Computer algebra independent integration tests

0-Independent-test-suites/Bondarenko-Problems

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [35]. This is test number [2].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 94.29 (33)	% 5.71 (2)
Mathematica	% 100. (35)	% 0. (0)
Maple	% 77.14 (27)	% 22.86 (8)
Maxima	% 42.86 (15)	% 57.14 (20)
Fricas	% 68.57 (24)	% 31.43 (11)
Sympy	% 20. (7)	% 80. (28)
Giac	% 37.14 (13)	% 62.86 (22)

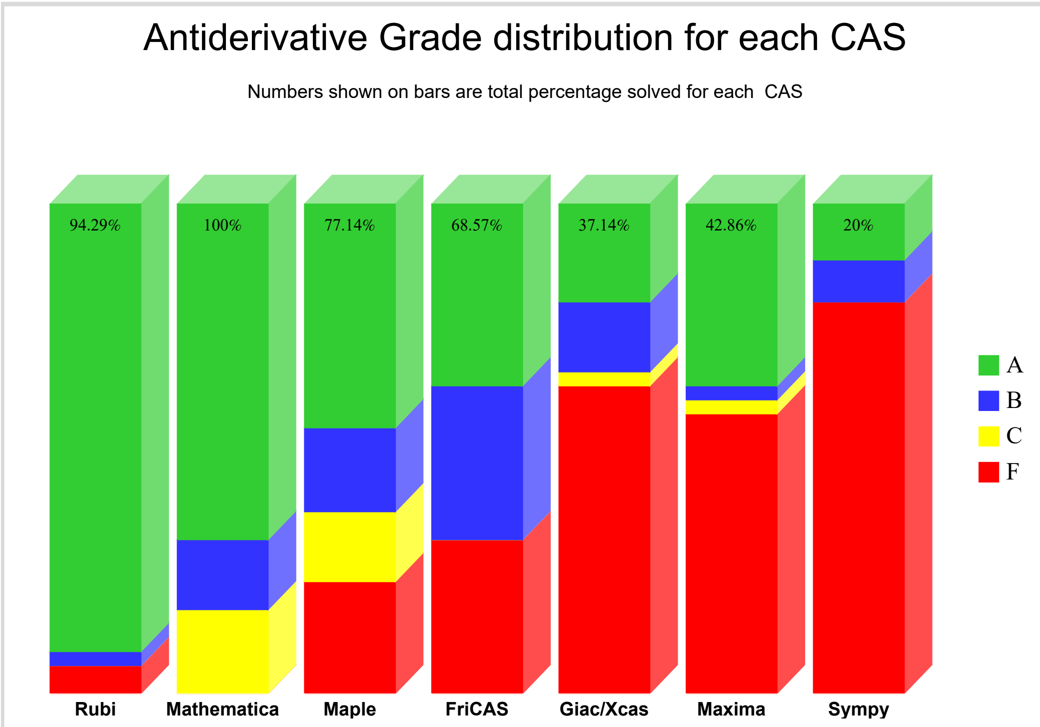
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

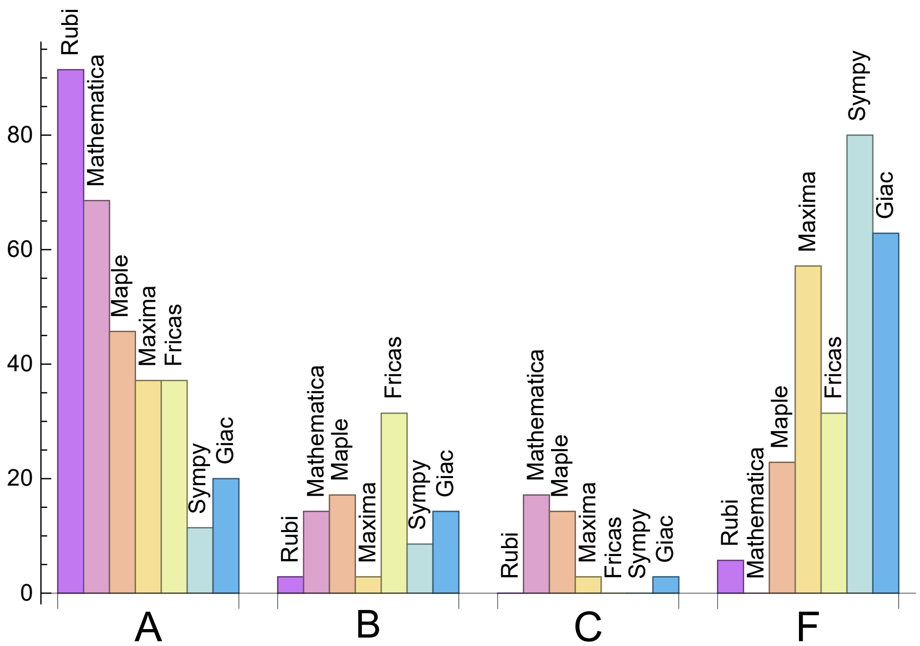
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	91.43	2.86	0.	5.71
Mathematica	68.57	14.29	17.14	0.
Maple	45.71	17.14	14.29	22.86
Maxima	37.14	2.86	2.86	57.14
Fricas	37.14	31.43	0.	31.43
Sympy	11.43	8.57	0.	80.
Giac	20.	14.29	2.86	62.86

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.3	183.58	1.23	83.	1.
Mathematica	0.58	271.77	1.74	102.	1.
Maple	0.05	138.19	1.21	83.	0.84
Maxima	1.44	127.33	1.62	69.	1.66
Fricas	8.69	1996.92	9.6	229.5	4.3
Sympy	5.03	179.86	4.15	65.	2.32
Giac	1.19	144.08	2.21	100.	1.41

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{21, 26\}$

Mathematica $\{7, 14, 21, 26, 28, 31, 35\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

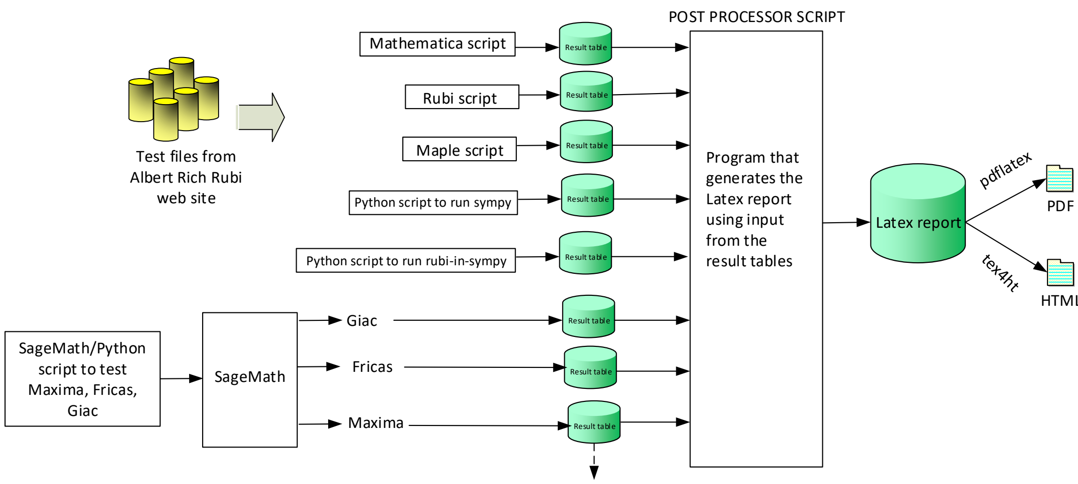
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35 }

B grade: { 21 }

C grade: { }

F grade: { 7, 8}

2.1.2 Mathematica

A grade: { 2, 3, 5, 7, 9, 10, 11, 12, 13, 15, 16, 17, 18, 22, 23, 24, 27, 28, 29, 30, 31, 32, 34, 35 }

B grade: { 8, 14, 19, 20, 33 }

C grade: { 1, 4, 6, 21, 25, 26 }

F grade: { }

2.1.3 Maple

A grade: { 1, 3, 4, 5, 10, 12, 20, 21, 22, 23, 25, 27, 30, 32, 33, 35 }

B grade: { 2, 11, 17, 19, 24, 26 }

C grade: { 6, 7, 8, 13, 14 }

F grade: { 9, 15, 16, 18, 28, 29, 31, 34 }

2.1.4 Maxima

A grade: { 1, 3, 5, 7, 8, 10, 11, 12, 19, 20, 22, 33, 34

B grade: { 23 }

C grade: { 4 }

F grade: { 2, 6, 9, 13, 14, 15, 16, 17, 18, 21, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 9, 12, 15, 16, 17, 18, 19, 22 }

B grade: { 6, 10, 11, 13, 14, 20, 21, 23, 24, 26, 33 }

C grade: { }

F grade: { 7, 8, 25, 27, 28, 29, 30, 31, 32, 34, 35 }

2.1.6 Sympy

A grade: { 3, 6, 10, 19 }

B grade: { 1, 5, 22 }

C grade: { }

F grade: { 2, 4, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35 }

2.1.7 Giac

A grade: { 1, 3, 5, 6, 21, 22, 23 }

B grade: { 2, 19, 20, 24, 26 }

C grade: { 4 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	77	21	27	82	51	24
normalized size	1	1.	3.5	0.95	1.23	3.73	2.32	1.09
time (sec)	N/A	0.135	0.061	0.047	1.437	1.028	6.845	1.118

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	24	50	0	116	0	198
normalized size	1	1.	0.75	1.56	0.	3.62	0.	6.19
time (sec)	N/A	0.156	0.017	0.009	0.	1.112	0.	1.149

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	16	16	31	69	14	20
normalized size	1	1.	0.64	0.64	1.24	2.76	0.56	0.8
time (sec)	N/A	0.014	0.009	0.005	0.965	1.119	0.408	1.101

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	C	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	68	42	151	182	0	58
normalized size	1	1.	1.17	0.72	2.6	3.14	0.	1.
time (sec)	N/A	0.082	0.016	0.012	1.55	1.138	0.	1.097

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	26	42	76	198	925	43
normalized size	1	1.	0.52	0.84	1.52	3.96	18.5	0.86
time (sec)	N/A	0.026	0.033	0.063	0.979	1.121	11.395	1.11

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	334	30	36	0	3152	26	335
normalized size	1	1.	0.09	0.11	0.	9.44	0.08	1.
time (sec)	N/A	0.347	0.004	0.015	0.	1.22	1.2	1.124

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	C	A	F(-2)	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	291	0	430	171	494	0	0	0
normalized size	1	0.	1.48	0.59	1.7	0.	0.	0.
time (sec)	N/A	0.049	0.92	0.096	1.62	0.	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	C	A	F(-2)	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	308	0	654	198	510	0	0	0
normalized size	1	0.	2.12	0.64	1.66	0.	0.	0.
time (sec)	N/A	0.043	0.51	0.008	1.614	0.	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	197	0	0
normalized size	1	1.	1.	0.	0.	2.35	0.	0.
time (sec)	N/A	0.059	0.041	0.017	0.	1.078	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	41	34	69	284	112	0
normalized size	1	1.	1.	0.83	1.68	6.93	2.73	0.
time (sec)	N/A	0.193	0.057	0.004	1.427	1.007	10.842	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	71	175	85	323	0	0
normalized size	1	1.	0.97	2.4	1.16	4.42	0.	0.
time (sec)	N/A	0.108	0.058	0.104	1.458	1.094	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	52	60	104	305	0	0
normalized size	1	1.	0.71	0.82	1.42	4.18	0.	0.
time (sec)	N/A	0.271	0.075	0.006	1.461	1.087	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	365	357	109	0	20056	0	0
normalized size	1	1.	0.98	0.3	0.	54.95	0.	0.
time (sec)	N/A	0.859	0.457	0.017	0.	56.984	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	B	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	337	337	2075	105	0	15946	0	0
normalized size	1	1.	6.16	0.31	0.	47.32	0.	0.
time (sec)	N/A	0.621	5.972	0.011	0.	44.925	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	74	0	0	165	0	0
normalized size	1	1.	0.96	0.	0.	2.14	0.	0.
time (sec)	N/A	0.071	0.038	0.014	0.	6.299	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	112	0	0	219	0	0
normalized size	1	1.	0.95	0.	0.	1.86	0.	0.
time (sec)	N/A	0.191	0.079	0.02	0.	11.04	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	85	298	0	240	0	0
normalized size	1	1.	1.02	3.59	0.	2.89	0.	0.
time (sec)	N/A	0.1	0.034	0.011	0.	29.436	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	98	0	0	297	0	0
normalized size	1	1.	1.02	0.	0.	3.09	0.	0.
time (sec)	N/A	0.084	0.168	0.028	0.	29.32	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	112	49	49	103	65	101
normalized size	1	1.	4.48	1.96	1.96	4.12	2.6	4.04
time (sec)	N/A	0.075	0.105	0.019	1.447	2.105	3.723	1.551

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	126	33	47	211	0	100
normalized size	1	1.	5.04	1.32	1.88	8.44	0.	4.
time (sec)	N/A	0.051	0.095	0.071	1.43	2.238	0.	1.269

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	C	A	F(-1)	B	F(-1)	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	108	786	478	95	0	718	0	140
normalized size	1	7.28	4.43	0.88	0.	6.65	0.	1.3
time (sec)	N/A	1.117	6.296	0.062	0.	2.92	0.	1.112

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	56	27	54	182	66	41
normalized size	1	1.	1.93	0.93	1.86	6.28	2.28	1.41
time (sec)	N/A	0.02	0.033	0.039	1.302	2.292	0.786	1.121

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	20	58	207	0	41
normalized size	1	1.	1.	0.77	2.23	7.96	0.	1.58
time (sec)	N/A	0.013	0.017	0.025	1.822	2.208	0.	1.089

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	147	121	366	0	2684	0	366
normalized size	1	1.34	1.1	3.33	0.	24.4	0.	3.33
time (sec)	N/A	0.607	0.118	0.113	0.	2.663	0.	1.365

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	54	75	0	0	0	0
normalized size	1	1.	1.35	1.88	0.	0.	0.	0.
time (sec)	N/A	0.081	1.789	0.1	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F	B
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	185	349	910	392	0	1277	0	406
normalized size	1	1.89	4.92	2.12	0.	6.9	0.	2.19
time (sec)	N/A	0.976	0.524	0.129	0.	2.734	0.	1.216

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	102	83	0	0	0	0
normalized size	1	1.	1.	0.81	0.	0.	0.	0.
time (sec)	N/A	0.146	0.017	0.028	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	159	159	122	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.203	0.086	2.18	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	395	347	0	0	0	0	0
normalized size	1	1.	0.88	0.	0.	0.	0.	0.
time (sec)	N/A	0.537	0.223	3.147	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	981	981	868	698	0	0	0	0
normalized size	1	1.	0.88	0.71	0.	0.	0.	0.
time (sec)	N/A	1.252	0.481	0.051	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	555	555	1076	0	0	0	0	0
normalized size	1	1.	1.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.712	1.427	0.01	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	303	252	0	0	0	0
normalized size	1	1.	0.97	0.81	0.	0.	0.	0.
time (sec)	N/A	0.377	0.086	0.017	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	262	113	113	713	0	0
normalized size	1	1.	3.28	1.41	1.41	8.91	0.	0.
time (sec)	N/A	0.083	0.236	0.097	1.483	2.474	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	44	0	42	0	0	0
normalized size	1	1.	0.77	0.	0.74	0.	0.	0.
time (sec)	N/A	0.076	0.054	0.35	1.63	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F(-2)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	121	121	131	171	0	0	0	0
normalized size	1	1.	1.08	1.41	0.	0.	0.	0.
time (sec)	N/A	0.109	0.195	0.266	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [29] had the largest ratio of [1.154]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	12	0.083
2	A	4	3	1.	19	0.158
3	A	2	2	1.	6	0.333
4	A	5	5	1.	10	0.5
5	A	3	2	1.	7	0.286
6	A	22	9	1.	8	1.125
7	F	0	0	N/A	0	N/A
8	F	0	0	N/A	0	N/A
9	A	4	3	1.	19	0.158
10	A	6	3	1.	25	0.12
11	A	5	2	1.	19	0.105
12	A	6	3	1.	21	0.143
13	A	20	8	1.	28	0.286
14	A	22	9	1.	21	0.429
15	A	2	1	1.	27	0.037

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
16	A	3	2	1.	36	0.056
17	A	7	5	1.	17	0.294
18	A	7	5	1.	17	0.294
19	A	6	6	1.	25	0.24
20	A	7	7	1.	14	0.5
21	B	45	7	7.28	9	0.778
22	A	3	3	1.	8	0.375
23	A	2	2	1.	10	0.2
24	A	11	7	1.34	16	0.438
25	A	5	5	1.	11	0.454
26	A	31	12	1.89	16	0.75
27	A	12	10	1.	16	0.625
28	A	13	12	1.	12	1.
29	A	28	15	1.	13	1.154
30	A	44	10	1.	18	0.556
31	A	35	16	1.	18	0.889
32	A	21	7	1.	14	0.5
33	A	7	4	1.	5	0.8
34	A	5	5	1.	8	0.625
35	A	10	7	1.	14	0.5

Chapter 3

Listing of integrals

3.1 $\int \frac{1}{\sqrt{2}+\cos(z)+\sin(z)}\,dz$

Optimal. Leaf size=22

$$-\frac{1-\sqrt{2}\sin(z)}{\cos(z)-\sin(z)}$$

```
[Out] -((1 - Sqrt[2]*Sin[z])/(Cos[z] - Sin[z]))
```

Rubi [A] time = 0.135323, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3114}

$$-\frac{1-\sqrt{2}\sin(z)}{\cos(z)-\sin(z)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[2] + Cos[z] + Sin[z])^(-1),z]
```

```
[Out] -((1 - Sqrt[2]*Sin[z])/(Cos[z] - Sin[z]))
```

Rule 3114

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> -Simp[(c - a*Sin[d + e*x])/(c*e*(c*Cos[d + e*x] - b*Sin[
```

```
d + e*x])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2 - c^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{2}+\cos(z)+\sin(z)}\,dz = -\frac{1-\sqrt{2}\sin(z)}{\cos(z)-\sin(z)}$$

Mathematica [C] time = 0.0607091, size = 77, normalized size = 3.5

$$\frac{\left((1+i)-i\sqrt{2}\right)\sin\left(\frac{z}{2}\right)-\left(\sqrt{2}+(1+3i)\right)\cos\left(\frac{z}{2}\right)}{i\left(\sqrt{2}+(-1-i)\right)\sin\left(\frac{z}{2}\right)+\left(\sqrt{2}+(1+i)\right)\cos\left(\frac{z}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[2] + Cos[z] + Sin[z])^(-1),z]
```

```
[Out] (-(((1 + 3*I) + Sqrt[2])*Cos[z/2]) + ((1 + I) - I*Sqrt[2])*Sin[z/2])/(((1 +
 I) + Sqrt[2])*Cos[z/2] + I*((-1 - I) + Sqrt[2])*Sin[z/2])
```

Maple [A] time = 0.047, size = 21, normalized size = 1.

$$-2\,\frac{1}{\left(\sqrt{2}-1\right)\left(\tan(z/2)+\sqrt{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(z)+sin(z)+2^(1/2)),z)
```

```
[Out] -2/(2^(1/2)-1)/(tan(1/2*z)+2^(1/2)+1)
```

Maxima [A] time = 1.43676, size = 27, normalized size = 1.23

$$-\frac{2}{\frac{\left(\sqrt{2}-1\right)\sin(z)}{\cos(z)+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(z)+sin(z)+2^(1/2)),z, algorithm="maxima")
```

```
[Out] -2/((sqrt(2) - 1)*sin(z)/(cos(z) + 1) + 1)
```

Fricas [A] time = 1.02825, size = 82, normalized size = 3.73

$$\frac{\sqrt{2}\cos(z) + \sqrt{2}\sin(z) - 2}{2\left(\cos(z) - \sin(z)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(z)+sin(z)+2^(1/2)),z, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(2)*cos(z) + sqrt(2)*sin(z) - 2)/(cos(z) - sin(z))
```

Sympy [B] time = 6.84537, size = 51, normalized size = 2.32

$$-\frac{2\tan\left(\frac{z}{2}\right)}{-\tan\left(\frac{z}{2}\right) + \sqrt{2}\tan\left(\frac{z}{2}\right) + 1} + \frac{2\sqrt{2}\tan\left(\frac{z}{2}\right)}{-\tan\left(\frac{z}{2}\right) + \sqrt{2}\tan\left(\frac{z}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(z)+sin(z)+2**(1/2)),z)
```

```
[Out] -2*tan(z/2)/(-tan(z/2) + sqrt(2)*tan(z/2) + 1) + 2*sqrt(2)*tan(z/2)/(-tan(z
/2) + sqrt(2)*tan(z/2) + 1)
```

Giac [A] time = 1.1178, size = 24, normalized size = 1.09

$$-\frac{2\left(\sqrt{2}+1\right)}{\sqrt{2} + \tan\left(\frac{1}{2}z\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(z)+sin(z)+2^(1/2)),z, algorithm="giac")
```

```
[Out] -2*(sqrt(2) + 1)/(sqrt(2) + tan(1/2*z) + 1)
```

3.2 $\int \frac{1}{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2} \, dx$

Optimal. Leaf size=32

$$\frac{\sqrt{1-x^2}}{2x}-\frac{1}{2x}+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] -1/(2*x) + Sqrt[1 - x^2]/(2*x) + ArcSin[x]/2
```

Rubi [A] time = 0.156235, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {6690, 277, 216}

$$\frac{\sqrt{1-x^2}}{2x}-\frac{1}{2x}+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] -1/(2*x) + Sqrt[1 - x^2]/(2*x) + ArcSin[x]/2
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*
(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rule 277

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\int \frac{1}{\left(\sqrt{1-x}+\sqrt{1+x}\right)^2}\,dx = \frac{1}{4}\int \left(\frac{2}{x^2}-\frac{2\sqrt{1-x^2}}{x^2}\right)dx$$
$$= -\frac{1}{2x}-\frac{1}{2}\int \frac{\sqrt{1-x^2}}{x^2}\,dx$$
$$= -\frac{1}{2x}+\frac{\sqrt{1-x^2}}{2x}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx$$
$$= -\frac{1}{2x}+\frac{\sqrt{1-x^2}}{2x}+\frac{1}{2}\sin^{-1}(x)$$

Mathematica [A] time = 0.0171405, size = 24, normalized size = 0.75

$$\frac{\sqrt{1-x^2}+x\sin^{-1}(x)-1}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^(-2),x]
```

```
[Out] (-1 + Sqrt[1 - x^2] + x*ArcSin[x])/(2*x)
```

Maple [B] time = 0.009, size = 50, normalized size = 1.6

$$-\frac{1}{2\,x}-\frac{1}{2\,x}\left(-\arcsin(x)\,x-\sqrt{-x^2+1}\right)\sqrt{1-x}\sqrt{1+x}\frac{1}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((1-x)^(1/2)+(1+x)^(1/2))^2,x)
```

```
[Out] -1/2/x-1/2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(1-x)^(1/2)*(1+x)^(1/2)/x/(-x^2+1)
^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\sqrt{x+1}+\sqrt{-x+1}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate((sqrt(x + 1) + sqrt(-x + 1))^(-2), x)
```

Fricas [A] time = 1.11214, size = 116, normalized size = 3.62

$$-\frac{2\,x\arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)-\sqrt{x+1}\sqrt{-x+1}+1}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x +
1) + 1)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(\sqrt{1-x}+\sqrt{x+1}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**(-2), x)
```

Giac [B] time = 1.14937, size = 198, normalized size = 6.19

$$\frac{1}{2}\pi + \frac{2\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2 - 4} - \frac{1}{2x} + \arctan\left(\frac{\sqrt{x+1}\left(\frac{\left(\sqrt{2}-\sqrt{-x+1}\right)^2}{x+1} - 1\right)}{2\left(\sqrt{2}-\sqrt{-x+1}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*pi + 2*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - s
qrt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2)
 - sqrt(-x + 1)))^2 - 4) - 1/2/x + arctan(1/2*sqrt(x + 1)*((sqrt(2) - sqrt(
-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1)))
```

3.3 $\int \frac{1}{(1+\cos(x))^2} \, dx$

Optimal. Leaf size=25

$$\frac{\sin(x)}{3(\cos(x)+1)}+\frac{\sin(x)}{3(\cos(x)+1)^2}$$

```
[Out] Sin[x]/(3*(1 + Cos[x])^2) + Sin[x]/(3*(1 + Cos[x]))
```

Rubi [A] time = 0.013607, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2650, 2648}

$$\frac{\sin(x)}{3(\cos(x)+1)}+\frac{\sin(x)}{3(\cos(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x])^(-2),x]
```

```
[Out] Sin[x]/(3*(1 + Cos[x])^2) + Sin[x]/(3*(1 + Cos[x]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(1+\cos(x))^2}\,dx &= \frac{\sin(x)}{3(1+\cos(x))^2} + \frac{1}{3}\int \frac{1}{1+\cos(x)}\,dx \\
&= \frac{\sin(x)}{3(1+\cos(x))^2} + \frac{\sin(x)}{3(1+\cos(x))}
\end{aligned}
$$

Mathematica [A] time = 0.0088695, size = 16, normalized size = 0.64

$$\frac{\sin(x)(\cos(x)+2)}{3(\cos(x)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x])^(-2),x]
```

```
[Out] ((2 + Cos[x])*Sin[x])/(3*(1 + Cos[x])^2)
```

Maple [A] time = 0.005, size = 16, normalized size = 0.6

$$\frac{1}{6}\left(\tan\left(\frac{x}{2}\right)\right)^3 + \frac{1}{2}\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x))^2,x)
```

```
[Out] 1/6*tan(1/2*x)^3+1/2*tan(1/2*x)
```

Maxima [A] time = 0.964996, size = 31, normalized size = 1.24

$$\frac{\sin(x)}{2(\cos(x)+1)} + \frac{\sin(x)^3}{6(\cos(x)+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*sin(x)/(cos(x) + 1) + 1/6*sin(x)^3/(cos(x) + 1)^3
```

Fricas [A] time = 1.11919, size = 69, normalized size = 2.76

$$\frac{(\cos(x) + 2)\sin(x)}{3\left(\cos(x)^2 + 2\cos(x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x) + 2)*sin(x)/(cos(x)^2 + 2*cos(x) + 1)
```

Sympy [A] time = 0.408357, size = 14, normalized size = 0.56

$$\frac{\tan^3\left(\frac{x}{2}\right)}{6} + \frac{\tan\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x))**2,x)
```

```
[Out] tan(x/2)**3/6 + tan(x/2)/2
```

Giac [A] time = 1.10102, size = 20, normalized size = 0.8

$$\frac{1}{6}\tan\left(\frac{1}{2}x\right)^3 + \frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x))^2,x, algorithm="giac")
```

```
[Out] 1/6*tan(1/2*x)^3 + 1/2*tan(1/2*x)
```

3.4 $\int \frac{\sin(x)}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=58

$$\sqrt{2\pi}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\sqrt{2\pi}\sin(1)\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)$$

```
[Out] Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]] - Sqrt[2*Pi]*FresnelC[Sq
rt[2/Pi]*Sqrt[1 + x]]*Sin[1]
```

Rubi [A] time = 0.0815101, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3306, 3305, 3351, 3304, 3352}

$$\sqrt{2\pi}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)-\sqrt{2\pi}\sin(1)\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/Sqrt[1 + x],x]
```

```
[Out] Sqrt[2*Pi]*Cos[1]*FresnelS[Sqrt[2/Pi]*Sqrt[1 + x]] - Sqrt[2*Pi]*FresnelC[Sq
rt[2/Pi]*Sqrt[1 + x]]*Sin[1]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{\sqrt{1+x}}\,dx &= \cos(1)\int \frac{\sin(1+x)}{\sqrt{1+x}}\,dx - \sin(1)\int \frac{\cos(1+x)}{\sqrt{1+x}}\,dx\\ &= (2\cos(1))\,\mathrm{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{1+x}\right) - (2\sin(1))\,\mathrm{Subst}\left(\int \cos\left(x^2\right)\,dx, x, \sqrt{1+x}\right)\\ &= \sqrt{2\pi}\cos(1)S\left(\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right) - \sqrt{2\pi}C\left(\sqrt{\frac{2}{\pi}}\sqrt{1+x}\right)\sin(1)\end{aligned}$$

Mathematica [C] time = 0.0161421, size = 68, normalized size = 1.17

$$-\frac{e^{-i}\left(\sqrt{-i(x+1)}\,\mathrm{Gamma}\left(\frac{1}{2},-i(x+1)\right)+e^{2i}\sqrt{i(x+1)}\,\mathrm{Gamma}\left(\frac{1}{2},i(x+1)\right)\right)}{2\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/Sqrt[1 + x],x]
```

```
[Out] -(Sqrt[(-I)*(1 + x)]*Gamma[1/2, (-I)*(1 + x)] + E^(2*I)*Sqrt[I*(1 + x)]*Gam
ma[1/2, I*(1 + x)])/(2*E^I*Sqrt[1 + x])
```

Maple [A] time = 0.012, size = 42, normalized size = 0.7

$$\sqrt{2}\sqrt{\pi}\left(\cos(1)\,\mathrm{FresnelS}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\sqrt{1+x}\right)-\sin(1)\,\mathrm{FresnelC}\left(\frac{\sqrt{2}}{\sqrt{\pi}}\sqrt{1+x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1+x)^(1/2),x)
```

```
[Out] 2^(1/2)*Pi^(1/2)*(cos(1)*FresnelS(2^(1/2)/Pi^(1/2)*(1+x)^(1/2))-sin(1)*Fres
nelC(2^(1/2)/Pi^(1/2)*(1+x)^(1/2)))
```

Maxima [C] time = 1.55022, size = 151, normalized size = 2.6

$$\frac{1}{8}\sqrt{\pi}\left(\left((i+1)\sqrt{2}\cos(1)+(i-1)\sqrt{2}\sin(1)\right)\operatorname{erf}\left(\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)+\left((i-1)\sqrt{2}\cos(1)+(i+1)\sqrt{2}\sin(1)\right)\operatorname{erf}\left(\left(\frac{1}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(pi)*(((I + 1)*sqrt(2)*cos(1) + (I - 1)*sqrt(2)*sin(1))*erf((1/2*I
+ 1/2)*sqrt(2)*sqrt(x + 1)) + ((I - 1)*sqrt(2)*cos(1) + (I + 1)*sqrt(2)*sin
(1))*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x + 1)) + (-(I - 1)*sqrt(2)*cos(1) - (I
 + 1)*sqrt(2)*sin(1))*erf(sqrt(-I)*sqrt(x + 1)) + ((I + 1)*sqrt(2)*cos(1) +
 (I - 1)*sqrt(2)*sin(1))*erf((-1)^(1/4)*sqrt(x + 1)))
```

Fricas [A] time = 1.13812, size = 182, normalized size = 3.14

$$\sqrt{2}\sqrt{\pi}\cos(1)\,\mathrm{S}\left(\frac{\sqrt{2}\sqrt{x+1}}{\sqrt{\pi}}\right)-\sqrt{2}\sqrt{\pi}\,\mathrm{C}\left(\frac{\sqrt{2}\sqrt{x+1}}{\sqrt{\pi}}\right)\sin(1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*sqrt(pi)*cos(1)*fresnel_sin(sqrt(2)*sqrt(x + 1)/sqrt(pi)) - sqrt(2)
*sqrt(pi)*fresnel_cos(sqrt(2)*sqrt(x + 1)/sqrt(pi))*sin(1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(x)}{\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+x)**(1/2),x)
```

```
[Out] Integral(sin(x)/sqrt(x + 1), x)
```

Giac [C] time = 1.09736, size = 58, normalized size = 1.

$$-\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{i}+\left(\frac{1}{4}i-\frac{1}{4}\right)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{x+1}\right)e^{(-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -(1/4*I + 1/4)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(x + 1))*e^I
 + (1/4*I - 1/4)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(x + 1))*e^
(-I)
```

3.5 $\int \frac{1}{(\cos(x)+\sin(x))^6}\,dx$

Optimal. Leaf size=50

$$-\frac{\cos(x)-\sin(x)}{15(\sin(x)+\cos(x))^3}-\frac{\cos(x)-\sin(x)}{10(\sin(x)+\cos(x))^5}+\frac{2\sin(x)}{15(\sin(x)+\cos(x))}$$

```
[Out] -(Cos[x] - Sin[x])/(10*(Cos[x] + Sin[x])^5) - (Cos[x] - Sin[x])/(15*(Cos[x]
 + Sin[x])^3) + (2*Sin[x])/(15*(Cos[x] + Sin[x]))
```

Rubi [A] time = 0.026043, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3076, 3075}

$$-\frac{\cos(x)-\sin(x)}{15(\sin(x)+\cos(x))^3}-\frac{\cos(x)-\sin(x)}{10(\sin(x)+\cos(x))^5}+\frac{2\sin(x)}{15(\sin(x)+\cos(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Sin[x])^(-6),x]
```

```
[Out] -(Cos[x] - Sin[x])/(10*(Cos[x] + Sin[x])^5) - (Cos[x] - Sin[x])/(15*(Cos[x]
 + Sin[x])^3) + (2*Sin[x])/(15*(Cos[x] + Sin[x]))
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(\cos(x)+\sin(x))^6}\,dx = -\frac{\cos(x)-\sin(x)}{10(\cos(x)+\sin(x))^5} + \frac{2}{5}\int \frac{1}{(\cos(x)+\sin(x))^4}\,dx$$
$$= -\frac{\cos(x)-\sin(x)}{10(\cos(x)+\sin(x))^5} - \frac{\cos(x)-\sin(x)}{15(\cos(x)+\sin(x))^3} + \frac{2}{15}\int \frac{1}{(\cos(x)+\sin(x))^2}\,dx$$
$$= -\frac{\cos(x)-\sin(x)}{10(\cos(x)+\sin(x))^5} - \frac{\cos(x)-\sin(x)}{15(\cos(x)+\sin(x))^3} + \frac{2\sin(x)}{15(\cos(x)+\sin(x))}$$

Mathematica [A] time = 0.0327017, size = 26, normalized size = 0.52

$$-\frac{-10\sin(x)+\sin(5x)+5\cos(3x)}{30(\sin(x)+\cos(x))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])^(-6),x]
```

```
[Out] -(5*Cos[3*x] - 10*Sin[x] + Sin[5*x])/(30*(Cos[x] + Sin[x])^5)
```

Maple [A] time = 0.063, size = 42, normalized size = 0.8

$$2\,(1+\tan(x))^{-4} - (1+\tan(x))^{-1} - \frac{8}{3\,(1+\tan(x))^3} + 2\,(1+\tan(x))^{-2} - \frac{4}{5\,(1+\tan(x))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)+sin(x))^6,x)
```

```
[Out] 2/(1+tan(x))^4-1/(1+tan(x))-8/3/(1+tan(x))^3+2/(1+tan(x))^2-4/5/(1+tan(x))^
5
```

Maxima [A] time = 0.979376, size = 76, normalized size = 1.52

$$-\frac{15\,\tan(x)^4+30\,\tan(x)^3+40\,\tan(x)^2+20\,\tan(x)+7}{15\left(\tan(x)^5+5\,\tan(x)^4+10\,\tan(x)^3+10\,\tan(x)^2+5\,\tan(x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x))^6,x, algorithm="maxima")
```

```
[Out] -1/15*(15*tan(x)^4 + 30*tan(x)^3 + 40*tan(x)^2 + 20*tan(x) + 7)/(tan(x)^5 +
 5*tan(x)^4 + 10*tan(x)^3 + 10*tan(x)^2 + 5*tan(x) + 1)
```

Fricas [A] time = 1.12067, size = 198, normalized size = 3.96

$$-\frac{8\cos(x)^5 - 20\cos(x)^3 - \left(8\cos(x)^4 + 4\cos(x)^2 - 7\right)\sin(x) + 5\cos(x)}{30\left(4\cos(x)^5 + \left(4\cos(x)^4 - 8\cos(x)^2 - 1\right)\sin(x) - 5\cos(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x))^6,x, algorithm="fricas")
```

```
[Out] -1/30*(8*cos(x)^5 - 20*cos(x)^3 - (8*cos(x)^4 + 4*cos(x)^2 - 7)*sin(x) + 5*
cos(x))/(4*cos(x)^5 + (4*cos(x)^4 - 8*cos(x)^2 - 1)*sin(x) - 5*cos(x))
```

Sympy [B] time = 11.3954, size = 925, normalized size = 18.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x))**6,x)
```

```
[Out] -59*tan(x/2)**10/(255*tan(x/2)**10 - 2550*tan(x/2)**9 + 8925*tan(x/2)**8 -
10200*tan(x/2)**7 - 7650*tan(x/2)**6 + 17340*tan(x/2)**5 + 7650*tan(x/2)**4
 - 10200*tan(x/2)**3 - 8925*tan(x/2)**2 - 2550*tan(x/2) - 255) + 80*tan(x/2
)**9/(255*tan(x/2)**10 - 2550*tan(x/2)**9 + 8925*tan(x/2)**8 - 10200*tan(x/
2)**7 - 7650*tan(x/2)**6 + 17340*tan(x/2)**5 + 7650*tan(x/2)**4 - 10200*tan
(x/2)**3 - 8925*tan(x/2)**2 - 2550*tan(x/2) - 255) - 25*tan(x/2)**8/(255*ta
n(x/2)**10 - 2550*tan(x/2)**9 + 8925*tan(x/2)**8 - 10200*tan(x/2)**7 - 7650
*tan(x/2)**6 + 17340*tan(x/2)**5 + 7650*tan(x/2)**4 - 10200*tan(x/2)**3 - 8
925*tan(x/2)**2 - 2550*tan(x/2) - 255) - 1040*tan(x/2)**7/(255*tan(x/2)**10
 - 2550*tan(x/2)**9 + 8925*tan(x/2)**8 - 10200*tan(x/2)**7 - 7650*tan(x/2)*
*6 + 17340*tan(x/2)**5 + 7650*tan(x/2)**4 - 10200*tan(x/2)**3 - 8925*tan(x/
2)**2 - 2550*tan(x/2) - 255) + 1090*tan(x/2)**6/(255*tan(x/2)**10 - 2550*ta
n(x/2)**9 + 8925*tan(x/2)**8 - 10200*tan(x/2)**7 - 7650*tan(x/2)**6 + 17340
```

```
*tan(x/2)**5 + 7650*tan(x/2)**4 - 10200*tan(x/2)**3 - 8925*tan(x/2)**2 - 25
50*tan(x/2) - 255) - 1090*tan(x/2)**4/(255*tan(x/2)**10 - 2550*tan(x/2)**9
+ 8925*tan(x/2)**8 - 10200*tan(x/2)**7 - 7650*tan(x/2)**6 + 17340*tan(x/2)*
*5 + 7650*tan(x/2)**4 - 10200*tan(x/2)**3 - 8925*tan(x/2)**2 - 2550*tan(x/2
) - 255) - 1040*tan(x/2)**3/(255*tan(x/2)**10 - 2550*tan(x/2)**9 + 8925*tan
(x/2)**8 - 10200*tan(x/2)**7 - 7650*tan(x/2)**6 + 17340*tan(x/2)**5 + 7650*
tan(x/2)**4 - 10200*tan(x/2)**3 - 8925*tan(x/2)**2 - 2550*tan(x/2) - 255) +
 25*tan(x/2)**2/(255*tan(x/2)**10 - 2550*tan(x/2)**9 + 8925*tan(x/2)**8 - 1
0200*tan(x/2)**7 - 7650*tan(x/2)**6 + 17340*tan(x/2)**5 + 7650*tan(x/2)**4
- 10200*tan(x/2)**3 - 8925*tan(x/2)**2 - 2550*tan(x/2) - 255) + 80*tan(x/2)
/(255*tan(x/2)**10 - 2550*tan(x/2)**9 + 8925*tan(x/2)**8 - 10200*tan(x/2)**
7 - 7650*tan(x/2)**6 + 17340*tan(x/2)**5 + 7650*tan(x/2)**4 - 10200*tan(x/2
)**3 - 8925*tan(x/2)**2 - 2550*tan(x/2) - 255) + 59/(255*tan(x/2)**10 - 255
0*tan(x/2)**9 + 8925*tan(x/2)**8 - 10200*tan(x/2)**7 - 7650*tan(x/2)**6 + 1
7340*tan(x/2)**5 + 7650*tan(x/2)**4 - 10200*tan(x/2)**3 - 8925*tan(x/2)**2
- 2550*tan(x/2) - 255)
```

Giac [A] time = 1.10987, size = 43, normalized size = 0.86

$$-\frac{15\,\tan\left(x\right)^{4}+30\,\tan\left(x\right)^{3}+40\,\tan\left(x\right)^{2}+20\,\tan\left(x\right)+7}{15\,{\left(\tan\left(x\right)+1\right)}^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x))^6,x, algorithm="giac")
```

```
[Out] -1/15*(15*tan(x)^4 + 30*tan(x)^3 + 40*tan(x)^2 + 20*tan(x) + 7)/(tan(x) + 1
)^5
```

3.6 $\int \log \left(\frac{1}{x^4}+x^4\right) dx$

Optimal. Leaf size=334

$$x \log \left(x^4+\frac{1}{x^4}\right)-\frac{1}{2}\sqrt{2-\sqrt{2}}\log \left(x^2-\sqrt{2-\sqrt{2}}x+1\right)+\frac{1}{2}\sqrt{2-\sqrt{2}}\log \left(x^2+\sqrt{2-\sqrt{2}}x+1\right)-\frac{1}{2}\sqrt{2+\sqrt{2}}\log \left(x^2-\sqrt{2\right.$$

```
[Out] -4*x - Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]
] - Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]] +
 Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]] + Sq
rt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]] - (Sqrt
[2 - Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/2 + (Sqrt[2 - Sqrt[2]]*Lo
g[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/2 - (Sqrt[2 + Sqrt[2]]*Log[1 - Sqrt[2 + S
qrt[2]]*x + x^2])/2 + (Sqrt[2 + Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2]
)/2 + x*Log[x^(-4) + x^4]
```

Rubi [A] time = 0.347468, antiderivative size = 334, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.125, Rules used = {2523, 12, 388, 213, 1169, 634, 618, 204, 628}

$$x \log \left(x^4+\frac{1}{x^4}\right)-\frac{1}{2}\sqrt{2-\sqrt{2}}\log \left(x^2-\sqrt{2-\sqrt{2}}x+1\right)+\frac{1}{2}\sqrt{2-\sqrt{2}}\log \left(x^2+\sqrt{2-\sqrt{2}}x+1\right)-\frac{1}{2}\sqrt{2+\sqrt{2}}\log \left(x^2-\sqrt{2\right.$$

Antiderivative was successfully verified.

