Computer algebra independent integration tests

0-Independent-test-suites/Moses-Problems

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [113]. This is test number [8].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (113)	% 0. (0)
Mathematica	% 99.12 (112)	% 0.88 (1)
Maple	% 100. (113)	% 0. (0)
Maxima	% 94.69 (107)	% 5.31 (6)
Fricas	% 99.12 (112)	% 0.88 (1)
Sympy	% 92.04 (104)	% 7.96 (9)
Giac	% 96.46 (109)	% 3.54 (4)

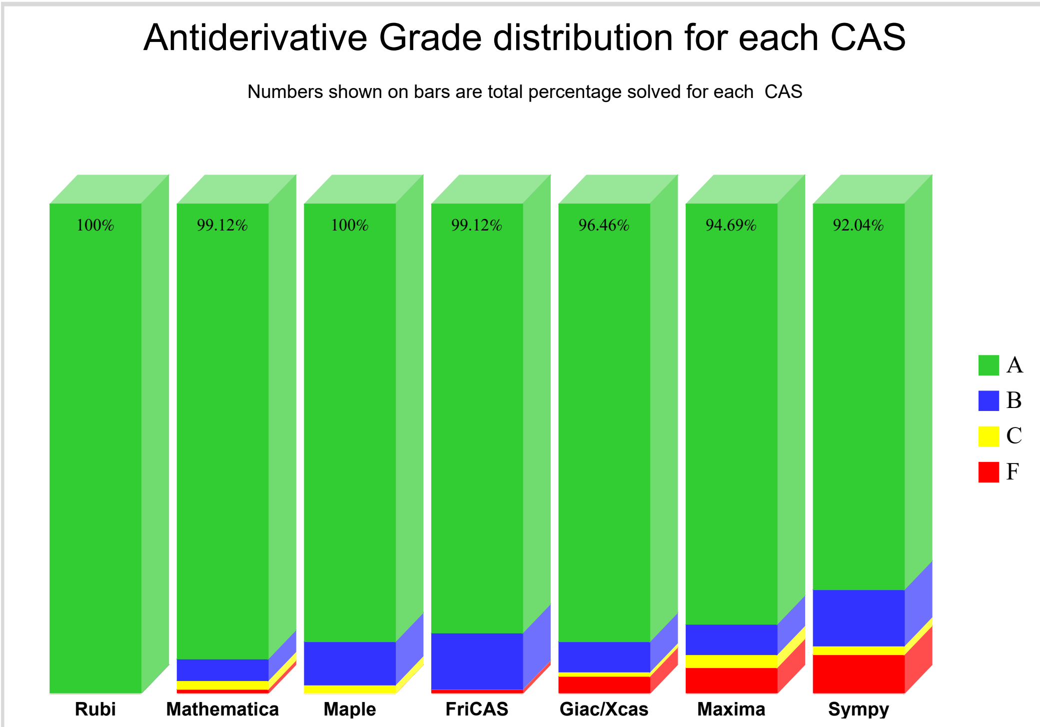
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

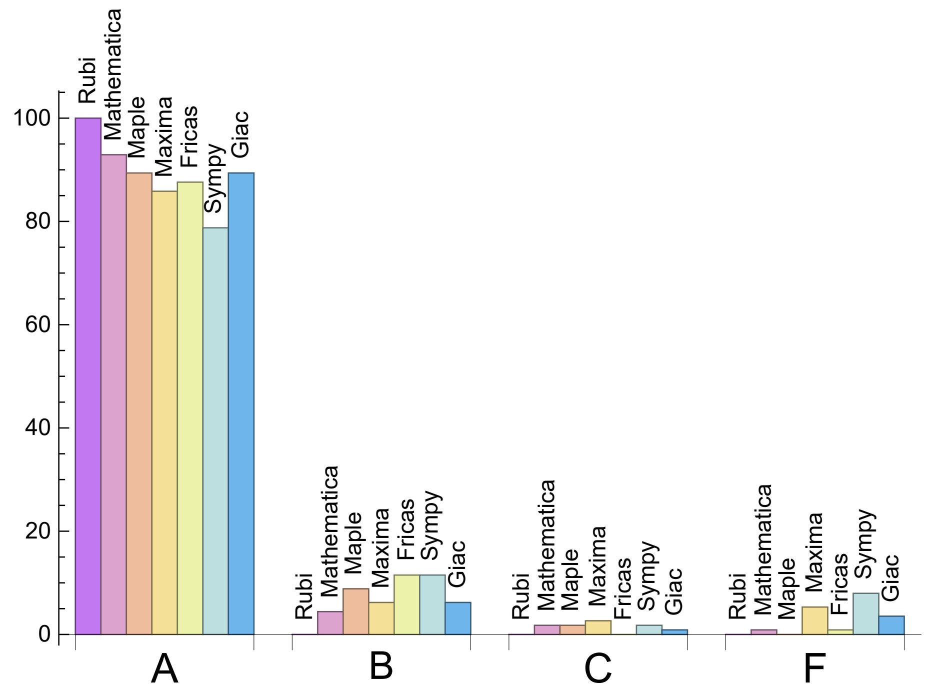
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	92.92	4.42	1.77	0.88
Maple	89.38	8.85	1.77	0.
Maxima	85.84	6.19	2.65	5.31
Fricas	87.61	11.5	0.	0.88
Sympy	78.76	11.5	1.77	7.96
Giac	89.38	6.19	0.88	3.54

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.02	20.14	1.01	16.	1.
Mathematica	0.01	24.32	1.11	16.	1.
Maple	0.01	25.83	1.28	14.	0.92
Maxima	1.09	28.51	1.54	18.	1.17
Fricas	1.94	66.79	3.59	46.	2.74
Sympy	0.49	29.38	1.59	15.	0.82
Giac	1.08	24.75	1.42	18.	1.14

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

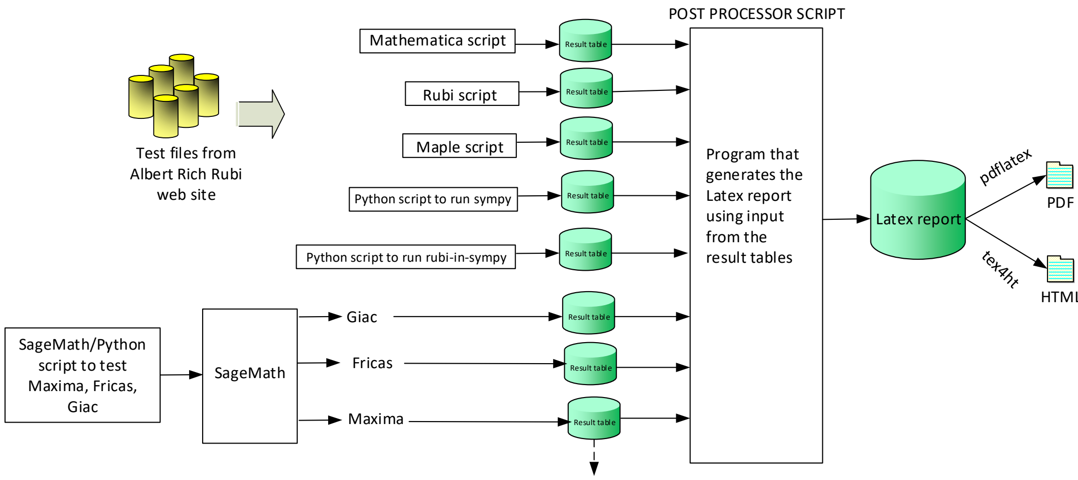
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 31, 42, 70, 71, 72 }

C grade: { 40, 69 }

F grade: { 57}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 37, 38, 39, 41, 43, 44, 45, 46, 47, 49, 50, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 35, 36, 40, 48, 51, 57, 69, 70, 71, 72 }

C grade: { 32, 42 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 33, 34, 35, 37, 39, 41, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113

B grade: { 36, 38, 68, 70, 71, 72, 87 }

C grade: { 10, 11, 47 }

F grade: { 27, 32, 40, 42, 67, 69 }

2.1.5 FriCAS

A grade: { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 38, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 1, 9, 37, 39, 40, 42, 69, 70, 73, 74, 83, 84, 100 }

C grade: { }

F grade: { 32 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 38, 41, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 74, 75, 76, 77, 78, 79, 80, 81, 85, 86, 88, 89, 91, 92, 93, 94, 95, 96, 97, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 7, 18, 37, 39, 50, 51, 56, 66, 73, 82, 83, 84, 100 }

C grade: { 71, 72 }

F grade: { 35, 36, 40, 42, 69, 70, 87, 90, 98 }

2.1.7 Giac

A grade: { 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 41, 44, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113 }

B grade: { 1, 25, 43, 45, 70, 71, 72 }

C grade: { 47 }

F grade: { 32, 40, 42, 69 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	14	22	126	19	46
normalized size	1	1.	1.5	1.17	1.83	10.5	1.58	3.83
time (sec)	N/A	0.01	0.003	0.	1.414	1.655	0.066	1.063

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	20	53	14	20
normalized size	1	1.	1.	0.92	1.54	4.08	1.08	1.54
time (sec)	N/A	0.005	0.004	0.005	1.413	1.506	0.105	1.079

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	11	15	38	15	15
normalized size	1	1.	0.74	0.58	0.79	2.	0.79	0.79
time (sec)	N/A	0.003	0.003	0.001	0.936	1.5	0.223	1.09

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	11	2	3
normalized size	1	1.	1.	1.5	1.5	5.5	1.	1.5
time (sec)	N/A	0.002	0.001	0.	0.927	1.582	0.053	1.093

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	8	18	5	8
normalized size	1	1.	1.	0.78	0.89	2.	0.56	0.89
time (sec)	N/A	0.006	0.001	0.	0.927	1.633	0.079	1.079

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	19	7	8
normalized size	1	1.	1.	0.88	1.	2.38	0.88	1.
time (sec)	N/A	0.011	0.004	0.005	0.936	1.599	0.066	1.066

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	28	22	12
normalized size	1	1.	1.	0.77	0.92	2.15	1.69	0.92
time (sec)	N/A	0.002	0.002	0.002	0.925	1.568	0.19	1.085

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	14	15	46	15	15
normalized size	1	1.	0.74	0.74	0.79	2.42	0.79	0.79
time (sec)	N/A	0.007	0.014	0.	0.934	1.641	0.298	1.098

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	39	8	8
normalized size	1	1.	1.	0.88	1.	4.88	1.	1.
time (sec)	N/A	0.012	0.004	0.013	0.94	1.607	0.064	1.072

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	20	26	3	4
normalized size	1	1.	1.	1.	5.	6.5	0.75	1.
time (sec)	N/A	0.012	0.005	0.004	1.065	1.931	0.641	1.06

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	18	23	2	3
normalized size	1	1.	1.	1.5	9.	11.5	1.	1.5
time (sec)	N/A	0.011	0.013	0.001	1.055	1.86	0.54	1.117

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	9	20	5	9
normalized size	1	1.	1.	1.	1.12	2.5	0.62	1.12
time (sec)	N/A	0.003	0.003	0.002	0.946	1.82	0.072	1.07

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	8	15	5	8
normalized size	1	1.	1.	1.	1.14	2.14	0.71	1.14
time (sec)	N/A	0.02	0.005	0.002	0.935	1.886	0.08	1.07

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	26	22	26	53	20	26
normalized size	1	1.	0.93	0.79	0.93	1.89	0.71	0.93
time (sec)	N/A	0.02	0.014	0.002	0.928	1.802	0.087	1.098

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	42	15	22
normalized size	1	1.	1.	0.77	1.	1.91	0.68	1.
time (sec)	N/A	0.005	0.002	0.001	0.933	1.878	0.083	1.074

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	20	5	8
normalized size	1	1.	1.	0.88	1.	2.5	0.62	1.
time (sec)	N/A	0.007	0.001	0.001	0.913	1.915	0.054	1.081

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	8	18	5	8
normalized size	1	1.	1.	0.78	0.89	2.	0.56	0.89
time (sec)	N/A	0.006	0.001	0.	0.925	1.913	0.075	1.07

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	28	22	12
normalized size	1	1.	1.	0.77	0.92	2.15	1.69	0.92
time (sec)	N/A	0.002	0.	0.	0.93	1.701	0.18	1.06

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	7	19	5	7
normalized size	1	1.	1.	1.	1.17	3.17	0.83	1.17
time (sec)	N/A	0.015	0.004	0.001	0.922	1.682	0.085	1.094

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	6	7	18	7	7
normalized size	1	1.	1.	0.67	0.78	2.	0.78	0.78
time (sec)	N/A	0.	0.001	0.001	0.927	1.739	0.052	1.081

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	24	7	11
normalized size	1	1.	1.	0.9	1.1	2.4	0.7	1.1
time (sec)	N/A	0.003	0.004	0.005	0.925	1.88	0.112	1.101

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	8	9	18	7	9
normalized size	1	1.	1.	1.	1.12	2.25	0.88	1.12
time (sec)	N/A	0.003	0.001	0.002	0.933	1.862	0.06	1.071

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	19	8	9	23	8	9
normalized size	1	1.	1.9	0.8	0.9	2.3	0.8	0.9
time (sec)	N/A	0.027	0.02	0.004	0.923	1.89	2.086	1.081

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	16	13	20	46	34	20
normalized size	1	1.	0.7	0.57	0.87	2.	1.48	0.87
time (sec)	N/A	0.004	0.004	0.003	0.925	1.819	0.848	1.1

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	23	68	17	26
normalized size	1	1.	1.92	0.77	1.77	5.23	1.31	2.
time (sec)	N/A	0.003	0.004	0.	1.418	1.883	0.115	1.087

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	14	18	50	15	18
normalized size	1	1.	1.	0.78	1.	2.78	0.83	1.
time (sec)	N/A	0.02	0.007	0.004	1.412	1.949	0.107	1.093

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	20	0	186	22	26
normalized size	1	1.	1.	0.65	0.	6.	0.71	0.84
time (sec)	N/A	0.035	0.008	0.007	0.	1.932	0.151	1.085

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	30	7	11
normalized size	1	1.	1.	1.12	1.38	3.75	0.88	1.38
time (sec)	N/A	0.034	0.005	0.001	0.932	1.947	0.096	1.082

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	27	8	16
normalized size	1	1.	1.	1.08	1.33	2.25	0.67	1.33
time (sec)	N/A	0.006	0.004	0.01	0.931	1.967	0.087	1.08

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	46	15	22
normalized size	1	1.	1.	0.85	1.1	2.3	0.75	1.1
time (sec)	N/A	0.019	0.002	0.	0.933	1.99	0.548	1.073

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	260	41	54	124	46	54
normalized size	1	1.	5.31	0.84	1.1	2.53	0.94	1.1
time (sec)	N/A	0.036	0.109	0.	1.407	1.843	0.159	1.096

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	142	41	0	0	54	0
normalized size	1	1.	1.23	0.36	0.	0.	0.47	0.
time (sec)	N/A	0.056	0.293	0.089	0.	0.	5.489	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	55	20	22
normalized size	1	1.	1.	0.77	1.	2.5	0.91	1.
time (sec)	N/A	0.01	0.014	0.01	0.931	1.83	0.295	1.079

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	16	13	20	46	34	20
normalized size	1	1.	0.7	0.57	0.87	2.	1.48	0.87
time (sec)	N/A	0.004	0.003	0.	0.927	1.79	0.84	1.092

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	92	32	76	0	32
normalized size	1	1.	1.	2.88	1.	2.38	0.	1.
time (sec)	N/A	0.014	0.013	0.	0.93	1.731	0.	1.076

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	71	75	108	151	0	82
normalized size	1	1.	1.61	1.7	2.45	3.43	0.	1.86
time (sec)	N/A	0.016	0.036	0.01	1.406	1.831	0.	1.105

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	59	146	105	39
normalized size	1	1.	0.74	0.86	1.69	4.17	3.	1.11
time (sec)	N/A	0.009	0.024	0.004	1.425	1.835	2.606	1.083

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	41	70	153	140	190	111
normalized size	1	1.	0.55	0.93	2.04	1.87	2.53	1.48
time (sec)	N/A	0.013	0.019	0.003	0.943	1.796	11.904	1.278

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	59	146	105	39
normalized size	1	1.	0.74	0.86	1.69	4.17	3.	1.11
time (sec)	N/A	0.008	0.004	0.	1.419	1.87	2.568	1.099

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	134	262	0	285	0	0
normalized size	1	1.	2.63	5.14	0.	5.59	0.	0.
time (sec)	N/A	0.029	0.029	0.022	0.	2.025	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	38	10	14
normalized size	1	1.	1.	0.79	1.	2.71	0.71	1.
time (sec)	N/A	0.006	0.002	0.	0.925	1.985	0.057	1.137

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	57	99	149	0	635	0	0
normalized size	1	1.16	2.02	3.04	0.	12.96	0.	0.
time (sec)	N/A	0.085	0.106	0.096	0.	2.998	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	12	28	3	41
normalized size	1	1.	0.67	0.56	1.33	3.11	0.33	4.56
time (sec)	N/A	0.008	0.004	0.	0.918	1.945	0.179	1.109

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	7	7	8	18	5	8
normalized size	1	1.	0.64	0.64	0.73	1.64	0.45	0.73
time (sec)	N/A	0.007	0.001	0.	0.929	1.931	0.074	1.092

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	11	18	5	26
normalized size	1	1.	1.	1.	1.22	2.	0.56	2.89
time (sec)	N/A	0.026	0.023	0.002	0.942	1.746	0.085	1.104

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	8	15	5	8
normalized size	1	1.	1.	1.	1.14	2.14	0.71	1.14
time (sec)	N/A	0.032	0.005	0.001	0.938	1.975	0.081	1.067

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	12	30	8	12
normalized size	1	1.	1.	0.73	1.09	2.73	0.73	1.09
time (sec)	N/A	0.002	0.002	0.	0.927	2.029	0.231	1.085

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	8	3	9	2	3
normalized size	1	1.	1.	4.	1.5	4.5	1.	1.5
time (sec)	N/A	0.01	0.004	0.002	1.013	1.869	0.736	1.076

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	40	35	46	112	41	47
normalized size	1	1.	0.98	0.85	1.12	2.73	1.	1.15
time (sec)	N/A	0.022	0.009	0.	1.405	1.981	0.122	1.072

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	73	75	66	88	230	83	90
normalized size	1	1.55	1.6	1.4	1.87	4.89	1.77	1.91
time (sec)	N/A	0.105	0.013	0.004	1.417	1.842	0.233	1.069

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	44	53	62	70	55
normalized size	1	1.	1.	2.1	2.52	2.95	3.33	2.62
time (sec)	N/A	0.009	0.004	0.005	0.931	1.813	0.271	1.076

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	35	12	18
normalized size	1	1.	1.	0.82	1.06	2.06	0.71	1.06
time (sec)	N/A	0.004	0.001	0.	0.924	2.002	0.08	1.091

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	29	34	45	68	32	51
normalized size	1	1.	0.72	0.85	1.12	1.7	0.8	1.27
time (sec)	N/A	0.024	0.012	0.	1.402	2.045	0.321	1.075

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	16	5	9
normalized size	1	1.	1.	1.14	1.29	2.29	0.71	1.29
time (sec)	N/A	0.001	0.001	0.002	0.925	2.021	0.071	1.084

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	22	5	11
normalized size	1	1.	1.	1.12	1.38	2.75	0.62	1.38
time (sec)	N/A	0.044	0.013	0.023	0.931	1.956	0.082	1.076

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	22	15	4
normalized size	1	1.	1.	1.33	1.33	7.33	5.	1.33
time (sec)	N/A	0.02	0.011	0.002	1.405	1.924	0.123	1.082

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	A	A	A	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	2	2	0	9	4	23	2	4
normalized size	1	1.	0.	4.5	2.	11.5	1.	2.
time (sec)	N/A	0.002	0.002	0.	1.022	1.801	0.439	1.12

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	46	15	19
normalized size	1	1.	1.	1.07	1.36	3.29	1.07	1.36
time (sec)	N/A	0.021	0.005	0.011	0.938	1.856	0.166	1.069

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	13	16	15	32	8	15
normalized size	1	1.	0.76	0.94	0.88	1.88	0.47	0.88
time (sec)	N/A	0.031	0.01	0.001	0.932	1.878	0.081	1.099

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	29	29	32	66	26	32
normalized size	1	1.	0.76	0.76	0.84	1.74	0.68	0.84
time (sec)	N/A	0.033	0.026	0.002	0.937	1.921	0.088	1.066

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	26	7	9
normalized size	1	1.	1.	1.14	1.29	3.71	1.	1.29
time (sec)	N/A	0.009	0.002	0.	0.935	1.905	0.167	1.061

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	55	20	22
normalized size	1	1.	1.	0.77	1.	2.5	0.91	1.
time (sec)	N/A	0.012	0.009	0.	0.941	2.054	0.293	1.07

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	26	7	9
normalized size	1	1.	1.	1.14	1.29	3.71	1.	1.29
time (sec)	N/A	0.009	0.002	0.	0.932	2.008	0.161	1.06

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	23	61	22	30
normalized size	1	1.	1.	0.82	0.82	2.18	0.79	1.07
time (sec)	N/A	0.01	0.001	0.002	0.926	1.817	0.095	1.059

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	51	8	12
normalized size	1	1.	1.	0.91	1.09	4.64	0.73	1.09
time (sec)	N/A	0.012	0.003	0.013	0.917	2.014	0.536	1.048

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	22	15	4
normalized size	1	1.	1.	1.33	1.33	7.33	5.	1.33
time (sec)	N/A	0.019	0.008	0.	1.417	1.798	0.124	1.04

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	0	26	3	4
normalized size	1	1.	1.	1.33	0.	8.67	1.	1.33
time (sec)	N/A	0.047	0.055	0.005	0.	2.154	0.418	1.064

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	21	72	51	12	28
normalized size	1	1.	1.	1.31	4.5	3.19	0.75	1.75
time (sec)	N/A	0.041	0.023	0.023	1.412	2.275	0.132	1.09

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	127	262	0	284	0	0
normalized size	1	1.	2.4	4.94	0.	5.36	0.	0.
time (sec)	N/A	0.065	0.055	0.006	0.	2.626	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	35	127	93	177	0	112
normalized size	1	1.	2.19	7.94	5.81	11.06	0.	7.
time (sec)	N/A	0.086	0.09	0.036	1.417	2.736	0.	1.117

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	35	121	92	76	422	111
normalized size	1	1.	2.19	7.56	5.75	4.75	26.38	6.94
time (sec)	N/A	0.124	0.018	0.007	1.41	2.162	1.301	1.097

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	35	121	88	76	422	107
normalized size	1	1.	2.19	7.56	5.5	4.75	26.38	6.69
time (sec)	N/A	0.019	0.013	0.005	1.41	2.432	1.275	1.076

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	59	146	105	39
normalized size	1	1.	0.74	0.86	1.69	4.17	3.	1.11
time (sec)	N/A	0.008	0.021	0.	1.413	2.289	2.558	1.065

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	13	16	74	19	16
normalized size	1	1.	1.29	0.93	1.14	5.29	1.36	1.14
time (sec)	N/A	0.009	0.007	0.007	1.418	2.252	0.061	1.066

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	34	8	15
normalized size	1	1.	1.	0.92	1.15	2.62	0.62	1.15
time (sec)	N/A	0.005	0.004	0.001	1.403	2.098	0.081	1.06

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	7	8	15	5	8
normalized size	1	1.	1.	1.	1.14	2.14	0.71	1.14
time (sec)	N/A	0.03	0.004	0.	0.989	2.099	0.08	1.055

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	20	24	31	55	20	41
normalized size	1	1.	0.83	1.	1.29	2.29	0.83	1.71
time (sec)	N/A	0.357	0.069	0.005	1.547	2.173	0.108	1.062

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	11	18	7	11
normalized size	1	1.	1.	1.	1.22	2.	0.78	1.22
time (sec)	N/A	0.003	0.003	0.	0.936	2.05	0.068	1.064

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	32	55	19	26
normalized size	1	1.	1.	0.8	1.28	2.2	0.76	1.04
time (sec)	N/A	0.045	0.002	0.002	0.927	2.145	0.085	1.051

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	23	5	8
normalized size	1	1.	1.	0.88	1.	2.88	0.62	1.
time (sec)	N/A	0.003	0.003	0.002	1.409	2.189	0.087	1.065

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	36	10	16
normalized size	1	1.	1.	0.81	1.	2.25	0.62	1.
time (sec)	N/A	0.008	0.003	0.002	1.408	2.053	0.09	1.059

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	17	22	62	36	22
normalized size	1	1.	1.	1.21	1.57	4.43	2.57	1.57
time (sec)	N/A	0.015	0.003	0.01	1.407	2.281	0.384	1.053

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	59	146	105	39
normalized size	1	1.	0.74	0.86	1.69	4.17	3.	1.11
time (sec)	N/A	0.011	0.003	0.	1.413	2.105	2.556	1.062

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	32	16	20	103	34	28
normalized size	1	1.	1.88	0.94	1.18	6.06	2.	1.65
time (sec)	N/A	0.005	0.007	0.004	1.41	2.169	0.526	1.071

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	14	15	46	15	15
normalized size	1	1.	0.74	0.74	0.79	2.42	0.79	0.79
time (sec)	N/A	0.008	0.014	0.	0.928	2.231	0.28	1.046

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	11	2	4
normalized size	1	1.	1.	1.5	1.5	5.5	1.	2.
time (sec)	N/A	0.	0.	0.	0.93	2.241	0.05	1.057

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	73	31	346	271	0	45
normalized size	1	1.	1.62	0.69	7.69	6.02	0.	1.
time (sec)	N/A	0.123	0.181	0.142	1.78	2.618	0.	1.104

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	36	10	22
normalized size	1	1.	1.	0.79	1.	2.57	0.71	1.57
time (sec)	N/A	0.006	0.002	0.002	0.928	2.206	0.061	1.065

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	14	11	15	31	14	15
normalized size	1	1.	0.82	0.65	0.88	1.82	0.82	0.88
time (sec)	N/A	0.003	0.003	0.003	0.924	1.775	0.203	1.059

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	25	20	26	77	0	36
normalized size	1	1.	1.09	0.87	1.13	3.35	0.	1.57
time (sec)	N/A	0.01	0.005	0.004	1.409	1.805	0.	1.059

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	38	5	8
normalized size	1	1.	1.	0.88	1.	4.75	0.62	1.
time (sec)	N/A	0.013	0.001	0.002	0.919	2.072	0.058	1.059

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	6	7	19	5	7
normalized size	1	1.	1.	1.	1.17	3.17	0.83	1.17
time (sec)	N/A	0.016	0.004	0.	0.934	2.045	0.072	1.063

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	27	8	14
normalized size	1	1.	1.	0.92	1.17	2.25	0.67	1.17
time (sec)	N/A	0.026	0.008	0.002	0.93	2.125	0.08	1.055

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	14	30	7	11
normalized size	1	1.	0.67	0.75	1.17	2.5	0.58	0.92
time (sec)	N/A	0.009	0.008	0.	0.927	2.21	0.354	1.055

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	19	7	8
normalized size	1	1.	1.	0.88	1.	2.38	0.88	1.
time (sec)	N/A	0.012	0.004	0.	0.927	1.911	0.064	1.06

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	35	12	18
normalized size	1	1.	1.	0.82	1.06	2.06	0.71	1.06
time (sec)	N/A	0.004	0.001	0.	0.936	1.806	0.082	1.059

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	20	5	8
normalized size	1	1.	1.	0.88	1.	2.5	0.62	1.
time (sec)	N/A	0.007	0.001	0.	0.923	1.909	0.054	1.068

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	27	21	30	68	0	27
normalized size	1	1.	1.12	0.88	1.25	2.83	0.	1.12
time (sec)	N/A	0.009	0.032	0.006	1.415	1.802	0.	1.077

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	14	18	72	15	18
normalized size	1	1.	1.	0.7	0.9	3.6	0.75	0.9
time (sec)	N/A	0.023	0.005	0.	1.421	1.922	0.106	1.066

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	59	146	105	39
normalized size	1	1.	0.74	0.86	1.69	4.17	3.	1.11
time (sec)	N/A	0.009	0.003	0.	1.407	1.876	2.557	1.078

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	15	19	49	24	19
normalized size	1	1.	0.9	0.75	0.95	2.45	1.2	0.95
time (sec)	N/A	0.024	0.008	0.003	1.416	1.919	0.111	1.048

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	35	103	31	35
normalized size	1	1.	1.	0.82	1.06	3.12	0.94	1.06
time (sec)	N/A	0.011	0.012	0.	1.412	1.894	0.126	1.049

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	26	55	15	26
normalized size	1	1.	1.	0.95	1.24	2.62	0.71	1.24
time (sec)	N/A	0.016	0.	0.002	0.932	1.74	0.052	1.046

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	32	74	19	31
normalized size	1	1.	1.	0.96	1.23	2.85	0.73	1.19
time (sec)	N/A	0.019	0.	0.002	0.936	1.789	0.062	1.053

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	26	34	85	22	34
normalized size	1	1.	1.	0.96	1.26	3.15	0.81	1.26
time (sec)	N/A	0.023	0.	0.	0.928	1.865	0.055	1.053

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	31	41	104	26	39
normalized size	1	1.	1.	0.97	1.28	3.25	0.81	1.22
time (sec)	N/A	0.026	0.	0.001	0.933	1.813	0.055	1.047

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	25	24	31	80	20	31
normalized size	1	1.	1.04	1.	1.29	3.33	0.83	1.29
time (sec)	N/A	0.027	0.	0.	0.931	1.875	0.053	1.242

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	30	29	38	99	24	36
normalized size	1	1.	1.03	1.	1.31	3.41	0.83	1.24
time (sec)	N/A	0.032	0.	0.	0.929	1.817	0.054	1.078

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	18	23	34	15	23
normalized size	1	1.	1.	0.95	1.21	1.79	0.79	1.21
time (sec)	N/A	0.009	0.	0.	0.93	1.807	0.051	1.049

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	30	42	19	28
normalized size	1	1.	1.	0.96	1.25	1.75	0.79	1.17
time (sec)	N/A	0.011	0.	0.002	0.937	1.817	0.053	1.05

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	85	22	31
normalized size	1	1.	1.	0.96	1.24	3.4	0.88	1.24
time (sec)	N/A	0.013	0.	0.002	0.931	1.881	0.053	1.06

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	29	38	101	26	36
normalized size	1	1.	1.	0.97	1.27	3.37	0.87	1.2
time (sec)	N/A	0.015	0.	0.	0.929	1.706	0.055	1.07

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	28	27	35	53	24	34
normalized size	1	1.	1.04	1.	1.3	1.96	0.89	1.26
time (sec)	N/A	0.006	0.	0.001	0.926	1.812	0.054	1.057

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [50] had the largest ratio of [0.8571]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.	4	0.5
2	A	3	2	1.	11	0.182
3	A	2	1	1.	11	0.091
4	A	1	1	1.	2	0.5
5	A	1	1	1.	7	0.143
6	A	2	2	1.	7	0.286
7	A	1	1	1.	11	0.091
8	A	1	1	1.	6	0.167
9	A	2	2	1.	7	0.286
10	A	2	2	1.	4	0.5
11	A	1	1	1.	6	0.167
12	A	3	2	1.	6	0.333
13	A	4	2	1.	16	0.125
14	A	5	3	1.	7	0.429
15	A	3	1	1.	14	0.071

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	2	2	1.	5	0.4
17	A	1	1	1.	7	0.143
18	A	1	1	1.	11	0.091
19	A	2	2	1.	11	0.182
20	A	1	1	1.	5	0.2
21	A	1	1	1.	6	0.167
22	A	2	2	1.	9	0.222
23	A	3	3	1.	14	0.214
24	A	2	1	1.	9	0.111
25	A	3	3	1.	7	0.429
26	A	2	2	1.	15	0.133
27	A	2	2	1.	17	0.118
28	A	3	3	1.	13	0.231
29	A	1	1	1.	5	0.2
30	A	3	3	1.	8	0.375
31	A	7	7	1.	11	0.636
32	A	3	2	1.	12	0.167
33	A	3	3	1.	6	0.5
34	A	2	1	1.	9	0.111
35	A	4	3	1.	13	0.231
36	A	4	4	1.	15	0.267
37	A	3	2	1.	15	0.133
38	A	6	3	1.	13	0.231
39	A	3	2	1.	15	0.133
40	A	5	5	1.	29	0.172
41	A	2	2	1.	4	0.5
42	A	6	6	1.16	19	0.316
43	A	1	1	1.	6	0.167
44	A	2	2	1.	5	0.4
45	A	1	1	1.	10	0.1
46	A	5	3	1.	13	0.231
47	A	1	1	1.	5	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
48	A	1	1	1.	7	0.143
49	A	6	6	1.	9	0.667
50	A	10	6	1.55	7	0.857
51	A	1	1	1.	27	0.037
52	A	1	1	1.	4	0.25
53	A	4	3	1.	6	0.5
54	A	2	2	1.	10	0.2
55	A	7	4	1.	9	0.444
56	A	2	1	1.	12	0.083
57	A	1	1	1.	4	0.25
58	A	6	4	1.	7	0.571
59	A	4	3	1.	11	0.273
60	A	6	3	1.	9	0.333
61	A	2	2	1.	4	0.5
62	A	3	3	1.	6	0.5
63	A	2	2	1.	4	0.5
64	A	2	2	1.	6	0.333
65	A	2	1	1.	9	0.111
66	A	2	1	1.	12	0.083
67	A	2	2	1.	20	0.1
68	A	2	2	1.	10	0.2
69	A	6	6	1.	30	0.2
70	A	5	5	1.	39	0.128
71	A	6	5	1.	45	0.111
72	A	4	4	1.	31	0.129
73	A	3	2	1.	15	0.133
74	A	3	2	1.	4	0.5
75	A	3	2	1.	11	0.182
76	A	5	3	1.	13	0.231
77	A	10	6	1.	33	0.182
78	A	1	1	1.	7	0.143
79	A	5	5	1.	8	0.625

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
80	A	2	2	1.	9	0.222
81	A	3	3	1.	11	0.273
82	A	3	3	1.	8	0.375
83	A	3	2	1.	15	0.133
84	A	2	2	1.	16	0.125
85	A	1	1	1.	6	0.167
86	A	1	1	1.	3	0.333
87	A	4	2	1.	17	0.118
88	A	2	2	1.	4	0.5
89	A	2	1	1.	11	0.091
90	A	3	3	1.	14	0.214
91	A	2	2	1.	7	0.286
92	A	2	2	1.	11	0.182
93	A	3	2	1.	13	0.154
94	A	1	1	1.	8	0.125
95	A	2	2	1.	7	0.286
96	A	1	1	1.	4	0.25
97	A	2	2	1.	5	0.4
98	A	3	3	1.	17	0.176
99	A	3	3	1.	16	0.188
100	A	3	2	1.	15	0.133
101	A	3	3	1.	15	0.2
102	A	3	3	1.	8	0.375
103	A	1	1	1.	20	0.05
104	A	1	1	1.	25	0.04
105	A	1	1	1.	26	0.038
106	A	1	1	1.	31	0.032
107	A	1	1	1.	24	0.042
108	A	1	1	1.	29	0.034
109	A	1	1	1.	18	0.056
110	A	1	1	1.	23	0.043
111	A	1	1	1.	24	0.042

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
112	A	1	1	1.	29	0.034
113	A	1	1	1.	27	0.037

Chapter 3

Listing of integrals

3.1 $\int \cot^4(x)\,dx$

Optimal. Leaf size=12

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

```
[Out] x + Cot[x] - Cot[x]^3/3
```

Rubi [A] time = 0.0099821, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 8}

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4,x]
```

```
[Out] x + Cot[x] - Cot[x]^3/3
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \cot^4(x)\,dx &= -\frac{1}{3}\cot^3(x) - \int \cot^2(x)\,dx \\ &= \cot(x) - \frac{\cot^3(x)}{3} + \int 1\,dx \\ &= x + \cot(x) - \frac{\cot^3(x)}{3} \end{aligned}$$

Mathematica [A] time = 0.0030371, size = 18, normalized size = 1.5

$$x + \frac{4\cot(x)}{3} - \frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4,x]
```

```
[Out] x + (4*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

Maple [A] time = 0., size = 14, normalized size = 1.2

$$-\frac{(\cot(x))^3}{3} + \cot(x) - \frac{\pi}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4,x)
```

