = 0.932546, size = 19, normalized size = 0.95

$$\frac{1}{4}\,x-\frac{1}{4}\,\cos\left(\frac{1}{6}\,\pi+2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="maxima")
```

```
[Out] 1/4*x - 1/4*cos(1/6*pi + 2*x)
```

Fricas [B] time = 1.98278, size = 105, normalized size = 5.25

$$-\frac{1}{4}\,\sqrt{3}\cos\left(\frac{1}{6}\,\pi+x\right)^{2}-\frac{1}{4}\,\cos\left(\frac{1}{6}\,\pi+x\right)\sin\left(\frac{1}{6}\,\pi+x\right)+\frac{1}{4}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(3)*cos(1/6*pi + x)^2 - 1/4*cos(1/6*pi + x)*sin(1/6*pi + x) + 1/4*
x
```

Sympy [B] time = 0.544632, size = 37, normalized size = 1.85

$$-\frac{x\sin(x)\cos\left(x+\frac{\pi}{6}\right)}{2}+\frac{x\sin\left(x+\frac{\pi}{6}\right)\cos(x)}{2}-\frac{\cos(x)\cos\left(x+\frac{\pi}{6}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x)
```

```
[Out] -x*sin(x)*cos(x + pi/6)/2 + x*sin(x + pi/6)*cos(x)/2 - cos(x)*cos(x + pi/6)
/2
```

Giac [A] time = 1.06048, size = 19, normalized size = 0.95

$$\frac{1}{4}x-\frac{1}{4}\cos\left(\frac{1}{6}\pi+2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(1/6*pi+x),x, algorithm="giac")
```

```
[Out] 1/4*x - 1/4*cos(1/6*pi + 2*x)
```

3.68 $\int \cos^5(x) \sin^5(x)\, dx$

Optimal. Leaf size=25

$$\frac{\sin^{10}(x)}{10} - \frac{\sin^8(x)}{4} + \frac{\sin^6(x)}{6}$$

```
[Out] Sin[x]^6/6 - Sin[x]^8/4 + Sin[x]^10/10
```

Rubi [A] time = 0.0299897, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2564, 266, 43}

$$\frac{\sin^{10}(x)}{10} - \frac{\sin^8(x)}{4} + \frac{\sin^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5*Sin[x]^5,x]
```

```
[Out] Sin[x]^6/6 - Sin[x]^8/4 + Sin[x]^10/10
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cos^5(x)\sin^5(x)\,dx &= \text{Subst}\left(\int x^5\left(1-x^2\right)^2\,dx,x,\sin(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int (1-x)^2x^2\,dx,x,\sin^2(x)\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(x^2-2x^3+x^4\right)\,dx,x,\sin^2(x)\right)\\
&= \frac{\sin^6(x)}{6}-\frac{\sin^8(x)}{4}+\frac{\sin^{10}(x)}{10}
\end{aligned}
$$

Mathematica [A] time = 0.0102496, size = 25, normalized size = 1.

$$
-\frac{5}{512}\cos(2x)+\frac{5\cos(6x)}{3072}-\frac{\cos(10x)}{5120}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5*Sin[x]^5,x]
```

```
[Out] (-5*Cos[2*x])/512 + (5*Cos[6*x])/3072 - Cos[10*x]/5120
```

Maple [A] time = 0.007, size = 28, normalized size = 1.1

$$
-\frac{(\cos(x))^6(\sin(x))^4}{10}-\frac{(\sin(x))^2(\cos(x))^6}{20}-\frac{(\cos(x))^6}{60}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5*sin(x)^5,x)
```

```
[Out] -1/10*cos(x)^6*sin(x)^4-1/20*sin(x)^2*cos(x)^6-1/60*cos(x)^6
```

Maxima [A] time = 0.927547, size = 26, normalized size = 1.04

$$
\frac{1}{10}\sin(x)^{10}-\frac{1}{4}\sin(x)^8+\frac{1}{6}\sin(x)^6
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5*sin(x)^5,x, algorithm="maxima")
```

```
[Out] 1/10*sin(x)^10 - 1/4*sin(x)^8 + 1/6*sin(x)^6
```

Fricas [A] time = 2.00251, size = 63, normalized size = 2.52

$$-\frac{1}{10}\cos\left(x\right)^{10}+\frac{1}{4}\cos\left(x\right)^{8}-\frac{1}{6}\cos\left(x\right)^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5*sin(x)^5,x, algorithm="fricas")
```

```
[Out] -1/10*cos(x)^10 + 1/4*cos(x)^8 - 1/6*cos(x)^6
```

Sympy [A] time = 0.067664, size = 19, normalized size = 0.76

$$\frac{\sin^{10}\left(x\right)}{10}-\frac{\sin^{8}\left(x\right)}{4}+\frac{\sin^{6}\left(x\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5*sin(x)**5,x)
```

```
[Out] sin(x)**10/10 - sin(x)**8/4 + sin(x)**6/6
```

Giac [A] time = 1.05139, size = 26, normalized size = 1.04

$$-\frac{1}{10}\cos\left(x\right)^{10}+\frac{1}{4}\cos\left(x\right)^{8}-\frac{1}{6}\cos\left(x\right)^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5*sin(x)^5,x, algorithm="giac")
```

```
[Out] -1/10*cos(x)^10 + 1/4*cos(x)^8 - 1/6*cos(x)^6
```

3.69 $\int \sin^6(x)\,dx$

Optimal. Leaf size=34

$$\frac{5x}{16}-\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{24}\sin^3(x)\cos(x)-\frac{5}{16}\sin(x)\cos(x)$$

```
[Out] (5*x)/16 - (5*Cos[x]*Sin[x])/16 - (5*Cos[x]*Sin[x]^3)/24 - (Cos[x]*Sin[x]^5
)/6
```

Rubi [A] time = 0.0167524, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{5x}{16}-\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{24}\sin^3(x)\cos(x)-\frac{5}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^6,x]
```


```
[Out] (5*x)/16 - (5*Cos[x]*Sin[x])/16 - (5*Cos[x]*Sin[x]^3)/24 - (Cos[x]*Sin[x]^5
)/6
```


Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^6(x)\,dx &= -\frac{1}{6}\cos(x)\sin^5(x) + \frac{5}{6}\int \sin^4(x)\,dx \\ &= -\frac{5}{24}\cos(x)\sin^3(x) - \frac{1}{6}\cos(x)\sin^5(x) + \frac{5}{8}\int \sin^2(x)\,dx \\ &= -\frac{5}{16}\cos(x)\sin(x) - \frac{5}{24}\cos(x)\sin^3(x) - \frac{1}{6}\cos(x)\sin^5(x) + \frac{5\int 1\,dx}{16} \\ &= \frac{5x}{16} - \frac{5}{16}\cos(x)\sin(x) - \frac{5}{24}\cos(x)\sin^3(x) - \frac{1}{6}\cos(x)\sin^5(x)\end{aligned}$$

Mathematica [A] time = 0.0021813, size = 30, normalized size = 0.88

$$\frac{5x}{16} - \frac{15}{64}\sin(2x) + \frac{3}{64}\sin(4x) - \frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^6,x]
```

```
[Out] (5*x)/16 - (15*Sin[2*x])/64 + (3*Sin[4*x])/64 - Sin[6*x]/192
```

Maple [A] time = 0., size = 24, normalized size = 0.7

$$-\frac{\cos(x)}{6}\left((\sin(x))^5 + \frac{5\,(\sin(x))^3}{4} + \frac{15\,\sin(x)}{8}\right) + \frac{5\,x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6,x)
```

```
[Out] -1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x
```

Maxima [A] time = 0.924352, size = 32, normalized size = 0.94

$$\frac{1}{48}\,\sin(2\,x)^3 + \frac{5}{16}\,x + \frac{3}{64}\,\sin(4\,x) - \frac{1}{4}\,\sin(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6,x, algorithm="maxima")
```

```
[Out] 1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) - 1/4*sin(2*x)
```

Fricas [A] time = 1.99774, size = 84, normalized size = 2.47

$$-\frac{1}{48}\left(8\cos\left(x\right)^{5}-26\cos\left(x\right)^{3}+33\cos\left(x\right)\right)\sin\left(x\right)+\frac{5}{16}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(x)^5 - 26*cos(x)^3 + 33*cos(x))*sin(x) + 5/16*x
```

Sympy [A] time = 0.061398, size = 36, normalized size = 1.06

$$\frac{5x}{16}-\frac{\sin^{5}\left(x\right)\cos\left(x\right)}{6}-\frac{5\sin^{3}\left(x\right)\cos\left(x\right)}{24}-\frac{5\sin\left(x\right)\cos\left(x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**6,x)
```

```
[Out] 5*x/16 - sin(x)**5*cos(x)/6 - 5*sin(x)**3*cos(x)/24 - 5*sin(x)*cos(x)/16
```

Giac [A] time = 1.05327, size = 30, normalized size = 0.88

$$\frac{5}{16}\,x-\frac{1}{192}\,\sin\left(6\,x\right)+\frac{3}{64}\,\sin\left(4\,x\right)-\frac{15}{64}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6,x, algorithm="giac")
```

```
[Out] 5/16*x - 1/192*sin(6*x) + 3/64*sin(4*x) - 15/64*sin(2*x)
```

3.70 $\int \cos^6(x)\,dx$

Optimal. Leaf size=34

$$\frac{5x}{16}+\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{24}\sin(x)\cos^3(x)+\frac{5}{16}\sin(x)\cos(x)$$

```
[Out] (5*x)/16 + (5*Cos[x]*Sin[x])/16 + (5*Cos[x]^3*Sin[x])/24 + (Cos[x]^5*Sin[x]
)/6
```

Rubi [A] time = 0.0168791, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{5x}{16}+\frac{1}{6}\sin(x)\cos^5(x)+\frac{5}{24}\sin(x)\cos^3(x)+\frac{5}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6,x]
```

```
[Out] (5*x)/16 + (5*Cos[x]*Sin[x])/16 + (5*Cos[x]^3*Sin[x])/24 + (Cos[x]^5*Sin[x]
)/6
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^6(x)\,dx &= \frac{1}{6}\cos^5(x)\sin(x) + \frac{5}{6}\int \cos^4(x)\,dx\\ &= \frac{5}{24}\cos^3(x)\sin(x) + \frac{1}{6}\cos^5(x)\sin(x) + \frac{5}{8}\int \cos^2(x)\,dx\\ &= \frac{5}{16}\cos(x)\sin(x) + \frac{5}{24}\cos^3(x)\sin(x) + \frac{1}{6}\cos^5(x)\sin(x) + \frac{5\int 1\,dx}{16}\\ &= \frac{5x}{16} + \frac{5}{16}\cos(x)\sin(x) + \frac{5}{24}\cos^3(x)\sin(x) + \frac{1}{6}\cos^5(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0018655, size = 30, normalized size = 0.88

$$\frac{5x}{16} + \frac{15}{64}\sin(2x) + \frac{3}{64}\sin(4x) + \frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6,x]
```

```
[Out] (5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192
```

Maple [A] time = 0.029, size = 24, normalized size = 0.7

$$\frac{\sin(x)}{6}\left((\cos(x))^5 + \frac{5\,(\cos(x))^3}{4} + \frac{15\,\cos(x)}{8}\right) + \frac{5\,x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6,x)
```

```
[Out] 1/6*(cos(x)^5+5/4*cos(x)^3+15/8*cos(x))*sin(x)+5/16*x
```

Maxima [A] time = 0.927446, size = 32, normalized size = 0.94

$$-\frac{1}{48}\sin(2\,x)^3 + \frac{5}{16}\,x + \frac{3}{64}\sin(4\,x) + \frac{1}{4}\sin(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6,x, algorithm="maxima")
```

```
[Out] -1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) + 1/4*sin(2*x)
```

Fricas [A] time = 2.05129, size = 82, normalized size = 2.41

$$\frac{1}{48}\left(8\cos(x)^5+10\cos(x)^3+15\cos(x)\right)\sin(x)+\frac{5}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6,x, algorithm="fricas")
```

```
[Out] 1/48*(8*cos(x)^5 + 10*cos(x)^3 + 15*cos(x))*sin(x) + 5/16*x
```

Sympy [A] time = 0.057272, size = 36, normalized size = 1.06

$$\frac{5x}{16}+\frac{\sin(x)\cos^5(x)}{6}+\frac{5\sin(x)\cos^3(x)}{24}+\frac{5\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6,x)
```

```
[Out] 5*x/16 + sin(x)*cos(x)**5/6 + 5*sin(x)*cos(x)**3/24 + 5*sin(x)*cos(x)/16
```

Giac [A] time = 1.04698, size = 30, normalized size = 0.88

$$\frac{5}{16}x+\frac{1}{192}\sin(6x)+\frac{3}{64}\sin(4x)+\frac{15}{64}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6,x, algorithm="giac")
```

```
[Out] 5/16*x + 1/192*sin(6*x) + 3/64*sin(4*x) + 15/64*sin(2*x)
```

3.71 $\int \cos^4(2x)\sin^2(2x)\,dx$

Optimal. Leaf size=46

$$\frac{x}{16}-\frac{1}{12}\sin(2x)\cos^5(2x)+\frac{1}{48}\sin(2x)\cos^3(2x)+\frac{1}{32}\sin(2x)\cos(2x)$$

```
[Out] x/16 + (Cos[2*x]*Sin[2*x])/32 + (Cos[2*x]^3*Sin[2*x])/48 - (Cos[2*x]^5*Sin[
2*x])/12
```

Rubi [A] time = 0.0360118, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2568, 2635, 8}

$$\frac{x}{16}-\frac{1}{12}\sin(2x)\cos^5(2x)+\frac{1}{48}\sin(2x)\cos^3(2x)+\frac{1}{32}\sin(2x)\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]^4*Sin[2*x]^2,x]
```

```
[Out] x/16 + (Cos[2*x]*Sin[2*x])/32 + (Cos[2*x]^3*Sin[2*x])/48 - (Cos[2*x]^5*Sin[
2*x])/12
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^4(2x)\sin^2(2x)\,dx &= -\frac{1}{12}\cos^5(2x)\sin(2x) + \frac{1}{6}\int \cos^4(2x)\,dx \\
&= \frac{1}{48}\cos^3(2x)\sin(2x) - \frac{1}{12}\cos^5(2x)\sin(2x) + \frac{1}{8}\int \cos^2(2x)\,dx \\
&= \frac{1}{32}\cos(2x)\sin(2x) + \frac{1}{48}\cos^3(2x)\sin(2x) - \frac{1}{12}\cos^5(2x)\sin(2x) + \frac{\int 1\,dx}{16} \\
&= \frac{x}{16} + \frac{1}{32}\cos(2x)\sin(2x) + \frac{1}{48}\cos^3(2x)\sin(2x) - \frac{1}{12}\cos^5(2x)\sin(2x)
\end{aligned}
$$

Mathematica [A] time = 0.0138663, size = 30, normalized size = 0.65

$$\frac{x}{16} + \frac{1}{128}\sin(4x) - \frac{1}{128}\sin(8x) - \frac{1}{384}\sin(12x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]^4*Sin[2*x]^2,x]
```

```
[Out] x/16 + Sin[4*x]/128 - Sin[8*x]/128 - Sin[12*x]/384
```

Maple [A] time = 0.011, size = 36, normalized size = 0.8

$$-\frac{(\cos(2x))^5 \sin(2x)}{12} + \frac{\sin(2x)}{48}\left((\cos(2x))^3 + \frac{3\cos(2x)}{2}\right) + \frac{x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)^4*sin(2*x)^2,x)
```

```
[Out] -1/12*cos(2*x)^5*sin(2*x)+1/48*(cos(2*x)^3+3/2*cos(2*x))*sin(2*x)+1/16*x
```

Maxima [A] time = 0.938778, size = 24, normalized size = 0.52

$$\frac{1}{96}\sin(4x)^3 + \frac{1}{16}x - \frac{1}{128}\sin(8x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^4*sin(2*x)^2,x, algorithm="maxima")
```

```
[Out] 1/96*sin(4*x)^3 + 1/16*x - 1/128*sin(8*x)
```

Fricas [A] time = 1.99528, size = 92, normalized size = 2.

$$-\frac{1}{96}\left(8\cos(2x)^5-2\cos(2x)^3-3\cos(2x)\right)\sin(2x)+\frac{1}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^4*sin(2*x)^2,x, algorithm="fricas")
```

```
[Out] -1/96*(8*cos(2*x)^5 - 2*cos(2*x)^3 - 3*cos(2*x))*sin(2*x) + 1/16*x
```

Sympy [A] time = 0.061795, size = 41, normalized size = 0.89

$$\frac{x}{16}-\frac{\sin(2x)\cos^5(2x)}{12}+\frac{\sin(2x)\cos^3(2x)}{48}+\frac{\sin(2x)\cos(2x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)**4*sin(2*x)**2,x)
```

```
[Out] x/16 - sin(2*x)*cos(2*x)**5/12 + sin(2*x)*cos(2*x)**3/48 + sin(2*x)*cos(2*x
)/32
```

Giac [A] time = 1.05166, size = 30, normalized size = 0.65

$$\frac{1}{16}x-\frac{1}{384}\sin(12x)-\frac{1}{128}\sin(8x)+\frac{1}{128}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)^4*sin(2*x)^2,x, algorithm="giac")
```

```
[Out] 1/16*x - 1/384*sin(12*x) - 1/128*sin(8*x) + 1/128*sin(4*x)
```

3.72 $\int \sin^5(x)\,dx$

Optimal. Leaf size=21

$$-\frac{1}{5}\cos^5(x)+\frac{2\cos^3(x)}{3}-\cos(x)$$

```
[Out] -Cos[x] + (2*Cos[x]^3)/3 - Cos[x]^5/5
```

Rubi [A] time = 0.0067568, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2633}

$$-\frac{1}{5}\cos^5(x)+\frac{2\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5,x]
```

```
[Out] -Cos[x] + (2*Cos[x]^3)/3 - Cos[x]^5/5
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^5(x)\,dx &= -\text{Subst}\left(\int \left(1-2x^2+x^4\right)dx, x, \cos(x)\right)\\ &= -\cos(x)+\frac{2\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.0017605, size = 23, normalized size = 1.1

$$-\frac{5\cos(x)}{8}+\frac{5}{48}\cos(3x)-\frac{1}{80}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5,x]
```

```
[Out] (-5*Cos[x])/8 + (5*Cos[3*x])/48 - Cos[5*x]/80
```

Maple [A] time = 0.004, size = 17, normalized size = 0.8

$$-\frac{\cos(x)}{5}\left(\frac{8}{3}+(\sin(x))^4+\frac{4\,(\sin(x))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5,x)
```

```
[Out] -1/5*(8/3+sin(x)^4+4/3*sin(x)^2)*cos(x)
```

Maxima [A] time = 0.928642, size = 23, normalized size = 1.1

$$-\frac{1}{5}\cos(x)^5+\frac{2}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5,x, algorithm="maxima")
```

```
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

Fricas [A] time = 1.97677, size = 53, normalized size = 2.52

$$-\frac{1}{5}\cos(x)^5+\frac{2}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5,x, algorithm="fricas")
```

```
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

Sympy [A] time = 0.06298, size = 17, normalized size = 0.81

$$-\frac{\cos^5(x)}{5} + \frac{2\cos^3(x)}{3} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**5,x)
```

```
[Out] -cos(x)**5/5 + 2*cos(x)**3/3 - cos(x)
```

Giac [A] time = 1.04548, size = 23, normalized size = 1.1

$$-\frac{1}{5}\cos(x)^5 + \frac{2}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5,x, algorithm="giac")
```

```
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

3.73 $\int \cos^4(x)\sin^4(x)\,dx$

Optimal. Leaf size=46

$$\frac{3x}{128}-\frac{1}{8}\sin^3(x)\cos^5(x)-\frac{1}{16}\sin(x)\cos^5(x)+\frac{1}{64}\sin(x)\cos^3(x)+\frac{3}{128}\sin(x)\cos(x)$$

```
[Out] (3*x)/128 + (3*Cos[x]*Sin[x])/128 + (Cos[x]^3*Sin[x])/64 - (Cos[x]^5*Sin[x]
)/16 - (Cos[x]^5*Sin[x]^3)/8
```

Rubi [A] time = 0.0516621, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{3x}{128}-\frac{1}{8}\sin^3(x)\cos^5(x)-\frac{1}{16}\sin(x)\cos^5(x)+\frac{1}{64}\sin(x)\cos^3(x)+\frac{3}{128}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4*Sin[x]^4,x]
```

```
[Out] (3*x)/128 + (3*Cos[x]*Sin[x])/128 + (Cos[x]^3*Sin[x])/64 - (Cos[x]^5*Sin[x]
)/16 - (Cos[x]^5*Sin[x]^3)/8
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \cos^4(x)\sin^4(x)\,dx &= -\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{3}{8}\int \cos^4(x)\sin^2(x)\,dx\\
&= -\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{1}{16}\int \cos^4(x)\,dx\\
&= \frac{1}{64}\cos^3(x)\sin(x)-\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{3}{64}\int \cos^2(x)\,dx\\
&= \frac{3}{128}\cos(x)\sin(x)+\frac{1}{64}\cos^3(x)\sin(x)-\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)+\frac{3\int 1\,dx}{128}\\
&= \frac{3x}{128}+\frac{3}{128}\cos(x)\sin(x)+\frac{1}{64}\cos^3(x)\sin(x)-\frac{1}{16}\cos^5(x)\sin(x)-\frac{1}{8}\cos^5(x)\sin^3(x)
\end{aligned}
$$

Mathematica [A] time = 0.0066726, size = 22, normalized size = 0.48

$$\frac{3x}{128}-\frac{1}{128}\sin(4x)+\frac{\sin(8x)}{1024}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4*Sin[x]^4,x]
```

```
[Out] (3*x)/128 - Sin[4*x]/128 + Sin[8*x]/1024
```

Maple [A] time = 0.005, size = 36, normalized size = 0.8

$$-\frac{(\cos(x))^5(\sin(x))^3}{8}-\frac{(\cos(x))^5\sin(x)}{16}+\frac{\sin(x)}{64}\left((\cos(x))^3+\frac{3\cos(x)}{2}\right)+\frac{3x}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*sin(x)^4,x)
```

```
[Out] -1/8*cos(x)^5*sin(x)^3-1/16*cos(x)^5*sin(x)+1/64*(cos(x)^3+3/2*cos(x))*sin(
x)+3/128*x
```

Maxima [A] time = 0.929238, size = 22, normalized size = 0.48

$$\frac{3}{128}\,x + \frac{1}{1024}\,\sin\left(8\,x\right) - \frac{1}{128}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="maxima")
```

```
[Out] 3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

Fricas [A] time = 2.10534, size = 103, normalized size = 2.24

$$\frac{1}{128}\,\left(16\,\cos\left(x\right)^{7} - 24\,\cos\left(x\right)^{5} + 2\,\cos\left(x\right)^{3} + 3\,\cos\left(x\right)\right)\sin\left(x\right) + \frac{3}{128}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(16*cos(x)^7 - 24*cos(x)^5 + 2*cos(x)^3 + 3*cos(x))*sin(x) + 3/128*x
```

Sympy [A] time = 0.067331, size = 31, normalized size = 0.67

$$\frac{3x}{128} - \frac{\sin^{3}\left(2x\right)\cos\left(2x\right)}{128} - \frac{3\sin\left(2x\right)\cos\left(2x\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4*sin(x)**4,x)
```

```
[Out] 3*x/128 - sin(2*x)**3*cos(2*x)/128 - 3*sin(2*x)*cos(2*x)/256
```

Giac [A] time = 1.04786, size = 22, normalized size = 0.48

$$\frac{3}{128}\,x + \frac{1}{1024}\,\sin\left(8\,x\right) - \frac{1}{128}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/128*x + 1/1024*sin(8*x) - 1/128*sin(4*x)
```

3.74 $\int \sqrt{\cos(x)} \sin^3(x)\,dx$

Optimal. Leaf size=21

$$\frac{2}{7}\cos^{\frac{7}{2}}(x)-\frac{2}{3}\cos^{\frac{3}{2}}(x)$$

```
[Out] (-2*Cos[x]^(3/2))/3 + (2*Cos[x]^(7/2))/7
```

Rubi [A] time = 0.0224993, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2565, 14}

$$\frac{2}{7}\cos^{\frac{7}{2}}(x)-\frac{2}{3}\cos^{\frac{3}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[x]]*Sin[x]^3,x]
```

```
[Out] (-2*Cos[x]^(3/2))/3 + (2*Cos[x]^(7/2))/7
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(x)}\sin^3(x)\,dx &= -\operatorname{Subst}\left(\int \sqrt{x}\left(1-x^2\right)\,dx, x, \cos(x)\right)\\ &= -\operatorname{Subst}\left(\int \left(\sqrt{x}-x^{5/2}\right)\,dx, x, \cos(x)\right)\\ &= -\frac{2}{3}\cos^{\frac{3}{2}}(x)+\frac{2}{7}\cos^{\frac{7}{2}}(x)\end{aligned}$$

Mathematica [A] time = 0.0634669, size = 34, normalized size = 1.62

$$\frac{(3\cos(2x)-11)\cos^2(x)+8\sqrt[4]{\cos^2(x)}}{21\sqrt{\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[x]]*Sin[x]^3,x]
```

```
[Out] (8*(Cos[x]^2)^(1/4) + Cos[x]^2*(-11 + 3*Cos[2*x]))/(21*Sqrt[Cos[x]])
```

Maple [B] time = 0.033, size = 39, normalized size = 1.9

$$-\frac{8}{21}\sqrt{-2\,(\sin(x/2))^2+1}\left(6\,(\sin(x/2))^6-9\,(\sin(x/2))^4+\left(\sin\left(\frac{x}{2}\right)\right)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3*cos(x)^(1/2),x)
```

```
[Out] -8/21*(-2*sin(1/2*x)^2+1)^(1/2)*(6*sin(1/2*x)^6-9*sin(1/2*x)^4+sin(1/2*x)^2
+1)
```

Maxima [A] time = 0.929643, size = 18, normalized size = 0.86

$$\frac{2}{7}\cos(x)^{\frac{7}{2}}-\frac{2}{3}\cos(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3*cos(x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*cos(x)^(7/2) - 2/3*cos(x)^(3/2)
```

Fricas [A] time = 2.06191, size = 58, normalized size = 2.76

$$\frac{2}{21}\left(3\,\cos\left(x\right)^{3}-7\,\cos\left(x\right)\right)\sqrt{\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3*cos(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*cos(x)^3 - 7*cos(x))*sqrt(cos(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3*cos(x)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.05324, size = 18, normalized size = 0.86

$$\frac{2}{7}\,\cos\left(x\right)^{\frac{7}{2}}-\frac{2}{3}\,\cos\left(x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3*cos(x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*cos(x)^(7/2) - 2/3*cos(x)^(3/2)
```

3.75 $\int \cos^3(x)\sqrt{\sin(x)}\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}\sin^{\frac{3}{2}}(x)-\frac{2}{7}\sin^{\frac{7}{2}}(x)$$

```
[Out] (2*Sin[x]^(3/2))/3 - (2*Sin[x]^(7/2))/7
```

Rubi [A] time = 0.0224468, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2564, 14}

$$\frac{2}{3}\sin^{\frac{3}{2}}(x)-\frac{2}{7}\sin^{\frac{7}{2}}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Sqrt[Sin[x]],x]
```

```
[Out] (2*Sin[x]^(3/2))/3 - (2*Sin[x]^(7/2))/7
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^3(x)\sqrt{\sin(x)}\,dx = \text{Subst}\left(\int \sqrt{x}\left(1-x^2\right)\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(\sqrt{x}-x^{5/2}\right)\,dx, x, \sin(x)\right)$$
$$= \frac{2}{3}\sin^{\frac{3}{2}}(x) - \frac{2}{7}\sin^{\frac{7}{2}}(x)$$

Mathematica [A] time = 0.0111494, size = 18, normalized size = 0.86

$$\frac{1}{21}\sin^{\frac{3}{2}}(x)(3\cos(2x)+11)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Sqrt[Sin[x]],x]
```

```
[Out] ((11 + 3*Cos[2*x])*Sin[x]^(3/2))/21
```

Maple [A] time = 0.02, size = 14, normalized size = 0.7

$$\frac{2}{3}(\sin(x))^{\frac{3}{2}} - \frac{2}{7}(\sin(x))^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^(1/2),x)
```

```
[Out] 2/3*sin(x)^(3/2)-2/7*sin(x)^(7/2)
```

Maxima [A] time = 0.933042, size = 18, normalized size = 0.86

$$-\frac{2}{7}\sin(x)^{\frac{7}{2}} + \frac{2}{3}\sin(x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/7*sin(x)^(7/2) + 2/3*sin(x)^(3/2)
```

Fricas [A] time = 2.06587, size = 49, normalized size = 2.33

$$\frac{2}{21} \left(3 \cos\left(x\right)^{2} + 4\right) \sin\left(x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/21*(3*cos(x)^2 + 4)*sin(x)^(3/2)
```

Sympy [B] time = 42.1372, size = 167, normalized size = 7.95

$$\frac{28\sqrt{2}\sqrt{\frac{1}{\tan^{2}\left(\frac{x}{2}\right)+1}}\tan^{\frac{11}{2}}\left(\frac{x}{2}\right)}{21\tan^{6}\left(\frac{x}{2}\right)+63\tan^{4}\left(\frac{x}{2}\right)+63\tan^{2}\left(\frac{x}{2}\right)+21}+\frac{8\sqrt{2}\sqrt{\frac{1}{\tan^{2}\left(\frac{x}{2}\right)+1}}\tan^{\frac{7}{2}}\left(\frac{x}{2}\right)}{21\tan^{6}\left(\frac{x}{2}\right)+63\tan^{4}\left(\frac{x}{2}\right)+63\tan^{2}\left(\frac{x}{2}\right)+21}+\frac{28\sqrt{2}\sqrt{\frac{1}{\tan^{2}\left(\frac{x}{2}\right.}}}{21\tan^{6}\left(\frac{x}{2}\right)+63\tan^{4}\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**(1/2),x)
```

```
[Out] 28*sqrt(2)*sqrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(11/2)/(21*tan(x/2)**6 + 63*
tan(x/2)**4 + 63*tan(x/2)**2 + 21) + 8*sqrt(2)*sqrt(1/(tan(x/2)**2 + 1))*ta
n(x/2)**(7/2)/(21*tan(x/2)**6 + 63*tan(x/2)**4 + 63*tan(x/2)**2 + 21) + 28*
sqrt(2)*sqrt(1/(tan(x/2)**2 + 1))*tan(x/2)**(3/2)/(21*tan(x/2)**6 + 63*tan(
x/2)**4 + 63*tan(x/2)**2 + 21)
```

Giac [A] time = 1.04997, size = 18, normalized size = 0.86

$$-\frac{2}{7} \sin\left(x\right)^{\frac{7}{2}} + \frac{2}{3} \sin\left(x\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^(1/2),x, algorithm="giac")
```

```
[Out] -2/7*sin(x)^(7/2) + 2/3*sin(x)^(3/2)
```

3.76 $\int \frac{\cos^2(\sqrt{x})}{\sqrt{x}}\,dx$

Optimal. Leaf size=19

$$\sqrt{x}+\sin\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)$$

```
[Out] Sqrt[x] + Cos[Sqrt[x]]*Sin[Sqrt[x]]
```

Rubi [A] time = 0.019536, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3380, 2635, 8}

$$\sqrt{x}+\sin\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[x]]^2/Sqrt[x],x]

[Out] Sqrt[x] + Cos[Sqrt[x]]*Sin[Sqrt[x]]
```

Rule 3380

```
Int[((a_.) + Cos[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cos[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^2\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \cos^2(x)\,dx, x, \sqrt{x}\right)$$
$$= \cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right) + \text{Subst}\left(\int 1\,dx, x, \sqrt{x}\right)$$
$$= \sqrt{x} + \cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.026684, size = 18, normalized size = 0.95

$$\sqrt{x} + \frac{1}{2}\sin\left(2\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[x]]^2/Sqrt[x],x]
```

```
[Out] Sqrt[x] + Sin[2*Sqrt[x]]/2
```

Maple [A] time = 0.009, size = 14, normalized size = 0.7

$$\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right) + \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2))^2/x^(1/2),x)
```

```
[Out] cos(x^(1/2))*sin(x^(1/2))+x^(1/2)
```

Maxima [A] time = 0.92742, size = 16, normalized size = 0.84

$$\sqrt{x} + \frac{1}{2}\sin\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))^2/x^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x) + 1/2*sin(2*sqrt(x))
```

Fricas [A] time = 1.87875, size = 50, normalized size = 2.63

$$\cos\left(\sqrt{x}\right)\sin\left(\sqrt{x}\right)+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))^2/x^(1/2),x, algorithm="fricas")
```

```
[Out] cos(sqrt(x))*sin(sqrt(x)) + sqrt(x)
```

Sympy [B] time = 0.320803, size = 39, normalized size = 2.05

$$\sqrt{x}\sin^2\left(\sqrt{x}\right)+\sqrt{x}\cos^2\left(\sqrt{x}\right)+\sin\left(\sqrt{x}\right)\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2))**2/x**(1/2),x)
```

```
[Out] sqrt(x)*sin(sqrt(x))**2 + sqrt(x)*cos(sqrt(x))**2 + sin(sqrt(x))*cos(sqrt(x
))
```

Giac [A] time = 1.04795, size = 16, normalized size = 0.84

$$\sqrt{x}+\frac{1}{2}\sin\left(2\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2))^2/x^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x) + 1/2*sin(2*sqrt(x))
```

3.77 $\int x\sin^3\left(x^2\right)\,dx$

Optimal. Leaf size=19

$$\frac{1}{6}\cos^3\left(x^2\right)-\frac{\cos\left(x^2\right)}{2}$$

```
[Out] -Cos[x^2]/2 + Cos[x^2]^3/6
```

Rubi [A] time = 0.0137001, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3379, 2633}

$$\frac{1}{6}\cos^3\left(x^2\right)-\frac{\cos\left(x^2\right)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x^2]^3,x]
```

```
[Out] -Cos[x^2]/2 + Cos[x^2]^3/6
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\sin^3\left(x^2\right)\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \sin^3(x)\,dx, x, x^2\right)\\
&= -\left(\frac{1}{2}\,\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos\left(x^2\right)\right)\right)\\
&= -\frac{1}{2}\cos\left(x^2\right)+\frac{1}{6}\cos^3\left(x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.01543, size = 19, normalized size = 1.

$$
\frac{1}{24}\cos\left(3x^2\right)-\frac{3\cos\left(x^2\right)}{8}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x^2]^3,x]
```

```
[Out] (-3*Cos[x^2])/8 + Cos[3*x^2]/24
```

Maple [A] time = 0.006, size = 15, normalized size = 0.8

$$
-\frac{\left(2+\left(\sin\left(x^2\right)\right)^2\right)\cos\left(x^2\right)}{6}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x^2)^3,x)
```

```
[Out] -1/6*(2+sin(x^2)^2)*cos(x^2)
```

Maxima [A] time = 0.931032, size = 20, normalized size = 1.05

$$
\frac{1}{24}\cos\left(3\,x^2\right)-\frac{3}{8}\cos\left(x^2\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*cos(3*x^2) - 3/8*cos(x^2)
```

Fricas [A] time = 2.06567, size = 42, normalized size = 2.21

$$\frac{1}{6}\cos\left(x^2\right)^3 - \frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2)^3,x, algorithm="fricas")
```

```
[Out] 1/6*cos(x^2)^3 - 1/2*cos(x^2)
```

Sympy [A] time = 0.572453, size = 22, normalized size = 1.16

$$-\frac{\sin^2\left(x^2\right)\cos\left(x^2\right)}{2} - \frac{\cos^3\left(x^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x**2)**3,x)
```

```
[Out] -sin(x**2)**2*cos(x**2)/2 - cos(x**2)**3/3
```

Giac [A] time = 1.05639, size = 20, normalized size = 1.05

$$\frac{1}{6}\cos\left(x^2\right)^3 - \frac{1}{2}\cos\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x^2)^3,x, algorithm="giac")
```

```
[Out] 1/6*cos(x^2)^3 - 1/2*cos(x^2)
```

3.78 $\int \sin^2(x)\tan(x)\,dx$

Optimal. Leaf size=14

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Rubi [A] time = 0.012949, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2590, 14}

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2*Tan[x],x]
```

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sin^2(x)\tan(x)\,dx = -\operatorname{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, \cos(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{1}{x}-x\right)dx, x, \cos(x)\right)$$
$$= \frac{\cos^2(x)}{2} - \log(\cos(x))$$

Mathematica [A] time = 0.0049839, size = 14, normalized size = 1.

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2*Tan[x],x]
```

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Maple [A] time = 0.011, size = 13, normalized size = 0.9

$$-\frac{(\sin(x))^2}{2} - \ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*tan(x)^3,x)
```

```
[Out] -1/2*sin(x)^2-ln(cos(x))
```

Maxima [A] time = 0.929918, size = 22, normalized size = 1.57

$$-\frac{1}{2}\sin(x)^2 - \frac{1}{2}\log\left(\sin(x)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/2*sin(x)^2 - 1/2*log(sin(x)^2 - 1)
```

Fricas [A] time = 2.11478, size = 39, normalized size = 2.79

$$\frac{1}{2}\cos(x)^2 - \log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*tan(x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)^2 - log(-cos(x))
```

Sympy [A] time = 0.079894, size = 10, normalized size = 0.71

$$-\log(\cos(x)) + \frac{\cos^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*tan(x)**3,x)
```

```
[Out] -log(cos(x)) + cos(x)**2/2
```

Giac [A] time = 1.07051, size = 19, normalized size = 1.36

$$\frac{1}{2}\cos(x)^2 - \frac{1}{2}\log\left(\cos(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*tan(x)^3,x, algorithm="giac")
```

```
[Out] 1/2*cos(x)^2 - 1/2*log(cos(x)^2)
```

3.79 $\int \cos^2(x)\cot^3(x)\,dx$

Optimal. Leaf size=22

$$\frac{\sin^2(x)}{2}-\frac{1}{2}\csc^2(x)-2\log(\sin(x))$$

```
[Out] -Csc[x]^2/2 - 2*Log[Sin[x]] + Sin[x]^2/2
```

Rubi [A] time = 0.0285206, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2590, 266, 43}

$$\frac{\sin^2(x)}{2}-\frac{1}{2}\csc^2(x)-2\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] -Csc[x]^2/2 - 2*Log[Sin[x]] + Sin[x]^2/2
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^2(x)\cot^3(x)\,dx = \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^3}\,dx, x, -\sin(x)\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, \sin^2(x)\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)dx, x, \sin^2(x)\right)$$
$$= -\frac{1}{2}\csc^2(x) - 2\log(\sin(x)) + \frac{\sin^2(x)}{2}$$

Mathematica [A] time = 0.0225152, size = 20, normalized size = 0.91

$$\frac{1}{2}\left(\sin^2(x) - \csc^2(x) - 4\log(\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (-Csc[x]^2 - 4*Log[Sin[x]] + Sin[x]^2)/2
```

Maple [A] time = 0.013, size = 29, normalized size = 1.3

$$-\frac{(\cos(x))^6}{2\,(\sin(x))^2} - \frac{(\cos(x))^4}{2} - (\cos(x))^2 - 2\,\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^5*sin(x)^2,x)
```

```
[Out] -1/2/sin(x)^2*cos(x)^6-1/2*cos(x)^4-cos(x)^2-2*ln(sin(x))
```

Maxima [A] time = 0.931084, size = 27, normalized size = 1.23

$$\frac{1}{2}\sin(x)^2 - \frac{1}{2\sin(x)^2} - \log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^5*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sin(x)^2 - 1/2/sin(x)^2 - log(sin(x)^2)
```

Fricas [B] time = 2.0689, size = 116, normalized size = 5.27

$$-\frac{2\cos(x)^4 - 3\cos(x)^2 + 8\left(\cos(x)^2 - 1\right)\log\left(\frac{1}{2}\sin(x)\right) - 1}{4\left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^5*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(x)^4 - 3*cos(x)^2 + 8*(cos(x)^2 - 1)*log(1/2*sin(x)) - 1)/(cos(
x)^2 - 1)
```

Sympy [A] time = 0.095172, size = 20, normalized size = 0.91

$$-2\log(\sin(x)) + \frac{\sin^2(x)}{2} - \frac{1}{2\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**5*sin(x)**2,x)
```

```
[Out] -2*log(sin(x)) + sin(x)**2/2 - 1/(2*sin(x)**2)
```

Giac [B] time = 1.07039, size = 49, normalized size = 2.23

$$-\frac{1}{2}\cos(x)^2 + \frac{2\cos(x)^2 - 1}{2\left(\cos(x)^2 - 1\right)} - \log\left(-\cos(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^5*sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2 + 1/2*(2*cos(x)^2 - 1)/(cos(x)^2 - 1) - log(-cos(x)^2 + 1)
```

3.80 $\int \sec(x)(1-\sin(x))\,dx$

Optimal. Leaf size=5

$$\log(\sin(x)+1)$$

```
[Out] Log[1 + Sin[x]]
```

Rubi [A] time = 0.013264, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2667, 31}

$$\log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*(1 - Sin[x]),x]
```

```
[Out] Log[1 + Sin[x]]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(x)(1-\sin(x))\,dx &= -\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, -\sin(x)\right)\\ &= \log(1+\sin(x))\end{aligned}$$

Mathematica [B] time = 0.0070203, size = 36, normalized size = 7.2

$$\log(\cos(x)) - \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*(1 - Sin[x]),x]
```

```
[Out] Log[Cos[x]] - Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

Maple [A] time = 0.021, size = 6, normalized size = 1.2

$$\ln(1 + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(x))/cos(x),x)
```

```
[Out] ln(1+sin(x))
```

Maxima [A] time = 0.935342, size = 7, normalized size = 1.4

$$\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))/cos(x),x, algorithm="maxima")
```

```
[Out] log(sin(x) + 1)
```

Fricas [A] time = 2.07607, size = 23, normalized size = 4.6

$$\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))/cos(x),x, algorithm="fricas")
```

```
[Out] log(sin(x) + 1)
```

Sympy [B] time = 0.317999, size = 19, normalized size = 3.8

$$2\log\left(\tan\left(\frac{x}{2}\right)+1\right)-\log\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))/cos(x),x)
```

```
[Out] 2*log(tan(x/2) + 1) - log(tan(x/2)**2 + 1)
```

Giac [A] time = 1.05386, size = 7, normalized size = 1.4

$$\log\left(\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))/cos(x),x, algorithm="giac")
```

```
[Out] log(sin(x) + 1)
```

3.81 $\int \frac{1}{1-\sin(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\cos(x)}{1-\sin(x)}$$

```
[Out] Cos[x]/(1 - Sin[x])
```

Rubi [A] time = 0.0078634, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2648}

$$\frac{\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x])^(-1),x]
```

```
[Out] Cos[x]/(1 - Sin[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\sin(x)}\,dx = \frac{\cos(x)}{1-\sin(x)}$$

Mathematica [B] time = 0.0118904, size = 25, normalized size = 2.27

$$\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x])^(-1),x]
```

```
[Out] (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

Maple [A] time = 0., size = 11, normalized size = 1.

$$-2\left(-1+\tan\left(x/2\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x)),x)
```

```
[Out] -2/(-1+tan(1/2*x))
```

Maxima [A] time = 0.932386, size = 20, normalized size = 1.82

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) - 1)
```

Fricas [A] time = 1.94193, size = 61, normalized size = 5.55

$$\frac{\cos\left(x\right)+\sin\left(x\right)+1}{\cos\left(x\right)-\sin\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="fricas")
```

```
[Out] (cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

Sympy [A] time = 0.380706, size = 8, normalized size = 0.73

$$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x)
```

```
[Out] -2/(tan(x/2) - 1)
```

Giac [A] time = 1.05628, size = 14, normalized size = 1.27

$$-\frac{2}{\tan\left(\frac{1}{2}x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*x) - 1)
```

3.82 $\int \tan^2(x)\,dx$

Optimal. Leaf size=6

$$\tan(x) - x$$

```
[Out] -x + Tan[x]
```

Rubi [A] time = 0.004091, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3473, 8}

$$\tan(x) - x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2,x]
```

```
[Out] -x + Tan[x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tan^2(x)\,dx &= \tan(x) - \int 1\,dx\\ &= -x + \tan(x)\end{aligned}$$

Mathematica [A] time = 0.0020183, size = 6, normalized size = 1.

$$\tan(x) - x$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2,x]
```

```
[Out] -x + Tan[x]
```

Maple [A] time = 0.002, size = 7, normalized size = 1.2

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2,x)
```

```
[Out] -x+tan(x)
```

Maxima [A] time = 1.40741, size = 8, normalized size = 1.33

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="maxima")
```

```
[Out] -x + tan(x)
```

Fricas [A] time = 1.97729, size = 18, normalized size = 3.

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="fricas")
```

```
[Out] -x + tan(x)
```

Sympy [B] time = 0.061568, size = 7, normalized size = 1.17

$$-x + \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2,x)
```

```
[Out] -x + sin(x)/cos(x)
```

Giac [A] time = 1.06425, size = 8, normalized size = 1.33

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="giac")
```

```
[Out] -x + tan(x)
```

3.83 $\int \tan^4(x)\,dx$

Optimal. Leaf size=14

$$x+\frac{\tan^3(x)}{3}-\tan(x)$$

```
[Out] x - Tan[x] + Tan[x]^3/3
```

Rubi [A] time = 0.008007, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 8}

$$x+\frac{\tan^3(x)}{3}-\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^4,x]
```

```
[Out] x - Tan[x] + Tan[x]^3/3
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tan^4(x)\,dx &= \frac{\tan^3(x)}{3}-\int \tan^2(x)\,dx\\ &= -\tan(x)+\frac{\tan^3(x)}{3}+\int 1\,dx\\ &= x-\tan(x)+\frac{\tan^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.0040879, size = 18, normalized size = 1.29

$$x-\frac{4\tan(x)}{3}+\frac{1}{3}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^4,x]
```

```
[Out] x - (4*Tan[x])/3 + (Sec[x]^2*Tan[x])/3
```

Maple [A] time = 0.001, size = 13, normalized size = 0.9

$$x-\tan(x)+\frac{(\tan(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4,x)
```

```
[Out] x-tan(x)+1/3*tan(x)^3
```

Maxima [A] time = 1.41, size = 16, normalized size = 1.14

$$\frac{1}{3}\tan(x)^3+x-\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Fricas [A] time = 1.9097, size = 36, normalized size = 2.57

$$\frac{1}{3}\tan(x)^3+x-\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Sympy [A] time = 0.065726, size = 19, normalized size = 1.36

$$x + \frac{\sin^3(x)}{3\cos^3(x)} - \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**4,x)
```

```
[Out] x + sin(x)**3/(3*cos(x)**3) - sin(x)/cos(x)
```

Giac [A] time = 1.05466, size = 16, normalized size = 1.14

$$\frac{1}{3}\tan(x)^3 + x - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4,x, algorithm="giac")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

3.84 $\int \sec^4(x)\,dx$

Optimal. Leaf size=11

$$\frac{\tan^3(x)}{3}+\tan(x)$$

```
[Out] Tan[x] + Tan[x]^3/3
```

Rubi [A] time = 0.006882, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3767}

$$\frac{\tan^3(x)}{3}+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4,x]
```

```
[Out] Tan[x] + Tan[x]^3/3
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(x)\,dx &= -\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(x)\right)\\ &= \tan(x)+\frac{\tan^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.0024551, size = 17, normalized size = 1.55

$$\frac{2\tan(x)}{3}+\frac{1}{3}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4,x]
```

```
[Out] (2*Tan[x])/3 + (Sec[x]^2*Tan[x])/3
```

Maple [A] time = 0.027, size = 13, normalized size = 1.2

$$-\left(-\frac{2}{3}-\frac{(\sec(x))^{2}}{3}\right)\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4,x)
```

```
[Out] -(-2/3-1/3*sec(x)^2)*tan(x)
```

Maxima [A] time = 0.923523, size = 12, normalized size = 1.09

$$\frac{1}{3}\tan(x)^{3}+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*tan(x)^3 + tan(x)
```

Fricas [A] time = 1.96521, size = 51, normalized size = 4.64

$$\frac{\left(2\cos(x)^{2}+1\right)\sin(x)}{3\cos(x)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(x)^2 + 1)*sin(x)/cos(x)^3
```

Sympy [B] time = 0.061384, size = 19, normalized size = 1.73

$$\frac{2\sin(x)}{3\cos(x)} + \frac{\sin(x)}{3\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4,x)
```

```
[Out] 2*sin(x)/(3*cos(x)) + sin(x)/(3*cos(x)**3)
```

Giac [A] time = 1.07935, size = 12, normalized size = 1.09

$$\frac{1}{3}\tan(x)^3 + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4,x, algorithm="giac")
```

```
[Out] 1/3*tan(x)^3 + tan(x)
```

3.85 $\int \sec^6(x)\,dx$

Optimal. Leaf size=19

$$\frac{\tan^5(x)}{5}+\frac{2\tan^3(x)}{3}+\tan(x)$$

```
[Out] Tan[x] + (2*Tan[x]^3)/3 + Tan[x]^5/5
```

Rubi [A] time = 0.0088159, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3767}

$$\frac{\tan^5(x)}{5}+\frac{2\tan^3(x)}{3}+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^6,x]
```

```
[Out] Tan[x] + (2*Tan[x]^3)/3 + Tan[x]^5/5
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(x)\,dx &= -\operatorname{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -\tan(x)\right)\\ &= \tan(x)+\frac{2\tan^3(x)}{3}+\frac{\tan^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.0027936, size = 27, normalized size = 1.42

$$\frac{8\tan(x)}{15}+\frac{1}{5}\tan(x)\sec^4(x)+\frac{4}{15}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^6,x]
```

```
[Out] (8*Tan[x])/15 + (4*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

Maple [A] time = 0.027, size = 19, normalized size = 1.

$$-\left(-\frac{8}{15}-\frac{(\sec(x))^4}{5}-\frac{4(\sec(x))^2}{15}\right)\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^6,x)
```

```
[Out] -(-8/15-1/5*sec(x)^4-4/15*sec(x)^2)*tan(x)
```

Maxima [A] time = 0.926333, size = 20, normalized size = 1.05

$$\frac{1}{5}\tan(x)^5+\frac{2}{3}\tan(x)^3+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6,x, algorithm="maxima")
```

```
[Out] 1/5*tan(x)^5 + 2/3*tan(x)^3 + tan(x)
```

Fricas [A] time = 1.94805, size = 70, normalized size = 3.68

$$\frac{\left(8\cos(x)^4+4\cos(x)^2+3\right)\sin(x)}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6,x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/cos(x)^5
```

Sympy [A] time = 0.062246, size = 31, normalized size = 1.63

$$\frac{8\sin(x)}{15\cos(x)} + \frac{4\sin(x)}{15\cos^3(x)} + \frac{\sin(x)}{5\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**6,x)
```

```
[Out] 8*sin(x)/(15*cos(x)) + 4*sin(x)/(15*cos(x)**3) + sin(x)/(5*cos(x)**5)
```

Giac [A] time = 1.05634, size = 20, normalized size = 1.05

$$\frac{1}{5}\tan(x)^5 + \frac{2}{3}\tan(x)^3 + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6,x, algorithm="giac")
```

```
[Out] 1/5*tan(x)^5 + 2/3*tan(x)^3 + tan(x)
```

3.86 $\int \sec^2(x)\tan^4(x)\,dx$

Optimal. Leaf size=8

$$\frac{\tan^5(x)}{5}$$

```
[Out] Tan[x]^5/5
```

Rubi [A] time = 0.0197036, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 30}

$$\frac{\tan^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Tan[x]^4,x]
```

```
[Out] Tan[x]^5/5
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\tan^4(x)\,dx &= \text{Subst}\left(\int x^4\,dx, x, \tan(x)\right)\\ &= \frac{\tan^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.0021575, size = 8, normalized size = 1.

$$\frac{\tan^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Tan[x]^4,x]
```

```
[Out] Tan[x]^5/5
```

Maple [A] time = 0.011, size = 11, normalized size = 1.4

$$\frac{(\sin(x))^5}{5\ (\cos(x))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*tan(x)^4,x)
```

```
[Out] 1/5*sin(x)^5/cos(x)^5
```

Maxima [A] time = 0.92368, size = 8, normalized size = 1.

$$\frac{1}{5}\tan(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^4,x, algorithm="maxima")
```

```
[Out] 1/5*tan(x)^5
```

Fricas [B] time = 1.97748, size = 66, normalized size = 8.25

$$\frac{\left(\cos(x)^4 - 2\ \cos(x)^2 + 1\right)\sin(x)}{5\ \cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^4,x, algorithm="fricas")
```

```
[Out] 1/5*(cos(x)^4 - 2*cos(x)^2 + 1)*sin(x)/cos(x)^5
```

Sympy [B] time = 0.063451, size = 29, normalized size = 3.62

$$\frac{\sin(x)}{5\cos(x)} - \frac{2\sin(x)}{5\cos^3(x)} + \frac{\sin(x)}{5\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*tan(x)**4,x)
```

```
[Out] sin(x)/(5*cos(x)) - 2*sin(x)/(5*cos(x)**3) + sin(x)/(5*cos(x)**5)
```

Giac [A] time = 1.05643, size = 8, normalized size = 1.

$$\frac{1}{5}\tan(x)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x)^4,x, algorithm="giac")
```

```
[Out] 1/5*tan(x)^5
```

3.87 $\int \sec^4(x)\tan^2(x)\,dx$

Optimal. Leaf size=17

$$\frac{\tan^5(x)}{5}+\frac{\tan^3(x)}{3}$$

```
[Out] Tan[x]^3/3 + Tan[x]^5/5
```

Rubi [A] time = 0.0216581, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 14}

$$\frac{\tan^5(x)}{5}+\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4*Tan[x]^2,x]
```

```
[Out] Tan[x]^3/3 + Tan[x]^5/5
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sec^4(x)\tan^2(x)\,dx &= \text{Subst}\left(\int x^2\left(1+x^2\right)\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(x^2+x^4\right)\,dx, x, \tan(x)\right)\\ &= \frac{\tan^3(x)}{3}+\frac{\tan^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.0185612, size = 27, normalized size = 1.59

$$-\frac{2\tan(x)}{15}+\frac{1}{5}\tan(x)\sec^4(x)-\frac{1}{15}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4*Tan[x]^2,x]
```

```
[Out] (-2*Tan[x])/15 - (Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

Maple [A] time = 0.012, size = 22, normalized size = 1.3

$$\frac{(\sin(x))^3}{5\,(\cos(x))^5}+\frac{2\,(\sin(x))^3}{15\,(\cos(x))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4*tan(x)^2,x)
```

```
[Out] 1/5*sin(x)^3/cos(x)^5+2/15*sin(x)^3/cos(x)^3
```

Maxima [A] time = 0.924341, size = 18, normalized size = 1.06

$$\frac{1}{5}\tan(x)^5+\frac{1}{3}\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

Fricas [A] time = 1.97466, size = 69, normalized size = 4.06

$$-\frac{\left(2\,\cos\left(x\right)^{4}+\cos\left(x\right)^{2}-3\right)\sin\left(x\right)}{15\,\cos\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(x)^4 + cos(x)^2 - 3)*sin(x)/cos(x)^5
```

Sympy [B] time = 0.064635, size = 29, normalized size = 1.71

$$-\frac{2\sin\left(x\right)}{15\cos\left(x\right)}-\frac{\sin\left(x\right)}{15\cos^{3}\left(x\right)}+\frac{\sin\left(x\right)}{5\cos^{5}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4*tan(x)**2,x)
```

```
[Out] -2*sin(x)/(15*cos(x)) - sin(x)/(15*cos(x)**3) + sin(x)/(5*cos(x)**5)
```

Giac [A] time = 1.06954, size = 18, normalized size = 1.06

$$\frac{1}{5}\,\tan\left(x\right)^{5}+\frac{1}{3}\,\tan\left(x\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="giac")
```

```
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

3.88 $\int \sec^3(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^3(x)}{3}$$

```
[Out] Sec[x]^3/3
```

Rubi [A] time = 0.011629, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2606, 30}

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3*Tan[x],x]
```

```
[Out] Sec[x]^3/3
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^3(x)\tan(x)\,dx &= \text{Subst}\left(\int x^2\,dx, x, \sec(x)\right)\\ &= \frac{\sec^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.0046055, size = 8, normalized size = 1.

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3*Tan[x],x]
```

```
[Out] Sec[x]^3/3
```

Maple [A] time = 0.006, size = 7, normalized size = 0.9

$$\frac{(\sec(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3*tan(x),x)
```

```
[Out] 1/3*sec(x)^3
```

Maxima [A] time = 0.927292, size = 8, normalized size = 1.

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="maxima")
```

```
[Out] 1/3/cos(x)^3
```

Fricas [A] time = 1.97207, size = 19, normalized size = 2.38

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="fricas")
```

```
[Out] 1/3/cos(x)^3
```

Sympy [A] time = 0.065122, size = 7, normalized size = 0.88

$$\frac{1}{3\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3*tan(x),x)
```

```
[Out] 1/(3*cos(x)**3)
```

Giac [A] time = 1.05396, size = 8, normalized size = 1.

$$\frac{1}{3\,\cos\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="giac")
```

```
[Out] 1/3/cos(x)^3
```

3.89 $\int \sec^3(x) \tan^3(x) \, dx$

Optimal. Leaf size=17

$$\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rubi [A] time = 0.0231833, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec^3(x)\tan^3(x)\,dx = \text{Subst}\left(\int x^2\left(-1+x^2\right)dx, x, \sec(x)\right)$$
$$= \text{Subst}\left(\int \left(-x^2+x^4\right)dx, x, \sec(x)\right)$$
$$= -\frac{1}{3}\sec^3(x) + \frac{\sec^5(x)}{5}$$

Mathematica [A] time = 0.0228923, size = 17, normalized size = 1.

$$\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Maple [B] time = 0.011, size = 42, normalized size = 2.5

$$\frac{(\sin(x))^4}{5(\cos(x))^5} + \frac{(\sin(x))^4}{15(\cos(x))^3} - \frac{(\sin(x))^4}{15\cos(x)} - \frac{\left(2+(\sin(x))^2\right)\cos(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3*tan(x)^3,x)
```

```
[Out] 1/5*sin(x)^4/cos(x)^5+1/15*sin(x)^4/cos(x)^3-1/15*sin(x)^4/cos(x)-1/15*(2+s
in(x)^2)*cos(x)
```

Maxima [A] time = 0.925088, size = 19, normalized size = 1.12

$$-\frac{5\cos(x)^2 - 3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Fricas [A] time = 2.23677, size = 45, normalized size = 2.65

$$-\frac{5\cos(x)^2 - 3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Sympy [A] time = 0.096607, size = 15, normalized size = 0.88

$$-\frac{5\cos^2(x) - 3}{15\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3*tan(x)**3,x)
```

```
[Out] -(5*cos(x)**2 - 3)/(15*cos(x)**5)
```

Giac [A] time = 1.05083, size = 19, normalized size = 1.12

$$-\frac{5\cos(x)^2 - 3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="giac")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

3.90 $\int \tan^5(x)\,dx$

Optimal. Leaf size=22

$$\frac{\tan^4(x)}{4}-\frac{\tan^2(x)}{2}-\log(\cos(x))$$

```
[Out] -Log[Cos[x]] - Tan[x]^2/2 + Tan[x]^4/4
```

Rubi [A] time = 0.0101662, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 3475}

$$\frac{\tan^4(x)}{4}-\frac{\tan^2(x)}{2}-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^5,x]
```

```
[Out] -Log[Cos[x]] - Tan[x]^2/2 + Tan[x]^4/4
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^5(x)\,dx &= \frac{\tan^4(x)}{4}-\int \tan^3(x)\,dx\\ &= -\frac{1}{2}\tan^2(x)+\frac{\tan^4(x)}{4}+\int \tan(x)\,dx\\ &= -\log(\cos(x))-\frac{\tan^2(x)}{2}+\frac{\tan^4(x)}{4}\end{aligned}$$

Mathematica [A] time = 0.0030332, size = 20, normalized size = 0.91

$$\frac{\sec^4(x)}{4}-\sec^2(x)-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^5,x]
```

```
[Out] -Log[Cos[x]] - Sec[x]^2 + Sec[x]^4/4
```

Maple [A] time = 0.002, size = 23, normalized size = 1.1

$$-\frac{(\tan(x))^2}{2}+\frac{(\tan(x))^4}{4}+\frac{\ln\left((\tan(x))^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^5,x)
```

```
[Out] -1/2*tan(x)^2+1/4*tan(x)^4+1/2*ln(tan(x)^2+1)
```

Maxima [A] time = 0.925153, size = 46, normalized size = 2.09

$$\frac{4\sin(x)^2-3}{4\left(\sin(x)^4-2\sin(x)^2+1\right)}-\frac{1}{2}\log\left(\sin(x)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^5,x, algorithm="maxima")
```

```
[Out] 1/4*(4*sin(x)^2 - 3)/(sin(x)^4 - 2*sin(x)^2 + 1) - 1/2*log(sin(x)^2 - 1)
```

Fricas [A] time = 2.2591, size = 77, normalized size = 3.5

$$\frac{1}{4}\tan(x)^4-\frac{1}{2}\tan(x)^2-\frac{1}{2}\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^5,x, algorithm="fricas")
```

```
[Out] 1/4*tan(x)^4 - 1/2*tan(x)^2 - 1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] time = 0.103746, size = 20, normalized size = 0.91

$$-\frac{4\cos^2(x)-1}{4\cos^4(x)}-\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**5,x)
```

```
[Out] -(4*cos(x)**2 - 1)/(4*cos(x)**4) - log(cos(x))
```

Giac [A] time = 1.05882, size = 30, normalized size = 1.36

$$\frac{1}{4}\tan(x)^4-\frac{1}{2}\tan(x)^2+\frac{1}{2}\log\left(\tan(x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^5,x, algorithm="giac")
```

```
[Out] 1/4*tan(x)^4 - 1/2*tan(x)^2 + 1/2*log(tan(x)^2 + 1)
```

3.91 $\int \tan^6(x)\,dx$

Optimal. Leaf size=22

$$-x+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}+\tan(x)$$

```
[Out] -x + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

Rubi [A] time = 0.0131138, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 8}

$$-x+\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}+\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^6,x]
```

```
[Out] -x + Tan[x] - Tan[x]^3/3 + Tan[x]^5/5
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tan^6(x)\,dx &= \frac{\tan^5(x)}{5} - \int \tan^4(x)\,dx \\ &= -\frac{1}{3}\tan^3(x) + \frac{\tan^5(x)}{5} + \int \tan^2(x)\,dx \\ &= \tan(x) - \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5} - \int 1\,dx \\ &= -x + \tan(x) - \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.003632, size = 30, normalized size = 1.36

$$-x + \frac{23\tan(x)}{15} + \frac{1}{5}\tan(x)\sec^4(x) - \frac{11}{15}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^6,x]
```

```
[Out] -x + (23*Tan[x])/15 - (11*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

Maple [A] time = 0.002, size = 19, normalized size = 0.9

$$-x + \tan(x) - \frac{(\tan(x))^3}{3} + \frac{(\tan(x))^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^6,x)
```

```
[Out] -x+tan(x)-1/3*tan(x)^3+1/5*tan(x)^5
```

Maxima [A] time = 1.41082, size = 24, normalized size = 1.09

$$\frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^6,x, algorithm="maxima")
```

```
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

Fricas [A] time = 2.03569, size = 57, normalized size = 2.59

$$\frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^6,x, algorithm="fricas")
```

```
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

Sympy [A] time = 0.067084, size = 31, normalized size = 1.41

$$-x + \frac{\sin^5(x)}{5\cos^5(x)} - \frac{\sin^3(x)}{3\cos^3(x)} + \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**6,x)
```

```
[Out] -x + sin(x)**5/(5*cos(x)**5) - sin(x)**3/(3*cos(x)**3) + sin(x)/cos(x)
```

Giac [A] time = 1.05793, size = 24, normalized size = 1.09

$$\frac{1}{5}\tan(x)^5 - \frac{1}{3}\tan(x)^3 - x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^6,x, algorithm="giac")
```

```
[Out] 1/5*tan(x)^5 - 1/3*tan(x)^3 - x + tan(x)
```

3.92 $\int \sec(x)\tan^5(x)\,dx$

Optimal. Leaf size=19

$$\frac{\sec^5(x)}{5}-\frac{2\sec^3(x)}{3}+\sec(x)$$

```
[Out] Sec[x] - (2*Sec[x]^3)/3 + Sec[x]^5/5
```

Rubi [A] time = 0.0163057, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 194}

$$\frac{\sec^5(x)}{5}-\frac{2\sec^3(x)}{3}+\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Tan[x]^5,x]
```

```
[Out] Sec[x] - (2*Sec[x]^3)/3 + Sec[x]^5/5
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(x)\tan^5(x)\,dx &= \text{Subst}\left(\int \left(-1+x^2\right)^2\,dx,x,\sec(x)\right)\\ &= \text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\sec(x)\right)\\ &= \sec(x)-\frac{2\sec^3(x)}{3}+\frac{\sec^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.0097963, size = 19, normalized size = 1.

$$\frac{\sec^5(x)}{5}-\frac{2\sec^3(x)}{3}+\sec(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Tan[x]^5,x]
```

```
[Out] Sec[x] - (2*Sec[x]^3)/3 + Sec[x]^5/5
```

Maple [B] time = 0.008, size = 48, normalized size = 2.5

$$\frac{(\sin(x))^6}{5(\cos(x))^5}-\frac{(\sin(x))^6}{15(\cos(x))^3}+\frac{(\sin(x))^6}{5\cos(x)}+\frac{\cos(x)}{5}\left(\frac{8}{3}+(\sin(x))^4+\frac{4(\sin(x))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)^5,x)
```

```
[Out] 1/5*sin(x)^6/cos(x)^5-1/15*sin(x)^6/cos(x)^3+1/5*sin(x)^6/cos(x)+1/5*(8/3+s
in(x)^4+4/3*sin(x)^2)*cos(x)
```

Maxima [A] time = 0.937615, size = 27, normalized size = 1.42

$$\frac{15\cos(x)^4-10\cos(x)^2+3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^5,x, algorithm="maxima")
```

```
[Out] 1/15*(15*cos(x)^4 - 10*cos(x)^2 + 3)/cos(x)^5
```

Fricas [A] time = 2.02845, size = 63, normalized size = 3.32

$$\frac{15\cos(x)^4 - 10\cos(x)^2 + 3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^5,x, algorithm="fricas")
```

```
[Out] 1/15*(15*cos(x)^4 - 10*cos(x)^2 + 3)/cos(x)^5
```

Sympy [A] time = 0.103827, size = 20, normalized size = 1.05

$$\frac{15\cos^4(x) - 10\cos^2(x) + 3}{15\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)**5,x)
```

```
[Out] (15*cos(x)**4 - 10*cos(x)**2 + 3)/(15*cos(x)**5)
```

Giac [A] time = 1.05896, size = 27, normalized size = 1.42

$$\frac{15\cos(x)^4 - 10\cos(x)^2 + 3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^5,x, algorithm="giac")
```

```
[Out] 1/15*(15*cos(x)^4 - 10*cos(x)^2 + 3)/cos(x)^5
```

3.93 $\int \sec^3(x) \tan^5(x) \, dx$

Optimal. Leaf size=25

$$\frac{\sec^7(x)}{7} - \frac{2\sec^5(x)}{5} + \frac{\sec^3(x)}{3}$$

```
[Out] Sec[x]^3/3 - (2*Sec[x]^5)/5 + Sec[x]^7/7
```

Rubi [A] time = 0.0291992, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 270}

$$\frac{\sec^7(x)}{7} - \frac{2\sec^5(x)}{5} + \frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3*Tan[x]^5,x]
```

```
[Out] Sec[x]^3/3 - (2*Sec[x]^5)/5 + Sec[x]^7/7
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sec^3(x)\tan^5(x)\,dx = \text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, \sec(x)\right)$$
$$= \text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \sec(x)\right)$$
$$= \frac{\sec^3(x)}{3} - \frac{2\sec^5(x)}{5} + \frac{\sec^7(x)}{7}$$

Mathematica [A] time = 0.011756, size = 25, normalized size = 1.

$$\frac{\sec^7(x)}{7} - \frac{2\sec^5(x)}{5} + \frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3*Tan[x]^5,x]
```

```
[Out] Sec[x]^3/3 - (2*Sec[x]^5)/5 + Sec[x]^7/7
```

Maple [B] time = 0.012, size = 58, normalized size = 2.3

$$\frac{(\sin(x))^6}{7(\cos(x))^7} + \frac{(\sin(x))^6}{35(\cos(x))^5} - \frac{(\sin(x))^6}{105(\cos(x))^3} + \frac{(\sin(x))^6}{35\cos(x)} + \frac{\cos(x)}{35}\left(\frac{8}{3} + (\sin(x))^4 + \frac{4(\sin(x))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3*tan(x)^5,x)
```

```
[Out] 1/7*sin(x)^6/cos(x)^7+1/35*sin(x)^6/cos(x)^5-1/105*sin(x)^6/cos(x)^3+1/35*s
in(x)^6/cos(x)+1/35*(8/3+sin(x)^4+4/3*sin(x)^2)*cos(x)
```

Maxima [A] time = 0.926329, size = 27, normalized size = 1.08

$$\frac{35\cos(x)^4 - 42\cos(x)^2 + 15}{105\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^5,x, algorithm="maxima")
```

```
[Out] 1/105*(35*cos(x)^4 - 42*cos(x)^2 + 15)/cos(x)^7
```

Fricas [A] time = 2.05767, size = 66, normalized size = 2.64

$$\frac{35\cos(x)^4 - 42\cos(x)^2 + 15}{105\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^5,x, algorithm="fricas")
```

```
[Out] 1/105*(35*cos(x)^4 - 42*cos(x)^2 + 15)/cos(x)^7
```

Sympy [A] time = 0.114735, size = 20, normalized size = 0.8

$$\frac{35\cos^4(x) - 42\cos^2(x) + 15}{105\cos^7(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3*tan(x)**5,x)
```

```
[Out] (35*cos(x)**4 - 42*cos(x)**2 + 15)/(105*cos(x)**7)
```

Giac [A] time = 1.06472, size = 27, normalized size = 1.08

$$\frac{35\cos(x)^4 - 42\cos(x)^2 + 15}{105\cos(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^5,x, algorithm="giac")
```

```
[Out] 1/105*(35*cos(x)^4 - 42*cos(x)^2 + 15)/cos(x)^7
```

3.94 $\int \sec^6(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^6(x)}{6}$$

```
[Out] Sec[x]^6/6
```

Rubi [A] time = 0.0134191, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 30}

$$\frac{\sec^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^6*Tan[x],x]
```

```
[Out] Sec[x]^6/6
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^6(x)\tan(x)\,dx &= \text{Subst}\left(\int x^5\,dx, x, \sec(x)\right)\\ &= \frac{\sec^6(x)}{6}\end{aligned}$$

Mathematica [A] time = 0.0040709, size = 8, normalized size = 1.

$$\frac{\sec^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^6*Tan[x],x]
```

```
[Out] Sec[x]^6/6
```

Maple [A] time = 0.007, size = 7, normalized size = 0.9

$$\frac{(\sec(x))^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^6*tan(x),x)
```

```
[Out] 1/6*sec(x)^6
```

Maxima [A] time = 0.924682, size = 14, normalized size = 1.75

$$-\frac{1}{6\left(\sin(x)^2-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6*tan(x),x, algorithm="maxima")
```

```
[Out] -1/6/(sin(x)^2 - 1)^3
```

Fricas [A] time = 2.00347, size = 19, normalized size = 2.38

$$\frac{1}{6\cos(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6*tan(x),x, algorithm="fricas")
```

```
[Out] 1/6/cos(x)^6
```

Sympy [A] time = 0.067629, size = 7, normalized size = 0.88

$$\frac{1}{6\cos^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**6*tan(x),x)
```

```
[Out] 1/(6*cos(x)**6)
```

Giac [A] time = 1.05836, size = 8, normalized size = 1.

$$\frac{1}{6\cos(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6*tan(x),x, algorithm="giac")
```

```
[Out] 1/6/cos(x)^6
```

3.95 $\int \sec^6(x)\tan^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sec^8(x)}{8}-\frac{\sec^6(x)}{6}$$

```
[Out] -Sec[x]^6/6 + Sec[x]^8/8
```

Rubi [A] time = 0.0276463, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$\frac{\sec^8(x)}{8}-\frac{\sec^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^6*Tan[x]^3,x]
```

```
[Out] -Sec[x]^6/6 + Sec[x]^8/8
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sec^6(x)\tan^3(x)\,dx &= \text{Subst}\left(\int x^5\left(-1+x^2\right)\,dx, x, \sec(x)\right)\\ &= \text{Subst}\left(\int \left(-x^5+x^7\right)\,dx, x, \sec(x)\right)\\ &= -\frac{1}{6}\sec^6(x)+\frac{\sec^8(x)}{8}\end{aligned}$$

Mathematica [A] time = 0.0125847, size = 17, normalized size = 1.

$$\frac{\sec^8(x)}{8}-\frac{\sec^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^6*Tan[x]^3,x]
```

```
[Out] -Sec[x]^6/6 + Sec[x]^8/8
```

Maple [B] time = 0.013, size = 32, normalized size = 1.9

$$\frac{(\sin(x))^4}{8\,(\cos(x))^8}+\frac{(\sin(x))^4}{12\,(\cos(x))^6}+\frac{(\sin(x))^4}{24\,(\cos(x))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^6*tan(x)^3,x)
```

```
[Out] 1/8*sin(x)^4/cos(x)^8+1/12*sin(x)^4/cos(x)^6+1/24*sin(x)^4/cos(x)^4
```

Maxima [B] time = 0.928485, size = 49, normalized size = 2.88

$$\frac{4\,\sin(x)^2-1}{24\left(\sin(x)^8-4\,\sin(x)^6+6\,\sin(x)^4-4\,\sin(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6*tan(x)^3,x, algorithm="maxima")
```

```
[Out] 1/24*(4*sin(x)^2 - 1)/(sin(x)^8 - 4*sin(x)^6 + 6*sin(x)^4 - 4*sin(x)^2 + 1)
```

Fricas [A] time = 1.94501, size = 45, normalized size = 2.65

$$-\frac{4\cos(x)^2-3}{24\cos(x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6*tan(x)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(4*cos(x)^2 - 3)/cos(x)^8
```

Sympy [A] time = 0.126972, size = 15, normalized size = 0.88

$$-\frac{4\cos^2(x)-3}{24\cos^8(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**6*tan(x)**3,x)
```

```
[Out] -(4*cos(x)**2 - 3)/(24*cos(x)**8)
```

Giac [A] time = 1.04927, size = 19, normalized size = 1.12

$$-\frac{4\cos(x)^2-3}{24\cos(x)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6*tan(x)^3,x, algorithm="giac")
```

```
[Out] -1/24*(4*cos(x)^2 - 3)/cos(x)^8
```

3.96 $\int \sec^2(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^2(x)}{2}$$

```
[Out] Sec[x]^2/2
```

Rubi [A] time = 0.0116286, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 30}

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \sec^2(x)\tan(x)\,dx &= \text{Subst}\left(\int x\,dx, x, \sec(x)\right)\\ &= \frac{\sec^2(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.0040826, size = 8, normalized size = 1.

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

Maple [A] time = 0.01, size = 7, normalized size = 0.9

$$\frac{(\sec(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/cot(x),x)
```

```
[Out] 1/2*sec(x)^2
```

Maxima [A] time = 0.924727, size = 14, normalized size = 1.75

$$-\frac{1}{2\left(\sin(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/cot(x),x, algorithm="maxima")
```

```
[Out] -1/2/(sin(x)^2 - 1)
```

Fricas [A] time = 2.04025, size = 19, normalized size = 2.38

$$\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/cot(x),x, algorithm="fricas")
```

```
[Out] 1/2/cos(x)^2
```

Sympy [A] time = 0.067394, size = 7, normalized size = 0.88

$$\frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/cot(x),x)
```

```
[Out] 1/(2*cos(x)**2)
```

Giac [A] time = 1.05376, size = 8, normalized size = 1.

$$\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/cot(x),x, algorithm="giac")
```

```
[Out] 1/2/cos(x)^2
```

3.97 $\int \sec(x)\tan^2(x)\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}\tan(x)\sec(x)-\frac{1}{2}\tanh^{-1}(\sin(x))$$

```
[Out] -ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2
```

Rubi [A] time = 0.0152064, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2611, 3770}

$$\frac{1}{2}\tan(x)\sec(x)-\frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Tan[x]^2,x]
```

```
[Out] -ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(x)\tan^2(x)\,dx&=\frac{1}{2}\sec(x)\tan(x)-\frac{1}{2}\int \sec(x)\,dx\\&=-\frac{1}{2}\tanh^{-1}(\sin(x))+\frac{1}{2}\sec(x)\tan(x)\end{aligned}$$

Mathematica [A] time = 0.005882, size = 16, normalized size = 1.

$$\frac{1}{2}\tan(x)\sec(x)-\frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Tan[x]^2,x]
```

```
[Out] -ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2
```

Maple [A] time = 0.008, size = 24, normalized size = 1.5

$$\frac{(\sin(x))^3}{2\ (\cos(x))^2}+\frac{\sin(x)}{2}-\frac{\ln(\sec(x)+\tan(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)^2,x)
```

```
[Out] 1/2*sin(x)^3/cos(x)^2+1/2*sin(x)-1/2*ln(sec(x)+tan(x))
```

Maxima [B] time = 0.928819, size = 36, normalized size = 2.25

$$-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)}-\frac{1}{4}\log(\sin(x)+1)+\frac{1}{4}\log(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sin(x)/(sin(x)^2 - 1) - 1/4*log(sin(x) + 1) + 1/4*log(sin(x) - 1)
```

Fricas [B] time = 2.17193, size = 109, normalized size = 6.81

$$-\frac{\cos(x)^2\log(\sin(x)+1)-\cos(x)^2\log(-\sin(x)+1)-2\ \sin(x)}{4\ \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) - 2*sin(x))/cos(
x)^2
```

Sympy [A] time = 0.109246, size = 27, normalized size = 1.69

$$\frac{\log\left(\sin\left(x\right)-1\right)}{4}-\frac{\log\left(\sin\left(x\right)+1\right)}{4}-\frac{\sin\left(x\right)}{2\sin^{2}\left(x\right)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)**2,x)
```

```
[Out] log(sin(x) - 1)/4 - log(sin(x) + 1)/4 - sin(x)/(2*sin(x)**2 - 2)
```

Giac [B] time = 1.05921, size = 39, normalized size = 2.44

$$-\frac{\sin\left(x\right)}{2\left(\sin\left(x\right)^{2}-1\right)}-\frac{1}{4}\log\left(\sin\left(x\right)+1\right)+\frac{1}{4}\log\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="giac")
```

```
[Out] -1/2*sin(x)/(sin(x)^2 - 1) - 1/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1)
```

3.98 $\int \cot^2(x)\,dx$

Optimal. Leaf size=8

$$-x-\cot(x)$$

```
[Out] -x - Cot[x]
```

Rubi [A] time = 0.0052417, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 8}

$$-x-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2,x]
```

```
[Out] -x - Cot[x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(x)\,dx &= -\cot(x)-\int 1\,dx\\ &= -x-\cot(x)\end{aligned}$$

Mathematica [A] time = 0.0022213, size = 8, normalized size = 1.

$$-x-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2,x]
```

```
[Out] -x - Cot[x]
```

Maple [A] time = 0.003, size = 12, normalized size = 1.5

$$-\cot(x)+\frac{\pi}{2}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2,x)
```

```
[Out] -cot(x)+1/2*Pi-x
```

Maxima [A] time = 1.40877, size = 14, normalized size = 1.75

$$-x-\frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2,x, algorithm="maxima")
```

```
[Out] -x - 1/tan(x)
```

Fricas [B] time = 2.09818, size = 53, normalized size = 6.62

$$-\frac{x\sin(2x)+\cos(2x)+1}{\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2,x, algorithm="fricas")
```

```
[Out] -(x*sin(2*x) + cos(2*x) + 1)/sin(2*x)
```

Sympy [A] time = 0.059427, size = 8, normalized size = 1.

$$-x - \frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2,x)
```

```
[Out] -x - cos(x)/sin(x)
```

Giac [B] time = 1.06771, size = 24, normalized size = 3.

$$-x - \frac{1}{2\tan\left(\frac{1}{2}x\right)} + \frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2,x, algorithm="giac")
```

```
[Out] -x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

3.99 $\int \cot^3(x)\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2}\cot^2(x)-\log(\sin(x))$$

```
[Out] -Cot[x]^2/2 - Log[Sin[x]]
```

Rubi [A] time = 0.0074278, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 3475}

$$-\frac{1}{2}\cot^2(x)-\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3,x]
```

```
[Out] -Cot[x]^2/2 - Log[Sin[x]]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^3(x)\,dx &= -\frac{1}{2}\cot^2(x)-\int \cot(x)\,dx\\ &= -\frac{1}{2}\cot^2(x)-\log(\sin(x))\end{aligned}$$

Mathematica [A] time = 0.0030025, size = 14, normalized size = 1.

$$-\frac{1}{2}\csc^2(x)-\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3,x]
```

```
[Out] -Csc[x]^2/2 - Log[Sin[x]]
```

Maple [A] time = 0.002, size = 17, normalized size = 1.2

$$-\frac{(\cot(x))^2}{2}+\frac{\ln\left((\cot(x))^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3,x)
```

```
[Out] -1/2*cot(x)^2+1/2*ln(cot(x)^2+1)
```

Maxima [A] time = 0.930429, size = 19, normalized size = 1.36

$$-\frac{1}{2\sin(x)^2}-\frac{1}{2}\log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/sin(x)^2 - 1/2*log(sin(x)^2)
```

Fricas [B] time = 1.97195, size = 90, normalized size = 6.43

$$-\frac{(\cos(2x)-1)\log\left(-\frac{1}{2}\cos(2x)+\frac{1}{2}\right)-2}{2(\cos(2x)-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((cos(2*x) - 1)*log(-1/2*cos(2*x) + 1/2) - 2)/(cos(2*x) - 1)
```

Sympy [A] time = 0.087574, size = 14, normalized size = 1.

$$-\log(\sin(x)) - \frac{1}{2\sin^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3,x)
```

```
[Out] -log(sin(x)) - 1/(2*sin(x)**2)
```

Giac [A] time = 1.06149, size = 30, normalized size = 2.14

$$\frac{1}{2\left(\cos(x)^2 - 1\right)} - \frac{1}{2}\log\left(-\cos(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3,x, algorithm="giac")
```

```
[Out] 1/2/(cos(x)^2 - 1) - 1/2*log(-cos(x)^2 + 1)
```

3.100 $\int \cot^4(x)\csc^4(x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{7}\cot^7(x)-\frac{\cot^5(x)}{5}$$

```
[Out] -Cot[x]^5/5 - Cot[x]^7/7
```

Rubi [A] time = 0.0260087, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 14}

$$-\frac{1}{7}\cot^7(x)-\frac{\cot^5(x)}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4*Csc[x]^4,x]

[Out] -Cot[x]^5/5 - Cot[x]^7/7
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cot^4(x)\csc^4(x)\,dx &= \text{Subst}\left(\int x^4\left(1+x^2\right)\,dx,x,-\cot(x)\right)\\ &= \text{Subst}\left(\int \left(x^4+x^6\right)\,dx,x,-\cot(x)\right)\\ &= -\frac{1}{5}\cot^5(x)-\frac{\cot^7(x)}{7}\end{aligned}$$

Mathematica [B] time = 0.0266781, size = 37, normalized size = 2.18

$$-\frac{2\cot(x)}{35}-\frac{1}{7}\cot(x)\csc^6(x)+\frac{8}{35}\cot(x)\csc^4(x)-\frac{1}{35}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4*Csc[x]^4,x]
```

```
[Out] (-2*Cot[x])/35 - (Cot[x]*Csc[x]^2)/35 + (8*Cot[x]*Csc[x]^4)/35 - (Cot[x]*Cs
c[x]^6)/7
```

Maple [A] time = 0.013, size = 22, normalized size = 1.3

$$-\frac{(\cos(x))^5}{7\,(\sin(x))^7}-\frac{2\,(\cos(x))^5}{35\,(\sin(x))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4*csc(x)^4,x)
```

```
[Out] -1/7/sin(x)^7*cos(x)^5-2/35/sin(x)^5*cos(x)^5
```

Maxima [A] time = 0.933675, size = 19, normalized size = 1.12

$$-\frac{7\,\tan(x)^2+5}{35\,\tan(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*csc(x)^4,x, algorithm="maxima")
```

```
[Out] -1/35*(7*tan(x)^2 + 5)/tan(x)^7
```

Fricas [B] time = 1.95796, size = 112, normalized size = 6.59

$$-\frac{2\cos(x)^7 - 7\cos(x)^5}{35\left(\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*csc(x)^4,x, algorithm="fricas")
```

```
[Out] -1/35*(2*cos(x)^7 - 7*cos(x)^5)/((cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)*s
in(x))
```

Sympy [B] time = 0.066887, size = 41, normalized size = 2.41

$$-\frac{2\cos(x)}{35\sin(x)} - \frac{\cos(x)}{35\sin^3(x)} + \frac{8\cos(x)}{35\sin^5(x)} - \frac{\cos(x)}{7\sin^7(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4*csc(x)**4,x)
```

```
[Out] -2*cos(x)/(35*sin(x)) - cos(x)/(35*sin(x)**3) + 8*cos(x)/(35*sin(x)**5) - c
os(x)/(7*sin(x)**7)
```

Giac [A] time = 1.07736, size = 19, normalized size = 1.12

$$-\frac{7\tan(x)^2 + 5}{35\tan(x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4*csc(x)^4,x, algorithm="giac")
```

```
[Out] -1/35*(7*tan(x)^2 + 5)/tan(x)^7
```

3.101 $\int \cot^3(x)\csc^4(x)\,dx$

Optimal. Leaf size=17

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Rubi [A] time = 0.0259722, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$\frac{\csc^4(x)}{4}-\frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3*Csc[x]^4,x]
```

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cot^3(x)\csc^4(x)\,dx = -\operatorname{Subst}\left(\int x^3\left(-1+x^2\right)dx, x, \csc(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(-x^3+x^5\right)dx, x, \csc(x)\right)$$
$$= \frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

Mathematica [A] time = 0.0073385, size = 17, normalized size = 1.

$$\frac{\csc^4(x)}{4} - \frac{\csc^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3*Csc[x]^4,x]
```

```
[Out] Csc[x]^4/4 - Csc[x]^6/6
```

Maple [A] time = 0.013, size = 22, normalized size = 1.3

$$-\frac{(\cos(x))^4}{6(\sin(x))^6} - \frac{(\cos(x))^4}{12(\sin(x))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3*csc(x)^4,x)
```

```
[Out] -1/6/sin(x)^6*cos(x)^4-1/12/sin(x)^4*cos(x)^4
```

Maxima [A] time = 0.940499, size = 19, normalized size = 1.12

$$\frac{3\sin(x)^2 - 2}{12\sin(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x)^4,x, algorithm="maxima")
```

```
[Out] 1/12*(3*sin(x)^2 - 2)/sin(x)^6
```

Fricas [B] time = 2.01787, size = 86, normalized size = 5.06

$$\frac{3\cos(x)^2 - 1}{12\left(\cos(x)^6 - 3\cos(x)^4 + 3\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x)^4,x, algorithm="fricas")
```

```
[Out] 1/12*(3*cos(x)^2 - 1)/(cos(x)^6 - 3*cos(x)^4 + 3*cos(x)^2 - 1)
```

Sympy [A] time = 0.103146, size = 14, normalized size = 0.82

$$\frac{3\sin^2(x) - 2}{12\sin^6(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3*csc(x)**4,x)
```

```
[Out] (3*sin(x)**2 - 2)/(12*sin(x)**6)
```

Giac [A] time = 1.05266, size = 24, normalized size = 1.41

$$\frac{3\cos(x)^2 - 1}{12\left(\cos(x)^2 - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3*csc(x)^4,x, algorithm="giac")
```

```
[Out] 1/12*(3*cos(x)^2 - 1)/(cos(x)^2 - 1)^3
```

3.102 $\int \csc(x)\,dx$

Optimal. Leaf size=5

$$-\tanh^{-1}(\cos(x))$$

```
[Out] -ArcTanh[Cos[x]]
```

Rubi [A] time = 0.0022161, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3770}

$$-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x],x]
```

```
[Out] -ArcTanh[Cos[x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(x)\,dx = -\tanh^{-1}(\cos(x))$$

Mathematica [B] time = 0.0025744, size = 17, normalized size = 3.4

$$\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x],x]
```

```
[Out] -Log[Cos[x/2]] + Log[Sin[x/2]]
```

Maple [A] time = 0.002, size = 9, normalized size = 1.8

$$-\ln(\csc(x) + \cot(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x),x)
```

```
[Out] -ln(csc(x)+cot(x))
```

Maxima [A] time = 0.930988, size = 11, normalized size = 2.2

$$-\log(\cot(x) + \csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x),x, algorithm="maxima")
```

```
[Out] -log(cot(x) + csc(x))
```

Fricas [B] time = 2.15113, size = 77, normalized size = 15.4

$$-\frac{1}{2}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [B] time = 0.093246, size = 15, normalized size = 3.

$$\frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x),x)
```

```
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2
```

Giac [A] time = 1.07252, size = 8, normalized size = 1.6

$$\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x),x, algorithm="giac")
```

```
[Out] log(abs(tan(1/2*x)))
```

3.103 $\int \csc^3(x)\,dx$

Optimal. Leaf size=16

$$-\frac{1}{2}\tanh^{-1}(\cos(x))-\frac{1}{2}\cot(x)\csc(x)$$

```
[Out] -ArcTanh[Cos[x]]/2 - (Cot[x]*Csc[x])/2
```

Rubi [A] time = 0.006974, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3768, 3770}

$$-\frac{1}{2}\tanh^{-1}(\cos(x))-\frac{1}{2}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3,x]
```

```
[Out] -ArcTanh[Cos[x]]/2 - (Cot[x]*Csc[x])/2
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3(x)\,dx &= -\frac{1}{2}\cot(x)\csc(x)+\frac{1}{2}\int \csc(x)\,dx\\ &= -\frac{1}{2}\tanh^{-1}(\cos(x))-\frac{1}{2}\cot(x)\csc(x)\end{aligned}$$

Mathematica [B] time = 0.0045829, size = 47, normalized size = 2.94

$$-\frac{1}{8}\csc^2\left(\frac{x}{2}\right)+\frac{1}{8}\sec^2\left(\frac{x}{2}\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3,x]
```

```
[Out] -Csc[x/2]^2/8 - Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2 + Sec[x/2]^2/8
```

Maple [A] time = 0.031, size = 18, normalized size = 1.1

$$-\frac{\cot(x)\csc(x)}{2}+\frac{\ln(\csc(x)-\cot(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3,x)
```

```
[Out] -1/2*cot(x)*csc(x)+1/2*ln(csc(x)-cot(x))
```

Maxima [B] time = 0.932211, size = 36, normalized size = 2.25

$$\frac{\cos(x)}{2\left(\cos(x)^2-1\right)}-\frac{1}{4}\log(\cos(x)+1)+\frac{1}{4}\log(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*cos(x)/(cos(x)^2 - 1) - 1/4*log(cos(x) + 1) + 1/4*log(cos(x) - 1)
```

Fricas [B] time = 2.04442, size = 150, normalized size = 9.38

$$-\frac{\left(\cos(x)^2-1\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-\left(\cos(x)^2-1\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2\cos(x)}{4\left(\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((cos(x)^2 - 1)*log(1/2*cos(x) + 1/2) - (cos(x)^2 - 1)*log(-1/2*cos(x)
 + 1/2) - 2*cos(x))/(cos(x)^2 - 1)
```

Sympy [A] time = 0.112021, size = 27, normalized size = 1.69

$$\frac{\log\left(\cos\left(x\right)-1\right)}{4}-\frac{\log\left(\cos\left(x\right)+1\right)}{4}+\frac{\cos\left(x\right)}{2\cos^{2}\left(x\right)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3,x)
```

```
[Out] log(cos(x) - 1)/4 - log(cos(x) + 1)/4 + cos(x)/(2*cos(x)**2 - 2)
```

Giac [B] time = 1.06475, size = 73, normalized size = 4.56

$$-\frac{\left(\frac{2\left(\cos(x)-1\right)}{\cos(x)+1}-1\right)\left(\cos\left(x\right)+1\right)}{8\left(\cos\left(x\right)-1\right)}-\frac{\cos\left(x\right)-1}{8\left(\cos\left(x\right)+1\right)}+\frac{1}{4}\log\left(-\frac{\cos\left(x\right)-1}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3,x, algorithm="giac")
```

```
[Out] -1/8*(2*(cos(x) - 1)/(cos(x) + 1) - 1)*(cos(x) + 1)/(cos(x) - 1) - 1/8*(cos
(x) - 1)/(cos(x) + 1) + 1/4*log(-(cos(x) - 1)/(cos(x) + 1))
```

3.104 $\int \cos(x)\cot(x)\,dx$

Optimal. Leaf size=8

$$\cos(x) - \tanh^{-1}(\cos(x))$$

```
[Out] -ArcTanh[Cos[x]] + Cos[x]
```

Rubi [A] time = 0.0107351, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {2592, 321, 206}

$$\cos(x) - \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cot[x],x]
```

```
[Out] -ArcTanh[Cos[x]] + Cos[x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \cos(x)\cot(x)\,dx &= -\operatorname{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \cos(x)\right)\\
&= \cos(x) - \operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(x)\right)\\
&= -\tanh^{-1}(\cos(x)) + \cos(x)
\end{aligned}
$$

Mathematica [B] time = 0.0041813, size = 19, normalized size = 2.38

$$\cos(x) + \log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cot[x],x]
```

```
[Out] Cos[x] - Log[Cos[x/2]] + Log[Sin[x/2]]
```

Maple [A] time = 0.007, size = 12, normalized size = 1.5

$$\cos(x) + \ln(\csc(x) - \cot(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/sin(x),x)
```

```
[Out] cos(x)+ln(csc(x)-cot(x))
```

Maxima [B] time = 0.923784, size = 23, normalized size = 2.88

$$\cos(x) - \frac{1}{2}\log(\cos(x) + 1) + \frac{1}{2}\log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/sin(x),x, algorithm="maxima")
```

```
[Out] cos(x) - 1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)
```

Fricas [B] time = 2.15307, size = 88, normalized size = 11.

$$\cos(x) - \frac{1}{2}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/sin(x),x, algorithm="fricas")
```

```
[Out] cos(x) - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [B] time = 0.093097, size = 19, normalized size = 2.38

$$\frac{\log(\cos(x) - 1)}{2} - \frac{\log(\cos(x) + 1)}{2} + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/sin(x),x)
```

```
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2 + cos(x)
```

Giac [B] time = 1.05468, size = 26, normalized size = 3.25

$$\cos(x) - \frac{1}{2}\log(\cos(x) + 1) + \frac{1}{2}\log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/sin(x),x, algorithm="giac")
```

```
[Out] cos(x) - 1/2*log(cos(x) + 1) + 1/2*log(-cos(x) + 1)
```

3.105 $\int \csc^4(x)\,dx$

Optimal. Leaf size=13

$$-\frac{1}{3}\cot^3(x)-\cot(x)$$

```
[Out] -Cot[x] - Cot[x]^3/3
```

Rubi [A] time = 0.0065964, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3767}

$$-\frac{1}{3}\cot^3(x)-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^4,x]
```

```
[Out] -Cot[x] - Cot[x]^3/3
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^4(x)\,dx &= -\operatorname{Subst}\left(\int \left(1+x^2\right)\,dx, x, \cot(x)\right)\\ &= -\cot(x)-\frac{\cot^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.0023914, size = 17, normalized size = 1.31

$$-\frac{2\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4,x]
```

```
[Out] (-2*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

Maple [A] time = 0.026, size = 12, normalized size = 0.9

$$\left(-\frac{2}{3}-\frac{\left(\csc\left(x\right)\right)^{2}}{3}\right)\cot\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x)^4,x)
```

```
[Out] (-2/3-1/3*csc(x)^2)*cot(x)
```

Maxima [A] time = 0.929989, size = 19, normalized size = 1.46

$$-\frac{3\,\tan\left(x\right)^{2}+1}{3\,\tan\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*tan(x)^2 + 1)/tan(x)^3
```

Fricas [B] time = 1.96192, size = 73, normalized size = 5.62

$$-\frac{2\,\cos\left(x\right)^{3}-3\,\cos\left(x\right)}{3\left(\cos\left(x\right)^{2}-1\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(2*cos(x)^3 - 3*cos(x))/((cos(x)^2 - 1)*sin(x))
```

Sympy [A] time = 0.057034, size = 20, normalized size = 1.54

$$-\frac{2\cos(x)}{3\sin(x)} - \frac{\cos(x)}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)**4,x)
```

```
[Out] -2*cos(x)/(3*sin(x)) - cos(x)/(3*sin(x)**3)
```

Giac [A] time = 1.05834, size = 19, normalized size = 1.46

$$-\frac{3\tan(x)^2 + 1}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x)^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*tan(x)^2 + 1)/tan(x)^3
```

3.106 $\int \sin(2x)\sin(5x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\sin(3x)-\frac{1}{14}\sin(7x)$$

```
[Out] Sin[3*x]/6 - Sin[7*x]/14
```

Rubi [A] time = 0.0084189, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4282}

$$\frac{1}{6}\sin(3x)-\frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]*Sin[5*x],x]
```

```
[Out] Sin[3*x]/6 - Sin[7*x]/14
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(2x)\sin(5x)\,dx=\frac{1}{6}\sin(3x)-\frac{1}{14}\sin(7x)$$

Mathematica [A] time = 0.0065277, size = 17, normalized size = 1.

$$\frac{1}{6}\sin(3x)-\frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]*Sin[5*x],x]
```

```
[Out] Sin[3*x]/6 - Sin[7*x]/14
```

Maple [A] time = 0.046, size = 14, normalized size = 0.8

$$\frac{\sin(3x)}{6} - \frac{\sin(7x)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*sin(5*x),x)
```

```
[Out] 1/6*sin(3*x)-1/14*sin(7*x)
```

Maxima [A] time = 0.933001, size = 18, normalized size = 1.06

$$-\frac{1}{14}\sin(7x) + \frac{1}{6}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*sin(5*x),x, algorithm="maxima")
```

```
[Out] -1/14*sin(7*x) + 1/6*sin(3*x)
```

Fricas [A] time = 2.08164, size = 81, normalized size = 4.76

$$-\frac{2}{21}\left(48\cos(x)^6 - 60\cos(x)^4 + 11\cos(x)^2 + 1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*sin(5*x),x, algorithm="fricas")
```

```
[Out] -2/21*(48*cos(x)^6 - 60*cos(x)^4 + 11*cos(x)^2 + 1)*sin(x)
```

Sympy [B] time = 0.503333, size = 26, normalized size = 1.53

$$-\frac{5\sin(2x)\cos(5x)}{21}+\frac{2\sin(5x)\cos(2x)}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*sin(5*x),x)
```

```
[Out] -5*sin(2*x)*cos(5*x)/21 + 2*sin(5*x)*cos(2*x)/21
```

Giac [A] time = 1.04559, size = 18, normalized size = 1.06

$$-\frac{1}{14}\sin(7x)+\frac{1}{6}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*sin(5*x),x, algorithm="giac")
```

```
[Out] -1/14*sin(7*x) + 1/6*sin(3*x)
```

3.107 $\int \cos(x)\sin(3x)\,dx$

Optimal. Leaf size=17

$$-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

```
[Out] -Cos[2*x]/4 - Cos[4*x]/8
```

Rubi [A] time = 0.0075752, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4284}

$$-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[3*x],x]
```

```
[Out] -Cos[2*x]/4 - Cos[4*x]/8
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\sin(3x)\,dx=-\frac{1}{4}\cos(2x)-\frac{1}{8}\cos(4x)$$

Mathematica [A] time = 0.0051436, size = 17, normalized size = 1.

$$-\frac{1}{2}\cos^2(x)-\frac{1}{8}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[3*x],x]
```

```
[Out] -Cos[x]^2/2 - Cos[4*x]/8
```

Maple [A] time = 0.029, size = 14, normalized size = 0.8

$$-(\cos(x))^4+\frac{(\cos(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(3*x),x)
```

```
[Out] -cos(x)^4+1/2*cos(x)^2
```

Maxima [A] time = 0.930472, size = 18, normalized size = 1.06

$$-\frac{1}{8}\cos(4x)-\frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] -1/8*cos(4*x) - 1/4*cos(2*x)
```

Fricas [A] time = 1.93429, size = 35, normalized size = 2.06

$$-\cos(x)^4+\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] -cos(x)^4 + 1/2*cos(x)^2
```

Sympy [A] time = 0.512396, size = 22, normalized size = 1.29

$$-\frac{\sin(x)\sin(3x)}{8}-\frac{3\cos(x)\cos(3x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x)
```

```
[Out] -sin(x)*sin(3*x)/8 - 3*cos(x)*cos(3*x)/8
```

Giac [A] time = 1.07167, size = 18, normalized size = 1.06

$$-\frac{1}{8}\cos(4x)-\frac{1}{4}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -1/8*cos(4*x) - 1/4*cos(2*x)
```

3.108 $\int \cos(3x)\cos(4x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

```
[Out] Sin[x]/2 + Sin[7*x]/14
```

Rubi [A] time = 0.00798, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4283}

$$\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]*Cos[4*x],x]

[Out] Sin[x]/2 + Sin[7*x]/14
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(3x)\cos(4x)\,dx=\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

Mathematica [A] time = 0.0055277, size = 15, normalized size = 1.

$$\frac{\sin(x)}{2}+\frac{1}{14}\sin(7x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]*Cos[4*x],x]
```

```
[Out] Sin[x]/2 + Sin[7*x]/14
```

Maple [A] time = 0.049, size = 12, normalized size = 0.8

$$\frac{\sin(x)}{2} + \frac{\sin(7x)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*cos(4*x),x)
```

```
[Out] 1/2*sin(x)+1/14*sin(7*x)
```

Maxima [A] time = 0.924837, size = 15, normalized size = 1.

$$\frac{1}{14}\sin(7x) + \frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="maxima")
```

```
[Out] 1/14*sin(7*x) + 1/2*sin(x)
```

Fricas [B] time = 2.12962, size = 78, normalized size = 5.2

$$\frac{1}{7}\left(32\cos(x)^6 - 40\cos(x)^4 + 12\cos(x)^2 + 3\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="fricas")
```

```
[Out] 1/7*(32*cos(x)^6 - 40*cos(x)^4 + 12*cos(x)^2 + 3)*sin(x)
```

Sympy [B] time = 0.493335, size = 26, normalized size = 1.73

$$-\frac{3\sin(3x)\cos(4x)}{7}+\frac{4\sin(4x)\cos(3x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x)
```

```
[Out] -3*sin(3*x)*cos(4*x)/7 + 4*sin(4*x)*cos(3*x)/7
```

Giac [A] time = 1.05369, size = 15, normalized size = 1.

$$\frac{1}{14}\sin(7x)+\frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(4*x),x, algorithm="giac")
```

```
[Out] 1/14*sin(7*x) + 1/2*sin(x)
```

3.109 $\int \sin(3x)\sin(6x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{6}\sin(3x)-\frac{1}{18}\sin(9x)$$

```
[Out] Sin[3*x]/6 - Sin[9*x]/18
```

Rubi [A] time = 0.0082184, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4282}

$$\frac{1}{6}\sin(3x)-\frac{1}{18}\sin(9x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[3*x]*Sin[6*x],x]
```

```
[Out] Sin[3*x]/6 - Sin[9*x]/18
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(3x)\sin(6x)\,dx=\frac{1}{6}\sin(3x)-\frac{1}{18}\sin(9x)$$

Mathematica [A] time = 0.0064867, size = 17, normalized size = 1.

$$\frac{1}{6}\sin(3x)-\frac{1}{18}\sin(9x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[3*x]*Sin[6*x],x]
```

```
[Out] Sin[3*x]/6 - Sin[9*x]/18
```

Maple [A] time = 0.01, size = 9, normalized size = 0.5

$$\frac{2\,(\sin(3\,x))^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)*sin(6*x),x)
```

```
[Out] 2/9*sin(3*x)^3
```

Maxima [A] time = 0.938837, size = 18, normalized size = 1.06

$$-\frac{1}{18}\,\sin(9\,x)+\frac{1}{6}\,\sin(3\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(6*x),x, algorithm="maxima")
```

```
[Out] -1/18*sin(9*x) + 1/6*sin(3*x)
```

Fricas [A] time = 2.10254, size = 43, normalized size = 2.53

$$-\frac{2}{9}\left(\cos(3\,x)^2-1\right)\sin(3\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(6*x),x, algorithm="fricas")
```

```
[Out] -2/9*(cos(3*x)^2 - 1)*sin(3*x)
```

Sympy [A] time = 0.53335, size = 24, normalized size = 1.41

$$-\frac{2\sin(3x)\cos(6x)}{9} + \frac{\sin(6x)\cos(3x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(6*x),x)
```

```
[Out] -2*sin(3*x)*cos(6*x)/9 + sin(6*x)*cos(3*x)/9
```

Giac [A] time = 1.05213, size = 18, normalized size = 1.06

$$-\frac{1}{18}\sin(9x) + \frac{1}{6}\sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(3*x)*sin(6*x),x, algorithm="giac")
```

```
[Out] -1/18*sin(9*x) + 1/6*sin(3*x)
```

3.110 $\int \cos^5(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$-\frac{1}{6}\cos^6(x)$$

[Out] -Cos[x]^6/6

Rubi [A] time = 0.0125415, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2565, 30}

$$-\frac{1}{6}\cos^6(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^5*Sin[x],x]

[Out] -Cos[x]^6/6

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^5(x)\sin(x)\,dx &= -\text{Subst}\left(\int x^5\,dx, x, \cos(x)\right)\\ &= -\frac{1}{6}\cos^6(x)\end{aligned}$$

Mathematica [A] time = 0.0012081, size = 8, normalized size = 1.

$$-\frac{1}{6}\cos^6(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5*Sin[x],x]
```

```
[Out] -Cos[x]^6/6
```

Maple [A] time = 0.001, size = 7, normalized size = 0.9

$$-\frac{(\cos(x))^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5*sin(x),x)
```

```
[Out] -1/6*cos(x)^6
```

Maxima [A] time = 0.927721, size = 8, normalized size = 1.

$$-\frac{1}{6}\cos(x)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5*sin(x),x, algorithm="maxima")
```

```
[Out] -1/6*cos(x)^6
```

Fricas [A] time = 1.95143, size = 20, normalized size = 2.5

$$-\frac{1}{6}\cos(x)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5*sin(x),x, algorithm="fricas")
```

```
[Out] -1/6*cos(x)^6
```

Sympy [A] time = 0.055917, size = 7, normalized size = 0.88

$$-\frac{\cos^6(x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5*sin(x),x)
```

```
[Out] -cos(x)**6/6
```

Giac [A] time = 1.05263, size = 8, normalized size = 1.

$$-\frac{1}{6}\cos(x)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5*sin(x),x, algorithm="giac")
```

```
[Out] -1/6*cos(x)^6
```

3.111 $\int \cos(x)\cos(2x)\cos(3x)\,dx$

Optimal. Leaf size=30

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rubi [A] time = 0.0306922, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2637}

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\cos(2x)\cos(3x)\,dx &= \int \left(\frac{1}{4}+\frac{1}{4}\cos(2x)+\frac{1}{4}\cos(4x)+\frac{1}{4}\cos(6x)\right)dx\\ &= \frac{x}{4}+\frac{1}{4}\int \cos(2x)\,dx+\frac{1}{4}\int \cos(4x)\,dx+\frac{1}{4}\int \cos(6x)\,dx\\ &= \frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)\end{aligned}$$

Mathematica [A] time = 0.0084162, size = 30, normalized size = 1.

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Maple [A] time = 0., size = 23, normalized size = 0.8

$$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Maxima [A] time = 0.931044, size = 30, normalized size = 1.

$$\frac{1}{4}x+\frac{1}{24}\sin(6x)+\frac{1}{16}\sin(4x)+\frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="maxima")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

Fricas [A] time = 1.97036, size = 81, normalized size = 2.7

$$\frac{1}{12}\left(16\cos\left(x\right)^{5}-10\cos\left(x\right)^{3}+3\cos\left(x\right)\right)\sin\left(x\right)+\frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="fricas")
```

```
[Out] 1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x
```

Sympy [B] time = 14.4865, size = 112, normalized size = 3.73

$$-\frac{x\sin\left(x\right)\sin\left(2x\right)\cos\left(3x\right)}{4}+\frac{x\sin\left(x\right)\sin\left(3x\right)\cos\left(2x\right)}{4}+\frac{x\sin\left(2x\right)\sin\left(3x\right)\cos\left(x\right)}{4}+\frac{x\cos\left(x\right)\cos\left(2x\right)\cos\left(3x\right)}{4}+\frac{\sin(x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*s
in(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + sin(x)*sin(2*x)*sin(3*x)/
24 - sin(2*x)*cos(x)*cos(3*x)/8 + sin(3*x)*cos(x)*cos(2*x)/3
```

Giac [A] time = 1.06672, size = 30, normalized size = 1.

$$\frac{1}{4}x+\frac{1}{24}\sin\left(6x\right)+\frac{1}{16}\sin\left(4x\right)+\frac{1}{8}\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="giac")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

3.112 $\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx$

Optimal. Leaf size=5

$$\sin(x)\cos(x)$$

```
[Out] Cos[x]*Sin[x]
```

Rubi [A] time = 0.0220327, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3675, 383}

$$\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*(1 - Tan[x]^2),x]
```

```
[Out] Cos[x]*Sin[x]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p
, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && In
tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
|| EqQ[n^2, 16])
```

Rule 383

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> S
imp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[
b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]
```

Rubi steps

$$\int \cos^2(x)\left(1-\tan^2(x)\right)\,dx = \text{Subst}\left(\int \frac{1-x^2}{\left(1+x^2\right)^2}\,dx, x, \tan(x)\right)$$
$$= \cos(x)\sin(x)$$

<u>Mathematica</u> [A] time = 0.0016832, size = 8, normalized size = 1.6

$$\frac{1}{2}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*(1 - Tan[x]^2),x]
```

```
[Out] Sin[2*x]/2
```

<u>Maple</u> [A] time = 0.016, size = 6, normalized size = 1.2

$$\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-tan(x)^2)/sec(x)^2,x)
```

```
[Out] cos(x)*sin(x)
```

<u>Maxima</u> [B] time = 0.930044, size = 15, normalized size = 3.

$$\frac{\tan(x)}{\tan(x)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tan(x)^2)/sec(x)^2,x, algorithm="maxima")
```

```
[Out] tan(x)/(tan(x)^2 + 1)
```

Fricas [A] time = 2.12409, size = 20, normalized size = 4.

$$\cos(x)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tan(x)^2)/sec(x)^2,x, algorithm="fricas")
```

```
[Out] cos(x)*sin(x)
```

Sympy [A] time = 0.414073, size = 7, normalized size = 1.4

$$\frac{\tan(x)}{\sec^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tan(x)**2)/sec(x)**2,x)
```

```
[Out] tan(x)/sec(x)**2
```

Giac [A] time = 1.06252, size = 12, normalized size = 2.4

$$\frac{1}{\frac{1}{\tan(x)} + \tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-tan(x)^2)/sec(x)^2,x, algorithm="giac")
```

```
[Out] 1/(1/tan(x) + tan(x))
```

3.113 $\int \csc(2x)(\cos(x) + \sin(x))\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\tanh^{-1}(\sin(x)) - \frac{1}{2}\tanh^{-1}(\cos(x))$$

```
[Out] -ArcTanh[Cos[x]]/2 + ArcTanh[Sin[x]]/2
```

Rubi [A] time = 0.05349, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {4401, 4287, 3770, 4288}

$$\frac{1}{2}\tanh^{-1}(\sin(x)) - \frac{1}{2}\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[2*x]*(Cos[x] + Sin[x]),x]
```

```
[Out] -ArcTanh[Cos[x]]/2 + ArcTanh[Sin[x]]/2
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4288

```
Int[((f_.)*sin[(a_.) + (b_.)*(x_)])^(n_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/f^p, Int[Cos[a + b*x]^p*(f*Sin[a + b*x])^(n + p), x], x
```

```
] /; FreeQ[{a, b, c, d, f, n}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \csc(2x)(\cos(x)+\sin(x))\,dx &= \int (\cos(x)\csc(2x)+\csc(2x)\sin(x))\,dx\\ &= \int \cos(x)\csc(2x)\,dx+\int \csc(2x)\sin(x)\,dx\\ &= \frac{1}{2}\int \csc(x)\,dx+\frac{1}{2}\int \sec(x)\,dx\\ &= -\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\tanh^{-1}(\sin(x))\end{aligned}$$

Mathematica [B] time = 0.0093158, size = 61, normalized size = 4.07

$$\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[2*x]*(Cos[x] + Sin[x]),x]
```

```
[Out] -Log[Cos[x/2]]/2 - Log[Cos[x/2] - Sin[x/2]]/2 + Log[Sin[x/2]]/2 + Log[Cos[x
/2] + Sin[x/2]]/2
```

Maple [A] time = 0.049, size = 20, normalized size = 1.3

$$\frac{\ln(\sec(x)+\tan(x))}{2}+\frac{\ln(\csc(x)-\cot(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+sin(x))/sin(2*x),x)
```

```
[Out] 1/2*ln(sec(x)+tan(x))+1/2*ln(csc(x)-cot(x))
```

Maxima [B] time = 1.43778, size = 93, normalized size = 6.2

$$-\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+2\cos(x)+1\right)+\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2-2\cos(x)+1\right)+\frac{1}{4}\log\left(\cos(x)^2+\sin(x)^2+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2
- 2*cos(x) + 1) + 1/4*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - 1/4*log(cos
(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

Fricas [B] time = 2.09555, size = 149, normalized size = 9.93

$$-\frac{1}{4}\log\left(-\frac{1}{2}(\cos(x)+1)\sin(x)+\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\frac{1}{4}\log\left(-\frac{1}{2}(\cos(x)-1)\sin(x)-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/sin(2*x),x, algorithm="fricas")
```

```
[Out] -1/4*log(-1/2*(cos(x) + 1)*sin(x) + 1/2*cos(x) + 1/2) + 1/4*log(-1/2*(cos(x
) - 1)*sin(x) - 1/2*cos(x) + 1/2)
```

Sympy [B] time = 1.05127, size = 32, normalized size = 2.13

$$-\frac{\log(\sin(x)-1)}{4}+\frac{\log(\sin(x)+1)}{4}+\frac{\log(\cos(x)-1)}{4}-\frac{\log(\cos(x)+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/sin(2*x),x)
```

```
[Out] -log(sin(x) - 1)/4 + log(sin(x) + 1)/4 + log(cos(x) - 1)/4 - log(cos(x) + 1
)/4
```

Giac [B] time = 1.10121, size = 39, normalized size = 2.6

$$\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right)-\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)+\frac{1}{2}\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/sin(2*x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(tan(1/2*x) + 1)) - 1/2*log(abs(tan(1/2*x) - 1)) + 1/2*log(abs(t
an(1/2*x)))
```

3.114 $\int \sin^2(x)\tan(x)\,dx$

Optimal. Leaf size=14

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Rubi [A] time = 0.0150215, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2590, 14}

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2*Tan[x],x]
```

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sin^2(x)\tan(x)\,dx = -\operatorname{Subst}\left(\int \frac{1-x^2}{x}\,dx, x, \cos(x)\right)$$
$$= -\operatorname{Subst}\left(\int \left(\frac{1}{x}-x\right)dx, x, \cos(x)\right)$$
$$= \frac{\cos^2(x)}{2} - \log(\cos(x))$$

Mathematica [A] time = 0.0052021, size = 14, normalized size = 1.

$$\frac{\cos^2(x)}{2} - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2*Tan[x],x]
```

```
[Out] Cos[x]^2/2 - Log[Cos[x]]
```

Maple [A] time = 0.011, size = 13, normalized size = 0.9

$$-\frac{(\sin(x))^2}{2} - \ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2*tan(x),x)
```

```
[Out] -1/2*sin(x)^2-ln(cos(x))
```

Maxima [A] time = 0.92659, size = 22, normalized size = 1.57

$$-\frac{1}{2}\sin(x)^2 - \frac{1}{2}\log\left(\sin(x)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="maxima")
```

```
[Out] -1/2*sin(x)^2 - 1/2*log(sin(x)^2 - 1)
```

Fricas [A] time = 2.16732, size = 39, normalized size = 2.79

$$\frac{1}{2}\cos(x)^2 - \log(-\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)^2 - log(-cos(x))
```

Sympy [A] time = 0.076593, size = 10, normalized size = 0.71

$$-\log(\cos(x)) + \frac{\cos^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2*tan(x),x)
```

```
[Out] -log(cos(x)) + cos(x)**2/2
```

Giac [A] time = 1.08055, size = 24, normalized size = 1.71

$$-\frac{1}{2}\sin(x)^2 - \frac{1}{2}\log\left(-\sin(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*tan(x),x, algorithm="giac")
```

```
[Out] -1/2*sin(x)^2 - 1/2*log(-sin(x)^2 + 1)
```

3.115 $\int \cos^2(x)\cot^3(x)\,dx$

Optimal. Leaf size=22

$$\frac{\sin^2(x)}{2}-\frac{1}{2}\csc^2(x)-2\log(\sin(x))$$

```
[Out] -Csc[x]^2/2 - 2*Log[Sin[x]] + Sin[x]^2/2
```

Rubi [A] time = 0.0521154, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2590, 266, 43}

$$\frac{\sin^2(x)}{2}-\frac{1}{2}\csc^2(x)-2\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] -Csc[x]^2/2 - 2*Log[Sin[x]] + Sin[x]^2/2
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\cot^3(x)\,dx &= \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^3}\,dx, x, -\sin(x)\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \frac{(1-x)^2}{x^2}\,dx, x, \sin^2(x)\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(1+\frac{1}{x^2}-\frac{2}{x}\right)dx, x, \sin^2(x)\right)\\ &= -\frac{1}{2}\csc^2(x) - 2\log(\sin(x)) + \frac{\sin^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0117264, size = 20, normalized size = 0.91

$$\frac{1}{2}\left(\sin^2(x) - \csc^2(x) - 4\log(\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (-Csc[x]^2 - 4*Log[Sin[x]] + Sin[x]^2)/2
```

Maple [A] time = 0.013, size = 29, normalized size = 1.3

$$-\frac{(\cos(x))^6}{2(\sin(x))^2} - \frac{(\cos(x))^4}{2} - (\cos(x))^2 - 2\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*cot(x)^3,x)
```

```
[Out] -1/2/sin(x)^2*cos(x)^6-1/2*cos(x)^4-cos(x)^2-2*ln(sin(x))
```

Maxima [A] time = 0.926457, size = 27, normalized size = 1.23

$$\frac{1}{2}\sin(x)^2 - \frac{1}{2\sin(x)^2} - \log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*sin(x)^2 - 1/2/sin(x)^2 - log(sin(x)^2)
```

Fricas [B] time = 2.07412, size = 116, normalized size = 5.27

$$-\frac{2\,\cos\left(x\right)^{4}-3\,\cos\left(x\right)^{2}+8\,\left(\cos\left(x\right)^{2}-1\right)\log\left(\frac{1}{2}\,\sin\left(x\right)\right)-1}{4\,\left(\cos\left(x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*cos(x)^4 - 3*cos(x)^2 + 8*(cos(x)^2 - 1)*log(1/2*sin(x)) - 1)/(cos(
x)^2 - 1)
```

Sympy [A] time = 0.094927, size = 20, normalized size = 0.91

$$-2\log\left(\sin\left(x\right)\right)+\frac{\sin^{2}\left(x\right)}{2}-\frac{1}{2\sin^{2}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*cot(x)**3,x)
```

```
[Out] -2*log(sin(x)) + sin(x)**2/2 - 1/(2*sin(x)**2)
```

Giac [B] time = 1.05987, size = 49, normalized size = 2.23

$$-\frac{1}{2}\,\cos\left(x\right)^{2}+\frac{2\,\cos\left(x\right)^{2}-1}{2\,\left(\cos\left(x\right)^{2}-1\right)}-\log\left(-\cos\left(x\right)^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2 + 1/2*(2*cos(x)^2 - 1)/(cos(x)^2 - 1) - log(-cos(x)^2 + 1)
```

3.116 $\int \sec^3(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^3(x)}{3}$$

```
[Out] Sec[x]^3/3
```

Rubi [A] time = 0.0153974, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 30}

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3*Tan[x],x]
```

```
[Out] Sec[x]^3/3
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^3(x)\tan(x)\,dx &= \text{Subst}\left(\int x^2\,dx, x, \sec(x)\right)\\ &= \frac{\sec^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.0033501, size = 8, normalized size = 1.

$$\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3*Tan[x],x]
```

```
[Out] Sec[x]^3/3
```

Maple [A] time = 0., size = 7, normalized size = 0.9

$$\frac{(\sec(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3*tan(x),x)
```

```
[Out] 1/3*sec(x)^3
```

Maxima [A] time = 0.934566, size = 8, normalized size = 1.

