

Computer algebra independent integration tests

0-Independent-test-suites/Moses-Problems

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Contents

1	Introduction	3
1.1	Listing of CAS systems tested	3
1.2	Results	3
1.3	Performance	5
1.4	list of integrals that has no closed form antiderivative	6
1.5	list of integrals solved by CAS but has no known antiderivative	6
1.6	list of integrals solved by CAS but failed verification	6
1.7	Timing	7
1.8	Verification	7
1.9	Important notes about some of the results	7
1.10	Design of the test system	8
2	detailed summary tables of results	11
2.1	List of integrals sorted by grade for each CAS	11
2.2	Detailed conclusion table per each integral for all CAS systems	12
2.3	Detailed conclusion table specific for Rubi results	29
3	Listing of integrals	33
3.1	$\int \cot^4(x) dx$	33
3.2	$\int \frac{1}{x^4(1+x^2)} dx$	36
3.3	$\int \frac{x+x^2}{\sqrt{x}} dx$	39
3.4	$\int \cos(x) dx$	41
3.5	$\int e^{x^2} x dx$	43
3.6	$\int \sec^2(x) \tan(x) dx$	45
3.7	$\int x\sqrt{1+x^2} dx$	48
3.8	$\int e^x \sin(x) dx$	50
3.9	$\int \cot(x) \csc^3(x) dx$	52
3.10	$\int \sin(e^x) dx$	55
3.11	$\int \frac{\sin(y)}{y} dy$	57
3.12	$\int (e^x + \sin(x)) dx$	59
3.13	$\int (e^{x^2} + 2e^{x^2} x^2) dx$	61
3.14	$\int (e^x + x)^2 dx$	64
3.15	$\int (2e^x + e^{2x} + x^2) dx$	67
3.16	$\int \cos(x) \sin(x) dx$	69

3.17	$\int e^{x^2} x dx$	72
3.18	$\int x\sqrt{1+x^2} dx$	74
3.19	$\int \frac{e^x}{1+e^x} dx$	76
3.20	$\int x^{3/2} dx$	78
3.21	$\int \cos(3+2x) dx$	80
3.22	$\int 2e^{2x} yz dx$	82
3.23	$\int e^x \cos^2(e^x) \sin(e^x) dx$	84
3.24	$\int x\sqrt{1+x} dx$	87
3.25	$\int \frac{1}{-1+x^4} dx$	89
3.26	$\int \frac{e^x}{2+3e^{2x}} dx$	92
3.27	$\int \frac{e^{2x}}{A+Be^{4x}} dx$	95
3.28	$\int \frac{e^{1+x}}{1+e^x} dx$	98
3.29	$\int (10e)^x dx$	101
3.30	$\int x^3 \sin(x^2) dx$	103
3.31	$\int \frac{x^7}{1+x^{12}} dx$	106
3.32	$\int x^{3a} \sin(x^{2a}) dx$	109
3.33	$\int \cos(\sqrt{x}) dx$	112
3.34	$\int x\sqrt{1+x} dx$	115
3.35	$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}} dx$	117
3.36	$\int \sqrt{\frac{1+x}{3+2x}} dx$	120
3.37	$\int \frac{x^4}{(1-x^2)^{5/2}} dx$	123
3.38	$\int \sqrt{x}(1+x)^{5/2} dx$	126
3.39	$\int \frac{x^4}{(1-x^2)^{5/2}} dx$	129
3.40	$\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2} dy$	132
3.41	$\int \sin^2(x) dx$	135
3.42	$\int \csc(x) \sqrt{A^2+B^2 \sin^2(x)} dx$	138
3.43	$\int \frac{1}{1+\cos(x)} dx$	141
3.44	$\int e^x x dx$	143
3.45	$\int \frac{e^x x}{(1+x)^2} dx$	145
3.46	$\int e^{x^2} (1+2x^2) dx$	147
3.47	$\int e^{x^2} dx$	150
3.48	$\int \frac{e^x}{x} dx$	152
3.49	$\int \frac{x}{1+x^3} dx$	154
3.50	$\int \frac{1}{-1+x^6} dx$	157
3.51	$\int \frac{1}{A^4-A^2B^2+(-A^2+B^2)x^2} dx$	160
3.52	$\int x \log(x) dx$	163
3.53	$\int x^2 \sin^{-1}(x) dx$	165
3.54	$\int \frac{1}{1+2x+x^2} dx$	168
3.55	$\int \frac{\log(x)}{(1+\log(x))^2} dx$	170
3.56	$\int \frac{1}{x(1+\log^2(x))} dx$	173
3.57	$\int \frac{1}{\log(x)} dx$	175
3.58	$\int x(\cos(x) + \sin(x)) dx$	177
3.59	$\int e^{-x} (e^x + x) dx$	180
3.60	$\int (1+e^x)^2 x dx$	183
3.61	$\int x \cos(x) dx$	186

3.62	$\int \cos(\sqrt{x}) dx$	188
3.63	$\int x \cos(x) dx$	191
3.64	$\int x \log^2(x) dx$	193
3.65	$\int \cos(x) (1 + \sin^3(x)) dx$	196
3.66	$\int \frac{1}{x(1+\log^2(x))} dx$	198
3.67	$\int \frac{1}{\sqrt{1-x^2}(1+\sin^{-1}(x^2))} dx$	200
3.68	$\int \frac{\sin(x)}{\cos(x)+\sin(x)} dx$	202
3.69	$\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2} dy$	205
3.70	$\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)} dz$	208
3.71	$\int -\frac{A^2+B^2}{B(1+w^2)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)} dw$	211
3.72	$\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)} dw$	214
3.73	$\int \frac{x^4}{(1-x^2)^{5/2}} dx$	217
3.74	$\int \tan^4(y) dy$	220
3.75	$\int \frac{z^4}{1+z^2} dz$	223
3.76	$\int e^{x^2} (1 + 2x^2) dx$	226
3.77	$\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2} dx$	229
3.78	$\int e^{-1-x} dx$	232
3.79	$\int \left(\frac{1}{x} + x\right) \log(x) dx$	234
3.80	$\int \frac{x}{1+x^4} dx$	237
3.81	$\int \frac{x^5}{1+x^4} dx$	240
3.82	$\int \frac{1}{1+\tan^2(x)} dx$	243
3.83	$\int \frac{x^4}{(1-x^2)^{5/2}} dx$	246
3.84	$\int -\frac{x^2}{(1-x^2)^{3/2}} dx$	249
3.85	$\int e^x \sin(x) dx$	252
3.86	$\int \frac{1}{x} dx$	254
3.87	$\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)} dt$	256
3.88	$\int \cos^2(x) dx$	259
3.89	$\int \frac{1+x^2}{\sqrt{x}} dx$	262
3.90	$\int \frac{x}{\sqrt{5+2x+x^2}} dx$	264
3.91	$\int \cos(x) \sin^2(x) dx$	267
3.92	$\int \frac{e^x}{1+e^x} dx$	270
3.93	$\int \frac{e^{2x}}{1+e^x} dx$	272
3.94	$\int \frac{1}{1-\cos(x)} dx$	275
3.95	$\int \sec^2(x) \tan(x) dx$	277
3.96	$\int x \log(x) dx$	280
3.97	$\int \cos(x) \sin(x) dx$	282
3.98	$\int \frac{1+x}{\sqrt{2x-x^2}} dx$	285
3.99	$\int \frac{2e^x}{2+3e^{2x}} dx$	288
3.100	$\int \frac{x^4}{(1-x^2)^{5/2}} dx$	291

3.101	$\int \frac{e^{6x}}{1+e^{4x}} dx$	294
3.102	$\int \log(2 + 3x^2) dx$	297
3.103	$\int \frac{1}{r\sqrt{-a^2+2Hr^2}} dx$	300
3.104	$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}} dx$	302
3.105	$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}} dx$	304
3.106	$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}} dx$	306
3.107	$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}} dx$	308
3.108	$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}} dx$	310
3.109	$\int \frac{1}{\sqrt{-a^2+2er^2}} dx$	312
3.110	$\int \frac{1}{\sqrt{-a^2-e^2+2er^2}} dx$	314
3.111	$\int \frac{1}{\sqrt{-a^2+2er^2-2Kr^4}} dx$	316
3.112	$\int \frac{1}{\sqrt{-a^2-e^2+2er^2-2Kr^4}} dx$	318
3.113	$\int \frac{1}{\sqrt{-a^2-e^2-2Kr+2Hr^2}} dx$	320

4 Listing of Grading functions

323

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [113]. This is test number [8].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (113)	% 0. (0)
Mathematica	% 99.12 (112)	% 0.88 (1)
Maple	% 100. (113)	% 0. (0)
Maxima	% 94.69 (107)	% 5.31 (6)
Fricas	% 99.12 (112)	% 0.88 (1)
Sympy	% 92.04 (104)	% 7.96 (9)
Giac	% 96.46 (109)	% 3.54 (4)

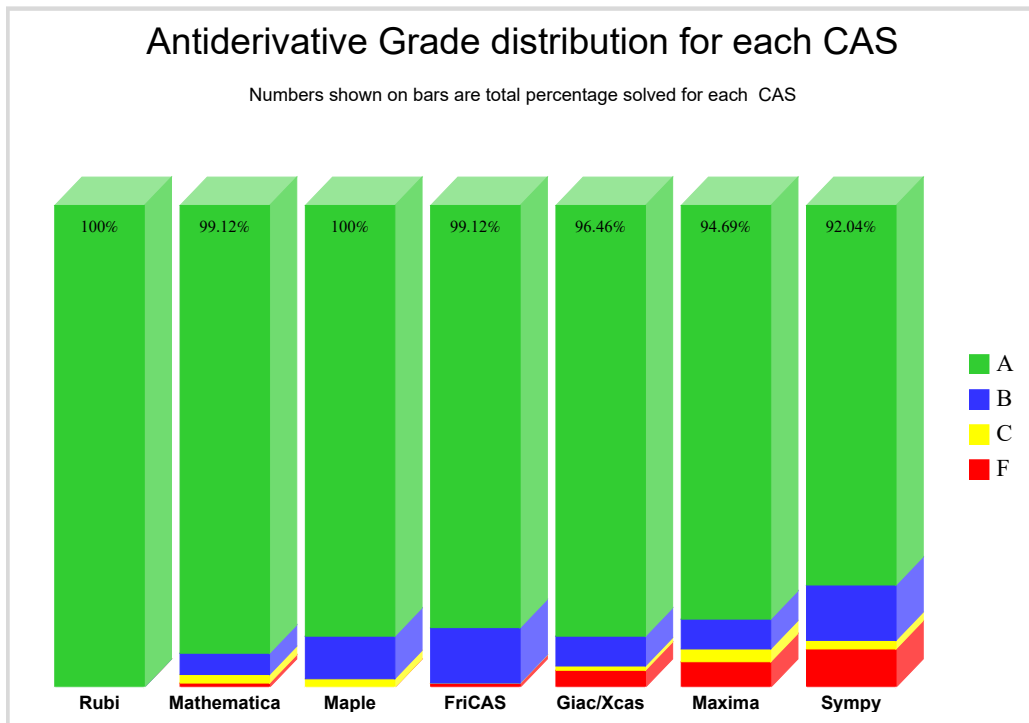
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

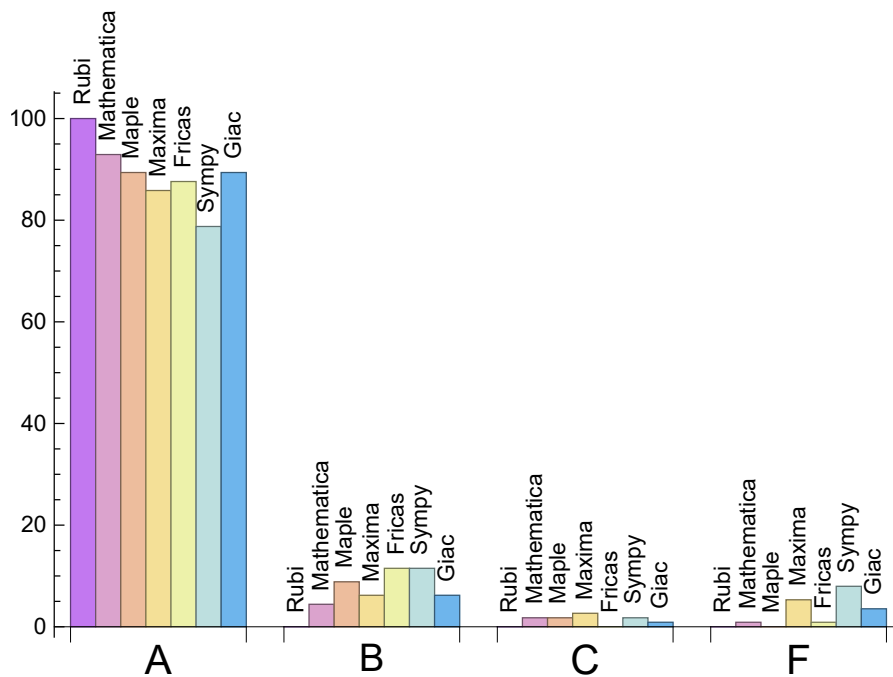
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	92.92	4.42	1.77	0.88
Maple	89.38	8.85	1.77	0.
Maxima	85.84	6.19	2.65	5.31
Fricas	87.61	11.5	0.	0.88
Sympy	78.76	11.5	1.77	7.96
Giac	89.38	6.19	0.88	3.54

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.02	20.14	1.01	16.	1.
Mathematica	0.01	24.32	1.11	16.	1.
Maple	0.01	25.83	1.28	14.	0.92
Maxima	1.09	28.51	1.54	18.	1.17
Fricas	1.94	66.79	3.59	46.	2.74
Sympy	0.49	29.38	1.59	15.	0.82
Giac	1.08	24.75	1.42	18.	1.14

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```

from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')

```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in->

```

def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

```

```

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1

```

For Sympy, called directly from Python, the following code is used

```

try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1

```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 31, 42, 70, 71, 72 }

C grade: { 40, 69 }

F grade: { 57 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 37, 38, 39, 41, 43, 44, 45, 46, 47, 49, 50, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 35, 36, 40, 48, 51, 57, 69, 70, 71, 72 }

C grade: { 32, 42 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 33, 34, 35, 37, 39, 41, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 36, 38, 68, 70, 71, 72, 87 }

C grade: { 10, 11, 47 }

F grade: { 27, 32, 40, 42, 67, 69 }

2.1.5 FriCAS

A grade: { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 38, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 1, 9, 37, 39, 40, 42, 69, 70, 73, 74, 83, 84, 100 }

C grade: { }

F grade: { 32 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 41, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 74, 75, 76, 77, 78, 79, 80, 81, 85, 86, 88, 89, 91, 92, 93, 94, 95, 96, 97, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 7, 18, 37, 39, 50, 51, 56, 66, 73, 82, 83, 84, 100 }

C grade: { 71, 72 }

F grade: { 35, 36, 40, 42, 69, 70, 87, 90, 98 }

2.1.7 Giac

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 41, 44, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 1, 25, 43, 45, 70, 71, 72 }

C grade: { 47 }

F grade: { 32, 40, 42, 69 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	14	22	126	19	46
normalized size	1	1.	1.5	1.17	1.83	10.5	1.58	3.83
time (sec)	N/A	0.01	0.003	0.	1.414	1.655	0.066	1.063

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	20	53	14	20
normalized size	1	1.	1.	0.92	1.54	4.08	1.08	1.54
time (sec)	N/A	0.005	0.004	0.005	1.413	1.506	0.105	1.079

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	11	15	38	15	15
normalized size	1	1.	0.74	0.58	0.79	2.	0.79	0.79
time (sec)	N/A	0.003	0.003	0.001	0.936	1.5	0.223	1.09

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	11	2	3
normalized size	1	1.	1.	1.5	1.5	5.5	1.	1.5
time (sec)	N/A	0.002	0.001	0.	0.927	1.582	0.053	1.093

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	8	18	5	8
normalized size	1	1.	1.	0.78	0.89	2.	0.56	0.89
time (sec)	N/A	0.006	0.001	0.	0.927	1.633	0.079	1.079

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	19	7	8
normalized size	1	1.	1.	0.88	1.	2.38	0.88	1.
time (sec)	N/A	0.011	0.004	0.005	0.936	1.599	0.066	1.066

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	28	22	12
normalized size	1	1.	1.	0.77	0.92	2.15	1.69	0.92
time (sec)	N/A	0.002	0.002	0.002	0.925	1.568	0.19	1.085

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	14	15	46	15	15
normalized size	1	1.	0.74	0.74	0.79	2.42	0.79	0.79
time (sec)	N/A	0.007	0.014	0.	0.934	1.641	0.298	1.098

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	39	8	8
normalized size	1	1.	1.	0.88	1.	4.88	1.	1.
time (sec)	N/A	0.012	0.004	0.013	0.94	1.607	0.064	1.072

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	20	26	3	4
normalized size	1	1.	1.	1.	5.	6.5	0.75	1.
time (sec)	N/A	0.012	0.005	0.004	1.065	1.931	0.641	1.06

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	18	23	2	3
normalized size	1	1.	1.	1.5	9.	11.5	1.	1.5
time (sec)	N/A	0.011	0.013	0.001	1.055	1.86	0.54	1.117

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	9	20	5	9
normalized size	1	1.	1.	1.	1.12	2.5	0.62	1.12
time (sec)	N/A	0.003	0.003	0.002	0.946	1.82	0.072	1.07

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	8	15	5	8
normalized size	1	1.	1.	1.	1.14	2.14	0.71	1.14
time (sec)	N/A	0.02	0.005	0.002	0.935	1.886	0.08	1.07

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	22	26	53	20	26
normalized size	1	1.	0.93	0.79	0.93	1.89	0.71	0.93
time (sec)	N/A	0.02	0.014	0.002	0.928	1.802	0.087	1.098

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	42	15	22
normalized size	1	1.	1.	0.77	1.	1.91	0.68	1.
time (sec)	N/A	0.005	0.002	0.001	0.933	1.878	0.083	1.074

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	20	5	8
normalized size	1	1.	1.	0.88	1.	2.5	0.62	1.
time (sec)	N/A	0.007	0.001	0.001	0.913	1.915	0.054	1.081

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	8	18	5	8
normalized size	1	1.	1.	0.78	0.89	2.	0.56	0.89
time (sec)	N/A	0.006	0.001	0.	0.925	1.913	0.075	1.07

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	28	22	12
normalized size	1	1.	1.	0.77	0.92	2.15	1.69	0.92
time (sec)	N/A	0.002	0.	0.	0.93	1.701	0.18	1.06

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	7	19	5	7
normalized size	1	1.	1.	1.	1.17	3.17	0.83	1.17
time (sec)	N/A	0.015	0.004	0.001	0.922	1.682	0.085	1.094

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	7	18	7	7
normalized size	1	1.	1.	0.67	0.78	2.	0.78	0.78
time (sec)	N/A	0.	0.001	0.001	0.927	1.739	0.052	1.081

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	24	7	11
normalized size	1	1.	1.	0.9	1.1	2.4	0.7	1.1
time (sec)	N/A	0.003	0.004	0.005	0.925	1.88	0.112	1.101

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	9	18	7	9
normalized size	1	1.	1.	1.	1.12	2.25	0.88	1.12
time (sec)	N/A	0.003	0.001	0.002	0.933	1.862	0.06	1.071

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	19	8	9	23	8	9
normalized size	1	1.	1.9	0.8	0.9	2.3	0.8	0.9
time (sec)	N/A	0.027	0.02	0.004	0.923	1.89	2.086	1.081

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	16	13	20	46	34	20
normalized size	1	1.	0.7	0.57	0.87	2.	1.48	0.87
time (sec)	N/A	0.004	0.004	0.003	0.925	1.819	0.848	1.1

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	23	68	17	26
normalized size	1	1.	1.92	0.77	1.77	5.23	1.31	2.
time (sec)	N/A	0.003	0.004	0.	1.418	1.883	0.115	1.087

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	14	18	50	15	18
normalized size	1	1.	1.	0.78	1.	2.78	0.83	1.
time (sec)	N/A	0.02	0.007	0.004	1.412	1.949	0.107	1.093

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	20	0	186	22	26
normalized size	1	1.	1.	0.65	0.	6.	0.71	0.84
time (sec)	N/A	0.035	0.008	0.007	0.	1.932	0.151	1.085

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	30	7	11
normalized size	1	1.	1.	1.12	1.38	3.75	0.88	1.38
time (sec)	N/A	0.034	0.005	0.001	0.932	1.947	0.096	1.082

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	27	8	16
normalized size	1	1.	1.	1.08	1.33	2.25	0.67	1.33
time (sec)	N/A	0.006	0.004	0.01	0.931	1.967	0.087	1.08

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	46	15	22
normalized size	1	1.	1.	0.85	1.1	2.3	0.75	1.1
time (sec)	N/A	0.019	0.002	0.	0.933	1.99	0.548	1.073

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	260	41	54	124	46	54
normalized size	1	1.	5.31	0.84	1.1	2.53	0.94	1.1
time (sec)	N/A	0.036	0.109	0.	1.407	1.843	0.159	1.096

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	142	41	0	0	54	0
normalized size	1	1.	1.23	0.36	0.	0.	0.47	0.
time (sec)	N/A	0.056	0.293	0.089	0.	0.	5.489	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	55	20	22
normalized size	1	1.	1.	0.77	1.	2.5	0.91	1.
time (sec)	N/A	0.01	0.014	0.01	0.931	1.83	0.295	1.079

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	16	13	20	46	34	20
normalized size	1	1.	0.7	0.57	0.87	2.	1.48	0.87
time (sec)	N/A	0.004	0.003	0.	0.927	1.79	0.84	1.092

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	92	32	76	0	32
normalized size	1	1.	1.	2.88	1.	2.38	0.	1.
time (sec)	N/A	0.014	0.013	0.	0.93	1.731	0.	1.076

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	71	75	108	151	0	82
normalized size	1	1.	1.61	1.7	2.45	3.43	0.	1.86
time (sec)	N/A	0.016	0.036	0.01	1.406	1.831	0.	1.105

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	59	146	105	39
normalized size	1	1.	0.74	0.86	1.69	4.17	3.	1.11
time (sec)	N/A	0.009	0.024	0.004	1.425	1.835	2.606	1.083

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	41	70	153	140	190	111
normalized size	1	1.	0.55	0.93	2.04	1.87	2.53	1.48
time (sec)	N/A	0.013	0.019	0.003	0.943	1.796	11.904	1.278

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	59	146	105	39
normalized size	1	1.	0.74	0.86	1.69	4.17	3.	1.11
time (sec)	N/A	0.008	0.004	0.	1.419	1.87	2.568	1.099

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	134	262	0	285	0	0
normalized size	1	1.	2.63	5.14	0.	5.59	0.	0.
time (sec)	N/A	0.029	0.029	0.022	0.	2.025	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	38	10	14
normalized size	1	1.	1.	0.79	1.	2.71	0.71	1.
time (sec)	N/A	0.006	0.002	0.	0.925	1.985	0.057	1.137

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	57	99	149	0	635	0	0
normalized size	1	1.16	2.02	3.04	0.	12.96	0.	0.
time (sec)	N/A	0.085	0.106	0.096	0.	2.998	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	12	28	3	41
normalized size	1	1.	0.67	0.56	1.33	3.11	0.33	4.56
time (sec)	N/A	0.008	0.004	0.	0.918	1.945	0.179	1.109

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	7	7	8	18	5	8
normalized size	1	1.	0.64	0.64	0.73	1.64	0.45	0.73
time (sec)	N/A	0.007	0.001	0.	0.929	1.931	0.074	1.092

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	11	18	5	26
normalized size	1	1.	1.	1.	1.22	2.	0.56	2.89
time (sec)	N/A	0.026	0.023	0.002	0.942	1.746	0.085	1.104

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	8	15	5	8
normalized size	1	1.	1.	1.	1.14	2.14	0.71	1.14
time (sec)	N/A	0.032	0.005	0.001	0.938	1.975	0.081	1.067

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	12	30	8	12
normalized size	1	1.	1.	0.73	1.09	2.73	0.73	1.09
time (sec)	N/A	0.002	0.002	0.	0.927	2.029	0.231	1.085

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	8	3	9	2	3
normalized size	1	1.	1.	4.	1.5	4.5	1.	1.5
time (sec)	N/A	0.01	0.004	0.002	1.013	1.869	0.736	1.076

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	40	35	46	112	41	47
normalized size	1	1.	0.98	0.85	1.12	2.73	1.	1.15
time (sec)	N/A	0.022	0.009	0.	1.405	1.981	0.122	1.072

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	73	75	66	88	230	83	90
normalized size	1	1.55	1.6	1.4	1.87	4.89	1.77	1.91
time (sec)	N/A	0.105	0.013	0.004	1.417	1.842	0.233	1.069

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	44	53	62	70	55
normalized size	1	1.	1.	2.1	2.52	2.95	3.33	2.62
time (sec)	N/A	0.009	0.004	0.005	0.931	1.813	0.271	1.076

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	35	12	18
normalized size	1	1.	1.	0.82	1.06	2.06	0.71	1.06
time (sec)	N/A	0.004	0.001	0.	0.924	2.002	0.08	1.091

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	29	34	45	68	32	51
normalized size	1	1.	0.72	0.85	1.12	1.7	0.8	1.27
time (sec)	N/A	0.024	0.012	0.	1.402	2.045	0.321	1.075

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	16	5	9
normalized size	1	1.	1.	1.14	1.29	2.29	0.71	1.29
time (sec)	N/A	0.001	0.001	0.002	0.925	2.021	0.071	1.084

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	22	5	11
normalized size	1	1.	1.	1.12	1.38	2.75	0.62	1.38
time (sec)	N/A	0.044	0.013	0.023	0.931	1.956	0.082	1.076

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	22	15	4
normalized size	1	1.	1.	1.33	1.33	7.33	5.	1.33
time (sec)	N/A	0.02	0.011	0.002	1.405	1.924	0.123	1.082

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	A	A	A	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	2	2	0	9	4	23	2	4
normalized size	1	1.	0.	4.5	2.	11.5	1.	2.
time (sec)	N/A	0.002	0.002	0.	1.022	1.801	0.439	1.12

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	46	15	19
normalized size	1	1.	1.	1.07	1.36	3.29	1.07	1.36
time (sec)	N/A	0.021	0.005	0.011	0.938	1.856	0.166	1.069

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	13	16	15	32	8	15
normalized size	1	1.	0.76	0.94	0.88	1.88	0.47	0.88
time (sec)	N/A	0.031	0.01	0.001	0.932	1.878	0.081	1.099

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	29	29	32	66	26	32
normalized size	1	1.	0.76	0.76	0.84	1.74	0.68	0.84
time (sec)	N/A	0.033	0.026	0.002	0.937	1.921	0.088	1.066

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	26	7	9
normalized size	1	1.	1.	1.14	1.29	3.71	1.	1.29
time (sec)	N/A	0.009	0.002	0.	0.935	1.905	0.167	1.061

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	55	20	22
normalized size	1	1.	1.	0.77	1.	2.5	0.91	1.
time (sec)	N/A	0.012	0.009	0.	0.941	2.054	0.293	1.07

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	26	7	9
normalized size	1	1.	1.	1.14	1.29	3.71	1.	1.29
time (sec)	N/A	0.009	0.002	0.	0.932	2.008	0.161	1.06

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	23	61	22	30
normalized size	1	1.	1.	0.82	0.82	2.18	0.79	1.07
time (sec)	N/A	0.01	0.001	0.002	0.926	1.817	0.095	1.059

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	51	8	12
normalized size	1	1.	1.	0.91	1.09	4.64	0.73	1.09
time (sec)	N/A	0.012	0.003	0.013	0.917	2.014	0.536	1.048

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	22	15	4
normalized size	1	1.	1.	1.33	1.33	7.33	5.	1.33
time (sec)	N/A	0.019	0.008	0.	1.417	1.798	0.124	1.04

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	0	26	3	4
normalized size	1	1.	1.	1.33	0.	8.67	1.	1.33
time (sec)	N/A	0.047	0.055	0.005	0.	2.154	0.418	1.064

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	21	72	51	12	28
normalized size	1	1.	1.	1.31	4.5	3.19	0.75	1.75
time (sec)	N/A	0.041	0.023	0.023	1.412	2.275	0.132	1.09

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	127	262	0	284	0	0
normalized size	1	1.	2.4	4.94	0.	5.36	0.	0.
time (sec)	N/A	0.065	0.055	0.006	0.	2.626	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	35	127	93	177	0	112
normalized size	1	1.	2.19	7.94	5.81	11.06	0.	7.
time (sec)	N/A	0.086	0.09	0.036	1.417	2.736	0.	1.117

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	35	121	92	76	422	111
normalized size	1	1.	2.19	7.56	5.75	4.75	26.38	6.94
time (sec)	N/A	0.124	0.018	0.007	1.41	2.162	1.301	1.097

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	35	121	88	76	422	107
normalized size	1	1.	2.19	7.56	5.5	4.75	26.38	6.69
time (sec)	N/A	0.019	0.013	0.005	1.41	2.432	1.275	1.076

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	59	146	105	39
normalized size	1	1.	0.74	0.86	1.69	4.17	3.	1.11
time (sec)	N/A	0.008	0.021	0.	1.413	2.289	2.558	1.065

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	13	16	74	19	16
normalized size	1	1.	1.29	0.93	1.14	5.29	1.36	1.14
time (sec)	N/A	0.009	0.007	0.007	1.418	2.252	0.061	1.066

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	34	8	15
normalized size	1	1.	1.	0.92	1.15	2.62	0.62	1.15
time (sec)	N/A	0.005	0.004	0.001	1.403	2.098	0.081	1.06

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	8	15	5	8
normalized size	1	1.	1.	1.	1.14	2.14	0.71	1.14
time (sec)	N/A	0.03	0.004	0.	0.989	2.099	0.08	1.055

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	20	24	31	55	20	41
normalized size	1	1.	0.83	1.	1.29	2.29	0.83	1.71
time (sec)	N/A	0.357	0.069	0.005	1.547	2.173	0.108	1.062

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	11	18	7	11
normalized size	1	1.	1.	1.	1.22	2.	0.78	1.22
time (sec)	N/A	0.003	0.003	0.	0.936	2.05	0.068	1.064

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	32	55	19	26
normalized size	1	1.	1.	0.8	1.28	2.2	0.76	1.04
time (sec)	N/A	0.045	0.002	0.002	0.927	2.145	0.085	1.051

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	23	5	8
normalized size	1	1.	1.	0.88	1.	2.88	0.62	1.
time (sec)	N/A	0.003	0.003	0.002	1.409	2.189	0.087	1.065

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	36	10	16
normalized size	1	1.	1.	0.81	1.	2.25	0.62	1.
time (sec)	N/A	0.008	0.003	0.002	1.408	2.053	0.09	1.059

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	17	22	62	36	22
normalized size	1	1.	1.	1.21	1.57	4.43	2.57	1.57
time (sec)	N/A	0.015	0.003	0.01	1.407	2.281	0.384	1.053

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	59	146	105	39
normalized size	1	1.	0.74	0.86	1.69	4.17	3.	1.11
time (sec)	N/A	0.011	0.003	0.	1.413	2.105	2.556	1.062

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	32	16	20	103	34	28
normalized size	1	1.	1.88	0.94	1.18	6.06	2.	1.65
time (sec)	N/A	0.005	0.007	0.004	1.41	2.169	0.526	1.071

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	14	15	46	15	15
normalized size	1	1.	0.74	0.74	0.79	2.42	0.79	0.79
time (sec)	N/A	0.008	0.014	0.	0.928	2.231	0.28	1.046

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	11	2	4
normalized size	1	1.	1.	1.5	1.5	5.5	1.	2.
time (sec)	N/A	0.	0.	0.	0.93	2.241	0.05	1.057

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	73	31	346	271	0	45
normalized size	1	1.	1.62	0.69	7.69	6.02	0.	1.
time (sec)	N/A	0.123	0.181	0.142	1.78	2.618	0.	1.104

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	36	10	22
normalized size	1	1.	1.	0.79	1.	2.57	0.71	1.57
time (sec)	N/A	0.006	0.002	0.002	0.928	2.206	0.061	1.065

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	14	11	15	31	14	15
normalized size	1	1.	0.82	0.65	0.88	1.82	0.82	0.88
time (sec)	N/A	0.003	0.003	0.003	0.924	1.775	0.203	1.059

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	25	20	26	77	0	36
normalized size	1	1.	1.09	0.87	1.13	3.35	0.	1.57
time (sec)	N/A	0.01	0.005	0.004	1.409	1.805	0.	1.059

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	38	5	8
normalized size	1	1.	1.	0.88	1.	4.75	0.62	1.
time (sec)	N/A	0.013	0.001	0.002	0.919	2.072	0.058	1.059

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	7	19	5	7
normalized size	1	1.	1.	1.	1.17	3.17	0.83	1.17
time (sec)	N/A	0.016	0.004	0.	0.934	2.045	0.072	1.063

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	27	8	14
normalized size	1	1.	1.	0.92	1.17	2.25	0.67	1.17
time (sec)	N/A	0.026	0.008	0.002	0.93	2.125	0.08	1.055

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	14	30	7	11
normalized size	1	1.	0.67	0.75	1.17	2.5	0.58	0.92
time (sec)	N/A	0.009	0.008	0.	0.927	2.21	0.354	1.055

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	19	7	8
normalized size	1	1.	1.	0.88	1.	2.38	0.88	1.
time (sec)	N/A	0.012	0.004	0.	0.927	1.911	0.064	1.06

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	35	12	18
normalized size	1	1.	1.	0.82	1.06	2.06	0.71	1.06
time (sec)	N/A	0.004	0.001	0.	0.936	1.806	0.082	1.059

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	20	5	8
normalized size	1	1.	1.	0.88	1.	2.5	0.62	1.
time (sec)	N/A	0.007	0.001	0.	0.923	1.909	0.054	1.068

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	27	21	30	68	0	27
normalized size	1	1.	1.12	0.88	1.25	2.83	0.	1.12
time (sec)	N/A	0.009	0.032	0.006	1.415	1.802	0.	1.077

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	14	18	72	15	18
normalized size	1	1.	1.	0.7	0.9	3.6	0.75	0.9
time (sec)	N/A	0.023	0.005	0.	1.421	1.922	0.106	1.066

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	59	146	105	39
normalized size	1	1.	0.74	0.86	1.69	4.17	3.	1.11
time (sec)	N/A	0.009	0.003	0.	1.407	1.876	2.557	1.078

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	15	19	49	24	19
normalized size	1	1.	0.9	0.75	0.95	2.45	1.2	0.95
time (sec)	N/A	0.024	0.008	0.003	1.416	1.919	0.111	1.048

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	35	103	31	35
normalized size	1	1.	1.	0.82	1.06	3.12	0.94	1.06
time (sec)	N/A	0.011	0.012	0.	1.412	1.894	0.126	1.049

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	26	55	15	26
normalized size	1	1.	1.	0.95	1.24	2.62	0.71	1.24
time (sec)	N/A	0.016	0.	0.002	0.932	1.74	0.052	1.046

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	32	74	19	31
normalized size	1	1.	1.	0.96	1.23	2.85	0.73	1.19
time (sec)	N/A	0.019	0.	0.002	0.936	1.789	0.062	1.053

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	34	85	22	34
normalized size	1	1.	1.	0.96	1.26	3.15	0.81	1.26
time (sec)	N/A	0.023	0.	0.	0.928	1.865	0.055	1.053

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	41	104	26	39
normalized size	1	1.	1.	0.97	1.28	3.25	0.81	1.22
time (sec)	N/A	0.026	0.	0.001	0.933	1.813	0.055	1.047

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	25	24	31	80	20	31
normalized size	1	1.	1.04	1.	1.29	3.33	0.83	1.29
time (sec)	N/A	0.027	0.	0.	0.931	1.875	0.053	1.242

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	30	29	38	99	24	36
normalized size	1	1.	1.03	1.	1.31	3.41	0.83	1.24
time (sec)	N/A	0.032	0.	0.	0.929	1.817	0.054	1.078

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	23	34	15	23
normalized size	1	1.	1.	0.95	1.21	1.79	0.79	1.21
time (sec)	N/A	0.009	0.	0.	0.93	1.807	0.051	1.049

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	30	42	19	28
normalized size	1	1.	1.	0.96	1.25	1.75	0.79	1.17
time (sec)	N/A	0.011	0.	0.002	0.937	1.817	0.053	1.05

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	85	22	31
normalized size	1	1.	1.	0.96	1.24	3.4	0.88	1.24
time (sec)	N/A	0.013	0.	0.002	0.931	1.881	0.053	1.06

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	29	38	101	26	36
normalized size	1	1.	1.	0.97	1.27	3.37	0.87	1.2
time (sec)	N/A	0.015	0.	0.	0.929	1.706	0.055	1.07

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	28	27	35	53	24	34
normalized size	1	1.	1.04	1.	1.3	1.96	0.89	1.26
time (sec)	N/A	0.006	0.	0.001	0.926	1.812	0.054	1.057

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [50] had the largest ratio of [0.8571]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.	4	0.5
2	A	3	2	1.	11	0.182
3	A	2	1	1.	11	0.091
4	A	1	1	1.	2	0.5
5	A	1	1	1.	7	0.143
6	A	2	2	1.	7	0.286
7	A	1	1	1.	11	0.091
8	A	1	1	1.	6	0.167
9	A	2	2	1.	7	0.286
10	A	2	2	1.	4	0.5
11	A	1	1	1.	6	0.167
12	A	3	2	1.	6	0.333
13	A	4	2	1.	16	0.125
14	A	5	3	1.	7	0.429
15	A	3	1	1.	14	0.071
16	A	2	2	1.	5	0.4
17	A	1	1	1.	7	0.143
18	A	1	1	1.	11	0.091
19	A	2	2	1.	11	0.182
20	A	1	1	1.	5	0.2
21	A	1	1	1.	6	0.167
22	A	2	2	1.	9	0.222
23	A	3	3	1.	14	0.214
24	A	2	1	1.	9	0.111
25	A	3	3	1.	7	0.429
26	A	2	2	1.	15	0.133
27	A	2	2	1.	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	3	3	1.	13	0.231
29	A	1	1	1.	5	0.2
30	A	3	3	1.	8	0.375
31	A	7	7	1.	11	0.636
32	A	3	2	1.	12	0.167
33	A	3	3	1.	6	0.5
34	A	2	1	1.	9	0.111
35	A	4	3	1.	13	0.231
36	A	4	4	1.	15	0.267
37	A	3	2	1.	15	0.133
38	A	6	3	1.	13	0.231
39	A	3	2	1.	15	0.133
40	A	5	5	1.	29	0.172
41	A	2	2	1.	4	0.5
42	A	6	6	1.16	19	0.316
43	A	1	1	1.	6	0.167
44	A	2	2	1.	5	0.4
45	A	1	1	1.	10	0.1
46	A	5	3	1.	13	0.231
47	A	1	1	1.	5	0.2
48	A	1	1	1.	7	0.143
49	A	6	6	1.	9	0.667
50	A	10	6	1.55	7	0.857
51	A	1	1	1.	27	0.037
52	A	1	1	1.	4	0.25
53	A	4	3	1.	6	0.5
54	A	2	2	1.	10	0.2
55	A	7	4	1.	9	0.444
56	A	2	1	1.	12	0.083
57	A	1	1	1.	4	0.25
58	A	6	4	1.	7	0.571
59	A	4	3	1.	11	0.273
60	A	6	3	1.	9	0.333
61	A	2	2	1.	4	0.5
62	A	3	3	1.	6	0.5
63	A	2	2	1.	4	0.5
64	A	2	2	1.	6	0.333
65	A	2	1	1.	9	0.111
66	A	2	1	1.	12	0.083
67	A	2	2	1.	20	0.1
68	A	2	2	1.	10	0.2
69	A	6	6	1.	30	0.2
70	A	5	5	1.	39	0.128
71	A	6	5	1.	45	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	A	4	4	1.	31	0.129
73	A	3	2	1.	15	0.133
74	A	3	2	1.	4	0.5
75	A	3	2	1.	11	0.182
76	A	5	3	1.	13	0.231
77	A	10	6	1.	33	0.182
78	A	1	1	1.	7	0.143
79	A	5	5	1.	8	0.625
80	A	2	2	1.	9	0.222
81	A	3	3	1.	11	0.273
82	A	3	3	1.	8	0.375
83	A	3	2	1.	15	0.133
84	A	2	2	1.	16	0.125
85	A	1	1	1.	6	0.167
86	A	1	1	1.	3	0.333
87	A	4	2	1.	17	0.118
88	A	2	2	1.	4	0.5
89	A	2	1	1.	11	0.091
90	A	3	3	1.	14	0.214
91	A	2	2	1.	7	0.286
92	A	2	2	1.	11	0.182
93	A	3	2	1.	13	0.154
94	A	1	1	1.	8	0.125
95	A	2	2	1.	7	0.286
96	A	1	1	1.	4	0.25
97	A	2	2	1.	5	0.4
98	A	3	3	1.	17	0.176
99	A	3	3	1.	16	0.188
100	A	3	2	1.	15	0.133
101	A	3	3	1.	15	0.2
102	A	3	3	1.	8	0.375
103	A	1	1	1.	20	0.05
104	A	1	1	1.	25	0.04
105	A	1	1	1.	26	0.038
106	A	1	1	1.	31	0.032
107	A	1	1	1.	24	0.042
108	A	1	1	1.	29	0.034
109	A	1	1	1.	18	0.056
110	A	1	1	1.	23	0.043
111	A	1	1	1.	24	0.042
112	A	1	1	1.	29	0.034
113	A	1	1	1.	27	0.037

Chapter 3

Listing of integrals

3.1 $\int \cot^4(x) dx$

Optimal. Leaf size=12

$$x - \frac{1}{3} \cot^3(x) + \cot(x)$$

[Out] x + Cot[x] - Cot[x]^3/3

Rubi [A] time = 0.0099821, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3473, 8}

$$x - \frac{1}{3} \cot^3(x) + \cot(x)$$

Antiderivative was successfully verified.

[In] Int[Cot[x]^4, x]

[Out] x + Cot[x] - Cot[x]^3/3

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \cot^4(x) dx &= -\frac{1}{3} \cot^3(x) - \int \cot^2(x) dx \\ &= \cot(x) - \frac{\cot^3(x)}{3} + \int 1 dx \\ &= x + \cot(x) - \frac{\cot^3(x)}{3} \end{aligned}$$

Mathematica [A] time = 0.0030371, size = 18, normalized size = 1.5

$$x + \frac{4 \cot(x)}{3} - \frac{1}{3} \cot(x) \csc^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]^4,x]

[Out] x + (4*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3

Maple [A] time = 0., size = 14, normalized size = 1.2

$$-\frac{(\cot(x))^3}{3} + \cot(x) - \frac{\pi}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cot(x)^4,x)

[Out] -1/3*cot(x)^3+cot(x)-1/2*Pi+x

Maxima [A] time = 1.41448, size = 22, normalized size = 1.83

$$x + \frac{3 \tan(x)^2 - 1}{3 \tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^4,x, algorithm="maxima")

[Out] x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3

Fricas [B] time = 1.65477, size = 126, normalized size = 10.5

$$\frac{4 \cos(2x)^2 + 3(x \cos(2x) - x) \sin(2x) + 2 \cos(2x) - 2}{3(\cos(2x) - 1) \sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^4,x, algorithm="fricas")

[Out] 1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)/((cos(2*x) - 1)*sin(2*x))

Sympy [A] time = 0.065953, size = 19, normalized size = 1.58

$$x + \frac{\cos(x)}{\sin(x)} - \frac{\cos^3(x)}{3 \sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)**4,x)

[Out] x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)

Giac [B] time = 1.06264, size = 46, normalized size = 3.83

$$\frac{1}{24} \tan\left(\frac{1}{2}x\right)^3 + x + \frac{15 \tan\left(\frac{1}{2}x\right)^2 - 1}{24 \tan\left(\frac{1}{2}x\right)^3} - \frac{5}{8} \tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cot(x)^4,x, algorithm="giac")

[Out] 1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 5/8*tan(1/2*x)

$$3.2 \quad \int \frac{1}{x^4(1+x^2)} dx$$

Optimal. Leaf size=13

$$-\frac{1}{3x^3} + \frac{1}{x} + \tan^{-1}(x)$$

[Out] -1/(3*x^3) + x^(-1) + ArcTan[x]

Rubi [A] time = 0.0048847, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {325, 203}

$$-\frac{1}{3x^3} + \frac{1}{x} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(1 + x^2)),x]

[Out] -1/(3*x^3) + x^(-1) + ArcTan[x]

Rule 325

```
Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^4(1+x^2)} dx &= -\frac{1}{3x^3} - \int \frac{1}{x^2(1+x^2)} dx \\ &= -\frac{1}{3x^3} + \frac{1}{x} + \int \frac{1}{1+x^2} dx \\ &= -\frac{1}{3x^3} + \frac{1}{x} + \tan^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.0038972, size = 13, normalized size = 1.

$$-\frac{1}{3x^3} + \frac{1}{x} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(1 + x^2)),x]

[Out] $-1/(3*x^3) + x^{-1} + \text{ArcTan}[x]$

Maple [A] time = 0.005, size = 12, normalized size = 0.9

$$-\frac{1}{3x^3} + x^{-1} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(x^2+1),x)`

[Out] $-1/3/x^3+1/x+\arctan(x)$

Maxima [A] time = 1.41326, size = 20, normalized size = 1.54

$$\frac{3x^2 - 1}{3x^3} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(x^2+1),x, algorithm="maxima")`

[Out] $1/3*(3*x^2 - 1)/x^3 + \arctan(x)$

Fricas [A] time = 1.50589, size = 53, normalized size = 4.08

$$\frac{3x^3 \arctan(x) + 3x^2 - 1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(x^2+1),x, algorithm="fricas")`

[Out] $1/3*(3*x^3*\arctan(x) + 3*x^2 - 1)/x^3$

Sympy [A] time = 0.105203, size = 14, normalized size = 1.08

$$\text{atan}(x) + \frac{3x^2 - 1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(x**2+1),x)`

[Out] $\text{atan}(x) + (3*x**2 - 1)/(3*x**3)$

Giac [A] time = 1.07896, size = 20, normalized size = 1.54

$$\frac{3x^2 - 1}{3x^3} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*(3*x^2 - 1)/x^3 + arctan(x)
```


3.3 $\int \frac{x+x^2}{\sqrt{x}} dx$

Optimal. Leaf size=19

$$\frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3}$$

[Out] $(2*x^{(3/2)})/3 + (2*x^{(5/2)})/5$

Rubi [A] time = 0.002675, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Int[(x + x^2)/Sqrt[x],x]

[Out] $(2*x^{(3/2)})/3 + (2*x^{(5/2)})/5$

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{x+x^2}{\sqrt{x}} dx &= \int (\sqrt{x} + x^{3/2}) dx \\ &= \frac{2x^{3/2}}{3} + \frac{2x^{5/2}}{5} \end{aligned}$$

Mathematica [A] time = 0.0029317, size = 14, normalized size = 0.74

$$\frac{2}{15}x^{3/2}(3x+5)$$

Antiderivative was successfully verified.

[In] Integrate[(x + x^2)/Sqrt[x],x]

[Out] $(2*x^{(3/2)}*(5 + 3*x))/15$

Maple [A] time = 0.001, size = 11, normalized size = 0.6

$$\frac{10 + 6x}{15}x^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+x)/x^(1/2),x)`

[Out] $2/15*x^{(3/2)}*(5+3*x)$

Maxima [A] time = 0.935709, size = 15, normalized size = 0.79

$$\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+x)/x^(1/2),x, algorithm="maxima")`

[Out] $2/5*x^{(5/2)} + 2/3*x^{(3/2)}$

Fricas [A] time = 1.49986, size = 38, normalized size = 2.

$$\frac{2}{15}(3x^2 + 5x)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+x)/x^(1/2),x, algorithm="fricas")`

[Out] $2/15*(3*x^2 + 5*x)*\text{sqrt}(x)$

Sympy [A] time = 0.223083, size = 15, normalized size = 0.79

$$\frac{2x^{\frac{5}{2}}}{5} + \frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+x)/x**(1/2),x)`

[Out] $2*x^{(5/2)}/5 + 2*x^{(3/2)}/3$

Giac [A] time = 1.09026, size = 15, normalized size = 0.79

$$\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+x)/x^(1/2),x, algorithm="giac")`

[Out] $2/5*x^{(5/2)} + 2/3*x^{(3/2)}$

3.4 $\int \cos(x) dx$

Optimal. Leaf size=2

$\sin(x)$

[Out] Sin[x]

Rubi [A] time = 0.0020616, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {2637}

$\sin(x)$

Antiderivative was successfully verified.

[In] Int[Cos[x], x]

[Out] Sin[x]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rubi steps

$$\int \cos(x) dx = \sin(x)$$

Mathematica [A] time = 0.0009164, size = 2, normalized size = 1.

$\sin(x)$

Antiderivative was successfully verified.

[In] Integrate[Cos[x], x]

[Out] Sin[x]

Maple [A] time = 0., size = 3, normalized size = 1.5

$\sin(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x), x)

[Out] sin(x)

Maxima [A] time = 0.926673, size = 3, normalized size = 1.5

$\sin(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x),x, algorithm="maxima")`

[Out] `sin(x)`

Fricas [A] time = 1.58175, size = 11, normalized size = 5.5

$\sin(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x),x, algorithm="fricas")`

[Out] `sin(x)`

Sympy [A] time = 0.05319, size = 2, normalized size = 1.

$\sin(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x),x)`

[Out] `sin(x)`

Giac [A] time = 1.0934, size = 3, normalized size = 1.5

$\sin(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x),x, algorithm="giac")`

[Out] `sin(x)`

3.5 $\int e^{x^2} x dx$

Optimal. Leaf size=9

$$\frac{e^{x^2}}{2}$$

[Out] $E^{x^2}/2$

Rubi [A] time = 0.0060194, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2209}

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Int [$E^{x^2}x$, x]

[Out] $E^{x^2}/2$

Rule 2209

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]

Rubi steps

$$\int e^{x^2} x dx = \frac{e^{x^2}}{2}$$

Mathematica [A] time = 0.0011679, size = 9, normalized size = 1.

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Integrate [$E^{x^2}x$, x]

[Out] $E^{x^2}/2$

Maple [A] time = 0., size = 7, normalized size = 0.8

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*x,x)
```

```
[Out] 1/2*exp(x^2)
```

Maxima [A] time = 0.926878, size = 8, normalized size = 0.89

$$\frac{1}{2}e^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*e^(x^2)
```

Fricas [A] time = 1.63268, size = 18, normalized size = 2.

$$\frac{1}{2}e^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*e^(x^2)
```

Sympy [A] time = 0.078546, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x,x)
```

```
[Out] exp(x**2)/2
```

Giac [A] time = 1.07871, size = 8, normalized size = 0.89

$$\frac{1}{2}e^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="giac")
```

```
[Out] 1/2*e^(x^2)
```

3.6 $\int \sec^2(x) \tan(x) dx$

Optimal. Leaf size=8

$$\frac{\sec^2(x)}{2}$$

[Out] Sec[x]^2/2

Rubi [A] time = 0.0106558, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2606, 30}

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2*Tan[x], x]

[Out] Sec[x]^2/2

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_.)]^(m_.))*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \sec^2(x) \tan(x) dx &= \text{Subst}\left(\int x dx, x, \sec(x)\right) \\ &= \frac{\sec^2(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.0041911, size = 8, normalized size = 1.