```
[Out] -1/3*cot(x)^3+cot(x)-1/2*Pi+x
```

Maxima [A] time = 1.41448, size = 22, normalized size = 1.83

$$x + \frac{3\,\tan\left(x\right)^{2} - 1}{3\,\tan\left(x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="maxima")
```

```
[Out] x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3
```

Fricas [B] time = 1.65477, size = 126, normalized size = 10.5

$$\frac{4\,\cos\left(2\,x\right)^{2} + 3\,{\left(x\cos\left(2\,x\right) - x\right)}\sin\left(2\,x\right) + 2\,\cos\left(2\,x\right) - 2}{3\,{\left(\cos\left(2\,x\right) - 1\right)}\sin\left(2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)/((cos(2*x
) - 1)*sin(2*x))
```

Sympy [A] time = 0.065953, size = 19, normalized size = 1.58

$$x + \frac{\cos{\left(x \right)}}{\sin{\left(x \right)}} - \frac{\cos^{3}{\left(x \right)}}{3 \sin^{3}{\left(x \right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4,x)
```

```
[Out] x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)
```

Giac [B] time = 1.06264, size = 46, normalized size = 3.83

$$\frac{1}{24}\,\tan\left(\frac{1}{2}\,x\right)^{3} + x + \frac{15\,\tan\left(\frac{1}{2}\,x\right)^{2} - 1}{24\,\tan\left(\frac{1}{2}\,x\right)^{3}} - \frac{5}{8}\,\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="giac")
```

```
[Out] 1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 5/8*tan(1
/2*x)
```

3.2 $\int \frac{1}{x^4(1+x^2)}\,dx$

Optimal. Leaf size=13

$$-\frac{1}{3x^3}+\frac{1}{x}+\tan^{-1}(x)$$

```
[Out] -1/(3*x^3) + x^(-1) + ArcTan[x]
```

Rubi [A] time = 0.0048847, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {325, 203}

$$-\frac{1}{3x^3}+\frac{1}{x}+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(1 + x^2)),x]
```

```
[Out] -1/(3*x^3) + x^(-1) + ArcTan[x]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x^4(1+x^2)}\,dx = -\frac{1}{3x^3} - \int \frac{1}{x^2(1+x^2)}\,dx$$
$$= -\frac{1}{3x^3} + \frac{1}{x} + \int \frac{1}{1+x^2}\,dx$$
$$= -\frac{1}{3x^3} + \frac{1}{x} + \tan^{-1}(x)$$

Mathematica [A] time = 0.0038972, size = 13, normalized size = 1.

$$-\frac{1}{3x^3} + \frac{1}{x} + \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(1 + x^2)),x]
```

```
[Out] -1/(3*x^3) + x^(-1) + ArcTan[x]
```

Maple [A] time = 0.005, size = 12, normalized size = 0.9

$$-\frac{1}{3\,x^3} + x^{-1} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(x^2+1),x)
```

```
[Out] -1/3/x^3+1/x+arctan(x)
```

Maxima [A] time = 1.41326, size = 20, normalized size = 1.54

$$\frac{3\,x^2 - 1}{3\,x^3} + \arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/3*(3*x^2 - 1)/x^3 + arctan(x)
```

Fricas [A] time = 1.50589, size = 53, normalized size = 4.08

$$\frac{3\,x^3\arctan\left(x\right)+3\,x^2-1}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/3*(3*x^3*arctan(x) + 3*x^2 - 1)/x^3
```

Sympy [A] time = 0.105203, size = 14, normalized size = 1.08

$$\operatorname{atan}\left(x\right)+\frac{3x^2-1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(x**2+1),x)
```

```
[Out] atan(x) + (3*x**2 - 1)/(3*x**3)
```

Giac [A] time = 1.07896, size = 20, normalized size = 1.54

$$\frac{3\,x^2-1}{3\,x^3}+\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/3*(3*x^2 - 1)/x^3 + arctan(x)
```

3.3 $\int \frac{x+x^2}{\sqrt{x}} \, dx$

Optimal. Leaf size=19

$$\frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3}$$

```
[Out] (2*x^(3/2))/3 + (2*x^(5/2))/5
```

Rubi [A] time = 0.002675, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{2x^{5/2}}{5} + \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(x + x^2)/Sqrt[x],x]
```

```
[Out] (2*x^(3/2))/3 + (2*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{x+x^2}{\sqrt{x}} \, dx &= \int \left(\sqrt{x}+x^{3/2}\right) dx \\ &= \frac{2x^{3/2}}{3} + \frac{2x^{5/2}}{5}\end{aligned}$$

Mathematica [A] time = 0.0029317, size = 14, normalized size = 0.74

$$\frac{2}{15} x^{3/2} (3x+5)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + x^2)/Sqrt[x],x]
```

```
[Out] (2*x^(3/2)*(5 + 3*x))/15
```

Maple [A] time = 0.001, size = 11, normalized size = 0.6

$$\frac{10+6x}{15}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)/x^(1/2),x)
```

```
[Out] 2/15*x^(3/2)*(5+3*x)
```

Maxima [A] time = 0.935709, size = 15, normalized size = 0.79

$$\frac{2}{5}x^{\frac{5}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2) + 2/3*x^(3/2)
```

Fricas [A] time = 1.49986, size = 38, normalized size = 2.

$$\frac{2}{15}\left(3x^2+5x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^2 + 5*x)*sqrt(x)
```

Sympy [A] time = 0.223083, size = 15, normalized size = 0.79

$$\frac{2x^{\frac{5}{2}}}{5} + \frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)/x**(1/2),x)
```

```
[Out] 2*x**(5/2)/5 + 2*x**(3/2)/3
```

Giac [A] time = 1.09026, size = 15, normalized size = 0.79

$$\frac{2}{5}\,x^{\frac{5}{2}} + \frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2) + 2/3*x^(3/2)
```

3.4 $\int \cos(x)\,dx$

Optimal. Leaf size=2

$$\sin(x)$$

```
[Out] Sin[x]
```

Rubi [A] time = 0.0020616, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2637}

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x],x]
```

```
[Out] Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(x)\,dx = \sin(x)$$

Mathematica [A] time = 0.0009164, size = 2, normalized size = 1.

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x],x]
```

```
[Out] Sin[x]
```

Maple [A] time = 0., size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x),x)
```

```
[Out] sin(x)
```

Maxima [A] time = 0.926673, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="maxima")
```

```
[Out] sin(x)
```

Fricas [A] time = 1.58175, size = 11, normalized size = 5.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="fricas")
```

```
[Out] sin(x)
```

Sympy [A] time = 0.05319, size = 2, normalized size = 1.

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x)
```

```
[Out] sin(x)
```

Giac [A] time = 1.0934, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="giac")
```

```
[Out] sin(x)
```

3.5 $\int e^{x^2} x \, dx$

Optimal. Leaf size=9

$$\frac{e^{x^2}}{2}$$

[Out] E^x^2/2

Rubi [A] time = 0.0060194, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2209}

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

[In] Int[E^x^2*x,x]

[Out] E^x^2/2

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

Mathematica [A] time = 0.0011679, size = 9, normalized size = 1.

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*x,x]
```

```
[Out] E^x^2/2
```

Maple [A] time = 0., size = 7, normalized size = 0.8

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*x,x)
```

```
[Out] 1/2*exp(x^2)
```

Maxima [A] time = 0.926878, size = 8, normalized size = 0.89

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*e^(x^2)
```

Fricas [A] time = 1.63268, size = 18, normalized size = 2.

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*e^(x^2)
```

Sympy [A] time = 0.078546, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x,x)
```

```
[Out] exp(x**2)/2
```

Giac [A] time = 1.07871, size = 8, normalized size = 0.89

$$\frac{1}{2} e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="giac")
```

```
[Out] 1/2*e^(x^2)
```

3.6 $\int \sec^2(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^2(x)}{2}$$

```
[Out] Sec[x]^2/2
```

Rubi [A] time = 0.0106558, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 30}

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \sec^2(x)\tan(x)\,dx &= \text{Subst}\left(\int x\,dx, x, \sec(x)\right) \\ &= \frac{\sec^2(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.0041911, size = 8, normalized size = 1.

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

Maple [A] time = 0.005, size = 7, normalized size = 0.9

$$\frac{(\sec(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*tan(x),x)
```

```
[Out] 1/2*sec(x)^2
```

Maxima [A] time = 0.935947, size = 8, normalized size = 1.

$$\frac{1}{2}\tan(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="maxima")
```

```
[Out] 1/2*tan(x)^2
```

Fricas [A] time = 1.59947, size = 19, normalized size = 2.38

$$\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="fricas")
```

```
[Out] 1/2/cos(x)^2
```

Sympy [A] time = 0.066424, size = 7, normalized size = 0.88

$$\frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*tan(x),x)
```

```
[Out] 1/(2*cos(x)**2)
```

Giac [A] time = 1.06617, size = 8, normalized size = 1.

$$\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="giac")
```

```
[Out] 1/2/cos(x)^2
```

3.7 $\int x\sqrt{1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] (1 + x^2)^(3/2)/3
```

Rubi [A] time = 0.0016729, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{1+x^2}\,dx=\frac{1}{3}\left(1+x^2\right)^{3/2}$$

Mathematica [A] time = 0.0019712, size = 13, normalized size = 1.

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3
```

Maple [A] time = 0.002, size = 10, normalized size = 0.8

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^(1/2),x)
```

```
[Out] 1/3*(x^2+1)^(3/2)
```

Maxima [A] time = 0.925371, size = 12, normalized size = 0.92

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

Fricas [A] time = 1.56769, size = 28, normalized size = 2.15

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

Sympy [B] time = 0.190332, size = 22, normalized size = 1.69

$$\frac{x^2\sqrt{x^2+1}}{3}+\frac{\sqrt{x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**(1/2),x)
```

```
[Out] x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

Giac [A] time = 1.0848, size = 12, normalized size = 0.92

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

3.8 $\int e^x \sin(x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

```
[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rubi [A] time = 0.0065227, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4432}

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[x],x]
```

```
[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x \sin(x)\,dx = -\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)$$

Mathematica [A] time = 0.013703, size = 14, normalized size = 0.74

$$\frac{1}{2}e^x(\sin(x) - \cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[x],x]
```

```
[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

Maple [A] time = 0., size = 14, normalized size = 0.7

$$-\frac{e^{x} \cos (x)}{2}+\frac{e^{x} \sin (x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x)
```

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Maxima [A] time = 0.93405, size = 15, normalized size = 0.79

$$-\frac{1}{2} (\cos (x)-\sin (x)) e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Fricas [A] time = 1.64102, size = 46, normalized size = 2.42

$$-\frac{1}{2} \cos (x) e^{x}+\frac{1}{2} e^{x} \sin (x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A] time = 0.298395, size = 15, normalized size = 0.79

$$\frac{e^x \sin(x)}{2} - \frac{e^x \cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x)
```

```
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

Giac [A] time = 1.0982, size = 15, normalized size = 0.79

$$-\frac{1}{2}(\cos(x) - \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*(cos(x) - sin(x))*e^x
```

3.9 $\int \cot(x)\csc^3(x)\,dx$

Optimal. Leaf size=8

$$-\frac{1}{3}\csc^3(x)$$

```
[Out] -Csc[x]^3/3
```

Rubi [A] time = 0.0124362, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 30}

$$-\frac{1}{3}\csc^3(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Csc[x]^3,x]
```

```
[Out] -Csc[x]^3/3
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot(x)\csc^3(x)\,dx &= -\text{Subst}\left(\int x^2\,dx, x, \csc(x)\right)\\ &= -\frac{1}{3}\csc^3(x)\end{aligned}$$

Mathematica [A] time = 0.0040954, size = 8, normalized size = 1.

$$-\frac{1}{3}\csc^3(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Csc[x]^3,x]
```

```
[Out] -Csc[x]^3/3
```

Maple [A] time = 0.013, size = 7, normalized size = 0.9

$$-\frac{1}{3\,(\sin(x))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*csc(x)^2/sin(x)^2,x)
```

```
[Out] -1/3/sin(x)^3
```

Maxima [A] time = 0.93987, size = 8, normalized size = 1.

$$-\frac{1}{3\,\sin(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/3/sin(x)^3
```

Fricas [B] time = 1.60745, size = 39, normalized size = 4.88

$$\frac{1}{3\left(\cos(x)^2-1\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/3/((cos(x)^2 - 1)*sin(x))
```

Sympy [A] time = 0.064042, size = 8, normalized size = 1.

$$-\frac{1}{3\sin^{3}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)**2/sin(x)**2,x)
```

```
[Out] -1/(3*sin(x)**3)
```

Giac [A] time = 1.07199, size = 8, normalized size = 1.

$$-\frac{1}{3\,\sin(x)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*csc(x)^2/sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/3/sin(x)^3
```

3.10 $\int \sin(e^x)\, dx$

Optimal. Leaf size=4

$$\text{Si}(e^x)$$

```
[Out] SinIntegral[E^x]
```

Rubi [A] time = 0.0117071, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2282, 3299}

$$\text{Si}(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[E^x],x]
```

```
[Out] SinIntegral[E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(e^x)\, dx &= \text{Subst}\left(\int \frac{\sin(x)}{x}\, dx, x, e^x\right)\\ &= \text{Si}(e^x)\end{aligned}$$

Mathematica [A] time = 0.005286, size = 4, normalized size = 1.

$$\mathrm{Si}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[E^x],x]
```

```
[Out] SinIntegral[E^x]
```

Maple [A] time = 0.004, size = 4, normalized size = 1.

$$\mathrm{Si}\left(\mathrm{e}^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(exp(x)),x)
```

```
[Out] Si(exp(x))
```

Maxima [C] time = 1.06478, size = 20, normalized size = 5.

$$-\frac{1}{2} i\,\mathrm{Ei}\left(i\,e^x\right) + \frac{1}{2} i\,\mathrm{Ei}\left(-i\,e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(exp(x)),x, algorithm="maxima")
```

```
[Out] -1/2*I*Ei(I*e^x) + 1/2*I*Ei(-I*e^x)
```

Fricas [A] time = 1.9311, size = 26, normalized size = 6.5

$$\mathrm{Si}\left(e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(exp(x)),x, algorithm="fricas")
```

```
[Out] sin_integral(e^x)
```

Sympy [A] time = 0.64135, size = 3, normalized size = 0.75

$$\mathrm{Si}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(exp(x)),x)
```

```
[Out] Si(exp(x))
```

Giac [A] time = 1.06027, size = 4, normalized size = 1.

$$\mathrm{Si}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(exp(x)),x, algorithm="giac")
```

```
[Out] sin_integral(e^x)
```

3.11 $\int \frac{\sin(y)}{y}\,dy$

Optimal. Leaf size=2

$$\mathrm{Si}(y)$$

```
[Out] SinIntegral[y]
```

Rubi [A] time = 0.0111784, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3299}

$$\mathrm{Si}(y)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[y]/y,y]
```

```
[Out] SinIntegral[y]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\sin(y)}{y}\,dy = \mathrm{Si}(y)$$

Mathematica [A] time = 0.0132604, size = 2, normalized size = 1.

$$\mathrm{Si}(y)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[y]/y,y]
```

```
[Out] SinIntegral[y]
```

Maple [A] time = 0.001, size = 3, normalized size = 1.5

$$\mathrm{Si}\left(y\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(y)/y,y)
```

```
[Out] Si(y)
```

Maxima [C] time = 1.05461, size = 18, normalized size = 9.

$$-\frac{1}{2} i \,\mathrm{Ei}\left(i y\right) + \frac{1}{2} i \,\mathrm{Ei}\left(-i y\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)/y,y, algorithm="maxima")
```

```
[Out] -1/2*I*Ei(I*y) + 1/2*I*Ei(-I*y)
```

Fricas [A] time = 1.85999, size = 23, normalized size = 11.5

$$\mathrm{Si}\left(y\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)/y,y, algorithm="fricas")
```

```
[Out] sin_integral(y)
```

Sympy [A] time = 0.539575, size = 2, normalized size = 1.

$$\mathrm{Si}\left(y\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)/y,y)
```

```
[Out] Si(y)
```

Giac [A] time = 1.11709, size = 3, normalized size = 1.5

$$\mathrm{Si}\left(y\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)/y,y, algorithm="giac")
```

```
[Out] sin_integral(y)
```

3.12 $\int (e^x + \sin(x)) \, dx$

Optimal. Leaf size=8

$$e^x - \cos(x)$$

```
[Out] E^x - Cos[x]
```

Rubi [A] time = 0.00333, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2194, 2638}

$$e^x - \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x + Sin[x],x]
```

```
[Out] E^x - Cos[x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (e^x + \sin(x)) \, dx &= \int e^x \, dx + \int \sin(x) \, dx \\ &= e^x - \cos(x) \end{aligned}$$

Mathematica [A] time = 0.0028654, size = 8, normalized size = 1.

$$e^x - \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x + Sin[x],x]
```

```
[Out] E^x - Cos[x]
```

Maple [A] time = 0.002, size = 8, normalized size = 1.

$$e^x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)+sin(x),x)
```

```
[Out] exp(x)-cos(x)
```

Maxima [A] time = 0.945783, size = 9, normalized size = 1.12

$$-\cos(x) + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)+sin(x),x, algorithm="maxima")
```

```
[Out] -cos(x) + e^x
```

Fricas [A] time = 1.82012, size = 20, normalized size = 2.5

$$-\cos(x) + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)+sin(x),x, algorithm="fricas")
```

```
[Out] -cos(x) + e^x
```

Sympy [A] time = 0.071743, size = 5, normalized size = 0.62

$$e^x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)+sin(x),x)
```

```
[Out] exp(x) - cos(x)
```

Giac [A] time = 1.06968, size = 9, normalized size = 1.12

$$-\cos(x) + e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)+sin(x),x, algorithm="giac")
```

```
[Out] -cos(x) + e^x
```

3.13 $\int \left(e^{x^2}+2e^{x^2}x^2\right) dx$

Optimal. Leaf size=7

$$e^{x^2}x$$

```
[Out] E^x^2*x
```

Rubi [A] time = 0.0199511, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2204, 2212}

$$e^{x^2}x$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2 + 2*E^x^2*x^2,x]
```

```
[Out] E^x^2*x
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\int \left(e^{x^2}+2e^{x^2}x^2\right)dx &= 2\int e^{x^2}x^2\,dx+\int e^{x^2}\,dx\\ &= e^{x^2}x+\frac{1}{2}\sqrt{\pi}\operatorname{erfi}(x)-\int e^{x^2}\,dx\\ &= e^{x^2}x\end{aligned}$$

Mathematica [A] time = 0.0052759, size = 7, normalized size = 1.

$$e^{x^2}x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2 + 2*E^x^2*x^2,x]
```

```
[Out] E^x^2*x
```

Maple [A] time = 0.002, size = 7, normalized size = 1.

$$e^{x^2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)+2*exp(x^2)*x^2,x)
```

```
[Out] exp(x^2)*x
```

Maxima [A] time = 0.935104, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="maxima")
```

```
[Out] x*e^(x^2)
```

Fricas [A] time = 1.88616, size = 15, normalized size = 2.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="fricas")
```

```
[Out] x*e^(x^2)
```

Sympy [A] time = 0.079765, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)+2*exp(x**2)*x**2,x)
```

```
[Out] x*exp(x**2)
```

Giac [A] time = 1.06987, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)+2*exp(x^2)*x^2,x, algorithm="giac")
```

```
[Out] x*e^(x^2)
```

3.14 $\int (e^x + x)^2 \, dx$

Optimal. Leaf size=28

$$\frac{x^3}{3} + 2e^x x - 2e^x + \frac{e^{2x}}{2}$$

```
[Out] -2*E^x + E^(2*x)/2 + 2*E^x*x + x^3/3
```

Rubi [A] time = 0.0196767, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6742, 2194, 2176}

$$\frac{x^3}{3} + 2e^x x - 2e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x + x)^2,x]
```

```
[Out] -2*E^x + E^(2*x)/2 + 2*E^x*x + x^3/3
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int (e^x + x)^2 \, dx &= \int \left(e^{2x} + 2e^x x + x^2\right) dx \\ &= \frac{x^3}{3} + 2\int e^x x \, dx + \int e^{2x} \, dx \\ &= \frac{e^{2x}}{2} + 2e^x x + \frac{x^3}{3} - 2\int e^x \, dx \\ &= -2e^x + \frac{e^{2x}}{2} + 2e^x x + \frac{x^3}{3}\end{aligned}$$

Mathematica [A] time = 0.0140445, size = 26, normalized size = 0.93

$$\frac{x^3}{3} + \frac{e^{2x}}{2} + e^x(2x - 2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x + x)^2,x]
```

```
[Out] E^(2*x)/2 + x^3/3 + E^x*(-2 + 2*x)
```

Maple [A] time = 0.002, size = 22, normalized size = 0.8

$$\frac{x^3}{3} + \frac{(e^x)^2}{2} + 2\,e^x x - 2\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x)+x)^2,x)
```

```
[Out] 1/3*x^3+1/2*exp(x)^2+2*exp(x)*x-2*exp(x)
```

Maxima [A] time = 0.928451, size = 26, normalized size = 0.93

$$\frac{1}{3}x^3 + 2\,(x - 1)e^x + \frac{1}{2}e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

Fricas [A] time = 1.80161, size = 53, normalized size = 1.89

$$\frac{1}{3}x^3 + 2(x-1)e^x + \frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

Sympy [A] time = 0.08726, size = 20, normalized size = 0.71

$$\frac{x^3}{3} + \frac{(4x-4)e^x}{2} + \frac{e^{2x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))**2,x)
```

```
[Out] x**3/3 + (4*x - 4)*exp(x)/2 + exp(2*x)/2
```

Giac [A] time = 1.09778, size = 26, normalized size = 0.93

$$\frac{1}{3}x^3 + 2(x-1)e^x + \frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 2*(x - 1)*e^x + 1/2*e^(2*x)
```

3.15 $\int \left(2e^x + e^{2x} + x^2\right) dx$

Optimal. Leaf size=22

$$\frac{x^3}{3} + 2e^x + \frac{e^{2x}}{2}$$

[Out] 2*E^x + E^(2*x)/2 + x^3/3

Rubi [A] time = 0.004773, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2194}

$$\frac{x^3}{3} + 2e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

[In] Int[2*E^x + E^(2*x) + x^2,x]

[Out] 2*E^x + E^(2*x)/2 + x^3/3

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int \left(2e^x + e^{2x} + x^2\right) dx &= \frac{x^3}{3} + 2\int e^x \, dx + \int e^{2x} \, dx \\ &= 2e^x + \frac{e^{2x}}{2} + \frac{x^3}{3} \end{aligned}$$

Mathematica [A] time = 0.0023558, size = 22, normalized size = 1.

$$\frac{x^3}{3} + 2e^x + \frac{e^{2x}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[2*E^x + E^(2*x) + x^2,x]
```

```
[Out] 2*E^x + E^(2*x)/2 + x^3/3
```

Maple [A] time = 0.001, size = 17, normalized size = 0.8

$$2\,\mathrm{e}^{x}+\frac{\mathrm{e}^{2\,x}}{2}+\frac{x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*exp(x)+exp(2*x)+x^2,x)
```

```
[Out] 2*exp(x)+1/2*exp(2*x)+1/3*x^3
```

Maxima [A] time = 0.933384, size = 22, normalized size = 1.

$$\frac{1}{3}\,x^{3}+\frac{1}{2}\,e^{(2\,x)}+2\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

Fricas [A] time = 1.87782, size = 42, normalized size = 1.91

$$\frac{1}{3}\,x^{3}+\frac{1}{2}\,e^{(2\,x)}+2\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

Sympy [A] time = 0.082634, size = 15, normalized size = 0.68

$$\frac{x^3}{3} + \frac{e^{2x}}{2} + 2e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)+exp(2*x)+x**2,x)
```

```
[Out] x**3/3 + exp(2*x)/2 + 2*exp(x)
```

Giac [A] time = 1.07362, size = 22, normalized size = 1.

$$\frac{1}{3}\,x^3 + \frac{1}{2}\,e^{(2\,x)} + 2\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)+exp(2*x)+x^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*e^(2*x) + 2*e^x
```

3.16 $\int \cos(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sin^2(x)}{2}$$

```
[Out] Sin[x]^2/2
```

Rubi [A] time = 0.0066878, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2564, 30}

$$\frac{\sin^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[x],x]
```

```
[Out] Sin[x]^2/2
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cos(x)\sin(x)\,dx = \text{Subst}\left(\int x\,dx, x, \sin(x)\right)$$
$$= \frac{\sin^2(x)}{2}$$

Mathematica [A] time = 0.0009839, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x],x]
```

```
[Out] -Cos[x]^2/2
```

Maple [A] time = 0.001, size = 7, normalized size = 0.9

$$\frac{(\sin(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x)
```

```
[Out] 1/2*sin(x)^2
```

Maxima [A] time = 0.912815, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*cos(x)^2
```

Fricas [A] time = 1.9149, size = 20, normalized size = 2.5

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)^2
```

Sympy [A] time = 0.053909, size = 5, normalized size = 0.62

$$\frac{\sin^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x)
```

```
[Out] sin(x)**2/2
```

Giac [A] time = 1.08135, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2
```

3.17 $\int e^{x^2} x \, dx$

Optimal. Leaf size=9

$$\frac{e^{x^2}}{2}$$

```
[Out] E^x^2/2
```

Rubi [A] time = 0.0062455, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2209}

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*x,x]
```

```
[Out] E^x^2/2
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

Mathematica [A] time = 0.000582, size = 9, normalized size = 1.

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*x,x]
```

```
[Out] E^x^2/2
```

Maple [A] time = 0., size = 7, normalized size = 0.8

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*x,x)
```

```
[Out] 1/2*exp(x^2)
```

Maxima [A] time = 0.925133, size = 8, normalized size = 0.89

$$\frac{1}{2} e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*e^(x^2)
```

Fricas [A] time = 1.91346, size = 18, normalized size = 2.

$$\frac{1}{2} e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*e^(x^2)
```

Sympy [A] time = 0.075075, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x,x)
```

```
[Out] exp(x**2)/2
```

Giac [A] time = 1.07024, size = 8, normalized size = 0.89

$$\frac{1}{2} e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="giac")
```

```
[Out] 1/2*e^(x^2)
```

3.18 $\int x\sqrt{1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] (1 + x^2)^(3/2)/3
```

Rubi [A] time = 0.0019663, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x^2],x]

[Out] (1 + x^2)^(3/2)/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{1+x^2}\,dx=\frac{1}{3}\left(1+x^2\right)^{3/2}$$

Mathematica [A] time = 0.0003708, size = 13, normalized size = 1.

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x^2],x]
```

```
[Out] (1 + x^2)^(3/2)/3
```

Maple [A] time = 0., size = 10, normalized size = 0.8

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^(1/2),x)
```

```
[Out] 1/3*(x^2+1)^(3/2)
```

Maxima [A] time = 0.930322, size = 12, normalized size = 0.92

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

Fricas [A] time = 1.70136, size = 28, normalized size = 2.15

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

Sympy [B] time = 0.179528, size = 22, normalized size = 1.69

$$\frac{x^2\sqrt{x^2+1}}{3}+\frac{\sqrt{x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**(1/2),x)
```

```
[Out] x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

Giac [A] time = 1.05952, size = 12, normalized size = 0.92

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 + 1)^(3/2)
```

3.19 $\int \frac{e^x}{1+e^x} \, dx$

Optimal. Leaf size=6

$$\log(e^x+1)$$

```
[Out] Log[1 + E^x]
```

Rubi [A] time = 0.014687, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2246, 31}

$$\log(e^x+1)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 + E^x),x]
```

```
[Out] Log[1 + E^x]
```

Rule 2246

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
 e, n, p}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{e^x}{1+e^x} \, dx &= \text{Subst}\left(\int \frac{1}{1+x} \, dx, x, e^x\right) \\ &= \log(1+e^x) \end{aligned}$$

Mathematica [A] time = 0.0040945, size = 6, normalized size = 1.

$$\log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 + E^x),x]
```

```
[Out] Log[1 + E^x]
```

Maple [A] time = 0.001, size = 6, normalized size = 1.

$$\ln(1 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(1+exp(x)),x)
```

```
[Out] ln(1+exp(x))
```

Maxima [A] time = 0.922343, size = 7, normalized size = 1.17

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="maxima")
```

```
[Out] log(e^x + 1)
```

Fricas [A] time = 1.68215, size = 19, normalized size = 3.17

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="fricas")
```

```
[Out] log(e^x + 1)
```

Sympy [A] time = 0.084821, size = 5, normalized size = 0.83

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x)
```

```
[Out] log(exp(x) + 1)
```

Giac [A] time = 1.09375, size = 7, normalized size = 1.17

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="giac")
```

```
[Out] log(e^x + 1)
```

3.20 $\int x^{3/2}\,dx$

Optimal. Leaf size=9

$$\frac{2x^{5/2}}{5}$$

```
[Out] (2*x^(5/2))/5
```

Rubi [A] time = 0.0004782, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {30}

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3/2),x]
```

```
[Out] (2*x^(5/2))/5
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x^{3/2}\,dx = \frac{2x^{5/2}}{5}$$

Mathematica [A] time = 0.0005174, size = 9, normalized size = 1.

$$\frac{2x^{5/2}}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3/2),x]
```

```
[Out] (2*x^(5/2))/5
```

Maple [A] time = 0.001, size = 6, normalized size = 0.7

$$\frac{2}{5}x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3/2),x)
```

```
[Out] 2/5*x^(5/2)
```

Maxima [A] time = 0.927467, size = 7, normalized size = 0.78

$$\frac{2}{5}\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2)
```

Fricas [A] time = 1.7388, size = 18, normalized size = 2.

$$\frac{2}{5}\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="fricas")
```

```
[Out] 2/5*x^(5/2)
```

Sympy [A] time = 0.052248, size = 7, normalized size = 0.78

$$\frac{2x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3/2),x)
```

```
[Out] 2*x**(5/2)/5
```

Giac [A] time = 1.0807, size = 7, normalized size = 0.78

$$\frac{2}{5}\,x^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3/2),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2)
```

3.21 $\int \cos(3+2x)\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sin(2x+3)$$

```
[Out] Sin[3 + 2*x]/2
```

Rubi [A] time = 0.0030981, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2637}

$$\frac{1}{2}\sin(2x+3)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3 + 2*x],x]
```

```
[Out] Sin[3 + 2*x]/2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(3+2x)\,dx = \frac{1}{2}\sin(3+2x)$$

Mathematica [A] time = 0.0042777, size = 10, normalized size = 1.

$$\frac{1}{2}\sin(2x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3 + 2*x],x]
```

```
[Out] Sin[3 + 2*x]/2
```

Maple [A] time = 0.005, size = 9, normalized size = 0.9

$$\frac{\sin\left(3+2\,x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3+2*x),x)
```

```
[Out] 1/2*sin(3+2*x)
```

Maxima [A] time = 0.925298, size = 11, normalized size = 1.1

$$\frac{1}{2}\,\sin\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3+2*x),x, algorithm="maxima")
```

```
[Out] 1/2*sin(2*x + 3)
```

Fricas [A] time = 1.87985, size = 24, normalized size = 2.4

$$\frac{1}{2}\,\sin\left(2\,x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3+2*x),x, algorithm="fricas")
```

```
[Out] 1/2*sin(2*x + 3)
```

Sympy [A] time = 0.111567, size = 7, normalized size = 0.7

$$\frac{\sin(2x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3+2*x),x)
```

```
[Out] sin(2*x + 3)/2
```

Giac [A] time = 1.101, size = 11, normalized size = 1.1

$$\frac{1}{2}\sin(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3+2*x),x, algorithm="giac")
```

```
[Out] 1/2*sin(2*x + 3)
```

3.22 $\int 2e^{2x}yz\,dx$

Optimal. Leaf size=8

$$e^{2x}yz$$

```
[Out] E^(2*x)*y*z
```

Rubi [A] time = 0.0032724, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {12, 2194}

$$e^{2x}yz$$

Antiderivative was successfully verified.

```
[In] Int[2*E^(2*x)*y*z,x]
```

```
[Out] E^(2*x)*y*z
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int 2e^{2x}yz\,dx &= (2yz)\int e^{2x}\,dx\\ &= e^{2x}yz\end{aligned}$$

Mathematica [A] time = 0.0005961, size = 8, normalized size = 1.

$$e^{2x}yz$$

Antiderivative was successfully verified.

```
[In] Integrate[2*E^(2*x)*y*z,x]
```

```
[Out] E^(2*x)*y*z
```

Maple [A] time = 0.002, size = 8, normalized size = 1.

$$e^{2x}yz$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*exp(2*x)*y*z,x)
```

```
[Out] exp(2*x)*y*z
```

Maxima [A] time = 0.932509, size = 9, normalized size = 1.12

$$yze^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(2*x)*y*z,x, algorithm="maxima")
```

```
[Out] y*z*e^(2*x)
```

Fricas [A] time = 1.86235, size = 18, normalized size = 2.25

$$yze^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(2*x)*y*z,x, algorithm="fricas")
```

```
[Out] y*z*e^(2*x)
```

Sympy [A] time = 0.06015, size = 7, normalized size = 0.88

$$yze^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(2*x)*y*z,x)
```

```
[Out] y*z*exp(2*x)
```

Giac [A] time = 1.07112, size = 9, normalized size = 1.12

$$yze^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(2*x)*y*z,x, algorithm="giac")
```

```
[Out] y*z*e^(2*x)
```

3.23 $\int e^x \cos^2(e^x) \sin(e^x)\, dx$

Optimal. Leaf size=10

$$-\frac{1}{3}\cos^3(e^x)$$

```
[Out] -Cos[E^x]^3/3
```

Rubi [A] time = 0.0274706, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2282, 2565, 30}

$$-\frac{1}{3}\cos^3(e^x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[E^x]^2*Sin[E^x],x]
```

```
[Out] -Cos[E^x]^3/3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int e^x \cos^2(e^x)\sin(e^x)\, dx &= \text{Subst}\left(\int \cos^2(x)\sin(x)\, dx, x, e^x\right)\\
&= -\text{Subst}\left(\int x^2\, dx, x, \cos(e^x)\right)\\
&= -\frac{1}{3}\cos^3(e^x)
\end{aligned}
$$

Mathematica [A] time = 0.0203797, size = 19, normalized size = 1.9

$$-\frac{1}{4}\cos(e^x) - \frac{1}{12}\cos(3e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[E^x]^2*Sin[E^x],x]
```

```
[Out] -Cos[E^x]/4 - Cos[3*E^x]/12
```

Maple [A] time = 0.004, size = 8, normalized size = 0.8

$$-\frac{(\cos(e^x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(exp(x))^2*sin(exp(x)),x)
```

```
[Out] -1/3*cos(exp(x))^3
```

Maxima [A] time = 0.922909, size = 9, normalized size = 0.9

$$-\frac{1}{3}\cos(e^x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="maxima")
```

```
[Out] -1/3*cos(e^x)^3
```

Fricas [A] time = 1.8898, size = 23, normalized size = 2.3

$$-\frac{1}{3}\cos\left(e^{x}\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="fricas")
```

```
[Out] -1/3*cos(e^x)^3
```

Sympy [A] time = 2.08616, size = 8, normalized size = 0.8

$$-\frac{\cos^{3}\left(e^{x}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x))**2*sin(exp(x)),x)
```

```
[Out] -cos(exp(x))**3/3
```

Giac [A] time = 1.08064, size = 9, normalized size = 0.9

$$-\frac{1}{3}\cos\left(e^{x}\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(exp(x))^2*sin(exp(x)),x, algorithm="giac")
```

```
[Out] -1/3*cos(e^x)^3
```

3.24 $\int x\sqrt{1+x}\,dx$

Optimal. Leaf size=23

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

```
[Out] (-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

Rubi [A] time = 0.0038989, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x],x]
```

```
[Out] (-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{1+x}\,dx &= \int\left(-\sqrt{1+x}+(1+x)^{3/2}\right)dx\\ &= -\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0042225, size = 16, normalized size = 0.7

$$\frac{2}{15}(x+1)^{3/2}(3x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x],x]
```

```
[Out] (2*(1 + x)^(3/2)*(-2 + 3*x))/15
```

Maple [A] time = 0.003, size = 13, normalized size = 0.6

$$\frac{-4+6x}{15}(1+x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^(1/2),x)
```

```
[Out] 2/15*(1+x)^(3/2)*(-2+3*x)
```

Maxima [A] time = 0.924545, size = 20, normalized size = 0.87

$$\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

Fricas [A] time = 1.81878, size = 46, normalized size = 2.

$$\frac{2}{15}\left(3x^2+x-2\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

Sympy [A] time = 0.847806, size = 34, normalized size = 1.48

$$\frac{2x^2\sqrt{x+1}}{5} + \frac{2x\sqrt{x+1}}{15} - \frac{4\sqrt{x+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(1/2),x)
```

```
[Out] 2*x**2*sqrt(x + 1)/5 + 2*x*sqrt(x + 1)/15 - 4*sqrt(x + 1)/15
```

Giac [A] time = 1.09969, size = 20, normalized size = 0.87

$$\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

3.25 $\int \frac{1}{-1+x^4} dx$

Optimal. Leaf size=13

$$-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] -ArcTan[x]/2 - ArcTanh[x]/2
```

Rubi [A] time = 0.0031654, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {212, 206, 203}

$$-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)^(-1),x]
```

```
[Out] -ArcTan[x]/2 - ArcTanh[x]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{-1+x^4}\,dx = -\left(\frac{1}{2}\int \frac{1}{1-x^2}\,dx\right) - \frac{1}{2}\int \frac{1}{1+x^2}\,dx$$
$$= -\frac{1}{2}\tan^{-1}(x) - \frac{1}{2}\tanh^{-1}(x)$$

Mathematica [A] time = 0.0035192, size = 25, normalized size = 1.92

$$\frac{1}{4}\log(1-x) - \frac{1}{4}\log(x+1) - \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)^(-1),x]
```

```
[Out] -ArcTan[x]/2 + Log[1 - x]/4 - Log[1 + x]/4
```

Maple [A] time = 0., size = 10, normalized size = 0.8

$$-\frac{\arctan(x)}{2} - \frac{\operatorname{Artanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-1),x)
```

```
[Out] -1/2*arctan(x)-1/2*arctanh(x)
```

Maxima [A] time = 1.41753, size = 23, normalized size = 1.77

$$-\frac{1}{2}\arctan(x) - \frac{1}{4}\log(x+1) + \frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Fricas [A] time = 1.88316, size = 68, normalized size = 5.23

$$-\frac{1}{2}\arctan\left(x\right) - \frac{1}{4}\log\left(x + 1\right) + \frac{1}{4}\log\left(x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Sympy [A] time = 0.114985, size = 17, normalized size = 1.31

$$\frac{\log\left(x - 1\right)}{4} - \frac{\log\left(x + 1\right)}{4} - \frac{\operatorname{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-1),x)
```

```
[Out] log(x - 1)/4 - log(x + 1)/4 - atan(x)/2
```

Giac [B] time = 1.08715, size = 26, normalized size = 2.

$$-\frac{1}{2}\arctan\left(x\right) - \frac{1}{4}\log\left(|x + 1|\right) + \frac{1}{4}\log\left(|x - 1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="giac")
```

```
[Out] -1/2*arctan(x) - 1/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

3.26 $\int \frac{e^x}{2+3e^{2x}}\,dx$

Optimal. Leaf size=18

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

[Out] ArcTan[Sqrt[3/2]*E^x]/Sqrt[6]

Rubi [A] time = 0.0204306, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2249, 203}

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[E^x/(2 + 3*E^(2*x)),x]

[Out] ArcTan[Sqrt[3/2]*E^x]/Sqrt[6]

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{e^x}{2+3e^{2x}}\,dx = \text{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, e^x\right)$$
$$= \frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

Mathematica [A] time = 0.0068214, size = 18, normalized size = 1.

$$\frac{\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)}{\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(2 + 3*E^(2*x)),x]
```

```
[Out] ArcTan[Sqrt[3/2]*E^x]/Sqrt[6]
```

Maple [A] time = 0.004, size = 14, normalized size = 0.8

$$\frac{\sqrt{6}}{6}\arctan\left(\frac{e^x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(2+3*exp(2*x)),x)
```

```
[Out] 1/6*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

Maxima [A] time = 1.41214, size = 18, normalized size = 1.