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="maxima")
```

```
[Out] 1/3/cos(x)^3
```

Fricas [A] time = 1.99591, size = 19, normalized size = 2.38

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="fricas")
```

```
[Out] 1/3/cos(x)^3
```

Sympy [A] time = 0.062272, size = 7, normalized size = 0.88

$$\frac{1}{3\cos^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3*tan(x),x)
```

```
[Out] 1/(3*cos(x)**3)
```

Giac [A] time = 1.04856, size = 8, normalized size = 1.

$$\frac{1}{3\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x),x, algorithm="giac")
```

```
[Out] 1/3/cos(x)^3
```

3.117 $\int \sec^3(x)\tan^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rubi [A] time = 0.0255822, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec^3(x)\tan^3(x)\,dx = \text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx, x, \sec(x)\right)$$
$$= \text{Subst}\left(\int \left(-x^2+x^4\right)\,dx, x, \sec(x)\right)$$
$$= -\frac{1}{3}\sec^3(x) + \frac{\sec^5(x)}{5}$$

Mathematica [A] time = 0.0121843, size = 17, normalized size = 1.

$$\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Maple [B] time = 0., size = 42, normalized size = 2.5

$$\frac{(\sin(x))^4}{5\,(\cos(x))^5} + \frac{(\sin(x))^4}{15\,(\cos(x))^3} - \frac{(\sin(x))^4}{15\,\cos(x)} - \frac{\left(2+(\sin(x))^2\right)\cos(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3*tan(x)^3,x)
```

```
[Out] 1/5*sin(x)^4/cos(x)^5+1/15*sin(x)^4/cos(x)^3-1/15*sin(x)^4/cos(x)-1/15*(2+s
in(x)^2)*cos(x)
```

Maxima [A] time = 0.935191, size = 19, normalized size = 1.12

$$-\frac{5\,\cos(x)^2 - 3}{15\,\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Fricas [A] time = 1.91428, size = 45, normalized size = 2.65

$$-\frac{5\cos\left(x\right)^{2}-3}{15\cos\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Sympy [A] time = 0.097077, size = 15, normalized size = 0.88

$$-\frac{5\cos^{2}\left(x\right)-3}{15\cos^{5}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3*tan(x)**3,x)
```

```
[Out] -(5*cos(x)**2 - 3)/(15*cos(x)**5)
```

Giac [A] time = 1.05615, size = 19, normalized size = 1.12

$$-\frac{5\cos\left(x\right)^{2}-3}{15\cos\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="giac")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

3.118 $\int \frac{\sqrt{9-x^2}}{x^2} \, dx$

Optimal. Leaf size=25

$$-\frac{\sqrt{9-x^2}}{x}-\sin^{-1}\left(\frac{x}{3}\right)$$

```
[Out] -(Sqrt[9 - x^2]/x) - ArcSin[x/3]
```

Rubi [A] time = 0.0042819, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {277, 216}

$$-\frac{\sqrt{9-x^2}}{x}-\sin^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 - x^2]/x) - ArcSin[x/3]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx = -\frac{\sqrt{9-x^2}}{x} - \int \frac{1}{\sqrt{9-x^2}}\,dx$$
$$= -\frac{\sqrt{9-x^2}}{x} - \sin^{-1}\left(\frac{x}{3}\right)$$

Mathematica [A] time = 0.0073502, size = 25, normalized size = 1.

$$-\frac{\sqrt{9-x^2}}{x} - \sin^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - x^2]/x^2,x]
```

```
[Out] -(Sqrt[9 - x^2]/x) - ArcSin[x/3]
```

Maple [A] time = 0.003, size = 34, normalized size = 1.4

$$-\frac{1}{9x}\left(-x^2+9\right)^{\frac{3}{2}} - \frac{x}{9}\sqrt{-x^2+9} - \arcsin\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+9)^(1/2)/x^2,x)
```

```
[Out] -1/9/x*(-x^2+9)^(3/2)-1/9*x*(-x^2+9)^(1/2)-arcsin(1/3*x)
```

Maxima [A] time = 1.40731, size = 28, normalized size = 1.12

$$-\frac{\sqrt{-x^2+9}}{x} - \arcsin\left(\frac{1}{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+9)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 9)/x - arcsin(1/3*x)
```

Fricas [A] time = 1.79534, size = 77, normalized size = 3.08

$$\frac{2\,x\arctan\left(\frac{\sqrt{-x^2+9}-3}{x}\right)-\sqrt{-x^2+9}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+9)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] (2*x*arctan((sqrt(-x^2 + 9) - 3)/x) - sqrt(-x^2 + 9))/x
```

Sympy [A] time = 0.226875, size = 15, normalized size = 0.6

$$-\operatorname{asin}\left(\frac{x}{3}\right)-\frac{\sqrt{9-x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+9)**(1/2)/x**2,x)
```

```
[Out] -asin(x/3) - sqrt(9 - x**2)/x
```

Giac [A] time = 1.06067, size = 53, normalized size = 2.12

$$\frac{x}{2\left(\sqrt{-x^2+9}-3\right)}-\frac{\sqrt{-x^2+9}-3}{2\,x}-\arcsin\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+9)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*x/(sqrt(-x^2 + 9) - 3) - 1/2*(sqrt(-x^2 + 9) - 3)/x - arcsin(1/3*x)
```

3.119 $\int \frac{1}{x^2\sqrt{4+x^2}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sqrt{x^2+4}}{4x}$$

[Out] -Sqrt[4 + x^2]/(4*x)

Rubi [A] time = 0.0027984, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {264}

$$-\frac{\sqrt{x^2+4}}{4x}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[4 + x^2]),x]

[Out] -Sqrt[4 + x^2]/(4*x)

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{4+x^2}}\,dx = -\frac{\sqrt{4+x^2}}{4x}$$

Mathematica [A] time = 0.0026909, size = 16, normalized size = 1.

$$-\frac{\sqrt{x^2+4}}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[4 + x^2]),x]
```

```
[Out] -Sqrt[4 + x^2]/(4*x)
```

Maple [A] time = 0.003, size = 13, normalized size = 0.8

$$-\frac{1}{4x}\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(x^2+4)^(1/2),x)
```

```
[Out] -1/4*(x^2+4)^(1/2)/x
```

Maxima [A] time = 1.39813, size = 16, normalized size = 1.

$$-\frac{\sqrt{x^2+4}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(x^2 + 4)/x
```

Fricas [A] time = 2.04558, size = 38, normalized size = 2.38

$$-\frac{x+\sqrt{x^2+4}}{4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(x + sqrt(x^2 + 4))/x
```

Sympy [A] time = 0.711962, size = 12, normalized size = 0.75

$$-\frac{\sqrt{1+\frac{4}{x^2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(x**2+4)**(1/2),x)
```

```
[Out] -sqrt(1 + 4/x**2)/4
```

Giac [A] time = 1.05554, size = 26, normalized size = 1.62

$$\frac{2}{\left(x-\sqrt{x^2+4}\right)^2-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] 2/((x - sqrt(x^2 + 4))^2 - 4)
```

3.120 $\int \frac{x}{\sqrt{4+x^2}} \, dx$

Optimal. Leaf size=9

$$\sqrt{x^2+4}$$

```
[Out] Sqrt[4 + x^2]
```

Rubi [A] time = 0.0016157, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\sqrt{x^2+4}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[4 + x^2],x]
```

```
[Out] Sqrt[4 + x^2]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{4+x^2}} \, dx = \sqrt{4+x^2}$$

Mathematica [A] time = 0.0009114, size = 9, normalized size = 1.

$$\sqrt{x^2+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[4 + x^2],x]
```

```
[Out] Sqrt[4 + x^2]
```

Maple [A] time = 0.003, size = 8, normalized size = 0.9

$$\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+4)^(1/2),x)
```

```
[Out] (x^2+4)^(1/2)
```

Maxima [A] time = 0.924973, size = 9, normalized size = 1.

$$\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 4)
```

Fricas [A] time = 1.79617, size = 20, normalized size = 2.22

$$\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 + 4)
```

Sympy [A] time = 0.125309, size = 7, normalized size = 0.78

$$\sqrt{x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+4)**(1/2),x)
```

```
[Out] sqrt(x**2 + 4)
```

Giac [A] time = 1.05176, size = 9, normalized size = 1.

$$\sqrt{x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 4)
```

3.121 $\int \frac{1}{\sqrt{-a^2+x^2}}\,dx$

Optimal. Leaf size=16

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-a^2}}\right)$$

```
[Out] ArcTanh[x/Sqrt[-a^2 + x^2]]
```

Rubi [A] time = 0.0024596, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {217, 206}

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-a^2 + x^2],x]
```

```
[Out] ArcTanh[x/Sqrt[-a^2 + x^2]]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-a^2+x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-a^2+x^2}}\right)\\ &= \tanh^{-1}\left(\frac{x}{\sqrt{-a^2+x^2}}\right)\end{aligned}$$

Mathematica [B] time = 0.0027682, size = 46, normalized size = 2.88

$$\frac{1}{2}\log\left(\frac{x}{\sqrt{x^2-a^2}}+1\right)-\frac{1}{2}\log\left(1-\frac{x}{\sqrt{x^2-a^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-a^2 + x^2],x]
```

```
[Out] -Log[1 - x/Sqrt[-a^2 + x^2]]/2 + Log[1 + x/Sqrt[-a^2 + x^2]]/2
```

Maple [A] time = 0.001, size = 15, normalized size = 0.9

$$\ln\left(x+\sqrt{-a^2+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2+x^2)^(1/2),x)
```

```
[Out] ln(x+(-a^2+x^2)^(1/2))
```

Maxima [A] time = 0.974328, size = 24, normalized size = 1.5

$$\log\left(2\,x+2\,\sqrt{-a^2+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(-a^2 + x^2))
```

Fricas [A] time = 1.88426, size = 39, normalized size = 2.44

$$-\log\left(-x+\sqrt{-a^2+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(-a^2 + x^2))
```

Sympy [A] time = 1.0204, size = 20, normalized size = 1.25

$$\begin{cases} \operatorname{acosh}\left(\frac{x}{a}\right) & \text{for}\ \frac{|x^2|}{|a^2|} > 1 \\ -i\operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2+x**2)**(1/2),x)
```

```
[Out] Piecewise((acosh(x/a), Abs(x**2)/Abs(a**2) > 1), (-I*asin(x/a), True))
```

Giac [A] time = 1.05721, size = 26, normalized size = 1.62

$$-\log\left(\left|-x+\sqrt{-a^2+x^2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-x + sqrt(-a^2 + x^2)))
```

3.122 $\int \frac{x^3}{(9+4x^2)^{3/2}} \, dx$

Optimal. Leaf size=31

$$\frac{1}{16}\sqrt{4x^2+9}+\frac{9}{16\sqrt{4x^2+9}}$$

```
[Out] 9/(16*Sqrt[9 + 4*x^2]) + Sqrt[9 + 4*x^2]/16
```

Rubi [A] time = 0.0146203, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{16}\sqrt{4x^2+9}+\frac{9}{16\sqrt{4x^2+9}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(9 + 4*x^2)^(3/2),x]
```

```
[Out] 9/(16*Sqrt[9 + 4*x^2]) + Sqrt[9 + 4*x^2]/16
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3}{\left(9+4x^2\right)^{3/2}}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x}{(9+4x)^{3/2}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{9}{4(9+4x)^{3/2}} + \frac{1}{4\sqrt{9+4x}}\right) dx, x, x^2\right)$$
$$= \frac{9}{16\sqrt{9+4x^2}} + \frac{1}{16}\sqrt{9+4x^2}$$

Mathematica [A] time = 0.0076352, size = 22, normalized size = 0.71

$$\frac{2x^2+9}{8\sqrt{4x^2+9}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(9 + 4*x^2)^(3/2),x]
```

```
[Out] (9 + 2*x^2)/(8*Sqrt[9 + 4*x^2])
```

Maple [A] time = 0.003, size = 19, normalized size = 0.6

$$\frac{2\,x^2+9}{8}\frac{1}{\sqrt{4\,x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(4*x^2+9)^(3/2),x)
```

```
[Out] 1/8*(2*x^2+9)/(4*x^2+9)^(1/2)
```

Maxima [A] time = 1.41136, size = 35, normalized size = 1.13

$$\frac{x^2}{4\,\sqrt{4\,x^2+9}} + \frac{9}{8\,\sqrt{4\,x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2+9)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*x^2/sqrt(4*x^2 + 9) + 9/8/sqrt(4*x^2 + 9)
```

Fricas [A] time = 1.82282, size = 45, normalized size = 1.45

$$\frac{2\,x^2+9}{8\,\sqrt{4\,x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2+9)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^2 + 9)/sqrt(4*x^2 + 9)
```

Sympy [A] time = 0.546797, size = 27, normalized size = 0.87

$$\frac{x^2}{4\sqrt{4x^2+9}}+\frac{9}{8\sqrt{4x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(4*x**2+9)**(3/2),x)
```

```
[Out] x**2/(4*sqrt(4*x**2 + 9)) + 9/(8*sqrt(4*x**2 + 9))
```

Giac [A] time = 1.04861, size = 31, normalized size = 1.

$$\frac{1}{16}\,\sqrt{4\,x^2+9}+\frac{9}{16\,\sqrt{4\,x^2+9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(4*x^2+9)^(3/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(4*x^2 + 9) + 9/16/sqrt(4*x^2 + 9)
```

3.123 $\int \frac{x}{\sqrt{3-2x-x^2}}\,dx$

Optimal. Leaf size=27

$$\sin^{-1}\left(\frac{1}{2}(-x-1)\right)-\sqrt{-x^2-2x+3}$$

```
[Out] -Sqrt[3 - 2*x - x^2] + ArcSin[(-1 - x)/2]
```

Rubi [A] time = 0.0104876, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {640, 619, 216}

$$\sin^{-1}\left(\frac{1}{2}(-x-1)\right)-\sqrt{-x^2-2x+3}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[3 - 2*x - x^2],x]
```

```
[Out] -Sqrt[3 - 2*x - x^2] + ArcSin[(-1 - x)/2]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{x}{\sqrt{3-2x-x^2}}\,dx = -\sqrt{3-2x-x^2} - \int \frac{1}{\sqrt{3-2x-x^2}}\,dx$$
$$= -\sqrt{3-2x-x^2} + \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx, x, -2-2x\right)$$
$$= -\sqrt{3-2x-x^2} + \sin^{-1}\left(\frac{1}{2}(-1-x)\right)$$

Mathematica [A] time = 0.0070841, size = 27, normalized size = 1.

$$\sin^{-1}\left(\frac{1}{4}(-2x-2)\right) - \sqrt{-x^2-2x+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[3 - 2*x - x^2],x]
```

```
[Out] -Sqrt[3 - 2*x - x^2] + ArcSin[(-2 - 2*x)/4]
```

Maple [A] time = 0.005, size = 24, normalized size = 0.9

$$-\arcsin\left(\frac{1}{2}+\frac{x}{2}\right) - \sqrt{-x^2-2\,x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2-2*x+3)^(1/2),x)
```

```
[Out] -arcsin(1/2+1/2*x)-(-x^2-2*x+3)^(1/2)
```

Maxima [A] time = 1.40455, size = 28, normalized size = 1.04

$$-\sqrt{-x^2-2\,x+3} + \arcsin\left(-\frac{1}{2}\,x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2-2*x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 - 2*x + 3) + arcsin(-1/2*x - 1/2)
```

Fricas [A] time = 2.02817, size = 105, normalized size = 3.89

$$-\sqrt{-x^2-2\,x+3}+\arctan\left(\frac{\sqrt{-x^2-2\,x+3}(x+1)}{x^2+2\,x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2-2*x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 - 2*x + 3) + arctan(sqrt(-x^2 - 2*x + 3)*(x + 1)/(x^2 + 2*x - 3)
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-\left(x-1\right)\left(x+3\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2-2*x+3)**(1/2),x)
```

```
[Out] Integral(x/sqrt(-(x - 1)*(x + 3)), x)
```

Giac [A] time = 1.06826, size = 31, normalized size = 1.15

$$-\sqrt{-x^2-2\,x+3}-\arcsin\left(\frac{1}{2}\,x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2-2*x+3)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 - 2*x + 3) - arcsin(1/2*x + 1/2)
```

3.124 $\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sqrt{1-x^2}}{x}$$

```
[Out] -(Sqrt[1 - x^2]/x)
```

Rubi [A] time = 0.0031218, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$-\frac{\sqrt{1-x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[1 - x^2]),x]

[Out] -(Sqrt[1 - x^2]/x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx=-\frac{\sqrt{1-x^2}}{x}$$

Mathematica [A] time = 0.002725, size = 16, normalized size = 1.

$$-\frac{\sqrt{1-x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[1 - x^2]),x]
```

```
[Out] -(Sqrt[1 - x^2]/x)
```

Maple [A]   time = 0.003, size = 20, normalized size = 1.3

$$\frac{(-1+x)(1+x)}{x}\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-x^2+1)^(1/2),x)
```

```
[Out] 1/x*(-1+x)*(1+x)/(-x^2+1)^(1/2)
```

Maxima [A]   time = 1.39644, size = 19, normalized size = 1.19

$$-\frac{\sqrt{-x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)/x
```

Fricas [A]   time = 1.84782, size = 26, normalized size = 1.62

$$-\frac{\sqrt{-x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 1)/x
```

Sympy [A] time = 0.708029, size = 27, normalized size = 1.69

$$\begin{cases} -\frac{i\sqrt{x^2-1}}{x} & \text{for } \left|x^2\right| > 1 \\ -\frac{\sqrt{1-x^2}}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-x**2+1)**(1/2),x)
```

```
[Out] Piecewise((-I*sqrt(x**2 - 1)/x, Abs(x**2) > 1), (-sqrt(1 - x**2)/x, True))
```

Giac [B] time = 1.06196, size = 45, normalized size = 2.81

$$\frac{x}{2\left(\sqrt{-x^2+1}-1\right)} - \frac{\sqrt{-x^2+1}-1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*x/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)/x
```

3.125 $\int x^3\sqrt{4-x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{5}\left(4-x^2\right)^{5/2}-\frac{4}{3}\left(4-x^2\right)^{3/2}$$

```
[Out] (-4*(4 - x^2)^(3/2))/3 + (4 - x^2)^(5/2)/5
```

Rubi [A] time = 0.0142777, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{5}\left(4-x^2\right)^{5/2}-\frac{4}{3}\left(4-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[4 - x^2],x]
```

```
[Out] (-4*(4 - x^2)^(3/2))/3 + (4 - x^2)^(5/2)/5
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^3\sqrt{4-x^2}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \sqrt{4-x}x\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(4\sqrt{4-x}-(4-x)^{3/2}\right)\,dx,x,x^2\right)\\ &= -\frac{4}{3}\left(4-x^2\right)^{3/2}+\frac{1}{5}\left(4-x^2\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0094979, size = 22, normalized size = 0.71

$$-\frac{1}{15}\left(4-x^2\right)^{3/2}\left(3x^2+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[4 - x^2],x]
```

```
[Out] -((4 - x^2)^(3/2)*(8 + 3*x^2))/15
```

Maple [A] time = 0.005, size = 25, normalized size = 0.8

$$\frac{(-2+x)\,(2+x)\left(3\,x^2+8\right)}{15}\sqrt{-x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-x^2+4)^(1/2),x)
```

```
[Out] 1/15*(-2+x)*(2+x)*(3*x^2+8)*(-x^2+4)^(1/2)
```

Maxima [A] time = 1.40397, size = 35, normalized size = 1.13

$$-\frac{1}{5}\left(-x^2+4\right)^{\frac{3}{2}}x^2-\frac{8}{15}\left(-x^2+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*(-x^2 + 4)^(3/2)*x^2 - 8/15*(-x^2 + 4)^(3/2)
```

Fricas [A] time = 1.89788, size = 57, normalized size = 1.84

$$\frac{1}{15}\left(3\,x^{4}-4\,x^{2}-32\right)\sqrt{-x^{2}+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^4 - 4*x^2 - 32)*sqrt(-x^2 + 4)
```

Sympy [A] time = 0.568048, size = 39, normalized size = 1.26

$$\frac{x^{4}\sqrt{4-x^{2}}}{5}-\frac{4x^{2}\sqrt{4-x^{2}}}{15}-\frac{32\sqrt{4-x^{2}}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-x**2+4)**(1/2),x)
```

```
[Out] x**4*sqrt(4 - x**2)/5 - 4*x**2*sqrt(4 - x**2)/15 - 32*sqrt(4 - x**2)/15
```

Giac [A] time = 1.05564, size = 41, normalized size = 1.32

$$\frac{1}{5}\left(x^{2}-4\right)^{2}\sqrt{-x^{2}+4}-\frac{4}{3}\left(-x^{2}+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*(x^2 - 4)^2*sqrt(-x^2 + 4) - 4/3*(-x^2 + 4)^(3/2)
```

3.126 $\int \frac{x}{\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=13

$$-\sqrt{1-x^2}$$

```
[Out] -Sqrt[1 - x^2]
```

Rubi [A] time = 0.0024715, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 - x^2],x]

[Out] -Sqrt[1 - x^2]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}$$

Mathematica [A] time = 0.0004805, size = 13, normalized size = 1.

$$-\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 - x^2],x]
```

```
[Out] -Sqrt[1 - x^2]
```

Maple [A] time = 0.003, size = 17, normalized size = 1.3

$$(-1+x)(1+x)\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^(1/2),x)
```

```
[Out] (-1+x)*(1+x)/(-x^2+1)^(1/2)
```

Maxima [A] time = 0.923715, size = 15, normalized size = 1.15

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)
```

Fricas [A] time = 1.81101, size = 23, normalized size = 1.77

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 1)
```

Sympy [A] time = 0.127718, size = 8, normalized size = 0.62

$$-\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(1 - x**2)
```

Giac [A] time = 1.05616, size = 15, normalized size = 1.15

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)
```

3.127 $\int x\sqrt{4-x^2}\,dx$

Optimal. Leaf size=15

$$-\frac{1}{3}\left(4-x^2\right)^{3/2}$$

```
[Out] -(4 - x^2)^(3/2)/3
```

Rubi [A] time = 0.0022245, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\frac{1}{3}\left(4-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[4 - x^2],x]

[Out] -(4 - x^2)^(3/2)/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{4-x^2}\,dx=-\frac{1}{3}\left(4-x^2\right)^{3/2}$$

Mathematica [A] time = 0.0015562, size = 15, normalized size = 1.

$$-\frac{1}{3}\left(4-x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[4 - x^2],x]
```

```
[Out] -(4 - x^2)^(3/2)/3
```

Maple [A] time = 0.003, size = 18, normalized size = 1.2

$$\frac{(-2+x)(2+x)}{3}\sqrt{-x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+4)^(1/2),x)
```

```
[Out] 1/3*(-2+x)*(2+x)*(-x^2+4)^(1/2)
```

Maxima [A] time = 0.919963, size = 15, normalized size = 1.

$$-\frac{1}{3}\left(-x^2+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-x^2 + 4)^(3/2)
```

Fricas [A] time = 2.00834, size = 41, normalized size = 2.73

$$\frac{1}{3}\left(x^2-4\right)\sqrt{-x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^2 - 4)*sqrt(-x^2 + 4)
```

Sympy [B] time = 0.180193, size = 24, normalized size = 1.6

$$\frac{x^2\sqrt{4-x^2}}{3} - \frac{4\sqrt{4-x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+4)**(1/2),x)
```

```
[Out] x**2*sqrt(4 - x**2)/3 - 4*sqrt(4 - x**2)/3
```

Giac [A] time = 1.07207, size = 15, normalized size = 1.

$$-\frac{1}{3}\left(-x^2+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(-x^2 + 4)^(3/2)
```

3.128 $\int \sqrt{1-4x^2}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}\sqrt{1-4x^2}\,x+\frac{1}{4}\sin^{-1}(2x)$$

```
[Out] (x*Sqrt[1 - 4*x^2])/2 + ArcSin[2*x]/4
```

Rubi [A] time = 0.0034196, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 216}

$$\frac{1}{2}\sqrt{1-4x^2}\,x+\frac{1}{4}\sin^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - 4*x^2],x]
```

```
[Out] (x*Sqrt[1 - 4*x^2])/2 + ArcSin[2*x]/4
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-4x^2}\,dx &= \frac{1}{2}x\sqrt{1-4x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1-4x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{1-4x^2}+\frac{1}{4}\sin^{-1}(2x)\end{aligned}$$

Mathematica [A] time = 0.0070851, size = 25, normalized size = 1.

$$\frac{1}{2}\sqrt{1-4x^2}x+\frac{1}{4}\sin^{-1}(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - 4*x^2],x]
```

```
[Out] (x*Sqrt[1 - 4*x^2])/2 + ArcSin[2*x]/4
```

Maple [A] time = 0.004, size = 20, normalized size = 0.8

$$\frac{\arcsin(2x)}{4}+\frac{x}{2}\sqrt{-4x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2+1)^(1/2),x)
```

```
[Out] 1/4*arcsin(2*x)+1/2*x*(-4*x^2+1)^(1/2)
```

Maxima [A] time = 1.40291, size = 26, normalized size = 1.04

$$\frac{1}{2}\sqrt{-4x^2+1}x+\frac{1}{4}\arcsin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-4*x^2 + 1)*x + 1/4*arcsin(2*x)
```

Fricas [A] time = 1.98043, size = 90, normalized size = 3.6

$$\frac{1}{2}\sqrt{-4x^2+1}x-\frac{1}{2}\arctan\left(\frac{\sqrt{-4x^2+1}-1}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-4*x^2 + 1)*x - 1/2*arctan(1/2*(sqrt(-4*x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.186725, size = 19, normalized size = 0.76

$$\frac{x\sqrt{1-4x^2}}{2}+\frac{\operatorname{asin}(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(1 - 4*x**2)/2 + asin(2*x)/4
```

Giac [A] time = 1.05781, size = 26, normalized size = 1.04

$$\frac{1}{2}\sqrt{-4\,x^2+1}x+\frac{1}{4}\,\arcsin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-4*x^2 + 1)*x + 1/4*arcsin(2*x)
```

3.129 $\int \frac{x^3}{\sqrt{4+x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{3}\left(x^2+4\right)^{3/2}-4\sqrt{x^2+4}$$

```
[Out] -4*Sqrt[4 + x^2] + (4 + x^2)^(3/2)/3
```

Rubi [A] time = 0.0111117, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{3}\left(x^2+4\right)^{3/2}-4\sqrt{x^2+4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[4 + x^2],x]
```

```
[Out] -4*Sqrt[4 + x^2] + (4 + x^2)^(3/2)/3
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{\sqrt{4+x^2}}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x}{\sqrt{4+x}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(-\frac{4}{\sqrt{4+x}}+\sqrt{4+x}\right)dx, x, x^2\right)\\ &= -4\sqrt{4+x^2}+\frac{1}{3}\left(4+x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0044458, size = 18, normalized size = 0.72

$$\frac{1}{3}\left(x^2-8\right)\sqrt{x^2+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[4 + x^2],x]
```

```
[Out] ((-8 + x^2)*Sqrt[4 + x^2])/3
```

Maple [A] time = 0.005, size = 15, normalized size = 0.6

$$\frac{x^2-8}{3}\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2+4)^(1/2),x)
```

```
[Out] 1/3*(x^2+4)^(1/2)*(x^2-8)
```

Maxima [A] time = 1.40537, size = 30, normalized size = 1.2

$$\frac{1}{3}\sqrt{x^2+4}x^2-\frac{8}{3}\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(x^2 + 4)*x^2 - 8/3*sqrt(x^2 + 4)
```

Fricas [A] time = 1.73206, size = 39, normalized size = 1.56

$$\frac{1}{3}\sqrt{x^2+4}\left(x^2-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(x^2 + 4)*(x^2 - 8)
```

Sympy [A] time = 0.341551, size = 24, normalized size = 0.96

$$\frac{x^2\sqrt{x^2+4}}{3}-\frac{8\sqrt{x^2+4}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**2+4)**(1/2),x)
```

```
[Out] x**2*sqrt(x**2 + 4)/3 - 8*sqrt(x**2 + 4)/3
```

Giac [A] time = 1.04427, size = 26, normalized size = 1.04

$$\frac{1}{3}\left(x^2+4\right)^{\frac{3}{2}}-4\sqrt{x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 + 4)^(3/2) - 4*sqrt(x^2 + 4)
```

3.130 $\int \frac{1}{\sqrt{9+x^2}} dx$

Optimal. Leaf size=6

$$\sinh^{-1}\left(\frac{x}{3}\right)$$

```
[Out] ArcSinh[x/3]
```

Rubi [A] time = 0.0011798, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {215}

$$\sinh^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 + x^2],x]

[Out] ArcSinh[x/3]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9+x^2}} dx = \sinh^{-1}\left(\frac{x}{3}\right)$$

Mathematica [A] time = 0.0034179, size = 6, normalized size = 1.

$$\sinh^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 + x^2],x]
```

```
[Out] ArcSinh[x/3]
```

Maple [A] time = 0.001, size = 5, normalized size = 0.8

$$\operatorname{Arcsinh}\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+9)^(1/2),x)
```

```
[Out] arcsinh(1/3*x)
```

Maxima [A] time = 1.40911, size = 5, normalized size = 0.83

$$\operatorname{arsinh}\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(1/3*x)
```

Fricas [B] time = 1.92847, size = 35, normalized size = 5.83

$$-\log\left(-x + \sqrt{x^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 9))
```

Sympy [A] time = 0.12484, size = 3, normalized size = 0.5

$$\operatorname{asinh}\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+9)**(1/2),x)
```

```
[Out] asinh(x/3)
```

Giac [B] time = 1.05858, size = 19, normalized size = 3.17

$$-\log\left(-x+\sqrt{x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-x + sqrt(x^2 + 9))
```

3.131 $\int \sqrt{1+x^2}\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\sinh^{-1}(x)$$

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rubi [A] time = 0.0026484, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {195, 215}

$$\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+x^2}\,dx &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1+x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{1+x^2}+\frac{1}{2}\sinh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0042448, size = 18, normalized size = 0.86

$$\frac{1}{2}\left(\sqrt{x^2+1}x+\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x^2],x]
```

```
[Out] (x*Sqrt[1 + x^2] + ArcSinh[x])/2
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$\frac{\text{Arcsinh}\,(x)}{2}+\frac{x}{2}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsinh(x)+1/2*x*(x^2+1)^(1/2)
```

Maxima [A] time = 1.39671, size = 20, normalized size = 0.95

$$\frac{1}{2}\sqrt{x^2+1}x+\frac{1}{2}\,\text{arsinh}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x + 1/2*arcsinh(x)
```

Fricas [A] time = 1.99301, size = 69, normalized size = 3.29

$$\frac{1}{2}\sqrt{x^2+1}x-\frac{1}{2}\,\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

Sympy [A] time = 0.176005, size = 15, normalized size = 0.71

$$\frac{x\sqrt{x^2+1}}{2} + \frac{\operatorname{asinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(x**2 + 1)/2 + asinh(x)/2
```

Giac [A] time = 1.05435, size = 34, normalized size = 1.62

$$\frac{1}{2}\sqrt{x^2+1}x - \frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 1)*x - 1/2*log(-x + sqrt(x^2 + 1))
```

3.132 $\int \frac{1}{x^3\sqrt{-16+x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{x^2-16}}{32x^2}+\frac{1}{128}\tan^{-1}\left(\frac{\sqrt{x^2-16}}{4}\right)$$

```
[Out] Sqrt[-16 + x^2]/(32*x^2) + ArcTan[Sqrt[-16 + x^2]/4]/128
```

Rubi [A] time = 0.0132875, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {266, 51, 63, 203}

$$\frac{\sqrt{x^2-16}}{32x^2}+\frac{1}{128}\tan^{-1}\left(\frac{\sqrt{x^2-16}}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*Sqrt[-16 + x^2]),x]
```

```
[Out] Sqrt[-16 + x^2]/(32*x^2) + ArcTan[Sqrt[-16 + x^2]/4]/128
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\sqrt{-16+x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-16+x}x^2}\,dx, x, x^2\right)\\ &= \frac{\sqrt{-16+x^2}}{32x^2}+\frac{1}{64}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-16+x}x}\,dx, x, x^2\right)\\ &= \frac{\sqrt{-16+x^2}}{32x^2}+\frac{1}{32}\operatorname{Subst}\left(\int \frac{1}{16+x^2}\,dx, x, \sqrt{-16+x^2}\right)\\ &= \frac{\sqrt{-16+x^2}}{32x^2}+\frac{1}{128}\tan^{-1}\left(\frac{1}{4}\sqrt{-16+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0203663, size = 46, normalized size = 1.31

$$\frac{1}{256}\sqrt{x^2-16}\left(\frac{8}{x^2}+\frac{2\tanh^{-1}\left(\sqrt{1-\frac{x^2}{16}}\right)}{\sqrt{16-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[-16 + x^2]),x]
```

```
[Out] (Sqrt[-16 + x^2]*(8/x^2 + (2*ArcTanh[Sqrt[1 - x^2/16]])/Sqrt[16 - x^2]))/25
6
```

Maple [A] time = 0.005, size = 26, normalized size = 0.7

$$\frac{1}{32\,x^2}\sqrt{x^2-16}-\frac{1}{128}\arctan\left(4\,\frac{1}{\sqrt{x^2-16}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(x^2-16)^(1/2),x)
```

```
[Out] 1/32*(x^2-16)^(1/2)/x^2-1/128*arctan(4/(x^2-16)^(1/2))
```

Maxima [A] time = 1.40181, size = 30, normalized size = 0.86

$$\frac{\sqrt{x^2-16}}{32\,x^2}-\frac{1}{128}\arcsin\left(\frac{4}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2-16)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/32*sqrt(x^2 - 16)/x^2 - 1/128*arcsin(4/abs(x))
```

Fricas [A] time = 1.87866, size = 96, normalized size = 2.74

$$\frac{x^2\arctan\left(-\frac{1}{4}\,x+\frac{1}{4}\,\sqrt{x^2-16}\right)+2\,\sqrt{x^2-16}}{64\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2-16)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/64*(x^2*arctan(-1/4*x + 1/4*sqrt(x^2 - 16)) + 2*sqrt(x^2 - 16))/x^2
```

Sympy [A] time = 2.23, size = 66, normalized size = 1.89

$$\begin{cases} \frac{i\operatorname{acosh}\left(\frac{4}{x}\right)}{128} + \frac{i\sqrt{-1+\frac{16}{x^2}}}{32x} & \text{for } \frac{16}{|x^2|} > 1 \\ -\frac{\operatorname{asin}\left(\frac{4}{x}\right)}{128} + \frac{1}{32x\sqrt{1-\frac{16}{x^2}}} - \frac{1}{2x^3\sqrt{1-\frac{16}{x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(x**2-16)**(1/2),x)
```

```
[Out] Piecewise((I*acosh(4/x)/128 + I*sqrt(-1 + 16/x**2)/(32*x), 16/Abs(x**2) > 1
), (-asin(4/x)/128 + 1/(32*x*sqrt(1 - 16/x**2)) - 1/(2*x**3*sqrt(1 - 16/x**
2)), True))
```

Giac [A] time = 1.05215, size = 34, normalized size = 0.97

$$\frac{\sqrt{x^2-16}}{32\,x^2} + \frac{1}{128}\,\arctan\left(\frac{1}{4}\,\sqrt{x^2-16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(x^2-16)^(1/2),x, algorithm="giac")
```

```
[Out] 1/32*sqrt(x^2 - 16)/x^2 + 1/128*arctan(1/4*sqrt(x^2 - 16))
```

3.133 $\int \frac{\sqrt{-a^2+x^2}}{x^4}\,dx$

Optimal. Leaf size=23

$$\frac{\left(x^2-a^2\right)^{3/2}}{3a^2x^3}$$

```
[Out] (-a^2 + x^2)^(3/2)/(3*a^2*x^3)
```

Rubi [A] time = 0.0040181, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {264}

$$\frac{\left(x^2-a^2\right)^{3/2}}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-a^2 + x^2]/x^4,x]

[Out] (-a^2 + x^2)^(3/2)/(3*a^2*x^3)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{-a^2+x^2}}{x^4}\,dx=\frac{\left(-a^2+x^2\right)^{3/2}}{3a^2x^3}$$

Mathematica [A] time = 0.0044987, size = 23, normalized size = 1.

$$\frac{\left(x^2-a^2\right)^{3/2}}{3a^2x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-a^2 + x^2]/x^4,x]
```

```
[Out] (-a^2 + x^2)^(3/2)/(3*a^2*x^3)
```

Maple [A] time = 0.004, size = 28, normalized size = 1.2

$$-\frac{(a+x)(a-x)}{3a^2x^3}\sqrt{-a^2+x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2+x^2)^(1/2)/x^4,x)
```

```
[Out] -1/3/x^3*(a+x)*(a-x)/a^2*(-a^2+x^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2+x^2)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.0589, size = 57, normalized size = 2.48

$$\frac{x^3+\left(-a^2+x^2\right)^{\frac{3}{2}}}{3a^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2+x^2)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] 1/3*(x^3 + (-a^2 + x^2)^(3/2))/(a^2*x^3)
```

Sympy [A] time = 0.628714, size = 78, normalized size = 3.39

$$\begin{cases} -\frac{i\sqrt{\frac{a^2}{x^2}-1}}{3x^2}+\frac{i\sqrt{\frac{a^2}{x^2}-1}}{3a^2} & \text{for } \frac{|a^2|}{|x^2|}>1 \\ -\frac{\sqrt{-\frac{a^2}{x^2}+1}}{3x^2}+\frac{\sqrt{-\frac{a^2}{x^2}+1}}{3a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2+x**2)**(1/2)/x**4,x)
```

```
[Out] Piecewise((-I*sqrt(a**2/x**2 - 1)/(3*x**2) + I*sqrt(a**2/x**2 - 1)/(3*a**2)
, Abs(a**2)/Abs(x**2) > 1), (-sqrt(-a**2/x**2 + 1)/(3*x**2) + sqrt(-a**2/x*
*2 + 1)/(3*a**2), True))
```

Giac [B] time = 1.06475, size = 65, normalized size = 2.83

$$\frac{2\left(a^4+3\left(x-\sqrt{-a^2+x^2}\right)^4\right)}{3\left(a^2+\left(x-\sqrt{-a^2+x^2}\right)^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2+x^2)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] 2/3*(a^4 + 3*(x - sqrt(-a^2 + x^2))^4)/(a^2 + (x - sqrt(-a^2 + x^2))^2)^3
```

3.134 $\int \frac{\sqrt{-4+9x^2}}{x} \, dx$

Optimal. Leaf size=30

$$\sqrt{9x^2-4}-2\tan^{-1}\left(\frac{1}{2}\sqrt{9x^2-4}\right)$$

```
[Out] Sqrt[-4 + 9*x^2] - 2*ArcTan[Sqrt[-4 + 9*x^2]/2]
```

Rubi [A] time = 0.015983, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 203}

$$\sqrt{9x^2-4}-2\tan^{-1}\left(\frac{1}{2}\sqrt{9x^2-4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-4 + 9*x^2]/x,x]
```

```
[Out] Sqrt[-4 + 9*x^2] - 2*ArcTan[Sqrt[-4 + 9*x^2]/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-4+9x^2}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{-4+9x}}{x}\,dx, x, x^2\right)\\ &= \sqrt{-4+9x^2} - 2\,\text{Subst}\left(\int \frac{1}{x\sqrt{-4+9x}}\,dx, x, x^2\right)\\ &= \sqrt{-4+9x^2} - \frac{4}{9}\,\text{Subst}\left(\int \frac{1}{\frac{4}{9}+\frac{x^2}{9}}\,dx, x, \sqrt{-4+9x^2}\right)\\ &= \sqrt{-4+9x^2} - 2\tan^{-1}\left(\frac{1}{2}\sqrt{-4+9x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0055626, size = 30, normalized size = 1.

$$\sqrt{9x^2-4} - 2\tan^{-1}\left(\frac{1}{2}\sqrt{9x^2-4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-4 + 9*x^2]/x,x]
```

```
[Out] Sqrt[-4 + 9*x^2] - 2*ArcTan[Sqrt[-4 + 9*x^2]/2]
```

Maple [A] time = 0.005, size = 25, normalized size = 0.8

$$\sqrt{9\,x^2-4} + 2\,\arctan\left(2\,\frac{1}{\sqrt{9\,x^2-4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2-4)^(1/2)/x,x)
```

```
[Out] (9*x^2-4)^(1/2)+2*arctan(2/(9*x^2-4)^(1/2))
```

Maxima [A] time = 1.40573, size = 26, normalized size = 0.87

$$\sqrt{9\,x^2-4}+2\,\arcsin\left(\frac{2}{3\,|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-4)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(9*x^2 - 4) + 2*arcsin(2/3/abs(x))
```

Fricas [A] time = 1.97538, size = 78, normalized size = 2.6

$$\sqrt{9\,x^2-4}-4\,\arctan\left(-\frac{3}{2}\,x+\frac{1}{2}\,\sqrt{9\,x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-4)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(9*x^2 - 4) - 4*arctan(-3/2*x + 1/2*sqrt(9*x^2 - 4))
```

Sympy [A] time = 1.29256, size = 92, normalized size = 3.07

$$\begin{cases} -\frac{3ix}{\sqrt{-1+\frac{4}{9x^2}}}-2i\,\operatorname{acosh}\left(\frac{2}{3x}\right)+\frac{4i}{3x\sqrt{-1+\frac{4}{9x^2}}} & \text{for}\ \frac{4}{9|x^2|}>1 \\ \frac{3x}{\sqrt{1-\frac{4}{9x^2}}}+2\,\operatorname{asin}\left(\frac{2}{3x}\right)-\frac{4}{3x\sqrt{1-\frac{4}{9x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x**2-4)**(1/2)/x,x)
```

```
[Out] Piecewise((-3*I*x/sqrt(-1 + 4/(9*x**2)) - 2*I*acosh(2/(3*x)) + 4*I/(3*x*sqr
t(-1 + 4/(9*x**2))), 4/(9*Abs(x**2)) > 1), (3*x/sqrt(1 - 4/(9*x**2)) + 2*as
in(2/(3*x)) - 4/(3*x*sqrt(1 - 4/(9*x**2))), True))
```

Giac [A] time = 1.06677, size = 32, normalized size = 1.07

$$\sqrt{9\,x^2-4}-2\,\arctan\left(\frac{1}{2}\,\sqrt{9\,x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-4)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(9*x^2 - 4) - 2*arctan(1/2*sqrt(9*x^2 - 4))
```

3.135 $\int \frac{1}{x^2\sqrt{-9+16x^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{16x^2-9}}{9x}$$

```
[Out] Sqrt[-9 + 16*x^2]/(9*x)
```

Rubi [A] time = 0.0030559, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {264}

$$\frac{\sqrt{16x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[-9 + 16*x^2]),x]
```

```
[Out] Sqrt[-9 + 16*x^2]/(9*x)
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{-9+16x^2}}\,dx=\frac{\sqrt{-9+16x^2}}{9x}$$

Mathematica [A] time = 0.0032676, size = 18, normalized size = 1.

$$\frac{\sqrt{16x^2-9}}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[-9 + 16*x^2]),x]
```

```
[Out] Sqrt[-9 + 16*x^2]/(9*x)
```

Maple [A] time = 0.003, size = 25, normalized size = 1.4

$$\frac{(4x-3)(3+4x)}{9x}\frac{1}{\sqrt{16x^2-9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(16*x^2-9)^(1/2),x)
```

```
[Out] 1/9/x*(4*x-3)*(3+4*x)/(16*x^2-9)^(1/2)
```

Maxima [A] time = 1.4093, size = 19, normalized size = 1.06

$$\frac{\sqrt{16x^2-9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(16*x^2-9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(16*x^2 - 9)/x
```

Fricas [A] time = 1.98513, size = 43, normalized size = 2.39

$$\frac{4x+\sqrt{16x^2-9}}{9x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(16*x^2-9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(4*x + sqrt(16*x^2 - 9))/x
```

Sympy [A] time = 0.743997, size = 37, normalized size = 2.06

$$\begin{cases} \frac{4i\sqrt{-1+\frac{9}{16x^2}}}{9} & \text{for}\ \frac{9}{16|x^2|} > 1 \\ \frac{4\sqrt{1-\frac{9}{16x^2}}}{9} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(16*x**2-9)**(1/2),x)
```

```
[Out] Piecewise((4*I*sqrt(-1 + 9/(16*x**2))/9, 9/(16*Abs(x**2)) > 1), (4*sqrt(1 -
 9/(16*x**2))/9, True))
```

Giac [A] time = 1.05403, size = 31, normalized size = 1.72

$$\frac{8}{\left(4\,x - \sqrt{16\,x^2 - 9}\right)^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(16*x^2-9)^(1/2),x, algorithm="giac")
```

```
[Out] 8/((4*x - sqrt(16*x^2 - 9))^2 + 9)
```

3.136 $\int \frac{x^2}{\left(a^2-x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=34

$$\frac{x}{\sqrt{a^2-x^2}}-\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[Out] x/Sqrt[a^2 - x^2] - ArcTan[x/Sqrt[a^2 - x^2]]
```

Rubi [A] time = 0.0063195, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {288, 217, 203}

$$\frac{x}{\sqrt{a^2-x^2}}-\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a^2 - x^2)^(3/2),x]

[Out] x/Sqrt[a^2 - x^2] - ArcTan[x/Sqrt[a^2 - x^2]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{\left(a^2-x^2\right)^{3/2}}\,dx = \frac{x}{\sqrt{a^2-x^2}} - \int \frac{1}{\sqrt{a^2-x^2}}\,dx$$
$$= \frac{x}{\sqrt{a^2-x^2}} - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{a^2-x^2}}\right)$$
$$= \frac{x}{\sqrt{a^2-x^2}} - \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Mathematica [A] time = 0.0301574, size = 39, normalized size = 1.15

$$\frac{x - a\sqrt{1-\frac{x^2}{a^2}}\sin^{-1}\left(\frac{x}{a}\right)}{\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a^2 - x^2)^(3/2),x]
```

```
[Out] (x - a*Sqrt[1 - x^2/a^2]*ArcSin[x/a])/Sqrt[a^2 - x^2]
```

Maple [A] time = 0.01, size = 31, normalized size = 0.9

$$-\arctan\left(x\frac{1}{\sqrt{a^2-x^2}}\right) + x\frac{1}{\sqrt{a^2-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2-x^2)^(3/2),x)
```

```
[Out] -arctan(x/(a^2-x^2)^(1/2))+x/(a^2-x^2)^(1/2)
```

Maxima [A] time = 1.40408, size = 32, normalized size = 0.94

$$\frac{x}{\sqrt{a^2-x^2}} - \arcsin\left(\frac{x}{\sqrt{a^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2-x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(a^2 - x^2) - arcsin(x/sqrt(a^2))
```

Fricas [A] time = 2.22168, size = 111, normalized size = 3.26

$$\frac{2\left(a^2-x^2\right)\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)+\sqrt{a^2-x^2}x}{a^2-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2-x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] (2*(a^2 - x^2)*arctan(-(a - sqrt(a^2 - x^2))/x) + sqrt(a^2 - x^2)*x)/(a^2 -
 x^2)
```

Sympy [A] time = 1.52178, size = 51, normalized size = 1.5

$$\begin{cases} i\operatorname{acosh}\left(\frac{x}{a}\right)-\frac{ix}{a\sqrt{-1+\frac{x^2}{a^2}}} & \text{for } \frac{|x^2|}{|a^2|}>1 \\ -\operatorname{asin}\left(\frac{x}{a}\right)+\frac{x}{a\sqrt{1-\frac{x^2}{a^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2-x**2)**(3/2),x)
```

```
[Out] Piecewise((I*acosh(x/a) - I*x/(a*sqrt(-1 + x**2/a**2)), Abs(x**2)/Abs(a**2)
 > 1), (-asin(x/a) + x/(a*sqrt(1 - x**2/a**2)), True))
```

Giac [A] time = 1.07619, size = 32, normalized size = 0.94

$$-\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a)+\frac{x}{\sqrt{a^2-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2-x^2)^(3/2),x, algorithm="giac")
```

```
[Out] -arcsin(x/a)*sgn(a) + x/sqrt(a^2 - x^2)
```

3.137 $\int \frac{x^2}{\sqrt{5-x^2}}\,dx$

Optimal. Leaf size=29

$$\frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)-\frac{1}{2}x\sqrt{5-x^2}$$

```
[Out] -(x*Sqrt[5 - x^2])/2 + (5*ArcSin[x/Sqrt[5]])/2
```

Rubi [A] time = 0.0061221, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 216}

$$\frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)-\frac{1}{2}x\sqrt{5-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[5 - x^2],x]

[Out] -(x*Sqrt[5 - x^2])/2 + (5*ArcSin[x/Sqrt[5]])/2
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{5-x^2}}\,dx = -\frac{1}{2}x\sqrt{5-x^2} + \frac{5}{2}\int \frac{1}{\sqrt{5-x^2}}\,dx$$
$$= -\frac{1}{2}x\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Mathematica [A] time = 0.0089389, size = 29, normalized size = 1.

$$\frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right) - \frac{1}{2}x\sqrt{5-x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[5 - x^2],x]
```

```
[Out] -(x*Sqrt[5 - x^2])/2 + (5*ArcSin[x/Sqrt[5]])/2
```

Maple [A] time = 0.005, size = 23, normalized size = 0.8

$$\frac{5}{2}\arcsin\left(\frac{x\sqrt{5}}{5}\right) - \frac{x}{2}\sqrt{-x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+5)^(1/2),x)
```

```
[Out] 5/2*arcsin(1/5*x*5^(1/2))-1/2*x*(-x^2+5)^(1/2)
```

Maxima [A] time = 1.40043, size = 30, normalized size = 1.03

$$-\frac{1}{2}\sqrt{-x^2+5}x + \frac{5}{2}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+5)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 5)*x + 5/2*arcsin(1/5*sqrt(5)*x)
```

Fricas [A] time = 2.27144, size = 73, normalized size = 2.52

$$-\frac{1}{2}\sqrt{-x^2+5}x-\frac{5}{2}\arctan\left(\frac{\sqrt{-x^2+5}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^2 + 5)*x - 5/2*arctan(sqrt(-x^2 + 5)/x)
```

Sympy [A] time = 0.202197, size = 24, normalized size = 0.83

$$-\frac{x\sqrt{5-x^2}}{2}+\frac{5\,\mathrm{asin}\left(\frac{\sqrt{5}x}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+5)**(1/2),x)
```

```
[Out] -x*sqrt(5 - x**2)/2 + 5*asin(sqrt(5)*x/5)/2
```

Giac [A] time = 1.08117, size = 30, normalized size = 1.03

$$-\frac{1}{2}\sqrt{-x^2+5}x+\frac{5}{2}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 5)*x + 5/2*arcsin(1/5*sqrt(5)*x)
```

3.138 $\int \frac{1}{x\sqrt{3+x^2}}\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+3}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTanh[Sqrt[3 + x^2]/Sqrt[3]]/Sqrt[3])
```

Rubi [A] time = 0.0110963, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 63, 207}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+3}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[3 + x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[3 + x^2]/Sqrt[3]]/Sqrt[3])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{3+x^2}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{3+x}}\,dx, x, x^2\right)\\ &= \operatorname{Subst}\left(\int \frac{1}{-3+x^2}\,dx, x, \sqrt{3+x^2}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{3+x^2}}{\sqrt{3}}\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0042559, size = 23, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x^2+3}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[3 + x^2]),x]
```

```
[Out] -(ArcTanh[Sqrt[3 + x^2]/Sqrt[3]]/Sqrt[3])
```

Maple [A] time = 0.004, size = 18, normalized size = 0.8

$$-\frac{\sqrt{3}}{3}\operatorname{Artanh}\left(\sqrt{3}\frac{1}{\sqrt{x^2+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+3)^(1/2),x)
```

```
[Out] -1/3*3^(1/2)*arctanh(3^(1/2)/(x^2+3)^(1/2))
```

Maxima [A] time = 1.40106, size = 19, normalized size = 0.83

$$-\frac{1}{3}\sqrt{3}\operatorname{arsinh}\left(\frac{\sqrt{3}}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arcsinh(sqrt(3)/abs(x))
```

Fricas [A] time = 2.2903, size = 63, normalized size = 2.74

$$\frac{1}{3}\sqrt{3}\log\left(-\frac{\sqrt{3}-\sqrt{x^2+3}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*log(-(sqrt(3) - sqrt(x^2 + 3))/x)
```

Sympy [A] time = 0.987104, size = 15, normalized size = 0.65

$$-\frac{\sqrt{3}\operatorname{asinh}\left(\frac{\sqrt{3}}{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+3)**(1/2),x)
```

```
[Out] -sqrt(3)*asinh(sqrt(3)/x)/3
```

Giac [B] time = 1.05048, size = 50, normalized size = 2.17

$$-\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}+\sqrt{x^2+3}\right)+\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}+\sqrt{x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*log(sqrt(3) + sqrt(x^2 + 3)) + 1/6*sqrt(3)*log(-sqrt(3) + sqrt
(x^2 + 3))
```

3.139 $\int \frac{x}{(4+x^2)^{5/2}}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{3\left(x^2+4\right)^{3/2}}$$

```
[Out] -1/(3*(4 + x^2)^(3/2))
```

Rubi [A] time = 0.0020326, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$-\frac{1}{3\left(x^2+4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(4 + x^2)^(5/2),x]
```

```
[Out] -1/(3*(4 + x^2)^(3/2))
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(4+x^2\right)^{5/2}}\,dx = -\frac{1}{3\left(4+x^2\right)^{3/2}}$$

Mathematica [A] time = 0.0021071, size = 13, normalized size = 1.

$$-\frac{1}{3\left(x^2+4\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(4 + x^2)^(5/2),x]
```

```
[Out] -1/(3*(4 + x^2)^(3/2))
```

Maple [A] time = 0.003, size = 10, normalized size = 0.8

$$-\frac{1}{3}\left(x^2+4\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+4)^(5/2),x)
```

```
[Out] -1/3/(x^2+4)^(3/2)
```

Maxima [A] time = 0.920904, size = 12, normalized size = 0.92

$$-\frac{1}{3\left(x^2+4\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3/(x^2 + 4)^(3/2)
```

Fricas [B] time = 2.36814, size = 53, normalized size = 4.08

$$-\frac{\sqrt{x^2+4}}{3\left(x^4+8\,x^2+16\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(x^2 + 4)/(x^4 + 8*x^2 + 16)
```

Sympy [B] time = 2.32755, size = 26, normalized size = 2.

$$-\frac{1}{3x^2\sqrt{x^2+4}+12\sqrt{x^2+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+4)**(5/2),x)
```

```
[Out] -1/(3*x**2*sqrt(x**2 + 4) + 12*sqrt(x**2 + 4))
```

Giac [A] time = 1.05664, size = 12, normalized size = 0.92

$$-\frac{1}{3\left(x^2+4\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3/(x^2 + 4)^(3/2)
```

3.140 $\int x^3\sqrt{4-9x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{405}\left(4-9x^2\right)^{5/2}-\frac{4}{243}\left(4-9x^2\right)^{3/2}$$

```
[Out] (-4*(4 - 9*x^2)^(3/2))/243 + (4 - 9*x^2)^(5/2)/405
```

Rubi [A] time = 0.0139225, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{1}{405}\left(4-9x^2\right)^{5/2}-\frac{4}{243}\left(4-9x^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[4 - 9*x^2],x]
```

```
[Out] (-4*(4 - 9*x^2)^(3/2))/243 + (4 - 9*x^2)^(5/2)/405
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^3\sqrt{4-9x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \sqrt{4-9x}x\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{4}{9}\sqrt{4-9x}-\frac{1}{9}(4-9x)^{3/2}\right)dx, x, x^2\right)\\
&= -\frac{4}{243}\left(4-9x^2\right)^{3/2}+\frac{1}{405}\left(4-9x^2\right)^{5/2}
\end{aligned}$$

Mathematica [A] time = 0.0105125, size = 22, normalized size = 0.71

$$-\frac{\left(4-9x^2\right)^{3/2}\left(27x^2+8\right)}{1215}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[4 - 9*x^2],x]
```

```
[Out] -((4 - 9*x^2)^(3/2)*(8 + 27*x^2))/1215
```

Maple [A] time = 0.004, size = 29, normalized size = 0.9

$$\frac{(-2+3x)(2+3x)\left(27x^2+8\right)}{1215}\sqrt{-9x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-9*x^2+4)^(1/2),x)
```

```
[Out] 1/1215*(-2+3*x)*(2+3*x)*(27*x^2+8)*(-9*x^2+4)^(1/2)
```

Maxima [A] time = 1.39839, size = 35, normalized size = 1.13

$$-\frac{1}{45}\left(-9x^2+4\right)^{\frac{3}{2}}x^2-\frac{8}{1215}\left(-9x^2+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-9*x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/45*(-9*x^2 + 4)^(3/2)*x^2 - 8/1215*(-9*x^2 + 4)^(3/2)
```

Fricas [A] time = 2.56603, size = 66, normalized size = 2.13

$$\frac{1}{1215}\left(243\,x^4 - 36\,x^2 - 32\right)\sqrt{-9\,x^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-9*x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1215*(243*x^4 - 36*x^2 - 32)*sqrt(-9*x^2 + 4)
```

Sympy [A] time = 0.592645, size = 44, normalized size = 1.42

$$\frac{x^4\sqrt{4-9x^2}}{5} - \frac{4x^2\sqrt{4-9x^2}}{135} - \frac{32\sqrt{4-9x^2}}{1215}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-9*x**2+4)**(1/2),x)
```

```
[Out] x**4*sqrt(4 - 9*x**2)/5 - 4*x**2*sqrt(4 - 9*x**2)/135 - 32*sqrt(4 - 9*x**2)
/1215
```

Giac [A] time = 1.06412, size = 43, normalized size = 1.39

$$\frac{1}{405}\left(9\,x^2 - 4\right)^2\sqrt{-9\,x^2 + 4} - \frac{4}{243}\left(-9\,x^2 + 4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-9*x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/405*(9*x^2 - 4)^2*sqrt(-9*x^2 + 4) - 4/243*(-9*x^2 + 4)^(3/2)
```

3.141 $\int x^2\sqrt{9-x^2}\,dx$

Optimal. Leaf size=45

$$\frac{1}{4}\sqrt{9-x^2}x^3-\frac{9}{8}\sqrt{9-x^2}x+\frac{81}{8}\sin^{-1}\left(\frac{x}{3}\right)$$

```
[Out] (-9*x*Sqrt[9 - x^2])/8 + (x^3*Sqrt[9 - x^2])/4 + (81*ArcSin[x/3])/8
```

Rubi [A] time = 0.0099616, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {279, 321, 216}

$$\frac{1}{4}\sqrt{9-x^2}x^3-\frac{9}{8}\sqrt{9-x^2}x+\frac{81}{8}\sin^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[9 - x^2],x]
```

```
[Out] (-9*x*Sqrt[9 - x^2])/8 + (x^3*Sqrt[9 - x^2])/4 + (81*ArcSin[x/3])/8
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{9-x^2}\,dx &= \frac{1}{4}x^3\sqrt{9-x^2}+\frac{9}{4}\int \frac{x^2}{\sqrt{9-x^2}}\,dx\\ &= -\frac{9}{8}x\sqrt{9-x^2}+\frac{1}{4}x^3\sqrt{9-x^2}+\frac{81}{8}\int \frac{1}{\sqrt{9-x^2}}\,dx\\ &= -\frac{9}{8}x\sqrt{9-x^2}+\frac{1}{4}x^3\sqrt{9-x^2}+\frac{81}{8}\sin^{-1}\left(\frac{x}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.0150719, size = 33, normalized size = 0.73

$$\frac{1}{8}\left(x\sqrt{9-x^2}\left(2x^2-9\right)+81\sin^{-1}\left(\frac{x}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[9 - x^2],x]

[Out] (x*Sqrt[9 - x^2]*(-9 + 2*x^2) + 81*ArcSin[x/3])/8
```

Maple [A] time = 0.003, size = 32, normalized size = 0.7

$$-\frac{x}{4}\left(-x^2+9\right)^{\frac{3}{2}}+\frac{9\,x}{8}\sqrt{-x^2+9}+\frac{81}{8}\arcsin\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+9)^(1/2),x)

[Out] -1/4*x*(-x^2+9)^(3/2)+9/8*x*(-x^2+9)^(1/2)+81/8*arcsin(1/3*x)
```

Maxima [A] time = 1.40555, size = 42, normalized size = 0.93

$$-\frac{1}{4}\left(-x^2+9\right)^{\frac{3}{2}}x+\frac{9}{8}\sqrt{-x^2+9}x+\frac{81}{8}\arcsin\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 9)^(3/2)*x + 9/8*sqrt(-x^2 + 9)*x + 81/8*arcsin(1/3*x)
```

Fricas [A] time = 2.41846, size = 97, normalized size = 2.16

$$\frac{1}{8}\left(2\,x^3-9\,x\right)\sqrt{-x^2+9}-\frac{81}{4}\,\arctan\left(\frac{\sqrt{-x^2+9}-3}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^3 - 9*x)*sqrt(-x^2 + 9) - 81/4*arctan((sqrt(-x^2 + 9) - 3)/x)
```

Sympy [A] time = 2.67154, size = 112, normalized size = 2.49

$$\begin{cases} \frac{ix^5}{4\sqrt{x^2-9}}-\frac{27ix^3}{8\sqrt{x^2-9}}+\frac{81ix}{8\sqrt{x^2-9}}-\frac{81i\operatorname{acosh}\left(\frac{x}{3}\right)}{8} & \text{for } \frac{|x^2|}{9}>1 \\ -\frac{x^5}{4\sqrt{9-x^2}}+\frac{27x^3}{8\sqrt{9-x^2}}-\frac{81x}{8\sqrt{9-x^2}}+\frac{81\operatorname{asin}\left(\frac{x}{3}\right)}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+9)**(1/2),x)
```

```
[Out] Piecewise((I*x**5/(4*sqrt(x**2 - 9)) - 27*I*x**3/(8*sqrt(x**2 - 9)) + 81*I*
x/(8*sqrt(x**2 - 9)) - 81*I*acosh(x/3)/8, Abs(x**2)/9 > 1), (-x**5/(4*sqrt(
9 - x**2)) + 27*x**3/(8*sqrt(9 - x**2)) - 81*x/(8*sqrt(9 - x**2)) + 81*asin
(x/3)/8, True))
```

Giac [A] time = 1.06193, size = 35, normalized size = 0.78

$$\frac{1}{8}\left(2\,x^2-9\right)\sqrt{-x^2+9}x+\frac{81}{8}\,\arcsin\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*x^2 - 9)*sqrt(-x^2 + 9)*x + 81/8*arcsin(1/3*x)
```

3.142 $\int 5x\sqrt{1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{5}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] (5*(1 + x^2)^(3/2))/3
```

Rubi [A] time = 0.0024651, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {12, 261}

$$\frac{5}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[5*x*Sqrt[1 + x^2],x]
```

```
[Out] (5*(1 + x^2)^(3/2))/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int 5x\sqrt{1+x^2}\,dx &= 5\int x\sqrt{1+x^2}\,dx\\ &= \frac{5}{3}\left(1+x^2\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0017622, size = 13, normalized size = 1.

$$\frac{5}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[5*x*Sqrt[1 + x^2],x]
```

```
[Out] (5*(1 + x^2)^(3/2))/3
```

Maple [A] time = 0.001, size = 10, normalized size = 0.8

$$\frac{5}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(5*x*(x^2+1)^(1/2),x)
```

```
[Out] 5/3*(x^2+1)^(3/2)
```

Maxima [A] time = 0.925844, size = 12, normalized size = 0.92

$$\frac{5}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x*(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/3*(x^2 + 1)^(3/2)
```

Fricas [A] time = 2.06477, size = 28, normalized size = 2.15

$$\frac{5}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 5/3*(x^2 + 1)^(3/2)
```

Sympy [B] time = 0.174944, size = 26, normalized size = 2.

$$\frac{5x^2\sqrt{x^2+1}}{3}+\frac{5\sqrt{x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x*(x**2+1)**(1/2),x)
```

```
[Out] 5*x**2*sqrt(x**2 + 1)/3 + 5*sqrt(x**2 + 1)/3
```

Giac [A] time = 1.04555, size = 12, normalized size = 0.92

$$\frac{5}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(5*x*(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 5/3*(x^2 + 1)^(3/2)
```

3.143 $\int \frac{1}{\left(-25+4x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=16

$$-\frac{x}{25\sqrt{4x^2-25}}$$

```
[Out] -x/(25*Sqrt[-25 + 4*x^2])
```

Rubi [A] time = 0.0014153, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {191}

$$-\frac{x}{25\sqrt{4x^2-25}}$$

Antiderivative was successfully verified.

```
[In] Int[(-25 + 4*x^2)^(-3/2),x]
```

```
[Out] -x/(25*Sqrt[-25 + 4*x^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(-25+4x^2\right)^{3/2}} \, dx = -\frac{x}{25\sqrt{-25+4x^2}}$$

Mathematica [A] time = 0.0038095, size = 16, normalized size = 1.

$$-\frac{x}{25\sqrt{4x^2-25}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-25 + 4*x^2)^(-3/2),x]
```

```
[Out] -x/(25*Sqrt[-25 + 4*x^2])
```

Maple [A] time = 0.002, size = 23, normalized size = 1.4

$$-\frac{(2x-5)(5+2x)x}{25}\left(4x^2-25\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2-25)^(3/2),x)
```

```
[Out] -1/25*(2*x-5)*(5+2*x)*x/(4*x^2-25)^(3/2)
```

Maxima [A] time = 0.920857, size = 16, normalized size = 1.

$$-\frac{x}{25\sqrt{4x^2-25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-25)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/25*x/sqrt(4*x^2 - 25)
```

Fricas [B] time = 2.06223, size = 76, normalized size = 4.75

$$-\frac{4x^2+2\sqrt{4x^2-25}x-25}{50\left(4x^2-25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-25)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/50*(4*x^2 + 2*sqrt(4*x^2 - 25)*x - 25)/(4*x^2 - 25)
```

Sympy [A] time = 0.721938, size = 36, normalized size = 2.25

$$\begin{cases} -\frac{x}{25\sqrt{4x^2-25}} & \text{for } \frac{4|x^2|}{25} > 1 \\ \frac{ix}{25\sqrt{25-4x^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**2-25)**(3/2),x)
```

```
[Out] Piecewise((-x/(25*sqrt(4*x**2 - 25)), 4*Abs(x**2)/25 > 1), (I*x/(25*sqrt(25
 - 4*x**2)), True))
```

Giac [A] time = 1.06364, size = 16, normalized size = 1.

$$-\frac{x}{25\sqrt{4x^2-25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2-25)^(3/2),x, algorithm="giac")
```

```
[Out] -1/25*x/sqrt(4*x^2 - 25)
```

3.144 $\int \sqrt{2x-x^2}\,dx$

Optimal. Leaf size=33

$$-\frac{1}{2}\sqrt{2x-x^2}(1-x)-\frac{1}{2}\sin^{-1}(1-x)$$

```
[Out] -((1 - x)*Sqrt[2*x - x^2])/2 - ArcSin[1 - x]/2
```

Rubi [A] time = 0.0076645, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {612, 619, 216}

$$-\frac{1}{2}\sqrt{2x-x^2}(1-x)-\frac{1}{2}\sin^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2*x - x^2],x]
```

```
[Out] -((1 - x)*Sqrt[2*x - x^2])/2 - ArcSin[1 - x]/2
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{2x-x^2}\,dx = -\frac{1}{2}(1-x)\sqrt{2x-x^2} + \frac{1}{2}\int \frac{1}{\sqrt{2x-x^2}}\,dx$$
$$= -\frac{1}{2}(1-x)\sqrt{2x-x^2} - \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, 2-2x\right)$$
$$= -\frac{1}{2}(1-x)\sqrt{2x-x^2} - \frac{1}{2}\sin^{-1}(1-x)$$

Mathematica [A] time = 0.0391544, size = 32, normalized size = 0.97

$$\frac{1}{2}(x-1)\sqrt{-(x-2)x} - \sin^{-1}\left(\sqrt{1-\frac{x}{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2*x - x^2],x]
```

```
[Out] ((-1 + x)*Sqrt[-((-2 + x)*x)])/2 - ArcSin[Sqrt[1 - x/2]]
```

Maple [A] time = 0.003, size = 26, normalized size = 0.8

$$-\frac{-2\,x+2}{4}\sqrt{-x^2+2\,x} + \frac{\arcsin\left(-1+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+2*x)^(1/2),x)
```

```
[Out] -1/4*(-2*x+2)*(-x^2+2*x)^(1/2)+1/2*arcsin(-1+x)
```

Maxima [A] time = 1.39716, size = 49, normalized size = 1.48

$$\frac{1}{2}\,\sqrt{-x^2+2\,x}x - \frac{1}{2}\,\sqrt{-x^2+2\,x} - \frac{1}{2}\,\arcsin\left(-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 2*x)*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*arcsin(-x + 1)
```

Fricas [A] time = 2.16882, size = 80, normalized size = 2.42

$$\frac{1}{2}\sqrt{-x^2+2x}(x-1)-\arctan\left(\frac{\sqrt{-x^2+2x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 2*x)*(x - 1) - arctan(sqrt(-x^2 + 2*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-x^2+2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+2*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**2 + 2*x), x)
```

Giac [A] time = 1.057, size = 31, normalized size = 0.94

$$\frac{1}{2}\sqrt{-x^2+2x}(x-1)+\frac{1}{2}\arcsin(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 2*x)*(x - 1) + 1/2*arcsin(x - 1)
```

3.145 $\int \frac{1}{\sqrt{8+4x+x^2}}\,dx$

Optimal. Leaf size=8

$$\sinh^{-1}\left(\frac{x+2}{2}\right)$$

```
[Out] ArcSinh[(2 + x)/2]
```

Rubi [A] time = 0.0062037, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {619, 215}

$$\sinh^{-1}\left(\frac{x+2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[8 + 4*x + x^2],x]
```

```
[Out] ArcSinh[(2 + x)/2]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{8+4x+x^2}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{16}}}\,dx, x, 4+2x\right)\\ &= \sinh^{-1}\left(\frac{2+x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0049086, size = 8, normalized size = 1.

$$\sinh^{-1}\left(\frac{x+2}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[8 + 4*x + x^2],x]
```

```
[Out] ArcSinh[(2 + x)/2]
```

Maple [A] time = 0.003, size = 7, normalized size = 0.9

$$\text{Arcsinh}\left(1+\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+4*x+8)^(1/2),x)
```

```
[Out] arcsinh(1+1/2*x)
```

Maxima [A] time = 1.40643, size = 8, normalized size = 1.

$$\text{arsinh}\left(\frac{1}{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x+8)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(1/2*x + 1)
```

Fricas [B] time = 2.10724, size = 49, normalized size = 6.12

$$-\log\left(-x+\sqrt{x^2+4\,x+8}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x+8)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 4*x + 8) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2 + 4x + 8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+4*x+8)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2 + 4*x + 8), x)
```

Giac [B] time = 1.06002, size = 24, normalized size = 3.

$$-\log\left(-x + \sqrt{x^2 + 4\,x + 8} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4*x+8)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-x + sqrt(x^2 + 4*x + 8) - 2)
```

3.146 $\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

```
[Out] ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x + 9*x^2]]/3
```

Rubi [A] time = 0.0058126, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {621, 206}

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-8 + 6*x + 9*x^2],x]
```

```
[Out] ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x + 9*x^2]]/3
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{36-x^2}\,dx, x, \frac{6+18x}{\sqrt{-8+6x+9x^2}}\right)\\ &= \frac{1}{3}\tanh^{-1}\left(\frac{1+3x}{\sqrt{-8+6x+9x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0059543, size = 24, normalized size = 0.96

$$\frac{1}{3}\log\left(\sqrt{9x^2+6x-8}+3x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-8 + 6*x + 9*x^2],x]
```

```
[Out] Log[1 + 3*x + Sqrt[-8 + 6*x + 9*x^2]]/3
```

Maple [A] time = 0.003, size = 30, normalized size = 1.2

$$\frac{\sqrt{9}}{9}\ln\left(\frac{(3+9x)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2+6*x-8)^(1/2),x)
```

```
[Out] 1/9*ln(1/9*(3+9*x)*9^(1/2)+(9*x^2+6*x-8)^(1/2))*9^(1/2)
```

Maxima [A] time = 1.40184, size = 30, normalized size = 1.2

$$\frac{1}{3}\log\left(18x+6\sqrt{9x^2+6x-8}+6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*log(18*x + 6*sqrt(9*x^2 + 6*x - 8) + 6)
```

Fricas [A] time = 2.08571, size = 59, normalized size = 2.36

$$-\frac{1}{3}\log\left(-3x+\sqrt{9x^2+6x-8}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*log(-3*x + sqrt(9*x^2 + 6*x - 8) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{9x^2 + 6x - 8}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2+6*x-8)**(1/2),x)
```

```
[Out] Integral(1/sqrt(9*x**2 + 6*x - 8), x)
```

Giac [A] time = 1.04942, size = 28, normalized size = 1.12

$$-\frac{1}{3}\log\left(\left|-3x + \sqrt{9x^2 + 6x - 8} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+6*x-8)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*log(abs(-3*x + sqrt(9*x^2 + 6*x - 8) - 1))
```

3.147 $\int \frac{x^2}{\sqrt{4x-x^2}}\,dx$

Optimal. Leaf size=44

$$-\frac{1}{2}\sqrt{4x-x^2}x-3\sqrt{4x-x^2}-6\sin^{-1}\left(1-\frac{x}{2}\right)$$

```
[Out] -3*Sqrt[4*x - x^2] - (x*Sqrt[4*x - x^2])/2 - 6*ArcSin[1 - x/2]
```

Rubi [A] time = 0.0170626, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {670, 640, 619, 216}

$$-\frac{1}{2}\sqrt{4x-x^2}x-3\sqrt{4x-x^2}-6\sin^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[4*x - x^2],x]

[Out] -3*Sqrt[4*x - x^2] - (x*Sqrt[4*x - x^2])/2 - 6*ArcSin[1 - x/2]
```

Rule 670

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p
+ 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(
m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p
 + 1, 0] && IntegerQ[2*p]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{4x-x^2}}\,dx &= -\frac{1}{2}x\sqrt{4x-x^2}+3\int \frac{x}{\sqrt{4x-x^2}}\,dx \\
&= -3\sqrt{4x-x^2}-\frac{1}{2}x\sqrt{4x-x^2}+6\int \frac{1}{\sqrt{4x-x^2}}\,dx \\
&= -3\sqrt{4x-x^2}-\frac{1}{2}x\sqrt{4x-x^2}-\frac{3}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}}\,dx, x, 4-2x\right) \\
&= -3\sqrt{4x-x^2}-\frac{1}{2}x\sqrt{4x-x^2}-6\sin^{-1}\left(1-\frac{x}{2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0525605, size = 47, normalized size = 1.07

$$\frac{1}{2}\left(-\sqrt{4-x}x^{3/2}-6\sqrt{-(x-4)x}-24\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[4*x - x^2],x]
```

```
[Out] (-(Sqrt[4 - x]*x^(3/2)) - 6*Sqrt[-((-4 + x)*x)] - 24*ArcSin[Sqrt[1 - x/4]])
/2
```

Maple [A] time = 0.003, size = 37, normalized size = 0.8

$$6\arcsin(-1+x/2)-3\sqrt{-x^2+4x}-\frac{x}{2}\sqrt{-x^2+4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+4*x)^(1/2),x)
```

```
[Out] 6*arcsin(-1+1/2*x)-3*(-x^2+4*x)^(1/2)-1/2*x*(-x^2+4*x)^(1/2)
```

Maxima [A] time = 1.40573, size = 49, normalized size = 1.11

$$-\frac{1}{2}\sqrt{-x^2+4\,x}x-3\sqrt{-x^2+4\,x}-6\arcsin\left(-\frac{1}{2}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+4*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-x^2 + 4*x)*x - 3*sqrt(-x^2 + 4*x) - 6*arcsin(-1/2*x + 1)
```

Fricas [A] time = 2.19813, size = 85, normalized size = 1.93

$$-\frac{1}{2}\sqrt{-x^2+4\,x}(x+6)-12\arctan\left(\frac{\sqrt{-x^2+4\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+4*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-x^2 + 4*x)*(x + 6) - 12*arctan(sqrt(-x^2 + 4*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-x\,(x-4)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+4*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(-x*(x - 4)), x)
```

Giac [A] time = 1.05737, size = 34, normalized size = 0.77

$$-\frac{1}{2}\sqrt{-x^2+4x}(x+6)+6\arcsin\left(\frac{1}{2}x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+4*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-x^2 + 4*x)*(x + 6) + 6*arcsin(1/2*x - 1)
```

3.148 $\int \frac{1}{\left(2+2x+x^2\right)^2}\,dx$

Optimal. Leaf size=26

$$\frac{x+1}{2\left(x^2+2x+2\right)}+\frac{1}{2}\tan^{-1}(x+1)$$

```
[Out] (1 + x)/(2*(2 + 2*x + x^2)) + ArcTan[1 + x]/2
```

Rubi [A] time = 0.0055748, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {614, 617, 204}

$$\frac{x+1}{2\left(x^2+2x+2\right)}+\frac{1}{2}\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x + x^2)^(-2),x]
```

```
[Out] (1 + x)/(2*(2 + 2*x + x^2)) + ArcTan[1 + x]/2
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(2+2x+x^2\right)^2}\,dx &= \frac{1+x}{2\left(2+2x+x^2\right)}+\frac{1}{2}\int \frac{1}{2+2x+x^2}\,dx\\ &= \frac{1+x}{2\left(2+2x+x^2\right)}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+x\right)\\ &= \frac{1+x}{2\left(2+2x+x^2\right)}+\frac{1}{2}\tan^{-1}(1+x)\end{aligned}$$

Mathematica [A] time = 0.007493, size = 23, normalized size = 0.88

$$\frac{1}{2}\left(\frac{x+1}{x^2+2x+2}+\tan^{-1}(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 2*x + x^2)^(-2),x]
```

```
[Out] ((1 + x)/(2 + 2*x + x^2) + ArcTan[1 + x])/2
```

Maple [A] time = 0.001, size = 25, normalized size = 1.

$$\frac{2x+2}{4x^2+8x+8}+\frac{\arctan(1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+2*x+2)^2,x)
```

```
[Out] 1/4*(2*x+2)/(x^2+2*x+2)+1/2*arctan(1+x)
```

Maxima [A] time = 1.4083, size = 30, normalized size = 1.15

$$\frac{x+1}{2\left(x^2+2\,x+2\right)}+\frac{1}{2}\,\arctan\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x + 1)/(x^2 + 2*x + 2) + 1/2*arctan(x + 1)
```

Fricas [A] time = 2.2187, size = 82, normalized size = 3.15

$$\frac{\left(x^2+2\,x+2\right)\arctan\left(x+1\right)+x+1}{2\left(x^2+2\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 2*x + 2)*arctan(x + 1) + x + 1)/(x^2 + 2*x + 2)
```

Sympy [A] time = 0.113337, size = 19, normalized size = 0.73

$$\frac{x+1}{2x^2+4x+4}+\frac{\operatorname{atan}\left(x+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+2*x+2)**2,x)
```

```
[Out] (x + 1)/(2*x**2 + 4*x + 4) + atan(x + 1)/2
```

Giac [A] time = 1.05064, size = 30, normalized size = 1.15

$$\frac{x+1}{2\left(x^2+2\,x+2\right)}+\frac{1}{2}\,\arctan\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x + 1)/(x^2 + 2*x + 2) + 1/2*arctan(x + 1)
```

3.149 $\int \frac{1}{\left(5-4x-x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2(x+2)}{243\sqrt{-x^2-4x+5}}+\frac{x+2}{27\left(-x^2-4x+5\right)^{3/2}}$$

```
[Out] (2 + x)/(27*(5 - 4*x - x^2)^(3/2)) + (2*(2 + x))/(243*Sqrt[5 - 4*x - x^2])
```

Rubi [A] time = 0.0075282, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {614, 613}

$$\frac{2(x+2)}{243\sqrt{-x^2-4x+5}}+\frac{x+2}{27\left(-x^2-4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 4*x - x^2)^(-5/2),x]
```

```
[Out] (2 + x)/(27*(5 - 4*x - x^2)^(3/2)) + (2*(2 + x))/(243*Sqrt[5 - 4*x - x^2])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{\left(5-4x-x^2\right)^{5/2}}\,dx = \frac{2+x}{27\left(5-4x-x^2\right)^{3/2}} + \frac{2}{27}\int \frac{1}{\left(5-4x-x^2\right)^{3/2}}\,dx$$
$$= \frac{2+x}{27\left(5-4x-x^2\right)^{3/2}} + \frac{2(2+x)}{243\sqrt{5-4x-x^2}}$$

Mathematica [A] time = 0.0093429, size = 31, normalized size = 0.72

$$-\frac{(x+2)\left(2x^2+8x-19\right)}{243\left(-x^2-4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x - x^2)^(-5/2),x]
```

```
[Out] -((2 + x)*(-19 + 8*x + 2*x^2))/(243*(5 - 4*x - x^2)^(3/2))
```

Maple [A] time = 0.001, size = 36, normalized size = 0.8

$$\frac{(5+x)(-1+x)\left(2x^3+12x^2-3x-38\right)}{243}\left(-x^2-4x+5\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-4*x+5)^(5/2),x)
```

```
[Out] 1/243*(5+x)*(-1+x)*(2*x^3+12*x^2-3*x-38)/(-x^2-4*x+5)^(5/2)
```

Maxima [A] time = 0.921316, size = 80, normalized size = 1.86

$$\frac{2x}{243\sqrt{-x^2-4x+5}} + \frac{4}{243\sqrt{-x^2-4x+5}} + \frac{x}{27\left(-x^2-4x+5\right)^{\frac{3}{2}}} + \frac{2}{27\left(-x^2-4x+5\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/243*x/sqrt(-x^2 - 4*x + 5) + 4/243/sqrt(-x^2 - 4*x + 5) + 1/27*x/(-x^2 -
4*x + 5)^(3/2) + 2/27/(-x^2 - 4*x + 5)^(3/2)
```

Fricas [A] time = 2.30562, size = 123, normalized size = 2.86

$$-\frac{\left(2\,x^{3}+12\,x^{2}-3\,x-38\right)\sqrt{-x^{2}-4\,x+5}}{243\,\left(x^{4}+8\,x^{3}+6\,x^{2}-40\,x+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/243*(2*x^3 + 12*x^2 - 3*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^4 + 8*x^3 + 6*x^
2 - 40*x + 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(-x^{2}-4x+5\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-4*x+5)**(5/2),x)
```

```
[Out] Integral((-x**2 - 4*x + 5)**(-5/2), x)
```

Giac [A] time = 1.07263, size = 49, normalized size = 1.14

$$-\frac{((2\,(x+6)x-3)x-38)\sqrt{-x^{2}-4\,x+5}}{243\,\left(x^{2}+4\,x-5\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(5/2),x, algorithm="giac")
```

```
[Out] -1/243*((2*(x + 6)*x - 3)*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^2 + 4*x - 5)^2
```

3.150 $\int e^t \sqrt{9-e^{2t}}\,dt$

Optimal. Leaf size=33

$$\frac{1}{2}e^t\sqrt{9-e^{2t}}+\frac{9}{2}\sin^{-1}\left(\frac{e^t}{3}\right)$$

```
[Out] (E^t*Sqrt[9 - E^(2*t)])/2 + (9*ArcSin[E^t/3])/2
```

Rubi [A] time = 0.0246832, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2249, 195, 216}

$$\frac{1}{2}e^t\sqrt{9-e^{2t}}+\frac{9}{2}\sin^{-1}\left(\frac{e^t}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^t*Sqrt[9 - E^(2*t)],t]
```

```
[Out] (E^t*Sqrt[9 - E^(2*t)])/2 + (9*ArcSin[E^t/3])/2
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int e^t\sqrt{9-e^{2t}}\,dt &= \text{Subst}\left(\int \sqrt{9-t^2}\,dt,t,e^t\right)\\ &= \frac{1}{2}e^t\sqrt{9-e^{2t}}+\frac{9}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{9-t^2}}\,dt,t,e^t\right)\\ &= \frac{1}{2}e^t\sqrt{9-e^{2t}}+\frac{9}{2}\sin^{-1}\left(\frac{e^t}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.0127705, size = 32, normalized size = 0.97

$$\frac{1}{2}\left(e^t\sqrt{9-e^{2t}}+9\sin^{-1}\left(\frac{e^t}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t*Sqrt[9 - E^(2*t)],t]
```

```
[Out] (E^t*Sqrt[9 - E^(2*t)] + 9*ArcSin[E^t/3])/2
```

Maple [A] time = 0.008, size = 23, normalized size = 0.7

$$\frac{\mathrm{e}^t}{2}\sqrt{9-(\mathrm{e}^t)^2}+\frac{9}{2}\arcsin\left(\frac{\mathrm{e}^t}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)*(9-exp(2*t))^(1/2),t)
```

```
[Out] 1/2*exp(t)*(9-exp(t)^2)^(1/2)+9/2*arcsin(1/3*exp(t))
```

Maxima [A] time = 1.40656, size = 30, normalized size = 0.91

$$\frac{1}{2}\sqrt{-e^{(2t)}+9}e^{t}+\frac{9}{2}\arcsin\left(\frac{1}{3}e^{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*(9-exp(2*t))^(1/2),t, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-e^(2*t) + 9)*e^t + 9/2*arcsin(1/3*e^t)
```

Fricas [A] time = 2.28943, size = 97, normalized size = 2.94

$$\frac{1}{2}\sqrt{-e^{(2t)}+9}e^{t}-9\arctan\left(\left(\sqrt{-e^{(2t)}+9}-3\right)e^{(-t)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*(9-exp(2*t))^(1/2),t, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-e^(2*t) + 9)*e^t - 9*arctan((sqrt(-e^(2*t) + 9) - 3)*e^(-t))
```

Sympy [A] time = 1.23372, size = 29, normalized size = 0.88

$$\begin{cases}\frac{\sqrt{9-e^{2t}}e^{t}}{2}+\frac{9\,\mathrm{asin}\left(\frac{e^{t}}{3}\right)}{2} & \text{for}\ e^{t}<\log(3)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*(9-exp(2*t))**(1/2),t)
```

```
[Out] Piecewise((sqrt(9 - exp(2*t))*exp(t)/2 + 9*asin(exp(t)/3)/2, exp(t) < log(3
)))
```

Giac [A] time = 1.05804, size = 30, normalized size = 0.91

$$\frac{1}{2}\sqrt{-e^{(2t)}+9}e^{t}+\frac{9}{2}\arcsin\left(\frac{1}{3}e^{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*(9-exp(2*t))^(1/2),t, algorithm="giac")
```

```
[Out] 1/2*sqrt(-e^(2*t) + 9)*e^t + 9/2*arcsin(1/3*e^t)
```

3.151 $\int \sqrt{-9+e^{2t}}\,dt$

Optimal. Leaf size=30

$$\sqrt{e^{2t}-9}-3\tan^{-1}\left(\frac{1}{3}\sqrt{e^{2t}-9}\right)$$

```
[Out] Sqrt[-9 + E^(2*t)] - 3*ArcTan[Sqrt[-9 + E^(2*t)]/3]
```

Rubi [A] time = 0.0153925, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2282, 50, 63, 203}

$$\sqrt{e^{2t}-9}-3\tan^{-1}\left(\frac{1}{3}\sqrt{e^{2t}-9}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-9 + E^(2*t)],t]
```

```
[Out] Sqrt[-9 + E^(2*t)] - 3*ArcTan[Sqrt[-9 + E^(2*t)]/3]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{-9+e^{2t}}\,dt &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{-9+t}}{t}\,dt, t, e^{2t}\right)\\ &= \sqrt{-9+e^{2t}} - \frac{9}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{-9+t}t}\,dt, t, e^{2t}\right)\\ &= \sqrt{-9+e^{2t}} - 9\,\text{Subst}\left(\int \frac{1}{9+t^2}\,dt, t, \sqrt{-9+e^{2t}}\right)\\ &= \sqrt{-9+e^{2t}} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{-9+e^{2t}}\right)\end{aligned}$$

Mathematica [A] time = 0.0069408, size = 30, normalized size = 1.

$$\sqrt{e^{2t}-9} - 3\tan^{-1}\left(\frac{1}{3}\sqrt{e^{2t}-9}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-9 + E^(2*t)],t]
```

```
[Out] Sqrt[-9 + E^(2*t)] - 3*ArcTan[Sqrt[-9 + E^(2*t)]/3]
```

Maple [A] time = 0.004, size = 23, normalized size = 0.8

$$-3\,\arctan\left(1/3\,\sqrt{-9+e^{2\,t}}\right)+\sqrt{-9+e^{2\,t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-9+exp(2*t))^(1/2),t)
```

```
[Out] -3*arctan(1/3*(-9+exp(2*t))^(1/2))+(-9+exp(2*t))^(1/2)
```

Maxima [A] time = 1.40053, size = 30, normalized size = 1.

$$\sqrt{e^{(2\,t)} - 9} - 3\,\arctan\left(\frac{1}{3}\,\sqrt{e^{(2\,t)} - 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9+exp(2*t))^(1/2),t, algorithm="maxima")
```

```
[Out] sqrt(e^(2*t) - 9) - 3*arctan(1/3*sqrt(e^(2*t) - 9))
```

Fricas [A] time = 2.21625, size = 72, normalized size = 2.4

$$\sqrt{e^{(2\,t)} - 9} - 3\,\arctan\left(\frac{1}{3}\,\sqrt{e^{(2\,t)} - 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9+exp(2*t))^(1/2),t, algorithm="fricas")
```

```
[Out] sqrt(e^(2*t) - 9) - 3*arctan(1/3*sqrt(e^(2*t) - 9))
```

Sympy [A] time = 1.1149, size = 22, normalized size = 0.73

$$\left\{\sqrt{e^{2t} - 9} - 3\operatorname{acos}\left(3e^{-t}\right) \quad \text{for}\ e^{t} < \log(3)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9+exp(2*t))**(1/2),t)
```

```
[Out] Piecewise((sqrt(exp(2*t) - 9) - 3*acos(3*exp(-t)), exp(t) < log(3)))
```

Giac [A] time = 1.05762, size = 30, normalized size = 1.

$$\sqrt{e^{(2t)} - 9} - 3 \arctan\left(\frac{1}{3}\sqrt{e^{(2t)} - 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-9+exp(2*t))^(1/2),t, algorithm="giac")
```

```
[Out] sqrt(e^(2*t) - 9) - 3*arctan(1/3*sqrt(e^(2*t) - 9))
```

3.152 $\int \frac{1}{\sqrt{a^2+x^2}}\,dx$

Optimal. Leaf size=14

$$\tanh^{-1}\left(\frac{x}{\sqrt{a^2+x^2}}\right)$$

```
[Out] ArcTanh[x/Sqrt[a^2 + x^2]]
```

Rubi [A] time = 0.0023598, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {217, 206}

$$\tanh^{-1}\left(\frac{x}{\sqrt{a^2+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 + x^2],x]
```

```
[Out] ArcTanh[x/Sqrt[a^2 + x^2]]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a^2+x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{a^2+x^2}}\right)\\ &= \tanh^{-1}\left(\frac{x}{\sqrt{a^2+x^2}}\right)\end{aligned}$$

Mathematica [B] time = 0.0025837, size = 42, normalized size = 3.

$$\frac{1}{2}\log\left(\frac{x}{\sqrt{a^2+x^2}}+1\right)-\frac{1}{2}\log\left(1-\frac{x}{\sqrt{a^2+x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 + x^2],x]
```

```
[Out] -Log[1 - x/Sqrt[a^2 + x^2]]/2 + Log[1 + x/Sqrt[a^2 + x^2]]/2
```

Maple [A] time = 0.002, size = 13, normalized size = 0.9

$$\ln\left(x+\sqrt{a^2+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+x^2)^(1/2),x)
```

```
[Out] ln(x+(a^2+x^2)^(1/2))
```

Maxima [A] time = 0.918816, size = 11, normalized size = 0.79

$$\operatorname{arsinh}\left(\frac{x}{\sqrt{a^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(x/sqrt(a^2))
```

Fricas [A] time = 2.02888, size = 38, normalized size = 2.71

$$-\log\left(-x+\sqrt{a^2+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(a^2 + x^2))
```