```
[In] Int[Log[x^(-4) + x^4],x]
```


```
[Out] -4*x - Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] - 2*x)/Sqrt[2 + Sqrt[2]]
] - Sqrt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] - 2*x)/Sqrt[2 - Sqrt[2]]] +
 Sqrt[2 + Sqrt[2]]*ArcTan[(Sqrt[2 - Sqrt[2]] + 2*x)/Sqrt[2 + Sqrt[2]]] + Sq
rt[2 - Sqrt[2]]*ArcTan[(Sqrt[2 + Sqrt[2]] + 2*x)/Sqrt[2 - Sqrt[2]]] - (Sqrt
[2 - Sqrt[2]]*Log[1 - Sqrt[2 - Sqrt[2]]*x + x^2])/2 + (Sqrt[2 - Sqrt[2]]*Lo
g[1 + Sqrt[2 - Sqrt[2]]*x + x^2])/2 - (Sqrt[2 + Sqrt[2]]*Log[1 - Sqrt[2 + S
qrt[2]]*x + x^2])/2 + (Sqrt[2 + Sqrt[2]]*Log[1 + Sqrt[2 + Sqrt[2]]*x + x^2]
)/2 + x*Log[x^(-4) + x^4]
```


Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
```

```
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Dist[r/(2*Sqrt[2]*a), Int[(Sqrt[2]*r -
s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Dist[r/(2*Sq
rt[2]*a), Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n
/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \log\left(\frac{1}{x^4}+x^4\right)dx &= x\log\left(\frac{1}{x^4}+x^4\right)-\int\frac{4\left(-1+x^8\right)}{1+x^8}dx\\ &= x\log\left(\frac{1}{x^4}+x^4\right)-4\int\frac{-1+x^8}{1+x^8}dx\\ &= -4x+x\log\left(\frac{1}{x^4}+x^4\right)+8\int\frac{1}{1+x^8}dx\\ &= -4x+x\log\left(\frac{1}{x^4}+x^4\right)+\left(2\sqrt{2}\right)\int\frac{\sqrt{2}-x^2}{1-\sqrt{2}x^2+x^4}dx+\left(2\sqrt{2}\right)\int\frac{\sqrt{2}+x^2}{1+\sqrt{2}x^2+x^4}dx\\ &= -4x+x\log\left(\frac{1}{x^4}+x^4\right)+\sqrt{2-\sqrt{2}}\int\frac{\sqrt{2\left(2+\sqrt{2}\right)}-\left(1+\sqrt{2}\right)x}{1-\sqrt{2+\sqrt{2}}x+x^2}dx+\sqrt{2-\sqrt{2}}\int\frac{\sqrt{2\left(2+\sqrt{2}\right)}+}{1+\sqrt{2+}}\\ &= -4x+x\log\left(\frac{1}{x^4}+x^4\right)-\frac{1}{2}\sqrt{2-\sqrt{2}}\int\frac{-\sqrt{2-\sqrt{2}}+2x}{1-\sqrt{2-\sqrt{2}}x+x^2}dx+\frac{1}{2}\sqrt{2-\sqrt{2}}\int\frac{\sqrt{2-\sqrt{2}}+2x}{1+\sqrt{2-\sqrt{2}}x+x^2}\\ &= -4x-\frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(1-\sqrt{2-\sqrt{2}}x+x^2\right)+\frac{1}{2}\sqrt{2-\sqrt{2}}\log\left(1+\sqrt{2-\sqrt{2}}x+x^2\right)-\frac{1}{2}\sqrt{2+\sqrt{2}}\,\mathrm{lo}\\ &= -4x-\sqrt{2+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2-\sqrt{2}}-2x}{\sqrt{2+\sqrt{2}}}\right)-\sqrt{2-\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{2}}-2x}{\sqrt{2-\sqrt{2}}}\right)+\sqrt{2+\sqrt{2}}\tan^{-1}\left(\right.\end{aligned}$$

Mathematica [C] time = 0.0041239, size = 30, normalized size = 0.09

$$8x\text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},-x^8\right)+x\log\left(x^4+\frac{1}{x^4}\right)-4x$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x^(-4) + x^4],x]
```

```
[Out] -4*x + 8*x*Hypergeometric2F1[1/8, 1, 9/8, -x^8] + x*Log[x^(-4) + x^4]
```

Maple [C] time = 0.015, size = 36, normalized size = 0.1

$$x \ln\left(\frac{x^8+1}{x^4}\right) - 4x + \sum_{\_R=\mathrm{RootOf}\left(\_Z^8+1\right)} \frac{\ln(x-\_R)}{\_R^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1/x^4+x^4),x)
```

```
[Out] x*ln((x^8+1)/x^4)-4*x+sum(1/_R^7*ln(x-_R),_R=RootOf(_Z^8+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x \log\left(x^8+1\right) - 4x\log(x) - 4x + 8\int \frac{1}{x^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x^4+x^4),x, algorithm="maxima")
```

```
[Out] x*log(x^8 + 1) - 4*x*log(x) - 4*x + 8*integrate(1/(x^8 + 1), x)
```

Fricas [B] time = 1.21965, size = 3152, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x^4+x^4),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*arctan(-(2*sq
rt(2)*x - sqrt(2)*sqrt(4*x^2 + 2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 2*sqrt(2)*x*
sqrt(-sqrt(2) + 2) + 4) + sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))/(sqrt(sqr
t(2) + 2) + sqrt(-sqrt(2) + 2))) - 1/2*(sqrt(2)*sqrt(sqrt(2) + 2) + sqrt(2)
*sqrt(-sqrt(2) + 2))*arctan(-(2*sqrt(2)*x - sqrt(2)*sqrt(4*x^2 - 2*sqrt(2)*
x*sqrt(sqrt(2) + 2) + 2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 4) - sqrt(sqrt(2) +
2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))) + 1/2*(s
qrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*arctan((2*sqrt(2)*x
- sqrt(2)*sqrt(4*x^2 + 2*sqrt(2)*x*sqrt(sqrt(2) + 2) + 2*sqrt(2)*x*sqrt(-sq
rt(2) + 2) + 4) + sqrt(sqrt(2) + 2) + sqrt(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2
) - sqrt(-sqrt(2) + 2))) + 1/2*(sqrt(2)*sqrt(sqrt(2) + 2) - sqrt(2)*sqrt(-s
qrt(2) + 2))*arctan((2*sqrt(2)*x - sqrt(2)*sqrt(4*x^2 - 2*sqrt(2)*x*sqrt(sq
rt(2) + 2) - 2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 4) - sqrt(sqrt(2) + 2) - sqrt
(-sqrt(2) + 2))/(sqrt(sqrt(2) + 2) - sqrt(-sqrt(2) + 2))) + 1/8*(sqrt(2)*sq
rt(sqrt(2) + 2) + sqrt(2)*sqrt(-sqrt(2) + 2))*log(4*x^2 + 2*sqrt(2)*x*sqrt(
sqrt(2) + 2) + 2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 4) + 1/8*(sqrt(2)*sqrt(sqrt
(2) + 2) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(4*x^2 + 2*sqrt(2)*x*sqrt(sqrt(2)
 + 2) - 2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 4) - 1/8*(sqrt(2)*sqrt(sqrt(2) + 2
) - sqrt(2)*sqrt(-sqrt(2) + 2))*log(4*x^2 - 2*sqrt(2)*x*sqrt(sqrt(2) + 2) +
 2*sqrt(2)*x*sqrt(-sqrt(2) + 2) + 4) - 1/8*(sqrt(2)*sqrt(sqrt(2) + 2) + sqr
t(2)*sqrt(-sqrt(2) + 2))*log(4*x^2 - 2*sqrt(2)*x*sqrt(sqrt(2) + 2) - 2*sqrt
(2)*x*sqrt(-sqrt(2) + 2) + 4) + x*log((x^8 + 1)/x^4) - sqrt(sqrt(2) + 2)*ar
ctan(-(2*x - 2*sqrt(x^2 + x*sqrt(-sqrt(2) + 2) + 1) + sqrt(-sqrt(2) + 2))/s
qrt(sqrt(2) + 2)) - sqrt(sqrt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 - x*sqrt(-s
qrt(2) + 2) + 1) - sqrt(-sqrt(2) + 2))/sqrt(sqrt(2) + 2)) - sqrt(-sqrt(2) +
 2)*arctan(-(2*x - 2*sqrt(x^2 + x*sqrt(sqrt(2) + 2) + 1) + sqrt(sqrt(2) + 2
))/sqrt(-sqrt(2) + 2)) - sqrt(-sqrt(2) + 2)*arctan(-(2*x - 2*sqrt(x^2 - x*s
qrt(sqrt(2) + 2) + 1) - sqrt(sqrt(2) + 2))/sqrt(-sqrt(2) + 2)) + 1/4*sqrt(s
qrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1/4*sqrt(sqrt(2) + 2)*log(
x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/4*sqrt(-sqrt(2) + 2)*log(x^2 + x*sqrt(-s
qrt(2) + 2) + 1) - 1/4*sqrt(-sqrt(2) + 2)*log(x^2 - x*sqrt(-sqrt(2) + 2) +
1) - 4*x
```

Sympy [A] time = 1.19951, size = 26, normalized size = 0.08

$$x\log\left(x^4+\frac{1}{x^4}\right)-4x-\operatorname{RootSum}\left(t^8+1,\left(t\mapsto t\log\left(-t+x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1/x**4+x**4),x)
```

```
[Out] x*log(x**4 + x**(-4)) - 4*x - RootSum(_t**8 + 1, Lambda(_t, _t*log(-_t + x)
))
```

<u>Giac</u> [A] time = 1.12418, size = 335, normalized size = 1.

$$x\log\left(x^4+\frac{1}{x^4}\right)+\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x+\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\sqrt{\sqrt{2}+2}\arctan\left(\frac{2x-\sqrt{-\sqrt{2}+2}}{\sqrt{\sqrt{2}+2}}\right)+\sqrt{-\sqrt{2}+2}\arctan\left(\frac{2x}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1/x^4+x^4),x, algorithm="giac")
```

```
[Out] x*log(x^4 + 1/x^4) + sqrt(sqrt(2) + 2)*arctan((2*x + sqrt(-sqrt(2) + 2))/sq
rt(sqrt(2) + 2)) + sqrt(sqrt(2) + 2)*arctan((2*x - sqrt(-sqrt(2) + 2))/sqrt
(sqrt(2) + 2)) + sqrt(-sqrt(2) + 2)*arctan((2*x + sqrt(sqrt(2) + 2))/sqrt(-
sqrt(2) + 2)) + sqrt(-sqrt(2) + 2)*arctan((2*x - sqrt(sqrt(2) + 2))/sqrt(-s
qrt(2) + 2)) + 1/2*sqrt(sqrt(2) + 2)*log(x^2 + x*sqrt(sqrt(2) + 2) + 1) - 1
/2*sqrt(sqrt(2) + 2)*log(x^2 - x*sqrt(sqrt(2) + 2) + 1) + 1/2*sqrt(-sqrt(2)
 + 2)*log(x^2 + x*sqrt(-sqrt(2) + 2) + 1) - 1/2*sqrt(-sqrt(2) + 2)*log(x^2
- x*sqrt(-sqrt(2) + 2) + 1) - 4*x
```

3.7 $\int \frac{\log(1+x)}{x\sqrt{1+\sqrt{1+x}}} \, dx$

Optimal. Leaf size=291

$$\sqrt{2}\text{PolyLog}\left(2,-\frac{\sqrt{2}\left(1-\sqrt{\sqrt{x+1}+1}\right)}{2-\sqrt{2}}\right)-\sqrt{2}\text{PolyLog}\left(2,\frac{\sqrt{2}\left(1-\sqrt{\sqrt{x+1}+1}\right)}{2+\sqrt{2}}\right)-\sqrt{2}\text{PolyLog}\left(2,-\frac{\sqrt{2}\left(\sqrt{\sqrt{x+1}+\right.}{2-\sqrt{2}}\right.$$

```
[Out] -8*ArcTanh[Sqrt[1 + Sqrt[1 + x]]] - (2*Log[1 + x])/Sqrt[1 + Sqrt[1 + x]] -
Sqrt[2]*ArcTanh[Sqrt[1 + Sqrt[1 + x]]/Sqrt[2]]*Log[1 + x] + 2*Sqrt[2]*ArcTa
nh[1/Sqrt[2]]*Log[1 - Sqrt[1 + Sqrt[1 + x]]] - 2*Sqrt[2]*ArcTanh[1/Sqrt[2]]
*Log[1 + Sqrt[1 + Sqrt[1 + x]]] + Sqrt[2]*PolyLog[2, -((Sqrt[2]*(1 - Sqrt[1
 + Sqrt[1 + x]]))/(2 - Sqrt[2]))] - Sqrt[2]*PolyLog[2, (Sqrt[2]*(1 - Sqrt[1
 + Sqrt[1 + x]]))/(2 + Sqrt[2])] - Sqrt[2]*PolyLog[2, -((Sqrt[2]*(1 + Sqrt[
1 + Sqrt[1 + x]]))/(2 - Sqrt[2]))] + Sqrt[2]*PolyLog[2, (Sqrt[2]*(1 + Sqrt[
1 + Sqrt[1 + x]]))/(2 + Sqrt[2])]
```

Rubi [F] time = 0.0486268, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\log(1+x)}{x\sqrt{1+\sqrt{1+x}}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Log[1 + x]/(x*Sqrt[1 + Sqrt[1 + x]]),x]
```

```
[Out] Defer[Int][Log[1 + x]/(x*Sqrt[1 + Sqrt[1 + x]]), x]
```

Rubi steps

$$\int \frac{\log(1+x)}{x\sqrt{1+\sqrt{1+x}}} \, dx = \int \frac{\log(1+x)}{x\sqrt{1+\sqrt{1+x}}} \, dx$$

Mathematica [A] time = 0.919844, size = 430, normalized size = 1.48

$$-\sqrt{2}\left(2\text{PolyLog}\left(2,\frac{\sqrt{2}}{\sqrt{\sqrt{x+1}+1}}\right)-\text{PolyLog}\left(2,-\left(\sqrt{2}-2\right)\left(\frac{1}{\sqrt{\sqrt{x+1}+1}}+1\right)\right)+\text{PolyLog}\left(2,\left(1+\sqrt{2}\right)\left(\frac{\sqrt{2}}{\sqrt{\sqrt{x+1}}}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[1 + x]/(x*Sqrt[1 + Sqrt[1 + x]]),x]
```

```
[Out] -8*ArcTanh[1/Sqrt[1 + Sqrt[1 + x]]] - (2*Log[1 + x])/Sqrt[1 + Sqrt[1 + x]]
+ (Log[1 + x]*Log[1 - Sqrt[2]/Sqrt[1 + Sqrt[1 + x]]])/Sqrt[2] - (Log[1 + x]
*Log[1 + Sqrt[2]/Sqrt[1 + Sqrt[1 + x]]])/Sqrt[2] - Sqrt[2]*(Log[1 + Sqrt[2]
]*Log[1 + 1/Sqrt[1 + Sqrt[1 + x]]] + Log[-((2 + Sqrt[2])*(-1 + 1/Sqrt[1 + S
qrt[1 + x]]))]*Log[1 - Sqrt[2]/Sqrt[1 + Sqrt[1 + x]]] + 2*PolyLog[2, Sqrt[2
]/Sqrt[1 + Sqrt[1 + x]]] - PolyLog[2, -((-2 + Sqrt[2])*(1 + 1/Sqrt[1 + Sqrt
[1 + x]]))] + PolyLog[2, (1 + Sqrt[2])*(-1 + Sqrt[2]/Sqrt[1 + Sqrt[1 + x]])
]) + Sqrt[2]*(Log[1 + Sqrt[2]]*Log[1 - 1/Sqrt[1 + Sqrt[1 + x]]] + Log[(2 +
Sqrt[2])*(1 + 1/Sqrt[1 + Sqrt[1 + x]])]*Log[1 + Sqrt[2]/Sqrt[1 + Sqrt[1 + x
]]] + 2*PolyLog[2, -(Sqrt[2]/Sqrt[1 + Sqrt[1 + x]])] - PolyLog[2, (-2 + Sqr
t[2])*(-1 + 1/Sqrt[1 + Sqrt[1 + x]])] + PolyLog[2, -((1 + Sqrt[2])*(1 + Sqr
t[2]/Sqrt[1 + Sqrt[1 + x]]))])
```

Maple [C] time = 0.096, size = 171, normalized size = 0.6

$$-2\frac{\ln(1+x)}{\sqrt{1+\sqrt{1+x}}}-8\,\text{Artanh}\left(\sqrt{1+\sqrt{1+x}}\right)+4\sum_{\_alpha=\text{RootOf}\left(\_Z^2-2\right)}1/8\left(\ln\left(\sqrt{1+\sqrt{1+x}}-\_alpha\right)\ln(1+x)-2\,\text{di}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+x)/x/(1+(1+x)^(1/2))^(1/2),x)
```

```
[Out] -2*ln(1+x)/(1+(1+x)^(1/2))^(1/2)-8*arctanh((1+(1+x)^(1/2))^(1/2))+4*Sum(1/8
*(ln((1+(1+x)^(1/2))^(1/2)-_alpha)*ln(1+x)-2*dilog((1+(1+(1+x)^(1/2))^(1/2)
)/(1+_alpha))-2*ln((1+(1+x)^(1/2))^(1/2)-_alpha)*ln((1+(1+(1+x)^(1/2))^(1/2
))/(1+_alpha))-2*dilog((-1+(1+(1+x)^(1/2))^(1/2))/(-1+_alpha))-2*ln((1+(1+x
)^(1/2))^(1/2)-_alpha)*ln((-1+(1+(1+x)^(1/2))^(1/2))/(-1+_alpha)))*_alpha,_
alpha=RootOf(_Z^2-2))
```

Maxima [A] time = 1.6204, size = 494, normalized size = 1.7

$$\frac{1}{2}\left(\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{\sqrt{x+1}+1}}{\sqrt{2}+\sqrt{\sqrt{x+1}+1}}\right)-\frac{4}{\sqrt{\sqrt{x+1}+1}}\right)\log\left(x+1\right)+\sqrt{2}\left(\log\left(\sqrt{2}+\sqrt{\sqrt{x+1}+1}\right)\log\left(-\frac{\sqrt{2}+\sqrt{\sqrt{x+1}+1}}{\sqrt{2}+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x)/x/(1+(1+x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(2)*log(-(sqrt(2) - sqrt(sqrt(x + 1) + 1))/(sqrt(2) + sqrt(sqrt(x
+ 1) + 1))) - 4/sqrt(sqrt(x + 1) + 1))*log(x + 1) + sqrt(2)*(log(sqrt(2) +
sqrt(sqrt(x + 1) + 1))*log(-(sqrt(2) + sqrt(sqrt(x + 1) + 1))/(sqrt(2) + 1)
 + 1) + dilog((sqrt(2) + sqrt(sqrt(x + 1) + 1))/(sqrt(2) + 1))) - sqrt(2)*(
log(-sqrt(2) + sqrt(sqrt(x + 1) + 1))*log(-(sqrt(2) - sqrt(sqrt(x + 1) + 1)
)/(sqrt(2) + 1) + 1) + dilog((sqrt(2) - sqrt(sqrt(x + 1) + 1))/(sqrt(2) + 1
))) + sqrt(2)*(log(sqrt(2) + sqrt(sqrt(x + 1) + 1))*log(-(sqrt(2) + sqrt(sq
rt(x + 1) + 1))/(sqrt(2) - 1) + 1) + dilog((sqrt(2) + sqrt(sqrt(x + 1) + 1)
)/(sqrt(2) - 1))) - sqrt(2)*(log(-sqrt(2) + sqrt(sqrt(x + 1) + 1))*log(-(sq
rt(2) - sqrt(sqrt(x + 1) + 1))/(sqrt(2) - 1) + 1) + dilog((sqrt(2) - sqrt(s
qrt(x + 1) + 1))/(sqrt(2) - 1))) - 4*log(sqrt(sqrt(x + 1) + 1) + 1) + 4*log
(sqrt(sqrt(x + 1) + 1) - 1)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x)/x/(1+(1+x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(x+1\right)}{x\sqrt{\sqrt{x+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+x)/x/(1+(1+x)**(1/2))**(1/2),x)
```

```
[Out] Integral(log(x + 1)/(x*sqrt(sqrt(x + 1) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x+1)}{x\sqrt{\sqrt{x+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x)/x/(1+(1+x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(log(x + 1)/(x*sqrt(sqrt(x + 1) + 1)), x)
```

3.8 $\int \frac{\sqrt{1+\sqrt{1+x}}\log(1+x)}{x}\,dx$

Optimal. Leaf size=308

$$2\sqrt{2}\text{PolyLog}\left(2,-\frac{\sqrt{2}\left(1-\sqrt{\sqrt{x+1}+1}\right)}{2-\sqrt{2}}\right)-2\sqrt{2}\text{PolyLog}\left(2,\frac{\sqrt{2}\left(1-\sqrt{\sqrt{x+1}+1}\right)}{2+\sqrt{2}}\right)-2\sqrt{2}\text{PolyLog}\left(2,-\frac{\sqrt{2}\left(\sqrt{\sqrt{x+}\right.}{2-}\right.$$

```
[Out] -16*Sqrt[1 + Sqrt[1 + x]] + 16*ArcTanh[Sqrt[1 + Sqrt[1 + x]]] + 4*Sqrt[1 +
Sqrt[1 + x]]*Log[1 + x] - 2*Sqrt[2]*ArcTanh[Sqrt[1 + Sqrt[1 + x]]/Sqrt[2]]*
Log[1 + x] + 4*Sqrt[2]*ArcTanh[1/Sqrt[2]]*Log[1 - Sqrt[1 + Sqrt[1 + x]]] -
4*Sqrt[2]*ArcTanh[1/Sqrt[2]]*Log[1 + Sqrt[1 + Sqrt[1 + x]]] + 2*Sqrt[2]*Pol
yLog[2, -((Sqrt[2]*(1 - Sqrt[1 + Sqrt[1 + x]]))/(2 - Sqrt[2]))] - 2*Sqrt[2]
*PolyLog[2, (Sqrt[2]*(1 - Sqrt[1 + Sqrt[1 + x]]))/(2 + Sqrt[2])] - 2*Sqrt[2
]*PolyLog[2, -((Sqrt[2]*(1 + Sqrt[1 + Sqrt[1 + x]]))/(2 - Sqrt[2]))] + 2*Sq
rt[2]*PolyLog[2, (Sqrt[2]*(1 + Sqrt[1 + Sqrt[1 + x]]))/(2 + Sqrt[2])]
```

Rubi [F] time = 0.0432186, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{\sqrt{1+\sqrt{1+x}}\log(1+x)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[1 + Sqrt[1 + x]]*Log[1 + x])/x,x]
```

```
[Out] Defer[Int][(Sqrt[1 + Sqrt[1 + x]]*Log[1 + x])/x, x]
```

Rubi steps

$$\int \frac{\sqrt{1+\sqrt{1+x}}\log(1+x)}{x}\,dx=\int \frac{\sqrt{1+\sqrt{1+x}}\log(1+x)}{x}\,dx$$

Mathematica [B] time = 0.510365, size = 654, normalized size = 2.12

$$-2\sqrt{2}\text{PolyLog}\left(2,-\left(\sqrt{2}-1\right)\left(\sqrt{\sqrt{x+1}+1}-1\right)\right)+2\sqrt{2}\text{PolyLog}\left(2,\left(1+\sqrt{2}\right)\left(\sqrt{\sqrt{x+1}+1}-1\right)\right)+2\sqrt{2}\text{PolyLog}\left(2\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[1 + Sqrt[1 + x]]*Log[1 + x])/x,x]
```

```
[Out] -16*Sqrt[1 + Sqrt[1 + x]] + 4*Sqrt[1 + Sqrt[1 + x]]*Log[1 + x] + Sqrt[2]*Lo
g[1 + x]*Log[Sqrt[2] - Sqrt[1 + Sqrt[1 + x]]] - 8*Log[-1 + Sqrt[1 + Sqrt[1
+ x]]] - 2*Sqrt[2]*Log[Sqrt[2] - Sqrt[1 + Sqrt[1 + x]]]*Log[-1 + Sqrt[1 + S
qrt[1 + x]]] + 8*Log[1 + Sqrt[1 + Sqrt[1 + x]]] - 2*Sqrt[2]*Log[Sqrt[2] - S
qrt[1 + Sqrt[1 + x]]]*Log[1 + Sqrt[1 + Sqrt[1 + x]]] - Sqrt[2]*Log[1 + x]*L
og[Sqrt[2] + Sqrt[1 + Sqrt[1 + x]]] + 2*Sqrt[2]*Log[-1 + Sqrt[1 + Sqrt[1 +
x]]]*Log[Sqrt[2] + Sqrt[1 + Sqrt[1 + x]]] + 2*Sqrt[2]*Log[1 + Sqrt[1 + Sqrt
[1 + x]]]*Log[Sqrt[2] + Sqrt[1 + Sqrt[1 + x]]] - 2*Sqrt[2]*Log[-1 + Sqrt[1
+ Sqrt[1 + x]]]*Log[(-1 + Sqrt[2])*(Sqrt[2] + Sqrt[1 + Sqrt[1 + x]])] - 2*S
qrt[2]*Log[1 + Sqrt[1 + Sqrt[1 + x]]]*Log[2 + Sqrt[2] + Sqrt[1 + Sqrt[1 + x
]] + Sqrt[2]*Sqrt[1 + Sqrt[1 + x]]] + 2*Sqrt[2]*Log[-1 + Sqrt[1 + Sqrt[1 +
x]]]*Log[1 - (1 + Sqrt[2])*(-1 + Sqrt[1 + Sqrt[1 + x]])] + 2*Sqrt[2]*Log[1
+ Sqrt[1 + Sqrt[1 + x]]]*Log[1 - (-1 + Sqrt[2])*(1 + Sqrt[1 + Sqrt[1 + x]])
] - 2*Sqrt[2]*PolyLog[2, -((-1 + Sqrt[2])*(-1 + Sqrt[1 + Sqrt[1 + x]]))] +
2*Sqrt[2]*PolyLog[2, (1 + Sqrt[2])*(-1 + Sqrt[1 + Sqrt[1 + x]])] + 2*Sqrt[2
]*PolyLog[2, (-1 + Sqrt[2])*(1 + Sqrt[1 + Sqrt[1 + x]])] - 2*Sqrt[2]*PolyLo
g[2, -((1 + Sqrt[2])*(1 + Sqrt[1 + Sqrt[1 + x]]))]
```

Maple [C] time = 0.008, size = 198, normalized size = 0.6

$$4\ln(1+x)\sqrt{1+\sqrt{1+x}}-16\sqrt{1+\sqrt{1+x}}-8\ln\left(-1+\sqrt{1+\sqrt{1+x}}\right)+8\ln\left(1+\sqrt{1+\sqrt{1+x}}\right)+4\sum_{\_alpha=RootOf(\_Z}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+x)*(1+(1+x)^(1/2))^(1/2)/x,x)
```

```
[Out] 4*ln(1+x)*(1+(1+x)^(1/2))^(1/2)-16*(1+(1+x)^(1/2))^(1/2)-8*ln(-1+(1+(1+x)^(
1/2))^(1/2))+8*ln(1+(1+(1+x)^(1/2))^(1/2))+4*Sum(1/4*(ln((1+(1+x)^(1/2))^(1
/2)-_alpha)*ln(1+x)-2*dilog((1+(1+(1+x)^(1/2))^(1/2))/(1+_alpha))-2*ln((1+(
1+x)^(1/2))^(1/2)-_alpha)*ln((1+(1+(1+x)^(1/2))^(1/2))/(1+_alpha))-2*dilog(
```

```
(-1+(1+(1+x)^(1/2))^(1/2))/(-1+_alpha))-2*ln((1+(1+x)^(1/2))^(1/2)-_alpha)*
ln((-1+(1+(1+x)^(1/2))^(1/2))/(-1+_alpha)))*_alpha,_alpha=RootOf(_Z^2-2))
```

Maxima [A] time = 1.61415, size = 510, normalized size = 1.66

$$\left(\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{\sqrt{x+1}+1}}{\sqrt{2}+\sqrt{\sqrt{x+1}+1}}\right)+4\sqrt{\sqrt{x+1}+1}\right)\log\left(x+1\right)+2\sqrt{2}\left(\log\left(\sqrt{2}+\sqrt{\sqrt{x+1}+1}\right)\log\left(-\frac{\sqrt{2}+\sqrt{\sqrt{x+1}+1}}{\sqrt{2}+1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x)*(1+(1+x)^(1/2))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] (sqrt(2)*log(-(sqrt(2) - sqrt(sqrt(x + 1) + 1))/(sqrt(2) + sqrt(sqrt(x + 1)
 + 1))) + 4*sqrt(sqrt(x + 1) + 1))*log(x + 1) + 2*sqrt(2)*(log(sqrt(2) + sq
rt(sqrt(x + 1) + 1))*log(-(sqrt(2) + sqrt(sqrt(x + 1) + 1))/(sqrt(2) + 1) +
 1) + dilog((sqrt(2) + sqrt(sqrt(x + 1) + 1))/(sqrt(2) + 1))) - 2*sqrt(2)*(
log(-sqrt(2) + sqrt(sqrt(x + 1) + 1))*log(-(sqrt(2) - sqrt(sqrt(x + 1) + 1)
)/(sqrt(2) + 1) + 1) + dilog((sqrt(2) - sqrt(sqrt(x + 1) + 1))/(sqrt(2) + 1
))) + 2*sqrt(2)*(log(sqrt(2) + sqrt(sqrt(x + 1) + 1))*log(-(sqrt(2) + sqrt(
sqrt(x + 1) + 1))/(sqrt(2) - 1) + 1) + dilog((sqrt(2) + sqrt(sqrt(x + 1) +
1))/(sqrt(2) - 1))) - 2*sqrt(2)*(log(-sqrt(2) + sqrt(sqrt(x + 1) + 1))*log(
-(sqrt(2) - sqrt(sqrt(x + 1) + 1))/(sqrt(2) - 1) + 1) + dilog((sqrt(2) - sq
rt(sqrt(x + 1) + 1))/(sqrt(2) - 1))) - 16*sqrt(sqrt(x + 1) + 1) + 8*log(sqr
t(sqrt(x + 1) + 1) + 1) - 8*log(sqrt(sqrt(x + 1) + 1) - 1)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x)*(1+(1+x)^(1/2))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sqrt{x+1}+1}\log{(x+1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+x)*(1+(1+x)**(1/2))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(sqrt(x + 1) + 1)*log(x + 1)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{\sqrt{x+1}+1}\log{(x+1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+x)*(1+(1+x)^(1/2))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(x + 1) + 1)*log(x + 1)/x, x)
```

3.9 $\int \frac{1}{1+\sqrt{x+\sqrt{1+x^2}}}\,dx$

Optimal. Leaf size=84

$$\sqrt{\sqrt{x^2+1}+x}+\frac{1}{\sqrt{\sqrt{x^2+1}+x}}-\frac{1}{2\left(\sqrt{x^2+1}+x\right)}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-2\log\left(\sqrt{\sqrt{x^2+1}+x}+1\right)$$

```
[Out] -1/(2*(x + Sqrt[1 + x^2])) + 1/Sqrt[x + Sqrt[1 + x^2]] + Sqrt[x + Sqrt[1 +
x^2]] + Log[x + Sqrt[1 + x^2]]/2 - 2*Log[1 + Sqrt[x + Sqrt[1 + x^2]]]
```

Rubi [A] time = 0.0586596, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {2117, 1821, 1620}

$$\sqrt{\sqrt{x^2+1}+x}+\frac{1}{\sqrt{\sqrt{x^2+1}+x}}-\frac{1}{2\left(\sqrt{x^2+1}+x\right)}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-2\log\left(\sqrt{\sqrt{x^2+1}+x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sqrt[x + Sqrt[1 + x^2]])^(-1),x]
```



```
[Out] -1/(2*(x + Sqrt[1 + x^2])) + 1/Sqrt[x + Sqrt[1 + x^2]] + Sqrt[x + Sqrt[1 +
x^2]] + Log[x + Sqrt[1 + x^2]]/2 - 2*Log[1 + Sqrt[x + Sqrt[1 + x^2]]]
```


Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(
n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^
2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; Fre
eQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rule 1821

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n,
 Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Si
mplify[(m + 1)/n]]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{1}{1+\sqrt{x+\sqrt{1+x^2}}}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1+x^2}{\left(1+\sqrt{x}\right)x^2}\,dx, x, x+\sqrt{1+x^2}\right)$$
$$= \text{Subst}\left(\int \frac{1+x^4}{x^3(1+x)}\,dx, x, \sqrt{x+\sqrt{1+x^2}}\right)$$
$$= \text{Subst}\left(\int \left(1+\frac{1}{x^3}-\frac{1}{x^2}+\frac{1}{x}-\frac{2}{1+x}\right)dx, x, \sqrt{x+\sqrt{1+x^2}}\right)$$
$$= -\frac{1}{2\left(x+\sqrt{1+x^2}\right)}+\frac{1}{\sqrt{x+\sqrt{1+x^2}}}+\sqrt{x+\sqrt{1+x^2}}+\frac{1}{2}\log\left(x+\sqrt{1+x^2}\right)-2\log\left(1+\sqrt{x\right.}$$

Mathematica [A] time = 0.0406454, size = 84, normalized size = 1.

$$\sqrt{\sqrt{x^2+1}+x}+\frac{1}{\sqrt{\sqrt{x^2+1}+x}}-\frac{1}{2\left(\sqrt{x^2+1}+x\right)}+\frac{1}{2}\log\left(\sqrt{x^2+1}+x\right)-2\log\left(\sqrt{\sqrt{x^2+1}+x}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x + Sqrt[1 + x^2]])^(-1),x]
```

```
[Out] -1/(2*(x + Sqrt[1 + x^2])) + 1/Sqrt[x + Sqrt[1 + x^2]] + Sqrt[x + Sqrt[1 +
x^2]] + Log[x + Sqrt[1 + x^2]]/2 - 2*Log[1 + Sqrt[x + Sqrt[1 + x^2]]]
```

Maple [F] time = 0.017, size = 0, normalized size = 0.

$$\int \left(1+\sqrt{x+\sqrt{x^2+1}}\right)^{-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+(x+(x^2+1)^(1/2))^(1/2)),x)
```

```
[Out] int(1/(1+(x+(x^2+1)^(1/2))^(1/2)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x + sqrt(x^2 + 1)) + 1), x)
```

Fricas [A] time = 1.07762, size = 197, normalized size = 2.35

$$-\sqrt{x+\sqrt{x^2+1}}\left(x-\sqrt{x^2+1}-1\right)+\frac{1}{2}\,x-\frac{1}{2}\,\sqrt{x^2+1}-2\,\log\left(\sqrt{x+\sqrt{x^2+1}}+1\right)+\log\left(\sqrt{x+\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(x + sqrt(x^2 + 1))*(x - sqrt(x^2 + 1) - 1) + 1/2*x - 1/2*sqrt(x^2 + 1
) - 2*log(sqrt(x + sqrt(x^2 + 1)) + 1) + log(sqrt(x + sqrt(x^2 + 1)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x+\sqrt{x^2+1}}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x+(x**2+1)**(1/2))**(1/2)),x)
```

```
[Out] Integral(1/(sqrt(x + sqrt(x**2 + 1)) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x + \sqrt{x^2 + 1}} + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+(x+(x^2+1)^(1/2))^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x + sqrt(x^2 + 1)) + 1), x)
```

3.10 $\int \frac{\sqrt{1+x}}{x+\sqrt{1+\sqrt{1+x}}}\,dx$

Optimal. Leaf size=41

$$2\sqrt{x+1}+\frac{8\tanh^{-1}\left(\frac{2\sqrt{\sqrt{x+1}+1}+1}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] 2*Sqrt[1 + x] + (8*ArcTanh[(1 + 2*Sqrt[1 + Sqrt[1 + x]])/Sqrt[5]])/Sqrt[5]
```

Rubi [A] time = 0.193243, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {800, 618, 206}

$$2\sqrt{x+1}+\frac{8\tanh^{-1}\left(\frac{2\sqrt{\sqrt{x+1}+1}+1}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x]/(x + Sqrt[1 + Sqrt[1 + x]]),x]
```


```
[Out] 2*Sqrt[1 + x] + (8*ArcTanh[(1 + 2*Sqrt[1 + Sqrt[1 + x]])/Sqrt[5]])/Sqrt[5]
```


Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x}}{x+\sqrt{1+\sqrt{1+x}}}\,dx &= 2\,\text{Subst}\left(\int \frac{x^2}{-1+x^2+\sqrt{1+x}}\,dx, x, \sqrt{1+x}\right)\\
&= 4\,\text{Subst}\left(\int \frac{(-1+x)(1+x)^2}{-1+x+x^2}\,dx, x, \sqrt{1+\sqrt{1+x}}\right)\\
&= 4\,\text{Subst}\left(\int \left(x-\frac{1}{-1+x+x^2}\right)dx, x, \sqrt{1+\sqrt{1+x}}\right)\\
&= 2\sqrt{1+x}-4\,\text{Subst}\left(\int \frac{1}{-1+x+x^2}\,dx, x, \sqrt{1+\sqrt{1+x}}\right)\\
&= 2\sqrt{1+x}+8\,\text{Subst}\left(\int \frac{1}{5-x^2}\,dx, x, 1+2\sqrt{1+\sqrt{1+x}}\right)\\
&= 2\sqrt{1+x}+\frac{8\tanh^{-1}\left(\frac{1+2\sqrt{1+\sqrt{1+x}}}{\sqrt{5}}\right)}{\sqrt{5}}
\end{aligned}$$

Mathematica [A] time = 0.05696, size = 41, normalized size = 1.

$$2\sqrt{x+1}+\frac{8\tanh^{-1}\left(\frac{2\sqrt{\sqrt{x+1}+1}+1}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]/(x + Sqrt[1 + Sqrt[1 + x]]),x]
```

```
[Out] 2*Sqrt[1 + x] + (8*ArcTanh[(1 + 2*Sqrt[1 + Sqrt[1 + x]])/Sqrt[5]])/Sqrt[5]
```

Maple [A] time = 0.004, size = 34, normalized size = 0.8

$$2+2\sqrt{1+x}+\frac{8\sqrt{5}}{5}\text{Artanh}\left(\frac{\sqrt{5}}{5}\left(1+2\sqrt{1+\sqrt{1+x}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)/(x+(1+(1+x)^(1/2))^(1/2)),x)
```

```
[Out] 2+2*(1+x)^(1/2)+8/5*arctanh(1/5*(1+2*(1+(1+x)^(1/2))^(1/2))*5^(1/2))*5^(1/2
)
```

Maxima [A] time = 1.42723, size = 69, normalized size = 1.68

$$-\frac{4}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{\sqrt{x+1}+1}-1}{\sqrt{5}+2\sqrt{\sqrt{x+1}+1}+1}\right)+2\sqrt{x+1}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(x+(1+(1+x)^(1/2))^(1/2)),x, algorithm="maxima")
```

```
[Out] -4/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(sqrt(x + 1) + 1) - 1)/(sqrt(5) + 2*sqrt
(sqrt(x + 1) + 1) + 1)) + 2*sqrt(x + 1) + 2
```

Fricas [B] time = 1.00712, size = 284, normalized size = 6.93

$$\frac{4}{5}\sqrt{5}\log\left(\frac{2x^2-\sqrt{5}(3x+1)-\left(\sqrt{5}(x+2)-5x\right)\sqrt{x+1}+\left(\sqrt{5}(x+2)+\left(\sqrt{5}(2x-1)-5\right)\sqrt{x+1}-5x\right)\sqrt{\sqrt{x+1}+1}+3}{x^2-x-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(x+(1+(1+x)^(1/2))^(1/2)),x, algorithm="fricas")
```

```
[Out] 4/5*sqrt(5)*log((2*x^2 - sqrt(5)*(3*x + 1) - (sqrt(5)*(x + 2) - 5*x)*sqrt(x
 + 1) + (sqrt(5)*(x + 2) + (sqrt(5)*(2*x - 1) - 5)*sqrt(x + 1) - 5*x)*sqrt(
sqrt(x + 1) + 1) + 3*x + 3)/(x^2 - x - 1)) + 2*sqrt(x + 1)
```

Sympy [A] time = 10.8422, size = 112, normalized size = 2.73

$$2\sqrt{x+1}-16\left(\begin{cases}-\frac{\sqrt{5}\operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)^{2}>\frac{5}{4}\\ -\frac{\sqrt{5}\operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)}{5}\right)}{10} & \text{for}\ \left(\sqrt{\sqrt{x+1}+1}+\frac{1}{2}\right)^{2}<\frac{5}{4}\end{cases}\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)/(x+(1+(1+x)**(1/2))**(1/2)),x)
```

```
[Out] 2*sqrt(x + 1) - 16*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(sqrt(x + 1) +
1) + 1/2)/5)/10, (sqrt(sqrt(x + 1) + 1) + 1/2)**2 > 5/4), (-sqrt(5)*atanh(2
*sqrt(5)*(sqrt(sqrt(x + 1) + 1) + 1/2)/5)/10, (sqrt(sqrt(x + 1) + 1) + 1/2)
**2 < 5/4)) + 2
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)^(1/2)/(x+(1+(1+x)^(1/2))^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.11 $\int \frac{1}{x-\sqrt{1+\sqrt{1+x}}}\,dx$

Optimal. Leaf size=73

$$\frac{2}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{\sqrt{x+1}+1}-\sqrt{5}+1\right)+\frac{2}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{\sqrt{x+1}+1}+\sqrt{5}+1\right)$$

```
[Out] (2*(5 + Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + Sqrt[1 + x]]])/5 + (2*(5 - Sq
rt[5])*Log[1 + Sqrt[5] - 2*Sqrt[1 + Sqrt[1 + x]]])/5
```

Rubi [A] time = 0.108042, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {632, 31}

$$\frac{2}{5}\left(5+\sqrt{5}\right)\log\left(-2\sqrt{\sqrt{x+1}+1}-\sqrt{5}+1\right)+\frac{2}{5}\left(5-\sqrt{5}\right)\log\left(-2\sqrt{\sqrt{x+1}+1}+\sqrt{5}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x - Sqrt[1 + Sqrt[1 + x]])^(-1),x]
```

```
[Out] (2*(5 + Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + Sqrt[1 + x]]])/5 + (2*(5 - Sq
rt[5])*Log[1 + Sqrt[5] - 2*Sqrt[1 + Sqrt[1 + x]]])/5
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{x-\sqrt{1+\sqrt{1+x}}}\,dx = 2\,\text{Subst}\left(\int \frac{x}{-1+x^2-\sqrt{1+x}}\,dx, x, \sqrt{1+x}\right)$$
$$= 4\,\text{Subst}\left(\int \frac{-1+x}{-1-x+x^2}\,dx, x, \sqrt{1+\sqrt{1+x}}\right)$$
$$= \frac{1}{5}\left(2\left(5-\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}-\frac{\sqrt{5}}{2}+x}\,dx, x, \sqrt{1+\sqrt{1+x}}\right) + \frac{1}{5}\left(2\left(5+\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{}{-\frac{1}{2}}\right.$$
$$= \frac{2}{5}\left(5+\sqrt{5}\right)\log\left(1-\sqrt{5}-2\sqrt{1+\sqrt{1+x}}\right) + \frac{2}{5}\left(5-\sqrt{5}\right)\log\left(1+\sqrt{5}-2\sqrt{1+\sqrt{1+x}}\right)$$

Mathematica [A] time = 0.0581307, size = 71, normalized size = 0.97

$$\frac{1}{5}\left(2\left(5+\sqrt{5}\right)\log\left(-2\sqrt{\sqrt{x+1}+1}-\sqrt{5}+1\right)-2\left(\sqrt{5}-5\right)\log\left(-2\sqrt{\sqrt{x+1}+1}+\sqrt{5}+1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[1 + Sqrt[1 + x]])^(-1),x]
```

```
[Out] (2*(5 + Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + Sqrt[1 + x]]] - 2*(-5 + Sqrt[
5])*Log[1 + Sqrt[5] - 2*Sqrt[1 + Sqrt[1 + x]]])/5
```

Maple [B] time = 0.104, size = 175, normalized size = 2.4

$$\frac{\sqrt{5}}{5}\text{Artanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)+\frac{\sqrt{5}}{5}\text{Artanh}\left(\frac{\sqrt{5}}{5}\left(2\sqrt{1+x}+1\right)\right)+\frac{2\sqrt{5}}{5}\text{Artanh}\left(\frac{\sqrt{5}}{5}\left(1+2\sqrt{1+\sqrt{1+x}}\right)\right)+\frac{\ln\left(x^2-x\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x-(1+(1+x)^(1/2))^(1/2)),x)
```

```
[Out] 1/5*5^(1/2)*arctanh(1/5*(2*x-1)*5^(1/2))+1/5*5^(1/2)*arctanh(1/5*(2*(1+x)^(
1/2)+1)*5^(1/2))+2/5*arctanh(1/5*(1+2*(1+(1+x)^(1/2))^(1/2))*5^(1/2))*5^(1/
2)+1/2*ln(x^2-x-1)+1/5*5^(1/2)*arctanh(1/5*(2*(1+x)^(1/2)-1)*5^(1/2))+2/5*5
^(1/2)*arctanh(1/5*(2*(1+(1+x)^(1/2))^(1/2)-1)*5^(1/2))+ln((1+x)^(1/2)-(1+(
1+x)^(1/2))^(1/2))-ln((1+x)^(1/2)+(1+(1+x)^(1/2))^(1/2))+1/2*ln(x-(1+x)^(1/
```

```
2))-1/2*ln(x+(1+x)^(1/2))
```

Maxima [A] time = 1.45819, size = 85, normalized size = 1.16

$$-\frac{2}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{\sqrt{x+1}+1}+1}{\sqrt{5}+2\sqrt{\sqrt{x+1}+1}-1}\right)+2\log\left(\sqrt{x+1}-\sqrt{\sqrt{x+1}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+(1+x)^(1/2))^(1/2)),x, algorithm="maxima")
```

