Computer algebra independent integration tests

0-Independent-test-suites/Hearn-Problems

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [284]. This is test number [5].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 98.24 (279)	% 1.76 (5)
Mathematica	% 99.65 (283)	% 0.35 (1)
Maple	% 99.3 (282)	% 0.7 (2)
Maxima	% 82.75 (235)	% 17.25 (49)
Fricas	% 98.59 (280)	% 1.41 (4)
Sympy	% 87.68 (249)	% 12.32 (35)
Giac	% 89.79 (255)	% 10.21 (29)

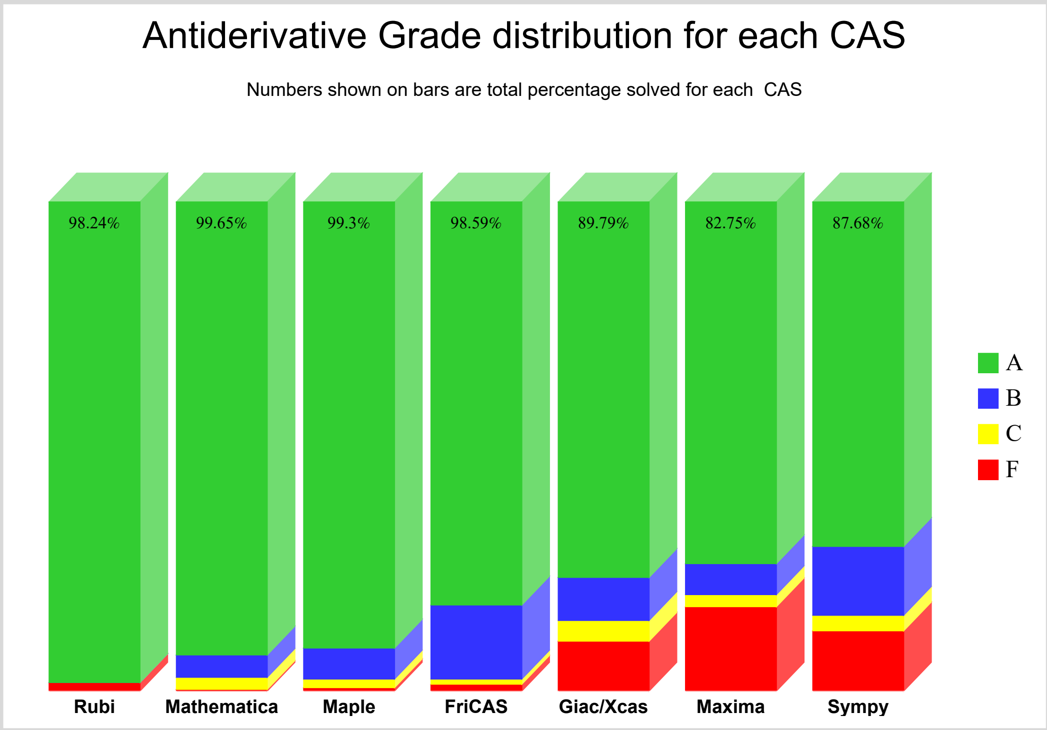
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

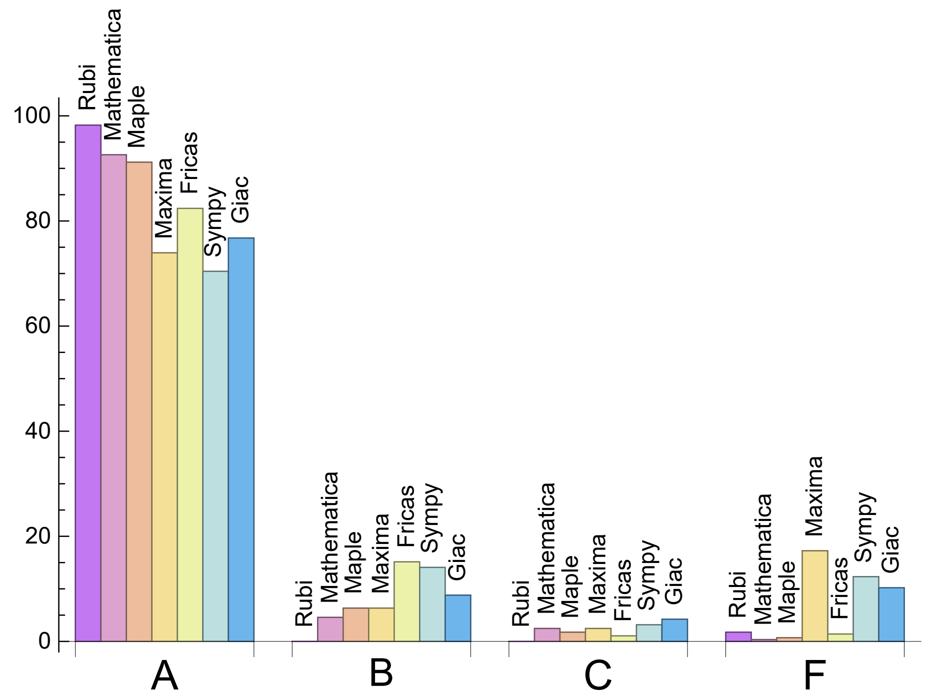
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	98.24	0.	0.	1.76
Mathematica	92.61	4.58	2.46	0.35
Maple	91.2	6.34	1.76	0.7
Maxima	73.94	6.34	2.46	17.25
Fricas	82.39	15.14	1.06	1.41
Sympy	70.42	14.08	3.17	12.32
Giac	76.76	8.8	4.23	10.21

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.04	39.11	0.99	22.	1.
Mathematica	0.13	65.92	1.27	23.	1.
Maple	0.02	4314.3	46.45	21.	0.94
Maxima	1.07	46.17	1.62	23.	1.17
Fricas	1.77	142.31	4.06	61.	2.83
Sympy	2.01	86.51	1.89	20.	1.
Giac	1.08	141.44	2.31	27.	1.26

1.4 list of integrals that has no closed form antiderivative

{75, 145, 170, 273}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {145}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {20, 279, 281}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

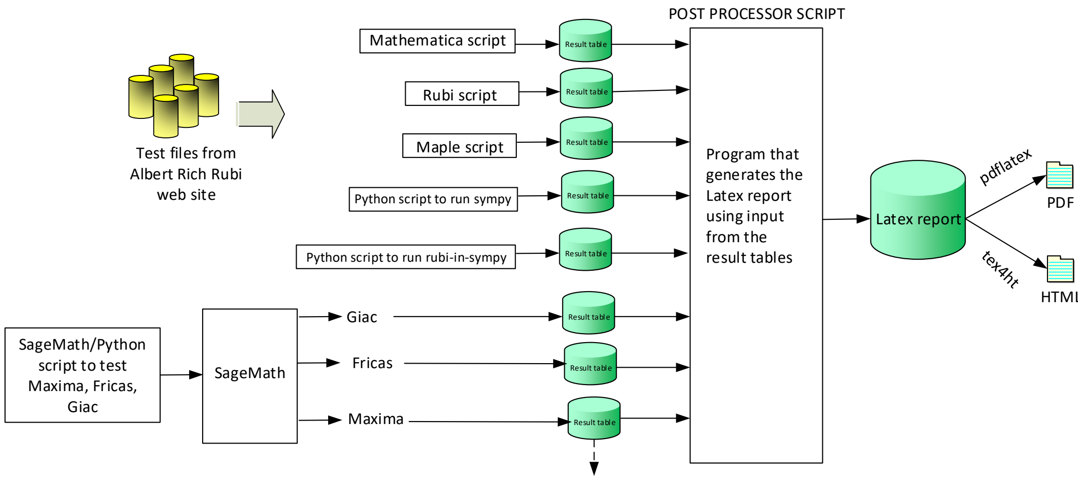
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 280, 282, 283 }

B grade: { }

C grade: { }

F grade: { 169, 278, 279, 281, 284}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48, 49, 50, 53, 54, 55, 56, 57, 58, 59, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 282, 283, 284 }

B grade: { 52, 81, 82, 108, 111, 120, 121, 190, 202, 235, 236, 256, 280 }

C grade: { 20, 44, 45, 51, 278, 279, 281 }

F grade: { 60}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48, 50, 52, 53, 54, 55, 56, 57, 58, 59, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 127, 128, 129, 130, 131, 132, 133, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 281, 283, 284 }

B grade: { 32, 44, 45, 60, 61, 110, 124, 126, 134, 135, 176, 204, 237, 242, 256, 257, 278, 280 }

C grade: { 49, 51, 203, 279, 282 }

F grade: { 86, 251 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 41, 46, 47, 48, 50, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 106, 107, 108, 109, 110, 113, 114, 115, 116, 117, 118, 119, 120, 121, 124, 125, 126, 127, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 162, 164, 165, 167, 169, 170, 171, 172, 173, 174, 177, 178, 179, 182, 183, 184, 188, 189, 190, 192, 193, 198, 199, 200, 201, 202, 209, 210, 215, 216, 217, 218, 219, 220, 221, 223, 227, 228, 229, 230, 231, 232, 233, 234, 237, 238, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 283

B grade: { 81, 82, 111, 112, 128, 129, 130, 146, 191, 194, 195, 204, 222, 225, 226, 241, 256, 280 }

C grade: { 102, 103, 104, 105, 166, 168, 197 }

F grade: { 8, 21, 35, 39, 40, 42, 43, 44, 45, 49, 51, 56, 64, 86, 122, 123, 147, 157, 160, 161, 163, 175, 176, 180, 181, 185, 186, 187, 196, 203, 205, 206, 207, 208, 211, 212, 213, 214, 224, 235, 236, 239, 251, 257, 278, 279, 281, 282, 284 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 38, 43, 46, 47, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 83, 84, 85, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 196, 198, 200, 201, 202, 203, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 226, 229, 230, 231, 232, 233, 234, 237, 238, 239, 240, 241, 242, 243, 248, 249, 250, 251, 252, 253, 255, 258, 259, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 274, 275, 276, 277, 279, 282, 283 }

B grade: { 36, 37, 39, 40, 41, 42, 44, 45, 49, 78, 79, 80, 81, 82, 90, 103, 110, 111, 112, 127, 133, 194, 195, 199, 204, 220, 222, 227, 228, 235, 236, 244, 245, 246, 247, 254, 256, 260, 268, 269, 278, 280, 284 }

C grade: { 128, 129, 197 }

F grade: { 86, 174, 257, 281 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 10, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 71, 74, 75, 76, 77, 78, 79, 83, 84, 85, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 130, 132, 134, 135, 136, 137, 144, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 177, 181, 182, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 199, 200, 201, 202, 205, 206, 209, 210, 215, 216, 217, 218, 219, 221, 223, 225, 226, 227, 230, 231, 232, 240, 241, 243, 244, 245, 248, 250, 252, 253, 255, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 273, 274, 276, 277, 282, 283, 284 }

B grade: { 7, 8, 12, 13, 32, 41, 80, 81, 82, 90, 103, 104, 131, 133, 138, 139, 140, 141, 142, 143, 175, 178, 179, 180, 183, 184, 189, 204, 220, 222, 228, 229, 233, 234, 239, 246, 247, 254, 256, 272 }

C grade: { 9, 14, 31, 50, 70, 72, 73, 203, 224 }

F grade: { 56, 86, 123, 128, 129, 145, 146, 147, 160, 162, 163, 176, 196, 197, 198, 207, 208, 211, 212, 213, 214, 235, 236, 237, 238, 242, 249, 251, 257, 265, 275, 278, 279, 280, 281 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 80, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 164, 165, 167, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 239, 240, 241, 242, 243, 248, 249, 250, 251, 252, 253, 254, 255, 258, 261, 262, 263, 264, 265, 266, 267, 270, 271, 272, 273, 275, 276, 277, 282, 283, 284 }

B grade: { 11, 23, 32, 79, 81, 82, 110, 111, 112, 118, 127, 195, 199, 220, 222, 228, 244, 245, 246, 247, 256, 260, 268, 269, 280 }

C grade: { 134, 135, 136, 137, 138, 139, 140, 141, 160, 161, 166, 197 }

F grade: { 33, 34, 44, 45, 47, 48, 56, 63, 86, 128, 129, 162, 163, 168, 169, 198, 200, 201, 203, 235, 236, 237, 238, 257, 259, 274, 278, 279, 281 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	31	10	16
normalized size	1	1.	1.	0.81	1.	1.94	0.62	1.
time (sec)	N/A	0.002	0.	0.001	0.941	1.573	0.051	1.091

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	39	17	22
normalized size	1	1.	1.	0.77	1.	1.77	0.77	1.
time (sec)	N/A	0.01	0.001	0.	0.95	1.605	0.053	1.101

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	39	15	22
normalized size	1	1.	1.	0.77	1.	1.77	0.68	1.
time (sec)	N/A	0.004	0.001	0.001	0.948	1.781	0.052	1.093

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	11	2	4
normalized size	1	1.	1.	1.5	1.5	5.5	1.	2.
time (sec)	N/A	0.	0.	0.	1.017	1.89	0.051	1.077

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	24	27	43	120	29	31
normalized size	1	1.	0.67	0.75	1.19	3.33	0.81	0.86
time (sec)	N/A	0.013	0.011	0.006	0.949	1.84	0.109	1.067

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	28	25	32	108	24	46
normalized size	1	1.	0.88	0.78	1.	3.38	0.75	1.44
time (sec)	N/A	0.012	0.011	0.008	0.948	1.772	0.134	1.082

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	34	66	54	77	144	57
normalized size	1	1.	0.85	1.65	1.35	1.92	3.6	1.42
time (sec)	N/A	0.027	0.017	0.007	0.937	1.941	0.655	1.068

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	38	35	0	277	124	46
normalized size	1	1.	1.12	1.03	0.	8.15	3.65	1.35
time (sec)	N/A	0.03	0.011	0.006	0.	2.004	0.198	1.082

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	46	26	19
normalized size	1	1.	1.	0.94	1.19	2.88	1.62	1.19
time (sec)	N/A	0.006	0.006	0.002	1.42	1.989	0.154	1.075

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	17	19	55	22	19
normalized size	1	1.	0.84	0.89	1.	2.89	1.16	1.
time (sec)	N/A	0.01	0.005	0.003	1.411	1.95	0.099	1.063

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	35	36	53	186	39	108
normalized size	1	1.	0.71	0.73	1.08	3.8	0.8	2.2
time (sec)	N/A	0.029	0.02	0.011	1.433	1.932	0.161	1.119

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	69	105	166	4216	109
normalized size	1	1.	0.91	1.01	1.54	2.44	62.	1.6
time (sec)	N/A	0.052	0.03	0.008	0.961	2.762	85.206	1.063

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	34	44	58	70	121	88
normalized size	1	1.	0.72	0.94	1.23	1.49	2.57	1.87
time (sec)	N/A	0.02	0.011	0.007	0.948	2.019	0.54	1.141

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	30	41	54	61	393	54
normalized size	1	1.	0.75	1.02	1.35	1.52	9.82	1.35
time (sec)	N/A	0.019	0.014	0.007	1.419	2.073	0.944	1.075

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	20	26	69	19	27
normalized size	1	1.	1.	0.74	0.96	2.56	0.7	1.
time (sec)	N/A	0.018	0.004	0.004	1.424	2.015	0.114	1.088

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	40	35	46	112	41	47
normalized size	1	1.	0.98	0.85	1.12	2.73	1.	1.15
time (sec)	N/A	0.02	0.008	0.006	1.417	1.829	0.121	1.104

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	34	32	42	124	31	49
normalized size	1	1.	0.79	0.74	0.98	2.88	0.72	1.14
time (sec)	N/A	0.13	0.019	0.006	1.41	1.845	0.131	1.104

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	64	58	97	302	73	97
normalized size	1	1.	0.75	0.68	1.14	3.55	0.86	1.14
time (sec)	N/A	0.041	0.018	0.	1.44	1.865	0.137	1.073

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	64	58	97	302	73	97
normalized size	1	1.	0.75	0.68	1.14	3.55	0.86	1.14
time (sec)	N/A	0.041	0.01	0.001	1.45	1.951	0.137	1.102

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	67	67	73	54	72	180	70	72
normalized size	1	1.	1.09	0.81	1.07	2.69	1.04	1.07
time (sec)	N/A	0.036	0.055	0.003	1.425	1.968	0.169	1.091

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	17	19	0	45	20	24
normalized size	1	1.	0.94	1.06	0.	2.5	1.11	1.33
time (sec)	N/A	0.004	0.01	0.002	0.	2.082	0.057	1.089

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	33	36	57	104	201	103
normalized size	1	1.	0.85	0.92	1.46	2.67	5.15	2.64
time (sec)	N/A	0.013	0.019	0.002	0.965	1.991	0.605	1.077

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	57	73	92	188	597	189
normalized size	1	1.	0.95	1.22	1.53	3.13	9.95	3.15
time (sec)	N/A	0.021	0.027	0.005	0.954	2.109	1.185	1.071

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	22	7	15
normalized size	1	1.	1.	1.1	1.4	2.2	0.7	1.5
time (sec)	N/A	0.001	0.001	0.002	0.95	1.927	0.057	1.078

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	16	24	10	16
normalized size	1	1.	1.	1.08	1.33	2.	0.83	1.33
time (sec)	N/A	0.002	0.003	0.002	0.931	1.997	0.284	1.084

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	38	14	26
normalized size	1	1.	1.	1.06	1.33	2.11	0.78	1.44
time (sec)	N/A	0.01	0.002	0.002	0.951	1.882	0.261	1.081

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	30	39	68	26	41
normalized size	1	1.	1.	0.97	1.26	2.19	0.84	1.32
time (sec)	N/A	0.016	0.003	0.002	0.943	2.017	0.267	1.072

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	38	10	27
normalized size	1	1.	1.	1.06	1.33	2.11	0.56	1.5
time (sec)	N/A	0.004	0.004	0.005	0.95	1.995	0.132	1.097

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	38	61	19	41
normalized size	1	1.	1.	1.04	1.36	2.18	0.68	1.46
time (sec)	N/A	0.013	0.004	0.009	0.959	1.995	0.325	1.079

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	61	138	36	70
normalized size	1	1.	0.83	1.02	1.45	3.29	0.86	1.67
time (sec)	N/A	0.023	0.038	0.01	0.931	1.845	0.404	1.064

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	20	20	14
normalized size	1	1.	1.	1.1	1.4	2.	2.	1.4
time (sec)	N/A	0.002	0.002	0.005	1.431	1.878	0.104	1.066

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	22	28	46	15	31
normalized size	1	1.	1.	2.2	2.8	4.6	1.5	3.1
time (sec)	N/A	0.003	0.003	0.004	0.946	1.905	0.112	1.072

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	66	58	89	209	78	0
normalized size	1	1.	0.85	0.74	1.14	2.68	1.	0.
time (sec)	N/A	0.044	0.022	0.003	1.451	1.931	0.309	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	65	54	76	221	71	0
normalized size	1	1.	0.88	0.73	1.03	2.99	0.96	0.
time (sec)	N/A	0.034	0.018	0.004	1.435	2.007	0.308	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	89	92	0	751	20	140
normalized size	1	1.	0.77	0.8	0.	6.53	0.17	1.22
time (sec)	N/A	0.066	0.029	0.003	0.	1.937	0.145	1.107

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	43	35	46	201	46	53
normalized size	1	1.	1.23	1.	1.31	5.74	1.31	1.51
time (sec)	N/A	0.011	0.018	0.002	1.448	2.071	0.317	1.119

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	43	36	55	182	48	53
normalized size	1	1.	1.23	1.03	1.57	5.2	1.37	1.51
time (sec)	N/A	0.014	0.015	0.003	1.439	1.927	0.323	1.111

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	120	111	204	621	151	128
normalized size	1	1.	0.7	0.65	1.19	3.63	0.88	0.75
time (sec)	N/A	0.108	0.048	0.005	1.464	1.984	0.397	1.114

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	68	56	0	475	24	100
normalized size	1	1.	0.93	0.77	0.	6.51	0.33	1.37
time (sec)	N/A	0.074	0.053	0.033	0.	1.983	0.35	1.148

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	68	56	0	475	24	100
normalized size	1	1.	0.93	0.77	0.	6.51	0.33	1.37
time (sec)	N/A	0.017	0.025	0.025	0.	2.165	0.359	1.137

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	83	54	101	255	158	109
normalized size	1	1.	1.15	0.75	1.4	3.54	2.19	1.51
time (sec)	N/A	0.065	0.035	0.006	1.446	1.893	0.414	1.086

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	60	0	408	24	136
normalized size	1	1.	1.	0.9	0.	6.09	0.36	2.03
time (sec)	N/A	0.052	0.031	0.03	0.	1.995	0.423	1.123

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	60	0	266	92	69
normalized size	1	1.	1.	0.9	0.	3.97	1.37	1.03
time (sec)	N/A	0.012	0.02	0.025	0.	2.084	0.189	1.082

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	91	386	0	988	24	0
normalized size	1	1.	0.46	1.97	0.	5.04	0.12	0.
time (sec)	N/A	0.148	0.062	0.064	0.	2.085	0.47	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	91	386	0	932	24	0
normalized size	1	1.	0.46	1.97	0.	4.76	0.12	0.
time (sec)	N/A	0.139	0.082	0.054	0.	2.159	0.487	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	75	66	88	230	83	90
normalized size	1	1.	1.03	0.9	1.21	3.15	1.14	1.23
time (sec)	N/A	0.1	0.016	0.008	1.432	1.965	0.229	1.079

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	122	111	151	603	14	0
normalized size	1	1.	0.88	0.8	1.09	4.37	0.1	0.
time (sec)	N/A	0.192	0.04	0.071	1.441	2.022	0.572	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	115	95	144	593	14	0
normalized size	1	1.	0.83	0.69	1.04	4.3	0.1	0.
time (sec)	N/A	0.281	0.027	0.098	1.435	2.133	0.256	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	339	339	209	22	0	3183	14	323
normalized size	1	1.	0.62	0.06	0.	9.39	0.04	0.95
time (sec)	N/A	0.235	0.006	0.005	0.	2.206	1.155	1.095

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	98	66	119	371	44	122
normalized size	1	1.	1.01	0.68	1.23	3.82	0.45	1.26
time (sec)	N/A	0.052	0.026	0.	1.433	2.119	128.354	1.086

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	42	30	0	608	165	277
normalized size	1	1.	0.15	0.11	0.	2.21	0.6	1.01
time (sec)	N/A	0.262	0.012	0.007	0.	2.138	0.203	1.098

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	260	41	54	124	46	54
normalized size	1	1.	5.31	0.84	1.1	2.53	0.94	1.1
time (sec)	N/A	0.037	0.11	0.005	1.446	2.041	0.167	1.098

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	19	5	11
normalized size	1	1.	1.	1.12	1.38	2.38	0.62	1.38
time (sec)	N/A	0.001	0.001	0.	0.965	1.889	0.08	1.066

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	35	12	18
normalized size	1	1.	1.	0.82	1.06	2.06	0.71	1.06
time (sec)	N/A	0.004	0.001	0.	0.94	1.895	0.084	1.092

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	35	12	18
normalized size	1	1.	1.	0.82	1.06	2.06	0.71	1.06
time (sec)	N/A	0.006	0.001	0.002	0.946	1.923	0.086	1.089

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	19	34	0	59	0	0
normalized size	1	1.	0.73	1.31	0.	2.27	0.	0.
time (sec)	N/A	0.01	0.007	0.007	0.	1.916	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	16	42	15	20
normalized size	1	1.	1.	1.07	1.07	2.8	1.	1.33
time (sec)	N/A	0.004	0.001	0.	0.943	2.065	0.091	1.095

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	127	104	96	410	133	139
normalized size	1	1.	1.	0.82	0.76	3.23	1.05	1.09
time (sec)	N/A	0.141	0.003	0.002	0.937	2.053	0.263	1.075

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	19	5	8
normalized size	1	1.	1.	0.88	1.	2.38	0.62	1.
time (sec)	N/A	0.012	0.001	0.	0.94	2.004	0.082	1.094

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	A	A	A	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	2	2	0	9	4	23	2	4
normalized size	1	1.	0.	4.5	2.	11.5	1.	2.
time (sec)	N/A	0.002	0.002	0.003	1.033	1.804	0.447	1.081

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	5	11	7	28	3	7
normalized size	1	1.	1.25	2.75	1.75	7.	0.75	1.75
time (sec)	N/A	0.004	0.01	0.002	1.035	1.854	0.471	1.08

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	18	3	5
normalized size	1	1.	1.	1.33	1.33	6.	1.	1.67
time (sec)	N/A	0.012	0.004	0.	0.955	2.004	0.086	1.074

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	15	8	62	14	0
normalized size	1	1.	1.	0.88	0.47	3.65	0.82	0.
time (sec)	N/A	0.035	0.016	0.003	1.027	1.937	0.636	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	0	34	15	16
normalized size	1	1.	1.	1.08	0.	2.83	1.25	1.33
time (sec)	N/A	0.016	0.002	0.001	0.	2.048	0.917	1.089

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	29	28	25	34	63	22	32
normalized size	1	1.04	1.	0.89	1.21	2.25	0.79	1.14
time (sec)	N/A	0.011	0.001	0.003	0.95	1.965	0.103	1.083

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	53	48	63	109	44	63
normalized size	1	1.	0.98	0.89	1.17	2.02	0.81	1.17
time (sec)	N/A	0.028	0.01	0.001	0.959	1.982	0.12	1.081

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	27	30	51	77	24	49
normalized size	1	1.	0.93	1.03	1.76	2.66	0.83	1.69
time (sec)	N/A	0.013	0.013	0.007	0.93	1.992	0.363	1.109

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	47	59	85	42	78
normalized size	1	1.	1.	1.02	1.28	1.85	0.91	1.7
time (sec)	N/A	0.023	0.015	0.003	0.956	1.979	0.321	1.076

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	59	58	77	111	54	127
normalized size	1	1.	1.	0.98	1.31	1.88	0.92	2.15
time (sec)	N/A	0.032	0.017	0.003	0.945	1.968	0.33	1.072

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	24	31	57	36	31
normalized size	1	1.	1.	1.04	1.35	2.48	1.57	1.35
time (sec)	N/A	0.007	0.002	0.003	1.443	2.056	0.305	1.181

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	26	29	38	58	31	38
normalized size	1	1.	0.96	1.07	1.41	2.15	1.15	1.41
time (sec)	N/A	0.02	0.003	0.001	0.966	2.021	0.307	1.084

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	37	49	93	53	49
normalized size	1	1.	1.	0.84	1.11	2.11	1.2	1.11
time (sec)	N/A	0.023	0.002	0.004	1.416	2.023	0.335	1.078

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	45	59	113	63	59
normalized size	1	1.	1.	0.83	1.09	2.09	1.17	1.09
time (sec)	N/A	0.025	0.003	0.003	1.429	2.049	0.349	1.089

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	42	76	29	45
normalized size	1	1.	1.	1.28	1.68	3.04	1.16	1.8
time (sec)	N/A	0.01	0.002	0.007	0.939	2.101	0.306	1.08

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	7	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.006	0.036	0.014	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	12	3	5
normalized size	1	1.	1.	1.25	1.25	3.	0.75	1.25
time (sec)	N/A	0.002	0.001	0.001	0.947	2.057	0.059	1.077

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	11	2	3
normalized size	1	1.	1.	1.5	1.5	5.5	1.	1.5
time (sec)	N/A	0.002	0.001	0.001	0.939	2.194	0.054	1.068

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	6	4	38	5	8
normalized size	1	1.	1.	1.2	0.8	7.6	1.	1.6
time (sec)	N/A	0.002	0.002	0.	0.931	2.127	0.061	1.097

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	46	3	23
normalized size	1	1.	1.	1.33	1.33	15.33	1.	7.67
time (sec)	N/A	0.002	0.002	0.002	0.935	1.977	0.066	1.081

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	27	26	34	124	75	35
normalized size	1	1.	1.29	1.24	1.62	5.9	3.57	1.67
time (sec)	N/A	0.026	0.04	0.014	1.426	2.061	0.365	1.1

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	33	7	20	59	15	23
normalized size	1	1.	11.	2.33	6.67	19.67	5.	7.67
time (sec)	N/A	0.003	0.003	0.004	0.942	2.098	0.098	1.088

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	17	9	20	77	15	23
normalized size	1	1.	3.4	1.8	4.	15.4	3.	4.6
time (sec)	N/A	0.002	0.003	0.005	0.942	2.082	0.097	1.102

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	38	10	14
normalized size	1	1.	1.	0.79	1.	2.71	0.71	1.
time (sec)	N/A	0.005	0.002	0.	0.935	1.982	0.058	1.075

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	22	46	15	22
normalized size	1	1.	1.	0.85	1.1	2.3	0.75	1.1
time (sec)	N/A	0.016	0.002	0.005	0.945	1.816	0.563	1.071

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	15	31	8	15
normalized size	1	1.	1.15	0.85	1.15	2.38	0.62	1.15
time (sec)	N/A	0.005	0.002	0.001	0.947	1.791	0.061	1.076

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.008	0.04	0.385	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	61	17	20
normalized size	1	1.	1.	0.84	1.05	3.21	0.89	1.05
time (sec)	N/A	0.019	0.004	0.01	0.956	1.635	1.094	1.069

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	36	10	14
normalized size	1	1.	1.	0.79	1.	2.57	0.71	1.
time (sec)	N/A	0.006	0.002	0.	0.94	1.719	0.059	1.098

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	15	11	12	36	8	12
normalized size	1	1.	1.36	1.	1.09	3.27	0.73	1.09
time (sec)	N/A	0.006	0.002	0.	0.939	1.684	0.062	1.102

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	20	5	3
normalized size	1	1.	1.	1.5	1.5	10.	2.5	1.5
time (sec)	N/A	0.006	0.002	0.029	0.941	1.601	0.058	1.082

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	7	15	38	20	15
normalized size	1	1.	1.	0.47	1.	2.53	1.33	1.
time (sec)	N/A	0.007	0.005	0.01	0.93	1.758	0.603	1.087

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	27	7	11
normalized size	1	1.	1.	1.12	1.38	3.38	0.88	1.38
time (sec)	N/A	0.008	0.002	0.	0.957	1.735	0.175	1.101

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	15	18	20	43	17	20
normalized size	1	1.	0.88	1.06	1.18	2.53	1.	1.18
time (sec)	N/A	0.02	0.015	0.	0.93	1.89	0.317	1.088

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	26	63	36	26
normalized size	1	1.	1.	1.	1.04	2.52	1.44	1.04
time (sec)	N/A	0.013	0.008	0.	0.957	1.662	0.333	1.094

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	29	37	35	89	56	35
normalized size	1	1.	0.71	0.9	0.85	2.17	1.37	0.85
time (sec)	N/A	0.03	0.037	0.	0.941	1.815	0.625	1.084

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	31	23	31	74	39	31
normalized size	1	1.	0.94	0.7	0.94	2.24	1.18	0.94
time (sec)	N/A	0.022	0.009	0.	0.962	1.821	0.603	1.085

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	9	26	7	9
normalized size	1	1.	1.	1.14	1.29	3.71	1.	1.29
time (sec)	N/A	0.009	0.002	0.004	0.932	1.698	0.169	1.077

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	14	17	19	42	17	19
normalized size	1	1.	0.88	1.06	1.19	2.62	1.06	1.19
time (sec)	N/A	0.021	0.013	0.001	0.947	1.682	0.319	1.087

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	26	62	36	26
normalized size	1	1.	1.	1.	1.04	2.48	1.44	1.04
time (sec)	N/A	0.014	0.011	0.005	0.952	1.664	0.336	1.072

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	29	37	35	89	56	35
normalized size	1	1.	0.71	0.9	0.85	2.17	1.37	0.85
time (sec)	N/A	0.032	0.036	0.017	0.952	1.778	0.641	1.094

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	31	23	31	80	39	31
normalized size	1	1.	0.94	0.7	0.94	2.42	1.18	0.94
time (sec)	N/A	0.023	0.008	0.011	0.952	1.775	0.602	1.065

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	18	23	2	3
normalized size	1	1.	1.	1.5	9.	11.5	1.	1.5
time (sec)	N/A	0.011	0.01	0.	1.054	1.822	0.576	1.073

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	18	59	12	3
normalized size	1	1.	1.	1.5	9.	29.5	6.	1.5
time (sec)	N/A	0.012	0.001	0.005	1.056	1.693	1.164	1.092

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	20	80	17	18
normalized size	1	1.	1.	1.1	2.	8.	1.7	1.8
time (sec)	N/A	0.025	0.002	0.004	1.068	1.816	1.602	1.072

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	23	84	10	15
normalized size	1	1.	1.	0.8	1.53	5.6	0.67	1.
time (sec)	N/A	0.038	0.012	0.007	1.062	1.861	1.086	1.082

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	27	57	12	22
normalized size	1	1.	1.	1.42	2.25	4.75	1.	1.83
time (sec)	N/A	0.006	0.003	0.002	0.95	1.859	0.087	1.092

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	22	12	15	23	14	15
normalized size	1	1.	2.	1.09	1.36	2.09	1.27	1.36
time (sec)	N/A	0.004	0.009	0.003	0.94	1.757	0.126	1.07

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	14	22	12	14
normalized size	1	1.	2.1	1.1	1.4	2.2	1.2	1.4
time (sec)	N/A	0.004	0.009	0.005	0.956	1.751	0.129	1.089

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	17	15	49	19	18
normalized size	1	1.	1.	1.42	1.25	4.08	1.58	1.5
time (sec)	N/A	0.004	0.009	0.002	0.936	1.809	0.123	1.071

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	19	29	15	65	29	76
normalized size	1	1.	1.73	2.64	1.36	5.91	2.64	6.91
time (sec)	N/A	0.004	0.01	0.004	0.964	1.765	0.338	1.134

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	38	21	35	93	17	69
normalized size	1	1.	3.17	1.75	2.92	7.75	1.42	5.75
time (sec)	N/A	0.004	0.017	0.006	0.964	1.89	0.538	1.085

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	19	35	76	34	38
normalized size	1	1.	1.	1.73	3.18	6.91	3.09	3.45
time (sec)	N/A	0.004	0.002	0.006	0.94	1.801	0.613	1.112

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	32	55	46	24
normalized size	1	1.	0.92	1.08	1.28	2.2	1.84	0.96
time (sec)	N/A	0.009	0.026	0.005	0.946	1.693	0.227	1.065

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	29	22	30	55	37	34
normalized size	1	1.	1.07	0.81	1.11	2.04	1.37	1.26
time (sec)	N/A	0.01	0.01	0.005	0.952	1.524	0.496	1.1

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	27	30	55	46	24
normalized size	1	1.	0.92	1.08	1.2	2.2	1.84	0.96
time (sec)	N/A	0.009	0.017	0.008	0.957	1.807	0.229	1.096

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	22	30	55	36	30
normalized size	1	1.	1.	0.85	1.15	2.12	1.38	1.15
time (sec)	N/A	0.011	0.007	0.009	0.943	1.613	0.467	1.073

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	42	58	14
normalized size	1	1.	1.	1.1	1.4	4.2	5.8	1.4
time (sec)	N/A	0.008	0.003	0.006	0.934	1.725	1.276	1.088

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	12	28	3	41
normalized size	1	1.	0.67	0.56	1.33	3.11	0.33	4.56
time (sec)	N/A	0.007	0.004	0.003	0.938	1.677	0.177	1.073

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	14	30	7	11
normalized size	1	1.	0.67	0.75	1.17	2.5	0.58	0.92
time (sec)	N/A	0.008	0.009	0.007	0.949	1.754	0.353	1.102

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	23	11	20	62	8	14
normalized size	1	1.	2.3	1.1	2.	6.2	0.8	1.4
time (sec)	N/A	0.006	0.01	0.006	0.939	1.666	0.376	1.073

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	25	11	20	61	8	14
normalized size	1	1.	2.27	1.	1.82	5.55	0.73	1.27
time (sec)	N/A	0.008	0.011	0.016	0.957	1.691	0.38	1.089

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	0	344	114	65
normalized size	1	1.	1.	0.98	0.	8.6	2.85	1.62
time (sec)	N/A	0.04	0.037	0.014	0.	1.846	8.443	1.096

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	44	43	0	707	0	81
normalized size	1	1.	0.94	0.91	0.	15.04	0.	1.72
time (sec)	N/A	0.056	0.06	0.039	0.	1.971	0.	1.093

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	47	158	158	134	105	61
normalized size	1	1.	0.64	2.16	2.16	1.84	1.44	0.84
time (sec)	N/A	0.043	0.128	0.005	0.966	1.777	1.102	1.101

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	39	20	15
normalized size	1	1.	1.	0.8	1.	2.6	1.33	1.
time (sec)	N/A	0.007	0.005	0.019	0.947	1.761	0.594	1.077

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	47	158	153	134	105	61
normalized size	1	1.	0.64	2.16	2.1	1.84	1.44	0.84
time (sec)	N/A	0.043	0.122	0.01	0.963	1.773	1.109	1.081

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	22	19	95	14	39
normalized size	1	1.	1.	1.57	1.36	6.79	1.	2.79
time (sec)	N/A	0.008	0.003	0.006	0.93	1.665	0.091	1.127

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	101	138	655	548	0	0
normalized size	1	1.	0.97	1.33	6.3	5.27	0.	0.
time (sec)	N/A	0.227	0.144	0.056	1.721	1.926	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	133	237	1034	683	0	0
normalized size	1	1.	0.87	1.55	6.76	4.46	0.	0.
time (sec)	N/A	0.407	0.311	0.069	3.097	1.842	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	20	144	66	19	31
normalized size	1	1.	1.	1.33	9.6	4.4	1.27	2.07
time (sec)	N/A	0.014	0.014	0.008	1.422	1.855	0.159	1.12

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	38	26	15
normalized size	1	1.	1.	0.8	1.	2.53	1.73	1.
time (sec)	N/A	0.008	0.006	0.036	0.931	1.905	0.525	1.092

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	14	19	14	58	14	14
normalized size	1	1.	0.58	0.79	0.58	2.42	0.58	0.58
time (sec)	N/A	0.025	0.004	0.005	0.948	1.742	0.065	1.08

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	6	15	12	47	12	12
normalized size	1	1.	0.86	2.14	1.71	6.71	1.71	1.71
time (sec)	N/A	0.021	0.007	0.03	0.946	1.675	0.069	1.068

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	22	69	34	61	104	443
normalized size	1	1.	0.69	2.16	1.06	1.91	3.25	13.84
time (sec)	N/A	0.011	0.018	0.018	0.976	1.829	1.095	1.131

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	71	32	61	107	444
normalized size	1	1.	0.65	2.29	1.03	1.97	3.45	14.32
time (sec)	N/A	0.008	0.016	0.016	0.959	1.879	1.08	1.123

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	50	137	81	177	308	1574
normalized size	1	1.	0.6	1.63	0.96	2.11	3.67	18.74
time (sec)	N/A	0.049	0.053	0.02	0.979	1.862	3.5	1.137

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	49	142	78	182	304	1573
normalized size	1	1.	0.59	1.71	0.94	2.19	3.66	18.95
time (sec)	N/A	0.046	0.045	0.02	0.978	1.775	3.552	1.156

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	94	225	144	333	668	3552
normalized size	1	1.	0.58	1.39	0.89	2.06	4.12	21.93
time (sec)	N/A	0.176	0.083	0.032	1.018	1.847	9.894	1.245

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	93	231	142	329	665	3552
normalized size	1	1.	0.58	1.43	0.88	2.04	4.13	22.06
time (sec)	N/A	0.171	0.074	0.028	1.02	1.783	9.793	1.209

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	169	431	251	570	1352	6857
normalized size	1	1.	0.65	1.65	0.96	2.18	5.18	26.27
time (sec)	N/A	0.435	0.146	0.039	1.088	1.883	27.707	1.275

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	168	441	248	578	1355	6851
normalized size	1	1.	0.65	1.7	0.95	2.22	5.21	26.35
time (sec)	N/A	0.416	0.122	0.039	1.097	1.788	27.026	1.316

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	20	26	54	114	18
normalized size	1	1.	1.	0.8	1.04	2.16	4.56	0.72
time (sec)	N/A	0.031	0.01	0.047	0.943	1.82	15.248	1.086

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	30	81	112	30
normalized size	1	1.	1.	0.77	1.	2.7	3.73	1.
time (sec)	N/A	0.032	0.008	0.041	0.937	1.878	14.623	1.078

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	55	64	74	144	100	81
normalized size	1	1.	0.65	0.75	0.87	1.69	1.18	0.95
time (sec)	N/A	0.066	0.079	0.026	0.973	1.802	2.194	1.103

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	324	0	0	0
normalized size	1	0.	0.	0.	24.92	0.	0.	0.
time (sec)	N/A	0.435	0.88	0.789	0.983	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	55	43	0	24
normalized size	1	1.	1.	0.92	4.58	3.58	0.	2.
time (sec)	N/A	0.03	0.013	0.044	0.951	1.703	0.	1.112

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	76	143	0	684	0	132
normalized size	1	1.	0.99	1.86	0.	8.88	0.	1.71
time (sec)	N/A	0.098	0.247	0.041	0.	1.925	0.	1.104

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	16	54	15	18
normalized size	1	1.	1.	0.82	0.94	3.18	0.88	1.06
time (sec)	N/A	0.003	0.003	0.001	0.938	1.754	0.414	1.075

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	53	15	18
normalized size	1	1.	1.	0.82	0.82	3.12	0.88	1.06
time (sec)	N/A	0.003	0.003	0.001	0.942	1.893	0.403	1.064

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	3	3	7	2	3
normalized size	1	1.	1.	1.	1.	2.33	0.67	1.
time (sec)	N/A	0.001	0.	0.	0.952	1.577	0.041	1.08

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	16	8	11
normalized size	1	1.	1.	1.12	1.38	2.	1.	1.38
time (sec)	N/A	0.003	0.001	0.003	0.931	1.69	0.084	1.079

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	9	11	15	7	11
normalized size	1	1.	1.	1.	1.22	1.67	0.78	1.22
time (sec)	N/A	0.002	0.001	0.001	0.956	1.561	0.055	1.071

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	9	5	12	3	5
normalized size	1	1.	1.	2.25	1.25	3.	0.75	1.25
time (sec)	N/A	0.011	0.005	0.003	1.049	1.655	0.964	1.074

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	31	31	46	15	32
normalized size	1	1.	1.	1.29	1.29	1.92	0.62	1.33
time (sec)	N/A	0.015	0.005	0.004	0.928	1.528	0.169	1.074

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	27	8	14
normalized size	1	1.	1.	0.92	1.17	2.25	0.67	1.17
time (sec)	N/A	0.021	0.009	0.002	0.95	1.819	0.098	1.072

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	19	31	14	19
normalized size	1	1.	1.	1.15	1.46	2.38	1.08	1.46
time (sec)	N/A	0.008	0.003	0.002	0.934	1.704	0.103	1.081

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	22	0	192	26	28
normalized size	1	1.	1.	0.71	0.	6.19	0.84	0.9
time (sec)	N/A	0.029	0.009	0.005	0.	1.73	0.177	1.093

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	14	14	18	31	19	18
normalized size	1	1.	0.67	0.67	0.86	1.48	0.9	0.86
time (sec)	N/A	0.009	0.006	0.002	0.942	1.68	0.101	1.095

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	102	102	136	535	102	136
normalized size	1	1.	0.63	0.63	0.83	3.28	0.63	0.83
time (sec)	N/A	0.242	0.01	0.003	0.949	1.584	0.128	1.083

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	0	39	0	292
normalized size	1	1.	1.	1.06	0.	2.17	0.	16.22
time (sec)	N/A	0.021	0.013	0.004	0.	1.557	0.	1.134

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	0	36	24	320
normalized size	1	1.	1.	1.07	0.	2.57	1.71	22.86
time (sec)	N/A	0.015	0.007	0.003	0.	1.673	0.651	1.163

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	21	14	45	0	0
normalized size	1	1.	1.	1.24	0.82	2.65	0.	0.
time (sec)	N/A	0.019	0.011	0.02	1.038	1.675	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	43	79	0	117	0	0
normalized size	1	1.	0.67	1.23	0.	1.83	0.	0.
time (sec)	N/A	0.097	0.127	0.032	0.	1.767	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	16	22	31	12	38
normalized size	1	1.	1.	1.	1.38	1.94	0.75	2.38
time (sec)	N/A	0.032	0.045	0.003	0.951	1.557	0.116	1.108

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	27	17	15
normalized size	1	1.	1.	0.92	1.15	2.08	1.31	1.15
time (sec)	N/A	0.007	0.002	0.003	0.941	1.733	0.095	1.081

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	8	12	30	8	12
normalized size	1	1.	1.	0.73	1.09	2.73	0.73	1.09
time (sec)	N/A	0.003	0.001	0.003	0.942	1.608	0.242	1.107

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	8	18	5	8
normalized size	1	1.	1.	0.78	0.89	2.	0.56	0.89
time (sec)	N/A	0.007	0.001	0.002	0.936	1.665	0.081	1.081

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	17	18	23	38	14	0
normalized size	1	1.	0.63	0.67	0.85	1.41	0.52	0.
time (sec)	N/A	0.105	0.012	0.002	1.025	1.504	0.096	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	A	A	A	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	25	0	25	23	30	73	31	0
normalized size	1	0.	1.	0.92	1.2	2.92	1.24	0.
time (sec)	N/A	0.982	0.165	0.06	1.271	1.792	0.407	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	11	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.019	0.016	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	14	27	8	14
normalized size	1	1.	1.	1.09	1.27	2.45	0.73	1.27
time (sec)	N/A	0.016	0.006	0.01	1.03	1.765	1.989	1.071

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	17	21	20	46	17	20
normalized size	1	1.	0.77	0.95	0.91	2.09	0.77	0.91
time (sec)	N/A	0.052	0.011	0.013	1.029	1.854	3.196	1.078

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	17	28	15	31	10	15
normalized size	1	1.	0.74	1.22	0.65	1.35	0.43	0.65
time (sec)	N/A	0.013	0.005	0.001	0.951	1.912	0.089	1.074

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	0	12	16
normalized size	1	1.	1.	0.81	1.	0.	0.75	1.
time (sec)	N/A	0.002	0.	0.001	1.417	0.	0.053	1.098

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	43	48	0	234	930	65
normalized size	1	1.	0.75	0.84	0.	4.11	16.32	1.14
time (sec)	N/A	0.03	0.023	0.009	0.	1.883	4.279	1.103

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	118	305	0	699	0	189
normalized size	1	1.	1.02	2.63	0.	6.03	0.	1.63
time (sec)	N/A	0.068	0.292	0.011	0.	1.932	0.	1.131

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	31	12	16
normalized size	1	1.	1.	0.81	1.	1.94	0.75	1.
time (sec)	N/A	0.001	0.004	0.002	0.943	1.559	0.055	1.061

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	24	21	35	70	202	34
normalized size	1	1.	0.71	0.62	1.03	2.06	5.94	1.
time (sec)	N/A	0.009	0.011	0.003	0.954	1.605	1.048	1.056

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	35	32	55	97	666	50
normalized size	1	1.	0.66	0.6	1.04	1.83	12.57	0.94
time (sec)	N/A	0.014	0.015	0.003	0.94	1.605	1.636	1.102

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	28	0	188	68	43
normalized size	1	1.	1.	0.8	0.	5.37	1.94	1.23
time (sec)	N/A	0.012	0.009	0.004	0.	1.658	1.437	1.068

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	47	37	0	225	44	55
normalized size	1	1.	1.21	0.95	0.	5.77	1.13	1.41
time (sec)	N/A	0.011	0.027	0.008	0.	1.607	1.905	1.075

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	26	10	16
normalized size	1	1.	1.	0.93	1.14	1.86	0.71	1.14
time (sec)	N/A	0.001	0.003	0.002	0.94	1.611	0.055	1.066

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	23	21	35	47	162	31
normalized size	1	1.	0.72	0.66	1.09	1.47	5.06	0.97
time (sec)	N/A	0.008	0.01	0.002	0.933	1.505	1.038	1.072

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	35	32	55	73	600	50
normalized size	1	1.	0.69	0.63	1.08	1.43	11.76	0.98
time (sec)	N/A	0.013	0.014	0.004	0.966	1.612	1.569	1.068

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	0	142	24	28
normalized size	1	1.	1.	0.78	0.	6.17	1.04	1.22
time (sec)	N/A	0.007	0.003	0.006	0.	1.733	1.094	1.11

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	47	40	0	232	44	63
normalized size	1	1.	1.15	0.98	0.	5.66	1.07	1.54
time (sec)	N/A	0.012	0.063	0.007	0.	1.756	2.185	1.106

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	24	25	0	55	26	28
normalized size	1	1.	1.04	1.09	0.	2.39	1.13	1.22
time (sec)	N/A	0.004	0.012	0.001	0.	1.701	0.058	1.076

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	38	43	61	117	216	116
normalized size	1	1.	0.79	0.9	1.27	2.44	4.5	2.42
time (sec)	N/A	0.014	0.02	0.002	0.968	1.819	0.655	1.063

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	48	42	70	111	230	70
normalized size	1	1.	0.87	0.76	1.27	2.02	4.18	1.27
time (sec)	N/A	0.028	0.034	0.003	1.426	1.91	1.301	1.075

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	38	11	19	35	2	20
normalized size	1	1.	3.17	0.92	1.58	2.92	0.17	1.67
time (sec)	N/A	0.002	0.003	0.002	0.955	1.609	0.125	1.093

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	31	50	96	105	119	42
normalized size	1	1.	0.72	1.16	2.23	2.44	2.77	0.98
time (sec)	N/A	0.005	0.012	0.003	0.947	1.716	2.596	1.148

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	57	20	22
normalized size	1	1.	1.	0.77	1.	2.59	0.91	1.
time (sec)	N/A	0.011	0.013	0.004	0.938	1.666	0.295	1.087

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	18	15	42	8	15
normalized size	1	1.	1.	1.38	1.15	3.23	0.62	1.15
time (sec)	N/A	0.002	0.002	0.002	0.957	1.521	1.073	1.086

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	22	47	19	8
normalized size	1	1.	1.	0.88	2.75	5.88	2.38	1.
time (sec)	N/A	0.003	0.002	0.008	1.421	1.554	0.964	1.113

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	34	78	8	34
normalized size	1	1.	1.	0.79	2.43	5.57	0.57	2.43
time (sec)	N/A	0.006	0.003	0.009	0.936	1.565	0.938	1.07

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	14	0	62	0	30
normalized size	1	1.	1.	0.78	0.	3.44	0.	1.67
time (sec)	N/A	0.016	0.003	0.011	0.	1.606	0.	1.105

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	23	23	155	0	20
normalized size	1	1.	0.93	0.77	0.77	5.17	0.	0.67
time (sec)	N/A	0.021	0.004	0.011	1.441	1.71	0.	1.126

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	16	15	19	46	0	0
normalized size	1	1.	0.94	0.88	1.12	2.71	0.	0.
time (sec)	N/A	0.009	0.007	0.003	1.42	1.635	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	35	2	19
normalized size	1	1.	1.	1.5	1.5	17.5	1.	9.5
time (sec)	N/A	0.001	0.003	0.002	1.415	1.616	0.126	1.076

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	30	15	19	50	17	0
normalized size	1	1.	1.5	0.75	0.95	2.5	0.85	0.
time (sec)	N/A	0.913	0.018	0.002	0.947	1.512	1.213	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	41	50	19	0
normalized size	1	1.	1.	0.88	1.71	2.08	0.79	0.
time (sec)	N/A	0.324	0.167	0.005	1.6	1.62	1.63	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	71	24	42	74	8	42
normalized size	1	1.	2.63	0.89	1.56	2.74	0.3	1.56
time (sec)	N/A	0.008	0.009	0.003	0.952	1.555	0.199	1.096

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	127	25	0	552	51	0
normalized size	1	1.	1.55	0.3	0.	6.73	0.62	0.
time (sec)	N/A	0.074	0.179	0.037	0.	1.682	1.492	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	39	54	95	48	14
normalized size	1	1.	1.	3.25	4.5	7.92	4.	1.17
time (sec)	N/A	0.053	0.021	0.009	1.46	1.636	1.726	1.105

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	33	0	235	68	46
normalized size	1	1.	1.	0.82	0.	5.88	1.7	1.15
time (sec)	N/A	0.011	0.01	0.003	0.	1.605	1.191	1.089

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	66	0	132	42	51
normalized size	1	1.	1.	1.43	0.	2.87	0.91	1.11
time (sec)	N/A	0.034	0.011	0.009	0.	1.642	1.268	1.081

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	39	52	0	140	0	54
normalized size	1	1.	1.05	1.41	0.	3.78	0.	1.46
time (sec)	N/A	0.018	0.018	0.007	0.	1.7	0.	1.119

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	65	74	0	325	0	69
normalized size	1	1.	1.07	1.21	0.	5.33	0.	1.13
time (sec)	N/A	0.024	0.029	0.005	0.	1.727	0.	1.19

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	42	29	26
normalized size	1	1.	1.	0.87	1.13	1.83	1.26	1.13
time (sec)	N/A	0.004	0.004	0.003	0.948	1.649	0.438	1.089

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	32	58	36	22
normalized size	1	1.	1.	0.89	1.14	2.07	1.29	0.79
time (sec)	N/A	0.005	0.005	0.004	0.95	1.68	0.543	1.089

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	0	381	0	81
normalized size	1	1.	1.	0.84	0.	6.8	0.	1.45
time (sec)	N/A	0.04	0.023	0.012	0.	1.697	0.	1.239

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	82	70	0	481	0	97
normalized size	1	1.	1.01	0.86	0.	5.94	0.	1.2
time (sec)	N/A	0.028	0.101	0.005	0.	1.654	0.	1.243

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	49	56	0	155	0	42
normalized size	1	1.	1.11	1.27	0.	3.52	0.	0.95
time (sec)	N/A	0.039	0.008	0.01	0.	1.62	0.	1.227

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	71	78	0	339	0	42
normalized size	1	1.	1.04	1.15	0.	4.99	0.	0.62
time (sec)	N/A	0.049	0.025	0.012	0.	1.899	0.	1.26

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	57	10	15
normalized size	1	1.	1.	0.92	1.15	4.38	0.77	1.15
time (sec)	N/A	0.017	0.007	0.006	0.944	1.65	0.304	1.107

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	34	17	20
normalized size	1	1.	1.	0.84	1.05	1.79	0.89	1.05
time (sec)	N/A	0.006	0.001	0.005	0.944	1.582	0.094	1.09

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	21	27	38	10	16
normalized size	1	1.	1.	1.75	2.25	3.17	0.83	1.33
time (sec)	N/A	0.009	0.024	0.005	0.944	1.684	0.169	1.093

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	11	14	23	7	14
normalized size	1	1.	1.	1.	1.27	2.09	0.64	1.27
time (sec)	N/A	0.025	0.008	0.013	1.09	1.577	0.283	1.079

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	31	49	20	32
normalized size	1	1.	1.	0.89	1.15	1.81	0.74	1.19
time (sec)	N/A	0.018	0.004	0.002	0.938	1.562	0.294	1.097

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	21	16	27	124	165	89
normalized size	1	1.	0.64	0.48	0.82	3.76	5.	2.7
time (sec)	N/A	0.003	0.006	0.003	0.944	1.61	9.954	1.119

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	30	7	15
normalized size	1	1.	1.	1.11	1.33	3.33	0.78	1.67
time (sec)	N/A	0.001	0.002	0.003	0.95	1.559	0.086	1.077

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	38	80	39	43
normalized size	1	1.	1.	0.74	2.	4.21	2.05	2.26
time (sec)	N/A	0.005	0.003	0.003	1.418	1.618	0.324	1.081

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	16	13	15	38	15	15
normalized size	1	1.	0.84	0.68	0.79	2.	0.79	0.79
time (sec)	N/A	0.003	0.003	0.002	0.931	1.581	0.963	1.074

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	112	85	0	309	153	139
normalized size	1	1.	1.27	0.97	0.	3.51	1.74	1.58
time (sec)	N/A	0.039	0.04	0.007	0.	1.607	1.745	1.839

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	46	28	7	23
normalized size	1	1.	1.	1.11	5.11	3.11	0.78	2.56
time (sec)	N/A	0.011	0.002	0.005	0.952	1.562	0.172	1.085

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	46	28	7	23
normalized size	1	1.	1.	1.11	5.11	3.11	0.78	2.56
time (sec)	N/A	0.011	0.002	0.003	0.96	1.591	0.185	1.077

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	8	9	69	7	18
normalized size	1	1.	1.	0.89	1.	7.67	0.78	2.
time (sec)	N/A	0.004	0.003	0.005	0.943	1.732	0.13	1.094

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	16	18	74	22	31
normalized size	1	1.	1.	1.33	1.5	6.17	1.83	2.58
time (sec)	N/A	0.033	0.083	0.011	0.967	1.722	1.331	1.087

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	30	116	75	30
normalized size	1	1.	1.	0.82	1.07	4.14	2.68	1.07
time (sec)	N/A	0.023	0.015	0.011	0.951	1.869	2.261	1.078

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	20	45	15	20
normalized size	1	1.	1.	0.76	0.95	2.14	0.71	0.95
time (sec)	N/A	0.008	0.003	0.001	1.431	1.713	0.247	1.076

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	20	45	15	23
normalized size	1	1.	1.	0.76	0.95	2.14	0.71	1.1
time (sec)	N/A	0.008	0.002	0.002	1.446	1.649	0.248	1.081

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	22	23	30	53	26	30
normalized size	1	1.	0.96	1.	1.3	2.3	1.13	1.3
time (sec)	N/A	0.019	0.003	0.002	0.965	1.603	0.297	1.059

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	103	32	19
normalized size	1	1.	1.	0.83	1.06	5.72	1.78	1.06
time (sec)	N/A	0.016	0.019	0.018	0.955	1.746	0.548	1.088

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	19	103	32	19
normalized size	1	1.	1.	0.83	1.06	5.72	1.78	1.06
time (sec)	N/A	0.013	0.012	0.015	0.948	1.788	0.544	1.069

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	40	17	0	88	0	0
normalized size	1	1.	3.33	1.42	0.	7.33	0.	0.
time (sec)	N/A	0.006	0.012	0.027	0.	1.659	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	42	23	0	88	0	0
normalized size	1	1.	3.	1.64	0.	6.29	0.	0.
time (sec)	N/A	0.009	0.011	0.035	0.	1.608	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	16	22	12	36	0	0
normalized size	1	1.	1.33	1.83	1.	3.	0.	0.
time (sec)	N/A	0.007	0.006	0.027	1.589	1.498	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	22	27	57	0	0
normalized size	1	1.	1.29	1.57	1.93	4.07	0.	0.
time (sec)	N/A	0.009	0.008	0.03	1.464	1.622	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	17	14	0	45	63	18
normalized size	1	1.	0.81	0.67	0.	2.14	3.	0.86
time (sec)	N/A	0.008	0.021	0.003	0.	1.628	0.386	1.075

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	31	24	55	20	26
normalized size	1	1.	1.	1.29	1.	2.29	0.83	1.08
time (sec)	N/A	0.01	0.01	0.005	0.933	1.652	0.118	1.12

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	45	45	7	22
normalized size	1	1.	1.	0.75	3.75	3.75	0.58	1.83
time (sec)	N/A	0.004	0.002	0.009	0.943	1.552	0.922	1.082

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	92	32	76	0	32
normalized size	1	1.	1.	2.88	1.	2.38	0.	1.
time (sec)	N/A	0.012	0.011	0.026	0.944	1.548	0.	1.086

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	27	35	103	31	35
normalized size	1	1.	1.	0.82	1.06	3.12	0.94	1.06
time (sec)	N/A	0.009	0.012	0.005	1.438	1.599	0.128	1.077

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	41	3	15
normalized size	1	1.	1.	1.33	1.33	13.67	1.	5.
time (sec)	N/A	0.002	0.002	0.	0.959	1.619	0.059	1.116

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	14	22	126	19	46
normalized size	1	1.	1.5	1.17	1.83	10.5	1.58	3.83
time (sec)	N/A	0.01	0.003	0.	1.431	1.634	0.066	1.097

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	55	7	15
normalized size	1	1.	1.	1.33	1.33	18.33	2.33	5.
time (sec)	N/A	0.003	0.002	0.001	0.931	1.539	0.123	1.094

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	55	12	16
normalized size	1	1.	1.	1.33	1.33	18.33	4.	5.33
time (sec)	N/A	0.003	0.002	0.001	0.955	1.536	0.294	1.079

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	16	8	11
normalized size	1	1.	1.	1.12	1.38	2.	1.	1.38
time (sec)	N/A	0.002	0.001	0.003	0.941	1.621	0.083	1.086

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	29	32	14	23	0	15
normalized size	1	1.	0.59	0.65	0.29	0.47	0.	0.31
time (sec)	N/A	0.015	0.009	0.004	1.433	1.438	0.	1.096

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	17	13	16	35	12	18
normalized size	1	1.	1.06	0.81	1.	2.19	0.75	1.12
time (sec)	N/A	0.006	0.003	0.001	0.947	1.511	0.069	1.079

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	36	0	0	84	0	54
normalized size	1	1.	0.9	0.	0.	2.1	0.	1.35
time (sec)	N/A	0.016	0.009	0.003	0.	1.639	0.	1.108

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	67	63	78	162	109	77
normalized size	1	1.	0.52	0.49	0.61	1.27	0.85	0.6
time (sec)	N/A	0.19	0.165	0.009	0.974	1.673	5.386	1.108

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	18	20	23	58	27	20
normalized size	1	1.	0.6	0.67	0.77	1.93	0.9	0.67
time (sec)	N/A	0.037	0.026	0.004	0.951	1.625	0.788	1.072

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	8	9	43	17	9
normalized size	1	1.	0.82	0.73	0.82	3.91	1.55	0.82
time (sec)	N/A	0.001	0.001	0.	0.974	1.499	0.095	1.075

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	33	43	112	41	45
normalized size	1	1.	1.	0.82	1.08	2.8	1.02	1.12
time (sec)	N/A	0.021	0.008	0.002	1.432	1.619	0.128	1.091

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	23	22	23	51	15	24
normalized size	1	1.	2.88	2.75	2.88	6.38	1.88	3.
time (sec)	N/A	0.003	0.003	0.003	0.941	1.579	0.089	1.085

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	194	1210	0	0	0	0
normalized size	1	1.	0.85	5.33	0.	0.	0.	0.
time (sec)	N/A	0.197	0.252	0.014	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	30	26	34	78	32	34
normalized size	1	1.	1.25	1.08	1.42	3.25	1.33	1.42
time (sec)	N/A	0.009	0.003	0.003	0.946	1.579	1.443	1.081

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	71	54	77	242	73	0
normalized size	1	1.	0.91	0.69	0.99	3.1	0.94	0.
time (sec)	N/A	0.046	0.023	0.004	1.423	1.662	0.304	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	3	3	35	2	19
normalized size	1	1.	1.	1.5	1.5	17.5	1.	9.5
time (sec)	N/A	0.001	0.003	0.	1.435	1.608	0.122	1.108

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	21	27	69	24	34
normalized size	1	1.	1.	0.78	1.	2.56	0.89	1.26
time (sec)	N/A	0.004	0.006	0.003	1.417	1.727	0.188	1.072

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	46	8	15
normalized size	1	1.	1.	1.1	1.4	4.6	0.8	1.5
time (sec)	N/A	0.004	0.003	0.	0.928	1.636	0.074	1.097

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	41	12	19
normalized size	1	1.	1.	0.94	1.19	2.56	0.75	1.19
time (sec)	N/A	0.004	0.002	0.	1.41	1.71	0.117	1.084

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	29	34	45	68	32	51
normalized size	1	1.	0.72	0.85	1.12	1.7	0.8	1.27
time (sec)	N/A	0.023	0.011	0.002	1.412	1.726	0.321	1.098

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	29	14	18	104	0	20
normalized size	1	1.	1.38	0.67	0.86	4.95	0.	0.95
time (sec)	N/A	0.115	0.028	0.053	0.924	1.81	0.	1.122

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	36	10	22
normalized size	1	1.	1.	0.79	1.	2.57	0.71	1.57
time (sec)	N/A	0.006	0.002	0.007	0.926	1.615	0.061	1.104

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	50	14	24
normalized size	1	1.	1.	0.94	1.22	2.78	0.78	1.33
time (sec)	N/A	0.014	0.004	0.007	0.938	1.558	0.104	1.063

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	7	8	46	7	22
normalized size	1	1.	1.	0.7	0.8	4.6	0.7	2.2
time (sec)	N/A	0.001	0.004	0.003	1.401	1.612	0.141	1.072

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	5	5	35	3	19
normalized size	1	1.	1.	0.83	0.83	5.83	0.5	3.17
time (sec)	N/A	0.001	0.003	0.003	1.409	1.538	0.128	1.091

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	17	22	59	22	22
normalized size	1	1.	1.	0.81	1.05	2.81	1.05	1.05
time (sec)	N/A	0.017	0.008	0.003	1.411	1.831	0.104	1.082

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	51	42	63	173	46	62
normalized size	1	1.	0.96	0.79	1.19	3.26	0.87	1.17
time (sec)	N/A	0.031	0.02	0.01	1.416	1.796	0.161	1.083

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	66	55	127	762	59
normalized size	1	1.	1.	1.35	1.12	2.59	15.55	1.2
time (sec)	N/A	0.078	0.026	0.006	0.937	1.976	25.138	1.082

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.004	2.811	0.028	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	28	36	128	26	0
normalized size	1	1.	1.	1.	1.29	4.57	0.93	0.
time (sec)	N/A	0.211	10.248	0.031	1.059	1.873	0.443	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	136	127	216	342	0	154
normalized size	1	1.	0.68	0.64	1.09	1.72	0.	0.77
time (sec)	N/A	0.101	0.208	0.035	1.029	2.175	0.	1.09

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	20	51	15	20
normalized size	1	1.	1.	0.89	1.11	2.83	0.83	1.11
time (sec)	N/A	0.005	0.007	0.006	0.929	1.803	0.308	1.077

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	22	28	88	36	50
normalized size	1	1.	1.	0.92	1.17	3.67	1.5	2.08
time (sec)	N/A	0.008	0.028	0.001	0.931	1.876	0.497	1.072

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	B	F	B	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	94	0	5137	1197351	0	464	0	0
normalized size	1	0.	54.65	12737.8	0.	4.94	0.	0.
time (sec)	N/A	1.84	6.419	0.914	0.	2.691	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	C	F	A	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	142	0	630	354	0	572	0	0
normalized size	1	0.	4.44	2.49	0.	4.03	0.	0.
time (sec)	N/A	3.324	1.524	0.213	0.	2.814	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	97	1088	231	452	0	105
normalized size	1	1.	4.62	51.81	11.	21.52	0.	5.
time (sec)	N/A	0.285	1.235	0.136	1.207	1.924	0.	1.388

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	C	A	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	4030	0	3168	4640	0	0	0	0
normalized size	1	0.	0.79	1.15	0.	0.	0.	0.
time (sec)	N/A	0.025	6.048	0.73	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	181	1356	0	590	330	637
normalized size	1	1.	0.55	4.11	0.	1.79	1.	1.93
time (sec)	N/A	0.872	0.14	0.144	0.	1.984	16.283	1.141

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	15	5	5
normalized size	1	1.	1.	1.25	1.25	3.75	1.25	1.25
time (sec)	N/A	0.012	0.001	0.009	0.927	1.812	0.707	1.098

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	F	B	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	71	0	71	102	0	319	76	132
normalized size	1	0.	1.	1.44	0.	4.49	1.07	1.86
time (sec)	N/A	0.754	0.036	0.022	0.	1.614	0.205	1.135

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [50] had the largest ratio of [1.429]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	0	1.	6	0.
2	A	3	2	1.	13	0.154
3	A	2	1	1.	10	0.1
4	A	1	1	1.	3	0.333
5	A	2	1	1.	11	0.091
6	A	2	1	1.	14	0.071
7	A	2	1	1.	20	0.05
8	A	2	2	1.	12	0.167
9	A	3	3	1.	13	0.231
10	A	2	2	1.	10	0.2
11	A	6	5	1.	13	0.385
12	A	2	1	1.	23	0.043
13	A	4	3	1.	20	0.15
14	A	3	2	1.	22	0.091
15	A	5	4	1.	14	0.286
16	A	6	6	1.	9	0.667
17	A	3	2	1.	16	0.125
18	A	9	6	1.	7	0.857
19	A	9	6	1.	11	0.546
20	A	9	5	1.	10	0.5
21	A	1	1	1.	7	0.143
22	A	2	1	1.	9	0.111
23	A	2	1	1.	11	0.091
24	A	1	1	1.	7	0.143
25	A	1	1	1.	7	0.143
26	A	2	1	1.	9	0.111
27	A	2	1	1.	11	0.091
28	A	3	3	1.	11	0.273
29	A	2	1	1.	11	0.091
30	A	2	1	1.	11	0.091
31	A	1	1	1.	9	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
32	A	1	1	1.	11	0.091
33	A	6	6	1.	9	0.667
34	A	6	6	1.	7	0.857
35	A	6	6	1.	11	0.546
36	A	3	3	1.	7	0.429
37	A	3	3	1.	9	0.333
38	A	9	6	1.	9	0.667
39	A	3	3	1.	12	0.25
40	A	3	3	1.	12	0.25
41	A	3	2	1.	12	0.167
42	A	3	2	1.	12	0.167
43	A	3	2	1.	12	0.167
44	A	9	5	1.	10	0.5
45	A	9	5	1.	12	0.417
46	A	10	6	1.	7	0.857
47	A	10	6	1.	7	0.857
48	A	10	6	1.	7	0.857
49	A	19	6	1.	7	0.857
50	A	13	10	1.	7	1.429
51	A	19	6	1.	12	0.5
52	A	7	7	1.	11	0.636
53	A	1	1	1.	2	0.5
54	A	1	1	1.	4	0.25
55	A	1	1	1.	6	0.167
56	A	1	1	1.	6	0.167
57	A	2	2	1.	4	0.5
58	A	11	2	1.	8	0.25
59	A	2	2	1.	8	0.25
60	A	1	1	1.	4	0.25
61	A	2	2	1.	6	0.333
62	A	2	2	1.	8	0.25
63	A	3	3	1.	8	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	2	2	1.	8	0.25
65	A	2	1	1.04	8	0.125
66	A	4	4	1.	10	0.4
67	A	2	2	1.	10	0.2
68	A	3	2	1.	8	0.25
69	A	3	2	1.	10	0.2
70	A	3	3	1.	8	0.375
71	A	3	3	1.	10	0.3
72	A	4	3	1.	12	0.25
73	A	4	3	1.	12	0.25
74	A	3	3	1.	10	0.3
75	A	0	0	0.	0	0.
76	A	1	1	1.	2	0.5
77	A	1	1	1.	2	0.5
78	A	1	1	1.	2	0.5
79	A	1	1	1.	2	0.5
80	A	2	2	1.	6	0.333
81	A	1	1	1.	2	0.5
82	A	1	1	1.	2	0.5
83	A	2	2	1.	4	0.5
84	A	3	3	1.	8	0.375
85	A	2	1	1.	4	0.25
86	A	1	1	1.	4	0.25
87	A	3	2	1.	11	0.182
88	A	2	2	1.	4	0.5
89	A	2	1	1.	4	0.25
90	A	2	2	1.	4	0.5
91	A	1	1	1.	7	0.143
92	A	2	2	1.	4	0.5
93	A	3	2	1.	6	0.333
94	A	2	2	1.	6	0.333
95	A	4	4	1.	8	0.5

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	3	3	1.	6	0.5
97	A	2	2	1.	4	0.5
98	A	3	2	1.	6	0.333
99	A	2	2	1.	6	0.333
100	A	4	4	1.	8	0.5
101	A	3	3	1.	6	0.5
102	A	1	1	1.	6	0.167
103	A	1	1	1.	6	0.167
104	A	2	2	1.	6	0.333
105	A	3	2	1.	8	0.25
106	A	2	2	1.	4	0.5
107	A	1	1	1.	6	0.167
108	A	1	1	1.	6	0.167
109	A	1	1	1.	6	0.167
110	A	1	1	1.	6	0.167
111	A	1	1	1.	6	0.167
112	A	1	1	1.	6	0.167
113	A	2	2	1.	8	0.25
114	A	2	1	1.	8	0.125
115	A	2	2	1.	8	0.25
116	A	2	1	1.	8	0.125
117	A	2	2	1.	8	0.25
118	A	1	1	1.	6	0.167
119	A	1	1	1.	8	0.125
120	A	1	1	1.	6	0.167
121	A	1	1	1.	8	0.125
122	A	3	3	1.	8	0.375
123	A	3	3	1.	10	0.3
124	A	4	4	1.	12	0.333
125	A	1	1	1.	7	0.143
126	A	4	4	1.	12	0.333
127	A	2	2	1.	4	0.5

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
128	A	17	8	1.	8	1.
129	A	34	9	1.	8	1.125
130	A	3	3	1.	6	0.5
131	A	1	1	1.	9	0.111
132	A	3	3	1.	9	0.333
133	A	3	2	1.	9	0.222
134	A	1	1	1.	6	0.167
135	A	1	1	1.	6	0.167
136	A	4	3	1.	7	0.429
137	A	4	3	1.	7	0.429
138	A	11	5	1.	9	0.556
139	A	11	5	1.	9	0.556
140	A	25	5	1.	9	0.556
141	A	25	5	1.	9	0.556
142	A	5	2	1.	11	0.182
143	A	5	2	1.	11	0.182
144	A	6	4	1.	10	0.4
145	A	0	0	0.	0	0.
146	A	4	3	1.	9	0.333
147	A	5	5	1.	15	0.333
148	A	1	1	1.	3	0.333
149	A	1	1	1.	3	0.333
150	A	1	1	1.	3	0.333
151	A	1	1	1.	3	0.333
152	A	1	1	1.	5	0.2
153	A	1	1	1.	9	0.111
154	A	4	4	1.	11	0.364
155	A	3	2	1.	13	0.154
156	A	2	2	1.	9	0.222
157	A	2	2	1.	18	0.111
158	A	2	2	1.	7	0.286
159	A	21	2	1.	7	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	2	2	1.	9	0.222
161	A	2	2	1.	7	0.286
162	A	2	2	1.	7	0.286
163	A	5	3	1.	12	0.25
164	A	1	1	1.	14	0.071
165	A	1	1	1.	7	0.143
166	A	1	1	1.	5	0.2
167	A	1	1	1.	7	0.143
168	A	7	3	1.	12	0.25
169	F	0	0	N/A	0	N/A
170	A	0	0	0.	0	0.
171	A	2	3	1.	6	0.5
172	A	5	5	1.	7	0.714
173	A	3	3	1.	10	0.3
174	A	1	0	1.	13	0.
175	A	4	4	1.	12	0.333
176	A	5	4	1.	19	0.21
177	A	1	1	1.	9	0.111
178	A	2	1	1.	11	0.091
179	A	2	1	1.	13	0.077
180	A	3	3	1.	13	0.231
181	A	3	3	1.	13	0.231
182	A	1	1	1.	9	0.111
183	A	2	1	1.	11	0.091
184	A	2	1	1.	13	0.077
185	A	2	2	1.	13	0.154
186	A	3	3	1.	13	0.231
187	A	1	1	1.	11	0.091
188	A	2	1	1.	13	0.077
189	A	6	6	1.	18	0.333
190	A	2	2	1.	9	0.222
191	A	4	3	1.	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
192	A	3	3	1.	6	0.5
193	A	1	1	1.	13	0.077
194	A	2	2	1.	13	0.154
195	A	3	3	1.	13	0.231
196	A	3	3	1.	14	0.214
197	A	3	3	1.	18	0.167
198	A	1	1	1.	20	0.05
199	A	1	1	1.	9	0.111
200	A	3	2	1.	65	0.031
201	A	5	4	1.	68	0.059
202	A	5	2	1.	21	0.095
203	A	6	6	1.	21	0.286
204	A	4	3	1.	23	0.13
205	A	2	2	1.	16	0.125
206	A	3	3	1.	25	0.12
207	A	2	2	1.	24	0.083
208	A	2	2	1.	29	0.069
209	A	1	1	1.	18	0.056
210	A	1	1	1.	23	0.043
211	A	3	3	1.	24	0.125
212	A	3	3	1.	22	0.136
213	A	3	3	1.	26	0.115
214	A	3	3	1.	31	0.097
215	A	3	3	1.	16	0.188
216	A	1	1	1.	8	0.125
217	A	2	2	1.	6	0.333
218	A	1	1	1.	20	0.05
219	A	3	2	1.	13	0.154
220	A	2	2	1.	13	0.154
221	A	3	3	1.	9	0.333
222	A	2	2	1.	13	0.154
223	A	2	1	1.	11	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
224	A	5	5	1.	13	0.385
225	A	2	2	1.	4	0.5
226	A	2	2	1.	4	0.5
227	A	1	1	1.	4	0.25
228	A	1	1	1.	28	0.036
229	A	4	2	1.	11	0.182
230	A	3	3	1.	4	0.75
231	A	3	3	1.	4	0.75
232	A	3	3	1.	8	0.375
233	A	3	2	1.	7	0.286
234	A	3	2	1.	7	0.286
235	A	1	1	1.	8	0.125
236	A	1	1	1.	10	0.1
237	A	1	1	1.	8	0.125
238	A	1	1	1.	10	0.1
239	A	3	3	1.	17	0.176
240	A	4	3	1.	13	0.231
241	A	2	2	1.	11	0.182
242	A	4	3	1.	13	0.231
243	A	3	3	1.	8	0.375
244	A	1	1	1.	2	0.5
245	A	3	2	1.	4	0.5
246	A	1	1	1.	2	0.5
247	A	1	1	1.	2	0.5
248	A	1	1	1.	3	0.333
249	A	4	3	1.	12	0.25
250	A	2	1	1.	13	0.077
251	A	3	3	1.	14	0.214
252	A	14	8	1.	12	0.667
253	A	4	3	1.	7	0.429
254	A	1	1	1.	5	0.2
255	A	6	6	1.	9	0.667

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
256	A	2	2	1.	9	0.222
257	A	10	3	1.	15	0.2
258	A	6	2	1.	11	0.182
259	A	6	6	1.	7	0.857
260	A	1	1	1.	9	0.111
261	A	2	2	1.	9	0.222
262	A	2	1	1.	7	0.143
263	A	2	2	1.	2	1.
264	A	4	3	1.	6	0.5
265	A	4	2	1.	17	0.118
266	A	2	2	1.	4	0.5
267	A	2	1	1.	19	0.053
268	A	1	1	1.	11	0.091
269	A	1	1	1.	9	0.111
270	A	2	2	1.	12	0.167
271	A	3	2	1.	24	0.083
272	A	2	1	1.	29	0.034
273	A	0	0	0.	0	0.
274	A	9	7	1.	54	0.13
275	A	4	3	1.	21	0.143
276	A	1	1	1.	2	0.5
277	A	1	1	1.	4	0.25
278	F	0	0	N/A	0	N/A
279	F	0	0	N/A	0	N/A
280	A	1	1	1.	85	0.012
281	F	0	0	N/A	0	N/A
282	A	20	8	1.	107	0.075
283	A	2	2	1.	7	0.286
284	F	0	0	N/A	0	N/A

Chapter 3

Listing of integrals

3.1 $\int (1 + x + x^2)\, dx$

Optimal. Leaf size=16

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

```
[Out] x + x^2/2 + x^3/3
```

Rubi [A] time = 0.0018128, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{x^3}{3} + \frac{x^2}{2} + x$$

Antiderivative was successfully verified.

```
[In] Int[1 + x + x^2,x]

[Out] x + x^2/2 + x^3/3
```

Rubi steps

$$\int (1 + x + x^2)\, dx = x + \frac{x^2}{2} + \frac{x^3}{3}$$

Mathematica [A] time = 0.0000346, size = 16, normalized size = 1.

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

Antiderivative was successfully verified.

```
[In] Integrate[1 + x + x^2,x]
```

```
[Out] x + x^2/2 + x^3/3
```

Maple [A] time = 0.001, size = 13, normalized size = 0.8

$$x+\frac{x^2}{2}+\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2+x+1,x)
```

```
[Out] x+1/2*x^2+1/3*x^3
```

Maxima [A] time = 0.940587, size = 16, normalized size = 1.

$$\frac{1}{3}\,x^3+\frac{1}{2}\,x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2+x+1,x, algorithm="maxima")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

Fricas [A] time = 1.57297, size = 31, normalized size = 1.94

$$\frac{1}{3}x^3+\frac{1}{2}x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2+x+1,x, algorithm="fricas")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

Sympy [A] time = 0.050605, size = 10, normalized size = 0.62

$$\frac{x^3}{3}+\frac{x^2}{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2+x+1,x)
```

```
[Out] x**3/3 + x**2/2 + x
```

Giac [A] time = 1.09147, size = 16, normalized size = 1.

$$\frac{1}{3}\,x^3+\frac{1}{2}\,x^2+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2+x+1,x, algorithm="giac")
```

```
[Out] 1/3*x^3 + 1/2*x^2 + x
```

3.2 $\int x^2 \left(x+2x^2\right)^2 dx$

Optimal. Leaf size=22

$$\frac{4x^7}{7}+\frac{2x^6}{3}+\frac{x^5}{5}$$

```
[Out] x^5/5 + (2*x^6)/3 + (4*x^7)/7
```

Rubi [A] time = 0.0103443, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {647, 43}

$$\frac{4x^7}{7}+\frac{2x^6}{3}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(x + 2*x^2)^2,x]
```

```
[Out] x^5/5 + (2*x^6)/3 + (4*x^7)/7
```

Rule 647

```
Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist
[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] &&
IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\left(x+2x^2\right)^2\,dx &= \int x^4(1+2x)^2\,dx\\ &= \int \left(x^4+4x^5+4x^6\right)dx\\ &= \frac{x^5}{5}+\frac{2x^6}{3}+\frac{4x^7}{7}\end{aligned}$$

Mathematica [A] time = 0.0012019, size = 22, normalized size = 1.

$$\frac{4x^7}{7}+\frac{2x^6}{3}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(x + 2*x^2)^2,x]
```

```
[Out] x^5/5 + (2*x^6)/3 + (4*x^7)/7
```

Maple [A] time = 0., size = 17, normalized size = 0.8

$$\frac{x^5}{5}+\frac{2\,x^6}{3}+\frac{4\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^2+x)^2,x)
```

```
[Out] 1/5*x^5+2/3*x^6+4/7*x^7
```

Maxima [A] time = 0.950201, size = 22, normalized size = 1.

$$\frac{4}{7}\,x^7+\frac{2}{3}\,x^6+\frac{1}{5}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^2+x)^2,x, algorithm="maxima")
```

```
[Out] 4/7*x^7 + 2/3*x^6 + 1/5*x^5
```

Fricas [A] time = 1.60464, size = 39, normalized size = 1.77

$$\frac{4}{7}x^7 + \frac{2}{3}x^6 + \frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^2+x)^2,x, algorithm="fricas")
```

```
[Out] 4/7*x^7 + 2/3*x^6 + 1/5*x^5
```

Sympy [A] time = 0.053199, size = 17, normalized size = 0.77

$$\frac{4x^7}{7} + \frac{2x^6}{3} + \frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**2+x)**2,x)
```

```
[Out] 4*x**7/7 + 2*x**6/3 + x**5/5
```

Giac [A] time = 1.10123, size = 22, normalized size = 1.

$$\frac{4}{7}x^7 + \frac{2}{3}x^6 + \frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^2+x)^2,x, algorithm="giac")
```

```
[Out] 4/7*x^7 + 2/3*x^6 + 1/5*x^5
```

3.3 $\int x\left(1+2x+x^2\right)\,dx$

Optimal. Leaf size=22

$$\frac{x^4}{4}+\frac{2x^3}{3}+\frac{x^2}{2}$$

```
[Out] x^2/2 + (2*x^3)/3 + x^4/4
```

Rubi [A] time = 0.0043658, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {14}

$$\frac{x^4}{4}+\frac{2x^3}{3}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + 2*x + x^2),x]
```

```
[Out] x^2/2 + (2*x^3)/3 + x^4/4
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int x\left(1+2x+x^2\right)\,dx &= \int \left(x+2x^2+x^3\right)\,dx\\ &= \frac{x^2}{2}+\frac{2x^3}{3}+\frac{x^4}{4}\end{aligned}$$

Mathematica [A] time = 0.000666, size = 22, normalized size = 1.

$$\frac{x^4}{4}+\frac{2x^3}{3}+\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + 2*x + x^2),x]
```

```
[Out] x^2/2 + (2*x^3)/3 + x^4/4
```

Maple [A] time = 0.001, size = 17, normalized size = 0.8

$$\frac{x^2}{2} + \frac{2\,x^3}{3} + \frac{x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+2*x+1),x)
```

```
[Out] 1/2*x^2+2/3*x^3+1/4*x^4
```

Maxima [A] time = 0.948026, size = 22, normalized size = 1.

$$\frac{1}{4}\,x^4 + \frac{2}{3}\,x^3 + \frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+1),x, algorithm="maxima")
```

```
[Out] 1/4*x^4 + 2/3*x^3 + 1/2*x^2
```

Fricas [A] time = 1.78135, size = 39, normalized size = 1.77

$$\frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+1),x, algorithm="fricas")
```

```
[Out] 1/4*x^4 + 2/3*x^3 + 1/2*x^2
```

Sympy [A] time = 0.051943, size = 15, normalized size = 0.68

$$\frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+2*x+1),x)
```

```
[Out] x**4/4 + 2*x**3/3 + x**2/2
```

Giac [A] time = 1.09264, size = 22, normalized size = 1.

$$\frac{1}{4}\,x^4 + \frac{2}{3}\,x^3 + \frac{1}{2}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2*x+1),x, algorithm="giac")
```

```
[Out] 1/4*x^4 + 2/3*x^3 + 1/2*x^2
```

3.4 $\int \frac{1}{x} \, dx$

Optimal. Leaf size=2

$$\log(x)$$

```
[Out] Log[x]
```

Rubi [A] time = 0.0001812, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {29}

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1),x]

[Out] Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \frac{1}{x} \, dx = \log(x)$$

Mathematica [A] time = 0.0000774, size = 2, normalized size = 1.

$$\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1),x]
```

```
[Out] Log[x]
```

Maple [A] time = 0., size = 3, normalized size = 1.5

$$\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x,x)
```

```
[Out] ln(x)
```

Maxima [A] time = 1.01651, size = 3, normalized size = 1.5

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="maxima")
```

```
[Out] log(x)
```

Fricas [A] time = 1.88984, size = 11, normalized size = 5.5

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="fricas")
```

```
[Out] log(x)
```

Sympy [A] time = 0.051384, size = 2, normalized size = 1.

$$\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x)
```

```
[Out] log(x)
```

Giac [A] time = 1.07666, size = 4, normalized size = 2.

$$\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x,x, algorithm="giac")
```

```
[Out] log(abs(x))
```

3.5 $\int \frac{(1+x)^3}{(-1+x)^4} dx$

Optimal. Leaf size=36

$$\frac{6}{1-x} - \frac{6}{(1-x)^2} + \frac{8}{3(1-x)^3} + \log(1-x)$$

```
[Out] 8/(3*(1 - x)^3) - 6/(1 - x)^2 + 6/(1 - x) + Log[1 - x]
```

Rubi [A] time = 0.0130342, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{6}{1-x} - \frac{6}{(1-x)^2} + \frac{8}{3(1-x)^3} + \log(1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)^3/(-1 + x)^4,x]
```

```
[Out] 8/(3*(1 - x)^3) - 6/(1 - x)^2 + 6/(1 - x) + Log[1 - x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(1+x)^3}{(-1+x)^4} dx &= \int \left(\frac{8}{(-1+x)^4} + \frac{12}{(-1+x)^3} + \frac{6}{(-1+x)^2} + \frac{1}{-1+x}\right) dx \\ &= \frac{8}{3(1-x)^3} - \frac{6}{(1-x)^2} + \frac{6}{1-x} + \log(1-x) \end{aligned}$$

Mathematica [A] time = 0.0113566, size = 24, normalized size = 0.67

$$\log(x-1)-\frac{2\left(9x^2-9x+4\right)}{3(x-1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)^3/(-1 + x)^4,x]
```

```
[Out] (-2*(4 - 9*x + 9*x^2))/(3*(-1 + x)^3) + Log[-1 + x]
```

Maple [A] time = 0.006, size = 27, normalized size = 0.8

$$-\frac{8}{3\,(-1+x)^3}-6\,(-1+x)^{-2}+\ln(-1+x)-6\,(-1+x)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^3/(-1+x)^4,x)
```

```
[Out] -8/3/(-1+x)^3-6/(-1+x)^2+ln(-1+x)-6/(-1+x)
```

Maxima [A] time = 0.948822, size = 43, normalized size = 1.19

$$-\frac{2\left(9\,x^2-9\,x+4\right)}{3\left(x^3-3\,x^2+3\,x-1\right)}+\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-1+x)^4,x, algorithm="maxima")
```

```
[Out] -2/3*(9*x^2 - 9*x + 4)/(x^3 - 3*x^2 + 3*x - 1) + log(x - 1)
```

Fricas [A] time = 1.84015, size = 120, normalized size = 3.33

$$-\frac{18\,x^2-3\left(x^3-3\,x^2+3\,x-1\right)\log(x-1)-18\,x+8}{3\left(x^3-3\,x^2+3\,x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-1+x)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(18*x^2 - 3*(x^3 - 3*x^2 + 3*x - 1)*log(x - 1) - 18*x + 8)/(x^3 - 3*x^
2 + 3*x - 1)
```

Sympy [A] time = 0.109386, size = 29, normalized size = 0.81

$$-\frac{18x^2 - 18x + 8}{3x^3 - 9x^2 + 9x - 3} + \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**3/(-1+x)**4,x)
```

```
[Out] -(18*x**2 - 18*x + 8)/(3*x**3 - 9*x**2 + 9*x - 3) + log(x - 1)
```

Giac [A] time = 1.06742, size = 31, normalized size = 0.86

$$-\frac{2\left(9\,x^2 - 9\,x + 4\right)}{3\,(x - 1)^3} + \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^3/(-1+x)^4,x, algorithm="giac")
```

```
[Out] -2/3*(9*x^2 - 9*x + 4)/(x - 1)^3 + log(abs(x - 1))
```

3.6 $\int \frac{1}{(-1+x)x(1+x)^2}\,dx$

Optimal. Leaf size=32

$$-\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)-\log(x)+\frac{3}{4}\log(x+1)$$

```
[Out] -1/(2*(1 + x)) + Log[1 - x]/4 - Log[x] + (3*Log[1 + x])/4
```

Rubi [A] time = 0.0123629, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {72}

$$-\frac{1}{2(x+1)}+\frac{1}{4}\log(1-x)-\log(x)+\frac{3}{4}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)*x*(1 + x)^2),x]
```

```
[Out] -1/(2*(1 + x)) + Log[1 - x]/4 - Log[x] + (3*Log[1 + x])/4
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(-1+x)x(1+x)^2}\,dx &= \int \left(\frac{1}{4(-1+x)}-\frac{1}{x}+\frac{1}{2(1+x)^2}+\frac{3}{4(1+x)}\right)dx\\ &= -\frac{1}{2(1+x)}+\frac{1}{4}\log(1-x)-\log(x)+\frac{3}{4}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0107612, size = 28, normalized size = 0.88

$$\frac{1}{4}\left(-\frac{2}{x+1}+\log(1-x)-4\log(x)+3\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)*x*(1 + x)^2),x]
```

```
[Out] (-2/(1 + x) + Log[1 - x] - 4*Log[x] + 3*Log[1 + x])/4
```

Maple [A] time = 0.008, size = 25, normalized size = 0.8

$$-\ln(x) - \frac{1}{2+2x} + \frac{3\ln(1+x)}{4} + \frac{\ln(-1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)/x/(1+x)^2,x)
```

```
[Out] -ln(x)-1/2/(1+x)+3/4*ln(1+x)+1/4*ln(-1+x)
```

Maxima [A] time = 0.948003, size = 32, normalized size = 1.

$$-\frac{1}{2(x+1)} + \frac{3}{4}\log(x+1) + \frac{1}{4}\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/x/(1+x)^2,x, algorithm="maxima")
```

```
[Out] -1/2/(x + 1) + 3/4*log(x + 1) + 1/4*log(x - 1) - log(x)
```

Fricas [A] time = 1.77226, size = 108, normalized size = 3.38

$$\frac{3(x+1)\log(x+1) + (x+1)\log(x-1) - 4(x+1)\log(x) - 2}{4(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/x/(1+x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(x + 1)*log(x + 1) + (x + 1)*log(x - 1) - 4*(x + 1)*log(x) - 2)/(x +
 1)
```

Sympy [A] time = 0.134233, size = 24, normalized size = 0.75

$$-\log(x) + \frac{\log(x-1)}{4} + \frac{3\log(x+1)}{4} - \frac{1}{2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/x/(1+x)**2,x)
```

```
[Out] -log(x) + log(x - 1)/4 + 3*log(x + 1)/4 - 1/(2*x + 2)
```

Giac [A] time = 1.08221, size = 46, normalized size = 1.44

$$-\frac{1}{2(x+1)} - \log\left(\left|-\frac{1}{x+1} + 1\right|\right) + \frac{1}{4}\log\left(\left|-\frac{2}{x+1} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)/x/(1+x)^2,x, algorithm="giac")
```

```
[Out] -1/2/(x + 1) - log(abs(-1/(x + 1) + 1)) + 1/4*log(abs(-2/(x + 1) + 1))
```

3.7 $\int \frac{b+ax}{(-p+x)(-q+x)} \, dx$

Optimal. Leaf size=40

$$\frac{(ap+b)\log(p-x)}{p-q} - \frac{(aq+b)\log(q-x)}{p-q}$$

```
[Out] ((b + a*p)*Log[p - x])/(p - q) - ((b + a*q)*Log[q - x])/(p - q)
```

Rubi [A] time = 0.0268245, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {72}

$$\frac{(ap+b)\log(p-x)}{p-q} - \frac{(aq+b)\log(q-x)}{p-q}$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x)/((-p + x)*(-q + x)),x]
```

```
[Out] ((b + a*p)*Log[p - x])/(p - q) - ((b + a*q)*Log[q - x])/(p - q)
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{b+ax}{(-p+x)(-q+x)} \, dx &= \int \left(\frac{-b-ap}{(p-q)(p-x)} + \frac{b+aq}{(p-q)(q-x)} \right) dx \\ &= \frac{(b+ap)\log(p-x)}{p-q} - \frac{(b+aq)\log(q-x)}{p-q}\end{aligned}$$

Mathematica [A] time = 0.0171396, size = 34, normalized size = 0.85

$$\frac{(ap+b)\log(x-p)-(aq+b)\log(x-q)}{p-q}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x)/((-p + x)*(-q + x)),x]
```

```
[Out] ((b + a*p)*Log[-p + x] - (b + a*q)*Log[-q + x])/(p - q)
```

Maple [A] time = 0.007, size = 66, normalized size = 1.7

$$-\frac{\ln\left(-q+x\right)aq}{p-q}-\frac{\ln\left(-q+x\right)b}{p-q}+\frac{\ln\left(-p+x\right)ap}{p-q}+\frac{\ln\left(-p+x\right)b}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)/(-p+x)/(-q+x),x)
```

```
[Out] -1/(p-q)*ln(-q+x)*a*q-1/(p-q)*ln(-q+x)*b+1/(p-q)*ln(-p+x)*a*p+1/(p-q)*ln(-p
+x)*b
```

Maxima [A] time = 0.936923, size = 54, normalized size = 1.35

$$\frac{\left(ap+b\right)\log\left(-p+x\right)}{p-q}-\frac{\left(aq+b\right)\log\left(-q+x\right)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(-p+x)/(-q+x),x, algorithm="maxima")
```

```
[Out] (a*p + b)*log(-p + x)/(p - q) - (a*q + b)*log(-q + x)/(p - q)
```

Fricas [A] time = 1.94076, size = 77, normalized size = 1.92

$$\frac{\left(ap+b\right)\log\left(-p+x\right)-\left(aq+b\right)\log\left(-q+x\right)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(-p+x)/(-q+x),x, algorithm="fricas")
```

```
[Out] ((a*p + b)*log(-p + x) - (a*q + b)*log(-q + x))/(p - q)
```

Sympy [B] time = 0.655328, size = 144, normalized size = 3.6

$$\frac{(ap+b)\log\left(x+\frac{-2apq-bp-bq-\frac{p^2(ap+b)}{p-q}+\frac{2pq(ap+b)}{p-q}-\frac{q^2(ap+b)}{p-q}}{ap+aq+2b}\right)}{p-q}-\frac{(aq+b)\log\left(x+\frac{-2apq-bp-bq+\frac{p^2(aq+b)}{p-q}-\frac{2pq(aq+b)}{p-q}+\frac{q^2(aq+b)}{p-q}}{ap+aq+2b}\right)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(-p+x)/(-q+x),x)
```

```
[Out] (a*p + b)*log(x + (-2*a*p*q - b*p - b*q - p**2*(a*p + b)/(p - q) + 2*p*q*(a
*p + b)/(p - q) - q**2*(a*p + b)/(p - q))/(a*p + a*q + 2*b))/(p - q) - (a*q
 + b)*log(x + (-2*a*p*q - b*p - b*q + p**2*(a*q + b)/(p - q) - 2*p*q*(a*q +
 b)/(p - q) + q**2*(a*q + b)/(p - q))/(a*p + a*q + 2*b))/(p - q)
```

Giac [A] time = 1.06803, size = 57, normalized size = 1.42

$$\frac{(ap+b)\log\left(|-p+x|\right)}{p-q}-\frac{(aq+b)\log\left(|-q+x|\right)}{p-q}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(-p+x)/(-q+x),x, algorithm="giac")
```

```
[Out] (a*p + b)*log(abs(-p + x))/(p - q) - (a*q + b)*log(abs(-q + x))/(p - q)
```

3.8 $\int \frac{1}{c+bx+ax^2}\,dx$

Optimal. Leaf size=34

$$-\frac{2\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] (-2*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rubi [A] time = 0.0300147, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + b*x + a*x^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(b + 2*a*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{c+bx+ax^2}\,dx = -\left(2\,\mathrm{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2ax\right)\right)$$
$$= -\frac{2\tanh^{-1}\left(\frac{b+2ax}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.0105421, size = 38, normalized size = 1.12

$$\frac{2\tan^{-1}\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + b*x + a*x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(b + 2*a*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c]
```

Maple [A] time = 0.006, size = 35, normalized size = 1.

$$2\,\frac{1}{\sqrt{4\,ac-b^2}}\arctan\left(\frac{2\,ax+b}{\sqrt{4\,ac-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^2+b*x+c),x)
```

```
[Out] 2/(4*a*c-b^2)^(1/2)*arctan((2*a*x+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.00397, size = 277, normalized size = 8.15

$$\left[\frac{\log\left(\frac{2\,a^2x^2+2\,abx+b^2-2\,ac-\sqrt{b^2-4\,ac}(2\,ax+b)}{ax^2+bx+c}\right)}{\sqrt{b^2-4\,ac}}, -\frac{2\,\sqrt{-b^2+4\,ac}\arctan\left(-\frac{\sqrt{-b^2+4\,ac}(2\,ax+b)}{b^2-4\,ac}\right)}{b^2-4\,ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x+c),x, algorithm="fricas")
```

```
[Out] [log((2*a^2*x^2 + 2*a*b*x + b^2 - 2*a*c - sqrt(b^2 - 4*a*c)*(2*a*x + b))/(a
*x^2 + b*x + c))/sqrt(b^2 - 4*a*c), -2*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2
 + 4*a*c)*(2*a*x + b)/(b^2 - 4*a*c))/(b^2 - 4*a*c)]
```

Sympy [B] time = 0.198148, size = 124, normalized size = 3.65

$$-\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{-4ac\sqrt{-\frac{1}{4ac-b^2}}+b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2a}\right)+\sqrt{-\frac{1}{4ac-b^2}}\log\left(x+\frac{4ac\sqrt{-\frac{1}{4ac-b^2}}-b^2\sqrt{-\frac{1}{4ac-b^2}}+b}{2a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**2+b*x+c),x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2))*log(x + (-4*a*c*sqrt(-1/(4*a*c - b**2)) + b**2*sqr
t(-1/(4*a*c - b**2)) + b)/(2*a)) + sqrt(-1/(4*a*c - b**2))*log(x + (4*a*c*s
qrt(-1/(4*a*c - b**2)) - b**2*sqrt(-1/(4*a*c - b**2)) + b)/(2*a))
```

Giac [A] time = 1.0815, size = 46, normalized size = 1.35

$$\frac{2\,\arctan\left(\frac{2\,ax+b}{\sqrt{-b^2+4\,ac}}\right)}{\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^2+b*x+c),x, algorithm="giac")
```

```
[Out] 2*arctan((2*a*x + b)/sqrt(-b^2 + 4*a*c))/sqrt(-b^2 + 4*a*c)
```

3.9 $\int \frac{b+ax}{1+x^2} \, dx$

Optimal. Leaf size=16

$$\frac{1}{2} a \log \left(x^2+1\right)+b \tan^{-1}(x)$$

```
[Out] b*ArcTan[x] + (a*Log[1 + x^2])/2
```

Rubi [A] time = 0.0063259, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {635, 203, 260}

$$\frac{1}{2} a \log \left(x^2+1\right)+b \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x)/(1 + x^2),x]
```

```
[Out] b*ArcTan[x] + (a*Log[1 + x^2])/2
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{b+ax}{1+x^2}\,dx = a\int \frac{x}{1+x^2}\,dx + b\int \frac{1}{1+x^2}\,dx$$
$$= b\tan^{-1}(x) + \frac{1}{2}a\log\left(1+x^2\right)$$

Mathematica [A] time = 0.0059073, size = 16, normalized size = 1.

$$\frac{1}{2}a\log\left(x^2+1\right) + b\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x)/(1 + x^2),x]
```

```
[Out] b*ArcTan[x] + (a*Log[1 + x^2])/2
```

Maple [A] time = 0.002, size = 15, normalized size = 0.9

$$b\arctan(x) + \frac{a\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)/(x^2+1),x)
```

```
[Out] b*arctan(x)+1/2*a*ln(x^2+1)
```

Maxima [A] time = 1.41959, size = 19, normalized size = 1.19

$$b\arctan(x) + \frac{1}{2}a\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(x^2+1),x, algorithm="maxima")
```

[Out] b*arctan(x) + 1/2*a*log(x^2 + 1)

Fricas [A] time = 1.98883, size = 46, normalized size = 2.88

$$b\arctan(x) + \frac{1}{2}a\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(x^2+1),x, algorithm="fricas")
```

```
[Out] b*arctan(x) + 1/2*a*log(x^2 + 1)
```

Sympy [C] time = 0.153833, size = 26, normalized size = 1.62

$$\left(\frac{a}{2}-\frac{ib}{2}\right)\log(x-i)+\left(\frac{a}{2}+\frac{ib}{2}\right)\log(x+i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(x**2+1),x)
```

```
[Out] (a/2 - I*b/2)*log(x - I) + (a/2 + I*b/2)*log(x + I)
```

Giac [A] time = 1.07504, size = 19, normalized size = 1.19

$$b\arctan(x) + \frac{1}{2}a\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)/(x^2+1),x, algorithm="giac")
```

```
[Out] b*arctan(x) + 1/2*a*log(x^2 + 1)
```

3.10 $\int \frac{1}{3-2x+x^2}\,dx$

Optimal. Leaf size=19

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTan[(1 - x)/Sqrt[2]]/Sqrt[2])
```

Rubi [A] time = 0.0100857, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {618, 204}

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - 2*x + x^2)^(-1),x]
```

```
[Out] -(ArcTan[(1 - x)/Sqrt[2]]/Sqrt[2])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{3-2x+x^2}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, -2+2x\right)\right)$$
$$= -\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Mathematica [A] time = 0.0050858, size = 16, normalized size = 0.84

$$\frac{\tan^{-1}\left(\frac{x-1}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - 2*x + x^2)^(-1),x]
```

```
[Out] ArcTan[(-1 + x)/Sqrt[2]]/Sqrt[2]
```

Maple [A] time = 0.003, size = 17, normalized size = 0.9

$$\frac{\sqrt{2}}{2}\arctan\left(\frac{(2x-2)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-2*x+3),x)
```

```
[Out] 1/2*2^(1/2)*arctan(1/4*(2*x-2)*2^(1/2))
```

Maxima [A] time = 1.41061, size = 19, normalized size = 1.

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))
```

Fricas [A] time = 1.9499, size = 55, normalized size = 2.89

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))
```

Sympy [A] time = 0.098663, size = 22, normalized size = 1.16

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}-\frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-2*x+3),x)
```

```
[Out] sqrt(2)*atan(sqrt(2)*x/2 - sqrt(2)/2)/2
```

Giac [A] time = 1.06335, size = 19, normalized size = 1.

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-2*x+3),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x - 1))
```

3.11 $\int \frac{1}{(-1+x)^2\left(1+x^2\right)^2}\,dx$

Optimal. Leaf size=49

$$-\frac{1}{4\left(x^2+1\right)}+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4(1-x)}-\frac{1}{2}\log(1-x)+\frac{1}{4}\tan^{-1}(x)$$

```
[Out] 1/(4*(1 - x)) - 1/(4*(1 + x^2)) + ArcTan[x]/4 - Log[1 - x]/2 + Log[1 + x^2]
/4
```

Rubi [A] time = 0.0288985, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {741, 801, 635, 203, 260}

$$-\frac{1}{4\left(x^2+1\right)}+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4(1-x)}-\frac{1}{2}\log(1-x)+\frac{1}{4}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x)^2*(1 + x^2)^2),x]
```

```
[Out] 1/(4*(1 - x)) - 1/(4*(1 + x^2)) + ArcTan[x]/4 - Log[1 - x]/2 + Log[1 + x^2]
/4
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(-1+x)^2\left(1+x^2\right)^2}\,dx &= -\frac{1}{4\left(1+x^2\right)}-\frac{1}{4}\int \frac{-4+2x}{(-1+x)^2\left(1+x^2\right)}\,dx\\
&= -\frac{1}{4\left(1+x^2\right)}-\frac{1}{4}\int \left(-\frac{1}{(-1+x)^2}+\frac{2}{-1+x}+\frac{-1-2x}{1+x^2}\right)dx\\
&= \frac{1}{4(1-x)}-\frac{1}{4\left(1+x^2\right)}-\frac{1}{2}\log(1-x)-\frac{1}{4}\int \frac{-1-2x}{1+x^2}\,dx\\
&= \frac{1}{4(1-x)}-\frac{1}{4\left(1+x^2\right)}-\frac{1}{2}\log(1-x)+\frac{1}{4}\int \frac{1}{1+x^2}\,dx+\frac{1}{2}\int \frac{x}{1+x^2}\,dx\\
&= \frac{1}{4(1-x)}-\frac{1}{4\left(1+x^2\right)}+\frac{1}{4}\tan^{-1}(x)-\frac{1}{2}\log(1-x)+\frac{1}{4}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0195791, size = 35, normalized size = 0.71

$$\frac{1}{4}\left(-\frac{1}{x^2+1}+\log\left(x^2+1\right)+\frac{1}{1-x}-2\log(x-1)+\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x)^2*(1 + x^2)^2),x]
```

```
[Out] ((1 - x)^(-1) - (1 + x^2)^(-1) + ArcTan[x] - 2*Log[-1 + x] + Log[1 + x^2])/
4
```

Maple [A] time = 0.011, size = 36, normalized size = 0.7

$$-\frac{1}{4x^2+4}+\frac{\ln\left(x^2+1\right)}{4}+\frac{\arctan(x)}{4}-\frac{1}{-4+4x}-\frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+x)^2/(x^2+1)^2,x)
```

```
[Out] -1/4/(x^2+1)+1/4*ln(x^2+1)+1/4*arctan(x)-1/4/(-1+x)-1/2*ln(-1+x)
```

Maxima [A] time = 1.43306, size = 53, normalized size = 1.08

$$-\frac{x^2+x}{4\left(x^3-x^2+x-1\right)}+\frac{1}{4}\arctan(x)+\frac{1}{4}\log\left(x^2+1\right)-\frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(x^2 + x)/(x^3 - x^2 + x - 1) + 1/4*arctan(x) + 1/4*log(x^2 + 1) - 1/2
*log(x - 1)
```

Fricas [A] time = 1.93164, size = 186, normalized size = 3.8

$$-\frac{x^2-\left(x^3-x^2+x-1\right)\arctan(x)-\left(x^3-x^2+x-1\right)\log\left(x^2+1\right)+2\left(x^3-x^2+x-1\right)\log(x-1)+x}{4\left(x^3-x^2+x-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(x^2 - (x^3 - x^2 + x - 1)*arctan(x) - (x^3 - x^2 + x - 1)*log(x^2 + 1
) + 2*(x^3 - x^2 + x - 1)*log(x - 1) + x)/(x^3 - x^2 + x - 1)
```

Sympy [A] time = 0.161398, size = 39, normalized size = 0.8

$$-\frac{x^2+x}{4x^3-4x^2+4x-4}-\frac{\log(x-1)}{2}+\frac{\log\left(x^2+1\right)}{4}+\frac{\operatorname{atan}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)**2/(x**2+1)**2,x)
```

```
[Out] -(x**2 + x)/(4*x**3 - 4*x**2 + 4*x - 4) - log(x - 1)/2 + log(x**2 + 1)/4 +
atan(x)/4
```

Giac [B] time = 1.11861, size = 108, normalized size = 2.2

$$\frac{1}{16}\pi-\frac{1}{4}\pi\left\lfloor\frac{\pi+4\arctan(x)}{4\pi}+\frac{1}{2}\right\rfloor+\frac{\frac{2}{x-1}+1}{8\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)}-\frac{1}{4(x-1)}+\frac{1}{4}\arctan(x)+\frac{1}{4}\log\left(\frac{2}{x-1}+\frac{2}{(x-1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+x)^2/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/16*pi - 1/4*pi*floor(1/4*(pi + 4*arctan(x))/pi + 1/2) + 1/8*(2/(x - 1) +
1)/(2/(x - 1) + 2/(x - 1)^2 + 1) - 1/4/(x - 1) + 1/4*arctan(x) + 1/4*log(2/
(x - 1) + 2/(x - 1)^2 + 1)
```

3.12 $\int \frac{x}{(-a+x)(-b+x)(-c+x)} dx$

Optimal. Leaf size=68

$$\frac{a\log(a-x)}{(a-b)(a-c)} - \frac{b\log(b-x)}{(a-b)(b-c)} + \frac{c\log(c-x)}{(a-c)(b-c)}$$

```
[Out] (a*Log[a - x])/((a - b)*(a - c)) - (b*Log[b - x])/((a - b)*(b - c)) + (c*Lo
g[c - x])/((a - c)*(b - c))
```

Rubi [A] time = 0.0515693, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {148}

$$\frac{a\log(a-x)}{(a-b)(a-c)} - \frac{b\log(b-x)}{(a-b)(b-c)} + \frac{c\log(c-x)}{(a-c)(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((-a + x)*(-b + x)*(-c + x)),x]
```

```
[Out] (a*Log[a - x])/((a - b)*(a - c)) - (b*Log[b - x])/((a - b)*(b - c)) + (c*Lo
g[c - x])/((a - c)*(b - c))
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(-a+x)(-b+x)(-c+x)} dx &= \int \left(-\frac{a}{(a-b)(a-c)(a-x)} + \frac{b}{(a-b)(b-c)(b-x)} + \frac{c}{(a-c)(-b+c)(c-x)}\right) dx\\ &= \frac{a\log(a-x)}{(a-b)(a-c)} - \frac{b\log(b-x)}{(a-b)(b-c)} + \frac{c\log(c-x)}{(a-c)(b-c)}\end{aligned}$$

Mathematica [A] time = 0.0300064, size = 62, normalized size = 0.91

$$\frac{a(b-c)\log(x-a)+b(c-a)\log(x-b)+c(a-b)\log(x-c)}{(a-b)(a-c)(b-c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-a + x)*(-b + x)*(-c + x)),x]
```

```
[Out] (a*(b - c)*Log[-a + x] + b*(-a + c)*Log[-b + x] + (a - b)*c*Log[-c + x])/((
a - b)*(a - c)*(b - c))
```

Maple [A] time = 0.008, size = 69, normalized size = 1.

$$-\frac{b\ln(-b+x)}{(a-b)(b-c)}+\frac{c\ln(-c+x)}{(b-c)(a-c)}+\frac{a\ln(-a+x)}{(a-b)(a-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-a+x)/(-b+x)/(-c+x),x)
```

```
[Out] -b/(a-b)/(b-c)*ln(-b+x)+c/(b-c)/(a-c)*ln(-c+x)+1/(a-b)/(a-c)*a*ln(-a+x)
```

Maxima [A] time = 0.961421, size = 105, normalized size = 1.54

$$\frac{a\log(-a+x)}{a^2-ab-(a-b)c}-\frac{b\log(-b+x)}{ab-b^2-(a-b)c}+\frac{c\log(-c+x)}{ab-(a+b)c+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a+x)/(-b+x)/(-c+x),x, algorithm="maxima")
```

```
[Out] a*log(-a + x)/(a^2 - a*b - (a - b)*c) - b*log(-b + x)/(a*b - b^2 - (a - b)*
c) + c*log(-c + x)/(a*b - (a + b)*c + c^2)
```

Fricas [A] time = 2.76227, size = 166, normalized size = 2.44

$$\frac{(a-b)c\log(-c+x)+(ab-ac)\log(-a+x)-(ab-bc)\log(-b+x)}{a^2b-ab^2+(a-b)c^2-\left(a^2-b^2\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a+x)/(-b+x)/(-c+x),x, algorithm="fricas")
```

```
[Out] ((a - b)*c*log(-c + x) + (a*b - a*c)*log(-a + x) - (a*b - b*c)*log(-b + x))
/(a^2*b - a*b^2 + (a - b)*c^2 - (a^2 - b^2)*c)
```

Sympy [B] time = 85.2064, size = 4216, normalized size = 62.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a+x)/(-b+x)/(-c+x),x)
```

```
[Out] a*log(x + (-a**8*b**3/((a - b)**2*(a - c)**2) + a**8*b**2*c/((a - b)**2*(a
- c)**2) + a**8*b*c**2/((a - b)**2*(a - c)**2) - a**8*c**3/((a - b)**2*(a -
 c)**2) + a**7*b**4/((a - b)**2*(a - c)**2) + 8*a**7*b**3*c/((a - b)**2*(a
- c)**2) - 18*a**7*b**2*c**2/((a - b)**2*(a - c)**2) + 8*a**7*b*c**3/((a -
b)**2*(a - c)**2) + a**7*c**4/((a - b)**2*(a - c)**2) + a**6*b**5/((a - b)*
*2*(a - c)**2) - 18*a**6*b**4*c/((a - b)**2*(a - c)**2) + 17*a**6*b**3*c**2
/((a - b)**2*(a - c)**2) - a**6*b**3/((a - b)*(a - c)) + 17*a**6*b**2*c**3/
((a - b)**2*(a - c)**2) + a**6*b**2*c/((a - b)*(a - c)) - 18*a**6*b*c**4/((
a - b)**2*(a - c)**2) + a**6*b*c**2/((a - b)*(a - c)) + a**6*c**5/((a - b)*
*2*(a - c)**2) - a**6*c**3/((a - b)*(a - c)) - a**5*b**6/((a - b)**2*(a - c
)**2) + 8*a**5*b**5*c/((a - b)**2*(a - c)**2) + 17*a**5*b**4*c**2/((a - b)*
*2*(a - c)**2) + 2*a**5*b**4/((a - b)*(a - c)) - 48*a**5*b**3*c**3/((a - b)
**2*(a - c)**2) - a**5*b**3*c/((a - b)*(a - c)) + 17*a**5*b**2*c**4/((a - b
)**2*(a - c)**2) - 2*a**5*b**2*c**2/((a - b)*(a - c)) + 8*a**5*b*c**5/((a -
 b)**2*(a - c)**2) - a**5*b*c**3/((a - b)*(a - c)) - a**5*c**6/((a - b)**2*
(a - c)**2) + 2*a**5*c**4/((a - b)*(a - c)) + a**4*b**6*c/((a - b)**2*(a -
c)**2) - 18*a**4*b**5*c**2/((a - b)**2*(a - c)**2) - a**4*b**5/((a - b)*(a
- c)) + 17*a**4*b**4*c**3/((a - b)**2*(a - c)**2) - a**4*b**4*c/((a - b)*(a
 - c)) + 17*a**4*b**3*c**4/((a - b)**2*(a - c)**2) + 2*a**4*b**3*c**2/((a -
 b)*(a - c)) - 18*a**4*b**2*c**5/((a - b)**2*(a - c)**2) + 2*a**4*b**2*c**3
/((a - b)*(a - c)) + a**4*b**2*c + a**4*b*c**6/((a - b)**2*(a - c)**2) - a*
*4*b*c**4/((a - b)*(a - c)) + a**4*b*c**2 - a**4*c**5/((a - b)*(a - c)) + a
**3*b**6*c**2/((a - b)**2*(a - c)**2) + 8*a**3*b**5*c**3/((a - b)**2*(a - c
)**2) + a**3*b**5*c/((a - b)*(a - c)) - 18*a**3*b**4*c**4/((a - b)**2*(a -
c)**2) - 2*a**3*b**4*c**2/((a - b)*(a - c)) + 8*a**3*b**3*c**5/((a - b)**2*
(a - c)**2) + 2*a**3*b**3*c**3/((a - b)*(a - c)) - 4*a**3*b**3*c + a**3*b**
2*c**6/((a - b)**2*(a - c)**2) - 2*a**3*b**2*c**4/((a - b)*(a - c)) + 2*a**
```

```
3*b**2*c**2 + a**3*b*c**5/((a - b)*(a - c)) - 4*a**3*b*c**3 - a**2*b**6*c**
3/((a - b)**2*(a - c)**2) + a**2*b**5*c**4/((a - b)**2*(a - c)**2) + a**2*b
**5*c**2/((a - b)*(a - c)) + a**2*b**4*c**5/((a - b)**2*(a - c)**2) - a**2*
b**4*c**3/((a - b)*(a - c)) + a**2*b**4*c - a**2*b**3*c**6/((a - b)**2*(a -
 c)**2) - a**2*b**3*c**4/((a - b)*(a - c)) + 2*a**2*b**3*c**2 + a**2*b**2*c
**5/((a - b)*(a - c)) + 2*a**2*b**2*c**3 + a**2*b*c**4 - a*b**5*c**3/((a -
b)*(a - c)) + 2*a*b**4*c**4/((a - b)*(a - c)) + a*b**4*c**2 - a*b**3*c**5/(
(a - b)*(a - c)) - 4*a*b**3*c**3 + a*b**2*c**4)/(2*a**3*b**3 - 3*a**3*b**2*
c - 3*a**3*b*c**2 + 2*a**3*c**3 - 3*a**2*b**3*c + 12*a**2*b**2*c**2 - 3*a**
2*b*c**3 - 3*a*b**3*c**2 - 3*a*b**2*c**3 + 2*b**3*c**3))/((a - b)*(a - c))
- b*log(x + (-a**6*b**5/((a - b)**2*(b - c)**2) + a**6*b**4*c/((a - b)**2*(
b - c)**2) + a**6*b**3*c**2/((a - b)**2*(b - c)**2) - a**6*b**2*c**3/((a -
b)**2*(b - c)**2) + a**5*b**6/((a - b)**2*(b - c)**2) + 8*a**5*b**5*c/((a -
 b)**2*(b - c)**2) - 18*a**5*b**4*c**2/((a - b)**2*(b - c)**2) + a**5*b**4/
((a - b)*(b - c)) + 8*a**5*b**3*c**3/((a - b)**2*(b - c)**2) - a**5*b**3*c/
((a - b)*(b - c)) + a**5*b**2*c**4/((a - b)**2*(b - c)**2) - a**5*b**2*c**2
/((a - b)*(b - c)) + a**5*b*c**3/((a - b)*(b - c)) + a**4*b**7/((a - b)**2*
(b - c)**2) - 18*a**4*b**6*c/((a - b)**2*(b - c)**2) + 17*a**4*b**5*c**2/((
a - b)**2*(b - c)**2) - 2*a**4*b**5/((a - b)*(b - c)) + 17*a**4*b**4*c**3/(
(a - b)**2*(b - c)**2) + a**4*b**4*c/((a - b)*(b - c)) - 18*a**4*b**3*c**4/
((a - b)**2*(b - c)**2) + 2*a**4*b**3*c**2/((a - b)*(b - c)) + a**4*b**2*c*
*5/((a - b)**2*(b - c)**2) + a**4*b**2*c**3/((a - b)*(b - c)) + a**4*b**2*c
 - 2*a**4*b*c**4/((a - b)*(b - c)) + a**4*b*c**2 - a**3*b**8/((a - b)**2*(b
 - c)**2) + 8*a**3*b**7*c/((a - b)**2*(b - c)**2) + 17*a**3*b**6*c**2/((a -
 b)**2*(b - c)**2) + a**3*b**6/((a - b)*(b - c)) - 48*a**3*b**5*c**3/((a -
b)**2*(b - c)**2) + a**3*b**5*c/((a - b)*(b - c)) + 17*a**3*b**4*c**4/((a -
 b)**2*(b - c)**2) - 2*a**3*b**4*c**2/((a - b)*(b - c)) + 8*a**3*b**3*c**5/
((a - b)**2*(b - c)**2) - 2*a**3*b**3*c**3/((a - b)*(b - c)) - 4*a**3*b**3*
c - a**3*b**2*c**6/((a - b)**2*(b - c)**2) + a**3*b**2*c**4/((a - b)*(b - c
)) + 2*a**3*b**2*c**2 + a**3*b*c**5/((a - b)*(b - c)) - 4*a**3*b*c**3 + a**
2*b**8*c/((a - b)**2*(b - c)**2) - 18*a**2*b**7*c**2/((a - b)**2*(b - c)**2
) + 17*a**2*b**6*c**3/((a - b)**2*(b - c)**2) - a**2*b**6*c/((a - b)*(b - c
)) + 17*a**2*b**5*c**4/((a - b)**2*(b - c)**2) + 2*a**2*b**5*c**2/((a - b)*
(b - c)) - 18*a**2*b**4*c**5/((a - b)**2*(b - c)**2) - 2*a**2*b**4*c**3/((a
 - b)*(b - c)) + a**2*b**4*c + a**2*b**3*c**6/((a - b)**2*(b - c)**2) + 2*a
**2*b**3*c**4/((a - b)*(b - c)) + 2*a**2*b**3*c**2 - a**2*b**2*c**5/((a - b
)*(b - c)) + 2*a**2*b**2*c**3 + a**2*b*c**4 + a*b**8*c**2/((a - b)**2*(b -
c)**2) + 8*a*b**7*c**3/((a - b)**2*(b - c)**2) - 18*a*b**6*c**4/((a - b)**2
*(b - c)**2) - a*b**6*c**2/((a - b)*(b - c)) + 8*a*b**5*c**5/((a - b)**2*(b
 - c)**2) + a*b**5*c**3/((a - b)*(b - c)) + a*b**4*c**6/((a - b)**2*(b - c)
**2) + a*b**4*c**4/((a - b)*(b - c)) + a*b**4*c**2 - a*b**3*c**5/((a - b)*(
b - c)) - 4*a*b**3*c**3 + a*b**2*c**4 - b**8*c**3/((a - b)**2*(b - c)**2) +
 b**7*c**4/((a - b)**2*(b - c)**2) + b**6*c**5/((a - b)**2*(b - c)**2) + b*
*6*c**3/((a - b)*(b - c)) - b**5*c**6/((a - b)**2*(b - c)**2) - 2*b**5*c**4
/((a - b)*(b - c)) + b**4*c**5/((a - b)*(b - c)))/(2*a**3*b**3 - 3*a**3*b**
```

```
2*c - 3*a**3*b*c**2 + 2*a**3*c**3 - 3*a**2*b**3*c + 12*a**2*b**2*c**2 - 3*a
**2*b*c**3 - 3*a*b**3*c**2 - 3*a*b**2*c**3 + 2*b**3*c**3))/((a - b)*(b - c)
) + c*log(x + (-a**6*b**3*c**2/((a - c)**2*(b - c)**2) + a**6*b**2*c**3/((a
 - c)**2*(b - c)**2) + a**6*b*c**4/((a - c)**2*(b - c)**2) - a**6*c**5/((a
- c)**2*(b - c)**2) + a**5*b**4*c**2/((a - c)**2*(b - c)**2) + 8*a**5*b**3*
c**3/((a - c)**2*(b - c)**2) - a**5*b**3*c/((a - c)*(b - c)) - 18*a**5*b**2
*c**4/((a - c)**2*(b - c)**2) + a**5*b**2*c**2/((a - c)*(b - c)) + 8*a**5*b
*c**5/((a - c)**2*(b - c)**2) + a**5*b*c**3/((a - c)*(b - c)) + a**5*c**6/(
(a - c)**2*(b - c)**2) - a**5*c**4/((a - c)*(b - c)) + a**4*b**5*c**2/((a -
 c)**2*(b - c)**2) - 18*a**4*b**4*c**3/((a - c)**2*(b - c)**2) + 2*a**4*b**
4*c/((a - c)*(b - c)) + 17*a**4*b**3*c**4/((a - c)**2*(b - c)**2) - a**4*b*
*3*c**2/((a - c)*(b - c)) + 17*a**4*b**2*c**5/((a - c)**2*(b - c)**2) - 2*a
**4*b**2*c**3/((a - c)*(b - c)) + a**4*b**2*c - 18*a**4*b*c**6/((a - c)**2*
(b - c)**2) - a**4*b*c**4/((a - c)*(b - c)) + a**4*b*c**2 + a**4*c**7/((a -
 c)**2*(b - c)**2) + 2*a**4*c**5/((a - c)*(b - c)) - a**3*b**6*c**2/((a - c
)**2*(b - c)**2) + 8*a**3*b**5*c**3/((a - c)**2*(b - c)**2) - a**3*b**5*c/(
(a - c)*(b - c)) + 17*a**3*b**4*c**4/((a - c)**2*(b - c)**2) - a**3*b**4*c*
*2/((a - c)*(b - c)) - 48*a**3*b**3*c**5/((a - c)**2*(b - c)**2) + 2*a**3*b
**3*c**3/((a - c)*(b - c)) - 4*a**3*b**3*c + 17*a**3*b**2*c**6/((a - c)**2*
(b - c)**2) + 2*a**3*b**2*c**4/((a - c)*(b - c)) + 2*a**3*b**2*c**2 + 8*a**
3*b*c**7/((a - c)**2*(b - c)**2) - a**3*b*c**5/((a - c)*(b - c)) - 4*a**3*b
*c**3 - a**3*c**8/((a - c)**2*(b - c)**2) - a**3*c**6/((a - c)*(b - c)) + a
**2*b**6*c**3/((a - c)**2*(b - c)**2) - 18*a**2*b**5*c**4/((a - c)**2*(b -
c)**2) + a**2*b**5*c**2/((a - c)*(b - c)) + 17*a**2*b**4*c**5/((a - c)**2*(
b - c)**2) - 2*a**2*b**4*c**3/((a - c)*(b - c)) + a**2*b**4*c + 17*a**2*b**
3*c**6/((a - c)**2*(b - c)**2) + 2*a**2*b**3*c**4/((a - c)*(b - c)) + 2*a**
2*b**3*c**2 - 18*a**2*b**2*c**7/((a - c)**2*(b - c)**2) - 2*a**2*b**2*c**5/
((a - c)*(b - c)) + 2*a**2*b**2*c**3 + a**2*b*c**8/((a - c)**2*(b - c)**2)
+ a**2*b*c**6/((a - c)*(b - c)) + a**2*b*c**4 + a*b**6*c**4/((a - c)**2*(b
- c)**2) + 8*a*b**5*c**5/((a - c)**2*(b - c)**2) + a*b**5*c**3/((a - c)*(b
- c)) - 18*a*b**4*c**6/((a - c)**2*(b - c)**2) - a*b**4*c**4/((a - c)*(b -
c)) + a*b**4*c**2 + 8*a*b**3*c**7/((a - c)**2*(b - c)**2) - a*b**3*c**5/((a
 - c)*(b - c)) - 4*a*b**3*c**3 + a*b**2*c**8/((a - c)**2*(b - c)**2) + a*b*
*2*c**6/((a - c)*(b - c)) + a*b**2*c**4 - b**6*c**5/((a - c)**2*(b - c)**2)
 + b**5*c**6/((a - c)**2*(b - c)**2) - b**5*c**4/((a - c)*(b - c)) + b**4*c
**7/((a - c)**2*(b - c)**2) + 2*b**4*c**5/((a - c)*(b - c)) - b**3*c**8/((a
 - c)**2*(b - c)**2) - b**3*c**6/((a - c)*(b - c)))/(2*a**3*b**3 - 3*a**3*b
**2*c - 3*a**3*b*c**2 + 2*a**3*c**3 - 3*a**2*b**3*c + 12*a**2*b**2*c**2 - 3
*a**2*b*c**3 - 3*a*b**3*c**2 - 3*a*b**2*c**3 + 2*b**3*c**3))/((a - c)*(b -
c))
```

<u>Giac</u> [A] time = 1.06321, size = 109, normalized size = 1.6

$$\frac{a\log\left(\left|-a+x\right|\right)}{a^2-ab-ac+bc}-\frac{b\log\left(\left|-b+x\right|\right)}{ab-b^2-ac+bc}+\frac{c\log\left(\left|-c+x\right|\right)}{ab-ac-bc+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-a+x)/(-b+x)/(-c+x),x, algorithm="giac")
```

```
[Out] a*log(abs(-a + x))/(a^2 - a*b - a*c + b*c) - b*log(abs(-b + x))/(a*b - b^2
- a*c + b*c) + c*log(abs(-c + x))/(a*b - a*c - b*c + c^2)
```

3.13 $\int \frac{x}{(a^2+x^2)(b^2+x^2)} \, dx$

Optimal. Leaf size=47

$$\frac{\log(b^2+x^2)}{2(a^2-b^2)} - \frac{\log(a^2+x^2)}{2(a^2-b^2)}$$

```
[Out] -Log[a^2 + x^2]/(2*(a^2 - b^2)) + Log[b^2 + x^2]/(2*(a^2 - b^2))
```

Rubi [A] time = 0.0197179, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.15, Rules used = {444, 36, 31}

$$\frac{\log(b^2+x^2)}{2(a^2-b^2)} - \frac{\log(a^2+x^2)}{2(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((a^2 + x^2)*(b^2 + x^2)),x]
```

```
[Out] -Log[a^2 + x^2]/(2*(a^2 - b^2)) + Log[b^2 + x^2]/(2*(a^2 - b^2))
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{x}{\left(a^2+x^2\right)\left(b^2+x^2\right)}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{\left(a^2+x\right)\left(b^2+x\right)}\,dx,x,x^2\right)$$
$$= -\frac{\mathrm{Subst}\left(\int \frac{1}{a^2+x}\,dx,x,x^2\right)}{2\left(a^2-b^2\right)}+\frac{\mathrm{Subst}\left(\int \frac{1}{b^2+x}\,dx,x,x^2\right)}{2\left(a^2-b^2\right)}$$
$$= -\frac{\log\left(a^2+x^2\right)}{2\left(a^2-b^2\right)}+\frac{\log\left(b^2+x^2\right)}{2\left(a^2-b^2\right)}$$

Mathematica [A] time = 0.0105171, size = 34, normalized size = 0.72

$$\frac{\log\left(b^2+x^2\right)-\log\left(a^2+x^2\right)}{2\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((a^2 + x^2)*(b^2 + x^2)),x]
```

```
[Out] (-Log[a^2 + x^2] + Log[b^2 + x^2])/(2*(a^2 - b^2))
```

Maple [A] time = 0.007, size = 44, normalized size = 0.9

$$-\frac{\ln\left(a^2+x^2\right)}{2\,a^2-2\,b^2}+\frac{\ln\left(b^2+x^2\right)}{2\,a^2-2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a^2+x^2)/(b^2+x^2),x)
```

```
[Out] -1/2*ln(a^2+x^2)/(a^2-b^2)+1/2*ln(b^2+x^2)/(a^2-b^2)
```

Maxima [A] time = 0.948073, size = 58, normalized size = 1.23

$$-\frac{\log\left(a^2+x^2\right)}{2\left(a^2-b^2\right)}+\frac{\log\left(b^2+x^2\right)}{2\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+x^2)/(b^2+x^2),x, algorithm="maxima")
```

```
[Out] -1/2*log(a^2 + x^2)/(a^2 - b^2) + 1/2*log(b^2 + x^2)/(a^2 - b^2)
```

Fricas [A] time = 2.01894, size = 70, normalized size = 1.49

$$-\frac{\log\left(a^2+x^2\right)-\log\left(b^2+x^2\right)}{2\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+x^2)/(b^2+x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(log(a^2 + x^2) - log(b^2 + x^2))/(a^2 - b^2)
```

Sympy [B] time = 0.540466, size = 121, normalized size = 2.57

$$\frac{\log\left(-\frac{a^4}{2(a-b)(a+b)}+\frac{a^2b^2}{(a-b)(a+b)}+\frac{a^2}{2}-\frac{b^4}{2(a-b)(a+b)}+\frac{b^2}{2}+x^2\right)}{2\,(a-b)\,(a+b)}-\frac{\log\left(\frac{a^4}{2(a-b)(a+b)}-\frac{a^2b^2}{(a-b)(a+b)}+\frac{a^2}{2}+\frac{b^4}{2(a-b)(a+b)}+\frac{b^2}{2}+x^2\right)}{2\,(a-b)\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a**2+x**2)/(b**2+x**2),x)
```

```
[Out] log(-a**4/(2*(a - b)*(a + b)) + a**2*b**2/((a - b)*(a + b)) + a**2/2 - b**4
/(2*(a - b)*(a + b)) + b**2/2 + x**2)/(2*(a - b)*(a + b)) - log(a**4/(2*(a
- b)*(a + b)) - a**2*b**2/((a - b)*(a + b)) + a**2/2 + b**4/(2*(a - b)*(a +
 b)) + b**2/2 + x**2)/(2*(a - b)*(a + b))
```

Giac [A] time = 1.14083, size = 88, normalized size = 1.87

$$\frac{\log\left(\frac{\left|a^2+b^2+2\,x^2-\left|a^2-b^2\right|\right|}{a^2+b^2+2\,x^2+\left|a^2-b^2\right|}\right)}{2\left|a^2-b^2\right|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a^2+x^2)/(b^2+x^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(a^2 + b^2 + 2*x^2 - abs(a^2 - b^2))/(a^2 + b^2 + 2*x^2 + abs(a^
2 - b^2)))/abs(a^2 - b^2)
```

3.14 $\int \frac{x^2}{\left(a^2+x^2\right)\left(b^2+x^2\right)}\,dx$

Optimal. Leaf size=40

$$\frac{a\tan^{-1}\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\tan^{-1}\left(\frac{x}{b}\right)}{a^2-b^2}$$

```
[Out] (a*ArcTan[x/a])/(a^2 - b^2) - (b*ArcTan[x/b])/(a^2 - b^2)
```

Rubi [A] time = 0.019227, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {481, 203}

$$\frac{a\tan^{-1}\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\tan^{-1}\left(\frac{x}{b}\right)}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((a^2 + x^2)*(b^2 + x^2)),x]

[Out] (a*ArcTan[x/a])/(a^2 - b^2) - (b*ArcTan[x/b])/(a^2 - b^2)
```

Rule 481

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x],
 x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; Fr
eeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m,
2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{x^2}{(a^2+x^2)(b^2+x^2)}\,dx = \frac{a^2\int \frac{1}{a^2+x^2}\,dx}{a^2-b^2} - \frac{b^2\int \frac{1}{b^2+x^2}\,dx}{a^2-b^2}$$
$$= \frac{a\tan^{-1}\left(\frac{x}{a}\right)}{a^2-b^2} - \frac{b\tan^{-1}\left(\frac{x}{b}\right)}{a^2-b^2}$$

Mathematica [A] time = 0.0140833, size = 30, normalized size = 0.75

$$\frac{a\tan^{-1}\left(\frac{x}{a}\right) - b\tan^{-1}\left(\frac{x}{b}\right)}{a^2-b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((a^2 + x^2)*(b^2 + x^2)),x]
```

```
[Out] (a*ArcTan[x/a] - b*ArcTan[x/b])/(a^2 - b^2)
```

Maple [A] time = 0.007, size = 41, normalized size = 1.

$$\frac{a}{a^2-b^2}\arctan\left(\frac{x}{a}\right) - \frac{b}{a^2-b^2}\arctan\left(\frac{x}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a^2+x^2)/(b^2+x^2),x)
```

```
[Out] a*arctan(x/a)/(a^2-b^2)-b*arctan(x/b)/(a^2-b^2)
```

Maxima [A] time = 1.41942, size = 54, normalized size = 1.35

$$\frac{a\arctan\left(\frac{x}{a}\right)}{a^2-b^2} - \frac{b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+x^2)/(b^2+x^2),x, algorithm="maxima")
```

```
[Out] a*arctan(x/a)/(a^2 - b^2) - b*arctan(x/b)/(a^2 - b^2)
```

Fricas [A] time = 2.07345, size = 61, normalized size = 1.52

$$\frac{a\arctan\left(\frac{x}{a}\right)-b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+x^2)/(b^2+x^2),x, algorithm="fricas")
```

```
[Out] (a*arctan(x/a) - b*arctan(x/b))/(a^2 - b^2)
```

Sympy [C] time = 0.943608, size = 393, normalized size = 9.82

$$-\frac{ia\log\left(-\frac{2ia^7}{(a-b)^3(a+b)^3}+\frac{4ia^5b^2}{(a-b)^3(a+b)^3}-\frac{2ia^3b^4}{(a-b)^3(a+b)^3}+\frac{ia^3}{(a-b)(a+b)}+\frac{iab^2}{(a-b)(a+b)}+x\right)}{2(a-b)(a+b)}+\frac{ia\log\left(\frac{2ia^7}{(a-b)^3(a+b)^3}-\frac{4ia^5b^2}{(a-b)^3(a+b)^3}+\frac{2ia^3b^4}{(a-b)^3(a+b)^3}\right.}{2(a-b)(a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a**2+x**2)/(b**2+x**2),x)
```

```
[Out] -I*a*log(-2*I*a**7/((a - b)**3*(a + b)**3) + 4*I*a**5*b**2/((a - b)**3*(a +
 b)**3) - 2*I*a**3*b**4/((a - b)**3*(a + b)**3) + I*a**3/((a - b)*(a + b))
+ I*a*b**2/((a - b)*(a + b)) + x)/(2*(a - b)*(a + b)) + I*a*log(2*I*a**7/((
a - b)**3*(a + b)**3) - 4*I*a**5*b**2/((a - b)**3*(a + b)**3) + 2*I*a**3*b*
*4/((a - b)**3*(a + b)**3) - I*a**3/((a - b)*(a + b)) - I*a*b**2/((a - b)*(
a + b)) + x)/(2*(a - b)*(a + b)) - I*b*log(-2*I*a**4*b**3/((a - b)**3*(a +
b)**3) + 4*I*a**2*b**5/((a - b)**3*(a + b)**3) + I*a**2*b/((a - b)*(a + b))
 - 2*I*b**7/((a - b)**3*(a + b)**3) + I*b**3/((a - b)*(a + b)) + x)/(2*(a -
 b)*(a + b)) + I*b*log(2*I*a**4*b**3/((a - b)**3*(a + b)**3) - 4*I*a**2*b**
5/((a - b)**3*(a + b)**3) - I*a**2*b/((a - b)*(a + b)) + 2*I*b**7/((a - b)*
*3*(a + b)**3) - I*b**3/((a - b)*(a + b)) + x)/(2*(a - b)*(a + b))
```

Giac [A] time = 1.07482, size = 54, normalized size = 1.35

$$\frac{a\arctan\left(\frac{x}{a}\right)}{a^2-b^2}-\frac{b\arctan\left(\frac{x}{b}\right)}{a^2-b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a^2+x^2)/(b^2+x^2),x, algorithm="giac")
```

```
[Out] a*arctan(x/a)/(a^2 - b^2) - b*arctan(x/b)/(a^2 - b^2)
```

3.15 $\int \frac{x}{(-1+x)\left(1+x^2\right)} dx$

Optimal. Leaf size=27

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(1-x)+\frac{1}{2}\tan^{-1}(x)$$

```
[Out] ArcTan[x]/2 + Log[1 - x]/2 - Log[1 + x^2]/4
```

Rubi [A] time = 0.0180382, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {801, 635, 203, 260}

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(1-x)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x/((-1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 + Log[1 - x]/2 - Log[1 + x^2]/4
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(-1+x)\left(1+x^2\right)}\,dx &= \int \left(\frac{1}{2(-1+x)}+\frac{1-x}{2\left(1+x^2\right)}\right)dx\\ &= \frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{1-x}{1+x^2}\,dx\\ &= \frac{1}{2}\log(1-x)+\frac{1}{2}\int \frac{1}{1+x^2}\,dx-\frac{1}{2}\int \frac{x}{1+x^2}\,dx\\ &= \frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log(1-x)-\frac{1}{4}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0043409, size = 27, normalized size = 1.

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2}\log(1-x)+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((-1 + x)*(1 + x^2)),x]
```

```
[Out] ArcTan[x]/2 + Log[1 - x]/2 - Log[1 + x^2]/4
```

Maple [A] time = 0.004, size = 20, normalized size = 0.7

$$-\frac{\ln\left(x^2+1\right)}{4}+\frac{\arctan(x)}{2}+\frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-1+x)/(x^2+1),x)
```

```
[Out] -1/4*ln(x^2+1)+1/2*arctan(x)+1/2*ln(-1+x)
```

Maxima [A] time = 1.42416, size = 26, normalized size = 0.96

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)/(x^2+1),x, algorithm="maxima")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x - 1)
```

Fricas [A] time = 2.01521, size = 69, normalized size = 2.56

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)/(x^2+1),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(x - 1)
```

Sympy [A] time = 0.113708, size = 19, normalized size = 0.7

$$\frac{\log(x-1)}{2} - \frac{\log\left(x^2+1\right)}{4} + \frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)/(x**2+1),x)
```

```
[Out] log(x - 1)/2 - log(x**2 + 1)/4 + atan(x)/2
```

Giac [A] time = 1.08794, size = 27, normalized size = 1.

$$\frac{1}{2}\arctan(x) - \frac{1}{4}\log\left(x^2+1\right) + \frac{1}{2}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-1+x)/(x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*arctan(x) - 1/4*log(x^2 + 1) + 1/2*log(abs(x - 1))
```

3.16 $\int \frac{x}{1+x^3}\,dx$

Optimal. Leaf size=41

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Rubi [A] time = 0.0201304, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {292, 31, 634, 618, 204, 628}

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x^3),x]
```

```
[Out] -(ArcTan[(1 - 2*x)/Sqrt[3]]/Sqrt[3]) - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{1+x^3}\,dx &= -\left(\frac{1}{3}\int \frac{1}{1+x}\,dx\right)+\frac{1}{3}\int \frac{1+x}{1-x+x^2}\,dx\\
&= -\frac{1}{3}\log(1+x)+\frac{1}{6}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1}{1-x+x^2}\,dx\\
&= -\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)-\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)\\
&= \frac{\tan^{-1}\left(\frac{-1+2x}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log(1+x)+\frac{1}{6}\log\left(1-x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0079071, size = 40, normalized size = 0.98

$$\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)+\frac{\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x^3),x]
```

```
[Out] ArcTan[(-1 + 2*x)/Sqrt[3]]/Sqrt[3] - Log[1 + x]/3 + Log[1 - x + x^2]/6
```

Maple [A] time = 0.006, size = 35, normalized size = 0.9

$$-\frac{\ln(1+x)}{3}+\frac{\ln\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}}{3}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3+1),x)
```

```
[Out] -1/3*ln(1+x)+1/6*ln(x^2-x+1)+1/3*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.41682, size = 46, normalized size = 1.12

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
x + 1)
```

Fricas [A] time = 1.82908, size = 112, normalized size = 2.73

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
x + 1)
```

Sympy [A] time = 0.120886, size = 41, normalized size = 1.

$$-\frac{\log(x+1)}{3}+\frac{\log\left(x^2-x+1\right)}{6}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3+1),x)
```

```
[Out] -log(x + 1)/3 + log(x**2 - x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/
3)/3
```

Giac [A] time = 1.1035, size = 47, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{6}\log\left(x^2-x+1\right)-\frac{1}{3}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3+1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1)) + 1/6*log(x^2 - x + 1) - 1/3*log(
abs(x + 1))
```

3.17 $\int \frac{x^3}{(-1+x)^2 (1+x^3)} \, dx$

Optimal. Leaf size=43

$$-\frac{1}{3} \log \left(x^2-x+1\right)+\frac{1}{2(1-x)}+\frac{3}{4} \log(1-x)-\frac{1}{12} \log(x+1)$$

```
[Out] 1/(2*(1 - x)) + (3*Log[1 - x])/4 - Log[1 + x]/12 - Log[1 - x + x^2]/3
```

Rubi [A] time = 0.130239, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6725, 628}

$$-\frac{1}{3} \log \left(x^2-x+1\right)+\frac{1}{2(1-x)}+\frac{3}{4} \log(1-x)-\frac{1}{12} \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((-1 + x)^2*(1 + x^3)),x]
```

```
[Out] 1/(2*(1 - x)) + (3*Log[1 - x])/4 - Log[1 + x]/12 - Log[1 - x + x^2]/3
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3}{(-1+x)^2\left(1+x^3\right)}\,dx &= \int\left(\frac{1}{2(-1+x)^2}+\frac{3}{4(-1+x)}-\frac{1}{12(1+x)}+\frac{1-2x}{3\left(1-x+x^2\right)}\right)dx\\ &= \frac{1}{2(1-x)}+\frac{3}{4}\log(1-x)-\frac{1}{12}\log(1+x)+\frac{1}{3}\int\frac{1-2x}{1-x+x^2}\,dx\\ &= \frac{1}{2(1-x)}+\frac{3}{4}\log(1-x)-\frac{1}{12}\log(1+x)-\frac{1}{3}\log\left(1-x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0194789, size = 34, normalized size = 0.79

$$\frac{1}{12}\left(-\frac{6}{x-1}+9\log(x-1)-\log(x+1)-4\log\left((x-1)^2+x\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((-1 + x)^2*(1 + x^3)),x]
```

```
[Out] (-6/(-1 + x) + 9*Log[-1 + x] - Log[1 + x] - 4*Log[(-1 + x)^2 + x])/12
```

Maple [A] time = 0.006, size = 32, normalized size = 0.7

$$-\frac{\ln(1+x)}{12}-\frac{\ln\left(x^2-x+1\right)}{3}-\frac{1}{2x-2}+\frac{3\ln(-1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-1+x)^2/(x^3+1),x)
```

```
[Out] -1/12*ln(1+x)-1/3*ln(x^2-x+1)-1/2/(-1+x)+3/4*ln(-1+x)
```

Maxima [A] time = 1.4097, size = 42, normalized size = 0.98

$$-\frac{1}{2(x-1)}-\frac{1}{3}\log\left(x^2-x+1\right)-\frac{1}{12}\log(x+1)+\frac{3}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-1+x)^2/(x^3+1),x, algorithm="maxima")
```

```
[Out] -1/2/(x - 1) - 1/3*log(x^2 - x + 1) - 1/12*log(x + 1) + 3/4*log(x - 1)
```

Fricas [A] time = 1.84451, size = 124, normalized size = 2.88

$$-\frac{4(x-1)\log\left(x^2-x+1\right)+(x-1)\log(x+1)-9(x-1)\log(x-1)+6}{12(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-1+x)^2/(x^3+1),x, algorithm="fricas")
```

```
[Out] -1/12*(4*(x - 1)*log(x^2 - x + 1) + (x - 1)*log(x + 1) - 9*(x - 1)*log(x -
1) + 6)/(x - 1)
```

Sympy [A] time = 0.130973, size = 31, normalized size = 0.72

$$\frac{3\log(x-1)}{4}-\frac{\log(x+1)}{12}-\frac{\log\left(x^2-x+1\right)}{3}-\frac{1}{2x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-1+x)**2/(x**3+1),x)
```

```
[Out] 3*log(x - 1)/4 - log(x + 1)/12 - log(x**2 - x + 1)/3 - 1/(2*x - 2)
```

Giac [A] time = 1.10436, size = 49, normalized size = 1.14

$$-\frac{1}{2(x-1)}-\frac{1}{3}\log\left(\frac{1}{x-1}+\frac{1}{(x-1)^2}+1\right)-\frac{1}{12}\log\left(\left|-\frac{2}{x-1}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-1+x)^2/(x^3+1),x, algorithm="giac")
```

```
[Out] -1/2/(x - 1) - 1/3*log(1/(x - 1) + 1/(x - 1)^2 + 1) - 1/12*log(abs(-2/(x -
1) - 1))
```

3.18 $\int \frac{1}{1+x^4}\,dx$

Optimal. Leaf size=85

$$-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) - Lo
g[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])
```

Rubi [A] time = 0.0413213, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}+\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) - Lo
g[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+x^4}\,dx &= \frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx+\frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx\\
&= \frac{1}{4}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx-\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{4\sqrt{2}}-\frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{4\sqrt{2}}\\
&= -\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}x\right)}{2\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\right.}{2\sqrt{2}}\\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}}-\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.0175107, size = 64, normalized size = 0.75

$$\frac{-\log\left(x^2-\sqrt{2}x+1\right)+\log\left(x^2+\sqrt{2}x+1\right)-2\tan^{-1}\left(1-\sqrt{2}x\right)+2\tan^{-1}\left(\sqrt{2}x+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqrt[2]*x + x
^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Maple [A] time = 0., size = 58, normalized size = 0.7

$$\frac{\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{4}+\frac{\arctan\left(-1+x\sqrt{2}\right)\sqrt{2}}{4}+\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1),x)
```

```
[Out] 1/4*arctan(1+x*2^(1/2))*2^(1/2)+1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/8*2^(1/2
)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))
```