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2*Tan[x], x]

[Out] Sec[x]^2/2

Maple [A] time = 0.005, size = 7, normalized size = 0.9

$$\frac{(\sec(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2*tan(x),x)

[Out] 1/2*sec(x)^2

Maxima [A] time = 0.935947, size = 8, normalized size = 1.

$$\frac{1}{2} \tan(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*tan(x),x, algorithm="maxima")

[Out] 1/2*tan(x)^2

Fricas [A] time = 1.59947, size = 19, normalized size = 2.38

$$\frac{1}{2 \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*tan(x),x, algorithm="fricas")

[Out] 1/2/cos(x)^2

Sympy [A] time = 0.066424, size = 7, normalized size = 0.88

$$\frac{1}{2 \cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2*tan(x),x)

[Out] 1/(2*cos(x)**2)

Giac [A] time = 1.06617, size = 8, normalized size = 1.

$$\frac{1}{2 \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(sec(x)^2*tan(x),x, algorithm="giac")
```

```
[Out] 1/2/cos(x)^2
```

3.7 $\int x\sqrt{1+x^2} dx$

Optimal. Leaf size=13

$$\frac{1}{3}(x^2+1)^{3/2}$$

[Out] $(1+x^2)^{(3/2)}/3$

Rubi [A] time = 0.0016729, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {261}

$$\frac{1}{3}(x^2+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[1 + x^2],x]

[Out] $(1+x^2)^{(3/2)}/3$

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int x\sqrt{1+x^2} dx = \frac{1}{3}(1+x^2)^{3/2}$$

Mathematica [A] time = 0.0019712, size = 13, normalized size = 1.

$$\frac{1}{3}(x^2+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[1 + x^2],x]

[Out] $(1+x^2)^{(3/2)}/3$

Maple [A] time = 0.002, size = 10, normalized size = 0.8

$$\frac{1}{3}(x^2+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(x^2+1)^(1/2),x)`

[Out] `1/3*(x^2+1)^(3/2)`

Maxima [A] time = 0.925371, size = 12, normalized size = 0.92

$$\frac{1}{3}(x^2 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x^2+1)^(1/2),x, algorithm="maxima")`

[Out] `1/3*(x^2 + 1)^(3/2)`

Fricas [A] time = 1.56769, size = 28, normalized size = 2.15

$$\frac{1}{3}(x^2 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x^2+1)^(1/2),x, algorithm="fricas")`

[Out] `1/3*(x^2 + 1)^(3/2)`

Sympy [B] time = 0.190332, size = 22, normalized size = 1.69

$$\frac{x^2\sqrt{x^2+1}}{3} + \frac{\sqrt{x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x**2+1)**(1/2),x)`

[Out] `x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3`

Giac [A] time = 1.0848, size = 12, normalized size = 0.92

$$\frac{1}{3}(x^2 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x^2+1)^(1/2),x, algorithm="giac")`

[Out] `1/3*(x^2 + 1)^(3/2)`

3.8 $\int e^x \sin(x) dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

[Out] $-(E^x \cos[x])/2 + (E^x \sin[x])/2$

Rubi [A] time = 0.0065227, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4432}

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

Antiderivative was successfully verified.

[In] Int[E^x*Sin[x],x]

[Out] $-(E^x \cos[x])/2 + (E^x \sin[x])/2$

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
  Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x \sin(x) dx = -\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)$$

Mathematica [A] time = 0.013703, size = 14, normalized size = 0.74

$$\frac{1}{2}e^x(\sin(x) - \cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[E^x*Sin[x],x]

[Out] $(E^x*(-\cos[x] + \sin[x]))/2$

Maple [A] time = 0., size = 14, normalized size = 0.7

$$-\frac{e^x \cos(x)}{2} + \frac{e^x \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)*sin(x),x)`

[Out] `-1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)`

Maxima [A] time = 0.93405, size = 15, normalized size = 0.79

$$-\frac{1}{2}(\cos(x) - \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*sin(x),x, algorithm="maxima")`

[Out] `-1/2*(cos(x) - sin(x))*e^x`

Fricas [A] time = 1.64102, size = 46, normalized size = 2.42

$$-\frac{1}{2}\cos(x)e^x + \frac{1}{2}e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*sin(x),x, algorithm="fricas")`

[Out] `-1/2*cos(x)*e^x + 1/2*e^x*sin(x)`

Sympy [A] time = 0.298395, size = 15, normalized size = 0.79

$$\frac{e^x \sin(x)}{2} - \frac{e^x \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*sin(x),x)`

[Out] `exp(x)*sin(x)/2 - exp(x)*cos(x)/2`

Giac [A] time = 1.0982, size = 15, normalized size = 0.79

$$-\frac{1}{2}(\cos(x) - \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*sin(x),x, algorithm="giac")`

[Out] `-1/2*(cos(x) - sin(x))*e^x`

3.9 $\int \cot(x) \csc^3(x) dx$

Optimal. Leaf size=8

$$-\frac{1}{3} \csc^3(x)$$

[Out] -Csc[x]^3/3

Rubi [A] time = 0.0124362, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2606, 30}

$$-\frac{1}{3} \csc^3(x)$$

Antiderivative was successfully verified.

[In] Int[Cot[x]*Csc[x]^3,x]

[Out] -Csc[x]^3/3

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_.)]^(m_.))*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \cot(x) \csc^3(x) dx &= -\text{Subst}\left(\int x^2 dx, x, \csc(x)\right) \\ &= -\frac{1}{3} \csc^3(x) \end{aligned}$$

Mathematica [A] time = 0.0040954, size = 8, normalized size = 1.

$$-\frac{1}{3} \csc^3(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cot[x]*Csc[x]^3,x]

[Out] -Csc[x]^3/3

Maple [A] time = 0.013, size = 7, normalized size = 0.9

$$-\frac{1}{3 (\sin(x))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*csc(x)^2/sin(x)^2,x)

[Out] -1/3/sin(x)^3

Maxima [A] time = 0.93987, size = 8, normalized size = 1.

$$-\frac{1}{3 \sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="maxima")

[Out] -1/3/sin(x)^3

Fricas [B] time = 1.60745, size = 39, normalized size = 4.88

$$\frac{1}{3 (\cos(x)^2 - 1) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="fricas")

[Out] 1/3/((cos(x)^2 - 1)*sin(x))

Sympy [A] time = 0.064042, size = 8, normalized size = 1.

$$-\frac{1}{3 \sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*csc(x)**2/sin(x)**2,x)

[Out] -1/(3*sin(x)**3)

Giac [A] time = 1.07199, size = 8, normalized size = 1.

$$-\frac{1}{3 \sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/3/sin(x)^3
```


3.10 $\int \sin(e^x) dx$

Optimal. Leaf size=4

$\text{Si}(e^x)$

[Out] SinIntegral[E^x]

Rubi [A] time = 0.0117071, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {2282, 3299}

$\text{Si}(e^x)$

Antiderivative was successfully verified.

[In] Int[Sin[E^x], x]

[Out] SinIntegral[E^x]

Rule 2282

```
Int[u_, x_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x],
Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :=> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \sin(e^x) dx = \text{Subst}\left(\int \frac{\sin(x)}{x} dx, x, e^x\right) = \text{Si}(e^x)$$

Mathematica [A] time = 0.005286, size = 4, normalized size = 1.

$\text{Si}(e^x)$

Antiderivative was successfully verified.

[In] Integrate[Sin[E^x], x]

[Out] SinIntegral[E^x]

Maple [A] time = 0.004, size = 4, normalized size = 1.

$$\text{Si}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(exp(x)),x)`

[Out] `Si(exp(x))`

Maxima [C] time = 1.06478, size = 20, normalized size = 5.

$$-\frac{1}{2}i\text{Ei}(ie^x) + \frac{1}{2}i\text{Ei}(-ie^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(exp(x)),x, algorithm="maxima")`

[Out] `-1/2*I*Ei(I*e^x) + 1/2*I*Ei(-I*e^x)`

Fricas [A] time = 1.9311, size = 26, normalized size = 6.5

$$\text{Si}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(exp(x)),x, algorithm="fricas")`

[Out] `sin_integral(e^x)`

Sympy [A] time = 0.64135, size = 3, normalized size = 0.75

$$\text{Si}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(exp(x)),x)`

[Out] `Si(exp(x))`

Giac [A] time = 1.06027, size = 4, normalized size = 1.

$$\text{Si}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(exp(x)),x, algorithm="giac")`

[Out] `sin_integral(e^x)`

$$3.11 \quad \int \frac{\sin(y)}{y} dy$$

Optimal. Leaf size=2

$$\text{Si}(y)$$

[Out] SinIntegral[y]

Rubi [A] time = 0.0111784, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3299}

$$\text{Si}(y)$$

Antiderivative was successfully verified.

[In] Int[Sin[y]/y,y]

[Out] SinIntegral[y]

Rule 3299

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinIntegral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]

Rubi steps

$$\int \frac{\sin(y)}{y} dy = \text{Si}(y)$$

Mathematica [A] time = 0.0132604, size = 2, normalized size = 1.

$$\text{Si}(y)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[y]/y,y]

[Out] SinIntegral[y]

Maple [A] time = 0.001, size = 3, normalized size = 1.5

$$\text{Si}(y)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(y)/y,y)

[Out] Si(y)

Maxima [C] time = 1.05461, size = 18, normalized size = 9.

$$-\frac{1}{2}i\text{Ei}(iy) + \frac{1}{2}i\text{Ei}(-iy)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(y)/y,y, algorithm="maxima")

[Out] -1/2*I*Ei(I*y) + 1/2*I*Ei(-I*y)

Fricas [A] time = 1.85999, size = 23, normalized size = 11.5

$$\text{Si}(y)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(y)/y,y, algorithm="fricas")

[Out] sin_integral(y)

Sympy [A] time = 0.539575, size = 2, normalized size = 1.

$$\text{Si}(y)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(y)/y,y)

[Out] Si(y)

Giac [A] time = 1.11709, size = 3, normalized size = 1.5

$$\text{Si}(y)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(y)/y,y, algorithm="giac")

[Out] sin_integral(y)

3.12 $\int (e^x + \sin(x)) dx$

Optimal. Leaf size=8

$$e^x - \cos(x)$$

[Out] $E^x - \text{Cos}[x]$

Rubi [A] time = 0.00333, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2194, 2638}

$$e^x - \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^x + \text{Sin}[x], x]$

[Out] $E^x - \text{Cos}[x]$

Rule 2194

$\text{Int}[(F_)^{((c_.) * (a_.) + (b_.) * (x_)))^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x)))^n / (b*c*n*\text{Log}[F])], x] /; \text{FreeQ}\{F, a, b, c, n\}, x]$

Rule 2638

$\text{Int}[\text{sin}[(c_.) + (d_.) * (x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x] / d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int (e^x + \sin(x)) dx &= \int e^x dx + \int \sin(x) dx \\ &= e^x - \cos(x) \end{aligned}$$

Mathematica [A] time = 0.0028654, size = 8, normalized size = 1.

$$e^x - \cos(x)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[E^x + \text{Sin}[x], x]$

[Out] $E^x - \text{Cos}[x]$

Maple [A] time = 0.002, size = 8, normalized size = 1.

$$e^x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)+sin(x),x)`

[Out] `exp(x)-cos(x)`

Maxima [A] time = 0.945783, size = 9, normalized size = 1.12

$$-\cos(x) + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)+sin(x),x, algorithm="maxima")`

[Out] `-cos(x) + e^x`

Fricas [A] time = 1.82012, size = 20, normalized size = 2.5

$$-\cos(x) + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)+sin(x),x, algorithm="fricas")`

[Out] `-cos(x) + e^x`

Sympy [A] time = 0.071743, size = 5, normalized size = 0.62

$$e^x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)+sin(x),x)`

[Out] `exp(x) - cos(x)`

Giac [A] time = 1.06968, size = 9, normalized size = 1.12

$$-\cos(x) + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)+sin(x),x, algorithm="giac")`

[Out] `-cos(x) + e^x`

3.13 $\int (e^{x^2} + 2e^{x^2}x^2) dx$

Optimal. Leaf size=7

$$e^{x^2}x$$

[Out] $E^{x^2}x$

Rubi [A] time = 0.0199511, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2204, 2212}

$$e^{x^2}x$$

Antiderivative was successfully verified.

[In] Int[$E^{x^2} + 2E^{x^2}x^2, x$]

[Out] $E^{x^2}x$

Rule 2204

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))²), x_Symbol] :> Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2212

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))ⁿ)*((c_.) + (d_.)*(x_))^m), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

Rubi steps

$$\begin{aligned} \int (e^{x^2} + 2e^{x^2}x^2) dx &= 2 \int e^{x^2}x^2 dx + \int e^{x^2} dx \\ &= e^{x^2}x + \frac{1}{2}\sqrt{\pi}\operatorname{erfi}(x) - \int e^{x^2} dx \\ &= e^{x^2}x \end{aligned}$$

Mathematica [A] time = 0.0052759, size = 7, normalized size = 1.

$$e^{x^2}x$$

Antiderivative was successfully verified.

[In] Integrate[$E^{x^2} + 2E^{x^2}x^2, x$]

[Out] $E^{x^2}x$

Maple [A] time = 0.002, size = 7, normalized size = 1.

$$e^{x^2}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x^2)+2*exp(x^2)*x^2,x)`

[Out] `exp(x^2)*x`

Maxima [A] time = 0.935104, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="maxima")`

[Out] `x*e^(x^2)`

Fricas [A] time = 1.88616, size = 15, normalized size = 2.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="fricas")`

[Out] `x*e^(x^2)`

Sympy [A] time = 0.079765, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x**2)+2*exp(x**2)*x**2,x)`

[Out] `x*exp(x**2)`

Giac [A] time = 1.06987, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="giac")
```

```
[Out] x*e^(x^2)
```

3.14 $\int (e^x + x)^2 dx$

Optimal. Leaf size=28

$$\frac{x^3}{3} + 2e^x x - 2e^x + \frac{e^{2x}}{2}$$

[Out] $-2 * E^x + E^{(2 * x) / 2} + 2 * E^x * x + x^3 / 3$

Rubi [A] time = 0.0196767, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6742, 2194, 2176}

$$\frac{x^3}{3} + 2e^x x - 2e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

[In] Int[(E^x + x)^2, x]

[Out] $-2 * E^x + E^{(2 * x) / 2} + 2 * E^x * x + x^3 / 3$

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 2194

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] := Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rule 2176

Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !\$UseGamma == True

Rubi steps

$$\begin{aligned} \int (e^x + x)^2 dx &= \int (e^{2x} + 2e^x x + x^2) dx \\ &= \frac{x^3}{3} + 2 \int e^x x dx + \int e^{2x} dx \\ &= \frac{e^{2x}}{2} + 2e^x x + \frac{x^3}{3} - 2 \int e^x dx \\ &= -2e^x + \frac{e^{2x}}{2} + 2e^x x + \frac{x^3}{3} \end{aligned}$$

Mathematica [A] time = 0.0140445, size = 26, normalized size = 0.93

$$\frac{x^3}{3} + \frac{e^{2x}}{2} + e^x(2x - 2)$$

Antiderivative was successfully verified.

[In] Integrate[(E^x + x)^2,x]

[Out] E^(2*x)/2 + x^3/3 + E^x*(-2 + 2*x)

Maple [A] time = 0.002, size = 22, normalized size = 0.8

$$\frac{x^3}{3} + \frac{(e^x)^2}{2} + 2e^xx - 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(x)+x)^2,x)

[Out] 1/3*x^3+1/2*exp(x)^2+2*exp(x)*x-2*exp(x)

Maxima [A] time = 0.928451, size = 26, normalized size = 0.93

$$\frac{1}{3}x^3 + 2(x-1)e^x + \frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+exp(x))^2,x, algorithm="maxima")

[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)

Fricas [A] time = 1.80161, size = 53, normalized size = 1.89

$$\frac{1}{3}x^3 + 2(x-1)e^x + \frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+exp(x))^2,x, algorithm="fricas")

[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)

Sympy [A] time = 0.08726, size = 20, normalized size = 0.71

$$\frac{x^3}{3} + \frac{(4x-4)e^x}{2} + \frac{e^{2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+exp(x))**2,x)

[Out] x**3/3 + (4*x - 4)*exp(x)/2 + exp(2*x)/2

Giac [A] time = 1.09778, size = 26, normalized size = 0.93

$$\frac{1}{3}x^3 + 2(x-1)e^x + \frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+exp(x))^2,x, algorithm="giac")

[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)

3.15 $\int (2e^x + e^{2x} + x^2) dx$

Optimal. Leaf size=22

$$\frac{x^3}{3} + 2e^x + \frac{e^{2x}}{2}$$

[Out] $2 * E^x + E^{(2 * x)} / 2 + x^3 / 3$

Rubi [A] time = 0.004773, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2194}

$$\frac{x^3}{3} + 2e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

[In] Int[2 * E^x + E^(2 * x) + x^2, x]

[Out] $2 * E^x + E^{(2 * x)} / 2 + x^3 / 3$

Rule 2194

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int (2e^x + e^{2x} + x^2) dx &= \frac{x^3}{3} + 2 \int e^x dx + \int e^{2x} dx \\ &= 2e^x + \frac{e^{2x}}{2} + \frac{x^3}{3} \end{aligned}$$

Mathematica [A] time = 0.0023558, size = 22, normalized size = 1.

$$\frac{x^3}{3} + 2e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

[In] Integrate[2 * E^x + E^(2 * x) + x^2, x]

[Out] $2 * E^x + E^{(2 * x)} / 2 + x^3 / 3$

Maple [A] time = 0.001, size = 17, normalized size = 0.8

$$2e^x + \frac{e^{2x}}{2} + \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2*exp(x)+exp(2*x)+x^2,x)`

[Out] `2*exp(x)+1/2*exp(2*x)+1/3*x^3`

Maxima [A] time = 0.933384, size = 22, normalized size = 1.

$$\frac{1}{3}x^3 + \frac{1}{2}e^{(2x)} + 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="maxima")`

[Out] `1/3*x^3 + 1/2*e^(2*x) + 2*e^x`

Fricas [A] time = 1.87782, size = 42, normalized size = 1.91

$$\frac{1}{3}x^3 + \frac{1}{2}e^{(2x)} + 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="fricas")`

[Out] `1/3*x^3 + 1/2*e^(2*x) + 2*e^x`

Sympy [A] time = 0.082634, size = 15, normalized size = 0.68

$$\frac{x^3}{3} + \frac{e^{2x}}{2} + 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*exp(x)+exp(2*x)+x**2,x)`

[Out] `x**3/3 + exp(2*x)/2 + 2*exp(x)`

Giac [A] time = 1.07362, size = 22, normalized size = 1.

$$\frac{1}{3}x^3 + \frac{1}{2}e^{(2x)} + 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="giac")`

[Out] `1/3*x^3 + 1/2*e^(2*x) + 2*e^x`

3.16 $\int \cos(x) \sin(x) dx$

Optimal. Leaf size=8

$$\frac{\sin^2(x)}{2}$$

[Out] Sin[x]^2/2

Rubi [A] time = 0.0066878, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {2564, 30}

$$\frac{\sin^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[x],x]

[Out] Sin[x]^2/2

Rule 2564

Int[cos[(e_.) + (f_.)*(x_.)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.), x_Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && LtQ[0, m, n])

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \cos(x) \sin(x) dx &= \text{Subst}\left(\int x dx, x, \sin(x)\right) \\ &= \frac{\sin^2(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.0009839, size = 8, normalized size = 1.

$$-\frac{1}{2} \cos^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sin[x],x]

[Out] -Cos[x]^2/2

Maple [A] time = 0.001, size = 7, normalized size = 0.9

$$\frac{(\sin(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*sin(x),x)`

[Out] `1/2*sin(x)^2`

Maxima [A] time = 0.912815, size = 8, normalized size = 1.

$$-\frac{1}{2} \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*sin(x),x, algorithm="maxima")`

[Out] `-1/2*cos(x)^2`

Fricas [A] time = 1.9149, size = 20, normalized size = 2.5

$$-\frac{1}{2} \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*sin(x),x, algorithm="fricas")`

[Out] `-1/2*cos(x)^2`

Sympy [A] time = 0.053909, size = 5, normalized size = 0.62

$$\frac{\sin^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*sin(x),x)`

[Out] `sin(x)**2/2`

Giac [A] time = 1.08135, size = 8, normalized size = 1.

$$-\frac{1}{2} \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(cos(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2
```

3.17 $\int e^{x^2} x dx$

Optimal. Leaf size=9

$$\frac{e^{x^2}}{2}$$

[Out] $E^{x^2}/2$

Rubi [A] time = 0.0062455, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2209}

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Int[E^{x^2}*x, x]

[Out] E^{x^2}/2

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol]
:= Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n*Log[F]), x]
/; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int e^{x^2} x dx = \frac{e^{x^2}}{2}$$

Mathematica [A] time = 0.000582, size = 9, normalized size = 1.

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Integrate[E^{x^2}*x, x]

[Out] E^{x^2}/2

Maple [A] time = 0., size = 7, normalized size = 0.8

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x^2)*x,x)`

[Out] `1/2*exp(x^2)`

Maxima [A] time = 0.925133, size = 8, normalized size = 0.89

$$\frac{1}{2}e^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2)*x,x, algorithm="maxima")`

[Out] `1/2*e^(x^2)`

Fricas [A] time = 1.91346, size = 18, normalized size = 2.

$$\frac{1}{2}e^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2)*x,x, algorithm="fricas")`

[Out] `1/2*e^(x^2)`

Sympy [A] time = 0.075075, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x**2)*x,x)`

[Out] `exp(x**2)/2`

Giac [A] time = 1.07024, size = 8, normalized size = 0.89

$$\frac{1}{2}e^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2)*x,x, algorithm="giac")`

[Out] `1/2*e^(x^2)`

3.18 $\int x\sqrt{1+x^2} dx$

Optimal. Leaf size=13

$$\frac{1}{3}(x^2+1)^{3/2}$$

[Out] $(1+x^2)^{(3/2)}/3$

Rubi [A] time = 0.0019663, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {261}

$$\frac{1}{3}(x^2+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[1 + x^2],x]

[Out] $(1+x^2)^{(3/2)}/3$

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\int x\sqrt{1+x^2} dx = \frac{1}{3}(1+x^2)^{3/2}$$

Mathematica [A] time = 0.0003708, size = 13, normalized size = 1.

$$\frac{1}{3}(x^2+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[1 + x^2],x]

[Out] $(1+x^2)^{(3/2)}/3$

Maple [A] time = 0., size = 10, normalized size = 0.8

$$\frac{1}{3}(x^2+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(x^2+1)^(1/2),x)`

[Out] `1/3*(x^2+1)^(3/2)`

Maxima [A] time = 0.930322, size = 12, normalized size = 0.92

$$\frac{1}{3}(x^2 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x^2+1)^(1/2),x, algorithm="maxima")`

[Out] `1/3*(x^2 + 1)^(3/2)`

Fricas [A] time = 1.70136, size = 28, normalized size = 2.15

$$\frac{1}{3}(x^2 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x^2+1)^(1/2),x, algorithm="fricas")`

[Out] `1/3*(x^2 + 1)^(3/2)`

Sympy [B] time = 0.179528, size = 22, normalized size = 1.69

$$\frac{x^2\sqrt{x^2 + 1}}{3} + \frac{\sqrt{x^2 + 1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x**2+1)**(1/2),x)`

[Out] `x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3`

Giac [A] time = 1.05952, size = 12, normalized size = 0.92

$$\frac{1}{3}(x^2 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x^2+1)^(1/2),x, algorithm="giac")`

[Out] `1/3*(x^2 + 1)^(3/2)`

3.19 $\int \frac{e^x}{1+e^x} dx$

Optimal. Leaf size=6

$$\log(e^x + 1)$$

[Out] Log[1 + E^x]

Rubi [A] time = 0.014687, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2246, 31}

$$\log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Int[E^x/(1 + E^x), x]

[Out] Log[1 + E^x]

Rule 2246

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.), x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{e^x}{1+e^x} dx = \text{Subst} \left(\int \frac{1}{1+x} dx, x, e^x \right) = \log(1 + e^x)$$

Mathematica [A] time = 0.0040945, size = 6, normalized size = 1.

$$\log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(1 + E^x), x]

[Out] Log[1 + E^x]

Maple [A] time = 0.001, size = 6, normalized size = 1.

$$\ln(1 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(1+exp(x)),x)`

[Out] `ln(1+exp(x))`

Maxima [A] time = 0.922343, size = 7, normalized size = 1.17

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(x)),x, algorithm="maxima")`

[Out] `log(e^x + 1)`

Fricas [A] time = 1.68215, size = 19, normalized size = 3.17

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(x)),x, algorithm="fricas")`

[Out] `log(e^x + 1)`

Sympy [A] time = 0.084821, size = 5, normalized size = 0.83

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(x)),x)`

[Out] `log(exp(x) + 1)`

Giac [A] time = 1.09375, size = 7, normalized size = 1.17

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(x)),x, algorithm="giac")`

[Out] `log(e^x + 1)`

3.20 $\int x^{3/2} dx$

Optimal. Leaf size=9

$$\frac{2x^{5/2}}{5}$$

[Out] (2*x^(5/2))/5

Rubi [A] time = 0.0004782, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {30}

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

[In] Int[x^(3/2), x]

[Out] (2*x^(5/2))/5

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\int x^{3/2} dx = \frac{2x^{5/2}}{5}$$

Mathematica [A] time = 0.0005174, size = 9, normalized size = 1.

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

[In] Integrate[x^(3/2), x]

[Out] (2*x^(5/2))/5

Maple [A] time = 0.001, size = 6, normalized size = 0.7

$$\frac{2}{5}x^{5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3/2), x)

[Out] $2/5*x^{(5/2)}$

Maxima [A] time = 0.927467, size = 7, normalized size = 0.78

$$\frac{2}{5}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2),x, algorithm="maxima")`

[Out] $2/5*x^{(5/2)}$

Fricas [A] time = 1.7388, size = 18, normalized size = 2.

$$\frac{2}{5}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2),x, algorithm="fricas")`

[Out] $2/5*x^{(5/2)}$

Sympy [A] time = 0.052248, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(3/2),x)`

[Out] $2*x^{(5/2)}/5$

Giac [A] time = 1.0807, size = 7, normalized size = 0.78

$$\frac{2}{5}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(3/2),x, algorithm="giac")`

[Out] $2/5*x^{(5/2)}$

3.21 $\int \cos(3 + 2x) dx$

Optimal. Leaf size=10

$$\frac{1}{2} \sin(2x + 3)$$

[Out] Sin[3 + 2*x]/2

Rubi [A] time = 0.0030981, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2637}

$$\frac{1}{2} \sin(2x + 3)$$

Antiderivative was successfully verified.

[In] Int[Cos[3 + 2*x], x]

[Out] Sin[3 + 2*x]/2

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rubi steps

$$\int \cos(3 + 2x) dx = \frac{1}{2} \sin(3 + 2x)$$

Mathematica [A] time = 0.0042777, size = 10, normalized size = 1.

$$\frac{1}{2} \sin(2x + 3)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[3 + 2*x], x]

[Out] Sin[3 + 2*x]/2

Maple [A] time = 0.005, size = 9, normalized size = 0.9

$$\frac{\sin(3 + 2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(3+2*x), x)

[Out] $1/2*\sin(3+2*x)$

Maxima [A] time = 0.925298, size = 11, normalized size = 1.1

$$\frac{1}{2} \sin(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(3+2*x),x, algorithm="maxima")`

[Out] $1/2*\sin(2*x + 3)$

Fricas [A] time = 1.87985, size = 24, normalized size = 2.4

$$\frac{1}{2} \sin(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(3+2*x),x, algorithm="fricas")`

[Out] $1/2*\sin(2*x + 3)$

Sympy [A] time = 0.111567, size = 7, normalized size = 0.7

$$\frac{\sin(2x + 3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(3+2*x),x)`

[Out] $\sin(2*x + 3)/2$

Giac [A] time = 1.101, size = 11, normalized size = 1.1

$$\frac{1}{2} \sin(2x + 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(3+2*x),x, algorithm="giac")`

[Out] $1/2*\sin(2*x + 3)$

3.22 $\int 2e^{2x}yz \, dx$

Optimal. Leaf size=8

$$e^{2x}yz$$

[Out] $E^{(2*x)*y*z}$

Rubi [A] time = 0.0032724, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {12, 2194}

$$e^{2x}yz$$

Antiderivative was successfully verified.

[In] $\text{Int}[2*E^{(2*x)*y*z}, x]$

[Out] $E^{(2*x)*y*z}$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 2194

$\text{Int}[(F_)^{((c_*)((a_*) + (b_*)(x_)))})^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(F^{(c*(a + b*x)))})^n / (b*c*n*\text{Log}[F]), x] /; \text{FreeQ}[\{F, a, b, c, n\}, x]$

Rubi steps

$$\begin{aligned} \int 2e^{2x}yz \, dx &= (2yz) \int e^{2x} \, dx \\ &= e^{2x}yz \end{aligned}$$

Mathematica [A] time = 0.0005961, size = 8, normalized size = 1.

$$e^{2x}yz$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[2*E^{(2*x)*y*z}, x]$

[Out] $E^{(2*x)*y*z}$

Maple [A] time = 0.002, size = 8, normalized size = 1.

$$e^{2x}yz$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(2*exp(2*x)*y*z,x)`

[Out] `exp(2*x)*y*z`

Maxima [A] time = 0.932509, size = 9, normalized size = 1.12

$$yze^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*exp(2*x)*y*z,x, algorithm="maxima")`

[Out] `y*z*e^(2*x)`

Fricas [A] time = 1.86235, size = 18, normalized size = 2.25

$$yze^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*exp(2*x)*y*z,x, algorithm="fricas")`

[Out] `y*z*e^(2*x)`

Sympy [A] time = 0.06015, size = 7, normalized size = 0.88

$$yze^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*exp(2*x)*y*z,x)`

[Out] `y*z*exp(2*x)`

Giac [A] time = 1.07112, size = 9, normalized size = 1.12

$$yze^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(2*exp(2*x)*y*z,x, algorithm="giac")`

[Out] `y*z*e^(2*x)`

3.23 $\int e^x \cos^2(e^x) \sin(e^x) dx$

Optimal. Leaf size=10

$$-\frac{1}{3} \cos^3(e^x)$$

[Out] -Cos[E^x]^3/3

Rubi [A] time = 0.0274706, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2282, 2565, 30}

$$-\frac{1}{3} \cos^3(e^x)$$

Antiderivative was successfully verified.

[In] Int[E^x*Cos[E^x]^2*Sin[E^x],x]

[Out] -Cos[E^x]^3/3

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int e^x \cos^2(e^x) \sin(e^x) dx &= \text{Subst} \left(\int \cos^2(x) \sin(x) dx, x, e^x \right) \\ &= -\text{Subst} \left(\int x^2 dx, x, \cos(e^x) \right) \\ &= -\frac{1}{3} \cos^3(e^x) \end{aligned}$$

Mathematica [A] time = 0.0203797, size = 19, normalized size = 1.9

$$-\frac{1}{4} \cos(e^x) - \frac{1}{12} \cos(3e^x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x*Cos[E^x]^2*Sin[E^x],x]

[Out] -Cos[E^x]/4 - Cos[3*E^x]/12

Maple [A] time = 0.004, size = 8, normalized size = 0.8

$$-\frac{(\cos(e^x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)*cos(exp(x))^2*sin(exp(x)),x)

[Out] -1/3*cos(exp(x))^3

Maxima [A] time = 0.922909, size = 9, normalized size = 0.9

$$-\frac{1}{3} \cos(e^x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="maxima")

[Out] -1/3*cos(e^x)^3

Fricas [A] time = 1.8898, size = 23, normalized size = 2.3

$$-\frac{1}{3} \cos(e^x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="fricas")

[Out] -1/3*cos(e^x)^3

Sympy [A] time = 2.08616, size = 8, normalized size = 0.8

$$-\frac{\cos^3(e^x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)*cos(exp(x))**2*sin(exp(x)),x)

[Out] -cos(exp(x))**3/3

Giac [A] time = 1.08064, size = 9, normalized size = 0.9

$$-\frac{1}{3} \cos(e^x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="giac")`

[Out] `-1/3*cos(e^x)^3`

3.24 $\int x\sqrt{1+x} dx$

Optimal. Leaf size=23

$$\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2}$$

[Out] $(-2*(1+x)^{(3/2)})/3 + (2*(1+x)^{(5/2)})/5$

Rubi [A] time = 0.0038989, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {43}

$$\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[1 + x], x]

[Out] $(-2*(1+x)^{(3/2)})/3 + (2*(1+x)^{(5/2)})/5$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int [ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int x\sqrt{1+x} dx &= \int \left(-\sqrt{1+x} + (1+x)^{3/2}\right) dx \\ &= -\frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} \end{aligned}$$

Mathematica [A] time = 0.0042225, size = 16, normalized size = 0.7

$$\frac{2}{15}(x+1)^{3/2}(3x-2)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[1 + x], x]

[Out] $(2*(1+x)^{(3/2)}*(-2+3*x))/15$

Maple [A] time = 0.003, size = 13, normalized size = 0.6

$$\frac{-4+6x}{15}(1+x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(1+x)^(1/2),x)`

[Out] `2/15*(1+x)^(3/2)*(-2+3*x)`

Maxima [A] time = 0.924545, size = 20, normalized size = 0.87

$$\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+x)^(1/2),x, algorithm="maxima")`

[Out] `2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)`

Fricas [A] time = 1.81878, size = 46, normalized size = 2.

$$\frac{2}{15}(3x^2 + x - 2)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+x)^(1/2),x, algorithm="fricas")`

[Out] `2/15*(3*x^2 + x - 2)*sqrt(x + 1)`

Sympy [A] time = 0.847806, size = 34, normalized size = 1.48

$$\frac{2x^2\sqrt{x+1}}{5} + \frac{2x\sqrt{x+1}}{15} - \frac{4\sqrt{x+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+x)**(1/2),x)`

[Out] `2*x**2*sqrt(x + 1)/5 + 2*x*sqrt(x + 1)/15 - 4*sqrt(x + 1)/15`

Giac [A] time = 1.09969, size = 20, normalized size = 0.87

$$\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+x)^(1/2),x, algorithm="giac")`

[Out] `2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)`

3.25 $\int \frac{1}{-1+x^4} dx$

Optimal. Leaf size=13

$$-\frac{1}{2} \tan^{-1}(x) - \frac{1}{2} \tanh^{-1}(x)$$

[Out] -ArcTan[x]/2 - ArcTanh[x]/2

Rubi [A] time = 0.0031654, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {212, 206, 203}

$$-\frac{1}{2} \tan^{-1}(x) - \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^4)^(-1), x]

[Out] -ArcTan[x]/2 - ArcTanh[x]/2

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{-1+x^4} dx &= -\left(\frac{1}{2} \int \frac{1}{1-x^2} dx\right) - \frac{1}{2} \int \frac{1}{1+x^2} dx \\ &= -\frac{1}{2} \tan^{-1}(x) - \frac{1}{2} \tanh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.0035192, size = 25, normalized size = 1.92

$$\frac{1}{4} \log(1-x) - \frac{1}{4} \log(x+1) - \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^4)^(-1), x]

[Out] -ArcTan[x]/2 + Log[1 - x]/4 - Log[1 + x]/4

Maple [A] time = 0., size = 10, normalized size = 0.8

$$-\frac{\arctan(x)}{2} - \frac{\operatorname{Artanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4-1), x)

[Out] -1/2*arctan(x)-1/2*arctanh(x)

Maxima [A] time = 1.41753, size = 23, normalized size = 1.77

$$-\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x + 1) + \frac{1}{4} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4-1), x, algorithm="maxima")

[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)

Fricas [A] time = 1.88316, size = 68, normalized size = 5.23

$$-\frac{1}{2} \arctan(x) - \frac{1}{4} \log(x + 1) + \frac{1}{4} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4-1), x, algorithm="fricas")

[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)

Sympy [A] time = 0.114985, size = 17, normalized size = 1.31

$$\frac{\log(x - 1)}{4} - \frac{\log(x + 1)}{4} - \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**4-1), x)

[Out] log(x - 1)/4 - log(x + 1)/4 - atan(x)/2

Giac [B] time = 1.08715, size = 26, normalized size = 2.

$$-\frac{1}{2} \arctan(x) - \frac{1}{4} \log(|x+1|) + \frac{1}{4} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="giac")
```

```
[Out] -1/2*arctan(x) - 1/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

$$3.26 \quad \int \frac{e^x}{2+3e^{2x}} dx$$

Optimal. Leaf size=18

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

[Out] ArcTan[Sqrt[3/2]*E^x]/Sqrt[6]

Rubi [A] time = 0.0204306, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2249, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[E^x/(2 + 3*E^(2*x)),x]

[Out] ArcTan[Sqrt[3/2]*E^x]/Sqrt[6]

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[(d*e*Log[F])/(g*h*Log[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{e^x}{2+3e^{2x}} dx &= \text{Subst} \left(\int \frac{1}{2+3x^2} dx, x, e^x \right) \\ &= \frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}} \end{aligned}$$

Mathematica [A] time = 0.0068214, size = 18, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(2 + 3*E^(2*x)),x]

[Out] ArcTan[Sqrt[3/2]*E^x]/Sqrt[6]

Maple [A] time = 0.004, size = 14, normalized size = 0.8

$$\frac{\sqrt{6}}{6} \arctan\left(\frac{e^x \sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/(2+3*exp(2*x)),x)

[Out] 1/6*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)

Maxima [A] time = 1.41214, size = 18, normalized size = 1.

$$\frac{1}{6} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6} e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="maxima")

[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)

Fricas [A] time = 1.94857, size = 50, normalized size = 2.78

$$\frac{1}{6} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6} e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="fricas")

[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)

Sympy [A] time = 0.106956, size = 15, normalized size = 0.83

$$\text{RootSum}\left(24z^2 + 1, (i \mapsto i \log(4i + e^x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(2+3*exp(2*x)),x)

[Out] RootSum(24*_z**2 + 1, Lambda(_i, _i*log(4*_i + exp(x))))

Giac [A] time = 1.0934, size = 18, normalized size = 1.

$$\frac{1}{6} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6} e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="giac")

[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)

$$3.27 \quad \int \frac{e^{2x}}{A+Be^{4x}} dx$$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

[Out] ArcTan[(Sqrt[B]*E^(2*x))/Sqrt[A]]/(2*Sqrt[A]*Sqrt[B])

Rubi [A] time = 0.0347668, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2249, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

Antiderivative was successfully verified.

[In] Int[E^(2*x)/(A + B*E^(4*x)), x]

[Out] ArcTan[(Sqrt[B]*E^(2*x))/Sqrt[A]]/(2*Sqrt[A]*Sqrt[B])

Rule 2249

Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[(d*e*Log[F])/(g*h*Log[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m]]^p, x], x, G^((h*(f + g*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{e^{2x}}{A+Be^{4x}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{A+Bx^2} dx, x, e^{2x} \right) \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}} \end{aligned}$$

Mathematica [A] time = 0.008076, size = 31, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

Antiderivative was successfully verified.

[In] Integrate[E^(2*x)/(A + B*E^(4*x)),x]

[Out] ArcTan[(Sqrt[B]*E^(2*x))/Sqrt[A]]/(2*Sqrt[A]*Sqrt[B])

Maple [A] time = 0.007, size = 20, normalized size = 0.7

$$\frac{1}{2} \arctan\left(B(e^x)^2 \frac{1}{\sqrt{AB}}\right) \frac{1}{\sqrt{AB}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(2*x)/(A+B*exp(4*x)),x)

[Out] 1/2/(A*B)^(1/2)*arctan(B*exp(x)^2/(A*B)^(1/2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 1.93193, size = 186, normalized size = 6.

$$\left[\frac{\sqrt{-AB} \log\left(\frac{Be^{4x} - 2\sqrt{-AB}e^{2x} - A}{Be^{4x} + A}\right)}{4AB}, \frac{\sqrt{AB} \arctan\left(\frac{\sqrt{AB}e^{-2x}}{B}\right)}{2AB} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="fricas")

[Out] [-1/4*sqrt(-A*B)*log((B*e^(4*x) - 2*sqrt(-A*B)*e^(2*x) - A)/(B*e^(4*x) + A))/(A*B), -1/2*sqrt(A*B)*arctan(sqrt(A*B)*e^(-2*x)/B)/(A*B)]

Sympy [A] time = 0.150645, size = 22, normalized size = 0.71

$$\text{RootSum}\left(16z^2AB + 1, \left(i \mapsto i \log(4iA + e^{2x})\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)/(A+B*exp(4*x)),x)

[Out] RootSum(16*_z**2*A*B + 1, Lambda(_i, _i*log(4*_i*A + exp(2*x))))

Giac [A] time = 1.08468, size = 26, normalized size = 0.84

$$\frac{\arctan\left(\frac{Be^{2x}}{\sqrt{AB}}\right)}{2\sqrt{AB}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="giac")

[Out] 1/2*arctan(B*e^(2*x)/sqrt(A*B))/sqrt(A*B)

3.28 $\int \frac{e^{1+x}}{1+e^x} dx$

Optimal. Leaf size=8

$$e \log(e^x + 1)$$

[Out] E*Log[1 + E^x]

Rubi [A] time = 0.0340458, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2247, 2246, 31}

$$e \log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Int[E^(1 + x)/(1 + E^x), x]

[Out] E*Log[1 + E^x]

Rule 2247

```
Int[((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.))^(p_.)*((G_)^((h_.)*((f_.) + (g_.)*(x_))))^(m_.), x_Symbol] := Dist[(G^(h*(f + g*x)))^m/(F^(e*(c + d*x)))^n, Int[(F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p, x], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d*e*n*Log[F], g*h*m*Log[G]]
```

Rule 2246

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{e^{1+x}}{1+e^x} dx &= e \int \frac{e^x}{1+e^x} dx \\ &= e \operatorname{Subst} \left(\int \frac{1}{1+x} dx, x, e^x \right) \\ &= e \log(1 + e^x) \end{aligned}$$

Mathematica [A] time = 0.0052211, size = 8, normalized size = 1.

$$e \log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[E^(1 + x)/(1 + E^x),x]

[Out] E*Log[1 + E^x]

Maple [A] time = 0.001, size = 9, normalized size = 1.1

$$e \ln(1 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(1+x)/(1+exp(x)),x)

[Out] exp(1)*ln(1+exp(x))

Maxima [A] time = 0.931826, size = 11, normalized size = 1.38

$$e \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1+x)/(1+exp(x)),x, algorithm="maxima")

[Out] e*log(e^x + 1)

Fricas [A] time = 1.94737, size = 30, normalized size = 3.75

$$e \log(e + e^{(x+1)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1+x)/(1+exp(x)),x, algorithm="fricas")

[Out] e*log(e + e^(x + 1))

Sympy [A] time = 0.096068, size = 7, normalized size = 0.88

$$e \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(1+x)/(1+exp(x)),x)

[Out] E*log(exp(x) + 1)

Giac [A] time = 1.0825, size = 11, normalized size = 1.38

$$e \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1+x)/(1+exp(x)),x, algorithm="giac")
```

```
[Out] e*log(e^x + 1)
```

3.29 $\int (10e)^x dx$

Optimal. Leaf size=12

$$\frac{(10e)^x}{1 + \log(10)}$$

[Out] $(10 * E)^x / (1 + \text{Log}[10])$

Rubi [A] time = 0.0055997, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {2194}

$$\frac{(10e)^x}{1 + \log(10)}$$

Antiderivative was successfully verified.