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

Fricas [A] time = 1.94857, size = 50, normalized size = 2.78

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

Sympy [A] time = 0.106956, size = 15, normalized size = 0.83

$$\mathrm{RootSum}\left(24z^{2}+1,\left(i\mapsto i\log\left(4i+e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(2+3*exp(2*x)),x)
```

```
[Out] RootSum(24*_z**2 + 1, Lambda(_i, _i*log(4*_i + exp(x))))
```

Giac [A] time = 1.0934, size = 18, normalized size = 1.

$$\frac{1}{6}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(2+3*exp(2*x)),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

3.27 $\int \frac{e^{2x}}{A+Be^{4x}}\,dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

```
[Out] ArcTan[(Sqrt[B]*E^(2*x))/Sqrt[A]]/(2*Sqrt[A]*Sqrt[B])
```

Rubi [A] time = 0.0347668, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2249, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(A + B*E^(4*x)),x]
```

```
[Out] ArcTan[(Sqrt[B]*E^(2*x))/Sqrt[A]]/(2*Sqrt[A]*Sqrt[B])
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{e^{2x}}{A+Be^{4x}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{A+Bx^2}\,dx, x, e^{2x}\right)$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

Mathematica [A] time = 0.008076, size = 31, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{B}e^{2x}}{\sqrt{A}}\right)}{2\sqrt{A}\sqrt{B}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(A + B*E^(4*x)),x]
```

```
[Out] ArcTan[(Sqrt[B]*E^(2*x))/Sqrt[A]]/(2*Sqrt[A]*Sqrt[B])
```

Maple [A] time = 0.007, size = 20, normalized size = 0.7

$$\frac{1}{2}\arctan\left(B\,(\mathrm{e}^{x})^{2}\,\frac{1}{\sqrt{AB}}\right)\frac{1}{\sqrt{AB}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(A+B*exp(4*x)),x)
```

```
[Out] 1/2/(A*B)^(1/2)*arctan(B*exp(x)^2/(A*B)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93193, size = 186, normalized size = 6.

$$\left[-\frac{\sqrt{-AB}\log\left(\frac{Be^{(4x)}-2\sqrt{-AB}e^{(2x)}-A}{Be^{(4x)}+A}\right)}{4AB}, -\frac{\sqrt{AB}\arctan\left(\frac{\sqrt{AB}e^{(-2x)}}{B}\right)}{2AB}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-A*B)*log((B*e^(4*x) - 2*sqrt(-A*B)*e^(2*x) - A)/(B*e^(4*x) + A)
)/(A*B), -1/2*sqrt(A*B)*arctan(sqrt(A*B)*e^(-2*x)/B)/(A*B)]
```

Sympy [A] time = 0.150645, size = 22, normalized size = 0.71

$$\operatorname{RootSum}\left(16z^2AB+1, \left(i \mapsto i\log\left(4iA+e^{2x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(A+B*exp(4*x)),x)
```

```
[Out] RootSum(16*_z**2*A*B + 1, Lambda(_i, _i*log(4*_i*A + exp(2*x))))
```

Giac [A] time = 1.08468, size = 26, normalized size = 0.84

$$\frac{\arctan\left(\frac{Be^{(2x)}}{\sqrt{AB}}\right)}{2\sqrt{AB}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(A+B*exp(4*x)),x, algorithm="giac")
```

```
[Out] 1/2*arctan(B*e^(2*x)/sqrt(A*B))/sqrt(A*B)
```

3.28 $\int \frac{e^{1+x}}{1+e^x} \, dx$

Optimal. Leaf size=8

$$e \log \left(e^x+1\right)$$

[Out] E*Log[1 + E^x]

Rubi [A] time = 0.0340458, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2247, 2246, 31}

$$e \log \left(e^x+1\right)$$

Antiderivative was successfully verified.

[In] Int[E^(1 + x)/(1 + E^x),x]

[Out] E*Log[1 + E^x]

Rule 2247

```
Int[((a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.))^(p_.)*((G_)^((
h_.)*((f_.) + (g_.)*(x_))))^(m_.), x_Symbol] :> Dist[(G^(h*(f + g*x)))^m/(F
^(e*(c + d*x)))^n, Int[(F^(e*(c + d*x)))^n*(a + b*(F^(e*(c + d*x)))^n)^p, x
], x] /; FreeQ[{F, G, a, b, c, d, e, f, g, h, m, n, p}, x] && EqQ[d*e*n*Log
[F], g*h*m*Log[G]]
```

Rule 2246

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
 e, n, p}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{e^{1+x}}{1+e^x}\,dx = e\int \frac{e^x}{1+e^x}\,dx$$
$$= e\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, e^x\right)$$
$$= e\log\left(1+e^x\right)$$

Mathematica [A] time = 0.0052211, size = 8, normalized size = 1.

$$e\log\left(e^x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(1 + x)/(1 + E^x),x]
```

```
[Out] E*Log[1 + E^x]
```

Maple [A] time = 0.001, size = 9, normalized size = 1.1

$$\mathrm{e}\ln\left(1+\mathrm{e}^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1+x)/(1+exp(x)),x)
```

```
[Out] exp(1)*ln(1+exp(x))
```

Maxima [A] time = 0.931826, size = 11, normalized size = 1.38

$$e\log\left(e^x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1+x)/(1+exp(x)),x, algorithm="maxima")
```

```
[Out] e*log(e^x + 1)
```

Fricas [A] time = 1.94737, size = 30, normalized size = 3.75

$$e\log\left(e+e^{(x+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1+x)/(1+exp(x)),x, algorithm="fricas")
```

```
[Out] e*log(e + e^(x + 1))
```

Sympy [A] time = 0.096068, size = 7, normalized size = 0.88

$$e\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1+x)/(1+exp(x)),x)
```

```
[Out] E*log(exp(x) + 1)
```

Giac [A] time = 1.0825, size = 11, normalized size = 1.38

$$e\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1+x)/(1+exp(x)),x, algorithm="giac")
```

```
[Out] e*log(e^x + 1)
```

3.29 $\int (10e)^x \, dx$

Optimal. Leaf size=12

$$\frac{(10e)^x}{1+\log(10)}$$

```
[Out] (10*E)^x/(1 + Log[10])
```

Rubi [A] time = 0.0055997, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2194}

$$\frac{(10e)^x}{1+\log(10)}$$

Antiderivative was successfully verified.

```
[In] Int[(10*E)^x,x]
```

```
[Out] (10*E)^x/(1 + Log[10])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int (10e)^x \, dx = \frac{(10e)^x}{1+\log(10)}$$

Mathematica [A] time = 0.0036176, size = 12, normalized size = 1.

$$\frac{(10e)^x}{\log(10e)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(10*E)^x,x]
```

```
[Out] (10*E)^x/Log[10*E]
```

Maple [A] time = 0.01, size = 13, normalized size = 1.1

$$\frac{(10\,E)^x}{\ln(10\,E)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((10*E)^x,x)
```

```
[Out] 1/ln(10*E)*(10*E)^x
```

Maxima [A] time = 0.930965, size = 16, normalized size = 1.33

$$\frac{(10\,E)^x}{\log(10\,E)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*E)^x,x, algorithm="maxima")
```

```
[Out] (10*E)^x/log(10*E)
```

Fricas [A] time = 1.96663, size = 27, normalized size = 2.25

$$\frac{(10\,E)^x}{\log(10\,E)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*E)^x,x, algorithm="fricas")
```

```
[Out] (10*E)^x/log(10*E)
```

Sympy [A] time = 0.087169, size = 8, normalized size = 0.67

$$\frac{(10e)^x}{1 + \log(10)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*E)**x,x)
```

```
[Out] (10*E)**x/(1 + log(10))
```

Giac [A] time = 1.08015, size = 16, normalized size = 1.33

$$\frac{(10\,E)^x}{\log(10\,E)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((10*E)^x,x, algorithm="giac")
```

```
[Out] (10*E)^x/log(10*E)
```

3.30 $\int x^3 \sin\left(x^2\right) dx$

Optimal. Leaf size=20

$$\frac{\sin\left(x^2\right)}{2} - \frac{1}{2}x^2\cos\left(x^2\right)$$

```
[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Rubi [A] time = 0.0189159, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3379, 3296, 2637}

$$\frac{\sin\left(x^2\right)}{2} - \frac{1}{2}x^2\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[x^2],x]
```

```
[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x^3 \sin\left(x^2\right) dx &= \frac{1}{2} \operatorname{Subst}\left(\int x \sin(x)\, dx, x, x^2\right) \\
&= -\frac{1}{2} x^2 \cos\left(x^2\right) + \frac{1}{2} \operatorname{Subst}\left(\int \cos(x)\, dx, x, x^2\right) \\
&= -\frac{1}{2} x^2 \cos\left(x^2\right) + \frac{\sin\left(x^2\right)}{2}
\end{aligned}
$$

Mathematica [A] time = 0.0017644, size = 20, normalized size = 1.

$$\frac{\sin\left(x^2\right)}{2} - \frac{1}{2} x^2 \cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x^2],x]

[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Maple [A] time = 0., size = 17, normalized size = 0.9

$$-\frac{x^2 \cos\left(x^2\right)}{2} + \frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x)

[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Maxima [A] time = 0.93255, size = 22, normalized size = 1.1

$$-\frac{1}{2} x^2 \cos\left(x^2\right) + \frac{1}{2} \sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Fricas [A] time = 1.98983, size = 46, normalized size = 2.3

$$-\frac{1}{2}x^2\cos\left(x^2\right) + \frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="fricas")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Sympy [A] time = 0.548402, size = 15, normalized size = 0.75

$$-\frac{x^2\cos\left(x^2\right)}{2} + \frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(x**2),x)
```

```
[Out] -x**2*cos(x**2)/2 + sin(x**2)/2
```

Giac [A] time = 1.07265, size = 22, normalized size = 1.1

$$-\frac{1}{2}x^2\cos\left(x^2\right) + \frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="giac")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

3.31 $\int \frac{x^7}{1+x^{12}} \, dx$

Optimal. Leaf size=49

$$-\frac{1}{12}\log\left(x^4+1\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4]/12 + Log[1 - x^4 +
x^8]/24
```

Rubi [A] time = 0.0360088, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {275, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{12}\log\left(x^4+1\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + x^12),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4]/12 + Log[1 - x^4 +
x^8]/24
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1+x^{12}}\,dx &= \frac{1}{4}\,\text{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, x^4\right) \\
&= -\left(\frac{1}{12}\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^4\right)\right) + \frac{1}{12}\,\text{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, x^4\right) \\
&= -\frac{1}{12}\log\left(1+x^4\right) + \frac{1}{24}\,\text{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right) + \frac{1}{8}\,\text{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right) \\
&= -\frac{1}{12}\log\left(1+x^4\right) + \frac{1}{24}\log\left(1-x^4+x^8\right) - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right) \\
&= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}} - \frac{1}{12}\log\left(1+x^4\right) + \frac{1}{24}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [B] time = 0.109162, size = 260, normalized size = 5.31

$$\frac{1}{24}\left(-2\log\left(x^2-\sqrt{2}x+1\right)-2\log\left(x^2+\sqrt{2}x+1\right)+\log\left(2x^2-\sqrt{6}x+\sqrt{2}x+2\right)+\log\left(2x^2+\sqrt{2}\left(\sqrt{3}-1\right)x+2\right)+\log\left(\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + x^12),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + Sqrt[3] - 2*Sqrt[2]*x)/(1 - Sqrt[3])] - 2*Sqrt[3]*Ar
cTan[(1 - Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] + 2*Sqrt[3]*ArcTan[(-1 + Sq
rt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] - 2*Sqrt[3]*ArcTan[(1 + Sqrt[3] + 2*Sqr
t[2]*x)/(-1 + Sqrt[3])] - 2*Log[1 - Sqrt[2]*x + x^2] - 2*Log[1 + Sqrt[2]*x
+ x^2] + Log[2 + Sqrt[2]*x - Sqrt[6]*x + 2*x^2] + Log[2 + Sqrt[2]*(-1 + Sqr
t[3])*x + 2*x^2] + Log[2 - (Sqrt[2] + Sqrt[6])*x + 2*x^2] + Log[2 + (Sqrt[2
] + Sqrt[6])*x + 2*x^2])/24
```

Maple [A] time = 0., size = 41, normalized size = 0.8

$$-\frac{\ln\left(x^4+1\right)}{12}+\frac{\ln\left(x^8-x^4+1\right)}{24}+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2x^4-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^12+1),x)
```

```
[Out] -1/12*ln(x^4+1)+1/24*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2
))
```

Maxima [A] time = 1.40679, size = 54, normalized size = 1.1

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^12+1),x, algorithm="maxima")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

Fricas [A] time = 1.8426, size = 124, normalized size = 2.53

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^12+1),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

Sympy [A] time = 0.158938, size = 46, normalized size = 0.94

$$-\frac{\log\left(x^4+1\right)}{12}+\frac{\log\left(x^8-x^4+1\right)}{24}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(x**12+1),x)
```

```
[Out] -log(x**4 + 1)/12 + log(x**8 - x**4 + 1)/24 + sqrt(3)*atan(2*sqrt(3)*x**4/3
 - sqrt(3)/3)/12
```

Giac [A] time = 1.09553, size = 54, normalized size = 1.1

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^12+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/
12*log(x^4 + 1)
```

3.32 $\int x^{3a}\sin\left(x^{2a}\right)\,dx$

Optimal. Leaf size=115

$$\frac{ix^{3a+1}\left(-ix^{2a}\right)^{-\frac{3a+1}{2a}}\,\mathrm{Gamma}\left(\frac{1}{2}\left(\frac{1}{a}+3\right),-ix^{2a}\right)}{4a}-\frac{ix^{3a+1}\left(ix^{2a}\right)^{-\frac{3a+1}{2a}}\,\mathrm{Gamma}\left(\frac{1}{2}\left(\frac{1}{a}+3\right),ix^{2a}\right)}{4a}$$

```
[Out] ((I/4)*x^(1 + 3*a)*Gamma[(3 + a^(-1))/2, (-I)*x^(2*a)])/(a*((-I)*x^(2*a))^(
(1 + 3*a)/(2*a))) - ((I/4)*x^(1 + 3*a)*Gamma[(3 + a^(-1))/2, I*x^(2*a)])/(a
*(I*x^(2*a))^((1 + 3*a)/(2*a)))
```

Rubi [A] time = 0.0560074, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3423, 2218}

$$\frac{ix^{3a+1}\left(-ix^{2a}\right)^{-\frac{3a+1}{2a}}\,\mathrm{Gamma}\left(\frac{1}{2}\left(\frac{1}{a}+3\right),-ix^{2a}\right)}{4a}-\frac{ix^{3a+1}\left(ix^{2a}\right)^{-\frac{3a+1}{2a}}\,\mathrm{Gamma}\left(\frac{1}{2}\left(\frac{1}{a}+3\right),ix^{2a}\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3*a)*Sin[x^(2*a)],x]
```

```
[Out] ((I/4)*x^(1 + 3*a)*Gamma[(3 + a^(-1))/2, (-I)*x^(2*a)])/(a*((-I)*x^(2*a))^(
(1 + 3*a)/(2*a))) - ((I/4)*x^(1 + 3*a)*Gamma[(3 + a^(-1))/2, I*x^(2*a)])/(a
*(I*x^(2*a))^((1 + 3*a)/(2*a)))
```

Rule 3423

```
Int[((e_.)*(x_))^(m_.)*Sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] :> Dist[I/2,
 Int[(e*x)^m*E^(-(c*I) - d*I*x^n), x], x] - Dist[I/2, Int[(e*x)^m*E^(c*I +
d*I*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 2218

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> -Simp[(F^a*(e + f*x)^(m + 1)*Gamma[(m + 1)/n, -(b*(c + d*x
)^n*Log[F])])/(f*n*(-(b*(c + d*x)^n*Log[F]))^((m + 1)/n)), x] /; FreeQ[{F,
a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int x^{3a}\sin\left(x^{2a}\right)dx = \frac{1}{2}i\int e^{-ix^{2a}}x^{3a}\,dx - \frac{1}{2}i\int e^{ix^{2a}}x^{3a}\,dx$$
$$= \frac{ix^{1+3a}\left(-ix^{2a}\right)^{-\frac{1+3a}{2a}}\Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right),-ix^{2a}\right)}{4a} - \frac{ix^{1+3a}\left(ix^{2a}\right)^{-\frac{1+3a}{2a}}\Gamma\left(\frac{1}{2}\left(3+\frac{1}{a}\right),ix^{2a}\right)}{4a}$$

Mathematica [A] time = 0.293061, size = 142, normalized size = 1.23

$$-\frac{x^{a+1}\left(x^{4a}\right)^{-\frac{a+1}{2a}}\left((a+1)\left(-ix^{2a}\right)^{\frac{a+1}{2a}}\text{Gamma}\left(\frac{a+1}{2a},ix^{2a}\right)+(a+1)\left(ix^{2a}\right)^{\frac{a+1}{2a}}\text{Gamma}\left(\frac{a+1}{2a},-ix^{2a}\right)+4a\left(x^{4a}\right)^{\frac{a+1}{2a}}\cos\left(x^{2a}\right)\right)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3*a)*Sin[x^(2*a)],x]
```

```
[Out] -(x^(1 + a)*(4*a*(x^(4*a))^((1 + a)/(2*a))*Cos[x^(2*a)] + (1 + a)*(I*x^(2*a
))^((1 + a)/(2*a))*Gamma[(1 + a)/(2*a), (-I)*x^(2*a)] + (1 + a)*((-I)*x^(2*
a))^((1 + a)/(2*a))*Gamma[(1 + a)/(2*a), I*x^(2*a)]))/(8*a^2*(x^(4*a))^((1
+ a)/(2*a)))
```

Maple [C] time = 0.089, size = 41, normalized size = 0.4

$$\frac{x^{5a+1}}{5a+1}\,{}_1F_2\left(\frac{5}{4}+\frac{1}{4a};\,\frac{3}{2},\frac{9}{4}+\frac{1}{4a};\,-\frac{x^{4a}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(3*a)*sin(x^(2*a)),x)
```

```
[Out] 1/(5*a+1)*x^(5*a+1)*hypergeom([5/4+1/4/a],[3/2,9/4+1/4/a],-1/4*x^(4*a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{xx^{a}\cos\left(x^{2a}\right)-(a+1)\int x^{a}\cos\left(x^{2a}\right)dx}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="maxima")
```

```
[Out] -1/2*(x*x^a*cos(x^(2*a)) - (a + 1)*integrate(x^a*cos(x^(2*a)), x))/a
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(x^{3\,a}\sin\left(x^{2\,a}\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="fricas")
```

```
[Out] integral(x^(3*a)*sin(x^(2*a)), x)
```

Sympy [A] time = 5.48852, size = 54, normalized size = 0.47

$$\frac{xx^{5a}\Gamma\left(\frac{5}{4}+\frac{1}{4a}\right){}_{1}F_{2}\left(\begin{matrix}\frac{5}{4}+\frac{1}{4a}\\ \frac{3}{2},\frac{9}{4}+\frac{1}{4a}\end{matrix}\middle|-\frac{x^{4a}}{4}\right)}{4a\Gamma\left(\frac{9}{4}+\frac{1}{4a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3*a)*sin(x**(2*a)),x)
```

```
[Out] x*x**(5*a)*gamma(5/4 + 1/(4*a))*hyper((5/4 + 1/(4*a),), (3/2, 9/4 + 1/(4*a)
), -x**(4*a)/4)/(4*a*gamma(9/4 + 1/(4*a)))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^{3\,a}\sin\left(x^{2\,a}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3*a)*sin(x^(2*a)),x, algorithm="giac")
```

```
[Out] integrate(x^(3*a)*sin(x^(2*a)), x)
```

3.33 $\int \cos \left(\sqrt{x}\right) dx$

Optimal. Leaf size=22

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rubi [A] time = 0.0100674, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3362, 3296, 2638}

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[x]],x]

[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rule 3362

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && IntegerQ[1/n]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos\left(\sqrt{x}\right)\,dx &= 2\,\text{Subst}\left(\int x\cos(x)\,dx, x, \sqrt{x}\right)\\ &= 2\sqrt{x}\sin\left(\sqrt{x}\right) - 2\,\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{x}\right)\\ &= 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\sin\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0138431, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Maple [A] time = 0.01, size = 17, normalized size = 0.8

$$2\,\cos\left(\sqrt{x}\right) + 2\,\sin\left(\sqrt{x}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2)),x)
```

```
[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Maxima [A] time = 0.930814, size = 22, normalized size = 1.

$$2\,\sqrt{x}\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Fricas [A] time = 1.83028, size = 55, normalized size = 2.5

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Sympy [A] time = 0.294918, size = 20, normalized size = 0.91

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Giac [A] time = 1.07938, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.34 $\int x\sqrt{1+x}\,dx$

Optimal. Leaf size=23

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

```
[Out] (-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

Rubi [A] time = 0.0039135, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{2}{5}(x+1)^{5/2}-\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x],x]

[Out] (-2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{1+x}\,dx &= \int\left(-\sqrt{1+x}+(1+x)^{3/2}\right)dx\\ &= -\frac{2}{3}(1+x)^{3/2}+\frac{2}{5}(1+x)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.0031991, size = 16, normalized size = 0.7

$$\frac{2}{15}(x+1)^{3/2}(3x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x],x]
```

```
[Out] (2*(1 + x)^(3/2)*(-2 + 3*x))/15
```

Maple [A] time = 0., size = 13, normalized size = 0.6

$$\frac{-4+6x}{15}(1+x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+x)^(1/2),x)
```

```
[Out] 2/15*(1+x)^(3/2)*(-2+3*x)
```

Maxima [A] time = 0.926539, size = 20, normalized size = 0.87

$$\frac{2}{5}(x+1)^{\frac{5}{2}}-\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

Fricas [A] time = 1.79048, size = 46, normalized size = 2.

$$\frac{2}{15}\left(3x^2+x-2\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*x^2 + x - 2)*sqrt(x + 1)
```

Sympy [A] time = 0.839955, size = 34, normalized size = 1.48

$$\frac{2x^2\sqrt{x+1}}{5} + \frac{2x\sqrt{x+1}}{15} - \frac{4\sqrt{x+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(1/2),x)
```

```
[Out] 2*x**2*sqrt(x + 1)/5 + 2*x*sqrt(x + 1)/15 - 4*sqrt(x + 1)/15
```

Giac [A] time = 1.09204, size = 20, normalized size = 0.87

$$\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2)
```

3.35 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}} \, dx$

Optimal. Leaf size=32

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rubi [A] time = 0.0136464, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 43}

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx = \int \frac{1}{\left(1+\sqrt[6]{x}\right)\sqrt[3]{x}}\,dx$$
$$= 6\,\text{Subst}\left(\int \frac{x^3}{1+x}\,dx, x, \sqrt[6]{x}\right)$$
$$= 6\,\text{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2\right)dx, x, \sqrt[6]{x}\right)$$
$$= 6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)$$

Mathematica [A] time = 0.0125579, size = 32, normalized size = 1.

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Maple [B] time = 0., size = 92, normalized size = 2.9

$$2\,\ln\left(-1+\sqrt[6]{x}\right)-\ln\left(\sqrt[3]{x}+\sqrt[6]{x}+1\right)-2\,\ln\left(1+\sqrt[6]{x}\right)+\ln\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)+2\,\sqrt{x}+\ln\left(\sqrt{x}-1\right)-\ln\left(\sqrt{x}+1\right)+6\,\sqrt[6]{x}-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)+x^(1/2)),x)
```

```
[Out] 2*ln(-1+x^(1/6))-ln(x^(1/3)+x^(1/6)+1)-2*ln(1+x^(1/6))+ln(x^(1/3)-x^(1/6)+1
)+2*x^(1/2)+ln(x^(1/2)-1)-ln(x^(1/2)+1)+6*x^(1/6)-ln(-1+x)-2*ln(-1+x^(1/3))
+ln(x^(2/3)+x^(1/3)+1)-3*x^(1/3)
```

Maxima [A] time = 0.930065, size = 32, normalized size = 1.

$$2\,\sqrt{x}-3\,x^{\frac{1}{3}}+6\,x^{\frac{1}{6}}-6\,\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Fricas [A] time = 1.73119, size = 76, normalized size = 2.38

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(1/(x**(1/3) + sqrt(x)), x)
```

Giac [A] time = 1.07626, size = 32, normalized size = 1.

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

3.36 $\int \sqrt{\frac{1+x}{3+2x}}\,dx$

Optimal. Leaf size=44

$$\frac{1}{2}\sqrt{x+1}\sqrt{2x+3}-\frac{\sinh^{-1}\left(\sqrt{2}\sqrt{x+1}\right)}{2\sqrt{2}}$$

```
[Out] (Sqrt[1 + x]*Sqrt[3 + 2*x])/2 - ArcSinh[Sqrt[2]*Sqrt[1 + x]]/(2*Sqrt[2])
```

Rubi [A] time = 0.0162064, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1958, 50, 54, 215}

$$\frac{1}{2}\sqrt{x+1}\sqrt{2x+3}-\frac{\sinh^{-1}\left(\sqrt{2}\sqrt{x+1}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(1 + x)/(3 + 2*x)],x]
```

```
[Out] (Sqrt[1 + x]*Sqrt[3 + 2*x])/2 - ArcSinh[Sqrt[2]*Sqrt[1 + x]]/(2*Sqrt[2])
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
```

```
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\frac{1+x}{3+2x}}\,dx &= \int \frac{\sqrt{1+x}}{\sqrt{3+2x}}\,dx \\
&= \frac{1}{2}\sqrt{1+x}\sqrt{3+2x} - \frac{1}{4}\int \frac{1}{\sqrt{1+x}\sqrt{3+2x}}\,dx \\
&= \frac{1}{2}\sqrt{1+x}\sqrt{3+2x} - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+2x^2}}\,dx, x, \sqrt{1+x}\right) \\
&= \frac{1}{2}\sqrt{1+x}\sqrt{3+2x} - \frac{\sinh^{-1}\left(\sqrt{2}\sqrt{1+x}\right)}{2\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0358298, size = 71, normalized size = 1.61

$$
\frac{2(x+1)\sqrt{2x+3} - \sqrt{2}\sqrt{x+1}\sinh^{-1}\left(\sqrt{2}\sqrt{x+1}\right)}{4\sqrt{\frac{x+1}{2x+3}}\sqrt{2x+3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(1 + x)/(3 + 2*x)],x]
```

```
[Out] (2*(1 + x)*Sqrt[3 + 2*x] - Sqrt[2]*Sqrt[1 + x]*ArcSinh[Sqrt[2]*Sqrt[1 + x]]
)/(4*Sqrt[(1 + x)/(3 + 2*x)]*Sqrt[3 + 2*x])
```

Maple [B] time = 0.01, size = 75, normalized size = 1.7

$$
-\frac{3+2\,x}{8}\sqrt{\frac{1+x}{3+2\,x}}\left(\ln\left(\frac{5\sqrt{2}}{4}+x\sqrt{2}+\sqrt{2\,x^2+5\,x+3}\right)\sqrt{2}-4\sqrt{2\,x^2+5\,x+3}\right)\frac{1}{\sqrt{(3+2\,x)(1+x)}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)/(3+2*x))^(1/2),x)
```

```
[Out] -1/8*((1+x)/(3+2*x))^(1/2)*(3+2*x)*(ln(5/4*2^(1/2)+x*2^(1/2)+(2*x^2+5*x+3)^
(1/2))*2^(1/2)-4*(2*x^2+5*x+3)^(1/2))/((3+2*x)*(1+x))^(1/2)
```

Maxima [B] time = 1.40605, size = 108, normalized size = 2.45

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\sqrt{\frac{x+1}{2x+3}}}{\sqrt{2}+2\sqrt{\frac{x+1}{2x+3}}}\right)-\frac{\sqrt{\frac{x+1}{2x+3}}}{2\left(\frac{2(x+1)}{2x+3}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(3+2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(-(sqrt(2) - 2*sqrt((x + 1)/(2*x + 3)))/(sqrt(2) + 2*sqrt((x
 + 1)/(2*x + 3)))) - 1/2*sqrt((x + 1)/(2*x + 3))/(2*(x + 1)/(2*x + 3) - 1)
```

Fricas [A] time = 1.83084, size = 151, normalized size = 3.43

$$\frac{1}{2}(2x+3)\sqrt{\frac{x+1}{2x+3}}+\frac{1}{8}\sqrt{2}\log\left(2\sqrt{2}(2x+3)\sqrt{\frac{x+1}{2x+3}}-4x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(3+2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x + 3)*sqrt((x + 1)/(2*x + 3)) + 1/8*sqrt(2)*log(2*sqrt(2)*(2*x + 3)
*sqrt((x + 1)/(2*x + 3)) - 4*x - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{x+1}{2x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(3+2*x))**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)/(2*x + 3)), x)
```

Giac [A] time = 1.10489, size = 82, normalized size = 1.86

$$\frac{1}{8}\sqrt{2}\log\left(\left|-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2+5x+3}\right)-5\right|\right)\operatorname{sgn}(2x+3)+\frac{1}{2}\sqrt{2x^2+5x+3}\operatorname{sgn}(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)/(3+2*x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*log(abs(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 + 5*x + 3)) - 5))*sg
n(2*x + 3) + 1/2*sqrt(2*x^2 + 5*x + 3)*sgn(2*x + 3)
```

3.37 $\int \frac{x^4}{\left(1-x^2\right)^{5/2}}\,dx$

<u>**Optimal**</u>. Leaf size=35

$$\frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

<u>**Rubi**</u> **[A]** time = 0.0086468, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {288, 216}

$$\frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

<u>Rule 288</u>

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

<u>Rule 216</u>

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

<u>Rubi steps</u>

$$\int \frac{x^4}{\left(1-x^2\right)^{5/2}}\,dx = \frac{x^3}{3\left(1-x^2\right)^{3/2}} - \int \frac{x^2}{\left(1-x^2\right)^{3/2}}\,dx$$
$$= \frac{x^3}{3\left(1-x^2\right)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \frac{x^3}{3\left(1-x^2\right)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$$

Mathematica [A] time = 0.0242349, size = 26, normalized size = 0.74

$$\frac{x\left(4x^2-3\right)}{3\left(1-x^2\right)^{3/2}} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]
```

Maple [A] time = 0.004, size = 30, normalized size = 0.9

$$\frac{x^3}{3}\left(-x^2+1\right)^{-\frac{3}{2}} + \arcsin(x) - x\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(5/2),x)
```

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.42494, size = 59, normalized size = 1.69

$$\frac{1}{3}x\left(\frac{3x^2}{\left(-x^2+1\right)^{\frac{3}{2}}} - \frac{2}{\left(-x^2+1\right)^{\frac{3}{2}}}\right) - \frac{x}{3\sqrt{-x^2+1}} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

Fricas [B] time = 1.83471, size = 146, normalized size = 4.17

$$-\frac{6\left(x^4-2x^2+1\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(4x^3-3x\right)\sqrt{-x^2+1}}{3\left(x^4-2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

Sympy [B] time = 2.60571, size = 105, normalized size = 3.

$$\frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3}+\frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3}-\frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3}-\frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3}+\frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

Giac [A] time = 1.08346, size = 39, normalized size = 1.11

$$\frac{\left(4x^2-3\right)\sqrt{-x^2+1}x}{3\left(x^2-1\right)^2}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

3.38 $\int \sqrt{x}(1+x)^{5/2}\,dx$

Optimal. Leaf size=75

$$\frac{1}{4}x^{3/2}(x+1)^{5/2}+\frac{5}{24}x^{3/2}(x+1)^{3/2}+\frac{5}{32}x^{3/2}\sqrt{x+1}+\frac{5}{64}\sqrt{x}\sqrt{x+1}-\frac{5}{64}\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] (5*Sqrt[x]*Sqrt[1 + x])/64 + (5*x^(3/2)*Sqrt[1 + x])/32 + (5*x^(3/2)*(1 + x
)^(3/2))/24 + (x^(3/2)*(1 + x)^(5/2))/4 - (5*ArcSinh[Sqrt[x]])/64
```

Rubi [A] time = 0.0130271, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 54, 215}

$$\frac{1}{4}x^{3/2}(x+1)^{5/2}+\frac{5}{24}x^{3/2}(x+1)^{3/2}+\frac{5}{32}x^{3/2}\sqrt{x+1}+\frac{5}{64}\sqrt{x}\sqrt{x+1}-\frac{5}{64}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(1 + x)^(5/2),x]
```