```
[Out] -2/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(sqrt(x + 1) + 1) + 1)/(sqrt(5) + 2*sqrt
(sqrt(x + 1) + 1) - 1)) + 2*log(sqrt(x + 1) - sqrt(sqrt(x + 1) + 1))
```

Fricas [B] time = 1.09437, size = 323, normalized size = 4.42

$$\frac{2}{5}\sqrt{5}\log\left(\frac{2x^2+\sqrt{5}(3x+1)+\left(\sqrt{5}(x+2)+5x\right)\sqrt{x+1}+\left(\sqrt{5}(x+2)+\left(\sqrt{5}(2x-1)+5\right)\sqrt{x+1}+5x\right)\sqrt{\sqrt{x+1}+1}+3}{x^2-x-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+(1+x)^(1/2))^(1/2)),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*log((2*x^2 + sqrt(5)*(3*x + 1) + (sqrt(5)*(x + 2) + 5*x)*sqrt(x
 + 1) + (sqrt(5)*(x + 2) + (sqrt(5)*(2*x - 1) + 5)*sqrt(x + 1) + 5*x)*sqrt(
sqrt(x + 1) + 1) + 3*x + 3)/(x^2 - x - 1)) + 2*log(sqrt(x + 1) - sqrt(sqrt(
x + 1) + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{x-\sqrt{\sqrt{x+1}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+(1+x)**(1/2))**(1/2)),x)
```

```
[Out] Integral(1/(x - sqrt(sqrt(x + 1) + 1)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(1+(1+x)^(1/2))^(1/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

3.12 $\int \frac{x}{x+\sqrt{1-\sqrt{1+x}}}\,dx$

Optimal. Leaf size=73

$$\left(1-\sqrt{x+1}\right)^2-4\sqrt{1-\sqrt{x+1}}+2\sqrt{x+1}+\frac{8\tanh^{-1}\left(\frac{2\sqrt{1-\sqrt{x+1}}+1}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] 2*Sqrt[1 + x] - 4*Sqrt[1 - Sqrt[1 + x]] + (1 - Sqrt[1 + x])^2 + (8*ArcTanh[
(1 + 2*Sqrt[1 - Sqrt[1 + x]])/Sqrt[5]])/Sqrt[5]
```

Rubi [A] time = 0.270541, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1628, 618, 206}

$$\left(1-\sqrt{x+1}\right)^2-4\sqrt{1-\sqrt{x+1}}+2\sqrt{x+1}+\frac{8\tanh^{-1}\left(\frac{2\sqrt{1-\sqrt{x+1}}+1}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(x + Sqrt[1 - Sqrt[1 + x]]),x]
```

```
[Out] 2*Sqrt[1 + x] - 4*Sqrt[1 - Sqrt[1 + x]] + (1 - Sqrt[1 + x])^2 + (8*ArcTanh[
(1 + 2*Sqrt[1 - Sqrt[1 + x]])/Sqrt[5]])/Sqrt[5]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x}{x+\sqrt{1-\sqrt{1+x}}}\,dx &= 2\,\text{Subst}\left(\int \frac{x\left(-1+x^2\right)}{-1+\sqrt{1-x}+x^2}\,dx, x, \sqrt{1+x}\right)\\
&= 4\,\text{Subst}\left(\int \frac{x^2(1+x)\left(-2+x^2\right)}{-1+x+x^2}\,dx, x, \sqrt{1-\sqrt{1+x}}\right)\\
&= 4\,\text{Subst}\left(\int \left(-1-x+x^3-\frac{1}{-1+x+x^2}\right)dx, x, \sqrt{1-\sqrt{1+x}}\right)\\
&= 2\sqrt{1+x}-4\sqrt{1-\sqrt{1+x}}+\left(1-\sqrt{1+x}\right)^2-4\,\text{Subst}\left(\int \frac{1}{-1+x+x^2}\,dx, x, \sqrt{1-\sqrt{1+x}}\right)\\
&= 2\sqrt{1+x}-4\sqrt{1-\sqrt{1+x}}+\left(1-\sqrt{1+x}\right)^2+8\,\text{Subst}\left(\int \frac{1}{5-x^2}\,dx, x, 1+2\sqrt{1-\sqrt{1+x}}\right)\\
&= 2\sqrt{1+x}-4\sqrt{1-\sqrt{1+x}}+\left(1-\sqrt{1+x}\right)^2+\frac{8\tanh^{-1}\left(\frac{1+2\sqrt{1-\sqrt{1+x}}}{\sqrt{5}}\right)}{\sqrt{5}}
\end{aligned}
$$

Mathematica [A] time = 0.0745484, size = 52, normalized size = 0.71

$$
x-4\sqrt{1-\sqrt{x+1}}+\frac{8\tanh^{-1}\left(\frac{2\sqrt{1-\sqrt{x+1}}+1}{\sqrt{5}}\right)}{\sqrt{5}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x + Sqrt[1 - Sqrt[1 + x]]),x]
```

```
[Out] x - 4*Sqrt[1 - Sqrt[1 + x]] + (8*ArcTanh[(1 + 2*Sqrt[1 - Sqrt[1 + x]])/Sqrt
[5]])/Sqrt[5]
```

Maple [A] time = 0.006, size = 60, normalized size = 0.8

$$\left(1-\sqrt{1+x}\right)^{2}-2+2\sqrt{1+x}-4\sqrt{1-\sqrt{1+x}}+\frac{8\sqrt{5}}{5}\mathrm{Artanh}\left(\frac{\sqrt{5}}{5}\left(1+2\sqrt{1-\sqrt{1+x}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+(1-(1+x)^(1/2))^(1/2)),x)
```

```
[Out] (1-(1+x)^(1/2))^2-2+2*(1+x)^(1/2)-4*(1-(1+x)^(1/2))^(1/2)+8/5*arctanh(1/5*(
1+2*(1-(1+x)^(1/2))^(1/2))*5^(1/2))*5^(1/2)
```

Maxima [A] time = 1.46052, size = 104, normalized size = 1.42

$$\left(\sqrt{x+1}-1\right)^{2}-\frac{4}{5}\sqrt{5}\log\left(-\frac{\sqrt{5}-2\sqrt{-\sqrt{x+1}+1}-1}{\sqrt{5}+2\sqrt{-\sqrt{x+1}+1}+1}\right)+2\sqrt{x+1}-4\sqrt{-\sqrt{x+1}+1}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(1-(1+x)^(1/2))^(1/2)),x, algorithm="maxima")
```

```
[Out] (sqrt(x + 1) - 1)^2 - 4/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(-sqrt(x + 1) + 1)
- 1)/(sqrt(5) + 2*sqrt(-sqrt(x + 1) + 1) + 1)) + 2*sqrt(x + 1) - 4*sqrt(-sq
rt(x + 1) + 1) - 2
```

Fricas [A] time = 1.08725, size = 305, normalized size = 4.18

$$\frac{4}{5}\sqrt{5}\log\left(\frac{2x^{2}-\sqrt{5}(3x+1)+\left(\sqrt{5}(x+2)-5x\right)\sqrt{x+1}+\left(\sqrt{5}(x+2)-\left(\sqrt{5}(2x-1)-5\right)\sqrt{x+1}-5x\right)\sqrt{-\sqrt{x+1}+1}+}{x^{2}-x-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(1-(1+x)^(1/2))^(1/2)),x, algorithm="fricas")
```

```
[Out] 4/5*sqrt(5)*log((2*x^2 - sqrt(5)*(3*x + 1) + (sqrt(5)*(x + 2) - 5*x)*sqrt(x
 + 1) + (sqrt(5)*(x + 2) - (sqrt(5)*(2*x - 1) - 5)*sqrt(x + 1) - 5*x)*sqrt(
```

```
-sqrt(x + 1) + 1) + 3*x + 3)/(x^2 - x - 1)) + x - 4*sqrt(-sqrt(x + 1) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{x+\sqrt{1-\sqrt{x+1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(1-(1+x)**(1/2))**(1/2)),x)
```

```
[Out] Integral(x/(x + sqrt(1 - sqrt(x + 1))), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x+(1-(1+x)^(1/2))^(1/2)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.13 $\int \frac{\sqrt{x+\sqrt{1+x}}}{\sqrt{1+x}\left(1+x^2\right)}\,dx$

Optimal. Leaf size=365

$$-\frac{i\tan^{-1}\left(\frac{-\left(1-2\sqrt{1-i}\right)\sqrt{x+1}+\sqrt{1-i}+2}{2\sqrt{i+\sqrt{1-i}}\sqrt{x+\sqrt{x+1}}}\right)}{2\sqrt{\frac{1-i}{i+\sqrt{1-i}}}}+\frac{i\tan^{-1}\left(\frac{-\left(1-2\sqrt{1+i}\right)\sqrt{x+1}+\sqrt{1+i}+2}{2\sqrt{\sqrt{1+i}-i}\sqrt{x+\sqrt{x+1}}}\right)}{2\sqrt{-\frac{1+i}{i-\sqrt{1+i}}}}+\frac{i\tanh^{-1}\left(\frac{-\left(1+2\sqrt{1-i}\right)\sqrt{x+1}-\sqrt{1-i}+2}{2\sqrt{\sqrt{1-i}-i}\sqrt{x+\sqrt{x+1}}}\right)}{2\sqrt{-\frac{1-i}{i-\sqrt{1-i}}}}-$$

```
[Out] ((-I/2)*ArcTan[(2 + Sqrt[1 - I] - (1 - 2*Sqrt[1 - I])*Sqrt[1 + x])/(2*Sqrt[
I + Sqrt[1 - I]]*Sqrt[x + Sqrt[1 + x]])])/Sqrt[(1 - I)/(I + Sqrt[1 - I])] +
 ((I/2)*ArcTan[(2 + Sqrt[1 + I] - (1 - 2*Sqrt[1 + I])*Sqrt[1 + x])/(2*Sqrt[
-I + Sqrt[1 + I]]*Sqrt[x + Sqrt[1 + x]])])/Sqrt[(-1 - I)/(I - Sqrt[1 + I])]
 + ((I/2)*ArcTanh[(2 - Sqrt[1 - I] - (1 + 2*Sqrt[1 - I])*Sqrt[1 + x])/(2*Sq
rt[-I + Sqrt[1 - I]]*Sqrt[x + Sqrt[1 + x]])])/Sqrt[(-1 + I)/(I - Sqrt[1 - I
])] - ((I/2)*ArcTanh[(2 - Sqrt[1 + I] - (1 + 2*Sqrt[1 + I])*Sqrt[1 + x])/(2
*Sqrt[I + Sqrt[1 + I]]*Sqrt[x + Sqrt[1 + x]])])/Sqrt[(1 + I)/(I + Sqrt[1 +
I])]
```

Rubi [A] time = 0.858984, antiderivative size = 365, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6741, 6728, 990, 621, 206, 1033, 724, 204}

$$-\frac{i\tan^{-1}\left(\frac{-\left(1-2\sqrt{1-i}\right)\sqrt{x+1}+\sqrt{1-i}+2}{2\sqrt{i+\sqrt{1-i}}\sqrt{x+\sqrt{x+1}}}\right)}{2\sqrt{\frac{1-i}{i+\sqrt{1-i}}}}+\frac{i\tan^{-1}\left(\frac{-\left(1-2\sqrt{1+i}\right)\sqrt{x+1}+\sqrt{1+i}+2}{2\sqrt{\sqrt{1+i}-i}\sqrt{x+\sqrt{x+1}}}\right)}{2\sqrt{-\frac{1+i}{i-\sqrt{1+i}}}}+\frac{i\tanh^{-1}\left(\frac{-\left(1+2\sqrt{1-i}\right)\sqrt{x+1}-\sqrt{1-i}+2}{2\sqrt{\sqrt{1-i}-i}\sqrt{x+\sqrt{x+1}}}\right)}{2\sqrt{-\frac{1-i}{i-\sqrt{1-i}}}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[1 + x]]/(Sqrt[1 + x]*(1 + x^2)),x]
```

```
[Out] ((-I/2)*ArcTan[(2 + Sqrt[1 - I] - (1 - 2*Sqrt[1 - I])*Sqrt[1 + x])/(2*Sqrt[
I + Sqrt[1 - I]]*Sqrt[x + Sqrt[1 + x]])])/Sqrt[(1 - I)/(I + Sqrt[1 - I])] +
 ((I/2)*ArcTan[(2 + Sqrt[1 + I] - (1 - 2*Sqrt[1 + I])*Sqrt[1 + x])/(2*Sqrt[
-I + Sqrt[1 + I]]*Sqrt[x + Sqrt[1 + x]])])/Sqrt[(-1 - I)/(I - Sqrt[1 + I])]
 + ((I/2)*ArcTanh[(2 - Sqrt[1 - I] - (1 + 2*Sqrt[1 - I])*Sqrt[1 + x])/(2*Sq
rt[-I + Sqrt[1 - I]]*Sqrt[x + Sqrt[1 + x]])])/Sqrt[(-1 + I)/(I - Sqrt[1 - I
])] - ((I/2)*ArcTanh[(2 - Sqrt[1 + I] - (1 + 2*Sqrt[1 + I])*Sqrt[1 + x])/(2
```

```
*Sqrt[I + Sqrt[1 + I]]*Sqrt[x + Sqrt[1 + x]])])/Sqrt[(1 + I)/(I + Sqrt[1 +
I])]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 990

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*
f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d
, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x+\sqrt{1+x}}}{\sqrt{1+x}\left(1+x^2\right)}\,dx &= 2\,\text{Subst}\left(\int \frac{\sqrt{-1+x+x^2}}{1+\left(-1+x^2\right)^2}\,dx, x, \sqrt{1+x}\right)\\
&= 2\,\text{Subst}\left(\int \frac{\sqrt{-1+x+x^2}}{2-2x^2+x^4}\,dx, x, \sqrt{1+x}\right)\\
&= 2\,\text{Subst}\left(\int \left(\frac{i\sqrt{-1+x+x^2}}{(2+2i)-2x^2}+\frac{i\sqrt{-1+x+x^2}}{(-2+2i)+2x^2}\right)dx, x, \sqrt{1+x}\right)\\
&= 2i\,\text{Subst}\left(\int \frac{\sqrt{-1+x+x^2}}{(2+2i)-2x^2}\,dx, x, \sqrt{1+x}\right)+2i\,\text{Subst}\left(\int \frac{\sqrt{-1+x+x^2}}{(-2+2i)+2x^2}\,dx, x, \sqrt{1+x}\right)\\
&= i\,\text{Subst}\left(\int \frac{2i+2x}{\left((2+2i)-2x^2\right)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)-i\,\text{Subst}\left(\int \frac{2i-2x}{\sqrt{-1+x+x^2}\left((-2+2i)\right.}\right.\\
&= -\left(\frac{1}{2}\left(i\left(-2-(1-i)^{3/2}\right)\right)\text{Subst}\left(\int \frac{1}{\left(-2\sqrt{1-i}+2x\right)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)\right)-\frac{1}{2}\left(i\left(-2+\left(\right.\right.\right.\\
&= \left(i\left(-2-(1-i)^{3/2}\right)\right)\text{Subst}\left(\int \frac{1}{-16i+16\sqrt{1-i}-x^2}\,dx, x, \frac{-4+2\sqrt{1-i}-\left(-2-4\sqrt{1-i}\right)\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right.\\
&= -\frac{i\sqrt{i+\sqrt{1-i}}\tan^{-1}\left(\frac{2+\sqrt{1-i}-\left(1-2\sqrt{1-i}\right)\sqrt{1+x}}{2\sqrt{i+\sqrt{1-i}}\sqrt{x+\sqrt{1+x}}}\right)}{2\sqrt{1-i}}+\frac{1}{4}(1+i)^{3/2}\sqrt{-i+\sqrt{1+i}}\tan^{-1}\left(\frac{2+\sqrt{1+i}-\left(1-\right.}{2\sqrt{-i+\sqrt{1+}}}\right.
\end{aligned}$$

Mathematica [A] time = 0.456813, size = 357, normalized size = 0.98

$$\frac{1}{4}(1-i)^{3/2}\sqrt{i+\sqrt{1-i}}\tan^{-1}\left(\frac{-\left(1-2\sqrt{1-i}\right)\sqrt{x+1}+\sqrt{1-i}+2}{2\sqrt{i+\sqrt{1-i}}\sqrt{x+\sqrt{x+1}}}\right)+\frac{1}{4}(1+i)^{3/2}\sqrt{\sqrt{1+i}-i}\tan^{-1}\left(\frac{-\left(1-2\sqrt{1+i}\right)\sqrt{x+\right.}{2\sqrt{\sqrt{1+i}-i}\sqrt{\right.}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[1 + x]]/(Sqrt[1 + x]*(1 + x^2)),x]
```

```
[Out] ((1 - I)^(3/2)*Sqrt[I + Sqrt[1 - I]]*ArcTan[(2 + Sqrt[1 - I] - (1 - 2*Sqrt[
1 - I])*Sqrt[1 + x])/(2*Sqrt[I + Sqrt[1 - I]]*Sqrt[x + Sqrt[1 + x]])])/4 +
((1 + I)^(3/2)*Sqrt[-I + Sqrt[1 + I]]*ArcTan[(2 + Sqrt[1 + I] - (1 - 2*Sqrt
[1 + I])*Sqrt[1 + x])/(2*Sqrt[-I + Sqrt[1 + I]]*Sqrt[x + Sqrt[1 + x]])])/4
- ((1 - I)^(3/2)*Sqrt[-I + Sqrt[1 - I]]*ArcTanh[(2 - Sqrt[1 - I] - (1 + 2*S
qrt[1 - I])*Sqrt[1 + x])/(2*Sqrt[-I + Sqrt[1 - I]]*Sqrt[x + Sqrt[1 + x]])])
/4 - ((1 + I)^(3/2)*Sqrt[I + Sqrt[1 + I]]*ArcTanh[(2 - Sqrt[1 + I] - (1 + 2
*Sqrt[1 + I])*Sqrt[1 + x])/(2*Sqrt[I + Sqrt[1 + I]]*Sqrt[x + Sqrt[1 + x]])]
)/4
```

Maple [C] time = 0.017, size = 109, normalized size = 0.3

$$-\frac{1}{2}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-4\_Z^6+8\_Z^5+20\_Z^4-48\_Z^3+40\_Z^2-8\_Z+1\right)}\frac{2\_R^5-5\_R^4+5\_R^2-1}{\_R^7-3\_R^5+5\_R^4+10\_R^3-18\_R^2+10\_R-1}\ln\left(\sqrt{x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(1+x)^(1/2))^(1/2)/(x^2+1)/(1+x)^(1/2),x)
```

```
[Out] -1/2*sum((2*_R^5-5*_R^4+5*_R^2-1)/(_R^7-3*_R^5+5*_R^4+10*_R^3-18*_R^2+10*_R
-1)*ln((x+(1+x)^(1/2))^(1/2)-(1+x)^(1/2)-_R),_R=RootOf(_Z^8-4*_Z^6+8*_Z^5+2
0*_Z^4-48*_Z^3+40*_Z^2-8*_Z+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{\left(x^2+1\right)\sqrt{x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1))/((x^2 + 1)*sqrt(x + 1)), x)
```

Fricas [B] time = 56.984, size = 20056, normalized size = 54.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(sqrt(1/8*I + 1/8) + sqrt(-1/8*I + 1/8) - 2*sqrt(-3/64*(4*sqrt(1/8
*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt
(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1
/8) + 1/8*I + 1/8) - 1/2)*log(1/4*(2*(((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*
sqrt(-1/8*I + 1/8) + I + 1) - 3*(3*x - 1)*sqrt(x + 1) - 7*x + 9)*sqrt(x + s
qrt(x + 1))*(4*sqrt(1/8*I + 1/8) - I + 1)^2 + 2*(((3*x - 1)*sqrt(x + 1) + 4
*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 4*((3*x - 1)*sqrt(x + 1) + 4*x -
 3)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 2*(7*x + 1)*sqrt(x + 1) + 12*x - 14)*s
qrt(x + sqrt(x + 1))*(4*sqrt(1/8*I + 1/8) - I + 1) + 16*((((3*x - 1)*sqrt(x
 + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 3*(3*x - 1)*sqrt(x + 1) -
 7*x + 9)*sqrt(x + sqrt(x + 1))*(4*sqrt(1/8*I + 1/8) - I + 1) - ((3*(3*x -
1)*sqrt(x + 1) + 7*x - 9)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 2*(11*x - 7)*sqr
t(x + 1) - 16*x + 22)*sqrt(x + sqrt(x + 1)))*sqrt(-3/64*(4*sqrt(1/8*I + 1/8
) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*I +
 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/
8*I + 1/8) - 2*((3*(3*x - 1)*sqrt(x + 1) + 7*x - 9)*(4*sqrt(-1/8*I + 1/8) +
 I + 1)^2 - 2*((7*x + 1)*sqrt(x + 1) + 6*x - 7)*(4*sqrt(-1/8*I + 1/8) + I +
 1) - 4*sqrt(x + 1)*(x - 7) - 12*x + 44)*sqrt(x + sqrt(x + 1)) + ((3*x^2 -
(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) +
2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(1/8*I + 1/8) - I + 1)^2 + (3*x
^2 + 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2
- 68*x^2 - ((x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8)
 + I + 1)^2 - 2*x^2 - 4*(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-
1/8*I + 1/8) + I + 1) + 4*(7*x + 6)*sqrt(x + 1) + 24*x + 30)*(4*sqrt(1/8*I
+ 1/8) - I + 1) + 2*(x^2 - 2*(7*x + 6)*sqrt(x + 1) - 12*x - 15)*(4*sqrt(-1/
8*I + 1/8) + I + 1) - 8*(14*x^2 - (3*x^2 - (x^2 + 2*(3*x + 4)*sqrt(x + 1) +
 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 2*(9*x + 7)*sqrt(x + 1) + 24*x +
 15)*(4*sqrt(1/8*I + 1/8) - I + 1) - (3*x^2 + 2*(9*x + 7)*sqrt(x + 1) + 24*
x + 15)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 4*(11*x + 8)*sqrt(x + 1) + 72*x +
30)*sqrt(-3/64*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8)
 + I + 1)^2 - 1/32*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I
- 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/8*I + 1/8) - 8*(11*x + 3)*sqrt(x + 1) - 6
4*x - 20)*sqrt(sqrt(1/8*I + 1/8) + sqrt(-1/8*I + 1/8) - 2*sqrt(-3/64*(4*sqr
t(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4
*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*
```

```
I + 1/8) + 1/8*I + 1/8) - 1/2))/(x^2 + 1)) + 1/4*sqrt(sqrt(1/8*I + 1/8) + s
qrt(-1/8*I + 1/8) - 2*sqrt(-3/64*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*
sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(
-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/8*I + 1/8) - 1/2)*log(1
/4*(2*(((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 3
*(3*x - 1)*sqrt(x + 1) - 7*x + 9)*sqrt(x + sqrt(x + 1))*(4*sqrt(1/8*I + 1/8
) - I + 1)^2 + 2*(((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) +
 I + 1)^2 - 4*((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I +
 1) + 2*(7*x + 1)*sqrt(x + 1) + 12*x - 14)*sqrt(x + sqrt(x + 1))*(4*sqrt(1/
8*I + 1/8) - I + 1) + 16*((((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I
 + 1/8) + I + 1) - 3*(3*x - 1)*sqrt(x + 1) - 7*x + 9)*sqrt(x + sqrt(x + 1))
*(4*sqrt(1/8*I + 1/8) - I + 1) - ((3*(3*x - 1)*sqrt(x + 1) + 7*x - 9)*(4*sq
rt(-1/8*I + 1/8) + I + 1) - 2*(11*x - 7)*sqrt(x + 1) - 16*x + 22)*sqrt(x +
sqrt(x + 1)))*sqrt(-3/64*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/
8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*I +
 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/8*I + 1/8) - 2*((3*(3*x - 1)*sq
rt(x + 1) + 7*x - 9)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 2*((7*x + 1)*sqrt(x
 + 1) + 6*x - 7)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 4*sqrt(x + 1)*(x - 7) - 1
2*x + 44)*sqrt(x + sqrt(x + 1)) - ((3*x^2 - (x^2 + 2*(3*x + 4)*sqrt(x + 1)
+ 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 2*(9*x + 7)*sqrt(x + 1) + 24*x
+ 15)*(4*sqrt(1/8*I + 1/8) - I + 1)^2 + (3*x^2 + 2*(9*x + 7)*sqrt(x + 1) +
24*x + 15)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 68*x^2 - ((x^2 + 2*(3*x + 4)*
sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 2*x^2 - 4*(x^2 +
2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 4*(7*x
+ 6)*sqrt(x + 1) + 24*x + 30)*(4*sqrt(1/8*I + 1/8) - I + 1) + 2*(x^2 - 2*(7
*x + 6)*sqrt(x + 1) - 12*x - 15)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 8*(14*x^2
 - (3*x^2 - (x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8)
 + I + 1) + 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(1/8*I + 1/8) - I +
 1) - (3*x^2 + 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(-1/8*I + 1/8) +
 I + 1) + 4*(11*x + 8)*sqrt(x + 1) + 72*x + 30)*sqrt(-3/64*(4*sqrt(1/8*I +
1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*
I + 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) +
 1/8*I + 1/8) - 8*(11*x + 3)*sqrt(x + 1) - 64*x - 20)*sqrt(sqrt(1/8*I + 1/8
) + sqrt(-1/8*I + 1/8) - 2*sqrt(-3/64*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 3/6
4*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*
sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/8*I + 1/8) - 1/2))
/(x^2 + 1)) - 1/4*sqrt(sqrt(1/8*I + 1/8) + sqrt(-1/8*I + 1/8) + 2*sqrt(-3/6
4*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 -
 1/32*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sq
rt(-1/8*I + 1/8) + 1/8*I + 1/8) - 1/2)*log(1/4*(2*(((3*x - 1)*sqrt(x + 1) +
 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 3*(3*x - 1)*sqrt(x + 1) - 7*x +
9)*sqrt(x + sqrt(x + 1))*(4*sqrt(1/8*I + 1/8) - I + 1)^2 + 2*(((3*x - 1)*sq
rt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 4*((3*x - 1)*sqrt(x
 + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 2*(7*x + 1)*sqrt(x + 1) +
 12*x - 14)*sqrt(x + sqrt(x + 1))*(4*sqrt(1/8*I + 1/8) - I + 1) - 16*((((3*
```

```
x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 3*(3*x - 1)*
sqrt(x + 1) - 7*x + 9)*sqrt(x + sqrt(x + 1))*(4*sqrt(1/8*I + 1/8) - I + 1)
- ((3*(3*x - 1)*sqrt(x + 1) + 7*x - 9)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 2*(
11*x - 7)*sqrt(x + 1) - 16*x + 22)*sqrt(x + sqrt(x + 1)))*sqrt(-3/64*(4*sqr
t(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4
*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*
I + 1/8) + 1/8*I + 1/8) - 2*((3*(3*x - 1)*sqrt(x + 1) + 7*x - 9)*(4*sqrt(-1
/8*I + 1/8) + I + 1)^2 - 2*((7*x + 1)*sqrt(x + 1) + 6*x - 7)*(4*sqrt(-1/8*I
 + 1/8) + I + 1) - 4*sqrt(x + 1)*(x - 7) - 12*x + 44)*sqrt(x + sqrt(x + 1))
 + ((3*x^2 - (x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8
) + I + 1) + 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(1/8*I + 1/8) - I
+ 1)^2 + (3*x^2 + 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(-1/8*I + 1/8
) + I + 1)^2 - 68*x^2 - ((x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(
-1/8*I + 1/8) + I + 1)^2 - 2*x^2 - 4*(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x +
 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 4*(7*x + 6)*sqrt(x + 1) + 24*x + 30)*(
4*sqrt(1/8*I + 1/8) - I + 1) + 2*(x^2 - 2*(7*x + 6)*sqrt(x + 1) - 12*x - 15
)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 8*(14*x^2 - (3*x^2 - (x^2 + 2*(3*x + 4)*
sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 2*(9*x + 7)*sqrt(x
+ 1) + 24*x + 15)*(4*sqrt(1/8*I + 1/8) - I + 1) - (3*x^2 + 2*(9*x + 7)*sqrt
(x + 1) + 24*x + 15)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 4*(11*x + 8)*sqrt(x +
 1) + 72*x + 30)*sqrt(-3/64*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(
-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*
I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/8*I + 1/8) - 8*(11*x + 3)*sq
rt(x + 1) - 64*x - 20)*sqrt(sqrt(1/8*I + 1/8) + sqrt(-1/8*I + 1/8) + 2*sqrt
(-3/64*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1
)^2 - 1/32*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1
/2*sqrt(-1/8*I + 1/8) + 1/8*I + 1/8) - 1/2))/(x^2 + 1)) + 1/4*sqrt(sqrt(1/8
*I + 1/8) + sqrt(-1/8*I + 1/8) + 2*sqrt(-3/64*(4*sqrt(1/8*I + 1/8) - I + 1)
^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*I + 1/8) - I
+ 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/8*I + 1/8)
 - 1/2)*log(1/4*(2*(((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8)
 + I + 1) - 3*(3*x - 1)*sqrt(x + 1) - 7*x + 9)*sqrt(x + sqrt(x + 1))*(4*sqr
t(1/8*I + 1/8) - I + 1)^2 + 2*(((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1
/8*I + 1/8) + I + 1)^2 - 4*((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I
 + 1/8) + I + 1) + 2*(7*x + 1)*sqrt(x + 1) + 12*x - 14)*sqrt(x + sqrt(x + 1
))*(4*sqrt(1/8*I + 1/8) - I + 1) - 16*((((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(
4*sqrt(-1/8*I + 1/8) + I + 1) - 3*(3*x - 1)*sqrt(x + 1) - 7*x + 9)*sqrt(x +
 sqrt(x + 1))*(4*sqrt(1/8*I + 1/8) - I + 1) - ((3*(3*x - 1)*sqrt(x + 1) + 7
*x - 9)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 2*(11*x - 7)*sqrt(x + 1) - 16*x +
22)*sqrt(x + sqrt(x + 1)))*sqrt(-3/64*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 3/6
4*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*
sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/8*I + 1/8) - 2*((3
*(3*x - 1)*sqrt(x + 1) + 7*x - 9)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 2*((7*
x + 1)*sqrt(x + 1) + 6*x - 7)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 4*sqrt(x + 1
)*(x - 7) - 12*x + 44)*sqrt(x + sqrt(x + 1)) - ((3*x^2 - (x^2 + 2*(3*x + 4)
```

```
*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) + 2*(9*x + 7)*sqrt(x
 + 1) + 24*x + 15)*(4*sqrt(1/8*I + 1/8) - I + 1)^2 + (3*x^2 + 2*(9*x + 7)*s
qrt(x + 1) + 24*x + 15)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 68*x^2 - ((x^2 +
 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 2*x^
2 - 4*(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I +
 1) + 4*(7*x + 6)*sqrt(x + 1) + 24*x + 30)*(4*sqrt(1/8*I + 1/8) - I + 1) +
2*(x^2 - 2*(7*x + 6)*sqrt(x + 1) - 12*x - 15)*(4*sqrt(-1/8*I + 1/8) + I + 1
) + 8*(14*x^2 - (3*x^2 - (x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(
-1/8*I + 1/8) + I + 1) + 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(1/8*I
 + 1/8) - I + 1) - (3*x^2 + 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(-1
/8*I + 1/8) + I + 1) + 4*(11*x + 8)*sqrt(x + 1) + 72*x + 30)*sqrt(-3/64*(4*
sqrt(1/8*I + 1/8) - I + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32
*(4*sqrt(1/8*I + 1/8) - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1
/8*I + 1/8) + 1/8*I + 1/8) - 8*(11*x + 3)*sqrt(x + 1) - 64*x - 20)*sqrt(sqr
t(1/8*I + 1/8) + sqrt(-1/8*I + 1/8) + 2*sqrt(-3/64*(4*sqrt(1/8*I + 1/8) - I
 + 1)^2 - 3/64*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 1/32*(4*sqrt(1/8*I + 1/8)
 - I + 1)*(4*sqrt(-1/8*I + 1/8) + I - 3) + 1/2*sqrt(-1/8*I + 1/8) + 1/8*I +
 1/8) - 1/2))/(x^2 + 1)) + 1/2*sqrt(-1/2*sqrt(1/8*I + 1/8) + 1/8*I - 1/8)*l
og(-((((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 3*
(3*x - 1)*sqrt(x + 1) - 7*x + 9)*sqrt(x + sqrt(x + 1))*(4*sqrt(1/8*I + 1/8)
 - I + 1)^2 + (((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I
+ 1)^2 - 4*((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1)
 + 2*(7*x + 1)*sqrt(x + 1) + 12*x - 14)*sqrt(x + sqrt(x + 1))*(4*sqrt(1/8*I
 + 1/8) - I + 1) + (((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8)
 + I + 1)^3 - 4*((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I
 + 1)^2 + 4*((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1
) + 2*(3*x - 1)*sqrt(x + 1) - 2*x + 14)*sqrt(x + sqrt(x + 1)) + ((x^2 + 2*(
3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^3 - (3*x^2 -
 (x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) +
 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(1/8*I + 1/8) - I + 1)^2 - 4*(
x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 -
 18*x^2 + ((x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8)
+ I + 1)^2 - 2*x^2 - 4*(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1
/8*I + 1/8) + I + 1) + 4*(7*x + 6)*sqrt(x + 1) + 24*x + 30)*(4*sqrt(1/8*I +
 1/8) - I + 1) + 4*(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I
 + 1/8) + I + 1) - 4*(7*x + 1)*sqrt(x + 1) + 16*x - 10)*sqrt(-1/2*sqrt(1/8*
I + 1/8) + 1/8*I - 1/8))/(x^2 + 1)) - 1/2*sqrt(-1/2*sqrt(1/8*I + 1/8) + 1/8
*I - 1/8)*log(-((((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) +
I + 1) - 3*(3*x - 1)*sqrt(x + 1) - 7*x + 9)*sqrt(x + sqrt(x + 1))*(4*sqrt(1
/8*I + 1/8) - I + 1)^2 + (((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I
+ 1/8) + I + 1)^2 - 4*((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/
8) + I + 1) + 2*(7*x + 1)*sqrt(x + 1) + 12*x - 14)*sqrt(x + sqrt(x + 1))*(4
*sqrt(1/8*I + 1/8) - I + 1) + (((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1
/8*I + 1/8) + I + 1)^3 - 4*((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I
 + 1/8) + I + 1)^2 + 4*((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1
```

```
/8) + I + 1) + 2*(3*x - 1)*sqrt(x + 1) - 2*x + 14)*sqrt(x + sqrt(x + 1)) -
((x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^3
 - (3*x^2 - (x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8)
 + I + 1) + 2*(9*x + 7)*sqrt(x + 1) + 24*x + 15)*(4*sqrt(1/8*I + 1/8) - I +
 1)^2 - 4*(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) +
 I + 1)^2 - 18*x^2 + ((x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/
8*I + 1/8) + I + 1)^2 - 2*x^2 - 4*(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)
*(4*sqrt(-1/8*I + 1/8) + I + 1) + 4*(7*x + 6)*sqrt(x + 1) + 24*x + 30)*(4*s
qrt(1/8*I + 1/8) - I + 1) + 4*(x^2 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*
sqrt(-1/8*I + 1/8) + I + 1) - 4*(7*x + 1)*sqrt(x + 1) + 16*x - 10)*sqrt(-1/
2*sqrt(1/8*I + 1/8) + 1/8*I - 1/8))/(x^2 + 1)) + 1/2*sqrt(-1/2*sqrt(-1/8*I
+ 1/8) - 1/8*I - 1/8)*log(((((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*
I + 1/8) + I + 1)^3 - ((3*x - 1)*sqrt(x + 1) + 9*x - 3)*(4*sqrt(-1/8*I + 1/
8) + I + 1)^2 - 2*((x + 3)*sqrt(x + 1) - 2*x - 1)*(4*sqrt(-1/8*I + 1/8) + I
 + 1) - 10*(3*x - 1)*sqrt(x + 1) - 30*x + 10)*sqrt(x + sqrt(x + 1)) + ((x^2
 + 2*(3*x + 4)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^3 - (x
^2 + 6*(x + 3)*sqrt(x + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 + 10
*x^2 + 2*(3*x^2 - 2*sqrt(x + 1)*(x - 2) + 4*x - 5)*(4*sqrt(-1/8*I + 1/8) +
I + 1) - 20*(x + 3)*sqrt(x + 1) - 80*x - 30)*sqrt(-1/2*sqrt(-1/8*I + 1/8) -
 1/8*I - 1/8))/(x^2 + 1)) - 1/2*sqrt(-1/2*sqrt(-1/8*I + 1/8) - 1/8*I - 1/8)
*log(((((3*x - 1)*sqrt(x + 1) + 4*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1)^3 -
 ((3*x - 1)*sqrt(x + 1) + 9*x - 3)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 - 2*((x
 + 3)*sqrt(x + 1) - 2*x - 1)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 10*(3*x - 1)*
sqrt(x + 1) - 30*x + 10)*sqrt(x + sqrt(x + 1)) - ((x^2 + 2*(3*x + 4)*sqrt(x
 + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^3 - (x^2 + 6*(x + 3)*sqrt(x
 + 1) + 8*x + 5)*(4*sqrt(-1/8*I + 1/8) + I + 1)^2 + 10*x^2 + 2*(3*x^2 - 2*s
qrt(x + 1)*(x - 2) + 4*x - 5)*(4*sqrt(-1/8*I + 1/8) + I + 1) - 20*(x + 3)*s
qrt(x + 1) - 80*x - 30)*sqrt(-1/2*sqrt(-1/8*I + 1/8) - 1/8*I - 1/8))/(x^2 +
 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{\sqrt{x+1}\left(x^2+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)**(1/2))**(1/2)/(x**2+1)/(1+x)**(1/2),x)
```

```
[Out] Integral(sqrt(x + sqrt(x + 1))/(sqrt(x + 1)*(x**2 + 1)), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(x^2+1)/(1+x)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.14 $\int \frac{\sqrt{x+\sqrt{1+x}}}{1+x^2}\,dx$

Optimal. Leaf size=337

$$\frac{1}{2}i\sqrt{i+\sqrt{1-i}}\tan^{-1}\left(\frac{-\left(1-2\sqrt{1-i}\right)\sqrt{x+1}+\sqrt{1-i}+2}{2\sqrt{i+\sqrt{1-i}}\sqrt{x+\sqrt{x+1}}}\right)-\frac{1}{2}i\sqrt{\sqrt{1+i}-i}\tan^{-1}\left(\frac{-\left(1-2\sqrt{1+i}\right)\sqrt{x+1}+\sqrt{1+i}+2}{2\sqrt{\sqrt{1+i}-i}\sqrt{x+\sqrt{x+1}}}\right.$$