Sympy [A] time = 0.938693, size = 3, normalized size = 0.21

$$\operatorname{asinh}\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+x**2)**(1/2),x)
```

```
[Out] asinh(x/a)
```

Giac [A] time = 1.06161, size = 22, normalized size = 1.57

$$-\log\left(-x + \sqrt{a^2 + x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-x + sqrt(a^2 + x^2))
```

3.153 $\int \frac{5+x}{-2+x+x^2} dx$

Optimal. Leaf size=15

$$2\log(1-x)-\log(x+2)$$

```
[Out] 2*Log[1 - x] - Log[2 + x]
```

Rubi [A] time = 0.0036326, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {632, 31}

$$2\log(1-x)-\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x)/(-2 + x + x^2),x]

[Out] 2*Log[1 - x] - Log[2 + x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{5+x}{-2+x+x^2}\,dx &= 2\int \frac{1}{-1+x}\,dx-\int \frac{1}{2+x}\,dx\\ &= 2\log(1-x)-\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.0033946, size = 15, normalized size = 1.

$$2\log(1-x)-\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x)/(-2 + x + x^2),x]
```

```
[Out] 2*Log[1 - x] - Log[2 + x]
```

Maple [A] time = 0.005, size = 14, normalized size = 0.9

$$-\ln(2+x)+2\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+x)/(x^2+x-2),x)
```

```
[Out] -ln(2+x)+2*ln(-1+x)
```

Maxima [A] time = 0.921607, size = 18, normalized size = 1.2

$$-\log(x+2)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x)/(x^2+x-2),x, algorithm="maxima")
```

```
[Out] -log(x + 2) + 2*log(x - 1)
```

Fricas [A] time = 2.12989, size = 38, normalized size = 2.53

$$-\log(x+2)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x)/(x^2+x-2),x, algorithm="fricas")
```

```
[Out] -log(x + 2) + 2*log(x - 1)
```

Sympy [A] time = 0.086555, size = 10, normalized size = 0.67

$$2\log(x-1) - \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x)/(x**2+x-2),x)
```

```
[Out] 2*log(x - 1) - log(x + 2)
```

Giac [A] time = 1.05226, size = 20, normalized size = 1.33

$$-\log(|x+2|) + 2\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+x)/(x^2+x-2),x, algorithm="giac")
```

```
[Out] -log(abs(x + 2)) + 2*log(abs(x - 1))
```

3.154 $\int \frac{x+x^3}{-1+x} \, dx$

Optimal. Leaf size=26

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\log(1-x)$$

```
[Out] 2*x + x^2/2 + x^3/3 + 2*Log[1 - x]
```

Rubi [A] time = 0.0163615, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 772}

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^3)/(-1 + x),x]
```

```
[Out] 2*x + x^2/2 + x^3/3 + 2*Log[1 - x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{x+x^3}{-1+x}\,dx = \int \frac{x\left(1+x^2\right)}{-1+x}\,dx$$
$$= \int \left(2+\frac{2}{-1+x}+x+x^2\right)dx$$
$$= 2x+\frac{x^2}{2}+\frac{x^3}{3}+2\log(1-x)$$

Mathematica [A] time = 0.0042414, size = 25, normalized size = 0.96

$$\frac{1}{6}\left(2x^3+3x^2+12x+12\log(x-1)-17\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^3)/(-1 + x),x]
```

```
[Out] (-17 + 12*x + 3*x^2 + 2*x^3 + 12*Log[-1 + x])/6
```

Maple [A] time = 0.002, size = 21, normalized size = 0.8

$$\frac{x^3}{3}+\frac{x^2}{2}+2\,x+2\,\ln\left(-1+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x)/(-1+x),x)
```

```
[Out] 1/3*x^3+1/2*x^2+2*x+2*ln(-1+x)
```

Maxima [A] time = 0.919088, size = 27, normalized size = 1.04

$$\frac{1}{3}\,x^3+\frac{1}{2}\,x^2+2\,x+2\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

Fricas [A] time = 2.04059, size = 54, normalized size = 2.08

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(x - 1)
```

Sympy [A] time = 0.06815, size = 19, normalized size = 0.73

$$\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x)/(-1+x),x)
```

```
[Out] x**3/3 + x**2/2 + 2*x + 2*log(x - 1)
```

Giac [A] time = 1.04923, size = 28, normalized size = 1.08

$$\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x + 2\log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x)/(-1+x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + 2*x + 2*log(abs(x - 1))
```

3.155 $\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx$

Optimal. Leaf size=25

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Rubi [A] time = 0.041823, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1594, 1628}

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
```

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+2x+x^2}{-2x+3x^2+2x^3}\,dx &= \int \frac{-1+2x+x^2}{x\left(-2+3x+2x^2\right)}\,dx\\ &= \int \left(\frac{1}{2x}-\frac{1}{10(2+x)}+\frac{1}{5(-1+2x)}\right)dx\\ &= \frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.0058306, size = 25, normalized size = 1.

$$\frac{1}{10}\log(1-2x)+\frac{\log(x)}{2}-\frac{1}{10}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x + x^2)/(-2*x + 3*x^2 + 2*x^3),x]
```

```
[Out] Log[1 - 2*x]/10 + Log[x]/2 - Log[2 + x]/10
```

Maple [A] time = 0.006, size = 20, normalized size = 0.8

$$-\frac{\ln(2+x)}{10}+\frac{\ln(x)}{2}+\frac{\ln(2x-1)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x)
```

```
[Out] -1/10*ln(2+x)+1/2*ln(x)+1/10*ln(2*x-1)
```

Maxima [A] time = 0.918427, size = 26, normalized size = 1.04

$$\frac{1}{10}\log(2x-1)-\frac{1}{10}\log(x+2)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="maxima")
```

```
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

Fricas [A] time = 2.13412, size = 68, normalized size = 2.72

$$\frac{1}{10}\log\left(2x-1\right)-\frac{1}{10}\log\left(x+2\right)+\frac{1}{2}\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="fricas")
```

```
[Out] 1/10*log(2*x - 1) - 1/10*log(x + 2) + 1/2*log(x)
```

Sympy [A] time = 0.122663, size = 19, normalized size = 0.76

$$\frac{\log\left(x\right)}{2}+\frac{\log\left(x-\frac{1}{2}\right)}{10}-\frac{\log\left(x+2\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x-1)/(2*x**3+3*x**2-2*x),x)
```

```
[Out] log(x)/2 + log(x - 1/2)/10 - log(x + 2)/10
```

Giac [A] time = 1.04813, size = 30, normalized size = 1.2

$$\frac{1}{10}\log\left(|2x-1|\right)-\frac{1}{10}\log\left(|x+2|\right)+\frac{1}{2}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x-1)/(2*x^3+3*x^2-2*x),x, algorithm="giac")
```

```
[Out] 1/10*log(abs(2*x - 1)) - 1/10*log(abs(x + 2)) + 1/2*log(abs(x))
```

3.156 $\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx$

Optimal. Leaf size=30

$$\frac{x^2}{2}+x+\frac{2}{1-x}+\log(1-x)-\log(x+1)$$

```
[Out] 2/(1 - x) + x + x^2/2 + Log[1 - x] - Log[1 + x]
```

Rubi [A] time = 0.0298122, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {2074}

$$\frac{x^2}{2}+x+\frac{2}{1-x}+\log(1-x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
```

```
[Out] 2/(1 - x) + x + x^2/2 + Log[1 - x] - Log[1 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+4x-2x^2+x^4}{1-x-x^2+x^3}\,dx &= \int \left(1+\frac{1}{-1-x}+\frac{2}{(-1+x)^2}+\frac{1}{-1+x}+x\right)dx\\ &= \frac{2}{1-x}+x+\frac{x^2}{2}+\log(1-x)-\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0158656, size = 29, normalized size = 0.97

$$\frac{1}{2}(x+1)^2-\frac{2}{x-1}+\log(1-x)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x - 2*x^2 + x^4)/(1 - x - x^2 + x^3),x]
```

```
[Out] -2/(-1 + x) + (1 + x)^2/2 + Log[1 - x] - Log[1 + x]
```

Maple [A] time = 0.008, size = 25, normalized size = 0.8

$$x+\frac{x^2}{2}-\ln(1+x)+\ln(-1+x)-2\,(-1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x)
```

```
[Out] x+1/2*x^2-ln(1+x)+ln(-1+x)-2/(-1+x)
```

Maxima [A] time = 0.925685, size = 32, normalized size = 1.07

$$\frac{1}{2}\,x^2+x-\frac{2}{x-1}-\log(x+1)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x - 2/(x - 1) - log(x + 1) + log(x - 1)
```

Fricas [A] time = 2.07323, size = 109, normalized size = 3.63

$$\frac{x^3+x^2-2\,(x-1)\log(x+1)+2\,(x-1)\log(x-1)-2\,x-4}{2\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="fricas")
```

```
[Out] 1/2*(x^3 + x^2 - 2*(x - 1)*log(x + 1) + 2*(x - 1)*log(x - 1) - 2*x - 4)/(x
- 1)
```

Sympy [A] time = 0.090176, size = 20, normalized size = 0.67

$$\frac{x^2}{2} + x + \log(x-1) - \log(x+1) - \frac{2}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-2*x**2+4*x+1)/(x**3-x**2-x+1),x)
```

```
[Out] x**2/2 + x + log(x - 1) - log(x + 1) - 2/(x - 1)
```

Giac [A] time = 1.05145, size = 35, normalized size = 1.17

$$\frac{1}{2}x^2 + x - \frac{2}{x-1} - \log(|x+1|) + \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-2*x^2+4*x+1)/(x^3-x^2-x+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x - 2/(x - 1) - log(abs(x + 1)) + log(abs(x - 1))
```

3.157 $\int \frac{4-x+2x^2}{4x+x^3}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

```
[Out] -ArcTan[x/2]/2 + Log[x] + Log[4 + x^2]/2
```

Rubi [A] time = 0.0382582, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1593, 1802, 635, 203, 260}

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - x + 2*x^2)/(4*x + x^3),x]
```

```
[Out] -ArcTan[x/2]/2 + Log[x] + Log[4 + x^2]/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{4-x+2x^2}{4x+x^3}\,dx &= \int \frac{4-x+2x^2}{x\left(4+x^2\right)}\,dx\\
&= \int \left(\frac{1}{x}+\frac{-1+x}{4+x^2}\right)dx\\
&= \log(x)+\int \frac{-1+x}{4+x^2}\,dx\\
&= \log(x)-\int \frac{1}{4+x^2}\,dx+\int \frac{x}{4+x^2}\,dx\\
&= -\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+\log(x)+\frac{1}{2}\log\left(4+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0046373, size = 23, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+4\right)+\log(x)-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - x + 2*x^2)/(4*x + x^3),x]
```

```
[Out] -ArcTan[x/2]/2 + Log[x] + Log[4 + x^2]/2
```

Maple [A] time = 0.005, size = 18, normalized size = 0.8

$$-\frac{1}{2}\arctan\left(\frac{x}{2}\right)+\ln(x)+\frac{\ln\left(x^2+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+4)/(x^3+4*x),x)
```

```
[Out] -1/2*arctan(1/2*x)+ln(x)+1/2*ln(x^2+4)
```

Maxima [A] time = 1.40942, size = 23, normalized size = 1.

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

Fricas [A] time = 2.05247, size = 65, normalized size = 2.83

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(x)
```

Sympy [A] time = 0.119764, size = 17, normalized size = 0.74

$$\log\left(x\right)+\frac{\log\left(x^2+4\right)}{2}-\frac{\operatorname{atan}\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+4)/(x**3+4*x),x)
```

```
[Out] log(x) + log(x**2 + 4)/2 - atan(x/2)/2
```

Giac [A] time = 1.09953, size = 24, normalized size = 1.04

$$-\frac{1}{2}\arctan\left(\frac{1}{2}x\right)+\frac{1}{2}\log\left(x^2+4\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+4)/(x^3+4*x),x, algorithm="giac")
```

```
[Out] -1/2*arctan(1/2*x) + 1/2*log(x^2 + 4) + log(abs(x))
```

3.158 $\int \frac{2-3x+4x^2}{3-4x+4x^2} \, dx$

Optimal. Leaf size=38

$$\frac{1}{8}\log\left(4x^2-4x+3\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[2]]/(4*Sqrt[2]) + Log[3 - 4*x + 4*x^2]/8
```

Rubi [A] time = 0.037902, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1657, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(4x^2-4x+3\right)+x+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 3*x + 4*x^2)/(3 - 4*x + 4*x^2),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[2]]/(4*Sqrt[2]) + Log[3 - 4*x + 4*x^2]/8
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{2-3x+4x^2}{3-4x+4x^2}\,dx &= \int \left(1-\frac{1-x}{3-4x+4x^2}\right)dx\\
&= x-\int \frac{1-x}{3-4x+4x^2}\,dx\\
&= x+\frac{1}{8}\int \frac{-4+8x}{3-4x+4x^2}\,dx-\frac{1}{2}\int \frac{1}{3-4x+4x^2}\,dx\\
&= x+\frac{1}{8}\log\left(3-4x+4x^2\right)+\text{Subst}\left(\int \frac{1}{-32-x^2}\,dx,x,-4+8x\right)\\
&= x-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{2}}\right)}{4\sqrt{2}}+\frac{1}{8}\log\left(3-4x+4x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0096372, size = 38, normalized size = 1.

$$\frac{1}{8}\log\left(4x^2-4x+3\right)+x-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 3*x + 4*x^2)/(3 - 4*x + 4*x^2),x]
```

```
[Out] x - ArcTan[(-1 + 2*x)/Sqrt[2]]/(4*Sqrt[2]) + Log[3 - 4*x + 4*x^2]/8
```

Maple [A] time = 0.004, size = 32, normalized size = 0.8

$$x+\frac{\ln\left(4x^{2}-4x+3\right)}{8}-\frac{\sqrt{2}}{8}\arctan\left(\frac{(8x-4)\sqrt{2}}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2-3*x+2)/(4*x^2-4*x+3),x)
```

```
[Out] x+1/8*ln(4*x^2-4*x+3)-1/8*2^(1/2)*arctan(1/8*(8*x-4)*2^(1/2))
```

Maxima [A] time = 1.40462, size = 42, normalized size = 1.11

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x-1)\right)+x+\frac{1}{8}\log\left(4x^{2}-4x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-3*x+2)/(4*x^2-4*x+3),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - 1)) + x + 1/8*log(4*x^2 - 4*x + 3)
```

Fricas [A] time = 2.15747, size = 101, normalized size = 2.66

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x-1)\right)+x+\frac{1}{8}\log\left(4x^{2}-4x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-3*x+2)/(4*x^2-4*x+3),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - 1)) + x + 1/8*log(4*x^2 - 4*x + 3)
```

Sympy [A] time = 0.10557, size = 34, normalized size = 0.89

$$x + \frac{\log\left(x^2 - x + \frac{3}{4}\right)}{8} - \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x - \frac{\sqrt{2}}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2-3*x+2)/(4*x**2-4*x+3),x)
```

```
[Out] x + log(x**2 - x + 3/4)/8 - sqrt(2)*atan(sqrt(2)*x - sqrt(2)/2)/8
```

Giac [A] time = 1.05783, size = 42, normalized size = 1.11

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(2x-1)\right) + x + \frac{1}{8}\log\left(4x^2 - 4x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-3*x+2)/(4*x^2-4*x+3),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x - 1)) + x + 1/8*log(4*x^2 - 4*x + 3)
```

3.159 $\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)} dx$

Optimal. Leaf size=103

$$-\frac{3(1-x)}{8\left(x^2+1\right)}+\frac{3x}{16\left(x^2+1\right)}+\frac{x+1}{8\left(x^2+1\right)^2}+\frac{15}{16}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{1}{8}\log(1-x)-\log(x)+\frac{7}{16}\tan^{-1}(x)-$$

```
[Out] (1 + x)/(8*(1 + x^2)^2) - (3*(1 - x))/(8*(1 + x^2)) + (3*x)/(16*(1 + x^2))
+ (7*ArcTan[x])/16 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/8 - Log
[x] + (15*Log[1 + x^2])/16 - Log[1 + x + x^2]/2
```

Rubi [A] time = 0.536711, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6728, 639, 199, 203, 635, 260, 634, 618, 204, 628}

$$-\frac{3(1-x)}{8\left(x^2+1\right)}+\frac{3x}{16\left(x^2+1\right)}+\frac{x+1}{8\left(x^2+1\right)^2}+\frac{15}{16}\log\left(x^2+1\right)-\frac{1}{2}\log\left(x^2+x+1\right)+\frac{1}{8}\log(1-x)-\log(x)+\frac{7}{16}\tan^{-1}(x)-$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] (1 + x)/(8*(1 + x^2)^2) - (3*(1 - x))/(8*(1 + x^2)) + (3*x)/(16*(1 + x^2))
+ (7*ArcTan[x])/16 - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/8 - Log
[x] + (15*Log[1 + x^2])/16 - Log[1 + x + x^2]/2
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
```

```
Q[p, -1] && NeQ[p, -3/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1+x^2+x^3}{(-1+x)x\left(1+x^2\right)^3\left(1+x+x^2\right)}\,dx = \int\left(\frac{1}{8(-1+x)} - \frac{1}{x} + \frac{1-x}{2\left(1+x^2\right)^3} + \frac{3(1+x)}{4\left(1+x^2\right)^2} + \frac{-1+15x}{8\left(1+x^2\right)} + \frac{-1-x}{1+x+x^2}\right)dx$$

$$= \frac{1}{8}\log(1-x) - \log(x) + \frac{1}{8}\int \frac{-1+15x}{1+x^2}\,dx + \frac{1}{2}\int \frac{1-x}{\left(1+x^2\right)^3}\,dx + \frac{3}{4}\int \frac{1+x}{\left(1+x^2\right)\ldots}$$

$$= \frac{1+x}{8\left(1+x^2\right)^2} - \frac{3(1-x)}{8\left(1+x^2\right)} + \frac{1}{8}\log(1-x) - \log(x) - \frac{1}{8}\int \frac{1}{1+x^2}\,dx + \frac{3}{8}\int \frac{1}{(1+\ldots}$$

$$= \frac{1+x}{8\left(1+x^2\right)^2} - \frac{3(1-x)}{8\left(1+x^2\right)} + \frac{3x}{16\left(1+x^2\right)} + \frac{1}{4}\tan^{-1}(x) + \frac{1}{8}\log(1-x) - \log(x) + \ldots$$

$$= \frac{1+x}{8\left(1+x^2\right)^2} - \frac{3(1-x)}{8\left(1+x^2\right)} + \frac{3x}{16\left(1+x^2\right)} + \frac{7}{16}\tan^{-1}(x) - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{8}\log\ldots$$

Mathematica [A] time = 0.0439117, size = 93, normalized size = 0.9

$$\frac{1}{48}\left(\frac{6(x+1)}{\left(x^2+1\right)^2} + \frac{9(3x-2)}{x^2+1} + 45\log\left(x^2+1\right) - 10\log\left(x^2+x+1\right) - 14\log\left(1-x^3\right) + 20\log(1-x) - 48\log(x) + 21\tan\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^3)/((-1 + x)*x*(1 + x^2)^3*(1 + x + x^2)),x]
```

```
[Out] ((6*(1 + x))/(1 + x^2)^2 + (9*(-2 + 3*x))/(1 + x^2) + 21*ArcTan[x] - 16*Sqr
t[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 20*Log[1 - x] - 48*Log[x] + 45*Log[1 + x^2
] - 10*Log[1 + x + x^2] - 14*Log[1 - x^3])/48
```

Maple [A] time = 0.01, size = 73, normalized size = 0.7

$$\frac{1}{8\left(x^{2}+1\right)^{2}}\left(\frac{9x^{3}}{2}-3x^{2}+\frac{11x}{2}-2\right)+\frac{15\ln\left(x^{2}+1\right)}{16}+\frac{7\arctan(x)}{16}-\ln(x)+\frac{\ln(-1+x)}{8}-\frac{\ln\left(x^{2}+x+1\right)}{2}-\frac{\sqrt{3}}{3}\mathrm{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x)
```

```
[Out] 1/8*(9/2*x^3-3*x^2+11/2*x-2)/(x^2+1)^2+15/16*ln(x^2+1)+7/16*arctan(x)-ln(x)
+1/8*ln(-1+x)-1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.41266, size = 104, normalized size = 1.01

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{9x^{3}-6x^{2}+11x-4}{16\left(x^{4}+2x^{2}+1\right)}+\frac{7}{16}\arctan(x)-\frac{1}{2}\log\left(x^{2}+x+1\right)+\frac{15}{16}\log\left(x^{2}+1\right)+\frac{1}{8}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4
)/(x^4 + 2*x^2 + 1) + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2
 + 1) + 1/8*log(x - 1) - log(x)
```

Fricas [A] time = 2.1198, size = 387, normalized size = 3.76

$$\frac{27x^{3}-16\sqrt{3}\left(x^{4}+2x^{2}+1\right)\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-18x^{2}+21\left(x^{4}+2x^{2}+1\right)\arctan(x)-24\left(x^{4}+2x^{2}+1\right)\log\left(x^{2}+\right.}{48\left(x^{4}+2x^{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/48*(27*x^3 - 16*sqrt(3)*(x^4 + 2*x^2 + 1)*arctan(1/3*sqrt(3)*(2*x + 1)) -
 18*x^2 + 21*(x^4 + 2*x^2 + 1)*arctan(x) - 24*(x^4 + 2*x^2 + 1)*log(x^2 + x
 + 1) + 45*(x^4 + 2*x^2 + 1)*log(x^2 + 1) + 6*(x^4 + 2*x^2 + 1)*log(x - 1)
```

```
- 48*(x^4 + 2*x^2 + 1)*log(x) + 33*x - 12)/(x^4 + 2*x^2 + 1)
```

Sympy [A] time = 0.460151, size = 88, normalized size = 0.85

$$-\log(x) + \frac{\log(x-1)}{8} + \frac{15\log(x^2+1)}{16} - \frac{\log(x^2+x+1)}{2} + \frac{7\,\mathrm{atan}(x)}{16} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{3} + \frac{9x^3 - 6x^2 + 11x - }{16x^4 + 32x^2 + 16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+1)/(-1+x)/x/(x**2+1)**3/(x**2+x+1),x)
```

```
[Out] -log(x) + log(x - 1)/8 + 15*log(x**2 + 1)/16 - log(x**2 + x + 1)/2 + 7*atan
(x)/16 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3 + (9*x**3 - 6*x**2 + 11*
x - 4)/(16*x**4 + 32*x**2 + 16)
```

Giac [A] time = 1.05815, size = 100, normalized size = 0.97

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \frac{9x^3 - 6x^2 + 11x - 4}{16(x^2+1)^2} + \frac{7}{16}\arctan(x) - \frac{1}{2}\log(x^2+x+1) + \frac{15}{16}\log(x^2+1) + \frac{1}{8}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(-1+x)/x/(x^2+1)^3/(x^2+x+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/16*(9*x^3 - 6*x^2 + 11*x - 4
)/(x^2 + 1)^2 + 7/16*arctan(x) - 1/2*log(x^2 + x + 1) + 15/16*log(x^2 + 1)
+ 1/8*log(abs(x - 1)) - log(abs(x))
```

3.160 $\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=33

$$-\frac{2x+1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-2\tan^{-1}(x)$$

```
[Out] -(1 + 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.047321, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1805, 801, 635, 203, 260}

$$-\frac{2x+1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]
```

```
[Out] -(1 + 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-3x+2x^2-x^3}{x\left(1+x^2\right)^2}\,dx &= -\frac{1+2x}{2\left(1+x^2\right)}-\frac{1}{2}\int \frac{-2+4x}{x\left(1+x^2\right)}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}-\frac{1}{2}\int \left(-\frac{2}{x}+\frac{2(2+x)}{1+x^2}\right)dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}+\log(x)-\int \frac{2+x}{1+x^2}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}+\log(x)-2\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\
&= -\frac{1+2x}{2\left(1+x^2\right)}-2\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0203084, size = 33, normalized size = 1.

$$\frac{-2x-1}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\log(x)-2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x + 2*x^2 - x^3)/(x*(1 + x^2)^2),x]

[Out] (-1 - 2*x)/(2*(1 + x^2)) - 2*ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.007, size = 28, normalized size = 0.9

$$-\frac{1}{x^2+1}\left(x+\frac{1}{2}\right)-\frac{\ln\left(x^2+1\right)}{2}-2\arctan\left(x\right)+\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x)
```

```
[Out] -(x+1/2)/(x^2+1)-1/2*ln(x^2+1)-2*arctan(x)+ln(x)
```

Maxima [A] time = 1.40388, size = 39, normalized size = 1.18

$$-\frac{2\,x+1}{2\left(x^2+1\right)}-2\arctan\left(x\right)-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Fricas [A] time = 1.90529, size = 130, normalized size = 3.94

$$-\frac{4\left(x^2+1\right)\arctan\left(x\right)+\left(x^2+1\right)\log\left(x^2+1\right)-2\left(x^2+1\right)\log\left(x\right)+2\,x+1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*(x^2 + 1)*arctan(x) + (x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1)*log(x) +
 2*x + 1)/(x^2 + 1)
```

Sympy [A] time = 0.131393, size = 27, normalized size = 0.82

$$-\frac{2x+1}{2x^2+2}+\log(x)-\frac{\log(x^2+1)}{2}-2\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+2*x**2-3*x+1)/x/(x**2+1)**2,x)
```

```
[Out] -(2*x + 1)/(2*x**2 + 2) + log(x) - log(x**2 + 1)/2 - 2*atan(x)
```

Giac [A] time = 1.06265, size = 41, normalized size = 1.24

$$-\frac{2\,x+1}{2\left(x^2+1\right)}-2\,\arctan(x)-\frac{1}{2}\,\log\left(x^2+1\right)+\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+2*x^2-3*x+1)/x/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*x + 1)/(x^2 + 1) - 2*arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

3.161 $\int \frac{1}{(1+x^2)^2} dx$

Optimal. Leaf size=19

$$\frac{x}{2(x^2+1)} + \frac{1}{2}\tan^{-1}(x)$$

```
[Out] x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rubi [A] time = 0.002733, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {199, 203}

$$\frac{x}{2(x^2+1)} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^(-2),x]
```

```
[Out] x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(1+x^2\right)^2}\,dx = \frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\int \frac{1}{1+x^2}\,dx$$
$$= \frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}(x)$$

Mathematica [A] time = 0.0048138, size = 16, normalized size = 0.84

$$\frac{1}{2}\left(\frac{x}{x^2+1} + \tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^(-2),x]
```

```
[Out] (x/(1 + x^2) + ArcTan[x])/2
```

Maple [A] time = 0.001, size = 16, normalized size = 0.8

$$\frac{x}{2x^2+2} + \frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^2,x)
```

```
[Out] 1/2*x/(x^2+1)+1/2*arctan(x)
```

Maxima [A] time = 1.40375, size = 20, normalized size = 1.05

$$\frac{x}{2\left(x^2+1\right)} + \frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Fricas [A] time = 1.76961, size = 55, normalized size = 2.89

$$\frac{(x^2+1)\arctan(x)+x}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) + x)/(x^2 + 1)
```

Sympy [A] time = 0.094896, size = 12, normalized size = 0.63

$$\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**2,x)
```

```
[Out] x/(2*x**2 + 2) + atan(x)/2
```

Giac [A] time = 1.05088, size = 20, normalized size = 1.05

$$\frac{x}{2(x^2+1)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

3.162 $\int \frac{1}{(-1+x)(2+x)}\,dx$

Optimal. Leaf size=19

$$\frac{1}{3}\log(1-x)-\frac{1}{3}\log(x+2)$$

```
[Out] Log[1 - x]/3 - Log[2 + x]/3
```

Rubi [A] time = 0.0029557, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {36, 31}

$$\frac{1}{3}\log(1-x)-\frac{1}{3}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)*(2 + x)),x]
```

```
[Out] Log[1 - x]/3 - Log[2 + x]/3
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-1+x)(2+x)}\,dx &= \frac{1}{3}\int \frac{1}{-1+x}\,dx-\frac{1}{3}\int \frac{1}{2+x}\,dx\\ &= \frac{1}{3}\log(1-x)-\frac{1}{3}\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.0024767, size = 19, normalized size = 1.

$$\frac{1}{3}\log(1-x)-\frac{1}{3}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)*(2 + x)),x]
```

```
[Out] Log[1 - x]/3 - Log[2 + x]/3
```

Maple [A] time = 0.004, size = 14, normalized size = 0.7

$$-\frac{\ln(2+x)}{3}+\frac{\ln(-1+x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)/(2+x),x)
```

```
[Out] -1/3*ln(2+x)+1/3*ln(-1+x)
```

Maxima [A] time = 0.924871, size = 18, normalized size = 0.95

$$-\frac{1}{3}\log(x+2)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(2+x),x, algorithm="maxima")
```

```
[Out] -1/3*log(x + 2) + 1/3*log(x - 1)
```

Fricas [A] time = 1.75034, size = 46, normalized size = 2.42

$$-\frac{1}{3}\log(x+2)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(2+x),x, algorithm="fricas")
```

```
[Out] -1/3*log(x + 2) + 1/3*log(x - 1)
```

Sympy [A] time = 0.090317, size = 12, normalized size = 0.63

$$\frac{\log(x-1)}{3}-\frac{\log(x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(2+x),x)
```

```
[Out] log(x - 1)/3 - log(x + 2)/3
```

Giac [A] time = 1.07272, size = 20, normalized size = 1.05

$$-\frac{1}{3}\log(|x+2|)+\frac{1}{3}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/(2+x),x, algorithm="giac")
```

```
[Out] -1/3*log(abs(x + 2)) + 1/3*log(abs(x - 1))
```

3.163 $\int \frac{7}{-12+5x+2x^2} \, dx$

Optimal. Leaf size=19

$$\frac{7}{11}\log(3-2x) - \frac{7}{11}\log(x+4)$$

```
[Out] (7*Log[3 - 2*x])/11 - (7*Log[4 + x])/11
```

Rubi [A] time = 0.0066137, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {12, 616, 31}

$$\frac{7}{11}\log(3-2x) - \frac{7}{11}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[7/(-12 + 5*x + 2*x^2),x]
```

```
[Out] (7*Log[3 - 2*x])/11 - (7*Log[4 + x])/11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{7}{-12+5x+2x^2}\,dx &= 7\int \frac{1}{-12+5x+2x^2}\,dx\\ &= \frac{14}{11}\int \frac{1}{-3+2x}\,dx - \frac{14}{11}\int \frac{1}{8+2x}\,dx\\ &= \frac{7}{11}\log(3-2x) - \frac{7}{11}\log(4+x)\end{aligned}$$

Mathematica [A] time = 0.0033927, size = 21, normalized size = 1.11

$$7\left(\frac{1}{11}\log(3-2x) - \frac{1}{11}\log(x+4)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[7/(-12 + 5*x + 2*x^2),x]
```

```
[Out] 7*(Log[3 - 2*x]/11 - Log[4 + x]/11)
```

Maple [A] time = 0.006, size = 16, normalized size = 0.8

$$\frac{7\,\ln(-3+2\,x)}{11} - \frac{7\,\ln(4+x)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(7/(2*x^2+5*x-12),x)
```

```
[Out] 7/11*ln(-3+2*x)-7/11*ln(4+x)
```

Maxima [A] time = 0.919794, size = 20, normalized size = 1.05

$$\frac{7}{11}\log(2\,x-3) - \frac{7}{11}\log(x+4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7/(2*x^2+5*x-12),x, algorithm="maxima")
```

```
[Out] 7/11*log(2*x - 3) - 7/11*log(x + 4)
```

Fricas [A] time = 1.88224, size = 50, normalized size = 2.63

$$\frac{7}{11}\,\log\left(2\,x-3\right)-\frac{7}{11}\,\log\left(x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7/(2*x^2+5*x-12),x, algorithm="fricas")
```

```
[Out] 7/11*log(2*x - 3) - 7/11*log(x + 4)
```

Sympy [A] time = 0.090972, size = 17, normalized size = 0.89

$$\frac{7\log\left(x-\frac{3}{2}\right)}{11}-\frac{7\log\left(x+4\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7/(2*x**2+5*x-12),x)
```

```
[Out] 7*log(x - 3/2)/11 - 7*log(x + 4)/11
```

Giac [A] time = 1.05879, size = 23, normalized size = 1.21

$$\frac{7}{11}\,\log\left(|2\,x-3|\right)-\frac{7}{11}\,\log\left(|x+4|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(7/(2*x^2+5*x-12),x, algorithm="giac")
```

```
[Out] 7/11*log(abs(2*x - 3)) - 7/11*log(abs(x + 4))
```

3.164 $\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)} dx$

Optimal. Leaf size=32

$$-\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

```
[Out] -9/(32*(1 - 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Rubi [A] time = 0.0258633, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {893}

$$-\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
```

```
[Out] -9/(32*(1 - 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{-4+3x+x^2}{(-1+2x)^2(3+2x)}\,dx &= \int \left(-\frac{9}{16(-1+2x)^2}+\frac{41}{64(-1+2x)}-\frac{25}{64(3+2x)}\right)dx\\ &= -\frac{9}{32(1-2x)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(3+2x)\end{aligned}$$

Mathematica [A] time = 0.0154008, size = 32, normalized size = 1.

$$\frac{9}{32(2x-1)}+\frac{41}{128}\log(1-2x)-\frac{25}{128}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 3*x + x^2)/((-1 + 2*x)^2*(3 + 2*x)),x]
```

```
[Out] 9/(32*(-1 + 2*x)) + (41*Log[1 - 2*x])/128 - (25*Log[3 + 2*x])/128
```

Maple [A] time = 0.008, size = 27, normalized size = 0.8

$$-\frac{25\ln(3+2x)}{128}+\frac{9}{64x-32}+\frac{41\ln(2x-1)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3*x-4)/(2*x-1)^2/(3+2*x),x)
```

```
[Out] -25/128*ln(3+2*x)+9/32/(2*x-1)+41/128*ln(2*x-1)
```

Maxima [A] time = 0.92048, size = 35, normalized size = 1.09

$$\frac{9}{32(2x-1)}-\frac{25}{128}\log(2x+3)+\frac{41}{128}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="maxima")
```

```
[Out] 9/32/(2*x - 1) - 25/128*log(2*x + 3) + 41/128*log(2*x - 1)
```

Fricas [A] time = 1.83746, size = 107, normalized size = 3.34

$$-\frac{25(2x-1)\log(2x+3)-41(2x-1)\log(2x-1)-36}{128(2x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="fricas")
```

```
[Out] -1/128*(25*(2*x - 1)*log(2*x + 3) - 41*(2*x - 1)*log(2*x - 1) - 36)/(2*x -
1)
```

Sympy [A] time = 0.12268, size = 26, normalized size = 0.81

$$\frac{41\log\left(x-\frac{1}{2}\right)}{128}-\frac{25\log\left(x+\frac{3}{2}\right)}{128}+\frac{9}{64x-32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3*x-4)/(-1+2*x)**2/(3+2*x),x)
```

```
[Out] 41*log(x - 1/2)/128 - 25*log(x + 3/2)/128 + 9/(64*x - 32)
```

Giac [A] time = 1.06433, size = 58, normalized size = 1.81

$$\frac{9}{32\,(2\,x-1)}-\frac{1}{8}\,\log\left(\frac{|2\,x-1|}{2\,(2\,x-1)^2}\right)-\frac{25}{128}\,\log\left(\left|-\frac{4}{2\,x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3*x-4)/(-1+2*x)^2/(3+2*x),x, algorithm="giac")
```

```
[Out] 9/32/(2*x - 1) - 1/8*log(1/2*abs(2*x - 1)/(2*x - 1)^2) - 25/128*log(abs(-4/
(2*x - 1) - 1))
```

3.165 $\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx$

Optimal. Leaf size=43

$$\frac{201}{15125(5x+3)}-\frac{12}{1375(5x+3)^2}+\frac{20\log(6-x)}{3993}+\frac{1493\log(5x+3)}{499125}$$

```
[Out] -12/(1375*(3 + 5*x)^2) + 201/(15125*(3 + 5*x)) + (20*Log[6 - x])/3993 + (14
93*Log[3 + 5*x])/499125
```

Rubi [A] time = 0.0422541, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1593, 148}

$$\frac{201}{15125(5x+3)}-\frac{12}{1375(5x+3)^2}+\frac{20\log(6-x)}{3993}+\frac{1493\log(5x+3)}{499125}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

```
[Out] -12/(1375*(3 + 5*x)^2) + 201/(15125*(3 + 5*x)) + (20*Log[6 - x])/3993 + (14
93*Log[3 + 5*x])/499125
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx = \int \frac{(-1+x)x^2}{(-6+x)(3+5x)^3}\,dx$$
$$= \int \left(\frac{20}{3993(-6+x)} + \frac{24}{275(3+5x)^3} - \frac{201}{3025(3+5x)^2} + \frac{1493}{99825(3+5x)}\right) dx$$
$$= -\frac{12}{1375(3+5x)^2} + \frac{201}{15125(3+5x)} + \frac{20\log(6-x)}{3993} + \frac{1493\log(3+5x)}{499125}$$

Mathematica [A] time = 0.0216479, size = 33, normalized size = 0.77

$$\frac{\frac{99(335x+157)}{(5x+3)^2} + 2500\log(x-6) + 1493\log(5x+3)}{499125}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

```
[Out] ((99*(157 + 335*x))/(3 + 5*x)^2 + 2500*Log[-6 + x] + 1493*Log[3 + 5*x])/499
125
```

Maple [A] time = 0.007, size = 34, normalized size = 0.8

$$-\frac{12}{1375\,(3+5\,x)^2} + \frac{201}{45375+75625\,x} + \frac{1493\,\ln(3+5\,x)}{499125} + \frac{20\,\ln(-6+x)}{3993}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x^2)/(-6+x)/(3+5*x)^3,x)
```

```
[Out] -12/1375/(3+5*x)^2+201/15125/(3+5*x)+1493/499125*ln(3+5*x)+20/3993*ln(-6+x)
```

Maxima [A] time = 0.92771, size = 46, normalized size = 1.07

$$\frac{3\,(335\,x+157)}{15125\left(25\,x^2+30\,x+9\right)} + \frac{1493}{499125}\,\log(5\,x+3) + \frac{20}{3993}\,\log(x-6)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="maxima")
```

```
[Out] 3/15125*(335*x + 157)/(25*x^2 + 30*x + 9) + 1493/499125*log(5*x + 3) + 20/3
993*log(x - 6)
```

Fricas [A] time = 1.92462, size = 170, normalized size = 3.95

$$\frac{1493\left(25\,x^2+30\,x+9\right)\log\left(5\,x+3\right)+2500\left(25\,x^2+30\,x+9\right)\log\left(x-6\right)+33165\,x+15543}{499125\left(25\,x^2+30\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="fricas")
```

```
[Out] 1/499125*(1493*(25*x^2 + 30*x + 9)*log(5*x + 3) + 2500*(25*x^2 + 30*x + 9)*
log(x - 6) + 33165*x + 15543)/(25*x^2 + 30*x + 9)
```

Sympy [A] time = 0.141443, size = 32, normalized size = 0.74

$$\frac{1005x+471}{378125x^2+453750x+136125}+\frac{20\log\left(x-6\right)}{3993}+\frac{1493\log\left(x+\frac{3}{5}\right)}{499125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x**2)/(-6+x)/(3+5*x)**3,x)
```

```
[Out] (1005*x + 471)/(378125*x**2 + 453750*x + 136125) + 20*log(x - 6)/3993 + 149
3*log(x + 3/5)/499125
```

Giac [A] time = 1.07588, size = 42, normalized size = 0.98

$$\frac{3\left(335\,x+157\right)}{15125\left(5\,x+3\right)^2}+\frac{1493}{499125}\log\left(\left|5\,x+3\right|\right)+\frac{20}{3993}\log\left(\left|x-6\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="giac")
```

```
[Out] 3/15125*(335*x + 157)/(5*x + 3)^2 + 1493/499125*log(abs(5*x + 3)) + 20/3993
*log(abs(x - 6))
```

3.166 $\int \frac{1}{-x^3+x^4}\,dx$

Optimal. Leaf size=21

$$\frac{1}{2x^2}+\frac{1}{x}+\log(1-x)-\log(x)$$

```
[Out] 1/(2*x^2) + x^(-1) + Log[1 - x] - Log[x]
```

Rubi [A] time = 0.0111957, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 44}

$$\frac{1}{2x^2}+\frac{1}{x}+\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + x^4)^(-1),x]
```

```
[Out] 1/(2*x^2) + x^(-1) + Log[1 - x] - Log[x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\int \frac{1}{-x^3+x^4}\,dx = \int \frac{1}{(-1+x)x^3}\,dx$$
$$= \int \left(\frac{1}{-1+x} - \frac{1}{x^3} - \frac{1}{x^2} - \frac{1}{x}\right) dx$$
$$= \frac{1}{2x^2} + \frac{1}{x} + \log(1-x) - \log(x)$$

Mathematica [A] time = 0.0019447, size = 21, normalized size = 1.

$$\frac{1}{2x^2} + \frac{1}{x} + \log(1-x) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + x^4)^(-1),x]
```

```
[Out] 1/(2*x^2) + x^(-1) + Log[1 - x] - Log[x]
```

Maple [A] time = 0.006, size = 18, normalized size = 0.9

$$\frac{1}{2\,x^2} + x^{-1} - \ln(x) + \ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-x^3),x)
```

```
[Out] 1/2/x^2+1/x-ln(x)+ln(-1+x)
```

Maxima [A] time = 0.923482, size = 26, normalized size = 1.24

$$\frac{2\,x+1}{2\,x^2} + \log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^3),x, algorithm="maxima")
```

```
[Out] 1/2*(2*x + 1)/x^2 + log(x - 1) - log(x)
```

Fricas [A] time = 1.86409, size = 72, normalized size = 3.43

$$\frac{2\,x^2\log\left(x-1\right)-2\,x^2\log\left(x\right)+2\,x+1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^2*log(x - 1) - 2*x^2*log(x) + 2*x + 1)/x^2
```

Sympy [A] time = 0.097411, size = 17, normalized size = 0.81

$$-\log\left(x\right)+\log\left(x-1\right)+\frac{2x+1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-x**3),x)
```

```
[Out] -log(x) + log(x - 1) + (2*x + 1)/(2*x**2)
```

Giac [A] time = 1.05542, size = 28, normalized size = 1.33

$$\frac{2\,x+1}{2\,x^2}+\log\left(|x-1|\right)-\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^3),x, algorithm="giac")
```

```
[Out] 1/2*(2*x + 1)/x^2 + log(abs(x - 1)) - log(abs(x))
```

3.167 $\int \frac{1-x-x^2+x^3+x^4}{-x+x^3} \, dx$

Optimal. Leaf size=25

$$\frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Rubi [A] time = 0.0501107, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1593, 1802, 260}

$$\frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
```

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1-x-x^2+x^3+x^4}{-x+x^3}\,dx &= \int \frac{1-x-x^2+x^3+x^4}{x\left(-1+x^2\right)}\,dx\\ &= \int \left(1-\frac{1}{x}+x+\frac{x}{-1+x^2}\right)dx\\ &= x+\frac{x^2}{2}-\log(x)+\int \frac{x}{-1+x^2}\,dx\\ &= x+\frac{x^2}{2}-\log(x)+\frac{1}{2}\log\left(1-x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0052752, size = 25, normalized size = 1.

$$\frac{x^2}{2}+\frac{1}{2}\log\left(1-x^2\right)+x-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x - x^2 + x^3 + x^4)/(-x + x^3),x]
```

```
[Out] x + x^2/2 - Log[x] + Log[1 - x^2]/2
```

Maple [A] time = 0.007, size = 24, normalized size = 1.

$$x+\frac{x^2}{2}-\ln(x)+\frac{\ln(1+x)}{2}+\frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^3-x^2-x+1)/(x^3-x),x)
```

```
[Out] x+1/2*x^2-ln(x)+1/2*ln(1+x)+1/2*ln(-1+x)
```

Maxima [A] time = 0.925505, size = 31, normalized size = 1.24

$$\frac{1}{2}x^2+x+\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x + 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

Fricas [A] time = 1.9284, size = 55, normalized size = 2.2

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log\left(x^2 - 1\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + x + 1/2*log(x^2 - 1) - log(x)
```

Sympy [A] time = 0.086741, size = 17, normalized size = 0.68

$$\frac{x^2}{2} + x - \log(x) + \frac{\log\left(x^2 - 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**3-x**2-x+1)/(x**3-x),x)
```

```
[Out] x**2/2 + x - log(x) + log(x**2 - 1)/2
```

Giac [A] time = 1.05335, size = 35, normalized size = 1.4

$$\frac{1}{2}x^2 + x + \frac{1}{2}\log(|x + 1|) + \frac{1}{2}\log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^3-x^2-x+1)/(x^3-x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + x + 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1)) - log(abs(x))
```

3.168 $\int \frac{-2+x^2}{x(2+x^2)} dx$

Optimal. Leaf size=11

$$\log(x^2+2) - \log(x)$$

```
[Out] -Log[x] + Log[2 + x^2]
```

Rubi [A] time = 0.0118638, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {446, 72}

$$\log(x^2+2) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^2)/(x*(2 + x^2)),x]
```

```
[Out] -Log[x] + Log[2 + x^2]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{-2+x^2}{x\left(2+x^2\right)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{-2+x}{x(2+x)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{1}{x}+\frac{2}{2+x}\right)dx, x, x^2\right)\\ &= -\log(x)+\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0035185, size = 11, normalized size = 1.

$$\log\left(x^2+2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^2)/(x*(2 + x^2)),x]
```

```
[Out] -Log[x] + Log[2 + x^2]
```

Maple [A] time = 0.004, size = 12, normalized size = 1.1

$$-\ln(x)+\ln\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2)/x/(x^2+2),x)
```

```
[Out] -ln(x)+ln(x^2+2)
```

Maxima [A] time = 0.924889, size = 18, normalized size = 1.64

$$\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="maxima")
```

```
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

Fricas [A] time = 1.83461, size = 31, normalized size = 2.82

$$\log\left(x^2+2\right)-\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="fricas")
```

```
[Out] log(x^2 + 2) - log(x)
```

Sympy [A] time = 0.087302, size = 8, normalized size = 0.73

$$-\log\left(x\right)+\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2)/x/(x**2+2),x)
```

```
[Out] -log(x) + log(x**2 + 2)
```

Giac [A] time = 1.06732, size = 18, normalized size = 1.64

$$\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2)/x/(x^2+2),x, algorithm="giac")
```

```
[Out] log(x^2 + 2) - 1/2*log(x^2)
```

3.169 $\int \frac{2-4x^2+x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=36

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

Rubi [A] time = 0.117578, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {6725, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{2-4x^2+x^3}{\left(1+x^2\right)\left(2+x^2\right)}\,dx &= \int \left(\frac{6-x}{1+x^2}+\frac{2(-5+x)}{2+x^2}\right)dx\\ &= 2\int \frac{-5+x}{2+x^2}\,dx+\int \frac{6-x}{1+x^2}\,dx\\ &= 2\int \frac{x}{2+x^2}\,dx+6\int \frac{1}{1+x^2}\,dx-10\int \frac{1}{2+x^2}\,dx-\int \frac{x}{1+x^2}\,dx\\ &= 6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-\frac{1}{2}\log\left(1+x^2\right)+\log\left(2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0152253, size = 36, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+1\right)+\log\left(x^2+2\right)+6\tan^{-1}(x)-5\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - 4*x^2 + x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] 6*ArcTan[x] - 5*Sqrt[2]*ArcTan[x/Sqrt[2]] - Log[1 + x^2]/2 + Log[2 + x^2]
```

Maple [A] time = 0.005, size = 32, normalized size = 0.9

$$6\arctan(x)-\frac{\ln\left(x^2+1\right)}{2}+\ln\left(x^2+2\right)-5\arctan\left(1/2\,x\sqrt{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x)
```

```
[Out] 6*arctan(x)-1/2*ln(x^2+1)+ln(x^2+2)-5*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.40449, size = 42, normalized size = 1.17

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan(x)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

Fricas [A] time = 1.88212, size = 111, normalized size = 3.08

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan(x)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="fricas")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

Sympy [A] time = 0.168758, size = 36, normalized size = 1.

$$-\frac{\log\left(x^2+1\right)}{2}+\log\left(x^2+2\right)+6\,\mathrm{atan}(x)-5\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-4*x**2+2)/(x**2+1)/(x**2+2),x)
```

```
[Out] -log(x**2 + 1)/2 + log(x**2 + 2) + 6*atan(x) - 5*sqrt(2)*atan(sqrt(2)*x/2)
```

Giac [A] time = 1.055, size = 42, normalized size = 1.17

$$-5\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+6\arctan\left(x\right)+\log\left(x^2+2\right)-\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-4*x^2+2)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] -5*sqrt(2)*arctan(1/2*sqrt(2)*x) + 6*arctan(x) + log(x^2 + 2) - 1/2*log(x^2
 + 1)
```

3.170 $\int \frac{1+x^2+x^4}{(1+x^2)(4+x^2)^2} \, dx$

Optimal. Leaf size=29

$$-\frac{13x}{24(x^2+4)} + \frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9}\tan^{-1}(x)$$

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Rubi [A] time = 0.113411, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {6725, 203, 199}

$$-\frac{13x}{24(x^2+4)} + \frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{9}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
```

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
```

```
ator[p + 1/n] < Denominator[p])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x^2+x^4}{\left(1+x^2\right)\left(4+x^2\right)^2}\,dx &= \int \left(\frac{1}{9\left(1+x^2\right)}-\frac{13}{3\left(4+x^2\right)^2}+\frac{8}{9\left(4+x^2\right)}\right)dx\\
&= \frac{1}{9}\int \frac{1}{1+x^2}\,dx+\frac{8}{9}\int \frac{1}{4+x^2}\,dx-\frac{13}{3}\int \frac{1}{\left(4+x^2\right)^2}\,dx\\
&= -\frac{13x}{24\left(4+x^2\right)}+\frac{4}{9}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)-\frac{13}{24}\int \frac{1}{4+x^2}\,dx\\
&= -\frac{13x}{24\left(4+x^2\right)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.0161005, size = 29, normalized size = 1.

$$-\frac{13x}{24\left(x^2+4\right)}+\frac{25}{144}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{9}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^4)/((1 + x^2)*(4 + x^2)^2),x]
```

```
[Out] (-13*x)/(24*(4 + x^2)) + (25*ArcTan[x/2])/144 + ArcTan[x]/9
```

Maple [A] time = 0.009, size = 22, normalized size = 0.8

$$-\frac{13\,x}{24\,x^2+96}+\frac{25}{144}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x)
```

```
[Out] -13/24*x/(x^2+4)+25/144*arctan(1/2*x)+1/9*arctan(x)
```

Maxima [A] time = 1.40065, size = 28, normalized size = 0.97

$$-\frac{13\,x}{24\,\left(x^2+4\right)}+\frac{25}{144}\,\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{9}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="maxima")
```

```
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

Fricas [A] time = 1.90528, size = 105, normalized size = 3.62

$$\frac{25\left(x^2+4\right)\arctan\left(\frac{1}{2}\,x\right)+16\left(x^2+4\right)\arctan\left(x\right)-78\,x}{144\left(x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(25*(x^2 + 4)*arctan(1/2*x) + 16*(x^2 + 4)*arctan(x) - 78*x)/(x^2 + 4
)
```

Sympy [A] time = 0.156617, size = 22, normalized size = 0.76

$$-\frac{13x}{24x^2+96}+\frac{25\operatorname{atan}\left(\frac{x}{2}\right)}{144}+\frac{\operatorname{atan}\left(x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+x**2+1)/(x**2+1)/(x**2+4)**2,x)
```

```
[Out] -13*x/(24*x**2 + 96) + 25*atan(x/2)/144 + atan(x)/9
```

Giac [A] time = 1.04939, size = 28, normalized size = 0.97

$$-\frac{13\,x}{24\left(x^2+4\right)}+\frac{25}{144}\,\arctan\left(\frac{1}{2}\,x\right)+\frac{1}{9}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+x^2+1)/(x^2+1)/(x^2+4)^2,x, algorithm="giac")
```

```
[Out] -13/24*x/(x^2 + 4) + 25/144*arctan(1/2*x) + 1/9*arctan(x)
```

3.171 $\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx$

Optimal. Leaf size=60

$$-\frac{481\log\left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(x+5)}{24843}+\frac{451\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

```
[Out] -79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]) + (200*L
og[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[1 + x + x^2])/5586
```

Rubi [A] time = 0.254217, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {6728, 634, 618, 204, 628}

$$-\frac{481\log\left(x^2+x+1\right)}{5586}-\frac{79}{273(x+5)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(x+5)}{24843}+\frac{451\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2793\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] -79/(273*(5 + x)) + (451*ArcTan[(1 + 2*x)/Sqrt[3]])/(2793*Sqrt[3]) + (200*L
og[3 - 2*x])/3211 + (2731*Log[5 + x])/24843 - (481*Log[1 + x + x^2])/5586
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1+16x}{(5+x)^2(-3+2x)\left(1+x+x^2\right)}\,dx = \int \left(\frac{79}{273(5+x)^2}+\frac{2731}{24843(5+x)}+\frac{400}{3211(-3+2x)}+\frac{-15-481x}{2793\left(1+x+x^2\right)}\right)dx$$

$$= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}+\frac{\int \frac{-15-481x}{1+x+x^2}\,dx}{2793}$$

$$= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}+\frac{451\int \frac{1}{1+x+x^2}\,dx}{5586}-\frac{481\int \frac{1+\cdots}{1+x\cdots}}{558\cdots}$$

$$= -\frac{79}{273(5+x)}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}-\frac{481\log\left(1+x+x^2\right)}{5586}-\frac{451\,\mathrm{S}\cdots}{\cdots}$$

$$= -\frac{79}{273(5+x)}+\frac{451\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2793\sqrt{3}}+\frac{200\log(3-2x)}{3211}+\frac{2731\log(5+x)}{24843}-\frac{481\log\cdots}{5\cdots}$$

Mathematica [A] time = 0.0522483, size = 54, normalized size = 0.9

$$\frac{-243867\log\left(x^2+x+1\right)-\frac{819546}{x+5}+176400\log(3-2x)+311334\log(x+5)+152438\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2832102}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 16*x)/((5 + x)^2*(-3 + 2*x)*(1 + x + x^2)),x]
```

```
[Out] (-819546/(5 + x) + 152438*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]] + 176400*Log[3
- 2*x] + 311334*Log[5 + x] - 243867*Log[1 + x + x^2])/2832102
```

Maple [A] time = 0.009, size = 48, normalized size = 0.8

$$-\frac{79}{1365+273\,x}+\frac{2731\,\ln\left(5+x\right)}{24843}+\frac{200\,\ln\left(-3+2\,x\right)}{3211}-\frac{481\,\ln\left(x^{2}+x+1\right)}{5586}+\frac{451\,\sqrt{3}}{8379}\arctan\left(\frac{\left(1+2\,x\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x)
```

```
[Out] -79/273/(5+x)+2731/24843*ln(5+x)+200/3211*ln(-3+2*x)-481/5586*ln(x^2+x+1)+4
51/8379*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.41567, size = 63, normalized size = 1.05

$$\frac{451}{8379}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-\frac{79}{273\,(x+5)}-\frac{481}{5586}\,\log\left(x^{2}+x+1\right)+\frac{200}{3211}\,\log\left(2\,x-3\right)+\frac{2731}{24843}\,\log\left(x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] 451/8379*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 79/273/(x + 5) - 481/5586*
log(x^2 + x + 1) + 200/3211*log(2*x - 3) + 2731/24843*log(x + 5)
```

Fricas [A] time = 2.00764, size = 236, normalized size = 3.93

$$\frac{152438\,\sqrt{3}(x+5)\arctan\left(\frac{1}{3}\,\sqrt{3}(2\,x+1)\right)-243867\,(x+5)\log\left(x^{2}+x+1\right)+176400\,(x+5)\log\left(2\,x-3\right)+311334\,(x-}{2832102\,(x+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] 1/2832102*(152438*sqrt(3)*(x + 5)*arctan(1/3*sqrt(3)*(2*x + 1)) - 243867*(x
 + 5)*log(x^2 + x + 1) + 176400*(x + 5)*log(2*x - 3) + 311334*(x + 5)*log(x
 + 5) - 819546)/(x + 5)
```

Sympy [A] time = 0.227584, size = 63, normalized size = 1.05

$$\frac{200\log\left(x-\frac{3}{2}\right)}{3211}+\frac{2731\log\left(x+5\right)}{24843}-\frac{481\log\left(x^2+x+1\right)}{5586}+\frac{451\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{8379}-\frac{79}{273x+1365}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)**2/(-3+2*x)/(x**2+x+1),x)
```

```
[Out] 200*log(x - 3/2)/3211 + 2731*log(x + 5)/24843 - 481*log(x**2 + x + 1)/5586
+ 451*sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/8379 - 79/(273*x + 1365)
```

Giac [A] time = 1.0654, size = 81, normalized size = 1.35

$$\frac{451}{8379}\sqrt{3}\arctan\left(-\sqrt{3}\left(\frac{14}{x+5}-3\right)\right)-\frac{79}{273\,(x+5)}-\frac{481}{5586}\log\left(-\frac{9}{x+5}+\frac{21}{(x+5)^2}+1\right)+\frac{200}{3211}\log\left(\left|-\frac{13}{x+5}+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+16*x)/(5+x)^2/(-3+2*x)/(x^2+x+1),x, algorithm="giac")
```

```
[Out] 451/8379*sqrt(3)*arctan(-sqrt(3)*(14/(x + 5) - 3)) - 79/273/(x + 5) - 481/5
586*log(-9/(x + 5) + 21/(x + 5)^2 + 1) + 200/3211*log(abs(-13/(x + 5) + 2))
```

3.172 $\int \frac{x^4}{(9+x^2)^3} \, dx$

Optimal. Leaf size=37

$$-\frac{x^3}{4(x^2+9)^2}-\frac{3x}{8(x^2+9)}+\frac{1}{8}\tan^{-1}\left(\frac{x}{3}\right)$$

```
[Out] -x^3/(4*(9 + x^2)^2) - (3*x)/(8*(9 + x^2)) + ArcTan[x/3]/8
```

Rubi [A] time = 0.0082532, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {288, 203}

$$-\frac{x^3}{4(x^2+9)^2}-\frac{3x}{8(x^2+9)}+\frac{1}{8}\tan^{-1}\left(\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(9 + x^2)^3,x]
```

```
[Out] -x^3/(4*(9 + x^2)^2) - (3*x)/(8*(9 + x^2)) + ArcTan[x/3]/8
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{\left(9+x^2\right)^3}\,dx = -\frac{x^3}{4\left(9+x^2\right)^2} + \frac{3}{4}\int \frac{x^2}{\left(9+x^2\right)^2}\,dx$$
$$= -\frac{x^3}{4\left(9+x^2\right)^2} - \frac{3x}{8\left(9+x^2\right)} + \frac{3}{8}\int \frac{1}{9+x^2}\,dx$$
$$= -\frac{x^3}{4\left(9+x^2\right)^2} - \frac{3x}{8\left(9+x^2\right)} + \frac{1}{8}\tan^{-1}\left(\frac{x}{3}\right)$$

Mathematica [A] time = 0.0127404, size = 28, normalized size = 0.76

$$\frac{1}{8}\left(\tan^{-1}\left(\frac{x}{3}\right) - \frac{x\left(5x^2+27\right)}{\left(x^2+9\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(9 + x^2)^3,x]
```

```
[Out] (-((x*(27 + 5*x^2))/(9 + x^2)^2) + ArcTan[x/3])/8
```

Maple [A] time = 0.007, size = 25, normalized size = 0.7

$$\frac{1}{\left(x^2+9\right)^2}\left(-\frac{5\,x^3}{8} - \frac{27\,x}{8}\right) + \frac{1}{8}\arctan\left(\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^2+9)^3,x)
```

```
[Out] (-5/8*x^3-27/8*x)/(x^2+9)^2+1/8*arctan(1/3*x)
```

Maxima [A] time = 1.41383, size = 41, normalized size = 1.11

$$-\frac{5\,x^3+27\,x}{8\left(x^4+18\,x^2+81\right)} + \frac{1}{8}\,\arctan\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^2+9)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(5*x^3 + 27*x)/(x^4 + 18*x^2 + 81) + 1/8*arctan(1/3*x)
```

Fricas [A] time = 1.92872, size = 104, normalized size = 2.81

$$-\frac{5\,x^3-\left(x^4+18\,x^2+81\right)\arctan\left(\frac{1}{3}\,x\right)+27\,x}{8\left(x^4+18\,x^2+81\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^2+9)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(5*x^3 - (x^4 + 18*x^2 + 81)*arctan(1/3*x) + 27*x)/(x^4 + 18*x^2 + 81)
```

Sympy [A] time = 0.116243, size = 26, normalized size = 0.7

$$-\frac{5x^3+27x}{8x^4+144x^2+648}+\frac{\operatorname{atan}\left(\frac{x}{3}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**2+9)**3,x)
```

```
[Out] -(5*x**3 + 27*x)/(8*x**4 + 144*x**2 + 648) + atan(x/3)/8
```

Giac [A] time = 1.0556, size = 34, normalized size = 0.92

$$-\frac{5\,x^3+27\,x}{8\left(x^2+9\right)^2}+\frac{1}{8}\,\arctan\left(\frac{1}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^2+9)^3,x, algorithm="giac")
```

```
[Out] -1/8*(5*x^3 + 27*x)/(x^2 + 9)^2 + 1/8*arctan(1/3*x)
```

3.173 $\int \frac{19x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx$

Optimal. Leaf size=97

$$\frac{19(44x+39)}{276(1-x)^2\left(4x^2+5x+3\right)}-\frac{209\log\left(4x^2+5x+3\right)}{4608}-\frac{1843}{4416(1-x)}-\frac{399}{736(1-x)^2}+\frac{209\log(1-x)}{2304}+\frac{114437\tan^{-1}\left(\frac{8x}{\sqrt{}}\right.}{52992\sqrt{23}}$$

```
[Out] -399/(736*(1 - x)^2) - 1843/(4416*(1 - x)) + (19*(39 + 44*x))/(276*(1 - x)^
2*(3 + 5*x + 4*x^2)) + (114437*ArcTan[(5 + 8*x)/Sqrt[23]])/(52992*Sqrt[23])
 + (209*Log[1 - x])/2304 - (209*Log[3 + 5*x + 4*x^2])/4608
```

Rubi [A] time = 0.0768091, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.35, Rules used = {12, 822, 800, 634, 618, 204, 628}

$$\frac{19(44x+39)}{276(1-x)^2\left(4x^2+5x+3\right)}-\frac{209\log\left(4x^2+5x+3\right)}{4608}-\frac{1843}{4416(1-x)}-\frac{399}{736(1-x)^2}+\frac{209\log(1-x)}{2304}+\frac{114437\tan^{-1}\left(\frac{8x}{\sqrt{}}\right.}{52992\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(19*x)/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] -399/(736*(1 - x)^2) - 1843/(4416*(1 - x)) + (19*(39 + 44*x))/(276*(1 - x)^
2*(3 + 5*x + 4*x^2)) + (114437*ArcTan[(5 + 8*x)/Sqrt[23]])/(52992*Sqrt[23])
 + (209*Log[1 - x])/2304 - (209*Log[3 + 5*x + 4*x^2])/4608
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
```

```
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{19x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx = 19\int \frac{x}{(-1+x)^3\left(3+5x+4x^2\right)^2}\,dx$$

$$= \frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)} + \frac{19}{276}\int \frac{57+132x}{(-1+x)^3\left(3+5x+4x^2\right)}\,dx$$

$$= \frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)} + \frac{19}{276}\int\left(\frac{63}{4(-1+x)^3} - \frac{97}{16(-1+x)^2} + \frac{253}{192(-1+x)} + \cdots\right.$$

$$= -\frac{399}{736(1-x)^2} - \frac{1843}{4416(1-x)} + \frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)} + \frac{209\log(1-x)}{2304} + \frac{19\int \cdots}{\cdots}$$

$$= -\frac{399}{736(1-x)^2} - \frac{1843}{4416(1-x)} + \frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)} + \frac{209\log(1-x)}{2304} - \frac{209\int \cdots}{\cdots}$$

$$= -\frac{399}{736(1-x)^2} - \frac{1843}{4416(1-x)} + \frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)} + \frac{209\log(1-x)}{2304} - \frac{209\log \cdots}{\cdots}$$

$$= -\frac{399}{736(1-x)^2} - \frac{1843}{4416(1-x)} + \frac{19(39+44x)}{276(1-x)^2\left(3+5x+4x^2\right)} + \frac{114437\tan^{-1}\left(\frac{5+8x}{\sqrt{23}}\right)}{52992\sqrt{23}} + \cdots$$

Mathematica [A] time = 0.0407205, size = 78, normalized size = 0.8

$$\frac{19\left(\frac{184(2204x+975)}{4x^2+5x+3} - 17457\log\left(4x^2+5x+3\right) + \frac{59248}{x-1} - \frac{25392}{(x-1)^2} + 34914\log(1-x) + 36138\sqrt{23}\tan^{-1}\left(\frac{8x+5}{\sqrt{23}}\right)\right)}{7312896}$$

Antiderivative was successfully verified.

```
[In] Integrate[(19*x)/((-1 + x)^3*(3 + 5*x + 4*x^2)^2),x]
```

```
[Out] (19*(-25392/(-1 + x)^2 + 59248/(-1 + x) + (184*(975 + 2204*x))/(3 + 5*x + 4
*x^2) + 36138*Sqrt[23]*ArcTan[(5 + 8*x)/Sqrt[23]] + 34914*Log[1 - x] - 1745
7*Log[3 + 5*x + 4*x^2]))/7312896
```

Maple [A] time = 0.01, size = 68, normalized size = 0.7

$$-\frac{19}{288\,(-1+x)^2} + \frac{133}{-864+864\,x} + \frac{209\,\ln(-1+x)}{2304} - \frac{19}{6912}\left(-\frac{2204\,x}{23} - \frac{975}{23}\right)\left(x^2+\frac{5\,x}{4}+\frac{3}{4}\right)^{-1} - \frac{209\,\ln\left(4\,x^2+5\,x+\cdots\right.}{4608}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(19*x/(-1+x)^3/(4*x^2+5*x+3)^2,x)
```

```
[Out] -19/288/(-1+x)^2+133/864/(-1+x)+209/2304*ln(-1+x)-19/6912*(-2204/23*x-975/2
3)/(x^2+5/4*x+3/4)-209/4608*ln(4*x^2+5*x+3)+114437/1218816*arctan(1/23*(5+8
*x)*23^(1/2))*23^(1/2)
```

Maxima [A] time = 1.41221, size = 101, normalized size = 1.04

$$\frac{114437}{1218816}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(8x+5)\right)+\frac{19\left(388x^3-407x^2-120x-45\right)}{4416\left(4x^4-3x^3-3x^2-x+3\right)}-\frac{209}{4608}\log\left(4x^2+5x+3\right)+\frac{209}{2304}\log(x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(19*x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="maxima")
```

```
[Out] 114437/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 19/4416*(388*x^3
- 407*x^2 - 120*x - 45)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3) - 209/4608*log(4*x^
2 + 5*x + 3) + 209/2304*log(x - 1)
```

Fricas [A] time = 1.97032, size = 378, normalized size = 3.9

$$\frac{19\left(214176x^3+12046\sqrt{23}\left(4x^4-3x^3-3x^2-x+3\right)\arctan\left(\frac{1}{23}\sqrt{23}(8x+5)\right)-224664x^2-5819\left(4x^4-3x^3-3x^2-\right.\right.}{2437632\left(4x^4-3x^3-3x^2-x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(19*x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="fricas")
```

```
[Out] 19/2437632*(214176*x^3 + 12046*sqrt(23)*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*arc
tan(1/23*sqrt(23)*(8*x + 5)) - 224664*x^2 - 5819*(4*x^4 - 3*x^3 - 3*x^2 - x
 + 3)*log(4*x^2 + 5*x + 3) + 11638*(4*x^4 - 3*x^3 - 3*x^2 - x + 3)*log(x -
1) - 66240*x - 24840)/(4*x^4 - 3*x^3 - 3*x^2 - x + 3)
```

Sympy [A] time = 0.205866, size = 88, normalized size = 0.91

$$\frac{19\left(388x^3-407x^2-120x-45\right)}{17664x^4-13248x^3-13248x^2-4416x+13248}+\frac{209\log\left(x-1\right)}{2304}-\frac{209\log\left(x^2+\frac{5x}{4}+\frac{3}{4}\right)}{4608}+\frac{114437\sqrt{23}\operatorname{atan}\left(\frac{8\sqrt{23}x}{23}\cdots\right.}{1218816}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(19*x/(-1+x)**3/(4*x**2+5*x+3)**2,x)
```

```
[Out] 19*(388*x**3 - 407*x**2 - 120*x - 45)/(17664*x**4 - 13248*x**3 - 13248*x**2
 - 4416*x + 13248) + 209*log(x - 1)/2304 - 209*log(x**2 + 5*x/4 + 3/4)/4608
 + 114437*sqrt(23)*atan(8*sqrt(23)*x/23 + 5*sqrt(23)/23)/1218816
```

Giac [A] time = 1.05541, size = 96, normalized size = 0.99

$$\frac{114437}{1218816}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(8x+5)\right)+\frac{19\left(388x^3-407x^2-120x-45\right)}{4416\left(4x^2+5x+3\right)(x-1)^2}-\frac{209}{4608}\log\left(4x^2+5x+3\right)+\frac{209}{2304}\log\left(|\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(19*x/(-1+x)^3/(4*x^2+5*x+3)^2,x, algorithm="giac")
```

```
[Out] 114437/1218816*sqrt(23)*arctan(1/23*sqrt(23)*(8*x + 5)) + 19/4416*(388*x^3
- 407*x^2 - 120*x - 45)/((4*x^2 + 5*x + 3)*(x - 1)^2) - 209/4608*log(4*x^2
+ 5*x + 3) + 209/2304*log(abs(x - 1))
```

3.174 $\int \frac{1+x^2+x^3}{2x^2+x^3+x^4} dx$

Optimal. Leaf size=46

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

```
[Out] -1/(2*x) + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

Rubi [A] time = 0.0592398, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1594, 1628, 634, 618, 204, 628}

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] -1/(2*x) + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^2+x^3}{2x^2+x^3+x^4}\,dx &= \int \frac{1+x^2+x^3}{x^2\left(2+x+x^2\right)}\,dx\\
&= \int \left(\frac{1}{2x^2}-\frac{1}{4x}+\frac{3+5x}{4\left(2+x+x^2\right)}\right)dx\\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{1}{4}\int \frac{3+5x}{2+x+x^2}\,dx\\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{1}{8}\int \frac{1}{2+x+x^2}\,dx+\frac{5}{8}\int \frac{1+2x}{2+x+x^2}\,dx\\
&= -\frac{1}{2x}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-7-x^2}\,dx,x,1+2x\right)\\
&= -\frac{1}{2x}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{7}}\right)}{4\sqrt{7}}-\frac{\log(x)}{4}+\frac{5}{8}\log\left(2+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0253046, size = 46, normalized size = 1.

$$\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{2x}-\frac{\log(x)}{4}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{4\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2 + x^3)/(2*x^2 + x^3 + x^4),x]
```

```
[Out] -1/(2*x) + ArcTan[(1 + 2*x)/Sqrt[7]]/(4*Sqrt[7]) - Log[x]/4 + (5*Log[2 + x
+ x^2])/8
```

Maple [A] time = 0.007, size = 36, normalized size = 0.8

$$-\frac{1}{2\,x}-\frac{\ln(x)}{4}+\frac{5\,\ln\left(x^2+x+2\right)}{8}+\frac{\sqrt{7}}{28}\arctan\left(\frac{(1+2\,x)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2+1)/(x^4+x^3+2*x^2),x)
```

```
[Out] -1/2/x-1/4*ln(x)+5/8*ln(x^2+x+2)+1/28*arctan(1/7*(1+2*x)*7^(1/2))*7^(1/2)
```

Maxima [A] time = 1.41136, size = 47, normalized size = 1.02

$$\frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2\,x+1)\right)-\frac{1}{2\,x}+\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{4}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="maxima")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) -
 1/4*log(x)
```

Fricas [A] time = 1.9602, size = 128, normalized size = 2.78

$$\frac{2\sqrt{7}x\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)+35x\log\left(x^2+x+2\right)-14x\log\left(x\right)-28}{56x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="fricas")
```

```
[Out] 1/56*(2*sqrt(7)*x*arctan(1/7*sqrt(7)*(2*x + 1)) + 35*x*log(x^2 + x + 2) - 1
4*x*log(x) - 28)/x
```

Sympy [A] time = 0.149009, size = 46, normalized size = 1.

$$-\frac{\log\left(x\right)}{4}+\frac{5\log\left(x^2+x+2\right)}{8}+\frac{\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x}{7}+\frac{\sqrt{7}}{7}\right)}{28}-\frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2+1)/(x**4+x**3+2*x**2),x)
```

```
[Out] -log(x)/4 + 5*log(x**2 + x + 2)/8 + sqrt(7)*atan(2*sqrt(7)*x/7 + sqrt(7)/7)
/28 - 1/(2*x)
```

Giac [A] time = 1.06356, size = 49, normalized size = 1.07

$$\frac{1}{28}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)-\frac{1}{2x}+\frac{5}{8}\log\left(x^2+x+2\right)-\frac{1}{4}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2+1)/(x^4+x^3+2*x^2),x, algorithm="giac")
```

```
[Out] 1/28*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) - 1/2/x + 5/8*log(x^2 + x + 2) -
 1/4*log(abs(x))
```

3.175 $\int \frac{1}{-x^3+x^6}\,dx$

Optimal. Leaf size=48

$$\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
x^2]/6
```

Rubi [A] time = 0.0239909, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {1593, 325, 200, 31, 634, 618, 204, 628}

$$\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
x^2]/6
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{-x^3+x^6}\,dx &= \int \frac{1}{x^3\left(-1+x^3\right)}\,dx \\
&= \frac{1}{2x^2} + \int \frac{1}{-1+x^3}\,dx \\
&= \frac{1}{2x^2} + \frac{1}{3}\int \frac{1}{-1+x}\,dx + \frac{1}{3}\int \frac{-2-x}{1+x+x^2}\,dx \\
&= \frac{1}{2x^2} + \frac{1}{3}\log(1-x) - \frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx - \frac{1}{2}\int \frac{1}{1+x+x^2}\,dx \\
&= \frac{1}{2x^2} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right) + \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right) \\
&= \frac{1}{2x^2} - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0120447, size = 48, normalized size = 1.

$$
\frac{1}{2x^2} - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{3}\log(1-x) - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^3 + x^6)^(-1),x]
```

```
[Out] 1/(2*x^2) - ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x +
x^2]/6
```

Maple [A] time = 0.004, size = 38, normalized size = 0.8

$$
\frac{\ln(-1+x)}{3} - \frac{\ln\left(x^2+x+1\right)}{6} - \frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right) + \frac{1}{2x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-x^3),x)
```

```
[Out] 1/3*ln(-1+x)-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)+1/2/x^
2
```

Maxima [A] time = 1.43038, size = 50, normalized size = 1.04

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1)
 + 1/3*log(x - 1)
```

Fricas [A] time = 1.84714, size = 138, normalized size = 2.88

$$-\frac{2\sqrt{3}x^2\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+x^2\log\left(x^2+x+1\right)-2x^2\log(x-1)-3}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="fricas")
```

```
[Out] -1/6*(2*sqrt(3)*x^2*arctan(1/3*sqrt(3)*(2*x + 1)) + x^2*log(x^2 + x + 1) -
2*x^2*log(x - 1) - 3)/x^2
```

Sympy [A] time = 0.129993, size = 48, normalized size = 1.

$$\frac{\log(x-1)}{3}-\frac{\log\left(x^2+x+1\right)}{6}-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}+\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-x**3),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3
)/3 + 1/(2*x**2)
```

Giac [A] time = 1.05633, size = 51, normalized size = 1.06

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2x^2}-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-x^3),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2/x^2 - 1/6*log(x^2 + x + 1)
 + 1/3*log(abs(x - 1))
```

3.176 $\int \frac{x^2}{1+x}\,dx$

Optimal. Leaf size=15

$$\frac{x^2}{2} - x + \log(x+1)$$

```
[Out] -x + x^2/2 + Log[1 + x]
```

Rubi [A] time = 0.00575, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{x^2}{2} - x + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x),x]

[Out] -x + x^2/2 + Log[1 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{1+x}\,dx &= \int \left(-1+x+\frac{1}{1+x}\right)dx\\ &= -x+\frac{x^2}{2}+\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0030243, size = 19, normalized size = 1.27

$$\frac{1}{2}(x+1)^2 - 2(x+1) + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x),x]
```

```
[Out] -2*(1 + x) + (1 + x)^2/2 + Log[1 + x]
```

Maple [A] time = 0.001, size = 14, normalized size = 0.9

$$-x+\frac{x^{2}}{2}+\ln\left(1+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+x),x)
```

```
[Out] -x+1/2*x^2+ln(1+x)
```

Maxima [A] time = 0.950728, size = 18, normalized size = 1.2

$$\frac{1}{2}\,x^{2}-x+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - x + log(x + 1)
```

Fricas [A] time = 1.80622, size = 35, normalized size = 2.33

$$\frac{1}{2}\,x^{2}-x+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - x + log(x + 1)
```

Sympy [A] time = 0.067622, size = 10, normalized size = 0.67

$$\frac{x^2}{2} - x + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+x),x)
```

```
[Out] x**2/2 - x + log(x + 1)
```

Giac [A] time = 1.04549, size = 19, normalized size = 1.27

$$\frac{1}{2}x^2 - x + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - x + log(abs(x + 1))
```

3.177 $\int \frac{x}{-5+x} \, dx$

Optimal. Leaf size=10

$$x + 5\log(5 - x)$$

```
[Out] x + 5*Log[5 - x]
```

Rubi [A] time = 0.0044039, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {43}

$$x + 5\log(5 - x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-5 + x),x]
```

```
[Out] x + 5*Log[5 - x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{-5+x} \, dx &= \int \left(1 + \frac{5}{-5+x}\right) dx \\ &= x + 5\log(5 - x) \end{aligned}$$

Mathematica [A] time = 0.0014074, size = 8, normalized size = 0.8

$$x + 5\log(x - 5)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-5 + x),x]
```

```
[Out] x + 5*Log[-5 + x]
```

Maple [A] time = 0.002, size = 9, normalized size = 0.9

$$x + 5 \ln(-5 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-5+x),x)
```

```
[Out] x+5*ln(-5+x)
```

Maxima [A] time = 0.927806, size = 11, normalized size = 1.1

$$x + 5 \log(x - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-5+x),x, algorithm="maxima")
```

```
[Out] x + 5*log(x - 5)
```

Fricas [A] time = 1.78642, size = 24, normalized size = 2.4

$$x + 5 \log(x - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-5+x),x, algorithm="fricas")
```

```
[Out] x + 5*log(x - 5)
```

Sympy [A] time = 0.067517, size = 7, normalized size = 0.7

$$x + 5\log(x - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-5+x),x)
```

```
[Out] x + 5*log(x - 5)
```

Giac [A] time = 1.05003, size = 12, normalized size = 1.2

$$x + 5\log(|x - 5|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-5+x),x, algorithm="giac")
```

```
[Out] x + 5*log(abs(x - 5))
```

3.178 $\int \frac{-1+4x}{(-1+x)(2+x)}\,dx$

Optimal. Leaf size=13

$$\log(1-x)+3\log(x+2)$$

```
[Out] Log[1 - x] + 3*Log[2 + x]
```

Rubi [A] time = 0.0057979, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {72}

$$\log(1-x)+3\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 4*x)/((-1 + x)*(2 + x)),x]
```

```
[Out] Log[1 - x] + 3*Log[2 + x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+4x}{(-1+x)(2+x)}\,dx &= \int \left(\frac{1}{-1+x}+\frac{3}{2+x}\right)dx\\ &= \log(1-x)+3\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.0038172, size = 13, normalized size = 1.

$$\log(1-x)+3\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 4*x)/((-1 + x)*(2 + x)),x]
```

```
[Out] Log[1 - x] + 3*Log[2 + x]
```

Maple [A] time = 0.004, size = 12, normalized size = 0.9

$$3 \ln(2 + x) + \ln(-1 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+4*x)/(-1+x)/(2+x),x)
```

```
[Out] 3*ln(2+x)+ln(-1+x)
```

Maxima [A] time = 0.918911, size = 15, normalized size = 1.15

$$3 \log(x + 2) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+4*x)/(-1+x)/(2+x),x, algorithm="maxima")
```

```
[Out] 3*log(x + 2) + log(x - 1)
```

Fricas [A] time = 1.9185, size = 36, normalized size = 2.77

$$3 \log(x + 2) + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+4*x)/(-1+x)/(2+x),x, algorithm="fricas")
```

```
[Out] 3*log(x + 2) + log(x - 1)
```

Sympy [A] time = 0.094259, size = 10, normalized size = 0.77

$$\log(x-1)+3\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+4*x)/(-1+x)/(2+x),x)
```

```
[Out] log(x - 1) + 3*log(x + 2)
```

Giac [A] time = 1.05401, size = 18, normalized size = 1.38

$$3\,\log(|x+2|)+\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+4*x)/(-1+x)/(2+x),x, algorithm="giac")
```

```
[Out] 3*log(abs(x + 2)) + log(abs(x - 1))
```

3.179 $\int \frac{1}{(1+x)(2+x)} dx$

Optimal. Leaf size=11

$$\log(x+1) - \log(x+2)$$

```
[Out] Log[1 + x] - Log[2 + x]
```

Rubi [A] time = 0.0014919, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {36, 31}

$$\log(x+1) - \log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + x)),x]
```

```
[Out] Log[1 + x] - Log[2 + x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)(2+x)} dx &= \int \frac{1}{1+x} dx - \int \frac{1}{2+x} dx\\ &= \log(1+x) - \log(2+x)\end{aligned}$$

Mathematica [A] time = 0.0028611, size = 11, normalized size = 1.

$$\log(x+1)-\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(2 + x)),x]
```

```
[Out] Log[1 + x] - Log[2 + x]
```

Maple [A] time = 0.004, size = 12, normalized size = 1.1

$$\ln(1+x)-\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(2+x),x)
```

```
[Out] ln(1+x)-ln(2+x)
```

Maxima [A] time = 0.92628, size = 15, normalized size = 1.36

$$-\log(x+2)+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x),x, algorithm="maxima")
```

```
[Out] -log(x + 2) + log(x + 1)
```

Fricas [A] time = 1.91272, size = 35, normalized size = 3.18

$$-\log(x+2)+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x),x, algorithm="fricas")
```

```
[Out] -log(x + 2) + log(x + 1)
```

Sympy [A] time = 0.090856, size = 8, normalized size = 0.73

$$\log(x+1) - \log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x),x)
```

```
[Out] log(x + 1) - log(x + 2)
```

Giac [A] time = 1.06296, size = 18, normalized size = 1.64

$$-\log(|x+2|) + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x),x, algorithm="giac")
```

```
[Out] -log(abs(x + 2)) + log(abs(x + 1))
```

3.180 $\int \frac{-5+6x}{3+2x} \, dx$

Optimal. Leaf size=12

$$3x - 7\log(2x + 3)$$

```
[Out] 3*x - 7*Log[3 + 2*x]
```

Rubi [A] time = 0.005847, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$3x - 7\log(2x + 3)$$

Antiderivative was successfully verified.

```
[In] Int[(-5 + 6*x)/(3 + 2*x),x]
```

```
[Out] 3*x - 7*Log[3 + 2*x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-5 + 6x}{3 + 2x} \, dx &= \int \left(3 - \frac{14}{3 + 2x}\right) dx \\ &= 3x - 7\log(3 + 2x)\end{aligned}$$

Mathematica [A] time = 0.0029611, size = 15, normalized size = 1.25

$$3x - 7\log(2x + 3) + \frac{9}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-5 + 6*x)/(3 + 2*x),x]
```

```
[Out] 9/2 + 3*x - 7*Log[3 + 2*x]
```

Maple [A] time = 0.002, size = 13, normalized size = 1.1

$$3x - 7\ln(3 + 2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-5+6*x)/(3+2*x),x)
```

```
[Out] 3*x-7*ln(3+2*x)
```

Maxima [A] time = 0.920511, size = 16, normalized size = 1.33

$$3x - 7\log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+6*x)/(3+2*x),x, algorithm="maxima")
```

```
[Out] 3*x - 7*log(2*x + 3)
```

Fricas [A] time = 1.81019, size = 30, normalized size = 2.5

$$3x - 7\log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+6*x)/(3+2*x),x, algorithm="fricas")
```

```
[Out] 3*x - 7*log(2*x + 3)
```

Sympy [A] time = 0.070079, size = 10, normalized size = 0.83

$$3x - 7\log(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+6*x)/(3+2*x),x)
```

```
[Out] 3*x - 7*log(2*x + 3)
```

Giac [A] time = 1.06254, size = 18, normalized size = 1.5

$$3\,x - 7\,\log(|2\,x + 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-5+6*x)/(3+2*x),x, algorithm="giac")
```

```
[Out] 3*x - 7*log(abs(2*x + 3))
```

3.181 $\int \frac{1}{(a+x)(b+x)} dx$

Optimal. Leaf size=26

$$\frac{\log(b+x)}{a-b}-\frac{\log(a+x)}{a-b}$$

```
[Out] -(Log[a + x]/(a - b)) + Log[b + x]/(a - b)
```

Rubi [A] time = 0.0057296, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {36, 31}

$$\frac{\log(b+x)}{a-b}-\frac{\log(a+x)}{a-b}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + x)*(b + x)),x]
```

```
[Out] -(Log[a + x]/(a - b)) + Log[b + x]/(a - b)
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+x)(b+x)} dx &= \frac{\int \frac{1}{a+x} dx}{-a+b}-\frac{\int \frac{1}{b+x} dx}{-a+b}\\ &= -\frac{\log(a+x)}{a-b}+\frac{\log(b+x)}{a-b}\end{aligned}$$

Mathematica [A] time = 0.0065485, size = 19, normalized size = 0.73

$$\frac{\log(b+x)-\log(a+x)}{a-b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + x)*(b + x)),x]
```

```
[Out] (-Log[a + x] + Log[b + x])/(a - b)
```

Maple [A] time = 0.006, size = 27, normalized size = 1.

$$-\frac{\ln(a+x)}{a-b}+\frac{\ln(b+x)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+x)/(b+x),x)
```

```
[Out] -ln(a+x)/(a-b)+ln(b+x)/(a-b)
```

Maxima [A] time = 0.928666, size = 35, normalized size = 1.35

$$-\frac{\log(a+x)}{a-b}+\frac{\log(b+x)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x)/(b+x),x, algorithm="maxima")
```

```
[Out] -log(a + x)/(a - b) + log(b + x)/(a - b)
```

Fricas [A] time = 1.8644, size = 49, normalized size = 1.88

$$-\frac{\log(a+x)-\log(b+x)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x)/(b+x),x, algorithm="fricas")
```

```
[Out] -(log(a + x) - log(b + x))/(a - b)
```

Sympy [B] time = 0.197396, size = 80, normalized size = 3.08

$$\frac{\log\left(-\frac{a^2}{2(a-b)}+\frac{ab}{a-b}+\frac{a}{2}-\frac{b^2}{2(a-b)}+\frac{b}{2}+x\right)}{a-b}-\frac{\log\left(\frac{a^2}{2(a-b)}-\frac{ab}{a-b}+\frac{a}{2}+\frac{b^2}{2(a-b)}+\frac{b}{2}+x\right)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x)/(b+x),x)
```

```
[Out] log(-a**2/(2*(a - b)) + a*b/(a - b) + a/2 - b**2/(2*(a - b)) + b/2 + x)/(a
- b) - log(a**2/(2*(a - b)) - a*b/(a - b) + a/2 + b**2/(2*(a - b)) + b/2 +
x)/(a - b)
```

Giac [A] time = 1.05028, size = 38, normalized size = 1.46

$$-\frac{\log\left(|a+x|\right)}{a-b}+\frac{\log\left(|b+x|\right)}{a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+x)/(b+x),x, algorithm="giac")
```

```
[Out] -log(abs(a + x))/(a - b) + log(abs(b + x))/(a - b)
```

3.182 $\int \frac{1+x^2}{-x+x^2} dx$

Optimal. Leaf size=14

$$x + 2\log(1 - x) - \log(x)$$

```
[Out] x + 2*Log[1 - x] - Log[x]
```

Rubi [A] time = 0.017737, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1593, 894}

$$x + 2\log(1 - x) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/(-x + x^2),x]
```

```
[Out] x + 2*Log[1 - x] - Log[x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{1+x^2}{-x+x^2}\,dx = \int \frac{1+x^2}{(-1+x)x}\,dx$$
$$= \int \left(1+\frac{2}{-1+x}-\frac{1}{x}\right)dx$$
$$= x+2\log(1-x)-\log(x)$$

Mathematica [A] time = 0.0031274, size = 14, normalized size = 1.

$$x+2\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/(-x + x^2),x]
```

```
[Out] x + 2*Log[1 - x] - Log[x]
```

Maple [A] time = 0.004, size = 13, normalized size = 0.9

$$x-\ln(x)+2\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/(x^2-x),x)
```

```
[Out] x-ln(x)+2*ln(-1+x)
```

Maxima [A] time = 0.926839, size = 16, normalized size = 1.14

$$x+2\log(x-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="maxima")
```

```
[Out] x + 2*log(x - 1) - log(x)
```

Fricas [A] time = 1.67985, size = 36, normalized size = 2.57

$$x + 2 \log(x - 1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="fricas")
```

```
[Out] x + 2*log(x - 1) - log(x)
```

Sympy [A] time = 0.08969, size = 10, normalized size = 0.71

$$x - \log(x) + 2\log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/(x**2-x),x)
```

```
[Out] x - log(x) + 2*log(x - 1)
```

Giac [A] time = 1.05026, size = 19, normalized size = 1.36

$$x + 2 \log(|x - 1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/(x^2-x),x, algorithm="giac")
```

```
[Out] x + 2*log(abs(x - 1)) - log(abs(x))
```

3.183 $\int \frac{1-12x+x^2+x^3}{-12+x+x^2} \, dx$

Optimal. Leaf size=26

$$\frac{x^2}{2} + \frac{1}{7}\log(3-x) - \frac{1}{7}\log(x+4)$$

```
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Rubi [A] time = 0.0159258, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {1657, 616, 31}

$$\frac{x^2}{2} + \frac{1}{7}\log(3-x) - \frac{1}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
```

```
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1-12x+x^2+x^3}{-12+x+x^2}\,dx = \int \left(x+\frac{1}{-12+x+x^2}\right)dx$$
$$= \frac{x^2}{2}+\int \frac{1}{-12+x+x^2}\,dx$$
$$= \frac{x^2}{2}+\frac{1}{7}\int \frac{1}{-3+x}\,dx-\frac{1}{7}\int \frac{1}{4+x}\,dx$$
$$= \frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(4+x)$$

Mathematica [A] time = 0.0050732, size = 26, normalized size = 1.

$$\frac{x^2}{2}+\frac{1}{7}\log(3-x)-\frac{1}{7}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 12*x + x^2 + x^3)/(-12 + x + x^2),x]
```

```
[Out] x^2/2 + Log[3 - x]/7 - Log[4 + x]/7
```

Maple [A] time = 0.005, size = 19, normalized size = 0.7

$$\frac{x^2}{2}-\frac{\ln(4+x)}{7}+\frac{\ln(-3+x)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x^2-12*x+1)/(x^2+x-12),x)
```

```
[Out] 1/2*x^2-1/7*ln(4+x)+1/7*ln(-3+x)
```

Maxima [A] time = 0.930975, size = 24, normalized size = 0.92

$$\frac{1}{2}\,x^2-\frac{1}{7}\,\log(x+4)+\frac{1}{7}\,\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

Fricas [A] time = 1.81161, size = 58, normalized size = 2.23

$$\frac{1}{2}x^2 - \frac{1}{7}\log(x+4) + \frac{1}{7}\log(x-3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/7*log(x + 4) + 1/7*log(x - 3)
```

Sympy [A] time = 0.088192, size = 17, normalized size = 0.65

$$\frac{x^2}{2} + \frac{\log(x-3)}{7} - \frac{\log(x+4)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2-12*x+1)/(x**2+x-12),x)
```

```
[Out] x**2/2 + log(x - 3)/7 - log(x + 4)/7
```

Giac [A] time = 1.05205, size = 27, normalized size = 1.04

$$\frac{1}{2}x^2 - \frac{1}{7}\log(|x+4|) + \frac{1}{7}\log(|x-3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x^2-12*x+1)/(x^2+x-12),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/7*log(abs(x + 4)) + 1/7*log(abs(x - 3))
```

3.184 $\int \frac{3+2x}{(1+x)^2}\,dx$

Optimal. Leaf size=14

$$2\log(x+1)-\frac{1}{x+1}$$

```
[Out] -(1 + x)^(-1) + 2*Log[1 + x]
```

Rubi [A] time = 0.0053683, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$2\log(x+1)-\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/(1 + x)^2,x]
```

```
[Out] -(1 + x)^(-1) + 2*Log[1 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x}{(1+x)^2}\,dx &= \int \left(\frac{1}{(1+x)^2}+\frac{2}{1+x}\right)dx\\ &= -\frac{1}{1+x}+2\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0036457, size = 14, normalized size = 1.

$$2\log(x+1)-\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/(1 + x)^2,x]
```

```
[Out] -(1 + x)^(-1) + 2*Log[1 + x]
```

Maple [A] time = 0.004, size = 15, normalized size = 1.1

$$-(1+x)^{-1}+2\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)/(1+x)^2,x)
```

```
[Out] -1/(1+x)+2*ln(1+x)
```

Maxima [A] time = 0.930263, size = 19, normalized size = 1.36

$$-\frac{1}{x+1}+2\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] -1/(x + 1) + 2*log(x + 1)
```

Fricas [A] time = 1.87043, size = 49, normalized size = 3.5

$$\frac{2(x+1)\log(x+1)-1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] (2*(x + 1)*log(x + 1) - 1)/(x + 1)
```

Sympy [A] time = 0.075777, size = 10, normalized size = 0.71

$$2\log(x+1) - \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(1+x)**2,x)
```

```
[Out] 2*log(x + 1) - 1/(x + 1)
```

Giac [A] time = 1.04253, size = 20, normalized size = 1.43

$$-\frac{1}{x+1} + 2\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(1+x)^2,x, algorithm="giac")
```

```
[Out] -1/(x + 1) + 2*log(abs(x + 1))
```

3.185 $\int \frac{1}{x(1+x)(3+2x)}\,dx$

Optimal. Leaf size=23

$$\frac{\log(x)}{3}-\log(x+1)+\frac{2}{3}\log(2x+3)$$

```
[Out] Log[x]/3 - Log[1 + x] + (2*Log[3 + 2*x])/3
```

Rubi [A] time = 0.0080618, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {72}

$$\frac{\log(x)}{3}-\log(x+1)+\frac{2}{3}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x)*(3 + 2*x)),x]
```

```
[Out] Log[x]/3 - Log[1 + x] + (2*Log[3 + 2*x])/3
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1+x)(3+2x)}\,dx &= \int \left(\frac{1}{-1-x}+\frac{1}{3x}+\frac{4}{3(3+2x)}\right)dx\\ &= \frac{\log(x)}{3}-\log(1+x)+\frac{2}{3}\log(3+2x)\end{aligned}$$

Mathematica [A] time = 0.0044875, size = 23, normalized size = 1.

$$\frac{\log(x)}{3}-\log(x+1)+\frac{2}{3}\log(2x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x)*(3 + 2*x)),x]
```

```
[Out] Log[x]/3 - Log[1 + x] + (2*Log[3 + 2*x])/3
```

Maple [A] time = 0.006, size = 20, normalized size = 0.9

$$\frac{\ln(x)}{3} - \ln(1 + x) + \frac{2\ln(3 + 2x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+x)/(3+2*x),x)
```

```
[Out] 1/3*ln(x)-ln(1+x)+2/3*ln(3+2*x)
```

Maxima [A] time = 0.934752, size = 26, normalized size = 1.13

$$\frac{2}{3}\log(2x + 3) - \log(x + 1) + \frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(3+2*x),x, algorithm="maxima")
```

```
[Out] 2/3*log(2*x + 3) - log(x + 1) + 1/3*log(x)
```

Fricas [A] time = 1.90344, size = 59, normalized size = 2.57

$$\frac{2}{3}\log(2x + 3) - \log(x + 1) + \frac{1}{3}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(3+2*x),x, algorithm="fricas")
```

```
[Out] 2/3*log(2*x + 3) - log(x + 1) + 1/3*log(x)
```

Sympy [A] time = 0.117938, size = 19, normalized size = 0.83

$$\frac{\log(x)}{3} - \log(x+1) + \frac{2\log\left(x+\frac{3}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(3+2*x),x)
```

```
[Out] log(x)/3 - log(x + 1) + 2*log(x + 3/2)/3
```

Giac [A] time = 1.05534, size = 30, normalized size = 1.3

$$\frac{2}{3}\log(|2x+3|) - \log(|x+1|) + \frac{1}{3}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)/(3+2*x),x, algorithm="giac")
```

```
[Out] 2/3*log(abs(2*x + 3)) - log(abs(x + 1)) + 1/3*log(abs(x))
```

3.186 $\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx$

Optimal. Leaf size=17

$$2\log(1-x)+\log(x)+3\log(x+3)$$

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Rubi [A] time = 0.0398629, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {1594, 1628}

$$2\log(1-x)+\log(x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+5x+6x^2}{-3x+2x^2+x^3}\,dx &= \int \frac{-3+5x+6x^2}{x\left(-3+2x+x^2\right)}\,dx\\ &= \int \left(\frac{2}{-1+x}+\frac{1}{x}+\frac{3}{3+x}\right)dx\\ &= 2\log(1-x)+\log(x)+3\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.0061575, size = 17, normalized size = 1.

$$2\log(1-x)+\log(x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 5*x + 6*x^2)/(-3*x + 2*x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x] + 3*Log[3 + x]
```

Maple [A] time = 0.008, size = 16, normalized size = 0.9

$$\ln(x)+3\ln(3+x)+2\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x)
```

```
[Out] ln(x)+3*ln(3+x)+2*ln(-1+x)
```

Maxima [A] time = 0.933435, size = 20, normalized size = 1.18

$$3\log(x+3)+2\log(x-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="maxima")
```

```
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

Fricas [A] time = 1.86743, size = 51, normalized size = 3.

$$3\log(x+3)+2\log(x-1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="fricas")
```

```
[Out] 3*log(x + 3) + 2*log(x - 1) + log(x)
```

Sympy [A] time = 0.120574, size = 15, normalized size = 0.88

$$\log(x) + 2\log(x-1) + 3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x**2+5*x-3)/(x**3+2*x**2-3*x),x)
```

```
[Out] log(x) + 2*log(x - 1) + 3*log(x + 3)
```

Giac [A] time = 1.06984, size = 24, normalized size = 1.41

$$3\log(|x+3|) + 2\log(|x-1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((6*x^2+5*x-3)/(x^3+2*x^2-3*x),x, algorithm="giac")
```

```
[Out] 3*log(abs(x + 3)) + 2*log(abs(x - 1)) + log(abs(x))
```

3.187 $\int \frac{x}{4+4x+x^2}\,dx$

Optimal. Leaf size=12

$$\frac{2}{x+2}+\log(x+2)$$

```
[Out] 2/(2 + x) + Log[2 + x]
```

Rubi [A] time = 0.005216, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {27, 43}

$$\frac{2}{x+2}+\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x/(4 + 4*x + x^2),x]
```

```
[Out] 2/(2 + x) + Log[2 + x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{4+4x+x^2}\,dx &= \int \frac{x}{(2+x)^2}\,dx \\ &= \int \left(-\frac{2}{(2+x)^2}+\frac{1}{2+x}\right)dx \\ &= \frac{2}{2+x}+\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.0031645, size = 12, normalized size = 1.

$$\frac{2}{x+2}+\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(4 + 4*x + x^2),x]
```

```
[Out] 2/(2 + x) + Log[2 + x]
```

Maple [A] time = 0.006, size = 13, normalized size = 1.1

$$2\,(2+x)^{-1}+\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+4*x+4),x)
```

```
[Out] 2/(2+x)+ln(2+x)
```

Maxima [A] time = 0.926493, size = 16, normalized size = 1.33

$$\frac{2}{x+2}+\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4*x+4),x, algorithm="maxima")
```

```
[Out] 2/(x + 2) + log(x + 2)
```

Fricas [A] time = 1.68812, size = 46, normalized size = 3.83

$$\frac{(x+2)\log(x+2)+2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4*x+4),x, algorithm="fricas")
```

```
[Out] ((x + 2)*log(x + 2) + 2)/(x + 2)
```

Sympy [A] time = 0.072204, size = 8, normalized size = 0.67

$$\log(x+2)+\frac{2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+4*x+4),x)
```

```
[Out] log(x + 2) + 2/(x + 2)
```

Giac [A] time = 1.06902, size = 18, normalized size = 1.5

$$\frac{2}{x+2}+\log(|x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+4*x+4),x, algorithm="giac")
```

```
[Out] 2/(x + 2) + log(abs(x + 2))
```

3.188 $\int \frac{1}{(-1+x)^2(4+x)}\,dx$

Optimal. Leaf size=30

$$\frac{1}{5(1-x)}-\frac{1}{25}\log(1-x)+\frac{1}{25}\log(x+4)$$

```
[Out] 1/(5*(1 - x)) - Log[1 - x]/25 + Log[4 + x]/25
```

Rubi [A] time = 0.009743, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$\frac{1}{5(1-x)}-\frac{1}{25}\log(1-x)+\frac{1}{25}\log(x+4)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)^2*(4 + x)),x]
```

```
[Out] 1/(5*(1 - x)) - Log[1 - x]/25 + Log[4 + x]/25
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-1+x)^2(4+x)}\,dx &= \int \left(\frac{1}{5(-1+x)^2}-\frac{1}{25(-1+x)}+\frac{1}{25(4+x)}\right)dx\\ &= \frac{1}{5(1-x)}-\frac{1}{25}\log(1-x)+\frac{1}{25}\log(4+x)\end{aligned}$$

Mathematica [A] time = 0.0072473, size = 22, normalized size = 0.73

$$\frac{1}{25}\left(-\frac{5}{x-1}-\log(x-1)+\log(x+4)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)^2*(4 + x)),x]
```

```
[Out] (-5/(-1 + x) - Log[-1 + x] + Log[4 + x])/25
```

Maple [A] time = 0.006, size = 21, normalized size = 0.7

$$-\frac{1}{-5+5\,x}-\frac{\ln\left(-1+x\right)}{25}+\frac{\ln\left(4+x\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)^2/(4+x),x)
```

```
[Out] -1/5/(-1+x)-1/25*ln(-1+x)+1/25*ln(4+x)
```

Maxima [A] time = 0.928554, size = 27, normalized size = 0.9

$$-\frac{1}{5\,(x-1)}+\frac{1}{25}\,\log\left(x+4\right)-\frac{1}{25}\,\log\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(4+x),x, algorithm="maxima")
```

```
[Out] -1/5/(x - 1) + 1/25*log(x + 4) - 1/25*log(x - 1)
```

Fricas [A] time = 1.93085, size = 81, normalized size = 2.7

$$\frac{(x-1)\log\left(x+4\right)-(x-1)\log\left(x-1\right)-5}{25\,(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(4+x),x, algorithm="fricas")
```

```
[Out] 1/25*((x - 1)*log(x + 4) - (x - 1)*log(x - 1) - 5)/(x - 1)
```

Sympy [A] time = 0.103599, size = 19, normalized size = 0.63

$$-\frac{\log(x-1)}{25}+\frac{\log(x+4)}{25}-\frac{1}{5x-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**2/(4+x),x)
```

```
[Out] -log(x - 1)/25 + log(x + 4)/25 - 1/(5*x - 5)
```

Giac [A] time = 1.05186, size = 28, normalized size = 0.93

$$-\frac{1}{5(x-1)}+\frac{1}{25}\log\left(\left|-\frac{5}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(4+x),x, algorithm="giac")
```

```
[Out] -1/5/(x - 1) + 1/25*log(abs(-5/(x - 1) - 1))
```

3.189 $\int \frac{x^2}{(-3+x)(2+x)^2} \, dx$

Optimal. Leaf size=28

$$\frac{4}{5(x+2)} + \frac{9}{25}\log(3-x) + \frac{16}{25}\log(x+2)$$

```
[Out] 4/(5*(2 + x)) + (9*Log[3 - x])/25 + (16*Log[2 + x])/25
```

Rubi [A] time = 0.0100071, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {88}

$$\frac{4}{5(x+2)} + \frac{9}{25}\log(3-x) + \frac{16}{25}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((-3 + x)*(2 + x)^2),x]
```

```
[Out] 4/(5*(2 + x)) + (9*Log[3 - x])/25 + (16*Log[2 + x])/25
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(-3+x)(2+x)^2} \, dx &= \int \left(\frac{9}{25(-3+x)} - \frac{4}{5(2+x)^2} + \frac{16}{25(2+x)}\right) dx \\
&= \frac{4}{5(2+x)} + \frac{9}{25}\log(3-x) + \frac{16}{25}\log(2+x)
\end{aligned}$$

Mathematica [A] time = 0.0158948, size = 26, normalized size = 0.93

$$\frac{4}{5(x+2)}+\frac{9}{25}\log(x-3)+\frac{16}{25}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((-3 + x)*(2 + x)^2),x]
```

```
[Out] 4/(5*(2 + x)) + (9*Log[-3 + x])/25 + (16*Log[2 + x])/25
```

Maple [A] time = 0.006, size = 21, normalized size = 0.8

$$\frac{4}{10+5\,x}+\frac{16\,\ln\left(2+x\right)}{25}+\frac{9\,\ln\left(-3+x\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-3+x)/(2+x)^2,x)
```

```
[Out] 4/5/(2+x)+16/25*ln(2+x)+9/25*ln(-3+x)
```

Maxima [A] time = 0.926318, size = 27, normalized size = 0.96

$$\frac{4}{5\,(x+2)}+\frac{16}{25}\,\log\left(x+2\right)+\frac{9}{25}\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3+x)/(2+x)^2,x, algorithm="maxima")
```

```
[Out] 4/5/(x + 2) + 16/25*log(x + 2) + 9/25*log(x - 3)
```

Fricas [A] time = 1.83164, size = 89, normalized size = 3.18

$$\frac{16\,(x+2)\log\left(x+2\right)+9\,(x+2)\log\left(x-3\right)+20}{25\,(x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3+x)/(2+x)^2,x, algorithm="fricas")
```

```
[Out] 1/25*(16*(x + 2)*log(x + 2) + 9*(x + 2)*log(x - 3) + 20)/(x + 2)
```