```
[Out] (5*Sqrt[x]*Sqrt[1 + x])/64 + (5*x^(3/2)*Sqrt[1 + x])/32 + (5*x^(3/2)*(1 + x
)^(3/2))/24 + (x^(3/2)*(1 + x)^(5/2))/4 - (5*ArcSinh[Sqrt[x]])/64
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}(1+x)^{5/2}\,dx &= \frac{1}{4}x^{3/2}(1+x)^{5/2}+\frac{5}{8}\int \sqrt{x}(1+x)^{3/2}\,dx\\
&= \frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}+\frac{5}{16}\int \sqrt{x}\sqrt{1+x}\,dx\\
&= \frac{5}{32}x^{3/2}\sqrt{1+x}+\frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}+\frac{5}{64}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\
&= \frac{5}{64}\sqrt{x}\sqrt{1+x}+\frac{5}{32}x^{3/2}\sqrt{1+x}+\frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}-\frac{5}{128}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx\\
&= \frac{5}{64}\sqrt{x}\sqrt{1+x}+\frac{5}{32}x^{3/2}\sqrt{1+x}+\frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}-\frac{5}{64}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{\right.\\
&= \frac{5}{64}\sqrt{x}\sqrt{1+x}+\frac{5}{32}x^{3/2}\sqrt{1+x}+\frac{5}{24}x^{3/2}(1+x)^{3/2}+\frac{1}{4}x^{3/2}(1+x)^{5/2}-\frac{5}{64}\sinh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0194251, size = 41, normalized size = 0.55

$$\frac{1}{192}\left(\sqrt{x}\sqrt{x+1}\left(48x^3+136x^2+118x+15\right)-15\sinh^{-1}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(1 + x)^(5/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(15 + 118*x + 136*x^2 + 48*x^3) - 15*ArcSinh[Sqrt[x]])
/192
```

Maple [A] time = 0.003, size = 70, normalized size = 0.9

$$\frac{1}{4}\sqrt{x}\,(1+x)^{\frac{7}{2}}-\frac{1}{24}\sqrt{x}\,(1+x)^{\frac{5}{2}}-\frac{5}{96}\sqrt{x}\,(1+x)^{\frac{3}{2}}-\frac{5}{64}\sqrt{x}\sqrt{1+x}-\frac{5}{128}\sqrt{x\,(1+x)}\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(1+x)^(5/2),x)
```

```
[Out] 1/4*x^(1/2)*(1+x)^(7/2)-1/24*x^(1/2)*(1+x)^(5/2)-5/96*x^(1/2)*(1+x)^(3/2)-5
/64*x^(1/2)*(1+x)^(1/2)-5/128*(x*(1+x))^(1/2)/(1+x)^(1/2)/x^(1/2)*ln(1/2+x+
```

```
(x^2+x)^(1/2))
```

Maxima [B] time = 0.943377, size = 153, normalized size = 2.04

$$\frac{\frac{15\,(x+1)^{\frac{7}{2}}}{x^{\frac{7}{2}}}+\frac{73\,(x+1)^{\frac{5}{2}}}{x^{\frac{5}{2}}}-\frac{55\,(x+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}}+\frac{15\,\sqrt{x+1}}{\sqrt{x}}}{192\left(\frac{(x+1)^4}{x^4}-\frac{4\,(x+1)^3}{x^3}+\frac{6\,(x+1)^2}{x^2}-\frac{4\,(x+1)}{x}+1\right)}-\frac{5}{128}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right)+\frac{5}{128}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/192*(15*(x + 1)^(7/2)/x^(7/2) + 73*(x + 1)^(5/2)/x^(5/2) - 55*(x + 1)^(3/
2)/x^(3/2) + 15*sqrt(x + 1)/sqrt(x))/((x + 1)^4/x^4 - 4*(x + 1)^3/x^3 + 6*(
x + 1)^2/x^2 - 4*(x + 1)/x + 1) - 5/128*log(sqrt(x + 1)/sqrt(x) + 1) + 5/12
8*log(sqrt(x + 1)/sqrt(x) - 1)
```

Fricas [A] time = 1.79587, size = 140, normalized size = 1.87

$$\frac{1}{192}\left(48\,x^3+136\,x^2+118\,x+15\right)\sqrt{x+1}\sqrt{x}+\frac{5}{128}\,\log\left(2\,\sqrt{x+1}\sqrt{x}-2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/192*(48*x^3 + 136*x^2 + 118*x + 15)*sqrt(x + 1)*sqrt(x) + 5/128*log(2*sqr
t(x + 1)*sqrt(x) - 2*x - 1)
```

Sympy [A] time = 11.9038, size = 190, normalized size = 2.53

$$\begin{cases} -\frac{5\operatorname{acosh}\left(\sqrt{x+1}\right)}{64}+\frac{(x+1)^{\frac{9}{2}}}{4\sqrt{x}}-\frac{7(x+1)^{\frac{7}{2}}}{24\sqrt{x}}-\frac{(x+1)^{\frac{5}{2}}}{96\sqrt{x}}-\frac{5(x+1)^{\frac{3}{2}}}{192\sqrt{x}}+\frac{5\sqrt{x+1}}{64\sqrt{x}} & \text{for } |x+1|>1 \\ \frac{5i\operatorname{asin}\left(\sqrt{x+1}\right)}{64}-\frac{i(x+1)^{\frac{9}{2}}}{4\sqrt{-x}}+\frac{7i(x+1)^{\frac{7}{2}}}{24\sqrt{-x}}+\frac{i(x+1)^{\frac{5}{2}}}{96\sqrt{-x}}+\frac{5i(x+1)^{\frac{3}{2}}}{192\sqrt{-x}}-\frac{5i\sqrt{x+1}}{64\sqrt{-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1+x)**(5/2),x)
```

```
[Out] Piecewise((-5*acosh(sqrt(x + 1))/64 + (x + 1)**(9/2)/(4*sqrt(x)) - 7*(x + 1
)**(7/2)/(24*sqrt(x)) - (x + 1)**(5/2)/(96*sqrt(x)) - 5*(x + 1)**(3/2)/(192
*sqrt(x)) + 5*sqrt(x + 1)/(64*sqrt(x)), Abs(x + 1) > 1), (5*I*asin(sqrt(x +
 1))/64 - I*(x + 1)**(9/2)/(4*sqrt(-x)) + 7*I*(x + 1)**(7/2)/(24*sqrt(-x))
+ I*(x + 1)**(5/2)/(96*sqrt(-x)) + 5*I*(x + 1)**(3/2)/(192*sqrt(-x)) - 5*I*
sqrt(x + 1)/(64*sqrt(-x)), True))
```

Giac [A] time = 1.27791, size = 111, normalized size = 1.48

$$\frac{1}{192}\left(2\left(4\left(6x-11\right)(x+1)+59\right)(x+1)-15\right)\sqrt{x+1}\sqrt{x}+\frac{1}{12}\left(2\left(4x-3\right)(x+1)+3\right)\sqrt{x+1}\sqrt{x}+\frac{1}{4}\left(2x+1\right)\sqrt{x+1}\sqrt{x}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/192*(2*(4*(6*x - 11)*(x + 1) + 59)*(x + 1) - 15)*sqrt(x + 1)*sqrt(x) + 1/
12*(2*(4*x - 3)*(x + 1) + 3)*sqrt(x + 1)*sqrt(x) + 1/4*(2*x + 1)*sqrt(x + 1
)*sqrt(x) + 5/64*log(abs(-sqrt(x + 1) + sqrt(x)))
```

3.39 $\int \frac{x^4}{\left(1-x^2\right)^{5/2}} \, dx$

Optimal. Leaf size=35

$$\frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rubi [A] time = 0.0076016, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {288, 216}

$$\frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{x^4}{\left(1-x^2\right)^{5/2}}\,dx = \frac{x^3}{3\left(1-x^2\right)^{3/2}} - \int \frac{x^2}{\left(1-x^2\right)^{3/2}}\,dx$$
$$= \frac{x^3}{3\left(1-x^2\right)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \frac{x^3}{3\left(1-x^2\right)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$$

Mathematica [A] time = 0.004046, size = 26, normalized size = 0.74

$$\frac{x\left(4x^2-3\right)}{3\left(1-x^2\right)^{3/2}} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]
```

Maple [A] time = 0., size = 30, normalized size = 0.9

$$\frac{x^3}{3}\left(-x^2+1\right)^{-\frac{3}{2}} + \arcsin(x) - x\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(5/2),x)
```

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.41915, size = 59, normalized size = 1.69

$$\frac{1}{3}x\left(\frac{3x^2}{\left(-x^2+1\right)^{\frac{3}{2}}} - \frac{2}{\left(-x^2+1\right)^{\frac{3}{2}}}\right) - \frac{x}{3\sqrt{-x^2+1}} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

Fricas [B] time = 1.86975, size = 146, normalized size = 4.17

$$-\frac{6\left(x^4-2x^2+1\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(4x^3-3x\right)\sqrt{-x^2+1}}{3\left(x^4-2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

Sympy [B] time = 2.56822, size = 105, normalized size = 3.

$$\frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3}+\frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3}-\frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3}-\frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3}+\frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

Giac [A] time = 1.09872, size = 39, normalized size = 1.11

$$\frac{\left(4x^2-3\right)\sqrt{-x^2+1}x}{3\left(x^2-1\right)^2}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

3.40 $\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy$

Optimal. Leaf size=51

$$B\tan^{-1}\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)+A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

```
[Out] B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]] + A*ArcTanh[(A*y)/Sqrt[A^2 + B^2
- B^2*y^2]]
```

Rubi [A] time = 0.0285577, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {402, 217, 203, 377, 206}

$$B\tan^{-1}\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)+A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[A^2 + B^2 - B^2*y^2]/(1 - y^2),y]
```

```
[Out] B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]] + A*ArcTanh[(A*y)/Sqrt[A^2 + B^2
- B^2*y^2]]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy &= A^2\int \frac{1}{(1-y^2)\sqrt{A^2+B^2-B^2y^2}}\,dy+B^2\int \frac{1}{\sqrt{A^2+B^2-B^2y^2}}\,dy\\ &= A^2\,\text{Subst}\left(\int \frac{1}{1-A^2y^2}\,dy,y,\frac{y}{\sqrt{A^2+B^2-B^2y^2}}\right)+B^2\,\text{Subst}\left(\int \frac{1}{1+B^2y^2}\,dy,y,\frac{y}{\sqrt{A^2+B^2-\ldots}}\right.\\ &= B\tan^{-1}\left(\frac{By}{\sqrt{A^2+B^2-B^2y^2}}\right)+A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2+B^2-B^2y^2}}\right)\end{aligned}$$

Mathematica [C] time = 0.0291717, size = 134, normalized size = 2.63

$$iB\log\left(2\sqrt{A^2-B^2y^2+B^2}-2iBy\right)+\frac{1}{2}A\log\left(A\sqrt{A^2-B^2y^2+B^2}+A^2-B^2y+B^2\right)-\frac{1}{2}A\log\left(A\sqrt{A^2-B^2y^2+B^2}+A^2\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[A^2 + B^2 - B^2*y^2]/(1 - y^2),y]
```

```
[Out] -(A*Log[1 - y])/2 + (A*Log[1 + y])/2 + I*B*Log[(-2*I)*B*y + 2*Sqrt[A^2 + B^
2 - B^2*y^2]] + (A*Log[A^2 + B^2 - B^2*y + A*Sqrt[A^2 + B^2 - B^2*y^2]])/2
- (A*Log[A^2 + B^2 + B^2*y + A*Sqrt[A^2 + B^2 - B^2*y^2]])/2
```

Maple [B] time = 0.022, size = 262, normalized size = 5.1

$$\frac{1}{2}\sqrt{-B^2\left(1+y\right)^2+2\,B^2\left(1+y\right)+A^2}+\frac{B^2}{2}\arctan\left(y\sqrt{B^2}\frac{1}{\sqrt{-B^2\left(1+y\right)^2+2\,B^2\left(1+y\right)+A^2}}\right)\frac{1}{\sqrt{B^2}}-\frac{A^2}{2}\ln\left(\frac{1}{1+y}\left(2\,A\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y)
```

```
[Out] 1/2*(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2)+1/2*B^2/(B^2)^(1/2)*arctan((B^2)^(
1/2)*y/(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2))-1/2*A^2/(A^2)^(1/2)*ln((2*A^2+
2*B^2*(1+y)+2*(A^2)^(1/2)*(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2))/(1+y))-1/2*
(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2)+1/2*B^2/(B^2)^(1/2)*arctan((B^2)^(1/2)
*y/(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2))+1/2*A^2/(A^2)^(1/2)*ln((2*A^2-2*B^
2*(y-1)+2*(A^2)^(1/2)*(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2))/(y-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.02532, size = 285, normalized size = 5.59

$$-B\arctan\left(\frac{\sqrt{-B^2y^2+A^2+B^2}}{By}\right)+\frac{1}{4}\,A\log\left(-\frac{\left(A^2-B^2\right)y^2+2\,\sqrt{-B^2y^2+A^2+B^2}Ay+A^2+B^2}{y^2}\right)-\frac{1}{4}\,A\log\left(-\frac{\left(A^2-B\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y, algorithm="fricas")
```

```
[Out] -B*arctan(sqrt(-B^2*y^2 + A^2 + B^2)/(B*y)) + 1/4*A*log(-((A^2 - B^2)*y^2 +
 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2) - 1/4*A*log(-((A^2 - B^
```

```
2)*y^2 - 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{A^2 - B^2y^2 + B^2}}{y^2 - 1}\, dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-B**2*y**2+A**2+B**2)**(1/2)/(-y**2+1),y)
```

```
[Out] -Integral(sqrt(A**2 - B**2*y**2 + B**2)/(y**2 - 1), y)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-B^2*y^2+A^2+B^2)^(1/2)/(-y^2+1),y, algorithm="giac")
```

```
[Out] Timed out
```

3.41 $\int \sin^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0057409, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2,x]
```

```
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^2(x)\,dx &= -\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0016437, size = 14, normalized size = 1.

$$\frac{x}{2}-\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2,x]
```

```
[Out] x/2 - Sin[2*x]/4
```

Maple [A] time = 0., size = 11, normalized size = 0.8

$$\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x)
```

```
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Maxima [A] time = 0.925094, size = 14, normalized size = 1.

$$\frac{1}{2}x-\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

Fricas [A] time = 1.98491, size = 38, normalized size = 2.71

$$-\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.057188, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2,x)
```

```
[Out] x/2 - sin(x)*cos(x)/2
```

Giac [A] time = 1.13738, size = 14, normalized size = 1.

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

3.42 $\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx$

Optimal. Leaf size=49

$$-B\tan^{-1}\left(\frac{B\cos(x)}{\sqrt{A^2+B^2\sin^2(x)}}\right)-A\tanh^{-1}\left(\frac{A\cos(x)}{\sqrt{A^2+B^2\sin^2(x)}}\right)$$

```
[Out] -(B*ArcTan[(B*Cos[x])/Sqrt[A^2 + B^2*Sin[x]^2]]) - A*ArcTanh[(A*Cos[x])/Sqr
t[A^2 + B^2*Sin[x]^2]]
```

Rubi [A] time = 0.0846305, antiderivative size = 57, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3186, 402, 217, 203, 377, 206}

$$-B\tan^{-1}\left(\frac{B\cos(x)}{\sqrt{A^2-B^2\cos^2(x)+B^2}}\right)-A\tanh^{-1}\left(\frac{A\cos(x)}{\sqrt{A^2-B^2\cos^2(x)+B^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*Sqrt[A^2 + B^2*Sin[x]^2],x]
```

```
[Out] -(B*ArcTan[(B*Cos[x])/Sqrt[A^2 + B^2 - B^2*Cos[x]^2]]) - A*ArcTanh[(A*Cos[x
])/Sqrt[A^2 + B^2 - B^2*Cos[x]^2]]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \csc(x)\sqrt{A^2+B^2\sin^2(x)}\,dx &= -\text{Subst}\left(\int \frac{\sqrt{A^2+B^2-B^2x^2}}{1-x^2}\,dx, x, \cos(x)\right)\\
&= -\left(A^2\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{A^2+B^2-B^2x^2}}\,dx, x, \cos(x)\right)\right) - B^2\,\text{Subst}\left(\int \frac{1}{\sqrt{A^2+B^2-\cdots}}\right.\\
&= -\left(A^2\,\text{Subst}\left(\int \frac{1}{1-A^2x^2}\,dx, x, \frac{\cos(x)}{\sqrt{A^2+B^2-B^2\cos^2(x)}}\right)\right) - B^2\,\text{Subst}\left(\int \frac{1}{1+B^2x^2}\,dx\cdots\right.\\
&= -B\tan^{-1}\left(\frac{B\cos(x)}{\sqrt{A^2+B^2-B^2\cos^2(x)}}\right) - A\tanh^{-1}\left(\frac{A\cos(x)}{\sqrt{A^2+B^2-B^2\cos^2(x)}}\right)
\end{aligned}
$$

Mathematica [B] time = 0.106308, size = 99, normalized size = 2.02

$$
\sqrt{-B^2}\log\left(\sqrt{2A^2-B^2\cos(2x)+B^2}+\sqrt{2}\sqrt{-B^2}\cos(x)\right)-\sqrt{A^2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{A^2}\cos(x)}{\sqrt{2A^2-B^2\cos(2x)+B^2}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*Sqrt[A^2 + B^2*Sin[x]^2],x]
```

```
[Out] -(Sqrt[A^2]*ArcTanh[(Sqrt[2]*Sqrt[A^2]*Cos[x])/Sqrt[2*A^2 + B^2 - B^2*Cos[2
*x]]]) + Sqrt[-B^2]*Log[Sqrt[2]*Sqrt[-B^2]*Cos[x] + Sqrt[2*A^2 + B^2 - B^2*
Cos[2*x]]]
```

Maple [C] time = 0.096, size = 149, normalized size = 3.

$$-\frac{1}{2\cos(x)}\sqrt{\left(A^2+B^2(\sin(x))^2\right)(\cos(x))^2}\left(A\mathrm{csgn}(A)\ln\left(-\frac{1}{(\sin(x))^2}\left(A^2(\sin(x))^2-B^2(\sin(x))^2-2\,\mathrm{csgn}(A)\,A\sqrt{\left(A^2\right.}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x)
```

```
[Out] -1/2*((A^2+B^2*sin(x)^2)*cos(x)^2)^(1/2)*(A*csgn(A)*ln(-(A^2*sin(x)^2-B^2*s
in(x)^2-2*csgn(A)*A*((A^2+B^2*sin(x)^2)*cos(x)^2)^(1/2)-2*A^2)/sin(x)^2)-B*
csgn(B)*arctan(1/2*csgn(B)/B*(2*B^2*sin(x)^2+A^2-B^2)/((A^2+B^2*sin(x)^2)*c
os(x)^2)^(1/2)))/cos(x)/(A^2+B^2*sin(x)^2)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.99838, size = 635, normalized size = 12.96

$$\frac{1}{2}B\arctan\left(-\frac{\left(A^4+2A^2B^2+B^4\right)\cos(x)\sin(x)-2\left(2B^3\cos(x)^3-\left(A^2B+B^3\right)\cos(x)\right)\sqrt{-B^2\cos(x)^2+A^2+B^2}}{4B^4\cos(x)^4+A^4+2A^2B^2+B^4-\left(A^4+6A^2B^2+5B^4\right)\cos(x)^2}\right)-\frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="fricas")
```

```
[Out] 1/2*B*arctan(-((A^4 + 2*A^2*B^2 + B^4)*cos(x)*sin(x) - 2*(2*B^3*cos(x)^3 -
(A^2*B + B^3)*cos(x))*sqrt(-B^2*cos(x)^2 + A^2 + B^2))/(4*B^4*cos(x)^4 + A^
4 + 2*A^2*B^2 + B^4 - (A^4 + 6*A^2*B^2 + 5*B^4)*cos(x)^2)) - 1/2*B*arctan(s
in(x)/cos(x)) - 1/2*A*log(-B^2*cos(x)^2 + A*B*cos(x)*sin(x) + A^2 + B^2 + s
qrt(-B^2*cos(x)^2 + A^2 + B^2)*(A*cos(x) + B*sin(x))) + 1/2*A*log(-B^2*cos(
x)^2 - A*B*cos(x)*sin(x) + A^2 + B^2 - sqrt(-B^2*cos(x)^2 + A^2 + B^2)*(A*c
os(x) - B*sin(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{A^2 + B^2 \sin^2(x)}}{\sin(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A**2+B**2*sin(x)**2)**(1/2)/sin(x),x)
```

```
[Out] Integral(sqrt(A**2 + B**2*sin(x)**2)/sin(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{B^2 \sin(x)^2 + A^2}}{\sin(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A^2+B^2*sin(x)^2)^(1/2)/sin(x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(B^2*sin(x)^2 + A^2)/sin(x), x)
```

3.43 $\int \frac{1}{1+\cos(x)} dx$

Optimal. Leaf size=9

$$\frac{\sin(x)}{\cos(x)+1}$$

```
[Out] Sin[x]/(1 + Cos[x])
```

Rubi [A] time = 0.0075139, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2648}

$$\frac{\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x])^(-1),x]

[Out] Sin[x]/(1 + Cos[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1+\cos(x)} dx = \frac{\sin(x)}{1+\cos(x)}$$

Mathematica [A] time = 0.0036081, size = 6, normalized size = 0.67

$$\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])^(-1),x]
```

```
[Out] Tan[x/2]
```

Maple [A] time = 0., size = 5, normalized size = 0.6

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)+1),x)
```

```
[Out] tan(1/2*x)
```

Maxima [A] time = 0.918008, size = 12, normalized size = 1.33

$$\frac{\sin\left(x\right)}{\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="maxima")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Fricas [A] time = 1.94506, size = 28, normalized size = 3.11

$$\frac{\sin\left(x\right)}{\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Sympy [A] time = 0.179115, size = 3, normalized size = 0.33

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x)
```

```
[Out] tan(x/2)
```

Giac [B] time = 1.10891, size = 41, normalized size = 4.56

$$-\frac{2\,\tan\left(\frac{1}{2}\,x\right)}{\left(x^2+1\right)\left(\frac{x^2-1}{x^2+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="giac")
```

```
[Out] -2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

3.44 $\int e^x x\,dx$

Optimal. Leaf size=11

$$e^x x - e^x$$

```
[Out] -E^x + E^x*x
```

Rubi [A] time = 0.007018, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2176, 2194}

$$e^x x - e^x$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x,x]
```

```
[Out] -E^x + E^x*x
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^x x\,dx &= e^x x - \int e^x\,dx\\ &= -e^x + e^x x\end{aligned}$$

Mathematica [A] time = 0.0009849, size = 7, normalized size = 0.64

$$e^x(x-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x,x]
```

```
[Out] E^x*(-1 + x)
```

Maple [A] time = 0., size = 7, normalized size = 0.6

$$(-1+x)\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x,x)
```

```
[Out] (-1+x)*exp(x)
```

Maxima [A] time = 0.928983, size = 8, normalized size = 0.73

$$(x-1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x, algorithm="maxima")
```

```
[Out] (x - 1)*e^x
```

Fricas [A] time = 1.93113, size = 18, normalized size = 1.64

$$(x-1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x, algorithm="fricas")
```

```
[Out] (x - 1)*e^x
```

Sympy [A] time = 0.073761, size = 5, normalized size = 0.45

$$(x-1)\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x)
```

```
[Out] (x - 1)*exp(x)
```

Giac [A] time = 1.09205, size = 8, normalized size = 0.73

$$(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x, algorithm="giac")
```

```
[Out] (x - 1)*e^x
```

3.45 $\int \frac{e^x x}{(1+x)^2} \, dx$

Optimal. Leaf size=9

$$\frac{e^x}{x+1}$$

```
[Out] E^x/(1 + x)
```

Rubi [A] time = 0.0256603, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2197}

$$\frac{e^x}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x*x)/(1 + x)^2,x]
```

```
[Out] E^x/(1 + x)
```

Rule 2197

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> With[{b = Coefficient[v
, x, 1], d = Coefficient[u, x, 0], e = Coefficient[u, x, 1], f = Coefficien
t[w, x, 0], g = Coefficient[w, x, 1]}, Simp[(g*u^(m + 1)*F^(c*v))/(b*c*e*Lo
g[F]), x] /; EqQ[e*g*(m + 1) - b*c*(e*f - d*g)*Log[F], 0]] /; FreeQ[{F, c,
m}, x] && LinearQ[{u, v, w}, x]
```

Rubi steps

$$\int \frac{e^x x}{(1+x)^2} \, dx = \frac{e^x}{1+x}$$

Mathematica [A] time = 0.0229454, size = 9, normalized size = 1.

$$\frac{e^x}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x*x)/(1 + x)^2,x]
```

```
[Out] E^x/(1 + x)
```

Maple [A] time = 0.002, size = 9, normalized size = 1.

$$\frac{e^x}{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x/(1+x)^2,x)
```

```
[Out] exp(x)/(1+x)
```

Maxima [A] time = 0.942497, size = 11, normalized size = 1.22

$$\frac{e^x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x/(1+x)^2,x, algorithm="maxima")
```

```
[Out] e^x/(x + 1)
```

Fricas [A] time = 1.74625, size = 18, normalized size = 2.

$$\frac{e^x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x/(1+x)^2,x, algorithm="fricas")
```

```
[Out] e^x/(x + 1)
```

Sympy [A] time = 0.085104, size = 5, normalized size = 0.56

$$\frac{e^x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x/(1+x)**2,x)
```

```
[Out] exp(x)/(x + 1)
```

Giac [B] time = 1.10392, size = 26, normalized size = 2.89

$$\frac{e^{\left(-(x+1)\left(\frac{1}{x+1}-1\right)\right)}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x/(1+x)^2,x, algorithm="giac")
```

```
[Out] e^(-(x + 1)*(1/(x + 1) - 1))/(x + 1)
```

3.46 $\int e^{x^2} \left(1+2x^2\right) \, dx$

Optimal. Leaf size=7

$$e^{x^2} x$$

```
[Out] E^x^2*x
```

Rubi [A] time = 0.031599, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2226, 2204, 2212}

$$e^{x^2} x$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*(1 + 2*x^2),x]
```

```
[Out] E^x^2*x
```

Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
, c, d, n}, x] && PolynomialQ[u, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$
\begin{aligned}
\int e^{x^2}\left(1+2x^2\right)\,dx &= \int \left(e^{x^2}+2e^{x^2}x^2\right)\,dx\\
&= 2\int e^{x^2}x^2\,dx+\int e^{x^2}\,dx\\
&= e^{x^2}x+\frac{1}{2}\sqrt{\pi}\,\mathrm{erfi}(x)-\int e^{x^2}\,dx\\
&= e^{x^2}x
\end{aligned}
$$

Mathematica [A] time = 0.0052642, size = 7, normalized size = 1.

$$e^{x^2}x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*(1 + 2*x^2),x]
```

```
[Out] E^x^2*x
```

Maple [A] time = 0.001, size = 7, normalized size = 1.

$$\mathrm{e}^{x^2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*(2*x^2+1),x)
```

```
[Out] exp(x^2)*x
```

Maxima [A] time = 0.937613, size = 8, normalized size = 1.14

$$xe^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="maxima")
```

```
[Out] x*e^(x^2)
```

Fricas [A] time = 1.9751, size = 15, normalized size = 2.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="fricas")
```

```
[Out] x*e^(x^2)
```

Sympy [A] time = 0.080686, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*(2*x**2+1),x)
```

```
[Out] x*exp(x**2)
```

Giac [A] time = 1.06662, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="giac")
```

```
[Out] x*e^(x^2)
```

3.47 $\int e^{x^2} \, dx$

Optimal. Leaf size=11

$$\frac{1}{2}\sqrt{\pi}\text{Erfi}(x)$$

```
[Out] (Sqrt[Pi]*Erfi[x])/2
```

Rubi [A] time = 0.0018481, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2204}

$$\frac{1}{2}\sqrt{\pi}\text{Erfi}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2,x]
```

```
[Out] (Sqrt[Pi]*Erfi[x])/2
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int e^{x^2} \, dx = \frac{1}{2}\sqrt{\pi}\text{erfi}(x)$$

Mathematica [A] time = 0.0017681, size = 11, normalized size = 1.

$$\frac{1}{2}\sqrt{\pi}\text{Erfi}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2,x]
```

```
[Out] (Sqrt[Pi]*Erfi[x])/2
```

Maple [A] time = 0., size = 8, normalized size = 0.7

$$\frac{\operatorname{erfi}(x)\sqrt{\pi}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2),x)
```

```
[Out] 1/2*erfi(x)*Pi^(1/2)
```

Maxima [C] time = 0.926658, size = 12, normalized size = 1.09

$$-\frac{1}{2}i\sqrt{\pi}\operatorname{erf}(ix)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*I*sqrt(pi)*erf(I*x)
```

Fricas [A] time = 2.02875, size = 30, normalized size = 2.73

$$\frac{1}{2}\sqrt{\pi}\operatorname{erfi}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(pi)*erfi(x)
```

Sympy [A] time = 0.231137, size = 8, normalized size = 0.73

$$\frac{\sqrt{\pi}\,\operatorname{erfi}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2),x)
```

```
[Out] sqrt(pi)*erfi(x)/2
```

Giac [C] time = 1.08518, size = 12, normalized size = 1.09

$$\frac{1}{2}i\,\sqrt{\pi}\,\operatorname{erf}(-i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2),x, algorithm="giac")
```

```
[Out] 1/2*I*sqrt(pi)*erf(-I*x)
```

3.48 $\int \frac{e^x}{x} dx$

Optimal. Leaf size=2

$$\text{ExpIntegralEi}(x)$$

```
[Out] ExpIntegralEi[x]
```

Rubi [A] time = 0.0099006, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2178}

$$\text{ExpIntegralEi}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/x,x]
```

```
[Out] ExpIntegralEi[x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\int \frac{e^x}{x} dx = \text{Ei}(x)$$

Mathematica [A] time = 0.0040816, size = 2, normalized size = 1.

$$\text{ExpIntegralEi}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/x,x]
```

```
[Out] ExpIntegralEi[x]
```

Maple [B] time = 0.002, size = 8, normalized size = 4.

$$-\mathrm{Ei}\left(1,-x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/x,x)
```

```
[Out] -Ei(1,-x)
```

Maxima [A] time = 1.01328, size = 3, normalized size = 1.5

$$\mathrm{Ei}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/x,x, algorithm="maxima")
```

```
[Out] Ei(x)
```

Fricas [A] time = 1.86946, size = 9, normalized size = 4.5

$$\mathrm{Ei}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/x,x, algorithm="fricas")
```

```
[Out] Ei(x)
```

Sympy [A] time = 0.735941, size = 2, normalized size = 1.

$$\mathrm{Ei}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/x,x)
```

```
[Out] Ei(x)
```

Giac [A] time = 1.07608, size = 3, normalized size = 1.5

$$\mathrm{Ei}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/x,x, algorithm="giac")
```

```
[Out] Ei(x)
```

3.49 $\int \frac{x}{1+x^3} \, dx$

Optimal. Leaf size=41

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Rubi [A] time = 0.0220315, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {292, 31, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{1+x^3}\,dx &= -\left(\frac{1}{3}\int \frac{1}{1+x}\,dx\right)+\frac{1}{3}\int \frac{1+x}{1-x+x^2}\,dx \\
&= -\frac{1}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx \\
&= -\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x\right) \\
&= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0085541, size = 40, normalized size = 0.98

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^3),x]
```

```
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Maple [A] time = 0., size = 35, normalized size = 0.9

$$-\frac{\ln(1+x)}{3}+\frac{\ln\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3+1),x)
```

```
[Out] -1/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.40549, size = 46, normalized size = 1.12

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
x + 1)
```

Fricas [A] time = 1.9814, size = 112, normalized size = 2.73

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
x + 1)
```

Sympy [A] time = 0.121567, size = 41, normalized size = 1.

$$-\frac{\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3+1),x)
```

```
[Out] -log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/
3)/3
```

Giac [A] time = 1.0722, size = 47, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
abs(x + 1))
```

3.50 $\int \frac{1}{-1+x^6} \, dx$

Optimal. Leaf size=47

$$-\frac{\tan^{-1}\left(\frac{\sqrt{3}x}{1-x^2}\right)}{2\sqrt{3}}-\frac{1}{6}\tanh^{-1}\left(\frac{x}{x^2+1}\right)-\frac{1}{3}\tanh^{-1}(x)$$

```
[Out] -ArcTan[(Sqrt[3]*x)/(1 - x^2)]/(2*Sqrt[3]) - ArcTanh[x]/3 - ArcTanh[x/(1 +
x^2)]/6
```

Rubi [A] time = 0.104898, antiderivative size = 73, normalized size of antiderivative = 1.55, number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {210, 634, 618, 204, 628, 206}

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)^(-1),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[3
]) - ArcTanh[x]/3 + Log[1 - x + x^2]/12 - Log[1 + x + x^2]/12
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+x^6}\,dx &= -\left(\frac{1}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx\right)-\frac{1}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx-\frac{1}{3}\int \frac{1}{1-x^2}\,dx\\ &= -\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{12}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx\\ &= -\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{12}\log\left(1+x+x^2\right)+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)+\frac{1}{2}\,\mathrm{S}\\ &= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{12}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.013066, size = 75, normalized size = 1.6

$$\frac{1}{12}\left(\log\left(x^2-x+1\right)-\log\left(x^2+x+1\right)+2\log(1-x)-2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)^(-1),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]
] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])/12
```

Maple [A] time = 0.004, size = 66, normalized size = 1.4

$$\frac{\ln(-1+x)}{6}-\frac{\ln\left(x^2+x+1\right)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln(1+x)}{6}+\frac{\ln\left(x^2-x+1\right)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-1),x)
```

```
[Out] 1/6*ln(-1+x)-1/12*ln(x^2+x+1)-1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/6*l
n(1+x)+1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.41661, size = 88, normalized size = 1.87

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log(x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1
) + 1/6*log(x - 1)
```

Fricas [A] time = 1.84237, size = 230, normalized size = 4.89

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log(x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1
) + 1/6*log(x - 1)
```

Sympy [B] time = 0.233098, size = 83, normalized size = 1.77

$$\frac{\log(x-1)}{6} - \frac{\log(x+1)}{6} + \frac{\log\left(x^2-x+1\right)}{12} - \frac{\log\left(x^2+x+1\right)}{12} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-1),x)
```

```
[Out] log(x - 1)/6 - log(x + 1)/6 + log(x**2 - x + 1)/12 - log(x**2 + x + 1)/12 -
 sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + s
qrt(3)/3)/6
```

Giac [A] time = 1.06863, size = 90, normalized size = 1.91

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{12}\log\left(x^2+x+1\right) + \frac{1}{12}\log\left(x^2-x+1\right) - \frac{1}{6}\log(|x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x
 + 1)) + 1/6*log(abs(x - 1))
```

3.51 $\int \frac{1}{A^4-A^2B^2+\left(-A^2+B^2\right)x^2}\,dx$

Optimal. Leaf size=21

$$\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A\left(A^2-B^2\right)}$$

[Out] ArcTanh[x/A]/(A*(A^2 - B^2))

Rubi [A] time = 0.0094333, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {208}

$$\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A\left(A^2-B^2\right)}$$

Antiderivative was successfully verified.

[In] Int[(A^4 - A^2*B^2 + (-A^2 + B^2)*x^2)^(-1),x]

[Out] ArcTanh[x/A]/(A*(A^2 - B^2))

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{A^4-A^2B^2+\left(-A^2+B^2\right)x^2}\,dx=\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A\left(A^2-B^2\right)}$$

Mathematica [A] time = 0.0041203, size = 21, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{x}{A}\right)}{A\left(A^2-B^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A^4 - A^2*B^2 + (-A^2 + B^2)*x^2)^(-1),x]
```

```
[Out] ArcTanh[x/A]/(A*(A^2 - B^2))
```

Maple [B] time = 0.005, size = 44, normalized size = 2.1

$$\frac{\ln(A+x)}{(2A^2-2B^2)A}-\frac{\ln(A-x)}{(2A^2-2B^2)A}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x)
```

```
[Out] 1/2/(A^2-B^2)/A*ln(A+x)-1/2/(A^2-B^2)/A*ln(A-x)
```

Maxima [A] time = 0.931059, size = 53, normalized size = 2.52

$$\frac{\log(A+x)}{2(A^3-AB^2)}-\frac{\log(-A+x)}{2(A^3-AB^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(A + x)/(A^3 - A*B^2) - 1/2*log(-A + x)/(A^3 - A*B^2)
```

Fricas [A] time = 1.81271, size = 62, normalized size = 2.95

$$\frac{\log(A+x)-\log(-A+x)}{2(A^3-AB^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(A + x) - log(-A + x))/(A^3 - A*B^2)
```

Sympy [B] time = 0.271437, size = 70, normalized size = 3.33

$$-\frac{\log\left(-\frac{A^3}{(A-B)(A+B)}+\frac{AB^2}{(A-B)(A+B)}+x\right)}{2A(A-B)(A+B)}+\frac{\log\left(\frac{A^3}{(A-B)(A+B)}-\frac{AB^2}{(A-B)(A+B)}+x\right)}{2A(A-B)(A+B)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A**4-A**2*B**2+(-A**2+B**2)*x**2),x)
```

```
[Out] -log(-A**3/((A - B)*(A + B)) + A*B**2/((A - B)*(A + B)) + x)/(2*A*(A - B)*(
A + B)) + log(A**3/((A - B)*(A + B)) - A*B**2/((A - B)*(A + B)) + x)/(2*A*(
A - B)*(A + B))
```

Giac [A] time = 1.07611, size = 55, normalized size = 2.62

$$\frac{\log(|A+x|)}{2\left(A^3-AB^2\right)}-\frac{\log(|-A+x|)}{2\left(A^3-AB^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A^4-A^2*B^2+(-A^2+B^2)*x^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(A + x))/(A^3 - A*B^2) - 1/2*log(abs(-A + x))/(A^3 - A*B^2)
```

3.52 $\int x \log(x)\, dx$

Optimal. Leaf size=17

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rubi [A] time = 0.0039045, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2304}

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x],x]
```

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x \log(x)\, dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

Mathematica [A] time = 0.0006047, size = 17, normalized size = 1.