```
[Out] (I/2)*Sqrt[I + Sqrt[1 - I]]*ArcTan[(2 + Sqrt[1 - I] - (1 - 2*Sqrt[1 - I])*S
qrt[1 + x])/(2*Sqrt[I + Sqrt[1 - I]]*Sqrt[x + Sqrt[1 + x]])] - (I/2)*Sqrt[-
I + Sqrt[1 + I]]*ArcTan[(2 + Sqrt[1 + I] - (1 - 2*Sqrt[1 + I])*Sqrt[1 + x])
/(2*Sqrt[-I + Sqrt[1 + I]]*Sqrt[x + Sqrt[1 + x]])] + (I/2)*Sqrt[-I + Sqrt[1
 - I]]*ArcTanh[(2 - Sqrt[1 - I] - (1 + 2*Sqrt[1 - I])*Sqrt[1 + x])/(2*Sqrt[
-I + Sqrt[1 - I]]*Sqrt[x + Sqrt[1 + x]])] - (I/2)*Sqrt[I + Sqrt[1 + I]]*Arc
Tanh[(2 - Sqrt[1 + I] - (1 + 2*Sqrt[1 + I])*Sqrt[1 + x])/(2*Sqrt[I + Sqrt[1
 + I]]*Sqrt[x + Sqrt[1 + x]])]
```

Rubi [A] time = 0.621143, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6741, 6728, 1021, 1078, 621, 206, 1033, 724, 204}

$$\frac{1}{2}i\sqrt{i+\sqrt{1-i}}\tan^{-1}\left(\frac{-\left(1-2\sqrt{1-i}\right)\sqrt{x+1}+\sqrt{1-i}+2}{2\sqrt{i+\sqrt{1-i}}\sqrt{x+\sqrt{x+1}}}\right)-\frac{1}{2}i\sqrt{\sqrt{1+i}-i}\tan^{-1}\left(\frac{-\left(1-2\sqrt{1+i}\right)\sqrt{x+1}+\sqrt{1+i}+2}{2\sqrt{\sqrt{1+i}-i}\sqrt{x+\sqrt{x+1}}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[1 + x]]/(1 + x^2),x]
```

```
[Out] (I/2)*Sqrt[I + Sqrt[1 - I]]*ArcTan[(2 + Sqrt[1 - I] - (1 - 2*Sqrt[1 - I])*S
qrt[1 + x])/(2*Sqrt[I + Sqrt[1 - I]]*Sqrt[x + Sqrt[1 + x]])] - (I/2)*Sqrt[-
I + Sqrt[1 + I]]*ArcTan[(2 + Sqrt[1 + I] - (1 - 2*Sqrt[1 + I])*Sqrt[1 + x])
/(2*Sqrt[-I + Sqrt[1 + I]]*Sqrt[x + Sqrt[1 + x]])] + (I/2)*Sqrt[-I + Sqrt[1
 - I]]*ArcTanh[(2 - Sqrt[1 - I] - (1 + 2*Sqrt[1 - I])*Sqrt[1 + x])/(2*Sqrt[
-I + Sqrt[1 - I]]*Sqrt[x + Sqrt[1 + x]])] - (I/2)*Sqrt[I + Sqrt[1 + I]]*Arc
Tanh[(2 - Sqrt[1 + I] - (1 + 2*Sqrt[1 + I])*Sqrt[1 + x])/(2*Sqrt[I + Sqrt[1
 + I]]*Sqrt[x + Sqrt[1 + x]])]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6728

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rule 1021

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(h*(a + b*x + c*x^2)^p*(d + f*x^2)^(q +
 1))/(2*f*(p + q + 1)), x] - Dist[1/(2*f*(p + q + 1)), Int[(a + b*x + c*x^2
)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(c
*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*(-(b*f)) + c*(-2*g*f)*(p + q +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*a
*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

Rule 1078

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
 (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqrt[d + e*x + f
*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x
 + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, A, B, C}, x] && NeQ[e^2 - 4*d*f
, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
```

```
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{x+\sqrt{1+x}}}{1+x^2}\,dx = 2\,\text{Subst}\left(\int \frac{x\sqrt{-1+x+x^2}}{1+\left(-1+x^2\right)^2}\,dx, x, \sqrt{1+x}\right)$$
$$= 2\,\text{Subst}\left(\int \frac{x\sqrt{-1+x+x^2}}{2-2x^2+x^4}\,dx, x, \sqrt{1+x}\right)$$
$$= 2\,\text{Subst}\left(\int \left(\frac{ix\sqrt{-1+x+x^2}}{(2+2i)-2x^2}+\frac{ix\sqrt{-1+x+x^2}}{(-2+2i)+2x^2}\right)dx, x, \sqrt{1+x}\right)$$
$$= 2i\,\text{Subst}\left(\int \frac{x\sqrt{-1+x+x^2}}{(2+2i)-2x^2}\,dx, x, \sqrt{1+x}\right)+2i\,\text{Subst}\left(\int \frac{x\sqrt{-1+x+x^2}}{(-2+2i)+2x^2}\,dx, x, \sqrt{1+x}\right)$$
$$= i\,\text{Subst}\left(\int \frac{(1+i)+2ix+x^2}{\left((2+2i)-2x^2\right)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)-i\,\text{Subst}\left(\int \frac{(-1+i)+2ix-x^2}{\sqrt{-1+x+x^2}\left((-2+2i)\right.}\right.$$
$$= -\left(\frac{1}{2}i\,\text{Subst}\left(\int \frac{(-4-4i)-4ix}{\left((2+2i)-2x^2\right)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)\right)-\frac{1}{2}i\,\text{Subst}\left(\int \frac{(-4+4i)}{\sqrt{-1+x+x^2}\left((-\right.}\right.$$
$$= -\left(\left(-1-i\sqrt{1-i}\right)\text{Subst}\left(\int \frac{1}{\left(-2\sqrt{1-i}+2x\right)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)\right)-\left(-1+i\sqrt{1-i}\right)\text{Sub}$$
$$= -\left(\left(2\left(1-i\sqrt{1-i}\right)\right)\text{Subst}\left(\int \frac{1}{-16i-16\sqrt{1-i}-x^2}\,dx, x, \frac{-4-2\sqrt{1-i}-\left(-2+4\sqrt{1-i}\right)\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right.\right.$$
$$= \frac{1}{2}i\sqrt{i+\sqrt{1-i}}\tan^{-1}\left(\frac{2+\sqrt{1-i}-\left(1-2\sqrt{1-i}\right)\sqrt{1+x}}{2\sqrt{i+\sqrt{1-i}}\sqrt{x+\sqrt{1+x}}}\right)-\frac{1}{2}i\sqrt{-i+\sqrt{1+i}}\tan^{-1}\left(\frac{2+\sqrt{1+i}-}{2\sqrt{-i+}}\right.$$

<u>Mathematica</u> [B] time = 5.97203, size = 2075, normalized size = 6.16

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[x + Sqrt[1 + x]]/(1 + x^2),x]
```

```
[Out] (1/8 + I/8)*(-2*Sqrt[(1 + I)/(I + Sqrt[1 + I])]*((1 - I) + Sqrt[1 + I])*Arc
Tan[(-10 - (10 - 5*I)*Sqrt[1 + I] - (1 - 2*I)*((5 + 2*I) + 5*Sqrt[1 + I])*x
 + (16 + 8*I)*Sqrt[1 + x] + (10 + 5*I)*Sqrt[1 + I]*Sqrt[1 + x] + (4 - 2*I)*
Sqrt[(-1 + I) + (1 + I)^(3/2)]*Sqrt[x + Sqrt[1 + x]] + (8 - 4*I)*Sqrt[(-1 +
 I) + (1 + I)^(3/2)]*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]] + 4*Sqrt[I + Sqrt[1
```

```
+ I]]*Sqrt[x + Sqrt[1 + x]]*(1 + 2*Sqrt[1 + x]))/((-15 - 5*I) - 10*Sqrt[1 +
 I] + ((-6 + 15*I) + (2 + 12*I)*Sqrt[1 + I])*x + (14 + 20*I)*Sqrt[1 + x] +
(22 + 12*I)*Sqrt[1 + I]*Sqrt[1 + x])] + 2*Sqrt[(-1 + I)/(-I + Sqrt[1 - I])]
*((1 + I) + Sqrt[1 - I])*ArcTan[((-2 + 4*I) - (4 - 3*I)*Sqrt[1 - I] + ((-5
+ 2*I) - 5*Sqrt[1 - I])*x - (8*I)*Sqrt[1 + x] - (5*I)*Sqrt[1 - I]*Sqrt[1 +
x] + (2*I)*Sqrt[(1 + I) - (1 - I)^(3/2)]*Sqrt[x + Sqrt[1 + x]] + (2 + 2*I)*
Sqrt[I - Sqrt[1 - I]]*Sqrt[x + Sqrt[1 + x]]*((1 + I) - (1 + 2*I)*Sqrt[1 + x
] - (2*I)*Sqrt[1 - I]*Sqrt[1 + x]))/((-8 + 6*I) - (6 - 2*I)*Sqrt[1 - I] + (
(-1 + 10*I) - (4 - 8*I)*Sqrt[1 - I])*x + (2 + 4*I)*Sqrt[1 + x] + (2 + 6*I)*
Sqrt[1 - I]*Sqrt[1 + x])] - 2*Sqrt[(1 + I)/(I - Sqrt[1 + I])]*((-1 + I) + S
qrt[1 + I])*ArcTan[(((9 - 8*I) - (5 - 10*I)*Sqrt[1 + I])*x - (2*I)*Sqrt[I -
 Sqrt[1 + I]]*(-2*I + (1 + 2*I)*Sqrt[1 + I])*Sqrt[x + Sqrt[1 + x]]*(1 + 2*S
qrt[1 + x]) + (2 + I)*((4 - 2*I) - (3 - 4*I)*Sqrt[1 + I] - 8*Sqrt[1 + x] +
5*Sqrt[1 + I]*Sqrt[1 + x]))/((15 + 5*I) - 10*Sqrt[1 + I] + ((6 - 15*I) + (2
 + 12*I)*Sqrt[1 + I])*x - (14 + 20*I)*Sqrt[1 + x] + (22 + 12*I)*Sqrt[1 + I]
*Sqrt[1 + x])] - (2*I)*((-1 - I) + Sqrt[1 - I])*Sqrt[(1 - I)/(I + Sqrt[1 -
I])]*ArcTanh[((4 + 2*I) - (3 + 4*I)*Sqrt[1 - I] + (2 + 5*I)*x - (5*I)*Sqrt[
1 - I]*x - 8*Sqrt[1 + x] + 5*Sqrt[1 - I]*Sqrt[1 + x] + 4*Sqrt[I + Sqrt[1 -
I]]*Sqrt[x + Sqrt[1 + x]] - 2*Sqrt[(1 + I) + (1 - I)^(3/2)]*Sqrt[x + Sqrt[1
 + x]] + (8*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]])/Sqrt[(1 - I)/(I + Sqrt[1 - I
])] - (6 + 2*I)*Sqrt[I + Sqrt[1 - I]]*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]])/((
8 - 6*I) - (6 - 2*I)*Sqrt[1 - I] + ((1 - 10*I) - (4 - 8*I)*Sqrt[1 - I])*x -
 (2 + 4*I)*Sqrt[1 + x] + (2 + 6*I)*Sqrt[1 - I]*Sqrt[1 + x])] + I*Sqrt[(-1 +
 I)/(-I + Sqrt[1 - I])]*((1 + I) + Sqrt[1 - I])*Log[(Sqrt[1 - I] - Sqrt[1 +
 x])^2] - I*Sqrt[(1 + I)/(I + Sqrt[1 + I])]*((1 - I) + Sqrt[1 + I])*Log[(Sq
rt[1 + I] - Sqrt[1 + x])^2] + I*((-1 - I) + Sqrt[1 - I])*Sqrt[(1 - I)/(I +
Sqrt[1 - I])]*Log[(Sqrt[1 - I] + Sqrt[1 + x])^2] + Sqrt[(1 + I)/(I - Sqrt[1
 + I])]*((1 + I) - I*Sqrt[1 + I])*Log[(Sqrt[1 + I] + Sqrt[1 + x])^2] + ((1
+ I) + I*Sqrt[1 + I])*Sqrt[(1 + I)/(I + Sqrt[1 + I])]*Log[(11 + 2*I) + (7 -
 I)*Sqrt[1 + I] + ((8 + 7*I) + (9 + 3*I)*Sqrt[1 + I])*x - (2 - 2*I)*Sqrt[1
+ x] - (1 - 3*I)*Sqrt[1 + I]*Sqrt[1 + x] + 8*Sqrt[(-1 + I) + (1 + I)^(3/2)]
*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]] - (2 + 2*I)*Sqrt[I + Sqrt[1 + I]]*(2 + S
qrt[1 + I] - (3 - I)*Sqrt[1 + x])*Sqrt[x + Sqrt[1 + x]]] - I*Sqrt[(-1 + I)/
(-I + Sqrt[1 - I])]*((1 + I) + Sqrt[1 - I])*Log[I*((15 + 20*I) + (5 + 15*I)
*Sqrt[1 - I] + ((-2 + 25*I) - (9 - 15*I)*Sqrt[1 - I])*x - (22 - 10*I)*Sqrt[
1 + x] - (19 + 5*I)*Sqrt[1 - I]*Sqrt[1 + x] + (2 + 6*I)*Sqrt[(1 + I) - (1 -
 I)^(3/2)]*Sqrt[x + Sqrt[1 + x]] + (4 + 12*I)*Sqrt[(1 + I) - (1 - I)^(3/2)]
*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]] + (4 + 4*I)*Sqrt[I - Sqrt[1 - I]]*Sqrt[x
 + Sqrt[1 + x]]*(1 + 2*Sqrt[1 + x]))] - I*((-1 - I) + Sqrt[1 - I])*Sqrt[(1
- I)/(I + Sqrt[1 - I])]*Log[(20 - 15*I) - (15 - 5*I)*Sqrt[1 - I] + ((25 + 2
*I) - (15 + 9*I)*Sqrt[1 - I])*x + (10 + 22*I)*Sqrt[1 + x] + (5 - 19*I)*Sqrt
[1 - I]*Sqrt[1 + x] - (6 - 2*I)*Sqrt[(1 + I) + (1 - I)^(3/2)]*Sqrt[x + Sqrt
[1 + x]] - (12 - 4*I)*Sqrt[(1 + I) + (1 - I)^(3/2)]*Sqrt[1 + x]*Sqrt[x + Sq
rt[1 + x]] + (4 - 4*I)*Sqrt[I + Sqrt[1 - I]]*Sqrt[x + Sqrt[1 + x]]*(1 + 2*S
qrt[1 + x])] + I*Sqrt[(1 + I)/(I - Sqrt[1 + I])]*((-1 + I) + Sqrt[1 + I])*L
```

```
og[(-11 - 2*I) + (7 - I)*Sqrt[1 + I] + ((-8 - 7*I) + (9 + 3*I)*Sqrt[1 + I])
*x + (2 - 2*I)*Sqrt[1 + x] - (1 - 3*I)*Sqrt[1 + I]*Sqrt[1 + x] + (2 + 2*I)*
Sqrt[I - Sqrt[1 + I]]*Sqrt[x + Sqrt[1 + x]]*(2 - Sqrt[1 + I] - (3 - I)*Sqrt
[1 + x] + (4*Sqrt[1 + x])/Sqrt[1 + I])])
```

Maple [C] time = 0.011, size = 105, normalized size = 0.3

$$\frac{1}{2} \sum_{\_R=\mathrm{RootOf}\left(\_Z^8-4\_Z^6+8\_Z^5+20\_Z^4-48\_Z^3+40\_Z^2-8\_Z+1\right)} \frac{\_R^6-2\_R^5+2\_R+1}{\_R^7-3\_R^5+5\_R^4+10\_R^3-18\_R^2+10\_R-1} \ln\left(\sqrt{x+\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(1+x)^(1/2))^(1/2)/(x^2+1),x)
```

```
[Out] 1/2*sum((_R^6-2*_R^5+2*_R+1)/(_R^7-3*_R^5+5*_R^4+10*_R^3-18*_R^2+10*_R-1)*l
n((x+(1+x)^(1/2))^(1/2)-(1+x)^(1/2)-_R),_R=RootOf(_Z^8-4*_Z^6+8*_Z^5+20*_Z^
4-48*_Z^3+40*_Z^2-8*_Z+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1))/(x^2 + 1), x)
```

Fricas [B] time = 44.9248, size = 15946, normalized size = 47.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(x^2+1),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(sqrt(1/4*I + 1/4) + sqrt(-1/4*I + 1/4) - 2*sqrt(-3/16*(2*sqrt(1/4
*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1/4) + I)*(2*sqrt(-1/4*I + 1/4) - I)
 - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2))*log(-1/4*(2*(((2*x + 1)*sqrt(x + 1)
- 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x + 1) - x - 8)*sqr
t(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I)^2 + 2*(((2*x + 1)*sqrt(x + 1)
 - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x - 16)*sqrt(x + 1) + 4*x - 3)
*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) + 8*((((2*x + 1)*sqrt(x +
 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x + 1) - x - 8)
*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) + ((4*(2*x + 1)*sqrt(x +
1) - x - 8)*(2*sqrt(1/4*I + 1/4) + I) - (3*x - 16)*sqrt(x + 1) - 4*x + 3)*s
qrt(x + sqrt(x + 1)))*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(
1/4*I + 1/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) -
 I)^2) + 2*((4*(2*x + 1)*sqrt(x + 1) - x - 8)*(2*sqrt(1/4*I + 1/4) + I)^2 +
 ((3*x - 16)*sqrt(x + 1) + 4*x - 3)*(2*sqrt(1/4*I + 1/4) + I) + 12*(2*x + 1
)*sqrt(x + 1) + 32*x + 46)*sqrt(x + sqrt(x + 1)) + ((3*x^2 + 8*sqrt(x + 1)*
(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x^2 - 2*(4*x^2 - sqrt(
x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sqrt(x + 1)*(x - 2)
 - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I)^2 + 44*x^2 - 2*(6*x^2 + (16*x + 3)*
sqrt(x + 1) + 3*x + 10)*(2*sqrt(1/4*I + 1/4) + I) - 2*((4*x^2 - sqrt(x + 1)
*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + 6*x^2 + (16*x + 3)*sqrt(x
 + 1) + 3*x + 10)*(2*sqrt(-1/4*I + 1/4) - I) + 4*(12*x^2 + (3*x^2 + 8*sqrt(
x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I) + (3*x^2 - 2*(4*x^2 -
sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sqrt(x + 1)*(x
 - 2) - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I) + 2*(16*x + 3)*sqrt(x + 1) + 6
*x + 20)*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1/4)
+ I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2) + 24*s
qrt(x + 1)*(x - 2) + 92*x - 20)*sqrt(sqrt(1/4*I + 1/4) + sqrt(-1/4*I + 1/4)
 - 2*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1/4) + I)
*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2)))/(x^2 + 1
)) + 1/4*sqrt(sqrt(1/4*I + 1/4) + sqrt(-1/4*I + 1/4) - 2*sqrt(-3/16*(2*sqrt
(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1/4) + I)*(2*sqrt(-1/4*I + 1/4)
- I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2))*log(-1/4*(2*(((2*x + 1)*sqrt(x +
 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x + 1) - x - 8)
*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I)^2 + 2*(((2*x + 1)*sqrt(x
+ 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x - 16)*sqrt(x + 1) + 4*x
- 3)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) + 8*((((2*x + 1)*sqrt
(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x + 1) - x
- 8)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) + ((4*(2*x + 1)*sqrt(
x + 1) - x - 8)*(2*sqrt(1/4*I + 1/4) + I) - (3*x - 16)*sqrt(x + 1) - 4*x +
3)*sqrt(x + sqrt(x + 1)))*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*s
qrt(1/4*I + 1/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/
4) - I)^2) + 2*((4*(2*x + 1)*sqrt(x + 1) - x - 8)*(2*sqrt(1/4*I + 1/4) + I)
^2 + ((3*x - 16)*sqrt(x + 1) + 4*x - 3)*(2*sqrt(1/4*I + 1/4) + I) + 12*(2*x
 + 1)*sqrt(x + 1) + 32*x + 46)*sqrt(x + sqrt(x + 1)) - ((3*x^2 + 8*sqrt(x +
 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x^2 - 2*(4*x^2 - s
```

```
qrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sqrt(x + 1)*(x
- 2) - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I)^2 + 44*x^2 - 2*(6*x^2 + (16*x +
 3)*sqrt(x + 1) + 3*x + 10)*(2*sqrt(1/4*I + 1/4) + I) - 2*((4*x^2 - sqrt(x
+ 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + 6*x^2 + (16*x + 3)*sq
rt(x + 1) + 3*x + 10)*(2*sqrt(-1/4*I + 1/4) - I) + 4*(12*x^2 + (3*x^2 + 8*s
qrt(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I) + (3*x^2 - 2*(4*x^
2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sqrt(x + 1
)*(x - 2) - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I) + 2*(16*x + 3)*sqrt(x + 1)
 + 6*x + 20)*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1
/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2) +
24*sqrt(x + 1)*(x - 2) + 92*x - 20)*sqrt(sqrt(1/4*I + 1/4) + sqrt(-1/4*I +
1/4) - 2*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1/4)
+ I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2)))/(x^2
 + 1)) - 1/4*sqrt(sqrt(1/4*I + 1/4) + sqrt(-1/4*I + 1/4) + 2*sqrt(-3/16*(2*
sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1/4) + I)*(2*sqrt(-1/4*I + 1
/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2))*log(-1/4*(2*(((2*x + 1)*sqrt
(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x + 1) - x
- 8)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I)^2 + 2*(((2*x + 1)*sqr
t(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x - 16)*sqrt(x + 1) +
4*x - 3)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) - 8*((((2*x + 1)*
sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x + 1)
- x - 8)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) + ((4*(2*x + 1)*s
qrt(x + 1) - x - 8)*(2*sqrt(1/4*I + 1/4) + I) - (3*x - 16)*sqrt(x + 1) - 4*
x + 3)*sqrt(x + sqrt(x + 1)))*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*
(2*sqrt(1/4*I + 1/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I
+ 1/4) - I)^2) + 2*((4*(2*x + 1)*sqrt(x + 1) - x - 8)*(2*sqrt(1/4*I + 1/4)
+ I)^2 + ((3*x - 16)*sqrt(x + 1) + 4*x - 3)*(2*sqrt(1/4*I + 1/4) + I) + 12*
(2*x + 1)*sqrt(x + 1) + 32*x + 46)*sqrt(x + sqrt(x + 1)) + ((3*x^2 + 8*sqrt
(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x^2 - 2*(4*x^2
 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sqrt(x + 1)
*(x - 2) - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I)^2 + 44*x^2 - 2*(6*x^2 + (16
*x + 3)*sqrt(x + 1) + 3*x + 10)*(2*sqrt(1/4*I + 1/4) + I) - 2*((4*x^2 - sqr
t(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + 6*x^2 + (16*x + 3
)*sqrt(x + 1) + 3*x + 10)*(2*sqrt(-1/4*I + 1/4) - I) - 4*(12*x^2 + (3*x^2 +
 8*sqrt(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I) + (3*x^2 - 2*(
4*x^2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sqrt(x
 + 1)*(x - 2) - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I) + 2*(16*x + 3)*sqrt(x
+ 1) + 6*x + 20)*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I
 + 1/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2
) + 24*sqrt(x + 1)*(x - 2) + 92*x - 20)*sqrt(sqrt(1/4*I + 1/4) + sqrt(-1/4*
I + 1/4) + 2*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1
/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2)))/
(x^2 + 1)) + 1/4*sqrt(sqrt(1/4*I + 1/4) + sqrt(-1/4*I + 1/4) + 2*sqrt(-3/16
*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I + 1/4) + I)*(2*sqrt(-1/4*I
 + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2))*log(-1/4*(2*(((2*x + 1)*
```

```
sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x + 1)
- x - 8)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I)^2 + 2*(((2*x + 1)
*sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x - 16)*sqrt(x + 1
) + 4*x - 3)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) - 8*((((2*x +
 1)*sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x +
 1) - x - 8)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) + ((4*(2*x +
1)*sqrt(x + 1) - x - 8)*(2*sqrt(1/4*I + 1/4) + I) - (3*x - 16)*sqrt(x + 1)
- 4*x + 3)*sqrt(x + sqrt(x + 1)))*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 -
1/8*(2*sqrt(1/4*I + 1/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/
4*I + 1/4) - I)^2) + 2*((4*(2*x + 1)*sqrt(x + 1) - x - 8)*(2*sqrt(1/4*I + 1
/4) + I)^2 + ((3*x - 16)*sqrt(x + 1) + 4*x - 3)*(2*sqrt(1/4*I + 1/4) + I) +
 12*(2*x + 1)*sqrt(x + 1) + 32*x + 46)*sqrt(x + sqrt(x + 1)) - ((3*x^2 + 8*
sqrt(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x^2 - 2*(4
*x^2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sqrt(x
+ 1)*(x - 2) - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I)^2 + 44*x^2 - 2*(6*x^2 +
 (16*x + 3)*sqrt(x + 1) + 3*x + 10)*(2*sqrt(1/4*I + 1/4) + I) - 2*((4*x^2 -
 sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + 6*x^2 + (16*x
 + 3)*sqrt(x + 1) + 3*x + 10)*(2*sqrt(-1/4*I + 1/4) - I) - 4*(12*x^2 + (3*x
^2 + 8*sqrt(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I) + (3*x^2 -
 2*(4*x^2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sq
rt(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I) + 2*(16*x + 3)*sqr
t(x + 1) + 6*x + 20)*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1
/4*I + 1/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) -
I)^2) + 24*sqrt(x + 1)*(x - 2) + 92*x - 20)*sqrt(sqrt(1/4*I + 1/4) + sqrt(-
1/4*I + 1/4) + 2*sqrt(-3/16*(2*sqrt(1/4*I + 1/4) + I)^2 - 1/8*(2*sqrt(1/4*I
 + 1/4) + I)*(2*sqrt(-1/4*I + 1/4) - I) - 3/16*(2*sqrt(-1/4*I + 1/4) - I)^2
)))/(x^2 + 1)) + 1/2*sqrt(-1/2*sqrt(-1/4*I + 1/4) + 1/4*I)*log(((((2*x + 1)
*sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I) + 4*(2*x + 1)*sqrt(x + 1)
 - x - 8)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I)^2 + (((2*x + 1)*
sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I)^2 + (3*x - 16)*sqrt(x + 1)
 + 4*x - 3)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4*I + 1/4) - I) + (((2*x + 1)*
sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I)^3 - 6*(2*x + 1)*sqrt(x + 1
) - 16*x - 23)*sqrt(x + sqrt(x + 1)) + (2*(4*x^2 - sqrt(x + 1)*(x - 2) + 2*
x - 5)*(2*sqrt(1/4*I + 1/4) + I)^3 - (3*x^2 - 2*(4*x^2 - sqrt(x + 1)*(x - 2
) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*sqrt(x + 1)*(x - 2) - 16*x + 5)*
(2*sqrt(-1/4*I + 1/4) - I)^2 + 22*x^2 + 2*((4*x^2 - sqrt(x + 1)*(x - 2) + 2
*x - 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + 6*x^2 + (16*x + 3)*sqrt(x + 1) + 3*x
+ 10)*(2*sqrt(-1/4*I + 1/4) - I) + 12*sqrt(x + 1)*(x - 2) + 46*x - 10)*sqrt
(-1/2*sqrt(-1/4*I + 1/4) + 1/4*I))/(x^2 + 1)) - 1/2*sqrt(-1/2*sqrt(-1/4*I +
 1/4) + 1/4*I)*log(((((2*x + 1)*sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4)
 + I) + 4*(2*x + 1)*sqrt(x + 1) - x - 8)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1/4
*I + 1/4) - I)^2 + (((2*x + 1)*sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4)
+ I)^2 + (3*x - 16)*sqrt(x + 1) + 4*x - 3)*sqrt(x + sqrt(x + 1))*(2*sqrt(-1
/4*I + 1/4) - I) + (((2*x + 1)*sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4)
+ I)^3 - 6*(2*x + 1)*sqrt(x + 1) - 16*x - 23)*sqrt(x + sqrt(x + 1)) - (2*(4
```

```
*x^2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I)^3 - (3*x^2
- 2*(4*x^2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I) + 8*s
qrt(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(-1/4*I + 1/4) - I)^2 + 22*x^2 + 2*((
4*x^2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I)^2 + 6*x^2
+ (16*x + 3)*sqrt(x + 1) + 3*x + 10)*(2*sqrt(-1/4*I + 1/4) - I) + 12*sqrt(x
 + 1)*(x - 2) + 46*x - 10)*sqrt(-1/2*sqrt(-1/4*I + 1/4) + 1/4*I))/(x^2 + 1)
) + 1/2*sqrt(-1/2*sqrt(1/4*I + 1/4) - 1/4*I)*log(-((((2*x + 1)*sqrt(x + 1)
- 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I)^3 - (4*(2*x + 1)*sqrt(x + 1) - x - 8)*
(2*sqrt(1/4*I + 1/4) + I)^2 - ((3*x - 16)*sqrt(x + 1) + 4*x - 3)*(2*sqrt(1/
4*I + 1/4) + I) + 10*(2*x + 1)*sqrt(x + 1) - 20*x + 15)*sqrt(x + sqrt(x + 1
)) + (2*(4*x^2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/4*I + 1/4) + I)^3
 + (3*x^2 + 8*sqrt(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*I + 1/4) + I)^2 +
 10*x^2 - 2*(6*x^2 + (16*x + 3)*sqrt(x + 1) + 3*x + 10)*(2*sqrt(1/4*I + 1/4
) + I) - 20*sqrt(x + 1)*(x - 2) - 30*x - 30)*sqrt(-1/2*sqrt(1/4*I + 1/4) -
1/4*I))/(x^2 + 1)) - 1/2*sqrt(-1/2*sqrt(1/4*I + 1/4) - 1/4*I)*log(-((((2*x
+ 1)*sqrt(x + 1) - 9*x - 2)*(2*sqrt(1/4*I + 1/4) + I)^3 - (4*(2*x + 1)*sqrt
(x + 1) - x - 8)*(2*sqrt(1/4*I + 1/4) + I)^2 - ((3*x - 16)*sqrt(x + 1) + 4*
x - 3)*(2*sqrt(1/4*I + 1/4) + I) + 10*(2*x + 1)*sqrt(x + 1) - 20*x + 15)*sq
rt(x + sqrt(x + 1)) - (2*(4*x^2 - sqrt(x + 1)*(x - 2) + 2*x - 5)*(2*sqrt(1/
4*I + 1/4) + I)^3 + (3*x^2 + 8*sqrt(x + 1)*(x - 2) - 16*x + 5)*(2*sqrt(1/4*
I + 1/4) + I)^2 + 10*x^2 - 2*(6*x^2 + (16*x + 3)*sqrt(x + 1) + 3*x + 10)*(2
*sqrt(1/4*I + 1/4) + I) - 20*sqrt(x + 1)*(x - 2) - 30*x - 30)*sqrt(-1/2*sqr
t(1/4*I + 1/4) - 1/4*I))/(x^2 + 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)**(1/2))**(1/2)/(x**2+1),x)
```

```
[Out] Integral(sqrt(x + sqrt(x + 1))/(x**2 + 1), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/(x^2+1),x, algorithm="giac")
```

```
[Out] Timed out
```

3.15 $\int \sqrt{1+\sqrt{x}+\sqrt{1+2\sqrt{x}+2x}}\,dx$

Optimal. Leaf size=77

$$\frac{2\sqrt{\sqrt{x}+\sqrt{2x+2\sqrt{x}+1}+1}\left(6x^{3/2}+\sqrt{x}-\left(2-\sqrt{x}\right)\sqrt{2x+2\sqrt{x}+1}+2\right)}{15\sqrt{x}}$$

```
[Out] (2*Sqrt[1 + Sqrt[x] + Sqrt[1 + 2*Sqrt[x] + 2*x]]*(2 + Sqrt[x] + 6*x^(3/2) -
 (2 - Sqrt[x])*Sqrt[1 + 2*Sqrt[x] + 2*x]))/(15*Sqrt[x])
```

Rubi [A] time = 0.0710915, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {2114}

$$\frac{2\sqrt{\sqrt{x}+\sqrt{2x+2\sqrt{x}+1}+1}\left(6x^{3/2}+\sqrt{x}-\left(2-\sqrt{x}\right)\sqrt{2x+2\sqrt{x}+1}+2\right)}{15\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Sqrt[x] + Sqrt[1 + 2*Sqrt[x] + 2*x]],x]
```

```
[Out] (2*Sqrt[1 + Sqrt[x] + Sqrt[1 + 2*Sqrt[x] + 2*x]]*(2 + Sqrt[x] + 6*x^(3/2) -
 (2 - Sqrt[x])*Sqrt[1 + 2*Sqrt[x] + 2*x]))/(15*Sqrt[x])
```

Rule 2114

```
Int[((g_.) + (h_.)*(x_))*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]], x_Symbol] :> Simp[(2*(f*(5*b*c*g^2 - 2*b^2*g*h - 3*
a*c*g*h + 2*a*b*h^2) + c*f*(10*c*g^2 - b*g*h + a*h^2)*x + 9*c^2*f*g*h*x^2 +
 3*c^2*f*h^2*x^3 - (e*g - d*h)*(5*c*g - 2*b*h + c*h*x)*Sqrt[a + b*x + c*x^2
])*Sqrt[d + e*x + f*Sqrt[a + b*x + c*x^2]])/(15*c^2*f*(g + h*x)), x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && EqQ[(e*g - d*h)^2 - f^2*(c*g^2 - b*g*h +
 a*h^2), 0] && EqQ[2*e^2*g - 2*d*e*h - f^2*(2*c*g - b*h), 0]
```

Rubi steps

$$\int \sqrt{1+\sqrt{x}+\sqrt{1+2\sqrt{x}+2x}}\,dx = 2\,\text{Subst}\left(\int x\sqrt{1+x+\sqrt{1+2x+2x^2}}\,dx, x, \sqrt{x}\right)$$
$$= \frac{2\sqrt{1+\sqrt{x}+\sqrt{1+2\sqrt{x}+2x}}\left(2+\sqrt{x}+6x^{3/2}-\left(2-\sqrt{x}\right)\sqrt{1+2\sqrt{x}+2x}\right)}{15\sqrt{x}}$$

Mathematica [A] time = 0.037998, size = 74, normalized size = 0.96

$$\frac{2\sqrt{\sqrt{x}+\sqrt{2x+2\sqrt{x}+1}+1}\left(6x^{3/2}+\sqrt{x}+\left(\sqrt{x}-2\right)\sqrt{2x+2\sqrt{x}+1}+2\right)}{15\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sqrt[x] + Sqrt[1 + 2*Sqrt[x] + 2*x]],x]
```

```
[Out] (2*Sqrt[1 + Sqrt[x] + Sqrt[1 + 2*Sqrt[x] + 2*x]]*(2 + Sqrt[x] + 6*x^(3/2) +
 (-2 + Sqrt[x])*Sqrt[1 + 2*Sqrt[x] + 2*x]))/(15*Sqrt[x])
```

Maple [F] time = 0.014, size = 0, normalized size = 0.

$$\int \sqrt{1+\sqrt{x}+\sqrt{1+2\,x+2\,\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x^(1/2)+(1+2*x+2*x^(1/2))^(1/2))^(1/2),x)
```

```
[Out] int((1+x^(1/2)+(1+2*x+2*x^(1/2))^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{2\,x+2\,\sqrt{x}+1}+\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2)+(1+2*x+2*x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(2*x + 2*sqrt(x) + 1) + sqrt(x) + 1), x)
```

Fricas [A] time = 6.29931, size = 165, normalized size = 2.14

$$\frac{2\left(6\,x^2+\sqrt{2\,x+2\,\sqrt{x}+1}\left(x-2\,\sqrt{x}\right)+x+2\,\sqrt{x}\right)\sqrt{\sqrt{2\,x+2\,\sqrt{x}+1}+\sqrt{x}+1}}{15\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2)+(1+2*x+2*x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/15*(6*x^2 + sqrt(2*x + 2*sqrt(x) + 1)*(x - 2*sqrt(x)) + x + 2*sqrt(x))*sq
rt(sqrt(2*x + 2*sqrt(x) + 1) + sqrt(x) + 1)/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{x}+\sqrt{2\sqrt{x}+2x+1}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2)+(1+2*x+2*x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) + sqrt(2*sqrt(x) + 2*x + 1) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{2\,x+2\,\sqrt{x}+1}+\sqrt{x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2)+(1+2*x+2*x^(1/2))^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(sqrt(2*x + 2*sqrt(x) + 1) + sqrt(x) + 1), x)
```

3.16 $\int \sqrt{\sqrt{2}+\sqrt{x}+\sqrt{2+2\sqrt{2}\sqrt{x}+2x}}\,dx$

Optimal. Leaf size=118

$$\frac{2\sqrt{2}\sqrt{\sqrt{x}+\sqrt{2}\sqrt{x+\sqrt{2}\sqrt{x}+1}+\sqrt{2}}\left(3\sqrt{2}x^{3/2}+\sqrt{2}\sqrt{x}-\sqrt{2}\left(2\sqrt{2}-\sqrt{x}\right)\sqrt{x+\sqrt{2}\sqrt{x}+1}+4\right)}{15\sqrt{x}}$$

```
[Out] (2*Sqrt[2]*Sqrt[Sqrt[2] + Sqrt[x] + Sqrt[2]*Sqrt[1 + Sqrt[2]*Sqrt[x] + x]]*
(4 + Sqrt[2]*Sqrt[x] + 3*Sqrt[2]*x^(3/2) - Sqrt[2]*(2*Sqrt[2] - Sqrt[x])*Sq
rt[1 + Sqrt[2]*Sqrt[x] + x]))/(15*Sqrt[x])
```

Rubi [A] time = 0.190937, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {2115, 2114}

$$\frac{2\sqrt{2}\sqrt{\sqrt{x}+\sqrt{2}\sqrt{x+\sqrt{2}\sqrt{x}+1}+\sqrt{2}}\left(3\sqrt{2}x^{3/2}+\sqrt{2}\sqrt{x}-\sqrt{2}\left(2\sqrt{2}-\sqrt{x}\right)\sqrt{x+\sqrt{2}\sqrt{x}+1}+4\right)}{15\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sqrt[2] + Sqrt[x] + Sqrt[2 + 2*Sqrt[2]*Sqrt[x] + 2*x]],x]
```

```
[Out] (2*Sqrt[2]*Sqrt[Sqrt[2] + Sqrt[x] + Sqrt[2]*Sqrt[1 + Sqrt[2]*Sqrt[x] + x]]*
(4 + Sqrt[2]*Sqrt[x] + 3*Sqrt[2]*x^(3/2) - Sqrt[2]*(2*Sqrt[2] - Sqrt[x])*Sq
rt[1 + Sqrt[2]*Sqrt[x] + x]))/(15*Sqrt[x])
```

Rule 2115

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*((g_.) + (h_.)*(x_))^(m_.
), x_Symbol] :> Int[(g + h*x)^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandT
oSum[v, x]])^n, x] /; FreeQ[{f, g, h, j, k, m, n}, x] && LinearQ[u, x] && Q
uadraticQ[v, x] &&  !(LinearMatchQ[u, x] && QuadraticMatchQ[v, x] && (EqQ[j
, 0] || EqQ[f, 1])) && EqQ[(Coefficient[u, x, 1]*g - h*(Coefficient[u, x, 0
] + f*j))^2 - f^2*k^2*(Coefficient[v, x, 2]*g^2 - Coefficient[v, x, 1]*g*h
+ Coefficient[v, x, 0]*h^2), 0]
```

Rule 2114

```
Int[((g_.) + (h_.)*(x_))*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]], x_Symbol] :> Simp[(2*(f*(5*b*c*g^2 - 2*b^2*g*h - 3*
a*c*g*h + 2*a*b*h^2) + c*f*(10*c*g^2 - b*g*h + a*h^2)*x + 9*c^2*f*g*h*x^2 +
 3*c^2*f*h^2*x^3 - (e*g - d*h)*(5*c*g - 2*b*h + c*h*x)*Sqrt[a + b*x + c*x^2
])*Sqrt[d + e*x + f*Sqrt[a + b*x + c*x^2]])/(15*c^2*f*(g + h*x)), x] /; Fre
eQ[{a, b, c, d, e, f, g, h}, x] && EqQ[(e*g - d*h)^2 - f^2*(c*g^2 - b*g*h +
 a*h^2), 0] && EqQ[2*e^2*g - 2*d*e*h - f^2*(2*c*g - b*h), 0]
```

Rubi steps

$$\int \sqrt{\sqrt{2}+\sqrt{x}+\sqrt{2+2\sqrt{2}\sqrt{x}+2x}}\,dx = 2\,\text{Subst}\left(\int x\sqrt{x+\sqrt{2}\left(1+\sqrt{1+\sqrt{2}x+x^2}\right)}\,dx, x, \sqrt{x}\right)$$

$$= 2\,\text{Subst}\left(\int x\sqrt{\sqrt{2}+x+\sqrt{2}\sqrt{1+\sqrt{2}x+x^2}}\,dx, x, \sqrt{x}\right)$$

$$= \frac{2\sqrt{2}\sqrt{\sqrt{2}+\sqrt{x}+\sqrt{2}\sqrt{1+\sqrt{2}\sqrt{x}+x}}\left(4+\sqrt{2}\sqrt{x}+3\sqrt{2}x^{3/2}-\sqrt{2}\left(2\sqrt{2}-\sqrt{x}\right.\right.}{15\sqrt{x}}$$

Mathematica [A] time = 0.0787935, size = 112, normalized size = 0.95

$$\frac{2\sqrt{2}\left(3\sqrt{2}x^{3/2}+\sqrt{2}\sqrt{x}+\sqrt{2}\left(\sqrt{x}-2\sqrt{2}\right)\sqrt{x+\sqrt{2}\sqrt{x}+1}+4\right)\sqrt{\sqrt{2}\left(\sqrt{x+\sqrt{2}\sqrt{x}+1}+1\right)+\sqrt{x}}}{15\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sqrt[2] + Sqrt[x] + Sqrt[2 + 2*Sqrt[2]*Sqrt[x] + 2*x]],x]
```

```
[Out] (2*Sqrt[2]*(4 + Sqrt[2]*Sqrt[x] + 3*Sqrt[2]*x^(3/2) + Sqrt[2]*(-2*Sqrt[2] +
 Sqrt[x])*Sqrt[1 + Sqrt[2]*Sqrt[x] + x])*Sqrt[Sqrt[x] + Sqrt[2]*(1 + Sqrt[1
 + Sqrt[2]*Sqrt[x] + x])]))/(15*Sqrt[x])
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{2}+\sqrt{x}+\sqrt{2+2\,x+2\,\sqrt{2}\sqrt{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2^(1/2)+x^(1/2)+(2+2*x+2*2^(1/2)*x^(1/2))^(1/2))^(1/2),x)
```

```
[Out] int((2^(1/2)+x^(1/2)+(2+2*x+2*2^(1/2)*x^(1/2))^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{2} + \sqrt{2\sqrt{2}\sqrt{x} + 2x + 2} + \sqrt{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(1/2)+x^(1/2)+(2+2*x+2*2^(1/2)*x^(1/2))^(1/2))^(1/2),x, algori
thm="maxima")
```

```
[Out] integrate(sqrt(sqrt(2) + sqrt(2*sqrt(2)*sqrt(x) + 2*x + 2) + sqrt(x)), x)
```

Fricas [A] time = 11.0405, size = 219, normalized size = 1.86

$$\frac{2\left(6x^2 + \left(\sqrt{2}x - 4\sqrt{x}\right)\sqrt{2\sqrt{2}\sqrt{x} + 2x + 2} + 4\sqrt{2}\sqrt{x} + 2x\right)\sqrt{\sqrt{2} + \sqrt{2\sqrt{2}\sqrt{x} + 2x + 2} + \sqrt{x}}}{15x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(1/2)+x^(1/2)+(2+2*x+2*2^(1/2)*x^(1/2))^(1/2))^(1/2),x, algori
thm="fricas")
```

```
[Out] 2/15*(6*x^2 + (sqrt(2)*x - 4*sqrt(x))*sqrt(2*sqrt(2)*sqrt(x) + 2*x + 2) + 4
*sqrt(2)*sqrt(x) + 2*x)*sqrt(sqrt(2) + sqrt(2*sqrt(2)*sqrt(x) + 2*x + 2) +
sqrt(x))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{x} + \sqrt{2\sqrt{2}\sqrt{x} + 2x + 2} + \sqrt{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2**(1/2)+x**(1/2)+(2+2*x+2*2**(1/2)*x**(1/2))**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(x) + sqrt(2*sqrt(2)*sqrt(x) + 2*x + 2) + sqrt(2)), x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2^(1/2)+x^(1/2)+(2+2*x+2*2^(1/2)*x^(1/2))^(1/2))^(1/2),x, algori
thm="giac")
```

```
[Out] Timed out
```

3.17 $\int \frac{\sqrt{x+\sqrt{1+x}}}{x^2}\,dx$

Optimal. Leaf size=83

$$-\frac{\sqrt{x+\sqrt{x+1}}}{x}-\frac{1}{4}\tan^{-1}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)+\frac{3}{4}\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)$$

```
[Out] -(Sqrt[x + Sqrt[1 + x]]/x) - ArcTan[(3 + Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 +
x]])]/4 + (3*ArcTanh[(1 - 3*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/4
```

Rubi [A] time = 0.100013, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1014, 1033, 724, 206, 204}

$$-\frac{\sqrt{x+\sqrt{x+1}}}{x}-\frac{1}{4}\tan^{-1}\left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}}\right)+\frac{3}{4}\tanh^{-1}\left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + Sqrt[1 + x]]/x^2,x]
```

```
[Out] -(Sqrt[x + Sqrt[1 + x]]/x) - ArcTan[(3 + Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 +
x]])]/4 + (3*ArcTanh[(1 - 3*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/4
```

Rule 1014

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a*h - g*c*x)*(a + c*x^2)^(p + 1)*(d +
 e*x + f*x^2)^q)/(2*a*c*(p + 1)), x] + Dist[2/(4*a*c*(p + 1)), Int[(a + c*x
^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[g*c*d*(2*p + 3) - a*(h*e*q) + (g
*c*e*(2*p + q + 3) - a*(2*h*f*q))*x + g*c*f*(2*p + 2*q + 3)*x^2, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] &&
 GtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
```

```
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{x+\sqrt{1+x}}}{x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x\sqrt{-1+x+x^2}}{\left(-1+x^2\right)^2}\,dx, x, \sqrt{1+x}\right)\\
&= -\frac{\sqrt{x+\sqrt{1+x}}}{x}+\text{Subst}\left(\int \frac{\frac{1}{2}+x}{\left(-1+x^2\right)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)\\
&= -\frac{\sqrt{x+\sqrt{1+x}}}{x}+\frac{1}{4}\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+x}\right)+\frac{3}{4}\,\text{Subst}\Bigg(\int \frac{1}{(-1+x)\sqrt{-1+}}\\
&= -\frac{\sqrt{x+\sqrt{1+x}}}{x}-\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3-\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}}\right)-\frac{3}{2}\,\text{Subst}\Bigg(\int \frac{1}{4-x^2}\,dx, x, \frac{-1+3\sqrt{1}}{\sqrt{x+\sqrt{1}}}\\
&= -\frac{\sqrt{x+\sqrt{1+x}}}{x}-\frac{1}{4}\tan^{-1}\left(\frac{3+\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)+\frac{3}{4}\tanh^{-1}\left(\frac{1-3\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0337009, size = 85, normalized size = 1.02

$$-\frac{\sqrt{x+\sqrt{x+1}}}{x}+\frac{1}{4}\tan^{-1}\left(\frac{-\sqrt{x+1}-3}{2\sqrt{x+\sqrt{x+1}}}\right)-\frac{3}{4}\tanh^{-1}\left(\frac{3\sqrt{x+1}-1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + Sqrt[1 + x]]/x^2,x]
```

```
[Out] -(Sqrt[x + Sqrt[1 + x]]/x) + ArcTan[(-3 - Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 +
 x]])]/4 - (3*ArcTanh[(-1 + 3*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])])/4
```

Maple [B] time = 0.011, size = 298, normalized size = 3.6

$$-\frac{1}{2}\left(\left(-1+\sqrt{1+x}\right)^{2}-2+3\sqrt{1+x}\right)^{\frac{3}{2}}\left(-1+\sqrt{1+x}\right)^{-1}+\frac{3}{4}\sqrt{\left(-1+\sqrt{1+x}\right)^{2}-2+3\sqrt{1+x}}+\frac{1}{2}\ln\left(\frac{1}{2}+\sqrt{1+x}+\sqrt{(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x+(1+x)^(1/2))^(1/2)/x^2,x)
```

```
[Out] -1/2/(-1+(1+x)^(1/2))*((-1+(1+x)^(1/2))^2-2+3*(1+x)^(1/2))^(3/2)+3/4*((-1+(
1+x)^(1/2))^2-2+3*(1+x)^(1/2))^(1/2)+1/2*ln(1/2+(1+x)^(1/2)+((-1+(1+x)^(1/2
))^2-2+3*(1+x)^(1/2))^(1/2))-3/4*arctanh(1/2*(-1+3*(1+x)^(1/2))/((-1+(1+x)^
(1/2))^2-2+3*(1+x)^(1/2))^(1/2))+1/4*(2*(1+x)^(1/2)+1)*((-1+(1+x)^(1/2))^2-
2+3*(1+x)^(1/2))^(1/2)-1/2/(1+(1+x)^(1/2))*((1+(1+x)^(1/2))^2-2-(1+x)^(1/2)
)^(3/2)-1/4*((1+(1+x)^(1/2))^2-2-(1+x)^(1/2))^(1/2)-1/2*ln(1/2+(1+x)^(1/2)+
((1+(1+x)^(1/2))^2-2-(1+x)^(1/2))^(1/2))+1/4*arctan(1/2*(-3-(1+x)^(1/2))/((
1+(1+x)^(1/2))^2-2-(1+x)^(1/2))^(1/2))+1/4*(2*(1+x)^(1/2)+1)*((1+(1+x)^(1/2
))^2-2-(1+x)^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x + sqrt(x + 1))/x^2, x)
```

Fricas [A] time = 29.4364, size = 240, normalized size = 2.89

$$\frac{x\arctan\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}-3\right)}{x-8}\right)+3\,x\log\left(\frac{2\sqrt{x+\sqrt{x+1}}\left(\sqrt{x+1}+1\right)-3\,x-2\sqrt{x+1}-2}{x}\right)-4\sqrt{x+\sqrt{x+1}}}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(x*arctan(2*sqrt(x + sqrt(x + 1))*(sqrt(x + 1) - 3)/(x - 8)) + 3*x*log(
(2*sqrt(x + sqrt(x + 1))*(sqrt(x + 1) + 1) - 3*x - 2*sqrt(x + 1) - 2)/x) -
4*sqrt(x + sqrt(x + 1)))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+\sqrt{x+1}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)**(1/2))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x + sqrt(x + 1))/x**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(1+x)^(1/2))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.18 $\int \sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\,dx$

Optimal. Leaf size=96

$$\sqrt{\sqrt{\frac{1}{x}+1}+\frac{1}{x}}\,x+\frac{1}{4}\tan^{-1}\left(\frac{\sqrt{\frac{1}{x}+1}+3}{2\sqrt{\sqrt{\frac{1}{x}+1}+\frac{1}{x}}}\right)-\frac{3}{4}\tanh^{-1}\left(\frac{1-3\sqrt{\frac{1}{x}+1}}{2\sqrt{\sqrt{\frac{1}{x}+1}+\frac{1}{x}}}\right)$$