Sympy [A] time = 0.113319, size = 22, normalized size = 0.79

$$\frac{9\log\left(x-3\right)}{25}+\frac{16\log\left(x+2\right)}{25}+\frac{4}{5x+10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-3+x)/(2+x)**2,x)
```

```
[Out] 9*log(x - 3)/25 + 16*log(x + 2)/25 + 4/(5*x + 10)
```

Giac [A] time = 1.04837, size = 35, normalized size = 1.25

$$\frac{4}{5\left(x+2\right)}+\log\left(\left|x+2\right|\right)+\frac{9}{25}\log\left(\left|-\frac{5}{x+2}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-3+x)/(2+x)^2,x, algorithm="giac")
```

```
[Out] 4/5/(x + 2) + log(abs(x + 2)) + 9/25*log(abs(-5/(x + 2) + 1))
```

3.190 $\int \frac{-2+3x+5x^2}{2x^2+x^3} \, dx$

Optimal. Leaf size=14

$$\frac{1}{x} + 2\log(x) + 3\log(x+2)$$

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Rubi [A] time = 0.0233708, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1593, 893}

$$\frac{1}{x} + 2\log(x) + 3\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\int \frac{-2+3x+5x^2}{2x^2+x^3}\,dx = \int \frac{-2+3x+5x^2}{x^2(2+x)}\,dx$$
$$= \int \left(-\frac{1}{x^2}+\frac{2}{x}+\frac{3}{2+x}\right)dx$$
$$= \frac{1}{x}+2\log(x)+3\log(2+x)$$

Mathematica [A] time = 0.0036714, size = 14, normalized size = 1.

$$\frac{1}{x}+2\log(x)+3\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 3*x + 5*x^2)/(2*x^2 + x^3),x]
```

```
[Out] x^(-1) + 2*Log[x] + 3*Log[2 + x]
```

Maple [A] time = 0.006, size = 15, normalized size = 1.1

$$x^{-1}+2\ln(x)+3\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x-2)/(x^3+2*x^2),x)
```

```
[Out] 1/x+2*ln(x)+3*ln(2+x)
```

Maxima [A] time = 0.927164, size = 19, normalized size = 1.36

$$\frac{1}{x}+3\log(x+2)+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="maxima")
```

```
[Out] 1/x + 3*log(x + 2) + 2*log(x)
```

Fricas [A] time = 1.74643, size = 50, normalized size = 3.57

$$\frac{3x\log(x+2)+2x\log(x)+1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="fricas")
```

```
[Out] (3*x*log(x + 2) + 2*x*log(x) + 1)/x
```

Sympy [A] time = 0.100243, size = 14, normalized size = 1.

$$2\log(x)+3\log(x+2)+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x-2)/(x**3+2*x**2),x)
```

```
[Out] 2*log(x) + 3*log(x + 2) + 1/x
```

Giac [A] time = 1.05714, size = 22, normalized size = 1.57

$$\frac{1}{x}+3\log(|x+2|)+2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x-2)/(x^3+2*x^2),x, algorithm="giac")
```

```
[Out] 1/x + 3*log(abs(x + 2)) + 2*log(abs(x))
```

3.191 $\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} \, dx$

Optimal. Leaf size=19

$$\log(1-x)-2\log(x+2)-3\log(x+3)$$

```
[Out] Log[1 - x] - 2*Log[2 + x] - 3*Log[3 + x]
```

Rubi [A] time = 0.0267347, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2074}

$$\log(1-x)-2\log(x+2)-3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
```

```
[Out] Log[1 - x] - 2*Log[2 + x] - 3*Log[3 + x]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{18-2x-4x^2}{-6+x+4x^2+x^3} \, dx &= \int \left(\frac{1}{-1+x}-\frac{2}{2+x}-\frac{3}{3+x}\right) dx\\ &= \log(1-x)-2\log(2+x)-3\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.0066508, size = 25, normalized size = 1.32

$$-2\left(-\frac{1}{2}\log(1-x)+\log(x+2)+\frac{3}{2}\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(18 - 2*x - 4*x^2)/(-6 + x + 4*x^2 + x^3),x]
```

```
[Out] -2*(-Log[1 - x]/2 + Log[2 + x] + (3*Log[3 + x])/2)
```

Maple [A] time = 0.007, size = 18, normalized size = 1.

$$-2\ln(2+x)-3\ln(3+x)+\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x)
```

```
[Out] -2*ln(2+x)-3*ln(3+x)+ln(-1+x)
```

Maxima [A] time = 0.929736, size = 23, normalized size = 1.21

$$-3\log(x+3)-2\log(x+2)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="maxima")
```

```
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

Fricas [A] time = 1.78051, size = 58, normalized size = 3.05

$$-3\log(x+3)-2\log(x+2)+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="fricas")
```

```
[Out] -3*log(x + 3) - 2*log(x + 2) + log(x - 1)
```

Sympy [A] time = 0.115153, size = 17, normalized size = 0.89

$$\log(x-1) - 2\log(x+2) - 3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x**2-2*x+18)/(x**3+4*x**2+x-6),x)
```

```
[Out] log(x - 1) - 2*log(x + 2) - 3*log(x + 3)
```

Giac [A] time = 1.05505, size = 27, normalized size = 1.42

$$-3\log(|x+3|) - 2\log(|x+2|) + \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-4*x^2-2*x+18)/(x^3+4*x^2+x-6),x, algorithm="giac")
```

```
[Out] -3*log(abs(x + 3)) - 2*log(abs(x + 2)) + log(abs(x - 1))
```

3.192 $\int \frac{2x+x^2}{4+3x^2+x^3} dx$

Optimal. Leaf size=15

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

Rubi [A] time = 0.0090738, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1587}

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{2x+x^2}{4+3x^2+x^3}\,dx=\frac{1}{3}\log\left(4+3x^2+x^3\right)$$

Mathematica [A] time = 0.0042837, size = 15, normalized size = 1.

$$\frac{1}{3}\log\left(x^3+3x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^2)/(4 + 3*x^2 + x^3),x]
```

```
[Out] Log[4 + 3*x^2 + x^3]/3
```

Maple [A] time = 0., size = 14, normalized size = 0.9

$$\frac{\ln\left(x^{3}+3x^{2}+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x)/(x^3+3*x^2+4),x)
```

```
[Out] 1/3*ln(x^3+3*x^2+4)
```

Maxima [A] time = 0.927364, size = 18, normalized size = 1.2

$$\frac{1}{3}\log\left(x^{3}+3x^{2}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="maxima")
```

```
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

Fricas [A] time = 1.84928, size = 35, normalized size = 2.33

$$\frac{1}{3}\log\left(x^{3}+3x^{2}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="fricas")
```

```
[Out] 1/3*log(x^3 + 3*x^2 + 4)
```

Sympy [A] time = 0.080973, size = 12, normalized size = 0.8

$$\frac{\log\left(x^3+3x^2+4\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x)/(x**3+3*x**2+4),x)
```

```
[Out] log(x**3 + 3*x**2 + 4)/3
```

Giac [A] time = 1.06531, size = 19, normalized size = 1.27

$$\frac{1}{3}\log\left(\left|x^3+3x^2+4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x)/(x^3+3*x^2+4),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(x^3 + 3*x^2 + 4))
```

3.193 $\int \frac{1}{(-1+x)^2x^2}\,dx$

Optimal. Leaf size=25

$$\frac{1}{1-x}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

```
[Out] (1 - x)^(-1) - x^(-1) - 2*Log[1 - x] + 2*Log[x]
```

Rubi [A] time = 0.0082479, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {44}

$$\frac{1}{1-x}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)^2*x^2),x]
```

```
[Out] (1 - x)^(-1) - x^(-1) - 2*Log[1 - x] + 2*Log[x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-1+x)^2x^2}\,dx &= \int \left(\frac{1}{(-1+x)^2}-\frac{2}{-1+x}+\frac{1}{x^2}+\frac{2}{x}\right)dx\\ &= \frac{1}{1-x}-\frac{1}{x}-2\log(1-x)+2\log(x)\end{aligned}$$

Mathematica [A] time = 0.0102419, size = 25, normalized size = 1.

$$-\frac{1}{x-1}-\frac{1}{x}-2\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)^2*x^2),x]
```

```
[Out] -(-1 + x)^(-1) - x^(-1) - 2*Log[1 - x] + 2*Log[x]
```

Maple [A] time = 0.008, size = 24, normalized size = 1.

$$-x^{-1}+2\ln(x)-(-1+x)^{-1}-2\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)^2/x^2,x)
```

```
[Out] -1/x+2*ln(x)-1/(-1+x)-2*ln(-1+x)
```

Maxima [A] time = 0.923035, size = 36, normalized size = 1.44

$$-\frac{2x-1}{x^2-x}-2\log(x-1)+2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/x^2,x, algorithm="maxima")
```

```
[Out] -(2*x - 1)/(x^2 - x) - 2*log(x - 1) + 2*log(x)
```

Fricas [A] time = 1.74229, size = 92, normalized size = 3.68

$$-\frac{2\left(x^2-x\right)\log(x-1)-2\left(x^2-x\right)\log(x)+2x-1}{x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/x^2,x, algorithm="fricas")
```

```
[Out] -(2*(x^2 - x)*log(x - 1) - 2*(x^2 - x)*log(x) + 2*x - 1)/(x^2 - x)
```

Sympy [A] time = 0.100422, size = 20, normalized size = 0.8

$$-\frac{2x-1}{x^2-x}+2\log(x)-2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**2/x**2,x)
```

```
[Out] -(2*x - 1)/(x**2 - x) + 2*log(x) - 2*log(x - 1)
```

Giac [A] time = 1.05045, size = 41, normalized size = 1.64

$$-\frac{1}{x-1}+\frac{1}{\frac{1}{x-1}+1}+2\log\left(\left|-\frac{1}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/x^2,x, algorithm="giac")
```

```
[Out] -1/(x - 1) + 1/(1/(x - 1) + 1) + 2*log(abs(-1/(x - 1) - 1))
```

3.194 $\int \frac{x^2}{(1+x)^3} \, dx$

Optimal. Leaf size=21

$$\frac{2}{x+1} - \frac{1}{2(x+1)^2} + \log(x+1)$$

```
[Out] -1/(2*(1 + x)^2) + 2/(1 + x) + Log[1 + x]
```

Rubi [A] time = 0.0072975, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{2}{x+1} - \frac{1}{2(x+1)^2} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x)^3,x]
```

```
[Out] -1/(2*(1 + x)^2) + 2/(1 + x) + Log[1 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(1+x)^3} \, dx &= \int \left(\frac{1}{(1+x)^3} - \frac{2}{(1+x)^2} + \frac{1}{1+x} \right) dx \\ &= -\frac{1}{2(1+x)^2} + \frac{2}{1+x} + \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.0083873, size = 21, normalized size = 1.

$$\frac{2}{x+1}-\frac{1}{2(x+1)^2}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x)^3,x]
```

```
[Out] -1/(2*(1 + x)^2) + 2/(1 + x) + Log[1 + x]
```

Maple [A] time = 0.005, size = 20, normalized size = 1.

$$-\frac{1}{2\ (1+x)^2}+2\ (1+x)^{-1}+\ln\left(1+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1+x)^3,x)
```

```
[Out] -1/2/(1+x)^2+2/(1+x)+ln(1+x)
```

Maxima [A] time = 0.922966, size = 30, normalized size = 1.43

$$\frac{4\,x+3}{2\left(x^2+2\,x+1\right)}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*x + 3)/(x^2 + 2*x + 1) + log(x + 1)
```

Fricas [A] time = 1.80456, size = 84, normalized size = 4.

$$\frac{2\left(x^2+2\,x+1\right)\log\left(x+1\right)+4\,x+3}{2\left(x^2+2\,x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x^2 + 2*x + 1)*log(x + 1) + 4*x + 3)/(x^2 + 2*x + 1)
```

Sympy [A] time = 0.086027, size = 19, normalized size = 0.9

$$\frac{4x+3}{2x^2+4x+2}+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(1+x)**3,x)
```

```
[Out] (4*x + 3)/(2*x**2 + 4*x + 2) + log(x + 1)
```

Giac [A] time = 1.05038, size = 24, normalized size = 1.14

$$\frac{4\,x+3}{2\,(x+1)^2}+\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(1+x)^3,x, algorithm="giac")
```

```
[Out] 1/2*(4*x + 3)/(x + 1)^2 + log(abs(x + 1))
```

3.195 $\int \frac{1}{-x^2+x^4} \, dx$

Optimal. Leaf size=8

$$\frac{1}{x} - \tanh^{-1}(x)$$

```
[Out] x^(-1) - ArcTanh[x]
```

Rubi [A] time = 0.0052763, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1593, 325, 207}

$$\frac{1}{x} - \tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + x^4)^(-1),x]
```

```
[Out] x^(-1) - ArcTanh[x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-x^2+x^4}\,dx &= \int \frac{1}{x^2\left(-1+x^2\right)}\,dx\\ &= \frac{1}{x}+\int \frac{1}{-1+x^2}\,dx\\ &= \frac{1}{x}-\tanh^{-1}(x)\end{aligned}$$

Mathematica [B] time = 0.0025107, size = 22, normalized size = 2.75

$$\frac{1}{x}+\frac{1}{2}\log(1-x)-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + x^4)^(-1),x]
```

```
[Out] x^(-1) + Log[1 - x]/2 - Log[1 + x]/2
```

Maple [A] time = 0.006, size = 17, normalized size = 2.1

$$-\frac{\ln(1+x)}{2}+\frac{\ln(-1+x)}{2}+x^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-x^2),x)
```

```
[Out] -1/2*ln(1+x)+1/2*ln(-1+x)+1/x
```

Maxima [A] time = 0.928798, size = 22, normalized size = 2.75

$$\frac{1}{x}-\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2),x, algorithm="maxima")
```

```
[Out] 1/x - 1/2*log(x + 1) + 1/2*log(x - 1)
```

Fricas [B] time = 1.77213, size = 57, normalized size = 7.12

$$-\frac{x\log\left(x+1\right)-x\log\left(x-1\right)-2}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(x*log(x + 1) - x*log(x - 1) - 2)/x
```

Sympy [B] time = 0.089384, size = 15, normalized size = 1.88

$$\frac{\log\left(x-1\right)}{2}-\frac{\log\left(x+1\right)}{2}+\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-x**2),x)
```

```
[Out] log(x - 1)/2 - log(x + 1)/2 + 1/x
```

Giac [B] time = 1.05507, size = 24, normalized size = 3.

$$\frac{1}{x}-\frac{1}{2}\,\log\left(|x+1|\right)+\frac{1}{2}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2),x, algorithm="giac")
```

```
[Out] 1/x - 1/2*log(abs(x + 1)) + 1/2*log(abs(x - 1))
```

3.196 $\int \frac{-x+2x^3}{1-x^2+x^4}\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

```
[Out] Log[1 - x^2 + x^4]/2
```

Rubi [A] time = 0.0092598, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {1587}

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + 2*x^3)/(1 - x^2 + x^4),x]
```

```
[Out] Log[1 - x^2 + x^4]/2
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{-x+2x^3}{1-x^2+x^4}\,dx=\frac{1}{2}\log\left(1-x^2+x^4\right)$$

Mathematica [A] time = 0.0061354, size = 15, normalized size = 1.

$$\frac{1}{2}\log\left(x^4-x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + 2*x^3)/(1 - x^2 + x^4),x]
```

```
[Out] Log[1 - x^2 + x^4]/2
```

Maple [A] time = 0.001, size = 14, normalized size = 0.9

$$\frac{\ln\left(x^4 - x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3-x)/(x^4-x^2+1),x)
```

```
[Out] 1/2*ln(x^4-x^2+1)
```

Maxima [A] time = 0.924786, size = 18, normalized size = 1.2

$$\frac{1}{2}\log\left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^4 - x^2 + 1)
```

Fricas [A] time = 1.81189, size = 32, normalized size = 2.13

$$\frac{1}{2}\log\left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^4 - x^2 + 1)
```

Sympy [A] time = 0.083604, size = 10, normalized size = 0.67

$$\frac{\log\left(x^4 - x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3-x)/(x**4-x**2+1),x)
```

```
[Out] log(x**4 - x**2 + 1)/2
```

Giac [A] time = 1.06469, size = 18, normalized size = 1.2

$$\frac{1}{2}\,\log\left(x^4 - x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3-x)/(x^4-x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*log(x^4 - x^2 + 1)
```

3.197 $\int \frac{x^3}{1+x^2}\,dx$

Optimal. Leaf size=18

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)$$

```
[Out] x^2/2 - Log[1 + x^2]/2
```

Rubi [A] time = 0.0075449, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {266, 43}

$$\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^2),x]
```

```
[Out] x^2/2 - Log[1 + x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{1+x^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{1+x}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, x^2\right) \\
&= \frac{x^2}{2} - \frac{1}{2}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0026922, size = 18, normalized size = 1.

$$\frac{x^2}{2} - \frac{1}{2}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^2),x]
```

```
[Out] x^2/2 - Log[1 + x^2]/2
```

Maple [A] time = 0.001, size = 15, normalized size = 0.8

$$\frac{x^2}{2} - \frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2+1),x)
```

```
[Out] 1/2*x^2-1/2*ln(x^2+1)
```

Maxima [A] time = 0.927049, size = 19, normalized size = 1.06

$$\frac{1}{2}\,x^2 - \frac{1}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/2*log(x^2 + 1)
```

Fricas [A] time = 1.81593, size = 38, normalized size = 2.11

$$\frac{1}{2}x^2 - \frac{1}{2}\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/2*log(x^2 + 1)
```

Sympy [A] time = 0.069481, size = 12, normalized size = 0.67

$$\frac{x^2}{2} - \frac{\log\left(x^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**2+1),x)
```

```
[Out] x**2/2 - log(x**2 + 1)/2
```

Giac [A] time = 1.05878, size = 19, normalized size = 1.06

$$\frac{1}{2}x^2 - \frac{1}{2}\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/2*log(x^2 + 1)
```

3.198 $\int \frac{-1+x}{2+2x+x^2} \, dx$

Optimal. Leaf size=20

$$\frac{1}{2}\log\left(x^2+2x+2\right)-2\tan^{-1}(x+1)$$

```
[Out] -2*ArcTan[1 + x] + Log[2 + 2*x + x^2]/2
```

Rubi [A] time = 0.0090319, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {634, 617, 204, 628}

$$\frac{1}{2}\log\left(x^2+2x+2\right)-2\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x)/(2 + 2*x + x^2),x]
```

```
[Out] -2*ArcTan[1 + x] + Log[2 + 2*x + x^2]/2
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{-1+x}{2+2x+x^2}\,dx &= \frac{1}{2}\int \frac{2+2x}{2+2x+x^2}\,dx - 2\int \frac{1}{2+2x+x^2}\,dx\\ &= \frac{1}{2}\log\left(2+2x+x^2\right) + 2\,\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+x\right)\\ &= -2\tan^{-1}(1+x) + \frac{1}{2}\log\left(2+2x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.005253, size = 20, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+2x+2\right) - 2\tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x)/(2 + 2*x + x^2),x]
```

```
[Out] -2*ArcTan[1 + x] + Log[2 + 2*x + x^2]/2
```

Maple [A] time = 0.001, size = 19, normalized size = 1.

$$-2\,\arctan(1+x) + \frac{\ln\left(x^2+2\,x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)/(x^2+2*x+2),x)
```

```
[Out] -2*arctan(1+x)+1/2*ln(x^2+2*x+2)
```

Maxima [A] time = 1.4061, size = 24, normalized size = 1.2

$$-2 \arctan(x+1) + \frac{1}{2} \log\left(x^2 + 2x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2+2*x+2),x, algorithm="maxima")
```

```
[Out] -2*arctan(x + 1) + 1/2*log(x^2 + 2*x + 2)
```

Fricas [A] time = 1.73515, size = 58, normalized size = 2.9

$$-2 \arctan(x+1) + \frac{1}{2} \log\left(x^2 + 2x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2+2*x+2),x, algorithm="fricas")
```

```
[Out] -2*arctan(x + 1) + 1/2*log(x^2 + 2*x + 2)
```

Sympy [A] time = 0.0911, size = 17, normalized size = 0.85

$$\frac{\log\left(x^2 + 2x + 2\right)}{2} - 2\,\mathrm{atan}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x**2+2*x+2),x)
```

```
[Out] log(x**2 + 2*x + 2)/2 - 2*atan(x + 1)
```

Giac [A] time = 1.06146, size = 24, normalized size = 1.2

$$-2 \arctan(x+1) + \frac{1}{2} \log\left(x^2 + 2x + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)/(x^2+2*x+2),x, algorithm="giac")
```

```
[Out] -2*arctan(x + 1) + 1/2*log(x^2 + 2*x + 2)
```

3.199 $\int \frac{x}{1+x+x^2}\,dx$

Optimal. Leaf size=31

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x + x^2]/2
```

Rubi [A] time = 0.0144767, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {634, 618, 204, 628}

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x + x^2),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x + x^2]/2
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{1+x+x^2}\,dx &= -\left(\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx\right)+\frac{1}{2}\int \frac{1+2x}{1+x+x^2}\,dx\\ &= \frac{1}{2}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{2}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0070553, size = 31, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x + x^2),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 + x + x^2]/2
```

Maple [A] time = 0.003, size = 27, normalized size = 0.9

$$\frac{\ln\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+x+1),x)
```

```
[Out] 1/2*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.40626, size = 35, normalized size = 1.13

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*log(x^2 + x + 1)
```

Fricas [A] time = 1.74668, size = 90, normalized size = 2.9

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*log(x^2 + x + 1)
```

Sympy [A] time = 0.092602, size = 34, normalized size = 1.1

$$\frac{\log\left(x^2+x+1\right)}{2}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+x+1),x)
```

```
[Out] log(x**2 + x + 1)/2 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/3
```

Giac [A] time = 1.05882, size = 35, normalized size = 1.13

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2}\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*log(x^2 + x + 1)
```

3.200 $\int \frac{7+5x+4x^2}{5+4x+4x^2} \, dx$

Optimal. Leaf size=27

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(x+\frac{1}{2}\right)$$

```
[Out] x + (3*ArcTan[1/2 + x])/8 + Log[5 + 4*x + 4*x^2]/8
```

Rubi [A] time = 0.0247256, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1657, 634, 618, 204, 628}

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(x+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
```

```
[Out] x + (3*ArcTan[1/2 + x])/8 + Log[5 + 4*x + 4*x^2]/8
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{7+5x+4x^2}{5+4x+4x^2}\,dx &= \int \left(1+\frac{2+x}{5+4x+4x^2}\right)dx\\
&= x+\int \frac{2+x}{5+4x+4x^2}\,dx\\
&= x+\frac{1}{8}\int \frac{4+8x}{5+4x+4x^2}\,dx+\frac{3}{2}\int \frac{1}{5+4x+4x^2}\,dx\\
&= x+\frac{1}{8}\log\left(5+4x+4x^2\right)-3\,\text{Subst}\left(\int \frac{1}{-64-x^2}\,dx,x,4+8x\right)\\
&= x+\frac{3}{8}\tan^{-1}\left(\frac{1}{2}+x\right)+\frac{1}{8}\log\left(5+4x+4x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0046953, size = 31, normalized size = 1.15

$$\frac{1}{8}\log\left(4x^2+4x+5\right)+x+\frac{3}{8}\tan^{-1}\left(\frac{1}{2}(2x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 5*x + 4*x^2)/(5 + 4*x + 4*x^2),x]
```

```
[Out] x + (3*ArcTan[(1 + 2*x)/2])/8 + Log[5 + 4*x + 4*x^2]/8
```

Maple [A] time = 0.004, size = 22, normalized size = 0.8

$$x+\frac{3}{8}\arctan\left(x+\frac{1}{2}\right)+\frac{\ln\left(4\,x^2+4\,x+5\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+5*x+7)/(4*x^2+4*x+5),x)
```

```
[Out] x+3/8*arctan(x+1/2)+1/8*ln(4*x^2+4*x+5)
```

Maxima [A] time = 1.40412, size = 28, normalized size = 1.04

$$x + \frac{3}{8} \arctan\left(x + \frac{1}{2}\right) + \frac{1}{8} \log\left(4 x^2 + 4 x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="maxima")
```

```
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

Fricas [A] time = 1.93485, size = 70, normalized size = 2.59

$$x + \frac{3}{8} \arctan\left(x + \frac{1}{2}\right) + \frac{1}{8} \log\left(4 x^2 + 4 x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="fricas")
```

```
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

Sympy [A] time = 0.098302, size = 22, normalized size = 0.81

$$x + \frac{\log\left(x^2 + x + \frac{5}{4}\right)}{8} + \frac{3 \operatorname{atan}\left(x + \frac{1}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+5*x+7)/(4*x**2+4*x+5),x)
```

```
[Out] x + log(x**2 + x + 5/4)/8 + 3*atan(x + 1/2)/8
```

Giac [A] time = 1.04893, size = 28, normalized size = 1.04

$$x + \frac{3}{8}\,\arctan\left(x + \frac{1}{2}\right) + \frac{1}{8}\,\log\left(4\,x^2 + 4\,x + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+5*x+7)/(4*x^2+4*x+5),x, algorithm="giac")
```

```
[Out] x + 3/8*arctan(x + 1/2) + 1/8*log(4*x^2 + 4*x + 5)
```

3.201 $\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)} dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+1\right)+2\log(1-x)-3\tan^{-1}(x)$$

```
[Out] -3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2
```

Rubi [A] time = 0.0346737, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1629, 635, 203, 260}

$$\frac{1}{2}\log\left(x^2+1\right)+2\log(1-x)-3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + 2*Log[1 - x] + Log[1 + x^2]/2
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{5-4x+3x^2}{(-1+x)\left(1+x^2\right)}\,dx &= \int \left(\frac{2}{-1+x}+\frac{-3+x}{1+x^2}\right)dx\\ &= 2\log(1-x)+\int \frac{-3+x}{1+x^2}\,dx\\ &= 2\log(1-x)-3\int \frac{1}{1+x^2}\,dx+\int \frac{x}{1+x^2}\,dx\\ &= -3\tan^{-1}(x)+2\log(1-x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.006862, size = 28, normalized size = 1.22

$$\frac{1}{2}\log\left((x-1)^2+2(x-1)+2\right)+2\log(x-1)-3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - 4*x + 3*x^2)/((-1 + x)*(1 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + Log[2 + 2*(-1 + x) + (-1 + x)^2]/2 + 2*Log[-1 + x]
```

Maple [A] time = 0.004, size = 20, normalized size = 0.9

$$\frac{\ln\left(x^2+1\right)}{2}-3\arctan(x)+2\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2-4*x+5)/(-1+x)/(x^2+1),x)
```

```
[Out] 1/2*ln(x^2+1)-3*arctan(x)+2*ln(-1+x)
```

Maxima [A] time = 1.41652, size = 26, normalized size = 1.13

$$-3 \arctan(x) + \frac{1}{2} \log\left(x^2+1\right) + 2 \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

Fricas [A] time = 1.75993, size = 65, normalized size = 2.83

$$-3 \arctan(x) + \frac{1}{2} \log\left(x^2+1\right) + 2 \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(x - 1)
```

Sympy [A] time = 0.11887, size = 19, normalized size = 0.83

$$2\log(x-1) + \frac{\log\left(x^2+1\right)}{2} - 3\,\mathrm{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2-4*x+5)/(-1+x)/(x**2+1),x)
```

```
[Out] 2*log(x - 1) + log(x**2 + 1)/2 - 3*atan(x)
```

Giac [A] time = 1.06216, size = 27, normalized size = 1.17

$$-3 \arctan(x) + \frac{1}{2} \log\left(x^2+1\right) + 2 \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2-4*x+5)/(-1+x)/(x^2+1),x, algorithm="giac")
```

```
[Out] -3*arctan(x) + 1/2*log(x^2 + 1) + 2*log(abs(x - 1))
```

3.202 $\int \frac{3+2x}{3x+x^3}\,dx$

Optimal. Leaf size=28

$$-\frac{1}{2}\log\left(x^2+3\right)+\log(x)+\frac{2\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (2*ArcTan[x/Sqrt[3]])/Sqrt[3] + Log[x] - Log[3 + x^2]/2
```

Rubi [A] time = 0.0224832, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1593, 801, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+3\right)+\log(x)+\frac{2\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/(3*x + x^3),x]
```

```
[Out] (2*ArcTan[x/Sqrt[3]])/Sqrt[3] + Log[x] - Log[3 + x^2]/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{3+2x}{3x+x^3}\,dx &= \int \frac{3+2x}{x\left(3+x^2\right)}\,dx\\
&= \int \left(\frac{1}{x}+\frac{2-x}{3+x^2}\right)dx\\
&= \log(x)+\int \frac{2-x}{3+x^2}\,dx\\
&= \log(x)+2\int \frac{1}{3+x^2}\,dx-\int \frac{x}{3+x^2}\,dx\\
&= \frac{2\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}+\log(x)-\frac{1}{2}\log\left(3+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0081235, size = 28, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+3\right)+\log(x)+\frac{2\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/(3*x + x^3),x]
```

```
[Out] (2*ArcTan[x/Sqrt[3]])/Sqrt[3] + Log[x] - Log[3 + x^2]/2
```

Maple [A] time = 0.004, size = 24, normalized size = 0.9

$$\ln(x) - \frac{\ln(x^2+3)}{2} + \frac{2\sqrt{3}}{3}\arctan\left(\frac{x\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)/(x^3+3*x),x)
```

```
[Out] ln(x)-1/2*ln(x^2+3)+2/3*arctan(1/3*x*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.41322, size = 31, normalized size = 1.11

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) - \frac{1}{2}\log\left(x^2+3\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x^3+3*x),x, algorithm="maxima")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/2*log(x^2 + 3) + log(x)
```

Fricas [A] time = 1.90635, size = 85, normalized size = 3.04

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right) - \frac{1}{2}\log\left(x^2+3\right) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x^3+3*x),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/2*log(x^2 + 3) + log(x)
```

Sympy [A] time = 0.11735, size = 29, normalized size = 1.04

$$\log(x) - \frac{\log(x^2+3)}{2} + \frac{2\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}x}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x**3+3*x),x)
```

```
[Out] log(x) - log(x**2 + 3)/2 + 2*sqrt(3)*atan(sqrt(3)*x/3)/3
```

Giac [A] time = 1.05555, size = 32, normalized size = 1.14

$$\frac{2}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}x\right)-\frac{1}{2}\log\left(x^2+3\right)+\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(x^3+3*x),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*arctan(1/3*sqrt(3)*x) - 1/2*log(x^2 + 3) + log(abs(x))
```

3.203 $\int \frac{1}{-1+x^3} dx$

Optimal. Leaf size=41

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

Rubi [A] time = 0.0199682, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {200, 31, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+x^3}\,dx &= \frac{1}{3}\int \frac{1}{-1+x}\,dx+\frac{1}{3}\int \frac{-2-x}{1+x+x^2}\,dx\\ &= \frac{1}{3}\log(1-x)-\frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1+x+x^2}\,dx\\ &= \frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2x\right)\\ &= -\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}}+\frac{1}{3}\log(1-x)-\frac{1}{6}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.005135, size = 41, normalized size = 1.

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3]) + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

Maple [A] time = 0.001, size = 33, normalized size = 0.8

$$\frac{\ln(-1+x)}{3}-\frac{\ln(x^2+x+1)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-1),x)
```

```
[Out] 1/3*ln(-1+x)-1/6*ln(x^2+x+1)-1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.41244, size = 43, normalized size = 1.05

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log
(x - 1)
```

Fricas [A] time = 1.76591, size = 113, normalized size = 2.76

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log
(x - 1)
```

Sympy [A] time = 0.117112, size = 41, normalized size = 1.

$$\frac{\log{(x-1)}}{3} - \frac{\log{\left(x^2+x+1\right)}}{6} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-1),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3
)/3
```

Giac [A] time = 1.06206, size = 45, normalized size = 1.1

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log
(abs(x - 1))
```

3.204 $\int \frac{x^3}{1+x^3}\,dx$

Optimal. Leaf size=41

$$\frac{1}{6}\log\left(x^2-x+1\right)+x-\frac{1}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Rubi [A] time = 0.0238205, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {321, 200, 31, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^2-x+1\right)+x-\frac{1}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^3),x]
```

```
[Out] x + ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{1+x^3}\,dx &= x-\int \frac{1}{1+x^3}\,dx\\
&= x-\frac{1}{3}\int \frac{1}{1+x}\,dx-\frac{1}{3}\int \frac{2-x}{1-x+x^2}\,dx\\
&= x-\frac{1}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\
&= x-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)+\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= x-\frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0073571, size = 42, normalized size = 1.02

$$\frac{1}{6}\log\left(x^2-x+1\right)+x-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^3),x]
```

```
[Out] x - ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Maple [A] time = 0.005, size = 36, normalized size = 0.9

$$x-\frac{\ln(1+x)}{3}+\frac{\ln\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^3+1),x)
```

```
[Out] x-1/3*ln(1+x)+1/6*ln(x^2-x+1)-1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.4118, size = 47, normalized size = 1.15

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x + 1/6*log(x^2 - x + 1) - 1/3
*log(x + 1)
```

Fricas [A] time = 1.88064, size = 119, normalized size = 2.9

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x + 1/6*log(x^2 - x + 1) - 1/3
*log(x + 1)
```

Sympy [A] time = 0.12089, size = 42, normalized size = 1.02

$$x-\frac{\log\left(x+1\right)}{3}+\frac{\log\left(x^2-x+1\right)}{6}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**3+1),x)
```

```
[Out] x - log(x + 1)/3 + log(x**2 - x + 1)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(
3)/3)/3
```

Giac [A] time = 1.0574, size = 49, normalized size = 1.2

$$-\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+x+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + x + 1/6*log(x^2 - x + 1) - 1/3
*log(abs(x + 1))
```

3.205 $\int \frac{-1-2x+x^2}{(-1+x)^2(1+x^2)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)+\tan^{-1}(x)$$

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0344154, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {1629, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(1-x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[1 - x] - Log[1 + x^2]/2
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1-2x+x^2}{(-1+x)^2\left(1+x^2\right)}\,dx &= \int \left(-\frac{1}{(-1+x)^2}+\frac{1}{-1+x}+\frac{1-x}{1+x^2}\right)dx \\
&= \frac{1}{-1+x}+\log(1-x)+\int \frac{1-x}{1+x^2}\,dx \\
&= \frac{1}{-1+x}+\log(1-x)+\int \frac{1}{1+x^2}\,dx-\int \frac{x}{1+x^2}\,dx \\
&= \frac{1}{-1+x}+\tan^{-1}(x)+\log(1-x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0137617, size = 22, normalized size = 0.92

$$-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{x-1}+\log(x-1)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 2*x + x^2)/((-1 + x)^2*(1 + x^2)),x]
```

```
[Out] (-1 + x)^(-1) + ArcTan[x] + Log[-1 + x] - Log[1 + x^2]/2
```

Maple [A] time = 0.006, size = 21, normalized size = 0.9

$$-\frac{\ln\left(x^2+1\right)}{2}+\arctan(x)+\ln(-1+x)+(-1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x-1)/(-1+x)^2/(x^2+1),x)
```

```
[Out] -1/2*ln(x^2+1)+arctan(x)+ln(-1+x)+1/(-1+x)
```

Maxima [A] time = 1.40285, size = 27, normalized size = 1.12

$$\frac{1}{x-1} + \arctan(x) - \frac{1}{2}\log\left(x^2+1\right) + \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/(x - 1) + arctan(x) - 1/2*log(x^2 + 1) + log(x - 1)
```

Fricas [A] time = 1.95175, size = 115, normalized size = 4.79

$$\frac{2(x-1)\arctan(x) - (x-1)\log\left(x^2+1\right) + 2(x-1)\log(x-1) + 2}{2(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(x - 1)*arctan(x) - (x - 1)*log(x^2 + 1) + 2*(x - 1)*log(x - 1) + 2)
/(x - 1)
```

Sympy [A] time = 0.125584, size = 20, normalized size = 0.83

$$\log(x-1) - \frac{\log\left(x^2+1\right)}{2} + \operatorname{atan}(x) + \frac{1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x-1)/(-1+x)**2/(x**2+1),x)
```

```
[Out] log(x - 1) - log(x**2 + 1)/2 + atan(x) + 1/(x - 1)
```

Giac [B] time = 1.06819, size = 63, normalized size = 2.62

$$\frac{1}{4}\pi - \pi\left\lfloor \frac{\pi + 4\arctan(x)}{4\pi} + \frac{1}{2}\right\rfloor + \frac{1}{x-1} + \arctan(x) - \frac{1}{2}\log\left(\frac{2}{x-1} + \frac{2}{(x-1)^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x-1)/(-1+x)^2/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/4*pi - pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/(x - 1) + arctan(x)
- 1/2*log(2/(x - 1) + 2/(x - 1)^2 + 1)
```

3.206 $\int \frac{x^4}{-1+x^4} dx$

Optimal. Leaf size=14

$$x-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] x - ArcTan[x]/2 - ArcTanh[x]/2
```

Rubi [A] time = 0.0063513, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {321, 212, 206, 203}

$$x-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(-1 + x^4),x]

[Out] x - ArcTan[x]/2 - ArcTanh[x]/2
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{-1+x^4}\,dx &= x+\int \frac{1}{-1+x^4}\,dx\\ &= x-\frac{1}{2}\int \frac{1}{1-x^2}\,dx-\frac{1}{2}\int \frac{1}{1+x^2}\,dx\\ &= x-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0041189, size = 26, normalized size = 1.86

$$x+\frac{1}{4}\log(1-x)-\frac{1}{4}\log(x+1)-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(-1 + x^4),x]
```

```
[Out] x - ArcTan[x]/2 + Log[1 - x]/4 - Log[1 + x]/4
```

Maple [A] time = 0.004, size = 19, normalized size = 1.4

$$x+\frac{\ln(-1+x)}{4}-\frac{\ln(1+x)}{4}-\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4-1),x)
```

```
[Out] x+1/4*ln(-1+x)-1/4*ln(1+x)-1/2*arctan(x)
```

Maxima [A] time = 1.40145, size = 24, normalized size = 1.71

$$x - \frac{1}{2}\arctan(x) - \frac{1}{4}\log(x+1) + \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1),x, algorithm="maxima")
```

```
[Out] x - 1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Fricas [A] time = 1.90322, size = 72, normalized size = 5.14

$$x - \frac{1}{2}\arctan(x) - \frac{1}{4}\log(x+1) + \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1),x, algorithm="fricas")
```

```
[Out] x - 1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Sympy [A] time = 0.113575, size = 19, normalized size = 1.36

$$x + \frac{\log(x-1)}{4} - \frac{\log(x+1)}{4} - \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4-1),x)
```

```
[Out] x + log(x - 1)/4 - log(x + 1)/4 - atan(x)/2
```

Giac [A] time = 1.05932, size = 27, normalized size = 1.93

$$x - \frac{1}{2}\arctan(x) - \frac{1}{4}\log(|x+1|) + \frac{1}{4}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4-1),x, algorithm="giac")
```

```
[Out] x - 1/2*arctan(x) - 1/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

3.207 $\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx$

Optimal. Leaf size=29

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Rubi [A] time = 0.113992, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {6725, 635, 203, 260}

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]
```

```
[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{-4+6x-x^2+3x^3}{(1+x^2)(2+x^2)}\,dx &= \int \left(\frac{3(-1+x)}{1+x^2}+\frac{2}{2+x^2}\right)dx\\ &= 2\int \frac{1}{2+x^2}\,dx+3\int \frac{-1+x}{1+x^2}\,dx\\ &= \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)-3\int \frac{1}{1+x^2}\,dx+3\int \frac{x}{1+x^2}\,dx\\ &= -3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+\frac{3}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.013264, size = 29, normalized size = 1.

$$\frac{3}{2}\log\left(x^2+1\right)-3\tan^{-1}(x)+\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4 + 6*x - x^2 + 3*x^3)/((1 + x^2)*(2 + x^2)),x]

[Out] -3*ArcTan[x] + Sqrt[2]*ArcTan[x/Sqrt[2]] + (3*Log[1 + x^2])/2
```

Maple [A] time = 0.003, size = 25, normalized size = 0.9

$$-3\arctan(x)+\frac{3\ln\left(x^2+1\right)}{2}+\arctan\left(\frac{x\sqrt{2}}{2}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x)
```

```
[Out] -3*arctan(x)+3/2*ln(x^2+1)+arctan(1/2*x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.40795, size = 32, normalized size = 1.1

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3\arctan(x) + \frac{3}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

Fricas [A] time = 1.81724, size = 86, normalized size = 2.97

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3\arctan(x) + \frac{3}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

Sympy [A] time = 0.16258, size = 29, normalized size = 1.

$$\frac{3\log\left(x^2+1\right)}{2} - 3\operatorname{atan}(x) + \sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**3-x**2+6*x-4)/(x**2+1)/(x**2+2),x)
```

```
[Out] 3*log(x**2 + 1)/2 - 3*atan(x) + sqrt(2)*atan(sqrt(2)*x/2)
```

Giac [A] time = 1.06316, size = 32, normalized size = 1.1

$$\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 3\arctan(x) + \frac{3}{2}\log\left(x^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^3-x^2+6*x-4)/(x^2+1)/(x^2+2),x, algorithm="giac")
```

```
[Out] sqrt(2)*arctan(1/2*sqrt(2)*x) - 3*arctan(x) + 3/2*log(x^2 + 1)
```

3.208 $\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4} \, dx$

Optimal. Leaf size=23

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

```
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Rubi [A] time = 0.0309383, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1673, 1166, 203, 1247, 626, 31}

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
```

```
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 && !PolyQ[Pq, x^2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 626

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, b, c, d
, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && Inte
gerQ[p]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+x-2x^2+x^3}{4+5x^2+x^4}\,dx &= \int \frac{1-2x^2}{4+5x^2+x^4}\,dx+\int \frac{x\left(1+x^2\right)}{4+5x^2+x^4}\,dx \\
&= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1+x}{4+5x+x^2}\,dx,x,x^2\right)-3\int \frac{1}{4+x^2}\,dx+\int \frac{1}{1+x^2}\,dx \\
&= -\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{4+x}\,dx,x,x^2\right) \\
&= -\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)+\frac{1}{2}\log\left(4+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0096138, size = 23, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+4\right)-\frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x - 2*x^2 + x^3)/(4 + 5*x^2 + x^4),x]
```

```
[Out] (-3*ArcTan[x/2])/2 + ArcTan[x] + Log[4 + x^2]/2
```

Maple [A] time = 0.003, size = 18, normalized size = 0.8

$$-\frac{3}{2}\arctan\left(\frac{x}{2}\right)+\arctan\left(x\right)+\frac{\ln\left(x^{2}+4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x)
```

```
[Out] -3/2*arctan(1/2*x)+arctan(x)+1/2*ln(x^2+4)
```

Maxima [A] time = 1.41103, size = 23, normalized size = 1.

$$-\frac{3}{2}\arctan\left(\frac{1}{2}x\right)+\arctan\left(x\right)+\frac{1}{2}\log\left(x^{2}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="maxima")
```

```
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

Fricas [A] time = 1.79896, size = 69, normalized size = 3.

$$-\frac{3}{2}\arctan\left(\frac{1}{2}x\right)+\arctan\left(x\right)+\frac{1}{2}\log\left(x^{2}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="fricas")
```

```
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

Sympy [A] time = 0.155445, size = 19, normalized size = 0.83

$$\frac{\log\left(x^2+4\right)}{2}-\frac{3\,\mathrm{atan}\left(\frac{x}{2}\right)}{2}+\mathrm{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-2*x**2+x+1)/(x**4+5*x**2+4),x)
```

```
[Out] log(x**2 + 4)/2 - 3*atan(x/2)/2 + atan(x)
```

Giac [A] time = 1.06224, size = 23, normalized size = 1.

$$-\frac{3}{2}\,\arctan\left(\frac{1}{2}\,x\right)+\arctan\left(x\right)+\frac{1}{2}\,\log\left(x^2+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-2*x^2+x+1)/(x^4+5*x^2+4),x, algorithm="giac")
```

```
[Out] -3/2*arctan(1/2*x) + arctan(x) + 1/2*log(x^2 + 4)
```

3.209 $\int \frac{-3+x}{\left(4+2x+x^2\right)^2} \, dx$

Optimal. Leaf size=39

$$-\frac{4x+7}{6\left(x^2+2x+4\right)}-\frac{2\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

```
[Out] -(7 + 4*x)/(6*(4 + 2*x + x^2)) - (2*ArcTan[(1 + x)/Sqrt[3]])/(3*Sqrt[3])
```

Rubi [A] time = 0.0141822, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {638, 618, 204}

$$-\frac{4x+7}{6\left(x^2+2x+4\right)}-\frac{2\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x)/(4 + 2*x + x^2)^2,x]
```

```
[Out] -(7 + 4*x)/(6*(4 + 2*x + x^2)) - (2*ArcTan[(1 + x)/Sqrt[3]])/(3*Sqrt[3])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{-3+x}{\left(4+2x+x^2\right)^2}\,dx &= -\frac{7+4x}{6\left(4+2x+x^2\right)}-\frac{2}{3}\int \frac{1}{4+2x+x^2}\,dx\\ &= -\frac{7+4x}{6\left(4+2x+x^2\right)}+\frac{4}{3}\,\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx,x,2+2x\right)\\ &= -\frac{7+4x}{6\left(4+2x+x^2\right)}-\frac{2\tan^{-1}\left(\frac{1+x}{\sqrt{3}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0224098, size = 39, normalized size = 1.

$$\frac{-4x-7}{6\left(x^2+2x+4\right)}-\frac{2\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x)/(4 + 2*x + x^2)^2,x]
```

```
[Out] (-7 - 4*x)/(6*(4 + 2*x + x^2)) - (2*ArcTan[(1 + x)/Sqrt[3]])/(3*Sqrt[3])
```

Maple [A] time = 0.004, size = 35, normalized size = 0.9

$$\frac{-8\,x-14}{12\,x^2+24\,x+48}-\frac{2\,\sqrt{3}}{9}\arctan\left(\frac{(2\,x+2)\,\sqrt{3}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+x)/(x^2+2*x+4)^2,x)
```

```
[Out] 1/12*(-8*x-14)/(x^2+2*x+4)-2/9*3^(1/2)*arctan(1/6*(2*x+2)*3^(1/2))
```

Maxima [A] time = 1.42095, size = 43, normalized size = 1.1

$$-\frac{2}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}(x+1)\right)-\frac{4\,x+7}{6\left(x^2+2\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^2+2*x+4)^2,x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/6*(4*x + 7)/(x^2 + 2*x + 4)
```

Fricas [A] time = 2.01193, size = 123, normalized size = 3.15

$$-\frac{4\,\sqrt{3}\left(x^2+2\,x+4\right)\arctan\left(\frac{1}{3}\,\sqrt{3}(x+1)\right)+12\,x+21}{18\left(x^2+2\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^2+2*x+4)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(4*sqrt(3)*(x^2 + 2*x + 4)*arctan(1/3*sqrt(3)*(x + 1)) + 12*x + 21)/(
x^2 + 2*x + 4)
```

Sympy [A] time = 0.115488, size = 41, normalized size = 1.05

$$-\frac{4x+7}{6x^2+12x+24}-\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x**2+2*x+4)**2,x)
```

```
[Out] -(4*x + 7)/(6*x**2 + 12*x + 24) - 2*sqrt(3)*atan(sqrt(3)*x/3 + sqrt(3)/3)/9
```

Giac [A] time = 1.0604, size = 43, normalized size = 1.1

$$-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right)-\frac{4x+7}{6\left(x^2+2x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^2+2*x+4)^2,x, algorithm="giac")
```

```
[Out] -2/9*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/6*(4*x + 7)/(x^2 + 2*x + 4)
```

3.210 $\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=10

$$\frac{1}{x^2+1}+\log(x)$$

```
[Out] (1 + x^2)^(-1) + Log[x]
```

Rubi [A] time = 0.0166967, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1252, 894}

$$\frac{1}{x^2+1}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(x*(1 + x^2)^2),x]
```

```
[Out] (1 + x^2)^(-1) + Log[x]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{1+x^4}{x\left(1+x^2\right)^2}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1+x^2}{x(1+x)^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(\frac{1}{x} - \frac{2}{(1+x)^2}\right)dx, x, x^2\right)$$
$$= \frac{1}{1+x^2} + \log(x)$$

Mathematica [A] time = 0.0064212, size = 10, normalized size = 1.

$$\frac{1}{x^2+1} + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(x*(1 + x^2)^2),x]
```

```
[Out] (1 + x^2)^(-1) + Log[x]
```

Maple [A] time = 0.005, size = 11, normalized size = 1.1

$$\left(x^2+1\right)^{-1} + \ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/x/(x^2+1)^2,x)
```

```
[Out] 1/(x^2+1)+ln(x)
```

Maxima [A] time = 0.925533, size = 19, normalized size = 1.9

$$\frac{1}{x^2+1} + \frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/(x^2 + 1) + 1/2*log(x^2)
```

Fricas [A] time = 1.91779, size = 46, normalized size = 4.6

$$\frac{\left(x^2+1\right)\log(x)+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] ((x^2 + 1)*log(x) + 1)/(x^2 + 1)
```

Sympy [A] time = 0.09167, size = 8, normalized size = 0.8

$$\log(x)+\frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/x/(x**2+1)**2,x)
```

```
[Out] log(x) + 1/(x**2 + 1)
```

Giac [A] time = 1.06447, size = 19, normalized size = 1.9

$$\frac{1}{x^2+1}+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/(x^2 + 1) + 1/2*log(x^2)
```

3.211 $\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)} dx$

Optimal. Leaf size=11

$$\log\left(\sin^2(x) - 3\sin(x) + 2\right)$$

```
[Out] Log[2 - 3*Sin[x] + Sin[x]^2]
```

Rubi [A] time = 0.047502, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {4334, 628}

$$\log\left(\sin^2(x) - 3\sin(x) + 2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*(-3 + 2*Sin[x]))/(2 - 3*Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[2 - 3*Sin[x] + Sin[x]^2]
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)(-3+2\sin(x))}{2-3\sin(x)+\sin^2(x)} dx &= \text{Subst}\left(\int \frac{-3+2x}{2-3x+x^2} dx, x, \sin(x)\right)\\ &= \log\left(2-3\sin(x)+\sin^2(x)\right)\end{aligned}$$

Mathematica [B] time = 0.08709, size = 26, normalized size = 2.36

$$\log(2-\sin(x))+2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*(-3 + 2*Sin[x]))/(2 - 3*Sin[x] + Sin[x]^2),x]
```

```
[Out] 2*Log[Cos[x/2] - Sin[x/2]] + Log[2 - Sin[x]]
```

Maple [A] time = 0.026, size = 12, normalized size = 1.1

$$\ln\left(2-3\sin(x)+(\sin(x))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x)
```

```
[Out] ln(2-3*sin(x)+sin(x)^2)
```

Maxima [A] time = 0.93726, size = 15, normalized size = 1.36

$$\log\left(\sin(x)^2-3\sin(x)+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="maxima")
```

```
[Out] log(sin(x)^2 - 3*sin(x) + 2)
```

Fricas [A] time = 2.11957, size = 55, normalized size = 5.

$$\log\left(-\frac{1}{2}\sin(x)+1\right)+\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="fricas")
```

```
[Out] log(-1/2*sin(x) + 1) + log(-sin(x) + 1)
```

Sympy [A] time = 0.215164, size = 12, normalized size = 1.09

$$\log(\sin(x)-2)+\log(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)**2),x)
```

```
[Out] log(sin(x) - 2) + log(sin(x) - 1)
```

Giac [A] time = 1.06162, size = 20, normalized size = 1.82

$$\log(-\sin(x)+2)+\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(-3+2*sin(x))/(2-3*sin(x)+sin(x)^2),x, algorithm="giac")
```

```
[Out] log(-sin(x) + 2) + log(-sin(x) + 1)
```

3.212 $\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx$

Optimal. Leaf size=20

$$\sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right)-\cos(x)$$

```
[Out] Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]] - Cos[x]
```

Rubi [A] time = 0.0517718, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4335, 321, 203}

$$\sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x])/(5 + Cos[x]^2),x]
```

```
[Out] Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]] - Cos[x]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(x)\sin(x)}{5+\cos^2(x)}\,dx &= -\operatorname{Subst}\left(\int \frac{x^2}{5+x^2}\,dx, x, \cos(x)\right)\\ &= -\cos(x)+5\operatorname{Subst}\left(\int \frac{1}{5+x^2}\,dx, x, \cos(x)\right)\\ &= \sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right)-\cos(x)\end{aligned}$$

Mathematica [B] time = 0.16288, size = 82, normalized size = 4.1

$$\frac{1}{20}\left(-20\cos(x)+21\sqrt{5}\tan^{-1}\left(\frac{1}{\sqrt{5}}-\sqrt{\frac{6}{5}}\tan\left(\frac{x}{2}\right)\right)+21\sqrt{5}\tan^{-1}\left(\sqrt{\frac{6}{5}}\tan\left(\frac{x}{2}\right)+\frac{1}{\sqrt{5}}\right)-\sqrt{5}\tan^{-1}\left(\frac{\cos(x)}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x])/(5 + Cos[x]^2),x]
```

```
[Out] (-(Sqrt[5]*ArcTan[Cos[x]/Sqrt[5]]) + 21*Sqrt[5]*ArcTan[1/Sqrt[5] - Sqrt[6/5
]*Tan[x/2]] + 21*Sqrt[5]*ArcTan[1/Sqrt[5] + Sqrt[6/5]*Tan[x/2]] - 20*Cos[x]
)/20
```

Maple [A] time = 0.015, size = 18, normalized size = 0.9

$$-\cos(x)+\arctan\left(\frac{\cos(x)\sqrt{5}}{5}\right)\sqrt{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)/(5+cos(x)^2),x)
```

```
[Out] -cos(x)+arctan(1/5*cos(x)*5^(1/2))*5^(1/2)
```

Maxima [A] time = 1.40769, size = 23, normalized size = 1.15

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos(x)\right)-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="maxima")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

Fricas [A] time = 1.99236, size = 61, normalized size = 3.05

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos(x)\right)-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="fricas")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

Sympy [A] time = 0.576645, size = 19, normalized size = 0.95

$$-\cos(x)+\sqrt{5}\,\mathrm{atan}\left(\frac{\sqrt{5}\cos(x)}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(5+cos(x)**2),x)
```

```
[Out] -cos(x) + sqrt(5)*atan(sqrt(5)*cos(x)/5)
```

Giac [A] time = 1.05913, size = 23, normalized size = 1.15

$$\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\cos(x)\right)-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(5+cos(x)^2),x, algorithm="giac")
```

```
[Out] sqrt(5)*arctan(1/5*sqrt(5)*cos(x)) - cos(x)
```

3.213 $\int \frac{1}{-3+2x+x^2} dx$

Optimal. Leaf size=19

$$\frac{1}{4}\log(1-x)-\frac{1}{4}\log(x+3)$$

```
[Out] Log[1 - x]/4 - Log[3 + x]/4
```

Rubi [A] time = 0.0037085, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {616, 31}

$$\frac{1}{4}\log(1-x)-\frac{1}{4}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 2*x + x^2)^(-1),x]
```

```
[Out] Log[1 - x]/4 - Log[3 + x]/4
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-3+2x+x^2}\,dx &= \frac{1}{4}\int \frac{1}{-1+x}\,dx-\frac{1}{4}\int \frac{1}{3+x}\,dx\\ &= \frac{1}{4}\log(1-x)-\frac{1}{4}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.0023606, size = 19, normalized size = 1.

$$\frac{1}{4}\log(1-x)-\frac{1}{4}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 2*x + x^2)^(-1),x]
```

```
[Out] Log[1 - x]/4 - Log[3 + x]/4
```

Maple [A] time = 0.005, size = 14, normalized size = 0.7

$$-\frac{\ln(3+x)}{4}+\frac{\ln(-1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+2*x-3),x)
```

```
[Out] -1/4*ln(3+x)+1/4*ln(-1+x)
```

Maxima [A] time = 0.926453, size = 18, normalized size = 0.95

$$-\frac{1}{4}\log(x+3)+\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x-3),x, algorithm="maxima")
```

```
[Out] -1/4*log(x + 3) + 1/4*log(x - 1)
```

Fricas [A] time = 1.80742, size = 46, normalized size = 2.42

$$-\frac{1}{4}\log(x+3)+\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x-3),x, algorithm="fricas")
```

```
[Out] -1/4*log(x + 3) + 1/4*log(x - 1)
```

Sympy [A] time = 0.088556, size = 12, normalized size = 0.63

$$\frac{\log(x-1)}{4} - \frac{\log(x+3)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+2*x-3),x)
```

```
[Out] log(x - 1)/4 - log(x + 3)/4
```

Giac [A] time = 1.05974, size = 20, normalized size = 1.05

$$-\frac{1}{4}\log(|x+3|) + \frac{1}{4}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x-3),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(x + 3)) + 1/4*log(abs(x - 1))
```

3.214 $\int \frac{1}{-2x+x^2} \, dx$

Optimal. Leaf size=17

$$\frac{1}{2}\log(2-x) - \frac{\log(x)}{2}$$

```
[Out] Log[2 - x]/2 - Log[x]/2
```

Rubi [A] time = 0.0020595, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {615}

$$\frac{1}{2}\log(2-x) - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*x + x^2)^(-1),x]
```

```
[Out] Log[2 - x]/2 - Log[x]/2
```

Rule 615

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[Log[x]/b, x] - Simp
[Log[RemoveContent[b + c*x, x]]/b, x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\int \frac{1}{-2x+x^2} \, dx = \frac{1}{2}\log(2-x) - \frac{\log(x)}{2}$$

Mathematica [A] time = 0.0017699, size = 17, normalized size = 1.

$$\frac{1}{2}\log(2-x) - \frac{\log(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*x + x^2)^(-1),x]
```

```
[Out] Log[2 - x]/2 - Log[x]/2
```

Maple [A] time = 0.004, size = 12, normalized size = 0.7

$$-\frac{\ln(x)}{2}+\frac{\ln(-2+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x),x)
```

```
[Out] -1/2*ln(x)+1/2*ln(-2+x)
```

Maxima [A] time = 0.92438, size = 15, normalized size = 0.88

$$\frac{1}{2}\log(x-2)-\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(x - 2) - 1/2*log(x)
```

Fricas [A] time = 1.86167, size = 39, normalized size = 2.29

$$\frac{1}{2}\log(x-2)-\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x),x, algorithm="fricas")
```

```
[Out] 1/2*log(x - 2) - 1/2*log(x)
```

Sympy [A] time = 0.087404, size = 10, normalized size = 0.59

$$-\frac{\log(x)}{2} + \frac{\log(x-2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x),x)
```

```
[Out] -log(x)/2 + log(x - 2)/2
```

Giac [A] time = 1.05437, size = 18, normalized size = 1.06

$$\frac{1}{2}\log(|x-2|) - \frac{1}{2}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x - 2)) - 1/2*log(abs(x))
```

3.215 $\int \frac{1+2x}{-7+12x+4x^2} \, dx$

Optimal. Leaf size=21

$$\frac{1}{8}\log(1-2x)+\frac{3}{8}\log(2x+7)$$

```
[Out] Log[1 - 2*x]/8 + (3*Log[7 + 2*x])/8
```

Rubi [A] time = 0.0054708, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {632, 31}

$$\frac{1}{8}\log(1-2x)+\frac{3}{8}\log(2x+7)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/(-7 + 12*x + 4*x^2),x]
```

```
[Out] Log[1 - 2*x]/8 + (3*Log[7 + 2*x])/8
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+2x}{-7+12x+4x^2}\,dx &= \frac{1}{2}\int \frac{1}{-2+4x}\,dx+\frac{3}{2}\int \frac{1}{14+4x}\,dx\\ &= \frac{1}{8}\log(1-2x)+\frac{3}{8}\log(7+2x)\end{aligned}$$

Mathematica [A] time = 0.0040953, size = 21, normalized size = 1.

$$\frac{1}{8}\log(1-2x)+\frac{3}{8}\log(2x+7)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/(-7 + 12*x + 4*x^2),x]
```

```
[Out] Log[1 - 2*x]/8 + (3*Log[7 + 2*x])/8
```

Maple [A] time = 0.005, size = 18, normalized size = 0.9

$$\frac{3\ln(7+2x)}{8}+\frac{\ln(2x-1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(4*x^2+12*x-7),x)
```

```
[Out] 3/8*ln(7+2*x)+1/8*ln(2*x-1)
```

Maxima [A] time = 0.943763, size = 23, normalized size = 1.1

$$\frac{3}{8}\log(2x+7)+\frac{1}{8}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(4*x^2+12*x-7),x, algorithm="maxima")
```

```
[Out] 3/8*log(2*x + 7) + 1/8*log(2*x - 1)
```

Fricas [A] time = 1.86837, size = 50, normalized size = 2.38

$$\frac{3}{8}\log(2x+7)+\frac{1}{8}\log(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(4*x^2+12*x-7),x, algorithm="fricas")
```

```
[Out] 3/8*log(2*x + 7) + 1/8*log(2*x - 1)
```

Sympy [A] time = 0.122183, size = 17, normalized size = 0.81

$$\frac{\log\left(x-\frac{1}{2}\right)}{8}+\frac{3\log\left(x+\frac{7}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(4*x**2+12*x-7),x)
```

```
[Out] log(x - 1/2)/8 + 3*log(x + 7/2)/8
```

Giac [A] time = 1.05618, size = 26, normalized size = 1.24

$$\frac{3}{8}\log\left(|2x+7|\right)+\frac{1}{8}\log\left(|2x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(4*x^2+12*x-7),x, algorithm="giac")
```

```
[Out] 3/8*log(abs(2*x + 7)) + 1/8*log(abs(2*x - 1))
```

3.216 $\int \frac{x}{-1+x+x^2} \, dx$

Optimal. Leaf size=49

$$\frac{1}{10}\left(5-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)+\frac{1}{10}\left(5+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/10 + ((5 + Sqrt[5])*Log[1 + Sqrt[5]
+ 2*x])/10
```

Rubi [A] time = 0.0117783, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {632, 31}

$$\frac{1}{10}\left(5-\sqrt{5}\right)\log\left(2x-\sqrt{5}+1\right)+\frac{1}{10}\left(5+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x + x^2),x]
```

```
[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] + 2*x])/10 + ((5 + Sqrt[5])*Log[1 + Sqrt[5]
+ 2*x])/10
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x}{-1+x+x^2}\,dx = \frac{1}{10}\left(5-\sqrt{5}\right)\int \frac{1}{\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\,dx + \frac{1}{10}\left(5+\sqrt{5}\right)\int \frac{1}{\frac{1}{2}+\frac{\sqrt{5}}{2}+x}\,dx$$
$$= \frac{1}{10}\left(5-\sqrt{5}\right)\log\left(1-\sqrt{5}+2x\right) + \frac{1}{10}\left(5+\sqrt{5}\right)\log\left(1+\sqrt{5}+2x\right)$$

Mathematica [A] time = 0.0169402, size = 44, normalized size = 0.9

$$\frac{1}{10}\left(\left(5+\sqrt{5}\right)\log\left(2x+\sqrt{5}+1\right)-\left(\sqrt{5}-5\right)\log\left(-2x+\sqrt{5}-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x + x^2),x]
```

```
[Out] (-((-5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x]) + (5 + Sqrt[5])*Log[1 + Sqrt[5]
+ 2*x])/10
```

Maple [A] time = 0.002, size = 27, normalized size = 0.6

$$\frac{\ln\left(x^2+x-1\right)}{2}+\frac{\sqrt{5}}{5}\operatorname{Artanh}\left(\frac{(1+2x)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+x-1),x)
```

```
[Out] 1/2*ln(x^2+x-1)+1/5*5^(1/2)*arctanh(1/5*(1+2*x)*5^(1/2))
```

Maxima [A] time = 1.40364, size = 50, normalized size = 1.02

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{2x-\sqrt{5}+1}{2x+\sqrt{5}+1}\right)+\frac{1}{2}\log\left(x^2+x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x-1),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) + 1/2*log(x^2 +
x - 1)
```

Fricas [A] time = 1.89853, size = 127, normalized size = 2.59

$$\frac{1}{10}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(2x+1)+2x+3}{x^2+x-1}\right)+\frac{1}{2}\log\left(x^2+x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x-1),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*log((2*x^2 + sqrt(5)*(2*x + 1) + 2*x + 3)/(x^2 + x - 1)) + 1/2
*log(x^2 + x - 1)
```

Sympy [A] time = 0.106135, size = 46, normalized size = 0.94

$$\left(\frac{\sqrt{5}}{10}+\frac{1}{2}\right)\log\left(x+\frac{1}{2}+\frac{\sqrt{5}}{2}\right)+\left(\frac{1}{2}-\frac{\sqrt{5}}{10}\right)\log\left(x-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+x-1),x)
```

```
[Out] (sqrt(5)/10 + 1/2)*log(x + 1/2 + sqrt(5)/2) + (1/2 - sqrt(5)/10)*log(x - sq
rt(5)/2 + 1/2)
```

Giac [A] time = 1.04988, size = 54, normalized size = 1.1

$$-\frac{1}{10}\sqrt{5}\log\left(\frac{\left|2x-\sqrt{5}+1\right|}{\left|2x+\sqrt{5}+1\right|}\right)+\frac{1}{2}\log\left(\left|x^2+x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x-1),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) + 1/2*log(
abs(x^2 + x - 1))
```

3.217 $\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx$

Optimal. Leaf size=63

$$\frac{11049\log\left(x^2+x+5\right)}{260015}-\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(2x+1)+\frac{4822\log(5x+2)}{4879}+\frac{3988\tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right)}{13685\sqrt{19}}$$

```
[Out] (3988*ArcTan[(1 + 2*x)/Sqrt[19]])/(13685*Sqrt[19]) - (3146*Log[7 - 3*x])/80
155 - (334*Log[1 + 2*x])/323 + (4822*Log[2 + 5*x])/4879 + (11049*Log[5 + x
+ x^2])/260015
```

Rubi [A] time = 0.0867445, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2074, 634, 618, 204, 628}

$$\frac{11049\log\left(x^2+x+5\right)}{260015}-\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(2x+1)+\frac{4822\log(5x+2)}{4879}+\frac{3988\tan^{-1}\left(\frac{2x+1}{\sqrt{19}}\right)}{13685\sqrt{19}}$$

Antiderivative was successfully verified.

```
[In] Int[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13*x^4 +
 30*x^5),x]
```

```
[Out] (3988*ArcTan[(1 + 2*x)/Sqrt[19]])/(13685*Sqrt[19]) - (3146*Log[7 - 3*x])/80
155 - (334*Log[1 + 2*x])/323 + (4822*Log[2 + 5*x])/4879 + (11049*Log[5 + x
+ x^2])/260015
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{-32+5x-27x^2+4x^3}{-70-299x-286x^2+50x^3-13x^4+30x^5}\,dx = \int\left(-\frac{668}{323(1+2x)}-\frac{9438}{80155(-7+3x)}+\frac{24110}{4879(2+5x)}+\frac{48935+}{260015\left(5-\right.}\right.$$

$$= -\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}+\frac{\int\frac{48935}{5+}}{26}$$

$$= -\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}+\frac{11049}{2}$$

$$= -\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\log(2+5x)}{4879}+\frac{11049}{}$$

$$= \frac{3988\tan^{-1}\left(\frac{1+2x}{\sqrt{19}}\right)}{13685\sqrt{19}}-\frac{3146\log(7-3x)}{80155}-\frac{334}{323}\log(1+2x)+\frac{4822\,\mathrm{lo}}{4}$$

Mathematica [A] time = 0.0265904, size = 57, normalized size = 0.9

$$\frac{453009\log\left(x^2+x+5\right)-418418\log(7-3x)-11023670\log(2x+1)+10536070\log(5x+2)+163508\sqrt{19}\tan^{-1}\left(\frac{2x}{\sqrt{1}}\right.}{10660615}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-32 + 5*x - 27*x^2 + 4*x^3)/(-70 - 299*x - 286*x^2 + 50*x^3 - 13
*x^4 + 30*x^5),x]
```

```
[Out] (163508*Sqrt[19]*ArcTan[(1 + 2*x)/Sqrt[19]] - 418418*Log[7 - 3*x] - 1102367
0*Log[1 + 2*x] + 10536070*Log[2 + 5*x] + 453009*Log[5 + x + x^2])/10660615
```

Maple [A] time = 0.012, size = 51, normalized size = 0.8

$$\frac{4822\,\ln(2+5\,x)}{4879}-\frac{3146\,\ln(3\,x-7)}{80155}-\frac{334\,\ln(1+2\,x)}{323}+\frac{11049\,\ln\left(x^2+x+5\right)}{260015}+\frac{3988\,\sqrt{19}}{260015}\arctan\left(\frac{(1+2\,x)\,\sqrt{19}}{19}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x)
```

```
[Out] 4822/4879*ln(2+5*x)-3146/80155*ln(3*x-7)-334/323*ln(1+2*x)+11049/260015*ln(
x^2+x+5)+3988/260015*arctan(1/19*(1+2*x)*19^(1/2))*19^(1/2)
```

Maxima [A] time = 1.41807, size = 68, normalized size = 1.08

$$\frac{3988}{260015}\,\sqrt{19}\arctan\left(\frac{1}{19}\,\sqrt{19}(2\,x+1)\right)+\frac{11049}{260015}\,\log\left(x^2+x+5\right)+\frac{4822}{4879}\,\log(5\,x+2)-\frac{3146}{80155}\,\log(3\,x-7)-\frac{334}{323}\,\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="maxima")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(
2*x + 1)
```

Fricas [A] time = 1.90173, size = 216, normalized size = 3.43

$$\frac{3988}{260015}\,\sqrt{19}\arctan\left(\frac{1}{19}\,\sqrt{19}(2\,x+1)\right)+\frac{11049}{260015}\,\log\left(x^2+x+5\right)+\frac{4822}{4879}\,\log(5\,x+2)-\frac{3146}{80155}\,\log(3\,x-7)-\frac{334}{323}\,\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="fricas")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(5*x + 2) - 3146/80155*log(3*x - 7) - 334/323*log(
2*x + 1)
```

Sympy [A] time = 0.317007, size = 68, normalized size = 1.08

$$-\frac{3146\log\left(x-\frac{7}{3}\right)}{80155}+\frac{4822\log\left(x+\frac{2}{5}\right)}{4879}-\frac{334\log\left(x+\frac{1}{2}\right)}{323}+\frac{11049\log\left(x^2+x+5\right)}{260015}+\frac{3988\sqrt{19}\operatorname{atan}\left(\frac{2\sqrt{19}x}{19}+\frac{\sqrt{19}}{19}\right)}{260015}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-27*x**2+5*x-32)/(30*x**5-13*x**4+50*x**3-286*x**2-299*x-7
0),x)
```

```
[Out] -3146*log(x - 7/3)/80155 + 4822*log(x + 2/5)/4879 - 334*log(x + 1/2)/323 +
11049*log(x**2 + x + 5)/260015 + 3988*sqrt(19)*atan(2*sqrt(19)*x/19 + sqrt(
19)/19)/260015
```

Giac [A] time = 1.06043, size = 72, normalized size = 1.14

$$\frac{3988}{260015}\sqrt{19}\arctan\left(\frac{1}{19}\sqrt{19}(2x+1)\right)+\frac{11049}{260015}\log\left(x^2+x+5\right)+\frac{4822}{4879}\log\left(|5x+2|\right)-\frac{3146}{80155}\log\left(|3x-7|\right)-\frac{334}{323}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-27*x^2+5*x-32)/(30*x^5-13*x^4+50*x^3-286*x^2-299*x-70),x,
algorithm="giac")
```

```
[Out] 3988/260015*sqrt(19)*arctan(1/19*sqrt(19)*(2*x + 1)) + 11049/260015*log(x^2
 + x + 5) + 4822/4879*log(abs(5*x + 2)) - 3146/80155*log(abs(3*x - 7)) - 33
4/323*log(abs(2*x + 1))
```

3.218 $\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx$

Optimal. Leaf size=86

$$-\frac{502x+313}{1452\left(2x^2+1\right)}+\frac{2843\log\left(2x^2+1\right)}{7986}+\frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{272\sqrt{2}\tan^{-1}\left(\sqrt{2}x\right)}{1331}-\frac{251\tan^{-1}\left(\sqrt{2}x\right)}{726\sqrt{2}}$$

```
[Out] 5828/(9075*(2 - 5*x)) - (313 + 502*x)/(1452*(1 + 2*x^2)) - (251*ArcTan[Sqrt
[2]*x])/(726*Sqrt[2]) + (272*Sqrt[2]*ArcTan[Sqrt[2]*x])/1331 - (59096*Log[2
 - 5*x])/99825 + (2843*Log[1 + 2*x^2])/7986
```

Rubi [A] time = 0.0978061, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 50, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2074, 639, 203, 635, 260}

$$-\frac{502x+313}{1452\left(2x^2+1\right)}+\frac{2843\log\left(2x^2+1\right)}{7986}+\frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{272\sqrt{2}\tan^{-1}\left(\sqrt{2}x\right)}{1331}-\frac{251\tan^{-1}\left(\sqrt{2}x\right)}{726\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x^4 - 8
0*x^5 + 100*x^6),x]
```

```
[Out] 5828/(9075*(2 - 5*x)) - (313 + 502*x)/(1452*(1 + 2*x^2)) - (251*ArcTan[Sqrt
[2]*x])/(726*Sqrt[2]) + (272*Sqrt[2]*ArcTan[Sqrt[2]*x])/1331 - (59096*Log[2
 - 5*x])/99825 + (2843*Log[1 + 2*x^2])/7986
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{8-13x^2-7x^3+12x^5}{4-20x+41x^2-80x^3+116x^4-80x^5+100x^6}\,dx = \int \left(\frac{5828}{1815(-2+5x)^2}-\frac{59096}{19965(-2+5x)}+\frac{-251+313x}{363\left(1+2x^2\right)^2}+\frac{2(8\ldots}{399\ldots}\right.$$

$$=\frac{5828}{9075(2-5x)}-\frac{59096\log(2-5x)}{99825}+\frac{2\int\frac{816+2843x}{1+2x^2}\,dx}{3993}+\frac{1}{363}\int\ldots$$

$$=\frac{5828}{9075(2-5x)}-\frac{313+502x}{1452\left(1+2x^2\right)}-\frac{59096\log(2-5x)}{99825}-\frac{251}{726}\int\ldots$$

$$=\frac{5828}{9075(2-5x)}-\frac{313+502x}{1452\left(1+2x^2\right)}-\frac{251\tan^{-1}\left(\sqrt{2}x\right)}{726\sqrt{2}}+\frac{272\sqrt{2}\,\mathrm{ta}\ldots}{1\ldots}$$

Mathematica [A] time = 0.0438645, size = 67, normalized size = 0.78

$$\frac{-\frac{33\left(36458x^2+4675x+2554\right)}{10x^3-4x^2+5x-2}+142150\log\left(2x^2+1\right)-236384\log(2-5x)+12575\sqrt{2}\tan^{-1}\left(\sqrt{2}x\right)}{399300}$$

Antiderivative was successfully verified.

```
[In] Integrate[(8 - 13*x^2 - 7*x^3 + 12*x^5)/(4 - 20*x + 41*x^2 - 80*x^3 + 116*x
^4 - 80*x^5 + 100*x^6),x]
```

```
[Out] ((-33*(2554 + 4675*x + 36458*x^2))/(-2 + 5*x - 4*x^2 + 10*x^3) + 12575*Sqrt
[2]*ArcTan[Sqrt[2]*x] - 236384*Log[2 - 5*x] + 142150*Log[1 + 2*x^2])/399300
```

Maple [A] time = 0.012, size = 54, normalized size = 0.6

$$\frac{1}{3993}\left(-\frac{2761\,x}{4}-\frac{3443}{8}\right)\left(x^{2}+\frac{1}{2}\right)^{-1}+\frac{2843\,\ln\left(2\,x^{2}+1\right)}{7986}+\frac{503\,\arctan\left(x\sqrt{2}\right)\sqrt{2}}{15972}-\frac{5828}{45375\,x-18150}-\frac{59096\,\ln(5\,x-\cdots}{99825}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*x+4),x
)
```

```
[Out] 1/3993*(-2761/4*x-3443/8)/(x^2+1/2)+2843/7986*ln(2*x^2+1)+503/15972*arctan(
x*2^(1/2))*2^(1/2)-5828/9075/(5*x-2)-59096/99825*ln(5*x-2)
```

Maxima [A] time = 1.42057, size = 80, normalized size = 0.93

$$\frac{503}{15972}\,\sqrt{2}\arctan\left(\sqrt{2}x\right)-\frac{36458\,x^{2}+4675\,x+2554}{12100\left(10\,x^{3}-4\,x^{2}+5\,x-2\right)}+\frac{2843}{7986}\,\log\left(2\,x^{2}+1\right)-\frac{59096}{99825}\,\log\left(5\,x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="maxima")
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/(
10*x^3 - 4*x^2 + 5*x - 2) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(5*x
- 2)
```

Fricas [A] time = 1.9806, size = 312, normalized size = 3.63

$$\frac{12575\,\sqrt{2}\left(10\,x^{3}-4\,x^{2}+5\,x-2\right)\arctan\left(\sqrt{2}x\right)-1203114\,x^{2}+142150\left(10\,x^{3}-4\,x^{2}+5\,x-2\right)\log\left(2\,x^{2}+1\right)-236384\left(1\cdots\right.}{399300\left(10\,x^{3}-4\,x^{2}+5\,x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="fricas")
```

```
[Out] 1/399300*(12575*sqrt(2)*(10*x^3 - 4*x^2 + 5*x - 2)*arctan(sqrt(2)*x) - 1203
114*x^2 + 142150*(10*x^3 - 4*x^2 + 5*x - 2)*log(2*x^2 + 1) - 236384*(10*x^3
 - 4*x^2 + 5*x - 2)*log(5*x - 2) - 154275*x - 84282)/(10*x^3 - 4*x^2 + 5*x
- 2)
```

Sympy [A] time = 0.203387, size = 63, normalized size = 0.73

$$-\frac{36458x^2 + 4675x + 2554}{121000x^3 - 48400x^2 + 60500x - 24200} - \frac{59096 \log\left(x - \frac{2}{5}\right)}{99825} + \frac{2843 \log\left(x^2 + \frac{1}{2}\right)}{7986} + \frac{503\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x\right)}{15972}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x**5-7*x**3-13*x**2+8)/(100*x**6-80*x**5+116*x**4-80*x**3+41*
x**2-20*x+4),x)
```

```
[Out] -(36458*x**2 + 4675*x + 2554)/(121000*x**3 - 48400*x**2 + 60500*x - 24200)
- 59096*log(x - 2/5)/99825 + 2843*log(x**2 + 1/2)/7986 + 503*sqrt(2)*atan(s
qrt(2)*x)/15972
```

Giac [A] time = 1.0753, size = 80, normalized size = 0.93

$$\frac{503}{15972}\sqrt{2}\arctan\left(\sqrt{2}x\right) - \frac{36458\,x^2 + 4675\,x + 2554}{12100\left(2\,x^2 + 1\right)(5\,x - 2)} + \frac{2843}{7986}\log\left(2\,x^2 + 1\right) - \frac{59096}{99825}\log\left(|5\,x - 2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((12*x^5-7*x^3-13*x^2+8)/(100*x^6-80*x^5+116*x^4-80*x^3+41*x^2-20*
x+4),x, algorithm="giac")
```

```
[Out] 503/15972*sqrt(2)*arctan(sqrt(2)*x) - 1/12100*(36458*x^2 + 4675*x + 2554)/(
(2*x^2 + 1)*(5*x - 2)) + 2843/7986*log(2*x^2 + 1) - 59096/99825*log(abs(5*x
 - 2))
```

3.219 $\int \frac{\sqrt{4+x}}{x} \, dx$

Optimal. Leaf size=24

$$2\sqrt{x+4} - 4\tanh^{-1}\left(\frac{\sqrt{x+4}}{2}\right)$$

```
[Out] 2*Sqrt[4 + x] - 4*ArcTanh[Sqrt[4 + x]/2]
```

Rubi [A] time = 0.0054734, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {50, 63, 207}

$$2\sqrt{x+4} - 4\tanh^{-1}\left(\frac{\sqrt{x+4}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 + x]/x,x]
```

```
[Out] 2*Sqrt[4 + x] - 4*ArcTanh[Sqrt[4 + x]/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{4+x}}{x}\,dx &= 2\sqrt{4+x}+4\int \frac{1}{x\sqrt{4+x}}\,dx\\ &= 2\sqrt{4+x}+8\,\text{Subst}\left(\int \frac{1}{-4+x^2}\,dx, x, \sqrt{4+x}\right)\\ &= 2\sqrt{4+x}-4\tanh^{-1}\left(\frac{\sqrt{4+x}}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0045924, size = 24, normalized size = 1.

$$2\sqrt{x+4}-4\tanh^{-1}\left(\frac{\sqrt{x+4}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 + x]/x,x]
```

```
[Out] 2*Sqrt[4 + x] - 4*ArcTanh[Sqrt[4 + x]/2]
```

Maple [A] time = 0.007, size = 29, normalized size = 1.2

$$2\sqrt{4+x}-2\ln\left(\sqrt{4+x}+2\right)+2\ln\left(\sqrt{4+x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4+x)^(1/2)/x,x)
```

```
[Out] 2*(4+x)^(1/2)-2*ln((4+x)^(1/2)+2)+2*ln((4+x)^(1/2)-2)
```

Maxima [A] time = 0.940056, size = 38, normalized size = 1.58

$$2\sqrt{x+4} - 2\log\left(\sqrt{x+4}+2\right) + 2\log\left(\sqrt{x+4}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 2*sqrt(x + 4) - 2*log(sqrt(x + 4) + 2) + 2*log(sqrt(x + 4) - 2)
```

Fricas [A] time = 1.79127, size = 88, normalized size = 3.67

$$2\sqrt{x+4} - 2\log\left(\sqrt{x+4}+2\right) + 2\log\left(\sqrt{x+4}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 2*sqrt(x + 4) - 2*log(sqrt(x + 4) + 2) + 2*log(sqrt(x + 4) - 2)
```

Sympy [A] time = 0.716193, size = 44, normalized size = 1.83

$$\begin{cases} 2\sqrt{x+4} - 4\operatorname{acoth}\left(\frac{\sqrt{x+4}}{2}\right) & \text{for}\ \frac{|x+4|}{4} > 1 \\ 2\sqrt{x+4} - 4\operatorname{atanh}\left(\frac{\sqrt{x+4}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)**(1/2)/x,x)
```

```
[Out] Piecewise((2*sqrt(x + 4) - 4*acoth(sqrt(x + 4)/2), Abs(x + 4)/4 > 1), (2*sq
rt(x + 4) - 4*atanh(sqrt(x + 4)/2), True))
```

Giac [A] time = 1.05978, size = 39, normalized size = 1.62

$$2\sqrt{x+4} - 2\log\left(\sqrt{x+4}+2\right) + 2\log\left(\left|\sqrt{x+4}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*sqrt(x + 4) - 2*log(sqrt(x + 4) + 2) + 2*log(abs(sqrt(x + 4) - 2))
```

3.220 $\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=200

$$2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)+\frac{3}{5}\sqrt{2(5$$

```
[Out] 2*Sqrt[x] + (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/Sqrt[
2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[5])/
10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 + Sqr
t[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 - Sqrt[5])
*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rubi [A] time = 0.377868, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {1593, 341, 321, 294, 634, 618, 204, 628, 31}

$$2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)+\frac{3}{5}\sqrt{2(5$$

Antiderivative was successfully verified.

```
[In] Int[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/Sqrt[
2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[5])/
10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 + Sqr
t[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 - Sqrt[5])
*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 294

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos
[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[
((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r^(m + 1)*Int[1/(r - s*x), x])/(a*n*s^
m) - Dist[(2*(-r)^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x], x]] /;
 FreeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && N
egQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx &= \int \frac{\sqrt[3]{x}}{-1+x^{5/6}}\,dx\\
&= 6\,\mathrm{Subst}\left(\int \frac{x^7}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)\\
&= 2\sqrt{x}+6\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^5}\,dx, x, \sqrt[6]{x}\right)\\
&= 2\sqrt{x}-\frac{6}{5}\,\mathrm{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sqrt[6]{x}\right)-\frac{12}{5}\,\mathrm{Subst}\left(\int \frac{\frac{1}{4}\left(-1-\sqrt{5}\right)+\frac{1}{4}\left(1+\sqrt{5}\right)x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)-\frac{12}{5}\,\mathrm{Sub}\\
&= 2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)+\frac{3\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{5}}-\frac{3\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{1}{2}\left(1+\sqrt{5}\right)x+x^2}\,dx, x, \sqrt[6]{x}\right)}{\sqrt{5}}\\
&= 2\sqrt{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\sqrt{5}\sqrt[6]{\right.\\
&= 2\sqrt{x}+6\sqrt{\frac{2}{5\left(5+\sqrt{5}\right)}}\tan^{-1}\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}+4\sqrt[6]{\right.\right.
\end{aligned}$$

Mathematica [C] time = 0.0053364, size = 22, normalized size = 0.11

$$-2\sqrt{x}\left(\mathrm{Hypergeometric2F1}\left(\frac{3}{5},1,\frac{8}{5},x^{5/6}\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(-1/3) + Sqrt[x])^(-1),x]
```

```
[Out] -2*Sqrt[x]*(-1 + Hypergeometric2F1[3/5, 1, 8/5, x^(5/6)])
```

Maple [A] time = 0.033, size = 175, normalized size = 0.9

$$2\sqrt{x}-\frac{3}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}+\sqrt[6]{x}\sqrt{5}\right)+\frac{3\sqrt{5}}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}+\sqrt[6]{x}\sqrt{5}\right)-\frac{12\sqrt{5}}{5\sqrt{10-2\sqrt{5}}}\arctan\left(\frac{1}{\sqrt{10-2\sqrt{5}}}(1+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1/x^(1/3)+x^(1/2)),x)
```

```
[Out] 2*x^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))+3/10*ln(2+x^(1/6)+2*
x^(1/3)+x^(1/6)*5^(1/2))*5^(1/2)-12/5/(10-2*5^(1/2))^(1/2)*arctan((1+4*x^(1
/6)+5^(1/2))/(10-2*5^(1/2))^(1/2))*5^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1
/6)*5^(1/2))*5^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))+12/5/(10+
2*5^(1/2))^(1/2)*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*5^(1/2)
+6/5*ln(-1+x^(1/6))
```

Maxima [B] time = 1.43988, size = 367, normalized size = 1.84

$$-\frac{6}{5}(-1)^{\frac{3}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right)-\frac{6\sqrt{5}(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}}+\frac{6\sqrt{5}(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-1}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-1}}\right.}{5\sqrt{-2\sqrt{5}-10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -6/5*(-1)^(3/5)*log((-1)^(1/5) + x^(1/6)) - 6/5*sqrt(5)*(-1)^(3/5)*log((sqr
t(5)*(-1)^(1/5) + (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))
/(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(
1/6)))/sqrt(2*sqrt(5) - 10) + 6/5*sqrt(5)*(-1)^(3/5)*log((sqrt(5)*(-1)^(1/5
) - (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1
)^(1/5) + (-1)^(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(
-2*sqrt(5) - 10) + 2*sqrt(x) + 6/5*log(-x^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(2/5) + (-1)^(2/5)) - 6/5*
log(x^(1/6)*(sqrt(5)*(-1)^(1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(
```

```
sqrt(5)*(-1)^(2/5) - (-1)^(2/5))
```

Fricas [B] time = 12.9079, size = 2130, normalized size = 10.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 +
9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) -
sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2
)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) -
 5) + sqrt(5) + 1)^2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 3*
sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(
sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/
4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5
) + 18*sqrt(5) - 90)*(sqrt(5) - 1) + 72*x^(1/6) + 36) + 1/10*(3*sqrt(5) + s
qrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(s
qrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4
*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5)
 + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 - 3*sqrt(-27/4*(sqrt(2)
*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqr
t(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90
)*(sqrt(5) - 1) + 72*x^(1/6) + 36) - 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt
(5) + 1)*log(-9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 36*x^(1/6))
 + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(-9/4*(sqrt(2)*sqrt(sq
rt(5) - 5) - sqrt(5) - 1)^2 + 36*x^(1/6)) + 2*sqrt(x) + 6/5*log(x^(1/6) - 1
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt[3]{x}}{\left(\sqrt[6]{x}-1\right)\left(\sqrt[6]{x}+x^{\frac{2}{3}}+\sqrt[3]{x}+\sqrt{x}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(x**(1/3)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3) + sqrt(x)
 + 1)), x)
```

<u>Giac</u> [A] time = 1.39809, size = 188, normalized size = 0.94

$$\frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right)+\frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 3/5*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
+ 10)) - 3/5*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2*
sqrt(5) + 10)) + 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1)
- 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + 2*sqrt(x) -
3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1/6) -
 1))
```

3.221 $\int \frac{1}{-4\cos(x)+3\sin(x)}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{5}\tanh^{-1}\left(\frac{1}{5}(4\sin(x)+3\cos(x))\right)$$

```
[Out] -ArcTanh[(3*Cos[x] + 4*Sin[x])/5]/5
```

Rubi [A] time = 0.0120571, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3074, 206}

$$-\frac{1}{5}\tanh^{-1}\left(\frac{1}{5}(4\sin(x)+3\cos(x))\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-4*Cos[x] + 3*Sin[x])^(-1),x]
```

```
[Out] -ArcTanh[(3*Cos[x] + 4*Sin[x])/5]/5
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-4\cos(x)+3\sin(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{25-x^2}\,dx, x, 3\cos(x)+4\sin(x)\right)\\ &= -\frac{1}{5}\tanh^{-1}\left(\frac{1}{5}(3\cos(x)+4\sin(x))\right)\end{aligned}$$

Mathematica [B] time = 0.0166594, size = 41, normalized size = 2.28

$$\frac{1}{5}\log\left(\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right)-\frac{1}{5}\log\left(\sin\left(\frac{x}{2}\right)+2\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-4*Cos[x] + 3*Sin[x])^(-1),x]
```

```
[Out] Log[Cos[x/2] - 2*Sin[x/2]]/5 - Log[2*Cos[x/2] + Sin[x/2]]/5
```

Maple [A] time = 0.031, size = 22, normalized size = 1.2

$$-\frac{1}{5}\ln\left(\tan\left(\frac{x}{2}\right)+2\right)+\frac{1}{5}\ln\left(2\tan\left(x/2\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-4*cos(x)+3*sin(x)),x)
```

```
[Out] -1/5*ln(tan(1/2*x)+2)+1/5*ln(2*tan(1/2*x)-1)
```

Maxima [B] time = 0.936192, size = 41, normalized size = 2.28

$$\frac{1}{5}\log\left(\frac{2\sin\left(x\right)}{\cos\left(x\right)+1}-1\right)-\frac{1}{5}\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*cos(x)+3*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/5*log(2*sin(x)/(cos(x) + 1) - 1) - 1/5*log(sin(x)/(cos(x) + 1) + 2)
```

Fricas [B] time = 1.92764, size = 109, normalized size = 6.06

$$-\frac{1}{10}\log\left(\frac{3}{2}\cos\left(x\right)+2\sin\left(x\right)+\frac{5}{2}\right)+\frac{1}{10}\log\left(-\frac{3}{2}\cos\left(x\right)-2\sin\left(x\right)+\frac{5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*cos(x)+3*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/10*log(3/2*cos(x) + 2*sin(x) + 5/2) + 1/10*log(-3/2*cos(x) - 2*sin(x) +
5/2)
```

Sympy [A] time = 0.250552, size = 20, normalized size = 1.11

$$\frac{\log\left(\tan\left(\frac{x}{2}\right)-\frac{1}{2}\right)}{5}-\frac{\log\left(\tan\left(\frac{x}{2}\right)+2\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*cos(x)+3*sin(x)),x)
```

```
[Out] log(tan(x/2) - 1/2)/5 - log(tan(x/2) + 2)/5
```

Giac [A] time = 1.07951, size = 31, normalized size = 1.72

$$\frac{1}{5}\log\left(\left|2\tan\left(\frac{1}{2}x\right)-1\right|\right)-\frac{1}{5}\log\left(\left|\tan\left(\frac{1}{2}x\right)+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-4*cos(x)+3*sin(x)),x, algorithm="giac")
```

```
[Out] 1/5*log(abs(2*tan(1/2*x) - 1)) - 1/5*log(abs(tan(1/2*x) + 2))
```

3.222 $\int \frac{1}{1+\sqrt{x}}\,dx$

Optimal. Leaf size=18

$$2\sqrt{x}-2\log\left(\sqrt{x}+1\right)$$

```
[Out] 2*Sqrt[x] - 2*Log[1 + Sqrt[x]]
```

Rubi [A] time = 0.0059158, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {190, 43}

$$2\sqrt{x}-2\log\left(\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] - 2*Log[1 + Sqrt[x]]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{1+\sqrt{x}}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x}{1+x}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\mathrm{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, \sqrt{x}\right)$$
$$= 2\sqrt{x} - 2\log\left(1+\sqrt{x}\right)$$

Mathematica [A] time = 0.0066791, size = 18, normalized size = 1.

$$2\sqrt{x} - 2\log\left(\sqrt{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])^(-1),x]

[Out] 2*Sqrt[x] - 2*Log[1 + Sqrt[x]]
```

Maple [A] time = 0.003, size = 27, normalized size = 1.5

$$2\sqrt{x} + \ln\left(\sqrt{x}-1\right) - \ln\left(\sqrt{x}+1\right) - \ln\left(-1+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)+1),x)

[Out] 2*x^(1/2)+ln(x^(1/2)-1)-ln(x^(1/2)+1)-ln(-1+x)
```

Maxima [A] time = 0.928509, size = 20, normalized size = 1.11

$$2\sqrt{x} - 2\log\left(\sqrt{x}+1\right) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1) + 2
```

Fricas [A] time = 1.79365, size = 43, normalized size = 2.39

$$2\sqrt{x} - 2\log\left(\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1)
```

Sympy [A] time = 0.109201, size = 15, normalized size = 0.83

$$2\sqrt{x} - 2\log\left(\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1)
```

Giac [A] time = 1.04502, size = 19, normalized size = 1.06

$$2\sqrt{x} - 2\log\left(\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) - 2*log(sqrt(x) + 1)
```

3.223 $\int \frac{1}{1+\frac{1}{\sqrt[3]{x}}} dx$

Optimal. Leaf size=32

$$-\frac{3x^{2/3}}{2} + x + 3\sqrt[3]{x} - 3\log\left(\frac{1}{\sqrt[3]{x}} + 1\right) - \log(x)$$

```
[Out] 3*x^(1/3) - (3*x^(2/3))/2 + x - 3*Log[1 + x^(-1/3)] - Log[x]
```

Rubi [A] time = 0.0132232, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {190, 44}

$$-\frac{3x^{2/3}}{2} + x + 3\sqrt[3]{x} - 3\log\left(\frac{1}{\sqrt[3]{x}} + 1\right) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(-1/3))^(-1),x]
```

```
[Out] 3*x^(1/3) - (3*x^(2/3))/2 + x - 3*Log[1 + x^(-1/3)] - Log[x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1+\frac{1}{\sqrt[3]{x}}}\,dx &= -\left(3\,\text{Subst}\left(\int \frac{1}{x^4(1+x)}\,dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\
&= -\left(3\,\text{Subst}\left(\int \left(\frac{1}{x^4}-\frac{1}{x^3}+\frac{1}{x^2}-\frac{1}{x}+\frac{1}{1+x}\right)dx, x, \frac{1}{\sqrt[3]{x}}\right)\right)\\
&= 3\sqrt[3]{x}-\frac{3x^{2/3}}{2}+x-3\log\left(1+\frac{1}{\sqrt[3]{x}}\right)-\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.010988, size = 28, normalized size = 0.88

$$-\frac{3x^{2/3}}{2}+x+3\sqrt[3]{x}-3\log\left(\sqrt[3]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(-1/3))^(-1),x]
```

```
[Out] 3*x^(1/3) - (3*x^(2/3))/2 + x - 3*Log[1 + x^(1/3)]
```

Maple [A] time = 0.002, size = 21, normalized size = 0.7

$$x-\frac{3}{2}x^{\frac{2}{3}}+3\sqrt[3]{x}-3\ln\left(\sqrt[3]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+1/x^(1/3)),x)
```

```
[Out] x-3/2*x^(2/3)+3*x^(1/3)-3*ln(x^(1/3)+1)
```

Maxima [A] time = 0.930783, size = 38, normalized size = 1.19

$$-\frac{1}{2}x\left(\frac{3}{x^{\frac{1}{3}}}-\frac{6}{x^{\frac{2}{3}}}-2\right)-\log(x)-3\log\left(\frac{1}{x^{\frac{1}{3}}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/x^(1/3)),x, algorithm="maxima")
```

```
[Out] -1/2*x*(3/x^(1/3) - 6/x^(2/3) - 2) - log(x) - 3*log(1/x^(1/3) + 1)
```

Fricas [A] time = 1.90929, size = 68, normalized size = 2.12

$$x - \frac{3}{2}x^{\frac{2}{3}} + 3x^{\frac{1}{3}} - 3\log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/x^(1/3)),x, algorithm="fricas")
```

```
[Out] x - 3/2*x^(2/3) + 3*x^(1/3) - 3*log(x^(1/3) + 1)
```

Sympy [A] time = 0.113406, size = 26, normalized size = 0.81

$$-\frac{3x^{\frac{2}{3}}}{2} + 3\sqrt[3]{x} + x - 3\log\left(\sqrt[3]{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/x**(1/3)),x)
```

```
[Out] -3*x**(2/3)/2 + 3*x**(1/3) + x - 3*log(x**(1/3) + 1)
```

Giac [A] time = 1.04935, size = 27, normalized size = 0.84

$$x - \frac{3}{2}x^{\frac{2}{3}} + 3x^{\frac{1}{3}} - 3\log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/x^(1/3)),x, algorithm="giac")
```

```
[Out] x - 3/2*x^(2/3) + 3*x^(1/3) - 3*log(x^(1/3) + 1)
```

3.224 $\int \frac{\sqrt{x}}{1+x}\,dx$

Optimal. Leaf size=16

$$2\sqrt{x} - 2\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*Sqrt[x] - 2*ArcTan[Sqrt[x]]
```

Rubi [A] time = 0.0030161, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {50, 63, 203}

$$2\sqrt{x} - 2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x),x]

[Out] 2*Sqrt[x] - 2*ArcTan[Sqrt[x]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{x}}{1+x}\,dx &= 2\sqrt{x}-\int \frac{1}{\sqrt{x}(1+x)}\,dx\\ &= 2\sqrt{x}-2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{x}\right)\\ &= 2\sqrt{x}-2\tan^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0033632, size = 16, normalized size = 1.

$$2\sqrt{x}-2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x),x]
```

```
[Out] 2*Sqrt[x] - 2*ArcTan[Sqrt[x]]
```

Maple [A] time = 0.003, size = 13, normalized size = 0.8

$$-2\arctan\left(\sqrt{x}\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1+x),x)
```

```
[Out] -2*arctan(x^(1/2))+2*x^(1/2)
```

Maxima [A] time = 1.40177, size = 16, normalized size = 1.

$$2\sqrt{x}-2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) - 2*arctan(sqrt(x))
```

Fricas [A] time = 1.91908, size = 42, normalized size = 2.62

$$2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) - 2*arctan(sqrt(x))
```

Sympy [A] time = 0.149241, size = 14, normalized size = 0.88

$$2\sqrt{x} - 2\operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x),x)
```

```
[Out] 2*sqrt(x) - 2*atan(sqrt(x))
```

Giac [A] time = 1.04868, size = 16, normalized size = 1.

$$2\sqrt{x} - 2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) - 2*arctan(sqrt(x))
```

3.225 $\int \frac{1}{x\sqrt{1+x}}\,dx$

Optimal. Leaf size=10

$$-2\tanh^{-1}\left(\sqrt{x+1}\right)$$

```
[Out] -2*ArcTanh[Sqrt[1 + x]]
```

Rubi [A] time = 0.0030611, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {63, 207}

$$-2\tanh^{-1}\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 + x]),x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{1+x}}\,dx = 2\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x}\right)$$
$$= -2\tanh^{-1}\left(\sqrt{1+x}\right)$$

Mathematica [A] time = 0.0019041, size = 10, normalized size = 1.

$$-2\tanh^{-1}\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[1 + x]),x]

[Out] -2*ArcTanh[Sqrt[1 + x]]
```

Maple [A] time = 0.003, size = 9, normalized size = 0.9

$$-2\,\mathrm{Artanh}\left(\sqrt{1+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+x)^(1/2),x)

[Out] -2*arctanh((1+x)^(1/2))
```

Maxima [B] time = 0.936395, size = 26, normalized size = 2.6

$$-\log\left(\sqrt{x+1}+1\right)+\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(sqrt(x + 1) + 1) + log(sqrt(x + 1) - 1)
```

Fricas [B] time = 1.87211, size = 62, normalized size = 6.2

$$-\log\left(\sqrt{x+1}+1\right)+\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(sqrt(x + 1) + 1) + log(sqrt(x + 1) - 1)
```

Sympy [A] time = 0.48158, size = 26, normalized size = 2.6

$$\begin{cases} -2\operatorname{acoth}\left(\sqrt{x+1}\right) & \text{for } |x+1|>1 \\ -2\operatorname{atanh}\left(\sqrt{x+1}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)**(1/2),x)
```

```
[Out] Piecewise((-2*acoth(sqrt(x + 1)), Abs(x + 1) > 1), (-2*atanh(sqrt(x + 1)),
True))
```

Giac [B] time = 1.05432, size = 27, normalized size = 2.7

$$-\log\left(\sqrt{x+1}+1\right)+\log\left(\left|\sqrt{x+1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -log(sqrt(x + 1) + 1) + log(abs(sqrt(x + 1) - 1))
```

3.226 $\int \frac{1}{-\sqrt[3]{x}+x} \, dx$

Optimal. Leaf size=14

$$\frac{3}{2}\log\left(1-x^{2/3}\right)$$

```
[Out] (3*Log[1 - x^(2/3)])/2
```

Rubi [A] time = 0.00433, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 260}

$$\frac{3}{2}\log\left(1-x^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-x^(1/3) + x)^(-1),x]
```

```
[Out] (3*Log[1 - x^(2/3)])/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-\sqrt[3]{x}+x} \, dx &= \int \frac{1}{\left(-1+x^{2/3}\right)\sqrt[3]{x}} \, dx\\ &= \frac{3}{2}\log\left(1-x^{2/3}\right)\end{aligned}$$

Mathematica [A] time = 0.0018832, size = 14, normalized size = 1.

$$\frac{3}{2}\log\left(1-x^{2/3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^(1/3) + x)^(-1),x]
```

```
[Out] (3*Log[1 - x^(2/3)])/2
```

Maple [B] time = 0.011, size = 50, normalized size = 3.6

$$\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}+\ln\left(-1+\sqrt[3]{x}\right)-\frac{1}{2}\ln\left(x^{\frac{2}{3}}+\sqrt[3]{x}+1\right)+\ln\left(\sqrt[3]{x}+1\right)-\frac{1}{2}\ln\left(x^{\frac{2}{3}}-\sqrt[3]{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^(1/3)+x),x)
```

```
[Out] 1/2*ln(-1+x)+1/2*ln(1+x)+ln(-1+x^(1/3))-1/2*ln(x^(2/3)+x^(1/3)+1)+ln(x^(1/3
)+1)-1/2*ln(x^(2/3)-x^(1/3)+1)
```

Maxima [A] time = 0.927172, size = 23, normalized size = 1.64

$$\frac{3}{2}\log\left(x^{\frac{1}{3}}+1\right)+\frac{3}{2}\log\left(x^{\frac{1}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/3)+x),x, algorithm="maxima")
```

```
[Out] 3/2*log(x^(1/3) + 1) + 3/2*log(x^(1/3) - 1)
```

Fricas [A] time = 1.90974, size = 30, normalized size = 2.14

$$\frac{3}{2}\log\left(x^{\frac{2}{3}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/3)+x),x, algorithm="fricas")
```

```
[Out] 3/2*log(x^(2/3) - 1)
```

Sympy [B] time = 0.178474, size = 22, normalized size = 1.57

$$\frac{3\log\left(\sqrt[3]{x}-1\right)}{2}+\frac{3\log\left(\sqrt[3]{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**(1/3)+x),x)
```

```
[Out] 3*log(x**(1/3) - 1)/2 + 3*log(x**(1/3) + 1)/2
```

Giac [A] time = 1.05442, size = 24, normalized size = 1.71

$$\frac{3}{2}\log\left(x^{\frac{1}{3}}+1\right)+\frac{3}{2}\log\left(\left|x^{\frac{1}{3}}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^(1/3)+x),x, algorithm="giac")
```

```
[Out] 3/2*log(x^(1/3) + 1) + 3/2*log(abs(x^(1/3) - 1))
```

3.227 $\int \frac{1}{x-\sqrt{2+x}}\,dx$

Optimal. Leaf size=31

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Rubi [A] time = 0.0225466, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {632, 31}

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right)+\frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[2 + x])^(-1),x]
```

```
[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x-\sqrt{2+x}}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x}{-2-x+x^2}\,dx, x, \sqrt{2+x}\right)$$
$$= \frac{2}{3}\,\mathrm{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt{2+x}\right) + \frac{4}{3}\,\mathrm{Subst}\left(\int \frac{1}{-2+x}\,dx, x, \sqrt{2+x}\right)$$
$$= \frac{4}{3}\log\left(2-\sqrt{2+x}\right) + \frac{2}{3}\log\left(1+\sqrt{2+x}\right)$$

Mathematica [A] time = 0.0101808, size = 31, normalized size = 1.

$$\frac{4}{3}\log\left(2-\sqrt{x+2}\right) + \frac{2}{3}\log\left(\sqrt{x+2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[2 + x])^(-1),x]

[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3
```

Maple [B] time = 0.013, size = 54, normalized size = 1.7

$$\frac{\ln(1+x)}{3} + \frac{2\,\ln(-2+x)}{3} - \frac{2}{3}\ln\left(\sqrt{2+x}+2\right) + \frac{1}{3}\ln\left(1+\sqrt{2+x}\right) - \frac{1}{3}\ln\left(-1+\sqrt{2+x}\right) + \frac{2}{3}\ln\left(\sqrt{2+x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(2+x)^(1/2)),x)

[Out] 1/3*ln(1+x)+2/3*ln(-2+x)-2/3*ln((2+x)^(1/2)+2)+1/3*ln(1+(2+x)^(1/2))-1/3*ln
(-1+(2+x)^(1/2))+2/3*ln((2+x)^(1/2)-2)
```

Maxima [A] time = 0.926835, size = 28, normalized size = 0.9

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right) + \frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

Fricas [A] time = 1.89868, size = 72, normalized size = 2.32

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\sqrt{x+2}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)
```