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x],x]
```

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Maple [A] time = 0., size = 14, normalized size = 0.8

$$-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x),x)
```

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Maxima [A] time = 0.924389, size = 18, normalized size = 1.06

$$\frac{1}{2} x^2 \log(x) - \frac{1}{4} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Fricas [A] time = 2.00152, size = 35, normalized size = 2.06

$$\frac{1}{2} x^2 \log(x) - \frac{1}{4} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A] time = 0.079624, size = 12, normalized size = 0.71

$$\frac{x^2\log(x)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x),x)
```

```
[Out] x**2*log(x)/2 - x**2/4
```

Giac [A] time = 1.09076, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x^2\log(x) - \frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

3.53 $\int x^2 \sin^{-1}(x)\,dx$

Optimal. Leaf size=40

$$-\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{\sqrt{1-x^2}}{3}+\frac{1}{3}x^3\sin^{-1}(x)$$

```
[Out] Sqrt[1 - x^2]/3 - (1 - x^2)^(3/2)/9 + (x^3*ArcSin[x])/3
```

Rubi [A] time = 0.0241101, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4627, 266, 43}

$$-\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{\sqrt{1-x^2}}{3}+\frac{1}{3}x^3\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSin[x],x]
```

```
[Out] Sqrt[1 - x^2]/3 - (1 - x^2)^(3/2)/9 + (x^3*ArcSin[x])/3
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^2 \sin^{-1}(x)\,dx &= \frac{1}{3}x^3\sin^{-1}(x) - \frac{1}{3}\int \frac{x^3}{\sqrt{1-x^2}}\,dx \\
&= \frac{1}{3}x^3\sin^{-1}(x) - \frac{1}{6}\,\text{Subst}\left(\int \frac{x}{\sqrt{1-x}}\,dx, x, x^2\right) \\
&= \frac{1}{3}x^3\sin^{-1}(x) - \frac{1}{6}\,\text{Subst}\left(\int \left(\frac{1}{\sqrt{1-x}} - \sqrt{1-x}\right)dx, x, x^2\right) \\
&= \frac{\sqrt{1-x^2}}{3} - \frac{1}{9}\left(1-x^2\right)^{3/2} + \frac{1}{3}x^3\sin^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.0117271, size = 29, normalized size = 0.72

$$\frac{1}{9}\left(\sqrt{1-x^2}\left(x^2+2\right)+3x^3\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSin[x],x]
```

```
[Out] (Sqrt[1 - x^2]*(2 + x^2) + 3*x^3*ArcSin[x])/9
```

Maple [A] time = 0., size = 34, normalized size = 0.9

$$\frac{x^3\arcsin(x)}{3}+\frac{x^2}{9}\sqrt{-x^2+1}+\frac{2}{9}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsin(x),x)
```

```
[Out] 1/3*x^3*arcsin(x)+1/9*x^2*(-x^2+1)^(1/2)+2/9*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.40179, size = 45, normalized size = 1.12

$$\frac{1}{3}x^3\arcsin(x)+\frac{1}{9}\sqrt{-x^2+1}x^2+\frac{2}{9}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsin(x) + 1/9*sqrt(-x^2 + 1)*x^2 + 2/9*sqrt(-x^2 + 1)
```

<u>Fricas</u> [A] time = 2.04512, size = 68, normalized size = 1.7

$$\frac{1}{3}\,x^{3}\arcsin\left(x\right)+\frac{1}{9}\,\left(x^{2}+2\right)\sqrt{-x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arcsin(x) + 1/9*(x^2 + 2)*sqrt(-x^2 + 1)
```

<u>Sympy</u> [A] time = 0.320597, size = 32, normalized size = 0.8

$$\frac{x^{3}\operatorname{asin}\left(x\right)}{3}+\frac{x^{2}\sqrt{1-x^{2}}}{9}+\frac{2\sqrt{1-x^{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asin(x),x)
```

```
[Out] x**3*asin(x)/3 + x**2*sqrt(1 - x**2)/9 + 2*sqrt(1 - x**2)/9
```

<u>Giac</u> [A] time = 1.07506, size = 51, normalized size = 1.27

$$\frac{1}{3}\,\left(x^{2}-1\right)x\arcsin\left(x\right)+\frac{1}{3}\,x\arcsin\left(x\right)-\frac{1}{9}\,\left(-x^{2}+1\right)^{\frac{3}{2}}+\frac{1}{3}\,\sqrt{-x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 - 1)*x*arcsin(x) + 1/3*x*arcsin(x) - 1/9*(-x^2 + 1)^(3/2) + 1/3*sq
rt(-x^2 + 1)
```

3.54 $\int \frac{1}{1+2x+x^2} dx$

Optimal. Leaf size=7

$$-\frac{1}{x+1}$$

```
[Out] -(1 + x)^(-1)
```

Rubi [A] time = 0.0010104, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {27, 32}

$$-\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x + x^2)^(-1),x]
```

```
[Out] -(1 + x)^(-1)
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{1+2x+x^2} dx &= \int \frac{1}{(1+x)^2} dx \\ &= -\frac{1}{1+x} \end{aligned}$$

Mathematica [A] time = 0.0008212, size = 7, normalized size = 1.

$$-\frac{1}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x + x^2)^(-1),x]
```

```
[Out] -(1 + x)^(-1)
```

Maple [A] time = 0.002, size = 8, normalized size = 1.1

$$-(1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+2*x+1),x)
```

```
[Out] -1/(1+x)
```

Maxima [A] time = 0.924597, size = 9, normalized size = 1.29

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+1),x, algorithm="maxima")
```

```
[Out] -1/(x + 1)
```

Fricas [A] time = 2.02132, size = 16, normalized size = 2.29

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+1),x, algorithm="fricas")
```

```
[Out] -1/(x + 1)
```

Sympy [A] time = 0.070916, size = 5, normalized size = 0.71

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+2*x+1),x)
```

```
[Out] -1/(x + 1)
```

Giac [A] time = 1.0837, size = 9, normalized size = 1.29

$$-\frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+1),x, algorithm="giac")
```

```
[Out] -1/(x + 1)
```

3.55 $\int \frac{\log(x)}{(1+\log(x))^2} \, dx$

Optimal. Leaf size=8

$$\frac{x}{\log(x)+1}$$

```
[Out] x/(1 + Log[x])
```

Rubi [A] time = 0.0441281, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {2360, 2297, 2299, 2178}

$$\frac{x}{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(1 + Log[x])^2,x]
```

```
[Out] x/(1 + Log[x])
```

Rule 2360

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(Log[(c_.)*(x_)^(n_.)]*(e_.
) + (d_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*x^n])^p*(d +
 e*Log[c*x^n])^q, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IntegerQ[p] &&
IntegerQ[q]
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log(x)}{(1+\log(x))^2}\,dx &= \int \left(-\frac{1}{(1+\log(x))^2}+\frac{1}{1+\log(x)}\right)dx \\
&= -\int \frac{1}{(1+\log(x))^2}\,dx+\int \frac{1}{1+\log(x)}\,dx \\
&= \frac{x}{1+\log(x)}-\int \frac{1}{1+\log(x)}\,dx+\text{Subst}\left(\int \frac{e^x}{1+x}\,dx,x,\log(x)\right) \\
&= \frac{\mathbf{Ei}(1+\log(x))}{e}+\frac{x}{1+\log(x)}-\text{Subst}\left(\int \frac{e^x}{1+x}\,dx,x,\log(x)\right) \\
&= \frac{x}{1+\log(x)}
\end{aligned}
$$

Mathematica [A] time = 0.0134753, size = 8, normalized size = 1.

$$\frac{x}{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(1 + Log[x])^2,x]
```

```
[Out] x/(1 + Log[x])
```

Maple [A] time = 0.023, size = 9, normalized size = 1.1

$$\frac{x}{1+\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(1+ln(x))^2,x)
```

```
[Out] x/(1+ln(x))
```

Maxima [A] time = 0.930617, size = 11, normalized size = 1.38

$$\frac{x}{\log(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(1+log(x))^2,x, algorithm="maxima")
```

```
[Out] x/(log(x) + 1)
```

Fricas [A] time = 1.95641, size = 22, normalized size = 2.75

$$\frac{x}{\log(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(1+log(x))^2,x, algorithm="fricas")
```

```
[Out] x/(log(x) + 1)
```

Sympy [A] time = 0.082002, size = 5, normalized size = 0.62

$$\frac{x}{\log(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(1+ln(x))**2,x)
```

```
[Out] x/(log(x) + 1)
```

Giac [A] time = 1.07624, size = 11, normalized size = 1.38

$$\frac{x}{\log(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(1+log(x))^2,x, algorithm="giac")
```

```
[Out] x/(log(x) + 1)
```

3.56 $\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\log(x))$$

```
[Out] ArcTan[Log[x]]
```

Rubi [A] time = 0.0196644, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {203}

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x\left(1+\log^2(x)\right)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \log(x)\right) \\ &= \tan^{-1}(\log(x)) \end{aligned}$$

Mathematica [A] time = 0.0108006, size = 3, normalized size = 1.

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Maple [A] time = 0.002, size = 4, normalized size = 1.3

$$\arctan(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+ln(x)^2),x)
```

```
[Out] arctan(ln(x))
```

Maxima [A] time = 1.40457, size = 4, normalized size = 1.33

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(log(x))
```

Fricas [A] time = 1.92429, size = 22, normalized size = 7.33

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(log(x))
```

Sympy [B] time = 0.123049, size = 15, normalized size = 5.

$$\text{RootSum}\left(4z^2+1,\left(i\mapsto i\log\left(2i+\log\left(x\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+ln(x)**2),x)
```

```
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

Giac [A] time = 1.08233, size = 4, normalized size = 1.33

$$\arctan\left(\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="giac")
```

```
[Out] arctan(log(x))
```

3.57 $\int \frac{1}{\log(x)} dx$

Optimal. Leaf size=2

$$\text{LogIntegral}(x)$$

```
[Out] LogIntegral[x]
```

Rubi [A] time = 0.001793, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {2298}

$$\text{LogIntegral}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^(-1),x]

[Out] LogIntegral[x]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\int \frac{1}{\log(x)} dx = \text{li}(x)$$

Mathematica [F] time = 0.0021106, size = 0, normalized size = 0.

$$\int \frac{1}{\log(x)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[x]^(-1),x]
```

```
[Out] Integrate[Log[x]^(-1), x]
```

Maple [B] time = 0., size = 9, normalized size = 4.5

$$-\mathrm{Ei}\left(1,-\ln\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(x),x)
```

```
[Out] -Ei(1,-ln(x))
```

Maxima [A] time = 1.0224, size = 4, normalized size = 2.

$$\mathrm{Ei}\left(\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="maxima")
```

```
[Out] Ei(log(x))
```

Fricas [A] time = 1.80083, size = 23, normalized size = 11.5

$$\mathrm{log\_integral}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="fricas")
```

```
[Out] log_integral(x)
```

Sympy [A] time = 0.439018, size = 2, normalized size = 1.

$$\text{li}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(x),x)
```

```
[Out] li(x)
```

Giac [A] time = 1.12029, size = 4, normalized size = 2.

$$\text{Ei}(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="giac")
```

```
[Out] Ei(log(x))
```

3.58 $\int x(\cos(x) + \sin(x))\,dx$

Optimal. Leaf size=14

$$x\sin(x) + \sin(x) - x\cos(x) + \cos(x)$$

```
[Out] Cos[x] - x*Cos[x] + Sin[x] + x*Sin[x]
```

Rubi [A] time = 0.0214771, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {14, 3296, 2638, 2637}

$$x\sin(x) + \sin(x) - x\cos(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*(Cos[x] + Sin[x]),x]
```

```
[Out] Cos[x] - x*Cos[x] + Sin[x] + x*Sin[x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int x(\cos(x)+\sin(x))\,dx &= \int (x\cos(x)+x\sin(x))\,dx\\
&= \int x\cos(x)\,dx+\int x\sin(x)\,dx\\
&= -x\cos(x)+x\sin(x)+\int \cos(x)\,dx-\int \sin(x)\,dx\\
&= \cos(x)-x\cos(x)+\sin(x)+x\sin(x)
\end{aligned}
$$

Mathematica [A] time = 0.0045986, size = 14, normalized size = 1.

$$x\sin(x)+\sin(x)-x\cos(x)+\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(Cos[x] + Sin[x]),x]
```

```
[Out] Cos[x] - x*Cos[x] + Sin[x] + x*Sin[x]
```

Maple [A] time = 0.011, size = 15, normalized size = 1.1

$$\cos(x)-x\cos(x)+\sin(x)+x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(cos(x)+sin(x)),x)
```

```
[Out] cos(x)-x*cos(x)+sin(x)+x*sin(x)
```

Maxima [A] time = 0.938204, size = 19, normalized size = 1.36

$$-x\cos(x)+x\sin(x)+\cos(x)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] -x*cos(x) + x*sin(x) + cos(x) + sin(x)
```

Fricas [A] time = 1.85592, size = 46, normalized size = 3.29

$$-(x-1)\cos(x)+(x+1)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] -(x - 1)*cos(x) + (x + 1)*sin(x)
```

Sympy [A] time = 0.165906, size = 15, normalized size = 1.07

$$x\sin(x)-x\cos(x)+\sin(x)+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x)+sin(x)),x)
```

```
[Out] x*sin(x) - x*cos(x) + sin(x) + cos(x)
```

Giac [A] time = 1.06894, size = 19, normalized size = 1.36

$$-x\cos(x)+x\sin(x)+\cos(x)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] -x*cos(x) + x*sin(x) + cos(x) + sin(x)
```

3.59 $\int e^{-x}(e^x + x)\,dx$

Optimal. Leaf size=17

$$-e^{-x}x + x - e^{-x}$$

```
[Out] -E^(-x) + x - x/E^x
```

Rubi [A] time = 0.0313499, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {6742, 2176, 2194}

$$-e^{-x}x + x - e^{-x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x + x)/E^x,x]
```

```
[Out] -E^(-x) + x - x/E^x
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int e^{-x}(e^x + x)\, dx &= \int (1 + e^{-x}x)\, dx \\
&= x + \int e^{-x}x\, dx \\
&= x - e^{-x}x + \int e^{-x}\, dx \\
&= -e^{-x} + x - e^{-x}x
\end{aligned}
$$

Mathematica [A] time = 0.0099566, size = 13, normalized size = 0.76

$$e^{-x}(-x-1) + x$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x + x)/E^x,x]
```

```
[Out] (-1 - x)/E^x + x
```

Maple [A] time = 0.001, size = 16, normalized size = 0.9

$$-\left(e^x\right)^{-1} + x - \frac{x}{e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((exp(x)+x)/exp(x),x)
```

```
[Out] -1/exp(x)+x-x/exp(x)
```

Maxima [A] time = 0.932095, size = 15, normalized size = 0.88

$$-(x+1)e^{(-x)} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))/exp(x),x, algorithm="maxima")
```

```
[Out] -(x + 1)*e^(-x) + x
```

Fricas [A] time = 1.87756, size = 32, normalized size = 1.88

$$(xe^x - x - 1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))/exp(x),x, algorithm="fricas")
```

```
[Out] (x*e^x - x - 1)*e^(-x)
```

Sympy [A] time = 0.081333, size = 8, normalized size = 0.47

$$x + (-x - 1)\, e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))/exp(x),x)
```

```
[Out] x + (-x - 1)*exp(-x)
```

Giac [A] time = 1.099, size = 15, normalized size = 0.88

$$-(x + 1)e^{(-x)} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+exp(x))/exp(x),x, algorithm="giac")
```

```
[Out] -(x + 1)*e^(-x) + x
```

3.60 $\int (1+e^x)^2 x\,dx$

Optimal. Leaf size=38

$$\frac{x^2}{2}+2e^x x+\frac{1}{2}e^{2x}x-2e^x-\frac{e^{2x}}{4}$$

```
[Out] -2*E^x - E^(2*x)/4 + 2*E^x*x + (E^(2*x)*x)/2 + x^2/2
```

Rubi [A] time = 0.0326818, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2183, 2176, 2194}

$$\frac{x^2}{2}+2e^x x+\frac{1}{2}e^{2x}x-2e^x-\frac{e^{2x}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + E^x)^2*x,x]
```

```
[Out] -2*E^x - E^(2*x)/4 + 2*E^x*x + (E^(2*x)*x)/2 + x^2/2
```

Rule 2183

```
Int[((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))^(p_.)*((c_.) +
 (d_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*(F
^(g*(e + f*x)))^n)^p, x], x] /; FreeQ[{F, a, b, c, d, e, f, g, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int (1+e^x)^2 x\,dx &= \int \left(x+2e^x x+e^{2x}x\right)\,dx \\
&= \frac{x^2}{2}+2\int e^x x\,dx+\int e^{2x}x\,dx \\
&= 2e^x x+\frac{1}{2}e^{2x}x+\frac{x^2}{2}-\frac{1}{2}\int e^{2x}\,dx-2\int e^x\,dx \\
&= -2e^x-\frac{e^{2x}}{4}+2e^x x+\frac{1}{2}e^{2x}x+\frac{x^2}{2}
\end{aligned}
$$

Mathematica [A] time = 0.0256854, size = 29, normalized size = 0.76

$$\frac{1}{4}\left(2x^2+8e^x(x-1)+e^{2x}(2x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + E^x)^2*x,x]
```

```
[Out] (8*E^x*(-1 + x) + 2*x^2 + E^(2*x)*(-1 + 2*x))/4
```

Maple [A] time = 0.002, size = 29, normalized size = 0.8

$$\frac{x^2}{2}+\frac{\left(e^x\right)^2 x}{2}-\frac{\left(e^x\right)^2}{4}+2\,e^x x-2\,e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+exp(x))^2*x,x)
```

```
[Out] 1/2*x^2+1/2*exp(x)^2*x-1/4*exp(x)^2+2*exp(x)*x-2*exp(x)
```

Maxima [A] time = 0.936797, size = 32, normalized size = 0.84

$$\frac{1}{2}\,x^2+\frac{1}{4}\,(2\,x-1)e^{(2\,x)}+2\,(x-1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))^2*x,x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

Fricas [A] time = 1.9211, size = 66, normalized size = 1.74

$$\frac{1}{2}x^2 + \frac{1}{4}(2x - 1)e^{(2x)} + 2(x - 1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))^2*x,x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

Sympy [A] time = 0.088345, size = 26, normalized size = 0.68

$$\frac{x^2}{2} + \frac{(2x - 1)e^{2x}}{4} + \frac{(8x - 8)e^x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))**2*x,x)
```

```
[Out] x**2/2 + (2*x - 1)*exp(2*x)/4 + (8*x - 8)*exp(x)/4
```

Giac [A] time = 1.06624, size = 32, normalized size = 0.84

$$\frac{1}{2}x^2 + \frac{1}{4}(2x - 1)e^{(2x)} + 2(x - 1)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(x))^2*x,x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 1/4*(2*x - 1)*e^(2*x) + 2*(x - 1)*e^x
```

3.61 $\int x\cos(x)\,dx$

Optimal. Leaf size=7

$$x\sin(x)+\cos(x)$$

```
[Out] Cos[x] + x*Sin[x]
```

Rubi [A] time = 0.0087321, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3296, 2638}

$$x\sin(x)+\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cos(x)\,dx &= x\sin(x)-\int\sin(x)\,dx\\ &= \cos(x)+x\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0020944, size = 7, normalized size = 1.

$$x\sin(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Maple [A] time = 0., size = 8, normalized size = 1.1

$$\cos(x) + x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)
```

```
[Out] cos(x)+x*sin(x)
```

Maxima [A] time = 0.935171, size = 9, normalized size = 1.29

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="maxima")
```

```
[Out] x*sin(x) + cos(x)
```

Fricas [A] time = 1.90491, size = 26, normalized size = 3.71

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="fricas")
```

```
[Out] x*sin(x) + cos(x)
```

Sympy [A] time = 0.167128, size = 7, normalized size = 1.

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x)
```

```
[Out] x*sin(x) + cos(x)
```

Giac [A] time = 1.06088, size = 9, normalized size = 1.29

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="giac")
```

```
[Out] x*sin(x) + cos(x)
```

3.62 $\int \cos\left(\sqrt{x}\right)\,dx$

Optimal. Leaf size=22

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rubi [A] time = 0.0117688, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3362, 3296, 2638}

$$2\sqrt{x}\sin\left(\sqrt{x}\right)+2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Sqrt[x]],x]
```

```
[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Rule 3362

```
Int[((a_.) + Cos[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)]*(b_.))^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Cos[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \cos\left(\sqrt{x}\right)\, dx = 2\,\text{Subst}\left(\int x\cos(x)\,dx, x, \sqrt{x}\right)$$
$$= 2\sqrt{x}\sin\left(\sqrt{x}\right) - 2\,\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{x}\right)$$
$$= 2\cos\left(\sqrt{x}\right) + 2\sqrt{x}\sin\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.008908, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Sqrt[x]],x]

[Out] 2*Cos[Sqrt[x]] + 2*Sqrt[x]*Sin[Sqrt[x]]
```

Maple [A] time = 0., size = 17, normalized size = 0.8

$$2\,\cos\left(\sqrt{x}\right) + 2\,\sin\left(\sqrt{x}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x^(1/2)),x)

[Out] 2*cos(x^(1/2))+2*sin(x^(1/2))*x^(1/2)
```

Maxima [A] time = 0.941327, size = 22, normalized size = 1.

$$2\,\sqrt{x}\sin\left(\sqrt{x}\right) + 2\,\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Fricas [A] time = 2.05377, size = 55, normalized size = 2.5

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Sympy [A] time = 0.292586, size = 20, normalized size = 0.91

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

Giac [A] time = 1.06991, size = 22, normalized size = 1.

$$2\sqrt{x}\sin\left(\sqrt{x}\right) + 2\cos\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x)*sin(sqrt(x)) + 2*cos(sqrt(x))
```

3.63 $\int x \cos(x)\,dx$

Optimal. Leaf size=7

$$x\sin(x) + \cos(x)$$

```
[Out] Cos[x] + x*Sin[x]
```

Rubi [A] time = 0.0090752, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3296, 2638}

$$x\sin(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cos(x)\,dx &= x\sin(x) - \int \sin(x)\,dx\\ &= \cos(x) + x\sin(x)\end{aligned}$$

Mathematica [A] time = 0.002008, size = 7, normalized size = 1.

$$x\sin(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Maple [A] time = 0., size = 8, normalized size = 1.1

$$\cos(x) + x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)
```

```
[Out] cos(x)+x*sin(x)
```

Maxima [A] time = 0.932358, size = 9, normalized size = 1.29

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="maxima")
```

```
[Out] x*sin(x) + cos(x)
```

Fricas [A] time = 2.00763, size = 26, normalized size = 3.71

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="fricas")
```

```
[Out] x*sin(x) + cos(x)
```

Sympy [A] time = 0.161085, size = 7, normalized size = 1.

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x)
```

```
[Out] x*sin(x) + cos(x)
```

Giac [A] time = 1.06041, size = 9, normalized size = 1.29

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="giac")
```

```
[Out] x*sin(x) + cos(x)
```

3.64 $\int x \log^2(x) \, dx$

Optimal. Leaf size=28

$$\frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rubi [A] time = 0.0097481, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2305, 2304}

$$\frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x \log^2(x) \, dx &= \frac{1}{2}x^2 \log^2(x) - \int x \log(x) \, dx \\
&= \frac{x^2}{4} - \frac{1}{2}x^2 \log(x) + \frac{1}{2}x^2 \log^2(x)
\end{aligned}$$

Mathematica [A] time = 0.0008745, size = 28, normalized size = 1.

$$\frac{x^2}{4}+\frac{1}{2}x^2\log^2(x)-\frac{1}{2}x^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Maple [A] time = 0.002, size = 23, normalized size = 0.8

$$\frac{x^2}{4}-\frac{x^2\ln(x)}{2}+\frac{x^2(\ln(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x)^2,x)
```

```
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Maxima [A] time = 0.925699, size = 23, normalized size = 0.82

$$\frac{1}{4}\left(2\log(x)^2-2\log(x)+1\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

Fricas [A] time = 1.81658, size = 61, normalized size = 2.18

$$\frac{1}{2}x^2\log(x)^2-\frac{1}{2}x^2\log(x)+\frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Sympy [A] time = 0.094922, size = 22, normalized size = 0.79

$$\frac{x^2 \log(x)^2}{2} - \frac{x^2 \log(x)}{2} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x)**2,x)
```

```
[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

Giac [A] time = 1.05854, size = 30, normalized size = 1.07

$$\frac{1}{2} x^2 \log(x)^2 - \frac{1}{2} x^2 \log(x) + \frac{1}{4} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

3.65 $\int \cos(x)\left(1+\sin^3(x)\right)\,dx$

Optimal. Leaf size=11

$$\frac{\sin^4(x)}{4}+\sin(x)$$

```
[Out] Sin[x] + Sin[x]^4/4
```

Rubi [A] time = 0.0120964, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3223}

$$\frac{\sin^4(x)}{4}+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(1 + Sin[x]^3),x]
```

```
[Out] Sin[x] + Sin[x]^4/4
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\begin{aligned}\int \cos(x)\left(1+\sin^3(x)\right)\,dx &= \text{Subst}\left(\int \left(1+x^3\right)\,dx,x,\sin(x)\right)\\ &= \sin(x)+\frac{\sin^4(x)}{4}\end{aligned}$$

Mathematica [A] time = 0.0029038, size = 11, normalized size = 1.

$$\frac{\sin^4(x)}{4} + \sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(1 + Sin[x]^3),x]
```

```
[Out] Sin[x] + Sin[x]^4/4
```

Maple [A] time = 0.013, size = 10, normalized size = 0.9

$$\sin(x) + \frac{(\sin(x))^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(1+sin(x)^3),x)
```

```
[Out] sin(x)+1/4*sin(x)^4
```

Maxima [A] time = 0.916528, size = 12, normalized size = 1.09

$$\frac{1}{4}\sin(x)^4 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^3),x, algorithm="maxima")
```

```
[Out] 1/4*sin(x)^4 + sin(x)
```

Fricas [A] time = 2.01435, size = 51, normalized size = 4.64

$$\frac{1}{4}\cos(x)^4 - \frac{1}{2}\cos(x)^2 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^3),x, algorithm="fricas")
```

```
[Out] 1/4*cos(x)^4 - 1/2*cos(x)^2 + sin(x)
```

Sympy [A] time = 0.536143, size = 8, normalized size = 0.73

$$\frac{\sin^4(x)}{4} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)**3),x)
```

```
[Out] sin(x)**4/4 + sin(x)
```

Giac [A] time = 1.04765, size = 12, normalized size = 1.09

$$\frac{1}{4}\sin(x)^4 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^3),x, algorithm="giac")
```

```
[Out] 1/4*sin(x)^4 + sin(x)
```

3.66 $\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}(\log(x))$$

```
[Out] ArcTan[Log[x]]
```

Rubi [A] time = 0.0190214, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {203}

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(1+\log^2(x)\right)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \log(x)\right)\\ &= \tan^{-1}(\log(x))\end{aligned}$$

Mathematica [A] time = 0.0078829, size = 3, normalized size = 1.

$$\tan^{-1}(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + Log[x]^2)),x]
```

```
[Out] ArcTan[Log[x]]
```

Maple [A] time = 0., size = 4, normalized size = 1.3

$$\arctan(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+ln(x)^2),x)
```

```
[Out] arctan(ln(x))
```

Maxima [A] time = 1.4175, size = 4, normalized size = 1.33

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(log(x))
```

Fricas [A] time = 1.79788, size = 22, normalized size = 7.33

$$\arctan(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="fricas")
```

```
[Out] arctan(log(x))
```

Sympy [B] time = 0.124024, size = 15, normalized size = 5.

$$\operatorname{RootSum}\left(4z^2+1,\left(i\mapsto i\log\left(2i+\log\left(x\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+ln(x)**2),x)
```

```
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + log(x))))
```

Giac [A] time = 1.03959, size = 4, normalized size = 1.33

$$\arctan\left(\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+log(x)^2),x, algorithm="giac")
```

```
[Out] arctan(log(x))
```

3.67 $\int \frac{1}{\sqrt{1-x^2}\left(1+\sin^{-1}(x)^2\right)}\,dx$

Optimal. Leaf size=3

$$\tan^{-1}\left(\sin^{-1}(x)\right)$$

```
[Out] ArcTan[ArcSin[x]]
```

Rubi [A] time = 0.0469675, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6696, 203}

$$\tan^{-1}\left(\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - x^2]*(1 + ArcSin[x]^2)),x]
```

```
[Out] ArcTan[ArcSin[x]]
```

Rule 6696

```
Int[(u_.)*((a_.) + (b_.)*(y_)^(n_))^(p_), x_Symbol] :> With[{q = Derivative
Divides[y, u, x]}, Dist[q, Subst[Int[(a + b*x^n)^p, x], x, y], x] /;  !Fals
eQ[q]] /; FreeQ[{a, b, n, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-x^2}\left(1+\sin^{-1}(x)^2\right)}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin^{-1}(x)\right)\\ &= \tan^{-1}\left(\sin^{-1}(x)\right)\end{aligned}$$

Mathematica [A] time = 0.05518, size = 3, normalized size = 1.

$$\tan^{-1}\left(\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - x^2]*(1 + ArcSin[x]^2)),x]
```

```
[Out] ArcTan[ArcSin[x]]
```

Maple [A] time = 0.005, size = 4, normalized size = 1.3

$$\arctan(\arcsin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x)
```

```
[Out] arctan(arcsin(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2+1}\left(\arcsin(x)^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-x^2 + 1)*(arcsin(x)^2 + 1)), x)
```

Fricas [A] time = 2.15427, size = 26, normalized size = 8.67

$$\arctan(\arcsin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(arcsin(x))
```

Sympy [A] time = 0.41762, size = 3, normalized size = 1.

$$\operatorname{atan}(\operatorname{asin}(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+asin(x)**2)/(-x**2+1)**(1/2),x)
```

```
[Out] atan(asin(x))
```

Giac [A] time = 1.06363, size = 4, normalized size = 1.33

$$\arctan(\arcsin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+arcsin(x)^2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(arcsin(x))
```

3.68 $\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=16

$$\frac{x}{2}-\frac{1}{2}\log(\sin(x)+\cos(x))$$

```
[Out] x/2 - Log[Cos[x] + Sin[x]]/2
```

Rubi [A] time = 0.0411408, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3097, 3133}

$$\frac{x}{2}-\frac{1}{2}\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(Cos[x] + Sin[x]),x]
```

```
[Out] x/2 - Log[Cos[x] + Sin[x]]/2
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\sin(x)}{\cos(x)+\sin(x)}\,dx = \frac{x}{2} - \frac{1}{2}\int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\,dx$$
$$= \frac{x}{2} - \frac{1}{2}\log(\cos(x)+\sin(x))$$

Mathematica [A] time = 0.0232694, size = 16, normalized size = 1.

$$\frac{x}{2} - \frac{1}{2}\log(\sin(x)+\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(Cos[x] + Sin[x]),x]
```

```
[Out] x/2 - Log[Cos[x] + Sin[x]]/2
```

Maple [A] time = 0.023, size = 21, normalized size = 1.3

$$\frac{\ln\left((\tan(x))^2+1\right)}{4} - \frac{\ln(1+\tan(x))}{2} + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)+sin(x)),x)
```

```
[Out] 1/4*ln(tan(x)^2+1)-1/2*ln(1+tan(x))+1/2*x
```

Maxima [B] time = 1.41169, size = 72, normalized size = 4.5

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) - \frac{1}{2}\log\left(-\frac{2\sin(x)}{\cos(x)+1} + \frac{\sin(x)^2}{(\cos(x)+1)^2} - 1\right) + \frac{1}{2}\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] arctan(sin(x)/(cos(x) + 1)) - 1/2*log(-2*sin(x)/(cos(x) + 1) + sin(x)^2/(co
s(x) + 1)^2 - 1) + 1/2*log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 2.27533, size = 51, normalized size = 3.19

$$\frac{1}{2}x - \frac{1}{4}\log\left(2\cos\left(x\right)\sin\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x - 1/4*log(2*cos(x)*sin(x) + 1)
```

Sympy [A] time = 0.132393, size = 12, normalized size = 0.75

$$\frac{x}{2} - \frac{\log\left(\sin\left(x\right) + \cos\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)+sin(x)),x)
```

```
[Out] x/2 - log(sin(x) + cos(x))/2
```

Giac [A] time = 1.08967, size = 28, normalized size = 1.75

$$\frac{1}{2}x + \frac{1}{4}\log\left(\tan\left(x\right)^{2} + 1\right) - \frac{1}{2}\log\left(\left|\tan\left(x\right) + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*log(tan(x)^2 + 1) - 1/2*log(abs(tan(x) + 1))
```

3.69 $\int -\frac{\sqrt{A^2+B^2(1-y^2)}}{1-y^2} \, dy$

Optimal. Leaf size=53

$$-B\tan^{-1}\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)-A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

```
[Out] -(B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]]) - A*ArcTanh[(A*y)/Sqrt[A^2 + B
^2 - B^2*y^2]]
```

Rubi [A] time = 0.0648788, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1974, 402, 217, 203, 377, 206}

$$-B\tan^{-1}\left(\frac{By}{\sqrt{A^2-B^2y^2+B^2}}\right)-A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2-B^2y^2+B^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[-(Sqrt[A^2 + B^2*(1 - y^2)]/(1 - y^2)),y]
```

```
[Out] -(B*ArcTan[(B*y)/Sqrt[A^2 + B^2 - B^2*y^2]]) - A*ArcTanh[(A*y)/Sqrt[A^2 + B
^2 - B^2*y^2]]
```

Rule 1974

```
Int[(u_)^(p_.)*(v_)^(q_.), x_Symbol] :> Int[ExpandToSum[u, x]^p*ExpandToSum
[v, x]^q, x] /; FreeQ[{p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDeg
ree[u, x] - BinomialDegree[v, x], 0] && !BinomialMatchQ[{u, v}, x]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int -\frac{\sqrt{A^2+B^2\left(1-y^2\right)}}{1-y^2}\,dy &= -\int \frac{\sqrt{A^2+B^2-B^2y^2}}{1-y^2}\,dy \\
&= -\left(A^2\int \frac{1}{\left(1-y^2\right)\sqrt{A^2+B^2-B^2y^2}}\,dy\right)-B^2\int \frac{1}{\sqrt{A^2+B^2-B^2y^2}}\,dy \\
&= -\left(A^2\,\text{Subst}\left(\int \frac{1}{1-A^2y^2}\,dy,y,\frac{y}{\sqrt{A^2+B^2-B^2y^2}}\right)\right)-B^2\,\text{Subst}\left(\int \frac{1}{1+B^2y^2}\,dy,y,\frac{}{\sqrt{A^2}}\right. \\
&= -B\tan^{-1}\left(\frac{By}{\sqrt{A^2+B^2-B^2y^2}}\right)-A\tanh^{-1}\left(\frac{Ay}{\sqrt{A^2+B^2-B^2y^2}}\right)
\end{aligned}
$$

Mathematica [C] time = 0.0549715, size = 127, normalized size = 2.4

$$\frac{1}{2}\left(-2iB\log\left(2\left(\sqrt{A^2-B^2y^2+B^2}-iBy\right)\right)-A\log\left(A\sqrt{A^2-B^2y^2+B^2}+A^2-B^2y+B^2\right)+A\log\left(A\sqrt{A^2-B^2y^2+B^2}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[-(Sqrt[A^2 + B^2*(1 - y^2)]/(1 - y^2)),y]
```

```
[Out] (A*Log[1 - y] - A*Log[1 + y] - (2*I)*B*Log[2*((-I)*B*y + Sqrt[A^2 + B^2 - B
^2*y^2])] - A*Log[A^2 + B^2 - B^2*y + A*Sqrt[A^2 + B^2 - B^2*y^2]] + A*Log[
A^2 + B^2*(1 + y) + A*Sqrt[A^2 + B^2 - B^2*y^2]])/2
```

Maple [B] time = 0.006, size = 262, normalized size = 4.9

$$-\frac{1}{2}\sqrt{-B^2\left(1+y\right)^2+2B^2\left(1+y\right)+A^2}-\frac{B^2}{2}\arctan\left(y\sqrt{B^2}\frac{1}{\sqrt{-B^2\left(1+y\right)^2+2B^2\left(1+y\right)+A^2}}\right)\frac{1}{\sqrt{B^2}}+\frac{A^2}{2}\ln\left(\frac{1}{1+y}\left(2A\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y)
```

```
[Out] -1/2*(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2)-1/2*B^2/(B^2)^(1/2)*arctan((B^2)^
(1/2)*y/(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2))+1/2*A^2/(A^2)^(1/2)*ln((2*A^2
+2*B^2*(1+y)+2*(A^2)^(1/2)*(-B^2*(1+y)^2+2*B^2*(1+y)+A^2)^(1/2))/(1+y))+1/2
*(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2)-1/2*B^2/(B^2)^(1/2)*arctan((B^2)^(1/2
)*y/(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2))-1/2*A^2/(A^2)^(1/2)*ln((2*A^2-2*B
^2*(y-1)+2*(A^2)^(1/2)*(-B^2*(y-1)^2-2*B^2*(y-1)+A^2)^(1/2))/(y-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.6256, size = 284, normalized size = 5.36

$$B\arctan\left(\frac{\sqrt{-B^2y^2+A^2+B^2}}{By}\right)-\frac{1}{4}A\log\left(-\frac{\left(A^2-B^2\right)y^2+2\sqrt{-B^2y^2+A^2+B^2}Ay+A^2+B^2}{y^2}\right)+\frac{1}{4}A\log\left(-\frac{\left(A^2-B^2\right)y\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="fricas")
```

```
[Out] B*arctan(sqrt(-B^2*y^2 + A^2 + B^2)/(B*y)) - 1/4*A*log(-((A^2 - B^2)*y^2 +
2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2) + 1/4*A*log(-((A^2 - B^2
)*y^2 - 2*sqrt(-B^2*y^2 + A^2 + B^2)*A*y + A^2 + B^2)/y^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{A^2 - B^2y^2 + B^2}}{(y-1)(y+1)}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(A**2+B**2*(-y**2+1))**(1/2)/(-y**2+1),y)
```

```
[Out] Integral(sqrt(A**2 - B**2*y**2 + B**2)/((y - 1)*(y + 1)), y)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(A^2+B^2*(-y^2+1))^(1/2)/(-y^2+1),y, algorithm="giac")
```

```
[Out] Timed out
```

3.70 $\int \frac{(-A^2-B^2)\cos^2(z)}{B\left(1-\frac{(A^2+B^2)\sin^2(z)}{B^2}\right)} dz$

Optimal. Leaf size=16

$$-A \tanh^{-1}\left(\frac{A \tan(z)}{B}\right) - Bz$$

```
[Out] -(B*z) - A*ArcTanh[(A*Tan[z])/B]
```

Rubi [A] time = 0.0859799, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {12, 3191, 391, 203, 206}

$$-A \tanh^{-1}\left(\frac{A \tan(z)}{B}\right) - Bz$$

Antiderivative was successfully verified.