[In] Int[(10 * E)^x, x]

[Out] $(10 * E)^x / (1 + \text{Log}[10])$

Rule 2194

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\int (10e)^x dx = \frac{(10e)^x}{1 + \log(10)}$$

Mathematica [A] time = 0.0036176, size = 12, normalized size = 1.

$$\frac{(10e)^x}{\log(10e)}$$

Antiderivative was successfully verified.

[In] Integrate[(10 * E)^x, x]

[Out] $(10 * E)^x / \text{Log}[10 * E]$

Maple [A] time = 0.01, size = 13, normalized size = 1.1

$$\frac{(10 E)^x}{\ln(10 E)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((10*E)^x,x)`

[Out] $1/\ln(10*E)*(10*E)^x$

Maxima [A] time = 0.930965, size = 16, normalized size = 1.33

$$\frac{(10E)^x}{\log(10E)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((10*E)^x,x, algorithm="maxima")`

[Out] $(10*E)^x/\log(10*E)$

Fricas [A] time = 1.96663, size = 27, normalized size = 2.25

$$\frac{(10E)^x}{\log(10E)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((10*E)^x,x, algorithm="fricas")`

[Out] $(10*E)^x/\log(10*E)$

Sympy [A] time = 0.087169, size = 8, normalized size = 0.67

$$\frac{(10e)^x}{1 + \log(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((10*E)**x,x)`

[Out] $(10*E)**x/(1 + \log(10))$

Giac [A] time = 1.08015, size = 16, normalized size = 1.33

$$\frac{(10E)^x}{\log(10E)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((10*E)^x,x, algorithm="giac")`

[Out] $(10*E)^x/\log(10*E)$

3.30 $\int x^3 \sin(x^2) dx$

Optimal. Leaf size=20

$$\frac{\sin(x^2)}{2} - \frac{1}{2}x^2 \cos(x^2)$$

[Out] $-(x^2 \cos[x^2])/2 + \sin[x^2]/2$

Rubi [A] time = 0.0189159, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3379, 3296, 2637}

$$\frac{\sin(x^2)}{2} - \frac{1}{2}x^2 \cos(x^2)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^3 \sin[x^2], x]$

[Out] $-(x^2 \cos[x^2])/2 + \sin[x^2]/2$

Rule 3379

$\text{Int}[(x_)^{(m_.)} * ((a_.) + (b_.) * \text{Sin}[(c_.) + (d_.) * (x_)^{(n_.)}])^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1) * (a + b * \text{Sin}[c + d * x])^p}, x], x, x^n], x] /;$ $\text{FreeQ}\{a, b, c, d, m, n, p\}, x] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]] \&\& (\text{EqQ}[p, 1] \mid \mid \text{EqQ}[m, n-1] \mid \mid (\text{IntegerQ}[p] \&\& \text{GtQ}[\text{Simplify}[(m+1)/n], 0]))$

Rule 3296

$\text{Int}[(c_.) + (d_.) * (x_)^{(m_.)} * \text{sin}[(e_.) + (f_.) * (x_)], x_Symbol] \rightarrow -\text{Simp}[(c + d * x)^m * \text{Cos}[e + f * x] / f, x] + \text{Dist}[(d * m) / f, \text{Int}[(c + d * x)^{(m-1)} * \text{Cos}[e + f * x], x], x] /;$ $\text{FreeQ}\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

Rule 2637

$\text{Int}[\text{sin}[\text{Pi}/2 + (c_.) + (d_.) * (x_)], x_Symbol] \rightarrow \text{Simp}[\text{Sin}[c + d * x] / d, x] /;$ $\text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int x^3 \sin(x^2) dx &= \frac{1}{2} \text{Subst} \left(\int x \sin(x) dx, x, x^2 \right) \\ &= -\frac{1}{2} x^2 \cos(x^2) + \frac{1}{2} \text{Subst} \left(\int \cos(x) dx, x, x^2 \right) \\ &= -\frac{1}{2} x^2 \cos(x^2) + \frac{\sin(x^2)}{2} \end{aligned}$$

Mathematica [A] time = 0.0017644, size = 20, normalized size = 1.

$$\frac{\sin(x^2)}{2} - \frac{1}{2}x^2 \cos(x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Sin[x^2],x]

[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2

Maple [A] time = 0., size = 17, normalized size = 0.9

$$-\frac{x^2 \cos(x^2)}{2} + \frac{\sin(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*sin(x^2),x)

[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)

Maxima [A] time = 0.93255, size = 22, normalized size = 1.1

$$-\frac{1}{2}x^2 \cos(x^2) + \frac{1}{2} \sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*sin(x^2),x, algorithm="maxima")

[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)

Fricas [A] time = 1.98983, size = 46, normalized size = 2.3

$$-\frac{1}{2}x^2 \cos(x^2) + \frac{1}{2} \sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*sin(x^2),x, algorithm="fricas")

[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)

Sympy [A] time = 0.548402, size = 15, normalized size = 0.75

$$-\frac{x^2 \cos(x^2)}{2} + \frac{\sin(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*sin(x**2),x)

[Out] -x**2*cos(x**2)/2 + sin(x**2)/2

Giac [A] time = 1.07265, size = 22, normalized size = 1.1

$$-\frac{1}{2}x^2 \cos(x^2) + \frac{1}{2} \sin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*sin(x^2),x, algorithm="giac")

[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)

3.31 $\int \frac{x^7}{1+x^{12}} dx$

Optimal. Leaf size=49

$$-\frac{1}{12} \log(x^4 + 1) + \frac{1}{24} \log(x^8 - x^4 + 1) - \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4]/12 + Log[1 - x^4 + x^8]/24

Rubi [A] time = 0.0360088, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {275, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{12} \log(x^4 + 1) + \frac{1}{24} \log(x^8 - x^4 + 1) - \frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x^7/(1 + x^12), x]

[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4]/12 + Log[1 - x^4 + x^8]/24

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :=> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :=> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :=> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :=> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] :=> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2] \cdot x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 628

$\text{Int}[(d_ + (e_ \cdot)(x_))/((a_ + (b_ \cdot)(x_) + (c_ \cdot)(x_)^2), x_Symbol] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

Rubi steps

$$\begin{aligned} \int \frac{x^7}{1+x^{12}} dx &= \frac{1}{4} \text{Subst} \left(\int \frac{x}{1+x^3} dx, x, x^4 \right) \\ &= -\left(\frac{1}{12} \text{Subst} \left(\int \frac{1}{1+x} dx, x, x^4 \right) \right) + \frac{1}{12} \text{Subst} \left(\int \frac{1+x}{1-x+x^2} dx, x, x^4 \right) \\ &= -\frac{1}{12} \log(1+x^4) + \frac{1}{24} \text{Subst} \left(\int \frac{-1+2x}{1-x+x^2} dx, x, x^4 \right) + \frac{1}{8} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^4 \right) \\ &= -\frac{1}{12} \log(1+x^4) + \frac{1}{24} \log(1-x^4+x^8) - \frac{1}{4} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2x^4 \right) \\ &= -\frac{\tan^{-1} \left(\frac{1-2x^4}{\sqrt{3}} \right)}{4\sqrt{3}} - \frac{1}{12} \log(1+x^4) + \frac{1}{24} \log(1-x^4+x^8) \end{aligned}$$

Mathematica [B] time = 0.109162, size = 260, normalized size = 5.31

$$\frac{1}{24} \left(-2 \log(x^2 - \sqrt{2}x + 1) - 2 \log(x^2 + \sqrt{2}x + 1) + \log(2x^2 - \sqrt{6}x + \sqrt{2}x + 2) + \log(2x^2 + \sqrt{2}(\sqrt{3}-1)x + 2) + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^7/(1 + x^12), x]

[Out] (2*Sqrt[3]*ArcTan[(1 + Sqrt[3] - 2*Sqrt[2]*x)/(1 - Sqrt[3])] - 2*Sqrt[3]*ArcTan[(1 - Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] + 2*Sqrt[3]*ArcTan[(-1 + Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] - 2*Sqrt[3]*ArcTan[(1 + Sqrt[3] + 2*Sqrt[2]*x)/(-1 + Sqrt[3])] - 2*Log[1 - Sqrt[2]*x + x^2] - 2*Log[1 + Sqrt[2]*x + x^2] + Log[2 + Sqrt[2]*x - Sqrt[6]*x + 2*x^2] + Log[2 + Sqrt[2]*(-1 + Sqrt[3])*x + 2*x^2] + Log[2 - (Sqrt[2] + Sqrt[6])*x + 2*x^2] + Log[2 + (Sqrt[2] + Sqrt[6])*x + 2*x^2])/24

Maple [A] time = 0., size = 41, normalized size = 0.8

$$-\frac{\ln(x^4 + 1)}{12} + \frac{\ln(x^8 - x^4 + 1)}{24} + \frac{\sqrt{3}}{12} \arctan\left(\frac{(2x^4 - 1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7/(x^12+1), x)

[Out] $-1/12*\ln(x^4+1)+1/24*\ln(x^8-x^4+1)+1/12*3^{(1/2)}*\arctan(1/3*(2*x^4-1)*3^{(1/2)})$

Maxima [A] time = 1.40679, size = 54, normalized size = 1.1

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x^4-1)\right) + \frac{1}{24} \log(x^8-x^4+1) - \frac{1}{12} \log(x^4+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(x^12+1),x, algorithm="maxima")`

[Out] $1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^4 - 1)) + 1/24*\log(x^8 - x^4 + 1) - 1/12*\log(x^4 + 1)$

Fricas [A] time = 1.8426, size = 124, normalized size = 2.53

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x^4-1)\right) + \frac{1}{24} \log(x^8-x^4+1) - \frac{1}{12} \log(x^4+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(x^12+1),x, algorithm="fricas")`

[Out] $1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^4 - 1)) + 1/24*\log(x^8 - x^4 + 1) - 1/12*\log(x^4 + 1)$

Sympy [A] time = 0.158938, size = 46, normalized size = 0.94

$$-\frac{\log(x^4+1)}{12} + \frac{\log(x^8-x^4+1)}{24} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3} - \frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7/(x**12+1),x)`

[Out] $-\log(x**4 + 1)/12 + \log(x**8 - x**4 + 1)/24 + \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x**4/3 - \sqrt{3}/3)/12$

Giac [A] time = 1.09553, size = 54, normalized size = 1.1

$$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x^4-1)\right) + \frac{1}{24} \log(x^8-x^4+1) - \frac{1}{12} \log(x^4+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7/(x^12+1),x, algorithm="giac")`

[Out] $1/12*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x^4 - 1)) + 1/24*\log(x^8 - x^4 + 1) - 1/12*\log(x^4 + 1)$

3.32 $\int x^{3a} \sin(x^{2a}) dx$

Optimal. Leaf size=115

$$\frac{ix^{3a+1}(-ix^{2a})^{-\frac{3a+1}{2a}} \Gamma\left(\frac{1}{2}\left(\frac{1}{a}+3\right), -ix^{2a}\right)}{4a} - \frac{ix^{3a+1}(ix^{2a})^{-\frac{3a+1}{2a}} \Gamma\left(\frac{1}{2}\left(\frac{1}{a}+3\right), ix^{2a}\right)}{4a}$$

[Out] $((I/4)*x^{(1+3*a)}*\Gamma[(3+a^{-1})/2, (-I)*x^{(2*a)}])/(a*((-I)*x^{(2*a)})^{(1+3*a)/(2*a)}) - ((I/4)*x^{(1+3*a)}*\Gamma[(3+a^{-1})/2, I*x^{(2*a)}])/(a*(I*x^{(2*a)})^{((1+3*a)/(2*a))})$

Rubi [A] time = 0.0560074, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3423, 2218}

$$\frac{ix^{3a+1}(-ix^{2a})^{-\frac{3a+1}{2a}} \Gamma\left(\frac{1}{2}\left(\frac{1}{a}+3\right), -ix^{2a}\right)}{4a} - \frac{ix^{3a+1}(ix^{2a})^{-\frac{3a+1}{2a}} \Gamma\left(\frac{1}{2}\left(\frac{1}{a}+3\right), ix^{2a}\right)}{4a}$$

Antiderivative was successfully verified.

[In] Int[x^(3*a)*Sin[x^(2*a)], x]

[Out] $((I/4)*x^{(1+3*a)}*\Gamma[(3+a^{-1})/2, (-I)*x^{(2*a)}])/(a*((-I)*x^{(2*a)})^{(1+3*a)/(2*a)}) - ((I/4)*x^{(1+3*a)}*\Gamma[(3+a^{-1})/2, I*x^{(2*a)}])/(a*(I*x^{(2*a)})^{((1+3*a)/(2*a))})$

Rule 3423

Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2, Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I + d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]

Rule 2218

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^(n_)))*((e_.) + (f_.)*(x_)^(m_.)), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]

Rubi steps

$$\begin{aligned} \int x^{3a} \sin(x^{2a}) dx &= \frac{1}{2}i \int e^{-ix^{2a}} x^{3a} dx - \frac{1}{2}i \int e^{ix^{2a}} x^{3a} dx \\ &= \frac{ix^{1+3a}(-ix^{2a})^{-\frac{1+3a}{2a}} \Gamma\left(\frac{1}{2}\left(3 + \frac{1}{a}\right), -ix^{2a}\right)}{4a} - \frac{ix^{1+3a}(ix^{2a})^{-\frac{1+3a}{2a}} \Gamma\left(\frac{1}{2}\left(3 + \frac{1}{a}\right), ix^{2a}\right)}{4a} \end{aligned}$$

Mathematica [A] time = 0.293061, size = 142, normalized size = 1.23

$$x^{a+1} (x^{4a})^{-\frac{a+1}{2a}} \left((a+1) (-ix^{2a})^{\frac{a+1}{2a}} \Gamma\left(\frac{a+1}{2a}, ix^{2a}\right) + (a+1) (ix^{2a})^{\frac{a+1}{2a}} \Gamma\left(\frac{a+1}{2a}, -ix^{2a}\right) + 4a (x^{4a})^{\frac{a+1}{2a}} \cos(x^{2a}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(3*a)*Sin[x^(2*a)],x]

[Out] $-(x^{(1+a)}(4a(x^{(4a)})^{((1+a)/(2a))}\cos[x^{(2a)}] + (1+a)(I x^{(2a)})^{((1+a)/(2a))}\Gamma[(1+a)/(2a), (-I)x^{(2a)}] + (1+a)((-I)x^{(2a)})^{((1+a)/(2a))}\Gamma[(1+a)/(2a), Ix^{(2a)}]))/(8a^2(x^{(4a)})^{((1+a)/(2a))})$

Maple [C] time = 0.089, size = 41, normalized size = 0.4

$$\frac{x^{5a+1}}{5a+1} {}_1F_2\left(\frac{5}{4} + \frac{1}{4a}; \frac{3}{2}, \frac{9}{4} + \frac{1}{4a}; -\frac{x^{4a}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(3*a)*sin(x^(2*a)),x)

[Out] $1/(5a+1)x^{(5a+1)}\text{hypergeom}([5/4+1/4/a], [3/2, 9/4+1/4/a], -1/4x^{(4a)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{xx^a \cos(x^{2a}) - (a+1) \int x^a \cos(x^{2a}) dx}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="maxima")

[Out] $-1/2*(x*x^a*\cos(x^{(2*a)}) - (a + 1)*\integrate(x^a*\cos(x^{(2*a)}), x))/a$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^{3a} \sin(x^{2a}), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="fricas")

[Out] integral(x^(3*a)*sin(x^(2*a)), x)

Sympy [A] time = 5.48852, size = 54, normalized size = 0.47

$$\frac{xx^{5a}\Gamma\left(\frac{5}{4} + \frac{1}{4a}\right) {}_1F_2\left(\frac{5}{4} + \frac{1}{4a}; \frac{3}{2}, \frac{9}{4} + \frac{1}{4a}; -\frac{x^{4a}}{4}\right)}{4a\Gamma\left(\frac{9}{4} + \frac{1}{4a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x**(3*a)*sin(x**(2*a)),x)
```

```
[Out] x*x**(5*a)*gamma(5/4 + 1/(4*a))*hyper((5/4 + 1/(4*a)), (3/2, 9/4 + 1/(4*a)), -x**(4*a)/4)/(4*a*gamma(9/4 + 1/(4*a)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{3a} \sin(x^{2a}) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="giac")
```

```
[Out] integrate(x^(3*a)*sin(x^(2*a)), x)
```

3.33 $\int \cos(\sqrt{x}) dx$

Optimal. Leaf size=22

$$2\sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]

Rubi [A] time = 0.0100674, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3362, 3296, 2638}

$$2\sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Cos[Sqrt[x]], x]

[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]

Rule 3362

Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[1/n]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \cos(\sqrt{x}) dx &= 2 \text{Subst} \left(\int x \cos(x) dx, x, \sqrt{x} \right) \\ &= 2\sqrt{x} \sin(\sqrt{x}) - 2 \text{Subst} \left(\int \sin(x) dx, x, \sqrt{x} \right) \\ &= 2 \cos(\sqrt{x}) + 2\sqrt{x} \sin(\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.0138431, size = 22, normalized size = 1.

$$2\sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[Cos[Sqrt[x]],x]

[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]

Maple [A] time = 0.01, size = 17, normalized size = 0.8

$$2 \cos(\sqrt{x}) + 2 \sin(\sqrt{x}) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x^(1/2)),x)

[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)

Maxima [A] time = 0.930814, size = 22, normalized size = 1.

$$2 \sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x^(1/2)),x, algorithm="maxima")

[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))

Fricas [A] time = 1.83028, size = 55, normalized size = 2.5

$$2 \sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x^(1/2)),x, algorithm="fricas")

[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))

Sympy [A] time = 0.294918, size = 20, normalized size = 0.91

$$2 \sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x**(1/2)),x)

[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))

Giac [A] time = 1.07938, size = 22, normalized size = 1.

$$2 \sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.34 $\int x\sqrt{1+x} dx$

Optimal. Leaf size=23

$$\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2}$$

[Out] $(-2*(1+x)^{(3/2)})/3 + (2*(1+x)^{(5/2)})/5$

Rubi [A] time = 0.0039135, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {43}

$$\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[1 + x], x]

[Out] $(-2*(1+x)^{(3/2)})/3 + (2*(1+x)^{(5/2)})/5$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int [ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int x\sqrt{1+x} dx &= \int \left(-\sqrt{1+x} + (1+x)^{3/2}\right) dx \\ &= -\frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} \end{aligned}$$

Mathematica [A] time = 0.0031991, size = 16, normalized size = 0.7

$$\frac{2}{15}(x+1)^{3/2}(3x-2)$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[1 + x], x]

[Out] $(2*(1+x)^{(3/2)}*(-2+3*x))/15$

Maple [A] time = 0., size = 13, normalized size = 0.6

$$\frac{-4+6x}{15}(1+x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(1+x)^(1/2),x)`

[Out] $2/15*(1+x)^{(3/2)}*(-2+3*x)$

Maxima [A] time = 0.926539, size = 20, normalized size = 0.87

$$\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+x)^(1/2),x, algorithm="maxima")`

[Out] $2/5*(x + 1)^{(5/2)} - 2/3*(x + 1)^{(3/2)}$

Fricas [A] time = 1.79048, size = 46, normalized size = 2.

$$\frac{2}{15}(3x^2 + x - 2)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+x)^(1/2),x, algorithm="fricas")`

[Out] $2/15*(3*x^2 + x - 2)*\text{sqrt}(x + 1)$

Sympy [A] time = 0.839955, size = 34, normalized size = 1.48

$$\frac{2x^2\sqrt{x+1}}{5} + \frac{2x\sqrt{x+1}}{15} - \frac{4\sqrt{x+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+x)**(1/2),x)`

[Out] $2*x**2*\text{sqrt}(x + 1)/5 + 2*x*\text{sqrt}(x + 1)/15 - 4*\text{sqrt}(x + 1)/15$

Giac [A] time = 1.09204, size = 20, normalized size = 0.87

$$\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+x)^(1/2),x, algorithm="giac")`

[Out] $2/5*(x + 1)^{(5/2)} - 2/3*(x + 1)^{(3/2)}$

$$3.35 \quad \int \frac{1}{\sqrt[3]{x} + \sqrt{x}} dx$$

Optimal. Leaf size=32

$$2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6 \log(\sqrt[6]{x} + 1)$$

[Out] $6*x^{(1/6)} - 3*x^{(1/3)} + 2*\text{Sqrt}[x] - 6*\text{Log}[1 + x^{(1/6)}]$

Rubi [A] time = 0.0136464, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1593, 266, 43}

$$2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6 \log(\sqrt[6]{x} + 1)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^{(1/3)} + \text{Sqrt}[x])^{(-1)}, x]$

[Out] $6*x^{(1/6)} - 3*x^{(1/3)} + 2*\text{Sqrt}[x] - 6*\text{Log}[1 + x^{(1/6)}]$

Rule 1593

$\text{Int}[(u_*)*((a_*)(x_*)^{(p_*)} + (b_*)(x_*)^{(q_*)})^{(n_*)}, x_Symbol] \rightarrow \text{Int}[u*x^{(n*p)}*(a + b*x^{(q-p)})^n, x] /;$ FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 266

$\text{Int}[(x_*)^{(m_*)}*((a_*) + (b_*)(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*x)^p, x}], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m+1)/n]]

Rule 43

$\text{Int}[(a_*) + (b_*)(x_*)^{(m_*)}*((c_*) + (d_*)(x_*)^{(n_*)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[3]{x} + \sqrt{x}} dx &= \int \frac{1}{(1 + \sqrt[6]{x}) \sqrt[3]{x}} dx \\ &= 6 \text{Subst} \left(\int \frac{x^3}{1+x} dx, x, \sqrt[6]{x} \right) \\ &= 6 \text{Subst} \left(\int \left(1 + \frac{1}{-1-x} - x + x^2 \right) dx, x, \sqrt[6]{x} \right) \\ &= 6\sqrt[6]{x} - 3\sqrt[3]{x} + 2\sqrt{x} - 6 \log(1 + \sqrt[6]{x}) \end{aligned}$$

Mathematica [A] time = 0.0125579, size = 32, normalized size = 1.

$$2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6 \log(\sqrt[6]{x} + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(x^(1/3) + Sqrt[x])^(-1),x]

[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]

Maple [B] time = 0., size = 92, normalized size = 2.9

$2 \ln(-1 + \sqrt[6]{x}) - \ln(\sqrt[3]{x} + \sqrt[6]{x} + 1) - 2 \ln(1 + \sqrt[6]{x}) + \ln(\sqrt[3]{x} - \sqrt[6]{x} + 1) + 2\sqrt{x} + \ln(\sqrt{x} - 1) - \ln(\sqrt{x} + 1) + 6\sqrt[6]{x} - \ln$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/3)+x^(1/2)),x)

[Out] 2*ln(-1+x^(1/6))-ln(x^(1/3)+x^(1/6)+1)-2*ln(1+x^(1/6))+ln(x^(1/3)-x^(1/6)+1)+2*x^(1/2)+ln(x^(1/2)-1)-ln(x^(1/2)+1)+6*x^(1/6)-ln(-1+x)-2*ln(-1+x^(1/3))+ln(x^(2/3)+x^(1/3)+1)-3*x^(1/3)

Maxima [A] time = 0.930065, size = 32, normalized size = 1.

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")

[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)

Fricas [A] time = 1.73119, size = 76, normalized size = 2.38

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")

[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**(1/3)+x**(1/2)),x)

[Out] Integral(1/(x**(1/3) + sqrt(x)), x)

Giac [A] time = 1.07626, size = 32, normalized size = 1.

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="giac")

[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)

$$3.36 \quad \int \sqrt{\frac{1+x}{3+2x}} dx$$

Optimal. Leaf size=44

$$\frac{1}{2}\sqrt{x+1}\sqrt{2x+3} - \frac{\sinh^{-1}(\sqrt{2}\sqrt{x+1})}{2\sqrt{2}}$$

[Out] (Sqrt[1 + x]*Sqrt[3 + 2*x])/2 - ArcSinh[Sqrt[2]*Sqrt[1 + x]]/(2*Sqrt[2])

Rubi [A] time = 0.0162064, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1958, 50, 54, 215}

$$\frac{1}{2}\sqrt{x+1}\sqrt{2x+3} - \frac{\sinh^{-1}(\sqrt{2}\sqrt{x+1})}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(1 + x)/(3 + 2*x)], x]

[Out] (Sqrt[1 + x]*Sqrt[3 + 2*x])/2 - ArcSinh[Sqrt[2]*Sqrt[1 + x]]/(2*Sqrt[2])

Rule 1958

Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_.) + (d_.)*(x_)^(n_.))]^(p_), x_Symbol] := Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]

Rule 50

Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 54

Int[1/(Sqrt[(a_.) + (b_.)*(x_)])*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{1+x}{3+2x}} dx &= \int \frac{\sqrt{1+x}}{\sqrt{3+2x}} dx \\
&= \frac{1}{2} \sqrt{1+x} \sqrt{3+2x} - \frac{1}{4} \int \frac{1}{\sqrt{1+x} \sqrt{3+2x}} dx \\
&= \frac{1}{2} \sqrt{1+x} \sqrt{3+2x} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1+2x^2}} dx, x, \sqrt{1+x} \right) \\
&= \frac{1}{2} \sqrt{1+x} \sqrt{3+2x} - \frac{\sinh^{-1}(\sqrt{2}\sqrt{1+x})}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.0358298, size = 71, normalized size = 1.61

$$\frac{2(x+1)\sqrt{2x+3} - \sqrt{2}\sqrt{x+1} \sinh^{-1}(\sqrt{2}\sqrt{x+1})}{4\sqrt{\frac{x+1}{2x+3}}\sqrt{2x+3}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(1 + x)/(3 + 2*x)], x]

[Out] (2*(1 + x)*Sqrt[3 + 2*x] - Sqrt[2]*Sqrt[1 + x]*ArcSinh[Sqrt[2]*Sqrt[1 + x]])/(4*Sqrt[(1 + x)/(3 + 2*x)]*Sqrt[3 + 2*x])

Maple [B] time = 0.01, size = 75, normalized size = 1.7

$$-\frac{3+2x}{8} \sqrt{\frac{1+x}{3+2x}} \left(\ln \left(\frac{5\sqrt{2}}{4} + x\sqrt{2} + \sqrt{2x^2+5x+3} \right) \sqrt{2} - 4\sqrt{2x^2+5x+3} \right) \frac{1}{\sqrt{(3+2x)(1+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((1+x)/(3+2*x))^(1/2), x)

[Out] -1/8*((1+x)/(3+2*x))^(1/2)*(3+2*x)*(ln(5/4*2^(1/2)+x*2^(1/2)+(2*x^2+5*x+3)^(1/2))*2^(1/2)-4*(2*x^2+5*x+3)^(1/2))/((3+2*x)*(1+x))^(1/2)

Maxima [B] time = 1.40605, size = 108, normalized size = 2.45

$$\frac{1}{8} \sqrt{2} \log \left(\frac{\sqrt{2} - 2\sqrt{\frac{x+1}{2x+3}}}{\sqrt{2} + 2\sqrt{\frac{x+1}{2x+3}}} \right) - \frac{\sqrt{\frac{x+1}{2x+3}}}{2 \left(\frac{2(x+1)}{2x+3} - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)/(3+2*x))^(1/2), x, algorithm="maxima")

[Out] 1/8*sqrt(2)*log(-(sqrt(2) - 2*sqrt((x + 1)/(2*x + 3)))/(sqrt(2) + 2*sqrt((x + 1)/(2*x + 3)))) - 1/2*sqrt((x + 1)/(2*x + 3))/(2*(x + 1)/(2*x + 3) - 1)

Fricas [A] time = 1.83084, size = 151, normalized size = 3.43

$$\frac{1}{2}(2x+3)\sqrt{\frac{x+1}{2x+3}} + \frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}(2x+3)\sqrt{\frac{x+1}{2x+3}} - 4x - 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)/(3+2*x))^(1/2),x, algorithm="fricas")

[Out] 1/2*(2*x + 3)*sqrt((x + 1)/(2*x + 3)) + 1/8*sqrt(2)*log(2*sqrt(2)*(2*x + 3)*sqrt((x + 1)/(2*x + 3)) - 4*x - 5)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{x+1}{2x+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)/(3+2*x))**(1/2),x)

[Out] Integral(sqrt((x + 1)/(2*x + 3)), x)

Giac [A] time = 1.10489, size = 82, normalized size = 1.86

$$\frac{1}{8}\sqrt{2}\log\left(\left|-2\sqrt{2}\left(\sqrt{2}x - \sqrt{2x^2 + 5x + 3}\right) - 5\right|\right)\operatorname{sgn}(2x+3) + \frac{1}{2}\sqrt{2x^2 + 5x + 3}\operatorname{sgn}(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)/(3+2*x))^(1/2),x, algorithm="giac")

[Out] 1/8*sqrt(2)*log(abs(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 + 5*x + 3)) - 5))*sgn(2*x + 3) + 1/2*sqrt(2*x^2 + 5*x + 3)*sgn(2*x + 3)

$$3.37 \quad \int \frac{x^4}{(1-x^2)^{5/2}} dx$$

Optimal. Leaf size=35

$$\frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$$

[Out] $x^3/(3*(1-x^2)^{(3/2)}) - x/\text{Sqrt}[1-x^2] + \text{ArcSin}[x]$

Rubi [A] time = 0.0086468, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {288, 216}

$$\frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^4/(1-x^2)^{(5/2)}, x]$

[Out] $x^3/(3*(1-x^2)^{(3/2)}) - x/\text{Sqrt}[1-x^2] + \text{ArcSin}[x]$

Rule 288

$\text{Int}[(c_.)*(x_)^{(m_.)*((a_.) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a+b*x^n)^{(p+1)})/(b*n*(p+1)), x] - \text{Dist}[(c^n*(m-n+1))/(b*n*(p+1)), \text{Int}[(c*x)^{(m-n)}*(a+b*x^n)^{(p+1)}, x], x] /;$ FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !ILtQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 216

$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /;$ FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned} \int \frac{x^4}{(1-x^2)^{5/2}} dx &= \frac{x^3}{3(1-x^2)^{3/2}} - \int \frac{x^2}{(1-x^2)^{3/2}} dx \\ &= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}} dx \\ &= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.0242349, size = 26, normalized size = 0.74

$$\frac{x(4x^2-3)}{3(1-x^2)^{3/2}} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(1 - x^2)^(5/2),x]

[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]

Maple [A] time = 0.004, size = 30, normalized size = 0.9

$$\frac{x^3}{3} (-x^2 + 1)^{-\frac{3}{2}} + \arcsin(x) - x \frac{1}{\sqrt{-x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(-x^2+1)^(5/2),x)

[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)

Maxima [A] time = 1.42494, size = 59, normalized size = 1.69

$$\frac{1}{3} x \left(\frac{3x^2}{(-x^2 + 1)^{\frac{3}{2}}} - \frac{2}{(-x^2 + 1)^{\frac{3}{2}}} \right) - \frac{x}{3\sqrt{-x^2 + 1}} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")

[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1) + arcsin(x)

Fricas [B] time = 1.83471, size = 146, normalized size = 4.17

$$\frac{6(x^4 - 2x^2 + 1) \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - (4x^3 - 3x)\sqrt{-x^2+1}}{3(x^4 - 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")

[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sqrt(-x^2 + 1))/(x^4 - 2*x^2 + 1)

Sympy [B] time = 2.60571, size = 105, normalized size = 3.

$$\frac{3x^4 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3} + \frac{4x^3 \sqrt{1 - x^2}}{3x^4 - 6x^2 + 3} - \frac{6x^2 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3} - \frac{3x \sqrt{1 - x^2}}{3x^4 - 6x^2 + 3} + \frac{3 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(-x**2+1)**(5/2),x)

[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x**2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)

Giac [A] time = 1.08346, size = 39, normalized size = 1.11

$$\frac{(4x^2 - 3)\sqrt{-x^2 + 1}x}{3(x^2 - 1)^2} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")

[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)

3.38 $\int \sqrt{x}(1+x)^{5/2} dx$

Optimal. Leaf size=75

$$\frac{1}{4}x^{3/2}(x+1)^{5/2} + \frac{5}{24}x^{3/2}(x+1)^{3/2} + \frac{5}{32}x^{3/2}\sqrt{x+1} + \frac{5}{64}\sqrt{x}\sqrt{x+1} - \frac{5}{64}\sinh^{-1}(\sqrt{x})$$

[Out] (5*Sqrt[x]*Sqrt[1 + x])/64 + (5*x^(3/2)*Sqrt[1 + x])/32 + (5*x^(3/2)*(1 + x)^(3/2))/24 + (x^(3/2)*(1 + x)^(5/2))/4 - (5*ArcSinh[Sqrt[x]])/64

Rubi [A] time = 0.0130271, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {50, 54, 215}

$$\frac{1}{4}x^{3/2}(x+1)^{5/2} + \frac{5}{24}x^{3/2}(x+1)^{3/2} + \frac{5}{32}x^{3/2}\sqrt{x+1} + \frac{5}{64}\sqrt{x}\sqrt{x+1} - \frac{5}{64}\sinh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]*(1 + x)^(5/2), x]

[Out] (5*Sqrt[x]*Sqrt[1 + x])/64 + (5*x^(3/2)*Sqrt[1 + x])/32 + (5*x^(3/2)*(1 + x)^(3/2))/24 + (x^(3/2)*(1 + x)^(5/2))/4 - (5*ArcSinh[Sqrt[x]])/64

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] := Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x}(1+x)^{5/2} dx &= \frac{1}{4}x^{3/2}(1+x)^{5/2} + \frac{5}{8} \int \sqrt{x}(1+x)^{3/2} dx \\
&= \frac{5}{24}x^{3/2}(1+x)^{3/2} + \frac{1}{4}x^{3/2}(1+x)^{5/2} + \frac{5}{16} \int \sqrt{x}\sqrt{1+x} dx \\
&= \frac{5}{32}x^{3/2}\sqrt{1+x} + \frac{5}{24}x^{3/2}(1+x)^{3/2} + \frac{1}{4}x^{3/2}(1+x)^{5/2} + \frac{5}{64} \int \frac{\sqrt{x}}{\sqrt{1+x}} dx \\
&= \frac{5}{64}\sqrt{x}\sqrt{1+x} + \frac{5}{32}x^{3/2}\sqrt{1+x} + \frac{5}{24}x^{3/2}(1+x)^{3/2} + \frac{1}{4}x^{3/2}(1+x)^{5/2} - \frac{5}{128} \int \frac{1}{\sqrt{x}\sqrt{1+x}} dx \\
&= \frac{5}{64}\sqrt{x}\sqrt{1+x} + \frac{5}{32}x^{3/2}\sqrt{1+x} + \frac{5}{24}x^{3/2}(1+x)^{3/2} + \frac{1}{4}x^{3/2}(1+x)^{5/2} - \frac{5}{64} \text{Subst} \left(\int \frac{1}{\sqrt{1+x^2}} dx, x \right) \\
&= \frac{5}{64}\sqrt{x}\sqrt{1+x} + \frac{5}{32}x^{3/2}\sqrt{1+x} + \frac{5}{24}x^{3/2}(1+x)^{3/2} + \frac{1}{4}x^{3/2}(1+x)^{5/2} - \frac{5}{64} \sinh^{-1}(\sqrt{x})
\end{aligned}$$

Mathematica [A] time = 0.0194251, size = 41, normalized size = 0.55

$$\frac{1}{192} \left(\sqrt{x}\sqrt{x+1} (48x^3 + 136x^2 + 118x + 15) - 15 \sinh^{-1}(\sqrt{x}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]*(1+x)^(5/2),x]

[Out] (Sqrt[x]*Sqrt[1+x]*(15+118*x+136*x^2+48*x^3)-15*ArcSinh[Sqrt[x]])/192

Maple [A] time = 0.003, size = 70, normalized size = 0.9

$$\frac{1}{4}\sqrt{x}(1+x)^{7/2} - \frac{1}{24}\sqrt{x}(1+x)^{5/2} - \frac{5}{96}\sqrt{x}(1+x)^{3/2} - \frac{5}{64}\sqrt{x}\sqrt{1+x} - \frac{5}{128}\sqrt{x(1+x)} \ln\left(\frac{1}{2} + x + \sqrt{x^2+x}\right) \frac{1}{\sqrt{x}} \frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)*(1+x)^(5/2),x)

[Out] 1/4*x^(1/2)*(1+x)^(7/2)-1/24*x^(1/2)*(1+x)^(5/2)-5/96*x^(1/2)*(1+x)^(3/2)-5/64*x^(1/2)*(1+x)^(1/2)-5/128*(x*(1+x))^(1/2)/(1+x)^(1/2)/x^(1/2)*ln(1/2+x+(x^2+x)^(1/2))

Maxima [B] time = 0.943377, size = 153, normalized size = 2.04

$$\frac{\frac{15(x+1)^{7/2}}{x^2} + \frac{73(x+1)^{5/2}}{x^2} - \frac{55(x+1)^{3/2}}{x^2} + \frac{15\sqrt{x+1}}{\sqrt{x}}}{192 \left(\frac{(x+1)^4}{x^4} - \frac{4(x+1)^3}{x^3} + \frac{6(x+1)^2}{x^2} - \frac{4(x+1)}{x} + 1 \right)} - \frac{5}{128} \log\left(\frac{\sqrt{x+1}}{\sqrt{x}} + 1\right) + \frac{5}{128} \log\left(\frac{\sqrt{x+1}}{\sqrt{x}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="maxima")

```
[Out] 1/192*(15*(x + 1)^(7/2)/x^(7/2) + 73*(x + 1)^(5/2)/x^(5/2) - 55*(x + 1)^(3/2)/x^(3/2) + 15*sqrt(x + 1)/sqrt(x))/((x + 1)^4/x^4 - 4*(x + 1)^3/x^3 + 6*(x + 1)^2/x^2 - 4*(x + 1)/x + 1) - 5/128*log(sqrt(x + 1)/sqrt(x) + 1) + 5/128*log(sqrt(x + 1)/sqrt(x) - 1)
```

Fricas [A] time = 1.79587, size = 140, normalized size = 1.87

$$\frac{1}{192} (48x^3 + 136x^2 + 118x + 15)\sqrt{x+1}\sqrt{x} + \frac{5}{128} \log\left(2\sqrt{x+1}\sqrt{x} - 2x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(48*x^3 + 136*x^2 + 118*x + 15)*sqrt(x + 1)*sqrt(x) + 5/128*log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

Sympy [A] time = 11.9038, size = 190, normalized size = 2.53

$$\begin{cases} -\frac{5 \operatorname{acosh}(\sqrt{x+1})}{64} + \frac{(x+1)^{\frac{9}{2}}}{4\sqrt{x}} - \frac{7(x+1)^{\frac{7}{2}}}{24\sqrt{x}} - \frac{(x+1)^{\frac{5}{2}}}{96\sqrt{x}} - \frac{5(x+1)^{\frac{3}{2}}}{192\sqrt{x}} + \frac{5\sqrt{x+1}}{64\sqrt{x}} & \text{for } |x+1| > 1 \\ \frac{5i \operatorname{asin}(\sqrt{x+1})}{64} - \frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{-x}} + \frac{7i(x+1)^{\frac{7}{2}}}{24\sqrt{-x}} + \frac{i(x+1)^{\frac{5}{2}}}{96\sqrt{-x}} + \frac{5i(x+1)^{\frac{3}{2}}}{192\sqrt{-x}} - \frac{5i\sqrt{x+1}}{64\sqrt{-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-5*acosh(sqrt(x + 1))/64 + (x + 1)**(9/2)/(4*sqrt(x)) - 7*(x + 1)**(7/2)/(24*sqrt(x)) - (x + 1)**(5/2)/(96*sqrt(x)) - 5*(x + 1)**(3/2)/(192*sqrt(x)) + 5*sqrt(x + 1)/(64*sqrt(x)), Abs(x + 1) > 1), (5*I*asin(sqrt(x + 1))/64 - I*(x + 1)**(9/2)/(4*sqrt(-x)) + 7*I*(x + 1)**(7/2)/(24*sqrt(-x)) + I*(x + 1)**(5/2)/(96*sqrt(-x)) + 5*I*(x + 1)**(3/2)/(192*sqrt(-x)) - 5*I*sqrt(x + 1)/(64*sqrt(-x)), True))
```