```
[Out] Sqrt[Sqrt[1 + x^(-1)] + x^(-1)]*x + ArcTan[(3 + Sqrt[1 + x^(-1)])/(2*Sqrt[S
qrt[1 + x^(-1)] + x^(-1)])]/4 - (3*ArcTanh[(1 - 3*Sqrt[1 + x^(-1)])/(2*Sqrt
[Sqrt[1 + x^(-1)] + x^(-1)])])/4
```

Rubi [A] time = 0.0838966, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1014, 1033, 724, 206, 204}

$$\sqrt{\sqrt{\frac{1}{x}+1}+\frac{1}{x}}\,x+\frac{1}{4}\tan^{-1}\left(\frac{\sqrt{\frac{1}{x}+1}+3}{2\sqrt{\sqrt{\frac{1}{x}+1}+\frac{1}{x}}}\right)-\frac{3}{4}\tanh^{-1}\left(\frac{1-3\sqrt{\frac{1}{x}+1}}{2\sqrt{\sqrt{\frac{1}{x}+1}+\frac{1}{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sqrt[1 + x^(-1)] + x^(-1)],x]
```

```
[Out] Sqrt[Sqrt[1 + x^(-1)] + x^(-1)]*x + ArcTan[(3 + Sqrt[1 + x^(-1)])/(2*Sqrt[S
qrt[1 + x^(-1)] + x^(-1)])]/4 - (3*ArcTanh[(1 - 3*Sqrt[1 + x^(-1)])/(2*Sqrt
[Sqrt[1 + x^(-1)] + x^(-1)])])/4
```

Rule 1014

```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a*h - g*c*x)*(a + c*x^2)^(p + 1)*(d +
 e*x + f*x^2)^q)/(2*a*c*(p + 1)), x] + Dist[2/(4*a*c*(p + 1)), Int[(a + c*x
^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[g*c*d*(2*p + 3) - a*(h*e*q) + (g
*c*e*(2*p + q + 3) - a*(2*h*f*q))*x + g*c*f*(2*p + 2*q + 3)*x^2, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] &&
 GtQ[q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\,dx = -\left(2\,\text{Subst}\left(\int \frac{x\sqrt{-1+x+x^2}}{\left(-1+x^2\right)^2}\,dx, x, \sqrt{1+\frac{1}{x}}\right)\right)$$

$$= \sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\,x - \text{Subst}\left(\int \frac{\frac{1}{2}+x}{\left(-1+x^2\right)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+\frac{1}{x}}\right)$$

$$= \sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\,x - \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{-1+x+x^2}}\,dx, x, \sqrt{1+\frac{1}{x}}\right) - \frac{3}{4}\,\text{Subst}\left(\int \frac{1}{(-1+x)\sqrt{-1}}\right.$$

$$= \sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\,x + \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3-\sqrt{1+\frac{1}{x}}}{\sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}}\right) + \frac{3}{2}\,\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{-1+}{\sqrt{\sqrt{1}}}\right.$$

$$= \sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\,x + \frac{1}{4}\tan^{-1}\left(\frac{3+\sqrt{1+\frac{1}{x}}}{2\sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}}\right) - \frac{3}{4}\tanh^{-1}\left(\frac{1-3\sqrt{1+\frac{1}{x}}}{2\sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}}\right)$$

Mathematica [A] time = 0.168488, size = 98, normalized size = 1.02

$$\sqrt{\sqrt{\frac{1}{x}+1}+\frac{1}{x}}\,x - \frac{1}{4}\tan^{-1}\left(\frac{-\sqrt{\frac{1}{x}+1}-3}{2\sqrt{\sqrt{\frac{1}{x}+1}+\frac{1}{x}}}\right) + \frac{3}{4}\tanh^{-1}\left(\frac{3\sqrt{\frac{1}{x}+1}-1}{2\sqrt{\sqrt{\frac{1}{x}+1}+\frac{1}{x}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sqrt[1 + x^(-1)] + x^(-1)],x]
```

```
[Out] Sqrt[Sqrt[1 + x^(-1)] + x^(-1)]*x - ArcTan[(-3 - Sqrt[1 + x^(-1)])/(2*Sqrt[
Sqrt[1 + x^(-1)] + x^(-1)])]/4 + (3*ArcTanh[(-1 + 3*Sqrt[1 + x^(-1)])/(2*Sq
rt[Sqrt[1 + x^(-1)] + x^(-1)])])/4
```

Maple [F] time = 0.028, size = 0, normalized size = 0.

$$\int \sqrt{x^{-1}+\sqrt{1+x^{-1}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x+(1+1/x)^(1/2))^(1/2),x)
```

```
[Out] int((1/x+(1+1/x)^(1/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{\frac{1}{x}+1}+\frac{1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+(1+1/x)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sqrt(1/x + 1) + 1/x), x)
```

Fricas [A] time = 29.3198, size = 297, normalized size = 3.09

$$x\sqrt{\frac{x\sqrt{\frac{x+1}{x}}+1}{x}}+\frac{1}{4}\arctan\left(\frac{2\left(x\sqrt{\frac{x+1}{x}}-3x\right)\sqrt{\frac{x\sqrt{\frac{x+1}{x}}+1}{x}}}{8x-1}\right)+\frac{3}{4}\log\left(2\left(x\sqrt{\frac{x+1}{x}}+x\right)\sqrt{\frac{x\sqrt{\frac{x+1}{x}}+1}{x}}+2x\sqrt{\frac{x+1}{x}}+2x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+(1+1/x)^(1/2))^(1/2),x, algorithm="fricas")
```

```
[Out] x*sqrt((x*sqrt((x + 1)/x) + 1)/x) + 1/4*arctan(2*(x*sqrt((x + 1)/x) - 3*x)*
sqrt((x*sqrt((x + 1)/x) + 1)/x)/(8*x - 1)) + 3/4*log(2*(x*sqrt((x + 1)/x) +
 x)*sqrt((x*sqrt((x + 1)/x) + 1)/x) + 2*x*sqrt((x + 1)/x) + 2*x + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{1+\frac{1}{x}}+\frac{1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+(1+1/x)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(sqrt(1 + 1/x) + 1/x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sqrt{\frac{1}{x}+1}+\frac{1}{x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/x+(1+1/x)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sqrt(1/x + 1) + 1/x), x)
```

3.19 $\int \frac{\sqrt{1+e^{-x}}}{-e^{-x}+e^x}\,dx$

Optimal. Leaf size=25

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{e^{-x}+1}}{\sqrt{2}}\right)$$

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[1 + E^(-x)]/Sqrt[2]])
```

Rubi [A] time = 0.0749026, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {2282, 1446, 1469, 627, 63, 206}

$$-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{e^{-x}+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + E^(-x)]/(-E^(-x) + E^x),x]
```

```
[Out] -(Sqrt[2]*ArcTanh[Sqrt[1 + E^(-x)]/Sqrt[2]])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 1446

```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_S
ymbol] :> Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x] /; FreeQ[{a,
c, d, e, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 1469

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
```

```
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+e^{-x}}}{-e^{-x}+e^x}\,dx &= \text{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}}{-1+x^2}\,dx, x, e^x\right)\\
&= \text{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}}{\left(1-\frac{1}{x^2}\right)x^2}\,dx, x, e^x\right)\\
&= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{1-x^2}\,dx, x, e^{-x}\right)\\
&= -\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{1+x}}\,dx, x, e^{-x}\right)\\
&= -\left(2\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+e^{-x}}\right)\right)\\
&= -\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+e^{-x}}}{\sqrt{2}}\right)
\end{aligned}
$$

Mathematica [B] time = 0.104817, size = 112, normalized size = 4.48

$$
\frac{e^{x/2}\sqrt{e^{-x}+1}\left(\log\left(1-e^{x/2}\right)-\log\left(e^{x/2}+1\right)+\log\left(\sqrt{2}\sqrt{e^x+1}-e^{x/2}+1\right)-\log\left(\sqrt{2}\sqrt{e^x+1}+e^{x/2}+1\right)\right)}{\sqrt{2}\sqrt{e^x+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + E^(-x)]/(-E^(-x) + E^x),x]
```

```
[Out] (E^(x/2)*Sqrt[1 + E^(-x)]*(Log[1 - E^(x/2)] - Log[1 + E^(x/2)] + Log[1 - E^
(x/2) + Sqrt[2]*Sqrt[1 + E^x]] - Log[1 + E^(x/2) + Sqrt[2]*Sqrt[1 + E^x]]))
/(Sqrt[2]*Sqrt[1 + E^x])
```

Maple [B] time = 0.019, size = 49, normalized size = 2.

$$
-\frac{e^x\sqrt{2}}{2}\sqrt{\frac{1+e^x}{e^x}}\,\text{Artanh}\left(\frac{(1+3\,e^x)\sqrt{2}}{4}\frac{1}{\sqrt{(e^x)^2+e^x}}\right)\frac{1}{\sqrt{(1+e^x)\,e^x}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+exp(-x))^(1/2)/(-exp(-x)+exp(x)),x)
```

```
[Out] -1/2*((1+exp(x))/exp(x))^(1/2)*exp(x)/((1+exp(x))*exp(x))^(1/2)*2^(1/2)*arc
tanh(1/4*(1+3*exp(x))*2^(1/2)/(exp(x)^2+exp(x))^(1/2))
```

Maxima [A] time = 1.44735, size = 49, normalized size = 1.96

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{e^{(-x)}+1}}{\sqrt{2}+\sqrt{e^{(-x)}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(-x))^(1/2)/(-exp(-x)+exp(x)),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - sqrt(e^(-x) + 1))/(sqrt(2) + sqrt(e^(-x) + 1)))
```

Fricas [A] time = 2.1053, size = 103, normalized size = 4.12

$$\frac{1}{2}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{e^{x}+1}e^{\left(\frac{1}{2}x\right)}-3e^{x}-1}{e^{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(-x))^(1/2)/(-exp(-x)+exp(x)),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log((2*sqrt(2)*sqrt(e^x + 1)*e^(1/2*x) - 3*e^x - 1)/(e^x - 1))
```

Sympy [A] time = 3.72265, size = 65, normalized size = 2.6

$$2\left(\begin{cases}-\frac{\sqrt{2}\operatorname{acoth}\left(\frac{\sqrt{2}\sqrt{1+e^{-x}}}{2}\right)}{2} & \text{for}\ 1+e^{-x}>2\\ -\frac{\sqrt{2}\operatorname{atanh}\left(\frac{\sqrt{2}\sqrt{1+e^{-x}}}{2}\right)}{2} & \text{for}\ 1+e^{-x}<2\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(-x))**(1/2)/(-exp(-x)+exp(x)),x)
```

```
[Out] 2*Piecewise((-sqrt(2)*acoth(sqrt(2)*sqrt(1 + exp(-x))/2)/2, 1 + exp(-x) > 2
), (-sqrt(2)*atanh(sqrt(2)*sqrt(1 + exp(-x))/2)/2, 1 + exp(-x) < 2))
```

Giac [B] time = 1.55128, size = 101, normalized size = 4.04

$$-\frac{1}{2}\,\sqrt{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)+\frac{1}{2}\,\sqrt{2}\log\left(\frac{\left|-2\,\sqrt{2}+2\,\sqrt{e^{(2\,x)}+e^{x}}-2\,e^{x}+2\right|}{\left|2\,\sqrt{2}+2\,\sqrt{e^{(2\,x)}+e^{x}}-2\,e^{x}+2\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(-x))^(1/2)/(-exp(-x)+exp(x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log((sqrt(2) - 1)/(sqrt(2) + 1)) + 1/2*sqrt(2)*log(abs(-2*sqrt
(2) + 2*sqrt(e^(2*x) + e^x) - 2*e^x + 2)/abs(2*sqrt(2) + 2*sqrt(e^(2*x) + e
^x) - 2*e^x + 2))
```

3.20 $\int \sqrt{1+e^{-x}}\,\mathrm{csch}(x)\,dx$

Optimal. Leaf size=25

$$-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{e^{-x}+1}}{\sqrt{2}}\right)$$

```
[Out] -2*Sqrt[2]*ArcTanh[Sqrt[1 + E^(-x)]/Sqrt[2]]
```

Rubi [A] time = 0.0510541, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2282, 12, 1446, 1469, 627, 63, 206}

$$-2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{e^{-x}+1}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + E^(-x)]*Csch[x],x]
```

```
[Out] -2*Sqrt[2]*ArcTanh[Sqrt[1 + E^(-x)]/Sqrt[2]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1446

```
Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_S
ymbol] :> Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x] /; FreeQ[{a,
c, d, e, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]
```

Rule 1469

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n
], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify
[m - n + 1], 0]
```

Rule 627

```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int
[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && Intege
rQ[m + p]))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1+e^{-x}}\,\mathrm{csch}(x)\,dx &= \mathrm{Subst}\left(\int \frac{2\sqrt{1+\frac{1}{x}}}{-1+x^2}\,dx, x, e^x\right)\\
&= 2\,\mathrm{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}}{-1+x^2}\,dx, x, e^x\right)\\
&= 2\,\mathrm{Subst}\left(\int \frac{\sqrt{1+\frac{1}{x}}}{\left(1-\frac{1}{x^2}\right)x^2}\,dx, x, e^x\right)\\
&= -\left(2\,\mathrm{Subst}\left(\int \frac{\sqrt{1+x}}{1-x^2}\,dx, x, e^{-x}\right)\right)\\
&= -\left(2\,\mathrm{Subst}\left(\int \frac{1}{(1-x)\sqrt{1+x}}\,dx, x, e^{-x}\right)\right)\\
&= -\left(4\,\mathrm{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+e^{-x}}\right)\right)\\
&= -2\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1+e^{-x}}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [B] time = 0.0949289, size = 126, normalized size = 5.04

$$\frac{\sqrt{2}e^{x/2}\sqrt{e^{-x}+1}\left(\log\left(1-e^{-x/2}\right)+\log\left(e^{-x/2}+1\right)-\log\left(e^{-x/2}\left(\sqrt{2}\sqrt{e^x+1}+e^{x/2}-1\right)\right)-\log\left(e^{-x/2}\left(\sqrt{2}\sqrt{e^x+1}+e^{x/2}+1\right)\right)\right)}{\sqrt{e^x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + E^(-x)]*Csch[x],x]
```

```
[Out] (Sqrt[2]*E^(x/2)*Sqrt[1 + E^(-x)]*(Log[1 - E^(-x/2)] + Log[1 + E^(-x/2)] -
Log[(-1 + E^(x/2) + Sqrt[2]*Sqrt[1 + E^x])/E^(x/2)] - Log[(1 + E^(x/2) + Sq
rt[2]*Sqrt[1 + E^x])/E^(x/2)]))/Sqrt[1 + E^x]
```

Maple [A] time = 0.071, size = 33, normalized size = 1.3

$$-2\sqrt{2}\sqrt{\left(\tanh\left(x/2\right)+1\right)^{-1}}\sqrt{\tanh\left(x/2\right)+1}\,\mathrm{Artanh}\left(\sqrt{\tanh\left(x/2\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+exp(-x))^(1/2)/sinh(x),x)
```

```
[Out] -2*2^(1/2)*(1/(tanh(1/2*x)+1))^(1/2)*(tanh(1/2*x)+1)^(1/2)*arctanh((tanh(1/
2*x)+1)^(1/2))
```

Maxima [A] time = 1.4304, size = 47, normalized size = 1.88

$$\sqrt{2}\log\left(-\frac{\sqrt{2}-\sqrt{e^{(-x)}+1}}{\sqrt{2}+\sqrt{e^{(-x)}+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(-x))^(1/2)/sinh(x),x, algorithm="maxima")
```

```
[Out] sqrt(2)*log(-(sqrt(2) - sqrt(e^(-x) + 1))/(sqrt(2) + sqrt(e^(-x) + 1)))
```

Fricas [B] time = 2.23759, size = 211, normalized size = 8.44

$$\sqrt{2}\log\left(\frac{2\left(\sqrt{2}\cosh\left(x\right)+\sqrt{2}\sinh\left(x\right)\right)\sqrt{\frac{\cosh(x)+\sinh(x)+1}{\cosh(x)+\sinh(x)}}-3\cosh\left(x\right)-3\sinh\left(x\right)-1}{\cosh\left(x\right)+\sinh\left(x\right)-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(-x))^(1/2)/sinh(x),x, algorithm="fricas")
```

```
[Out] sqrt(2)*log((2*(sqrt(2)*cosh(x) + sqrt(2)*sinh(x))*sqrt((cosh(x) + sinh(x)
+ 1)/(cosh(x) + sinh(x))) - 3*cosh(x) - 3*sinh(x) - 1)/(cosh(x) + sinh(x) -
 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{1+e^{-x}}}{\sinh\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(-x))**(1/2)/sinh(x),x)
```

```
[Out] Integral(sqrt(1 + exp(-x))/sinh(x), x)
```

Giac [B] time = 1.26869, size = 100, normalized size = 4.

$$-\sqrt{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)+\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+2\sqrt{e^{(2x)}+e^{x}}-2e^{x}+2\right|}{\left|2\sqrt{2}+2\sqrt{e^{(2x)}+e^{x}}-2e^{x}+2\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+exp(-x))^(1/2)/sinh(x),x, algorithm="giac")
```

```
[Out] -sqrt(2)*log((sqrt(2) - 1)/(sqrt(2) + 1)) + sqrt(2)*log(abs(-2*sqrt(2) + 2*
sqrt(e^(2*x) + e^x) - 2*e^x + 2)/abs(2*sqrt(2) + 2*sqrt(e^(2*x) + e^x) - 2*
e^x + 2))
```

3.21 $\int \frac{1}{(\cos(x)+\cos(3x))^5}\,dx$

Optimal. Leaf size=108

$$-\frac{437\sin(x)}{512\left(1-2\sin^2(x)\right)}+\frac{203\sin(x)}{768\left(1-2\sin^2(x)\right)^2}-\frac{17\sin(x)}{192\left(1-2\sin^2(x)\right)^3}+\frac{\sin(x)}{32\left(1-2\sin^2(x)\right)^4}-\frac{523}{256}\tanh^{-1}(\sin(x))+\frac{1483\,t}{}$$

```
[Out] (-523*ArcTanh[Sin[x]])/256 + (1483*ArcTanh[Sqrt[2]*Sin[x]])/(512*Sqrt[2]) +
 Sin[x]/(32*(1 - 2*Sin[x]^2)^4) - (17*Sin[x])/(192*(1 - 2*Sin[x]^2)^3) + (2
03*Sin[x])/(768*(1 - 2*Sin[x]^2)^2) - (437*Sin[x])/(512*(1 - 2*Sin[x]^2)) -
 (43*Sec[x]*Tan[x])/256 - (Sec[x]^3*Tan[x])/128
```

Rubi [B] time = 1.11659, antiderivative size = 786, normalized size of antiderivative = 7.28, number of steps used = 45, number of rules used = 7, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {12, 2073, 207, 638, 614, 618, 206}

$$\frac{451\left(\tan\left(\frac{x}{2}\right)+1\right)}{512\left(-\tan^2\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)+1\right)}-\frac{15\tan\left(\frac{x}{2}\right)+89}{64\left(-\tan^2\left(\frac{x}{2}\right)-2\tan\left(\frac{x}{2}\right)+1\right)}+\frac{89-15\tan\left(\frac{x}{2}\right)}{64\left(-\tan^2\left(\frac{x}{2}\right)+2\tan\left(\frac{x}{2}\right)+1\right)}-\frac{451\,(1}{512\left(-\tan^2\left(\frac{x}{2}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Cos[x] + Cos[3*x])^(-5),x]
```

```
[Out] (-523*ArcTanh[Sin[x]])/256 - (1483*Log[2 + Sqrt[2] + Cos[x] + Sqrt[2]*Cos[x
] - Sin[x] - Sqrt[2]*Sin[x]])/(2048*Sqrt[2]) - (1483*Log[2 - Sqrt[2] + Cos[
x] - Sqrt[2]*Cos[x] + Sin[x] - Sqrt[2]*Sin[x]])/(2048*Sqrt[2]) + (1483*Log[
2 - Sqrt[2] + Cos[x] - Sqrt[2]*Cos[x] - Sin[x] + Sqrt[2]*Sin[x]])/(2048*Sqr
t[2]) + (1483*Log[2 + Sqrt[2] + Cos[x] + Sqrt[2]*Cos[x] + Sin[x] + Sqrt[2]*
Sin[x]])/(2048*Sqrt[2]) - 1/(128*(1 - Tan[x/2])^4) + 1/(64*(1 - Tan[x/2])^3
) - 47/(256*(1 - Tan[x/2])^2) + 45/(256*(1 - Tan[x/2])) + 1/(128*(1 + Tan[x
/2])^4) - 1/(64*(1 + Tan[x/2])^3) + 47/(256*(1 + Tan[x/2])^2) - 45/(256*(1
+ Tan[x/2])) - (7 - 17*Tan[x/2])/(4*(1 - 2*Tan[x/2] - Tan[x/2]^2)^4) + (119
*(1 + Tan[x/2]))/(48*(1 - 2*Tan[x/2] - Tan[x/2]^2)^3) - (11*(1 + 3*Tan[x/2]
))/(12*(1 - 2*Tan[x/2] - Tan[x/2]^2)^3) - (1 - 43*Tan[x/2])/(32*(1 - 2*Tan[
x/2] - Tan[x/2]^2)^2) - (65*(1 + Tan[x/2]))/(384*(1 - 2*Tan[x/2] - Tan[x/2]
^2)^2) + (451*(1 + Tan[x/2]))/(512*(1 - 2*Tan[x/2] - Tan[x/2]^2)) - (89 + 1
5*Tan[x/2])/(64*(1 - 2*Tan[x/2] - Tan[x/2]^2)) + (7 + 17*Tan[x/2])/(4*(1 +
2*Tan[x/2] - Tan[x/2]^2)^4) + (11*(1 - 3*Tan[x/2]))/(12*(1 + 2*Tan[x/2] - T
an[x/2]^2)^3) - (119*(1 - Tan[x/2]))/(48*(1 + 2*Tan[x/2] - Tan[x/2]^2)^3) +
 (65*(1 - Tan[x/2]))/(384*(1 + 2*Tan[x/2] - Tan[x/2]^2)^2) + (1 + 43*Tan[x/
```

```
2])/(32*(1 + 2*Tan[x/2] - Tan[x/2]^2)^2) + (89 - 15*Tan[x/2])/(64*(1 + 2*Ta
n[x/2] - Tan[x/2]^2)) - (451*(1 - Tan[x/2]))/(512*(1 + 2*Tan[x/2] - Tan[x/2
]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2073

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P /. x -> Sqrt[x]]
}, Int[ExpandIntegrand[(PP /. x -> x^2)^p*Q^q, x], x] /;  !SumQ[NonfreeFact
ors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x^2] && PolyQ[Q, x] && ILtQ[p, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(\cos(x)+\cos(3x))^5}\,dx = 2\,\text{Subst}\left(\int \frac{\left(1+x^2\right)^{14}}{32\left(1-7x^2+7x^4-x^6\right)^5}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$

$$= \frac{1}{16}\,\text{Subst}\left(\int \frac{\left(1+x^2\right)^{14}}{\left(1-7x^2+7x^4-x^6\right)^5}\,dx, x, \tan\left(\frac{x}{2}\right)\right)$$

$$= \frac{1}{16}\,\text{Subst}\left(\int \left(\frac{1}{2(-1+x)^5}+\frac{3}{4(-1+x)^4}+\frac{47}{8(-1+x)^3}+\frac{45}{16(-1+x)^2}-\frac{1}{2(1+x)^5}+\frac{3}{4(1+\ldots}\right.\right.$$

$$= -\frac{1}{128\left(1-\tan\left(\frac{x}{2}\right)\right)^4}+\frac{1}{64\left(1-\tan\left(\frac{x}{2}\right)\right)^3}-\frac{47}{256\left(1-\tan\left(\frac{x}{2}\right)\right)^2}+\frac{45}{256\left(1-\tan\left(\frac{x}{2}\right)\right)}+\frac{\ldots}{128\ldots}$$

$$= -\frac{523}{256}\tanh^{-1}(\sin(x))-\frac{1}{128\left(1-\tan\left(\frac{x}{2}\right)\right)^4}+\frac{1}{64\left(1-\tan\left(\frac{x}{2}\right)\right)^3}-\frac{47}{256\left(1-\tan\left(\frac{x}{2}\right)\right)^2}+\frac{\ldots}{25\ldots}$$

$$= -\frac{523}{256}\tanh^{-1}(\sin(x))-\frac{9\log\left(2+\sqrt{2}+\cos(x)+\sqrt{2}\cos(x)-\sin(x)-\sqrt{2}\sin(x)\right)}{16\sqrt{2}}-\frac{9\log\ldots}{\ldots}$$

$$= -\frac{523}{256}\tanh^{-1}(\sin(x))-\frac{129\log\left(2+\sqrt{2}+\cos(x)+\sqrt{2}\cos(x)-\sin(x)-\sqrt{2}\sin(x)\right)}{256\sqrt{2}}-\frac{129\ldots}{\ldots}$$

$$= -\frac{523}{256}\tanh^{-1}(\sin(x))-\frac{387\log\left(2+\sqrt{2}+\cos(x)+\sqrt{2}\cos(x)-\sin(x)-\sqrt{2}\sin(x)\right)}{512\sqrt{2}}-\frac{387\ldots}{\ldots}$$

$$= -\frac{523}{256}\tanh^{-1}(\sin(x))-\frac{111\log\left(2+\sqrt{2}+\cos(x)+\sqrt{2}\cos(x)-\sin(x)-\sqrt{2}\sin(x)\right)}{256\sqrt{2}}-\frac{111\ldots}{\ldots}$$

$$= -\frac{523}{256}\tanh^{-1}(\sin(x))-\frac{1483\log\left(2+\sqrt{2}+\cos(x)+\sqrt{2}\cos(x)-\sin(x)-\sqrt{2}\sin(x)\right)}{2048\sqrt{2}}-\frac{14\ldots}{\ldots}$$

Mathematica [C] time = 6.29602, size = 478, normalized size = 4.43

$$\frac{1483\log\left(2\sin(x)+\sqrt{2}\right)}{1024\sqrt{2}}+\frac{83\sin(x)}{512(\cos(x)-\sin(x))^2}+\frac{\sin(x)}{128(\cos(x)-\sin(x))^4}-\frac{437}{1024(\cos(x)-\sin(x))}+\frac{437}{1024(\sin(x)+\cos}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[x] + Cos[3*x])^(-5),x]
```

```
[Out] (((-1483*I)/1024)*ArcTan[(Cos[x/2] - Sin[x/2] - Sqrt[2]*Sin[x/2])/(-Cos[x/2
] + Sqrt[2]*Cos[x/2] - Sin[x/2])])/Sqrt[2] + ((1483/2048 + (1483*I)/2048)*(
(-1 - I) + Sqrt[2])*ArcTan[(Cos[x/2] + Sin[x/2] - Sqrt[2]*Sin[x/2])/(Cos[x/
2] + Sqrt[2]*Cos[x/2] - Sin[x/2])])/((-1 + I) + Sqrt[2]) + (523*Log[Cos[x/2
] - Sin[x/2]])/256 - (523*Log[Cos[x/2] + Sin[x/2]])/256 + (1483*Log[Sqrt[2]
 + 2*Sin[x]])/(1024*Sqrt[2]) - (1483*Log[2 - Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x
]])/(2048*Sqrt[2]) + ((1483/4096 - (1483*I)/4096)*((-1 - I) + Sqrt[2])*Log[
2 + Sqrt[2]*Cos[x] - Sqrt[2]*Sin[x]])/((-1 + I) + Sqrt[2]) - 1/(512*(Cos[x/
2] - Sin[x/2])^4) - 43/(512*(Cos[x/2] - Sin[x/2])^2) + 1/(512*(Cos[x/2] + S
in[x/2])^4) + 43/(512*(Cos[x/2] + Sin[x/2])^2) - 17/(768*(Cos[x] - Sin[x])^
3) - 437/(1024*(Cos[x] - Sin[x])) + Sin[x]/(128*(Cos[x] - Sin[x])^4) + (83*
Sin[x])/(512*(Cos[x] - Sin[x])^2) + Sin[x]/(128*(Cos[x] + Sin[x])^4) + 17/(
768*(Cos[x] + Sin[x])^3) + (83*Sin[x])/(512*(Cos[x] + Sin[x])^2) + 437/(102
4*(Cos[x] + Sin[x]))
```

Maple [A] time = 0.062, size = 95, normalized size = 0.9

$$-4\,\frac{1}{\left(2\,(\sin(x))^2-1\right)^4}\left(-\frac{437\,(\sin(x))^7}{256}+\frac{3527\,(\sin(x))^5}{1536}-\frac{3257\,(\sin(x))^3}{3072}+\frac{331\,\sin(x)}{2048}\right)+\frac{1483\,\mathrm{Artanh}\left(\sin(x)\sqrt{2}\right)}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)+cos(3*x))^5,x)
```

```
[Out] -4*(-437/256*sin(x)^7+3527/1536*sin(x)^5-3257/3072*sin(x)^3+331/2048*sin(x)
)/(2*sin(x)^2-1)^4+1483/1024*arctanh(sin(x)*2^(1/2))*2^(1/2)-1/512/(-1+sin(
x))^2+43/512/(-1+sin(x))+523/512*ln(-1+sin(x))+1/512/(1+sin(x))^2+43/512/(1
+sin(x))-523/512*ln(1+sin(x))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+cos(3*x))^5,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 2.92037, size = 718, normalized size = 6.65

$$4449\left(16\sqrt{2}\cos(x)^{12} - 32\sqrt{2}\cos(x)^{10} + 24\sqrt{2}\cos(x)^{8} - 8\sqrt{2}\cos(x)^{6} + \sqrt{2}\cos(x)^{4}\right)\log\left(-\frac{2\cos(x)^{2} - 2\sqrt{2}\sin(x) - 3}{2\cos(x)^{2} - 1}\right) - 62$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+cos(3*x))^5,x, algorithm="fricas")
```

```
[Out] 1/6144*(4449*(16*sqrt(2)*cos(x)^12 - 32*sqrt(2)*cos(x)^10 + 24*sqrt(2)*cos(
x)^8 - 8*sqrt(2)*cos(x)^6 + sqrt(2)*cos(x)^4)*log(-(2*cos(x)^2 - 2*sqrt(2)*
sin(x) - 3)/(2*cos(x)^2 - 1)) - 6276*(16*cos(x)^12 - 32*cos(x)^10 + 24*cos(
x)^8 - 8*cos(x)^6 + cos(x)^4)*log(sin(x) + 1) + 6276*(16*cos(x)^12 - 32*cos
(x)^10 + 24*cos(x)^8 - 8*cos(x)^6 + cos(x)^4)*log(-sin(x) + 1) - 4*(14616*c
os(x)^10 - 25420*cos(x)^8 + 15570*cos(x)^6 - 3677*cos(x)^4 + 162*cos(x)^2 +
 12)*sin(x))/(16*cos(x)^12 - 32*cos(x)^10 + 24*cos(x)^8 - 8*cos(x)^6 + cos(
x)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+cos(3*x))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11205, size = 140, normalized size = 1.3

$$-\frac{1483}{2048}\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(x\right)\right|}{\left|2\sqrt{2}+4\sin\left(x\right)\right|}\right)+\frac{43\sin\left(x\right)^{3}-45\sin\left(x\right)}{256\left(\sin\left(x\right)^{2}-1\right)^{2}}+\frac{10488\sin\left(x\right)^{7}-14108\sin\left(x\right)^{5}+6514\sin\left(x\right)^{3}-993\ldots}{1536\left(2\sin\left(x\right)^{2}-1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+cos(3*x))^5,x, algorithm="giac")
```

```
[Out] -1483/2048*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(x))/abs(2*sqrt(2) + 4*sin(x))
) + 1/256*(43*sin(x)^3 - 45*sin(x))/(sin(x)^2 - 1)^2 + 1/1536*(10488*sin(x)
^7 - 14108*sin(x)^5 + 6514*sin(x)^3 - 993*sin(x))/(2*sin(x)^2 - 1)^4 - 523/
512*log(sin(x) + 1) + 523/512*log(-sin(x) + 1)
```

3.22 $\int \frac{1}{(1+\cos(x)+\sin(x))^2}\,dx$

Optimal. Leaf size=29

$$-\log\left(\tan\left(\frac{x}{2}\right)+1\right)-\frac{\cos(x)-\sin(x)}{\sin(x)+\cos(x)+1}$$

```
[Out] -Log[1 + Tan[x/2]] - (Cos[x] - Sin[x])/(1 + Cos[x] + Sin[x])
```

Rubi [A] time = 0.0201917, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3129, 3124, 31}

$$-\log\left(\tan\left(\frac{x}{2}\right)+1\right)-\frac{\cos(x)-\sin(x)}{\sin(x)+\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Cos[x] + Sin[x])^(-2),x]
```

```
[Out] -Log[1 + Tan[x/2]] - (Cos[x] - Sin[x])/(1 + Cos[x] + Sin[x])
```

Rule 3129

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(n_), x_Symbol] :> Simp[((-(c*Cos[d + e*x]) + b*Sin[d + e*x])*(a + b*Cos[d
+ e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[
1/((n + 1)*(a^2 - b^2 - c^2)), Int[(a*(n + 1) - b*(n + 2)*Cos[d + e*x] - c*
(n + 2)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0] && LtQ[n, -1] && N
eQ[n, -3/2]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+\cos(x)+\sin(x))^2}\,dx &= -\frac{\cos(x)-\sin(x)}{1+\cos(x)+\sin(x)}-\int \frac{1}{1+\cos(x)+\sin(x)}\,dx\\ &= -\frac{\cos(x)-\sin(x)}{1+\cos(x)+\sin(x)}-2\,\mathrm{Subst}\left(\int \frac{1}{2+2x}\,dx,x,\tan\left(\frac{x}{2}\right)\right)\\ &= -\log\left(1+\tan\left(\frac{x}{2}\right)\right)-\frac{\cos(x)-\sin(x)}{1+\cos(x)+\sin(x)}\end{aligned}$$

Mathematica [A] time = 0.0327142, size = 56, normalized size = 1.93

$$\frac{1}{2}\tan\left(\frac{x}{2}\right)+\log\left(\cos\left(\frac{x}{2}\right)\right)+\frac{\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Cos[x] + Sin[x])^(-2),x]
```

```
[Out] Log[Cos[x/2]] - Log[Cos[x/2] + Sin[x/2]] + Sin[x/2]/(Cos[x/2] + Sin[x/2]) +
 Tan[x/2]/2
```

Maple [A] time = 0.039, size = 27, normalized size = 0.9

$$\frac{1}{2}\tan\left(\frac{x}{2}\right)-\left(1+\tan\left(\frac{x}{2}\right)\right)^{-1}-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+cos(x)+sin(x))^2,x)
```

```
[Out] 1/2*tan(1/2*x)-1/(1+tan(1/2*x))-ln(1+tan(1/2*x))
```

Maxima [A] time = 1.30164, size = 54, normalized size = 1.86

$$\frac{\sin(x)}{2(\cos(x)+1)} - \frac{1}{\frac{\sin(x)}{\cos(x)+1}+1} - \log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)+sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*sin(x)/(cos(x) + 1) - 1/(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(cos(x)
+ 1) + 1)
```

Fricas [A] time = 2.29235, size = 182, normalized size = 6.28

$$\frac{(\cos(x)+\sin(x)+1)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)-(\cos(x)+\sin(x)+1)\log(\sin(x)+1)-2\cos(x)+2\sin(x)}{2(\cos(x)+\sin(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)+sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((cos(x) + sin(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + sin(x) + 1)*lo
g(sin(x) + 1) - 2*cos(x) + 2*sin(x))/(cos(x) + sin(x) + 1)
```

Sympy [B] time = 0.785549, size = 66, normalized size = 2.28

$$-\frac{2\log\left(\tan\left(\frac{x}{2}\right)+1\right)\tan\left(\frac{x}{2}\right)}{2\tan\left(\frac{x}{2}\right)+2} - \frac{2\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{2\tan\left(\frac{x}{2}\right)+2} + \frac{\tan^2\left(\frac{x}{2}\right)}{2\tan\left(\frac{x}{2}\right)+2} - \frac{3}{2\tan\left(\frac{x}{2}\right)+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)+sin(x))**2,x)
```

```
[Out] -2*log(tan(x/2) + 1)*tan(x/2)/(2*tan(x/2) + 2) - 2*log(tan(x/2) + 1)/(2*tan
(x/2) + 2) + tan(x/2)**2/(2*tan(x/2) + 2) - 3/(2*tan(x/2) + 2)
```

Giac [A] time = 1.12091, size = 41, normalized size = 1.41

$$\frac{\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)+1} - \log\left(\left|\tan\left(\frac{1}{2}x\right)+1\right|\right) + \frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+cos(x)+sin(x))^2,x, algorithm="giac")
```

```
[Out] tan(1/2*x)/(tan(1/2*x) + 1) - log(abs(tan(1/2*x) + 1)) + 1/2*tan(1/2*x)
```

3.23 $\int \sqrt{1+\tanh(4x)}\,dx$

Optimal. Leaf size=26

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(4x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] ArcTanh[Sqrt[1 + Tanh[4*x]]/Sqrt[2]]/(2*Sqrt[2])
```

Rubi [A] time = 0.0134693, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3480, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(4x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + Tanh[4*x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[4*x]]/Sqrt[2]]/(2*Sqrt[2])
```

Rule 3480

```
Int[Sqrt[(a_) + (b_.)*tan[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[(-2*b)/d,
 Subst[Int[1/(2*a - x^2), x], x, Sqrt[a + b*Tan[c + d*x]]], x] /; FreeQ[{a,
 b, c, d}, x] && EqQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{1+\tanh(4x)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \sqrt{1+\tanh(4x)}\right)$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{1+\tanh(4x)}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Mathematica [A] time = 0.0167125, size = 26, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\tanh(4x)+1}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Tanh[4*x]],x]
```

```
[Out] ArcTanh[Sqrt[1 + Tanh[4*x]]/Sqrt[2]]/(2*Sqrt[2])
```

Maple [A] time = 0.025, size = 20, normalized size = 0.8

$$\frac{\sqrt{2}}{4}\text{Artanh}\left(\frac{\sqrt{2}}{2}\sqrt{1+\tanh(4x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+tanh(4*x))^(1/2),x)
```

```
[Out] 1/4*arctanh(1/2*(1+tanh(4*x))^(1/2)*2^(1/2))*2^(1/2)
```

Maxima [B] time = 1.82197, size = 58, normalized size = 2.23

$$-\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sqrt{2}}{\sqrt{e^{(-8x)}+1}}}{\sqrt{2}+\frac{\sqrt{2}}{\sqrt{e^{(-8x)}+1}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(4*x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*sqrt(2)*log(-(sqrt(2) - sqrt(2)/sqrt(e^(-8*x) + 1))/(sqrt(2) + sqrt(2)
/sqrt(e^(-8*x) + 1)))
```

Fricas [B] time = 2.20844, size = 207, normalized size = 7.96

$$\frac{1}{8}\sqrt{2}\log\left(-2\sqrt{2}\sqrt{\frac{\cosh(4x)}{\cosh(4x)-\sinh(4x)}}(\cosh(4x)+\sinh(4x))-2\cosh(4x)^2-4\cosh(4x)\sinh(4x)-2\sinh(4x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(4*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*log(-2*sqrt(2)*sqrt(cosh(4*x)/(cosh(4*x) - sinh(4*x)))*(cosh(4*
x) + sinh(4*x)) - 2*cosh(4*x)^2 - 4*cosh(4*x)*sinh(4*x) - 2*sinh(4*x)^2 - 1
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\tanh(4x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(4*x))**(1/2),x)
```

```
[Out] Integral(sqrt(tanh(4*x) + 1), x)
```

Giac [A] time = 1.08865, size = 41, normalized size = 1.58

$$\frac{1}{8}\sqrt{2}\left(\log\left(\sqrt{e^{(-8x)}+1}+1\right)-\log\left(\sqrt{e^{(-8x)}+1}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+tanh(4*x))^(1/2),x, algorithm="giac")

[Out] 1/8*sqrt(2)*(log(sqrt(e^(-8*x) + 1) + 1) - log(sqrt(e^(-8*x) + 1) - 1))
```

3.24 $\int \frac{\tanh(x)}{\sqrt{e^x+e^{2x}}}\,dx$

Optimal. Leaf size=110

$$2e^{-x}\sqrt{e^x+e^{2x}}-\frac{\tan^{-1}\left(\frac{i-(1-2i)e^x}{2\sqrt{1+i}\sqrt{e^x+e^{2x}}}\right)}{\sqrt{1+i}}+\frac{\tan^{-1}\left(\frac{(1+2i)e^x+i}{2\sqrt{1-i}\sqrt{e^x+e^{2x}}}\right)}{\sqrt{1-i}}$$

```
[Out] (2*Sqrt[E^x + E^(2*x)])/E^x - ArcTan[(I - (1 - 2*I)*E^x)/(2*Sqrt[1 + I]*Sqr
t[E^x + E^(2*x)])]/Sqrt[1 + I] + ArcTan[(I + (1 + 2*I)*E^x)/(2*Sqrt[1 - I]*
Sqrt[E^x + E^(2*x)])]/Sqrt[1 - I]
```

Rubi [A] time = 0.606929, antiderivative size = 147, normalized size of antiderivative = 1.34, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {2282, 6724, 1586, 6725, 94, 93, 208}

$$\frac{2\left(e^x+1\right)}{\sqrt{e^x+e^{2x}}}-\frac{(1-i)^{3/2}\sqrt{e^x}\sqrt{e^x+1}\tanh^{-1}\left(\frac{\sqrt{1-i}\sqrt{e^x}}{\sqrt{e^x+1}}\right)}{\sqrt{e^x+e^{2x}}}-\frac{(1+i)^{3/2}\sqrt{e^x}\sqrt{e^x+1}\tanh^{-1}\left(\frac{\sqrt{1+i}\sqrt{e^x}}{\sqrt{e^x+1}}\right)}{\sqrt{e^x+e^{2x}}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[x]/Sqrt[E^x + E^(2*x)],x]
```