Maxima [A] time = 1.44, size = 97, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A] time = 1.86509, size = 302, normalized size = 3.55

$$-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 1
/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 1/8
*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Sympy [A] time = 0.136687, size = 73, normalized size = 0.86

$$-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8
+ sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

Giac [A] time = 1.07284, size = 97, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

3.19 $\int \frac{x^2}{1+x^4} \, dx$

Optimal. Leaf size=85

$$\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) + Lo
g[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])
```

Rubi [A] time = 0.0412805, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {297, 1162, 617, 204, 1165, 628}

$$\frac{\log\left(x^2-\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x^4),x]
```

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) + Lo
g[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) - Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])
```

Rule 297

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4
), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{1+x^4}\,dx &= -\left(\frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx\\
&= \frac{1}{4}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx+\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{4\sqrt{2}}+\frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{4\sqrt{2}}\\
&= \frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}-\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}x\right)}{2\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1+\sqrt{2}x\right)}{2\sqrt{2}}\\
&= -\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}-\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.0095085, size = 64, normalized size = 0.75

$$
\frac{\log\left(x^2-\sqrt{2}x+1\right)-\log\left(x^2+\sqrt{2}x+1\right)-2\tan^{-1}\left(1-\sqrt{2}x\right)+2\tan^{-1}\left(\sqrt{2}x+1\right)}{4\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x^4),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] + Log[1 - Sqrt[2]*x + x
^2] - Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Maple [A] time = 0.001, size = 58, normalized size = 0.7

$$\frac{\arctan\left(-1+x\sqrt{2}\right)\sqrt{2}}{4}+\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2-x\sqrt{2}}{1+x^2+x\sqrt{2}}\right)+\frac{\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^4+1),x)
```

```
[Out] 1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/8*2^(1/2)*ln((1+x^2-x*2^(1/2))/(1+x^2+x*
2^(1/2)))+1/4*arctan(1+x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.44976, size = 97, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A] time = 1.95137, size = 302, normalized size = 3.55

$$-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)-\frac{1}{2}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 1
/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - 1/8
*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Sympy [A] time = 0.137043, size = 73, normalized size = 0.86

$$\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8}-\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**4+1),x)
```

```
[Out] sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 - sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8 +
 sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

Giac [A] time = 1.10219, size = 97, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) - 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

3.20 $\int \frac{1}{1+x^2+x^4}\,dx$

Optimal. Leaf size=67

$$-\frac{1}{4}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) - Log[1 - x + x^2]/4 + Log[1 + x + x^2]/4
```

Rubi [A] time = 0.0364459, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{1}{4}\log\left(x^2-x+1\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) + ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[
3]) - Log[1 - x + x^2]/4 + Log[1 + x + x^2]/4
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x^2+x^4}\,dx &= \frac{1}{2}\int \frac{1-x}{1-x+x^2}\,dx+\frac{1}{2}\int \frac{1+x}{1+x+x^2}\,dx\\ &= \frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{-1+2x}{1-x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx+\frac{1}{4}\int \frac{1+2x}{1+x+x^2}\,dx\\ &= -\frac{1}{4}\log\left(1-x+x^2\right)+\frac{1}{4}\log\left(1+x+x^2\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{-3-\right.\\ &= -\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}+\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{4}\log\left(1-x+x^2\right)+\frac{1}{4}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [C] time = 0.055464, size = 73, normalized size = 1.09

$$\frac{i\left(\sqrt{1-i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}-i\right)x\right)-\sqrt{1+i\sqrt{3}}\tan^{-1}\left(\frac{1}{2}\left(\sqrt{3}+i\right)x\right)\right)}{\sqrt{6}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + x^2 + x^4)^(-1),x]
```

```
[Out] (I*(Sqrt[1 - I*Sqrt[3]]*ArcTan[((-I + Sqrt[3])*x)/2] - Sqrt[1 + I*Sqrt[3]]*
ArcTan[((I + Sqrt[3])*x)/2]))/Sqrt[6]
```

Maple [A] time = 0.003, size = 54, normalized size = 0.8

$$-\frac{\ln\left(x^2-x+1\right)}{4}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)+\frac{\ln\left(x^2+x+1\right)}{4}+\frac{\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+x^2+1),x)
```

```
[Out] -1/4*ln(x^2-x+1)+1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))+1/4*ln(x^2+x+1)+1/
6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.42515, size = 72, normalized size = 1.07

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

Fricas [A] time = 1.96776, size = 180, normalized size = 2.69

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

Sympy [A] time = 0.169291, size = 70, normalized size = 1.04

$$-\frac{\log\left(x^2-x+1\right)}{4}+\frac{\log\left(x^2+x+1\right)}{4}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}-\frac{\sqrt{3}}{3}\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+x**2+1),x)
```

```
[Out] -log(x**2 - x + 1)/4 + log(x**2 + x + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x/3 - s
qrt(3)/3)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3)/6
```

Giac [A] time = 1.09133, size = 72, normalized size = 1.07

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)-\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/6*sqrt(3)*arctan(1/3*sqrt(3)*
(2*x - 1)) + 1/4*log(x^2 + x + 1) - 1/4*log(x^2 - x + 1)
```

3.21 $\int (a+bx)^p \, dx$

Optimal. Leaf size=18

$$\frac{(a+bx)^{p+1}}{b(p+1)}$$

```
[Out] (a + b*x)^(1 + p)/(b*(1 + p))
```

Rubi [A] time = 0.0040883, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$\frac{(a+bx)^{p+1}}{b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^p,x]
```

```
[Out] (a + b*x)^(1 + p)/(b*(1 + p))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^p \, dx = \frac{(a+bx)^{1+p}}{b(1+p)}$$

Mathematica [A] time = 0.0097329, size = 17, normalized size = 0.94

$$\frac{(a+bx)^{p+1}}{bp+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^p,x]
```

```
[Out] (a + b*x)^(1 + p)/(b + b*p)
```

Maple [A] time = 0.002, size = 19, normalized size = 1.1

$$\frac{(bx+a)^{1+p}}{b\left(1+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^p,x)
```

```
[Out] (b*x+a)^(1+p)/b/(1+p)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.08196, size = 45, normalized size = 2.5

$$\frac{(bx+a)(bx+a)^{p}}{bp+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^p,x, algorithm="fricas")
```

```
[Out] (b*x + a)*(b*x + a)^p/(b*p + b)
```

Sympy [A] time = 0.056502, size = 20, normalized size = 1.11

$$\frac{\begin{cases} \frac{(a+bx)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(a+bx) & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**p,x)
```

```
[Out] Piecewise(((a + b*x)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x), True))/b
```

Giac [A] time = 1.08902, size = 24, normalized size = 1.33

$$\frac{(bx+a)^{p+1}}{b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^p,x, algorithm="giac")
```

```
[Out] (b*x + a)^(p + 1)/(b*(p + 1))
```

3.22 $\int x(a+bx)^p\,dx$

Optimal. Leaf size=39

$$\frac{(a+bx)^{p+2}}{b^2(p+2)}-\frac{a(a+bx)^{p+1}}{b^2(p+1)}$$

```
[Out] -((a*(a + b*x)^(1 + p))/(b^2*(1 + p))) + (a + b*x)^(2 + p)/(b^2*(2 + p))
```

Rubi [A] time = 0.0129684, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{(a+bx)^{p+2}}{b^2(p+2)}-\frac{a(a+bx)^{p+1}}{b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^p,x]
```

```
[Out] -((a*(a + b*x)^(1 + p))/(b^2*(1 + p))) + (a + b*x)^(2 + p)/(b^2*(2 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^p\,dx &= \int\left(-\frac{a(a+bx)^p}{b}+\frac{(a+bx)^{1+p}}{b}\right)dx\\ &= -\frac{a(a+bx)^{1+p}}{b^2(1+p)}+\frac{(a+bx)^{2+p}}{b^2(2+p)}\end{aligned}$$

Mathematica [A] time = 0.0188862, size = 33, normalized size = 0.85

$$\frac{(a+bx)^{p+1}(b(p+1)x-a)}{b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^p,x]
```

```
[Out] ((a + b*x)^(1 + p)*(-a + b*(1 + p)*x))/(b^2*(1 + p)*(2 + p))
```

Maple [A] time = 0.002, size = 36, normalized size = 0.9

$$-\frac{(bx+a)^{1+p}\left(-xpb-bx+a\right)}{b^2\left(p^2+3p+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^p,x)
```

```
[Out] -(b*x+a)^(1+p)*(-b*p*x-b*x+a)/b^2/(p^2+3*p+2)
```

Maxima [A] time = 0.965327, size = 57, normalized size = 1.46

$$\frac{\left(b^2\left(p+1\right)x^2+abpx-a^2\right)(bx+a)^p}{\left(p^2+3p+2\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^p,x, algorithm="maxima")
```

```
[Out] (b^2*(p + 1)*x^2 + a*b*p*x - a^2)*(b*x + a)^p/((p^2 + 3*p + 2)*b^2)
```

Fricas [A] time = 1.99131, size = 104, normalized size = 2.67

$$\frac{\left(abpx+\left(b^2p+b^2\right)x^2-a^2\right)(bx+a)^p}{b^2p^2+3b^2p+2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^p,x, algorithm="fricas")
```

```
[Out] (a*b*p*x + (b^2*p + b^2)*x^2 - a^2)*(b*x + a)^p/(b^2*p^2 + 3*b^2*p + 2*b^2)
```

Sympy [A] time = 0.605047, size = 201, normalized size = 5.15

$$\begin{cases} \frac{a^p x^2}{2} & \text{for } b = 0 \\ \frac{a \log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} + \frac{a}{ab^2+b^3x} + \frac{bx \log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for } p = -2 \\ -\frac{a \log\left(\frac{a}{b}+x\right)}{b^2} + \frac{x}{b} & \text{for } p = -1 \\ -\frac{a^2(a+bx)^p}{b^2p^2+3b^2p+2b^2} + \frac{abpx(a+bx)^p}{b^2p^2+3b^2p+2b^2} + \frac{b^2px^2(a+bx)^p}{b^2p^2+3b^2p+2b^2} + \frac{b^2x^2(a+bx)^p}{b^2p^2+3b^2p+2b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**p,x)
```

```
[Out] Piecewise((a**p*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) + a/(a
*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(p, -2)), (-a*log(a
/b + x)/b**2 + x/b, Eq(p, -1)), (-a**2*(a + b*x)**p/(b**2*p**2 + 3*b**2*p +
 2*b**2) + a*b*p*x*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2) + b**2*p*x*
*2*(a + b*x)**p/(b**2*p**2 + 3*b**2*p + 2*b**2) + b**2*x**2*(a + b*x)**p/(b
**2*p**2 + 3*b**2*p + 2*b**2), True))
```

Giac [A] time = 1.07666, size = 103, normalized size = 2.64

$$\frac{(bx+a)^p b^2px^2 + (bx+a)^p abpx + (bx+a)^p b^2x^2 - (bx+a)^p a^2}{b^2p^2 + 3\,b^2p + 2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^p,x, algorithm="giac")
```

```
[Out] ((b*x + a)^p*b^2*p*x^2 + (b*x + a)^p*a*b*p*x + (b*x + a)^p*b^2*x^2 - (b*x +
 a)^p*a^2)/(b^2*p^2 + 3*b^2*p + 2*b^2)
```

3.23 $\int x^2(a+bx)^p\,dx$

Optimal. Leaf size=60

$$\frac{a^2(a+bx)^{p+1}}{b^3(p+1)}-\frac{2a(a+bx)^{p+2}}{b^3(p+2)}+\frac{(a+bx)^{p+3}}{b^3(p+3)}$$

```
[Out] (a^2*(a + b*x)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x)^(2 + p))/(b^3*(2 + p
)) + (a + b*x)^(3 + p)/(b^3*(3 + p))
```

Rubi [A] time = 0.0213359, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2(a+bx)^{p+1}}{b^3(p+1)}-\frac{2a(a+bx)^{p+2}}{b^3(p+2)}+\frac{(a+bx)^{p+3}}{b^3(p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^p,x]
```

```
[Out] (a^2*(a + b*x)^(1 + p))/(b^3*(1 + p)) - (2*a*(a + b*x)^(2 + p))/(b^3*(2 + p
)) + (a + b*x)^(3 + p)/(b^3*(3 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+bx)^p\,dx &= \int\left(\frac{a^2(a+bx)^p}{b^2}-\frac{2a(a+bx)^{1+p}}{b^2}+\frac{(a+bx)^{2+p}}{b^2}\right)dx\\ &= \frac{a^2(a+bx)^{1+p}}{b^3(1+p)}-\frac{2a(a+bx)^{2+p}}{b^3(2+p)}+\frac{(a+bx)^{3+p}}{b^3(3+p)}\end{aligned}$$

Mathematica [A] time = 0.0267356, size = 57, normalized size = 0.95

$$\frac{(a+bx)^{p+1}\left(2a^2-2ab(p+1)x+b^2\left(p^2+3p+2\right)x^2\right)}{b^3(p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^p,x]
```

```
[Out] ((a + b*x)^(1 + p)*(2*a^2 - 2*a*b*(1 + p)*x + b^2*(2 + 3*p + p^2)*x^2))/(b^
3*(1 + p)*(2 + p)*(3 + p))
```

Maple [A] time = 0.005, size = 73, normalized size = 1.2

$$\frac{(bx+a)^{1+p}\left(b^2p^2x^2+3\,b^2px^2-2\,abpx+2\,x^2b^2-2\,axb+2\,a^2\right)}{b^3\left(p^3+6\,p^2+11\,p+6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^p,x)
```

```
[Out] (b*x+a)^(1+p)*(b^2*p^2*x^2+3*b^2*p*x^2-2*a*b*p*x+2*b^2*x^2-2*a*b*x+2*a^2)/b
^3/(p^3+6*p^2+11*p+6)
```

Maxima [A] time = 0.954102, size = 92, normalized size = 1.53

$$\frac{\left(\left(p^2+3\,p+2\right)b^3x^3+\left(p^2+p\right)ab^2x^2-2\,a^2bpx+2\,a^3\right)(bx+a)^p}{\left(p^3+6\,p^2+11\,p+6\right)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^p,x, algorithm="maxima")
```

```
[Out] ((p^2 + 3*p + 2)*b^3*x^3 + (p^2 + p)*a*b^2*x^2 - 2*a^2*b*p*x + 2*a^3)*(b*x
+ a)^p/((p^3 + 6*p^2 + 11*p + 6)*b^3)
```

Fricas [A] time = 2.10934, size = 188, normalized size = 3.13

$$-\frac{\left(2\,a^2bpx-\left(b^3p^2+3\,b^3p+2\,b^3\right)x^3-2\,a^3-\left(ab^2p^2+ab^2p\right)x^2\right)(bx+a)^p}{b^3p^3+6\,b^3p^2+11\,b^3p+6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^p,x, algorithm="fricas")
```

```
[Out] -(2*a^2*b*p*x - (b^3*p^2 + 3*b^3*p + 2*b^3)*x^3 - 2*a^3 - (a*b^2*p^2 + a*b^
2*p)*x^2)*(b*x + a)^p/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

Sympy [A] time = 1.18536, size = 597, normalized size = 9.95

$$\begin{cases} \frac{a^px^3}{3} \\ \frac{2a^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{a^2}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{4abx\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}+\frac{2b^2x^2\log\left(\frac{a}{b}+x\right)}{2a^2b^3+4ab^4x+2b^5x^2}-\frac{2b^2x^2}{2a^2b^3+4ab^4x+2b^5x^2} \\ -\frac{2a^2\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x}-\frac{2a^2}{ab^3+b^4x}-\frac{2abx\log\left(\frac{a}{b}+x\right)}{ab^3+b^4x}+\frac{b^2x^2}{ab^3+b^4x} \\ \frac{a^2\log\left(\frac{a}{b}+x\right)}{b^3}-\frac{ax}{b^2}+\frac{x^2}{2b} \\ \frac{2a^3(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3}-\frac{2a^2bpx(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3}+\frac{ab^2p^2x^2(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3}+\frac{ab^2px^2(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3}+\frac{b^3p^2x^3(a+bx)^p}{b^3p^3+6b^3p^2+11b^3p+6b^3}+\frac{3b^3px^3(\cdots}{b^3p^3+6b^3p^2+\cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**p,x)
```

```
[Out] Piecewise((a**p*x**3/3, Eq(b, 0)), (2*a**2*log(a/b + x)/(2*a**2*b**3 + 4*a*
b**4*x + 2*b**5*x**2) + a**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a
*b*x*log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*x**2*lo
g(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - 2*b**2*x**2/(2*a**2*b
**3 + 4*a*b**4*x + 2*b**5*x**2), Eq(p, -3)), (-2*a**2*log(a/b + x)/(a*b**3
+ b**4*x) - 2*a**2/(a*b**3 + b**4*x) - 2*a*b*x*log(a/b + x)/(a*b**3 + b**4*
x) + b**2*x**2/(a*b**3 + b**4*x), Eq(p, -2)), (a**2*log(a/b + x)/b**3 - a*x
/b**2 + x**2/(2*b), Eq(p, -1)), (2*a**3*(a + b*x)**p/(b**3*p**3 + 6*b**3*p*
*2 + 11*b**3*p + 6*b**3) - 2*a**2*b*p*x*(a + b*x)**p/(b**3*p**3 + 6*b**3*p*
*2 + 11*b**3*p + 6*b**3) + a*b**2*p**2*x**2*(a + b*x)**p/(b**3*p**3 + 6*b**
3*p**2 + 11*b**3*p + 6*b**3) + a*b**2*p*x**2*(a + b*x)**p/(b**3*p**3 + 6*b*
*3*p**2 + 11*b**3*p + 6*b**3) + b**3*p**2*x**3*(a + b*x)**p/(b**3*p**3 + 6*
b**3*p**2 + 11*b**3*p + 6*b**3) + 3*b**3*p*x**3*(a + b*x)**p/(b**3*p**3 + 6
*b**3*p**2 + 11*b**3*p + 6*b**3) + 2*b**3*x**3*(a + b*x)**p/(b**3*p**3 + 6*
```

```
b**3*p**2 + 11*b**3*p + 6*b**3), True))
```

Giac [B] time = 1.0708, size = 189, normalized size = 3.15

$$\frac{(bx+a)^p b^3p^2x^3 + (bx+a)^p ab^2p^2x^2 + 3\,(bx+a)^p b^3px^3 + (bx+a)^p ab^2px^2 + 2\,(bx+a)^p b^3x^3 - 2\,(bx+a)^p a^2bpx + 2\,(bx+}{b^3p^3 + 6\,b^3p^2 + 11\,b^3p + 6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^p,x, algorithm="giac")
```

```
[Out] ((b*x + a)^p*b^3*p^2*x^3 + (b*x + a)^p*a*b^2*p^2*x^2 + 3*(b*x + a)^p*b^3*p*
x^3 + (b*x + a)^p*a*b^2*p*x^2 + 2*(b*x + a)^p*b^3*x^3 - 2*(b*x + a)^p*a^2*b
*p*x + 2*(b*x + a)^p*a^3)/(b^3*p^3 + 6*b^3*p^2 + 11*b^3*p + 6*b^3)
```

3.24 $\int \frac{1}{a+bx}\,dx$

Optimal. Leaf size=10

$$\frac{\log(a+bx)}{b}$$

```
[Out] Log[a + b*x]/b
```

Rubi [A] time = 0.0014255, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{a+bx}\,dx = \frac{\log(a+bx)}{b}$$

Mathematica [A] time = 0.0010603, size = 10, normalized size = 1.

$$\frac{\log(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-1),x]
```

```
[Out] Log[a + b*x]/b
```

Maple [A] time = 0.002, size = 11, normalized size = 1.1

$$\frac{\ln\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a),x)
```

```
[Out] ln(b*x+a)/b
```

Maxima [A] time = 0.950428, size = 14, normalized size = 1.4

$$\frac{\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="maxima")
```

```
[Out] log(b*x + a)/b
```

Fricas [A] time = 1.92743, size = 22, normalized size = 2.2

$$\frac{\log\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="fricas")
```

```
[Out] log(b*x + a)/b
```

Sympy [A] time = 0.05728, size = 7, normalized size = 0.7

$$\frac{\log\left(a+bx\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x)
```

```
[Out] log(a + b*x)/b
```

Giac [A] time = 1.07847, size = 15, normalized size = 1.5

$$\frac{\log\left(|bx+a|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a),x, algorithm="giac")
```

```
[Out] log(abs(b*x + a))/b
```

3.25 $\int \frac{1}{(a+bx)^2} \, dx$

Optimal. Leaf size=12

$$-\frac{1}{b(a+bx)}$$

[Out] -(1/(b*(a + b*x)))

Rubi [A] time = 0.0016221, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {32}

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^(-2),x]

[Out] -(1/(b*(a + b*x)))

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(a+bx)^2} \, dx = -\frac{1}{b(a+bx)}$$

Mathematica [A] time = 0.0026913, size = 12, normalized size = 1.

$$-\frac{1}{b(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(-2),x]
```

```
[Out] -(1/(b*(a + b*x)))
```

Maple [A] time = 0.002, size = 13, normalized size = 1.1

$$-\frac{1}{b(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^2,x)
```

```
[Out] -1/b/(b*x+a)
```

Maxima [A] time = 0.930961, size = 16, normalized size = 1.33

$$-\frac{1}{(bx+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/((b*x + a)*b)
```

Fricas [A] time = 1.99723, size = 24, normalized size = 2.

$$-\frac{1}{b^2x+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/(b^2*x + a*b)
```

Sympy [A] time = 0.283601, size = 10, normalized size = 0.83

$$-\frac{1}{ab + b^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**2,x)
```

```
[Out] -1/(a*b + b**2*x)
```

Giac [A] time = 1.08407, size = 16, normalized size = 1.33

$$-\frac{1}{(bx + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/((b*x + a)*b)
```

3.26 $\int \frac{x}{a+bx} \, dx$

Optimal. Leaf size=18

$$\frac{x}{b} - \frac{a \log(a + bx)}{b^2}$$

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Rubi [A] time = 0.0099383, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{x}{b} - \frac{a \log(a + bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + b*x),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{a + bx} \, dx &= \int \left(\frac{1}{b} - \frac{a}{b(a + bx)} \right) dx \\ &= \frac{x}{b} - \frac{a \log(a + bx)}{b^2} \end{aligned}$$

Mathematica [A] time = 0.0023684, size = 18, normalized size = 1.

$$\frac{x}{b} - \frac{a \log(a + bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + b*x),x]
```

```
[Out] x/b - (a*Log[a + b*x])/b^2
```

Maple [A] time = 0.002, size = 19, normalized size = 1.1

$$\frac{x}{b} - \frac{a \ln (bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a),x)
```

```
[Out] x/b-a*ln(b*x+a)/b^2
```

Maxima [A] time = 0.950947, size = 24, normalized size = 1.33

$$\frac{x}{b} - \frac{a \log (bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="maxima")
```

```
[Out] x/b - a*log(b*x + a)/b^2
```

Fricas [A] time = 1.88243, size = 38, normalized size = 2.11

$$\frac{bx - a \log (bx + a)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x - a*log(b*x + a))/b^2
```

Sympy [A] time = 0.260511, size = 14, normalized size = 0.78

$$-\frac{a\log\left(a+bx\right)}{b^2}+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x)
```

```
[Out] -a*log(a + b*x)/b**2 + x/b
```

Giac [A] time = 1.08058, size = 26, normalized size = 1.44

$$\frac{x}{b}-\frac{a\log\left(|bx+a|\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a),x, algorithm="giac")
```

```
[Out] x/b - a*log(abs(b*x + a))/b^2
```

3.27 $\int \frac{x^2}{a+bx} \, dx$

Optimal. Leaf size=31

$$\frac{a^2 \log(a+bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rubi [A] time = 0.015678, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{a^2 \log(a+bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + b*x),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{a+bx} \, dx &= \int \left(-\frac{a}{b^2} + \frac{x}{b} + \frac{a^2}{b^2(a+bx)}\right) dx \\
&= -\frac{ax}{b^2} + \frac{x^2}{2b} + \frac{a^2 \log(a+bx)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.0032206, size = 31, normalized size = 1.

$$\frac{a^2 \log(a+bx)}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + b*x),x]
```

```
[Out] -((a*x)/b^2) + x^2/(2*b) + (a^2*Log[a + b*x])/b^3
```

Maple [A] time = 0.002, size = 30, normalized size = 1.

$$-\frac{ax}{b^2} + \frac{x^2}{2\,b} + \frac{a^2 \ln(bx+a)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a),x)
```

```
[Out] -a*x/b^2+1/2*x^2/b+a^2*ln(b*x+a)/b^3
```

Maxima [A] time = 0.94316, size = 39, normalized size = 1.26

$$\frac{a^2 \log(bx+a)}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] a^2*log(b*x + a)/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

Fricas [A] time = 2.01677, size = 68, normalized size = 2.19

$$\frac{b^2x^2 - 2\,abx + 2\,a^2 \log(bx+a)}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*x^2 - 2*a*b*x + 2*a^2*log(b*x + a))/b^3
```

Sympy [A] time = 0.267409, size = 26, normalized size = 0.84

$$\frac{a^2 \log{(a + bx)}}{b^3} - \frac{ax}{b^2} + \frac{x^2}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a),x)
```

```
[Out] a**2*log(a + b*x)/b**3 - a*x/b**2 + x**2/(2*b)
```

Giac [A] time = 1.07159, size = 41, normalized size = 1.32

$$\frac{a^2 \log{(|bx + a|)}}{b^3} + \frac{bx^2 - 2\,ax}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] a^2*log(abs(b*x + a))/b^3 + 1/2*(b*x^2 - 2*a*x)/b^2
```

3.28 $\int \frac{1}{x(a+bx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Rubi [A] time = 0.0035481, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {36, 29, 31}

$$\frac{\log(x)}{a}-\frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(a + b*x)),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x(a+bx)}\,dx = \frac{\int \frac{1}{x}\,dx}{a} - \frac{b\int \frac{1}{a+bx}\,dx}{a}$$
$$= \frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

Mathematica [A] time = 0.0040577, size = 18, normalized size = 1.

$$\frac{\log(x)}{a} - \frac{\log(a+bx)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(a + b*x)),x]
```

```
[Out] Log[x]/a - Log[a + b*x]/a
```

Maple [A] time = 0.005, size = 19, normalized size = 1.1

$$\frac{\ln(x)}{a} - \frac{\ln(bx+a)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a),x)
```

```
[Out] ln(x)/a-ln(b*x+a)/a
```

Maxima [A] time = 0.95042, size = 24, normalized size = 1.33

$$-\frac{\log(bx+a)}{a} + \frac{\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] -log(b*x + a)/a + log(x)/a
```

Fricas [A] time = 1.99473, size = 38, normalized size = 2.11

$$-\frac{\log(bx+a)-\log(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] -(log(b*x + a) - log(x))/a
```

Sympy [A] time = 0.131829, size = 10, normalized size = 0.56

$$\frac{\log(x)-\log\left(\frac{a}{b}+x\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x)
```

```
[Out] (log(x) - log(a/b + x))/a
```

Giac [A] time = 1.09712, size = 27, normalized size = 1.5

$$-\frac{\log(|bx+a|)}{a}+\frac{\log(|x|)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a),x, algorithm="giac")
```

```
[Out] -log(abs(b*x + a))/a + log(abs(x))/a
```

3.29 $\int \frac{1}{x^2(a+bx)}\,dx$

Optimal. Leaf size=28

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rubi [A] time = 0.012942, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)),x]

[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)}\,dx &= \int \left(\frac{1}{ax^2}-\frac{b}{a^2x}+\frac{b^2}{a^2(a+bx)}\right)dx\\ &= -\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.0041583, size = 28, normalized size = 1.

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)),x]
```

```
[Out] -(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Maple [A] time = 0.009, size = 29, normalized size = 1.

$$-\frac{1}{ax}-\frac{b\ln\left(x\right)}{a^{2}}+\frac{b\ln\left(bx+a\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a),x)
```

```
[Out] -1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Maxima [A] time = 0.958913, size = 38, normalized size = 1.36

$$\frac{b\log\left(bx+a\right)}{a^{2}}-\frac{b\log\left(x\right)}{a^{2}}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="maxima")
```

```
[Out] b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

Fricas [A] time = 1.99516, size = 61, normalized size = 2.18

$$\frac{bx\log\left(bx+a\right)-bx\log\left(x\right)-a}{a^{2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] (b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [A] time = 0.325009, size = 19, normalized size = 0.68

$$-\frac{1}{ax}+\frac{b\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a),x)
```

```
[Out] -1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

Giac [A] time = 1.07897, size = 41, normalized size = 1.46

$$\frac{b\log\left(|bx+a|\right)}{a^2}-\frac{b\log\left(|x|\right)}{a^2}-\frac{1}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a),x, algorithm="giac")
```

```
[Out] b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

3.30 $\int \frac{1}{x^2(a+bx)^2} dx$

Optimal. Leaf size=42

$$-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{1}{a^2x}$$

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^
3
```

Rubi [A] time = 0.0230598, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {44}

$$-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}-\frac{1}{a^2x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*x)^2),x]
```

```
[Out] -(1/(a^2*x)) - b/(a^2*(a + b*x)) - (2*b*Log[x])/a^3 + (2*b*Log[a + b*x])/a^
3
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2(a+bx)^2} dx &= \int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right) dx\\ &= -\frac{1}{a^2x}-\frac{b}{a^2(a+bx)}-\frac{2b\log(x)}{a^3}+\frac{2b\log(a+bx)}{a^3}\end{aligned}$$

Mathematica [A] time = 0.0382501, size = 35, normalized size = 0.83

$$-\frac{a\left(\frac{b}{a+bx}+\frac{1}{x}\right)-2b\log(a+bx)+2b\log(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*x)^2),x]
```

```
[Out] -((a*(x^(-1) + b/(a + b*x)) + 2*b*Log[x] - 2*b*Log[a + b*x])/a^3)
```

Maple [A] time = 0.01, size = 43, normalized size = 1.

$$-\frac{1}{a^2x}-\frac{b}{a^2\left(bx+a\right)}-2\,\frac{b\ln\left(x\right)}{a^3}+2\,\frac{b\ln\left(bx+a\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^2,x)
```

```
[Out] -1/a^2/x-b/a^2/(b*x+a)-2*b*ln(x)/a^3+2*b*ln(b*x+a)/a^3
```

Maxima [A] time = 0.931402, size = 61, normalized size = 1.45

$$-\frac{2\,bx+a}{a^2bx^2+a^3x}+\frac{2\,b\log\left(bx+a\right)}{a^3}-\frac{2\,b\log\left(x\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*b*x + a)/(a^2*b*x^2 + a^3*x) + 2*b*log(b*x + a)/a^3 - 2*b*log(x)/a^3
```

Fricas [A] time = 1.84507, size = 138, normalized size = 3.29

$$-\frac{2\,abx+a^2-2\left(b^2x^2+abx\right)\log\left(bx+a\right)+2\left(b^2x^2+abx\right)\log\left(x\right)}{a^3bx^2+a^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*a*b*x + a^2 - 2*(b^2*x^2 + a*b*x)*log(b*x + a) + 2*(b^2*x^2 + a*b*x)*lo
g(x))/(a^3*b*x^2 + a^4*x)
```

Sympy [A] time = 0.404427, size = 36, normalized size = 0.86

$$-\frac{a+2bx}{a^3x+a^2bx^2}+\frac{2b\left(-\log{(x)}+\log{\left(\frac{a}{b}+x\right)}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**2,x)
```

```
[Out] -(a + 2*b*x)/(a**3*x + a**2*b*x**2) + 2*b*(-log(x) + log(a/b + x))/a**3
```

Giac [A] time = 1.06443, size = 70, normalized size = 1.67

$$-\frac{2\,b\log\left(\left|-\frac{a}{bx+a}+1\right|\right)}{a^3}-\frac{b}{(bx+a)a^2}+\frac{b}{a^3\left(\frac{a}{bx+a}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -2*b*log(abs(-a/(b*x + a) + 1))/a^3 - b/((b*x + a)*a^2) + b/(a^3*(a/(b*x +
a) - 1))
```

3.31 $\int \frac{1}{c^2+x^2} \, dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

```
[Out] ArcTan[x/c]/c
```

Rubi [A] time = 0.0024157, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {203}

$$\frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/c]/c
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{c^2+x^2} \, dx = \frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

Mathematica [A] time = 0.0023078, size = 10, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/c]/c
```

Maple [A] time = 0.005, size = 11, normalized size = 1.1

$$\frac{1}{c}\arctan\left(\frac{x}{c}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2+x^2),x)
```

```
[Out] arctan(x/c)/c
```

Maxima [A] time = 1.4306, size = 14, normalized size = 1.4

$$\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2+x^2),x, algorithm="maxima")
```

```
[Out] arctan(x/c)/c
```

Fricas [A] time = 1.87814, size = 20, normalized size = 2.

$$\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2+x^2),x, algorithm="fricas")
```

```
[Out] arctan(x/c)/c
```

Sympy [C] time = 0.104491, size = 20, normalized size = 2.

$$\frac{-\frac{i\log\left(-ic+x\right)}{2}+\frac{i\log\left(ic+x\right)}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2+x**2),x)
```

```
[Out] (-I*log(-I*c + x)/2 + I*log(I*c + x)/2)/c
```

Giac [A] time = 1.06632, size = 14, normalized size = 1.4

$$\frac{\arctan\left(\frac{x}{c}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2+x^2),x, algorithm="giac")
```

```
[Out] arctan(x/c)/c
```

3.32 $\int \frac{1}{c^2-x^2}\,dx$

Optimal. Leaf size=10

$$\frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

```
[Out] ArcTanh[x/c]/c
```

Rubi [A] time = 0.0028164, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {206}

$$\frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(c^2 - x^2)^(-1),x]
```

```
[Out] ArcTanh[x/c]/c
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{c^2-x^2}\,dx=\frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

Mathematica [A] time = 0.0025066, size = 10, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{x}{c}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c^2 - x^2)^(-1),x]
```

```
[Out] ArcTanh[x/c]/c
```

Maple [B] time = 0.004, size = 22, normalized size = 2.2

$$-\frac{\ln\left(-c+x\right)}{2\,c}+\frac{\ln\left(c+x\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c^2-x^2),x)
```

```
[Out] -1/2/c*ln(-c+x)+1/2/c*ln(c+x)
```

Maxima [A] time = 0.946434, size = 28, normalized size = 2.8

$$\frac{\log\left(c+x\right)}{2\,c}-\frac{\log\left(-c+x\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2-x^2),x, algorithm="maxima")
```

```
[Out] 1/2*log(c + x)/c - 1/2*log(-c + x)/c
```

Fricas [A] time = 1.90468, size = 46, normalized size = 4.6

$$\frac{\log\left(c+x\right)-\log\left(-c+x\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2-x^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(c + x) - log(-c + x))/c
```

Sympy [B] time = 0.112406, size = 15, normalized size = 1.5

$$-\frac{\frac{\log(-c+x)}{2}-\frac{\log(c+x)}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c**2-x**2),x)
```

```
[Out] -(log(-c + x)/2 - log(c + x)/2)/c
```

Giac [B] time = 1.07247, size = 31, normalized size = 3.1

$$\frac{\log(|c+x|)}{2c}-\frac{\log(|-c+x|)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c^2-x^2),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(c + x))/c - 1/2*log(abs(-c + x))/c
```

3.33 $\int \frac{1}{-1+2x^3}\,dx$

Optimal. Leaf size=78

$$-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2}x+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{2}x+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

```
[Out] -(ArcTan[(1 + 2*2^(1/3)*x)/Sqrt[3]]/(2^(1/3)*Sqrt[3])) + Log[1 - 2^(1/3)*x]
/(3*2^(1/3)) - Log[1 + 2^(1/3)*x + 2^(2/3)*x^2]/(6*2^(1/3))
```

Rubi [A] time = 0.0440628, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2}x+1\right)}{6\sqrt[3]{2}}+\frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{2}x+1}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(1 + 2*2^(1/3)*x)/Sqrt[3]]/(2^(1/3)*Sqrt[3])) + Log[1 - 2^(1/3)*x]
/(3*2^(1/3)) - Log[1 + 2^(1/3)*x + 2^(2/3)*x^2]/(6*2^(1/3))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-1+2x^3}\,dx &= \frac{1}{3}\int \frac{1}{-1+\sqrt[3]{2}x}\,dx+\frac{1}{3}\int \frac{-2-\sqrt[3]{2}x}{1+\sqrt[3]{2}x+2^{2/3}x^2}\,dx\\
&= \frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-\frac{1}{2}\int \frac{1}{1+\sqrt[3]{2}x+2^{2/3}x^2}\,dx-\frac{\int \frac{\sqrt[3]{2}+2\ 2^{2/3}x}{1+\sqrt[3]{2}x+2^{2/3}x^2}\,dx}{6\sqrt[3]{2}}\\
&= \frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-\frac{\log\left(1+\sqrt[3]{2}x+2^{2/3}x^2\right)}{6\sqrt[3]{2}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2\sqrt[3]{2}x\right)}{\sqrt[3]{2}}\\
&= -\frac{\tan^{-1}\left(\frac{1+2\sqrt[3]{2}x}{\sqrt{3}}\right)}{\sqrt[3]{2}\sqrt{3}}+\frac{\log\left(1-\sqrt[3]{2}x\right)}{3\sqrt[3]{2}}-\frac{\log\left(1+\sqrt[3]{2}x+2^{2/3}x^2\right)}{6\sqrt[3]{2}}
\end{aligned}$$

Mathematica [A] time = 0.0221478, size = 66, normalized size = 0.85

$$-\frac{\log\left(2^{2/3}x^2+\sqrt[3]{2}x+1\right)-2\log\left(1-\sqrt[3]{2}x\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\sqrt[3]{2}x+1}{\sqrt{3}}\right)}{6\sqrt[3]{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*x^3)^(-1),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(1 + 2*2^(1/3)*x)/Sqrt[3]] - 2*Log[1 - 2^(1/3)*x] + Log[
1 + 2^(1/3)*x + 2^(2/3)*x^2])/(6*2^(1/3))
```

Maple [A] time = 0.003, size = 58, normalized size = 0.7

$$\frac{2^{\frac{2}{3}}}{6}\ln\left(x-\frac{2^{\frac{2}{3}}}{2}\right)-\frac{2^{\frac{2}{3}}}{12}\ln\left(x^2+\frac{2^{\frac{2}{3}}x}{2}+\frac{\sqrt[3]{2}}{2}\right)-\frac{2^{\frac{2}{3}}\sqrt{3}}{6}\arctan\left(\frac{\left(1+2\sqrt[3]{2}x\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^3-1),x)
```

```
[Out] 1/6*2^(2/3)*ln(x-1/2*2^(2/3))-1/12*2^(2/3)*ln(x^2+1/2*2^(2/3)*x+1/2*2^(1/3)
)-1/6*arctan(1/3*(1+2*2^(1/3)*x)*3^(1/2))*2^(2/3)*3^(1/2)
```

Maxima [A] time = 1.45058, size = 89, normalized size = 1.14

$$-\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2\cdot 2^{\frac{2}{3}}x+2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2^{\frac{2}{3}}x^2+2^{\frac{1}{3}}x+1\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(\frac{1}{2}\cdot 2^{\frac{2}{3}}\left(2^{\frac{1}{3}}x-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^3-1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(2/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2*2^(2/3)*x + 2^(1/3))) -
1/12*2^(2/3)*log(2^(2/3)*x^2 + 2^(1/3)*x + 1) + 1/6*2^(2/3)*log(1/2*2^(2/3)
*(2^(1/3)*x - 1))
```

Fricas [A] time = 1.93149, size = 209, normalized size = 2.68

$$-\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{6}2^{\frac{1}{6}}\left(2\cdot 2^{\frac{2}{3}}x+2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{2}{3}}\log\left(2x^2+2^{\frac{2}{3}}x+2^{\frac{1}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{2}{3}}\log\left(2x-2^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^3-1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(6)*2^(1/6)*arctan(1/6*sqrt(6)*2^(1/6)*(2*2^(2/3)*x + 2^(1/3))) -
1/12*2^(2/3)*log(2*x^2 + 2^(2/3)*x + 2^(1/3)) + 1/6*2^(2/3)*log(2*x - 2^(2/
3))
```

Sympy [A] time = 0.308794, size = 78, normalized size = 1.

$$\frac{2^{\frac{2}{3}}\log\left(x-\frac{2^{\frac{2}{3}}}{2}\right)}{6}-\frac{2^{\frac{2}{3}}\log\left(x^2+\frac{2^{\frac{2}{3}}x}{2}+\frac{\sqrt[3]{2}}{2}\right)}{12}-\frac{2^{\frac{2}{3}}\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt[3]{2}\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**3-1),x)
```

```
[Out] 2**(2/3)*log(x - 2**(2/3)/2)/6 - 2**(2/3)*log(x**2 + 2**(2/3)*x/2 + 2**(1/3
)/2)/12 - 2**(2/3)*sqrt(3)*atan(2*2**(1/3)*sqrt(3)*x/3 + sqrt(3)/3)/6
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^3-1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.34 $\int \frac{1}{-2+x^3} \, dx$

Optimal. Leaf size=74

$$-\frac{\log \left(x^2+\sqrt[3]{2} x+2^{2/3}\right)}{6\ 2^{2/3}}+\frac{\log \left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\tan ^{-1}\left(\frac{2^{2/3} x+1}{\sqrt{3}}\right)}{2^{2/3} \sqrt{3}}$$

```
[Out] -(ArcTan[(1 + 2^(2/3)*x)/Sqrt[3]]/(2^(2/3)*Sqrt[3])) + Log[2^(1/3) - x]/(3*
2^(2/3)) - Log[2^(2/3) + 2^(1/3)*x + x^2]/(6*2^(2/3))
```

Rubi [A] time = 0.034405, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log \left(x^2+\sqrt[3]{2} x+2^{2/3}\right)}{6\ 2^{2/3}}+\frac{\log \left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\tan ^{-1}\left(\frac{2^{2/3} x+1}{\sqrt{3}}\right)}{2^{2/3} \sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(1 + 2^(2/3)*x)/Sqrt[3]]/(2^(2/3)*Sqrt[3])) + Log[2^(1/3) - x]/(3*
2^(2/3)) - Log[2^(2/3) + 2^(1/3)*x + x^2]/(6*2^(2/3))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{-2+x^3}\,dx &= \frac{\int \frac{1}{-\sqrt[3]{2}+x}\,dx}{3\ 2^{2/3}}+\frac{\int \frac{-2\sqrt[3]{2}-x}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx}{3\ 2^{2/3}}\\
&= \frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\int \frac{\sqrt[3]{2}+2x}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx}{6\ 2^{2/3}}-\frac{\int \frac{1}{2^{2/3}+\sqrt[3]{2}x+x^2}\,dx}{2\sqrt[3]{2}}\\
&= \frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\log\left(2^{2/3}+\sqrt[3]{2}x+x^2\right)}{6\ 2^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+2^{2/3}x\right)}{2^{2/3}}\\
&= -\frac{\tan^{-1}\left(\frac{1+2^{2/3}x}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}}+\frac{\log\left(\sqrt[3]{2}-x\right)}{3\ 2^{2/3}}-\frac{\log\left(2^{2/3}+\sqrt[3]{2}x+x^2\right)}{6\ 2^{2/3}}
\end{aligned}
$$

Mathematica [A] time = 0.0177549, size = 65, normalized size = 0.88

$$
-\frac{\log\left(\sqrt[3]{2}x^2+2^{2/3}x+2\right)-2\log\left(2-2^{2/3}x\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{2/3}x+1}{\sqrt{3}}\right)}{6\ 2^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^3)^(-1),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(1 + 2^(2/3)*x)/Sqrt[3]] - 2*Log[2 - 2^(2/3)*x] + Log[2
+ 2^(2/3)*x + 2^(1/3)*x^2])/(6*2^(2/3))
```

Maple [A] time = 0.004, size = 54, normalized size = 0.7

$$\frac{\sqrt[3]{2}\ln\left(x-\sqrt[3]{2}\right)}{6}-\frac{\ln\left(2^{\frac{2}{3}}+\sqrt[3]{2}x+x^{2}\right)\sqrt[3]{2}}{12}-\frac{\sqrt[3]{2}\sqrt{3}}{6}\arctan\left(\frac{\left(1+2^{\frac{2}{3}}x\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-2),x)
```

```
[Out] 1/6*2^(1/3)*ln(x-2^(1/3))-1/12*ln(2^(2/3)+2^(1/3)*x+x^2)*2^(1/3)-1/6*arctan
(1/3*(1+2^(2/3)*x)*3^(1/2))*2^(1/3)*3^(1/2)
```

Maxima [A] time = 1.4354, size = 76, normalized size = 1.03

$$-\frac{1}{6}\sqrt{3}2^{\frac{1}{3}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{2}{3}}\left(2x+2^{\frac{1}{3}}\right)\right)-\frac{1}{12}\cdot 2^{\frac{1}{3}}\log\left(x^{2}+2^{\frac{1}{3}}x+2^{\frac{2}{3}}\right)+\frac{1}{6}\cdot 2^{\frac{1}{3}}\log\left(x-2^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-2),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*2^(1/3)*arctan(1/6*sqrt(3)*2^(2/3)*(2*x + 2^(1/3))) - 1/12*2^(
1/3)*log(x^2 + 2^(1/3)*x + 2^(2/3)) + 1/6*2^(1/3)*log(x - 2^(1/3))
```

Fricas [A] time = 2.00709, size = 221, normalized size = 2.99

$$-\frac{1}{6}\cdot 4^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 4^{\frac{1}{6}}\left(4^{\frac{2}{3}}\sqrt{3}x+4^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{24}\cdot 4^{\frac{2}{3}}\log\left(2x^{2}+4^{\frac{2}{3}}x+2\cdot 4^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 4^{\frac{2}{3}}\log\left(2x-4^{\frac{2}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-2),x, algorithm="fricas")
```

```
[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*(4^(2/3)*sqrt(3)*x + 4^(1/3)*sqrt(3
))) - 1/24*4^(2/3)*log(2*x^2 + 4^(2/3)*x + 2*4^(1/3)) + 1/12*4^(2/3)*log(2*
x - 4^(2/3))
```

Sympy [A] time = 0.308081, size = 71, normalized size = 0.96

$$\frac{\sqrt[3]{2}\log\left(x-\sqrt[3]{2}\right)}{6}-\frac{\sqrt[3]{2}\log\left(x^2+\sqrt[3]{2}x+2^{\frac{2}{3}}\right)}{12}-\frac{\sqrt[3]{2}\sqrt{3}\operatorname{atan}\left(\frac{2^{\frac{2}{3}}\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-2),x)
```

```
[Out] 2**(1/3)*log(x - 2**(1/3))/6 - 2**(1/3)*log(x**2 + 2**(1/3)*x + 2**(2/3))/1
2 - 2**(1/3)*sqrt(3)*atan(2**(2/3)*sqrt(3)*x/3 + sqrt(3)/3)/6
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.35 $\int \frac{1}{-b+ax^3}\,dx$

Optimal. Leaf size=115

$$-\frac{\log\left(a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{6\sqrt[3]{a}b^{2/3}}+\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a}x+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}$$

```
[Out] -(ArcTan[(b^(1/3) + 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^(1/3)*b^(2/3
))) + Log[b^(1/3) - a^(1/3)*x]/(3*a^(1/3)*b^(2/3)) - Log[b^(2/3) + a^(1/3)*
b^(1/3)*x + a^(2/3)*x^2]/(6*a^(1/3)*b^(2/3))
```

Rubi [A] time = 0.066438, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)}{6\sqrt[3]{a}b^{2/3}}+\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\tan^{-1}\left(\frac{2\sqrt[3]{a}x+\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-b + a*x^3)^(-1),x]
```

```
[Out] -(ArcTan[(b^(1/3) + 2*a^(1/3)*x)/(Sqrt[3]*b^(1/3))]/(Sqrt[3]*a^(1/3)*b^(2/3
))) + Log[b^(1/3) - a^(1/3)*x]/(3*a^(1/3)*b^(2/3)) - Log[b^(2/3) + a^(1/3)*
b^(1/3)*x + a^(2/3)*x^2]/(6*a^(1/3)*b^(2/3))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{-b+ax^3}\,dx &= \frac{\int \frac{1}{-\sqrt[3]{b}+\sqrt[3]{a}x}\,dx}{3b^{2/3}}+\frac{\int \frac{-2\sqrt[3]{b}-\sqrt[3]{a}x}{b^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{3b^{2/3}}\\
&= \frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\int \frac{\sqrt[3]{a}\sqrt[3]{b}+2a^{2/3}x}{b^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{6\sqrt[3]{a}b^{2/3}}-\frac{\int \frac{1}{b^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2}\,dx}{2\sqrt[3]{b}}\\
&= \frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\log\left(b^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}+\frac{\mathrm{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1+\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}\right)}{\sqrt[3]{a}b^{2/3}}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{b}+2\sqrt[3]{a}x}{\sqrt{3}\sqrt[3]{b}}\right)}{\sqrt{3}\sqrt[3]{a}b^{2/3}}+\frac{\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\log\left(b^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+a^{2/3}x^2\right)}{6\sqrt[3]{a}b^{2/3}}
\end{aligned}
$$

Mathematica [A] time = 0.0293769, size = 89, normalized size = 0.77

$$-\frac{\log\left(a^{2/3}x^2+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}\right)-2\log\left(\sqrt[3]{b}-\sqrt[3]{a}x\right)+2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{a}x}{\sqrt[3]{b}}+1}{\sqrt{3}}\right)}{6\sqrt[3]{a}b^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-b + a*x^3)^(-1),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(1 + (2*a^(1/3)*x)/b^(1/3))/Sqrt[3]] - 2*Log[b^(1/3) - a
^(1/3)*x] + Log[b^(2/3) + a^(1/3)*b^(1/3)*x + a^(2/3)*x^2])/(6*a^(1/3)*b^(2
/3))
```

Maple [A] time = 0.003, size = 92, normalized size = 0.8

$$\frac{1}{3a}\ln\left(x-\sqrt[3]{\frac{b}{a}}\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}}-\frac{1}{6a}\ln\left(x^2+\sqrt[3]{\frac{b}{a}}x+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}}-\frac{\sqrt{3}}{3a}\arctan\left(\frac{\sqrt{3}}{3}\left(2x\frac{1}{\sqrt[3]{\frac{b}{a}}}+1\right)\right)\left(\frac{b}{a}\right)^{-\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x^3-b),x)
```

```
[Out] 1/3/a/(b/a)^(2/3)*ln(x-(b/a)^(1/3))-1/6/a/(b/a)^(2/3)*ln(x^2+(b/a)^(1/3)*x+
(b/a)^(2/3))-1/3/a/(b/a)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(b/a)^(1/3)*x+
1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3-b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.9372, size = 751, normalized size = 6.53

$$\left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}\log\left(\frac{2\,abx^3-3\left(ab^2\right)^{\frac{1}{3}}bx+b^2-3\sqrt{\frac{1}{3}}\left(2\,abx^2-\left(ab^2\right)^{\frac{2}{3}}x-\left(ab^2\right)^{\frac{1}{3}}b\right)\sqrt{-\frac{\left(ab^2\right)^{\frac{1}{3}}}{a}}}{ax^3-b}\right)-\left(ab^2\right)^{\frac{2}{3}}\log\left(abx^2+\left(ab^2\right)^{\frac{2}{3}}x+\left(ab^2\right)^{\frac{1}{3}}b\right)}{6\,ab^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3-b),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a*b^2)^(1/3)/a)*log((2*a*b*x^3 - 3*(a*b^2)^(1/
3)*b*x + b^2 - 3*sqrt(1/3)*(2*a*b*x^2 - (a*b^2)^(2/3)*x - (a*b^2)^(1/3)*b)*
sqrt(-(a*b^2)^(1/3)/a))/(a*x^3 - b)) - (a*b^2)^(2/3)*log(a*b*x^2 + (a*b^2)^
(2/3)*x + (a*b^2)^(1/3)*b) + 2*(a*b^2)^(2/3)*log(a*b*x - (a*b^2)^(2/3)))/(a
*b^2), -1/6*(6*sqrt(1/3)*a*b*sqrt((a*b^2)^(1/3)/a)*arctan(sqrt(1/3)*(2*(a*b
^2)^(2/3)*x + (a*b^2)^(1/3)*b)*sqrt((a*b^2)^(1/3)/a)/b^2) + (a*b^2)^(2/3)*l
og(a*b*x^2 + (a*b^2)^(2/3)*x + (a*b^2)^(1/3)*b) - 2*(a*b^2)^(2/3)*log(a*b*x
 - (a*b^2)^(2/3)))/(a*b^2)]
```

Sympy [A] time = 0.145288, size = 20, normalized size = 0.17

$$\operatorname{RootSum}\left(27t^3ab^2-1,\left(t\mapsto t\log\left(-3tb+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x**3-b),x)
```

```
[Out] RootSum(27*_t**3*a*b**2 - 1, Lambda(_t, _t*log(-3*_t*b + x)))
```

Giac [A] time = 1.10683, size = 140, normalized size = 1.22

$$\frac{\left(\frac{b}{a}\right)^{\frac{1}{3}}\log\left(\left|x-\left(\frac{b}{a}\right)^{\frac{1}{3}}\right|\right)}{3\,b}-\frac{\sqrt{3}\left(a^{2}b\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(\frac{b}{a}\right)^{\frac{1}{3}}\right)}{3\left(\frac{b}{a}\right)^{\frac{1}{3}}}\right)}{3\,ab}-\frac{\left(a^{2}b\right)^{\frac{1}{3}}\log\left(x^{2}+x\left(\frac{b}{a}\right)^{\frac{1}{3}}+\left(\frac{b}{a}\right)^{\frac{2}{3}}\right)}{6\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*x^3-b),x, algorithm="giac")
```

```
[Out] 1/3*(b/a)^(1/3)*log(abs(x - (b/a)^(1/3)))/b - 1/3*sqrt(3)*(a^2*b)^(1/3)*arc
tan(1/3*sqrt(3)*(2*x + (b/a)^(1/3))/(b/a)^(1/3))/(a*b) - 1/6*(a^2*b)^(1/3)*
log(x^2 + x*(b/a)^(1/3) + (b/a)^(2/3))/(a*b)
```

3.36 $\int \frac{1}{-2+x^4}\,dx$

Optimal. Leaf size=35

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\ 2^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\ 2^{3/4}}$$

```
[Out] -ArcTan[x/2^(1/4)]/(2*2^(3/4)) - ArcTanh[x/2^(1/4)]/(2*2^(3/4))
```

Rubi [A] time = 0.0107502, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {212, 206, 203}

$$-\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\ 2^{3/4}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^4)^(-1),x]
```

```
[Out] -ArcTan[x/2^(1/4)]/(2*2^(3/4)) - ArcTanh[x/2^(1/4)]/(2*2^(3/4))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{-2+x^4}\,dx = -\frac{\int \frac{1}{\sqrt{2}-x^2}\,dx}{2\sqrt{2}} - \frac{\int \frac{1}{\sqrt{2}+x^2}\,dx}{2\sqrt{2}}$$
$$= -\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\ 2^{3/4}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{2\ 2^{3/4}}$$

Mathematica [A] time = 0.0177323, size = 43, normalized size = 1.23

$$-\frac{-\log\left(2-2^{3/4}x\right)+\log\left(2^{3/4}x+2\right)+2\tan^{-1}\left(\frac{x}{\sqrt[4]{2}}\right)}{4\ 2^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^4)^(-1),x]
```

```
[Out] -(2*ArcTan[x/2^(1/4)] - Log[2 - 2^(3/4)*x] + Log[2 + 2^(3/4)*x])/(4*2^(3/4)
)
```

Maple [A] time = 0.002, size = 35, normalized size = 1.

$$-\frac{\sqrt[4]{2}}{4}\arctan\left(\frac{x2^{\frac{3}{4}}}{2}\right)-\frac{\sqrt[4]{2}}{8}\ln\left(\frac{x+\sqrt[4]{2}}{x-\sqrt[4]{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-2),x)
```

```
[Out] -1/4*arctan(1/2*x*2^(3/4))*2^(1/4)-1/8*2^(1/4)*ln((x+2^(1/4))/(x-2^(1/4)))
```

Maxima [A] time = 1.44758, size = 46, normalized size = 1.31

$$-\frac{1}{4}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}x\right)+\frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\frac{x-2^{\frac{1}{4}}}{x+2^{\frac{1}{4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2),x, algorithm="maxima")
```

```
[Out] -1/4*2^(1/4)*arctan(1/2*2^(3/4)*x) + 1/8*2^(1/4)*log((x - 2^(1/4))/(x + 2^(
1/4)))
```

Fricas [B] time = 2.07077, size = 201, normalized size = 5.74

$$\frac{1}{8}\cdot 8^{\frac{3}{4}}\arctan\left(\frac{1}{4}\cdot 8^{\frac{1}{4}}\sqrt{2}\sqrt{2\,x^2+2\,\sqrt{2}}-\frac{1}{2}\cdot 8^{\frac{1}{4}}x\right)-\frac{1}{32}\cdot 8^{\frac{3}{4}}\log\left(4\,x+8^{\frac{3}{4}}\right)+\frac{1}{32}\cdot 8^{\frac{3}{4}}\log\left(4\,x-8^{\frac{3}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2),x, algorithm="fricas")
```

```
[Out] 1/8*8^(3/4)*arctan(1/4*8^(1/4)*sqrt(2)*sqrt(2*x^2 + 2*sqrt(2)) - 1/2*8^(1/4
)*x) - 1/32*8^(3/4)*log(4*x + 8^(3/4)) + 1/32*8^(3/4)*log(4*x - 8^(3/4))
```

Sympy [A] time = 0.317221, size = 46, normalized size = 1.31

$$\frac{\sqrt[4]{2}\log\left(x-\sqrt[4]{2}\right)}{8}-\frac{\sqrt[4]{2}\log\left(x+\sqrt[4]{2}\right)}{8}-\frac{\sqrt[4]{2}\operatorname{atan}\left(\frac{2^{\frac{3}{4}}x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-2),x)
```

```
[Out] 2**(1/4)*log(x - 2**(1/4))/8 - 2**(1/4)*log(x + 2**(1/4))/8 - 2**(1/4)*atan
(2**(3/4)*x/2)/4
```

Giac [A] time = 1.11874, size = 53, normalized size = 1.51

$$-\frac{1}{4}\cdot 2^{\frac{1}{4}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{3}{4}}x\right)-\frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\left|x+2^{\frac{1}{4}}\right|\right)+\frac{1}{8}\cdot 2^{\frac{1}{4}}\log\left(\left|x-2^{\frac{1}{4}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2),x, algorithm="giac")
```

```
[Out] -1/4*2^(1/4)*arctan(1/2*2^(3/4)*x) - 1/8*2^(1/4)*log(abs(x + 2^(1/4))) + 1/
8*2^(1/4)*log(abs(x - 2^(1/4)))
```

3.37 $\int \frac{1}{-1+5x^4} dx$

Optimal. Leaf size=35

$$-\frac{\tan^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}$$

```
[Out] -ArcTan[5^(1/4)*x]/(2*5^(1/4)) - ArcTanh[5^(1/4)*x]/(2*5^(1/4))
```

Rubi [A] time = 0.0143085, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {212, 206, 203}

$$-\frac{\tan^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}-\frac{\tanh^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 5*x^4)^(-1),x]
```

```
[Out] -ArcTan[5^(1/4)*x]/(2*5^(1/4)) - ArcTanh[5^(1/4)*x]/(2*5^(1/4))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{-1+5x^4}\,dx = -\left(\frac{1}{2}\int \frac{1}{1-\sqrt{5}x^2}\,dx\right) - \frac{1}{2}\int \frac{1}{1+\sqrt{5}x^2}\,dx$$
$$= -\frac{\tan^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}} - \frac{\tanh^{-1}\left(\sqrt[4]{5}x\right)}{2\sqrt[4]{5}}$$

Mathematica [A] time = 0.0146024, size = 43, normalized size = 1.23

$$-\frac{-\log\left(1-\sqrt[4]{5}x\right)+\log\left(\sqrt[4]{5}x+1\right)+2\tan^{-1}\left(\sqrt[4]{5}x\right)}{4\sqrt[4]{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 5*x^4)^(-1),x]
```

```
[Out] -(2*ArcTan[5^(1/4)*x] - Log[1 - 5^(1/4)*x] + Log[1 + 5^(1/4)*x])/(4*5^(1/4)
)
```

Maple [A] time = 0.003, size = 36, normalized size = 1.

$$-\frac{\arctan\left(\sqrt[4]{5}x\right)5^{\frac{3}{4}}}{10} - \frac{5^{\frac{3}{4}}}{20}\ln\left(\left(x+\frac{5^{\frac{3}{4}}}{5}\right)\left(x-\frac{5^{\frac{3}{4}}}{5}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^4-1),x)
```

```
[Out] -1/10*arctan(5^(1/4)*x)*5^(3/4)-1/20*5^(3/4)*ln((x+1/5*5^(3/4))/(x-1/5*5^(3
/4)))
```

Maxima [A] time = 1.43936, size = 55, normalized size = 1.57

$$-\frac{1}{10}\cdot 5^{\frac{3}{4}}\arctan\left(5^{\frac{1}{4}}x\right)+\frac{1}{20}\cdot 5^{\frac{3}{4}}\log\left(\frac{\sqrt{5}x-5^{\frac{1}{4}}}{\sqrt{5}x+5^{\frac{1}{4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4-1),x, algorithm="maxima")
```

```
[Out] -1/10*5^(3/4)*arctan(5^(1/4)*x) + 1/20*5^(3/4)*log((sqrt(5)*x - 5^(1/4))/(s
qrt(5)*x + 5^(1/4)))
```

Fricas [B] time = 1.9271, size = 182, normalized size = 5.2

$$\frac{1}{5}\cdot 5^{\frac{3}{4}}\arctan\left(\frac{1}{5}\cdot 5^{\frac{3}{4}}\sqrt{5x^{2}+\sqrt{5}}-5^{\frac{1}{4}}x\right)-\frac{1}{20}\cdot 5^{\frac{3}{4}}\log\left(5x+5^{\frac{3}{4}}\right)+\frac{1}{20}\cdot 5^{\frac{3}{4}}\log\left(5x-5^{\frac{3}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4-1),x, algorithm="fricas")
```

```
[Out] 1/5*5^(3/4)*arctan(1/5*5^(3/4)*sqrt(5*x^2 + sqrt(5)) - 5^(1/4)*x) - 1/20*5^
(3/4)*log(5*x + 5^(3/4)) + 1/20*5^(3/4)*log(5*x - 5^(3/4))
```

Sympy [A] time = 0.323315, size = 48, normalized size = 1.37

$$\frac{5^{\frac{3}{4}}\log\left(x-\frac{5^{\frac{3}{4}}}{5}\right)}{20}-\frac{5^{\frac{3}{4}}\log\left(x+\frac{5^{\frac{3}{4}}}{5}\right)}{20}-\frac{5^{\frac{3}{4}}\operatorname{atan}\left(\sqrt[4]{5}x\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**4-1),x)
```

```
[Out] 5**(3/4)*log(x - 5**(3/4)/5)/20 - 5**(3/4)*log(x + 5**(3/4)/5)/20 - 5**(3/4
)*atan(5**(1/4)*x)/10
```

Giac [A] time = 1.11051, size = 53, normalized size = 1.51

$$-\frac{1}{10}\cdot 5^{\frac{3}{4}}\arctan\left(5\left(\frac{1}{5}\right)^{\frac{3}{4}}x\right)-\frac{1}{20}\cdot 5^{\frac{3}{4}}\log\left(\left|x+\left(\frac{1}{5}\right)^{\frac{1}{4}}\right|\right)+\frac{1}{20}\cdot 5^{\frac{3}{4}}\log\left(\left|x-\left(\frac{1}{5}\right)^{\frac{1}{4}}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^4-1),x, algorithm="giac")
```

```
[Out] -1/10*5^(3/4)*arctan(5*(1/5)^(3/4)*x) - 1/20*5^(3/4)*log(abs(x + (1/5)^(1/4
))) + 1/20*5^(3/4)*log(abs(x - (1/5)^(1/4)))
```

3.38 $\int \frac{1}{7+3x^4}\,dx$

Optimal. Leaf size=171

$$-\frac{\log\left(3x^2-\sqrt{2}3^{3/4}\sqrt[4]{7}x+\sqrt{21}\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}}+\frac{\log\left(3x^2+\sqrt{2}3^{3/4}\sqrt[4]{7}x+\sqrt{21}\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}}-\frac{\tan^{-1}\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{2\sqrt{2}\sqrt[4]{3}7^{3/4}}+\frac{\tan^{-1}\left(\sqrt[4]{\frac{3}{7}}\sqrt{2}x+1\right)}{2\sqrt{2}\sqrt[4]{3}7^{3/4}}$$

```
[Out] -ArcTan[1 - (3/7)^(1/4)*Sqrt[2]*x]/(2*Sqrt[2]*3^(1/4)*7^(3/4)) + ArcTan[1 +
 (3/7)^(1/4)*Sqrt[2]*x]/(2*Sqrt[2]*3^(1/4)*7^(3/4)) - Log[Sqrt[21] - Sqrt[2
]*3^(3/4)*7^(1/4)*x + 3*x^2]/(4*Sqrt[2]*3^(1/4)*7^(3/4)) + Log[Sqrt[21] + S
qrt[2]*3^(3/4)*7^(1/4)*x + 3*x^2]/(4*Sqrt[2]*3^(1/4)*7^(3/4))
```

Rubi [A] time = 0.108253, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {211, 1165, 628, 1162, 617, 204}

$$-\frac{\log\left(3x^2-\sqrt{2}3^{3/4}\sqrt[4]{7}x+\sqrt{21}\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}}+\frac{\log\left(3x^2+\sqrt{2}3^{3/4}\sqrt[4]{7}x+\sqrt{21}\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}}-\frac{\tan^{-1}\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{2\sqrt{2}\sqrt[4]{3}7^{3/4}}+\frac{\tan^{-1}\left(\sqrt[4]{\frac{3}{7}}\sqrt{2}x+1\right)}{2\sqrt{2}\sqrt[4]{3}7^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 3*x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - (3/7)^(1/4)*Sqrt[2]*x]/(2*Sqrt[2]*3^(1/4)*7^(3/4)) + ArcTan[1 +
 (3/7)^(1/4)*Sqrt[2]*x]/(2*Sqrt[2]*3^(1/4)*7^(3/4)) - Log[Sqrt[21] - Sqrt[2
]*3^(3/4)*7^(1/4)*x + 3*x^2]/(4*Sqrt[2]*3^(1/4)*7^(3/4)) + Log[Sqrt[21] + S
qrt[2]*3^(3/4)*7^(1/4)*x + 3*x^2]/(4*Sqrt[2]*3^(1/4)*7^(3/4))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
```

```
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{7+3x^4}\,dx = \frac{\int \frac{\sqrt{7}-\sqrt{3}x^2}{7+3x^4}\,dx}{2\sqrt{7}} + \frac{\int \frac{\sqrt{7}+\sqrt{3}x^2}{7+3x^4}\,dx}{2\sqrt{7}}$$

$$= -\frac{\int \frac{\sqrt{2}\sqrt[4]{\frac{7}{3}}+2x}{-\sqrt{\frac{7}{3}}-\sqrt{2}\sqrt[4]{\frac{7}{3}}x-x^2}\,dx}{4\sqrt{2}\sqrt[4]{3}7^{3/4}} - \frac{\int \frac{\sqrt{2}\sqrt[4]{\frac{7}{3}}-2x}{-\sqrt{\frac{7}{3}}+\sqrt{2}\sqrt[4]{\frac{7}{3}}x-x^2}\,dx}{4\sqrt{2}\sqrt[4]{3}7^{3/4}} + \frac{\int \frac{1}{\sqrt{\frac{7}{3}}-\sqrt{2}\sqrt[4]{\frac{7}{3}}x+x^2}\,dx}{4\sqrt{21}} + \frac{\int \frac{1}{\sqrt{\frac{7}{3}}+\sqrt{2}\sqrt[4]{\frac{7}{3}}x+x^2}\,dx}{4\sqrt{21}}$$

$$= -\frac{\log\left(\sqrt{21}-\sqrt{2}3^{3/4}\sqrt[4]{7}x+3x^2\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}} + \frac{\log\left(\sqrt{21}+\sqrt{2}3^{3/4}\sqrt[4]{7}x+3x^2\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}} + \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right.}{2\sqrt{2}\sqrt[4]{3}7^{3/4}}$$

$$= -\frac{\tan^{-1}\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{2\sqrt{2}\sqrt[4]{3}7^{3/4}} + \frac{\tan^{-1}\left(1+\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)}{2\sqrt{2}\sqrt[4]{3}7^{3/4}} - \frac{\log\left(\sqrt{21}-\sqrt{2}3^{3/4}\sqrt[4]{7}x+3x^2\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}} + \frac{\log\left(\sqrt{21}+\sqrt{2}3^{3/4}\sqrt[4]{\ }\right.}{4\sqrt{2}\sqrt[4]{3}7^{3/}}$$

Mathematica [A] time = 0.0481946, size = 120, normalized size = 0.7

$$\frac{-\log\left(\sqrt{21}x^2-\sqrt{2}\sqrt[4]{3}7^{3/4}x+7\right)+\log\left(\sqrt{21}x^2+\sqrt{2}\sqrt[4]{3}7^{3/4}x+7\right)-2\tan^{-1}\left(1-\sqrt[4]{\frac{3}{7}}\sqrt{2}x\right)+2\tan^{-1}\left(\sqrt[4]{\frac{3}{7}}\sqrt{2}x+1\right)}{4\sqrt{2}\sqrt[4]{3}7^{3/4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 3*x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - (3/7)^(1/4)*Sqrt[2]*x] + 2*ArcTan[1 + (3/7)^(1/4)*Sqrt[2]*x]
 - Log[7 - Sqrt[2]*3^(1/4)*7^(3/4)*x + Sqrt[21]*x^2] + Log[7 + Sqrt[2]*3^(1
/4)*7^(3/4)*x + Sqrt[21]*x^2])/(4*Sqrt[2]*3^(1/4)*7^(3/4))
```

Maple [A] time = 0.005, size = 111, normalized size = 0.7

$$\frac{\sqrt{3}\sqrt[4]{21}\sqrt{2}}{84}\arctan\left(\frac{\sqrt{2}\sqrt{3}21^{\frac{3}{4}}x}{21}+1\right)+\frac{\sqrt{3}\sqrt[4]{21}\sqrt{2}}{84}\arctan\left(\frac{\sqrt{2}\sqrt{3}21^{\frac{3}{4}}x}{21}-1\right)+\frac{\sqrt{3}\sqrt[4]{21}\sqrt{2}}{168}\ln\left(\left(x^2+\frac{\sqrt{3}\sqrt[4]{21}x\sqrt{2}}{3}+\frac{\sqrt{2}}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(3*x^4+7),x)
```

```
[Out] 1/84*3^(1/2)*21^(1/4)*2^(1/2)*arctan(1/21*2^(1/2)*3^(1/2)*21^(3/4)*x+1)+1/8
4*3^(1/2)*21^(1/4)*2^(1/2)*arctan(1/21*2^(1/2)*3^(1/2)*21^(3/4)*x-1)+1/168*
3^(1/2)*21^(1/4)*2^(1/2)*ln((x^2+1/3*3^(1/2)*21^(1/4)*x*2^(1/2)+1/3*21^(1/2
))/(x^2-1/3*3^(1/2)*21^(1/4)*x*2^(1/2)+1/3*21^(1/2)))
```

Maxima [A] time = 1.46404, size = 204, normalized size = 1.19

$$\frac{1}{84} \cdot 7^{\frac{1}{4}} 3^{\frac{3}{4}} \sqrt{2} \arctan\left(\frac{1}{42} \cdot 7^{\frac{3}{4}} 3^{\frac{3}{4}} \sqrt{2}\left(2\sqrt{3}x + 7^{\frac{1}{4}} 3^{\frac{1}{4}} \sqrt{2}\right)\right) + \frac{1}{84} \cdot 7^{\frac{1}{4}} 3^{\frac{3}{4}} \sqrt{2} \arctan\left(\frac{1}{42} \cdot 7^{\frac{3}{4}} 3^{\frac{3}{4}} \sqrt{2}\left(2\sqrt{3}x - 7^{\frac{1}{4}} 3^{\frac{1}{4}} \sqrt{2}\right)\right) + \frac{1}{168}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+7),x, algorithm="maxima")
```

```
[Out] 1/84*7^(1/4)*3^(3/4)*sqrt(2)*arctan(1/42*7^(3/4)*3^(3/4)*sqrt(2)*(2*sqrt(3)
*x + 7^(1/4)*3^(1/4)*sqrt(2))) + 1/84*7^(1/4)*3^(3/4)*sqrt(2)*arctan(1/42*7
^(3/4)*3^(3/4)*sqrt(2)*(2*sqrt(3)*x - 7^(1/4)*3^(1/4)*sqrt(2))) + 1/168*7^(
1/4)*3^(3/4)*sqrt(2)*log(sqrt(3)*x^2 + 7^(1/4)*3^(1/4)*sqrt(2)*x + sqrt(7))
 - 1/168*7^(1/4)*3^(3/4)*sqrt(2)*log(sqrt(3)*x^2 - 7^(1/4)*3^(1/4)*sqrt(2)*
x + sqrt(7))
```

Fricas [A] time = 1.98422, size = 621, normalized size = 3.63

$$-\frac{1}{2058} \cdot 1029^{\frac{3}{4}} \sqrt{2} \arctan\left(\frac{1}{147} \cdot 1029^{\frac{1}{4}} \sqrt{3}\sqrt{2}\sqrt{1029^{\frac{3}{4}} \sqrt{2}x + 147x^2 + 49\sqrt{21}} - \frac{1}{7} \cdot 1029^{\frac{1}{4}} \sqrt{2}x - 1\right) - \frac{1}{2058} \cdot 1029^{\frac{3}{4}} \sqrt{2} \text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+7),x, algorithm="fricas")
```

```
[Out] -1/2058*1029^(3/4)*sqrt(2)*arctan(1/147*1029^(1/4)*sqrt(3)*sqrt(2)*sqrt(102
9^(3/4)*sqrt(2)*x + 147*x^2 + 49*sqrt(21)) - 1/7*1029^(1/4)*sqrt(2)*x - 1)
- 1/2058*1029^(3/4)*sqrt(2)*arctan(1/147*1029^(1/4)*sqrt(3)*sqrt(2)*sqrt(-1
029^(3/4)*sqrt(2)*x + 147*x^2 + 49*sqrt(21)) - 1/7*1029^(1/4)*sqrt(2)*x + 1
) + 1/8232*1029^(3/4)*sqrt(2)*log(1029^(3/4)*sqrt(2)*x + 147*x^2 + 49*sqrt(
21)) - 1/8232*1029^(3/4)*sqrt(2)*log(-1029^(3/4)*sqrt(2)*x + 147*x^2 + 49*s
qrt(21))
```

Sympy [A] time = 0.397105, size = 151, normalized size = 0.88

$$-\frac{\sqrt[4]{189}\sqrt{2}\log\left(x^2-\frac{\sqrt[4]{189}\sqrt{2}x}{3}+\frac{\sqrt{21}}{3}\right)}{168}+\frac{\sqrt[4]{189}\sqrt{2}\log\left(x^2+\frac{\sqrt[4]{189}\sqrt{2}x}{3}+\frac{\sqrt{21}}{3}\right)}{168}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt[4]{7}\,\mathrm{atan}\left(\frac{\sqrt{2}\sqrt[4]{3}\cdot 7^{\frac{3}{4}}x}{7}-1\right)}{84}+\frac{\sqrt{2}\cdot 3^{\frac{3}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x**4+7),x)
```

```
[Out] -189**(1/4)*sqrt(2)*log(x**2 - 189**(1/4)*sqrt(2)*x/3 + sqrt(21)/3)/168 + 1
89**(1/4)*sqrt(2)*log(x**2 + 189**(1/4)*sqrt(2)*x/3 + sqrt(21)/3)/168 + sqr
t(2)*3**(3/4)*7**(1/4)*atan(sqrt(2)*3**(1/4)*7**(3/4)*x/7 - 1)/84 + sqrt(2)
*3**(3/4)*7**(1/4)*atan(sqrt(2)*3**(1/4)*7**(3/4)*x/7 + 1)/84
```

Giac [A] time = 1.11397, size = 128, normalized size = 0.75

$$\frac{1}{84}\cdot 756^{\frac{1}{4}}\arctan\left(\frac{3}{14}\left(\frac{7}{3}\right)^{\frac{3}{4}}\sqrt{2}\left(2x+\left(\frac{7}{3}\right)^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{84}\cdot 756^{\frac{1}{4}}\arctan\left(\frac{3}{14}\left(\frac{7}{3}\right)^{\frac{3}{4}}\sqrt{2}\left(2x-\left(\frac{7}{3}\right)^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{168}\cdot 756^{\frac{1}{4}}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3*x^4+7),x, algorithm="giac")
```

```
[Out] 1/84*756^(1/4)*arctan(3/14*(7/3)^(3/4)*sqrt(2)*(2*x + (7/3)^(1/4)*sqrt(2)))
 + 1/84*756^(1/4)*arctan(3/14*(7/3)^(3/4)*sqrt(2)*(2*x - (7/3)^(1/4)*sqrt(2
))) + 1/168*756^(1/4)*log(x^2 + (7/3)^(1/4)*sqrt(2)*x + sqrt(7/3)) - 1/168*
756^(1/4)*log(x^2 - (7/3)^(1/4)*sqrt(2)*x + sqrt(7/3))
```

3.39 $\int \frac{1}{-1+3x^2+x^4}\,dx$

Optimal. Leaf size=73

$$-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)-\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)$$

```
[Out] -(Sqrt[2/(13*(3 + Sqrt[13]))]*ArcTan[Sqrt[2/(3 + Sqrt[13])]*x]) - Sqrt[(3 +
 Sqrt[13])/26]*ArcTanh[Sqrt[2/(-3 + Sqrt[13])]*x]
```

Rubi [A] time = 0.0737312, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1093, 207, 203}

$$-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)-\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 3*x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[2/(13*(3 + Sqrt[13]))]*ArcTan[Sqrt[2/(3 + Sqrt[13])]*x]) - Sqrt[(3 +
 Sqrt[13])/26]*ArcTanh[Sqrt[2/(-3 + Sqrt[13])]*x]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{-1+3x^2+x^4}\,dx = \frac{\int \frac{1}{\frac{3}{2}-\frac{\sqrt{13}}{2}+x^2}\,dx}{\sqrt{13}} - \frac{\int \frac{1}{\frac{3}{2}+\frac{\sqrt{13}}{2}+x^2}\,dx}{\sqrt{13}}$$
$$= -\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right) - \sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tanh^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{13}}}x\right)$$

Mathematica [A] time = 0.0531403, size = 68, normalized size = 0.93

$$-\frac{\sqrt{\sqrt{13}-3}\tan^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)+\sqrt{3+\sqrt{13}}\tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)}{\sqrt{26}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 3*x^2 + x^4)^(-1),x]
```

```
[Out] -((Sqrt[-3 + Sqrt[13]]*ArcTan[Sqrt[2/(3 + Sqrt[13])]*x] + Sqrt[3 + Sqrt[13]
]*ArcTanh[Sqrt[2/(-3 + Sqrt[13])]*x])/Sqrt[26])
```

Maple [A] time = 0.033, size = 56, normalized size = 0.8

$$-\frac{2\sqrt{13}}{13\sqrt{-6+2\sqrt{13}}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{-6+2\sqrt{13}}}\right)-\frac{2\sqrt{13}}{13\sqrt{6+2\sqrt{13}}}\arctan\left(2\frac{x}{\sqrt{6+2\sqrt{13}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+3*x^2-1),x)
```

```
[Out] -2/13*13^(1/2)/(-6+2*13^(1/2))^(1/2)*arctanh(2*x/(-6+2*13^(1/2))^(1/2))-2/1
3*13^(1/2)/(6+2*13^(1/2))^(1/2)*arctan(2*x/(6+2*13^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 + 3x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2-1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 3*x^2 - 1), x)
```

Fricas [B] time = 1.98269, size = 475, normalized size = 6.51

$$\frac{1}{13}\sqrt{26}\sqrt{\sqrt{13}-3}\arctan\left(\frac{1}{52}\sqrt{26}\sqrt{13}\sqrt{2}\sqrt{2x^2+\sqrt{13}+3}\sqrt{\sqrt{13}-3}-\frac{1}{26}\sqrt{26}\sqrt{13}x\sqrt{\sqrt{13}-3}\right)+\frac{1}{52}\sqrt{26}\sqrt{\sqrt{13}+3}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2-1),x, algorithm="fricas")
```

```
[Out] 1/13*sqrt(26)*sqrt(sqrt(13) - 3)*arctan(1/52*sqrt(26)*sqrt(13)*sqrt(2)*sqrt
(2*x^2 + sqrt(13) + 3)*sqrt(sqrt(13) - 3) - 1/26*sqrt(26)*sqrt(13)*x*sqrt(s
qrt(13) - 3)) + 1/52*sqrt(26)*sqrt(sqrt(13) + 3)*log(sqrt(26)*(3*sqrt(13) -
 13)*sqrt(sqrt(13) + 3) + 52*x) - 1/52*sqrt(26)*sqrt(sqrt(13) + 3)*log(-sqr
t(26)*(3*sqrt(13) - 13)*sqrt(sqrt(13) + 3) + 52*x)
```

Sympy [A] time = 0.350315, size = 24, normalized size = 0.33

$$\mathrm{RootSum}\left(2704t^4 - 156t^2 - 1, \left(t \mapsto t\log\left(312t^3 - 22t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+3*x**2-1),x)
```

```
[Out] RootSum(2704*_t**4 - 156*_t**2 - 1, Lambda(_t, _t*log(312*_t**3 - 22*_t + x
)))
```

Giac [A] time = 1.14783, size = 100, normalized size = 1.37

$$-\frac{1}{26}\sqrt{26\sqrt{13}-78}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{13}+\frac{3}{2}}}\right)-\frac{1}{52}\sqrt{26\sqrt{13}+78}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{13}-\frac{3}{2}}\right|\right)+\frac{1}{52}\sqrt{26\sqrt{13}+78}\log\left(\left|x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+3*x^2-1),x, algorithm="giac")
```

```
[Out] -1/26*sqrt(26*sqrt(13) - 78)*arctan(x/sqrt(1/2*sqrt(13) + 3/2)) - 1/52*sqrt
(26*sqrt(13) + 78)*log(abs(x + sqrt(1/2*sqrt(13) - 3/2))) + 1/52*sqrt(26*sq
rt(13) + 78)*log(abs(x - sqrt(1/2*sqrt(13) - 3/2)))
```

3.40 $\int \frac{1}{-1-3x^2+x^4}\,dx$

Optimal. Leaf size=73

$$-\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)$$

```
[Out] -(Sqrt[(3 + Sqrt[13])/26]*ArcTan[Sqrt[2/(-3 + Sqrt[13])]*x]) - Sqrt[2/(13*(
3 + Sqrt[13]))]*ArcTanh[Sqrt[2/(3 + Sqrt[13])]*x]
```

Rubi [A] time = 0.01737, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {1093, 207, 203}

$$-\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)-\sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - 3*x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[(3 + Sqrt[13])/26]*ArcTan[Sqrt[2/(-3 + Sqrt[13])]*x]) - Sqrt[2/(13*(
3 + Sqrt[13]))]*ArcTanh[Sqrt[2/(3 + Sqrt[13])]*x]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{-1-3x^2+x^4}\,dx = \frac{\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{13}}{2}+x^2}\,dx}{\sqrt{13}} - \frac{\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{13}}{2}+x^2}\,dx}{\sqrt{13}}$$
$$= -\sqrt{\frac{1}{26}\left(3+\sqrt{13}\right)}\tan^{-1}\left(\sqrt{\frac{2}{-3+\sqrt{13}}}x\right) - \sqrt{\frac{2}{13\left(3+\sqrt{13}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)$$

Mathematica [A] time = 0.0251605, size = 68, normalized size = 0.93

$$-\frac{\sqrt{3+\sqrt{13}}\tan^{-1}\left(\sqrt{\frac{2}{\sqrt{13}-3}}x\right)+\sqrt{\sqrt{13}-3}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{13}}}x\right)}{\sqrt{26}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - 3*x^2 + x^4)^(-1),x]
```

```
[Out] -((Sqrt[3 + Sqrt[13]]*ArcTan[Sqrt[2/(-3 + Sqrt[13])]*x] + Sqrt[-3 + Sqrt[13
]]*ArcTanh[Sqrt[2/(3 + Sqrt[13])]*x])/Sqrt[26])
```

Maple [A] time = 0.025, size = 56, normalized size = 0.8

$$-\frac{2\sqrt{13}}{13\sqrt{6+2\sqrt{13}}}\text{Artanh}\left(2\frac{x}{\sqrt{6+2\sqrt{13}}}\right) - \frac{2\sqrt{13}}{13\sqrt{-6+2\sqrt{13}}}\arctan\left(2\frac{x}{\sqrt{-6+2\sqrt{13}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-3*x^2-1),x)
```

```
[Out] -2/13*13^(1/2)/(6+2*13^(1/2))^(1/2)*arctanh(2*x/(6+2*13^(1/2))^(1/2))-2/13*
13^(1/2)/(-6+2*13^(1/2))^(1/2)*arctan(2*x/(-6+2*13^(1/2))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 - 3x^2 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2-1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 - 3*x^2 - 1), x)
```

Fricas [B] time = 2.16484, size = 475, normalized size = 6.51

$$\frac{1}{13}\sqrt{26}\sqrt{\sqrt{13}+3}\arctan\left(\frac{1}{52}\sqrt{26}\sqrt{13}\sqrt{2}\sqrt{2x^2+\sqrt{13}-3}\sqrt{\sqrt{13}+3}-\frac{1}{26}\sqrt{26}\sqrt{13}x\sqrt{\sqrt{13}+3}\right)-\frac{1}{52}\sqrt{26}\sqrt{\sqrt{13}-3}\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2-1),x, algorithm="fricas")
```

```
[Out] 1/13*sqrt(26)*sqrt(sqrt(13) + 3)*arctan(1/52*sqrt(26)*sqrt(13)*sqrt(2)*sqrt
(2*x^2 + sqrt(13) - 3)*sqrt(sqrt(13) + 3) - 1/26*sqrt(26)*sqrt(13)*x*sqrt(s
qrt(13) + 3)) - 1/52*sqrt(26)*sqrt(sqrt(13) - 3)*log(sqrt(26)*(3*sqrt(13) +
 13)*sqrt(sqrt(13) - 3) + 52*x) + 1/52*sqrt(26)*sqrt(sqrt(13) - 3)*log(-sqr
t(26)*(3*sqrt(13) + 13)*sqrt(sqrt(13) - 3) + 52*x)
```

Sympy [A] time = 0.358659, size = 24, normalized size = 0.33

$$\mathrm{RootSum}\left(2704t^4 + 156t^2 - 1, \left(t \mapsto t\log\left(-312t^3 - 22t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-3*x**2-1),x)
```

```
[Out] RootSum(2704*_t**4 + 156*_t**2 - 1, Lambda(_t, _t*log(-312*_t**3 - 22*_t +
x)))
```

Giac [A] time = 1.1372, size = 100, normalized size = 1.37

$$-\frac{1}{26}\sqrt{26\sqrt{13}+78}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{13}-\frac{3}{2}}}\right)-\frac{1}{52}\sqrt{26\sqrt{13}-78}\log\left(\left|x+\sqrt{\frac{1}{2}\sqrt{13}+\frac{3}{2}}\right|\right)+\frac{1}{52}\sqrt{26\sqrt{13}-78}\log\left(\left|x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2-1),x, algorithm="giac")
```

```
[Out] -1/26*sqrt(26*sqrt(13) + 78)*arctan(x/sqrt(1/2*sqrt(13) - 3/2)) - 1/52*sqrt
(26*sqrt(13) - 78)*log(abs(x + sqrt(1/2*sqrt(13) + 3/2))) + 1/52*sqrt(26*sq
rt(13) - 78)*log(abs(x - sqrt(1/2*sqrt(13) + 3/2)))
```

3.41 $\int \frac{1}{1-3x^2+x^4}\,dx$

Optimal. Leaf size=72

$$\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)-\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

```
[Out] -(Sqrt[2/(5*(3 + Sqrt[5]))]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x]) + Sqrt[(3 + S
qrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x]
```

Rubi [A] time = 0.0653318, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1093, 207}

$$\sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)-\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 3*x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[2/(5*(3 + Sqrt[5]))]*ArcTanh[Sqrt[2/(3 + Sqrt[5])]*x]) + Sqrt[(3 + S
qrt[5])/10]*ArcTanh[Sqrt[(3 + Sqrt[5])/2]*x]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1-3x^2+x^4}\,dx = \frac{\int \frac{1}{-\frac{3}{2}-\frac{\sqrt{5}}{2}+x^2}\,dx}{\sqrt{5}} - \frac{\int \frac{1}{-\frac{3}{2}+\frac{\sqrt{5}}{2}+x^2}\,dx}{\sqrt{5}}$$
$$= -\sqrt{\frac{2}{5\left(3+\sqrt{5}\right)}}\tanh^{-1}\left(\sqrt{\frac{2}{3+\sqrt{5}}}x\right) + \sqrt{\frac{1}{10}\left(3+\sqrt{5}\right)}\tanh^{-1}\left(\sqrt{\frac{1}{2}\left(3+\sqrt{5}\right)}x\right)$$

Mathematica [A] time = 0.0351519, size = 83, normalized size = 1.15

$$\frac{1}{20}\left(-\left(5+\sqrt{5}\right)\log\left(-2x+\sqrt{5}-1\right)-\left(\sqrt{5}-5\right)\log\left(-2x+\sqrt{5}+1\right)+\left(5+\sqrt{5}\right)\log\left(2x+\sqrt{5}-1\right)+\left(\sqrt{5}-5\right)\log\left(2x+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 3*x^2 + x^4)^(-1),x]
```

```
[Out] (-((5 + Sqrt[5])*Log[-1 + Sqrt[5] - 2*x]) - (-5 + Sqrt[5])*Log[1 + Sqrt[5]
- 2*x] + (5 + Sqrt[5])*Log[-1 + Sqrt[5] + 2*x] + (-5 + Sqrt[5])*Log[1 + Sqr
t[5] + 2*x])/20
```

Maple [A] time = 0.006, size = 54, normalized size = 0.8

$$\frac{\ln\left(x^2-x-1\right)}{4}+\frac{\sqrt{5}}{10}\mathrm{Artanh}\left(\frac{(2\,x-1)\sqrt{5}}{5}\right)-\frac{\ln\left(x^2+x-1\right)}{4}+\frac{\sqrt{5}}{10}\mathrm{Artanh}\left(\frac{(1+2\,x)\sqrt{5}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-3*x^2+1),x)
```

```
[Out] 1/4*ln(x^2-x-1)+1/10*5^(1/2)*arctanh(1/5*(2*x-1)*5^(1/2))-1/4*ln(x^2+x-1)+1
/10*5^(1/2)*arctanh(1/5*(1+2*x)*5^(1/2))
```

Maxima [A] time = 1.44582, size = 101, normalized size = 1.4

$$-\frac{1}{20}\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}+1}{2\,x+\sqrt{5}+1}\right)-\frac{1}{20}\sqrt{5}\log\left(\frac{2\,x-\sqrt{5}-1}{2\,x+\sqrt{5}-1}\right)-\frac{1}{4}\log\left(x^2+x-1\right)+\frac{1}{4}\log\left(x^2-x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2+1),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(5)*log((2*x - sqrt(5) + 1)/(2*x + sqrt(5) + 1)) - 1/20*sqrt(5)*l
og((2*x - sqrt(5) - 1)/(2*x + sqrt(5) - 1)) - 1/4*log(x^2 + x - 1) + 1/4*lo
g(x^2 - x - 1)
```

Fricas [B] time = 1.89264, size = 255, normalized size = 3.54

$$\frac{1}{20}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(2x+1)+2x+3}{x^2+x-1}\right)+\frac{1}{20}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(2x-1)-2x+3}{x^2-x-1}\right)-\frac{1}{4}\log\left(x^2+x-1\right)+\frac{1}{4}\log\left(x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2+1),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(5)*log((2*x^2 + sqrt(5)*(2*x + 1) + 2*x + 3)/(x^2 + x - 1)) + 1/2
0*sqrt(5)*log((2*x^2 + sqrt(5)*(2*x - 1) - 2*x + 3)/(x^2 - x - 1)) - 1/4*lo
g(x^2 + x - 1) + 1/4*log(x^2 - x - 1)
```

Sympy [B] time = 0.413806, size = 158, normalized size = 2.19

$$\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)\log\left(x-\frac{7}{2}-\frac{7\sqrt{5}}{10}+120\left(\frac{\sqrt{5}}{20}+\frac{1}{4}\right)^3\right)+\left(\frac{1}{4}-\frac{\sqrt{5}}{20}\right)\log\left(x-\frac{7}{2}+120\left(\frac{1}{4}-\frac{\sqrt{5}}{20}\right)^3+\frac{7\sqrt{5}}{10}\right)+\left(-\frac{1}{4}+\frac{\sqrt{5}}{20}\right)\log\left(x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-3*x**2+1),x)
```

```
[Out] (sqrt(5)/20 + 1/4)*log(x - 7/2 - 7*sqrt(5)/10 + 120*(sqrt(5)/20 + 1/4)**3)
+ (1/4 - sqrt(5)/20)*log(x - 7/2 + 120*(1/4 - sqrt(5)/20)**3 + 7*sqrt(5)/10
) + (-1/4 + sqrt(5)/20)*log(x - 7*sqrt(5)/10 + 120*(-1/4 + sqrt(5)/20)**3 +
 7/2) + (-1/4 - sqrt(5)/20)*log(x + 120*(-1/4 - sqrt(5)/20)**3 + 7*sqrt(5)/
10 + 7/2)
```

<u>Giac</u> [A] time = 1.08602, size = 109, normalized size = 1.51

$$-\frac{1}{20}\sqrt{5}\log\left(\frac{\left|2x-\sqrt{5}+1\right|}{\left|2x+\sqrt{5}+1\right|}\right)-\frac{1}{20}\sqrt{5}\log\left(\frac{\left|2x-\sqrt{5}-1\right|}{\left|2x+\sqrt{5}-1\right|}\right)-\frac{1}{4}\log\left(\left|x^2+x-1\right|\right)+\frac{1}{4}\log\left(\left|x^2-x-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-3*x^2+1),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(5)*log(abs(2*x - sqrt(5) + 1)/abs(2*x + sqrt(5) + 1)) - 1/20*sqr
t(5)*log(abs(2*x - sqrt(5) - 1)/abs(2*x + sqrt(5) - 1)) - 1/4*log(abs(x^2 +
 x - 1)) + 1/4*log(abs(x^2 - x - 1))
```

3.42 $\int \frac{1}{1-4x^2+x^4}\,dx$

Optimal. Leaf size=67

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] ArcTanh[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTanh[x/Sqrt[2 +
 Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Rubi [A] time = 0.0521472, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1093, 207}

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTanh[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTanh[x/Sqrt[2 +
 Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1-4x^2+x^4}\,dx = \frac{\int \frac{1}{-2-\sqrt{3}+x^2}\,dx}{2\sqrt{3}} - \frac{\int \frac{1}{-2+\sqrt{3}+x^2}\,dx}{2\sqrt{3}}$$
$$= \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Mathematica [A] time = 0.0305019, size = 67, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTanh[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTanh[x/Sqrt[2 +
 Sqrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Maple [A] time = 0.03, size = 60, normalized size = 0.9

$$\frac{\sqrt{3}}{3\sqrt{6}-3\sqrt{2}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{6}-\sqrt{2}}\right) - \frac{\sqrt{3}}{3\sqrt{6}+3\sqrt{2}}\mathrm{Artanh}\left(2\frac{x}{\sqrt{6}+\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-4*x^2+1),x)
```

```
[Out] 1/3*3^(1/2)/(6^(1/2)-2^(1/2))*arctanh(2*x/(6^(1/2)-2^(1/2)))-1/3*3^(1/2)/(6
^(1/2)+2^(1/2))*arctanh(2*x/(6^(1/2)+2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 - 4x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-4*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 - 4*x^2 + 1), x)
```

Fricas [B] time = 1.99463, size = 408, normalized size = 6.09

$$-\frac{1}{12}\sqrt{3}\sqrt{\sqrt{3}+2}\log\left(\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+x\right)+\frac{1}{12}\sqrt{3}\sqrt{\sqrt{3}+2}\log\left(-\sqrt{\sqrt{3}+2}\left(\sqrt{3}-2\right)+x\right)-\frac{1}{12}\sqrt{3}\sqrt{-\sqrt{3}+2}\log\left(\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-4*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*sqrt(sqrt(3) + 2)*log(sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + x) +
1/12*sqrt(3)*sqrt(sqrt(3) + 2)*log(-sqrt(sqrt(3) + 2)*(sqrt(3) - 2) + x) -
1/12*sqrt(3)*sqrt(-sqrt(3) + 2)*log((sqrt(3) + 2)*sqrt(-sqrt(3) + 2) + x) +
 1/12*sqrt(3)*sqrt(-sqrt(3) + 2)*log(-(sqrt(3) + 2)*sqrt(-sqrt(3) + 2) + x)
```

Sympy [A] time = 0.42284, size = 24, normalized size = 0.36

$$\mathrm{RootSum}\left(2304t^4 - 192t^2 + 1, \left(t \mapsto t\log\left(384t^3 - 28t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-4*x**2+1),x)
```

```
[Out] RootSum(2304*_t**4 - 192*_t**2 + 1, Lambda(_t, _t*log(384*_t**3 - 28*_t + x
)))
```

Giac [A] time = 1.12312, size = 136, normalized size = 2.03

$$\frac{1}{24}\left(\sqrt{6}-3\sqrt{2}\right)\log\left(\left|x+\frac{1}{2}\sqrt{6}+\frac{1}{2}\sqrt{2}\right|\right)+\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(\left|x+\frac{1}{2}\sqrt{6}-\frac{1}{2}\sqrt{2}\right|\right)-\frac{1}{24}\left(\sqrt{6}+3\sqrt{2}\right)\log\left(\left|x-\frac{1}{2}\sqrt{6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/24*(sqrt(6) - 3*sqrt(2))*log(abs(x + 1/2*sqrt(6) + 1/2*sqrt(2))) + 1/24*(
sqrt(6) + 3*sqrt(2))*log(abs(x + 1/2*sqrt(6) - 1/2*sqrt(2))) - 1/24*(sqrt(6
) + 3*sqrt(2))*log(abs(x - 1/2*sqrt(6) + 1/2*sqrt(2))) - 1/24*(sqrt(6) - 3*
sqrt(2))*log(abs(x - 1/2*sqrt(6) - 1/2*sqrt(2)))
```

3.43 $\int \frac{1}{1+4x^2+x^4}\,dx$

Optimal. Leaf size=67

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[x/Sqrt[2 + S
qrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Rubi [A] time = 0.0119304, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1093, 203}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[x/Sqrt[2 + S
qrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1+4x^2+x^4}\,dx = \frac{\int \frac{1}{2-\sqrt{3}+x^2}\,dx}{2\sqrt{3}} - \frac{\int \frac{1}{2+\sqrt{3}+x^2}\,dx}{2\sqrt{3}}$$
$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Mathematica [A] time = 0.0198654, size = 67, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{3\left(2-\sqrt{3}\right)}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{3\left(2+\sqrt{3}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 4*x^2 + x^4)^(-1),x]
```

```
[Out] ArcTan[x/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[3*(2 - Sqrt[3])]) - ArcTan[x/Sqrt[2 + S
qrt[3]]]/(2*Sqrt[3*(2 + Sqrt[3])])
```

Maple [A] time = 0.025, size = 60, normalized size = 0.9

$$\frac{\sqrt{3}}{3\sqrt{6}-3\sqrt{2}}\arctan\left(2\frac{x}{\sqrt{6}-\sqrt{2}}\right) - \frac{\sqrt{3}}{3\sqrt{6}+3\sqrt{2}}\arctan\left(2\frac{x}{\sqrt{6}+\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+4*x^2+1),x)
```

```
[Out] 1/3*3^(1/2)/(6^(1/2)-2^(1/2))*arctan(2*x/(6^(1/2)-2^(1/2)))-1/3*3^(1/2)/(6^
(1/2)+2^(1/2))*arctan(2*x/(6^(1/2)+2^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 + 4x^2 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + 4*x^2 + 1), x)
```

Fricas [A] time = 2.08446, size = 266, normalized size = 3.97

$$-\frac{1}{3}\sqrt{3}\sqrt{\sqrt{3}+2}\arctan\left(-\left(x-\sqrt{x^2-\sqrt{3}+2}\right)\sqrt{\sqrt{3}+2}\right)+\frac{1}{3}\sqrt{3}\sqrt{-\sqrt{3}+2}\arctan\left(-x\sqrt{-\sqrt{3}+2}+\sqrt{x^2+\sqrt{3}+2}\sqrt{-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+1),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(3)*sqrt(sqrt(3) + 2)*arctan(-(x - sqrt(x^2 - sqrt(3) + 2))*sqrt(s
qrt(3) + 2)) + 1/3*sqrt(3)*sqrt(-sqrt(3) + 2)*arctan(-x*sqrt(-sqrt(3) + 2)
+ sqrt(x^2 + sqrt(3) + 2)*sqrt(-sqrt(3) + 2))
```

Sympy [A] time = 0.189128, size = 92, normalized size = 1.37

$$-2\sqrt{\frac{1}{24}-\frac{\sqrt{3}}{48}}\operatorname{atan}\left(\frac{x}{\sqrt{3}\sqrt{2-\sqrt{3}}+2\sqrt{2-\sqrt{3}}}\right)-2\sqrt{\frac{\sqrt{3}}{48}+\frac{1}{24}}\operatorname{atan}\left(\frac{x}{-2\sqrt{\sqrt{3}+2}+\sqrt{3}\sqrt{\sqrt{3}+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+4*x**2+1),x)
```

```
[Out] -2*sqrt(1/24 - sqrt(3)/48)*atan(x/(sqrt(3)*sqrt(2 - sqrt(3)) + 2*sqrt(2 - s
qrt(3)))) - 2*sqrt(sqrt(3)/48 + 1/24)*atan(x/(-2*sqrt(sqrt(3) + 2) + sqrt(3
)*sqrt(sqrt(3) + 2)))
```

Giac [A] time = 1.08174, size = 69, normalized size = 1.03

$$\frac{1}{12}\left(\sqrt{6}-3\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\left(\sqrt{6}+3\sqrt{2}\right)\arctan\left(\frac{2x}{\sqrt{6}-\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+4*x^2+1),x, algorithm="giac")
```

```
[Out] 1/12*(sqrt(6) - 3*sqrt(2))*arctan(2*x/(sqrt(6) + sqrt(2))) + 1/12*(sqrt(6)
+ 3*sqrt(2))*arctan(2*x/(sqrt(6) - sqrt(2)))
```

3.44 $\int \frac{1}{2+x^2+x^4}\,dx$

Optimal. Leaf size=196

$$-\frac{\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\log\left(x^2+\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{1+2\sqrt{2}}}\right)+\frac{1}{2}\sqrt{\frac{1}{14}\left(2\right.}$$

```
[Out] -(Sqrt[(-1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] - 2*x)/Sqrt[1 + 2*
Sqrt[2]]])/2 + (Sqrt[(-1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*
x)/Sqrt[1 + 2*Sqrt[2]]])/2 - Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2]/(4
*Sqrt[2*(-1 + 2*Sqrt[2])]) + Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2]/(4
*Sqrt[2*(-1 + 2*Sqrt[2])])
```

Rubi [A] time = 0.148191, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}+\frac{\log\left(x^2+\sqrt{2\sqrt{2}-1}x+\sqrt{2}\right)}{4\sqrt{2\left(2\sqrt{2}-1\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(2\sqrt{2}-1\right)}\tan^{-1}\left(\frac{\sqrt{2\sqrt{2}-1}-2x}{\sqrt{1+2\sqrt{2}}}\right)+\frac{1}{2}\sqrt{\frac{1}{14}\left(2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[(-1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] - 2*x)/Sqrt[1 + 2*
Sqrt[2]]])/2 + (Sqrt[(-1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[-1 + 2*Sqrt[2]] + 2*
x)/Sqrt[1 + 2*Sqrt[2]]])/2 - Log[Sqrt[2] - Sqrt[-1 + 2*Sqrt[2]]*x + x^2]/(4
*Sqrt[2*(-1 + 2*Sqrt[2])]) + Log[Sqrt[2] + Sqrt[-1 + 2*Sqrt[2]]*x + x^2]/(4
*Sqrt[2*(-1 + 2*Sqrt[2])])
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{2+x^2+x^4}\,dx = \frac{\int \frac{\sqrt{-1+2\sqrt{2}}-x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{2\sqrt{2\left(-1+2\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}}+x}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{2\sqrt{2\left(-1+2\sqrt{2}\right)}}$$

$$= \frac{\int \frac{1}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{4\sqrt{2}} + \frac{\int \frac{1}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{4\sqrt{2}} - \frac{\int \frac{-\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{4\sqrt{2\left(-1+2\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2}\,dx}{4\sqrt{2\left(-1+2\sqrt{2}\right)}}$$

$$= -\frac{\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}} + \frac{\log\left(\sqrt{2}+\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}} - \frac{\text{Subst}\left(\int \frac{1}{-1-2\sqrt{2}-x^2}\,dx, x, -\sqrt{\cdots}\right.}{2\sqrt{2}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}-2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}} + \frac{\tan^{-1}\left(\frac{\sqrt{-1+2\sqrt{2}}+2x}{\sqrt{1+2\sqrt{2}}}\right)}{2\sqrt{2\left(1+2\sqrt{2}\right)}} - \frac{\log\left(\sqrt{2}-\sqrt{-1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(-1+2\sqrt{2}\right)}} + \frac{\log\left(\sqrt{2}+\sqrt{-1+\cdots}\right.}{4\sqrt{2\left(-1+\cdots\right.}}$$

Mathematica [C] time = 0.0616323, size = 91, normalized size = 0.46

$$\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1+i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(1+i\sqrt{7}\right)}}-\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(1-i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(1-i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^2 + x^4)^(-1),x]
```

```
[Out] ((-I)*ArcTan[x/Sqrt[(1 - I*Sqrt[7])/2]])/Sqrt[(7*(1 - I*Sqrt[7]))/2] + (I*A
rcTan[x/Sqrt[(1 + I*Sqrt[7])/2]])/Sqrt[(7*(1 + I*Sqrt[7]))/2]
```

Maple [B] time = 0.064, size = 386, normalized size = 2.

$$\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)\sqrt{-1+2\sqrt{2}}\sqrt{2}}{56}+\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{-1+2\sqrt{2}}\right)\sqrt{-1+2\sqrt{2}}}{14}-\frac{\left(-1+2\sqrt{2}\right)\sqrt{2}}{28\sqrt{1+2\sqrt{2}}}\arctan\left(\frac{2}{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+x^2+2),x)
```

```
[Out] 1/56*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*2^(1/2)+1/
14*ln(x^2+2^(1/2)+x*(-1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)-1/28/(1+2*2^
(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2
^(1/2))*2^(1/2)-1/7/(1+2*2^(1/2))^(1/2)*arctan((2*x+(-1+2*2^(1/2))^(1/2))/(
1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))+1/2/(1+2*2^(1/2))^(1/2)*arctan((2*x+(-1+
2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*2^(1/2)-1/56*ln(x^2+2^(1/2)-x*(-1+2*
2^(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)*2^(1/2)-1/14*ln(x^2+2^(1/2)-x*(-1+2*2^
(1/2))^(1/2))*(-1+2*2^(1/2))^(1/2)-1/28/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1
+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2^(1/2))*2^(1/2)-1/7/(1+2*2^(
1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/2))^(1/2))*(-1+2*2^
(1/2))+1/2/(1+2*2^(1/2))^(1/2)*arctan((2*x-(-1+2*2^(1/2))^(1/2))/(1+2*2^(1/
2))^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 + x^2 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 + x^2 + 2), x)
```

Fricas [B] time = 2.08491, size = 988, normalized size = 5.04

$$-\frac{1}{56}\cdot 8^{\frac{1}{4}}\sqrt{7}\sqrt{2}\sqrt{-4\sqrt{2}+16}\arctan\left(-\frac{1}{56}\cdot 8^{\frac{3}{4}}\sqrt{7}\sqrt{2}x\sqrt{-4\sqrt{2}+16}+\frac{1}{112}\cdot 8^{\frac{3}{4}}\sqrt{7}\sqrt{2}\sqrt{4x^2+8^{\frac{1}{4}}x\sqrt{-4\sqrt{2}+16}+4\sqrt{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+2),x, algorithm="fricas")
```

```
[Out] -1/56*8^(1/4)*sqrt(7)*sqrt(2)*sqrt(-4*sqrt(2) + 16)*arctan(-1/56*8^(3/4)*sq
rt(7)*sqrt(2)*x*sqrt(-4*sqrt(2) + 16) + 1/112*8^(3/4)*sqrt(7)*sqrt(2)*sqrt(
4*x^2 + 8^(1/4)*x*sqrt(-4*sqrt(2) + 16) + 4*sqrt(2))*sqrt(-4*sqrt(2) + 16)
- 1/7*sqrt(7)*(2*sqrt(2) - 1)) - 1/56*8^(1/4)*sqrt(7)*sqrt(2)*sqrt(-4*sqrt(
2) + 16)*arctan(-1/56*8^(3/4)*sqrt(7)*sqrt(2)*x*sqrt(-4*sqrt(2) + 16) + 1/1
12*8^(3/4)*sqrt(7)*sqrt(2)*sqrt(4*x^2 - 8^(1/4)*x*sqrt(-4*sqrt(2) + 16) + 4
*sqrt(2))*sqrt(-4*sqrt(2) + 16) + 1/7*sqrt(7)*(2*sqrt(2) - 1)) + 1/224*8^(1
/4)*(sqrt(2) + 4)*sqrt(-4*sqrt(2) + 16)*log(4*x^2 + 8^(1/4)*x*sqrt(-4*sqrt(
2) + 16) + 4*sqrt(2)) - 1/224*8^(1/4)*(sqrt(2) + 4)*sqrt(-4*sqrt(2) + 16)*l
og(4*x^2 - 8^(1/4)*x*sqrt(-4*sqrt(2) + 16) + 4*sqrt(2))
```

Sympy [A] time = 0.469528, size = 24, normalized size = 0.12

$$\operatorname{RootSum}\left(1568t^4 - 28t^2 + 1, \left(t \mapsto t\log\left(112t^3 + 6t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+x**2+2),x)
```

```
[Out] RootSum(1568*_t**4 - 28*_t**2 + 1, Lambda(_t, _t*log(112*_t**3 + 6*_t + x))
)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 + x^2 + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+x^2+2),x, algorithm="giac")
```

```
[Out] integrate(1/(x^4 + x^2 + 2), x)
```

3.45 $\int \frac{1}{2-x^2+x^4}\,dx$

Optimal. Leaf size=196

$$-\frac{\log\left(x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\log\left(x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(1+2\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{1+2\sqrt{2}}-2x}{\sqrt{2\sqrt{2}-1}}\right)+\frac{1}{2}\sqrt{\frac{1}{14}\left(\right.}$$

```
[Out] -(Sqrt[(1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[1 + 2*Sqrt[2]] - 2*x)/Sqrt[-1 + 2*S
qrt[2]]])/2 + (Sqrt[(1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[1 + 2*Sqrt[2]] + 2*x)/
Sqrt[-1 + 2*Sqrt[2]]])/2 - Log[Sqrt[2] - Sqrt[1 + 2*Sqrt[2]]*x + x^2]/(4*Sq
rt[2*(1 + 2*Sqrt[2])]) + Log[Sqrt[2] + Sqrt[1 + 2*Sqrt[2]]*x + x^2]/(4*Sqrt
[2*(1 + 2*Sqrt[2])])
```

Rubi [A] time = 0.138953, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1094, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{1+2\sqrt{2}}x+\sqrt{2}\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\log\left(x^2+\sqrt{1+2\sqrt{2}}x+\sqrt{2}\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}-\frac{1}{2}\sqrt{\frac{1}{14}\left(1+2\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{1+2\sqrt{2}}-2x}{\sqrt{2\sqrt{2}-1}}\right)+\frac{1}{2}\sqrt{\frac{1}{14}\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x^2 + x^4)^(-1),x]
```

```
[Out] -(Sqrt[(1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[1 + 2*Sqrt[2]] - 2*x)/Sqrt[-1 + 2*S
qrt[2]]])/2 + (Sqrt[(1 + 2*Sqrt[2])/14]*ArcTan[(Sqrt[1 + 2*Sqrt[2]] + 2*x)/
Sqrt[-1 + 2*Sqrt[2]]])/2 - Log[Sqrt[2] - Sqrt[1 + 2*Sqrt[2]]*x + x^2]/(4*Sq
rt[2*(1 + 2*Sqrt[2])]) + Log[Sqrt[2] + Sqrt[1 + 2*Sqrt[2]]*x + x^2]/(4*Sqrt
[2*(1 + 2*Sqrt[2])])
```

Rule 1094

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[a/
c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x
 + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /
; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{2-x^2+x^4}\,dx = \frac{\int \frac{\sqrt{1+2\sqrt{2}}-x}{\sqrt{2}-\sqrt{1+2\sqrt{2}}x+x^2}\,dx}{2\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\int \frac{\sqrt{1+2\sqrt{2}}+x}{\sqrt{2}+\sqrt{1+2\sqrt{2}}x+x^2}\,dx}{2\sqrt{2\left(1+2\sqrt{2}\right)}}$$

$$=\frac{\int \frac{1}{\sqrt{2}-\sqrt{1+2\sqrt{2}}x+x^2}\,dx}{4\sqrt{2}}+\frac{\int \frac{1}{\sqrt{2}+\sqrt{1+2\sqrt{2}}x+x^2}\,dx}{4\sqrt{2}}-\frac{\int \frac{-\sqrt{1+2\sqrt{2}}+2x}{\sqrt{2}-\sqrt{1+2\sqrt{2}}x+x^2}\,dx}{4\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\int \frac{\sqrt{1+2\sqrt{2}}+2x}{\sqrt{2}+\sqrt{1+2\sqrt{2}}x+x^2}\,dx}{4\sqrt{2\left(1+2\sqrt{2}\right)}}$$

$$=-\frac{\log\left(\sqrt{2}-\sqrt{1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\log\left(\sqrt{2}+\sqrt{1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}-\frac{\text{Subst}\left(\int \frac{1}{1-2\sqrt{2}-x^2}\,dx,x,-\sqrt{1+2\sqrt{}}\right.}{2\sqrt{2}}$$

$$=-\frac{\tan^{-1}\left(\frac{\sqrt{1+2\sqrt{2}}-2x}{\sqrt{-1+2\sqrt{2}}}\right)}{2\sqrt{2\left(-1+2\sqrt{2}\right)}}+\frac{\tan^{-1}\left(\frac{\sqrt{1+2\sqrt{2}}+2x}{\sqrt{-1+2\sqrt{2}}}\right)}{2\sqrt{2\left(-1+2\sqrt{2}\right)}}-\frac{\log\left(\sqrt{2}-\sqrt{1+2\sqrt{2}}x+x^2\right)}{4\sqrt{2\left(1+2\sqrt{2}\right)}}+\frac{\log\left(\sqrt{2}+\sqrt{1+2\sqrt{2}}x\right.}{4\sqrt{2\left(1+2\sqrt{2}\right)}}$$

Mathematica [C] time = 0.0815613, size = 91, normalized size = 0.46

$$\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(-1+i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(-1+i\sqrt{7}\right)}}-\frac{i\tan^{-1}\left(\frac{x}{\sqrt{\frac{1}{2}\left(-1-i\sqrt{7}\right)}}\right)}{\sqrt{\frac{7}{2}\left(-1-i\sqrt{7}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x^2 + x^4)^(-1),x]
```

```
[Out] ((-I)*ArcTan[x/Sqrt[(-1 - I*Sqrt[7])/2]])/Sqrt[(7*(-1 - I*Sqrt[7]))/2] + (I
*ArcTan[x/Sqrt[(-1 + I*Sqrt[7])/2]])/Sqrt[(7*(-1 + I*Sqrt[7]))/2]
```

Maple [B] time = 0.054, size = 386, normalized size = 2.

$$-\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{1+2\sqrt{2}}\right)\sqrt{1+2\sqrt{2}}\sqrt{2}}{56}+\frac{\ln\left(x^2+\sqrt{2}+x\sqrt{1+2\sqrt{2}}\right)\sqrt{1+2\sqrt{2}}}{14}+\frac{\left(1+2\sqrt{2}\right)\sqrt{2}}{28\sqrt{-1+2\sqrt{2}}}\arctan\left(\frac{2x+}{\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-x^2+2),x)
```

```
[Out] -1/56*ln(x^2+2^(1/2)+x*(1+2*2^(1/2))^(1/2))*(1+2*2^(1/2))^(1/2)*2^(1/2)+1/1
4*ln(x^2+2^(1/2)+x*(1+2*2^(1/2))^(1/2))*(1+2*2^(1/2))^(1/2)+1/28/(-1+2*2^(1
/2))^(1/2)*arctan((2*x+(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*(1+2*2^(1
/2))*2^(1/2)-1/7/(-1+2*2^(1/2))^(1/2)*arctan((2*x+(1+2*2^(1/2))^(1/2))/(-1+
2*2^(1/2))^(1/2))*(1+2*2^(1/2))+1/2/(-1+2*2^(1/2))^(1/2)*arctan((2*x+(1+2*2
^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*2^(1/2)+1/56*ln(x^2+2^(1/2)-x*(1+2*2^(
1/2))^(1/2))*(1+2*2^(1/2))^(1/2)*2^(1/2)-1/14*ln(x^2+2^(1/2)-x*(1+2*2^(1/2)
)^(1/2))*(1+2*2^(1/2))^(1/2)+1/28/(-1+2*2^(1/2))^(1/2)*arctan((2*x-(1+2*2^(
1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*(1+2*2^(1/2))*2^(1/2)-1/7/(-1+2*2^(1/2))
^(1/2)*arctan((2*x-(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(1/2))*(1+2*2^(1/2))
+1/2/(-1+2*2^(1/2))^(1/2)*arctan((2*x-(1+2*2^(1/2))^(1/2))/(-1+2*2^(1/2))^(
1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 - x^2 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+2),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^4 - x^2 + 2), x)
```

Fricas [B] time = 2.15945, size = 932, normalized size = 4.76

$$-\frac{1}{28}\cdot 8^{\frac{1}{4}}\sqrt{7}\sqrt{2}\sqrt{\sqrt{2}+4}\arctan\left(-\frac{1}{28}\cdot 8^{\frac{3}{4}}\sqrt{7}\sqrt{2}x\sqrt{\sqrt{2}+4}+\frac{1}{56}\cdot 8^{\frac{3}{4}}\sqrt{7}\sqrt{2}\sqrt{4\,x^2+2\cdot 8^{\frac{1}{4}}x\sqrt{\sqrt{2}+4}+4\,\sqrt{2}}\sqrt{\sqrt{2}+4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+2),x, algorithm="fricas")
```

```
[Out] -1/28*8^(1/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(2) + 4)*arctan(-1/28*8^(3/4)*sqrt(7
)*sqrt(2)*x*sqrt(sqrt(2) + 4) + 1/56*8^(3/4)*sqrt(7)*sqrt(2)*sqrt(4*x^2 + 2
*8^(1/4)*x*sqrt(sqrt(2) + 4) + 4*sqrt(2))*sqrt(sqrt(2) + 4) - 1/7*sqrt(7)*(
2*sqrt(2) + 1)) - 1/28*8^(1/4)*sqrt(7)*sqrt(2)*sqrt(sqrt(2) + 4)*arctan(-1/
28*8^(3/4)*sqrt(7)*sqrt(2)*x*sqrt(sqrt(2) + 4) + 1/56*8^(3/4)*sqrt(7)*sqrt(
2)*sqrt(4*x^2 - 2*8^(1/4)*x*sqrt(sqrt(2) + 4) + 4*sqrt(2))*sqrt(sqrt(2) + 4
) + 1/7*sqrt(7)*(2*sqrt(2) + 1)) - 1/112*8^(1/4)*sqrt(sqrt(2) + 4)*(sqrt(2)
 - 4)*log(4*x^2 + 2*8^(1/4)*x*sqrt(sqrt(2) + 4) + 4*sqrt(2)) + 1/112*8^(1/4
)*sqrt(sqrt(2) + 4)*(sqrt(2) - 4)*log(4*x^2 - 2*8^(1/4)*x*sqrt(sqrt(2) + 4)
 + 4*sqrt(2))
```

Sympy [A] time = 0.486539, size = 24, normalized size = 0.12

$$\operatorname{RootSum}\left(1568t^4 + 28t^2 + 1, \left(t \mapsto t\log\left(-112t^3 + 6t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-x**2+2),x)
```

```
[Out] RootSum(1568*_t**4 + 28*_t**2 + 1, Lambda(_t, _t*log(-112*_t**3 + 6*_t + x)
))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 - x^2 + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-x^2+2),x, algorithm="giac")
```

```
[Out] integrate(1/(x^4 - x^2 + 2), x)
```

3.46 $\int \frac{1}{-1+x^6} \, dx$

Optimal. Leaf size=73

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{3}\tanh^{-1}(x)$$

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[3
]) - ArcTanh[x]/3 + Log[1 - x + x^2]/12 - Log[1 + x + x^2]/12
```

Rubi [A] time = 0.0999681, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {210, 634, 618, 204, 628, 206}

$$\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{3}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^6)^(-1),x]
```

```
[Out] ArcTan[(1 - 2*x)/Sqrt[3]]/(2*Sqrt[3]) - ArcTan[(1 + 2*x)/Sqrt[3]]/(2*Sqrt[3
]) - ArcTanh[x]/3 + Log[1 - x + x^2]/12 - Log[1 + x + x^2]/12
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+x^6}\,dx &= -\left(\frac{1}{3}\int \frac{1-\frac{x}{2}}{1-x+x^2}\,dx\right)-\frac{1}{3}\int \frac{1+\frac{x}{2}}{1+x+x^2}\,dx-\frac{1}{3}\int \frac{1}{1-x^2}\,dx\\ &= -\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\int \frac{-1+2x}{1-x+x^2}\,dx-\frac{1}{12}\int \frac{1+2x}{1+x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1-x+x^2}\,dx-\frac{1}{4}\int \frac{1}{1+x+x^2}\,dx\\ &= -\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{12}\log\left(1+x+x^2\right)+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,-1+2x\right)+\frac{1}{2}\,\text{S}\\ &= \frac{\tan^{-1}\left(\frac{1-2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{2\sqrt{3}}-\frac{1}{3}\tanh^{-1}(x)+\frac{1}{12}\log\left(1-x+x^2\right)-\frac{1}{12}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0161625, size = 75, normalized size = 1.03

$$\frac{1}{12}\left(\log\left(x^2-x+1\right)-\log\left(x^2+x+1\right)+2\log(1-x)-2\log(x+1)-2\sqrt{3}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)-2\sqrt{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^6)^(-1),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(-1 + 2*x)/Sqrt[3]] - 2*Sqrt[3]*ArcTan[(1 + 2*x)/Sqrt[3]
] + 2*Log[1 - x] - 2*Log[1 + x] + Log[1 - x + x^2] - Log[1 + x + x^2])/12
```

Maple [A] time = 0.008, size = 66, normalized size = 0.9

$$\frac{\ln(-1+x)}{6}-\frac{\ln\left(x^2+x+1\right)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)-\frac{\ln(1+x)}{6}+\frac{\ln\left(x^2-x+1\right)}{12}-\frac{\sqrt{3}}{6}\arctan\left(\frac{(2x-1)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-1),x)
```

```
[Out] 1/6*ln(-1+x)-1/12*ln(x^2+x+1)-1/6*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)-1/6*l
n(1+x)+1/12*ln(x^2-x+1)-1/6*3^(1/2)*arctan(1/3*(2*x-1)*3^(1/2))
```

Maxima [A] time = 1.43174, size = 88, normalized size = 1.21

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log(x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1
) + 1/6*log(x - 1)
```

Fricas [A] time = 1.96531, size = 230, normalized size = 3.15

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)-\frac{1}{12}\log\left(x^2+x+1\right)+\frac{1}{12}\log\left(x^2-x+1\right)-\frac{1}{6}\log(x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(x + 1
) + 1/6*log(x - 1)
```

Sympy [A] time = 0.229185, size = 83, normalized size = 1.14

$$\frac{\log(x-1)}{6} - \frac{\log(x+1)}{6} + \frac{\log\left(x^2-x+1\right)}{12} - \frac{\log\left(x^2+x+1\right)}{12} - \frac{\sqrt{3}\,\text{atan}\left(\frac{2\sqrt{3}x}{3} - \frac{\sqrt{3}}{3}\right)}{6} - \frac{\sqrt{3}\,\text{atan}\left(\frac{2\sqrt{3}x}{3} + \frac{\sqrt{3}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-1),x)
```

```
[Out] log(x - 1)/6 - log(x + 1)/6 + log(x**2 - x + 1)/12 - log(x**2 + x + 1)/12 -
 sqrt(3)*atan(2*sqrt(3)*x/3 - sqrt(3)/3)/6 - sqrt(3)*atan(2*sqrt(3)*x/3 + s
qrt(3)/3)/6
```

Giac [A] time = 1.07887, size = 90, normalized size = 1.23

$$-\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) - \frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right) - \frac{1}{12}\log\left(x^2+x+1\right) + \frac{1}{12}\log\left(x^2-x+1\right) - \frac{1}{6}\log(|x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-1),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) - 1/12*log(x^2 + x + 1) + 1/12*log(x^2 - x + 1) - 1/6*log(abs(x
 + 1)) + 1/6*log(abs(x - 1))
```

3.47 $\int \frac{1}{-2+x^6}\,dx$

Optimal. Leaf size=138

$$\frac{\log\left(x^2-\sqrt[6]{2}x+\sqrt[3]{2}\right)}{12\ 2^{5/6}}-\frac{\log\left(x^2+\sqrt[6]{2}x+\sqrt[3]{2}\right)}{12\ 2^{5/6}}+\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{5/6}x}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{5/6}x}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

```
[Out] ArcTan[1/Sqrt[3] - (2^(5/6)*x)/Sqrt[3]]/(2*2^(5/6)*Sqrt[3]) - ArcTan[1/Sqrt
[3] + (2^(5/6)*x)/Sqrt[3]]/(2*2^(5/6)*Sqrt[3]) - ArcTanh[x/2^(1/6)]/(3*2^(5
/6)) + Log[2^(1/3) - 2^(1/6)*x + x^2]/(12*2^(5/6)) - Log[2^(1/3) + 2^(1/6)*
x + x^2]/(12*2^(5/6))
```

Rubi [A] time = 0.191552, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {210, 634, 618, 204, 628, 206}

$$\frac{\log\left(x^2-\sqrt[6]{2}x+\sqrt[3]{2}\right)}{12\ 2^{5/6}}-\frac{\log\left(x^2+\sqrt[6]{2}x+\sqrt[3]{2}\right)}{12\ 2^{5/6}}+\frac{\tan^{-1}\left(\frac{1}{\sqrt{3}}-\frac{2^{5/6}x}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\tan^{-1}\left(\frac{2^{5/6}x}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}}-\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + x^6)^(-1),x]
```

```
[Out] ArcTan[1/Sqrt[3] - (2^(5/6)*x)/Sqrt[3]]/(2*2^(5/6)*Sqrt[3]) - ArcTan[1/Sqrt
[3] + (2^(5/6)*x)/Sqrt[3]]/(2*2^(5/6)*Sqrt[3]) - ArcTanh[x/2^(1/6)]/(3*2^(5
/6)) + Log[2^(1/3) - 2^(1/6)*x + x^2]/(12*2^(5/6)) - Log[2^(1/3) + 2^(1/6)*
x + x^2]/(12*2^(5/6))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-(
a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r - s*Cos[(
2*k*Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Co
s[(2*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; (2*r^2*Int[
1/(r^2 - s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 2)/4}],
 x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{-2+x^6}\,dx = -\frac{\int \frac{\sqrt[6]{2}-\frac{x}{2}}{\sqrt[3]{2}-\sqrt[6]{2}x+x^2}\,dx}{3\ 2^{5/6}} - \frac{\int \frac{\sqrt[6]{2}+\frac{x}{2}}{\sqrt[3]{2}+\sqrt[6]{2}x+x^2}\,dx}{3\ 2^{5/6}} - \frac{\int \frac{1}{\sqrt[3]{2}-x^2}\,dx}{3\ 2^{2/3}}$$

$$= -\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}} + \frac{\int \frac{-\sqrt[6]{2}+2x}{\sqrt[3]{2}-\sqrt[6]{2}x+x^2}\,dx}{12\ 2^{5/6}} - \frac{\int \frac{\sqrt[6]{2}+2x}{\sqrt[3]{2}+\sqrt[6]{2}x+x^2}\,dx}{12\ 2^{5/6}} - \frac{\int \frac{1}{\sqrt[3]{2}-\sqrt[6]{2}x+x^2}\,dx}{4\ 2^{2/3}} - \frac{\int \frac{1}{\sqrt[3]{2}+\sqrt[6]{2}x+x^2}\,dx}{4\ 2^{2/3}}$$

$$= -\frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}x+x^2\right)}{12\ 2^{5/6}} - \frac{\log\left(\sqrt[3]{2}+\sqrt[6]{2}x+x^2\right)}{12\ 2^{5/6}} - \frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-2^{5/6}x\right)}{2\ 2^{5/6}} + \cdots$$

$$= \frac{\tan^{-1}\left(\frac{1-2^{5/6}x}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}} - \frac{\tan^{-1}\left(\frac{1+2^{5/6}x}{\sqrt{3}}\right)}{2\ 2^{5/6}\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}} + \frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}x+x^2\right)}{12\ 2^{5/6}} - \frac{\log\left(\sqrt[3]{2}+\sqrt[6]{2}x+x^2\right)}{12\ 2^{5/6}}$$

Mathematica [A] time = 0.0401925, size = 122, normalized size = 0.88

$$-\frac{-\log\left(2^{2/3}x^2-2^{5/6}x+2\right)+\log\left(2^{2/3}x^2+2^{5/6}x+2\right)-2\log\left(2-2^{5/6}x\right)+2\log\left(2^{5/6}x+2\right)+2\sqrt{3}\tan^{-1}\left(\frac{2^{5/6}x-1}{\sqrt{3}}\right)+2\sqrt{3}}{12\ 2^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + x^6)^(-1),x]
```

```
[Out] -(2*Sqrt[3]*ArcTan[(-1 + 2^(5/6)*x)/Sqrt[3]] + 2*Sqrt[3]*ArcTan[(1 + 2^(5/6
)*x)/Sqrt[3]] - 2*Log[2 - 2^(5/6)*x] + 2*Log[2 + 2^(5/6)*x] - Log[2 - 2^(5/
6)*x + 2^(2/3)*x^2] + Log[2 + 2^(5/6)*x + 2^(2/3)*x^2])/(12*2^(5/6))
```

Maple [A] time = 0.071, size = 111, normalized size = 0.8

$$-\frac{\sqrt[6]{2}\ln\left(x+\sqrt[6]{2}\right)}{12}+\frac{\ln\left(\sqrt[3]{2}-\sqrt[6]{2}x+x^2\right)\sqrt[6]{2}}{24}-\frac{\sqrt{3}\sqrt[6]{2}}{12}\arctan\left(-\frac{\sqrt{3}}{3}+\frac{2^{\frac{5}{6}}x\sqrt{3}}{3}\right)-\frac{\ln\left(\sqrt[3]{2}+\sqrt[6]{2}x+x^2\right)\sqrt[6]{2}}{24}-\frac{\sqrt{3}\sqrt[6]{2}}{12}\text{arct}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6-2),x)
```

```
[Out] -1/12*2^(1/6)*ln(x+2^(1/6))+1/24*ln(2^(1/3)-2^(1/6)*x+x^2)*2^(1/6)-1/12*arc
tan(-1/3*3^(1/2)+1/3*2^(5/6)*x*3^(1/2))*2^(1/6)*3^(1/2)-1/24*ln(2^(1/3)+2^(
1/6)*x+x^2)*2^(1/6)-1/12*arctan(1/3*3^(1/2)+1/3*2^(5/6)*x*3^(1/2))*2^(1/6)*
3^(1/2)+1/12*2^(1/6)*ln(x-2^(1/6))
```

Maxima [A] time = 1.4415, size = 151, normalized size = 1.09

$$-\frac{1}{12}\sqrt{3}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{5}{6}}\left(2x+2^{\frac{1}{6}}\right)\right)-\frac{1}{12}\sqrt{3}2^{\frac{1}{6}}\arctan\left(\frac{1}{6}\sqrt{3}2^{\frac{5}{6}}\left(2x-2^{\frac{1}{6}}\right)\right)-\frac{1}{24}\cdot 2^{\frac{1}{6}}\log\left(x^2+2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)+\frac{1}{24}\cdot 2^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-2),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x + 2^(1/6))) - 1/12*sq
rt(3)*2^(1/6)*arctan(1/6*sqrt(3)*2^(5/6)*(2*x - 2^(1/6))) - 1/24*2^(1/6)*lo
```

```
g(x^2 + 2^(1/6)*x + 2^(1/3)) + 1/24*2^(1/6)*log(x^2 - 2^(1/6)*x + 2^(1/3))
- 1/12*2^(1/6)*log(x + 2^(1/6)) + 1/12*2^(1/6)*log(x - 2^(1/6))
```

Fricas [A] time = 2.02239, size = 603, normalized size = 4.37

$$\frac{1}{96}\cdot 32^{\frac{5}{6}}\sqrt{3}\arctan\left(-\frac{1}{3}\cdot 32^{\frac{1}{6}}\sqrt{3}x+\frac{1}{12}\cdot 32^{\frac{1}{6}}\sqrt{3}\sqrt{16\,x^2+32^{\frac{5}{6}}x+8\cdot 4^{\frac{2}{3}}}-\frac{1}{3}\sqrt{3}\right)+\frac{1}{96}\cdot 32^{\frac{5}{6}}\sqrt{3}\arctan\left(-\frac{1}{3}\cdot 32^{\frac{1}{6}}\sqrt{3}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-2),x, algorithm="fricas")
```

```
[Out] 1/96*32^(5/6)*sqrt(3)*arctan(-1/3*32^(1/6)*sqrt(3)*x + 1/12*32^(1/6)*sqrt(3
)*sqrt(16*x^2 + 32^(5/6)*x + 8*4^(2/3)) - 1/3*sqrt(3)) + 1/96*32^(5/6)*sqrt
(3)*arctan(-1/3*32^(1/6)*sqrt(3)*x + 1/12*32^(1/6)*sqrt(3)*sqrt(16*x^2 - 32
^(5/6)*x + 8*4^(2/3)) + 1/3*sqrt(3)) - 1/384*32^(5/6)*log(16*x^2 + 32^(5/6)
*x + 8*4^(2/3)) + 1/384*32^(5/6)*log(16*x^2 - 32^(5/6)*x + 8*4^(2/3)) - 1/1
92*32^(5/6)*log(16*x + 32^(5/6)) + 1/192*32^(5/6)*log(16*x - 32^(5/6))
```

Sympy [A] time = 0.571694, size = 14, normalized size = 0.1

$$\operatorname{RootSum}\left(1492992t^6-1,\left(t\mapsto t\log\left(-12t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6-2),x)
```

```
[Out] RootSum(1492992*_t**6 - 1, Lambda(_t, _t*log(-12*_t + x)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6-2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.48 $\int \frac{1}{2+x^6} \, dx$

Optimal. Leaf size=138

$$-\frac{\log\left(x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}{4\ 2^{5/6}\sqrt{3}}+\frac{\log\left(x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}{4\ 2^{5/6}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}-\frac{\tan^{-1}\left(\sqrt{3}-2^{5/6}x\right)}{6\ 2^{5/6}}+\frac{\tan^{-1}\left(2^{5/6}x+\sqrt{3}\right)}{6\ 2^{5/6}}$$

```
[Out] ArcTan[x/2^(1/6)]/(3*2^(5/6)) - ArcTan[Sqrt[3] - 2^(5/6)*x]/(6*2^(5/6)) + A
rcTan[Sqrt[3] + 2^(5/6)*x]/(6*2^(5/6)) - Log[2^(1/3) - 2^(1/6)*Sqrt[3]*x +
x^2]/(4*2^(5/6)*Sqrt[3]) + Log[2^(1/3) + 2^(1/6)*Sqrt[3]*x + x^2]/(4*2^(5/6
)*Sqrt[3])
```

Rubi [A] time = 0.281148, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {209, 634, 618, 204, 628, 203}

$$-\frac{\log\left(x^2-\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}{4\ 2^{5/6}\sqrt{3}}+\frac{\log\left(x^2+\sqrt[6]{2}\sqrt{3}x+\sqrt[3]{2}\right)}{4\ 2^{5/6}\sqrt{3}}+\frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}}-\frac{\tan^{-1}\left(\sqrt{3}-2^{5/6}x\right)}{6\ 2^{5/6}}+\frac{\tan^{-1}\left(2^{5/6}x+\sqrt{3}\right)}{6\ 2^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x^6)^(-1),x]
```

```
[Out] ArcTan[x/2^(1/6)]/(3*2^(5/6)) - ArcTan[Sqrt[3] - 2^(5/6)*x]/(6*2^(5/6)) + A
rcTan[Sqrt[3] + 2^(5/6)*x]/(6*2^(5/6)) - Log[2^(1/3) - 2^(1/6)*Sqrt[3]*x +
x^2]/(4*2^(5/6)*Sqrt[3]) + Log[2^(1/3) + 2^(1/6)*Sqrt[3]*x + x^2]/(4*2^(5/6
)*Sqrt[3])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[((2*
k - 1)*Pi)/n]*x)/(r^2 - 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x] + Int[
(r + s*Cos[((2*k - 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*
x^2), x]; (2*r^2*Int[1/(r^2 + s^2*x^2), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u
, {k, 1, (n - 2)/4}], x], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] &&
PosQ[a/b]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{2+x^6}\,dx = \frac{\int \frac{\sqrt[6]{2}-\frac{\sqrt{3}x}{2}}{\sqrt[3]{2}-\sqrt[6]{2}\sqrt{3}x+x^2}\,dx}{3\ 2^{5/6}} + \frac{\int \frac{\sqrt[6]{2}+\frac{\sqrt{3}x}{2}}{\sqrt[3]{2}+\sqrt[6]{2}\sqrt{3}x+x^2}\,dx}{3\ 2^{5/6}} + \frac{\int \frac{1}{\sqrt[3]{2}+x^2}\,dx}{3\ 2^{2/3}}$$

$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}} + \frac{\int \frac{1}{\sqrt[3]{2}-\sqrt[6]{2}\sqrt{3}x+x^2}\,dx}{12\ 2^{2/3}} + \frac{\int \frac{1}{\sqrt[3]{2}+\sqrt[6]{2}\sqrt{3}x+x^2}\,dx}{12\ 2^{2/3}} - \frac{\int \frac{-\sqrt[6]{2}\sqrt{3}+2x}{\sqrt[3]{2}-\sqrt[6]{2}\sqrt{3}x+x^2}\,dx}{4\ 2^{5/6}\sqrt{3}} + \frac{\int \frac{\sqrt[6]{2}\sqrt{3}+2x}{\sqrt[3]{2}+\sqrt[6]{2}\sqrt{3}x+x^2}\,dx}{4\ 2^{5/6}\sqrt{3}}$$

$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}\sqrt{3}x+x^2\right)}{4\ 2^{5/6}\sqrt{3}} + \frac{\log\left(\sqrt[3]{2}+\sqrt[6]{2}\sqrt{3}x+x^2\right)}{4\ 2^{5/6}\sqrt{3}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{1}{3}-x^2}\,dx, x, 1-\frac{2^{5/6}x}{\sqrt{3}}\right)}{6\ 2^{5/6}\sqrt{3}} - $$

$$= \frac{\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)}{3\ 2^{5/6}} - \frac{\tan^{-1}\left(\sqrt{3}-2^{5/6}x\right)}{6\ 2^{5/6}} + \frac{\tan^{-1}\left(\sqrt{3}+2^{5/6}x\right)}{6\ 2^{5/6}} - \frac{\log\left(\sqrt[3]{2}-\sqrt[6]{2}\sqrt{3}x+x^2\right)}{4\ 2^{5/6}\sqrt{3}} + \frac{\log\left(\sqrt[3]{2}+\sqrt[6]{2}\sqrt{3}x\right.}{4\ 2^{5/6}\sqrt{3}}$$

Mathematica [A] time = 0.0270939, size = 115, normalized size = 0.83

$$\frac{-\sqrt{3}\log\left(2^{2/3}x^2-2^{5/6}\sqrt{3}x+2\right)+\sqrt{3}\log\left(2^{2/3}x^2+2^{5/6}\sqrt{3}x+2\right)+4\tan^{-1}\left(\frac{x}{\sqrt[6]{2}}\right)-2\tan^{-1}\left(\sqrt{3}-2^{5/6}x\right)+2\tan^{-1}\left(2^{5/6}x\right.}{12\ 2^{5/6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x^6)^(-1),x]
```

```
[Out] (4*ArcTan[x/2^(1/6)] - 2*ArcTan[Sqrt[3] - 2^(5/6)*x] + 2*ArcTan[Sqrt[3] + 2
^(5/6)*x] - Sqrt[3]*Log[2 - 2^(5/6)*Sqrt[3]*x + 2^(2/3)*x^2] + Sqrt[3]*Log[
2 + 2^(5/6)*Sqrt[3]*x + 2^(2/3)*x^2])/(12*2^(5/6))
```

Maple [A] time = 0.098, size = 95, normalized size = 0.7

$$\frac{\sqrt[6]{2}}{6}\arctan\left(\frac{x2^{\frac{5}{6}}}{2}\right) + \frac{\arctan\left(x2^{\frac{5}{6}}-\sqrt{3}\right)\sqrt[6]{2}}{12} + \frac{\arctan\left(x2^{\frac{5}{6}}+\sqrt{3}\right)\sqrt[6]{2}}{12} - \frac{\ln\left(\sqrt[3]{2}+x^2-\sqrt[6]{2}x\sqrt{3}\right)\sqrt[6]{2}\sqrt{3}}{24} + \frac{\ln\left(\sqrt[3]{2}+x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6+2),x)
```

```
[Out] 1/6*arctan(1/2*x*2^(5/6))*2^(1/6)+1/12*arctan(x*2^(5/6)-3^(1/2))*2^(1/6)+1/
12*arctan(x*2^(5/6)+3^(1/2))*2^(1/6)-1/24*ln(2^(1/3)+x^2-2^(1/6)*x*3^(1/2))
*2^(1/6)*3^(1/2)+1/24*ln(2^(1/3)+x^2+2^(1/6)*x*3^(1/2))*2^(1/6)*3^(1/2)
```

Maxima [A] time = 1.43452, size = 144, normalized size = 1.04

$$\frac{1}{24}\sqrt{3}2^{\frac{1}{6}}\log\left(x^2+\sqrt{3}2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)-\frac{1}{24}\sqrt{3}2^{\frac{1}{6}}\log\left(x^2-\sqrt{3}2^{\frac{1}{6}}x+2^{\frac{1}{3}}\right)+\frac{1}{12}\cdot 2^{\frac{1}{6}}\arctan\left(\frac{1}{2}\cdot 2^{\frac{5}{6}}\left(2x+\sqrt{3}2^{\frac{1}{6}}\right)\right)+\frac{1}{12}\cdot 2^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2),x, algorithm="maxima")
```

```
[Out] 1/24*sqrt(3)*2^(1/6)*log(x^2 + sqrt(3)*2^(1/6)*x + 2^(1/3)) - 1/24*sqrt(3)*
2^(1/6)*log(x^2 - sqrt(3)*2^(1/6)*x + 2^(1/3)) + 1/12*2^(1/6)*arctan(1/2*2^
(5/6)*(2*x + sqrt(3)*2^(1/6))) + 1/12*2^(1/6)*arctan(1/2*2^(5/6)*(2*x - sqr
t(3)*2^(1/6))) + 1/6*2^(1/6)*arctan(1/2*2^(5/6)*x)
```

Fricas [A] time = 2.13277, size = 593, normalized size = 4.3

$$\frac{1}{384}\cdot 32^{\frac{5}{6}}\sqrt{3}\log\left(32^{\frac{5}{6}}\sqrt{3}x+16x^2+8\cdot 4^{\frac{2}{3}}\right)-\frac{1}{384}\cdot 32^{\frac{5}{6}}\sqrt{3}\log\left(-32^{\frac{5}{6}}\sqrt{3}x+16x^2+8\cdot 4^{\frac{2}{3}}\right)-\frac{1}{48}\cdot 32^{\frac{5}{6}}\arctan\left(\frac{1}{4}\cdot 32^{\frac{1}{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2),x, algorithm="fricas")
```

```
[Out] 1/384*32^(5/6)*sqrt(3)*log(32^(5/6)*sqrt(3)*x + 16*x^2 + 8*4^(2/3)) - 1/384
*32^(5/6)*sqrt(3)*log(-32^(5/6)*sqrt(3)*x + 16*x^2 + 8*4^(2/3)) - 1/48*32^(
5/6)*arctan(1/4*32^(1/6)*sqrt(2)*sqrt(2*x^2 + 4^(2/3)) - 1/2*32^(1/6)*x) -
1/96*32^(5/6)*arctan(-32^(1/6)*x + 1/4*32^(1/6)*sqrt(32^(5/6)*sqrt(3)*x + 1
6*x^2 + 8*4^(2/3)) - sqrt(3)) - 1/96*32^(5/6)*arctan(-32^(1/6)*x + 1/4*32^(
1/6)*sqrt(-32^(5/6)*sqrt(3)*x + 16*x^2 + 8*4^(2/3)) + sqrt(3))
```

Sympy [A] time = 0.255566, size = 14, normalized size = 0.1

$$\operatorname{RootSum}\left(1492992t^6+1,\left(t\mapsto t\log\left(12t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**6+2),x)
```

```
[Out] RootSum(1492992*_t**6 + 1, Lambda(_t, _t*log(12*_t + x)))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^6+2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.49 $\int \frac{1}{1+x^8}\,dx$

Optimal. Leaf size=339

$$-\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)+\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)-\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right.$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2 - Sqrt[2]
)]) - ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2*(2 + Sq
rt[2])]) + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2
 - Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt
[2*(2 + Sqrt[2])]) - (Sqrt[2 - Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])
/16 + (Sqrt[2 - Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/16 - (Sqrt[2 +
 Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/16 + (Sqrt[2 + Sqrt[2]]*Log[1
 + Sqrt[2 + Sqrt[2]]*x + x^2])/16
```

Rubi [A] time = 0.23485, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {213, 1169, 634, 618, 204, 628}

$$-\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(x^2-\sqrt{2-\sqrt{2}}x+1\right)+\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(x^2+\sqrt{2-\sqrt{2}}x+1\right)-\frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(x^2-\sqrt{2+\sqrt{2}}x+1\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^8)^(-1),x]
```


```
[Out] -ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2 - Sqrt[2]
)]) - ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt[2*(2 + Sq
rt[2])]) + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]]/(4*Sqrt[2*(2
 - Sqrt[2])]) + ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]]/(4*Sqrt
[2*(2 + Sqrt[2])]) - (Sqrt[2 - Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])
/16 + (Sqrt[2 - Sqrt[2]]*Log[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/16 - (Sqrt[2 +
 Sqrt[2]]*Log[1 - Sqrt[2 + Sqrt[2]]*x + x^2])/16 + (Sqrt[2 + Sqrt[2]]*Log[1
 + Sqrt[2 + Sqrt[2]]*x + x^2])/16
```


Rule 213

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Dist[r/(2*Sqrt[2]*a), Int[(Sqrt[2]*r -
s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Dist[r/(2*Sq
rt[2]*a), Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n
/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{1+x^8}\,dx = \frac{\int \frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}\,dx}{2\sqrt{2}} + \frac{\int \frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}\,dx}{2\sqrt{2}}$$

$$= \frac{\int \frac{\sqrt{2(2-\sqrt{2})}-(-1+\sqrt{2})x}{1-\sqrt{2-\sqrt{2}}x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2(2-\sqrt{2})}+(-1+\sqrt{2})x}{1+\sqrt{2-\sqrt{2}}x+x^2}\,dx}{4\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2(2+\sqrt{2})}-(1+\sqrt{2})x}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}} + \frac{\int \frac{\sqrt{2(2+\sqrt{2})}+(1+\sqrt{2})x}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx}{4\sqrt{2\left(2+\sqrt{2}\right)}}$$

$$= \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\int \frac{1}{1-\sqrt{2+\sqrt{2}}x+x^2}\,dx + \frac{1}{8}\sqrt{\frac{1}{2}\left(3-2\sqrt{2}\right)}\int \frac{1}{1+\sqrt{2+\sqrt{2}}x+x^2}\,dx - \frac{1}{16}\sqrt{2-\sqrt{2}}\cdots$$

$$= -\frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right) + \frac{1}{16}\sqrt{2-\sqrt{2}}\log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right) - \frac{1}{16}\sqrt{2+\sqrt{2}}\log\left(1-\cdots\right.$$

$$= -\frac{1}{8}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right) - \frac{1}{8}\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right) + \frac{1}{8}\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\cdots}}{\sqrt{2+\cdots}}\right.$$

Mathematica [A] time = 0.0055191, size = 209, normalized size = 0.62

$$-\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2-2x\sin\left(\frac{\pi}{8}\right)+1\right)+\frac{1}{8}\sin\left(\frac{\pi}{8}\right)\log\left(x^2+2x\sin\left(\frac{\pi}{8}\right)+1\right)-\frac{1}{8}\cos\left(\frac{\pi}{8}\right)\log\left(x^2-2x\cos\left(\frac{\pi}{8}\right)+1\right)+\cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^8)^(-1),x]
```

```
[Out] (ArcTan[Sec[Pi/8]*(x - Sin[Pi/8])]*Cos[Pi/8])/4 + (ArcTan[Sec[Pi/8]*(x + Si
n[Pi/8])]*Cos[Pi/8])/4 - (Cos[Pi/8]*Log[1 + x^2 - 2*x*Cos[Pi/8]])/8 + (Cos[
Pi/8]*Log[1 + x^2 + 2*x*Cos[Pi/8]])/8 + (ArcTan[(x - Cos[Pi/8])*Csc[Pi/8]]*
Sin[Pi/8])/4 + (ArcTan[(x + Cos[Pi/8])*Csc[Pi/8]]*Sin[Pi/8])/4 - (Log[1 + x
^2 - 2*x*Sin[Pi/8]]*Sin[Pi/8])/8 + (Log[1 + x^2 + 2*x*Sin[Pi/8]]*Sin[Pi/8])
/8
```