Giac [A] time = 1.27791, size = 111, normalized size = 1.48

$$\frac{1}{192} (2(4(6x - 11)(x + 1) + 59)(x + 1) - 15)\sqrt{x+1}\sqrt{x} + \frac{1}{12} (2(4x - 3)(x + 1) + 3)\sqrt{x+1}\sqrt{x} + \frac{1}{4} (2x + 1)\sqrt{x+1}\sqrt{x} + \frac{1}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/192*(2*(4*(6*x - 11)*(x + 1) + 59)*(x + 1) - 15)*sqrt(x + 1)*sqrt(x) + 1/12*(2*(4*x - 3)*(x + 1) + 3)*sqrt(x + 1)*sqrt(x) + 1/4*(2*x + 1)*sqrt(x + 1)*sqrt(x) + 5/64*log(abs(-sqrt(x + 1) + sqrt(x)))
```

$$3.39 \quad \int \frac{x^4}{(1-x^2)^{5/2}} dx$$

Optimal. Leaf size=35

$$\frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$$

[Out] $x^3/(3*(1-x^2)^{(3/2)}) - x/\text{Sqrt}[1-x^2] + \text{ArcSin}[x]$

Rubi [A] time = 0.0076016, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {288, 216}

$$\frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^4/(1-x^2)^{(5/2)}, x]$

[Out] $x^3/(3*(1-x^2)^{(3/2)}) - x/\text{Sqrt}[1-x^2] + \text{ArcSin}[x]$

Rule 288

$\text{Int}[(c_.)*(x_)^{(m_.)*((a_.) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a+b*x^n)^{(p+1)})/(b*n*(p+1)), x] - \text{Dist}[(c^n*(m-n+1))/(b*n*(p+1)), \text{Int}[(c*x)^{(m-n)}*(a+b*x^n)^{(p+1)}, x], x] /;$ FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !ILtQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 216

$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /;$ FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned} \int \frac{x^4}{(1-x^2)^{5/2}} dx &= \frac{x^3}{3(1-x^2)^{3/2}} - \int \frac{x^2}{(1-x^2)^{3/2}} dx \\ &= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}} dx \\ &= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.004046, size = 26, normalized size = 0.74

$$\frac{x(4x^2-3)}{3(1-x^2)^{3/2}} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(1 - x^2)^(5/2),x]

[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]

Maple [A] time = 0., size = 30, normalized size = 0.9

$$\frac{x^3}{3} (-x^2 + 1)^{-\frac{3}{2}} + \arcsin(x) - x \frac{1}{\sqrt{-x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(-x^2+1)^(5/2),x)

[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)

Maxima [A] time = 1.41915, size = 59, normalized size = 1.69

$$\frac{1}{3} x \left(\frac{3x^2}{(-x^2 + 1)^{\frac{3}{2}}} - \frac{2}{(-x^2 + 1)^{\frac{3}{2}}} \right) - \frac{x}{3\sqrt{-x^2 + 1}} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")

[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1) + arcsin(x)

Fricas [B] time = 1.86975, size = 146, normalized size = 4.17

$$\frac{6(x^4 - 2x^2 + 1) \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - (4x^3 - 3x)\sqrt{-x^2+1}}{3(x^4 - 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")

[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sqrt(-x^2 + 1))/(x^4 - 2*x^2 + 1)

Sympy [B] time = 2.56822, size = 105, normalized size = 3.

$$\frac{3x^4 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3} + \frac{4x^3 \sqrt{1 - x^2}}{3x^4 - 6x^2 + 3} - \frac{6x^2 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3} - \frac{3x \sqrt{1 - x^2}}{3x^4 - 6x^2 + 3} + \frac{3 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(-x**2+1)**(5/2),x)

[Out] $3x^{**4}\operatorname{asin}(x)/(3x^{**4} - 6x^{**2} + 3) + 4x^{**3}\sqrt{1 - x^{**2}}/(3x^{**4} - 6x^{**2} + 3) - 6x^{**2}\operatorname{asin}(x)/(3x^{**4} - 6x^{**2} + 3) - 3x\sqrt{1 - x^{**2}}/(3x^{**4} - 6x^{**2} + 3) + 3\operatorname{asin}(x)/(3x^{**4} - 6x^{**2} + 3)$

Giac [A] time = 1.09872, size = 39, normalized size = 1.11

$$\frac{(4x^2 - 3)\sqrt{-x^2 + 1}x}{3(x^2 - 1)^2} + \operatorname{arcsin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")

[Out] $1/3*(4*x^2 - 3)*\sqrt{-x^2 + 1}*x/(x^2 - 1)^2 + \operatorname{arcsin}(x)$

$$3.40 \quad \int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2} dy$$

Optimal. Leaf size=51

$$B \tan^{-1}\left(\frac{By}{\sqrt{A^2 - B^2y^2 + B^2}}\right) + A \tanh^{-1}\left(\frac{Ay}{\sqrt{A^2 - B^2y^2 + B^2}}\right)$$

[Out] B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]] + A*ArcTanh[(A*y)/Sqrt[A^2 + B^2 - B^2*y^2]]

Rubi [A] time = 0.0285577, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {402, 217, 203, 377, 206}

$$B \tan^{-1}\left(\frac{By}{\sqrt{A^2 - B^2y^2 + B^2}}\right) + A \tanh^{-1}\left(\frac{Ay}{\sqrt{A^2 - B^2y^2 + B^2}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[A^2 + B^2 - B^2*y^2]/(1 - y^2),y]

[Out] B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]] + A*ArcTanh[(A*y)/Sqrt[A^2 + B^2 - B^2*y^2]]

Rule 402

Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)^2), x_Symbol] := Dist[b/d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{A^2 + B^2 - B^2 y^2}}{1 - y^2} dy &= A^2 \int \frac{1}{(1 - y^2) \sqrt{A^2 + B^2 - B^2 y^2}} dy + B^2 \int \frac{1}{\sqrt{A^2 + B^2 - B^2 y^2}} dy \\ &= A^2 \operatorname{Subst} \left(\int \frac{1}{1 - A^2 y^2} dy, y, \frac{y}{\sqrt{A^2 + B^2 - B^2 y^2}} \right) + B^2 \operatorname{Subst} \left(\int \frac{1}{1 + B^2 y^2} dy, y, \frac{y}{\sqrt{A^2 + B^2 - B^2 y^2}} \right) \\ &= B \tan^{-1} \left(\frac{By}{\sqrt{A^2 + B^2 - B^2 y^2}} \right) + A \tanh^{-1} \left(\frac{Ay}{\sqrt{A^2 + B^2 - B^2 y^2}} \right) \end{aligned}$$

Mathematica [C] time = 0.0291717, size = 134, normalized size = 2.63

$$iB \log \left(2\sqrt{A^2 - B^2 y^2 + B^2} - 2iBy \right) + \frac{1}{2} A \log \left(A\sqrt{A^2 - B^2 y^2 + B^2} + A^2 - B^2 y + B^2 \right) - \frac{1}{2} A \log \left(A\sqrt{A^2 - B^2 y^2 + B^2} + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[A^2 + B^2 - B^2*y^2]/(1 - y^2), y]

[Out] -(A*Log[1 - y])/2 + (A*Log[1 + y])/2 + I*B*Log[(-2*I)*B*y + 2*Sqrt[A^2 + B^2 - B^2*y^2]] + (A*Log[A^2 + B^2 - B^2*y + A*Sqrt[A^2 + B^2 - B^2*y^2]])/2 - (A*Log[A^2 + B^2 + B^2*y + A*Sqrt[A^2 + B^2 - B^2*y^2]])/2

Maple [B] time = 0.022, size = 262, normalized size = 5.1

$$\frac{1}{2} \sqrt{-B^2(1+y)^2 + 2B^2(1+y) + A^2} + \frac{B^2}{2} \arctan \left(y\sqrt{B^2} \frac{1}{\sqrt{-B^2(1+y)^2 + 2B^2(1+y) + A^2}} \right) \frac{1}{\sqrt{B^2}} - \frac{A^2}{2} \ln \left(\frac{1}{1+y} \left(2 \dots \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1), y)

[Out] 1/2*(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2)+1/2*B^2/(B^2)^(1/2)*arctan((B^2)^(1/2)*y/(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2))-1/2*A^2/(A^2)^(1/2)*ln((2*A^2+2*B^2*(1+y)+2*(A^2)^(1/2)*(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2))/(1+y))-1/2*(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2)+1/2*B^2/(B^2)^(1/2)*arctan((B^2)^(1/2)*y/(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2))+1/2*A^2/(A^2)^(1/2)*ln((2*A^2-2*B^2*(y-1)+2*(A^2)^(1/2)*(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2))/(y-1))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1), y, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.02532, size = 285, normalized size = 5.59

$$-B \arctan\left(\frac{\sqrt{-B^2y^2 + A^2 + B^2}}{By}\right) + \frac{1}{4}A \log\left(-\frac{(A^2 - B^2)y^2 + 2\sqrt{-B^2y^2 + A^2 + B^2}Ay + A^2 + B^2}{y^2}\right) - \frac{1}{4}A \log\left(-\frac{(A^2 - B^2)y^2 - 2\sqrt{-B^2y^2 + A^2 + B^2}Ay + A^2 + B^2}{y^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y, algorithm="fricas")

[Out] -B*arctan(sqrt(-B^2*y^2 + A^2 + B^2)/(B*y)) + 1/4*A*log(-((A^2 - B^2)*y^2 + 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2) - 1/4*A*log(-((A^2 - B^2)*y^2 - 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{A^2 - B^2y^2 + B^2}}{y^2 - 1} dy$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-B**2*y**2+A**2+B**2)**(1/2)/(-y**2+1),y)

[Out] -Integral(sqrt(A**2 - B**2*y**2 + B**2)/(y**2 - 1), y)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y, algorithm="giac")

[Out] Timed out

3.41 $\int \sin^2(x) dx$

Optimal. Leaf size=14

$$\frac{x}{2} - \frac{1}{2} \sin(x) \cos(x)$$

[Out] x/2 - (Cos[x]*Sin[x])/2

Rubi [A] time = 0.0057409, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {2635, 8}

$$\frac{x}{2} - \frac{1}{2} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^2,x]

[Out] x/2 - (Cos[x]*Sin[x])/2

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x] *(b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \sin^2(x) dx &= -\frac{1}{2} \cos(x) \sin(x) + \frac{\int 1 dx}{2} \\ &= \frac{x}{2} - \frac{1}{2} \cos(x) \sin(x) \end{aligned}$$

Mathematica [A] time = 0.0016437, size = 14, normalized size = 1.

$$\frac{x}{2} - \frac{1}{4} \sin(2x)$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^2,x]

[Out] x/2 - Sin[2*x]/4

Maple [A] time = 0., size = 11, normalized size = 0.8

$$\frac{x}{2} - \frac{\cos(x) \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^2,x)

[Out] 1/2*x-1/2*cos(x)*sin(x)

Maxima [A] time = 0.925094, size = 14, normalized size = 1.

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2,x, algorithm="maxima")

[Out] 1/2*x - 1/4*sin(2*x)

Fricas [A] time = 1.98491, size = 38, normalized size = 2.71

$$-\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2,x, algorithm="fricas")

[Out] -1/2*cos(x)*sin(x) + 1/2*x

Sympy [A] time = 0.057188, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(x) \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**2,x)

[Out] x/2 - sin(x)*cos(x)/2

Giac [A] time = 1.13738, size = 14, normalized size = 1.

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

3.42 $\int \csc(x) \sqrt{A^2 + B^2 \sin^2(x)} dx$

Optimal. Leaf size=49

$$-B \tan^{-1} \left(\frac{B \cos(x)}{\sqrt{A^2 + B^2 \sin^2(x)}} \right) - A \tanh^{-1} \left(\frac{A \cos(x)}{\sqrt{A^2 + B^2 \sin^2(x)}} \right)$$

[Out] $-(B \cdot \text{ArcTan}[(B \cdot \text{Cos}[x]) / \text{Sqrt}[A^2 + B^2 \cdot \text{Sin}[x]^2]]) - A \cdot \text{ArcTanh}[(A \cdot \text{Cos}[x]) / \text{Sqrt}[A^2 + B^2 \cdot \text{Sin}[x]^2]]$

Rubi [A] time = 0.0846305, antiderivative size = 57, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3186, 402, 217, 203, 377, 206}

$$-B \tan^{-1} \left(\frac{B \cos(x)}{\sqrt{A^2 - B^2 \cos^2(x) + B^2}} \right) - A \tanh^{-1} \left(\frac{A \cos(x)}{\sqrt{A^2 - B^2 \cos^2(x) + B^2}} \right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[x] \cdot \text{Sqrt}[A^2 + B^2 \cdot \text{Sin}[x]^2], x]$

[Out] $-(B \cdot \text{ArcTan}[(B \cdot \text{Cos}[x]) / \text{Sqrt}[A^2 + B^2 - B^2 \cdot \text{Cos}[x]^2]]) - A \cdot \text{ArcTanh}[(A \cdot \text{Cos}[x]) / \text{Sqrt}[A^2 + B^2 - B^2 \cdot \text{Cos}[x]^2]]$

Rule 3186

$\text{Int}[\sin[(e \cdot) + (f \cdot)(x)]^{(m \cdot)} \cdot ((a \cdot) + (b \cdot) \cdot \sin[(e \cdot) + (f \cdot)(x)]^2)^{(p \cdot)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Cos}[e + f \cdot x], x]\}, -\text{Dist}[ff/f, \text{Subst}[\text{Int}[(1 - ff^2 \cdot x^2)^{(m-1)/2} \cdot (a + b - b \cdot ff^2 \cdot x^2)^p, x], x, \text{Cos}[e + f \cdot x]/ff], x] /; \text{FreeQ}\{a, b, e, f, p\}, x \ \&\& \ \text{IntegerQ}[(m-1)/2]$

Rule 402

$\text{Int}[(a \cdot) + (b \cdot)(x)^2]^{(p \cdot)} / ((c \cdot) + (d \cdot)(x)^2), x_Symbol] \rightarrow \text{Dist}[b/d, \text{Int}[(a + b \cdot x^2)^{(p-1)}, x], x] - \text{Dist}[(b \cdot c - a \cdot d)/d, \text{Int}[(a + b \cdot x^2)^{(p-1)} / (c + d \cdot x^2), x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{GtQ}[p, 0] \ \&\& \ (\text{EqQ}[p, 1/2] \ || \ \text{EqQ}[\text{Denominator}[p], 4])$

Rule 217

$\text{Int}[1/\text{Sqrt}[(a \cdot) + (b \cdot)(x)^2], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b \cdot x^2), x], x, x/\text{Sqrt}[a + b \cdot x^2]] /; \text{FreeQ}\{a, b\}, x \ \&\& \ !\text{GtQ}[a, 0]$

Rule 203

$\text{Int}[(a \cdot) + (b \cdot)(x)^2]^{(-1)}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTan}[(\text{Rt}[b, 2] \cdot x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 377

$\text{Int}[(a \cdot) + (b \cdot)(x)^{(n \cdot)}]^{(p \cdot)} / ((c \cdot) + (d \cdot)(x)^{(n \cdot)}), x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(c - (b \cdot c - a \cdot d) \cdot x^n), x], x, x/(a + b \cdot x^n)^{(1/n)}] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{EqQ}[n \cdot p + 1, 0] \ \&\& \ \text{IntegerQ}[n]$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \csc(x) \sqrt{A^2 + B^2 \sin^2(x)} dx &= -\text{Subst} \left(\int \frac{\sqrt{A^2 + B^2 - B^2 x^2}}{1 - x^2} dx, x, \cos(x) \right) \\ &= - \left(A^2 \text{Subst} \left(\int \frac{1}{(1 - x^2) \sqrt{A^2 + B^2 - B^2 x^2}} dx, x, \cos(x) \right) \right) - B^2 \text{Subst} \left(\int \frac{1}{\sqrt{A^2 + B^2 - B^2 x^2}} dx, x, \cos(x) \right) \\ &= - \left(A^2 \text{Subst} \left(\int \frac{1}{1 - A^2 x^2} dx, x, \frac{\cos(x)}{\sqrt{A^2 + B^2 - B^2 \cos^2(x)}} \right) \right) - B^2 \text{Subst} \left(\int \frac{1}{1 + B^2 x^2} dx, x, \frac{\cos(x)}{\sqrt{A^2 + B^2 - B^2 \cos^2(x)}} \right) \\ &= -B \tan^{-1} \left(\frac{B \cos(x)}{\sqrt{A^2 + B^2 - B^2 \cos^2(x)}} \right) - A \tanh^{-1} \left(\frac{A \cos(x)}{\sqrt{A^2 + B^2 - B^2 \cos^2(x)}} \right) \end{aligned}$$

Mathematica [B] time = 0.106308, size = 99, normalized size = 2.02

$$\sqrt{-B^2} \log \left(\sqrt{2A^2 - B^2 \cos(2x) + B^2} + \sqrt{2} \sqrt{-B^2} \cos(x) \right) - \sqrt{A^2} \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{A^2} \cos(x)}{\sqrt{2A^2 - B^2 \cos(2x) + B^2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]*Sqrt[A^2 + B^2*Sin[x]^2],x]

[Out] -(Sqrt[A^2]*ArcTanh[(Sqrt[2]*Sqrt[A^2]*Cos[x])/Sqrt[2*A^2 + B^2 - B^2*Cos[2*x]]) + Sqrt[-B^2]*Log[Sqrt[2]*Sqrt[-B^2]*Cos[x] + Sqrt[2*A^2 + B^2 - B^2*Cos[2*x]])]

Maple [C] time = 0.096, size = 149, normalized size = 3.

$$-\frac{1}{2 \cos(x)} \sqrt{(A^2 + B^2 (\sin(x))^2) (\cos(x))^2} \left(A \operatorname{csgn}(A) \ln \left(-\frac{1}{(\sin(x))^2} \left(A^2 (\sin(x))^2 - B^2 (\sin(x))^2 - 2 \operatorname{csgn}(A) A \sqrt{(A^2 + B^2 (\sin(x))^2) (\cos(x))^2} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x)

[Out] -1/2*((A^2+B^2*sin(x)^2)*cos(x)^2)^(1/2)*(A*csgn(A)*ln(-(A^2*sin(x)^2-B^2*sin(x)^2-2*csgn(A)*A*((A^2+B^2*sin(x)^2)*cos(x)^2)^(1/2)-2*A^2)/sin(x)^2)-B*csgn(B)*arctan(1/2*csgn(B)/B*(2*B^2*sin(x)^2+A^2-B^2)/((A^2+B^2*sin(x)^2)*cos(x)^2)^(1/2)))/cos(x)/(A^2+B^2*sin(x)^2)^(1/2)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.99838, size = 635, normalized size = 12.96

$$\frac{1}{2} B \arctan \left(- \frac{(A^4 + 2 A^2 B^2 + B^4) \cos(x) \sin(x) - 2 (2 B^3 \cos(x)^3 - (A^2 B + B^3) \cos(x)) \sqrt{-B^2 \cos(x)^2 + A^2 + B^2}}{4 B^4 \cos(x)^4 + A^4 + 2 A^2 B^2 + B^4 - (A^4 + 6 A^2 B^2 + 5 B^4) \cos(x)^2} \right) - \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="fricas")

[Out] 1/2*B*arctan(-((A^4 + 2*A^2*B^2 + B^4)*cos(x)*sin(x) - 2*(2*B^3*cos(x)^3 - (A^2*B + B^3)*cos(x))*sqrt(-B^2*cos(x)^2 + A^2 + B^2))/(4*B^4*cos(x)^4 + A^4 + 2*A^2*B^2 + B^4 - (A^4 + 6*A^2*B^2 + 5*B^4)*cos(x)^2)) - 1/2*B*arctan(sin(x)/cos(x)) - 1/2*A*log(-B^2*cos(x)^2 + A*B*cos(x)*sin(x) + A^2 + B^2 + sqrt(-B^2*cos(x)^2 + A^2 + B^2)*(A*cos(x) + B*sin(x))) + 1/2*A*log(-B^2*cos(x)^2 - A*B*cos(x)*sin(x) + A^2 + B^2 - sqrt(-B^2*cos(x)^2 + A^2 + B^2)*(A*cos(x) - B*sin(x)))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{A^2 + B^2 \sin^2(x)}}{\sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A**2+B**2*sin(x)**2)**(1/2)/sin(x),x)

[Out] Integral(sqrt(A**2 + B**2*sin(x)**2)/sin(x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{B^2 \sin(x)^2 + A^2}}{\sin(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="giac")

[Out] integrate(sqrt(B^2*sin(x)^2 + A^2)/sin(x), x)

$$3.43 \quad \int \frac{1}{1+\cos(x)} dx$$

Optimal. Leaf size=9

$$\frac{\sin(x)}{\cos(x)+1}$$

[Out] Sin[x]/(1 + Cos[x])

Rubi [A] time = 0.0075139, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2648}

$$\frac{\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[x])^(-1), x]

[Out] Sin[x]/(1 + Cos[x])

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \frac{1}{1+\cos(x)} dx = \frac{\sin(x)}{1+\cos(x)}$$

Mathematica [A] time = 0.0036081, size = 6, normalized size = 0.67

$$\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Cos[x])^(-1), x]

[Out] Tan[x/2]

Maple [A] time = 0., size = 5, normalized size = 0.6

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cos(x)+1),x)`

[Out] `tan(1/2*x)`

Maxima [A] time = 0.918008, size = 12, normalized size = 1.33

$$\frac{\sin(x)}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+cos(x)),x, algorithm="maxima")`

[Out] `sin(x)/(cos(x) + 1)`

Fricas [A] time = 1.94506, size = 28, normalized size = 3.11

$$\frac{\sin(x)}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+cos(x)),x, algorithm="fricas")`

[Out] `sin(x)/(cos(x) + 1)`

Sympy [A] time = 0.179115, size = 3, normalized size = 0.33

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+cos(x)),x)`

[Out] `tan(x/2)`

Giac [B] time = 1.10891, size = 41, normalized size = 4.56

$$-\frac{2 \tan\left(\frac{1}{2} x\right)}{\left(x^2 + 1\right)\left(\frac{x^2 - 1}{x^2 + 1} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+cos(x)),x, algorithm="giac")`

[Out] `-2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))`

3.44 $\int e^x x dx$

Optimal. Leaf size=11

$$e^x x - e^x$$

[Out] $-E^x + E^{x*x}$

Rubi [A] time = 0.007018, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {2176, 2194}

$$e^x x - e^x$$

Antiderivative was successfully verified.

[In] Int[E^{x*x}, x]

[Out] $-E^x + E^{x*x}$

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x]
/; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !$UseGamma == True
```

Rule 2194

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))^(n_.), x_Symbol]
:> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int e^x x dx &= e^x x - \int e^x dx \\ &= -e^x + e^x x \end{aligned}$$

Mathematica [A] time = 0.0009849, size = 7, normalized size = 0.64

$$e^x(x - 1)$$

Antiderivative was successfully verified.

[In] Integrate[E^{x*x}, x]

[Out] $E^x*(-1 + x)$

Maple [A] time = 0., size = 7, normalized size = 0.6

$$(-1 + x)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)*x,x)`

[Out] `(-1+x)*exp(x)`

Maxima [A] time = 0.928983, size = 8, normalized size = 0.73

$$(x - 1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x,x, algorithm="maxima")`

[Out] `(x - 1)*e^x`

Fricas [A] time = 1.93113, size = 18, normalized size = 1.64

$$(x - 1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x,x, algorithm="fricas")`

[Out] `(x - 1)*e^x`

Sympy [A] time = 0.073761, size = 5, normalized size = 0.45

$$(x - 1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x,x)`

[Out] `(x - 1)*exp(x)`

Giac [A] time = 1.09205, size = 8, normalized size = 0.73

$$(x - 1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x,x, algorithm="giac")`

[Out] `(x - 1)*e^x`

$$3.45 \quad \int \frac{e^x x}{(1+x)^2} dx$$

Optimal. Leaf size=9

$$\frac{e^x}{x+1}$$

[Out] E^x/(1 + x)

Rubi [A] time = 0.0256603, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {2197}

$$\frac{e^x}{x+1}$$

Antiderivative was successfully verified.

[In] Int[(E^x*x)/(1 + x)²,x]

[Out] E^x/(1 + x)

Rule 2197

Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> With[{b = Coefficient[v, x, 1], d = Coefficient[u, x, 0], e = Coefficient[u, x, 1], f = Coefficient[w, x, 0], g = Coefficient[w, x, 1]}, Simp[(g*u^(m + 1)*F^(c*v))/(b*c*e*Log[F]), x] /; EqQ[e*g*(m + 1) - b*c*(e*f - d*g)*Log[F], 0] /; FreeQ[{F, c, m}, x] && LinearQ[{u, v, w}, x]

Rubi steps

$$\int \frac{e^x x}{(1+x)^2} dx = \frac{e^x}{1+x}$$

Mathematica [A] time = 0.0229454, size = 9, normalized size = 1.

$$\frac{e^x}{x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(E^x*x)/(1 + x)²,x]

[Out] E^x/(1 + x)

Maple [A] time = 0.002, size = 9, normalized size = 1.

$$\frac{e^x}{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)*x/(1+x)^2,x)`

[Out] `exp(x)/(1+x)`

Maxima [A] time = 0.942497, size = 11, normalized size = 1.22

$$\frac{e^x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x/(1+x)^2,x, algorithm="maxima")`

[Out] `e^x/(x + 1)`

Fricas [A] time = 1.74625, size = 18, normalized size = 2.

$$\frac{e^x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x/(1+x)^2,x, algorithm="fricas")`

[Out] `e^x/(x + 1)`

Sympy [A] time = 0.085104, size = 5, normalized size = 0.56

$$\frac{e^x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x/(1+x)**2,x)`

[Out] `exp(x)/(x + 1)`

Giac [B] time = 1.10392, size = 26, normalized size = 2.89

$$\frac{e^{-(x+1)\left(\frac{1}{x+1}-1\right)}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*x/(1+x)^2,x, algorithm="giac")`

[Out] `e^(-(x + 1)*(1/(x + 1) - 1))/(x + 1)`

$$3.46 \quad \int e^{x^2} (1 + 2x^2) dx$$

Optimal. Leaf size=7

$$e^{x^2}x$$

[Out] E^{x^2}x

Rubi [A] time = 0.031599, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2226, 2204, 2212}

$$e^{x^2}x$$

Antiderivative was successfully verified.

[In] Int[E^{x^2}*(1 + 2*x^2), x]

[Out] E^{x^2}x

Rule 2226

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] := Int[ExpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b, c, d, n}, x] && PolynomialQ[u, x]

Rule 2204

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2)), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2212

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n, 0])

Rubi steps

$$\begin{aligned} \int e^{x^2} (1 + 2x^2) dx &= \int (e^{x^2} + 2e^{x^2}x^2) dx \\ &= 2 \int e^{x^2}x^2 dx + \int e^{x^2} dx \\ &= e^{x^2}x + \frac{1}{2}\sqrt{\pi}\operatorname{erfi}(x) - \int e^{x^2} dx \\ &= e^{x^2}x \end{aligned}$$

Mathematica [A] time = 0.0052642, size = 7, normalized size = 1.

$$e^{x^2}x$$

Antiderivative was successfully verified.

[In] Integrate[E^x^2*(1 + 2*x^2),x]

[Out] E^x^2*x

Maple [A] time = 0.001, size = 7, normalized size = 1.

$$e^{x^2}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x^2)*(2*x^2+1),x)

[Out] exp(x^2)*x

Maxima [A] time = 0.937613, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="maxima")

[Out] x*e^(x^2)

Fricas [A] time = 1.9751, size = 15, normalized size = 2.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="fricas")

[Out] x*e^(x^2)

Sympy [A] time = 0.080686, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x**2)*(2*x**2+1),x)

[Out] x*exp(x**2)

Giac [A] time = 1.06662, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="giac")
```

```
[Out] x*e^(x^2)
```

3.47 $\int e^{x^2} dx$

Optimal. Leaf size=11

$$\frac{1}{2}\sqrt{\pi}\operatorname{Erfi}(x)$$

[Out] (Sqrt[Pi]*Erfi[x])/2

Rubi [A] time = 0.0018481, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {2204}

$$\frac{1}{2}\sqrt{\pi}\operatorname{Erfi}(x)$$

Antiderivative was successfully verified.

[In] Int[E^x^2,x]

[Out] (Sqrt[Pi]*Erfi[x])/2

Rule 2204

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))²), x_Symbol] := Simp[(F^a*Sqrt[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rubi steps

$$\int e^{x^2} dx = \frac{1}{2}\sqrt{\pi}\operatorname{erfi}(x)$$

Mathematica [A] time = 0.0017681, size = 11, normalized size = 1.

$$\frac{1}{2}\sqrt{\pi}\operatorname{Erfi}(x)$$

Antiderivative was successfully verified.

[In] Integrate[E^x^2,x]

[Out] (Sqrt[Pi]*Erfi[x])/2

Maple [A] time = 0., size = 8, normalized size = 0.7

$$\frac{\operatorname{erfi}(x)\sqrt{\pi}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x^2),x)`

[Out] `1/2*erfi(x)*Pi^(1/2)`

Maxima [C] time = 0.926658, size = 12, normalized size = 1.09

$$-\frac{1}{2}i\sqrt{\pi}\operatorname{erf}(ix)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2),x, algorithm="maxima")`

[Out] `-1/2*I*sqrt(pi)*erf(I*x)`

Fricas [A] time = 2.02875, size = 30, normalized size = 2.73

$$\frac{1}{2}\sqrt{\pi}\operatorname{erfi}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2),x, algorithm="fricas")`

[Out] `1/2*sqrt(pi)*erfi(x)`

Sympy [A] time = 0.231137, size = 8, normalized size = 0.73

$$\frac{\sqrt{\pi}\operatorname{erfi}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x**2),x)`

[Out] `sqrt(pi)*erfi(x)/2`

Giac [C] time = 1.08518, size = 12, normalized size = 1.09

$$\frac{1}{2}i\sqrt{\pi}\operatorname{erf}(-ix)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x^2),x, algorithm="giac")`

[Out] `1/2*I*sqrt(pi)*erf(-I*x)`

$$3.48 \quad \int \frac{e^x}{x} dx$$

Optimal. Leaf size=2

ExpIntegralEi(x)

[Out] ExpIntegralEi[x]

Rubi [A] time = 0.0099006, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2178}

ExpIntegralEi(x)

Antiderivative was successfully verified.

[In] Int[E^x/x,x]

[Out] ExpIntegralEi[x]

Rule 2178

Int[(F_)^((g_)*((e_.) + (f_)*(x_)))/((c_.) + (d_)*(x_)), x_Symbol] :> Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !UseGamma == True

Rubi steps

$$\int \frac{e^x}{x} dx = \text{Ei}(x)$$

Mathematica [A] time = 0.0040816, size = 2, normalized size = 1.

ExpIntegralEi(x)

Antiderivative was successfully verified.

[In] Integrate[E^x/x,x]

[Out] ExpIntegralEi[x]

Maple [B] time = 0.002, size = 8, normalized size = 4.

-Ei(1, -x)

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x)/x,x)

[Out] $-Ei(1, -x)$

Maxima [A] time = 1.01328, size = 3, normalized size = 1.5

$Ei(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/x,x, algorithm="maxima")`

[Out] $Ei(x)$

Fricas [A] time = 1.86946, size = 9, normalized size = 4.5

$Ei(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/x,x, algorithm="fricas")`

[Out] $Ei(x)$

Sympy [A] time = 0.735941, size = 2, normalized size = 1.

$Ei(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/x,x)`

[Out] $Ei(x)$

Giac [A] time = 1.07608, size = 3, normalized size = 1.5

$Ei(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/x,x, algorithm="giac")`

[Out] $Ei(x)$

3.49 $\int \frac{x}{1+x^3} dx$

Optimal. Leaf size=41

$$\frac{1}{6} \log(x^2 - x + 1) - \frac{1}{3} \log(x + 1) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] $-(\text{ArcTan}[(1 - 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3]) - \text{Log}[1 + x]/3 + \text{Log}[1 - x + x^2]/6$

Rubi [A] time = 0.0220315, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {292, 31, 634, 618, 204, 628}

$$\frac{1}{6} \log(x^2 - x + 1) - \frac{1}{3} \log(x + 1) - \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x/(1 + x^3), x]

[Out] $-(\text{ArcTan}[(1 - 2*x)/\text{Sqrt}[3]]/\text{Sqrt}[3]) - \text{Log}[1 + x]/3 + \text{Log}[1 - x + x^2]/6$

Rule 292

```
Int[(x_)/((a_) + (b_)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^-1, x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{1+x^3} dx &= -\left(\frac{1}{3} \int \frac{1}{1+x} dx\right) + \frac{1}{3} \int \frac{1+x}{1-x+x^2} dx \\ &= -\frac{1}{3} \log(1+x) + \frac{1}{6} \int \frac{-1+2x}{1-x+x^2} dx + \frac{1}{2} \int \frac{1}{1-x+x^2} dx \\ &= -\frac{1}{3} \log(1+x) + \frac{1}{6} \log(1-x+x^2) - \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right) \\ &= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3} \log(1+x) + \frac{1}{6} \log(1-x+x^2) \end{aligned}$$

Mathematica [A] time = 0.0085541, size = 40, normalized size = 0.98

$$\frac{1}{6} \log(x^2 - x + 1) - \frac{1}{3} \log(x + 1) + \frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^3), x]
```

```
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Maple [A] time = 0., size = 35, normalized size = 0.9

$$-\frac{\ln(1+x)}{3} + \frac{\ln(x^2-x+1)}{6} + \frac{\sqrt{3}}{3} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3+1), x)
```

```
[Out] -1/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.40549, size = 46, normalized size = 1.12

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) + \frac{1}{6} \log(x^2-x+1) - \frac{1}{3} \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1), x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(x + 1)
```

Fricas [A] time = 1.9814, size = 112, normalized size = 2.73

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) + \frac{1}{6} \log(x^2 - x + 1) - \frac{1}{3} \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^3+1),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(x + 1)

Sympy [A] time = 0.121567, size = 41, normalized size = 1.

$$-\frac{\log(x+1)}{3} + \frac{\log(x^2-x+1)}{6} + \frac{\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x**3+1),x)

[Out] -log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/3

Giac [A] time = 1.0722, size = 47, normalized size = 1.15

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) + \frac{1}{6} \log(x^2 - x + 1) - \frac{1}{3} \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^3+1),x, algorithm="giac")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(abs(x + 1))

3.50 $\int \frac{1}{-1+x^6} dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{1-x^2}\right)}{2\sqrt{3}} - \frac{1}{6} \tanh^{-1}\left(\frac{x}{x^2+1}\right) - \frac{1}{3} \tanh^{-1}(x)$$

[Out] -ArcTan[(Sqrt[3]*x)/(1 - x^2)]/(2*Sqrt[3]) - ArcTanh[x]/3 - ArcTanh[x/(1 + x^2)]/6

Rubi [A] time = 0.104898, antiderivative size = 73, normalized size of antiderivative = 1.55, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {210, 634, 618, 204, 628, 206}

$$\frac{1}{12} \log(x^2 - x + 1) - \frac{1}{12} \log(x^2 + x + 1) + \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{3} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^6)^(-1), x]

[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTanh[x]/3 + Log[1 - x + x^2]/12 - Log[1 + x + x^2]/12

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(n_ - 1), x_Symbol] := Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}], x, x] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_ - 1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(n_ - 1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{-1+x^6} dx &= -\left(\frac{1}{3} \int \frac{1-\frac{x}{2}}{1-x+x^2} dx\right) - \frac{1}{3} \int \frac{1+\frac{x}{2}}{1+x+x^2} dx - \frac{1}{3} \int \frac{1}{1-x^2} dx \\ &= -\frac{1}{3} \tanh^{-1}(x) + \frac{1}{12} \int \frac{-1+2x}{1-x+x^2} dx - \frac{1}{12} \int \frac{1+2x}{1+x+x^2} dx - \frac{1}{4} \int \frac{1}{1-x+x^2} dx - \frac{1}{4} \int \frac{1}{1+x+x^2} dx \\ &= -\frac{1}{3} \tanh^{-1}(x) + \frac{1}{12} \log(1-x+x^2) - \frac{1}{12} \log(1+x+x^2) + \frac{1}{2} \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, -1+2x\right) + \frac{1}{2} \text{Subst}\left(\int \frac{1}{-3-x^2} dx, x, 1+2x\right) \\ &= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}} - \frac{1}{3} \tanh^{-1}(x) + \frac{1}{12} \log(1-x+x^2) - \frac{1}{12} \log(1+x+x^2) \end{aligned}$$

Mathematica [A] time = 0.013066, size = 75, normalized size = 1.6

$$\frac{1}{12} \left(\log(x^2 - x + 1) - \log(x^2 + x + 1) + 2 \log(1 - x) - 2 \log(x + 1) - 2\sqrt{3} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) - 2\sqrt{3} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)^(-1), x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]]
] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])/12
```

Maple [A] time = 0.004, size = 66, normalized size = 1.4

$$\frac{\ln(-1+x)}{6} - \frac{\ln(x^2+x+1)}{12} - \frac{\sqrt{3}}{6} \arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right) - \frac{\ln(1+x)}{6} + \frac{\ln(x^2-x+1)}{12} - \frac{\sqrt{3}}{6} \arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-1), x)
```

```
[Out] 1/6*ln(-1+x)-1/12*ln(x^2+x+1)-1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/6*ln(1+x)+1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.41661, size = 88, normalized size = 1.87

$$-\frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x+1)\right) - \frac{1}{6} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(2x-1)\right) - \frac{1}{12} \log(x^2+x+1) + \frac{1}{12} \log(x^2-x+1) - \frac{1}{6} \log(x+1) + \frac{1}{6} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-1),x, algorithm="maxima")

[Out] $-1/6*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/6*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) - 1/12*\log(x^2 + x + 1) + 1/12*\log(x^2 - x + 1) - 1/6*\log(x + 1) + 1/6*\log(x - 1)$

Fricas [A] time = 1.84237, size = 230, normalized size = 4.89

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{12}\log(x^2+x+1) + \frac{1}{12}\log(x^2-x+1) - \frac{1}{6}\log(x+1) + \frac{1}{6}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-1),x, algorithm="fricas")

[Out] $-1/6*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/6*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) - 1/12*\log(x^2 + x + 1) + 1/12*\log(x^2 - x + 1) - 1/6*\log(x + 1) + 1/6*\log(x - 1)$

Sympy [B] time = 0.233098, size = 83, normalized size = 1.77

$$\frac{\log(x-1)}{6} - \frac{\log(x+1)}{6} + \frac{\log(x^2-x+1)}{12} - \frac{\log(x^2+x+1)}{12} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**6-1),x)

[Out] $\log(x - 1)/6 - \log(x + 1)/6 + \log(x**2 - x + 1)/12 - \log(x**2 + x + 1)/12 - \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 - \sqrt{3}/3)/6 - \sqrt{3}*\operatorname{atan}(2*\sqrt{3}*x/3 + \sqrt{3}/3)/6$

Giac [A] time = 1.06863, size = 90, normalized size = 1.91

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{12}\log(x^2+x+1) + \frac{1}{12}\log(x^2-x+1) - \frac{1}{6}\log(x+1) + \frac{1}{6}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^6-1),x, algorithm="giac")

[Out] $-1/6*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x + 1)) - 1/6*\sqrt{3}*\arctan(1/3*\sqrt{3}*(2*x - 1)) - 1/12*\log(x^2 + x + 1) + 1/12*\log(x^2 - x + 1) - 1/6*\log(\operatorname{abs}(x + 1)) + 1/6*\log(\operatorname{abs}(x - 1))$

$$3.51 \quad \int \frac{1}{A^4 - A^2 B^2 + (-A^2 + B^2)x^2} dx$$

Optimal. Leaf size=21

$$\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A(A^2 - B^2)}$$

[Out] ArcTanh[x/A]/(A*(A^2 - B^2))

Rubi [A] time = 0.0094333, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {208}

$$\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A(A^2 - B^2)}$$

Antiderivative was successfully verified.

[In] Int[(A^4 - A^2*B^2 + (-A^2 + B^2)*x^2)^(-1), x]

[Out] ArcTanh[x/A]/(A*(A^2 - B^2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\int \frac{1}{A^4 - A^2 B^2 + (-A^2 + B^2)x^2} dx = \frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A(A^2 - B^2)}$$

Mathematica [A] time = 0.0041203, size = 21, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A(A^2 - B^2)}$$

Antiderivative was successfully verified.