```
[In] Int[((-A^2 - B^2)*Cos[z]^2)/(B*(1 - ((A^2 + B^2)*Sin[z]^2)/B^2)),z]
```

```
[Out] -(B*z) - A*ArcTanh[(A*Tan[z])/B]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
```

```
d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\left(-A^2-B^2\right)\cos^2(z)}{B\left(1-\frac{\left(A^2+B^2\right)\sin^2(z)}{B^2}\right)}\,dz &= -\frac{\left(A^2+B^2\right)\int \frac{\cos^2(z)}{1-\frac{\left(A^2+B^2\right)\sin^2(z)}{B^2}}\,dz}{B} \\
&= -\frac{\left(A^2+B^2\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+z^2\right)\left(1+\left(1-\frac{A^2+B^2}{B^2}\right)z^2\right)}\,dz, z, \tan(z)\right)}{B} \\
&= -\frac{A^2\operatorname{Subst}\left(\int \frac{1}{1+\left(1-\frac{A^2+B^2}{B^2}\right)z^2}\,dz, z, \tan(z)\right)}{B} - B\operatorname{Subst}\left(\int \frac{1}{1+z^2}\,dz, z, \tan(z)\right) \\
&= -Bz - A\tanh^{-1}\left(\frac{A\tan(z)}{B}\right)
\end{aligned}
$$

Mathematica [B] time = 0.0902899, size = 35, normalized size = 2.19

$$
-\frac{B\left(A^2+B^2\right)\left(A\tanh^{-1}\left(\frac{A\tan(z)}{B}\right)+Bz\right)}{A^2B+B^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((-A^2 - B^2)*Cos[z]^2)/(B*(1 - ((A^2 + B^2)*Sin[z]^2)/B^2)),z]
```

```
[Out] -((B*(A^2 + B^2)*(B*z + A*ArcTanh[(A*Tan[z])/B]))/(A^2*B + B^3))
```

Maple [B] time = 0.036, size = 127, normalized size = 7.9

$$\frac{A^3\ln(A\tan(z)-B)}{2A^2+2B^2}+\frac{AB^2\ln(A\tan(z)-B)}{2A^2+2B^2}-\frac{B\arctan(\tan(z))A^2}{A^2+B^2}-\frac{\arctan(\tan(z))B^3}{A^2+B^2}-\frac{A^3\ln(A\tan(z)+B)}{2A^2+2B^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z)
```

```
[Out] 1/2*A^3/(A^2+B^2)*ln(A*tan(z)-B)+1/2*A*B^2/(A^2+B^2)*ln(A*tan(z)-B)-B/(A^2+
B^2)*arctan(tan(z))*A^2-1/(A^2+B^2)*arctan(tan(z))*B^3-1/2*A^3/(A^2+B^2)*ln
(A*tan(z)+B)-1/2*A*B^2/(A^2+B^2)*ln(A*tan(z)+B)
```

Maxima [B] time = 1.41742, size = 93, normalized size = 5.81

$$-\frac{\left(A^2+B^2\right)\left(\frac{2B^2z}{A^2+B^2}+\frac{AB\log(A\tan(z)+B)}{A^2+B^2}-\frac{AB\log(A\tan(z)-B)}{A^2+B^2}\right)}{2B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="ma
xima")
```

```
[Out] -1/2*(A^2 + B^2)*(2*B^2*z/(A^2 + B^2) + A*B*log(A*tan(z) + B)/(A^2 + B^2) -
 A*B*log(A*tan(z) - B)/(A^2 + B^2))/B
```

Fricas [B] time = 2.73631, size = 177, normalized size = 11.06

$$-Bz-\frac{1}{4}A\log\left(2AB\cos(z)\sin(z)-\left(A^2-B^2\right)\cos(z)^2+A^2\right)+\frac{1}{4}A\log\left(-2AB\cos(z)\sin(z)-\left(A^2-B^2\right)\cos(z)^2+A^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="fr
icas")
```

```
[Out] -B*z - 1/4*A*log(2*A*B*cos(z)*sin(z) - (A^2 - B^2)*cos(z)^2 + A^2) + 1/4*A*
log(-2*A*B*cos(z)*sin(z) - (A^2 - B^2)*cos(z)^2 + A^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A**2-B**2)*cos(z)**2/B/(1-(A**2+B**2)*sin(z)**2/B**2),z)
```

```
[Out] Timed out
```

Giac [B] time = 1.11659, size = 112, normalized size = 7.

$$-\frac{\left(\frac{A^3B\log(|A\tan(z)+B|)}{A^4+A^2B^2} - \frac{A^3B\log(|A\tan(z)-B|)}{A^4+A^2B^2} + \frac{2B^2z}{A^2+B^2}\right)\left(A^2+B^2\right)}{2B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)*cos(z)^2/B/(1-(A^2+B^2)*sin(z)^2/B^2),z, algorithm="gi
ac")
```

```
[Out] -1/2*(A^3*B*log(abs(A*tan(z) + B))/(A^4 + A^2*B^2) - A^3*B*log(abs(A*tan(z)
 - B))/(A^4 + A^2*B^2) + 2*B^2*z/(A^2 + B^2))*(A^2 + B^2)/B
```

3.71 $\int -\frac{A^2+B^2}{B\left(1+w^2\right)^2\left(1-\frac{\left(A^2+B^2\right)w^2}{B^2\left(1+w^2\right)}\right)}\,dw$

Optimal. Leaf size=16

$$-A\tanh^{-1}\left(\frac{Aw}{B}\right)-B\tan^{-1}(w)$$

```
[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]
```

Rubi [A] time = 0.124371, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {12, 6688, 391, 203, 208}

$$-A\tanh^{-1}\left(\frac{Aw}{B}\right)-B\tan^{-1}(w)$$

Antiderivative was successfully verified.

```
[In] Int[-((A^2 + B^2)/(B*(1 + w^2)^2*(1 - ((A^2 + B^2)*w^2)/(B^2*(1 + w^2))))),
w]
```

```
[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int -\frac{A^2+B^2}{B\left(1+w^2\right)^2\left(1-\frac{\left(A^2+B^2\right)w^2}{B^2\left(1+w^2\right)}\right)}\,dw &= -\frac{\left(A^2+B^2\right)\int \frac{1}{\left(1+w^2\right)^2\left(1-\frac{\left(A^2+B^2\right)w^2}{B^2\left(1+w^2\right)}\right)}\,dw}{B} \\
&= -\frac{\left(A^2+B^2\right)\int \frac{B^2}{\left(1+w^2\right)\left(B^2-A^2w^2\right)}\,dw}{B} \\
&= -\left(\left(B\left(A^2+B^2\right)\right)\int \frac{1}{\left(1+w^2\right)\left(B^2-A^2w^2\right)}\,dw\right) \\
&= -\left(B\int \frac{1}{1+w^2}\,dw\right)-\left(A^2B\right)\int \frac{1}{B^2-A^2w^2}\,dw \\
&= -B\tan^{-1}(w)-A\tanh^{-1}\left(\frac{Aw}{B}\right)
\end{aligned}
$$

Mathematica [B] time = 0.01813, size = 35, normalized size = 2.19

$$
-\frac{B\left(A^2+B^2\right)\left(A\tanh^{-1}\left(\frac{Aw}{B}\right)+B\tan^{-1}(w)\right)}{A^2B+B^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[-((A^2 + B^2)/(B*(1 + w^2)^2*(1 - ((A^2 + B^2)*w^2)/(B^2*(1 + w^2
))))),w]
```

```
[Out] -((B*(A^2 + B^2)*(B*ArcTan[w] + A*ArcTanh[(A*w)/B]))/(A^2*B + B^3))
```

Maple [B] time = 0.007, size = 121, normalized size = 7.6

$$\frac{A^3 \ln (Aw - B)}{2A^2 + 2B^2} + \frac{AB^2 \ln (Aw - B)}{2A^2 + 2B^2} - \frac{B \arctan (w) A^2}{A^2 + B^2} - \frac{\arctan (w) B^3}{A^2 + B^2} - \frac{A^3 \ln (Aw + B)}{2A^2 + 2B^2} - \frac{AB^2 \ln (Aw + B)}{2A^2 + 2B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w)
```

```
[Out] 1/2*A^3/(A^2+B^2)*ln(A*w-B)+1/2*A*B^2/(A^2+B^2)*ln(A*w-B)-B/(A^2+B^2)*arcta
n(w)*A^2-1/(A^2+B^2)*arctan(w)*B^3-1/2*A^3/(A^2+B^2)*ln(A*w+B)-1/2*A*B^2/(A
^2+B^2)*ln(A*w+B)
```

Maxima [B] time = 1.41031, size = 92, normalized size = 5.75

$$-\frac{\left(A^2 + B^2\right)\left(\frac{2B^2 \arctan(w)}{A^2+B^2} + \frac{AB \log(Aw+B)}{A^2+B^2} - \frac{AB \log(Aw-B)}{A^2+B^2}\right)}{2B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorithm
="maxima")
```

```
[Out] -1/2*(A^2 + B^2)*(2*B^2*arctan(w)/(A^2 + B^2) + A*B*log(A*w + B)/(A^2 + B^2
) - A*B*log(A*w - B)/(A^2 + B^2))/B
```

Fricas [A] time = 2.16152, size = 76, normalized size = 4.75

$$-B \arctan (w) - \frac{1}{2} A \log (Aw + B) + \frac{1}{2} A \log (Aw - B)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorithm
="fricas")
```

```
[Out] -B*arctan(w) - 1/2*A*log(A*w + B) + 1/2*A*log(A*w - B)
```

Sympy [C] time = 1.30127, size = 422, normalized size = 26.38

$$\left(A^{2}B+B^{3}\right)\left(-\frac{A\log\left(w+\frac{-\frac{A^{9}}{B\left(A^{2}+B^{2}\right)^{3}}-\frac{A^{7}B}{\left(A^{2}+B^{2}\right)^{3}}+\frac{A^{5}B^{3}}{\left(A^{2}+B^{2}\right)^{3}}+\frac{A^{5}}{B\left(A^{2}+B^{2}\right)}+\frac{A^{3}B^{5}}{\left(A^{2}+B^{2}\right)^{3}}+\frac{AB^{3}}{A^{2}+B^{2}}}{A^{2}}\right)}{2B\left(A^{2}+B^{2}\right)}+\frac{A\log\left(w+\frac{\frac{A^{9}}{B\left(A^{2}+B^{2}\right)^{3}}+\frac{A^{7}B}{\left(A^{2}+B^{2}\right)^{3}}-\frac{A^{5}B^{3}}{\left(A^{2}+B^{2}\right)^{3}}-\cdots}{A^{2}}\right.}{2B\left(A^{2}+B^{2}\right)}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A**2-B**2)/B/(w**2+1)**2/(1-(A**2+B**2)*w**2/B**2/(w**2+1)),w)
```

```
[Out] (A**2*B + B**3)*(-A*log(w + (-A**9/(B*(A**2 + B**2)**3) - A**7*B/(A**2 + B*
*2)**3 + A**5*B**3/(A**2 + B**2)**3 + A**5/(B*(A**2 + B**2)) + A**3*B**5/(A
**2 + B**2)**3 + A*B**3/(A**2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + A*log(w
+ (A**9/(B*(A**2 + B**2)**3) + A**7*B/(A**2 + B**2)**3 - A**5*B**3/(A**2 +
B**2)**3 - A**5/(B*(A**2 + B**2)) - A**3*B**5/(A**2 + B**2)**3 - A*B**3/(A*
*2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + I*log(w + (-I*A**6*B**2/(A**2 + B**
2)**3 - I*A**4*B**4/(A**2 + B**2)**3 - I*A**4/(A**2 + B**2) + I*A**2*B**6/(
A**2 + B**2)**3 + I*B**8/(A**2 + B**2)**3 - I*B**4/(A**2 + B**2))/A**2)/(2*
(A**2 + B**2)) - I*log(w + (I*A**6*B**2/(A**2 + B**2)**3 + I*A**4*B**4/(A**
2 + B**2)**3 + I*A**4/(A**2 + B**2) - I*A**2*B**6/(A**2 + B**2)**3 - I*B**8
/(A**2 + B**2)**3 + I*B**4/(A**2 + B**2))/A**2)/(2*(A**2 + B**2)))
```

Giac [B] time = 1.09746, size = 111, normalized size = 6.94

$$-\frac{\left(\frac{A^{3}B\log\left(|Aw+B|\right)}{A^{4}+A^{2}B^{2}}-\frac{A^{3}B\log\left(|Aw-B|\right)}{A^{4}+A^{2}B^{2}}+\frac{2\,B^{2}\arctan\left(w\right)}{A^{2}+B^{2}}\right)\left(A^{2}+B^{2}\right)}{2\,B}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-A^2-B^2)/B/(w^2+1)^2/(1-(A^2+B^2)*w^2/B^2/(w^2+1)),w, algorithm
="giac")
```

```
[Out] -1/2*(A^3*B*log(abs(A*w + B))/(A^4 + A^2*B^2) - A^3*B*log(abs(A*w - B))/(A^
4 + A^2*B^2) + 2*B^2*arctan(w)/(A^2 + B^2))*(A^2 + B^2)/B
```

3.72 $\int -\frac{B(A^2+B^2)}{(1+w^2)(B^2-A^2w^2)}\,dw$

Optimal. Leaf size=16

$$-A\tanh^{-1}\left(\frac{Aw}{B}\right)-B\tan^{-1}(w)$$

```
[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]
```

Rubi [A] time = 0.0186529, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {12, 391, 203, 208}

$$-A\tanh^{-1}\left(\frac{Aw}{B}\right)-B\tan^{-1}(w)$$

Antiderivative was successfully verified.

```
[In] Int[-((B*(A^2 + B^2))/((1 + w^2)*(B^2 - A^2*w^2))),w]

[Out] -(B*ArcTan[w]) - A*ArcTanh[(A*w)/B]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int -\frac{B\left(A^2+B^2\right)}{\left(1+w^2\right)\left(B^2-A^2w^2\right)}\,dw &= -\left(\left(B\left(A^2+B^2\right)\right)\int \frac{1}{\left(1+w^2\right)\left(B^2-A^2w^2\right)}\,dw\right)\\ &= -\left(B\int \frac{1}{1+w^2}\,dw\right)-\left(A^2B\right)\int \frac{1}{B^2-A^2w^2}\,dw\\ &= -B\tan^{-1}(w)-A\tanh^{-1}\left(\frac{Aw}{B}\right)\end{aligned}$$

Mathematica [B] time = 0.0130182, size = 35, normalized size = 2.19

$$-\frac{B\left(A^2+B^2\right)\left(A\tanh^{-1}\left(\frac{Aw}{B}\right)+B\tan^{-1}(w)\right)}{A^2B+B^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[-((B*(A^2 + B^2))/((1 + w^2)*(B^2 - A^2*w^2))),w]
```

```
[Out] -((B*(A^2 + B^2)*(B*ArcTan[w] + A*ArcTanh[(A*w)/B]))/(A^2*B + B^3))
```

Maple [B] time = 0.005, size = 121, normalized size = 7.6

$$\frac{A^3\ln(Aw-B)}{2A^2+2B^2}+\frac{AB^2\ln(Aw-B)}{2A^2+2B^2}-\frac{B\arctan(w)\,A^2}{A^2+B^2}-\frac{\arctan(w)\,B^3}{A^2+B^2}-\frac{A^3\ln(Aw+B)}{2A^2+2B^2}-\frac{AB^2\ln(Aw+B)}{2A^2+2B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w)
```

```
[Out] 1/2*A^3/(A^2+B^2)*ln(A*w-B)+1/2*A*B^2/(A^2+B^2)*ln(A*w-B)-B/(A^2+B^2)*arcta
n(w)*A^2-1/(A^2+B^2)*arctan(w)*B^3-1/2*A^3/(A^2+B^2)*ln(A*w+B)-1/2*A*B^2/(A
^2+B^2)*ln(A*w+B)
```

Maxima [B] time = 1.41032, size = 88, normalized size = 5.5

$$-\frac{1}{2}\left(A^2+B^2\right)B\left(\frac{A\log\left(Aw+B\right)}{A^2B+B^3}-\frac{A\log\left(Aw-B\right)}{A^2B+B^3}+\frac{2\,\arctan\left(w\right)}{A^2+B^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w, algorithm="maxima")
```

```
[Out] -1/2*(A^2 + B^2)*B*(A*log(A*w + B)/(A^2*B + B^3) - A*log(A*w - B)/(A^2*B +
B^3) + 2*arctan(w)/(A^2 + B^2))
```

Fricas [A] time = 2.43177, size = 76, normalized size = 4.75

$$-B\arctan\left(w\right)-\frac{1}{2}\,A\log\left(Aw+B\right)+\frac{1}{2}\,A\log\left(Aw-B\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w, algorithm="fricas")
```

```
[Out] -B*arctan(w) - 1/2*A*log(A*w + B) + 1/2*A*log(A*w - B)
```

Sympy [C] time = 1.27477, size = 422, normalized size = 26.38

$$\left(A^2B+B^3\right)\left(-\frac{A\log\left(w+\frac{-\frac{A^9}{B\left(A^2+B^2\right)^3}-\frac{A^7B}{\left(A^2+B^2\right)^3}+\frac{A^5B^3}{\left(A^2+B^2\right)^3}+\frac{A^5}{B\left(A^2+B^2\right)}+\frac{A^3B^5}{\left(A^2+B^2\right)^3}+\frac{AB^3}{A^2+B^2}}{A^2}\right)}{2B\left(A^2+B^2\right)}+\frac{A\log\left(w+\frac{\frac{A^9}{B\left(A^2+B^2\right)^3}+\frac{A^7B}{\left(A^2+B^2\right)^3}-\frac{A^5B^3}{\left(A^2+B^2\right)^3}-\cdots}{A^2}\right)}{2B\left(A^2+B^2\right)}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-B*(A**2+B**2)/(w**2+1)/(-A**2*w**2+B**2),w)
```

```
[Out] (A**2*B + B**3)*(-A*log(w + (-A**9/(B*(A**2 + B**2)**3) - A**7*B/(A**2 + B*
*2)**3 + A**5*B**3/(A**2 + B**2)**3 + A**5/(B*(A**2 + B**2)) + A**3*B**5/(A
```

```
**2 + B**2)**3 + A*B**3/(A**2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + A*log(w
+ (A**9/(B*(A**2 + B**2)**3) + A**7*B/(A**2 + B**2)**3 - A**5*B**3/(A**2 +
B**2)**3 - A**5/(B*(A**2 + B**2)) - A**3*B**5/(A**2 + B**2)**3 - A*B**3/(A*
*2 + B**2))/A**2)/(2*B*(A**2 + B**2)) + I*log(w + (-I*A**6*B**2/(A**2 + B**
2)**3 - I*A**4*B**4/(A**2 + B**2)**3 - I*A**4/(A**2 + B**2) + I*A**2*B**6/(
A**2 + B**2)**3 + I*B**8/(A**2 + B**2)**3 - I*B**4/(A**2 + B**2))/A**2)/(2*
(A**2 + B**2)) - I*log(w + (I*A**6*B**2/(A**2 + B**2)**3 + I*A**4*B**4/(A**
2 + B**2)**3 + I*A**4/(A**2 + B**2) - I*A**2*B**6/(A**2 + B**2)**3 - I*B**8
/(A**2 + B**2)**3 + I*B**4/(A**2 + B**2))/A**2)/(2*(A**2 + B**2)))
```

Giac [B] time = 1.07565, size = 107, normalized size = 6.69

$$-\frac{1}{2}\left(\frac{A^3\log\left(|Aw+B|\right)}{A^4B+A^2B^3}-\frac{A^3\log\left(|Aw-B|\right)}{A^4B+A^2B^3}+\frac{2\arctan\left(w\right)}{A^2+B^2}\right)\left(A^2+B^2\right)B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-B*(A^2+B^2)/(w^2+1)/(-A^2*w^2+B^2),w, algorithm="giac")
```

```
[Out] -1/2*(A^3*log(abs(A*w + B))/(A^4*B + A^2*B^3) - A^3*log(abs(A*w - B))/(A^4*
B + A^2*B^3) + 2*arctan(w)/(A^2 + B^2))*(A^2 + B^2)*B
```

3.73 $\int \frac{x^4}{(1-x^2)^{5/2}}\,dx$

Optimal. Leaf size=35

$$\frac{x^3}{3(1-x^2)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rubi [A] time = 0.0080682, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {288, 216}

$$\frac{x^3}{3(1-x^2)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{x^4}{(1-x^2)^{5/2}}\,dx = \frac{x^3}{3(1-x^2)^{3/2}} - \int \frac{x^2}{(1-x^2)^{3/2}}\,dx$$
$$= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= \frac{x^3}{3(1-x^2)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$$

Mathematica [A] time = 0.020909, size = 26, normalized size = 0.74

$$\frac{x(4x^2-3)}{3(1-x^2)^{3/2}} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]
```

Maple [A] time = 0., size = 30, normalized size = 0.9

$$\frac{x^3}{3}\left(-x^2+1\right)^{-\frac{3}{2}} + \arcsin(x) - x\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(5/2),x)
```

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.41289, size = 59, normalized size = 1.69

$$\frac{1}{3}x\left(\frac{3x^2}{(-x^2+1)^{\frac{3}{2}}} - \frac{2}{(-x^2+1)^{\frac{3}{2}}}\right) - \frac{x}{3\sqrt{-x^2+1}} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

Fricas [B] time = 2.28929, size = 146, normalized size = 4.17

$$-\frac{6\left(x^4-2\,x^2+1\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(4\,x^3-3\,x\right)\sqrt{-x^2+1}}{3\left(x^4-2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

Sympy [B] time = 2.55844, size = 105, normalized size = 3.

$$\frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3}+\frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3}-\frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3}-\frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3}+\frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

Giac [A] time = 1.06539, size = 39, normalized size = 1.11

$$\frac{\left(4\,x^2-3\right)\sqrt{-x^2+1}x}{3\left(x^2-1\right)^2}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

3.74 $\int \tan^4(y)\,dy$

Optimal. Leaf size=14

$$y+\frac{\tan^3(y)}{3}-\tan(y)$$

```
[Out] y - Tan[y] + Tan[y]^3/3
```

Rubi [A] time = 0.0094383, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 8}

$$y+\frac{\tan^3(y)}{3}-\tan(y)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[y]^4,y]
```

```
[Out] y - Tan[y] + Tan[y]^3/3
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tan^4(y)\,dy &= \frac{\tan^3(y)}{3}-\int \tan^2(y)\,dy\\ &= -\tan(y)+\frac{\tan^3(y)}{3}+\int 1\,dy\\ &= y-\tan(y)+\frac{\tan^3(y)}{3}\end{aligned}$$

Mathematica [A] time = 0.006699, size = 18, normalized size = 1.29

$$y-\frac{4\tan(y)}{3}+\frac{1}{3}\tan(y)\sec^2(y)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[y]^4,y]
```

```
[Out] y - (4*Tan[y])/3 + (Sec[y]^2*Tan[y])/3
```

Maple [A] time = 0.007, size = 13, normalized size = 0.9

$$y-\tan\left(y\right)+\frac{\left(\tan\left(y\right)\right)^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(y)^4/cos(y)^4,y)
```

```
[Out] y-tan(y)+1/3*tan(y)^3
```

Maxima [A] time = 1.41807, size = 16, normalized size = 1.14

$$\frac{1}{3}\tan\left(y\right)^{3}+y-\tan\left(y\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)^4/cos(y)^4,y, algorithm="maxima")
```

```
[Out] 1/3*tan(y)^3 + y - tan(y)
```

Fricas [B] time = 2.2519, size = 74, normalized size = 5.29

$$\frac{3\,y\cos\left(y\right)^{3}-\left(4\,\cos\left(y\right)^{2}-1\right)\sin\left(y\right)}{3\,\cos\left(y\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)^4/cos(y)^4,y, algorithm="fricas")
```

```
[Out] 1/3*(3*y*cos(y)^3 - (4*cos(y)^2 - 1)*sin(y))/cos(y)^3
```

Sympy [A] time = 0.060681, size = 19, normalized size = 1.36

$$y + \frac{\sin^3(y)}{3\cos^3(y)} - \frac{\sin(y)}{\cos(y)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)**4/cos(y)**4,y)
```

```
[Out] y + sin(y)**3/(3*cos(y)**3) - sin(y)/cos(y)
```

Giac [A] time = 1.06633, size = 16, normalized size = 1.14

$$\frac{1}{3}\tan(y)^3 + y - \tan(y)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(y)^4/cos(y)^4,y, algorithm="giac")
```

```
[Out] 1/3*tan(y)^3 + y - tan(y)
```

3.75 $\int \frac{z^4}{1+z^2} \, dz$

Optimal. Leaf size=13

$$\frac{z^3}{3} - z + \tan^{-1}(z)$$

```
[Out] -z + z^3/3 + ArcTan[z]
```

Rubi [A] time = 0.0053858, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {302, 203}

$$\frac{z^3}{3} - z + \tan^{-1}(z)$$

Antiderivative was successfully verified.

```
[In] Int[z^4/(1 + z^2),z]
```

```
[Out] -z + z^3/3 + ArcTan[z]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{z^4}{1+z^2}\,dz = \int \left(-1+z^2+\frac{1}{1+z^2}\right) dz$$
$$= -z+\frac{z^3}{3}+\int \frac{1}{1+z^2}\,dz$$
$$= -z+\frac{z^3}{3}+\tan^{-1}(z)$$

Mathematica [A] time = 0.0039374, size = 13, normalized size = 1.

$$\frac{z^3}{3}-z+\tan^{-1}(z)$$

Antiderivative was successfully verified.

```
[In] Integrate[z^4/(1 + z^2),z]
```

```
[Out] -z + z^3/3 + ArcTan[z]
```

Maple [A] time = 0.001, size = 12, normalized size = 0.9

$$-z+\frac{z^3}{3}+\arctan(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(z^4/(z^2+1),z)
```

```
[Out] -z+1/3*z^3+arctan(z)
```

Maxima [A] time = 1.40311, size = 15, normalized size = 1.15

$$\frac{1}{3}z^3-z+\arctan(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^4/(z^2+1),z, algorithm="maxima")
```

```
[Out] 1/3*z^3 - z + arctan(z)
```

Fricas [A] time = 2.09771, size = 34, normalized size = 2.62

$$\frac{1}{3}z^3 - z + \arctan(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^4/(z^2+1),z, algorithm="fricas")
```

```
[Out] 1/3*z^3 - z + arctan(z)
```

Sympy [A] time = 0.080637, size = 8, normalized size = 0.62

$$\frac{z^3}{3} - z + \operatorname{atan}(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z**4/(z**2+1),z)
```

```
[Out] z**3/3 - z + atan(z)
```

Giac [A] time = 1.05979, size = 15, normalized size = 1.15

$$\frac{1}{3}z^3 - z + \arctan(z)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(z^4/(z^2+1),z, algorithm="giac")
```

```
[Out] 1/3*z^3 - z + arctan(z)
```

3.76 $\int e^{x^2}\left(1+2x^2\right)\,dx$

Optimal. Leaf size=7

$$e^{x^2}x$$

```
[Out] E^x^2*x
```

Rubi [A] time = 0.0298354, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2226, 2204, 2212}

$$e^{x^2}x$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*(1 + 2*x^2),x]
```

```
[Out] E^x^2*x
```

Rule 2226

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*(u_), x_Symbol] :> Int[E
xpandLinearProduct[F^(a + b*(c + d*x)^n), u, c, d, x], x] /; FreeQ[{F, a, b
, c, d, n}, x] && PolynomialQ[u, x]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rubi steps

$$\begin{aligned}\int e^{x^2}\left(1+2x^2\right)dx &= \int \left(e^{x^2}+2e^{x^2}x^2\right)dx \\ &= 2\int e^{x^2}x^2\,dx+\int e^{x^2}\,dx \\ &= e^{x^2}x+\frac{1}{2}\sqrt{\pi}\text{erfi}(x)-\int e^{x^2}\,dx \\ &= e^{x^2}x\end{aligned}$$

Mathematica [A] time = 0.0043513, size = 7, normalized size = 1.

$$e^{x^2}x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*(1 + 2*x^2),x]
```

```
[Out] E^x^2*x
```

Maple [A] time = 0., size = 7, normalized size = 1.

$$e^{x^2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*(2*x^2+1),x)
```

```
[Out] exp(x^2)*x
```

Maxima [A] time = 0.989386, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="maxima")
```

```
[Out] x*e^(x^2)
```

Fricas [A] time = 2.09897, size = 15, normalized size = 2.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="fricas")
```

```
[Out] x*e^(x^2)
```

Sympy [A] time = 0.080341, size = 5, normalized size = 0.71

$$xe^{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*(2*x**2+1),x)
```

```
[Out] x*exp(x**2)
```

Giac [A] time = 1.05528, size = 8, normalized size = 1.14

$$xe^{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^2+1),x, algorithm="giac")
```

```
[Out] x*e^(x^2)
```

3.77 $\int \frac{e^{x^2}(1+4x^2+x^3+5x^4+2x^6)}{(1+x^2)^2}\,dx$

Optimal. Leaf size=24

$$e^{x^2}x+\frac{e^{x^2}}{2(x^2+1)}$$

```
[Out] E^x^2*x + E^x^2/(2*(1 + x^2))
```

Rubi [A] time = 0.35703, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {6742, 2204, 2212, 6715, 2177, 2178}

$$e^{x^2}x+\frac{e^{x^2}}{2(x^2+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x^2*(1 + 4*x^2 + x^3 + 5*x^4 + 2*x^6))/(1 + x^2)^2,x]
```

```
[Out] E^x^2*x + E^x^2/(2*(1 + x^2))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
```

```
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !$UseGamma === True
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\int \frac{e^{x^2}\left(1+4x^2+x^3+5x^4+2x^6\right)}{\left(1+x^2\right)^2}\,dx = \int \left(e^{x^2}+2e^{x^2}x^2-\frac{e^{x^2}x}{\left(1+x^2\right)^2}+\frac{e^{x^2}x}{1+x^2}\right)dx$$

$$= 2\int e^{x^2}x^2\,dx+\int e^{x^2}\,dx-\int \frac{e^{x^2}x}{\left(1+x^2\right)^2}\,dx+\int \frac{e^{x^2}x}{1+x^2}\,dx$$

$$= e^{x^2}x+\frac{1}{2}\sqrt{\pi}\,\mathbf{erfi}(x)-\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{e^x}{(1+x)^2}\,dx,x,x^2\right)+\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{e^x}{1+x}\,dx,x,x$$

$$= e^{x^2}x+\frac{e^{x^2}}{2\left(1+x^2\right)}+\frac{\mathbf{Ei}\left(1+x^2\right)}{2e}-\frac{1}{2}\,\mathrm{Subst}\left(\int \frac{e^x}{1+x}\,dx,x,x^2\right)$$

$$= e^{x^2}x+\frac{e^{x^2}}{2\left(1+x^2\right)}$$

Mathematica [A] time = 0.068602, size = 20, normalized size = 0.83

$$\frac{1}{2}e^{x^2}\left(\frac{1}{x^2+1}+2x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x^2*(1 + 4*x^2 + x^3 + 5*x^4 + 2*x^6))/(1 + x^2)^2,x]
```

```
[Out] (E^x^2*(2*x + (1 + x^2)^(-1)))/2
```

Maple [A] time = 0.005, size = 24, normalized size = 1.

$$\frac{\left(2\,x^3+2\,x+1\right)\mathrm{e}^{x^2}}{2\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x)
```

```
[Out] 1/2*(2*x^3+2*x+1)*exp(x^2)/(x^2+1)
```

Maxima [A] time = 1.54735, size = 31, normalized size = 1.29

$$\frac{\left(2\,x^3+2\,x+1\right)e^{\left(x^2\right)}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="maxima
")
```

```
[Out] 1/2*(2*x^3 + 2*x + 1)*e^(x^2)/(x^2 + 1)
```

Fricas [A] time = 2.17282, size = 55, normalized size = 2.29

$$\frac{\left(2\,x^3+2\,x+1\right)e^{\left(x^2\right)}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="fricas
")
```

```
[Out] 1/2*(2*x^3 + 2*x + 1)*e^(x^2)/(x^2 + 1)
```

Sympy [A] time = 0.108063, size = 20, normalized size = 0.83

$$\frac{\left(2x^3+2x+1\right)e^{x^2}}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*(2*x**6+5*x**4+x**3+4*x**2+1)/(x**2+1)**2,x)
```