Maple [C] time = 0.005, size = 22, normalized size = 0.1

$$\frac{1}{8}\sum_{\_R=\mathrm{RootOf}(\_Z^8+1)}\frac{\ln(x-\_R)}{\_R^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8+1),x)
```

```
[Out] 1/8*sum(1/_R^7*ln(x-_R),_R=RootOf(_Z^8+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 + 1), x)
```

Fricas [B] time = 2.20618, size = 3183, normalized size = 9.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+1),x, algorithm="fricas")
```

```
[Out] -1/16*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*s
qrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(
2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqr
t(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) + s
qrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2*
sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt(
sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))
) + 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((2
*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 + 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqr
t(2)*x*sqrt(-sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(s
qrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 1/16*(sqrt(2)*sqrt(sqrt(2) + 2) -
 sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((2*sqrt(2)*x - 2*sqrt(2)*sqrt(x^2 - 1/2
*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - sqrt
(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2)
)) + 1/64*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2
```

```
+ 1/2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) +
 1/64*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2 + 1/
2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/6
4*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2 - 1/2*sq
rt(2)*x*sqrt(sqrt(2) + 2) + 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/64*(s
qrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(x^2 - 1/2*sqrt(2
)*x*sqrt(sqrt(2) + 2) - 1/2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 1) - 1/8*sqrt(sq
rt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + sqrt(-sq
rt(2) + 2))/sqrt(sqrt(2) + 2)) - 1/8*sqrt(sqrt(2) + 2)*arctan(-(2*x - 2*sqr
t(x^2 - x*sqrt(-sqrt(2) + 2) + 1) - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))
- 1/8*sqrt(-sqrt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(sqrt(2) + 2) +
1) + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) - 1/8*sqrt(-sqrt(2) + 2)*arctan
(-(2*x - 2*sqrt(x^2 - x*sqrt(sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2))/sqrt(-s
qrt(2) + 2)) + 1/32*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) -
1/32*sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/32*sqrt(-sqrt
(2) + 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/32*sqrt(-sqrt(2) + 2)*log(
x^2 - x*sqrt(-sqrt(2) + 2) + 1)
```

Sympy [A] time = 1.15493, size = 14, normalized size = 0.04

$$\operatorname{RootSum}\left(16777216t^8 + 1, \left(t \mapsto t \log\left(8t + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8+1),x)
```

```
[Out] RootSum(16777216*_t**8 + 1, Lambda(_t, _t*log(8*_t + x)))
```

Giac [A] time = 1.09509, size = 323, normalized size = 0.95

$$\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\frac{1}{8}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{\sqrt{2}+}}{\sqrt{-\sqrt{2}+2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8+1),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2))
+ 1/8*sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)
```

```
) + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) +
 2)) + 1/8*sqrt(-sqrt(2) + 2)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2
) + 2)) + 1/16*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/16*
sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/16*sqrt(-sqrt(2) +
 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/16*sqrt(-sqrt(2) + 2)*log(x^2 -
 x*sqrt(-sqrt(2) + 2) + 1)
```

3.50 $\int \frac{1}{-1+x^8}\,dx$

Optimal. Leaf size=97

$$\frac{\log\left(x^2-\sqrt{2}x+1\right)}{8\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{8\sqrt{2}}-\frac{1}{4}\tan^{-1}(x)+\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{1}{4}\tanh^{-1}(x)$$

```
[Out] -ArcTan[x]/4 + ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) - ArcTan[1 + Sqrt[2]*x]/(4
*Sqrt[2]) - ArcTanh[x]/4 + Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2]) - Log[1 + S
qrt[2]*x + x^2]/(8*Sqrt[2])
```

Rubi [A] time = 0.0520014, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.429, Rules used = {214, 212, 206, 203, 211, 1165, 628, 1162, 617, 204}

$$\frac{\log\left(x^2-\sqrt{2}x+1\right)}{8\sqrt{2}}-\frac{\log\left(x^2+\sqrt{2}x+1\right)}{8\sqrt{2}}-\frac{1}{4}\tan^{-1}(x)+\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{4\sqrt{2}}-\frac{\tan^{-1}\left(\sqrt{2}x+1\right)}{4\sqrt{2}}-\frac{1}{4}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^8)^(-1),x]
```

```
[Out] -ArcTan[x]/4 + ArcTan[1 - Sqrt[2]*x]/(4*Sqrt[2]) - ArcTan[1 + Sqrt[2]*x]/(4
*Sqrt[2]) - ArcTanh[x]/4 + Log[1 - Sqrt[2]*x + x^2]/(8*Sqrt[2]) - Log[1 + S
qrt[2]*x + x^2]/(8*Sqrt[2])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/
b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^(n/2
)), x], x] + Dist[r/(2*a), Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b},
x] && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{-1+x^8}\,dx &= -\left(\frac{1}{2}\int \frac{1}{1-x^4}\,dx\right) - \frac{1}{2}\int \frac{1}{1+x^4}\,dx \\
&= -\left(\frac{1}{4}\int \frac{1}{1-x^2}\,dx\right) - \frac{1}{4}\int \frac{1}{1+x^2}\,dx - \frac{1}{4}\int \frac{1-x^2}{1+x^4}\,dx - \frac{1}{4}\int \frac{1+x^2}{1+x^4}\,dx \\
&= -\frac{1}{4}\tan^{-1}(x) - \frac{1}{4}\tanh^{-1}(x) - \frac{1}{8}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx - \frac{1}{8}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx + \frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{8\sqrt{2}} + \cdots \\
&= -\frac{1}{4}\tan^{-1}(x) - \frac{1}{4}\tanh^{-1}(x) + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{8\sqrt{2}} - \frac{\log\left(1+\sqrt{2}x+x^2\right)}{8\sqrt{2}} - \frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1-\cdots\right.}{4\sqrt{2}} \cdots \\
&= -\frac{1}{4}\tan^{-1}(x) + \frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{4\sqrt{2}} - \frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{4\sqrt{2}} - \frac{1}{4}\tanh^{-1}(x) + \frac{\log\left(1-\sqrt{2}x+x^2\right)}{8\sqrt{2}} - \frac{\log\left(1+\cdots\right.}{8\cdots}
\end{aligned}
$$

Mathematica [A] time = 0.0257536, size = 98, normalized size = 1.01

$$\frac{1}{16}\left(\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)-\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+2\log(1-x)-2\log(x+1)-4\tan^{-1}(x)+2\sqrt{2}\tan^{-1}\left(1-\sqrt{2}x\right)\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^8)^(-1),x]
```

```
[Out] (-4*ArcTan[x] + 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*x] - 2*Sqrt[2]*ArcTan[1 + Sqrt
[2]*x] + 2*Log[1 - x] - 2*Log[1 + x] + Sqrt[2]*Log[1 - Sqrt[2]*x + x^2] - S
qrt[2]*Log[1 + Sqrt[2]*x + x^2])/16
```

Maple [A] time = 0., size = 66, normalized size = 0.7

$$-\frac{\operatorname{Artanh}(x)}{4}-\frac{\arctan(x)}{4}-\frac{\sqrt{2}}{16}\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)-\frac{\arctan\left(-1+x\sqrt{2}\right)\sqrt{2}}{8}-\frac{\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-1),x)
```

```
[Out] -1/4*arctanh(x)-1/4*arctan(x)-1/16*2^(1/2)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^
(1/2)))-1/8*arctan(-1+x*2^(1/2))*2^(1/2)-1/8*arctan(1+x*2^(1/2))*2^(1/2)
```

Maxima [A] time = 1.43254, size = 119, normalized size = 1.23

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)+\frac{1}{16}\sqrt{2}\log\left(x^2-\sqrt{2}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-1),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt
(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x) - 1/8*log(x + 1) + 1/8*log(x -
 1)
```

Fricas [A] time = 2.11882, size = 371, normalized size = 3.82

$$\frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\right)+\frac{1}{4}\sqrt{2}\arctan\left(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\right)-\frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-1),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) + 1/
4*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) - 1/16
*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt(2)*log(x^2 - sqrt(2)*x + 1) -
```

```
1/4*arctan(x) - 1/8*log(x + 1) + 1/8*log(x - 1)
```

Sympy [C] time = 128.354, size = 44, normalized size = 0.45

$$\frac{\log(x-1)}{8} - \frac{\log(x+1)}{8} + \frac{i\log(x-i)}{8} - \frac{i\log(x+i)}{8} + \operatorname{RootSum}\left(4096t^4+1,(t\mapsto t\log(-8t+x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-1),x)
```

```
[Out] log(x - 1)/8 - log(x + 1)/8 + I*log(x - I)/8 - I*log(x + I)/8 + RootSum(409
6*_t**4 + 1, Lambda(_t, _t*log(-8*_t + x)))
```

Giac [A] time = 1.08627, size = 122, normalized size = 1.26

$$-\frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right) - \frac{1}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right) - \frac{1}{16}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right) + \frac{1}{16}\sqrt{2}\log\left(x^2-\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-1),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) - 1/8*sqrt(2)*arctan(1/2*s
qrt(2)*(2*x - sqrt(2))) - 1/16*sqrt(2)*log(x^2 + sqrt(2)*x + 1) + 1/16*sqrt
(2)*log(x^2 - sqrt(2)*x + 1) - 1/4*arctan(x) - 1/8*log(abs(x + 1)) + 1/8*lo
g(abs(x - 1))
```

3.51 $\int \frac{1}{1-x^4+x^8}\,dx$

Optimal. Leaf size=275

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\tan^{-}}{}$$

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(
Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]]
+ 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/
(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*
Sqrt[6])
```

Rubi [A] time = 0.262004, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {1346, 1169, 634, 618, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2-\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\log\left(x^2-\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}+\frac{\log\left(x^2+\sqrt{2+\sqrt{3}}x+1\right)}{4\sqrt{6}}-\frac{\tan^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^4 + x^8)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[2 - Sqrt[3]] - 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) - ArcTan[(
Sqrt[2 + Sqrt[3]] - 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 -
Sqrt[3]] + 2*x)/Sqrt[2 + Sqrt[3]]]/(2*Sqrt[6]) + ArcTan[(Sqrt[2 + Sqrt[3]]
+ 2*x)/Sqrt[2 - Sqrt[3]]]/(2*Sqrt[6]) - Log[1 - Sqrt[2 - Sqrt[3]]*x + x^2]/
(4*Sqrt[6]) + Log[1 + Sqrt[2 - Sqrt[3]]*x + x^2]/(4*Sqrt[6]) - Log[1 - Sqrt
[2 + Sqrt[3]]*x + x^2]/(4*Sqrt[6]) + Log[1 + Sqrt[2 + Sqrt[3]]*x + x^2]/(4*
Sqrt[6])
```

Rule 1346

```
Int[((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_))^(-1), x_Symbol] :> With[{q
= Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x^(n
```

```
/2))/(q - r*x^(n/2) + x^n), x], x] + Dist[1/(2*c*q*r), Int[(r + x^(n/2))/(q
 + r*x^(n/2) + x^n), x], x]]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[n/2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{1-x^4+x^8}\,dx = \frac{\int \frac{\sqrt{3}-x^2}{1-\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}} + \frac{\int \frac{\sqrt{3}+x^2}{1+\sqrt{3}x^2+x^4}\,dx}{2\sqrt{3}}$$

$$= \frac{\int \frac{\sqrt{3(2-\sqrt{3})}-(-1+\sqrt{3})x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3(2-\sqrt{3})}+(-1+\sqrt{3})x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2-\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3(2+\sqrt{3})}-(1+\sqrt{3})x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}} + \frac{\int \frac{\sqrt{3(2+\sqrt{3})}+(1+\sqrt{3})x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{3\left(2+\sqrt{3}\right)}}$$

$$= -\frac{\int \frac{-\sqrt{2-\sqrt{3}}+2x}{1-\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2-\sqrt{3}}+2x}{1+\sqrt{2-\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} - \frac{\int \frac{-\sqrt{2+\sqrt{3}}+2x}{1-\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{\sqrt{2+\sqrt{3}}+2x}{1+\sqrt{2+\sqrt{3}}x+x^2}\,dx}{4\sqrt{6}} + \frac{\int \frac{1}{1-\sqrt{2-\sqrt{3}}x+x^2}\,d}{4\sqrt{6\left(2-\sqrt{3}\right)}}$$

$$= -\frac{\log\left(1-\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2-\sqrt{3}}x+x^2\right)}{4\sqrt{6}} - \frac{\log\left(1-\sqrt{2+\sqrt{3}}x+x^2\right)}{4\sqrt{6}} + \frac{\log\left(1+\sqrt{2}\right.}{4}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}-2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}-2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2-\sqrt{3}}+2x}{\sqrt{2+\sqrt{3}}}\right)}{2\sqrt{6}} + \frac{\tan^{-1}\left(\frac{\sqrt{2+\sqrt{3}}+2x}{\sqrt{2-\sqrt{3}}}\right)}{2\sqrt{6}} - \frac{\log\left(1-\sqrt{2-}\right.}{4\sqrt{6}}$$

Mathematica [C] time = 0.0117178, size = 42, normalized size = 0.15

$$\frac{1}{4}\text{RootSum}\left[\#1^8-\#1^4+1\&,\frac{\log(x-\#1)}{2\#1^7-\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^4 + x^8)^(-1),x]
```

```
[Out] RootSum[1 - #1^4 + #1^8 & , Log[x - #1]/(-#1^3 + 2*#1^7) & ]/4
```

Maple [C] time = 0.007, size = 30, normalized size = 0.1

$$\frac{\sum_{\_R=\text{RootOf}\left(9\_Z^4+1\right)} \_R \ln\left(3\_R^2+3\_R\,x+x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-x^4+1),x)
```

```
[Out] 1/4*sum(_R*ln(3*_R^2+3*_R*x+x^2),_R=RootOf(9*_Z^4+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^8 - x^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="maxima")
```

```
[Out] integrate(1/(x^8 - x^4 + 1), x)
```

Fricas [A] time = 2.13769, size = 608, normalized size = 2.21

$$-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}\left(x^3-x\right)+x^2-\sqrt{x^4+\sqrt{3}\sqrt{2}\left(x^3+x\right)+3\,x^2+1}\left(\sqrt{3}\sqrt{2}x-2\right)}{3\,x^2-2}\right)-\frac{1}{6}\sqrt{3}\sqrt{2}\arctan\left(-\frac{\sqrt{3}\sqrt{2}(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) + x^2 - sqrt(x^4 +
sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x - 2))/(3*x^2 - 2)
) - 1/6*sqrt(3)*sqrt(2)*arctan(-(sqrt(3)*sqrt(2)*(x^3 - x) - x^2 - sqrt(x^4
 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)*(sqrt(3)*sqrt(2)*x + 2))/(3*x^2 -
 2)) + 1/24*sqrt(3)*sqrt(2)*log(x^4 + sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1
) - 1/24*sqrt(3)*sqrt(2)*log(x^4 - sqrt(3)*sqrt(2)*(x^3 + x) + 3*x^2 + 1)
```

Sympy [A] time = 0.203264, size = 165, normalized size = 0.6

$$\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}-\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3-4x^2+2\sqrt{6}x-3\right)\right)}{24}+\frac{\sqrt{6}\left(2\,\mathrm{atan}\left(\frac{\sqrt{6}x}{3}+\frac{1}{3}\right)+2\,\mathrm{atan}\left(\sqrt{6}x^3+4x^2+2\sqrt{6}x+3\right.\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-x**4+1),x)
```

```
[Out] sqrt(6)*(2*atan(sqrt(6)*x/3 - 1/3) + 2*atan(sqrt(6)*x**3 - 4*x**2 + 2*sqrt(
6)*x - 3))/24 + sqrt(6)*(2*atan(sqrt(6)*x/3 + 1/3) + 2*atan(sqrt(6)*x**3 +
4*x**2 + 2*sqrt(6)*x + 3))/24 - sqrt(6)*log(x**4 - sqrt(6)*x**3 + 3*x**2 -
sqrt(6)*x + 1)/24 + sqrt(6)*log(x**4 + sqrt(6)*x**3 + 3*x**2 + sqrt(6)*x +
1)/24
```

<u>Giac</u> [A] time = 1.09782, size = 277, normalized size = 1.01

$$\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x-\sqrt{6}+\sqrt{2}}{\sqrt{6}+\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(\frac{4x+\sqrt{6}+\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right)+\frac{1}{12}\sqrt{6}\arctan\left(4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-x^4+1),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(6)*arctan((4*x + sqrt(6) - sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*s
qrt(6)*arctan((4*x - sqrt(6) + sqrt(2))/(sqrt(6) + sqrt(2))) + 1/12*sqrt(6)
*arctan((4*x + sqrt(6) + sqrt(2))/(sqrt(6) - sqrt(2))) + 1/12*sqrt(6)*arcta
n((4*x - sqrt(6) - sqrt(2))/(sqrt(6) - sqrt(2))) + 1/24*sqrt(6)*log(x^2 + 1
/2*x*(sqrt(6) + sqrt(2)) + 1) - 1/24*sqrt(6)*log(x^2 - 1/2*x*(sqrt(6) + sqr
t(2)) + 1) + 1/24*sqrt(6)*log(x^2 + 1/2*x*(sqrt(6) - sqrt(2)) + 1) - 1/24*s
qrt(6)*log(x^2 - 1/2*x*(sqrt(6) - sqrt(2)) + 1)
```

3.52 $\int \frac{x^7}{1+x^{12}} \, dx$

Optimal. Leaf size=49

$$-\frac{1}{12}\log\left(x^4+1\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4]/12 + Log[1 - x^4 +
x^8]/24
```

Rubi [A] time = 0.037024, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {275, 292, 31, 634, 618, 204, 628}

$$-\frac{1}{12}\log\left(x^4+1\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/(1 + x^12),x]
```

```
[Out] -ArcTan[(1 - 2*x^4)/Sqrt[3]]/(4*Sqrt[3]) - Log[1 + x^4]/12 + Log[1 - x^4 +
x^8]/24
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{1+x^{12}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{x}{1+x^3}\,dx, x, x^4\right)\\
&= -\left(\frac{1}{12}\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^4\right)\right)+\frac{1}{12}\operatorname{Subst}\left(\int \frac{1+x}{1-x+x^2}\,dx, x, x^4\right)\\
&= -\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\operatorname{Subst}\left(\int \frac{-1+2x}{1-x+x^2}\,dx, x, x^4\right)+\frac{1}{8}\operatorname{Subst}\left(\int \frac{1}{1-x+x^2}\,dx, x, x^4\right)\\
&= -\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\log\left(1-x^4+x^8\right)-\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, -1+2x^4\right)\\
&= -\frac{\tan^{-1}\left(\frac{1-2x^4}{\sqrt{3}}\right)}{4\sqrt{3}}-\frac{1}{12}\log\left(1+x^4\right)+\frac{1}{24}\log\left(1-x^4+x^8\right)
\end{aligned}$$

Mathematica [B] time = 0.110216, size = 260, normalized size = 5.31

$$\frac{1}{24}\left(-2\log\left(x^2-\sqrt{2}x+1\right)-2\log\left(x^2+\sqrt{2}x+1\right)+\log\left(2x^2-\sqrt{6}x+\sqrt{2}x+2\right)+\log\left(2x^2+\sqrt{2}\left(\sqrt{3}-1\right)x+2\right)+\log\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/(1 + x^12),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(1 + Sqrt[3] - 2*Sqrt[2]*x)/(1 - Sqrt[3])] - 2*Sqrt[3]*Ar
cTan[(1 - Sqrt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] + 2*Sqrt[3]*ArcTan[(-1 + Sq
rt[3] + 2*Sqrt[2]*x)/(1 + Sqrt[3])] - 2*Sqrt[3]*ArcTan[(1 + Sqrt[3] + 2*Sqr
t[2]*x)/(-1 + Sqrt[3])] - 2*Log[1 - Sqrt[2]*x + x^2] - 2*Log[1 + Sqrt[2]*x
+ x^2] + Log[2 + Sqrt[2]*x - Sqrt[6]*x + 2*x^2] + Log[2 + Sqrt[2]*(-1 + Sqr
t[3])*x + 2*x^2] + Log[2 - (Sqrt[2] + Sqrt[6])*x + 2*x^2] + Log[2 + (Sqrt[2
] + Sqrt[6])*x + 2*x^2])/24
```

Maple [A] time = 0.005, size = 41, normalized size = 0.8

$$-\frac{\ln\left(x^4+1\right)}{12}+\frac{\ln\left(x^8-x^4+1\right)}{24}+\frac{\sqrt{3}}{12}\arctan\left(\frac{\left(2\,x^4-1\right)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(x^12+1),x)
```

```
[Out] -1/12*ln(x^4+1)+1/24*ln(x^8-x^4+1)+1/12*3^(1/2)*arctan(1/3*(2*x^4-1)*3^(1/2
))
```

Maxima [A] time = 1.44634, size = 54, normalized size = 1.1

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^4-1\right)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(x^12+1),x, algorithm="maxima")
```

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/12*log(x^4 + 1)

Fricas [A] time = 2.04143, size = 124, normalized size = 2.53

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^12+1),x, algorithm="fricas")

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/12*log(x^4 + 1)

Sympy [A] time = 0.166582, size = 46, normalized size = 0.94

$$-\frac{\log\left(x^4+1\right)}{12}+\frac{\log\left(x^8-x^4+1\right)}{24}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^4}{3}-\frac{\sqrt{3}}{3}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7/(x**12+1),x)

[Out] -log(x**4 + 1)/12 + log(x**8 - x**4 + 1)/24 + sqrt(3)*atan(2*sqrt(3)*x**4/3 - sqrt(3)/3)/12

Giac [A] time = 1.0982, size = 54, normalized size = 1.1

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^4-1\right)\right)+\frac{1}{24}\log\left(x^8-x^4+1\right)-\frac{1}{12}\log\left(x^4+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/(x^12+1),x, algorithm="giac")

[Out] 1/12*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^4 - 1)) + 1/24*log(x^8 - x^4 + 1) - 1/12*log(x^4 + 1)

3.53 $\int \log(x)\,dx$

Optimal. Leaf size=8

$$x\log(x) - x$$

```
[Out] -x + x*Log[x]
```

Rubi [A] time = 0.001005, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {2295}

$$x\log(x) - x$$

Antiderivative was successfully verified.

```
[In] Int[Log[x],x]
```

```
[Out] -x + x*Log[x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int \log(x)\,dx = -x + x\log(x)$$

Mathematica [A] time = 0.000539, size = 8, normalized size = 1.

$$x\log(x) - x$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x],x]
```

```
[Out] -x + x*Log[x]
```

Maple [A] time = 0., size = 9, normalized size = 1.1

$$-x + x\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x),x)
```

```
[Out] -x+x*ln(x)
```

Maxima [A] time = 0.964622, size = 11, normalized size = 1.38

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="maxima")
```

```
[Out] x*log(x) - x
```

Fricas [A] time = 1.88868, size = 19, normalized size = 2.38

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="fricas")
```

```
[Out] x*log(x) - x
```

Sympy [A] time = 0.079962, size = 5, normalized size = 0.62

$$x\log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x),x)
```

```
[Out] x*log(x) - x
```

Giac [A] time = 1.06649, size = 11, normalized size = 1.38

$$x \log(x) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x),x, algorithm="giac")
```

```
[Out] x*log(x) - x
```

3.54 $\int x \log(x) \, dx$

Optimal. Leaf size=17

$$\frac{1}{2}x^2\log(x) - \frac{x^2}{4}$$

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rubi [A] time = 0.0039296, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2304}

$$\frac{1}{2}x^2\log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x],x]
```

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x\log(x)\,dx = -\frac{x^2}{4} + \frac{1}{2}x^2\log(x)$$

Mathematica [A] time = 0.0007304, size = 17, normalized size = 1.

$$\frac{1}{2}x^2\log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x],x]
```

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Maple [A] time = 0., size = 14, normalized size = 0.8

$$-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x),x)
```

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Maxima [A] time = 0.939588, size = 18, normalized size = 1.06

$$\frac{1}{2}x^2 \log(x) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Fricas [A] time = 1.89497, size = 35, normalized size = 2.06

$$\frac{1}{2}x^2 \log(x) - \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A] time = 0.084395, size = 12, normalized size = 0.71

$$\frac{x^2 \log(x)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x),x)
```

```
[Out] x**2*log(x)/2 - x**2/4
```

Giac [A] time = 1.09176, size = 18, normalized size = 1.06

$$\frac{1}{2} x^2 \log(x) - \frac{1}{4} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

3.55 $\int x^2 \log(x) \, dx$

Optimal. Leaf size=17

$$\frac{1}{3}x^3 \log(x) - \frac{x^3}{9}$$

```
[Out] -x^3/9 + (x^3*Log[x])/3
```

Rubi [A] time = 0.0063653, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2304}

$$\frac{1}{3}x^3 \log(x) - \frac{x^3}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[x],x]
```

```
[Out] -x^3/9 + (x^3*Log[x])/3
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \log(x) \, dx = -\frac{x^3}{9} + \frac{1}{3}x^3 \log(x)$$

Mathematica [A] time = 0.0008194, size = 17, normalized size = 1.

$$\frac{1}{3}x^3 \log(x) - \frac{x^3}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[x],x]
```

```
[Out] -x^3/9 + (x^3*Log[x])/3
```

Maple [A] time = 0.002, size = 14, normalized size = 0.8

$$-\frac{x^3}{9}+\frac{x^3\ln\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(x),x)
```

```
[Out] -1/9*x^3+1/3*x^3*ln(x)
```

Maxima [A] time = 0.945992, size = 18, normalized size = 1.06

$$\frac{1}{3}\,x^3\log\left(x\right)-\frac{1}{9}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(x) - 1/9*x^3
```

Fricas [A] time = 1.9229, size = 35, normalized size = 2.06

$$\frac{1}{3}\,x^3\log\left(x\right)-\frac{1}{9}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*log(x) - 1/9*x^3
```

Sympy [A] time = 0.085538, size = 12, normalized size = 0.71

$$\frac{x^3 \log(x)}{3} - \frac{x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(x),x)
```

```
[Out] x**3*log(x)/3 - x**3/9
```

Giac [A] time = 1.08936, size = 18, normalized size = 1.06

$$\frac{1}{3} x^3 \log(x) - \frac{1}{9} x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(x) - 1/9*x^3
```

3.56 $\int x^p \log(x) \, dx$

Optimal. Leaf size=26

$$\frac{x^{p+1}\log(x)}{p+1} - \frac{x^{p+1}}{(p+1)^2}$$

```
[Out] -(x^(1 + p)/(1 + p)^2) + (x^(1 + p)*Log[x])/(1 + p)
```

Rubi [A] time = 0.0098902, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2304}

$$\frac{x^{p+1}\log(x)}{p+1} - \frac{x^{p+1}}{(p+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^p*Log[x],x]
```

```
[Out] -(x^(1 + p)/(1 + p)^2) + (x^(1 + p)*Log[x])/(1 + p)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^p \log(x) \, dx = -\frac{x^{1+p}}{(1+p)^2} + \frac{x^{1+p}\log(x)}{1+p}$$

Mathematica [A] time = 0.0074999, size = 19, normalized size = 0.73

$$\frac{x^{p+1}((p+1)\log(x)-1)}{(p+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^p*Log[x],x]
```

```
[Out] (x^(1 + p)*(-1 + (1 + p)*Log[x]))/(1 + p)^2
```

Maple [A] time = 0.007, size = 34, normalized size = 1.3

$$\frac{x\ln(x)\,\mathrm{e}^{\ln(x)p}}{1+p}-\frac{x\mathrm{e}^{\ln(x)p}}{p^2+2p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^p*ln(x),x)
```

```
[Out] 1/(1+p)*x*ln(x)*exp(ln(x)*p)-1/(p^2+2*p+1)*x*exp(ln(x)*p)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*log(x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.91599, size = 59, normalized size = 2.27

$$\frac{\left(\left(p+1\right)x\log\left(x\right)-x\right)x^p}{p^2+2p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*log(x),x, algorithm="fricas")
```

```
[Out] ((p + 1)*x*log(x) - x)*x^p/(p^2 + 2*p + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**p*ln(x),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^p \log(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^p*log(x),x, algorithm="giac")
```

```
[Out] integrate(x^p*log(x), x)
```

3.57 $\int \log^2(x)\,dx$

Optimal. Leaf size=15

$$2x + x\log^2(x) - 2x\log(x)$$

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Rubi [A] time = 0.0036137, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2296, 2295}

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^2,x]
```

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \log^2(x)\,dx &= x\log^2(x) - 2\int \log(x)\,dx\\ &= 2x - 2x\log(x) + x\log^2(x)\end{aligned}$$

Mathematica [A] time = 0.0008687, size = 15, normalized size = 1.

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^2,x]
```

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Maple [A] time = 0., size = 16, normalized size = 1.1

$$2\,x - 2\,x\ln(x) + x(\ln(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2,x)
```

```
[Out] 2*x-2*x*ln(x)+x*ln(x)^2
```

Maxima [A] time = 0.943169, size = 16, normalized size = 1.07

$$\left(\log(x)^2 - 2\,\log(x) + 2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="maxima")
```

```
[Out] (log(x)^2 - 2*log(x) + 2)*x
```

Fricas [A] time = 2.06514, size = 42, normalized size = 2.8

$$x\log(x)^2 - 2\,x\log(x) + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="fricas")
```

```
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

Sympy [A] time = 0.091383, size = 15, normalized size = 1.

$$x\log(x)^2 - 2x\log(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2,x)
```

```
[Out] x*log(x)**2 - 2*x*log(x) + 2*x
```

Giac [A] time = 1.09479, size = 20, normalized size = 1.33

$$x\log(x)^2 - 2\,x\log(x) + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="giac")
```

```
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

3.58 $\int x^9 \log^{11}(x)\,dx$

Optimal. Leaf size=127

$$-\frac{6237x^{10}}{156250000}+\frac{1}{10}x^{10}\log^{11}(x)-\frac{11}{100}x^{10}\log^{10}(x)+\frac{11}{100}x^{10}\log^{9}(x)-\frac{99x^{10}\log^{8}(x)}{1000}+\frac{99x^{10}\log^{7}(x)}{1250}-\frac{693x^{10}\log^{6}(x)}{12500}+$$

```
[Out] (-6237*x^10)/156250000 + (6237*x^10*Log[x])/15625000 - (6237*x^10*Log[x]^2)
/3125000 + (2079*x^10*Log[x]^3)/312500 - (2079*x^10*Log[x]^4)/125000 + (207
9*x^10*Log[x]^5)/62500 - (693*x^10*Log[x]^6)/12500 + (99*x^10*Log[x]^7)/125
0 - (99*x^10*Log[x]^8)/1000 + (11*x^10*Log[x]^9)/100 - (11*x^10*Log[x]^10)/
100 + (x^10*Log[x]^11)/10
```

Rubi [A] time = 0.141166, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2305, 2304}

$$-\frac{6237x^{10}}{156250000}+\frac{1}{10}x^{10}\log^{11}(x)-\frac{11}{100}x^{10}\log^{10}(x)+\frac{11}{100}x^{10}\log^{9}(x)-\frac{99x^{10}\log^{8}(x)}{1000}+\frac{99x^{10}\log^{7}(x)}{1250}-\frac{693x^{10}\log^{6}(x)}{12500}+$$

Antiderivative was successfully verified.

```
[In] Int[x^9*Log[x]^11,x]
```


```
[Out] (-6237*x^10)/156250000 + (6237*x^10*Log[x])/15625000 - (6237*x^10*Log[x]^2)
/3125000 + (2079*x^10*Log[x]^3)/312500 - (2079*x^10*Log[x]^4)/125000 + (207
9*x^10*Log[x]^5)/62500 - (693*x^10*Log[x]^6)/12500 + (99*x^10*Log[x]^7)/125
0 - (99*x^10*Log[x]^8)/1000 + (11*x^10*Log[x]^9)/100 - (11*x^10*Log[x]^10)/
100 + (x^10*Log[x]^11)/10
```


Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
```

```
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int x^9 \log^{11}(x)\,dx &= \frac{1}{10}x^{10}\log^{11}(x) - \frac{11}{10}\int x^9\log^{10}(x)\,dx \\
&= -\frac{11}{100}x^{10}\log^{10}(x) + \frac{1}{10}x^{10}\log^{11}(x) + \frac{11}{10}\int x^9\log^9(x)\,dx \\
&= \frac{11}{100}x^{10}\log^9(x) - \frac{11}{100}x^{10}\log^{10}(x) + \frac{1}{10}x^{10}\log^{11}(x) - \frac{99}{100}\int x^9\log^8(x)\,dx \\
&= -\frac{99x^{10}\log^8(x)}{1000} + \frac{11}{100}x^{10}\log^9(x) - \frac{11}{100}x^{10}\log^{10}(x) + \frac{1}{10}x^{10}\log^{11}(x) + \frac{99}{125}\int x^9\log^7(x)\,dx \\
&= \frac{99x^{10}\log^7(x)}{1250} - \frac{99x^{10}\log^8(x)}{1000} + \frac{11}{100}x^{10}\log^9(x) - \frac{11}{100}x^{10}\log^{10}(x) + \frac{1}{10}x^{10}\log^{11}(x) - \frac{693\int x^9\log^{6}}{1250} \\
&= -\frac{693x^{10}\log^6(x)}{12500} + \frac{99x^{10}\log^7(x)}{1250} - \frac{99x^{10}\log^8(x)}{1000} + \frac{11}{100}x^{10}\log^9(x) - \frac{11}{100}x^{10}\log^{10}(x) + \frac{1}{10}x^{10}\log^{1} \\
&= \frac{2079x^{10}\log^5(x)}{62500} - \frac{693x^{10}\log^6(x)}{12500} + \frac{99x^{10}\log^7(x)}{1250} - \frac{99x^{10}\log^8(x)}{1000} + \frac{11}{100}x^{10}\log^9(x) - \frac{11}{100}x^{10}\log^{1} \\
&= -\frac{2079x^{10}\log^4(x)}{125000} + \frac{2079x^{10}\log^5(x)}{62500} - \frac{693x^{10}\log^6(x)}{12500} + \frac{99x^{10}\log^7(x)}{1250} - \frac{99x^{10}\log^8(x)}{1000} + \frac{11}{100}x^{10}\mathrm{l} \\
&= \frac{2079x^{10}\log^3(x)}{312500} - \frac{2079x^{10}\log^4(x)}{125000} + \frac{2079x^{10}\log^5(x)}{62500} - \frac{693x^{10}\log^6(x)}{12500} + \frac{99x^{10}\log^7(x)}{1250} - \frac{99x^{10}\mathrm{lo}}{100} \\
&= -\frac{6237x^{10}\log^2(x)}{3125000} + \frac{2079x^{10}\log^3(x)}{312500} - \frac{2079x^{10}\log^4(x)}{125000} + \frac{2079x^{10}\log^5(x)}{62500} - \frac{693x^{10}\log^6(x)}{12500} + 99x \\
&= -\frac{6237x^{10}}{156250000} + \frac{6237x^{10}\log(x)}{15625000} - \frac{6237x^{10}\log^2(x)}{3125000} + \frac{2079x^{10}\log^3(x)}{312500} - \frac{2079x^{10}\log^4(x)}{125000} + \frac{2079x^{10}}{625}
\end{aligned}
$$

Mathematica [A] time = 0.0032175, size = 127, normalized size = 1.

$$-\frac{6237x^{10}}{156250000} + \frac{1}{10}x^{10}\log^{11}(x) - \frac{11}{100}x^{10}\log^{10}(x) + \frac{11}{100}x^{10}\log^9(x) - \frac{99x^{10}\log^8(x)}{1000} + \frac{99x^{10}\log^7(x)}{1250} - \frac{693x^{10}\log^6(x)}{12500} +$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9*Log[x]^11,x]
```

```
[Out] (-6237*x^10)/156250000 + (6237*x^10*Log[x])/15625000 - (6237*x^10*Log[x]^2)
/3125000 + (2079*x^10*Log[x]^3)/312500 - (2079*x^10*Log[x]^4)/125000 + (207
9*x^10*Log[x]^5)/62500 - (693*x^10*Log[x]^6)/12500 + (99*x^10*Log[x]^7)/125
```

```
0 - (99*x^10*Log[x]^8)/1000 + (11*x^10*Log[x]^9)/100 - (11*x^10*Log[x]^10)/
100 + (x^10*Log[x]^11)/10
```

Maple [A] time = 0.002, size = 104, normalized size = 0.8

$$-\frac{6237\,x^{10}}{156250000}+\frac{6237\,x^{10}\ln(x)}{15625000}-\frac{6237\,x^{10}(\ln(x))^{2}}{3125000}+\frac{2079\,x^{10}(\ln(x))^{3}}{312500}-\frac{2079\,x^{10}(\ln(x))^{4}}{125000}+\frac{2079\,x^{10}(\ln(x))^{5}}{62500}-\frac{693\,x}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*ln(x)^11,x)
```

```
[Out] -6237/156250000*x^10+6237/15625000*x^10*ln(x)-6237/3125000*x^10*ln(x)^2+207
9/312500*x^10*ln(x)^3-2079/125000*x^10*ln(x)^4+2079/62500*x^10*ln(x)^5-693/
12500*x^10*ln(x)^6+99/1250*x^10*ln(x)^7-99/1000*x^10*ln(x)^8+11/100*x^10*ln
(x)^9-11/100*x^10*ln(x)^10+1/10*x^10*ln(x)^11
```

Maxima [A] time = 0.936561, size = 96, normalized size = 0.76

$$\frac{1}{156250000}\left(15625000\,\log(x)^{11}-17187500\,\log(x)^{10}+17187500\,\log(x)^{9}-15468750\,\log(x)^{8}+12375000\,\log(x)^{7}-8\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*log(x)^11,x, algorithm="maxima")
```

```
[Out] 1/156250000*(15625000*log(x)^11 - 17187500*log(x)^10 + 17187500*log(x)^9 -
15468750*log(x)^8 + 12375000*log(x)^7 - 8662500*log(x)^6 + 5197500*log(x)^5
 - 2598750*log(x)^4 + 1039500*log(x)^3 - 311850*log(x)^2 + 62370*log(x) - 6
237)*x^10
```

Fricas [A] time = 2.05302, size = 410, normalized size = 3.23

$$\frac{1}{10}\,x^{10}\log(x)^{11}-\frac{11}{100}\,x^{10}\log(x)^{10}+\frac{11}{100}\,x^{10}\log(x)^{9}-\frac{99}{1000}\,x^{10}\log(x)^{8}+\frac{99}{1250}\,x^{10}\log(x)^{7}-\frac{693}{12500}\,x^{10}\log(x)^{6}+\frac{207}{625}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*log(x)^11,x, algorithm="fricas")
```

```
[Out] 1/10*x^10*log(x)^11 - 11/100*x^10*log(x)^10 + 11/100*x^10*log(x)^9 - 99/100
0*x^10*log(x)^8 + 99/1250*x^10*log(x)^7 - 693/12500*x^10*log(x)^6 + 2079/62
500*x^10*log(x)^5 - 2079/125000*x^10*log(x)^4 + 2079/312500*x^10*log(x)^3 -
 6237/3125000*x^10*log(x)^2 + 6237/15625000*x^10*log(x) - 6237/156250000*x^
10
```

Sympy [A] time = 0.262542, size = 133, normalized size = 1.05

$$\frac{x^{10}\log(x)^{11}}{10} - \frac{11x^{10}\log(x)^{10}}{100} + \frac{11x^{10}\log(x)^{9}}{100} - \frac{99x^{10}\log(x)^{8}}{1000} + \frac{99x^{10}\log(x)^{7}}{1250} - \frac{693x^{10}\log(x)^{6}}{12500} + \frac{2079x^{10}\log(x)^{5}}{62500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*ln(x)**11,x)
```

```
[Out] x**10*log(x)**11/10 - 11*x**10*log(x)**10/100 + 11*x**10*log(x)**9/100 - 99
*x**10*log(x)**8/1000 + 99*x**10*log(x)**7/1250 - 693*x**10*log(x)**6/12500
 + 2079*x**10*log(x)**5/62500 - 2079*x**10*log(x)**4/125000 + 2079*x**10*lo
g(x)**3/312500 - 6237*x**10*log(x)**2/3125000 + 6237*x**10*log(x)/15625000
- 6237*x**10/156250000
```

Giac [A] time = 1.07493, size = 139, normalized size = 1.09

$$\frac{1}{10}x^{10}\log(x)^{11} - \frac{11}{100}x^{10}\log(x)^{10} + \frac{11}{100}x^{10}\log(x)^{9} - \frac{99}{1000}x^{10}\log(x)^{8} + \frac{99}{1250}x^{10}\log(x)^{7} - \frac{693}{12500}x^{10}\log(x)^{6} + \frac{20}{62}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*log(x)^11,x, algorithm="giac")
```

```
[Out] 1/10*x^10*log(x)^11 - 11/100*x^10*log(x)^10 + 11/100*x^10*log(x)^9 - 99/100
0*x^10*log(x)^8 + 99/1250*x^10*log(x)^7 - 693/12500*x^10*log(x)^6 + 2079/62
500*x^10*log(x)^5 - 2079/125000*x^10*log(x)^4 + 2079/312500*x^10*log(x)^3 -
 6237/3125000*x^10*log(x)^2 + 6237/15625000*x^10*log(x) - 6237/156250000*x^
10
```

3.59 $\int \frac{\log^2(x)}{x} \, dx$

Optimal. Leaf size=8

$$\frac{\log^3(x)}{3}$$

```
[Out] Log[x]^3/3
```

Rubi [A] time = 0.0116141, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2302, 30}

$$\frac{\log^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^2/x,x]

[Out] Log[x]^3/3
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\log^2(x)}{x} \, dx &= \text{Subst}\left(\int x^2 \, dx, x, \log(x)\right) \\ &= \frac{\log^3(x)}{3} \end{aligned}$$

Mathematica [A] time = 0.0009859, size = 8, normalized size = 1.

$$\frac{\log^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^2/x,x]
```

```
[Out] Log[x]^3/3
```

Maple [A] time = 0., size = 7, normalized size = 0.9

$$\frac{(\ln (x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2/x,x)
```

```
[Out] 1/3*ln(x)^3
```

Maxima [A] time = 0.940217, size = 8, normalized size = 1.

$$\frac{1}{3} \log (x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2/x,x, algorithm="maxima")
```

```
[Out] 1/3*log(x)^3
```

Fricas [A] time = 2.00398, size = 19, normalized size = 2.38

$$\frac{1}{3} \log (x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2/x,x, algorithm="fricas")
```

```
[Out] 1/3*log(x)^3
```

Sympy [A] time = 0.08174, size = 5, normalized size = 0.62

$$\frac{\log(x)^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2/x,x)
```

```
[Out] log(x)**3/3
```

Giac [A] time = 1.09391, size = 8, normalized size = 1.

$$\frac{1}{3} \log(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2/x,x, algorithm="giac")
```

```
[Out] 1/3*log(x)^3
```

3.60 $\int \frac{1}{\log(x)} \, dx$

Optimal. Leaf size=2

$$\text{LogIntegral}(x)$$

```
[Out] LogIntegral[x]
```

Rubi [A] time = 0.0018131, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2298}

$$\text{LogIntegral}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^(-1),x]
```

```
[Out] LogIntegral[x]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\int \frac{1}{\log(x)} \, dx = \text{li}(x)$$

Mathematica [F] time = 0.0021084, size = 0, normalized size = 0.

$$\int \frac{1}{\log(x)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[x]^(-1),x]
```

```
[Out] Integrate[Log[x]^(-1), x]
```

Maple [B] time = 0.003, size = 9, normalized size = 4.5

$$-\mathrm{Ei}\left(1,-\ln\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(x),x)
```

```
[Out] -Ei(1,-ln(x))
```

Maxima [A] time = 1.03323, size = 4, normalized size = 2.

$$\mathrm{Ei}\left(\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="maxima")
```

```
[Out] Ei(log(x))
```

Fricas [A] time = 1.80419, size = 23, normalized size = 11.5

$$\mathrm{log\_integral}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="fricas")
```

```
[Out] log_integral(x)
```

Sympy [A] time = 0.447003, size = 2, normalized size = 1.

$$\mathrm{li}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(x),x)
```

```
[Out] li(x)
```

Giac [A] time = 1.08115, size = 4, normalized size = 2.

$$\mathrm{Ei}\,(\log\,(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(x),x, algorithm="giac")
```

```
[Out] Ei(log(x))
```

3.61 $\int \frac{1}{\log(1+x)}\,dx$

Optimal. Leaf size=4

$$\text{LogIntegral}(x+1)$$

```
[Out] LogIntegral[1 + x]
```

Rubi [A] time = 0.0037252, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2389, 2298}

$$\text{LogIntegral}(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + x]^(-1),x]
```

```
[Out] LogIntegral[1 + x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\log(1+x)}\,dx &= \text{Subst}\left(\int \frac{1}{\log(x)}\,dx, x, 1+x\right)\\ &= \text{li}(1+x)\end{aligned}$$

Mathematica [A] time = 0.0101527, size = 5, normalized size = 1.25

$$\text{ExpIntegralEi}(\log(x+1))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + x]^(-1),x]
```

```
[Out] ExpIntegralEi[Log[1 + x]]
```

Maple [B] time = 0.002, size = 11, normalized size = 2.8

$$-\text{Ei}\left(1,-\ln\left(1+x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(1+x),x)
```

```
[Out] -Ei(1,-ln(1+x))
```

Maxima [A] time = 1.03548, size = 7, normalized size = 1.75

$$\text{Ei}\left(\log\left(x+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(1+x),x, algorithm="maxima")
```

```
[Out] Ei(log(x + 1))
```

Fricas [A] time = 1.85379, size = 28, normalized size = 7.

$$\text{log\_integral}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(1+x),x, algorithm="fricas")
```

```
[Out] log_integral(x + 1)
```

Sympy [A] time = 0.471035, size = 3, normalized size = 0.75

$$\mathrm{li}\,(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(1+x),x)
```

```
[Out] li(x + 1)
```

Giac [A] time = 1.08028, size = 7, normalized size = 1.75

$$\mathrm{Ei}\,(\log\,(x+1))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(1+x),x, algorithm="giac")
```

```
[Out] Ei(log(x + 1))
```

3.62 $\int \frac{1}{x\log(x)}\,dx$

Optimal. Leaf size=3

$$\log(\log(x))$$

```
[Out] Log[Log[x]]
```

Rubi [A] time = 0.01205, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {2302, 29}

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Log[x]),x]
```

```
[Out] Log[Log[x]]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\log(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, \log(x)\right)\\ &= \log(\log(x))\end{aligned}$$

Mathematica [A] time = 0.0040431, size = 3, normalized size = 1.

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Log[x]),x]
```

```
[Out] Log[Log[x]]
```

Maple [A] time = 0., size = 4, normalized size = 1.3

$$\ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x),x)
```

```
[Out] ln(ln(x))
```

Maxima [A] time = 0.954723, size = 4, normalized size = 1.33

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="maxima")
```

```
[Out] log(log(x))
```

Fricas [A] time = 2.00407, size = 18, normalized size = 6.

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="fricas")
```

```
[Out] log(log(x))
```

Sympy [A] time = 0.086409, size = 3, normalized size = 1.

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x),x)
```

```
[Out] log(log(x))
```

Giac [A] time = 1.07431, size = 5, normalized size = 1.67

$$\log(|\log(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="giac")
```

```
[Out] log(abs(log(x)))
```

3.63 $\int \frac{1}{x^2 \log^2(x)} \, dx$

Optimal. Leaf size=17

$$-\text{ExpIntegralEi}(-\log(x)) - \frac{1}{x\log(x)}$$

```
[Out] -ExpIntegralEi[-Log[x]] - 1/(x*Log[x])
```

Rubi [A] time = 0.0348469, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2306, 2309, 2178}

$$-\text{ExpIntegralEi}(-\log(x)) - \frac{1}{x\log(x)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Log[x]^2),x]

[Out] -ExpIntegralEi[-Log[x]] - 1/(x*Log[x])
```

Rule 2306

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^(p + 1))/(b*d*n*(p + 1)), x] -
Dist[(m + 1)/(b*n*(p + 1)), Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
/; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

Rule 2309

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :> Dist[1/c^
(m + 1), Subst[Int[E^((m + 1)*x)*(a + b*x)^p, x], x, Log[c*x]], x] /; FreeQ
[{a, b, c, p}, x] && IntegerQ[m]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2 \log^2(x)}\,dx &= -\frac{1}{x\log(x)} - \int \frac{1}{x^2\log(x)}\,dx\\ &= -\frac{1}{x\log(x)} - \text{Subst}\left(\int \frac{e^{-x}}{x}\,dx, x, \log(x)\right)\\ &= -\textbf{Ei}(-\log(x)) - \frac{1}{x\log(x)}\end{aligned}$$

Mathematica [A] time = 0.0159956, size = 17, normalized size = 1.

$$-\text{ExpIntegralEi}(-\log(x)) - \frac{1}{x\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Log[x]^2),x]
```

```
[Out] -ExpIntegralEi[-Log[x]] - 1/(x*Log[x])
```

Maple [A] time = 0.003, size = 15, normalized size = 0.9

$$-\frac{1}{x\ln(x)} + \text{Ei}(1, \ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/ln(x)^2,x)
```

```
[Out] -1/x/ln(x)+Ei(1,ln(x))
```

Maxima [A] time = 1.02696, size = 8, normalized size = 0.47

$$-\Gamma(-1, \log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(x)^2,x, algorithm="maxima")
```

```
[Out] -gamma(-1, log(x))
```

Fricas [A] time = 1.93701, size = 62, normalized size = 3.65

$$-\frac{x\log(x)\,\mathrm{log\_integral}\left(\frac{1}{x}\right)+1}{x\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(x)^2,x, algorithm="fricas")
```

```
[Out] -(x*log(x)*log_integral(1/x) + 1)/(x*log(x))
```

Sympy [A] time = 0.636319, size = 14, normalized size = 0.82

$$-\operatorname{Ei}\left(-\log(x)\right)-\frac{1}{x\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/ln(x)**2,x)
```

```
[Out] -Ei(-log(x)) - 1/(x*log(x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\log(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/log(x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*log(x)^2), x)
```

3.64 $\int \frac{\log^p(x)}{x}\,dx$

Optimal. Leaf size=12

$$\frac{\log^{p+1}(x)}{p+1}$$

```
[Out] Log[x]^(1 + p)/(1 + p)
```

Rubi [A] time = 0.0162275, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2302, 30}

$$\frac{\log^{p+1}(x)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^p/x,x]
```

```
[Out] Log[x]^(1 + p)/(1 + p)
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log^p(x)}{x}\,dx &= \text{Subst}\left(\int x^p\,dx, x, \log(x)\right)\\ &= \frac{\log^{1+p}(x)}{1+p}\end{aligned}$$

Mathematica [A] time = 0.002378, size = 12, normalized size = 1.

$$\frac{\log^{p+1}(x)}{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^p/x,x]
```

```
[Out] Log[x]^(1 + p)/(1 + p)
```

Maple [A] time = 0.001, size = 13, normalized size = 1.1

$$\frac{(\ln(x))^{1+p}}{1+p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^p/x,x)
```

```
[Out] ln(x)^(1+p)/(1+p)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^p/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.04835, size = 34, normalized size = 2.83

$$\frac{\log(x)^p \log(x)}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^p/x,x, algorithm="fricas")
```

```
[Out] log(x)^p*log(x)/(p + 1)
```

Sympy [A] time = 0.916745, size = 15, normalized size = 1.25

$$\begin{cases} \frac{\log{(x)}^{p+1}}{p+1} & \text{for}\: p \neq -1 \\ \log{(\log{(x)})} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**p/x,x)
```

```
[Out] Piecewise((log(x)**(p + 1)/(p + 1), Ne(p, -1)), (log(log(x)), True))
```

Giac [A] time = 1.08916, size = 16, normalized size = 1.33

$$\frac{\log{(x)}^{p+1}}{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^p/x,x, algorithm="giac")
```

```
[Out] log(x)^(p + 1)/(p + 1)
```

3.65 $\int (b+ax)\log(x)\,dx$

Optimal. Leaf size=28

$$-\frac{ax^2}{4}+\frac{1}{2}ax^2\log(x)-bx+bx\log(x)$$

```
[Out] -(b*x) - (a*x^2)/4 + b*x*Log[x] + (a*x^2*Log[x])/2
```

Rubi [A] time = 0.0109207, antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2313}

$$\frac{1}{2}\log(x)\left(ax^2+2bx\right)-\frac{ax^2}{4}-bx$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x)*Log[x],x]
```

```
[Out] -(b*x) - (a*x^2)/4 + ((2*b*x + a*x^2)*Log[x])/2
```

Rule 2313

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]
), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
 d, e, n, r}, x] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (b+ax)\log(x)\,dx &= \frac{1}{2}\left(2bx+ax^2\right)\log(x)-\int\left(b+\frac{ax}{2}\right)dx\\ &= -bx-\frac{ax^2}{4}+\frac{1}{2}\left(2bx+ax^2\right)\log(x)\end{aligned}$$

Mathematica [A] time = 0.001392, size = 28, normalized size = 1.

$$-\frac{ax^2}{4}+\frac{1}{2}ax^2\log(x)-bx+bx\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x)*Log[x],x]
```

```
[Out] -(b*x) - (a*x^2)/4 + b*x*Log[x] + (a*x^2*Log[x])/2
```

Maple [A] time = 0.003, size = 25, normalized size = 0.9

$$-bx - \frac{ax^2}{4} + bx\ln(x) + \frac{ax^2\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)*ln(x),x)
```

```
[Out] -b*x-1/4*a*x^2+b*x*ln(x)+1/2*a*x^2*ln(x)
```

Maxima [A] time = 0.950142, size = 34, normalized size = 1.21

$$-\frac{1}{4}ax^2 - bx + \frac{1}{2}\left(ax^2 + 2bx\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*log(x),x, algorithm="maxima")
```

```
[Out] -1/4*a*x^2 - b*x + 1/2*(a*x^2 + 2*b*x)*log(x)
```

Fricas [A] time = 1.9649, size = 63, normalized size = 2.25

$$-\frac{1}{4}ax^2 - bx + \frac{1}{2}\left(ax^2 + 2bx\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*log(x),x, algorithm="fricas")
```

```
[Out] -1/4*a*x^2 - b*x + 1/2*(a*x^2 + 2*b*x)*log(x)
```

Sympy [A] time = 0.103396, size = 22, normalized size = 0.79

$$-\frac{ax^2}{4} - bx + \left(\frac{ax^2}{2} + bx\right)\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*ln(x),x)
```

```
[Out] -a*x**2/4 - b*x + (a*x**2/2 + b*x)*log(x)
```

Giac [A] time = 1.08335, size = 32, normalized size = 1.14

$$\frac{1}{2}ax^2\log(x) - \frac{1}{4}ax^2 + bx\log(x) - bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)*log(x),x, algorithm="giac")
```

```
[Out] 1/2*a*x^2*log(x) - 1/4*a*x^2 + b*x*log(x) - b*x
```

3.66 $\int (b+ax)^2 \log(x)\,dx$

Optimal. Leaf size=54

$$-\frac{a^2x^3}{9}-\frac{b^3\log(x)}{3a}-\frac{1}{2}abx^2+\frac{\log(x)(ax+b)^3}{3a}-b^2x$$

```
[Out] -(b^2*x) - (a*b*x^2)/2 - (a^2*x^3)/9 - (b^3*Log[x])/(3*a) + ((b + a*x)^3*Lo
g[x])/(3*a)
```

Rubi [A] time = 0.0278243, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {32, 2313, 12, 43}

$$-\frac{a^2x^3}{9}-\frac{b^3\log(x)}{3a}-\frac{1}{2}abx^2+\frac{\log(x)(ax+b)^3}{3a}-b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(b + a*x)^2*Log[x],x]
```

```
[Out] -(b^2*x) - (a*b*x^2)/2 - (a^2*x^3)/9 - (b^3*Log[x])/(3*a) + ((b + a*x)^3*Lo
g[x])/(3*a)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2313

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_.),
x_Symbol] :> With[{u = IntHide[(d + e*x^r)^q, x]}, Simp[u*(a + b*Log[c*x^n]
), x] - Dist[b*n, Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[{a, b, c,
 d, e, n, r}, x] && IGtQ[q, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (b+ax)^2\log(x)\,dx &= \frac{(b+ax)^3\log(x)}{3a}-\int \frac{(b+ax)^3}{3ax}\,dx\\ &= \frac{(b+ax)^3\log(x)}{3a}-\frac{\int \frac{(b+ax)^3}{x}\,dx}{3a}\\ &= \frac{(b+ax)^3\log(x)}{3a}-\frac{\int \left(3ab^2+\frac{b^3}{x}+3a^2bx+a^3x^2\right)dx}{3a}\\ &= -b^2x-\frac{1}{2}abx^2-\frac{a^2x^3}{9}-\frac{b^3\log(x)}{3a}+\frac{(b+ax)^3\log(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.0103252, size = 53, normalized size = 0.98

$$-\frac{1}{9}a^2x^3+\frac{1}{3}a^2x^3\log(x)-\frac{1}{2}abx^2+abx^2\log(x)-b^2x+b^2x\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(b + a*x)^2*Log[x],x]
```

```
[Out] -(b^2*x) - (a*b*x^2)/2 - (a^2*x^3)/9 + b^2*x*Log[x] + a*b*x^2*Log[x] + (a^2
*x^3*Log[x])/3
```

Maple [A] time = 0.001, size = 48, normalized size = 0.9

$$\frac{a^2x^3\ln(x)}{3}-\frac{a^2x^3}{9}+abx^2\ln(x)-\frac{abx^2}{2}+\ln(x)\,xb^2-b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+b)^2*ln(x),x)
```

```
[Out] 1/3*a^2*x^3*ln(x)-1/9*a^2*x^3+a*b*x^2*ln(x)-1/2*a*b*x^2+ln(x)*x*b^2-b^2*x
```

Maxima [A] time = 0.958832, size = 63, normalized size = 1.17

$$-\frac{1}{9}\,a^2x^3 - \frac{1}{2}\,abx^2 - b^2x + \frac{1}{3}\left(a^2x^3 + 3\,abx^2 + 3\,b^2x\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^2*log(x),x, algorithm="maxima")
```

```
[Out] -1/9*a^2*x^3 - 1/2*a*b*x^2 - b^2*x + 1/3*(a^2*x^3 + 3*a*b*x^2 + 3*b^2*x)*lo
g(x)
```

Fricas [A] time = 1.98242, size = 109, normalized size = 2.02

$$-\frac{1}{9}\,a^2x^3 - \frac{1}{2}\,abx^2 - b^2x + \frac{1}{3}\left(a^2x^3 + 3\,abx^2 + 3\,b^2x\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^2*log(x),x, algorithm="fricas")
```

```
[Out] -1/9*a^2*x^3 - 1/2*a*b*x^2 - b^2*x + 1/3*(a^2*x^3 + 3*a*b*x^2 + 3*b^2*x)*lo
g(x)
```

Sympy [A] time = 0.120427, size = 44, normalized size = 0.81

$$-\frac{a^2x^3}{9} - \frac{abx^2}{2} - b^2x + \left(\frac{a^2x^3}{3} + abx^2 + b^2x\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)**2*ln(x),x)
```

```
[Out] -a**2*x**3/9 - a*b*x**2/2 - b**2*x + (a**2*x**3/3 + a*b*x**2 + b**2*x)*log(
x)
```

Giac [A] time = 1.08132, size = 63, normalized size = 1.17

$$\frac{1}{3}\,a^{2}x^{3}\log\left(x\right)-\frac{1}{9}\,a^{2}x^{3}+abx^{2}\log\left(x\right)-\frac{1}{2}\,abx^{2}+b^{2}x\log\left(x\right)-b^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+b)^2*log(x),x, algorithm="giac")
```

```
[Out] 1/3*a^2*x^3*log(x) - 1/9*a^2*x^3 + a*b*x^2*log(x) - 1/2*a*b*x^2 + b^2*x*log
(x) - b^2*x
```

3.67 $\int \frac{\log(x)}{(b+ax)^2}\,dx$

Optimal. Leaf size=29

$$\frac{x\log(x)}{b(ax+b)}-\frac{\log(ax+b)}{ab}$$

```
[Out] (x*Log[x])/(b*(b + a*x)) - Log[b + a*x]/(a*b)
```

Rubi [A] time = 0.0130004, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2314, 31}

$$\frac{x\log(x)}{b(ax+b)}-\frac{\log(ax+b)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(b + a*x)^2,x]
```

```
[Out] (x*Log[x])/(b*(b + a*x)) - Log[b + a*x]/(a*b)
```

Rule 2314

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[(x*(d + e*x^r)^(q + 1)*(a + b*Log[c*x^n]))/d, x] - Dist[(b
*n)/d, Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r}, x
] && EqQ[r*(q + 1) + 1, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(x)}{(b+ax)^2}\,dx &= \frac{x\log(x)}{b(b+ax)}-\frac{\int \frac{1}{b+ax}\,dx}{b}\\ &= \frac{x\log(x)}{b(b+ax)}-\frac{\log(b+ax)}{ab}\end{aligned}$$

Mathematica [A] time = 0.0125981, size = 27, normalized size = 0.93

$$\frac{\frac{x\log(x)}{ax+b}-\frac{\log(ax+b)}{a}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(b + a*x)^2,x]
```

```
[Out] ((x*Log[x])/(b + a*x) - Log[b + a*x]/a)/b
```

Maple [A] time = 0.007, size = 30, normalized size = 1.

$$\frac{x\ln(x)}{b(ax+b)}-\frac{\ln(ax+b)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(a*x+b)^2,x)
```

```
[Out] x*ln(x)/b/(a*x+b)-ln(a*x+b)/a/b
```

Maxima [A] time = 0.929854, size = 51, normalized size = 1.76

$$-\frac{\frac{\log(ax+b)}{b}-\frac{\log(x)}{b}}{a}-\frac{\log(x)}{(ax+b)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^2,x, algorithm="maxima")
```

```
[Out] -(log(a*x + b)/b - log(x)/b)/a - log(x)/((a*x + b)*a)
```

Fricas [A] time = 1.99178, size = 77, normalized size = 2.66

$$\frac{ax\log(x)-(ax+b)\log(ax+b)}{a^2bx+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^2,x, algorithm="fricas")
```

```
[Out] (a*x*log(x) - (a*x + b)*log(a*x + b))/(a^2*b*x + a*b^2)
```

Sympy [A] time = 0.362573, size = 24, normalized size = 0.83

$$-\frac{\log(x)}{a^2x+ab}+\frac{\log(x)-\log\left(x+\frac{b}{a}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(a*x+b)**2,x)
```

```
[Out] -log(x)/(a**2*x + a*b) + (log(x) - log(x + b/a))/(a*b)
```

Giac [A] time = 1.10867, size = 49, normalized size = 1.69

$$-\frac{\log(x)}{(ax+b)a}+\frac{\log\left(\left|-\frac{b}{ax+b}+1\right|\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^2,x, algorithm="giac")
```

```
[Out] -log(x)/((a*x + b)*a) + log(abs(-b/(a*x + b) + 1))/(a*b)
```

3.68 $\int x \log(b + ax)\, dx$

Optimal. Leaf size=46

$$-\frac{b^2 \log(ax+b)}{2a^2} + \frac{1}{2}x^2 \log(ax+b) + \frac{bx}{2a} - \frac{x^2}{4}$$

```
[Out] (b*x)/(2*a) - x^2/4 - (b^2*Log[b + a*x])/(2*a^2) + (x^2*Log[b + a*x])/2
```

Rubi [A] time = 0.0231036, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2395, 43}

$$-\frac{b^2 \log(ax+b)}{2a^2} + \frac{1}{2}x^2 \log(ax+b) + \frac{bx}{2a} - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[b + a*x],x]
```

```
[Out] (b*x)/(2*a) - x^2/4 - (b^2*Log[b + a*x])/(2*a^2) + (x^2*Log[b + a*x])/2
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x\log(b+ax)\,dx = \frac{1}{2}x^2\log(b+ax) - \frac{1}{2}a\int\frac{x^2}{b+ax}\,dx$$
$$= \frac{1}{2}x^2\log(b+ax) - \frac{1}{2}a\int\left(-\frac{b}{a^2}+\frac{x}{a}+\frac{b^2}{a^2(b+ax)}\right)dx$$
$$= \frac{bx}{2a} - \frac{x^2}{4} - \frac{b^2\log(b+ax)}{2a^2} + \frac{1}{2}x^2\log(b+ax)$$

Mathematica [A] time = 0.0146575, size = 46, normalized size = 1.

$$-\frac{b^2\log(ax+b)}{2a^2} + \frac{1}{2}x^2\log(ax+b) + \frac{bx}{2a} - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[b + a*x],x]
```

```
[Out] (b*x)/(2*a) - x^2/4 - (b^2*Log[b + a*x])/(2*a^2) + (x^2*Log[b + a*x])/2
```

Maple [A] time = 0.003, size = 47, normalized size = 1.

$$-\frac{b^2\ln(ax+b)}{2a^2} + \frac{bx}{2a} + \frac{3b^2}{4a^2} + \frac{x^2\ln(ax+b)}{2} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(a*x+b),x)
```

```
[Out] -1/2*b^2*ln(a*x+b)/a^2+1/2*b*x/a+3/4*b^2/a^2+1/2*x^2*ln(a*x+b)-1/4*x^2
```

Maxima [A] time = 0.955528, size = 59, normalized size = 1.28

$$\frac{1}{2}x^2\log(ax+b) - \frac{1}{4}a\left(\frac{2b^2\log(ax+b)}{a^3} + \frac{ax^2-2bx}{a^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a*x+b),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(a*x + b) - 1/4*a*(2*b^2*log(a*x + b)/a^3 + (a*x^2 - 2*b*x)/a^2)
```

Fricas [A] time = 1.97919, size = 85, normalized size = 1.85

$$-\frac{a^2x^2 - 2\,abx - 2\left(a^2x^2 - b^2\right)\log\left(ax + b\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a*x+b),x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*x^2 - 2*a*b*x - 2*(a^2*x^2 - b^2)*log(a*x + b))/a^2
```

Sympy [A] time = 0.320611, size = 42, normalized size = 0.91

$$-a\left(\frac{x^2}{4a} - \frac{bx}{2a^2} + \frac{b^2\log\left(ax + b\right)}{2a^3}\right) + \frac{x^2\log\left(ax + b\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(a*x+b),x)
```

```
[Out] -a*(x**2/(4*a) - b*x/(2*a**2) + b**2*log(a*x + b)/(2*a**3)) + x**2*log(a*x
+ b)/2
```

Giac [A] time = 1.07616, size = 78, normalized size = 1.7

$$\frac{(ax + b)^2\log\left(ax + b\right)}{2\,a^2} - \frac{(ax + b)b\log\left(ax + b\right)}{a^2} - \frac{(ax + b)^2}{4\,a^2} + \frac{(ax + b)b}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a*x+b),x, algorithm="giac")
```

```
[Out] 1/2*(a*x + b)^2*log(a*x + b)/a^2 - (a*x + b)*b*log(a*x + b)/a^2 - 1/4*(a*x
+ b)^2/a^2 + (a*x + b)*b/a^2
```

3.69 $\int x^2 \log(b+ax)\,dx$

Optimal. Leaf size=59

$$-\frac{b^2x}{3a^2}+\frac{b^3\log(ax+b)}{3a^3}+\frac{bx^2}{6a}+\frac{1}{3}x^3\log(ax+b)-\frac{x^3}{9}$$

```
[Out] -(b^2*x)/(3*a^2) + (b*x^2)/(6*a) - x^3/9 + (b^3*Log[b + a*x])/(3*a^3) + (x^
3*Log[b + a*x])/3
```

Rubi [A] time = 0.0316671, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2395, 43}

$$-\frac{b^2x}{3a^2}+\frac{b^3\log(ax+b)}{3a^3}+\frac{bx^2}{6a}+\frac{1}{3}x^3\log(ax+b)-\frac{x^3}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[b + a*x],x]
```

```
[Out] -(b^2*x)/(3*a^2) + (b*x^2)/(6*a) - x^3/9 + (b^3*Log[b + a*x])/(3*a^3) + (x^
3*Log[b + a*x])/3
```

Rule 2395

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + (g_.)*(x_
))^(q_.), x_Symbol] :> Simp[((f + g*x)^(q + 1)*(a + b*Log[c*(d + e*x)^n]))/
(g*(q + 1)), x] - Dist[(b*e*n)/(g*(q + 1)), Int[(f + g*x)^(q + 1)/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, q}, x] && NeQ[e*f - d*g, 0] && N
eQ[q, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int x^2\log(b+ax)\,dx = \frac{1}{3}x^3\log(b+ax) - \frac{1}{3}a\int \frac{x^3}{b+ax}\,dx$$
$$= \frac{1}{3}x^3\log(b+ax) - \frac{1}{3}a\int\left(\frac{b^2}{a^3} - \frac{bx}{a^2} + \frac{x^2}{a} - \frac{b^3}{a^3(b+ax)}\right)dx$$
$$= -\frac{b^2x}{3a^2} + \frac{bx^2}{6a} - \frac{x^3}{9} + \frac{b^3\log(b+ax)}{3a^3} + \frac{1}{3}x^3\log(b+ax)$$

Mathematica [A] time = 0.0172276, size = 59, normalized size = 1.

$$-\frac{b^2x}{3a^2} + \frac{b^3\log(ax+b)}{3a^3} + \frac{bx^2}{6a} + \frac{1}{3}x^3\log(ax+b) - \frac{x^3}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[b + a*x],x]
```

```
[Out] -(b^2*x)/(3*a^2) + (b*x^2)/(6*a) - x^3/9 + (b^3*Log[b + a*x])/(3*a^3) + (x^
3*Log[b + a*x])/3
```

Maple [A] time = 0.003, size = 58, normalized size = 1.

$$\frac{x^3\ln(ax+b)}{3} + \frac{b^3\ln(ax+b)}{3a^3} - \frac{x^3}{9} + \frac{bx^2}{6a} - \frac{b^2x}{3a^2} - \frac{11b^3}{18a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(a*x+b),x)
```

```
[Out] 1/3*x^3*ln(a*x+b)+1/3*b^3*ln(a*x+b)/a^3-1/9*x^3+1/6*b*x^2/a-1/3*b^2*x/a^2-1
1/18*b^3/a^3
```

Maxima [A] time = 0.945145, size = 77, normalized size = 1.31

$$\frac{1}{3}x^3\log(ax+b) + \frac{1}{18}a\left(\frac{6b^3\log(ax+b)}{a^4} - \frac{2a^2x^3 - 3abx^2 + 6b^2x}{a^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a*x+b),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*log(a*x + b) + 1/18*a*(6*b^3*log(a*x + b)/a^4 - (2*a^2*x^3 - 3*a*b*
x^2 + 6*b^2*x)/a^3)
```

Fricas [A] time = 1.96776, size = 111, normalized size = 1.88

$$-\frac{2\,a^3x^3 - 3\,a^2bx^2 + 6\,ab^2x - 6\left(a^3x^3 + b^3\right)\log\left(ax+b\right)}{18\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a*x+b),x, algorithm="fricas")
```

```
[Out] -1/18*(2*a^3*x^3 - 3*a^2*b*x^2 + 6*a*b^2*x - 6*(a^3*x^3 + b^3)*log(a*x + b)
)/a^3
```

Sympy [A] time = 0.330298, size = 54, normalized size = 0.92

$$-a\left(\frac{x^3}{9a} - \frac{bx^2}{6a^2} + \frac{b^2x}{3a^3} - \frac{b^3\log\left(ax+b\right)}{3a^4}\right) + \frac{x^3\log\left(ax+b\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(a*x+b),x)
```

```
[Out] -a*(x**3/(9*a) - b*x**2/(6*a**2) + b**2*x/(3*a**3) - b**3*log(a*x + b)/(3*a
**4)) + x**3*log(a*x + b)/3
```

Giac [A] time = 1.07197, size = 127, normalized size = 2.15

$$\frac{(ax+b)^3\log\left(ax+b\right)}{3\,a^3} - \frac{(ax+b)^2b\log\left(ax+b\right)}{a^3} + \frac{(ax+b)b^2\log\left(ax+b\right)}{a^3} - \frac{(ax+b)^3}{9\,a^3} + \frac{(ax+b)^2b}{2\,a^3} - \frac{(ax+b)b^2}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a*x+b),x, algorithm="giac")
```

```
[Out] 1/3*(a*x + b)^3*log(a*x + b)/a^3 - (a*x + b)^2*b*log(a*x + b)/a^3 + (a*x +
b)*b^2*log(a*x + b)/a^3 - 1/9*(a*x + b)^3/a^3 + 1/2*(a*x + b)^2*b/a^3 - (a*
x + b)*b^2/a^3
```

3.70 $\int \log\left(a^2+x^2\right) dx$

Optimal. Leaf size=23

$$x\log\left(a^2+x^2\right)+2a\tan^{-1}\left(\frac{x}{a}\right)-2x$$

```
[Out] -2*x + 2*a*ArcTan[x/a] + x*Log[a^2 + x^2]
```

Rubi [A] time = 0.0074804, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2448, 321, 203}

$$x\log\left(a^2+x^2\right)+2a\tan^{-1}\left(\frac{x}{a}\right)-2x$$

Antiderivative was successfully verified.

```
[In] Int[Log[a^2 + x^2],x]
```

```
[Out] -2*x + 2*a*ArcTan[x/a] + x*Log[a^2 + x^2]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \log\left(a^2+x^2\right)\,dx &= x\log\left(a^2+x^2\right)-2\int\frac{x^2}{a^2+x^2}\,dx\\ &= -2x+x\log\left(a^2+x^2\right)+\left(2a^2\right)\int\frac{1}{a^2+x^2}\,dx\\ &= -2x+2a\tan^{-1}\left(\frac{x}{a}\right)+x\log\left(a^2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0016114, size = 23, normalized size = 1.

$$x\log\left(a^2+x^2\right)+2a\tan^{-1}\left(\frac{x}{a}\right)-2x$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[a^2 + x^2],x]
```

```
[Out] -2*x + 2*a*ArcTan[x/a] + x*Log[a^2 + x^2]
```

Maple [A] time = 0.003, size = 24, normalized size = 1.

$$-2\,x+2\,a\arctan\left(\frac{x}{a}\right)+x\ln\left(a^2+x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(a^2+x^2),x)
```

```
[Out] -2*x+2*a*arctan(x/a)+x*ln(a^2+x^2)
```

Maxima [A] time = 1.44333, size = 31, normalized size = 1.35

$$2\,a\arctan\left(\frac{x}{a}\right)+x\log\left(a^2+x^2\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a^2+x^2),x, algorithm="maxima")
```

```
[Out] 2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x
```

Fricas [A] time = 2.05649, size = 57, normalized size = 2.48

$$2\,a\arctan\left(\frac{x}{a}\right)+x\log\left(a^2+x^2\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a^2+x^2),x, algorithm="fricas")
```

```
[Out] 2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x
```

Sympy [C] time = 0.304679, size = 36, normalized size = 1.57

$$-2a\left(\frac{i\log\left(-ia+x\right)}{2}-\frac{i\log\left(ia+x\right)}{2}\right)+x\log\left(a^2+x^2\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(a**2+x**2),x)
```

```
[Out] -2*a*(I*log(-I*a + x)/2 - I*log(I*a + x)/2) + x*log(a**2 + x**2) - 2*x
```

Giac [A] time = 1.18137, size = 31, normalized size = 1.35

$$2\,a\arctan\left(\frac{x}{a}\right)+x\log\left(a^2+x^2\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(a^2+x^2),x, algorithm="giac")
```

```
[Out] 2*a*arctan(x/a) + x*log(a^2 + x^2) - 2*x
```

3.71 $\int x \log \left(a^2+x^2\right) dx$

Optimal. Leaf size=27

$$\frac{1}{2}\left(a^2+x^2\right)\log\left(a^2+x^2\right)-\frac{x^2}{2}$$

```
[Out] -x^2/2 + ((a^2 + x^2)*Log[a^2 + x^2])/2
```

Rubi [A] time = 0.0204105, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2454, 2389, 2295}

$$\frac{1}{2}\left(a^2+x^2\right)\log\left(a^2+x^2\right)-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[a^2 + x^2],x]
```

```
[Out] -x^2/2 + ((a^2 + x^2)*Log[a^2 + x^2])/2
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
 !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\int x \log \left(a^2+x^2\right) dx &= \frac{1}{2} \operatorname{Subst}\left(\int \log \left(a^2+x\right) dx, x, x^2\right)\\ &= \frac{1}{2} \operatorname{Subst}\left(\int \log(x)\, dx, x, a^2+x^2\right)\\ &= -\frac{x^2}{2}+\frac{1}{2}\left(a^2+x^2\right) \log \left(a^2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0029061, size = 26, normalized size = 0.96

$$\frac{1}{2}\left(\left(a^2+x^2\right) \log \left(a^2+x^2\right)-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[a^2 + x^2],x]
```

```
[Out] (-x^2 + (a^2 + x^2)*Log[a^2 + x^2])/2
```

Maple [A] time = 0.001, size = 29, normalized size = 1.1

$$\frac{\left(a^2+x^2\right) \ln \left(a^2+x^2\right)}{2}-\frac{x^2}{2}-\frac{a^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(a^2+x^2),x)
```

```
[Out] 1/2*(a^2+x^2)*ln(a^2+x^2)-1/2*x^2-1/2*a^2
```

Maxima [A] time = 0.965512, size = 38, normalized size = 1.41

$$-\frac{1}{2} a^2-\frac{1}{2} x^2+\frac{1}{2}\left(a^2+x^2\right) \log \left(a^2+x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a^2+x^2),x, algorithm="maxima")
```

```
[Out] -1/2*a^2 - 1/2*x^2 + 1/2*(a^2 + x^2)*log(a^2 + x^2)
```

Fricas [A] time = 2.02125, size = 58, normalized size = 2.15

$$-\frac{1}{2}x^2 + \frac{1}{2}\left(a^2 + x^2\right)\log\left(a^2 + x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a^2+x^2),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + 1/2*(a^2 + x^2)*log(a^2 + x^2)
```

Sympy [A] time = 0.307117, size = 31, normalized size = 1.15

$$\frac{a^2\log\left(a^2 + x^2\right)}{2} + \frac{x^2\log\left(a^2 + x^2\right)}{2} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(a**2+x**2),x)
```

```
[Out] a**2*log(a**2 + x**2)/2 + x**2*log(a**2 + x**2)/2 - x**2/2
```

Giac [A] time = 1.08371, size = 38, normalized size = 1.41

$$-\frac{1}{2}a^2 - \frac{1}{2}x^2 + \frac{1}{2}\left(a^2 + x^2\right)\log\left(a^2 + x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(a^2+x^2),x, algorithm="giac")
```

```
[Out] -1/2*a^2 - 1/2*x^2 + 1/2*(a^2 + x^2)*log(a^2 + x^2)
```

3.72 $\int x^2 \log \left(a^2+x^2\right) dx$

Optimal. Leaf size=44

$$\frac{1}{3}x^3\log\left(a^2+x^2\right)+\frac{2a^2x}{3}-\frac{2}{3}a^3\tan^{-1}\left(\frac{x}{a}\right)-\frac{2x^3}{9}$$

```
[Out] (2*a^2*x)/3 - (2*x^3)/9 - (2*a^3*ArcTan[x/a])/3 + (x^3*Log[a^2 + x^2])/3
```

Rubi [A] time = 0.0228396, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2455, 302, 203}

$$\frac{1}{3}x^3\log\left(a^2+x^2\right)+\frac{2a^2x}{3}-\frac{2}{3}a^3\tan^{-1}\left(\frac{x}{a}\right)-\frac{2x^3}{9}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[a^2 + x^2],x]
```

```
[Out] (2*a^2*x)/3 - (2*x^3)/9 - (2*a^3*ArcTan[x/a])/3 + (x^3*Log[a^2 + x^2])/3
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x^2 \log\left(a^2+x^2\right) dx &= \frac{1}{3}x^3 \log\left(a^2+x^2\right) - \frac{2}{3}\int \frac{x^4}{a^2+x^2}\, dx \\ &= \frac{1}{3}x^3 \log\left(a^2+x^2\right) - \frac{2}{3}\int \left(-a^2+x^2+\frac{a^4}{a^2+x^2}\right) dx \\ &= \frac{2a^2x}{3} - \frac{2x^3}{9} + \frac{1}{3}x^3 \log\left(a^2+x^2\right) - \frac{1}{3}\left(2a^4\right)\int \frac{1}{a^2+x^2}\, dx \\ &= \frac{2a^2x}{3} - \frac{2x^3}{9} - \frac{2}{3}a^3 \tan^{-1}\left(\frac{x}{a}\right) + \frac{1}{3}x^3 \log\left(a^2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0020295, size = 44, normalized size = 1.

$$\frac{1}{3}x^3 \log\left(a^2+x^2\right) + \frac{2a^2x}{3} - \frac{2}{3}a^3 \tan^{-1}\left(\frac{x}{a}\right) - \frac{2x^3}{9}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[a^2 + x^2],x]
```

```
[Out] (2*a^2*x)/3 - (2*x^3)/9 - (2*a^3*ArcTan[x/a])/3 + (x^3*Log[a^2 + x^2])/3
```

Maple [A] time = 0.004, size = 37, normalized size = 0.8

$$\frac{2\,a^2x}{3} - \frac{2\,x^3}{9} - \frac{2\,a^3}{3}\arctan\left(\frac{x}{a}\right) + \frac{x^3 \ln\left(a^2+x^2\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(a^2+x^2),x)
```

```
[Out] 2/3*a^2*x-2/9*x^3-2/3*a^3*arctan(x/a)+1/3*x^3*ln(a^2+x^2)
```

Maxima [A] time = 1.41594, size = 49, normalized size = 1.11

$$-\frac{2}{3}\,a^3 \arctan\left(\frac{x}{a}\right) + \frac{1}{3}\,x^3 \log\left(a^2+x^2\right) + \frac{2}{3}\,a^2x - \frac{2}{9}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a^2+x^2),x, algorithm="maxima")
```

```
[Out] -2/3*a^3*arctan(x/a) + 1/3*x^3*log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3
```

Fricas [A] time = 2.02291, size = 93, normalized size = 2.11

$$-\frac{2}{3}\,a^3\arctan\left(\frac{x}{a}\right)+\frac{1}{3}\,x^3\log\left(a^2+x^2\right)+\frac{2}{3}\,a^2x-\frac{2}{9}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a^2+x^2),x, algorithm="fricas")
```

```
[Out] -2/3*a^3*arctan(x/a) + 1/3*x^3*log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3
```

Sympy [C] time = 0.335372, size = 53, normalized size = 1.2

$$-2a^3\left(-\frac{i\log\left(-ia+x\right)}{6}+\frac{i\log\left(ia+x\right)}{6}\right)+\frac{2a^2x}{3}+\frac{x^3\log\left(a^2+x^2\right)}{3}-\frac{2x^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(a**2+x**2),x)
```

```
[Out] -2*a**3*(-I*log(-I*a + x)/6 + I*log(I*a + x)/6) + 2*a**2*x/3 + x**3*log(a**
2 + x**2)/3 - 2*x**3/9
```

Giac [A] time = 1.07825, size = 49, normalized size = 1.11

$$-\frac{2}{3}\,a^3\arctan\left(\frac{x}{a}\right)+\frac{1}{3}\,x^3\log\left(a^2+x^2\right)+\frac{2}{3}\,a^2x-\frac{2}{9}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(a^2+x^2),x, algorithm="giac")
```

```
[Out] -2/3*a^3*arctan(x/a) + 1/3*x^3*log(a^2 + x^2) + 2/3*a^2*x - 2/9*x^3
```

3.73 $\int x^4 \log\left(a^2 + x^2\right) dx$

Optimal. Leaf size=54

$$\frac{2a^2x^3}{15} + \frac{1}{5}x^5 \log\left(a^2 + x^2\right) - \frac{2a^4x}{5} + \frac{2}{5}a^5 \tan^{-1}\left(\frac{x}{a}\right) - \frac{2x^5}{25}$$

```
[Out] (-2*a^4*x)/5 + (2*a^2*x^3)/15 - (2*x^5)/25 + (2*a^5*ArcTan[x/a])/5 + (x^5*L
og[a^2 + x^2])/5
```

Rubi [A] time = 0.02527, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2455, 302, 203}

$$\frac{2a^2x^3}{15} + \frac{1}{5}x^5 \log\left(a^2 + x^2\right) - \frac{2a^4x}{5} + \frac{2}{5}a^5 \tan^{-1}\left(\frac{x}{a}\right) - \frac{2x^5}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Log[a^2 + x^2],x]
```

```
[Out] (-2*a^4*x)/5 + (2*a^2*x^3)/15 - (2*x^5)/25 + (2*a^5*ArcTan[x/a])/5 + (x^5*L
og[a^2 + x^2])/5
```

Rule 2455

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))*((f_.)*(x_))^
(m_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(a + b*Log[c*(d + e*x^n)^p]))/(f*(m
 + 1)), x] - Dist[(b*e*n*p)/(f*(m + 1)), Int[(x^(n - 1)*(f*x)^(m + 1))/(d +
 e*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && NeQ[m, -1]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^4 \log\left(a^2+x^2\right)\,dx &= \frac{1}{5}x^5\log\left(a^2+x^2\right)-\frac{2}{5}\int \frac{x^6}{a^2+x^2}\,dx\\
&= \frac{1}{5}x^5\log\left(a^2+x^2\right)-\frac{2}{5}\int \left(a^4-a^2x^2+x^4-\frac{a^6}{a^2+x^2}\right)dx\\
&= -\frac{2a^4x}{5}+\frac{2a^2x^3}{15}-\frac{2x^5}{25}+\frac{1}{5}x^5\log\left(a^2+x^2\right)+\frac{1}{5}\left(2a^6\right)\int \frac{1}{a^2+x^2}\,dx\\
&= -\frac{2a^4x}{5}+\frac{2a^2x^3}{15}-\frac{2x^5}{25}+\frac{2}{5}a^5\tan^{-1}\left(\frac{x}{a}\right)+\frac{1}{5}x^5\log\left(a^2+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0026944, size = 54, normalized size = 1.

$$\frac{2a^2x^3}{15}+\frac{1}{5}x^5\log\left(a^2+x^2\right)-\frac{2a^4x}{5}+\frac{2}{5}a^5\tan^{-1}\left(\frac{x}{a}\right)-\frac{2x^5}{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*Log[a^2 + x^2],x]
```

```
[Out] (-2*a^4*x)/5 + (2*a^2*x^3)/15 - (2*x^5)/25 + (2*a^5*ArcTan[x/a])/5 + (x^5*L
og[a^2 + x^2])/5
```

Maple [A] time = 0.003, size = 45, normalized size = 0.8

$$-\frac{2\,a^4x}{5}+\frac{2\,a^2x^3}{15}-\frac{2\,x^5}{25}+\frac{2\,a^5}{5}\arctan\left(\frac{x}{a}\right)+\frac{x^5\ln\left(a^2+x^2\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*ln(a^2+x^2),x)
```

```
[Out] -2/5*a^4*x+2/15*a^2*x^3-2/25*x^5+2/5*a^5*arctan(x/a)+1/5*x^5*ln(a^2+x^2)
```

Maxima [A] time = 1.42888, size = 59, normalized size = 1.09

$$\frac{2}{5}\,a^5\arctan\left(\frac{x}{a}\right)+\frac{1}{5}\,x^5\log\left(a^2+x^2\right)-\frac{2}{5}\,a^4x+\frac{2}{15}\,a^2x^3-\frac{2}{25}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(a^2+x^2),x, algorithm="maxima")
```

```
[Out] 2/5*a^5*arctan(x/a) + 1/5*x^5*log(a^2 + x^2) - 2/5*a^4*x + 2/15*a^2*x^3 - 2
/25*x^5
```

Fricas [A] time = 2.04898, size = 113, normalized size = 2.09

$$\frac{2}{5}\,a^5\arctan\left(\frac{x}{a}\right)+\frac{1}{5}\,x^5\log\left(a^2+x^2\right)-\frac{2}{5}\,a^4x+\frac{2}{15}\,a^2x^3-\frac{2}{25}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(a^2+x^2),x, algorithm="fricas")
```

```
[Out] 2/5*a^5*arctan(x/a) + 1/5*x^5*log(a^2 + x^2) - 2/5*a^4*x + 2/15*a^2*x^3 - 2
/25*x^5
```

Sympy [C] time = 0.348756, size = 63, normalized size = 1.17

$$-2a^5\left(\frac{i\log\left(-ia+x\right)}{10}-\frac{i\log\left(ia+x\right)}{10}\right)-\frac{2a^4x}{5}+\frac{2a^2x^3}{15}+\frac{x^5\log\left(a^2+x^2\right)}{5}-\frac{2x^5}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*ln(a**2+x**2),x)
```

```
[Out] -2*a**5*(I*log(-I*a + x)/10 - I*log(I*a + x)/10) - 2*a**4*x/5 + 2*a**2*x**3
/15 + x**5*log(a**2 + x**2)/5 - 2*x**5/25
```

Giac [A] time = 1.08867, size = 59, normalized size = 1.09

$$\frac{2}{5}\,a^{5}\arctan\left(\frac{x}{a}\right)+\frac{1}{5}\,x^{5}\log\left(a^{2}+x^{2}\right)-\frac{2}{5}\,a^{4}x+\frac{2}{15}\,a^{2}x^{3}-\frac{2}{25}\,x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*log(a^2+x^2),x, algorithm="giac")
```

```
[Out] 2/5*a^5*arctan(x/a) + 1/5*x^5*log(a^2 + x^2) - 2/5*a^4*x + 2/15*a^2*x^3 - 2
/25*x^5
```

3.74 $\int \log\left(-a^2+x^2\right) dx$

Optimal. Leaf size=25

$$x\log\left(x^2-a^2\right)+2a\tanh^{-1}\left(\frac{x}{a}\right)-2x$$

```
[Out] -2*x + 2*a*ArcTanh[x/a] + x*Log[-a^2 + x^2]
```

Rubi [A] time = 0.0099368, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2448, 321, 207}

$$x\log\left(x^2-a^2\right)+2a\tanh^{-1}\left(\frac{x}{a}\right)-2x$$

Antiderivative was successfully verified.

```
[In] Int[Log[-a^2 + x^2],x]
```

```
[Out] -2*x + 2*a*ArcTanh[x/a] + x*Log[-a^2 + x^2]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \log\left(-a^2+x^2\right)\,dx &= x\log\left(-a^2+x^2\right)-2\int \frac{x^2}{-a^2+x^2}\,dx\\ &= -2x+x\log\left(-a^2+x^2\right)-\left(2a^2\right)\int \frac{1}{-a^2+x^2}\,dx\\ &= -2x+2a\tanh^{-1}\left(\frac{x}{a}\right)+x\log\left(-a^2+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0023323, size = 25, normalized size = 1.

$$x\log\left(x^2-a^2\right)+2a\tanh^{-1}\left(\frac{x}{a}\right)-2x$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[-a^2 + x^2],x]
```

```
[Out] -2*x + 2*a*ArcTanh[x/a] + x*Log[-a^2 + x^2]
```

Maple [A] time = 0.007, size = 32, normalized size = 1.3

$$x\ln\left(-a^2+x^2\right)-2\,x-a\ln\left(-a+x\right)+a\ln\left(a+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(-a^2+x^2),x)
```

```
[Out] x*ln(-a^2+x^2)-2*x-a*ln(-a+x)+a*ln(a+x)
```

Maxima [A] time = 0.938799, size = 42, normalized size = 1.68

$$x\log\left(-a^2+x^2\right)+a\log\left(a+x\right)-a\log\left(-a+x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-a^2+x^2),x, algorithm="maxima")
```

```
[Out] x*log(-a^2 + x^2) + a*log(a + x) - a*log(-a + x) - 2*x
```

Fricas [A] time = 2.1013, size = 76, normalized size = 3.04

$$x\log\left(-a^2+x^2\right)+a\log\left(a+x\right)-a\log\left(-a+x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-a^2+x^2),x, algorithm="fricas")
```

```
[Out] x*log(-a^2 + x^2) + a*log(a + x) - a*log(-a + x) - 2*x
```

Sympy [A] time = 0.306389, size = 29, normalized size = 1.16

$$-2a\left(\frac{\log\left(-a+x\right)}{2}-\frac{\log\left(a+x\right)}{2}\right)+x\log\left(-a^2+x^2\right)-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(-a**2+x**2),x)
```

```
[Out] -2*a*(log(-a + x)/2 - log(a + x)/2) + x*log(-a**2 + x**2) - 2*x
```

Giac [A] time = 1.07971, size = 45, normalized size = 1.8

$$x\log\left(-a^2+x^2\right)+a\log\left(|a+x|\right)-a\log\left(|-a+x|\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(-a^2+x^2),x, algorithm="giac")
```

```
[Out] x*log(-a^2 + x^2) + a*log(abs(a + x)) - a*log(abs(-a + x)) - 2*x
```

3.75 $\int \log(\log(\log(\log(x))))\,dx$

Optimal. Leaf size=7

$$\text{CannotIntegrate}(\log(\log(\log(\log(x)))),x)$$

```
[Out] Defer[Int][Log[Log[Log[Log[x]]]], x]
```

Rubi [A] time = 0.0058479, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \log(\log(\log(\log(x))))\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[Log[Log[Log[x]]]],x]
```

```
[Out] Defer[Int][Log[Log[Log[Log[x]]]], x]
```

Rubi steps

$$\int \log(\log(\log(\log(x))))\,dx = \int \log(\log(\log(\log(x))))\,dx$$

Mathematica [A] time = 0.0363875, size = 0, normalized size = 0.

$$\int \log(\log(\log(\log(x))))\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Log[Log[Log[Log[x]]]],x]
```

```
[Out] Integrate[Log[Log[Log[Log[x]]]], x]
```

Maple [A] time = 0.014, size = 0, normalized size = 0.

$$\int \ln(\ln(\ln(\ln(x)))) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(ln(ln(ln(x)))),x)
```

```
[Out] int(ln(ln(ln(ln(x)))),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$x\log(\log(\log(\log(x)))) - \int \frac{1}{\log(x)\log(\log(x))\log(\log(\log(x)))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(log(log(x)))),x, algorithm="maxima")
```

```
[Out] x*log(log(log(log(x)))) - integrate(1/(log(x)*log(log(x))*log(log(log(x))))
, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}(\log(\log(\log(\log(x)))), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(log(log(x)))),x, algorithm="fricas")
```

```
[Out] integral(log(log(log(log(x)))), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$x\log(\log(\log(\log(x)))) - \int \frac{1}{\log(x)\log(\log(x))\log(\log(\log(x)))} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(ln(ln(ln(x)))),x)
```

```
[Out] x*log(log(log(log(x)))) - Integral(1/(log(x)*log(log(x))*log(log(log(x)))),
 x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \log \left(\log \left(\log \left(\log \left(x\right)\right)\right)\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(log(log(log(x)))),x, algorithm="giac")
```

```
[Out] integrate(log(log(log(log(x)))), x)
```

3.76 $\int \sin(x)\,dx$

Optimal. Leaf size=4

$$-\cos(x)$$

```
[Out] -Cos[x]
```

Rubi [A] time = 0.0020246, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2638}

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x],x]

[Out] -Cos[x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sin(x)\,dx = -\cos(x)$$

Mathematica [A] time = 0.0009246, size = 4, normalized size = 1.

$$-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x],x]
```

```
[Out] -Cos[x]
```

Maple [A] time = 0.001, size = 5, normalized size = 1.3

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x),x)
```

```
[Out] -cos(x)
```

Maxima [A] time = 0.946543, size = 5, normalized size = 1.25

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x, algorithm="maxima")
```

```
[Out] -cos(x)
```

Fricas [A] time = 2.05707, size = 12, normalized size = 3.

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x, algorithm="fricas")
```

```
[Out] -cos(x)
```

Sympy [A] time = 0.058617, size = 3, normalized size = 0.75

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x)
```

```
[Out] -cos(x)
```

Giac [A] time = 1.07726, size = 5, normalized size = 1.25

$$-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x),x, algorithm="giac")
```

```
[Out] -cos(x)
```

3.77 $\int \cos(x)\,dx$

Optimal. Leaf size=2

$$\sin(x)$$

```
[Out] Sin[x]
```

Rubi [A] time = 0.0020367, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2637}

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x],x]

[Out] Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(x)\,dx = \sin(x)$$

Mathematica [A] time = 0.0008623, size = 2, normalized size = 1.

$$\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x],x]
```

```
[Out] Sin[x]
```

Maple [A] time = 0.001, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x),x)
```

```
[Out] sin(x)
```

Maxima [A] time = 0.93906, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="maxima")
```

```
[Out] sin(x)
```

Fricas [A] time = 2.1938, size = 11, normalized size = 5.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="fricas")
```

```
[Out] sin(x)
```

Sympy [A] time = 0.054347, size = 2, normalized size = 1.

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x)
```

```
[Out] sin(x)
```

Giac [A] time = 1.06838, size = 3, normalized size = 1.5

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x),x, algorithm="giac")
```

```
[Out] sin(x)
```

3.78 $\int \tan(x)\,dx$

Optimal. Leaf size=5

$$-\log(\cos(x))$$

```
[Out] -Log[Cos[x]]
```

Rubi [A] time = 0.0016492, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3475}

$$-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x],x]

[Out] -Log[Cos[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tan(x)\,dx = -\log(\cos(x))$$

Mathematica [A] time = 0.0018955, size = 5, normalized size = 1.

$$-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x],x]
```

```
[Out] -Log[Cos[x]]
```

Maple [A] time = 0., size = 6, normalized size = 1.2

$$-\ln(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x),x)
```

```
[Out] -ln(cos(x))
```

Maxima [A] time = 0.93052, size = 4, normalized size = 0.8

$$\log(\sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x, algorithm="maxima")
```

```
[Out] log(sec(x))
```

Fricas [B] time = 2.12655, size = 38, normalized size = 7.6

$$-\frac{1}{2}\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] time = 0.060989, size = 5, normalized size = 1.

$$-\log(\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x)
```

```
[Out] -log(cos(x))
```

<u>Giac</u> [A] time = 1.09744, size = 8, normalized size = 1.6

$$-\log\left(|\cos\left(x\right)|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x),x, algorithm="giac")
```

```
[Out] -log(abs(cos(x)))
```

3.79 $\int \cot(x)\,dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

```
[Out] Log[Sin[x]]
```

Rubi [A] time = 0.0021964, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3475}

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(x)\,dx = \log(\sin(x))$$

Mathematica [A] time = 0.0019737, size = 3, normalized size = 1.

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Maple [A] time = 0.002, size = 4, normalized size = 1.3

$$\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(x),x)
```

```
[Out] ln(sin(x))
```

Maxima [A] time = 0.935286, size = 4, normalized size = 1.33

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x),x, algorithm="maxima")
```

```
[Out] log(sin(x))
```

Fricas [B] time = 1.97721, size = 46, normalized size = 15.33

$$\frac{1}{2}\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(tan(x)^2/(tan(x)^2 + 1))
```

Sympy [A] time = 0.066143, size = 3, normalized size = 1.

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x),x)
```

```
[Out] log(sin(x))
```

Giac [B] time = 1.08098, size = 23, normalized size = 7.67

$$-\frac{1}{2}\log\left(\tan(x)^2+1\right)+\frac{1}{2}\log\left(\tan(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x),x, algorithm="giac")
```

```
[Out] -1/2*log(tan(x)^2 + 1) + 1/2*log(tan(x)^2)
```

3.80 $\int \frac{1}{(1+\tan(x))^2} \, dx$

Optimal. Leaf size=21

$$\frac{1}{2}\log(\sin(x)+\cos(x))-\frac{1}{2(\tan(x)+1)}$$

```
[Out] Log[Cos[x] + Sin[x]]/2 - 1/(2*(1 + Tan[x]))
```

Rubi [A] time = 0.0256371, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3483, 3530}

$$\frac{1}{2}\log(\sin(x)+\cos(x))-\frac{1}{2(\tan(x)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Tan[x])^(-2),x]
```

```
[Out] Log[Cos[x] + Sin[x]]/2 - 1/(2*(1 + Tan[x]))
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{1}{(1+\tan(x))^2}\,dx = -\frac{1}{2(1+\tan(x))} + \frac{1}{2}\int \frac{1-\tan(x)}{1+\tan(x)}\,dx$$
$$= \frac{1}{2}\log(\cos(x)+\sin(x)) - \frac{1}{2(1+\tan(x))}$$

Mathematica [A] time = 0.0404895, size = 27, normalized size = 1.29

$$\frac{\tan(x)+\log(\sin(x)+\cos(x))+\tan(x)\log(\sin(x)+\cos(x))}{2\tan(x)+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Tan[x])^(-2),x]
```

```
[Out] (Log[Cos[x] + Sin[x]] + Tan[x] + Log[Cos[x] + Sin[x]]*Tan[x])/(2 + 2*Tan[x]
)
```

Maple [A] time = 0.014, size = 26, normalized size = 1.2

$$-\frac{\ln\left((\tan(x))^2+1\right)}{4} - \frac{1}{2+2\tan(x)} + \frac{\ln(1+\tan(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+tan(x))^2,x)
```

```
[Out] -1/4*ln(tan(x)^2+1)-1/2/(1+tan(x))+1/2*ln(1+tan(x))
```

Maxima [A] time = 1.42644, size = 34, normalized size = 1.62

$$-\frac{1}{2(\tan(x)+1)} - \frac{1}{4}\log\left(\tan(x)^2+1\right) + \frac{1}{2}\log(\tan(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x))^2,x, algorithm="maxima")
```

```
[Out] -1/2/(tan(x) + 1) - 1/4*log(tan(x)^2 + 1) + 1/2*log(tan(x) + 1)
```

Fricas [B] time = 2.06109, size = 124, normalized size = 5.9

$$\frac{(\tan(x)+1)\log\left(\frac{\tan(x)^2+2\tan(x)+1}{\tan(x)^2+1}\right)+\tan(x)-1}{4(\tan(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((tan(x) + 1)*log((tan(x)^2 + 2*tan(x) + 1)/(tan(x)^2 + 1)) + tan(x) -
1)/(tan(x) + 1)
```

Sympy [B] time = 0.365338, size = 75, normalized size = 3.57

$$\frac{2\log(\tan(x)+1)\tan(x)}{4\tan(x)+4}+\frac{2\log(\tan(x)+1)}{4\tan(x)+4}-\frac{\log\left(\tan^2(x)+1\right)\tan(x)}{4\tan(x)+4}-\frac{\log\left(\tan^2(x)+1\right)}{4\tan(x)+4}-\frac{2}{4\tan(x)+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x))**2,x)
```

```
[Out] 2*log(tan(x) + 1)*tan(x)/(4*tan(x) + 4) + 2*log(tan(x) + 1)/(4*tan(x) + 4)
- log(tan(x)**2 + 1)*tan(x)/(4*tan(x) + 4) - log(tan(x)**2 + 1)/(4*tan(x) +
 4) - 2/(4*tan(x) + 4)
```

Giac [A] time = 1.10039, size = 35, normalized size = 1.67

$$-\frac{1}{2(\tan(x)+1)}-\frac{1}{4}\log\left(\tan(x)^2+1\right)+\frac{1}{2}\log(|\tan(x)+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+tan(x))^2,x, algorithm="giac")
```

```
[Out] -1/2/(tan(x) + 1) - 1/4*log(tan(x)^2 + 1) + 1/2*log(abs(tan(x) + 1))
```

3.81 $\int \sec(x)\,dx$

Optimal. Leaf size=3

$$\tanh^{-1}(\sin(x))$$

[Out] ArcTanh[Sin[x]]

Rubi [A] time = 0.0025583, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3770}

$$\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

[In] Int[Sec[x],x]

[Out] ArcTanh[Sin[x]]

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(x)\,dx = \tanh^{-1}(\sin(x))$$

Mathematica [B] time = 0.0029171, size = 33, normalized size = 11.

$$\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x],x]

```
[Out] -Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

Maple [A] time = 0.004, size = 7, normalized size = 2.3

$$\ln(\sec(x) + \tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x),x)
```

```
[Out] ln(sec(x)+tan(x))
```

Maxima [B] time = 0.941983, size = 20, normalized size = 6.67

$$\frac{1}{2}\log(\sin(x) + 1) - \frac{1}{2}\log(\sin(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x),x, algorithm="maxima")
```

```
[Out] 1/2*log(sin(x) + 1) - 1/2*log(sin(x) - 1)
```

Fricas [B] time = 2.09825, size = 59, normalized size = 19.67

$$\frac{1}{2}\log(\sin(x) + 1) - \frac{1}{2}\log(-\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Sympy [B] time = 0.097763, size = 15, normalized size = 5.

$$-\frac{\log(\sin(x)-1)}{2}+\frac{\log(\sin(x)+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x),x)
```

```
[Out] -log(sin(x) - 1)/2 + log(sin(x) + 1)/2
```

Giac [B] time = 1.08769, size = 23, normalized size = 7.67

$$\frac{1}{2}\log(\sin(x)+1)-\frac{1}{2}\log(-\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x),x, algorithm="giac")
```

```
[Out] 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

3.82 $\int \csc(x)\,dx$

Optimal. Leaf size=5

$$-\tanh^{-1}(\cos(x))$$

```
[Out] -ArcTanh[Cos[x]]
```

Rubi [A] time = 0.0016223, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3770}

$$-\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x],x]

[Out] -ArcTanh[Cos[x]]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(x)\,dx = -\tanh^{-1}(\cos(x))$$

Mathematica [B] time = 0.0029174, size = 17, normalized size = 3.4

$$\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x],x]
```

```
[Out] -Log[Cos[x/2]] + Log[Sin[x/2]]
```

Maple [A] time = 0.005, size = 9, normalized size = 1.8

$$\ln(\csc(x) - \cot(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(x),x)
```

```
[Out] ln(csc(x)-cot(x))
```

Maxima [B] time = 0.941772, size = 20, normalized size = 4.

$$-\frac{1}{2}\log(\cos(x) + 1) + \frac{1}{2}\log(\cos(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*log(cos(x) + 1) + 1/2*log(cos(x) - 1)
```

Fricas [B] time = 2.08161, size = 77, normalized size = 15.4

$$-\frac{1}{2}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [B] time = 0.097254, size = 15, normalized size = 3.

$$\frac{\log(\cos(x)-1)}{2}-\frac{\log(\cos(x)+1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x),x)
```

```
[Out] log(cos(x) - 1)/2 - log(cos(x) + 1)/2
```

Giac [B] time = 1.10185, size = 23, normalized size = 4.6

$$-\frac{1}{2}\log(\cos(x)+1)+\frac{1}{2}\log(-\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(x),x, algorithm="giac")
```

```
[Out] -1/2*log(cos(x) + 1) + 1/2*log(-cos(x) + 1)
```

3.83 $\int \sin^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0045568, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2,x]
```

```
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^2(x)\,dx &= -\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0015989, size = 14, normalized size = 1.

$$\frac{x}{2}-\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2,x]
```

```
[Out] x/2 - Sin[2*x]/4
```

Maple [A] time = 0., size = 11, normalized size = 0.8

$$\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x)
```

```
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Maxima [A] time = 0.934652, size = 14, normalized size = 1.

$$\frac{1}{2}x-\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

Fricas [A] time = 1.98156, size = 38, normalized size = 2.71

$$-\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.058461, size = 10, normalized size = 0.71

$$\frac{x}{2}-\frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2,x)
```

```
[Out] x/2 - sin(x)*cos(x)/2
```

Giac [A] time = 1.07458, size = 14, normalized size = 1.

$$\frac{1}{2}\,x-\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

3.84 $\int x^3 \sin\left(x^2\right) dx$

Optimal. Leaf size=20

$$\frac{\sin\left(x^2\right)}{2} - \frac{1}{2}x^2\cos\left(x^2\right)$$

```
[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Rubi [A] time = 0.0156995, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3379, 3296, 2637}

$$\frac{\sin\left(x^2\right)}{2} - \frac{1}{2}x^2\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[x^2],x]
```

```
[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int x^3 \sin\left(x^2\right) dx = \frac{1}{2}\text{Subst}\left(\int x\sin(x)\,dx, x, x^2\right)$$
$$= -\frac{1}{2}x^2\cos\left(x^2\right) + \frac{1}{2}\text{Subst}\left(\int \cos(x)\,dx, x, x^2\right)$$
$$= -\frac{1}{2}x^2\cos\left(x^2\right) + \frac{\sin\left(x^2\right)}{2}$$

Mathematica [A] time = 0.0021866, size = 20, normalized size = 1.

$$\frac{\sin\left(x^2\right)}{2} - \frac{1}{2}x^2\cos\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x^2],x]
```

```
[Out] -(x^2*Cos[x^2])/2 + Sin[x^2]/2
```

Maple [A] time = 0.005, size = 17, normalized size = 0.9

$$-\frac{x^2\cos\left(x^2\right)}{2} + \frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x^2),x)
```

```
[Out] -1/2*x^2*cos(x^2)+1/2*sin(x^2)
```

Maxima [A] time = 0.945259, size = 22, normalized size = 1.1

$$-\frac{1}{2}x^2\cos\left(x^2\right) + \frac{1}{2}\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Fricas [A] time = 1.81623, size = 46, normalized size = 2.3

$$-\frac{1}{2}\,x^2\cos\left(x^2\right)+\frac{1}{2}\,\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="fricas")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

Sympy [A] time = 0.562769, size = 15, normalized size = 0.75

$$-\frac{x^2\cos\left(x^2\right)}{2}+\frac{\sin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(x**2),x)
```

```
[Out] -x**2*cos(x**2)/2 + sin(x**2)/2
```

Giac [A] time = 1.0713, size = 22, normalized size = 1.1

$$-\frac{1}{2}\,x^2\cos\left(x^2\right)+\frac{1}{2}\,\sin\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x^2),x, algorithm="giac")
```

```
[Out] -1/2*x^2*cos(x^2) + 1/2*sin(x^2)
```

3.85 $\int \sin^3(x)\,dx$

Optimal. Leaf size=13

$$\frac{\cos^3(x)}{3}-\cos(x)$$

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rubi [A] time = 0.0053879, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2633}

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3,x]
```

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(x)\,dx &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(x)\right)\\ &= -\cos(x)+\frac{\cos^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.0017832, size = 15, normalized size = 1.15

$$\frac{1}{12}\cos(3x)-\frac{3\cos(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3,x]
```

```
[Out] (-3*Cos[x])/4 + Cos[3*x]/12
```

Maple [A] time = 0.001, size = 11, normalized size = 0.9

$$-\frac{\left(2+(\sin(x))^2\right)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x)
```

```
[Out] -1/3*(2+sin(x)^2)*cos(x)
```

Maxima [A] time = 0.946961, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

Fricas [A] time = 1.79082, size = 31, normalized size = 2.38

$$\frac{1}{3}\cos(x)^3-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

Sympy [A] time = 0.060588, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3,x)
```

```
[Out] cos(x)**3/3 - cos(x)
```

Giac [A] time = 1.07568, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

3.86 $\int \sin^p(x)\,dx$

Optimal. Leaf size=44

$$\frac{\cos(x)\sin^{p+1}(x)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{p+1}{2},\frac{p+3}{2},\sin^2(x)\right)}{(p+1)\sqrt{\cos^2(x)}}$$

```
[Out] (Cos[x]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[x]^2]*Sin[x]^(1 +
p))/((1 + p)*Sqrt[Cos[x]^2])
```

Rubi [A] time = 0.0076611, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2643}

$$\frac{\cos(x)\sin^{p+1}(x)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{p+1}{2},\frac{p+3}{2},\sin^2(x)\right)}{(p+1)\sqrt{\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^p,x]
```

```
[Out] (Cos[x]*Hypergeometric2F1[1/2, (1 + p)/2, (3 + p)/2, Sin[x]^2]*Sin[x]^(1 +
p))/((1 + p)*Sqrt[Cos[x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int \sin^p(x)\,dx = \frac{\cos(x)\,{}_2F_1\left(\frac{1}{2},\frac{1+p}{2};\frac{3+p}{2};\sin^2(x)\right)\sin^{1+p}(x)}{(1+p)\sqrt{\cos^2(x)}}$$

Mathematica [A] time = 0.0395527, size = 44, normalized size = 1.

$$-\cos(x)\sin^{p+1}(x)\sin^2(x)^{\frac{1}{2}(-p-1)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1-p}{2},\frac{3}{2},\cos^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^p,x]
```

```
[Out] -(Cos[x]*Hypergeometric2F1[1/2, (1 - p)/2, 3/2, Cos[x]^2]*Sin[x]^(1 + p)*(S
in[x]^2)^((-1 - p)/2))
```

Maple [F] time = 0.385, size = 0, normalized size = 0.

$$\int (\sin(x))^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^p,x)
```

```
[Out] int(sin(x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(x)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^p,x, algorithm="maxima")
```

```
[Out] integrate(sin(x)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sin(x)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^p,x, algorithm="fricas")
```

```
[Out] integral(sin(x)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sin^{p}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**p,x)
```

```
[Out] Integral(sin(x)**p, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sin(x)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^p,x, algorithm="giac")
```

```
[Out] integrate(sin(x)^p, x)
```

3.87 $\int \cos(x)\left(1+\sin^2(x)\right)^2\,dx$

Optimal. Leaf size=19

$$\frac{\sin^5(x)}{5}+\frac{2\sin^3(x)}{3}+\sin(x)$$

```
[Out] Sin[x] + (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Rubi [A] time = 0.0191771, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3190, 194}

$$\frac{\sin^5(x)}{5}+\frac{2\sin^3(x)}{3}+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*(1 + Sin[x]^2)^2,x]
```

```
[Out] Sin[x] + (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos(x)\left(1+\sin^2(x)\right)^2 dx = \text{Subst}\left(\int \left(1+x^2\right)^2 dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(1+2x^2+x^4\right) dx, x, \sin(x)\right)$$
$$= \sin(x) + \frac{2\sin^3(x)}{3} + \frac{\sin^5(x)}{5}$$

Mathematica [A] time = 0.0035859, size = 19, normalized size = 1.

$$\frac{\sin^5(x)}{5} + \frac{2\sin^3(x)}{3} + \sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*(1 + Sin[x]^2)^2,x]
```

```
[Out] Sin[x] + (2*Sin[x]^3)/3 + Sin[x]^5/5
```

Maple [A] time = 0.01, size = 16, normalized size = 0.8

$$\sin(x) + \frac{2(\sin(x))^3}{3} + \frac{(\sin(x))^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*(1+sin(x)^2)^2,x)
```

```
[Out] sin(x)+2/3*sin(x)^3+1/5*sin(x)^5
```

Maxima [A] time = 0.955533, size = 20, normalized size = 1.05

$$\frac{1}{5}\sin(x)^5 + \frac{2}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/5*sin(x)^5 + 2/3*sin(x)^3 + sin(x)
```

Fricas [A] time = 1.63509, size = 61, normalized size = 3.21

$$\frac{1}{15}\left(3\cos(x)^4 - 16\cos(x)^2 + 28\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(x)^4 - 16*cos(x)^2 + 28)*sin(x)
```

Sympy [A] time = 1.09388, size = 17, normalized size = 0.89

$$\frac{\sin^5(x)}{5} + \frac{2\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)**2)**2,x)
```

```
[Out] sin(x)**5/5 + 2*sin(x)**3/3 + sin(x)
```

Giac [A] time = 1.06927, size = 20, normalized size = 1.05

$$\frac{1}{5}\sin(x)^5 + \frac{2}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*(1+sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/5*sin(x)^5 + 2/3*sin(x)^3 + sin(x)
```

3.88 $\int \cos^2(x)\, dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0058477, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\, dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\, dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.002088, size = 14, normalized size = 1.

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2,x]
```

```
[Out] x/2 + Sin[2*x]/4
```

Maple [A] time = 0., size = 11, normalized size = 0.8

$$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
```

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A] time = 0.939514, size = 14, normalized size = 1.

$$\frac{1}{2}x+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A] time = 1.71917, size = 36, normalized size = 2.57

$$\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.059192, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

Giac [A] time = 1.09755, size = 14, normalized size = 1.

$$\frac{1}{2}\,x+\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

3.89 $\int \cos^3(x)\,dx$

Optimal. Leaf size=11

$$\sin(x)-\frac{\sin^3(x)}{3}$$

```
[Out] Sin[x] - Sin[x]^3/3
```

Rubi [A] time = 0.0063308, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2633}

$$\sin(x)-\frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3,x]
```

```
[Out] Sin[x] - Sin[x]^3/3
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\,dx &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(x)\right)\\ &= \sin(x)-\frac{\sin^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.001777, size = 15, normalized size = 1.36

$$\frac{3\sin(x)}{4}+\frac{1}{12}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3,x]
```

```
[Out] (3*Sin[x])/4 + Sin[3*x]/12
```

Maple [A] time = 0., size = 11, normalized size = 1.

$$\frac{\left(2+(\cos(x))^2\right)\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3,x)
```

```
[Out] 1/3*(2+cos(x)^2)*sin(x)
```

Maxima [A] time = 0.938879, size = 12, normalized size = 1.09

$$-\frac{1}{3}\sin(x)^3+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="maxima")
```

```
[Out] -1/3*sin(x)^3 + sin(x)
```

Fricas [A] time = 1.68354, size = 36, normalized size = 3.27

$$\frac{1}{3}\left(\cos(x)^2+2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + 2)*sin(x)
```

Sympy [A] time = 0.061609, size = 8, normalized size = 0.73

$$-\frac{\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3,x)
```

```
[Out] -sin(x)**3/3 + sin(x)
```

Giac [A] time = 1.10237, size = 12, normalized size = 1.09

$$-\frac{1}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="giac")
```

```
[Out] -1/3*sin(x)^3 + sin(x)
```

3.90 $\int \sec^2(x)\,dx$

Optimal. Leaf size=2

$$\tan(x)$$

```
[Out] Tan[x]
```

Rubi [A] time = 0.0058249, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3767, 8}

$$\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2,x]
```

```
[Out] Tan[x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(x)\,dx &= -\operatorname{Subst}\left(\int 1\,dx, x, -\tan(x)\right)\\ &= \tan(x)\end{aligned}$$

Mathematica [A] time = 0.001646, size = 2, normalized size = 1.

$$\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2,x]
```

```
[Out] Tan[x]
```

Maple [A] time = 0.029, size = 3, normalized size = 1.5

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^2,x)
```

```
[Out] tan(x)
```

Maxima [A] time = 0.940791, size = 3, normalized size = 1.5

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2,x, algorithm="maxima")
```

```
[Out] tan(x)
```

Fricas [B] time = 1.60085, size = 20, normalized size = 10.

$$\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2,x, algorithm="fricas")
```

```
[Out] sin(x)/cos(x)
```

Sympy [B] time = 0.057754, size = 5, normalized size = 2.5

$$\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**2,x)
```

```
[Out] sin(x)/cos(x)
```

Giac [A] time = 1.0819, size = 3, normalized size = 1.5

$$\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2,x, algorithm="giac")
```

```
[Out] tan(x)
```

3.91 $\int \sin(x)\sin(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

```
[Out] Sin[x]/2 - Sin[3*x]/6
```

Rubi [A] time = 0.0074203, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4282}

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[2*x],x]
```

```
[Out] Sin[x]/2 - Sin[3*x]/6
```

Rule 4282

```
Int[sin[(a_.) + (b_.)*(x_)]*sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \sin(x)\sin(2x)\,dx=\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

Mathematica [A] time = 0.0045002, size = 15, normalized size = 1.

$$\frac{\sin(x)}{2}-\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[2*x],x]
```

```
[Out] Sin[x]/2 - Sin[3*x]/6
```

Maple [A] time = 0.01, size = 7, normalized size = 0.5

$$\frac{2 (\sin(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(2*x),x)
```

```
[Out] 2/3*sin(x)^3
```

Maxima [A] time = 0.930142, size = 15, normalized size = 1.

$$-\frac{1}{6} \sin(3x) + \frac{1}{2} \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/6*sin(3*x) + 1/2*sin(x)
```

Fricas [A] time = 1.75803, size = 38, normalized size = 2.53

$$-\frac{2}{3} \left(\cos(x)^2 - 1\right) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -2/3*(cos(x)^2 - 1)*sin(x)
```

Sympy [A] time = 0.602697, size = 20, normalized size = 1.33

$$-\frac{2\sin(x)\cos(2x)}{3}+\frac{\sin(2x)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x)
```

```
[Out] -2*sin(x)*cos(2*x)/3 + sin(2*x)*cos(x)/3
```

Giac [A] time = 1.08732, size = 15, normalized size = 1.

$$-\frac{1}{6}\sin(3x)+\frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x),x, algorithm="giac")
```

```
[Out] -1/6*sin(3*x) + 1/2*sin(x)
```

3.92 $\int x\sin(x)\,dx$

Optimal. Leaf size=8

$$\sin(x) - x\cos(x)$$

```
[Out] -(x*Cos[x]) + Sin[x]
```

Rubi [A] time = 0.0083124, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3296, 2637}

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x],x]
```

```
[Out] -(x*Cos[x]) + Sin[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sin(x)\,dx &= -x\cos(x) + \int \cos(x)\,dx\\ &= -x\cos(x) + \sin(x)\end{aligned}$$

Mathematica [A] time = 0.0020384, size = 8, normalized size = 1.

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x],x]
```

```
[Out] -(x*Cos[x]) + Sin[x]
```

Maple [A] time = 0., size = 9, normalized size = 1.1

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x),x)
```

```
[Out] -x*cos(x)+sin(x)
```

Maxima [A] time = 0.956942, size = 11, normalized size = 1.38

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="maxima")
```

```
[Out] -x*cos(x) + sin(x)
```

Fricas [A] time = 1.73467, size = 27, normalized size = 3.38

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="fricas")
```

```
[Out] -x*cos(x) + sin(x)
```

Sympy [A] time = 0.174753, size = 7, normalized size = 0.88

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x)
```

```
[Out] -x*cos(x) + sin(x)
```

Giac [A] time = 1.10093, size = 11, normalized size = 1.38

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="giac")
```

```
[Out] -x*cos(x) + sin(x)
```

3.93 $\int x^2 \sin(x)\, dx$

Optimal. Leaf size=17

$$x^2(-\cos(x)) + 2x\sin(x) + 2\cos(x)$$

```
[Out] 2*Cos[x] - x^2*Cos[x] + 2*x*Sin[x]
```

Rubi [A] time = 0.0204533, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2638}

$$x^2(-\cos(x)) + 2x\sin(x) + 2\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[x],x]
```

```
[Out] 2*Cos[x] - x^2*Cos[x] + 2*x*Sin[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^2 \sin(x)\, dx &= -x^2\cos(x) + 2\int x\cos(x)\, dx\\ &= -x^2\cos(x) + 2x\sin(x) - 2\int \sin(x)\, dx\\ &= 2\cos(x) - x^2\cos(x) + 2x\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0147528, size = 15, normalized size = 0.88

$$2x\sin(x) - \left(x^2 - 2\right)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[x],x]
```

```
[Out] -((-2 + x^2)*Cos[x]) + 2*x*Sin[x]
```

Maple [A] time = 0., size = 18, normalized size = 1.1

$$2\cos(x) - x^2\cos(x) + 2x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x),x)
```

```
[Out] 2*cos(x)-x^2*cos(x)+2*x*sin(x)
```

Maxima [A] time = 0.930093, size = 20, normalized size = 1.18

$$-\left(x^2 - 2\right)\cos(x) + 2x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="maxima")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

Fricas [A] time = 1.88995, size = 43, normalized size = 2.53

$$-\left(x^2 - 2\right)\cos(x) + 2x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

Sympy [A] time = 0.316653, size = 17, normalized size = 1.

$$-x^2\cos(x) + 2x\sin(x) + 2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x),x)
```

```
[Out] -x**2*cos(x) + 2*x*sin(x) + 2*cos(x)
```

Giac [A] time = 1.08849, size = 20, normalized size = 1.18

$$-\left(x^2 - 2\right)\cos(x) + 2\,x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="giac")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

3.94 $\int x \sin^2(x)\,dx$

Optimal. Leaf size=25

$$\frac{x^2}{4}+\frac{\sin^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$

```
[Out] x^2/4 - (x*Cos[x]*Sin[x])/2 + Sin[x]^2/4
```

Rubi [A] time = 0.013332, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3310, 30}

$$\frac{x^2}{4}+\frac{\sin^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x]^2,x]
```

```
[Out] x^2/4 - (x*Cos[x]*Sin[x])/2 + Sin[x]^2/4
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\sin^2(x)\,dx &= -\frac{1}{2}x\cos(x)\sin(x)+\frac{\sin^2(x)}{4}+\frac{\int x\,dx}{2}\\ &= \frac{x^2}{4}-\frac{1}{2}x\cos(x)\sin(x)+\frac{\sin^2(x)}{4}\end{aligned}$$

Mathematica [A] time = 0.0083994, size = 25, normalized size = 1.

$$\frac{x^2}{4}-\frac{1}{4}x\sin(2x)-\frac{1}{8}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x]^2,x]
```

```
[Out] x^2/4 - Cos[2*x]/8 - (x*Sin[2*x])/4
```

Maple [A] time = 0., size = 25, normalized size = 1.

$$x\left(\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}\right)-\frac{x^2}{4}+\frac{(\sin(x))^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^2,x)
```

```
[Out] x*(1/2*x-1/2*cos(x)*sin(x))-1/4*x^2+1/4*sin(x)^2
```

Maxima [A] time = 0.957372, size = 26, normalized size = 1.04

$$\frac{1}{4}x^2-\frac{1}{4}x\sin(2x)-\frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

Fricas [A] time = 1.66212, size = 63, normalized size = 2.52

$$-\frac{1}{2}x\cos(x)\sin(x)+\frac{1}{4}x^2-\frac{1}{4}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*x*cos(x)*sin(x) + 1/4*x^2 - 1/4*cos(x)^2
```

Sympy [A] time = 0.333076, size = 36, normalized size = 1.44

$$\frac{x^2\sin^2(x)}{4}+\frac{x^2\cos^2(x)}{4}-\frac{x\sin(x)\cos(x)}{2}+\frac{\sin^2(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**2,x)
```

```
[Out] x**2*sin(x)**2/4 + x**2*cos(x)**2/4 - x*sin(x)*cos(x)/2 + sin(x)**2/4
```

Giac [A] time = 1.09378, size = 26, normalized size = 1.04

$$\frac{1}{4}x^2-\frac{1}{4}x\sin(2x)-\frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

3.95 $\int x^2 \sin^2(x) \, dx$

Optimal. Leaf size=41

$$\frac{x^3}{6} - \frac{1}{2}x^2\sin(x)\cos(x) - \frac{x}{4} + \frac{1}{2}x\sin^2(x) + \frac{1}{4}\sin(x)\cos(x)$$

```
[Out] -x/4 + x^3/6 + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin[x]^2)/2
```

Rubi [A] time = 0.0304728, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3311, 30, 2635, 8}

$$\frac{x^3}{6} - \frac{1}{2}x^2\sin(x)\cos(x) - \frac{x}{4} + \frac{1}{2}x\sin^2(x) + \frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[x]^2,x]
```

```
[Out] -x/4 + x^3/6 + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin[x]^2)/2
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sin^2(x)\,dx &= -\frac{1}{2}x^2\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)+\frac{\int x^2\,dx}{2}-\frac{1}{2}\int\sin^2(x)\,dx\\ &= \frac{x^3}{6}+\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x^2\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)-\frac{\int 1\,dx}{4}\\ &= -\frac{x}{4}+\frac{x^3}{6}+\frac{1}{4}\cos(x)\sin(x)-\frac{1}{2}x^2\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)\end{aligned}$$

Mathematica [A] time = 0.0369686, size = 29, normalized size = 0.71

$$\frac{1}{24}\left(4x^3+\left(3-6x^2\right)\sin(2x)-6x\cos(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[x]^2,x]
```

```
[Out] (4*x^3 - 6*x*Cos[2*x] + (3 - 6*x^2)*Sin[2*x])/24
```

Maple [A] time = 0., size = 37, normalized size = 0.9

$$x^2\left(\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}\right)-\frac{x(\cos(x))^2}{2}+\frac{\cos(x)\sin(x)}{4}+\frac{x}{4}-\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x)^2,x)
```

```
[Out] x^2*(1/2*x-1/2*cos(x)*sin(x))-1/2*x*cos(x)^2+1/4*cos(x)*sin(x)+1/4*x-1/3*x^
3
```

Maxima [A] time = 0.941102, size = 35, normalized size = 0.85

$$\frac{1}{6}\,x^3 - \frac{1}{4}\,x\cos\left(2\,x\right) - \frac{1}{8}\left(2\,x^2 - 1\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

Fricas [A] time = 1.81543, size = 89, normalized size = 2.17

$$\frac{1}{6}\,x^3 - \frac{1}{2}\,x\cos\left(x\right)^2 - \frac{1}{4}\left(2\,x^2 - 1\right)\cos\left(x\right)\sin\left(x\right) + \frac{1}{4}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^3 - 1/2*x*cos(x)^2 - 1/4*(2*x^2 - 1)*cos(x)*sin(x) + 1/4*x
```

Sympy [A] time = 0.624869, size = 56, normalized size = 1.37

$$\frac{x^3\sin^2\left(x\right)}{6} + \frac{x^3\cos^2\left(x\right)}{6} - \frac{x^2\sin\left(x\right)\cos\left(x\right)}{2} + \frac{x\sin^2\left(x\right)}{4} - \frac{x\cos^2\left(x\right)}{4} + \frac{\sin\left(x\right)\cos\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x)**2,x)
```

```
[Out] x**3*sin(x)**2/6 + x**3*cos(x)**2/6 - x**2*sin(x)*cos(x)/2 + x*sin(x)**2/4
- x*cos(x)**2/4 + sin(x)*cos(x)/4
```

Giac [A] time = 1.08424, size = 35, normalized size = 0.85

$$\frac{1}{6}\,x^3 - \frac{1}{4}\,x\cos\left(2\,x\right) - \frac{1}{8}\left(2\,x^2 - 1\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

3.96 $\int x\sin^3(x)\,dx$

Optimal. Leaf size=33

$$\frac{\sin^3(x)}{9}+\frac{2\sin(x)}{3}-\frac{2}{3}x\cos(x)-\frac{1}{3}x\sin^2(x)\cos(x)$$

```
[Out] (-2*x*Cos[x])/3 + (2*Sin[x])/3 - (x*Cos[x]*Sin[x]^2)/3 + Sin[x]^3/9
```

Rubi [A] time = 0.0216979, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3310, 3296, 2637}

$$\frac{\sin^3(x)}{9}+\frac{2\sin(x)}{3}-\frac{2}{3}x\cos(x)-\frac{1}{3}x\sin^2(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x]^3,x]
```

```
[Out] (-2*x*Cos[x])/3 + (2*Sin[x])/3 - (x*Cos[x]*Sin[x]^2)/3 + Sin[x]^3/9
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sin^3(x)\,dx &= -\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}+\frac{2}{3}\int x\sin(x)\,dx\\ &= -\frac{2}{3}x\cos(x)-\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}+\frac{2}{3}\int\cos(x)\,dx\\ &= -\frac{2}{3}x\cos(x)+\frac{2\sin(x)}{3}-\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}\end{aligned}$$

Mathematica [A] time = 0.0085734, size = 31, normalized size = 0.94

$$\frac{3\sin(x)}{4}-\frac{1}{36}\sin(3x)-\frac{3}{4}x\cos(x)+\frac{1}{12}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x]^3,x]
```

```
[Out] (-3*x*Cos[x])/4 + (x*Cos[3*x])/12 + (3*Sin[x])/4 - Sin[3*x]/36
```

Maple [A] time = 0., size = 23, normalized size = 0.7

$$-\frac{x\left(2+(\sin(x))^2\right)\cos(x)}{3}+\frac{(\sin(x))^3}{9}+\frac{2\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^3,x)
```

```
[Out] -1/3*x*(2+sin(x)^2)*cos(x)+1/9*sin(x)^3+2/3*sin(x)
```

Maxima [A] time = 0.962111, size = 31, normalized size = 0.94

$$\frac{1}{12}x\cos(3x)-\frac{3}{4}x\cos(x)-\frac{1}{36}\sin(3x)+\frac{3}{4}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

Fricas [A] time = 1.82112, size = 74, normalized size = 2.24

$$\frac{1}{3}\,x\cos\left(x\right)^{3} - x\cos\left(x\right) - \frac{1}{9}\left(\cos\left(x\right)^{2} - 7\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*x*cos(x)^3 - x*cos(x) - 1/9*(cos(x)^2 - 7)*sin(x)
```

Sympy [A] time = 0.603218, size = 39, normalized size = 1.18

$$-x\sin^{2}\left(x\right)\cos\left(x\right) - \frac{2x\cos^{3}\left(x\right)}{3} + \frac{7\sin^{3}\left(x\right)}{9} + \frac{2\sin\left(x\right)\cos^{2}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**3,x)
```

```
[Out] -x*sin(x)**2*cos(x) - 2*x*cos(x)**3/3 + 7*sin(x)**3/9 + 2*sin(x)*cos(x)**2/
3
```

Giac [A] time = 1.08484, size = 31, normalized size = 0.94

$$\frac{1}{12}\,x\cos\left(3\,x\right) - \frac{3}{4}\,x\cos\left(x\right) - \frac{1}{36}\,\sin\left(3\,x\right) + \frac{3}{4}\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

3.97 $\int x \cos(x) \, dx$

Optimal. Leaf size=7

$$x\sin(x) + \cos(x)$$

```
[Out] Cos[x] + x*Sin[x]
```

Rubi [A] time = 0.0088408, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3296, 2638}

$$x\sin(x) + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int x\cos(x)\,dx &= x\sin(x) - \int \sin(x)\,dx \\ &= \cos(x) + x\sin(x) \end{aligned}$$

Mathematica [A] time = 0.0022053, size = 7, normalized size = 1.

$$x\sin(x)+\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x],x]
```

```
[Out] Cos[x] + x*Sin[x]
```

Maple [A] time = 0.004, size = 8, normalized size = 1.1

$$\cos(x)+x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x),x)
```

```
[Out] cos(x)+x*sin(x)
```

Maxima [A] time = 0.931511, size = 9, normalized size = 1.29

$$x\sin(x)+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="maxima")
```

```
[Out] x*sin(x) + cos(x)
```

Fricas [A] time = 1.69831, size = 26, normalized size = 3.71

$$x\sin(x)+\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="fricas")
```

```
[Out] x*sin(x) + cos(x)
```

Sympy [A] time = 0.169327, size = 7, normalized size = 1.

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x)
```

```
[Out] x*sin(x) + cos(x)
```

Giac [A] time = 1.07695, size = 9, normalized size = 1.29

$$x\sin(x) + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x),x, algorithm="giac")
```

```
[Out] x*sin(x) + cos(x)
```

3.98 $\int x^2 \cos(x) \, dx$

Optimal. Leaf size=16

$$x^2 \sin(x) - 2 \sin(x) + 2x \cos(x)$$

```
[Out] 2*x*Cos[x] - 2*Sin[x] + x^2*Sin[x]
```

Rubi [A] time = 0.0212841, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2637}

$$x^2 \sin(x) - 2 \sin(x) + 2x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x],x]
```

```
[Out] 2*x*Cos[x] - 2*Sin[x] + x^2*Sin[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^2 \cos(x) \, dx &= x^2 \sin(x) - 2 \int x \sin(x) \, dx \\
&= 2x \cos(x) + x^2 \sin(x) - 2 \int \cos(x) \, dx \\
&= 2x \cos(x) - 2 \sin(x) + x^2 \sin(x)
\end{aligned}$$

Mathematica [A] time = 0.012978, size = 14, normalized size = 0.88

$$\left(x^2-2\right)\sin(x)+2x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x],x]
```

```
[Out] 2*x*Cos[x] + (-2 + x^2)*Sin[x]
```

Maple [A] time = 0.001, size = 17, normalized size = 1.1

$$2\,x\cos\left(x\right)-2\,\sin\left(x\right)+x^{2}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x),x)
```

```
[Out] 2*x*cos(x)-2*sin(x)+x^2*sin(x)
```

Maxima [A] time = 0.947238, size = 19, normalized size = 1.19

$$2\,x\cos\left(x\right)+\left(x^{2}-2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x),x, algorithm="maxima")
```

```
[Out] 2*x*cos(x) + (x^2 - 2)*sin(x)
```

Fricas [A] time = 1.68208, size = 42, normalized size = 2.62

$$2\,x\cos\left(x\right)+\left(x^{2}-2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x),x, algorithm="fricas")
```

```
[Out] 2*x*cos(x) + (x^2 - 2)*sin(x)
```

Sympy [A] time = 0.318554, size = 17, normalized size = 1.06

$$x^2 \sin(x) + 2x \cos(x) - 2 \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x),x)
```

```
[Out] x**2*sin(x) + 2*x*cos(x) - 2*sin(x)
```

Giac [A] time = 1.08699, size = 19, normalized size = 1.19

$$2\,x \cos(x) + \left(x^2 - 2\right) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x),x, algorithm="giac")
```

```
[Out] 2*x*cos(x) + (x^2 - 2)*sin(x)
```

3.99 $\int x \cos^2(x) \, dx$

Optimal. Leaf size=25

$$\frac{x^2}{4} + \frac{\cos^2(x)}{4} + \frac{1}{2} x \sin(x) \cos(x)$$

```
[Out] x^2/4 + Cos[x]^2/4 + (x*Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.014378, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3310, 30}

$$\frac{x^2}{4} + \frac{\cos^2(x)}{4} + \frac{1}{2} x \sin(x) \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]^2,x]
```

```
[Out] x^2/4 + Cos[x]^2/4 + (x*Cos[x]*Sin[x])/2
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x \cos^2(x) \, dx &= \frac{\cos^2(x)}{4} + \frac{1}{2} x \cos(x) \sin(x) + \frac{\int x \, dx}{2}\\ &= \frac{x^2}{4} + \frac{\cos^2(x)}{4} + \frac{1}{2} x \cos(x) \sin(x)\end{aligned}$$

Mathematica [A] time = 0.0106562, size = 25, normalized size = 1.

$$\frac{x^2}{4}+\frac{1}{4}x\sin(2x)+\frac{1}{8}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]^2,x]
```

```
[Out] x^2/4 + Cos[2*x]/8 + (x*Sin[2*x])/4
```

Maple [A] time = 0.005, size = 25, normalized size = 1.

$$x\left(\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}\right)-\frac{x^2}{4}-\frac{(\sin(x))^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2,x)
```

```
[Out] x*(1/2*x+1/2*cos(x)*sin(x))-1/4*x^2-1/4*sin(x)^2
```

Maxima [A] time = 0.952112, size = 26, normalized size = 1.04

$$\frac{1}{4}x^2+\frac{1}{4}x\sin(2x)+\frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^2 + 1/4*x*sin(2*x) + 1/8*cos(2*x)
```

Fricas [A] time = 1.66428, size = 62, normalized size = 2.48

$$\frac{1}{2}x\cos(x)\sin(x)+\frac{1}{4}x^2+\frac{1}{4}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(x)*sin(x) + 1/4*x^2 + 1/4*cos(x)^2
```

Sympy [A] time = 0.336439, size = 36, normalized size = 1.44

$$\frac{x^2\sin^2{(x)}}{4} + \frac{x^2\cos^2{(x)}}{4} + \frac{x\sin{(x)}\cos{(x)}}{2} - \frac{\sin^2{(x)}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)**2,x)
```

```
[Out] x**2*sin(x)**2/4 + x**2*cos(x)**2/4 + x*sin(x)*cos(x)/2 - sin(x)**2/4
```

Giac [A] time = 1.07154, size = 26, normalized size = 1.04

$$\frac{1}{4}\,x^2 + \frac{1}{4}\,x\sin\left(2\,x\right) + \frac{1}{8}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^2 + 1/4*x*sin(2*x) + 1/8*cos(2*x)
```

3.100 $\int x^2 \cos^2(x)\,dx$

Optimal. Leaf size=41

$$\frac{x^3}{6}+\frac{1}{2}x^2\sin(x)\cos(x)-\frac{x}{4}+\frac{1}{2}x\cos^2(x)-\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] -x/4 + x^3/6 + (x*Cos[x]^2)/2 - (Cos[x]*Sin[x])/4 + (x^2*Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0319042, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3311, 30, 2635, 8}

$$\frac{x^3}{6}+\frac{1}{2}x^2\sin(x)\cos(x)-\frac{x}{4}+\frac{1}{2}x\cos^2(x)-\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x]^2,x]
```

```
[Out] -x/4 + x^3/6 + (x*Cos[x]^2)/2 - (Cos[x]*Sin[x])/4 + (x^2*Cos[x]*Sin[x])/2
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int x^2\cos^2(x)\,dx &= \frac{1}{2}x\cos^2(x)+\frac{1}{2}x^2\cos(x)\sin(x)+\frac{\int x^2\,dx}{2}-\frac{1}{2}\int\cos^2(x)\,dx\\ &= \frac{x^3}{6}+\frac{1}{2}x\cos^2(x)-\frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x^2\cos(x)\sin(x)-\frac{\int 1\,dx}{4}\\ &= -\frac{x}{4}+\frac{x^3}{6}+\frac{1}{2}x\cos^2(x)-\frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x^2\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0358696, size = 29, normalized size = 0.71

$$\frac{1}{24}\left(4x^3+\left(6x^2-3\right)\sin(2x)+6x\cos(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x]^2,x]
```

```
[Out] (4*x^3 + 6*x*Cos[2*x] + (-3 + 6*x^2)*Sin[2*x])/24
```

Maple [A] time = 0.017, size = 37, normalized size = 0.9

$$x^2\left(\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}\right)+\frac{x(\cos(x))^2}{2}-\frac{\cos(x)\sin(x)}{4}-\frac{x}{4}-\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2,x)
```

```
[Out] x^2*(1/2*x+1/2*cos(x)*sin(x))+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x-1/3*x^
3
```

Maxima [A] time = 0.951689, size = 35, normalized size = 0.85

$$\frac{1}{6}x^3 + \frac{1}{4}x\cos(2x) + \frac{1}{8}\left(2x^2 - 1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^3 + 1/4*x*cos(2*x) + 1/8*(2*x^2 - 1)*sin(2*x)
```

Fricas [A] time = 1.7778, size = 89, normalized size = 2.17

$$\frac{1}{6}x^3 + \frac{1}{2}x\cos(x)^2 + \frac{1}{4}\left(2x^2 - 1\right)\cos(x)\sin(x) - \frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^3 + 1/2*x*cos(x)^2 + 1/4*(2*x^2 - 1)*cos(x)*sin(x) - 1/4*x
```

Sympy [A] time = 0.640509, size = 56, normalized size = 1.37

$$\frac{x^3\sin^2(x)}{6} + \frac{x^3\cos^2(x)}{6} + \frac{x^2\sin(x)\cos(x)}{2} - \frac{x\sin^2(x)}{4} + \frac{x\cos^2(x)}{4} - \frac{\sin(x)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x)**2,x)
```

```
[Out] x**3*sin(x)**2/6 + x**3*cos(x)**2/6 + x**2*sin(x)*cos(x)/2 - x*sin(x)**2/4
+ x*cos(x)**2/4 - sin(x)*cos(x)/4
```

Giac [A] time = 1.09371, size = 35, normalized size = 0.85

$$\frac{1}{6}x^3 + \frac{1}{4}x\cos(2x) + \frac{1}{8}\left(2x^2 - 1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 + 1/4*x*cos(2*x) + 1/8*(2*x^2 - 1)*sin(2*x)
```

3.101 $\int x\cos^3(x)\,dx$

Optimal. Leaf size=33

$$\frac{2}{3}x\sin(x)+\frac{\cos^3(x)}{9}+\frac{2\cos(x)}{3}+\frac{1}{3}x\sin(x)\cos^2(x)$$

```
[Out] (2*Cos[x])/3 + Cos[x]^3/9 + (2*x*Sin[x])/3 + (x*Cos[x]^2*Sin[x])/3
```

Rubi [A] time = 0.0230636, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3310, 3296, 2638}

$$\frac{2}{3}x\sin(x)+\frac{\cos^3(x)}{9}+\frac{2\cos(x)}{3}+\frac{1}{3}x\sin(x)\cos^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]^3,x]
```

```
[Out] (2*Cos[x])/3 + Cos[x]^3/9 + (2*x*Sin[x])/3 + (x*Cos[x]^2*Sin[x])/3
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cos^3(x)\,dx &= \frac{\cos^3(x)}{9} + \frac{1}{3}x\cos^2(x)\sin(x) + \frac{2}{3}\int x\cos(x)\,dx \\ &= \frac{\cos^3(x)}{9} + \frac{2}{3}x\sin(x) + \frac{1}{3}x\cos^2(x)\sin(x) - \frac{2}{3}\int \sin(x)\,dx \\ &= \frac{2\cos(x)}{3} + \frac{\cos^3(x)}{9} + \frac{2}{3}x\sin(x) + \frac{1}{3}x\cos^2(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00813, size = 31, normalized size = 0.94

$$\frac{3}{4}x\sin(x) + \frac{1}{12}x\sin(3x) + \frac{3\cos(x)}{4} + \frac{1}{36}\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]^3,x]
```

```
[Out] (3*Cos[x])/4 + Cos[3*x]/36 + (3*x*Sin[x])/4 + (x*Sin[3*x])/12
```

Maple [A] time = 0.011, size = 23, normalized size = 0.7

$$\frac{x\left(2+(\cos(x))^2\right)\sin(x)}{3} + \frac{(\cos(x))^3}{9} + \frac{2\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^3,x)
```

```
[Out] 1/3*x*(2+cos(x)^2)*sin(x)+1/9*cos(x)^3+2/3*cos(x)
```

Maxima [A] time = 0.951987, size = 31, normalized size = 0.94

$$\frac{1}{12}x\sin(3x) + \frac{3}{4}x\sin(x) + \frac{1}{36}\cos(3x) + \frac{3}{4}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^3,x, algorithm="maxima")
```

```
[Out] 1/12*x*sin(3*x) + 3/4*x*sin(x) + 1/36*cos(3*x) + 3/4*cos(x)
```

Fricas [A] time = 1.77494, size = 80, normalized size = 2.42

$$\frac{1}{9}\cos(x)^3+\frac{1}{3}\left(x\cos(x)^2+2x\right)\sin(x)+\frac{2}{3}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^3,x, algorithm="fricas")
```

```
[Out] 1/9*cos(x)^3 + 1/3*(x*cos(x)^2 + 2*x)*sin(x) + 2/3*cos(x)
```

Sympy [A] time = 0.60164, size = 39, normalized size = 1.18

$$\frac{2x\sin^3(x)}{3}+x\sin(x)\cos^2(x)+\frac{2\sin^2(x)\cos(x)}{3}+\frac{7\cos^3(x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)**3,x)
```

```
[Out] 2*x*sin(x)**3/3 + x*sin(x)*cos(x)**2 + 2*sin(x)**2*cos(x)/3 + 7*cos(x)**3/9
```

Giac [A] time = 1.06452, size = 31, normalized size = 0.94

$$\frac{1}{12}x\sin(3x)+\frac{3}{4}x\sin(x)+\frac{1}{36}\cos(3x)+\frac{3}{4}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^3,x, algorithm="giac")
```

```
[Out] 1/12*x*sin(3*x) + 3/4*x*sin(x) + 1/36*cos(3*x) + 3/4*cos(x)
```

3.102 $\int \frac{\sin(x)}{x} \, dx$

Optimal. Leaf size=2

$$\text{Si}(x)$$

```
[Out] SinIntegral[x]
```

Rubi [A] time = 0.0108873, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3299}

$$\text{Si}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/x,x]

[Out] SinIntegral[x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\sin(x)}{x} \, dx = \text{Si}(x)$$

Mathematica [A] time = 0.0102894, size = 2, normalized size = 1.

$$\text{Si}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/x,x]
```

```
[Out] SinIntegral[x]
```

Maple [A] time = 0., size = 3, normalized size = 1.5

$$\mathrm{Si}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/x,x)
```

```
[Out] Si(x)
```

Maxima [C] time = 1.0543, size = 18, normalized size = 9.