[In] Integrate[(A^4 - A^2*B^2 + (-A^2 + B^2)*x^2)^(-1), x]

[Out] ArcTanh[x/A]/(A*(A^2 - B^2))

Maple [B] time = 0.005, size = 44, normalized size = 2.1

$$\frac{\ln(A + x)}{(2A^2 - 2B^2)A} - \frac{\ln(A - x)}{(2A^2 - 2B^2)A}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x)`

[Out] $1/2/(A^2-B^2)/A*\ln(A+x)-1/2/(A^2-B^2)/A*\ln(A-x)$

Maxima [A] time = 0.931059, size = 53, normalized size = 2.52

$$\frac{\log(A+x)}{2(A^3-AB^2)} - \frac{\log(-A+x)}{2(A^3-AB^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="maxima")`

[Out] $1/2*\log(A+x)/(A^3-A*B^2) - 1/2*\log(-A+x)/(A^3-A*B^2)$

Fricas [A] time = 1.81271, size = 62, normalized size = 2.95

$$\frac{\log(A+x) - \log(-A+x)}{2(A^3-AB^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="fricas")`

[Out] $1/2*(\log(A+x) - \log(-A+x))/(A^3 - A*B^2)$

Sympy [B] time = 0.271437, size = 70, normalized size = 3.33

$$-\frac{\log\left(-\frac{A^3}{(A-B)(A+B)} + \frac{AB^2}{(A-B)(A+B)} + x\right)}{2A(A-B)(A+B)} + \frac{\log\left(\frac{A^3}{(A-B)(A+B)} - \frac{AB^2}{(A-B)(A+B)} + x\right)}{2A(A-B)(A+B)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A**4-A**2*B**2+(-A**2+B**2)*x**2),x)`

[Out] $-\log(-A**3/((A-B)*(A+B)) + A*B**2/((A-B)*(A+B)) + x)/(2*A*(A-B)*(A+B)) + \log(A**3/((A-B)*(A+B)) - A*B**2/((A-B)*(A+B)) + x)/(2*A*(A-B)*(A+B))$

Giac [A] time = 1.07611, size = 55, normalized size = 2.62

$$\frac{\log(|A+x|)}{2(A^3-AB^2)} - \frac{\log(|-A+x|)}{2(A^3-AB^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(A + x))/(A^3 - A*B^2) - 1/2*log(abs(-A + x))/(A^3 - A*B^2)
```

3.52 $\int x \log(x) dx$

Optimal. Leaf size=17

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

[Out] $-x^2/4 + (x^2*\text{Log}[x])/2$

Rubi [A] time = 0.0039045, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {2304}

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Int[x*Log[x],x]

[Out] $-x^2/4 + (x^2*\text{Log}[x])/2$

Rule 2304

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int x \log(x) dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

Mathematica [A] time = 0.0006047, size = 17, normalized size = 1.

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Integrate[x*Log[x],x]

[Out] $-x^2/4 + (x^2*\text{Log}[x])/2$

Maple [A] time = 0., size = 14, normalized size = 0.8

$$-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*ln(x),x)`

[Out] $-1/4*x^2+1/2*x^2*\ln(x)$

Maxima [A] time = 0.924389, size = 18, normalized size = 1.06

$$\frac{1}{2}x^2 \log(x) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(x),x, algorithm="maxima")`

[Out] $1/2*x^2*\log(x) - 1/4*x^2$

Fricas [A] time = 2.00152, size = 35, normalized size = 2.06

$$\frac{1}{2}x^2 \log(x) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(x),x, algorithm="fricas")`

[Out] $1/2*x^2*\log(x) - 1/4*x^2$

Sympy [A] time = 0.079624, size = 12, normalized size = 0.71

$$\frac{x^2 \log(x)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*ln(x),x)`

[Out] $x**2*\log(x)/2 - x**2/4$

Giac [A] time = 1.09076, size = 18, normalized size = 1.06

$$\frac{1}{2}x^2 \log(x) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(x),x, algorithm="giac")`

[Out] $1/2*x^2*\log(x) - 1/4*x^2$

3.53 $\int x^2 \sin^{-1}(x) dx$

Optimal. Leaf size=40

$$-\frac{1}{9}(1-x^2)^{3/2} + \frac{\sqrt{1-x^2}}{3} + \frac{1}{3}x^3 \sin^{-1}(x)$$

[Out] Sqrt[1 - x^2]/3 - (1 - x^2)^(3/2)/9 + (x^3*ArcSin[x])/3

Rubi [A] time = 0.0241101, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {4627, 266, 43}

$$-\frac{1}{9}(1-x^2)^{3/2} + \frac{\sqrt{1-x^2}}{3} + \frac{1}{3}x^3 \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x^2*ArcSin[x],x]

[Out] Sqrt[1 - x^2]/3 - (1 - x^2)^(3/2)/9 + (x^3*ArcSin[x])/3

Rule 4627

Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n_.*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned} \int x^2 \sin^{-1}(x) dx &= \frac{1}{3}x^3 \sin^{-1}(x) - \frac{1}{3} \int \frac{x^3}{\sqrt{1-x^2}} dx \\ &= \frac{1}{3}x^3 \sin^{-1}(x) - \frac{1}{6} \text{Subst} \left(\int \frac{x}{\sqrt{1-x}} dx, x, x^2 \right) \\ &= \frac{1}{3}x^3 \sin^{-1}(x) - \frac{1}{6} \text{Subst} \left(\int \left(\frac{1}{\sqrt{1-x}} - \sqrt{1-x} \right) dx, x, x^2 \right) \\ &= \frac{\sqrt{1-x^2}}{3} - \frac{1}{9}(1-x^2)^{3/2} + \frac{1}{3}x^3 \sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.0117271, size = 29, normalized size = 0.72

$$\frac{1}{9} \left(\sqrt{1-x^2} (x^2+2) + 3x^3 \sin^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*ArcSin[x],x]

[Out] (Sqrt[1 - x^2]*(2 + x^2) + 3*x^3*ArcSin[x])/9

Maple [A] time = 0., size = 34, normalized size = 0.9

$$\frac{x^3 \arcsin(x)}{3} + \frac{x^2}{9} \sqrt{-x^2+1} + \frac{2}{9} \sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*arcsin(x),x)

[Out] 1/3*x^3*arcsin(x)+1/9*x^2*(-x^2+1)^(1/2)+2/9*(-x^2+1)^(1/2)

Maxima [A] time = 1.40179, size = 45, normalized size = 1.12

$$\frac{1}{3} x^3 \arcsin(x) + \frac{1}{9} \sqrt{-x^2+1} x^2 + \frac{2}{9} \sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arcsin(x),x, algorithm="maxima")

[Out] 1/3*x^3*arcsin(x) + 1/9*sqrt(-x^2 + 1)*x^2 + 2/9*sqrt(-x^2 + 1)

Fricas [A] time = 2.04512, size = 68, normalized size = 1.7

$$\frac{1}{3} x^3 \arcsin(x) + \frac{1}{9} (x^2+2) \sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arcsin(x),x, algorithm="fricas")

[Out] 1/3*x^3*arcsin(x) + 1/9*(x^2 + 2)*sqrt(-x^2 + 1)

Sympy [A] time = 0.320597, size = 32, normalized size = 0.8

$$\frac{x^3 \operatorname{asin}(x)}{3} + \frac{x^2 \sqrt{1-x^2}}{9} + \frac{2\sqrt{1-x^2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*asin(x),x)

[Out] x**3*asin(x)/3 + x**2*sqrt(1 - x**2)/9 + 2*sqrt(1 - x**2)/9

Giac [A] time = 1.07506, size = 51, normalized size = 1.27

$$\frac{1}{3}(x^2 - 1)x \arcsin(x) + \frac{1}{3}x \arcsin(x) - \frac{1}{9}(-x^2 + 1)^{\frac{3}{2}} + \frac{1}{3}\sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arcsin(x),x, algorithm="giac")

[Out] 1/3*(x^2 - 1)*x*arcsin(x) + 1/3*x*arcsin(x) - 1/9*(-x^2 + 1)^(3/2) + 1/3*sqrt(-x^2 + 1)

$$3.54 \quad \int \frac{1}{1+2x+x^2} dx$$

Optimal. Leaf size=7

$$-\frac{1}{x+1}$$

[Out] $-(1 + x)^{-1}$

Rubi [A] time = 0.0010104, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {27, 32}

$$-\frac{1}{x+1}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x + x^2)^(-1), x]

[Out] $-(1 + x)^{-1}$

Rule 27

Int[(u_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[u*Cancel[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{1+2x+x^2} dx &= \int \frac{1}{(1+x)^2} dx \\ &= -\frac{1}{1+x} \end{aligned}$$

Mathematica [A] time = 0.0008212, size = 7, normalized size = 1.

$$-\frac{1}{x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x + x^2)^(-1), x]

[Out] $-(1 + x)^{-1}$

Maple [A] time = 0.002, size = 8, normalized size = 1.1

$$-(1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+2*x+1),x)

[Out] -1/(1+x)

Maxima [A] time = 0.924597, size = 9, normalized size = 1.29

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+2*x+1),x, algorithm="maxima")

[Out] -1/(x + 1)

Fricas [A] time = 2.02132, size = 16, normalized size = 2.29

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+2*x+1),x, algorithm="fricas")

[Out] -1/(x + 1)

Sympy [A] time = 0.070916, size = 5, normalized size = 0.71

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+2*x+1),x)

[Out] -1/(x + 1)

Giac [A] time = 1.0837, size = 9, normalized size = 1.29

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+2*x+1),x, algorithm="giac")

[Out] -1/(x + 1)

$$3.55 \quad \int \frac{\log(x)}{(1+\log(x))^2} dx$$

Optimal. Leaf size=8

$$\frac{x}{\log(x)+1}$$

[Out] x/(1 + Log[x])

Rubi [A] time = 0.0441281, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2360, 2297, 2299, 2178}

$$\frac{x}{\log(x)+1}$$

Antiderivative was successfully verified.

[In] Int[Log[x]/(1 + Log[x])^2,x]

[Out] x/(1 + Log[x])

Rule 2360

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(Log[(c_.)*(x_)^(n_.)]*(e_.) + (d_.))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*Log[c*x^n])^p*(d + e*Log[c*x^n])^q, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IntegerQ[p] && IntegerQ[q]

Rule 2297

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] := Simp[(x*(a + b*Log[c*x^n])^(p + 1)/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && IntegerQ[2*p]

Rule 2299

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] := Dist[1/(n*c^(1/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, p}, x] && IntegerQ[1/n]

Rule 2178

Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rubi steps

$$\begin{aligned}
\int \frac{\log(x)}{(1 + \log(x))^2} dx &= \int \left(-\frac{1}{(1 + \log(x))^2} + \frac{1}{1 + \log(x)} \right) dx \\
&= -\int \frac{1}{(1 + \log(x))^2} dx + \int \frac{1}{1 + \log(x)} dx \\
&= \frac{x}{1 + \log(x)} - \int \frac{1}{1 + \log(x)} dx + \text{Subst} \left(\int \frac{e^x}{1 + x} dx, x, \log(x) \right) \\
&= \frac{\text{Ei}(1 + \log(x))}{e} + \frac{x}{1 + \log(x)} - \text{Subst} \left(\int \frac{e^x}{1 + x} dx, x, \log(x) \right) \\
&= \frac{x}{1 + \log(x)}
\end{aligned}$$

Mathematica [A] time = 0.0134753, size = 8, normalized size = 1.

$$\frac{x}{\log(x) + 1}$$

Antiderivative was successfully verified.

[In] Integrate[Log[x]/(1 + Log[x])^2,x]

[Out] x/(1 + Log[x])

Maple [A] time = 0.023, size = 9, normalized size = 1.1

$$\frac{x}{1 + \ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(x)/(1+ln(x))^2,x)

[Out] x/(1+ln(x))

Maxima [A] time = 0.930617, size = 11, normalized size = 1.38

$$\frac{x}{\log(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(x)/(1+log(x))^2,x, algorithm="maxima")

[Out] x/(log(x) + 1)

Fricas [A] time = 1.95641, size = 22, normalized size = 2.75

$$\frac{x}{\log(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(1+log(x))^2,x, algorithm="fricas")
```

```
[Out] x/(log(x) + 1)
```

Sympy [A] time = 0.082002, size = 5, normalized size = 0.62

$$\frac{x}{\log(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(1+ln(x))**2,x)
```

```
[Out] x/(log(x) + 1)
```

Giac [A] time = 1.07624, size = 11, normalized size = 1.38

$$\frac{x}{\log(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(1+log(x))^2,x, algorithm="giac")
```

```
[Out] x/(log(x) + 1)
```

$$3.56 \quad \int \frac{1}{x(1+\log^2(x))} dx$$

Optimal. Leaf size=3

$$\tan^{-1}(\log(x))$$

[Out] ArcTan[Log[x]]

Rubi [A] time = 0.0196644, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {203}

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 + Log[x]^2)),x]

[Out] ArcTan[Log[x]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\int \frac{1}{x(1+\log^2(x))} dx = \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \log(x)\right) \\ = \tan^{-1}(\log(x))$$

Mathematica [A] time = 0.0108006, size = 3, normalized size = 1.

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 + Log[x]^2)),x]

[Out] ArcTan[Log[x]]

Maple [A] time = 0.002, size = 4, normalized size = 1.3

$$\arctan(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(1+ln(x)^2),x)`

[Out] `arctan(ln(x))`

Maxima [A] time = 1.40457, size = 4, normalized size = 1.33

`arctan(log(x))`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+log(x)^2),x, algorithm="maxima")`

[Out] `arctan(log(x))`

Fricas [A] time = 1.92429, size = 22, normalized size = 7.33

`arctan(log(x))`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+log(x)^2),x, algorithm="fricas")`

[Out] `arctan(log(x))`

Sympy [B] time = 0.123049, size = 15, normalized size = 5.

`RootSum(4z2 + 1, (i ↦ i log(2i + log(x))))`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+ln(x)**2),x)`

[Out] `RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))`

Giac [A] time = 1.08233, size = 4, normalized size = 1.33

`arctan(log(x))`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+log(x)^2),x, algorithm="giac")`

[Out] `arctan(log(x))`

$$3.57 \quad \int \frac{1}{\log(x)} dx$$

Optimal. Leaf size=2

LogIntegral(x)

[Out] LogIntegral[x]

Rubi [A] time = 0.001793, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {2298}

LogIntegral(x)

Antiderivative was successfully verified.

[In] Int[Log[x]^(-1), x]

[Out] LogIntegral[x]

Rule 2298

Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ[c, x]

Rubi steps

$$\int \frac{1}{\log(x)} dx = \text{li}(x)$$

Mathematica [F] time = 0.0021106, size = 0, normalized size = 0.

$$\int \frac{1}{\log(x)} dx$$

Verification is Not applicable to the result.

[In] Integrate[Log[x]^(-1), x]

[Out] Integrate[Log[x]^(-1), x]

Maple [B] time = 0., size = 9, normalized size = 4.5

-Ei(1, -ln(x))

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/ln(x), x)

[Out] $-Ei(1, -\ln(x))$

Maxima [A] time = 1.0224, size = 4, normalized size = 2.

$Ei(\log(x))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/log(x),x, algorithm="maxima")`

[Out] $Ei(\log(x))$

Fricas [A] time = 1.80083, size = 23, normalized size = 11.5

$\log_integral(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/log(x),x, algorithm="fricas")`

[Out] $\log_integral(x)$

Sympy [A] time = 0.439018, size = 2, normalized size = 1.

$li(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/ln(x),x)`

[Out] $li(x)$

Giac [A] time = 1.12029, size = 4, normalized size = 2.

$Ei(\log(x))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/log(x),x, algorithm="giac")`

[Out] $Ei(\log(x))$

3.58 $\int x(\cos(x) + \sin(x)) dx$

Optimal. Leaf size=14

$$x \sin(x) + \sin(x) - x \cos(x) + \cos(x)$$

[Out] Cos[x] - x*Cos[x] + Sin[x] + x*Sin[x]

Rubi [A] time = 0.0214771, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {14, 3296, 2638, 2637}

$$x \sin(x) + \sin(x) - x \cos(x) + \cos(x)$$

Antiderivative was successfully verified.

[In] Int[x*(Cos[x] + Sin[x]),x]

[Out] Cos[x] - x*Cos[x] + Sin[x] + x*Sin[x]

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 3296

Int[((c_) + (d_)*(x_))^(m_)*sin[(e_) + (f_)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c_) + (d_)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2637

Int[sin[Pi/2 + (c_) + (d_)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int x(\cos(x) + \sin(x)) dx &= \int (x \cos(x) + x \sin(x)) dx \\ &= \int x \cos(x) dx + \int x \sin(x) dx \\ &= -x \cos(x) + x \sin(x) + \int \cos(x) dx - \int \sin(x) dx \\ &= \cos(x) - x \cos(x) + \sin(x) + x \sin(x) \end{aligned}$$

Mathematica [A] time = 0.0045986, size = 14, normalized size = 1.

$$x \sin(x) + \sin(x) - x \cos(x) + \cos(x)$$

Antiderivative was successfully verified.

[In] Integrate[x*(Cos[x] + Sin[x]),x]

[Out] Cos[x] - x*Cos[x] + Sin[x] + x*Sin[x]

Maple [A] time = 0.011, size = 15, normalized size = 1.1

$$\cos(x) - x \cos(x) + \sin(x) + x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(cos(x)+sin(x)),x)

[Out] cos(x)-x*cos(x)+sin(x)+x*sin(x)

Maxima [A] time = 0.938204, size = 19, normalized size = 1.36

$$-x \cos(x) + x \sin(x) + \cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(cos(x)+sin(x)),x, algorithm="maxima")

[Out] -x*cos(x) + x*sin(x) + cos(x) + sin(x)

Fricas [A] time = 1.85592, size = 46, normalized size = 3.29

$$-(x - 1) \cos(x) + (x + 1) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(cos(x)+sin(x)),x, algorithm="fricas")

[Out] -(x - 1)*cos(x) + (x + 1)*sin(x)

Sympy [A] time = 0.165906, size = 15, normalized size = 1.07

$$x \sin(x) - x \cos(x) + \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(cos(x)+sin(x)),x)

[Out] $x*\sin(x) - x*\cos(x) + \sin(x) + \cos(x)$

Giac [A] time = 1.06894, size = 19, normalized size = 1.36

$$-x \cos(x) + x \sin(x) + \cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(cos(x)+sin(x)),x, algorithm="giac")`

[Out] $-x*\cos(x) + x*\sin(x) + \cos(x) + \sin(x)$

3.59 $\int e^{-x} (e^x + x) dx$

Optimal. Leaf size=17

$$-e^{-x}x + x - e^{-x}$$

[Out] $-E^{(-x)} + x - x/E^x$

Rubi [A] time = 0.0313499, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6742, 2176, 2194}

$$-e^{-x}x + x - e^{-x}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(E^x + x)/E^x, x]$

[Out] $-E^{(-x)} + x - x/E^x$

Rule 6742

$\text{Int}[u_, x_Symbol] \text{ :> With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] \text{ /; SumQ}[v]$
]

Rule 2176

$\text{Int}[(b_)*(F_)^{(g_)*((e_)+(f_)*(x_)))^{(n_)*((c_)+(d_)*(x_))^{(m_)}, x_Symbol] \text{ :> Simp}[(c + d*x)^m*(b*F^{(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - \text{Dist}[(d*m)/(f*g*n*Log[F]), \text{Int}[(c + d*x)^{(m - 1)}*(b*F^{(g*(e + f*x)))^n}, x], x] \text{ /; FreeQ}[\{F, b, c, d, e, f, g, n\}, x] \&\& \text{GtQ}[m, 0] \&\& \text{IntegerQ}[2*m] \&\& !\$UseGamma == True$

Rule 2194

$\text{Int}[(F_)^{(c_)*((a_)+(b_)*(x_)))^{(n_)}, x_Symbol] \text{ :> Simp}[(F^{(c*(a + b*x)))^n/(b*c*n*Log[F]), x] \text{ /; FreeQ}[\{F, a, b, c, n\}, x]$

Rubi steps

$$\begin{aligned} \int e^{-x} (e^x + x) dx &= \int (1 + e^{-x}x) dx \\ &= x + \int e^{-x}x dx \\ &= x - e^{-x}x + \int e^{-x} dx \\ &= -e^{-x} + x - e^{-x}x \end{aligned}$$

Mathematica [A] time = 0.0099566, size = 13, normalized size = 0.76

$$e^{-x}(-x - 1) + x$$

Antiderivative was successfully verified.

[In] Integrate[(E^x + x)/E^x,x]

[Out] (-1 - x)/E^x + x

Maple [A] time = 0.001, size = 16, normalized size = 0.9

$$-(e^x)^{-1} + x - \frac{x}{e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((exp(x)+x)/exp(x),x)

[Out] -1/exp(x)+x-x/exp(x)

Maxima [A] time = 0.932095, size = 15, normalized size = 0.88

$$-(x + 1)e^{(-x)} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+exp(x))/exp(x),x, algorithm="maxima")

[Out] -(x + 1)*e^(-x) + x

Fricas [A] time = 1.87756, size = 32, normalized size = 1.88

$$(xe^x - x - 1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+exp(x))/exp(x),x, algorithm="fricas")

[Out] (x*e^x - x - 1)*e^(-x)

Sympy [A] time = 0.081333, size = 8, normalized size = 0.47

$$x + (-x - 1)e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+exp(x))/exp(x),x)

[Out] x + (-x - 1)*exp(-x)

Giac [A] time = 1.099, size = 15, normalized size = 0.88

$$-(x + 1)e^{(-x)} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))/exp(x),x, algorithm="giac")
```

```
[Out] -(x + 1)*e^(-x) + x
```


3.60 $\int (1 + e^x)^2 x dx$

Optimal. Leaf size=38

$$\frac{x^2}{2} + 2e^x x + \frac{1}{2}e^{2x} x - 2e^x - \frac{e^{2x}}{4}$$

[Out] $-2E^x - E^{(2*x)}/4 + 2E^x*x + (E^{(2*x)*x})/2 + x^2/2$

Rubi [A] time = 0.0326818, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2183, 2176, 2194}

$$\frac{x^2}{2} + 2e^x x + \frac{1}{2}e^{2x} x - 2e^x - \frac{e^{2x}}{4}$$

Antiderivative was successfully verified.

[In] Int[(1 + E^x)^2*x, x]

[Out] $-2E^x - E^{(2*x)}/4 + 2E^x*x + (E^{(2*x)*x})/2 + x^2/2$

Rule 2183

Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F^(g*(e + f*x))))^n]^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, 0]

Rule 2176

Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]), x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m] && !\$UseGamma == True

Rule 2194

Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a + b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]

Rubi steps

$$\begin{aligned} \int (1 + e^x)^2 x dx &= \int (x + 2e^x x + e^{2x} x) dx \\ &= \frac{x^2}{2} + 2 \int e^x x dx + \int e^{2x} x dx \\ &= 2e^x x + \frac{1}{2}e^{2x} x + \frac{x^2}{2} - \frac{1}{2} \int e^{2x} dx - 2 \int e^x dx \\ &= -2e^x - \frac{e^{2x}}{4} + 2e^x x + \frac{1}{2}e^{2x} x + \frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.0256854, size = 29, normalized size = 0.76

$$\frac{1}{4} (2x^2 + 8e^x(x-1) + e^{2x}(2x-1))$$

Antiderivative was successfully verified.

[In] Integrate[(1 + E^x)^2*x, x]

[Out] (8*E^x*(-1 + x) + 2*x^2 + E^(2*x)*(-1 + 2*x))/4

Maple [A] time = 0.002, size = 29, normalized size = 0.8

$$\frac{x^2}{2} + \frac{(e^x)^2 x}{2} - \frac{(e^x)^2}{4} + 2e^x x - 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+exp(x))^2*x, x)

[Out] 1/2*x^2+1/2*exp(x)^2*x-1/4*exp(x)^2+2*exp(x)*x-2*exp(x)

Maxima [A] time = 0.936797, size = 32, normalized size = 0.84

$$\frac{1}{2}x^2 + \frac{1}{4}(2x-1)e^{(2x)} + 2(x-1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+exp(x))^2*x, x, algorithm="maxima")

[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x

Fricas [A] time = 1.9211, size = 66, normalized size = 1.74

$$\frac{1}{2}x^2 + \frac{1}{4}(2x-1)e^{(2x)} + 2(x-1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+exp(x))^2*x, x, algorithm="fricas")

[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x

Sympy [A] time = 0.088345, size = 26, normalized size = 0.68

$$\frac{x^2}{2} + \frac{(2x-1)e^{2x}}{4} + \frac{(8x-8)e^x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))**2*x,x)
```

```
[Out] x**2/2 + (2*x - 1)*exp(2*x)/4 + (8*x - 8)*exp(x)/4
```

Giac [A] time = 1.06624, size = 32, normalized size = 0.84

$$\frac{1}{2}x^2 + \frac{1}{4}(2x-1)e^{(2x)} + 2(x-1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))^2*x,x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

3.61 $\int x \cos(x) dx$

Optimal. Leaf size=7

$$x \sin(x) + \cos(x)$$

[Out] Cos[x] + x*Sin[x]

Rubi [A] time = 0.0087321, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3296, 2638}

$$x \sin(x) + \cos(x)$$

Antiderivative was successfully verified.

[In] Int[x*Cos[x],x]

[Out] Cos[x] + x*Sin[x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int x \cos(x) dx &= x \sin(x) - \int \sin(x) dx \\ &= \cos(x) + x \sin(x) \end{aligned}$$

Mathematica [A] time = 0.0020944, size = 7, normalized size = 1.

$$x \sin(x) + \cos(x)$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[x],x]

[Out] Cos[x] + x*Sin[x]

Maple [A] time = 0., size = 8, normalized size = 1.1

$$\cos(x) + x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*cos(x),x)`

[Out] `cos(x)+x*sin(x)`

Maxima [A] time = 0.935171, size = 9, normalized size = 1.29

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x),x, algorithm="maxima")`

[Out] `x*sin(x) + cos(x)`

Fricas [A] time = 1.90491, size = 26, normalized size = 3.71

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x),x, algorithm="fricas")`

[Out] `x*sin(x) + cos(x)`

Sympy [A] time = 0.167128, size = 7, normalized size = 1.

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x),x)`

[Out] `x*sin(x) + cos(x)`

Giac [A] time = 1.06088, size = 9, normalized size = 1.29

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*cos(x),x, algorithm="giac")`

[Out] `x*sin(x) + cos(x)`

3.62 $\int \cos(\sqrt{x}) dx$

Optimal. Leaf size=22

$$2\sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]

Rubi [A] time = 0.0117688, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3362, 3296, 2638}

$$2\sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Cos[Sqrt[x]],x]

[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]

Rule 3362

Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[1/n]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[(c + d*x)^m*Cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \cos(\sqrt{x}) dx &= 2 \text{Subst} \left(\int x \cos(x) dx, x, \sqrt{x} \right) \\ &= 2\sqrt{x} \sin(\sqrt{x}) - 2 \text{Subst} \left(\int \sin(x) dx, x, \sqrt{x} \right) \\ &= 2 \cos(\sqrt{x}) + 2\sqrt{x} \sin(\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.008908, size = 22, normalized size = 1.

$$2\sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[Cos[Sqrt[x]],x]

[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]

Maple [A] time = 0., size = 17, normalized size = 0.8

$$2 \cos(\sqrt{x}) + 2 \sin(\sqrt{x}) \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x^(1/2)),x)

[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)

Maxima [A] time = 0.941327, size = 22, normalized size = 1.

$$2 \sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x^(1/2)),x, algorithm="maxima")

[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))

Fricas [A] time = 2.05377, size = 55, normalized size = 2.5

$$2 \sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x^(1/2)),x, algorithm="fricas")

[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))

Sympy [A] time = 0.292586, size = 20, normalized size = 0.91

$$2 \sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x**(1/2)),x)

[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))

Giac [A] time = 1.06991, size = 22, normalized size = 1.

$$2 \sqrt{x} \sin(\sqrt{x}) + 2 \cos(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```


3.63 $\int x \cos(x) dx$

Optimal. Leaf size=7

$$x \sin(x) + \cos(x)$$

[Out] Cos[x] + x*Sin[x]

Rubi [A] time = 0.0090752, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3296, 2638}

$$x \sin(x) + \cos(x)$$

Antiderivative was successfully verified.

[In] Int[x*Cos[x],x]

[Out] Cos[x] + x*Sin[x]

Rule 3296

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int x \cos(x) dx &= x \sin(x) - \int \sin(x) dx \\ &= \cos(x) + x \sin(x) \end{aligned}$$

Mathematica [A] time = 0.002008, size = 7, normalized size = 1.

$$x \sin(x) + \cos(x)$$

Antiderivative was successfully verified.

[In] Integrate[x*Cos[x],x]

[Out] Cos[x] + x*Sin[x]

Maple [A] time = 0., size = 8, normalized size = 1.1

$$\cos(x) + x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)
```

```
[Out] cos(x)+x*sin(x)
```

Maxima [A] time = 0.932358, size = 9, normalized size = 1.29

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="maxima")
```

```
[Out] x*sin(x) + cos(x)
```

Fricas [A] time = 2.00763, size = 26, normalized size = 3.71

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="fricas")
```

```
[Out] x*sin(x) + cos(x)
```

Sympy [A] time = 0.161085, size = 7, normalized size = 1.

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x)
```

```
[Out] x*sin(x) + cos(x)
```

Giac [A] time = 1.06041, size = 9, normalized size = 1.29

$$x \sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="giac")
```

```
[Out] x*sin(x) + cos(x)
```

3.64 $\int x \log^2(x) dx$

Optimal. Leaf size=28

$$\frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

[Out] $x^2/4 - (x^2*\text{Log}[x])/2 + (x^2*\text{Log}[x]^2)/2$

Rubi [A] time = 0.0097481, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2305, 2304}

$$\frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

Antiderivative was successfully verified.

[In] Int[x*Log[x]^2,x]

[Out] $x^2/4 - (x^2*\text{Log}[x])/2 + (x^2*\text{Log}[x]^2)/2$

Rule 2305

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]

Rule 2304

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int x \log^2(x) dx &= \frac{1}{2}x^2 \log^2(x) - \int x \log(x) dx \\ &= \frac{x^2}{4} - \frac{1}{2}x^2 \log(x) + \frac{1}{2}x^2 \log^2(x) \end{aligned}$$

Mathematica [A] time = 0.0008745, size = 28, normalized size = 1.

$$\frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[x*Log[x]^2,x]

[Out] $x^2/4 - (x^2*\text{Log}[x])/2 + (x^2*\text{Log}[x]^2)/2$

Maple [A] time = 0.002, size = 23, normalized size = 0.8

$$\frac{x^2}{4} - \frac{x^2 \ln(x)}{2} + \frac{x^2 (\ln(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*ln(x)^2,x)

[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2

Maxima [A] time = 0.925699, size = 23, normalized size = 0.82

$$\frac{1}{4} (2 \log(x)^2 - 2 \log(x) + 1) x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*log(x)^2,x, algorithm="maxima")

[Out] 1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2

Fricas [A] time = 1.81658, size = 61, normalized size = 2.18

$$\frac{1}{2} x^2 \log(x)^2 - \frac{1}{2} x^2 \log(x) + \frac{1}{4} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*log(x)^2,x, algorithm="fricas")

[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2

Sympy [A] time = 0.094922, size = 22, normalized size = 0.79

$$\frac{x^2 \log(x)^2}{2} - \frac{x^2 \log(x)}{2} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*ln(x)**2,x)

[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4

Giac [A] time = 1.05854, size = 30, normalized size = 1.07

$$\frac{1}{2} x^2 \log(x)^2 - \frac{1}{2} x^2 \log(x) + \frac{1}{4} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

3.65 $\int \cos(x) (1 + \sin^3(x)) dx$

Optimal. Leaf size=11

$$\frac{\sin^4(x)}{4} + \sin(x)$$

[Out] Sin[x] + Sin[x]^4/4

Rubi [A] time = 0.0120964, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {3223}

$$\frac{\sin^4(x)}{4} + \sin(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*(1 + Sin[x]^3), x]

[Out] Sin[x] + Sin[x]^4/4

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\begin{aligned} \int \cos(x) (1 + \sin^3(x)) dx &= \text{Subst} \left(\int (1 + x^3) dx, x, \sin(x) \right) \\ &= \sin(x) + \frac{\sin^4(x)}{4} \end{aligned}$$

Mathematica [A] time = 0.0029038, size = 11, normalized size = 1.

$$\frac{\sin^4(x)}{4} + \sin(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*(1 + Sin[x]^3), x]

[Out] Sin[x] + Sin[x]^4/4

Maple [A] time = 0.013, size = 10, normalized size = 0.9

$$\sin(x) + \frac{(\sin(x))^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(x)*(1+sin(x)^3),x)`

[Out] `sin(x)+1/4*sin(x)^4`

Maxima [A] time = 0.916528, size = 12, normalized size = 1.09

$$\frac{1}{4} \sin(x)^4 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(1+sin(x)^3),x, algorithm="maxima")`

[Out] `1/4*sin(x)^4 + sin(x)`

Fricas [A] time = 2.01435, size = 51, normalized size = 4.64

$$\frac{1}{4} \cos(x)^4 - \frac{1}{2} \cos(x)^2 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(1+sin(x)^3),x, algorithm="fricas")`

[Out] `1/4*cos(x)^4 - 1/2*cos(x)^2 + sin(x)`

Sympy [A] time = 0.536143, size = 8, normalized size = 0.73

$$\frac{\sin^4(x)}{4} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(1+sin(x)**3),x)`

[Out] `sin(x)**4/4 + sin(x)`

Giac [A] time = 1.04765, size = 12, normalized size = 1.09

$$\frac{1}{4} \sin(x)^4 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(x)*(1+sin(x)^3),x, algorithm="giac")`

[Out] `1/4*sin(x)^4 + sin(x)`

$$3.66 \quad \int \frac{1}{x(1+\log^2(x))} dx$$

Optimal. Leaf size=3

$$\tan^{-1}(\log(x))$$

[Out] ArcTan[Log[x]]

Rubi [A] time = 0.0190214, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {203}

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 + Log[x]^2)),x]

[Out] ArcTan[Log[x]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{x(1+\log^2(x))} dx &= \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \log(x)\right) \\ &= \tan^{-1}(\log(x)) \end{aligned}$$

Mathematica [A] time = 0.0078829, size = 3, normalized size = 1.

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(1 + Log[x]^2)),x]

[Out] ArcTan[Log[x]]

Maple [A] time = 0., size = 4, normalized size = 1.3

$$\arctan(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(1+ln(x)^2),x)`

[Out] `arctan(ln(x))`

Maxima [A] time = 1.4175, size = 4, normalized size = 1.33

`arctan(log(x))`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+log(x)^2),x, algorithm="maxima")`

[Out] `arctan(log(x))`

Fricas [A] time = 1.79788, size = 22, normalized size = 7.33

`arctan(log(x))`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+log(x)^2),x, algorithm="fricas")`

[Out] `arctan(log(x))`

Sympy [B] time = 0.124024, size = 15, normalized size = 5.

`RootSum(4z2 + 1, (i ↦ i log(2i + log(x))))`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+ln(x)**2),x)`

[Out] `RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))`

Giac [A] time = 1.03959, size = 4, normalized size = 1.33

`arctan(log(x))`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+log(x)^2),x, algorithm="giac")`

[Out] `arctan(log(x))`

$$3.67 \quad \int \frac{1}{\sqrt{1-x^2}(1+\sin^{-1}(x)^2)} dx$$

Optimal. Leaf size=3

$$\tan^{-1}(\sin^{-1}(x))$$

[Out] ArcTan[ArcSin[x]]

Rubi [A] time = 0.0469675, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {6696, 203}

$$\tan^{-1}(\sin^{-1}(x))$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 - x^2]*(1 + ArcSin[x]^2)),x]

[Out] ArcTan[ArcSin[x]]

Rule 6696

Int[(u_.)*((a_.) + (b_.)*(y_)^(n_))^(p_), x_Symbol] :> With[{q = Derivative Divides[y, u, x]}, Dist[q, Subst[Int[(a + b*x^n)^p, x], x, y], x] /; !FalseQ[q]] /; FreeQ[{a, b, n, p}, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}(1+\sin^{-1}(x)^2)} dx = \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sin^{-1}(x) \right) = \tan^{-1}(\sin^{-1}(x))$$

Mathematica [A] time = 0.05518, size = 3, normalized size = 1.

$$\tan^{-1}(\sin^{-1}(x))$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[1 - x^2]*(1 + ArcSin[x]^2)),x]

[Out] ArcTan[ArcSin[x]]

Maple [A] time = 0.005, size = 4, normalized size = 1.3

$\arctan(\arcsin(x))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x)`

[Out] `arctan(arcsin(x))`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2 + 1}(\arcsin(x)^2 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(-x^2 + 1)*(arcsin(x)^2 + 1)), x)`

Fricas [A] time = 2.15427, size = 26, normalized size = 8.67

$\arctan(\arcsin(x))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="fricas")`

[Out] `arctan(arcsin(x))`

Sympy [A] time = 0.41762, size = 3, normalized size = 1.

$\operatorname{atan}(\operatorname{asin}(x))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+asin(x)**2)/(-x**2+1)**(1/2),x)`

[Out] `atan(asin(x))`

Giac [A] time = 1.06363, size = 4, normalized size = 1.33

$\arctan(\arcsin(x))$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="giac")`

[Out] `arctan(arcsin(x))`

$$3.68 \quad \int \frac{\sin(x)}{\cos(x)+\sin(x)} dx$$

Optimal. Leaf size=16

$$\frac{x}{2} - \frac{1}{2} \log(\sin(x) + \cos(x))$$

[Out] x/2 - Log[Cos[x] + Sin[x]]/2

Rubi [A] time = 0.0411408, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3097, 3133}

$$\frac{x}{2} - \frac{1}{2} \log(\sin(x) + \cos(x))$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(Cos[x] + Sin[x]),x]

[Out] x/2 - Log[Cos[x] + Sin[x]]/2

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]) / ((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a + b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c, d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C), 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sin(x)}{\cos(x) + \sin(x)} dx &= \frac{x}{2} - \frac{1}{2} \int \frac{\cos(x) - \sin(x)}{\cos(x) + \sin(x)} dx \\ &= \frac{x}{2} - \frac{1}{2} \log(\cos(x) + \sin(x)) \end{aligned}$$

Mathematica [A] time = 0.0232694, size = 16, normalized size = 1.

$$\frac{x}{2} - \frac{1}{2} \log(\sin(x) + \cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/(Cos[x] + Sin[x]),x]

[Out] $x/2 - \text{Log}[\text{Cos}[x] + \text{Sin}[x]]/2$

Maple [A] time = 0.023, size = 21, normalized size = 1.3

$$\frac{\ln((\tan(x))^2 + 1)}{4} - \frac{\ln(1 + \tan(x))}{2} + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/(cos(x)+sin(x)),x)`

[Out] $1/4*\ln(\tan(x)^2+1)-1/2*\ln(1+\tan(x))+1/2*x$

Maxima [B] time = 1.41169, size = 72, normalized size = 4.5

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) - \frac{1}{2} \log\left(-\frac{2\sin(x)}{\cos(x)+1} + \frac{\sin(x)^2}{(\cos(x)+1)^2} - 1\right) + \frac{1}{2} \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="maxima")`

[Out] $\arctan(\sin(x)/(\cos(x)+1)) - 1/2*\log(-2*\sin(x)/(\cos(x)+1) + \sin(x)^2/(\cos(x)+1)^2 - 1) + 1/2*\log(\sin(x)^2/(\cos(x)+1)^2 + 1)$

Fricas [A] time = 2.27533, size = 51, normalized size = 3.19

$$\frac{1}{2}x - \frac{1}{4} \log(2 \cos(x) \sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="fricas")`

[Out] $1/2*x - 1/4*\log(2*\cos(x)*\sin(x) + 1)$

Sympy [A] time = 0.132393, size = 12, normalized size = 0.75

$$\frac{x}{2} - \frac{\log(\sin(x) + \cos(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/(cos(x)+sin(x)),x)`

[Out] $x/2 - \log(\sin(x) + \cos(x))/2$

Giac [A] time = 1.08967, size = 28, normalized size = 1.75

$$\frac{1}{2}x + \frac{1}{4}\log(\tan(x)^2 + 1) - \frac{1}{2}\log(|\tan(x) + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="giac")

[Out] 1/2*x + 1/4*log(tan(x)^2 + 1) - 1/2*log(abs(tan(x) + 1))

$$3.69 \quad \int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2} dy$$

Optimal. Leaf size=53

$$-B \tan^{-1}\left(\frac{By}{\sqrt{A^2 - B^2y^2 + B^2}}\right) - A \tanh^{-1}\left(\frac{Ay}{\sqrt{A^2 - B^2y^2 + B^2}}\right)$$

[Out] $-(B \cdot \text{ArcTan}[(B \cdot y) / \text{Sqrt}[A^2 + B^2 - B^2 \cdot y^2]]) - A \cdot \text{ArcTanh}[(A \cdot y) / \text{Sqrt}[A^2 + B^2 - B^2 \cdot y^2]]$

Rubi [A] time = 0.0648788, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {1974, 402, 217, 203, 377, 206}

$$-B \tan^{-1}\left(\frac{By}{\sqrt{A^2 - B^2y^2 + B^2}}\right) - A \tanh^{-1}\left(\frac{Ay}{\sqrt{A^2 - B^2y^2 + B^2}}\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[-(\text{Sqrt}[A^2 + B^2 \cdot (1 - y^2)]) / (1 - y^2), y]$

[Out] $-(B \cdot \text{ArcTan}[(B \cdot y) / \text{Sqrt}[A^2 + B^2 - B^2 \cdot y^2]]) - A \cdot \text{ArcTanh}[(A \cdot y) / \text{Sqrt}[A^2 + B^2 - B^2 \cdot y^2]]$

Rule 1974

$\text{Int}[(u_)^{(p_)} \cdot (v_)^{(q_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandToSum}[u, x]^{p_} \cdot \text{ExpandToSum}[v, x]^{q_}, x] /;$ FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !BinomialMatchQ[{u, v}, x]

Rule 402

$\text{Int}[(a_ + (b_)\cdot(x_)^2)^{(p_)} / ((c_ + (d_)\cdot(x_)^2), x_Symbol] \rightarrow \text{Dist}[b/d, \text{Int}[(a + b \cdot x^2)^{(p-1)}, x], x] - \text{Dist}[(b \cdot c - a \cdot d)/d, \text{Int}[(a + b \cdot x^2)^{(p-1)} / (c + d \cdot x^2), x], x] /;$ FreeQ[{a, b, c, d}, x] && NeQ[b \cdot c - a \cdot d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_ + (b_)\cdot(x_)^2), x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b \cdot x^2), x], x, x/\text{Sqrt}[a + b \cdot x^2]] /;$ FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 203

$\text{Int}[(a_ + (b_)\cdot(x_)^2)^{(-1)}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTan}[(\text{Rt}[b, 2] \cdot x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[b, 2]), x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 377

$\text{Int}[(a_ + (b_)\cdot(x_)^n)^{(p_)} / ((c_ + (d_)\cdot(x_)^n), x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(c - (b \cdot c - a \cdot d) \cdot x^n), x], x, x/(a + b \cdot x^n)^{(1/n)}] /;$ FreeQ[{a, b, c, d}, x] && NeQ[b \cdot c - a \cdot d, 0] && EqQ[n \cdot p + 1, 0] && IntegerQ[n]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/
Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int -\frac{\sqrt{A^2 + B^2(1 - y^2)}}{1 - y^2} dy &= -\int \frac{\sqrt{A^2 + B^2 - B^2y^2}}{1 - y^2} dy \\ &= -\left(A^2 \int \frac{1}{(1 - y^2)\sqrt{A^2 + B^2 - B^2y^2}} dy \right) - B^2 \int \frac{1}{\sqrt{A^2 + B^2 - B^2y^2}} dy \\ &= -\left(A^2 \operatorname{Subst}\left(\int \frac{1}{1 - A^2y^2} dy, y, \frac{y}{\sqrt{A^2 + B^2 - B^2y^2}} \right) \right) - B^2 \operatorname{Subst}\left(\int \frac{1}{1 + B^2y^2} dy, y, \frac{y}{\sqrt{A^2 + B^2 - B^2y^2}} \right) \\ &= -B \tan^{-1}\left(\frac{By}{\sqrt{A^2 + B^2 - B^2y^2}} \right) - A \tanh^{-1}\left(\frac{Ay}{\sqrt{A^2 + B^2 - B^2y^2}} \right) \end{aligned}$$

Mathematica [C] time = 0.0549715, size = 127, normalized size = 2.4

$$\frac{1}{2} \left(-2iB \log\left(2 \left(\sqrt{A^2 - B^2y^2 + B^2} - iBy \right) \right) - A \log\left(A \sqrt{A^2 - B^2y^2 + B^2} + A^2 - B^2y + B^2 \right) + A \log\left(A \sqrt{A^2 - B^2y^2 + B^2} - A^2 + B^2y + B^2 \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[-(Sqrt[A^2 + B^2*(1 - y^2)]/(1 - y^2)), y]
```

```
[Out] (A*Log[1 - y] - A*Log[1 + y] - (2*I)*B*Log[2*((-I)*B*y + Sqrt[A^2 + B^2 - B^2*y^2]]) - A*Log[A^2 + B^2 - B^2*y + A*Sqrt[A^2 + B^2 - B^2*y^2]] + A*Log[A^2 + B^2*(1 + y) + A*Sqrt[A^2 + B^2 - B^2*y^2]])/2
```

Maple [B] time = 0.006, size = 262, normalized size = 4.9

$$-\frac{1}{2} \sqrt{-B^2(1+y)^2 + 2B^2(1+y) + A^2} - \frac{B^2}{2} \arctan\left(y\sqrt{B^2} \frac{1}{\sqrt{-B^2(1+y)^2 + 2B^2(1+y) + A^2}} \right) \frac{1}{\sqrt{B^2}} + \frac{A^2}{2} \ln\left(\frac{1}{1+y} \left(2\sqrt{-B^2(1+y)^2 + 2B^2(1+y) + A^2} + A \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1), y)
```

```
[Out] -1/2*(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2)-1/2*B^2/(B^2)^(1/2)*arctan((B^2)^(1/2)*y/(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2))+1/2*A^2/(A^2)^(1/2)*ln((2*A^2+2*B^2*(1+y)+2*(A^2)^(1/2)*(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2))/(1+y))+1/2*(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2)-1/2*B^2/(B^2)^(1/2)*arctan((B^2)^(1/2)*y/(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2))-1/2*A^2/(A^2)^(1/2)*ln((2*A^2-2*B^2*(y-1)+2*(A^2)^(1/2)*(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2))/(y-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.6256, size = 284, normalized size = 5.36

$$B \arctan\left(\frac{\sqrt{-B^2y^2 + A^2 + B^2}}{By}\right) - \frac{1}{4} A \log\left(-\frac{(A^2 - B^2)y^2 + 2\sqrt{-B^2y^2 + A^2 + B^2}Ay + A^2 + B^2}{y^2}\right) + \frac{1}{4} A \log\left(-\frac{(A^2 - B^2)y^2 + 2\sqrt{-B^2y^2 + A^2 + B^2}Ay + A^2 + B^2}{y^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="fricas")

[Out] B*arctan(sqrt(-B^2*y^2 + A^2 + B^2)/(B*y)) - 1/4*A*log(-((A^2 - B^2)*y^2 + 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2) + 1/4*A*log(-((A^2 - B^2)*y^2 - 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{A^2 - B^2y^2 + B^2}}{(y-1)(y+1)} dy$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(A**2+B**2*(-y**2+1))**(1/2)/(-y**2+1),y)

[Out] Integral(sqrt(A**2 - B**2*y**2 + B**2)/((y - 1)*(y + 1)), y)

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="giac")

[Out] Timed out

$$3.70 \quad \int \frac{(-A^2 - B^2) \cos^2(z)}{B \left(1 - \frac{(A^2 + B^2) \sin^2(z)}{B^2} \right)} dz$$

Optimal. Leaf size=16

$$-A \tanh^{-1} \left(\frac{A \tan(z)}{B} \right) - Bz$$

[Out] $-(B*z) - A*\text{ArcTanh}[(A*\text{Tan}[z])/B]$

Rubi [A] time = 0.0859799, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.128$, Rules used = {12, 3191, 391, 203, 206}

$$-A \tanh^{-1} \left(\frac{A \tan(z)}{B} \right) - Bz$$

Antiderivative was successfully verified.