```
[Out] (2*(1 + E^x))/Sqrt[E^x + E^(2*x)] - ((1 - I)^(3/2)*Sqrt[E^x]*Sqrt[1 + E^x]*
ArcTanh[(Sqrt[1 - I]*Sqrt[E^x])/Sqrt[1 + E^x]])/Sqrt[E^x + E^(2*x)] - ((1 +
 I)^(3/2)*Sqrt[E^x]*Sqrt[1 + E^x]*ArcTanh[(Sqrt[1 + I]*Sqrt[E^x])/Sqrt[1 +
E^x]])/Sqrt[E^x + E^(2*x)]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6724

```
Int[(u_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(m_), x_Symbol] :> With[{v
= (a*x^r + b*x^s)^FracPart[m]/(x^(r*FracPart[m])*(a + b*x^(s - r))^FracPart
[m])}, Dist[v, Int[u*x^(m*r)*(a + b*x^(s - r))^m, x], x] /; NeQ[Simplify[v]
```

```
, 1]] /; FreeQ[{a, b, m, r, s}, x] &&  !IntegerQ[m] && PosQ[s - r]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh(x)}{\sqrt{e^x+e^{2x}}}\,dx = \text{Subst}\left(\int \frac{-1+x^2}{x\left(1+x^2\right)\sqrt{x+x^2}}\,dx, x, e^x\right)$$

$$= \frac{\left(\sqrt{e^x}\sqrt{1+e^x}\right)\text{Subst}\left(\int \frac{-1+x^2}{x^{3/2}\sqrt{1+x}\left(1+x^2\right)}\,dx, x, e^x\right)}{\sqrt{e^x+e^{2x}}}$$

$$= \frac{\left(\sqrt{e^x}\sqrt{1+e^x}\right)\text{Subst}\left(\int \frac{(-1+x)\sqrt{1+x}}{x^{3/2}\left(1+x^2\right)}\,dx, x, e^x\right)}{\sqrt{e^x+e^{2x}}}$$

$$= \frac{\left(\sqrt{e^x}\sqrt{1+e^x}\right)\text{Subst}\left(\int \left(-\frac{\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{1+x}}{(i-x)x^{3/2}}+\frac{\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{1+x}}{x^{3/2}(i+x)}\right)dx, x, e^x\right)}{\sqrt{e^x+e^{2x}}}$$

$$= -\frac{\left(\left(\frac{1}{2}+\frac{i}{2}\right)\sqrt{e^x}\sqrt{1+e^x}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{(i-x)x^{3/2}}\,dx, x, e^x\right)}{\sqrt{e^x+e^{2x}}} + \frac{\left(\left(\frac{1}{2}-\frac{i}{2}\right)\sqrt{e^x}\sqrt{1+e^x}\right)\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^{3/2}(i+x)}\,dx, x, e\right.}{\sqrt{e^x+e^{2x}}}$$

$$= \frac{2\left(1+e^x\right)}{\sqrt{e^x+e^{2x}}} - \frac{\left(\sqrt{e^x}\sqrt{1+e^x}\right)\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{x}\sqrt{1+x}}\,dx, x, e^x\right)}{\sqrt{e^x+e^{2x}}} + \frac{\left(\sqrt{e^x}\sqrt{1+e^x}\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}(i+x)\sqrt{1+x}}\,dx, x\right.}{\sqrt{e^x+e^{2x}}}$$

$$= \frac{2\left(1+e^x\right)}{\sqrt{e^x+e^{2x}}} - \frac{\left(2\sqrt{e^x}\sqrt{1+e^x}\right)\text{Subst}\left(\int \frac{1}{i-(1+i)x^2}\,dx, x, \frac{\sqrt{e^x}}{\sqrt{1+e^x}}\right)}{\sqrt{e^x+e^{2x}}} + \frac{\left(2\sqrt{e^x}\sqrt{1+e^x}\right)\text{Subst}\left(\int \frac{1}{i+(1-i)x^2}\,dx, x\right.}{\sqrt{e^x+e^{2x}}}$$

$$= \frac{2\left(1+e^x\right)}{\sqrt{e^x+e^{2x}}} - \frac{(1-i)^{3/2}\sqrt{e^x}\sqrt{1+e^x}\tanh^{-1}\left(\frac{\sqrt{1-i}\sqrt{e^x}}{\sqrt{1+e^x}}\right)}{\sqrt{e^x+e^{2x}}} - \frac{(1+i)^{3/2}\sqrt{e^x}\sqrt{1+e^x}\tanh^{-1}\left(\frac{\sqrt{1+i}\sqrt{e^x}}{\sqrt{1+e^x}}\right)}{\sqrt{e^x+e^{2x}}}$$

Mathematica [A] time = 0.118181, size = 121, normalized size = 1.1

$$\frac{2e^x-(1-i)^{3/2}e^{x/2}\sqrt{e^x+1}\tanh^{-1}\left(\frac{\sqrt{1-i}e^{x/2}}{\sqrt{e^x+1}}\right)-(1+i)^{3/2}e^{x/2}\sqrt{e^x+1}\tanh^{-1}\left(\frac{\sqrt{1+i}e^{x/2}}{\sqrt{e^x+1}}\right)+2}{\sqrt{e^x\left(e^x+1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/Sqrt[E^x + E^(2*x)],x]
```

```
[Out] (2 + 2*E^x - (1 - I)^(3/2)*E^(x/2)*Sqrt[1 + E^x]*ArcTanh[(Sqrt[1 - I]*E^(x/
2))/Sqrt[1 + E^x]] - (1 + I)^(3/2)*E^(x/2)*Sqrt[1 + E^x]*ArcTanh[(Sqrt[1 +
I]*E^(x/2))/Sqrt[1 + E^x]])/Sqrt[E^x*(1 + E^x)]
```

Maple [B] time = 0.113, size = 366, normalized size = 3.3

$$-\frac{\sqrt{2}}{4\sqrt{-2+2\sqrt{2}}}\left(\sqrt{\tanh\left(\frac{x}{2}\right)+1}\sqrt{2+2\sqrt{2}}\ln\left(\tanh\left(\frac{x}{2}\right)+1-\sqrt{\tanh\left(\frac{x}{2}\right)+1}\sqrt{2+2\sqrt{2}}+\sqrt{2}\right)\sqrt{-2+2\sqrt{2}}\sqrt{2}-\sqrt{t}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(exp(x)+exp(2*x))^(1/2),x)
```

```
[Out] -1/4*2^(1/2)*((tanh(1/2*x)+1)^(1/2)*(2+2*2^(1/2))^(1/2)*ln(tanh(1/2*x)+1-(t
anh(1/2*x)+1)^(1/2)*(2+2*2^(1/2))^(1/2)+2^(1/2))*(-2+2*2^(1/2))^(1/2)*2^(1/
2)-(tanh(1/2*x)+1)^(1/2)*(2+2*2^(1/2))^(1/2)*ln(tanh(1/2*x)+1+(tanh(1/2*x)+
1)^(1/2)*(2+2*2^(1/2))^(1/2)+2^(1/2))*(-2+2*2^(1/2))^(1/2)*2^(1/2)-(tanh(1/
2*x)+1)^(1/2)*(2+2*2^(1/2))^(1/2)*ln(tanh(1/2*x)+1-(tanh(1/2*x)+1)^(1/2)*(2
+2*2^(1/2))^(1/2)+2^(1/2))*(-2+2*2^(1/2))^(1/2)+(tanh(1/2*x)+1)^(1/2)*(2+2*
2^(1/2))^(1/2)*ln(tanh(1/2*x)+1+(tanh(1/2*x)+1)^(1/2)*(2+2*2^(1/2))^(1/2)+2
^(1/2))*(-2+2*2^(1/2))^(1/2)+4*(tanh(1/2*x)+1)^(1/2)*arctan((2*(tanh(1/2*x)
+1)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))+4*(tanh(1/2*x)+1)^(1/2
)*arctan((2*(tanh(1/2*x)+1)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2)
)+8*(-2+2*2^(1/2))^(1/2))/((tanh(1/2*x)+1)/(tanh(1/2*x)-1)^2)^(1/2)/(tanh(1
/2*x)-1)/(-2+2*2^(1/2))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{e^{(2x)}+e^{x}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(exp(x)+exp(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/sqrt(e^(2*x) + e^x), x)
```

Fricas [B] time = 2.66269, size = 2684, normalized size = 24.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(exp(x)+exp(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*e^x*log(2*(8^(1/4)*(sqrt(2)
 - 1)*e^x - 8^(1/4))*sqrt(2*sqrt(2) + 4) - 2*(8^(1/4)*sqrt(2*sqrt(2) + 4)*(
sqrt(2) - 1) + 4*e^x)*sqrt(e^(2*x) + e^x) + 4*sqrt(2) + 8*e^(2*x) + 4*e^x +
 4) - 8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*e^x*log(-2*(8^(1/4)*(sqrt(2
) - 1)*e^x - 8^(1/4))*sqrt(2*sqrt(2) + 4) + 2*(8^(1/4)*sqrt(2*sqrt(2) + 4)*
(sqrt(2) - 1) - 4*e^x)*sqrt(e^(2*x) + e^x) + 4*sqrt(2) + 8*e^(2*x) + 4*e^x
+ 4) + 4*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(1/7*(sqrt(2)*(5*sqrt(2)
 + 6) + 8*sqrt(2) + 4)*e^x + 1/112*(8*sqrt(2)*(5*sqrt(2) + 6) + (8^(3/4)*(5
*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1))*sqrt(2*sqrt(2) + 4) + 64*sqrt(2)
 + 32)*sqrt(2*(8^(1/4)*(sqrt(2) - 1)*e^x - 8^(1/4))*sqrt(2*sqrt(2) + 4) - 2
*(8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1) + 4*e^x)*sqrt(e^(2*x) + e^x) +
4*sqrt(2) + 8*e^(2*x) + 4*e^x + 4) + 1/56*((8^(3/4)*(5*sqrt(2) + 6) + 8*8^(
1/4)*(2*sqrt(2) + 1))*e^x + 8^(3/4)*(sqrt(2) + 4) - 8*8^(1/4)*(sqrt(2) - 3)
)*sqrt(2*sqrt(2) + 4) + 1/7*sqrt(2)*(sqrt(2) + 4) - 1/56*(8*sqrt(2)*(5*sqrt
(2) + 6) + (8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1))*sqrt(2*sqr
t(2) + 4) + 64*sqrt(2) + 32)*sqrt(e^(2*x) + e^x) + 3/7*sqrt(2) + 5/7)*e^x +
 4*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(-1/7*(sqrt(2)*(5*sqrt(2) + 6)
 + 8*sqrt(2) + 4)*e^x - 1/112*(8*sqrt(2)*(5*sqrt(2) + 6) - (8^(3/4)*(5*sqrt
(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1))*sqrt(2*sqrt(2) + 4) + 64*sqrt(2) + 32
)*sqrt(-2*(8^(1/4)*(sqrt(2) - 1)*e^x - 8^(1/4))*sqrt(2*sqrt(2) + 4) + 2*(8^
(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1) - 4*e^x)*sqrt(e^(2*x) + e^x) + 4*sq
rt(2) + 8*e^(2*x) + 4*e^x + 4) + 1/56*((8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)
*(2*sqrt(2) + 1))*e^x + 8^(3/4)*(sqrt(2) + 4) - 8*8^(1/4)*(sqrt(2) - 3))*sq
rt(2*sqrt(2) + 4) - 1/7*sqrt(2)*(sqrt(2) + 4) + 1/56*(8*sqrt(2)*(5*sqrt(2)
+ 6) - (8^(3/4)*(5*sqrt(2) + 6) + 8*8^(1/4)*(2*sqrt(2) + 1))*sqrt(2*sqrt(2)
 + 4) + 64*sqrt(2) + 32)*sqrt(e^(2*x) + e^x) - 3/7*sqrt(2) - 5/7)*e^x - 16*
sqrt(e^(2*x) + e^x) - 16*e^x)*e^(-x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(x)}{\sqrt{(e^x + 1)\,e^x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(exp(x)+exp(2*x))**(1/2),x)
```

```
[Out] Integral(tanh(x)/sqrt((exp(x) + 1)*exp(x)), x)
```

Giac [B] time = 1.36504, size = 366, normalized size = 3.33

$$-\left(\frac{1}{4}i+\frac{1}{4}\right)\sqrt{2\sqrt{2}-2}\left(\frac{i}{\sqrt{2}-1}+1\right)\log\left(2\sqrt{10\sqrt{2}-14}\left(-\frac{i}{5\sqrt{2}-7}+1\right)+(4i+8)\sqrt{e^{(2x)}+e^{x}}-(4i+8)e^{x}+8i-4\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(exp(x)+exp(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] -(1/4*I + 1/4)*sqrt(2*sqrt(2) - 2)*(I/(sqrt(2) - 1) + 1)*log(2*sqrt(10*sqrt
(2) - 14)*(-I/(5*sqrt(2) - 7) + 1) + (4*I + 8)*sqrt(e^(2*x) + e^x) - (4*I +
 8)*e^x + 8*I - 4) + (1/4*I + 1/4)*sqrt(2*sqrt(2) - 2)*(I/(sqrt(2) - 1) + 1
)*log(-2*sqrt(10*sqrt(2) - 14)*(-I/(5*sqrt(2) - 7) + 1) + (4*I + 8)*sqrt(e^
(2*x) + e^x) - (4*I + 8)*e^x + 8*I - 4) - (1/4*I + 1/4)*sqrt(2*sqrt(2) + 2)
*(I/(sqrt(2) + 1) + 1)*log(2*sqrt(2*sqrt(2) - 2)*(-I/(sqrt(2) - 1) + 1) + 4
*sqrt(e^(2*x) + e^x) - 4*e^x - 4*I) + (1/4*I + 1/4)*sqrt(2*sqrt(2) + 2)*(I/
(sqrt(2) + 1) + 1)*log(-2*sqrt(2*sqrt(2) - 2)*(-I/(sqrt(2) - 1) + 1) + 4*sq
rt(e^(2*x) + e^x) - 4*e^x - 4*I) + 2/(sqrt(e^(2*x) + e^x) - e^x)
```

3.25 $\int \sqrt{\textbf{sech}(x)\sinh(2x)}\,dx$

Optimal. Leaf size=40

$$\frac{2i\sqrt{2}\sqrt{\sinh(x)}E\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)}{\sqrt{i\sinh(x)}}$$

```
[Out] ((2*I)*Sqrt[2]*EllipticE[Pi/4 - (I/2)*x, 2]*Sqrt[Sinh[x]])/Sqrt[I*Sinh[x]]
```

Rubi [A] time = 0.0812508, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {4398, 4400, 4221, 4309, 2639}

$$\frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{\sinh(2x)\textbf{sech}(x)}}{\sqrt{i\sinh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sech[x]*Sinh[2*x]],x]
```

```
[Out] ((2*I)*EllipticE[Pi/4 - (I/2)*x, 2]*Sqrt[Sech[x]*Sinh[2*x]])/Sqrt[I*Sinh[x]
]
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[
v]
```

Rule 4400

```
Int[(u_.)*((v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> With[{uu = ActivateTri
g[u], vv = ActivateTrig[v], ww = ActivateTrig[w]}, Dist[(vv^m*ww^n)^FracPar
t[p]/(vv^(m*FracPart[p])*ww^(n*FracPart[p])), Int[uu*vv^(m*p)*ww^(n*p), x],
 x]] /; FreeQ[{m, n, p}, x] && !IntegerQ[p] && ( !InertTrigFreeQ[v] || !I
nertTrigFreeQ[w])
```

Rule 4221

```
Int[(u_)*((c_.)*sec[(a_.) + (b_.)*(x_)])^(m_.), x_Symbol] :> Dist[(c*Sec[a
+ b*x])^m*(c*Cos[a + b*x])^m, Int[ActivateTrig[u]/(c*Cos[a + b*x])^m, x], x
] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[m] && KnownSineIntegrandQ[u, x]
```

Rule 4309

```
Int[(cos[(a_.) + (b_.)*(x_)]*(e_.))^(m_.)*((g_.)*sin[(c_.) + (d_.)*(x_)])^(
p_), x_Symbol] :> Dist[(g*Sin[c + d*x])^p/((e*Cos[a + b*x])^p*Sin[a + b*x]^
p), Int[(e*Cos[a + b*x])^(m + p)*Sin[a + b*x]^p, x], x] /; FreeQ[{a, b, c,
d, e, g, m, p}, x] && EqQ[b*c - a*d, 0] && EqQ[d/b, 2] &&  !IntegerQ[p]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\operatorname{sech}(x)\sinh(2x)}\,dx &= \frac{\sqrt{\operatorname{sech}(x)\sinh(2x)}\int \sqrt{i\operatorname{sech}(x)\sinh(2x)}\,dx}{\sqrt{i\operatorname{sech}(x)\sinh(2x)}}\\
&= \frac{\sqrt{\operatorname{sech}(x)\sinh(2x)}\int \sqrt{\operatorname{sech}(x)}\sqrt{i\sinh(2x)}\,dx}{\sqrt{\operatorname{sech}(x)}\sqrt{i\sinh(2x)}}\\
&= \frac{\left(\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)\sinh(2x)}\right)\int \frac{\sqrt{i\sinh(2x)}}{\sqrt{\cosh(x)}}\,dx}{\sqrt{i\sinh(2x)}}\\
&= \frac{\sqrt{\operatorname{sech}(x)\sinh(2x)}\int \sqrt{i\sinh(x)}\,dx}{\sqrt{i\sinh(x)}}\\
&= \frac{2iE\left(\frac{\pi}{4}-\frac{ix}{2}\middle|2\right)\sqrt{\operatorname{sech}(x)\sinh(2x)}}{\sqrt{i\sinh(x)}}
\end{aligned}$$

Mathematica [C] time = 1.78887, size = 54, normalized size = 1.35

$$-\frac{2}{3}\sqrt{2}\sqrt{\sinh(x)}\tanh\left(\frac{x}{2}\right)\left(\sqrt{\operatorname{sech}^2\left(\frac{x}{2}\right)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},\tanh^2\left(\frac{x}{2}\right)\right)-3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sech[x]*Sinh[2*x]],x]
```

```
[Out] (-2*Sqrt[2]*(-3 + Hypergeometric2F1[1/2, 3/4, 7/4, Tanh[x/2]^2]*Sqrt[Sech[x
/2]^2])*Sqrt[Sinh[x]]*Tanh[x/2])/3
```

Maple [A] time = 0.1, size = 75, normalized size = 1.9

$$2\,\frac{\sqrt{-i\left(\sinh\left(x\right)+i\right)}\sqrt{-i\left(-\sinh\left(x\right)+i\right)}\sqrt{i\sinh\left(x\right)}\left(2\,\mathrm{EllipticE}\left(\sqrt{1-i\sinh\left(x\right)},1/2\,\sqrt{2}\right)-\mathrm{EllipticF}\left(\sqrt{1-i\sinh\left(x\right)},1/\right.\right.}{\cosh\left(x\right)\sqrt{\sinh\left(x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sinh(2*x)/cosh(x))^(1/2),x)
```

```
[Out] 2*(-I*(sinh(x)+I))^(1/2)*(-I*(-sinh(x)+I))^(1/2)*(I*sinh(x))^(1/2)*(2*Ellip
ticE((1-I*sinh(x))^(1/2),1/2*2^(1/2))-EllipticF((1-I*sinh(x))^(1/2),1/2*2^(
1/2)))/cosh(x)/sinh(x)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\frac{\sinh\left(2\,x\right)}{\cosh\left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(2*x)/cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sinh(2*x)/cosh(x)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{\frac{\sinh\left(2\,x\right)}{\cosh\left(x\right)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(2*x)/cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sinh(2*x)/cosh(x)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{\sinh(2x)}{\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(2*x)/cosh(x))**(1/2),x)
```

```
[Out] Integral(sqrt(sinh(2*x)/cosh(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{\sinh(2\,x)}{\cosh(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((sinh(2*x)/cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sinh(2*x)/cosh(x)), x)
```

3.26 $\int \log\left(x^2+\sqrt{1-x^2}\right)\,dx$

Optimal. Leaf size=185

$$x\log\left(x^2+\sqrt{1-x^2}\right)+\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x}{\sqrt{1-x^2}}\right)-\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}x}{\sqrt{1-x^2}}\right)-2x-\sin^{-1}(x)+$$

```
[Out] -2*x - ArcSin[x] + Sqrt[(1 + Sqrt[5])/2]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] +
Sqrt[(1 + Sqrt[5])/2]*ArcTan[(Sqrt[(1 + Sqrt[5])/2]*x)/Sqrt[1 - x^2]] + Sqr
t[(-1 + Sqrt[5])/2]*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x] - Sqrt[(-1 + Sqrt[5])
/2]*ArcTanh[(Sqrt[(-1 + Sqrt[5])/2]*x)/Sqrt[1 - x^2]] + x*Log[x^2 + Sqrt[1
- x^2]]
```

Rubi [A] time = 0.976096, antiderivative size = 349, normalized size of antiderivative = 1.89, number of steps used = 31, number of rules used = 12, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {2548, 6742, 1293, 216, 1692, 377, 207, 203, 1166, 1130, 1174, 402}

$$x\log\left(x^2+\sqrt{1-x^2}\right)+2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x}{\sqrt{1-x^2}}\right)-\sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}x}{\sqrt{1-x^2}}\right)-\sqrt{\frac{1}{10}\left(\sqrt{5}-1\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Int[Log[x^2 + Sqrt[1 - x^2]],x]
```

```
[Out] -2*x - ArcSin[x] - Sqrt[(1 + Sqrt[5])/10]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] +
 2*Sqrt[(2 + Sqrt[5])/5]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sqrt[(1 + Sqrt[5
])/10]*ArcTan[(Sqrt[(1 + Sqrt[5])/2]*x)/Sqrt[1 - x^2]] + 2*Sqrt[(2 + Sqrt[5
])/5]*ArcTan[(Sqrt[(1 + Sqrt[5])/2]*x)/Sqrt[1 - x^2]] + 2*Sqrt[(-2 + Sqrt[5
])/5]*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x] + Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[S
qrt[2/(-1 + Sqrt[5])]*x] - 2*Sqrt[(-2 + Sqrt[5])/5]*ArcTanh[(Sqrt[(-1 + Sqr
t[5])/2]*x)/Sqrt[1 - x^2]] - Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[(Sqrt[(-1 + Sq
rt[5])/2]*x)/Sqrt[1 - x^2]] + x*Log[x^2 + Sqrt[1 - x^2]]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 1293

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a
*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1692

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1174

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rubi steps

$$\int \log\left(x^2+\sqrt{1-x^2}\right)dx = x\log\left(x^2+\sqrt{1-x^2}\right) - \int \frac{x^2\left(2-\frac{1}{\sqrt{1-x^2}}\right)}{x^2+\sqrt{1-x^2}}\,dx$$
$$= x\log\left(x^2+\sqrt{1-x^2}\right) - \int \left(\frac{2x^2}{x^2+\sqrt{1-x^2}} - \frac{x^2}{1-x^2+x^2\sqrt{1-x^2}}\right)dx$$
$$= x\log\left(x^2+\sqrt{1-x^2}\right) - 2\int \frac{x^2}{x^2+\sqrt{1-x^2}}\,dx + \int \frac{x^2}{1-x^2+x^2\sqrt{1-x^2}}\,dx$$
$$= x\log\left(x^2+\sqrt{1-x^2}\right) - 2\int \left(1-\frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}+\frac{1-x^2}{-1+x^2+x^4}\right)dx + \int \left(\frac{1}{\sqrt{1-x^2}} - \frac{x^2}{-1+x^2}\right.$$
$$= -2x + x\log\left(x^2+\sqrt{1-x^2}\right) + 2\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx - 2\int \frac{1-x^2}{-1+x^2+x^4}\,dx + \int \frac{1}{\sqrt{1-x^2}}\,dx -$$
$$= -2x + \sin^{-1}(x) + x\log\left(x^2+\sqrt{1-x^2}\right) - 2\int \frac{1}{\sqrt{1-x^2}}\,dx - 2\int \frac{1-2x^2}{\sqrt{1-x^2}\left(-1+x^2+x^4\right)}\,dx +$$
$$= -2x - \sin^{-1}(x) - \sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right) + 2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right) +$$
$$= -2x - \sin^{-1}(x) - \sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right) + 2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right) +$$
$$= -2x - \sin^{-1}(x) - \sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right) + 2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right) -$$
$$= -2x - \sin^{-1}(x) - \sqrt{\frac{1}{10}\left(1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right) + 2\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right) -$$

Mathematica [C] time = 0.524187, size = 910, normalized size = 4.92

$$4\sqrt{5}\log\left(x^2+\sqrt{1-x^2}\right)x - 8\sqrt{5}x - 4\sqrt{5}\sin^{-1}(x) + \sqrt{10\left(-1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right) + 5\sqrt{2\left(-1+\sqrt{5}\right)}\tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}}x\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[x^2 + Sqrt[1 - x^2]],x]
```

```
[Out] (-8*Sqrt[5]*x - 4*Sqrt[5]*ArcSin[x] + 5*Sqrt[2*(-1 + Sqrt[5])]*ArcTan[Sqrt[
2/(1 + Sqrt[5])]*x] + Sqrt[10*(-1 + Sqrt[5])]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*
x] - (-5 + Sqrt[5])*Sqrt[2*(1 + Sqrt[5])]*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x]
 - 5*Sqrt[2 + Sqrt[5]]*Log[-Sqrt[(-1 + Sqrt[5])/2] + x] + 3*Sqrt[5*(2 + Sqr
t[5])]*Log[-Sqrt[(-1 + Sqrt[5])/2] + x] + 5*Sqrt[2 + Sqrt[5]]*Log[Sqrt[(-1
+ Sqrt[5])/2] + x] - 3*Sqrt[5*(2 + Sqrt[5])]*Log[Sqrt[(-1 + Sqrt[5])/2] + x
] - (5*I)*Sqrt[-2 + Sqrt[5]]*Log[(-I)*Sqrt[(1 + Sqrt[5])/2] + x] - (3*I)*Sq
rt[5*(-2 + Sqrt[5])]*Log[(-I)*Sqrt[(1 + Sqrt[5])/2] + x] + (5*I)*Sqrt[-2 +
Sqrt[5]]*Log[I*Sqrt[(1 + Sqrt[5])/2] + x] + (3*I)*Sqrt[5*(-2 + Sqrt[5])]*Lo
g[I*Sqrt[(1 + Sqrt[5])/2] + x] + 4*Sqrt[5]*x*Log[x^2 + Sqrt[1 - x^2]] + (5*
I)*Sqrt[-2 + Sqrt[5]]*Log[2 - I*Sqrt[2*(1 + Sqrt[5])]*x + Sqrt[2*(3 + Sqrt[
5])]*Sqrt[1 - x^2]] + (3*I)*Sqrt[5*(-2 + Sqrt[5])]*Log[2 - I*Sqrt[2*(1 + Sq
rt[5])]*x + Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 - x^2]] - (5*I)*Sqrt[-2 + Sqrt[5]]
*Log[2 + I*Sqrt[2*(1 + Sqrt[5])]*x + Sqrt[2*(3 + Sqrt[5])]*Sqrt[1 - x^2]] -
 (3*I)*Sqrt[5*(-2 + Sqrt[5])]*Log[2 + I*Sqrt[2*(1 + Sqrt[5])]*x + Sqrt[2*(3
 + Sqrt[5])]*Sqrt[1 - x^2]] + 5*Sqrt[2 + Sqrt[5]]*Log[2 - Sqrt[2*(-1 + Sqrt
[5])]*x + Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)]] - 3*Sqrt[5*(2 + Sqrt[5])
]*Log[2 - Sqrt[2*(-1 + Sqrt[5])]*x + Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)
]] - 5*Sqrt[2 + Sqrt[5]]*Log[2 + Sqrt[2*(-1 + Sqrt[5])]*x + Sqrt[2]*Sqrt[(-
3 + Sqrt[5])*(-1 + x^2)]] + 3*Sqrt[5*(2 + Sqrt[5])]*Log[2 + Sqrt[2*(-1 + Sq
rt[5])]*x + Sqrt[2]*Sqrt[(-3 + Sqrt[5])*(-1 + x^2)]])/(4*Sqrt[5])
```

Maple [B] time = 0.129, size = 392, normalized size = 2.1

$$x\ln\left(x^2+\sqrt{-x^2+1}\right)+\frac{1}{\sqrt{2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right)+\frac{\sqrt{5}}{\sqrt{2+2\sqrt{5}}}\arctan\left(2\frac{x}{\sqrt{2+2\sqrt{5}}}\right)-\frac{1}{\sqrt{-2+2\sqrt{5}}}\mathrm{Artanl}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x^2+(-x^2+1)^(1/2)),x)
```

```
[Out] x*ln(x^2+(-x^2+1)^(1/2))+1/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/
2))+5^(1/2)/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))-1/(-2+2*5^(
1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))+5^(1/2)/(-2+2*5^(1/2))^(1/2)*
arctanh(2*x/(-2+2*5^(1/2))^(1/2))-2*x-3/2/(2+5^(1/2))^(1/2)*arctan(((-x^2+1
)^(1/2)-1)/x/(2+5^(1/2))^(1/2))-1/2*5^(1/2)/(2+5^(1/2))^(1/2)*arctan(((-x^2
+1)^(1/2)-1)/x/(2+5^(1/2))^(1/2))+3/2/(-2+5^(1/2))^(1/2)*arctanh(((-x^2+1)^
(1/2)-1)/x/(-2+5^(1/2))^(1/2))-1/2*5^(1/2)/(-2+5^(1/2))^(1/2)*arctanh(((-x^
2+1)^(1/2)-1)/x/(-2+5^(1/2))^(1/2))-1/2*5^(1/2)/(-2+5^(1/2))^(1/2)*arctan((
(-x^2+1)^(1/2)-1)/x/(-2+5^(1/2))^(1/2))+1/2/(-2+5^(1/2))^(1/2)*arctan(((-x^
2+1)^(1/2)-1)/x/(-2+5^(1/2))^(1/2))-1/2*5^(1/2)/(2+5^(1/2))^(1/2)*arctanh((
```

```
(-x^2+1)^(1/2)-1)/x/(2+5^(1/2))^(1/2))-1/2/(2+5^(1/2))^(1/2)*arctanh(((-x^2
+1)^(1/2)-1)/x/(2+5^(1/2))^(1/2))-arcsin(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x\log\left(x^2+\sqrt{x+1}\sqrt{-x+1}\right)-x-\int\frac{x^4-2\,x^2}{x^4-x^2+\left(x^2-1\right)e^{\left(\frac{1}{2}\,\log(x+1)+\frac{1}{2}\,\log(-x+1)\right)}}\,dx+\frac{1}{2}\,\log\left(x+1\right)-\frac{1}{2}\,\log\left(-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] x*log(x^2 + sqrt(x + 1)*sqrt(-x + 1)) - x - integrate((x^4 - 2*x^2)/(x^4 -
x^2 + (x^2 - 1)*e^(1/2*log(x + 1) + 1/2*log(-x + 1))), x) + 1/2*log(x + 1)
- 1/2*log(-x + 1)
```

Fricas [B] time = 2.73356, size = 1277, normalized size = 6.9

$$-\sqrt{2}\sqrt{\sqrt{5}+1}\arctan\left(\frac{1}{8}\,\sqrt{4\,x^2+2\,\sqrt{5}+2}\left(\sqrt{5}\sqrt{2}-\sqrt{2}\right)\sqrt{\sqrt{5}+1}-\frac{1}{4}\left(\sqrt{5}\sqrt{2}x-\sqrt{2}x\right)\sqrt{\sqrt{5}+1}\right)-\sqrt{2}\sqrt{\sqrt{5}+1}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] -sqrt(2)*sqrt(sqrt(5) + 1)*arctan(1/8*sqrt(4*x^2 + 2*sqrt(5) + 2)*(sqrt(5)*
sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 1) - 1/4*(sqrt(5)*sqrt(2)*x - sqrt(2)*x)*
sqrt(sqrt(5) + 1)) - sqrt(2)*sqrt(sqrt(5) + 1)*arctan(1/8*(sqrt(2)*(sqrt(-x
^2 + 1)*(sqrt(5)*sqrt(2) - sqrt(2)) + sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(
5) + 1)*sqrt((x^4 - 4*x^2 - sqrt(5)*(x^4 - 2*x^2) - 2*(sqrt(5)*x^2 - x^2 +
2)*sqrt(-x^2 + 1) + 4)/x^4) + 2*sqrt(-x^2 + 1)*(sqrt(5)*sqrt(2) - sqrt(2))*
sqrt(sqrt(5) + 1))/x) + x*log(x^2 + sqrt(-x^2 + 1)) + 1/4*sqrt(2)*sqrt(sqrt
(5) - 1)*log(2*x + sqrt(2)*sqrt(sqrt(5) - 1)) - 1/4*sqrt(2)*sqrt(sqrt(5) -
1)*log(2*x - sqrt(2)*sqrt(sqrt(5) - 1)) + 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*log
(-(2*x^2 + (sqrt(2)*sqrt(-x^2 + 1)*x - sqrt(2)*x)*sqrt(sqrt(5) - 1) + 2*sqr
t(-x^2 + 1) - 2)/x^2) - 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*log(-(2*x^2 - (sqrt(2
)*sqrt(-x^2 + 1)*x - sqrt(2)*x)*sqrt(sqrt(5) - 1) + 2*sqrt(-x^2 + 1) - 2)/x
```

```
^2) - 2*x + 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(x^2 + \sqrt{1 - x^2}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x**2+(-x**2+1)**(1/2)),x)
```

```
[Out] Integral(log(x**2 + sqrt(1 - x**2)), x)
```

Giac [B] time = 1.2163, size = 406, normalized size = 2.19

$$x\log\left(x^2 + \sqrt{-x^2+1}\right) - \frac{1}{2}\pi\text{sgn}\left(x\right) + \frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right) - \frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(-\frac{\frac{x}{\sqrt{-x^2+1}-1} - \frac{\sqrt{-x^2+1-}}{x}}{\sqrt{2\sqrt{5}+2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x^2+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] x*log(x^2 + sqrt(-x^2 + 1)) - 1/2*pi*sgn(x) + 1/2*sqrt(2*sqrt(5) + 2)*arcta
n(x/sqrt(1/2*sqrt(5) + 1/2)) - 1/2*sqrt(2*sqrt(5) + 2)*arctan(-(x/(sqrt(-x^
2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt(2*sqrt(5) + 2)) + 1/4*sqrt(2*sqr
t(5) - 2)*log(abs(x + sqrt(1/2*sqrt(5) - 1/2))) - 1/4*sqrt(2*sqrt(5) - 2)*l
og(abs(x - sqrt(1/2*sqrt(5) - 1/2))) - 1/4*sqrt(2*sqrt(5) - 2)*log(abs(sqrt
(2*sqrt(5) - 2) - x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x)) + 1/4*s
qrt(2*sqrt(5) - 2)*log(abs(-sqrt(2*sqrt(5) - 2) - x/(sqrt(-x^2 + 1) - 1) +
(sqrt(-x^2 + 1) - 1)/x)) - 2*x - arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2
- 1)/(sqrt(-x^2 + 1) - 1))
```

3.27 $\int \frac{\log(1+e^x)}{1+e^{2x}}\,dx$

Optimal. Leaf size=102

$$-\text{PolyLog}(2,-e^x)-\frac{1}{2}\text{PolyLog}\left(2,\left(\frac{1}{2}-\frac{i}{2}\right)(e^x+1)\right)-\frac{1}{2}\text{PolyLog}\left(2,\left(\frac{1}{2}+\frac{i}{2}\right)(e^x+1)\right)-\frac{1}{2}\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(-e^x+i)\right)\log$$

```
[Out] -(Log[(1/2 - I/2)*(I - E^x)]*Log[1 + E^x])/2 - (Log[(-1/2 - I/2)*(I + E^x)]
*Log[1 + E^x])/2 - PolyLog[2, -E^x] - PolyLog[2, (1/2 - I/2)*(1 + E^x)]/2 -
 PolyLog[2, (1/2 + I/2)*(1 + E^x)]/2
```

Rubi [A] time = 0.146247, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {2282, 266, 36, 29, 31, 2416, 2391, 260, 2394, 2393}

$$-\text{PolyLog}(2,-e^x)-\frac{1}{2}\text{PolyLog}\left(2,\left(\frac{1}{2}-\frac{i}{2}\right)(e^x+1)\right)-\frac{1}{2}\text{PolyLog}\left(2,\left(\frac{1}{2}+\frac{i}{2}\right)(e^x+1)\right)-\frac{1}{2}\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(-e^x+i)\right)\log$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 + E^x]/(1 + E^(2*x)),x]
```

```
[Out] -(Log[(1/2 - I/2)*(I - E^x)]*Log[1 + E^x])/2 - (Log[(-1/2 - I/2)*(I + E^x)]
*Log[1 + E^x])/2 - PolyLog[2, -E^x] - PolyLog[2, (1/2 - I/2)*(1 + E^x)]/2 -
 PolyLog[2, (1/2 + I/2)*(1 + E^x)]/2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2416

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\log(1+e^x)}{1+e^{2x}}\,dx &= \text{Subst}\left(\int \frac{\log(1+x)}{x\left(1+x^2\right)}\,dx,x,e^x\right)\\
&= \text{Subst}\left(\int \left(\frac{\log(1+x)}{x}-\frac{x\log(1+x)}{1+x^2}\right)dx,x,e^x\right)\\
&= \text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^x\right)-\text{Subst}\left(\int \frac{x\log(1+x)}{1+x^2}\,dx,x,e^x\right)\\
&= -\mathbf{Li}_2\left(-e^x\right)-\text{Subst}\left(\int \left(-\frac{\log(1+x)}{2(i-x)}+\frac{\log(1+x)}{2(i+x)}\right)dx,x,e^x\right)\\
&= -\mathbf{Li}_2\left(-e^x\right)+\frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{i-x}\,dx,x,e^x\right)-\frac{1}{2}\text{Subst}\left(\int \frac{\log(1+x)}{i+x}\,dx,x,e^x\right)\\
&= -\frac{1}{2}\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(i-e^x)\right)\log\left(1+e^x\right)-\frac{1}{2}\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(i+e^x)\right)\log\left(1+e^x\right)-\mathbf{Li}_2\left(-e^x\right)+\frac{1}{2}\text{Subst}\\
&= -\frac{1}{2}\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(i-e^x)\right)\log\left(1+e^x\right)-\frac{1}{2}\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(i+e^x)\right)\log\left(1+e^x\right)-\mathbf{Li}_2\left(-e^x\right)+\frac{1}{2}\text{Subst}\\
&= -\frac{1}{2}\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)(i-e^x)\right)\log\left(1+e^x\right)-\frac{1}{2}\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)(i+e^x)\right)\log\left(1+e^x\right)-\mathbf{Li}_2\left(-e^x\right)-\frac{1}{2}\mathbf{Li}_2\left(\left(\frac{1}{2}\right.\right.
\end{aligned}
$$

Mathematica [A] time = 0.0171325, size = 102, normalized size = 1.

$$
-\text{PolyLog}\left(2,-e^x\right)-\frac{1}{2}\text{PolyLog}\left(2,\left(\frac{1}{2}-\frac{i}{2}\right)\left(e^x+1\right)\right)-\frac{1}{2}\text{PolyLog}\left(2,\left(\frac{1}{2}+\frac{i}{2}\right)\left(e^x+1\right)\right)-\frac{1}{2}\log\left(\left(\frac{1}{2}-\frac{i}{2}\right)\left(-e^x+i\right)\right)\log
$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 + E^x]/(1 + E^(2*x)),x]
```

```
[Out] -(Log[(1/2 - I/2)*(I - E^x)]*Log[1 + E^x])/2 - (Log[(-1/2 - I/2)*(I + E^x)]
*Log[1 + E^x])/2 - PolyLog[2, -E^x] - PolyLog[2, (1/2 - I/2)*(1 + E^x)]/2 -
 PolyLog[2, (1/2 + I/2)*(1 + E^x)]/2
```

Maple [A] time = 0.028, size = 83, normalized size = 0.8

$$
-\frac{\ln\left(1+e^x\right)}{2}\ln\left(\frac{1}{2}-\frac{e^x}{2}+\frac{i}{2}\left(1+e^x\right)\right)-\frac{\ln\left(1+e^x\right)}{2}\ln\left(\frac{1}{2}-\frac{e^x}{2}-\frac{i}{2}\left(1+e^x\right)\right)-\frac{1}{2}\text{dilog}\left(\frac{1}{2}-\frac{e^x}{2}+\frac{i}{2}\left(1+e^x\right)\right)-\frac{1}{2}\text{dilog}\left(\frac{1}{2}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1+exp(x))/(1+exp(2*x)),x)
```

```
[Out] -1/2*ln(1+exp(x))*ln(1/2-1/2*exp(x)+1/2*I*(1+exp(x)))-1/2*ln(1+exp(x))*ln(1
/2-1/2*exp(x)-1/2*I*(1+exp(x)))-1/2*dilog(1/2-1/2*exp(x)+1/2*I*(1+exp(x)))-
1/2*dilog(1/2-1/2*exp(x)-1/2*I*(1+exp(x)))-dilog(1+exp(x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(e^{x}+1\right)}{e^{(2\,x)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+exp(x))/(1+exp(2*x)),x, algorithm="maxima")
```

```
[Out] integrate(log(e^x + 1)/(e^(2*x) + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(e^{x}+1\right)}{e^{(2\,x)}+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+exp(x))/(1+exp(2*x)),x, algorithm="fricas")
```

```
[Out] integral(log(e^x + 1)/(e^(2*x) + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(e^{x}+1\right)}{e^{2x}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1+exp(x))/(1+exp(2*x)),x)
```

```
[Out] Integral(log(exp(x) + 1)/(exp(2*x) + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(e^{x}+1\right)}{e^{(2x)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+exp(x))/(1+exp(2*x)),x, algorithm="giac")
```

```
[Out] integrate(log(e^x + 1)/(e^(2*x) + 1), x)
```

3.28 $\int \cosh(x)\log^2\left(1+\cosh^2(x)\right)\,dx$

Optimal. Leaf size=159

$$4i\sqrt{2}\text{PolyLog}\left(2,1-\frac{2\sqrt{2}}{\sqrt{2}+i\sinh(x)}\right)+8\sinh(x)+\sinh(x)\log^2\left(\sinh^2(x)+2\right)-4\sinh(x)\log\left(\sinh^2(x)+2\right)+4i\sqrt{2}$$

```
[Out] -8*Sqrt[2]*ArcTan[Sinh[x]/Sqrt[2]] + (4*I)*Sqrt[2]*ArcTan[Sinh[x]/Sqrt[2]]^
2 + 8*Sqrt[2]*ArcTan[Sinh[x]/Sqrt[2]]*Log[(2*Sqrt[2])/(Sqrt[2] + I*Sinh[x])
] + 4*Sqrt[2]*ArcTan[Sinh[x]/Sqrt[2]]*Log[2 + Sinh[x]^2] + (4*I)*Sqrt[2]*Po
lyLog[2, 1 - (2*Sqrt[2])/(Sqrt[2] + I*Sinh[x])] + 8*Sinh[x] - 4*Log[2 + Sin
h[x]^2]*Sinh[x] + Log[2 + Sinh[x]^2]^2*Sinh[x]
```

Rubi [A] time = 0.202866, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 12, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {4358, 2450, 2476, 2448, 321, 203, 2470, 12, 4920, 4854, 2402, 2315}

$$4i\sqrt{2}\text{PolyLog}\left(2,1-\frac{2\sqrt{2}}{\sqrt{2}+i\sinh(x)}\right)+8\sinh(x)+\sinh(x)\log^2\left(\sinh^2(x)+2\right)-4\sinh(x)\log\left(\sinh^2(x)+2\right)+4i\sqrt{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Log[1 + Cosh[x]^2]^2,x]
```

```
[Out] -8*Sqrt[2]*ArcTan[Sinh[x]/Sqrt[2]] + (4*I)*Sqrt[2]*ArcTan[Sinh[x]/Sqrt[2]]^
2 + 8*Sqrt[2]*ArcTan[Sinh[x]/Sqrt[2]]*Log[(2*Sqrt[2])/(Sqrt[2] + I*Sinh[x])
] + 4*Sqrt[2]*ArcTan[Sinh[x]/Sqrt[2]]*Log[2 + Sinh[x]^2] + (4*I)*Sqrt[2]*Po
lyLog[2, 1 - (2*Sqrt[2])/(Sqrt[2] + I*Sinh[x])] + 8*Sinh[x] - 4*Log[2 + Sin
h[x]^2]*Sinh[x] + Log[2 + Sinh[x]^2]^2*Sinh[x]
```

Rule 4358

```
Int[Cosh[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sinh[c*(a +
b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a +
b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x]
```

Rule 2450

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_), x_Symbo
l] :> Simp[x*(a + b*Log[c*(d + e*x^n)^p])^q, x] - Dist[b*e*n*p*q, Int[(x^n*
```

```
(a + b*Log[c*(d + e*x^n)^p])^(q - 1))/(d + e*x^n), x], x] /; FreeQ[{a, b, c
, d, e, n, p}, x] && IGtQ[q, 0] && (EqQ[q, 1] || IntegerQ[n])
```

Rule 2476

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b
*Log[c*(d + e*x^n)^p])^q, x^m*(f + g*x^s)^r, x], x] /; FreeQ[{a, b, c, d, e
, f, g, m, n, p, q, r, s}, x] && IGtQ[q, 0] && IntegerQ[m] && IntegerQ[r] &
& IntegerQ[s]
```

Rule 2448

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)], x_Symbol] :> Simp[x*Log[c*(d
 + e*x^n)^p], x] - Dist[e*n*p, Int[x^n/(d + e*x^n), x], x] /; FreeQ[{c, d,
e, n, p}, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2470

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))/((f_) + (g_.)
*(x_)^2), x_Symbol] :> With[{u = IntHide[1/(f + g*x^2), x]}, Simp[u*(a + b*
Log[c*(d + e*x^n)^p]), x] - Dist[b*e*n*p, Int[(u*x^(n - 1))/(d + e*x^n), x]
, x]] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && IntegerQ[n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 4920

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> -Simp[(I*(a + b*ArcTan[c*x])^(p + 1))/(b*e*(p + 1)), x] - Dist
[1/(c*d), Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c, d,
 e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