Sympy [A] time = 1.31173, size = 36, normalized size = 1.16

$$\log\left(x-\sqrt{x+2}\right)+\frac{\log\left(2\sqrt{x+2}-4\right)}{3}-\frac{\log\left(2\sqrt{x+2}+2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)**(1/2)),x)
```

```
[Out] log(x - sqrt(x + 2)) + log(2*sqrt(x + 2) - 4)/3 - log(2*sqrt(x + 2) + 2)/3
```

Giac [A] time = 1.07866, size = 30, normalized size = 0.97

$$\frac{2}{3}\log\left(\sqrt{x+2}+1\right)+\frac{4}{3}\log\left(\left|\sqrt{x+2}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(abs(sqrt(x + 2) - 2))
```

3.228 $\int \frac{x^2}{\sqrt{-1+x}} dx$

Optimal. Leaf size=32

$$\frac{2}{5}(x-1)^{5/2}+\frac{4}{3}(x-1)^{3/2}+2\sqrt{x-1}$$

```
[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 + (2*(-1 + x)^(5/2))/5
```

Rubi [A] time = 0.0054953, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2}{5}(x-1)^{5/2}+\frac{4}{3}(x-1)^{3/2}+2\sqrt{x-1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[-1 + x],x]
```

```
[Out] 2*Sqrt[-1 + x] + (4*(-1 + x)^(3/2))/3 + (2*(-1 + x)^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{-1+x}}\,dx &= \int \left(\frac{1}{\sqrt{-1+x}}+2\sqrt{-1+x}+(-1+x)^{3/2}\right)dx\\ &= 2\sqrt{-1+x}+\frac{4}{3}(-1+x)^{3/2}+\frac{2}{5}(-1+x)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0055076, size = 21, normalized size = 0.66

$$\frac{2}{15}\sqrt{x-1}\left(3x^2+4x+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[-1 + x],x]
```

```
[Out] (2*Sqrt[-1 + x]*(8 + 4*x + 3*x^2))/15
```

Maple [A] time = 0.003, size = 18, normalized size = 0.6

$$\frac{6 x^2 + 8 x + 16}{15} \sqrt{-1 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-1+x)^(1/2),x)
```

```
[Out] 2/15*(-1+x)^(1/2)*(3*x^2+4*x+8)
```

Maxima [A] time = 0.925678, size = 30, normalized size = 0.94

$$\frac{2}{5} (x - 1)^{\frac{5}{2}} + \frac{4}{3} (x - 1)^{\frac{3}{2}} + 2 \sqrt{x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(x - 1)^(5/2) + 4/3*(x - 1)^(3/2) + 2*sqrt(x - 1)
```

Fricas [A] time = 1.79603, size = 49, normalized size = 1.53

$$\frac{2}{15} \left(3 x^2 + 4 x + 8\right) \sqrt{x - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^2 + 4*x + 8)*sqrt(x - 1)
```

Sympy [A] time = 1.07552, size = 76, normalized size = 2.38

$$\begin{cases} \frac{2x^2\sqrt{x-1}}{5} + \frac{8x\sqrt{x-1}}{15} + \frac{16\sqrt{x-1}}{15} & \text{for } |x| > 1 \\ \frac{2ix^2\sqrt{1-x}}{5} + \frac{8ix\sqrt{1-x}}{15} + \frac{16i\sqrt{1-x}}{15} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-1+x)**(1/2),x)
```

```
[Out] Piecewise((2*x**2*sqrt(x - 1)/5 + 8*x*sqrt(x - 1)/15 + 16*sqrt(x - 1)/15, A
bs(x) > 1), (2*I*x**2*sqrt(1 - x)/5 + 8*I*x*sqrt(1 - x)/15 + 16*I*sqrt(1 -
x)/15, True))
```

Giac [A] time = 1.05024, size = 30, normalized size = 0.94

$$\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{4}{3}(x-1)^{\frac{3}{2}} + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*(x - 1)^(5/2) + 4/3*(x - 1)^(3/2) + 2*sqrt(x - 1)
```

3.229 $\int \frac{\sqrt{-1+x}}{1+x}\,dx$

Optimal. Leaf size=31

$$2\sqrt{x-1}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)$$

```
[Out] 2*Sqrt[-1 + x] - 2*Sqrt[2]*ArcTan[Sqrt[-1 + x]/Sqrt[2]]
```

Rubi [A] time = 0.0080735, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 203}

$$2\sqrt{x-1}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + x]/(1 + x),x]
```

```
[Out] 2*Sqrt[-1 + x] - 2*Sqrt[2]*ArcTan[Sqrt[-1 + x]/Sqrt[2]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-1+x}}{1+x}\,dx &= 2\sqrt{-1+x}-2\int \frac{1}{\sqrt{-1+x}(1+x)}\,dx\\ &= 2\sqrt{-1+x}-4\,\mathrm{Subst}\left(\int \frac{1}{2+x^2}\,dx,x,\sqrt{-1+x}\right)\\ &= 2\sqrt{-1+x}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{-1+x}}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0068531, size = 31, normalized size = 1.

$$2\sqrt{x-1}-2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + x]/(1 + x),x]
```

```
[Out] 2*Sqrt[-1 + x] - 2*Sqrt[2]*ArcTan[Sqrt[-1 + x]/Sqrt[2]]
```

Maple [A] time = 0.005, size = 25, normalized size = 0.8

$$-2\arctan\left(1/2\sqrt{-1+x}\sqrt{2}\right)\sqrt{2}+2\sqrt{-1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+x)^(1/2)/(1+x),x)
```

```
[Out] -2*arctan(1/2*(-1+x)^(1/2)*2^(1/2))*2^(1/2)+2*(-1+x)^(1/2)
```

Maxima [A] time = 1.40636, size = 32, normalized size = 1.03

$$-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)+2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x),x, algorithm="maxima")
```

```
[Out] -2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 2*sqrt(x - 1)
```

Fricas [A] time = 1.87988, size = 81, normalized size = 2.61

$$-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)+2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x),x, algorithm="fricas")
```

```
[Out] -2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 2*sqrt(x - 1)
```

Sympy [A] time = 1.21423, size = 76, normalized size = 2.45

$$\begin{cases} 2\sqrt{x-1}+2\sqrt{2}\operatorname{asin}\left(\frac{\sqrt{2}}{\sqrt{x+1}}\right) & \text{for } \frac{|x+1|}{2}>1 \\ 2i\sqrt{1-x}+\sqrt{2}i\log\left(x+1\right)-2\sqrt{2}i\log\left(\sqrt{\frac{1}{2}-\frac{x}{2}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)**(1/2)/(1+x),x)
```

```
[Out] Piecewise((2*sqrt(x - 1) + 2*sqrt(2)*asin(sqrt(2)/sqrt(x + 1)), Abs(x + 1)/
2 > 1), (2*I*sqrt(1 - x) + sqrt(2)*I*log(x + 1) - 2*sqrt(2)*I*log(sqrt(1/2
- x/2) + 1), True))
```

<u>Giac</u> [A] time = 1.04958, size = 32, normalized size = 1.03

$$-2\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x-1}\right)+2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+x)^(1/2)/(1+x),x, algorithm="giac")
```

```
[Out] -2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x - 1)) + 2*sqrt(x - 1)
```

3.230 $\int \frac{1}{\sqrt{1+\sqrt{x}}} dx$

Optimal. Leaf size=29

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{3/2}-4\sqrt{\sqrt{x}+1}$$

```
[Out] -4*Sqrt[1 + Sqrt[x]] + (4*(1 + Sqrt[x])^(3/2))/3
```

Rubi [A] time = 0.0073292, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {190, 43}

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{3/2}-4\sqrt{\sqrt{x}+1}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Sqrt[x]],x]
```

```
[Out] -4*Sqrt[1 + Sqrt[x]] + (4*(1 + Sqrt[x])^(3/2))/3
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\sqrt{1+\sqrt{x}}}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx, x, \sqrt{x}\right) \\
&= 2\,\text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right)dx, x, \sqrt{x}\right) \\
&= -4\sqrt{1+\sqrt{x}}+\frac{4}{3}\left(1+\sqrt{x}\right)^{3/2}
\end{aligned}
$$

Mathematica [A] time = 0.00741, size = 22, normalized size = 0.76

$$\frac{4}{3}\left(\sqrt{x}-2\right)\sqrt{\sqrt{x}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Sqrt[x]],x]
```

```
[Out] (4*(-2 + Sqrt[x])*Sqrt[1 + Sqrt[x]])/3
```

Maple [A] time = 0.005, size = 20, normalized size = 0.7

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}-4\sqrt{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)+1)^(1/2),x)
```

```
[Out] 4/3*(x^(1/2)+1)^(3/2)-4*(x^(1/2)+1)^(1/2)
```

Maxima [A] time = 0.928268, size = 26, normalized size = 0.9

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}-4\sqrt{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3*(sqrt(x) + 1)^(3/2) - 4*sqrt(sqrt(x) + 1)
```

Fricas [A] time = 2.05516, size = 50, normalized size = 1.72

$$\frac{4}{3}\sqrt{\sqrt{x}+1}\left(\sqrt{x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3*sqrt(sqrt(x) + 1)*(sqrt(x) - 2)
```

Sympy [B] time = 0.760373, size = 117, normalized size = 4.03

$$-\frac{4x^{\frac{5}{2}}\sqrt{\sqrt{x}+1}}{3x^{\frac{5}{2}}+3x^{2}}+\frac{8x^{\frac{5}{2}}}{3x^{\frac{5}{2}}+3x^{2}}+\frac{4x^{3}\sqrt{\sqrt{x}+1}}{3x^{\frac{5}{2}}+3x^{2}}-\frac{8x^{2}\sqrt{\sqrt{x}+1}}{3x^{\frac{5}{2}}+3x^{2}}+\frac{8x^{2}}{3x^{\frac{5}{2}}+3x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x**(1/2))**(1/2),x)
```

```
[Out] -4*x**(5/2)*sqrt(sqrt(x) + 1)/(3*x**(5/2) + 3*x**2) + 8*x**(5/2)/(3*x**(5/2
) + 3*x**2) + 4*x**3*sqrt(sqrt(x) + 1)/(3*x**(5/2) + 3*x**2) - 8*x**2*sqrt(
sqrt(x) + 1)/(3*x**(5/2) + 3*x**2) + 8*x**2/(3*x**(5/2) + 3*x**2)
```

Giac [A] time = 1.06352, size = 26, normalized size = 0.9

$$\frac{4}{3}\left(\sqrt{x}+1\right)^{\frac{3}{2}}-4\sqrt{\sqrt{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/3*(sqrt(x) + 1)^(3/2) - 4*sqrt(sqrt(x) + 1)
```

3.231 $\int \frac{\sqrt{x}}{x+x^2} \, dx$

Optimal. Leaf size=8

$$2\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] 2*ArcTan[Sqrt[x]]
```

Rubi [A] time = 0.0049472, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {647, 63, 203}

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(x + x^2),x]

[Out] 2*ArcTan[Sqrt[x]]
```

Rule 647

```
Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] &&
IntegerQ[p]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{x}}{x+x^2}\,dx = \int \frac{1}{\sqrt{x}(1+x)}\,dx$$
$$= 2\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{x}\right)$$
$$= 2\tan^{-1}\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.0016925, size = 8, normalized size = 1.

$$2\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(x + x^2),x]
```

```
[Out] 2*ArcTan[Sqrt[x]]
```

Maple [A] time = 0.003, size = 7, normalized size = 0.9

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(x^2+x),x)
```

```
[Out] 2*arctan(x^(1/2))
```

Maxima [A] time = 1.41381, size = 8, normalized size = 1.

$$2\arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="maxima")
```

```
[Out] 2*arctan(sqrt(x))
```

Fricas [A] time = 2.29497, size = 26, normalized size = 3.25

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="fricas")
```

```
[Out] 2*arctan(sqrt(x))
```

Sympy [A] time = 0.29128, size = 7, normalized size = 0.88

$$2 \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(x**2+x),x)
```

```
[Out] 2*atan(sqrt(x))
```

Giac [A] time = 1.05882, size = 8, normalized size = 1.

$$2 \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(x^2+x),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(x))
```

3.232 $\int \frac{1+\sqrt{x}}{-1+\sqrt{x}} dx$

Optimal. Leaf size=21

$$x + 4\sqrt{x} + 4\log\left(1 - \sqrt{x}\right)$$

```
[Out] 4*Sqrt[x] + x + 4*Log[1 - Sqrt[x]]
```

Rubi [A] time = 0.0112809, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {376, 77}

$$x + 4\sqrt{x} + 4\log\left(1 - \sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
```

```
[Out] 4*Sqrt[x] + x + 4*Log[1 - Sqrt[x]]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f])))))
```

Rubi steps

$$\int \frac{1+\sqrt{x}}{-1+\sqrt{x}}\,dx = 2\,\text{Subst}\left(\int \frac{x(1+x)}{-1+x}\,dx, x, \sqrt{x}\right)$$
$$= 2\,\text{Subst}\left(\int \left(2+\frac{2}{-1+x}+x\right)dx, x, \sqrt{x}\right)$$
$$= 4\sqrt{x}+x+4\log\left(1-\sqrt{x}\right)$$

Mathematica [A] time = 0.0079634, size = 20, normalized size = 0.95

$$x+4\left(\sqrt{x}+\log\left(1-\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]
```

```
[Out] x + 4*(Sqrt[x] + Log[1 - Sqrt[x]])
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$x+4\sqrt{x}+4\ln\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)+1)/(x^(1/2)-1),x)
```

```
[Out] x+4*x^(1/2)+4*ln(x^(1/2)-1)
```

Maxima [A] time = 0.950811, size = 20, normalized size = 0.95

$$x+4\sqrt{x}+4\log\left(\sqrt{x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

Fricas [A] time = 2.19142, size = 49, normalized size = 2.33

$$x + 4\sqrt{x} + 4\log\left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="fricas")
```

```
[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)
```

Sympy [A] time = 0.136053, size = 17, normalized size = 0.81

$$4\sqrt{x} + x + 4\log\left(\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))/(-1+x**(1/2)),x)
```

```
[Out] 4*sqrt(x) + x + 4*log(sqrt(x) - 1)
```

Giac [A] time = 1.05136, size = 22, normalized size = 1.05

$$x + 4\sqrt{x} + 4\log\left(\left|\sqrt{x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="giac")
```

```
[Out] x + 4*sqrt(x) + 4*log(abs(sqrt(x) - 1))
```

3.233 $\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx$

Optimal. Leaf size=30

$$-3x^{2/3}-x-6\sqrt[3]{x}-6\log\left(1-\sqrt[3]{x}\right)$$

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

Rubi [A] time = 0.0170899, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {374, 376, 77}

$$-3x^{2/3}-x-6\sqrt[3]{x}-6\log\left(1-\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

Rule 374

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[x^(n*(p + q))*(b + a/x^n)^p*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[b*c - a*d, 0] && IntegersQ[p, q] && NegQ[n]
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))
^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
&& NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
```

```
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+\frac{1}{\sqrt[3]{x}}}{-1+\frac{1}{\sqrt[3]{x}}}\,dx &= \int \frac{1+\sqrt[3]{x}}{1-\sqrt[3]{x}}\,dx \\
&= 3\,\text{Subst}\left(\int \frac{x^2(1+x)}{1-x}\,dx, x, \sqrt[3]{x}\right) \\
&= 3\,\text{Subst}\left(\int \left(-2-\frac{2}{-1+x}-2x-x^2\right)dx, x, \sqrt[3]{x}\right) \\
&= -6\sqrt[3]{x}-3x^{2/3}-x-6\log\left(1-\sqrt[3]{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0128185, size = 30, normalized size = 1.

$$-3x^{2/3}-x-6\sqrt[3]{x}-6\log\left(1-\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]
```

```
[Out] -6*x^(1/3) - 3*x^(2/3) - x - 6*Log[1 - x^(1/3)]
```

Maple [A] time = 0.003, size = 23, normalized size = 0.8

$$-x-3\,x^{2/3}-6\,\sqrt[3]{x}-6\,\ln\left(-1+\sqrt[3]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/x^(1/3))/(-1+1/x^(1/3)),x)
```

```
[Out] -x-3*x^(2/3)-6*x^(1/3)-6*ln(-1+x^(1/3))
```

Maxima [A] time = 0.944397, size = 30, normalized size = 1.

$$-x - 3\,x^{\frac{2}{3}} - 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="maxima")
```

```
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

Fricas [A] time = 2.12879, size = 66, normalized size = 2.2

$$-x - 3\,x^{\frac{2}{3}} - 6\,x^{\frac{1}{3}} - 6\,\log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="fricas")
```

```
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(x^(1/3) - 1)
```

Sympy [A] time = 0.150008, size = 26, normalized size = 0.87

$$-3x^{\frac{2}{3}} - 6\sqrt[3]{x} - x - 6\log\left(\sqrt[3]{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x**(1/3))/(-1+1/x**(1/3)),x)
```

```
[Out] -3*x**(2/3) - 6*x**(1/3) - x - 6*log(x**(1/3) - 1)
```

Giac [A] time = 1.05281, size = 31, normalized size = 1.03

$$-x - 3\,x^{\frac{2}{3}} - 6\,x^{\frac{1}{3}} - 6\,\log\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="giac")
```

```
[Out] -x - 3*x^(2/3) - 6*x^(1/3) - 6*log(abs(x^(1/3) - 1))
```

3.234 $\int \frac{x^3}{\sqrt[3]{1+x^2}}\,dx$

Optimal. Leaf size=27

$$\frac{3}{10}\left(x^2+1\right)^{5/3}-\frac{3}{4}\left(x^2+1\right)^{2/3}$$

```
[Out] (-3*(1 + x^2)^(2/3))/4 + (3*(1 + x^2)^(5/3))/10
```

Rubi [A] time = 0.0103061, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{3}{10}\left(x^2+1\right)^{5/3}-\frac{3}{4}\left(x^2+1\right)^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x^2)^(1/3),x]
```

```
[Out] (-3*(1 + x^2)^(2/3))/4 + (3*(1 + x^2)^(5/3))/10
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3}{\sqrt[3]{1+x^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt[3]{1+x}}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(-\frac{1}{\sqrt[3]{1+x}} + (1+x)^{2/3}\right) dx, x, x^2\right)$$
$$= -\frac{3}{4}\left(1+x^2\right)^{2/3} + \frac{3}{10}\left(1+x^2\right)^{5/3}$$

Mathematica [A] time = 0.0074507, size = 20, normalized size = 0.74

$$\frac{3}{20}\left(x^2+1\right)^{2/3}\left(2x^2-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x^2)^(1/3),x]
```

```
[Out] (3*(1 + x^2)^(2/3)*(-3 + 2*x^2))/20
```

Maple [A] time = 0.003, size = 17, normalized size = 0.6

$$\frac{6\,x^2-9}{20}\left(x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(x^2+1)^(1/3),x)
```

```
[Out] 3/20*(x^2+1)^(2/3)*(2*x^2-3)
```

Maxima [A] time = 0.945974, size = 26, normalized size = 0.96

$$\frac{3}{10}\left(x^2+1\right)^{\frac{5}{3}} - \frac{3}{4}\left(x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/10*(x^2 + 1)^(5/3) - 3/4*(x^2 + 1)^(2/3)
```

Fricas [A] time = 2.09868, size = 46, normalized size = 1.7

$$\frac{3}{20}\left(2x^2-3\right)\left(x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1)^(1/3),x, algorithm="fricas")
```

```
[Out] 3/20*(2*x^2 - 3)*(x^2 + 1)^(2/3)
```

Sympy [A] time = 0.798445, size = 26, normalized size = 0.96

$$\frac{3x^2\left(x^2+1\right)^{\frac{2}{3}}}{10}-\frac{9\left(x^2+1\right)^{\frac{2}{3}}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(x**2+1)**(1/3),x)
```

```
[Out] 3*x**2*(x**2 + 1)**(2/3)/10 - 9*(x**2 + 1)**(2/3)/20
```

Giac [A] time = 1.06786, size = 26, normalized size = 0.96

$$\frac{3}{10}\left(x^2+1\right)^{\frac{5}{3}}-\frac{3}{4}\left(x^2+1\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(x^2+1)^(1/3),x, algorithm="giac")
```

```
[Out] 3/10*(x^2 + 1)^(5/3) - 3/4*(x^2 + 1)^(2/3)
```

3.235 $\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=201

$$x+6\sqrt[6]{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{5}$$

```
[Out] 6*x^(1/6) + x - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/S
qrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[
5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 -
 Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 + Sqrt
[5])*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rubi [A] time = 0.216354, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1584, 341, 302, 202, 634, 618, 204, 628, 31}

$$x+6\sqrt[6]{x}+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2\sqrt[3]{x}-\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2\sqrt[3]{x}+\sqrt{5}\sqrt[6]{x}+\sqrt[6]{x}+2\right)-\frac{3}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + x - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/S
qrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[
5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 -
 Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 + Sqrt
[5])*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 202

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r + s*Cos[(
(2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r
*Int[1/(r - s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}],
x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}}\,dx=\int \frac{x^{5/6}}{-1+x^{5/6}}\,dx$$
$$=6\,\text{Subst}\left(\int \frac{x^{10}}{-1+x^5}\,dx,x,\sqrt[6]{x}\right)$$
$$=6\,\text{Subst}\left(\int \left(1+x^5+\frac{1}{-1+x^5}\right)dx,x,\sqrt[6]{x}\right)$$
$$=6\sqrt[6]{x}+x+6\,\text{Subst}\left(\int \frac{1}{-1+x^5}\,dx,x,\sqrt[6]{x}\right)$$
$$=6\sqrt[6]{x}+x-\frac{6}{5}\,\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sqrt[6]{x}\right)-\frac{12}{5}\,\text{Subst}\left(\int \frac{1+\frac{1}{4}\left(1-\sqrt{5}\right)x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx,x,\sqrt[6]{x}\right)-\frac{12}{5}\,\text{Sub}$$
$$=6\sqrt[6]{x}+x+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{1}{10}\left(3\left(1-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{\frac{1}{2}\left(1-\sqrt{5}\right)+2x}{1+\frac{1}{2}\left(1-\sqrt{5}\right)x+x^2}\,dx,x,\sqrt[6]{x}\right)-\frac{1}{10}\left(3\left(5\right.\right.$$
$$=6\sqrt[6]{x}+x+\frac{6}{5}\log\left(1-\sqrt[6]{x}\right)-\frac{3}{10}\left(1-\sqrt{5}\right)\log\left(2+\sqrt[6]{x}-\sqrt{5}\sqrt[6]{x}+2\sqrt[3]{x}\right)-\frac{3}{10}\left(1+\sqrt{5}\right)\log\left(2+\sqrt[6]{x}+\right.$$
$$=6\sqrt[6]{x}+x-\frac{3}{5}\sqrt{2\left(5+\sqrt{5}\right)}\tan^{-1}\left(\frac{1-\sqrt{5}+4\sqrt[6]{x}}{\sqrt{2\left(5+\sqrt{5}\right)}}\right)-\frac{3}{5}\sqrt{2\left(5-\sqrt{5}\right)}\tan^{-1}\left(\frac{1}{2}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\left(1+\sqrt{5}\right.\right.$$

Mathematica [C] time = 0.0067922, size = 29, normalized size = 0.14

$$-6\sqrt[6]{x}\text{Hypergeometric2F1}\left(\frac{1}{5},1,\frac{6}{5},x^{5/6}\right)+x+6\sqrt[6]{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]
```

```
[Out] 6*x^(1/6) + x - 6*x^(1/6)*Hypergeometric2F1[1/5, 1, 6/5, x^(5/6)]
```

Maple [A] time = 0.013, size = 242, normalized size = 1.2

$$x+6\sqrt[6]{x}-\frac{3\sqrt{5}}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}+\sqrt[6]{x}\sqrt{5}\right)-\frac{3}{10}\ln\left(2+\sqrt[6]{x}+2\sqrt[3]{x}+\sqrt[6]{x}\sqrt{5}\right)-6\frac{1}{\sqrt{10-2\sqrt{5}}}\arctan\left(\frac{1+4\sqrt[6]{x}+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x)
```

```
[Out] x+6*x^(1/6)-3/10*ln(2+x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))*5^(1/2)-3/10*ln(2+
x^(1/6)+2*x^(1/3)+x^(1/6)*5^(1/2))-6/(10-2*5^(1/2))^(1/2)*arctan((1+4*x^(1/
6)+5^(1/2))/(10-2*5^(1/2))^(1/2))+6/5/(10-2*5^(1/2))^(1/2)*arctan((1+4*x^(1
/6)+5^(1/2))/(10-2*5^(1/2))^(1/2))*5^(1/2)-3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1
/6)*5^(1/2))+3/10*ln(2+x^(1/6)+2*x^(1/3)-x^(1/6)*5^(1/2))*5^(1/2)-6/(10+2*5
^(1/2))^(1/2)*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))-6/5/(10+2*
5^(1/2))^(1/2)*arctan((1+4*x^(1/6)-5^(1/2))/(10+2*5^(1/2))^(1/2))*5^(1/2)+6
/5*ln(-1+x^(1/6))
```

Maxima [B] time = 1.47636, size = 396, normalized size = 1.97

$$-\frac{3\sqrt{5}(-1)^{\frac{1}{5}}\left(\sqrt{5}-1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}}-\frac{3\sqrt{5}(-1)^{\frac{1}{5}}\left(\sqrt{5}+1\right)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{-2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}\right.}{5\sqrt{-2\sqrt{5}-10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) - 1)*log((sqrt(5)*(-1)^(1/5) + (-1)^(1/5)*
sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) - (-1)^(
1/5)*sqrt(2*sqrt(5) - 10) + (-1)^(1/5) - 4*x^(1/6)))/sqrt(2*sqrt(5) - 10) -
 3/5*sqrt(5)*(-1)^(1/5)*(sqrt(5) + 1)*log((sqrt(5)*(-1)^(1/5) - (-1)^(1/5)*
sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6))/(sqrt(5)*(-1)^(1/5) + (-1)^
(1/5)*sqrt(-2*sqrt(5) - 10) - (-1)^(1/5) + 4*x^(1/6)))/sqrt(-2*sqrt(5) - 10
) - 6/5*(-1)^(1/5)*log((-1)^(1/5) + x^(1/6)) + x - 3/5*(sqrt(5) + 3)*log(-x
^(1/6)*(sqrt(5)*(-1)^(1/5) + (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(
5)*(-1)^(4/5) + (-1)^(4/5)) - 3/5*(sqrt(5) - 3)*log(x^(1/6)*(sqrt(5)*(-1)^(
1/5) - (-1)^(1/5)) + 2*(-1)^(2/5) + 2*x^(1/3))/(sqrt(5)*(-1)^(4/5) - (-1)^(
```

```
4/5)) + 6*x^(1/6)
```

Fricas [B] time = 14.3074, size = 1858, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(3/2*sqrt(2)*sqrt(sqrt(5
) - 5) + 3/2*sqrt(5) + 6*x^(1/6) + 3/2) + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) -
 sqrt(5) - 1)*log(-3/2*sqrt(2)*sqrt(sqrt(5) - 5) + 3/2*sqrt(5) + 6*x^(1/6)
+ 3/2) + 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5)
+ 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5
) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 +
18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(-3*sqrt(5) + sqrt(
-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(
5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sq
rt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 1
8*sqrt(5) - 90) + 12*x^(1/6) + 3) + 1/10*(3*sqrt(5) + sqrt(-27/4*(sqrt(2)*s
qrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5
) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(
5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90)
- 3)*log(-3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^
2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5
) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sq
rt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) + 12*x^(1/6) + 3) + x + 6*x^(1/6
) + 6/5*log(x^(1/6) - 1)
```

Sympy [B] time = 36.2715, size = 561, normalized size = 2.79

$$-\frac{60\sqrt[6]{x}}{-10+10\sqrt{5}}+\frac{60\sqrt{5}\sqrt[6]{x}}{-10+10\sqrt{5}}-\frac{10x}{-10+10\sqrt{5}}+\frac{10\sqrt{5}x}{-10+10\sqrt{5}}-\frac{12\log\left(\sqrt[6]{x}-1\right)}{-10+10\sqrt{5}}+\frac{12\sqrt{5}\log\left(\sqrt[6]{x}-1\right)}{-10+10\sqrt{5}}-\frac{12\log\left(8\sqrt[6]{x}+8\right.}{-10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(-1/x**(1/3)+x**(1/2)),x)
```

```
[Out] -60*x**(1/6)/(-10 + 10*sqrt(5)) + 60*sqrt(5)*x**(1/6)/(-10 + 10*sqrt(5)) -
10*x/(-10 + 10*sqrt(5)) + 10*sqrt(5)*x/(-10 + 10*sqrt(5)) - 12*log(x**(1/6)
 - 1)/(-10 + 10*sqrt(5)) + 12*sqrt(5)*log(x**(1/6) - 1)/(-10 + 10*sqrt(5))
- 12*log(8*x**(1/6) + 8*sqrt(5)*x**(1/6) + 16*x**(1/3) + 16)/(-10 + 10*sqrt
(5)) - 6*sqrt(5)*log(-8*sqrt(5)*x**(1/6) + 8*x**(1/6) + 16*x**(1/3) + 16)/(
-10 + 10*sqrt(5)) + 18*log(-8*sqrt(5)*x**(1/6) + 8*x**(1/6) + 16*x**(1/3) +
 16)/(-10 + 10*sqrt(5)) - 6*sqrt(10)*sqrt(5 - sqrt(5))*atan(2*sqrt(2)*x**(1
/6)/sqrt(5 - sqrt(5)) + sqrt(2)/(2*sqrt(5 - sqrt(5))) + sqrt(10)/(2*sqrt(5
- sqrt(5))))/(-10 + 10*sqrt(5)) + 6*sqrt(2)*sqrt(5 - sqrt(5))*atan(2*sqrt(2
)*x**(1/6)/sqrt(5 - sqrt(5)) + sqrt(2)/(2*sqrt(5 - sqrt(5))) + sqrt(10)/(2*
sqrt(5 - sqrt(5))))/(-10 + 10*sqrt(5)) - 6*sqrt(10)*sqrt(sqrt(5) + 5)*atan(
2*sqrt(2)*x**(1/6)/sqrt(sqrt(5) + 5) - sqrt(10)/(2*sqrt(sqrt(5) + 5)) + sqr
t(2)/(2*sqrt(sqrt(5) + 5)))/(-10 + 10*sqrt(5)) + 6*sqrt(2)*sqrt(sqrt(5) + 5
)*atan(2*sqrt(2)*x**(1/6)/sqrt(sqrt(5) + 5) - sqrt(10)/(2*sqrt(sqrt(5) + 5)
) + sqrt(2)/(2*sqrt(sqrt(5) + 5)))/(-10 + 10*sqrt(5))
```

Giac [A] time = 1.40893, size = 189, normalized size = 0.94

$$-\frac{3}{5}\sqrt{2\sqrt{5}+10}\arctan\left(-\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right)-\frac{3}{5}\sqrt{-2\sqrt{5}+10}\arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right)-\frac{3}{10}\sqrt{5}\log\left(\frac{1}{2}x^{\frac{1}{6}}\left(\sqrt{5}+1\right)+x^{\frac{1}{3}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] -3/5*sqrt(2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5)
+ 10)) - 3/5*sqrt(-2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2
*sqrt(5) + 10)) - 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1)
 + 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + x + 6*x^(1/
6) - 3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1
/6) - 1))
```

3.236 $\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx$

Optimal. Leaf size=62

$$2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)+\frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] 2*Sqrt[x] + (4*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log[1 + x^(1/4
)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3
```

Rubi [A] time = 0.037231, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {1593, 341, 321, 292, 31, 634, 618, 204, 628}

$$2\sqrt{x}+\frac{4}{3}\log\left(\sqrt[4]{x}+1\right)-\frac{2}{3}\log\left(\sqrt{x}-\sqrt[4]{x}+1\right)+\frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/4) + Sqrt[x])^(-1),x]
```

```
[Out] 2*Sqrt[x] + (4*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log[1 + x^(1/4
)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}}\,dx = \int \frac{\sqrt[4]{x}}{1+x^{3/4}}\,dx$$
$$= 4\,\mathrm{Subst}\left(\int \frac{x^4}{1+x^3}\,dx, x, \sqrt[4]{x}\right)$$
$$= 2\sqrt{x} - 4\,\mathrm{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, \sqrt[4]{x}\right)$$
$$= 2\sqrt{x} + \frac{4}{3}\,\mathrm{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt[4]{x}\right) - \frac{4}{3}\,\mathrm{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right)$$
$$= 2\sqrt{x} + \frac{4}{3}\log\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\,\mathrm{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right) - 2\,\mathrm{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, \sqrt[4]{x}\right)$$
$$= 2\sqrt{x} + \frac{4}{3}\log\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right) + 4\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2\sqrt[4]{x}\right)$$
$$= 2\sqrt{x} + \frac{4\tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{4}{3}\log\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\log\left(1-\sqrt[4]{x}+\sqrt{x}\right)$$

Mathematica [C] time = 0.0066814, size = 24, normalized size = 0.39

$$-2\sqrt{x}\left(\mathrm{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-x^{3/4}\right)-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/4) + Sqrt[x])^(-1),x]
```

```
[Out] -2*Sqrt[x]*(-1 + Hypergeometric2F1[2/3, 1, 5/3, -x^(3/4)])
```

Maple [A] time = 0.007, size = 46, normalized size = 0.7

$$2\sqrt{x} + \frac{4}{3}\ln\left(1+\sqrt[4]{x}\right) - \frac{2}{3}\ln\left(1-\sqrt[4]{x}+\sqrt{x}\right) - \frac{4\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\sqrt[4]{x}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/4)+x^(1/2)),x)
```

```
[Out] 2*x^(1/2)+4/3*ln(1+x^(1/4))-2/3*ln(1-x^(1/4)+x^(1/2))-4/3*3^(1/2)*arctan(1/
3*(2*x^(1/4)-1)*3^(1/2))
```

Maxima [A] time = 1.44284, size = 61, normalized size = 0.98

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt
(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

Fricas [A] time = 2.15339, size = 167, normalized size = 2.69

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{4}}-\frac{1}{3}\sqrt{3}\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] -4/3*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/4) - 1/3*sqrt(3)) + 2*sqrt(x) - 2/3*lo
g(sqrt(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

Sympy [A] time = 0.701258, size = 68, normalized size = 1.1

$$2\sqrt{x}+\frac{4\log\left(\sqrt[4]{x}+1\right)}{3}-\frac{2\log\left(-4\sqrt[4]{x}+4\sqrt{x}+4\right)}{3}-\frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[4]{x}}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/4)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x) + 4*log(x**(1/4) + 1)/3 - 2*log(-4*x**(1/4) + 4*sqrt(x) + 4)/3 -
4*sqrt(3)*atan(2*sqrt(3)*x**(1/4)/3 - sqrt(3)/3)/3
```

Giac [A] time = 1.05391, size = 61, normalized size = 0.98

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/4)+x^(1/2)),x, algorithm="giac")
```

```
[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt
(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)
```

3.237 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx$

Optimal. Leaf size=130

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{}$$

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5 - 2*Sqrt[
x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*
x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^(7/6))/7 + (4*x^(5/4))/5 - 12*
Log[1 + x^(1/12)]
```

Rubi [A] time = 0.0462393, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 43}

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12\sqrt[12]{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5 - 2*Sqrt[
x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*
x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^(7/6))/7 + (4*x^(5/4))/5 - 12*
Log[1 + x^(1/12)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}}\,dx = \int \frac{\sqrt[3]{x}}{1+\sqrt[12]{x}}\,dx$$

$$= 12\,\text{Subst}\left(\int \frac{x^{15}}{1+x}\,dx, x, \sqrt[12]{x}\right)$$

$$= 12\,\text{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2-x^3+x^4-x^5+x^6-x^7+x^8-x^9+x^{10}-x^{11}+x^{12}-x^{13}+x^{14}\right) d$$

$$= 12\sqrt[12]{x}-6\sqrt[6]{x}+4\sqrt[4]{x}-3\sqrt[3]{x}+\frac{12x^{5/12}}{5}-2\sqrt{x}+\frac{12x^{7/12}}{7}-\frac{3x^{2/3}}{2}+\frac{4x^{3/4}}{3}-\frac{6x^{5/6}}{5}+\frac{12x^{11/12}}{11}-x+\frac{12x}{1}$$

Mathematica [A] time = 0.0366835, size = 130, normalized size = 1.

$$\frac{4x^{5/4}}{5}-\frac{6x^{7/6}}{7}+\frac{12x^{13/12}}{13}+\frac{12x^{11/12}}{11}-\frac{6x^{5/6}}{5}+\frac{4x^{3/4}}{3}-\frac{3x^{2/3}}{2}+\frac{12x^{7/12}}{7}+\frac{12x^{5/12}}{5}-x-2\sqrt{x}-3\sqrt[3]{x}+4\sqrt[4]{x}-6\sqrt[6]{x}+12$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/3) + x^(-1/4))^(-1),x]
```

```
[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5 - 2*Sqrt[
x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*
x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^(7/6))/7 + (4*x^(5/4))/5 - 12*
Log[1 + x^(1/12)]
```

Maple [A] time = 0.004, size = 83, normalized size = 0.6

$$12\,x^{1/12}-6\,\sqrt[6]{x}+4\,\sqrt[4]{x}-3\,\sqrt[3]{x}+\frac{12}{5}x^{\frac{5}{12}}+\frac{12}{7}x^{\frac{7}{12}}-\frac{3}{2}x^{\frac{2}{3}}+\frac{4}{3}x^{\frac{3}{4}}-\frac{6}{5}x^{\frac{5}{6}}+\frac{12}{11}x^{\frac{11}{12}}-x+\frac{12}{13}x^{\frac{13}{12}}-\frac{6}{7}x^{\frac{7}{6}}+\frac{4}{5}x^{\frac{5}{4}}-12\,\ln\left(1+x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/3)+1/x^(1/4)),x)
```

```
[Out] 12*x^(1/12)-6*x^(1/6)+4*x^(1/4)-3*x^(1/3)+12/5*x^(5/12)+12/7*x^(7/12)-3/2*x
^(2/3)+4/3*x^(3/4)-6/5*x^(5/6)+12/11*x^(11/12)-x+12/13*x^(13/12)-6/7*x^(7/6
)+4/5*x^(5/4)-12*ln(1+x^(1/12))-2*x^(1/2)
```

Maxima [A] time = 0.946955, size = 111, normalized size = 0.85

$$\frac{4}{5}\,x^{\frac{5}{4}} - \frac{6}{7}\,x^{\frac{7}{6}} + \frac{12}{13}\,x^{\frac{13}{12}} - x + \frac{12}{11}\,x^{\frac{11}{12}} - \frac{6}{5}\,x^{\frac{5}{6}} + \frac{4}{3}\,x^{\frac{3}{4}} - \frac{3}{2}\,x^{\frac{2}{3}} + \frac{12}{7}\,x^{\frac{7}{12}} - 2\,\sqrt{x} + \frac{12}{5}\,x^{\frac{5}{12}} - 3\,x^{\frac{1}{3}} + 4\,x^{\frac{1}{4}} - 6\,x^{\frac{1}{6}} + 12\,x^{\frac{1}{12}} - 12\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="maxima")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12
) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) + 1)
```

Fricas [A] time = 2.12897, size = 286, normalized size = 2.2

$$\frac{4}{5}\,(x+5)x^{\frac{1}{4}} - \frac{6}{7}\,(x+7)x^{\frac{1}{6}} + \frac{12}{13}\,(x+13)x^{\frac{1}{12}} - x + \frac{12}{11}\,x^{\frac{11}{12}} - \frac{6}{5}\,x^{\frac{5}{6}} + \frac{4}{3}\,x^{\frac{3}{4}} - \frac{3}{2}\,x^{\frac{2}{3}} + \frac{12}{7}\,x^{\frac{7}{12}} - 2\,\sqrt{x} + \frac{12}{5}\,x^{\frac{5}{12}} - 3\,x^{\frac{1}{3}} - 12\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="fricas")
```

```
[Out] 4/5*(x + 5)*x^(1/4) - 6/7*(x + 7)*x^(1/6) + 12/13*(x + 13)*x^(1/12) - x + 1
2/11*x^(11/12) - 6/5*x^(5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) -
2*sqrt(x) + 12/5*x^(5/12) - 3*x^(1/3) - 12*log(x^(1/12) + 1)
```

Sympy [A] time = 2.02183, size = 121, normalized size = 0.93

$$\frac{12x^{\frac{13}{12}}}{13} + \frac{12x^{\frac{11}{12}}}{11} + \frac{12x^{\frac{7}{12}}}{7} + \frac{12x^{\frac{5}{12}}}{5} + 12\,\sqrt[12]{x} - \frac{6x^{\frac{7}{6}}}{7} - \frac{6x^{\frac{5}{6}}}{5} - 6\sqrt[6]{x} + \frac{4x^{\frac{5}{4}}}{5} + \frac{4x^{\frac{3}{4}}}{3} + 4\sqrt[4]{x} - \frac{3x^{\frac{2}{3}}}{2} - 3\sqrt[3]{x} - 2\sqrt{x} - x - 12\log\big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/3)+1/x**(1/4)),x)
```

```
[Out] 12*x**(13/12)/13 + 12*x**(11/12)/11 + 12*x**(7/12)/7 + 12*x**(5/12)/5 + 12*
x**(1/12) - 6*x**(7/6)/7 - 6*x**(5/6)/5 - 6*x**(1/6) + 4*x**(5/4)/5 + 4*x**
(3/4)/3 + 4*x**(1/4) - 3*x**(2/3)/2 - 3*x**(1/3) - 2*sqrt(x) - x - 12*log(x
**(1/12) + 1)
```

Giac [A] time = 1.06202, size = 111, normalized size = 0.85

$$\frac{4}{5}x^{\frac{5}{4}} - \frac{6}{7}x^{\frac{7}{6}} + \frac{12}{13}x^{\frac{13}{12}} - x + \frac{12}{11}x^{\frac{11}{12}} - \frac{6}{5}x^{\frac{5}{6}} + \frac{4}{3}x^{\frac{3}{4}} - \frac{3}{2}x^{\frac{2}{3}} + \frac{12}{7}x^{\frac{7}{12}} - 2\sqrt{x} + \frac{12}{5}x^{\frac{5}{12}} - 3x^{\frac{1}{3}} + 4x^{\frac{1}{4}} - 6x^{\frac{1}{6}} + 12x^{\frac{1}{12}} - 12$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="giac")
```

```
[Out] 4/5*x^(5/4) - 6/7*x^(7/6) + 12/13*x^(13/12) - x + 12/11*x^(11/12) - 6/5*x^(
5/6) + 4/3*x^(3/4) - 3/2*x^(2/3) + 12/7*x^(7/12) - 2*sqrt(x) + 12/5*x^(5/12
) - 3*x^(1/3) + 4*x^(1/4) - 6*x^(1/6) + 12*x^(1/12) - 12*log(x^(1/12) + 1)
```

3.238 $\int \sqrt{\frac{1-x}{x}}\,dx$

Optimal. Leaf size=24

$$\sqrt{\frac{1}{x}-1}x-\tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

```
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Rubi [A] time = 0.0088508, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1972, 242, 47, 63, 203}

$$\sqrt{\frac{1}{x}-1}x-\tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 - x)/x],x]
```

```
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] &&  !BinomialMatchQ[u, x]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{1-x}{x}}\,dx &= \int \sqrt{-1+\frac{1}{x}}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{\sqrt{-1+x}}{x^2}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{-1+\frac{1}{x}}x - \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x}x}\,dx, x, \frac{1}{x}\right) \\
&= \sqrt{-1+\frac{1}{x}}x - \operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+\frac{1}{x}}\right) \\
&= \sqrt{-1+\frac{1}{x}}x - \tan^{-1}\left(\sqrt{-1+\frac{1}{x}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0075932, size = 24, normalized size = 1.

$$
\sqrt{\frac{1}{x}-1}x - \tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 - x)/x],x]
```

```
[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]
```

Maple [A] time = 0.007, size = 40, normalized size = 1.7

$$\frac{x}{2}\sqrt{-\frac{-1+x}{x}}\left(2\sqrt{-x^2+x}+\arcsin(2x-1)\right)\frac{1}{\sqrt{-x(-1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1-x)/x)^(1/2),x)
```

```
[Out] 1/2*(-(-1+x)/x)^(1/2)*x*(2*(-x^2+x)^(1/2)+arcsin(2*x-1))/(-x*(-1+x))^(1/2)
```

Maxima [A] time = 1.42513, size = 50, normalized size = 2.08

$$-\frac{\sqrt{-\frac{x-1}{x}}}{\frac{x-1}{x}-1}-\arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-(x - 1)/x)/((x - 1)/x - 1) - arctan(sqrt(-(x - 1)/x))
```

Fricas [A] time = 2.21482, size = 63, normalized size = 2.62

$$x\sqrt{-\frac{x-1}{x}}-\arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="fricas")
```

```
[Out] x*sqrt(-(x - 1)/x) - arctan(sqrt(-(x - 1)/x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{1-x}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)**(1/2),x)
```

```
[Out] Integral(sqrt((1 - x)/x), x)
```

Giac [A] time = 1.06681, size = 38, normalized size = 1.58

$$\frac{1}{4}\,\pi \mathrm{sgn}\left(x\right) + \frac{1}{2}\,\arcsin\left(2\,x - 1\right)\mathrm{sgn}\left(x\right) + \sqrt{-x^2 + x}\mathrm{sgn}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*pi*sgn(x) + 1/2*arcsin(2*x - 1)*sgn(x) + sqrt(-x^2 + x)*sgn(x)
```

3.239 $\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx$

Optimal. Leaf size=11

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Rubi [A] time = 0.0208168, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3258, 615}

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Rule 3258

```
Int[cos[(d_.) + (e_.)*(x_)]^(m_.)*((a_.) + (b_.)*((f_.)*sin[(d_.) + (e_.)*(
x_)])^(n_.) + (c_.)*((f_.)*sin[(d_.) + (e_.)*(x_)])^(n2_.))^(p_.), x_Symbol
] :> Module[{g = FreeFactors[Sin[d + e*x], x]}, Dist[g/e, Subst[Int[(1 - g^
2*x^2)^((m - 1)/2)*(a + b*(f*g*x)^n + c*(f*g*x)^(2*n))^p, x], x, Sin[d + e*
x]/g], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && EqQ[n2, 2*n] && Integer
Q[(m - 1)/2]
```

Rule 615

```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[Log[x]/b, x] - Simp
[Log[RemoveContent[b + c*x, x]]/b, x] /; FreeQ[{b, c}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\sin(x)+\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x+x^2}\,dx, x, \sin(x)\right)\\ &= \log(\sin(x)) - \log(1+\sin(x))\end{aligned}$$

Mathematica [A] time = 0.0072482, size = 11, normalized size = 1.

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(Sin[x] + Sin[x]^2),x]
```

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Maple [A] time = 0.031, size = 12, normalized size = 1.1

$$\ln(\sin(x)) - \ln(1 + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sin(x)+sin(x)^2),x)
```

```
[Out] ln(sin(x))-ln(1+sin(x))
```

Maxima [A] time = 0.938084, size = 15, normalized size = 1.36

$$-\log(\sin(x) + 1) + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="maxima")
```

```
[Out] -log(sin(x) + 1) + log(sin(x))
```

Fricas [A] time = 2.33544, size = 47, normalized size = 4.27

$$\log\left(\frac{1}{2}\sin(x)\right) - \log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="fricas")
```

```
[Out] log(1/2*sin(x)) - log(sin(x) + 1)
```

Sympy [A] time = 0.195589, size = 10, normalized size = 0.91

$$-\log(\sin(x)+1)+\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)**2),x)
```

```
[Out] -log(sin(x) + 1) + log(sin(x))
```

Giac [A] time = 1.05639, size = 16, normalized size = 1.45

$$-\log(\sin(x)+1)+\log(|\sin(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sin(x)+sin(x)^2),x, algorithm="giac")
```

```
[Out] -log(sin(x) + 1) + log(abs(sin(x)))
```

3.240 $\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx$

Optimal. Leaf size=17

$$2\log\left(e^x+2\right)-\log\left(e^x+1\right)$$

```
[Out] -Log[1 + E^x] + 2*Log[2 + E^x]
```

Rubi [A] time = 0.0318288, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {2282, 632, 31}

$$2\log\left(e^x+2\right)-\log\left(e^x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(2 + 3*E^x + E^(2*x)),x]
```

```
[Out] -Log[1 + E^x] + 2*Log[2 + E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{e^{2x}}{2+3e^x+e^{2x}}\,dx &= \text{Subst}\left(\int \frac{x}{2+3x+x^2}\,dx, x, e^x\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{2+x}\,dx, x, e^x\right) - \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, e^x\right)\\ &= -\log\left(1+e^x\right) + 2\log\left(2+e^x\right)\end{aligned}$$

Mathematica [A] time = 0.0134487, size = 17, normalized size = 1.

$$2\log\left(e^x+2\right) - \log\left(e^x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(2 + 3*E^x + E^(2*x)),x]
```

```
[Out] -Log[1 + E^x] + 2*Log[2 + E^x]
```

Maple [A] time = 0.007, size = 16, normalized size = 0.9

$$-\ln\left(1+e^{x}\right) + 2\,\ln\left(2+e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(2+3*exp(x)+exp(2*x)),x)
```

```
[Out] -ln(1+exp(x))+2*ln(2+exp(x))
```

Maxima [A] time = 0.949909, size = 20, normalized size = 1.18

$$2\,\log\left(e^x+2\right) - \log\left(e^x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x, algorithm="maxima")
```

```
[Out] 2*log(e^x + 2) - log(e^x + 1)
```

Fricas [A] time = 2.25475, size = 42, normalized size = 2.47

$$2\log\left(e^{x}+2\right)-\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x, algorithm="fricas")
```

```
[Out] 2*log(e^x + 2) - log(e^x + 1)
```

Sympy [A] time = 0.112225, size = 14, normalized size = 0.82

$$-\log\left(e^{x}+1\right)+2\log\left(e^{x}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x)
```

```
[Out] -log(exp(x) + 1) + 2*log(exp(x) + 2)
```

Giac [A] time = 1.0665, size = 20, normalized size = 1.18

$$2\log\left(e^{x}+2\right)-\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(2+3*exp(x)+exp(2*x)),x, algorithm="giac")
```

```
[Out] 2*log(e^x + 2) - log(e^x + 1)
```

3.241 $\int \frac{1}{\sqrt{1+e^x}} \, dx$

Optimal. Leaf size=12

$$-2\tanh^{-1}\left(\sqrt{e^x+1}\right)$$

```
[Out] -2*ArcTanh[Sqrt[1 + E^x]]
```

Rubi [A] time = 0.0080599, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2282, 63, 207}

$$-2\tanh^{-1}\left(\sqrt{e^x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + E^x],x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + E^x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+e^x}}\,dx &= \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, e^x\right)\\ &= 2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+e^x}\right)\\ &= -2\tanh^{-1}\left(\sqrt{1+e^x}\right)\end{aligned}$$

Mathematica [A] time = 0.0032689, size = 12, normalized size = 1.

$$-2\tanh^{-1}\left(\sqrt{e^x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + E^x],x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + E^x]]
```

Maple [A] time = 0.005, size = 10, normalized size = 0.8

$$-2\,\text{Artanh}\left(\sqrt{1+e^x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+exp(x))^(1/2),x)
```

```
[Out] -2*arctanh((1+exp(x))^(1/2))
```

Maxima [B] time = 0.935081, size = 28, normalized size = 2.33

$$-\log\left(\sqrt{e^x+1}+1\right)+\log\left(\sqrt{e^x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -log(sqrt(e^x + 1) + 1) + log(sqrt(e^x + 1) - 1)
```

Fricas [B] time = 2.06284, size = 68, normalized size = 5.67

$$-\log\left(\sqrt{e^x+1}+1\right)+\log\left(\sqrt{e^x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -log(sqrt(e^x + 1) + 1) + log(sqrt(e^x + 1) - 1)
```

Sympy [B] time = 1.04146, size = 26, normalized size = 2.17

$$\log\left(-1+\frac{1}{\sqrt{e^x+1}}\right)-\log\left(1+\frac{1}{\sqrt{e^x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x))**(1/2),x)
```

```
[Out] log(-1 + 1/sqrt(exp(x) + 1)) - log(1 + 1/sqrt(exp(x) + 1))
```

Giac [B] time = 1.05047, size = 28, normalized size = 2.33

$$-\log\left(\sqrt{e^x+1}+1\right)+\log\left(\sqrt{e^x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x))^(1/2),x, algorithm="giac")
```

```
[Out] -log(sqrt(e^x + 1) + 1) + log(sqrt(e^x + 1) - 1)
```

3.242 $\int \sqrt{1-e^x}\,dx$

Optimal. Leaf size=28

$$2\sqrt{1-e^x}-2\tanh^{-1}\left(\sqrt{1-e^x}\right)$$

```
[Out] 2*Sqrt[1 - E^x] - 2*ArcTanh[Sqrt[1 - E^x]]
```

Rubi [A] time = 0.0129139, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2282, 50, 63, 206}

$$2\sqrt{1-e^x}-2\tanh^{-1}\left(\sqrt{1-e^x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - E^x],x]
```

```
[Out] 2*Sqrt[1 - E^x] - 2*ArcTanh[Sqrt[1 - E^x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-e^x}\,dx &= \text{Subst}\left(\int \frac{\sqrt{1-x}}{x}\,dx, x, e^x\right)\\ &= 2\sqrt{1-e^x} + \text{Subst}\left(\int \frac{1}{\sqrt{1-x}x}\,dx, x, e^x\right)\\ &= 2\sqrt{1-e^x} - 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-e^x}\right)\\ &= 2\sqrt{1-e^x} - 2\tanh^{-1}\left(\sqrt{1-e^x}\right)\end{aligned}$$

Mathematica [A] time = 0.0087402, size = 28, normalized size = 1.

$$2\sqrt{1-e^x} - 2\tanh^{-1}\left(\sqrt{1-e^x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - E^x],x]
```

```
[Out] 2*Sqrt[1 - E^x] - 2*ArcTanh[Sqrt[1 - E^x]]
```

Maple [A] time = 0.002, size = 36, normalized size = 1.3

$$2\sqrt{1-e^x} + \ln\left(-1+\sqrt{1-e^x}\right) - \ln\left(1+\sqrt{1-e^x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-exp(x))^(1/2),x)
```

```
[Out] 2*(1-exp(x))^(1/2)+ln(-1+(1-exp(x))^(1/2))-ln(1+(1-exp(x))^(1/2))
```

Maxima [A] time = 0.951086, size = 47, normalized size = 1.68

$$2\sqrt{-e^{x}+1}-\log\left(\sqrt{-e^{x}+1}+1\right)+\log\left(\sqrt{-e^{x}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-exp(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(-e^x + 1) - log(sqrt(-e^x + 1) + 1) + log(sqrt(-e^x + 1) - 1)
```

Fricas [A] time = 2.02845, size = 95, normalized size = 3.39

$$2\sqrt{-e^{x}+1}-\log\left(\sqrt{-e^{x}+1}+1\right)+\log\left(\sqrt{-e^{x}+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-exp(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-e^x + 1) - log(sqrt(-e^x + 1) + 1) + log(sqrt(-e^x + 1) - 1)
```

Sympy [A] time = 1.24318, size = 32, normalized size = 1.14

$$2\sqrt{1-e^{x}}+\log\left(\sqrt{1-e^{x}}-1\right)-\log\left(\sqrt{1-e^{x}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-exp(x))**(1/2),x)
```

```
[Out] 2*sqrt(1 - exp(x)) + log(sqrt(1 - exp(x)) - 1) - log(sqrt(1 - exp(x)) + 1)
```

<u>Giac</u> [A] time = 1.06754, size = 50, normalized size = 1.79

$$2\sqrt{-e^x+1}-\log\left(\sqrt{-e^x+1}+1\right)+\log\left(-\sqrt{-e^x+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-exp(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(-e^x + 1) - log(sqrt(-e^x + 1) + 1) + log(-sqrt(-e^x + 1) + 1)
```

3.243 $\int \frac{1}{3-5\sin(x)}\,dx$

Optimal. Leaf size=43

$$\frac{1}{4}\log\left(3\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{4}\log\left(\cos\left(\frac{x}{2}\right)-3\sin\left(\frac{x}{2}\right)\right)$$

```
[Out] -Log[Cos[x/2] - 3*Sin[x/2]]/4 + Log[3*Cos[x/2] - Sin[x/2]]/4
```

Rubi [A] time = 0.0187309, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2660, 616, 31}

$$\frac{1}{4}\log\left(3\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{4}\log\left(\cos\left(\frac{x}{2}\right)-3\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 5*Sin[x])^(-1),x]
```

```
[Out] -Log[Cos[x/2] - 3*Sin[x/2]]/4 + Log[3*Cos[x/2] - Sin[x/2]]/4
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{3-5\sin(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{3-10x+3x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{-9+3x}\,dx, x, \tan\left(\frac{x}{2}\right)\right) - \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{-1+3x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\
&= -\frac{1}{4}\log\left(1-3\tan\left(\frac{x}{2}\right)\right) + \frac{1}{4}\log\left(3-\tan\left(\frac{x}{2}\right)\right)
\end{aligned}
$$

Mathematica [A] time = 0.0113205, size = 43, normalized size = 1.

$$
\frac{1}{4}\log\left(3\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{4}\log\left(\cos\left(\frac{x}{2}\right)-3\sin\left(\frac{x}{2}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 5*Sin[x])^(-1),x]
```

```
[Out] -Log[Cos[x/2] - 3*Sin[x/2]]/4 + Log[3*Cos[x/2] - Sin[x/2]]/4
```

Maple [A] time = 0.011, size = 22, normalized size = 0.5

$$
-\frac{1}{4}\ln\left(3\,\tan\left(x/2\right)-1\right)+\frac{1}{4}\ln\left(\tan\left(\frac{x}{2}\right)-3\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3-5*sin(x)),x)
```

```
[Out] -1/4*ln(3*tan(1/2*x)-1)+1/4*ln(tan(1/2*x)-3)
```

Maxima [A] time = 0.927992, size = 41, normalized size = 0.95

$$
-\frac{1}{4}\,\log\left(\frac{3\,\sin\left(x\right)}{\cos\left(x\right)+1}-1\right)+\frac{1}{4}\,\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}-3\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-5*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/4*log(3*sin(x)/(cos(x) + 1) - 1) + 1/4*log(sin(x)/(cos(x) + 1) - 3)
```

Fricas [A] time = 2.30637, size = 95, normalized size = 2.21

$$\frac{1}{8}\log\left(4\cos\left(x\right) - 3\sin\left(x\right) + 5\right) - \frac{1}{8}\log\left(-4\cos\left(x\right) - 3\sin\left(x\right) + 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-5*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/8*log(4*cos(x) - 3*sin(x) + 5) - 1/8*log(-4*cos(x) - 3*sin(x) + 5)
```

Sympy [A] time = 0.20946, size = 20, normalized size = 0.47

$$\frac{\log\left(\tan\left(\frac{x}{2}\right) - 3\right)}{4} - \frac{\log\left(\tan\left(\frac{x}{2}\right) - \frac{1}{3}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-5*sin(x)),x)
```

```
[Out] log(tan(x/2) - 3)/4 - log(tan(x/2) - 1/3)/4
```

Giac [A] time = 1.0834, size = 31, normalized size = 0.72

$$-\frac{1}{4}\log\left(\left|3\tan\left(\frac{1}{2}x\right) - 1\right|\right) + \frac{1}{4}\log\left(\left|\tan\left(\frac{1}{2}x\right) - 3\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-5*sin(x)),x, algorithm="giac")
```

```
[Out] -1/4*log(abs(3*tan(1/2*x) - 1)) + 1/4*log(abs(tan(1/2*x) - 3))
```

3.244 $\int \frac{1}{\cos(x)+\sin(x)} dx$

Optimal. Leaf size=21

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2])
```

Rubi [A] time = 0.0096234, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3074, 206}

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Sin[x])^(-1),x]

[Out] -(ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\cos(x)+\sin(x)}\,dx = -\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \cos(x)-\sin(x)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Mathematica [C] time = 0.0202798, size = 24, normalized size = 1.14

$$(-1-i)(-1)^{3/4}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])^(-1),x]
```

```
[Out] (-1 - I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]]
```

Maple [A] time = 0.019, size = 19, normalized size = 0.9

$$\sqrt{2}\operatorname{Artanh}\left(\frac{\sqrt{2}}{4}\left(2\tan\left(x/2\right)-2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)+sin(x)),x)
```

```
[Out] 2^(1/2)*arctanh(1/4*(2*tan(1/2*x)-2)*2^(1/2))
```

Maxima [B] time = 1.4082, size = 53, normalized size = 2.52

$$-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}+1}{\sqrt{2}+\frac{\sin(x)}{\cos(x)+1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) + 1)/(sqrt(2) + sin(x)/(co
s(x) + 1) - 1))
```

Fricas [B] time = 2.22559, size = 126, normalized size = 6.

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}-\cos\left(x\right)\right)\sin\left(x\right)-2\sqrt{2}\cos\left(x\right)+3}{2\cos\left(x\right)\sin\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*(sqrt(2) - cos(x))*sin(x) - 2*sqrt(2)*cos(x) + 3)/(2*cos
(x)*sin(x) + 1))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.12603, size = 50, normalized size = 2.38

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}{\left|2\sqrt{2}+2\tan\left(\frac{1}{2}x\right)-2\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) - 2)/abs(2*sqrt(2) + 2*tan(1
/2*x) - 2))
```

3.245 $\int \frac{1}{1-\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=11

$$-\log\left(\cot\left(\frac{x}{2}\right)+1\right)$$

```
[Out] -Log[1 + Cot[x/2]]
```

Rubi [A] time = 0.0107476, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3121, 31}

$$-\log\left(\cot\left(\frac{x}{2}\right)+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x] + Sin[x])^(-1),x]
```

```
[Out] -Log[1 + Cot[x/2]]
```

Rule 3121

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Cot[(d + e*x)/2], x]}, -Dist[f/e
, Subst[Int[1/(a + c*f*x), x], x, Cot[(d + e*x)/2]/f], x]] /; FreeQ[{a, b,
c, d, e}, x] && EqQ[a + b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\cos(x)+\sin(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cot\left(\frac{x}{2}\right)\right)\\ &= -\log\left(1+\cot\left(\frac{x}{2}\right)\right)\end{aligned}$$

Mathematica [B] time = 0.0143862, size = 24, normalized size = 2.18

$$\log \left(\sin \left(\frac{x}{2}\right)\right)-\log \left(\sin \left(\frac{x}{2}\right)+\cos \left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x] + Sin[x])^(-1),x]
```

```
[Out] Log[Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]]
```

Maple [A] time = 0.027, size = 16, normalized size = 1.5

$$\ln \left(\tan \left(\frac{x}{2}\right)\right)-\ln \left(1+\tan \left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)+sin(x)),x)
```

```
[Out] ln(tan(1/2*x))-ln(1+tan(1/2*x))
```

Maxima [B] time = 0.9437, size = 34, normalized size = 3.09

$$-\log \left(\frac{\sin (x)}{\cos (x)+1}+1\right)+\log \left(\frac{\sin (x)}{\cos (x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] -log(sin(x)/(cos(x) + 1) + 1) + log(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 2.25173, size = 68, normalized size = 6.18

$$\frac{1}{2} \log \left(-\frac{1}{2} \cos (x)+\frac{1}{2}\right)-\frac{1}{2} \log (\sin (x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-1/2*cos(x) + 1/2) - 1/2*log(sin(x) + 1)
```

Sympy [A] time = 0.237113, size = 14, normalized size = 1.27

$$-\log\left(\tan\left(\frac{x}{2}\right)+1\right)+\log\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)+sin(x)),x)
```

```
[Out] -log(tan(x/2) + 1) + log(tan(x/2))
```

Giac [A] time = 1.09904, size = 23, normalized size = 2.09

$$-\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)+\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] -log(abs(tan(1/2*x) + 1)) + log(abs(tan(1/2*x)))
```

3.246 $\int \frac{1}{4\cos(x)+3\sin(x)}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{5}\tanh^{-1}\left(\frac{1}{5}(3\cos(x)-4\sin(x))\right)$$

```
[Out] -ArcTanh[(3*Cos[x] - 4*Sin[x])/5]/5
```

Rubi [A] time = 0.0121786, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3074, 206}

$$-\frac{1}{5}\tanh^{-1}\left(\frac{1}{5}(3\cos(x)-4\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4*Cos[x] + 3*Sin[x])^(-1),x]
```

```
[Out] -ArcTanh[(3*Cos[x] - 4*Sin[x])/5]/5
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4\cos(x)+3\sin(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{25-x^2}\,dx, x, 3\cos(x)-4\sin(x)\right)\\ &= -\frac{1}{5}\tanh^{-1}\left(\frac{1}{5}(3\cos(x)-4\sin(x))\right)\end{aligned}$$

Mathematica [B] time = 0.0161083, size = 43, normalized size = 2.39

$$\frac{1}{5}\log\left(2\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\frac{1}{5}\log\left(2\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4*Cos[x] + 3*Sin[x])^(-1),x]
```

```
[Out] -Log[2*Cos[x/2] - Sin[x/2]]/5 + Log[Cos[x/2] + 2*Sin[x/2]]/5
```

Maple [A] time = 0.031, size = 22, normalized size = 1.2

$$\frac{1}{5}\ln(2\tan(x/2)+1)-\frac{1}{5}\ln\left(\tan\left(\frac{x}{2}\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*cos(x)+3*sin(x)),x)
```

```
[Out] 1/5*ln(2*tan(1/2*x)+1)-1/5*ln(tan(1/2*x)-2)
```

Maxima [B] time = 0.943065, size = 41, normalized size = 2.28

$$\frac{1}{5}\log\left(\frac{2\sin(x)}{\cos(x)+1}+1\right)-\frac{1}{5}\log\left(\frac{\sin(x)}{\cos(x)+1}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(x)+3*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/5*log(2*sin(x)/(cos(x) + 1) + 1) - 1/5*log(sin(x)/(cos(x) + 1) - 2)
```

Fricas [B] time = 2.22224, size = 109, normalized size = 6.06

$$-\frac{1}{10}\log\left(\frac{3}{2}\cos(x)-2\sin(x)+\frac{5}{2}\right)+\frac{1}{10}\log\left(-\frac{3}{2}\cos(x)+2\sin(x)+\frac{5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(x)+3*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/10*log(3/2*cos(x) - 2*sin(x) + 5/2) + 1/10*log(-3/2*cos(x) + 2*sin(x) +
5/2)
```

Sympy [A] time = 0.248526, size = 20, normalized size = 1.11

$$-\frac{\log\left(\tan\left(\frac{x}{2}\right)-2\right)}{5}+\frac{\log\left(\tan\left(\frac{x}{2}\right)+\frac{1}{2}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(x)+3*sin(x)),x)
```

```
[Out] -log(tan(x/2) - 2)/5 + log(tan(x/2) + 1/2)/5
```

Giac [A] time = 1.08077, size = 31, normalized size = 1.72

$$\frac{1}{5}\log\left(\left|2\tan\left(\frac{1}{2}x\right)+1\right|\right)-\frac{1}{5}\log\left(\left|\tan\left(\frac{1}{2}x\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*cos(x)+3*sin(x)),x, algorithm="giac")
```

```
[Out] 1/5*log(abs(2*tan(1/2*x) + 1)) - 1/5*log(abs(tan(1/2*x) - 2))
```

3.247 $\int \frac{1}{\sin(x)+\tan(x)}\,dx$

Optimal. Leaf size=24

$$-\frac{1}{2}\csc^2(x)-\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\cot(x)\csc(x)$$

```
[Out] -ArcTanh[Cos[x]]/2 + (Cot[x]*Csc[x])/2 - Csc[x]^2/2
```

Rubi [A] time = 0.0523432, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {4397, 2706, 2606, 30, 2611, 3770}

$$-\frac{1}{2}\csc^2(x)-\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\cot(x)\csc(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sin[x] + Tan[x])^(-1),x]
```

```
[Out] -ArcTanh[Cos[x]]/2 + (Cot[x]*Csc[x])/2 - Csc[x]^2/2
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sin(x)+\tan(x)}\,dx &= \int \frac{\cot(x)}{1+\cos(x)}\,dx\\ &= -\int \cot^2(x)\csc(x)\,dx+\int \cot(x)\csc^2(x)\,dx\\ &= \frac{1}{2}\cot(x)\csc(x)+\frac{1}{2}\int \csc(x)\,dx-\text{Subst}\left(\int x\,dx,x,\csc(x)\right)\\ &= -\frac{1}{2}\tanh^{-1}(\cos(x))+\frac{1}{2}\cot(x)\csc(x)-\frac{\csc^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.020161, size = 35, normalized size = 1.46

$$-\frac{1}{4}\sec^2\left(\frac{x}{2}\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)\right)-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sin[x] + Tan[x])^(-1),x]
```

```
[Out] -Log[Cos[x/2]]/2 + Log[Sin[x/2]]/2 - Sec[x/2]^2/4
```

Maple [A] time = 0.036, size = 24, normalized size = 1.

$$-\frac{1}{2\cos(x)+2}-\frac{\ln(\cos(x)+1)}{4}+\frac{\ln(\cos(x)-1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)+tan(x)),x)
```

```
[Out] -1/2/(cos(x)+1)-1/4*ln(cos(x)+1)+1/4*ln(cos(x)-1)
```

Maxima [A] time = 0.95352, size = 34, normalized size = 1.42

$$-\frac{\sin(x)^2}{4(\cos(x)+1)^2}+\frac{1}{2}\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] -1/4*sin(x)^2/(cos(x) + 1)^2 + 1/2*log(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 2.31762, size = 132, normalized size = 5.5

$$-\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)+2}{4(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) + 1
/2) + 2)/(cos(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sin(x)+\tan(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x)
```

```
[Out] Integral(1/(sin(x) + tan(x)), x)
```

Giac [A] time = 1.06748, size = 38, normalized size = 1.58

$$\frac{\cos(x) - 1}{4(\cos(x) + 1)} + \frac{1}{4}\log\left(-\frac{\cos(x) - 1}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sin(x)+tan(x)),x, algorithm="giac")
```

```
[Out] 1/4*(cos(x) - 1)/(cos(x) + 1) + 1/4*log(-(cos(x) - 1)/(cos(x) + 1))
```

3.248 $\int \frac{1}{2\sin(x)+\sin(2x)}\,dx$

Optimal. Leaf size=24

$$\frac{1}{8}\tan^2\left(\frac{x}{2}\right)+\frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)$$

```
[Out] Log[Tan[x/2]]/4 + Tan[x/2]^2/8
```

Rubi [A] time = 0.0268354, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {12, 14}

$$\frac{1}{8}\tan^2\left(\frac{x}{2}\right)+\frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*Sin[x] + Sin[2*x])^(-1),x]
```

```
[Out] Log[Tan[x/2]]/4 + Tan[x/2]^2/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{1}{2\sin(x)+\sin(2x)}\,dx = 2\,\text{Subst}\left(\int \frac{1+x^2}{8x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= \frac{1}{4}\,\text{Subst}\left(\int \frac{1+x^2}{x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= \frac{1}{4}\,\text{Subst}\left(\int \left(\frac{1}{x}+x\right)dx, x, \tan\left(\frac{x}{2}\right)\right)$$
$$= \frac{1}{4}\log\left(\tan\left(\frac{x}{2}\right)\right) + \frac{1}{8}\tan^2\left(\frac{x}{2}\right)$$

Mathematica [A] time = 0.0286321, size = 39, normalized size = 1.62

$$\frac{1-2\cos^2\left(\frac{x}{2}\right)\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{4(\cos(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*Sin[x] + Sin[2*x])^(-1),x]
```

```
[Out] (1 - 2*Cos[x/2]^2*(Log[Cos[x/2]] - Log[Sin[x/2]]))/(4*(1 + Cos[x]))
```

Maple [A] time = 0.052, size = 24, normalized size = 1.

$$\frac{1}{4\,\cos(x)+4} - \frac{\ln(\cos(x)+1)}{8} + \frac{\ln(\cos(x)-1)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*sin(x)+sin(2*x)),x)
```

```
[Out] 1/4/(cos(x)+1)-1/8*ln(cos(x)+1)+1/8*ln(cos(x)-1)
```

Maxima [B] time = 0.963876, size = 297, normalized size = 12.38

$$4\,\cos(2\,x)\cos(x) + 8\,\cos(x)^2 - \left(2\,(2\,\cos(x)+1)\cos(2\,x) + \cos(2\,x)^2 + 4\,\cos(x)^2 + \sin(2\,x)^2 + 4\,\sin(2\,x)\sin(x) + \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="maxima")
```

```
[Out] 1/8*(4*cos(2*x)*cos(x) + 8*cos(x)^2 - (2*(2*cos(x) + 1)*cos(2*x) + cos(2*x)
^2 + 4*cos(x)^2 + sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) +
1)*log(cos(x)^2 + sin(x)^2 + 2*cos(x) + 1) + (2*(2*cos(x) + 1)*cos(2*x) + c
os(2*x)^2 + 4*cos(x)^2 + sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*co
s(x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + 4*sin(2*x)*sin(x) + 8*s
in(x)^2 + 4*cos(x))/(2*(2*cos(x) + 1)*cos(2*x) + cos(2*x)^2 + 4*cos(x)^2 +
sin(2*x)^2 + 4*sin(2*x)*sin(x) + 4*sin(x)^2 + 4*cos(x) + 1)
```

Fricas [B] time = 2.34319, size = 132, normalized size = 5.5

$$-\frac{(\cos(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+1)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)-2}{8(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="fricas")
```

```
[Out] -1/8*((cos(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + 1)*log(-1/2*cos(x) + 1
/2) - 2)/(cos(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{2\sin(x)+\sin(2x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x)
```

```
[Out] Integral(1/(2*sin(x) + sin(2*x)), x)
```

<u>Giac</u> [A] time = 1.06128, size = 38, normalized size = 1.58

$$-\frac{\cos\left(x\right)-1}{8\left(\cos\left(x\right)+1\right)}+\frac{1}{8}\log\left(-\frac{\cos\left(x\right)-1}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*sin(x)+sin(2*x)),x, algorithm="giac")
```

```
[Out] -1/8*(cos(x) - 1)/(cos(x) + 1) + 1/8*log(-(cos(x) - 1)/(cos(x) + 1))
```

3.249 $\int \frac{\sec(x)}{1+\sin(x)} dx$

Optimal. Leaf size=18

$$\frac{1}{2}\tanh^{-1}(\sin(x)) - \frac{1}{2(\sin(x)+1)}$$

```
[Out] ArcTanh[Sin[x]]/2 - 1/(2*(1 + Sin[x]))
```

Rubi [A] time = 0.030173, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2667, 44, 207}

$$\frac{1}{2}\tanh^{-1}(\sin(x)) - \frac{1}{2(\sin(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(1 + Sin[x]),x]
```

```
[Out] ArcTanh[Sin[x]]/2 - 1/(2*(1 + Sin[x]))
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(x)}{1+\sin(x)}\,dx &= \text{Subst}\left(\int \frac{1}{(1-x)(1+x)^2}\,dx, x, \sin(x)\right)\\
&= \text{Subst}\left(\int \left(\frac{1}{2(1+x)^2}-\frac{1}{2\left(-1+x^2\right)}\right)dx, x, \sin(x)\right)\\
&= -\frac{1}{2(1+\sin(x))}-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sin(x)\right)\\
&= \frac{1}{2}\tanh^{-1}(\sin(x))-\frac{1}{2(1+\sin(x))}
\end{aligned}
$$

Mathematica [A] time = 0.0173726, size = 18, normalized size = 1.

$$\frac{1}{2}\tanh^{-1}(\sin(x))-\frac{1}{2(\sin(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(1 + Sin[x]),x]
```

```
[Out] ArcTanh[Sin[x]]/2 - 1/(2*(1 + Sin[x]))
```

Maple [A] time = 0.03, size = 24, normalized size = 1.3

$$-\frac{1}{2+2\sin(x)}+\frac{\ln(1+\sin(x))}{4}-\frac{\ln(-1+\sin(x))}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(1+sin(x)),x)
```

```
[Out] -1/2/(1+sin(x))+1/4*ln(1+sin(x))-1/4*ln(-1+sin(x))
```

Maxima [A] time = 0.930043, size = 31, normalized size = 1.72

$$-\frac{1}{2(\sin(x)+1)}+\frac{1}{4}\log(\sin(x)+1)-\frac{1}{4}\log(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(1+sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2/(sin(x) + 1) + 1/4*log(sin(x) + 1) - 1/4*log(sin(x) - 1)
```

Fricas [B] time = 2.35832, size = 115, normalized size = 6.39

$$\frac{(\sin(x)+1)\log(\sin(x)+1)-(\sin(x)+1)\log(-\sin(x)+1)-2}{4(\sin(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(1+sin(x)),x, algorithm="fricas")
```

```
[Out] 1/4*((sin(x) + 1)*log(sin(x) + 1) - (sin(x) + 1)*log(-sin(x) + 1) - 2)/(sin
(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec(x)}{\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(1+sin(x)),x)
```

```
[Out] Integral(sec(x)/(sin(x) + 1), x)
```

Giac [A] time = 1.05734, size = 34, normalized size = 1.89

$$-\frac{1}{2(\sin(x)+1)}+\frac{1}{4}\log(\sin(x)+1)-\frac{1}{4}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(1+sin(x)),x, algorithm="giac")
```

```
[Out] -1/2/(sin(x) + 1) + 1/4*log(sin(x) + 1) - 1/4*log(-sin(x) + 1)
```

3.250 $\int \frac{1}{b\cos(x)+a\sin(x)}\,dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] -(ArcTanh[(a*Cos[x] - b*Sin[x])/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2])
```

Rubi [A] time = 0.0223097, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3074, 206}

$$-\frac{\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[x] + a*Sin[x])^(-1),x]
```

```
[Out] -(ArcTanh[(a*Cos[x] - b*Sin[x])/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{b\cos(x)+a\sin(x)}\,dx = -\,\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, a\cos(x)-b\sin(x)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Mathematica [A] time = 0.0427802, size = 38, normalized size = 1.06

$$\frac{2\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-a}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[x] + a*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTanh[(-a + b*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
```

Maple [A] time = 0.046, size = 35, normalized size = 1.

$$2\,\frac{1}{\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,b\tan\left(x/2\right)-2\,a}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(x)+a*sin(x)),x)
```

```
[Out] 2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*a)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.25021, size = 242, normalized size = 6.72

$$\frac{\log\left(-\frac{2\,ab\cos\left(x\right)\sin\left(x\right)-\left(a^2-b^2\right)\cos\left(x\right)^2-a^2-2\,b^2+2\,\sqrt{a^2+b^2}\left(a\cos\left(x\right)-b\sin\left(x\right)\right)}{2\,ab\cos\left(x\right)\sin\left(x\right)-\left(a^2-b^2\right)\cos\left(x\right)^2+a^2}\right)}{2\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x)^2 - a^2 - 2*b^2 + 2*sqrt
(a^2 + b^2)*(a*cos(x) - b*sin(x)))/(2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x
)^2 + a^2))/sqrt(a^2 + b^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.11633, size = 82, normalized size = 2.28

$$-\frac{\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x)),x, algorithm="giac")
```

```
[Out] -log(abs(2*b*tan(1/2*x) - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*tan(1/2*x) - 2*a
 + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

3.251 $\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Rubi [A] time = 0.024756, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]

[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{b^2\cos^2(x)+a^2\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{b^2+a^2x^2}\,dx, x, \tan(x)\right)$$
$$= \frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Mathematica [A] time = 0.0417648, size = 15, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Maple [A] time = 0., size = 16, normalized size = 1.1

$$\frac{1}{ab}\arctan\left(\frac{a\tan(x)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*cos(x)^2+a^2*sin(x)^2),x)
```

```
[Out] arctan(a*tan(x)/b)/a/b
```

Maxima [A] time = 1.44672, size = 20, normalized size = 1.33

$$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(a*tan(x)/b)/(a*b)
```

Fricas [B] time = 2.52294, size = 99, normalized size = 6.6

$$-\frac{\arctan\left(\frac{(a^2+b^2)\cos(x)^2-a^2}{2\,ab\cos(x)\sin(x)}\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*((a^2 + b^2)*cos(x)^2 - a^2)/(a*b*cos(x)*sin(x)))/(a*b)
```

Sympy [A] time = 40.2914, size = 2866, normalized size = 191.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*cos(x)**2+a**2*sin(x)**2),x)
```

```
[Out] Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), ((tan(x/2)
/2 - 1/(2*tan(x/2)))/a**2, Eq(b, 0)), (-2*tan(x/2)/(b**2*(tan(x/2)**2 - 1))
, Eq(a, 0)), (-16*a**5*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a*
*5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(
a**2 - b**2) + 2*a*b**6) + 16*a**5*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x
/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2
*b**4*sqrt(a**2 - b**2) + 2*a*b**6) + 16*a**4*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 -
 b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - 16*a**4
*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b*
*2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2
- b**2) + 2*a*b**6) + 20*a**3*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(
x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**
2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - 20*a**3*b**2*sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a
**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - 4*a**3*b**2*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**
2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) + 4*a*
*3*b**2*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**
4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2
```

```
*a*b**6) - 12*a**2*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4
+ 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) + 12*a**2*b**2*sqrt(a**2 - b**2
)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2
*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**
6) + 4*a**2*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(
x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**
2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - 4*a**2*b**2*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*s
qrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a*
*2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - 5*a
*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4
*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*
a*b**6) + 5*a*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b*
*2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2
- b**2) + 2*a*b**6) + 3*a*b**4*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2)
)/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b*
*4*sqrt(a**2 - b**2) + 2*a*b**6) - 3*a*b**4*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4
 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) + b**4*sqrt(a**2 - b**2)*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(
a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - b
**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*l
og(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5
*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a*
*2 - b**2) + 2*a*b**6) - b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3
*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) + b**4*sqrt(a**2 - b**2)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 +
 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*s
qrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6)
, True))
```

Giac [A] time = 1.06591, size = 35, normalized size = 2.33

$$\frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor + \arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2) + arctan(a*tan(x)/b))/(a*b)
```

3.252 $\int \frac{x}{-1+x^2} dx$

Optimal. Leaf size=12

$$\frac{1}{2}\log\left(1-x^2\right)$$

```
[Out] Log[1 - x^2]/2
```

Rubi [A] time = 0.0018597, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {260}

$$\frac{1}{2}\log\left(1-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x^2),x]
```

```
[Out] Log[1 - x^2]/2
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{x}{-1+x^2}\,dx = \frac{1}{2}\log\left(1-x^2\right)$$

Mathematica [A] time = 0.001481, size = 10, normalized size = 0.83

$$\frac{1}{2}\log\left(x^2-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x^2),x]
```

```
[Out] Log[-1 + x^2]/2
```

Maple [A] time = 0.001, size = 14, normalized size = 1.2

$$\frac{\ln\left(-1+x\right)}{2}+\frac{\ln\left(1+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-1),x)
```

```
[Out] 1/2*ln(-1+x)+1/2*ln(1+x)
```

Maxima [A] time = 0.917003, size = 11, normalized size = 0.92

$$\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="maxima")
```

```
[Out] 1/2*log(x^2 - 1)
```

Fricas [A] time = 2.01126, size = 24, normalized size = 2.

$$\frac{1}{2}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - 1)
```

Sympy [A] time = 0.071522, size = 7, normalized size = 0.58

$$\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-1),x)
```

```
[Out] log(x**2 - 1)/2
```

Giac [A] time = 1.05911, size = 12, normalized size = 1.

$$\frac{1}{2}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-1),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(x^2 - 1))
```

3.253 $\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx$

Optimal. Leaf size=17

$$\frac{x^2}{2}+\frac{2x^{3/2}}{3}$$

```
[Out] (2*x^(3/2))/3 + x^2/2
```

Rubi [A] time = 0.0030238, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {14}

$$\frac{x^2}{2}+\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])*Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3 + x^2/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \left(1+\sqrt{x}\right)\sqrt{x}\,dx &= \int \left(\sqrt{x}+x\right)\,dx\\ &= \frac{2x^{3/2}}{3}+\frac{x^2}{2}\end{aligned}$$

Mathematica [A] time = 0.0024605, size = 17, normalized size = 1.

$$\frac{x^2}{2}+\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])*Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3 + x^2/2
```

Maple [A] time = 0.001, size = 12, normalized size = 0.7

$$\frac{2}{3}x^{\frac{3}{2}}+\frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^(1/2)+1),x)
```

```
[Out] 2/3*x^(3/2)+1/2*x^2
```

Maxima [B] time = 0.926366, size = 35, normalized size = 2.06

$$\frac{1}{2}\left(\sqrt{x}+1\right)^4-\frac{4}{3}\left(\sqrt{x}+1\right)^3+\left(\sqrt{x}+1\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(x) + 1)^4 - 4/3*(sqrt(x) + 1)^3 + (sqrt(x) + 1)^2
```

Fricas [A] time = 2.09504, size = 31, normalized size = 1.82

$$\frac{1}{2}x^2+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="fricas")
```

[Out] 1/2*x^2 + 2/3*x^(3/2)

Sympy [A] time = 0.120932, size = 12, normalized size = 0.71

$$\frac{2x^{\frac{3}{2}}}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1+x**(1/2)),x)
```

```
[Out] 2*x**(3/2)/3 + x**2/2
```

Giac [A] time = 1.04057, size = 15, normalized size = 0.88

$$\frac{1}{2}\,x^2 + \frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 2/3*x^(3/2)
```

3.254 $\int \frac{1}{1-\cos(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rubi [A] time = 0.008591, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2648}

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x])^(-1),x]

[Out] -(Sin[x]/(1 - Cos[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A] time = 0.0081957, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x])^(-1),x]
```

```
[Out] -Cot[x/2]
```

Maple [A] time = 0.006, size = 9, normalized size = 0.8

$$-\left(\tan\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)),x)
```

```
[Out] -1/tan(1/2*x)
```

Maxima [A] time = 0.920649, size = 14, normalized size = 1.17

$$-\frac{\cos\left(x\right)+1}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="maxima")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Fricas [A] time = 2.08231, size = 30, normalized size = 2.5

$$-\frac{\cos\left(x\right)+1}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Sympy [A] time = 0.348639, size = 7, normalized size = 0.58

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x)
```

```
[Out] -1/tan(x/2)
```

Giac [A] time = 1.06795, size = 11, normalized size = 0.92

$$-\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="giac")
```

```
[Out] -1/tan(1/2*x)
```

3.255 $\int \sec(x)\tan^2(x)\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}\tan(x)\sec(x)-\frac{1}{2}\tanh^{-1}(\sin(x))$$

```
[Out] -ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2
```

Rubi [A] time = 0.0141955, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2611, 3770}

$$\frac{1}{2}\tan(x)\sec(x)-\frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]*Tan[x]^2,x]
```

```
[Out] -ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(x)\tan^2(x)\,dx &= \frac{1}{2}\sec(x)\tan(x)-\frac{1}{2}\int \sec(x)\,dx\\ &= -\frac{1}{2}\tanh^{-1}(\sin(x))+\frac{1}{2}\sec(x)\tan(x)\end{aligned}$$

Mathematica [A] time = 0.0070759, size = 16, normalized size = 1.

$$\frac{1}{2}\tan(x)\sec(x)-\frac{1}{2}\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Tan[x]^2,x]
```

```
[Out] -ArcTanh[Sin[x]]/2 + (Sec[x]*Tan[x])/2
```

Maple [A] time = 0., size = 24, normalized size = 1.5

$$\frac{(\sin(x))^3}{2\,(\cos(x))^2}+\frac{\sin(x)}{2}-\frac{\ln(\sec(x)+\tan(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)*tan(x)^2,x)
```

```
[Out] 1/2*sin(x)^3/cos(x)^2+1/2*sin(x)-1/2*ln(sec(x)+tan(x))
```

Maxima [B] time = 0.935764, size = 36, normalized size = 2.25

$$-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)}-\frac{1}{4}\log(\sin(x)+1)+\frac{1}{4}\log(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sin(x)/(sin(x)^2 - 1) - 1/4*log(sin(x) + 1) + 1/4*log(sin(x) - 1)
```

Fricas [B] time = 1.97302, size = 109, normalized size = 6.81

$$-\frac{\cos(x)^2\log(\sin(x)+1)-\cos(x)^2\log(-\sin(x)+1)-2\,\sin(x)}{4\,\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) - 2*sin(x))/cos(
x)^2
```

Sympy [A] time = 0.110642, size = 27, normalized size = 1.69

$$\frac{\log(\sin(x) - 1)}{4} - \frac{\log(\sin(x) + 1)}{4} - \frac{\sin(x)}{2\sin^2(x) - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)**2,x)
```

```
[Out] log(sin(x) - 1)/4 - log(sin(x) + 1)/4 - sin(x)/(2*sin(x)**2 - 2)
```

Giac [B] time = 1.0672, size = 39, normalized size = 2.44

$$-\frac{\sin(x)}{2\left(\sin(x)^2 - 1\right)} - \frac{1}{4}\log(\sin(x) + 1) + \frac{1}{4}\log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)*tan(x)^2,x, algorithm="giac")
```

```
[Out] -1/2*sin(x)/(sin(x)^2 - 1) - 1/4*log(sin(x) + 1) + 1/4*log(-sin(x) + 1)
```

3.256 $\int \sec^3(x) \tan^3(x) \, dx$

Optimal. Leaf size=17

$$\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rubi [A] time = 0.0252672, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$\frac{\sec^5(x)}{5} - \frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sec^3(x)\tan^3(x)\,dx &= \text{Subst}\left(\int x^2\left(-1+x^2\right)dx, x, \sec(x)\right)\\ &= \text{Subst}\left(\int\left(-x^2+x^4\right)dx, x, \sec(x)\right)\\ &= -\frac{1}{3}\sec^3(x)+\frac{\sec^5(x)}{5}\end{aligned}$$

<u>Mathematica</u> [A] time = 0.0161254, size = 17, normalized size = 1.

$$\frac{\sec^5(x)}{5}-\frac{\sec^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3*Tan[x]^3,x]
```

```
[Out] -Sec[x]^3/3 + Sec[x]^5/5
```

<u>Maple</u> [B] time = 0., size = 42, normalized size = 2.5

$$\frac{(\sin(x))^4}{5\,(\cos(x))^5}+\frac{(\sin(x))^4}{15\,(\cos(x))^3}-\frac{(\sin(x))^4}{15\cos(x)}-\frac{\left(2+(\sin(x))^2\right)\cos(x)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3*tan(x)^3,x)
```

```
[Out] 1/5*sin(x)^4/cos(x)^5+1/15*sin(x)^4/cos(x)^3-1/15*sin(x)^4/cos(x)-1/15*(2+s
in(x)^2)*cos(x)
```

<u>Maxima</u> [A] time = 0.944727, size = 19, normalized size = 1.12

$$-\frac{5\cos(x)^2-3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Fricas [A] time = 1.85028, size = 45, normalized size = 2.65

$$-\frac{5\cos(x)^2-3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

Sympy [A] time = 0.096019, size = 15, normalized size = 0.88

$$-\frac{5\cos^2(x)-3}{15\cos^5(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3*tan(x)**3,x)
```

```
[Out] -(5*cos(x)**2 - 3)/(15*cos(x)**5)
```

Giac [A] time = 1.06395, size = 19, normalized size = 1.12

$$-\frac{5\cos(x)^2-3}{15\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3*tan(x)^3,x, algorithm="giac")
```

```
[Out] -1/15*(5*cos(x)^2 - 3)/cos(x)^5
```

3.257 $\int e^{\sqrt{x}}\,dx$

Optimal. Leaf size=24

$$2e^{\sqrt{x}}\sqrt{x}-2e^{\sqrt{x}}$$

```
[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rubi [A] time = 0.0086377, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2207, 2176, 2194}

$$2e^{\sqrt{x}}\sqrt{x}-2e^{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[E^Sqrt[x],x]
```

```
[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rule 2207

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Dist[k/d, Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (c
 + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !Inte
gerQ[n]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{\sqrt{x}}\,dx &= 2\,\mathrm{Subst}\left(\int e^{x}x\,dx, x, \sqrt{x}\right)\\ &= 2e^{\sqrt{x}}\sqrt{x} - 2\,\mathrm{Subst}\left(\int e^{x}\,dx, x, \sqrt{x}\right)\\ &= -2e^{\sqrt{x}} + 2e^{\sqrt{x}}\sqrt{x}\end{aligned}$$

Mathematica [A] time = 0.0061248, size = 16, normalized size = 0.67

$$2e^{\sqrt{x}}\left(\sqrt{x}-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sqrt[x],x]
```

```
[Out] 2*E^Sqrt[x]*(-1 + Sqrt[x])
```

Maple [A] time = 0., size = 17, normalized size = 0.7

$$-2\,\mathrm{e}^{\sqrt{x}} + 2\,\mathrm{e}^{\sqrt{x}}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^(1/2)),x)
```

```
[Out] -2*exp(x^(1/2))+2*exp(x^(1/2))*x^(1/2)
```

Maxima [A] time = 0.941537, size = 15, normalized size = 0.62

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Fricas [A] time = 1.86672, size = 36, normalized size = 1.5

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Sympy [A] time = 0.175594, size = 20, normalized size = 0.83

$$2\sqrt{x}e^{\sqrt{x}}-2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*exp(sqrt(x)) - 2*exp(sqrt(x))
```

Giac [A] time = 1.06289, size = 15, normalized size = 0.62

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

3.258 $\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx$

Optimal. Leaf size=42

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

Rubi [A] time = 0.0426682, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {1594, 1628}

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{1+x^5}{-10x-3x^2+x^3}\,dx = \int \frac{1+x^5}{x\left(-10-3x+x^2\right)}\,dx$$
$$= \int \left(19+\frac{3126}{35(-5+x)}-\frac{1}{10x}+3x+x^2-\frac{31}{14(2+x)}\right)dx$$
$$= 19x+\frac{3x^2}{2}+\frac{x^3}{3}+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(2+x)$$

Mathematica [A] time = 0.0067155, size = 42, normalized size = 1.

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x+\frac{3126}{35}\log(5-x)-\frac{\log(x)}{10}-\frac{31}{14}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^5)/(-10*x - 3*x^2 + x^3),x]
```

```
[Out] 19*x + (3*x^2)/2 + x^3/3 + (3126*Log[5 - x])/35 - Log[x]/10 - (31*Log[2 + x
])/14
```

Maple [A] time = 0.007, size = 31, normalized size = 0.7

$$\frac{x^3}{3}+\frac{3x^2}{2}+19x-\frac{31\ln(2+x)}{14}-\frac{\ln(x)}{10}+\frac{3126\ln(-5+x)}{35}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5+1)/(x^3-3*x^2-10*x),x)
```

```
[Out] 1/3*x^3+3/2*x^2+19*x-31/14*ln(2+x)-1/10*ln(x)+3126/35*ln(-5+x)
```

Maxima [A] time = 0.970599, size = 41, normalized size = 0.98

$$\frac{1}{3}x^3+\frac{3}{2}x^2+19x-\frac{31}{14}\log(x+2)+\frac{3126}{35}\log(x-5)-\frac{1}{10}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log
(x)
```

Fricas [A] time = 1.70858, size = 108, normalized size = 2.57

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 19x - \frac{31}{14}\log(x+2) + \frac{3126}{35}\log(x-5) - \frac{1}{10}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(x + 2) + 3126/35*log(x - 5) - 1/10*log
(x)
```

Sympy [A] time = 0.129621, size = 36, normalized size = 0.86

$$\frac{x^3}{3} + \frac{3x^2}{2} + 19x - \frac{\log(x)}{10} + \frac{3126\log(x-5)}{35} - \frac{31\log(x+2)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**5+1)/(x**3-3*x**2-10*x),x)
```

```
[Out] x**3/3 + 3*x**2/2 + 19*x - log(x)/10 + 3126*log(x - 5)/35 - 31*log(x + 2)/1
4
```

Giac [A] time = 1.06358, size = 45, normalized size = 1.07

$$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 19x - \frac{31}{14}\log(|x+2|) + \frac{3126}{35}\log(|x-5|) - \frac{1}{10}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^5+1)/(x^3-3*x^2-10*x),x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 3/2*x^2 + 19*x - 31/14*log(abs(x + 2)) + 3126/35*log(abs(x - 5))
- 1/10*log(abs(x))
```

3.259 $\int \frac{1}{x\sqrt{\log(x)}}\,dx$

Optimal. Leaf size=8

$$2\sqrt{\log(x)}$$

```
[Out] 2*Sqrt[Log[x]]
```

Rubi [A] time = 0.013675, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2302, 30}

$$2\sqrt{\log(x)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[Log[x]]),x]
```

```
[Out] 2*Sqrt[Log[x]]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{\log(x)}}\,dx &= \text{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, \log(x)\right)\\ &= 2\sqrt{\log(x)}\end{aligned}$$

Mathematica [A] time = 0.0014124, size = 8, normalized size = 1.

$$2\sqrt{\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[Log[x]]),x]
```

```
[Out] 2*Sqrt[Log[x]]
```

Maple [A] time = 0.003, size = 7, normalized size = 0.9

$$2\sqrt{\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x)^(1/2),x)
```

```
[Out] 2*ln(x)^(1/2)
```

Maxima [A] time = 0.956284, size = 8, normalized size = 1.

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(log(x))
```

Fricas [A] time = 1.8706, size = 22, normalized size = 2.75

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(log(x))
```

Sympy [A] time = 0.410009, size = 7, normalized size = 0.88

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x)**(1/2),x)
```

```
[Out] 2*sqrt(log(x))
```

Giac [A] time = 1.05744, size = 8, normalized size = 1.

$$2\sqrt{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(log(x))
```

3.260 $\int \frac{5+2x}{-3+x} \, dx$

Optimal. Leaf size=12

$$2x + 11 \log(3 - x)$$

```
[Out] 2*x + 11*Log[3 - x]
```

Rubi [A] time = 0.0063038, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$2x + 11 \log(3 - x)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x)/(-3 + x),x]

[Out] 2*x + 11*Log[3 - x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{5 + 2x}{-3 + x} \, dx &= \int \left(2 + \frac{11}{-3 + x}\right) dx \\ &= 2x + 11 \log(3 - x) \end{aligned}$$

Mathematica [A] time = 0.0031168, size = 12, normalized size = 1.

$$2(x - 3) + 11 \log(x - 3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x)/(-3 + x),x]
```

```
[Out] 2*(-3 + x) + 11*Log[-3 + x]
```

Maple [A] time = 0.003, size = 11, normalized size = 0.9

$$2x + 11\ln(-3 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)/(-3+x),x)
```

```
[Out] 2*x+11*ln(-3+x)
```

Maxima [A] time = 0.958498, size = 14, normalized size = 1.17

$$2x + 11\log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(-3+x),x, algorithm="maxima")
```

```
[Out] 2*x + 11*log(x - 3)
```

Fricas [A] time = 1.79746, size = 28, normalized size = 2.33

$$2x + 11\log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(-3+x),x, algorithm="fricas")
```

```
[Out] 2*x + 11*log(x - 3)
```

Sympy [A] time = 0.068852, size = 8, normalized size = 0.67

$$2x + 11\log(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(-3+x),x)
```

```
[Out] 2*x + 11*log(x - 3)
```

Giac [A] time = 1.046, size = 15, normalized size = 1.25

$$2\,x + 11\,\log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(-3+x),x, algorithm="giac")
```

```
[Out] 2*x + 11*log(abs(x - 3))
```

3.261 $\int e^{e^x+x}\,dx$

Optimal. Leaf size=5

$$e^{e^x}$$

```
[Out] E^E^x
```

Rubi [A] time = 0.0055676, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2282, 2194}

$$e^{e^x}$$

Antiderivative was successfully verified.