[In] $\text{Int}[((-A^2 - B^2) * \text{Cos}[z]^2) / (B * (1 - ((A^2 + B^2) * \text{Sin}[z]^2) / B^2)), z]$

[Out] $-(B*z) - A*\text{ArcTanh}[(A*\text{Tan}[z])/B]$

Rule 12

$\text{Int}[(a_)*(u_), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

Rule 3191

$\text{Int}[\cos[(e_.) + (f_.)*(x_.)]^{(m_.)} * ((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[ff/f, \text{Subst}[\text{Int}[(a + (a + b)*ff^2*x^2)^p / (1 + ff^2*x^2)^{(m/2 + p + 1)}, x], x, \text{Tan}[e + f*x]/ff], x] /; \text{FreeQ}\{a, b, e, f\}, x] \ \&\& \ \text{IntegerQ}[m/2] \ \&\& \ \text{IntegerQ}[p]$

Rule 391

$\text{Int}[1/(((a_.) + (b_.)*(x_.)^{(n_.)})) * ((c_.) + (d_.)*(x_.)^{(n_.)}), x_Symbol] \rightarrow \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*x^n), x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*x^n), x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 203

$\text{Int}(((a_.) + (b_.)*(x_.)^2)^{-1}), x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 206

$\text{Int}(((a_.) + (b_.)*(x_.)^2)^{-1}), x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{(-A^2 - B^2) \cos^2(z)}{B \left(1 - \frac{(A^2 + B^2) \sin^2(z)}{B^2}\right)} dz &= - \frac{(A^2 + B^2) \int \frac{\cos^2(z)}{1 - \frac{(A^2 + B^2) \sin^2(z)}{B^2}} dz}{B} \\
&= - \frac{(A^2 + B^2) \operatorname{Subst} \left(\int \frac{1}{(1+z^2) \left(1 + \left(1 - \frac{A^2 + B^2}{B^2}\right) z^2\right)} dz, z, \tan(z) \right)}{B} \\
&= - \frac{A^2 \operatorname{Subst} \left(\int \frac{1}{1 + \left(1 - \frac{A^2 + B^2}{B^2}\right) z^2} dz, z, \tan(z) \right)}{B} - B \operatorname{Subst} \left(\int \frac{1}{1 + z^2} dz, z, \tan(z) \right) \\
&= -Bz - A \tanh^{-1} \left(\frac{A \tan(z)}{B} \right)
\end{aligned}$$

Mathematica [B] time = 0.0902899, size = 35, normalized size = 2.19

$$\frac{B(A^2 + B^2) \left(A \tanh^{-1} \left(\frac{A \tan(z)}{B} \right) + Bz \right)}{A^2 B + B^3}$$

Antiderivative was successfully verified.

[In] Integrate[((-A^2 - B^2)*Cos[z]^2)/(B*(1 - ((A^2 + B^2)*Sin[z]^2)/B^2)),z]

[Out] -((B*(A^2 + B^2)*(B*z + A*ArcTanh[(A*Tan[z])/B]))/(A^2*B + B^3))

Maple [B] time = 0.036, size = 127, normalized size = 7.9

$$\frac{A^3 \ln(A \tan(z) - B)}{2A^2 + 2B^2} + \frac{AB^2 \ln(A \tan(z) - B)}{2A^2 + 2B^2} - \frac{B \arctan(\tan(z)) A^2}{A^2 + B^2} - \frac{\arctan(\tan(z)) B^3}{A^2 + B^2} - \frac{A^3 \ln(A \tan(z) + B)}{2A^2 + 2B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z)

[Out] 1/2*A^3/(A^2+B^2)*ln(A*tan(z)-B)+1/2*A*B^2/(A^2+B^2)*ln(A*tan(z)-B)-B/(A^2+B^2)*arctan(tan(z))*A^2-1/(A^2+B^2)*arctan(tan(z))*B^3-1/2*A^3/(A^2+B^2)*ln(A*tan(z)+B)-1/2*A*B^2/(A^2+B^2)*ln(A*tan(z)+B)

Maxima [B] time = 1.41742, size = 93, normalized size = 5.81

$$\frac{(A^2 + B^2) \left(\frac{2B^2 z}{A^2 + B^2} + \frac{AB \log(A \tan(z) + B)}{A^2 + B^2} - \frac{AB \log(A \tan(z) - B)}{A^2 + B^2} \right)}{2B}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="maxima")

[Out] $-1/2*(A^2 + B^2)*(2*B^2*z/(A^2 + B^2) + A*B*\log(A*\tan(z) + B)/(A^2 + B^2) - A*B*\log(A*\tan(z) - B)/(A^2 + B^2))/B$

Fricas [B] time = 2.73631, size = 177, normalized size = 11.06

$-Bz - \frac{1}{4} A \log(2AB \cos(z) \sin(z) - (A^2 - B^2) \cos(z)^2 + A^2) + \frac{1}{4} A \log(-2AB \cos(z) \sin(z) - (A^2 - B^2) \cos(z)^2 + A^2)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="fricas")

[Out] $-B*z - 1/4*A*\log(2*A*B*\cos(z)*\sin(z) - (A^2 - B^2)*\cos(z)^2 + A^2) + 1/4*A*\log(-2*A*B*\cos(z)*\sin(z) - (A^2 - B^2)*\cos(z)^2 + A^2)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-A**2-B**2)*cos(z)**2/B/(1-(A**2+B**2)*sin(z)**2/B**2),z)

[Out] Timed out

Giac [B] time = 1.11659, size = 112, normalized size = 7.

$$\frac{\left(\frac{A^3 B \log(|A \tan(z) + B|)}{A^4 + A^2 B^2} - \frac{A^3 B \log(|A \tan(z) - B|)}{A^4 + A^2 B^2} + \frac{2 B^2 z}{A^2 + B^2}\right) (A^2 + B^2)}{2 B}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="giac")

[Out] $-1/2*(A^3*B*\log(\text{abs}(A*\tan(z) + B))/(A^4 + A^2*B^2) - A^3*B*\log(\text{abs}(A*\tan(z) - B))/(A^4 + A^2*B^2) + 2*B^2*z/(A^2 + B^2))*(A^2 + B^2)/B$

$$3.71 \quad \int -\frac{A^2+B^2}{B(1+w^2)^2\left(1-\frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)} dw$$

Optimal. Leaf size=16

$$-A \tanh^{-1}\left(\frac{Aw}{B}\right) - B \tan^{-1}(w)$$

[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]

Rubi [A] time = 0.124371, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {12, 6688, 391, 203, 208}

$$-A \tanh^{-1}\left(\frac{Aw}{B}\right) - B \tan^{-1}(w)$$

Antiderivative was successfully verified.

[In] Int[-((A^2 + B^2)/(B*(1 + w^2)^2*(1 - ((A^2 + B^2)*w^2)/(B^2*(1 + w^2))))), w]

[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 6688

Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

Rule 391

Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int -\frac{A^2 + B^2}{B(1+w^2)^2 \left(1 - \frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)} dw &= -\frac{(A^2 + B^2) \int \frac{1}{(1+w^2)^2 \left(1 - \frac{(A^2+B^2)w^2}{B^2(1+w^2)}\right)} dw}{B} \\
&= -\frac{(A^2 + B^2) \int \frac{B^2}{(1+w^2)(B^2 - A^2w^2)} dw}{B} \\
&= -\left((B(A^2 + B^2)) \int \frac{1}{(1+w^2)(B^2 - A^2w^2)} dw \right) \\
&= -\left(B \int \frac{1}{1+w^2} dw \right) - (A^2B) \int \frac{1}{B^2 - A^2w^2} dw \\
&= -B \tan^{-1}(w) - A \tanh^{-1}\left(\frac{Aw}{B}\right)
\end{aligned}$$

Mathematica [B] time = 0.01813, size = 35, normalized size = 2.19

$$-\frac{B(A^2 + B^2) \left(A \tanh^{-1}\left(\frac{Aw}{B}\right) + B \tan^{-1}(w) \right)}{A^2B + B^3}$$

Antiderivative was successfully verified.

[In] Integrate[-((A^2 + B^2)/(B*(1 + w^2)^2*(1 - ((A^2 + B^2)*w^2)/(B^2*(1 + w^2))))),w]

[Out] -((B*(A^2 + B^2)*(B*ArcTan[w] + A*ArcTanh[(A*w)/B]))/(A^2*B + B^3))

Maple [B] time = 0.007, size = 121, normalized size = 7.6

$$\frac{A^3 \ln(Aw - B)}{2A^2 + 2B^2} + \frac{AB^2 \ln(Aw - B)}{2A^2 + 2B^2} - \frac{B \arctan(w) A^2}{A^2 + B^2} - \frac{\arctan(w) B^3}{A^2 + B^2} - \frac{A^3 \ln(Aw + B)}{2A^2 + 2B^2} - \frac{AB^2 \ln(Aw + B)}{2A^2 + 2B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w)

[Out] 1/2*A^3/(A^2+B^2)*ln(A*w-B)+1/2*A*B^2/(A^2+B^2)*ln(A*w-B)-B/(A^2+B^2)*arctan(w)*A^2-1/(A^2+B^2)*arctan(w)*B^3-1/2*A^3/(A^2+B^2)*ln(A*w+B)-1/2*A*B^2/(A^2+B^2)*ln(A*w+B)

Maxima [B] time = 1.41031, size = 92, normalized size = 5.75

$$-\frac{(A^2 + B^2) \left(\frac{2B^2 \arctan(w)}{A^2+B^2} + \frac{AB \log(Aw+B)}{A^2+B^2} - \frac{AB \log(Aw-B)}{A^2+B^2} \right)}{2B}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorithm="maxima")

[Out] $-1/2*(A^2 + B^2)*(2*B^2*\arctan(w)/(A^2 + B^2) + A*B*\log(A*w + B)/(A^2 + B^2) - A*B*\log(A*w - B)/(A^2 + B^2))/B$

Fricas [A] time = 2.16152, size = 76, normalized size = 4.75

$$-B \arctan(w) - \frac{1}{2} A \log(Aw + B) + \frac{1}{2} A \log(Aw - B)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorithm="fricas")`

[Out] $-B*\arctan(w) - 1/2*A*\log(A*w + B) + 1/2*A*\log(A*w - B)$

Sympy [C] time = 1.30127, size = 422, normalized size = 26.38

$$(A^2B + B^3) \left(\frac{A \log \left(w + \frac{-\frac{A^9}{B(A^2+B^2)^3} - \frac{A^7B}{(A^2+B^2)^3} + \frac{A^5B^3}{(A^2+B^2)^3} + \frac{A^5}{B(A^2+B^2)} + \frac{A^3B^5}{(A^2+B^2)^3} + \frac{AB^3}{A^2+B^2}}{A^2} \right)}{2B(A^2 + B^2)} \right) + \frac{A \log \left(w + \frac{\frac{A^9}{B(A^2+B^2)^3} + \frac{A^7B}{(A^2+B^2)^3} - \frac{A^5B^3}{(A^2+B^2)^3}}{A^2} \right)}{2B(A^2 + B^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-A**2-B**2)/B/(w**2+1)**2/(1-(A**2+B**2)*w**2/B**2/(w**2+1)),w)`

[Out] $(A**2*B + B**3)*(-A*\log(w + (-A**9/(B*(A**2 + B**2)**3) - A**7*B/(A**2 + B**2)**3 + A**5*B**3/(A**2 + B**2)**3 + A**5/(B*(A**2 + B**2)) + A**3*B**5/(A**2 + B**2)**3 + A*B**3/(A**2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + A*\log(w + (A**9/(B*(A**2 + B**2)**3) + A**7*B/(A**2 + B**2)**3 - A**5*B**3/(A**2 + B**2)**3 - A**5/(B*(A**2 + B**2)) - A**3*B**5/(A**2 + B**2)**3 - A*B**3/(A**2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + I*\log(w + (-I*A**6*B**2/(A**2 + B**2)**3 - I*A**4*B**4/(A**2 + B**2)**3 - I*A**4/(A**2 + B**2) + I*A**2*B**6/(A**2 + B**2)**3 + I*B**8/(A**2 + B**2)**3 - I*B**4/(A**2 + B**2))/A**2)/(2*(A**2 + B**2)) - I*\log(w + (I*A**6*B**2/(A**2 + B**2)**3 + I*A**4*B**4/(A**2 + B**2)**3 + I*A**4/(A**2 + B**2) - I*A**2*B**6/(A**2 + B**2)**3 - I*B**8/(A**2 + B**2)**3 + I*B**4/(A**2 + B**2))/A**2)/(2*(A**2 + B**2)))$

Giac [B] time = 1.09746, size = 111, normalized size = 6.94

$$\frac{\left(\frac{A^3B \log(|Aw+B|)}{A^4+A^2B^2} - \frac{A^3B \log(|Aw-B|)}{A^4+A^2B^2} + \frac{2B^2 \arctan(w)}{A^2+B^2} \right) (A^2 + B^2)}{2B}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorithm="giac")`

[Out] $-1/2*(A^3*B*\log(\text{abs}(A*w + B))/(A^4 + A^2*B^2) - A^3*B*\log(\text{abs}(A*w - B))/(A^4 + A^2*B^2) + 2*B^2*\arctan(w)/(A^2 + B^2))*(A^2 + B^2)/B$

$$3.72 \quad \int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)} dw$$

Optimal. Leaf size=16

$$-A \tanh^{-1}\left(\frac{Aw}{B}\right) - B \tan^{-1}(w)$$

[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]

Rubi [A] time = 0.0186529, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {12, 391, 203, 208}

$$-A \tanh^{-1}\left(\frac{Aw}{B}\right) - B \tan^{-1}(w)$$

Antiderivative was successfully verified.

[In] Int[-((B*(A^2 + B^2))/((1 + w^2)*(B^2 - A^2*w^2))),w]

[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 391

Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)} dw &= -\left((B(A^2+B^2)) \int \frac{1}{(1+w^2)(B^2-A^2w^2)} dw \right) \\ &= -\left(B \int \frac{1}{1+w^2} dw \right) - (A^2B) \int \frac{1}{B^2-A^2w^2} dw \\ &= -B \tan^{-1}(w) - A \tanh^{-1}\left(\frac{Aw}{B}\right) \end{aligned}$$

Mathematica [B] time = 0.0130182, size = 35, normalized size = 2.19

$$\frac{B(A^2 + B^2) \left(A \tanh^{-1} \left(\frac{Aw}{B} \right) + B \tan^{-1}(w) \right)}{A^2 B + B^3}$$

Antiderivative was successfully verified.

[In] Integrate[-((B*(A^2 + B^2))/((1 + w^2)*(B^2 - A^2*w^2))),w]

[Out] -((B*(A^2 + B^2)*(B*ArcTan[w] + A*ArcTanh[(A*w)/B]))/(A^2*B + B^3))

Maple [B] time = 0.005, size = 121, normalized size = 7.6

$$\frac{A^3 \ln(Aw - B)}{2A^2 + 2B^2} + \frac{AB^2 \ln(Aw - B)}{2A^2 + 2B^2} - \frac{B \arctan(w) A^2}{A^2 + B^2} - \frac{\arctan(w) B^3}{A^2 + B^2} - \frac{A^3 \ln(Aw + B)}{2A^2 + 2B^2} - \frac{AB^2 \ln(Aw + B)}{2A^2 + 2B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w)

[Out] 1/2*A^3/(A^2+B^2)*ln(A*w-B)+1/2*A*B^2/(A^2+B^2)*ln(A*w-B)-B/(A^2+B^2)*arctan(w)*A^2-1/(A^2+B^2)*arctan(w)*B^3-1/2*A^3/(A^2+B^2)*ln(A*w+B)-1/2*A*B^2/(A^2+B^2)*ln(A*w+B)

Maxima [B] time = 1.41032, size = 88, normalized size = 5.5

$$-\frac{1}{2} (A^2 + B^2) B \left(\frac{A \log(Aw + B)}{A^2 B + B^3} - \frac{A \log(Aw - B)}{A^2 B + B^3} + \frac{2 \arctan(w)}{A^2 + B^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w, algorithm="maxima")

[Out] -1/2*(A^2 + B^2)*B*(A*log(A*w + B)/(A^2*B + B^3) - A*log(A*w - B)/(A^2*B + B^3) + 2*arctan(w)/(A^2 + B^2))

Fricas [A] time = 2.43177, size = 76, normalized size = 4.75

$$-B \arctan(w) - \frac{1}{2} A \log(Aw + B) + \frac{1}{2} A \log(Aw - B)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w, algorithm="fricas")

[Out] -B*arctan(w) - 1/2*A*log(A*w + B) + 1/2*A*log(A*w - B)

Sympy [C] time = 1.27477, size = 422, normalized size = 26.38

$$(A^2B + B^3) \left(\frac{A \log \left(w + \frac{-\frac{A^9}{B(A^2+B^2)^3} - \frac{A^7B}{(A^2+B^2)^3} + \frac{A^5B^3}{(A^2+B^2)^3} + \frac{A^5}{B(A^2+B^2)} + \frac{A^3B^5}{(A^2+B^2)^3} + \frac{AB^3}{A^2+B^2}}{A^2} \right)}{2B(A^2 + B^2)} + \frac{A \log \left(w + \frac{\frac{A^9}{B(A^2+B^2)^3} + \frac{A^7B}{(A^2+B^2)^3} - \frac{A^5B^3}{(A^2+B^2)^3} - \frac{A^5}{B(A^2+B^2)} - \frac{A^3B^5}{(A^2+B^2)^3} - \frac{AB^3}{A^2+B^2}}{A^2} \right)}{2B(A^2 + B^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-B*(A**2+B**2)/(w**2+1)/(-A**2*w**2+B**2), w)

[Out] (A**2*B + B**3)*(-A*log(w + (-A**9/(B*(A**2 + B**2)**3) - A**7*B/(A**2 + B**2)**3 + A**5*B**3/(A**2 + B**2)**3 + A**5/(B*(A**2 + B**2))) + A**3*B**5/(A**2 + B**2)**3 + A*B**3/(A**2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + A*log(w + (A**9/(B*(A**2 + B**2)**3) + A**7*B/(A**2 + B**2)**3 - A**5*B**3/(A**2 + B**2)**3 - A**5/(B*(A**2 + B**2))) - A**3*B**5/(A**2 + B**2)**3 - A*B**3/(A**2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + I*log(w + (-I*A**6*B**2/(A**2 + B**2)**3 - I*A**4*B**4/(A**2 + B**2)**3 - I*A**4/(A**2 + B**2) + I*A**2*B**6/(A**2 + B**2)**3 + I*B**8/(A**2 + B**2)**3 - I*B**4/(A**2 + B**2))/A**2)/(2*(A**2 + B**2)) - I*log(w + (I*A**6*B**2/(A**2 + B**2)**3 + I*A**4*B**4/(A**2 + B**2)**3 + I*A**4/(A**2 + B**2) - I*A**2*B**6/(A**2 + B**2)**3 - I*B**8/(A**2 + B**2)**3 + I*B**4/(A**2 + B**2))/A**2)/(2*(A**2 + B**2)))

Giac [B] time = 1.07565, size = 107, normalized size = 6.69

$$-\frac{1}{2} \left(\frac{A^3 \log(|Aw + B|)}{A^4B + A^2B^3} - \frac{A^3 \log(|Aw - B|)}{A^4B + A^2B^3} + \frac{2 \arctan(w)}{A^2 + B^2} \right) (A^2 + B^2)B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2), w, algorithm="giac")

[Out] -1/2*(A^3*log(abs(A*w + B))/(A^4*B + A^2*B^3) - A^3*log(abs(A*w - B))/(A^4*B + A^2*B^3) + 2*arctan(w)/(A^2 + B^2))*(A^2 + B^2)*B

$$3.73 \quad \int \frac{x^4}{(1-x^2)^{5/2}} dx$$

Optimal. Leaf size=35

$$\frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$$

[Out] $x^3/(3*(1-x^2)^{(3/2)}) - x/\text{Sqrt}[1-x^2] + \text{ArcSin}[x]$

Rubi [A] time = 0.0080682, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {288, 216}

$$\frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^4/(1-x^2)^{(5/2)}, x]$

[Out] $x^3/(3*(1-x^2)^{(3/2)}) - x/\text{Sqrt}[1-x^2] + \text{ArcSin}[x]$

Rule 288

$\text{Int}[(c_.)*(x_)^{(m_.)*((a_.) + (b_.)*(x_)^{(n_)})^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a+b*x^n)^{(p+1)})/(b*n*(p+1)), x] - \text{Dist}[(c^n*(m-n+1))/(b*n*(p+1)), \text{Int}[(c*x)^{(m-n)}*(a+b*x^n)^{(p+1)}, x], x] /;$ FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !ILtQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 216

$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /;$ FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned} \int \frac{x^4}{(1-x^2)^{5/2}} dx &= \frac{x^3}{3(1-x^2)^{3/2}} - \int \frac{x^2}{(1-x^2)^{3/2}} dx \\ &= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}} dx \\ &= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.020909, size = 26, normalized size = 0.74

$$\frac{x(4x^2-3)}{3(1-x^2)^{3/2}} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(1 - x^2)^(5/2),x]

[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]

Maple [A] time = 0., size = 30, normalized size = 0.9

$$\frac{x^3}{3} (-x^2 + 1)^{-\frac{3}{2}} + \arcsin(x) - x \frac{1}{\sqrt{-x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(-x^2+1)^(5/2),x)

[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)

Maxima [A] time = 1.41289, size = 59, normalized size = 1.69

$$\frac{1}{3} x \left(\frac{3x^2}{(-x^2 + 1)^{\frac{3}{2}}} - \frac{2}{(-x^2 + 1)^{\frac{3}{2}}} \right) - \frac{x}{3\sqrt{-x^2 + 1}} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")

[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1) + arcsin(x)

Fricas [B] time = 2.28929, size = 146, normalized size = 4.17

$$\frac{6(x^4 - 2x^2 + 1) \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - (4x^3 - 3x)\sqrt{-x^2 + 1}}{3(x^4 - 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")

[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sqrt(-x^2 + 1))/(x^4 - 2*x^2 + 1)

Sympy [B] time = 2.55844, size = 105, normalized size = 3.

$$\frac{3x^4 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3} + \frac{4x^3 \sqrt{1 - x^2}}{3x^4 - 6x^2 + 3} - \frac{6x^2 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3} - \frac{3x \sqrt{1 - x^2}}{3x^4 - 6x^2 + 3} + \frac{3 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(-x**2+1)**(5/2),x)

[Out] $3x^{**4}\operatorname{asin}(x)/(3x^{**4} - 6x^{**2} + 3) + 4x^{**3}\sqrt{1 - x^{**2}}/(3x^{**4} - 6x^{**2} + 3) - 6x^{**2}\operatorname{asin}(x)/(3x^{**4} - 6x^{**2} + 3) - 3x\sqrt{1 - x^{**2}}/(3x^{**4} - 6x^{**2} + 3) + 3\operatorname{asin}(x)/(3x^{**4} - 6x^{**2} + 3)$

Giac [A] time = 1.06539, size = 39, normalized size = 1.11

$$\frac{(4x^2 - 3)\sqrt{-x^2 + 1}x}{3(x^2 - 1)^2} + \operatorname{arcsin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")

[Out] $1/3*(4*x^2 - 3)*\sqrt{-x^2 + 1}*x/(x^2 - 1)^2 + \operatorname{arcsin}(x)$

3.74 $\int \tan^4(y) dy$

Optimal. Leaf size=14

$$y + \frac{\tan^3(y)}{3} - \tan(y)$$

[Out] $y - \tan(y) + \tan(y)^3/3$

Rubi [A] time = 0.0094383, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3473, 8}

$$y + \frac{\tan^3(y)}{3} - \tan(y)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\tan(y)^4, y]$

[Out] $y - \tan(y) + \tan(y)^3/3$

Rule 3473

$\text{Int}[(b \cdot \tan(c + d \cdot x))^n, x_Symbol] \rightarrow \text{Simp}[(b \cdot \tan(c + d \cdot x))^{n-1} / (d \cdot (n-1)), x] - \text{Dist}[b^2, \text{Int}[(b \cdot \tan(c + d \cdot x))^{n-2}, x], x] /;$ $\text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 1]$

Rule 8

$\text{Int}[a, x_Symbol] \rightarrow \text{Simp}[a \cdot x, x] /;$ $\text{FreeQ}[a, x]$

Rubi steps

$$\begin{aligned} \int \tan^4(y) dy &= \frac{\tan^3(y)}{3} - \int \tan^2(y) dy \\ &= -\tan(y) + \frac{\tan^3(y)}{3} + \int 1 dy \\ &= y - \tan(y) + \frac{\tan^3(y)}{3} \end{aligned}$$

Mathematica [A] time = 0.006699, size = 18, normalized size = 1.29

$$y - \frac{4 \tan(y)}{3} + \frac{1}{3} \tan(y) \sec^2(y)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\tan(y)^4, y]$

[Out] $y - (4 \cdot \tan(y))/3 + (\sec(y)^2 \cdot \tan(y))/3$

Maple [A] time = 0.007, size = 13, normalized size = 0.9

$$y - \tan(y) + \frac{(\tan(y))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(y)^4/cos(y)^4,y)

[Out] y-tan(y)+1/3*tan(y)^3

Maxima [A] time = 1.41807, size = 16, normalized size = 1.14

$$\frac{1}{3} \tan(y)^3 + y - \tan(y)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(y)^4/cos(y)^4,y, algorithm="maxima")

[Out] 1/3*tan(y)^3 + y - tan(y)

Fricas [B] time = 2.2519, size = 74, normalized size = 5.29

$$\frac{3y \cos(y)^3 - (4 \cos(y)^2 - 1) \sin(y)}{3 \cos(y)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(y)^4/cos(y)^4,y, algorithm="fricas")

[Out] 1/3*(3*y*cos(y)^3 - (4*cos(y)^2 - 1)*sin(y))/cos(y)^3

Sympy [A] time = 0.060681, size = 19, normalized size = 1.36

$$y + \frac{\sin^3(y)}{3 \cos^3(y)} - \frac{\sin(y)}{\cos(y)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(y)**4/cos(y)**4,y)

[Out] y + sin(y)**3/(3*cos(y)**3) - sin(y)/cos(y)

Giac [A] time = 1.06633, size = 16, normalized size = 1.14

$$\frac{1}{3} \tan(y)^3 + y - \tan(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)^4/cos(y)^4,y, algorithm="giac")
```

```
[Out] 1/3*tan(y)^3 + y - tan(y)
```


$$3.75 \quad \int \frac{z^4}{1+z^2} dz$$

Optimal. Leaf size=13

$$\frac{z^3}{3} - z + \tan^{-1}(z)$$

[Out] -z + z^3/3 + ArcTan[z]

Rubi [A] time = 0.0053858, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {302, 203}

$$\frac{z^3}{3} - z + \tan^{-1}(z)$$

Antiderivative was successfully verified.

[In] Int[z^4/(1 + z^2),z]

[Out] -z + z^3/3 + ArcTan[z]

Rule 302

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{z^4}{1+z^2} dz &= \int \left(-1 + z^2 + \frac{1}{1+z^2} \right) dz \\ &= -z + \frac{z^3}{3} + \int \frac{1}{1+z^2} dz \\ &= -z + \frac{z^3}{3} + \tan^{-1}(z) \end{aligned}$$

Mathematica [A] time = 0.0039374, size = 13, normalized size = 1.

$$\frac{z^3}{3} - z + \tan^{-1}(z)$$

Antiderivative was successfully verified.

[In] Integrate[z^4/(1 + z^2),z]

[Out] $-z + z^3/3 + \text{ArcTan}[z]$

Maple [A] time = 0.001, size = 12, normalized size = 0.9

$$-z + \frac{z^3}{3} + \arctan(z)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(z^4/(z^2+1),z)`

[Out] $-z+1/3*z^3+\arctan(z)$

Maxima [A] time = 1.40311, size = 15, normalized size = 1.15

$$\frac{1}{3}z^3 - z + \arctan(z)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(z^4/(z^2+1),z, algorithm="maxima")`

[Out] $1/3*z^3 - z + \arctan(z)$

Fricas [A] time = 2.09771, size = 34, normalized size = 2.62

$$\frac{1}{3}z^3 - z + \arctan(z)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(z^4/(z^2+1),z, algorithm="fricas")`

[Out] $1/3*z^3 - z + \arctan(z)$

Sympy [A] time = 0.080637, size = 8, normalized size = 0.62

$$\frac{z^3}{3} - z + \text{atan}(z)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(z**4/(z**2+1),z)`

[Out] $z**3/3 - z + \text{atan}(z)$

Giac [A] time = 1.05979, size = 15, normalized size = 1.15

$$\frac{1}{3}z^3 - z + \arctan(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^4/(z^2+1),z, algorithm="giac")
```

```
[Out] 1/3*z^3 - z + arctan(z)
```

3.76 $\int e^{x^2} (1 + 2x^2) dx$

Optimal. Leaf size=7

$$e^{x^2} x$$

[Out] $E^{x^2} x$

Rubi [A] time = 0.0298354, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2226, 2204, 2212}

$$e^{x^2} x$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{x^2}(1 + 2x^2), x]$

[Out] $E^{x^2} x$

Rule 2226

$\text{Int}[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] \text{ :> Int}[E\text{x}pandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; \text{FreeQ}\{F, a, b, c, d, n\}, x] \ \&\& \ \text{PolynomialQ}[u, x]$

Rule 2204

$\text{Int}[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2)), x_Symbol] \text{ :> Simp}[(F^a*\text{Sqrt}[\text{Pi}]*\text{Erfi}[(c + d*x)*\text{Rt}[b*\text{Log}[F], 2]])/(2*d*\text{Rt}[b*\text{Log}[F], 2]), x] /; \text{FreeQ}\{F, a, b, c, d\}, x] \ \&\& \ \text{PosQ}[b]$

Rule 2212

$\text{Int}[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] \text{ :> Simp}[(c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n)/(b*d*n*\text{Log}[F]), x] - \text{Dist}[(m - n + 1)/(b*n*\text{Log}[F]), \text{Int}[(c + d*x)^(m - n)*F^(a + b*(c + d*x)^n), x], x] /; \text{FreeQ}\{F, a, b, c, d\}, x] \ \&\& \ \text{IntegerQ}[(2*(m + 1))/n] \ \&\& \ \text{LtQ}[0, (m + 1)/n, 5] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ (\text{LtQ}[0, n, m + 1] \ || \ \text{LtQ}[m, n, 0])$

Rubi steps

$$\begin{aligned} \int e^{x^2} (1 + 2x^2) dx &= \int (e^{x^2} + 2e^{x^2} x^2) dx \\ &= 2 \int e^{x^2} x^2 dx + \int e^{x^2} dx \\ &= e^{x^2} x + \frac{1}{2} \sqrt{\pi} \text{erfi}(x) - \int e^{x^2} dx \\ &= e^{x^2} x \end{aligned}$$

Mathematica [A] time = 0.0043513, size = 7, normalized size = 1.

$$e^{x^2} x$$

Antiderivative was successfully verified.

[In] Integrate[E^x^2*(1 + 2*x^2),x]

[Out] E^x^2*x

Maple [A] time = 0., size = 7, normalized size = 1.

$$e^{x^2}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x^2)*(2*x^2+1),x)

[Out] exp(x^2)*x

Maxima [A] time = 0.989386, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="maxima")

[Out] x*e^(x^2)

Fricas [A] time = 2.09897, size = 15, normalized size = 2.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="fricas")

[Out] x*e^(x^2)

Sympy [A] time = 0.080341, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(x**2)*(2*x**2+1),x)

[Out] x*exp(x**2)

Giac [A] time = 1.05528, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="giac")
```

```
[Out] x*e^(x^2)
```

$$3.77 \quad \int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2} dx$$

Optimal. Leaf size=24

$$e^{x^2}x + \frac{e^{x^2}}{2(x^2+1)}$$

[Out] $E^{x^2}x + E^{x^2}/(2*(1 + x^2))$

Rubi [A] time = 0.35703, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {6742, 2204, 2212, 6715, 2177, 2178}

$$e^{x^2}x + \frac{e^{x^2}}{2(x^2+1)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(E^{x^2}(1 + 4x^2 + x^3 + 5x^4 + 2x^6))/(1 + x^2)^2, x]$

[Out] $E^{x^2}x + E^{x^2}/(2*(1 + x^2))$

Rule 6742

$\text{Int}[u_, x_Symbol] := \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

Rule 2204

$\text{Int}[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2}), x_Symbol] := \text{Simp}[(F^{a*\text{Sqrt}[\text{Pi}]*\text{Erfi}[(c + d*x)*\text{Rt}[b*\text{Log}[F], 2]]}/(2*d*\text{Rt}[b*\text{Log}[F], 2])), x] /; \text{FreeQ}\{F, a, b, c, d\}, x] \&\& \text{PosQ}[b]$

Rule 2212

$\text{Int}[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{n_}))*((c_.) + (d_.)*(x_))^{(m_.)}, x_Symbol] := \text{Simp}[(c + d*x)^{(m - n + 1)}*F^{(a + b*(c + d*x)^n)}/(b*d*n*\text{Log}[F]), x] - \text{Dist}[(m - n + 1)/(b*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m - n)}*F^{(a + b*(c + d*x)^n)}, x], x] /; \text{FreeQ}\{F, a, b, c, d\}, x] \&\& \text{IntegerQ}[(2*(m + 1))/n] \&\& \text{LtQ}[0, (m + 1)/n, 5] \&\& \text{IntegerQ}[n] \&\& (\text{LtQ}[0, n, m + 1] || \text{LtQ}[m, n, 0])$

Rule 6715

$\text{Int}[(u_)*(x_)^{(m_.)}, x_Symbol] := \text{Dist}[1/(m + 1), \text{Subst}[\text{Int}[\text{SubstFor}[x^{(m + 1)}, u, x], x, x^{(m + 1)}], x] /; \text{FreeQ}[m, x] \&\& \text{NeQ}[m, -1] \&\& \text{FunctionOf}[x^{(m + 1)}, u, x]$

Rule 2177

$\text{Int}[(b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_)))^{(n_.)*((c_.) + (d_.)*(x_))^{(m_.)}, x_Symbol] := \text{Simp}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x))})^n]/(d*(m + 1)), x] - \text{Dist}[(f*g*n*\text{Log}[F])/d*(m + 1), \text{Int}[(c + d*x)^{(m + 1)}*(b*F^{(g*(e + f*x))})^n, x], x] /; \text{FreeQ}\{F, b, c, d, e, f, g, n\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{Int}$

egerQ[2*m] && !\$UseGamma == True

Rule 2178

Int[(F_)^((g_.)*(e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; FreeQ[{F, c, d, e, f, g}, x] && !\$UseGamma == True

Rubi steps

$$\begin{aligned}
 \int \frac{e^{x^2} (1 + 4x^2 + x^3 + 5x^4 + 2x^6)}{(1 + x^2)^2} dx &= \int \left(e^{x^2} + 2e^{x^2}x^2 - \frac{e^{x^2}x}{(1 + x^2)^2} + \frac{e^{x^2}x}{1 + x^2} \right) dx \\
 &= 2 \int e^{x^2}x^2 dx + \int e^{x^2} dx - \int \frac{e^{x^2}x}{(1 + x^2)^2} dx + \int \frac{e^{x^2}x}{1 + x^2} dx \\
 &= e^{x^2}x + \frac{1}{2}\sqrt{\pi}\operatorname{erfi}(x) - \frac{1}{2} \operatorname{Subst} \left(\int \frac{e^x}{(1 + x)^2} dx, x, x^2 \right) + \frac{1}{2} \operatorname{Subst} \left(\int \frac{e^x}{1 + x} dx, x, x^2 \right) \\
 &= e^{x^2}x + \frac{e^{x^2}}{2(1 + x^2)} + \frac{\operatorname{Ei}(1 + x^2)}{2e} - \frac{1}{2} \operatorname{Subst} \left(\int \frac{e^x}{1 + x} dx, x, x^2 \right) \\
 &= e^{x^2}x + \frac{e^{x^2}}{2(1 + x^2)}
 \end{aligned}$$

Mathematica [A] time = 0.068602, size = 20, normalized size = 0.83

$$\frac{1}{2}e^{x^2} \left(\frac{1}{x^2 + 1} + 2x \right)$$

Antiderivative was successfully verified.

[In] Integrate[(E^x^2*(1 + 4*x^2 + x^3 + 5*x^4 + 2*x^6))/(1 + x^2)^2,x]

[Out] (E^x^2*(2*x + (1 + x^2)^(-1)))/2

Maple [A] time = 0.005, size = 24, normalized size = 1.

$$\frac{(2x^3 + 2x + 1)e^{x^2}}{2x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x)

[Out] 1/2*(2*x^3+2*x+1)*exp(x^2)/(x^2+1)

Maxima [A] time = 1.54735, size = 31, normalized size = 1.29

$$\frac{(2x^3 + 2x + 1)e^{x^2}}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x^3 + 2*x + 1)*e^(x^2)/(x^2 + 1)
```

Fricas [A] time = 2.17282, size = 55, normalized size = 2.29

$$\frac{(2x^3 + 2x + 1)e^{x^2}}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*x^3 + 2*x + 1)*e^(x^2)/(x^2 + 1)
```

Sympy [A] time = 0.108063, size = 20, normalized size = 0.83

$$\frac{(2x^3 + 2x + 1)e^{x^2}}{2x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*(2*x**6+5*x**4+x**3+4*x**2+1)/(x**2+1)**2,x)
```

```
[Out] (2*x**3 + 2*x + 1)*exp(x**2)/(2*x**2 + 2)
```

Giac [A] time = 1.06218, size = 41, normalized size = 1.71

$$\frac{2x^3e^{x^2} + 2xe^{x^2} + e^{x^2}}{2(x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*x^3*e^(x^2) + 2*x*e^(x^2) + e^(x^2))/(x^2 + 1)
```

3.78 $\int e^{-1-x} dx$

Optimal. Leaf size=9

$$-e^{-x-1}$$

[Out] $-E^{(-1 - x)}$

Rubi [A] time = 0.003023, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2194}

$$-e^{-x-1}$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(-1 - x)}, x]$

[Out] $-E^{(-1 - x)}$

Rule 2194

$\text{Int}[(F_)^((c_.)*((a_.) + (b_.)*(x_)))^{(n_.)}, x_Symbol] \text{ :> Simp}[(F^{(c*(a + b*x)))^n/(b*c*n*\text{Log}[F]), x] \text{ /; FreeQ}\{F, a, b, c, n\}, x]$

Rubi steps

$$\int e^{-1-x} dx = -e^{-1-x}$$

Mathematica [A] time = 0.0025931, size = 9, normalized size = 1.

$$-e^{-x-1}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[E^{(-1 - x)}, x]$

[Out] $-E^{(-1 - x)}$

Maple [A] time = 0., size = 9, normalized size = 1.

$$-e^{-1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\exp(-1-x), x)$

[Out] $-\exp(-1-x)$

Maxima [A] time = 0.935708, size = 11, normalized size = 1.22

$$-e^{(-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-1-x),x, algorithm="maxima")

[Out] -e^(-x - 1)

Fricas [A] time = 2.0505, size = 18, normalized size = 2.

$$-e^{(-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-1-x),x, algorithm="fricas")

[Out] -e^(-x - 1)

Sympy [A] time = 0.067781, size = 7, normalized size = 0.78

$$-e^{-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-1-x),x)

[Out] -exp(-x - 1)

Giac [A] time = 1.06434, size = 11, normalized size = 1.22

$$-e^{(-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(-1-x),x, algorithm="giac")

[Out] -e^(-x - 1)

3.79 $\int \left(\frac{1}{x} + x\right) \log(x) dx$

Optimal. Leaf size=25

$$-\frac{x^2}{4} + \frac{1}{2}x^2 \log(x) + \frac{\log^2(x)}{2}$$

[Out] $-x^2/4 + (x^2*\text{Log}[x])/2 + \text{Log}[x]^2/2$

Rubi [A] time = 0.0449937, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {1593, 14, 2351, 2301, 2304}

$$-\frac{x^2}{4} + \frac{1}{2}x^2 \log(x) + \frac{\log^2(x)}{2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^{(-1)} + x)*\text{Log}[x], x]$

[Out] $-x^2/4 + (x^2*\text{Log}[x])/2 + \text{Log}[x]^2/2$

Rule 1593

$\text{Int}[(u_.)*((a_.)*(x_)^{(p_.)} + (b_.)*(x_)^{(q_.)})^{(n_.)}, x_Symbol] \rightarrow \text{Int}[u*x^{(n*p)}*(a + b*x^{(q - p)})^n, x] /;$ FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 14

$\text{Int}[(u_.)*((c_.)*(x_)^{(m_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^m*u, x], x] /;$ FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 2351

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}]*(b_.))*((f_.)*(x_)^{(m_.)}*((d_) + (e_.)*(x_)^{(r_.)})^{(q_.)}), x_Symbol] \rightarrow \text{With}[\{u = \text{ExpandIntegrand}[a + b*\text{Log}[c*x^n], (f*x)^m*(d + e*x^r)^q, x]\}, \text{Int}[u, x] /; \text{SumQ}[u] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n, q, r\}, x] \&\& \text{IntegerQ}[q] \&\& (\text{GtQ}[q, 0] \|\| (\text{IntegerQ}[m] \&\& \text{IntegerQ}[r]))]$

Rule 2301

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}]*(b_.)]/(x_), x_Symbol] \rightarrow \text{Simp}[(a + b*\text{Log}[c*x^n])^2/(2*b*n), x] /;$ FreeQ[{a, b, c, n}, x]

Rule 2304

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_)^{(n_.)}]*(b_.))*((d_.)*(x_)^{(m_.)}), x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{Log}[c*x^n])/(d*(m+1)), x] - \text{Simp}[(b*n*(d*x)^{(m+1)})/(d*(m+1)^2), x] /;$ FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned}
 \int \left(\frac{1}{x} + x \right) \log(x) dx &= \int \frac{(1+x^2) \log(x)}{x} dx \\
 &= \int \left(\frac{\log(x)}{x} + x \log(x) \right) dx \\
 &= \int \frac{\log(x)}{x} dx + \int x \log(x) dx \\
 &= -\frac{x^2}{4} + \frac{1}{2} x^2 \log(x) + \frac{\log^2(x)}{2}
 \end{aligned}$$

Mathematica [A] time = 0.0017662, size = 25, normalized size = 1.

$$-\frac{x^2}{4} + \frac{1}{2} x^2 \log(x) + \frac{\log^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1) + x)*Log[x],x]

[Out] -x^2/4 + (x^2*Log[x])/2 + Log[x]^2/2

Maple [A] time = 0.002, size = 20, normalized size = 0.8

$$-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2} + \frac{(\ln(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/x+x)*ln(x),x)

[Out] -1/4*x^2+1/2*x^2*ln(x)+1/2*ln(x)^2

Maxima [A] time = 0.926879, size = 32, normalized size = 1.28

$$-\frac{1}{4} x^2 + \frac{1}{2} (x^2 + 2 \log(x)) \log(x) - \frac{1}{2} \log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/x+x)*log(x),x, algorithm="maxima")

[Out] -1/4*x^2 + 1/2*(x^2 + 2*log(x))*log(x) - 1/2*log(x)^2

Fricas [A] time = 2.14503, size = 55, normalized size = 2.2

$$\frac{1}{2} x^2 \log(x) - \frac{1}{4} x^2 + \frac{1}{2} \log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2 + 1/2*log(x)^2
```

Sympy [A] time = 0.084918, size = 19, normalized size = 0.76

$$\frac{x^2 \log(x)}{2} - \frac{x^2}{4} + \frac{\log(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)*ln(x),x)
```

```
[Out] x**2*log(x)/2 - x**2/4 + log(x)**2/2
```

Giac [A] time = 1.05138, size = 26, normalized size = 1.04

$$\frac{1}{2} x^2 \log(x) - \frac{1}{4} x^2 + \frac{1}{2} \log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2 + 1/2*log(x)^2
```

$$3.80 \quad \int \frac{x}{1+x^4} dx$$

Optimal. Leaf size=8

$$\frac{1}{2} \tan^{-1}(x^2)$$

[Out] ArcTan[x^2]/2

Rubi [A] time = 0.0031274, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {275, 203}

$$\frac{1}{2} \tan^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Int[x/(1 + x^4), x]

[Out] ArcTan[x^2]/2

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{x}{1+x^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, x^2 \right) \\ &= \frac{1}{2} \tan^{-1}(x^2) \end{aligned}$$

Mathematica [A] time = 0.003168, size = 8, normalized size = 1.

$$\frac{1}{2} \tan^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 + x^4), x]

[Out] ArcTan[x^2]/2

Maple [A] time = 0.002, size = 7, normalized size = 0.9

$$\frac{\arctan(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^4+1),x)

[Out] 1/2*arctan(x^2)

Maxima [A] time = 1.4093, size = 8, normalized size = 1.

$$\frac{1}{2} \arctan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^4+1),x, algorithm="maxima")

[Out] 1/2*arctan(x^2)

Fricas [A] time = 2.18864, size = 23, normalized size = 2.88

$$\frac{1}{2} \arctan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^4+1),x, algorithm="fricas")

[Out] 1/2*arctan(x^2)

Sympy [A] time = 0.086513, size = 5, normalized size = 0.62

$$\frac{\operatorname{atan}(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x**4+1),x)

[Out] atan(x**2)/2

Giac [A] time = 1.0647, size = 8, normalized size = 1.