$$-\frac{1}{2}i\,\mathrm{Ei}\,(i\,x)+\frac{1}{2}i\,\mathrm{Ei}\,(-i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x,x, algorithm="maxima")
```

```
[Out] -1/2*I*Ei(I*x) + 1/2*I*Ei(-I*x)
```

Fricas [A] time = 1.8218, size = 23, normalized size = 11.5

$$\mathrm{Si}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x,x, algorithm="fricas")
```

```
[Out] sin_integral(x)
```

Sympy [A] time = 0.576056, size = 2, normalized size = 1.

$$\mathrm{Si}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x,x)
```

```
[Out] Si(x)
```

Giac [A] time = 1.07321, size = 3, normalized size = 1.5

$$\mathrm{Si}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x,x, algorithm="giac")
```

```
[Out] sin_integral(x)
```

3.103 $\int \frac{\cos(x)}{x} \, dx$

Optimal. Leaf size=2

$$\text{CosIntegral}(x)$$

```
[Out] CosIntegral[x]
```

Rubi [A] time = 0.0124961, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3302}

$$\text{CosIntegral}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/x,x]
```

```
[Out] CosIntegral[x]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\cos(x)}{x} \, dx = \text{Ci}(x)$$

Mathematica [A] time = 0.0014025, size = 2, normalized size = 1.

$$\text{CosIntegral}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/x,x]
```

```
[Out] CosIntegral[x]
```

Maple [A] time = 0.005, size = 3, normalized size = 1.5

$$\mathrm{Ci}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/x,x)
```

```
[Out] Ci(x)
```

Maxima [C] time = 1.05578, size = 18, normalized size = 9.

$$\frac{1}{2}\,\mathrm{Ei}\,(i\,x) + \frac{1}{2}\,\mathrm{Ei}\,(-i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x,x, algorithm="maxima")
```

```
[Out] 1/2*Ei(I*x) + 1/2*Ei(-I*x)
```

Fricas [B] time = 1.69277, size = 59, normalized size = 29.5

$$\frac{1}{2}\,\mathrm{Ci}\,(-x) + \frac{1}{2}\,\mathrm{Ci}\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x,x, algorithm="fricas")
```

```
[Out] 1/2*cos_integral(-x) + 1/2*cos_integral(x)
```

Sympy [B] time = 1.16375, size = 12, normalized size = 6.

$$-\log(x) + \frac{\log(x^2)}{2} + \mathrm{Ci}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x,x)
```

```
[Out] -log(x) + log(x**2)/2 + Ci(x)
```

Giac [A] time = 1.09193, size = 3, normalized size = 1.5

$$\mathrm{Ci}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/x,x, algorithm="giac")
```

```
[Out] cos_integral(x)
```

3.104 $\int \frac{\sin(x)}{x^2}\,dx$

Optimal. Leaf size=10

$$\text{CosIntegral}(x) - \frac{\sin(x)}{x}$$

```
[Out] CosIntegral[x] - Sin[x]/x
```

Rubi [A] time = 0.0254586, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3297, 3302}

$$\text{CosIntegral}(x) - \frac{\sin(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/x^2,x]
```

```
[Out] CosIntegral[x] - Sin[x]/x
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{x^2}\,dx &= -\frac{\sin(x)}{x} + \int \frac{\cos(x)}{x}\,dx\\ &= \text{Ci}(x) - \frac{\sin(x)}{x}\end{aligned}$$

Mathematica [A] time = 0.0020751, size = 10, normalized size = 1.

$$\text{CosIntegral}(x)-\frac{\sin(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/x^2,x]
```

```
[Out] CosIntegral[x] - Sin[x]/x
```

Maple [A] time = 0.004, size = 11, normalized size = 1.1

$$\text{Ci}\left(x\right)-\frac{\sin\left(x\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/x^2,x)
```

```
[Out] Ci(x)-sin(x)/x
```

Maxima [C] time = 1.06833, size = 20, normalized size = 2.

$$\frac{1}{2}\,\Gamma\left(-1,i\,x\right)+\frac{1}{2}\,\Gamma\left(-1,-i\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*gamma(-1, I*x) + 1/2*gamma(-1, -I*x)
```

Fricas [A] time = 1.81588, size = 80, normalized size = 8.

$$\frac{x\,\text{Ci}\left(-x\right)+x\,\text{Ci}\left(x\right)-2\,\sin\left(x\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x*cos_integral(-x) + x*cos_integral(x) - 2*sin(x))/x
```

Sympy [B] time = 1.60248, size = 17, normalized size = 1.7

$$-\log(x) + \frac{\log(x^2)}{2} + \mathrm{Ci}(x) - \frac{\sin(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x**2,x)
```

```
[Out] -log(x) + log(x**2)/2 + Ci(x) - sin(x)/x
```

Giac [A] time = 1.07237, size = 18, normalized size = 1.8

$$\frac{x\,\mathrm{Ci}(x) - \sin(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/x^2,x, algorithm="giac")
```

```
[Out] (x*cos_integral(x) - sin(x))/x
```

3.105 $\int \frac{\sin^2(x)}{x} \, dx$

Optimal. Leaf size=15

$$\frac{\log(x)}{2} - \frac{1}{2}\text{CosIntegral}(2x)$$

```
[Out] -CosIntegral[2*x]/2 + Log[x]/2
```

Rubi [A] time = 0.0382972, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3312, 3302}

$$\frac{\log(x)}{2} - \frac{1}{2}\text{CosIntegral}(2x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/x,x]
```

```
[Out] -CosIntegral[2*x]/2 + Log[x]/2
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rubi steps

$$\int \frac{\sin^2(x)}{x}\,dx = \int \left(\frac{1}{2x} - \frac{\cos(2x)}{2x}\right) dx$$
$$= \frac{\log(x)}{2} - \frac{1}{2}\int \frac{\cos(2x)}{x}\,dx$$
$$= -\frac{\text{Ci}(2x)}{2} + \frac{\log(x)}{2}$$

Mathematica [A] time = 0.0115446, size = 15, normalized size = 1.

$$\frac{\log(x)}{2} - \frac{1}{2}\text{CosIntegral}(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/x,x]
```

```
[Out] -CosIntegral[2*x]/2 + Log[x]/2
```

Maple [A] time = 0.007, size = 12, normalized size = 0.8

$$-\frac{\text{Ci}(2x)}{2} + \frac{\ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/x,x)
```

```
[Out] -1/2*Ci(2*x)+1/2*ln(x)
```

Maxima [C] time = 1.06165, size = 23, normalized size = 1.53

$$-\frac{1}{4}\,\text{Ei}(2ix) - \frac{1}{4}\,\text{Ei}(-2ix) + \frac{1}{2}\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/x,x, algorithm="maxima")
```

```
[Out] -1/4*Ei(2*I*x) - 1/4*Ei(-2*I*x) + 1/2*log(x)
```

Fricas [A] time = 1.86062, size = 84, normalized size = 5.6

$$-\frac{1}{4}\,\mathrm{Ci}\,(2\,x)-\frac{1}{4}\,\mathrm{Ci}\,(-2\,x)+\frac{1}{2}\,\log\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/x,x, algorithm="fricas")
```

```
[Out] -1/4*cos_integral(2*x) - 1/4*cos_integral(-2*x) + 1/2*log(x)
```

Sympy [A] time = 1.0865, size = 10, normalized size = 0.67

$$\frac{\log\,(x)}{2}-\frac{\mathrm{Ci}\,(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/x,x)
```

```
[Out] log(x)/2 - Ci(2*x)/2
```

Giac [A] time = 1.08195, size = 15, normalized size = 1.

$$-\frac{1}{2}\,\mathrm{Ci}\,(2\,x)+\frac{1}{2}\,\log\,(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/x,x, algorithm="giac")
```

```
[Out] -1/2*cos_integral(2*x) + 1/2*log(x)
```

3.106 $\int \tan^3(x)\,dx$

Optimal. Leaf size=12

$$\frac{\tan^2(x)}{2}+\log(\cos(x))$$

```
[Out] Log[Cos[x]] + Tan[x]^2/2
```

Rubi [A] time = 0.00568, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 3475}

$$\frac{\tan^2(x)}{2}+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^3,x]
```

```
[Out] Log[Cos[x]] + Tan[x]^2/2
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \tan^3(x)\,dx &= \frac{\tan^2(x)}{2}-\int \tan(x)\,dx\\ &= \log(\cos(x))+\frac{\tan^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0029812, size = 12, normalized size = 1.

$$\frac{\sec^2(x)}{2}+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^3,x]
```

```
[Out] Log[Cos[x]] + Sec[x]^2/2
```

Maple [A] time = 0.002, size = 17, normalized size = 1.4

$$\frac{(\tan(x))^2}{2}-\frac{\ln\left((\tan(x))^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^3,x)
```

```
[Out] 1/2*tan(x)^2-1/2*ln(tan(x)^2+1)
```

Maxima [A] time = 0.950209, size = 27, normalized size = 2.25

$$-\frac{1}{2\left(\sin(x)^2-1\right)}+\frac{1}{2}\log\left(\sin(x)^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/(sin(x)^2 - 1) + 1/2*log(sin(x)^2 - 1)
```

Fricas [A] time = 1.85927, size = 57, normalized size = 4.75

$$\frac{1}{2}\tan(x)^2+\frac{1}{2}\log\left(\frac{1}{\tan(x)^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3,x, algorithm="fricas")
```

```
[Out] 1/2*tan(x)^2 + 1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] time = 0.087172, size = 12, normalized size = 1.

$$\log(\cos(x)) + \frac{1}{2\cos^2(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**3,x)
```

```
[Out] log(cos(x)) + 1/(2*cos(x)**2)
```

Giac [A] time = 1.09183, size = 22, normalized size = 1.83

$$\frac{1}{2}\tan(x)^2 - \frac{1}{2}\log\left(\tan(x)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^3,x, algorithm="giac")
```

```
[Out] 1/2*tan(x)^2 - 1/2*log(tan(x)^2 + 1)
```

3.107 $\int \sin(a+bx)\,dx$

Optimal. Leaf size=11

$$-\frac{\cos(a+bx)}{b}$$

[Out] -(Cos[a + b*x]/b)

Rubi [A] time = 0.0035687, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2638}

$$-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sin[a + b*x],x]

[Out] -(Cos[a + b*x]/b)

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \sin(a+bx)\,dx = -\frac{\cos(a+bx)}{b}$$

Mathematica [A] time = 0.0092239, size = 22, normalized size = 2.

$$\frac{\sin(a)\sin(bx)}{b}-\frac{\cos(a)\cos(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x],x]
```

```
[Out] -((Cos[a]*Cos[b*x])/b) + (Sin[a]*Sin[b*x])/b
```

Maple [A] time = 0.003, size = 12, normalized size = 1.1

$$-\frac{\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a),x)
```

```
[Out] -cos(b*x+a)/b
```

Maxima [A] time = 0.940373, size = 15, normalized size = 1.36

$$-\frac{\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -cos(b*x + a)/b
```

Fricas [A] time = 1.75709, size = 23, normalized size = 2.09

$$-\frac{\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -cos(b*x + a)/b
```

Sympy [A] time = 0.125937, size = 14, normalized size = 1.27

$$\begin{cases} -\frac{\cos(a+bx)}{b} & \text{for } b \neq 0 \\ x \sin(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x)
```

```
[Out] Piecewise((-cos(a + b*x)/b, Ne(b, 0)), (x*sin(a), True))
```

Giac [A] time = 1.07024, size = 15, normalized size = 1.36

$$-\frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a),x, algorithm="giac")
```

```
[Out] -cos(b*x + a)/b
```

3.108 $\int \cos(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\sin(a+bx)}{b}$$

```
[Out] Sin[a + b*x]/b
```

Rubi [A] time = 0.0035237, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2637}

$$\frac{\sin(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x],x]
```

```
[Out] Sin[a + b*x]/b
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(a+bx)\,dx=\frac{\sin(a+bx)}{b}$$

Mathematica [B] time = 0.0088529, size = 21, normalized size = 2.1

$$\frac{\sin(a)\cos(bx)}{b}+\frac{\cos(a)\sin(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x],x]
```

```
[Out] (Cos[b*x]*Sin[a])/b + (Cos[a]*Sin[b*x])/b
```

Maple [A] time = 0.005, size = 11, normalized size = 1.1

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a),x)
```

```
[Out] sin(b*x+a)/b
```

Maxima [A] time = 0.956322, size = 14, normalized size = 1.4

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x, algorithm="maxima")
```

```
[Out] sin(b*x + a)/b
```

Fricas [A] time = 1.7505, size = 22, normalized size = 2.2

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x, algorithm="fricas")
```

```
[Out] sin(b*x + a)/b
```

Sympy [A] time = 0.129336, size = 12, normalized size = 1.2

$$\begin{cases} \frac{\sin(a+bx)}{b} & \text{for } b \neq 0 \\ x\cos(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x)
```

```
[Out] Piecewise((sin(a + b*x)/b, Ne(b, 0)), (x*cos(a), True))
```

Giac [A] time = 1.08862, size = 14, normalized size = 1.4

$$\frac{\sin(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a),x, algorithm="giac")
```

```
[Out] sin(b*x + a)/b
```

3.109 $\int \tan(a+bx)\,dx$

Optimal. Leaf size=12

$$-\frac{\log(\cos(a+bx))}{b}$$

```
[Out] -(Log[Cos[a + b*x]]/b)
```

Rubi [A] time = 0.0040621, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3475}

$$-\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[a + b*x],x]
```

```
[Out] -(Log[Cos[a + b*x]]/b)
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tan(a+bx)\,dx = -\frac{\log(\cos(a+bx))}{b}$$

Mathematica [A] time = 0.0088248, size = 12, normalized size = 1.

$$-\frac{\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[a + b*x],x]
```

```
[Out] -(Log[Cos[a + b*x]]/b)
```

Maple [A] time = 0.002, size = 17, normalized size = 1.4

$$\frac{\ln\left(1+(\tan(bx+a))^2\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a),x)
```

```
[Out] 1/2/b*ln(1+tan(b*x+a)^2)
```

Maxima [A] time = 0.936232, size = 15, normalized size = 1.25

$$\frac{\log(\sec(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a),x, algorithm="maxima")
```

```
[Out] log(sec(b*x + a))/b
```

Fricas [A] time = 1.80857, size = 49, normalized size = 4.08

$$-\frac{\log\left(\frac{1}{\tan(bx+a)^2+1}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*log(1/(tan(b*x + a)^2 + 1))/b
```

Sympy [A] time = 0.123411, size = 19, normalized size = 1.58

$$\begin{cases} \frac{\log\left(\tan^{2}(a+bx)+1\right)}{2b} & \text{for } b \neq 0 \\ x\tan(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a),x)
```

```
[Out] Piecewise((log(tan(a + b*x)**2 + 1)/(2*b), Ne(b, 0)), (x*tan(a), True))
```

Giac [A] time = 1.07097, size = 18, normalized size = 1.5

$$-\frac{\log(|\cos(bx+a)|)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a),x, algorithm="giac")
```

```
[Out] -log(abs(cos(b*x + a)))/b
```

3.110 $\int \cot(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\log(\sin(a+bx))}{b}$$

```
[Out] Log[Sin[a + b*x]]/b
```

Rubi [A] time = 0.0038209, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3475}

$$\frac{\log(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[a + b*x],x]
```

```
[Out] Log[Sin[a + b*x]]/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(a+bx)\,dx = \frac{\log(\sin(a+bx))}{b}$$

Mathematica [A] time = 0.0097907, size = 19, normalized size = 1.73

$$\frac{\log(\tan(a+bx))+\log(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[a + b*x],x]
```

```
[Out] (Log[Cos[a + b*x]] + Log[Tan[a + b*x]])/b
```

Maple [B] time = 0.004, size = 29, normalized size = 2.6

$$-\frac{\ln\left(1+(\tan(bx+a))^2\right)}{2b}+\frac{\ln(\tan(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(b*x+a),x)
```

```
[Out] -1/2/b*ln(1+tan(b*x+a)^2)+1/b*ln(tan(b*x+a))
```

Maxima [A] time = 0.963702, size = 15, normalized size = 1.36

$$\frac{\log(\sin(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(b*x+a),x, algorithm="maxima")
```

```
[Out] log(sin(b*x + a))/b
```

Fricas [B] time = 1.76488, size = 65, normalized size = 5.91

$$\frac{\log\left(\frac{\tan(bx+a)^2}{\tan(bx+a)^2+1}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*log(tan(b*x + a)^2/(tan(b*x + a)^2 + 1))/b
```

Sympy [A] time = 0.33813, size = 29, normalized size = 2.64

$$\begin{cases} -\frac{\log\left(\tan^{2}(a+bx)+1\right)}{2b} + \frac{\log(\tan(a+bx))}{b} & \text{for } b \neq 0 \\ \frac{x}{\tan(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(b*x+a),x)
```

```
[Out] Piecewise((-log(tan(a + b*x)**2 + 1)/(2*b) + log(tan(a + b*x))/b, Ne(b, 0))
, (x/tan(a), True))
```

Giac [B] time = 1.13425, size = 76, normalized size = 6.91

$$\frac{\log\left(\frac{|-\cos(bx+a)+1|}{|\cos(bx+a)+1|}\right) - 2\log\left(\left|-\frac{\cos(bx+a)-1}{\cos(bx+a)+1}+1\right|\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(-cos(b*x + a) + 1)/abs(cos(b*x + a) + 1)) - 2*log(abs(-(cos(b*
x + a) - 1)/(cos(b*x + a) + 1) + 1)))/b
```

3.111 $\int \csc(a+bx)\,dx$

Optimal. Leaf size=12

$$-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

```
[Out] -(ArcTanh[Cos[a + b*x]]/b)
```

Rubi [A] time = 0.0035878, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3770}

$$-\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[a + b*x],x]
```

```
[Out] -(ArcTanh[Cos[a + b*x]]/b)
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \csc(a+bx)\,dx = -\frac{\tanh^{-1}(\cos(a+bx))}{b}$$

Mathematica [B] time = 0.0168323, size = 38, normalized size = 3.17

$$\frac{\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}-\frac{\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[a + b*x],x]
```

```
[Out] -(Log[Cos[a/2 + (b*x)/2]]/b) + Log[Sin[a/2 + (b*x)/2]]/b
```

Maple [A] time = 0.006, size = 21, normalized size = 1.8

$$\frac{\ln(\csc(bx+a)-\cot(bx+a))}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sin(b*x+a),x)
```

```
[Out] 1/b*ln(csc(b*x+a)-cot(b*x+a))
```

Maxima [B] time = 0.964039, size = 35, normalized size = 2.92

$$-\frac{\log(\cos(bx+a)+1)-\log(\cos(bx+a)-1)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b
```

Fricas [B] time = 1.89001, size = 93, normalized size = 7.75

$$-\frac{\log\left(\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)-\log\left(-\frac{1}{2}\cos(bx+a)+\frac{1}{2}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(log(1/2*cos(b*x + a) + 1/2) - log(-1/2*cos(b*x + a) + 1/2))/b
```

Sympy [A] time = 0.538475, size = 17, normalized size = 1.42

$$\begin{cases} \frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b} & \text{for } b \neq 0 \\ \frac{x}{\sin(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a),x)
```

```
[Out] Piecewise((log(tan(a/2 + b*x/2))/b, Ne(b, 0)), (x/sin(a), True))
```

Giac [B] time = 1.08453, size = 69, normalized size = 5.75

$$\frac{\log\left(\left|-\frac{\cos(bx+a)}{b}+\frac{1}{|b|}\right|\right)}{2\,|b|}-\frac{\log\left(\left|-\frac{\cos(bx+a)}{b}-\frac{1}{|b|}\right|\right)}{2\,|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sin(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*log(abs(-cos(b*x + a)/b + 1/abs(b)))/abs(b) - 1/2*log(abs(-cos(b*x + a)
/b - 1/abs(b)))/abs(b)
```

3.112 $\int \sec(a+bx)\,dx$

Optimal. Leaf size=11

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

```
[Out] ArcTanh[Sin[a + b*x]]/b
```

Rubi [A] time = 0.0039425, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3770}

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[a + b*x],x]

[Out] ArcTanh[Sin[a + b*x]]/b
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(a+bx)\,dx=\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Mathematica [A] time = 0.0015468, size = 11, normalized size = 1.

$$\frac{\tanh^{-1}(\sin(a+bx))}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x],x]
```

```
[Out] ArcTanh[Sin[a + b*x]]/b
```

Maple [A] time = 0.006, size = 19, normalized size = 1.7

$$\frac{\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a),x)
```

```
[Out] 1/b*ln(sec(b*x+a)+tan(b*x+a))
```

Maxima [B] time = 0.939586, size = 35, normalized size = 3.18

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(\sin\left(bx+a\right)-1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b
```

Fricas [B] time = 1.80147, size = 76, normalized size = 6.91

$$\frac{\log\left(\sin\left(bx+a\right)+1\right)-\log\left(-\sin\left(bx+a\right)+1\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(log(sin(b*x + a) + 1) - log(-sin(b*x + a) + 1))/b
```

Sympy [A] time = 0.613017, size = 34, normalized size = 3.09

$$\begin{cases} -\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)-1\right)}{b}+\frac{\log\left(\tan\left(\frac{a}{2}+\frac{bx}{2}\right)+1\right)}{b} & \text{for } b\neq 0 \\ \frac{x}{\cos(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a),x)
```

```
[Out] Piecewise((-log(tan(a/2 + b*x/2) - 1)/b + log(tan(a/2 + b*x/2) + 1)/b, Ne(b
, 0)), (x/cos(a), True))
```

Giac [B] time = 1.11236, size = 38, normalized size = 3.45

$$\frac{\log\left(\left|\sin\left(bx+a\right)+1\right|\right)-\log\left(\left|\sin\left(bx+a\right)-1\right|\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(sin(b*x + a) + 1)) - log(abs(sin(b*x + a) - 1)))/b
```

3.113 $\int \sin^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

```
[Out] x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rubi [A] time = 0.0086649, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$\frac{x}{2}-\frac{\sin(a+bx)\cos(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^2,x]
```

```
[Out] x/2 - (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^2(a+bx)\,dx &= -\frac{\cos(a+bx)\sin(a+bx)}{2b}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0256389, size = 23, normalized size = 0.92

$$-\frac{\sin(2(a+bx))-2(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^2,x]
```

```
[Out] -(-2*(a + b*x) + Sin[2*(a + b*x)])/(4*b)
```

Maple [A] time = 0.005, size = 27, normalized size = 1.1

$$\frac{1}{b}\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^2,x)
```

```
[Out] 1/b*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
```

Maxima [A] time = 0.94602, size = 32, normalized size = 1.28

$$\frac{2\,bx+2\,a-\sin(2\,bx+2\,a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a - sin(2*b*x + 2*a))/b
```

Fricas [A] time = 1.69346, size = 55, normalized size = 2.2

$$\frac{bx-\cos(bx+a)\sin(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x - cos(b*x + a)*sin(b*x + a))/b
```

Sympy [A] time = 0.227452, size = 46, normalized size = 1.84

$$\begin{cases} \frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} - \frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\sin^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 - sin(a + b*x)*cos(a +
 b*x)/(2*b), Ne(b, 0)), (x*sin(a)**2, True))
```

Giac [A] time = 1.06503, size = 24, normalized size = 0.96

$$\frac{1}{2}x - \frac{\sin(2bx + 2a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x - 1/4*sin(2*b*x + 2*a)/b
```

3.114 $\int \sin^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\frac{\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

```
[Out] -(Cos[a + b*x]/b) + Cos[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0100385, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\cos^3(a+bx)}{3b}-\frac{\cos(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a + b*x]^3,x]
```

```
[Out] -(Cos[a + b*x]/b) + Cos[a + b*x]^3/(3*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(a+bx)\right)}{b}\\ &= -\frac{\cos(a+bx)}{b}+\frac{\cos^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0096789, size = 29, normalized size = 1.07

$$\frac{\cos(3(a+bx))}{12b}-\frac{3\cos(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a + b*x]^3,x]
```

```
[Out] (-3*Cos[a + b*x])/(4*b) + Cos[3*(a + b*x)]/(12*b)
```

Maple [A] time = 0.005, size = 22, normalized size = 0.8

$$-\frac{\left(2+\left(\sin\left(bx+a\right)\right)^{2}\right)\cos\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)^3,x)
```

```
[Out] -1/3/b*(2+sin(b*x+a)^2)*cos(b*x+a)
```

Maxima [A] time = 0.952396, size = 30, normalized size = 1.11

$$\frac{\cos\left(bx+a\right)^{3}-3\,\cos\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

Fricas [A] time = 1.52438, size = 55, normalized size = 2.04

$$\frac{\cos\left(bx+a\right)^{3}-3\,\cos\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(b*x + a)^3 - 3*cos(b*x + a))/b
```

Sympy [A] time = 0.495649, size = 37, normalized size = 1.37

$$\begin{cases} -\frac{\sin^2(a+bx)\cos(a+bx)}{b} - \frac{2\cos^3(a+bx)}{3b} & \text{for } b \neq 0 \\ x\sin^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-sin(a + b*x)**2*cos(a + b*x)/b - 2*cos(a + b*x)**3/(3*b), Ne(b,
 0)), (x*sin(a)**3, True))
```

Giac [A] time = 1.10015, size = 34, normalized size = 1.26

$$\frac{\cos(bx+a)^3}{3\,b} - \frac{\cos(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/3*cos(b*x + a)^3/b - cos(b*x + a)/b
```

3.115 $\int \cos^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}$$

```
[Out] x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rubi [A] time = 0.0085367, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2635, 8}

$$\frac{\sin(a+bx)\cos(a+bx)}{2b}+\frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^2,x]
```

```
[Out] x/2 + (Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(a+bx)\,dx &= \frac{\cos(a+bx)\sin(a+bx)}{2b}+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{\cos(a+bx)\sin(a+bx)}{2b}\end{aligned}$$

Mathematica [A] time = 0.0167231, size = 23, normalized size = 0.92

$$\frac{2(a+bx)+\sin(2(a+bx))}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^2,x]
```

```
[Out] (2*(a + b*x) + Sin[2*(a + b*x)])/(4*b)
```

Maple [A] time = 0.008, size = 27, normalized size = 1.1

$$\frac{1}{b}\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2,x)
```

```
[Out] 1/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
```

Maxima [A] time = 0.95676, size = 30, normalized size = 1.2

$$\frac{2\,bx+2\,a+\sin(2\,bx+2\,a)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*x + 2*a + sin(2*b*x + 2*a))/b
```

Fricas [A] time = 1.80681, size = 55, normalized size = 2.2

$$\frac{bx+\cos(bx+a)\sin(bx+a)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b*x + cos(b*x + a)*sin(b*x + a))/b
```

Sympy [A] time = 0.228971, size = 46, normalized size = 1.84

$$\begin{cases} \frac{x\sin^2(a+bx)}{2} + \frac{x\cos^2(a+bx)}{2} + \frac{\sin(a+bx)\cos(a+bx)}{2b} & \text{for } b \neq 0 \\ x\cos^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2,x)
```

```
[Out] Piecewise((x*sin(a + b*x)**2/2 + x*cos(a + b*x)**2/2 + sin(a + b*x)*cos(a +
 b*x)/(2*b), Ne(b, 0)), (x*cos(a)**2, True))
```

Giac [A] time = 1.09624, size = 24, normalized size = 0.96

$$\frac{1}{2}x + \frac{\sin(2bx + 2a)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*b*x + 2*a)/b
```

3.116 $\int \cos^3(a+bx)\,dx$

Optimal. Leaf size=26

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

```
[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Rubi [A] time = 0.0106401, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2633}

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + b*x]^3,x]

[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(a+bx)\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(a+bx)\right)}{b}\\ &= \frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.0074745, size = 26, normalized size = 1.

$$\frac{\sin(a+bx)}{b}-\frac{\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + b*x]^3,x]
```

```
[Out] Sin[a + b*x]/b - Sin[a + b*x]^3/(3*b)
```

Maple [A] time = 0.009, size = 22, normalized size = 0.9

$$\frac{\left(2+\left(\cos\left(bx+a\right)\right)^{2}\right)\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3,x)
```

```
[Out] 1/3/b*(2+cos(b*x+a)^2)*sin(b*x+a)
```

Maxima [A] time = 0.943241, size = 30, normalized size = 1.15

$$-\frac{\sin\left(bx+a\right)^{3}-3\,\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

Fricas [A] time = 1.61269, size = 55, normalized size = 2.12

$$\frac{\left(\cos\left(bx+a\right)^{2}+2\right)\sin\left(bx+a\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(b*x + a)^2 + 2)*sin(b*x + a)/b
```

Sympy [A] time = 0.466847, size = 36, normalized size = 1.38

$$\begin{cases} \frac{2\sin^3(a+bx)}{3b} + \frac{\sin(a+bx)\cos^2(a+bx)}{b} & \text{for } b \neq 0 \\ x\cos^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3,x)
```

```
[Out] Piecewise((2*sin(a + b*x)**3/(3*b) + sin(a + b*x)*cos(a + b*x)**2/b, Ne(b,
0)), (x*cos(a)**3, True))
```

Giac [A] time = 1.0726, size = 30, normalized size = 1.15

$$-\frac{\sin(bx+a)^3 - 3\sin(bx+a)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/3*(sin(b*x + a)^3 - 3*sin(b*x + a))/b
```

3.117 $\int \sec^2(a+bx)\,dx$

Optimal. Leaf size=10

$$\frac{\tan(a+bx)}{b}$$

[Out] Tan[a + b*x]/b

Rubi [A] time = 0.0084205, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3767, 8}

$$\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sec[a + b*x]^2,x]

[Out] Tan[a + b*x]/b

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(a+bx)\,dx &= -\frac{\text{Subst}\left(\int 1\,dx, x, -\tan(a+bx)\right)}{b}\\ &= \frac{\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.0029589, size = 10, normalized size = 1.

$$\frac{\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[a + b*x]^2,x]
```

```
[Out] Tan[a + b*x]/b
```

Maple [A] time = 0.006, size = 11, normalized size = 1.1

$$\frac{\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(b*x+a)^2,x)
```

```
[Out] tan(b*x+a)/b
```

Maxima [A] time = 0.934283, size = 14, normalized size = 1.4

$$\frac{\tan(bx+a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] tan(b*x + a)/b
```

Fricas [A] time = 1.7253, size = 42, normalized size = 4.2

$$\frac{\sin(bx+a)}{b\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] sin(b*x + a)/(b*cos(b*x + a))
```

Sympy [A] time = 1.27562, size = 58, normalized size = 5.8

$$\begin{cases} \tilde{\infty} x & \text{for}\: \left(a = -\frac{\pi}{2} \vee a = - b x - \frac{\pi}{2}\right) \wedge \left(a = - b x - \frac{\pi}{2} \vee b = 0\right) \\ \frac{x}{\cos^{2}{\left(a \right)}} & \text{for}\: b = 0 \\ - \frac{2 \tan{\left(\frac{a}{2} + \frac{b x}{2} \right)}}{b \tan^{2}{\left(\frac{a}{2} + \frac{b x}{2} \right)} - b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*x, (Eq(b, 0) | Eq(a, -b*x - pi/2)) & (Eq(a, -pi/2) | Eq(a, -
b*x - pi/2))), (x/cos(a)**2, Eq(b, 0)), (-2*tan(a/2 + b*x/2)/(b*tan(a/2 + b
*x/2)**2 - b), True))
```

Giac [A] time = 1.08762, size = 14, normalized size = 1.4

$$\frac{\tan\left(bx + a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] tan(b*x + a)/b
```

3.118 $\int \frac{1}{1+\cos(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\sin(x)}{\cos(x)+1}$$

```
[Out] Sin[x]/(1 + Cos[x])
```

Rubi [A] time = 0.0065632, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2648}

$$\frac{\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x])^(-1),x]
```

```
[Out] Sin[x]/(1 + Cos[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1+\cos(x)}\,dx = \frac{\sin(x)}{1+\cos(x)}$$

Mathematica [A] time = 0.0037188, size = 6, normalized size = 0.67

$$\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])^(-1),x]
```

```
[Out] Tan[x/2]
```

Maple [A] time = 0.003, size = 5, normalized size = 0.6

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)+1),x)
```

```
[Out] tan(1/2*x)
```

Maxima [A] time = 0.93782, size = 12, normalized size = 1.33

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="maxima")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Fricas [A] time = 1.67681, size = 28, normalized size = 3.11

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="fricas")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Sympy [A] time = 0.177133, size = 3, normalized size = 0.33

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x)
```

```
[Out] tan(x/2)
```

Giac [B] time = 1.07276, size = 41, normalized size = 4.56

$$-\frac{2\tan\left(\frac{1}{2}x\right)}{\left(x^2+1\right)\left(\frac{x^2-1}{x^2+1}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)),x, algorithm="giac")
```

```
[Out] -2*tan(1/2*x)/((x^2 + 1)*((x^2 - 1)/(x^2 + 1) - 1))
```

3.119 $\int \frac{1}{1-\cos(x)} dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rubi [A] time = 0.0079359, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2648}

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Cos[x])^(-1),x]

[Out] -(Sin[x]/(1 - Cos[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\cos(x)} dx = -\frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A] time = 0.0087344, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Cos[x])^(-1),x]
```

```
[Out] -Cot[x/2]
```

Maple [A] time = 0.007, size = 9, normalized size = 0.8

$$-\left(\tan\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-cos(x)),x)
```

```
[Out] -1/tan(1/2*x)
```

Maxima [A] time = 0.949286, size = 14, normalized size = 1.17

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="maxima")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Fricas [A] time = 1.75446, size = 30, normalized size = 2.5

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Sympy [A] time = 0.353359, size = 7, normalized size = 0.58

$$-\frac{1}{\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x)
```

```
[Out] -1/tan(x/2)
```

Giac [A] time = 1.10178, size = 11, normalized size = 0.92

$$-\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-cos(x)),x, algorithm="giac")
```

```
[Out] -1/tan(1/2*x)
```

3.120 $\int \frac{1}{1+\sin(x)} \, dx$

Optimal. Leaf size=10

$$-\frac{\cos(x)}{\sin(x)+1}$$

```
[Out] -(Cos[x]/(1 + Sin[x]))
```

Rubi [A] time = 0.005755, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2648}

$$-\frac{\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x])^(-1),x]
```

```
[Out] -(Cos[x]/(1 + Sin[x]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1+\sin(x)} \, dx = -\frac{\cos(x)}{1+\sin(x)}$$

Mathematica [B] time = 0.009546, size = 23, normalized size = 2.3

$$\frac{2\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x])^(-1),x]
```

```
[Out] (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Maple [A] time = 0.006, size = 11, normalized size = 1.1

$$-2\left(1+\tan\left(x/2\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(x)),x)
```

```
[Out] -2/(1+tan(1/2*x))
```

Maxima [A] time = 0.938928, size = 20, normalized size = 2.

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) + 1)
```

Fricas [A] time = 1.66611, size = 62, normalized size = 6.2

$$-\frac{\cos\left(x\right)-\sin\left(x\right)+1}{\cos\left(x\right)+\sin\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) - sin(x) + 1)/(cos(x) + sin(x) + 1)
```

Sympy [A] time = 0.376199, size = 8, normalized size = 0.8

$$-\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x)
```

```
[Out] -2/(tan(x/2) + 1)
```

Giac [A] time = 1.07258, size = 14, normalized size = 1.4

$$-\frac{2}{\tan\left(\frac{1}{2}x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*x) + 1)
```

3.121 $\int \frac{1}{1-\sin(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\cos(x)}{1-\sin(x)}$$

```
[Out] Cos[x]/(1 - Sin[x])
```

Rubi [A] time = 0.0075248, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2648}

$$\frac{\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x])^(-1),x]
```

```
[Out] Cos[x]/(1 - Sin[x])
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{1}{1-\sin(x)}\,dx=\frac{\cos(x)}{1-\sin(x)}$$

Mathematica [B] time = 0.0105548, size = 25, normalized size = 2.27

$$\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x])^(-1),x]
```

```
[Out] (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

Maple [A] time = 0.016, size = 11, normalized size = 1.

$$-2\left(-1+\tan\left(x/2\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x)),x)
```

```
[Out] -2/(-1+tan(1/2*x))
```

Maxima [A] time = 0.956683, size = 20, normalized size = 1.82

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) - 1)
```

Fricas [A] time = 1.69065, size = 61, normalized size = 5.55

$$\frac{\cos\left(x\right)+\sin\left(x\right)+1}{\cos\left(x\right)-\sin\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="fricas")
```

```
[Out] (cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

Sympy [A] time = 0.379989, size = 8, normalized size = 0.73

$$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x)
```

```
[Out] -2/(tan(x/2) - 1)
```

Giac [A] time = 1.08851, size = 14, normalized size = 1.27

$$-\frac{2}{\tan\left(\frac{1}{2}\,x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*x) - 1)
```

3.122 $\int \frac{1}{a+b\sin(x)}\,dx$

Optimal. Leaf size=40

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Rubi [A] time = 0.0399785, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\sin(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{a+2bx+ax^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\left(4\,\text{Subst}\left(\int \frac{1}{-4\left(a^2-b^2\right)-x^2}\,dx, x, 2b+2a\tan\left(\frac{x}{2}\right)\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\end{aligned}$$

Mathematica [A] time = 0.0374633, size = 40, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + a*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]
```

Maple [A] time = 0.014, size = 39, normalized size = 1.

$$2\frac{1}{\sqrt{a^2-b^2}}\arctan\left(1/2\,\frac{2\,a\tan\left(x/2\right)+2\,b}{\sqrt{a^2-b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)),x)
```

```
[Out] 2/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*x)+2*b)/(a^2-b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.8461, size = 344, normalized size = 8.6

$$\left[-\frac{\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-b^2)\cos(x)^2-2ab\sin(x)-a^2-b^2+2(a\cos(x)\sin(x)+b\cos(x))\sqrt{-a^2+b^2}}{b^2\cos(x)^2-2ab\sin(x)-a^2-b^2}\right)}{2(a^2-b^2)}, -\frac{\arctan\left(-\frac{a\sin(x)+b}{\sqrt{a^2-b^2}\cos(x)}\right)}{\sqrt{a^2-b^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(x)^2 - 2*a*b*sin(x) - a^2 - b
^2 + 2*(a*cos(x)*sin(x) + b*cos(x))*sqrt(-a^2 + b^2))/(b^2*cos(x)^2 - 2*a*b
*sin(x) - a^2 - b^2))/(a^2 - b^2), -arctan(-(a*sin(x) + b)/(sqrt(a^2 - b^2)
*cos(x)))/sqrt(a^2 - b^2)]
```

Sympy [A] time = 8.44262, size = 114, normalized size = 2.85

$$\begin{cases} -\frac{2}{b-\sqrt{b^2}\tan\left(\frac{x}{2}\right)} & \text{for } a=-\sqrt{b^2} \\ -\frac{2}{b+\sqrt{b^2}\tan\left(\frac{x}{2}\right)} & \text{for } a=\sqrt{b^2} \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)\right)}{b} & \text{for } a=0 \\ \frac{\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}-\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}} - \frac{\log\left(\tan\left(\frac{x}{2}\right)+\frac{b}{a}+\frac{\sqrt{-a^2+b^2}}{a}\right)}{\sqrt{-a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x)
```

```
[Out] Piecewise((-2/(b - sqrt(b**2)*tan(x/2)), Eq(a, -sqrt(b**2))), (-2/(b + sqrt
(b**2)*tan(x/2)), Eq(a, sqrt(b**2))), (log(tan(x/2))/b, Eq(a, 0)), (log(tan
(x/2) + b/a - sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2) - log(tan(x/2) + b/a
 + sqrt(-a**2 + b**2)/a)/sqrt(-a**2 + b**2), True))
```

Giac [A] time = 1.09591, size = 65, normalized size = 1.62

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*x) + b)/sqrt(a^2 - b
^2)))/sqrt(a^2 - b^2)
```

3.123 $\int \frac{1}{a+\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=47

$$-\frac{2\tanh^{-1}\left(\frac{b-(1-a)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+1}}\right)}{\sqrt{-a^2+b^2+1}}$$

```
[Out] (-2*ArcTanh[(b - (1 - a)*Tan[x/2])/Sqrt[1 - a^2 + b^2]])/Sqrt[1 - a^2 + b^2
]
```

Rubi [A] time = 0.0555467, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3124, 618, 206}

$$-\frac{2\tanh^{-1}\left(\frac{b-(1-a)\tan\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2+1}}\right)}{\sqrt{-a^2+b^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + Cos[x] + b*Sin[x])^(-1),x]
```

```
[Out] (-2*ArcTanh[(b - (1 - a)*Tan[x/2])/Sqrt[1 - a^2 + b^2]])/Sqrt[1 - a^2 + b^2
]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+\cos(x)+b\sin(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1+a+2bx+(-1+a)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\left(4\,\text{Subst}\left(\int \frac{1}{4\left(1-a^2+b^2\right)-x^2}\,dx, x, 2b+2(-1+a)\tan\left(\frac{x}{2}\right)\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{b-(1-a)\tan\left(\frac{x}{2}\right)}{\sqrt{1-a^2+b^2}}\right)}{\sqrt{1-a^2+b^2}}\end{aligned}$$

Mathematica [A] time = 0.0603158, size = 44, normalized size = 0.94

$$\frac{2\tan^{-1}\left(\frac{(a-1)\tan\left(\frac{x}{2}\right)+b}{\sqrt{a^2-b^2-1}}\right)}{\sqrt{a^2-b^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + Cos[x] + b*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTan[(b + (-1 + a)*Tan[x/2])/Sqrt[-1 + a^2 - b^2]])/Sqrt[-1 + a^2 - b^
2]
```

Maple [A] time = 0.039, size = 43, normalized size = 0.9

$$2\,\frac{1}{\sqrt{a^2-b^2-1}}\arctan\left(1/2\,\frac{2\,(a-1)\tan(x/2)+2\,b}{\sqrt{a^2-b^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+cos(x)+b*sin(x)),x)
```

```
[Out] 2/(a^2-b^2-1)^(1/2)*arctan(1/2*(2*(a-1)*tan(1/2*x)+2*b)/(a^2-b^2-1)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.97115, size = 707, normalized size = 15.04

$$\left[-\frac{\sqrt{-a^2+b^2+1}\log\left(-\frac{b^4+(a^2+3)b^2-(2a^2b^2-b^4-2a^2+1)\cos(x)^2-a^2+2(ab^2+a)\cos(x)+2(ab^3+ab-(b^3-(2a^2-1)b)\cos(x))\sin(x)-2(2ab\cos(x)^2-ab}{(b^2-1)\cos(x)^2-a^2-b^2-2a\cos(x)-2(ab+b\cos(x))\sin(x)}\right.}{2(a^2-b^2-1)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a^2 + b^2 + 1)*log(-(b^4 + (a^2 + 3)*b^2 - (2*a^2*b^2 - b^4 - 2
*a^2 + 1)*cos(x)^2 - a^2 + 2*(a*b^2 + a)*cos(x) + 2*(a*b^3 + a*b - (b^3 - (
2*a^2 - 1)*b)*cos(x))*sin(x) - 2*(2*a*b*cos(x)^2 - a*b + (b^3 + b)*cos(x) -
 (b^2 - (a*b^2 - a)*cos(x) + 1)*sin(x))*sqrt(-a^2 + b^2 + 1) + 2)/((b^2 - 1
)*cos(x)^2 - a^2 - b^2 - 2*a*cos(x) - 2*(a*b + b*cos(x))*sin(x)))/(a^2 - b^
2 - 1), arctan(-(a*b*sin(x) + b^2 + a*cos(x) + 1)*sqrt(a^2 - b^2 - 1)/((b^3
 - (a^2 - 1)*b)*cos(x) + (a^2 - b^2 - 1)*sin(x)))/sqrt(a^2 - b^2 - 1)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09334, size = 81, normalized size = 1.72

$$\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)+b-\tan\left(\frac{1}{2}x\right)}{\sqrt{a^2-b^2-1}}\right)\right)}{\sqrt{a^2-b^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*a - 2) + arctan((a*tan(1/2*x) + b - tan(1
/2*x))/sqrt(a^2 - b^2 - 1)))/sqrt(a^2 - b^2 - 1)
```

3.124 $\int x^2 \sin^2(a+bx)\,dx$

Optimal. Leaf size=73

$$\frac{x\sin^2(a+bx)}{2b^2}+\frac{\sin(a+bx)\cos(a+bx)}{4b^3}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{4b^2}+\frac{x^3}{6}$$

```
[Out] -x/(4*b^2) + x^3/6 + (Cos[a + b*x]*Sin[a + b*x])/(4*b^3) - (x^2*Cos[a + b*x
]*Sin[a + b*x])/(2*b) + (x*Sin[a + b*x]^2)/(2*b^2)
```

Rubi [A] time = 0.0432089, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3311, 30, 2635, 8}

$$\frac{x\sin^2(a+bx)}{2b^2}+\frac{\sin(a+bx)\cos(a+bx)}{4b^3}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{x}{4b^2}+\frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[a + b*x]^2,x]
```

```
[Out] -x/(4*b^2) + x^3/6 + (Cos[a + b*x]*Sin[a + b*x])/(4*b^3) - (x^2*Cos[a + b*x
]*Sin[a + b*x])/(2*b) + (x*Sin[a + b*x]^2)/(2*b^2)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int x^2\sin^2(a+bx)\,dx &= -\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}+\frac{x\sin^2(a+bx)}{2b^2}+\frac{\int x^2\,dx}{2}-\frac{\int \sin^2(a+bx)\,dx}{2b^2}\\ &= \frac{x^3}{6}+\frac{\cos(a+bx)\sin(a+bx)}{4b^3}-\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}+\frac{x\sin^2(a+bx)}{2b^2}-\frac{\int 1\,dx}{4b^2}\\ &= -\frac{x}{4b^2}+\frac{x^3}{6}+\frac{\cos(a+bx)\sin(a+bx)}{4b^3}-\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}+\frac{x\sin^2(a+bx)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.128293, size = 47, normalized size = 0.64

$$\frac{\left(3-6b^2x^2\right)\sin(2(a+bx))-6bx\cos(2(a+bx))+4b^3x^3}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[a + b*x]^2,x]
```

```
[Out] (4*b^3*x^3 - 6*b*x*Cos[2*(a + b*x)] + (3 - 6*b^2*x^2)*Sin[2*(a + b*x)])/(24
*b^3)
```

Maple [B] time = 0.005, size = 158, normalized size = 2.2

$$\frac{1}{b^3}\left((bx+a)^2\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)(\cos(bx+a))^2}{2}+\frac{\cos(bx+a)\sin(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}-\frac{(b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b^3*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/2*(b*x+a)*cos
(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a-1/3*(b*x+a)^3-2*a*((b*x+a
)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2+1/4*sin(b*x+a)^2
)+a^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [A] time = 0.965575, size = 158, normalized size = 2.16

$$\frac{4\,(bx+a)^3+6\,(2\,bx+2\,a-\sin(2\,bx+2\,a))a^2-6\left(2\,(bx+a)^2-2\,(bx+a)\sin(2\,bx+2\,a)-\cos(2\,bx+2\,a)\right)a-6\,(bx+}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(4*(b*x + a)^3 + 6*(2*b*x + 2*a - sin(2*b*x + 2*a))*a^2 - 6*(2*(b*x +
a)^2 - 2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a))*a - 6*(b*x + a)*cos
(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))/b^3
```

Fricas [A] time = 1.77741, size = 134, normalized size = 1.84

$$\frac{2\,b^3x^3-6\,bx\cos(bx+a)^2-3\left(2\,b^2x^2-1\right)\cos(bx+a)\sin(bx+a)+3\,bx}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 - 6*b*x*cos(b*x + a)^2 - 3*(2*b^2*x^2 - 1)*cos(b*x + a)*sin
(b*x + a) + 3*b*x)/b^3
```

Sympy [A] time = 1.10156, size = 105, normalized size = 1.44

$$\begin{cases} \frac{x^3\sin^2(a+bx)}{6}+\frac{x^3\cos^2(a+bx)}{6}-\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}+\frac{x\sin^2(a+bx)}{4b^2}-\frac{x\cos^2(a+bx)}{4b^2}+\frac{\sin(a+bx)\cos(a+bx)}{4b^3} & \text{for } b\neq 0 \\ \frac{x^3\sin^2(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((x**3*sin(a + b*x)**2/6 + x**3*cos(a + b*x)**2/6 - x**2*sin(a + b
*x)*cos(a + b*x)/(2*b) + x*sin(a + b*x)**2/(4*b**2) - x*cos(a + b*x)**2/(4*
b**2) + sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne(b, 0)), (x**3*sin(a)**2/3, T
rue))
```

Giac [A] time = 1.10103, size = 61, normalized size = 0.84

$$\frac{1}{6}x^3 - \frac{x\cos(2bx + 2a)}{4b^2} - \frac{(2b^2x^2 - 1)\sin(2bx + 2a)}{8b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*b*x + 2*a)/b^2 - 1/8*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a)
/b^3
```

3.125 $\int \cos(x)\cos(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

```
[Out] Sin[x]/2 + Sin[3*x]/6
```

Rubi [A] time = 0.0074209, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4283}

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[2*x],x]
```

```
[Out] Sin[x]/2 + Sin[3*x]/6
```

Rule 4283

```
Int[cos[(a_.) + (b_.)*(x_)]*cos[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[
a - c + (b - d)*x]/(2*(b - d)), x] + Simp[Sin[a + c + (b + d)*x]/(2*(b + d)
), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(x)\cos(2x)\,dx=\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Mathematica [A] time = 0.0045822, size = 15, normalized size = 1.

$$\frac{\sin(x)}{2}+\frac{1}{6}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[2*x],x]
```

```
[Out] Sin[x]/2 + Sin[3*x]/6
```

Maple [A] time = 0.019, size = 12, normalized size = 0.8

$$\frac{\sin(x)}{2} + \frac{\sin(3x)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x),x)
```

```
[Out] 1/2*sin(x)+1/6*sin(3*x)
```

Maxima [A] time = 0.947356, size = 15, normalized size = 1.

$$\frac{1}{6}\sin(3x) + \frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x, algorithm="maxima")
```

```
[Out] 1/6*sin(3*x) + 1/2*sin(x)
```

Fricas [A] time = 1.76068, size = 39, normalized size = 2.6

$$\frac{1}{3}\left(2\cos(x)^2 + 1\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(x)^2 + 1)*sin(x)
```

Sympy [A] time = 0.593521, size = 20, normalized size = 1.33

$$-\frac{\sin(x)\cos(2x)}{3}+\frac{2\sin(2x)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x)
```

```
[Out] -sin(x)*cos(2*x)/3 + 2*sin(2*x)*cos(x)/3
```

Giac [A] time = 1.077, size = 15, normalized size = 1.

$$\frac{1}{6}\sin(3x)+\frac{1}{2}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x),x, algorithm="giac")
```

```
[Out] 1/6*sin(3*x) + 1/2*sin(x)
```

3.126 $\int x^2 \cos^2(a + bx)\, dx$

Optimal. Leaf size=73

$$\frac{x \cos^2(a + bx)}{2b^2} - \frac{\sin(a + bx)\cos(a + bx)}{4b^3} + \frac{x^2 \sin(a + bx)\cos(a + bx)}{2b} - \frac{x}{4b^2} + \frac{x^3}{6}$$

```
[Out] -x/(4*b^2) + x^3/6 + (x*Cos[a + b*x]^2)/(2*b^2) - (Cos[a + b*x]*Sin[a + b*x
])/(4*b^3) + (x^2*Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rubi [A] time = 0.0428616, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3311, 30, 2635, 8}

$$\frac{x \cos^2(a + bx)}{2b^2} - \frac{\sin(a + bx)\cos(a + bx)}{4b^3} + \frac{x^2 \sin(a + bx)\cos(a + bx)}{2b} - \frac{x}{4b^2} + \frac{x^3}{6}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[a + b*x]^2,x]
```

```
[Out] -x/(4*b^2) + x^3/6 + (x*Cos[a + b*x]^2)/(2*b^2) - (Cos[a + b*x]*Sin[a + b*x
])/(4*b^3) + (x^2*Cos[a + b*x]*Sin[a + b*x])/(2*b)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int x^2\cos^2(a+bx)\,dx &= \frac{x\cos^2(a+bx)}{2b^2}+\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}+\frac{\int x^2\,dx}{2}-\frac{\int\cos^2(a+bx)\,dx}{2b^2}\\
&= \frac{x^3}{6}+\frac{x\cos^2(a+bx)}{2b^2}-\frac{\cos(a+bx)\sin(a+bx)}{4b^3}+\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}-\frac{\int 1\,dx}{4b^2}\\
&= -\frac{x}{4b^2}+\frac{x^3}{6}+\frac{x\cos^2(a+bx)}{2b^2}-\frac{\cos(a+bx)\sin(a+bx)}{4b^3}+\frac{x^2\cos(a+bx)\sin(a+bx)}{2b}
\end{aligned}
$$

Mathematica [A] time = 0.121593, size = 47, normalized size = 0.64

$$
\frac{\left(6b^2x^2-3\right)\sin(2(a+bx))+6bx\cos(2(a+bx))+4b^3x^3}{24b^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[a + b*x]^2,x]
```

```
[Out] (4*b^3*x^3 + 6*b*x*Cos[2*(a + b*x)] + (-3 + 6*b^2*x^2)*Sin[2*(a + b*x)])/(2
4*b^3)
```

Maple [B] time = 0.01, size = 158, normalized size = 2.2

$$
\frac{1}{b^3}\left((bx+a)^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{(bx+a)(\cos(bx+a))^2}{2}-\frac{\cos(bx+a)\sin(bx+a)}{4}-\frac{bx}{4}-\frac{a}{4}-\frac{(b
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(b*x+a)^2,x)
```

```
[Out] 1/b^3*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(
b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3-2*a*((b*x+a)
*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+
a^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [A] time = 0.962896, size = 153, normalized size = 2.1

$$\frac{4\,(bx+a)^3+6\,(2\,bx+2\,a+\sin\left(2\,bx+2\,a\right))a^2-6\left(2\,(bx+a)^2+2\,(bx+a)\sin\left(2\,bx+2\,a\right)+\cos\left(2\,bx+2\,a\right)\right)a+6\,(bx+}{24\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/24*(4*(b*x + a)^3 + 6*(2*b*x + 2*a + sin(2*b*x + 2*a))*a^2 - 6*(2*(b*x +
a)^2 + 2*(b*x + a)*sin(2*b*x + 2*a) + cos(2*b*x + 2*a))*a + 6*(b*x + a)*cos
(2*b*x + 2*a) + 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))/b^3
```

Fricas [A] time = 1.77345, size = 134, normalized size = 1.84

$$\frac{2\,b^3x^3+6\,bx\cos\left(bx+a\right)^2+3\left(2\,b^2x^2-1\right)\cos\left(bx+a\right)\sin\left(bx+a\right)-3\,bx}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*x^3 + 6*b*x*cos(b*x + a)^2 + 3*(2*b^2*x^2 - 1)*cos(b*x + a)*sin
(b*x + a) - 3*b*x)/b^3
```

Sympy [A] time = 1.10929, size = 105, normalized size = 1.44

$$\begin{cases} \frac{x^3\sin^2(a+bx)}{6}+\frac{x^3\cos^2(a+bx)}{6}+\frac{x^2\sin(a+bx)\cos(a+bx)}{2b}-\frac{x\sin^2(a+bx)}{4b^2}+\frac{x\cos^2(a+bx)}{4b^2}-\frac{\sin(a+bx)\cos(a+bx)}{4b^3} & \text{for}\ b\neq 0 \\ \frac{x^3\cos^2(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(b*x+a)**2,x)
```

```
[Out] Piecewise((x**3*sin(a + b*x)**2/6 + x**3*cos(a + b*x)**2/6 + x**2*sin(a + b
*x)*cos(a + b*x)/(2*b) - x*sin(a + b*x)**2/(4*b**2) + x*cos(a + b*x)**2/(4*
b**2) - sin(a + b*x)*cos(a + b*x)/(4*b**3), Ne(b, 0)), (x**3*cos(a)**2/3, T
rue))
```

Giac [A] time = 1.08096, size = 61, normalized size = 0.84

$$\frac{1}{6}x^3 + \frac{x\cos(2\,bx + 2\,a)}{4\,b^2} + \frac{(2\,b^2x^2 - 1)\sin(2\,bx + 2\,a)}{8\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 + 1/4*x*cos(2*b*x + 2*a)/b^2 + 1/8*(2*b^2*x^2 - 1)*sin(2*b*x + 2*a)
/b^3
```

3.127 $\int \cot^3(x)\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2}\cot^2(x)-\log(\sin(x))$$

```
[Out] -Cot[x]^2/2 - Log[Sin[x]]
```

Rubi [A] time = 0.0075916, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 3475}

$$-\frac{1}{2}\cot^2(x)-\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3,x]

[Out] -Cot[x]^2/2 - Log[Sin[x]]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cot^3(x)\,dx &= -\frac{1}{2}\cot^2(x)-\int \cot(x)\,dx\\ &= -\frac{1}{2}\cot^2(x)-\log(\sin(x))\end{aligned}$$

Mathematica [A] time = 0.0029982, size = 14, normalized size = 1.

$$-\frac{1}{2}\csc^2(x)-\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3,x]
```

```
[Out] -Csc[x]^2/2 - Log[Sin[x]]
```

Maple [A] time = 0.006, size = 22, normalized size = 1.6

$$\frac{\ln\left((\tan(x))^2+1\right)}{2}-\frac{1}{2(\tan(x))^2}-\ln(\tan(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/tan(x)^3,x)
```

```
[Out] 1/2*ln(tan(x)^2+1)-1/2/tan(x)^2-ln(tan(x))
```

Maxima [A] time = 0.929571, size = 19, normalized size = 1.36

$$-\frac{1}{2\sin(x)^2}-\frac{1}{2}\log\left(\sin(x)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/sin(x)^2 - 1/2*log(sin(x)^2)
```

Fricas [B] time = 1.66543, size = 95, normalized size = 6.79

$$-\frac{\log\left(\frac{\tan(x)^2}{\tan(x)^2+1}\right)\tan(x)^2+\tan(x)^2+1}{2\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(log(tan(x)^2/(tan(x)^2 + 1))*tan(x)^2 + tan(x)^2 + 1)/tan(x)^2
```

Sympy [A] time = 0.090889, size = 14, normalized size = 1.

$$-\log\left(\sin\left(x\right)\right) - \frac{1}{2\sin^{2}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x)**3,x)
```

```
[Out] -log(sin(x)) - 1/(2*sin(x)**2)
```

Giac [B] time = 1.12662, size = 39, normalized size = 2.79

$$\frac{\tan\left(x\right)^{2} - 1}{2\,\tan\left(x\right)^{2}} + \frac{1}{2}\,\log\left(\tan\left(x\right)^{2} + 1\right) - \frac{1}{2}\,\log\left(\tan\left(x\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/tan(x)^3,x, algorithm="giac")
```

```
[Out] 1/2*(tan(x)^2 - 1)/tan(x)^2 + 1/2*log(tan(x)^2 + 1) - 1/2*log(tan(x)^2)
```

3.128 $\int x^3 \tan^4(x)\,dx$

Optimal. Leaf size=104

$$4ix\text{PolyLog}\left(2,-e^{2ix}\right)-2\text{PolyLog}\left(3,-e^{2ix}\right)+\frac{x^4}{4}+\frac{4ix^3}{3}-\frac{x^2}{2}-4x^2\log\left(1+e^{2ix}\right)+\frac{1}{3}x^3\tan^3(x)-\frac{1}{2}x^2\tan^2(x)-x^3\text{ta}$$

```
[Out] -x^2/2 + ((4*I)/3)*x^3 + x^4/4 - 4*x^2*Log[1 + E^((2*I)*x)] + Log[Cos[x]] +
 (4*I)*x*PolyLog[2, -E^((2*I)*x)] - 2*PolyLog[3, -E^((2*I)*x)] + x*Tan[x] -
 x^3*Tan[x] - (x^2*Tan[x]^2)/2 + (x^3*Tan[x]^3)/3
```

Rubi [A] time = 0.227076, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {3720, 3475, 30, 3719, 2190, 2531, 2282, 6589}

$$4ix\text{PolyLog}\left(2,-e^{2ix}\right)-2\text{PolyLog}\left(3,-e^{2ix}\right)+\frac{x^4}{4}+\frac{4ix^3}{3}-\frac{x^2}{2}-4x^2\log\left(1+e^{2ix}\right)+\frac{1}{3}x^3\tan^3(x)-\frac{1}{2}x^2\tan^2(x)-x^3\text{ta}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tan[x]^4,x]
```

```
[Out] -x^2/2 + ((4*I)/3)*x^3 + x^4/4 - 4*x^2*Log[1 + E^((2*I)*x)] + Log[Cos[x]] +
 (4*I)*x*PolyLog[2, -E^((2*I)*x)] - 2*PolyLog[3, -E^((2*I)*x)] + x*Tan[x] -
 x^3*Tan[x] - (x^2*Tan[x]^2)/2 + (x^3*Tan[x]^3)/3
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3\tan^4(x)\,dx = \frac{1}{3}x^3\tan^3(x) - \int x^3\tan^2(x)\,dx - \int x^2\tan^3(x)\,dx$$
$$= -x^3\tan(x) - \frac{1}{2}x^2\tan^2(x) + \frac{1}{3}x^3\tan^3(x) + 3\int x^2\tan(x)\,dx + \int x^3\,dx + \int x^2\tan(x)\,dx + \int x\tan$$
$$= \frac{4ix^3}{3} + \frac{x^4}{4} + x\tan(x) - x^3\tan(x) - \frac{1}{2}x^2\tan^2(x) + \frac{1}{3}x^3\tan^3(x) - 2i\int \frac{e^{2ix}x^2}{1+e^{2ix}}\,dx - 6i\int \frac{e^{2ix}x^2}{1+e^{2ix}}\,dx$$
$$= -\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2\log\left(1+e^{2ix}\right) + \log(\cos(x)) + x\tan(x) - x^3\tan(x) - \frac{1}{2}x^2\tan^2(x) + \frac{1}{3}x^3\tan$$
$$= -\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2\log\left(1+e^{2ix}\right) + \log(\cos(x)) + 4ix\text{Li}_2\left(-e^{2ix}\right) + x\tan(x) - x^3\tan(x) - \frac{1}{2}x^2\text{ta}$$
$$= -\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2\log\left(1+e^{2ix}\right) + \log(\cos(x)) + 4ix\text{Li}_2\left(-e^{2ix}\right) + x\tan(x) - x^3\tan(x) - \frac{1}{2}x^2\text{ta}$$
$$= -\frac{x^2}{2} + \frac{4ix^3}{3} + \frac{x^4}{4} - 4x^2\log\left(1+e^{2ix}\right) + \log(\cos(x)) + 4ix\text{Li}_2\left(-e^{2ix}\right) - 2\text{Li}_3\left(-e^{2ix}\right) + x\tan(x) - x^3\text{t}$$

Mathematica [A] time = 0.14389, size = 101, normalized size = 0.97

$$4ix\text{PolyLog}\left(2,-e^{2ix}\right) - 2\text{PolyLog}\left(3,-e^{2ix}\right) + \frac{x^4}{4} + \frac{4ix^3}{3} - 4x^2\log\left(1+e^{2ix}\right) - \frac{4}{3}x^3\tan(x) - \frac{1}{2}x^2\sec^2(x) + \frac{1}{3}x^3\tan(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tan[x]^4,x]
```

```
[Out] ((4*I)/3)*x^3 + x^4/4 - 4*x^2*Log[1 + E^((2*I)*x)] + Log[Cos[x]] + (4*I)*x*
PolyLog[2, -E^((2*I)*x)] - 2*PolyLog[3, -E^((2*I)*x)] - (x^2*Sec[x]^2)/2 +
x*Tan[x] - (4*x^3*Tan[x])/3 + (x^3*Sec[x]^2*Tan[x])/3
```

Maple [A] time = 0.056, size = 138, normalized size = 1.3

$$\frac{x^4}{4} - \frac{\frac{2i}{3}x\left(6x^2e^{4ix} + 6x^2e^{2ix} - 3e^{4ix} - 3ixe^{4ix} + 4x^2 - 6e^{2ix} - 3ixe^{2ix} - 3\right)}{\left(1+e^{2ix}\right)^3} - 2\ln\left(e^{ix}\right) + \ln\left(1+e^{2ix}\right) + \frac{8i}{3}x^3 - 4x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(x)^4,x)
```

```
[Out] 1/4*x^4-2/3*I*x*(6*x^2*exp(4*I*x)+6*x^2*exp(2*I*x)-3*exp(4*I*x)-3*I*x*exp(4
*I*x)+4*x^2-6*exp(2*I*x)-3*I*x*exp(2*I*x)-3)/(1+exp(2*I*x))^3-2*ln(exp(I*x)
)+ln(1+exp(2*I*x))+8/3*I*x^3-4*x^2*ln(1+exp(2*I*x))+4*I*x*polylog(2,-exp(2*
I*x))-2*polylog(3,-exp(2*I*x))
```

Maxima [B] time = 1.72096, size = 655, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^4,x, algorithm="maxima")
```

```
[Out] -(3*I*x^4 + (48*x^2 + 12*(4*x^2 - 1)*cos(6*x) + 36*(4*x^2 - 1)*cos(4*x) + 3
6*(4*x^2 - 1)*cos(2*x) + (48*I*x^2 - 12*I)*sin(6*x) + (144*I*x^2 - 36*I)*si
n(4*x) + (144*I*x^2 - 36*I)*sin(2*x) - 12)*arctan2(sin(2*x), cos(2*x) + 1)
+ (3*I*x^4 - 32*x^3 + 24*x)*cos(6*x) + (9*I*x^4 - 48*x^3 - 24*I*x^2 + 48*x)
*cos(4*x) + (9*I*x^4 - 48*x^3 - 24*I*x^2 + 24*x)*cos(2*x) - (48*x*cos(6*x)
+ 144*x*cos(4*x) + 144*x*cos(2*x) + 48*I*x*sin(6*x) + 144*I*x*sin(4*x) + 14
4*I*x*sin(2*x) + 48*x)*dilog(-e^(2*I*x)) + (-24*I*x^2 + (-24*I*x^2 + 6*I)*c
os(6*x) + (-72*I*x^2 + 18*I)*cos(4*x) + (-72*I*x^2 + 18*I)*cos(2*x) + 6*(4*
x^2 - 1)*sin(6*x) + 18*(4*x^2 - 1)*sin(4*x) + 18*(4*x^2 - 1)*sin(2*x) + 6*I
)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + (-24*I*cos(6*x) - 72*I*co
s(4*x) - 72*I*cos(2*x) + 24*sin(6*x) + 72*sin(4*x) + 72*sin(2*x) - 24*I)*po
lylog(3, -e^(2*I*x)) - (3*x^4 + 32*I*x^3 - 24*I*x)*sin(6*x) - (9*x^4 + 48*I
*x^3 - 24*x^2 - 48*I*x)*sin(4*x) - (9*x^4 + 48*I*x^3 - 24*x^2 - 24*I*x)*sin
(2*x))/(-12*I*cos(6*x) - 36*I*cos(4*x) - 36*I*cos(2*x) + 12*sin(6*x) + 36*s
in(4*x) + 36*sin(2*x) - 12*I)
```

Fricas [C] time = 1.92639, size = 548, normalized size = 5.27

$$\frac{1}{3}x^3\tan(x)^3+\frac{1}{4}x^4-\frac{1}{2}x^2\tan(x)^2-\frac{1}{2}x^2-2ix\mathrm{Li}_2\left(\frac{2(i\tan(x)-1)}{\tan(x)^2+1}+1\right)+2ix\mathrm{Li}_2\left(\frac{2(-i\tan(x)-1)}{\tan(x)^2+1}+1\right)-\frac{1}{2}\left(4x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*tan(x)^3 + 1/4*x^4 - 1/2*x^2*tan(x)^2 - 1/2*x^2 - 2*I*x*dilog(2*(I*
tan(x) - 1)/(tan(x)^2 + 1) + 1) + 2*I*x*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 +
```

```
 1) + 1) - 1/2*(4*x^2 - 1)*log(-2*(I*tan(x) - 1)/(tan(x)^2 + 1)) - 1/2*(4*x
^2 - 1)*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) - (x^3 - x)*tan(x) - polylog
(3, (tan(x)^2 + 2*I*tan(x) - 1)/(tan(x)^2 + 1)) - polylog(3, (tan(x)^2 - 2*
I*tan(x) - 1)/(tan(x)^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \tan^4(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tan(x)**4,x)
```

```
[Out] Integral(x**3*tan(x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \tan(x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^4,x, algorithm="giac")
```

```
[Out] integrate(x^3*tan(x)^4, x)
```

3.129 $\int x^3 \tan^6(x)\,dx$

Optimal. Leaf size=153

$$-\frac{23}{5}ix\text{PolyLog}\left(2,-e^{2ix}\right)+\frac{23}{10}\text{PolyLog}\left(3,-e^{2ix}\right)-\frac{x^4}{4}-\frac{23ix^3}{15}+\frac{19x^2}{20}+\frac{23}{5}x^2\log\left(1+e^{2ix}\right)+\frac{1}{5}x^3\tan^5(x)-\frac{3}{20}x^2\tan$$

```
[Out] (19*x^2)/20 - ((23*I)/15)*x^3 - x^4/4 + (23*x^2*Log[1 + E^((2*I)*x)])/5 - 2
*Log[Cos[x]] - ((23*I)/5)*x*PolyLog[2, -E^((2*I)*x)] + (23*PolyLog[3, -E^((
2*I)*x)])/10 - (19*x*Tan[x])/10 + x^3*Tan[x] - Tan[x]^2/20 + (4*x^2*Tan[x]^
2)/5 + (x*Tan[x]^3)/10 - (x^3*Tan[x]^3)/3 - (3*x^2*Tan[x]^4)/20 + (x^3*Tan[
x]^5)/5
```

Rubi [A] time = 0.40749, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 34, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {3720, 3473, 3475, 30, 3719, 2190, 2531, 2282, 6589}

$$-\frac{23}{5}ix\text{PolyLog}\left(2,-e^{2ix}\right)+\frac{23}{10}\text{PolyLog}\left(3,-e^{2ix}\right)-\frac{x^4}{4}-\frac{23ix^3}{15}+\frac{19x^2}{20}+\frac{23}{5}x^2\log\left(1+e^{2ix}\right)+\frac{1}{5}x^3\tan^5(x)-\frac{3}{20}x^2\tan$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Tan[x]^6,x]
```

```
[Out] (19*x^2)/20 - ((23*I)/15)*x^3 - x^4/4 + (23*x^2*Log[1 + E^((2*I)*x)])/5 - 2
*Log[Cos[x]] - ((23*I)/5)*x*PolyLog[2, -E^((2*I)*x)] + (23*PolyLog[3, -E^((
2*I)*x)])/10 - (19*x*Tan[x])/10 + x^3*Tan[x] - Tan[x]^2/20 + (4*x^2*Tan[x]^
2)/5 + (x*Tan[x]^3)/10 - (x^3*Tan[x]^3)/3 - (3*x^2*Tan[x]^4)/20 + (x^3*Tan[
x]^5)/5
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
```

```
x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^3 \tan^6(x)\,dx = \frac{1}{5}x^3\tan^5(x) - \frac{3}{5}\int x^2\tan^5(x)\,dx - \int x^3\tan^4(x)\,dx$$
$$= -\frac{1}{3}x^3\tan^3(x) - \frac{3}{20}x^2\tan^4(x) + \frac{1}{5}x^3\tan^5(x) + \frac{3}{10}\int x\tan^4(x)\,dx + \frac{3}{5}\int x^2\tan^3(x)\,dx + \int x^3\tan^2($$
$$= x^3\tan(x) + \frac{4}{5}x^2\tan^2(x) + \frac{1}{10}x\tan^3(x) - \frac{1}{3}x^3\tan^3(x) - \frac{3}{20}x^2\tan^4(x) + \frac{1}{5}x^3\tan^5(x) - \frac{1}{10}\int \tan^3(x)$$
$$= -\frac{23ix^3}{15} - \frac{x^4}{4} - \frac{19}{10}x\tan(x) + x^3\tan(x) - \frac{\tan^2(x)}{20} + \frac{4}{5}x^2\tan^2(x) + \frac{1}{10}x\tan^3(x) - \frac{1}{3}x^3\tan^3(x) - \frac{3}{20}x$$
$$= \frac{19x^2}{20} - \frac{23ix^3}{15} - \frac{x^4}{4} + \frac{23}{5}x^2\log\left(1+e^{2ix}\right) - 2\log(\cos(x)) - \frac{19}{10}x\tan(x) + x^3\tan(x) - \frac{\tan^2(x)}{20} + \frac{4}{5}x^2\,t$$
$$= \frac{19x^2}{20} - \frac{23ix^3}{15} - \frac{x^4}{4} + \frac{23}{5}x^2\log\left(1+e^{2ix}\right) - 2\log(\cos(x)) - \frac{23}{5}ix\text{Li}_2\left(-e^{2ix}\right) - \frac{19}{10}x\tan(x) + x^3\tan(x)$$
$$= \frac{19x^2}{20} - \frac{23ix^3}{15} - \frac{x^4}{4} + \frac{23}{5}x^2\log\left(1+e^{2ix}\right) - 2\log(\cos(x)) - \frac{23}{5}ix\text{Li}_2\left(-e^{2ix}\right) - \frac{19}{10}x\tan(x) + x^3\tan(x)$$
$$= \frac{19x^2}{20} - \frac{23ix^3}{15} - \frac{x^4}{4} + \frac{23}{5}x^2\log\left(1+e^{2ix}\right) - 2\log(\cos(x)) - \frac{23}{5}ix\text{Li}_2\left(-e^{2ix}\right) + \frac{23}{10}\text{Li}_3\left(-e^{2ix}\right) - \frac{19}{10}x\tan$$

Mathematica [A] time = 0.310905, size = 133, normalized size = 0.87

$$\frac{1}{60}\left(-276ix\text{PolyLog}\left(2,-e^{2ix}\right) + 138\text{PolyLog}\left(3,-e^{2ix}\right) - 15x^4 - 92ix^3 + 276x^2\log\left(1+e^{2ix}\right) + 92x^3\tan(x) - 9x^2\sec^4(x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Tan[x]^6,x]
```

```
[Out] ((-92*I)*x^3 - 15*x^4 + 276*x^2*Log[1 + E^((2*I)*x)] - 120*Log[Cos[x]] - (2
76*I)*x*PolyLog[2, -E^((2*I)*x)] + 138*PolyLog[3, -E^((2*I)*x)] - 3*Sec[x]^
2 + 66*x^2*Sec[x]^2 - 9*x^2*Sec[x]^4 - 120*x*Tan[x] + 92*x^3*Tan[x] + 6*x*S
ec[x]^2*Tan[x] - 44*x^3*Sec[x]^2*Tan[x] + 12*x^3*Sec[x]^4*Tan[x])/60
```

Maple [A] time = 0.069, size = 237, normalized size = 1.6

$$-\frac{x^4}{4} + \frac{\frac{i}{15}\left(-66\, i x^2 e^{2\,ix} + 90\, x^3 e^{8\,ix} - 162\, i x^2 e^{6\,ix} - 162\, i x^2 e^{4\,ix} + 180\, x^3 e^{6\,ix} - 66\, x e^{8\,ix} + 3\, i e^{2\,ix} + 9\, i e^{6\,ix} + 280\, x^3 e^{4\,ix} - 2\right.}{\left(1 + e^{2\,ix}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*tan(x)^6,x)
```

```
[Out] -1/4*x^4+1/15*I*(-66*I*x^2*exp(2*I*x)+90*x^3*exp(8*I*x)-162*I*x^2*exp(6*I*x
)-162*I*x^2*exp(4*I*x)+180*x^3*exp(6*I*x)-66*x*exp(8*I*x)+3*I*exp(2*I*x)+9*
I*exp(6*I*x)+280*x^3*exp(4*I*x)-246*x*exp(6*I*x)+3*I*exp(8*I*x)+9*I*exp(4*I
*x)+140*x^3*exp(2*I*x)-354*x*exp(4*I*x)-66*I*x^2*exp(8*I*x)+46*x^3-234*x*ex
p(2*I*x)-60*x)/(1+exp(2*I*x))^5+4*ln(exp(I*x))-2*ln(1+exp(2*I*x))-46/15*I*x
^3+23/5*x^2*ln(1+exp(2*I*x))-23/5*I*x*polylog(2,-exp(2*I*x))+23/10*polylog(
3,-exp(2*I*x))
```

Maxima [B] time = 3.09721, size = 1034, normalized size = 6.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^6,x, algorithm="maxima")
```

```
[Out] (15*I*x^4 + (276*x^2 + 12*(23*x^2 - 10)*cos(10*x) + 60*(23*x^2 - 10)*cos(8*
x) + 120*(23*x^2 - 10)*cos(6*x) + 120*(23*x^2 - 10)*cos(4*x) + 60*(23*x^2 -
 10)*cos(2*x) + (276*I*x^2 - 120*I)*sin(10*x) + (1380*I*x^2 - 600*I)*sin(8*
x) + (2760*I*x^2 - 1200*I)*sin(6*x) + (2760*I*x^2 - 1200*I)*sin(4*x) + (138
0*I*x^2 - 600*I)*sin(2*x) - 120)*arctan2(sin(2*x), cos(2*x) + 1) + (15*I*x^
4 - 184*x^3 + 240*x)*cos(10*x) + (75*I*x^4 - 560*x^3 - 264*I*x^2 + 936*x +
12*I)*cos(8*x) + (150*I*x^4 - 1120*x^3 - 648*I*x^2 + 1416*x + 36*I)*cos(6*x
) + (150*I*x^4 - 720*x^3 - 648*I*x^2 + 984*x + 36*I)*cos(4*x) + (75*I*x^4 -
 360*x^3 - 264*I*x^2 + 264*x + 12*I)*cos(2*x) - (276*x*cos(10*x) + 1380*x*c
os(8*x) + 2760*x*cos(6*x) + 2760*x*cos(4*x) + 1380*x*cos(2*x) + 276*I*x*sin
(10*x) + 1380*I*x*sin(8*x) + 2760*I*x*sin(6*x) + 2760*I*x*sin(4*x) + 1380*I
*x*sin(2*x) + 276*x)*dilog(-e^(2*I*x)) + (-138*I*x^2 + (-138*I*x^2 + 60*I)*
cos(10*x) + (-690*I*x^2 + 300*I)*cos(8*x) + (-1380*I*x^2 + 600*I)*cos(6*x)
+ (-1380*I*x^2 + 600*I)*cos(4*x) + (-690*I*x^2 + 300*I)*cos(2*x) + 6*(23*x^
2 - 10)*sin(10*x) + 30*(23*x^2 - 10)*sin(8*x) + 60*(23*x^2 - 10)*sin(6*x) +
 60*(23*x^2 - 10)*sin(4*x) + 30*(23*x^2 - 10)*sin(2*x) + 60*I)*log(cos(2*x)
```

```
^2 + sin(2*x)^2 + 2*cos(2*x) + 1) + (-138*I*cos(10*x) - 690*I*cos(8*x) - 13
80*I*cos(6*x) - 1380*I*cos(4*x) - 690*I*cos(2*x) + 138*sin(10*x) + 690*sin(
8*x) + 1380*sin(6*x) + 1380*sin(4*x) + 690*sin(2*x) - 138*I)*polylog(3, -e^
(2*I*x)) - (15*x^4 + 184*I*x^3 - 240*I*x)*sin(10*x) - (75*x^4 + 560*I*x^3 -
 264*x^2 - 936*I*x + 12)*sin(8*x) - (150*x^4 + 1120*I*x^3 - 648*x^2 - 1416*
I*x + 36)*sin(6*x) - (150*x^4 + 720*I*x^3 - 648*x^2 - 984*I*x + 36)*sin(4*x
) - (75*x^4 + 360*I*x^3 - 264*x^2 - 264*I*x + 12)*sin(2*x))/(-60*I*cos(10*x
) - 300*I*cos(8*x) - 600*I*cos(6*x) - 600*I*cos(4*x) - 300*I*cos(2*x) + 60*
sin(10*x) + 300*sin(8*x) + 600*sin(6*x) + 600*sin(4*x) + 300*sin(2*x) - 60*
I)
```

Fricas [C] time = 1.84195, size = 683, normalized size = 4.46

$$\frac{1}{5}x^3\tan(x)^5 - \frac{3}{20}x^2\tan(x)^4 - \frac{1}{4}x^4 - \frac{1}{30}\left(10x^3 - 3x\right)\tan(x)^3 + \frac{1}{20}\left(16x^2 - 1\right)\tan(x)^2 + \frac{19}{20}x^2 + \frac{23}{10}ix\mathrm{Li}_2\left(\frac{2(i\tan(x)}{\tan(x)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^6,x, algorithm="fricas")
```

```
[Out] 1/5*x^3*tan(x)^5 - 3/20*x^2*tan(x)^4 - 1/4*x^4 - 1/30*(10*x^3 - 3*x)*tan(x)
^3 + 1/20*(16*x^2 - 1)*tan(x)^2 + 19/20*x^2 + 23/10*I*x*dilog(2*(I*tan(x) -
 1)/(tan(x)^2 + 1) + 1) - 23/10*I*x*dilog(2*(-I*tan(x) - 1)/(tan(x)^2 + 1)
+ 1) + 1/10*(23*x^2 - 10)*log(-2*(I*tan(x) - 1)/(tan(x)^2 + 1)) + 1/10*(23*
x^2 - 10)*log(-2*(-I*tan(x) - 1)/(tan(x)^2 + 1)) + 1/10*(10*x^3 - 19*x)*tan
(x) + 23/20*polylog(3, (tan(x)^2 + 2*I*tan(x) - 1)/(tan(x)^2 + 1)) + 23/20*
polylog(3, (tan(x)^2 - 2*I*tan(x) - 1)/(tan(x)^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3\tan^6(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*tan(x)**6,x)
```

```
[Out] Integral(x**3*tan(x)**6, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \tan(x)^6 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*tan(x)^6,x, algorithm="giac")
```

```
[Out] integrate(x^3*tan(x)^6, x)
```

3.130 $\int x \tan^2(x)\,dx$

Optimal. Leaf size=15

$$-\frac{x^2}{2}+x\tan(x)+\log(\cos(x))$$

```
[Out] -x^2/2 + Log[Cos[x]] + x*Tan[x]
```

Rubi [A] time = 0.0135304, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3720, 3475, 30}

$$-\frac{x^2}{2}+x\tan(x)+\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Tan[x]^2,x]
```

```
[Out] -x^2/2 + Log[Cos[x]] + x*Tan[x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int x \tan^2(x) \, dx = x \tan(x) - \int x \, dx - \int \tan(x) \, dx$$
$$= -\frac{x^2}{2} + \log(\cos(x)) + x \tan(x)$$

Mathematica [A] time = 0.0143505, size = 15, normalized size = 1.

$$-\frac{x^2}{2} + x \tan(x) + \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Tan[x]^2,x]
```

```
[Out] -x^2/2 + Log[Cos[x]] + x*Tan[x]
```

Maple [A] time = 0.008, size = 20, normalized size = 1.3

$$x \tan(x) - \frac{x^2}{2} - \frac{\ln\left((\tan(x))^2 + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*tan(x)^2,x)
```

```
[Out] x*tan(x)-1/2*x^2-1/2*ln(tan(x)^2+1)
```

Maxima [B] time = 1.42228, size = 144, normalized size = 9.6

$$-\frac{x^2 \cos(2x)^2 + x^2 \sin(2x)^2 + 2x^2 \cos(2x) + x^2 - \left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right) \log\left(\cos(2x)^2 + \sin(2x)^2\right.}{2\left(\cos(2x)^2 + \sin(2x)^2 + 2\cos(2x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x^2*cos(2*x)^2 + x^2*sin(2*x)^2 + 2*x^2*cos(2*x) + x^2 - (cos(2*x)^2
+ sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1
) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

Fricas [A] time = 1.85542, size = 66, normalized size = 4.4

$$-\frac{1}{2}x^2 + x\tan(x) + \frac{1}{2}\log\left(\frac{1}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + x*tan(x) + 1/2*log(1/(tan(x)^2 + 1))
```

Sympy [A] time = 0.159422, size = 19, normalized size = 1.27

$$-\frac{x^2}{2} + x\tan(x) - \frac{\log\left(\tan^2(x) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)**2,x)
```

```
[Out] -x**2/2 + x*tan(x) - log(tan(x)**2 + 1)/2
```

Giac [A] time = 1.11988, size = 31, normalized size = 2.07

$$-\frac{1}{2}x^2 + x\tan(x) + \frac{1}{2}\log\left(\frac{4}{\tan(x)^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*tan(x)^2,x, algorithm="giac")
```

```
[Out] -1/2*x^2 + x*tan(x) + 1/2*log(4/(tan(x)^2 + 1))
```

3.131 $\int \cos(3x)\sin(2x)\,dx$

Optimal. Leaf size=15

$$\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

```
[Out] Cos[x]/2 - Cos[5*x]/10
```

Rubi [A] time = 0.0076803, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4284}

$$\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[3*x]*Sin[2*x],x]
```

```
[Out] Cos[x]/2 - Cos[5*x]/10
```

Rule 4284

```
Int[cos[(c_.) + (d_.)*(x_)]*sin[(a_.) + (b_.)*(x_)], x_Symbol] :> -Simp[Cos
[a - c + (b - d)*x]/(2*(b - d)), x] - Simp[Cos[a + c + (b + d)*x]/(2*(b + d
)), x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - d^2, 0]
```

Rubi steps

$$\int \cos(3x)\sin(2x)\,dx=\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

Mathematica [A] time = 0.0061278, size = 15, normalized size = 1.

$$\frac{\cos(x)}{2}-\frac{1}{10}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[3*x]*Sin[2*x],x]
```

```
[Out] Cos[x]/2 - Cos[5*x]/10
```

Maple [A] time = 0.036, size = 12, normalized size = 0.8

$$\frac{\cos(x)}{2} - \frac{\cos(5x)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(3*x)*sin(2*x),x)
```

```
[Out] 1/2*cos(x)-1/10*cos(5*x)
```

Maxima [A] time = 0.931286, size = 15, normalized size = 1.

$$-\frac{1}{10}\cos(5x) + \frac{1}{2}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(2*x),x, algorithm="maxima")
```

```
[Out] -1/10*cos(5*x) + 1/2*cos(x)
```

Fricas [A] time = 1.90517, size = 38, normalized size = 2.53

$$-\frac{8}{5}\cos(x)^5 + 2\cos(x)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(2*x),x, algorithm="fricas")
```

```
[Out] -8/5*cos(x)^5 + 2*cos(x)^3
```

Sympy [B] time = 0.525338, size = 26, normalized size = 1.73

$$\frac{3\sin(2x)\sin(3x)}{5} + \frac{2\cos(2x)\cos(3x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(2*x),x)
```

```
[Out] 3*sin(2*x)*sin(3*x)/5 + 2*cos(2*x)*cos(3*x)/5
```

Giac [A] time = 1.09231, size = 15, normalized size = 1.

$$-\frac{1}{10}\cos(5x) + \frac{1}{2}\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(3*x)*sin(2*x),x, algorithm="giac")
```

```
[Out] -1/10*cos(5*x) + 1/2*cos(x)
```

3.132 $\int \cos^2(x)\sin^2(x)\,dx$

Optimal. Leaf size=24

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rubi [A] time = 0.025102, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2568, 2635, 8}

$$\frac{x}{8}-\frac{1}{4}\sin(x)\cos^3(x)+\frac{1}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 + (Cos[x]*Sin[x])/8 - (Cos[x]^3*Sin[x])/4
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\sin^2(x)\,dx &= -\frac{1}{4}\cos^3(x)\sin(x) + \frac{1}{4}\int \cos^2(x)\,dx\\ &= \frac{1}{8}\cos(x)\sin(x) - \frac{1}{4}\cos^3(x)\sin(x) + \frac{\int 1\,dx}{8}\\ &= \frac{x}{8} + \frac{1}{8}\cos(x)\sin(x) - \frac{1}{4}\cos^3(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0044485, size = 14, normalized size = 0.58

$$\frac{x}{8} - \frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sin[x]^2,x]
```

```
[Out] x/8 - Sin[4*x]/32
```

Maple [A] time = 0.005, size = 19, normalized size = 0.8

$$\frac{x}{8} + \frac{\cos(x)\sin(x)}{8} - \frac{(\cos(x))^3\sin(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2,x)
```

```
[Out] 1/8*x+1/8*cos(x)*sin(x)-1/4*cos(x)^3*sin(x)
```

Maxima [A] time = 0.948059, size = 14, normalized size = 0.58

$$\frac{1}{8}x - \frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

Fricas [A] time = 1.74223, size = 58, normalized size = 2.42

$$-\frac{1}{8}\left(2\cos(x)^3-\cos(x)\right)\sin(x)+\frac{1}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*cos(x)^3 - cos(x))*sin(x) + 1/8*x
```

Sympy [A] time = 0.065233, size = 14, normalized size = 0.58

$$\frac{x}{8}-\frac{\sin(2x)\cos(2x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**2,x)
```

```
[Out] x/8 - sin(2*x)*cos(2*x)/16
```

Giac [A] time = 1.08004, size = 14, normalized size = 0.58

$$\frac{1}{8}x-\frac{1}{32}\sin(4x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/8*x - 1/32*sin(4*x)
```

3.133 $\int \csc^2(x) \sec^2(x) \, dx$

Optimal. Leaf size=7

$$\tan(x) - \cot(x)$$

```
[Out] -Cot[x] + Tan[x]
```

Rubi [A] time = 0.021431, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2620, 14}

$$\tan(x) - \cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2*Sec[x]^2,x]
```

```
[Out] -Cot[x] + Tan[x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(x) \sec^2(x) \, dx &= \text{Subst}\left(\int \frac{1+x^2}{x^2} \, dx, x, \tan(x)\right) \\
&= \text{Subst}\left(\int \left(1 + \frac{1}{x^2}\right) dx, x, \tan(x)\right) \\
&= -\cot(x) + \tan(x)
\end{aligned}$$

Mathematica [A] time = 0.0069747, size = 6, normalized size = 0.86

$$-2\cot(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2*Sec[x]^2,x]
```

```
[Out] -2*Cot[2*x]
```

Maple [A] time = 0.03, size = 15, normalized size = 2.1

$$\frac{1}{\cos(x)\sin(x)} - 2\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/cos(x)^2/sin(x)^2,x)
```

```
[Out] 1/sin(x)/cos(x)-2*cot(x)
```

Maxima [A] time = 0.946459, size = 12, normalized size = 1.71

$$-\frac{1}{\tan(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/tan(x) + tan(x)
```

Fricas [B] time = 1.67464, size = 47, normalized size = 6.71

$$-\frac{2\cos(x)^2 - 1}{\cos(x)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="fricas")
```

```
[Out] -(2*cos(x)^2 - 1)/(cos(x)*sin(x))
```

Sympy [B] time = 0.068981, size = 12, normalized size = 1.71

$$-\frac{2\cos(2x)}{\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)**2/sin(x)**2,x)
```

```
[Out] -2*cos(2*x)/sin(2*x)
```

Giac [A] time = 1.06829, size = 12, normalized size = 1.71

$$-\frac{1}{\tan(x)} + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/cos(x)^2/sin(x)^2,x, algorithm="giac")
```

```
[Out] -1/tan(x) + tan(x)
```

3.134 $\int d^x \sin(x)\, dx$

Optimal. Leaf size=32

$$\frac{d^x \log(d)\sin(x)}{\log^2(d)+1} - \frac{d^x \cos(x)}{\log^2(d)+1}$$

```
[Out] -((d^x*Cos[x])/(1 + Log[d]^2)) + (d^x*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rubi [A] time = 0.0109728, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4432}

$$\frac{d^x \log(d)\sin(x)}{\log^2(d)+1} - \frac{d^x \cos(x)}{\log^2(d)+1}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*Sin[x],x]
```

```
[Out] -((d^x*Cos[x])/(1 + Log[d]^2)) + (d^x*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int d^x \sin(x)\, dx = -\frac{d^x \cos(x)}{1+\log^2(d)} + \frac{d^x \log(d)\sin(x)}{1+\log^2(d)}$$

Mathematica [A] time = 0.0183267, size = 22, normalized size = 0.69

$$\frac{d^x(\log(d)\sin(x) - \cos(x))}{\log^2(d)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*Sin[x],x]
```

```
[Out] (d^x*(-Cos[x] + Log[d]*Sin[x]))/(1 + Log[d]^2)
```

Maple [B] time = 0.018, size = 69, normalized size = 2.2

$$\left(\frac{e^{x\ln(d)}}{1+(\ln(d))^2}\left(\tan\left(\frac{x}{2}\right)\right)^2-\frac{e^{x\ln(d)}}{1+(\ln(d))^2}+2\,\frac{\ln(d)\,e^{x\ln(d)}\tan(x/2)}{1+(\ln(d))^2}\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*sin(x),x)
```

```
[Out] (1/(1+ln(d)^2)*exp(x*ln(d))*tan(1/2*x)^2-1/(1+ln(d)^2)*exp(x*ln(d))+2/(1+ln
(d)^2)*ln(d)*exp(x*ln(d))*tan(1/2*x))/(tan(1/2*x)^2+1)
```

Maxima [A] time = 0.976485, size = 34, normalized size = 1.06

$$\frac{d^x\log(d)\sin(x)-d^x\cos(x)}{\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*sin(x),x, algorithm="maxima")
```

```
[Out] (d^x*log(d)*sin(x) - d^x*cos(x))/(log(d)^2 + 1)
```

Fricas [A] time = 1.82861, size = 61, normalized size = 1.91

$$\frac{(\log(d)\sin(x)-\cos(x))d^x}{\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*sin(x),x, algorithm="fricas")
```

```
[Out] (log(d)*sin(x) - cos(x))*d^x/(log(d)^2 + 1)
```

Sympy [A] time = 1.09542, size = 104, normalized size = 3.25

$$\begin{cases} \frac{xe^{-ix}\sin(x)}{2} - \frac{ixe^{-ix}\cos(x)}{2} - \frac{e^{-ix}\cos(x)}{2} & \text{for } d = e^{-i} \\ \frac{xe^{ix}\sin(x)}{2} + \frac{ixe^{ix}\cos(x)}{2} - \frac{e^{ix}\cos(x)}{2} & \text{for } d = e^{i} \\ \frac{d^x\log(d)\sin(x)}{\log(d)^2+1} - \frac{d^x\cos(x)}{\log(d)^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*sin(x),x)
```

```
[Out] Piecewise((x*exp(-I*x)*sin(x)/2 - I*x*exp(-I*x)*cos(x)/2 - exp(-I*x)*cos(x)
/2, Eq(d, exp(-I))), (x*exp(I*x)*sin(x)/2 + I*x*exp(I*x)*cos(x)/2 - exp(I*x
)*cos(x)/2, Eq(d, exp(I))), (d**x*log(d)*sin(x)/(log(d)**2 + 1) - d**x*cos(
x)/(log(d)**2 + 1), True))
```

Giac [C] time = 1.13138, size = 443, normalized size = 13.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*sin(x),x, algorithm="giac")
```

```
[Out] abs(d)^x*((pi - pi*sgn(d) - 2)*cos(1/2*pi*x*sgn(d) - 1/2*pi*x + x)/((pi - p
i*sgn(d) - 2)^2 + 4*log(abs(d))^2) + 2*log(abs(d))*sin(1/2*pi*x*sgn(d) - 1/
2*pi*x + x)/((pi - pi*sgn(d) - 2)^2 + 4*log(abs(d))^2)) - abs(d)^x*((pi - p
i*sgn(d) + 2)*cos(1/2*pi*x*sgn(d) - 1/2*pi*x - x)/((pi - pi*sgn(d) + 2)^2 +
 4*log(abs(d))^2) + 2*log(abs(d))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x)/((pi
- pi*sgn(d) + 2)^2 + 4*log(abs(d))^2)) + 1/2*abs(d)^x*(2*I*e^(1/2*I*pi*x*sg
n(d) - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I) +
2*I*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4*l
og(abs(d)) - 4*I)) + 1/2*abs(d)^x*(-2*I*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x -
 I*x)/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) - 4*I) - 2*I*e^(-1/2*I*pi*x*
sgn(d) + 1/2*I*pi*x + I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I))
```

3.135 $\int d^x \cos(x)\,dx$

Optimal. Leaf size=31

$$\frac{d^x \sin(x)}{\log^2(d)+1}+\frac{d^x \log(d)\cos(x)}{\log^2(d)+1}$$

```
[Out] (d^x*Cos[x]*Log[d])/(1 + Log[d]^2) + (d^x*Sin[x])/(1 + Log[d]^2)
```

Rubi [A] time = 0.0082266, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4433}

$$\frac{d^x \sin(x)}{\log^2(d)+1}+\frac{d^x \log(d)\cos(x)}{\log^2(d)+1}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*Cos[x],x]
```

```
[Out] (d^x*Cos[x]*Log[d])/(1 + Log[d]^2) + (d^x*Sin[x])/(1 + Log[d]^2)
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int d^x \cos(x)\,dx=\frac{d^x \cos(x)\log(d)}{1+\log^2(d)}+\frac{d^x \sin(x)}{1+\log^2(d)}$$

Mathematica [A] time = 0.0162946, size = 20, normalized size = 0.65

$$\frac{d^x(\log(d)\cos(x)+\sin(x))}{\log^2(d)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*Cos[x],x]
```

```
[Out] (d^x*(Cos[x]*Log[d] + Sin[x]))/(1 + Log[d]^2)
```

Maple [B] time = 0.016, size = 71, normalized size = 2.3

$$\left(\frac{\ln(d)\,e^{x\ln(d)}}{1+(\ln(d))^2}+2\,\frac{e^{x\ln(d)}\tan(x/2)}{1+(\ln(d))^2}-\frac{\ln(d)\,e^{x\ln(d)}}{1+(\ln(d))^2}\left(\tan\left(\frac{x}{2}\right)\right)^2\right)\left(\left(\tan\left(\frac{x}{2}\right)\right)^2+1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*cos(x),x)
```

```
[Out] (1/(1+ln(d)^2)*ln(d)*exp(x*ln(d))+2/(1+ln(d)^2)*exp(x*ln(d))*tan(1/2*x)-1/(
1+ln(d)^2)*ln(d)*exp(x*ln(d))*tan(1/2*x)^2)/(tan(1/2*x)^2+1)
```

Maxima [A] time = 0.958547, size = 32, normalized size = 1.03

$$\frac{d^x\cos(x)\log(d)+d^x\sin(x)}{\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*cos(x),x, algorithm="maxima")
```

```
[Out] (d^x*cos(x)*log(d) + d^x*sin(x))/(log(d)^2 + 1)
```

Fricas [A] time = 1.87864, size = 61, normalized size = 1.97

$$\frac{(\cos(x)\log(d)+\sin(x))d^x}{\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*cos(x),x, algorithm="fricas")
```

```
[Out] (cos(x)*log(d) + sin(x))*d^x/(log(d)^2 + 1)
```

Sympy [A] time = 1.08018, size = 107, normalized size = 3.45

$$\begin{cases} \frac{ixe^{-ix}\sin(x)}{2} + \frac{xe^{-ix}\cos(x)}{2} + \frac{ie^{-ix}\cos(x)}{2} & \text{for } d = e^{-i} \\ -\frac{ixe^{ix}\sin(x)}{2} + \frac{xe^{ix}\cos(x)}{2} - \frac{ie^{ix}\cos(x)}{2} & \text{for } d = e^{i} \\ \frac{d^x\log(d)\cos(x)}{\log(d)^2+1} + \frac{d^x\sin(x)}{\log(d)^2+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*cos(x),x)
```

```
[Out] Piecewise((I*x*exp(-I*x)*sin(x)/2 + x*exp(-I*x)*cos(x)/2 + I*exp(-I*x)*cos(
x)/2, Eq(d, exp(-I))), (-I*x*exp(I*x)*sin(x)/2 + x*exp(I*x)*cos(x)/2 - I*ex
p(I*x)*cos(x)/2, Eq(d, exp(I))), (d**x*log(d)*cos(x)/(log(d)**2 + 1) + d**x
*sin(x)/(log(d)**2 + 1), True))
```

Giac [C] time = 1.12309, size = 444, normalized size = 14.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*cos(x),x, algorithm="giac")
```

```
[Out] abs(d)^x*(2*cos(1/2*pi*x*sgn(d) - 1/2*pi*x + x)*log(abs(d))/((pi - pi*sgn(d
) - 2)^2 + 4*log(abs(d))^2) - (pi - pi*sgn(d) - 2)*sin(1/2*pi*x*sgn(d) - 1/
2*pi*x + x)/((pi - pi*sgn(d) - 2)^2 + 4*log(abs(d))^2)) + abs(d)^x*(2*cos(1
/2*pi*x*sgn(d) - 1/2*pi*x - x)*log(abs(d))/((pi - pi*sgn(d) + 2)^2 + 4*log(
abs(d))^2) - (pi - pi*sgn(d) + 2)*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x)/((pi
- pi*sgn(d) + 2)^2 + 4*log(abs(d))^2)) - 1/2*I*abs(d)^x*(-2*I*e^(1/2*I*pi*x
*sgn(d) - 1/2*I*pi*x + I*x)/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) + 4*I)
 + 2*I*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(2*I*pi - 2*I*pi*sgn(d) +
4*log(abs(d)) - 4*I)) - 1/2*I*abs(d)^x*(-2*I*e^(1/2*I*pi*x*sgn(d) - 1/2*I*p
i*x - I*x)/(-2*I*pi + 2*I*pi*sgn(d) + 4*log(abs(d)) - 4*I) + 2*I*e^(-1/2*I*
pi*x*sgn(d) + 1/2*I*pi*x + I*x)/(2*I*pi - 2*I*pi*sgn(d) + 4*log(abs(d)) + 4
*I))
```

3.136 $\int d^x x \sin(x)\,dx$

Optimal. Leaf size=84

$$\frac{xd^x\log(d)\sin(x)}{\log^2(d)+1}-\frac{d^x\log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{d^x\sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{xd^x\cos(x)}{\log^2(d)+1}+\frac{2d^x\log(d)\cos(x)}{\left(\log^2(d)+1\right)^2}$$

```
[Out] (2*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (d^x*x*Cos[x])/(1 + Log[d]^2) + (d
^x*Sin[x])/(1 + Log[d]^2)^2 - (d^x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^2 + (d^x
*x*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rubi [A] time = 0.04918, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4432, 4465, 4433}

$$\frac{xd^x\log(d)\sin(x)}{\log^2(d)+1}-\frac{d^x\log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{d^x\sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{xd^x\cos(x)}{\log^2(d)+1}+\frac{2d^x\log(d)\cos(x)}{\left(\log^2(d)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x*Sin[x],x]
```

```
[Out] (2*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (d^x*x*Cos[x])/(1 + Log[d]^2) + (d
^x*Sin[x])/(1 + Log[d]^2)^2 - (d^x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^2 + (d^x
*x*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int d^x x \sin(x)\,dx &= -\frac{d^x x\cos(x)}{1+\log^2(d)}+\frac{d^x x\log(d)\sin(x)}{1+\log^2(d)}-\int\left(-\frac{d^x\cos(x)}{1+\log^2(d)}+\frac{d^x\log(d)\sin(x)}{1+\log^2(d)}\right)dx\\ &= -\frac{d^x x\cos(x)}{1+\log^2(d)}+\frac{d^x x\log(d)\sin(x)}{1+\log^2(d)}+\frac{\int d^x\cos(x)\,dx}{1+\log^2(d)}-\frac{\log(d)\int d^x\sin(x)\,dx}{1+\log^2(d)}\\ &= \frac{2d^x\cos(x)\log(d)}{\left(1+\log^2(d)\right)^2}-\frac{d^x x\cos(x)}{1+\log^2(d)}+\frac{d^x\sin(x)}{\left(1+\log^2(d)\right)^2}-\frac{d^x\log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x\log(d)\sin(x)}{1+\log^2(d)}\end{aligned}$$

Mathematica [A] time = 0.0525414, size = 50, normalized size = 0.6

$$\frac{d^x\left(\sin(x)\left(x\log^3(d)+x\log(d)-\log^2(d)+1\right)-\cos(x)\left(x\log^2(d)-2\log(d)+x\right)\right)}{\left(\log^2(d)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x*Sin[x],x]
```

```
[Out] (d^x*(-(Cos[x]*(x - 2*Log[d] + x*Log[d]^2)) + (1 + x*Log[d] - Log[d]^2 + x*
Log[d]^3)*Sin[x]))/(1 + Log[d]^2)^2
```

Maple [A] time = 0.02, size = 137, normalized size = 1.6

$$\left(\frac{x e^{x\ln(d)}}{1+(\ln(d))^2}\left(\tan\left(\frac{x}{2}\right)\right)^2+2\frac{\ln(d)e^{x\ln(d)}}{\left(1+(\ln(d))^2\right)^2}-\frac{x e^{x\ln(d)}}{1+(\ln(d))^2}-2\frac{\ln(d)e^{x\ln(d)}(\tan(x/2))^2}{\left(1+(\ln(d))^2\right)^2}-2\frac{\left((\ln(d))^2-1\right)e^{x\ln(d)}\tan}{\left(1+(\ln(d))^2\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x*sin(x),x)
```

```
[Out] (1/(1+ln(d)^2)*x*exp(x*ln(d))*tan(1/2*x)^2+2/(1+ln(d)^2)^2*ln(d)*exp(x*ln(d
))-1/(1+ln(d)^2)*x*exp(x*ln(d))-2/(1+ln(d)^2)^2*ln(d)*exp(x*ln(d))*tan(1/2*
x)^2-2*(ln(d)^2-1)/(1+ln(d)^2)^2*exp(x*ln(d))*tan(1/2*x)+2/(1+ln(d)^2)*ln(d
)*x*exp(x*ln(d))*tan(1/2*x))/(tan(1/2*x)^2+1)
```

Maxima [A] time = 0.979189, size = 81, normalized size = 0.96

$$-\frac{\left(\left(\log(d)^2+1\right)x-2\log(d)\right)d^x\cos(x)-\left(\left(\log(d)^3+\log(d)\right)x-\log(d)^2+1\right)d^x\sin(x)}{\log(d)^4+2\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*sin(x),x, algorithm="maxima")
```

```
[Out] -(((log(d)^2 + 1)*x - 2*log(d))*d^x*cos(x) - ((log(d)^3 + log(d))*x - log(d
)^2 + 1)*d^x*sin(x))/(log(d)^4 + 2*log(d)^2 + 1)
```

Fricas [A] time = 1.86185, size = 177, normalized size = 2.11

$$-\frac{\left(x\cos(x)\log(d)^2+x\cos(x)-2\cos(x)\log(d)-\left(x\log(d)^3+x\log(d)-\log(d)^2+1\right)\sin(x)\right)d^x}{\log(d)^4+2\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*sin(x),x, algorithm="fricas")
```

```
[Out] -(x*cos(x)*log(d)^2 + x*cos(x) - 2*cos(x)*log(d) - (x*log(d)^3 + x*log(d) -
 log(d)^2 + 1)*sin(x))*d^x/(log(d)^4 + 2*log(d)^2 + 1)
```

Sympy [A] time = 3.50037, size = 308, normalized size = 3.67

$$\begin{cases} \frac{x^2e^{-ix}\sin(x)}{4}-\frac{ix^2e^{-ix}\cos(x)}{4}+\frac{ixe^{-ix}\sin(x)}{4}-\frac{xe^{-ix}\cos(x)}{4}+\frac{ie^{-ix}\cos(x)}{4} \\ \frac{x^2e^{ix}\sin(x)}{4}+\frac{ix^2e^{ix}\cos(x)}{4}-\frac{ixe^{ix}\sin(x)}{4}-\frac{xe^{ix}\cos(x)}{4}-\frac{ie^{ix}\cos(x)}{4} \\ \frac{d^xx\log(d)^3\sin(x)}{\log(d)^4+2\log(d)^2+1}-\frac{d^xx\log(d)^2\cos(x)}{\log(d)^4+2\log(d)^2+1}+\frac{d^xx\log(d)\sin(x)}{\log(d)^4+2\log(d)^2+1}-\frac{d^xx\cos(x)}{\log(d)^4+2\log(d)^2+1}-\frac{d^x\log(d)^2\sin(x)}{\log(d)^4+2\log(d)^2+1}+\frac{2d^x\log(d)\cos(x)}{\log(d)^4+2\log(d)^2+1}+\frac{}{\log} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x*sin(x),x)
```

```
[Out] Piecewise((x**2*exp(-I*x)*sin(x)/4 - I*x**2*exp(-I*x)*cos(x)/4 + I*x*exp(-I
*x)*sin(x)/4 - x*exp(-I*x)*cos(x)/4 + I*exp(-I*x)*cos(x)/4, Eq(d, exp(-I)))
, (x**2*exp(I*x)*sin(x)/4 + I*x**2*exp(I*x)*cos(x)/4 - I*x*exp(I*x)*sin(x)/
4 - x*exp(I*x)*cos(x)/4 - I*exp(I*x)*cos(x)/4, Eq(d, exp(I))), (d**x*x*log(
d)**3*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*x*log(d)**2*cos(x)/(log(d
)**4 + 2*log(d)**2 + 1) + d**x*x*log(d)*sin(x)/(log(d)**4 + 2*log(d)**2 + 1
) - d**x*x*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*log(d)**2*sin(x)/(lo
g(d)**4 + 2*log(d)**2 + 1) + 2*d**x*log(d)*cos(x)/(log(d)**4 + 2*log(d)**2
+ 1) + d**x*sin(x)/(log(d)**4 + 2*log(d)**2 + 1), True))
```

Giac [C] time = 1.13661, size = 1574, normalized size = 18.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*sin(x),x, algorithm="giac")
```

```
[Out] 1/2*(((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)*(pi*x
*sgn(d) - pi*x + 2*x)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*
sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^
2) - 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))*(x*log(abs(
d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2
 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^2))*cos(1/2*p
i*x*sgn(d) - 1/2*pi*x + x) + 2*((pi*x*sgn(d) - pi*x + 2*x)*(pi*log(abs(d))*
sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))/((2*pi + pi^2*sgn(d) - pi^2 + 2*lo
g(abs(d))^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)
) + 2*log(abs(d)))^2) + (2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi
*sgn(d) - 2)*(x*log(abs(d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d)
)^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*lo
g(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d)^x + 1/2*(((2*pi
- pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)*(pi*x*sgn(d) - pi
*x - 2*x)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)^
2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2) + 4*(pi*l
og(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))*(x*log(abs(d)) - 1)/((2
*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)^2 + 4*(pi*log
(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d) -
 1/2*pi*x - x) - 2*((pi*x*sgn(d) - pi*x - 2*x)*(pi*log(abs(d))*sgn(d) - pi*
```

```
log(abs(d)) - 2*log(abs(d)))/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2
- 2*pi*sgn(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(ab
s(d)))^2) - (2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)
*(x*log(abs(d)) - 1)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*s
gn(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2
))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x))*abs(d)^x - 1/2*abs(d)^x*((2*pi*x*sg
n(d) - 2*pi*x - 4*I*x*log(abs(d)) + 4*x + 4*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I
*pi*x + I*x)/(8*pi + 4*pi^2*sgn(d) + 8*I*pi*log(abs(d))*sgn(d) - 4*pi^2 - 8
*I*pi*log(abs(d)) + 8*log(abs(d))^2 - 8*pi*sgn(d) + 16*I*log(abs(d)) - 8) -
 (2*pi*x*sgn(d) - 2*pi*x + 4*I*x*log(abs(d)) + 4*x - 4*I)*e^(-1/2*I*pi*x*sg
n(d) + 1/2*I*pi*x - I*x)/(8*pi + 4*pi^2*sgn(d) - 8*I*pi*log(abs(d))*sgn(d)
- 4*pi^2 + 8*I*pi*log(abs(d)) + 8*log(abs(d))^2 - 8*pi*sgn(d) - 16*I*log(ab
s(d)) - 8)) - 1/2*abs(d)^x*((2*pi*x*sgn(d) - 2*pi*x - 4*I*x*log(abs(d)) - 4
*x + 4*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x - I*x)/(8*pi - 4*pi^2*sgn(d) -
8*I*pi*log(abs(d))*sgn(d) + 4*pi^2 + 8*I*pi*log(abs(d)) - 8*log(abs(d))^2 -
 8*pi*sgn(d) + 16*I*log(abs(d)) + 8) - (2*pi*x*sgn(d) - 2*pi*x + 4*I*x*log(
abs(d)) - 4*x - 4*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x + I*x)/(8*pi - 4*pi
^2*sgn(d) + 8*I*pi*log(abs(d))*sgn(d) + 4*pi^2 - 8*I*pi*log(abs(d)) - 8*log
(abs(d))^2 - 8*pi*sgn(d) - 16*I*log(abs(d)) + 8))
```

3.137 $\int d^x x \cos(x)\, dx$

Optimal. Leaf size=83

$$\frac{x d^x \sin(x)}{\log^2(d)+1}-\frac{2 d^x \log(d) \sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{x d^x \log(d) \cos(x)}{\log^2(d)+1}-\frac{d^x \log^2(d) \cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{d^x \cos(x)}{\left(\log^2(d)+1\right)^2}$$

```
[Out] (d^x*Cos[x])/(1 + Log[d]^2)^2 - (d^x*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^2 + (d
^x*x*Cos[x]*Log[d])/(1 + Log[d]^2) - (2*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^2
 + (d^x*x*Sin[x])/(1 + Log[d]^2)
```

Rubi [A] time = 0.0455074, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4433, 4466, 4432}

$$\frac{x d^x \sin(x)}{\log^2(d)+1}-\frac{2 d^x \log(d) \sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{x d^x \log(d) \cos(x)}{\log^2(d)+1}-\frac{d^x \log^2(d) \cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{d^x \cos(x)}{\left(\log^2(d)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x*Cos[x],x]
```

```
[Out] (d^x*Cos[x])/(1 + Log[d]^2)^2 - (d^x*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^2 + (d
^x*x*Cos[x]*Log[d])/(1 + Log[d]^2) - (2*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^2
 + (d^x*x*Sin[x])/(1 + Log[d]^2)
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int d^x x\cos(x)\,dx &= \frac{d^x x\cos(x)\log(d)}{1+\log^2(d)}+\frac{d^x x\sin(x)}{1+\log^2(d)}-\int\left(\frac{d^x\cos(x)\log(d)}{1+\log^2(d)}+\frac{d^x\sin(x)}{1+\log^2(d)}\right)dx\\ &= \frac{d^x x\cos(x)\log(d)}{1+\log^2(d)}+\frac{d^x x\sin(x)}{1+\log^2(d)}-\frac{\int d^x\sin(x)\,dx}{1+\log^2(d)}-\frac{\log(d)\int d^x\cos(x)\,dx}{1+\log^2(d)}\\ &= \frac{d^x\cos(x)}{\left(1+\log^2(d)\right)^2}-\frac{d^x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x\cos(x)\log(d)}{1+\log^2(d)}-\frac{2d^x\log(d)\sin(x)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x\sin(x)}{1+\log^2(d)}\end{aligned}$$

Mathematica [A] time = 0.0447757, size = 49, normalized size = 0.59

$$\frac{d^x\left(\sin(x)\left(x\log^2(d)-2\log(d)+x\right)+\cos(x)\left(x\log^3(d)+x\log(d)-\log^2(d)+1\right)\right)}{\left(\log^2(d)+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x*Cos[x],x]
```

```
[Out] (d^x*(Cos[x]*(1 + x*Log[d] - Log[d]^2 + x*Log[d]^3) + (x - 2*Log[d] + x*Log
[d]^2)*Sin[x]))/(1 + Log[d]^2)^2
```

Maple [A] time = 0.02, size = 142, normalized size = 1.7

$$\left(\frac{x\ln(d)\,\mathrm{e}^{x\ln(d)}}{1+(\ln(d))^2}+\frac{\left((\ln(d))^2-1\right)\mathrm{e}^{x\ln(d)}}{\left(1+(\ln(d))^2\right)^2}\left(\tan\left(\frac{x}{2}\right)\right)^2-\frac{\left((\ln(d))^2-1\right)\mathrm{e}^{x\ln(d)}}{\left(1+(\ln(d))^2\right)^2}-4\,\frac{\ln(d)\,\mathrm{e}^{x\ln(d)}\tan(x/2)}{\left(1+(\ln(d))^2\right)^2}+2\,\frac{x\mathrm{e}^{x\ln(d)}\tan(x/}{1+(\ln(d))^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x*cos(x),x)
```

```
[Out] (1/(1+ln(d)^2)*ln(d)*x*exp(x*ln(d))+(ln(d)^2-1)/(1+ln(d)^2)^2*exp(x*ln(d))*
tan(1/2*x)^2-(ln(d)^2-1)/(1+ln(d)^2)^2*exp(x*ln(d))-4/(1+ln(d)^2)^2*ln(d)*e
xp(x*ln(d))*tan(1/2*x)+2/(1+ln(d)^2)*x*exp(x*ln(d))*tan(1/2*x)-1/(1+ln(d)^2
)*ln(d)*x*exp(x*ln(d))*tan(1/2*x)^2)/(tan(1/2*x)^2+1)
```

Maxima [A] time = 0.978334, size = 78, normalized size = 0.94

$$\frac{\left(\left(\log\left(d\right)^{3}+\log\left(d\right)\right)x-\log\left(d\right)^{2}+1\right)d^{x}\cos\left(x\right)+\left(\left(\log\left(d\right)^{2}+1\right)x-2\,\log\left(d\right)\right)d^{x}\sin\left(x\right)}{\log\left(d\right)^{4}+2\,\log\left(d\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*cos(x),x, algorithm="maxima")
```

```
[Out] (((log(d)^3 + log(d))*x - log(d)^2 + 1)*d^x*cos(x) + ((log(d)^2 + 1)*x - 2*
log(d))*d^x*sin(x))/(log(d)^4 + 2*log(d)^2 + 1)
```

Fricas [A] time = 1.77531, size = 182, normalized size = 2.19

$$\frac{\left(x\cos\left(x\right)\log\left(d\right)^{3}+x\cos\left(x\right)\log\left(d\right)-\cos\left(x\right)\log\left(d\right)^{2}+\left(x\log\left(d\right)^{2}+x-2\,\log\left(d\right)\right)\sin\left(x\right)+\cos\left(x\right)\right)d^{x}}{\log\left(d\right)^{4}+2\,\log\left(d\right)^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*cos(x),x, algorithm="fricas")
```

```
[Out] (x*cos(x)*log(d)^3 + x*cos(x)*log(d) - cos(x)*log(d)^2 + (x*log(d)^2 + x -
2*log(d))*sin(x) + cos(x))*d^x/(log(d)^4 + 2*log(d)^2 + 1)
```

Sympy [A] time = 3.552, size = 304, normalized size = 3.66

$$\begin{cases} \frac{ix^{2}e^{-ix}\sin\left(x\right)}{4}+\frac{x^{2}e^{-ix}\cos\left(x\right)}{4}+\frac{xe^{-ix}\sin\left(x\right)}{4}+\frac{ixe^{-ix}\cos\left(x\right)}{4}+\frac{e^{-ix}\cos\left(x\right)}{4} \\ -\frac{ix^{2}e^{ix}\sin\left(x\right)}{4}+\frac{x^{2}e^{ix}\cos\left(x\right)}{4}+\frac{xe^{ix}\sin\left(x\right)}{4}-\frac{ixe^{ix}\cos\left(x\right)}{4}+\frac{e^{ix}\cos\left(x\right)}{4} \\ \frac{d^{x}x\log\left(d\right)^{3}\cos\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}+\frac{d^{x}x\log\left(d\right)^{2}\sin\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}+\frac{d^{x}x\log\left(d\right)\cos\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}+\frac{d^{x}x\sin\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}-\frac{d^{x}\log\left(d\right)^{2}\cos\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}-\frac{2d^{x}\log\left(d\right)\sin\left(x\right)}{\log\left(d\right)^{4}+2\log\left(d\right)^{2}+1}+ \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x*cos(x),x)
```

```
[Out] Piecewise((I*x**2*exp(-I*x)*sin(x)/4 + x**2*exp(-I*x)*cos(x)/4 + x*exp(-I*x
)*sin(x)/4 + I*x*exp(-I*x)*cos(x)/4 + exp(-I*x)*cos(x)/4, Eq(d, exp(-I))),
(-I*x**2*exp(I*x)*sin(x)/4 + x**2*exp(I*x)*cos(x)/4 + x*exp(I*x)*sin(x)/4 -
 I*x*exp(I*x)*cos(x)/4 + exp(I*x)*cos(x)/4, Eq(d, exp(I))), (d**x*x*log(d)*
*3*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) + d**x*x*log(d)**2*sin(x)/(log(d)**
4 + 2*log(d)**2 + 1) + d**x*x*log(d)*cos(x)/(log(d)**4 + 2*log(d)**2 + 1) +
 d**x*x*sin(x)/(log(d)**4 + 2*log(d)**2 + 1) - d**x*log(d)**2*cos(x)/(log(d
)**4 + 2*log(d)**2 + 1) - 2*d**x*log(d)*sin(x)/(log(d)**4 + 2*log(d)**2 + 1
) + d**x*cos(x)/(log(d)**4 + 2*log(d)**2 + 1), True))
```

Giac [C] time = 1.15571, size = 1573, normalized size = 18.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x*cos(x),x, algorithm="giac")
```

```
[Out] 1/2*(2*((pi*x*sgn(d) - pi*x + 2*x)*(pi*log(abs(d))*sgn(d) - pi*log(abs(d))
+ 2*log(abs(d)))/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d
) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^2) +
(2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)*(x*log(abs(
d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2
 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*log(abs(d)))^2))*cos(1/2*p
i*x*sgn(d) - 1/2*pi*x + x) - ((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d))^2
- 2*pi*sgn(d) - 2)*(pi*x*sgn(d) - pi*x + 2*x)/((2*pi + pi^2*sgn(d) - pi^2 +
 2*log(abs(d))^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(a
bs(d)) + 2*log(abs(d)))^2) - 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*
log(abs(d)))*(x*log(abs(d)) - 1)/((2*pi + pi^2*sgn(d) - pi^2 + 2*log(abs(d)
)^2 - 2*pi*sgn(d) - 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) + 2*lo
g(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d)^x + 1/2*(2*((pi*
x*sgn(d) - pi*x - 2*x)*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(
d)))/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)^2 + 4
*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2) - (2*pi - pi^2
*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)*(x*log(abs(d)) - 1)/((2
*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d) + 2)^2 + 4*(pi*log
(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d) -
 1/2*pi*x - x) + ((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*sgn(d
```

```
) + 2)*(pi*x*sgn(d) - pi*x - 2*x)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d
))^2 - 2*pi*sgn(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*l
og(abs(d)))^2) + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))
*(x*log(abs(d)) - 1)/((2*pi - pi^2*sgn(d) + pi^2 - 2*log(abs(d))^2 - 2*pi*s
gn(d) + 2)^2 + 4*(pi*log(abs(d))*sgn(d) - pi*log(abs(d)) - 2*log(abs(d)))^2
))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x))*abs(d)^x - 1/2*I*abs(d)^x*((2*pi*x*
sgn(d) - 2*pi*x - 4*I*x*log(abs(d)) + 4*x + 4*I)*e^(1/2*I*pi*x*sgn(d) - 1/2
*I*pi*x + I*x)/(8*pi + 4*pi^2*sgn(d) + 8*I*pi*log(abs(d))*sgn(d) - 4*pi^2 -
 8*I*pi*log(abs(d)) + 8*log(abs(d))^2 - 8*pi*sgn(d) + 16*I*log(abs(d)) - 8)
 + (2*pi*x*sgn(d) - 2*pi*x + 4*I*x*log(abs(d)) + 4*x - 4*I)*e^(-1/2*I*pi*x*
sgn(d) + 1/2*I*pi*x - I*x)/(8*pi + 4*pi^2*sgn(d) - 8*I*pi*log(abs(d))*sgn(d
) - 4*pi^2 + 8*I*pi*log(abs(d)) + 8*log(abs(d))^2 - 8*pi*sgn(d) - 16*I*log(
abs(d)) - 8)) + 1/2*I*abs(d)^x*((2*pi*x*sgn(d) - 2*pi*x - 4*I*x*log(abs(d))
 - 4*x + 4*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x - I*x)/(8*pi - 4*pi^2*sgn(d
) - 8*I*pi*log(abs(d))*sgn(d) + 4*pi^2 + 8*I*pi*log(abs(d)) - 8*log(abs(d))
^2 - 8*pi*sgn(d) + 16*I*log(abs(d)) + 8) + (2*pi*x*sgn(d) - 2*pi*x + 4*I*x*
log(abs(d)) - 4*x - 4*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x + I*x)/(8*pi -
4*pi^2*sgn(d) + 8*I*pi*log(abs(d))*sgn(d) + 4*pi^2 - 8*I*pi*log(abs(d)) - 8
*log(abs(d))^2 - 8*pi*sgn(d) - 16*I*log(abs(d)) + 8))
```

3.138 $\int d^x x^2 \sin(x)\,dx$

Optimal. Leaf size=162

$$\frac{x^2 d^x \log(d)\sin(x)}{\log^2(d)+1}-\frac{x^2 d^x \cos(x)}{\log^2(d)+1}-\frac{2xd^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{2xd^x \sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{2d^x \log^3(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{6d^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^3}+\frac{4xd^x}{(\text{lo}}$$

```
[Out] (2*d^x*Cos[x])/(1 + Log[d]^2)^3 - (6*d^x*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^3
+ (4*d^x*x*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (d^x*x^2*Cos[x])/(1 + Log[d]^2
) - (6*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^3 + (2*d^x*Log[d]^3*Sin[x])/(1 + L
og[d]^2)^3 + (2*d^x*x*Sin[x])/(1 + Log[d]^2)^2 - (2*d^x*x*Log[d]^2*Sin[x])/
(1 + Log[d]^2)^2 + (d^x*x^2*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rubi [A] time = 0.175857, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4432, 4465, 14, 4433, 4466}

$$\frac{x^2 d^x \log(d)\sin(x)}{\log^2(d)+1}-\frac{x^2 d^x \cos(x)}{\log^2(d)+1}-\frac{2xd^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{2xd^x \sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{2d^x \log^3(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{6d^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^3}+\frac{4xd^x}{(\text{lo}}$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x^2*Sin[x],x]
```

```
[Out] (2*d^x*Cos[x])/(1 + Log[d]^2)^3 - (6*d^x*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^3
+ (4*d^x*x*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (d^x*x^2*Cos[x])/(1 + Log[d]^2
) - (6*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^3 + (2*d^x*Log[d]^3*Sin[x])/(1 + L
og[d]^2)^3 + (2*d^x*x*Sin[x])/(1 + Log[d]^2)^2 - (2*d^x*x*Log[d]^2*Sin[x])/
(1 + Log[d]^2)^2 + (d^x*x^2*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
```

```
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int d^x x^2 \sin(x)\,dx &= -\frac{d^x x^2\cos(x)}{1+\log^2(d)}+\frac{d^x x^2\log(d)\sin(x)}{1+\log^2(d)}-2\int x\left(-\frac{d^x\cos(x)}{1+\log^2(d)}+\frac{d^x\log(d)\sin(x)}{1+\log^2(d)}\right)dx\\
&= -\frac{d^x x^2\cos(x)}{1+\log^2(d)}+\frac{d^x x^2\log(d)\sin(x)}{1+\log^2(d)}-2\int\left(-\frac{d^x x\cos(x)}{1+\log^2(d)}+\frac{d^x x\log(d)\sin(x)}{1+\log^2(d)}\right)dx\\
&= -\frac{d^x x^2\cos(x)}{1+\log^2(d)}+\frac{d^x x^2\log(d)\sin(x)}{1+\log^2(d)}+\frac{2\int d^x x\cos(x)\,dx}{1+\log^2(d)}-\frac{(2\log(d))\int d^x x\sin(x)\,dx}{1+\log^2(d)}\\
&= \frac{4d^x x\cos(x)\log(d)}{\left(1+\log^2(d)\right)^2}-\frac{d^x x^2\cos(x)}{1+\log^2(d)}+\frac{2d^x x\sin(x)}{\left(1+\log^2(d)\right)^2}-\frac{2d^x x\log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x^2\log(d)\sin(x)}{1+\log^2(d)}-\frac{2\int}{}\\
&= \frac{4d^x x\cos(x)\log(d)}{\left(1+\log^2(d)\right)^2}-\frac{d^x x^2\cos(x)}{1+\log^2(d)}+\frac{2d^x x\sin(x)}{\left(1+\log^2(d)\right)^2}-\frac{2d^x x\log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2}+\frac{d^x x^2\log(d)\sin(x)}{1+\log^2(d)}-\frac{2\int}{(1}\\
&= \frac{2d^x\cos(x)}{\left(1+\log^2(d)\right)^3}-\frac{2d^x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^3}+\frac{4d^x x\cos(x)\log(d)}{\left(1+\log^2(d)\right)^2}-\frac{d^x x^2\cos(x)}{1+\log^2(d)}-\frac{2d^x\log(d)\sin(x)}{\left(1+\log^2(d)\right)^3}+\frac{2d^x\,\mathrm{l}}{(1}
\end{aligned}$$

Mathematica [A] time = 0.082667, size = 94, normalized size = 0.58

$$\frac{d^x\left(\sin(x)\left(x^2\log^5(d)+2\left(x^2+1\right)\log^3(d)+\left(x^2-6\right)\log(d)-2x\log^4(d)+2x\right)-\cos(x)\left(x^2\log^4(d)+2\left(x^2+3\right)\log^2(d)-\right.\right.}{\left(\log^2(d)+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x^2*Sin[x],x]
```

```
[Out] (d^x*(-(Cos[x]*(-2 + x^2 - 4*x*Log[d] + 2*(3 + x^2)*Log[d]^2 - 4*x*Log[d]^3
 + x^2*Log[d]^4)) + (2*x + (-6 + x^2)*Log[d] + 2*(1 + x^2)*Log[d]^3 - 2*x*L
og[d]^4 + x^2*Log[d]^5)*Sin[x]))/(1 + Log[d]^2)^3
```

Maple [A] time = 0.032, size = 225, normalized size = 1.4

$$\left(\frac{x^2e^{x\ln(d)}}{1+(\ln(d))^2}\left(\tan\left(\frac{x}{2}\right)\right)^2-\frac{x^2e^{x\ln(d)}}{1+(\ln(d))^2}-2\frac{\left(3(\ln(d))^2-1\right)e^{x\ln(d)}}{\left(1+(\ln(d))^2\right)^3}+4\frac{x\ln(d)e^{x\ln(d)}}{\left(1+(\ln(d))^2\right)^2}+2\frac{\left(3(\ln(d))^2-1\right)e^{x\ln(d)}(\tan}{\left(1+(\ln(d))^2\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^2*sin(x),x)
```

```
[Out] (1/(1+ln(d)^2)*x^2*exp(x*ln(d))*tan(1/2*x)^2-1/(1+ln(d)^2)*x^2*exp(x*ln(d))
-2*(3*ln(d)^2-1)/(1+ln(d)^2)^3*exp(x*ln(d))+4/(1+ln(d)^2)^2*ln(d)*x*exp(x*l
n(d))+2*(3*ln(d)^2-1)/(1+ln(d)^2)^3*exp(x*ln(d))*tan(1/2*x)^2-4/(1+ln(d)^2)
^2*ln(d)*x*exp(x*ln(d))*tan(1/2*x)^2+2/(1+ln(d)^2)*ln(d)*x^2*exp(x*ln(d))*t
an(1/2*x)-4*(ln(d)^2-1)/(1+ln(d)^2)^2*x*exp(x*ln(d))*tan(1/2*x)+4*ln(d)*(ln
(d)^2-3)/(1+ln(d)^2)^3*exp(x*ln(d))*tan(1/2*x))/(tan(1/2*x)^2+1)
```

Maxima [A] time = 1.01808, size = 144, normalized size = 0.89

$$-\frac{\left(\left(\log(d)^4+2\log(d)^2+1\right)x^2-4\left(\log(d)^3+\log(d)\right)x+6\log(d)^2-2\right)d^x\cos(x)-\left(\left(\log(d)^5+2\log(d)^3+\log(d)\right)x^2\right.}{\log(d)^6+3\log(d)^4+3\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*sin(x),x, algorithm="maxima")
```

```
[Out] -(((log(d)^4 + 2*log(d)^2 + 1)*x^2 - 4*(log(d)^3 + log(d))*x + 6*log(d)^2 -
 2)*d^x*cos(x) - ((log(d)^5 + 2*log(d)^3 + log(d))*x^2 + 2*log(d)^3 - 2*(lo
g(d)^4 - 1)*x - 6*log(d))*d^x*sin(x))/(log(d)^6 + 3*log(d)^4 + 3*log(d)^2 +
 1)
```

Fricas [A] time = 1.84666, size = 333, normalized size = 2.06

$$-\frac{\left(x^2\cos(x)\log(d)^4 - 4\,x\cos(x)\log(d)^3 + 2\left(x^2+3\right)\cos(x)\log(d)^2 - 4\,x\cos(x)\log(d) + \left(x^2-2\right)\cos(x) - \left(x^2\log(d\right.\right.}{\log(d)^6 + 3\,\log(d)^4 + 3\,\log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^2*cos(x)*log(d)^4 - 4*x*cos(x)*log(d)^3 + 2*(x^2 + 3)*cos(x)*log(d)^2 -
 4*x*cos(x)*log(d) + (x^2 - 2)*cos(x) - (x^2*log(d)^5 - 2*x*log(d)^4 + 2*(x
^2 + 1)*log(d)^3 + (x^2 - 6)*log(d) + 2*x)*sin(x))*d^x/(log(d)^6 + 3*log(d)
^4 + 3*log(d)^2 + 1)
```

Sympy [B] time = 9.89357, size = 668, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x**2*sin(x),x)
```

```
[Out] Piecewise((x**3*exp(-I*x)*sin(x)/6 - I*x**3*exp(-I*x)*cos(x)/6 + I*x**2*exp
(-I*x)*sin(x)/4 - x**2*exp(-I*x)*cos(x)/4 + x*exp(-I*x)*sin(x)/4 + I*x*exp(
-I*x)*cos(x)/4 - I*exp(-I*x)*sin(x)/4, Eq(d, exp(-I))), (x**3*exp(I*x)*sin(
x)/6 + I*x**3*exp(I*x)*cos(x)/6 - I*x**2*exp(I*x)*sin(x)/4 - x**2*exp(I*x)*
cos(x)/4 + x*exp(I*x)*sin(x)/4 - I*x*exp(I*x)*cos(x)/4 + I*exp(I*x)*sin(x)/
4, Eq(d, exp(I))), (d**x*x**2*log(d)**5*sin(x)/(log(d)**6 + 3*log(d)**4 + 3
*log(d)**2 + 1) - d**x*x**2*log(d)**4*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*l
og(d)**2 + 1) + 2*d**x*x**2*log(d)**3*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*l
og(d)**2 + 1) - 2*d**x*x**2*log(d)**2*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*l
og(d)**2 + 1) + d**x*x**2*log(d)*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)
```

```
**2 + 1) - d**x*x**2*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 2
*d**x*x*log(d)**4*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 4*d*
*x*x*log(d)**3*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 4*d**x*
x*log(d)*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*x*sin(
x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*log(d)**3*sin(x)/(l
og(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)**2*cos(x)/(log(d)
**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)*sin(x)/(log(d)**6 + 3*
log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*l
og(d)**2 + 1), True))
```

Giac [C] time = 1.24486, size = 3552, normalized size = 21.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*(((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(ab
s(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)*(pi^2
*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d) + 2*pi*x^2 -
 2*x^2 - 4*x*log(abs(d)) + 4)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn
(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2
- 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*
log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))
^2) - 2*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^2*log(abs(d))
 - pi*x*sgn(d) + pi*x - 2*x)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)
) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(ab
s(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(a
bs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)^2 +
(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 - 6*pi*lo
g(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d
) - 1/2*pi*x + x) - (2*(3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi
^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*s
gn(d) - 2)*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^2*log(abs(
d)) - pi*x*sgn(d) + pi*x - 2*x)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*s
gn(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^
2 - 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) +
2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)
))^2) + (pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d)
 + 2*pi*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)*(3*pi^2*log(abs(d))*sgn(d) - 3*p
i^2*log(abs(d)) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(
```

d)) - 6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^
3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sg
n(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d
))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*sin(
1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d)^x + 1/2*(((3*pi - pi^3*sgn(d) + 3*p
i*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sgn(d) + 3*pi^2
 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)*(pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*l
og(abs(d))^2 + 2*pi*x^2*sgn(d) - 2*pi*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)/((
3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2
 - 3*pi^2*sgn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*l
og(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))
*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))^2) - 2*(pi*x^2*log(abs(d))*sgn(
d) - pi*x^2*log(abs(d)) - 2*x^2*log(abs(d)) - pi*x*sgn(d) + pi*x + 2*x)*(3*
pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(a
bs(d))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*
pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sgn(d) + 3*pi^
2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*p
i^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))*sgn(d) - 6*pi*log(abs(
d)) - 6*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d) - 1/2*pi*x - x) - (2*(3*pi - p
i^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2
*sgn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)*(pi*x^2*log(abs(d))*s
gn(d) - pi*x^2*log(abs(d)) - 2*x^2*log(abs(d)) - pi*x*sgn(d) + pi*x + 2*x)/
((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^
2 - 3*pi^2*sgn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2
*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d
))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))^2) + (pi^2*x^2*sgn(d) - pi^2*
x^2 + 2*x^2*log(abs(d))^2 + 2*pi*x^2*sgn(d) - 2*pi*x^2 - 2*x^2 - 4*x*log(ab
s(d)) + 4)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^
3 + 6*pi*log(abs(d))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))/((3*pi - pi
^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*
sgn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*log(abs(d)
)*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))*sgn(d) -
 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x))
*abs(d)^x + 1/2*abs(d)^x*((4*I*pi^2*x^2*sgn(d) - 8*pi*x^2*log(abs(d))*sgn(d
) - 4*I*pi^2*x^2 + 8*pi*x^2*log(abs(d)) + 8*I*x^2*log(abs(d))^2 - 8*I*pi*x^
2*sgn(d) + 8*I*pi*x^2 - 16*x^2*log(abs(d)) + 8*pi*x*sgn(d) - 8*pi*x - 8*I*x
^2 - 16*I*x*log(abs(d)) + 16*x + 16*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x +
I*x)/(24*I*pi - 8*I*pi^3*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) + 24*I*pi*log(
abs(d))^2*sgn(d) + 8*I*pi^3 - 24*pi^2*log(abs(d)) - 24*I*pi*log(abs(d))^2 +
 16*log(abs(d))^3 + 24*I*pi^2*sgn(d) - 48*pi*log(abs(d))*sgn(d) - 24*I*pi^2
 + 48*pi*log(abs(d)) + 48*I*log(abs(d))^2 - 24*I*pi*sgn(d) - 48*log(abs(d))
 - 16*I) + (4*I*pi^2*x^2*sgn(d) + 8*pi*x^2*log(abs(d))*sgn(d) - 4*I*pi^2*x^
2 - 8*pi*x^2*log(abs(d)) + 8*I*x^2*log(abs(d))^2 - 8*I*pi*x^2*sgn(d) + 8*I*
pi*x^2 + 16*x^2*log(abs(d)) - 8*pi*x*sgn(d) + 8*pi*x - 8*I*x^2 - 16*I*x*log
(abs(d)) - 16*x + 16*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(-24*I*pi

```
 + 8*I*pi^3*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) - 24*I*pi*log(abs(d))^2*sgn
(d) - 8*I*pi^3 - 24*pi^2*log(abs(d)) + 24*I*pi*log(abs(d))^2 + 16*log(abs(d
))^3 - 24*I*pi^2*sgn(d) - 48*pi*log(abs(d))*sgn(d) + 24*I*pi^2 + 48*pi*log(
abs(d)) - 48*I*log(abs(d))^2 + 24*I*pi*sgn(d) - 48*log(abs(d)) + 16*I)) + 1
/2*abs(d)^x*((-4*I*pi^2*x^2*sgn(d) + 8*pi*x^2*log(abs(d))*sgn(d) + 4*I*pi^2
*x^2 - 8*pi*x^2*log(abs(d)) - 8*I*x^2*log(abs(d))^2 - 8*I*pi*x^2*sgn(d) + 8
*I*pi*x^2 - 16*x^2*log(abs(d)) - 8*pi*x*sgn(d) + 8*pi*x + 8*I*x^2 + 16*I*x*
log(abs(d)) + 16*x - 16*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x - I*x)/(24*I*p
i - 8*I*pi^3*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) + 24*I*pi*log(abs(d))^2*sg
n(d) + 8*I*pi^3 - 24*pi^2*log(abs(d)) - 24*I*pi*log(abs(d))^2 + 16*log(abs(
d))^3 - 24*I*pi^2*sgn(d) + 48*pi*log(abs(d))*sgn(d) + 24*I*pi^2 - 48*pi*log
(abs(d)) - 48*I*log(abs(d))^2 - 24*I*pi*sgn(d) - 48*log(abs(d)) + 16*I) + (
-4*I*pi^2*x^2*sgn(d) - 8*pi*x^2*log(abs(d))*sgn(d) + 4*I*pi^2*x^2 + 8*pi*x^
2*log(abs(d)) - 8*I*x^2*log(abs(d))^2 - 8*I*pi*x^2*sgn(d) + 8*I*pi*x^2 + 16
*x^2*log(abs(d)) + 8*pi*x*sgn(d) - 8*pi*x + 8*I*x^2 + 16*I*x*log(abs(d)) -
16*x - 16*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x + I*x)/(-24*I*pi + 8*I*pi^3
*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) - 24*I*pi*log(abs(d))^2*sgn(d) - 8*I*p
i^3 - 24*pi^2*log(abs(d)) + 24*I*pi*log(abs(d))^2 + 16*log(abs(d))^3 + 24*I
*pi^2*sgn(d) + 48*pi*log(abs(d))*sgn(d) - 24*I*pi^2 - 48*pi*log(abs(d)) + 4
8*I*log(abs(d))^2 + 24*I*pi*sgn(d) - 48*log(abs(d)) - 16*I))
```

3.139 $\int d^x x^2 \cos(x)\,dx$

Optimal. Leaf size=161

$$\frac{x^2 d^x \sin(x)}{\log^2(d)+1}+\frac{x^2 d^x \log(d)\cos(x)}{\log^2(d)+1}-\frac{4xd^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{6d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{2d^x \sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{2xd^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+$$

```
[Out] (-6*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^3 + (2*d^x*Cos[x]*Log[d]^3)/(1 + Log[
d]^2)^3 + (2*d^x*x*Cos[x])/(1 + Log[d]^2)^2 - (2*d^x*x*Cos[x]*Log[d]^2)/(1
+ Log[d]^2)^2 + (d^x*x^2*Cos[x]*Log[d])/(1 + Log[d]^2) - (2*d^x*Sin[x])/(1
+ Log[d]^2)^3 + (6*d^x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^3 - (4*d^x*x*Log[d]*
Sin[x])/(1 + Log[d]^2)^2 + (d^x*x^2*Sin[x])/(1 + Log[d]^2)
```

Rubi [A] time = 0.170726, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4433, 4466, 14, 4432, 4465}

$$\frac{x^2 d^x \sin(x)}{\log^2(d)+1}+\frac{x^2 d^x \log(d)\cos(x)}{\log^2(d)+1}-\frac{4xd^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{6d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{2d^x \sin(x)}{\left(\log^2(d)+1\right)^3}-\frac{2xd^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x^2*Cos[x],x]
```


```
[Out] (-6*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^3 + (2*d^x*Cos[x]*Log[d]^3)/(1 + Log[
d]^2)^3 + (2*d^x*x*Cos[x])/(1 + Log[d]^2)^2 - (2*d^x*x*Cos[x]*Log[d]^2)/(1
+ Log[d]^2)^2 + (d^x*x^2*Cos[x]*Log[d])/(1 + Log[d]^2) - (2*d^x*Sin[x])/(1
+ Log[d]^2)^3 + (6*d^x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^3 - (4*d^x*x*Log[d]*
Sin[x])/(1 + Log[d]^2)^2 + (d^x*x^2*Sin[x])/(1 + Log[d]^2)
```


Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
```

```
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int d^x x^2 \cos(x)\,dx &= \frac{d^x x^2 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^2 \sin(x)}{1+\log^2(d)} - 2\int x\left(\frac{d^x \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x \sin(x)}{1+\log^2(d)}\right)dx \\
&= \frac{d^x x^2 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^2 \sin(x)}{1+\log^2(d)} - 2\int \left(\frac{d^x x\cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x\sin(x)}{1+\log^2(d)}\right)dx \\
&= \frac{d^x x^2 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^2 \sin(x)}{1+\log^2(d)} - \frac{2\int d^x x\sin(x)\,dx}{1+\log^2(d)} - \frac{(2\log(d))\int d^x x\cos(x)\,dx}{1+\log^2(d)} \\
&= \frac{2d^x x\cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{2d^x x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^2\cos(x)\log(d)}{1+\log^2(d)} - \frac{4d^x x\log(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^2\sin(x)}{1+\log^2(d)} + \frac{2\int \cdots}{\cdots} \\
&= \frac{2d^x x\cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{2d^x x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^2\cos(x)\log(d)}{1+\log^2(d)} - \frac{4d^x x\log(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^2\sin(x)}{1+\log^2(d)} - \frac{2\int \cdots}{(1 \cdots} \\
&= -\frac{2d^x\cos(x)\log(d)}{\left(1+\log^2(d)\right)^3} + \frac{2d^x\cos(x)\log^3(d)}{\left(1+\log^2(d)\right)^3} + \frac{2d^x x\cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{2d^x x\cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^2\cos(x)\log(d \cdots}{1+\log^2(d)}
\end{aligned}$$

Mathematica [A] time = 0.073773, size = 93, normalized size = 0.58

$$\frac{d^x\left(\sin(x)\left(x^2\log^4(d)+2\left(x^2+3\right)\log^2(d)-4x\log^3(d)-4x\log(d)+x^2-2\right)+\cos(x)\left(x^2\log^5(d)+2\left(x^2+1\right)\log^3(d)+\right.}{\left(\log^2(d)+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x^2*Cos[x],x]
```

```
[Out] (d^x*(Cos[x]*(2*x + (-6 + x^2)*Log[d] + 2*(1 + x^2)*Log[d]^3 - 2*x*Log[d]^4
 + x^2*Log[d]^5) + (-2 + x^2 - 4*x*Log[d] + 2*(3 + x^2)*Log[d]^2 - 4*x*Log[
d]^3 + x^2*Log[d]^4)*Sin[x]))/(1 + Log[d]^2)^3
```

Maple [A] time = 0.028, size = 231, normalized size = 1.4

$$\left(\frac{\ln(d)\,x^2e^{x\ln(d)}}{1+(\ln(d))^2}+2\frac{x^2e^{x\ln(d)}\tan(x/2)}{1+(\ln(d))^2}-2\frac{\left((\ln(d))^2-1\right)xe^{x\ln(d)}}{\left(1+(\ln(d))^2\right)^2}+4\frac{\left(3\,(\ln(d))^2-1\right)e^{x\ln(d)}\tan(x/2)}{\left(1+(\ln(d))^2\right)^3}+2\frac{\ln(d)\left((\ln(\right.}{\left(1+(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^2*cos(x),x)
```

```
[Out] (1/(1+ln(d)^2)*ln(d)*x^2*exp(x*ln(d))+2/(1+ln(d)^2)*x^2*exp(x*ln(d))*tan(1/
2*x)-2*(ln(d)^2-1)/(1+ln(d)^2)^2*x*exp(x*ln(d))+4*(3*ln(d)^2-1)/(1+ln(d)^2)
^3*exp(x*ln(d))*tan(1/2*x)+2*ln(d)*(ln(d)^2-3)/(1+ln(d)^2)^3*exp(x*ln(d))-8
/(1+ln(d)^2)^2*ln(d)*x*exp(x*ln(d))*tan(1/2*x)-1/(1+ln(d)^2)*ln(d)*x^2*exp(
x*ln(d))*tan(1/2*x)^2+2*(ln(d)^2-1)/(1+ln(d)^2)^2*x*exp(x*ln(d))*tan(1/2*x)
^2-2*ln(d)*(ln(d)^2-3)/(1+ln(d)^2)^3*exp(x*ln(d))*tan(1/2*x)^2)/(tan(1/2*x)
^2+1)
```

Maxima [A] time = 1.0198, size = 142, normalized size = 0.88

$$\frac{\left(\left(\log(d)^5+2\,\log(d)^3+\log(d)\right)x^2+2\,\log(d)^3-2\left(\log(d)^4-1\right)x-6\,\log(d)\right)d^x\cos(x)+\left(\left(\log(d)^4+2\,\log(d)^2+1\right)x\right.}{\log(d)^6+3\,\log(d)^4+3\,\log(d)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*cos(x),x, algorithm="maxima")
```

```
[Out] (((log(d)^5 + 2*log(d)^3 + log(d))*x^2 + 2*log(d)^3 - 2*(log(d)^4 - 1)*x -
6*log(d))*d^x*cos(x) + ((log(d)^4 + 2*log(d)^2 + 1)*x^2 - 4*(log(d)^3 + log
(d))*x + 6*log(d)^2 - 2)*d^x*sin(x))/(log(d)^6 + 3*log(d)^4 + 3*log(d)^2 +
1)
```

Fricas [A] time = 1.78349, size = 329, normalized size = 2.04

$$\frac{\left(x^2\cos(x)\log(d)^5 - 2\,x\cos(x)\log(d)^4 + 2\left(x^2+1\right)\cos(x)\log(d)^3 + \left(x^2-6\right)\cos(x)\log(d) + 2\,x\cos(x) + \left(x^2\log(d)^4 \cdots\right.}{\log(d)^6 + 3\,\log(d)^4 + 3\,\log(d)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*cos(x),x, algorithm="fricas")
```

```
[Out] (x^2*cos(x)*log(d)^5 - 2*x*cos(x)*log(d)^4 + 2*(x^2 + 1)*cos(x)*log(d)^3 +
(x^2 - 6)*cos(x)*log(d) + 2*x*cos(x) + (x^2*log(d)^4 - 4*x*log(d)^3 + 2*(x^
2 + 3)*log(d)^2 + x^2 - 4*x*log(d) - 2)*sin(x))*d^x/(log(d)^6 + 3*log(d)^4
+ 3*log(d)^2 + 1)
```