```
[Out] (2*x**3 + 2*x + 1)*exp(x**2)/(2*x**2 + 2)
```

Giac [A] time = 1.06218, size = 41, normalized size = 1.71

$$\frac{2\,x^3e^{\left(x^2\right)}+2\,xe^{\left(x^2\right)}+e^{\left(x^2\right)}}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*(2*x^6+5*x^4+x^3+4*x^2+1)/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*x^3*e^(x^2) + 2*x*e^(x^2) + e^(x^2))/(x^2 + 1)
```

3.78 $\int e^{-1-x}\,dx$

Optimal. Leaf size=9

$$-e^{-x-1}$$

```
[Out] -E^(-1 - x)
```

Rubi [A] time = 0.003023, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2194}

$$-e^{-x-1}$$

Antiderivative was successfully verified.

```
[In] Int[E^(-1 - x),x]
```

```
[Out] -E^(-1 - x)
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{-1-x}\,dx = -e^{-1-x}$$

Mathematica [A] time = 0.0025931, size = 9, normalized size = 1.

$$-e^{-x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(-1 - x),x]
```

```
[Out] -E^(-1 - x)
```

Maple [A] time = 0., size = 9, normalized size = 1.

$$-e^{-1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(-1-x),x)
```

```
[Out] -exp(-1-x)
```

Maxima [A] time = 0.935708, size = 11, normalized size = 1.22

$$-e^{(-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1-x),x, algorithm="maxima")
```

```
[Out] -e^(-x - 1)
```

Fricas [A] time = 2.0505, size = 18, normalized size = 2.

$$-e^{(-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1-x),x, algorithm="fricas")
```

```
[Out] -e^(-x - 1)
```

Sympy [A] time = 0.067781, size = 7, normalized size = 0.78

$$-e^{-x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1-x),x)
```

```
[Out] -exp(-x - 1)
```

Giac [A] time = 1.06434, size = 11, normalized size = 1.22

$$-e^{(-x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(-1-x),x, algorithm="giac")
```

```
[Out] -e^(-x - 1)
```

3.79 $\int \left(\frac{1}{x}+x\right)\log(x)\,dx$

Optimal. Leaf size=25

$$-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)+\frac{\log^2(x)}{2}$$

```
[Out] -x^2/4 + (x^2*Log[x])/2 + Log[x]^2/2
```

Rubi [A] time = 0.0449937, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {1593, 14, 2351, 2301, 2304}

$$-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)+\frac{\log^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^(-1) + x)*Log[x],x]
```

```
[Out] -x^2/4 + (x^2*Log[x])/2 + Log[x]^2/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2351

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{1}{x}+x\right)\log(x)\,dx &= \int \frac{\left(1+x^2\right)\log(x)}{x}\,dx \\
&= \int \left(\frac{\log(x)}{x}+x\log(x)\right)dx \\
&= \int \frac{\log(x)}{x}\,dx+\int x\log(x)\,dx \\
&= -\frac{x^2}{4}+\frac{1}{2}x^2\log(x)+\frac{\log^2(x)}{2}
\end{aligned}$$

Mathematica [A] time = 0.0017662, size = 25, normalized size = 1.

$$-\frac{x^2}{4}+\frac{1}{2}x^2\log(x)+\frac{\log^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1) + x)*Log[x],x]
```

```
[Out] -x^2/4 + (x^2*Log[x])/2 + Log[x]^2/2
```

Maple [A] time = 0.002, size = 20, normalized size = 0.8

$$-\frac{x^2}{4}+\frac{x^2\ln(x)}{2}+\frac{(\ln(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x+x)*ln(x),x)
```

```
[Out] -1/4*x^2+1/2*x^2*ln(x)+1/2*ln(x)^2
```

Maxima [A] time = 0.926879, size = 32, normalized size = 1.28

$$-\frac{1}{4}x^2 + \frac{1}{2}\left(x^2 + 2\log(x)\right)\log(x) - \frac{1}{2}\log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)*log(x),x, algorithm="maxima")
```

```
[Out] -1/4*x^2 + 1/2*(x^2 + 2*log(x))*log(x) - 1/2*log(x)^2
```

Fricas [A] time = 2.14503, size = 55, normalized size = 2.2

$$\frac{1}{2}x^2\log(x) - \frac{1}{4}x^2 + \frac{1}{2}\log(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2 + 1/2*log(x)^2
```

Sympy [A] time = 0.084918, size = 19, normalized size = 0.76

$$\frac{x^2\log(x)}{2} - \frac{x^2}{4} + \frac{\log(x)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)*ln(x),x)
```

```
[Out] x**2*log(x)/2 - x**2/4 + log(x)**2/2
```

Giac [A] time = 1.05138, size = 26, normalized size = 1.04

$$\frac{1}{2}\,x^2\log\left(x\right) - \frac{1}{4}\,x^2 + \frac{1}{2}\,\log\left(x\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+x)*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2 + 1/2*log(x)^2
```

3.80 $\int \frac{x}{1+x^4} \, dx$

Optimal. Leaf size=8

$$\frac{1}{2}\tan^{-1}\left(x^2\right)$$

```
[Out] ArcTan[x^2]/2
```

Rubi [A] time = 0.0031274, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {275, 203}

$$\frac{1}{2}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^4),x]
```

```
[Out] ArcTan[x^2]/2
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{1+x^4} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x^2} \, dx, x, x^2\right) \\ &= \frac{1}{2}\tan^{-1}\left(x^2\right) \end{aligned}$$

Mathematica [A] time = 0.003168, size = 8, normalized size = 1.

$$\frac{1}{2}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^4),x]
```

```
[Out] ArcTan[x^2]/2
```

Maple [A] time = 0.002, size = 7, normalized size = 0.9

$$\frac{\arctan\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+1),x)
```

```
[Out] 1/2*arctan(x^2)
```

Maxima [A] time = 1.4093, size = 8, normalized size = 1.

$$\frac{1}{2}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x^2)
```

Fricas [A] time = 2.18864, size = 23, normalized size = 2.88

$$\frac{1}{2}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x^2)
```

Sympy [A] time = 0.086513, size = 5, normalized size = 0.62

$$\frac{\operatorname{atan}\left(x^{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+1),x)
```

```
[Out] atan(x**2)/2
```

Giac [A] time = 1.0647, size = 8, normalized size = 1.

$$\frac{1}{2} \arctan\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x^2)
```

3.81 $\int \frac{x^5}{1+x^4} \, dx$

Optimal. Leaf size=16

$$\frac{x^2}{2} - \frac{1}{2}\tan^{-1}\left(x^2\right)$$

```
[Out] x^2/2 - ArcTan[x^2]/2
```

Rubi [A] time = 0.0082944, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {275, 321, 203}

$$\frac{x^2}{2} - \frac{1}{2}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(1 + x^4),x]
```

```
[Out] x^2/2 - ArcTan[x^2]/2
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{1+x^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,x^2\right)\\ &= \frac{x^2}{2}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,x^2\right)\\ &= \frac{x^2}{2}-\frac{1}{2}\tan^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.00335, size = 16, normalized size = 1.

$$\frac{x^2}{2}-\frac{1}{2}\tan^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(1 + x^4),x]
```

```
[Out] x^2/2 - ArcTan[x^2]/2
```

Maple [A] time = 0.002, size = 13, normalized size = 0.8

$$\frac{x^2}{2}-\frac{\arctan\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^4+1),x)
```

```
[Out] 1/2*x^2-1/2*arctan(x^2)
```

Maxima [A] time = 1.40815, size = 16, normalized size = 1.

$$\frac{1}{2}\,x^2-\frac{1}{2}\,\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - 1/2*arctan(x^2)
```

Fricas [A] time = 2.05275, size = 36, normalized size = 2.25

$$\frac{1}{2}x^2 - \frac{1}{2}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - 1/2*arctan(x^2)
```

Sympy [A] time = 0.090205, size = 10, normalized size = 0.62

$$\frac{x^2}{2} - \frac{\operatorname{atan}\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**4+1),x)
```

```
[Out] x**2/2 - atan(x**2)/2
```

Giac [A] time = 1.05893, size = 16, normalized size = 1.

$$\frac{1}{2}x^2 - \frac{1}{2}\arctan\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/2*arctan(x^2)
```

3.82 $\int \frac{1}{1+\tan^2(x)} \, dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0153262, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3657, 2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[x]^2)^(-1),x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 3657

```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{1+\tan^2(x)}\,dx &= \int \cos^2(x)\,dx \\
&= \frac{1}{2}\cos(x)\sin(x) + \frac{\int 1\,dx}{2} \\
&= \frac{x}{2} + \frac{1}{2}\cos(x)\sin(x)
\end{aligned}
$$

Mathematica [A] time = 0.0025164, size = 14, normalized size = 1.

$$\frac{x}{2} + \frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[x]^2)^(-1),x]
```

```
[Out] x/2 + Sin[2*x]/4
```

Maple [A] time = 0.01, size = 17, normalized size = 1.2

$$\frac{\tan(x)}{2(\tan(x))^2+2} + \frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x)^2+1),x)
```

```
[Out] 1/2/(tan(x)^2+1)*tan(x)+1/2*x
```

Maxima [A] time = 1.40717, size = 22, normalized size = 1.57

$$\frac{1}{2}x + \frac{\tan(x)}{2\left(\tan(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

Fricas [A] time = 2.28108, size = 62, normalized size = 4.43

$$\frac{x\tan(x)^2 + x + \tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(x*tan(x)^2 + x + tan(x))/(tan(x)^2 + 1)
```

Sympy [B] time = 0.383732, size = 36, normalized size = 2.57

$$\frac{x\tan^2(x)}{2\tan^2(x) + 2} + \frac{x}{2\tan^2(x) + 2} + \frac{\tan(x)}{2\tan^2(x) + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)**2),x)
```

```
[Out] x*tan(x)**2/(2*tan(x)**2 + 2) + x/(2*tan(x)**2 + 2) + tan(x)/(2*tan(x)**2 +
 2)
```

Giac [A] time = 1.05259, size = 22, normalized size = 1.57

$$\frac{1}{2}x + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

3.83 $\int \frac{x^4}{\left(1-x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=35

$$\frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rubi [A] time = 0.0110509, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {288, 216}

$$\frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4}{\left(1-x^2\right)^{5/2}}\,dx &= \frac{x^3}{3\left(1-x^2\right)^{3/2}} - \int \frac{x^2}{\left(1-x^2\right)^{3/2}}\,dx \\
&= \frac{x^3}{3\left(1-x^2\right)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}}\,dx \\
&= \frac{x^3}{3\left(1-x^2\right)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)
\end{aligned}
$$

Mathematica [A] time = 0.0027465, size = 26, normalized size = 0.74

$$\frac{x\left(4x^2-3\right)}{3\left(1-x^2\right)^{3/2}} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]
```

Maple [A] time = 0., size = 30, normalized size = 0.9

$$\frac{x^3}{3}\left(-x^2+1\right)^{-\frac{3}{2}} + \arcsin(x) - x\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(5/2),x)
```

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.41346, size = 59, normalized size = 1.69

$$\frac{1}{3}x\left(\frac{3x^2}{\left(-x^2+1\right)^{\frac{3}{2}}} - \frac{2}{\left(-x^2+1\right)^{\frac{3}{2}}}\right) - \frac{x}{3\sqrt{-x^2+1}} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

Fricas [B] time = 2.10504, size = 146, normalized size = 4.17

$$-\frac{6\left(x^4-2\,x^2+1\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(4\,x^3-3\,x\right)\sqrt{-x^2+1}}{3\left(x^4-2\,x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

Sympy [B] time = 2.55635, size = 105, normalized size = 3.

$$\frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3}+\frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3}-\frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3}-\frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3}+\frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

Giac [A] time = 1.06232, size = 39, normalized size = 1.11

$$\frac{\left(4\,x^2-3\right)\sqrt{-x^2+1}x}{3\left(x^2-1\right)^2}+\arcsin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

3.84 $\int -\frac{x^2}{\left(1-x^2\right)^{3/2}} \, dx$

Optimal. Leaf size=17

$$\sin^{-1}(x) - \frac{x}{\sqrt{1-x^2}}$$

```
[Out] -(x/Sqrt[1 - x^2]) + ArcSin[x]
```

Rubi [A] time = 0.0050743, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {288, 216}

$$\sin^{-1}(x) - \frac{x}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[-(x^2/(1 - x^2)^(3/2)),x]
```

```
[Out] -(x/Sqrt[1 - x^2]) + ArcSin[x]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int -\frac{x^2}{\left(1-x^2\right)^{3/2}}\,dx = -\frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)$$

Mathematica [A] time = 0.0068797, size = 32, normalized size = 1.88

$$\frac{\sqrt{1-x^2}x + x^2\sin^{-1}(x) - \sin^{-1}(x)}{x^2-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[-(x^2/(1 - x^2)^(3/2)),x]
```

```
[Out] (x*Sqrt[1 - x^2] - ArcSin[x] + x^2*ArcSin[x])/(-1 + x^2)
```

Maple [A] time = 0.004, size = 16, normalized size = 0.9

$$\arcsin(x) - x\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/(-x^2+1)^(3/2),x)
```

```
[Out] arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.41049, size = 20, normalized size = 1.18

$$-\frac{x}{\sqrt{-x^2+1}} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] -x/sqrt(-x^2 + 1) + arcsin(x)
```

Fricas [B] time = 2.16875, size = 103, normalized size = 6.06

$$-\frac{2\left(x^2-1\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\sqrt{-x^2+1}x}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2*(x^2 - 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - sqrt(-x^2 + 1)*x)/(x^2 - 1)
```

Sympy [B] time = 0.5255, size = 34, normalized size = 2.

$$\frac{x^2\operatorname{asin}(x)}{x^2-1}+\frac{x\sqrt{1-x^2}}{x^2-1}-\frac{\operatorname{asin}(x)}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x**2/(-x**2+1)**(3/2),x)
```

```
[Out] x**2*asin(x)/(x**2 - 1) + x*sqrt(1 - x**2)/(x**2 - 1) - asin(x)/(x**2 - 1)
```

Giac [A] time = 1.07129, size = 28, normalized size = 1.65

$$\frac{\sqrt{-x^2+1}x}{x^2-1}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-x^2/(-x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(-x^2 + 1)*x/(x^2 - 1) + arcsin(x)
```

3.85 $\int e^x \sin(x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

```
[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rubi [A] time = 0.007656, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4432}

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[x],x]
```

```
[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x \sin(x)\,dx = -\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)$$

Mathematica [A] time = 0.0140425, size = 14, normalized size = 0.74

$$\frac{1}{2}e^x(\sin(x) - \cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[x],x]
```

```
[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

Maple [A] time = 0., size = 14, normalized size = 0.7

$$-\frac{e^{x}\cos(x)}{2}+\frac{e^{x}\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x)
```

```
[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Maxima [A] time = 0.92849, size = 15, normalized size = 0.79

$$-\frac{1}{2}\,(\cos(x)-\sin(x))e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*(cos(x) - sin(x))*e^x
```

Fricas [A] time = 2.23052, size = 46, normalized size = 2.42

$$-\frac{1}{2}\,\cos(x)\,e^{x}+\frac{1}{2}\,e^{x}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A] time = 0.279987, size = 15, normalized size = 0.79

$$\frac{e^x\sin(x)}{2} - \frac{e^x\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x)
```

```
[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

Giac [A] time = 1.04556, size = 15, normalized size = 0.79

$$-\frac{1}{2}(\cos(x) - \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*(cos(x) - sin(x))*e^x
```

3.86 $\int \frac{1}{x} \, dx$

Optimal. Leaf size=2

$$\log(x)$$

```
[Out] Log[x]
```

Rubi [A] time = 0.0002056, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {29}

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1),x]

[Out] Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x} \, dx = \log(x)$$

Mathematica [A] time = 0.0001045, size = 2, normalized size = 1.

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1),x]
```

```
[Out] Log[x]
```

Maple [A] time = 0., size = 3, normalized size = 1.5

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
```

```
[Out] ln(x)
```

Maxima [A] time = 0.930007, size = 3, normalized size = 1.5

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="maxima")
```

```
[Out] log(x)
```

Fricas [A] time = 2.24052, size = 11, normalized size = 5.5

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="fricas")
```

```
[Out] log(x)
```

Sympy [A] time = 0.049597, size = 2, normalized size = 1.

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x)
```

```
[Out] log(x)
```

Giac [A] time = 1.05741, size = 4, normalized size = 2.

$$\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="giac")
```

```
[Out] log(abs(x))
```

3.87 $\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt$

Optimal. Leaf size=45

$$-\frac{1}{2(\tan(t)+1)}-\frac{1}{12}\log(\cos(t)-\sin(t))-\frac{1}{4}\log(\sin(t)+\cos(t))+\frac{1}{3}\log(\sin(t)+2\cos(t))$$

```
[Out] -Log[Cos[t] - Sin[t]]/12 - Log[Cos[t] + Sin[t]]/4 + Log[2*Cos[t] + Sin[t]]/
3 - 1/(2*(1 + Tan[t]))
```

Rubi [A] time = 0.123094, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {709, 800}

$$-\frac{1}{2(\tan(t)+1)}-\frac{1}{12}\log(\cos(t)-\sin(t))-\frac{1}{4}\log(\sin(t)+\cos(t))+\frac{1}{3}\log(\sin(t)+2\cos(t))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[2*t]/(1 + Sec[t]^2 + 3*Tan[t]),t]
```

```
[Out] -Log[Cos[t] - Sin[t]]/12 - Log[Cos[t] + Sin[t]]/4 + Log[2*Cos[t] + Sin[t]]/
3 - 1/(2*(1 + Tan[t]))
```

Rule 709

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m + 1))/((m + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dis
t[1/(c*d^2 - b*d*e + a*e^2), Int[((d + e*x)^(m + 1)*Simp[c*d - b*e - c*e*x,
 x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 -
 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[m
, -1]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec(2t)}{1+\sec^2(t)+3\tan(t)}\,dt = \text{Subst}\left(\int \frac{1}{(1+t)^2\left(2-t-t^2\right)}\,dt, t, \tan(t)\right)$$
$$= -\frac{1}{2(1+\tan(t))}+\frac{1}{2}\text{Subst}\left(\int \frac{t}{(1+t)\left(2-t-t^2\right)}\,dt, t, \tan(t)\right)$$
$$= -\frac{1}{2(1+\tan(t))}+\frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{6(-1+t)}-\frac{1}{2(1+t)}+\frac{2}{3(2+t)}\right)dt, t, \tan(t)\right)$$
$$= -\frac{1}{12}\log(\cos(t)-\sin(t))-\frac{1}{4}\log(\cos(t)+\sin(t))+\frac{1}{3}\log(2\cos(t)+\sin(t))-\frac{1}{2(1+\tan(t$$

Mathematica [A] time = 0.180955, size = 73, normalized size = 1.62

$$-\frac{\cos(t)(\log(\cos(t)-\sin(t))+3\log(\sin(t)+\cos(t))-4\log(\sin(t)+2\cos(t)))+\sin(t)(\log(\cos(t)-\sin(t))+3\log(\sin(}{12(\sin(t)+\cos(t))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[2*t]/(1 + Sec[t]^2 + 3*Tan[t]),t]
```

```
[Out] -(Cos[t]*(Log[Cos[t] - Sin[t]] + 3*Log[Cos[t] + Sin[t]] - 4*Log[2*Cos[t] +
Sin[t]]) + (-6 + Log[Cos[t] - Sin[t]] + 3*Log[Cos[t] + Sin[t]] - 4*Log[2*Co
s[t] + Sin[t]])*Sin[t])/(12*(Cos[t] + Sin[t]))
```

Maple [A] time = 0.142, size = 31, normalized size = 0.7

$$\frac{\ln(\tan(t)+2)}{3}-\frac{1}{2+2\tan(t)}-\frac{\ln(1+\tan(t))}{4}-\frac{\ln(-1+\tan(t))}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(2*t)/(1+sec(t)^2+3*tan(t)),t)
```

```
[Out] 1/3*ln(tan(t)+2)-1/2/(1+tan(t))-1/4*ln(1+tan(t))-1/12*ln(-1+tan(t))
```

Maxima [B] time = 1.78015, size = 346, normalized size = 7.69

$$\frac{3 \left(\cos(2t)^2 + \sin(2t)^2 + 2\sin(2t) + 1\right)\log\left(953674316406250\,(3\cos(2t) + \sin(2t) + 4)\cos(4t) + 238418579101562\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="maxima")
```

```
[Out] 1/48*(3*(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)*log(953674316406250*(3*c
os(2*t) + sin(2*t) + 4)*cos(4*t) + 2384185791015625*cos(4*t)^2 + 9536743164
06250*cos(2*t)^2 - 953674316406250*(cos(2*t) - 3*sin(2*t) + 3)*sin(4*t) + 2
384185791015625*sin(4*t)^2 + 953674316406250*sin(2*t)^2 + 2861022949218750*
cos(2*t) - 953674316406250*sin(2*t) + 2384185791015625) - 6*(cos(2*t)^2 + s
in(2*t)^2 + 2*sin(2*t) + 1)*log(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1) +
 5*(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)*log(1/5*(5*cos(2*t)^2 + 5*sin
(2*t)^2 + 6*cos(2*t) + 8*sin(2*t) + 5)/(cos(2*t)^2 + sin(2*t)^2 - 2*sin(2*t
) + 1)) - 24*cos(2*t))/(cos(2*t)^2 + sin(2*t)^2 + 2*sin(2*t) + 1)
```

Fricas [A] time = 2.61796, size = 271, normalized size = 6.02

$$\frac{4\,(\cos(t) + \sin(t))\log\left(\frac{3}{4}\cos(t)^2 + \cos(t)\sin(t) + \frac{1}{4}\right) - 3\,(\cos(t) + \sin(t))\log(2\cos(t)\sin(t) + 1) - (\cos(t) + \sin(t))}{24\,(\cos(t) + \sin(t))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="fricas")
```

```
[Out] 1/24*(4*(cos(t) + sin(t))*log(3/4*cos(t)^2 + cos(t)*sin(t) + 1/4) - 3*(cos(
t) + sin(t))*log(2*cos(t)*sin(t) + 1) - (cos(t) + sin(t))*log(-2*cos(t)*sin
(t) + 1) - 6*cos(t) + 6*sin(t))/(cos(t) + sin(t))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(2t)}{3\tan(t) + \sec^2(t) + 1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*t)/(1+sec(t)**2+3*tan(t)),t)
```

```
[Out] Integral(sec(2*t)/(3*tan(t) + sec(t)**2 + 1), t)
```

Giac [A] time = 1.10377, size = 45, normalized size = 1.

$$-\frac{1}{2(\tan(t)+1)}+\frac{1}{3}\log(|\tan(t)+2|)-\frac{1}{4}\log(|\tan(t)+1|)-\frac{1}{12}\log(|\tan(t)-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(2*t)/(1+sec(t)^2+3*tan(t)),t, algorithm="giac")
```

```
[Out] -1/2/(tan(t) + 1) + 1/3*log(abs(tan(t) + 2)) - 1/4*log(abs(tan(t) + 1)) - 1
/12*log(abs(tan(t) - 1))
```

3.88 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0064947, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\,dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0017493, size = 14, normalized size = 1.

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2,x]
```

```
[Out] x/2 + Sin[2*x]/4
```

Maple [A] time = 0.002, size = 11, normalized size = 0.8

$$\frac{x}{2}+\frac{\cos\left(x\right)\sin\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(x)^2,x)
```

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A] time = 0.928115, size = 14, normalized size = 1.

$$\frac{1}{2}\,x+\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A] time = 2.2063, size = 36, normalized size = 2.57

$$\frac{1}{2}\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.061051, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

Giac [A] time = 1.06457, size = 22, normalized size = 1.57

$$\frac{1}{2}x + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

3.89 $\int \frac{1+x^2}{\sqrt{x}}\,dx$

Optimal. Leaf size=17

$$\frac{2x^{5/2}}{5}+2\sqrt{x}$$

```
[Out] 2*Sqrt[x] + (2*x^(5/2))/5
```

Rubi [A] time = 0.0032497, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{2x^{5/2}}{5}+2\sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)/Sqrt[x],x]
```

```
[Out] 2*Sqrt[x] + (2*x^(5/2))/5
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x^2}{\sqrt{x}}\,dx &= \int \left(\frac{1}{\sqrt{x}}+x^{3/2}\right)dx\\ &= 2\sqrt{x}+\frac{2x^{5/2}}{5}\end{aligned}$$

Mathematica [A] time = 0.0028835, size = 14, normalized size = 0.82

$$\frac{2}{5}\sqrt{x}\left(x^2+5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)/Sqrt[x],x]
```

```
[Out] (2*Sqrt[x]*(5 + x^2))/5
```

Maple [A] time = 0.003, size = 11, normalized size = 0.7

$$\frac{2x^2+10}{5}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)/x^(1/2),x)
```

```
[Out] 2/5*x^(1/2)*(x^2+5)
```

Maxima [A] time = 0.923614, size = 15, normalized size = 0.88

$$\frac{2}{5}x^{\frac{5}{2}}+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*x^(5/2) + 2*sqrt(x)
```

Fricas [A] time = 1.77471, size = 31, normalized size = 1.82

$$\frac{2}{5}\left(x^2+5\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2/5*(x^2 + 5)*sqrt(x)
```

Sympy [A] time = 0.202541, size = 14, normalized size = 0.82

$$\frac{2x^{\frac{5}{2}}}{5} + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)/x**(1/2),x)
```

```
[Out] 2*x**(5/2)/5 + 2*sqrt(x)
```

Giac [A] time = 1.05947, size = 15, normalized size = 0.88

$$\frac{2}{5} x^{\frac{5}{2}} + 2 \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)/x^(1/2),x, algorithm="giac")
```

```
[Out] 2/5*x^(5/2) + 2*sqrt(x)
```

3.90 $\int \frac{x}{\sqrt{5+2x+x^2}}\,dx$

Optimal. Leaf size=23

$$\sqrt{x^2+2x+5}-\sinh^{-1}\left(\frac{x+1}{2}\right)$$

```
[Out] Sqrt[5 + 2*x + x^2] - ArcSinh[(1 + x)/2]
```

Rubi [A] time = 0.0098206, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {640, 619, 215}

$$\sqrt{x^2+2x+5}-\sinh^{-1}\left(\frac{x+1}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[5 + 2*x + x^2],x]
```

```
[Out] Sqrt[5 + 2*x + x^2] - ArcSinh[(1 + x)/2]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{\sqrt{5+2x+x^2}}\,dx &= \sqrt{5+2x+x^2}-\int \frac{1}{\sqrt{5+2x+x^2}}\,dx \\
&= \sqrt{5+2x+x^2}-\frac{1}{4}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{16}}}\,dx, x, 2+2x\right) \\
&= \sqrt{5+2x+x^2}-\sinh^{-1}\left(\frac{1+x}{2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0052585, size = 25, normalized size = 1.09

$$\sqrt{x^2+2x+5}-\sinh^{-1}\left(\frac{1}{4}(2x+2)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[5 + 2*x + x^2],x]
```

```
[Out] Sqrt[5 + 2*x + x^2] - ArcSinh[(2 + 2*x)/4]
```

Maple [A] time = 0.004, size = 20, normalized size = 0.9

$$-\mathrm{Arcsinh}\left(\frac{1}{2}+\frac{x}{2}\right)+\sqrt{x^2+2\,x+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+2*x+5)^(1/2),x)
```

```
[Out] -arcsinh(1/2+1/2*x)+(x^2+2*x+5)^(1/2)
```

Maxima [A] time = 1.40927, size = 26, normalized size = 1.13

$$\sqrt{x^2+2\,x+5}-\mathrm{arsinh}\left(\frac{1}{2}\,x+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + 2*x + 5) - arcsinh(1/2*x + 1/2)
```

Fricas [A] time = 1.80516, size = 77, normalized size = 3.35

$$\sqrt{x^2+2\,x+5}+\log\left(-x+\sqrt{x^2+2\,x+5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 + 2*x + 5) + log(-x + sqrt(x^2 + 2*x + 5) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+2*x+5)**(1/2),x)
```

```
[Out] Integral(x/sqrt(x**2 + 2*x + 5), x)
```

Giac [A] time = 1.05923, size = 36, normalized size = 1.57

$$\sqrt{x^2+2\,x+5}+\log\left(-x+\sqrt{x^2+2\,x+5}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(x^2 + 2*x + 5) + log(-x + sqrt(x^2 + 2*x + 5) - 1)
```

3.91 $\int \cos(x)\sin^2(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sin^3(x)}{3}$$

```
[Out] Sin[x]^3/3
```

Rubi [A] time = 0.0127217, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2564, 30}

$$\frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[x]^2,x]
```

```
[Out] Sin[x]^3/3
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \cos(x)\sin^2(x)\,dx &= \text{Subst}\left(\int x^2\,dx, x, \sin(x)\right)\\ &= \frac{\sin^3(x)}{3} \end{aligned}$$

Mathematica [A] time = 0.0010916, size = 8, normalized size = 1.

$$\frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x]^2,x]
```

```
[Out] Sin[x]^3/3
```

Maple [A] time = 0.002, size = 7, normalized size = 0.9

$$\frac{(\sin(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^2,x)
```

```
[Out] 1/3*sin(x)^3
```

Maxima [A] time = 0.919453, size = 8, normalized size = 1.

$$\frac{1}{3}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*sin(x)^3
```

Fricas [A] time = 2.07203, size = 38, normalized size = 4.75

$$-\frac{1}{3}\left(\cos(x)^2-1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(cos(x)^2 - 1)*sin(x)
```

Sympy [A] time = 0.057543, size = 5, normalized size = 0.62

$$\frac{\sin^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**2,x)
```

```
[Out] sin(x)**3/3
```

Giac [A] time = 1.05907, size = 8, normalized size = 1.

$$\frac{1}{3}\sin(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/3*sin(x)^3
```

3.92 $\int \frac{e^x}{1+e^x} \, dx$

Optimal. Leaf size=6

$$\log\left(e^x+1\right)$$

```
[Out] Log[1 + E^x]
```

Rubi [A] time = 0.0164506, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2246, 31}

$$\log\left(e^x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 + E^x),x]

[Out] Log[1 + E^x]
```

Rule 2246

```
Int[((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)*((a_) + (b_.)*((F_)^((e_.)*((
c_.) + (d_.)*(x_))))^(n_.))^(p_.), x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subs
t[Int[(a + b*x)^p, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d,
 e, n, p}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{e^x}{1+e^x} \, dx &= \text{Subst}\left(\int \frac{1}{1+x} \, dx, x, e^x\right) \\ &= \log\left(1+e^x\right) \end{aligned}$$

Mathematica [A] time = 0.0042869, size = 6, normalized size = 1.

$$\log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 + E^x),x]
```

```
[Out] Log[1 + E^x]
```

Maple [A] time = 0., size = 6, normalized size = 1.

$$\ln(1 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(1+exp(x)),x)
```

```
[Out] ln(1+exp(x))
```

Maxima [A] time = 0.933567, size = 7, normalized size = 1.17

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="maxima")
```

```
[Out] log(e^x + 1)
```

Fricas [A] time = 2.04549, size = 19, normalized size = 3.17

$$\log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="fricas")
```

```
[Out] log(e^x + 1)
```

Sympy [A] time = 0.071965, size = 5, normalized size = 0.83

$$\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x)
```

```
[Out] log(exp(x) + 1)
```

Giac [A] time = 1.06321, size = 7, normalized size = 1.17

$$\log\left(e^{x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(x)),x, algorithm="giac")
```

```
[Out] log(e^x + 1)
```

3.93 $\int \frac{e^{2x}}{1+e^x} \, dx$

Optimal. Leaf size=12

$$e^x - \log(e^x + 1)$$

```
[Out] E^x - Log[1 + E^x]
```

Rubi [A] time = 0.0264504, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2248, 43}

$$e^x - \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(1 + E^x),x]

[Out] E^x - Log[1 + E^x]
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{2x}}{1+e^x}\,dx = \text{Subst}\left(\int \frac{x}{1+x}\,dx, x, e^x\right)$$
$$= \text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, e^x\right)$$
$$= e^x - \log\left(1+e^x\right)$$

Mathematica [A] time = 0.0075345, size = 12, normalized size = 1.

$$e^x - \log\left(e^x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(1 + E^x),x]
```

```
[Out] E^x - Log[1 + E^x]
```

Maple [A] time = 0.002, size = 11, normalized size = 0.9

$$e^x - \ln\left(1+e^x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(1+exp(x)),x)
```

```
[Out] exp(x)-ln(1+exp(x))
```

Maxima [A] time = 0.929565, size = 14, normalized size = 1.17

$$e^x - \log\left(e^x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="maxima")
```

```
[Out] e^x - log(e^x + 1)
```

Fricas [A] time = 2.12464, size = 27, normalized size = 2.25

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="fricas")
```

```
[Out] e^x - log(e^x + 1)
```

Sympy [A] time = 0.080449, size = 8, normalized size = 0.67

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x)
```

```
[Out] exp(x) - log(exp(x) + 1)
```

Giac [A] time = 1.05476, size = 14, normalized size = 1.17

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="giac")
```

```
[Out] e^x - log(e^x + 1)
```

3.94 $\int \frac{1}{1-\cos(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rubi [A] time = 0.0090661, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2648}

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x])^(-1),x]

[Out] -(Sin[x]/(1 - Cos[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\cos(x)}\,dx = -\frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A] time = 0.008356, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x])^(-1),x]
```

```
[Out] -Cot[x/2]
```

Maple [A] time = 0., size = 9, normalized size = 0.8

$$-\left(\tan\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)),x)
```

```
[Out] -1/tan(1/2*x)
```

Maxima [A] time = 0.927184, size = 14, normalized size = 1.17

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="maxima")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Fricas [A] time = 2.21, size = 30, normalized size = 2.5

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Sympy [A] time = 0.353726, size = 7, normalized size = 0.58

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x)
```

```
[Out] -1/tan(x/2)
```

Giac [A] time = 1.05536, size = 11, normalized size = 0.92

$$-\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="giac")
```

```
[Out] -1/tan(1/2*x)
```

3.95 $\int \sec^2(x)\tan(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sec^2(x)}{2}$$

```
[Out] Sec[x]^2/2
```

Rubi [A] time = 0.0119479, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 30}

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\tan(x)\,dx &= \text{Subst}\left(\int x\,dx, x, \sec(x)\right)\\ &= \frac{\sec^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.004168, size = 8, normalized size = 1.

$$\frac{\sec^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2*Tan[x],x]
```

```
[Out] Sec[x]^2/2
```

Maple [A] time = 0., size = 7, normalized size = 0.9

$$\frac{(\sec(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2*tan(x),x)
```

```
[Out] 1/2*sec(x)^2
```

Maxima [A] time = 0.926987, size = 8, normalized size = 1.

$$\frac{1}{2}\tan(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="maxima")
```

```
[Out] 1/2*tan(x)^2
```

Fricas [A] time = 1.9112, size = 19, normalized size = 2.38

$$\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="fricas")
```

```
[Out] 1/2/cos(x)^2
```

Sympy [A] time = 0.06361, size = 7, normalized size = 0.88

$$\frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2*tan(x),x)
```

```
[Out] 1/(2*cos(x)**2)
```

Giac [A] time = 1.05978, size = 8, normalized size = 1.

$$\frac{1}{2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2*tan(x),x, algorithm="giac")
```

```
[Out] 1/2/cos(x)^2
```

3.96 $\int x \log(x)\, dx$

Optimal. Leaf size=17

$$\frac{1}{2}x^2\log(x) - \frac{x^2}{4}$$

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rubi [A] time = 0.0042968, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2304}

$$\frac{1}{2}x^2\log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x],x]
```

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\log(x)\, dx = -\frac{x^2}{4} + \frac{1}{2}x^2\log(x)$$

Mathematica [A] time = 0.0007928, size = 17, normalized size = 1.

$$\frac{1}{2}x^2\log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x],x]
```

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Maple [A] time = 0., size = 14, normalized size = 0.8

$$-\frac{x^2}{4} + \frac{x^2 \ln (x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x),x)
```

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Maxima [A] time = 0.935629, size = 18, normalized size = 1.06

$$\frac{1}{2} x^2 \log (x) - \frac{1}{4} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Fricas [A] time = 1.80642, size = 35, normalized size = 2.06

$$\frac{1}{2} x^2 \log (x) - \frac{1}{4} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A] time = 0.082428, size = 12, normalized size = 0.71

$$\frac{x^2 \log(x)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x),x)
```