Rule 4854

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> -Simp[((a + b*ArcTan[c*x])^p*Log[2/(1 + (e*x)/d)])/e, x] + Dist[(b*c*p)
/e, Int[((a + b*ArcTan[c*x])^(p - 1)*Log[2/(1 + (e*x)/d)])/(1 + c^2*x^2), x
], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
```

Rule 2402

```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> -Dis
t[e/g, Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[{
c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\int \cosh(x)\log^2\left(1+\cosh^2(x)\right)\,dx = \text{Subst}\left(\int \log^2\left(2+x^2\right)\,dx, x, \sinh(x)\right)$$

$$= \log^2\left(2+\sinh^2(x)\right)\sinh(x) - 4\,\text{Subst}\left(\int \frac{x^2\log\left(2+x^2\right)}{2+x^2}\,dx, x, \sinh(x)\right)$$

$$= \log^2\left(2+\sinh^2(x)\right)\sinh(x) - 4\,\text{Subst}\left(\int\left(\log\left(2+x^2\right) - \frac{2\log\left(2+x^2\right)}{2+x^2}\right)dx, x, \text{sinh}$$

$$= \log^2\left(2+\sinh^2(x)\right)\sinh(x) - 4\,\text{Subst}\left(\int \log\left(2+x^2\right)\,dx, x, \sinh(x)\right) + 8\,\text{Subst}\left(\int\right.$$

$$= 4\sqrt{2}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)\log\left(2+\sinh^2(x)\right) - 4\log\left(2+\sinh^2(x)\right)\sinh(x) + \log^2\left(2+\right.$$

$$= 4\sqrt{2}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)\log\left(2+\sinh^2(x)\right) + 8\sinh(x) - 4\log\left(2+\sinh^2(x)\right)\sinh(x)$$

$$= -8\sqrt{2}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right) + 4i\sqrt{2}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)^2 + 4\sqrt{2}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)\log\left(2+\text{si}\right.$$

$$= -8\sqrt{2}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right) + 4i\sqrt{2}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)^2 + 8\sqrt{2}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)\log\left(\frac{}{\sqrt{2}+}\right.$$

$$= -8\sqrt{2}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right) + 4i\sqrt{2}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)^2 + 8\sqrt{2}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)\log\left(\frac{}{\sqrt{2}+}\right.$$

$$= -8\sqrt{2}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right) + 4i\sqrt{2}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)^2 + 8\sqrt{2}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)\log\left(\frac{}{\sqrt{2}+}\right.$$

<u>Mathematica</u> [A] time = 0.0862541, size = 122, normalized size = 0.77

$$4i\sqrt{2}\text{PolyLog}\left(2, \frac{\sqrt{2}\sinh(x)+2i}{\sqrt{2}\sinh(x)-2i}\right) + \sinh(x)\left(\log^2\left(\sinh^2(x)+2\right) - 4\log\left(\sinh^2(x)+2\right) + 8\right) + 4\sqrt{2}\tan^{-1}\left(\frac{\sinh(x)}{\sqrt{2}}\right)\left(\text{l}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cosh[x]*Log[1 + Cosh[x]^2]^2,x]
```

```
[Out] 4*Sqrt[2]*ArcTan[Sinh[x]/Sqrt[2]]*(-2 + I*ArcTan[Sinh[x]/Sqrt[2]] + 2*Log[(
4*I)/(2*I - Sqrt[2]*Sinh[x])] + Log[2 + Sinh[x]^2]) + (4*I)*Sqrt[2]*PolyLog
[2, (2*I + Sqrt[2]*Sinh[x])/(-2*I + Sqrt[2]*Sinh[x])] + (8 - 4*Log[2 + Sinh
[x]^2] + Log[2 + Sinh[x]^2]^2)*Sinh[x]
```

Maple [F] time = 2.18, size = 0, normalized size = 0.

$$\int \cosh(x)\left(\ln\left(1+(\cosh(x))^2\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*ln(1+cosh(x)^2)^2,x)
```

```
[Out] int(cosh(x)*ln(1+cosh(x)^2)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*log(1+cosh(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x)*log(e^(4*x) + 6*e^(2*x) + 1)^2 - 2*(e^(-x) + integ
rate((e^(2*x) + 6)*e^x/(e^(4*x) + 6*e^(2*x) + 1), x))*log(2)^2 + 2*(e^x - i
ntegrate((6*e^(2*x) + 1)*e^x/(e^(4*x) + 6*e^(2*x) + 1), x))*log(2)^2 + 14*i
ntegrate(e^(3*x)/(e^(4*x) + 6*e^(2*x) + 1), x)*log(2)^2 + 14*integrate(e^x/
(e^(4*x) + 6*e^(2*x) + 1), x)*log(2)^2 + 4*integrate(x*e^(6*x)/(e^(5*x) + 6
*e^(3*x) + e^x), x)*log(2) + 28*integrate(x*e^(4*x)/(e^(5*x) + 6*e^(3*x) +
e^x), x)*log(2) + 28*integrate(x*e^(2*x)/(e^(5*x) + 6*e^(3*x) + e^x), x)*lo
g(2) - 2*integrate(e^(6*x)*log(e^(4*x) + 6*e^(2*x) + 1)/(e^(5*x) + 6*e^(3*x
) + e^x), x)*log(2) - 14*integrate(e^(4*x)*log(e^(4*x) + 6*e^(2*x) + 1)/(e^
(5*x) + 6*e^(3*x) + e^x), x)*log(2) - 14*integrate(e^(2*x)*log(e^(4*x) + 6*
e^(2*x) + 1)/(e^(5*x) + 6*e^(3*x) + e^x), x)*log(2) + 4*integrate(x/(e^(5*x
) + 6*e^(3*x) + e^x), x)*log(2) - 2*integrate(log(e^(4*x) + 6*e^(2*x) + 1)/
(e^(5*x) + 6*e^(3*x) + e^x), x)*log(2) + 2*integrate(x^2*e^(6*x)/(e^(5*x) +
 6*e^(3*x) + e^x), x) + 14*integrate(x^2*e^(4*x)/(e^(5*x) + 6*e^(3*x) + e^x
), x) + 14*integrate(x^2*e^(2*x)/(e^(5*x) + 6*e^(3*x) + e^x), x) - 2*integr
ate(x*e^(6*x)*log(e^(4*x) + 6*e^(2*x) + 1)/(e^(5*x) + 6*e^(3*x) + e^x), x)
```

```
- 14*integrate(x*e^(4*x)*log(e^(4*x) + 6*e^(2*x) + 1)/(e^(5*x) + 6*e^(3*x)
+ e^x), x) - 14*integrate(x*e^(2*x)*log(e^(4*x) + 6*e^(2*x) + 1)/(e^(5*x) +
 6*e^(3*x) + e^x), x) + 2*integrate(x^2/(e^(5*x) + 6*e^(3*x) + e^x), x) - 2
*integrate(x*log(e^(4*x) + 6*e^(2*x) + 1)/(e^(5*x) + 6*e^(3*x) + e^x), x) -
 4*integrate(e^(6*x)*log(e^(4*x) + 6*e^(2*x) + 1)/(e^(5*x) + 6*e^(3*x) + e^
x), x) - 8*integrate(e^(4*x)*log(e^(4*x) + 6*e^(2*x) + 1)/(e^(5*x) + 6*e^(3
*x) + e^x), x) + 12*integrate(e^(2*x)*log(e^(4*x) + 6*e^(2*x) + 1)/(e^(5*x)
 + 6*e^(3*x) + e^x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cosh\left(x\right)\log\left(\cosh\left(x\right)^{2}+1\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*log(1+cosh(x)^2)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(x)*log(cosh(x)^2 + 1)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \log\left(\cosh^{2}\left(x\right)+1\right)^{2}\cosh\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*ln(1+cosh(x)**2)**2,x)
```

```
[Out] Integral(log(cosh(x)**2 + 1)**2*cosh(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \cosh\left(x\right)\log\left(\cosh\left(x\right)^{2}+1\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*log(1+cosh(x)^2)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(x)*log(cosh(x)^2 + 1)^2, x)
```

3.29 $\int \cosh(x)\log^2\left(\cosh^2(x)+\sinh(x)\right)\,dx$

Optimal. Leaf size=395

$$-\left(1+i\sqrt{3}\right)\text{PolyLog}\left(2,-\frac{2i\sinh(x)-\sqrt{3}+i}{2\sqrt{3}}\right)-\left(1-i\sqrt{3}\right)\text{PolyLog}\left(2,\frac{2i\sinh(x)+\sqrt{3}+i}{2\sqrt{3}}\right)+8\sinh(x)+\sinh(x)\log^2$$

```
[Out] -4*Sqrt[3]*ArcTan[(1 + 2*Sinh[x])/Sqrt[3]] - ((1 - I*Sqrt[3])*Log[1 - I*Sqr
t[3] + 2*Sinh[x]]^2)/2 - (1 + I*Sqrt[3])*Log[((I/2)*(1 - I*Sqrt[3] + 2*Sinh
[x]))/Sqrt[3]]*Log[1 + I*Sqrt[3] + 2*Sinh[x]] - ((1 + I*Sqrt[3])*Log[1 + I*
Sqrt[3] + 2*Sinh[x]]^2)/2 - (1 - I*Sqrt[3])*Log[1 - I*Sqrt[3] + 2*Sinh[x]]*
Log[((-I/2)*(1 + I*Sqrt[3] + 2*Sinh[x]))/Sqrt[3]] - 2*Log[1 + Sinh[x] + Sin
h[x]^2] + (1 - I*Sqrt[3])*Log[1 - I*Sqrt[3] + 2*Sinh[x]]*Log[1 + Sinh[x] +
Sinh[x]^2] + (1 + I*Sqrt[3])*Log[1 + I*Sqrt[3] + 2*Sinh[x]]*Log[1 + Sinh[x]
 + Sinh[x]^2] - (1 + I*Sqrt[3])*PolyLog[2, -(I - Sqrt[3] + (2*I)*Sinh[x])/(
2*Sqrt[3])] - (1 - I*Sqrt[3])*PolyLog[2, (I + Sqrt[3] + (2*I)*Sinh[x])/(2*S
qrt[3])] + 8*Sinh[x] - 4*Log[1 + Sinh[x] + Sinh[x]^2]*Sinh[x] + Log[1 + Sin
h[x] + Sinh[x]^2]^2*Sinh[x]
```

Rubi [A] time = 0.536977, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 28, number of rules used = 15, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.154, Rules used = {4358, 2523, 2528, 773, 634, 618, 204, 628, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\left(1+i\sqrt{3}\right)\text{PolyLog}\left(2,-\frac{2i\sinh(x)-\sqrt{3}+i}{2\sqrt{3}}\right)-\left(1-i\sqrt{3}\right)\text{PolyLog}\left(2,\frac{2i\sinh(x)+\sqrt{3}+i}{2\sqrt{3}}\right)+8\sinh(x)+\sinh(x)\log^2$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]*Log[Cosh[x]^2 + Sinh[x]]^2,x]
```


```
[Out] -4*Sqrt[3]*ArcTan[(1 + 2*Sinh[x])/Sqrt[3]] - ((1 - I*Sqrt[3])*Log[1 - I*Sqr
t[3] + 2*Sinh[x]]^2)/2 - (1 + I*Sqrt[3])*Log[((I/2)*(1 - I*Sqrt[3] + 2*Sinh
[x]))/Sqrt[3]]*Log[1 + I*Sqrt[3] + 2*Sinh[x]] - ((1 + I*Sqrt[3])*Log[1 + I*
Sqrt[3] + 2*Sinh[x]]^2)/2 - (1 - I*Sqrt[3])*Log[1 - I*Sqrt[3] + 2*Sinh[x]]*
Log[((-I/2)*(1 + I*Sqrt[3] + 2*Sinh[x]))/Sqrt[3]] - 2*Log[1 + Sinh[x] + Sin
h[x]^2] + (1 - I*Sqrt[3])*Log[1 - I*Sqrt[3] + 2*Sinh[x]]*Log[1 + Sinh[x] +
Sinh[x]^2] + (1 + I*Sqrt[3])*Log[1 + I*Sqrt[3] + 2*Sinh[x]]*Log[1 + Sinh[x]
 + Sinh[x]^2] - (1 + I*Sqrt[3])*PolyLog[2, -(I - Sqrt[3] + (2*I)*Sinh[x])/(
2*Sqrt[3])] - (1 - I*Sqrt[3])*PolyLog[2, (I + Sqrt[3] + (2*I)*Sinh[x])/(2*S
qrt[3])] + 8*Sinh[x] - 4*Log[1 + Sinh[x] + Sinh[x]^2]*Sinh[x] + Log[1 + Sin
```

```
h[x] + Sinh[x]^2]^2*Sinh[x]
```

Rule 4358

```
Int[Cosh[(c_.)*((a_.) + (b_.)*(x_))]*(u_), x_Symbol] :> With[{d = FreeFacto
rs[Sinh[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sinh[c*(a +
b*x)]/d, u, x], x], x, Sinh[c*(a + b*x)]/d], x] /; FunctionOfQ[Sinh[c*(a +
b*x)]/d, u, x]] /; FreeQ[{a, b, c}, x]
```

Rule 2523

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a +
 b*Log[c*RFx^p])^n, x] - Dist[b*n*p, Int[SimplifyIntegrand[(x*(a + b*Log[c*
RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, p}, x] && Rat
ionalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \cosh(x)\log^2\left(\cosh^2(x)+\sinh(x)\right)dx = \text{Subst}\left(\int \log^2\left(1+x+x^2\right)dx, x, \sinh(x)\right)$$

$$= \log^2\left(1+\sinh(x)+\sinh^2(x)\right)\sinh(x) - 2\,\text{Subst}\left(\int \frac{x(1+2x)\log\left(1+x+x^2\right.}{1+x+x^2}\right.$$

$$= \log^2\left(1+\sinh(x)+\sinh^2(x)\right)\sinh(x) - 2\,\text{Subst}\left(\int\left(2\log\left(1+x+x^2\right)-\frac{(2}{}\right.\right.$$

$$= \log^2\left(1+\sinh(x)+\sinh^2(x)\right)\sinh(x) + 2\,\text{Subst}\left(\int \frac{(2+x)\log\left(1+x+x^2\right)}{1+x+x^2}\right.$$

$$= -4\log\left(1+\sinh(x)+\sinh^2(x)\right)\sinh(x) + \log^2\left(1+\sinh(x)+\sinh^2(x)\right)\sinh$$

$$= 8\sinh(x) - 4\log\left(1+\sinh(x)+\sinh^2(x)\right)\sinh(x) + \log^2\left(1+\sinh(x)+\sinh\right.$$

$$= \left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2\sinh(x)\right)\log\left(1+\sinh(x)+\sinh^2(x)\right) + \left(1+i\sqrt{3}\right)$$

$$= -2\log\left(1+\sinh(x)+\sinh^2(x)\right) + \left(1-i\sqrt{3}\right)\log\left(1-i\sqrt{3}+2\sinh(x)\right)\log\left(1\right.$$

$$= -4\sqrt{3}\tan^{-1}\left(\frac{1+2\sinh(x)}{\sqrt{3}}\right) - 2\log\left(1+\sinh(x)+\sinh^2(x)\right) + \left(1-i\sqrt{3}\right)\log$$

$$= -4\sqrt{3}\tan^{-1}\left(\frac{1+2\sinh(x)}{\sqrt{3}}\right) - \left(1+i\sqrt{3}\right)\log\left(\frac{i\left(1-i\sqrt{3}+2\sinh(x)\right)}{2\sqrt{3}}\right)\log\left(1\right.$$

$$= -4\sqrt{3}\tan^{-1}\left(\frac{1+2\sinh(x)}{\sqrt{3}}\right) - \frac{1}{2}\left(1-i\sqrt{3}\right)\log^2\left(1-i\sqrt{3}+2\sinh(x)\right) - \left(1+\right.$$

$$= -4\sqrt{3}\tan^{-1}\left(\frac{1+2\sinh(x)}{\sqrt{3}}\right) - \frac{1}{2}\left(1-i\sqrt{3}\right)\log^2\left(1-i\sqrt{3}+2\sinh(x)\right) - \left(1+\right.$$

Mathematica [A] time = 0.222666, size = 347, normalized size = 0.88

$$-\frac{1}{2}i\left(\sqrt{3}-i\right)\left(2\text{PolyLog}\left(2,\frac{-2i\sinh(x)+\sqrt{3}-i}{2\sqrt{3}}\right)+\log\left(2\sinh(x)+i\sqrt{3}+1\right)\left(2\log\left(\frac{2i\sinh(x)+\sqrt{3}+i}{2\sqrt{3}}\right)+\log\left(2\sinh\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]*Log[Cosh[x]^2 + Sinh[x]]^2,x]
```

```
[Out] -4*Sqrt[3]*ArcTan[(1 + 2*Sinh[x])/Sqrt[3]] - 2*Log[1 + Sinh[x] + Sinh[x]^2]
 + (1 - I*Sqrt[3])*Log[1 - I*Sqrt[3] + 2*Sinh[x]]*Log[1 + Sinh[x] + Sinh[x]
^2] + (1 + I*Sqrt[3])*Log[1 + I*Sqrt[3] + 2*Sinh[x]]*Log[1 + Sinh[x] + Sinh
[x]^2] - (I/2)*(-I + Sqrt[3])*(Log[1 + I*Sqrt[3] + 2*Sinh[x]]*(2*Log[(I + S
qrt[3] + (2*I)*Sinh[x])/(2*Sqrt[3])] + Log[1 + I*Sqrt[3] + 2*Sinh[x]]) + 2*
PolyLog[2, (-I + Sqrt[3] - (2*I)*Sinh[x])/(2*Sqrt[3])]) + (I/2)*(I + Sqrt[3
])*(Log[1 - I*Sqrt[3] + 2*Sinh[x]]*(2*Log[(-I + Sqrt[3] - (2*I)*Sinh[x])/(2
*Sqrt[3])] + Log[1 - I*Sqrt[3] + 2*Sinh[x]]) + 2*PolyLog[2, (I + Sqrt[3] +
(2*I)*Sinh[x])/(2*Sqrt[3])]) + 8*Sinh[x] - 4*Log[1 + Sinh[x] + Sinh[x]^2]*S
inh[x] + Log[1 + Sinh[x] + Sinh[x]^2]^2*Sinh[x]
```

Maple [F] time = 3.147, size = 0, normalized size = 0.

$$\int \cosh(x) \left(\ln\left((\cosh(x))^2 + \sinh(x)\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)*ln(cosh(x)^2+sinh(x))^2,x)
```

```
[Out] int(cosh(x)*ln(cosh(x)^2+sinh(x))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*log(cosh(x)^2+sinh(x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*x) - 1)*e^(-x)*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)^2
 + 2*(2*x - e^(-x) - integrate((2*e^(3*x) + 5*e^(2*x) + 6*e^x - 2)*e^x/(e^(
4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1), x))*log(2)^2 - 4*(x - integrate(
(e^(3*x) + 2*e^(2*x) + 2*e^x - 2)*e^x/(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*
e^x + 1), x))*log(2)^2 + 2*(e^x - integrate((2*e^(3*x) + 2*e^(2*x) - 2*e^x
+ 1)*e^x/(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1), x))*log(2)^2 + 4*in
tegrate(e^(4*x)/(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1), x)*log(2)^2
```

```
+ 6*integrate(e^(3*x)/(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1), x)*log
(2)^2 + 6*integrate(e^x/(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1), x)*l
og(2)^2 + 4*integrate(x*e^(6*x)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x
) + e^x), x)*log(2) + 8*integrate(x*e^(5*x)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x
) - 2*e^(2*x) + e^x), x)*log(2) + 12*integrate(x*e^(4*x)/(e^(5*x) + 2*e^(4*
x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x)*log(2) + 12*integrate(x*e^(2*x)/(e^(5
*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x)*log(2) - 8*integrate(x*e
^x/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x)*log(2) - 2*integ
rate(e^(6*x)*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*
e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x)*log(2) - 4*integrate(e^(5*x)*log
(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3
*x) - 2*e^(2*x) + e^x), x)*log(2) - 6*integrate(e^(4*x)*log(e^(4*x) + 2*e^(
3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x)
+ e^x), x)*log(2) - 6*integrate(e^(2*x)*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x)
 - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x)*log(2
) + 4*integrate(e^x*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*
x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x)*log(2) + 4*integrate(x/(e
^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x)*log(2) - 2*integrate(
log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) + 2*e
^(3*x) - 2*e^(2*x) + e^x), x)*log(2) + 2*integrate(x^2*e^(6*x)/(e^(5*x) + 2
*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) + 4*integrate(x^2*e^(5*x)/(e^(5
*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) + 6*integrate(x^2*e^(4*x
)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) + 6*integrate(x^2
*e^(2*x)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) - 4*integr
ate(x^2*e^x/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) - 2*int
egrate(x*e^(6*x)*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x)
+ 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) - 4*integrate(x*e^(5*x)*log(
e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*
x) - 2*e^(2*x) + e^x), x) - 6*integrate(x*e^(4*x)*log(e^(4*x) + 2*e^(3*x) +
 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x)
, x) - 6*integrate(x*e^(2*x)*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x +
1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) + 4*integrate(x*
e^x*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) +
 2*e^(3*x) - 2*e^(2*x) + e^x), x) + 2*integrate(x^2/(e^(5*x) + 2*e^(4*x) +
2*e^(3*x) - 2*e^(2*x) + e^x), x) - 2*integrate(x*log(e^(4*x) + 2*e^(3*x) +
2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x),
 x) - 4*integrate(e^(6*x)*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/
(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) - 6*integrate(e^(5*
x)*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) +
2*e^(3*x) - 2*e^(2*x) + e^x), x) + 8*integrate(e^(3*x)*log(e^(4*x) + 2*e^(3
*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) +
 e^x), x) + 4*integrate(e^(2*x)*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x
 + 1)/(e^(5*x) + 2*e^(4*x) + 2*e^(3*x) - 2*e^(2*x) + e^x), x) - 2*integrate
(e^x*log(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1)/(e^(5*x) + 2*e^(4*x)
+ 2*e^(3*x) - 2*e^(2*x) + e^x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\cosh\left(x\right)\log\left(\cosh\left(x\right)^{2}+\sinh\left(x\right)\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*log(cosh(x)^2+sinh(x))^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(x)*log(cosh(x)^2 + sinh(x))^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*ln(cosh(x)**2+sinh(x))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\text{sage}_{0}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)*log(cosh(x)^2+sinh(x))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

3.30 $\int \frac{\log\left(x+\sqrt{1+x}\right)}{1+x^2}\,dx$

Optimal. Leaf size=981

result too large to display

```
[Out] (I/2)*Log[Sqrt[1 - I] - Sqrt[1 + x]]*Log[x + Sqrt[1 + x]] - (I/2)*Log[Sqrt[
1 + I] - Sqrt[1 + x]]*Log[x + Sqrt[1 + x]] + (I/2)*Log[Sqrt[1 - I] + Sqrt[1
 + x]]*Log[x + Sqrt[1 + x]] - (I/2)*Log[Sqrt[1 + I] + Sqrt[1 + x]]*Log[x +
Sqrt[1 + x]] - (I/2)*Log[Sqrt[1 - I] + Sqrt[1 + x]]*Log[(1 - Sqrt[5] + 2*Sq
rt[1 + x])/(1 - 2*Sqrt[1 - I] - Sqrt[5])] - (I/2)*Log[Sqrt[1 - I] - Sqrt[1
+ x]]*Log[(1 - Sqrt[5] + 2*Sqrt[1 + x])/(1 + 2*Sqrt[1 - I] - Sqrt[5])] + (I
/2)*Log[Sqrt[1 + I] + Sqrt[1 + x]]*Log[(1 - Sqrt[5] + 2*Sqrt[1 + x])/(1 - 2
*Sqrt[1 + I] - Sqrt[5])] + (I/2)*Log[Sqrt[1 + I] - Sqrt[1 + x]]*Log[(1 - Sq
rt[5] + 2*Sqrt[1 + x])/(1 + 2*Sqrt[1 + I] - Sqrt[5])] - (I/2)*Log[Sqrt[1 -
I] + Sqrt[1 + x]]*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 - 2*Sqrt[1 - I] + Sq
rt[5])] - (I/2)*Log[Sqrt[1 - I] - Sqrt[1 + x]]*Log[(1 + Sqrt[5] + 2*Sqrt[1
+ x])/(1 + 2*Sqrt[1 - I] + Sqrt[5])] + (I/2)*Log[Sqrt[1 + I] + Sqrt[1 + x]]
*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 - 2*Sqrt[1 + I] + Sqrt[5])] + (I/2)*L
og[Sqrt[1 + I] - Sqrt[1 + x]]*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 + 2*Sqrt
[1 + I] + Sqrt[5])] - (I/2)*PolyLog[2, (2*(Sqrt[1 - I] - Sqrt[1 + x]))/(1 +
 2*Sqrt[1 - I] - Sqrt[5])] - (I/2)*PolyLog[2, (2*(Sqrt[1 - I] - Sqrt[1 + x]
))/(1 + 2*Sqrt[1 - I] + Sqrt[5])] + (I/2)*PolyLog[2, (2*(Sqrt[1 + I] - Sqrt
[1 + x]))/(1 + 2*Sqrt[1 + I] - Sqrt[5])] + (I/2)*PolyLog[2, (2*(Sqrt[1 + I]
 - Sqrt[1 + x]))/(1 + 2*Sqrt[1 + I] + Sqrt[5])] - (I/2)*PolyLog[2, (-2*(Sqr
t[1 - I] + Sqrt[1 + x]))/(1 - 2*Sqrt[1 - I] - Sqrt[5])] - (I/2)*PolyLog[2,
(-2*(Sqrt[1 - I] + Sqrt[1 + x]))/(1 - 2*Sqrt[1 - I] + Sqrt[5])] + (I/2)*Pol
yLog[2, (-2*(Sqrt[1 + I] + Sqrt[1 + x]))/(1 - 2*Sqrt[1 + I] - Sqrt[5])] + (
I/2)*PolyLog[2, (-2*(Sqrt[1 + I] + Sqrt[1 + x]))/(1 - 2*Sqrt[1 + I] + Sqrt[
5])]
```

Rubi [A] time = 1.25169, antiderivative size = 981, normalized size of antiderivative = 1., number of steps used = 44, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {2530, 1591, 203, 6741, 2528, 2524, 2418, 2394, 2393, 2391}

$$\frac{1}{2}i\log\left(\sqrt{1-i}-\sqrt{x+1}\right)\log\left(x+\sqrt{x+1}\right)-\frac{1}{2}i\log\left(\sqrt{1+i}-\sqrt{x+1}\right)\log\left(x+\sqrt{x+1}\right)+\frac{1}{2}i\log\left(\sqrt{x+1}+\sqrt{1-i}\right)\log\left(x\right.$$

Antiderivative was successfully verified.

```
[In] Int[Log[x + Sqrt[1 + x]]/(1 + x^2),x]
```

```
[Out] (I/2)*Log[Sqrt[1 - I] - Sqrt[1 + x]]*Log[x + Sqrt[1 + x]] - (I/2)*Log[Sqrt[
1 + I] - Sqrt[1 + x]]*Log[x + Sqrt[1 + x]] + (I/2)*Log[Sqrt[1 - I] + Sqrt[1
 + x]]*Log[x + Sqrt[1 + x]] - (I/2)*Log[Sqrt[1 + I] + Sqrt[1 + x]]*Log[x +
Sqrt[1 + x]] - (I/2)*Log[Sqrt[1 - I] + Sqrt[1 + x]]*Log[(1 - Sqrt[5] + 2*Sq
rt[1 + x])/(1 - 2*Sqrt[1 - I] - Sqrt[5])] - (I/2)*Log[Sqrt[1 - I] - Sqrt[1
+ x]]*Log[(1 - Sqrt[5] + 2*Sqrt[1 + x])/(1 + 2*Sqrt[1 - I] - Sqrt[5])] + (I
/2)*Log[Sqrt[1 + I] + Sqrt[1 + x]]*Log[(1 - Sqrt[5] + 2*Sqrt[1 + x])/(1 - 2
*Sqrt[1 + I] - Sqrt[5])] + (I/2)*Log[Sqrt[1 + I] - Sqrt[1 + x]]*Log[(1 - Sq
rt[5] + 2*Sqrt[1 + x])/(1 + 2*Sqrt[1 + I] - Sqrt[5])] - (I/2)*Log[Sqrt[1 -
I] + Sqrt[1 + x]]*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 - 2*Sqrt[1 - I] + Sq
rt[5])] - (I/2)*Log[Sqrt[1 - I] - Sqrt[1 + x]]*Log[(1 + Sqrt[5] + 2*Sqrt[1
+ x])/(1 + 2*Sqrt[1 - I] + Sqrt[5])] + (I/2)*Log[Sqrt[1 + I] + Sqrt[1 + x]]
*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 - 2*Sqrt[1 + I] + Sqrt[5])] + (I/2)*L
og[Sqrt[1 + I] - Sqrt[1 + x]]*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 + 2*Sqrt
[1 + I] + Sqrt[5])] - (I/2)*PolyLog[2, (2*(Sqrt[1 - I] - Sqrt[1 + x]))/(1 +
 2*Sqrt[1 - I] - Sqrt[5])] - (I/2)*PolyLog[2, (2*(Sqrt[1 - I] - Sqrt[1 + x]
))/(1 + 2*Sqrt[1 - I] + Sqrt[5])] + (I/2)*PolyLog[2, (2*(Sqrt[1 + I] - Sqrt
[1 + x]))/(1 + 2*Sqrt[1 + I] - Sqrt[5])] + (I/2)*PolyLog[2, (2*(Sqrt[1 + I]
 - Sqrt[1 + x]))/(1 + 2*Sqrt[1 + I] + Sqrt[5])] - (I/2)*PolyLog[2, (-2*(Sqr
t[1 - I] + Sqrt[1 + x]))/(1 - 2*Sqrt[1 - I] - Sqrt[5])] - (I/2)*PolyLog[2,
(-2*(Sqrt[1 - I] + Sqrt[1 + x]))/(1 - 2*Sqrt[1 - I] + Sqrt[5])] + (I/2)*Pol
yLog[2, (-2*(Sqrt[1 + I] + Sqrt[1 + x]))/(1 - 2*Sqrt[1 + I] - Sqrt[5])] + (
I/2)*PolyLog[2, (-2*(Sqrt[1 + I] + Sqrt[1 + x]))/(1 - 2*Sqrt[1 + I] + Sqrt[
5])]
```

<u>Rule 2530</u>

```
Int[((a_.) + Log[u_]*(b_.))*(RFx_), x_Symbol] :> With[{lst = SubstForFracti
onalPowerOfLinear[RFx*(a + b*Log[u]), x]}, Dist[lst[[2]]*lst[[4]], Subst[In
t[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x] /;  !FalseQ[lst]] /; FreeQ[{a
, b}, x] && RationalFunctionQ[RFx, x]
```

<u>Rule 1591</u>

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
 PolyQ[Qr, x]
```

<u>Rule 203</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\log\left(x+\sqrt{1+x}\right)}{1+x^2}\,dx &= 2\,\text{Subst}\left(\int \frac{x\log\left(-1+x+x^2\right)}{1+\left(-1+x^2\right)^2}\,dx, x, \sqrt{1+x}\right)\\
&= 2\,\text{Subst}\left(\int \frac{x\log\left(-1+x+x^2\right)}{2-2x^2+x^4}\,dx, x, \sqrt{1+x}\right)\\
&= 2\,\text{Subst}\left(\int \left(\frac{ix\log\left(-1+x+x^2\right)}{(2+2i)-2x^2}+\frac{ix\log\left(-1+x+x^2\right)}{(-2+2i)+2x^2}\right)dx, x, \sqrt{1+x}\right)\\
&= 2i\,\text{Subst}\left(\int \frac{x\log\left(-1+x+x^2\right)}{(2+2i)-2x^2}\,dx, x, \sqrt{1+x}\right)+2i\,\text{Subst}\left(\int \frac{x\log\left(-1+x+x^2\right)}{(-2+2i)+2x^2}\,dx, x, \sqrt{1 \cdots}\right.\\
&= 2i\,\text{Subst}\left(\int \left(-\frac{\log\left(-1+x+x^2\right)}{4\left(\sqrt{1-i}-x\right)}+\frac{\log\left(-1+x+x^2\right)}{4\left(\sqrt{1-i}+x\right)}\right)dx, x, \sqrt{1+x}\right)+2i\,\text{Subst}\left(\int\left(\frac{\log\left(\cdots\right.}{4\left(\sqrt{\cdots}\right.}\right.\right.\\
&= -\left(\frac{1}{2}i\,\text{Subst}\left(\int \frac{\log\left(-1+x+x^2\right)}{\sqrt{1-i}-x}\,dx, x, \sqrt{1+x}\right)\right)+\frac{1}{2}i\,\text{Subst}\left(\int \frac{\log\left(-1+x+x^2\right)}{\sqrt{1+i}-x}\,dx, x, \sqrt{\cdots}\right.\\
&= \frac{1}{2}i\log\left(\sqrt{1-i}-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right)-\frac{1}{2}i\log\left(\sqrt{1+i}-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right)+\frac{1}{2}i\,\cdots\\
&= \frac{1}{2}i\log\left(\sqrt{1-i}-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right)-\frac{1}{2}i\log\left(\sqrt{1+i}-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right)+\frac{1}{2}i\,\cdots\\
&= \frac{1}{2}i\log\left(\sqrt{1-i}-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right)-\frac{1}{2}i\log\left(\sqrt{1+i}-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right)+\frac{1}{2}i\,\cdots\\
&= \frac{1}{2}i\log\left(\sqrt{1-i}-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right)-\frac{1}{2}i\log\left(\sqrt{1+i}-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right)+\frac{1}{2}i\,\cdots\\
&= \frac{1}{2}i\log\left(\sqrt{1-i}-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right)-\frac{1}{2}i\log\left(\sqrt{1+i}-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right)+\frac{1}{2}i\,\cdots\\
&= \frac{1}{2}i\log\left(\sqrt{1-i}-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right)-\frac{1}{2}i\log\left(\sqrt{1+i}-\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right)+\frac{1}{2}i\,\cdots
\end{aligned}$$

Mathematica [A] time = 0.481055, size = 868, normalized size = 0.88

$$\frac{1}{2}i\left(2i\tan^{-1}(x)\log\left(\sqrt{x+1}-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)+\log\left(\frac{2\left(\sqrt{1-i}-\sqrt{x+1}\right)}{1+2\sqrt{1-i}-\sqrt{5}}\right)\log\left(\sqrt{x+1}-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)-\log\left(\frac{2\left(\sqrt{1+i}-\sqrt{x+1}\right)}{1+2\sqrt{1+i}-\sqrt{5}}\right)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x + Sqrt[1 + x]]/(1 + x^2),x]
```

```
[Out] (I/2)*((2*I)*ArcTan[x]*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]] + Log[(2*(Sqrt[1
- I] - Sqrt[1 + x]))/(1 + 2*Sqrt[1 - I] - Sqrt[5])]*Log[1/2 - Sqrt[5]/2 + S
qrt[1 + x]] - Log[(2*(Sqrt[1 + I] - Sqrt[1 + x]))/(1 + 2*Sqrt[1 + I] - Sqrt
[5])]*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]] + Log[(2*(Sqrt[1 - I] + Sqrt[1 + x
]))/(-1 + 2*Sqrt[1 - I] + Sqrt[5])]*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]] - Lo
g[(2*(Sqrt[1 + I] + Sqrt[1 + x]))/(-1 + 2*Sqrt[1 + I] + Sqrt[5])]*Log[1/2 -
 Sqrt[5]/2 + Sqrt[1 + x]] + (2*I)*ArcTan[x]*Log[(1 + Sqrt[5])/2 + Sqrt[1 +
x]] + Log[(2*(Sqrt[1 - I] - Sqrt[1 + x]))/(1 + 2*Sqrt[1 - I] + Sqrt[5])]*Lo
g[(1 + Sqrt[5])/2 + Sqrt[1 + x]] - Log[(2*(Sqrt[1 + I] - Sqrt[1 + x]))/(1 +
 2*Sqrt[1 + I] + Sqrt[5])]*Log[(1 + Sqrt[5])/2 + Sqrt[1 + x]] + Log[(2*(Sqr
t[1 - I] + Sqrt[1 + x]))/(-1 + 2*Sqrt[1 - I] - Sqrt[5])]*Log[(1 + Sqrt[5])/
2 + Sqrt[1 + x]] - Log[(2*(Sqrt[1 + I] + Sqrt[1 + x]))/(-1 + 2*Sqrt[1 + I]
- Sqrt[5])]*Log[(1 + Sqrt[5])/2 + Sqrt[1 + x]] - (2*I)*ArcTan[x]*Log[x + Sq
rt[1 + x]] + PolyLog[2, (-1 + Sqrt[5] - 2*Sqrt[1 + x])/(-1 + 2*Sqrt[1 - I]
+ Sqrt[5])] - PolyLog[2, (-1 + Sqrt[5] - 2*Sqrt[1 + x])/(-1 + 2*Sqrt[1 + I]
 + Sqrt[5])] + PolyLog[2, (1 - Sqrt[5] + 2*Sqrt[1 + x])/(1 + 2*Sqrt[1 - I]
- Sqrt[5])] - PolyLog[2, (1 - Sqrt[5] + 2*Sqrt[1 + x])/(1 + 2*Sqrt[1 + I] -
 Sqrt[5])] + PolyLog[2, (1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 - 2*Sqrt[1 - I] +
Sqrt[5])] + PolyLog[2, (1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 + 2*Sqrt[1 - I] + S
qrt[5])] - PolyLog[2, (1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 - 2*Sqrt[1 + I] + Sq
rt[5])] - PolyLog[2, (1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 + 2*Sqrt[1 + I] + Sqr
t[5])])
```

Maple [A] time = 0.051, size = 698, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+(1+x)^(1/2))/(x^2+1),x)
```

```
[Out] -1/2*I*ln((1+x)^(1/2)-(1-I)^(1/2))*ln((-1+5^(1/2)-2*(1+x)^(1/2))/(-2*(1-I)^
(1/2)-1+5^(1/2)))-1/2*I*dilog((-1+5^(1/2)-2*(1+x)^(1/2))/(2*(1-I)^(1/2)-1+5
```

```
^(1/2)))+1/2*I*ln((1+x)^(1/2)-(1+I)^(1/2))*ln((-1+5^(1/2)-2*(1+x)^(1/2))/(-
2*(1+I)^(1/2)-1+5^(1/2)))-1/2*I*ln(x+(1+x)^(1/2))*ln((1+x)^(1/2)-(1+I)^(1/2
))+1/2*I*ln((1+x)^(1/2)-(1+I)^(1/2))*ln((1+5^(1/2)+2*(1+x)^(1/2))/(1+2*(1+I
)^(1/2)+5^(1/2)))+1/2*I*ln((1+I)^(1/2)+(1+x)^(1/2))*ln((-1+5^(1/2)-2*(1+x)^
(1/2))/(2*(1+I)^(1/2)-1+5^(1/2)))+1/2*I*ln(x+(1+x)^(1/2))*ln((1-I)^(1/2)+(1
+x)^(1/2))-1/2*I*dilog((-1+5^(1/2)-2*(1+x)^(1/2))/(-2*(1-I)^(1/2)-1+5^(1/2)
))+1/2*I*dilog((-1+5^(1/2)-2*(1+x)^(1/2))/(2*(1+I)^(1/2)-1+5^(1/2)))-1/2*I*
dilog((1+5^(1/2)+2*(1+x)^(1/2))/(1+2*(1-I)^(1/2)+5^(1/2)))+1/2*I*ln((1+I)^(
1/2)+(1+x)^(1/2))*ln((1+5^(1/2)+2*(1+x)^(1/2))/(1-2*(1+I)^(1/2)+5^(1/2)))-1
/2*I*ln((1+x)^(1/2)-(1-I)^(1/2))*ln((1+5^(1/2)+2*(1+x)^(1/2))/(1+2*(1-I)^(1
/2)+5^(1/2)))-1/2*I*ln((1-I)^(1/2)+(1+x)^(1/2))*ln((1+5^(1/2)+2*(1+x)^(1/2)
)/(1-2*(1-I)^(1/2)+5^(1/2)))-1/2*I*dilog((1+5^(1/2)+2*(1+x)^(1/2))/(1-2*(1-
I)^(1/2)+5^(1/2)))-1/2*I*ln(x+(1+x)^(1/2))*ln((1+I)^(1/2)+(1+x)^(1/2))+1/2*
I*dilog((-1+5^(1/2)-2*(1+x)^(1/2))/(-2*(1+I)^(1/2)-1+5^(1/2)))+1/2*I*dilog(
(1+5^(1/2)+2*(1+x)^(1/2))/(1-2*(1+I)^(1/2)+5^(1/2)))+1/2*I*ln(x+(1+x)^(1/2)
)*ln((1+x)^(1/2)-(1-I)^(1/2))+1/2*I*dilog((1+5^(1/2)+2*(1+x)^(1/2))/(1+2*(1
+I)^(1/2)+5^(1/2)))-1/2*I*ln((1-I)^(1/2)+(1+x)^(1/2))*ln((-1+5^(1/2)-2*(1+x
)^(1/2))/(2*(1-I)^(1/2)-1+5^(1/2)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(x+\sqrt{x+1}\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(1+x)^(1/2))/(x^2+1),x, algorithm="maxima")
```

```
[Out] integrate(log(x + sqrt(x + 1))/(x^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(x+\sqrt{x+1}\right)}{x^2+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(1+x)^(1/2))/(x^2+1),x, algorithm="fricas")
```

```
[Out] integral(log(x + sqrt(x + 1))/(x^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(x+\sqrt{x+1}\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x+(1+x)**(1/2))/(x**2+1),x)
```

```
[Out] Integral(log(x + sqrt(x + 1))/(x**2 + 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(x+\sqrt{x+1}\right)}{x^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(1+x)^(1/2))/(x^2+1),x, algorithm="giac")
```

```
[Out] integrate(log(x + sqrt(x + 1))/(x^2 + 1), x)
```

3.31 $\int \frac{\log^2\left(x+\sqrt{1+x}\right)}{(1+x)^2}\,dx$

Optimal. Leaf size=555

$$6\text{PolyLog}\left(2,-\frac{2\sqrt{x+1}}{1+\sqrt{5}}\right)-\left(3+\sqrt{5}\right)\text{PolyLog}\left(2,-\frac{2\sqrt{x+1}-\sqrt{5}+1}{2\sqrt{5}}\right)-\left(3-\sqrt{5}\right)\text{PolyLog}\left(2,\frac{2\sqrt{x+1}+\sqrt{5}+1}{2\sqrt{5}}\right)-$$

```
[Out] Log[1 + x] + (2*Log[x + Sqrt[1 + x]])/Sqrt[1 + x] - 6*Log[Sqrt[1 + x]]*Log[
x + Sqrt[1 + x]] - Log[x + Sqrt[1 + x]]^2/(1 + x) - (1 + Sqrt[5])*Log[1 - S
qrt[5] + 2*Sqrt[1 + x]] + 6*Log[(-1 + Sqrt[5])/2]*Log[1 - Sqrt[5] + 2*Sqrt[
1 + x]] + (3 + Sqrt[5])*Log[x + Sqrt[1 + x]]*Log[1 - Sqrt[5] + 2*Sqrt[1 + x
]] - ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 + x]]^2)/2 - (1 - Sqrt[5])*L
og[1 + Sqrt[5] + 2*Sqrt[1 + x]] + (3 - Sqrt[5])*Log[x + Sqrt[1 + x]]*Log[1
+ Sqrt[5] + 2*Sqrt[1 + x]] - (3 - Sqrt[5])*Log[-(1 - Sqrt[5] + 2*Sqrt[1 + x
])/(2*Sqrt[5])]*Log[1 + Sqrt[5] + 2*Sqrt[1 + x]] - ((3 - Sqrt[5])*Log[1 + S
qrt[5] + 2*Sqrt[1 + x]]^2)/2 - (3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 + x
]]*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])/(2*Sqrt[5])] + 6*Log[Sqrt[1 + x]]*Log[
1 + (2*Sqrt[1 + x])/(1 + Sqrt[5])] + 6*PolyLog[2, (-2*Sqrt[1 + x])/(1 + Sqr
t[5])] - (3 + Sqrt[5])*PolyLog[2, -(1 - Sqrt[5] + 2*Sqrt[1 + x])/(2*Sqrt[5]
)] - (3 - Sqrt[5])*PolyLog[2, (1 + Sqrt[5] + 2*Sqrt[1 + x])/(2*Sqrt[5])] -
6*PolyLog[2, 1 + (2*Sqrt[1 + x])/(1 - Sqrt[5])]
```

Rubi [A] time = 0.711506, antiderivative size = 555, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 16, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {2525, 2528, 800, 632, 31, 2524, 2357, 2316, 2315, 2317, 2391, 2418, 2390, 2301, 2394, 2393}

$$6\text{PolyLog}\left(2,-\frac{2\sqrt{x+1}}{1+\sqrt{5}}\right)-\left(3+\sqrt{5}\right)\text{PolyLog}\left(2,-\frac{2\sqrt{x+1}-\sqrt{5}+1}{2\sqrt{5}}\right)-\left(3-\sqrt{5}\right)\text{PolyLog}\left(2,\frac{2\sqrt{x+1}+\sqrt{5}+1}{2\sqrt{5}}\right)-$$

Antiderivative was successfully verified.