$$\frac{1}{2} \arctan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x^2)
```

$$3.81 \quad \int \frac{x^5}{1+x^4} dx$$

Optimal. Leaf size=16

$$\frac{x^2}{2} - \frac{1}{2} \tan^{-1}(x^2)$$

[Out] x²/2 - ArcTan[x²]/2

Rubi [A] time = 0.0082944, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {275, 321, 203}

$$\frac{x^2}{2} - \frac{1}{2} \tan^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Int[x⁵/(1 + x⁴),x]

[Out] x²/2 - ArcTan[x²]/2

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{x^5}{1+x^4} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{1+x^2} dx, x, x^2 \right) \\ &= \frac{x^2}{2} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, x^2 \right) \\ &= \frac{x^2}{2} - \frac{1}{2} \tan^{-1}(x^2) \end{aligned}$$

Mathematica [A] time = 0.00335, size = 16, normalized size = 1.

$$\frac{x^2}{2} - \frac{1}{2} \tan^{-1}(x^2)$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(1 + x^4),x]

[Out] x^2/2 - ArcTan[x^2]/2

Maple [A] time = 0.002, size = 13, normalized size = 0.8

$$\frac{x^2}{2} - \frac{\arctan(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(x^4+1),x)

[Out] 1/2*x^2-1/2*arctan(x^2)

Maxima [A] time = 1.40815, size = 16, normalized size = 1.

$$\frac{1}{2} x^2 - \frac{1}{2} \arctan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^4+1),x, algorithm="maxima")

[Out] 1/2*x^2 - 1/2*arctan(x^2)

Fricas [A] time = 2.05275, size = 36, normalized size = 2.25

$$\frac{1}{2} x^2 - \frac{1}{2} \arctan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(x^4+1),x, algorithm="fricas")

[Out] 1/2*x^2 - 1/2*arctan(x^2)

Sympy [A] time = 0.090205, size = 10, normalized size = 0.62

$$\frac{x^2}{2} - \frac{\operatorname{atan}(x^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**4+1),x)
```

```
[Out] x**2/2 - atan(x**2)/2
```

Giac [A] time = 1.05893, size = 16, normalized size = 1.

$$\frac{1}{2}x^2 - \frac{1}{2}\arctan(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/2*arctan(x^2)
```

$$3.82 \quad \int \frac{1}{1+\tan^2(x)} dx$$

Optimal. Leaf size=14

$$\frac{x}{2} + \frac{1}{2} \sin(x) \cos(x)$$

[Out] x/2 + (Cos[x]*Sin[x])/2

Rubi [A] time = 0.0153262, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3657, 2635, 8}

$$\frac{x}{2} + \frac{1}{2} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + Tan[x]^2)^(-1), x]

[Out] x/2 + (Cos[x]*Sin[x])/2

Rule 3657

Int[(u_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]^2)^p], x_Symbol] :> Int[ActivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a, b]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^n], x_Symbol] :> -Simp[(b*Cos[c + d*x])*(b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{1+\tan^2(x)} dx &= \int \cos^2(x) dx \\ &= \frac{1}{2} \cos(x) \sin(x) + \frac{\int 1 dx}{2} \\ &= \frac{x}{2} + \frac{1}{2} \cos(x) \sin(x) \end{aligned}$$

Mathematica [A] time = 0.0025164, size = 14, normalized size = 1.

$$\frac{x}{2} + \frac{1}{4} \sin(2x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Tan[x]^2)^(-1),x]

[Out] x/2 + Sin[2*x]/4

Maple [A] time = 0.01, size = 17, normalized size = 1.2

$$\frac{\tan(x)}{2(\tan(x))^2 + 2} + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(tan(x)^2+1),x)

[Out] 1/2/(tan(x)^2+1)*tan(x)+1/2*x

Maxima [A] time = 1.40717, size = 22, normalized size = 1.57

$$\frac{1}{2}x + \frac{\tan(x)}{2(\tan(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+tan(x)^2),x, algorithm="maxima")

[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)

Fricas [A] time = 2.28108, size = 62, normalized size = 4.43

$$\frac{x \tan(x)^2 + x + \tan(x)}{2(\tan(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+tan(x)^2),x, algorithm="fricas")

[Out] 1/2*(x*tan(x)^2 + x + tan(x))/(tan(x)^2 + 1)

Sympy [B] time = 0.383732, size = 36, normalized size = 2.57

$$\frac{x \tan^2(x)}{2 \tan^2(x) + 2} + \frac{x}{2 \tan^2(x) + 2} + \frac{\tan(x)}{2 \tan^2(x) + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+tan(x)**2),x)

```
[Out] x*tan(x)**2/(2*tan(x)**2 + 2) + x/(2*tan(x)**2 + 2) + tan(x)/(2*tan(x)**2 + 2)
```

Giac [A] time = 1.05259, size = 22, normalized size = 1.57

$$\frac{1}{2}x + \frac{\tan(x)}{2(\tan(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

$$3.83 \quad \int \frac{x^4}{(1-x^2)^{5/2}} dx$$

Optimal. Leaf size=35

$$\frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$$

[Out] $x^3/(3*(1-x^2)^{(3/2)}) - x/\text{Sqrt}[1-x^2] + \text{ArcSin}[x]$

Rubi [A] time = 0.0110509, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {288, 216}

$$\frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^4/(1-x^2)^{(5/2)}, x]$

[Out] $x^3/(3*(1-x^2)^{(3/2)}) - x/\text{Sqrt}[1-x^2] + \text{ArcSin}[x]$

Rule 288

$\text{Int}[(c_.)*(x_)^{(m_.)*((a_) + (b_.)*(x_)^{(n_)})^{(p_)}, x_Symbol] := \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a+b*x^n)^{(p+1)})/(b*n*(p+1)), x] - \text{Dist}[(c^n*(m-n+1))/(b*n*(p+1)), \text{Int}[(c*x)^{(m-n)}*(a+b*x^n)^{(p+1)}, x], x] /;$
 $\text{FreeQ}\{a, b, c\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{GtQ}[m+1, n] \&\& \text{!} \text{LtQ}[m+n*(p+1)+1, n, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 216

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^2], x_Symbol] := \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /;$
 $\text{FreeQ}\{a, b\}, x] \&\& \text{GtQ}[a, 0] \&\& \text{NegQ}[b]$

Rubi steps

$$\begin{aligned} \int \frac{x^4}{(1-x^2)^{5/2}} dx &= \frac{x^3}{3(1-x^2)^{3/2}} - \int \frac{x^2}{(1-x^2)^{3/2}} dx \\ &= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}} dx \\ &= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.0027465, size = 26, normalized size = 0.74

$$\frac{x(4x^2-3)}{3(1-x^2)^{3/2}} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(1 - x^2)^(5/2), x]

[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]

Maple [A] time = 0., size = 30, normalized size = 0.9

$$\frac{x^3}{3} (-x^2 + 1)^{-\frac{3}{2}} + \arcsin(x) - x \frac{1}{\sqrt{-x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(-x^2+1)^(5/2), x)

[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)

Maxima [A] time = 1.41346, size = 59, normalized size = 1.69

$$\frac{1}{3} x \left(\frac{3x^2}{(-x^2 + 1)^{\frac{3}{2}}} - \frac{2}{(-x^2 + 1)^{\frac{3}{2}}} \right) - \frac{x}{3\sqrt{-x^2 + 1}} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^2+1)^(5/2), x, algorithm="maxima")

[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1) + arcsin(x)

Fricas [B] time = 2.10504, size = 146, normalized size = 4.17

$$\frac{6(x^4 - 2x^2 + 1) \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - (4x^3 - 3x)\sqrt{-x^2 + 1}}{3(x^4 - 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^2+1)^(5/2), x, algorithm="fricas")

[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sqrt(-x^2 + 1))/(x^4 - 2*x^2 + 1)

Sympy [B] time = 2.55635, size = 105, normalized size = 3.

$$\frac{3x^4 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3} + \frac{4x^3 \sqrt{1 - x^2}}{3x^4 - 6x^2 + 3} - \frac{6x^2 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3} - \frac{3x \sqrt{1 - x^2}}{3x^4 - 6x^2 + 3} + \frac{3 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(-x**2+1)**(5/2),x)

[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x**2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)

Giac [A] time = 1.06232, size = 39, normalized size = 1.11

$$\frac{(4x^2 - 3)\sqrt{-x^2 + 1}x}{3(x^2 - 1)^2} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")

[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)

$$3.84 \quad \int -\frac{x^2}{(1-x^2)^{3/2}} dx$$

Optimal. Leaf size=17

$$\sin^{-1}(x) - \frac{x}{\sqrt{1-x^2}}$$

[Out] -(x/Sqrt[1 - x^2]) + ArcSin[x]

Rubi [A] time = 0.0050743, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {288, 216}

$$\sin^{-1}(x) - \frac{x}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

[In] Int[-(x^2/(1 - x^2)^(3/2)),x]

[Out] -(x/Sqrt[1 - x^2]) + ArcSin[x]

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !IntegerQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned} \int -\frac{x^2}{(1-x^2)^{3/2}} dx &= -\frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}} dx \\ &= -\frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.0068797, size = 32, normalized size = 1.88

$$\frac{\sqrt{1-x^2}x + x^2 \sin^{-1}(x) - \sin^{-1}(x)}{x^2 - 1}$$

Antiderivative was successfully verified.

[In] Integrate[-(x^2/(1 - x^2)^(3/2)),x]

[Out] $(x*\text{Sqrt}[1 - x^2] - \text{ArcSin}[x] + x^2*\text{ArcSin}[x])/(-1 + x^2)$

Maple [A] time = 0.004, size = 16, normalized size = 0.9

$$\arcsin(x) - x \frac{1}{\sqrt{-x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-x^2/(-x^2+1)^(3/2),x)`

[Out] $\arcsin(x) - x/(-x^2+1)^{(1/2)}$

Maxima [A] time = 1.41049, size = 20, normalized size = 1.18

$$-\frac{x}{\sqrt{-x^2 + 1}} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="maxima")`

[Out] $-x/\text{sqrt}(-x^2 + 1) + \arcsin(x)$

Fricas [B] time = 2.16875, size = 103, normalized size = 6.06

$$\frac{2(x^2 - 1) \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - \sqrt{-x^2 + 1}x}{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="fricas")`

[Out] $-(2*(x^2 - 1)*\arctan((\text{sqrt}(-x^2 + 1) - 1)/x) - \text{sqrt}(-x^2 + 1)*x)/(x^2 - 1)$

Sympy [B] time = 0.5255, size = 34, normalized size = 2.

$$\frac{x^2 \arcsin(x)}{x^2 - 1} + \frac{x\sqrt{1 - x^2}}{x^2 - 1} - \frac{\arcsin(x)}{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-x**2/(-x**2+1)**(3/2),x)`

[Out] $x**2*\arcsin(x)/(x**2 - 1) + x*\text{sqrt}(1 - x**2)/(x**2 - 1) - \arcsin(x)/(x**2 - 1)$

Giac [A] time = 1.07129, size = 28, normalized size = 1.65

$$\frac{\sqrt{-x^2 + 1}x}{x^2 - 1} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(-x^2 + 1)*x/(x^2 - 1) + arcsin(x)
```

3.85 $\int e^x \sin(x) dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

[Out] $-(E^x \cos[x])/2 + (E^x \sin[x])/2$

Rubi [A] time = 0.007656, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4432}

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

Antiderivative was successfully verified.

[In] Int[E^x*Sin[x],x]

[Out] $-(E^x \cos[x])/2 + (E^x \sin[x])/2$

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
  Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x \sin(x) dx = -\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)$$

Mathematica [A] time = 0.0140425, size = 14, normalized size = 0.74

$$\frac{1}{2}e^x(\sin(x) - \cos(x))$$

Antiderivative was successfully verified.

[In] Integrate[E^x*Sin[x],x]

[Out] $(E^x*(-\cos[x] + \sin[x]))/2$

Maple [A] time = 0., size = 14, normalized size = 0.7

$$-\frac{e^x \cos(x)}{2} + \frac{e^x \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)*sin(x),x)`

[Out] `-1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)`

Maxima [A] time = 0.92849, size = 15, normalized size = 0.79

$$-\frac{1}{2}(\cos(x) - \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*sin(x),x, algorithm="maxima")`

[Out] `-1/2*(cos(x) - sin(x))*e^x`

Fricas [A] time = 2.23052, size = 46, normalized size = 2.42

$$-\frac{1}{2}\cos(x)e^x + \frac{1}{2}e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*sin(x),x, algorithm="fricas")`

[Out] `-1/2*cos(x)*e^x + 1/2*e^x*sin(x)`

Sympy [A] time = 0.279987, size = 15, normalized size = 0.79

$$\frac{e^x \sin(x)}{2} - \frac{e^x \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*sin(x),x)`

[Out] `exp(x)*sin(x)/2 - exp(x)*cos(x)/2`

Giac [A] time = 1.04556, size = 15, normalized size = 0.79

$$-\frac{1}{2}(\cos(x) - \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)*sin(x),x, algorithm="giac")`

[Out] `-1/2*(cos(x) - sin(x))*e^x`

$$3.86 \quad \int \frac{1}{x} dx$$

Optimal. Leaf size=2

$\log(x)$

[Out] Log[x]

Rubi [A] time = 0.0002056, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {29}

$\log(x)$

Antiderivative was successfully verified.

[In] Int[x⁽⁻¹⁾, x]

[Out] Log[x]

Rule 29

Int[(x_)⁽⁻¹⁾, x_Symbol] :> Simp[Log[x], x]

Rubi steps

$$\int \frac{1}{x} dx = \log(x)$$

Mathematica [A] time = 0.0001045, size = 2, normalized size = 1.

$\log(x)$

Antiderivative was successfully verified.

[In] Integrate[x⁽⁻¹⁾, x]

[Out] Log[x]

Maple [A] time = 0., size = 3, normalized size = 1.5

$\ln(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x, x)

[Out] ln(x)

Maxima [A] time = 0.930007, size = 3, normalized size = 1.5

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="maxima")
```

```
[Out] log(x)
```

Fricas [A] time = 2.24052, size = 11, normalized size = 5.5

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="fricas")
```

```
[Out] log(x)
```

Sympy [A] time = 0.049597, size = 2, normalized size = 1.

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x)
```

```
[Out] log(x)
```

Giac [A] time = 1.05741, size = 4, normalized size = 2.

$$\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="giac")
```

```
[Out] log(abs(x))
```

$$3.87 \quad \int \frac{\sec(2t)}{1+\sec^2(t)+3 \tan(t)} dt$$

Optimal. Leaf size=45

$$-\frac{1}{2(\tan(t)+1)} - \frac{1}{12} \log(\cos(t) - \sin(t)) - \frac{1}{4} \log(\sin(t) + \cos(t)) + \frac{1}{3} \log(\sin(t) + 2 \cos(t))$$

[Out] -Log[Cos[t] - Sin[t]]/12 - Log[Cos[t] + Sin[t]]/4 + Log[2*Cos[t] + Sin[t]]/3 - 1/(2*(1 + Tan[t]))

Rubi [A] time = 0.123094, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {709, 800}

$$-\frac{1}{2(\tan(t)+1)} - \frac{1}{12} \log(\cos(t) - \sin(t)) - \frac{1}{4} \log(\sin(t) + \cos(t)) + \frac{1}{3} \log(\sin(t) + 2 \cos(t))$$

Antiderivative was successfully verified.

[In] Int[Sec[2*t]/(1 + Sec[t]^2 + 3*Tan[t]),t]

[Out] -Log[Cos[t] - Sin[t]]/12 - Log[Cos[t] + Sin[t]]/4 + Log[2*Cos[t] + Sin[t]]/3 - 1/(2*(1 + Tan[t]))

Rule 709

Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m, -1]

Rule 800

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{\sec(2t)}{1+\sec^2(t)+3 \tan(t)} dt &= \text{Subst} \left(\int \frac{1}{(1+t)^2(2-t-t^2)} dt, t, \tan(t) \right) \\ &= -\frac{1}{2(1+\tan(t))} + \frac{1}{2} \text{Subst} \left(\int \frac{t}{(1+t)(2-t-t^2)} dt, t, \tan(t) \right) \\ &= -\frac{1}{2(1+\tan(t))} + \frac{1}{2} \text{Subst} \left(\int \left(-\frac{1}{6(-1+t)} - \frac{1}{2(1+t)} + \frac{2}{3(2+t)} \right) dt, t, \tan(t) \right) \\ &= -\frac{1}{12} \log(\cos(t) - \sin(t)) - \frac{1}{4} \log(\cos(t) + \sin(t)) + \frac{1}{3} \log(2 \cos(t) + \sin(t)) - \frac{1}{2(1+\tan(t))} \end{aligned}$$

Mathematica [A] time = 0.180955, size = 73, normalized size = 1.62

$$\frac{\cos(t)(\log(\cos(t) - \sin(t)) + 3 \log(\sin(t) + \cos(t)) - 4 \log(\sin(t) + 2 \cos(t))) + \sin(t)(\log(\cos(t) - \sin(t)) + 3 \log(\sin(t) + \cos(t)))}{12(\sin(t) + \cos(t))}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[2*t]/(1 + Sec[t]^2 + 3*Tan[t]),t]

[Out] -(Cos[t]*(Log[Cos[t] - Sin[t]] + 3*Log[Cos[t] + Sin[t]] - 4*Log[2*Cos[t] + Sin[t]]) + (-6 + Log[Cos[t] - Sin[t]] + 3*Log[Cos[t] + Sin[t]] - 4*Log[2*Cos[t] + Sin[t]])*Sin[t])/(12*(Cos[t] + Sin[t]))

Maple [A] time = 0.142, size = 31, normalized size = 0.7

$$\frac{\ln(\tan(t) + 2)}{3} - \frac{1}{2 + 2 \tan(t)} - \frac{\ln(1 + \tan(t))}{4} - \frac{\ln(-1 + \tan(t))}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(2*t)/(1+sec(t)^2+3*tan(t)),t)

[Out] 1/3*ln(tan(t)+2)-1/2/(1+tan(t))-1/4*ln(1+tan(t))-1/12*ln(-1+tan(t))

Maxima [B] time = 1.78015, size = 346, normalized size = 7.69

$$3(\cos(2t)^2 + \sin(2t)^2 + 2 \sin(2t) + 1) \log(953674316406250(3 \cos(2t) + \sin(2t) + 4) \cos(4t) + 2384185791015625 \cos(4t)^2 + 953674316406250 \cos(2t)^2 - 953674316406250(\cos(2t) - 3 \sin(2t) + 3) \sin(4t) + 2384185791015625 \sin(4t)^2 + 953674316406250 \sin(2t)^2 + 2861022949218750 \cos(2t) - 953674316406250 \sin(2t) + 2384185791015625) - 6(\cos(2t)^2 + \sin(2t)^2 + 2 \sin(2t) + 1) \log(\cos(2t)^2 + \sin(2t)^2 + 2 \sin(2t) + 1) + 5(\cos(2t)^2 + \sin(2t)^2 + 2 \sin(2t) + 1) \log(1/5(5 \cos(2t)^2 + 5 \sin(2t)^2 + 6 \cos(2t) + 8 \sin(2t) + 5)/(\cos(2t)^2 + \sin(2t)^2 - 2 \sin(2t) + 1)) - 24 \cos(2t)/(\cos(2t)^2 + \sin(2t)^2 + 2 \sin(2t) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="maxima")

[Out] 1/48*(3*(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)*log(953674316406250*(3*cos(2*t) + sin(2*t) + 4)*cos(4*t) + 2384185791015625*cos(4*t)^2 + 953674316406250*cos(2*t)^2 - 953674316406250*(cos(2*t) - 3*sin(2*t) + 3)*sin(4*t) + 2384185791015625*sin(4*t)^2 + 953674316406250*sin(2*t)^2 + 2861022949218750*cos(2*t) - 953674316406250*sin(2*t) + 2384185791015625) - 6*(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)*log(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1) + 5*(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)*log(1/5*(5*cos(2*t)^2 + 5*sin(2*t)^2 + 6*cos(2*t) + 8*sin(2*t) + 5)/(\cos(2*t)^2 + \sin(2*t)^2 - 2*sin(2*t) + 1)) - 24*cos(2*t)/(\cos(2*t)^2 + \sin(2*t)^2 + 2*sin(2*t) + 1)

Fricas [A] time = 2.61796, size = 271, normalized size = 6.02

$$\frac{4(\cos(t) + \sin(t)) \log\left(\frac{3}{4} \cos(t)^2 + \cos(t) \sin(t) + \frac{1}{4}\right) - 3(\cos(t) + \sin(t)) \log(2 \cos(t) \sin(t) + 1) - (\cos(t) + \sin(t)) \log\left(\frac{1}{5} (5 \cos(2t)^2 + 5 \sin(2t)^2 + 6 \cos(2t) + 8 \sin(2t) + 5)\right)}{24(\cos(t) + \sin(t))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="fricas")
```

```
[Out] 1/24*(4*(cos(t) + sin(t))*log(3/4*cos(t)^2 + cos(t)*sin(t) + 1/4) - 3*(cos(t) + sin(t))*log(2*cos(t)*sin(t) + 1) - (cos(t) + sin(t))*log(-2*cos(t)*sin(t) + 1) - 6*cos(t) + 6*sin(t))/(cos(t) + sin(t))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(2t)}{3 \tan(t) + \sec^2(t) + 1} dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*t)/(1+sec(t)**2+3*tan(t)),t)
```

```
[Out] Integral(sec(2*t)/(3*tan(t) + sec(t)**2 + 1), t)
```

Giac [A] time = 1.10377, size = 45, normalized size = 1.

$$-\frac{1}{2(\tan(t) + 1)} + \frac{1}{3} \log(|\tan(t) + 2|) - \frac{1}{4} \log(|\tan(t) + 1|) - \frac{1}{12} \log(|\tan(t) - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="giac")
```

```
[Out] -1/2/(tan(t) + 1) + 1/3*log(abs(tan(t) + 2)) - 1/4*log(abs(tan(t) + 1)) - 1/12*log(abs(tan(t) - 1))
```

3.88 $\int \cos^2(x) dx$

Optimal. Leaf size=14

$$\frac{x}{2} + \frac{1}{2} \sin(x) \cos(x)$$

[Out] x/2 + (Cos[x]*Sin[x])/2

Rubi [A] time = 0.0064947, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {2635, 8}

$$\frac{x}{2} + \frac{1}{2} \sin(x) \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2,x]

[Out] x/2 + (Cos[x]*Sin[x])/2

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x] * (b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \cos^2(x) dx &= \frac{1}{2} \cos(x) \sin(x) + \frac{\int 1 dx}{2} \\ &= \frac{x}{2} + \frac{1}{2} \cos(x) \sin(x) \end{aligned}$$

Mathematica [A] time = 0.0017493, size = 14, normalized size = 1.

$$\frac{x}{2} + \frac{1}{4} \sin(2x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^2,x]

[Out] x/2 + Sin[2*x]/4

Maple [A] time = 0.002, size = 11, normalized size = 0.8

$$\frac{x}{2} + \frac{\cos(x) \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/sec(x)^2,x)

[Out] 1/2*x+1/2*cos(x)*sin(x)

Maxima [A] time = 0.928115, size = 14, normalized size = 1.

$$\frac{1}{2}x + \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(x)^2,x, algorithm="maxima")

[Out] 1/2*x + 1/4*sin(2*x)

Fricas [A] time = 2.2063, size = 36, normalized size = 2.57

$$\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(x)^2,x, algorithm="fricas")

[Out] 1/2*cos(x)*sin(x) + 1/2*x

Sympy [A] time = 0.061051, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(x) \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/sec(x)**2,x)

[Out] x/2 + sin(x)*cos(x)/2

Giac [A] time = 1.06457, size = 22, normalized size = 1.57

$$\frac{1}{2}x + \frac{\tan(x)}{2(\tan(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

$$3.89 \quad \int \frac{1+x^2}{\sqrt{x}} dx$$

Optimal. Leaf size=17

$$\frac{2x^{5/2}}{5} + 2\sqrt{x}$$

[Out] 2*Sqrt[x] + (2*x^(5/2))/5

Rubi [A] time = 0.0032497, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {14}

$$\frac{2x^{5/2}}{5} + 2\sqrt{x}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^2)/Sqrt[x], x]

[Out] 2*Sqrt[x] + (2*x^(5/2))/5

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{1+x^2}{\sqrt{x}} dx &= \int \left(\frac{1}{\sqrt{x}} + x^{3/2} \right) dx \\ &= 2\sqrt{x} + \frac{2x^{5/2}}{5} \end{aligned}$$

Mathematica [A] time = 0.0028835, size = 14, normalized size = 0.82

$$\frac{2}{5}\sqrt{x}(x^2 + 5)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^2)/Sqrt[x], x]

[Out] (2*Sqrt[x]*(5 + x^2))/5

Maple [A] time = 0.003, size = 11, normalized size = 0.7

$$\frac{2x^2 + 10}{5}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+1)/x^(1/2),x)`

[Out] `2/5*x^(1/2)*(x^2+5)`

Maxima [A] time = 0.923614, size = 15, normalized size = 0.88

$$\frac{2}{5}x^{\frac{5}{2}} + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)/x^(1/2),x, algorithm="maxima")`

[Out] `2/5*x^(5/2) + 2*sqrt(x)`

Fricas [A] time = 1.77471, size = 31, normalized size = 1.82

$$\frac{2}{5}(x^2 + 5)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)/x^(1/2),x, algorithm="fricas")`

[Out] `2/5*(x^2 + 5)*sqrt(x)`

Sympy [A] time = 0.202541, size = 14, normalized size = 0.82

$$\frac{2x^{\frac{5}{2}}}{5} + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)/x**(1/2),x)`

[Out] `2*x**(5/2)/5 + 2*sqrt(x)`

Giac [A] time = 1.05947, size = 15, normalized size = 0.88

$$\frac{2}{5}x^{\frac{5}{2}} + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)/x^(1/2),x, algorithm="giac")`

[Out] `2/5*x^(5/2) + 2*sqrt(x)`

$$3.90 \quad \int \frac{x}{\sqrt{5+2x+x^2}} dx$$

Optimal. Leaf size=23

$$\sqrt{x^2 + 2x + 5} - \sinh^{-1}\left(\frac{x+1}{2}\right)$$

[Out] Sqrt[5 + 2*x + x^2] - ArcSinh[(1 + x)/2]

Rubi [A] time = 0.0098206, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {640, 619, 215}

$$\sqrt{x^2 + 2x + 5} - \sinh^{-1}\left(\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[5 + 2*x + x^2],x]

[Out] Sqrt[5 + 2*x + x^2] - ArcSinh[(1 + x)/2]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{5+2x+x^2}} dx &= \sqrt{5+2x+x^2} - \int \frac{1}{\sqrt{5+2x+x^2}} dx \\ &= \sqrt{5+2x+x^2} - \frac{1}{4} \text{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{16}}} dx, x, 2+2x \right) \\ &= \sqrt{5+2x+x^2} - \sinh^{-1}\left(\frac{1+x}{2}\right) \end{aligned}$$

Mathematica [A] time = 0.0052585, size = 25, normalized size = 1.09

$$\sqrt{x^2 + 2x + 5} - \sinh^{-1}\left(\frac{1}{4}(2x + 2)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[5 + 2*x + x^2], x]

[Out] Sqrt[5 + 2*x + x^2] - ArcSinh[(2 + 2*x)/4]

Maple [A] time = 0.004, size = 20, normalized size = 0.9

$$-\text{Arcsinh}\left(\frac{1}{2} + \frac{x}{2}\right) + \sqrt{x^2 + 2x + 5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^2+2*x+5)^(1/2), x)

[Out] -arcsinh(1/2+1/2*x)+(x^2+2*x+5)^(1/2)

Maxima [A] time = 1.40927, size = 26, normalized size = 1.13

$$\sqrt{x^2 + 2x + 5} - \text{arsinh}\left(\frac{1}{2}x + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2+2*x+5)^(1/2), x, algorithm="maxima")

[Out] sqrt(x^2 + 2*x + 5) - arcsinh(1/2*x + 1/2)

Fricas [A] time = 1.80516, size = 77, normalized size = 3.35

$$\sqrt{x^2 + 2x + 5} + \log\left(-x + \sqrt{x^2 + 2x + 5} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2+2*x+5)^(1/2), x, algorithm="fricas")

[Out] sqrt(x^2 + 2*x + 5) + log(-x + sqrt(x^2 + 2*x + 5) - 1)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2 + 2x + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+2*x+5)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2 + 2*x + 5), x)
```

Giac [A] time = 1.05923, size = 36, normalized size = 1.57

$$\sqrt{x^2 + 2x + 5} + \log\left(-x + \sqrt{x^2 + 2x + 5} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 2*x + 5) + log(-x + sqrt(x^2 + 2*x + 5) - 1)
```

3.91 $\int \cos(x) \sin^2(x) dx$

Optimal. Leaf size=8

$$\frac{\sin^3(x)}{3}$$

[Out] Sin[x]^3/3

Rubi [A] time = 0.0127217, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2564, 30}

$$\frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[x]^2,x]

[Out] Sin[x]^3/3

Rule 2564

Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && LtQ[0, m, n])

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \cos(x) \sin^2(x) dx &= \text{Subst} \left(\int x^2 dx, x, \sin(x) \right) \\ &= \frac{\sin^3(x)}{3} \end{aligned}$$

Mathematica [A] time = 0.0010916, size = 8, normalized size = 1.

$$\frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sin[x]^2,x]

[Out] Sin[x]^3/3

Maple [A] time = 0.002, size = 7, normalized size = 0.9

$$\frac{(\sin(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(x)^2,x)

[Out] 1/3*sin(x)^3

Maxima [A] time = 0.919453, size = 8, normalized size = 1.

$$\frac{1}{3} \sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)^2,x, algorithm="maxima")

[Out] 1/3*sin(x)^3

Fricas [A] time = 2.07203, size = 38, normalized size = 4.75

$$-\frac{1}{3} (\cos(x)^2 - 1) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)^2,x, algorithm="fricas")

[Out] -1/3*(cos(x)^2 - 1)*sin(x)

Sympy [A] time = 0.057543, size = 5, normalized size = 0.62

$$\frac{\sin^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x)**2,x)

[Out] sin(x)**3/3

Giac [A] time = 1.05907, size = 8, normalized size = 1.

$$\frac{1}{3} \sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/3*sin(x)^3
```

3.92 $\int \frac{e^x}{1+e^x} dx$

Optimal. Leaf size=6

$$\log(e^x + 1)$$

[Out] Log[1 + E^x]

Rubi [A] time = 0.0164506, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2246, 31}

$$\log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Int[E^x/(1 + E^x), x]

[Out] Log[1 + E^x]

Rule 2246

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.), x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n, p}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{e^x}{1+e^x} dx = \text{Subst} \left(\int \frac{1}{1+x} dx, x, e^x \right) = \log(1 + e^x)$$

Mathematica [A] time = 0.0042869, size = 6, normalized size = 1.

$$\log(e^x + 1)$$

Antiderivative was successfully verified.

[In] Integrate[E^x/(1 + E^x), x]

[Out] Log[1 + E^x]

Maple [A] time = 0., size = 6, normalized size = 1.

$$\ln(1 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(x)/(1+exp(x)),x)`

[Out] `ln(1+exp(x))`

Maxima [A] time = 0.933567, size = 7, normalized size = 1.17

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(x)),x, algorithm="maxima")`

[Out] `log(e^x + 1)`

Fricas [A] time = 2.04549, size = 19, normalized size = 3.17

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(x)),x, algorithm="fricas")`

[Out] `log(e^x + 1)`

Sympy [A] time = 0.071965, size = 5, normalized size = 0.83

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(x)),x)`

[Out] `log(exp(x) + 1)`

Giac [A] time = 1.06321, size = 7, normalized size = 1.17

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(x)/(1+exp(x)),x, algorithm="giac")`

[Out] `log(e^x + 1)`

3.93 $\int \frac{e^{2x}}{1+e^x} dx$

Optimal. Leaf size=12

$$e^x - \log(e^x + 1)$$

[Out] $E^x - \text{Log}[1 + E^x]$

Rubi [A] time = 0.0264504, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2248, 43}

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(2*x)/(1 + E^x)}, x]$

[Out] $E^x - \text{Log}[1 + E^x]$

Rule 2248

$\text{Int}[(a_ + (b_)*(F_)^{(e_)*((c_ + (d_)*(x_))})^{(p_)*(G_)^{(h_)*((f_ + (g_)*(x_))})})], x_Symbol] \rightarrow \text{With}[\{m = \text{FullSimplify}[(g*h*\text{Log}[G])/(d*e*\text{Log}[F])]\}, \text{Dist}[(\text{Denominator}[m]*G^{(f*h - (c*g*h)/d})/(d*e*\text{Log}[F]), \text{Subst}[\text{Int}[x^{(\text{Numerator}[m] - 1)*(a + b*x^{\text{Denominator}[m]})^p}, x], x, F^{(e*(c + d*x))/\text{Denominator}[m])}], x] /; \text{LeQ}[m, -1] \ || \ \text{GeQ}[m, 1] /; \text{FreeQ}[\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

Rule 43

$\text{Int}[(a_ + (b_)*(x_))^{(m_)*((c_ + (d_)*(x_))^{(n_)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

Rubi steps

$$\begin{aligned} \int \frac{e^{2x}}{1+e^x} dx &= \text{Subst} \left(\int \frac{x}{1+x} dx, x, e^x \right) \\ &= \text{Subst} \left(\int \left(1 + \frac{1}{-1-x} \right) dx, x, e^x \right) \\ &= e^x - \log(1 + e^x) \end{aligned}$$

Mathematica [A] time = 0.0075345, size = 12, normalized size = 1.

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[E^{(2*x)/(1 + E^x)}, x]$

[Out] $E^x - \text{Log}[1 + E^x]$

Maple [A] time = 0.002, size = 11, normalized size = 0.9

$$e^x - \ln(1 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(exp(2*x)/(1+exp(x)),x)`

[Out] `exp(x)-ln(1+exp(x))`

Maxima [A] time = 0.929565, size = 14, normalized size = 1.17

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(2*x)/(1+exp(x)),x, algorithm="maxima")`

[Out] `e^x - log(e^x + 1)`

Fricas [A] time = 2.12464, size = 27, normalized size = 2.25

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(2*x)/(1+exp(x)),x, algorithm="fricas")`

[Out] `e^x - log(e^x + 1)`

Sympy [A] time = 0.080449, size = 8, normalized size = 0.67

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(exp(2*x)/(1+exp(x)),x)`

[Out] `exp(x) - log(exp(x) + 1)`

Giac [A] time = 1.05476, size = 14, normalized size = 1.17

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="giac")
```

```
[Out] e^x - log(e^x + 1)
```

$$3.94 \quad \int \frac{1}{1-\cos(x)} dx$$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

[Out] -(Sin[x]/(1 - Cos[x]))

Rubi [A] time = 0.0090661, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2648}

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

[In] Int[(1 - Cos[x])^(-1), x]

[Out] -(Sin[x]/(1 - Cos[x]))

Rule 2648

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \frac{1}{1-\cos(x)} dx = -\frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A] time = 0.008356, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Cos[x])^(-1), x]

[Out] -Cot[x/2]

Maple [A] time = 0., size = 9, normalized size = 0.8

$$-\left(\tan\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1-cos(x)),x)`

[Out] `-1/tan(1/2*x)`

Maxima [A] time = 0.927184, size = 14, normalized size = 1.17

$$-\frac{\cos(x) + 1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-cos(x)),x, algorithm="maxima")`

[Out] `-(cos(x) + 1)/sin(x)`

Fricas [A] time = 2.21, size = 30, normalized size = 2.5

$$-\frac{\cos(x) + 1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-cos(x)),x, algorithm="fricas")`

[Out] `-(cos(x) + 1)/sin(x)`

Sympy [A] time = 0.353726, size = 7, normalized size = 0.58

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-cos(x)),x)`

[Out] `-1/tan(x/2)`

Giac [A] time = 1.05536, size = 11, normalized size = 0.92

$$-\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-cos(x)),x, algorithm="giac")`

[Out] `-1/tan(1/2*x)`

3.95 $\int \sec^2(x) \tan(x) dx$

Optimal. Leaf size=8

$$\frac{\sec^2(x)}{2}$$

[Out] Sec[x]^2/2

Rubi [A] time = 0.0119479, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2606, 30}

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2*Tan[x], x]

[Out] Sec[x]^2/2

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_.)]^(m_.))*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \sec^2(x) \tan(x) dx &= \text{Subst}\left(\int x dx, x, \sec(x)\right) \\ &= \frac{\sec^2(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.004168, size = 8, normalized size = 1.

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2*Tan[x], x]

[Out] Sec[x]^2/2

Maple [A] time = 0., size = 7, normalized size = 0.9

$$\frac{(\sec(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(x)^2*tan(x),x)

[Out] 1/2*sec(x)^2

Maxima [A] time = 0.926987, size = 8, normalized size = 1.

$$\frac{1}{2} \tan(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*tan(x),x, algorithm="maxima")

[Out] 1/2*tan(x)^2

Fricas [A] time = 1.9112, size = 19, normalized size = 2.38

$$\frac{1}{2 \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)^2*tan(x),x, algorithm="fricas")

[Out] 1/2/cos(x)^2

Sympy [A] time = 0.06361, size = 7, normalized size = 0.88

$$\frac{1}{2 \cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(x)**2*tan(x),x)

[Out] 1/(2*cos(x)**2)

Giac [A] time = 1.05978, size = 8, normalized size = 1.

$$\frac{1}{2 \cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(sec(x)^2*tan(x),x, algorithm="giac")
```

```
[Out] 1/2/cos(x)^2
```

3.96 $\int x \log(x) dx$

Optimal. Leaf size=17

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

[Out] $-x^2/4 + (x^2*\text{Log}[x])/2$

Rubi [A] time = 0.0042968, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {2304}

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Int[x*Log[x],x]

[Out] $-x^2/4 + (x^2*\text{Log}[x])/2$

Rule 2304

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_)^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]

Rubi steps

$$\int x \log(x) dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

Mathematica [A] time = 0.0007928, size = 17, normalized size = 1.

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

[In] Integrate[x*Log[x],x]

[Out] $-x^2/4 + (x^2*\text{Log}[x])/2$

Maple [A] time = 0., size = 14, normalized size = 0.8

$$-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*ln(x),x)`

[Out] `-1/4*x^2+1/2*x^2*ln(x)`

Maxima [A] time = 0.935629, size = 18, normalized size = 1.06

$$\frac{1}{2}x^2 \log(x) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(x),x, algorithm="maxima")`

[Out] `1/2*x^2*log(x) - 1/4*x^2`

Fricas [A] time = 1.80642, size = 35, normalized size = 2.06

$$\frac{1}{2}x^2 \log(x) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(x),x, algorithm="fricas")`

[Out] `1/2*x^2*log(x) - 1/4*x^2`

Sympy [A] time = 0.082428, size = 12, normalized size = 0.71

$$\frac{x^2 \log(x)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*ln(x),x)`

[Out] `x**2*log(x)/2 - x**2/4`

Giac [A] time = 1.05853, size = 18, normalized size = 1.06

$$\frac{1}{2}x^2 \log(x) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*log(x),x, algorithm="giac")`

[Out] `1/2*x^2*log(x) - 1/4*x^2`

3.97 $\int \cos(x) \sin(x) dx$

Optimal. Leaf size=8

$$\frac{\sin^2(x)}{2}$$

[Out] Sin[x]^2/2

Rubi [A] time = 0.007369, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {2564, 30}

$$\frac{\sin^2(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]*Sin[x],x]

[Out] Sin[x]^2/2

Rule 2564

Int[cos[(e_.) + (f_.)*(x_.)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.), x_Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[(m - 1)/2] && LtQ[0, m, n])

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \cos(x) \sin(x) dx &= \text{Subst}\left(\int x dx, x, \sin(x)\right) \\ &= \frac{\sin^2(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.0013897, size = 8, normalized size = 1.

$$-\frac{1}{2} \cos^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]*Sin[x],x]

[Out] -Cos[x]^2/2

Maple [A] time = 0., size = 7, normalized size = 0.9

$$\frac{(\sin(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(x)*sin(x),x)

[Out] 1/2*sin(x)^2

Maxima [A] time = 0.923263, size = 8, normalized size = 1.

$$-\frac{1}{2} \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x),x, algorithm="maxima")

[Out] -1/2*cos(x)^2

Fricas [A] time = 1.9094, size = 20, normalized size = 2.5

$$-\frac{1}{2} \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x),x, algorithm="fricas")

[Out] -1/2*cos(x)^2

Sympy [A] time = 0.053841, size = 5, normalized size = 0.62

$$\frac{\sin^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)*sin(x),x)

[Out] sin(x)**2/2

Giac [A] time = 1.06816, size = 8, normalized size = 1.

$$-\frac{1}{2} \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2
```

$$3.98 \quad \int \frac{1+x}{\sqrt{2x-x^2}} dx$$

Optimal. Leaf size=24

$$-\sqrt{2x-x^2} - 2 \sin^{-1}(1-x)$$

[Out] -Sqrt[2*x - x^2] - 2*ArcSin[1 - x]

Rubi [A] time = 0.0092363, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {640, 619, 216}

$$-\sqrt{2x-x^2} - 2 \sin^{-1}(1-x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/Sqrt[2*x - x^2], x]

[Out] -Sqrt[2*x - x^2] - 2*ArcSin[1 - x]

Rule 640

Int[((d_.) + (e_.)*(x_.))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned} \int \frac{1+x}{\sqrt{2x-x^2}} dx &= -\sqrt{2x-x^2} + 2 \int \frac{1}{\sqrt{2x-x^2}} dx \\ &= -\sqrt{2x-x^2} - \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}} dx, x, 2-2x \right) \\ &= -\sqrt{2x-x^2} - 2 \sin^{-1}(1-x) \end{aligned}$$

Mathematica [A] time = 0.0316691, size = 27, normalized size = 1.12

$$-\sqrt{-(x-2)x} - 4 \sin^{-1} \left(\sqrt{1-\frac{x}{2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)/Sqrt[2*x - x^2],x]

[Out] -Sqrt[-((-2 + x)*x)] - 4*ArcSin[Sqrt[1 - x/2]]

Maple [A] time = 0.006, size = 21, normalized size = 0.9

$$2 \arcsin(-1 + x) - \sqrt{-x^2 + 2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)/(-x^2+2*x)^(1/2),x)

[Out] 2*arcsin(-1+x)-(-x^2+2*x)^(1/2)

Maxima [A] time = 1.4145, size = 30, normalized size = 1.25

$$-\sqrt{-x^2 + 2x} - 2 \arcsin(-x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="maxima")

[Out] -sqrt(-x^2 + 2*x) - 2*arcsin(-x + 1)

Fricas [A] time = 1.80234, size = 68, normalized size = 2.83

$$-\sqrt{-x^2 + 2x} - 4 \arctan\left(\frac{\sqrt{-x^2 + 2x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="fricas")

[Out] -sqrt(-x^2 + 2*x) - 4*arctan(sqrt(-x^2 + 2*x)/x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x + 1}{\sqrt{-x(x - 2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-x**2+2*x)**(1/2),x)

[Out] Integral((x + 1)/sqrt(-x*(x - 2)), x)

Giac [A] time = 1.07746, size = 27, normalized size = 1.12

$$-\sqrt{-x^2 + 2x} + 2 \arcsin(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="giac")

[Out] -sqrt(-x^2 + 2*x) + 2*arcsin(x - 1)

$$3.99 \quad \int \frac{2e^x}{2+3e^{2x}} dx$$

Optimal. Leaf size=20

$$\sqrt{\frac{2}{3}} \tan^{-1} \left(\sqrt{\frac{3}{2}} e^x \right)$$

[Out] Sqrt[2/3]*ArcTan[Sqrt[3/2]*E^x]

Rubi [A] time = 0.0226488, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {12, 2249, 203}

$$\sqrt{\frac{2}{3}} \tan^{-1} \left(\sqrt{\frac{3}{2}} e^x \right)$$

Antiderivative was successfully verified.