```
[In] Int[E^(E^x + x),x]
```

```
[Out] E^E^x
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{e^x+x}\,dx &= \text{Subst}\left(\int e^x\,dx, x, e^x\right)\\ &= e^{e^x}\end{aligned}$$

Mathematica [A] time = 0.0047036, size = 5, normalized size = 1.

$$e^{e^x}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(E^x + x),x]
```

```
[Out] E^E^x
```

Maple [A] time = 0.002, size = 4, normalized size = 0.8

$$\mathrm{e}^{\mathrm{e}^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(exp(x)+x),x)
```

```
[Out] exp(exp(x))
```

Maxima [A] time = 0.970828, size = 4, normalized size = 0.8

$$e^{(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x+exp(x)),x, algorithm="maxima")
```

```
[Out] e^(e^x)
```

Fricas [A] time = 1.94064, size = 12, normalized size = 2.4

$$e^{(e^x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x+exp(x)),x, algorithm="fricas")
```

```
[Out] e^(e^x)
```

Sympy [A] time = 0.623809, size = 3, normalized size = 0.6

$$e^{e^{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x+exp(x)),x)
```

```
[Out] exp(exp(x))
```

Giac [A] time = 1.05456, size = 4, normalized size = 0.8

$$e^{\left(e^{x}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x+exp(x)),x, algorithm="giac")
```

```
[Out] e^(e^x)
```

3.262 $\int \cos^2(x)\sin^2(x)\,dx$

Optimal. Leaf size=24

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rubi [A] time = 0.0282671, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\sin^2(x)\,dx &= -\frac{1}{4}\cos^3(x)\sin(x) + \frac{1}{4}\int \cos^2(x)\,dx\\ &= \frac{1}{8}\cos(x)\sin(x) - \frac{1}{4}\cos^3(x)\sin(x) + \frac{\int 1\,dx}{8}\\ &= \frac{x}{8} + \frac{1}{8}\cos(x)\sin(x) - \frac{1}{4}\cos^3(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0042057, size = 14, normalized size = 0.58

$$\frac{x}{8} - \frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 - Sin[4*x]/32
```

Maple [A] time = 0., size = 19, normalized size = 0.8

$$\frac{x}{8} + \frac{\cos(x)\sin(x)}{8} - \frac{(\cos(x))^3\sin(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2,x)
```

```
[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

Maxima [A] time = 0.968631, size = 14, normalized size = 0.58

$$\frac{1}{8}x - \frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

Fricas [A] time = 1.99608, size = 58, normalized size = 2.42

$$-\frac{1}{8}\left(2\cos\left(x\right)^{3}-\cos\left(x\right)\right)\sin\left(x\right)+\frac{1}{8}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

Sympy [A] time = 0.062355, size = 14, normalized size = 0.58

$$\frac{x}{8}-\frac{\sin\left(2x\right)\cos\left(2x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**2,x)
```

```
[Out] x/8 - sin(2*x)*cos(2*x)/16
```

Giac [A] time = 1.05061, size = 14, normalized size = 0.58

$$\frac{1}{8}\,x-\frac{1}{32}\,\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

3.263 $\int \frac{-\cos(x)+\sin(x)}{\cos(x)+\sin(x)} \, dx$

Optimal. Leaf size=8

$$-\log(\sin(x)+\cos(x))$$

```
[Out] -Log[Cos[x] + Sin[x]]
```

Rubi [A] time = 0.0225275, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {3133}

$$-\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[(-Cos[x] + Sin[x])/(Cos[x] + Sin[x]),x]
```

```
[Out] -Log[Cos[x] + Sin[x]]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{-\cos(x)+\sin(x)}{\cos(x)+\sin(x)} \, dx = -\log(\cos(x)+\sin(x))$$

Mathematica [A] time = 0.0233293, size = 8, normalized size = 1.

$$-\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Cos[x] + Sin[x])/(Cos[x] + Sin[x]),x]
```

```
[Out] -Log[Cos[x] + Sin[x]]
```

Maple [A] time = 0.029, size = 9, normalized size = 1.1

$$-\ln(\cos(x) + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-cos(x)+sin(x))/(cos(x)+sin(x)),x)
```

```
[Out] -ln(cos(x)+sin(x))
```

Maxima [A] time = 0.953654, size = 11, normalized size = 1.38

$$-\log(\cos(x) + \sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))/(cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] -log(cos(x) + sin(x))
```

Fricas [A] time = 2.05828, size = 42, normalized size = 5.25

$$-\frac{1}{2}\log(2\cos(x)\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))/(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*cos(x)*sin(x) + 1)
```

Sympy [A] time = 0.129102, size = 8, normalized size = 1.

$$-\log\left(\sin\left(x\right)+\cos\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))/(cos(x)+sin(x)),x)
```

```
[Out] -log(sin(x) + cos(x))
```

Giac [B] time = 1.08499, size = 24, normalized size = 3.

$$\frac{1}{2}\log\left(\tan\left(x\right)^{2}+1\right)-\log\left(\left|\tan\left(x\right)+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-cos(x)+sin(x))/(cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*log(tan(x)^2 + 1) - log(abs(tan(x) + 1))
```

3.264 $\int \frac{x}{\sqrt{1-x^2}} dx$

Optimal. Leaf size=13

$$-\sqrt{1-x^2}$$

```
[Out] -Sqrt[1 - x^2]
```

Rubi [A] time = 0.0022397, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 - x^2],x]

[Out] -Sqrt[1 - x^2]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-x^2}} dx = -\sqrt{1-x^2}$$

Mathematica [A] time = 0.0013003, size = 13, normalized size = 1.

$$-\sqrt{1-x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 - x^2],x]
```

```
[Out] -Sqrt[1 - x^2]
```

Maple [A] time = 0., size = 17, normalized size = 1.3

$$(-1 + x)(1 + x)\frac{1}{\sqrt{-x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^(1/2),x)
```

```
[Out] (-1+x)*(1+x)/(-x^2+1)^(1/2)
```

Maxima [A] time = 0.956916, size = 15, normalized size = 1.15

$$-\sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 1)
```

Fricas [A] time = 1.87824, size = 23, normalized size = 1.77

$$-\sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 1)
```

Sympy [A] time = 0.125356, size = 8, normalized size = 0.62

$$-\sqrt{1-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(1/2),x)
```

```
[Out] -sqrt(1 - x**2)
```

Giac [A] time = 1.05779, size = 15, normalized size = 1.15

$$-\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 1)
```

3.265 $\int x^3 \log(x) \, dx$

Optimal. Leaf size=17

$$\frac{1}{4}x^4 \log(x) - \frac{x^4}{16}$$

```
[Out] -x^4/16 + (x^4*Log[x])/4
```

Rubi [A] time = 0.0066092, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2304}

$$\frac{1}{4}x^4 \log(x) - \frac{x^4}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[x],x]

[Out] -x^4/16 + (x^4*Log[x])/4
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^3 \log(x) \, dx = -\frac{x^4}{16} + \frac{1}{4}x^4 \log(x)$$

Mathematica [A] time = 0.0008132, size = 17, normalized size = 1.

$$\frac{1}{4}x^4 \log(x) - \frac{x^4}{16}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[x],x]
```

```
[Out] -x^4/16 + (x^4*Log[x])/4
```

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$-\frac{x^4}{16}+\frac{x^4\ln\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(x),x)
```

```
[Out] -1/16*x^4+1/4*x^4*ln(x)
```

Maxima [A] time = 0.973134, size = 18, normalized size = 1.06

$$\frac{1}{4}\,x^4\log\left(x\right)-\frac{1}{16}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(x),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*log(x) - 1/16*x^4
```

Fricas [A] time = 1.88875, size = 36, normalized size = 2.12

$$\frac{1}{4}\,x^4\log\left(x\right)-\frac{1}{16}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(x),x, algorithm="fricas")
```

```
[Out] 1/4*x^4*log(x) - 1/16*x^4
```

Sympy [A] time = 0.08583, size = 12, normalized size = 0.71

$$\frac{x^4\log(x)}{4} - \frac{x^4}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(x),x)
```

```
[Out] x**4*log(x)/4 - x**4/16
```

Giac [A] time = 1.05809, size = 18, normalized size = 1.06

$$\frac{1}{4}\,x^4\log(x) - \frac{1}{16}\,x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(x),x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(x) - 1/16*x^4
```

3.266 $\int \frac{\sqrt{-2+x}}{2+x} dx$

Optimal. Leaf size=24

$$2\sqrt{x-2} - 4\tan^{-1}\left(\frac{\sqrt{x-2}}{2}\right)$$

```
[Out] 2*Sqrt[-2 + x] - 4*ArcTan[Sqrt[-2 + x]/2]
```

Rubi [A] time = 0.0057144, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 203}

$$2\sqrt{x-2} - 4\tan^{-1}\left(\frac{\sqrt{x-2}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-2 + x]/(2 + x),x]
```

```
[Out] 2*Sqrt[-2 + x] - 4*ArcTan[Sqrt[-2 + x]/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{-2+x}}{2+x}\,dx &= 2\sqrt{-2+x}-4\int \frac{1}{\sqrt{-2+x}(2+x)}\,dx\\ &= 2\sqrt{-2+x}-8\,\text{Subst}\left(\int \frac{1}{4+x^2}\,dx,x,\sqrt{-2+x}\right)\\ &= 2\sqrt{-2+x}-4\tan^{-1}\left(\frac{\sqrt{-2+x}}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0055719, size = 24, normalized size = 1.

$$2\sqrt{x-2}-4\tan^{-1}\left(\frac{\sqrt{x-2}}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-2 + x]/(2 + x),x]
```

```
[Out] 2*Sqrt[-2 + x] - 4*ArcTan[Sqrt[-2 + x]/2]
```

Maple [A] time = 0.006, size = 19, normalized size = 0.8

$$-4\,\arctan\left(1/2\,\sqrt{-2+x}\right)+2\,\sqrt{-2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2+x)^(1/2)/(2+x),x)
```

```
[Out] -4*arctan(1/2*(-2+x)^(1/2))+2*(-2+x)^(1/2)
```

Maxima [A] time = 1.42182, size = 24, normalized size = 1.

$$2\sqrt{x-2}-4\arctan\left(\frac{1}{2}\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)^(1/2)/(2+x),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x - 2) - 4*arctan(1/2*sqrt(x - 2))
```

Fricas [A] time = 1.90211, size = 58, normalized size = 2.42

$$2\sqrt{x-2}-4\arctan\left(\frac{1}{2}\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)^(1/2)/(2+x),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x - 2) - 4*arctan(1/2*sqrt(x - 2))
```

Sympy [B] time = 1.2493, size = 107, normalized size = 4.46

$$\begin{cases} -4i\operatorname{acosh}\left(\frac{2}{\sqrt{x+2}}\right)-\frac{2i\sqrt{x+2}}{\sqrt{-1+\frac{4}{x+2}}}+\frac{8i}{\sqrt{-1+\frac{4}{x+2}}\sqrt{x+2}} & \text{for } \frac{4}{|x+2|}>1 \\ 4\operatorname{asin}\left(\frac{2}{\sqrt{x+2}}\right)+\frac{2\sqrt{x+2}}{\sqrt{1-\frac{4}{x+2}}}-\frac{8}{\sqrt{1-\frac{4}{x+2}}\sqrt{x+2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)**(1/2)/(2+x),x)
```

```
[Out] Piecewise((-4*I*acosh(2/sqrt(x + 2)) - 2*I*sqrt(x + 2)/sqrt(-1 + 4/(x + 2))
 + 8*I/(sqrt(-1 + 4/(x + 2))*sqrt(x + 2)), 4/Abs(x + 2) > 1), (4*asin(2/sqr
t(x + 2)) + 2*sqrt(x + 2)/sqrt(1 - 4/(x + 2)) - 8/(sqrt(1 - 4/(x + 2))*sqrt
(x + 2)), True))
```

Giac [A] time = 1.05946, size = 24, normalized size = 1.

$$2\sqrt{x-2} - 4\arctan\left(\frac{1}{2}\sqrt{x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2+x)^(1/2)/(2+x),x, algorithm="giac")
```

```
[Out] 2*sqrt(x - 2) - 4*arctan(1/2*sqrt(x - 2))
```

3.267 $\int \frac{x}{(2+x)^2}\,dx$

Optimal. Leaf size=12

$$\frac{2}{x+2}+\log(x+2)$$

[Out] 2/(2 + x) + Log[2 + x]

Rubi [A] time = 0.0048786, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {43}

$$\frac{2}{x+2}+\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x/(2 + x)^2,x]
```

```
[Out] 2/(2 + x) + Log[2 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(2+x)^2}\,dx &= \int \left(-\frac{2}{(2+x)^2}+\frac{1}{2+x}\right)dx\\ &= \frac{2}{2+x}+\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.0030157, size = 12, normalized size = 1.

$$\frac{2}{x+2}+\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 + x)^2,x]
```

```
[Out] 2/(2 + x) + Log[2 + x]
```

Maple [A] time = 0.004, size = 13, normalized size = 1.1

$$2\,(2+x)^{-1}+\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2+x)^2,x)
```

```
[Out] 2/(2+x)+ln(2+x)
```

Maxima [A] time = 0.935116, size = 16, normalized size = 1.33

$$\frac{2}{x+2}+\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)^2,x, algorithm="maxima")
```

```
[Out] 2/(x + 2) + log(x + 2)
```

Fricas [A] time = 1.81843, size = 46, normalized size = 3.83

$$\frac{(x+2)\log(x+2)+2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)^2,x, algorithm="fricas")
```

```
[Out] ((x + 2)*log(x + 2) + 2)/(x + 2)
```

Sympy [A] time = 0.077507, size = 8, normalized size = 0.67

$$\log(x+2) + \frac{2}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)**2,x)
```

```
[Out] log(x + 2) + 2/(x + 2)
```

Giac [A] time = 1.05102, size = 18, normalized size = 1.5

$$\frac{2}{x+2} + \log(|x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2+x)^2,x, algorithm="giac")
```

```
[Out] 2/(x + 2) + log(abs(x + 2))
```

3.268 $\int \log\left(1+x^2\right) dx$

Optimal. Leaf size=16

$$x\log\left(x^2+1\right)-2x+2\tan^{-1}(x)$$

```
[Out] -2*x + 2*ArcTan[x] + x*Log[1 + x^2]
```

Rubi [A] time = 0.0047707, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2448, 321, 203}

$$x\log\left(x^2+1\right)-2x+2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x^2],x]
```

```
[Out] -2*x + 2*ArcTan[x] + x*Log[1 + x^2]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \log\left(1+x^2\right) dx &= x\log\left(1+x^2\right) - 2\int \frac{x^2}{1+x^2}\, dx \\ &= -2x + x\log\left(1+x^2\right) + 2\int \frac{1}{1+x^2}\, dx \\ &= -2x + 2\tan^{-1}(x) + x\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0015108, size = 16, normalized size = 1.

$$x\log\left(x^2+1\right) - 2x + 2\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x^2],x]
```

```
[Out] -2*x + 2*ArcTan[x] + x*Log[1 + x^2]
```

Maple [A] time = 0.002, size = 17, normalized size = 1.1

$$-2\,x + 2\,\arctan(x) + x\ln\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2+1),x)
```

```
[Out] -2*x+2*arctan(x)+x*ln(x^2+1)
```

Maxima [A] time = 1.43056, size = 22, normalized size = 1.38

$$x\log\left(x^2+1\right) - 2\,x + 2\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+1),x, algorithm="maxima")
```

```
[Out] x*log(x^2 + 1) - 2*x + 2*arctan(x)
```

Fricas [A] time = 1.93475, size = 49, normalized size = 3.06

$$x \log \left(x^2+1\right) - 2\,x + 2\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+1),x, algorithm="fricas")
```

```
[Out] x*log(x^2 + 1) - 2*x + 2*arctan(x)
```

Sympy [A] time = 0.117136, size = 15, normalized size = 0.94

$$x \log \left(x^2+1\right) - 2x + 2\,\operatorname{atan}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2+1),x)
```

```
[Out] x*log(x**2 + 1) - 2*x + 2*atan(x)
```

Giac [A] time = 1.05528, size = 22, normalized size = 1.38

$$x \log \left(x^2+1\right) - 2\,x + 2\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+1),x, algorithm="giac")
```

```
[Out] x*log(x^2 + 1) - 2*x + 2*arctan(x)
```

3.269 $\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx$

Optimal. Leaf size=22

$$2\sqrt{\log(x)+1}-2\tanh^{-1}\left(\sqrt{\log(x)+1}\right)$$

```
[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Rubi [A] time = 0.0561296, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2365, 50, 63, 207}

$$2\sqrt{\log(x)+1}-2\tanh^{-1}\left(\sqrt{\log(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Log[x]]/(x*Log[x]),x]

[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Rule 2365

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d + e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+\log(x)}}{x\log(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{1+x}}{x}\,dx, x, \log(x)\right)\\
&= 2\sqrt{1+\log(x)} + \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, \log(x)\right)\\
&= 2\sqrt{1+\log(x)} + 2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+\log(x)}\right)\\
&= -2\tanh^{-1}\left(\sqrt{1+\log(x)}\right) + 2\sqrt{1+\log(x)}
\end{aligned}$$

Mathematica [A] time = 0.021054, size = 22, normalized size = 1.

$$2\sqrt{\log(x)+1} - 2\tanh^{-1}\left(\sqrt{\log(x)+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Log[x]]/(x*Log[x]),x]
```

```
[Out] -2*ArcTanh[Sqrt[1 + Log[x]]] + 2*Sqrt[1 + Log[x]]
```

Maple [A] time = 0.002, size = 30, normalized size = 1.4

$$2\sqrt{1+\ln(x)} + \ln\left(-1+\sqrt{1+\ln(x)}\right) - \ln\left(1+\sqrt{1+\ln(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+ln(x))^(1/2)/x/ln(x),x)
```

```
[Out] 2*(1+ln(x))^(1/2)+ln(-1+(1+ln(x))^(1/2))-ln(1+(1+ln(x))^(1/2))
```

Maxima [A] time = 0.949866, size = 39, normalized size = 1.77

$$2\sqrt{\log(x)+1} - \log\left(\sqrt{\log(x)+1}+1\right) + \log\left(\sqrt{\log(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="maxima")
```

```
[Out] 2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x) + 1) - 1)
```

Fricas [A] time = 1.97212, size = 103, normalized size = 4.68

$$2\sqrt{\log(x)+1} - \log\left(\sqrt{\log(x)+1}+1\right) + \log\left(\sqrt{\log(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="fricas")
```

```
[Out] 2*sqrt(log(x) + 1) - log(sqrt(log(x) + 1) + 1) + log(sqrt(log(x) + 1) - 1)
```

Sympy [A] time = 1.79135, size = 32, normalized size = 1.45

$$2\sqrt{\log(x)+1} + \log\left(\sqrt{\log(x)+1}-1\right) - \log\left(\sqrt{\log(x)+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+ln(x))**(1/2)/x/ln(x),x)
```

```
[Out] 2*sqrt(log(x) + 1) + log(sqrt(log(x) + 1) - 1) - log(sqrt(log(x) + 1) + 1)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+log(x))^(1/2)/x/log(x),x, algorithm="giac")
```

```
[Out] Timed out
```

3.270 $\int \left(1+\sqrt{x}\right)^8 dx$

Optimal. Leaf size=27

$$\frac{1}{5}\left(\sqrt{x}+1\right)^{10}-\frac{2}{9}\left(\sqrt{x}+1\right)^{9}$$

```
[Out] (-2*(1 + Sqrt[x])^9)/9 + (1 + Sqrt[x])^10/5
```

Rubi [A] time = 0.0062772, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {190, 43}

$$\frac{1}{5}\left(\sqrt{x}+1\right)^{10}-\frac{2}{9}\left(\sqrt{x}+1\right)^{9}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x])^8,x]
```

```
[Out] (-2*(1 + Sqrt[x])^9)/9 + (1 + Sqrt[x])^10/5
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \left(1+\sqrt{x}\right)^8 dx = 2\,\mathrm{Subst}\left(\int x(1+x)^8\,dx, x, \sqrt{x}\right)$$
$$= 2\,\mathrm{Subst}\left(\int \left(-(1+x)^8+(1+x)^9\right)dx, x, \sqrt{x}\right)$$
$$= -\frac{2}{9}\left(1+\sqrt{x}\right)^9+\frac{1}{5}\left(1+\sqrt{x}\right)^{10}$$

Mathematica [A] time = 0.0172791, size = 22, normalized size = 0.81

$$\frac{1}{45}\left(\sqrt{x}+1\right)^9\left(9\sqrt{x}-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])^8,x]
```

```
[Out] ((1 + Sqrt[x])^9*(-1 + 9*Sqrt[x]))/45
```

Maple [B] time = 0.001, size = 43, normalized size = 1.6

$$\frac{x^5}{5}+\frac{16}{9}x^{\frac{9}{2}}+7\,x^4+16\,x^{7/2}+\frac{70\,x^3}{3}+\frac{112}{5}x^{\frac{5}{2}}+14\,x^2+\frac{16}{3}x^{\frac{3}{2}}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^(1/2)+1)^8,x)
```

```
[Out] 1/5*x^5+16/9*x^(9/2)+7*x^4+16*x^(7/2)+70/3*x^3+112/5*x^(5/2)+14*x^2+16/3*x^
(3/2)+x
```

Maxima [A] time = 0.929772, size = 26, normalized size = 0.96

$$\frac{1}{5}\left(\sqrt{x}+1\right)^{10}-\frac{2}{9}\left(\sqrt{x}+1\right)^9$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^8,x, algorithm="maxima")
```

```
[Out] 1/5*(sqrt(x) + 1)^10 - 2/9*(sqrt(x) + 1)^9
```

Fricas [B] time = 1.81559, size = 122, normalized size = 4.52

$$\frac{1}{5}x^5 + 7x^4 + \frac{70}{3}x^3 + 14x^2 + \frac{16}{45}\left(5x^4 + 45x^3 + 63x^2 + 15x\right)\sqrt{x} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^8,x, algorithm="fricas")
```

```
[Out] 1/5*x^5 + 7*x^4 + 70/3*x^3 + 14*x^2 + 16/45*(5*x^4 + 45*x^3 + 63*x^2 + 15*x
)*sqrt(x) + x
```

Sympy [B] time = 0.808166, size = 54, normalized size = 2.

$$\frac{16x^{\frac{9}{2}}}{9} + 16x^{\frac{7}{2}} + \frac{112x^{\frac{5}{2}}}{5} + \frac{16x^{\frac{3}{2}}}{3} + \frac{x^5}{5} + 7x^4 + \frac{70x^3}{3} + 14x^2 + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))**8,x)
```

```
[Out] 16*x**(9/2)/9 + 16*x**(7/2) + 112*x**(5/2)/5 + 16*x**(3/2)/3 + x**5/5 + 7*x
**4 + 70*x**3/3 + 14*x**2 + x
```

Giac [B] time = 1.09836, size = 57, normalized size = 2.11

$$\frac{1}{5}x^5 + \frac{16}{9}x^{\frac{9}{2}} + 7x^4 + 16x^{\frac{7}{2}} + \frac{70}{3}x^3 + \frac{112}{5}x^{\frac{5}{2}} + 14x^2 + \frac{16}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^8,x, algorithm="giac")
```

```
[Out] 1/5*x^5 + 16/9*x^(9/2) + 7*x^4 + 16*x^(7/2) + 70/3*x^3 + 112/5*x^(5/2) + 14
*x^2 + 16/3*x^(3/2) + x
```

3.271 $\int \sec^4(x)\tan^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sec^6(x)}{6}-\frac{\sec^4(x)}{4}$$

```
[Out] -Sec[x]^4/4 + Sec[x]^6/6
```

Rubi [A] time = 0.0245593, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2606, 14}

$$\frac{\sec^6(x)}{6}-\frac{\sec^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4*Tan[x]^3,x]
```

```
[Out] -Sec[x]^4/4 + Sec[x]^6/6
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sec^4(x)\tan^3(x)\,dx &= \text{Subst}\left(\int x^3\left(-1+x^2\right)\,dx, x, \sec(x)\right)\\ &= \text{Subst}\left(\int \left(-x^3+x^5\right)\,dx, x, \sec(x)\right)\\ &= -\frac{1}{4}\sec^4(x)+\frac{\sec^6(x)}{6}\end{aligned}$$

Mathematica [A] time = 0.0109855, size = 17, normalized size = 1.

$$\frac{\sec^6(x)}{6}-\frac{\sec^4(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4*Tan[x]^3,x]
```

```
[Out] -Sec[x]^4/4 + Sec[x]^6/6
```

Maple [A] time = 0.012, size = 22, normalized size = 1.3

$$\frac{(\sin(x))^4}{6\,(\cos(x))^6}+\frac{(\sin(x))^4}{12\,(\cos(x))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4*tan(x)^3,x)
```

```
[Out] 1/6*sin(x)^4/cos(x)^6+1/12*sin(x)^4/cos(x)^4
```

Maxima [B] time = 0.942643, size = 41, normalized size = 2.41

$$-\frac{3\,\sin(x)^2-1}{12\left(\sin(x)^6-3\,\sin(x)^4+3\,\sin(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/12*(3*sin(x)^2 - 1)/(sin(x)^6 - 3*sin(x)^4 + 3*sin(x)^2 - 1)
```

Fricas [A] time = 2.04556, size = 45, normalized size = 2.65

$$-\frac{3\,\cos\left(x\right)^{2}-2}{12\,\cos\left(x\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*cos(x)^2 - 2)/cos(x)^6
```

Sympy [A] time = 0.1049, size = 15, normalized size = 0.88

$$-\frac{3\cos^{2}\left(x\right)-2}{12\cos^{6}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4*tan(x)**3,x)
```

```
[Out] -(3*cos(x)**2 - 2)/(12*cos(x)**6)
```

Giac [A] time = 1.07188, size = 19, normalized size = 1.12

$$-\frac{3\,\cos\left(x\right)^{2}-2}{12\,\cos\left(x\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*cos(x)^2 - 2)/cos(x)^6
```

3.272 $\int \frac{x}{2-2x+x^2}\,dx$

Optimal. Leaf size=22

$$\frac{1}{2}\log\left(x^2-2x+2\right)-\tan^{-1}(1-x)$$

```
[Out] -ArcTan[1 - x] + Log[2 - 2*x + x^2]/2
```

Rubi [A] time = 0.0075908, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {634, 617, 204, 628}

$$\frac{1}{2}\log\left(x^2-2x+2\right)-\tan^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[x/(2 - 2*x + x^2),x]
```

```
[Out] -ArcTan[1 - x] + Log[2 - 2*x + x^2]/2
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{2-2x+x^2}\,dx &= \frac{1}{2}\int \frac{-2+2x}{2-2x+x^2}\,dx+\int \frac{1}{2-2x+x^2}\,dx\\ &= \frac{1}{2}\log\left(2-2x+x^2\right)+\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-x\right)\\ &= -\tan^{-1}(1-x)+\frac{1}{2}\log\left(2-2x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0032798, size = 22, normalized size = 1.

$$\frac{1}{2}\log\left(x^2-2x+2\right)-\tan^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 - 2*x + x^2),x]
```

```
[Out] -ArcTan[1 - x] + Log[2 - 2*x + x^2]/2
```

Maple [A] time = 0.003, size = 17, normalized size = 0.8

$$\arctan(-1+x)+\frac{\ln\left(x^2-2\,x+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-2*x+2),x)
```

```
[Out] arctan(-1+x)+1/2*ln(x^2-2*x+2)
```

Maxima [A] time = 1.41361, size = 22, normalized size = 1.

$$\arctan\left(x-1\right)+\frac{1}{2}\,\log\left(x^{2}-2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-2*x+2),x, algorithm="maxima")
```

```
[Out] arctan(x - 1) + 1/2*log(x^2 - 2*x + 2)
```

Fricas [A] time = 1.76874, size = 54, normalized size = 2.45

$$\arctan\left(x-1\right)+\frac{1}{2}\,\log\left(x^{2}-2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-2*x+2),x, algorithm="fricas")
```

```
[Out] arctan(x - 1) + 1/2*log(x^2 - 2*x + 2)
```

Sympy [A] time = 0.091334, size = 15, normalized size = 0.68

$$\frac{\log\left(x^{2}-2x+2\right)}{2}+\operatorname{atan}\left(x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-2*x+2),x)
```

```
[Out] log(x**2 - 2*x + 2)/2 + atan(x - 1)
```

Giac [A] time = 1.07189, size = 22, normalized size = 1.

$$\arctan\left(x-1\right)+\frac{1}{2}\,\log\left(x^{2}-2\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-2*x+2),x, algorithm="giac")
```

```
[Out] arctan(x - 1) + 1/2*log(x^2 - 2*x + 2)
```

3.273 $\int x\sin^{-1}(x)\,dx$

Optimal. Leaf size=32

$$\frac{1}{4}\sqrt{1-x^2}x+\frac{1}{2}x^2\sin^{-1}(x)-\frac{1}{4}\sin^{-1}(x)$$

```
[Out] (x*Sqrt[1 - x^2])/4 - ArcSin[x]/4 + (x^2*ArcSin[x])/2
```

Rubi [A] time = 0.0101239, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {4627, 321, 216}

$$\frac{1}{4}\sqrt{1-x^2}x+\frac{1}{2}x^2\sin^{-1}(x)-\frac{1}{4}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSin[x],x]
```

```
[Out] (x*Sqrt[1 - x^2])/4 - ArcSin[x]/4 + (x^2*ArcSin[x])/2
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int x\sin^{-1}(x)\,dx &= \frac{1}{2}x^2\sin^{-1}(x)-\frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{4}x\sqrt{1-x^2}+\frac{1}{2}x^2\sin^{-1}(x)-\frac{1}{4}\int\frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{4}x\sqrt{1-x^2}-\frac{1}{4}\sin^{-1}(x)+\frac{1}{2}x^2\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0091866, size = 28, normalized size = 0.88

$$\frac{1}{4}\left(\sqrt{1-x^2}x+\left(2x^2-1\right)\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSin[x],x]
```

```
[Out] (x*Sqrt[1 - x^2] + (-1 + 2*x^2)*ArcSin[x])/4
```

Maple [A] time = 0.003, size = 25, normalized size = 0.8

$$-\frac{\arcsin(x)}{4}+\frac{x^2\arcsin(x)}{2}+\frac{x}{4}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsin(x),x)
```

```
[Out] -1/4*arcsin(x)+1/2*x^2*arcsin(x)+1/4*x*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.4477, size = 32, normalized size = 1.

$$\frac{1}{2}\,x^2\arcsin(x)+\frac{1}{4}\,\sqrt{-x^2+1}x-\frac{1}{4}\,\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsin(x) + 1/4*sqrt(-x^2 + 1)*x - 1/4*arcsin(x)
```

Fricas [A] time = 1.98438, size = 68, normalized size = 2.12

$$\frac{1}{4}\left(2x^2-1\right)\arcsin(x)+\frac{1}{4}\sqrt{-x^2+1}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x^2 - 1)*arcsin(x) + 1/4*sqrt(-x^2 + 1)*x
```

Sympy [A] time = 0.181605, size = 24, normalized size = 0.75

$$\frac{x^2\operatorname{asin}(x)}{2}+\frac{x\sqrt{1-x^2}}{4}-\frac{\operatorname{asin}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asin(x),x)
```

```
[Out] x**2*asin(x)/2 + x*sqrt(1 - x**2)/4 - asin(x)/4
```

Giac [A] time = 1.07066, size = 35, normalized size = 1.09

$$\frac{1}{2}\left(x^2-1\right)\arcsin(x)+\frac{1}{4}\sqrt{-x^2+1}x+\frac{1}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x),x, algorithm="giac")
```

```
[Out] 1/2*(x^2 - 1)*arcsin(x) + 1/4*sqrt(-x^2 + 1)*x + 1/4*arcsin(x)
```

3.274 $\int \frac{\sqrt{9-x^2}}{x} \, dx$

Optimal. Leaf size=30

$$\sqrt{9-x^2}-3\tanh^{-1}\left(\frac{\sqrt{9-x^2}}{3}\right)$$

```
[Out] Sqrt[9 - x^2] - 3*ArcTanh[Sqrt[9 - x^2]/3]
```

Rubi [A] time = 0.0149736, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 206}

$$\sqrt{9-x^2}-3\tanh^{-1}\left(\frac{\sqrt{9-x^2}}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[9 - x^2]/x,x]
```

```
[Out] Sqrt[9 - x^2] - 3*ArcTanh[Sqrt[9 - x^2]/3]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{9-x^2}}{x}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{9-x}}{x}\,dx, x, x^2\right)\\ &= \sqrt{9-x^2}+\frac{9}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{9-x}x}\,dx, x, x^2\right)\\ &= \sqrt{9-x^2}-9\,\text{Subst}\left(\int \frac{1}{9-x^2}\,dx, x, \sqrt{9-x^2}\right)\\ &= \sqrt{9-x^2}-3\tanh^{-1}\left(\frac{\sqrt{9-x^2}}{3}\right)\end{aligned}$$

Mathematica [A] time = 0.0051874, size = 30, normalized size = 1.

$$\sqrt{9-x^2}-3\tanh^{-1}\left(\frac{\sqrt{9-x^2}}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[9 - x^2]/x,x]
```

```
[Out] Sqrt[9 - x^2] - 3*ArcTanh[Sqrt[9 - x^2]/3]
```

Maple [A] time = 0.004, size = 25, normalized size = 0.8

$$\sqrt{-x^2+9}-3\,\text{Artanh}\left(3\,\frac{1}{\sqrt{-x^2+9}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+9)^(1/2)/x,x)
```

```
[Out] (-x^2+9)^(1/2)-3*arctanh(3/(-x^2+9)^(1/2))
```

Maxima [A] time = 1.42481, size = 47, normalized size = 1.57

$$\sqrt{-x^2+9} - 3\,\log\left(\frac{6\,\sqrt{-x^2+9}}{|x|} + \frac{18}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+9)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(-x^2 + 9) - 3*log(6*sqrt(-x^2 + 9)/abs(x) + 18/abs(x))
```

Fricas [A] time = 1.87756, size = 65, normalized size = 2.17

$$\sqrt{-x^2+9} + 3\,\log\left(\frac{\sqrt{-x^2+9}-3}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+9)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(-x^2 + 9) + 3*log((sqrt(-x^2 + 9) - 3)/x)
```

Sympy [A] time = 1.30047, size = 73, normalized size = 2.43

$$\begin{cases} -\frac{x}{\sqrt{-1+\frac{9}{x^2}}} - 3\,\operatorname{acosh}\left(\frac{3}{x}\right) + \frac{9}{x\sqrt{-1+\frac{9}{x^2}}} & \text{for } \frac{9}{|x^2|} > 1 \\ \frac{ix}{\sqrt{1-\frac{9}{x^2}}} + 3i\,\operatorname{asin}\left(\frac{3}{x}\right) - \frac{9i}{x\sqrt{1-\frac{9}{x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+9)**(1/2)/x,x)
```

```
[Out] Piecewise((-x/sqrt(-1 + 9/x**2) - 3*acosh(3/x) + 9/(x*sqrt(-1 + 9/x**2)), 9
/Abs(x**2) > 1), (I*x/sqrt(1 - 9/x**2) + 3*I*asin(3/x) - 9*I/(x*sqrt(1 - 9/
x**2)), True))
```

Giac [A] time = 1.06007, size = 54, normalized size = 1.8

$$\sqrt{-x^2+9}-\frac{3}{2}\log\left(\sqrt{-x^2+9}+3\right)+\frac{3}{2}\log\left(-\sqrt{-x^2+9}+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+9)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(-x^2 + 9) - 3/2*log(sqrt(-x^2 + 9) + 3) + 3/2*log(-sqrt(-x^2 + 9) + 3)
```

3.275 $\int \frac{x}{2+3x+x^2} dx$

Optimal. Leaf size=13

$$2\log(x+2) - \log(x+1)$$

```
[Out] -Log[1 + x] + 2*Log[2 + x]
```

Rubi [A] time = 0.0034389, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {632, 31}

$$2\log(x+2) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(2 + 3*x + x^2),x]
```

```
[Out] -Log[1 + x] + 2*Log[2 + x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{2+3x+x^2} dx &= 2\int \frac{1}{2+x} dx - \int \frac{1}{1+x} dx \\ &= -\log(1+x) + 2\log(2+x) \end{aligned}$$

Mathematica [A] time = 0.0034576, size = 13, normalized size = 1.

$$2\log(x+2)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 + 3*x + x^2),x]
```

```
[Out] -Log[1 + x] + 2*Log[2 + x]
```

Maple [A] time = 0.001, size = 14, normalized size = 1.1

$$-\ln(1+x)+2\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+3*x+2),x)
```

```
[Out] -ln(1+x)+2*ln(2+x)
```

Maxima [A] time = 0.948218, size = 18, normalized size = 1.38

$$2\log(x+2)-\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 2*log(x + 2) - log(x + 1)
```

Fricas [A] time = 1.70567, size = 36, normalized size = 2.77

$$2\log(x+2)-\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 2*log(x + 2) - log(x + 1)
```

Sympy [A] time = 0.088606, size = 10, normalized size = 0.77

$$-\log(x+1)+2\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+3*x+2),x)
```

```
[Out] -log(x + 1) + 2*log(x + 2)
```

Giac [A] time = 1.08687, size = 20, normalized size = 1.54

$$2\log(|x+2|)-\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 2*log(abs(x + 2)) - log(abs(x + 1))
```

3.276 $\int x^2 \cosh(x)\, dx$

Optimal. Leaf size=16

$$x^2 \sinh(x) + 2 \sinh(x) - 2x \cosh(x)$$

```
[Out] -2*x*Cosh[x] + 2*Sinh[x] + x^2*Sinh[x]
```

Rubi [A] time = 0.0271148, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2637}

$$x^2 \sinh(x) + 2 \sinh(x) - 2x \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[x],x]
```

```
[Out] -2*x*Cosh[x] + 2*Sinh[x] + x^2*Sinh[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^2 \cosh(x)\, dx &= x^2 \sinh(x) - 2 \int x \sinh(x)\, dx \\
&= -2x \cosh(x) + x^2 \sinh(x) + 2 \int \cosh(x)\, dx \\
&= -2x \cosh(x) + 2 \sinh(x) + x^2 \sinh(x)
\end{aligned}$$

Mathematica [A] time = 0.0129836, size = 14, normalized size = 0.88

$$(x^2+2)\sinh(x)-2x\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[x],x]
```

```
[Out] -2*x*Cosh[x] + (2 + x^2)*Sinh[x]
```

Maple [A] time = 0.004, size = 17, normalized size = 1.1

$$-2\,x\cosh(x)+2\,\sinh(x)+x^2\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(x),x)
```

```
[Out] -2*x*cosh(x)+2*sinh(x)+x^2*sinh(x)
```

Maxima [B] time = 0.95329, size = 59, normalized size = 3.69

$$\frac{1}{3}\,x^3\cosh(x)-\frac{1}{6}\left(x^3+3\,x^2+6\,x+6\right)e^{(-x)}-\frac{1}{6}\left(x^3-3\,x^2+6\,x-6\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*cosh(x) - 1/6*(x^3 + 3*x^2 + 6*x + 6)*e^(-x) - 1/6*(x^3 - 3*x^2 + 6
*x - 6)*e^x
```

Fricas [A] time = 1.8497, size = 46, normalized size = 2.88

$$-2\,x\cosh(x)+(x^2+2)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x),x, algorithm="fricas")
```

```
[Out] -2*x*cosh(x) + (x^2 + 2)*sinh(x)
```

Sympy [A] time = 0.326455, size = 17, normalized size = 1.06

$$x^2 \sinh(x) - 2x \cosh(x) + 2 \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(x),x)
```

```
[Out] x**2*sinh(x) - 2*x*cosh(x) + 2*sinh(x)
```

Giac [A] time = 1.09227, size = 36, normalized size = 2.25

$$-\frac{1}{2}\left(x^2 + 2x + 2\right)e^{(-x)} + \frac{1}{2}\left(x^2 - 2x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(x),x, algorithm="giac")
```

```
[Out] -1/2*(x^2 + 2*x + 2)*e^(-x) + 1/2*(x^2 - 2*x + 2)*e^x
```

3.277 $\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

```
[Out] Log[4*x + 2*x^2 + x^4]/4
```

Rubi [A] time = 0.0092228, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1587}

$$\frac{1}{4}\log\left(x^4+2x^2+4x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

```
[Out] Log[4*x + 2*x^2 + x^4]/4
```

Rule 1587

```
Int[(Pp_)/(Qq_), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Si
mp[(Coeff[Pp, x, p]*Log[RemoveContent[Qq, x]])/(q*Coeff[Qq, x, q]), x] /; E
qQ[p, q - 1] && EqQ[Pp, Simplify[(Coeff[Pp, x, p]*D[Qq, x])/(q*Coeff[Qq, x,
 q])]]] /; PolyQ[Pp, x] && PolyQ[Qq, x]
```

Rubi steps

$$\int \frac{1+x+x^3}{4x+2x^2+x^4}\,dx=\frac{1}{4}\log\left(4x+2x^2+x^4\right)$$

Mathematica [A] time = 0.005459, size = 20, normalized size = 1.18

$$\frac{1}{4}\log\left(x^3+2x+4\right)+\frac{\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^3)/(4*x + 2*x^2 + x^4),x]
```

```
[Out] Log[x]/4 + Log[4 + 2*x + x^3]/4
```

Maple [A] time = 0.003, size = 14, normalized size = 0.8

$$\frac{\ln\left(x\left(x^{3}+2x+4\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+x+1)/(x^4+2*x^2+4*x),x)
```

```
[Out] 1/4*ln(x*(x^3+2*x+4))
```

Maxima [A] time = 0.937277, size = 20, normalized size = 1.18

$$\frac{1}{4}\log\left(x^{4}+2x^{2}+4x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="maxima")
```

```
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

Fricas [A] time = 1.88942, size = 38, normalized size = 2.24

$$\frac{1}{4}\log\left(x^{4}+2x^{2}+4x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="fricas")
```

```
[Out] 1/4*log(x^4 + 2*x^2 + 4*x)
```

Sympy [A] time = 0.091942, size = 14, normalized size = 0.82

$$\frac{\log\left(x^4+2x^2+4x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x+1)/(x**4+2*x**2+4*x),x)
```

```
[Out] log(x**4 + 2*x**2 + 4*x)/4
```

Giac [A] time = 1.06831, size = 24, normalized size = 1.41

$$\frac{1}{4}\log\left(\left|x^3+2x+4\right|\right)+\frac{1}{4}\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+x+1)/(x^4+2*x^2+4*x),x, algorithm="giac")
```

```
[Out] 1/4*log(abs(x^3 + 2*x + 4)) + 1/4*log(abs(x))
```

3.278 $\int \frac{\cos(x)}{1+\sin^2(x)} \, dx$

Optimal. Leaf size=3

$$\tan^{-1}(\sin(x))$$

```
[Out] ArcTan[Sin[x]]
```

Rubi [A] time = 0.0179629, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3190, 203}

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(1 + Sin[x]^2),x]
```

```
[Out] ArcTan[Sin[x]]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{1+\sin^2(x)} \, dx &= \text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, \sin(x)\right)\\ &= \tan^{-1}(\sin(x))\end{aligned}$$

Mathematica [A] time = 0.0041201, size = 3, normalized size = 1.

$$\tan^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(1 + Sin[x]^2),x]
```

```
[Out] ArcTan[Sin[x]]
```

Maple [A] time = 0.01, size = 4, normalized size = 1.3

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(1+sin(x)^2),x)
```

```
[Out] arctan(sin(x))
```

Maxima [A] time = 1.42151, size = 4, normalized size = 1.33

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(sin(x))
```

Fricas [A] time = 1.94026, size = 22, normalized size = 7.33

$$\arctan(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(sin(x))
```

Sympy [A] time = 0.223876, size = 3, normalized size = 1.

$$\operatorname{atan}\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)**2),x)
```

```
[Out] atan(sin(x))
```

Giac [A] time = 1.06824, size = 4, normalized size = 1.33

$$\arctan\left(\sin\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(1+sin(x)^2),x, algorithm="giac")
```

```
[Out] arctan(sin(x))
```

3.279 $\int \cos \left(\sqrt{x}\right) dx$

Optimal. Leaf size=22

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rubi [A] time = 0.0114538, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3362, 3296, 2638}

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rule 3362

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cos\left(\sqrt{x}\right) dx = 2\,\text{Subst}\left(\int x\cos(x)\,dx, x, \sqrt{x}\right)$$
$$= 2\sqrt{x}\sin\left(\sqrt{x}\right) - 2\,\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{x}\right)$$
$$= 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\sin\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.0132581, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Maple [A] time = 0., size = 17, normalized size = 0.8

$$2\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2)),x)
```

```
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Maxima [A] time = 0.940533, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Fricas [A] time = 2.03848, size = 55, normalized size = 2.5

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Sympy [A] time = 0.294535, size = 20, normalized size = 0.91

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Giac [A] time = 1.06754, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.280 $\int \sin(\pi x) \, dx$

Optimal. Leaf size=9

$$-\frac{\cos(\pi x)}{\pi}$$

```
[Out] -(Cos[Pi*x]/Pi)
```

Rubi [A] time = 0.0037205, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2638}

$$-\frac{\cos(\pi x)}{\pi}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Pi*x],x]

[Out] -(Cos[Pi*x]/Pi)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sin(\pi x) \, dx = -\frac{\cos(\pi x)}{\pi}$$

Mathematica [A] time = 0.0036049, size = 9, normalized size = 1.

$$-\frac{\cos(\pi x)}{\pi}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Pi*x],x]
```

```
[Out] -(Cos[Pi*x]/Pi)
```

Maple [A] time = 0.003, size = 10, normalized size = 1.1

$$-\frac{\cos(\pi x)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(Pi*x),x)
```

```
[Out] -cos(Pi*x)/Pi
```

Maxima [A] time = 0.967854, size = 12, normalized size = 1.33

$$-\frac{\cos(\pi x)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*x),x, algorithm="maxima")
```

```
[Out] -cos(pi*x)/pi
```

Fricas [A] time = 2.02815, size = 20, normalized size = 2.22

$$-\frac{\cos(\pi x)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*x),x, algorithm="fricas")
```

```
[Out] -cos(pi*x)/pi
```

Sympy [A] time = 0.060021, size = 7, normalized size = 0.78

$$-\frac{\cos(\pi x)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*x),x)
```

```
[Out] -cos(pi*x)/pi
```

Giac [A] time = 1.04805, size = 12, normalized size = 1.33

$$-\frac{\cos(\pi x)}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(pi*x),x, algorithm="giac")
```

```
[Out] -cos(pi*x)/pi
```

3.281 $\int \frac{e^{2x}}{1+e^x} dx$

Optimal. Leaf size=12

$$e^x - \log(e^x + 1)$$

```
[Out] E^x - Log[1 + E^x]
```

Rubi [A] time = 0.0196965, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2248, 43}

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(1 + E^x),x]

[Out] E^x - Log[1 + E^x]
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{2x}}{1+e^x} dx = \text{Subst}\left(\int \frac{x}{1+x} dx, x, e^x\right)$$
$$= \text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right) dx, x, e^x\right)$$
$$= e^x - \log\left(1+e^x\right)$$

Mathematica [A] time = 0.0074975, size = 12, normalized size = 1.

$$e^x - \log\left(e^x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(1 + E^x),x]
```

```
[Out] E^x - Log[1 + E^x]
```

Maple [A] time = 0., size = 11, normalized size = 0.9

$$e^x - \ln\left(1+e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(1+exp(x)),x)
```

```
[Out] exp(x)-ln(1+exp(x))
```

Maxima [A] time = 0.954929, size = 14, normalized size = 1.17

$$e^x - \log\left(e^x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="maxima")
```

```
[Out] e^x - log(e^x + 1)
```

Fricas [A] time = 2.00244, size = 27, normalized size = 2.25

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="fricas")
```

```
[Out] e^x - log(e^x + 1)
```

Sympy [A] time = 0.082269, size = 8, normalized size = 0.67

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x)
```

```
[Out] exp(x) - log(exp(x) + 1)
```

Giac [A] time = 1.0726, size = 14, normalized size = 1.17

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="giac")
```

```
[Out] e^x - log(e^x + 1)
```

3.282 $\int e^{3x}\cos(5x)\,dx$

Optimal. Leaf size=27

$$\frac{5}{34}e^{3x}\sin(5x)+\frac{3}{34}e^{3x}\cos(5x)$$

```
[Out] (3*E^(3*x)*Cos[5*x])/34 + (5*E^(3*x)*Sin[5*x])/34
```

Rubi [A] time = 0.0096329, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4433}

$$\frac{5}{34}e^{3x}\sin(5x)+\frac{3}{34}e^{3x}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*x)*Cos[5*x],x]
```

```
[Out] (3*E^(3*x)*Cos[5*x])/34 + (5*E^(3*x)*Sin[5*x])/34
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{3x}\cos(5x)\,dx=\frac{3}{34}e^{3x}\cos(5x)+\frac{5}{34}e^{3x}\sin(5x)$$

Mathematica [A] time = 0.0270942, size = 22, normalized size = 0.81

$$\frac{1}{34}e^{3x}(5\sin(5x)+3\cos(5x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*x)*Cos[5*x],x]
```

```
[Out] (E^(3*x)*(3*Cos[5*x] + 5*Sin[5*x]))/34
```

Maple [A] time = 0.007, size = 22, normalized size = 0.8

$$\frac{3\,\mathrm{e}^{3\,x}\cos\left(5\,x\right)}{34}+\frac{5\,\mathrm{e}^{3\,x}\sin\left(5\,x\right)}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(3*x)*cos(5*x),x)
```

```
[Out] 3/34*exp(3*x)*cos(5*x)+5/34*exp(3*x)*sin(5*x)
```

Maxima [A] time = 0.953807, size = 26, normalized size = 0.96

$$\frac{1}{34}\,(3\,\cos\left(5\,x\right)+5\,\sin\left(5\,x\right))e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*cos(5*x),x, algorithm="maxima")
```

```
[Out] 1/34*(3*cos(5*x) + 5*sin(5*x))*e^(3*x)
```

Fricas [A] time = 1.96981, size = 63, normalized size = 2.33

$$\frac{3}{34}\,\cos\left(5\,x\right)e^{(3\,x)}+\frac{5}{34}\,e^{(3\,x)}\sin\left(5\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*cos(5*x),x, algorithm="fricas")
```

```
[Out] 3/34*cos(5*x)*e^(3*x) + 5/34*e^(3*x)*sin(5*x)
```

Sympy [A] time = 0.296948, size = 26, normalized size = 0.96

$$\frac{5e^{3x}\sin(5x)}{34}+\frac{3e^{3x}\cos(5x)}{34}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*cos(5*x),x)
```

```
[Out] 5*exp(3*x)*sin(5*x)/34 + 3*exp(3*x)*cos(5*x)/34
```

Giac [A] time = 1.05933, size = 26, normalized size = 0.96

$$\frac{1}{34}\,(3\,\cos(5\,x)+5\,\sin(5\,x))e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*cos(5*x),x, algorithm="giac")
```

```
[Out] 1/34*(3*cos(5*x) + 5*sin(5*x))*e^(3*x)
```

3.283 $\int \cos(3x)\cos(5x)\,dx$

Optimal. Leaf size=17

$$\frac{1}{4}\sin(2x)+\frac{1}{16}\sin(8x)$$

```
[Out] Sin[2*x]/4 + Sin[8*x]/16
```

Rubi [A] time = 0.0078319, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4283}

$$\frac{1}{4}\sin(2x)+\frac{1}{16}\sin(8x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]*Cos[5*x],x]
```

```
[Out] Sin[2*x]/4 + Sin[8*x]/16
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(3x)\cos(5x)\,dx=\frac{1}{4}\sin(2x)+\frac{1}{16}\sin(8x)$$

Mathematica [A] time = 0.0070379, size = 17, normalized size = 1.

$$\frac{1}{4}\sin(2x)+\frac{1}{16}\sin(8x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]*Cos[5*x],x]
```

```
[Out] Sin[2*x]/4 + Sin[8*x]/16
```

Maple [A] time = 0.041, size = 14, normalized size = 0.8

$$\frac{\sin(2x)}{4} + \frac{\sin(8x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*cos(5*x),x)
```

```
[Out] 1/4*sin(2*x)+1/16*sin(8*x)
```

Maxima [A] time = 0.948322, size = 18, normalized size = 1.06

$$\frac{1}{16}\sin(8x) + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(5*x),x, algorithm="maxima")
```

```
[Out] 1/16*sin(8*x) + 1/4*sin(2*x)
```

Fricas [A] time = 2.00115, size = 65, normalized size = 3.82

$$\left(8\cos(x)^7 - 12\cos(x)^5 + 5\cos(x)^3\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(5*x),x, algorithm="fricas")
```

```
[Out] (8*cos(x)^7 - 12*cos(x)^5 + 5*cos(x)^3)*sin(x)
```

Sympy [B] time = 0.532001, size = 26, normalized size = 1.53

$$-\frac{3\sin(3x)\cos(5x)}{16}+\frac{5\sin(5x)\cos(3x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(5*x),x)
```

```
[Out] -3*sin(3*x)*cos(5*x)/16 + 5*sin(5*x)*cos(3*x)/16
```

Giac [A] time = 1.05692, size = 18, normalized size = 1.06

$$\frac{1}{16}\sin(8x)+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*cos(5*x),x, algorithm="giac")
```

```
[Out] 1/16*sin(8*x) + 1/4*sin(2*x)
```

3.284 $\int \frac{1}{1+x+x^2+x^3}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rubi [A] time = 0.0136644, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2058, 635, 203, 260}

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + x^2 + x^3)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x+x^2+x^3}\,dx &= \int \left(\frac{1}{2(1+x)}+\frac{1-x}{2\left(1+x^2\right)}\right)dx\\ &= \frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1-x}{1+x^2}\,dx\\ &= \frac{1}{2}\log(1+x)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx-\frac{1}{2}\int \frac{x}{1+x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log(1+x)-\frac{1}{4}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0050561, size = 25, normalized size = 1.

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + x^2 + x^3)^(-1),x]
```

```
[Out] ArcTan[x]/2 + Log[1 + x]/2 - Log[1 + x^2]/4
```

Maple [A] time = 0.004, size = 20, normalized size = 0.8

$$\frac{\arctan(x)}{2}+\frac{\ln(1+x)}{2}-\frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+x^2+x+1),x)
```

```
[Out] 1/2*arctan(x)+1/2*ln(1+x)-1/4*ln(x^2+1)
```

Maxima [A] time = 1.44708, size = 26, normalized size = 1.04

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="maxima")

[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Fricas [A] time = 1.81518, size = 69, normalized size = 2.76

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="fricas")

[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x + 1)
```

Sympy [A] time = 0.11245, size = 19, normalized size = 0.76

$$\frac{\log(x+1)}{2} - \frac{\log\left(x^2+1\right)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+x**2+x+1),x)

[Out] log(x + 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

Giac [A] time = 1.06655, size = 27, normalized size = 1.08

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+x^2+x+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x + 1))
```

3.285 $\int x^2 \log(1+x)\,dx$

Optimal. Leaf size=39

$$-\frac{x^3}{9}+\frac{x^2}{6}+\frac{1}{3}x^3\log(x+1)-\frac{x}{3}+\frac{1}{3}\log(x+1)$$

```
[Out] -x/3 + x^2/6 - x^3/9 + Log[1 + x]/3 + (x^3*Log[1 + x])/3
```

Rubi [A] time = 0.0150372, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2395, 43}

$$-\frac{x^3}{9}+\frac{x^2}{6}+\frac{1}{3}x^3\log(x+1)-\frac{x}{3}+\frac{1}{3}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[1 + x],x]
```

```
[Out] -x/3 + x^2/6 - x^3/9 + Log[1 + x]/3 + (x^3*Log[1 + x])/3
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2 \log(1+x)\,dx = \frac{1}{3}x^3\log(1+x) - \frac{1}{3}\int \frac{x^3}{1+x}\,dx$$
$$= \frac{1}{3}x^3\log(1+x) - \frac{1}{3}\int \left(1 + \frac{1}{-1-x} - x + x^2\right)dx$$
$$= -\frac{x}{3} + \frac{x^2}{6} - \frac{x^3}{9} + \frac{1}{3}\log(1+x) + \frac{1}{3}x^3\log(1+x)$$

Mathematica [A] time = 0.0092566, size = 28, normalized size = 0.72

$$\frac{1}{18}\left(x\left(-2x^2+3x-6\right)+6\left(x^3+1\right)\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[1 + x],x]
```

```
[Out] (x*(-6 + 3*x - 2*x^2) + 6*(1 + x^3)*Log[1 + x])/18
```

Maple [A] time = 0.002, size = 46, normalized size = 1.2

$$\frac{(1+x)^3\ln(1+x)}{3} - \frac{x^3}{9} + \frac{x^2}{6} - \frac{x}{3} - \frac{11}{18} - (1+x)^2\ln(1+x) + (1+x)\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(1+x),x)
```

```
[Out] 1/3*(1+x)^3*ln(1+x)-1/9*x^3+1/6*x^2-1/3*x-11/18-(1+x)^2*ln(1+x)+(1+x)*ln(1+
x)
```

Maxima [A] time = 0.953137, size = 39, normalized size = 1.

$$\frac{1}{3}x^3\log(x+1) - \frac{1}{9}x^3 + \frac{1}{6}x^2 - \frac{1}{3}x + \frac{1}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(1+x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(x + 1) - 1/9*x^3 + 1/6*x^2 - 1/3*x + 1/3*log(x + 1)
```

Fricas [A] time = 1.91516, size = 74, normalized size = 1.9

$$-\frac{1}{9}x^3 + \frac{1}{6}x^2 + \frac{1}{3}\left(x^3+1\right)\log\left(x+1\right) - \frac{1}{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(1+x),x, algorithm="fricas")
```

```
[Out] -1/9*x^3 + 1/6*x^2 + 1/3*(x^3 + 1)*log(x + 1) - 1/3*x
```

Sympy [A] time = 0.107155, size = 29, normalized size = 0.74

$$\frac{x^3\log\left(x+1\right)}{3} - \frac{x^3}{9} + \frac{x^2}{6} - \frac{x}{3} + \frac{\log\left(x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(1+x),x)
```

```
[Out] x**3*log(x + 1)/3 - x**3/9 + x**2/6 - x/3 + log(x + 1)/3
```

Giac [A] time = 1.04495, size = 66, normalized size = 1.69

$$\frac{1}{3}\left(x+1\right)^3\log\left(x+1\right) - \frac{1}{9}\left(x+1\right)^3 - \left(x+1\right)^2\log\left(x+1\right) + \frac{1}{2}\left(x+1\right)^2 + \left(x+1\right)\log\left(x+1\right) - x - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(1+x),x, algorithm="giac")
```

```
[Out] 1/3*(x + 1)^3*log(x + 1) - 1/9*(x + 1)^3 - (x + 1)^2*log(x + 1) + 1/2*(x +
1)^2 + (x + 1)*log(x + 1) - x - 1
```

3.286 $\int e^{-x^3} x^5 \, dx$

Optimal. Leaf size=26

$$-\frac{1}{3}e^{-x^3}x^3 - \frac{e^{-x^3}}{3}$$

```
[Out] -1/(3*E^x^3) - x^3/(3*E^x^3)
```

Rubi [A] time = 0.0266444, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2212, 2209}

$$-\frac{1}{3}e^{-x^3}x^3 - \frac{e^{-x^3}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/E^x^3,x]
```

```
[Out] -1/(3*E^x^3) - x^3/(3*E^x^3)
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int e^{-x^3}x^5\,dx = -\frac{1}{3}e^{-x^3}x^3 + \int e^{-x^3}x^2\,dx$$
$$= -\frac{e^{-x^3}}{3} - \frac{1}{3}e^{-x^3}x^3$$

Mathematica [A] time = 0.0030772, size = 16, normalized size = 0.62

$$-\frac{1}{3}e^{-x^3}\left(x^3+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/E^x^3,x]
```

```
[Out] -(1 + x^3)/(3*E^x^3)
```

Maple [A] time = 0.002, size = 14, normalized size = 0.5

$$-\frac{x^3+1}{3\,\mathrm{e}^{x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/exp(x^3),x)
```

```
[Out] -1/3*(x^3+1)/exp(x^3)
```

Maxima [A] time = 0.96886, size = 18, normalized size = 0.69

$$-\frac{1}{3}\left(x^3+1\right)e^{\left(-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/exp(x^3),x, algorithm="maxima")
```

```
[Out] -1/3*(x^3 + 1)*e^(-x^3)
```

Fricas [A] time = 1.92135, size = 34, normalized size = 1.31

$$-\frac{1}{3}\left(x^3+1\right)e^{\left(-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/exp(x^3),x, algorithm="fricas")
```

```
[Out] -1/3*(x^3 + 1)*e^(-x^3)
```

Sympy [A] time = 0.08548, size = 12, normalized size = 0.46

$$\frac{\left(-x^3-1\right)e^{-x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/exp(x**3),x)
```

```
[Out] (-x**3 - 1)*exp(-x**3)/3
```

Giac [A] time = 1.04592, size = 18, normalized size = 0.69

$$-\frac{1}{3}\left(x^3+1\right)e^{\left(-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/exp(x^3),x, algorithm="giac")
```

```
[Out] -1/3*(x^3 + 1)*e^(-x^3)
```

3.287 $\int \tan^2(4x)\,dx$

Optimal. Leaf size=12

$$\frac{1}{4}\tan(4x) - x$$

```
[Out] -x + Tan[4*x]/4
```

Rubi [A] time = 0.0053044, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3473, 8}

$$\frac{1}{4}\tan(4x) - x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[4*x]^2,x]
```

```
[Out] -x + Tan[4*x]/4
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \tan^2(4x)\,dx &= \frac{1}{4}\tan(4x) - \int 1\,dx \\ &= -x + \frac{1}{4}\tan(4x) \end{aligned}$$

Mathematica [A] time = 0.0081634, size = 18, normalized size = 1.5

$$\frac{1}{4}\tan(4x) - \frac{1}{4}\tan^{-1}(\tan(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[4*x]^2,x]
```

```
[Out] -ArcTan[Tan[4*x]]/4 + Tan[4*x]/4
```

Maple [A] time = 0.003, size = 11, normalized size = 0.9

$$-x + \frac{\tan(4x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(4*x)^2,x)
```

```
[Out] -x+1/4*tan(4*x)
```

Maxima [A] time = 1.46556, size = 14, normalized size = 1.17

$$-x + \frac{1}{4}\tan(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(4*x)^2,x, algorithm="maxima")
```

```
[Out] -x + 1/4*tan(4*x)
```

Fricas [A] time = 2.01629, size = 26, normalized size = 2.17

$$-x + \frac{1}{4}\tan(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(4*x)^2,x, algorithm="fricas")
```

```
[Out] -x + 1/4*tan(4*x)
```

Sympy [A] time = 0.06253, size = 12, normalized size = 1.

$$-x+\frac{\sin(4x)}{4\cos(4x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(4*x)**2,x)
```

```
[Out] -x + sin(4*x)/(4*cos(4*x))
```

Giac [A] time = 1.06407, size = 14, normalized size = 1.17

$$-x+\frac{1}{4}\tan(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(4*x)^2,x, algorithm="giac")
```

```
[Out] -x + 1/4*tan(4*x)
```

3.288 $\int \frac{1}{\sqrt{-5+12x+9x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+2}{\sqrt{9x^2+12x-5}}\right)$$

```
[Out] ArcTanh[(2 + 3*x)/Sqrt[-5 + 12*x + 9*x^2]]/3
```

Rubi [A] time = 0.0054843, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {621, 206}

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+2}{\sqrt{9x^2+12x-5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-5 + 12*x + 9*x^2],x]
```

```
[Out] ArcTanh[(2 + 3*x)/Sqrt[-5 + 12*x + 9*x^2]]/3
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-5+12x+9x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{36-x^2}\,dx, x, \frac{12+18x}{\sqrt{-5+12x+9x^2}}\right)\\ &= \frac{1}{3}\tanh^{-1}\left(\frac{2+3x}{\sqrt{-5+12x+9x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0058602, size = 24, normalized size = 0.96

$$\frac{1}{3}\log\left(\sqrt{9x^2+12x-5}+3x+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-5 + 12*x + 9*x^2],x]
```

```
[Out] Log[2 + 3*x + Sqrt[-5 + 12*x + 9*x^2]]/3
```

Maple [A] time = 0.004, size = 30, normalized size = 1.2

$$\frac{\sqrt{9}}{9}\ln\left(\frac{(6+9x)\sqrt{9}}{9}+\sqrt{9x^2+12x-5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2+12*x-5)^(1/2),x)
```

```
[Out] 1/9*ln(1/9*(6+9*x)*9^(1/2)+(9*x^2+12*x-5)^(1/2))*9^(1/2)
```

Maxima [A] time = 1.4183, size = 30, normalized size = 1.2

$$\frac{1}{3}\log\left(18x+6\sqrt{9x^2+12x-5}+12\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x-5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*log(18*x + 6*sqrt(9*x^2 + 12*x - 5) + 12)
```

Fricas [A] time = 1.98372, size = 61, normalized size = 2.44

$$-\frac{1}{3}\log\left(-3x+\sqrt{9x^2+12x-5}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x-5)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*log(-3*x + sqrt(9*x^2 + 12*x - 5) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{9x^2 + 12x - 5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2+12*x-5)**(1/2),x)
```

```
[Out] Integral(1/sqrt(9*x**2 + 12*x - 5), x)
```

Giac [A] time = 1.07072, size = 28, normalized size = 1.12

$$-\frac{1}{3} \log\left(\left|-3\,x + \sqrt{9\,x^2 + 12\,x - 5} - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2+12*x-5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*log(abs(-3*x + sqrt(9*x^2 + 12*x - 5) - 2))
```

3.289 $\int x^2 \tan^{-1}(x)\, dx$

Optimal. Leaf size=27

$$-\frac{x^2}{6}+\frac{1}{6}\log\left(x^2+1\right)+\frac{1}{3}x^3\tan^{-1}(x)$$

```
[Out] -x^2/6 + (x^3*ArcTan[x])/3 + Log[1 + x^2]/6
```

Rubi [A] time = 0.0154581, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4852, 266, 43}

$$-\frac{x^2}{6}+\frac{1}{6}\log\left(x^2+1\right)+\frac{1}{3}x^3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[x],x]
```

```
[Out] -x^2/6 + (x^3*ArcTan[x])/3 + Log[1 + x^2]/6
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2 \tan^{-1}(x)\,dx &= \frac{1}{3}x^3\tan^{-1}(x)-\frac{1}{3}\int \frac{x^3}{1+x^2}\,dx\\
&= \frac{1}{3}x^3\tan^{-1}(x)-\frac{1}{6}\,\text{Subst}\left(\int \frac{x}{1+x}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\tan^{-1}(x)-\frac{1}{6}\,\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx,x,x^2\right)\\
&= -\frac{x^2}{6}+\frac{1}{3}x^3\tan^{-1}(x)+\frac{1}{6}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0044701, size = 23, normalized size = 0.85

$$\frac{1}{6}\left(-x^2+\log\left(x^2+1\right)+2x^3\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[x],x]
```

```
[Out] (-x^2 + 2*x^3*ArcTan[x] + Log[1 + x^2])/6
```

Maple [A] time = 0.002, size = 22, normalized size = 0.8

$$-\frac{x^2}{6}+\frac{x^3\arctan(x)}{3}+\frac{\ln\left(x^2+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x),x)
```

```
[Out] -1/6*x^2+1/3*x^3*arctan(x)+1/6*ln(x^2+1)
```

Maxima [A] time = 0.938885, size = 28, normalized size = 1.04

$$\frac{1}{3}x^3\arctan(x)-\frac{1}{6}x^2+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

Fricas [A] time = 1.97857, size = 65, normalized size = 2.41

$$\frac{1}{3}\,x^3\arctan\left(x\right)-\frac{1}{6}\,x^2+\frac{1}{6}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

Sympy [A] time = 0.340051, size = 20, normalized size = 0.74

$$\frac{x^3\operatorname{atan}\left(x\right)}{3}-\frac{x^2}{6}+\frac{\log\left(x^2+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x),x)
```

```
[Out] x**3*atan(x)/3 - x**2/6 + log(x**2 + 1)/6
```

Giac [A] time = 1.06126, size = 28, normalized size = 1.04

$$\frac{1}{3}\,x^3\arctan\left(x\right)-\frac{1}{6}\,x^2+\frac{1}{6}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

3.290 $\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=19

$$\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}$$

```
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Rubi [A] time = 0.003367, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {14}

$$\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[x])/x^(1/3),x]
```

```
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{1-\sqrt{x}}{\sqrt[3]{x}}\,dx &= \int \left(\frac{1}{\sqrt[3]{x}}-\sqrt[6]{x}\right)dx\\ &= \frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}\end{aligned}$$

Mathematica [A] time = 0.0034718, size = 19, normalized size = 1.

$$\frac{3x^{2/3}}{2}-\frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[x])/x^(1/3),x]
```

```
[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7
```

Maple [A] time = 0.002, size = 12, normalized size = 0.6

$$\frac{3}{2}x^{\frac{2}{3}} - \frac{6}{7}x^{\frac{7}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x^(1/2))/x^(1/3),x)
```

```
[Out] 3/2*x^(2/3)-6/7*x^(7/6)
```

Maxima [A] time = 0.939876, size = 15, normalized size = 0.79

$$-\frac{6}{7}x^{\frac{7}{6}} + \frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="maxima")
```

```
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

Fricas [A] time = 1.90134, size = 38, normalized size = 2.

$$-\frac{6}{7}x^{\frac{7}{6}} + \frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="fricas")
```

```
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

Sympy [A] time = 1.85478, size = 15, normalized size = 0.79

$$-\frac{6x^{\frac{7}{6}}}{7}+\frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x**(1/2))/x**(1/3),x)
```

```
[Out] -6*x**(7/6)/7 + 3*x**(2/3)/2
```

Giac [A] time = 1.05121, size = 15, normalized size = 0.79

$$-\frac{6}{7}\,x^{\frac{7}{6}}+\frac{3}{2}\,x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="giac")
```

```
[Out] -6/7*x^(7/6) + 3/2*x^(2/3)
```

3.291 $\int \frac{1}{-e^{-x}+e^x} \, dx$

Optimal. Leaf size=6

$$-\tanh^{-1}\left(e^x\right)$$

```
[Out] -ArcTanh[E^x]
```

Rubi [A] time = 0.0095504, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2282, 207}

$$-\tanh^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-E^(-x) + E^x)^(-1),x]
```

```
[Out] -ArcTanh[E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-e^{-x}+e^x} \, dx &= \text{Subst}\left(\int \frac{1}{-1+x^2} \, dx,x,e^x\right)\\ &= -\tanh^{-1}\left(e^x\right)\end{aligned}$$

Mathematica [A] time = 0.0027007, size = 6, normalized size = 1.

$$-\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-E^(-x) + E^x)^(-1),x]
```

```
[Out] -ArcTanh[E^x]
```

Maple [A] time = 0.002, size = 6, normalized size = 1.

$$-\mathrm{Artanh}(\mathrm{e}^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1/exp(x)+exp(x)),x)
```

```
[Out] -arctanh(exp(x))
```

Maxima [B] time = 0.935972, size = 26, normalized size = 4.33

$$-\frac{1}{2}\log\left(e^{(-x)}+1\right)+\frac{1}{2}\log\left(e^{(-x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/exp(x)+exp(x)),x, algorithm="maxima")
```

```
[Out] -1/2*log(e^(-x) + 1) + 1/2*log(e^(-x) - 1)
```

Fricas [B] time = 1.90714, size = 51, normalized size = 8.5

$$-\frac{1}{2}\log(e^x+1)+\frac{1}{2}\log(e^x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/exp(x)+exp(x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Sympy [B] time = 0.101435, size = 19, normalized size = 3.17

$$\frac{\log(-1+e^{-x})}{2}-\frac{\log(1+e^{-x})}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/exp(x)+exp(x)),x)
```

```
[Out] log(-1 + exp(-x))/2 - log(1 + exp(-x))/2
```

Giac [B] time = 1.05135, size = 22, normalized size = 3.67

$$-\frac{1}{2}\log(e^x+1)+\frac{1}{2}\log(|e^x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1/exp(x)+exp(x)),x, algorithm="giac")
```

```
[Out] -1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

3.292 $\int \frac{x}{10+2x^2+x^4}\,dx$

Optimal. Leaf size=14

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

```
[Out] ArcTan[(1 + x^2)/3]/6
```

Rubi [A] time = 0.013418, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1107, 618, 204}

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(10 + 2*x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + x^2)/3]/6
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{10+2x^2+x^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{10+2x+x^2}\,dx,x,x^2\right)\\
&= -\,\text{Subst}\left(\int \frac{1}{-36-x^2}\,dx,x,2\left(1+x^2\right)\right)\\
&= \frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(1+x^2\right)\right)
\end{aligned}
$$

Mathematica [A] time = 0.0043375, size = 14, normalized size = 1.

$$\frac{1}{6}\tan^{-1}\left(\frac{1}{3}\left(x^2+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(10 + 2*x^2 + x^4),x]
```

```
[Out] ArcTan[(1 + x^2)/3]/6
```

Maple [A] time = 0.002, size = 11, normalized size = 0.8

$$\frac{1}{6}\arctan\left(\frac{x^2}{3}+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+2*x^2+10),x)
```

```
[Out] 1/6*arctan(1/3*x^2+1/3)
```

Maxima [A] time = 1.41837, size = 14, normalized size = 1.

$$\frac{1}{6}\arctan\left(\frac{1}{3}x^2+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="maxima")
```

```
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

Fricas [A] time = 1.83923, size = 36, normalized size = 2.57

$$\frac{1}{6} \arctan\left(\frac{1}{3} x^2 + \frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="fricas")
```

```
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

Sympy [A] time = 0.095847, size = 10, normalized size = 0.71

$$\frac{\operatorname{atan}\left(\frac{x^2}{3} + \frac{1}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+2*x**2+10),x)
```

```
[Out] atan(x**2/3 + 1/3)/6
```

Giac [A] time = 1.05179, size = 14, normalized size = 1.

$$\frac{1}{6} \arctan\left(\frac{1}{3} x^2 + \frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+2*x^2+10),x, algorithm="giac")
```

```
[Out] 1/6*arctan(1/3*x^2 + 1/3)
```

3.293 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} \, dx$

Optimal. Leaf size=12

$$\frac{3}{4} \log \left(x^{4/3}+1\right)$$

```
[Out] (3*Log[1 + x^(4/3)])/4
```

Rubi [A] time = 0.0034391, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1593, 260}

$$\frac{3}{4} \log \left(x^{4/3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1/3) + x)^(-1),x]
```

```
[Out] (3*Log[1 + x^(4/3)])/4
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\frac{1}{\sqrt[3]{x}}+x} \, dx &= \int \frac{\sqrt[3]{x}}{1+x^{4/3}} \, dx \\ &= \frac{3}{4} \log \left(1+x^{4/3}\right) \end{aligned}$$

Mathematica [A] time = 0.0017443, size = 12, normalized size = 1.

$$\frac{3}{4}\log\left(x^{4/3}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1/3) + x)^(-1),x]
```

```
[Out] (3*Log[1 + x^(4/3)])/4
```

Maple [A] time = 0.003, size = 9, normalized size = 0.8

$$\frac{3}{4}\ln\left(1+x^{\frac{4}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1/x^(1/3)+x),x)
```

```
[Out] 3/4*ln(1+x^(4/3))
```

Maxima [A] time = 1.41533, size = 11, normalized size = 0.92

$$\frac{3}{4}\log\left(x^{\frac{4}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="maxima")
```

```
[Out] 3/4*log(x^(4/3) + 1)
```

Fricas [A] time = 2.1519, size = 30, normalized size = 2.5

$$\frac{3}{4}\log\left(x^{\frac{4}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="fricas")
```

```
[Out] 3/4*log(x^(4/3) + 1)
```

Sympy [A] time = 0.217919, size = 10, normalized size = 0.83

$$\frac{3\log\left(x^{\frac{4}{3}}+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x**(1/3)+x),x)
```

```
[Out] 3*log(x**(4/3) + 1)/4
```

Giac [B] time = 1.05467, size = 43, normalized size = 3.58

$$\frac{3}{4}\log\left(\sqrt{2}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)+\frac{3}{4}\log\left(-\sqrt{2}x^{\frac{1}{3}}+x^{\frac{2}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1/x^(1/3)+x),x, algorithm="giac")
```

```
[Out] 3/4*log(sqrt(2)*x^(1/3) + x^(2/3) + 1) + 3/4*log(-sqrt(2)*x^(1/3) + x^(2/3)
 + 1)
```

3.294 $\int \cos^4(x)\sin^2(x)\,dx$

Optimal. Leaf size=34

$$\frac{x}{16}-\frac{1}{6}\sin(x)\cos^5(x)+\frac{1}{24}\sin(x)\cos^3(x)+\frac{1}{16}\sin(x)\cos(x)$$

```
[Out] x/16 + (Cos[x]*Sin[x])/16 + (Cos[x]^3*Sin[x])/24 - (Cos[x]^5*Sin[x])/6
```

Rubi [A] time = 0.0326611, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{x}{16}-\frac{1}{6}\sin(x)\cos^5(x)+\frac{1}{24}\sin(x)\cos^3(x)+\frac{1}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4*Sin[x]^2,x]
```

```
[Out] x/16 + (Cos[x]*Sin[x])/16 + (Cos[x]^3*Sin[x])/24 - (Cos[x]^5*Sin[x])/6
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(x)\sin^2(x)\,dx &= -\frac{1}{6}\cos^5(x)\sin(x)+\frac{1}{6}\int \cos^4(x)\,dx\\ &= \frac{1}{24}\cos^3(x)\sin(x)-\frac{1}{6}\cos^5(x)\sin(x)+\frac{1}{8}\int \cos^2(x)\,dx\\ &= \frac{1}{16}\cos(x)\sin(x)+\frac{1}{24}\cos^3(x)\sin(x)-\frac{1}{6}\cos^5(x)\sin(x)+\frac{\int 1\,dx}{16}\\ &= \frac{x}{16}+\frac{1}{16}\cos(x)\sin(x)+\frac{1}{24}\cos^3(x)\sin(x)-\frac{1}{6}\cos^5(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0078148, size = 30, normalized size = 0.88

$$\frac{x}{16}+\frac{1}{64}\sin(2x)-\frac{1}{64}\sin(4x)-\frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4*Sin[x]^2,x]
```

```
[Out] x/16 + Sin[2*x]/64 - Sin[4*x]/64 - Sin[6*x]/192
```

Maple [A] time = 0.001, size = 26, normalized size = 0.8

$$-\frac{(\cos(x))^5\sin(x)}{6}+\frac{\sin(x)}{24}\left((\cos(x))^3+\frac{3\cos(x)}{2}\right)+\frac{x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4*sin(x)^2,x)
```

```
[Out] -1/6*cos(x)^5*sin(x)+1/24*(cos(x)^3+3/2*cos(x))*sin(x)+1/16*x
```

Maxima [A] time = 0.937827, size = 24, normalized size = 0.71

$$\frac{1}{48}\sin(2x)^3+\frac{1}{16}x-\frac{1}{64}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/48*sin(2*x)^3 + 1/16*x - 1/64*sin(4*x)
```

Fricas [A] time = 2.30492, size = 81, normalized size = 2.38

$$-\frac{1}{48}\left(8\cos\left(x\right)^{5}-2\cos\left(x\right)^{3}-3\cos\left(x\right)\right)\sin\left(x\right)+\frac{1}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(x)^5 - 2*cos(x)^3 - 3*cos(x))*sin(x) + 1/16*x
```

Sympy [A] time = 0.060285, size = 31, normalized size = 0.91

$$\frac{x}{16}-\frac{\sin\left(x\right)\cos^{5}\left(x\right)}{6}+\frac{\sin\left(x\right)\cos^{3}\left(x\right)}{24}+\frac{\sin\left(x\right)\cos\left(x\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4*sin(x)**2,x)
```

```
[Out] x/16 - sin(x)*cos(x)**5/6 + sin(x)*cos(x)**3/24 + sin(x)*cos(x)/16
```

Giac [A] time = 1.06386, size = 30, normalized size = 0.88

$$\frac{1}{16}x-\frac{1}{192}\sin\left(6x\right)-\frac{1}{64}\sin\left(4x\right)+\frac{1}{64}\sin\left(2x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/16*x - 1/192*sin(6*x) - 1/64*sin(4*x) + 1/64*sin(2*x)
```

3.295 $\int \frac{1}{\sqrt{5-4x-x^2}}\,dx$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{3}(-x-2)\right)$$

```
[Out] -ArcSin[(-2 - x)/3]
```

Rubi [A] time = 0.0066545, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$-\sin^{-1}\left(\frac{1}{3}(-x-2)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[5 - 4*x - x^2],x]
```

```
[Out] -ArcSin[(-2 - x)/3]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{5-4x-x^2}}\,dx &= -\left(\frac{1}{6}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx, x, -4-2x\right)\right)\\ &= -\sin^{-1}\left(\frac{1}{3}(-2-x)\right)\end{aligned}$$

Mathematica [A] time = 0.0060933, size = 12, normalized size = 1.

$$-\sin^{-1}\left(\frac{1}{3}(-x-2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[5 - 4*x - x^2],x]
```

```
[Out] -ArcSin[(-2 - x)/3]
```

Maple [A] time = 0.003, size = 7, normalized size = 0.6

$$\arcsin\left(\frac{2}{3}+\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-4*x+5)^(1/2),x)
```

```
[Out] arcsin(2/3+1/3*x)
```

Maxima [A] time = 1.40417, size = 11, normalized size = 0.92

$$-\arcsin\left(-\frac{1}{3}x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/3*x - 2/3)
```

Fricas [B] time = 2.10494, size = 74, normalized size = 6.17

$$-\arctan\left(\frac{\sqrt{-x^2-4x+5}(x+2)}{x^2+4x-5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(sqrt(-x^2 - 4*x + 5)*(x + 2)/(x^2 + 4*x - 5))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2 - 4x + 5}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-4*x+5)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 - 4*x + 5), x)
```

Giac [A] time = 1.06393, size = 8, normalized size = 0.67

$$\arcsin\left(\frac{1}{3}\, x + \frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-4*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(1/3*x + 2/3)
```

3.296 $\int \frac{x}{1-x^2+\sqrt{1-x^2}}\,dx$

Optimal. Leaf size=16

$$-\log\left(\sqrt{1-x^2}+1\right)$$

```
[Out] -Log[1 + Sqrt[1 - x^2]]
```

Rubi [A] time = 0.04629, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2155, 31}

$$-\log\left(\sqrt{1-x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^2 + Sqrt[1 - x^2]),x]
```

```
[Out] -Log[1 + Sqrt[1 - x^2]]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x}{1-x^2+\sqrt{1-x^2}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+\sqrt{1-x}-x}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt{1-x^2}\right)$$
$$= -\log\left(1+\sqrt{1-x^2}\right)$$

Mathematica [A] time = 0.0302715, size = 16, normalized size = 1.

$$-\log\left(\sqrt{1-x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^2 + Sqrt[1 - x^2]),x]
```

```
[Out] -Log[1 + Sqrt[1 - x^2]]
```

Maple [B] time = 0.016, size = 59, normalized size = 3.7

$$-\ln(x)+\sqrt{-x^2+1}-\text{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)-\frac{1}{2}\sqrt{-(1+x)^2+2+2x}-\frac{1}{2}\sqrt{-(-1+x)^2+2-2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1-x^2+(-x^2+1)^(1/2)),x)
```

```
[Out] -ln(x)+(-x^2+1)^(1/2)-arctanh(1/(-x^2+1)^(1/2))-1/2*(-(1+x)^2+2+2*x)^(1/2)-
1/2*(-(-1+x)^2+2-2*x)^(1/2)
```

Maxima [A] time = 0.928118, size = 19, normalized size = 1.19

$$-\log\left(\sqrt{-x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x^2+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] -log(sqrt(-x^2 + 1) + 1)
```

Fricas [A] time = 2.02024, size = 53, normalized size = 3.31

$$-\log(x) + \log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x^2+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -log(x) + log((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [B] time = 2.74743, size = 44, normalized size = 2.75

$$\frac{\log\left(2\sqrt{1-x^2}\right)}{2} - \frac{\log\left(2\sqrt{1-x^2}+2\right)}{2} - \frac{\log\left(x^2-\sqrt{1-x^2}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x**2+(-x**2+1)**(1/2)),x)
```

```
[Out] log(2*sqrt(1 - x**2))/2 - log(2*sqrt(1 - x**2) + 2)/2 - log(x**2 - sqrt(1 -
 x**2) - 1)/2
```

Giac [A] time = 1.05177, size = 19, normalized size = 1.19

$$-\log\left(\sqrt{-x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1-x^2+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] -log(sqrt(-x^2 + 1) + 1)
```

3.297 $\int (1+\cos(x))\csc(x)\,dx$

Optimal. Leaf size=7

$$\log(1-\cos(x))$$

[Out] Log[1 - Cos[x]]

Rubi [A] time = 0.013242, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2667, 31}

$$\log(1-\cos(x))$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[x])*Csc[x],x]

[Out] Log[1 - Cos[x]]

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int (1+\cos(x))\csc(x)\,dx &= -\text{Subst}\left(\int \frac{1}{1-x}\,dx, x, \cos(x)\right)\\ &= \log(1-\cos(x))\end{aligned}$$

Mathematica [B] time = 0.0054386, size = 20, normalized size = 2.86

$$\log\left(\sin\left(\frac{x}{2}\right)\right)+\log(\sin(x))-\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])*Csc[x],x]
```

```
[Out] -Log[Cos[x/2]] + Log[Sin[x/2]] + Log[Sin[x]]
```

Maple [A] time = 0.014, size = 6, normalized size = 0.9

$$\ln(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+1)*csc(x),x)
```

```
[Out] ln(cos(x)-1)
```

Maxima [A] time = 0.928996, size = 7, normalized size = 1.

$$\log(\cos(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="maxima")
```

```
[Out] log(cos(x) - 1)
```

Fricas [A] time = 2.14859, size = 32, normalized size = 4.57

$$\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="fricas")
```

```
[Out] log(-1/2*cos(x) + 1/2)
```

Sympy [B] time = 2.02163, size = 12, normalized size = 1.71

$$-\log(\cot(x) + \csc(x)) + \log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x)
```

```
[Out] -log(cot(x) + csc(x)) + log(sin(x))
```

Giac [A] time = 1.05414, size = 9, normalized size = 1.29

$$\log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))*csc(x),x, algorithm="giac")
```

```
[Out] log(-cos(x) + 1)
```

3.298 $\int \frac{e^x}{-1+e^{2x}} \, dx$

Optimal. Leaf size=6

$$-\tanh^{-1}(e^x)$$

[Out] -ArcTanh[E^x]

Rubi [A] time = 0.0167741, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2249, 207}

$$-\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(-1 + E^(2*x)),x]
```

```
[Out] -ArcTanh[E^x]
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{e^x}{-1+e^{2x}} \, dx &= \text{Subst}\left(\int \frac{1}{-1+x^2} \, dx, x, e^x\right) \\ &= -\tanh^{-1}(e^x) \end{aligned}$$

Mathematica [A] time = 0.002085, size = 6, normalized size = 1.

$$-\tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(-1 + E^(2*x)),x]
```

```
[Out] -ArcTanh[E^x]
```

Maple [A] time = 0.003, size = 6, normalized size = 1.

$$-\mathrm{Artanh}(\mathrm{e}^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(-1+exp(2*x)),x)
```

```
[Out] -arctanh(exp(x))
```

Maxima [B] time = 0.949424, size = 20, normalized size = 3.33

$$-\frac{1}{2}\log(e^x+1)+\frac{1}{2}\log(e^x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="maxima")
```

```
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Fricas [B] time = 1.9015, size = 51, normalized size = 8.5

$$-\frac{1}{2}\log(e^x+1)+\frac{1}{2}\log(e^x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="fricas")
```

```
[Out] -1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Sympy [B] time = 0.095842, size = 15, normalized size = 2.5

$$\frac{\log\left(e^{x}-1\right)}{2}-\frac{\log\left(e^{x}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-1+exp(2*x)),x)
```

```
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2
```

Giac [B] time = 1.05029, size = 22, normalized size = 3.67

$$-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(-1+exp(2*x)),x, algorithm="giac")
```

```
[Out] -1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

3.299 $\int \frac{1}{-8+x^3}\,dx$

Optimal. Leaf size=43

$$-\frac{1}{24}\log\left(x^2+2x+4\right)+\frac{1}{12}\log(2-x)-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] -ArcTan[(1 + x)/Sqrt[3]]/(4*Sqrt[3]) + Log[2 - x]/12 - Log[4 + 2*x + x^2]/2
4
```

Rubi [A] time = 0.0203541, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {200, 31, 634, 618, 204, 628}

$$-\frac{1}{24}\log\left(x^2+2x+4\right)+\frac{1}{12}\log(2-x)-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-8 + x^3)^(-1),x]
```

```
[Out] -ArcTan[(1 + x)/Sqrt[3]]/(4*Sqrt[3]) + Log[2 - x]/12 - Log[4 + 2*x + x^2]/2
4
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-8+x^3}\,dx &= \frac{1}{12}\int \frac{1}{-2+x}\,dx+\frac{1}{12}\int \frac{-4-x}{4+2x+x^2}\,dx\\
&= \frac{1}{12}\log(2-x)-\frac{1}{24}\int \frac{2+2x}{4+2x+x^2}\,dx-\frac{1}{4}\int \frac{1}{4+2x+x^2}\,dx\\
&= \frac{1}{12}\log(2-x)-\frac{1}{24}\log\left(4+2x+x^2\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-12-x^2}\,dx,x,2+2x\right)\\
&= -\frac{\tan^{-1}\left(\frac{1+x}{\sqrt{3}}\right)}{4\sqrt{3}}+\frac{1}{12}\log(2-x)-\frac{1}{24}\log\left(4+2x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0083778, size = 43, normalized size = 1.

$$-\frac{1}{24}\log\left(x^2+2x+4\right)+\frac{1}{12}\log(2-x)-\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-8 + x^3)^(-1),x]
```

```
[Out] -ArcTan[(1 + x)/Sqrt[3]]/(4*Sqrt[3]) + Log[2 - x]/12 - Log[4 + 2*x + x^2]/2
4
```

Maple [A] time = 0.005, size = 35, normalized size = 0.8

$$-\frac{\ln\left(x^2+2x+4\right)}{24}-\frac{\sqrt{3}}{12}\arctan\left(\frac{(2x+2)\sqrt{3}}{6}\right)+\frac{\ln(-2+x)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-8),x)
```

```
[Out] -1/24*ln(x^2+2*x+4)-1/12*3^(1/2)*arctan(1/6*(2*x+2)*3^(1/2))+1/12*ln(-2+x)
```

Maxima [A] time = 1.4085, size = 43, normalized size = 1.

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right)-\frac{1}{24}\log\left(x^2+2x+4\right)+\frac{1}{12}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-8),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/24*log(x^2 + 2*x + 4) + 1/12*
log(x - 2)
```

Fricas [A] time = 1.987, size = 117, normalized size = 2.72

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right)-\frac{1}{24}\log\left(x^2+2x+4\right)+\frac{1}{12}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-8),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/24*log(x^2 + 2*x + 4) + 1/12*
log(x - 2)
```

Sympy [A] time = 0.122397, size = 41, normalized size = 0.95

$$\frac{\log(x-2)}{12} - \frac{\log\left(x^2+2x+4\right)}{24} - \frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-8),x)
```

```
[Out] log(x - 2)/12 - log(x**2 + 2*x + 4)/24 - sqrt(3)*atan(sqrt(3)*x/3 + sqrt(3)
/3)/12
```

Giac [A] time = 1.04449, size = 45, normalized size = 1.05

$$-\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(x+1)\right) - \frac{1}{24}\log\left(x^2+2\,x+4\right) + \frac{1}{12}\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-8),x, algorithm="giac")
```

```
[Out] -1/12*sqrt(3)*arctan(1/3*sqrt(3)*(x + 1)) - 1/24*log(x^2 + 2*x + 4) + 1/12*
log(abs(x - 2))
```

3.300 $\int x^5 \cosh(x)\,dx$

Optimal. Leaf size=37

$$x^5 \sinh(x) + 20x^3 \sinh(x) - 5x^4 \cosh(x) - 60x^2 \cosh(x) + 120x \sinh(x) - 120 \cosh(x)$$

```
[Out] -120*Cosh[x] - 60*x^2*Cosh[x] - 5*x^4*Cosh[x] + 120*x*Sinh[x] + 20*x^3*Sinh
[x] + x^5*Sinh[x]
```

Rubi [A] time = 0.0764606, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2638}

$$x^5 \sinh(x) + 20x^3 \sinh(x) - 5x^4 \cosh(x) - 60x^2 \cosh(x) + 120x \sinh(x) - 120 \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Cosh[x],x]
```

```
[Out] -120*Cosh[x] - 60*x^2*Cosh[x] - 5*x^4*Cosh[x] + 120*x*Sinh[x] + 20*x^3*Sinh
[x] + x^5*Sinh[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^5 \cosh(x)\,dx &= x^5 \sinh(x) - 5\int x^4 \sinh(x)\,dx \\
&= -5x^4 \cosh(x) + x^5 \sinh(x) + 20\int x^3 \cosh(x)\,dx \\
&= -5x^4 \cosh(x) + 20x^3 \sinh(x) + x^5 \sinh(x) - 60\int x^2 \sinh(x)\,dx \\
&= -60x^2 \cosh(x) - 5x^4 \cosh(x) + 20x^3 \sinh(x) + x^5 \sinh(x) + 120\int x \cosh(x)\,dx \\
&= -60x^2 \cosh(x) - 5x^4 \cosh(x) + 120x \sinh(x) + 20x^3 \sinh(x) + x^5 \sinh(x) - 120\int \sinh(x)\,dx \\
&= -120\cosh(x) - 60x^2 \cosh(x) - 5x^4 \cosh(x) + 120x \sinh(x) + 20x^3 \sinh(x) + x^5 \sinh(x)
\end{aligned}
$$

Mathematica [A] time = 0.0158831, size = 29, normalized size = 0.78

$$x\left(x^4+20x^2+120\right)\sinh(x)-5\left(x^4+12x^2+24\right)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Cosh[x],x]
```

```
[Out] -5*(24 + 12*x^2 + x^4)*Cosh[x] + x*(120 + 20*x^2 + x^4)*Sinh[x]
```

Maple [A] time = 0.005, size = 38, normalized size = 1.

$$-120\cosh(x)-60x^2\cosh(x)-5x^4\cosh(x)+120x\sinh(x)+20x^3\sinh(x)+x^5\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*cosh(x),x)
```

```
[Out] -120*cosh(x)-60*x^2*cosh(x)-5*x^4*cosh(x)+120*x*sinh(x)+20*x^3*sinh(x)+x^5*
sinh(x)
```

Maxima [A] time = 0.969414, size = 100, normalized size = 2.7

$$\frac{1}{6}x^6\cosh(x)-\frac{1}{12}\left(x^6+6x^5+30x^4+120x^3+360x^2+720x+720\right)e^{(-x)}-\frac{1}{12}\left(x^6-6x^5+30x^4-120x^3+360x^2-7\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(x),x, algorithm="maxima")
```

```
[Out] 1/6*x^6*cosh(x) - 1/12*(x^6 + 6*x^5 + 30*x^4 + 120*x^3 + 360*x^2 + 720*x +
720)*e^(-x) - 1/12*(x^6 - 6*x^5 + 30*x^4 - 120*x^3 + 360*x^2 - 720*x + 720)
*e^x
```

Fricas [A] time = 1.82199, size = 88, normalized size = 2.38

$$-5\left(x^4+12\,x^2+24\right)\cosh\left(x\right)+\left(x^5+20\,x^3+120\,x\right)\sinh\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(x),x, algorithm="fricas")
```

```
[Out] -5*(x^4 + 12*x^2 + 24)*cosh(x) + (x^5 + 20*x^3 + 120*x)*sinh(x)
```

Sympy [A] time = 2.09731, size = 42, normalized size = 1.14

$$x^5\sinh\left(x\right)-5x^4\cosh\left(x\right)+20x^3\sinh\left(x\right)-60x^2\cosh\left(x\right)+120x\sinh\left(x\right)-120\cosh\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*cosh(x),x)
```

```
[Out] x**5*sinh(x) - 5*x**4*cosh(x) + 20*x**3*sinh(x) - 60*x**2*cosh(x) + 120*x*s
inh(x) - 120*cosh(x)
```

Giac [A] time = 1.04601, size = 77, normalized size = 2.08

$$-\frac{1}{2}\left(x^5+5\,x^4+20\,x^3+60\,x^2+120\,x+120\right)e^{(-x)}+\frac{1}{2}\left(x^5-5\,x^4+20\,x^3-60\,x^2+120\,x-120\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*cosh(x),x, algorithm="giac")
```

```
[Out] -1/2*(x^5 + 5*x^4 + 20*x^3 + 60*x^2 + 120*x + 120)*e^(-x) + 1/2*(x^5 - 5*x^
4 + 20*x^3 - 60*x^2 + 120*x - 120)*e^x
```

3.301 $\int \csc(x)\log(\tan(x))\sec(x)\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log^2(\tan(x))$$

```
[Out] Log[Tan[x]]^2/2
```

Rubi [A] time = 0.0232566, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2620, 29, 6686}

$$\frac{1}{2}\log^2(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*Log[Tan[x]]*Sec[x],x]

[Out] Log[Tan[x]]^2/2
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \csc(x)\log(\tan(x))\sec(x)\,dx = \frac{1}{2}\log^2(\tan(x))$$

Mathematica [A] time = 0.0052715, size = 9, normalized size = 1.

$$\frac{1}{2}\log^2(\tan(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*Log[Tan[x]]*Sec[x],x]
```

```
[Out] Log[Tan[x]]^2/2
```

Maple [A] time = 0.02, size = 8, normalized size = 0.9

$$\frac{(\ln(\tan(x)))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(tan(x))/cos(x)/sin(x),x)
```

```
[Out] 1/2*ln(tan(x))^2
```

Maxima [A] time = 0.932059, size = 9, normalized size = 1.

$$\frac{1}{2}\log(\tan(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tan(x))/cos(x)/sin(x),x, algorithm="maxima")
```

```
[Out] 1/2*log(tan(x))^2
```

Fricas [A] time = 2.03166, size = 35, normalized size = 3.89

$$\frac{1}{2}\log\left(\frac{\sin(x)}{\cos(x)}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tan(x))/cos(x)/sin(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(x)/cos(x))^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log{(\tan{(x)})}}{\sin{(x)}\cos{(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(tan(x))/cos(x)/sin(x),x)
```

```
[Out] Integral(log(tan(x))/(sin(x)*cos(x)), x)
```

Giac [A] time = 1.05607, size = 9, normalized size = 1.

$$\frac{1}{2}\log{(\tan{(x)})}^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(tan(x))/cos(x)/sin(x),x, algorithm="giac")
```

```
[Out] 1/2*log(tan(x))^2
```

3.302 $\int \left(-2x + x^2 + x^3\right) dx$

Optimal. Leaf size=20

$$\frac{x^4}{4} + \frac{x^3}{3} - x^2$$

```
[Out] -x^2 + x^3/3 + x^4/4
```

Rubi [A] time = 0.0024438, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{x^4}{4} + \frac{x^3}{3} - x^2$$

Antiderivative was successfully verified.

```
[In] Int[-2*x + x^2 + x^3,x]

[Out] -x^2 + x^3/3 + x^4/4
```

Rubi steps

$$\int \left(-2x + x^2 + x^3\right) dx = -x^2 + \frac{x^3}{3} + \frac{x^4}{4}$$

Mathematica [A] time = 0.0000315, size = 20, normalized size = 1.

$$\frac{x^4}{4} + \frac{x^3}{3} - x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[-2*x + x^2 + x^3,x]

[Out] -x^2 + x^3/3 + x^4/4
```

Maple [A] time = 0., size = 17, normalized size = 0.9

$$-x^2+\frac{x^3}{3}+\frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3+x^2-2*x,x)
```

```
[Out] -x^2+1/3*x^3+1/4*x^4
```

Maxima [A] time = 0.936531, size = 22, normalized size = 1.1

$$\frac{1}{4}\,x^4+\frac{1}{3}\,x^3-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

Fricas [A] time = 1.59752, size = 34, normalized size = 1.7

$$\frac{1}{4}x^4+\frac{1}{3}x^3-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

Sympy [A] time = 0.048737, size = 12, normalized size = 0.6

$$\frac{x^4}{4}+\frac{x^3}{3}-x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3+x**2-2*x,x)
```

```
[Out] x**4/4 + x**3/3 - x**2
```

Giac [A] time = 1.04368, size = 22, normalized size = 1.1

$$\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3+x^2-2*x,x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 1/3*x^3 - x^2
```

3.303 $\int \frac{1+e^x}{1-e^x} \, dx$

Optimal. Leaf size=12

$$x - 2\log(1 - e^x)$$

```
[Out] x - 2*Log[1 - E^x]
```

Rubi [A] time = 0.0210943, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2282, 72}

$$x - 2\log(1 - e^x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + E^x)/(1 - E^x),x]
```

```
[Out] x - 2*Log[1 - E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1+e^x}{1-e^x} dx = \text{Subst}\left(\int \frac{1+x}{(1-x)x} dx, x, e^x\right)$$
$$= \text{Subst}\left(\int \left(-\frac{2}{-1+x}+\frac{1}{x}\right) dx, x, e^x\right)$$
$$= x - 2\log\left(1-e^x\right)$$

Mathematica [A] time = 0.0084745, size = 12, normalized size = 1.

$$x - 2\log\left(1-e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + E^x)/(1 - E^x),x]
```

```
[Out] x - 2*Log[1 - E^x]
```

Maple [A] time = 0.005, size = 12, normalized size = 1.

$$\ln\left(e^x\right) - 2\ \ln\left(-1+e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+exp(x))/(1-exp(x)),x)
```

```
[Out] ln(exp(x))-2*ln(-1+exp(x))
```

Maxima [A] time = 0.942234, size = 12, normalized size = 1.

$$x - 2\ \log\left(e^x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))/(1-exp(x)),x, algorithm="maxima")
```

```
[Out] x - 2*log(e^x - 1)
```

Fricas [A] time = 1.91797, size = 27, normalized size = 2.25

$$x - 2 \log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))/(1-exp(x)),x, algorithm="fricas")
```

```
[Out] x - 2*log(e^x - 1)
```

Sympy [A] time = 0.079297, size = 8, normalized size = 0.67

$$x - 2\log(e^x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))/(1-exp(x)),x)
```

```
[Out] x - 2*log(exp(x) - 1)
```

Giac [A] time = 1.05154, size = 14, normalized size = 1.17

$$x - 2 \log(|e^x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))/(1-exp(x)),x, algorithm="giac")
```

```
[Out] x - 2*log(abs(e^x - 1))
```

3.304 $\int \frac{x}{(1+x^2)(4+x^2)}\,dx$

Optimal. Leaf size=21

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

```
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rubi [A] time = 0.0114482, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {444, 36, 31}

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\left(1+x^2\right)\left(4+x^2\right)}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(1+x)(4+x)}\,dx,x,x^2\right)\\ &= \frac{1}{6}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right)-\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{4+x}\,dx,x,x^2\right)\\ &= \frac{1}{6}\log\left(1+x^2\right)-\frac{1}{6}\log\left(4+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0046765, size = 21, normalized size = 1.

$$\frac{1}{6}\log\left(x^2+1\right)-\frac{1}{6}\log\left(x^2+4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x^2)*(4 + x^2)),x]
```

```
[Out] Log[1 + x^2]/6 - Log[4 + x^2]/6
```

Maple [A] time = 0.004, size = 18, normalized size = 0.9

$$\frac{\ln\left(x^2+1\right)}{6}-\frac{\ln\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+1)/(x^2+4),x)
```

```
[Out] 1/6*ln(x^2+1)-1/6*ln(x^2+4)
```

Maxima [A] time = 0.928231, size = 23, normalized size = 1.1

$$-\frac{1}{6}\,\log\left(x^2+4\right)+\frac{1}{6}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="maxima")
```

```
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Fricas [A] time = 1.81432, size = 51, normalized size = 2.43

$$-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="fricas")
```

```
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

Sympy [A] time = 0.096426, size = 15, normalized size = 0.71

$$\frac{\log\left(x^2+1\right)}{6}-\frac{\log\left(x^2+4\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+1)/(x**2+4),x)
```

```
[Out] log(x**2 + 1)/6 - log(x**2 + 4)/6
```

Giac [A] time = 1.05022, size = 23, normalized size = 1.1

$$-\frac{1}{6}\log\left(x^2+4\right)+\frac{1}{6}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)/(x^2+4),x, algorithm="giac")
```

```
[Out] -1/6*log(x^2 + 4) + 1/6*log(x^2 + 1)
```

3.305 $\int \frac{1}{4-5\sin(x)}\,dx$

Optimal. Leaf size=43

$$\frac{1}{3}\log\left(2\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{3}\log\left(\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right)$$

```
[Out] -Log[Cos[x/2] - 2*Sin[x/2]]/3 + Log[2*Cos[x/2] - Sin[x/2]]/3
```

Rubi [A] time = 0.0176698, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2660, 616, 31}

$$\frac{1}{3}\log\left(2\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{3}\log\left(\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Int[(4 - 5*Sin[x])^(-1),x]
```

```
[Out] -Log[Cos[x/2] - 2*Sin[x/2]]/3 + Log[2*Cos[x/2] - Sin[x/2]]/3
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{4-5\sin(x)}\,dx &= 2\,\mathrm{Subst}\left(\int \frac{1}{4-10x+4x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= \frac{4}{3}\,\mathrm{Subst}\left(\int \frac{1}{-8+4x}\,dx, x, \tan\left(\frac{x}{2}\right)\right) - \frac{4}{3}\,\mathrm{Subst}\left(\int \frac{1}{-2+4x}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\frac{1}{3}\log\left(1-2\tan\left(\frac{x}{2}\right)\right) + \frac{1}{3}\log\left(2-\tan\left(\frac{x}{2}\right)\right)\end{aligned}$$

Mathematica [A] time = 0.0118386, size = 43, normalized size = 1.

$$\frac{1}{3}\log\left(2\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\frac{1}{3}\log\left(\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 - 5*Sin[x])^(-1),x]
```

```
[Out] -Log[Cos[x/2] - 2*Sin[x/2]]/3 + Log[2*Cos[x/2] - Sin[x/2]]/3
```

Maple [A] time = 0.012, size = 22, normalized size = 0.5

$$-\frac{1}{3}\ln\left(2\,\tan\left(x/2\right)-1\right)+\frac{1}{3}\ln\left(\tan\left(\frac{x}{2}\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4-5*sin(x)),x)
```

```
[Out] -1/3*ln(2*tan(1/2*x)-1)+1/3*ln(tan(1/2*x)-2)
```

Maxima [A] time = 0.936871, size = 41, normalized size = 0.95

$$-\frac{1}{3}\,\log\left(\frac{2\,\sin\left(x\right)}{\cos\left(x\right)+1}-1\right)+\frac{1}{3}\,\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-5*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/3*log(2*sin(x)/(cos(x) + 1) - 1) + 1/3*log(sin(x)/(cos(x) + 1) - 2)
```

Fricas [A] time = 2.29027, size = 105, normalized size = 2.44

$$\frac{1}{6}\log\left(\frac{3}{2}\cos\left(x\right)-2\sin\left(x\right)+\frac{5}{2}\right)-\frac{1}{6}\log\left(-\frac{3}{2}\cos\left(x\right)-2\sin\left(x\right)+\frac{5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-5*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/6*log(3/2*cos(x) - 2*sin(x) + 5/2) - 1/6*log(-3/2*cos(x) - 2*sin(x) + 5/2
)
```

Sympy [A] time = 0.222954, size = 20, normalized size = 0.47

$$\frac{\log\left(\tan\left(\frac{x}{2}\right)-2\right)}{3}-\frac{\log\left(\tan\left(\frac{x}{2}\right)-\frac{1}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-5*sin(x)),x)
```

```
[Out] log(tan(x/2) - 2)/3 - log(tan(x/2) - 1/2)/3
```

Giac [A] time = 1.08024, size = 31, normalized size = 0.72

$$-\frac{1}{3}\log\left(\left|2\tan\left(\frac{1}{2}x\right)-1\right|\right)+\frac{1}{3}\log\left(\left|\tan\left(\frac{1}{2}x\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4-5*sin(x)),x, algorithm="giac")
```

```
[Out] -1/3*log(abs(2*tan(1/2*x) - 1)) + 1/3*log(abs(tan(1/2*x) - 2))
```

3.306 $\int x\sqrt[3]{c+x}\,dx$

Optimal. Leaf size=24

$$\frac{3}{7}(c+x)^{7/3}-\frac{3}{4}c(c+x)^{4/3}$$

```
[Out] (-3*c*(c + x)^(4/3))/4 + (3*(c + x)^(7/3))/7
```

Rubi [A] time = 0.0044251, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{3}{7}(c+x)^{7/3}-\frac{3}{4}c(c+x)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(c + x)^(1/3),x]
```

```
[Out] (-3*c*(c + x)^(4/3))/4 + (3*(c + x)^(7/3))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt[3]{c+x}\,dx &= \int\left(-c\sqrt[3]{c+x}+(c+x)^{4/3}\right)dx\\ &= -\frac{3}{4}c(c+x)^{4/3}+\frac{3}{7}(c+x)^{7/3}\end{aligned}$$

Mathematica [A] time = 0.0064194, size = 18, normalized size = 0.75

$$\frac{3}{28}(c+x)^{4/3}(4x-3c)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(c + x)^(1/3),x]
```

```
[Out] (3*(c + x)^(4/3)*(-3*c + 4*x))/28
```

Maple [A] time = 0.003, size = 15, normalized size = 0.6

$$-\frac{9c-12x}{28}(c+x)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c+x)^(1/3),x)
```

```
[Out] -3/28*(c+x)^(4/3)*(3*c-4*x)
```

Maxima [A] time = 0.936892, size = 22, normalized size = 0.92

$$\frac{3}{7}(c+x)^{\frac{7}{3}}-\frac{3}{4}(c+x)^{\frac{4}{3}}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+x)^(1/3),x, algorithm="maxima")
```

```
[Out] 3/7*(c + x)^(7/3) - 3/4*(c + x)^(4/3)*c
```

Fricas [A] time = 1.99004, size = 58, normalized size = 2.42

$$-\frac{3}{28}\left(3c^2-cx-4x^2\right)(c+x)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+x)^(1/3),x, algorithm="fricas")
```

```
[Out] -3/28*(3*c^2 - c*x - 4*x^2)*(c + x)^(1/3)
```

Sympy [B] time = 0.926045, size = 144, normalized size = 6.

$$-\frac{9c^{\frac{13}{3}}\sqrt[3]{1+\frac{x}{c}}}{28c^2+28cx}+\frac{9c^{\frac{13}{3}}}{28c^2+28cx}-\frac{6c^{\frac{10}{3}}x\sqrt[3]{1+\frac{x}{c}}}{28c^2+28cx}+\frac{9c^{\frac{10}{3}}x}{28c^2+28cx}+\frac{15c^{\frac{7}{3}}x^2\sqrt[3]{1+\frac{x}{c}}}{28c^2+28cx}+\frac{12c^{\frac{4}{3}}x^3\sqrt[3]{1+\frac{x}{c}}}{28c^2+28cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+x)**(1/3),x)
```

```
[Out] -9*c**(13/3)*(1 + x/c)**(1/3)/(28*c**2 + 28*c*x) + 9*c**(13/3)/(28*c**2 + 2
8*c*x) - 6*c**(10/3)*x*(1 + x/c)**(1/3)/(28*c**2 + 28*c*x) + 9*c**(10/3)*x/
(28*c**2 + 28*c*x) + 15*c**(7/3)*x**2*(1 + x/c)**(1/3)/(28*c**2 + 28*c*x) +
 12*c**(4/3)*x**3*(1 + x/c)**(1/3)/(28*c**2 + 28*c*x)
```

Giac [A] time = 1.05171, size = 22, normalized size = 0.92

$$\frac{3}{7}\left(c+x\right)^{\frac{7}{3}}-\frac{3}{4}\left(c+x\right)^{\frac{4}{3}}c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c+x)^(1/3),x, algorithm="giac")
```

```
[Out] 3/7*(c + x)^(7/3) - 3/4*(c + x)^(4/3)*c
```

3.307 $\int e^{\sqrt[3]{x}}\,dx$

Optimal. Leaf size=38

$$3e^{\sqrt[3]{x}}x^{2/3}-6e^{\sqrt[3]{x}}\sqrt[3]{x}+6e^{\sqrt[3]{x}}$$

```
[Out] 6*E^x^(1/3) - 6*E^x^(1/3)*x^(1/3) + 3*E^x^(1/3)*x^(2/3)
```

Rubi [A] time = 0.018782, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2207, 2176, 2194}

$$3e^{\sqrt[3]{x}}x^{2/3}-6e^{\sqrt[3]{x}}\sqrt[3]{x}+6e^{\sqrt[3]{x}}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^(1/3),x]
```

```
[Out] 6*E^x^(1/3) - 6*E^x^(1/3)*x^(1/3) + 3*E^x^(1/3)*x^(2/3)
```

Rule 2207

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Dist[k/d, Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (c
 + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !Inte
gerQ[n]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{\sqrt[3]{x}}\,dx &= 3\,\text{Subst}\left(\int e^x x^2\,dx, x, \sqrt[3]{x}\right)\\ &= 3e^{\sqrt[3]{x}}x^{2/3} - 6\,\text{Subst}\left(\int e^x x\,dx, x, \sqrt[3]{x}\right)\\ &= -6e^{\sqrt[3]{x}}\sqrt[3]{x} + 3e^{\sqrt[3]{x}}x^{2/3} + 6\,\text{Subst}\left(\int e^x\,dx, x, \sqrt[3]{x}\right)\\ &= 6e^{\sqrt[3]{x}} - 6e^{\sqrt[3]{x}}\sqrt[3]{x} + 3e^{\sqrt[3]{x}}x^{2/3}\end{aligned}$$

Mathematica [A] time = 0.0080055, size = 24, normalized size = 0.63

$$e^{\sqrt[3]{x}}\left(3x^{2/3} - 6\sqrt[3]{x} + 6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^(1/3),x]
```

```
[Out] E^x^(1/3)*(6 - 6*x^(1/3) + 3*x^(2/3))
```

Maple [A] time = 0.002, size = 26, normalized size = 0.7

$$6\,\mathrm{e}^{\sqrt[3]{x}} - 6\,\mathrm{e}^{\sqrt[3]{x}}\sqrt[3]{x} + 3\,\mathrm{e}^{\sqrt[3]{x}}x^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^(1/3)),x)
```

```
[Out] 6*exp(x^(1/3))-6*exp(x^(1/3))*x^(1/3)+3*exp(x^(1/3))*x^(2/3)
```

Maxima [A] time = 0.936532, size = 22, normalized size = 0.58

$$3\left(x^{\frac{2}{3}} - 2\,x^{\frac{1}{3}} + 2\right)e^{\left(x^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/3)),x, algorithm="maxima")
```

```
[Out] 3*(x^(2/3) - 2*x^(1/3) + 2)*e^(x^(1/3))
```

Fricas [A] time = 2.19803, size = 55, normalized size = 1.45

$$3\left(x^{\frac{2}{3}} - 2\,x^{\frac{1}{3}} + 2\right)e^{\left(x^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/3)),x, algorithm="fricas")
```

```
[Out] 3*(x^(2/3) - 2*x^(1/3) + 2)*e^(x^(1/3))
```

Sympy [A] time = 0.318136, size = 34, normalized size = 0.89

$$3x^{\frac{2}{3}}e^{\sqrt[3]{x}} - 6\sqrt[3]{x}e^{\sqrt[3]{x}} + 6e^{\sqrt[3]{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**(1/3)),x)
```

```
[Out] 3*x**(2/3)*exp(x**(1/3)) - 6*x**(1/3)*exp(x**(1/3)) + 6*exp(x**(1/3))
```

Giac [A] time = 1.05636, size = 22, normalized size = 0.58

$$3\left(x^{\frac{2}{3}} - 2\,x^{\frac{1}{3}} + 2\right)e^{\left(x^{\frac{1}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/3)),x, algorithm="giac")
```

```
[Out] 3*(x^(2/3) - 2*x^(1/3) + 2)*e^(x^(1/3))
```

3.308 $\int \frac{1}{4+x+\sqrt{1+x}}\,dx$

Optimal. Leaf size=37

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

Rubi [A] time = 0.0357258, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {634, 618, 204, 628}

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{4+x+\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)\\
&= -\text{Subst}\left(\int \frac{1}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)+\text{Subst}\left(\int \frac{1+2x}{3+x+x^2}\,dx, x, \sqrt{1+x}\right)\\
&= \log\left(4+x+\sqrt{1+x}\right)+2\,\text{Subst}\left(\int \frac{1}{-11-x^2}\,dx, x, 1+2\sqrt{1+x}\right)\\
&= -\frac{2\tan^{-1}\left(\frac{1+2\sqrt{1+x}}{\sqrt{11}}\right)}{\sqrt{11}}+\log\left(4+x+\sqrt{1+x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0133128, size = 37, normalized size = 1.