Sympy [B] time = 9.79313, size = 665, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x**2*cos(x),x)
```

```
[Out] Piecewise((I*x**3*exp(-I*x)*sin(x)/6 + x**3*exp(-I*x)*cos(x)/6 + x**2*exp(-
I*x)*sin(x)/4 + I*x**2*exp(-I*x)*cos(x)/4 - I*x*exp(-I*x)*sin(x)/4 + x*exp(
-I*x)*cos(x)/4 - exp(-I*x)*sin(x)/4, Eq(d, exp(-I))), (-I*x**3*exp(I*x)*sin
(x)/6 + x**3*exp(I*x)*cos(x)/6 + x**2*exp(I*x)*sin(x)/4 - I*x**2*exp(I*x)*c
os(x)/4 + I*x*exp(I*x)*sin(x)/4 + x*exp(I*x)*cos(x)/4 - exp(I*x)*sin(x)/4,
Eq(d, exp(I))), (d**x*x**2*log(d)**5*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*lo
g(d)**2 + 1) + d**x*x**2*log(d)**4*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(
d)**2 + 1) + 2*d**x*x**2*log(d)**3*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(
d)**2 + 1) + 2*d**x*x**2*log(d)**2*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(
d)**2 + 1) + d**x*x**2*log(d)*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2
```

```
 + 1) + d**x*x**2*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 2*d*
*x*x*log(d)**4*cos(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 4*d**x*
x*log(d)**3*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) - 4*d**x*x*l
og(d)*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*x*cos(x)/
(log(d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 2*d**x*log(d)**3*cos(x)/(log(
d)**6 + 3*log(d)**4 + 3*log(d)**2 + 1) + 6*d**x*log(d)**2*sin(x)/(log(d)**6
 + 3*log(d)**4 + 3*log(d)**2 + 1) - 6*d**x*log(d)*cos(x)/(log(d)**6 + 3*log
(d)**4 + 3*log(d)**2 + 1) - 2*d**x*sin(x)/(log(d)**6 + 3*log(d)**4 + 3*log(
d)**2 + 1), True))
```

Giac [C] time = 1.20871, size = 3552, normalized size = 22.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^2*cos(x),x, algorithm="giac")
```

```
[Out] 1/2*((2*(3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(a
bs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)*(pi*
x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^2*log(abs(d)) - pi*x*sgn(
d) + pi*x - 2*x)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 -
3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d)
 - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3
 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2) + (pi^2*x
^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d) + 2*pi*x^2 - 2
*x^2 - 4*x*log(abs(d)) + 4)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d))
 + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs
(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(ab
s(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(d) - 2)^2 + (
3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 - 6*pi*log
(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d)
 - 1/2*pi*x + x) + ((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3
- 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn(
d) - 2)*(pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log(abs(d))^2 - 2*pi*x^2*sgn(d)
 + 2*pi*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)/((3*pi - pi^3*sgn(d) + 3*pi*log(
abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*l
og(abs(d))^2 - 3*pi*sgn(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log
(abs(d)) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6
*log(abs(d)))^2) - 2*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*log(abs(d)) + 2*x^
2*log(abs(d)) - pi*x*sgn(d) + pi*x - 2*x)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi
^2*log(abs(d)) + 2*log(abs(d))^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d
```

```
)) - 6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3
 - 3*pi*log(abs(d))^2 + 3*pi^2*sgn(d) - 3*pi^2 + 6*log(abs(d))^2 - 3*pi*sgn
(d) - 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d)
)^3 - 6*pi*log(abs(d))*sgn(d) + 6*pi*log(abs(d)) - 6*log(abs(d)))^2))*sin(1
/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d)^x + 1/2*((2*(3*pi - pi^3*sgn(d) + 3*
pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sgn(d) + 3*pi^
2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)*(pi*x^2*log(abs(d))*sgn(d) - pi*x^2*
log(abs(d)) - 2*x^2*log(abs(d)) - pi*x*sgn(d) + pi*x + 2*x)/((3*pi - pi^3*s
gn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sgn(
d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*log(abs(d))*sg
n(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))*sgn(d) - 6*p
i*log(abs(d)) - 6*log(abs(d)))^2) + (pi^2*x^2*sgn(d) - pi^2*x^2 + 2*x^2*log
(abs(d))^2 + 2*pi*x^2*sgn(d) - 2*pi*x^2 - 2*x^2 - 4*x*log(abs(d)) + 4)*(3*p
i^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(ab
s(d))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))/((3*pi - pi^3*sgn(d) + 3*p
i*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sgn(d) + 3*pi^2
 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi
^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))*sgn(d) - 6*pi*log(abs(d
)) - 6*log(abs(d)))^2))*cos(1/2*pi*x*sgn(d) - 1/2*pi*x - x) + ((3*pi - pi^3
*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*sg
n(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)*(pi^2*x^2*sgn(d) - pi^2*
x^2 + 2*x^2*log(abs(d))^2 + 2*pi*x^2*sgn(d) - 2*pi*x^2 - 2*x^2 - 4*x*log(ab
s(d)) + 4)/((3*pi - pi^3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*l
og(abs(d))^2 - 3*pi^2*sgn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^
2 + (3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*p
i*log(abs(d))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))^2) - 2*(pi*x^2*log
(abs(d))*sgn(d) - pi*x^2*log(abs(d)) - 2*x^2*log(abs(d)) - pi*x*sgn(d) + pi
*x + 2*x)*(3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3
 + 6*pi*log(abs(d))*sgn(d) - 6*pi*log(abs(d)) - 6*log(abs(d)))/((3*pi - pi^
3*sgn(d) + 3*pi*log(abs(d))^2*sgn(d) + pi^3 - 3*pi*log(abs(d))^2 - 3*pi^2*s
gn(d) + 3*pi^2 - 6*log(abs(d))^2 - 3*pi*sgn(d) + 2)^2 + (3*pi^2*log(abs(d))
*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 6*pi*log(abs(d))*sgn(d) -
6*pi*log(abs(d)) - 6*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x))*
abs(d)^x - 1/2*I*abs(d)^x*((-4*I*pi^2*x^2*sgn(d) + 8*pi*x^2*log(abs(d))*sgn
(d) + 4*I*pi^2*x^2 - 8*pi*x^2*log(abs(d)) - 8*I*x^2*log(abs(d))^2 + 8*I*pi*
x^2*sgn(d) - 8*I*pi*x^2 + 16*x^2*log(abs(d)) - 8*pi*x*sgn(d) + 8*pi*x + 8*I
*x^2 + 16*I*x*log(abs(d)) - 16*x - 16*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x
+ I*x)/(24*I*pi - 8*I*pi^3*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) + 24*I*pi*lo
g(abs(d))^2*sgn(d) + 8*I*pi^3 - 24*pi^2*log(abs(d)) - 24*I*pi*log(abs(d))^2
 + 16*log(abs(d))^3 + 24*I*pi^2*sgn(d) - 48*pi*log(abs(d))*sgn(d) - 24*I*pi
^2 + 48*pi*log(abs(d)) + 48*I*log(abs(d))^2 - 24*I*pi*sgn(d) - 48*log(abs(d
)) - 16*I) - (-4*I*pi^2*x^2*sgn(d) - 8*pi*x^2*log(abs(d))*sgn(d) + 4*I*pi^2
*x^2 + 8*pi*x^2*log(abs(d)) - 8*I*x^2*log(abs(d))^2 + 8*I*pi*x^2*sgn(d) - 8
*I*pi*x^2 - 16*x^2*log(abs(d)) + 8*pi*x*sgn(d) - 8*pi*x + 8*I*x^2 + 16*I*x*
log(abs(d)) + 16*x - 16*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(-24*I
```

```
*pi + 8*I*pi^3*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) - 24*I*pi*log(abs(d))^2*
sgn(d) - 8*I*pi^3 - 24*pi^2*log(abs(d)) + 24*I*pi*log(abs(d))^2 + 16*log(ab
s(d))^3 - 24*I*pi^2*sgn(d) - 48*pi*log(abs(d))*sgn(d) + 24*I*pi^2 + 48*pi*l
og(abs(d)) - 48*I*log(abs(d))^2 + 24*I*pi*sgn(d) - 48*log(abs(d)) + 16*I))
- 1/2*I*abs(d)^x*((-4*I*pi^2*x^2*sgn(d) + 8*pi*x^2*log(abs(d))*sgn(d) + 4*I
*pi^2*x^2 - 8*pi*x^2*log(abs(d)) - 8*I*x^2*log(abs(d))^2 - 8*I*pi*x^2*sgn(d
) + 8*I*pi*x^2 - 16*x^2*log(abs(d)) - 8*pi*x*sgn(d) + 8*pi*x + 8*I*x^2 + 16
*I*x*log(abs(d)) + 16*x - 16*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x - I*x)/(2
4*I*pi - 8*I*pi^3*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) + 24*I*pi*log(abs(d))
^2*sgn(d) + 8*I*pi^3 - 24*pi^2*log(abs(d)) - 24*I*pi*log(abs(d))^2 + 16*log
(abs(d))^3 - 24*I*pi^2*sgn(d) + 48*pi*log(abs(d))*sgn(d) + 24*I*pi^2 - 48*p
i*log(abs(d)) - 48*I*log(abs(d))^2 - 24*I*pi*sgn(d) - 48*log(abs(d)) + 16*I
) - (-4*I*pi^2*x^2*sgn(d) - 8*pi*x^2*log(abs(d))*sgn(d) + 4*I*pi^2*x^2 + 8*
pi*x^2*log(abs(d)) - 8*I*x^2*log(abs(d))^2 - 8*I*pi*x^2*sgn(d) + 8*I*pi*x^2
 + 16*x^2*log(abs(d)) + 8*pi*x*sgn(d) - 8*pi*x + 8*I*x^2 + 16*I*x*log(abs(d
)) - 16*x - 16*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x + I*x)/(-24*I*pi + 8*I
*pi^3*sgn(d) + 24*pi^2*log(abs(d))*sgn(d) - 24*I*pi*log(abs(d))^2*sgn(d) -
8*I*pi^3 - 24*pi^2*log(abs(d)) + 24*I*pi*log(abs(d))^2 + 16*log(abs(d))^3 +
 24*I*pi^2*sgn(d) + 48*pi*log(abs(d))*sgn(d) - 24*I*pi^2 - 48*pi*log(abs(d)
) + 48*I*log(abs(d))^2 + 24*I*pi*sgn(d) - 48*log(abs(d)) - 16*I))
```

3.140 $\int d^x x^3 \sin(x)\,dx$

Optimal. Leaf size=261

$$\frac{x^3 d^x \log(d)\sin(x)}{\log^2(d)+1}+\frac{3x^2 d^x \sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{3x^2 d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{x^3 d^x \cos(x)}{\log^2(d)+1}+\frac{6x^2 d^x \log(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{6x d^x \log^3(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-$$

```
[Out] (-24*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^4 + (24*d^x*Cos[x]*Log[d]^3)/(1 + Lo
g[d]^2)^4 + (6*d^x*x*Cos[x])/(1 + Log[d]^2)^3 - (18*d^x*x*Cos[x]*Log[d]^2)/
(1 + Log[d]^2)^3 + (6*d^x*x^2*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (d^x*x^3*Co
s[x])/(1 + Log[d]^2) - (6*d^x*Sin[x])/(1 + Log[d]^2)^4 + (36*d^x*Log[d]^2*S
in[x])/(1 + Log[d]^2)^4 - (6*d^x*Log[d]^4*Sin[x])/(1 + Log[d]^2)^4 - (18*d^
x*x*Log[d]*Sin[x])/(1 + Log[d]^2)^3 + (6*d^x*x*Log[d]^3*Sin[x])/(1 + Log[d]
^2)^3 + (3*d^x*x^2*Sin[x])/(1 + Log[d]^2)^2 - (3*d^x*x^2*Log[d]^2*Sin[x])/(
1 + Log[d]^2)^2 + (d^x*x^3*Log[d]*Sin[x])/(1 + Log[d]^2)
```

Rubi [A] time = 0.434687, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4432, 4465, 14, 4433, 4466}

$$\frac{x^3 d^x \log(d)\sin(x)}{\log^2(d)+1}+\frac{3x^2 d^x \sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{3x^2 d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^2}-\frac{x^3 d^x \cos(x)}{\log^2(d)+1}+\frac{6x^2 d^x \log(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{6x d^x \log^3(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x^3*Sin[x],x]
```


```
[Out] (-24*d^x*Cos[x]*Log[d])/(1 + Log[d]^2)^4 + (24*d^x*Cos[x]*Log[d]^3)/(1 + Lo
g[d]^2)^4 + (6*d^x*x*Cos[x])/(1 + Log[d]^2)^3 - (18*d^x*x*Cos[x]*Log[d]^2)/
(1 + Log[d]^2)^3 + (6*d^x*x^2*Cos[x]*Log[d])/(1 + Log[d]^2)^2 - (d^x*x^3*Co
s[x])/(1 + Log[d]^2) - (6*d^x*Sin[x])/(1 + Log[d]^2)^4 + (36*d^x*Log[d]^2*S
in[x])/(1 + Log[d]^2)^4 - (6*d^x*Log[d]^4*Sin[x])/(1 + Log[d]^2)^4 - (18*d^
x*x*Log[d]*Sin[x])/(1 + Log[d]^2)^3 + (6*d^x*x*Log[d]^3*Sin[x])/(1 + Log[d]
^2)^3 + (3*d^x*x^2*Sin[x])/(1 + Log[d]^2)^2 - (3*d^x*x^2*Log[d]^2*Sin[x])/(
1 + Log[d]^2)^2 + (d^x*x^3*Log[d]*Sin[x])/(1 + Log[d]^2)
```


Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
```

```
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\int d^x x^3 \sin(x)\,dx = -\frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{d^x x^3 \log(d)\sin(x)}{1+\log^2(d)} - 3\int x^2\left(-\frac{d^x \cos(x)}{1+\log^2(d)} + \frac{d^x \log(d)\sin(x)}{1+\log^2(d)}\right)dx$$

$$= -\frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{d^x x^3 \log(d)\sin(x)}{1+\log^2(d)} - 3\int\left(-\frac{d^x x^2 \cos(x)}{1+\log^2(d)} + \frac{d^x x^2 \log(d)\sin(x)}{1+\log^2(d)}\right)dx$$

$$= -\frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{d^x x^3 \log(d)\sin(x)}{1+\log^2(d)} + \frac{3\int d^x x^2 \cos(x)\,dx}{1+\log^2(d)} - \frac{(3\log(d))\int d^x x^2 \sin(x)\,dx}{1+\log^2(d)}$$

$$= \frac{6d^x x^2 \cos(x)\log(d)}{\left(1+\log^2(d)\right)^2} - \frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{3d^x x^2 \sin(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \log(d)\sin(x)}{1+\log^2(d)} - \cdots$$

$$= \frac{6d^x x^2 \cos(x)\log(d)}{\left(1+\log^2(d)\right)^2} - \frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{3d^x x^2 \sin(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \log(d)\sin(x)}{1+\log^2(d)} - \cdots$$

$$= \frac{6d^x x^2 \cos(x)\log(d)}{\left(1+\log^2(d)\right)^2} - \frac{d^x x^3 \cos(x)}{1+\log^2(d)} + \frac{3d^x x^2 \sin(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \log^2(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \log(d)\sin(x)}{1+\log^2(d)} - \cdots$$

$$= \frac{6d^x x \cos(x)}{\left(1+\log^2(d)\right)^3} - \frac{6d^x x \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^3} + \frac{6d^x x^2 \cos(x)\log(d)}{\left(1+\log^2(d)\right)^2} - \frac{d^x x^3 \cos(x)}{1+\log^2(d)} - \frac{6d^x x \log(d)\sin(x)}{\left(1+\log^2(d)\right)^3} + \cdots$$

$$= \frac{6d^x x \cos(x)}{\left(1+\log^2(d)\right)^3} - \frac{6d^x x \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^3} + \frac{6d^x x^2 \cos(x)\log(d)}{\left(1+\log^2(d)\right)^2} - \frac{d^x x^3 \cos(x)}{1+\log^2(d)} - \frac{6d^x x \log(d)\sin(x)}{\left(1+\log^2(d)\right)^3} + \cdots$$

$$= -\frac{12d^x \cos(x)\log(d)}{\left(1+\log^2(d)\right)^4} + \frac{12d^x \cos(x)\log^3(d)}{\left(1+\log^2(d)\right)^4} + \frac{6d^x x \cos(x)}{\left(1+\log^2(d)\right)^3} - \frac{6d^x x \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^3} + \frac{6d^x x^2 \cos(x)\log\cdots}{\left(1+\log^2(d)\right)\cdots}$$

Mathematica [A] time = 0.146113, size = 169, normalized size = 0.65

$$\frac{d^x\left(\sin(x)\left(x^3\log^7(d) - 3x^2\log^6(d) + 3x\left(x^2+2\right)\log^5(d) - 3\left(x^2+2\right)\log^4(d) + 3x\left(x^2-4\right)\log^3(d) + 3\left(x^2+12\right)\log^2(d) \cdots\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x^3*Sin[x],x]
```

```
[Out] (d^x*(-(Cos[x]*(x*(-6 + x^2) - 6*(-4 + x^2)*Log[d] + 3*x*(4 + x^2)*Log[d]^2
 - 12*(2 + x^2)*Log[d]^3 + 3*x*(6 + x^2)*Log[d]^4 - 6*x^2*Log[d]^5 + x^3*Lo
g[d]^6)) + (3*(-2 + x^2) + x*(-18 + x^2)*Log[d] + 3*(12 + x^2)*Log[d]^2 + 3
```

```
*x*(-4 + x^2)*Log[d]^3 - 3*(2 + x^2)*Log[d]^4 + 3*x*(2 + x^2)*Log[d]^5 - 3*
x^2*Log[d]^6 + x^3*Log[d]^7)*Sin[x]))/(1 + Log[d]^2)^4
```

Maple [A] time = 0.039, size = 431, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^3*sin(x),x)
```

```
[Out] (1/(1+ln(d)^2)*x^3*exp(x*ln(d))*tan(1/2*x)^2-1/(1+ln(d)^2)*x^3*exp(x*ln(d))
+6*ln(d)/(ln(d)^4+2*ln(d)^2+1)*x^2*exp(x*ln(d))+2/(1+ln(d)^2)*ln(d)*x^3*exp
(x*ln(d))*tan(1/2*x)-6*(ln(d)^2-1)/(ln(d)^4+2*ln(d)^2+1)*x^2*exp(x*ln(d))*t
an(1/2*x)-6*(3*ln(d)^2-1)/(1+ln(d)^2)/(ln(d)^4+2*ln(d)^2+1)*x*exp(x*ln(d))-
12*(ln(d)^4-6*ln(d)^2+1)/(ln(d)^4+2*ln(d)^2+1)/(1+ln(d)^2)^2*exp(x*ln(d))*t
an(1/2*x)+24/(ln(d)^6+3*ln(d)^4+3*ln(d)^2+1)*ln(d)*(ln(d)^2-1)/(1+ln(d)^2)*
exp(x*ln(d))-6*ln(d)/(ln(d)^4+2*ln(d)^2+1)*x^2*exp(x*ln(d))*tan(1/2*x)^2+6*
(3*ln(d)^2-1)/(1+ln(d)^2)/(ln(d)^4+2*ln(d)^2+1)*x*exp(x*ln(d))*tan(1/2*x)^2
-24/(ln(d)^6+3*ln(d)^4+3*ln(d)^2+1)*ln(d)*(ln(d)^2-1)/(1+ln(d)^2)*exp(x*ln(
d))*tan(1/2*x)^2+12*ln(d)*(ln(d)^2-3)/(1+ln(d)^2)/(ln(d)^4+2*ln(d)^2+1)*x*e
xp(x*ln(d))*tan(1/2*x))/(tan(1/2*x)^2+1)
```

Maxima [A] time = 1.08774, size = 251, normalized size = 0.96

$$-\frac{\left(\left(\log\left(d\right)^{6}+3\,\log\left(d\right)^{4}+3\,\log\left(d\right)^{2}+1\right)x^{3}-6\left(\log\left(d\right)^{5}+2\,\log\left(d\right)^{3}+\log\left(d\right)\right)x^{2}-24\,\log\left(d\right)^{3}+6\left(3\,\log\left(d\right)^{4}+2\,\log\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*sin(x),x, algorithm="maxima")
```

```
[Out] -(((log(d)^6 + 3*log(d)^4 + 3*log(d)^2 + 1)*x^3 - 6*(log(d)^5 + 2*log(d)^3
 + log(d))*x^2 - 24*log(d)^3 + 6*(3*log(d)^4 + 2*log(d)^2 - 1)*x + 24*log(d)
)*d^x*cos(x) - ((log(d)^7 + 3*log(d)^5 + 3*log(d)^3 + log(d))*x^3 - 6*log(d
)^4 - 3*(log(d)^6 + log(d)^4 - log(d)^2 - 1)*x^2 + 6*(log(d)^5 - 2*log(d)^3
 - 3*log(d))*x + 36*log(d)^2 - 6)*d^x*sin(x))/(log(d)^8 + 4*log(d)^6 + 6*lo
g(d)^4 + 4*log(d)^2 + 1)
```

Fricas [A] time = 1.88263, size = 570, normalized size = 2.18

$$-\frac{\left(x^3\cos(x)\log(d)^6 - 6x^2\cos(x)\log(d)^5 + 3\left(x^3+6x\right)\cos(x)\log(d)^4 - 12\left(x^2+2\right)\cos(x)\log(d)^3 + 3\left(x^3+4x\right)\cos(x)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^3*cos(x)*log(d)^6 - 6*x^2*cos(x)*log(d)^5 + 3*(x^3 + 6*x)*cos(x)*log(d)
^4 - 12*(x^2 + 2)*cos(x)*log(d)^3 + 3*(x^3 + 4*x)*cos(x)*log(d)^2 - 6*(x^2
- 4)*cos(x)*log(d) + (x^3 - 6*x)*cos(x) - (x^3*log(d)^7 - 3*x^2*log(d)^6 +
3*(x^3 + 2*x)*log(d)^5 - 3*(x^2 + 2)*log(d)^4 + 3*(x^3 - 4*x)*log(d)^3 + 3*
(x^2 + 12)*log(d)^2 + 3*x^2 + (x^3 - 18*x)*log(d) - 6)*sin(x))*d^x/(log(d)^
8 + 4*log(d)^6 + 6*log(d)^4 + 4*log(d)^2 + 1)
```

Sympy [B] time = 27.7074, size = 1352, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x**3*sin(x),x)
```

```
[Out] Piecewise((x**4*exp(-I*x)*sin(x)/8 - I*x**4*exp(-I*x)*cos(x)/8 + I*x**3*exp
(-I*x)*sin(x)/4 - x**3*exp(-I*x)*cos(x)/4 + 3*x**2*exp(-I*x)*sin(x)/8 + 3*I
*x**2*exp(-I*x)*cos(x)/8 - 3*I*x*exp(-I*x)*sin(x)/8 + 3*x*exp(-I*x)*cos(x)/
8 - 3*exp(-I*x)*sin(x)/8, Eq(d, exp(-I))), (x**4*exp(I*x)*sin(x)/8 + I*x**4
*exp(I*x)*cos(x)/8 - I*x**3*exp(I*x)*sin(x)/4 - x**3*exp(I*x)*cos(x)/4 + 3*
x**2*exp(I*x)*sin(x)/8 - 3*I*x**2*exp(I*x)*cos(x)/8 + 3*I*x*exp(I*x)*sin(x)
/8 + 3*x*exp(I*x)*cos(x)/8 - 3*exp(I*x)*sin(x)/8, Eq(d, exp(I))), (d**x*x**
3*log(d)**7*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1
) - d**x*x**3*log(d)**6*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*l
og(d)**2 + 1) + 3*d**x*x**3*log(d)**5*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*l
og(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**3*log(d)**4*cos(x)/(log(d)**8 + 4*l
og(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**3*sin(x)/(l
og(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**3*log(d
)**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + d**
x*x**3*log(d)*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 +
 1) - d**x*x**3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2
```

```
 + 1) - 3*d**x*x**2*log(d)**6*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4
 + 4*log(d)**2 + 1) + 6*d**x*x**2*log(d)**5*cos(x)/(log(d)**8 + 4*log(d)**6
 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**4*sin(x)/(log(d)**8
 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 12*d**x*x**2*log(d)**3*co
s(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**
2*log(d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1
) + 6*d**x*x**2*log(d)*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*lo
g(d)**2 + 1) + 3*d**x*x**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 +
4*log(d)**2 + 1) + 6*d**x*x*log(d)**5*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*l
og(d)**4 + 4*log(d)**2 + 1) - 18*d**x*x*log(d)**4*cos(x)/(log(d)**8 + 4*log
(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x*log(d)**3*sin(x)/(log(d
)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x*log(d)**2*c
os(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 18*d**x*x
*log(d)*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) +
6*d**x*x*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) -
 6*d**x*log(d)**4*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)*
*2 + 1) + 24*d**x*log(d)**3*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 +
 4*log(d)**2 + 1) + 36*d**x*log(d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*l
og(d)**4 + 4*log(d)**2 + 1) - 24*d**x*log(d)*cos(x)/(log(d)**8 + 4*log(d)**
6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*sin(x)/(log(d)**8 + 4*log(d)**6
 + 6*log(d)**4 + 4*log(d)**2 + 1), True))
```

Giac [C] time = 1.27547, size = 6857, normalized size = 26.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*sin(x),x, algorithm="giac")
```

```
[Out] 1/2*(((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log
(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) +
 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 -
 4*pi*sgn(d) - 2)*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(abs(d))^2*sgn(d) - pi^3*x
^3 + 3*pi*x^3*log(abs(d))^2 - 3*pi^2*x^3*sgn(d) + 3*pi^2*x^3 - 6*x^3*log(ab
s(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*pi*x^3 - 6*pi*x
^2*log(abs(d)) + 2*x^3 + 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x - 12*x
)/((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log(ab
s(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*
pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*
pi*sgn(d) - 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) -
pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*lo
```

g(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) +
2*log(abs(d)))^2) + 4*(3*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi^2*x^3*log(abs(d
)) + 2*x^3*log(abs(d))^3 - 6*pi*x^3*log(abs(d))*sgn(d) + 6*pi*x^3*log(abs(d
)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*log(abs(d)) - 6*x^2*log(abs(d))
^2 + 6*pi*x^2*sgn(d) - 6*pi*x^2 + 6*x^2 + 12*x*log(abs(d)) - 12)*(pi^3*log(
abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d)
)^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*
pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))/((4*pi + pi^4*sgn
(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log(abs
(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(ab
s(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)^2 +
16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) +
pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*log(a
bs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))^2))*
cos(1/2*pi*x*sgn(d) - 1/2*pi*x + x) - ((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs
(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(
d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn
(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)*(3*pi^2*x^3*log(abs(d))*
sgn(d) - 3*pi^2*x^3*log(abs(d)) + 2*x^3*log(abs(d))^3 - 6*pi*x^3*log(abs(d)
)*sgn(d) + 6*pi*x^3*log(abs(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*lo
g(abs(d)) - 6*x^2*log(abs(d))^2 + 6*pi*x^2*sgn(d) - 6*pi*x^2 + 6*x^2 + 12*x
*log(abs(d)) - 12)/((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^
4 + 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(
d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*l
og(abs(d))^2 - 4*pi*sgn(d) - 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(ab
s(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*s
gn(d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*
pi*log(abs(d)) + 2*log(abs(d)))^2) - 4*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(abs(
d))^2*sgn(d) - pi^3*x^3 + 3*pi*x^3*log(abs(d))^2 - 3*pi^2*x^3*sgn(d) + 3*pi
^2*x^3 - 6*x^3*log(abs(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d
) - 3*pi*x^3 - 6*pi*x^2*log(abs(d)) + 2*x^3 + 12*x^2*log(abs(d)) - 6*pi*x*s
gn(d) + 6*pi*x - 12*x)*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) -
pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*l
og(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) +
2*log(abs(d)))/((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 +
6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))
^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(
abs(d))^2 - 4*pi*sgn(d) - 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d
))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(
d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*
log(abs(d)) + 2*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(d
)^x + 1/2*(((4*pi - pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*pi
^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sg
n(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(d
))^2 - 4*pi*sgn(d) + 2)*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(abs(d))^2*sgn(d) -

```
pi^3*x^3 + 3*pi*x^3*log(abs(d))^2 + 3*pi^2*x^3*sgn(d) - 3*pi^2*x^3 + 6*x^3*
log(abs(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*pi*x^3 -
6*pi*x^2*log(abs(d)) - 2*x^3 - 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x
+ 12*x)/((4*pi - pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*
log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d
) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(d))^
2 - 4*pi*sgn(d) + 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn
(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*p
i^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(
d)) - 2*log(abs(d)))^2) - 4*(3*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi^2*x^3*log
(abs(d)) + 2*x^3*log(abs(d))^3 + 6*pi*x^3*log(abs(d))*sgn(d) - 6*pi*x^3*log
(abs(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*log(abs(d)) - 6*x^2*log(a
bs(d))^2 - 6*pi*x^2*sgn(d) + 6*pi*x^2 + 6*x^2 + 12*x*log(abs(d)) - 12)*(pi^
3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(
abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^
3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) - 2*log(abs(d)))/((4*pi - pi
^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*log(abs(d))^2 + 2*l
og(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*
log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(d))^2 - 4*pi*sgn(d) + 2
)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(
d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2
*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) - 2*log(abs(d))
)^2))*cos(1/2*pi*x*sgn(d) - 1/2*pi*x - x) - ((4*pi - pi^4*sgn(d) + 6*pi^2*l
og(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^
3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi
^2*sgn(d) + 6*pi^2 - 12*log(abs(d))^2 - 4*pi*sgn(d) + 2)*(3*pi^2*x^3*log(ab
s(d))*sgn(d) - 3*pi^2*x^3*log(abs(d)) + 2*x^3*log(abs(d))^3 + 6*pi*x^3*log(
abs(d))*sgn(d) - 6*pi*x^3*log(abs(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*
x^3*log(abs(d)) - 6*x^2*log(abs(d))^2 - 6*pi*x^2*sgn(d) + 6*pi*x^2 + 6*x^2
+ 12*x*log(abs(d)) - 12)/((4*pi - pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d)
 + pi^4 - 6*pi^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*lo
g(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2
- 12*log(abs(d))^2 - 4*pi*sgn(d) + 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*
log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs
(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d
) - 3*pi*log(abs(d)) - 2*log(abs(d)))^2) + 4*(pi^3*x^3*sgn(d) - 3*pi*x^3*lo
g(abs(d))^2*sgn(d) - pi^3*x^3 + 3*pi*x^3*log(abs(d))^2 + 3*pi^2*x^3*sgn(d)
- 3*pi^2*x^3 + 6*x^3*log(abs(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))
*sgn(d) - 3*pi*x^3 - 6*pi*x^2*log(abs(d)) - 2*x^3 - 12*x^2*log(abs(d)) - 6*
pi*x*sgn(d) + 6*pi*x + 12*x)*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sg
n(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*
pi^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs
(d)) - 2*log(abs(d)))/((4*pi - pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) +
pi^4 - 6*pi^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(a
bs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 1
```

```
2*log(abs(d))^2 - 4*pi*sgn(d) + 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log
(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d)
)*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) -
 3*pi*log(abs(d)) - 2*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x))
*abs(d)^x - 1/2*abs(d)^x*((8*pi^3*x^3*sgn(d) + 24*I*pi^2*x^3*log(abs(d))*sg
n(d) - 24*pi*x^3*log(abs(d))^2*sgn(d) - 8*pi^3*x^3 - 24*I*pi^2*x^3*log(abs(
d)) + 24*pi*x^3*log(abs(d))^2 + 16*I*x^3*log(abs(d))^3 - 24*pi^2*x^3*sgn(d)
 - 48*I*pi*x^3*log(abs(d))*sgn(d) + 24*pi^2*x^3 + 48*I*pi*x^3*log(abs(d)) -
 48*x^3*log(abs(d))^2 - 24*I*pi^2*x^2*sgn(d) + 24*pi*x^3*sgn(d) + 48*pi*x^2
*log(abs(d))*sgn(d) + 24*I*pi^2*x^2 - 24*pi*x^3 - 48*pi*x^2*log(abs(d)) - 4
8*I*x^3*log(abs(d)) - 48*I*x^2*log(abs(d))^2 + 48*I*pi*x^2*sgn(d) - 48*I*pi
*x^2 + 16*x^3 + 96*x^2*log(abs(d)) - 48*pi*x*sgn(d) + 48*pi*x + 48*I*x^2 +
96*I*x*log(abs(d)) - 96*x - 96*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x + I*x)/
(64*pi + 16*pi^4*sgn(d) + 64*I*pi^3*log(abs(d))*sgn(d) - 96*pi^2*log(abs(d)
)^2*sgn(d) - 64*I*pi*log(abs(d))^3*sgn(d) - 16*pi^4 - 64*I*pi^3*log(abs(d))
 + 96*pi^2*log(abs(d))^2 + 64*I*pi*log(abs(d))^3 - 32*log(abs(d))^4 - 64*pi
^3*sgn(d) - 192*I*pi^2*log(abs(d))*sgn(d) + 192*pi*log(abs(d))^2*sgn(d) + 6
4*pi^3 + 192*I*pi^2*log(abs(d)) - 192*pi*log(abs(d))^2 - 128*I*log(abs(d))^
3 + 96*pi^2*sgn(d) + 192*I*pi*log(abs(d))*sgn(d) - 96*pi^2 - 192*I*pi*log(a
bs(d)) + 192*log(abs(d))^2 - 64*pi*sgn(d) + 128*I*log(abs(d)) - 32) - (8*pi
^3*x^3*sgn(d) - 24*I*pi^2*x^3*log(abs(d))*sgn(d) - 24*pi*x^3*log(abs(d))^2*
sgn(d) - 8*pi^3*x^3 + 24*I*pi^2*x^3*log(abs(d)) + 24*pi*x^3*log(abs(d))^2 -
 16*I*x^3*log(abs(d))^3 - 24*pi^2*x^3*sgn(d) + 48*I*pi*x^3*log(abs(d))*sgn(
d) + 24*pi^2*x^3 - 48*I*pi*x^3*log(abs(d)) - 48*x^3*log(abs(d))^2 + 24*I*pi
^2*x^2*sgn(d) + 24*pi*x^3*sgn(d) + 48*pi*x^2*log(abs(d))*sgn(d) - 24*I*pi^2
*x^2 - 24*pi*x^3 - 48*pi*x^2*log(abs(d)) + 48*I*x^3*log(abs(d)) + 48*I*x^2*
log(abs(d))^2 - 48*I*pi*x^2*sgn(d) + 48*I*pi*x^2 + 16*x^3 + 96*x^2*log(abs(
d)) - 48*pi*x*sgn(d) + 48*pi*x - 48*I*x^2 - 96*I*x*log(abs(d)) - 96*x + 96*
I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(64*pi + 16*pi^4*sgn(d) - 64*I
*pi^3*log(abs(d))*sgn(d) - 96*pi^2*log(abs(d))^2*sgn(d) + 64*I*pi*log(abs(d
))^3*sgn(d) - 16*pi^4 + 64*I*pi^3*log(abs(d)) + 96*pi^2*log(abs(d))^2 - 64*
I*pi*log(abs(d))^3 - 32*log(abs(d))^4 - 64*pi^3*sgn(d) + 192*I*pi^2*log(abs
(d))*sgn(d) + 192*pi*log(abs(d))^2*sgn(d) + 64*pi^3 - 192*I*pi^2*log(abs(d)
) - 192*pi*log(abs(d))^2 + 128*I*log(abs(d))^3 + 96*pi^2*sgn(d) - 192*I*pi*
log(abs(d))*sgn(d) - 96*pi^2 + 192*I*pi*log(abs(d)) + 192*log(abs(d))^2 - 6
4*pi*sgn(d) - 128*I*log(abs(d)) - 32)) - 1/2*abs(d)^x*((8*pi^3*x^3*sgn(d) +
 24*I*pi^2*x^3*log(abs(d))*sgn(d) - 24*pi*x^3*log(abs(d))^2*sgn(d) - 8*pi^3
*x^3 - 24*I*pi^2*x^3*log(abs(d)) + 24*pi*x^3*log(abs(d))^2 + 16*I*x^3*log(a
bs(d))^3 + 24*pi^2*x^3*sgn(d) + 48*I*pi*x^3*log(abs(d))*sgn(d) - 24*pi^2*x^
3 - 48*I*pi*x^3*log(abs(d)) + 48*x^3*log(abs(d))^2 - 24*I*pi^2*x^2*sgn(d) +
 24*pi*x^3*sgn(d) + 48*pi*x^2*log(abs(d))*sgn(d) + 24*I*pi^2*x^2 - 24*pi*x^
3 - 48*pi*x^2*log(abs(d)) - 48*I*x^3*log(abs(d)) - 48*I*x^2*log(abs(d))^2 -
 48*I*pi*x^2*sgn(d) + 48*I*pi*x^2 - 16*x^3 - 96*x^2*log(abs(d)) - 48*pi*x*s
gn(d) + 48*pi*x + 48*I*x^2 + 96*I*x*log(abs(d)) + 96*x - 96*I)*e^(1/2*I*pi*
x*sgn(d) - 1/2*I*pi*x - I*x)/(64*pi - 16*pi^4*sgn(d) - 64*I*pi^3*log(abs(d)
```

```
)*sgn(d) + 96*pi^2*log(abs(d))^2*sgn(d) + 64*I*pi*log(abs(d))^3*sgn(d) + 16
*pi^4 + 64*I*pi^3*log(abs(d)) - 96*pi^2*log(abs(d))^2 - 64*I*pi*log(abs(d))
^3 + 32*log(abs(d))^4 - 64*pi^3*sgn(d) - 192*I*pi^2*log(abs(d))*sgn(d) + 19
2*pi*log(abs(d))^2*sgn(d) + 64*pi^3 + 192*I*pi^2*log(abs(d)) - 192*pi*log(a
bs(d))^2 - 128*I*log(abs(d))^3 - 96*pi^2*sgn(d) - 192*I*pi*log(abs(d))*sgn(
d) + 96*pi^2 + 192*I*pi*log(abs(d)) - 192*log(abs(d))^2 - 64*pi*sgn(d) + 12
8*I*log(abs(d)) + 32) - (8*pi^3*x^3*sgn(d) - 24*I*pi^2*x^3*log(abs(d))*sgn(
d) - 24*pi*x^3*log(abs(d))^2*sgn(d) - 8*pi^3*x^3 + 24*I*pi^2*x^3*log(abs(d)
) + 24*pi*x^3*log(abs(d))^2 - 16*I*x^3*log(abs(d))^3 + 24*pi^2*x^3*sgn(d) -
 48*I*pi*x^3*log(abs(d))*sgn(d) - 24*pi^2*x^3 + 48*I*pi*x^3*log(abs(d)) + 4
8*x^3*log(abs(d))^2 + 24*I*pi^2*x^2*sgn(d) + 24*pi*x^3*sgn(d) + 48*pi*x^2*l
og(abs(d))*sgn(d) - 24*I*pi^2*x^2 - 24*pi*x^3 - 48*pi*x^2*log(abs(d)) + 48*
I*x^3*log(abs(d)) + 48*I*x^2*log(abs(d))^2 + 48*I*pi*x^2*sgn(d) - 48*I*pi*x
^2 - 16*x^3 - 96*x^2*log(abs(d)) - 48*pi*x*sgn(d) + 48*pi*x - 48*I*x^2 - 96
*I*x*log(abs(d)) + 96*x + 96*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x + I*x)/(
64*pi - 16*pi^4*sgn(d) + 64*I*pi^3*log(abs(d))*sgn(d) + 96*pi^2*log(abs(d))
^2*sgn(d) - 64*I*pi*log(abs(d))^3*sgn(d) + 16*pi^4 - 64*I*pi^3*log(abs(d))
- 96*pi^2*log(abs(d))^2 + 64*I*pi*log(abs(d))^3 + 32*log(abs(d))^4 - 64*pi^
3*sgn(d) + 192*I*pi^2*log(abs(d))*sgn(d) + 192*pi*log(abs(d))^2*sgn(d) + 64
*pi^3 - 192*I*pi^2*log(abs(d)) - 192*pi*log(abs(d))^2 + 128*I*log(abs(d))^3
 - 96*pi^2*sgn(d) + 192*I*pi*log(abs(d))*sgn(d) + 96*pi^2 - 192*I*pi*log(ab
s(d)) - 192*log(abs(d))^2 - 64*pi*sgn(d) - 128*I*log(abs(d)) + 32))
```

3.141 $\int d^x x^3 \cos(x)\, dx$

Optimal. Leaf size=260

$$\frac{x^3 d^x \sin(x)}{\log^2(d)+1}-\frac{6x^2 d^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{x^3 d^x \log(d)\cos(x)}{\log^2(d)+1}-\frac{3x^2 d^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{3x^2 d^x \cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{18x d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-$$

```
[Out] (-6*d^x*Cos[x])/(1 + Log[d]^2)^4 + (36*d^x*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^
4 - (6*d^x*Cos[x]*Log[d]^4)/(1 + Log[d]^2)^4 - (18*d^x*x*Cos[x]*Log[d])/(1
+ Log[d]^2)^3 + (6*d^x*x*Cos[x]*Log[d]^3)/(1 + Log[d]^2)^3 + (3*d^x*x^2*Cos
[x])/(1 + Log[d]^2)^2 - (3*d^x*x^2*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^2 + (d^x
*x^3*Cos[x]*Log[d])/(1 + Log[d]^2) + (24*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^
4 - (24*d^x*Log[d]^3*Sin[x])/(1 + Log[d]^2)^4 - (6*d^x*x*Sin[x])/(1 + Log[d
]^2)^3 + (18*d^x*x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^3 - (6*d^x*x^2*Log[d]*Si
n[x])/(1 + Log[d]^2)^2 + (d^x*x^3*Sin[x])/(1 + Log[d]^2)
```

Rubi [A] time = 0.416488, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {4433, 4466, 14, 4432, 4465}

$$\frac{x^3 d^x \sin(x)}{\log^2(d)+1}-\frac{6x^2 d^x \log(d)\sin(x)}{\left(\log^2(d)+1\right)^2}+\frac{x^3 d^x \log(d)\cos(x)}{\log^2(d)+1}-\frac{3x^2 d^x \log^2(d)\cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{3x^2 d^x \cos(x)}{\left(\log^2(d)+1\right)^2}+\frac{18x d^x \log^2(d)\sin(x)}{\left(\log^2(d)+1\right)^3}-$$

Antiderivative was successfully verified.

```
[In] Int[d^x*x^3*Cos[x],x]
```


```
[Out] (-6*d^x*Cos[x])/(1 + Log[d]^2)^4 + (36*d^x*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^
4 - (6*d^x*Cos[x]*Log[d]^4)/(1 + Log[d]^2)^4 - (18*d^x*x*Cos[x]*Log[d])/(1
+ Log[d]^2)^3 + (6*d^x*x*Cos[x]*Log[d]^3)/(1 + Log[d]^2)^3 + (3*d^x*x^2*Cos
[x])/(1 + Log[d]^2)^2 - (3*d^x*x^2*Cos[x]*Log[d]^2)/(1 + Log[d]^2)^2 + (d^x
*x^3*Cos[x]*Log[d])/(1 + Log[d]^2) + (24*d^x*Log[d]*Sin[x])/(1 + Log[d]^2)^
4 - (24*d^x*Log[d]^3*Sin[x])/(1 + Log[d]^2)^4 - (6*d^x*x*Sin[x])/(1 + Log[d
]^2)^3 + (18*d^x*x*Log[d]^2*Sin[x])/(1 + Log[d]^2)^3 - (6*d^x*x^2*Log[d]*Si
n[x])/(1 + Log[d]^2)^2 + (d^x*x^3*Sin[x])/(1 + Log[d]^2)
```


Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
```

```
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rubi steps

$$\int d^x x^3 \cos(x)\,dx = \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^3 \sin(x)}{1+\log^2(d)} - 3\int x^2\left(\frac{d^x \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x \sin(x)}{1+\log^2(d)}\right)dx$$

$$= \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^3 \sin(x)}{1+\log^2(d)} - 3\int\left(\frac{d^x x^2 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^2 \sin(x)}{1+\log^2(d)}\right)dx$$

$$= \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} + \frac{d^x x^3 \sin(x)}{1+\log^2(d)} - \frac{3\int d^x x^2 \sin(x)\,dx}{1+\log^2(d)} - \frac{(3\log(d))\int d^x x^2 \cos(x)\,dx}{1+\log^2(d)}$$

$$= \frac{3d^x x^2 \cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} - \frac{6d^x x^2 \log(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \sin(x)}{1+\log^2(d)} + \frac{6}{}$$

$$= \frac{3d^x x^2 \cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} - \frac{6d^x x^2 \log(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \sin(x)}{1+\log^2(d)} + \frac{6}{}$$

$$= \frac{3d^x x^2 \cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \cos(x)\log(d)}{1+\log^2(d)} - \frac{6d^x x^2 \log(d)\sin(x)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \sin(x)}{1+\log^2(d)} - \frac{6}{(}$$

$$= -\frac{6d^x x \cos(x)\log(d)}{\left(1+\log^2(d)\right)^3} + \frac{6d^x x \cos(x)\log^3(d)}{\left(1+\log^2(d)\right)^3} + \frac{3d^x x^2 \cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \cos(x)\,\mathrm{lo}}{1+\log^2(d}$$

$$= -\frac{6d^x x \cos(x)\log(d)}{\left(1+\log^2(d)\right)^3} + \frac{6d^x x \cos(x)\log^3(d)}{\left(1+\log^2(d)\right)^3} + \frac{3d^x x^2 \cos(x)}{\left(1+\log^2(d)\right)^2} - \frac{3d^x x^2 \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^2} + \frac{d^x x^3 \cos(x)\,\mathrm{lo}}{1+\log^2(d}$$

$$= -\frac{6d^x \cos(x)}{\left(1+\log^2(d)\right)^4} + \frac{12d^x \cos(x)\log^2(d)}{\left(1+\log^2(d)\right)^4} - \frac{6d^x \cos(x)\log^4(d)}{\left(1+\log^2(d)\right)^4} - \frac{6d^x x \cos(x)\log(d)}{\left(1+\log^2(d)\right)^3} + \frac{6d^x x \cos(x)\log^3}{\left(1+\log^2(d)\right)^3}$$

Mathematica [A] time = 0.12184, size = 168, normalized size = 0.65

$$d^x\left(\sin(x)\left(x^3\log^6(d) - 6x^2\log^5(d) + 3x\left(x^2+6\right)\log^4(d) - 12\left(x^2+2\right)\log^3(d) + 3x\left(x^2+4\right)\log^2(d) - 6\left(x^2-4\right)\log(d) +\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[d^x*x^3*Cos[x],x]
```

```
[Out] (d^x*(Cos[x]*(3*(-2 + x^2) + x*(-18 + x^2)*Log[d] + 3*(12 + x^2)*Log[d]^2 +
 3*x*(-4 + x^2)*Log[d]^3 - 3*(2 + x^2)*Log[d]^4 + 3*x*(2 + x^2)*Log[d]^5 -
3*x^2*Log[d]^6 + x^3*Log[d]^7) + (x*(-6 + x^2) - 6*(-4 + x^2)*Log[d] + 3*x*
```

```
(4 + x^2)*Log[d]^2 - 12*(2 + x^2)*Log[d]^3 + 3*x*(6 + x^2)*Log[d]^4 - 6*x^2
*Log[d]^5 + x^3*Log[d]^6)*Sin[x]))/(1 + Log[d]^2)^4
```

Maple [A] time = 0.039, size = 441, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(d^x*x^3*cos(x),x)
```

```
[Out] (1/(1+ln(d)^2)*ln(d)*x^3*exp(x*ln(d))+2/(1+ln(d)^2)*x^3*exp(x*ln(d))*tan(1/
2*x)-3*(ln(d)^2-1)/(ln(d)^4+2*ln(d)^2+1)*x^2*exp(x*ln(d))-6/(ln(d)^6+3*ln(d
)^4+3*ln(d)^2+1)*(ln(d)^4-6*ln(d)^2+1)/(1+ln(d)^2)*exp(x*ln(d))-1/(1+ln(d)^
2)*ln(d)*x^3*exp(x*ln(d))*tan(1/2*x)^2+3*(ln(d)^2-1)/(ln(d)^4+2*ln(d)^2+1)*
x^2*exp(x*ln(d))*tan(1/2*x)^2+6/(ln(d)^6+3*ln(d)^4+3*ln(d)^2+1)*(ln(d)^4-6*
ln(d)^2+1)/(1+ln(d)^2)*exp(x*ln(d))*tan(1/2*x)^2-12*ln(d)/(ln(d)^4+2*ln(d)^
2+1)*x^2*exp(x*ln(d))*tan(1/2*x)-48*(ln(d)^2-1)*ln(d)/(ln(d)^4+2*ln(d)^2+1)
/(1+ln(d)^2)^2*exp(x*ln(d))*tan(1/2*x)+12*(3*ln(d)^2-1)/(1+ln(d)^2)/(ln(d)^
4+2*ln(d)^2+1)*x*exp(x*ln(d))*tan(1/2*x)+6*ln(d)*(ln(d)^2-3)/(1+ln(d)^2)/(l
n(d)^4+2*ln(d)^2+1)*x*exp(x*ln(d))-6*ln(d)*(ln(d)^2-3)/(1+ln(d)^2)/(ln(d)^4
+2*ln(d)^2+1)*x*exp(x*ln(d))*tan(1/2*x)^2)/(tan(1/2*x)^2+1)
```

Maxima [A] time = 1.09693, size = 248, normalized size = 0.95

$$\frac{\left(\left(\log\left(d\right)^{7} + 3\,\log\left(d\right)^{5} + 3\,\log\left(d\right)^{3} + \log\left(d\right)\right)x^{3} - 6\,\log\left(d\right)^{4} - 3\left(\log\left(d\right)^{6} + \log\left(d\right)^{4} - \log\left(d\right)^{2} - 1\right)x^{2} + 6\left(\log\left(d\right)^{5} - 2\,\mathrm{l}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*cos(x),x, algorithm="maxima")
```

```
[Out] (((log(d)^7 + 3*log(d)^5 + 3*log(d)^3 + log(d))*x^3 - 6*log(d)^4 - 3*(log(d
)^6 + log(d)^4 - log(d)^2 - 1)*x^2 + 6*(log(d)^5 - 2*log(d)^3 - 3*log(d))*x
 + 36*log(d)^2 - 6)*d^x*cos(x) + ((log(d)^6 + 3*log(d)^4 + 3*log(d)^2 + 1)*
x^3 - 6*(log(d)^5 + 2*log(d)^3 + log(d))*x^2 - 24*log(d)^3 + 6*(3*log(d)^4
+ 2*log(d)^2 - 1)*x + 24*log(d))*d^x*sin(x))/(log(d)^8 + 4*log(d)^6 + 6*log
(d)^4 + 4*log(d)^2 + 1)
```

Fricas [A] time = 1.78822, size = 578, normalized size = 2.22

$$\frac{\left(x^3 \cos(x) \log(d)^7 - 3x^2 \cos(x) \log(d)^6 + 3\left(x^3 + 2x\right) \cos(x) \log(d)^5 - 3\left(x^2 + 2\right) \cos(x) \log(d)^4 + 3\left(x^3 - 4x\right) \cos(x) \mathrm{l}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*cos(x),x, algorithm="fricas")
```

```
[Out] (x^3*cos(x)*log(d)^7 - 3*x^2*cos(x)*log(d)^6 + 3*(x^3 + 2*x)*cos(x)*log(d)^
5 - 3*(x^2 + 2)*cos(x)*log(d)^4 + 3*(x^3 - 4*x)*cos(x)*log(d)^3 + 3*(x^2 +
12)*cos(x)*log(d)^2 + (x^3 - 18*x)*cos(x)*log(d) + 3*(x^2 - 2)*cos(x) + (x^
3*log(d)^6 - 6*x^2*log(d)^5 + 3*(x^3 + 6*x)*log(d)^4 - 12*(x^2 + 2)*log(d)^
3 + x^3 + 3*(x^3 + 4*x)*log(d)^2 - 6*(x^2 - 4)*log(d) - 6*x)*sin(x))*d^x/(l
og(d)^8 + 4*log(d)^6 + 6*log(d)^4 + 4*log(d)^2 + 1)
```

Sympy [B] time = 27.0257, size = 1355, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d**x*x**3*cos(x),x)
```

```
[Out] Piecewise((I*x**4*exp(-I*x)*sin(x)/8 + x**4*exp(-I*x)*cos(x)/8 + x**3*exp(-
I*x)*sin(x)/4 + I*x**3*exp(-I*x)*cos(x)/4 - 3*I*x**2*exp(-I*x)*sin(x)/8 + 3
*x**2*exp(-I*x)*cos(x)/8 - 3*x*exp(-I*x)*sin(x)/8 - 3*I*x*exp(-I*x)*cos(x)/
8 + 3*I*exp(-I*x)*sin(x)/8, Eq(d, exp(-I))), (-I*x**4*exp(I*x)*sin(x)/8 + x
**4*exp(I*x)*cos(x)/8 + x**3*exp(I*x)*sin(x)/4 - I*x**3*exp(I*x)*cos(x)/4 +
 3*I*x**2*exp(I*x)*sin(x)/8 + 3*x**2*exp(I*x)*cos(x)/8 - 3*x*exp(I*x)*sin(x
)/8 + 3*I*x*exp(I*x)*cos(x)/8 - 3*I*exp(I*x)*sin(x)/8, Eq(d, exp(I))), (d**
x*x**3*log(d)**7*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**
2 + 1) + d**x*x**3*log(d)**6*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4
+ 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**5*cos(x)/(log(d)**8 + 4*log(d)**6
+ 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**4*sin(x)/(log(d)**8
+ 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*log(d)**3*cos(
x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**x*x**3*
log(d)**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1)
+ d**x*x**3*log(d)*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)
**2 + 1) + d**x*x**3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(
```

```
d)**2 + 1) - 3*d**x*x**2*log(d)**6*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(
d)**4 + 4*log(d)**2 + 1) - 6*d**x*x**2*log(d)**5*sin(x)/(log(d)**8 + 4*log(
d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 3*d**x*x**2*log(d)**4*cos(x)/(log(
d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x**2*log(d)*
*3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 3*d**
x*x**2*log(d)**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**
2 + 1) - 6*d**x*x**2*log(d)*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 +
 4*log(d)**2 + 1) + 3*d**x*x**2*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)*
*4 + 4*log(d)**2 + 1) + 6*d**x*x*log(d)**5*cos(x)/(log(d)**8 + 4*log(d)**6
+ 6*log(d)**4 + 4*log(d)**2 + 1) + 18*d**x*x*log(d)**4*sin(x)/(log(d)**8 +
4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 12*d**x*x*log(d)**3*cos(x)/(
log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) + 12*d**x*x*log(d)
**2*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 18*d
**x*x*log(d)*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 +
1) - 6*d**x*x*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*log(d)**2 +
 1) - 6*d**x*log(d)**4*cos(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)**4 + 4*lo
g(d)**2 + 1) - 24*d**x*log(d)**3*sin(x)/(log(d)**8 + 4*log(d)**6 + 6*log(d)
**4 + 4*log(d)**2 + 1) + 36*d**x*log(d)**2*cos(x)/(log(d)**8 + 4*log(d)**6
+ 6*log(d)**4 + 4*log(d)**2 + 1) + 24*d**x*log(d)*sin(x)/(log(d)**8 + 4*log
(d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1) - 6*d**x*cos(x)/(log(d)**8 + 4*log(
d)**6 + 6*log(d)**4 + 4*log(d)**2 + 1), True))
```

Giac [C] time = 1.31646, size = 6851, normalized size = 26.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d^x*x^3*cos(x),x, algorithm="giac")
```

```
[Out] -1/2*(((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*lo
g(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d)
+ 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2
- 4*pi*sgn(d) - 2)*(3*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi^2*x^3*log(abs(d))
+ 2*x^3*log(abs(d))^3 - 6*pi*x^3*log(abs(d))*sgn(d) + 6*pi*x^3*log(abs(d))
- 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*log(abs(d)) - 6*x^2*log(abs(d))^2
+ 6*pi*x^2*sgn(d) - 6*pi*x^2 + 6*x^2 + 12*x*log(abs(d)) - 12)/((4*pi + pi^4
*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log
(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*lo
g(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)^
2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)
) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*l
```

```
og(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))^
2) - 4*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(abs(d))^2*sgn(d) - pi^3*x^3 + 3*pi*x
^3*log(abs(d))^2 - 3*pi^2*x^3*sgn(d) + 3*pi^2*x^3 - 6*x^3*log(abs(d))^2 + 3
*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*pi*x^3 - 6*pi*x^2*log(abs(
d)) + 2*x^3 + 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x - 12*x)*(pi^3*log
(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d
))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3
*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))/((4*pi + pi^4*sg
n(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log(ab
s(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(a
bs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)^2 +
 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) +
 pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*log(
abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))^2))
*cos(1/2*pi*x*sgn(d) - 1/2*pi*x + x) + ((4*pi + pi^4*sgn(d) - 6*pi^2*log(ab
s(d))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn
(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sg
n(d) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)*(pi^3*x^3*sgn(d) - 3*pi
*x^3*log(abs(d))^2*sgn(d) - pi^3*x^3 + 3*pi*x^3*log(abs(d))^2 - 3*pi^2*x^3*
sgn(d) + 3*pi^2*x^3 - 6*x^3*log(abs(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(
abs(d))*sgn(d) - 3*pi*x^3 - 6*pi*x^2*log(abs(d)) + 2*x^3 + 12*x^2*log(abs(d
)) - 6*pi*x*sgn(d) + 6*pi*x - 12*x)/((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d
))^2*sgn(d) - pi^4 + 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d)
 + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d
) - 6*pi^2 + 12*log(abs(d))^2 - 4*pi*sgn(d) - 2)^2 + 16*(pi^3*log(abs(d))*s
gn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*p
i^2*log(abs(d))*sgn(d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(ab
s(d))*sgn(d) - 3*pi*log(abs(d)) + 2*log(abs(d)))^2) + 4*(3*pi^2*x^3*log(abs
(d))*sgn(d) - 3*pi^2*x^3*log(abs(d)) + 2*x^3*log(abs(d))^3 - 6*pi*x^3*log(a
bs(d))*sgn(d) + 6*pi*x^3*log(abs(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x
^3*log(abs(d)) - 6*x^2*log(abs(d))^2 + 6*pi*x^2*sgn(d) - 6*pi*x^2 + 6*x^2 +
 12*x*log(abs(d)) - 12)*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d)
- pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn(d) + 3*pi^2*
log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d))
+ 2*log(abs(d)))/((4*pi + pi^4*sgn(d) - 6*pi^2*log(abs(d))^2*sgn(d) - pi^4
+ 6*pi^2*log(abs(d))^2 - 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d)
)^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 + 6*pi^2*sgn(d) - 6*pi^2 + 12*log
(abs(d))^2 - 4*pi*sgn(d) - 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(
d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 - 3*pi^2*log(abs(d))*sgn
(d) + 3*pi^2*log(abs(d)) - 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi
*log(abs(d)) + 2*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x + x))*abs(
d)^x + 1/2*(((4*pi - pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*p
i^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*s
gn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(
d))^2 - 4*pi*sgn(d) + 2)*(3*pi^2*x^3*log(abs(d))*sgn(d) - 3*pi^2*x^3*log(ab
```

```
s(d)) + 2*x^3*log(abs(d))^3 + 6*pi*x^3*log(abs(d))*sgn(d) - 6*pi*x^3*log(ab
s(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2 - 6*x^3*log(abs(d)) - 6*x^2*log(abs(
d))^2 - 6*pi*x^2*sgn(d) + 6*pi*x^2 + 6*x^2 + 12*x*log(abs(d)) - 12)/((4*pi
- pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*log(abs(d))^2 +
 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12
*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(d))^2 - 4*pi*sgn(d)
 + 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(
abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d))
 + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) - 2*log(abs
(d)))^2) + 4*(pi^3*x^3*sgn(d) - 3*pi*x^3*log(abs(d))^2*sgn(d) - pi^3*x^3 +
3*pi*x^3*log(abs(d))^2 + 3*pi^2*x^3*sgn(d) - 3*pi^2*x^3 + 6*x^3*log(abs(d))
^2 + 3*pi*x^3*sgn(d) + 6*pi*x^2*log(abs(d))*sgn(d) - 3*pi*x^3 - 6*pi*x^2*lo
g(abs(d)) - 2*x^3 - 12*x^2*log(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x + 12*x)*(pi
^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log
(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))
^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) - 2*log(abs(d)))/((4*pi - p
i^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*log(abs(d))^2 + 2*
log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi
*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 - 12*log(abs(d))^2 - 4*pi*sgn(d) +
2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs
(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) +
2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(abs(d)) - 2*log(abs(d)
))^2))*cos(1/2*pi*x*sgn(d) - 1/2*pi*x - x) + ((4*pi - pi^4*sgn(d) + 6*pi^2*
log(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi
^3*sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*p
i^2*sgn(d) + 6*pi^2 - 12*log(abs(d))^2 - 4*pi*sgn(d) + 2)*(pi^3*x^3*sgn(d)
- 3*pi*x^3*log(abs(d))^2*sgn(d) - pi^3*x^3 + 3*pi*x^3*log(abs(d))^2 + 3*pi^
2*x^3*sgn(d) - 3*pi^2*x^3 + 6*x^3*log(abs(d))^2 + 3*pi*x^3*sgn(d) + 6*pi*x^
2*log(abs(d))*sgn(d) - 3*pi*x^3 - 6*pi*x^2*log(abs(d)) - 2*x^3 - 12*x^2*log
(abs(d)) - 6*pi*x*sgn(d) + 6*pi*x + 12*x)/((4*pi - pi^4*sgn(d) + 6*pi^2*log
(abs(d))^2*sgn(d) + pi^4 - 6*pi^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*
sgn(d) + 12*pi*log(abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2
*sgn(d) + 6*pi^2 - 12*log(abs(d))^2 - 4*pi*sgn(d) + 2)^2 + 16*(pi^3*log(abs
(d))*sgn(d) - pi*log(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3
 + 3*pi^2*log(abs(d))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*
log(abs(d))*sgn(d) - 3*pi*log(abs(d)) - 2*log(abs(d)))^2) - 4*(3*pi^2*x^3*l
og(abs(d))*sgn(d) - 3*pi^2*x^3*log(abs(d)) + 2*x^3*log(abs(d))^3 + 6*pi*x^3
*log(abs(d))*sgn(d) - 6*pi*x^3*log(abs(d)) - 3*pi^2*x^2*sgn(d) + 3*pi^2*x^2
 - 6*x^3*log(abs(d)) - 6*x^2*log(abs(d))^2 - 6*pi*x^2*sgn(d) + 6*pi*x^2 + 6
*x^2 + 12*x*log(abs(d)) - 12)*(pi^3*log(abs(d))*sgn(d) - pi*log(abs(d))^3*s
gn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d))*sgn(d) - 3
*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d) - 3*pi*log(ab
s(d)) - 2*log(abs(d)))/((4*pi - pi^4*sgn(d) + 6*pi^2*log(abs(d))^2*sgn(d) +
 pi^4 - 6*pi^2*log(abs(d))^2 + 2*log(abs(d))^4 - 4*pi^3*sgn(d) + 12*pi*log(
abs(d))^2*sgn(d) + 4*pi^3 - 12*pi*log(abs(d))^2 - 6*pi^2*sgn(d) + 6*pi^2 -
```

12*log(abs(d))^2 - 4*pi*sgn(d) + 2)^2 + 16*(pi^3*log(abs(d))*sgn(d) - pi*lo
g(abs(d))^3*sgn(d) - pi^3*log(abs(d)) + pi*log(abs(d))^3 + 3*pi^2*log(abs(d
))*sgn(d) - 3*pi^2*log(abs(d)) + 2*log(abs(d))^3 + 3*pi*log(abs(d))*sgn(d)
- 3*pi*log(abs(d)) - 2*log(abs(d)))^2))*sin(1/2*pi*x*sgn(d) - 1/2*pi*x - x)
)*abs(d)^x - 1/2*I*abs(d)^x*((8*pi^3*x^3*sgn(d) + 24*I*pi^2*x^3*log(abs(d))
*sgn(d) - 24*pi*x^3*log(abs(d))^2*sgn(d) - 8*pi^3*x^3 - 24*I*pi^2*x^3*log(a
bs(d)) + 24*pi*x^3*log(abs(d))^2 + 16*I*x^3*log(abs(d))^3 - 24*pi^2*x^3*sgn
(d) - 48*I*pi*x^3*log(abs(d))*sgn(d) + 24*pi^2*x^3 + 48*I*pi*x^3*log(abs(d)
) - 48*x^3*log(abs(d))^2 - 24*I*pi^2*x^2*sgn(d) + 24*pi*x^3*sgn(d) + 48*pi*
x^2*log(abs(d))*sgn(d) + 24*I*pi^2*x^2 - 24*pi*x^3 - 48*pi*x^2*log(abs(d))
- 48*I*x^3*log(abs(d)) - 48*I*x^2*log(abs(d))^2 + 48*I*pi*x^2*sgn(d) - 48*I
*pi*x^2 + 16*x^3 + 96*x^2*log(abs(d)) - 48*pi*x*sgn(d) + 48*pi*x + 48*I*x^2
 + 96*I*x*log(abs(d)) - 96*x - 96*I)*e^(1/2*I*pi*x*sgn(d) - 1/2*I*pi*x + I*
x)/(64*pi + 16*pi^4*sgn(d) + 64*I*pi^3*log(abs(d))*sgn(d) - 96*pi^2*log(abs
(d))^2*sgn(d) - 64*I*pi*log(abs(d))^3*sgn(d) - 16*pi^4 - 64*I*pi^3*log(abs(
d)) + 96*pi^2*log(abs(d))^2 + 64*I*pi*log(abs(d))^3 - 32*log(abs(d))^4 - 64
*pi^3*sgn(d) - 192*I*pi^2*log(abs(d))*sgn(d) + 192*pi*log(abs(d))^2*sgn(d)
+ 64*pi^3 + 192*I*pi^2*log(abs(d)) - 192*pi*log(abs(d))^2 - 128*I*log(abs(d
))^3 + 96*pi^2*sgn(d) + 192*I*pi*log(abs(d))*sgn(d) - 96*pi^2 - 192*I*pi*lo
g(abs(d)) + 192*log(abs(d))^2 - 64*pi*sgn(d) + 128*I*log(abs(d)) - 32) + (8
*pi^3*x^3*sgn(d) - 24*I*pi^2*x^3*log(abs(d))*sgn(d) - 24*pi*x^3*log(abs(d))
^2*sgn(d) - 8*pi^3*x^3 + 24*I*pi^2*x^3*log(abs(d)) + 24*pi*x^3*log(abs(d))^
2 - 16*I*x^3*log(abs(d))^3 - 24*pi^2*x^3*sgn(d) + 48*I*pi*x^3*log(abs(d))*s
gn(d) + 24*pi^2*x^3 - 48*I*pi*x^3*log(abs(d)) - 48*x^3*log(abs(d))^2 + 24*I
*pi^2*x^2*sgn(d) + 24*pi*x^3*sgn(d) + 48*pi*x^2*log(abs(d))*sgn(d) - 24*I*p
i^2*x^2 - 24*pi*x^3 - 48*pi*x^2*log(abs(d)) + 48*I*x^3*log(abs(d)) + 48*I*x
^2*log(abs(d))^2 - 48*I*pi*x^2*sgn(d) + 48*I*pi*x^2 + 16*x^3 + 96*x^2*log(a
bs(d)) - 48*pi*x*sgn(d) + 48*pi*x - 48*I*x^2 - 96*I*x*log(abs(d)) - 96*x +
96*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x - I*x)/(64*pi + 16*pi^4*sgn(d) - 6
4*I*pi^3*log(abs(d))*sgn(d) - 96*pi^2*log(abs(d))^2*sgn(d) + 64*I*pi*log(ab
s(d))^3*sgn(d) - 16*pi^4 + 64*I*pi^3*log(abs(d)) + 96*pi^2*log(abs(d))^2 -
64*I*pi*log(abs(d))^3 - 32*log(abs(d))^4 - 64*pi^3*sgn(d) + 192*I*pi^2*log(
abs(d))*sgn(d) + 192*pi*log(abs(d))^2*sgn(d) + 64*pi^3 - 192*I*pi^2*log(abs
(d)) - 192*pi*log(abs(d))^2 + 128*I*log(abs(d))^3 + 96*pi^2*sgn(d) - 192*I*
pi*log(abs(d))*sgn(d) - 96*pi^2 + 192*I*pi*log(abs(d)) + 192*log(abs(d))^2
- 64*pi*sgn(d) - 128*I*log(abs(d)) - 32)) + 1/2*I*abs(d)^x*((8*pi^3*x^3*sgn
(d) + 24*I*pi^2*x^3*log(abs(d))*sgn(d) - 24*pi*x^3*log(abs(d))^2*sgn(d) - 8
*pi^3*x^3 - 24*I*pi^2*x^3*log(abs(d)) + 24*pi*x^3*log(abs(d))^2 + 16*I*x^3*
log(abs(d))^3 + 24*pi^2*x^3*sgn(d) + 48*I*pi*x^3*log(abs(d))*sgn(d) - 24*pi
^2*x^3 - 48*I*pi*x^3*log(abs(d)) + 48*x^3*log(abs(d))^2 - 24*I*pi^2*x^2*sgn
(d) + 24*pi*x^3*sgn(d) + 48*pi*x^2*log(abs(d))*sgn(d) + 24*I*pi^2*x^2 - 24*
pi*x^3 - 48*pi*x^2*log(abs(d)) - 48*I*x^3*log(abs(d)) - 48*I*x^2*log(abs(d)
)^2 - 48*I*pi*x^2*sgn(d) + 48*I*pi*x^2 - 16*x^3 - 96*x^2*log(abs(d)) - 48*p
i*x*sgn(d) + 48*pi*x + 48*I*x^2 + 96*I*x*log(abs(d)) + 96*x - 96*I)*e^(1/2*
I*pi*x*sgn(d) - 1/2*I*pi*x - I*x)/(64*pi - 16*pi^4*sgn(d) - 64*I*pi^3*log(a

```
bs(d))*sgn(d) + 96*pi^2*log(abs(d))^2*sgn(d) + 64*I*pi*log(abs(d))^3*sgn(d)
 + 16*pi^4 + 64*I*pi^3*log(abs(d)) - 96*pi^2*log(abs(d))^2 - 64*I*pi*log(ab
s(d))^3 + 32*log(abs(d))^4 - 64*pi^3*sgn(d) - 192*I*pi^2*log(abs(d))*sgn(d)
 + 192*pi*log(abs(d))^2*sgn(d) + 64*pi^3 + 192*I*pi^2*log(abs(d)) - 192*pi*
log(abs(d))^2 - 128*I*log(abs(d))^3 - 96*pi^2*sgn(d) - 192*I*pi*log(abs(d))
*sgn(d) + 96*pi^2 + 192*I*pi*log(abs(d)) - 192*log(abs(d))^2 - 64*pi*sgn(d)
 + 128*I*log(abs(d)) + 32) + (8*pi^3*x^3*sgn(d) - 24*I*pi^2*x^3*log(abs(d))
*sgn(d) - 24*pi*x^3*log(abs(d))^2*sgn(d) - 8*pi^3*x^3 + 24*I*pi^2*x^3*log(a
bs(d)) + 24*pi*x^3*log(abs(d))^2 - 16*I*x^3*log(abs(d))^3 + 24*pi^2*x^3*sgn
(d) - 48*I*pi*x^3*log(abs(d))*sgn(d) - 24*pi^2*x^3 + 48*I*pi*x^3*log(abs(d)
) + 48*x^3*log(abs(d))^2 + 24*I*pi^2*x^2*sgn(d) + 24*pi*x^3*sgn(d) + 48*pi*
x^2*log(abs(d))*sgn(d) - 24*I*pi^2*x^2 - 24*pi*x^3 - 48*pi*x^2*log(abs(d))
+ 48*I*x^3*log(abs(d)) + 48*I*x^2*log(abs(d))^2 + 48*I*pi*x^2*sgn(d) - 48*I
*pi*x^2 - 16*x^3 - 96*x^2*log(abs(d)) - 48*pi*x*sgn(d) + 48*pi*x - 48*I*x^2
 - 96*I*x*log(abs(d)) + 96*x + 96*I)*e^(-1/2*I*pi*x*sgn(d) + 1/2*I*pi*x + I
*x)/(64*pi - 16*pi^4*sgn(d) + 64*I*pi^3*log(abs(d))*sgn(d) + 96*pi^2*log(ab
s(d))^2*sgn(d) - 64*I*pi*log(abs(d))^3*sgn(d) + 16*pi^4 - 64*I*pi^3*log(abs
(d)) - 96*pi^2*log(abs(d))^2 + 64*I*pi*log(abs(d))^3 + 32*log(abs(d))^4 - 6
4*pi^3*sgn(d) + 192*I*pi^2*log(abs(d))*sgn(d) + 192*pi*log(abs(d))^2*sgn(d)
 + 64*pi^3 - 192*I*pi^2*log(abs(d)) - 192*pi*log(abs(d))^2 + 128*I*log(abs(
d))^3 - 96*pi^2*sgn(d) + 192*I*pi*log(abs(d))*sgn(d) + 96*pi^2 - 192*I*pi*l
og(abs(d)) - 192*log(abs(d))^2 - 64*pi*sgn(d) - 128*I*log(abs(d)) + 32))
```

3.142 $\int \sin(x)\sin(2x)\sin(3x)\,dx$

Optimal. Leaf size=25

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

```
[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Rubi [A] time = 0.0308244, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2638}

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sin(x)\sin(2x)\sin(3x)\,dx &= \int \left(\frac{1}{4}\sin(2x)+\frac{1}{4}\sin(4x)-\frac{1}{4}\sin(6x)\right)dx\\ &= \frac{1}{4}\int \sin(2x)\,dx+\frac{1}{4}\int \sin(4x)\,dx-\frac{1}{4}\int \sin(6x)\,dx\\ &= -\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)\end{aligned}$$

Mathematica [A] time = 0.0095652, size = 25, normalized size = 1.

$$-\frac{1}{8}\cos(2x)-\frac{1}{16}\cos(4x)+\frac{1}{24}\cos(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sin[2*x]*Sin[3*x],x]
```

```
[Out] -Cos[2*x]/8 - Cos[4*x]/16 + Cos[6*x]/24
```

Maple [A] time = 0.047, size = 20, normalized size = 0.8

$$-\frac{\cos(2x)}{8}-\frac{\cos(4x)}{16}+\frac{\cos(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] -1/8*cos(2*x)-1/16*cos(4*x)+1/24*cos(6*x)
```

Maxima [A] time = 0.943437, size = 26, normalized size = 1.04

$$\frac{1}{24}\cos(6x)-\frac{1}{16}\cos(4x)-\frac{1}{8}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 1/24*cos(6*x) - 1/16*cos(4*x) - 1/8*cos(2*x)
```

Fricas [A] time = 1.8199, size = 54, normalized size = 2.16

$$\frac{4}{3}\cos(x)^6 - \frac{5}{2}\cos(x)^4 + \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 4/3*cos(x)^6 - 5/2*cos(x)^4 + cos(x)^2
```

Sympy [B] time = 15.2476, size = 114, normalized size = 4.56

$$\frac{x\sin(x)\sin(2x)\sin(3x)}{4} + \frac{x\sin(x)\cos(2x)\cos(3x)}{4} + \frac{x\sin(2x)\cos(x)\cos(3x)}{4} - \frac{x\sin(3x)\cos(x)\cos(2x)}{4} - \frac{5\sin(x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x)
```

```
[Out] x*sin(x)*sin(2*x)*sin(3*x)/4 + x*sin(x)*cos(2*x)*cos(3*x)/4 + x*sin(2*x)*co
s(x)*cos(3*x)/4 - x*sin(3*x)*cos(x)*cos(2*x)/4 - 5*sin(x)*sin(2*x)*cos(3*x)
/24 - sin(2*x)*sin(3*x)*cos(x)/8 - cos(x)*cos(2*x)*cos(3*x)/6
```

Giac [A] time = 1.0863, size = 18, normalized size = 0.72

$$-\frac{4}{3}\sin(x)^6 + \frac{3}{2}\sin(x)^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*sin(2*x)*sin(3*x),x, algorithm="giac")
```

```
[Out] -4/3*sin(x)^6 + 3/2*sin(x)^4
```

3.143 $\int \cos(x)\cos(2x)\cos(3x)\,dx$

Optimal. Leaf size=30

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rubi [A] time = 0.0315283, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4355, 2637}

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Rule 4355

```
Int[(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) + (d_.)*(x_)]^(q_.)*(H_)[(e_.
) + (f_.)*(x_)]^(r_.), x_Symbol] :> Int[ExpandTrigReduce[ActivateTrig[F[a +
 b*x]^p*G[c + d*x]^q*H[e + f*x]^r], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& (EqQ[F, sin] || EqQ[F, cos]) && (EqQ[G, sin] || EqQ[G, cos]) && (EqQ[H,
sin] || EqQ[H, cos]) && IGtQ[p, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(x)\cos(2x)\cos(3x)\,dx = \int \left(\frac{1}{4}+\frac{1}{4}\cos(2x)+\frac{1}{4}\cos(4x)+\frac{1}{4}\cos(6x)\right)dx$$
$$= \frac{x}{4}+\frac{1}{4}\int \cos(2x)\,dx+\frac{1}{4}\int \cos(4x)\,dx+\frac{1}{4}\int \cos(6x)\,dx$$
$$= \frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Mathematica [A] time = 0.0084235, size = 30, normalized size = 1.

$$\frac{x}{4}+\frac{1}{8}\sin(2x)+\frac{1}{16}\sin(4x)+\frac{1}{24}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Cos[2*x]*Cos[3*x],x]
```

```
[Out] x/4 + Sin[2*x]/8 + Sin[4*x]/16 + Sin[6*x]/24
```

Maple [A] time = 0.041, size = 23, normalized size = 0.8

$$\frac{x}{4}+\frac{\sin(2x)}{8}+\frac{\sin(4x)}{16}+\frac{\sin(6x)}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] 1/4*x+1/8*sin(2*x)+1/16*sin(4*x)+1/24*sin(6*x)
```

Maxima [A] time = 0.936522, size = 30, normalized size = 1.

$$\frac{1}{4}x+\frac{1}{24}\sin(6x)+\frac{1}{16}\sin(4x)+\frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="maxima")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

Fricas [A] time = 1.87783, size = 81, normalized size = 2.7

$$\frac{1}{12}\left(16\cos(x)^5 - 10\cos(x)^3 + 3\cos(x)\right)\sin(x) + \frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="fricas")
```

```
[Out] 1/12*(16*cos(x)^5 - 10*cos(x)^3 + 3*cos(x))*sin(x) + 1/4*x
```

Sympy [B] time = 14.6232, size = 112, normalized size = 3.73

$$-\frac{x\sin(x)\sin(2x)\cos(3x)}{4} + \frac{x\sin(x)\sin(3x)\cos(2x)}{4} + \frac{x\sin(2x)\sin(3x)\cos(x)}{4} + \frac{x\cos(x)\cos(2x)\cos(3x)}{4} + \frac{\sin(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x)
```

```
[Out] -x*sin(x)*sin(2*x)*cos(3*x)/4 + x*sin(x)*sin(3*x)*cos(2*x)/4 + x*sin(2*x)*s
in(3*x)*cos(x)/4 + x*cos(x)*cos(2*x)*cos(3*x)/4 + sin(x)*sin(2*x)*sin(3*x)/
24 - sin(2*x)*cos(x)*cos(3*x)/8 + sin(3*x)*cos(x)*cos(2*x)/3
```

Giac [A] time = 1.07845, size = 30, normalized size = 1.

$$\frac{1}{4}x + \frac{1}{24}\sin(6x) + \frac{1}{16}\sin(4x) + \frac{1}{8}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*cos(2*x)*cos(3*x),x, algorithm="giac")
```

```
[Out] 1/4*x + 1/24*sin(6*x) + 1/16*sin(4*x) + 1/8*sin(2*x)
```

3.144 $\int x^2 \sin^3(kx)\,dx$

Optimal. Leaf size=85

$$\frac{2x\sin^3(kx)}{9k^2}+\frac{4x\sin(kx)}{3k^2}-\frac{2\cos^3(kx)}{27k^3}+\frac{14\cos(kx)}{9k^3}-\frac{2x^2\cos(kx)}{3k}-\frac{x^2\sin^2(kx)\cos(kx)}{3k}$$

```
[Out] (14*Cos[k*x])/(9*k^3) - (2*x^2*Cos[k*x])/(3*k) - (2*Cos[k*x]^3)/(27*k^3) +
(4*x*Sin[k*x])/(3*k^2) - (x^2*Cos[k*x]*Sin[k*x]^2)/(3*k) + (2*x*Sin[k*x]^3)
/(9*k^2)
```

Rubi [A] time = 0.0663495, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3311, 3296, 2638, 2633}

$$\frac{2x\sin^3(kx)}{9k^2}+\frac{4x\sin(kx)}{3k^2}-\frac{2\cos^3(kx)}{27k^3}+\frac{14\cos(kx)}{9k^3}-\frac{2x^2\cos(kx)}{3k}-\frac{x^2\sin^2(kx)\cos(kx)}{3k}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[k*x]^3,x]
```

```
[Out] (14*Cos[k*x])/(9*k^3) - (2*x^2*Cos[k*x])/(3*k) - (2*Cos[k*x]^3)/(27*k^3) +
(4*x*Sin[k*x])/(3*k^2) - (x^2*Cos[k*x]*Sin[k*x]^2)/(3*k) + (2*x*Sin[k*x]^3)
/(9*k^2)
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\sin^3(kx)\,dx &= -\frac{x^2\cos(kx)\sin^2(kx)}{3k}+\frac{2x\sin^3(kx)}{9k^2}+\frac{2}{3}\int x^2\sin(kx)\,dx-\frac{2\int\sin^3(kx)\,dx}{9k^2}\\ &= -\frac{2x^2\cos(kx)}{3k}-\frac{x^2\cos(kx)\sin^2(kx)}{3k}+\frac{2x\sin^3(kx)}{9k^2}+\frac{2\,\mathrm{Subst}\left(\int\left(1-x^2\right)dx,x,\cos(kx)\right)}{9k^3}+\frac{4\int x\cos\\ &= \frac{2\cos(kx)}{9k^3}-\frac{2x^2\cos(kx)}{3k}-\frac{2\cos^3(kx)}{27k^3}+\frac{4x\sin(kx)}{3k^2}-\frac{x^2\cos(kx)\sin^2(kx)}{3k}+\frac{2x\sin^3(kx)}{9k^2}-\frac{4\int\sin(\\ &= \frac{14\cos(kx)}{9k^3}-\frac{2x^2\cos(kx)}{3k}-\frac{2\cos^3(kx)}{27k^3}+\frac{4x\sin(kx)}{3k^2}-\frac{x^2\cos(kx)\sin^2(kx)}{3k}+\frac{2x\sin^3(kx)}{9k^2}\end{aligned}$$

Mathematica [A] time = 0.0790781, size = 55, normalized size = 0.65

$$\frac{-81\left(k^2x^2-2\right)\cos(kx)+\left(9k^2x^2-2\right)\cos(3kx)-6kx(\sin(3kx)-27\sin(kx))}{108k^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[k*x]^3,x]
```

```
[Out] (-81*(-2 + k^2*x^2)*Cos[k*x] + (-2 + 9*k^2*x^2)*Cos[3*k*x] - 6*k*x*(-27*Sin
[k*x] + Sin[3*k*x]))/(108*k^3)
```

Maple [A] time = 0.026, size = 64, normalized size = 0.8

$$\frac{1}{k^3}\left(-\frac{k^2x^2\left(2+(\sin(kx))^2\right)\cos(kx)}{3}+\frac{4\cos(kx)}{3}+\frac{4kx\sin(kx)}{3}+\frac{2kx(\sin(kx))^3}{9}+\frac{\left(4+2(\sin(kx))^2\right)\cos(kx)}{27}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(k*x)^3,x)
```

```
[Out] 1/k^3*(-1/3*k^2*x^2*(2+sin(k*x)^2)*cos(k*x)+4/3*cos(k*x)+4/3*k*x*sin(k*x)+2
/9*k*x*sin(k*x)^3+2/27*(2+sin(k*x)^2)*cos(k*x))
```

Maxima [A] time = 0.972621, size = 74, normalized size = 0.87

$$-\frac{6\,kx\sin\left(3\,kx\right)-162\,kx\sin\left(kx\right)-\left(9\,k^{2}x^{2}-2\right)\cos\left(3\,kx\right)+81\,\left(k^{2}x^{2}-2\right)\cos\left(kx\right)}{108\,k^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(k*x)^3,x, algorithm="maxima")
```

```
[Out] -1/108*(6*k*x*sin(3*k*x) - 162*k*x*sin(k*x) - (9*k^2*x^2 - 2)*cos(3*k*x) +
81*(k^2*x^2 - 2)*cos(k*x))/k^3
```

Fricas [A] time = 1.80226, size = 144, normalized size = 1.69

$$\frac{\left(9\,k^{2}x^{2}-2\right)\cos\left(kx\right)^{3}-3\left(9\,k^{2}x^{2}-14\right)\cos\left(kx\right)-6\left(kx\cos\left(kx\right)^{2}-7\,kx\right)\sin\left(kx\right)}{27\,k^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(k*x)^3,x, algorithm="fricas")
```

```
[Out] 1/27*((9*k^2*x^2 - 2)*cos(k*x)^3 - 3*(9*k^2*x^2 - 14)*cos(k*x) - 6*(k*x*cos
(k*x)^2 - 7*k*x)*sin(k*x))/k^3
```

Sympy [A] time = 2.19431, size = 100, normalized size = 1.18

$$\begin{cases} -\frac{x^{2}\sin^{2}\left(kx\right)\cos\left(kx\right)}{k}-\frac{2x^{2}\cos^{3}\left(kx\right)}{3k}+\frac{14x\sin^{3}\left(kx\right)}{9k^{2}}+\frac{4x\sin\left(kx\right)\cos^{2}\left(kx\right)}{3k^{2}}+\frac{14\sin^{2}\left(kx\right)\cos\left(kx\right)}{9k^{3}}+\frac{40\cos^{3}\left(kx\right)}{27k^{3}} & \text{for}\: k\neq 0\\0 & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(k*x)**3,x)
```

```
[Out] Piecewise((-x**2*sin(k*x)**2*cos(k*x)/k - 2*x**2*cos(k*x)**3/(3*k) + 14*x*s
in(k*x)**3/(9*k**2) + 4*x*sin(k*x)*cos(k*x)**2/(3*k**2) + 14*sin(k*x)**2*co
s(k*x)/(9*k**3) + 40*cos(k*x)**3/(27*k**3), Ne(k, 0)), (0, True))
```

Giac [A] time = 1.10263, size = 81, normalized size = 0.95

$$-\frac{x\sin\left(3\,kx\right)}{18\,k^{2}}+\frac{3\,x\sin\left(kx\right)}{2\,k^{2}}+\frac{\left(9\,k^{2}x^{2}-2\right)\cos\left(3\,kx\right)}{108\,k^{3}}-\frac{3\,\left(k^{2}x^{2}-2\right)\cos\left(kx\right)}{4\,k^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(k*x)^3,x, algorithm="giac")
```

```
[Out] -1/18*x*sin(3*k*x)/k^2 + 3/2*x*sin(k*x)/k^2 + 1/108*(9*k^2*x^2 - 2)*cos(3*k
*x)/k^3 - 3/4*(k^2*x^2 - 2)*cos(k*x)/k^3
```

3.145 $\int x \cos(k \csc(x)) \cot(x) \csc(x) \, dx$

Optimal. Leaf size=13

$$\text{CannotIntegrate}(x \cot(x) \csc(x) \cos(k \csc(x)), x)$$

```
[Out] Defer[Int][x*Cos[k*Csc[x]]*Cot[x]*Csc[x], x]
```

Rubi [A] time = 0.435403, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[x*Cos[k*Csc[x]]*Cot[x]*Csc[x],x]
```

```
[Out] Defer[Int][x*Cos[k*Csc[x]]*Cot[x]*Csc[x], x]
```

Rubi steps

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) \, dx = \int x \cos(k \csc(x)) \cot(x) \csc(x) \, dx$$

Mathematica [A] time = 0.8798, size = 0, normalized size = 0.

$$\int x \cos(k \csc(x)) \cot(x) \csc(x) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x*Cos[k*Csc[x]]*Cot[x]*Csc[x],x]
```

```
[Out] Integrate[x*Cos[k*Csc[x]]*Cot[x]*Csc[x], x]
```

Maple [A] time = 0.789, size = 0, normalized size = 0.

$$\int \frac{x\cos(x)}{(\sin(x))^2}\cos\left(\frac{k}{\sin(x)}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)*cos(k/sin(x))/sin(x)^2,x)
```

```
[Out] int(x*cos(x)*cos(k/sin(x))/sin(x)^2,x)
```

Maxima [A] time = 0.983046, size = 324, normalized size = 24.92

$$-\frac{\left(xe^{\left(\frac{4\,k\cos(2\,x)\cos(x)}{\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1}+\frac{4\,k\sin(2\,x)\sin(x)}{\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1}\right)}+xe^{\left(\frac{4\,k\cos(x)}{\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1}\right)}\right)e^{\left(-\frac{2\,k\cos(2\,x)\cos(x)}{\cos(2\,x)^2+\sin(2\,x)^2-2\,\cos(2\,x)+1}-\frac{2\,k\sin}{\cos(2\,x)^2+\sin(}\right.}}{2\,k}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(x*e^(4*k*cos(2*x)*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) +
 4*k*sin(2*x)*sin(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)) + x*e^(4*k
*cos(x)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1)))*e^(-2*k*cos(2*x)*cos(x
)/(cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) - 2*k*sin(2*x)*sin(x)/(cos(2*x
)^2 + sin(2*x)^2 - 2*cos(2*x) + 1) - 2*k*cos(x)/(cos(2*x)^2 + sin(2*x)^2 -
2*cos(2*x) + 1))*sin(2*(k*cos(x)*sin(2*x) - k*cos(2*x)*sin(x) + k*sin(x))/(
cos(2*x)^2 + sin(2*x)^2 - 2*cos(2*x) + 1))/k
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{x\cos(x)\cos\left(\frac{k}{\sin(x)}\right)}{\cos(x)^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="fricas")
```

```
[Out] integral(-x*cos(x)*cos(k/sin(x))/(cos(x)^2 - 1), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*cos(k/sin(x))/sin(x)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x\cos(x)\cos\left(\frac{k}{\sin(x)}\right)}{\sin(x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2,x, algorithm="giac")
```

```
[Out] integrate(x*cos(x)*cos(k/sin(x))/sin(x)^2, x)
```

3.146 $\int \cot\left(\frac{x}{2}\right)\cot(x)\,dx$

Optimal. Leaf size=12

$$-x-\cot\left(\frac{x}{2}\right)$$

```
[Out] -x - Cot[x/2]
```

Rubi [A] time = 0.0304232, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {12, 453, 203}

$$-x-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x/2]*Cot[x],x]
```

```
[Out] -x - Cot[x/2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \cot\left(\frac{x}{2}\right)\cot(x)\,dx &= 2\,\text{Subst}\left(\int \frac{1-x^2}{2x^2\left(1+x^2\right)}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= \text{Subst}\left(\int \frac{1-x^2}{x^2\left(1+x^2\right)}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\cot\left(\frac{x}{2}\right) - 2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -x - \cot\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A] time = 0.0128978, size = 12, normalized size = 1.

$$-x - \cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x/2]*Cot[x],x]
```

```
[Out] -x - Cot[x/2]
```

Maple [A] time = 0.044, size = 11, normalized size = 0.9

$$-x - \cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(x)/tan(1/2*x),x)
```

```
[Out] -x-cot(1/2*x)
```

Maxima [B] time = 0.951178, size = 55, normalized size = 4.58

$$-\frac{x\cos(x)^2 + x\sin(x)^2 - 2\,x\cos(x) + x + 2\,\sin(x)}{\cos(x)^2 + \sin(x)^2 - 2\,\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/tan(1/2*x),x, algorithm="maxima")
```

```
[Out] -(x*cos(x)^2 + x*sin(x)^2 - 2*x*cos(x) + x + 2*sin(x))/(cos(x)^2 + sin(x)^2
 - 2*cos(x) + 1)
```

Fricas [A] time = 1.70271, size = 43, normalized size = 3.58

$$-\frac{x \tan\left(\frac{1}{2}\,x\right)+1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/tan(1/2*x),x, algorithm="fricas")
```

```
[Out] -(x*tan(1/2*x) + 1)/tan(1/2*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)}{\sin(x)\tan\left(\frac{x}{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/tan(1/2*x),x)
```

```
[Out] Integral(cos(x)/(sin(x)*tan(x/2)), x)
```

Giac [A] time = 1.11236, size = 24, normalized size = 2.

$$-x-\frac{1}{2\,\tan\left(\frac{1}{4}\,x\right)}+\frac{1}{2}\,\tan\left(\frac{1}{4}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)/tan(1/2*x),x, algorithm="giac")
```

```
[Out] -x - 1/2/tan(1/4*x) + 1/2*tan(1/4*x)
```

3.147 $\int \frac{\sin(ax)}{(b+c\sin(ax))^2}\,dx$

Optimal. Leaf size=77

$$-\frac{2c\tan^{-1}\left(\frac{b\tan\left(\frac{ax}{2}\right)+c}{\sqrt{b^2-c^2}}\right)}{a\left(b^2-c^2\right)^{3/2}}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)(c\sin(ax)+b)}$$

```
[Out] (-2*c*ArcTan[(c + b*Tan[(a*x)/2])/Sqrt[b^2 - c^2]])/(a*(b^2 - c^2)^(3/2)) -
 (b*Cos[a*x])/(a*(b^2 - c^2)*(b + c*Sin[a*x]))
```

Rubi [A] time = 0.0977043, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2754, 12, 2660, 618, 204}

$$-\frac{2c\tan^{-1}\left(\frac{b\tan\left(\frac{ax}{2}\right)+c}{\sqrt{b^2-c^2}}\right)}{a\left(b^2-c^2\right)^{3/2}}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)(c\sin(ax)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[a*x]/(b + c*Sin[a*x])^2,x]
```

```
[Out] (-2*c*ArcTan[(c + b*Tan[(a*x)/2])/Sqrt[b^2 - c^2]])/(a*(b^2 - c^2)^(3/2)) -
 (b*Cos[a*x])/(a*(b^2 - c^2)*(b + c*Sin[a*x]))
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(ax)}{(b+c\sin(ax))^2}\,dx &= -\frac{b\cos(ax)}{a\left(b^2-c^2\right)(b+c\sin(ax))}+\frac{\int \frac{c}{b+c\sin(ax)}\,dx}{-b^2+c^2}\\
&= -\frac{b\cos(ax)}{a\left(b^2-c^2\right)(b+c\sin(ax))}-\frac{c\int \frac{1}{b+c\sin(ax)}\,dx}{b^2-c^2}\\
&= -\frac{b\cos(ax)}{a\left(b^2-c^2\right)(b+c\sin(ax))}-\frac{(2c)\operatorname{Subst}\left(\int \frac{1}{b+2cx+bx^2}\,dx,x,\tan\left(\frac{ax}{2}\right)\right)}{a\left(b^2-c^2\right)}\\
&= -\frac{b\cos(ax)}{a\left(b^2-c^2\right)(b+c\sin(ax))}+\frac{(4c)\operatorname{Subst}\left(\int \frac{1}{-4\left(b^2-c^2\right)-x^2}\,dx,x,2c+2b\tan\left(\frac{ax}{2}\right)\right)}{a\left(b^2-c^2\right)}\\
&= -\frac{2c\tan^{-1}\left(\frac{c+b\tan\left(\frac{ax}{2}\right)}{\sqrt{b^2-c^2}}\right)}{a\left(b^2-c^2\right)^{3/2}}-\frac{b\cos(ax)}{a\left(b^2-c^2\right)(b+c\sin(ax))}
\end{aligned}
$$

Mathematica [A] time = 0.246516, size = 76, normalized size = 0.99

$$-\frac{\frac{2c\tan^{-1}\left(\frac{b\tan\left(\frac{ax}{2}\right)+c}{\sqrt{b^2-c^2}}\right)}{\left(b^2-c^2\right)^{3/2}}+\frac{b\cos(ax)}{(b-c)(b+c)(c\sin(ax)+b)}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[a*x]/(b + c*Sin[a*x])^2,x]
```

```
[Out] -(((2*c*ArcTan[(c + b*Tan[(a*x)/2])/Sqrt[b^2 - c^2]])/(b^2 - c^2)^(3/2) + (
b*Cos[a*x])/((b - c)*(b + c)*(b + c*Sin[a*x])))/a)
```

Maple [A] time = 0.041, size = 143, normalized size = 1.9

$$-8\frac{c\tan(1/2\,ax)}{a\left(4\,b^2-4\,c^2\right)\left(b\left(\tan(1/2\,ax)\right)^2+2\,c\tan(1/2\,ax)+b\right)}-8\frac{b}{a\left(4\,b^2-4\,c^2\right)\left(b\left(\tan(1/2\,ax)\right)^2+2\,c\tan(1/2\,ax)+b\right)}-8$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a*x)/(b+c*sin(a*x))^2,x)
```

```
[Out] -8/a/(4*b^2-4*c^2)/(b*tan(1/2*a*x)^2+2*c*tan(1/2*a*x)+b)*c*tan(1/2*a*x)-8/a
/(4*b^2-4*c^2)/(b*tan(1/2*a*x)^2+2*c*tan(1/2*a*x)+b)*b-8/a*c/(4*b^2-4*c^2)/
(b^2-c^2)^(1/2)*arctan(1/2*(2*b*tan(1/2*a*x)+2*c)/(b^2-c^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)/(b+c*sin(a*x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92501, size = 684, normalized size = 8.88

$$\left[\frac{\left(c^{2} \sin\left(ax\right) + bc\right)\sqrt{-b^{2} + c^{2}} \log\left(\frac{\left(2\,b^{2}-c^{2}\right)\cos(ax)^{2}-2\,bc\sin(ax)-b^{2}-c^{2}+2\left(b\cos(ax)\sin(ax)+c\cos(ax)\right)\sqrt{-b^{2}+c^{2}}}{c^{2}\cos(ax)^{2}-2\,bc\sin(ax)-b^{2}-c^{2}}\right) - 2\left(b^{3} - bc^{2}\right)\cos\left(ax\right)}{2\left(ab^{5} - 2\,ab^{3}c^{2} + abc^{4} + \left(ab^{4}c - 2\,ab^{2}c^{3} + ac^{5}\right)\sin\left(ax\right)\right)}, -\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)/(b+c*sin(a*x))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((c^2*sin(a*x) + b*c)*sqrt(-b^2 + c^2)*log(((2*b^2 - c^2)*cos(a*x)^2 -
 2*b*c*sin(a*x) - b^2 - c^2 + 2*(b*cos(a*x)*sin(a*x) + c*cos(a*x))*sqrt(-b^
2 + c^2))/(c^2*cos(a*x)^2 - 2*b*c*sin(a*x) - b^2 - c^2)) - 2*(b^3 - b*c^2)*
cos(a*x))/(a*b^5 - 2*a*b^3*c^2 + a*b*c^4 + (a*b^4*c - 2*a*b^2*c^3 + a*c^5)*
sin(a*x)), ((c^2*sin(a*x) + b*c)*sqrt(b^2 - c^2)*arctan(-(b*sin(a*x) + c)/(
sqrt(b^2 - c^2)*cos(a*x))) - (b^3 - b*c^2)*cos(a*x))/(a*b^5 - 2*a*b^3*c^2 +
 a*b*c^4 + (a*b^4*c - 2*a*b^2*c^3 + a*c^5)*sin(a*x))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)/(b+c*sin(a*x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.10437, size = 132, normalized size = 1.71

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{ax}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(b)+\arctan\left(\frac{b\tan\left(\frac{1}{2}ax\right)+c}{\sqrt{b^{2}-c^{2}}}\right)\right)c}{\left(b^{2}-c^{2}\right)^{\frac{3}{2}}}+\frac{c\tan\left(\frac{1}{2}ax\right)+b}{\left(b\tan\left(\frac{1}{2}ax\right)^{2}+2\,c\tan\left(\frac{1}{2}ax\right)+b\right)\left(b^{2}-c^{2}\right)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(a*x)/(b+c*sin(a*x))^2,x, algorithm="giac")
```

```
[Out] -2*((pi*floor(1/2*a*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*a*x) + c)/sqrt(b
^2 - c^2)))*c/(b^2 - c^2)^(3/2) + (c*tan(1/2*a*x) + b)/((b*tan(1/2*a*x)^2 +
 2*c*tan(1/2*a*x) + b)*(b^2 - c^2)))/a
```

3.148 $\int \sin(\log(x))\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x\sin(\log(x)) - \frac{1}{2}x\cos(\log(x))$$

```
[Out] -(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rubi [A] time = 0.0031254, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4475}

$$\frac{1}{2}x\sin(\log(x)) - \frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Log[x]],x]
```

```
[Out] -(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rule 4475

```
Int[Sin[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(x*
Sin[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] - Simp[(b*d*n*x*Cos[d*(a +
 b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\int \sin(\log(x))\,dx = -\frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

Mathematica [A] time = 0.0027508, size = 17, normalized size = 1.

$$\frac{1}{2}x\sin(\log(x)) - \frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Log[x]],x]
```

```
[Out] -(x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$-\frac{x\cos\left(\ln\left(x\right)\right)}{2}+\frac{x\sin\left(\ln\left(x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(ln(x)),x)
```

```
[Out] -1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Maxima [A] time = 0.93794, size = 16, normalized size = 0.94

$$-\frac{1}{2}\,x(\cos\left(\log\left(x\right)\right)-\sin\left(\log\left(x\right)\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x)),x, algorithm="maxima")
```

```
[Out] -1/2*x*(cos(log(x)) - sin(log(x)))
```

Fricas [A] time = 1.75441, size = 54, normalized size = 3.18

$$-\frac{1}{2}\,x\cos\left(\log\left(x\right)\right)+\frac{1}{2}\,x\sin\left(\log\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x)),x, algorithm="fricas")
```

```
[Out] -1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A] time = 0.414042, size = 15, normalized size = 0.88

$$\frac{x\sin(\log(x))}{2} - \frac{x\cos(\log(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(ln(x)),x)
```

```
[Out] x*sin(log(x))/2 - x*cos(log(x))/2
```

Giac [A] time = 1.07497, size = 18, normalized size = 1.06

$$-\frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(log(x)),x, algorithm="giac")
```

```
[Out] -1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

3.149 $\int \cos(\log(x))\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rubi [A] time = 0.0030925, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4476}

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[Log[x]],x]
```

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Rule 4476

```
Int[Cos[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(x*
Cos[d*(a + b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] + Simp[(b*d*n*x*Sin[d*(a +
 b*Log[c*x^n])])/(b^2*d^2*n^2 + 1), x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[
b^2*d^2*n^2 + 1, 0]
```

Rubi steps

$$\int \cos(\log(x))\,dx=\frac{1}{2}x\cos(\log(x))+\frac{1}{2}x\sin(\log(x))$$

Mathematica [A] time = 0.0027359, size = 17, normalized size = 1.

$$\frac{1}{2}x\sin(\log(x))+\frac{1}{2}x\cos(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[Log[x]],x]
```

```
[Out] (x*Cos[Log[x]])/2 + (x*Sin[Log[x]])/2
```

Maple [A] time = 0.001, size = 14, normalized size = 0.8

$$\frac{x\cos(\ln(x))}{2}+\frac{x\sin(\ln(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(ln(x)),x)
```

```
[Out] 1/2*x*cos(ln(x))+1/2*x*sin(ln(x))
```

Maxima [A] time = 0.942229, size = 14, normalized size = 0.82

$$\frac{1}{2}\,x(\cos(\log(x))+\sin(\log(x)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="maxima")
```

```
[Out] 1/2*x*(cos(log(x)) + sin(log(x)))
```

Fricas [A] time = 1.89331, size = 53, normalized size = 3.12

$$\frac{1}{2}\,x\cos(\log(x))+\frac{1}{2}\,x\sin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

Sympy [A] time = 0.402949, size = 15, normalized size = 0.88

$$\frac{x\sin(\log(x))}{2} + \frac{x\cos(\log(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(ln(x)),x)
```

```
[Out] x*sin(log(x))/2 + x*cos(log(x))/2
```

Giac [A] time = 1.06364, size = 18, normalized size = 1.06

$$\frac{1}{2}x\cos(\log(x)) + \frac{1}{2}x\sin(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(log(x)),x, algorithm="giac")
```

```
[Out] 1/2*x*cos(log(x)) + 1/2*x*sin(log(x))
```

3.150 $\int e^x \, dx$

Optimal. Leaf size=3

$$e^x$$

```
[Out] E^x
```

Rubi [A] time = 0.0009167, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2194}

$$e^x$$

Antiderivative was successfully verified.

```
[In] Int[E^x,x]
```

```
[Out] E^x
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^x \, dx = e^x$$

Mathematica [A] time = 0.0001061, size = 3, normalized size = 1.

$$e^x$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x,x]
```

```
[Out] E^x
```

Maple [A] time = 0., size = 3, normalized size = 1.

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x),x)
```

```
[Out] exp(x)
```

Maxima [A] time = 0.952167, size = 3, normalized size = 1.

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x, algorithm="maxima")
```

```
[Out] e^x
```

Fricas [A] time = 1.57722, size = 7, normalized size = 2.33

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x, algorithm="fricas")
```

```
[Out] e^x
```

Sympy [A] time = 0.041012, size = 2, normalized size = 0.67

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x)
```

```
[Out] exp(x)
```

Giac [A] time = 1.07981, size = 3, normalized size = 1.

$$e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x),x, algorithm="giac")
```

```
[Out] e^x
```

3.151 $\int a^x\,dx$

Optimal. Leaf size=8

$$\frac{a^x}{\log(a)}$$

```
[Out] a^x/Log[a]
```

Rubi [A] time = 0.0025112, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2194}

$$\frac{a^x}{\log(a)}$$

Antiderivative was successfully verified.

```
[In] Int[a^x,x]
```

```
[Out] a^x/Log[a]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int a^x\,dx = \frac{a^x}{\log(a)}$$

Mathematica [A] time = 0.0005415, size = 8, normalized size = 1.

$$\frac{a^x}{\log(a)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^x,x]
```

```
[Out] a^x/Log[a]
```

Maple [A] time = 0.003, size = 9, normalized size = 1.1

$$\frac{a^x}{\ln(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x,x)
```

```
[Out] a^x/ln(a)
```

Maxima [A] time = 0.930795, size = 11, normalized size = 1.38

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x,x, algorithm="maxima")
```

```
[Out] a^x/log(a)
```

Fricas [A] time = 1.6903, size = 16, normalized size = 2.

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x,x, algorithm="fricas")
```

```
[Out] a^x/log(a)
```

Sympy [A] time = 0.084479, size = 8, normalized size = 1.

$$\begin{cases} \frac{a^x}{\log(a)} & \text{for } \log(a) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x,x)
```

```
[Out] Piecewise((a**x/log(a), Ne(log(a), 0)), (x, True))
```

Giac [A] time = 1.07918, size = 11, normalized size = 1.38

$$\frac{a^x}{\log(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x,x, algorithm="giac")
```

```
[Out] a^x/log(a)
```

3.152 $\int e^{ax}\,dx$

Optimal. Leaf size=9

$$\frac{e^{ax}}{a}$$

```
[Out] E^(a*x)/a
```

Rubi [A] time = 0.0017289, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2194}

$$\frac{e^{ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a*x),x]
```

```
[Out] E^(a*x)/a
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{ax}\,dx = \frac{e^{ax}}{a}$$

Mathematica [A] time = 0.0011267, size = 9, normalized size = 1.

$$\frac{e^{ax}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a*x),x]
```

```
[Out] E^(a*x)/a
```

Maple [A] time = 0.001, size = 9, normalized size = 1.

$$\frac{\mathrm{e}^{ax}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x),x)
```

```
[Out] exp(a*x)/a
```

Maxima [A] time = 0.955901, size = 11, normalized size = 1.22

$$\frac{e^{(ax)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x),x, algorithm="maxima")
```

```
[Out] e^(a*x)/a
```

Fricas [A] time = 1.56077, size = 15, normalized size = 1.67

$$\frac{e^{(ax)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x),x, algorithm="fricas")
```

```
[Out] e^(a*x)/a
```

Sympy [A] time = 0.054832, size = 7, normalized size = 0.78

$$\begin{cases} \frac{e^{ax}}{a} & \text{for } a \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x),x)
```

```
[Out] Piecewise((exp(a*x)/a, Ne(a, 0)), (x, True))
```

Giac [A] time = 1.07132, size = 11, normalized size = 1.22

$$\frac{e^{(ax)}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x),x, algorithm="giac")
```

```
[Out] e^(a*x)/a
```

3.153 $\int \frac{e^{ax}}{x}\,dx$

Optimal. Leaf size=4

$$\text{ExpIntegralEi}(ax)$$

```
[Out] ExpIntegralEi[a*x]
```

Rubi [A] time = 0.0112163, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2178}

$$\text{ExpIntegralEi}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[E^(a*x)/x,x]
```

```
[Out] ExpIntegralEi[a*x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\int \frac{e^{ax}}{x}\,dx = \text{Ei}(ax)$$

Mathematica [A] time = 0.0054589, size = 4, normalized size = 1.

$$\text{ExpIntegralEi}(ax)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a*x)/x,x]
```

```
[Out] ExpIntegralEi[a*x]
```

Maple [A] time = 0.003, size = 9, normalized size = 2.3

$$-\mathrm{Ei}\,(1, -ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)/x,x)
```

```
[Out] -Ei(1,-a*x)
```

Maxima [A] time = 1.04897, size = 5, normalized size = 1.25

$$\mathrm{Ei}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)/x,x, algorithm="maxima")
```

```
[Out] Ei(a*x)
```

Fricas [A] time = 1.65542, size = 12, normalized size = 3.

$$\mathrm{Ei}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)/x,x, algorithm="fricas")
```

```
[Out] Ei(a*x)
```

Sympy [A] time = 0.963768, size = 3, normalized size = 0.75

$$\mathrm{Ei}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)/x,x)
```

```
[Out] Ei(a*x)
```

Giac [A] time = 1.07437, size = 5, normalized size = 1.25

$$\mathrm{Ei}\left(ax\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)/x,x, algorithm="giac")
```

```
[Out] Ei(a*x)
```

3.154 $\int \frac{1}{a+be^{mx}}\,dx$

Optimal. Leaf size=24

$$\frac{x}{a}-\frac{\log\left(a+be^{mx}\right)}{am}$$

```
[Out] x/a - Log[a + b*E^(m*x)]/(a*m)
```

Rubi [A] time = 0.0154089, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2282, 36, 29, 31}

$$\frac{x}{a}-\frac{\log\left(a+be^{mx}\right)}{am}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*E^(m*x))^(-1),x]
```

```
[Out] x/a - Log[a + b*E^(m*x)]/(a*m)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+be^{mx}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx, x, e^{mx}\right)}{m}\\ &= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx, x, e^{mx}\right)}{am} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx, x, e^{mx}\right)}{am}\\ &= \frac{x}{a} - \frac{\log\left(a+be^{mx}\right)}{am}\end{aligned}$$

Mathematica [A] time = 0.0054045, size = 24, normalized size = 1.

$$\frac{x}{a} - \frac{\log\left(a+be^{mx}\right)}{am}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*E^(m*x))^(-1),x]

[Out] x/a - Log[a + b*E^(m*x)]/(a*m)
```

Maple [A] time = 0.004, size = 31, normalized size = 1.3

$$\frac{\ln\left(e^{mx}\right)}{ma} - \frac{\ln\left(a+be^{mx}\right)}{ma}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*exp(m*x)),x)

[Out] 1/m/a*ln(exp(m*x))-ln(a+b*exp(m*x))/a/m
```

Maxima [A] time = 0.928376, size = 31, normalized size = 1.29

$$\frac{x}{a} - \frac{\log\left(be^{(mx)} + a\right)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(m*x)),x, algorithm="maxima")
```

```
[Out] x/a - log(b*e^(m*x) + a)/(a*m)
```

Fricas [A] time = 1.52804, size = 46, normalized size = 1.92

$$\frac{mx - \log\left(be^{(mx)} + a\right)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(m*x)),x, algorithm="fricas")
```

```
[Out] (m*x - log(b*e^(m*x) + a))/(a*m)
```

Sympy [A] time = 0.168962, size = 15, normalized size = 0.62

$$\frac{x}{a} - \frac{\log\left(\frac{a}{b} + e^{mx}\right)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(m*x)),x)
```

```
[Out] x/a - log(a/b + exp(m*x))/(a*m)
```

Giac [A] time = 1.07444, size = 32, normalized size = 1.33

$$\frac{x}{a} - \frac{\log\left(\left|be^{(mx)} + a\right|\right)}{am}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*exp(m*x)),x, algorithm="giac")
```

```
[Out] x/a - log(abs(b*e^(m*x) + a))/(a*m)
```

3.155 $\int \frac{e^{2x}}{1+e^x} \, dx$

Optimal. Leaf size=12

$$e^x - \log\left(e^x + 1\right)$$

```
[Out] E^x - Log[1 + E^x]
```

Rubi [A] time = 0.0209917, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2248, 43}

$$e^x - \log\left(e^x + 1\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(1 + E^x),x]

[Out] E^x - Log[1 + E^x]
```

Rule 2248

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(g*h*Log[G])/(d*e*Lo
g[F])]}, Dist[(Denominator[m]*G^(f*h - (c*g*h)/d))/(d*e*Log[F]), Subst[Int[
x^(Numerator[m] - 1)*(a + b*x^Denominator[m])^p, x], x, F^((e*(c + d*x))/De
nominator[m])], x] /; LeQ[m, -1] || GeQ[m, 1]] /; FreeQ[{F, G, a, b, c, d,
e, f, g, h, p}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{e^{2x}}{1+e^x}\,dx = \text{Subst}\left(\int \frac{x}{1+x}\,dx, x, e^x\right)$$
$$= \text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, e^x\right)$$
$$= e^x - \log\left(1+e^x\right)$$

Mathematica [A] time = 0.0085164, size = 12, normalized size = 1.

$$e^x - \log\left(e^x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(1 + E^x),x]
```

```
[Out] E^x - Log[1 + E^x]
```

Maple [A] time = 0.002, size = 11, normalized size = 0.9

$$\mathrm{e}^{x} - \ln\left(1+\mathrm{e}^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(1+exp(x)),x)
```

```
[Out] exp(x)-ln(1+exp(x))
```

Maxima [A] time = 0.950254, size = 14, normalized size = 1.17

$$e^x - \log\left(e^x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="maxima")
```

```
[Out] e^x - log(e^x + 1)
```

Fricas [A] time = 1.81852, size = 27, normalized size = 2.25

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="fricas")
```

```
[Out] e^x - log(e^x + 1)
```

Sympy [A] time = 0.098359, size = 8, normalized size = 0.67

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x)
```

```
[Out] exp(x) - log(exp(x) + 1)
```

Giac [A] time = 1.07216, size = 14, normalized size = 1.17

$$e^x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(1+exp(x)),x, algorithm="giac")
```

```
[Out] e^x - log(e^x + 1)
```

3.156 $\int e^{2x+ax}\,dx$

Optimal. Leaf size=13

$$\frac{e^{(a+2)x}}{a+2}$$

[Out] E^((2 + a)*x)/(2 + a)

Rubi [A] time = 0.0075806, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2227, 2194}

$$\frac{e^{(a+2)x}}{a+2}$$

Antiderivative was successfully verified.

[In] Int[E^(2*x + a*x),x]

[Out] E^((2 + a)*x)/(2 + a)

Rule 2227

```
Int[(u_.)*(F_)^((a_.) + (b_.)*(v_)), x_Symbol] :> Int[u*F^(a + b*NormalizeP
owerOfLinear[v, x]), x] /; FreeQ[{F, a, b}, x] && PolynomialQ[u, x] && Powe
rOfLinearQ[v, x] &&  !PowerOfLinearMatchQ[v, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{2x+ax}\,dx &= \int e^{(2+a)x}\,dx\\ &= \frac{e^{(2+a)x}}{2+a}\end{aligned}$$

Mathematica [A] time = 0.0033436, size = 13, normalized size = 1.

$$\frac{e^{(a+2)x}}{a+2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x + a*x),x]
```

```
[Out] E^((2 + a)*x)/(2 + a)
```

Maple [A] time = 0.002, size = 15, normalized size = 1.2

$$\frac{\mathrm{e}^{ax+2x}}{2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x+2*x),x)
```

```
[Out] 1/(2+a)*exp(a*x+2*x)
```

Maxima [A] time = 0.933516, size = 19, normalized size = 1.46

$$\frac{e^{(ax+2x)}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x+2*x),x, algorithm="maxima")
```

```
[Out] e^(a*x + 2*x)/(a + 2)
```

Fricas [A] time = 1.70432, size = 31, normalized size = 2.38

$$\frac{e^{((a+2)x)}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x+2*x),x, algorithm="fricas")
```

```
[Out] e^((a + 2)*x)/(a + 2)
```

Sympy [A] time = 0.103013, size = 14, normalized size = 1.08

$$\begin{cases} \frac{e^{ax+2x}}{a+2} & \text{for } a+2 \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x+2*x),x)
```

```
[Out] Piecewise((exp(a*x + 2*x)/(a + 2), Ne(a + 2, 0)), (x, True))
```

Giac [A] time = 1.08114, size = 19, normalized size = 1.46

$$\frac{e^{(ax+2x)}}{a+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x+2*x),x, algorithm="giac")
```

```
[Out] e^(a*x + 2*x)/(a + 2)
```

3.157 $\int \frac{1}{be^{-mx}+ae^{mx}}\,dx$

Optimal. Leaf size=31

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}m}$$

```
[Out] ArcTan[(Sqrt[a]*E^(m*x))/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*m)
```

Rubi [A] time = 0.0287608, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2282, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}m}$$

Antiderivative was successfully verified.

```
[In] Int[(b/E^(m*x) + a*E^(m*x))^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*E^(m*x))/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*m)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{be^{-mx}+ae^{mx}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{b+ax^2}\,dx, x, e^{mx}\right)}{m}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a}e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}m}$$

Mathematica [A] time = 0.0091206, size = 31, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{a}e^{mx}}{\sqrt{b}}\right)}{\sqrt{a}\sqrt{b}m}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b/E^(m*x) + a*E^(m*x))^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a]*E^(m*x))/Sqrt[b]]/(Sqrt[a]*Sqrt[b]*m)
```

Maple [A] time = 0.005, size = 22, normalized size = 0.7

$$\frac{1}{m}\arctan\left(ae^{mx}\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b/exp(m*x)+a*exp(m*x)),x)
```

```
[Out] 1/m/(a*b)^(1/2)*arctan(a*exp(m*x)/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(m*x)+a*exp(m*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73, size = 192, normalized size = 6.19

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{ae^{(2\,mx)}-2\,\sqrt{-ab}e^{(mx)}-b}{ae^{(2\,mx)}+b}\right)}{2\,abm}, -\frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}e^{(-mx)}}{a}\right)}{abm}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(m*x)+a*exp(m*x)),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((a*e^(2*m*x) - 2*sqrt(-a*b)*e^(m*x) - b)/(a*e^(2*m*x)
+ b))/(a*b*m), -sqrt(a*b)*arctan(sqrt(a*b)*e^(-m*x)/a)/(a*b*m)]
```

Sympy [A] time = 0.176954, size = 26, normalized size = 0.84

$$\frac{\operatorname{RootSum}\left(4z^2ab+1, \left(i \mapsto i\log\left(-2ia+e^{-mx}\right)\right)\right)}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(m*x)+a*exp(m*x)),x)
```

```
[Out] RootSum(4*_z**2*a*b + 1, Lambda(_i, _i*log(-2*_i*a + exp(-m*x))))/m
```

Giac [A] time = 1.09318, size = 28, normalized size = 0.9

$$\frac{\arctan\left(\frac{ae^{(mx)}}{\sqrt{ab}}\right)}{\sqrt{ab}m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b/exp(m*x)+a*exp(m*x)),x, algorithm="giac")
```

```
[Out] arctan(a*e^(m*x)/sqrt(a*b))/(sqrt(a*b)*m)
```

3.158 $\int e^{ax}x\,dx$

Optimal. Leaf size=21

$$\frac{xe^{ax}}{a}-\frac{e^{ax}}{a^2}$$

```
[Out] -(E^(a*x)/a^2) + (E^(a*x)*x)/a
```

Rubi [A] time = 0.0086599, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2176, 2194}

$$\frac{xe^{ax}}{a}-\frac{e^{ax}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a*x)*x,x]
```

```
[Out] -(E^(a*x)/a^2) + (E^(a*x)*x)/a
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{ax}x\,dx&=\frac{e^{ax}x}{a}-\frac{\int e^{ax}\,dx}{a}\\&=-\frac{e^{ax}}{a^2}+\frac{e^{ax}x}{a}\end{aligned}$$

Mathematica [A] time = 0.0056161, size = 14, normalized size = 0.67

$$\frac{e^{ax}(ax-1)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a*x)*x,x]
```

```
[Out] (E^(a*x)*(-1 + a*x))/a^2
```

Maple [A] time = 0.002, size = 14, normalized size = 0.7

$$\frac{(ax-1)\,\mathrm{e}^{ax}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)*x,x)
```

```
[Out] (a*x-1)*exp(a*x)/a^2
```

Maxima [A] time = 0.942147, size = 18, normalized size = 0.86

$$\frac{(ax-1)e^{(ax)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x,x, algorithm="maxima")
```

```
[Out] (a*x - 1)*e^(a*x)/a^2
```

Fricas [A] time = 1.6796, size = 31, normalized size = 1.48

$$\frac{(ax-1)e^{(ax)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x,x, algorithm="fricas")
```

```
[Out] (a*x - 1)*e^(a*x)/a^2
```

Sympy [A] time = 0.101134, size = 19, normalized size = 0.9

$$\begin{cases} \frac{(ax-1)e^{ax}}{a^2} & \text{for } a^2 \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x,x)
```

```
[Out] Piecewise(((a*x - 1)*exp(a*x)/a**2, Ne(a**2, 0)), (x**2/2, True))
```

Giac [A] time = 1.09533, size = 18, normalized size = 0.86

$$\frac{(ax-1)e^{(ax)}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x,x, algorithm="giac")
```

```
[Out] (a*x - 1)*e^(a*x)/a^2
```

3.159 $\int e^x x^{20} \, dx$

Optimal. Leaf size=163

$$e^x x^{20} - 20 e^x x^{19} + 380 e^x x^{18} - 6840 e^x x^{17} + 116280 e^x x^{16} - 1860480 e^x x^{15} + 27907200 e^x x^{14} - 390700800 e^x x^{13} + 5079110$$

```
[Out] 2432902008176640000*E^x - 2432902008176640000*E^x*x + 1216451004088320000*E
^x*x^2 - 405483668029440000*E^x*x^3 + 101370917007360000*E^x*x^4 - 20274183
401472000*E^x*x^5 + 3379030566912000*E^x*x^6 - 482718652416000*E^x*x^7 + 60
339831552000*E^x*x^8 - 6704425728000*E^x*x^9 + 670442572800*E^x*x^10 - 6094
9324800*E^x*x^11 + 5079110400*E^x*x^12 - 390700800*E^x*x^13 + 27907200*E^x*
x^14 - 1860480*E^x*x^15 + 116280*E^x*x^16 - 6840*E^x*x^17 + 380*E^x*x^18 -
20*E^x*x^19 + E^x*x^20
```

Rubi [A] time = 0.241771, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2176, 2194}

$$e^x x^{20} - 20 e^x x^{19} + 380 e^x x^{18} - 6840 e^x x^{17} + 116280 e^x x^{16} - 1860480 e^x x^{15} + 27907200 e^x x^{14} - 390700800 e^x x^{13} + 5079110$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x^20,x]
```

```
[Out] 2432902008176640000*E^x - 2432902008176640000*E^x*x + 1216451004088320000*E
^x*x^2 - 405483668029440000*E^x*x^3 + 101370917007360000*E^x*x^4 - 20274183
401472000*E^x*x^5 + 3379030566912000*E^x*x^6 - 482718652416000*E^x*x^7 + 60
339831552000*E^x*x^8 - 6704425728000*E^x*x^9 + 670442572800*E^x*x^10 - 6094
9324800*E^x*x^11 + 5079110400*E^x*x^12 - 390700800*E^x*x^13 + 27907200*E^x*
x^14 - 1860480*E^x*x^15 + 116280*E^x*x^16 - 6840*E^x*x^17 + 380*E^x*x^18 -
20*E^x*x^19 + E^x*x^20
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^x x^{20}\,dx = e^x x^{20} - 20\int e^x x^{19}\,dx$$

$$= -20e^x x^{19} + e^x x^{20} + 380\int e^x x^{18}\,dx$$

$$= 380e^x x^{18} - 20e^x x^{19} + e^x x^{20} - 6840\int e^x x^{17}\,dx$$

$$= -6840e^x x^{17} + 380e^x x^{18} - 20e^x x^{19} + e^x x^{20} + 116280\int e^x x^{16}\,dx$$

$$= 116280e^x x^{16} - 6840e^x x^{17} + 380e^x x^{18} - 20e^x x^{19} + e^x x^{20} - 1860480\int e^x x^{15}\,dx$$

$$= -1860480e^x x^{15} + 116280e^x x^{16} - 6840e^x x^{17} + 380e^x x^{18} - 20e^x x^{19} + e^x x^{20} + 27907200\int e^x x^{14}\,dx$$

$$= 27907200e^x x^{14} - 1860480e^x x^{15} + 116280e^x x^{16} - 6840e^x x^{17} + 380e^x x^{18} - 20e^x x^{19} + e^x x^{20} - 390700800\int e$$

$$= -390700800e^x x^{13} + 27907200e^x x^{14} - 1860480e^x x^{15} + 116280e^x x^{16} - 6840e^x x^{17} + 380e^x x^{18} - 20e^x x^{19} + e^x$$

$$= 5079110400e^x x^{12} - 390700800e^x x^{13} + 27907200e^x x^{14} - 1860480e^x x^{15} + 116280e^x x^{16} - 6840e^x x^{17} + 380e^x$$

$$= -60949324800e^x x^{11} + 5079110400e^x x^{12} - 390700800e^x x^{13} + 27907200e^x x^{14} - 1860480e^x x^{15} + 116280e^x x$$

$$= 670442572800e^x x^{10} - 60949324800e^x x^{11} + 5079110400e^x x^{12} - 390700800e^x x^{13} + 27907200e^x x^{14} - 18604$$

$$= -6704425728000e^x x^{9} + 670442572800e^x x^{10} - 60949324800e^x x^{11} + 5079110400e^x x^{12} - 390700800e^x x^{13} +$$

$$= 60339831552000e^x x^{8} - 6704425728000e^x x^{9} + 670442572800e^x x^{10} - 60949324800e^x x^{11} + 5079110400e^x x$$

$$= -482718652416000e^x x^{7} + 60339831552000e^x x^{8} - 6704425728000e^x x^{9} + 670442572800e^x x^{10} - 60949324$$

$$= 3379030566912000e^x x^{6} - 482718652416000e^x x^{7} + 60339831552000e^x x^{8} - 6704425728000e^x x^{9} + 670442$$

$$= -20274183401472000e^x x^{5} + 3379030566912000e^x x^{6} - 482718652416000e^x x^{7} + 60339831552000e^x x^{8} - 6$$

$$= 101370917007360000e^x x^{4} - 20274183401472000e^x x^{5} + 3379030566912000e^x x^{6} - 482718652416000e^x x^{7}$$

$$= -405483668029440000e^x x^{3} + 101370917007360000e^x x^{4} - 20274183401472000e^x x^{5} + 337903056691200$$

$$= 1216451004088320000e^x x^{2} - 405483668029440000e^x x^{3} + 101370917007360000e^x x^{4} - 202741834014720$$

$$= -2432902008176640000e^x x + 1216451004088320000e^x x^{2} - 405483668029440000e^x x^{3} + 10137091700736$$

$$= 2432902008176640000e^x - 2432902008176640000e^x x + 1216451004088320000e^x x^{2} - 405483668029440$$

Mathematica [A] time = 0.0100452, size = 102, normalized size = 0.63

$e^x \left(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 5079110400x^{12} - 60\right.$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x^20,x]
```

```
[Out] E^x*(2432902008176640000 - 2432902008176640000*x + 1216451004088320000*x^2
- 405483668029440000*x^3 + 101370917007360000*x^4 - 20274183401472000*x^5 +
 3379030566912000*x^6 - 482718652416000*x^7 + 60339831552000*x^8 - 67044257
28000*x^9 + 670442572800*x^10 - 60949324800*x^11 + 5079110400*x^12 - 390700
800*x^13 + 27907200*x^14 - 1860480*x^15 + 116280*x^16 - 6840*x^17 + 380*x^1
8 - 20*x^19 + x^20)
```

Maple [A] time = 0.003, size = 102, normalized size = 0.6

$\left(x^{20} - 20\,x^{19} + 380\,x^{18} - 6840\,x^{17} + 116280\,x^{16} - 1860480\,x^{15} + 27907200\,x^{14} - 390700800\,x^{13} + 5079110400\,x^{12} - 60\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x^20,x)
```

```
[Out] (x^20-20*x^19+380*x^18-6840*x^17+116280*x^16-1860480*x^15+27907200*x^14-390
700800*x^13+5079110400*x^12-60949324800*x^11+670442572800*x^10-670442572800
0*x^9+60339831552000*x^8-482718652416000*x^7+3379030566912000*x^6-202741834
01472000*x^5+101370917007360000*x^4-405483668029440000*x^3+1216451004088320
000*x^2-2432902008176640000*x+2432902008176640000)*exp(x)
```

Maxima [A] time = 0.948502, size = 136, normalized size = 0.83

$\left(x^{20} - 20\,x^{19} + 380\,x^{18} - 6840\,x^{17} + 116280\,x^{16} - 1860480\,x^{15} + 27907200\,x^{14} - 390700800\,x^{13} + 5079110400\,x^{12} - 60\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^20,x, algorithm="maxima")
```

```
[Out] (x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^15 + 27907
200*x^14 - 390700800*x^13 + 5079110400*x^12 - 60949324800*x^11 + 6704425728
00*x^10 - 6704425728000*x^9 + 60339831552000*x^8 - 482718652416000*x^7 + 33
79030566912000*x^6 - 20274183401472000*x^5 + 101370917007360000*x^4 - 40548
3668029440000*x^3 + 1216451004088320000*x^2 - 2432902008176640000*x + 24329
02008176640000)*e^x
```

Fricas [A] time = 1.5842, size = 535, normalized size = 3.28

$\big(x^{20} - 20\,x^{19} + 380\,x^{18} - 6840\,x^{17} + 116280\,x^{16} - 1860480\,x^{15} + 27907200\,x^{14} - 390700800\,x^{13} + 5079110400\,x^{12} - 609$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^20,x, algorithm="fricas")
```

```
[Out] (x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^15 + 27907
200*x^14 - 390700800*x^13 + 5079110400*x^12 - 60949324800*x^11 + 6704425728
00*x^10 - 6704425728000*x^9 + 60339831552000*x^8 - 482718652416000*x^7 + 33
79030566912000*x^6 - 20274183401472000*x^5 + 101370917007360000*x^4 - 40548
3668029440000*x^3 + 1216451004088320000*x^2 - 2432902008176640000*x + 24329
02008176640000)*e^x
```

Sympy [A] time = 0.127938, size = 102, normalized size = 0.63

$\big(x^{20} - 20x^{19} + 380x^{18} - 6840x^{17} + 116280x^{16} - 1860480x^{15} + 27907200x^{14} - 390700800x^{13} + 5079110400x^{12} - 609493$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x**20,x)
```

```
[Out] (x**20 - 20*x**19 + 380*x**18 - 6840*x**17 + 116280*x**16 - 1860480*x**15 +
 27907200*x**14 - 390700800*x**13 + 5079110400*x**12 - 60949324800*x**11 +
670442572800*x**10 - 6704425728000*x**9 + 60339831552000*x**8 - 48271865241
6000*x**7 + 3379030566912000*x**6 - 20274183401472000*x**5 + 10137091700736
0000*x**4 - 405483668029440000*x**3 + 1216451004088320000*x**2 - 2432902008
176640000*x + 2432902008176640000)*exp(x)
```

Giac [A] time = 1.08302, size = 136, normalized size = 0.83

$$\left(x^{20} - 20\,x^{19} + 380\,x^{18} - 6840\,x^{17} + 116280\,x^{16} - 1860480\,x^{15} + 27907200\,x^{14} - 390700800\,x^{13} + 5079110400\,x^{12} - 60$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^20,x, algorithm="giac")
```

```
[Out] (x^20 - 20*x^19 + 380*x^18 - 6840*x^17 + 116280*x^16 - 1860480*x^15 + 27907
200*x^14 - 390700800*x^13 + 5079110400*x^12 - 60949324800*x^11 + 6704425728
00*x^10 - 6704425728000*x^9 + 60339831552000*x^8 - 482718652416000*x^7 + 33
79030566912000*x^6 - 20274183401472000*x^5 + 101370917007360000*x^4 - 40548
3668029440000*x^3 + 1216451004088320000*x^2 - 2432902008176640000*x + 24329
02008176640000)*e^x
```

3.160 $\int a^x b^{-x} \, dx$

Optimal. Leaf size=18

$$\frac{a^x b^{-x}}{\log(a) - \log(b)}$$

```
[Out] a^x/(b^x*(Log[a] - Log[b]))
```

Rubi [A] time = 0.02053, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2287, 2194}

$$\frac{a^x b^{-x}}{\log(a) - \log(b)}$$

Antiderivative was successfully verified.

```
[In] Int[a^x/b^x,x]
```

```
[Out] a^x/(b^x*(Log[a] - Log[b]))
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int a^x b^{-x} \, dx &= \int e^{x(\log(a)-\log(b))} \, dx \\ &= \frac{a^x b^{-x}}{\log(a) - \log(b)} \end{aligned}$$

Mathematica [A] time = 0.0130107, size = 18, normalized size = 1.

$$\frac{a^x b^{-x}}{\log(a) - \log(b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^x/b^x,x]
```

```
[Out] a^x/(b^x*(Log[a] - Log[b]))
```

Maple [A] time = 0.004, size = 19, normalized size = 1.1

$$\frac{a^x}{b^x (\ln(a) - \ln(b))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x/(b^x),x)
```

```
[Out] a^x/(b^x)/(ln(a)-ln(b))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/(b^x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.55677, size = 39, normalized size = 2.17

$$\frac{a^x}{b^x(\log(a) - \log(b))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/(b^x),x, algorithm="fricas")
```

```
[Out] a^x/(b^x*(log(a) - log(b)))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x/(b**x),x)
```

```
[Out] Exception raised: TypeError
```

Giac [C] time = 1.13389, size = 292, normalized size = 16.22

$$2\left(\frac{2\left(\log\left(|a|\right)-\log\left(|b|\right)\right)\cos\left(-\frac{1}{2}\pi x\operatorname{sgn}\left(a\right)+\frac{1}{2}\pi x\operatorname{sgn}\left(b\right)\right)}{\left(\pi\operatorname{sgn}\left(a\right)-\pi\operatorname{sgn}\left(b\right)\right)^{2}+4\left(\log\left(|a|\right)-\log\left(|b|\right)\right)^{2}}-\frac{\left(\pi\operatorname{sgn}\left(a\right)-\pi\operatorname{sgn}\left(b\right)\right)\sin\left(-\frac{1}{2}\pi x\operatorname{sgn}\left(a\right)+\frac{1}{2}\pi x\operatorname{sgn}\left(b\right)\right)}{\left(\pi\operatorname{sgn}\left(a\right)-\pi\operatorname{sgn}\left(b\right)\right)^{2}+4\left(\log\left(|a|\right)-\log\left(|b|\right)\right)^{2}}\right)e^{(x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/(b^x),x, algorithm="giac")
```

```
[Out] 2*(2*(log(abs(a)) - log(abs(b)))*cos(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((
pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) - log(abs(b)))^2) - (pi*sgn(a) -
pi*sgn(b))*sin(-1/2*pi*x*sgn(a) + 1/2*pi*x*sgn(b))/((pi*sgn(a) - pi*sgn(b))
^2 + 4*(log(abs(a)) - log(abs(b)))^2))*e^(x*(log(abs(a)) - log(abs(b)))) -
1/2*I*(-2*I*e^(1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b))/(I*pi*sgn(a) - I*pi*s
gn(b) + 2*log(abs(a)) - 2*log(abs(b))) + 2*I*e^(-1/2*I*pi*x*sgn(a) + 1/2*I*
pi*x*sgn(b))/(-I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a)) - 2*log(abs(b))))*
e^(x*(log(abs(a)) - log(abs(b))))
```

3.161 $\int a^x b^x \, dx$

Optimal. Leaf size=14

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

```
[Out] (a^x*b^x)/(Log[a] + Log[b])
```

Rubi [A] time = 0.0151084, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2287, 2194}

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

Antiderivative was successfully verified.

```
[In] Int[a^x*b^x,x]
```

```
[Out] (a^x*b^x)/(Log[a] + Log[b])
```

Rule 2287

```
Int[(u_.)*(F_)^(v_)*(G_)^(w_), x_Symbol] :> With[{z = v*Log[F] + w*Log[G]},
 Int[u*NormalizeIntegrand[E^z, x], x] /; BinomialQ[z, x] || (PolynomialQ[z,
 x] && LeQ[Exponent[z, x], 2])] /; FreeQ[{F, G}, x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int a^x b^x \, dx &= \int e^{x(\log(a)+\log(b))} \, dx \\ &= \frac{a^x b^x}{\log(a) + \log(b)} \end{aligned}$$

Mathematica [A] time = 0.0068867, size = 14, normalized size = 1.

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^x*b^x,x]
```

```
[Out] (a^x*b^x)/(Log[a] + Log[b])
```

Maple [A] time = 0.003, size = 15, normalized size = 1.1

$$\frac{a^x b^x}{\ln(a) + \ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x*b^x,x)
```

```
[Out] a^x*b^x/(ln(a)+ln(b))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*b^x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.67334, size = 36, normalized size = 2.57

$$\frac{a^x b^x}{\log(a) + \log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*b^x,x, algorithm="fricas")
```

```
[Out] a^x*b^x/(log(a) + log(b))
```

Sympy [A] time = 0.651024, size = 24, normalized size = 1.71

$$\begin{cases} \frac{a^x b^x}{\log(a)+\log(b)} & \text{for } a \neq \frac{1}{b} \\ \tilde{\infty} b^x \left(\frac{1}{b}\right)^x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x*b**x,x)
```

```
[Out] Piecewise((a**x*b**x/(log(a) + log(b)), Ne(a, 1/b)), (zoo*b**x*(1/b)**x, Tr
ue))
```

Giac [C] time = 1.16279, size = 320, normalized size = 22.86

$$2\left(\frac{2\left(\log\left(|a|\right)+\log\left(|b|\right)\right)\cos\left(-\frac{1}{2}\pi x\operatorname{sgn}\left(a\right)-\frac{1}{2}\pi x\operatorname{sgn}\left(b\right)+\pi x\right)}{\left(2\pi-\pi\operatorname{sgn}\left(a\right)-\pi\operatorname{sgn}\left(b\right)\right)^2+4\left(\log\left(|a|\right)+\log\left(|b|\right)\right)^2}+\frac{\left(2\pi-\pi\operatorname{sgn}\left(a\right)-\pi\operatorname{sgn}\left(b\right)\right)\sin\left(-\frac{1}{2}\pi x\operatorname{sgn}\left(a\right)-\frac{1}{2}\pi\right.}{\left(2\pi-\pi\operatorname{sgn}\left(a\right)-\pi\operatorname{sgn}\left(b\right)\right)^2+4\left(\log\left(|a|\right)+\log\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*b^x,x, algorithm="giac")
```

```
[Out] 2*(2*(log(abs(a)) + log(abs(b)))*cos(-1/2*pi*x*sgn(a) - 1/2*pi*x*sgn(b) + p
i*x)/((2*pi - pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) + log(abs(b)))^2) +
 (2*pi - pi*sgn(a) - pi*sgn(b))*sin(-1/2*pi*x*sgn(a) - 1/2*pi*x*sgn(b) + pi
*x)/((2*pi - pi*sgn(a) - pi*sgn(b))^2 + 4*(log(abs(a)) + log(abs(b)))^2))*e
^(x*(log(abs(a)) + log(abs(b)))) - 1/2*I*(-2*I*e^(1/2*I*pi*x*sgn(a) + 1/2*I
*pi*x*sgn(b) - I*pi*x)/(-2*I*pi + I*pi*sgn(a) + I*pi*sgn(b) + 2*log(abs(a))
 + 2*log(abs(b))) + 2*I*e^(-1/2*I*pi*x*sgn(a) - 1/2*I*pi*x*sgn(b) + I*pi*x)
/(2*I*pi - I*pi*sgn(a) - I*pi*sgn(b) + 2*log(abs(a)) + 2*log(abs(b))))*e^(x
*(log(abs(a)) + log(abs(b))))
```

3.162 $\int \frac{a^x}{x^2}\,dx$

Optimal. Leaf size=17

$$\log(a)\text{ExpIntegralEi}(x\log(a)) - \frac{a^x}{x}$$

```
[Out] -(a^x/x) + ExpIntegralEi[x*Log[a]]*Log[a]
```

Rubi [A] time = 0.0190059, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2177, 2178}

$$\log(a)\text{ExpIntegralEi}(x\log(a)) - \frac{a^x}{x}$$

Antiderivative was successfully verified.

```
[In] Int[a^x/x^2,x]
```

```
[Out] -(a^x/x) + ExpIntegralEi[x*Log[a]]*Log[a]
```

Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !$UseGamma === True
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\int \frac{a^x}{x^2}\,dx = -\frac{a^x}{x} + \log(a)\int \frac{a^x}{x}\,dx$$
$$= -\frac{a^x}{x} + \mathbf{Ei}(x\log(a))\log(a)$$

Mathematica [A] time = 0.0107686, size = 17, normalized size = 1.

$$\log(a)\mathbf{ExpIntegralEi}(x\log(a)) - \frac{a^x}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[a^x/x^2,x]
```

```
[Out] -(a^x/x) + ExpIntegralEi[x*Log[a]]*Log[a]
```

Maple [A] time = 0.02, size = 21, normalized size = 1.2

$$-\frac{a^x}{x} - \ln(a)\,\mathrm{Ei}(1, -x\ln(a))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x/x^2,x)
```

```
[Out] -a^x/x-ln(a)*Ei(1,-x*ln(a))
```

Maxima [A] time = 1.03775, size = 14, normalized size = 0.82

$$\Gamma(-1, -x\log(a))\log(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/x^2,x, algorithm="maxima")
```

```
[Out] gamma(-1, -x*log(a))*log(a)
```

Fricas [A] time = 1.67461, size = 45, normalized size = 2.65

$$\frac{x\text{Ei}\left(x\log\left(a\right)\right)\log\left(a\right)-a^{x}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/x^2,x, algorithm="fricas")
```

```
[Out] (x*Ei(x*log(a))*log(a) - a^x)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a^{x}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x/x**2,x)
```

```
[Out] Integral(a**x/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a^{x}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x/x^2,x, algorithm="giac")
```

```
[Out] integrate(a^x/x^2, x)
```

3.163 $\int \frac{a^x x}{(1+bx)^2} dx$

Optimal. Leaf size=64

$$\frac{a^{-1/b}\text{ExpIntegralEi}\left(\frac{\log(a)(bx+1)}{b}\right)}{b^2}-\frac{a^{-1/b}\log(a)\text{ExpIntegralEi}\left(\frac{\log(a)(bx+1)}{b}\right)}{b^3}+\frac{a^x}{b^2(bx+1)}$$

```
[Out] a^x/(b^2*(1 + b*x)) + ExpIntegralEi[((1 + b*x)*Log[a])/b]/(a^b^(-1)*b^2) -
(ExpIntegralEi[((1 + b*x)*Log[a])/b]*Log[a])/(a^b^(-1)*b^3)
```

Rubi [A] time = 0.0974545, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2199, 2177, 2178}

$$\frac{a^{-1/b}\text{ExpIntegralEi}\left(\frac{\log(a)(bx+1)}{b}\right)}{b^2}-\frac{a^{-1/b}\log(a)\text{ExpIntegralEi}\left(\frac{\log(a)(bx+1)}{b}\right)}{b^3}+\frac{a^x}{b^2(bx+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a^x*x)/(1 + b*x)^2,x]
```

```
[Out] a^x/(b^2*(1 + b*x)) + ExpIntegralEi[((1 + b*x)*Log[a])/b]/(a^b^(-1)*b^2) -
(ExpIntegralEi[((1 + b*x)*Log[a])/b]*Log[a])/(a^b^(-1)*b^3)
```

Rule 2199

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
 c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] &&  !$UseGamma === True
```

Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !$UseGamma === True
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}
\int \frac{a^x x}{(1+bx)^2}\,dx &= \int \left(-\frac{a^x}{b(1+bx)^2}+\frac{a^x}{b(1+bx)}\right)dx\\
&= -\frac{\int \frac{a^x}{(1+bx)^2}\,dx}{b}+\frac{\int \frac{a^x}{1+bx}\,dx}{b}\\
&= \frac{a^x}{b^2(1+bx)}+\frac{a^{-1/b}\text{Ei}\left(\frac{(1+bx)\log(a)}{b}\right)}{b^2}-\frac{\log(a)\int \frac{a^x}{1+bx}\,dx}{b^2}\\
&= \frac{a^x}{b^2(1+bx)}+\frac{a^{-1/b}\text{Ei}\left(\frac{(1+bx)\log(a)}{b}\right)}{b^2}-\frac{a^{-1/b}\text{Ei}\left(\frac{(1+bx)\log(a)}{b}\right)\log(a)}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.126643, size = 43, normalized size = 0.67

$$\frac{a^{-1/b}(b-\log(a))\text{ExpIntegralEi}\left(\frac{\log(a)(bx+1)}{b}\right)+\frac{ba^x}{bx+1}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^x*x)/(1 + b*x)^2,x]
```

```
[Out] ((a^x*b)/(1 + b*x) + (ExpIntegralEi[((1 + b*x)*Log[a])/b]*(b - Log[a]))/a^b
^(-1))/b^3
```

Maple [A] time = 0.032, size = 79, normalized size = 1.2

$$-\frac{1}{b^2}a^{-b^{-1}}\text{Ei}\left(1,-x\ln(a)-\frac{\ln(a)}{b}\right)+\frac{a^x\ln(a)}{b^3}\left(x\ln(a)+\frac{\ln(a)}{b}\right)^{-1}+\frac{\ln(a)}{b^3}a^{-b^{-1}}\text{Ei}\left(1,-x\ln(a)-\frac{\ln(a)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a^x*x/(b*x+1)^2,x)
```

```
[Out] -1/b^2*a^(-1/b)*Ei(1,-x*ln(a)-ln(a)/b)+ln(a)/b^3*a^x/(x*ln(a)+ln(a)/b)+ln(a
)/b^3*a^(-1/b)*Ei(1,-x*ln(a)-ln(a)/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{a^x x}{b^2x^2 \log(a) + 2\,bx \log(a) + \log(a)} + \int \frac{(bx-1)a^x}{b^3x^3 \log(a) + 3\,b^2x^2 \log(a) + 3\,bx \log(a) + \log(a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*x/(b*x+1)^2,x, algorithm="maxima")
```

```
[Out] a^x*x/(b^2*x^2*log(a) + 2*b*x*log(a) + log(a)) + integrate((b*x - 1)*a^x/(b
^3*x^3*log(a) + 3*b^2*x^2*log(a) + 3*b*x*log(a) + log(a)), x)
```

Fricas [A] time = 1.76654, size = 117, normalized size = 1.83

$$\frac{a^x b + \frac{\left(b^2x-(bx+1)\log(a)+b\right)\mathrm{Ei}\left(\frac{(bx+1)\log(a)}{b}\right)}{a^{\left(\frac{1}{b}\right)}}}{b^4x + b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*x/(b*x+1)^2,x, algorithm="fricas")
```

```
[Out] (a^x*b + (b^2*x - (b*x + 1)*log(a) + b)*Ei((b*x + 1)*log(a)/b)/a^(1/b))/(b^
4*x + b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^x x}{(bx+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a**x*x/(b*x+1)**2,x)
```

```
[Out] Integral(a**x*x/(b*x + 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a^x x}{(bx+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a^x*x/(b*x+1)^2,x, algorithm="giac")
```

```
[Out] integrate(a^x*x/(b*x + 1)^2, x)
```

3.164 $\int \frac{e^{ax}x}{(1+ax)^2}\,dx$

Optimal. Leaf size=16

$$\frac{e^{ax}}{a^2(ax+1)}$$

```
[Out] E^(a*x)/(a^2*(1 + a*x))
```

Rubi [A] time = 0.0315516, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2197}

$$\frac{e^{ax}}{a^2(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(E^(a*x)*x)/(1 + a*x)^2,x]
```

```
[Out] E^(a*x)/(a^2*(1 + a*x))
```

Rule 2197

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> With[{b = Coefficient[v
, x, 1], d = Coefficient[u, x, 0], e = Coefficient[u, x, 1], f = Coefficien
t[w, x, 0], g = Coefficient[w, x, 1]}, Simp[(g*u^(m + 1)*F^(c*v))/(b*c*e*Lo
g[F]), x] /; EqQ[e*g*(m + 1) - b*c*(e*f - d*g)*Log[F], 0]] /; FreeQ[{F, c,
m}, x] && LinearQ[{u, v, w}, x]
```

Rubi steps

$$\int \frac{e^{ax}x}{(1+ax)^2}\,dx = \frac{e^{ax}}{a^2(1+ax)}$$

Mathematica [A] time = 0.045405, size = 16, normalized size = 1.

$$\frac{e^{ax}}{a^2(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(a*x)*x)/(1 + a*x)^2,x]
```

```
[Out] E^(a*x)/(a^2*(1 + a*x))
```

Maple [A] time = 0.003, size = 16, normalized size = 1.

$$\frac{e^{ax}}{a^2 (ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)*x/(a*x+1)^2,x)
```

```
[Out] exp(a*x)/a^2/(a*x+1)
```

Maxima [A] time = 0.950543, size = 22, normalized size = 1.38

$$\frac{e^{(ax)}}{a^3 x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x/(a*x+1)^2,x, algorithm="maxima")
```

```
[Out] e^(a*x)/(a^3*x + a^2)
```

Fricas [A] time = 1.55689, size = 31, normalized size = 1.94

$$\frac{e^{(ax)}}{a^3 x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x/(a*x+1)^2,x, algorithm="fricas")
```

```
[Out] e^(a*x)/(a^3*x + a^2)
```

Sympy [A] time = 0.116405, size = 12, normalized size = 0.75

$$\frac{e^{ax}}{a^3x + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x/(a*x+1)**2,x)
```

```
[Out] exp(a*x)/(a**3*x + a**2)
```

Giac [A] time = 1.10811, size = 38, normalized size = 2.38

$$\frac{e^{\left(-(ax+1)\left(\frac{1}{ax+1}-1\right)\right)}}{(ax+1)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*x/(a*x+1)^2,x, algorithm="giac")
```

```
[Out] e^(-(a*x + 1)*(1/(a*x + 1) - 1))/((a*x + 1)*a^2)
```

3.165 $\int k^{x^2} x \, dx$

Optimal. Leaf size=13

$$\frac{k^{x^2}}{2\log(k)}$$

```
[Out] k^x^2/(2*Log[k])
```

Rubi [A] time = 0.0072021, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2209}

$$\frac{k^{x^2}}{2\log(k)}$$

Antiderivative was successfully verified.

```
[In] Int[k^x^2*x,x]
```

```
[Out] k^x^2/(2*Log[k])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int k^{x^2} x \, dx = \frac{k^{x^2}}{2\log(k)}$$

Mathematica [A] time = 0.0016624, size = 13, normalized size = 1.

$$\frac{k^{x^2}}{2\log(k)}$$

Antiderivative was successfully verified.