[In] Int[(2*E^x)/(2 + 3*E^(2*x)),x]

[Out] Sqrt[2/3]*ArcTan[Sqrt[3/2]*E^x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 2249

Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_.) + (g_.)*(x_))), x_Symbol] := With[{m = FullSimplify[(d*e*Log[F])/(g*h*Log[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m]]^p, x], x, G^((h*(f + g*x))/Denominator[m])], x] /; LtQ[m, -1] || GtQ[m, 1] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, p}, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{2e^x}{2+3e^{2x}} dx &= 2 \int \frac{e^x}{2+3e^{2x}} dx \\ &= 2 \text{Subst} \left(\int \frac{1}{2+3x^2} dx, x, e^x \right) \\ &= \sqrt{\frac{2}{3}} \tan^{-1} \left(\sqrt{\frac{3}{2}} e^x \right) \end{aligned}$$

Mathematica [A] time = 0.0051819, size = 20, normalized size = 1.

$$\sqrt{\frac{2}{3}} \tan^{-1} \left(\sqrt{\frac{3}{2}} e^x \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2*E^x)/(2 + 3*E^(2*x)), x]

[Out] Sqrt[2/3]*ArcTan[Sqrt[3/2]*E^x]

Maple [A] time = 0., size = 14, normalized size = 0.7

$$\frac{\sqrt{6}}{3} \arctan \left(\frac{e^x \sqrt{6}}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(2*exp(x)/(2+3*exp(2*x)), x)

[Out] 1/3*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)

Maxima [A] time = 1.421, size = 18, normalized size = 0.9

$$\frac{1}{3} \sqrt{6} \arctan \left(\frac{1}{2} \sqrt{6} e^x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*exp(x)/(2+3*exp(2*x)), x, algorithm="maxima")

[Out] 1/3*sqrt(6)*arctan(1/2*sqrt(6)*e^x)

Fricas [A] time = 1.92223, size = 72, normalized size = 3.6

$$\frac{1}{3} \sqrt{3} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{3} \sqrt{2} e^x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(2*exp(x)/(2+3*exp(2*x)), x, algorithm="fricas")

[Out] 1/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*e^x)

Sympy [A] time = 0.106325, size = 15, normalized size = 0.75

$$\text{RootSum} \left(6z^2 + 1, (i \mapsto i \log(2i + e^x)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x)
```

```
[Out] RootSum(6*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))
```

Giac [A] time = 1.06551, size = 18, normalized size = 0.9

$$\frac{1}{3} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6} e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

$$3.100 \quad \int \frac{x^4}{(1-x^2)^{5/2}} dx$$

Optimal. Leaf size=35

$$\frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$$

[Out] $x^3/(3*(1-x^2)^{(3/2)}) - x/\text{Sqrt}[1-x^2] + \text{ArcSin}[x]$

Rubi [A] time = 0.0088799, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {288, 216}

$$\frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^4/(1-x^2)^{(5/2)}, x]$

[Out] $x^3/(3*(1-x^2)^{(3/2)}) - x/\text{Sqrt}[1-x^2] + \text{ArcSin}[x]$

Rule 288

$\text{Int}[(c_.)*(x_)^{(m_.)*((a_.) + (b_.)*(x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a+b*x^n)^{(p+1)})/(b*n*(p+1)), x] - \text{Dist}[(c^n*(m-n+1))/(b*n*(p+1)), \text{Int}[(c*x)^{(m-n)}*(a+b*x^n)^{(p+1)}, x], x] /;$ FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !ILtQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 216

$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /;$ FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned} \int \frac{x^4}{(1-x^2)^{5/2}} dx &= \frac{x^3}{3(1-x^2)^{3/2}} - \int \frac{x^2}{(1-x^2)^{3/2}} dx \\ &= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}} dx \\ &= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.0029117, size = 26, normalized size = 0.74

$$\frac{x(4x^2-3)}{3(1-x^2)^{3/2}} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(1 - x^2)^(5/2),x]

[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]

Maple [A] time = 0., size = 30, normalized size = 0.9

$$\frac{x^3}{3} (-x^2 + 1)^{-\frac{3}{2}} + \arcsin(x) - x \frac{1}{\sqrt{-x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(-x^2+1)^(5/2),x)

[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)

Maxima [A] time = 1.40746, size = 59, normalized size = 1.69

$$\frac{1}{3} x \left(\frac{3x^2}{(-x^2 + 1)^{\frac{3}{2}}} - \frac{2}{(-x^2 + 1)^{\frac{3}{2}}} \right) - \frac{x}{3\sqrt{-x^2 + 1}} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")

[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1) + arcsin(x)

Fricas [B] time = 1.8756, size = 146, normalized size = 4.17

$$\frac{6(x^4 - 2x^2 + 1) \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - (4x^3 - 3x)\sqrt{-x^2+1}}{3(x^4 - 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")

[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sqrt(-x^2 + 1))/(x^4 - 2*x^2 + 1)

Sympy [B] time = 2.55683, size = 105, normalized size = 3.

$$\frac{3x^4 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3} + \frac{4x^3 \sqrt{1 - x^2}}{3x^4 - 6x^2 + 3} - \frac{6x^2 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3} - \frac{3x \sqrt{1 - x^2}}{3x^4 - 6x^2 + 3} + \frac{3 \operatorname{asin}(x)}{3x^4 - 6x^2 + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/(-x**2+1)**(5/2),x)

[Out] $3x^{**4}\operatorname{asin}(x)/(3x^{**4} - 6x^{**2} + 3) + 4x^{**3}\sqrt{1 - x^{**2}}/(3x^{**4} - 6x^{**2} + 3) - 6x^{**2}\operatorname{asin}(x)/(3x^{**4} - 6x^{**2} + 3) - 3x\sqrt{1 - x^{**2}}/(3x^{**4} - 6x^{**2} + 3) + 3\operatorname{asin}(x)/(3x^{**4} - 6x^{**2} + 3)$

Giac [A] time = 1.07752, size = 39, normalized size = 1.11

$$\frac{(4x^2 - 3)\sqrt{-x^2 + 1}x}{3(x^2 - 1)^2} + \operatorname{arcsin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")

[Out] $1/3*(4*x^2 - 3)*\sqrt{-x^2 + 1}*x/(x^2 - 1)^2 + \operatorname{arcsin}(x)$

3.101 $\int \frac{e^{6x}}{1+e^{4x}} dx$

Optimal. Leaf size=20

$$\frac{e^{2x}}{2} - \frac{1}{2} \tan^{-1}(e^{2x})$$

[Out] $E^{(2*x)}/2 - \text{ArcTan}[E^{(2*x)}]/2$

Rubi [A] time = 0.0239091, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {2248, 321, 203}

$$\frac{e^{2x}}{2} - \frac{1}{2} \tan^{-1}(e^{2x})$$

Antiderivative was successfully verified.

[In] $\text{Int}[E^{(6*x)}/(1 + E^{(4*x)}), x]$

[Out] $E^{(2*x)}/2 - \text{ArcTan}[E^{(2*x)}]/2$

Rule 2248

$\text{Int}[(a_ + (b_)*(F_)^{((e_)*((c_ + (d_)*(x_))))^{(p_)}*(G_)^{((h_)*((f_ + (g_)*(x_))))}, x_Symbol] :> \text{With}[\{m = \text{FullSimplify}[(g*h*\text{Log}[G])/(\text{d}*e*\text{Log}[F])]\}, \text{Dist}[(\text{Denominator}[m]*G^{(f*h - (c*g*h)/d})/(\text{d}*e*\text{Log}[F]), \text{Subst}[\text{Int}[x^{(\text{Numerator}[m] - 1)*(a + b*x^{\text{Denominator}[m]})^p}, x], x, F^{((e*(c + d*x))/\text{Denominator}[m])}], x] /; \text{LeQ}[m, -1] || \text{GeQ}[m, 1]] /; \text{FreeQ}[\{F, G, a, b, c, d, e, f, g, h, p\}, x]$

Rule 321

$\text{Int}[(c_)*(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}), x_Symbol] :> \text{Simp}[(c^{(n - 1)}*(c*x)^{(m - n + 1)}*(a + b*x^n)^{(p + 1)})/(b*(m + n*p + 1)), x] - \text{Dist}[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), \text{Int}[(c*x)^{(m - n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, p\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[m, n - 1] \&\& \text{NeQ}[m + n*p + 1, 0] \&\& \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{(-1)}, x_Symbol] :> \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] || \text{GtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{e^{6x}}{1+e^{4x}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{1+x^2} dx, x, e^{2x} \right) \\ &= \frac{e^{2x}}{2} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, e^{2x} \right) \\ &= \frac{e^{2x}}{2} - \frac{1}{2} \tan^{-1}(e^{2x}) \end{aligned}$$

Mathematica [A] time = 0.007745, size = 18, normalized size = 0.9

$$\frac{1}{2} (e^{2x} - \tan^{-1}(e^{2x}))$$

Antiderivative was successfully verified.

[In] Integrate[E^(6*x)/(1 + E^(4*x)), x]

[Out] (E^(2*x) - ArcTan[E^(2*x)])/2

Maple [A] time = 0.003, size = 15, normalized size = 0.8

$$\frac{(e^x)^2}{2} - \frac{\arctan((e^x)^2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(exp(6*x)/(1+exp(4*x)), x)

[Out] 1/2*exp(x)^2-1/2*arctan(exp(x)^2)

Maxima [A] time = 1.41559, size = 19, normalized size = 0.95

$$-\frac{1}{2} \arctan(e^{2x}) + \frac{1}{2} e^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(6*x)/(1+exp(4*x)), x, algorithm="maxima")

[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)

Fricas [A] time = 1.91873, size = 49, normalized size = 2.45

$$-\frac{1}{2} \arctan(e^{2x}) + \frac{1}{2} e^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(6*x)/(1+exp(4*x)), x, algorithm="fricas")

[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)

Sympy [A] time = 0.110742, size = 24, normalized size = 1.2

$$\frac{e^{2x}}{2} + \text{RootSum}(16z^2 + 1, (i \mapsto i \log(-4i + e^{2x})))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(6*x)/(1+exp(4*x)),x)

[Out] exp(2*x)/2 + RootSum(16*_z**2 + 1, Lambda(_i, _i*log(-4*_i + exp(2*x))))

Giac [A] time = 1.04763, size = 19, normalized size = 0.95

$$-\frac{1}{2} \arctan(e^{2x}) + \frac{1}{2} e^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(exp(6*x)/(1+exp(4*x)),x, algorithm="giac")

[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)

3.102 $\int \log(2 + 3x^2) dx$

Optimal. Leaf size=33

$$x \log(3x^2 + 2) - 2x + 2\sqrt{\frac{2}{3}} \tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

[Out] $-2*x + 2*\text{Sqrt}[2/3]*\text{ArcTan}[\text{Sqrt}[3/2]*x] + x*\text{Log}[2 + 3*x^2]$

Rubi [A] time = 0.0111443, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2448, 321, 203}

$$x \log(3x^2 + 2) - 2x + 2\sqrt{\frac{2}{3}} \tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Log}[2 + 3*x^2], x]$

[Out] $-2*x + 2*\text{Sqrt}[2/3]*\text{ArcTan}[\text{Sqrt}[3/2]*x] + x*\text{Log}[2 + 3*x^2]$

Rule 2448

$\text{Int}[\text{Log}[(c_)*(d_)+(e_)*(x_)^{(n_)}]^{(p_)}], x_Symbol] \rightarrow \text{Simp}[x*\text{Log}[c*(d + e*x^n)^p], x] - \text{Dist}[e*n*p, \text{Int}[x^n/(d + e*x^n), x], x] /;$ FreeQ[{c, d, e, n, p}, x]

Rule 321

$\text{Int}[(c_)*(x_)^{(m_)*((a_)+(b_)*(x_)^{(n_)}])^{(p_)}], x_Symbol] \rightarrow \text{Simp}[(c^{(n-1)}*(c*x)^{(m-n+1)}*(a+b*x^n)^{(p+1)})/(b*(m+n*p+1)), x] - \text{Dist}[(a*c^n*(m-n+1))/(b*(m+n*p+1)), \text{Int}[(c*x)^{(m-n)}*(a+b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 203

$\text{Int}[(a_)+(b_)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \log(2 + 3x^2) dx &= x \log(2 + 3x^2) - 6 \int \frac{x^2}{2 + 3x^2} dx \\ &= -2x + x \log(2 + 3x^2) + 4 \int \frac{1}{2 + 3x^2} dx \\ &= -2x + 2\sqrt{\frac{2}{3}} \tan^{-1}\left(\sqrt{\frac{3}{2}}x\right) + x \log(2 + 3x^2) \end{aligned}$$

Mathematica [A] time = 0.0121088, size = 33, normalized size = 1.

$$x \log(3x^2 + 2) - 2x + 2\sqrt{\frac{2}{3}} \tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

[In] Integrate[Log[2 + 3*x^2], x]

[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]

Maple [A] time = 0., size = 27, normalized size = 0.8

$$-2x + x \ln(3x^2 + 2) + \frac{2\sqrt{6}}{3} \arctan\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(3*x^2+2), x)

[Out] -2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)

Maxima [A] time = 1.41188, size = 35, normalized size = 1.06

$$x \log(3x^2 + 2) + \frac{2}{3} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6}x\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(3*x^2+2), x, algorithm="maxima")

[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x

Fricas [A] time = 1.89351, size = 103, normalized size = 3.12

$$\frac{2}{3} \sqrt{3}\sqrt{2} \arctan\left(\frac{1}{2} \sqrt{3}\sqrt{2}x\right) + x \log(3x^2 + 2) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(3*x^2+2), x, algorithm="fricas")

[Out] 2/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x) + x*log(3*x^2 + 2) - 2*x

Sympy [A] time = 0.125984, size = 31, normalized size = 0.94

$$x \log(3x^2 + 2) - 2x + \frac{2\sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(3*x**2+2),x)

[Out] x*log(3*x**2 + 2) - 2*x + 2*sqrt(6)*atan(sqrt(6)*x/2)/3

Giac [A] time = 1.04873, size = 35, normalized size = 1.06

$$x \log(3x^2 + 2) + \frac{2}{3} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6}x\right) - 2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(3*x^2+2),x, algorithm="giac")

[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x

$$3.103 \quad \int \frac{1}{r\sqrt{-a^2+2Hr^2}} dx$$

Optimal. Leaf size=21

$$\frac{x}{r\sqrt{2Hr^2 - a^2}}$$

[Out] x/(r*Sqrt[-a^2 + 2*H*r^2])

Rubi [A] time = 0.0161468, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$, Rules used = {8}

$$\frac{x}{r\sqrt{2Hr^2 - a^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(r*Sqrt[-a^2 + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 + 2*H*r^2])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2 + 2Hr^2}} dx = \frac{x}{r\sqrt{-a^2 + 2Hr^2}}$$

Mathematica [A] time = 0.0000386, size = 21, normalized size = 1.

$$\frac{x}{r\sqrt{2Hr^2 - a^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(r*Sqrt[-a^2 + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 + 2*H*r^2])

Maple [A] time = 0.002, size = 20, normalized size = 1.

$$\frac{x}{r\sqrt{2Hr^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/r/(2*H*r^2-a^2)^(1/2),x)

[Out] $x/r/(2*H*r^2-a^2)^{(1/2)}$

Maxima [A] time = 0.931835, size = 26, normalized size = 1.24

$$\frac{x}{\sqrt{2Hr^2 - a^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="maxima")`

[Out] $x/(\text{sqrt}(2*H*r^2 - a^2)*r)$

Fricas [A] time = 1.73989, size = 55, normalized size = 2.62

$$\frac{\sqrt{2Hr^2 - a^2}x}{2Hr^3 - a^2r}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="fricas")`

[Out] $\text{sqrt}(2*H*r^2 - a^2)*x/(2*H*r^3 - a^2*r)$

Sympy [A] time = 0.052351, size = 15, normalized size = 0.71

$$\frac{x}{r\sqrt{2Hr^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*H*r**2-a**2)**(1/2),x)`

[Out] $x/(r*\text{sqrt}(2*H*r**2 - a**2))$

Giac [A] time = 1.04571, size = 26, normalized size = 1.24

$$\frac{x}{\sqrt{2Hr^2 - a^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="giac")`

[Out] $x/(\text{sqrt}(2*H*r^2 - a^2)*r)$

$$3.104 \quad \int \frac{1}{r\sqrt{-a^2 - e^2 + 2Hr^2}} dx$$

Optimal. Leaf size=26

$$\frac{x}{r\sqrt{-a^2 - e^2 + 2Hr^2}}$$

[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2])

Rubi [A] time = 0.0194002, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$, Rules used = {8}

$$\frac{x}{r\sqrt{-a^2 - e^2 + 2Hr^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2 - e^2 + 2Hr^2}} dx = \frac{x}{r\sqrt{-a^2 - e^2 + 2Hr^2}}$$

Mathematica [A] time = 0.0000479, size = 26, normalized size = 1.

$$\frac{x}{r\sqrt{-a^2 - e^2 + 2Hr^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2])

Maple [A] time = 0.002, size = 25, normalized size = 1.

$$\frac{x}{r\sqrt{2Hr^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/r/(2*H*r^2-a^2-e^2)^(1/2),x)

[Out] $x/r/(2*H*r^2-a^2-e^2)^{(1/2)}$

Maxima [A] time = 0.935708, size = 32, normalized size = 1.23

$$\frac{x}{\sqrt{2Hr^2 - a^2 - e^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")`

[Out] $x/(\text{sqrt}(2*H*r^2 - a^2 - e^2)*r)$

Fricas [A] time = 1.78943, size = 74, normalized size = 2.85

$$\frac{\sqrt{2Hr^2 - a^2 - e^2}x}{2Hr^3 - (a^2 + e^2)r}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")`

[Out] $\text{sqrt}(2*H*r^2 - a^2 - e^2)*x/(2*H*r^3 - (a^2 + e^2)*r)$

Sympy [A] time = 0.062354, size = 19, normalized size = 0.73

$$\frac{x}{r\sqrt{2Hr^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*H*r**2-a**2-e**2)**(1/2),x)`

[Out] $x/(r*\text{sqrt}(2*H*r**2 - a**2 - e**2))$

Giac [A] time = 1.05253, size = 31, normalized size = 1.19

$$\frac{x}{\sqrt{2Hr^2 - a^2 - e^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="giac")`

[Out] $x/(\text{sqrt}(2*H*r^2 - a^2 - e^2)*r)$

$$3.105 \quad \int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}} dx$$

Optimal. Leaf size=27

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

[Out] x/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4])

Rubi [A] time = 0.0227884, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {8}

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

[In] Int[1/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4]),x]

[Out] x/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}} dx = \frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

Mathematica [A] time = 0.0000354, size = 27, normalized size = 1.

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4]),x]

[Out] x/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4])

Maple [A] time = 0., size = 26, normalized size = 1.

$$\frac{x}{r\sqrt{-2Kr^4+2Hr^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x)

[Out] $x/r/(-2*K*r^4+2*H*r^2-a^2)^{(1/2)}$

Maxima [A] time = 0.927586, size = 34, normalized size = 1.26

$$\frac{x}{\sqrt{-2Kr^4 + 2Hr^2 - a^2r}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="maxima")`

[Out] $x/(\text{sqrt}(-2*K*r^4 + 2*H*r^2 - a^2)*r)$

Fricas [A] time = 1.8654, size = 85, normalized size = 3.15

$$-\frac{\sqrt{-2Kr^4 + 2Hr^2 - a^2}x}{2Kr^5 - 2Hr^3 + a^2r}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="fricas")`

[Out] $-\text{sqrt}(-2*K*r^4 + 2*H*r^2 - a^2)*x/(2*K*r^5 - 2*H*r^3 + a^2*r)$

Sympy [A] time = 0.054584, size = 22, normalized size = 0.81

$$\frac{x}{r\sqrt{2Hr^2 - 2Kr^4 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(-2*K*r**4+2*H*r**2-a**2)**(1/2),x)`

[Out] $x/(r*\text{sqrt}(2*H*r**2 - 2*K*r**4 - a**2))$

Giac [A] time = 1.05276, size = 34, normalized size = 1.26

$$\frac{x}{\sqrt{-2Kr^4 + 2Hr^2 - a^2r}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="giac")`

[Out] $x/(\text{sqrt}(-2*K*r^4 + 2*H*r^2 - a^2)*r)$

$$3.106 \quad \int \frac{1}{r\sqrt{-a^2 - e^2 + 2Hr^2 - 2Kr^4}} dx$$

Optimal. Leaf size=32

$$\frac{x}{r\sqrt{-a^2 - e^2 + 2Hr^2 - 2Kr^4}}$$

[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4])

Rubi [A] time = 0.0256325, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {8}

$$\frac{x}{r\sqrt{-a^2 - e^2 + 2Hr^2 - 2Kr^4}}$$

Antiderivative was successfully verified.

[In] Int[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4]),x]

[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2 - e^2 + 2Hr^2 - 2Kr^4}} dx = \frac{x}{r\sqrt{-a^2 - e^2 + 2Hr^2 - 2Kr^4}}$$

Mathematica [A] time = 0.0000345, size = 32, normalized size = 1.

$$\frac{x}{r\sqrt{-a^2 - e^2 + 2Hr^2 - 2Kr^4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4]),x]

[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4])

Maple [A] time = 0.001, size = 31, normalized size = 1.

$$\frac{x}{r\sqrt{-2Kr^4 + 2Hr^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x)

[Out] $x/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^{(1/2)}$

Maxima [A] time = 0.932703, size = 41, normalized size = 1.28

$$\frac{x}{\sqrt{-2Kr^4 + 2Hr^2 - a^2 - e^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")`

[Out] $x/(\text{sqrt}(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r)$

Fricas [A] time = 1.81301, size = 104, normalized size = 3.25

$$-\frac{\sqrt{-2Kr^4 + 2Hr^2 - a^2 - e^2}x}{2Kr^5 - 2Hr^3 + (a^2 + e^2)r}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")`

[Out] $-\text{sqrt}(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*x/(2*K*r^5 - 2*H*r^3 + (a^2 + e^2)*r)$

Sympy [A] time = 0.055048, size = 26, normalized size = 0.81

$$\frac{x}{r\sqrt{2Hr^2 - 2Kr^4 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(-2*K*r**4+2*H*r**2-a**2-e**2)**(1/2),x)`

[Out] $x/(r*\text{sqrt}(2*H*r**2 - 2*K*r**4 - a**2 - e**2))$

Giac [A] time = 1.04741, size = 39, normalized size = 1.22

$$\frac{x}{\sqrt{-2Kr^4 + 2Hr^2 - a^2 - e^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="giac")`

[Out] $x/(\text{sqrt}(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r)$

$$3.107 \quad \int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}} dx$$

Optimal. Leaf size=24

$$\frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

[Out] x/(r*Sqrt[-a^2 - 2*r*(K - H*r)])

Rubi [A] time = 0.0265996, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {8}

$$\frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

Antiderivative was successfully verified.

[In] Int[1/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 - 2*r*(K - H*r)])

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}} dx = \frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

Mathematica [A] time = 0.0000408, size = 25, normalized size = 1.04

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2])

Maple [A] time = 0., size = 24, normalized size = 1.

$$\frac{x}{r\sqrt{2Hr^2-2Kr-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x)

[Out] $1/r/(2*H*r^2-2*K*r-a^2)^{(1/2)}*x$

Maxima [A] time = 0.930996, size = 31, normalized size = 1.29

$$\frac{x}{\sqrt{2Hr^2 - a^2 - 2Krr}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="maxima")`

[Out] $x/(\text{sqrt}(2*H*r^2 - a^2 - 2*K*r)*r)$

Fricas [A] time = 1.87546, size = 80, normalized size = 3.33

$$\frac{\sqrt{2Hr^2 - a^2 - 2Krx}}{2Hr^3 - a^2r - 2Kr^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="fricas")`

[Out] $\text{sqrt}(2*H*r^2 - a^2 - 2*K*r)*x/(2*H*r^3 - a^2*r - 2*K*r^2)$

Sympy [A] time = 0.053192, size = 20, normalized size = 0.83

$$\frac{x}{r\sqrt{2Hr^2 - 2Kr - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*H*r**2-2*K*r-a**2)**(1/2),x)`

[Out] $x/(r*\text{sqrt}(2*H*r**2 - 2*K*r - a**2))$

Giac [A] time = 1.24152, size = 31, normalized size = 1.29

$$\frac{x}{\sqrt{2Hr^2 - a^2 - 2Krr}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="giac")`

[Out] $x/(\text{sqrt}(2*H*r^2 - a^2 - 2*K*r)*r)$

$$3.108 \quad \int \frac{1}{r\sqrt{-a^2 - e^2 - 2Kr + 2Hr^2}} dx$$

Optimal. Leaf size=29

$$\frac{x}{r\sqrt{-a^2 - e^2 - 2r(K - Hr)}}$$

[Out] x/(r*Sqrt[-a^2 - e^2 - 2*r*(K - H*r)])

Rubi [A] time = 0.0316036, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {8}

$$\frac{x}{r\sqrt{-a^2 - e^2 - 2r(K - Hr)}}$$

Antiderivative was successfully verified.

[In] Int[1/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 - e^2 - 2*r*(K - H*r)])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2 - e^2 - 2Kr + 2Hr^2}} dx = \frac{x}{r\sqrt{-a^2 - e^2 - 2r(K - Hr)}}$$

Mathematica [A] time = 0.000036, size = 30, normalized size = 1.03

$$\frac{x}{r\sqrt{-a^2 - e^2 + 2Hr^2 - 2Kr}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2])

Maple [A] time = 0., size = 29, normalized size = 1.

$$\frac{x}{r\sqrt{2Hr^2 - 2Kr - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x)

[Out] $x/r/(2*H*r^2-2*K*r-a^2-e^2)^{(1/2)}$

Maxima [A] time = 0.928842, size = 38, normalized size = 1.31

$$\frac{x}{\sqrt{2Hr^2 - a^2 - e^2 - 2Krr}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="maxima")`

[Out] `x/(sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)*r)`

Fricas [A] time = 1.81653, size = 99, normalized size = 3.41

$$\frac{\sqrt{2Hr^2 - a^2 - e^2 - 2Krx}}{2Hr^3 - 2Kr^2 - (a^2 + e^2)r}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="fricas")`

[Out] `sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)*x/(2*H*r^3 - 2*K*r^2 - (a^2 + e^2)*r)`

Sympy [A] time = 0.054356, size = 24, normalized size = 0.83

$$\frac{x}{r\sqrt{2Hr^2 - 2Kr - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*H*r**2-2*K*r-a**2-e**2)**(1/2),x)`

[Out] `x/(r*sqrt(2*H*r**2 - 2*K*r - a**2 - e**2))`

Giac [A] time = 1.07776, size = 36, normalized size = 1.24

$$\frac{x}{\sqrt{2Hr^2 - a^2 - 2Kr - e^2r}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="giac")`

[Out] `x/(sqrt(2*H*r^2 - a^2 - 2*K*r - e^2)*r)`

$$3.109 \quad \int \frac{r}{\sqrt{-a^2+2er^2}} dx$$

Optimal. Leaf size=19

$$\frac{rx}{\sqrt{2er^2 - a^2}}$$

[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2]

Rubi [A] time = 0.0094807, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {8}

$$\frac{rx}{\sqrt{2er^2 - a^2}}$$

Antiderivative was successfully verified.

[In] Int[r/Sqrt[-a^2 + 2*E*r^2], x]

[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{r}{\sqrt{-a^2 + 2er^2}} dx = \frac{rx}{\sqrt{-a^2 + 2er^2}}$$

Mathematica [A] time = 0.0000577, size = 19, normalized size = 1.

$$\frac{rx}{\sqrt{2er^2 - a^2}}$$

Antiderivative was successfully verified.

[In] Integrate[r/Sqrt[-a^2 + 2*E*r^2], x]

[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2]

Maple [A] time = 0., size = 18, normalized size = 1.

$$rx \frac{1}{\sqrt{2Er^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(r/(2*E*r^2-a^2)^(1/2), x)

[Out] $r*x/(2*E*r^2-a^2)^{(1/2)}$

Maxima [A] time = 0.929918, size = 23, normalized size = 1.21

$$\frac{rx}{\sqrt{2Er^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*E*r^2-a^2)^(1/2),x, algorithm="maxima")`

[Out] $r*x/\text{sqrt}(2*E*r^2 - a^2)$

Fricas [A] time = 1.80718, size = 34, normalized size = 1.79

$$\frac{rx}{\sqrt{2Er^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*E*r^2-a^2)^(1/2),x, algorithm="fricas")`

[Out] $r*x/\text{sqrt}(2*E*r^2 - a^2)$

Sympy [A] time = 0.050965, size = 15, normalized size = 0.79

$$\frac{rx}{\sqrt{-a^2 + 2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*E*r**2-a**2)**(1/2),x)`

[Out] $r*x/\text{sqrt}(-a**2 + 2*E*r**2)$

Giac [A] time = 1.04923, size = 23, normalized size = 1.21

$$\frac{rx}{\sqrt{2Er^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*E*r^2-a^2)^(1/2),x, algorithm="giac")`

[Out] $r*x/\text{sqrt}(2*E*r^2 - a^2)$

$$3.110 \quad \int \frac{r}{\sqrt{-a^2 - e^2 + 2er^2}} dx$$

Optimal. Leaf size=24

$$\frac{rx}{\sqrt{-a^2 - e^2 + 2er^2}}$$

[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2]

Rubi [A] time = 0.0111275, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {8}

$$\frac{rx}{\sqrt{-a^2 - e^2 + 2er^2}}$$

Antiderivative was successfully verified.

[In] Int[r/Sqrt[-a^2 - e^2 + 2*E*r^2], x]

[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{r}{\sqrt{-a^2 - e^2 + 2er^2}} dx = \frac{rx}{\sqrt{-a^2 - e^2 + 2er^2}}$$

Mathematica [A] time = 0.0000289, size = 24, normalized size = 1.

$$\frac{rx}{\sqrt{-a^2 - e^2 + 2er^2}}$$

Antiderivative was successfully verified.

[In] Integrate[r/Sqrt[-a^2 - e^2 + 2*E*r^2], x]

[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2]

Maple [A] time = 0.002, size = 23, normalized size = 1.

$$rx \frac{1}{\sqrt{2Er^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(r/(2*E*r^2-a^2-e^2)^(1/2), x)

[Out] $r*x/(2*E*r^2-a^2-e^2)^{(1/2)}$

Maxima [A] time = 0.937238, size = 30, normalized size = 1.25

$$\frac{rx}{\sqrt{2Er^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*E*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")`

[Out] $r*x/\text{sqrt}(2*E*r^2 - a^2 - e^2)$

Fricas [A] time = 1.81658, size = 42, normalized size = 1.75

$$\frac{rx}{\sqrt{2Er^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*E*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")`

[Out] $r*x/\text{sqrt}(2*E*r^2 - a^2 - e^2)$

Sympy [A] time = 0.05323, size = 19, normalized size = 0.79

$$\frac{rx}{\sqrt{-a^2 - e^2 + 2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*E*r**2-a**2-e**2)**(1/2),x)`

[Out] $r*x/\text{sqrt}(-a**2 - e**2 + 2*E*r**2)$

Giac [A] time = 1.0504, size = 28, normalized size = 1.17

$$\frac{rx}{\sqrt{2Er^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*E*r^2-a^2-e^2)^(1/2),x, algorithm="giac")`

[Out] $r*x/\text{sqrt}(2*E*r^2 - a^2 - e^2)$

$$3.111 \quad \int \frac{r}{\sqrt{-a^2 + 2er^2 - 2Kr^4}} dx$$

Optimal. Leaf size=25

$$\frac{rx}{\sqrt{-a^2 - 2Kr^4 + 2er^2}}$$

[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4]

Rubi [A] time = 0.0127465, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {8}

$$\frac{rx}{\sqrt{-a^2 - 2Kr^4 + 2er^2}}$$

Antiderivative was successfully verified.

[In] Int[r/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4], x]

[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{r}{\sqrt{-a^2 + 2er^2 - 2Kr^4}} dx = \frac{rx}{\sqrt{-a^2 + 2er^2 - 2Kr^4}}$$

Mathematica [A] time = 0.0000295, size = 25, normalized size = 1.

$$\frac{rx}{\sqrt{-a^2 - 2Kr^4 + 2er^2}}$$

Antiderivative was successfully verified.

[In] Integrate[r/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4], x]

[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4]

Maple [A] time = 0.002, size = 24, normalized size = 1.

$$rx \frac{1}{\sqrt{-2Kr^4 + 2Er^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(r/(-2*K*r^4+2*E*r^2-a^2)^(1/2), x)

[Out] $r*x/(-2*K*r^4+2*E*r^2-a^2)^{(1/2)}$

Maxima [A] time = 0.930996, size = 31, normalized size = 1.24

$$\frac{rx}{\sqrt{-2Kr^4 + 2Er^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(-2*K*r^4+2*E*r^2-a^2)^(1/2),x, algorithm="maxima")`

[Out] $r*x/\text{sqrt}(-2*K*r^4 + 2*E*r^2 - a^2)$

Fricas [A] time = 1.88055, size = 85, normalized size = 3.4

$$-\frac{\sqrt{-2Kr^4 + 2Er^2 - a^2}rx}{2Kr^4 - 2Er^2 + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(-2*K*r^4+2*E*r^2-a^2)^(1/2),x, algorithm="fricas")`

[Out] $-\text{sqrt}(-2*K*r^4 + 2*E*r^2 - a^2)*r*x/(2*K*r^4 - 2*E*r^2 + a^2)$

Sympy [A] time = 0.053411, size = 22, normalized size = 0.88

$$\frac{rx}{\sqrt{-2Kr^4 - a^2 + 2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(-2*K*r**4+2*E*r**2-a**2)**(1/2),x)`

[Out] $r*x/\text{sqrt}(-2*K*r**4 - a**2 + 2*E*r**2)$

Giac [A] time = 1.06028, size = 31, normalized size = 1.24

$$\frac{rx}{\sqrt{-2Kr^4 + 2Er^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(-2*K*r^4+2*E*r^2-a^2)^(1/2),x, algorithm="giac")`

[Out] $r*x/\text{sqrt}(-2*K*r^4 + 2*E*r^2 - a^2)$

$$3.112 \quad \int \frac{r}{\sqrt{-a^2 - e^2 + 2er^2 - 2Kr^4}} dx$$

Optimal. Leaf size=30

$$\frac{rx}{\sqrt{-a^2 - e^2 - 2Kr^4 + 2er^2}}$$

[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4]

Rubi [A] time = 0.0153597, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {8}

$$\frac{rx}{\sqrt{-a^2 - e^2 - 2Kr^4 + 2er^2}}$$

Antiderivative was successfully verified.

[In] Int[r/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4], x]

[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{r}{\sqrt{-a^2 - e^2 + 2er^2 - 2Kr^4}} dx = \frac{rx}{\sqrt{-a^2 - e^2 + 2er^2 - 2Kr^4}}$$

Mathematica [A] time = 0.0000365, size = 30, normalized size = 1.

$$\frac{rx}{\sqrt{-a^2 - e^2 - 2Kr^4 + 2er^2}}$$

Antiderivative was successfully verified.

[In] Integrate[r/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4], x]

[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4]

Maple [A] time = 0., size = 29, normalized size = 1.

$$rx \frac{1}{\sqrt{-2Kr^4 + 2Er^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(r/(-2*K*r^4+2*E*r^2-a^2-e^2)^(1/2), x)

[Out] $r*x/(-2*K*r^4+2*E*r^2-a^2-e^2)^{(1/2)}$

Maxima [A] time = 0.929396, size = 38, normalized size = 1.27

$$\frac{rx}{\sqrt{-2Kr^4 + 2Er^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(-2*K*r^4+2*E*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")`

[Out] $r*x/\text{sqrt}(-2*K*r^4 + 2*E*r^2 - a^2 - e^2)$

Fricas [A] time = 1.70583, size = 101, normalized size = 3.37

$$-\frac{\sqrt{-2Kr^4 + 2Er^2 - a^2 - e^2}rx}{2Kr^4 - 2Er^2 + a^2 + e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(-2*K*r^4+2*E*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")`

[Out] $-\text{sqrt}(-2*K*r^4 + 2*E*r^2 - a^2 - e^2)*r*x/(2*K*r^4 - 2*E*r^2 + a^2 + e^2)$

Sympy [A] time = 0.054855, size = 26, normalized size = 0.87

$$\frac{rx}{\sqrt{-2Kr^4 - a^2 - e^2 + 2Er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(-2*K*r**4+2*E*r**2-a**2-e**2)**(1/2),x)`

[Out] $r*x/\text{sqrt}(-2*K*r**4 - a**2 - e**2 + 2*E*r**2)$

Giac [A] time = 1.06962, size = 36, normalized size = 1.2

$$\frac{rx}{\sqrt{-2Kr^4 + 2Er^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(-2*K*r^4+2*E*r^2-a^2-e^2)^(1/2),x, algorithm="giac")`

[Out] $r*x/\text{sqrt}(-2*K*r^4 + 2*E*r^2 - a^2 - e^2)$

$$3.113 \quad \int \frac{r}{\sqrt{-a^2 - e^2 - 2Kr + 2Hr^2}} dx$$

Optimal. Leaf size=27

$$\frac{rx}{\sqrt{-a^2 - e^2 - 2r(K - Hr)}}$$

[Out] (r*x)/Sqrt[-a^2 - e^2 - 2*r*(K - H*r)]

Rubi [A] time = 0.0056563, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {8}

$$\frac{rx}{\sqrt{-a^2 - e^2 - 2r(K - Hr)}}$$

Antiderivative was successfully verified.

[In] Int[r/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2], x]

[Out] (r*x)/Sqrt[-a^2 - e^2 - 2*r*(K - H*r)]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{r}{\sqrt{-a^2 - e^2 - 2Kr + 2Hr^2}} dx = \frac{rx}{\sqrt{-a^2 - e^2 - 2r(K - Hr)}}$$

Mathematica [A] time = 0.0000306, size = 28, normalized size = 1.04

$$\frac{rx}{\sqrt{-a^2 - e^2 + 2Hr^2 - 2Kr}}$$

Antiderivative was successfully verified.

[In] Integrate[r/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2], x]

[Out] (r*x)/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]

Maple [A] time = 0.001, size = 27, normalized size = 1.

$$rx \frac{1}{\sqrt{2Hr^2 - 2Kr - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2), x)

[Out] $r*x/(2*H*r^2-2*K*r-a^2-e^2)^{(1/2)}$

Maxima [A] time = 0.925849, size = 35, normalized size = 1.3

$$\frac{rx}{\sqrt{2Hr^2 - a^2 - e^2 - 2Kr}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="maxima")`

[Out] $r*x/\text{sqrt}(2*H*r^2 - a^2 - e^2 - 2*K*r)$

Fricas [A] time = 1.81226, size = 53, normalized size = 1.96

$$\frac{rx}{\sqrt{2Hr^2 - a^2 - e^2 - 2Kr}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="fricas")`

[Out] $r*x/\text{sqrt}(2*H*r^2 - a^2 - e^2 - 2*K*r)$

Sympy [A] time = 0.05365, size = 24, normalized size = 0.89

$$\frac{rx}{\sqrt{2Hr^2 - 2Kr - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*H*r**2-2*K*r-a**2-e**2)**(1/2),x)`

[Out] $r*x/\text{sqrt}(2*H*r**2 - 2*K*r - a**2 - e**2)$

Giac [A] time = 1.05683, size = 34, normalized size = 1.26

$$\frac{rx}{\sqrt{2Hr^2 - a^2 - 2Kr - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="giac")`

[Out] $r*x/\text{sqrt}(2*H*r^2 - a^2 - 2*K*r - e^2)$

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*   is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*   antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
22       If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25       "C"],
26     If[FreeQ[result,Integrate] && FreeQ[result,Int],
27       "C",
28       "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
```

```

38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46   If[AtomQ[expn],
47     1,
48     If[ListQ[expn],
49       Max[Map[ExpnType,expn]],
50       If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52           ExpnType[expn[[1]]],
53           If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55               1,
56               Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58             If[Head[expn]===Plus || Head[expn]===Times,
59               Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60             If[ElementaryFunctionQ[Head[expn]],
61               Max[3,ExpnType[expn[[1]]],
62             If[SpecialFunctionQ[Head[expn]],
63               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64             If[HypergeometricFunctionQ[Head[expn]],
65               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66             If[AppellFunctionQ[Head[expn]],
67               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68             If[Head[expn]===RootSum,
69               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
70             If[Head[expn]===Integrate || Head[expn]===Int,
71               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
72             9]]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp,Log,
78     Sin,Cos,Tan,Cot,Sec,Csc,
79     ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
80     Sinh,Cosh,Tanh,Coth,Sech,Csch,
81     ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
82   },func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf,Erfc,Erfi,
88     FresnelS,FresnelC,
89     ExpIntegralE,ExpIntegralEi,LogIntegral,
90     SinIntegral,CosIntegral,SinhIntegral,CoshIntegral,
91     Gamma,LogGamma,PolyGamma,
92     Zeta,PolyLog,ProductLog,
93     EllipticF,EllipticE,EllipticPi
94   },func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
99
100

```

```

101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1},func]

```

4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 # if leaf size is "too large". Set at 500,000
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 # see problem 156, file Apostol_Problems
11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
14     debug:=false;
15
16     leaf_count_result:=leafcount(result);
17     #do NOT call ExpnType() if leaf size is too large. Recursion problem
18     if leaf_count_result > 500000 then
19         return "B";
20     fi;
21
22     leaf_count_optimal:=leafcount(optimal);
23
24     ExpnType_result:=ExpnType(result);
25     ExpnType_optimal:=ExpnType(optimal);
26
27     if debug then
28         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
29             ExpnType_optimal);
30     fi;
31
32 # If result and optimal are mathematical expressions,
33 # GradeAntiderivative[result,optimal] returns
34 # "F" if the result fails to integrate an expression that
35 # is integrable
36 # "C" if result involves higher level functions than necessary
37 # "B" if result is more than twice the size of the optimal
38 # antiderivative
39 # "A" if result can be considered optimal
40
41 #This check below actually is not needed, since I only
42 #call this grading only for passed integrals. i.e. I check
43 #for "F" before calling this. But no harm of keeping it here.
44 #just in case.
45
46 if not type(result,freeof('int')) then
47     return "F";
48 end if;
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then
56             if debug then

```

```

57         print("both result and optimal complex");
58         fi;
59         #both result and optimal complex
60         if leaf_count_result<=2*leaf_count_optimal then
61             return "A";
62         else
63             return "B";
64         end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do
not as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
102 is_contains_complex:= proc(expression)
103     return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)

```



```

119   if type(expn,'atomic') then
120       1
121   elif type(expn,'list') then
122       apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124       if type(op(1,expn),'rational') then
125           1
126       else
127           max(2,ExpnType(op(1,expn)))
128       end if
129   elif type(expn,'^^') then
130       if type(op(2,expn),'integer') then
131           ExpnType(op(1,expn))
132       elif type(op(2,expn),'rational') then
133           if type(op(1,expn),'rational') then
134               1
135           else
136               max(2,ExpnType(op(1,expn)))
137           end if
138       else
139           max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140       end if
141   elif type(expn,'+`') or type(expn,'*`') then
142       max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144       max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146       max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148       max(5,apply(max,map(ExpnType,[op(expn)])))
149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159     member(func,[
160         exp,log,ln,
161         sin,cos,tan,cot,sec,csc,
162         arcsin,arccos,arctan,arccot,arcsec,arccsc,
163         sinh,cosh,tanh,coth,sech,csch,
164         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168     member(func,[
169         erf,erfc,erfi,
170         FresnelS,FresnelC,
171         Ei,Ei,Li,Si,Ci,Shi,Chi,
172         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173         EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181     member(func,[AppellF1])

```

```

182 end proc:
183
184 # u is a sum or product.  rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:
42         if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43             return True

```

```

44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,``^`)
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn
72 )))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or
77 type(expn,``*`)
78     m1 = expnType(expn.args[0])
79     m2 = expnType(list(expn.args[1:]))
80     return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82     return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84     m1 = max(map(expnType, list(expn.args)))
85     return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88     m1 = max(map(expnType, list(expn.args)))
89     return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
90     elif is_appell_function(expn.func):
91     m1 = max(map(expnType, list(expn.args)))
92     return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
93     elif isinstance(expn,RootSum):
94     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
95 ,Apply[List,expn]],7]],
96     return max(7,m1)
97     elif str(expn).find("Integral") != -1:
98     m1 = max(map(expnType, list(expn.args)))
99     return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
100     else:
101     return 9
102
103 #main function
104 def grade_antiderivative(result,optimal):
105
106     leaf_count_result = leaf_count(result)

```

```

102 leaf_count_optimal = leaf_count(optimal)
103
104 expnType_result = expnType(result)
105 expnType_optimal = expnType(optimal)
106
107 if str(result).find("Integral") != -1:
108     return "F"
109
110 if expnType_result <= expnType_optimal:
111     if result.has(I):
112         if optimal.has(I): #both result and optimal complex
113             if leaf_count_result <= 2*leaf_count_optimal:
114                 return "A"
115             else:
116                 return "B"
117         else: #result contains complex but optimal is not
118             return "C"
119     else: # result do not contain complex, this assumes optimal do not as
well
120         if leaf_count_result <= 2*leaf_count_optimal:
121             return "A"
122         else:
123             return "B"
124     else:
125         return "C"

```

4.0.4 SageMath grading function

```

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by
2 #     Albert Rich to use with Sagemath. This is used to
3 #     grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #     'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
(flatten(tree(anti))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34             #since this estimate of leaf count is bit lower than

```

```

35         #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow: #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func , " is special_function")
83         else:
84             print ("func ", func , " is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','
91     hypergeometric_U']
92
93 def is_appell_function(func):
94     return func.name() in ['hypergeometric'] #[appellf1] can't find this in
95     sagemath

```

```

95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
104             return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list: #isinstance(expn,list):
121         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
127     elif expn.operator() == operator.pow: #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
129             return expnType(expn.operands()[0]) #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
132                 return 1
133             else:
134                 return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138         m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139         m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
141     elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
142         return max(3,expnType(expn.operands()[0]))
143     elif is_special_function(expn.operator()): #is_special_function(expn.func)
144         m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

145     return max(4,m1)    #max(4,m1)
146     elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
148         return max(5,m1)    #max(5,m1)
149     elif is_appell_function(expn.operator()):
150         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
151         return max(6,m1)    #max(6,m1)
152     elif str(expn).find("Integral") != -1: #this will never happen, since it
153         #is checked before calling the grading function that is passed.
154         #but kept it here.
155         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
156         return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157     else:
158         return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"

```