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x
]]
```

Maple [B] time = 0.012, size = 93, normalized size = 2.5

$$-\frac{1}{2}\ln\left(x+4-\sqrt{1+x}\right)-\frac{\sqrt{11}}{11}\arctan\left(\frac{\sqrt{11}}{11}\left(2\sqrt{1+x}-1\right)\right)+\frac{1}{2}\ln\left(4+x+\sqrt{1+x}\right)-\frac{\sqrt{11}}{11}\arctan\left(\frac{\sqrt{11}}{11}\left(1+2\sqrt{1+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+x+(1+x)^(1/2)),x)
```

```
[Out] -1/2*ln(x+4-(1+x)^(1/2))-1/11*11^(1/2)*arctan(1/11*(2*(1+x)^(1/2)-1)*11^(1/
2))+1/2*ln(4+x+(1+x)^(1/2))-1/11*arctan(1/11*(1+2*(1+x)^(1/2))*11^(1/2))*11
^(1/2)+1/11*11^(1/2)*arctan(1/11*(7+2*x)*11^(1/2))+1/2*ln(x^2+7*x+15)
```

Maxima [A] time = 1.41183, size = 41, normalized size = 1.11

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x +
 1) + 4)
```

Fricas [A] time = 2.08235, size = 126, normalized size = 3.41

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{2}{11}\sqrt{11}\sqrt{x+1}+\frac{1}{11}\sqrt{11}\right)+\log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2/11*sqrt(11)*arctan(2/11*sqrt(11)*sqrt(x + 1) + 1/11*sqrt(11)) + log(x +
sqrt(x + 1) + 4)
```

Sympy [A] time = 1.26376, size = 39, normalized size = 1.05

$$\log\left(x+\sqrt{x+1}+4\right)-\frac{2\sqrt{11}\operatorname{atan}\left(\frac{2\sqrt{11}\left(\sqrt{x+1}+\frac{1}{2}\right)}{11}\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)**(1/2)),x)
```

```
[Out] log(x + sqrt(x + 1) + 4) - 2*sqrt(11)*atan(2*sqrt(11)*(sqrt(x + 1) + 1/2)/1
1)/11
```

Giac [A] time = 1.05178, size = 41, normalized size = 1.11

$$-\frac{2}{11}\sqrt{11}\arctan\left(\frac{1}{11}\sqrt{11}\left(2\sqrt{x+1}+1\right)\right)+\log\left(x+\sqrt{x+1}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x +
 1) + 4)
```

3.309 $\int \frac{1+x^3}{-x^2+x^3} \, dx$

Optimal. Leaf size=17

$$x + \frac{1}{x} + 2\log(1-x) - \log(x)$$

```
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Rubi [A] time = 0.0292751, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1593, 1620}

$$x + \frac{1}{x} + 2\log(1-x) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^3)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^3}{-x^2+x^3}\,dx &= \int \frac{1+x^3}{(-1+x)x^2}\,dx\\ &= \int \left(1+\frac{2}{-1+x}-\frac{1}{x^2}-\frac{1}{x}\right)dx\\ &= \frac{1}{x}+x+2\log(1-x)-\log(x)\end{aligned}$$

Mathematica [A] time = 0.0038988, size = 17, normalized size = 1.

$$x+\frac{1}{x}+2\log(1-x)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^3)/(-x^2 + x^3),x]
```

```
[Out] x^(-1) + x + 2*Log[1 - x] - Log[x]
```

Maple [A] time = 0.007, size = 16, normalized size = 0.9

$$x+x^{-1}-\ln(x)+2\ \ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)/(x^3-x^2),x)
```

```
[Out] x+1/x-ln(x)+2*ln(-1+x)
```

Maxima [A] time = 0.936249, size = 20, normalized size = 1.18

$$x+\frac{1}{x}+2\ \log(x-1)-\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="maxima")
```

```
[Out] x + 1/x + 2*log(x - 1) - log(x)
```

Fricas [A] time = 2.14304, size = 55, normalized size = 3.24

$$\frac{x^2 + 2\,x\log(x-1) - x\log(x) + 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="fricas")
```

```
[Out] (x^2 + 2*x*log(x - 1) - x*log(x) + 1)/x
```

Sympy [A] time = 0.097483, size = 14, normalized size = 0.82

$$x - \log(x) + 2\log(x-1) + \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)/(x**3-x**2),x)
```

```
[Out] x - log(x) + 2*log(x - 1) + 1/x
```

Giac [A] time = 1.05698, size = 23, normalized size = 1.35

$$x + \frac{1}{x} + 2\,\log(|x-1|) - \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)/(x^3-x^2),x, algorithm="giac")
```

```
[Out] x + 1/x + 2*log(abs(x - 1)) - log(abs(x))
```

3.310 $\int \left(-3+4x+x^2\right) \sin(2x) \, dx$

Optimal. Leaf size=40

$$-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\sin(2x)-2x\cos(2x)+\frac{7}{4}\cos(2x)$$

```
[Out] (7*Cos[2*x])/4 - 2*x*Cos[2*x] - (x^2*Cos[2*x])/2 + Sin[2*x] + (x*Sin[2*x])/
2
```

Rubi [A] time = 0.0667267, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {6742, 2638, 3296, 2637}

$$-\frac{1}{2}x^2\cos(2x)+\frac{1}{2}x\sin(2x)+\sin(2x)-2x\cos(2x)+\frac{7}{4}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + 4*x + x^2)*Sin[2*x],x]
```

```
[Out] (7*Cos[2*x])/4 - 2*x*Cos[2*x] - (x^2*Cos[2*x])/2 + Sin[2*x] + (x*Sin[2*x])/
2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int\left(-3+4x+x^2\right)\sin(2x)\,dx&=\int\left(-3\sin(2x)+4x\sin(2x)+x^2\sin(2x)\right)\,dx\\&=-(3\int\sin(2x)\,dx)+4\int x\sin(2x)\,dx+\int x^2\sin(2x)\,dx\\&=\frac{3}{2}\cos(2x)-2x\cos(2x)-\frac{1}{2}x^2\cos(2x)+2\int\cos(2x)\,dx+\int x\cos(2x)\,dx\\&=\frac{3}{2}\cos(2x)-2x\cos(2x)-\frac{1}{2}x^2\cos(2x)+\sin(2x)+\frac{1}{2}x\sin(2x)-\frac{1}{2}\int\sin(2x)\,dx\\&=\frac{7}{4}\cos(2x)-2x\cos(2x)-\frac{1}{2}x^2\cos(2x)+\sin(2x)+\frac{1}{2}x\sin(2x)\end{aligned}$$

Mathematica [A] time = 0.0374874, size = 29, normalized size = 0.72

$$\frac{1}{4}\left(\left(-2x^2-8x+7\right)\cos(2x)+2(x+2)\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + 4*x + x^2)*Sin[2*x],x]
```

```
[Out] ((7 - 8*x - 2*x^2)*Cos[2*x] + 2*(2 + x)*Sin[2*x])/4
```

Maple [A] time = 0.007, size = 35, normalized size = 0.9

$$\frac{7\cos(2x)}{4}-2x\cos(2x)-\frac{x^2\cos(2x)}{2}+\sin(2x)+\frac{x\sin(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+4*x-3)*sin(2*x),x)
```

```
[Out] 7/4*cos(2*x)-2*x*cos(2*x)-1/2*x^2*cos(2*x)+sin(2*x)+1/2*x*sin(2*x)
```

Maxima [A] time = 0.95162, size = 51, normalized size = 1.27

$$-\frac{1}{4}\left(2x^2-1\right)\cos(2x)-2x\cos(2x)+\frac{1}{2}x\sin(2x)+\frac{3}{2}\cos(2x)+\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^2 - 1)*cos(2*x) - 2*x*cos(2*x) + 1/2*x*sin(2*x) + 3/2*cos(2*x) +
sin(2*x)
```

Fricas [A] time = 2.26788, size = 76, normalized size = 1.9

$$-\frac{1}{4}\left(2x^2+8x-7\right)\cos(2x)+\frac{1}{2}(x+2)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)
```

Sympy [A] time = 0.31514, size = 39, normalized size = 0.98

$$-\frac{x^2\cos(2x)}{2}+\frac{x\sin(2x)}{2}-2x\cos(2x)+\sin(2x)+\frac{7\cos(2x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+4*x-3)*sin(2*x),x)
```

```
[Out] -x**2*cos(2*x)/2 + x*sin(2*x)/2 - 2*x*cos(2*x) + sin(2*x) + 7*cos(2*x)/4
```

Giac [A] time = 1.04826, size = 35, normalized size = 0.88

$$-\frac{1}{4}\left(2x^2+8x-7\right)\cos(2x)+\frac{1}{2}(x+2)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+4*x-3)*sin(2*x),x, algorithm="giac")
```

```
[Out] -1/4*(2*x^2 + 8*x - 7)*cos(2*x) + 1/2*(x + 2)*sin(2*x)
```

3.311 $\int \cos(\cos(x))\sin(x)\,dx$

Optimal. Leaf size=5

$$-\sin(\cos(x))$$

```
[Out] -Sin[Cos[x]]
```

Rubi [A] time = 0.0088673, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4335, 2637}

$$-\sin(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Cos[x]]*Sin[x],x]

[Out] -Sin[Cos[x]]
```

Rule 4335

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Cos[c*(a + b*x)], x]}, -Dist[d/(b*c), Subst[Int[SubstFor[1, Cos[c*(a + b
*x)]/d, u, x], x], x, Cos[c*(a + b*x)]/d], x] /; FunctionOfQ[Cos[c*(a + b*x
)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(\cos(x))\sin(x)\,dx &= -\text{Subst}\left(\int \cos(x)\,dx, x, \cos(x)\right)\\ &= -\sin(\cos(x))\end{aligned}$$

Mathematica [A] time = 2.47877, size = 5, normalized size = 1.

$$-\sin(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Cos[x]]*Sin[x],x]
```

```
[Out] -Sin[Cos[x]]
```

Maple [A] time = 0.007, size = 6, normalized size = 1.2

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(cos(x))*sin(x),x)
```

```
[Out] -sin(cos(x))
```

Maxima [A] time = 0.926392, size = 7, normalized size = 1.4

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="maxima")
```

```
[Out] -sin(cos(x))
```

Fricas [B] time = 2.28435, size = 59, normalized size = 11.8

$$\sin\left(\frac{\tan\left(\frac{1}{2}x\right)^2 - 1}{\tan\left(\frac{1}{2}x\right)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="fricas")
```

```
[Out] sin((tan(1/2*x)^2 - 1)/(tan(1/2*x)^2 + 1))
```

Sympy [A] time = 0.505605, size = 5, normalized size = 1.

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x)
```

```
[Out] -sin(cos(x))
```

Giac [A] time = 1.0521, size = 7, normalized size = 1.4

$$-\sin(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(cos(x))*sin(x),x, algorithm="giac")
```

```
[Out] -sin(cos(x))
```

3.312 $\int \frac{1}{\sqrt{16-x^2}} \, dx$

Optimal. Leaf size=6

$$\sin^{-1}\left(\frac{x}{4}\right)$$

```
[Out] ArcSin[x/4]
```

Rubi [A] time = 0.0012241, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {216}

$$\sin^{-1}\left(\frac{x}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[16 - x^2],x]

[Out] ArcSin[x/4]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{16-x^2}} \, dx = \sin^{-1}\left(\frac{x}{4}\right)$$

Mathematica [A] time = 0.0041155, size = 6, normalized size = 1.

$$\sin^{-1}\left(\frac{x}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[16 - x^2],x]
```

```
[Out] ArcSin[x/4]
```

Maple [A] time = 0.003, size = 5, normalized size = 0.8

$$\arcsin\left(\frac{x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2+16)^(1/2),x)
```

```
[Out] arcsin(1/4*x)
```

Maxima [A] time = 1.40092, size = 5, normalized size = 0.83

$$\arcsin\left(\frac{1}{4}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+16)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(1/4*x)
```

Fricas [B] time = 2.20828, size = 49, normalized size = 8.17

$$-2\,\arctan\left(\frac{\sqrt{-x^2+16}-4}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+16)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-x^2 + 16) - 4)/x)
```

Sympy [A] time = 0.131268, size = 3, normalized size = 0.5

$$\operatorname{asin}\left(\frac{x}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2+16)**(1/2),x)
```

```
[Out] asin(x/4)
```

Giac [A] time = 1.07072, size = 5, normalized size = 0.83

$$\arcsin\left(\frac{1}{4}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2+16)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(1/4*x)
```

3.313 $\int \frac{x^3}{(1+x)^{10}}\,dx$

Optimal. Leaf size=37

$$-\frac{1}{6(x+1)^6}+\frac{3}{7(x+1)^7}-\frac{3}{8(x+1)^8}+\frac{1}{9(x+1)^9}$$

```
[Out] 1/(9*(1 + x)^9) - 3/(8*(1 + x)^8) + 3/(7*(1 + x)^7) - 1/(6*(1 + x)^6)
```

Rubi [A] time = 0.0114004, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$-\frac{1}{6(x+1)^6}+\frac{3}{7(x+1)^7}-\frac{3}{8(x+1)^8}+\frac{1}{9(x+1)^9}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(1 + x)^10,x]
```

```
[Out] 1/(9*(1 + x)^9) - 3/(8*(1 + x)^8) + 3/(7*(1 + x)^7) - 1/(6*(1 + x)^6)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(1+x)^{10}}\,dx &= \int \left(-\frac{1}{(1+x)^{10}}+\frac{3}{(1+x)^9}-\frac{3}{(1+x)^8}+\frac{1}{(1+x)^7}\right)dx\\ &= \frac{1}{9(1+x)^9}-\frac{3}{8(1+x)^8}+\frac{3}{7(1+x)^7}-\frac{1}{6(1+x)^6}\end{aligned}$$

Mathematica [A] time = 0.0065565, size = 24, normalized size = 0.65

$$-\frac{84x^3+36x^2+9x+1}{504(x+1)^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(1 + x)^10,x]
```

```
[Out] -(1 + 9*x + 36*x^2 + 84*x^3)/(504*(1 + x)^9)
```

Maple [A] time = 0.005, size = 30, normalized size = 0.8

$$\frac{1}{9(1+x)^9}-\frac{3}{8(1+x)^8}+\frac{3}{7(1+x)^7}-\frac{1}{6(1+x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(1+x)^10,x)
```

```
[Out] 1/9/(1+x)^9-3/8/(1+x)^8+3/7/(1+x)^7-1/6/(1+x)^6
```

Maxima [B] time = 0.944489, size = 84, normalized size = 2.27

$$-\frac{84x^3+36x^2+9x+1}{504\left(x^9+9x^8+36x^7+84x^6+126x^5+126x^4+84x^3+36x^2+9x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^10,x, algorithm="maxima")
```

```
[Out] -1/504*(84*x^3 + 36*x^2 + 9*x + 1)/(x^9 + 9*x^8 + 36*x^7 + 84*x^6 + 126*x^5
 + 126*x^4 + 84*x^3 + 36*x^2 + 9*x + 1)
```

Fricas [B] time = 2.03816, size = 157, normalized size = 4.24

$$-\frac{84x^3+36x^2+9x+1}{504\left(x^9+9x^8+36x^7+84x^6+126x^5+126x^4+84x^3+36x^2+9x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^10,x, algorithm="fricas")
```

```
[Out] -1/504*(84*x^3 + 36*x^2 + 9*x + 1)/(x^9 + 9*x^8 + 36*x^7 + 84*x^6 + 126*x^5
 + 126*x^4 + 84*x^3 + 36*x^2 + 9*x + 1)
```

Sympy [A] time = 0.139897, size = 61, normalized size = 1.65

$$-\frac{84x^3+36x^2+9x+1}{504x^9+4536x^8+18144x^7+42336x^6+63504x^5+63504x^4+42336x^3+18144x^2+4536x+504}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(1+x)**10,x)
```

```
[Out] -(84*x**3 + 36*x**2 + 9*x + 1)/(504*x**9 + 4536*x**8 + 18144*x**7 + 42336*x
**6 + 63504*x**5 + 63504*x**4 + 42336*x**3 + 18144*x**2 + 4536*x + 504)
```

Giac [A] time = 1.06152, size = 30, normalized size = 0.81

$$-\frac{84\,x^3+36\,x^2+9\,x+1}{504\,(x+1)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(1+x)^10,x, algorithm="giac")
```

```
[Out] -1/504*(84*x^3 + 36*x^2 + 9*x + 1)/(x + 1)^9
```

3.314 $\int \cot^3(2x)\csc^3(2x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{6}\csc^3(2x)-\frac{1}{10}\csc^5(2x)$$

```
[Out] Csc[2*x]^3/6 - Csc[2*x]^5/10
```

Rubi [A] time = 0.0294961, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2606, 14}

$$\frac{1}{6}\csc^3(2x)-\frac{1}{10}\csc^5(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[2*x]^3*Csc[2*x]^3,x]
```

```
[Out] Csc[2*x]^3/6 - Csc[2*x]^5/10
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$
\begin{aligned}
\int \cot^3(2x)\csc^3(2x)\,dx &= -\left(\frac{1}{2}\operatorname{Subst}\left(\int x^2\left(-1+x^2\right)\,dx,x,\csc(2x)\right)\right)\\
&= -\left(\frac{1}{2}\operatorname{Subst}\left(\int \left(-x^2+x^4\right)\,dx,x,\csc(2x)\right)\right)\\
&= \frac{1}{6}\csc^3(2x)-\frac{1}{10}\csc^5(2x)
\end{aligned}
$$

Mathematica [A] time = 0.0233481, size = 21, normalized size = 1.

$$\frac{1}{6}\csc^3(2x)-\frac{1}{10}\csc^5(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[2*x]^3*Csc[2*x]^3,x]
```

```
[Out] Csc[2*x]^3/6 - Csc[2*x]^5/10
```

Maple [B] time = 0.028, size = 58, normalized size = 2.8

$$-\frac{(\cos(2x))^4}{10(\sin(2x))^5}-\frac{(\cos(2x))^4}{30(\sin(2x))^3}+\frac{(\cos(2x))^4}{30\sin(2x)}+\frac{\left(2+(\cos(2x))^2\right)\sin(2x)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(2*x)^3*csc(2*x)^3,x)
```

```
[Out] -1/10/sin(2*x)^5*cos(2*x)^4-1/30/sin(2*x)^3*cos(2*x)^4+1/30/sin(2*x)*cos(2*
x)^4+1/30*(2+cos(2*x)^2)*sin(2*x)
```

Maxima [A] time = 0.931975, size = 24, normalized size = 1.14

$$\frac{5\sin(2x)^2-3}{30\sin(2x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)^3*csc(2*x)^3,x, algorithm="maxima")
```

```
[Out] 1/30*(5*sin(2*x)^2 - 3)/sin(2*x)^5
```

Fricas [B] time = 2.32517, size = 93, normalized size = 4.43

$$-\frac{5 \cos\left(2 x\right)^{2} - 2}{30 \left(\cos\left(2 x\right)^{4} - 2 \cos\left(2 x\right)^{2} + 1\right) \sin\left(2 x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)^3*csc(2*x)^3,x, algorithm="fricas")
```

```
[Out] -1/30*(5*cos(2*x)^2 - 2)/((cos(2*x)^4 - 2*cos(2*x)^2 + 1)*sin(2*x))
```

Sympy [A] time = 0.099444, size = 17, normalized size = 0.81

$$\frac{5 \sin^{2}{\left(2x \right)} - 3}{30 \sin^{5}{\left(2x \right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)**3*csc(2*x)**3,x)
```

```
[Out] (5*sin(2*x)**2 - 3)/(30*sin(2*x)**5)
```

Giac [A] time = 1.08467, size = 24, normalized size = 1.14

$$\frac{5 \sin\left(2 x\right)^{2} - 3}{30 \sin\left(2 x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(2*x)^3*csc(2*x)^3,x, algorithm="giac")
```

```
[Out] 1/30*(5*sin(2*x)^2 - 3)/sin(2*x)^5
```

3.315 $\int (x+\sin(x))^2 \, dx$

Optimal. Leaf size=30

$$\frac{x^3}{3}+\frac{x}{2}+2\sin(x)-2x\cos(x)-\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + x^3/3 - 2*x*Cos[x] + 2*Sin[x] - (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0340623, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6742, 3296, 2637, 2635, 8}

$$\frac{x^3}{3}+\frac{x}{2}+2\sin(x)-2x\cos(x)-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x + Sin[x])^2,x]
```

```
[Out] x/2 + x^3/3 - 2*x*Cos[x] + 2*Sin[x] - (Cos[x]*Sin[x])/2
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (x+\sin(x))^2\,dx &= \int \left(x^2+2x\sin(x)+\sin^2(x)\right)\,dx\\ &= \frac{x^3}{3}+2\int x\sin(x)\,dx+\int \sin^2(x)\,dx\\ &= \frac{x^3}{3}-2x\cos(x)-\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}+2\int \cos(x)\,dx\\ &= \frac{x}{2}+\frac{x^3}{3}-2x\cos(x)+2\sin(x)-\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0578726, size = 30, normalized size = 1.

$$\frac{1}{6}x\left(2x^2+3\right)+2\sin(x)-\frac{1}{4}\sin(2x)-2x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sin[x])^2,x]
```

```
[Out] (x*(3 + 2*x^2))/6 - 2*x*Cos[x] + 2*Sin[x] - Sin[2*x]/4
```

Maple [A] time = 0.008, size = 25, normalized size = 0.8

$$\frac{x}{2}+\frac{x^3}{3}-2\,x\cos(x)+2\,\sin(x)-\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+sin(x))^2,x)
```

```
[Out] 1/2*x+1/3*x^3-2*x*cos(x)+2*sin(x)-1/2*cos(x)*sin(x)
```

Maxima [A] time = 0.935999, size = 32, normalized size = 1.07

$$\frac{1}{3}x^3 - 2x\cos(x) + \frac{1}{2}x - \frac{1}{4}\sin(2x) + 2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)
```

Fricas [A] time = 2.17985, size = 76, normalized size = 2.53

$$\frac{1}{3}x^3 - 2x\cos(x) - \frac{1}{2}(\cos(x) - 4)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 - 2*x*cos(x) - 1/2*(cos(x) - 4)*sin(x) + 1/2*x
```

Sympy [A] time = 0.192206, size = 41, normalized size = 1.37

$$\frac{x^3}{3} + \frac{x\sin^2(x)}{2} + \frac{x\cos^2(x)}{2} - 2x\cos(x) - \frac{\sin(x)\cos(x)}{2} + 2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))**2,x)
```

```
[Out] x**3/3 + x*sin(x)**2/2 + x*cos(x)**2/2 - 2*x*cos(x) - sin(x)*cos(x)/2 + 2*s
in(x)
```

Giac [A] time = 1.05383, size = 32, normalized size = 1.07

$$\frac{1}{3}x^3 - 2x\cos(x) + \frac{1}{2}x - \frac{1}{4}\sin(2x) + 2\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+sin(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 2*x*cos(x) + 1/2*x - 1/4*sin(2*x) + 2*sin(x)
```

3.316 $\int \frac{e^{\tan^{-1}(x)}}{1+x^2}\,dx$

Optimal. Leaf size=4

$$e^{\tan^{-1}(x)}$$

```
[Out] E^ArcTan[x]
```

Rubi [A] time = 0.0205325, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5071}

$$e^{\tan^{-1}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[x]/(1 + x^2),x]
```

```
[Out] E^ArcTan[x]
```

Rule 5071

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[E
^(n*ArcTan[a*x])/(a*c*n), x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c]
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(x)}}{1+x^2}\,dx = e^{\tan^{-1}(x)}$$

Mathematica [C] time = 0.0042299, size = 27, normalized size = 6.75

$$(1-ix)^{\frac{i}{2}}(1+ix)^{-\frac{i}{2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[E^ArcTan[x]/(1 + x^2),x]
```

```
[Out] (1 - I*x)^(I/2)/(1 + I*x)^(I/2)
```

Maple [A] time = 0.003, size = 4, normalized size = 1.

$$e^{\arctan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(x))/(x^2+1),x)
```

```
[Out] exp(arctan(x))
```

Maxima [A] time = 0.925476, size = 4, normalized size = 1.

$$e^{\arctan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))/(x^2+1),x, algorithm="maxima")
```

```
[Out] e^arctan(x)
```

Fricas [A] time = 2.18507, size = 18, normalized size = 4.5

$$e^{\arctan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))/(x^2+1),x, algorithm="fricas")
```

```
[Out] e^arctan(x)
```

Sympy [A] time = 0.866199, size = 3, normalized size = 0.75

$$e^{\operatorname{atan}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(x))/(x**2+1),x)
```

```
[Out] exp(atan(x))
```

Giac [A] time = 1.04797, size = 4, normalized size = 1.

$$e^{\arctan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))/(x^2+1),x, algorithm="giac")
```

```
[Out] e^arctan(x)
```

3.317 $\int \frac{1}{x(1+x^4)}\,dx$

Optimal. Leaf size=13

$$\log(x)-\frac{1}{4}\log\left(x^4+1\right)$$

```
[Out] Log[x] - Log[1 + x^4]/4
```

Rubi [A] time = 0.0049037, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {266, 36, 29, 31}

$$\log(x)-\frac{1}{4}\log\left(x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^4)),x]
```

```
[Out] Log[x] - Log[1 + x^4]/4
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1+x^4\right)}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,x^4\right)\\ &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x}\,dx,x,x^4\right)-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^4\right)\\ &= \log(x)-\frac{1}{4}\log\left(1+x^4\right)\end{aligned}$$

Mathematica [A] time = 0.0030564, size = 13, normalized size = 1.

$$\log(x)-\frac{1}{4}\log\left(x^4+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^4)),x]

[Out] Log[x] - Log[1 + x^4]/4
```

Maple [A] time = 0.004, size = 12, normalized size = 0.9

$$\ln(x)-\frac{\ln\left(x^4+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4+1),x)

[Out] ln(x)-1/4*ln(x^4+1)
```

Maxima [A] time = 0.929044, size = 20, normalized size = 1.54

$$-\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1),x, algorithm="maxima")
```

```
[Out] -1/4*log(x^4 + 1) + 1/4*log(x^4)
```

Fricas [A] time = 2.02057, size = 38, normalized size = 2.92

$$-\frac{1}{4}\log\left(x^4+1\right)+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/4*log(x^4 + 1) + log(x)
```

Sympy [A] time = 0.09011, size = 10, normalized size = 0.77

$$\log\left(x\right)-\frac{\log\left(x^4+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4+1),x)
```

```
[Out] log(x) - log(x**4 + 1)/4
```

Giac [A] time = 1.04758, size = 20, normalized size = 1.54

$$-\frac{1}{4}\log\left(x^4+1\right)+\frac{1}{4}\log\left(x^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1),x, algorithm="giac")
```

```
[Out] -1/4*log(x^4 + 1) + 1/4*log(x^4)
```

3.318 $\int e^{-2t} t^3 \, dt$

Optimal. Leaf size=44

$$-\frac{1}{2}e^{-2t}t^3 - \frac{3}{4}e^{-2t}t^2 - \frac{3}{4}e^{-2t}t - \frac{3e^{-2t}}{8}$$

```
[Out] -3/(8*E^(2*t)) - (3*t)/(4*E^(2*t)) - (3*t^2)/(4*E^(2*t)) - t^3/(2*E^(2*t))
```

Rubi [A] time = 0.0385098, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$-\frac{1}{2}e^{-2t}t^3 - \frac{3}{4}e^{-2t}t^2 - \frac{3}{4}e^{-2t}t - \frac{3e^{-2t}}{8}$$

Antiderivative was successfully verified.

```
[In] Int[t^3/E^(2*t),t]
```

```
[Out] -3/(8*E^(2*t)) - (3*t)/(4*E^(2*t)) - (3*t^2)/(4*E^(2*t)) - t^3/(2*E^(2*t))
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int e^{-2t}t^3\,dt &= -\frac{1}{2}e^{-2t}t^3 + \frac{3}{2}\int e^{-2t}t^2\,dt \\
&= -\frac{3}{4}e^{-2t}t^2 - \frac{1}{2}e^{-2t}t^3 + \frac{3}{2}\int e^{-2t}t\,dt \\
&= -\frac{3}{4}e^{-2t}t - \frac{3}{4}e^{-2t}t^2 - \frac{1}{2}e^{-2t}t^3 + \frac{3}{4}\int e^{-2t}\,dt \\
&= -\frac{3}{8}e^{-2t} - \frac{3}{4}e^{-2t}t - \frac{3}{4}e^{-2t}t^2 - \frac{1}{2}e^{-2t}t^3
\end{aligned}
$$

Mathematica [A] time = 0.0087274, size = 24, normalized size = 0.55

$$-\frac{1}{8}e^{-2t}\left(4t^3 + 6t^2 + 6t + 3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[t^3/E^(2*t),t]
```

```
[Out] -(3 + 6*t + 6*t^2 + 4*t^3)/(8*E^(2*t))
```

Maple [A] time = 0.001, size = 24, normalized size = 0.6

$$-\frac{4\,t^3 + 6\,t^2 + 6\,t + 3}{8\,e^{2\,t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^3/exp(2*t),t)
```

```
[Out] -1/8*(4*t^3+6*t^2+6*t+3)/exp(2*t)
```

Maxima [A] time = 0.93674, size = 28, normalized size = 0.64

$$-\frac{1}{8}\left(4\,t^3 + 6\,t^2 + 6\,t + 3\right)e^{(-2\,t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/exp(2*t),t, algorithm="maxima")
```

```
[Out] -1/8*(4*t^3 + 6*t^2 + 6*t + 3)*e^(-2*t)
```

Fricas [A] time = 2.10728, size = 55, normalized size = 1.25

$$-\frac{1}{8}\left(4\,t^3+6\,t^2+6\,t+3\right)e^{(-2\,t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/exp(2*t),t, algorithm="fricas")
```

```
[Out] -1/8*(4*t^3 + 6*t^2 + 6*t + 3)*e^(-2*t)
```

Sympy [A] time = 0.082717, size = 22, normalized size = 0.5

$$\frac{\left(-4t^3-6t^2-6t-3\right)e^{-2t}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t**3/exp(2*t),t)
```

```
[Out] (-4*t**3 - 6*t**2 - 6*t - 3)*exp(-2*t)/8
```

Giac [A] time = 1.0501, size = 28, normalized size = 0.64

$$-\frac{1}{8}\left(4\,t^3+6\,t^2+6\,t+3\right)e^{(-2\,t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/exp(2*t),t, algorithm="giac")
```

```
[Out] -1/8*(4*t^3 + 6*t^2 + 6*t + 3)*e^(-2*t)
```

3.319 $\int \frac{\sqrt{t}}{1+\sqrt[3]{t}}\,dt$

Optimal. Leaf size=41

$$\frac{6t^{7/6}}{7}-\frac{6t^{5/6}}{5}+2\sqrt{t}-6\sqrt[6]{t}+6\tan^{-1}\left(\sqrt[6]{t}\right)$$

```
[Out] -6*t^(1/6) + 2*Sqrt[t] - (6*t^(5/6))/5 + (6*t^(7/6))/7 + 6*ArcTan[t^(1/6)]
```

Rubi [A] time = 0.0119473, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {341, 50, 63, 203}

$$\frac{6t^{7/6}}{7}-\frac{6t^{5/6}}{5}+2\sqrt{t}-6\sqrt[6]{t}+6\tan^{-1}\left(\sqrt[6]{t}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[t]/(1 + t^(1/3)),t]
```

```
[Out] -6*t^(1/6) + 2*Sqrt[t] - (6*t^(5/6))/5 + (6*t^(7/6))/7 + 6*ArcTan[t^(1/6)]
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^
(1/k)], x]] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{t}}{1+\sqrt[3]{t}}\,dt &= 3\,\text{Subst}\left(\int \frac{t^{7/2}}{1+t}\,dt, t, \sqrt[3]{t}\right)\\
&= \frac{6t^{7/6}}{7} - 3\,\text{Subst}\left(\int \frac{t^{5/2}}{1+t}\,dt, t, \sqrt[3]{t}\right)\\
&= -\frac{6t^{5/6}}{5} + \frac{6t^{7/6}}{7} + 3\,\text{Subst}\left(\int \frac{t^{3/2}}{1+t}\,dt, t, \sqrt[3]{t}\right)\\
&= 2\sqrt{t} - \frac{6t^{5/6}}{5} + \frac{6t^{7/6}}{7} - 3\,\text{Subst}\left(\int \frac{\sqrt{t}}{1+t}\,dt, t, \sqrt[3]{t}\right)\\
&= -6\sqrt[6]{t} + 2\sqrt{t} - \frac{6t^{5/6}}{5} + \frac{6t^{7/6}}{7} + 3\,\text{Subst}\left(\int \frac{1}{\sqrt{t}(1+t)}\,dt, t, \sqrt[3]{t}\right)\\
&= -6\sqrt[6]{t} + 2\sqrt{t} - \frac{6t^{5/6}}{5} + \frac{6t^{7/6}}{7} + 6\,\text{Subst}\left(\int \frac{1}{1+t^2}\,dt, t, \sqrt[6]{t}\right)\\
&= -6\sqrt[6]{t} + 2\sqrt{t} - \frac{6t^{5/6}}{5} + \frac{6t^{7/6}}{7} + 6\tan^{-1}\left(\sqrt[6]{t}\right)
\end{aligned}$$

Mathematica [A] time = 0.0095159, size = 41, normalized size = 1.

$$\frac{6t^{7/6}}{7} - \frac{6t^{5/6}}{5} + 2\sqrt{t} - 6\sqrt[6]{t} + 6\tan^{-1}\left(\sqrt[6]{t}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[t]/(1 + t^(1/3)),t]
```

```
[Out] -6*t^(1/6) + 2*Sqrt[t] - (6*t^(5/6))/5 + (6*t^(7/6))/7 + 6*ArcTan[t^(1/6)]
```

Maple [A] time = 0.005, size = 28, normalized size = 0.7

$$-6\sqrt[6]{t}-\frac{6}{5}t^{\frac{5}{6}}+\frac{6}{7}t^{\frac{7}{6}}+6\arctan\left(\sqrt[6]{t}\right)+2\sqrt{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^(1/2)/(1+t^(1/3)),t)
```

```
[Out] -6*t^(1/6)-6/5*t^(5/6)+6/7*t^(7/6)+6*arctan(t^(1/6))+2*t^(1/2)
```

Maxima [A] time = 1.40737, size = 36, normalized size = 0.88

$$\frac{6}{7}t^{\frac{7}{6}}-\frac{6}{5}t^{\frac{5}{6}}+2\sqrt{t}-6t^{\frac{1}{6}}+6\arctan\left(t^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^(1/2)/(1+t^(1/3)),t, algorithm="maxima")
```

```
[Out] 6/7*t^(7/6) - 6/5*t^(5/6) + 2*sqrt(t) - 6*t^(1/6) + 6*arctan(t^(1/6))
```

Fricas [A] time = 2.12846, size = 90, normalized size = 2.2

$$\frac{6}{7}(t-7)t^{\frac{1}{6}}-\frac{6}{5}t^{\frac{5}{6}}+2\sqrt{t}+6\arctan\left(t^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^(1/2)/(1+t^(1/3)),t, algorithm="fricas")
```

```
[Out] 6/7*(t - 7)*t^(1/6) - 6/5*t^(5/6) + 2*sqrt(t) + 6*arctan(t^(1/6))
```

Sympy [A] time = 2.93056, size = 37, normalized size = 0.9

$$\frac{6t^{\frac{7}{6}}}{7}-\frac{6t^{\frac{5}{6}}}{5}-6\sqrt[6]{t}+2\sqrt{t}+6\,\mathrm{atan}\left(\sqrt[6]{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t**(1/2)/(1+t**(1/3)),t)
```

```
[Out] 6*t**(7/6)/7 - 6*t**(5/6)/5 - 6*t**(1/6) + 2*sqrt(t) + 6*atan(t**(1/6))
```

Giac [A] time = 1.05334, size = 36, normalized size = 0.88

$$\frac{6}{7}\,t^{\frac{7}{6}} - \frac{6}{5}\,t^{\frac{5}{6}} + 2\,\sqrt{t} - 6\,t^{\frac{1}{6}} + 6\,\arctan\left(t^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^(1/2)/(1+t^(1/3)),t, algorithm="giac")
```

```
[Out] 6/7*t^(7/6) - 6/5*t^(5/6) + 2*sqrt(t) - 6*t^(1/6) + 6*arctan(t^(1/6))
```

3.320 $\int \sin(x)\sin(2x)\sin(3x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

```
[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Rubi [A] time = 0.0296515, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2638}

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sin(x)\sin(2x)\sin(3x)\,dx = \int \left(\frac{1}{4}\sin(2x) + \frac{1}{4}\sin(4x) - \frac{1}{4}\sin(6x)\right)dx$$
$$= \frac{1}{4}\int \sin(2x)\,dx + \frac{1}{4}\int \sin(4x)\,dx - \frac{1}{4}\int \sin(6x)\,dx$$
$$= -\frac{1}{8}\cos(2x) - \frac{1}{16}\cos(4x) + \frac{1}{24}\cos(6x)$$

Mathematica [A] time = 0.0096997, size = 25, normalized size = 1.

$$-\frac{1}{8}\cos(2x) - \frac{1}{16}\cos(4x) + \frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Maple [A] time = 0.046, size = 20, normalized size = 0.8

$$-\frac{\cos(2x)}{8} - \frac{\cos(4x)}{16} + \frac{\cos(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Maxima [A] time = 0.928093, size = 26, normalized size = 1.04

$$\frac{1}{24}\cos(6x) - \frac{1}{16}\cos(4x) - \frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)
```

Fricas [A] time = 2.309, size = 54, normalized size = 2.16

$$\frac{4}{3}\cos(x)^6 - \frac{5}{2}\cos(x)^4 + \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2
```

Sympy [B] time = 15.0418, size = 114, normalized size = 4.56

$$\frac{x\sin(x)\sin(2x)\sin(3x)}{4} + \frac{x\sin(x)\cos(2x)\cos(3x)}{4} + \frac{x\sin(2x)\cos(x)\cos(3x)}{4} - \frac{x\sin(3x)\cos(x)\cos(2x)}{4} - \frac{5\sin(x)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*co
s(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - 5*sin(x)*sin(2*x)*cos(3*x)
/24 - sin(2*x)*sin(3*x)*cos(x)/8 - cos(x)*cos(2*x)*cos(3*x)/6
```

Giac [A] time = 1.05124, size = 18, normalized size = 0.72

$$-\frac{4}{3}\sin(x)^6 + \frac{3}{2}\sin(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -4/3*sin(x)^6 + 3/2*sin(x)^4
```

3.321 $\int \log\left(\frac{x}{2}\right)\,dx$

Optimal. Leaf size=12

$$x\log\left(\frac{x}{2}\right)-x$$

```
[Out] -x + x*Log[x/2]
```

Rubi [A] time = 0.0013106, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2295}

$$x\log\left(\frac{x}{2}\right)-x$$

Antiderivative was successfully verified.

```
[In] Int[Log[x/2],x]
```

```
[Out] -x + x*Log[x/2]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int \log\left(\frac{x}{2}\right)\,dx=-x+x\log\left(\frac{x}{2}\right)$$

Mathematica [A] time = 0.0006773, size = 12, normalized size = 1.

$$x\log\left(\frac{x}{2}\right)-x$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x/2],x]
```

```
[Out] -x + x*Log[x/2]
```

Maple [A] time = 0.001, size = 11, normalized size = 0.9

$$-x + x\ln\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1/2*x),x)
```

```
[Out] -x+x*ln(1/2*x)
```

Maxima [A] time = 0.937229, size = 14, normalized size = 1.17

$$x\log\left(\frac{1}{2}\,x\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/2*x),x, algorithm="maxima")
```

```
[Out] x*log(1/2*x) - x
```

Fricas [A] time = 2.3514, size = 24, normalized size = 2.

$$x\log\left(\frac{1}{2}\,x\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/2*x),x, algorithm="fricas")
```

```
[Out] x*log(1/2*x) - x
```

Sympy [A] time = 0.081452, size = 7, normalized size = 0.58

$$x\log\left(\frac{x}{2}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1/2*x),x)
```

```
[Out] x*log(x/2) - x
```

Giac [A] time = 1.04918, size = 14, normalized size = 1.17

$$x\log\left(\frac{1}{2}\,x\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/2*x),x, algorithm="giac")
```

```
[Out] x*log(1/2*x) - x
```

3.322 $\int \sqrt{\frac{1+x}{1-x}}\,dx$

Optimal. Leaf size=41

$$2\tan^{-1}\left(\sqrt{\frac{x+1}{1-x}}\right)-(1-x)\sqrt{\frac{x+1}{1-x}}$$

```
[Out] -((1 - x)*Sqrt[(1 + x)/(1 - x)]) + 2*ArcTan[Sqrt[(1 + x)/(1 - x)]]
```

Rubi [A] time = 0.0130075, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1959, 288, 203}

$$2\tan^{-1}\left(\sqrt{\frac{x+1}{1-x}}\right)-(1-x)\sqrt{\frac{x+1}{1-x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x)/(1 - x)],x]
```

```
[Out] -((1 - x)*Sqrt[(1 + x)/(1 - x)]) + 2*ArcTan[Sqrt[(1 + x)/(1 - x)]]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int\sqrt{\frac{1+x}{1-x}}\,dx &= 4\,\text{Subst}\left(\int\frac{x^2}{\left(1+x^2\right)^2}\,dx,x,\sqrt{\frac{1+x}{1-x}}\right)\\ &= -(1-x)\sqrt{\frac{1+x}{1-x}}+2\,\text{Subst}\left(\int\frac{1}{1+x^2}\,dx,x,\sqrt{\frac{1+x}{1-x}}\right)\\ &= -(1-x)\sqrt{\frac{1+x}{1-x}}+2\tan^{-1}\left(\sqrt{\frac{1+x}{1-x}}\right)\end{aligned}$$

Mathematica [A] time = 0.0228433, size = 62, normalized size = 1.51

$$\frac{\sqrt{\frac{x+1}{1-x}}\left((x-1)\sqrt{x+1}-2\sqrt{1-x}\sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)\right)}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x)/(1 - x)],x]
```

```
[Out] (Sqrt[(1 + x)/(1 - x)]*((-1 + x)*Sqrt[1 + x] - 2*Sqrt[1 - x]*ArcSin[Sqrt[1
- x]/Sqrt[2]]))/Sqrt[1 + x]
```

Maple [A] time = 0.008, size = 41, normalized size = 1.

$$(-1+x)\sqrt{-\frac{1+x}{-1+x}}\left(\sqrt{-x^2+1}-\arcsin(x)\right)\frac{1}{\sqrt{-(-1+x)(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(1-x))^(1/2),x)
```

```
[Out] (-(1+x)/(-1+x))^(1/2)*(-1+x)/(-(-1+x)*(1+x))^(1/2)*((-x^2+1)^(1/2)-arcsin(x
))
```

Maxima [A] time = 1.41076, size = 58, normalized size = 1.41

$$\frac{2\sqrt{-\frac{x+1}{x-1}}}{\frac{x+1}{x-1}-1}+2\arctan\left(\sqrt{-\frac{x+1}{x-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(1-x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(-(x + 1)/(x - 1))/((x + 1)/(x - 1) - 1) + 2*arctan(sqrt(-(x + 1)/(x
- 1)))
```

Fricas [A] time = 2.27597, size = 90, normalized size = 2.2

$$(x-1)\sqrt{-\frac{x+1}{x-1}}+2\arctan\left(\sqrt{-\frac{x+1}{x-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(1-x))^(1/2),x, algorithm="fricas")
```

```
[Out] (x - 1)*sqrt(-(x + 1)/(x - 1)) + 2*arctan(sqrt(-(x + 1)/(x - 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{x+1}{1-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(1-x))**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)/(1 - x)), x)
```

Giac [A] time = 1.07219, size = 41, normalized size = 1.

$$\frac{1}{2}\,\pi\mathrm{sgn}\,(x-1) - \arcsin\,(x)\,\mathrm{sgn}\,(x-1) + \sqrt{-x^2+1}\mathrm{sgn}\,(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(1-x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*pi*sgn(x - 1) - arcsin(x)*sgn(x - 1) + sqrt(-x^2 + 1)*sgn(x - 1)
```

3.323 $\int \frac{x\log(x)}{\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=34

$$-\sqrt{x^2-1}+\sqrt{x^2-1}\log(x)+\tan^{-1}\left(\sqrt{x^2-1}\right)$$

```
[Out] -Sqrt[-1 + x^2] + ArcTan[Sqrt[-1 + x^2]] + Sqrt[-1 + x^2]*Log[x]
```

Rubi [A] time = 0.0365319, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2338, 266, 50, 63, 203}

$$-\sqrt{x^2-1}+\sqrt{x^2-1}\log(x)+\tan^{-1}\left(\sqrt{x^2-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*Log[x])/Sqrt[-1 + x^2],x]
```

```
[Out] -Sqrt[-1 + x^2] + ArcTan[Sqrt[-1 + x^2]] + Sqrt[-1 + x^2]*Log[x]
```

Rule 2338

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_))^(q_.), x_Symbol] :> Simp[(f^m*(d + e*x^r)^(q + 1)*(a + b*L
og[c*x^n])^p)/(e*r*(q + 1)), x] - Dist[(b*f^m*n*p)/(e*r*(q + 1)), Int[((d +
 e*x^r)^(q + 1)*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d,
 e, f, m, n, q, r}, x] && EqQ[m, r - 1] && IGtQ[p, 0] && (IntegerQ[m] || Gt
Q[f, 0]) && NeQ[r, n] && NeQ[q, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
```

```
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\log(x)}{\sqrt{-1+x^2}}\,dx &= \sqrt{-1+x^2}\log(x)-\int \frac{\sqrt{-1+x^2}}{x}\,dx\\
&= \sqrt{-1+x^2}\log(x)-\frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx,x,x^2\right)\\
&= -\sqrt{-1+x^2}+\sqrt{-1+x^2}\log(x)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{-1+xx}}\,dx,x,x^2\right)\\
&= -\sqrt{-1+x^2}+\sqrt{-1+x^2}\log(x)+\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sqrt{-1+x^2}\right)\\
&= -\sqrt{-1+x^2}+\tan^{-1}\left(\sqrt{-1+x^2}\right)+\sqrt{-1+x^2}\log(x)
\end{aligned}
$$

Mathematica [A] time = 0.0203238, size = 27, normalized size = 0.79

$$\sqrt{x^2-1}(\log(x)-1)-\tan^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Log[x])/Sqrt[-1 + x^2],x]
```

```
[Out] -ArcTan[1/Sqrt[-1 + x^2]] + Sqrt[-1 + x^2]*(-1 + Log[x])
```

Maple [C] time = 0., size = 119, normalized size = 3.5

$$-\frac{1}{4}\sqrt{-\mathrm{signum}\left(x^2-1\right)}\left(2-2\sqrt{-x^2+1}\right)\frac{1}{\sqrt{\mathrm{signum}\left(x^2-1\right)}}+\frac{\ln(x)}{2}\sqrt{-\mathrm{signum}\left(x^2-1\right)}\left(2-2\sqrt{-x^2+1}\right)\frac{1}{\sqrt{\mathrm{signum}\left(x\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x)/(x^2-1)^(1/2),x)
```

```
[Out] -1/4/signum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*(2-2*(-x^2+1)^(1/2))+1/2/si
gnum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*ln(x)*(2-2*(-x^2+1)^(1/2))+1/32/si
gnum(x^2-1)^(1/2)*(-signum(x^2-1))^(1/2)*(-16+16*(-x^2+1)^(1/2)-32*ln(1/2+1
/2*(-x^2+1)^(1/2)))
```

Maxima [A] time = 1.41712, size = 36, normalized size = 1.06

$$\sqrt{x^2-1}\log(x)-\sqrt{x^2-1}-\arcsin\left(\frac{1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 1)*log(x) - sqrt(x^2 - 1) - arcsin(1/abs(x))
```

Fricas [A] time = 2.35498, size = 80, normalized size = 2.35

$$\sqrt{x^2-1}(\log(x)-1)+2\arctan\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 - 1)*(log(x) - 1) + 2*arctan(-x + sqrt(x^2 - 1))
```

Sympy [A] time = 2.89017, size = 29, normalized size = 0.85

$$\sqrt{x^2-1}\log(x) - \begin{cases}\sqrt{x^2-1} - \operatorname{acos}\left(\frac{1}{x}\right) & \text{for } x > -1 \wedge x < 1\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x)/(x**2-1)**(1/2),x)
```

```
[Out] sqrt(x**2 - 1)*log(x) - Piecewise((sqrt(x**2 - 1) - acos(1/x), (x > -1) & (
x < 1)))
```

Giac [A] time = 1.06846, size = 38, normalized size = 1.12

$$\sqrt{x^2-1}\log(x) - \sqrt{x^2-1} + \arctan\left(\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 - 1)*log(x) - sqrt(x^2 - 1) + arctan(sqrt(x^2 - 1))
```

3.324 $\int \frac{a+x}{a^2+x^2} dx$

Optimal. Leaf size=19

$$\frac{1}{2}\log\left(a^2+x^2\right)+\tan^{-1}\left(\frac{x}{a}\right)$$

```
[Out] ArcTan[x/a] + Log[a^2 + x^2]/2
```

Rubi [A] time = 0.0062446, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {635, 203, 260}

$$\frac{1}{2}\log\left(a^2+x^2\right)+\tan^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + x)/(a^2 + x^2),x]
```

```
[Out] ArcTan[x/a] + Log[a^2 + x^2]/2
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{a+x}{a^2+x^2}\,dx = a\int \frac{1}{a^2+x^2}\,dx + \int \frac{x}{a^2+x^2}\,dx$$
$$= \tan^{-1}\left(\frac{x}{a}\right) + \frac{1}{2}\log\left(a^2+x^2\right)$$

Mathematica [A] time = 0.0042114, size = 19, normalized size = 1.

$$\frac{1}{2}\log\left(a^2+x^2\right) + \tan^{-1}\left(\frac{x}{a}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + x)/(a^2 + x^2),x]
```

```
[Out] ArcTan[x/a] + Log[a^2 + x^2]/2
```

Maple [A] time = 0.001, size = 18, normalized size = 1.

$$\arctan\left(\frac{x}{a}\right) + \frac{\ln\left(a^2+x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+x)/(a^2+x^2),x)
```

```
[Out] arctan(x/a)+1/2*ln(a^2+x^2)
```

Maxima [A] time = 1.42015, size = 23, normalized size = 1.21

$$\arctan\left(\frac{x}{a}\right) + \frac{1}{2}\log\left(a^2+x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(a^2+x^2),x, algorithm="maxima")
```

```
[Out] arctan(x/a) + 1/2*log(a^2 + x^2)
```

Fricas [A] time = 2.09241, size = 46, normalized size = 2.42

$$\arctan\left(\frac{x}{a}\right) + \frac{1}{2}\,\log\left(a^2 + x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(a^2+x^2),x, algorithm="fricas")
```

```
[Out] arctan(x/a) + 1/2*log(a^2 + x^2)
```

Sympy [C] time = 0.101728, size = 42, normalized size = 2.21

$$\left(\frac{1}{2} - \frac{i}{2}\right)\log\left(-a + 2a\left(\frac{1}{2} - \frac{i}{2}\right) + x\right) + \left(\frac{1}{2} + \frac{i}{2}\right)\log\left(-a + 2a\left(\frac{1}{2} + \frac{i}{2}\right) + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(a**2+x**2),x)
```

```
[Out] (1/2 - I/2)*log(-a + 2*a*(1/2 - I/2) + x) + (1/2 + I/2)*log(-a + 2*a*(1/2 +
 I/2) + x)
```

Giac [A] time = 1.05321, size = 23, normalized size = 1.21

$$\arctan\left(\frac{x}{a}\right) + \frac{1}{2}\,\log\left(a^2 + x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+x)/(a^2+x^2),x, algorithm="giac")
```

```
[Out] arctan(x/a) + 1/2*log(a^2 + x^2)
```

3.325 $\int \sqrt{1+x-x^2}\,dx$

Optimal. Leaf size=38

$$-\frac{1}{4}\sqrt{-x^2+x+1}(1-2x)-\frac{5}{8}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)$$

```
[Out] -((1 - 2*x)*Sqrt[1 + x - x^2])/4 - (5*ArcSin[(1 - 2*x)/Sqrt[5]])/8
```

Rubi [A] time = 0.0117996, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {612, 619, 216}

$$-\frac{1}{4}\sqrt{-x^2+x+1}(1-2x)-\frac{5}{8}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x - x^2],x]
```

```
[Out] -((1 - 2*x)*Sqrt[1 + x - x^2])/4 - (5*ArcSin[(1 - 2*x)/Sqrt[5]])/8
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{1+x-x^2}\,dx = -\frac{1}{4}(1-2x)\sqrt{1+x-x^2}+\frac{5}{8}\int \frac{1}{\sqrt{1+x-x^2}}\,dx$$
$$= -\frac{1}{4}(1-2x)\sqrt{1+x-x^2}-\frac{1}{8}\sqrt{5}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{5}}}\,dx, x, 1-2x\right)$$
$$= -\frac{1}{4}(1-2x)\sqrt{1+x-x^2}-\frac{5}{8}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)$$

Mathematica [A] time = 0.0146953, size = 39, normalized size = 1.03

$$\left(\frac{x}{2}-\frac{1}{4}\right)\sqrt{-x^2+x+1}-\frac{5}{8}\sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x - x^2],x]
```

```
[Out] (-1/4 + x/2)*Sqrt[1 + x - x^2] - (5*ArcSin[(1 - 2*x)/Sqrt[5]])/8
```

Maple [A] time = 0.003, size = 30, normalized size = 0.8

$$-\frac{1-2\,x}{4}\sqrt{-x^2+x+1}+\frac{5}{8}\arcsin\left(\frac{2\sqrt{5}}{5}\left(x-\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+x+1)^(1/2),x)
```

```
[Out] -1/4*(1-2*x)*(-x^2+x+1)^(1/2)+5/8*arcsin(2/5*5^(1/2)*(x-1/2))
```

Maxima [A] time = 1.41514, size = 53, normalized size = 1.39

$$\frac{1}{2}\sqrt{-x^2+x+1}x-\frac{1}{4}\sqrt{-x^2+x+1}-\frac{5}{8}\arcsin\left(-\frac{1}{5}\sqrt{5}(2\,x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + x + 1)*x - 1/4*sqrt(-x^2 + x + 1) - 5/8*arcsin(-1/5*sqrt(5)
*(2*x - 1))
```

Fricas [A] time = 2.12894, size = 101, normalized size = 2.66

$$\frac{1}{4}\sqrt{-x^2+x+1}(2x-1)-\frac{5}{4}\arctan\left(\frac{\sqrt{-x^2+x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-x^2 + x + 1)*(2*x - 1) - 5/4*arctan((sqrt(-x^2 + x + 1) - 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^2+x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+x+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**2 + x + 1), x)
```

Giac [A] time = 1.05773, size = 42, normalized size = 1.11

$$\frac{1}{4}\sqrt{-x^2+x+1}(2x-1)+\frac{5}{8}\arcsin\left(\frac{1}{5}\sqrt{5}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(-x^2 + x + 1)*(2*x - 1) + 5/8*arcsin(1/5*sqrt(5)*(2*x - 1))
```

3.326 $\int \frac{x^4}{16+x^{10}} \, dx$

Optimal. Leaf size=12

$$\frac{1}{20} \tan^{-1}\left(\frac{x^5}{4}\right)$$

```
[Out] ArcTan[x^5/4]/20
```

Rubi [A] time = 0.0050937, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {275, 203}

$$\frac{1}{20} \tan^{-1}\left(\frac{x^5}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(16 + x^10),x]
```

```
[Out] ArcTan[x^5/4]/20
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^4}{16+x^{10}} \, dx &= \frac{1}{5} \text{Subst}\left(\int \frac{1}{16+x^2} \, dx, x, x^5\right) \\ &= \frac{1}{20} \tan^{-1}\left(\frac{x^5}{4}\right) \end{aligned}$$

Mathematica [A] time = 0.0036869, size = 12, normalized size = 1.

$$\frac{1}{20}\tan^{-1}\left(\frac{x^5}{4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(16 + x^10),x]
```

```
[Out] ArcTan[x^5/4]/20
```

Maple [A] time = 0.001, size = 9, normalized size = 0.8

$$\frac{1}{20}\arctan\left(\frac{x^5}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^10+16),x)
```

```
[Out] 1/20*arctan(1/4*x^5)
```

Maxima [A] time = 1.40738, size = 11, normalized size = 0.92

$$\frac{1}{20}\arctan\left(\frac{1}{4}x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+16),x, algorithm="maxima")
```

```
[Out] 1/20*arctan(1/4*x^5)
```

Fricas [A] time = 2.14365, size = 30, normalized size = 2.5

$$\frac{1}{20}\arctan\left(\frac{1}{4}x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+16),x, algorithm="fricas")
```

```
[Out] 1/20*arctan(1/4*x^5)
```

Sympy [A] time = 0.111918, size = 7, normalized size = 0.58

$$\frac{\operatorname{atan}\left(\frac{x^5}{4}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**10+16),x)
```

```
[Out] atan(x**5/4)/20
```

Giac [A] time = 1.06897, size = 11, normalized size = 0.92

$$\frac{1}{20} \arctan\left(\frac{1}{4} x^5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10+16),x, algorithm="giac")
```

```
[Out] 1/20*arctan(1/4*x^5)
```

3.327 $\int \frac{2+x}{2+x+x^2} dx$

Optimal. Leaf size=31

$$\frac{1}{2}\log\left(x^2+x+2\right)+\frac{3\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] (3*ArcTan[(1 + 2*x)/Sqrt[7]])/Sqrt[7] + Log[2 + x + x^2]/2
```

Rubi [A] time = 0.0145056, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {634, 618, 204, 628}

$$\frac{1}{2}\log\left(x^2+x+2\right)+\frac{3\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(2 + x + x^2),x]
```

```
[Out] (3*ArcTan[(1 + 2*x)/Sqrt[7]])/Sqrt[7] + Log[2 + x + x^2]/2
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2+x}{2+x+x^2}\,dx &= \frac{1}{2}\int \frac{1+2x}{2+x+x^2}\,dx + \frac{3}{2}\int \frac{1}{2+x+x^2}\,dx\\ &= \frac{1}{2}\log\left(2+x+x^2\right) - 3\,\mathrm{Subst}\left(\int \frac{1}{-7-x^2}\,dx, x, 1+2x\right)\\ &= \frac{3\tan^{-1}\left(\frac{1+2x}{\sqrt{7}}\right)}{\sqrt{7}} + \frac{1}{2}\log\left(2+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0098404, size = 31, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+x+2\right) + \frac{3\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(2 + x + x^2),x]
```

```
[Out] (3*ArcTan[(1 + 2*x)/Sqrt[7]])/Sqrt[7] + Log[2 + x + x^2]/2
```

Maple [A] time = 0.001, size = 27, normalized size = 0.9

$$\frac{\ln\left(x^2+x+2\right)}{2} + \frac{3\sqrt{7}}{7}\arctan\left(\frac{(1+2x)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(x^2+x+2),x)
```

```
[Out] 1/2*ln(x^2+x+2)+3/7*arctan(1/7*(1+2*x)*7^(1/2))*7^(1/2)
```

Maxima [A] time = 1.40508, size = 35, normalized size = 1.13

$$\frac{3}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)+\frac{1}{2}\log\left(x^2+x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+2),x, algorithm="maxima")
```

```
[Out] 3/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) + 1/2*log(x^2 + x + 2)
```

Fricas [A] time = 2.11347, size = 89, normalized size = 2.87

$$\frac{3}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)+\frac{1}{2}\log\left(x^2+x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+2),x, algorithm="fricas")
```

```
[Out] 3/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) + 1/2*log(x^2 + x + 2)
```

Sympy [A] time = 0.099768, size = 36, normalized size = 1.16

$$\frac{\log\left(x^2+x+2\right)}{2}+\frac{3\sqrt{7}\operatorname{atan}\left(\frac{2\sqrt{7}x}{7}+\frac{\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+x+2),x)
```

```
[Out] log(x**2 + x + 2)/2 + 3*sqrt(7)*atan(2*sqrt(7)*x/7 + sqrt(7)/7)/7
```

Giac [A] time = 1.05416, size = 35, normalized size = 1.13

$$\frac{3}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x+1)\right)+\frac{1}{2}\log\left(x^2+x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x+2),x, algorithm="giac")
```

```
[Out] 3/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x + 1)) + 1/2*log(x^2 + x + 2)
```

3.328 $\int x \sec(x) \tan(x) \, dx$

Optimal. Leaf size=10

$$x \sec(x) - \tanh^{-1}(\sin(x))$$

```
[Out] -ArcTanh[Sin[x]] + x*Sec[x]
```

Rubi [A] time = 0.0099904, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3757, 3770}

$$x \sec(x) - \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[x]*Tan[x],x]
```

```
[Out] -ArcTanh[Sin[x]] + x*Sec[x]
```

Rule 3757

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tan[(a_.) + (b_.)*(x_)^(
n_.)]^(q_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^p)/(b*n*p), x]
 - Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sec[a + b*x^n]^p, x], x] /; Free
Q[{a, b, p}, x] && IntegerQ[n] && GeQ[m, n] && EqQ[q, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int x \sec(x) \tan(x) \, dx &= x \sec(x) - \int \sec(x) \, dx \\ &= -\tanh^{-1}(\sin(x)) + x \sec(x) \end{aligned}$$

Mathematica [B] time = 0.0103341, size = 37, normalized size = 3.7

$$x\sec(x) + \log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[x]*Tan[x],x]
```

```
[Out] Log[Cos[x/2] - Sin[x/2]] - Log[Cos[x/2] + Sin[x/2]] + x*Sec[x]
```

Maple [A] time = 0.004, size = 16, normalized size = 1.6

$$\frac{x}{\cos(x)} - \ln(\sec(x) + \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(x)*tan(x),x)
```

```
[Out] x/cos(x)-ln(sec(x)+tan(x))
```

Maxima [B] time = 1.42178, size = 163, normalized size = 16.3

$$\frac{4\,x\cos(2\,x)\cos(x) + 4\,x\sin(2\,x)\sin(x) + 4\,x\cos(x) - \left(\cos(2\,x)^2 + \sin(2\,x)^2 + 2\,\cos(2\,x) + 1\right)\log\left(\cos(x)^2 + \sin(x)\right.}{2\left(\cos(2\,x)^2 + \sin(2\,x)^2 + 2\,\mathrm{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*tan(x),x, algorithm="maxima")
```

```
[Out] 1/2*(4*x*cos(2*x)*cos(x) + 4*x*sin(2*x)*sin(x) + 4*x*cos(x) - (cos(2*x)^2 +
 sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) + (co
s(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x)
+ 1))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [B] time = 2.11584, size = 95, normalized size = 9.5

$$-\frac{\cos(x)\log(\sin(x)+1)-\cos(x)\log(-\sin(x)+1)-2x}{2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*tan(x),x, algorithm="fricas")
```

```
[Out] -1/2*(cos(x)*log(sin(x) + 1) - cos(x)*log(-sin(x) + 1) - 2*x)/cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\tan(x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*tan(x),x)
```

```
[Out] Integral(x*tan(x)*sec(x), x)
```

Giac [B] time = 1.16014, size = 203, normalized size = 20.3

$$-\frac{2x\tan\left(\frac{1}{2}x\right)^2+\log\left(\frac{2\left(\tan\left(\frac{1}{2}x\right)^2+2\tan\left(\frac{1}{2}x\right)+1\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)\tan\left(\frac{1}{2}x\right)^2-\log\left(\frac{2\left(\tan\left(\frac{1}{2}x\right)^2-2\tan\left(\frac{1}{2}x\right)+1\right)}{\tan\left(\frac{1}{2}x\right)^2+1}\right)\tan\left(\frac{1}{2}x\right)^2+2x-\log\left(\frac{2\left(\tan\left(\frac{1}{2}\right.\right.}{\mathrm{ta}}\right.}{2\left(\tan\left(\frac{1}{2}x\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(x)*tan(x),x, algorithm="giac")
```

```
[Out] -1/2*(2*x*tan(1/2*x)^2 + log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan(1/2*x
)^2 + 1))*tan(1/2*x)^2 - log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x
)^2 + 1))*tan(1/2*x)^2 + 2*x - log(2*(tan(1/2*x)^2 + 2*tan(1/2*x) + 1)/(tan
(1/2*x)^2 + 1)) + log(2*(tan(1/2*x)^2 - 2*tan(1/2*x) + 1)/(tan(1/2*x)^2 + 1
)))/(tan(1/2*x)^2 - 1)
```

3.329 $\int \frac{x}{-a^4+x^4} \, dx$

Optimal. Leaf size=15

$$-\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

```
[Out] -ArcTanh[x^2/a^2]/(2*a^2)
```

Rubi [A] time = 0.0070744, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {275, 207}

$$-\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(-a^4 + x^4),x]
```

```
[Out] -ArcTanh[x^2/a^2]/(2*a^2)
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{-a^4+x^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-a^4+x^2}\,dx, x, x^2\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Mathematica [A] time = 0.0039054, size = 15, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{x^2}{a^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-a^4 + x^4),x]
```

```
[Out] -ArcTanh[x^2/a^2]/(2*a^2)
```

Maple [B] time = 0.006, size = 30, normalized size = 2.

$$-\frac{\ln\left(a^2+x^2\right)}{4\,a^2}+\frac{\ln\left(-a^2+x^2\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a^4+x^4),x)
```

```
[Out] -1/4/a^2*ln(a^2+x^2)+1/4/a^2*ln(-a^2+x^2)
```

Maxima [B] time = 0.926207, size = 39, normalized size = 2.6

$$-\frac{\log\left(a^2+x^2\right)}{4\,a^2}+\frac{\log\left(-a^2+x^2\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^4+x^4),x, algorithm="maxima")
```

```
[Out] -1/4*log(a^2 + x^2)/a^2 + 1/4*log(-a^2 + x^2)/a^2
```

Fricas [A] time = 1.8419, size = 61, normalized size = 4.07

$$-\frac{\log\left(a^2+x^2\right)-\log\left(-a^2+x^2\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^4+x^4),x, algorithm="fricas")
```

```
[Out] -1/4*(log(a^2 + x^2) - log(-a^2 + x^2))/a^2
```

Sympy [A] time = 0.132057, size = 22, normalized size = 1.47

$$\frac{\frac{\log\left(-a^2+x^2\right)}{4}-\frac{\log\left(a^2+x^2\right)}{4}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a**4+x**4),x)
```

```
[Out] (log(-a**2 + x**2)/4 - log(a**2 + x**2)/4)/a**2
```

Giac [B] time = 1.07081, size = 41, normalized size = 2.73

$$-\frac{\log\left(a^2+x^2\right)}{4\,a^2}+\frac{\log\left(\left|-a^2+x^2\right|\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a^4+x^4),x, algorithm="giac")
```

```
[Out] -1/4*log(a^2 + x^2)/a^2 + 1/4*log(abs(-a^2 + x^2))/a^2
```

3.330 $\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx$

Optimal. Leaf size=21

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Rubi [A] time = 0.0055358, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2106, 30, 32}

$$\frac{2}{3}(x+1)^{3/2}-\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Rule 2106

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{x}+\sqrt{1+x}}\,dx = -\int \sqrt{x}\,dx + \int \sqrt{1+x}\,dx$$
$$= -\frac{2x^{3/2}}{3} + \frac{2}{3}(1+x)^{3/2}$$

Mathematica [A] time = 0.0158995, size = 21, normalized size = 1.

$$\frac{2}{3}(x+1)^{3/2} - \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + Sqrt[1 + x])^(-1),x]
```

```
[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3
```

Maple [A] time = 0.002, size = 14, normalized size = 0.7

$$-\frac{2}{3}x^{\frac{3}{2}} + \frac{2}{3}(1+x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/2)+(1+x)^(1/2)),x)
```

```
[Out] -2/3*x^(3/2)+2/3*(1+x)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x + 1) + sqrt(x)), x)
```

Fricas [A] time = 1.84066, size = 45, normalized size = 2.14

$$\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

Sympy [B] time = 0.351887, size = 63, normalized size = 3.

$$\frac{2\sqrt{x}\sqrt{x+1}}{3\sqrt{x}+3\sqrt{x+1}} + \frac{4x}{3\sqrt{x}+3\sqrt{x+1}} + \frac{2}{3\sqrt{x}+3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x + 1)/(3*sqrt(x) + 3*sqrt(x + 1)) + 4*x/(3*sqrt(x) + 3*sqrt
(x + 1)) + 2/(3*sqrt(x) + 3*sqrt(x + 1))
```

Giac [A] time = 1.05009, size = 18, normalized size = 0.86

$$\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*(x + 1)^(3/2) - 2/3*x^(3/2)
```

3.331 $\int \frac{1}{1-e^{-x}+2e^x}\,dx$

Optimal. Leaf size=23

$$\frac{1}{3}\log\left(1-2e^x\right)-\frac{1}{3}\log\left(e^x+1\right)$$

```
[Out] Log[1 - 2*E^x]/3 - Log[1 + E^x]/3
```

Rubi [A] time = 0.0168223, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2282, 616, 31}

$$\frac{1}{3}\log\left(1-2e^x\right)-\frac{1}{3}\log\left(e^x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - E^(-x) + 2*E^x)^(-1),x]
```

```
[Out] Log[1 - 2*E^x]/3 - Log[1 + E^x]/3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{1-e^{-x}+2e^x}\,dx = \text{Subst}\left(\int \frac{1}{-1+x+2x^2}\,dx,x,e^x\right)$$
$$= \frac{2}{3}\text{Subst}\left(\int \frac{1}{-1+2x}\,dx,x,e^x\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{2+2x}\,dx,x,e^x\right)$$
$$= \frac{1}{3}\log(1-2e^x) - \frac{1}{3}\log(1+e^x)$$

Mathematica [A] time = 0.0112246, size = 16, normalized size = 0.7

$$-\frac{2}{3}\tanh^{-1}\left(\frac{1}{3}(4e^x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - E^(-x) + 2*E^x)^(-1),x]
```

```
[Out] (-2*ArcTanh[(1 + 4*E^x)/3])/3
```

Maple [A] time = 0.005, size = 18, normalized size = 0.8

$$-\frac{\ln(1+e^x)}{3} + \frac{\ln(2e^x-1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-1/exp(x)+2*exp(x)),x)
```

```
[Out] -1/3*ln(1+exp(x))+1/3*ln(2*exp(x)-1)
```

Maxima [A] time = 0.933128, size = 26, normalized size = 1.13

$$-\frac{1}{3}\log\left(e^{(-x)}+1\right) + \frac{1}{3}\log\left(e^{(-x)}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-1/exp(x)+2*exp(x)),x, algorithm="maxima")
```

```
[Out] -1/3*log(e^(-x) + 1) + 1/3*log(e^(-x) - 2)
```

Fricas [A] time = 1.93344, size = 53, normalized size = 2.3

$$\frac{1}{3}\log\left(2e^{x}-1\right)-\frac{1}{3}\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-1/exp(x)+2*exp(x)),x, algorithm="fricas")
```

```
[Out] 1/3*log(2*e^x - 1) - 1/3*log(e^x + 1)
```

Sympy [A] time = 0.108245, size = 19, normalized size = 0.83

$$\frac{\log\left(-2+e^{-x}\right)}{3}-\frac{\log\left(1+e^{-x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-1/exp(x)+2*exp(x)),x)
```

```
[Out] log(-2 + exp(-x))/3 - log(1 + exp(-x))/3
```

Giac [A] time = 1.04985, size = 24, normalized size = 1.04

$$-\frac{1}{3}\log\left(e^{x}+1\right)+\frac{1}{3}\log\left(|2e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-1/exp(x)+2*exp(x)),x, algorithm="giac")
```

```
[Out] -1/3*log(e^x + 1) + 1/3*log(abs(2*e^x - 1))
```

3.332 $\int \frac{\tan^{-1}(\sqrt{x})}{\sqrt{x}} dx$

Optimal. Leaf size=20

$$2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right)-\log(x+1)$$

```
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Rubi [A] time = 0.0075875, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5033, 31}

$$2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Rule 5033

```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :
> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x^n]))/(d*(m + 1)), x] - Dist[(b*c*n)
/(d*(m + 1)), Int[(x^(n - 1)*(d*x)^(m + 1))/(1 + c^2*x^(2*n)), x], x] /; Fr
eeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx &= 2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right)-\int \frac{1}{1+x}\,dx\\ &= 2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right)-\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0068592, size = 20, normalized size = 1.

$$2\sqrt{x}\tan^{-1}\left(\sqrt{x}\right)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x]*ArcTan[Sqrt[x]] - Log[1 + x]
```

Maple [A] time = 0.004, size = 17, normalized size = 0.9

$$-\ln(1+x)+2\arctan\left(\sqrt{x}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/x^(1/2),x)
```

```
[Out] -ln(1+x)+2*arctan(x^(1/2))*x^(1/2)
```

Maxima [A] time = 0.926823, size = 22, normalized size = 1.1

$$2\sqrt{x}\arctan\left(\sqrt{x}\right)-\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Fricas [A] time = 2.0133, size = 54, normalized size = 2.7

$$2\sqrt{x}\arctan\left(\sqrt{x}\right)-\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

Sympy [A] time = 0.480636, size = 17, normalized size = 0.85

$$2\sqrt{x}\operatorname{atan}\left(\sqrt{x}\right) - \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*sqrt(x)*atan(sqrt(x)) - log(x + 1)
```

Giac [A] time = 1.04905, size = 22, normalized size = 1.1

$$2\sqrt{x}\arctan\left(\sqrt{x}\right) - \log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*arctan(sqrt(x)) - log(x + 1)
```

3.333 $\int \frac{\log(1+x)}{x^2} dx$

Optimal. Leaf size=18

$$\log(x) - \frac{\log(x+1)}{x} - \log(x+1)$$

```
[Out] Log[x] - Log[1 + x] - Log[1 + x]/x
```

Rubi [A] time = 0.0083889, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2395, 36, 29, 31}

$$\log(x) - \frac{\log(x+1)}{x} - \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x]/x^2,x]
```

```
[Out] Log[x] - Log[1 + x] - Log[1 + x]/x
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(1+x)}{x^2}\,dx &= -\frac{\log(1+x)}{x}+\int \frac{1}{x(1+x)}\,dx\\ &= -\frac{\log(1+x)}{x}+\int \frac{1}{x}\,dx-\int \frac{1}{1+x}\,dx\\ &= \log(x)-\log(1+x)-\frac{\log(1+x)}{x}\end{aligned}$$

Mathematica [A] time = 0.0033289, size = 18, normalized size = 1.

$$\log(x)-\frac{\log(x+1)}{x}-\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x]/x^2,x]
```

```
[Out] Log[x] - Log[1 + x] - Log[1 + x]/x
```

Maple [A] time = 0.006, size = 16, normalized size = 0.9

$$\ln(x)-\frac{(1+x)\ln(1+x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+x)/x^2,x)
```

```
[Out] ln(x)-ln(1+x)*(1+x)/x
```

Maxima [A] time = 0.923736, size = 24, normalized size = 1.33

$$-\frac{\log(x+1)}{x}-\log(x+1)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x)/x^2,x, algorithm="maxima")
```

```
[Out] -log(x + 1)/x - log(x + 1) + log(x)
```

Fricas [A] time = 1.81763, size = 49, normalized size = 2.72

$$-\frac{(x+1)\log(x+1) - x\log(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x)/x^2,x, algorithm="fricas")
```

```
[Out] -((x + 1)*log(x + 1) - x*log(x))/x
```

Sympy [A] time = 0.120995, size = 14, normalized size = 0.78

$$\log(x) - \log(x+1) - \frac{\log(x+1)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+x)/x**2,x)
```

```
[Out] log(x) - log(x + 1) - log(x + 1)/x
```

Giac [A] time = 1.0526, size = 27, normalized size = 1.5

$$-\frac{\log(x+1)}{x} - \log(|x+1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x)/x^2,x, algorithm="giac")
```

```
[Out] -log(x + 1)/x - log(abs(x + 1)) + log(abs(x))
```

3.334 $\int \frac{1}{-e^x+e^{3x}}\,dx$

Optimal. Leaf size=12

$$e^{-x} - \tanh^{-1}(e^x)$$

```
[Out] E^(-x) - ArcTanh[E^x]
```

Rubi [A] time = 0.0118939, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2282, 325, 207}

$$e^{-x} - \tanh^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[(-E^x + E^(3*x))^(-1),x]
```

```
[Out] E^(-x) - ArcTanh[E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{-e^x+e^{3x}}\,dx = \text{Subst}\left(\int \frac{1}{x^2\left(-1+x^2\right)}\,dx, x, e^x\right)$$
$$= e^{-x} + \text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, e^x\right)$$
$$= e^{-x} - \tanh^{-1}\left(e^x\right)$$

Mathematica [C] time = 0.0053248, size = 19, normalized size = 1.58

$$e^{-x}\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-E^x + E^(3*x))^(-1),x]
```

```
[Out] Hypergeometric2F1[-1/2, 1, 1/2, E^(2*x)]/E^x
```

Maple [A] time = 0.005, size = 20, normalized size = 1.7

$$-\frac{\ln\left(1+e^x\right)}{2}+\frac{\ln\left(-1+e^x\right)}{2}+\left(e^x\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-exp(x)+exp(3*x)),x)
```

```
[Out] -1/2*ln(1+exp(x))+1/2*ln(-1+exp(x))+1/exp(x)
```

Maxima [A] time = 0.935188, size = 26, normalized size = 2.17

$$e^{(-x)} - \frac{1}{2}\log\left(e^x+1\right) + \frac{1}{2}\log\left(e^x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-exp(x)+exp(3*x)),x, algorithm="maxima")
```

```
[Out] e^(-x) - 1/2*log(e^x + 1) + 1/2*log(e^x - 1)
```

Fricas [B] time = 1.98467, size = 74, normalized size = 6.17

$$-\frac{1}{2}\left(e^{x}\log\left(e^{x}+1\right)-e^{x}\log\left(e^{x}-1\right)-2\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-exp(x)+exp(3*x)),x, algorithm="fricas")
```

```
[Out] -1/2*(e^x*log(e^x + 1) - e^x*log(e^x - 1) - 2)*e^(-x)
```

Sympy [B] time = 0.109092, size = 20, normalized size = 1.67

$$\frac{\log\left(e^{x}-1\right)}{2}-\frac{\log\left(e^{x}+1\right)}{2}+e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-exp(x)+exp(3*x)),x)
```

```
[Out] log(exp(x) - 1)/2 - log(exp(x) + 1)/2 + exp(-x)
```

Giac [A] time = 1.05866, size = 27, normalized size = 2.25

$$e^{(-x)}-\frac{1}{2}\log\left(e^{x}+1\right)+\frac{1}{2}\log\left(|e^{x}-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-exp(x)+exp(3*x)),x, algorithm="giac")
```

```
[Out] e^(-x) - 1/2*log(e^x + 1) + 1/2*log(abs(e^x - 1))
```

3.335 $\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx$

Optimal. Leaf size=8

$$-x-2\cot(x)$$

```
[Out] -x - 2*Cot[x]
```

Rubi [A] time = 0.0371258, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3171, 3175, 3767, 8}

$$-x-2\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x]^2)/(1 - Cos[x]^2),x]
```

```
[Out] -x - 2*Cot[x]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1+\cos^2(x)}{1-\cos^2(x)}\,dx &= -x+2\int \frac{1}{1-\cos^2(x)}\,dx\\ &= -x+2\int \csc^2(x)\,dx\\ &= -x-2\,\text{Subst}\left(\int 1\,dx,x,\cot(x)\right)\\ &= -x-2\cot(x)\end{aligned}$$

Mathematica [A] time = 0.007607, size = 8, normalized size = 1.

$$-x-2\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x]^2)/(1 - Cos[x]^2),x]
```

```
[Out] -x - 2*Cot[x]
```

Maple [A] time = 0.032, size = 11, normalized size = 1.4

$$-2\,(\tan(x))^{-1}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+cos(x)^2)/(-cos(x)^2+1),x)
```

```
[Out] -2/tan(x)-x
```

Maxima [A] time = 1.40983, size = 14, normalized size = 1.75

$$-x-\frac{2}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(-cos(x)^2+1),x, algorithm="maxima")
```

```
[Out] -x - 2/tan(x)
```

Fricas [A] time = 2.15822, size = 42, normalized size = 5.25

$$-\frac{x\sin(x) + 2\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(-cos(x)^2+1),x, algorithm="fricas")
```

```
[Out] -(x*sin(x) + 2*cos(x))/sin(x)
```

Sympy [A] time = 1.31058, size = 12, normalized size = 1.5

$$-x + \tan\left(\frac{x}{2}\right) - \frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)**2)/(-cos(x)**2+1),x)
```

```
[Out] -x + tan(x/2) - 1/tan(x/2)
```

Giac [A] time = 1.06638, size = 22, normalized size = 2.75

$$-x - \frac{1}{\tan\left(\frac{1}{2}x\right)} + \tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x)^2)/(-cos(x)^2+1),x, algorithm="giac")
```

```
[Out] -x - 1/tan(1/2*x) + tan(1/2*x)
```

3.336 $\int \frac{1}{x\sqrt{-25+2x}}\,dx$

Optimal. Leaf size=18

$$\frac{2}{5}\tan^{-1}\left(\frac{1}{5}\sqrt{2x-25}\right)$$

```
[Out] (2*ArcTan[Sqrt[-25 + 2*x]/5])/5
```

Rubi [A] time = 0.002827, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 203}

$$\frac{2}{5}\tan^{-1}\left(\frac{1}{5}\sqrt{2x-25}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-25 + 2*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[-25 + 2*x]/5])/5
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{-25+2x}}\,dx = \text{Subst}\left(\int \frac{1}{\frac{25}{2}+\frac{x^2}{2}}\,dx, x, \sqrt{-25+2x}\right)$$
$$= \frac{2}{5}\tan^{-1}\left(\frac{1}{5}\sqrt{-25+2x}\right)$$

Mathematica [A] time = 0.0024075, size = 18, normalized size = 1.

$$\frac{2}{5}\tan^{-1}\left(\frac{1}{5}\sqrt{2x-25}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-25 + 2*x]),x]
```

```
[Out] (2*ArcTan[Sqrt[-25 + 2*x]/5])/5
```

Maple [A] time = 0.006, size = 13, normalized size = 0.7

$$\frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{-25+2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-25+2*x)^(1/2),x)
```

```
[Out] 2/5*arctan(1/5*(-25+2*x)^(1/2))
```

Maxima [A] time = 1.40573, size = 16, normalized size = 0.89

$$\frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{2\,x-25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-25+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*arctan(1/5*sqrt(2*x - 25))
```

Fricas [A] time = 1.85513, size = 43, normalized size = 2.39

$$\frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{2x-25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-25+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*arctan(1/5*sqrt(2*x - 25))
```

Sympy [A] time = 0.979219, size = 44, normalized size = 2.44

$$\begin{cases} \frac{2i\operatorname{acosh}\left(\frac{5\sqrt{2}}{2\sqrt{x}}\right)}{5} & \text{for}\ \frac{25}{2|x|} > 1 \\ -\frac{2\operatorname{asin}\left(\frac{5\sqrt{2}}{2\sqrt{x}}\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-25+2*x)**(1/2),x)
```

```
[Out] Piecewise((2*I*acosh(5*sqrt(2)/(2*sqrt(x)))/5, 25/(2*Abs(x)) > 1), (-2*asin
(5*sqrt(2)/(2*sqrt(x)))/5, True))
```

Giac [A] time = 1.0588, size = 16, normalized size = 0.89

$$\frac{2}{5}\arctan\left(\frac{1}{5}\sqrt{2x-25}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-25+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*arctan(1/5*sqrt(2*x - 25))
```

3.337 $\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}} dx$

Optimal. Leaf size=11

$$-\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

```
[Out] -ArcSin[Cos[x]^2/3]
```

Rubi [A] time = 0.0500845, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {12, 1107, 619, 216}

$$-\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]/Sqrt[9 - Cos[x]^4],x]

[Out] -ArcSin[Cos[x]^2/3]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(2x)}{\sqrt{9-\cos^4(x)}}\,dx &= \text{Subst}\left(\int \frac{2x}{\sqrt{8+2x^2-x^4}}\,dx, x, \sin(x)\right) \\
&= 2\,\text{Subst}\left(\int \frac{x}{\sqrt{8+2x^2-x^4}}\,dx, x, \sin(x)\right) \\
&= \text{Subst}\left(\int \frac{1}{\sqrt{8+2x-x^2}}\,dx, x, \sin^2(x)\right) \\
&= -\left(\frac{1}{6}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx, x, 2\cos^2(x)\right)\right) \\
&= -\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0157304, size = 11, normalized size = 1.

$$-\sin^{-1}\left(\frac{\cos^2(x)}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]/Sqrt[9 - Cos[x]^4],x]
```

```
[Out] -ArcSin[Cos[x]^2/3]
```

Maple [A] time = 0.03, size = 10, normalized size = 0.9

$$-\arcsin\left(\frac{(\cos(x))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/(9-cos(x)^4)^(1/2),x)
```

```
[Out] -arcsin(1/3*cos(x)^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(2x)}{\sqrt{-\cos(x)^4 + 9}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(2*x)/sqrt(-cos(x)^4 + 9), x)
```

Fricas [B] time = 2.55318, size = 72, normalized size = 6.55

$$\arctan\left(\frac{\sqrt{-\cos(x)^4 + 9}\cos(x)^2}{\cos(x)^4 - 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(-cos(x)^4 + 9)*cos(x)^2/(cos(x)^4 - 9))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)**4)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(2x)}{\sqrt{-\cos(x)^4 + 9}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/(9-cos(x)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(2*x)/sqrt(-cos(x)^4 + 9), x)
```

3.338 $\int \frac{x^2}{\sqrt{5-4x^2}}\,dx$

Optimal. Leaf size=30

$$\frac{5}{16}\sin^{-1}\left(\frac{2x}{\sqrt{5}}\right)-\frac{1}{8}x\sqrt{5-4x^2}$$

```
[Out] -(x*Sqrt[5 - 4*x^2])/8 + (5*ArcSin[(2*x)/Sqrt[5]])/16
```

Rubi [A] time = 0.0062046, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {321, 216}

$$\frac{5}{16}\sin^{-1}\left(\frac{2x}{\sqrt{5}}\right)-\frac{1}{8}x\sqrt{5-4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[5 - 4*x^2],x]
```

```
[Out] -(x*Sqrt[5 - 4*x^2])/8 + (5*ArcSin[(2*x)/Sqrt[5]])/16
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{5-4x^2}}\,dx = -\frac{1}{8}x\sqrt{5-4x^2} + \frac{5}{8}\int \frac{1}{\sqrt{5-4x^2}}\,dx$$
$$= -\frac{1}{8}x\sqrt{5-4x^2} + \frac{5}{16}\sin^{-1}\left(\frac{2x}{\sqrt{5}}\right)$$

Mathematica [A] time = 0.008429, size = 30, normalized size = 1.

$$\frac{5}{16}\sin^{-1}\left(\frac{2x}{\sqrt{5}}\right) - \frac{1}{8}x\sqrt{5-4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[5 - 4*x^2],x]
```

```
[Out] -(x*Sqrt[5 - 4*x^2])/8 + (5*ArcSin[(2*x)/Sqrt[5]])/16
```

Maple [A] time = 0.004, size = 23, normalized size = 0.8

$$\frac{5}{16}\arcsin\left(\frac{2\,x\sqrt{5}}{5}\right) - \frac{x}{8}\sqrt{-4\,x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-4*x^2+5)^(1/2),x)
```

```
[Out] 5/16*arcsin(2/5*x*5^(1/2))-1/8*x*(-4*x^2+5)^(1/2)
```

Maxima [A] time = 1.44639, size = 30, normalized size = 1.

$$-\frac{1}{8}\sqrt{-4\,x^2+5}x + \frac{5}{16}\arcsin\left(\frac{2}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+5)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(-4*x^2 + 5)*x + 5/16*arcsin(2/5*sqrt(5)*x)
```

Fricas [A] time = 1.9948, size = 85, normalized size = 2.83

$$-\frac{1}{8}\sqrt{-4x^2+5}x-\frac{5}{16}\arctan\left(\frac{\sqrt{-4x^2+5}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+5)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-4*x^2 + 5)*x - 5/16*arctan(1/2*sqrt(-4*x^2 + 5)/x)
```

Sympy [A] time = 0.206622, size = 27, normalized size = 0.9

$$-\frac{x\sqrt{5-4x^2}}{8}+\frac{5\operatorname{asin}\left(\frac{2\sqrt{5}x}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-4*x**2+5)**(1/2),x)
```

```
[Out] -x*sqrt(5 - 4*x**2)/8 + 5*asin(2*sqrt(5)*x/5)/16
```

Giac [A] time = 1.07222, size = 30, normalized size = 1.

$$-\frac{1}{8}\sqrt{-4x^2+5}x+\frac{5}{16}\arcsin\left(\frac{2}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-4*x^2+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(-4*x^2 + 5)*x + 5/16*arcsin(2/5*sqrt(5)*x)
```

3.339 $\int x^3 \sin(x)\, dx$

Optimal. Leaf size=24

$$3x^2\sin(x) + x^3(-\cos(x)) - 6\sin(x) + 6x\cos(x)$$

```
[Out] 6*x*Cos[x] - x^3*Cos[x] - 6*Sin[x] + 3*x^2*Sin[x]
```

Rubi [A] time = 0.0371059, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2637}

$$3x^2\sin(x) + x^3(-\cos(x)) - 6\sin(x) + 6x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[x],x]
```

```
[Out] 6*x*Cos[x] - x^3*Cos[x] - 6*Sin[x] + 3*x^2*Sin[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^3\sin(x)\,dx &= -x^3\cos(x) + 3\int x^2\cos(x)\,dx\\
&= -x^3\cos(x) + 3x^2\sin(x) - 6\int x\sin(x)\,dx\\
&= 6x\cos(x) - x^3\cos(x) + 3x^2\sin(x) - 6\int \cos(x)\,dx\\
&= 6x\cos(x) - x^3\cos(x) - 6\sin(x) + 3x^2\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.0149542, size = 20, normalized size = 0.83

$$3\left(x^2-2\right)\sin(x)-x\left(x^2-6\right)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x],x]
```

```
[Out] -(x*(-6 + x^2)*Cos[x]) + 3*(-2 + x^2)*Sin[x]
```

Maple [A] time = 0.004, size = 25, normalized size = 1.

$$6\,x\cos\left(x\right)-x^{3}\cos\left(x\right)-6\,\sin\left(x\right)+3\,x^{2}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x),x)
```

```
[Out] 6*x*cos(x)-x^3*cos(x)-6*sin(x)+3*x^2*sin(x)
```

Maxima [A] time = 0.952481, size = 28, normalized size = 1.17

$$-\left(x^{3}-6\,x\right)\cos\left(x\right)+3\left(x^{2}-2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x),x, algorithm="maxima")
```

```
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

Fricas [A] time = 1.96472, size = 57, normalized size = 2.38

$$-\left(x^{3}-6\,x\right)\cos\left(x\right)+3\left(x^{2}-2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

Sympy [A] time = 0.554758, size = 26, normalized size = 1.08

$$-x^3 \cos(x) + 3x^2 \sin(x) + 6x \cos(x) - 6 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(x),x)
```

```
[Out] -x**3*cos(x) + 3*x**2*sin(x) + 6*x*cos(x) - 6*sin(x)
```

Giac [A] time = 1.08333, size = 28, normalized size = 1.17

$$-\left(x^3 - 6\,x\right)\cos(x) + 3\left(x^2 - 2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x),x, algorithm="giac")
```

```
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

3.340 $\int x\sqrt{4+2x+x^2}\,dx$

Optimal. Leaf size=50

$$\frac{1}{3}\left(x^2+2x+4\right)^{3/2}-\frac{1}{2}(x+1)\sqrt{x^2+2x+4}-\frac{3}{2}\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$$

```
[Out] -((1 + x)*Sqrt[4 + 2*x + x^2])/2 + (4 + 2*x + x^2)^(3/2)/3 - (3*ArcSinh[(1
+ x)/Sqrt[3]])/2
```

Rubi [A] time = 0.0145362, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {640, 612, 619, 215}

$$\frac{1}{3}\left(x^2+2x+4\right)^{3/2}-\frac{1}{2}(x+1)\sqrt{x^2+2x+4}-\frac{3}{2}\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[4 + 2*x + x^2],x]
```

```
[Out] -((1 + x)*Sqrt[4 + 2*x + x^2])/2 + (4 + 2*x + x^2)^(3/2)/3 - (3*ArcSinh[(1
+ x)/Sqrt[3]])/2
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{4+2x+x^2}\,dx &= \frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\int \sqrt{4+2x+x^2}\,dx\\
&= -\frac{1}{2}(1+x)\sqrt{4+2x+x^2}+\frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\frac{3}{2}\int \frac{1}{\sqrt{4+2x+x^2}}\,dx\\
&= -\frac{1}{2}(1+x)\sqrt{4+2x+x^2}+\frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\frac{1}{4}\sqrt{3}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{12}}}\,dx,x,2+2x\right)\\
&= -\frac{1}{2}(1+x)\sqrt{4+2x+x^2}+\frac{1}{3}\left(4+2x+x^2\right)^{3/2}-\frac{3}{2}\sinh^{-1}\left(\frac{1+x}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0182636, size = 38, normalized size = 0.76

$$\frac{1}{6}\left(\sqrt{x^2+2x+4}\left(2x^2+x+5\right)-9\sinh^{-1}\left(\frac{x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[4 + 2*x + x^2],x]
```

```
[Out] (Sqrt[4 + 2*x + x^2]*(5 + x + 2*x^2) - 9*ArcSinh[(1 + x)/Sqrt[3]])/6
```

Maple [A] time = 0.004, size = 42, normalized size = 0.8

$$\frac{1}{3}\left(x^2+2x+4\right)^{\frac{3}{2}}-\frac{2x+2}{4}\sqrt{x^2+2x+4}-\frac{3}{2}\text{Arcsinh}\left(\frac{(1+x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+2*x+4)^(1/2),x)
```

```
[Out] 1/3*(x^2+2*x+4)^(3/2)-1/4*(2*x+2)*(x^2+2*x+4)^(1/2)-3/2*arcsinh(1/3*(1+x)*3
^(1/2))
```

Maxima [A] time = 1.42293, size = 66, normalized size = 1.32

$$\frac{1}{3}\left(x^2+2x+4\right)^{\frac{3}{2}}-\frac{1}{2}\sqrt{x^2+2x+4}x-\frac{1}{2}\sqrt{x^2+2x+4}-\frac{3}{2}\operatorname{arsinh}\left(\frac{1}{3}\sqrt{3}(x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 + 2*x + 4)^(3/2) - 1/2*sqrt(x^2 + 2*x + 4)*x - 1/2*sqrt(x^2 + 2*x
+ 4) - 3/2*arcsinh(1/3*sqrt(3)*(x + 1))
```

Fricas [A] time = 1.94938, size = 109, normalized size = 2.18

$$\frac{1}{6}\left(2x^2+x+5\right)\sqrt{x^2+2x+4}+\frac{3}{2}\log\left(-x+\sqrt{x^2+2x+4}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(2*x^2 + x + 5)*sqrt(x^2 + 2*x + 4) + 3/2*log(-x + sqrt(x^2 + 2*x + 4)
- 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\sqrt{x^2+2x+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+2*x+4)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x**2 + 2*x + 4), x)
```

Giac [A] time = 1.05913, size = 54, normalized size = 1.08

$$\frac{1}{6}\left((2x+1)x+5\right)\sqrt{x^2+2x+4}+\frac{3}{2}\log\left(-x+\sqrt{x^2+2x+4}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*((2*x + 1)*x + 5)*sqrt(x^2 + 2*x + 4) + 3/2*log(-x + sqrt(x^2 + 2*x + 4
) - 1)
```

3.341 $\int x\left(5+x^2\right)^8\,dx$

Optimal. Leaf size=11

$$\frac{1}{18}\left(x^2+5\right)^9$$

```
[Out] (5 + x^2)^9/18
```

Rubi [A] time = 0.0014835, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {261}

$$\frac{1}{18}\left(x^2+5\right)^9$$

Antiderivative was successfully verified.

```
[In] Int[x*(5 + x^2)^8,x]
```

```
[Out] (5 + x^2)^9/18
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(5+x^2\right)^8\,dx=\frac{1}{18}\left(5+x^2\right)^9$$

Mathematica [A] time = 0.0019411, size = 11, normalized size = 1.

$$\frac{1}{18}\left(x^2+5\right)^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(5 + x^2)^8,x]
```

```
[Out] (5 + x^2)^9/18
```

Maple [B] time = 0., size = 47, normalized size = 4.3

$$\frac{x^{18}}{18}+\frac{5\,x^{16}}{2}+50\,x^{14}+\frac{1750\,x^{12}}{3}+4375\,x^{10}+21875\,x^{8}+\frac{218750\,x^{6}}{3}+156250\,x^{4}+\frac{390625\,x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+5)^8,x)
```

```
[Out] 1/18*x^18+5/2*x^16+50*x^14+1750/3*x^12+4375*x^10+21875*x^8+218750/3*x^6+156
250*x^4+390625/2*x^2
```

Maxima [A] time = 0.931201, size = 12, normalized size = 1.09

$$\frac{1}{18}\left(x^{2}+5\right)^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+5)^8,x, algorithm="maxima")
```

```
[Out] 1/18*(x^2 + 5)^9
```

Fricas [B] time = 1.57474, size = 153, normalized size = 13.91

$$\frac{1}{18}x^{18}+\frac{5}{2}x^{16}+50x^{14}+\frac{1750}{3}x^{12}+4375x^{10}+21875x^{8}+\frac{218750}{3}x^{6}+156250x^{4}+\frac{390625}{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+5)^8,x, algorithm="fricas")
```

```
[Out] 1/18*x^18 + 5/2*x^16 + 50*x^14 + 1750/3*x^12 + 4375*x^10 + 21875*x^8 + 2187
50/3*x^6 + 156250*x^4 + 390625/2*x^2
```

Sympy [B] time = 0.055313, size = 51, normalized size = 4.64

$$\frac{x^{18}}{18} + \frac{5x^{16}}{2} + 50x^{14} + \frac{1750x^{12}}{3} + 4375x^{10} + 21875x^{8} + \frac{218750x^{6}}{3} + 156250x^{4} + \frac{390625x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+5)**8,x)
```

```
[Out] x**18/18 + 5*x**16/2 + 50*x**14 + 1750*x**12/3 + 4375*x**10 + 21875*x**8 +
218750*x**6/3 + 156250*x**4 + 390625*x**2/2
```

Giac [A] time = 1.05683, size = 12, normalized size = 1.09

$$\frac{1}{18}\left(x^{2} + 5\right)^{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+5)^8,x, algorithm="giac")
```

```
[Out] 1/18*(x^2 + 5)^9
```

3.342 $\int \cos^2(x)\sin^5(x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{7}\cos^7(x)+\frac{2\cos^5(x)}{5}-\frac{\cos^3(x)}{3}$$

```
[Out] -Cos[x]^3/3 + (2*Cos[x]^5)/5 - Cos[x]^7/7
```

Rubi [A] time = 0.0265598, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2565, 270}

$$-\frac{1}{7}\cos^7(x)+\frac{2\cos^5(x)}{5}-\frac{\cos^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sin[x]^5,x]
```

```
[Out] -Cos[x]^3/3 + (2*Cos[x]^5)/5 - Cos[x]^7/7
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\sin^5(x)\,dx &= -\operatorname{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx, x, \cos(x)\right)\\ &= -\operatorname{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \cos(x)\right)\\ &= -\frac{1}{3}\cos^3(x)+\frac{2\cos^5(x)}{5}-\frac{\cos^7(x)}{7}\end{aligned}$$

Mathematica [A] time = 0.0099994, size = 31, normalized size = 1.24

$$-\frac{5\cos(x)}{64}-\frac{1}{192}\cos(3x)+\frac{3}{320}\cos(5x)-\frac{1}{448}\cos(7x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sin[x]^5,x]
```

```
[Out] (-5*Cos[x])/64 - Cos[3*x]/192 + (3*Cos[5*x])/320 - Cos[7*x]/448
```

Maple [A] time = 0.006, size = 28, normalized size = 1.1

$$-\frac{(\cos(x))^3(\sin(x))^4}{7}-\frac{4\,(\sin(x))^2(\cos(x))^3}{35}-\frac{8\,(\cos(x))^3}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^5,x)
```

```
[Out] -1/7*cos(x)^3*sin(x)^4-4/35*sin(x)^2*cos(x)^3-8/105*cos(x)^3
```

Maxima [A] time = 0.949246, size = 26, normalized size = 1.04

$$-\frac{1}{7}\cos(x)^7+\frac{2}{5}\cos(x)^5-\frac{1}{3}\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^5,x, algorithm="maxima")
```

```
[Out] -1/7*cos(x)^7 + 2/5*cos(x)^5 - 1/3*cos(x)^3
```

Fricas [A] time = 2.07527, size = 61, normalized size = 2.44

$$-\frac{1}{7}\cos(x)^7 + \frac{2}{5}\cos(x)^5 - \frac{1}{3}\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^5,x, algorithm="fricas")
```

```
[Out] -1/7*cos(x)^7 + 2/5*cos(x)^5 - 1/3*cos(x)^3
```

Sympy [A] time = 0.062005, size = 20, normalized size = 0.8

$$-\frac{\cos^7(x)}{7} + \frac{2\cos^5(x)}{5} - \frac{\cos^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**5,x)
```

```
[Out] -cos(x)**7/7 + 2*cos(x)**5/5 - cos(x)**3/3
```

Giac [A] time = 1.0583, size = 26, normalized size = 1.04

$$-\frac{1}{7}\cos(x)^7 + \frac{2}{5}\cos(x)^5 - \frac{1}{3}\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^5,x, algorithm="giac")
```

```
[Out] -1/7*cos(x)^7 + 2/5*cos(x)^5 - 1/3*cos(x)^3
```

3.343 $\int e^{-3x} \cos(4x)\, dx$

Optimal. Leaf size=27

$$\frac{4}{25}e^{-3x}\sin(4x) - \frac{3}{25}e^{-3x}\cos(4x)$$

```
[Out] (-3*Cos[4*x])/(25*E^(3*x)) + (4*Sin[4*x])/(25*E^(3*x))
```

Rubi [A] time = 0.0096246, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4433}

$$\frac{4}{25}e^{-3x}\sin(4x) - \frac{3}{25}e^{-3x}\cos(4x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[4*x]/E^(3*x),x]
```

```
[Out] (-3*Cos[4*x])/(25*E^(3*x)) + (4*Sin[4*x])/(25*E^(3*x))
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{-3x}\cos(4x)\, dx = -\frac{3}{25}e^{-3x}\cos(4x) + \frac{4}{25}e^{-3x}\sin(4x)$$

Mathematica [A] time = 0.0266588, size = 22, normalized size = 0.81

$$\frac{1}{25}e^{-3x}(4\sin(4x) - 3\cos(4x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[4*x]/E^(3*x),x]
```

```
[Out] (-3*Cos[4*x] + 4*Sin[4*x])/(25*E^(3*x))
```

Maple [A] time = 0.006, size = 22, normalized size = 0.8

$$-\frac{3\,\mathrm{e}^{-3\,x}\cos\left(4\,x\right)}{25}+\frac{4\,\mathrm{e}^{-3\,x}\sin\left(4\,x\right)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(4*x)/exp(3*x),x)
```

```
[Out] -3/25*exp(-3*x)*cos(4*x)+4/25*exp(-3*x)*sin(4*x)
```

Maxima [A] time = 0.93744, size = 26, normalized size = 0.96

$$-\frac{1}{25}\left(3\,\cos\left(4\,x\right)-4\,\sin\left(4\,x\right)\right)e^{\left(-3\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="maxima")
```

```
[Out] -1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)
```

Fricas [A] time = 1.87236, size = 68, normalized size = 2.52

$$-\frac{3}{25}\,\cos\left(4\,x\right)e^{\left(-3\,x\right)}+\frac{4}{25}\,e^{\left(-3\,x\right)}\sin\left(4\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="fricas")
```

```
[Out] -3/25*cos(4*x)*e^(-3*x) + 4/25*e^(-3*x)*sin(4*x)
```

Sympy [A] time = 0.458763, size = 26, normalized size = 0.96

$$\frac{4e^{-3x}\sin(4x)}{25} - \frac{3e^{-3x}\cos(4x)}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x)
```

```
[Out] 4*exp(-3*x)*sin(4*x)/25 - 3*exp(-3*x)*cos(4*x)/25
```

Giac [A] time = 1.05248, size = 26, normalized size = 0.96

$$-\frac{1}{25}\left(3\cos(4x) - 4\sin(4x)\right)e^{(-3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(4*x)/exp(3*x),x, algorithm="giac")
```

```
[Out] -1/25*(3*cos(4*x) - 4*sin(4*x))*e^(-3*x)
```

3.344 $\int \csc^3\left(\frac{x}{2}\right)\,dx$

Optimal. Leaf size=24

$$-\tanh^{-1}\left(\cos\left(\frac{x}{2}\right)\right)-\cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)$$

```
[Out] -ArcTanh[Cos[x/2]] - Cot[x/2]*Csc[x/2]
```

Rubi [A] time = 0.0113921, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3768, 3770}

$$-\tanh^{-1}\left(\cos\left(\frac{x}{2}\right)\right)-\cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x/2]^3,x]
```

```
[Out] -ArcTanh[Cos[x/2]] - Cot[x/2]*Csc[x/2]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3\left(\frac{x}{2}\right)dx &= -\cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)+\frac{1}{2}\int \csc\left(\frac{x}{2}\right)dx\\ &= -\tanh^{-1}\left(\cos\left(\frac{x}{2}\right)\right)-\cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0085372, size = 41, normalized size = 1.71

$$-\frac{1}{4}\csc^2\left(\frac{x}{4}\right)+\frac{1}{4}\sec^2\left(\frac{x}{4}\right)+\log\left(\sin\left(\frac{x}{4}\right)\right)-\log\left(\cos\left(\frac{x}{4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x/2]^3,x]
```

```
[Out] -Csc[x/4]^2/4 - Log[Cos[x/4]] + Log[Sin[x/4]] + Sec[x/4]^2/4
```

Maple [A] time = 0.009, size = 24, normalized size = 1.

$$-\cot\left(\frac{x}{2}\right)\csc\left(\frac{x}{2}\right)+\ln\left(\csc\left(\frac{x}{2}\right)-\cot\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(1/2*x)^3,x)
```

```
[Out] -cot(1/2*x)*csc(1/2*x)+ln(csc(1/2*x)-cot(1/2*x))
```

Maxima [A] time = 0.980341, size = 46, normalized size = 1.92

$$\frac{\cos\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)^2-1}-\frac{1}{2}\log\left(\cos\left(\frac{1}{2}x\right)+1\right)+\frac{1}{2}\log\left(\cos\left(\frac{1}{2}x\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/2*x)^3,x, algorithm="maxima")
```

```
[Out] cos(1/2*x)/(cos(1/2*x)^2 - 1) - 1/2*log(cos(1/2*x) + 1) + 1/2*log(cos(1/2*x
) - 1)
```

Fricas [B] time = 2.02372, size = 182, normalized size = 7.58

$$-\frac{\left(\cos\left(\frac{1}{2}\,x\right)^{2}-1\right)\log\left(\frac{1}{2}\,\cos\left(\frac{1}{2}\,x\right)+\frac{1}{2}\right)-\left(\cos\left(\frac{1}{2}\,x\right)^{2}-1\right)\log\left(-\frac{1}{2}\,\cos\left(\frac{1}{2}\,x\right)+\frac{1}{2}\right)-2\,\cos\left(\frac{1}{2}\,x\right)}{2\left(\cos\left(\frac{1}{2}\,x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/2*x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((cos(1/2*x)^2 - 1)*log(1/2*cos(1/2*x) + 1/2) - (cos(1/2*x)^2 - 1)*log
(-1/2*cos(1/2*x) + 1/2) - 2*cos(1/2*x))/(cos(1/2*x)^2 - 1)
```

Sympy [B] time = 0.116014, size = 36, normalized size = 1.5

$$\frac{\log\left(\cos\left(\frac{x}{2}\right)-1\right)}{2}-\frac{\log\left(\cos\left(\frac{x}{2}\right)+1\right)}{2}+\frac{2\cos\left(\frac{x}{2}\right)}{2\cos^{2}\left(\frac{x}{2}\right)-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/2*x)**3,x)
```

```
[Out] log(cos(x/2) - 1)/2 - log(cos(x/2) + 1)/2 + 2*cos(x/2)/(2*cos(x/2)**2 - 2)
```

Giac [B] time = 1.06559, size = 95, normalized size = 3.96

$$-\frac{\left(\frac{2\left(\cos\left(\frac{1}{2}\,x\right)-1\right)}{\cos\left(\frac{1}{2}\,x\right)+1}-1\right)\left(\cos\left(\frac{1}{2}\,x\right)+1\right)}{4\left(\cos\left(\frac{1}{2}\,x\right)-1\right)}-\frac{\cos\left(\frac{1}{2}\,x\right)-1}{4\left(\cos\left(\frac{1}{2}\,x\right)+1\right)}+\frac{1}{2}\,\log\left(-\frac{\cos\left(\frac{1}{2}\,x\right)-1}{\cos\left(\frac{1}{2}\,x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(1/2*x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(2*(cos(1/2*x) - 1)/(cos(1/2*x) + 1) - 1)*(cos(1/2*x) + 1)/(cos(1/2*x)
 - 1) - 1/4*(cos(1/2*x) - 1)/(cos(1/2*x) + 1) + 1/2*log(-(cos(1/2*x) - 1)/(
cos(1/2*x) + 1))
```

3.345 $\int \frac{\sqrt{-1+9x^2}}{x^2}\,dx$

Optimal. Leaf size=34

$$3\tanh^{-1}\left(\frac{3x}{\sqrt{9x^2-1}}\right)-\frac{\sqrt{9x^2-1}}{x}$$

```
[Out] -(Sqrt[-1 + 9*x^2]/x) + 3*ArcTanh[(3*x)/Sqrt[-1 + 9*x^2]]
```

Rubi [A] time = 0.007611, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {277, 217, 206}

$$3\tanh^{-1}\left(\frac{3x}{\sqrt{9x^2-1}}\right)-\frac{\sqrt{9x^2-1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + 9*x^2]/x^2,x]
```

```
[Out] -(Sqrt[-1 + 9*x^2]/x) + 3*ArcTanh[(3*x)/Sqrt[-1 + 9*x^2]]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{-1+9x^2}}{x^2}\,dx = -\frac{\sqrt{-1+9x^2}}{x} + 9\int \frac{1}{\sqrt{-1+9x^2}}\,dx$$
$$= -\frac{\sqrt{-1+9x^2}}{x} + 9\,\text{Subst}\left(\int \frac{1}{1-9x^2}\,dx, x, \frac{x}{\sqrt{-1+9x^2}}\right)$$
$$= -\frac{\sqrt{-1+9x^2}}{x} + 3\tanh^{-1}\left(\frac{3x}{\sqrt{-1+9x^2}}\right)$$

Mathematica [A] time = 0.0152595, size = 35, normalized size = 1.03

$$\sqrt{9x^2-1}\left(-\frac{3\sin^{-1}(3x)}{\sqrt{1-9x^2}}-\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + 9*x^2]/x^2,x]
```

```
[Out] Sqrt[-1 + 9*x^2]*(-x^(-1) - (3*ArcSin[3*x])/Sqrt[1 - 9*x^2])
```

Maple [A] time = 0.005, size = 47, normalized size = 1.4

$$\frac{1}{x}\left(9\,x^2-1\right)^{\frac{3}{2}}-9\,x\sqrt{9\,x^2-1}+\ln\left(x\sqrt{9}+\sqrt{9\,x^2-1}\right)\sqrt{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((9*x^2-1)^(1/2)/x^2,x)
```

```
[Out] 1/x*(9*x^2-1)^(3/2)-9*x*(9*x^2-1)^(1/2)+ln(x*9^(1/2)+(9*x^2-1)^(1/2))*9^(1/
2)
```

Maxima [A] time = 1.4237, size = 45, normalized size = 1.32

$$-\frac{\sqrt{9\,x^2-1}}{x}+3\,\log\left(18\,x+6\,\sqrt{9\,x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-1)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] -sqrt(9*x^2 - 1)/x + 3*log(18*x + 6*sqrt(9*x^2 - 1))
```

Fricas [A] time = 1.89075, size = 84, normalized size = 2.47

$$-\frac{3\,x\log\left(-3\,x+\sqrt{9\,x^2-1}\right)+3\,x+\sqrt{9\,x^2-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-1)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] -(3*x*log(-3*x + sqrt(9*x^2 - 1)) + 3*x + sqrt(9*x^2 - 1))/x
```

Sympy [A] time = 0.224803, size = 17, normalized size = 0.5

$$3\operatorname{acosh}(3x)-\frac{\sqrt{9x^2-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x**2-1)**(1/2)/x**2,x)
```

```
[Out] 3*acosh(3*x) - sqrt(9*x**2 - 1)/x
```

Giac [A] time = 1.073, size = 59, normalized size = 1.74

$$-\frac{6}{\left(3\,x-\sqrt{9\,x^2-1}\right)^2+1}-\frac{3}{2}\,\log\left(\left(3\,x-\sqrt{9\,x^2-1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9*x^2-1)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -6/((3*x - sqrt(9*x^2 - 1))^2 + 1) - 3/2*log((3*x - sqrt(9*x^2 - 1))^2)
```

3.346 $\int \frac{\sqrt{4-3x^2}}{x} \, dx$

Optimal. Leaf size=30

$$\sqrt{4-3x^2}-2\tanh^{-1}\left(\frac{1}{2}\sqrt{4-3x^2}\right)$$

```
[Out] Sqrt[4 - 3*x^2] - 2*ArcTanh[Sqrt[4 - 3*x^2]/2]
```

Rubi [A] time = 0.0165422, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 206}

$$\sqrt{4-3x^2}-2\tanh^{-1}\left(\frac{1}{2}\sqrt{4-3x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[4 - 3*x^2]/x,x]
```

```
[Out] Sqrt[4 - 3*x^2] - 2*ArcTanh[Sqrt[4 - 3*x^2]/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{4-3x^2}}{x}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{\sqrt{4-3x}}{x}\,dx, x, x^2\right)\\ &= \sqrt{4-3x^2}+2\operatorname{Subst}\left(\int \frac{1}{\sqrt{4-3x}x}\,dx, x, x^2\right)\\ &= \sqrt{4-3x^2}-\frac{4}{3}\operatorname{Subst}\left(\int \frac{1}{\frac{4}{3}-\frac{x^2}{3}}\,dx, x, \sqrt{4-3x^2}\right)\\ &= \sqrt{4-3x^2}-2\tanh^{-1}\left(\frac{1}{2}\sqrt{4-3x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0052576, size = 30, normalized size = 1.

$$\sqrt{4-3x^2}-2\tanh^{-1}\left(\frac{1}{2}\sqrt{4-3x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[4 - 3*x^2]/x,x]
```

```
[Out] Sqrt[4 - 3*x^2] - 2*ArcTanh[Sqrt[4 - 3*x^2]/2]
```

Maple [A] time = 0.004, size = 25, normalized size = 0.8

$$\sqrt{-3\,x^2+4}-2\operatorname{Artanh}\left(2\,\frac{1}{\sqrt{-3\,x^2+4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3*x^2+4)^(1/2)/x,x)
```

```
[Out] (-3*x^2+4)^(1/2)-2*arctanh(2/(-3*x^2+4)^(1/2))
```

Maxima [A] time = 1.42568, size = 47, normalized size = 1.57

$$\sqrt{-3\,x^2+4}-2\,\log\left(\frac{4\,\sqrt{-3\,x^2+4}}{|x|}+\frac{8}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(-3*x^2 + 4) - 2*log(4*sqrt(-3*x^2 + 4)/abs(x) + 8/abs(x))
```

Fricas [A] time = 1.96684, size = 70, normalized size = 2.33

$$\sqrt{-3\,x^2+4}+2\,\log\left(\frac{\sqrt{-3\,x^2+4}-2}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(-3*x^2 + 4) + 2*log((sqrt(-3*x^2 + 4) - 2)/x)
```

Sympy [A] time = 1.48456, size = 75, normalized size = 2.5

$$\begin{cases} i\sqrt{3x^2-4}-2\log\left(x\right)+\log\left(x^2\right)+2i\operatorname{asin}\left(\frac{2\sqrt{3}}{3x}\right) & \text{for}\ \frac{3|x^2|}{4}>1\\ \sqrt{4-3x^2}+\log\left(x^2\right)-2\log\left(\sqrt{1-\frac{3x^2}{4}}+1\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x**2+4)**(1/2)/x,x)
```

```
[Out] Piecewise((I*sqrt(3*x**2 - 4) - 2*log(x) + log(x**2) + 2*I*asin(2*sqrt(3)/(
3*x)), 3*Abs(x**2)/4 > 1), (sqrt(4 - 3*x**2) + log(x**2) - 2*log(sqrt(1 - 3
*x**2/4) + 1), True))
```

Giac [A] time = 1.04958, size = 51, normalized size = 1.7

$$\sqrt{-3\,x^2+4}-\log\left(\sqrt{-3\,x^2+4}+2\right)+\log\left(-\sqrt{-3\,x^2+4}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3*x^2+4)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(-3*x^2 + 4) - log(sqrt(-3*x^2 + 4) + 2) + log(-sqrt(-3*x^2 + 4) + 2)
```

3.347 $\int e^{3x}x^2\,dx$

Optimal. Leaf size=32

$$\frac{1}{3}e^{3x}x^2-\frac{2}{9}e^{3x}x+\frac{2e^{3x}}{27}$$

```
[Out] (2*E^(3*x))/27 - (2*E^(3*x)*x)/9 + (E^(3*x)*x^2)/3
```

Rubi [A] time = 0.0194959, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$\frac{1}{3}e^{3x}x^2-\frac{2}{9}e^{3x}x+\frac{2e^{3x}}{27}$$

Antiderivative was successfully verified.