```
[In] Integrate[k^x^2*x,x]
```

```
[Out] k^x^2/(2*Log[k])
```

Maple [A] time = 0.003, size = 12, normalized size = 0.9

$$\frac{k^{x^2}}{2\ln(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(k^(x^2)*x,x)
```

```
[Out] 1/2*k^(x^2)/ln(k)
```

Maxima [A] time = 0.941191, size = 15, normalized size = 1.15

$$\frac{k^{(x^2)}}{2\log(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(k^(x^2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*k^(x^2)/log(k)
```

Fricas [A] time = 1.73341, size = 27, normalized size = 2.08

$$\frac{k^{(x^2)}}{2\log(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(k^(x^2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*k^(x^2)/log(k)
```

Sympy [A] time = 0.095293, size = 17, normalized size = 1.31

$$\begin{cases} \frac{k^{x^2}}{2\log(k)} & \text{for } 2\log(k) \neq 0 \\ \frac{x^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(k**(x**2)*x,x)
```

```
[Out] Piecewise((k**(x**2)/(2*log(k)), Ne(2*log(k), 0)), (x**2/2, True))
```

Giac [A] time = 1.08108, size = 15, normalized size = 1.15

$$\frac{k^{(x^2)}}{2\log(k)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(k^(x^2)*x,x, algorithm="giac")
```

```
[Out] 1/2*k^(x^2)/log(k)
```

3.166 $\int e^{x^2} dx$

Optimal. Leaf size=11

$$\frac{1}{2}\sqrt{\pi}\text{Erfi}(x)$$

```
[Out] (Sqrt[Pi]*Erfi[x])/2
```

Rubi [A] time = 0.0027621, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2204}

$$\frac{1}{2}\sqrt{\pi}\text{Erfi}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2,x]
```

```
[Out] (Sqrt[Pi]*Erfi[x])/2
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rubi steps

$$\int e^{x^2} dx = \frac{1}{2}\sqrt{\pi}\text{erfi}(x)$$

Mathematica [A] time = 0.0014612, size = 11, normalized size = 1.

$$\frac{1}{2}\sqrt{\pi}\text{Erfi}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2,x]
```

```
[Out] (Sqrt[Pi]*Erfi[x])/2
```

Maple [A] time = 0.003, size = 8, normalized size = 0.7

$$\frac{\operatorname{erfi}(x)\sqrt{\pi}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2),x)
```

```
[Out] 1/2*erfi(x)*Pi^(1/2)
```

Maxima [C] time = 0.94179, size = 12, normalized size = 1.09

$$-\frac{1}{2}i\sqrt{\pi}\operatorname{erf}(ix)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*I*sqrt(pi)*erf(I*x)
```

Fricas [A] time = 1.60826, size = 30, normalized size = 2.73

$$\frac{1}{2}\sqrt{\pi}\operatorname{erfi}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(pi)*erfi(x)
```

Sympy [A] time = 0.242371, size = 8, normalized size = 0.73

$$\frac{\sqrt{\pi}\,\mathrm{erfi}\,(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2),x)
```

```
[Out] sqrt(pi)*erfi(x)/2
```

Giac [C] time = 1.10665, size = 12, normalized size = 1.09

$$\frac{1}{2}i\,\sqrt{\pi}\,\mathrm{erf}\,(-i\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2),x, algorithm="giac")
```

```
[Out] 1/2*I*sqrt(pi)*erf(-I*x)
```

3.167 $\int e^{x^2} x \, dx$

Optimal. Leaf size=9

$$\frac{e^{x^2}}{2}$$

```
[Out] E^x^2/2
```

Rubi [A] time = 0.0065963, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2209}

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2*x,x]

[Out] E^x^2/2
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int e^{x^2} x \, dx = \frac{e^{x^2}}{2}$$

Mathematica [A] time = 0.0010756, size = 9, normalized size = 1.

$$\frac{e^{x^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2*x,x]
```

```
[Out] E^x^2/2
```

Maple [A] time = 0.002, size = 7, normalized size = 0.8

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)*x,x)
```

```
[Out] 1/2*exp(x^2)
```

Maxima [A] time = 0.935613, size = 8, normalized size = 0.89

$$\frac{1}{2}e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="maxima")
```

```
[Out] 1/2*e^(x^2)
```

Fricas [A] time = 1.66456, size = 18, normalized size = 2.

$$\frac{1}{2}e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="fricas")
```

```
[Out] 1/2*e^(x^2)
```

Sympy [A] time = 0.081193, size = 5, normalized size = 0.56

$$\frac{e^{x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)*x,x)
```

```
[Out] exp(x**2)/2
```

Giac [A] time = 1.08081, size = 8, normalized size = 0.89

$$\frac{1}{2}\,e^{\left(x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)*x,x, algorithm="giac")
```

```
[Out] 1/2*e^(x^2)
```

3.168 $\int \frac{e^{\frac{1}{x}}(1+x)}{x^4} \, dx$

Optimal. Leaf size=27

$$-\frac{e^{\frac{1}{x}}}{x^2}-e^{\frac{1}{x}}+\frac{e^{\frac{1}{x}}}{x}$$

```
[Out] -E^x^(-1) - E^x^(-1)/x^2 + E^x^(-1)/x
```

Rubi [A] time = 0.105, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6742, 2212, 2209}

$$-\frac{e^{\frac{1}{x}}}{x^2}-e^{\frac{1}{x}}+\frac{e^{\frac{1}{x}}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(E^x^(-1)*(1 + x))/x^4,x]
```

```
[Out] -E^x^(-1) - E^x^(-1)/x^2 + E^x^(-1)/x
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
```

```
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{\frac{1}{x}}(1+x)}{x^4}\,dx &= \int \left(\frac{e^{\frac{1}{x}}}{x^4}+\frac{e^{\frac{1}{x}}}{x^3}\right)dx \\
&= \int \frac{e^{\frac{1}{x}}}{x^4}\,dx+\int \frac{e^{\frac{1}{x}}}{x^3}\,dx \\
&= -\frac{e^{\frac{1}{x}}}{x^2}-\frac{e^{\frac{1}{x}}}{x}-2\int \frac{e^{\frac{1}{x}}}{x^3}\,dx-\int \frac{e^{\frac{1}{x}}}{x^2}\,dx \\
&= e^{\frac{1}{x}}-\frac{e^{\frac{1}{x}}}{x^2}+\frac{e^{\frac{1}{x}}}{x}+2\int \frac{e^{\frac{1}{x}}}{x^2}\,dx \\
&= -e^{\frac{1}{x}}-\frac{e^{\frac{1}{x}}}{x^2}+\frac{e^{\frac{1}{x}}}{x}
\end{aligned}$$

Mathematica [A] time = 0.0115183, size = 17, normalized size = 0.63

$$\frac{e^{\frac{1}{x}}\left(-x^2+x-1\right)}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^x^(-1)*(1 + x))/x^4,x]
```

```
[Out] (E^x^(-1)*(-1 + x - x^2))/x^2
```

Maple [A] time = 0.002, size = 18, normalized size = 0.7

$$-\frac{\left(x^2-x+1\right)e^{x^{-1}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1/x)*(1+x)/x^4,x)
```

```
[Out] -(x^2-x+1)*exp(1/x)/x^2
```

Maxima [C] time = 1.02522, size = 23, normalized size = 0.85

$$-\Gamma\left(3, -\frac{1}{x}\right) + \Gamma\left(2, -\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/x)*(1+x)/x^4,x, algorithm="maxima")
```

```
[Out] -gamma(3, -1/x) + gamma(2, -1/x)
```

Fricas [A] time = 1.50407, size = 38, normalized size = 1.41

$$-\frac{\left(x^2 - x + 1\right)e^{\frac{1}{x}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/x)*(1+x)/x^4,x, algorithm="fricas")
```

```
[Out] -(x^2 - x + 1)*e^(1/x)/x^2
```

Sympy [A] time = 0.095526, size = 14, normalized size = 0.52

$$\frac{\left(-x^2 + x - 1\right)e^{\frac{1}{x}}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/x)*(1+x)/x**4,x)
```

```
[Out] (-x**2 + x - 1)*exp(1/x)/x**2
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)e^{\frac{1}{x}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/x)*(1+x)/x^4,x, algorithm="giac")
```

```
[Out] integrate((x + 1)*e^(1/x)/x^4, x)
```

3.169 $\int \frac{e^{1-e^{x^2}x+2x^2}(x+2x^3)}{\left(1-e^{x^2}x\right)^2}\,dx$

Optimal. Leaf size=25

$$-\frac{e^{1-e^{x^2}x}}{e^{x^2}x-1}$$

```
[Out] -(E^(1 - E^x^2*x)/(-1 + E^x^2*x))
```

Rubi [F] time = 0.981599, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{e^{1-e^{x^2}x+2x^2}\left(x+2x^3\right)}{\left(1-e^{x^2}x\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(E^(1 - E^x^2*x + 2*x^2)*(x + 2*x^3))/(1 - E^x^2*x)^2,x]
```

```
[Out] Defer[Int][(E^(1 - E^x^2*x + 2*x^2)*x)/(-1 + E^x^2*x)^2, x] + 2*Defer[Int][
(E^(1 - E^x^2*x + 2*x^2)*x^3)/(-1 + E^x^2*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{1-e^{x^2}x+2x^2}\left(x+2x^3\right)}{\left(1-e^{x^2}x\right)^2}\,dx &= \int \frac{e^{1-e^{x^2}x+2x^2}x\left(1+2x^2\right)}{\left(1-e^{x^2}x\right)^2}\,dx\\
&= \int \left(\frac{e^{1-e^{x^2}x+2x^2}x}{\left(-1+e^{x^2}x\right)^2}+\frac{2e^{1-e^{x^2}x+2x^2}x^3}{\left(-1+e^{x^2}x\right)^2}\right)dx\\
&= 2\int \frac{e^{1-e^{x^2}x+2x^2}x^3}{\left(-1+e^{x^2}x\right)^2}\,dx+\int \frac{e^{1-e^{x^2}x+2x^2}x}{\left(-1+e^{x^2}x\right)^2}\,dx
\end{aligned}$$

Mathematica [A] time = 0.164539, size = 25, normalized size = 1.

$$-\frac{e^{1-e^{x^2}x}}{e^{x^2}x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^(1 - E^x^2*x + 2*x^2)*(x + 2*x^3))/(1 - E^x^2*x)^2,x]
```

```
[Out] -(E^(1 - E^x^2*x)/(-1 + E^x^2*x))
```

Maple [A] time = 0.06, size = 23, normalized size = 0.9

$$-\frac{e^{1-e^{x^2}x}}{-1+e^{x^2}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x)
```

```
[Out] -exp(1-exp(x^2)*x)/(-1+exp(x^2)*x)
```

Maxima [A] time = 1.27126, size = 30, normalized size = 1.2

$$-\frac{e^{\left(-xe^{\left(x^2\right)}+1\right)}}{xe^{\left(x^2\right)}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x, algorithm="
maxima")
```

```
[Out] -e^(-x*e^(x^2) + 1)/(x*e^(x^2) - 1)
```

Fricas [A] time = 1.79165, size = 73, normalized size = 2.92

$$-\frac{e^{\left(2\,x^2-xe^{\left(x^2\right)}+1\right)}}{xe^{\left(3\,x^2\right)}-e^{\left(2\,x^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x, algorithm="
fricas")
```

```
[Out] -e^(2*x^2 - x*e^(x^2) + 1)/(x*e^(3*x^2) - e^(2*x^2))
```

Sympy [A] time = 0.406817, size = 31, normalized size = 1.24

$$-\frac{e^{2x^2-xe^{x^2}+1}}{xe^{3x^2}-e^{2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1-exp(x**2)*x+2*x**2)*(2*x**3+x)/(1-exp(x**2)*x)**2,x)
```

```
[Out] -exp(2*x**2 - x*exp(x**2) + 1)/(x*exp(3*x**2) - exp(2*x**2))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,x^3+x\right)e^{\left(2\,x^2-xe^{\left(x^2\right)}+1\right)}}{\left(xe^{\left(x^2\right)}-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1-exp(x^2)*x+2*x^2)*(2*x^3+x)/(1-exp(x^2)*x)^2,x, algorithm="
giac")
```

```
[Out] integrate((2*x^3 + x)*e^(2*x^2 - x*e^(x^2) + 1)/(x*e^(x^2) - 1)^2, x)
```

3.170 $\int e^{e^{e^{e^x}}}\,dx$

Optimal. Leaf size=11

$$\text{CannotIntegrate}\left(e^{e^{e^{e^x}}},x\right)$$

```
[Out] Defer[Int][E^E^E^E^x, x]
```

Rubi [A] time = 0.0463063, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int e^{e^{e^{e^x}}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[E^E^E^E^x,x]
```

```
[Out] Defer[Subst][Defer[Int][E^E^E^x/x, x], x, E^x]
```

Rubi steps

$$\int e^{e^{e^{e^x}}}\,dx=\text{Subst}\left(\int \frac{e^{e^{e^x}}}{x}\,dx,x,e^x\right)$$

Mathematica [A] time = 0.019356, size = 0, normalized size = 0.

$$\int e^{e^{e^{e^x}}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[E^E^E^E^x,x]
```

```
[Out] Integrate[E^E^E^E^x, x]
```

Maple [A] time = 0.016, size = 0, normalized size = 0.

$$\int \mathrm{e}^{\mathrm{e}^{\mathrm{e}^{\mathrm{e}^{x}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(exp(exp(exp(x)))),x)
```

```
[Out] int(exp(exp(exp(exp(x)))),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(e^{\left(e^{\left(e^{x}\right)}\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(exp(exp(x)))),x, algorithm="maxima")
```

```
[Out] integrate(e^(e^(e^(e^x))), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(e^{\left(e^{\left(e^{\left(e^{x}\right)}\right)}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(exp(exp(x)))),x, algorithm="fricas")
```

```
[Out] integral(e^(e^(e^(e^x))), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int e^{e^{e^{e^x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(exp(exp(x)))),x)
```

```
[Out] Integral(exp(exp(exp(exp(x)))), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int e^{\left(e^{\left(e^{\left(e^x\right)}\right)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(exp(exp(exp(x)))),x, algorithm="giac")
```

```
[Out] integrate(e^(e^(e^(e^x))), x)
```

3.171 $\int e^x \log(x) \, dx$

Optimal. Leaf size=11

$$e^x \log(x) - \text{ExpIntegralEi}(x)$$

```
[Out] -ExpIntegralEi[x] + E^x*Log[x]
```

Rubi [A] time = 0.0159023, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2194, 2554, 2178}

$$e^x \log(x) - \text{ExpIntegralEi}(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Log[x],x]
```

```
[Out] -ExpIntegralEi[x] + E^x*Log[x]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\int e^x \log(x)\,dx = e^x \log(x) - \int \frac{e^x}{x}\,dx$$
$$= -\mathrm{Ei}(x) + e^x \log(x)$$

Mathematica [A] time = 0.0055287, size = 11, normalized size = 1.

$$e^x \log(x) - \mathrm{ExpIntegralEi}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Log[x],x]
```

```
[Out] -ExpIntegralEi[x] + E^x*Log[x]
```

Maple [A] time = 0.01, size = 12, normalized size = 1.1

$$e^x \ln(x) + \mathrm{Ei}(1, -x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*ln(x),x)
```

```
[Out] exp(x)*ln(x)+Ei(1,-x)
```

Maxima [A] time = 1.0295, size = 14, normalized size = 1.27

$$e^x \log(x) - \mathrm{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(x),x, algorithm="maxima")
```

```
[Out] e^x*log(x) - Ei(x)
```

Fricas [A] time = 1.7651, size = 27, normalized size = 2.45

$$e^x \log(x) - \text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(x),x, algorithm="fricas")
```

```
[Out] e^x*log(x) - Ei(x)
```

Sympy [A] time = 1.98923, size = 8, normalized size = 0.73

$$e^x \log(x) - \text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*ln(x),x)
```

```
[Out] exp(x)*log(x) - Ei(x)
```

Giac [A] time = 1.07051, size = 14, normalized size = 1.27

$$e^x \log(x) - \text{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*log(x),x, algorithm="giac")
```

```
[Out] e^x*log(x) - Ei(x)
```

3.172 $\int e^x x \log(x)\,dx$

Optimal. Leaf size=22

$$\text{ExpIntegralEi}(x) - e^x - e^x \log(x) + e^x x \log(x)$$

```
[Out] -E^x + ExpIntegralEi[x] - E^x*Log[x] + E^x*x*Log[x]
```

Rubi [A] time = 0.0521265, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {2176, 2194, 2554, 2199, 2178}

$$\text{ExpIntegralEi}(x) - e^x - e^x \log(x) + e^x x \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x*Log[x],x]
```

```
[Out] -E^x + ExpIntegralEi[x] - E^x*Log[x] + E^x*x*Log[x]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2199

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
 c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] &&  !$UseGamma === True
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int e^x x \log(x)\,dx &= -e^x \log(x) + e^x x \log(x) - \int \frac{e^x(-1+x)}{x}\,dx\\ &= -e^x \log(x) + e^x x \log(x) - \int \left(e^x - \frac{e^x}{x}\right)dx\\ &= -e^x \log(x) + e^x x \log(x) - \int e^x\,dx + \int \frac{e^x}{x}\,dx\\ &= -e^x + \mathrm{Ei}(x) - e^x \log(x) + e^x x \log(x)\end{aligned}$$

Mathematica [A] time = 0.0110205, size = 17, normalized size = 0.77

$$\text{ExpIntegralEi}(x) - e^x + e^x(x-1)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x*Log[x],x]
```

```
[Out] -E^x + ExpIntegralEi[x] + E^x*(-1 + x)*Log[x]
```

Maple [A] time = 0.013, size = 21, normalized size = 1.

$$(-1+x)\,\mathrm{e}^{x}\ln(x) - \mathrm{Ei}(1,-x) - \mathrm{e}^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x*ln(x),x)
```

```
[Out] (-1+x)*exp(x)*ln(x)-Ei(1,-x)-exp(x)
```

Maxima [A] time = 1.02932, size = 20, normalized size = 0.91

$$(x-1)e^{x}\log(x)+\mathrm{Ei}(x)-e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*log(x),x, algorithm="maxima")
```

```
[Out] (x - 1)*e^x*log(x) + Ei(x) - e^x
```

Fricas [A] time = 1.85356, size = 46, normalized size = 2.09

$$(x-1)e^{x}\log(x)+\mathrm{Ei}(x)-e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*log(x),x, algorithm="fricas")
```

```
[Out] (x - 1)*e^x*log(x) + Ei(x) - e^x
```

Sympy [A] time = 3.19639, size = 17, normalized size = 0.77

$$(xe^{x}-e^{x})\log(x)-e^{x}+\mathrm{Ei}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*ln(x),x)
```

```
[Out] (x*exp(x) - exp(x))*log(x) - exp(x) + Ei(x)
```

Giac [A] time = 1.07831, size = 20, normalized size = 0.91

$$(x-1)e^{x}\log(x)+\mathrm{Ei}(x)-e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*log(x),x, algorithm="giac")
```

```
[Out] (x - 1)*e^x*log(x) + Ei(x) - e^x
```

3.173 $\int e^{2x} \log(e^x)\, dx$

Optimal. Leaf size=23

$$\frac{1}{2}e^{2x}\log\left(e^x\right)-\frac{e^{2x}}{4}$$

```
[Out] -E^(2*x)/4 + (E^(2*x)*Log[E^x])/2
```

Rubi [A] time = 0.012565, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {2194, 2554, 12}

$$\frac{1}{2}e^{2x}\log\left(e^x\right)-\frac{e^{2x}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)*Log[E^x],x]
```

```
[Out] -E^(2*x)/4 + (E^(2*x)*Log[E^x])/2
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int e^{2x}\log(e^x)\,dx = \frac{1}{2}e^{2x}\log(e^x) - \int \frac{e^{2x}}{2}\,dx$$
$$= \frac{1}{2}e^{2x}\log(e^x) - \frac{1}{2}\int e^{2x}\,dx$$
$$= -\frac{e^{2x}}{4} + \frac{1}{2}e^{2x}\log(e^x)$$

Mathematica [A] time = 0.004753, size = 17, normalized size = 0.74

$$\frac{1}{4}e^{2x}(2\log(e^x)-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)*Log[E^x],x]
```

```
[Out] (E^(2*x)*(-1 + 2*Log[E^x]))/4
```

Maple [A] time = 0.001, size = 28, normalized size = 1.2

$$\frac{e^{2x}x}{2} - \frac{e^{2x}}{4} + \frac{e^{2x}(\ln(e^x)-x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)*ln(exp(x)),x)
```

```
[Out] 1/2*exp(2*x)*x-1/4*exp(2*x)+1/2*exp(2*x)*(ln(exp(x))-x)
```

Maxima [A] time = 0.951424, size = 15, normalized size = 0.65

$$\frac{1}{4}(2x-1)e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*log(exp(x)),x, algorithm="maxima")
```

```
[Out] 1/4*(2*x - 1)*e^(2*x)
```

Fricas [A] time = 1.91165, size = 31, normalized size = 1.35

$$\frac{1}{4}\,(2\,x-1)e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*log(exp(x)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x - 1)*e^(2*x)
```

Sympy [A] time = 0.088962, size = 10, normalized size = 0.43

$$\frac{(2x-1)\,e^{2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*ln(exp(x)),x)
```

```
[Out] (2*x - 1)*exp(2*x)/4
```

Giac [A] time = 1.07369, size = 15, normalized size = 0.65

$$\frac{1}{4}\,(2\,x-1)e^{(2\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)*log(exp(x)),x, algorithm="giac")
```

```
[Out] 1/4*(2*x - 1)*e^(2*x)
```

3.174 $\int \left(2x+\sqrt{2}x^2\right) dx$

Optimal. Leaf size=16

$$\frac{\sqrt{2}x^3}{3}+x^2$$

```
[Out] x^2 + (Sqrt[2]*x^3)/3
```

Rubi [A] time = 0.0024867, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\frac{\sqrt{2}x^3}{3}+x^2$$

Antiderivative was successfully verified.

```
[In] Int[2*x + Sqrt[2]*x^2,x]
```

```
[Out] x^2 + (Sqrt[2]*x^3)/3
```

Rubi steps

$$\int \left(2x+\sqrt{2}x^2\right) dx = x^2+\frac{\sqrt{2}x^3}{3}$$

Mathematica [A] time = 0.0000348, size = 16, normalized size = 1.

$$\frac{\sqrt{2}x^3}{3}+x^2$$

Antiderivative was successfully verified.

```
[In] Integrate[2*x + Sqrt[2]*x^2,x]
```

```
[Out] x^2 + (Sqrt[2]*x^3)/3
```

Maple [A] time = 0.001, size = 13, normalized size = 0.8

$$x^2 + \frac{x^3\sqrt{2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2*x+x^2*2^(1/2),x)
```

```
[Out] x^2+1/3*x^3*2^(1/2)
```

Maxima [A] time = 1.41658, size = 16, normalized size = 1.

$$\frac{1}{3}\sqrt{2}x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x+x^2*2^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(2)*x^3 + x^2
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x+x^2*2^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [A] time = 0.053283, size = 12, normalized size = 0.75

$$\frac{\sqrt{2}x^3}{3} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x+x**2*2**(1/2),x)
```

```
[Out] sqrt(2)*x**3/3 + x**2
```

Giac [A] time = 1.09769, size = 16, normalized size = 1.

$$\frac{1}{3}\sqrt{2}x^3 + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2*x+x^2*2^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(2)*x^3 + x^2
```

3.175 $\int \frac{\log(x)}{\sqrt{b+ax}} \, dx$

Optimal. Leaf size=57

$$-\frac{4\sqrt{ax+b}}{a}+\frac{2\log(x)\sqrt{ax+b}}{a}+\frac{4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$$

```
[Out] (-4*Sqrt[b + a*x])/a + (4*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/Sqrt[b]])/a + (2*Sq
rt[b + a*x]*Log[x])/a
```

Rubi [A] time = 0.0296218, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2319, 50, 63, 208}

$$-\frac{4\sqrt{ax+b}}{a}+\frac{2\log(x)\sqrt{ax+b}}{a}+\frac{4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/Sqrt[b + a*x],x]
```

```
[Out] (-4*Sqrt[b + a*x])/a + (4*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/Sqrt[b]])/a + (2*Sq
rt[b + a*x]*Log[x])/a
```

Rule 2319

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p)/(e*(q + 1)), x]
- Dist[(b*n*p)/(e*(q + 1)), Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
-1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &&
 NeQ[q, 1]))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
```

```
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(x)}{\sqrt{b+ax}}\,dx &= \frac{2\sqrt{b+ax}\log(x)}{a} - \frac{2\int \frac{\sqrt{b+ax}}{x}\,dx}{a} \\
&= -\frac{4\sqrt{b+ax}}{a} + \frac{2\sqrt{b+ax}\log(x)}{a} - \frac{(2b)\int \frac{1}{x\sqrt{b+ax}}\,dx}{a} \\
&= -\frac{4\sqrt{b+ax}}{a} + \frac{2\sqrt{b+ax}\log(x)}{a} - \frac{(4b)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{b}{a}+\frac{x^2}{a}}\,dx, x, \sqrt{b+ax}\right)}{a^2} \\
&= -\frac{4\sqrt{b+ax}}{a} + \frac{4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b+ax}}{\sqrt{b}}\right)}{a} + \frac{2\sqrt{b+ax}\log(x)}{a}
\end{aligned}$$

Mathematica [A] time = 0.0226035, size = 43, normalized size = 0.75

$$\frac{2(\log(x)-2)\sqrt{ax+b}+4\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{ax+b}}{\sqrt{b}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/Sqrt[b + a*x],x]
```

```
[Out] (4*Sqrt[b]*ArcTanh[Sqrt[b + a*x]/Sqrt[b]] + 2*Sqrt[b + a*x]*(-2 + Log[x]))/
a
```

Maple [A] time = 0.009, size = 48, normalized size = 0.8

$$4\,{\frac {\sqrt {b}}{a}{\rm Artanh} \left({\frac {\sqrt {ax+b}}{\sqrt {b}}}\right)}-4\,{\frac {\sqrt {ax+b}}{a}}+2\,{\frac {\ln \left(x \right) \sqrt {ax+b}}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/(a*x+b)^(1/2),x)
```

```
[Out] 4*arctanh((a*x+b)^(1/2)/b^(1/2))*b^(1/2)/a-4*(a*x+b)^(1/2)/a+2*ln(x)*(a*x+b
)^(1/2)/a
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.88307, size = 234, normalized size = 4.11

$$\left[\frac{2\left(\sqrt{ax+b}(\log\left(x\right)-2)+\sqrt{b}\log\left(\frac{ax+2\,\sqrt{ax+b}\sqrt{b}+2\,b}{x}\right)\right)}{a}, \frac{2\left(\sqrt{ax+b}(\log\left(x\right)-2)-2\,\sqrt{-b}\arctan\left(\frac{\sqrt{ax+b}\sqrt{-b}}{b}\right)\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^(1/2),x, algorithm="fricas")
```

```
[Out] [2*(sqrt(a*x + b)*(log(x) - 2) + sqrt(b)*log((a*x + 2*sqrt(a*x + b)*sqrt(b)
 + 2*b)/x))/a, 2*(sqrt(a*x + b)*(log(x) - 2) - 2*sqrt(-b)*arctan(sqrt(a*x +
```

```
b)*sqrt(-b)/b))/a]
```

Sympy [B] time = 4.27925, size = 930, normalized size = 16.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/(a*x+b)**(1/2),x)
```

```
[Out] Piecewise((4*sqrt(b)*acoth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x +
b/a)*log(b/a)/sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sq
rt(x + b/a)*log(-1 + b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) + 2
*I*pi*sqrt(x + b/a)/sqrt(a), (Abs(x + b/a) < 1) & (Abs(b)/(Abs(a)*Abs(x + b
/a)) > 1)), (4*sqrt(b)*atanh(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*sqrt(x
+ b/a)*log(b/a)/sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*
sqrt(x + b/a)*log(1 - b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a), A
bs(x + b/a) < 1), (4*sqrt(b)*acoth(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a + 2*s
qrt(x + b/a)*log(b/a)/sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a
) + 2*sqrt(x + b/a)*log(-1 + b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqr
t(a) + 2*I*pi*sqrt(x + b/a)/sqrt(a), (1/Abs(x + b/a) < 1) & (Abs(b)/(Abs(a)
*Abs(x + b/a)) > 1)), (4*sqrt(b)*atanh(sqrt(b)/(sqrt(a)*sqrt(x + b/a)))/a +
 2*sqrt(x + b/a)*log(b/a)/sqrt(a) - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sq
rt(a) + 2*sqrt(x + b/a)*log(1 - b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/
sqrt(a), 1/Abs(x + b/a) < 1), (4*sqrt(b)*acoth(sqrt(b)/(sqrt(a)*sqrt(x + b/
a)))/a - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sqrt(a) + 2*sqrt(x + b/a)*log
(-1 + b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/sqrt(a) + meijerg(((1,), (
3/2,)), ((1/2,), (0,)), x + b/a)*log(b/a)/sqrt(a) + I*pi*meijerg(((1,), (3/
2,)), ((1/2,), (0,)), x + b/a)/sqrt(a) + meijerg(((3/2, 1), ()), ((), (1/2,
 0)), x + b/a)*log(b/a)/sqrt(a) + I*pi*meijerg(((3/2, 1), ()), ((), (1/2, 0
)), x + b/a)/sqrt(a), Abs(b)/(Abs(a)*Abs(x + b/a)) > 1), (4*sqrt(b)*atanh(s
qrt(b)/(sqrt(a)*sqrt(x + b/a)))/a - 2*sqrt(x + b/a)*log(b/(a*(x + b/a)))/sq
rt(a) + 2*sqrt(x + b/a)*log(1 - b/(a*(x + b/a)))/sqrt(a) - 4*sqrt(x + b/a)/
sqrt(a) - 2*I*pi*sqrt(x + b/a)/sqrt(a) + meijerg(((1,), (3/2,)), ((1/2,), (
0,)), x + b/a)*log(b/a)/sqrt(a) + I*pi*meijerg(((1,), (3/2,)), ((1/2,), (0,
)), x + b/a)/sqrt(a) + meijerg(((3/2, 1), ()), ((), (1/2, 0)), x + b/a)*log
(b/a)/sqrt(a) + I*pi*meijerg(((3/2, 1), ()), ((), (1/2, 0)), x + b/a)/sqrt(
a), True))
```

Giac [A] time = 1.10269, size = 65, normalized size = 1.14

$$-\frac{2\left(\frac{2\,b\arctan\left(\frac{\sqrt{ax+b}}{\sqrt{-b}}\right)}{\sqrt{-b}}-\sqrt{ax+b}\log\left(x\right)+2\,\sqrt{ax+b}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/(a*x+b)^(1/2),x, algorithm="giac")
```

```
[Out] -2*(2*b*arctan(sqrt(a*x + b)/sqrt(-b))/sqrt(-b) - sqrt(a*x + b)*log(x) + 2*
sqrt(a*x + b))/a
```

3.176 $\int \sqrt{a+bx}\sqrt{c+dx}\,dx$

Optimal. Leaf size=116

$$-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}+\frac{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

```
[Out] ((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b*d) + ((a + b*x)^(3/2)*Sqrt[c
 + d*x])/(2*b) - ((b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sq
rt[c + d*x])])/(4*b^(3/2)*d^(3/2))
```

Rubi [A] time = 0.0682431, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {50, 63, 217, 206}

$$-\frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}+\frac{\sqrt{a+bx}\sqrt{c+dx}(bc-ad)}{4bd}+\frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] ((b*c - a*d)*Sqrt[a + b*x]*Sqrt[c + d*x])/(4*b*d) + ((a + b*x)^(3/2)*Sqrt[c
 + d*x])/(2*b) - ((b*c - a*d)^2*ArcTanh[(Sqrt[d]*Sqrt[a + b*x])/(Sqrt[b]*Sq
rt[c + d*x])])/(4*b^(3/2)*d^(3/2))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+bx}\sqrt{c+dx}\,dx = \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} + \frac{(bc-ad)\int \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\,dx}{4b}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\int \frac{1}{\sqrt{a+bx}\sqrt{c+dx}}\,dx}{8bd}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{c-\frac{ad}{b}+\frac{dx^2}{b}}}\,dx, x, \sqrt{a+bx}\right)}{4b^2d}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{dx^2}{b}}\,dx, x, \frac{\sqrt{a+bx}}{\sqrt{c+dx}}\right)}{4b^2d}$$

$$= \frac{(bc-ad)\sqrt{a+bx}\sqrt{c+dx}}{4bd} + \frac{(a+bx)^{3/2}\sqrt{c+dx}}{2b} - \frac{(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{b}\sqrt{c+dx}}\right)}{4b^{3/2}d^{3/2}}$$

Mathematica [A] time = 0.291796, size = 118, normalized size = 1.02

$$\frac{b\sqrt{d}\sqrt{a+bx}(c+dx)(ad+b(c+2dx)) - (bc-ad)^{5/2}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{4b^2d^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]*Sqrt[c + d*x],x]
```

```
[Out] (b*Sqrt[d]*Sqrt[a + b*x]*(c + d*x)*(a*d + b*(c + 2*d*x)) - (b*c - a*d)^(5/2
)*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c
- a*d]])/(4*b^2*d^(3/2)*Sqrt[c + d*x])
```

Maple [B] time = 0.011, size = 305, normalized size = 2.6

$$\frac{1}{2\,d}\sqrt{bx+a}\,(dx+c)^{\frac{3}{2}}+\frac{a}{4\,b}\sqrt{bx+a}\sqrt{dx+c}-\frac{c}{4\,d}\sqrt{bx+a}\sqrt{dx+c}-\frac{da^{2}}{8\,b}\sqrt{(bx+a)\,(dx+c)}\ln\left(\left(\frac{ad}{2}+\frac{bc}{2}+bdx\right)\frac{1}{\sqrt{bd}}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)*(d*x+c)^(1/2),x)
```

```
[Out] 1/2/d*(b*x+a)^(1/2)*(d*x+c)^(3/2)+1/4/b*(d*x+c)^(1/2)*(b*x+a)^(1/2)*a-1/4/d
*(d*x+c)^(1/2)*(b*x+a)^(1/2)*c-1/8*d/b*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2
)/(b*x+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x
+a*c)^(1/2))/(b*d)^(1/2)*a^2+1/4*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x
+a)^(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^
(1/2))/(b*d)^(1/2)*a*c-1/8/d*((b*x+a)*(d*x+c))^(1/2)/(d*x+c)^(1/2)/(b*x+a)^
(1/2)*ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(b*d*x^2+(a*d+b*c)*x+a*c)^(1/2
))/(b*d)^(1/2)*c^2*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.93164, size = 699, normalized size = 6.03

$$\left[\frac{\left(b^{2}c^{2}-2\,abcd+a^{2}d^{2}\right)\sqrt{bd}\log\left(8\,b^{2}d^{2}x^{2}+b^{2}c^{2}+6\,abcd+a^{2}d^{2}-4\,(2\,bdx+bc+ad)\sqrt{bd}\sqrt{bx+a}\sqrt{dx+c}+8\,\left(b^{2}cd+\right.\right.}{16\,b^{2}d^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^
2 + 6*a*b*c*d + a^2*d^2 - 4*(2*b*d*x + b*c + a*d)*sqrt(b*d)*sqrt(b*x + a)*s
qrt(d*x + c) + 8*(b^2*c*d + a*b*d^2)*x) + 4*(2*b^2*d^2*x + b^2*c*d + a*b*d^
2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2), 1/8*((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*sqrt(-b*d)*arctan(1/2*(2*b*d*x + b*c + a*d)*sqrt(-b*d)*sqrt(b*x + a)*s
qrt(d*x + c)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)) + 2*(2*b^2*d^
2*x + b^2*c*d + a*b*d^2)*sqrt(b*x + a)*sqrt(d*x + c))/(b^2*d^2)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+bx}\sqrt{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x)*sqrt(c + d*x), x)
```

Giac [A] time = 1.1314, size = 189, normalized size = 1.63

$$\frac{\left(\sqrt{b^2c+(bx+a)bd-abd}\sqrt{bx+a}\left(\frac{2\,(bx+a)}{b^4d^2}+\frac{bcd-ad^2}{b^4d^4}\right)+\frac{\left(b^2c^2-2\,abcd+a^2d^2\right)\log\left(\left|-\sqrt{bd}\sqrt{bx+a}+\sqrt{b^2c+(bx+a)bd-abd}\right|\right)}{\sqrt{bd}b^3d^3}\right)|b|}{96\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(sqrt(b^2*c + (b*x + a)*b*d - a*b*d)*sqrt(b*x + a)*(2*(b*x + a)/(b^4*d
^2) + (b*c*d - a*d^2)/(b^4*d^4)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(abs(
-sqrt(b*d)*sqrt(b*x + a) + sqrt(b^2*c + (b*x + a)*b*d - a*b*d)))/(sqrt(b*d)
*b^3*d^3))*abs(b)/b^3
```

3.177 $\int \sqrt{a+bx}\,dx$

Optimal. Leaf size=16

$$\frac{2(a+bx)^{3/2}}{3b}$$

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Rubi [A] time = 0.001468, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{a+bx}\,dx = \frac{2(a+bx)^{3/2}}{3b}$$

Mathematica [A] time = 0.0044021, size = 16, normalized size = 1.

$$\frac{2(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2))/(3*b)
```

Maple [A] time = 0.002, size = 13, normalized size = 0.8

$$\frac{2}{3b}(bx+a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2),x)
```

```
[Out] 2/3*(b*x+a)^(3/2)/b
```

Maxima [A] time = 0.942817, size = 16, normalized size = 1.

$$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

Fricas [A] time = 1.55855, size = 31, normalized size = 1.94

$$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

Sympy [A] time = 0.054944, size = 12, normalized size = 0.75

$$\frac{2(a+bx)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2),x)
```

```
[Out] 2*(a + b*x)**(3/2)/(3*b)
```

Giac [A] time = 1.06098, size = 16, normalized size = 1.

$$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b
```

3.178 $\int x\sqrt{a+bx}\,dx$

Optimal. Leaf size=34

$$\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}$$

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2)
```

Rubi [A] time = 0.0086508, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{5/2}}{5b^2}-\frac{2a(a+bx)^{3/2}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + b*x],x]
```

```
[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2) + (2*(a + b*x)^(5/2))/(5*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+bx}\,dx &= \int \left(-\frac{a\sqrt{a+bx}}{b}+\frac{(a+bx)^{3/2}}{b}\right)dx\\ &= -\frac{2a(a+bx)^{3/2}}{3b^2}+\frac{2(a+bx)^{5/2}}{5b^2}\end{aligned}$$

Mathematica [A] time = 0.0113039, size = 24, normalized size = 0.71

$$\frac{2(a+bx)^{3/2}(3bx-2a)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2)*(-2*a + 3*b*x))/(15*b^2)
```

Maple [A] time = 0.003, size = 21, normalized size = 0.6

$$-\frac{-6\,bx+4\,a}{15\,b^2}(bx+a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^(1/2),x)
```

```
[Out] -2/15*(b*x+a)^(3/2)*(-3*b*x+2*a)/b^2
```

Maxima [A] time = 0.954078, size = 35, normalized size = 1.03

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^2}-\frac{2\,(bx+a)^{\frac{3}{2}}a}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^2 - 2/3*(b*x + a)^(3/2)*a/b^2
```

Fricas [A] time = 1.6055, size = 70, normalized size = 2.06

$$\frac{2\left(3\,b^2x^2+abx-2\,a^2\right)\sqrt{bx+a}}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

Sympy [B] time = 1.04818, size = 202, normalized size = 5.94

$$-\frac{4a^{\frac{9}{2}}\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{9}{2}}}{15a^2b^2+15ab^3x}-\frac{2a^{\frac{7}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{4a^{\frac{7}{2}}bx}{15a^2b^2+15ab^3x}+\frac{8a^{\frac{5}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}+\frac{6a^{\frac{3}{2}}b^3x^3\sqrt{1+\frac{bx}{a}}}{15a^2b^2+15ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)**(1/2),x)
```

```
[Out] -4*a**(9/2)*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 4*a**(9/2)/(15*a
**2*b**2 + 15*a*b**3*x) - 2*a**(7/2)*b*x*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15
*a*b**3*x) + 4*a**(7/2)*b*x/(15*a**2*b**2 + 15*a*b**3*x) + 8*a**(5/2)*b**2*
x**2*sqrt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x) + 6*a**(3/2)*b**3*x**3*sq
rt(1 + b*x/a)/(15*a**2*b**2 + 15*a*b**3*x)
```

Giac [A] time = 1.05599, size = 34, normalized size = 1.

$$\frac{2\left(3\,(bx+a)^{\frac{5}{2}}-5\,(bx+a)^{\frac{3}{2}}a\right)}{15\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)/b^2
```

3.179 $\int x^2\sqrt{a+bx}\,dx$

Optimal. Leaf size=53

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}$$

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) + (2*(a + b
*x)^(7/2))/(7*b^3)
```

Rubi [A] time = 0.0137814, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{7/2}}{7b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3) - (4*a*(a + b*x)^(5/2))/(5*b^3) + (2*(a + b
*x)^(7/2))/(7*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{a+bx}\,dx &= \int\left(\frac{a^2\sqrt{a+bx}}{b^2}-\frac{2a(a+bx)^{3/2}}{b^2}+\frac{(a+bx)^{5/2}}{b^2}\right)dx\\ &= \frac{2a^2(a+bx)^{3/2}}{3b^3}-\frac{4a(a+bx)^{5/2}}{5b^3}+\frac{2(a+bx)^{7/2}}{7b^3}\end{aligned}$$

Mathematica [A] time = 0.0152363, size = 35, normalized size = 0.66

$$\frac{2(a+bx)^{3/2}\left(8a^2-12abx+15b^2x^2\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + b*x],x]
```

```
[Out] (2*(a + b*x)^(3/2)*(8*a^2 - 12*a*b*x + 15*b^2*x^2))/(105*b^3)
```

Maple [A] time = 0.003, size = 32, normalized size = 0.6

$$\frac{30\,b^2x^2-24\,axb+16\,a^2}{105\,b^3}\,(bx+a)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^(1/2),x)
```

```
[Out] 2/105*(b*x+a)^(3/2)*(15*b^2*x^2-12*a*b*x+8*a^2)/b^3
```

Maxima [A] time = 0.940389, size = 55, normalized size = 1.04

$$\frac{2\,(bx+a)^{\frac{7}{2}}}{7\,b^3}-\frac{4\,(bx+a)^{\frac{5}{2}}a}{5\,b^3}+\frac{2\,(bx+a)^{\frac{3}{2}}a^2}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*(b*x + a)^(7/2)/b^3 - 4/5*(b*x + a)^(5/2)*a/b^3 + 2/3*(b*x + a)^(3/2)*a
^2/b^3
```

Fricas [A] time = 1.6045, size = 97, normalized size = 1.83

$$\frac{2\left(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3\right)\sqrt{bx+a}}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)/b^3
```

Sympy [B] time = 1.63627, size = 666, normalized size = 12.57

$$\frac{16a^{\frac{23}{2}}\sqrt{1+\frac{bx}{a}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}-\frac{16a^{\frac{23}{2}}}{105a^8b^3+315a^7b^4x+315a^6b^5x^2+105a^5b^6x^3}+\frac{40a^{\frac{21}{2}}}{105a^8b^3+315a^7b^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**(1/2),x)
```

```
[Out] 16*a**(23/2)*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b*
*5*x**2 + 105*a**5*b**6*x**3) - 16*a**(23/2)/(105*a**8*b**3 + 315*a**7*b**4
*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 40*a**(21/2)*b*x*sqrt(1 + b
*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6
*x**3) - 48*a**(21/2)*b*x/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*
x**2 + 105*a**5*b**6*x**3) + 30*a**(19/2)*b**2*x**2*sqrt(1 + b*x/a)/(105*a*
*8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 48*a
**(19/2)*b**2*x**2/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 +
105*a**5*b**6*x**3) + 40*a**(17/2)*b**3*x**3*sqrt(1 + b*x/a)/(105*a**8*b**3
 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) - 16*a**(17/2
)*b**3*x**3/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**
5*b**6*x**3) + 100*a**(15/2)*b**4*x**4*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315
*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3) + 96*a**(13/2)*b**5
*x**5*sqrt(1 + b*x/a)/(105*a**8*b**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2
 + 105*a**5*b**6*x**3) + 30*a**(11/2)*b**6*x**6*sqrt(1 + b*x/a)/(105*a**8*b
**3 + 315*a**7*b**4*x + 315*a**6*b**5*x**2 + 105*a**5*b**6*x**3)
```

Giac [A] time = 1.10159, size = 50, normalized size = 0.94

$$\frac{2\left(15\,(bx+a)^{\frac{7}{2}}-42\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2\right)}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/105*(15*(b*x + a)^(7/2) - 42*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2)/
b^3
```

3.180 $\int \frac{\sqrt{a+bx}}{x} \, dx$

Optimal. Leaf size=35

$$2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rubi [A] time = 0.0115886, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 63, 208}

$$2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx}}{x}\,dx &= 2\sqrt{a+bx} + a\int \frac{1}{x\sqrt{a+bx}}\,dx \\
&= 2\sqrt{a+bx} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b} \\
&= 2\sqrt{a+bx} - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0091192, size = 35, normalized size = 1.

$$2\sqrt{a+bx} - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x,x]
```

```
[Out] 2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Maple [A] time = 0.004, size = 28, normalized size = 0.8

$$-2\,\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{a} + 2\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x,x)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)+2*(b*x+a)^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65777, size = 188, normalized size = 5.37

$$\left[\sqrt{a}\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a},2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+2\sqrt{bx+a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a), 2*
sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)]
```

Sympy [B] time = 1.43705, size = 68, normalized size = 1.94

$$-2\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)+\frac{2a}{\sqrt{b}\sqrt{x}\sqrt{\frac{a}{bx}+1}}+\frac{2\sqrt{b}\sqrt{x}}{\sqrt{\frac{a}{bx}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x,x)
```

```
[Out] -2*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*sqrt(x))) + 2*a/(sqrt(b)*sqrt(x)*sqrt(a/(
b*x) + 1)) + 2*sqrt(b)*sqrt(x)/sqrt(a/(b*x) + 1)
```

Giac [A] time = 1.06783, size = 43, normalized size = 1.23

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + 2\,\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + 2*sqrt(b*x + a)
```

3.181 $\int \frac{\sqrt{a+bx}}{x^2} dx$

Optimal. Leaf size=39

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.0106959, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {47, 63, 208}

$$-\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x]/x^2,x]
```

```
[Out] -(Sqrt[a + b*x]/x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}}{x^2}\,dx &= -\frac{\sqrt{a+bx}}{x}+\frac{1}{2}b\int \frac{1}{x\sqrt{a+bx}}\,dx\\ &= -\frac{\sqrt{a+bx}}{x}+\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)\\ &= -\frac{\sqrt{a+bx}}{x}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}\end{aligned}$$

Mathematica [A] time = 0.026963, size = 47, normalized size = 1.21

$$-\frac{bx\sqrt{\frac{bx}{a}+1}\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx}{x\sqrt{a+bx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x]/x^2,x]
```

```
[Out] -((a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/(x*Sqrt[a +
b*x]))
```

Maple [A] time = 0.008, size = 37, normalized size = 1.

$$2\,b\left(-1/2\,\frac{\sqrt{bx+a}}{bx}-1/2\,\frac{1}{\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^(1/2)/x^2,x)
```

```
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60688, size = 225, normalized size = 5.77

$$\left[\frac{\sqrt{a}bx\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}a}{2ax}, \frac{\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)-\sqrt{bx+a}a}{ax}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(b*x +
 a)*a)/(a*x)]
```

Sympy [A] time = 1.90541, size = 44, normalized size = 1.13

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{\sqrt{x}}-\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**(1/2)/x**2,x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/sqrt(x) - b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqr
t(a)
```

Giac [A] time = 1.07493, size = 55, normalized size = 1.41

$$\frac{\frac{b^2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\sqrt{bx+a}b}{x}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] (b^2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - sqrt(b*x + a)*b/x)/b
```

3.182 $\int \frac{1}{\sqrt{a+bx}} dx$

Optimal. Leaf size=14

$$\frac{2\sqrt{a+bx}}{b}$$

```
[Out] (2*Sqrt[a + b*x])/b
```

Rubi [A] time = 0.0013685, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x])/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx}} dx = \frac{2\sqrt{a+bx}}{b}$$

Mathematica [A] time = 0.0034517, size = 14, normalized size = 1.

$$\frac{2\sqrt{a+bx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x])/b
```

Maple [A] time = 0.002, size = 13, normalized size = 0.9

$$2\frac{\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)^(1/2),x)
```

```
[Out] 2*(b*x+a)^(1/2)/b
```

Maxima [A] time = 0.939872, size = 16, normalized size = 1.14

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Fricas [A] time = 1.61068, size = 26, normalized size = 1.86

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(b*x + a)/b
```

Sympy [A] time = 0.054909, size = 10, normalized size = 0.71

$$\frac{2\sqrt{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)**(1/2),x)
```

```
[Out] 2*sqrt(a + b*x)/b
```

Giac [A] time = 1.06621, size = 16, normalized size = 1.14

$$\frac{2\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(b*x + a)/b
```

3.183 $\int \frac{x}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=32

$$\frac{2(a+bx)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx}}{b^2}$$

```
[Out] (-2*a*Sqrt[a + b*x])/b^2 + (2*(a + b*x)^(3/2))/(3*b^2)
```

Rubi [A] time = 0.0080337, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2(a+bx)^{3/2}}{3b^2}-\frac{2a\sqrt{a+bx}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + b*x],x]
```

```
[Out] (-2*a*Sqrt[a + b*x])/b^2 + (2*(a + b*x)^(3/2))/(3*b^2)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{a+bx}}\,dx &= \int \left(-\frac{a}{b\sqrt{a+bx}}+\frac{\sqrt{a+bx}}{b}\right)dx\\ &= -\frac{2a\sqrt{a+bx}}{b^2}+\frac{2(a+bx)^{3/2}}{3b^2}\end{aligned}$$

Mathematica [A] time = 0.0099597, size = 23, normalized size = 0.72

$$\frac{2(bx-2a)\sqrt{a+bx}}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + b*x],x]
```

```
[Out] (2*(-2*a + b*x)*Sqrt[a + b*x])/(3*b^2)
```

Maple [A] time = 0.002, size = 21, normalized size = 0.7

$$-\frac{-2\,bx+4\,a}{3\,b^2}\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(b*x+a)^(1/2),x)
```

```
[Out] -2/3*(b*x+a)^(1/2)*(-b*x+2*a)/b^2
```

Maxima [A] time = 0.932934, size = 35, normalized size = 1.09

$$\frac{2\,(bx+a)^{\frac{3}{2}}}{3\,b^2}-\frac{2\,\sqrt{bx+a}a}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(b*x + a)^(3/2)/b^2 - 2*sqrt(b*x + a)*a/b^2
```

Fricas [A] time = 1.50508, size = 47, normalized size = 1.47

$$\frac{2\,\sqrt{bx+a}(bx-2\,a)}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(b*x + a)*(b*x - 2*a)/b^2
```

Sympy [B] time = 1.03835, size = 162, normalized size = 5.06

$$-\frac{4a^{\frac{7}{2}}\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}+\frac{4a^{\frac{7}{2}}}{3a^2b^2+3ab^3x}-\frac{2a^{\frac{5}{2}}bx\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}+\frac{4a^{\frac{5}{2}}bx}{3a^2b^2+3ab^3x}+\frac{2a^{\frac{3}{2}}b^2x^2\sqrt{1+\frac{bx}{a}}}{3a^2b^2+3ab^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)**(1/2),x)
```

```
[Out] -4*a**(7/2)*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x) + 4*a**(7/2)/(3*a**2
*b**2 + 3*a*b**3*x) - 2*a**(5/2)*b*x*sqrt(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**
3*x) + 4*a**(5/2)*b*x/(3*a**2*b**2 + 3*a*b**3*x) + 2*a**(3/2)*b**2*x**2*sqr
t(1 + b*x/a)/(3*a**2*b**2 + 3*a*b**3*x)
```

Giac [A] time = 1.07206, size = 31, normalized size = 0.97

$$\frac{2\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)/b^2
```

3.184 $\int \frac{x^2}{\sqrt{a+bx}}\,dx$

Optimal. Leaf size=51

$$\frac{2a^2\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}$$

```
[Out] (2*a^2*Sqrt[a + b*x])/b^3 - (4*a*(a + b*x)^(3/2))/(3*b^3) + (2*(a + b*x)^(5
/2))/(5*b^3)
```

Rubi [A] time = 0.012585, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2a^2\sqrt{a+bx}}{b^3}+\frac{2(a+bx)^{5/2}}{5b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*a^2*Sqrt[a + b*x])/b^3 - (4*a*(a + b*x)^(3/2))/(3*b^3) + (2*(a + b*x)^(5
/2))/(5*b^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+bx}}\,dx &= \int \left(\frac{a^2}{b^2\sqrt{a+bx}}-\frac{2a\sqrt{a+bx}}{b^2}+\frac{(a+bx)^{3/2}}{b^2}\right)dx\\ &= \frac{2a^2\sqrt{a+bx}}{b^3}-\frac{4a(a+bx)^{3/2}}{3b^3}+\frac{2(a+bx)^{5/2}}{5b^3}\end{aligned}$$

Mathematica [A] time = 0.0140645, size = 35, normalized size = 0.69

$$\frac{2\sqrt{a+bx}\left(8a^2-4abx+3b^2x^2\right)}{15b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + b*x],x]
```

```
[Out] (2*Sqrt[a + b*x]*(8*a^2 - 4*a*b*x + 3*b^2*x^2))/(15*b^3)
```

Maple [A] time = 0.004, size = 32, normalized size = 0.6

$$\frac{6\,b^2x^2-8\,axb+16\,a^2}{15\,b^3}\sqrt{bx+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(b*x+a)^(1/2),x)
```

```
[Out] 2/15*(b*x+a)^(1/2)*(3*b^2*x^2-4*a*b*x+8*a^2)/b^3
```

Maxima [A] time = 0.965589, size = 55, normalized size = 1.08

$$\frac{2\,(bx+a)^{\frac{5}{2}}}{5\,b^3}-\frac{4\,(bx+a)^{\frac{3}{2}}a}{3\,b^3}+\frac{2\,\sqrt{bx+a}a^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/5*(b*x + a)^(5/2)/b^3 - 4/3*(b*x + a)^(3/2)*a/b^3 + 2*sqrt(b*x + a)*a^2/b
^3
```

Fricas [A] time = 1.61231, size = 73, normalized size = 1.43

$$\frac{2\left(3\,b^2x^2 - 4\,abx + 8\,a^2\right)\sqrt{bx+a}}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(3*b^2*x^2 - 4*a*b*x + 8*a^2)*sqrt(b*x + a)/b^3
```

Sympy [B] time = 1.56892, size = 600, normalized size = 11.76

$$\frac{16a^{\frac{21}{2}}\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} - \frac{16a^{\frac{21}{2}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2+15a^5b^6x^3} + \frac{40a^{\frac{19}{2}}bx\sqrt{1+\frac{bx}{a}}}{15a^8b^3+45a^7b^4x+45a^6b^5x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(b*x+a)**(1/2),x)
```

```
[Out] 16*a**(21/2)*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*
x**2 + 15*a**5*b**6*x**3) - 16*a**(21/2)/(15*a**8*b**3 + 45*a**7*b**4*x + 4
5*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 40*a**(19/2)*b*x*sqrt(1 + b*x/a)/(1
5*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*
a**(19/2)*b*x/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*
b**6*x**3) + 30*a**(17/2)*b**2*x**2*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7
*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) - 48*a**(17/2)*b**2*x**2/(
15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10
*a**(15/2)*b**3*x**3*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a*
*6*b**5*x**2 + 15*a**5*b**6*x**3) - 16*a**(15/2)*b**3*x**3/(15*a**8*b**3 +
45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3) + 10*a**(13/2)*b**4
*x**4*sqrt(1 + b*x/a)/(15*a**8*b**3 + 45*a**7*b**4*x + 45*a**6*b**5*x**2 +
15*a**5*b**6*x**3) + 6*a**(11/2)*b**5*x**5*sqrt(1 + b*x/a)/(15*a**8*b**3 +
45*a**7*b**4*x + 45*a**6*b**5*x**2 + 15*a**5*b**6*x**3)
```

Giac [A] time = 1.06844, size = 50, normalized size = 0.98

$$\frac{2\left(3\,(bx+a)^{\frac{5}{2}} - 10\,(bx+a)^{\frac{3}{2}}a + 15\,\sqrt{bx+a}a^2\right)}{15\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2/15*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)/b^3
```

3.185 $\int \frac{1}{x\sqrt{a+bx}}\,dx$

Optimal. Leaf size=23

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] time = 0.007127, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+bx}}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{b}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [A] time = 0.0032239, size = 23, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[a + b*x]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]
```

Maple [A] time = 0.006, size = 18, normalized size = 0.8

$$-2\,\frac{1}{\sqrt{a}}\mathrm{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(b*x+a)^(1/2),x)
```

```
[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.73259, size = 142, normalized size = 6.17

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sq
rt(b*x + a)*sqrt(-a)/a)/a]
```

Sympy [A] time = 1.09362, size = 24, normalized size = 1.04

$$-\frac{2\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)**(1/2),x)
```

```
[Out] -2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)
```

Giac [A] time = 1.11005, size = 28, normalized size = 1.22

$$\frac{2\,\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)
```

3.186 $\int \frac{1}{x^2\sqrt{a+bx}}\,dx$

Optimal. Leaf size=41

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

```
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rubi [A] time = 0.0117272, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {51, 63, 208}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\sqrt{a+bx}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[a + b*x]),x]
```

```
[Out] -(Sqrt[a + b*x]/(a*x)) + (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/a^(3/2)
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{a+bx}}\,dx &= -\frac{\sqrt{a+bx}}{ax}-\frac{b\int \frac{1}{x\sqrt{a+bx}}\,dx}{2a}\\ &= -\frac{\sqrt{a+bx}}{ax}-\frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{a}\\ &= -\frac{\sqrt{a+bx}}{ax}+\frac{b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.062887, size = 47, normalized size = 1.15

$$\frac{\sqrt{a+bx}\left(\frac{b\tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)}{\sqrt{\frac{bx}{a}+1}}-\frac{a}{x}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*x]),x]
```

```
[Out] (Sqrt[a + b*x]*(-(a/x) + (b*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[1 + (b*x)/a]))
/a^2
```

Maple [A] time = 0.007, size = 40, normalized size = 1.

$$2\,b\left(-1/2\,\frac{\sqrt{bx+a}}{abx}+1/2\,\frac{1}{a^{3/2}}\text{Artanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(b*x+a)^(1/2),x)
```

```
[Out] 2*b*(-1/2*(b*x+a)^(1/2)/a/b/x+1/2/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.7556, size = 232, normalized size = 5.66

$$\left[\frac{\sqrt{a}bx\log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)-2\sqrt{bx+a}a}{2a^2x}, -\frac{\sqrt{-a}bx\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\sqrt{bx+a}a}{a^2x}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x
 + a)*a)/(a^2*x), -(sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) + sqrt(b*
x + a)*a)/(a^2*x)]
```

Sympy [A] time = 2.18542, size = 44, normalized size = 1.07

$$-\frac{\sqrt{b}\sqrt{\frac{a}{bx}+1}}{a\sqrt{x}}+\frac{b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(b*x+a)**(1/2),x)
```

```
[Out] -sqrt(b)*sqrt(a/(b*x) + 1)/(a*sqrt(x)) + b*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))
/a**(3/2)
```

Giac [A] time = 1.10643, size = 63, normalized size = 1.54

$$-\frac{\frac{b^2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{\sqrt{bx+a}b}{ax}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] -(b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a) + sqrt(b*x + a)*b/(a*x))/
b
```

3.187 $\int (a+bx)^{p/2}\,dx$

Optimal. Leaf size=23

$$\frac{2(a+bx)^{\frac{p+2}{2}}}{b(p+2)}$$

```
[Out] (2*(a + b*x)^((2 + p)/2))/(b*(2 + p))
```

Rubi [A] time = 0.0036895, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {32}

$$\frac{2(a+bx)^{\frac{p+2}{2}}}{b(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^(p/2),x]
```

```
[Out] (2*(a + b*x)^((2 + p)/2))/(b*(2 + p))
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (a+bx)^{p/2}\,dx=\frac{2(a+bx)^{\frac{2+p}{2}}}{b(2+p)}$$

Mathematica [A] time = 0.0118408, size = 24, normalized size = 1.04

$$\frac{2(a+bx)^{\frac{p}{2}+1}}{bp+2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^(p/2),x]
```

```
[Out] (2*(a + b*x)^(1 + p/2))/(2*b + b*p)
```

Maple [A] time = 0.001, size = 25, normalized size = 1.1

$$2\,\frac{(bx+a)\left(\sqrt{bx+a}\right)^{p}}{b\left(2+p\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x+a)^(1/2))^p,x)
```

```
[Out] 2*(b*x+a)*((b*x+a)^(1/2))^p/b/(2+p)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.70099, size = 55, normalized size = 2.39

$$\frac{2\,(bx+a)\sqrt{bx+a}^{p}}{bp+2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] 2*(b*x + a)*sqrt(b*x + a)^p/(b*p + 2*b)
```

Sympy [A] time = 0.058297, size = 26, normalized size = 1.13

$$\frac{\begin{cases} \frac{(a+bx)^{\frac{p}{2}+1}}{\frac{p}{2}+1} & \text{for } \frac{p}{2} \neq -1 \\ \log(a+bx) & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)**(1/2))**p,x)
```

```
[Out] Piecewise(((a + b*x)**(p/2 + 1)/(p/2 + 1), Ne(p/2, -1)), (log(a + b*x), Tru
e))/b
```

Giac [A] time = 1.07583, size = 28, normalized size = 1.22

$$\frac{2\,(bx+a)^{\frac{1}{2}\,p+1}}{b\left(p+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b*x+a)^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*(b*x + a)^(1/2*p + 1)/(b*(p + 2))
```

3.188 $\int x(a+bx)^{p/2}\,dx$

Optimal. Leaf size=48

$$\frac{2(a+bx)^{\frac{p+4}{2}}}{b^2(p+4)}-\frac{2a(a+bx)^{\frac{p+2}{2}}}{b^2(p+2)}$$

```
[Out] (-2*a*(a + b*x)^((2 + p)/2))/(b^2*(2 + p)) + (2*(a + b*x)^((4 + p)/2))/(b^2
*(4 + p))
```

Rubi [A] time = 0.0142299, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2(a+bx)^{\frac{p+4}{2}}}{b^2(p+4)}-\frac{2a(a+bx)^{\frac{p+2}{2}}}{b^2(p+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^(p/2),x]
```

```
[Out] (-2*a*(a + b*x)^((2 + p)/2))/(b^2*(2 + p)) + (2*(a + b*x)^((4 + p)/2))/(b^2
*(4 + p))
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x(a+bx)^{p/2}\,dx &= \int \left(\frac{(a+bx)^{1+\frac{p}{2}}}{b}-\frac{a(a+bx)^{p/2}}{b}\right)dx\\ &= -\frac{2a(a+bx)^{\frac{2+p}{2}}}{b^2(2+p)}+\frac{2(a+bx)^{\frac{4+p}{2}}}{b^2(4+p)}\end{aligned}$$

Mathematica [A] time = 0.0199537, size = 38, normalized size = 0.79

$$\frac{2(a+bx)^{\frac{p}{2}+1}(b(p+2)x-2a)}{b^2(p+2)(p+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^(p/2),x]
```

```
[Out] (2*(a + b*x)^(1 + p/2)*(-2*a + b*(2 + p)*x))/(b^2*(2 + p)*(4 + p))
```

Maple [A] time = 0.002, size = 43, normalized size = 0.9

$$-2\,\frac{\left(\sqrt{bx+a}\right)^{p}\left(-xpb-2\,bx+2\,a\right)(bx+a)}{b^2\left(p^2+6\,p+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*((b*x+a)^(1/2))^p,x)
```

```
[Out] -2*((b*x+a)^(1/2))^p*(-b*p*x-2*b*x+2*a)*(b*x+a)/b^2/(p^2+6*p+8)
```

Maxima [A] time = 0.967557, size = 61, normalized size = 1.27

$$\frac{2\left(b^2\left(p+2\right)x^2+abpx-2\,a^2\right)(bx+a)^{\frac{1}{2}\,p}}{\left(p^2+6\,p+8\right)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^(1/2))^p,x, algorithm="maxima")
```

```
[Out] 2*(b^2*(p + 2)*x^2 + a*b*p*x - 2*a^2)*(b*x + a)^(1/2*p)/((p^2 + 6*p + 8)*b^
2)
```

Fricas [A] time = 1.81903, size = 117, normalized size = 2.44

$$\frac{2\left(abpx + \left(b^2p + 2\,b^2\right)x^2 - 2\,a^2\right)\sqrt{bx + a}^p}{b^2p^2 + 6\,b^2p + 8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^(1/2))^p,x, algorithm="fricas")
```

```
[Out] 2*(a*b*p*x + (b^2*p + 2*b^2)*x^2 - 2*a^2)*sqrt(b*x + a)^p/(b^2*p^2 + 6*b^2*
p + 8*b^2)
```

Sympy [A] time = 0.654637, size = 216, normalized size = 4.5

$$\begin{cases} \frac{a^{\frac{p}{2}}x^2}{2} & \text{for}\ b = 0 \\ \frac{a\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} + \frac{a}{ab^2+b^3x} + \frac{bx\log\left(\frac{a}{b}+x\right)}{ab^2+b^3x} & \text{for}\ p = -4 \\ -\frac{a\log\left(\frac{a}{b}+x\right)}{b^2} + \frac{x}{b} & \text{for}\ p = -2 \\ -\frac{4a^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} + \frac{2abpx(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} + \frac{2b^2px^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} + \frac{4b^2x^2(a+bx)^{\frac{p}{2}}}{b^2p^2+6b^2p+8b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)**(1/2))**p,x)
```

```
[Out] Piecewise((a**(p/2)*x**2/2, Eq(b, 0)), (a*log(a/b + x)/(a*b**2 + b**3*x) +
a/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b**3*x), Eq(p, -4)), (-a*l
og(a/b + x)/b**2 + x/b, Eq(p, -2)), (-4*a**2*(a + b*x)**(p/2)/(b**2*p**2 +
6*b**2*p + 8*b**2) + 2*a*b*p*x*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b
**2) + 2*b**2*p*x**2*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2) + 4*b
**2*x**2*(a + b*x)**(p/2)/(b**2*p**2 + 6*b**2*p + 8*b**2), True))
```

Giac [A] time = 1.06266, size = 116, normalized size = 2.42

$$\frac{2\left((bx + a)^{\frac{1}{2}\,p}b^2px^2 + (bx + a)^{\frac{1}{2}\,p}abpx + 2\,(bx + a)^{\frac{1}{2}\,p}b^2x^2 - 2\,(bx + a)^{\frac{1}{2}\,p}a^2\right)}{b^2p^2 + 6\,b^2p + 8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((b*x+a)^(1/2))^p,x, algorithm="giac")
```

```
[Out] 2*((b*x + a)^(1/2*p)*b^2*p*x^2 + (b*x + a)^(1/2*p)*a*b*p*x + 2*(b*x + a)^(1
/2*p)*b^2*x^2 - 2*(b*x + a)^(1/2*p)*a^2)/(b^2*p^2 + 6*b^2*p + 8*b^2)
```

3.189 $\int \tan^{-1}\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right) dx$

Optimal. Leaf size=55

$$-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}-x\tan^{-1}\left(1-\sqrt{2}x\right)+\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}$$

```
[Out] ArcTan[1 - Sqrt[2]*x]/Sqrt[2] - x*ArcTan[1 - Sqrt[2]*x] - Log[1 - Sqrt[2]*x
 + x^2]/(2*Sqrt[2])
```

Rubi [A] time = 0.0277273, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5203, 12, 634, 617, 204, 628}

$$-\frac{\log\left(x^2-\sqrt{2}x+1\right)}{2\sqrt{2}}-x\tan^{-1}\left(1-\sqrt{2}x\right)+\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[(-Sqrt[2] + 2*x)/Sqrt[2]],x]
```

```
[Out] ArcTan[1 - Sqrt[2]*x]/Sqrt[2] - x*ArcTan[1 - Sqrt[2]*x] - Log[1 - Sqrt[2]*x
 + x^2]/(2*Sqrt[2])
```

Rule 5203

```
Int[ArcTan[u_], x_Symbol] :> Simp[x*ArcTan[u], x] - Int[SimplifyIntegrand[(
x*D[u, x])/(1 + u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \tan^{-1}\left(\frac{-\sqrt{2}+2x}{\sqrt{2}}\right)dx &= -x\tan^{-1}\left(1-\sqrt{2}x\right)-\int \frac{x}{\sqrt{2}\left(1-\sqrt{2}x+x^2\right)}\,dx\\
&= -x\tan^{-1}\left(1-\sqrt{2}x\right)-\frac{\int \frac{x}{1-\sqrt{2}x+x^2}\,dx}{\sqrt{2}}\\
&= -x\tan^{-1}\left(1-\sqrt{2}x\right)-\frac{1}{2}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx-\frac{\int \frac{-\sqrt{2}+2x}{1-\sqrt{2}x+x^2}\,dx}{2\sqrt{2}}\\
&= -x\tan^{-1}\left(1-\sqrt{2}x\right)-\frac{\log\left(1-\sqrt{2}x+x^2\right)}{2\sqrt{2}}-\frac{\mathrm{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}x\right)}{\sqrt{2}}\\
&= \frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{\sqrt{2}}-x\tan^{-1}\left(1-\sqrt{2}x\right)-\frac{\log\left(1-\sqrt{2}x+x^2\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.0337808, size = 48, normalized size = 0.87

$$\frac{1}{4}\left(2\left(\sqrt{2}-2x\right)\tan^{-1}\left(1-\sqrt{2}x\right)-\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[(-Sqrt[2] + 2*x)/Sqrt[2]],x]
```

```
[Out] (2*(Sqrt[2] - 2*x)*ArcTan[1 - Sqrt[2]*x] - Sqrt[2]*Log[1 - Sqrt[2]*x + x^2]
)/4
```

Maple [A] time = 0.003, size = 42, normalized size = 0.8

$$x \arctan\left(-1 + x\sqrt{2}\right) - \frac{\arctan\left(-1 + x\sqrt{2}\right)\sqrt{2}}{2} - \frac{\sqrt{2}\ln\left(\left(-1 + x\sqrt{2}\right)^2 + 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x)
```

```
[Out] x*arctan(-1+x*2^(1/2))-1/2*arctan(-1+x*2^(1/2))*2^(1/2)-1/4*2^(1/2)*ln((-1+
x*2^(1/2))^2+1)
```

Maxima [A] time = 1.42633, size = 70, normalized size = 1.27

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2}\left(2x - \sqrt{2}\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x - \sqrt{2}\right)\right) - \log\left(\frac{1}{2}\left(2x - \sqrt{2}\right)^2 + 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*(2*x - sqrt(2))*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) -
log(1/2*(2*x - sqrt(2))^2 + 1))
```

Fricas [A] time = 1.91039, size = 111, normalized size = 2.02

$$\frac{1}{2}\left(2x - \sqrt{2}\right)\arctan\left(\sqrt{2}x - 1\right) - \frac{1}{4}\sqrt{2}\log\left(x^2 - \sqrt{2}x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*x - sqrt(2))*arctan(sqrt(2)*x - 1) - 1/4*sqrt(2)*log(x^2 - sqrt(2)*x
 + 1)
```

Sympy [B] time = 1.3012, size = 230, normalized size = 4.18

$$\frac{4x^3 \operatorname{atan}\left(\sqrt{2}x-1\right)}{4x^2-4\sqrt{2}x+4}-\frac{\sqrt{2}x^2\log\left(x^2-\sqrt{2}x+1\right)}{4x^2-4\sqrt{2}x+4}-\frac{6\sqrt{2}x^2\operatorname{atan}\left(\sqrt{2}x-1\right)}{4x^2-4\sqrt{2}x+4}+\frac{2x\log\left(x^2-\sqrt{2}x+1\right)}{4x^2-4\sqrt{2}x+4}+\frac{8x\operatorname{atan}\left(\sqrt{2}x-1\right.}{4x^2-4\sqrt{2}x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(1/2*(2*x-2**(1/2))*2**(1/2)),x)
```

```
[Out] 4*x**3*atan(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4) - sqrt(2)*x**2*log(x*
*2 - sqrt(2)*x + 1)/(4*x**2 - 4*sqrt(2)*x + 4) - 6*sqrt(2)*x**2*atan(sqrt(2
)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4) + 2*x*log(x**2 - sqrt(2)*x + 1)/(4*x**2
 - 4*sqrt(2)*x + 4) + 8*x*atan(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4) -
sqrt(2)*log(x**2 - sqrt(2)*x + 1)/(4*x**2 - 4*sqrt(2)*x + 4) - 2*sqrt(2)*at
an(sqrt(2)*x - 1)/(4*x**2 - 4*sqrt(2)*x + 4)
```

Giac [A] time = 1.07475, size = 70, normalized size = 1.27

$$\frac{1}{4}\sqrt{2}\left(\sqrt{2}\left(2x-\sqrt{2}\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)-\log\left(\frac{1}{2}\left(2x-\sqrt{2}\right)^2+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(1/2*(2*x-2^(1/2))*2^(1/2)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(sqrt(2)*(2*x - sqrt(2))*arctan(1/2*sqrt(2)*(2*x - sqrt(2))) -
log(1/2*(2*x - sqrt(2))^2 + 1))
```

3.190 $\int \frac{1}{\sqrt{-1+x^2}}\,dx$

Optimal. Leaf size=12

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

```
[Out] ArcTanh[x/Sqrt[-1 + x^2]]
```

Rubi [A] time = 0.0023553, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {217, 206}

$$\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + x^2],x]
```

```
[Out] ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{-1+x^2}}\right)\\ &= \tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)\end{aligned}$$

Mathematica [B] time = 0.00269, size = 38, normalized size = 3.17

$$\frac{1}{2}\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)-\frac{1}{2}\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + x^2],x]
```

```
[Out] -Log[1 - x/Sqrt[-1 + x^2]]/2 + Log[1 + x/Sqrt[-1 + x^2]]/2
```

Maple [A] time = 0.002, size = 11, normalized size = 0.9

$$\ln\left(x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^(1/2),x)
```

```
[Out] ln(x+(x^2-1)^(1/2))
```

Maxima [A] time = 0.954707, size = 19, normalized size = 1.58

$$\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 - 1))
```

Fricas [A] time = 1.60869, size = 35, normalized size = 2.92

$$-\log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 - 1))
```

Sympy [A] time = 0.124899, size = 2, normalized size = 0.17

$$\operatorname{acosh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**(1/2),x)
```

```
[Out] acosh(x)
```

Giac [A] time = 1.09333, size = 20, normalized size = 1.67

$$-\log\left(\left|-x+\sqrt{x^2-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-x + sqrt(x^2 - 1)))
```

3.191 $\int \sqrt{x}\sqrt{1+x}\,dx$

Optimal. Leaf size=43

$$\frac{1}{2}\sqrt{x+1}x^{3/2}+\frac{1}{4}\sqrt{x+1}\sqrt{x}-\frac{1}{4}\sinh^{-1}\left(\sqrt{x}\right)$$

```
[Out] (Sqrt[x]*Sqrt[1 + x])/4 + (x^(3/2)*Sqrt[1 + x])/2 - ArcSinh[Sqrt[x]]/4
```

Rubi [A] time = 0.0054499, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {50, 54, 215}

$$\frac{1}{2}\sqrt{x+1}x^{3/2}+\frac{1}{4}\sqrt{x+1}\sqrt{x}-\frac{1}{4}\sinh^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x])/4 + (x^(3/2)*Sqrt[1 + x])/2 - ArcSinh[Sqrt[x]]/4
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\sqrt{1+x}\,dx &= \frac{1}{2}x^{3/2}\sqrt{1+x}+\frac{1}{4}\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx\\ &= \frac{1}{4}\sqrt{x}\sqrt{1+x}+\frac{1}{2}x^{3/2}\sqrt{1+x}-\frac{1}{8}\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx\\ &= \frac{1}{4}\sqrt{x}\sqrt{1+x}+\frac{1}{2}x^{3/2}\sqrt{1+x}-\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}}\,dx,x,\sqrt{x}\right)\\ &= \frac{1}{4}\sqrt{x}\sqrt{1+x}+\frac{1}{2}x^{3/2}\sqrt{1+x}-\frac{1}{4}\sinh^{-1}\left(\sqrt{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0117258, size = 31, normalized size = 0.72

$$\frac{1}{4}\left(\sqrt{x}\sqrt{x+1}(2x+1)-\sinh^{-1}\left(\sqrt{x}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*Sqrt[1 + x],x]
```

```
[Out] (Sqrt[x]*Sqrt[1 + x]*(1 + 2*x) - ArcSinh[Sqrt[x]])/4
```

Maple [A] time = 0.003, size = 50, normalized size = 1.2

$$\frac{1}{2}\sqrt{x}\,(1+x)^{\frac{3}{2}}-\frac{1}{4}\sqrt{x}\sqrt{1+x}-\frac{1}{8}\sqrt{x\,(1+x)}\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\frac{1}{\sqrt{x}}\frac{1}{\sqrt{1+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(1+x)^(1/2),x)
```

```
[Out] 1/2*x^(1/2)*(1+x)^(3/2)-1/4*x^(1/2)*(1+x)^(1/2)-1/8*(x*(1+x))^(1/2)/(1+x)^(
1/2)/x^(1/2)*ln(1/2+x+(x^2+x)^(1/2))
```

Maxima [B] time = 0.946795, size = 96, normalized size = 2.23

$$\frac{\frac{(x+1)^{\frac{3}{2}}}{x^{\frac{3}{2}}}+\frac{\sqrt{x+1}}{\sqrt{x}}}{4\left(\frac{(x+1)^{2}}{x^{2}}-\frac{2\,(x+1)}{x}+1\right)}-\frac{1}{8}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right)+\frac{1}{8}\,\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*((x + 1)^(3/2)/x^(3/2) + sqrt(x + 1)/sqrt(x))/((x + 1)^2/x^2 - 2*(x + 1
)/x + 1) - 1/8*log(sqrt(x + 1)/sqrt(x) + 1) + 1/8*log(sqrt(x + 1)/sqrt(x) -
 1)
```

Fricas [A] time = 1.71554, size = 105, normalized size = 2.44

$$\frac{1}{4}\,(2\,x+1)\sqrt{x+1}\sqrt{x}+\frac{1}{8}\,\log\left(2\,\sqrt{x+1}\sqrt{x}-2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x + 1)*sqrt(x + 1)*sqrt(x) + 1/8*log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1
)
```

Sympy [A] time = 2.59574, size = 119, normalized size = 2.77

$$\begin{cases} -\frac{\operatorname{acosh}\left(\sqrt{x+1}\right)}{4}+\frac{(x+1)^{\frac{5}{2}}}{2\sqrt{x}}-\frac{3(x+1)^{\frac{3}{2}}}{4\sqrt{x}}+\frac{\sqrt{x+1}}{4\sqrt{x}} & \text{for}\ |x+1|>1 \\ \frac{i\operatorname{asin}\left(\sqrt{x+1}\right)}{4}-\frac{i(x+1)^{\frac{5}{2}}}{2\sqrt{-x}}+\frac{3i(x+1)^{\frac{3}{2}}}{4\sqrt{-x}}-\frac{i\sqrt{x+1}}{4\sqrt{-x}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(1+x)**(1/2),x)
```

```
[Out] Piecewise((-acosh(sqrt(x + 1))/4 + (x + 1)**(5/2)/(2*sqrt(x)) - 3*(x + 1)**
(3/2)/(4*sqrt(x)) + sqrt(x + 1)/(4*sqrt(x)), Abs(x + 1) > 1), (I*asin(sqrt(
```

```
x + 1))/4 - I*(x + 1)**(5/2)/(2*sqrt(-x)) + 3*I*(x + 1)**(3/2)/(4*sqrt(-x))
 - I*sqrt(x + 1)/(4*sqrt(-x)), True))
```

Giac [A] time = 1.1477, size = 42, normalized size = 0.98

$$\frac{1}{4}\left(2x+1\right)\sqrt{x+1}\sqrt{x}+\frac{1}{4}\log\left(\left|-\sqrt{x+1}+\sqrt{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(2*x + 1)*sqrt(x + 1)*sqrt(x) + 1/4*log(abs(-sqrt(x + 1) + sqrt(x)))
```

3.192 $\int \sin\left(\sqrt{x}\right) dx$

Optimal. Leaf size=22

$$2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rubi [A] time = 0.0108578, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3361, 3296, 2637}

$$2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[x]],x]
```

```
[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sin\left(\sqrt{x}\right) dx = 2\,\text{Subst}\left(\int x\sin(x)\,dx, x, \sqrt{x}\right)$$
$$= -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\,\text{Subst}\left(\int \cos(x)\,dx, x, \sqrt{x}\right)$$
$$= -2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

Mathematica [A] time = 0.0129006, size = 22, normalized size = 1.

$$2\sin\left(\sqrt{x}\right) - 2\sqrt{x}\cos\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[x]],x]

[Out] -2*Sqrt[x]*Cos[Sqrt[x]] + 2*Sin[Sqrt[x]]
```

Maple [A] time = 0.004, size = 17, normalized size = 0.8

$$2\sin\left(\sqrt{x}\right) - 2\cos\left(\sqrt{x}\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x^(1/2)),x)

[Out] 2*sin(x^(1/2))-2*cos(x^(1/2))*x^(1/2)
```

Maxima [A] time = 0.938029, size = 22, normalized size = 1.

$$-2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Fricas [A] time = 1.66608, size = 57, normalized size = 2.59

$$-2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Sympy [A] time = 0.295031, size = 20, normalized size = 0.91

$$-2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x**(1/2)),x)
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

Giac [A] time = 1.08711, size = 22, normalized size = 1.

$$-2\sqrt{x}\cos\left(\sqrt{x}\right) + 2\sin\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x^(1/2)),x, algorithm="giac")
```

```
[Out] -2*sqrt(x)*cos(sqrt(x)) + 2*sin(sqrt(x))
```

3.193 $\int \frac{x}{\left(1-x^2\right)^{9/8}} \, dx$

Optimal. Leaf size=13

$$\frac{4}{\sqrt[8]{1-x^2}}$$

```
[Out] 4/(1 - x^2)^(1/8)
```

Rubi [A] time = 0.0024579, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$\frac{4}{\sqrt[8]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 - x^2)^(9/8),x]
```

```
[Out] 4/(1 - x^2)^(1/8)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\left(1-x^2\right)^{9/8}} \, dx = \frac{4}{\sqrt[8]{1-x^2}}$$

Mathematica [A] time = 0.0021493, size = 13, normalized size = 1.

$$\frac{4}{\sqrt[8]{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 - x^2)^(9/8),x]
```

```
[Out] 4/(1 - x^2)^(1/8)
```

Maple [A] time = 0.002, size = 18, normalized size = 1.4

$$-4\left(-1+x\right)\left(1+x\right)\left(-x^{2}+1\right)^{-\frac{9}{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^(9/8),x)
```

```
[Out] -4*(-1+x)*(1+x)/(-x^2+1)^(9/8)
```

Maxima [A] time = 0.957097, size = 15, normalized size = 1.15

$$\frac{4}{\left(-x^{2}+1\right)^{\frac{1}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(9/8),x, algorithm="maxima")
```

```
[Out] 4/(-x^2 + 1)^(1/8)
```

Fricas [A] time = 1.52099, size = 42, normalized size = 3.23

$$-\frac{4\left(-x^{2}+1\right)^{\frac{7}{8}}}{x^{2}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(9/8),x, algorithm="fricas")
```

```
[Out] -4*(-x^2 + 1)^(7/8)/(x^2 - 1)
```

Sympy [A] time = 1.07254, size = 8, normalized size = 0.62

$$\frac{4}{\sqrt[8]{1-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(9/8),x)
```

```
[Out] 4/(1 - x**2)**(1/8)
```

Giac [A] time = 1.08615, size = 15, normalized size = 1.15

$$\frac{4}{\left(-x^2+1\right)^{\frac{1}{8}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(9/8),x, algorithm="giac")
```

```
[Out] 4/(-x^2 + 1)^(1/8)
```

3.194 $\int \frac{x}{\sqrt{1-x^4}} \, dx$

Optimal. Leaf size=8

$$\frac{1}{2}\sin^{-1}\left(x^2\right)$$

```
[Out] ArcSin[x^2]/2
```

Rubi [A] time = 0.0027417, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {275, 216}

$$\frac{1}{2}\sin^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 - x^4],x]
```

```
[Out] ArcSin[x^2]/2
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{1-x^4}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} \, dx, x, x^2\right)\\ &= \frac{1}{2}\sin^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [A] time = 0.002432, size = 8, normalized size = 1.

$$\frac{1}{2}\sin^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 - x^4],x]
```

```
[Out] ArcSin[x^2]/2
```

Maple [A] time = 0.008, size = 7, normalized size = 0.9

$$\frac{\arcsin\left(x^2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^4+1)^(1/2),x)
```

```
[Out] 1/2*arcsin(x^2)
```

Maxima [B] time = 1.42137, size = 22, normalized size = 2.75

$$-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^4+1}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(sqrt(-x^4 + 1)/x^2)
```

Fricas [B] time = 1.55369, size = 47, normalized size = 5.88

$$-\arctan\left(\frac{\sqrt{-x^4+1}-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan((sqrt(-x^4 + 1) - 1)/x^2)
```

Sympy [A] time = 0.964442, size = 19, normalized size = 2.38

$$\begin{cases} -\frac{i \operatorname{acosh}\left(x^{2}\right)}{2} & \text{for } \left|x^{4}\right| > 1 \\ \frac{\operatorname{asin}\left(x^{2}\right)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**4+1)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(x**2)/2, Abs(x**4) > 1), (asin(x**2)/2, True))
```

Giac [A] time = 1.11262, size = 8, normalized size = 1.

$$\frac{1}{2} \arcsin\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arcsin(x^2)
```

3.195 $\int \frac{1}{x\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=14

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

```
[Out] -ArcTanh[Sqrt[1 + x^4]]/2
```

Rubi [A] time = 0.0063204, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {266, 63, 207}

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[1 + x^4]),x]
```

```
[Out] -ArcTanh[Sqrt[1 + x^4]]/2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{1+x^4}}\,dx &= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x}}\,dx, x, x^4\right)\\ &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1+x^4}\right)\\ &= -\frac{1}{2}\tanh^{-1}\left(\sqrt{1+x^4}\right)\end{aligned}$$

Mathematica [A] time = 0.0026209, size = 14, normalized size = 1.

$$-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^4+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[1 + x^4]),x]
```

```
[Out] -ArcTanh[Sqrt[1 + x^4]]/2
```

Maple [A] time = 0.009, size = 11, normalized size = 0.8

$$-\frac{1}{2}\operatorname{Artanh}\left(\frac{1}{\sqrt{x^4+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^4+1)^(1/2),x)
```

```
[Out] -1/2*arctanh(1/(x^4+1)^(1/2))
```

Maxima [B] time = 0.935516, size = 34, normalized size = 2.43

$$-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

Fricas [B] time = 1.56532, size = 78, normalized size = 5.57

$$-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

Sympy [A] time = 0.938144, size = 8, normalized size = 0.57

$$-\frac{\operatorname{asinh}\left(\frac{1}{x^2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**4+1)**(1/2),x)
```

```
[Out] -asinh(x**(-2))/2
```

Giac [B] time = 1.07049, size = 34, normalized size = 2.43

$$-\frac{1}{4}\log\left(\sqrt{x^4+1}+1\right)+\frac{1}{4}\log\left(\sqrt{x^4+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*log(sqrt(x^4 + 1) + 1) + 1/4*log(sqrt(x^4 + 1) - 1)
```

3.196 $\int \frac{x}{\sqrt{1+x^2+x^4}}\,dx$

Optimal. Leaf size=18

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

```
[Out] ArcSinh[(1 + 2*x^2)/Sqrt[3]]/2
```

Rubi [A] time = 0.0162554, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1107, 619, 215}

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] ArcSinh[(1 + 2*x^2)/Sqrt[3]]/2
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^2+x^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+x+x^2}}\,dx, x, x^2\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx, x, 1+2x^2\right)}{2\sqrt{3}}$$
$$= \frac{1}{2}\sinh^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)$$

Mathematica [A] time = 0.0029637, size = 18, normalized size = 1.

$$\frac{1}{2}\sinh^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 + x^2 + x^4],x]
```

```
[Out] ArcSinh[(1 + 2*x^2)/Sqrt[3]]/2
```

Maple [A] time = 0.011, size = 14, normalized size = 0.8

$$\frac{1}{2}\text{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(x^2+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsinh(2/3*3^(1/2)*(x^2+1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^4+x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(x^4 + x^2 + 1), x)
```

Fricas [A] time = 1.6057, size = 62, normalized size = 3.44

$$-\frac{1}{2}\log\left(-2x^2+2\sqrt{x^4+x^2+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\left(x^2-x+1\right)\left(x^2+x+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+x**2+1)**(1/2),x)
```

```
[Out] Integral(x/sqrt((x**2 - x + 1)*(x**2 + x + 1)), x)
```

Giac [A] time = 1.10486, size = 30, normalized size = 1.67

$$-\frac{1}{2}\log\left(-2x^2+2\sqrt{x^4+x^2+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(-2*x^2 + 2*sqrt(x^4 + x^2 + 1) - 1)
```

3.197 $\int \frac{1}{x\sqrt{-1+x^2-x^4}}\,dx$

Optimal. Leaf size=30

$$-\frac{1}{2}\tan^{-1}\left(\frac{2-x^2}{2\sqrt{-x^4+x^2-1}}\right)$$

```
[Out] -ArcTan[(2 - x^2)/(2*Sqrt[-1 + x^2 - x^4])]/2
```

Rubi [A] time = 0.0210465, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1114, 724, 204}

$$-\frac{1}{2}\tan^{-1}\left(\frac{2-x^2}{2\sqrt{-x^4+x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[-1 + x^2 - x^4]),x]
```

```
[Out] -ArcTan[(2 - x^2)/(2*Sqrt[-1 + x^2 - x^4])]/2
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{x\sqrt{-1+x^2-x^4}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x\sqrt{-1+x-x^2}}\,dx, x, x^2\right)$$
$$= -\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-2+x^2}{\sqrt{-1+x^2-x^4}}\right)$$
$$= \frac{1}{2}\tan^{-1}\left(\frac{-2+x^2}{2\sqrt{-1+x^2-x^4}}\right)$$

Mathematica [A] time = 0.0040188, size = 28, normalized size = 0.93

$$\frac{1}{2}\tan^{-1}\left(\frac{x^2-2}{2\sqrt{-x^4+x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Sqrt[-1 + x^2 - x^4]),x]
```

```
[Out] ArcTan[(-2 + x^2)/(2*Sqrt[-1 + x^2 - x^4])]/2
```

Maple [A] time = 0.011, size = 23, normalized size = 0.8

$$\frac{1}{2}\arctan\left(\frac{x^2-2}{2}\frac{1}{\sqrt{-x^4+x^2-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-x^4+x^2-1)^(1/2),x)
```

```
[Out] 1/2*arctan(1/2*(x^2-2)/(-x^4+x^2-1)^(1/2))
```

Maxima [C] time = 1.44107, size = 23, normalized size = 0.77

$$-\frac{1}{2}i\,\text{arsinh}\left(-\frac{1}{3}\sqrt{3}+\frac{2\sqrt{3}}{3x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*I*arcsinh(-1/3*sqrt(3) + 2/3*sqrt(3)/x^2)
```

Fricas [C] time = 1.70981, size = 155, normalized size = 5.17

$$\frac{1}{4}i\log\left(\frac{x^2+2i\sqrt{-x^4+x^2-1}-2}{2x^2}\right)-\frac{1}{4}i\log\left(\frac{x^2-2i\sqrt{-x^4+x^2-1}-2}{2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*I*log(1/2*(x^2 + 2*I*sqrt(-x^4 + x^2 - 1) - 2)/x^2) - 1/4*I*log(1/2*(x^
2 - 2*I*sqrt(-x^4 + x^2 - 1) - 2)/x^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x\sqrt{-x^4+x^2-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x**4+x**2-1)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-x**4 + x**2 - 1)), x)
```

Giac [C] time = 1.12593, size = 20, normalized size = 0.67

$$\frac{1}{2}i\arcsin\left(\frac{1}{3}\sqrt{3}\left(\frac{2i}{x^2}-i\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-x^4+x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*I*arcsin(1/3*sqrt(3)*(2*I/x^2 - I))
```

3.198 $\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=17

$$\frac{\sqrt{x^2+1}}{1-x}$$

```
[Out] Sqrt[1 + x^2]/(1 - x)
```

Rubi [A] time = 0.0090941, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {803}

$$\frac{\sqrt{x^2+1}}{1-x}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/((1 - x)^2*Sqrt[1 + x^2]),x]

[Out] Sqrt[1 + x^2]/(1 - x)
```

Rule 803

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && N
eQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && EqQ[c*d*f + a*e*g,
 0]
```

Rubi steps

$$\int \frac{1+x}{(1-x)^2\sqrt{1+x^2}}\,dx = \frac{\sqrt{1+x^2}}{1-x}$$

Mathematica [A] time = 0.0069062, size = 16, normalized size = 0.94

$$-\frac{\sqrt{x^2+1}}{x-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/((1 - x)^2*Sqrt[1 + x^2]),x]
```

```
[Out] -(Sqrt[1 + x^2]/(-1 + x))
```

Maple [A] time = 0.003, size = 15, normalized size = 0.9

$$-\frac{1}{-1+x}\sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(1-x)^2/(x^2+1)^(1/2),x)
```

```
[Out] -(x^2+1)^(1/2)/(-1+x)
```

Maxima [A] time = 1.41986, size = 19, normalized size = 1.12

$$-\frac{\sqrt{x^2+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(x^2 + 1)/(x - 1)
```

Fricas [A] time = 1.63456, size = 46, normalized size = 2.71

$$-\frac{x+\sqrt{x^2+1}-1}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(x + sqrt(x^2 + 1) - 1)/(x - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{(x-1)^2\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1-x)**2/(x**2+1)**(1/2),x)
```

```
[Out] Integral((x + 1)/((x - 1)**2*sqrt(x**2 + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x+1}{\sqrt{x^2+1}(x-1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(1-x)^2/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((x + 1)/(sqrt(x^2 + 1)*(x - 1)^2), x)
```

3.199 $\int \frac{1}{\sqrt{1+x^2}} \, dx$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

```
[Out] ArcSinh[x]
```

Rubi [A] time = 0.0008872, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {215}

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcSinh[x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}} \, dx = \sinh^{-1}(x)$$

Mathematica [A] time = 0.0029345, size = 2, normalized size = 1.

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcSinh[x]
```

Maple [A] time = 0.002, size = 3, normalized size = 1.5

$$\operatorname{Arcsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2),x)
```

```
[Out] arcsinh(x)
```

Maxima [A] time = 1.41454, size = 3, normalized size = 1.5

$$\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(x)
```

Fricas [B] time = 1.61621, size = 35, normalized size = 17.5

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 1))
```

Sympy [A] time = 0.125512, size = 2, normalized size = 1.

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2),x)
```

```
[Out] asinh(x)
```

Giac [B] time = 1.07613, size = 19, normalized size = 9.5

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-x + sqrt(x^2 + 1))
```

3.200 $\int \frac{\sqrt{x}\sqrt{1+x}+\sqrt{x}\sqrt{2+x}+\sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}} dx$

Optimal. Leaf size=20

$$\sqrt{x}+\sqrt{x+1}+\sqrt{x+2}$$

```
[Out] Sqrt[x] + Sqrt[1 + x] + Sqrt[2 + x]
```

Rubi [A] time = 0.912736, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 65, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {12, 6688}

$$\sqrt{x}+\sqrt{x+1}+\sqrt{x+2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[x]*Sqrt[1 + x] + Sqrt[x]*Sqrt[2 + x] + Sqrt[1 + x]*Sqrt[2 + x])/(
2*Sqrt[x]*Sqrt[1 + x]*Sqrt[2 + x]),x]
```

```
[Out] Sqrt[x] + Sqrt[1 + x] + Sqrt[2 + x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}\sqrt{1+x}+\sqrt{x}\sqrt{2+x}+\sqrt{1+x}\sqrt{2+x}}{2\sqrt{x}\sqrt{1+x}\sqrt{2+x}}\,dx &= \frac{1}{2}\int \frac{\sqrt{x}\sqrt{1+x}+\sqrt{x}\sqrt{2+x}+\sqrt{1+x}\sqrt{2+x}}{\sqrt{x}\sqrt{1+x}\sqrt{2+x}}\,dx\\
&= \frac{1}{2}\int \left(\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{1+x}}+\frac{1}{\sqrt{2+x}}\right)dx\\
&= \sqrt{x}+\sqrt{1+x}+\sqrt{2+x}
\end{aligned}$$

Mathematica [A] time = 0.0176992, size = 30, normalized size = 1.5

$$\frac{1}{2}\left(2\sqrt{x}+2\sqrt{x+1}+2\sqrt{x+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x]*Sqrt[1 + x] + Sqrt[x]*Sqrt[2 + x] + Sqrt[1 + x]*Sqrt[2 +
 x])/(2*Sqrt[x]*Sqrt[1 + x]*Sqrt[2 + x]),x]
```

```
[Out] (2*Sqrt[x] + 2*Sqrt[1 + x] + 2*Sqrt[2 + x])/2
```

Maple [A] time = 0.002, size = 15, normalized size = 0.8

$$\sqrt{x}+\sqrt{1+x}+\sqrt{2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1/2))/x
^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x)
```

```
[Out] x^(1/2)+(1+x)^(1/2)+(2+x)^(1/2)
```

Maxima [A] time = 0.946515, size = 19, normalized size = 0.95

$$\sqrt{x+2}+\sqrt{x+1}+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1
/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x + 2) + sqrt(x + 1) + sqrt(x)
```

Fricas [A] time = 1.51155, size = 50, normalized size = 2.5

$$\sqrt{x+2}+\sqrt{x+1}+\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1
/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x + 2) + sqrt(x + 1) + sqrt(x)
```

Sympy [A] time = 1.21279, size = 17, normalized size = 0.85

$$\sqrt{x} + \sqrt{x+1} + \sqrt{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(x**(1/2)*(1+x)**(1/2)+x**(1/2)*(2+x)**(1/2)+(1+x)**(1/2)*(2+
x)**(1/2))/x**(1/2)/(1+x)**(1/2)/(2+x)**(1/2),x)
```

```
[Out] sqrt(x) + sqrt(x + 1) + sqrt(x + 2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+2}\sqrt{x+1} + \sqrt{x+2}\sqrt{x} + \sqrt{x+1}\sqrt{x}}{2\sqrt{x+2}\sqrt{x+1}\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(x^(1/2)*(1+x)^(1/2)+x^(1/2)*(2+x)^(1/2)+(1+x)^(1/2)*(2+x)^(1
/2))/x^(1/2)/(1+x)^(1/2)/(2+x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/2*(sqrt(x + 2)*sqrt(x + 1) + sqrt(x + 2)*sqrt(x) + sqrt(x + 1)*
sqrt(x))/(sqrt(x + 2)*sqrt(x + 1)*sqrt(x)), x)
```

3.201 $\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx$

Optimal. Leaf size=24

$$\sqrt{x^5-2x+1}-\sqrt{x^3+1}$$

```
[Out] -Sqrt[1 + x^3] + Sqrt[1 - 2*x + x^5]
```

Rubi [A] time = 0.323708, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 68, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {12, 6688, 261, 2099}

$$\sqrt{x^5-2x+1}-\sqrt{x^3+1}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*Sqrt[1 + x^3] + 5*x^4*Sqrt[1 + x^3] - 3*x^2*Sqrt[1 - 2*x + x^5])/(2
*Sqrt[1 + x^3]*Sqrt[1 - 2*x + x^5]),x]
```

```
[Out] -Sqrt[1 + x^3] + Sqrt[1 - 2*x + x^5]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2099

```
Int[(Pm_)*(Qn_)^(p_), x_Symbol] :> With[{m = Expon[Pm, x], n = Expon[Qn, x]
}, Simp[(Coeff[Pm, x, m]*Qn^(p + 1))/(n*(p + 1)*Coeff[Qn, x, n]), x] + Dist
```

```
[Simplify[Pm - (Coeff[Pm, x, m]*D[Qn, x])/(n*Coeff[Qn, x, n])], Int[Qn^p, x
], x] /; EqQ[m, n - 1] && EqQ[D[Simplify[Pm - (Coeff[Pm, x, m]*D[Qn, x])/(n
*Coeff[Qn, x, n])], x], 0]] /; FreeQ[p, x] && PolyQ[Pm, x] && PolyQ[Qn, x]
&& NeQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{2\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx &= \frac{1}{2}\int \frac{-2\sqrt{1+x^3}+5x^4\sqrt{1+x^3}-3x^2\sqrt{1-2x+x^5}}{\sqrt{1+x^3}\sqrt{1-2x+x^5}}\,dx \\
&= \frac{1}{2}\int \left(-\frac{3x^2}{\sqrt{1+x^3}}-\frac{2}{\sqrt{1-2x+x^5}}+\frac{5x^4}{\sqrt{1-2x+x^5}}\right) dx \\
&= -\left(\frac{3}{2}\int \frac{x^2}{\sqrt{1+x^3}}\,dx\right)+\frac{5}{2}\int \frac{x^4}{\sqrt{1-2x+x^5}}\,dx-\int \frac{1}{\sqrt{1-2x+x^5}}\,dx \\
&= -\sqrt{1+x^3}+\sqrt{1-2x+x^5}
\end{aligned}
$$

Mathematica [A] time = 0.16667, size = 24, normalized size = 1.

$$\sqrt{x^5-2x+1}-\sqrt{x^3+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*Sqrt[1 + x^3] + 5*x^4*Sqrt[1 + x^3] - 3*x^2*Sqrt[1 - 2*x + x^
5])/(2*Sqrt[1 + x^3]*Sqrt[1 - 2*x + x^5]),x]
```

```
[Out] -Sqrt[1 + x^3] + Sqrt[1 - 2*x + x^5]
```

Maple [A] time = 0.005, size = 21, normalized size = 0.9

$$-\sqrt{x^3+1}+\sqrt{x^5-2\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2))/(x^3
+1)^(1/2)/(x^5-2*x+1)^(1/2),x)
```

```
[Out] -(x^3+1)^(1/2)+(x^5-2*x+1)^(1/2)
```

Maxima [A] time = 1.60047, size = 41, normalized size = 1.71

$$\sqrt{x^4 + x^3 + x^2 + x - 1}\sqrt{x - 1} - \sqrt{x^3 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2)
)/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^4 + x^3 + x^2 + x - 1)*sqrt(x - 1) - sqrt(x^3 + 1)
```

Fricas [A] time = 1.6198, size = 50, normalized size = 2.08

$$\sqrt{x^5 - 2\,x + 1} - \sqrt{x^3 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2)
)/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^5 - 2*x + 1) - sqrt(x^3 + 1)
```

Sympy [A] time = 1.6297, size = 19, normalized size = 0.79

$$-\sqrt{x^3 + 1} + \sqrt{x^5 - 2x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(-2*(x**3+1)**(1/2)+5*x**4*(x**3+1)**(1/2)-3*x**2*(x**5-2*x+1
)**(1/2))/(x**3+1)**(1/2)/(x**5-2*x+1)**(1/2),x)
```

```
[Out] -sqrt(x**3 + 1) + sqrt(x**5 - 2*x + 1)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{5\sqrt{x^3+1}x^4 - 3\sqrt{x^5-2x+1}x^2 - 2\sqrt{x^3+1}}{2\sqrt{x^5-2x+1}\sqrt{x^3+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/2*(-2*(x^3+1)^(1/2)+5*x^4*(x^3+1)^(1/2)-3*x^2*(x^5-2*x+1)^(1/2)
)/(x^3+1)^(1/2)/(x^5-2*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/2*(5*sqrt(x^3 + 1)*x^4 - 3*sqrt(x^5 - 2*x + 1)*x^2 - 2*sqrt(x^3
 + 1))/(sqrt(x^5 - 2*x + 1)*sqrt(x^3 + 1)), x)
```

3.202 $\int \left(\frac{10}{\sqrt{-4+x^2}}+\frac{1}{\sqrt{-1+x^2}}\right) dx$

Optimal. Leaf size=27

$$10 \tanh^{-1}\left(\frac{x}{\sqrt{x^2-4}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

```
[Out] 10*ArcTanh[x/Sqrt[-4 + x^2]] + ArcTanh[x/Sqrt[-1 + x^2]]
```

Rubi [A] time = 0.0080556, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {217, 206}

$$10 \tanh^{-1}\left(\frac{x}{\sqrt{x^2-4}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[10/Sqrt[-4 + x^2] + 1/Sqrt[-1 + x^2],x]
```

```
[Out] 10*ArcTanh[x/Sqrt[-4 + x^2]] + ArcTanh[x/Sqrt[-1 + x^2]]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], 
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(\frac{10}{\sqrt{-4+x^2}}+\frac{1}{\sqrt{-1+x^2}}\right)dx = 10\int \frac{1}{\sqrt{-4+x^2}}\,dx+\int \frac{1}{\sqrt{-1+x^2}}\,dx$$
$$= 10\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-4+x^2}}\right)+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\frac{x}{\sqrt{-1+x^2}}\right)$$
$$= 10\tanh^{-1}\left(\frac{x}{\sqrt{-4+x^2}}\right)+\tanh^{-1}\left(\frac{x}{\sqrt{-1+x^2}}\right)$$

Mathematica [B] time = 0.0088356, size = 71, normalized size = 2.63

$$-5\log\left(1-\frac{x}{\sqrt{x^2-4}}\right)+5\log\left(\frac{x}{\sqrt{x^2-4}}+1\right)-\frac{1}{2}\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)+\frac{1}{2}\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[10/Sqrt[-4 + x^2] + 1/Sqrt[-1 + x^2],x]
```

```
[Out] -5*Log[1 - x/Sqrt[-4 + x^2]] + 5*Log[1 + x/Sqrt[-4 + x^2]] - Log[1 - x/Sqrt
[-1 + x^2]]/2 + Log[1 + x/Sqrt[-1 + x^2]]/2
```

Maple [A] time = 0.003, size = 24, normalized size = 0.9

$$\ln\left(x+\sqrt{x^2-1}\right)+10\,\ln\left(x+\sqrt{x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x)
```

```
[Out] ln(x+(x^2-1)^(1/2))+10*ln(x+(x^2-4)^(1/2))
```

Maxima [A] time = 0.951793, size = 42, normalized size = 1.56

$$\log\left(2\,x+2\,\sqrt{x^2-1}\right)+10\,\log\left(2\,x+2\,\sqrt{x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 - 1)) + 10*log(2*x + 2*sqrt(x^2 - 4))
```

Fricas [A] time = 1.55508, size = 74, normalized size = 2.74

$$-\log\left(-x+\sqrt{x^2-1}\right)-10\,\log\left(-x+\sqrt{x^2-4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 - 1)) - 10*log(-x + sqrt(x^2 - 4))
```

Sympy [A] time = 0.198941, size = 8, normalized size = 0.3

$$10\operatorname{acosh}\left(\frac{x}{2}\right)+\operatorname{acosh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10/(x**2-4)**(1/2)+1/(x**2-1)**(1/2),x)
```

```
[Out] 10*acosh(x/2) + acosh(x)
```

Giac [A] time = 1.09578, size = 42, normalized size = 1.56

$$-\log\left(\left|-x+\sqrt{x^2-1}\right|\right)-10\,\log\left(\left|-x+\sqrt{x^2-4}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(10/(x^2-4)^(1/2)+1/(x^2-1)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-x + sqrt(x^2 - 1))) - 10*log(abs(-x + sqrt(x^2 - 4)))
```

3.203 $\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$

Optimal. Leaf size=82

$$2\sqrt{\sqrt{a^2+x^2}+x}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]
```

Rubi [A] time = 0.0743947, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2119, 459, 329, 212, 206, 203}

$$2\sqrt{\sqrt{a^2+x^2}+x}-2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] 2*Sqrt[x + Sqrt[a^2 + x^2]] - 2*Sqrt[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sq
rt[a]] - 2*Sqrt[a]*ArcTanh[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]]
```

Rule 2119

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^
2])^(n_.), x_Symbol] :> Dist[1/(2^(m + 1)*e^(m + 1)), Subst[Int[x^(n - m -
2)*(a*f^2 + x^2)*(-(a*f^2*h) + 2*e*g*x + h*x^2)^m, x], x, e*x + f*Sqrt[a +
c*x^2]], x] /; FreeQ[{a, c, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && In
tegerQ[m]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
```

```
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx = \text{Subst}\left(\int \frac{a^2+x^2}{\sqrt{x}\left(-a^2+x^2\right)}\,dx, x, x+\sqrt{a^2+x^2}\right)$$
$$= 2\sqrt{x+\sqrt{a^2+x^2}} + \left(2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(-a^2+x^2\right)}\,dx, x, x+\sqrt{a^2+x^2}\right)$$
$$= 2\sqrt{x+\sqrt{a^2+x^2}} + \left(4a^2\right)\text{Subst}\left(\int \frac{1}{-a^2+x^4}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right)$$
$$= 2\sqrt{x+\sqrt{a^2+x^2}} - (2a)\,\text{Subst}\left(\int \frac{1}{a-x^2}\,dx, x, \sqrt{x+\sqrt{a^2+x^2}}\right) - (2a)\,\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \sqrt{\right.$$
$$= 2\sqrt{x+\sqrt{a^2+x^2}} - 2\sqrt{a}\tan^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right) - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)$$

Mathematica [A] time = 0.17944, size = 127, normalized size = 1.55

$$-\frac{2\sqrt{a^2+x^2}\left(\sqrt{a^2+x^2}+x\right)\left(-\sqrt{\sqrt{a^2+x^2}+x}+\sqrt{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)+\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a^2+x^2}+x}}{\sqrt{a}}\right)\right)}{x\left(\sqrt{a^2+x^2}+x\right)+a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[a^2 + x^2]]/x,x]
```

```
[Out] (-2*Sqrt[a^2 + x^2]*(x + Sqrt[a^2 + x^2])*(-Sqrt[x + Sqrt[a^2 + x^2]] + Sqr
t[a]*ArcTan[Sqrt[x + Sqrt[a^2 + x^2]]/Sqrt[a]] + Sqrt[a]*ArcTanh[Sqrt[x + S
qrt[a^2 + x^2]]/Sqrt[a]]))/(a^2 + x*(x + Sqrt[a^2 + x^2]))
```

Maple [C] time = 0.037, size = 25, normalized size = 0.3

$$2\sqrt{2}\sqrt{x}\,{}_3F_2(-1/4,-1/4,1/4;\,1/2,3/4;\,-\frac{a^2}{x^2})$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(a^2+x^2)^(1/2))^(1/2)/x,x)
```

```
[Out] 2*2^(1/2)*x^(1/2)*hypergeom([-1/4,-1/4,1/4],[1/2,3/4],-1/x^2*a^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

Fricas [A] time = 1.68155, size = 552, normalized size = 6.73

$$\left[-2\sqrt{a}\arctan\left(\frac{\sqrt{x+\sqrt{a^2+x^2}}}{\sqrt{a}}\right)+\sqrt{a}\log\left(\frac{a^2+\sqrt{a^2+x^2}a-\left((a-x)\sqrt{a}+\sqrt{a^2+x^2}\sqrt{a}\right)\sqrt{x+\sqrt{a^2+x^2}}}{x}\right)+2\sqrt{x+\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [-2*sqrt(a)*arctan(sqrt(x + sqrt(a^2 + x^2))/sqrt(a)) + sqrt(a)*log((a^2 +
sqrt(a^2 + x^2)*a - ((a - x)*sqrt(a) + sqrt(a^2 + x^2)*sqrt(a))*sqrt(x + sq
rt(a^2 + x^2)))/x) + 2*sqrt(x + sqrt(a^2 + x^2)), 2*sqrt(-a)*arctan(sqrt(-a
)*sqrt(x + sqrt(a^2 + x^2))/a) + sqrt(-a)*log(-(a^2 - sqrt(a^2 + x^2)*a + (
sqrt(-a)*(a + x) - sqrt(a^2 + x^2)*sqrt(-a))*sqrt(x + sqrt(a^2 + x^2)))/x)
+ 2*sqrt(x + sqrt(a^2 + x^2))]
```

Sympy [C] time = 1.49162, size = 51, normalized size = 0.62

$$\frac{\sqrt{x}\Gamma^2\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right){}_3F_2\left(\begin{matrix}-\frac{1}{4},-\frac{1}{4},\frac{1}{4}\\ \frac{1}{2},\frac{3}{4}\end{matrix}\middle|\frac{a^2e^{i\pi}}{x^2}\right)}{8\pi\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a**2+x**2)**(1/2))**(1/2)/x,x)
```

```
[Out] sqrt(x)*gamma(-1/4)**2*gamma(1/4)*hyper((-1/4, -1/4, 1/4), (1/2, 3/4), a**2
*exp_polar(I*pi)/x**2)/(8*pi*gamma(3/4))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+\sqrt{a^2+x^2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(a^2+x^2)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x + sqrt(a^2 + x^2))/x, x)
```

3.204 $\int \frac{3x^2}{2\left(1+x^3+\sqrt{1+x^3}\right)}\,dx$

Optimal. Leaf size=12

$$\log\left(\sqrt{x^3+1}+1\right)$$

```
[Out] Log[1 + Sqrt[1 + x^3]]
```

Rubi [A] time = 0.0533348, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {12, 2155, 31}

$$\log\left(\sqrt{x^3+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(3*x^2)/(2*(1 + x^3 + Sqrt[1 + x^3])),x]
```

```
[Out] Log[1 + Sqrt[1 + x^3]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{3x^2}{2\left(1+x^3+\sqrt{1+x^3}\right)}\,dx = \frac{3}{2}\int \frac{x^2}{1+x^3+\sqrt{1+x^3}}\,dx$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{1+x+\sqrt{1+x}}\,dx, x, x^3\right)$$
$$= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt{1+x^3}\right)$$
$$= \log\left(1+\sqrt{1+x^3}\right)$$

Mathematica [A] time = 0.0213799, size = 12, normalized size = 1.

$$\log\left(\sqrt{x^3+1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x^2)/(2*(1 + x^3 + Sqrt[1 + x^3])),x]
```

```
[Out] Log[1 + Sqrt[1 + x^3]]
```

Maple [B] time = 0.009, size = 39, normalized size = 3.3

$$\frac{3\ln(x)}{2} - \frac{\ln(1+x)}{2} - \frac{\ln\left(x^2-x+1\right)}{2} + \frac{\ln\left(x^3+1\right)}{2} + \text{Artanh}\left(\sqrt{x^3+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x)
```

```
[Out] 3/2*ln(x)-1/2*ln(1+x)-1/2*ln(x^2-x+1)+1/2*ln(x^3+1)+arctanh((x^3+1)^(1/2))
```

Maxima [B] time = 1.46015, size = 54, normalized size = 4.5

$$-\frac{1}{2}\log\left(x^2-x+1\right) + \log\left(\frac{x^3+\sqrt{x^2-x+1}\sqrt{x+1}+1}{\sqrt{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] -1/2*log(x^2 - x + 1) + log((x^3 + sqrt(x^2 - x + 1)*sqrt(x + 1) + 1)/sqrt(
x + 1))
```

Fricas [B] time = 1.63561, size = 95, normalized size = 7.92

$$\frac{3}{2}\log(x) + \frac{1}{2}\log\left(\sqrt{x^3+1}+1\right) - \frac{1}{2}\log\left(\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 3/2*log(x) + 1/2*log(sqrt(x^3 + 1) + 1) - 1/2*log(sqrt(x^3 + 1) - 1)
```

Sympy [B] time = 1.72642, size = 48, normalized size = 4.

$$-\frac{\log\left(2\sqrt{x^3+1}\right)}{2} + \frac{\log\left(2\sqrt{x^3+1}+2\right)}{2} + \frac{\log\left(3x^3+3\sqrt{x^3+1}+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3/2*x**2/(1+x**3+(x**3+1)**(1/2)),x)
```

```
[Out] -log(2*sqrt(x**3 + 1))/2 + log(2*sqrt(x**3 + 1) + 2)/2 + log(3*x**3 + 3*sqr
t(x**3 + 1) + 3)/2
```

Giac [A] time = 1.10482, size = 14, normalized size = 1.17

$$\log\left(\sqrt{x^3+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(3/2*x^2/(1+x^3+(x^3+1)^(1/2)),x, algorithm="giac")
```

```
[Out] log(sqrt(x^3 + 1) + 1)
```

3.205 $\int \frac{1}{\sqrt{-\alpha^2+2hr^2}}\,dr$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{h}r}{\sqrt{2hr^2-\alpha^2}}\right)}{\sqrt{2}\sqrt{h}}$$

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[h]*r)/Sqrt[-alpha^2 + 2*h*r^2]]/(Sqrt[2]*Sqrt[h])
```

Rubi [A] time = 0.0114905, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{h}r}{\sqrt{2hr^2-\alpha^2}}\right)}{\sqrt{2}\sqrt{h}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-alpha^2 + 2*h*r^2],r]
```



```
[Out] ArcTanh[(Sqrt[2]*Sqrt[h]*r)/Sqrt[-alpha^2 + 2*h*r^2]]/(Sqrt[2]*Sqrt[h])
```


Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{-\alpha^2+2hr^2}}\,dr = \text{Subst}\left(\int \frac{1}{1-2hr^2}\,dr, r, \frac{r}{\sqrt{-\alpha^2+2hr^2}}\right) = \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{h}r}{\sqrt{-\alpha^2+2hr^2}}\right)}{\sqrt{2}\sqrt{h}}$$

Mathematica [A] time = 0.0098647, size = 40, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{h}r}{\sqrt{2hr^2-\alpha^2}}\right)}{\sqrt{2}\sqrt{h}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-alpha^2 + 2*h*r^2],r]
```

```
[Out] ArcTanh[(Sqrt[2]*Sqrt[h]*r)/Sqrt[-alpha^2 + 2*h*r^2]]/(Sqrt[2]*Sqrt[h])
```

Maple [A] time = 0.003, size = 33, normalized size = 0.8

$$\frac{\sqrt{2}}{2}\ln\left(\sqrt{h}r\sqrt{2}+\sqrt{2\,hr^2-\alpha^2}\right)\frac{1}{\sqrt{h}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*h*r^2-alpha^2)^(1/2),r)
```

```
[Out] 1/2*ln(h^(1/2)*r*2^(1/2)+(2*h*r^2-alpha^2)^(1/2))*2^(1/2)/h^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*h*r^2-alpha^2)^(1/2),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60488, size = 235, normalized size = 5.88

$$\left[\frac{\sqrt{2}\log\left(4hr^2+2\sqrt{2}\sqrt{2hr^2-\alpha^2}\sqrt{h}r-\alpha^2\right)}{4\sqrt{h}}, -\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{h}}\arctan\left(\frac{\sqrt{2}hr\sqrt{-\frac{1}{h}}}{\sqrt{2hr^2-\alpha^2}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*h*r^2-alpha^2)^(1/2),r, algorithm="fricas")
```

```
[Out] [1/4*sqrt(2)*log(4*h*r^2 + 2*sqrt(2)*sqrt(2*h*r^2 - alpha^2)*sqrt(h)*r - al
pha^2)/sqrt(h), -1/2*sqrt(2)*sqrt(-1/h)*arctan(sqrt(2)*h*r*sqrt(-1/h)/sqrt(
2*h*r^2 - alpha^2))]
```

Sympy [A] time = 1.19104, size = 68, normalized size = 1.7

$$\begin{cases} \frac{\sqrt{2}\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{h}r}{\alpha}\right)}{2\sqrt{h}} & \text{for}\ \frac{2|hr^2|}{|\alpha^2|} > 1 \\ -\frac{\sqrt{2}i\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{h}r}{\alpha}\right)}{2\sqrt{h}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*h*r**2-alpha**2)**(1/2),r)
```

```
[Out] Piecewise((sqrt(2)*acosh(sqrt(2)*sqrt(h)*r/alpha)/(2*sqrt(h)), 2*Abs(h*r**2
)/Abs(alpha**2) > 1), (-sqrt(2)*I*asin(sqrt(2)*sqrt(h)*r/alpha)/(2*sqrt(h))
, True))
```

Giac [A] time = 1.08898, size = 46, normalized size = 1.15

$$-\frac{\sqrt{2}\log\left(\left|-\sqrt{2}\sqrt{h}r+\sqrt{2hr^2-\alpha^2}\right|\right)}{2\sqrt{h}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*h*r^2-alpha^2)^(1/2),r, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-sqrt(2)*sqrt(h)*r + sqrt(2*h*r^2 - alpha^2)))/sqrt(h)
```

3.206 $\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2}}\,dr$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

```
[Out] ArcTan[Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]/Sqrt[alpha^2 + epsilon^2]]/Sqrt
[alpha^2 + epsilon^2]
```

Rubi [A] time = 0.0335206, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {266, 63, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]),r]
```

```
[Out] ArcTan[Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]/Sqrt[alpha^2 + epsilon^2]]/Sqrt
[alpha^2 + epsilon^2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2}}\,dr &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr}}\,dr, r, r^2\right)\\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{-\alpha^2-\epsilon^2}{2h}+\frac{r^2}{2h}}\,dr, r, \sqrt{-\alpha^2-\epsilon^2+2hr^2}\right)}{2h}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}\end{aligned}$$

Mathematica [A] time = 0.0112394, size = 46, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{-\alpha^2-\epsilon^2+2hr^2}}{\sqrt{\alpha^2+\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]),r]
```

```
[Out] ArcTan[Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2]/Sqrt[alpha^2 + epsilon^2]]/Sqrt
[alpha^2 + epsilon^2]
```

Maple [A] time = 0.009, size = 66, normalized size = 1.4

$$-\ln\left(\frac{1}{r}\left(-2\,\alpha^2-2\,\epsilon^2+2\,\sqrt{-\alpha^2-\epsilon^2}\sqrt{2\,hr^2-\alpha^2-\epsilon^2}\right)\right)\frac{1}{\sqrt{-\alpha^2-\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r)
```

```
[Out] -1/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2+2*(-alpha^2-epsilo
n^2)^(1/2)*(2*h*r^2-alpha^2-epsilon^2)^(1/2))/r)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.64199, size = 132, normalized size = 2.87

$$-\frac{\arctan\left(\frac{\sqrt{\alpha^2+\epsilon^2}}{\sqrt{2\,hr^2-\alpha^2-\epsilon^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fricas")
```

```
[Out] -arctan(sqrt(alpha^2 + epsilon^2)/sqrt(2*h*r^2 - alpha^2 - epsilon^2))/sqrt
(alpha^2 + epsilon^2)
```

Sympy [A] time = 1.26772, size = 42, normalized size = 0.91

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{\operatorname{polar\_lift}\left(-\alpha^2-\epsilon^2\right)}}{2\sqrt{h}r}\right)}{\sqrt{\operatorname{polar\_lift}\left(-\alpha^2-\epsilon^2\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r**2-alpha**2-epsilon**2)**(1/2),r)
```

```
[Out] -asinh(sqrt(2)*sqrt(polar_lift(-alpha**2 - epsilon**2))/(2*sqrt(h)*r))/sqrt
(polar_lift(-alpha**2 - epsilon**2))
```

Giac [A] time = 1.08102, size = 51, normalized size = 1.11

$$\frac{1000000000000.0 \arctan\left(\frac{1000000000000.0 \sqrt{1.9999999999999998 \, hr^2 - 0.9999999999999999 \, \alpha^2 - 1 \times 10^{-24}}}{\sqrt{1 \times 10^{24} \, \alpha^2 + 1.0}}\right)}{\sqrt{1 \times 10^{24} \, \alpha^2 + 1.0}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="giac")
```

```
[Out] 1000000000000.0*arctan(1000000000000.0*sqrt(1.9999999999999998*h*r^2 - 0.99
99999999999999*alpha^2 - 1e-24)/sqrt((1e+24)*alpha^2 + 1.0))/sqrt((1e+24)*a
lpha^2 + 1.0)
```

3.207 $\int \frac{1}{r\sqrt{-\alpha^2-2kr+2hr^2}}\,dr$

Optimal. Leaf size=37

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+kr}{\alpha\sqrt{-\alpha^2+2hr^2-2kr}}\right)}{\alpha}$$

```
[Out] -(ArcTan[(alpha^2 + k*r)/(alpha*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2])]/alpha)
```

Rubi [A] time = 0.0176463, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {724, 204}

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+kr}{\alpha\sqrt{-\alpha^2+2hr^2-2kr}}\right)}{\alpha}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2]),r]
```

```
[Out] -(ArcTan[(alpha^2 + k*r)/(alpha*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2])]/alpha)
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{r\sqrt{-\alpha^2-2kr+2hr^2}}\,dr = -\left(2\,\text{Subst}\left(\int \frac{1}{-4\alpha^2-r^2}\,dr, r, \frac{-2\alpha^2-2kr}{\sqrt{-\alpha^2-2kr+2hr^2}}\right)\right)$$
$$= -\frac{\tan^{-1}\left(\frac{\alpha^2+kr}{\alpha\sqrt{-\alpha^2-2kr+2hr^2}}\right)}{\alpha}$$

Mathematica [A] time = 0.0183538, size = 39, normalized size = 1.05

$$\frac{\tan^{-1}\left(\frac{-\alpha^2-kr}{\alpha\sqrt{2r(hr-k)-\alpha^2}}\right)}{\alpha}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-alpha^2 - 2*k*r + 2*h*r^2]),r]
```

```
[Out] ArcTan[(-alpha^2 - k*r)/(alpha*Sqrt[-alpha^2 + 2*r*(-k + h*r)])]/alpha
```

Maple [A] time = 0.007, size = 52, normalized size = 1.4

$$-\ln\left(\frac{1}{r}\left(-2\,\alpha^2-2\,kr+2\,\sqrt{-\alpha^2}\sqrt{2\,hr^2-\alpha^2-2\,kr}\right)\right)\frac{1}{\sqrt{-\alpha^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r)
```

```
[Out] -1/(-alpha^2)^(1/2)*ln((-2*alpha^2-2*k*r+2*(-alpha^2)^(1/2)*(2*h*r^2-alpha^
2-2*k*r)^(1/2))/r)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69996, size = 140, normalized size = 3.78

$$-\frac{\arctan\left(\frac{\sqrt{2\,hr^2-\alpha^2-2\,kr}\left(\alpha^2+kr\right)}{2\,\alpha hr^2-\alpha^3-2\,\alpha kr}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="fricas")
```

```
[Out] -arctan(sqrt(2*h*r^2 - alpha^2 - 2*k*r)*(alpha^2 + k*r)/(2*alpha*h*r^2 - al
pha^3 - 2*alpha*k*r))/alpha
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r**2-alpha**2-2*k*r)**(1/2),r)
```

```
[Out] Integral(1/(r*sqrt(-alpha**2 + 2*h*r**2 - 2*k*r)), r)
```

Giac [A] time = 1.11944, size = 54, normalized size = 1.46

$$\frac{2\,\arctan\left(-\frac{\sqrt{2}\sqrt{h}r-\sqrt{2\,hr^2-\alpha^2-2\,kr}}{\alpha}\right)}{\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="giac")
```

```
[Out] 2*arctan(-(sqrt(2)*sqrt(h)*r - sqrt(2*h*r^2 - alpha^2 - 2*k*r))/alpha)/alph
a
```

3.208 $\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\,dr$

Optimal. Leaf size=61

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+\epsilon^2+kr}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

```
[Out] -(ArcTan[(alpha^2 + epsilon^2 + k*r)/(Sqrt[alpha^2 + epsilon^2]*Sqrt[-alpha
^2 - epsilon^2 - 2*k*r + 2*h*r^2])]/Sqrt[alpha^2 + epsilon^2])
```

Rubi [A] time = 0.0237135, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {724, 204}

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+\epsilon^2+kr}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2]),r]
```

```
[Out] -(ArcTan[(alpha^2 + epsilon^2 + k*r)/(Sqrt[alpha^2 + epsilon^2]*Sqrt[-alpha
^2 - epsilon^2 - 2*k*r + 2*h*r^2])]/Sqrt[alpha^2 + epsilon^2])
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\,dr = -\left(2\,\text{Subst}\left(\int \frac{1}{4\left(-\alpha^2-\epsilon^2\right)-r^2}\,dr, r, \frac{2\left(-\alpha^2-\epsilon^2\right)-2kr}{\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\right)\right)$$
$$= -\frac{\tan^{-1}\left(\frac{\alpha^2+\epsilon^2+kr}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2-2kr+2hr^2}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Mathematica [A] time = 0.0292344, size = 65, normalized size = 1.07

$$\frac{\tan^{-1}\left(\frac{-\alpha^2-\epsilon^2-kr}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2r(hr-k)}}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 - 2*k*r + 2*h*r^2]),r]
```

```
[Out] ArcTan[(-alpha^2 - epsilon^2 - k*r)/(Sqrt[alpha^2 + epsilon^2]*Sqrt[-alpha^
2 - epsilon^2 + 2*r*(-k + h*r)])]/Sqrt[alpha^2 + epsilon^2]
```

Maple [A] time = 0.005, size = 74, normalized size = 1.2

$$-\ln\left(\frac{1}{r}\left(-2\,\alpha^2-2\,\epsilon^2-2\,kr+2\,\sqrt{-\alpha^2-\epsilon^2}\sqrt{2\,hr^2-\alpha^2-\epsilon^2-2\,kr}\right)\right)\frac{1}{\sqrt{-\alpha^2-\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r)
```

```
[Out] -1/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2-2*k*r+2*(-alpha^2-
epsilon^2)^(1/2)*(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2))/r)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r, algorithm="maxima"
)
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.72665, size = 325, normalized size = 5.33

$$-\frac{\arctan\left(-\frac{\sqrt{2\,hr^2-\alpha^2-\epsilon^2-2\,kr}\left(\alpha^2+\epsilon^2+kr\right)\sqrt{\alpha^2+\epsilon^2}}{\alpha^4+2\,\alpha^2\epsilon^2+\epsilon^4-2\,\left(\alpha^2+\epsilon^2\right)hr^2+2\,\left(\alpha^2+\epsilon^2\right)kr}\right)}{\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r, algorithm="fricas"
)
```

```
[Out] -arctan(-sqrt(2*h*r^2 - alpha^2 - epsilon^2 - 2*k*r)*(alpha^2 + epsilon^2 +
 k*r)*sqrt(alpha^2 + epsilon^2)/(alpha^4 + 2*alpha^2*epsilon^2 + epsilon^4
- 2*(alpha^2 + epsilon^2)*h*r^2 + 2*(alpha^2 + epsilon^2)*k*r))/sqrt(alpha^
2 + epsilon^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r**2-alpha**2-epsilon**2-2*k*r)**(1/2),r)
```

```
[Out] Integral(1/(r*sqrt(-alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r)), r)
```

Giac [A] time = 1.18963, size = 69, normalized size = 1.13

$$\frac{2000000000000.0 \arctan\left(\frac{\left(6.5536\times10^{-08}\right)\left(-2.157918643757774\times10^{19}\sqrt{h}r+1.52587890625\times10^{19}\sqrt{2.0\,hr^2-\alpha^2-2.0\,kr-1\times10^{-24}}\right)}{\sqrt{1\times10^{24}\,\alpha^2+1.0}}\right)}{\sqrt{1\times10^{24}\,\alpha^2+1.0}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(2*h*r^2-alpha^2-epsilon^2-2*k*r)^(1/2),r, algorithm="giac")
```

```
[Out] 2000000000000.0*arctan((6.5536e-08)*(-(2.157918643757774e+19)*sqrt(h)*r + (
1.52587890625e+19)*sqrt(2.0*h*r^2 - alpha^2 - 2.0*k*r - 1e-24))/sqrt((1e+24
)*alpha^2 + 1.0))/sqrt((1e+24)*alpha^2 + 1.0)
```

3.209 $\int \frac{r}{\sqrt{-\alpha^2+2er^2}}\,dr$

Optimal. Leaf size=23

$$\frac{\sqrt{2er^2-\alpha^2}}{2e}$$

```
[Out] Sqrt[-alpha^2 + 2*e*r^2]/(2*e)
```

Rubi [A] time = 0.0041856, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {261}

$$\frac{\sqrt{2er^2-\alpha^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-alpha^2 + 2*e*r^2],r]
```

```
[Out] Sqrt[-alpha^2 + 2*e*r^2]/(2*e)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2}}\,dr=\frac{\sqrt{-\alpha^2+2er^2}}{2e}$$

Mathematica [A] time = 0.004312, size = 23, normalized size = 1.

$$\frac{\sqrt{2er^2-\alpha^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-alpha^2 + 2*e*r^2],r]
```

```
[Out] Sqrt[-alpha^2 + 2*e*r^2]/(2*e)
```

Maple [A] time = 0.003, size = 20, normalized size = 0.9

$$\frac{1}{2e}\sqrt{2\,er^2-\alpha^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*e*r^2-alpha^2)^(1/2),r)
```

```
[Out] 1/2*(2*e*r^2-alpha^2)^(1/2)/e
```

Maxima [A] time = 0.947773, size = 26, normalized size = 1.13

$$\frac{\sqrt{2\,er^2-\alpha^2}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2)^(1/2),r, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2*e*r^2 - alpha^2)/e
```

Fricas [A] time = 1.64869, size = 42, normalized size = 1.83

$$\frac{\sqrt{2\,er^2-\alpha^2}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2)^(1/2),r, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2*e*r^2 - alpha^2)/e
```

Sympy [A] time = 0.438046, size = 29, normalized size = 1.26

$$\begin{cases} \frac{\sqrt{-\alpha^2+2er^2}}{2e} & \text{for } e \neq 0 \\ \frac{r^2}{2\sqrt{-\alpha^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r**2-alpha**2)**(1/2),r)
```

```
[Out] Piecewise((sqrt(-alpha**2 + 2*e*r**2)/(2*e), Ne(e, 0)), (r**2/(2*sqrt(-alph
a**2)), True))
```

Giac [A] time = 1.0891, size = 26, normalized size = 1.13

$$\frac{1}{2}\sqrt{2\,r^2 e-\alpha^2}e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2)^(1/2),r, algorithm="giac")
```

```
[Out] 1/2*sqrt(2*r^2*e - alpha^2)*e^(-1)
```

3.210 $\int \frac{r}{\sqrt{-\alpha^2-\epsilon^2+2er^2}}\,dr$

Optimal. Leaf size=28

$$\frac{\sqrt{-\alpha^2+2er^2-\epsilon^2}}{2e}$$

```
[Out] Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2]/(2*e)
```

Rubi [A] time = 0.0046793, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {261}

$$\frac{\sqrt{-\alpha^2+2er^2-\epsilon^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2],r]
```

```
[Out] Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2]/(2*e)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{r}{\sqrt{-\alpha^2-\epsilon^2+2er^2}}\,dr=\frac{\sqrt{-\alpha^2-\epsilon^2+2er^2}}{2e}$$

Mathematica [A] time = 0.0053698, size = 28, normalized size = 1.

$$\frac{\sqrt{-\alpha^2+2er^2-\epsilon^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2],r]
```

```
[Out] Sqrt[-alpha^2 - epsilon^2 + 2*e*r^2]/(2*e)
```

Maple [A] time = 0.004, size = 25, normalized size = 0.9

$$\frac{1}{2e}\sqrt{2\,er^2-\alpha^2-\epsilon^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r)
```

```
[Out] 1/2*(2*e*r^2-alpha^2-epsilon^2)^(1/2)/e
```

Maxima [A] time = 0.949937, size = 32, normalized size = 1.14

$$\frac{\sqrt{2\,er^2-\alpha^2-\epsilon^2}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2*e*r^2 - alpha^2 - epsilon^2)/e
```

Fricas [A] time = 1.68034, size = 58, normalized size = 2.07

$$\frac{\sqrt{2\,er^2-\alpha^2-\epsilon^2}}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2*e*r^2 - alpha^2 - epsilon^2)/e
```

Sympy [A] time = 0.543385, size = 36, normalized size = 1.29

$$\begin{cases} \frac{\sqrt{-\alpha^2+2er^2-\epsilon^2}}{2e} & \text{for } e \neq 0 \\ \frac{r^2}{2\sqrt{-\alpha^2-\epsilon^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r**2-alpha**2-epsilon**2)**(1/2),r)
```

```
[Out] Piecewise((sqrt(-alpha**2 + 2*e*r**2 - epsilon**2)/(2*e), Ne(e, 0)), (r**2/
(2*sqrt(-alpha**2 - epsilon**2)), True))
```

Giac [A] time = 1.08926, size = 22, normalized size = 0.79

$$0.183939720586\sqrt{-\alpha^2 + 5.43656365692\,r^2 - 1 \times 10^{-24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="giac")
```

```
[Out] 0.183939720586*sqrt(-alpha^2 + 5.43656365692*r^2 - 1e-24)
```

3.211 $\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr^4}}\,dr$

Optimal. Leaf size=56

$$-\frac{\tan^{-1}\left(\frac{e-2kr^2}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}$$

```
[Out] -ArcTan[(e - 2*k*r^2)/(Sqrt[2]*Sqrt[k]*Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4])]
/(2*Sqrt[2]*Sqrt[k])
```

Rubi [A] time = 0.039761, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1107, 621, 204}

$$-\frac{\tan^{-1}\left(\frac{e-2kr^2}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4],r]
```


```
[Out] -ArcTan[(e - 2*k*r^2)/(Sqrt[2]*Sqrt[k]*Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4])]
/(2*Sqrt[2]*Sqrt[k])
```


Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr^4}}\,dr &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{-\alpha^2+2er-2kr^2}}\,dr, r, r^2\right)\\ &= \text{Subst}\left(\int \frac{1}{-8k-r^2}\,dr, r, \frac{2\left(e-2kr^2\right)}{\sqrt{-\alpha^2+2er^2-2kr^4}}\right)\\ &= -\frac{\tan^{-1}\left(\frac{e-2kr^2}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}\end{aligned}$$

Mathematica [A] time = 0.0230724, size = 56, normalized size = 1.

$$-\frac{\tan^{-1}\left(\frac{e-2kr^2}{\sqrt{2}\sqrt{k}\sqrt{-\alpha^2+2er^2-2kr^4}}\right)}{2\sqrt{2}\sqrt{k}}$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4],r]
```

```
[Out] -ArcTan[(e - 2*k*r^2)/(Sqrt[2]*Sqrt[k]*Sqrt[-alpha^2 + 2*e*r^2 - 2*k*r^4])]
/(2*Sqrt[2]*Sqrt[k])
```

Maple [A] time = 0.012, size = 47, normalized size = 0.8

$$\frac{\sqrt{2}}{4}\arctan\left(\sqrt{2}\sqrt{k}\left(r^2-\frac{e}{2\,k}\right)\frac{1}{\sqrt{-2\,kr^4+2\,er^2-\alpha^2}}\right)\frac{1}{\sqrt{k}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r)
```

```
[Out] 1/4*2^(1/2)/k^(1/2)*arctan(2^(1/2)*k^(1/2)*(r^2-1/2*e/k)/(-2*k*r^4+2*e*r^2-
alpha^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.69661, size = 381, normalized size = 6.8

$$\left[-\frac{\sqrt{2}\sqrt{-k}\log\left(-8\,k^2r^4+8\,ekr^2-2\,\alpha^2k+2\,\sqrt{2}\sqrt{-2\,kr^4+2\,er^2-\alpha^2}\left(2\,kr^2-e\right)\sqrt{-k}-e^2\right)}{8\,k}, -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{-2\,kr^4+2\,er^2-\alpha^2}\right.}{2\left(2\,k^2r^4-2\,ek\right.}}{4\,\sqrt{k}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(2)*sqrt(-k)*log(-8*k^2*r^4 + 8*e*k*r^2 - 2*alpha^2*k + 2*sqrt(2)
*sqrt(-2*k*r^4 + 2*e*r^2 - alpha^2)*(2*k*r^2 - e)*sqrt(-k) - e^2)/k, -1/4*s
qrt(2)*arctan(1/2*sqrt(2)*sqrt(-2*k*r^4 + 2*e*r^2 - alpha^2)*(2*k*r^2 - e)*
sqrt(k)/(2*k^2*r^4 - 2*e*k*r^2 + alpha^2*k))/sqrt(k)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{r}{\sqrt{-\alpha^2+2er^2-2kr^4}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*k*r**4+2*e*r**2-alpha**2)**(1/2),r)
```

```
[Out] Integral(r/sqrt(-alpha**2 + 2*e*r**2 - 2*k*r**4), r)
```

Giac [A] time = 1.23936, size = 81, normalized size = 1.45

$$-\frac{\sqrt{2}\log\left(\left|\sqrt{2}\left(\sqrt{2}\sqrt{-k}r^2-\sqrt{-2\,kr^4+2\,r^2e-\alpha^2}\right)\sqrt{-k}+e\right|\right)}{4\,\sqrt{-k}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(-2*k*r^4+2*e*r^2-alpha^2)^(1/2),r, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(abs(sqrt(2)*(sqrt(2)*sqrt(-k)*r^2 - sqrt(-2*k*r^4 + 2*r^2*
e - alpha^2))*sqrt(-k) + e))/sqrt(-k)
```

3.212 $\int \frac{r}{\sqrt{-\alpha^2-2kr+2er^2}}\,dr$

Optimal. Leaf size=81

$$\frac{\sqrt{-\alpha^2+2er^2-2kr}}{2e}-\frac{k\tanh^{-1}\left(\frac{k-2er}{\sqrt{2}\sqrt{e}\sqrt{-\alpha^2+2er^2-2kr}}\right)}{2\sqrt{2}e^{3/2}}$$

```
[Out] Sqrt[-alpha^2 - 2*k*r + 2*e*r^2]/(2*e) - (k*ArcTanh[(k - 2*e*r)/(Sqrt[2]*Sq
rt[e]*Sqrt[-alpha^2 - 2*k*r + 2*e*r^2])])/(2*Sqrt[2]*e^(3/2))
```

Rubi [A] time = 0.0275925, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {640, 621, 206}

$$\frac{\sqrt{-\alpha^2+2er^2-2kr}}{2e}-\frac{k\tanh^{-1}\left(\frac{k-2er}{\sqrt{2}\sqrt{e}\sqrt{-\alpha^2+2er^2-2kr}}\right)}{2\sqrt{2}e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[r/Sqrt[-alpha^2 - 2*k*r + 2*e*r^2],r]
```

```
[Out] Sqrt[-alpha^2 - 2*k*r + 2*e*r^2]/(2*e) - (k*ArcTanh[(k - 2*e*r)/(Sqrt[2]*Sq
rt[e]*Sqrt[-alpha^2 - 2*k*r + 2*e*r^2])])/(2*Sqrt[2]*e^(3/2))
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{r}{\sqrt{-\alpha^2-2kr+2er^2}}\,dr &= \frac{\sqrt{-\alpha^2-2kr+2er^2}}{2e}+\frac{k\int \frac{1}{\sqrt{-\alpha^2-2kr+2er^2}}\,dr}{2e}\\
&= \frac{\sqrt{-\alpha^2-2kr+2er^2}}{2e}+\frac{k\,\mathrm{Subst}\left(\int \frac{1}{8e-r^2}\,dr,r,\frac{-2k+4er}{\sqrt{-\alpha^2-2kr+2er^2}}\right)}{e}\\
&= \frac{\sqrt{-\alpha^2-2kr+2er^2}}{2e}-\frac{k\tanh^{-1}\left(\frac{k-2er}{\sqrt{2}\sqrt{e}\sqrt{-\alpha^2-2kr+2er^2}}\right)}{2\sqrt{2}e^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.100761, size = 82, normalized size = 1.01

$$\frac{1}{4}\left(\frac{\sqrt{2}k\tanh^{-1}\left(\frac{2er-k}{\sqrt{2}\sqrt{e}\sqrt{2r(er-k)-\alpha^2}}\right)}{e^{3/2}}+\frac{2\sqrt{2r(er-k)-\alpha^2}}{e}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[r/Sqrt[-alpha^2 - 2*k*r + 2*e*r^2],r]
```

```
[Out] ((2*Sqrt[-alpha^2 + 2*r*(-k + e*r)])/e + (Sqrt[2]*k*ArcTanh[(-k + 2*e*r)/(S
qrt[2]*Sqrt[e]*Sqrt[-alpha^2 + 2*r*(-k + e*r)])])/e^(3/2))/4
```

Maple [A] time = 0.005, size = 70, normalized size = 0.9

$$\frac{1}{2e}\sqrt{2er^2-\alpha^2-2kr}+\frac{k\sqrt{2}}{4}\ln\left(\frac{(2er-k)\sqrt{2}}{2}\frac{1}{\sqrt{e}}+\sqrt{2er^2-\alpha^2-2kr}\right)e^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r)
```

```
[Out] 1/2*(2*e*r^2-alpha^2-2*k*r)^(1/2)/e+1/4*k/e^(3/2)*ln(1/2*(2*e*r-k)*2^(1/2)/
e^(1/2)+(2*e*r^2-alpha^2-2*k*r)^(1/2))*2^(1/2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.65361, size = 481, normalized size = 5.94

$$\left[\frac{\sqrt{2}\sqrt{e}k\log\left(8\,e^2r^2 - 2\,\alpha^2e - 8\,ekr + 2\,\sqrt{2}\sqrt{2\,er^2 - \alpha^2 - 2\,kr}(2\,er - k)\sqrt{e} + k^2\right) + 4\,\sqrt{2\,er^2 - \alpha^2 - 2\,kr}e}{8\,e^2}, -\frac{\sqrt{2}\sqrt{-e}k\,\mathrm{arcta}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*sqrt(e)*k*log(8*e^2*r^2 - 2*alpha^2*e - 8*e*k*r + 2*sqrt(2)*s
qrt(2*e*r^2 - alpha^2 - 2*k*r)*(2*e*r - k)*sqrt(e) + k^2) + 4*sqrt(2*e*r^2
- alpha^2 - 2*k*r)*e)/e^2, -1/4*(sqrt(2)*sqrt(-e)*k*arctan(1/2*sqrt(2)*sqrt
(2*e*r^2 - alpha^2 - 2*k*r)*(2*e*r - k)*sqrt(-e)/(2*e^2*r^2 - alpha^2*e - 2
*e*k*r)) - 2*sqrt(2*e*r^2 - alpha^2 - 2*k*r)*e)/e^2]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{r}{\sqrt{-\alpha^2 + 2er^2 - 2kr}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r**2-alpha**2-2*k*r)**(1/2),r)
```

```
[Out] Integral(r/sqrt(-alpha**2 + 2*e*r**2 - 2*k*r), r)
```

