Computer algebra independent integration tests

0-Independent-test-suites/Apostol-Problems

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [175]. This is test number [1].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (175)	% 0. (0)
Mathematica	% 100. (175)	% 0. (0)
Maple	% 98.86 (173)	% 1.14 (2)
Maxima	% 93.71 (164)	% 6.29 (11)
Fricas	% 98.29 (172)	% 1.71 (3)
Sympy	% 89.14 (156)	% 10.86 (19)
Giac	% 94.29 (165)	% 5.71 (10)

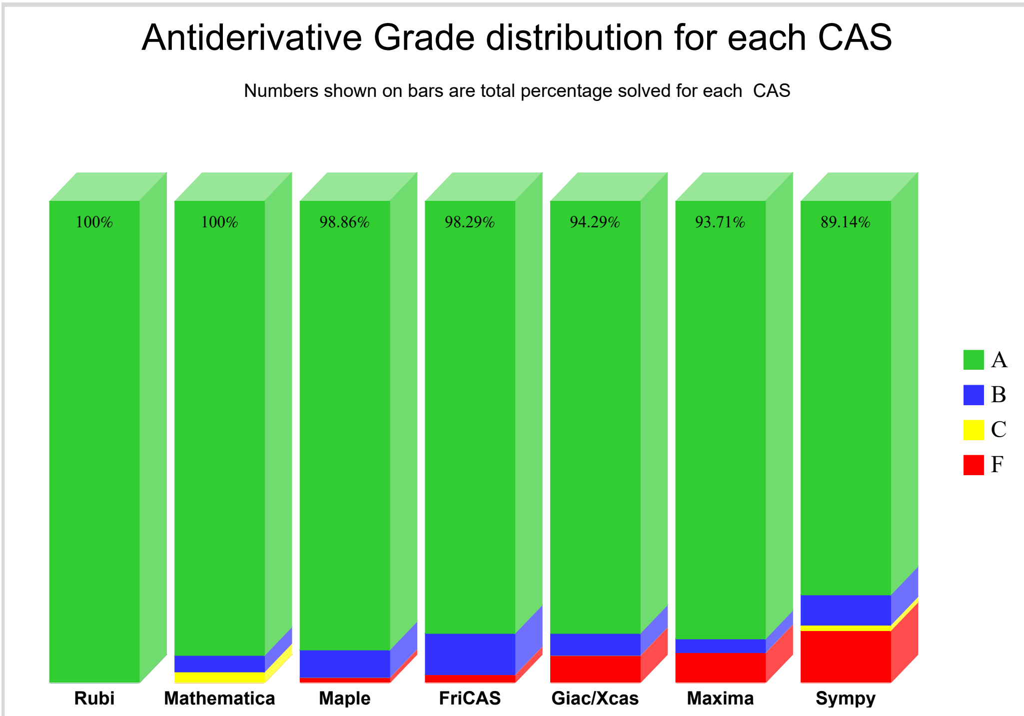
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

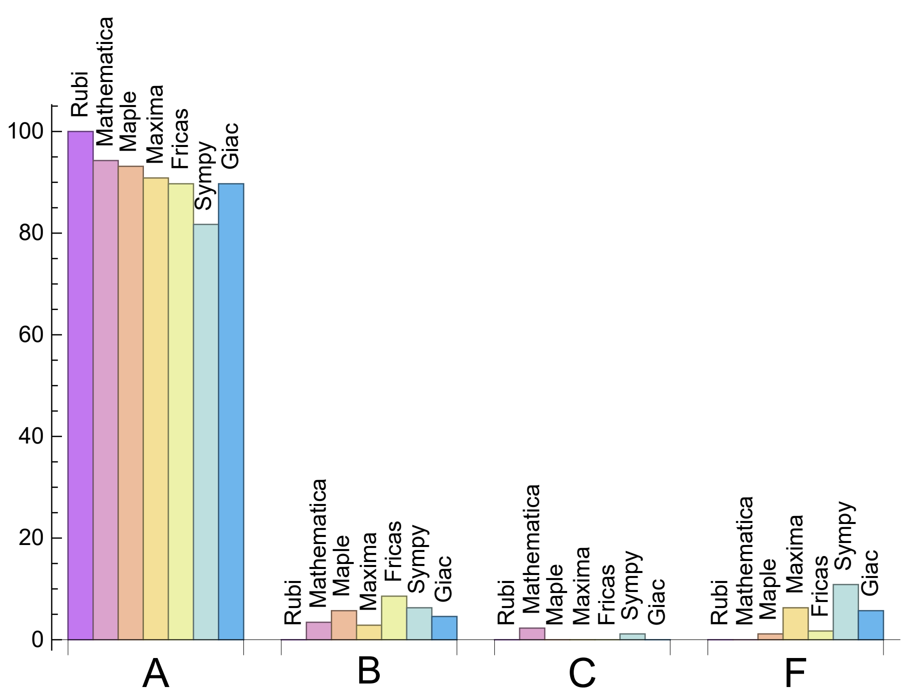
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	94.29	3.43	2.29	0.
Maple	93.14	5.71	0.	1.14
Maxima	90.86	2.86	0.	6.29
Fricas	89.71	8.57	0.	1.71
Sympy	81.71	6.29	1.14	10.86
Giac	89.71	4.57	0.	5.71