```
[Out] x**2*log(x)/2 - x**2/4
```

Giac [A] time = 1.05853, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x^2 \log(x) - \frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

3.97 $\int \cos(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sin^2(x)}{2}$$

```
[Out] Sin[x]^2/2
```

Rubi [A] time = 0.007369, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2564, 30}

$$\frac{\sin^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[x],x]
```

```
[Out] Sin[x]^2/2
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin(x)\,dx &= \text{Subst}\left(\int x\,dx, x, \sin(x)\right)\\ &= \frac{\sin^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0013897, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x],x]
```

```
[Out] -Cos[x]^2/2
```

Maple [A] time = 0., size = 7, normalized size = 0.9

$$\frac{(\sin(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x)
```

```
[Out] 1/2*sin(x)^2
```

Maxima [A] time = 0.923263, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*cos(x)^2
```

Fricas [A] time = 1.9094, size = 20, normalized size = 2.5

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)^2
```

Sympy [A] time = 0.053841, size = 5, normalized size = 0.62

$$\frac{\sin^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x)
```

```
[Out] sin(x)**2/2
```

Giac [A] time = 1.06816, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2
```

3.98 $\int \frac{1+x}{\sqrt{2x-x^2}}\,dx$

Optimal. Leaf size=24

$$-\sqrt{2x-x^2}-2\sin^{-1}(1-x)$$

```
[Out] -Sqrt[2*x - x^2] - 2*ArcSin[1 - x]
```

Rubi [A] time = 0.0092363, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {640, 619, 216}

$$-\sqrt{2x-x^2}-2\sin^{-1}(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/Sqrt[2*x - x^2],x]
```

```
[Out] -Sqrt[2*x - x^2] - 2*ArcSin[1 - x]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1+x}{\sqrt{2x-x^2}}\,dx = -\sqrt{2x-x^2} + 2\int \frac{1}{\sqrt{2x-x^2}}\,dx$$
$$= -\sqrt{2x-x^2} - \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, 2-2x\right)$$
$$= -\sqrt{2x-x^2} - 2\sin^{-1}(1-x)$$

Mathematica [A] time = 0.0316691, size = 27, normalized size = 1.12

$$-\sqrt{-(x-2)x} - 4\sin^{-1}\left(\sqrt{1-\frac{x}{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/Sqrt[2*x - x^2],x]
```

```
[Out] -Sqrt[-((-2 + x)*x)] - 4*ArcSin[Sqrt[1 - x/2]]
```

Maple [A] time = 0.006, size = 21, normalized size = 0.9

$$2 \arcsin(-1+x) - \sqrt{-x^2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(-x^2+2*x)^(1/2),x)
```

```
[Out] 2*arcsin(-1+x)-(-x^2+2*x)^(1/2)
```

Maxima [A] time = 1.4145, size = 30, normalized size = 1.25

$$-\sqrt{-x^2+2x} - 2 \arcsin(-x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 2*x) - 2*arcsin(-x + 1)
```

Fricas [A] time = 1.80234, size = 68, normalized size = 2.83

$$-\sqrt{-x^2 + 2x} - 4\arctan\left(\frac{\sqrt{-x^2 + 2x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 2*x) - 4*arctan(sqrt(-x^2 + 2*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{-x(x-2)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x**2+2*x)**(1/2),x)
```

```
[Out] Integral((x + 1)/sqrt(-x*(x - 2)), x)
```

Giac [A] time = 1.07746, size = 27, normalized size = 1.12

$$-\sqrt{-x^2 + 2x} + 2\arcsin(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(-x^2+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 2*x) + 2*arcsin(x - 1)
```

3.99 $\int \frac{2e^x}{2+3e^{2x}}\,dx$

Optimal. Leaf size=20

$$\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)$$

```
[Out] Sqrt[2/3]*ArcTan[Sqrt[3/2]*E^x]
```

Rubi [A] time = 0.0226488, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {12, 2249, 203}

$$\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2*E^x)/(2 + 3*E^(2*x)),x]
```

```
[Out] Sqrt[2/3]*ArcTan[Sqrt[3/2]*E^x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{2e^x}{2+3e^{2x}}\,dx &= 2\int \frac{e^x}{2+3e^{2x}}\,dx\\ &= 2\,\mathrm{Subst}\left(\int \frac{1}{2+3x^2}\,dx, x, e^x\right)\\ &= \sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)\end{aligned}$$

Mathematica [A] time = 0.0051819, size = 20, normalized size = 1.

$$\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*E^x)/(2 + 3*E^(2*x)),x]
```

```
[Out] Sqrt[2/3]*ArcTan[Sqrt[3/2]*E^x]
```

Maple [A] time = 0., size = 14, normalized size = 0.7

$$\frac{\sqrt{6}}{3}\arctan\left(\frac{e^x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*exp(x)/(2+3*exp(2*x)),x)
```

```
[Out] 1/3*arctan(1/2*exp(x)*6^(1/2))*6^(1/2)
```

Maxima [A] time = 1.421, size = 18, normalized size = 0.9

$$\frac{1}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

Fricas [A] time = 1.92223, size = 72, normalized size = 3.6

$$\frac{1}{3}\sqrt{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{3}\sqrt{2}e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*e^x)
```

Sympy [A] time = 0.106325, size = 15, normalized size = 0.75

$$\operatorname{RootSum}\left(6z^{2}+1,\left(i\mapsto i\log\left(2i+e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x)
```

```
[Out] RootSum(6*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))
```

Giac [A] time = 1.06551, size = 18, normalized size = 0.9

$$\frac{1}{3}\sqrt{6}\arctan\left(\frac{1}{2}\sqrt{6}e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*exp(x)/(2+3*exp(2*x)),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(6)*arctan(1/2*sqrt(6)*e^x)
```

3.100 $\int \frac{x^4}{(1-x^2)^{5/2}}\,dx$

Optimal. Leaf size=35

$$\frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rubi [A] time = 0.0088799, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {288, 216}

$$\frac{x^3}{3\left(1-x^2\right)^{3/2}}-\frac{x}{\sqrt{1-x^2}}+\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] x^3/(3*(1 - x^2)^(3/2)) - x/Sqrt[1 - x^2] + ArcSin[x]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{\left(1-x^2\right)^{5/2}}\,dx &= \frac{x^3}{3\left(1-x^2\right)^{3/2}} - \int \frac{x^2}{\left(1-x^2\right)^{3/2}}\,dx \\ &= \frac{x^3}{3\left(1-x^2\right)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}}\,dx \\ &= \frac{x^3}{3\left(1-x^2\right)^{3/2}} - \frac{x}{\sqrt{1-x^2}} + \sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0029117, size = 26, normalized size = 0.74

$$\frac{x\left(4x^2-3\right)}{3\left(1-x^2\right)^{3/2}} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(1 - x^2)^(5/2),x]
```

```
[Out] (x*(-3 + 4*x^2))/(3*(1 - x^2)^(3/2)) + ArcSin[x]
```

Maple [A] time = 0., size = 30, normalized size = 0.9

$$\frac{x^3}{3}\left(-x^2+1\right)^{-\frac{3}{2}} + \arcsin(x) - x\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-x^2+1)^(5/2),x)
```

```
[Out] 1/3*x^3/(-x^2+1)^(3/2)+arcsin(x)-x/(-x^2+1)^(1/2)
```

Maxima [A] time = 1.40746, size = 59, normalized size = 1.69

$$\frac{1}{3}x\left(\frac{3x^2}{\left(-x^2+1\right)^{\frac{3}{2}}} - \frac{2}{\left(-x^2+1\right)^{\frac{3}{2}}}\right) - \frac{x}{3\sqrt{-x^2+1}} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*x*(3*x^2/(-x^2 + 1)^(3/2) - 2/(-x^2 + 1)^(3/2)) - 1/3*x/sqrt(-x^2 + 1)
+ arcsin(x)
```

Fricas [B] time = 1.8756, size = 146, normalized size = 4.17

$$-\frac{6\left(x^4-2x^2+1\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\left(4x^3-3x\right)\sqrt{-x^2+1}}{3\left(x^4-2x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*(6*(x^4 - 2*x^2 + 1)*arctan((sqrt(-x^2 + 1) - 1)/x) - (4*x^3 - 3*x)*sq
rt(-x^2 + 1))/(x^4 - 2*x^2 + 1)
```

Sympy [B] time = 2.55683, size = 105, normalized size = 3.

$$\frac{3x^4\operatorname{asin}(x)}{3x^4-6x^2+3}+\frac{4x^3\sqrt{1-x^2}}{3x^4-6x^2+3}-\frac{6x^2\operatorname{asin}(x)}{3x^4-6x^2+3}-\frac{3x\sqrt{1-x^2}}{3x^4-6x^2+3}+\frac{3\operatorname{asin}(x)}{3x^4-6x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-x**2+1)**(5/2),x)
```

```
[Out] 3*x**4*asin(x)/(3*x**4 - 6*x**2 + 3) + 4*x**3*sqrt(1 - x**2)/(3*x**4 - 6*x*
*2 + 3) - 6*x**2*asin(x)/(3*x**4 - 6*x**2 + 3) - 3*x*sqrt(1 - x**2)/(3*x**4
 - 6*x**2 + 3) + 3*asin(x)/(3*x**4 - 6*x**2 + 3)
```

Giac [A] time = 1.07752, size = 39, normalized size = 1.11

$$\frac{\left(4x^2-3\right)\sqrt{-x^2+1}x}{3\left(x^2-1\right)^2}+\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-x^2+1)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(4*x^2 - 3)*sqrt(-x^2 + 1)*x/(x^2 - 1)^2 + arcsin(x)
```

3.101 $\int \frac{e^{6x}}{1+e^{4x}}\,dx$

Optimal. Leaf size=20

$$\frac{e^{2x}}{2}-\frac{1}{2}\tan^{-1}\left(e^{2x}\right)$$

```
[Out] E^(2*x)/2 - ArcTan[E^(2*x)]/2
```

Rubi [A] time = 0.0239091, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2248, 321, 203}

$$\frac{e^{2x}}{2}-\frac{1}{2}\tan^{-1}\left(e^{2x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(6*x)/(1 + E^(4*x)),x]
```

```
[Out] E^(2*x)/2 - ArcTan[E^(2*x)]/2
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^{6x}}{1+e^{4x}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, e^{2x}\right)\\ &= \frac{e^{2x}}{2}-\frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^{2x}\right)\\ &= \frac{e^{2x}}{2}-\frac{1}{2}\tan^{-1}\left(e^{2x}\right)\end{aligned}$$

Mathematica [A] time = 0.007745, size = 18, normalized size = 0.9

$$\frac{1}{2}\left(e^{2x}-\tan^{-1}\left(e^{2x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(6*x)/(1 + E^(4*x)),x]
```

```
[Out] (E^(2*x) - ArcTan[E^(2*x)])/2
```

Maple [A] time = 0.003, size = 15, normalized size = 0.8

$$\frac{\left(e^{x}\right)^{2}}{2}-\frac{\arctan\left(\left(e^{x}\right)^{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(6*x)/(1+exp(4*x)),x)
```

```
[Out] 1/2*exp(x)^2-1/2*arctan(exp(x)^2)
```

Maxima [A] time = 1.41559, size = 19, normalized size = 0.95

$$-\frac{1}{2}\arctan\left(e^{(2x)}\right)+\frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(6*x)/(1+exp(4*x)),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

Fricas [A] time = 1.91873, size = 49, normalized size = 2.45

$$-\frac{1}{2}\arctan\left(e^{(2x)}\right)+\frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(6*x)/(1+exp(4*x)),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

Sympy [A] time = 0.110742, size = 24, normalized size = 1.2

$$\frac{e^{2x}}{2}+\operatorname{RootSum}\left(16z^2+1,\left(i\mapsto i\log\left(-4i+e^{2x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(6*x)/(1+exp(4*x)),x)
```

```
[Out] exp(2*x)/2 + RootSum(16*_z**2 + 1, Lambda(_i, _i*log(-4*_i + exp(2*x))))
```

Giac [A] time = 1.04763, size = 19, normalized size = 0.95

$$-\frac{1}{2}\arctan\left(e^{(2x)}\right)+\frac{1}{2}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(6*x)/(1+exp(4*x)),x, algorithm="giac")
```

```
[Out] -1/2*arctan(e^(2*x)) + 1/2*e^(2*x)
```

3.102 $\int \log\left(2+3x^2\right)\,dx$

Optimal. Leaf size=33

$$x\log\left(3x^2+2\right)-2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

```
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Rubi [A] time = 0.0111443, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2448, 321, 203}

$$x\log\left(3x^2+2\right)-2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[2 + 3*x^2],x]
```

```
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \log\left(2+3x^2\right)\,dx &= x\log\left(2+3x^2\right)-6\int\frac{x^2}{2+3x^2}\,dx\\ &= -2x+x\log\left(2+3x^2\right)+4\int\frac{1}{2+3x^2}\,dx\\ &= -2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)+x\log\left(2+3x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0121088, size = 33, normalized size = 1.

$$x\log\left(3x^2+2\right)-2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[2 + 3*x^2],x]
```

```
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Maple [A] time = 0., size = 27, normalized size = 0.8

$$-2\,x+x\ln\left(3\,x^2+2\right)+\frac{2\,\sqrt{6}}{3}\arctan\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(3*x^2+2),x)
```

```
[Out] -2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Maxima [A] time = 1.41188, size = 35, normalized size = 1.06

$$x \log\left(3 x^2 + 2\right) + \frac{2}{3} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6} x\right) - 2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="maxima")
```

```
[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

Fricas [A] time = 1.89351, size = 103, normalized size = 3.12

$$\frac{2}{3} \sqrt{3}\sqrt{2} \arctan\left(\frac{1}{2} \sqrt{3}\sqrt{2} x\right) + x \log\left(3 x^2 + 2\right) - 2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x) + x*log(3*x^2 + 2) - 2*x
```

Sympy [A] time = 0.125984, size = 31, normalized size = 0.94

$$x \log\left(3x^2 + 2\right) - 2x + \frac{2\sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6}x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(3*x**2+2),x)
```

```
[Out] x*log(3*x**2 + 2) - 2*x + 2*sqrt(6)*atan(sqrt(6)*x/2)/3
```

Giac [A] time = 1.04873, size = 35, normalized size = 1.06

$$x \log\left(3 x^2 + 2\right) + \frac{2}{3} \sqrt{6} \arctan\left(\frac{1}{2} \sqrt{6} x\right) - 2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="giac")
```

```
[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

3.103 $\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx$

Optimal. Leaf size=21

$$\frac{x}{r\sqrt{2Hr^2-a^2}}$$

```
[Out] x/(r*Sqrt[-a^2 + 2*H*r^2])
```

Rubi [A] time = 0.0161468, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {8}

$$\frac{x}{r\sqrt{2Hr^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 + 2*H*r^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2}}\,dx=\frac{x}{r\sqrt{-a^2+2Hr^2}}$$

Mathematica [A] time = 0.0000386, size = 21, normalized size = 1.

$$\frac{x}{r\sqrt{2Hr^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 + 2*H*r^2]),x]
```

```
[Out] x/(r*Sqrt[-a^2 + 2*H*r^2])
```

Maple [A] time = 0.002, size = 20, normalized size = 1.

$$\frac{x}{r}\frac{1}{\sqrt{2\,H r^{2}-a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*H*r^2-a^2)^(1/2),x)
```

```
[Out] x/r/(2*H*r^2-a^2)^(1/2)
```

Maxima [A] time = 0.931835, size = 26, normalized size = 1.24

$$\frac{x}{\sqrt{2\,H r^{2}-a^{2}}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2)*r)
```

Fricas [A] time = 1.73989, size = 55, normalized size = 2.62

$$\frac{\sqrt{2\,H r^{2}-a^{2}}x}{2\,H r^{3}-a^{2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2*H*r^2 - a^2)*x/(2*H*r^3 - a^2*r)
```

Sympy [A] time = 0.052351, size = 15, normalized size = 0.71

$$\frac{x}{r\sqrt{2Hr^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r**2-a**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - a**2))
```

Giac [A] time = 1.04571, size = 26, normalized size = 1.24

$$\frac{x}{\sqrt{2\,Hr^2 - a^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2)*r)
```

3.104 $\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx$

Optimal. Leaf size=26

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

```
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2])
```

Rubi [A] time = 0.0194002, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {8}

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2]),x]
```

```
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

Mathematica [A] time = 0.0000479, size = 26, normalized size = 1.

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2]),x]
```

```
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2])
```

Maple [A] time = 0.002, size = 25, normalized size = 1.

$$\frac{x}{r}\frac{1}{\sqrt{2\,H r^{2}-a^{2}-e^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*H*r^2-a^2-e^2)^(1/2),x)
```

```
[Out] x/r/(2*H*r^2-a^2-e^2)^(1/2)
```

Maxima [A] time = 0.935708, size = 32, normalized size = 1.23

$$\frac{x}{\sqrt{2\,H r^{2}-a^{2}-e^{2}}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - e^2)*r)
```

Fricas [A] time = 1.78943, size = 74, normalized size = 2.85

$$\frac{\sqrt{2\,H r^{2}-a^{2}-e^{2}}x}{2\,H r^{3}-\left(a^{2}+e^{2}\right)r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2*H*r^2 - a^2 - e^2)*x/(2*H*r^3 - (a^2 + e^2)*r)
```

Sympy [A] time = 0.062354, size = 19, normalized size = 0.73

$$\frac{x}{r\sqrt{2Hr^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r**2-a**2-e**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - a**2 - e**2))
```

Giac [A] time = 1.05253, size = 31, normalized size = 1.19

$$\frac{x}{\sqrt{2\,Hr^2 - a^2 - e^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-a^2-e^2)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - e^2)*r)
```

3.105 $\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx$

Optimal. Leaf size=27

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

```
[Out] x/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4])
```

Rubi [A] time = 0.0227884, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {8}

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4]),x]

[Out] x/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2+2Hr^2-2Kr^4}}\,dx=\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

Mathematica [A] time = 0.0000354, size = 27, normalized size = 1.

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4]),x]

[Out] x/(r*Sqrt[-a^2 + 2*H*r^2 - 2*K*r^4])
```

Maple [A] time = 0., size = 26, normalized size = 1.

$$\frac{x}{r}\frac{1}{\sqrt{-2\,K r^{4}+2\,H r^{2}-a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x)

[Out] x/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2)
```

Maxima [A] time = 0.927586, size = 34, normalized size = 1.26

$$\frac{x}{\sqrt{-2\,K r^{4}+2\,H r^{2}-a^{2}}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="maxima")

[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*r)
```

Fricas [A] time = 1.8654, size = 85, normalized size = 3.15

$$-\frac{\sqrt{-2\,K r^{4}+2\,H r^{2}-a^{2}}x}{2\,K r^{5}-2\,H r^{3}+a^{2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="fricas")

[Out] -sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*x/(2*K*r^5 - 2*H*r^3 + a^2*r)
```

Sympy [A] time = 0.054584, size = 22, normalized size = 0.81

$$\frac{x}{r\sqrt{2Hr^2 - 2Kr^4 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r**4+2*H*r**2-a**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r**4 - a**2))
```

Giac [A] time = 1.05276, size = 34, normalized size = 1.26

$$\frac{x}{\sqrt{-2\,Kr^4 + 2\,Hr^2 - a^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2)*r)
```

3.106 $\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx$

Optimal. Leaf size=32

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

```
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4])
```

Rubi [A] time = 0.0256325, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.032, Rules used = {8}

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4]),x]

[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}\,dx = \frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

Mathematica [A] time = 0.0000345, size = 32, normalized size = 1.

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4]),x]
```

```
[Out] x/(r*Sqrt[-a^2 - e^2 + 2*H*r^2 - 2*K*r^4])
```

Maple [A] time = 0.001, size = 31, normalized size = 1.

$$\frac{x}{r}\frac{1}{\sqrt{-2\,K r^4+2\,H r^2-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x)
```

```
[Out] x/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2)
```

Maxima [A] time = 0.932703, size = 41, normalized size = 1.28

$$\frac{x}{\sqrt{-2\,K r^4+2\,H r^2-a^2-e^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r)
```

Fricas [A] time = 1.81301, size = 104, normalized size = 3.25

$$-\frac{\sqrt{-2\,K r^4+2\,H r^2-a^2-e^2}x}{2\,K r^5-2\,H r^3+\left(a^2+e^2\right)r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*x/(2*K*r^5 - 2*H*r^3 + (a^2 + e^2)*r)
```

Sympy [A] time = 0.055048, size = 26, normalized size = 0.81

$$\frac{x}{r\sqrt{2Hr^2-2Kr^4-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r**4+2*H*r**2-a**2-e**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r**4 - a**2 - e**2))
```

Giac [A] time = 1.04741, size = 39, normalized size = 1.22

$$\frac{x}{\sqrt{-2\,Kr^4+2\,Hr^2-a^2-e^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*K*r^4+2*H*r^2-a^2-e^2)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(-2*K*r^4 + 2*H*r^2 - a^2 - e^2)*r)
```

3.107 $\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx$

Optimal. Leaf size=24

$$\frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

```
[Out] x/(r*Sqrt[-a^2 - 2*r*(K - H*r)])
```

Rubi [A] time = 0.0265996, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {8}

$$\frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 - 2*r*(K - H*r)])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2-2Kr+2Hr^2}}\,dx = \frac{x}{r\sqrt{-a^2-2r(K-Hr)}}$$

Mathematica [A] time = 0.0000408, size = 25, normalized size = 1.04

$$\frac{x}{r\sqrt{-a^2+2Hr^2-2Kr}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2]),x]
```

```
[Out] x/(r*Sqrt[-a^2 - 2*K*r + 2*H*r^2])
```

Maple [A] time = 0., size = 24, normalized size = 1.

$$\frac{x}{r}\frac{1}{\sqrt{2\,H r^{2}-2\,K r-a^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x)
```

```
[Out] 1/r/(2*H*r^2-2*K*r-a^2)^(1/2)*x
```

Maxima [A] time = 0.930996, size = 31, normalized size = 1.29

$$\frac{x}{\sqrt{2\,H r^{2}-a^{2}-2\,K r}\,r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - 2*K*r)*r)
```

Fricas [A] time = 1.87546, size = 80, normalized size = 3.33

$$\frac{\sqrt{2\,H r^{2}-a^{2}-2\,K r}\,x}{2\,H r^{3}-a^{2} r-2\,K r^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2*H*r^2 - a^2 - 2*K*r)*x/(2*H*r^3 - a^2*r - 2*K*r^2)
```

Sympy [A] time = 0.053192, size = 20, normalized size = 0.83

$$\frac{x}{r\sqrt{2Hr^2 - 2Kr - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r**2-2*K*r-a**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r - a**2))
```

Giac [A] time = 1.24152, size = 31, normalized size = 1.29

$$\frac{x}{\sqrt{2\,Hr^2 - a^2 - 2\,Kr}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - 2*K*r)*r)
```

3.108 $\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx$

Optimal. Leaf size=29

$$\frac{x}{r\sqrt{-a^2-e^2-2r(K-Hr)}}$$

```
[Out] x/(r*Sqrt[-a^2 - e^2 - 2*r*(K - H*r)])
```

Rubi [A] time = 0.0316036, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {8}

$$\frac{x}{r\sqrt{-a^2-e^2-2r(K-Hr)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]),x]

[Out] x/(r*Sqrt[-a^2 - e^2 - 2*r*(K - H*r)])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{r\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx=\frac{x}{r\sqrt{-a^2-e^2-2r(K-Hr)}}$$

Mathematica [A] time = 0.000036, size = 30, normalized size = 1.03

$$\frac{x}{r\sqrt{-a^2-e^2+2Hr^2-2Kr}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]),x]
```

```
[Out] x/(r*Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2])
```

Maple [A] time = 0., size = 29, normalized size = 1.

$$\frac{x}{r}\frac{1}{\sqrt{2\,H r^2-2\,K r-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x)
```

```
[Out] x/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)
```

Maxima [A] time = 0.928842, size = 38, normalized size = 1.31

$$\frac{x}{\sqrt{2\,H r^2-a^2-e^2-2\,K r}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)*r)
```

Fricas [A] time = 1.81653, size = 99, normalized size = 3.41

$$\frac{\sqrt{2\,H r^2-a^2-e^2-2\,K r}x}{2\,H r^3-2\,K r^2-\left(a^2+e^2\right)r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)*x/(2*H*r^3 - 2*K*r^2 - (a^2 + e^2)*r)
```

Sympy [A] time = 0.054356, size = 24, normalized size = 0.83

$$\frac{x}{r\sqrt{2Hr^2 - 2Kr - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r**2-2*K*r-a**2-e**2)**(1/2),x)
```

```
[Out] x/(r*sqrt(2*H*r**2 - 2*K*r - a**2 - e**2))
```

Giac [A] time = 1.07776, size = 36, normalized size = 1.24

$$\frac{x}{\sqrt{2\,Hr^2 - a^2 - 2\,Kr - e^2}r}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(2*H*r^2 - a^2 - 2*K*r - e^2)*r)
```

3.109 $\int \frac{r}{\sqrt{-a^2+2er^2}} \, dx$

Optimal. Leaf size=19

$$\frac{rx}{\sqrt{2er^2-a^2}}$$

```
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2]
```

Rubi [A] time = 0.0094807, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {8}

$$\frac{rx}{\sqrt{2er^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-a^2 + 2*E*r^2],x]

[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{r}{\sqrt{-a^2+2er^2}} \, dx = \frac{rx}{\sqrt{-a^2+2er^2}}$$

Mathematica [A] time = 0.0000577, size = 19, normalized size = 1.

$$\frac{rx}{\sqrt{2er^2-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-a^2 + 2*E*r^2],x]
```

```
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2]
```

Maple [A] time = 0., size = 18, normalized size = 1.

$$rx\frac{1}{\sqrt{2\,Er^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*E*r^2-a^2)^(1/2),x)
```

```
[Out] r*x/(2*E*r^2-a^2)^(1/2)
```

Maxima [A] time = 0.929918, size = 23, normalized size = 1.21

$$\frac{rx}{\sqrt{2\,Er^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*E*r^2-a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] r*x/sqrt(2*E*r^2 - a^2)
```

Fricas [A] time = 1.80718, size = 34, normalized size = 1.79

$$\frac{rx}{\sqrt{2\,Er^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*E*r^2-a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] r*x/sqrt(2*E*r^2 - a^2)
```

Sympy [A] time = 0.050965, size = 15, normalized size = 0.79

$$\frac{rx}{\sqrt{-a^2 + 2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*E*r**2-a**2)**(1/2),x)
```

```
[Out] r*x/sqrt(-a**2 + 2*E*r**2)
```

Giac [A] time = 1.04923, size = 23, normalized size = 1.21

$$\frac{rx}{\sqrt{2\,Er^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*E*r^2-a^2)^(1/2),x, algorithm="giac")
```

```
[Out] r*x/sqrt(2*E*r^2 - a^2)
```

3.110 $\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx$

Optimal. Leaf size=24

$$\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

```
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2]
```

Rubi [A] time = 0.0111275, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {8}

$$\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-a^2 - e^2 + 2*E*r^2],x]
```

```
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2}}\,dx = \frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

Mathematica [A] time = 0.0000289, size = 24, normalized size = 1.

$$\frac{rx}{\sqrt{-a^2-e^2+2er^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-a^2 - e^2 + 2*E*r^2],x]
```

```
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2]
```

Maple [A] time = 0.002, size = 23, normalized size = 1.

$$rx\frac{1}{\sqrt{2\,Er^2-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*E*r^2-a^2-e^2)^(1/2),x)
```

```
[Out] r*x/(2*E*r^2-a^2-e^2)^(1/2)
```

Maxima [A] time = 0.937238, size = 30, normalized size = 1.25

$$\frac{rx}{\sqrt{2\,Er^2-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*E*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")
```

```
[Out] r*x/sqrt(2*E*r^2 - a^2 - e^2)
```

Fricas [A] time = 1.81658, size = 42, normalized size = 1.75

$$\frac{rx}{\sqrt{2\,Er^2-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*E*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")
```

```
[Out] r*x/sqrt(2*E*r^2 - a^2 - e^2)
```

Sympy [A] time = 0.05323, size = 19, normalized size = 0.79

$$\frac{rx}{\sqrt{-a^2 - e^2 + 2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*E*r**2-a**2-e**2)**(1/2),x)
```

```
[Out] r*x/sqrt(-a**2 - e**2 + 2*E*r**2)
```

Giac [A] time = 1.0504, size = 28, normalized size = 1.17

$$\frac{rx}{\sqrt{2\,Er^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*E*r^2-a^2-e^2)^(1/2),x, algorithm="giac")
```

```
[Out] r*x/sqrt(2*E*r^2 - a^2 - e^2)
```

3.111 $\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}} dx$

Optimal. Leaf size=25

$$\frac{rx}{\sqrt{-a^2-2Kr^4+2er^2}}$$

```
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4]
```

Rubi [A] time = 0.0127465, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {8}

$$\frac{rx}{\sqrt{-a^2-2Kr^4+2er^2}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4],x]

[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{r}{\sqrt{-a^2+2er^2-2Kr^4}} dx = \frac{rx}{\sqrt{-a^2+2er^2-2Kr^4}}$$

Mathematica [A] time = 0.0000295, size = 25, normalized size = 1.

$$\frac{rx}{\sqrt{-a^2-2Kr^4+2er^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4],x]
```

```
[Out] (r*x)/Sqrt[-a^2 + 2*E*r^2 - 2*K*r^4]
```

Maple [A] time = 0.002, size = 24, normalized size = 1.

$$rx\frac{1}{\sqrt{-2\,Kr^4+2\,Er^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(-2*K*r^4+2*E*r^2-a^2)^(1/2),x)
```

```
[Out] r*x/(-2*K*r^4+2*E*r^2-a^2)^(1/2)
```

Maxima [A] time = 0.930996, size = 31, normalized size = 1.24

$$\frac{rx}{\sqrt{-2\,Kr^4+2\,Er^2-a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*K*r^4+2*E*r^2-a^2)^(1/2),x, algorithm="maxima")
```

```
[Out] r*x/sqrt(-2*K*r^4 + 2*E*r^2 - a^2)
```

Fricas [A] time = 1.88055, size = 85, normalized size = 3.4

$$-\frac{\sqrt{-2\,Kr^4+2\,Er^2-a^2}rx}{2\,Kr^4-2\,Er^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*K*r^4+2*E*r^2-a^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-2*K*r^4 + 2*E*r^2 - a^2)*r*x/(2*K*r^4 - 2*E*r^2 + a^2)
```

Sympy [A] time = 0.053411, size = 22, normalized size = 0.88

$$\frac{rx}{\sqrt{-2Kr^4 - a^2 + 2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*K*r**4+2*E*r**2-a**2)**(1/2),x)
```

```
[Out] r*x/sqrt(-2*K*r**4 - a**2 + 2*E*r**2)
```

Giac [A] time = 1.06028, size = 31, normalized size = 1.24

$$\frac{rx}{\sqrt{-2\,Kr^4 + 2\,Er^2 - a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*K*r^4+2*E*r^2-a^2)^(1/2),x, algorithm="giac")
```

```
[Out] r*x/sqrt(-2*K*r^4 + 2*E*r^2 - a^2)
```

3.112 $\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx$

Optimal. Leaf size=30

$$\frac{rx}{\sqrt{-a^2-e^2-2Kr^4+2er^2}}$$

```
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4]
```

Rubi [A] time = 0.0153597, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {8}

$$\frac{rx}{\sqrt{-a^2-e^2-2Kr^4+2er^2}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4],x]
```

```
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{r}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}\,dx = \frac{rx}{\sqrt{-a^2-e^2+2er^2-2Kr^4}}$$

Mathematica [A] time = 0.0000365, size = 30, normalized size = 1.

$$\frac{rx}{\sqrt{-a^2-e^2-2Kr^4+2er^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4],x]
```

```
[Out] (r*x)/Sqrt[-a^2 - e^2 + 2*E*r^2 - 2*K*r^4]
```

Maple [A] time = 0., size = 29, normalized size = 1.

$$rx\frac{1}{\sqrt{-2\,Kr^4+2\,Er^2-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(-2*K*r^4+2*E*r^2-a^2-e^2)^(1/2),x)
```

```
[Out] r*x/(-2*K*r^4+2*E*r^2-a^2-e^2)^(1/2)
```

Maxima [A] time = 0.929396, size = 38, normalized size = 1.27

$$\frac{rx}{\sqrt{-2\,Kr^4+2\,Er^2-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*K*r^4+2*E*r^2-a^2-e^2)^(1/2),x, algorithm="maxima")
```

```
[Out] r*x/sqrt(-2*K*r^4 + 2*E*r^2 - a^2 - e^2)
```

Fricas [A] time = 1.70583, size = 101, normalized size = 3.37

$$-\frac{\sqrt{-2\,Kr^4+2\,Er^2-a^2-e^2}rx}{2\,Kr^4-2\,Er^2+a^2+e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*K*r^4+2*E*r^2-a^2-e^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-2*K*r^4 + 2*E*r^2 - a^2 - e^2)*r*x/(2*K*r^4 - 2*E*r^2 + a^2 + e^2)
```

Sympy [A] time = 0.054855, size = 26, normalized size = 0.87

$$\frac{rx}{\sqrt{-2Kr^4 - a^2 - e^2 + 2er^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*K*r**4+2*E*r**2-a**2-e**2)**(1/2),x)
```

```
[Out] r*x/sqrt(-2*K*r**4 - a**2 - e**2 + 2*E*r**2)
```

Giac [A] time = 1.06962, size = 36, normalized size = 1.2

$$\frac{rx}{\sqrt{-2\,Kr^4 + 2\,Er^2 - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*K*r^4+2*E*r^2-a^2-e^2)^(1/2),x, algorithm="giac")
```

```
[Out] r*x/sqrt(-2*K*r^4 + 2*E*r^2 - a^2 - e^2)
```

3.113 $\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx$

Optimal. Leaf size=27

$$\frac{rx}{\sqrt{-a^2-e^2-2r(K-Hr)}}$$

```
[Out] (r*x)/Sqrt[-a^2 - e^2 - 2*r*(K - H*r)]
```

Rubi [A] time = 0.0056563, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {8}

$$\frac{rx}{\sqrt{-a^2-e^2-2r(K-Hr)}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2],x]

[Out] (r*x)/Sqrt[-a^2 - e^2 - 2*r*(K - H*r)]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{r}{\sqrt{-a^2-e^2-2Kr+2Hr^2}}\,dx=\frac{rx}{\sqrt{-a^2-e^2-2r(K-Hr)}}$$

Mathematica [A] time = 0.0000306, size = 28, normalized size = 1.04

$$\frac{rx}{\sqrt{-a^2-e^2+2Hr^2-2Kr}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2],x]
```

```
[Out] (r*x)/Sqrt[-a^2 - e^2 - 2*K*r + 2*H*r^2]
```

Maple [A] time = 0.001, size = 27, normalized size = 1.

$$rx\frac{1}{\sqrt{2\,Hr^2-2\,Kr-a^2-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x)
```

```
[Out] r*x/(2*H*r^2-2*K*r-a^2-e^2)^(1/2)
```

Maxima [A] time = 0.925849, size = 35, normalized size = 1.3

$$\frac{rx}{\sqrt{2\,Hr^2-a^2-e^2-2\,Kr}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="maxima")
```

```
[Out] r*x/sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)
```

Fricas [A] time = 1.81226, size = 53, normalized size = 1.96

$$\frac{rx}{\sqrt{2\,Hr^2-a^2-e^2-2\,Kr}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="fricas")
```

```
[Out] r*x/sqrt(2*H*r^2 - a^2 - e^2 - 2*K*r)
```

Sympy [A] time = 0.05365, size = 24, normalized size = 0.89

$$\frac{rx}{\sqrt{2Hr^2 - 2Kr - a^2 - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*H*r**2-2*K*r-a**2-e**2)**(1/2),x)
```

```
[Out] r*x/sqrt(2*H*r**2 - 2*K*r - a**2 - e**2)
```

Giac [A] time = 1.05683, size = 34, normalized size = 1.26

$$\frac{rx}{\sqrt{2\,Hr^2 - a^2 - 2\,Kr - e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*H*r^2-2*K*r-a^2-e^2)^(1/2),x, algorithm="giac")
```

```
[Out] r*x/sqrt(2*H*r^2 - a^2 - 2*K*r - e^2)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=", ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: # isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```
    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```