```
[In] Int[E^(3*x)*x^2,x]
```

```
[Out] (2*E^(3*x))/27 - (2*E^(3*x)*x)/9 + (E^(3*x)*x^2)/3
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{3x}x^2\,dx &= \frac{1}{3}e^{3x}x^2 - \frac{2}{3}\int e^{3x}x\,dx\\ &= -\frac{2}{9}e^{3x}x + \frac{1}{3}e^{3x}x^2 + \frac{2}{9}\int e^{3x}\,dx\\ &= \frac{2e^{3x}}{27} - \frac{2}{9}e^{3x}x + \frac{1}{3}e^{3x}x^2\end{aligned}$$

Mathematica [A] time = 0.00593, size = 19, normalized size = 0.59

$$\frac{1}{27}e^{3x}\left(9x^2-6x+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(3*x)*x^2,x]
```

```
[Out] (E^(3*x)*(2 - 6*x + 9*x^2))/27
```

Maple [A] time = 0.001, size = 17, normalized size = 0.5

$$\frac{\left(9x^2-6x+2\right)e^{3x}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(3*x)*x^2,x)
```

```
[Out] 1/27*(9*x^2-6*x+2)*exp(3*x)
```

Maxima [A] time = 0.941502, size = 22, normalized size = 0.69

$$\frac{1}{27}\left(9x^2-6x+2\right)e^{(3x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*x^2,x, algorithm="maxima")
```

```
[Out] 1/27*(9*x^2 - 6*x + 2)*e^(3*x)
```

Fricas [A] time = 1.73333, size = 43, normalized size = 1.34

$$\frac{1}{27}\left(9\,x^2-6\,x+2\right)e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*x^2,x, algorithm="fricas")
```

```
[Out] 1/27*(9*x^2 - 6*x + 2)*e^(3*x)
```

Sympy [A] time = 0.079068, size = 15, normalized size = 0.47

$$\frac{\left(9x^2-6x+2\right)e^{3x}}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*x**2,x)
```

```
[Out] (9*x**2 - 6*x + 2)*exp(3*x)/27
```

Giac [A] time = 1.06645, size = 22, normalized size = 0.69

$$\frac{1}{27}\left(9\,x^2-6\,x+2\right)e^{(3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(3*x)*x^2,x, algorithm="giac")
```

```
[Out] 1/27*(9*x^2 - 6*x + 2)*e^(3*x)
```

3.348 $\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}(\sin(x)+1)^{3/2}-2\sqrt{\sin(x)+1}$$

```
[Out] -2*Sqrt[1 + Sin[x]] + (2*(1 + Sin[x])^(3/2))/3
```

Rubi [A] time = 0.0367756, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2833, 43}

$$\frac{2}{3}(\sin(x)+1)^{3/2}-2\sqrt{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]
```

```
[Out] -2*Sqrt[1 + Sin[x]] + (2*(1 + Sin[x])^(3/2))/3
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(x)\sin(x)}{\sqrt{1+\sin(x)}}\,dx = \text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}} + \sqrt{1+x}\right)dx, x, \sin(x)\right)$$
$$= -2\sqrt{1+\sin(x)} + \frac{2}{3}(1+\sin(x))^{3/2}$$

Mathematica [A] time = 0.0224933, size = 31, normalized size = 1.35

$$\frac{2(\sin(x)-2)\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2}{3\sqrt{\sin(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/Sqrt[1 + Sin[x]],x]
```

```
[Out] (2*(Cos[x/2] + Sin[x/2])^2*(-2 + Sin[x]))/(3*Sqrt[1 + Sin[x]])
```

Maple [A] time = 0.006, size = 18, normalized size = 0.8

$$\frac{2}{3}(1+\sin(x))^{\frac{3}{2}} - 2\sqrt{1+\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/(1+sin(x))^(1/2),x)
```

```
[Out] 2/3*(1+sin(x))^(3/2)-2*(1+sin(x))^(1/2)
```

Maxima [A] time = 0.958319, size = 23, normalized size = 1.

$$\frac{2}{3}(\sin(x)+1)^{\frac{3}{2}} - 2\sqrt{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(sin(x) + 1)^(3/2) - 2*sqrt(sin(x) + 1)
```

Fricas [A] time = 2.04638, size = 47, normalized size = 2.04

$$\frac{2}{3}\sqrt{\sin(x)+1}(\sin(x)-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(sin(x) + 1)*(sin(x) - 2)
```

Sympy [A] time = 0.322213, size = 26, normalized size = 1.13

$$\frac{2\sqrt{\sin(x)+1}\sin(x)}{3}-\frac{4\sqrt{\sin(x)+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))**(1/2),x)
```

```
[Out] 2*sqrt(sin(x) + 1)*sin(x)/3 - 4*sqrt(sin(x) + 1)/3
```

Giac [A] time = 1.05914, size = 23, normalized size = 1.

$$\frac{2}{3}(\sin(x)+1)^{\frac{3}{2}}-2\sqrt{\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(1+sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(sin(x) + 1)^(3/2) - 2*sqrt(sin(x) + 1)
```

3.349 $\int x \sin^{-1}\left(x^2\right) dx$

Optimal. Leaf size=27

$$\frac{\sqrt{1-x^4}}{2}+\frac{1}{2}x^2\sin^{-1}\left(x^2\right)$$

```
[Out] Sqrt[1 - x^4]/2 + (x^2*ArcSin[x^2])/2
```

Rubi [A] time = 0.0152655, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6715, 4619, 261}

$$\frac{\sqrt{1-x^4}}{2}+\frac{1}{2}x^2\sin^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcSin[x^2],x]
```

```
[Out] Sqrt[1 - x^4]/2 + (x^2*ArcSin[x^2])/2
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\sin^{-1}\left(x^2\right)\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \sin^{-1}(x)\,dx,x,x^2\right)\\ &= \frac{1}{2}x^2\sin^{-1}\left(x^2\right)-\frac{1}{2}\,\text{Subst}\left(\int \frac{x}{\sqrt{1-x^2}}\,dx,x,x^2\right)\\ &= \frac{\sqrt{1-x^4}}{2}+\frac{1}{2}x^2\sin^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0044751, size = 24, normalized size = 0.89

$$\frac{1}{2}\left(\sqrt{1-x^4}+x^2\sin^{-1}\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcSin[x^2],x]
```

```
[Out] (Sqrt[1 - x^4] + x^2*ArcSin[x^2])/2
```

Maple [A] time = 0.002, size = 22, normalized size = 0.8

$$\frac{x^2\arcsin\left(x^2\right)}{2}+\frac{1}{2}\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arcsin(x^2),x)
```

```
[Out] 1/2*x^2*arcsin(x^2)+1/2*(-x^4+1)^(1/2)
```

Maxima [A] time = 1.41565, size = 28, normalized size = 1.04

$$\frac{1}{2}\,x^2\arcsin\left(x^2\right)+\frac{1}{2}\,\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x^2),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arcsin(x^2) + 1/2*sqrt(-x^4 + 1)
```

Fricas [A] time = 2.05932, size = 57, normalized size = 2.11

$$\frac{1}{2}x^2\arcsin\left(x^2\right) + \frac{1}{2}\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*arcsin(x^2) + 1/2*sqrt(-x^4 + 1)
```

Sympy [A] time = 0.17522, size = 19, normalized size = 0.7

$$\frac{x^2\operatorname{asin}\left(x^2\right)}{2} + \frac{\sqrt{1-x^4}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*asin(x**2),x)
```

```
[Out] x**2*asin(x**2)/2 + sqrt(1 - x**4)/2
```

Giac [A] time = 1.05492, size = 28, normalized size = 1.04

$$\frac{1}{2}x^2\arcsin\left(x^2\right) + \frac{1}{2}\sqrt{-x^4+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arcsin(x^2),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arcsin(x^2) + 1/2*sqrt(-x^4 + 1)
```

3.350 $\int x^3 \sin^{-1}\left(x^2\right) dx$

Optimal. Leaf size=38

$$\frac{1}{8}\sqrt{1-x^4}x^2+\frac{1}{4}x^4\sin^{-1}\left(x^2\right)-\frac{1}{8}\sin^{-1}\left(x^2\right)$$

```
[Out] (x^2*Sqrt[1 - x^4])/8 - ArcSin[x^2]/8 + (x^4*ArcSin[x^2])/4
```

Rubi [A] time = 0.0233579, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4842, 12, 275, 321, 216}

$$\frac{1}{8}\sqrt{1-x^4}x^2+\frac{1}{4}x^4\sin^{-1}\left(x^2\right)-\frac{1}{8}\sin^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*ArcSin[x^2],x]
```

```
[Out] (x^2*Sqrt[1 - x^4])/8 - ArcSin[x^2]/8 + (x^4*ArcSin[x^2])/4
```

Rule 4842

```
Int[((a_.) + ArcSin[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Sim
p[((c + d*x)^(m + 1)*(a + b*ArcSin[u]))/(d*(m + 1)), x] - Dist[b/(d*(m + 1)
), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/Sqrt[1 - u^2], x], x],
 x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x
] &&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] &&  !FunctionOfExponentialQ[u,
x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \sin^{-1}\left(x^2\right) dx &= \frac{1}{4}x^4 \sin^{-1}\left(x^2\right) - \frac{1}{4}\int \frac{2x^5}{\sqrt{1-x^4}}\,dx \\
&= \frac{1}{4}x^4 \sin^{-1}\left(x^2\right) - \frac{1}{2}\int \frac{x^5}{\sqrt{1-x^4}}\,dx \\
&= \frac{1}{4}x^4 \sin^{-1}\left(x^2\right) - \frac{1}{4}\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^2}}\,dx, x, x^2\right) \\
&= \frac{1}{8}x^2\sqrt{1-x^4} + \frac{1}{4}x^4 \sin^{-1}\left(x^2\right) - \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}}\,dx, x, x^2\right) \\
&= \frac{1}{8}x^2\sqrt{1-x^4} - \frac{1}{8}\sin^{-1}\left(x^2\right) + \frac{1}{4}x^4 \sin^{-1}\left(x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.011075, size = 32, normalized size = 0.84

$$\frac{1}{8}\left(\sqrt{1-x^4}x^2 + \left(2x^4-1\right)\sin^{-1}\left(x^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*ArcSin[x^2],x]
```

```
[Out] (x^2*Sqrt[1 - x^4] + (-1 + 2*x^4)*ArcSin[x^2])/8
```

Maple [A] time = 0.002, size = 31, normalized size = 0.8

$$-\frac{\arcsin\left(x^{2}\right)}{8}+\frac{x^{4}\arcsin\left(x^{2}\right)}{4}+\frac{x^{2}}{8}\sqrt{-x^{4}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arcsin(x^2),x)
```

```
[Out] -1/8*arcsin(x^2)+1/4*x^4*arcsin(x^2)+1/8*x^2*(-x^4+1)^(1/2)
```

Maxima [A] time = 1.43772, size = 72, normalized size = 1.89

$$\frac{1}{4}\,x^{4}\arcsin\left(x^{2}\right)-\frac{\sqrt{-x^{4}+1}}{8\,x^{2}\left(\frac{x^{4}-1}{x^{4}}-1\right)}+\frac{1}{8}\,\arctan\left(\frac{\sqrt{-x^{4}+1}}{x^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsin(x^2),x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arcsin(x^2) - 1/8*sqrt(-x^4 + 1)/(x^2*((x^4 - 1)/x^4 - 1)) + 1/8*ar
ctan(sqrt(-x^4 + 1)/x^2)
```

Fricas [A] time = 2.18518, size = 73, normalized size = 1.92

$$\frac{1}{8}\,\sqrt{-x^{4}+1}x^{2}+\frac{1}{8}\left(2\,x^{4}-1\right)\arcsin\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsin(x^2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(-x^4 + 1)*x^2 + 1/8*(2*x^4 - 1)*arcsin(x^2)
```

Sympy [A] time = 0.562222, size = 29, normalized size = 0.76

$$\frac{x^{4}\,\mathrm{asin}\left(x^{2}\right)}{4}+\frac{x^{2}\sqrt{1-x^{4}}}{8}-\frac{\mathrm{asin}\left(x^{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*asin(x**2),x)
```

```
[Out] x**4*asin(x**2)/4 + x**2*sqrt(1 - x**4)/8 - asin(x**2)/8
```

Giac [A] time = 1.07989, size = 43, normalized size = 1.13

$$\frac{1}{8}\sqrt{-x^4+1}x^2 + \frac{1}{4}\left(x^4-1\right)\arcsin\left(x^2\right) + \frac{1}{8}\arcsin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arcsin(x^2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(-x^4 + 1)*x^2 + 1/4*(x^4 - 1)*arcsin(x^2) + 1/8*arcsin(x^2)
```

3.351 $\int e^x \textbf{sech}(e^x)\, dx$

Optimal. Leaf size=5

$$\tan^{-1}(\sinh(e^x))$$

```
[Out] ArcTan[Sinh[E^x]]
```

Rubi [A] time = 0.009588, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {2282, 3770}

$$\tan^{-1}(\sinh(e^x))$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sech[E^x],x]

[Out] ArcTan[Sinh[E^x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int e^x \text{sech}(e^x)\, dx &= \text{Subst}\left(\int \text{sech}(x)\, dx, x, e^x\right)\\ &= \tan^{-1}(\sinh(e^x))\end{aligned}$$

Mathematica [A] time = 0.0049734, size = 5, normalized size = 1.

$$\tan^{-1}(\sinh(e^x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sech[E^x],x]
```

```
[Out] ArcTan[Sinh[E^x]]
```

Maple [A] time = 0.002, size = 5, normalized size = 1.

$$\arctan(\sinh(e^x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sech(exp(x)),x)
```

```
[Out] arctan(sinh(exp(x)))
```

Maxima [A] time = 0.953006, size = 5, normalized size = 1.

$$\arctan(\sinh(e^x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(exp(x)),x, algorithm="maxima")
```

```
[Out] arctan(sinh(e^x))
```

Fricas [B] time = 1.92324, size = 82, normalized size = 16.4

$$2\arctan(\cosh(\cosh(x)+\sinh(x))+\sinh(\cosh(x)+\sinh(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(exp(x)),x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(cosh(x) + sinh(x)) + sinh(cosh(x) + sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^x \operatorname{sech}(e^x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(exp(x)),x)
```

```
[Out] Integral(exp(x)*sech(exp(x)), x)
```

Giac [A] time = 1.06723, size = 8, normalized size = 1.6

$$2\arctan\left(e^{(e^x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sech(exp(x)),x, algorithm="giac")
```

```
[Out] 2*arctan(e^(e^x))
```

3.352 $\int x^2 \cos(3x)\,dx$

Optimal. Leaf size=29

$$\frac{1}{3}x^2\sin(3x)-\frac{2}{27}\sin(3x)+\frac{2}{9}x\cos(3x)$$

```
[Out] (2*x*Cos[3*x])/9 - (2*Sin[3*x])/27 + (x^2*Sin[3*x])/3
```

Rubi [A] time = 0.0283989, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3296, 2637}

$$\frac{1}{3}x^2\sin(3x)-\frac{2}{27}\sin(3x)+\frac{2}{9}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[3*x],x]
```

```
[Out] (2*x*Cos[3*x])/9 - (2*Sin[3*x])/27 + (x^2*Sin[3*x])/3
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^2\cos(3x)\,dx&=\frac{1}{3}x^2\sin(3x)-\frac{2}{3}\int x\sin(3x)\,dx\\&=\frac{2}{9}x\cos(3x)+\frac{1}{3}x^2\sin(3x)-\frac{2}{9}\int\cos(3x)\,dx\\&=\frac{2}{9}x\cos(3x)-\frac{2}{27}\sin(3x)+\frac{1}{3}x^2\sin(3x)\end{aligned}$$

Mathematica [A] time = 0.0237247, size = 25, normalized size = 0.86

$$\frac{1}{27}\left(9x^2-2\right)\sin(3x)+\frac{2}{9}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[3*x],x]
```

```
[Out] (2*x*Cos[3*x])/9 + ((-2 + 9*x^2)*Sin[3*x])/27
```

Maple [A] time = 0., size = 24, normalized size = 0.8

$$\frac{2\,x\cos\left(3\,x\right)}{9}-\frac{2\,\sin\left(3\,x\right)}{27}+\frac{x^2\sin\left(3\,x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(3*x),x)
```

```
[Out] 2/9*x*cos(3*x)-2/27*sin(3*x)+1/3*x^2*sin(3*x)
```

Maxima [A] time = 0.94075, size = 28, normalized size = 0.97

$$\frac{2}{9}\,x\cos\left(3\,x\right)+\frac{1}{27}\left(9\,x^2-2\right)\sin\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x),x, algorithm="maxima")
```

```
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

Fricas [A] time = 1.92203, size = 59, normalized size = 2.03

$$\frac{2}{9}\,x\cos\left(3\,x\right)+\frac{1}{27}\left(9\,x^2-2\right)\sin\left(3\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x),x, algorithm="fricas")
```

```
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

Sympy [A] time = 0.31606, size = 27, normalized size = 0.93

$$\frac{x^2 \sin(3x)}{3} + \frac{2x \cos(3x)}{9} - \frac{2 \sin(3x)}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(3*x),x)
```

```
[Out] x**2*sin(3*x)/3 + 2*x*cos(3*x)/9 - 2*sin(3*x)/27
```

Giac [A] time = 1.06166, size = 28, normalized size = 0.97

$$\frac{2}{9} x \cos(3x) + \frac{1}{27} \left(9x^2 - 2\right) \sin(3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(3*x),x, algorithm="giac")
```

```
[Out] 2/9*x*cos(3*x) + 1/27*(9*x^2 - 2)*sin(3*x)
```

3.353 $\int \sqrt{5-4x-x^2}\,dx$

Optimal. Leaf size=36

$$\frac{1}{2}(x+2)\sqrt{-x^2-4x+5}-\frac{9}{2}\sin^{-1}\left(\frac{1}{3}(-x-2)\right)$$

```
[Out] ((2 + x)*Sqrt[5 - 4*x - x^2])/2 - (9*ArcSin[(-2 - x)/3])/2
```

Rubi [A] time = 0.0102789, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {612, 619, 216}

$$\frac{1}{2}(x+2)\sqrt{-x^2-4x+5}-\frac{9}{2}\sin^{-1}\left(\frac{1}{3}(-x-2)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5 - 4*x - x^2],x]
```

```
[Out] ((2 + x)*Sqrt[5 - 4*x - x^2])/2 - (9*ArcSin[(-2 - x)/3])/2
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \sqrt{5-4x-x^2}\,dx = \frac{1}{2}(2+x)\sqrt{5-4x-x^2} + \frac{9}{2}\int \frac{1}{\sqrt{5-4x-x^2}}\,dx$$
$$= \frac{1}{2}(2+x)\sqrt{5-4x-x^2} - \frac{3}{4}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{36}}}\,dx, x, -4-2x\right)$$
$$= \frac{1}{2}(2+x)\sqrt{5-4x-x^2} - \frac{9}{2}\sin^{-1}\left(\frac{1}{3}(-2-x)\right)$$

Mathematica [A] time = 0.015808, size = 33, normalized size = 0.92

$$\frac{1}{2}\left(\sqrt{-x^2-4x+5}(x+2)+9\sin^{-1}\left(\frac{x+2}{3}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5 - 4*x - x^2],x]
```

```
[Out] ((2 + x)*Sqrt[5 - 4*x - x^2] + 9*ArcSin[(2 + x)/3])/2
```

Maple [A] time = 0.003, size = 29, normalized size = 0.8

$$-\frac{-2\,x-4}{4}\sqrt{-x^2-4\,x+5}+\frac{9}{2}\arcsin\left(\frac{2}{3}+\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-4*x+5)^(1/2),x)
```

```
[Out] -1/4*(-2*x-4)*(-x^2-4*x+5)^(1/2)+9/2*arcsin(2/3+1/3*x)
```

Maxima [A] time = 1.43099, size = 49, normalized size = 1.36

$$\frac{1}{2}\sqrt{-x^2-4\,x+5}x+\sqrt{-x^2-4\,x+5}-\frac{9}{2}\arcsin\left(-\frac{1}{3}\,x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 - 4*x + 5)*x + sqrt(-x^2 - 4*x + 5) - 9/2*arcsin(-1/3*x - 2/3
)
```

Fricas [A] time = 2.05362, size = 126, normalized size = 3.5

$$\frac{1}{2}\sqrt{-x^2-4x+5}(x+2)-\frac{9}{2}\arctan\left(\frac{\sqrt{-x^2-4x+5}(x+2)}{x^2+4x-5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 - 4*x + 5)*(x + 2) - 9/2*arctan(sqrt(-x^2 - 4*x + 5)*(x + 2)/
(x^2 + 4*x - 5))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-x^2-4x+5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-4*x+5)**(1/2),x)
```

```
[Out] Integral(sqrt(-x**2 - 4*x + 5), x)
```

Giac [A] time = 1.06128, size = 35, normalized size = 0.97

$$\frac{1}{2}\sqrt{-x^2-4x+5}(x+2)+\frac{9}{2}\arcsin\left(\frac{1}{3}x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 - 4*x + 5)*(x + 2) + 9/2*arcsin(1/3*x + 2/3)
```

3.354 $\int \frac{x^5}{\sqrt{2}+x^2}\,dx$

Optimal. Leaf size=28

$$\frac{x^4}{4}-\frac{x^2}{\sqrt{2}}+\log\left(x^2+\sqrt{2}\right)$$

```
[Out] -(x^2/Sqrt[2]) + x^4/4 + Log[Sqrt[2] + x^2]
```

Rubi [A] time = 0.0191322, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {266, 43}

$$\frac{x^4}{4}-\frac{x^2}{\sqrt{2}}+\log\left(x^2+\sqrt{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[2] + x^2),x]
```

```
[Out] -(x^2/Sqrt[2]) + x^4/4 + Log[Sqrt[2] + x^2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^5}{\sqrt{2}+x^2}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^2}{\sqrt{2}+x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\mathrm{Subst}\left(\int \left(-\sqrt{2}+x+\frac{2}{\sqrt{2}+x}\right)dx, x, x^2\right)$$
$$= -\frac{x^2}{\sqrt{2}}+\frac{x^4}{4}+\log\left(\sqrt{2}+x^2\right)$$

Mathematica [A] time = 0.0105252, size = 31, normalized size = 1.11

$$\frac{1}{4}\left(x^4-2\sqrt{2}x^2+4\log\left(x^2+\sqrt{2}\right)-6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[2] + x^2),x]
```

```
[Out] (-6 - 2*Sqrt[2]*x^2 + x^4 + 4*Log[Sqrt[2] + x^2])/4
```

Maple [A] time = 0.007, size = 23, normalized size = 0.8

$$\frac{x^4}{4}+\ln\left(x^2+\sqrt{2}\right)-\frac{x^2\sqrt{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^2+2^(1/2)),x)
```

```
[Out] 1/4*x^4+ln(x^2+2^(1/2))-1/2*x^2*2^(1/2)
```

Maxima [A] time = 1.44337, size = 30, normalized size = 1.07

$$\frac{1}{4}x^4-\frac{1}{2}\sqrt{2}x^2+\log\left(x^2+\sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2+2^(1/2)),x, algorithm="maxima")

[Out] 1/4*x^4 - 1/2*sqrt(2)*x^2 + log(x^2 + sqrt(2))
```

Fricas [A] time = 1.87988, size = 65, normalized size = 2.32

$$\frac{1}{4}x^4 - \frac{1}{2}\sqrt{2}x^2 + \log\left(x^2 + \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2+2^(1/2)),x, algorithm="fricas")

[Out] 1/4*x^4 - 1/2*sqrt(2)*x^2 + log(x^2 + sqrt(2))
```

Sympy [A] time = 0.114682, size = 24, normalized size = 0.86

$$\frac{x^4}{4} - \frac{\sqrt{2}x^2}{2} + \log\left(x^2 + \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**2+2**(1/2)),x)

[Out] x**4/4 - sqrt(2)*x**2/2 + log(x**2 + sqrt(2))
```

Giac [A] time = 1.05794, size = 30, normalized size = 1.07

$$\frac{1}{4}x^4 - \frac{1}{2}\sqrt{2}x^2 + \log\left(x^2 + \sqrt{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2+2^(1/2)),x, algorithm="giac")

[Out] 1/4*x^4 - 1/2*sqrt(2)*x^2 + log(x^2 + sqrt(2))
```

3.355 $\int \sec^5(x)\,dx$

Optimal. Leaf size=26

$$\frac{3}{8}\tanh^{-1}(\sin(x))+\frac{1}{4}\tan(x)\sec^3(x)+\frac{3}{8}\tan(x)\sec(x)$$

```
[Out] (3*ArcTanh[Sin[x]])/8 + (3*Sec[x]*Tan[x])/8 + (Sec[x]^3*Tan[x])/4
```

Rubi [A] time = 0.0143809, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3768, 3770}

$$\frac{3}{8}\tanh^{-1}(\sin(x))+\frac{1}{4}\tan(x)\sec^3(x)+\frac{3}{8}\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^5,x]
```

```
[Out] (3*ArcTanh[Sin[x]])/8 + (3*Sec[x]*Tan[x])/8 + (Sec[x]^3*Tan[x])/4
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^5(x)\,dx &= \frac{1}{4}\sec^3(x)\tan(x)+\frac{3}{4}\int \sec^3(x)\,dx\\ &= \frac{3}{8}\sec(x)\tan(x)+\frac{1}{4}\sec^3(x)\tan(x)+\frac{3}{8}\int \sec(x)\,dx\\ &= \frac{3}{8}\tanh^{-1}(\sin(x))+\frac{3}{8}\sec(x)\tan(x)+\frac{1}{4}\sec^3(x)\tan(x)\end{aligned}$$

Mathematica [B] time = 0.113231, size = 58, normalized size = 2.23

$$\frac{1}{16}\left(\frac{1}{2}(11\sin(x)+3\sin(3x))\sec^4(x)-6\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+6\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^5,x]
```

```
[Out] (-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2] + Sin[x/2]] + (Sec[x]^4*(11*S
in[x] + 3*Sin[3*x]))/2)/16
```

Maple [A] time = 0.032, size = 25, normalized size = 1.

$$-\left(-\frac{(\sec(x))^3}{4}-\frac{3\sec(x)}{8}\right)\tan(x)+\frac{3\ln(\sec(x)+\tan(x))}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^5,x)
```

```
[Out] -(-1/4*sec(x)^3-3/8*sec(x))*tan(x)+3/8*ln(sec(x)+tan(x))
```

Maxima [B] time = 0.946055, size = 57, normalized size = 2.19

$$-\frac{3\sin(x)^3-5\sin(x)}{8\left(\sin(x)^4-2\sin(x)^2+1\right)}+\frac{3}{16}\log(\sin(x)+1)-\frac{3}{16}\log(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5,x, algorithm="maxima")
```

```
[Out] -1/8*(3*sin(x)^3 - 5*sin(x))/(sin(x)^4 - 2*sin(x)^2 + 1) + 3/16*log(sin(x)
+ 1) - 3/16*log(sin(x) - 1)
```

Fricas [B] time = 2.16144, size = 138, normalized size = 5.31

$$\frac{3 \cos\left(x\right)^{4} \log\left(\sin\left(x\right) + 1\right) - 3 \cos\left(x\right)^{4} \log\left(-\sin\left(x\right) + 1\right) + 2 \left(3 \cos\left(x\right)^{2} + 2\right) \sin\left(x\right)}{16 \cos\left(x\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5,x, algorithm="fricas")
```

```
[Out] 1/16*(3*cos(x)^4*log(sin(x) + 1) - 3*cos(x)^4*log(-sin(x) + 1) + 2*(3*cos(x
)^2 + 2)*sin(x))/cos(x)^4
```

Sympy [A] time = 0.135195, size = 46, normalized size = 1.77

$$-\frac{3 \sin^{3}{\left(x \right)} - 5 \sin{\left(x \right)}}{8 \sin^{4}{\left(x \right)} - 16 \sin^{2}{\left(x \right)} + 8} - \frac{3 \log{\left(\sin{\left(x \right)} - 1 \right)}}{16} + \frac{3 \log{\left(\sin{\left(x \right)} + 1 \right)}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**5,x)
```

```
[Out] -(3*sin(x)**3 - 5*sin(x))/(8*sin(x)**4 - 16*sin(x)**2 + 8) - 3*log(sin(x) -
 1)/16 + 3*log(sin(x) + 1)/16
```

Giac [A] time = 1.08199, size = 51, normalized size = 1.96

$$-\frac{3 \sin\left(x\right)^{3} - 5 \sin\left(x\right)}{8 \left(\sin\left(x\right)^{2} - 1\right)^{2}} + \frac{3}{16} \log\left(\sin\left(x\right) + 1\right) - \frac{3}{16} \log\left(-\sin\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5,x, algorithm="giac")
```

```
[Out] -1/8*(3*sin(x)^3 - 5*sin(x))/(sin(x)^2 - 1)^2 + 3/16*log(sin(x) + 1) - 3/16
*log(-sin(x) + 1)
```

3.356 $\int \sin^6(2x)\, dx$

Optimal. Leaf size=46

$$\frac{5x}{16} - \frac{1}{12}\sin^5(2x)\cos(2x) - \frac{5}{48}\sin^3(2x)\cos(2x) - \frac{5}{32}\sin(2x)\cos(2x)$$

```
[Out] (5*x)/16 - (5*Cos[2*x]*Sin[2*x])/32 - (5*Cos[2*x]*Sin[2*x]^3)/48 - (Cos[2*x
]*Sin[2*x]^5)/12
```

Rubi [A] time = 0.0197261, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2635, 8}

$$\frac{5x}{16} - \frac{1}{12}\sin^5(2x)\cos(2x) - \frac{5}{48}\sin^3(2x)\cos(2x) - \frac{5}{32}\sin(2x)\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[2*x]^6,x]
```

```
[Out] (5*x)/16 - (5*Cos[2*x]*Sin[2*x])/32 - (5*Cos[2*x]*Sin[2*x]^3)/48 - (Cos[2*x
]*Sin[2*x]^5)/12
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \sin^6(2x)\,dx &= -\frac{1}{12}\cos(2x)\sin^5(2x) + \frac{5}{6}\int \sin^4(2x)\,dx \\
&= -\frac{5}{48}\cos(2x)\sin^3(2x) - \frac{1}{12}\cos(2x)\sin^5(2x) + \frac{5}{8}\int \sin^2(2x)\,dx \\
&= -\frac{5}{32}\cos(2x)\sin(2x) - \frac{5}{48}\cos(2x)\sin^3(2x) - \frac{1}{12}\cos(2x)\sin^5(2x) + \frac{5\int 1\,dx}{16} \\
&= \frac{5x}{16} - \frac{5}{32}\cos(2x)\sin(2x) - \frac{5}{48}\cos(2x)\sin^3(2x) - \frac{1}{12}\cos(2x)\sin^5(2x)
\end{aligned}
$$

Mathematica [A] time = 0.0064757, size = 30, normalized size = 0.65

$$\frac{5x}{16} - \frac{15}{128}\sin(4x) + \frac{3}{128}\sin(8x) - \frac{1}{384}\sin(12x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[2*x]^6,x]
```

```
[Out] (5*x)/16 - (15*Sin[4*x])/128 + (3*Sin[8*x])/128 - Sin[12*x]/384
```

Maple [A] time = 0.007, size = 32, normalized size = 0.7

$$-\frac{\cos(2x)}{12}\left((\sin(2x))^5 + \frac{5(\sin(2x))^3}{4} + \frac{15\sin(2x)}{8}\right) + \frac{5x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)^6,x)
```

```
[Out] -1/12*(sin(2*x)^5+5/4*sin(2*x)^3+15/8*sin(2*x))*cos(2*x)+5/16*x
```

Maxima [A] time = 0.949246, size = 32, normalized size = 0.7

$$\frac{1}{96}\sin(4x)^3 + \frac{5}{16}x + \frac{3}{128}\sin(8x) - \frac{1}{8}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)^6,x, algorithm="maxima")
```

```
[Out] 1/96*sin(4*x)^3 + 5/16*x + 3/128*sin(8*x) - 1/8*sin(4*x)
```

Fricas [A] time = 1.9265, size = 95, normalized size = 2.07

$$-\frac{1}{96}\left(8\cos\left(2x\right)^{5}-26\cos\left(2x\right)^{3}+33\cos\left(2x\right)\right)\sin\left(2x\right)+\frac{5}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)^6,x, algorithm="fricas")
```

```
[Out] -1/96*(8*cos(2*x)^5 - 26*cos(2*x)^3 + 33*cos(2*x))*sin(2*x) + 5/16*x
```

Sympy [A] time = 0.059177, size = 46, normalized size = 1.

$$\frac{5x}{16}-\frac{\sin^{5}(2x)\cos(2x)}{12}-\frac{5\sin^{3}(2x)\cos(2x)}{48}-\frac{5\sin(2x)\cos(2x)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)**6,x)
```

```
[Out] 5*x/16 - sin(2*x)**5*cos(2*x)/12 - 5*sin(2*x)**3*cos(2*x)/48 - 5*sin(2*x)*c
os(2*x)/32
```

Giac [A] time = 1.05715, size = 30, normalized size = 0.65

$$\frac{5}{16}x-\frac{1}{384}\sin\left(12x\right)+\frac{3}{128}\sin\left(8x\right)-\frac{15}{128}\sin\left(4x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)^6,x, algorithm="giac")
```

```
[Out] 5/16*x - 1/384*sin(12*x) + 3/128*sin(8*x) - 15/128*sin(4*x)
```

3.357 $\int \cos(x)\log(\sin(x))\sin^2(x)\,dx$

Optimal. Leaf size=20

$$\frac{1}{3}\sin^3(x)\log(\sin(x))-\frac{\sin^3(x)}{9}$$

```
[Out] -Sin[x]^3/9 + (Log[Sin[x]]*Sin[x]^3)/3
```

Rubi [A] time = 0.0348471, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2564, 30, 2554, 12}

$$\frac{1}{3}\sin^3(x)\log(\sin(x))-\frac{\sin^3(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]
```

```
[Out] -Sin[x]^3/9 + (Log[Sin[x]]*Sin[x]^3)/3
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\log(\sin(x))\sin^2(x)\,dx &= \frac{1}{3}\log(\sin(x))\sin^3(x) - \int \frac{1}{3}\cos(x)\sin^2(x)\,dx\\ &= \frac{1}{3}\log(\sin(x))\sin^3(x) - \frac{1}{3}\int \cos(x)\sin^2(x)\,dx\\ &= \frac{1}{3}\log(\sin(x))\sin^3(x) - \frac{1}{3}\operatorname{Subst}\left(\int x^2\,dx, x, \sin(x)\right)\\ &= -\frac{1}{9}\sin^3(x) + \frac{1}{3}\log(\sin(x))\sin^3(x)\end{aligned}$$

Mathematica [A] time = 0.0150786, size = 15, normalized size = 0.75

$$\frac{1}{9}\sin^3(x)(3\log(\sin(x)) - 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Log[Sin[x]]*Sin[x]^2,x]
```

```
[Out] ((-1 + 3*Log[Sin[x]])*Sin[x]^3)/9
```

Maple [A] time = 0.006, size = 17, normalized size = 0.9

$$-\frac{(\sin(x))^3}{9} + \frac{\ln(\sin(x))(\sin(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*ln(sin(x))*sin(x)^2,x)
```

```
[Out] -1/9*sin(x)^3+1/3*ln(sin(x))*sin(x)^3
```

Maxima [A] time = 0.946016, size = 22, normalized size = 1.1

$$\frac{1}{3}\log\left(\sin\left(x\right)\right)\sin\left(x\right)^{3}-\frac{1}{9}\sin\left(x\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*log(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

Fricas [A] time = 2.09145, size = 92, normalized size = 4.6

$$-\frac{1}{3}\left(\cos\left(x\right)^{2}-1\right)\log\left(\sin\left(x\right)\right)\sin\left(x\right)+\frac{1}{9}\left(\cos\left(x\right)^{2}-1\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(x)^2 - 1)*log(sin(x))*sin(x) + 1/9*(cos(x)^2 - 1)*sin(x)
```

Sympy [A] time = 6.153, size = 17, normalized size = 0.85

$$\frac{\log\left(\sin\left(x\right)\right)\sin^{3}\left(x\right)}{3}-\frac{\sin^{3}\left(x\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*ln(sin(x))*sin(x)**2,x)
```

```
[Out] log(sin(x))*sin(x)**3/3 - sin(x)**3/9
```

Giac [A] time = 1.05062, size = 22, normalized size = 1.1

$$\frac{1}{3}\log\left(\sin\left(x\right)\right)\sin\left(x\right)^{3}-\frac{1}{9}\sin\left(x\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*log(sin(x))*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/3*log(sin(x))*sin(x)^3 - 1/9*sin(x)^3
```

3.358 $\int \frac{e^{-x}}{1+2e^x} \, dx$

Optimal. Leaf size=21

$$-2x - e^{-x} + 2\log(2e^x + 1)$$

```
[Out] -E^(-x) - 2*x + 2*Log[1 + 2*E^x]
```

Rubi [A] time = 0.0259144, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2248, 44}

$$-2x - e^{-x} + 2\log(2e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^x*(1 + 2*E^x)),x]
```

```
[Out] -E^(-x) - 2*x + 2*Log[1 + 2*E^x]
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{-x}}{1+2e^x}\,dx = \text{Subst}\left(\int \frac{1}{x^2(1+2x)}\,dx, x, e^x\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{x^2} - \frac{2}{x} + \frac{4}{1+2x}\right) dx, x, e^x\right)$$
$$= -e^{-x} - 2x + 2\log\left(1+2e^x\right)$$

Mathematica [A] time = 0.0156914, size = 21, normalized size = 1.

$$-2x - e^{-x} + 2\log\left(2e^x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^x*(1 + 2*E^x)),x]
```

```
[Out] -E^(-x) - 2*x + 2*Log[1 + 2*E^x]
```

Maple [A] time = 0.007, size = 22, normalized size = 1.1

$$-\left(e^x\right)^{-1} - 2\ln\left(e^x\right) + 2\ln\left(1+2e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(x)/(1+2*exp(x)),x)
```

```
[Out] -1/exp(x)-2*ln(exp(x))+2*ln(1+2*exp(x))
```

Maxima [A] time = 0.925633, size = 22, normalized size = 1.05

$$-e^{(-x)} + 2\log\left(e^{(-x)}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(x)/(1+2*exp(x)),x, algorithm="maxima")
```

```
[Out] -e^(-x) + 2*log(e^(-x) + 2)
```

Fricas [A] time = 2.02538, size = 62, normalized size = 2.95

$$-(2xe^{x} - 2e^{x}\log(2e^{x} + 1) + 1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(x)/(1+2*exp(x)),x, algorithm="fricas")
```

```
[Out] -(2*x*e^x - 2*e^x*log(2*e^x + 1) + 1)*e^(-x)
```

Sympy [A] time = 0.089545, size = 14, normalized size = 0.67

$$2\log(2 + e^{-x}) - e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(x)/(1+2*exp(x)),x)
```

```
[Out] 2*log(2 + exp(-x)) - exp(-x)
```

Giac [A] time = 1.052, size = 26, normalized size = 1.24

$$-2x - e^{(-x)} + 2\log(2e^{x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(x)/(1+2*exp(x)),x, algorithm="giac")
```

```
[Out] -2*x - e^(-x) + 2*log(2*e^x + 1)
```

3.359 $\int \sqrt{2+3\cos(x)}\tan(x)\,dx$

Optimal. Leaf size=37

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3\cos(x)+2}}{\sqrt{2}}\right)-2\sqrt{3\cos(x)+2}$$

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[2 + 3*Cos[x]]/Sqrt[2]] - 2*Sqrt[2 + 3*Cos[x]]
```

Rubi [A] time = 0.0405188, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2721, 50, 63, 207}

$$2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{3\cos(x)+2}}{\sqrt{2}}\right)-2\sqrt{3\cos(x)+2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + 3*Cos[x]]*Tan[x],x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[2 + 3*Cos[x]]/Sqrt[2]] - 2*Sqrt[2 + 3*Cos[x]]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{2+3\cos(x)}\tan(x)\,dx &= -\operatorname{Subst}\left(\int \frac{\sqrt{2+x}}{x}\,dx, x, 3\cos(x)\right)\\
&= -2\sqrt{2+3\cos(x)} - 2\operatorname{Subst}\left(\int \frac{1}{x\sqrt{2+x}}\,dx, x, 3\cos(x)\right)\\
&= -2\sqrt{2+3\cos(x)} - 4\operatorname{Subst}\left(\int \frac{1}{-2+x^2}\,dx, x, \sqrt{2+3\cos(x)}\right)\\
&= 2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{2+3\cos(x)}}{\sqrt{2}}\right) - 2\sqrt{2+3\cos(x)}
\end{aligned}$$

Mathematica [A] time = 0.0191128, size = 33, normalized size = 0.89

$$2\sqrt{2}\tanh^{-1}\left(\sqrt{\frac{3\cos(x)}{2}+1}\right) - 2\sqrt{3\cos(x)+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + 3*Cos[x]]*Tan[x],x]
```

```
[Out] 2*Sqrt[2]*ArcTanh[Sqrt[1 + (3*Cos[x])/2]] - 2*Sqrt[2 + 3*Cos[x]]
```

Maple [A] time = 0.016, size = 31, normalized size = 0.8

$$2\,\mathrm{Artanh}\left(1/2\,\sqrt{2+3\,\cos(x)}\sqrt{2}\right)\sqrt{2} - 2\,\sqrt{2+3\,\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*cos(x))^(1/2)*tan(x),x)
```

```
[Out] 2*arctanh(1/2*(2+3*cos(x))^(1/2)*2^(1/2))*2^(1/2)-2*(2+3*cos(x))^(1/2)
```

Maxima [A] time = 1.40633, size = 63, normalized size = 1.7

$$-\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{3\cos(x)+2}}{\sqrt{2}+\sqrt{3\cos(x)+2}}\right)-2\sqrt{3\cos(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(x))^(1/2)*tan(x),x, algorithm="maxima")
```

```
[Out] -sqrt(2)*log(-(sqrt(2) - sqrt(3*cos(x) + 2))/(sqrt(2) + sqrt(3*cos(x) + 2))
) - 2*sqrt(3*cos(x) + 2)
```

Fricas [A] time = 3.00829, size = 182, normalized size = 4.92

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{9\cos(x)^2+4\left(3\sqrt{2}\cos(x)+4\sqrt{2}\right)\sqrt{3\cos(x)+2}+48\cos(x)+32}{\cos(x)^2}\right)-2\sqrt{3\cos(x)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(x))^(1/2)*tan(x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(9*cos(x)^2 + 4*(3*sqrt(2)*cos(x) + 4*sqrt(2))*sqrt(3*cos(
x) + 2) + 48*cos(x) + 32)/cos(x)^2) - 2*sqrt(3*cos(x) + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{3\cos(x)+2}\tan(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(x))**(1/2)*tan(x),x)
```

```
[Out] Integral(sqrt(3*cos(x) + 2)*tan(x), x)
```

Giac [A] time = 1.06913, size = 68, normalized size = 1.84

$$-\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{3\cos\left(x\right)+2}\right|}{2\left(\sqrt{2}+\sqrt{3\cos\left(x\right)+2}\right)}\right)-2\sqrt{3\cos\left(x\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*cos(x))^(1/2)*tan(x),x, algorithm="giac")
```

```
[Out] -sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*sqrt(3*cos(x) + 2))/(sqrt(2) + sqrt(3*c
os(x) + 2))) - 2*sqrt(3*cos(x) + 2)
```

3.360 $\int \frac{x}{\sqrt{-4x+x^2}} \, dx$

Optimal. Leaf size=28

$$\sqrt{x^2-4x}+4\tanh^{-1}\left(\frac{x}{\sqrt{x^2-4x}}\right)$$

```
[Out] Sqrt[-4*x + x^2] + 4*ArcTanh[x/Sqrt[-4*x + x^2]]
```

Rubi [A] time = 0.0077534, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {640, 620, 206}

$$\sqrt{x^2-4x}+4\tanh^{-1}\left(\frac{x}{\sqrt{x^2-4x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-4*x + x^2],x]
```

```
[Out] Sqrt[-4*x + x^2] + 4*ArcTanh[x/Sqrt[-4*x + x^2]]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x}{\sqrt{-4x+x^2}}\,dx = \sqrt{-4x+x^2} + 2\int \frac{1}{\sqrt{-4x+x^2}}\,dx$$
$$= \sqrt{-4x+x^2} + 4\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-4x+x^2}}\right)$$
$$= \sqrt{-4x+x^2} + 4\tanh^{-1}\left(\frac{x}{\sqrt{-4x+x^2}}\right)$$

Mathematica [A] time = 0.0285069, size = 40, normalized size = 1.43

$$\frac{(x-4)x - 4\sqrt{-(x-4)x}\,\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)}{\sqrt{(x-4)x}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[-4*x + x^2],x]
```

```
[Out] ((-4 + x)*x - 4*Sqrt[-((-4 + x)*x)]*ArcSin[Sqrt[1 - x/4]])/Sqrt[(-4 + x)*x]
```

Maple [A] time = 0.003, size = 26, normalized size = 0.9

$$\sqrt{x^2-4x} + 2\ln\left(x - 2 + \sqrt{x^2-4x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2-4*x)^(1/2),x)
```

```
[Out] (x^2-4*x)^(1/2)+2*ln(x-2+(x^2-4*x)^(1/2))
```

Maxima [A] time = 0.929766, size = 39, normalized size = 1.39

$$\sqrt{x^2-4x} + 2\log\left(2x + 2\sqrt{x^2-4x} - 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-4*x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 - 4*x) + 2*log(2*x + 2*sqrt(x^2 - 4*x) - 4)
```

Fricas [A] time = 1.96705, size = 69, normalized size = 2.46

$$\sqrt{x^2-4\,x}-2\,\log\left(-x+\sqrt{x^2-4\,x}+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-4*x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 - 4*x) - 2*log(-x + sqrt(x^2 - 4*x) + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x\,(x-4)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2-4*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x*(x - 4)), x)
```

Giac [A] time = 1.06386, size = 38, normalized size = 1.36

$$\sqrt{x^2-4\,x}-2\,\log\left(\left|-x+\sqrt{x^2-4\,x}+2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2-4*x)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 - 4*x) - 2*log(abs(-x + sqrt(x^2 - 4*x) + 2))
```

3.361 $\int \cos^5(x)\,dx$

Optimal. Leaf size=19

$$\frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

```
[Out] Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Rubi [A] time = 0.0086871, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2633}

$$\frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5,x]
```

```
[Out] Sin[x] - (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(x)\,dx &= -\operatorname{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, -\sin(x)\right)\\ &= \sin(x) - \frac{2\sin^3(x)}{3} + \frac{\sin^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.0019552, size = 23, normalized size = 1.21

$$\frac{5\sin(x)}{8} + \frac{5}{48}\sin(3x) + \frac{1}{80}\sin(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5,x]
```

```
[Out] (5*Sin[x])/8 + (5*Sin[3*x])/48 + Sin[5*x]/80
```

Maple [A] time = 0.027, size = 17, normalized size = 0.9

$$\frac{\sin(x)}{5}\left(\frac{8}{3}+(\cos(x))^{4}+\frac{4\,(\cos(x))^{2}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5,x)
```

```
[Out] 1/5*(8/3+cos(x)^4+4/3*cos(x)^2)*sin(x)
```

Maxima [A] time = 0.942157, size = 20, normalized size = 1.05

$$\frac{1}{5}\sin(x)^{5}-\frac{2}{3}\sin(x)^{3}+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5,x, algorithm="maxima")
```

```
[Out] 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

Fricas [A] time = 1.9634, size = 58, normalized size = 3.05

$$\frac{1}{15}\left(3\cos(x)^{4}+4\cos(x)^{2}+8\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5,x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)
```

Sympy [A] time = 0.058083, size = 17, normalized size = 0.89

$$\frac{\sin^5(x)}{5} - \frac{2\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5,x)
```

```
[Out] sin(x)**5/5 - 2*sin(x)**3/3 + sin(x)
```

Giac [A] time = 1.07823, size = 20, normalized size = 1.05

$$\frac{1}{5}\sin(x)^5 - \frac{2}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5,x, algorithm="giac")
```

```
[Out] 1/5*sin(x)^5 - 2/3*sin(x)^3 + sin(x)
```

3.362 $\int e^{-x} x^4 \, dx$

Optimal. Leaf size=46

$$-e^{-x}x^4 - 4e^{-x}x^3 - 12e^{-x}x^2 - 24e^{-x}x - 24e^{-x}$$

```
[Out] -24/E^x - (24*x)/E^x - (12*x^2)/E^x - (4*x^3)/E^x - x^4/E^x
```

Rubi [A] time = 0.0403911, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$-e^{-x}x^4 - 4e^{-x}x^3 - 12e^{-x}x^2 - 24e^{-x}x - 24e^{-x}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/E^x,x]
```

```
[Out] -24/E^x - (24*x)/E^x - (12*x^2)/E^x - (4*x^3)/E^x - x^4/E^x
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-x}x^4\,dx &= -e^{-x}x^4 + 4\int e^{-x}x^3\,dx\\ &= -4e^{-x}x^3 - e^{-x}x^4 + 12\int e^{-x}x^2\,dx\\ &= -12e^{-x}x^2 - 4e^{-x}x^3 - e^{-x}x^4 + 24\int e^{-x}x\,dx\\ &= -24e^{-x}x - 12e^{-x}x^2 - 4e^{-x}x^3 - e^{-x}x^4 + 24\int e^{-x}\,dx\\ &= -24e^{-x} - 24e^{-x}x - 12e^{-x}x^2 - 4e^{-x}x^3 - e^{-x}x^4\end{aligned}$$

Mathematica [A] time = 0.0072785, size = 26, normalized size = 0.57

$$e^{-x}\left(-x^4 - 4x^3 - 12x^2 - 24x - 24\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/E^x,x]
```

```
[Out] (-24 - 24*x - 12*x^2 - 4*x^3 - x^4)/E^x
```

Maple [A] time = 0.002, size = 25, normalized size = 0.5

$$-\frac{x^4 + 4\,x^3 + 12\,x^2 + 24\,x + 24}{e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/exp(x),x)
```

```
[Out] -(x^4+4*x^3+12*x^2+24*x+24)/exp(x)
```

Maxima [A] time = 0.924995, size = 32, normalized size = 0.7

$$-\left(x^4 + 4\,x^3 + 12\,x^2 + 24\,x + 24\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/exp(x),x, algorithm="maxima")
```

```
[Out] -(x^4 + 4*x^3 + 12*x^2 + 24*x + 24)*e^(-x)
```

Fricas [A] time = 1.96542, size = 59, normalized size = 1.28

$$-\left(x^4 + 4\,x^3 + 12\,x^2 + 24\,x + 24\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/exp(x),x, algorithm="fricas")
```

```
[Out] -(x^4 + 4*x^3 + 12*x^2 + 24*x + 24)*e^(-x)
```

Sympy [A] time = 0.082029, size = 22, normalized size = 0.48

$$\left(-x^4 - 4x^3 - 12x^2 - 24x - 24\right)e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/exp(x),x)
```

```
[Out] (-x**4 - 4*x**3 - 12*x**2 - 24*x - 24)*exp(-x)
```

Giac [A] time = 1.05508, size = 32, normalized size = 0.7

$$-\left(x^4 + 4\,x^3 + 12\,x^2 + 24\,x + 24\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/exp(x),x, algorithm="giac")
```

```
[Out] -(x^4 + 4*x^3 + 12*x^2 + 24*x + 24)*e^(-x)
```

3.363 $\int \frac{x^4}{\sqrt{-2+x^{10}}}\,dx$

Optimal. Leaf size=18

$$\frac{1}{5}\tanh^{-1}\left(\frac{x^5}{\sqrt{x^{10}-2}}\right)$$

```
[Out] ArcTanh[x^5/Sqrt[-2 + x^10]]/5
```

Rubi [A] time = 0.007273, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {275, 217, 206}

$$\frac{1}{5}\tanh^{-1}\left(\frac{x^5}{\sqrt{x^{10}-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/Sqrt[-2 + x^10],x]
```

```
[Out] ArcTanh[x^5/Sqrt[-2 + x^10]]/5
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{x^4}{\sqrt{-2+x^{10}}}\,dx = \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{\sqrt{-2+x^2}}\,dx, x, x^5\right)$$
$$= \frac{1}{5}\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x^5}{\sqrt{-2+x^{10}}}\right)$$
$$= \frac{1}{5}\tanh^{-1}\left(\frac{x^5}{\sqrt{-2+x^{10}}}\right)$$

Mathematica [A] time = 0.0030268, size = 18, normalized size = 1.

$$\frac{1}{5}\tanh^{-1}\left(\frac{x^5}{\sqrt{x^{10}-2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/Sqrt[-2 + x^10],x]
```

```
[Out] ArcTanh[x^5/Sqrt[-2 + x^10]]/5
```

Maple [C] time = 0.043, size = 34, normalized size = 1.9

$$\frac{1}{5}\sqrt{-\text{signum}\left(-1+\frac{x^{10}}{2}\right)}\arcsin\left(\frac{x^5\sqrt{2}}{2}\right)\frac{1}{\sqrt{\text{signum}\left(-1+\frac{x^{10}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^10-2)^(1/2),x)
```

```
[Out] 1/5/signum(-1+1/2*x^10)^(1/2)*(-signum(-1+1/2*x^10))^(1/2)*arcsin(1/2*x^5*2
^(1/2))
```

Maxima [B] time = 0.931524, size = 45, normalized size = 2.5

$$\frac{1}{10}\,\log\left(\frac{\sqrt{x^{10}-2}}{x^5}+1\right)-\frac{1}{10}\,\log\left(\frac{\sqrt{x^{10}-2}}{x^5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10-2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/10*log(sqrt(x^10 - 2)/x^5 + 1) - 1/10*log(sqrt(x^10 - 2)/x^5 - 1)
```

Fricas [A] time = 1.81069, size = 45, normalized size = 2.5

$$-\frac{1}{5}\log\left(-x^5+\sqrt{x^{10}-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10-2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/5*log(-x^5 + sqrt(x^10 - 2))
```

Sympy [A] time = 1.01159, size = 34, normalized size = 1.89

$$\begin{cases} \frac{\operatorname{acosh}\left(\frac{\sqrt{2}x^5}{2}\right)}{5} & \text{for}\ \frac{|x^{10}|}{2} > 1 \\ -\frac{i\operatorname{asin}\left(\frac{\sqrt{2}x^5}{2}\right)}{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**10-2)**(1/2),x)
```

```
[Out] Piecewise((acosh(sqrt(2)*x**5/2)/5, Abs(x**10)/2 > 1), (-I*asin(sqrt(2)*x**
5/2)/5, True))
```

Giac [A] time = 1.07437, size = 23, normalized size = 1.28

$$-\frac{1}{5}\log\left(\left|-x^5+\sqrt{x^{10}-2}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^10-2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/5*log(abs(-x^5 + sqrt(x^10 - 2)))
```

3.364 $\int e^x \cos(4+3x)\,dx$

Optimal. Leaf size=27

$$\frac{3}{10}e^x\sin(3x+4)+\frac{1}{10}e^x\cos(3x+4)$$

```
[Out] (E^x*Cos[4 + 3*x])/10 + (3*E^x*Sin[4 + 3*x])/10
```

Rubi [A] time = 0.0089086, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4433}

$$\frac{3}{10}e^x\sin(3x+4)+\frac{1}{10}e^x\cos(3x+4)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[4 + 3*x],x]

[Out] (E^x*Cos[4 + 3*x])/10 + (3*E^x*Sin[4 + 3*x])/10
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x\cos(4+3x)\,dx=\frac{1}{10}e^x\cos(4+3x)+\frac{3}{10}e^x\sin(4+3x)$$

Mathematica [A] time = 0.0421079, size = 22, normalized size = 0.81

$$\frac{1}{10}e^x(3\sin(3x+4)+\cos(3x+4))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[4 + 3*x],x]
```

```
[Out] (E^x*(Cos[4 + 3*x] + 3*Sin[4 + 3*x]))/10
```

Maple [A] time = 0.007, size = 22, normalized size = 0.8

$$\frac{e^{x} \cos \left(3 x+4\right)}{10}+\frac{3 e^{x} \sin \left(3 x+4\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(3*x+4),x)
```

```
[Out] 1/10*exp(x)*cos(3*x+4)+3/10*exp(x)*sin(3*x+4)
```

Maxima [A] time = 0.943301, size = 26, normalized size = 0.96

$$\frac{1}{10} \left(\cos \left(3 x+4\right)+3 \sin \left(3 x+4\right)\right) e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(4+3*x),x, algorithm="maxima")
```

```
[Out] 1/10*(cos(3*x + 4) + 3*sin(3*x + 4))*e^x
```

Fricas [A] time = 1.91943, size = 63, normalized size = 2.33

$$\frac{1}{10} \cos \left(3 x+4\right) e^{x}+\frac{3}{10} e^{x} \sin \left(3 x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(4+3*x),x, algorithm="fricas")
```

```
[Out] 1/10*cos(3*x + 4)*e^x + 3/10*e^x*sin(3*x + 4)
```

Sympy [A] time = 0.287545, size = 24, normalized size = 0.89

$$\frac{3e^x \sin(3x+4)}{10} + \frac{e^x \cos(3x+4)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(4+3*x),x)
```

```
[Out] 3*exp(x)*sin(3*x + 4)/10 + exp(x)*cos(3*x + 4)/10
```

Giac [A] time = 1.05335, size = 26, normalized size = 0.96

$$\frac{1}{10} (\cos(3x+4) + 3\sin(3x+4))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(4+3*x),x, algorithm="giac")
```

```
[Out] 1/10*(cos(3*x + 4) + 3*sin(3*x + 4))*e^x
```

3.365 $\int e^x \log(1+e^x)\, dx$

Optimal. Leaf size=18

$$(e^x+1)\log(e^x+1)-e^x$$

```
[Out] -E^x + (1 + E^x)*Log[1 + E^x]
```

Rubi [A] time = 0.0295601, antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2194, 2554, 2248, 43}

$$-e^x+e^x\log(e^x+1)+\log(e^x+1)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Log[1 + E^x],x]
```

```
[Out] -E^x + Log[1 + E^x] + E^x*Log[1 + E^x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int e^x \log(1+e^x)\,dx &= e^x \log(1+e^x) - \int \frac{e^{2x}}{1+e^x}\,dx \\
&= e^x \log(1+e^x) - \text{Subst}\left(\int \frac{x}{1+x}\,dx, x, e^x\right) \\
&= e^x \log(1+e^x) - \text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, e^x\right) \\
&= -e^x + \log(1+e^x) + e^x \log(1+e^x)
\end{aligned}$$

Mathematica [A] time = 0.005832, size = 18, normalized size = 1.

$$(e^x+1)\log(e^x+1) - e^x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Log[1 + E^x],x]
```

```
[Out] -E^x + (1 + E^x)*Log[1 + E^x]
```

Maple [A] time = 0.002, size = 17, normalized size = 0.9

$$(1+e^x)\ln(1+e^x) - 1 - e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*ln(1+exp(x)),x)
```

```
[Out] (1+exp(x))*ln(1+exp(x))-1-exp(x)
```

Maxima [A] time = 0.931435, size = 22, normalized size = 1.22

$$(e^x + 1)\log(e^x + 1) - e^x - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(1+exp(x)),x, algorithm="maxima")
```

```
[Out] (e^x + 1)*log(e^x + 1) - e^x - 1
```

Fricas [A] time = 1.92823, size = 41, normalized size = 2.28

$$(e^x + 1)\log(e^x + 1) - e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(1+exp(x)),x, algorithm="fricas")
```

```
[Out] (e^x + 1)*log(e^x + 1) - e^x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*ln(1+exp(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.04817, size = 22, normalized size = 1.22

$$(e^x + 1)\log(e^x + 1) - e^x - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(1+exp(x)),x, algorithm="giac")
```

```
[Out] (e^x + 1)*log(e^x + 1) - e^x - 1
```

3.366 $\int x^2 \tan^{-1}(x)\,dx$

Optimal. Leaf size=27

$$-\frac{x^2}{6}+\frac{1}{6}\log\left(x^2+1\right)+\frac{1}{3}x^3\tan^{-1}(x)$$

```
[Out] -x^2/6 + (x^3*ArcTan[x])/3 + Log[1 + x^2]/6
```

Rubi [A] time = 0.0162793, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4852, 266, 43}

$$-\frac{x^2}{6}+\frac{1}{6}\log\left(x^2+1\right)+\frac{1}{3}x^3\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcTan[x],x]
```

```
[Out] -x^2/6 + (x^3*ArcTan[x])/3 + Log[1 + x^2]/6
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2 \tan^{-1}(x)\,dx &= \frac{1}{3}x^3\tan^{-1}(x) - \frac{1}{3}\int \frac{x^3}{1+x^2}\,dx \\
&= \frac{1}{3}x^3\tan^{-1}(x) - \frac{1}{6}\,\text{Subst}\left(\int \frac{x}{1+x}\,dx, x, x^2\right) \\
&= \frac{1}{3}x^3\tan^{-1}(x) - \frac{1}{6}\,\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, x^2\right) \\
&= -\frac{x^2}{6} + \frac{1}{3}x^3\tan^{-1}(x) + \frac{1}{6}\log\left(1+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0042958, size = 23, normalized size = 0.85

$$\frac{1}{6}\left(-x^2+\log\left(x^2+1\right)+2x^3\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcTan[x],x]
```

```
[Out] (-x^2 + 2*x^3*ArcTan[x] + Log[1 + x^2])/6
```

Maple [A] time = 0., size = 22, normalized size = 0.8

$$-\frac{x^2}{6}+\frac{x^3\arctan(x)}{3}+\frac{\ln\left(x^2+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arctan(x),x)
```

```
[Out] -1/6*x^2+1/3*x^3*arctan(x)+1/6*ln(x^2+1)
```

Maxima [A] time = 0.920129, size = 28, normalized size = 1.04

$$\frac{1}{3}\,x^3\arctan(x)-\frac{1}{6}\,x^2+\frac{1}{6}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

Fricas [A] time = 2.03142, size = 65, normalized size = 2.41

$$\frac{1}{3}\,x^3\arctan\left(x\right)-\frac{1}{6}\,x^2+\frac{1}{6}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

Sympy [A] time = 0.325198, size = 20, normalized size = 0.74

$$\frac{x^3\operatorname{atan}\left(x\right)}{3}-\frac{x^2}{6}+\frac{\log\left(x^2+1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*atan(x),x)
```

```
[Out] x**3*atan(x)/3 - x**2/6 + log(x**2 + 1)/6
```

Giac [A] time = 1.06491, size = 28, normalized size = 1.04

$$\frac{1}{3}\,x^3\arctan\left(x\right)-\frac{1}{6}\,x^2+\frac{1}{6}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arctan(x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arctan(x) - 1/6*x^2 + 1/6*log(x^2 + 1)
```

3.367 $\int \sqrt{-1+e^{2x}}\,dx$

Optimal. Leaf size=26

$$\sqrt{e^{2x}-1}-\tan^{-1}\left(\sqrt{e^{2x}-1}\right)$$

```
[Out] Sqrt[-1 + E^(2*x)] - ArcTan[Sqrt[-1 + E^(2*x)]]
```

Rubi [A] time = 0.0119735, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2282, 50, 63, 203}

$$\sqrt{e^{2x}-1}-\tan^{-1}\left(\sqrt{e^{2x}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + E^(2*x)],x]

[Out] Sqrt[-1 + E^(2*x)] - ArcTan[Sqrt[-1 + E^(2*x)]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{-1+e^{2x}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x}\,dx, x, e^{2x}\right)\\
&= \sqrt{-1+e^{2x}} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{-1+x}x}\,dx, x, e^{2x}\right)\\
&= \sqrt{-1+e^{2x}} - \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+e^{2x}}\right)\\
&= \sqrt{-1+e^{2x}} - \tan^{-1}\left(\sqrt{-1+e^{2x}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0061245, size = 26, normalized size = 1.

$$\sqrt{e^{2x}-1} - \tan^{-1}\left(\sqrt{e^{2x}-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + E^(2*x)],x]
```

```
[Out] Sqrt[-1 + E^(2*x)] - ArcTan[Sqrt[-1 + E^(2*x)]]
```

Maple [A] time = 0.004, size = 21, normalized size = 0.8

$$-\arctan\left(\sqrt{-1+\mathrm{e}^{2x}}\right) + \sqrt{-1+\mathrm{e}^{2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+exp(2*x))^(1/2),x)
```

```
[Out] -arctan((-1+exp(2*x))^(1/2))+(-1+exp(2*x))^(1/2)
```

Maxima [A] time = 1.41183, size = 27, normalized size = 1.04

$$\sqrt{e^{(2\,x)}-1}-\arctan\left(\sqrt{e^{(2\,x)}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(e^(2*x) - 1) - arctan(sqrt(e^(2*x) - 1))
```

Fricas [A] time = 1.86274, size = 63, normalized size = 2.42

$$\sqrt{e^{(2\,x)}-1}-\arctan\left(\sqrt{e^{(2\,x)}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(e^(2*x) - 1) - arctan(sqrt(e^(2*x) - 1))
```

Sympy [A] time = 1.0767, size = 19, normalized size = 0.73

$$\begin{cases}\sqrt{e^{2x}-1}-\operatorname{acos}\left(e^{-x}\right) & \text{for}\: e^{x}<0\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(2*x))**(1/2),x)
```

```
[Out] Piecewise((sqrt(exp(2*x) - 1) - acos(exp(-x)), exp(x) < 0))
```

Giac [A] time = 1.06856, size = 27, normalized size = 1.04

$$\sqrt{e^{(2x)}-1}-\arctan\left(\sqrt{e^{(2x)}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+exp(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(e^(2*x) - 1) - arctan(sqrt(e^(2*x) - 1))
```

3.368 $\int e^{\sin(x)} \sin(2x)\, dx$

Optimal. Leaf size=15

$$2e^{\sin(x)} \sin(x) - 2e^{\sin(x)}$$

```
[Out] -2*E^Sin[x] + 2*E^Sin[x]*Sin[x]
```

Rubi [A] time = 0.0165931, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 2176, 2194}

$$2e^{\sin(x)} \sin(x) - 2e^{\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[E^Sin[x]*Sin[2*x],x]
```

```
[Out] -2*E^Sin[x] + 2*E^Sin[x]*Sin[x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int e^{\sin(x)}\sin(2x)\,dx &= \text{Subst}\left(\int 2e^x x\,dx, x, \sin(x)\right)\\
&= 2\,\text{Subst}\left(\int e^x x\,dx, x, \sin(x)\right)\\
&= 2e^{\sin(x)}\sin(x) - 2\,\text{Subst}\left(\int e^x\,dx, x, \sin(x)\right)\\
&= -2e^{\sin(x)} + 2e^{\sin(x)}\sin(x)
\end{aligned}
$$

Mathematica [A] time = 0.0153761, size = 11, normalized size = 0.73

$$e^{\sin(x)}(2\sin(x) - 2)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sin[x]*Sin[2*x],x]
```

```
[Out] E^Sin[x]*(-2 + 2*Sin[x])
```

Maple [A] time = 0.009, size = 14, normalized size = 0.9

$$-2\,e^{\sin(x)} + 2\,e^{\sin(x)}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(sin(x))*sin(2*x),x)
```

```
[Out] -2*exp(sin(x))+2*exp(sin(x))*sin(x)
```

Maxima [A] time = 0.94368, size = 12, normalized size = 0.8

$$2\,(\sin(x) - 1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sin(2*x),x, algorithm="maxima")
```

```
[Out] 2*(sin(x) - 1)*e^sin(x)
```

Fricas [A] time = 1.97079, size = 34, normalized size = 2.27

$$2 (\sin (x) - 1)e^{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sin(2*x),x, algorithm="fricas")
```

```
[Out] 2*(sin(x) - 1)*e^sin(x)
```

Sympy [A] time = 2.2503, size = 15, normalized size = 1.

$$2e^{\sin (x)} \sin (x) - 2e^{\sin (x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sin(2*x),x)
```

```
[Out] 2*exp(sin(x))*sin(x) - 2*exp(sin(x))
```

Giac [B] time = 1.09381, size = 721, normalized size = 48.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(sin(x))*sin(2*x),x, algorithm="giac")
```

```
[Out] 2*(e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^6*tan(x)^2 - 2*e^(2*tan(1
/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^5*tan(x)^2 - e^(2*tan(1/2*x)/(tan(1/2*
x)^2 + 1))*tan(1/2*x)^6 + 8*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^
5*tan(x) - 5*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4*tan(x)^2 + 2*
e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^5 - 16*e^(2*tan(1/2*x)/(tan(
1/2*x)^2 + 1))*tan(1/2*x)^4*tan(x) + 12*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))
*tan(1/2*x)^3*tan(x)^2 + 5*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^4
```

```
 - 5*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2*tan(x)^2 - 12*e^(2*ta
n(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^3 + 16*e^(2*tan(1/2*x)/(tan(1/2*x)^
2 + 1))*tan(1/2*x)^2*tan(x) - 2*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2
*x)*tan(x)^2 + 5*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - 8*e^(2*
tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x)*tan(x) + e^(2*tan(1/2*x)/(tan(1/2
*x)^2 + 1))*tan(x)^2 + 2*e^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1))*tan(1/2*x) - e
^(2*tan(1/2*x)/(tan(1/2*x)^2 + 1)))/(tan(1/2*x)^6*tan(x)^2 + tan(1/2*x)^6 +
 3*tan(1/2*x)^4*tan(x)^2 + 3*tan(1/2*x)^4 + 3*tan(1/2*x)^2*tan(x)^2 + 3*tan
(1/2*x)^2 + tan(x)^2 + 1)
```

3.369 $\int x^2\sqrt{5-x^2}\,dx$

Optimal. Leaf size=47

$$\frac{1}{4}\sqrt{5-x^2}x^3-\frac{5}{8}\sqrt{5-x^2}x+\frac{25}{8}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

```
[Out] (-5*x*Sqrt[5 - x^2])/8 + (x^3*Sqrt[5 - x^2])/4 + (25*ArcSin[x/Sqrt[5]])/8
```

Rubi [A] time = 0.0098578, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {279, 321, 216}

$$\frac{1}{4}\sqrt{5-x^2}x^3-\frac{5}{8}\sqrt{5-x^2}x+\frac{25}{8}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[5 - x^2],x]
```

```
[Out] (-5*x*Sqrt[5 - x^2])/8 + (x^3*Sqrt[5 - x^2])/4 + (25*ArcSin[x/Sqrt[5]])/8
```

Rule 279

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p +
1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
 p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int x^2\sqrt{5-x^2}\,dx = \frac{1}{4}x^3\sqrt{5-x^2}+\frac{5}{4}\int\frac{x^2}{\sqrt{5-x^2}}\,dx$$
$$= -\frac{5}{8}x\sqrt{5-x^2}+\frac{1}{4}x^3\sqrt{5-x^2}+\frac{25}{8}\int\frac{1}{\sqrt{5-x^2}}\,dx$$
$$= -\frac{5}{8}x\sqrt{5-x^2}+\frac{1}{4}x^3\sqrt{5-x^2}+\frac{25}{8}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Mathematica [A] time = 0.017908, size = 35, normalized size = 0.74

$$\frac{1}{8}\left(x\sqrt{5-x^2}\left(2x^2-5\right)+25\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[5 - x^2],x]
```

```
[Out] (x*Sqrt[5 - x^2]*(-5 + 2*x^2) + 25*ArcSin[x/Sqrt[5]])/8
```

Maple [A] time = 0.005, size = 35, normalized size = 0.7

$$-\frac{x}{4}\left(-x^2+5\right)^{\frac{3}{2}}+\frac{5\,x}{8}\sqrt{-x^2+5}+\frac{25}{8}\arcsin\left(\frac{x\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+5)^(1/2),x)
```

```
[Out] -1/4*x*(-x^2+5)^(3/2)+5/8*x*(-x^2+5)^(1/2)+25/8*arcsin(1/5*x*5^(1/2))
```

Maxima [A] time = 1.40881, size = 46, normalized size = 0.98

$$-\frac{1}{4}\left(-x^2+5\right)^{\frac{3}{2}}x+\frac{5}{8}\sqrt{-x^2+5}x+\frac{25}{8}\arcsin\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+5)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(-x^2 + 5)^(3/2)*x + 5/8*sqrt(-x^2 + 5)*x + 25/8*arcsin(1/5*sqrt(5)*x)
```

Fricas [A] time = 2.02214, size = 89, normalized size = 1.89

$$\frac{1}{8}\left(2\,x^{3}-5\,x\right)\sqrt{-x^{2}+5}-\frac{25}{8}\,\arctan\left(\frac{\sqrt{-x^{2}+5}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^3 - 5*x)*sqrt(-x^2 + 5) - 25/8*arctan(sqrt(-x^2 + 5)/x)
```

Sympy [A] time = 2.63583, size = 122, normalized size = 2.6

$$\begin{cases} \frac{ix^{5}}{4\sqrt{x^{2}-5}}-\frac{15ix^{3}}{8\sqrt{x^{2}-5}}+\frac{25ix}{8\sqrt{x^{2}-5}}-\frac{25i\operatorname{acosh}\left(\frac{\sqrt{5}x}{5}\right)}{8} & \text{for}\ \frac{\left|x^{2}\right|}{5}>1 \\ -\frac{x^{5}}{4\sqrt{5-x^{2}}}+\frac{15x^{3}}{8\sqrt{5-x^{2}}}-\frac{25x}{8\sqrt{5-x^{2}}}+\frac{25\operatorname{asin}\left(\frac{\sqrt{5}x}{5}\right)}{8} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+5)**(1/2),x)
```

```
[Out] Piecewise((I*x**5/(4*sqrt(x**2 - 5)) - 15*I*x**3/(8*sqrt(x**2 - 5)) + 25*I*
x/(8*sqrt(x**2 - 5)) - 25*I*acosh(sqrt(5)*x/5)/8, Abs(x**2)/5 > 1), (-x**5/
(4*sqrt(5 - x**2)) + 15*x**3/(8*sqrt(5 - x**2)) - 25*x/(8*sqrt(5 - x**2)) +
 25*asin(sqrt(5)*x/5)/8, True))
```

Giac [A] time = 1.06833, size = 39, normalized size = 0.83

$$\frac{1}{8}\left(2\,x^{2}-5\right)\sqrt{-x^{2}+5}x+\frac{25}{8}\,\arcsin\left(\frac{1}{5}\,\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*x^2 - 5)*sqrt(-x^2 + 5)*x + 25/8*arcsin(1/5*sqrt(5)*x)
```

3.370 $\int x^2 \left(1+x^3\right)^4 \, dx$

Optimal. Leaf size=11

$$\frac{1}{15}\left(x^3+1\right)^5$$

[Out] (1 + x^3)^5/15

Rubi [A] time = 0.0017565, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{1}{15}\left(x^3+1\right)^5$$

Antiderivative was successfully verified.

[In] Int[x^2*(1 + x^3)^4,x]

[Out] (1 + x^3)^5/15

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \left(1+x^3\right)^4 \, dx = \frac{1}{15}\left(1+x^3\right)^5$$

Mathematica [B] time = 0.0014124, size = 36, normalized size = 3.27

$$\frac{x^{15}}{15}+\frac{x^{12}}{3}+\frac{2x^9}{3}+\frac{2x^6}{3}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 + x^3)^4,x]
```

```
[Out] x^3/3 + (2*x^6)/3 + (2*x^9)/3 + x^12/3 + x^15/15
```

Maple [B] time = 0.001, size = 27, normalized size = 2.5

$$\frac{x^{15}}{15} + \frac{x^{12}}{3} + \frac{2\,x^{9}}{3} + \frac{2\,x^{6}}{3} + \frac{x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^3+1)^4,x)
```

```
[Out] 1/15*x^15+1/3*x^12+2/3*x^9+2/3*x^6+1/3*x^3
```

Maxima [A] time = 0.929509, size = 12, normalized size = 1.09

$$\frac{1}{15}\left(x^{3}+1\right)^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^4,x, algorithm="maxima")
```

```
[Out] 1/15*(x^3 + 1)^5
```

Fricas [B] time = 1.56919, size = 70, normalized size = 6.36

$$\frac{1}{15}x^{15} + \frac{1}{3}x^{12} + \frac{2}{3}x^{9} + \frac{2}{3}x^{6} + \frac{1}{3}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^4,x, algorithm="fricas")
```

```
[Out] 1/15*x^15 + 1/3*x^12 + 2/3*x^9 + 2/3*x^6 + 1/3*x^3
```

Sympy [B] time = 0.050945, size = 27, normalized size = 2.45

$$\frac{x^{15}}{15} + \frac{x^{12}}{3} + \frac{2x^9}{3} + \frac{2x^6}{3} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**3+1)**4,x)
```

```
[Out] x**15/15 + x**12/3 + 2*x**9/3 + 2*x**6/3 + x**3/3
```

Giac [A] time = 1.05947, size = 12, normalized size = 1.09

$$\frac{1}{15}\left(x^3+1\right)^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^3+1)^4,x, algorithm="giac")
```

```
[Out] 1/15*(x^3 + 1)^5
```

3.371 $\int \cos^3(x)\sin^3(x)\,dx$

Optimal. Leaf size=17

$$\frac{\sin^4(x)}{4}-\frac{\sin^6(x)}{6}$$

```
[Out] Sin[x]^4/4 - Sin[x]^6/6
```

Rubi [A] time = 0.0222371, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2564, 14}

$$\frac{\sin^4(x)}{4}-\frac{\sin^6(x)}{6}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3*Sin[x]^3,x]
```

```
[Out] Sin[x]^4/4 - Sin[x]^6/6
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\sin^3(x)\,dx &= \text{Subst}\left(\int x^3\left(1-x^2\right)\,dx,x,\sin(x)\right)\\ &= \text{Subst}\left(\int \left(x^3-x^5\right)\,dx,x,\sin(x)\right)\\ &= \frac{\sin^4(x)}{4}-\frac{\sin^6(x)}{6}\end{aligned}$$

Mathematica [A] time = 0.006301, size = 17, normalized size = 1.

$$\frac{1}{192}\cos(6x)-\frac{3}{64}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3*Sin[x]^3,x]
```

```
[Out] (-3*Cos[2*x])/64 + Cos[6*x]/192
```

Maple [A] time = 0.006, size = 18, normalized size = 1.1

$$-\frac{(\cos(x))^4(\sin(x))^2}{6}-\frac{(\cos(x))^4}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^3,x)
```

```
[Out] -1/6*cos(x)^4*sin(x)^2-1/12*cos(x)^4
```

Maxima [A] time = 0.923391, size = 18, normalized size = 1.06

$$-\frac{1}{6}\sin(x)^6+\frac{1}{4}\sin(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3,x, algorithm="maxima")
```

```
[Out] -1/6*sin(x)^6 + 1/4*sin(x)^4
```

Fricas [A] time = 2.00521, size = 39, normalized size = 2.29

$$\frac{1}{6}\cos(x)^6 - \frac{1}{4}\cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/6*cos(x)^6 - 1/4*cos(x)^4
```

Sympy [A] time = 0.058797, size = 12, normalized size = 0.71

$$-\frac{\sin^6(x)}{6} + \frac{\sin^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**3,x)
```

```
[Out] -sin(x)**6/6 + sin(x)**4/4
```

Giac [A] time = 1.05459, size = 18, normalized size = 1.06

$$\frac{1}{6}\cos(x)^6 - \frac{1}{4}\cos(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/6*cos(x)^6 - 1/4*cos(x)^4
```

3.372 $\int \sec^4(x)\tan^2(x)\,dx$

Optimal. Leaf size=17

$$\frac{\tan^5(x)}{5}+\frac{\tan^3(x)}{3}$$

```
[Out] Tan[x]^3/3 + Tan[x]^5/5
```

Rubi [A] time = 0.0232369, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2607, 14}

$$\frac{\tan^5(x)}{5}+\frac{\tan^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4*Tan[x]^2,x]
```

```
[Out] Tan[x]^3/3 + Tan[x]^5/5
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec^4(x)\tan^2(x)\,dx = \text{Subst}\left(\int x^2\left(1+x^2\right)\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(x^2+x^4\right)\,dx, x, \tan(x)\right)$$
$$= \frac{\tan^3(x)}{3} + \frac{\tan^5(x)}{5}$$

Mathematica [A] time = 0.0157197, size = 27, normalized size = 1.59

$$-\frac{2\tan(x)}{15} + \frac{1}{5}\tan(x)\sec^4(x) - \frac{1}{15}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4*Tan[x]^2,x]
```

```
[Out] (-2*Tan[x])/15 - (Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5
```

Maple [A] time = 0., size = 22, normalized size = 1.3

$$\frac{(\sin(x))^3}{5\,(\cos(x))^5} + \frac{2\,(\sin(x))^3}{15\,(\cos(x))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4*tan(x)^2,x)
```

```
[Out] 1/5*sin(x)^3/cos(x)^5+2/15*sin(x)^3/cos(x)^3
```

Maxima [A] time = 0.923833, size = 18, normalized size = 1.06

$$\frac{1}{5}\,\tan(x)^5 + \frac{1}{3}\,\tan(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

Fricas [A] time = 2.22829, size = 69, normalized size = 4.06

$$-\frac{\left(2\,\cos\left(x\right)^{4}+\cos\left(x\right)^{2}-3\right)\sin\left(x\right)}{15\,\cos\left(x\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(x)^4 + cos(x)^2 - 3)*sin(x)/cos(x)^5
```

Sympy [B] time = 0.06336, size = 29, normalized size = 1.71

$$-\frac{2\sin\left(x\right)}{15\cos\left(x\right)}-\frac{\sin\left(x\right)}{15\cos^{3}\left(x\right)}+\frac{\sin\left(x\right)}{5\cos^{5}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4*tan(x)**2,x)
```

```
[Out] -2*sin(x)/(15*cos(x)) - sin(x)/(15*cos(x)**3) + sin(x)/(5*cos(x)**5)
```

Giac [A] time = 1.06102, size = 18, normalized size = 1.06

$$\frac{1}{5}\,\tan\left(x\right)^{5}+\frac{1}{3}\,\tan\left(x\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4*tan(x)^2,x, algorithm="giac")
```

```
[Out] 1/5*tan(x)^5 + 1/3*tan(x)^3
```

3.373 $\int x\sqrt{1+2x}\,dx$

Optimal. Leaf size=27

$$\frac{1}{10}(2x+1)^{5/2}-\frac{1}{6}(2x+1)^{3/2}$$

```
[Out] -(1 + 2*x)^(3/2)/6 + (1 + 2*x)^(5/2)/10
```

Rubi [A] time = 0.0048999, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{1}{10}(2x+1)^{5/2}-\frac{1}{6}(2x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + 2*x],x]
```

```
[Out] -(1 + 2*x)^(3/2)/6 + (1 + 2*x)^(5/2)/10
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{1+2x}\,dx &= \int\left(-\frac{1}{2}\sqrt{1+2x}+\frac{1}{2}(1+2x)^{3/2}\right)dx\\ &= -\frac{1}{6}(1+2x)^{3/2}+\frac{1}{10}(1+2x)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0057128, size = 18, normalized size = 0.67

$$\frac{1}{15}(2x+1)^{3/2}(3x-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + 2*x],x]
```

```
[Out] ((1 + 2*x)^(3/2)*(-1 + 3*x))/15
```

Maple [A] time = 0.002, size = 15, normalized size = 0.6

$$\frac{3x-1}{15}(1+2x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+2*x)^(1/2),x)
```

```
[Out] 1/15*(1+2*x)^(3/2)*(3*x-1)
```

Maxima [A] time = 0.924912, size = 26, normalized size = 0.96

$$\frac{1}{10}(2x+1)^{\frac{5}{2}} - \frac{1}{6}(2x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/10*(2*x + 1)^(5/2) - 1/6*(2*x + 1)^(3/2)
```

Fricas [A] time = 2.09458, size = 49, normalized size = 1.81

$$\frac{1}{15}\left(6x^2 + x - 1\right)\sqrt{2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(6*x^2 + x - 1)*sqrt(2*x + 1)
```

Sympy [A] time = 0.844147, size = 36, normalized size = 1.33

$$\frac{2x^2\sqrt{2x+1}}{5}+\frac{x\sqrt{2x+1}}{15}-\frac{\sqrt{2x+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)**(1/2),x)
```

```
[Out] 2*x**2*sqrt(2*x + 1)/5 + x*sqrt(2*x + 1)/15 - sqrt(2*x + 1)/15
```

Giac [A] time = 1.05702, size = 26, normalized size = 0.96

$$\frac{1}{10}\,(2\,x+1)^{\frac{5}{2}}-\frac{1}{6}\,(2\,x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/10*(2*x + 1)^(5/2) - 1/6*(2*x + 1)^(3/2)
```

3.374 $\int \sin^4(x)\,dx$

Optimal. Leaf size=24

$$\frac{3x}{8}-\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{8}\sin(x)\cos(x)$$

```
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rubi [A] time = 0.009457, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{3x}{8}-\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^4(x)\,dx &= -\frac{1}{4}\cos(x)\sin^3(x)+\frac{3}{4}\int \sin^2(x)\,dx\\ &= -\frac{3}{8}\cos(x)\sin(x)-\frac{1}{4}\cos(x)\sin^3(x)+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}-\frac{3}{8}\cos(x)\sin(x)-\frac{1}{4}\cos(x)\sin^3(x)\end{aligned}$$

Mathematica [A] time = 0.0016437, size = 22, normalized size = 0.92

$$\frac{3x}{8}-\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32
```

Maple [A] time = 0.004, size = 18, normalized size = 0.8

$$-\frac{\cos(x)}{4}\left((\sin(x))^{3}+\frac{3\sin(x)}{2}\right)+\frac{3x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4,x)
```

```
[Out] -1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x
```

Maxima [A] time = 0.919824, size = 22, normalized size = 0.92

$$\frac{3}{8}x+\frac{1}{32}\sin(4x)-\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

Fricas [A] time = 1.92061, size = 59, normalized size = 2.46

$$\frac{1}{8}\left(2\cos(x)^{3}-5\cos(x)\right)\sin(x)+\frac{3}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(2*cos(x)^3 - 5*cos(x))*sin(x) + 3/8*x
```

Sympy [A] time = 0.056392, size = 24, normalized size = 1.

$$\frac{3x}{8} - \frac{\sin^3(x)\cos(x)}{4} - \frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4,x)
```

```
[Out] 3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

Giac [A] time = 1.06319, size = 22, normalized size = 0.92

$$\frac{3}{8}x + \frac{1}{32}\sin(4x) - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

3.375 $\int \tan^3(x)\,dx$

Optimal. Leaf size=12

$$\frac{\tan^2(x)}{2}+\log(\cos(x))$$

```
[Out] Log[Cos[x]] + Tan[x]^2/2
```

Rubi [A] time = 0.0063899, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 3475}

$$\frac{\tan^2(x)}{2}+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3,x]
```

```
[Out] Log[Cos[x]] + Tan[x]^2/2
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^3(x)\,dx&=\frac{\tan^2(x)}{2}-\int \tan(x)\,dx\\&=\log(\cos(x))+\frac{\tan^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0029327, size = 12, normalized size = 1.

$$\frac{\sec^2(x)}{2}+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3,x]
```

```
[Out] Log[Cos[x]] + Sec[x]^2/2
```

Maple [A] time = 0.001, size = 17, normalized size = 1.4

$$\frac{(\tan(x))^2}{2}-\frac{\ln\left((\tan(x))^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3,x)
```

```
[Out] 1/2*tan(x)^2-1/2*ln(tan(x)^2+1)
```

Maxima [A] time = 0.924025, size = 27, normalized size = 2.25

$$-\frac{1}{2\left(\sin(x)^2-1\right)}+\frac{1}{2}\log\left(\sin(x)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(sin(x)^2 - 1) + 1/2*log(sin(x)^2 - 1)
```

Fricas [A] time = 2.07571, size = 57, normalized size = 4.75

$$\frac{1}{2}\tan(x)^2+\frac{1}{2}\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*tan(x)^2 + 1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] time = 0.086282, size = 12, normalized size = 1.

$$\log(\cos(x)) + \frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3,x)
```

```
[Out] log(cos(x)) + 1/(2*cos(x)**2)
```

Giac [A] time = 1.07701, size = 22, normalized size = 1.83

$$\frac{1}{2}\tan(x)^2 - \frac{1}{2}\log\left(\tan(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3,x, algorithm="giac")
```

```
[Out] 1/2*tan(x)^2 - 1/2*log(tan(x)^2 + 1)
```

3.376 $\int x^5\sqrt{1+x^2}\,dx$

Optimal. Leaf size=40

$$\frac{1}{7}\left(x^2+1\right)^{7/2}-\frac{2}{5}\left(x^2+1\right)^{5/2}+\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] (1 + x^2)^(3/2)/3 - (2*(1 + x^2)^(5/2))/5 + (1 + x^2)^(7/2)/7
```

Rubi [A] time = 0.0135769, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{7}\left(x^2+1\right)^{7/2}-\frac{2}{5}\left(x^2+1\right)^{5/2}+\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3 - (2*(1 + x^2)^(5/2))/5 + (1 + x^2)^(7/2)/7
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^5\sqrt{1+x^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2\sqrt{1+x}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int\left(\sqrt{1+x}-2(1+x)^{3/2}+(1+x)^{5/2}\right)dx,x,x^2\right)\\ &= \frac{1}{3}\left(1+x^2\right)^{3/2}-\frac{2}{5}\left(1+x^2\right)^{5/2}+\frac{1}{7}\left(1+x^2\right)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0074132, size = 25, normalized size = 0.62

$$\frac{1}{105}\left(x^2+1\right)^{3/2}\left(15x^4-12x^2+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[1 + x^2],x]
```

```
[Out] ((1 + x^2)^(3/2)*(8 - 12*x^2 + 15*x^4))/105
```

Maple [A] time = 0.005, size = 22, normalized size = 0.6

$$\frac{15\,x^4-12\,x^2+8}{105}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^2+1)^(1/2),x)
```

```
[Out] 1/105*(x^2+1)^(3/2)*(15*x^4-12*x^2+8)
```

Maxima [A] time = 1.40403, size = 46, normalized size = 1.15

$$\frac{1}{7}\left(x^2+1\right)^{\frac{3}{2}}x^4-\frac{4}{35}\left(x^2+1\right)^{\frac{3}{2}}x^2+\frac{8}{105}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/7*(x^2 + 1)^(3/2)*x^4 - 4/35*(x^2 + 1)^(3/2)*x^2 + 8/105*(x^2 + 1)^(3/2)
```

Fricas [A] time = 1.82029, size = 68, normalized size = 1.7

$$\frac{1}{105}\left(15\,x^{6}+3\,x^{4}-4\,x^{2}+8\right)\sqrt{x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*(15*x^6 + 3*x^4 - 4*x^2 + 8)*sqrt(x^2 + 1)
```

Sympy [A] time = 2.02928, size = 53, normalized size = 1.32

$$\frac{x^{6}\sqrt{x^{2}+1}}{7}+\frac{x^{4}\sqrt{x^{2}+1}}{35}-\frac{4x^{2}\sqrt{x^{2}+1}}{105}+\frac{8\sqrt{x^{2}+1}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**2+1)**(1/2),x)
```

```
[Out] x**6*sqrt(x**2 + 1)/7 + x**4*sqrt(x**2 + 1)/35 - 4*x**2*sqrt(x**2 + 1)/105
+ 8*sqrt(x**2 + 1)/105
```

Giac [A] time = 1.05398, size = 38, normalized size = 0.95

$$\frac{1}{7}\left(x^{2}+1\right)^{\frac{7}{2}}-\frac{2}{5}\left(x^{2}+1\right)^{\frac{5}{2}}+\frac{1}{3}\left(x^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7*(x^2 + 1)^(7/2) - 2/5*(x^2 + 1)^(5/2) + 1/3*(x^2 + 1)^(3/2)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
     as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)
```

```
    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
   expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```