Giac [A] time = 1.24285, size = 97, normalized size = 1.2

$$-\frac{1}{4}\sqrt{2}ke^{\left(-\frac{3}{2}\right)}\log\left(\left|-\sqrt{2}\left(\sqrt{2}re^{\frac{1}{2}}-\sqrt{2r^{2}e-\alpha^{2}-2kr}\right)e^{\frac{1}{2}}+k\right|\right)+\frac{1}{2}\sqrt{2r^{2}e-\alpha^{2}-2kr}e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(r/(2*e*r^2-alpha^2-2*k*r)^(1/2),r, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*k*e^(-3/2)*log(abs(-sqrt(2)*(sqrt(2)*r*e^(1/2) - sqrt(2*r^2*e
- alpha^2 - 2*k*r))*e^(1/2) + k)) + 1/2*sqrt(2*r^2*e - alpha^2 - 2*k*r)*e^(
-1)
```

3.213 $\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr$

Optimal. Leaf size=44

$$-\frac{\tan^{-1}\left(\frac{\alpha^2-hr^2}{\alpha\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)}{2\alpha}$$

```
[Out] -ArcTan[(alpha^2 - h*r^2)/(alpha*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4])]/(2*al
pha)
```

Rubi [A] time = 0.0394127, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {1114, 724, 204}

$$-\frac{\tan^{-1}\left(\frac{\alpha^2-hr^2}{\alpha\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)}{2\alpha}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]),r]
```

```
[Out] -ArcTan[(alpha^2 - h*r^2)/(alpha*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4])]/(2*al
pha)
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{r\sqrt{-\alpha^2+2hr-2kr^2}}\,dr, r, r^2\right)\\ &= -\operatorname{Subst}\left(\int \frac{1}{-4\alpha^2-r^2}\,dr, r, \frac{2\left(-\alpha^2+hr^2\right)}{\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)\\ &= \frac{\tan^{-1}\left(\frac{-\alpha^2+hr^2}{\alpha\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)}{2\alpha}\end{aligned}$$

Mathematica [A] time = 0.0084803, size = 49, normalized size = 1.11

$$\frac{\tan^{-1}\left(\frac{2hr^2-2\alpha^2}{2\alpha\sqrt{-\alpha^2+2hr^2-2kr^4}}\right)}{2\alpha}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4]),r]
```

```
[Out] ArcTan[(-2*alpha^2 + 2*h*r^2)/(2*alpha*Sqrt[-alpha^2 + 2*h*r^2 - 2*k*r^4])]
/(2*alpha)
```

Maple [A] time = 0.01, size = 56, normalized size = 1.3

$$-\frac{1}{2}\ln\left(\frac{1}{r^2}\left(-2\,\alpha^2+2\,hr^2+2\,\sqrt{-\alpha^2}\sqrt{-2\,kr^4+2\,hr^2-\alpha^2}\right)\right)\frac{1}{\sqrt{-\alpha^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r)
```

```
[Out] -1/2/(-alpha^2)^(1/2)*ln((-2*alpha^2+2*h*r^2+2*(-alpha^2)^(1/2)*(-2*k*r^4+2
*h*r^2-alpha^2)^(1/2))/r^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.61988, size = 155, normalized size = 3.52

$$-\frac{\arctan\left(\frac{\sqrt{-2\,kr^4+2\,hr^2-\alpha^2}\left(hr^2-\alpha^2\right)}{2\,\alpha kr^4-2\,\alpha hr^2+\alpha^3}\right)}{2\,\alpha}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r, algorithm="fricas")
```

```
[Out] -1/2*arctan(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2)*(h*r^2 - alpha^2)/(2*alpha*k
*r^4 - 2*alpha*h*r^2 + alpha^3))/alpha
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{r\sqrt{-\alpha^2+2hr^2-2kr^4}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r**4+2*h*r**2-alpha**2)**(1/2),r)
```

```
[Out] Integral(1/(r*sqrt(-alpha**2 + 2*h*r**2 - 2*k*r**4)), r)
```

Giac [A] time = 1.22695, size = 42, normalized size = 0.95

$$-\frac{\arcsin\left(-\frac{h-\frac{\alpha^2}{r^2}}{\sqrt{-2\,\alpha^2k+h^2}}\right)}{2\,|\alpha|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2)^(1/2),r, algorithm="giac")
```

```
[Out] -1/2*arcsin(-(h - alpha^2/r^2)/sqrt(-2*alpha^2*k + h^2))/abs(alpha)
```

3.214 $\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr$

Optimal. Leaf size=68

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+\epsilon^2-hr^2}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

```
[Out] -ArcTan[(alpha^2 + epsilon^2 - h*r^2)/(Sqrt[alpha^2 + epsilon^2]*Sqrt[-alph
a^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4])]/(2*Sqrt[alpha^2 + epsilon^2])
```

Rubi [A] time = 0.0488941, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1114, 724, 204}

$$-\frac{\tan^{-1}\left(\frac{\alpha^2+\epsilon^2-hr^2}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4]),r]
```

```
[Out] -ArcTan[(alpha^2 + epsilon^2 - h*r^2)/(Sqrt[alpha^2 + epsilon^2]*Sqrt[-alph
a^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4])]/(2*Sqrt[alpha^2 + epsilon^2])
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr-2kr^2}}\,dr, r, r^2\right)$$

$$= -\,\mathrm{Subst}\left(\int \frac{1}{4\left(-\alpha^2-\epsilon^2\right)-r^2}\,dr, r, \frac{2\left(-\alpha^2-\epsilon^2+hr^2\right)}{\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)$$

$$= \frac{\tan^{-1}\left(\frac{-\alpha^2-\epsilon^2+hr^2}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

Mathematica [A] time = 0.025217, size = 71, normalized size = 1.04

$$\frac{\tan^{-1}\left(\frac{-\alpha^2-\epsilon^2+hr^2}{\sqrt{\alpha^2+\epsilon^2}\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(r*Sqrt[-alpha^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4]),r]
```

```
[Out] ArcTan[(-alpha^2 - epsilon^2 + h*r^2)/(Sqrt[alpha^2 + epsilon^2]*Sqrt[-alph
a^2 - epsilon^2 + 2*h*r^2 - 2*k*r^4])]/(2*Sqrt[alpha^2 + epsilon^2])
```

Maple [A] time = 0.012, size = 78, normalized size = 1.2

$$-\frac{1}{2}\ln\left(\frac{1}{r^2}\left(-2\,\alpha^2-2\,\epsilon^2+2\,hr^2+2\,\sqrt{-\alpha^2-\epsilon^2}\sqrt{-2\,kr^4+2\,hr^2-\alpha^2-\epsilon^2}\right)\right)\frac{1}{\sqrt{-\alpha^2-\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r)
```

```
[Out] -1/2/(-alpha^2-epsilon^2)^(1/2)*ln((-2*alpha^2-2*epsilon^2+2*h*r^2+2*(-alph
a^2-epsilon^2)^(1/2)*(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2))/r^2)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.89919, size = 339, normalized size = 4.99

$$-\frac{\arctan\left(\frac{\sqrt{-2\,kr^4+2\,hr^2-\alpha^2-\epsilon^2}\left(hr^2-\alpha^2-\epsilon^2\right)\sqrt{\alpha^2+\epsilon^2}}{2\left(\alpha^2+\epsilon^2\right)kr^4+\alpha^4+2\,\alpha^2\epsilon^2+\epsilon^4-2\left(\alpha^2+\epsilon^2\right)hr^2}\right)}{2\sqrt{\alpha^2+\epsilon^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="fric
as")
```

```
[Out] -1/2*arctan(sqrt(-2*k*r^4 + 2*h*r^2 - alpha^2 - epsilon^2)*(h*r^2 - alpha^2
 - epsilon^2)*sqrt(alpha^2 + epsilon^2)/(2*(alpha^2 + epsilon^2)*k*r^4 + al
pha^4 + 2*alpha^2*epsilon^2 + epsilon^4 - 2*(alpha^2 + epsilon^2)*h*r^2))/s
qrt(alpha^2 + epsilon^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{r\sqrt{-\alpha^2-\epsilon^2+2hr^2-2kr^4}}\,dr$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r**4+2*h*r**2-alpha**2-epsilon**2)**(1/2),r)
```

```
[Out] Integral(1/(r*sqrt(-alpha**2 - epsilon**2 + 2*h*r**2 - 2*k*r**4)), r)
```

Giac [A] time = 1.25976, size = 42, normalized size = 0.62

$$-\frac{\arcsin\left(-\frac{h-\frac{\alpha^2}{r^2}}{\sqrt{-2\,\alpha^2k+h^2}}\right)}{2\,|\alpha|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/r/(-2*k*r^4+2*h*r^2-alpha^2-epsilon^2)^(1/2),r, algorithm="giac
")
```

```
[Out] -1/2*arcsin(-(h - alpha^2/r^2)/sqrt(-2*alpha^2*k + h^2))/abs(alpha)
```

3.215 $\int a\cos(5+3x)\sin^2(5+3x)\,dx$

Optimal. Leaf size=13

$$\frac{1}{9}a\sin^3(3x+5)$$

[Out] (a*Sin[5 + 3*x]^3)/9

Rubi [A] time = 0.0174402, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {12, 2564, 30}

$$\frac{1}{9}a\sin^3(3x+5)$$

Antiderivative was successfully verified.

[In] Int[a*Cos[5 + 3*x]*Sin[5 + 3*x]^2,x]

[Out] (a*Sin[5 + 3*x]^3)/9

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int a\cos(5+3x)\sin^2(5+3x)\,dx &= a\int\cos(5+3x)\sin^2(5+3x)\,dx\\ &= \frac{1}{3}a\,\mathrm{Subst}\left(\int x^2\,dx,x,\sin(5+3x)\right)\\ &= \frac{1}{9}a\sin^3(5+3x)\end{aligned}$$

Mathematica [A] time = 0.0070728, size = 13, normalized size = 1.

$$\frac{1}{9}a\sin^3(3x+5)$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cos[5 + 3*x]*Sin[5 + 3*x]^2,x]
```

```
[Out] (a*Sin[5 + 3*x]^3)/9
```

Maple [A] time = 0.006, size = 12, normalized size = 0.9

$$\frac{a\left(\sin\left(5+3\,x\right)\right)^3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(5+3*x)*sin(5+3*x)^2,x)
```

```
[Out] 1/9*a*sin(5+3*x)^3
```

Maxima [A] time = 0.943559, size = 15, normalized size = 1.15

$$\frac{1}{9}\,a\sin\left(3\,x+5\right)^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(5+3*x)*sin(5+3*x)^2,x, algorithm="maxima")
```

```
[Out] 1/9*a*sin(3*x + 5)^3
```

Fricas [A] time = 1.65026, size = 57, normalized size = 4.38

$$-\frac{1}{9}\left(a\cos\left(3\,x+5\right)^{2}-a\right)\sin\left(3\,x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(5+3*x)*sin(5+3*x)^2,x, algorithm="fricas")
```

```
[Out] -1/9*(a*cos(3*x + 5)^2 - a)*sin(3*x + 5)
```

Sympy [A] time = 0.303857, size = 10, normalized size = 0.77

$$\frac{a\sin^{3}\left(3x+5\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(5+3*x)*sin(5+3*x)**2,x)
```

```
[Out] a*sin(3*x + 5)**3/9
```

Giac [A] time = 1.10725, size = 15, normalized size = 1.15

$$\frac{1}{9}\,a\sin\left(3\,x+5\right)^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(5+3*x)*sin(5+3*x)^2,x, algorithm="giac")
```

```
[Out] 1/9*a*sin(3*x + 5)^3
```

3.216 $\int \frac{\log(x^2)}{x^3} dx$

Optimal. Leaf size=19

$$-\frac{1}{2x^2}-\frac{\log\left(x^2\right)}{2x^2}$$

```
[Out] -1/(2*x^2) - Log[x^2]/(2*x^2)
```

Rubi [A] time = 0.0064493, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2304}

$$-\frac{1}{2x^2}-\frac{\log\left(x^2\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x^2]/x^3,x]
```

```
[Out] -1/(2*x^2) - Log[x^2]/(2*x^2)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\log\left(x^2\right)}{x^3} dx=-\frac{1}{2x^2}-\frac{\log\left(x^2\right)}{2x^2}$$

Mathematica [A] time = 0.0011705, size = 19, normalized size = 1.

$$-\frac{1}{2x^2}-\frac{\log\left(x^2\right)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x^2]/x^3,x]
```

```
[Out] -1/(2*x^2) - Log[x^2]/(2*x^2)
```

Maple [A] time = 0.005, size = 16, normalized size = 0.8

$$-\frac{1}{2\,x^{2}}-\frac{\ln\left(x^{2}\right)}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2)/x^3,x)
```

```
[Out] -1/2/x^2-1/2*ln(x^2)/x^2
```

Maxima [A] time = 0.943673, size = 20, normalized size = 1.05

$$-\frac{\log\left(x^{2}\right)}{2\,x^{2}}-\frac{1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*log(x^2)/x^2 - 1/2/x^2
```

Fricas [A] time = 1.58228, size = 34, normalized size = 1.79

$$-\frac{\log\left(x^{2}\right)+1}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(log(x^2) + 1)/x^2
```

Sympy [A] time = 0.093514, size = 17, normalized size = 0.89

$$-\frac{\log\left(x^2\right)}{2x^2}-\frac{1}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2)/x**3,x)
```

```
[Out] -log(x**2)/(2*x**2) - 1/(2*x**2)
```

Giac [A] time = 1.0904, size = 20, normalized size = 1.05

$$-\frac{\log\left(x^2\right)}{2\,x^2}-\frac{1}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2)/x^3,x, algorithm="giac")
```

```
[Out] -1/2*log(x^2)/x^2 - 1/2/x^2
```

3.217 $\int x \sin(a+x)\,dx$

Optimal. Leaf size=12

$$\sin(a+x)-x\cos(a+x)$$

```
[Out] -(x*Cos[a + x]) + Sin[a + x]
```

Rubi [A] time = 0.0092589, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2637}

$$\sin(a+x)-x\cos(a+x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[a + x],x]
```

```
[Out] -(x*Cos[a + x]) + Sin[a + x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sin(a+x)\,dx &= -x\cos(a+x)+\int \cos(a+x)\,dx\\ &= -x\cos(a+x)+\sin(a+x)\end{aligned}$$

Mathematica [A] time = 0.0242601, size = 12, normalized size = 1.

$$\sin(a + x) - x\cos(a + x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[a + x],x]
```

```
[Out] -(x*Cos[a + x]) + Sin[a + x]
```

Maple [A] time = 0.005, size = 21, normalized size = 1.8

$$a\cos(a + x) + \sin(a + x) - (a + x)\cos(a + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a+x),x)
```

```
[Out] a*cos(a+x)+sin(a+x)-(a+x)*cos(a+x)
```

Maxima [A] time = 0.943976, size = 27, normalized size = 2.25

$$-(a + x)\cos(a + x) + a\cos(a + x) + \sin(a + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+x),x, algorithm="maxima")
```

```
[Out] -(a + x)*cos(a + x) + a*cos(a + x) + sin(a + x)
```

Fricas [A] time = 1.68374, size = 38, normalized size = 3.17

$$-x\cos(a + x) + \sin(a + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+x),x, algorithm="fricas")
```

```
[Out] -x*cos(a + x) + sin(a + x)
```

Sympy [A] time = 0.168637, size = 10, normalized size = 0.83

$$-x\cos(a+x)+\sin(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+x),x)
```

```
[Out] -x*cos(a + x) + sin(a + x)
```

Giac [A] time = 1.09323, size = 16, normalized size = 1.33

$$-x\cos(a+x)+\sin(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(a+x),x, algorithm="giac")
```

```
[Out] -x*cos(a + x) + sin(a + x)
```

3.218 $\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)}\,dx$

Optimal. Leaf size=11

$$\frac{e^{-x}x}{\log(x)}$$

```
[Out] x/(E^x*Log[x])
```

Rubi [A] time = 0.0245364, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {2201}

$$\frac{e^{-x}x}{\log(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + (1 - x)*Log[x])/(E^x*Log[x]^2),x]
```

```
[Out] x/(E^x*Log[x])
```

Rule 2201

```
Int[Log[(d_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((e_) + Log[(d_
.)*(x_)]*(h_.)*((f_.) + (g_.)*(x_))), x_Symbol] :> Simp[(e*x*F^(c*(a + b*x)
)*Log[d*x]^(n + 1))/(n + 1), x] /; FreeQ[{F, a, b, c, d, e, f, g, h, n}, x]
 && EqQ[e - f*h*(n + 1), 0] && EqQ[g*h*(n + 1) - b*c*e*Log[F], 0] && NeQ[n,
 -1]
```

Rubi steps

$$\int \frac{e^{-x}(-1+(1-x)\log(x))}{\log^2(x)}\,dx = \frac{e^{-x}x}{\log(x)}$$

Mathematica [A] time = 0.0078425, size = 11, normalized size = 1.

$$\frac{e^{-x}x}{\log(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + (1 - x)*Log[x])/(E^x*Log[x]^2),x]
```

```
[Out] x/(E^x*Log[x])
```

Maple [A] time = 0.013, size = 11, normalized size = 1.

$$\frac{x}{e^{x} \ln \left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+(1-x)*ln(x))/exp(x)/ln(x)^2,x)
```

```
[Out] x/exp(x)/ln(x)
```

Maxima [A] time = 1.08985, size = 14, normalized size = 1.27

$$\frac{xe^{(-x)}}{\log \left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x, algorithm="maxima")
```

```
[Out] x*e^(-x)/log(x)
```

Fricas [A] time = 1.57698, size = 23, normalized size = 2.09

$$\frac{xe^{(-x)}}{\log \left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x, algorithm="fricas")
```

```
[Out] x*e^(-x)/log(x)
```

Sympy [A] time = 0.283352, size = 7, normalized size = 0.64

$$\frac{xe^{-x}}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(1-x)*ln(x))/exp(x)/ln(x)**2,x)
```

```
[Out] x*exp(-x)/log(x)
```

Giac [A] time = 1.07873, size = 14, normalized size = 1.27

$$\frac{xe^{(-x)}}{\log(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(1-x)*log(x))/exp(x)/log(x)^2,x, algorithm="giac")
```

```
[Out] x*e^(-x)/log(x)
```

3.219 $\int \frac{x^3}{b+ax^2} dx$

Optimal. Leaf size=27

$$\frac{x^2}{2a} - \frac{b \log\left(ax^2 + b\right)}{2a^2}$$

```
[Out] x^2/(2*a) - (b*Log[b + a*x^2])/(2*a^2)
```

Rubi [A] time = 0.0182291, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{x^2}{2a} - \frac{b \log\left(ax^2 + b\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(b + a*x^2),x]
```

```
[Out] x^2/(2*a) - (b*Log[b + a*x^2])/(2*a^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{x^3}{b+ax^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x}{b+ax}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{1}{a} - \frac{b}{a(b+ax)}\right)dx, x, x^2\right)$$
$$= \frac{x^2}{2a} - \frac{b\log\left(b+ax^2\right)}{2a^2}$$

Mathematica [A] time = 0.0042536, size = 27, normalized size = 1.

$$\frac{x^2}{2a} - \frac{b\log\left(ax^2+b\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(b + a*x^2),x]
```

```
[Out] x^2/(2*a) - (b*Log[b + a*x^2])/(2*a^2)
```

Maple [A] time = 0.002, size = 24, normalized size = 0.9

$$\frac{x^2}{2\,a} - \frac{b\ln\left(ax^2+b\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a*x^2+b),x)
```

```
[Out] 1/2*x^2/a-1/2*b*ln(a*x^2+b)/a^2
```

Maxima [A] time = 0.938308, size = 31, normalized size = 1.15

$$\frac{x^2}{2\,a} - \frac{b\log\left(ax^2+b\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^2+b),x, algorithm="maxima")
```

```
[Out] 1/2*x^2/a - 1/2*b*log(a*x^2 + b)/a^2
```

Fricas [A] time = 1.56157, size = 49, normalized size = 1.81

$$\frac{ax^2 - b\log\left(ax^2 + b\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^2+b),x, algorithm="fricas")
```

```
[Out] 1/2*(a*x^2 - b*log(a*x^2 + b))/a^2
```

Sympy [A] time = 0.293708, size = 20, normalized size = 0.74

$$\frac{x^2}{2a} - \frac{b\log\left(ax^2 + b\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a*x**2+b),x)
```

```
[Out] x**2/(2*a) - b*log(a*x**2 + b)/(2*a**2)
```

Giac [A] time = 1.09707, size = 32, normalized size = 1.19

$$\frac{x^2}{2\,a} - \frac{b\log\left(\left|ax^2 + b\right|\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a*x^2+b),x, algorithm="giac")
```

```
[Out] 1/2*x^2/a - 1/2*b*log(abs(a*x^2 + b))/a^2
```

3.220 $\int \frac{\sqrt{x}}{(1+x)^{7/2}}\,dx$

Optimal. Leaf size=33

$$\frac{4x^{3/2}}{15(x+1)^{3/2}}+\frac{2x^{3/2}}{5(x+1)^{5/2}}$$

```
[Out] (2*x^(3/2))/(5*(1 + x)^(5/2)) + (4*x^(3/2))/(15*(1 + x)^(3/2))
```

Rubi [A] time = 0.0030038, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {45, 37}

$$\frac{4x^{3/2}}{15(x+1)^{3/2}}+\frac{2x^{3/2}}{5(x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + x)^(7/2),x]
```

```
[Out] (2*x^(3/2))/(5*(1 + x)^(5/2)) + (4*x^(3/2))/(15*(1 + x)^(3/2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sqrt{x}}{(1+x)^{7/2}}\,dx = \frac{2x^{3/2}}{5(1+x)^{5/2}} + \frac{2}{5}\int \frac{\sqrt{x}}{(1+x)^{5/2}}\,dx$$
$$= \frac{2x^{3/2}}{5(1+x)^{5/2}} + \frac{4x^{3/2}}{15(1+x)^{3/2}}$$

Mathematica [A] time = 0.0062169, size = 21, normalized size = 0.64

$$\frac{2x^{3/2}(2x+5)}{15(x+1)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]/(1 + x)^(7/2),x]
```

```
[Out] (2*x^(3/2)*(5 + 2*x))/(15*(1 + x)^(5/2))
```

Maple [A] time = 0.003, size = 16, normalized size = 0.5

$$\frac{10+4\,x}{15}x^{\frac{3}{2}}\,(1+x)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)/(1+x)^(7/2),x)
```

```
[Out] 2/15*x^(3/2)*(5+2*x)/(1+x)^(5/2)
```

Maxima [A] time = 0.943993, size = 27, normalized size = 0.82

$$\frac{2\,x^{\frac{5}{2}}\left(\frac{5\,(x+1)}{x}-3\right)}{15\,(x+1)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(7/2),x, algorithm="maxima")
```

```
[Out] 2/15*x^(5/2)*(5*(x + 1)/x - 3)/(x + 1)^(5/2)
```

Fricas [B] time = 1.60976, size = 124, normalized size = 3.76

$$\frac{2\left(2x^3+\left(2x^2+5x\right)\sqrt{x+1}\sqrt{x}+6x^2+6x+2\right)}{15\left(x^3+3x^2+3x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(7/2),x, algorithm="fricas")
```

```
[Out] 2/15*(2*x^3 + (2*x^2 + 5*x)*sqrt(x + 1)*sqrt(x) + 6*x^2 + 6*x + 2)/(x^3 + 3
*x^2 + 3*x + 1)
```

Sympy [B] time = 9.95409, size = 165, normalized size = 5.

$$\begin{cases} \frac{4i\sqrt{-1+\frac{1}{x+1}}}{15}+\frac{2i\sqrt{-1+\frac{1}{x+1}}}{15(x+1)}-\frac{2i\sqrt{-1+\frac{1}{x+1}}}{5(x+1)^2} & \text{for } \frac{1}{|x+1|}>1 \\ \frac{4\sqrt{1-\frac{1}{x+1}}(x+1)^2}{-15x+15(x+1)^2-15}-\frac{2\sqrt{1-\frac{1}{x+1}}(x+1)}{-15x+15(x+1)^2-15}-\frac{8\sqrt{1-\frac{1}{x+1}}}{-15x+15(x+1)^2-15}+\frac{6\sqrt{1-\frac{1}{x+1}}}{(x+1)\left(-15x+15(x+1)^2-15\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)/(1+x)**(7/2),x)
```

```
[Out] Piecewise((4*I*sqrt(-1 + 1/(x + 1))/15 + 2*I*sqrt(-1 + 1/(x + 1))/(15*(x +
1)) - 2*I*sqrt(-1 + 1/(x + 1))/(5*(x + 1)**2), 1/Abs(x + 1) > 1), (4*sqrt(1
 - 1/(x + 1))*(x + 1)**2/(-15*x + 15*(x + 1)**2 - 15) - 2*sqrt(1 - 1/(x + 1
))*(x + 1)/(-15*x + 15*(x + 1)**2 - 15) - 8*sqrt(1 - 1/(x + 1))/(-15*x + 15
*(x + 1)**2 - 15) + 6*sqrt(1 - 1/(x + 1))/((x + 1)*(-15*x + 15*(x + 1)**2 -
 15)), True))
```

Giac [B] time = 1.11898, size = 89, normalized size = 2.7

$$\frac{8\left(15\left(\sqrt{x+1}-\sqrt{x}\right)^{6}-5\left(\sqrt{x+1}-\sqrt{x}\right)^{4}+5\left(\sqrt{x+1}-\sqrt{x}\right)^{2}+1\right)}{15\left(\left(\sqrt{x+1}-\sqrt{x}\right)^{2}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)/(1+x)^(7/2),x, algorithm="giac")
```

```
[Out] 8/15*(15*(sqrt(x + 1) - sqrt(x))^6 - 5*(sqrt(x + 1) - sqrt(x))^4 + 5*(sqrt(
x + 1) - sqrt(x))^2 + 1)/((sqrt(x + 1) - sqrt(x))^2 + 1)^5
```

3.221 $\int \frac{1}{x(1+x)}\,dx$

Optimal. Leaf size=9

$$\log(x) - \log(x+1)$$

[Out] Log[x] - Log[1 + x]

Rubi [A] time = 0.0009658, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {36, 29, 31}

$$\log(x) - \log(x+1)$$

Antiderivative was successfully verified.

[In] Int[1/(x*(1 + x)),x]

[Out] Log[x] - Log[1 + x]

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(1+x)}\,dx &= \int \frac{1}{x}\,dx - \int \frac{1}{1+x}\,dx\\ &= \log(x) - \log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0020154, size = 9, normalized size = 1.

$$\log(x) - \log(x + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x)),x]
```

```
[Out] Log[x] - Log[1 + x]
```

Maple [A] time = 0.003, size = 10, normalized size = 1.1

$$\ln(x) - \ln(1 + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(1+x),x)
```

```
[Out] ln(x)-ln(1+x)
```

Maxima [A] time = 0.949721, size = 12, normalized size = 1.33

$$-\log(x + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x),x, algorithm="maxima")
```

```
[Out] -log(x + 1) + log(x)
```

Fricas [A] time = 1.55904, size = 30, normalized size = 3.33

$$-\log(x + 1) + \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x),x, algorithm="fricas")
```

```
[Out] -log(x + 1) + log(x)
```

Sympy [A] time = 0.085563, size = 7, normalized size = 0.78

$$\log(x) - \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x),x)
```

```
[Out] log(x) - log(x + 1)
```

Giac [A] time = 1.07695, size = 15, normalized size = 1.67

$$-\log(|x + 1|) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(1+x),x, algorithm="giac")
```

```
[Out] -log(abs(x + 1)) + log(abs(x))
```

3.222 $\int \frac{1}{\sqrt{x}(-1+2x)}\,dx$

Optimal. Leaf size=19

$$-\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sqrt{x}\right)$$

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[2]*Sqrt[x]])
```

Rubi [A] time = 0.0052372, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {63, 207}

$$-\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[x]*(-1 + 2*x)),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[2]*Sqrt[x]])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{x}(-1+2x)}\,dx = 2\,\text{Subst}\left(\int \frac{1}{-1+2x^2}\,dx, x, \sqrt{x}\right)$$
$$= -\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sqrt{x}\right)$$

Mathematica [A] time = 0.0028453, size = 19, normalized size = 1.

$$-\sqrt{2}\tanh^{-1}\left(\sqrt{2}\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[x]*(-1 + 2*x)),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[2]*Sqrt[x]])
```

Maple [A] time = 0.003, size = 14, normalized size = 0.7

$$-\text{Artanh}\left(\sqrt{2}\sqrt{x}\right)\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^(1/2)/(2*x-1),x)
```

```
[Out] -arctanh(2^(1/2)*x^(1/2))*2^(1/2)
```

Maxima [B] time = 1.41823, size = 38, normalized size = 2.

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\sqrt{x}}{\sqrt{2}+2\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+2*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - 2*sqrt(x))/(sqrt(2) + 2*sqrt(x)))
```

Fricas [B] time = 1.61785, size = 80, normalized size = 4.21

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{2\sqrt{2}\sqrt{x}-2x-1}{2x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+2*x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(2*sqrt(2)*sqrt(x) - 2*x - 1)/(2*x - 1))
```

Sympy [B] time = 0.323808, size = 39, normalized size = 2.05

$$\frac{\sqrt{2}\log\left(\sqrt{x}-\frac{\sqrt{2}}{2}\right)}{2}-\frac{\sqrt{2}\log\left(\sqrt{x}+\frac{\sqrt{2}}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**(1/2)/(-1+2*x),x)
```

```
[Out] sqrt(2)*log(sqrt(x) - sqrt(2)/2)/2 - sqrt(2)*log(sqrt(x) + sqrt(2)/2)/2
```

Giac [B] time = 1.08088, size = 43, normalized size = 2.26

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{1}{2}\sqrt{2}+\sqrt{x}\right)+\frac{1}{2}\sqrt{2}\log\left(\left|-\frac{1}{2}\sqrt{2}+\sqrt{x}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^(1/2)/(-1+2*x),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(1/2*sqrt(2) + sqrt(x)) + 1/2*sqrt(2)*log(abs(-1/2*sqrt(2)
+ sqrt(x)))
```

3.223 $\int \sqrt{x}\left(1+x^2\right) dx$

Optimal. Leaf size=19

$$\frac{2x^{7/2}}{7}+\frac{2x^{3/2}}{3}$$

```
[Out] (2*x^(3/2))/3 + (2*x^(7/2))/7
```

Rubi [A] time = 0.0025534, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {14}

$$\frac{2x^{7/2}}{7}+\frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]*(1 + x^2),x]
```



```
[Out] (2*x^(3/2))/3 + (2*x^(7/2))/7
```


Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \sqrt{x}\left(1+x^2\right) dx &= \int \left(\sqrt{x}+x^{5/2}\right) dx\\ &= \frac{2x^{3/2}}{3}+\frac{2x^{7/2}}{7}\end{aligned}$$

Mathematica [A] time = 0.0027283, size = 16, normalized size = 0.84

$$\frac{2}{21}x^{3/2}\left(3x^2+7\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x]*(1 + x^2),x]
```

```
[Out] (2*x^(3/2)*(7 + 3*x^2))/21
```

Maple [A] time = 0.002, size = 13, normalized size = 0.7

$$\frac{6x^2+14}{21}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1/2)*(x^2+1),x)
```

```
[Out] 2/21*x^(3/2)*(3*x^2+7)
```

Maxima [A] time = 0.930852, size = 15, normalized size = 0.79

$$\frac{2}{7}x^{\frac{7}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x^2+1),x, algorithm="maxima")
```

```
[Out] 2/7*x^(7/2) + 2/3*x^(3/2)
```

Fricas [A] time = 1.58067, size = 38, normalized size = 2.

$$\frac{2}{21}\left(3x^3+7x\right)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x^2+1),x, algorithm="fricas")
```

```
[Out] 2/21*(3*x^3 + 7*x)*sqrt(x)
```

Sympy [A] time = 0.962552, size = 15, normalized size = 0.79

$$\frac{2x^{\frac{7}{2}}}{7} + \frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1/2)*(x**2+1),x)
```

```
[Out] 2*x**(7/2)/7 + 2*x**(3/2)/3
```

Giac [A] time = 1.07425, size = 15, normalized size = 0.79

$$\frac{2}{7}\,x^{\frac{7}{2}} + \frac{2}{3}\,x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(x^2+1),x, algorithm="giac")
```

```
[Out] 2/7*x^(7/2) + 2/3*x^(3/2)
```

3.224 $\int \frac{\sqrt[3]{-a+x}}{x} \, dx$

Optimal. Leaf size=88

$$3\sqrt[3]{x-a}+\frac{1}{2}\sqrt[3]{a}\log(x)-\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{x-a}+\sqrt[3]{a}\right)+\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{a}}\right)$$

```
[Out] 3*(-a + x)^(1/3) + Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) - 2*(-a + x)^(1/3))/(Sqr
t[3]*a^(1/3))] + (a^(1/3)*Log[x])/2 - (3*a^(1/3)*Log[a^(1/3) + (-a + x)^(1/
3)])/2
```

Rubi [A] time = 0.0386913, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {50, 58, 617, 204, 31}

$$3\sqrt[3]{x-a}+\frac{1}{2}\sqrt[3]{a}\log(x)-\frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{x-a}+\sqrt[3]{a}\right)+\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{x-a}}{\sqrt{3}\sqrt[3]{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-a + x)^(1/3)/x,x]
```

```
[Out] 3*(-a + x)^(1/3) + Sqrt[3]*a^(1/3)*ArcTan[(a^(1/3) - 2*(-a + x)^(1/3))/(Sqr
t[3]*a^(1/3))] + (a^(1/3)*Log[x])/2 - (3*a^(1/3)*Log[a^(1/3) + (-a + x)^(1/
3)])/2
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 58

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[
{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^
2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(
1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)],
```

```
x])] /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt[3]{-a+x}}{x}\,dx &= 3\sqrt[3]{-a+x} - a\int \frac{1}{x(-a+x)^{2/3}}\,dx \\
&= 3\sqrt[3]{-a+x} + \frac{1}{2}\sqrt[3]{a}\log(x) - \frac{1}{2}\left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+x}\,dx, x, \sqrt[3]{-a+x}\right) - \frac{1}{2}\left(3a^{2/3}\right)\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\right. \\
&= 3\sqrt[3]{-a+x} + \frac{1}{2}\sqrt[3]{a}\log(x) - \frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{-a+x}\right) - \left(3\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1-\frac{2\sqrt[3]{-a+x}}{\sqrt[3]{a}}\right) \\
&= 3\sqrt[3]{-a+x} + \sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{-a+x}}{\sqrt[3]{a}}}{\sqrt{3}}\right) + \frac{1}{2}\sqrt[3]{a}\log(x) - \frac{3}{2}\sqrt[3]{a}\log\left(\sqrt[3]{a}+\sqrt[3]{-a+x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.039665, size = 112, normalized size = 1.27

$$
\frac{1}{2}\sqrt[3]{a}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{x-a}+(x-a)^{2/3}\right)+3\sqrt[3]{x-a}-\sqrt[3]{a}\log\left(\sqrt[3]{x-a}+\sqrt[3]{a}\right)+\sqrt{3}\sqrt[3]{a}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{x-a}}{\sqrt[3]{a}}}{\sqrt{3}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-a + x)^(1/3)/x,x]
```

```
[Out] 3*(-a + x)^(1/3) + Sqrt[3]*a^(1/3)*ArcTan[(1 - (2*(-a + x)^(1/3))/a^(1/3))/
Sqrt[3]] - a^(1/3)*Log[a^(1/3) + (-a + x)^(1/3)] + (a^(1/3)*Log[a^(2/3) - a
^(1/3)*(-a + x)^(1/3) + (-a + x)^(2/3)])/2
```

Maple [A] time = 0.007, size = 85, normalized size = 1.

$$3\sqrt[3]{-a+x}-\sqrt[3]{a}\ln\left(\sqrt[3]{a}+\sqrt[3]{-a+x}\right)+\frac{1}{2}\sqrt[3]{a}\ln\left((-a+x)^{\frac{2}{3}}-\sqrt[3]{a}\sqrt[3]{-a+x}+a^{\frac{2}{3}}\right)-\sqrt[3]{a}\sqrt{3}\arctan\left(\frac{\sqrt{3}}{3}\left(2\frac{\sqrt[3]{-a+x}}{\sqrt[3]{a}}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a+x)^(1/3)/x,x)
```

```
[Out] 3*(-a+x)^(1/3)-a^(1/3)*ln(a^(1/3)+(-a+x)^(1/3))+1/2*a^(1/3)*ln((-a+x)^(2/3)
-a^(1/3)*(-a+x)^(1/3)+a^(2/3))-a^(1/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/a^(1/3
)*(-a+x)^(1/3)-1))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)^(1/3)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.60695, size = 309, normalized size = 3.51

$$\sqrt{3}\,(-a)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}a-2\sqrt{3}\,(-a)^{\frac{2}{3}}\,(-a+x)^{\frac{1}{3}}}{3\,a}\right)-\frac{1}{2}\,(-a)^{\frac{1}{3}}\log\left((-a)^{\frac{2}{3}}+(-a)^{\frac{1}{3}}\,(-a+x)^{\frac{1}{3}}+(-a+x)^{\frac{2}{3}}\right)+(-a)^{\frac{1}{3}}\log\left(-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)^(1/3)/x,x, algorithm="fricas")
```

```
[Out] sqrt(3)*(-a)^(1/3)*arctan(-1/3*(sqrt(3)*a - 2*sqrt(3)*(-a)^(2/3)*(-a + x)^(
1/3))/a) - 1/2*(-a)^(1/3)*log((-a)^(2/3) + (-a)^(1/3)*(-a + x)^(1/3) + (-a
+ x)^(2/3)) + (-a)^(1/3)*log(-(-a)^(1/3) + (-a + x)^(1/3)) + 3*(-a + x)^(1/
3)
```

Sympy [C] time = 1.74508, size = 153, normalized size = 1.74

$$\frac{4\sqrt[3]{a}e^{-\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{-a+x}e^{\frac{i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}-\frac{4\sqrt[3]{a}\log\left(1-\frac{\sqrt[3]{-a+x}e^{i\pi}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{a}e^{\frac{i\pi}{3}}\log\left(1-\frac{\sqrt[3]{-a+x}e^{\frac{5i\pi}{3}}}{\sqrt[3]{a}}\right)\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}+\frac{4\sqrt[3]{-a+x}\Gamma\left(\frac{4}{3}\right)}{\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)**(1/3)/x,x)
```

```
[Out] 4*a**(1/3)*exp(-I*pi/3)*log(1 - (-a + x)**(1/3)*exp_polar(I*pi/3)/a**(1/3))
*gamma(4/3)/(3*gamma(7/3)) - 4*a**(1/3)*log(1 - (-a + x)**(1/3)*exp_polar(I
*pi)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*a**(1/3)*exp(I*pi/3)*log(1 - (
-a + x)**(1/3)*exp_polar(5*I*pi/3)/a**(1/3))*gamma(4/3)/(3*gamma(7/3)) + 4*
(-a + x)**(1/3)*gamma(4/3)/gamma(7/3)
```

Giac [A] time = 1.83854, size = 139, normalized size = 1.58

$$-\sqrt{3}\left(-a\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-a\right)^{\frac{1}{3}}+2\left(-a+x\right)^{\frac{1}{3}}\right)}{3\left(-a\right)^{\frac{1}{3}}}\right)-\frac{1}{2}\left(-a\right)^{\frac{1}{3}}\log\left(\left(-a\right)^{\frac{2}{3}}+\left(-a\right)^{\frac{1}{3}}\left(-a+x\right)^{\frac{1}{3}}+\left(-a+x\right)^{\frac{2}{3}}\right)+\left(-a\right)^{\frac{1}{3}}\log\left(\left|-(-a\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+x)^(1/3)/x,x, algorithm="giac")
```

```
[Out] -sqrt(3)*(-a)^(1/3)*arctan(1/3*sqrt(3)*((-a)^(1/3) + 2*(-a + x)^(1/3))/(-a)
^(1/3)) - 1/2*(-a)^(1/3)*log((-a)^(2/3) + (-a)^(1/3)*(-a + x)^(1/3) + (-a +
 x)^(2/3)) + (-a)^(1/3)*log(abs(-(-a)^(1/3) + (-a + x)^(1/3))) + 3*(-a + x)
^(1/3)
```

3.225 $\int x \sinh(x) \, dx$

Optimal. Leaf size=9

$$x \cosh(x) - \sinh(x)$$

```
[Out] x*Cosh[x] - Sinh[x]
```

Rubi [A] time = 0.0109491, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3296, 2637}

$$x \cosh(x) - \sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sinh[x],x]
```

```
[Out] x*Cosh[x] - Sinh[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int x \sinh(x) \, dx &= x \cosh(x) - \int \cosh(x) \, dx \\ &= x \cosh(x) - \sinh(x) \end{aligned}$$

Mathematica [A] time = 0.0023825, size = 9, normalized size = 1.

$$x\cosh(x) - \sinh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sinh[x],x]
```

```
[Out] x*Cosh[x] - Sinh[x]
```

Maple [A] time = 0.005, size = 10, normalized size = 1.1

$$x\cosh(x) - \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sinh(x),x)
```

```
[Out] x*cosh(x)-sinh(x)
```

Maxima [B] time = 0.951741, size = 46, normalized size = 5.11

$$\frac{1}{2}x^2\sinh(x) + \frac{1}{4}\left(x^2 + 2x + 2\right)e^{(-x)} - \frac{1}{4}\left(x^2 - 2x + 2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*sinh(x) + 1/4*(x^2 + 2*x + 2)*e^(-x) - 1/4*(x^2 - 2*x + 2)*e^x
```

Fricas [A] time = 1.56237, size = 28, normalized size = 3.11

$$x\cosh(x) - \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x),x, algorithm="fricas")
```

```
[Out] x*cosh(x) - sinh(x)
```

Sympy [A] time = 0.172455, size = 7, normalized size = 0.78

$$x\cosh(x) - \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x),x)
```

```
[Out] x*cosh(x) - sinh(x)
```

Giac [A] time = 1.08537, size = 23, normalized size = 2.56

$$\frac{1}{2}(x+1)e^{(-x)} + \frac{1}{2}(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sinh(x),x, algorithm="giac")
```

```
[Out] 1/2*(x + 1)*e^(-x) + 1/2*(x - 1)*e^x
```

3.226 $\int x \cosh(x)\,dx$

Optimal. Leaf size=9

$$x\sinh(x) - \cosh(x)$$

```
[Out] -Cosh[x] + x*Sinh[x]
```

Rubi [A] time = 0.0105901, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3296, 2638}

$$x\sinh(x) - \cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[x],x]
```

```
[Out] -Cosh[x] + x*Sinh[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\cosh(x)\,dx &= x\sinh(x) - \int \sinh(x)\,dx\\ &= -\cosh(x) + x\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.0020097, size = 9, normalized size = 1.

$$x\sinh(x)-\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[x],x]
```

```
[Out] -Cosh[x] + x*Sinh[x]
```

Maple [A] time = 0.003, size = 10, normalized size = 1.1

$$-\cosh(x)+x\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(x),x)
```

```
[Out] -cosh(x)+x*sinh(x)
```

Maxima [B] time = 0.960057, size = 46, normalized size = 5.11

$$\frac{1}{2}x^2\cosh(x)-\frac{1}{4}\left(x^2+2x+2\right)e^{(-x)}-\frac{1}{4}\left(x^2-2x+2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*cosh(x) - 1/4*(x^2 + 2*x + 2)*e^(-x) - 1/4*(x^2 - 2*x + 2)*e^x
```

Fricas [A] time = 1.59057, size = 28, normalized size = 3.11

$$x\sinh(x)-\cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x),x, algorithm="fricas")
```

```
[Out] x*sinh(x) - cosh(x)
```

Sympy [A] time = 0.18511, size = 7, normalized size = 0.78

$$x\sinh(x) - \cosh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x),x)
```

```
[Out] x*sinh(x) - cosh(x)
```

Giac [A] time = 1.0772, size = 23, normalized size = 2.56

$$-\frac{1}{2}(x+1)e^{(-x)} + \frac{1}{2}(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(x),x, algorithm="giac")
```

```
[Out] -1/2*(x + 1)*e^(-x) + 1/2*(x - 1)*e^x
```

3.227 $\int \tanh(2x)\,dx$

Optimal. Leaf size=9

$$\frac{1}{2}\log(\cosh(2x))$$

```
[Out] Log[Cosh[2*x]]/2
```

Rubi [A] time = 0.0037429, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3475}

$$\frac{1}{2}\log(\cosh(2x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[2*x],x]
```

```
[Out] Log[Cosh[2*x]]/2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tanh(2x)\,dx = \frac{1}{2}\log(\cosh(2x))$$

Mathematica [A] time = 0.0029506, size = 9, normalized size = 1.

$$\frac{1}{2}\log(\cosh(2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[2*x],x]
```

```
[Out] Log[Cosh[2*x]]/2
```

Maple [A] time = 0.005, size = 8, normalized size = 0.9

$$\frac{\ln(\cosh(2x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(2*x)/cosh(2*x),x)
```

```
[Out] 1/2*ln(cosh(2*x))
```

Maxima [A] time = 0.942742, size = 9, normalized size = 1.

$$\frac{1}{2}\log(\cosh(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(2*x)/cosh(2*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(cosh(2*x))
```

Fricas [B] time = 1.73173, size = 69, normalized size = 7.67

$$-x+\frac{1}{2}\log\left(\frac{2\cosh(2x)}{\cosh(2x)-\sinh(2x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(2*x)/cosh(2*x),x, algorithm="fricas")
```

```
[Out] -x + 1/2*log(2*cosh(2*x)/(cosh(2*x) - sinh(2*x)))
```

Sympy [A] time = 0.130381, size = 7, normalized size = 0.78

$$\frac{\log\left(\cosh\left(2x\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(2*x)/cosh(2*x),x)
```

```
[Out] log(cosh(2*x))/2
```

Giac [A] time = 1.09376, size = 18, normalized size = 2.

$$-x + \frac{1}{2}\log\left(e^{(4x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(2*x)/cosh(2*x),x, algorithm="giac")
```

```
[Out] -x + 1/2*log(e^(4*x) + 1)
```

3.228 $\int \frac{-1+i\mathbf{eps}\sinh(x)}{ia-x+i\mathbf{eps}\cosh(x)}\,dx$

Optimal. Leaf size=12

$$\log(a+\text{eps}\cosh(x)+ix)$$

```
[Out] Log[a + I*x + eps*Cosh[x]]
```

Rubi [A] time = 0.0325908, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {6684}

$$\log(a+\text{eps}\cosh(x)+ix)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + I*eps*Sinh[x])/(I*a - x + I*eps*Cosh[x]),x]

[Out] Log[a + I*x + eps*Cosh[x]]
```

Rule 6684

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rubi steps

$$\int \frac{-1+i\text{eps}\sinh(x)}{ia-x+i\text{eps}\cosh(x)}\,dx=\log(a+ix+\text{eps}\cosh(x))$$

Mathematica [A] time = 0.0827899, size = 12, normalized size = 1.

$$\log(a+\text{eps}\cosh(x)+ix)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + I*eps*Sinh[x])/(I*a - x + I*eps*Cosh[x]),x]
```

```
[Out] Log[a + I*x + eps*Cosh[x]]
```

Maple [A] time = 0.011, size = 16, normalized size = 1.3

$$\ln(ia - x + i\text{eps}\ \cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x)
```

```
[Out] ln(I*a-x+I*eps*cosh(x))
```

Maxima [A] time = 0.966939, size = 18, normalized size = 1.5

$$\log(i\,\text{eps}\cosh(x) + i\,a - x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x, algorithm="maxima")
```

```
[Out] log(I*eps*cosh(x) + I*a - x)
```

Fricas [B] time = 1.72218, size = 74, normalized size = 6.17

$$-x + \log\left(\frac{\text{eps}e^{(2\,x)} + (2\,a + 2i\,x)e^{x} + \text{eps}}{\text{eps}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x, algorithm="fricas")
```

```
[Out] -x + log((eps*e^(2*x) + (2*a + 2*I*x)*e^x + eps)/eps)
```

Sympy [B] time = 1.33109, size = 22, normalized size = 1.83

$$-x + \log\left(e^{2x} + 1 + \frac{(2a + 2ix)\, e^{x}}{eps}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x)
```

```
[Out] -x + log(exp(2*x) + 1 + (2*a + 2*I*x)*exp(x)/eps)
```

Giac [B] time = 1.08743, size = 31, normalized size = 2.58

$$-x + \log\left(\mathrm{eps} e^{(2\,x)} + 2\,a e^{x} + 2i\,x e^{x} + \mathrm{eps}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+I*eps*sinh(x))/(I*a-x+I*eps*cosh(x)),x, algorithm="giac")
```

```
[Out] -x + log(eps*e^(2*x) + 2*a*e^x + 2*I*x*e^x + eps)
```

3.229 $\int \cos^2(x) \sin(3+2x)\,dx$

Optimal. Leaf size=28

$$\frac{1}{4}x\sin(3)-\frac{1}{4}\cos(2x+3)-\frac{1}{16}\cos(4x+3)$$

```
[Out] -Cos[3 + 2*x]/4 - Cos[3 + 4*x]/16 + (x*Sin[3])/4
```

Rubi [A] time = 0.0231948, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4574, 2638}

$$\frac{1}{4}x\sin(3)-\frac{1}{4}\cos(2x+3)-\frac{1}{16}\cos(4x+3)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2*Sin[3 + 2*x],x]
```

```
[Out] -Cos[3 + 2*x]/4 - Cos[3 + 4*x]/16 + (x*Sin[3])/4
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\sin(3+2x)\,dx &= \int \left(\frac{\sin(3)}{4}+\frac{1}{2}\sin(3+2x)+\frac{1}{4}\sin(3+4x)\right)dx\\ &= \frac{1}{4}x\sin(3)+\frac{1}{4}\int \sin(3+4x)\,dx+\frac{1}{2}\int \sin(3+2x)\,dx\\ &= -\frac{1}{4}\cos(3+2x)-\frac{1}{16}\cos(3+4x)+\frac{1}{4}x\sin(3)\end{aligned}$$

Mathematica [A] time = 0.0152, size = 28, normalized size = 1.

$$\frac{1}{4}x\sin(3)-\frac{1}{4}\cos(2x+3)-\frac{1}{16}\cos(4x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2*Sin[3 + 2*x],x]
```

```
[Out] -Cos[3 + 2*x]/4 - Cos[3 + 4*x]/16 + (x*Sin[3])/4
```

Maple [A] time = 0.011, size = 23, normalized size = 0.8

$$-\frac{\cos(3+2x)}{4}-\frac{\cos(3+4x)}{16}+\frac{x\sin(3)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(3+2*x),x)
```

```
[Out] -1/4*cos(3+2*x)-1/16*cos(3+4*x)+1/4*x*sin(3)
```

Maxima [A] time = 0.95125, size = 30, normalized size = 1.07

$$\frac{1}{4}x\sin(3)-\frac{1}{16}\cos(4x+3)-\frac{1}{4}\cos(2x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(3+2*x),x, algorithm="maxima")
```

```
[Out] 1/4*x*sin(3) - 1/16*cos(4*x + 3) - 1/4*cos(2*x + 3)
```

Fricas [A] time = 1.86913, size = 116, normalized size = 4.14

$$-\frac{1}{2}\cos(3)\cos(x)^4+\frac{1}{4}x\sin(3)+\frac{1}{4}\left(2\cos(x)^3\sin(3)+\cos(x)\sin(3)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(3+2*x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(3)*cos(x)^4 + 1/4*x*sin(3) + 1/4*(2*cos(x)^3*sin(3) + cos(x)*sin(3
))*sin(x)
```

Sympy [B] time = 2.26138, size = 75, normalized size = 2.68

$$-\frac{x\sin^2(x)\sin(2x+3)}{4}-\frac{x\sin(x)\cos(x)\cos(2x+3)}{2}+\frac{x\sin(2x+3)\cos^2(x)}{4}-\frac{\sin(x)\sin(2x+3)\cos(x)}{4}-\frac{\cos^2(x)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(3+2*x),x)
```

```
[Out] -x*sin(x)**2*sin(2*x + 3)/4 - x*sin(x)*cos(x)*cos(2*x + 3)/2 + x*sin(2*x +
3)*cos(x)**2/4 - sin(x)*sin(2*x + 3)*cos(x)/4 - cos(x)**2*cos(2*x + 3)/2
```

Giac [A] time = 1.07769, size = 30, normalized size = 1.07

$$\frac{1}{4}\,x\sin(3)-\frac{1}{16}\,\cos(4\,x+3)-\frac{1}{4}\,\cos(2\,x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(3+2*x),x, algorithm="giac")
```

```
[Out] 1/4*x*sin(3) - 1/16*cos(4*x + 3) - 1/4*cos(2*x + 3)
```

3.230 $\int x \tan^{-1}(x) \, dx$

Optimal. Leaf size=21

$$\frac{1}{2}x^2\tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rubi [A] time = 0.0078855, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {4852, 321, 203}

$$\frac{1}{2}x^2\tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x],x]
```

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}(x)\,dx &= \frac{1}{2}x^2\tan^{-1}(x)-\frac{1}{2}\int\frac{x^2}{1+x^2}\,dx\\ &= -\frac{x}{2}+\frac{1}{2}x^2\tan^{-1}(x)+\frac{1}{2}\int\frac{1}{1+x^2}\,dx\\ &= -\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}x^2\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0025254, size = 21, normalized size = 1.

$$\frac{1}{2}x^2\tan^{-1}(x)-\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x],x]
```

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Maple [A] time = 0.001, size = 16, normalized size = 0.8

$$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x),x)
```

```
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Maxima [A] time = 1.43141, size = 20, normalized size = 0.95

$$\frac{1}{2}x^2\arctan(x)-\frac{1}{2}x+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Fricas [A] time = 1.71301, size = 45, normalized size = 2.14

$$\frac{1}{2}\left(x^{2}+1\right)\arctan\left(x\right)-\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

Sympy [A] time = 0.246741, size = 15, normalized size = 0.71

$$\frac{x^{2}\,\mathrm{atan}\left(x\right)}{2}-\frac{x}{2}+\frac{\mathrm{atan}\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x),x)
```

```
[Out] x**2*atan(x)/2 - x/2 + atan(x)/2
```

Giac [A] time = 1.07561, size = 20, normalized size = 0.95

$$\frac{1}{2}\,x^{2}\arctan\left(x\right)-\frac{1}{2}\,x+\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

3.231 $\int x \cot^{-1}(x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}x^2\cot^{-1}(x)+\frac{x}{2}-\frac{1}{2}\tan^{-1}(x)$$

```
[Out] x/2 + (x^2*ArcCot[x])/2 - ArcTan[x]/2
```

Rubi [A] time = 0.0082851, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {4853, 321, 203}

$$\frac{1}{2}x^2\cot^{-1}(x)+\frac{x}{2}-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcCot[x],x]
```

```
[Out] x/2 + (x^2*ArcCot[x])/2 - ArcTan[x]/2
```

Rule 4853

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCot[c*x])^p)/(d*(m + 1)), x] + Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCot[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int x\cot^{-1}(x)\,dx &= \frac{1}{2}x^2\cot^{-1}(x)+\frac{1}{2}\int\frac{x^2}{1+x^2}\,dx\\ &= \frac{x}{2}+\frac{1}{2}x^2\cot^{-1}(x)-\frac{1}{2}\int\frac{1}{1+x^2}\,dx\\ &= \frac{x}{2}+\frac{1}{2}x^2\cot^{-1}(x)-\frac{1}{2}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0023428, size = 21, normalized size = 1.

$$\frac{1}{2}x^2\cot^{-1}(x)+\frac{x}{2}-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcCot[x],x]
```

```
[Out] x/2 + (x^2*ArcCot[x])/2 - ArcTan[x]/2
```

Maple [A] time = 0.002, size = 16, normalized size = 0.8

$$\frac{x}{2}+\frac{x^2\operatorname{arccot}(x)}{2}-\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccot(x),x)
```

```
[Out] 1/2*x+1/2*x^2*arccot(x)-1/2*arctan(x)
```

Maxima [A] time = 1.44582, size = 20, normalized size = 0.95

$$\frac{1}{2}x^2\operatorname{arccot}(x)+\frac{1}{2}x-\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arccot(x) + 1/2*x - 1/2*arctan(x)
```

Fricas [A] time = 1.64884, size = 45, normalized size = 2.14

$$\frac{1}{2}\left(x^2+1\right)\operatorname{arccot}(x)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 1)*arccot(x) + 1/2*x
```

Sympy [A] time = 0.247543, size = 15, normalized size = 0.71

$$\frac{x^2\operatorname{acot}(x)}{2}+\frac{x}{2}+\frac{\operatorname{acot}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acot(x),x)
```

```
[Out] x**2*acot(x)/2 + x/2 + acot(x)/2
```

Giac [A] time = 1.08124, size = 23, normalized size = 1.1

$$\frac{1}{2}\,x^2\arctan\left(\frac{1}{x}\right)+\frac{1}{2}\,x-\frac{1}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccot(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(1/x) + 1/2*x - 1/2*arctan(x)
```

3.232 $\int x \log \left(a+x^2\right) dx$

Optimal. Leaf size=23

$$\frac{1}{2} \left(a+x^2\right) \log \left(a+x^2\right)-\frac{x^2}{2}$$

```
[Out] -x^2/2 + ((a + x^2)*Log[a + x^2])/2
```

Rubi [A] time = 0.0185245, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2454, 2389, 2295}

$$\frac{1}{2} \left(a+x^2\right) \log \left(a+x^2\right)-\frac{x^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[a + x^2],x]
```

```
[Out] -x^2/2 + ((a + x^2)*Log[a + x^2])/2
```

Rule 2454

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Lo
g[c*(d + e*x)^p])^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q},
x] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0] || IGtQ[q, 0]) &&
  !(EqQ[q, 1] && ILtQ[n, 0] && IGtQ[m, 0])
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\int x \log \left(a+x^2\right) dx = \frac{1}{2} \operatorname{Subst}\left(\int \log(a+x)\, dx, x, x^2\right)$$
$$= \frac{1}{2} \operatorname{Subst}\left(\int \log(x)\, dx, x, a+x^2\right)$$
$$= -\frac{x^2}{2} + \frac{1}{2}\left(a+x^2\right) \log \left(a+x^2\right)$$

Mathematica [A] time = 0.0032048, size = 22, normalized size = 0.96

$$\frac{1}{2}\left(\left(a+x^2\right) \log \left(a+x^2\right) - x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[a + x^2],x]
```

```
[Out] (-x^2 + (a + x^2)*Log[a + x^2])/2
```

Maple [A] time = 0.002, size = 23, normalized size = 1.

$$\frac{\left(x^2+a\right) \ln \left(x^2+a\right)}{2} - \frac{x^2}{2} - \frac{a}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x^2+a),x)
```

```
[Out] 1/2*(x^2+a)*ln(x^2+a)-1/2*x^2-1/2*a
```

Maxima [A] time = 0.965159, size = 30, normalized size = 1.3

$$-\frac{1}{2}\, x^2 + \frac{1}{2}\left(x^2+a\right) \log \left(x^2+a\right) - \frac{1}{2}\, a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^2+a),x, algorithm="maxima")
```

```
[Out] -1/2*x^2 + 1/2*(x^2 + a)*log(x^2 + a) - 1/2*a
```

Fricas [A] time = 1.60257, size = 53, normalized size = 2.3

$$-\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 + a\right)\log\left(x^2 + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*x^2 + 1/2*(x^2 + a)*log(x^2 + a)
```

Sympy [A] time = 0.29671, size = 26, normalized size = 1.13

$$\frac{a\log\left(a + x^2\right)}{2} + \frac{x^2\log\left(a + x^2\right)}{2} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x**2+a),x)
```

```
[Out] a*log(a + x**2)/2 + x**2*log(a + x**2)/2 - x**2/2
```

Giac [A] time = 1.05921, size = 30, normalized size = 1.3

$$-\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 + a\right)\log\left(x^2 + a\right) - \frac{1}{2}a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*x^2 + 1/2*(x^2 + a)*log(x^2 + a) - 1/2*a
```

3.233 $\int \cos(x)\sin(a+x)\,dx$

Optimal. Leaf size=18

$$\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

```
[Out] -Cos[a + 2*x]/4 + (x*Sin[a])/2
```

Rubi [A] time = 0.0159437, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4574, 2638}

$$\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[a + x],x]
```

```
[Out] -Cos[a + 2*x]/4 + (x*Sin[a])/2
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin(a+x)\,dx &= \int \left(\frac{\sin(a)}{2}+\frac{1}{2}\sin(a+2x)\right)dx\\ &= \frac{1}{2}x\sin(a)+\frac{1}{2}\int \sin(a+2x)\,dx\\ &= -\frac{1}{4}\cos(a+2x)+\frac{1}{2}x\sin(a)\end{aligned}$$

Mathematica [A] time = 0.0191674, size = 18, normalized size = 1.

$$\frac{1}{4}(2x\sin(a)-\cos(a+2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[a + x],x]
```

```
[Out] (-Cos[a + 2*x] + 2*x*Sin[a])/4
```

Maple [A] time = 0.018, size = 15, normalized size = 0.8

$$-\frac{\cos\left(a+2\,x\right)}{4}+\frac{x\sin\left(a\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(a+x),x)
```

```
[Out] -1/4*cos(a+2*x)+1/2*x*sin(a)
```

Maxima [A] time = 0.954662, size = 19, normalized size = 1.06

$$\frac{1}{2}\,x\sin\left(a\right)-\frac{1}{4}\,\cos\left(a+2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(a+x),x, algorithm="maxima")
```

```
[Out] 1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

Fricas [A] time = 1.74644, size = 103, normalized size = 5.72

$$-\frac{1}{2}\,\cos\left(a+x\right)^{2}\cos\left(a\right)-\frac{1}{2}\,\cos\left(a+x\right)\sin\left(a+x\right)\sin\left(a\right)+\frac{1}{2}\,x\sin\left(a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(a+x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(a + x)^2*cos(a) - 1/2*cos(a + x)*sin(a + x)*sin(a) + 1/2*x*sin(a)
```

Sympy [B] time = 0.548398, size = 32, normalized size = 1.78

$$-\frac{x\sin(x)\cos(a+x)}{2}+\frac{x\sin(a+x)\cos(x)}{2}+\frac{\sin(x)\sin(a+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(a+x),x)
```

```
[Out] -x*sin(x)*cos(a + x)/2 + x*sin(a + x)*cos(x)/2 + sin(x)*sin(a + x)/2
```

Giac [A] time = 1.0879, size = 19, normalized size = 1.06

$$\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(a+x),x, algorithm="giac")
```

```
[Out] 1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

3.234 $\int \cos(a+x)\sin(x)\,dx$

Optimal. Leaf size=18

$$-\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

```
[Out] -Cos[a + 2*x]/4 - (x*Sin[a])/2
```

Rubi [A] time = 0.0128961, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4574, 2638}

$$-\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[a + x]*Sin[x],x]
```

```
[Out] -Cos[a + 2*x]/4 - (x*Sin[a])/2
```

Rule 4574

```
Int[Cos[w_]^(q_.)*Sin[v_]^(p_.), x_Symbol] :> Int[ExpandTrigReduce[Sin[v]^p
*Cos[w]^q, x], x] /; IGtQ[p, 0] && IGtQ[q, 0] && ((PolynomialQ[v, x] && Pol
ynomialQ[w, x]) || (BinomialQ[{v, w}, x] && IndependentQ[Cancel[v/w], x]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(a+x)\sin(x)\,dx &= \int \left(-\frac{\sin(a)}{2}+\frac{1}{2}\sin(a+2x)\right)dx\\ &= -\frac{1}{2}x\sin(a)+\frac{1}{2}\int \sin(a+2x)\,dx\\ &= -\frac{1}{4}\cos(a+2x)-\frac{1}{2}x\sin(a)\end{aligned}$$

Mathematica [A] time = 0.0123611, size = 18, normalized size = 1.

$$\frac{1}{4}(-2x\sin(a)-\cos(a+2x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[a + x]*Sin[x],x]
```

```
[Out] (-Cos[a + 2*x] - 2*x*Sin[a])/4
```

Maple [A] time = 0.015, size = 15, normalized size = 0.8

$$-\frac{\cos(a+2x)}{4}-\frac{x\sin(a)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a+x)*sin(x),x)
```

```
[Out] -1/4*cos(a+2*x)-1/2*x*sin(a)
```

Maxima [A] time = 0.947506, size = 19, normalized size = 1.06

$$-\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

Fricas [A] time = 1.78751, size = 103, normalized size = 5.72

$$-\frac{1}{2}\cos(a+x)^2\cos(a)-\frac{1}{2}\cos(a+x)\sin(a+x)\sin(a)-\frac{1}{2}x\sin(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(a + x)^2*cos(a) - 1/2*cos(a + x)*sin(a + x)*sin(a) - 1/2*x*sin(a)
```

Sympy [B] time = 0.544237, size = 32, normalized size = 1.78

$$\frac{x\sin(x)\cos(a+x)}{2}-\frac{x\sin(a+x)\cos(x)}{2}+\frac{\sin(x)\sin(a+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+x)*sin(x),x)
```

```
[Out] x*sin(x)*cos(a + x)/2 - x*sin(a + x)*cos(x)/2 + sin(x)*sin(a + x)/2
```

Giac [A] time = 1.06884, size = 19, normalized size = 1.06

$$-\frac{1}{2}x\sin(a)-\frac{1}{4}\cos(a+2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(a+x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*x*sin(a) - 1/4*cos(a + 2*x)
```

3.235 $\int \sqrt{1+\sin(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{2\cos(x)}{\sqrt{\sin(x)+1}}$$

```
[Out] (-2*Cos[x])/Sqrt[1 + Sin[x]]
```

Rubi [A] time = 0.0063663, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2646}

$$-\frac{2\cos(x)}{\sqrt{\sin(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sin[x]],x]
```

```
[Out] (-2*Cos[x])/Sqrt[1 + Sin[x]]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{1+\sin(x)}\,dx = -\frac{2\cos(x)}{\sqrt{1+\sin(x)}}$$

Mathematica [B] time = 0.011964, size = 40, normalized size = 3.33

$$\frac{2\sqrt{\sin(x)+1}\left(\sin\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sin[x]],x]
```

```
[Out] (2*(-Cos[x/2] + Sin[x/2])*Sqrt[1 + Sin[x]])/(Cos[x/2] + Sin[x/2])
```

Maple [A] time = 0.027, size = 17, normalized size = 1.4

$$2\,\frac{(-1+\sin(x))\sqrt{1+\sin(x)}}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sin(x))^(1/2),x)
```

```
[Out] 2*(-1+sin(x))*(1+sin(x))^(1/2)/cos(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sin(x) + 1), x)
```

Fricas [B] time = 1.65851, size = 88, normalized size = 7.33

$$-\frac{2\,(\cos(x)-\sin(x)+1)\sqrt{\sin(x)+1}}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(cos(x) - sin(x) + 1)*sqrt(sin(x) + 1)/(cos(x) + sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x))**(1/2),x)
```

```
[Out] Integral(sqrt(sin(x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sin(x) + 1), x)
```

3.236 $\int \sqrt{1-\sin(x)}\,dx$

Optimal. Leaf size=14

$$\frac{2\cos(x)}{\sqrt{1-\sin(x)}}$$

```
[Out] (2*Cos[x])/Sqrt[1 - Sin[x]]
```

Rubi [A] time = 0.0088184, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2646}

$$\frac{2\cos(x)}{\sqrt{1-\sin(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sin[x]],x]
```

```
[Out] (2*Cos[x])/Sqrt[1 - Sin[x]]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{1-\sin(x)}\,dx = \frac{2\cos(x)}{\sqrt{1-\sin(x)}}$$

Mathematica [B] time = 0.0111571, size = 42, normalized size = 3.

$$\frac{2\sqrt{1-\sin(x)}\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sin[x]],x]
```

```
[Out] (2*(Cos[x/2] + Sin[x/2])*Sqrt[1 - Sin[x]])/(Cos[x/2] - Sin[x/2])
```

Maple [A] time = 0.035, size = 23, normalized size = 1.6

$$-2\,\frac{(-1+\sin(x))\,(1+\sin(x))}{\cos(x)\,\sqrt{1-\sin(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sin(x))^(1/2),x)
```

```
[Out] -2*(-1+sin(x))*(1+sin(x))/cos(x)/(1-sin(x))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-sin(x) + 1), x)
```

Fricas [B] time = 1.60813, size = 88, normalized size = 6.29

$$\frac{2\,(\cos(x)+\sin(x)+1)\sqrt{-\sin(x)+1}}{\cos(x)-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(cos(x) + sin(x) + 1)*sqrt(-sin(x) + 1)/(cos(x) - sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{1-\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))**(1/2),x)
```

```
[Out] Integral(sqrt(1 - sin(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\sin(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sin(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-sin(x) + 1), x)
```

3.237 $\int \sqrt{1+\cos(x)}\,dx$

Optimal. Leaf size=12

$$\frac{2\sin(x)}{\sqrt{\cos(x)+1}}$$

```
[Out] (2*Sin[x])/Sqrt[1 + Cos[x]]
```

Rubi [A] time = 0.007023, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2646}

$$\frac{2\sin(x)}{\sqrt{\cos(x)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Cos[x]],x]
```

```
[Out] (2*Sin[x])/Sqrt[1 + Cos[x]]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{1+\cos(x)}\,dx = \frac{2\sin(x)}{\sqrt{1+\cos(x)}}$$

Mathematica [A] time = 0.0062206, size = 16, normalized size = 1.33

$$2\sqrt{\cos(x)+1}\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Cos[x]],x]
```

```
[Out] 2*Sqrt[1 + Cos[x]]*Tan[x/2]
```

Maple [B] time = 0.027, size = 22, normalized size = 1.8

$$2\,\frac{\cos\left(x/2\right)\sin\left(x/2\right)\sqrt{2}}{\sqrt{\left(\cos\left(x/2\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+1)^(1/2),x)
```

```
[Out] 2*cos(1/2*x)*sin(1/2*x)*2^(1/2)/(cos(1/2*x)^2)^(1/2)
```

Maxima [A] time = 1.589, size = 12, normalized size = 1.

$$2\,\sqrt{2}\sin\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*sin(1/2*x)
```

Fricas [A] time = 1.49828, size = 36, normalized size = 3.

$$\frac{2\,\sin\left(x\right)}{\sqrt{\cos\left(x\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sin(x)/sqrt(cos(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cos(x) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))**(1/2),x)
```

```
[Out] Integral(sqrt(cos(x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\cos(x) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(cos(x) + 1), x)
```

3.238 $\int \sqrt{1-\cos(x)}\,dx$

Optimal. Leaf size=14

$$-\frac{2\sin(x)}{\sqrt{1-\cos(x)}}$$

```
[Out] (-2*Sin[x])/Sqrt[1 - Cos[x]]
```

Rubi [A] time = 0.0086146, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2646}

$$-\frac{2\sin(x)}{\sqrt{1-\cos(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Cos[x]],x]
```

```
[Out] (-2*Sin[x])/Sqrt[1 - Cos[x]]
```

Rule 2646

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(-2*b*Cos
[c + d*x])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{1-\cos(x)}\,dx = -\frac{2\sin(x)}{\sqrt{1-\cos(x)}}$$

Mathematica [A] time = 0.0077716, size = 18, normalized size = 1.29

$$-2\sqrt{1-\cos(x)}\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Cos[x]],x]
```

```
[Out] -2*Sqrt[1 - Cos[x]]*Cot[x/2]
```

Maple [A] time = 0.03, size = 22, normalized size = 1.6

$$-2\,{\frac {\sin \left(x/2 \right) \cos \left(x/2 \right) \sqrt {2}}{\sqrt { \left(\sin \left(x/2 \right) \right) ^{2}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-cos(x))^(1/2),x)
```

```
[Out] -2*sin(1/2*x)*cos(1/2*x)*2^(1/2)/(sin(1/2*x)^2)^(1/2)
```

Maxima [A] time = 1.46405, size = 27, normalized size = 1.93

$$-\frac{2\,\sqrt{2}}{\sqrt{\frac{\sin\left(x\right)^{2}}{{\left(\cos\left(x\right) + 1\right)}^{2}} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(2)/sqrt(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 1.62238, size = 57, normalized size = 4.07

$$-\frac{2 \,{\left(\cos\left(x\right) + 1\right)} \sqrt{-\cos\left(x\right) + 1}}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(cos(x) + 1)*sqrt(-cos(x) + 1)/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{1-\cos(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))**(1/2),x)
```

```
[Out] Integral(sqrt(1 - cos(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\cos(x)+1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-cos(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-cos(x) + 1), x)
```

3.239 $\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}}\,dx$

Optimal. Leaf size=21

$$\frac{2x^{3/2}}{3}+\frac{2}{3}(x-1)^{3/2}$$

```
[Out] (2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Rubi [A] time = 0.0077088, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2106, 30, 32}

$$\frac{2x^{3/2}}{3}+\frac{2}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(-Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

```
[Out] (2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3
```

Rule 2106

```
Int[(u_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symb
ol] :> -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^
(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d
^2, 0]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{-\sqrt{-1+x}+\sqrt{x}}\,dx = \int \sqrt{-1+x}\,dx + \int \sqrt{x}\,dx$$
$$= \frac{2}{3}(-1+x)^{3/2} + \frac{2x^{3/2}}{3}$$

Mathematica [A] time = 0.0209015, size = 17, normalized size = 0.81

$$\frac{2}{3}\left(x^{3/2}+(x-1)^{3/2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-Sqrt[-1 + x] + Sqrt[x])^(-1),x]
```

```
[Out] (2*((-1 + x)^(3/2) + x^(3/2)))/3
```

Maple [A] time = 0.003, size = 14, normalized size = 0.7

$$\frac{2}{3}(-1+x)^{\frac{3}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(-1+x)^(1/2)+x^(1/2)),x)
```

```
[Out] 2/3*(-1+x)^(3/2)+2/3*x^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{x-1}-\sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-1+x)^(1/2)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] -integrate(1/(sqrt(x - 1) - sqrt(x)), x)
```

Fricas [A] time = 1.62778, size = 45, normalized size = 2.14

$$\frac{2}{3}(x-1)^{\frac{3}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-1+x)^(1/2)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

Sympy [B] time = 0.386084, size = 63, normalized size = 3.

$$\frac{2\sqrt{x}\sqrt{x-1}}{-3\sqrt{x}+3\sqrt{x-1}}-\frac{4x}{-3\sqrt{x}+3\sqrt{x-1}}+\frac{2}{-3\sqrt{x}+3\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-1+x)**(1/2)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x - 1)/(-3*sqrt(x) + 3*sqrt(x - 1)) - 4*x/(-3*sqrt(x) + 3*sq
rt(x - 1)) + 2/(-3*sqrt(x) + 3*sqrt(x - 1))
```

Giac [A] time = 1.07475, size = 18, normalized size = 0.86

$$\frac{2}{3}(x-1)^{\frac{3}{2}}+\frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-(-1+x)^(1/2)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*(x - 1)^(3/2) + 2/3*x^(3/2)
```

3.240 $\int \frac{1}{1-\sqrt{1+x}}\,dx$

Optimal. Leaf size=24

$$-2\sqrt{x+1}-2\log\left(1-\sqrt{x+1}\right)$$

```
[Out] -2*Sqrt[1 + x] - 2*Log[1 - Sqrt[1 + x]]
```

Rubi [A] time = 0.010346, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {247, 190, 43}

$$-2\sqrt{x+1}-2\log\left(1-\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sqrt[1 + x])^(-1),x]
```

```
[Out] -2*Sqrt[1 + x] - 2*Log[1 - Sqrt[1 + x]]
```

Rule 247

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1
], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && Line
arQ[v, x] && NeQ[v, x]
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\sqrt{1+x}}\,dx &= \text{Subst}\left(\int \frac{1}{1-\sqrt{x}}\,dx, x, 1+x\right)\\ &= 2\,\text{Subst}\left(\int \frac{x}{1-x}\,dx, x, \sqrt{1+x}\right)\\ &= 2\,\text{Subst}\left(\int \left(-1+\frac{1}{1-x}\right)dx, x, \sqrt{1+x}\right)\\ &= -2\sqrt{1+x}-2\log\left(1-\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A] time = 0.0100692, size = 24, normalized size = 1.

$$-2\sqrt{x+1}-2\log\left(1-\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sqrt[1 + x])^(-1),x]
```

```
[Out] -2*Sqrt[1 + x] - 2*Log[1 - Sqrt[1 + x]]
```

Maple [A] time = 0.005, size = 31, normalized size = 1.3

$$-\ln(x)-2\sqrt{1+x}-\ln\left(-1+\sqrt{1+x}\right)+\ln\left(1+\sqrt{1+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-(1+x)^(1/2)),x)
```

```
[Out] -ln(x)-2*(1+x)^(1/2)-ln(-1+(1+x)^(1/2))+ln(1+(1+x)^(1/2))
```

Maxima [A] time = 0.93336, size = 24, normalized size = 1.

$$-2\sqrt{x+1}-2\log\left(\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^(1/2)),x, algorithm="maxima")
```

```
[Out] -2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)
```

Fricas [A] time = 1.65158, size = 55, normalized size = 2.29

$$-2\sqrt{x+1} - 2\log\left(\sqrt{x+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^(1/2)),x, algorithm="fricas")
```

```
[Out] -2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)
```

Sympy [A] time = 0.118422, size = 20, normalized size = 0.83

$$-2\sqrt{x+1} - 2\log\left(\sqrt{x+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)**(1/2)),x)
```

```
[Out] -2*sqrt(x + 1) - 2*log(sqrt(x + 1) - 1)
```

Giac [A] time = 1.12016, size = 26, normalized size = 1.08

$$-2\sqrt{x+1} - 2\log\left(\left|\sqrt{x+1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-(1+x)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*sqrt(x + 1) - 2*log(abs(sqrt(x + 1) - 1))
```

3.241 $\int \frac{x}{\sqrt{36+x^4}}\,dx$

Optimal. Leaf size=12

$$\frac{1}{2}\sinh^{-1}\left(\frac{x^2}{6}\right)$$

```
[Out] ArcSinh[x^2/6]/2
```

Rubi [A] time = 0.003783, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {275, 215}

$$\frac{1}{2}\sinh^{-1}\left(\frac{x^2}{6}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[36 + x^4],x]
```

```
[Out] ArcSinh[x^2/6]/2
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{36+x^4}}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{36+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}\sinh^{-1}\left(\frac{x^2}{6}\right)\end{aligned}$$

Mathematica [A] time = 0.0020782, size = 12, normalized size = 1.

$$\frac{1}{2}\sinh^{-1}\left(\frac{x^2}{6}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[36 + x^4],x]
```

```
[Out] ArcSinh[x^2/6]/2
```

Maple [A] time = 0.009, size = 9, normalized size = 0.8

$$\frac{1}{2}\mathrm{Arcsinh}\left(\frac{x^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4+36)^(1/2),x)
```

```
[Out] 1/2*arcsinh(1/6*x^2)
```

Maxima [B] time = 0.943175, size = 45, normalized size = 3.75

$$\frac{1}{4}\log\left(\frac{\sqrt{x^4+36}}{x^2}+1\right)-\frac{1}{4}\log\left(\frac{\sqrt{x^4+36}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+36)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*log(sqrt(x^4 + 36)/x^2 + 1) - 1/4*log(sqrt(x^4 + 36)/x^2 - 1)
```

Fricas [A] time = 1.55247, size = 45, normalized size = 3.75

$$-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+36)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(-x^2 + sqrt(x^4 + 36))
```

Sympy [A] time = 0.922305, size = 7, normalized size = 0.58

$$\frac{\operatorname{asinh}\left(\frac{x^2}{6}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4+36)**(1/2),x)
```

```
[Out] asinh(x**2/6)/2
```

Giac [A] time = 1.08182, size = 22, normalized size = 1.83

$$-\frac{1}{2}\log\left(-x^2+\sqrt{x^4+36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4+36)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(-x^2 + sqrt(x^4 + 36))
```

3.242 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx$

Optimal. Leaf size=32

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rubi [A] time = 0.0122561, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1593, 266, 43}

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt[3]{x}+\sqrt{x}}\,dx &= \int \frac{1}{\left(1+\sqrt[6]{x}\right)\sqrt[3]{x}}\,dx\\ &= 6\,\mathrm{Subst}\left(\int \frac{x^3}{1+x}\,dx, x, \sqrt[6]{x}\right)\\ &= 6\,\mathrm{Subst}\left(\int \left(1+\frac{1}{-1-x}-x+x^2\right)dx, x, \sqrt[6]{x}\right)\\ &= 6\sqrt[6]{x}-3\sqrt[3]{x}+2\sqrt{x}-6\log\left(1+\sqrt[6]{x}\right)\end{aligned}$$

Mathematica [A] time = 0.0112853, size = 32, normalized size = 1.

$$2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\log\left(\sqrt[6]{x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^(1/3) + Sqrt[x])^(-1),x]
```

```
[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]
```

Maple [B] time = 0.026, size = 92, normalized size = 2.9

$$2\,\ln\left(-1+\sqrt[6]{x}\right)-\ln\left(\sqrt[3]{x}+\sqrt[6]{x}+1\right)-2\,\ln\left(1+\sqrt[6]{x}\right)+\ln\left(\sqrt[3]{x}-\sqrt[6]{x}+1\right)+2\,\sqrt{x}+\ln\left(\sqrt{x}-1\right)-\ln\left(\sqrt{x}+1\right)+6\,\sqrt[6]{x}-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^(1/3)+x^(1/2)),x)
```

```
[Out] 2*ln(-1+x^(1/6))-ln(x^(1/3)+x^(1/6)+1)-2*ln(1+x^(1/6))+ln(x^(1/3)-x^(1/6)+1
)+2*x^(1/2)+ln(x^(1/2)-1)-ln(x^(1/2)+1)+6*x^(1/6)-ln(-1+x)-2*ln(-1+x^(1/3))
+ln(x^(2/3)+x^(1/3)+1)-3*x^(1/3)
```

Maxima [A] time = 0.943711, size = 32, normalized size = 1.

$$2\,\sqrt{x}-3\,x^{\frac{1}{3}}+6\,x^{\frac{1}{6}}-6\,\log\left(x^{\frac{1}{6}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Fricas [A] time = 1.54806, size = 76, normalized size = 2.38

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/3)+x**(1/2)),x)
```

```
[Out] Integral(1/(x**(1/3) + sqrt(x)), x)
```

Giac [A] time = 1.08606, size = 32, normalized size = 1.

$$2\sqrt{x} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^(1/3)+x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*sqrt(x) - 3*x^(1/3) + 6*x^(1/6) - 6*log(x^(1/6) + 1)
```

3.243 $\int \log\left(2+3x^2\right)\,dx$

Optimal. Leaf size=33

$$x\log\left(3x^2+2\right)-2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

```
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Rubi [A] time = 0.0094059, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2448, 321, 203}

$$x\log\left(3x^2+2\right)-2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[2 + 3*x^2],x]
```

```
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \log\left(2+3x^2\right)\,dx &= x\log\left(2+3x^2\right)-6\int\frac{x^2}{2+3x^2}\,dx\\
&= -2x+x\log\left(2+3x^2\right)+4\int\frac{1}{2+3x^2}\,dx\\
&= -2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)+x\log\left(2+3x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0115271, size = 33, normalized size = 1.

$$x\log\left(3x^2+2\right)-2x+2\sqrt{\frac{2}{3}}\tan^{-1}\left(\sqrt{\frac{3}{2}}x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[2 + 3*x^2],x]
```

```
[Out] -2*x + 2*Sqrt[2/3]*ArcTan[Sqrt[3/2]*x] + x*Log[2 + 3*x^2]
```

Maple [A] time = 0.005, size = 27, normalized size = 0.8

$$-2\,x+x\ln\left(3\,x^2+2\right)+\frac{2\,\sqrt{6}}{3}\arctan\left(\frac{x\sqrt{6}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(3*x^2+2),x)
```

```
[Out] -2*x+x*ln(3*x^2+2)+2/3*arctan(1/2*x*6^(1/2))*6^(1/2)
```

Maxima [A] time = 1.43805, size = 35, normalized size = 1.06

$$x \log \left(3 x^2+2\right)+\frac{2}{3} \sqrt{6} \arctan \left(\frac{1}{2} \sqrt{6} x\right)-2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="maxima")
```

```
[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

Fricas [A] time = 1.59939, size = 103, normalized size = 3.12

$$\frac{2}{3} \sqrt{3} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{3} \sqrt{2} x\right)+x \log \left(3 x^2+2\right)-2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(3)*sqrt(2)*arctan(1/2*sqrt(3)*sqrt(2)*x) + x*log(3*x^2 + 2) - 2*x
```

Sympy [A] time = 0.127987, size = 31, normalized size = 0.94

$$x \log \left(3x^2+2\right)-2x+\frac{2 \sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6} x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(3*x**2+2),x)
```

```
[Out] x*log(3*x**2 + 2) - 2*x + 2*sqrt(6)*atan(sqrt(6)*x/2)/3
```

Giac [A] time = 1.07727, size = 35, normalized size = 1.06

$$x \log \left(3 x^2+2\right)+\frac{2}{3} \sqrt{6} \arctan \left(\frac{1}{2} \sqrt{6} x\right)-2 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(3*x^2+2),x, algorithm="giac")
```

```
[Out] x*log(3*x^2 + 2) + 2/3*sqrt(6)*arctan(1/2*sqrt(6)*x) - 2*x
```

3.244 $\int \cot(x)\,dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

```
[Out] Log[Sin[x]]
```

Rubi [A] time = 0.0024754, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3475}

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(x)\,dx = \log(\sin(x))$$

Mathematica [A] time = 0.0019405, size = 3, normalized size = 1.

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Maple [A] time = 0., size = 4, normalized size = 1.3

$$\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x),x)
```

```
[Out] ln(sin(x))
```

Maxima [A] time = 0.959389, size = 4, normalized size = 1.33

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="maxima")
```

```
[Out] log(sin(x))
```

Fricas [B] time = 1.6188, size = 41, normalized size = 13.67

$$\frac{1}{2}\log\left(-\frac{1}{2}\cos(2x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(-1/2*cos(2*x) + 1/2)
```

Sympy [A] time = 0.059446, size = 3, normalized size = 1.

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x)
```

```
[Out] log(sin(x))
```

Giac [B] time = 1.11588, size = 15, normalized size = 5.

$$\frac{1}{2}\log\left(-\cos\left(x\right)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="giac")
```

```
[Out] 1/2*log(-cos(x)^2 + 1)
```

3.245 $\int \cot^4(x)\,dx$

Optimal. Leaf size=12

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

```
[Out] x + Cot[x] - Cot[x]^3/3
```

Rubi [A] time = 0.0095483, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 8}

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4,x]
```

```
[Out] x + Cot[x] - Cot[x]^3/3
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cot^4(x)\,dx &= -\frac{1}{3}\cot^3(x)-\int \cot^2(x)\,dx\\ &= \cot(x)-\frac{\cot^3(x)}{3}+\int 1\,dx\\ &= x+\cot(x)-\frac{\cot^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.0029604, size = 18, normalized size = 1.5

$$x+\frac{4\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4,x]
```

```
[Out] x + (4*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

Maple [A] time = 0., size = 14, normalized size = 1.2

$$-\frac{(\cot(x))^3}{3}+\cot(x)-\frac{\pi}{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4,x)
```

```
[Out] -1/3*cot(x)^3+cot(x)-1/2*Pi+x
```

Maxima [A] time = 1.43083, size = 22, normalized size = 1.83

$$x+\frac{3\tan(x)^2-1}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="maxima")
```

```
[Out] x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3
```

Fricas [B] time = 1.63421, size = 126, normalized size = 10.5

$$\frac{4\cos(2x)^2+3(x\cos(2x)-x)\sin(2x)+2\cos(2x)-2}{3(\cos(2x)-1)\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)/((cos(2*x
) - 1)*sin(2*x))
```

Sympy [A] time = 0.065766, size = 19, normalized size = 1.58

$$x+\frac{\cos(x)}{\sin(x)}-\frac{\cos^3(x)}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4,x)
```

```
[Out] x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)
```

Giac [B] time = 1.09733, size = 46, normalized size = 3.83

$$\frac{1}{24}\tan\left(\frac{1}{2}x\right)^3+x+\frac{15\tan\left(\frac{1}{2}x\right)^2-1}{24\tan\left(\frac{1}{2}x\right)^3}-\frac{5}{8}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="giac")
```

```
[Out] 1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 5/8*tan(1
/2*x)
```

3.246 $\int \tanh(x)\,dx$

Optimal. Leaf size=3

$$\log(\cosh(x))$$

```
[Out] Log[Cosh[x]]
```

Rubi [A] time = 0.0032031, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3475}

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x],x]
```

```
[Out] Log[Cosh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \tanh(x)\,dx = \log(\cosh(x))$$

Mathematica [A] time = 0.0018261, size = 3, normalized size = 1.

$$\log(\cosh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x],x]
```

```
[Out] Log[Cosh[x]]
```

Maple [A] time = 0.001, size = 4, normalized size = 1.3

$$\ln(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x),x)
```

```
[Out] ln(cosh(x))
```

Maxima [A] time = 0.930621, size = 4, normalized size = 1.33

$$\log(\cosh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="maxima")
```

```
[Out] log(cosh(x))
```

Fricas [B] time = 1.53863, size = 55, normalized size = 18.33

$$-x + \log\left(\frac{2\cosh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="fricas")
```

```
[Out] -x + log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [B] time = 0.12288, size = 7, normalized size = 2.33

$$x - \log(\tanh(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x)
```

```
[Out] x - log(tanh(x) + 1)
```

Giac [B] time = 1.0941, size = 15, normalized size = 5.

$$-x + \log\left(e^{(2x)} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x),x, algorithm="giac")
```

```
[Out] -x + log(e^(2*x) + 1)
```

3.247 $\int \coth(x)\,dx$

Optimal. Leaf size=3

$$\log(\sinh(x))$$

```
[Out] Log[Sinh[x]]
```

Rubi [A] time = 0.0030312, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3475}

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x],x]
```

```
[Out] Log[Sinh[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \coth(x)\,dx = \log(\sinh(x))$$

Mathematica [A] time = 0.001822, size = 3, normalized size = 1.

$$\log(\sinh(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x],x]
```

```
[Out] Log[Sinh[x]]
```

Maple [A] time = 0.001, size = 4, normalized size = 1.3

$$\ln(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x),x)
```

```
[Out] ln(sinh(x))
```

Maxima [A] time = 0.95489, size = 4, normalized size = 1.33

$$\log(\sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x, algorithm="maxima")
```

```
[Out] log(sinh(x))
```

Fricas [B] time = 1.53577, size = 55, normalized size = 18.33

$$-x + \log\left(\frac{2\sinh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x, algorithm="fricas")
```

```
[Out] -x + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [B] time = 0.293818, size = 12, normalized size = 4.

$$x - \log(\tanh(x) + 1) + \log(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x)
```

```
[Out] x - log(tanh(x) + 1) + log(tanh(x))
```

Giac [B] time = 1.07867, size = 16, normalized size = 5.33

$$-x + \log\left(\left|e^{(2x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x),x, algorithm="giac")
```

```
[Out] -x + log(abs(e^(2*x) - 1))
```

3.248 $\int b^x \, dx$

Optimal. Leaf size=8

$$\frac{b^x}{\log(b)}$$

```
[Out] b^x/Log[b]
```

Rubi [A] time = 0.0020969, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2194}

$$\frac{b^x}{\log(b)}$$

Antiderivative was successfully verified.

```
[In] Int[b^x,x]
```

```
[Out] b^x/Log[b]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int b^x \, dx = \frac{b^x}{\log(b)}$$

Mathematica [A] time = 0.0005588, size = 8, normalized size = 1.

$$\frac{b^x}{\log(b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[b^x,x]
```

```
[Out] b^x/Log[b]
```

Maple [A] time = 0.003, size = 9, normalized size = 1.1

$$\frac{b^x}{\ln(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(b^x,x)
```

```
[Out] b^x/ln(b)
```

Maxima [A] time = 0.940985, size = 11, normalized size = 1.38

$$\frac{b^x}{\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^x,x, algorithm="maxima")
```

```
[Out] b^x/log(b)
```

Fricas [A] time = 1.62097, size = 16, normalized size = 2.

$$\frac{b^x}{\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^x,x, algorithm="fricas")
```

```
[Out] b^x/log(b)
```

Sympy [A] time = 0.082876, size = 8, normalized size = 1.

$$\begin{cases} \frac{b^x}{\log(b)} & \text{for } \log(b) \neq 0 \\ x & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b**x,x)
```

```
[Out] Piecewise((b**x/log(b), Ne(log(b), 0)), (x, True))
```

Giac [A] time = 1.08596, size = 11, normalized size = 1.38

$$\frac{b^x}{\log(b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(b^x,x, algorithm="giac")
```

```
[Out] b^x/log(b)
```

3.249 $\int \sqrt{2+\frac{1}{x^4}+x^4}\,dx$

Optimal. Leaf size=49

$$\frac{x^5\sqrt{x^4+\frac{1}{x^4}+2}}{3\left(x^4+1\right)}-\frac{x\sqrt{x^4+\frac{1}{x^4}+2}}{x^4+1}$$

```
[Out] -((x*Sqrt[2 + x^(-4) + x^4])/(1 + x^4)) + (x^5*Sqrt[2 + x^(-4) + x^4])/(3*(
1 + x^4))
```

Rubi [A] time = 0.0148724, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {1351, 1355, 14}

$$\frac{x^5\sqrt{x^4+\frac{1}{x^4}+2}}{3\left(x^4+1\right)}-\frac{x\sqrt{x^4+\frac{1}{x^4}+2}}{x^4+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + x^(-4) + x^4],x]
```


```
[Out] -((x*Sqrt[2 + x^(-4) + x^4])/(1 + x^4)) + (x^5*Sqrt[2 + x^(-4) + x^4])/(3*(
1 + x^4))
```


Rule 1351

```
Int[((a_) + (c_.)*(x_)^(n_.) + (b_.)*(x_)^(mn_))^(p_), x_Symbol] :> Dist[(x
^(n*FracPart[p])*(a + b/x^n + c*x^n)^FracPart[p])/(b + a*x^n + c*x^(2*n))^F
racPart[p], Int[(b + a*x^n + c*x^(2*n))^p/x^(n*p), x], x] /; FreeQ[{a, b, c
, n, p}, x] && EqQ[mn, -n] &&  !IntegerQ[p] && PosQ[n]
```

Rule 1355

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.))^(p_),
x_Symbol] :> Dist[(a + b*x^n + c*x^(2*n))^FracPart[p]/(c^IntPart[p]*(b/2 +
c*x^n)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{
a, b, c, d, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ
[p - 1/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{2+\frac{1}{x^4}+x^4}\,dx &= \frac{\left(x^2\sqrt{2+\frac{1}{x^4}+x^4}\right)\int \frac{\sqrt{1+2x^4+x^8}}{x^2}\,dx}{\sqrt{1+2x^4+x^8}} \\
&= \frac{\left(x^2\sqrt{2+\frac{1}{x^4}+x^4}\right)\int \frac{1+x^4}{x^2}\,dx}{1+x^4} \\
&= \frac{\left(x^2\sqrt{2+\frac{1}{x^4}+x^4}\right)\int \left(\frac{1}{x^2}+x^2\right)dx}{1+x^4} \\
&= -\frac{x\sqrt{2+\frac{1}{x^4}+x^4}}{1+x^4}+\frac{x^5\sqrt{2+\frac{1}{x^4}+x^4}}{3\left(1+x^4\right)}
\end{aligned}$$

Mathematica [A] time = 0.0088593, size = 29, normalized size = 0.59

$$\frac{x\left(x^4-3\right)\sqrt{x^4+\frac{1}{x^4}+2}}{3\left(x^4+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 + x^(-4) + x^4],x]
```

```
[Out] (x*(-3 + x^4)*Sqrt[2 + x^(-4) + x^4])/(3*(1 + x^4))
```

Maple [A] time = 0.004, size = 32, normalized size = 0.7

$$\frac{\left(x^4-3\right)x}{3\,x^4+3}\sqrt{\frac{x^8+2\,x^4+1}{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+1/x^4+x^4)^(1/2),x)
```

```
[Out] 1/3*x*(x^4-3)*((x^8+2*x^4+1)/x^4)^(1/2)/(x^4+1)
```

Maxima [A] time = 1.4332, size = 14, normalized size = 0.29

$$\frac{x^4-3}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+1/x^4+x^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 - 3)/x
```

Fricas [A] time = 1.4382, size = 23, normalized size = 0.47

$$\frac{x^4-3}{3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+1/x^4+x^4)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(x^4 - 3)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^4+2+\frac{1}{x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+1/x**4+x**4)**(1/2),x)
```

```
[Out] Integral(sqrt(x**4 + 2 + x**(-4)), x)
```

Giac [A] time = 1.09628, size = 15, normalized size = 0.31

$$\frac{1}{3}x^3 - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+1/x^4+x^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*x^3 - 1/x
```

3.250 $\int \frac{1+2x}{2+3x} \, dx$

Optimal. Leaf size=16

$$\frac{2x}{3} - \frac{1}{9}\log(3x+2)$$

```
[Out] (2*x)/3 - Log[2 + 3*x]/9
```

Rubi [A] time = 0.0063008, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {43}

$$\frac{2x}{3} - \frac{1}{9}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x)/(2 + 3*x),x]
```

```
[Out] (2*x)/3 - Log[2 + 3*x]/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1+2x}{2+3x} \, dx &= \int \left(\frac{2}{3} - \frac{1}{3(2+3x)}\right) dx \\ &= \frac{2x}{3} - \frac{1}{9}\log(2+3x) \end{aligned}$$

Mathematica [A] time = 0.0027664, size = 17, normalized size = 1.06

$$\frac{1}{9}(6x - \log(3x+2) + 4)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x)/(2 + 3*x),x]
```

```
[Out] (4 + 6*x - Log[2 + 3*x])/9
```

Maple [A] time = 0.001, size = 13, normalized size = 0.8

$$\frac{2x}{3} - \frac{\ln(2+3x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)/(2+3*x),x)
```

```
[Out] 2/3*x-1/9*ln(2+3*x)
```

Maxima [A] time = 0.947144, size = 16, normalized size = 1.

$$\frac{2}{3}x - \frac{1}{9}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(2+3*x),x, algorithm="maxima")
```

```
[Out] 2/3*x - 1/9*log(3*x + 2)
```

Fricas [A] time = 1.51052, size = 35, normalized size = 2.19

$$\frac{2}{3}x - \frac{1}{9}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(2+3*x),x, algorithm="fricas")
```

```
[Out] 2/3*x - 1/9*log(3*x + 2)
```

Sympy [A] time = 0.069165, size = 12, normalized size = 0.75

$$\frac{2x}{3} - \frac{\log(3x+2)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(2+3*x),x)
```

```
[Out] 2*x/3 - log(3*x + 2)/9
```

Giac [A] time = 1.07865, size = 18, normalized size = 1.12

$$\frac{2}{3}\,x - \frac{1}{9}\,\log(|3\,x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)/(2+3*x),x, algorithm="giac")
```

```
[Out] 2/3*x - 1/9*log(abs(3*x + 2))
```

3.251 $\int x \log\left(x+\sqrt{1+x^2}\right) dx$

Optimal. Leaf size=40

$$-\frac{1}{4}\sqrt{x^2+1}x+\frac{1}{2}x^2\log\left(\sqrt{x^2+1}+x\right)+\frac{1}{4}\sinh^{-1}(x)$$

```
[Out] -(x*Sqrt[1 + x^2])/4 + ArcSinh[x]/4 + (x^2*Log[x + Sqrt[1 + x^2]])/2
```

Rubi [A] time = 0.0157841, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2536, 321, 215}

$$-\frac{1}{4}\sqrt{x^2+1}x+\frac{1}{2}x^2\log\left(\sqrt{x^2+1}+x\right)+\frac{1}{4}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x + Sqrt[1 + x^2]],x]
```

```
[Out] -(x*Sqrt[1 + x^2])/4 + ArcSinh[x]/4 + (x^2*Log[x + Sqrt[1 + x^2]])/2
```

Rule 2536

```
Int[Log[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (c_.)*(x_)^2]]*((g_.)*(x_))
^(m_.), x_Symbol] :> Simp[((g*x)^(m + 1)*Log[d + e*x + f*Sqrt[a + c*x^2]])/
(g*(m + 1)), x] - Dist[(a*c*f^2)/(g*(m + 1)), Int[(g*x)^(m + 1)/(d*e*(a + c
*x^2) + f*(a*e - c*d*x)*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g
, m}, x] && EqQ[e^2 - c*f^2, 0] && NeQ[m, -1] && IntegerQ[2*m]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int x\log\left(x+\sqrt{1+x^2}\right)dx &= \frac{1}{2}x^2\log\left(x+\sqrt{1+x^2}\right)-\frac{1}{2}\int\frac{x^2}{\sqrt{1+x^2}}\,dx\\ &= -\frac{1}{4}x\sqrt{1+x^2}+\frac{1}{2}x^2\log\left(x+\sqrt{1+x^2}\right)+\frac{1}{4}\int\frac{1}{\sqrt{1+x^2}}\,dx\\ &= -\frac{1}{4}x\sqrt{1+x^2}+\frac{1}{4}\sinh^{-1}(x)+\frac{1}{2}x^2\log\left(x+\sqrt{1+x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.0092454, size = 36, normalized size = 0.9

$$\frac{1}{4}\left(-\sqrt{x^2+1}x+2x^2\log\left(\sqrt{x^2+1}+x\right)+\sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x + Sqrt[1 + x^2]],x]
```

```
[Out] (-(x*Sqrt[1 + x^2]) + ArcSinh[x] + 2*x^2*Log[x + Sqrt[1 + x^2]])/4
```

Maple [F] time = 0.003, size = 0, normalized size = 0.

$$\int x\ln\left(x+\sqrt{x^2+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x+(x^2+1)^(1/2)),x)
```

```
[Out] int(x*ln(x+(x^2+1)^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}x^2\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}x^2-\int\frac{x^2}{2\left(x^3+\left(x^2+1\right)^{\frac{3}{2}}+x\right)}\,dx+\frac{1}{4}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(x + sqrt(x^2 + 1)) - 1/4*x^2 - integrate(1/2*x^2/(x^3 + (x^2 +
1)^(3/2) + x), x) + 1/4*log(x^2 + 1)
```

Fricas [A] time = 1.63882, size = 84, normalized size = 2.1

$$\frac{1}{4}\left(2x^2+1\right)\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}\sqrt{x^2+1}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*x^2 + 1)*log(x + sqrt(x^2 + 1)) - 1/4*sqrt(x^2 + 1)*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x\log\left(x+\sqrt{x^2+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x+(x**2+1)**(1/2)),x)
```

```
[Out] Integral(x*log(x + sqrt(x**2 + 1)), x)
```

Giac [A] time = 1.1079, size = 54, normalized size = 1.35

$$\frac{1}{2}x^2\log\left(x+\sqrt{x^2+1}\right)-\frac{1}{4}\sqrt{x^2+1}x-\frac{1}{4}\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x+(x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x + sqrt(x^2 + 1)) - 1/4*sqrt(x^2 + 1)*x - 1/4*log(-x + sqrt(x^
2 + 1))
```

3.252 $\int x\left(1+e^x\sin(x)\right)^2\,dx$

Optimal. Leaf size=128

$$\frac{x^2}{2}+\frac{1}{8}e^{2x}x-\frac{3e^{2x}}{32}+\frac{1}{4}e^{2x}x\sin^2(x)-\frac{1}{16}e^{2x}\sin^2(x)+e^xx\sin(x)+\frac{1}{32}e^{2x}\sin(2x)-e^xx\cos(x)+e^x\cos(x)-\frac{1}{32}e^{2x}\cos(2x$$

```
[Out] (-3*E^(2*x))/32 + (E^(2*x)*x)/8 + x^2/2 + E^x*Cos[x] - E^x*x*Cos[x] - (E^(2
*x)*Cos[2*x])/32 + E^x*x*Sin[x] + (E^(2*x)*Cos[x]*Sin[x])/16 - (E^(2*x)*x*C
os[x]*Sin[x])/4 - (E^(2*x)*Sin[x]^2)/16 + (E^(2*x)*x*Sin[x]^2)/4 + (E^(2*x)
*Sin[2*x])/32
```

Rubi [A] time = 0.189774, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6742, 4432, 4465, 4433, 4434, 2194, 4469, 12}

$$\frac{x^2}{2}+\frac{1}{8}e^{2x}x-\frac{3e^{2x}}{32}+\frac{1}{4}e^{2x}x\sin^2(x)-\frac{1}{16}e^{2x}\sin^2(x)+e^xx\sin(x)+\frac{1}{32}e^{2x}\sin(2x)-e^xx\cos(x)+e^x\cos(x)-\frac{1}{32}e^{2x}\cos(2x$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + E^x*Sin[x])^2,x]
```

```
[Out] (-3*E^(2*x))/32 + (E^(2*x)*x)/8 + x^2/2 + E^x*Cos[x] - E^x*x*Cos[x] - (E^(2
*x)*Cos[2*x])/32 + E^x*x*Sin[x] + (E^(2*x)*Cos[x]*Sin[x])/16 - (E^(2*x)*x*C
os[x]*Sin[x])/4 - (E^(2*x)*Sin[x]^2)/16 + (E^(2*x)*x*Sin[x]^2)/4 + (E^(2*x)
*Sin[2*x])/32
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4465

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*(x_))^(m_.)*Sin[(d_.) + (e_.)*
(x_)]^(n_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Sin[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4434

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x]^n)/(e^2*n^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[(e*n*F^(c*(a + b*x))*Cos[d
+ e*x]*Sin[d + e*x]^(n - 1))/(e^2*n^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 4469

```
Int[Cos[(f_.) + (g_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_
.) + (e_.)*(x_)]^(m_.), x_Symbol] :> Int[ExpandTrigReduce[F^(c*(a + b*x)),
Sin[d + e*x]^m*Cos[f + g*x]^n, x], x] /; FreeQ[{F, a, b, c, d, e, f, g}, x]
 && IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int x(1+e^x\sin(x))^2\,dx = \int \left(x+2e^x x\sin(x)+e^{2x}x\sin^2(x)\right)dx$$
$$= \frac{x^2}{2}+2\int e^x x\sin(x)\,dx+\int e^{2x}x\sin^2(x)\,dx$$
$$= \frac{1}{8}e^{2x}x+\frac{x^2}{2}-e^x x\cos(x)+e^x x\sin(x)-\frac{1}{4}e^{2x}x\cos(x)\sin(x)+\frac{1}{4}e^{2x}x\sin^2(x)-2\int\left(-\frac{1}{2}e^x\cos(\right.$$
$$= \frac{1}{8}e^{2x}x+\frac{x^2}{2}-e^x x\cos(x)+e^x x\sin(x)-\frac{1}{4}e^{2x}x\cos(x)\sin(x)+\frac{1}{4}e^{2x}x\sin^2(x)-\frac{1}{8}\int e^{2x}\,dx+\frac{1}{4}$$
$$= -\frac{e^{2x}}{16}+\frac{1}{8}e^{2x}x+\frac{x^2}{2}+e^x\cos(x)-e^x x\cos(x)+e^x x\sin(x)+\frac{1}{16}e^{2x}\cos(x)\sin(x)-\frac{1}{4}e^{2x}x\cos(x)$$
$$= -\frac{3e^{2x}}{32}+\frac{1}{8}e^{2x}x+\frac{x^2}{2}+e^x\cos(x)-e^x x\cos(x)+e^x x\sin(x)+\frac{1}{16}e^{2x}\cos(x)\sin(x)-\frac{1}{4}e^{2x}x\cos(x$$
$$= -\frac{3e^{2x}}{32}+\frac{1}{8}e^{2x}x+\frac{x^2}{2}+e^x\cos(x)-e^x x\cos(x)-\frac{1}{32}e^{2x}\cos(2x)+e^x x\sin(x)+\frac{1}{16}e^{2x}\cos(x)\sin($$

Mathematica [A] time = 0.164996, size = 67, normalized size = 0.52

$$\frac{1}{8}\left(4x^2+e^{2x}(2x-1)+8e^x x\sin(x)-e^{2x}x\cos(2x)-8e^x(x-1)\cos(x)-e^{2x}(2x-1)\sin(x)\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + E^x*Sin[x])^2,x]
```

```
[Out] (4*x^2 + E^(2*x)*(-1 + 2*x) - 8*E^x*(-1 + x)*Cos[x] - E^(2*x)*x*Cos[2*x] +
8*E^x*x*Sin[x] - E^(2*x)*(-1 + 2*x)*Cos[x]*Sin[x])/8
```

Maple [A] time = 0.009, size = 63, normalized size = 0.5

$$\frac{x^2}{2}+2\,(-x/2+1/2)\,e^x\cos(x)+e^x x\sin(x)+\frac{(e^x)^2 x}{4}-\frac{(e^x)^2}{8}-\frac{xe^{2x}\cos(2x)}{8}+\frac{e^{2x}\sin(2x)}{2}\left(-\frac{x}{4}+\frac{1}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+exp(x)*sin(x))^2,x)
```

```
[Out] 1/2*x^2+2*(-1/2*x+1/2)*exp(x)*cos(x)+exp(x)*x*sin(x)+1/4*exp(x)^2*x-1/8*exp
(x)^2-1/8*x*exp(2*x)*cos(2*x)+1/2*(-1/4*x+1/8)*exp(2*x)*sin(2*x)
```

Maxima [A] time = 0.97353, size = 78, normalized size = 0.61

$$-\frac{1}{8}\,x\cos\left(2\,x\right)e^{\left(2\,x\right)}-\left(x-1\right)\cos\left(x\right)e^{x}-\frac{1}{16}\,\left(2\,x-1\right)e^{\left(2\,x\right)}\sin\left(2\,x\right)+xe^{x}\sin\left(x\right)+\frac{1}{2}\,x^{2}+\frac{1}{8}\,\left(2\,x-1\right)e^{\left(2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+exp(x)*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -1/8*x*cos(2*x)*e^(2*x) - (x - 1)*cos(x)*e^x - 1/16*(2*x - 1)*e^(2*x)*sin(2
*x) + x*e^x*sin(x) + 1/2*x^2 + 1/8*(2*x - 1)*e^(2*x)
```

Fricas [A] time = 1.67328, size = 162, normalized size = 1.27

$$-\left(x-1\right)\cos\left(x\right)e^{x}+\frac{1}{2}\,x^{2}-\frac{1}{8}\,\left(2\,x\cos\left(x\right)^{2}-3\,x+1\right)e^{\left(2\,x\right)}-\frac{1}{8}\,\left(\left(2\,x-1\right)\cos\left(x\right)e^{\left(2\,x\right)}-8\,xe^{x}\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+exp(x)*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -(x - 1)*cos(x)*e^x + 1/2*x^2 - 1/8*(2*x*cos(x)^2 - 3*x + 1)*e^(2*x) - 1/8*
((2*x - 1)*cos(x)*e^(2*x) - 8*x*e^x)*sin(x)
```

Sympy [A] time = 5.38626, size = 109, normalized size = 0.85

$$\frac{x^{2}}{2}+\frac{3xe^{2x}\sin^{2}\left(x\right)}{8}-\frac{xe^{2x}\sin\left(x\right)\cos\left(x\right)}{4}+\frac{xe^{2x}\cos^{2}\left(x\right)}{8}+xe^{x}\sin\left(x\right)-xe^{x}\cos\left(x\right)-\frac{e^{2x}\sin^{2}\left(x\right)}{8}+\frac{e^{2x}\sin\left(x\right)\cos\left(x\right)}{8}-\frac{e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+exp(x)*sin(x))**2,x)
```

```
[Out] x**2/2 + 3*x*exp(2*x)*sin(x)**2/8 - x*exp(2*x)*sin(x)*cos(x)/4 + x*exp(2*x)
*cos(x)**2/8 + x*exp(x)*sin(x) - x*exp(x)*cos(x) - exp(2*x)*sin(x)**2/8 + e
xp(2*x)*sin(x)*cos(x)/8 - exp(2*x)*cos(x)**2/8 + exp(x)*cos(x)
```

Giac [A] time = 1.10757, size = 77, normalized size = 0.6

$$\frac{1}{2}x^2 - \frac{1}{16}(2x\cos(2x) + (2x-1)\sin(2x))e^{(2x)} + \frac{1}{8}(2x-1)e^{(2x)} - ((x-1)\cos(x) - x\sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+exp(x)*sin(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2 - 1/16*(2*x*cos(2*x) + (2*x - 1)*sin(2*x))*e^(2*x) + 1/8*(2*x - 1)*
e^(2*x) - ((x - 1)*cos(x) - x*sin(x))*e^x
```

3.253 $\int e^x x \cos(x)\, dx$

Optimal. Leaf size=30

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

```
[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

Rubi [A] time = 0.0370662, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4433, 4466, 4432}

$$-\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x x \sin(x) + \frac{1}{2}e^x x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x*Cos[x],x]
```

```
[Out] (E^x*x*Cos[x])/2 - (E^x*Sin[x])/2 + (E^x*x*Sin[x])/2
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rule 4466

```
Int[Cos[(d_.) + (e_.)*(x_)]^(n_.)*(F_)^((c_.)*((a_.) + (b_.)*(x_)))*((f_.)*
(x_))^(m_.), x_Symbol] :> Module[{u = IntHide[F^(c*(a + b*x))*Cos[d + e*x]^
n, x]}, Dist[(f*x)^m, u, x] - Dist[f*m, Int[(f*x)^(m - 1)*u, x], x]] /; Fre
eQ[{F, a, b, c, d, e, f}, x] && IGtQ[n, 0] && GtQ[m, 0]
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\begin{aligned}\int e^x x\cos(x)\,dx &= \frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)-\int\left(\frac{1}{2}e^x\cos(x)+\frac{1}{2}e^x\sin(x)\right)dx\\ &= \frac{1}{2}e^x x\cos(x)+\frac{1}{2}e^x x\sin(x)-\frac{1}{2}\int e^x\cos(x)\,dx-\frac{1}{2}\int e^x\sin(x)\,dx\\ &= \frac{1}{2}e^x x\cos(x)-\frac{1}{2}e^x\sin(x)+\frac{1}{2}e^x x\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0257486, size = 18, normalized size = 0.6

$$\frac{1}{2}e^x((x-1)\sin(x)+x\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x*Cos[x],x]
```

```
[Out] (E^x*(x*Cos[x] + (-1 + x)*Sin[x]))/2
```

Maple [A] time = 0.004, size = 20, normalized size = 0.7

$$\frac{e^x x\cos(x)}{2}-\left(-\frac{x}{2}+\frac{1}{2}\right)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x*cos(x),x)
```

```
[Out] 1/2*exp(x)*x*cos(x)-(-1/2*x+1/2)*exp(x)*sin(x)
```

Maxima [A] time = 0.951059, size = 23, normalized size = 0.77

$$\frac{1}{2}x\cos(x)e^x+\frac{1}{2}(x-1)e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="maxima")
```

```
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Fricas [A] time = 1.62456, size = 58, normalized size = 1.93

$$\frac{1}{2}\,x\cos\left(x\right)e^{x} + \frac{1}{2}\,(x-1)e^{x}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*x*cos(x)*e^x + 1/2*(x - 1)*e^x*sin(x)
```

Sympy [A] time = 0.788038, size = 27, normalized size = 0.9

$$\frac{xe^{x}\sin\left(x\right)}{2} + \frac{xe^{x}\cos\left(x\right)}{2} - \frac{e^{x}\sin\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x)
```

```
[Out] x*exp(x)*sin(x)/2 + x*exp(x)*cos(x)/2 - exp(x)*sin(x)/2
```

Giac [A] time = 1.07229, size = 20, normalized size = 0.67

$$\frac{1}{2}\,(x\cos\left(x\right) + (x-1)\sin\left(x\right))e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x*cos(x),x, algorithm="giac")
```

```
[Out] 1/2*(x*cos(x) + (x - 1)*sin(x))*e^x
```

3.254 $\int \frac{1}{(-3+x)^4}\,dx$

Optimal. Leaf size=11

$$\frac{1}{3(3-x)^3}$$

```
[Out] 1/(3*(3 - x)^3)
```

Rubi [A] time = 0.0006913, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {32}

$$\frac{1}{3(3-x)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x)^(-4),x]
```

```
[Out] 1/(3*(3 - x)^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{1}{(-3+x)^4}\,dx = \frac{1}{3(3-x)^3}$$

Mathematica [A] time = 0.0013664, size = 9, normalized size = 0.82

$$-\frac{1}{3(x-3)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x)^(-4),x]
```

```
[Out] -1/(3*(-3 + x)^3)
```

Maple [A] time = 0., size = 8, normalized size = 0.7

$$-\frac{1}{3\,(-3+x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-3+x)^4,x)
```

```
[Out] -1/3/(-3+x)^3
```

Maxima [A] time = 0.973595, size = 9, normalized size = 0.82

$$-\frac{1}{3\,(x-3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)^4,x, algorithm="maxima")
```

```
[Out] -1/3/(x - 3)^3
```

Fricas [B] time = 1.49917, size = 43, normalized size = 3.91

$$-\frac{1}{3\left(x^3-9\,x^2+27\,x-27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)^4,x, algorithm="fricas")
```

```
[Out] -1/3/(x^3 - 9*x^2 + 27*x - 27)
```

Sympy [B] time = 0.095338, size = 17, normalized size = 1.55

$$-\frac{1}{3x^3 - 27x^2 + 81x - 81}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)**4,x)
```

```
[Out] -1/(3*x**3 - 27*x**2 + 81*x - 81)
```

Giac [A] time = 1.07494, size = 9, normalized size = 0.82

$$-\frac{1}{3\,(x-3)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3+x)^4,x, algorithm="giac")
```

```
[Out] -1/3/(x - 3)^3
```

3.255 $\int \frac{x}{-1+x^3} \, dx$

Optimal. Leaf size=40

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

Rubi [A] time = 0.0211335, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {292, 31, 634, 618, 204, 628}

$$-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(1-x)+\frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x^3),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

Rule 292

```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-
1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), I
nt[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x
^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{-1+x^3}\,dx &= \frac{1}{3}\int \frac{1}{-1+x}\,dx - \frac{1}{3}\int \frac{-1+x}{1+x+x^2}\,dx \\
&= \frac{1}{3}\log(1-x) - \frac{1}{6}\int \frac{1+2x}{1+x+x^2}\,dx + \frac{1}{2}\int \frac{1}{1+x+x^2}\,dx \\
&= \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right) - \text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x\right) \\
&= \frac{\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \frac{1}{3}\log(1-x) - \frac{1}{6}\log\left(1+x+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0083162, size = 40, normalized size = 1.

$$-\frac{1}{6}\log\left(x^2+x+1\right) + \frac{1}{3}\log(1-x) + \frac{\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x^3),x]
```

```
[Out] ArcTan[(1 + 2*x)/Sqrt[3]]/Sqrt[3] + Log[1 - x]/3 - Log[1 + x + x^2]/6
```

Maple [A] time = 0.002, size = 33, normalized size = 0.8

$$\frac{\ln(-1+x)}{3}-\frac{\ln\left(x^2+x+1\right)}{6}+\frac{\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3-1),x)
```

```
[Out] 1/3*ln(-1+x)-1/6*ln(x^2+x+1)+1/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.43189, size = 43, normalized size = 1.08

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-1),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log(
x - 1)
```

Fricas [A] time = 1.61868, size = 112, normalized size = 2.8

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-1),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log(
x - 1)
```

Sympy [A] time = 0.128426, size = 41, normalized size = 1.02

$$\frac{\log(x-1)}{3}-\frac{\log\left(x^2+x+1\right)}{6}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x}{3}+\frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3-1),x)
```

```
[Out] log(x - 1)/3 - log(x**2 + x + 1)/6 + sqrt(3)*atan(2*sqrt(3)*x/3 + sqrt(3)/3
)/3
```

Giac [A] time = 1.09073, size = 45, normalized size = 1.12

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)-\frac{1}{6}\log\left(x^2+x+1\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-1),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) - 1/6*log(x^2 + x + 1) + 1/3*log(
abs(x - 1))
```

3.256 $\int \frac{x}{-1+x^4} \, dx$

Optimal. Leaf size=8

$$-\frac{1}{2}\tanh^{-1}\left(x^2\right)$$

```
[Out] -ArcTanh[x^2]/2
```

Rubi [A] time = 0.0029564, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {275, 207}

$$-\frac{1}{2}\tanh^{-1}\left(x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(-1 + x^4),x]
```

```
[Out] -ArcTanh[x^2]/2
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{-1+x^4} \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{-1+x^2} \, dx, x, x^2\right)\\ &= -\frac{1}{2}\tanh^{-1}\left(x^2\right)\end{aligned}$$

Mathematica [B] time = 0.0027506, size = 23, normalized size = 2.88

$$\frac{1}{4}\log\left(1-x^2\right)-\frac{1}{4}\log\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(-1 + x^4),x]
```

```
[Out] Log[1 - x^2]/4 - Log[1 + x^2]/4
```

Maple [B] time = 0.003, size = 22, normalized size = 2.8

$$\frac{\ln\left(-1+x\right)}{4}+\frac{\ln\left(1+x\right)}{4}-\frac{\ln\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^4-1),x)
```

```
[Out] 1/4*ln(-1+x)+1/4*ln(1+x)-1/4*ln(x^2+1)
```

Maxima [B] time = 0.941232, size = 23, normalized size = 2.88

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-1),x, algorithm="maxima")
```

```
[Out] -1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)
```

Fricas [B] time = 1.57931, size = 51, normalized size = 6.38

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4}\log\left(x^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-1),x, algorithm="fricas")
```

```
[Out] -1/4*log(x^2 + 1) + 1/4*log(x^2 - 1)
```

Sympy [B] time = 0.089208, size = 15, normalized size = 1.88

$$\frac{\log\left(x^2-1\right)}{4}-\frac{\log\left(x^2+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**4-1),x)
```

```
[Out] log(x**2 - 1)/4 - log(x**2 + 1)/4
```

Giac [B] time = 1.08494, size = 24, normalized size = 3.

$$-\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{4}\log\left(\left|x^2-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^4-1),x, algorithm="giac")
```

```
[Out] -1/4*log(x^2 + 1) + 1/4*log(abs(x^2 - 1))
```

3.257 $\int \frac{(1+x^3)\log(x)}{2+x^4}\,dx$

Optimal. Leaf size=227

$$\frac{1}{16}\left(4+(1-i)2^{3/4}\right)\text{PolyLog}\left(2,-\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2+i\sqrt[4]{-2}\right)\text{PolyLog}\left(2,\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2-\sqrt[4]{-2}\right)\text{PolyLog}\left(2,-\frac{(-1)^{3/4}}{\sqrt[4]{2}}\right.$$

```
[Out] ((2 + I*(-2)^(1/4))*Log[x]*Log[1 - ((1 + I)*x)/2^(3/4)])/8 + ((4 + (1 - I)*
2^(3/4))*Log[x]*Log[1 + ((1 + I)*x)/2^(3/4)])/16 + ((2 + (-2)^(1/4))*Log[x]
*Log[1 - ((-1)^(3/4)*x)/2^(1/4)])/8 + ((2 - (-2)^(1/4))*Log[x]*Log[1 + ((-1
)^(3/4)*x)/2^(1/4)])/8 + ((4 + (1 - I)*2^(3/4))*PolyLog[2, ((-1 - I)*x)/2^(
3/4)])/16 + ((2 + I*(-2)^(1/4))*PolyLog[2, ((1 + I)*x)/2^(3/4)])/8 + ((2 -
(-2)^(1/4))*PolyLog[2, -(((-1)^(3/4)*x)/2^(1/4))])/8 + ((2 + (-2)^(1/4))*Po
lyLog[2, ((-1)^(3/4)*x)/2^(1/4)])/8
```

Rubi [A] time = 0.197169, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {2357, 2317, 2391}

$$\frac{1}{16}\left(4+(1-i)2^{3/4}\right)\text{PolyLog}\left(2,-\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2+i\sqrt[4]{-2}\right)\text{PolyLog}\left(2,\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2-\sqrt[4]{-2}\right)\text{PolyLog}\left(2,-\frac{(-1)^{3/4}}{\sqrt[4]{2}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^3)*Log[x])/(2 + x^4),x]
```



```
[Out] ((2 + I*(-2)^(1/4))*Log[x]*Log[1 - ((1 + I)*x)/2^(3/4)])/8 + ((4 + (1 - I)*
2^(3/4))*Log[x]*Log[1 + ((1 + I)*x)/2^(3/4)])/16 + ((2 + (-2)^(1/4))*Log[x]
*Log[1 - ((-1)^(3/4)*x)/2^(1/4)])/8 + ((2 - (-2)^(1/4))*Log[x]*Log[1 + ((-1
)^(3/4)*x)/2^(1/4)])/8 + ((4 + (1 - I)*2^(3/4))*PolyLog[2, ((-1 - I)*x)/2^(
3/4)])/16 + ((2 + I*(-2)^(1/4))*PolyLog[2, ((1 + I)*x)/2^(3/4)])/8 + ((2 -
(-2)^(1/4))*PolyLog[2, -(((-1)^(3/4)*x)/2^(1/4))])/8 + ((2 + (-2)^(1/4))*Po
lyLog[2, ((-1)^(3/4)*x)/2^(1/4)])/8
```


Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{(1+x^3)\log(x)}{2+x^4}\,dx = \int \left(\frac{\left(-2+\sqrt[4]{-2}\right)\log(x)}{8\left(\sqrt[4]{-2}-x\right)}+\frac{\left(-2i+\sqrt[4]{-2}\right)\log(x)}{8\left(\sqrt[4]{-2}-ix\right)}+\frac{\left(2i+\sqrt[4]{-2}\right)\log(x)}{8\left(\sqrt[4]{-2}+ix\right)}+\frac{\left(2+\sqrt[4]{-2}\right)\log(x)}{8\left(\sqrt[4]{-2}+x\right)}\right)dx$$

$$= \frac{1}{8}\left(-2+\sqrt[4]{-2}\right)\int \frac{\log(x)}{\sqrt[4]{-2}-x}\,dx+\frac{1}{8}\left(-2i+\sqrt[4]{-2}\right)\int \frac{\log(x)}{\sqrt[4]{-2}-ix}\,dx+\frac{1}{8}\left(2i+\sqrt[4]{-2}\right)\int \frac{\log(x)}{\sqrt[4]{-2}+ix}\,dx$$

$$= \frac{1}{8}\left(2+i\sqrt[4]{-2}\right)\log(x)\log\left(1-\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2-i\sqrt[4]{-2}\right)\log(x)\log\left(1+\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2+\sqrt[4]{-2}\right)\mathrm{l}$$

$$= \frac{1}{8}\left(2+i\sqrt[4]{-2}\right)\log(x)\log\left(1-\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2-i\sqrt[4]{-2}\right)\log(x)\log\left(1+\frac{(1+i)x}{2^{3/4}}\right)+\frac{1}{8}\left(2+\sqrt[4]{-2}\right)\mathrm{l}$$

Mathematica [A] time = 0.251529, size = 194, normalized size = 0.85

$$\frac{1}{8}\left(\left(2+\frac{1-i}{\sqrt[4]{2}}\right)\mathrm{PolyLog}\left(2,-\frac{(1+i)x}{2^{3/4}}\right)+\left(2+\sqrt[4]{-2}\right)\mathrm{PolyLog}\left(2,-\frac{(1-i)x}{2^{3/4}}\right)-\left(\sqrt[4]{-2}-2\right)\mathrm{PolyLog}\left(2,\frac{(1-i)x}{2^{3/4}}\right)+\left(2+i\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^3)*Log[x])/(2 + x^4),x]
```

```
[Out] ((2 + I*(-2)^(1/4))*Log[x]*Log[1 - (-1/2)^(1/4)*x] + (2 + (1 - I)/2^(1/4))*
Log[x]*Log[1 + (-1/2)^(1/4)*x] - (-2 + (-2)^(1/4))*Log[x]*Log[1 - ((1 - I)*
x)/2^(3/4)] + (2 + (-2)^(1/4))*Log[x]*Log[1 + ((1 - I)*x)/2^(3/4)] + (2 + (
1 - I)/2^(1/4))*PolyLog[2, ((-1 - I)*x)/2^(3/4)] + (2 + (-2)^(1/4))*PolyLog
[2, ((-1 + I)*x)/2^(3/4)] - (-2 + (-2)^(1/4))*PolyLog[2, ((1 - I)*x)/2^(3/4
)] + (2 + I*(-2)^(1/4))*PolyLog[2, ((1 + I)*x)/2^(3/4)])/8
```

Maple [B] time = 0.014, size = 1210, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+1)*ln(x)/(x^4+2),x)
```

```
[Out] -1/4/(1/2*2^(3/4)+1/2*I*2^(3/4))^3*ln(x)*ln((1/2*2^(3/4)+1/2*I*2^(3/4)-x)/(
1/2*2^(3/4)+1/2*I*2^(3/4)))*2^(1/4)-1/4/(1/2*2^(3/4)+1/2*I*2^(3/4))^3*dilog
((1/2*2^(3/4)+1/2*I*2^(3/4)-x)/(1/2*2^(3/4)+1/2*I*2^(3/4)))*2^(1/4)+1/4*I/(
1/2*I*2^(3/4)-1/2*2^(3/4))^3*dilog((1/2*I*2^(3/4)-1/2*2^(3/4)-x)/(1/2*I*2^(
3/4)-1/2*2^(3/4)))*2^(1/4)-1/4*I/(-1/2*2^(3/4)-1/2*I*2^(3/4))^3*dilog((-1/2
*2^(3/4)-1/2*I*2^(3/4)-x)/(-1/2*2^(3/4)-1/2*I*2^(3/4)))*2^(1/4)+1/4/(1/2*2^
(3/4)+1/2*I*2^(3/4))^3*ln(x)*ln((1/2*2^(3/4)+1/2*I*2^(3/4)-x)/(1/2*2^(3/4)+
1/2*I*2^(3/4)))+1/4/(1/2*2^(3/4)+1/2*I*2^(3/4))^3*dilog((1/2*2^(3/4)+1/2*I*
2^(3/4)-x)/(1/2*2^(3/4)+1/2*I*2^(3/4)))-1/4*I/(-1/2*I*2^(3/4)+1/2*2^(3/4))^
3*dilog((-1/2*I*2^(3/4)+1/2*2^(3/4)-x)/(-1/2*I*2^(3/4)+1/2*2^(3/4)))*2^(1/4
)-1/4*I/(-1/2*I*2^(3/4)+1/2*2^(3/4))^3*ln(x)*ln((-1/2*I*2^(3/4)+1/2*2^(3/4)
-x)/(-1/2*I*2^(3/4)+1/2*2^(3/4)))*2^(1/4)+1/4/(1/2*I*2^(3/4)-1/2*2^(3/4))^3
*ln(x)*ln((1/2*I*2^(3/4)-1/2*2^(3/4)-x)/(1/2*I*2^(3/4)-1/2*2^(3/4)))*2^(1/4
)+1/4/(1/2*I*2^(3/4)-1/2*2^(3/4))^3*dilog((1/2*I*2^(3/4)-1/2*2^(3/4)-x)/(1/
2*I*2^(3/4)-1/2*2^(3/4)))*2^(1/4)+1/4/(1/2*I*2^(3/4)-1/2*2^(3/4))^3*ln(x)*l
n((1/2*I*2^(3/4)-1/2*2^(3/4)-x)/(1/2*I*2^(3/4)-1/2*2^(3/4)))+1/4/(1/2*I*2^(
3/4)-1/2*2^(3/4))^3*dilog((1/2*I*2^(3/4)-1/2*2^(3/4)-x)/(1/2*I*2^(3/4)-1/2*
2^(3/4)))+1/4/(-1/2*2^(3/4)-1/2*I*2^(3/4))^3*ln(x)*ln((-1/2*2^(3/4)-1/2*I*2
^(3/4)-x)/(-1/2*2^(3/4)-1/2*I*2^(3/4)))*2^(1/4)+1/4/(-1/2*2^(3/4)-1/2*I*2^(
3/4))^3*dilog((-1/2*2^(3/4)-1/2*I*2^(3/4)-x)/(-1/2*2^(3/4)-1/2*I*2^(3/4)))*
2^(1/4)+1/4*I/(1/2*2^(3/4)+1/2*I*2^(3/4))^3*dilog((1/2*2^(3/4)+1/2*I*2^(3/4
)-x)/(1/2*2^(3/4)+1/2*I*2^(3/4)))*2^(1/4)+1/4*I/(1/2*I*2^(3/4)-1/2*2^(3/4))
^3*ln(x)*ln((1/2*I*2^(3/4)-1/2*2^(3/4)-x)/(1/2*I*2^(3/4)-1/2*2^(3/4)))*2^(1
/4)+1/4/(-1/2*2^(3/4)-1/2*I*2^(3/4))^3*ln(x)*ln((-1/2*2^(3/4)-1/2*I*2^(3/4)
-x)/(-1/2*2^(3/4)-1/2*I*2^(3/4)))+1/4/(-1/2*2^(3/4)-1/2*I*2^(3/4))^3*dilog(
(-1/2*2^(3/4)-1/2*I*2^(3/4)-x)/(-1/2*2^(3/4)-1/2*I*2^(3/4)))-1/4*I/(-1/2*2^
(3/4)-1/2*I*2^(3/4))^3*ln(x)*ln((-1/2*2^(3/4)-1/2*I*2^(3/4)-x)/(-1/2*2^(3/4
)-1/2*I*2^(3/4)))*2^(1/4)+1/4*I/(1/2*2^(3/4)+1/2*I*2^(3/4))^3*ln(x)*ln((1/2
*2^(3/4)+1/2*I*2^(3/4)-x)/(1/2*2^(3/4)+1/2*I*2^(3/4)))*2^(1/4)-1/4/(-1/2*I*
2^(3/4)+1/2*2^(3/4))^3*ln(x)*ln((-1/2*I*2^(3/4)+1/2*2^(3/4)-x)/(-1/2*I*2^(3
/4)+1/2*2^(3/4)))*2^(1/4)-1/4/(-1/2*I*2^(3/4)+1/2*2^(3/4))^3*dilog((-1/2*I*
2^(3/4)+1/2*2^(3/4)-x)/(-1/2*I*2^(3/4)+1/2*2^(3/4)))*2^(1/4)+1/4/(-1/2*I*2^
(3/4)+1/2*2^(3/4))^3*ln(x)*ln((-1/2*I*2^(3/4)+1/2*2^(3/4)-x)/(-1/2*I*2^(3/4
)+1/2*2^(3/4)))+1/4/(-1/2*I*2^(3/4)+1/2*2^(3/4))^3*dilog((-1/2*I*2^(3/4)+1/
```

```
2*2^(3/4)-x)/(-1/2*I*2^(3/4)+1/2*2^(3/4)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^3+1)\log(x)}{x^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*log(x)/(x^4+2),x, algorithm="maxima")
```

```
[Out] integrate((x^3 + 1)*log(x)/(x^4 + 2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(x^3+1)\log(x)}{x^4+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*log(x)/(x^4+2),x, algorithm="fricas")
```

```
[Out] integral((x^3 + 1)*log(x)/(x^4 + 2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)(x^2-x+1)\log(x)}{x^4+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+1)*ln(x)/(x**4+2),x)
```

```
[Out] Integral((x + 1)*(x**2 - x + 1)*log(x)/(x**4 + 2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^3 + 1)\log(x)}{x^4 + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+1)*log(x)/(x^4+2),x, algorithm="giac")
```

```
[Out] integrate((x^3 + 1)*log(x)/(x^4 + 2), x)
```

3.258 $\int (\log(x)+\log(1+x)+\log(2+x))\,dx$

Optimal. Leaf size=24

$$-3x+x\log(x)+(x+1)\log(x+1)+(x+2)\log(x+2)$$

```
[Out] -3*x + x*Log[x] + (1 + x)*Log[1 + x] + (2 + x)*Log[2 + x]
```

Rubi [A] time = 0.0087903, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2295, 2389}

$$-3x+x\log(x)+(x+1)\log(x+1)+(x+2)\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x] + Log[1 + x] + Log[2 + x],x]
```

```
[Out] -3*x + x*Log[x] + (1 + x)*Log[1 + x] + (2 + x)*Log[2 + x]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rule 2389

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.), x_Symbol] :
> Dist[1/e, Subst[Int[(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a
, b, c, d, e, n, p}, x]
```

Rubi steps

$$\begin{aligned}\int (\log(x)+\log(1+x)+\log(2+x))\,dx &= \int \log(x)\,dx+\int \log(1+x)\,dx+\int \log(2+x)\,dx\\ &= -x+x\log(x)+\text{Subst}\left(\int \log(x)\,dx,x,1+x\right)+\text{Subst}\left(\int \log(x)\,dx,x,2+x\right)\\ &= -3x+x\log(x)+(1+x)\log(1+x)+(2+x)\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.0029841, size = 30, normalized size = 1.25

$$-3x + x\log(x) + x\log(x+1) + x\log(x+2) + \log(x+1) + 2\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x] + Log[1 + x] + Log[2 + x],x]
```

```
[Out] -3*x + x*Log[x] + Log[1 + x] + x*Log[1 + x] + 2*Log[2 + x] + x*Log[2 + x]
```

Maple [A] time = 0.003, size = 26, normalized size = 1.1

$$x\ln(x) - 3x + (1+x)\ln(1+x) - 3 + (2+x)\ln(2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)+ln(1+x)+ln(2+x),x)
```

```
[Out] x*ln(x)-3*x+(1+x)*ln(1+x)-3+(2+x)*ln(2+x)
```

Maxima [A] time = 0.946145, size = 34, normalized size = 1.42

$$(x+2)\log(x+2) + (x+1)\log(x+1) + x\log(x) - 3x - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)+log(1+x)+log(2+x),x, algorithm="maxima")
```

```
[Out] (x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x - 3
```

Fricas [A] time = 1.57867, size = 78, normalized size = 3.25

$$(x+2)\log(x+2) + (x+1)\log(x+1) + x\log(x) - 3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)+log(1+x)+log(2+x),x, algorithm="fricas")
```

```
[Out] (x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x
```

Sympy [A] time = 1.44325, size = 32, normalized size = 1.33

$$x\log(x) + x\log(x+1) + x\log(x+2) - 3x + \log(x+1) + 2\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)+ln(1+x)+ln(2+x),x)
```

```
[Out] x*log(x) + x*log(x + 1) + x*log(x + 2) - 3*x + log(x + 1) + 2*log(x + 2)
```

Giac [A] time = 1.08147, size = 34, normalized size = 1.42

$$(x+2)\log(x+2) + (x+1)\log(x+1) + x\log(x) - 3x - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)+log(1+x)+log(2+x),x, algorithm="giac")
```

```
[Out] (x + 2)*log(x + 2) + (x + 1)*log(x + 1) + x*log(x) - 3*x - 3
```

3.259 $\int \frac{1}{5+x^3} dx$

Optimal. Leaf size=78

$$-\frac{\log\left(x^2-\sqrt[3]{5}x+5^{2/3}\right)}{6\ 5^{2/3}}+\frac{\log\left(x+\sqrt[3]{5}\right)}{3\ 5^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{5}-2x}{\sqrt{3}\sqrt[3]{5}}\right)}{\sqrt{3}5^{2/3}}$$

```
[Out] -(ArcTan[(5^(1/3) - 2*x)/(Sqrt[3]*5^(1/3))]/(Sqrt[3]*5^(2/3))) + Log[5^(1/3
) + x]/(3*5^(2/3)) - Log[5^(2/3) - 5^(1/3)*x + x^2]/(6*5^(2/3))
```

Rubi [A] time = 0.0458739, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(x^2-\sqrt[3]{5}x+5^{2/3}\right)}{6\ 5^{2/3}}+\frac{\log\left(x+\sqrt[3]{5}\right)}{3\ 5^{2/3}}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{5}-2x}{\sqrt{3}\sqrt[3]{5}}\right)}{\sqrt{3}5^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 + x^3)^(-1),x]
```

```
[Out] -(ArcTan[(5^(1/3) - 2*x)/(Sqrt[3]*5^(1/3))]/(Sqrt[3]*5^(2/3))) + Log[5^(1/3
) + x]/(3*5^(2/3)) - Log[5^(2/3) - 5^(1/3)*x + x^2]/(6*5^(2/3))
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{5+x^3}\,dx &= \frac{\int \frac{1}{\sqrt[3]{5}+x}\,dx}{3\ 5^{2/3}}+\frac{\int \frac{2\sqrt[3]{5}-x}{5^{2/3}-\sqrt[3]{5}x+x^2}\,dx}{3\ 5^{2/3}}\\
&= \frac{\log\left(\sqrt[3]{5}+x\right)}{3\ 5^{2/3}}-\frac{\int \frac{-\sqrt[3]{5}+2x}{5^{2/3}-\sqrt[3]{5}x+x^2}\,dx}{6\ 5^{2/3}}+\frac{\int \frac{1}{5^{2/3}-\sqrt[3]{5}x+x^2}\,dx}{2\sqrt[3]{5}}\\
&= \frac{\log\left(\sqrt[3]{5}+x\right)}{3\ 5^{2/3}}-\frac{\log\left(5^{2/3}-\sqrt[3]{5}x+x^2\right)}{6\ 5^{2/3}}+\frac{\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx,x,1-\frac{2x}{\sqrt[3]{5}}\right)}{5^{2/3}}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[3]{5}-2x}{\sqrt{3}\sqrt[3]{5}}\right)}{\sqrt{3}5^{2/3}}+\frac{\log\left(\sqrt[3]{5}+x\right)}{3\ 5^{2/3}}-\frac{\log\left(5^{2/3}-\sqrt[3]{5}x+x^2\right)}{6\ 5^{2/3}}
\end{aligned}
$$

Mathematica [A] time = 0.0226852, size = 71, normalized size = 0.91

$$
\frac{-\log\left(\sqrt[3]{5}x^2-5^{2/3}x+5\right)+2\log\left(5^{2/3}x+5\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\ 5^{2/3}x-5}{5\sqrt{3}}\right)}{6\ 5^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + x^3)^(-1),x]
```

```
[Out] (2*Sqrt[3]*ArcTan[(-5 + 2*5^(2/3)*x)/(5*Sqrt[3])] + 2*Log[5 + 5^(2/3)*x] -
Log[5 - 5^(2/3)*x + 5^(1/3)*x^2])/(6*5^(2/3))
```

Maple [A] time = 0.004, size = 54, normalized size = 0.7

$$\frac{\ln\left(\sqrt[3]{5}+x\right)\sqrt[3]{5}}{15}-\frac{\ln\left(5^{\frac{2}{3}}-\sqrt[3]{5}x+x^{2}\right)\sqrt[3]{5}}{30}+\frac{\sqrt[3]{5}\sqrt{3}}{15}\arctan\left(\frac{\sqrt{3}}{3}\left(\frac{2\,5^{2/3}x}{5}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3+5),x)
```

```
[Out] 1/15*ln(5^(1/3)+x)*5^(1/3)-1/30*ln(5^(2/3)-5^(1/3)*x+x^2)*5^(1/3)+1/15*5^(1
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/5*5^(2/3)*x-1))
```

Maxima [A] time = 1.42263, size = 77, normalized size = 0.99

$$\frac{1}{15}\cdot 5^{\frac{1}{3}}\sqrt{3}\arctan\left(\frac{1}{15}\cdot 5^{\frac{2}{3}}\sqrt{3}\left(2\,x-5^{\frac{1}{3}}\right)\right)-\frac{1}{30}\cdot 5^{\frac{1}{3}}\log\left(x^{2}-5^{\frac{1}{3}}x+5^{\frac{2}{3}}\right)+\frac{1}{15}\cdot 5^{\frac{1}{3}}\log\left(x+5^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+5),x, algorithm="maxima")
```

```
[Out] 1/15*5^(1/3)*sqrt(3)*arctan(1/15*5^(2/3)*sqrt(3)*(2*x - 5^(1/3))) - 1/30*5^
(1/3)*log(x^2 - 5^(1/3)*x + 5^(2/3)) + 1/15*5^(1/3)*log(x + 5^(1/3))
```

Fricas [A] time = 1.66226, size = 242, normalized size = 3.1

$$\frac{1}{15}\cdot 25^{\frac{1}{6}}\sqrt{3}\arctan\left(\frac{1}{75}\cdot 25^{\frac{1}{6}}\left(2\cdot 25^{\frac{2}{3}}\sqrt{3}x-5\cdot 25^{\frac{1}{3}}\sqrt{3}\right)\right)-\frac{1}{150}\cdot 25^{\frac{2}{3}}\log\left(5\,x^{2}-25^{\frac{2}{3}}x+5\cdot 25^{\frac{1}{3}}\right)+\frac{1}{75}\cdot 25^{\frac{2}{3}}\log\left(5\,x+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+5),x, algorithm="fricas")
```

```
[Out] 1/15*25^(1/6)*sqrt(3)*arctan(1/75*25^(1/6)*(2*25^(2/3)*sqrt(3)*x - 5*25^(1/
3)*sqrt(3))) - 1/150*25^(2/3)*log(5*x^2 - 25^(2/3)*x + 5*25^(1/3)) + 1/75*2
5^(2/3)*log(5*x + 25^(2/3))
```

Sympy [A] time = 0.304137, size = 73, normalized size = 0.94

$$\frac{\sqrt[3]{5}\log\left(x+\sqrt[3]{5}\right)}{15}-\frac{\sqrt[3]{5}\log\left(x^2-\sqrt[3]{5}x+5^{\frac{2}{3}}\right)}{30}+\frac{\sqrt{3}\sqrt[3]{5}\,\mathrm{atan}\left(\frac{2\sqrt{3}\cdot 5^{\frac{2}{3}}x}{15}-\frac{\sqrt{3}}{3}\right)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3+5),x)
```

```
[Out] 5**(1/3)*log(x + 5**(1/3))/15 - 5**(1/3)*log(x**2 - 5**(1/3)*x + 5**(2/3))/
30 + sqrt(3)*5**(1/3)*atan(2*sqrt(3)*5**(2/3)*x/15 - sqrt(3)/3)/15
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3+5),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.260 $\int \frac{1}{\sqrt{1+x^2}}\,dx$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

```
[Out] ArcSinh[x]
```

Rubi [A] time = 0.0008121, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {215}

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcSinh[x]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \sinh^{-1}(x)$$

Mathematica [A] time = 0.003484, size = 2, normalized size = 1.

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + x^2],x]
```

```
[Out] ArcSinh[x]
```

Maple [A] time = 0., size = 3, normalized size = 1.5

$$\text{Arcsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(1/2),x)
```

```
[Out] arcsinh(x)
```

Maxima [A] time = 1.43465, size = 3, normalized size = 1.5

$$\text{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(x)
```

Fricas [B] time = 1.60789, size = 35, normalized size = 17.5

$$-\log\left(-x + \sqrt{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 1))
```

Sympy [A] time = 0.121992, size = 2, normalized size = 1.

$$\text{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(1/2),x)
```

```
[Out] asinh(x)
```

Giac [B] time = 1.10777, size = 19, normalized size = 9.5

$$-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-x + sqrt(x^2 + 1))
```

3.261 $\int \sqrt{3+x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}\sqrt{x^2+3}x+\frac{3}{2}\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

```
[Out] (x*Sqrt[3 + x^2])/2 + (3*ArcSinh[x/Sqrt[3]])/2
```

Rubi [A] time = 0.0035726, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {195, 215}

$$\frac{1}{2}\sqrt{x^2+3}x+\frac{3}{2}\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 + x^2],x]

[Out] (x*Sqrt[3 + x^2])/2 + (3*ArcSinh[x/Sqrt[3]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{3+x^2}\,dx = \frac{1}{2}x\sqrt{3+x^2} + \frac{3}{2}\int \frac{1}{\sqrt{3+x^2}}\,dx$$
$$= \frac{1}{2}x\sqrt{3+x^2} + \frac{3}{2}\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Mathematica [A] time = 0.006495, size = 27, normalized size = 1.

$$\frac{1}{2}\sqrt{x^2+3}x + \frac{3}{2}\sinh^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 + x^2],x]
```

```
[Out] (x*Sqrt[3 + x^2])/2 + (3*ArcSinh[x/Sqrt[3]])/2
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$\frac{3}{2}\text{Arcsinh}\left(\frac{x\sqrt{3}}{3}\right) + \frac{x}{2}\sqrt{x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+3)^(1/2),x)
```

```
[Out] 3/2*arcsinh(1/3*x*3^(1/2))+1/2*x*(x^2+3)^(1/2)
```

Maxima [A] time = 1.41678, size = 27, normalized size = 1.

$$\frac{1}{2}\sqrt{x^2+3}x + \frac{3}{2}\,\text{arsinh}\left(\frac{1}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 + 3)*x + 3/2*arcsinh(1/3*sqrt(3)*x)
```

Fricas [A] time = 1.7269, size = 69, normalized size = 2.56

$$\frac{1}{2}\sqrt{x^2+3}x - \frac{3}{2}\log\left(-x+\sqrt{x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 + 3)*x - 3/2*log(-x + sqrt(x^2 + 3))
```