```
[In] Int[Log[x + Sqrt[1 + x]]^2/(1 + x)^2,x]
```

```
[Out] Log[1 + x] + (2*Log[x + Sqrt[1 + x]])/Sqrt[1 + x] - 6*Log[Sqrt[1 + x]]*Log[
x + Sqrt[1 + x]] - Log[x + Sqrt[1 + x]]^2/(1 + x) - (1 + Sqrt[5])*Log[1 - S
qrt[5] + 2*Sqrt[1 + x]] + 6*Log[(-1 + Sqrt[5])/2]*Log[1 - Sqrt[5] + 2*Sqrt[
1 + x]] + (3 + Sqrt[5])*Log[x + Sqrt[1 + x]]*Log[1 - Sqrt[5] + 2*Sqrt[1 + x
]] - ((3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 + x]]^2)/2 - (1 - Sqrt[5])*L
og[1 + Sqrt[5] + 2*Sqrt[1 + x]] + (3 - Sqrt[5])*Log[x + Sqrt[1 + x]]*Log[1
```

```
+ Sqrt[5] + 2*Sqrt[1 + x]] - (3 - Sqrt[5])*Log[-(1 - Sqrt[5] + 2*Sqrt[1 + x
])/(2*Sqrt[5])]*Log[1 + Sqrt[5] + 2*Sqrt[1 + x]] - ((3 - Sqrt[5])*Log[1 + S
qrt[5] + 2*Sqrt[1 + x]]^2)/2 - (3 + Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 + x
]]*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])/(2*Sqrt[5])] + 6*Log[Sqrt[1 + x]]*Log[
1 + (2*Sqrt[1 + x])/(1 + Sqrt[5])] + 6*PolyLog[2, (-2*Sqrt[1 + x])/(1 + Sqr
t[5])] - (3 + Sqrt[5])*PolyLog[2, -(1 - Sqrt[5] + 2*Sqrt[1 + x])/(2*Sqrt[5]
)] - (3 - Sqrt[5])*PolyLog[2, (1 + Sqrt[5] + 2*Sqrt[1 + x])/(2*Sqrt[5])] -
6*PolyLog[2, 1 + (2*Sqrt[1 + x])/(1 - Sqrt[5])]
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2357

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] :> With[{
u = ExpandIntegrand[(a + b*Log[c*x^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /
; FreeQ[{a, b, c, n}, x] && RationalFunctionQ[RFx, x] && IGtQ[p, 0]
```

Rule 2316

```
Int[((a_.) + Log[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[
((a + b*Log[-((c*d)/e)])*Log[d + e*x])/e, x] + Dist[b, Int[Log[-((e*x)/d)]/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[-((c*d)/e), 0]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rubi steps

$$\int \frac{\log^2\left(x+\sqrt{1+x}\right)}{(1+x)^2}\,dx = 2\,\text{Subst}\left(\int \frac{\log^2\left(-1+x+x^2\right)}{x^3}\,dx, x, \sqrt{1+x}\right)$$

$$= -\frac{\log^2\left(x+\sqrt{1+x}\right)}{1+x} + 2\,\text{Subst}\left(\int \frac{(1+2x)\log\left(-1+x+x^2\right)}{x^2\left(-1+x+x^2\right)}\,dx, x, \sqrt{1+x}\right)$$

$$= -\frac{\log^2\left(x+\sqrt{1+x}\right)}{1+x} + 2\,\text{Subst}\left(\int\left(-\frac{\log\left(-1+x+x^2\right)}{x^2} - \frac{3\log\left(-1+x+x^2\right)}{x} + \frac{(4+3x)\log}{-1}\right.\right.$$

$$= -\frac{\log^2\left(x+\sqrt{1+x}\right)}{1+x} - 2\,\text{Subst}\left(\int \frac{\log\left(-1+x+x^2\right)}{x^2}\,dx, x, \sqrt{1+x}\right) + 2\,\text{Subst}\left(\int \frac{(4+3x)\log}{-1}\right.$$

$$= \frac{2\log\left(x+\sqrt{1+x}\right)}{\sqrt{1+x}} - 6\log\left(\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \frac{\log^2\left(x+\sqrt{1+x}\right)}{1+x} - 2\,\text{Subst}\left(\int \frac{}{x(}\right.$$

$$= \frac{2\log\left(x+\sqrt{1+x}\right)}{\sqrt{1+x}} - 6\log\left(\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \frac{\log^2\left(x+\sqrt{1+x}\right)}{1+x} - 2\,\text{Subst}\left(\int\left(-\right.\right.$$

$$= \log(1+x) + \frac{2\log\left(x+\sqrt{1+x}\right)}{\sqrt{1+x}} - 6\log\left(\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \frac{\log^2\left(x+\sqrt{1+x}\right)}{1+x} + (3$$

$$= \log(1+x) + \frac{2\log\left(x+\sqrt{1+x}\right)}{\sqrt{1+x}} - 6\log\left(\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \frac{\log^2\left(x+\sqrt{1+x}\right)}{1+x} + 6$$

$$= \log(1+x) + \frac{2\log\left(x+\sqrt{1+x}\right)}{\sqrt{1+x}} - 6\log\left(\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \frac{\log^2\left(x+\sqrt{1+x}\right)}{1+x} - (1$$

$$= \log(1+x) + \frac{2\log\left(x+\sqrt{1+x}\right)}{\sqrt{1+x}} - 6\log\left(\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \frac{\log^2\left(x+\sqrt{1+x}\right)}{1+x} - (1$$

$$= \log(1+x) + \frac{2\log\left(x+\sqrt{1+x}\right)}{\sqrt{1+x}} - 6\log\left(\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \frac{\log^2\left(x+\sqrt{1+x}\right)}{1+x} - (1$$

$$= \log(1+x) + \frac{2\log\left(x+\sqrt{1+x}\right)}{\sqrt{1+x}} - 6\log\left(\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \frac{\log^2\left(x+\sqrt{1+x}\right)}{1+x} - (1$$

Mathematica [A] time = 1.42651, size = 1076, normalized size = 1.94

$$\frac{1}{2}\left(\sqrt{5}\log^2\left(\sqrt{x+1}-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)+3\log^2\left(\sqrt{x+1}-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)-12\log\left(\frac{2\sqrt{x+1}}{-1+\sqrt{5}}\right)\log\left(\sqrt{x+1}-\frac{\sqrt{5}}{2}+\frac{1}{2}\right)+6\log(x+1)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Log[x + Sqrt[1 + x]]^2/(1 + x)^2,x]
```

```
[Out] (2*Log[1 + x] - 6*Log[(1 + Sqrt[5])/2]*Log[1 + x] - 12*Log[(2*Sqrt[1 + x])/
(-1 + Sqrt[5])]*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]] + 6*Log[1 + x]*Log[1/2 -
 Sqrt[5]/2 + Sqrt[1 + x]] - 6*Log[-1 + Sqrt[5] - 2*Sqrt[1 + x]]*Log[1/2 - S
qrt[5]/2 + Sqrt[1 + x]] - 2*Sqrt[5]*Log[-1 + Sqrt[5] - 2*Sqrt[1 + x]]*Log[1
/2 - Sqrt[5]/2 + Sqrt[1 + x]] + 3*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]]^2 + Sq
rt[5]*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]]^2 + 6*Log[1 + x]*Log[(1 + Sqrt[5])
/2 + Sqrt[1 + x]] - 6*Log[-1 + Sqrt[5] - 2*Sqrt[1 + x]]*Log[(1 + Sqrt[5])/2
 + Sqrt[1 + x]] - 2*Sqrt[5]*Log[-1 + Sqrt[5] - 2*Sqrt[1 + x]]*Log[(1 + Sqrt
[5])/2 + Sqrt[1 + x]] + 3*Log[(1 + Sqrt[5])/2 + Sqrt[1 + x]]^2 - Sqrt[5]*Lo
g[(1 + Sqrt[5])/2 + Sqrt[1 + x]]^2 + (4*Log[x + Sqrt[1 + x]])/Sqrt[1 + x] -
 6*Log[1 + x]*Log[x + Sqrt[1 + x]] + 6*Log[-1 + Sqrt[5] - 2*Sqrt[1 + x]]*Lo
g[x + Sqrt[1 + x]] + 2*Sqrt[5]*Log[-1 + Sqrt[5] - 2*Sqrt[1 + x]]*Log[x + Sq
rt[1 + x]] - (2*Log[x + Sqrt[1 + x]]^2)/(1 + x) - 2*Log[1 - Sqrt[5] + 2*Sqr
t[1 + x]] - 2*Sqrt[5]*Log[1 - Sqrt[5] + 2*Sqrt[1 + x]] + 3*Log[5]*Log[1 - S
qrt[5] + 2*Sqrt[1 + x]] + Sqrt[5]*Log[5]*Log[1 - Sqrt[5] + 2*Sqrt[1 + x]] -
 2*Log[1 + Sqrt[5] + 2*Sqrt[1 + x]] + 2*Sqrt[5]*Log[1 + Sqrt[5] + 2*Sqrt[1
+ x]] - 6*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]]*Log[1 + Sqrt[5] + 2*Sqrt[1 + x
]] + 2*Sqrt[5]*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]]*Log[1 + Sqrt[5] + 2*Sqrt[
1 + x]] - 6*Log[(1 + Sqrt[5])/2 + Sqrt[1 + x]]*Log[1 + Sqrt[5] + 2*Sqrt[1 +
 x]] + 2*Sqrt[5]*Log[(1 + Sqrt[5])/2 + Sqrt[1 + x]]*Log[1 + Sqrt[5] + 2*Sqr
t[1 + x]] + 6*Log[x + Sqrt[1 + x]]*Log[1 + Sqrt[5] + 2*Sqrt[1 + x]] - 2*Sqr
t[5]*Log[x + Sqrt[1 + x]]*Log[1 + Sqrt[5] + 2*Sqrt[1 + x]] + 6*Log[1/2 - Sq
rt[5]/2 + Sqrt[1 + x]]*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])/(2*Sqrt[5])] - 2*S
qrt[5]*Log[1/2 - Sqrt[5]/2 + Sqrt[1 + x]]*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])
/(2*Sqrt[5])] + 12*PolyLog[2, (-2*Sqrt[1 + x])/(1 + Sqrt[5])] - 4*Sqrt[5]*P
olyLog[2, (-1 + Sqrt[5] - 2*Sqrt[1 + x])/(2*Sqrt[5])] - 12*PolyLog[2, (-1 +
 Sqrt[5] - 2*Sqrt[1 + x])/(-1 + Sqrt[5])])/2
```

Maple [F] time = 0.01, size = 0, normalized size = 0.

$$\int \frac{1}{(1+x)^2} \left(\ln\left(x+\sqrt{1+x}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+(1+x)^(1/2))^2/(1+x)^2,x)
```

```
[Out] int(ln(x+(1+x)^(1/2))^2/(1+x)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(x+\sqrt{x+1}\right)^{2}}{x+1}+\int\frac{\left(2\,x+\sqrt{x+1}+2\right)\log\left(x+\sqrt{x+1}\right)}{x^{3}+2\,x^{2}+\left(x^{2}+2\,x+1\right)\sqrt{x+1}+x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(1+x)^(1/2))^2/(1+x)^2,x, algorithm="maxima")
```

```
[Out] -log(x + sqrt(x + 1))^2/(x + 1) + integrate((2*x + sqrt(x + 1) + 2)*log(x +
 sqrt(x + 1))/(x^3 + 2*x^2 + (x^2 + 2*x + 1)*sqrt(x + 1) + x), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log\left(x+\sqrt{x+1}\right)^{2}}{x^{2}+2\,x+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(1+x)^(1/2))^2/(1+x)^2,x, algorithm="fricas")
```

```
[Out] integral(log(x + sqrt(x + 1))^2/(x^2 + 2*x + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\log\left(x+\sqrt{x+1}\right)^{2}}{\left(x+1\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x+(1+x)**(1/2))**2/(1+x)**2,x)
```

```
[Out] Integral(log(x + sqrt(x + 1))**2/(x + 1)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(x+\sqrt{x+1}\right)^2}{(x+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(1+x)^(1/2))^2/(1+x)^2,x, algorithm="giac")
```

```
[Out] integrate(log(x + sqrt(x + 1))^2/(x + 1)^2, x)
```

3.32 $\int \frac{\log\left(x+\sqrt{1+x}\right)}{x}\,dx$

Optimal. Leaf size=313

$$-\text{PolyLog}\left(2,\frac{2\left(1-\sqrt{x+1}\right)}{3-\sqrt{5}}\right)-\text{PolyLog}\left(2,\frac{2\left(1-\sqrt{x+1}\right)}{3+\sqrt{5}}\right)-\text{PolyLog}\left(2,\frac{2\left(\sqrt{x+1}+1\right)}{1-\sqrt{5}}\right)-\text{PolyLog}\left(2,\frac{2\left(\sqrt{x+1}\right.}{1+\sqrt{}}\right.$$

```
[Out] Log[-1 + Sqrt[1 + x]]*Log[x + Sqrt[1 + x]] + Log[1 + Sqrt[1 + x]]*Log[x + S
qrt[1 + x]] - Log[-1 + Sqrt[1 + x]]*Log[(1 - Sqrt[5] + 2*Sqrt[1 + x])/(3 -
Sqrt[5])] - Log[1 + Sqrt[1 + x]]*Log[-((1 - Sqrt[5] + 2*Sqrt[1 + x])/(1 + S
qrt[5]))] - Log[1 + Sqrt[1 + x]]*Log[-((1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 - S
qrt[5]))] - Log[-1 + Sqrt[1 + x]]*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])/(3 + Sq
rt[5])] - PolyLog[2, (2*(1 - Sqrt[1 + x]))/(3 - Sqrt[5])] - PolyLog[2, (2*(
1 - Sqrt[1 + x]))/(3 + Sqrt[5])] - PolyLog[2, (2*(1 + Sqrt[1 + x]))/(1 - Sq
rt[5])] - PolyLog[2, (2*(1 + Sqrt[1 + x]))/(1 + Sqrt[5])]
```

Rubi [A] time = 0.377384, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2530, 2528, 2524, 2418, 2394, 2393, 2391}

$$-\text{PolyLog}\left(2,\frac{2\left(1-\sqrt{x+1}\right)}{3-\sqrt{5}}\right)-\text{PolyLog}\left(2,\frac{2\left(1-\sqrt{x+1}\right)}{3+\sqrt{5}}\right)-\text{PolyLog}\left(2,\frac{2\left(\sqrt{x+1}+1\right)}{1-\sqrt{5}}\right)-\text{PolyLog}\left(2,\frac{2\left(\sqrt{x+1}\right.}{1+\sqrt{}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Log[x + Sqrt[1 + x]]/x,x]
```

```
[Out] Log[-1 + Sqrt[1 + x]]*Log[x + Sqrt[1 + x]] + Log[1 + Sqrt[1 + x]]*Log[x + S
qrt[1 + x]] - Log[-1 + Sqrt[1 + x]]*Log[(1 - Sqrt[5] + 2*Sqrt[1 + x])/(3 -
Sqrt[5])] - Log[1 + Sqrt[1 + x]]*Log[-((1 - Sqrt[5] + 2*Sqrt[1 + x])/(1 + S
qrt[5]))] - Log[1 + Sqrt[1 + x]]*Log[-((1 + Sqrt[5] + 2*Sqrt[1 + x])/(1 - S
qrt[5]))] - Log[-1 + Sqrt[1 + x]]*Log[(1 + Sqrt[5] + 2*Sqrt[1 + x])/(3 + Sq
rt[5])] - PolyLog[2, (2*(1 - Sqrt[1 + x]))/(3 - Sqrt[5])] - PolyLog[2, (2*(
1 - Sqrt[1 + x]))/(3 + Sqrt[5])] - PolyLog[2, (2*(1 + Sqrt[1 + x]))/(1 - Sq
rt[5])] - PolyLog[2, (2*(1 + Sqrt[1 + x]))/(1 + Sqrt[5])]
```

Rule 2530

```
Int[((a_.) + Log[u_]*(b_.))*(RFx_), x_Symbol] :> With[{lst = SubstForFracti
onalPowerOfLinear[RFx*(a + b*Log[u]), x]}, Dist[lst[[2]]*lst[[4]], Subst[In
```

```
t[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x] /;  !FalseQ[lst]] /; FreeQ[{a
, b}, x] && RationalFunctionQ[RFx, x]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\log\left(x+\sqrt{1+x}\right)}{x}\,dx = 2\,\mathrm{Subst}\left(\int \frac{x\log\left(-1+x+x^2\right)}{-1+x^2}\,dx, x, \sqrt{1+x}\right)$$
$$= 2\,\mathrm{Subst}\left(\int\left(\frac{\log\left(-1+x+x^2\right)}{2(-1+x)} + \frac{\log\left(-1+x+x^2\right)}{2(1+x)}\right)dx, x, \sqrt{1+x}\right)$$
$$= \mathrm{Subst}\left(\int \frac{\log\left(-1+x+x^2\right)}{-1+x}\,dx, x, \sqrt{1+x}\right) + \mathrm{Subst}\left(\int \frac{\log\left(-1+x+x^2\right)}{1+x}\,dx, x, \sqrt{1+x}\right)$$
$$= \log\left(-1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) + \log\left(1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \mathrm{Subst}\left(\int \frac{(1+2x}{-1}\right.$$
$$= \log\left(-1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) + \log\left(1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \mathrm{Subst}\left(\int\left(\frac{2\log(}{1-\sqrt{}}\right.\right.$$
$$= \log\left(-1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) + \log\left(1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - 2\,\mathrm{Subst}\left(\int \frac{\log(-}{1-\sqrt{}}\right.$$
$$= \log\left(-1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) + \log\left(1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \log\left(-1+\sqrt{1+x}\right)$$
$$= \log\left(-1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) + \log\left(1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \log\left(-1+\sqrt{1+x}\right)$$
$$= \log\left(-1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) + \log\left(1+\sqrt{1+x}\right)\log\left(x+\sqrt{1+x}\right) - \log\left(-1+\sqrt{1+x}\right)$$

Mathematica [A] time = 0.0863093, size = 303, normalized size = 0.97

$$-\mathrm{PolyLog}\left(2, \frac{2\left(\sqrt{x+1}+1\right)}{1-\sqrt{5}}\right) + \mathrm{PolyLog}\left(2, \frac{2\sqrt{x+1}-\sqrt{5}+1}{3-\sqrt{5}}\right) + \mathrm{PolyLog}\left(2, -\frac{2\sqrt{x+1}-\sqrt{5}+1}{1+\sqrt{5}}\right) + \mathrm{PolyLog}\left(2,\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x + Sqrt[1 + x]]/x,x]
```

```
[Out] Log[1 - Sqrt[1 + x]]*Log[x + Sqrt[1 + x]] + Log[1 + Sqrt[1 + x]]*Log[x + Sq
rt[1 + x]] - Log[(3 - Sqrt[5])/2]*Log[1 - Sqrt[5] + 2*Sqrt[1 + x]] - Log[(1
 + Sqrt[5])/2]*Log[1 - Sqrt[5] + 2*Sqrt[1 + x]] - Log[(3 + Sqrt[5])/2]*Log[
1 + Sqrt[5] + 2*Sqrt[1 + x]] - Log[1 + Sqrt[1 + x]]*Log[-((1 + Sqrt[5] + 2*
Sqrt[1 + x])/(1 - Sqrt[5]))] - PolyLog[2, (2*(1 + Sqrt[1 + x]))/(1 - Sqrt[5
```

```
])] + PolyLog[2, (1 - Sqrt[5] + 2*Sqrt[1 + x])/(3 - Sqrt[5])] + PolyLog[2,
-((1 - Sqrt[5] + 2*Sqrt[1 + x])/(1 + Sqrt[5]))] + PolyLog[2, (1 + Sqrt[5] +
 2*Sqrt[1 + x])/(3 + Sqrt[5])]
```

Maple [A] time = 0.017, size = 252, normalized size = 0.8

$$\ln\left(-1+\sqrt{1+x}\right)\ln\left(x+\sqrt{1+x}\right)-\ln\left(-1+\sqrt{1+x}\right)\ln\left(\frac{1}{\sqrt{5}-3}\left(-1+\sqrt{5}-2\sqrt{1+x}\right)\right)-\ln\left(-1+\sqrt{1+x}\right)\ln\left(\frac{1}{3+\sqrt{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x+(1+x)^(1/2))/x,x)
```

```
[Out] ln(-1+(1+x)^(1/2))*ln(x+(1+x)^(1/2))-ln(-1+(1+x)^(1/2))*ln((-1+5^(1/2)-2*(1
+x)^(1/2))/(5^(1/2)-3))-ln(-1+(1+x)^(1/2))*ln((1+5^(1/2)+2*(1+x)^(1/2))/(3+
5^(1/2)))-dilog((-1+5^(1/2)-2*(1+x)^(1/2))/(5^(1/2)-3))-dilog((1+5^(1/2)+2*
(1+x)^(1/2))/(3+5^(1/2)))+ln(1+(1+x)^(1/2))*ln(x+(1+x)^(1/2))-ln(1+(1+x)^(1
/2))*ln((-1+5^(1/2)-2*(1+x)^(1/2))/(5^(1/2)+1))-ln(1+(1+x)^(1/2))*ln((1+5^(
1/2)+2*(1+x)^(1/2))/(5^(1/2)-1))-dilog((-1+5^(1/2)-2*(1+x)^(1/2))/(5^(1/2)+
1))-dilog((1+5^(1/2)+2*(1+x)^(1/2))/(5^(1/2)-1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(x+\sqrt{x+1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(1+x)^(1/2))/x,x, algorithm="maxima")
```

```
[Out] integrate(log(x + sqrt(x + 1))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\log\left(x+\sqrt{x+1}\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(1+x)^(1/2))/x,x, algorithm="fricas")
```

```
[Out] integral(log(x + sqrt(x + 1))/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(x + \sqrt{x+1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x+(1+x)**(1/2))/x,x)
```

```
[Out] Integral(log(x + sqrt(x + 1))/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(x + \sqrt{x+1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x+(1+x)^(1/2))/x,x, algorithm="giac")
```

```
[Out] integrate(log(x + sqrt(x + 1))/x, x)
```

3.33 $\int \tan^{-1}(2\tan(x))\,dx$

Optimal. Leaf size=80

$$-\frac{1}{4}\text{PolyLog}\left(2,\frac{1}{3}e^{2ix}\right)+\frac{1}{4}\text{PolyLog}\left(2,3e^{2ix}\right)+\frac{1}{2}ix\log\left(1-3e^{2ix}\right)-\frac{1}{2}ix\log\left(1-\frac{1}{3}e^{2ix}\right)+x\tan^{-1}(2\tan(x))$$

```
[Out] x*ArcTan[2*Tan[x]] + (I/2)*x*Log[1 - 3*E^((2*I)*x)] - (I/2)*x*Log[1 - E^((2
*I)*x)/3] - PolyLog[2, E^((2*I)*x)/3]/4 + PolyLog[2, 3*E^((2*I)*x)]/4
```

Rubi [A] time = 0.0832127, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {5167, 2190, 2279, 2391}

$$-\frac{1}{4}\text{PolyLog}\left(2,\frac{1}{3}e^{2ix}\right)+\frac{1}{4}\text{PolyLog}\left(2,3e^{2ix}\right)+\frac{1}{2}ix\log\left(1-3e^{2ix}\right)-\frac{1}{2}ix\log\left(1-\frac{1}{3}e^{2ix}\right)+x\tan^{-1}(2\tan(x))$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[2*Tan[x]],x]
```

```
[Out] x*ArcTan[2*Tan[x]] + (I/2)*x*Log[1 - 3*E^((2*I)*x)] - (I/2)*x*Log[1 - E^((2
*I)*x)/3] - PolyLog[2, E^((2*I)*x)/3]/4 + PolyLog[2, 3*E^((2*I)*x)]/4
```

Rule 5167

```
Int[ArcTan[(c_.) + (d_.)*Tan[(a_.) + (b_.)*(x_)]], x_Symbol] :> Simp[x*ArcT
an[c + d*Tan[a + b*x]], x] + (Dist[b*(1 - I*c - d), Int[(x*E^(2*I*a + 2*I*b
*x))/(1 - I*c + d + (1 - I*c - d)*E^(2*I*a + 2*I*b*x)), x], x] - Dist[b*(1
+ I*c + d), Int[(x*E^(2*I*a + 2*I*b*x))/(1 + I*c - d + (1 + I*c + d)*E^(2*I
*a + 2*I*b*x)), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[(c + I*d)^2, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \tan^{-1}(2\tan(x))\,dx = x\tan^{-1}(2\tan(x)) - 3\int \frac{e^{2ix}x}{-1+3e^{2ix}}\,dx - \int \frac{e^{2ix}x}{3-e^{2ix}}\,dx$$
$$= x\tan^{-1}(2\tan(x)) + \frac{1}{2}ix\log\left(1-3e^{2ix}\right) - \frac{1}{2}ix\log\left(1-\frac{1}{3}e^{2ix}\right) - \frac{1}{2}i\int \log\left(1-3e^{2ix}\right)\,dx + \frac{1}{2}i\int$$
$$= x\tan^{-1}(2\tan(x)) + \frac{1}{2}ix\log\left(1-3e^{2ix}\right) - \frac{1}{2}ix\log\left(1-\frac{1}{3}e^{2ix}\right) - \frac{1}{4}\,\mathrm{Subst}\left(\int \frac{\log(1-3x)}{x}\,dx, x, e\right.$$
$$= x\tan^{-1}(2\tan(x)) + \frac{1}{2}ix\log\left(1-3e^{2ix}\right) - \frac{1}{2}ix\log\left(1-\frac{1}{3}e^{2ix}\right) - \frac{1}{4}\mathrm{Li}_2\left(\frac{1}{3}e^{2ix}\right) + \frac{1}{4}\mathrm{Li}_2\left(3e^{2ix}\right)$$

Mathematica [B] time = 0.235709, size = 262, normalized size = 3.28

$$x\tan^{-1}(2\tan(x)) - \frac{1}{4}i\left(i\left(\mathrm{PolyLog}\left(2,\frac{2\tan(x)-i}{6\tan(x)+3i}\right) - \mathrm{PolyLog}\left(2,\frac{6\tan(x)-3i}{2\tan(x)+i}\right)\right) + 2i\cos^{-1}\left(\frac{5}{3}\right)\tan^{-1}(2\tan(x)) + \right.$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[2*Tan[x]],x]
```

```
[Out] x*ArcTan[2*Tan[x]] - (I/4)*((4*I)*x*ArcTan[Cot[x]/2] + (2*I)*ArcCos[5/3]*Ar
cTan[2*Tan[x]] + (ArcCos[5/3] + 2*ArcTan[Cot[x]/2] + 2*ArcTan[2*Tan[x]])*Lo
g[((2*I)*Sqrt[2/3])/(E^(I*x)*Sqrt[-5 + 3*Cos[2*x]])] + (ArcCos[5/3] - 2*Arc
Tan[Cot[x]/2] - 2*ArcTan[2*Tan[x]])*Log[((2*I)*Sqrt[2/3]*E^(I*x))/Sqrt[-5 +
 3*Cos[2*x]]] - (ArcCos[5/3] - 2*ArcTan[2*Tan[x]])*Log[(4*I - 4*Tan[x])/(I
+ 2*Tan[x])] - (ArcCos[5/3] + 2*ArcTan[2*Tan[x]])*Log[(4*(I + Tan[x]))/(3*I
 + 6*Tan[x])] + I*(-PolyLog[2, (-3*I + 6*Tan[x])/(I + 2*Tan[x])] + PolyLog[
2, (-I + 2*Tan[x])/(3*I + 6*Tan[x])]))
```

Maple [A] time = 0.097, size = 113, normalized size = 1.4

$$\arctan(2\tan(x))\arctan(\tan(x)) - \frac{i}{2}\arctan(\tan(x))\ln\left(1 - \frac{(1+i\tan(x))^2}{3(\tan(x))^2+3}\right) - \frac{1}{4}\mathrm{polylog}\left(2, \frac{(1+i\tan(x))^2}{3(\tan(x))^2+3}\right) + \frac{i}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(2*tan(x)),x)
```

```
[Out] arctan(2*tan(x))*arctan(tan(x))-1/2*I*arctan(tan(x))*ln(1-1/3*(1+I*tan(x))^
2/(tan(x)^2+1))-1/4*polylog(2,1/3*(1+I*tan(x))^2/(tan(x)^2+1))+1/2*I*arctan
(tan(x))*ln(1-3*(1+I*tan(x))^2/(tan(x)^2+1))+1/4*polylog(2,3*(1+I*tan(x))^2
/(tan(x)^2+1))
```

Maxima [A] time = 1.48328, size = 113, normalized size = 1.41

$$x\arctan(2\tan(x)) - \frac{1}{8}\log\left(4\tan(x)^2+4\right)\log\left(4\tan(x)^2+1\right) + \frac{1}{8}\log\left(4\tan(x)^2+1\right)\log\left(\frac{4}{9}\tan(x)^2+\frac{4}{9}\right) - \frac{1}{4}\mathrm{Li}_2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(2*tan(x)),x, algorithm="maxima")
```

```
[Out] x*arctan(2*tan(x)) - 1/8*log(4*tan(x)^2 + 4)*log(4*tan(x)^2 + 1) + 1/8*log(
4*tan(x)^2 + 1)*log(4/9*tan(x)^2 + 4/9) - 1/4*dilog(2*I*tan(x) - 1) + 1/4*d
ilog(2/3*I*tan(x) + 1/3) + 1/4*dilog(-2/3*I*tan(x) + 1/3) - 1/4*dilog(-2*I*
tan(x) - 1)
```

Fricas [B] time = 2.47376, size = 713, normalized size = 8.91

$$x\arctan(2\tan(x)) - \frac{1}{4}ix\log\left(\frac{2\left(2\tan(x)^2+3i\tan(x)-1\right)}{\tan(x)^2+1}\right) + \frac{1}{4}ix\log\left(\frac{2\left(2\tan(x)^2+i\tan(x)+1\right)}{3\left(\tan(x)^2+1\right)}\right) - \frac{1}{4}ix\log\left(\frac{2}{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(2*tan(x)),x, algorithm="fricas")
```

```
[Out] x*arctan(2*tan(x)) - 1/4*I*x*log(2*(2*tan(x)^2 + 3*I*tan(x) - 1)/(tan(x)^2
+ 1)) + 1/4*I*x*log(2/3*(2*tan(x)^2 + I*tan(x) + 1)/(tan(x)^2 + 1)) - 1/4*I
*x*log(2/3*(2*tan(x)^2 - I*tan(x) + 1)/(tan(x)^2 + 1)) + 1/4*I*x*log(2*(2*t
an(x)^2 - 3*I*tan(x) - 1)/(tan(x)^2 + 1)) + 1/8*dilog(-2*(2*tan(x)^2 + 3*I*
tan(x) - 1)/(tan(x)^2 + 1) + 1) - 1/8*dilog(-2/3*(2*tan(x)^2 + I*tan(x) + 1
)/(tan(x)^2 + 1) + 1) - 1/8*dilog(-2/3*(2*tan(x)^2 - I*tan(x) + 1)/(tan(x)^
2 + 1) + 1) + 1/8*dilog(-2*(2*tan(x)^2 - 3*I*tan(x) - 1)/(tan(x)^2 + 1) + 1
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{atan}(2\tan(x))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(2*tan(x)),x)
```

```
[Out] Integral(atan(2*tan(x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \arctan\left(2\tan(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(2*tan(x)),x, algorithm="giac")
```

```
[Out] integrate(arctan(2*tan(x)), x)
```

3.34 $\int \frac{\tan^{-1}(x)\log(x)}{x}\,dx$

Optimal. Leaf size=57

$$-\frac{1}{2}i\text{PolyLog}(3,-ix)+\frac{1}{2}i\text{PolyLog}(3,ix)+\frac{1}{2}i\log(x)\text{PolyLog}(2,-ix)-\frac{1}{2}i\log(x)\text{PolyLog}(2,ix)$$

```
[Out] (I/2)*Log[x]*PolyLog[2, (-I)*x] - (I/2)*Log[x]*PolyLog[2, I*x] - (I/2)*Poly
Log[3, (-I)*x] + (I/2)*PolyLog[3, I*x]
```

Rubi [A] time = 0.075564, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4848, 2391, 5005, 2374, 6589}

$$-\frac{1}{2}i\text{PolyLog}(3,-ix)+\frac{1}{2}i\text{PolyLog}(3,ix)+\frac{1}{2}i\log(x)\text{PolyLog}(2,-ix)-\frac{1}{2}i\log(x)\text{PolyLog}(2,ix)$$

Antiderivative was successfully verified.

```
[In] Int[(ArcTan[x]*Log[x])/x,x]
```

```
[Out] (I/2)*Log[x]*PolyLog[2, (-I)*x] - (I/2)*Log[x]*PolyLog[2, I*x] - (I/2)*Poly
Log[3, (-I)*x] + (I/2)*PolyLog[3, I*x]
```

Rule 4848

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Dist[(I*b)/2, Int[Log[1 - I*c*x]/x, x], x] - Dist[(I*b)/2, Int[Log[1 +
I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5005

```
Int[(ArcTan[(c_.)*(x_)^(n_.)]*Log[(d_.)*(x_)^(m_.)])/(x_), x_Symbol] :> Dis
t[I/2, Int[(Log[d*x^m]*Log[1 - I*c*x^n])/x, x], x] - Dist[I/2, Int[(Log[d*x
^m]*Log[1 + I*c*x^n])/x, x], x] /; FreeQ[{c, d, m, n}, x]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^{-1}(x)\log(x)}{x}\,dx &= \frac{1}{2}i\int \frac{\log(1-ix)\log(x)}{x}\,dx-\frac{1}{2}i\int \frac{\log(1+ix)\log(x)}{x}\,dx\\ &= \frac{1}{2}i\log(x)\text{Li}_2(-ix)-\frac{1}{2}i\log(x)\text{Li}_2(ix)-\frac{1}{2}i\int \frac{\text{Li}_2(-ix)}{x}\,dx+\frac{1}{2}i\int \frac{\text{Li}_2(ix)}{x}\,dx\\ &= \frac{1}{2}i\log(x)\text{Li}_2(-ix)-\frac{1}{2}i\log(x)\text{Li}_2(ix)-\frac{1}{2}i\text{Li}_3(-ix)+\frac{1}{2}i\text{Li}_3(ix)\end{aligned}$$

Mathematica [A] time = 0.0536139, size = 44, normalized size = 0.77

$$\frac{1}{2}i(-\text{PolyLog}(3,-ix)+\text{PolyLog}(3,ix)+\log(x)\text{PolyLog}(2,-ix)-\log(x)\text{PolyLog}(2,ix))$$

Antiderivative was successfully verified.

```
[In] Integrate[(ArcTan[x]*Log[x])/x,x]
```

```
[Out] (I/2)*(Log[x]*PolyLog[2, (-I)*x] - Log[x]*PolyLog[2, I*x] - PolyLog[3, (-I)
*x] + PolyLog[3, I*x])
```

Maple [F] time = 0.35, size = 0, normalized size = 0.

$$\int \frac{\arctan(x)\ln(x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)*ln(x)/x,x)
```

```
[Out] int(arctan(x)*ln(x)/x,x)
```

Maxima [A] time = 1.62968, size = 42, normalized size = 0.74

$$-\frac{1}{2} i \operatorname{Li}_2(i x) \log(x) + \frac{1}{2} i \operatorname{Li}_2(-i x) \log(x) + \frac{1}{2} i \operatorname{Li}_3(i x) - \frac{1}{2} i \operatorname{Li}_3(-i x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x)/x,x, algorithm="maxima")
```

```
[Out] -1/2*I*dilog(I*x)*log(x) + 1/2*I*dilog(-I*x)*log(x) + 1/2*I*polylog(3, I*x)
 - 1/2*I*polylog(3, -I*x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\arctan(x) \log(x)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x)/x,x, algorithm="fricas")
```

```
[Out] integral(arctan(x)*log(x)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(x) \operatorname{atan}(x)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)*ln(x)/x,x)
```

```
[Out] Integral(log(x)*atan(x)/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\arctan(x)\log(x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)*log(x)/x,x, algorithm="giac")
```

```
[Out] integrate(arctan(x)*log(x)/x, x)
```

3.35 $\int \sqrt{1+x^2} \tan^{-1}(x)^2 \, dx$

Optimal. Leaf size=121

$$i \tan^{-1}(x) \text{PolyLog}\left(2, -ie^{i \tan^{-1}(x)}\right) - i \tan^{-1}(x) \text{PolyLog}\left(2, ie^{i \tan^{-1}(x)}\right) - \text{PolyLog}\left(3, -ie^{i \tan^{-1}(x)}\right) + \text{PolyLog}\left(3, ie^{i \tan^{-1}(x)}\right)$$

```
[Out] ArcSinh[x] - Sqrt[1 + x^2]*ArcTan[x] + (x*Sqrt[1 + x^2]*ArcTan[x]^2)/2 - I*
ArcTan[E^(I*ArcTan[x])]*ArcTan[x]^2 + I*ArcTan[x]*PolyLog[2, (-I)*E^(I*ArcT
an[x])] - I*ArcTan[x]*PolyLog[2, I*E^(I*ArcTan[x])] - PolyLog[3, (-I)*E^(I*
ArcTan[x])] + PolyLog[3, I*E^(I*ArcTan[x])]
```

Rubi [A] time = 0.109459, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4880, 4888, 4181, 2531, 2282, 6589, 215}

$$i \tan^{-1}(x) \text{PolyLog}\left(2, -ie^{i \tan^{-1}(x)}\right) - i \tan^{-1}(x) \text{PolyLog}\left(2, ie^{i \tan^{-1}(x)}\right) - \text{PolyLog}\left(3, -ie^{i \tan^{-1}(x)}\right) + \text{PolyLog}\left(3, ie^{i \tan^{-1}(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + x^2]*ArcTan[x]^2,x]
```

```
[Out] ArcSinh[x] - Sqrt[1 + x^2]*ArcTan[x] + (x*Sqrt[1 + x^2]*ArcTan[x]^2)/2 - I*
ArcTan[E^(I*ArcTan[x])]*ArcTan[x]^2 + I*ArcTan[x]*PolyLog[2, (-I)*E^(I*ArcT
an[x])] - I*ArcTan[x]*PolyLog[2, I*E^(I*ArcTan[x])] - PolyLog[3, (-I)*E^(I*
ArcTan[x])] + PolyLog[3, I*E^(I*ArcTan[x])]
```

Rule 4880

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> -Simp[(b*p*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1))/(2*c*q*(2*
q + 1)), x] + (Dist[(2*d*q)/(2*q + 1), Int[(d + e*x^2)^(q - 1)*(a + b*ArcTa
n[c*x])^p, x], x] + Dist[(b^2*d*p*(p - 1))/(2*q*(2*q + 1)), Int[(d + e*x^2)
^(q - 1)*(a + b*ArcTan[c*x])^(p - 2), x], x] + Simp[(x*(d + e*x^2)^q*(a + b
*ArcTan[c*x])^p)/(2*q + 1), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*
d] && GtQ[q, 0] && GtQ[p, 1]
```

Rule 4888

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Dist[1/(c*Sqrt[d]), Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[c
```

```
*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && GtQ
[d, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{1+x^2}\tan^{-1}(x)^2\,dx = -\sqrt{1+x^2}\tan^{-1}(x) + \frac{1}{2}x\sqrt{1+x^2}\tan^{-1}(x)^2 + \frac{1}{2}\int \frac{\tan^{-1}(x)^2}{\sqrt{1+x^2}}\,dx + \int \frac{1}{\sqrt{1+x^2}}\,dx$$
$$= \sinh^{-1}(x) - \sqrt{1+x^2}\tan^{-1}(x) + \frac{1}{2}x\sqrt{1+x^2}\tan^{-1}(x)^2 + \frac{1}{2}\,\text{Subst}\left(\int x^2\sec(x)\,dx, x, \tan^{-1}(x)\right.$$
$$= \sinh^{-1}(x) - \sqrt{1+x^2}\tan^{-1}(x) + \frac{1}{2}x\sqrt{1+x^2}\tan^{-1}(x)^2 - i\tan^{-1}\left(e^{i\tan^{-1}(x)}\right)\tan^{-1}(x)^2 - \text{Subst}$$
$$= \sinh^{-1}(x) - \sqrt{1+x^2}\tan^{-1}(x) + \frac{1}{2}x\sqrt{1+x^2}\tan^{-1}(x)^2 - i\tan^{-1}\left(e^{i\tan^{-1}(x)}\right)\tan^{-1}(x)^2 + i\tan^{-}$$
$$= \sinh^{-1}(x) - \sqrt{1+x^2}\tan^{-1}(x) + \frac{1}{2}x\sqrt{1+x^2}\tan^{-1}(x)^2 - i\tan^{-1}\left(e^{i\tan^{-1}(x)}\right)\tan^{-1}(x)^2 + i\tan^{-}$$
$$= \sinh^{-1}(x) - \sqrt{1+x^2}\tan^{-1}(x) + \frac{1}{2}x\sqrt{1+x^2}\tan^{-1}(x)^2 - i\tan^{-1}\left(e^{i\tan^{-1}(x)}\right)\tan^{-1}(x)^2 + i\tan^{-}$$

Mathematica [A] time = 0.194612, size = 131, normalized size = 1.08

$$i\tan^{-1}(x)\text{PolyLog}\left(2, -ie^{i\tan^{-1}(x)}\right) - i\tan^{-1}(x)\text{PolyLog}\left(2, ie^{i\tan^{-1}(x)}\right) - \text{PolyLog}\left(3, -ie^{i\tan^{-1}(x)}\right) + \text{PolyLog}\left(3, ie^{i\tan^{-}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 + x^2]*ArcTan[x]^2,x]
```

```
[Out] -(Sqrt[1 + x^2]*ArcTan[x]) + (x*Sqrt[1 + x^2]*ArcTan[x]^2)/2 - I*ArcTan[E^(
I*ArcTan[x])]*ArcTan[x]^2 + ArcTanh[x/Sqrt[1 + x^2]] + I*ArcTan[x]*PolyLog[
2, (-I)*E^(I*ArcTan[x])] - I*ArcTan[x]*PolyLog[2, I*E^(I*ArcTan[x])] - Poly
Log[3, (-I)*E^(I*ArcTan[x])] + PolyLog[3, I*E^(I*ArcTan[x])]
```

Maple [A] time = 0.266, size = 171, normalized size = 1.4

$$\frac{\arctan(x)\,(x\arctan(x)-2)}{2}\sqrt{x^2+1} + \frac{(\arctan(x))^2}{2}\ln\left(1 - i(1+ix)\frac{1}{\sqrt{x^2+1}}\right) - \frac{(\arctan(x))^2}{2}\ln\left(1 + i(1+ix)\frac{1}{\sqrt{x^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x)^2*(x^2+1)^(1/2),x)
```

```
[Out] 1/2*(x^2+1)^(1/2)*arctan(x)*(x*arctan(x)-2)+1/2*arctan(x)^2*ln(1-I*(1+I*x)/
(x^2+1)^(1/2))-1/2*arctan(x)^2*ln(1+I*(1+I*x)/(x^2+1)^(1/2))-I*arctan(x)*po
```

```
lylog(2,I*(1+I*x)/(x^2+1)^(1/2))+polylog(3,I*(1+I*x)/(x^2+1)^(1/2))+I*arcta
n(x)*polylog(2,-I*(1+I*x)/(x^2+1)^(1/2))-polylog(3,-I*(1+I*x)/(x^2+1)^(1/2)
)-2*I*arctan((1+I*x)/(x^2+1)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2+1}\arctan(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2*(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 + 1)*arctan(x)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x^2+1}\arctan(x)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2*(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 1)*arctan(x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2+1}\,\text{atan}^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x)**2*(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 + 1)*atan(x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x)^2*(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=", ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0], Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1], Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0], Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: # isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): # is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```