Sympy [A] time = 0.187642, size = 24, normalized size = 0.89

$$\frac{x\sqrt{x^2+3}}{2} + \frac{3\operatorname{asinh}\left(\frac{\sqrt{3}x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+3)**(1/2),x)
```

```
[Out] x*sqrt(x**2 + 3)/2 + 3*asinh(sqrt(3)*x/3)/2
```

Giac [A] time = 1.07199, size = 34, normalized size = 1.26

$$\frac{1}{2}\sqrt{x^2+3}x - \frac{3}{2}\log\left(-x+\sqrt{x^2+3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 3)*x - 3/2*log(-x + sqrt(x^2 + 3))
```

3.262 $\int \frac{x}{(1+x)^2} dx$

Optimal. Leaf size=10

$$\frac{1}{x+1} + \log(x+1)$$

```
[Out] (1 + x)^(-1) + Log[1 + x]
```

Rubi [A] time = 0.0043905, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {43}

$$\frac{1}{x+1} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x)^2,x]
```

```
[Out] (1 + x)^(-1) + Log[1 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{(1+x)^2} dx &= \int \left(-\frac{1}{(1+x)^2} + \frac{1}{1+x}\right) dx \\ &= \frac{1}{1+x} + \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.0027193, size = 10, normalized size = 1.

$$\frac{1}{x+1} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x)^2,x]
```

```
[Out] (1 + x)^(-1) + Log[1 + x]
```

Maple [A] time = 0., size = 11, normalized size = 1.1

$$(1+x)^{-1}+\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)^2,x)
```

```
[Out] 1/(1+x)+ln(1+x)
```

Maxima [A] time = 0.927928, size = 14, normalized size = 1.4

$$\frac{1}{x+1}+\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/(x + 1) + log(x + 1)
```

Fricas [A] time = 1.63569, size = 46, normalized size = 4.6

$$\frac{(x+1)\log(x+1)+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="fricas")
```

```
[Out] ((x + 1)*log(x + 1) + 1)/(x + 1)
```

Sympy [A] time = 0.074338, size = 8, normalized size = 0.8

$$\log(x+1) + \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**2,x)
```

```
[Out] log(x + 1) + 1/(x + 1)
```

Giac [A] time = 1.0972, size = 15, normalized size = 1.5

$$\frac{1}{x+1} + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/(x + 1) + log(abs(x + 1))
```

3.263 $\int \sin^{-1}(x)\,dx$

Optimal. Leaf size=16

$$\sqrt{1-x^2}+x\sin^{-1}(x)$$

```
[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

Rubi [A] time = 0.0041649, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {4619, 261}

$$\sqrt{1-x^2}+x\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x],x]
```

```
[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 -
 c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sin^{-1}(x)\,dx &= x\sin^{-1}(x)-\int \frac{x}{\sqrt{1-x^2}}\,dx\\ &= \sqrt{1-x^2}+x\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0020293, size = 16, normalized size = 1.

$$\sqrt{1-x^2}+x\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x],x]
```

```
[Out] Sqrt[1 - x^2] + x*ArcSin[x]
```

Maple [A] time = 0., size = 15, normalized size = 0.9

$$x\arcsin(x)+\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x),x)
```

```
[Out] x*arcsin(x)+(-x^2+1)^(1/2)
```

Maxima [A] time = 1.41038, size = 19, normalized size = 1.19

$$x\arcsin(x)+\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x),x, algorithm="maxima")
```

```
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

Fricas [A] time = 1.70955, size = 41, normalized size = 2.56

$$x\arcsin(x)+\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x),x, algorithm="fricas")
```

```
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

Sympy [A] time = 0.116579, size = 12, normalized size = 0.75

$$x \operatorname{asin}(x) + \sqrt{1 - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x),x)
```

```
[Out] x*asin(x) + sqrt(1 - x**2)
```

Giac [A] time = 1.08368, size = 19, normalized size = 1.19

$$x \arcsin(x) + \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x),x, algorithm="giac")
```

```
[Out] x*arcsin(x) + sqrt(-x^2 + 1)
```

3.264 $\int x^2 \sin^{-1}(x)\,dx$

Optimal. Leaf size=40

$$-\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{\sqrt{1-x^2}}{3}+\frac{1}{3}x^3\sin^{-1}(x)$$

```
[Out] Sqrt[1 - x^2]/3 - (1 - x^2)^(3/2)/9 + (x^3*ArcSin[x])/3
```

Rubi [A] time = 0.0230503, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4627, 266, 43}

$$-\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{\sqrt{1-x^2}}{3}+\frac{1}{3}x^3\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSin[x],x]
```

```
[Out] Sqrt[1 - x^2]/3 - (1 - x^2)^(3/2)/9 + (x^3*ArcSin[x])/3
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sin^{-1}(x)\,dx &= \frac{1}{3}x^3\sin^{-1}(x)-\frac{1}{3}\int\frac{x^3}{\sqrt{1-x^2}}\,dx\\&=\frac{1}{3}x^3\sin^{-1}(x)-\frac{1}{6}\,\text{Subst}\left(\int\frac{x}{\sqrt{1-x}}\,dx,x,x^2\right)\\&=\frac{1}{3}x^3\sin^{-1}(x)-\frac{1}{6}\,\text{Subst}\left(\int\left(\frac{1}{\sqrt{1-x}}-\sqrt{1-x}\right)dx,x,x^2\right)\\&=\frac{\sqrt{1-x^2}}{3}-\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{1}{3}x^3\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0113738, size = 29, normalized size = 0.72

$$\frac{1}{9}\left(\sqrt{1-x^2}\left(x^2+2\right)+3x^3\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcSin[x],x]
```

```
[Out] (Sqrt[1 - x^2]*(2 + x^2) + 3*x^3*ArcSin[x])/9
```

Maple [A] time = 0.002, size = 34, normalized size = 0.9

$$\frac{x^3\arcsin(x)}{3}+\frac{x^2}{9}\sqrt{-x^2+1}+\frac{2}{9}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arcsin(x),x)
```

```
[Out] 1/3*x^3*arcsin(x)+1/9*x^2*(-x^2+1)^(1/2)+2/9*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.412, size = 45, normalized size = 1.12

$$\frac{1}{3}x^3\arcsin(x)+\frac{1}{9}\sqrt{-x^2+1}x^2+\frac{2}{9}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arcsin(x) + 1/9*sqrt(-x^2 + 1)*x^2 + 2/9*sqrt(-x^2 + 1)
```

Fricas [A] time = 1.72575, size = 68, normalized size = 1.7

$$\frac{1}{3}x^3\arcsin(x) + \frac{1}{9}\left(x^2+2\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arcsin(x) + 1/9*(x^2 + 2)*sqrt(-x^2 + 1)
```

Sympy [A] time = 0.321265, size = 32, normalized size = 0.8

$$\frac{x^3\operatorname{asin}(x)}{3} + \frac{x^2\sqrt{1-x^2}}{9} + \frac{2\sqrt{1-x^2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*asin(x),x)
```

```
[Out] x**3*asin(x)/3 + x**2*sqrt(1 - x**2)/9 + 2*sqrt(1 - x**2)/9
```

Giac [A] time = 1.0983, size = 51, normalized size = 1.27

$$\frac{1}{3}\left(x^2-1\right)x\arcsin(x) + \frac{1}{3}x\arcsin(x) - \frac{1}{9}\left(-x^2+1\right)^{\frac{3}{2}} + \frac{1}{3}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arcsin(x),x, algorithm="giac")
```

```
[Out] 1/3*(x^2 - 1)*x*arcsin(x) + 1/3*x*arcsin(x) - 1/9*(-x^2 + 1)^(3/2) + 1/3*sq
rt(-x^2 + 1)
```

3.265 $\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx$

Optimal. Leaf size=21

$$\log(2\cos(x)-\sin(x))-\log(\cos(x)-\sin(x))$$

```
[Out] -Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]
```

Rubi [A] time = 0.114805, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {616, 31}

$$\log(2\cos(x)-\sin(x))-\log(\cos(x)-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]
```

```
[Out] -Log[Cos[x] - Sin[x]] + Log[2*Cos[x] - Sin[x]]
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(x)}{1+\sec^2(x)-3\tan(x)}\,dx &= \text{Subst}\left(\int \frac{1}{2-3x+x^2}\,dx, x, \tan(x)\right)\\
&= \text{Subst}\left(\int \frac{1}{-2+x}\,dx, x, \tan(x)\right)-\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, \tan(x)\right)\\
&= -\log(1-\tan(x))+\log(2-\tan(x))
\end{aligned}$$

Mathematica [A] time = 0.0279755, size = 29, normalized size = 1.38

$$2\left(\frac{1}{2}\log(2\cos(x)-\sin(x))-\frac{1}{2}\log(\cos(x)-\sin(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(1 + Sec[x]^2 - 3*Tan[x]),x]
```

```
[Out] 2*(-Log[Cos[x] - Sin[x]]/2 + Log[2*Cos[x] - Sin[x]]/2)
```

Maple [A] time = 0.053, size = 14, normalized size = 0.7

$$-\ln\left(-1+\tan\left(x\right)\right)+\ln\left(\tan\left(x\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(1+sec(x)^2-3*tan(x)),x)
```

```
[Out] -ln(-1+tan(x))+ln(tan(x)-2)
```

Maxima [A] time = 0.924135, size = 18, normalized size = 0.86

$$-\log\left(\tan\left(x\right)-1\right)+\log\left(\tan\left(x\right)-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="maxima")
```

```
[Out] -log(tan(x) - 1) + log(tan(x) - 2)
```

Fricas [A] time = 1.80975, size = 104, normalized size = 4.95

$$\frac{1}{2}\log\left(\frac{3}{4}\cos\left(x\right)^{2}-\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}\right)-\frac{1}{2}\log\left(-2\cos\left(x\right)\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(3/4*cos(x)^2 - cos(x)*sin(x) + 1/4) - 1/2*log(-2*cos(x)*sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(x)}{-3\tan(x)+\sec^2(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(1+sec(x)**2-3*tan(x)),x)
```

```
[Out] Integral(sec(x)**2/(-3*tan(x) + sec(x)**2 + 1), x)
```

Giac [A] time = 1.1223, size = 20, normalized size = 0.95

$$-\log(|\tan(x)-1|)+\log(|\tan(x)-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(1+sec(x)^2-3*tan(x)),x, algorithm="giac")
```

```
[Out] -log(abs(tan(x) - 1)) + log(abs(tan(x) - 2))
```

3.266 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0059712, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\,dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0018286, size = 14, normalized size = 1.

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2,x]
```

```
[Out] x/2 + Sin[2*x]/4
```

Maple [A] time = 0.007, size = 11, normalized size = 0.8

$$\frac{x}{2}+\frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/sec(x)^2,x)
```

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A] time = 0.926108, size = 14, normalized size = 1.

$$\frac{1}{2}x+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A] time = 1.61454, size = 36, normalized size = 2.57

$$\frac{1}{2}\cos(x)\sin(x)+\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.060909, size = 10, normalized size = 0.71

$$\frac{x}{2} + \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

Giac [A] time = 1.10386, size = 22, normalized size = 1.57

$$\frac{1}{2}x + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/sec(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*tan(x)/(tan(x)^2 + 1)
```

3.267 $\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx$

Optimal. Leaf size=18

$$-\frac{1}{x}+3\log(2-x)+2\log(x)$$

```
[Out] -x^(-1) + 3*Log[2 - x] + 2*Log[x]
```

Rubi [A] time = 0.0143458, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {893}

$$-\frac{1}{x}+3\log(2-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-2 - 3*x + 5*x^2)/((-2 + x)*x^2),x]
```

```
[Out] -x^(-1) + 3*Log[2 - x] + 2*Log[x]
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{-2-3x+5x^2}{(-2+x)x^2}\,dx &= \int \left(\frac{3}{-2+x}+\frac{1}{x^2}+\frac{2}{x}\right)dx\\ &= -\frac{1}{x}+3\log(2-x)+2\log(x)\end{aligned}$$

Mathematica [A] time = 0.0040948, size = 18, normalized size = 1.

$$-\frac{1}{x} + 3\log(2-x) + 2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 - 3*x + 5*x^2)/((-2 + x)*x^2),x]
```

```
[Out] -x^(-1) + 3*Log[2 - x] + 2*Log[x]
```

Maple [A] time = 0.007, size = 17, normalized size = 0.9

$$-x^{-1} + 2\ln(x) + 3\ln(-2+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2-3*x-2)/(-2+x)/x^2,x)
```

```
[Out] -1/x+2*ln(x)+3*ln(-2+x)
```

Maxima [A] time = 0.938333, size = 22, normalized size = 1.22

$$-\frac{1}{x} + 3\log(x-2) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="maxima")
```

```
[Out] -1/x + 3*log(x - 2) + 2*log(x)
```

Fricas [A] time = 1.5578, size = 50, normalized size = 2.78

$$\frac{3x\log(x-2) + 2x\log(x) - 1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="fricas")
```

```
[Out] (3*x*log(x - 2) + 2*x*log(x) - 1)/x
```

Sympy [A] time = 0.103647, size = 14, normalized size = 0.78

$$2\log(x) + 3\log(x-2) - \frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2-3*x-2)/(-2+x)/x**2,x)
```

```
[Out] 2*log(x) + 3*log(x - 2) - 1/x
```

Giac [A] time = 1.06285, size = 24, normalized size = 1.33

$$-\frac{1}{x} + 3\log(|x-2|) + 2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2-3*x-2)/(-2+x)/x^2,x, algorithm="giac")
```

```
[Out] -1/x + 3*log(abs(x - 2)) + 2*log(abs(x))
```

3.268 $\int \frac{1}{\sqrt{9+4x^2}} \, dx$

Optimal. Leaf size=10

$$\frac{1}{2} \sinh^{-1}\left(\frac{2x}{3}\right)$$

```
[Out] ArcSinh[(2*x)/3]/2
```

Rubi [A] time = 0.0014627, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {215}

$$\frac{1}{2} \sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[9 + 4*x^2],x]
```

```
[Out] ArcSinh[(2*x)/3]/2
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{9+4x^2}} \, dx = \frac{1}{2} \sinh^{-1}\left(\frac{2x}{3}\right)$$

Mathematica [A] time = 0.0040377, size = 10, normalized size = 1.

$$\frac{1}{2} \sinh^{-1}\left(\frac{2x}{3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[9 + 4*x^2],x]
```

```
[Out] ArcSinh[(2*x)/3]/2
```

Maple [A] time = 0.003, size = 7, normalized size = 0.7

$$\frac{1}{2}\mathrm{Arcsinh}\left(\frac{2\,x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4*x^2+9)^(1/2),x)
```

```
[Out] 1/2*arcsinh(2/3*x)
```

Maxima [A] time = 1.40079, size = 8, normalized size = 0.8

$$\frac{1}{2}\,\mathrm{arsinh}\left(\frac{2}{3}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*arcsinh(2/3*x)
```

Fricas [B] time = 1.61173, size = 46, normalized size = 4.6

$$-\frac{1}{2}\,\log\left(-2\,x + \sqrt{4\,x^2 + 9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*log(-2*x + sqrt(4*x^2 + 9))
```

Sympy [A] time = 0.140751, size = 7, normalized size = 0.7

$$\frac{\operatorname{asinh}\left(\frac{2x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x**2+9)**(1/2),x)
```

```
[Out] asinh(2*x/3)/2
```

Giac [B] time = 1.07223, size = 22, normalized size = 2.2

$$-\frac{1}{2}\log\left(-2x+\sqrt{4x^2+9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(4*x^2+9)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*log(-2*x + sqrt(4*x^2 + 9))
```

3.269 $\int \frac{1}{\sqrt{4+x^2}} \, dx$

Optimal. Leaf size=6

$$\sinh^{-1}\left(\frac{x}{2}\right)$$

```
[Out] ArcSinh[x/2]
```

Rubi [A] time = 0.0011517, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {215}

$$\sinh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[4 + x^2],x]
```

```
[Out] ArcSinh[x/2]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{1}{\sqrt{4+x^2}} \, dx = \sinh^{-1}\left(\frac{x}{2}\right)$$

Mathematica [A] time = 0.0034716, size = 6, normalized size = 1.

$$\sinh^{-1}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[4 + x^2],x]
```

```
[Out] ArcSinh[x/2]
```

Maple [A] time = 0.003, size = 5, normalized size = 0.8

$$\operatorname{Arcsinh}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+4)^(1/2),x)
```

```
[Out] arcsinh(1/2*x)
```

Maxima [A] time = 1.4087, size = 5, normalized size = 0.83

$$\operatorname{arsinh}\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(1/2*x)
```

Fricas [B] time = 1.53824, size = 35, normalized size = 5.83

$$-\log\left(-x+\sqrt{x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 4))
```

Sympy [A] time = 0.127756, size = 3, normalized size = 0.5

$$\operatorname{asinh}\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+4)**(1/2),x)
```

```
[Out] asinh(x/2)
```

Giac [B] time = 1.09132, size = 19, normalized size = 3.17

$$-\log\left(-x+\sqrt{x^2+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+4)^(1/2),x, algorithm="giac")
```

```
[Out] -log(-x + sqrt(x^2 + 4))
```

3.270 $\int \frac{1}{10-12x+9x^2} \, dx$

Optimal. Leaf size=21

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{6}}\right)}{3\sqrt{6}}$$

```
[Out] -ArcTan[(2 - 3*x)/Sqrt[6]]/(3*Sqrt[6])
```

Rubi [A] time = 0.0170595, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {618, 204}

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{6}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Int[(10 - 12*x + 9*x^2)^(-1),x]
```

```
[Out] -ArcTan[(2 - 3*x)/Sqrt[6]]/(3*Sqrt[6])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{10-12x+9x^2}\,dx = -\left(2\,\text{Subst}\left(\int \frac{1}{-216-x^2}\,dx, x, -12+18x\right)\right)$$
$$= -\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{6}}\right)}{3\sqrt{6}}$$

Mathematica [A] time = 0.008068, size = 21, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{3x-2}{\sqrt{6}}\right)}{3\sqrt{6}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(10 - 12*x + 9*x^2)^(-1),x]
```

```
[Out] ArcTan[(-2 + 3*x)/Sqrt[6]]/(3*Sqrt[6])
```

Maple [A] time = 0.003, size = 17, normalized size = 0.8

$$\frac{\sqrt{6}}{18}\arctan\left(\frac{(18\,x-12)\sqrt{6}}{36}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(9*x^2-12*x+10),x)
```

```
[Out] 1/18*6^(1/2)*arctan(1/36*(18*x-12)*6^(1/2))
```

Maxima [A] time = 1.41101, size = 22, normalized size = 1.05

$$\frac{1}{18}\sqrt{6}\arctan\left(\frac{1}{6}\sqrt{6}(3\,x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-12*x+10),x, algorithm="maxima")
```

```
[Out] 1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))
```

Fricas [A] time = 1.83148, size = 59, normalized size = 2.81

$$\frac{1}{18}\sqrt{6}\arctan\left(\frac{1}{6}\sqrt{6}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-12*x+10),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))
```

Sympy [A] time = 0.104321, size = 22, normalized size = 1.05

$$\frac{\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}x}{2}-\frac{\sqrt{6}}{3}\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x**2-12*x+10),x)
```

```
[Out] sqrt(6)*atan(sqrt(6)*x/2 - sqrt(6)/3)/18
```

Giac [A] time = 1.0824, size = 22, normalized size = 1.05

$$\frac{1}{18}\sqrt{6}\arctan\left(\frac{1}{6}\sqrt{6}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(9*x^2-12*x+10),x, algorithm="giac")
```

```
[Out] 1/18*sqrt(6)*arctan(1/6*sqrt(6)*(3*x - 2))
```

3.271 $\int \frac{1}{x^4-2x^5+2x^6-2x^7+x^8}\,dx$

Optimal. Leaf size=53

$$-\frac{1}{x^2}-\frac{1}{3x^3}+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2(1-x)}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)$$

```
[Out] 1/(2*(1 - x)) - 1/(3*x^3) - x^(-2) - 2/x - (5*Log[1 - x])/2 + 2*Log[x] + Lo
g[1 + x^2]/4
```

Rubi [A] time = 0.0312722, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2058, 260}

$$-\frac{1}{x^2}-\frac{1}{3x^3}+\frac{1}{4}\log\left(x^2+1\right)+\frac{1}{2(1-x)}-\frac{2}{x}-\frac{5}{2}\log(1-x)+2\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8)^(-1),x]
```

```
[Out] 1/(2*(1 - x)) - 1/(3*x^3) - x^(-2) - 2/x - (5*Log[1 - x])/2 + 2*Log[x] + Lo
g[1 + x^2]/4
```

Rule 2058

```
Int[(P_)^(p_), x_Symbol] :> With[{u = Factor[P]}, Int[ExpandIntegrand[u^p,
x], x] /;  !SumQ[NonfreeFactors[u, x]]] /; PolyQ[P, x] && ILtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{1}{x^4-2x^5+2x^6-2x^7+x^8}\,dx = \int \left(\frac{1}{2(-1+x)^2} - \frac{5}{2(-1+x)} + \frac{1}{x^4} + \frac{2}{x^3} + \frac{2}{x^2} + \frac{2}{x} + \frac{x}{2\left(1+x^2\right)}\right) dx$$

$$= \frac{1}{2(1-x)} - \frac{1}{3x^3} - \frac{1}{x^2} - \frac{2}{x} - \frac{5}{2}\log(1-x) + 2\log(x) + \frac{1}{2}\int \frac{x}{1+x^2}\,dx$$

$$= \frac{1}{2(1-x)} - \frac{1}{3x^3} - \frac{1}{x^2} - \frac{2}{x} - \frac{5}{2}\log(1-x) + 2\log(x) + \frac{1}{4}\log\left(1+x^2\right)$$

Mathematica [A] time = 0.0197673, size = 51, normalized size = 0.96

$$-\frac{1}{x^2} - \frac{1}{3x^3} + \frac{1}{4}\log\left(x^2+1\right) - \frac{1}{2(x-1)} - \frac{2}{x} - \frac{5}{2}\log(1-x) + 2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4 - 2*x^5 + 2*x^6 - 2*x^7 + x^8)^(-1),x]
```

```
[Out] -1/(2*(-1 + x)) - 1/(3*x^3) - x^(-2) - 2/x - (5*Log[1 - x])/2 + 2*Log[x] +
Log[1 + x^2]/4
```

Maple [A] time = 0.01, size = 42, normalized size = 0.8

$$\frac{\ln\left(x^2+1\right)}{4} - \frac{1}{3\,x^3} - x^{-2} - 2\,x^{-1} + 2\,\ln(x) - \frac{1}{2\,x-2} - \frac{5\,\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x)
```

```
[Out] 1/4*ln(x^2+1)-1/3/x^3-1/x^2-2/x+2*ln(x)-1/2/(-1+x)-5/2*ln(-1+x)
```

Maxima [A] time = 1.41625, size = 63, normalized size = 1.19

$$-\frac{15\,x^3-6\,x^2-4\,x-2}{6\left(x^4-x^3\right)} + \frac{1}{4}\,\log\left(x^2+1\right) - \frac{5}{2}\,\log(x-1) + 2\,\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x, algorithm="maxima")
```

```
[Out] -1/6*(15*x^3 - 6*x^2 - 4*x - 2)/(x^4 - x^3) + 1/4*log(x^2 + 1) - 5/2*log(x
- 1) + 2*log(x)
```

Fricas [A] time = 1.79574, size = 173, normalized size = 3.26

$$-\frac{30\,x^3-12\,x^2-3\left(x^4-x^3\right)\log\left(x^2+1\right)+30\left(x^4-x^3\right)\log\left(x-1\right)-24\left(x^4-x^3\right)\log\left(x\right)-8\,x-4}{12\left(x^4-x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x, algorithm="fricas")
```

```
[Out] -1/12*(30*x^3 - 12*x^2 - 3*(x^4 - x^3)*log(x^2 + 1) + 30*(x^4 - x^3)*log(x
- 1) - 24*(x^4 - x^3)*log(x) - 8*x - 4)/(x^4 - x^3)
```

Sympy [A] time = 0.160552, size = 46, normalized size = 0.87

$$2\log\left(x\right)-\frac{5\log\left(x-1\right)}{2}+\frac{\log\left(x^2+1\right)}{4}-\frac{15x^3-6x^2-4x-2}{6x^4-6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**8-2*x**7+2*x**6-2*x**5+x**4),x)
```

```
[Out] 2*log(x) - 5*log(x - 1)/2 + log(x**2 + 1)/4 - (15*x**3 - 6*x**2 - 4*x - 2)/
(6*x**4 - 6*x**3)
```

Giac [A] time = 1.08327, size = 62, normalized size = 1.17

$$-\frac{15\,x^3-6\,x^2-4\,x-2}{6\,(x-1)x^3}+\frac{1}{4}\,\log\left(x^2+1\right)-\frac{5}{2}\,\log\left(|x-1|\right)+2\,\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^8-2*x^7+2*x^6-2*x^5+x^4),x, algorithm="giac")
```

```
[Out] -1/6*(15*x^3 - 6*x^2 - 4*x - 2)/((x - 1)*x^3) + 1/4*log(x^2 + 1) - 5/2*log(
abs(x - 1)) + 2*log(abs(x))
```

3.272 $\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)}\,dx$

Optimal. Leaf size=49

$$\frac{1}{12}\log(3-x)(27a+9b+3c+d)-\frac{1}{4}\log(x+1)(a-b+c-d)+ax-\frac{1}{3}d\log(x)$$

```
[Out] a*x + ((27*a + 9*b + 3*c + d)*Log[3 - x])/12 - (d*Log[x])/3 - ((a - b + c -
 d)*Log[1 + x])/4
```

Rubi [A] time = 0.0778306, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {1612}

$$\frac{1}{12}\log(3-x)(27a+9b+3c+d)-\frac{1}{4}\log(x+1)(a-b+c-d)+ax-\frac{1}{3}d\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(d + c*x + b*x^2 + a*x^3)/((-3 + x)*x*(1 + x)),x]
```

```
[Out] a*x + ((27*a + 9*b + 3*c + d)*Log[3 - x])/12 - (d*Log[x])/3 - ((a - b + c -
 d)*Log[1 + x])/4
```

Rule 1612

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

Rubi steps

$$\begin{aligned}\int \frac{d+cx+bx^2+ax^3}{(-3+x)x(1+x)}\,dx &= \int \left(a+\frac{27a+9b+3c+d}{12(-3+x)}-\frac{d}{3x}+\frac{-a+b-c+d}{4(1+x)}\right)dx\\ &= ax+\frac{1}{12}(27a+9b+3c+d)\log(3-x)-\frac{1}{3}d\log(x)-\frac{1}{4}(a-b+c-d)\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0260068, size = 49, normalized size = 1.

$$\frac{1}{12}\log(3-x)(27a+9b+3c+d)+\frac{1}{4}\log(x+1)(-a+b-c+d)+ax-\frac{1}{3}d\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + c*x + b*x^2 + a*x^3)/((-3 + x)*x*(1 + x)),x]
```

```
[Out] a*x + ((27*a + 9*b + 3*c + d)*Log[3 - x])/12 - (d*Log[x])/3 + ((-a + b - c
+ d)*Log[1 + x])/4
```

Maple [A] time = 0.006, size = 66, normalized size = 1.4

$$ax-\frac{d\ln(x)}{3}-\frac{\ln(1+x)a}{4}+\frac{\ln(1+x)b}{4}-\frac{\ln(1+x)c}{4}+\frac{\ln(1+x)d}{4}+\frac{9\ln(-3+x)a}{4}+\frac{3\ln(-3+x)b}{4}+\frac{\ln(-3+x)c}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x)
```

```
[Out] a*x-1/3*d*ln(x)-1/4*ln(1+x)*a+1/4*ln(1+x)*b-1/4*ln(1+x)*c+1/4*ln(1+x)*d+9/4
*ln(-3+x)*a+3/4*ln(-3+x)*b+1/4*ln(-3+x)*c+1/12*ln(-3+x)*d
```

Maxima [A] time = 0.936996, size = 55, normalized size = 1.12

$$ax-\frac{1}{4}(a-b+c-d)\log(x+1)+\frac{1}{12}(27a+9b+3c+d)\log(x-3)-\frac{1}{3}d\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x, algorithm="maxima")
```

```
[Out] a*x - 1/4*(a - b + c - d)*log(x + 1) + 1/12*(27*a + 9*b + 3*c + d)*log(x -
3) - 1/3*d*log(x)
```

Fricas [A] time = 1.97565, size = 127, normalized size = 2.59

$$ax-\frac{1}{4}(a-b+c-d)\log(x+1)+\frac{1}{12}(27a+9b+3c+d)\log(x-3)-\frac{1}{3}d\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x, algorithm="fricas")
```

```
[Out] a*x - 1/4*(a - b + c - d)*log(x + 1) + 1/12*(27*a + 9*b + 3*c + d)*log(x -
3) - 1/3*d*log(x)
```

Sympy [B] time = 25.1378, size = 762, normalized size = 15.55

$$ax - \frac{d\log(x)}{3} - \frac{(a-b+c-d)\log\left(x + \frac{-1512a^2d+1134a^2(a-b+c-d)-864abd+648ab(a-b+c-d)-432acd+324ac(a-b+c-d)-144ad^2+81a(a-b+c-}{1215a^3-567a^2b+1593a^2c-2691a^2d-567a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**3+b*x**2+c*x+d)/(-3+x)/x/(1+x),x)
```

```
[Out] a*x - d*log(x)/3 - (a - b + c - d)*log(x + (-1512*a**2*d + 1134*a**2*(a - b
 + c - d) - 864*a*b*d + 648*a*b*(a - b + c - d) - 432*a*c*d + 324*a*c*(a -
b + c - d) - 144*a*d**2 + 81*a*(a - b + c - d)**2 - 216*b**2*d + 162*b**2*(
a - b + c - d) - 288*b*d**2 + 108*b*d*(a - b + c - d) + 81*b*(a - b + c - d
)**2 - 72*c**2*d + 54*c**2*(a - b + c - d) + 144*c*d**2 - 72*c*d*(a - b + c
 - d) - 27*c*(a - b + c - d)**2 - 136*d**3 - 54*d**2*(a - b + c - d) + 117*
d*(a - b + c - d)**2)/(1215*a**3 - 567*a**2*b + 1593*a**2*c - 2691*a**2*d -
 567*a*b**2 + 378*a*b*c - 1638*a*b*d + 405*a*c**2 - 702*a*c*d - 351*a*d**2
- 81*b**3 - 27*b**2*c - 207*b**2*d + 81*b*c**2 - 270*b*c*d - 27*b*d**2 + 27
*c**3 - 27*c**2*d - 99*c*d**2 + 35*d**3))/4 + (27*a + 9*b + 3*c + d)*log(x
+ (-1512*a**2*d - 378*a**2*(27*a + 9*b + 3*c + d) - 864*a*b*d - 216*a*b*(27
*a + 9*b + 3*c + d) - 432*a*c*d - 108*a*c*(27*a + 9*b + 3*c + d) - 144*a*d*
*2 + 9*a*(27*a + 9*b + 3*c + d)**2 - 216*b**2*d - 54*b**2*(27*a + 9*b + 3*c
 + d) - 288*b*d**2 - 36*b*d*(27*a + 9*b + 3*c + d) + 9*b*(27*a + 9*b + 3*c
+ d)**2 - 72*c**2*d - 18*c**2*(27*a + 9*b + 3*c + d) + 144*c*d**2 + 24*c*d*
(27*a + 9*b + 3*c + d) - 3*c*(27*a + 9*b + 3*c + d)**2 - 136*d**3 + 18*d**2
*(27*a + 9*b + 3*c + d) + 13*d*(27*a + 9*b + 3*c + d)**2)/(1215*a**3 - 567*
a**2*b + 1593*a**2*c - 2691*a**2*d - 567*a*b**2 + 378*a*b*c - 1638*a*b*d +
405*a*c**2 - 702*a*c*d - 351*a*d**2 - 81*b**3 - 27*b**2*c - 207*b**2*d + 81
*b*c**2 - 270*b*c*d - 27*b*d**2 + 27*c**3 - 27*c**2*d - 99*c*d**2 + 35*d**3
))/12
```

Giac [A] time = 1.0817, size = 59, normalized size = 1.2

$$ax - \frac{1}{4}(a - b + c - d)\log(|x + 1|) + \frac{1}{12}(27\,a + 9\,b + 3\,c + d)\log(|x - 3|) - \frac{1}{3}\,d\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x^3+b*x^2+c*x+d)/(-3+x)/x/(1+x),x, algorithm="giac")
```

```
[Out] a*x - 1/4*(a - b + c - d)*log(abs(x + 1)) + 1/12*(27*a + 9*b + 3*c + d)*log
(abs(x - 3)) - 1/3*d*log(abs(x))
```

3.273 $\int \frac{1}{\left(2-\log\left(1+x^2\right)\right)^5} dx$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{1}{\left(2-\log\left(x^2+1\right)\right)^5},x\right)$$

```
[Out] Defer[Int][(2 - Log[1 + x^2])^(-5), x]
```

Rubi [A] time = 0.0038601, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{1}{\left(2-\log\left(1+x^2\right)\right)^5} dx$$

Verification is Not applicable to the result.

```
[In] Int[(2 - Log[1 + x^2])^(-5),x]
```

```
[Out] Defer[Int][(2 - Log[1 + x^2])^(-5), x]
```

Rubi steps

$$\int \frac{1}{\left(2-\log\left(1+x^2\right)\right)^5} dx = \int \frac{1}{\left(2-\log\left(1+x^2\right)\right)^5} dx$$

Mathematica [A] time = 2.81093, size = 0, normalized size = 0.

$$\int \frac{1}{\left(2-\log\left(1+x^2\right)\right)^5} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(2 - Log[1 + x^2])^(-5),x]
```

```
[Out] Integrate[(2 - Log[1 + x^2])^(-5), x]
```

Maple [A] time = 0.028, size = 0, normalized size = 0.

$$\int \left(2-\ln\left(x^2+1\right)\right)^{-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2-ln(x^2+1))^5,x)
```

```
[Out] int(1/(2-ln(x^2+1))^5,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{32\,x^8+56\,x^6+120\,x^4+\left(x^8-10\,x^4-24\,x^2-15\right)\log\left(x^2+1\right)^3-2\left(2\,x^8-x^6-33\,x^4-75\,x^2-45\right)\log\left(x^2+1\right)^2+216\,x^2}{384\left(x^7\log\left(x^2+1\right)^4-8\,x^7\log\left(x^2+1\right)^3+24\,x^7\log\left(x^2+1\right)^2-32\,x^7\log\left(x^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-log(x^2+1))^5,x, algorithm="maxima")
```

```
[Out] 1/384*(32*x^8 + 56*x^6 + 120*x^4 + (x^8 - 10*x^4 - 24*x^2 - 15)*log(x^2 + 1
)^3 - 2*(2*x^8 - x^6 - 33*x^4 - 75*x^2 - 45)*log(x^2 + 1)^2 + 216*x^2 + 4*(
3*x^8 - 2*x^6 - 38*x^4 - 78*x^2 - 45)*log(x^2 + 1) + 120)/(x^7*log(x^2 + 1)
^4 - 8*x^7*log(x^2 + 1)^3 + 24*x^7*log(x^2 + 1)^2 - 32*x^7*log(x^2 + 1) + 1
6*x^7) - integrate(1/384*(x^8 + 30*x^4 + 120*x^2 + 105)/(x^8*log(x^2 + 1) -
 2*x^8), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{1}{\log\left(x^2+1\right)^5-10\,\log\left(x^2+1\right)^4+40\,\log\left(x^2+1\right)^3-80\,\log\left(x^2+1\right)^2+80\,\log\left(x^2+1\right)-32},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-log(x^2+1))^5,x, algorithm="fricas")
```

```
[Out] integral(-1/(log(x^2 + 1)^5 - 10*log(x^2 + 1)^4 + 40*log(x^2 + 1)^3 - 80*lo
g(x^2 + 1)^2 + 80*log(x^2 + 1) - 32), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$-\frac{\int \frac{120x^2}{x^8\log\left(x^2+1\right)-2x^8}\,dx + \int \frac{30x^4}{x^8\log\left(x^2+1\right)-2x^8}\,dx + \int \frac{x^8}{x^8\log\left(x^2+1\right)-2x^8}\,dx + \int \frac{105}{x^8\log\left(x^2+1\right)-2x^8}\,dx}{384} + \frac{\frac{2x^8}{3}+\frac{7x^6}{6}+\frac{5x^4}{2}+\frac{9x^2}{2}+\left(\frac{x^8}{48}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-ln(x**2+1))**5,x)
```

```
[Out] -(Integral(120*x**2/(x**8*log(x**2 + 1) - 2*x**8), x) + Integral(30*x**4/(x
**8*log(x**2 + 1) - 2*x**8), x) + Integral(x**8/(x**8*log(x**2 + 1) - 2*x**
8), x) + Integral(105/(x**8*log(x**2 + 1) - 2*x**8), x))/384 + (2*x**8/3 +
7*x**6/6 + 5*x**4/2 + 9*x**2/2 + (x**8/48 - 5*x**4/24 - x**2/2 - 5/16)*log(
x**2 + 1)**3 + (-x**8/12 + x**6/24 + 11*x**4/8 + 25*x**2/8 + 15/8)*log(x**2
 + 1)**2 + (x**8/4 - x**6/6 - 19*x**4/6 - 13*x**2/2 - 15/4)*log(x**2 + 1) +
 5/2)/(8*x**7*log(x**2 + 1)**4 - 64*x**7*log(x**2 + 1)**3 + 192*x**7*log(x*
*2 + 1)**2 - 256*x**7*log(x**2 + 1) + 128*x**7)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\left(\log\left(x^2+1\right)-2\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-log(x^2+1))^5,x, algorithm="giac")
```

```
[Out] integrate(-1/(log(x^2 + 1) - 2)^5, x)
```

3.274 $\int \left(\frac{e^{x^2}}{x}+2e^{x^2}x\log(x)+\frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2}+\frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)}\right)dx$

Optimal. Leaf size=28

$$e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}+\log\left(x+\log^2(x)\right)$$

```
[Out] E^x^2*Log[x] - Log[x]/(x + Log[x]^2) + Log[x + Log[x]^2]
```

Rubi [A] time = 0.210937, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {2210, 2209, 2554, 12, 2547, 6742, 2538}

$$e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}+\log\left(x+\log^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x^2/x + 2*E^x^2*x*Log[x] + (-2 + Log[x])/(x + Log[x]^2)^2 + (1 + x^(-1) + (2*Log[x])/x)/(x + Log[x]^2),x]
```

```
[Out] E^x^2*Log[x] - Log[x]/(x + Log[x]^2) + Log[x + Log[x]^2]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ[d*e - c*f, 0]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2547

```
Int[(Log[(c_.)*(x_)^(n_.)]*(e_.) + (d_))/(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.)
+ (a_.)*(x_))^2, x_Symbol] :> -Simp[(e*Log[c*x^n])/(a*(a*x + b*Log[c*x^n]^q
)), x] + Dist[(d + e*n)/a, Int[1/(x*(a*x + b*Log[c*x^n]^q)), x], x] /; Free
Q[{a, b, c, d, e, n, q}, x] && EqQ[d + e*n*q, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2538

```
Int[Log[(c_.)*(x_)^(n_.)]^(r_.)/((x_)*(Log[(c_.)*(x_)^(n_.)]^(q_)*(b_.) + (
a_.)*(x_)^(m_.))), x_Symbol] :> Simp[Log[a*x^m + b*Log[c*x^n]^q]/(b*n*q), x
] - Dist[(a*m)/(b*n*q), Int[x^(m - 1)/(a*x^m + b*Log[c*x^n]^q), x], x] /; F
reeQ[{a, b, c, m, n, q, r}, x] && EqQ[r, q - 1]
```

Rubi steps

$$\int\left(\frac{e^{x^2}}{x}+2e^{x^2}x\log(x)+\frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2}+\frac{1+\frac{1}{x}+\frac{2\log(x)}{x}}{x+\log^2(x)}\right)dx=2\int e^{x^2}x\log(x)\,dx+\int\frac{e^{x^2}}{x}\,dx+\int\frac{-2+\log(x)}{\left(x+\log^2(x)\right)^2}\,dx$$

$$=\frac{\text{Ei}\left(x^2\right)}{2}+e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}-2\int\frac{e^{x^2}}{2x}\,dx-\int$$

$$=\frac{\text{Ei}\left(x^2\right)}{2}+e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}+2\int\frac{\log(x)}{x\left(x+\log^2(x)\right)}$$

$$=e^{x^2}\log(x)-\frac{\log(x)}{x+\log^2(x)}+\log\left(x+\log^2(x)\right)$$

Mathematica [A] time = 10.2476, size = 28, normalized size = 1.

$$e^{x^2}\log(x) - \frac{\log(x)}{x+\log^2(x)} + \log\left(x+\log^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^2/x + 2*E^x^2*x*Log[x] + (-2 + Log[x])/(x + Log[x]^2)^2 + (1
+ x^(-1) + (2*Log[x])/x)/(x + Log[x]^2),x]
```

```
[Out] E^x^2*Log[x] - Log[x]/(x + Log[x]^2) + Log[x + Log[x]^2]
```

Maple [A] time = 0.031, size = 28, normalized size = 1.

$$e^{x^2}\ln(x) - \frac{\ln(x)}{x+(\ln(x))^2} + \ln\left(x+(\ln(x))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^2)/x+2*exp(x^2)*x*ln(x)+(-2+ln(x))/(x+ln(x)^2)^2+(1+1/x+2*ln(x)/x
)/(x+ln(x)^2),x)
```

```
[Out] exp(x^2)*ln(x)-ln(x)/(x+ln(x)^2)+ln(x+ln(x)^2)
```

Maxima [A] time = 1.05943, size = 36, normalized size = 1.29

$$e^{\left(x^2\right)}\log(x) - \frac{\log(x)}{\log(x)^2+x} + \log\left(\log(x)^2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)^2+(1+1/x+
2*log(x)/x)/(x+log(x)^2),x, algorithm="maxima")
```

```
[Out] e^(x^2)*log(x) - log(x)/(log(x)^2 + x) + log(log(x)^2 + x)
```

Fricas [A] time = 1.87339, size = 128, normalized size = 4.57

$$\frac{e^{\left(x^2\right)}\log\left(x\right)^3+\left(\log\left(x\right)^2+x\right)\log\left(\log\left(x\right)^2+x\right)+\left(xe^{\left(x^2\right)}-1\right)\log\left(x\right)}{\log\left(x\right)^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)^2+(1+1/x+
2*log(x)/x)/(x+log(x)^2),x, algorithm="fricas")
```

```
[Out] (e^(x^2)*log(x)^3 + (log(x)^2 + x)*log(log(x)^2 + x) + (x*e^(x^2) - 1)*log(
x))/(log(x)^2 + x)
```

Sympy [A] time = 0.442609, size = 26, normalized size = 0.93

$$e^{x^2}\log\left(x\right)+\log\left(x+\log\left(x\right)^2\right)-\frac{\log\left(x\right)}{x+\log\left(x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**2)/x+2*exp(x**2)*x*ln(x)+(-2+ln(x))/(x+ln(x)**2)**2+(1+1/x
+2*ln(x)/x)/(x+ln(x)**2),x)
```

```
[Out] exp(x**2)*log(x) + log(x + log(x)**2) - log(x)/(x + log(x)**2)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int 2\,xe^{\left(x^2\right)}\log\left(x\right)+\frac{\frac{2\log(x)}{x}+\frac{1}{x}+1}{\log\left(x\right)^2+x}+\frac{e^{\left(x^2\right)}}{x}+\frac{\log\left(x\right)-2}{\left(\log\left(x\right)^2+x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^2)/x+2*exp(x^2)*x*log(x)+(-2+log(x))/(x+log(x)^2)^2+(1+1/x+
2*log(x)/x)/(x+log(x)^2),x, algorithm="giac")
```

```
[Out] integrate(2*x*e^(x^2)*log(x) + (2*log(x)/x + 1/x + 1)/(log(x)^2 + x) + e^(x
^2)/x + (log(x) - 2)/(log(x)^2 + x)^2, x)
```

3.275 $\int e^{\frac{x}{2}+xz}x^4\sin^4(\pi z)\,dz$

Optimal. Leaf size=199

$$\frac{24\pi^4x^3e^{xz+\frac{x}{2}}}{x^4+20\pi^2x^2+64\pi^4}+\frac{x^5e^{xz+\frac{x}{2}}\sin^4(\pi z)}{x^2+16\pi^2}+\frac{12\pi^2x^5e^{xz+\frac{x}{2}}\sin^2(\pi z)}{x^4+20\pi^2x^2+64\pi^4}-\frac{4\pi x^4e^{xz+\frac{x}{2}}\sin^3(\pi z)\cos(\pi z)}{x^2+16\pi^2}-\frac{24\pi^3x^4e^{xz+\frac{x}{2}}\sin(\pi z)\cos(\pi z)}{x^4+20\pi^2x^2+64\pi^4}$$

```
[Out] (24*E^(x/2 + x*z)*Pi^4*x^3)/(64*Pi^4 + 20*Pi^2*x^2 + x^4) - (24*E^(x/2 + x*
z)*Pi^3*x^4*Cos[Pi*z]*Sin[Pi*z])/(64*Pi^4 + 20*Pi^2*x^2 + x^4) + (12*E^(x/2
 + x*z)*Pi^2*x^5*Sin[Pi*z]^2)/(64*Pi^4 + 20*Pi^2*x^2 + x^4) - (4*E^(x/2 + x
*z)*Pi*x^4*Cos[Pi*z]*Sin[Pi*z]^3)/(16*Pi^2 + x^2) + (E^(x/2 + x*z)*x^5*Sin[
Pi*z]^4)/(16*Pi^2 + x^2)
```

Rubi [A] time = 0.101416, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {12, 4434, 2194}

$$\frac{24\pi^4x^3e^{xz+\frac{x}{2}}}{x^4+20\pi^2x^2+64\pi^4}+\frac{x^5e^{xz+\frac{x}{2}}\sin^4(\pi z)}{x^2+16\pi^2}+\frac{12\pi^2x^5e^{xz+\frac{x}{2}}\sin^2(\pi z)}{x^4+20\pi^2x^2+64\pi^4}-\frac{4\pi x^4e^{xz+\frac{x}{2}}\sin^3(\pi z)\cos(\pi z)}{x^2+16\pi^2}-\frac{24\pi^3x^4e^{xz+\frac{x}{2}}\sin(\pi z)\cos(\pi z)}{x^4+20\pi^2x^2+64\pi^4}$$

Antiderivative was successfully verified.

```
[In] Int[E^(x/2 + x*z)*x^4*Sin[Pi*z]^4,z]
```


```
[Out] (24*E^(x/2 + x*z)*Pi^4*x^3)/(64*Pi^4 + 20*Pi^2*x^2 + x^4) - (24*E^(x/2 + x*
z)*Pi^3*x^4*Cos[Pi*z]*Sin[Pi*z])/(64*Pi^4 + 20*Pi^2*x^2 + x^4) + (12*E^(x/2
 + x*z)*Pi^2*x^5*Sin[Pi*z]^2)/(64*Pi^4 + 20*Pi^2*x^2 + x^4) - (4*E^(x/2 + x
*z)*Pi*x^4*Cos[Pi*z]*Sin[Pi*z]^3)/(16*Pi^2 + x^2) + (E^(x/2 + x*z)*x^5*Sin[
Pi*z]^4)/(16*Pi^2 + x^2)
```


Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4434

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)]^(n_), x_Symbo
l] :> Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x]^n)/(e^2*n^2 + b^2*c^2*L
og[F]^2), x] + (Dist[(n*(n - 1)*e^2)/(e^2*n^2 + b^2*c^2*Log[F]^2), Int[F^(c
*(a + b*x))*Sin[d + e*x]^(n - 2), x], x] - Simp[(e*n*F^(c*(a + b*x))*Cos[d
```

```
+ e*x]*Sin[d + e*x]^(n - 1))/(e^2*n^2 + b^2*c^2*Log[F]^2), x]) /; FreeQ[{F,
 a, b, c, d, e}, x] && NeQ[e^2*n^2 + b^2*c^2*Log[F]^2, 0] && GtQ[n, 1]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\int e^{\frac{x}{2}+xz}x^4\sin^4(\pi z)\,dz = x^4\int e^{\frac{x}{2}+xz}\sin^4(\pi z)\,dz$$
$$= -\frac{4e^{\frac{x}{2}+xz}\pi x^4\cos(\pi z)\sin^3(\pi z)}{16\pi^2+x^2}+\frac{e^{\frac{x}{2}+xz}x^5\sin^4(\pi z)}{16\pi^2+x^2}+\frac{\left(12\pi^2x^4\right)\int e^{\frac{x}{2}+xz}\sin^2(\pi z)\,dz}{16\pi^2+x^2}$$
$$= -\frac{24e^{\frac{x}{2}+xz}\pi^3x^4\cos(\pi z)\sin(\pi z)}{64\pi^4+20\pi^2x^2+x^4}+\frac{12e^{\frac{x}{2}+xz}\pi^2x^5\sin^2(\pi z)}{64\pi^4+20\pi^2x^2+x^4}-\frac{4e^{\frac{x}{2}+xz}\pi x^4\cos(\pi z)\sin^3(\pi z)}{16\pi^2+x^2}+\frac{e^{\frac{x}{2}+x}}{1}$$
$$= \frac{24e^{\frac{x}{2}+xz}\pi^4x^3}{64\pi^4+20\pi^2x^2+x^4}-\frac{24e^{\frac{x}{2}+xz}\pi^3x^4\cos(\pi z)\sin(\pi z)}{64\pi^4+20\pi^2x^2+x^4}+\frac{12e^{\frac{x}{2}+xz}\pi^2x^5\sin^2(\pi z)}{64\pi^4+20\pi^2x^2+x^4}-\frac{4e^{\frac{x}{2}+xz}\pi x^4\text{co}}{16\pi^2}$$

Mathematica [A] time = 0.208029, size = 136, normalized size = 0.68

$$\frac{x^4e^{x\left(z+\frac{1}{2}\right)}\left(-8\pi x^3\sin(2\pi z)+4\pi x^3\sin(4\pi z)-4\left(x^2+16\pi^2\right)x^2\cos(2\pi z)+\left(x^2+4\pi^2\right)x^2\cos(4\pi z)+3x^4+60\pi^2x^2-12\right.}{8\left(x^5+20\pi^2x^3+64\pi^4x\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(x/2 + x*z)*x^4*Sin[Pi*z]^4,z]
```

```
[Out] (E^(x*(1/2 + z))*x^4*(192*Pi^4 + 60*Pi^2*x^2 + 3*x^4 - 4*x^2*(16*Pi^2 + x^2
)*Cos[2*Pi*z] + x^2*(4*Pi^2 + x^2)*Cos[4*Pi*z] - 128*Pi^3*x*Sin[2*Pi*z] - 8
*Pi*x^3*Sin[2*Pi*z] + 16*Pi^3*x*Sin[4*Pi*z] + 4*Pi*x^3*Sin[4*Pi*z]))/(8*(64
*Pi^4*x + 20*Pi^2*x^3 + x^5))
```

Maple [A] time = 0.035, size = 127, normalized size = 0.6

$$\frac{x^4}{8}\left(3\,\frac{e^{x/2+xz}}{x}+\frac{x\cos(4\,\pi\,z)}{16\,\pi^2+x^2}e^{\frac{x}{2}+xz}+4\,\frac{\pi\,e^{x/2+xz}\sin(4\,\pi\,z)}{16\,\pi^2+x^2}-4\,\frac{xe^{x/2+xz}\cos(2\,\pi\,z)}{4\,\pi^2+x^2}-8\,\frac{\pi\,e^{x/2+xz}\sin(2\,\pi\,z)}{4\,\pi^2+x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*exp(1/2*x+x*z)*sin(Pi*z)^4,z)
```

```
[Out] 1/8*x^4*(3/x*exp(1/2*x+x*z)+x/(16*Pi^2+x^2)*exp(1/2*x+x*z)*cos(4*Pi*z)+4*Pi
/(16*Pi^2+x^2)*exp(1/2*x+x*z)*sin(4*Pi*z)-4*x/(4*Pi^2+x^2)*exp(1/2*x+x*z)*c
os(2*Pi*z)-8*Pi/(4*Pi^2+x^2)*exp(1/2*x+x*z)*sin(2*Pi*z))
```

Maxima [A] time = 1.02902, size = 216, normalized size = 1.09

$$\frac{\left(\left(4\,\pi^2x^2+x^4\right)\cos\left(4\,\pi z\right)e^{\left(xz+\frac{1}{2}\,x\right)}-4\left(16\,\pi^2x^2+x^4\right)\cos\left(2\,\pi z\right)e^{\left(xz+\frac{1}{2}\,x\right)}+4\left(4\,\pi^3x+\pi x^3\right)e^{\left(xz+\frac{1}{2}\,x\right)}\sin\left(4\,\pi z\right)-8\left(16\,\pi^3x\right.\right.}{8\left(64\,\pi^4x+20\,\pi^2x^3+x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z, algorithm="maxima")
```

```
[Out] 1/8*((4*pi^2*x^2 + x^4)*cos(4*pi*z)*e^(x*z + 1/2*x) - 4*(16*pi^2*x^2 + x^4)
*cos(2*pi*z)*e^(x*z + 1/2*x) + 4*(4*pi^3*x + pi*x^3)*e^(x*z + 1/2*x)*sin(4*
pi*z) - 8*(16*pi^3*x + pi*x^3)*e^(x*z + 1/2*x)*sin(2*pi*z) + 3*(64*pi^4 + 2
0*pi^2*x^2 + x^4)*e^(x*z + 1/2*x))*x^4/(64*pi^4*x + 20*pi^2*x^3 + x^5)
```

Fricas [A] time = 2.1752, size = 342, normalized size = 1.72

$$\frac{4\left(\left(\left(4\,\pi^3x^4+\pi x^6\right)\cos\left(\pi z\right)^3-\left(10\,\pi^3x^4+\pi x^6\right)\cos\left(\pi z\right)\right)e^{\left(xz+\frac{1}{2}\,x\right)}\sin\left(\pi z\right)+\left(24\,\pi^4x^3+16\,\pi^2x^5+x^7+\left(4\,\pi^2x^5+x^7\right)\cos\right.\right.}{64\,\pi^4+20\,\pi^2x^2+x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z, algorithm="fricas")
```

```
[Out] (4*((4*pi^3*x^4 + pi*x^6)*cos(pi*z)^3 - (10*pi^3*x^4 + pi*x^6)*cos(pi*z))*e
^(x*z + 1/2*x)*sin(pi*z) + (24*pi^4*x^3 + 16*pi^2*x^5 + x^7 + (4*pi^2*x^5 +
 x^7)*cos(pi*z)^4 - 2*(10*pi^2*x^5 + x^7)*cos(pi*z)^2)*e^(x*z + 1/2*x))/(64
*pi^4 + 20*pi^2*x^2 + x^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*exp(1/2*x+x*z)*sin(pi*z)**4,z)
```

```
[Out] Timed out
```

Giac [A] time = 1.09004, size = 154, normalized size = 0.77

$$\frac{1}{8}\left(\left(\frac{x\cos(4\pi z)}{16\pi^2+x^2}+\frac{4\pi\sin(4\pi z)}{16\pi^2+x^2}\right)e^{\left(xz+\frac{1}{2}x\right)}-4\left(\frac{x\cos(2\pi z)}{4\pi^2+x^2}+\frac{2\pi\sin(2\pi z)}{4\pi^2+x^2}\right)e^{\left(xz+\frac{1}{2}x\right)}+\frac{3e^{\left(xz+\frac{1}{2}x\right)}}{x}\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*exp(1/2*x+x*z)*sin(pi*z)^4,z, algorithm="giac")
```

```
[Out] 1/8*((x*cos(4*pi*z)/(16*pi^2 + x^2) + 4*pi*sin(4*pi*z)/(16*pi^2 + x^2))*e^(
x*z + 1/2*x) - 4*(x*cos(2*pi*z)/(4*pi^2 + x^2) + 2*pi*sin(2*pi*z)/(4*pi^2 +
 x^2))*e^(x*z + 1/2*x) + 3*e^(x*z + 1/2*x)/x)*x^4
```

3.276 $\int \mathbf{Erf}(x)\,dx$

Optimal. Leaf size=18

$$x\text{Erf}(x)+\frac{e^{-x^2}}{\sqrt{\pi}}$$

```
[Out] 1/(E^x^2*Sqrt[Pi]) + x*Erf[x]
```

Rubi [A] time = 0.0053284, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {6349}

$$x\text{Erf}(x)+\frac{e^{-x^2}}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[x],x]
```

```
[Out] 1/(E^x^2*Sqrt[Pi]) + x*Erf[x]
```

Rule 6349

```
Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Erf[a + b*x])/b,
x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \text{erf}(x)\,dx=\frac{e^{-x^2}}{\sqrt{\pi}}+x\text{erf}(x)$$

Mathematica [A] time = 0.0066766, size = 18, normalized size = 1.

$$x\text{Erf}(x)+\frac{e^{-x^2}}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[x],x]
```

```
[Out] 1/(E^x^2*Sqrt[Pi]) + x*Erf[x]
```

Maple [A] time = 0.006, size = 16, normalized size = 0.9

$$x\mathrm{Erf}\left(x\right)+\frac{\mathrm{e}^{-x^{2}}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(x),x)
```

```
[Out] x*erf(x)+1/Pi^(1/2)*exp(-x^2)
```

Maxima [A] time = 0.928914, size = 20, normalized size = 1.11

$$x\,\mathrm{erf}\left(x\right)+\frac{e^{\left(-x^{2}\right)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(x),x, algorithm="maxima")
```

```
[Out] x*erf(x) + e^(-x^2)/sqrt(pi)
```

Fricas [A] time = 1.80279, size = 51, normalized size = 2.83

$$\frac{\pi x\,\mathrm{erf}\left(x\right)+\sqrt{\pi}e^{\left(-x^{2}\right)}}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(x),x, algorithm="fricas")
```

```
[Out] (pi*x*erf(x) + sqrt(pi)*e^(-x^2))/pi
```

Sympy [A] time = 0.307698, size = 15, normalized size = 0.83

$$x\operatorname{erf}(x) + \frac{e^{-x^2}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(x),x)
```

```
[Out] x*erf(x) + exp(-x**2)/sqrt(pi)
```

Giac [A] time = 1.07719, size = 20, normalized size = 1.11

$$x\operatorname{erf}(x) + \frac{e^{\left(-x^2\right)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(x),x, algorithm="giac")
```

```
[Out] x*erf(x) + e^(-x^2)/sqrt(pi)
```

3.277 $\int \mathbf{Erf}(a+x)\,dx$

Optimal. Leaf size=24

$$(a+x)\text{Erf}(a+x)+\frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

```
[Out] 1/(E^(a + x)^2*Sqrt[Pi]) + (a + x)*Erf[a + x]
```

Rubi [A] time = 0.0076567, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6349}

$$(a+x)\text{Erf}(a+x)+\frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Int[Erf[a + x],x]
```

```
[Out] 1/(E^(a + x)^2*Sqrt[Pi]) + (a + x)*Erf[a + x]
```

Rule 6349

```
Int[Erf[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((a + b*x)*Erf[a + b*x])/b,
x] + Simp[1/(b*Sqrt[Pi]*E^(a + b*x)^2), x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \text{erf}(a+x)\,dx=\frac{e^{-(a+x)^2}}{\sqrt{\pi}}+(a+x)\text{erf}(a+x)$$

Mathematica [A] time = 0.0276084, size = 24, normalized size = 1.

$$(a+x)\text{Erf}(a+x)+\frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Erf[a + x],x]
```

```
[Out] 1/(E^(a + x)^2*Sqrt[Pi]) + (a + x)*Erf[a + x]
```

Maple [A] time = 0.001, size = 22, normalized size = 0.9

$$(a+x)\operatorname{Erf}(a+x)+\frac{e^{-(a+x)^2}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(erf(a+x),x)
```

```
[Out] (a+x)*erf(a+x)+1/Pi^(1/2)*exp(-(a+x)^2)
```

Maxima [A] time = 0.931253, size = 28, normalized size = 1.17

$$(a+x)\operatorname{erf}(a+x)+\frac{e^{\left(-(a+x)^2\right)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(a+x),x, algorithm="maxima")
```

```
[Out] (a + x)*erf(a + x) + e^(-(a + x)^2)/sqrt(pi)
```

Fricas [A] time = 1.87621, size = 88, normalized size = 3.67

$$\frac{(\pi a+\pi x)\operatorname{erf}(a+x)+\sqrt{\pi}e^{\left(-a^2-2\,ax-x^2\right)}}{\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(a+x),x, algorithm="fricas")
```

```
[Out] ((pi*a + pi*x)*erf(a + x) + sqrt(pi)*e^(-a^2 - 2*a*x - x^2))/pi
```

Sympy [A] time = 0.497251, size = 36, normalized size = 1.5

$$a \operatorname{erf}(a+x) + x \operatorname{erf}(a+x) + \frac{e^{-a^2}e^{-x^2}e^{-2ax}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(a+x),x)
```

```
[Out] a*erf(a + x) + x*erf(a + x) + exp(-a**2)*exp(-x**2)*exp(-2*a*x)/sqrt(pi)
```

Giac [A] time = 1.07223, size = 50, normalized size = 2.08

$$x \operatorname{erf}(a+x) + \frac{\sqrt{\pi} a \operatorname{erf}(a+x) + e^{\left(-a^2-2\,ax-x^2\right)}}{\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(erf(a+x),x, algorithm="giac")
```

```
[Out] x*erf(a + x) + (sqrt(pi)*a*erf(a + x) + e^(-a^2 - 2*a*x - x^2))/sqrt(pi)
```

3.278 $\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx$

Optimal. Leaf size=94

$$\frac{(2x+1)\sqrt{x^4+4x^3+2x^2+1}}{2(2x^2-1)}-\tanh^{-1}\left(\frac{x(x+2)(33x^3+27x^2-x+7)}{(31x^3+37x^2+2)\sqrt{x^4+4x^3+2x^2+1}}\right)$$

```
[Out] ((1 + 2*x)*Sqrt[1 + 2*x^2 + 4*x^3 + x^4])/(2*(-1 + 2*x^2)) - ArcTanh[(x*(2
+ x)*(7 - x + 27*x^2 + 33*x^3))/((2 + 37*x^2 + 31*x^3)*Sqrt[1 + 2*x^2 + 4*x
^3 + x^4])]
```

Rubi [F] time = 1.83995, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{(-1+2x^2)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(-8 - 8*x - x^2 - 3*x^3 + 7*x^4 + 4*x^5 + 2*x^6)/((-1 + 2*x^2)^2*Sqrt[1
 + 2*x^2 + 4*x^3 + x^4]),x]
```

```
[Out] (9*Defer[Int][1/Sqrt[1 + 2*x^2 + 4*x^3 + x^4], x])/4 - (13*Defer[Int][1/((S
qrt[2] - 2*x)^2*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]), x])/4 + Defer[Int][x/Sqrt[1
 + 2*x^2 + 4*x^3 + x^4], x] + Defer[Int][x^2/Sqrt[1 + 2*x^2 + 4*x^3 + x^4],
 x]/2 - (13*Defer[Int][1/((Sqrt[2] + 2*x)^2*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]),
 x])/4 - (13*Defer[Int][1/((1 - Sqrt[2]*x)*Sqrt[1 + 2*x^2 + 4*x^3 + x^4]),
x])/8 - ((15 + Sqrt[2])*Defer[Int][1/((1 - Sqrt[2]*x)*Sqrt[1 + 2*x^2 + 4*x^
3 + x^4]), x])/8 - (13*Defer[Int][1/((1 + Sqrt[2]*x)*Sqrt[1 + 2*x^2 + 4*x^3
 + x^4]), x])/8 - ((15 - Sqrt[2])*Defer[Int][1/((1 + Sqrt[2]*x)*Sqrt[1 + 2*
x^2 + 4*x^3 + x^4]), x])/8 - (17*Defer[Int][x/((-1 + 2*x^2)^2*Sqrt[1 + 2*x^
2 + 4*x^3 + x^4]), x])/2
```

Rubi steps

$$\int \frac{-8-8x-x^2-3x^3+7x^4+4x^5+2x^6}{\left(-1+2x^2\right)^2\sqrt{1+2x^2+4x^3+x^4}}\,dx = \int \left(\frac{9}{4\sqrt{1+2x^2+4x^3+x^4}}+\frac{x}{\sqrt{1+2x^2+4x^3+x^4}}+\frac{x^2}{2\sqrt{1+2x^2+4x^3+\cdots}}\right.$$

$$= \frac{1}{4}\int \frac{15+2x}{\left(-1+2x^2\right)\sqrt{1+2x^2+4x^3+x^4}}\,dx+\frac{1}{2}\int \frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}\,dx \cdots$$

$$= \frac{1}{4}\int \left(-\frac{15+\sqrt{2}}{2\left(1-\sqrt{2}x\right)\sqrt{1+2x^2+4x^3+x^4}}-\frac{15-\sqrt{2}}{2\left(1+\sqrt{2}x\right)\sqrt{1+2x^2+4x^3 \cdots}}\right.$$

$$= \frac{1}{2}\int \frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}\,dx+\frac{9}{4}\int \frac{1}{\sqrt{1+2x^2+4x^3+x^4}}\,dx-\frac{13}{2}\int \cdots$$

$$= \frac{1}{2}\int \frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}\,dx+\frac{9}{4}\int \frac{1}{\sqrt{1+2x^2+4x^3+x^4}}\,dx-\frac{13}{2}\int \Bigg(\cdots$$

$$= \frac{1}{2}\int \frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}\,dx+\frac{9}{4}\int \frac{1}{\sqrt{1+2x^2+4x^3+x^4}}\,dx-\frac{13}{4}\int \cdots$$

$$= \frac{1}{2}\int \frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}\,dx+\frac{9}{4}\int \frac{1}{\sqrt{1+2x^2+4x^3+x^4}}\,dx-\frac{13}{4}\int \cdots$$

$$= \frac{1}{2}\int \frac{x^2}{\sqrt{1+2x^2+4x^3+x^4}}\,dx-\frac{13}{8}\int \frac{1}{\left(1-\sqrt{2}x\right)\sqrt{1+2x^2+4x^3+x^4}}\,dx \cdots$$

Mathematica [C] time = 6.41877, size = 5137, normalized size = 54.65

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(-8 - 8*x - x^2 - 3*x^3 + 7*x^4 + 4*x^5 + 2*x^6)/((-1 + 2*x^2)^2*
Sqrt[1 + 2*x^2 + 4*x^3 + x^4]),x]
```

```
[Out] Result too large to show
```

Maple [B] time = 0.914, size = 1197351, normalized size = 12737.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+1)^(1/
2),x)
```

```
[Out] result too large to display
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2\,x^{6} + 4\,x^{5} + 7\,x^{4} - 3\,x^{3} - x^{2} - 8\,x - 8}{\sqrt{x^{4} + 4\,x^{3} + 2\,x^{2} + 1}\left(2\,x^{2} - 1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+
1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^6 + 4*x^5 + 7*x^4 - 3*x^3 - x^2 - 8*x - 8)/(sqrt(x^4 + 4*x^3
 + 2*x^2 + 1)*(2*x^2 - 1)^2), x)
```

Fricas [B] time = 2.69053, size = 464, normalized size = 4.94

$$\frac{\left(2\,x^{2} - 1\right)\log\left(\frac{1025\,x^{10} + 6138\,x^{9} + 12307\,x^{8} + 10188\,x^{7} + 4503\,x^{6} + 3134\,x^{5} + 1589\,x^{4} + 140\,x^{3} + 176\,x^{2} - \left(1023\,x^{8} + 4104\,x^{7} + 5084\,x^{6} + 2182\,x^{5} + 805\,x^{4} + 624\,x^{3} + 10\,x \cdots}{32\,x^{10} - 80\,x^{8} + 80\,x^{6} - 40\,x^{4} + 10\,x^{2} - 1}\right. \cdots}{2\left(2\,x^{2} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+
1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((2*x^2 - 1)*log((1025*x^10 + 6138*x^9 + 12307*x^8 + 10188*x^7 + 4503*x
^6 + 3134*x^5 + 1589*x^4 + 140*x^3 + 176*x^2 - (1023*x^8 + 4104*x^7 + 5084*
x^6 + 2182*x^5 + 805*x^4 + 624*x^3 + 10*x^2 + 28*x)*sqrt(x^4 + 4*x^3 + 2*x^
2 + 1) + 2)/(32*x^10 - 80*x^8 + 80*x^6 - 40*x^4 + 10*x^2 - 1)) + sqrt(x^4 +
 4*x^3 + 2*x^2 + 1)*(2*x + 1))/(2*x^2 - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^6+4x^5+7x^4-3x^3-x^2-8x-8}{\sqrt{(x+1)\left(x^3+3x^2-x+1\right)}\left(2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**6+4*x**5+7*x**4-3*x**3-x**2-8*x-8)/(2*x**2-1)**2/(x**4+4*x*
*3+2*x**2+1)**(1/2),x)
```

```
[Out] Integral((2*x**6 + 4*x**5 + 7*x**4 - 3*x**3 - x**2 - 8*x - 8)/(sqrt((x + 1)
*(x**3 + 3*x**2 - x + 1))*(2*x**2 - 1)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{2x^6+4x^5+7x^4-3x^3-x^2-8x-8}{\sqrt{x^4+4x^3+2x^2+1}\left(2x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^6+4*x^5+7*x^4-3*x^3-x^2-8*x-8)/(2*x^2-1)^2/(x^4+4*x^3+2*x^2+
1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*x^6 + 4*x^5 + 7*x^4 - 3*x^3 - x^2 - 8*x - 8)/(sqrt(x^4 + 4*x^3
 + 2*x^2 + 1)*(2*x^2 - 1)^2), x)
```

3.279 $\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy$

Optimal. Leaf size=142

$$-\frac{1}{4}\tanh^{-1}\left(\frac{(1-3y)\sqrt{-5y^2-5y+1}}{(1-5y)\sqrt{-y^2-y+1}}\right)-\frac{1}{2}\tanh^{-1}\left(\frac{(3y+4)\sqrt{-5y^2-5y+1}}{(5y+6)\sqrt{-y^2-y+1}}\right)+\frac{9}{4}\tanh^{-1}\left(\frac{(7y+11)\sqrt{-5y^2-5y+1}}{3(5y+7)\sqrt{-y^2-y+1}}\right)$$

```
[Out] -ArcTanh[((1 - 3*y)*Sqrt[1 - 5*y - 5*y^2])/((1 - 5*y)*Sqrt[1 - y - y^2])]/4
 - ArcTanh[((4 + 3*y)*Sqrt[1 - 5*y - 5*y^2])/((6 + 5*y)*Sqrt[1 - y - y^2])]
/2 + (9*ArcTanh[((11 + 7*y)*Sqrt[1 - 5*y - 5*y^2])/(3*(7 + 5*y)*Sqrt[1 - y
- y^2])])/4
```

Rubi [F] time = 3.3236, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy$$

Verification is Not applicable to the result.

```
[In] Int[((1 + 2*y)*Sqrt[1 - 5*y - 5*y^2])/(y*(1 + y)*(2 + y)*Sqrt[1 - y - y^2])
,y]
```

```
[Out] Defer[Int][Sqrt[1 - 5*y - 5*y^2]/(y*Sqrt[1 - y - y^2]), y]/2 + Defer[Int][S
qrt[1 - 5*y - 5*y^2]/((1 + y)*Sqrt[1 - y - y^2]), y] - (3*Defer[Int][Sqrt[1
 - 5*y - 5*y^2]/((2 + y)*Sqrt[1 - y - y^2]), y])/2
```

Rubi steps

$$\begin{aligned}\int \frac{(1+2y)\sqrt{1-5y-5y^2}}{y(1+y)(2+y)\sqrt{1-y-y^2}}\,dy &= \int \left(\frac{\sqrt{1-5y-5y^2}}{2y\sqrt{1-y-y^2}}+\frac{\sqrt{1-5y-5y^2}}{(1+y)\sqrt{1-y-y^2}}-\frac{3\sqrt{1-5y-5y^2}}{2(2+y)\sqrt{1-y-y^2}}\right)dy\\ &= \frac{1}{2}\int \frac{\sqrt{1-5y-5y^2}}{y\sqrt{1-y-y^2}}\,dy-\frac{3}{2}\int \frac{\sqrt{1-5y-5y^2}}{(2+y)\sqrt{1-y-y^2}}\,dy+\int \frac{\sqrt{1-5y-5y^2}}{(1+y)\sqrt{1-y-y^2}}\,dy\end{aligned}$$

Mathematica [C] time = 1.52404, size = 630, normalized size = 4.44

$$\left(-1-\frac{2}{\sqrt{5}}\right)\left(2y+\sqrt{5}+1\right)^2\sqrt{\frac{10y+3\sqrt{5}+5}{10y+5\sqrt{5}+5}}\left(20\left(\sqrt{5}\sqrt{\frac{-10y+3\sqrt{5}-5}{2y+\sqrt{5}+1}}\sqrt{\frac{-2y+\sqrt{5}-1}{2y+\sqrt{5}+1}}-4\sqrt{\frac{-10y+3\sqrt{5}-5}{2y+\sqrt{5}+1}}\sqrt{\frac{-2y+\sqrt{5}-1}{2y+\sqrt{5}+1}}-2\sqrt{5}\sqrt{-\frac{2\sqrt{5}y+\sqrt{5}}{2y+\sqrt{5}+}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((1 + 2*y)*Sqrt[1 - 5*y - 5*y^2])/(y*(1 + y)*(2 + y)*Sqrt[1 - y -
 y^2]),y]
```

```
[Out] ((-1 - 2/Sqrt[5])*(1 + Sqrt[5] + 2*y)^2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(5 + 5*
Sqrt[5] + 10*y)]*(20*(-4*Sqrt[(-5 + 3*Sqrt[5] - 10*y)/(1 + Sqrt[5] + 2*y)]*
Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*y)] + Sqrt[5]*Sqrt[(-5 + 3*Sqrt[
5] - 10*y)/(1 + Sqrt[5] + 2*y)]*Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*
y)] + 5*Sqrt[-((-5 + Sqrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))]*Sqrt[-((-
3 + Sqrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))] - 2*Sqrt[5]*Sqrt[-((-5 + S
qrt[5] + 2*Sqrt[5]*y)/(1 + Sqrt[5] + 2*y))]*Sqrt[-((-3 + Sqrt[5] + 2*Sqrt[5
]*y)/(1 + Sqrt[5] + 2*y))])*EllipticF[ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)
/(1 + Sqrt[5] + 2*y)])/Sqrt[15]], 15/16] + Sqrt[(-5 + 3*Sqrt[5] - 10*y)/(1
+ Sqrt[5] + 2*y)]*Sqrt[(-1 + Sqrt[5] - 2*y)/(1 + Sqrt[5] + 2*y)]*(9*Sqrt[5]
*EllipticPi[5/8 - Sqrt[5]/8, ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqr
t[5] + 2*y)])/Sqrt[15]], 15/16] + (-20 + 9*Sqrt[5])*EllipticPi[(-3*(-5 + Sq
rt[5]))/8, ArcSin[(2*Sqrt[(5 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y)])/Sqrt
[15]], 15/16] + 2*Sqrt[5]*EllipticPi[(3*(5 + Sqrt[5]))/8, ArcSin[(2*Sqrt[(5
 + 3*Sqrt[5] + 10*y)/(1 + Sqrt[5] + 2*y)])/Sqrt[15]], 15/16])))/(16*Sqrt[1
- 5*y - 5*y^2]*Sqrt[1 - y - y^2])
```

Maple [C] time = 0.213, size = 354, normalized size = 2.5

$$300\,\frac{\sqrt{-5\,y^2-5\,y+1}\sqrt{-y^2-y+1}\left(-10\,y-5+3\,\sqrt{5}\right)^2\sqrt{5}}{\sqrt{5\,y^4+10\,y^3-y^2-6\,y+1}\sqrt{\left(10\,y+5+3\,\sqrt{5}\right)\left(-10\,y-5+3\,\sqrt{5}\right)\left(\sqrt{5}-1-2\,y\right)\left(2\,y+1+\sqrt{5}\right)}\left(5+\sqrt{5}\right)\left(\sqrt{5}-5\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y)
```

```
[Out] 300*(-5*y^2-5*y+1)^(1/2)*(-y^2-y+1)^(1/2)*(-(10*y+5+3*5^(1/2))/(-10*y-5+3*5
^(1/2)))^(1/2)*(-10*y-5+3*5^(1/2))^2*((5^(1/2)-1-2*y)/(-10*y-5+3*5^(1/2)))^
```

```
(1/2)*5^(1/2)*((2*y+1+5^(1/2))/(-10*y-5+3*5^(1/2)))^(1/2)*(3*EllipticPi(2*(
-(10*y+5+3*5^(1/2))/(-10*y-5+3*5^(1/2)))^(1/2),-1/4*(5+5^(1/2))/(5^(1/2)-5)
,1/4)-EllipticPi(2*(-(10*y+5+3*5^(1/2))/(-10*y-5+3*5^(1/2)))^(1/2),-1/4*(3*
5^(1/2)-5)/(5+3*5^(1/2)),1/4)-2*EllipticPi(2*(-(10*y+5+3*5^(1/2))/(-10*y-5+
3*5^(1/2)))^(1/2),-1/4*(5+3*5^(1/2))/(3*5^(1/2)-5),1/4))/(5*y^4+10*y^3-y^2-
6*y+1)^(1/2)/((10*y+5+3*5^(1/2))*(-10*y-5+3*5^(1/2))*(5^(1/2)-1-2*y)*(2*y+1
+5^(1/2)))^(1/2)/(5+5^(1/2))/(5^(1/2)-5)/(3*5^(1/2)-5)/(5+3*5^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-5\,y^2-5\,y+1}\left(2\,y+1\right)}{\sqrt{-y^2-y+1}\left(y+2\right)\left(y+1\right)y}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y, al
gorithm="maxima")
```

```
[Out] integrate(sqrt(-5*y^2 - 5*y + 1)*(2*y + 1)/(sqrt(-y^2 - y + 1)*(y + 2)*(y +
 1)*y), y)
```

Fricas [A] time = 2.8141, size = 572, normalized size = 4.03

$$\frac{9}{8}\,\log\left(-\frac{235\,y^4+935\,y^3-3\left(35\,y^2+104\,y+77\right)\sqrt{-y^2-y+1}\sqrt{-5\,y^2-5\,y+1}+1086\,y^2+131\,y-281}{y^4+8\,y^3+24\,y^2+32\,y+16}\right)+\frac{1}{4}\,\log\left(\frac{35\,y^4}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y, al
gorithm="fricas")
```

```
[Out] 9/8*log(-(235*y^4 + 935*y^3 - 3*(35*y^2 + 104*y + 77)*sqrt(-y^2 - y + 1)*sq
rt(-5*y^2 - 5*y + 1) + 1086*y^2 + 131*y - 281)/(y^4 + 8*y^3 + 24*y^2 + 32*y
 + 16)) + 1/4*log((35*y^4 + 125*y^3 + (15*y^2 + 38*y + 24)*sqrt(-y^2 - y +
1)*sqrt(-5*y^2 - 5*y + 1) + 131*y^2 + 16*y - 26)/(y^4 + 4*y^3 + 6*y^2 + 4*y
 + 1)) + 1/8*log((35*y^4 + 15*y^3 + (15*y^2 - 8*y + 1)*sqrt(-y^2 - y + 1)*s
qrt(-5*y^2 - 5*y + 1) - 34*y^2 + 11*y - 1)/y^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2y+1\right)\sqrt{-5y^2-5y+1}}{y\left(y+1\right)\left(y+2\right)\sqrt{-y^2-y+1}}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*y)*(-5*y**2-5*y+1)**(1/2)/y/(1+y)/(2+y)/(-y**2-y+1)**(1/2),y
)
```

```
[Out] Integral((2*y + 1)*sqrt(-5*y**2 - 5*y + 1)/(y*(y + 1)*(y + 2)*sqrt(-y**2 -
y + 1)), y)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-5\,y^2-5\,y+1}\left(2\,y+1\right)}{\sqrt{-y^2-y+1}\left(y+2\right)\left(y+1\right)y}\,dy$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*y)*(-5*y^2-5*y+1)^(1/2)/y/(1+y)/(2+y)/(-y^2-y+1)^(1/2),y, al
gorithm="giac")
```

```
[Out] integrate(sqrt(-5*y^2 - 5*y + 1)*(2*y + 1)/(sqrt(-y^2 - y + 1)*(y + 2)*(y +
 1)*y), y)
```

3.280 $\int \frac{x\left(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2}\right)}{\left(4-5x^2+x^4\right)\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)}\,dx$

Optimal. Leaf size=21

$$\log\left(\sqrt{x^2-4}+\sqrt{x^2-1}+1\right)$$

```
[Out] Log[1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2]]
```

Rubi [A] time = 0.285303, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 85, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.012, Rules used = {6684}

$$\log\left(\sqrt{x^2-4}+\sqrt{x^2-1}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-Sqrt[-4 + x^2] + x^2*Sqrt[-4 + x^2] - 4*Sqrt[-1 + x^2] + x^2*Sqrt[
-1 + x^2]))/((4 - 5*x^2 + x^4)*(1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2])),x]
```

```
[Out] Log[1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2]]
```

Rule 6684

```
Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*L
og[RemoveContent[y, x]], x] /;  !FalseQ[q]]
```

Rubi steps

$$\int \frac{x\left(-\sqrt{-4+x^2}+x^2\sqrt{-4+x^2}-4\sqrt{-1+x^2}+x^2\sqrt{-1+x^2}\right)}{\left(4-5x^2+x^4\right)\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)}\,dx=\log\left(1+\sqrt{-4+x^2}+\sqrt{-1+x^2}\right)$$

Mathematica [B] time = 1.23474, size = 97, normalized size = 4.62

$$\frac{1}{4}\log\left(-5x^2-4\sqrt{x^2-4}\sqrt{x^2-1}+17\right)+\frac{1}{4}\log\left(-2x^2-2\sqrt{x^2-4}\sqrt{x^2-1}+5\right)-\frac{1}{2}\tanh^{-1}\left(\sqrt{x^2-4}\right)+\frac{1}{2}\tanh^{-1}\left(\frac{\sqrt{x^2-}}{2}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-Sqrt[-4 + x^2] + x^2*Sqrt[-4 + x^2] - 4*Sqrt[-1 + x^2] + x^2
*Sqrt[-1 + x^2]))/((4 - 5*x^2 + x^4)*(1 + Sqrt[-4 + x^2] + Sqrt[-1 + x^2]))
,x]
```

```
[Out] -ArcTanh[Sqrt[-4 + x^2]]/2 + ArcTanh[Sqrt[-1 + x^2]/2]/2 + Log[17 - 5*x^2 -
 4*Sqrt[-4 + x^2]*Sqrt[-1 + x^2]]/4 + Log[5 - 2*x^2 - 2*Sqrt[-4 + x^2]*Sqrt
[-1 + x^2]]/4
```

Maple [B] time = 0.136, size = 1088, normalized size = 51.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(1/2))/
(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x)
```

```
[Out] 1/(-2+5^(1/2))/(5^(1/2)-1)/(5^(1/2)+1)/(2+5^(1/2))*5^(1/2)*ln(x+((x-5^(1/2)
)^2+2*5^(1/2)*(x-5^(1/2))+1)^(1/2))-1/(-2+5^(1/2))/(5^(1/2)-1)/(5^(1/2)+1)/
(2+5^(1/2))*5^(1/2)*ln(x+((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+1)^(1/2))-1/2
/(5^(1/2)+1)/(5^(1/2)-1)*ln(x+((1+x)^2-2-2*x)^(1/2))+1/2/(5^(1/2)+1)/(5^(1/
2)-1)*((-1+x)^2-2+2*x)^(1/2)+1/2/(5^(1/2)+1)/(5^(1/2)-1)*ln(x+((-1+x)^2-2+2
*x)^(1/2))+1/2/(5^(1/2)+1)/(5^(1/2)-1)*((1+x)^2-2-2*x)^(1/2)-1/4/(2+5^(1/2)
)/(-2+5^(1/2))*((-2+x)^2+4*x-8)^(1/2)-1/2/(2+5^(1/2))/(-2+5^(1/2))*ln(x+((-
2+x)^2+4*x-8)^(1/2))-1/4/(2+5^(1/2))/(-2+5^(1/2))*((2+x)^2-4*x-8)^(1/2)+1/2
/(2+5^(1/2))/(-2+5^(1/2))*ln(x+((2+x)^2-4*x-8)^(1/2))-1/2/(-2+5^(1/2))/(5^(
1/2)-1)/(5^(1/2)+1)/(2+5^(1/2))*5^(1/2)*ln(x+((x-5^(1/2))^2+2*5^(1/2)*(x-5^
(1/2))+4)^(1/2))+1/2/(-2+5^(1/2))/(5^(1/2)-1)/(5^(1/2)+1)/(2+5^(1/2))*5^(1/
2)*ln(x+((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+4)^(1/2))+1/4*ln(x^2-5)-1/(-2+
5^(1/2))/(5^(1/2)-1)/(5^(1/2)+1)/(2+5^(1/2))*arctanh(1/2*(2+2*5^(1/2)*(x-5^
(1/2)))/((x-5^(1/2))^2+2*5^(1/2)*(x-5^(1/2))+1)^(1/2))+1/(-2+5^(1/2))/(5^(1
/2)-1)/(5^(1/2)+1)/(2+5^(1/2))*((x-5^(1/2))^2+2*5^(1/2)*(x-5^(1/2))+1)^(1/2
)-1/(-2+5^(1/2))/(5^(1/2)-1)/(5^(1/2)+1)/(2+5^(1/2))*arctanh(1/2*(2-2*5^(1/
2)*(x+5^(1/2)))/((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+1)^(1/2))+1/(-2+5^(1/2
))/(5^(1/2)-1)/(5^(1/2)+1)/(2+5^(1/2))*((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))
+1)^(1/2)+1/(-2+5^(1/2))/(5^(1/2)-1)/(5^(1/2)+1)/(2+5^(1/2))*arctanh(1/4*(8
+2*5^(1/2)*(x-5^(1/2)))/((x-5^(1/2))^2+2*5^(1/2)*(x-5^(1/2))+4)^(1/2))-1/2/
(-2+5^(1/2))/(5^(1/2)-1)/(5^(1/2)+1)/(2+5^(1/2))*((x-5^(1/2))^2+2*5^(1/2)*(
x-5^(1/2))+4)^(1/2)+1/(-2+5^(1/2))/(5^(1/2)-1)/(5^(1/2)+1)/(2+5^(1/2))*arct
anh(1/4*(8-2*5^(1/2)*(x+5^(1/2)))/((x+5^(1/2))^2-2*5^(1/2)*(x+5^(1/2))+4)^(
```

```
1/2))-1/2/(-2+5^(1/2))/(5^(1/2)-1)/(5^(1/2)+1)/(2+5^(1/2))*((x+5^(1/2))^2-2
*5^(1/2)*(x+5^(1/2))+4)^(1/2)+7/8*(x^2-4)^(1/2)*(x^2-1)^(1/2)/(x^4-5*x^2+4)
^(1/2)*arctanh(1/4*(5*x^2-17)/(x^4-5*x^2+4)^(1/2))+1/8*(x^2-4)^(1/2)*(x^2-1
)^(1/2)*(2*ln(-5/2+x^2+(x^4-5*x^2+4)^(1/2))-5*arctanh(1/4*(5*x^2-17)/(x^4-5
*x^2+4)^(1/2)))/(x^4-5*x^2+4)^(1/2)
```

Maxima [B] time = 1.20735, size = 231, normalized size = 11.

$$\frac{1}{4}\log(x+1)+\frac{3}{8}\log(x-1)+\frac{1}{8}\log(x-2)+\frac{1}{4}\log\left(\frac{2x^4+4\left(x^2-3\right)\sqrt{x+1}\sqrt{x-1}-7x^2+2\left(\left(x^2-1\right)\sqrt{x+1}\sqrt{x-1}\right.}{2\left(\left(x^2-1\right)\sqrt{x+1}\sqrt{x-1}\sqrt{x-2}+\left(2x^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(
1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/4*log(x + 1) + 3/8*log(x - 1) + 1/8*log(x - 2) + 1/4*log(1/2*(2*x^4 + 4*(
x^2 - 3)*sqrt(x + 1)*sqrt(x - 1) - 7*x^2 + 2*((x^2 - 1)*sqrt(x + 1)*sqrt(x
- 1)*sqrt(x - 2) + (2*x^2 - 3)*sqrt(x - 2))*sqrt(x + 2) + 3)/((x^2 - 1)*sqr
t(x + 1)*sqrt(x - 1)*sqrt(x - 2) + (2*x^2 - 3)*sqrt(x - 2))) + 1/4*log(((x^
2 - 1)*sqrt(x + 1)*sqrt(x - 1) + 2*x^2 - 3)/((x^2 - 1)*sqrt(x - 1)))
```

Fricas [B] time = 1.92392, size = 452, normalized size = 21.52

$$-\frac{1}{4}\log\left(4x^4-\left(4x^2-11\right)\sqrt{x^2-1}\sqrt{x^2-4}-21x^2+23\right)-\frac{1}{4}\log\left(x^2-\sqrt{x^2-1}(x+2)+2x-1\right)+\frac{1}{4}\log\left(x^2-\sqrt{x^2-4}(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(
1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/4*log(4*x^4 - (4*x^2 - 11)*sqrt(x^2 - 1)*sqrt(x^2 - 4) - 21*x^2 + 23) -
1/4*log(x^2 - sqrt(x^2 - 1)*(x + 2) + 2*x - 1) + 1/4*log(x^2 - sqrt(x^2 - 4
)*(x + 1) + x - 4) - 1/4*log(x^2 - sqrt(x^2 - 4)*(x - 1) - x - 4) + 1/4*log
(x^2 - sqrt(x^2 - 1)*(x - 2) - 2*x - 1) + 1/4*log(x^2 - 5) + 1/4*log(-x^2 +
 sqrt(x^2 - 1)*sqrt(x^2 - 4) + 7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(x**2-4)**(1/2)+x**2*(x**2-4)**(1/2)-4*(x**2-1)**(1/2)+x**2*(
x**2-1)**(1/2))/(x**4-5*x**2+4)/(1+(x**2-4)**(1/2)+(x**2-1)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38784, size = 105, normalized size = 5.

$$\frac{1}{2}\log\left(\sqrt{x^2-1}+2\right)-\frac{1}{2}\log\left(\left|-\sqrt{x^2-1}+\sqrt{x^2-4}\right|\right)-\frac{1}{2}\log\left(\left|-\sqrt{x^2-1}+\sqrt{x^2-4}-1\right|\right)+\frac{1}{2}\log\left(\left|-\sqrt{x^2-1}+\sqrt{x^2-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-(x^2-4)^(1/2)+x^2*(x^2-4)^(1/2)-4*(x^2-1)^(1/2)+x^2*(x^2-1)^(
1/2))/(x^4-5*x^2+4)/(1+(x^2-4)^(1/2)+(x^2-1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*log(sqrt(x^2 - 1) + 2) - 1/2*log(abs(-sqrt(x^2 - 1) + sqrt(x^2 - 4))) -
 1/2*log(abs(-sqrt(x^2 - 1) + sqrt(x^2 - 4) - 1)) + 1/2*log(abs(-sqrt(x^2 -
 1) + sqrt(x^2 - 4) - 3))
```

3.281 $\int \left(\sqrt{9-4\sqrt{2}}x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right) dx$

Optimal. Leaf size=4030

result too large to display

```
[Out] (Sqrt[9 - 4*Sqrt[2]]*x^2)/2 - Sqrt[2]*(-Sqrt[1 + 4*x + 2*x^2 + x^4]/3 + ((1
 + x)*Sqrt[1 + 4*x + 2*x^2 + x^4])/3 + ((4*I)*(-13 + 3*Sqrt[33])^(1/3)*Sqrt
[1 + 4*x + 2*x^2 + x^4])/(4*2^(2/3)*(-I + Sqrt[3]) - (2*I)*(-13 + 3*Sqrt[33
])^(1/3) + 2^(1/3)*(I + Sqrt[3])*(-13 + 3*Sqrt[33])^(2/3) + (6*I)*(-13 + 3*
Sqrt[33])^(1/3)*x) - (8*2^(2/3)*Sqrt[3/(-13 + 3*Sqrt[33] + 4*(-26 + 6*Sqrt[
33])^(1/3))]*Sqrt[(I*(-19899 + 3445*Sqrt[33] + (-26 + 6*Sqrt[33])^(2/3)*(-2
574 + 466*Sqrt[33]) + (-26 + 6*Sqrt[33])^(1/3)*(-19899 + 3445*Sqrt[33]) + (
59697 - 10335*Sqrt[33])*x))/((-39 - (13*I)*Sqrt[3] + (9*I)*Sqrt[11] + 9*Sqr
t[33] + (4*I)*(3*I + Sqrt[3])*(-26 + 6*Sqrt[33])^(1/3))*(26 - 6*Sqrt[33] +
(-13 + (13*I)*Sqrt[3] - (9*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/
3) + (-4 - (4*I)*Sqrt[3])*(-26 + 6*Sqrt[33])^(2/3) + 6*(-13 + 3*Sqrt[33])*x
))]*Sqrt[1 + 4*x + 2*x^2 + x^4]*EllipticE[ArcSin[Sqrt[26 - 6*Sqrt[33] + (-1
3 - (13*I)*Sqrt[3] + (9*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3)
+ (4*I)*(I + Sqrt[3])*(-26 + 6*Sqrt[33])^(2/3) + 6*(-13 + 3*Sqrt[33])*x]/(S
qrt[(39 + (13*I)*Sqrt[3] - (9*I)*Sqrt[11] - 9*Sqrt[33] + 4*(3 - I*Sqrt[3])*
(-26 + 6*Sqrt[33])^(1/3))/(39 - (13*I)*Sqrt[3] + (9*I)*Sqrt[11] - 9*Sqrt[33
] + 4*(3 + I*Sqrt[3])*(-26 + 6*Sqrt[33])^(1/3))]*Sqrt[26 - 6*Sqrt[33] + (-1
3 + (13*I)*Sqrt[3] - (9*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3)
+ (-4 - (4*I)*Sqrt[3])*(-26 + 6*Sqrt[33])^(2/3) + 6*(-13 + 3*Sqrt[33])*x])]
, (4*(21 + (7*I)*Sqrt[3] - (3*I)*Sqrt[11] - 3*Sqrt[33]) + (3 - I*Sqrt[3] -
(3*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3))/(4*(21 - (7*I)*Sqrt[
3] + (3*I)*Sqrt[11] - 3*Sqrt[33]) + (3 + I*Sqrt[3] + (3*I)*Sqrt[11] + 3*Sqr
t[33])*(-26 + 6*Sqrt[33])^(1/3))])/((4*2^(2/3) - (-13 + 3*Sqrt[33])^(1/3) -
 2^(1/3)*(-13 + 3*Sqrt[33])^(2/3) + 3*(-13 + 3*Sqrt[33])^(1/3)*x)*Sqrt[(I*(
1 + x))/((104 - 24*Sqrt[33] + (-13 - (13*I)*Sqrt[3] + (9*I)*Sqrt[11] + 3*Sq
rt[33])*(-26 + 6*Sqrt[33])^(1/3) + (4*I)*(I + Sqrt[3])*(-26 + 6*Sqrt[33])^(
2/3))*(26 - 6*Sqrt[33] + (-13 + (13*I)*Sqrt[3] - (9*I)*Sqrt[11] + 3*Sqrt[33
])*(-26 + 6*Sqrt[33])^(1/3) + (-4 - (4*I)*Sqrt[3])*(-26 + 6*Sqrt[33])^(2/3)
 + 6*(-13 + 3*Sqrt[33])*x))]*Sqrt[26 - 6*Sqrt[33] + (-13 + (13*I)*Sqrt[3] -
 (9*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3) + (-4 - (4*I)*Sqrt[3
])*(-26 + 6*Sqrt[33])^(2/3) + 6*(-13 + 3*Sqrt[33])*x]*Sqrt[26 - 6*Sqrt[33]
+ (-13 - (13*I)*Sqrt[3] + (9*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(
1/3) + (4*I)*(I + Sqrt[3])*(-26 + 6*Sqrt[33])^(2/3) + 6*(-13 + 3*Sqrt[33])*
x]) + ((2^(1/3)*(13 - (13*I)*Sqrt[3] + (9*I)*Sqrt[11] - 3*Sqrt[33]) + 4*2^(
2/3)*(1 + I*Sqrt[3])*(-13 + 3*Sqrt[33])^(1/3) + 20*(-13 + 3*Sqrt[33])^(2/3)
)*(4*2^(2/3)*(I + Sqrt[3]) + (8*I)*(-13 + 3*Sqrt[33])^(1/3) + 2^(1/3)*(-I +
 Sqrt[3])*(-13 + 3*Sqrt[33])^(2/3))*Sqrt[(52 - 12*Sqrt[33] - 2^(1/3)*(-13 +
```

```
 3*Sqrt[33])^(4/3) + 4*(-26 + 6*Sqrt[33])^(2/3))/(-13 + 3*Sqrt[33] + 4*(-26
 + 6*Sqrt[33])^(1/3))]*Sqrt[((-8*I)*(-13 + 3*Sqrt[33]) + (-43*I - 13*Sqrt[3
] + 9*Sqrt[11] + (5*I)*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3) + (2*I + 4*Sqrt[3
] - (2*I)*Sqrt[33])*(-26 + 6*Sqrt[33])^(2/3) + ((8*I)*(-13 + 3*Sqrt[33]) +
(13*I - 13*Sqrt[3] + 9*Sqrt[11] - (3*I)*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3)
+ 4*(I + Sqrt[3])*(-26 + 6*Sqrt[33])^(2/3))*x)/(1 + x)]*Sqrt[1 + 4*x + 2*x^
2 + x^4]*EllipticF[ArcSin[(Sqrt[52 - 12*Sqrt[33] - 2^(1/3)*(-13 + 3*Sqrt[33
])^(4/3) + 4*(-26 + 6*Sqrt[33])^(2/3)]*Sqrt[26 - 6*Sqrt[33] + (-13 - (13*I)
*Sqrt[3] + (9*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3) + (4*I)*(I
 + Sqrt[3])*(-26 + 6*Sqrt[33])^(2/3) + 6*(-13 + 3*Sqrt[33])*x])/(2^(1/6)*Sq
rt[3]*(-13 + 3*Sqrt[33])^(2/3)*Sqrt[39 + (13*I)*Sqrt[3] - (9*I)*Sqrt[11] -
9*Sqrt[33] + 4*(3 - I*Sqrt[3])*(-26 + 6*Sqrt[33])^(1/3)]*Sqrt[1 + x])], (4*
(21*I - 7*Sqrt[3] + 3*Sqrt[11] - (3*I)*Sqrt[33]) + (3*I + Sqrt[3] + 3*Sqrt[
11] + (3*I)*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3))/(-56*Sqrt[3] + 24*Sqrt[11]
+ 2*(Sqrt[3] + 3*Sqrt[11])*(-26 + 6*Sqrt[33])^(1/3))])/(3*2^(2/3)*3^(3/4)*(
-13 + 3*Sqrt[33])^(1/3)*Sqrt[39 + (13*I)*Sqrt[3] - (9*I)*Sqrt[11] - 9*Sqrt[
33] + 4*(3 - I*Sqrt[3])*(-26 + 6*Sqrt[33])^(1/3)]*Sqrt[1 + x]*(4*2^(2/3)*(-
I + Sqrt[3]) - (2*I)*(-13 + 3*Sqrt[33])^(1/3) + 2^(1/3)*(I + Sqrt[3])*(-13
+ 3*Sqrt[33])^(2/3) + (6*I)*(-13 + 3*Sqrt[33])^(1/3)*x)*Sqrt[26 - 6*Sqrt[33
] + (-13 - (13*I)*Sqrt[3] + (9*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])
^(1/3) + (4*I)*(I + Sqrt[3])*(-26 + 6*Sqrt[33])^(2/3) + 6*(-13 + 3*Sqrt[33]
)*x]*Sqrt[(8*(-13 + 3*Sqrt[33]) - (5 - (3*I)*Sqrt[3] + (3*I)*Sqrt[11] + Sqr
t[33])*(-26 + 6*Sqrt[33])^(2/3) + (-26 + 6*Sqrt[33])^(1/3)*(-41 + (15*I)*Sq
rt[3] - (3*I)*Sqrt[11] + 7*Sqrt[33]) + (104 - 24*Sqrt[33] + (-13 - (13*I)*S
qrt[3] + (9*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3) + (4*I)*(I +
 Sqrt[3])*(-26 + 6*Sqrt[33])^(2/3))*x)/((-39 - (13*I)*Sqrt[3] + (9*I)*Sqrt[
11] + 9*Sqrt[33] + (4*I)*(3*I + Sqrt[3])*(-26 + 6*Sqrt[33])^(1/3))*(1 + x))
]) + ((4*2^(2/3) + 2*(-13 + 3*Sqrt[33])^(1/3) - 2^(1/3)*(-13 + 3*Sqrt[33])^
(2/3))*(4*2^(2/3)*(I + Sqrt[3]) - (4*I)*(-13 + 3*Sqrt[33])^(1/3) + 2^(1/3)*
(-I + Sqrt[3])*(-13 + 3*Sqrt[33])^(2/3))*(4*2^(2/3)*(-I + Sqrt[3]) + (4*I)*
(-13 + 3*Sqrt[33])^(1/3) + 2^(1/3)*(I + Sqrt[3])*(-13 + 3*Sqrt[33])^(2/3))*
Sqrt[(-39 + (13*I)*Sqrt[3] - (9*I)*Sqrt[11] + 9*Sqrt[33] - (4*I)*(-3*I + Sq
rt[3])*(-26 + 6*Sqrt[33])^(1/3))/(104 - 24*Sqrt[33] + (-13 + (13*I)*Sqrt[3]
 - (9*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3) + (-4 - (4*I)*Sqrt
[3])*(-26 + 6*Sqrt[33])^(2/3))]*Sqrt[1 + x]*Sqrt[(104 - 24*Sqrt[33] + 2*(1
+ (14*I)*Sqrt[3] - (6*I)*Sqrt[11] + Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3) + (-
7 - I*Sqrt[3] - (3*I)*Sqrt[11] + Sqrt[33])*(-26 + 6*Sqrt[33])^(2/3) + 2*(-5
2 + 12*Sqrt[33] + 2^(1/3)*(-13 + 3*Sqrt[33])^(4/3) - 4*(-26 + 6*Sqrt[33])^(
2/3))*x)/((-39 + (13*I)*Sqrt[3] - (9*I)*Sqrt[11] + 9*Sqrt[33] - (4*I)*(-3*I
 + Sqrt[3])*(-26 + 6*Sqrt[33])^(1/3))*(1 + x))]*Sqrt[(104 - 24*Sqrt[33] + 2
*(1 - (14*I)*Sqrt[3] + (6*I)*Sqrt[11] + Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3)
+ (-7 + I*Sqrt[3] + (3*I)*Sqrt[11] + Sqrt[33])*(-26 + 6*Sqrt[33])^(2/3) + 2
*(-52 + 12*Sqrt[33] + 2^(1/3)*(-13 + 3*Sqrt[33])^(4/3) - 4*(-26 + 6*Sqrt[33
])^(2/3))*x)/((-39 - (13*I)*Sqrt[3] + (9*I)*Sqrt[11] + 9*Sqrt[33] + (4*I)*(
3*I + Sqrt[3])*(-26 + 6*Sqrt[33])^(1/3))*(1 + x))]*Sqrt[1 + 4*x + 2*x^2 + x
```

```
^4]*EllipticPi[(2^(1/3)*(4*2^(1/3)*(-3*I + Sqrt[3]) + (3*I + Sqrt[3])*(-13
+ 3*Sqrt[33])^(2/3)))/(4*2^(2/3)*(-I + Sqrt[3]) - (8*I)*(-13 + 3*Sqrt[33])^
(1/3) + 2^(1/3)*(I + Sqrt[3])*(-13 + 3*Sqrt[33])^(2/3)), ArcSin[Sqrt[13 - 3
*Sqrt[33] - 2^(1/3)*(-13 + 3*Sqrt[33])^(4/3) + 4*(-26 + 6*Sqrt[33])^(2/3) +
 (-39 + 9*Sqrt[33])*x]/(2^(1/6)*Sqrt[3]*(-13 + 3*Sqrt[33])^(2/3)*Sqrt[(-39
+ (13*I)*Sqrt[3] - (9*I)*Sqrt[11] + 9*Sqrt[33] - (4*I)*(-3*I + Sqrt[3])*(-2
6 + 6*Sqrt[33])^(1/3))/(104 - 24*Sqrt[33] + (-13 + (13*I)*Sqrt[3] - (9*I)*S
qrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3) + (-4 - (4*I)*Sqrt[3])*(-26
+ 6*Sqrt[33])^(2/3))]*Sqrt[1 + x])], (4*(21 - (7*I)*Sqrt[3] + (3*I)*Sqrt[11
] - 3*Sqrt[33]) + (3 + I*Sqrt[3] + (3*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sq
rt[33])^(1/3))/(4*(21 + (7*I)*Sqrt[3] - (3*I)*Sqrt[11] - 3*Sqrt[33]) + (3 -
 I*Sqrt[3] - (3*I)*Sqrt[11] + 3*Sqrt[33])*(-26 + 6*Sqrt[33])^(1/3))])/(2^(1
/6)*Sqrt[3]*(4*2^(2/3)*(I + Sqrt[3]) + (2*I)*(-13 + 3*Sqrt[33])^(1/3) + 2^(
1/3)*(-I + Sqrt[3])*(-13 + 3*Sqrt[33])^(2/3) - (6*I)*(-13 + 3*Sqrt[33])^(1/
3)*x)*(4*2^(2/3)*(-I + Sqrt[3]) - (2*I)*(-13 + 3*Sqrt[33])^(1/3) + 2^(1/3)*
(I + Sqrt[3])*(-13 + 3*Sqrt[33])^(2/3) + (6*I)*(-13 + 3*Sqrt[33])^(1/3)*x)*
Sqrt[13 - 3*Sqrt[33] - 2^(1/3)*(-13 + 3*Sqrt[33])^(4/3) + 4*(-26 + 6*Sqrt[3
3])^(2/3) + (-39 + 9*Sqrt[33])*x]))
```

Rubi [F] time = 0.0250234, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \left(\sqrt{9-4\sqrt{2}}x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[9 - 4*Sqrt[2]]*x - Sqrt[2]*Sqrt[1 + 4*x + 2*x^2 + x^4],x]
```

```
[Out] (Sqrt[9 - 4*Sqrt[2]]*x^2)/2 - Sqrt[2]*Defer[Int][Sqrt[1 + 4*x + 2*x^2 + x^4
], x]
```

Rubi steps

$$\int \left(\sqrt{9-4\sqrt{2}}x-\sqrt{2}\sqrt{1+4x+2x^2+x^4}\right)dx=\frac{1}{2}\sqrt{9-4\sqrt{2}}x^2-\sqrt{2}\int \sqrt{1+4x+2x^2+x^4}\,dx$$

Mathematica [C] time = 6.04772, size = 3168, normalized size = 0.79

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[9 - 4*Sqrt[2]]*x - Sqrt[2]*Sqrt[1 + 4*x + 2*x^2 + x^4],x]
```

```
[Out] (Sqrt[9 - 4*Sqrt[2]]*x^2)/2 - (Sqrt[2]*x*Sqrt[1 + 4*x + 2*x^2 + x^4])/3 - (
2*Sqrt[2]*((6*(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])^2*(-(EllipticF[Ar
cSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1
 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1
+ Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]], ((Root[1 + 3*#1 - #1^2 + #1^3
 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*#1 - #1
^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(Root[1
+ 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]*R
oot[1 + 3*#1 - #1^2 + #1^3 & , 1, 0]) + EllipticPi[(1 + Root[1 + 3*#1 - #1^
2 + #1^3 & , 3, 0])/(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1
 - #1^2 + #1^3 & , 3, 0]), ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #
1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1
 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]],
((Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2
, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1
^2 + #1^3 & , 2, 0])*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1
 - #1^2 + #1^3 & , 3, 0]))]*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0]))*Sq
rt[(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])/((x - Root[1 + 3*#1 - #1^2 +
 #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0]))]*(-1 - Root[1
 + 3*#1 - #1^2 + #1^3 & , 3, 0])*Sqrt[(x - Root[1 + 3*#1 - #1^2 + #1^3 & ,
3, 0])/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1
^2 + #1^3 & , 3, 0]))]*Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1,
0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 + #
1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))])/(Sqrt[1 + 4*
x + 2*x^2 + x^4]*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #
1^2 + #1^3 & , 3, 0])) + (2*EllipticF[ArcSin[Sqrt[((1 + x)*(-Root[1 + 3*#1
- #1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Roo
t[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3,
 0]))]], ((Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #
1^3 & , 2, 0])*(-1 - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((-1 - Root[1
+ 3*#1 - #1^2 + #1^3 & , 2, 0])*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Ro
ot[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]*(x - Root[1 + 3*#1 - #1^2 + #1^3 & ,
 1, 0])^2*Sqrt[(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])/((x - Root[1 + 3
*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0]))]*
(-1 - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])*Sqrt[(x - Root[1 + 3*#1 - #1^2
 + #1^3 & , 3, 0])/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1
```

```
 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]*Sqrt[((1 + x)*(-Root[1 + 3*#1 - #1^2 + #
1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1
 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))])/(S
qrt[1 + 4*x + 2*x^2 + x^4]*(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1
 + 3*#1 - #1^2 + #1^3 & , 3, 0])) + ((1 + x)*(x - Root[1 + 3*#1 - #1^2 + #1
^3 & , 2, 0])*(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]) + (x - Root[1 + 3
*#1 - #1^2 + #1^3 & , 1, 0])^2*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*
Sqrt[(x - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])/((x - Root[1 + 3*#1 - #1^2
 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0]))]*Sqrt[(x -
Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])/((x - Root[1 + 3*#1 - #1^2 + #1^3 &
, 1, 0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]*Sqrt[-(((1 + x)*(Roo
t[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])
)/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(1 + Root[1 + 3*#1 - #1^2 +
#1^3 & , 3, 0])))]*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])*((EllipticE[
ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*
#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(
1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]], ((Root[1 + 3*#1 - #1^2 + #1
^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*#1 -
#1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(Root[
1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]
*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0]))/(1 + Root[1 + 3*#1 - #1^2 + #
1^3 & , 1, 0]) - (EllipticPi[(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])/(-
Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 3,
0]), ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1
 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1,
0])*(1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]], ((Root[1 + 3*#1 - #1^2
 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*
#1 - #1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(
Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3,
0]))]*(1 - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #
1^3 & , 2, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/(-Root[1 + 3*#1 - #
1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]) + (EllipticF[
ArcSin[Sqrt[-(((1 + x)*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*
#1 - #1^2 + #1^3 & , 3, 0]))/((x - Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0])*(
1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0])))]], ((Root[1 + 3*#1 - #1^2 + #1
^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(1 + Root[1 + 3*#1 -
#1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 - #1^2 + #1^3 & , 2, 0])*(Root[
1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))]
*(Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1 + 3*#1 - #1^2 + #1^3 & , 1
, 0]*(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] - Root[1 + 3*#1 - #1^2 + #1^3
& , 3, 0]) - Root[1 + 3*#1 - #1^2 + #1^3 & , 3, 0]))/((1 + Root[1 + 3*#1 -
#1^2 + #1^3 & , 1, 0])*(-Root[1 + 3*#1 - #1^2 + #1^3 & , 1, 0] + Root[1 + 3
*#1 - #1^2 + #1^3 & , 3, 0]))))/Sqrt[1 + 4*x + 2*x^2 + x^4]))/3
```

Maple [A] time = 0.73, size = 4640, normalized size = 1.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x)
```

```
[Out] 1/2*x^2*(-1+2*2^(1/2))-2^(1/2)*(1/3*x*(x^4+2*x^2+4*x+1)^(1/2)+4/3*(-4/3-1/6
*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*
33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*((1/2*(26+6*33^(1/2))^(1/3)-4/(
26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33
^(1/2))^(1/3)))*(1+x)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+
4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(
x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*(x+1/3*(2
6+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3)^2*((-1/3*(26+6*33^(1/2))
^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+
6*33^(1/2))^(1/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*3
3^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3+1
/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3
*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*((-1/3*(26+6*3
3^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+
4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3
/(26+6*33^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/
3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))
)/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)/(1/2*(
26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(
1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/3*(26+6*33^(1/2))^(1/3)+8/3/(26
+6*33^(1/2))^(1/3)+4/3)/((1+x)*(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1
/2))^(1/3)-1/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3-
1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(x-1/
6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(
26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2)*EllipticF(((1/2*(26
+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/
2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(1+x)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/
(26+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26
+6*33^(1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)
-1/3))^(1/2),((-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(
1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(-4/3-1/6*(26+
6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1
/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/
(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*3
3^(1/2))^(1/3)))/(-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1/2))^(1/3)+1/2*I*
```

```
3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2))+4*(
-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/
3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*((1/2*(26+6*33^(1/2))^(
1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/
(26+6*33^(1/2))^(1/3)))*(1+x)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2)
)^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(
1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*(
x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3)^2*((-1/3*(26+6*3
3^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+
4/3/(26+6*33^(1/2))^(1/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3
/(26+6*33^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/
3)+4/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))
)/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*((-1/3
*(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2)
)^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(
1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1
/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2)
)^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2
)/(1/2*(26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(2
6+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/3*(26+6*33^(1/2))^(1/3)
+8/3/(26+6*33^(1/2))^(1/3)+4/3)/((1+x)*(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26
+6*33^(1/2))^(1/3)-1/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1
/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)
))*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)
*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2)*((-1/3*(26+
6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+1/3)*EllipticF(((1/2*(26+6*33^(
1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/
3)-8/3/(26+6*33^(1/2))^(1/3)))*(1+x)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*3
3^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(
1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^
(1/2),((-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-
1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(-4/3-1/6*(26+6*33^(1
/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1
/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*3
3^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2)
)^(1/3)))/(-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)
*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2))+(-4/3+1/3*
(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))*EllipticPi(((1/2*(26+6*33^
(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1
/3)-8/3/(26+6*33^(1/2))^(1/3)))*(1+x)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*
33^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^
(1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))
^(1/2),(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1/
2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(1/2*(26+6*33^(1
/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3
```

```
)-8/3/(26+6*33^(1/2))^(1/3))),((-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1/2)
)^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)
))*(-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*
(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-4/3-1/6*(26+6*33^
(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^
(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*33^(1
/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1
/3))))^(1/2))+2/3*((1+x)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^
(1/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/
3)))*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/
2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3))))+(-4/3-1/6*(26+6*
33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2
))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*((1/2*(26+6*33^(1/2))^(1/3)-4/(26+6*33
^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))
^(1/3)))*(1+x)/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-1/2
*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3*(
26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*(x+1/3*(26+6*33^
(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3)^2*((-1/3*(26+6*33^(1/2))^(1/3)+
8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1
/2))^(1/3)-1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2)
)^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3+1/2*I*3^
(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3*(26+6*
33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*((-1/3*(26+6*33^(1/2)
)^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+4/3)*(x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26
+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*
33^(1/2))^(1/3)))/(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-
1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/
3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2)*((1/2*(26+6*3
3^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^
(1/3)-8/3/(26+6*33^(1/2))^(1/3))+(1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1
/2))^(1/3)+1/3-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2)
)^(1/3)))*(-1/3*(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+1/3)+(-1/3*
(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+1/3)^2)/(1/2*(26+6*33^(1/2)
)^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8
/3/(26+6*33^(1/2))^(1/3)))/(-1/3*(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^
(1/3)+4/3)*EllipticF(((1/2*(26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/
2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(1+x)/(
1/6*(26+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3
*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(
1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2),((-1/2*(26+6*33^(1/2))^(1/3)+4/(
26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33
^(1/2))^(1/3)))*(-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1
/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-4/3-
1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26
+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/2*(26+6*33^(1/2))^(1/3)+
```

```
4/(26+6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6
*33^(1/2))^(1/3))))^(1/2))+(-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/
2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/
3)))*EllipticE(((1/2*(26+6*33^(1/2))^(1/3)-4/(26+6*33^(1/2))^(1/3)-1/2*I*3^
(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(1+x)/(1/6*(2
6+6*33^(1/2))^(1/3)-4/3/(26+6*33^(1/2))^(1/3)+4/3-1/2*I*3^(1/2)*(-1/3*(26+6
*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(x+1/3*(26+6*33^(1/2))^(1/3)-8
/3/(26+6*33^(1/2))^(1/3)-1/3))^(1/2),((-1/2*(26+6*33^(1/2))^(1/3)+4/(26+6*3
3^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2)
)^(1/3)))*(-4/3-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)+1/2*I*3
^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-4/3-1/6*(2
6+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/2*I*3^(1/2)*(-1/3*(26+6*33^
(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))/(-1/2*(26+6*33^(1/2))^(1/3)+4/(26+
6*33^(1/2))^(1/3)+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1
/2))^(1/3))))^(1/2))/(-1/3*(26+6*33^(1/2))^(1/3)+8/3/(26+6*33^(1/2))^(1/3)+
4/3)))/((1+x)*(x+1/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)-1/3)*(
x-1/6*(26+6*33^(1/2))^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3-1/2*I*3^(1/2)*(-1
/3*(26+6*33^(1/2))^(1/3)-8/3/(26+6*33^(1/2))^(1/3)))*(x-1/6*(26+6*33^(1/2))
^(1/3)+4/3/(26+6*33^(1/2))^(1/3)-1/3+1/2*I*3^(1/2)*(-1/3*(26+6*33^(1/2))^(1
/3)-8/3/(26+6*33^(1/2))^(1/3))))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}\,x^2\left(2\,\sqrt{2}-1\right)-\sqrt{2}\int\sqrt{x^3-x^2+3\,x+1}\sqrt{x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x, algorithm="m
axima")
```

```
[Out] 1/2*x^2*(2*sqrt(2) - 1) - sqrt(2)*integrate(sqrt(x^3 - x^2 + 3*x + 1)*sqrt(
x + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(2\,\sqrt{2}x-\sqrt{2}\sqrt{x^4+2\,x^2+4\,x+1}-x,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x, algorithm="f
ricas")
```

```
[Out] integral(2*sqrt(2)*x - sqrt(2)*sqrt(x^4 + 2*x^2 + 4*x + 1) - x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x\left(-1+2\sqrt{2}\right)-\sqrt{2}\sqrt{x^4+2x^2+4x+1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2**(1/2)*(x**4+2*x**2+4*x+1)**(1/2)+x*(-1+2*2**(1/2)),x)
```

```
[Out] Integral(x*(-1 + 2*sqrt(2)) - sqrt(2)*sqrt(x**4 + 2*x**2 + 4*x + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x\left(2\sqrt{2}-1\right)-\sqrt{2}\sqrt{x^4+2x^2+4x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-2^(1/2)*(x^4+2*x^2+4*x+1)^(1/2)+x*(-1+2*2^(1/2)),x, algorithm="g
iac")
```

```
[Out] integrate(x*(2*sqrt(2) - 1) - sqrt(2)*sqrt(x^4 + 2*x^2 + 4*x + 1), x)
```

3.282

$$\int \frac{e^{-\frac{x}{y}}\left(\pi^2\left(-3\mathbf{mc}^8+4\mathbf{mc}^9+24\mathbf{mc}^6x-48\mathbf{mc}^7x-144\mathbf{mc}^5x^2-24\mathbf{mc}^2x^3+176\mathbf{mc}^3x^3\right.\right.}{384x^2}$$

Optimal. Leaf size=330

$$\frac{\pi^2(3-4\text{mc})\text{mc}^8\text{ExpIntegralEi}\left(-\frac{x}{y}\right)}{384y}+\frac{1}{16}\pi^2(1-2\text{mc})\text{mc}^6\text{ExpIntegralEi}\left(-\frac{x}{y}\right)+\frac{1}{32}\pi^2\text{mc}^3y\left(-12\text{mc}^2+3\text{mc}-8y\right)$$

```
[Out] ((3 - 4*mc)*mc^8*Pi^2)/(384*E^(x/y)*x) + (3*mc^5*Pi^2*y)/(8*E^(x/y)) + ((3
- 22*mc)*mc^2*Pi^2*x*y)/(48*E^(x/y)) - ((1 + 4*mc)*Pi^2*x^2*y)/(128*E^(x/y)
) + ((3 - 22*mc)*mc^2*Pi^2*y^2)/(48*E^(x/y)) + (mc^3*Pi^2*y^2)/(4*E^(x/y))
- ((1 + 4*mc)*Pi^2*x*y^2)/(64*E^(x/y)) - ((1 + 4*mc)*Pi^2*y^3)/(64*E^(x/y))
 + ((1 - 2*mc)*mc^6*Pi^2*ExpIntegralEi[-(x/y)])/16 + ((3 - 4*mc)*mc^8*Pi^2*
ExpIntegralEi[-(x/y)])/(384*y) + (mc^3*Pi^2*(3*mc - 12*mc^2 - 8*y)*y*ExpInt
egralEi[-(x/y)])/32 - (mc^3*Pi^2*(3*(1 - 4*mc)*mc - 8*x)*y*Log[x/mc^2])/(32
*E^(x/y)) + (mc^3*Pi^2*y^2*Log[x/mc^2])/(4*E^(x/y))
```

Rubi [A] time = 0.871562, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 107, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {12, 6742, 2199, 2194, 2177, 2178, 2176, 2554}

$$\frac{\pi^2(3-4\text{mc})\text{mc}^8\text{ExpIntegralEi}\left(-\frac{x}{y}\right)}{384y}+\frac{1}{16}\pi^2(1-2\text{mc})\text{mc}^6\text{ExpIntegralEi}\left(-\frac{x}{y}\right)+\frac{1}{32}\pi^2\text{mc}^3y\left(-12\text{mc}^2+3\text{mc}-8y\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Pi^2*(-3*mc^8 + 4*mc^9 + 24*mc^6*x - 48*mc^7*x - 144*mc^5*x^2 - 24*mc^
2*x^3 + 176*mc^3*x^3 + 3*x^4 + 12*mc*x^4) + 12*mc^3*Pi^2*(3*mc - 12*mc^2 -
8*x)*x^2*Log[x/mc^2])/(384*E^(x/y)*x^2),x]
```


```
[Out] ((3 - 4*mc)*mc^8*Pi^2)/(384*E^(x/y)*x) + (3*mc^5*Pi^2*y)/(8*E^(x/y)) + ((3
- 22*mc)*mc^2*Pi^2*x*y)/(48*E^(x/y)) - ((1 + 4*mc)*Pi^2*x^2*y)/(128*E^(x/y)
) + ((3 - 22*mc)*mc^2*Pi^2*y^2)/(48*E^(x/y)) + (mc^3*Pi^2*y^2)/(4*E^(x/y))
- ((1 + 4*mc)*Pi^2*x*y^2)/(64*E^(x/y)) - ((1 + 4*mc)*Pi^2*y^3)/(64*E^(x/y))
 + ((1 - 2*mc)*mc^6*Pi^2*ExpIntegralEi[-(x/y)])/16 + ((3 - 4*mc)*mc^8*Pi^2*
ExpIntegralEi[-(x/y)])/(384*y) + (mc^3*Pi^2*(3*mc - 12*mc^2 - 8*y)*y*ExpInt
egralEi[-(x/y)])/32 - (mc^3*Pi^2*(3*(1 - 4*mc)*mc - 8*x)*y*Log[x/mc^2])/(32
*E^(x/y)) + (mc^3*Pi^2*y^2*Log[x/mc^2])/(4*E^(x/y))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2199

```
Int[(F_)^((c_.)*(v_))*(u_)^(m_.)*(w_), x_Symbol] :> Int[ExpandIntegrand[F^(
c*ExpandToSum[v, x]), w*NormalizePowerOfLinear[u, x]^m, x], x] /; FreeQ[{F,
 c}, x] && PolynomialQ[w, x] && LinearQ[v, x] && PowerOfLinearQ[u, x] && In
tegerQ[m] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !$UseGamma === True
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int \frac{e^{-\frac{x}{y}}\left(\pi^2\left(-3\text{mc}^8+4\text{mc}^9+24\text{mc}^6x-48\text{mc}^7x-144\text{mc}^5x^2-24\text{mc}^2x^3+176\text{mc}^3x^3+3x^4+12\text{mc}x^4\right)+12\text{mc}^3\pi^2\left(3\text{r}\right.\right.}{384x^2}$$

Mathematica [A] time = 0.139802, size = 181, normalized size = 0.55

$$\frac{1}{384}\pi^2\left(\frac{e^{-\frac{x}{y}}\left(144\text{mc}^5xy-16\text{mc}^3xy(11x+5y)+24\text{mc}^2xy(x+y)+12\text{mc}^3xy\left(12\text{mc}^2-3\text{mc}+8(x+y)\right)\log\left(\frac{x}{\text{mc}^2}\right)-4\text{m}\right.}{x}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Pi^2*(-3*mc^8 + 4*mc^9 + 24*mc^6*x - 48*mc^7*x - 144*mc^5*x^2 -
24*mc^2*x^3 + 176*mc^3*x^3 + 3*x^4 + 12*mc*x^4) + 12*mc^3*Pi^2*(3*mc - 12*m
```

```
c^2 - 8*x)*x^2*Log[x/mc^2])/(384*E^(x/y)*x^2),x]
```

```
[Out] (Pi^2*(-((mc^3*(-3*mc^5 + 4*mc^6 - 24*mc^3*y + 48*mc^4*y - 36*mc*y^2 + 144*
mc^2*y^2 + 96*y^3)*ExpIntegralEi[-(x/y)])/y) + (3*mc^8 - 4*mc^9 + 144*mc^5*
x*y + 24*mc^2*x*y*(x + y) - 16*mc^3*x*y*(11*x + 5*y) - 3*x*y*(x^2 + 2*x*y +
 2*y^2) - 12*mc*x*y*(x^2 + 2*x*y + 2*y^2) + 12*mc^3*x*y*(-3*mc + 12*mc^2 +
8*(x + y))*Log[x/mc^2])/(E^(x/y)*x)))/384
```

Maple [C] time = 0.144, size = 1356, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/384*(Pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*mc^3*x^
3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*Pi^2*(-12*mc^2+3*mc-8*x)*x^2*ln(x/mc
^2))/exp(x/y)/x^2,x)
```

```
[Out] 1/4*Pi^2*mc^3*y^2*Ei(1,x/y)-3/32*Pi^2*mc^4*y*Ei(1,x/y)+3/8*Pi^2*mc^5*y*Ei(1
,x/y)-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I*x)*csgn(I/mc^2*x)^2+1/8*I*y^2*Pi^
3*mc^3*csgn(I*x)*csgn(I/mc^2*x)^2*exp(-x/y)+1/8*I*y^2*Pi^3*mc^3*csgn(I/mc^2
)*csgn(I/mc^2*x)^2*exp(-x/y)-1/8*I*y*Pi^3*mc^3*csgn(I/mc^2*x)^3*x*exp(-x/y)
+1/8*I*y*Pi^3*mc^3*csgn(I*mc^2)^3*x*exp(-x/y)-1/4*I*y^2*Pi^3*mc^3*csgn(I*mc
)*csgn(I*mc^2)^2*exp(-x/y)+1/8*I*y^2*Pi^3*mc^3*csgn(I*mc)^2*csgn(I*mc^2)*ex
p(-x/y)+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I/mc^2)*csgn(I/mc^2*x)^2+3/16*I*y
*Pi^3*exp(-x/y)*mc^5*csgn(I*mc)^2*csgn(I*mc^2)-3/8*I*y*Pi^3*exp(-x/y)*mc^5*
csgn(I*mc)*csgn(I*mc^2)^2+3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I*x)*csgn(I/mc^
2*x)^2-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I/mc^2)*csgn(I/mc^2*x)^2-1/16*Pi^2
*mc^6*Ei(1,x/y)+1/8*Pi^2*mc^7*Ei(1,x/y)-1/64*y^3*Pi^2*exp(-x/y)+1/384*(144*
Pi^2*mc^5*y-36*Pi^2*mc^4*y+96*Pi^2*mc^3*x*y+96*Pi^2*mc^3*y^2)*exp(-x/y)*ln(
x)-1/8*I*y*Pi^3*mc^3*csgn(I/mc^2)*csgn(I*x)*csgn(I/mc^2*x)*x*exp(-x/y)-1/2*
y*Pi^2*ln(mc)*mc^3*x*exp(-x/y)-1/8*I*y^2*Pi^3*mc^3*csgn(I/mc^2*x)^3*exp(-x/
y)+1/8*I*y^2*Pi^3*mc^3*csgn(I*mc^2)^3*exp(-x/y)-3/64*I*y*Pi^3*exp(-x/y)*mc^
4*csgn(I*mc^2)^3+3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I/mc^2*x)^3+3/16*I*y*Pi^
3*exp(-x/y)*mc^5*csgn(I*mc^2)^3-3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I/mc^2*x)
^3-3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I*mc)^2*csgn(I*mc^2)+3/32*I*y*Pi^3*exp
(-x/y)*mc^4*csgn(I*mc)*csgn(I*mc^2)^2-1/128*y*Pi^2*exp(-x/y)*x^2-1/64*y^2*P
i^2*x*exp(-x/y)-1/16*y^3*Pi^2*mc*exp(-x/y)+1/16*y^2*Pi^2*mc^2*exp(-x/y)+3/8
*y*Pi^2*exp(-x/y)*mc^5-1/128/y*Pi^2*mc^8*Ei(1,x/y)+1/96/y*Pi^2*mc^9*Ei(1,x/
y)-1/96*Pi^2*mc^9/x*exp(-x/y)+1/128*Pi^2*mc^8/x*exp(-x/y)-5/24*mc^3*Pi^2*y^
2*exp(-x/y)+1/8*I*y*Pi^3*mc^3*csgn(I*mc)^2*csgn(I*mc^2)*x*exp(-x/y)-1/4*I*y
*Pi^3*mc^3*csgn(I*mc)*csgn(I*mc^2)^2*x*exp(-x/y)+1/8*I*y*Pi^3*mc^3*csgn(I*x
```

```
)*csgn(I/mc^2*x)^2*x*exp(-x/y)+1/8*I*y*Pi^3*mc^3*csgn(I/mc^2)*csgn(I/mc^2*x
)^2*x*exp(-x/y)-1/8*I*y^2*Pi^3*mc^3*csgn(I/mc^2)*csgn(I*x)*csgn(I/mc^2*x)*e
xp(-x/y)-3/16*I*y*Pi^3*exp(-x/y)*mc^5*csgn(I/mc^2)*csgn(I*x)*csgn(I/mc^2*x)
+3/64*I*y*Pi^3*exp(-x/y)*mc^4*csgn(I/mc^2)*csgn(I*x)*csgn(I/mc^2*x)+1/16*y*
Pi^2*mc^2*x*exp(-x/y)-11/24*y*Pi^2*mc^3*x*exp(-x/y)-1/2*y^2*Pi^2*ln(mc)*mc^
3*exp(-x/y)-1/32*y*Pi^2*mc*exp(-x/y)*x^2-1/16*y^2*Pi^2*mc*x*exp(-x/y)-3/4*y
*Pi^2*exp(-x/y)*ln(mc)*mc^5+3/16*y*Pi^2*exp(-x/y)*ln(mc)*mc^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\pi^2 \mathrm{mc}^9 \Gamma\left(-1, \frac{x}{y}\right)}{96\,y} - \frac{1}{8}\,\pi^2 \mathrm{mc}^7 \mathrm{Ei}\left(-\frac{x}{y}\right) + \frac{\pi^2 \mathrm{mc}^8 \Gamma\left(-1, \frac{x}{y}\right)}{128\,y} + \frac{3}{8}\,\pi^2 \mathrm{mc}^5 y e^{\left(-\frac{x}{y}\right)} \log\left(\frac{x}{\mathrm{mc}^2}\right) + \frac{1}{16}\,\pi^2 \mathrm{mc}^6 \mathrm{Ei}\left(-\frac{x}{y}\right) - \frac{3}{8}\,\pi^2 \mathrm{mc}^5 y$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*m
c^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2*l
og(x/mc^2))/exp(x/y)/x^2,x, algorithm="maxima")
```

```
[Out] -1/96*pi^2*mc^9*gamma(-1, x/y)/y - 1/8*pi^2*mc^7*Ei(-x/y) + 1/128*pi^2*mc^8
*gamma(-1, x/y)/y + 3/8*pi^2*mc^5*y*e^(-x/y)*log(x/mc^2) + 1/16*pi^2*mc^6*E
i(-x/y) - 3/8*pi^2*mc^5*y*Ei(-x/y) + 3/8*pi^2*mc^5*y*e^(-x/y) - 3/32*pi^2*m
c^4*y*e^(-x/y)*log(x/mc^2) + 3/32*pi^2*mc^4*y*Ei(-x/y) - 11/24*pi^2*(x*y +
y^2)*mc^3*e^(-x/y) + 1/4*pi^2*((x*y + y^2)*e^(-x/y)*log(x) + integrate((2*x
^2*log(mc) - x*y - y^2)*e^(-x/y)/x, x))*mc^3 + 1/16*pi^2*(x*y + y^2)*mc^2*e
^(-x/y) - 1/32*pi^2*(x^2*y + 2*x*y^2 + 2*y^3)*mc*e^(-x/y) - 1/128*pi^2*(x^2
*y + 2*x*y^2 + 2*y^3)*e^(-x/y)
```

Fricas [A] time = 1.98407, size = 590, normalized size = 1.79

$$\frac{12\left(8\,\pi^2 \mathrm{mc}^3 x y^3 + \left(8\,\pi^2 \mathrm{mc}^3 x^2 + 3\,\pi^2\left(4\,\mathrm{mc}^5 - \mathrm{mc}^4\right)x\right)y^2\right)e^{\left(-\frac{x}{y}\right)} \log\left(\frac{x}{\mathrm{mc}^2}\right) - \left(96\,\pi^2 \mathrm{mc}^3 x y^3 + 36\,\pi^2\left(4\,\mathrm{mc}^5 - \mathrm{mc}^4\right)x y^2 + 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*m
c^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2*l
```

```
og(x/mc^2))/exp(x/y)/x^2,x, algorithm="fricas")
```

```
[Out] 1/384*(12*(8*pi^2*mc^3*x*y^3 + (8*pi^2*mc^3*x^2 + 3*pi^2*(4*mc^5 - mc^4)*x)
*y^2)*e^(-x/y)*log(x/mc^2) - (96*pi^2*mc^3*x*y^3 + 36*pi^2*(4*mc^5 - mc^4)*
x*y^2 + 24*pi^2*(2*mc^7 - mc^6)*x*y + pi^2*(4*mc^9 - 3*mc^8)*x)*Ei(-x/y) -
(6*pi^2*(4*mc + 1)*x*y^4 + pi^2*(4*mc^9 - 3*mc^8)*y + 2*(3*pi^2*(4*mc + 1)*
x^2 + 4*pi^2*(10*mc^3 - 3*mc^2)*x)*y^3 - (144*pi^2*mc^5*x - 3*pi^2*(4*mc +
1)*x^3 - 8*pi^2*(22*mc^3 - 3*mc^2)*x^2)*y^2)*e^(-x/y))/(x*y)
```

Sympy [A] time = 16.2833, size = 330, normalized size = 1.

$$-\frac{\pi^2 mc^9 \operatorname{E}_2\left(\frac{x}{y}\right)}{96x}+\frac{\pi^2 mc^8 \operatorname{E}_2\left(\frac{x}{y}\right)}{128x}-\frac{\pi^2 mc^7 \operatorname{Ei}\left(-\frac{x}{y}\right)}{8}+\frac{\pi^2 mc^6 \operatorname{Ei}\left(-\frac{x}{y}\right)}{16}+\frac{3\pi^2 mc^5 ye^{-\frac{x}{y}}}{8}-\frac{3\pi^2 mc^5\left(y\operatorname{Ei}\left(-\frac{x}{y}\right)-ye^{-\frac{x}{y}}\log\left(\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/384*(pi**2*(4*mc**9-3*mc**8-48*mc**7*x+24*mc**6*x-144*mc**5*x**
2+176*mc**3*x**3-24*mc**2*x**3+12*mc*x**4+3*x**4)+12*mc**3*pi**2*(-12*mc**2
+3*mc-8*x)*x**2*ln(x/mc**2))/exp(x/y)/x**2,x)
```

```
[Out] -pi**2*mc**9*expint(2, x/y)/(96*x) + pi**2*mc**8*expint(2, x/y)/(128*x) - p
i**2*mc**7*Ei(-x/y)/8 + pi**2*mc**6*Ei(-x/y)/16 + 3*pi**2*mc**5*y*exp(-x/y)
/8 - 3*pi**2*mc**5*(y*Ei(-x/y) - y*exp(-x/y)*log(x/mc**2))/8 + 3*pi**2*mc**
4*(y*Ei(-x/y) - y*exp(-x/y)*log(x/mc**2))/32 + 11*pi**2*mc**3*(-x*y*exp(-x/
y) - y**2*exp(-x/y))/24 - pi**2*mc**3*(y**2*Ei(-x/y) - y**2*exp(-x/y) + (-x
*y*exp(-x/y) - y**2*exp(-x/y))*log(x/mc**2))/4 - pi**2*mc**2*(-x*y*exp(-x/y
) - y**2*exp(-x/y))/16 + pi**2*mc*(-x**2*y*exp(-x/y) - 2*x*y**2*exp(-x/y) -
 2*y**3*exp(-x/y))/32 + pi**2*(-x**2*y*exp(-x/y) - 2*x*y**2*exp(-x/y) - 2*y
**3*exp(-x/y))/128
```

Giac [A] time = 1.14092, size = 637, normalized size = 1.93

$$-\frac{4\pi^2 \mathrm{mc}^9 x\operatorname{Ei}\left(-\frac{x}{y}\right)+4\pi^2 \mathrm{mc}^9 ye^{\left(-\frac{x}{y}\right)}-3\pi^2 \mathrm{mc}^8 x\operatorname{Ei}\left(-\frac{x}{y}\right)+48\pi^2 \mathrm{mc}^7 xy\operatorname{Ei}\left(-\frac{x}{y}\right)-3\pi^2 \mathrm{mc}^8 ye^{\left(-\frac{x}{y}\right)}-144\pi^2 \mathrm{mc}^5 xy^2 e^{\left(-\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/384*(pi^2*(4*mc^9-3*mc^8-48*mc^7*x+24*mc^6*x-144*mc^5*x^2+176*m
c^3*x^3-24*mc^2*x^3+12*mc*x^4+3*x^4)+12*mc^3*pi^2*(-12*mc^2+3*mc-8*x)*x^2*l
```

```
og(x/mc^2))/exp(x/y)/x^2,x, algorithm="giac")
```

```
[Out] -1/384*(4*pi^2*mc^9*x*Ei(-x/y) + 4*pi^2*mc^9*y*e^(-x/y) - 3*pi^2*mc^8*x*Ei(
-x/y) + 48*pi^2*mc^7*x*y*Ei(-x/y) - 3*pi^2*mc^8*y*e^(-x/y) - 144*pi^2*mc^5*
x*y^2*e^(-x/y)*log(x/mc^2) - 24*pi^2*mc^6*x*y*Ei(-x/y) + 144*pi^2*mc^5*x*y^
2*Ei(-x/y) - 144*pi^2*mc^5*x*y^2*e^(-x/y) + 36*pi^2*mc^4*x*y^2*e^(-x/y)*log
(x/mc^2) - 96*pi^2*mc^3*x^2*y^2*e^(-x/y)*log(x/mc^2) - 96*pi^2*mc^3*x*y^3*e
^(-x/y)*log(x/mc^2) - 36*pi^2*mc^4*x*y^2*Ei(-x/y) + 96*pi^2*mc^3*x*y^3*Ei(-
x/y) + 176*pi^2*mc^3*x^2*y^2*e^(-x/y) + 80*pi^2*mc^3*x*y^3*e^(-x/y) - 24*pi
^2*mc^2*x^2*y^2*e^(-x/y) + 12*pi^2*mc*x^3*y^2*e^(-x/y) - 24*pi^2*mc^2*x*y^3
*e^(-x/y) + 24*pi^2*mc*x^2*y^3*e^(-x/y) + 24*pi^2*mc*x*y^4*e^(-x/y) + 3*pi^
2*x^3*y^2*e^(-x/y) + 6*pi^2*x^2*y^3*e^(-x/y) + 6*pi^2*x*y^4*e^(-x/y))/(x*y)
```

3.283 $\int \sec(x)\sin(2x)\,dx$

Optimal. Leaf size=4

$$-2\cos(x)$$

[Out] -2*Cos[x]

Rubi [A] time = 0.0118598, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4287, 2638}

$$-2\cos(x)$$

Antiderivative was successfully verified.

[In] Int[Sec[x]*Sin[2*x],x]

[Out] -2*Cos[x]

Rule 4287

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*sin[(c_.) + (d_.)*(x_)]^(p_.), x_
Symbol] :> Dist[2^p/e^p, Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] && In
tegerQ[p]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(x)\sin(2x)\,dx &= 2\int \sin(x)\,dx\\ &= -2\cos(x)\end{aligned}$$

Mathematica [A] time = 0.0014547, size = 4, normalized size = 1.

$$-2\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]*Sin[2*x],x]
```

```
[Out] -2*Cos[x]
```

Maple [A] time = 0.009, size = 5, normalized size = 1.3

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)/cos(x),x)
```

```
[Out] -2*cos(x)
```

Maxima [A] time = 0.927243, size = 5, normalized size = 1.25

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/cos(x),x, algorithm="maxima")
```

```
[Out] -2*cos(x)
```

Fricas [A] time = 1.81245, size = 15, normalized size = 3.75

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/cos(x),x, algorithm="fricas")
```

```
[Out] -2*cos(x)
```

Sympy [A] time = 0.707154, size = 5, normalized size = 1.25

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/cos(x),x)
```

```
[Out] -2*cos(x)
```

Giac [A] time = 1.09796, size = 5, normalized size = 1.25

$$-2\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)/cos(x),x, algorithm="giac")
```

```
[Out] -2*cos(x)
```

3.284 $\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}} dx$

Optimal. Leaf size=71

$$\frac{1}{2}\left(\left(1+\sqrt{2}\right)\log\left(-x^7+\sqrt{2}x^2+\sqrt{2}x+x+1\right)-\left(\sqrt{2}-1\right)\log\left(x^7+\sqrt{2}x^2+\left(\sqrt{2}-1\right)x-1\right)\right)$$

```
[Out] ((1 + Sqrt[2])*Log[1 + x + Sqrt[2]*x + Sqrt[2]*x^2 - x^7] - (-1 + Sqrt[2])*
Log[-1 + (-1 + Sqrt[2])*x + Sqrt[2]*x^2 + x^7])/2
```

Rubi [F] time = 0.754187, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}} dx$$

Verification is Not applicable to the result.

```
[In] Int[(3 + 3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^13)/(1 + 2*x -
x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14),x]
```

```
[Out] Log[1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14]/2 + 2*Defer[Int][
(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14)^(-1), x] + 4*Defer[I
nt][x/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14), x] + 2*Defer[
Int][x^2/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14), x] + 12*De
fer[Int][x^7/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14), x] + 1
0*Defer[Int][x^8/(1 + 2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14), x]
```

Rubi steps

$$\int \frac{3+3x-4x^2-4x^3-7x^6+4x^7+10x^8+7x^{13}}{1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}}\,dx = \frac{1}{2}\log\left(1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right)+\frac{1}{14}\int \frac{2}{1+2}$$

$$= \frac{1}{2}\log\left(1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right)+\frac{1}{14}\int \frac{}{1+2}$$

$$= \frac{1}{2}\log\left(1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right)+2\int \frac{}{1+2x}$$

$$= \frac{1}{2}\log\left(1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right)+2\int \left(\frac{}{1+2x}\right.$$

$$= \frac{1}{2}\log\left(1+2x-x^2-4x^3-2x^4-2x^7-2x^8+x^{14}\right)+2\int \frac{}{1+2x}$$

Mathematica [A] time = 0.036176, size = 71, normalized size = 1.

$$\frac{1}{2}\left(\left(1+\sqrt{2}\right)\log\left(-x^7+\sqrt{2}x^2+\sqrt{2}x+x+1\right)-\left(\sqrt{2}-1\right)\log\left(x^7+\sqrt{2}x^2+\left(\sqrt{2}-1\right)x-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 3*x - 4*x^2 - 4*x^3 - 7*x^6 + 4*x^7 + 10*x^8 + 7*x^13)/(1 +
2*x - x^2 - 4*x^3 - 2*x^4 - 2*x^7 - 2*x^8 + x^14),x]
```

```
[Out] ((1 + Sqrt[2])*Log[1 + x + Sqrt[2]*x + Sqrt[2]*x^2 - x^7] - (-1 + Sqrt[2])*
Log[-1 + (-1 + Sqrt[2])*x + Sqrt[2]*x^2 + x^7])/2
```

Maple [A] time = 0.022, size = 102, normalized size = 1.4

$$\frac{\ln\left(x^7-x^2\sqrt{2}+\left(-\sqrt{2}-1\right)x-1\right)}{2}+\frac{\ln\left(x^7-x^2\sqrt{2}+\left(-\sqrt{2}-1\right)x-1\right)\sqrt{2}}{2}+\frac{\ln\left(-1+x^7+x\left(\sqrt{2}-1\right)+x^2\sqrt{2}\right)}{2}-\frac{\ln\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2*x^4-4
*x^3-x^2+2*x+1),x)
```

```
[Out] 1/2*ln(x^7-x^2*2^(1/2)+(-2^(1/2)-1)*x-1)+1/2*ln(x^7-x^2*2^(1/2)+(-2^(1/2)-1
)*x-1)*2^(1/2)+1/2*ln(-1+x^7+x*(2^(1/2)-1)+x^2*2^(1/2))-1/2*ln(-1+x^7+x*(2^
(1/2)-1)+x^2*2^(1/2))*2^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{7\,x^{13}+10\,x^{8}+4\,x^{7}-7\,x^{6}-4\,x^{3}-4\,x^{2}+3\,x+3}{x^{14}-2\,x^{8}-2\,x^{7}-2\,x^{4}-4\,x^{3}-x^{2}+2\,x+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2
*x^4-4*x^3-x^2+2*x+1),x, algorithm="maxima")
```

```
[Out] integrate((7*x^13 + 10*x^8 + 4*x^7 - 7*x^6 - 4*x^3 - 4*x^2 + 3*x + 3)/(x^14
 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1), x)
```

Fricas [B] time = 1.61445, size = 319, normalized size = 4.49

$$\frac{1}{2}\,\sqrt{2}\log\left(\frac{x^{14}-2\,x^{8}-2\,x^{7}+2\,x^{4}+4\,x^{3}+3\,x^{2}-2\,\sqrt{2}\left(x^{9}+x^{8}-x^{3}-2\,x^{2}-x\right)+2\,x+1}{x^{14}-2\,x^{8}-2\,x^{7}-2\,x^{4}-4\,x^{3}-x^{2}+2\,x+1}\right)+\frac{1}{2}\,\log\left(x^{14}-2\,x^{8}-2\,x^{7}-2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2
*x^4-4*x^3-x^2+2*x+1),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((x^14 - 2*x^8 - 2*x^7 + 2*x^4 + 4*x^3 + 3*x^2 - 2*sqrt(2)*(
x^9 + x^8 - x^3 - 2*x^2 - x) + 2*x + 1)/(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x
^3 - x^2 + 2*x + 1)) + 1/2*log(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 +
 2*x + 1)
```

Sympy [A] time = 0.204742, size = 76, normalized size = 1.07

$$\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)\log\left(x^{7}-\sqrt{2}x^{2}-2x\left(\frac{1}{2}+\frac{\sqrt{2}}{2}\right)-1\right)+\left(\frac{1}{2}-\frac{\sqrt{2}}{2}\right)\log\left(x^{7}+\sqrt{2}x^{2}-2x\left(\frac{1}{2}-\frac{\sqrt{2}}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x**13+10*x**8+4*x**7-7*x**6-4*x**3-4*x**2+3*x+3)/(x**14-2*x**8
-2*x**7-2*x**4-4*x**3-x**2+2*x+1),x)
```

```
[Out] (1/2 + sqrt(2)/2)*log(x**7 - sqrt(2)*x**2 - 2*x*(1/2 + sqrt(2)/2) - 1) + (1
/2 - sqrt(2)/2)*log(x**7 + sqrt(2)*x**2 - 2*x*(1/2 - sqrt(2)/2) - 1)
```

Giac [A] time = 1.13533, size = 132, normalized size = 1.86

$$-\frac{1}{2}\sqrt{2}\log\left(\left|14\,x^7+14\,\sqrt{2}x^2+14\,x\left(\sqrt{2}-1\right)-14\right|\right)+\frac{1}{2}\sqrt{2}\log\left(\left|14\,x^7-14\,\sqrt{2}x^2-14\,x\left(\sqrt{2}+1\right)-14\right|\right)+\frac{1}{2}\log\left(\left|x^{14}-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((7*x^13+10*x^8+4*x^7-7*x^6-4*x^3-4*x^2+3*x+3)/(x^14-2*x^8-2*x^7-2
*x^4-4*x^3-x^2+2*x+1),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(14*x^7 + 14*sqrt(2)*x^2 + 14*x*(sqrt(2) - 1) - 14)) +
1/2*sqrt(2)*log(abs(14*x^7 - 14*sqrt(2)*x^2 - 14*x*(sqrt(2) + 1) - 14)) + 1
/2*log(abs(x^14 - 2*x^8 - 2*x^7 - 2*x^4 - 4*x^3 - x^2 + 2*x + 1))
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
    ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
    ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
    (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType, Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
   gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
   Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
   args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
   Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
   Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```