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.02	23.09	1.	19.	1.
Mathematica	0.01	21.66	1.01	18.	1.
Maple	0.01	23.2	1.11	16.	0.88
Maxima	1.08	24.66	1.16	19.	1.09
Fricas	0.69	64.1	3.17	51.	2.59
Sympy	3.66	49.14	2.51	17.	0.83
Giac	1.1	28.26	1.34	20.	1.12

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {98}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

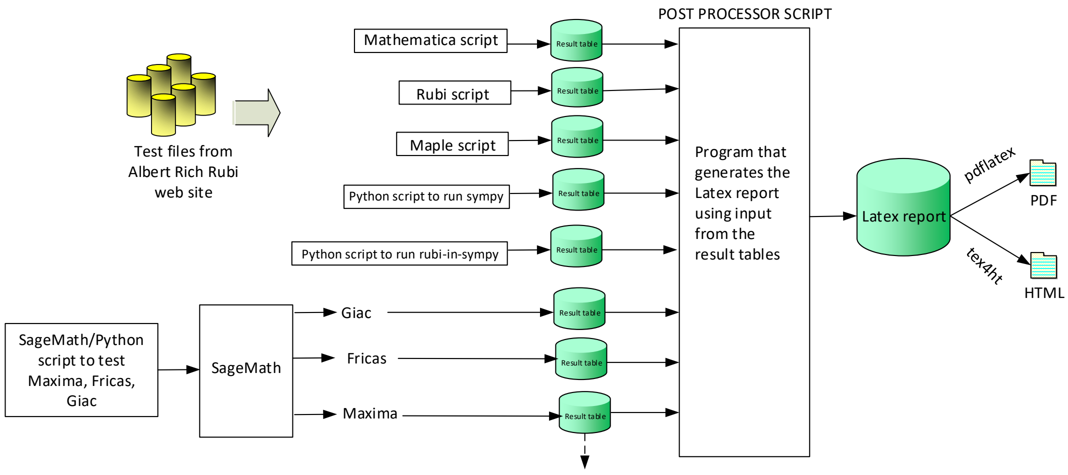
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade: { 50, 51, 83, 84, 105, 154 }

C grade: { 41, 98, 113, 175 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 171, 173, 174, 175 }

B grade: { 14, 17, 48, 50, 51, 114, 139, 158, 169, 170 }

C grade: { }

F grade: { 19, 172 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 171, 172, 173

B grade: { 51, 83, 84, 113, 169 }

C grade: { }

F grade: { 19, 41, 62, 90, 98, 99, 104, 105, 141, 174, 175 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 46, 47, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade: { 16, 44, 45, 48, 50, 51, 61, 84, 88, 113, 114, 124, 131, 145, 146 }

C grade: { }

F grade: { 41, 156, 175 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 85, 86, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 171, 172, 175 }

B grade: { 9, 17, 42, 47, 48, 50, 51, 90, 101, 114, 144 }

C grade: { 89, 156 }

F grade: { 19, 62, 83, 84, 88, 103, 104, 105, 113, 151, 153, 154, 155, 162, 163, 164, 165, 173, 174 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 42, 43, 46, 47, 48, 49, 50, 52, 53, 54, 56, 57, 58, 59, 60, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 157, 158, 159, 160, 162, 163, 165, 166, 167, 168, 169, 170, 171, 173, 174 }

B grade: { 44, 45, 51, 55, 61, 113, 136, 155 }

C grade: { }

F grade: { 21, 41, 62, 98, 99, 156, 161, 164, 172, 175 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	28	8	12
normalized size	1	1.	1.	0.77	0.92	2.15	0.62	0.92
time (sec)	N/A	0.001	0.037	0.011	0.945	0.417	0.051	1.063

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	26	54	39	26
normalized size	1	1.	0.67	0.56	0.96	2.	1.44	0.96
time (sec)	N/A	0.004	0.006	0.002	0.95	0.412	0.855	1.099

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	21	18	30	62	48	30
normalized size	1	1.	0.62	0.53	0.88	1.82	1.41	0.88
time (sec)	N/A	0.005	0.006	0.003	0.938	0.418	1.197	1.066

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	18	15	26	43	61	26
normalized size	1	1.	0.67	0.56	0.96	1.59	2.26	0.96
time (sec)	N/A	0.005	0.005	0.004	0.948	0.417	0.82	1.064

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	16	51	22	16
normalized size	1	1.	1.	0.93	1.14	3.64	1.57	1.14
time (sec)	N/A	0.003	0.004	0.007	0.983	0.404	0.106	1.055

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	15	31	8	15
normalized size	1	1.	1.15	0.85	1.15	2.38	0.62	1.15
time (sec)	N/A	0.006	0.004	0.062	0.98	0.455	0.058	1.104

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	16	13	20	49	92	20
normalized size	1	1.	0.7	0.57	0.87	2.13	4.	0.87
time (sec)	N/A	0.004	0.005	0.002	0.932	0.415	0.855	1.141

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	27	8	14
normalized size	1	1.	1.	0.88	1.	3.38	1.	1.75
time (sec)	N/A	0.013	0.005	0.007	0.942	0.435	0.06	1.081

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	54	29	16
normalized size	1	1.	1.	0.81	1.	3.38	1.81	1.
time (sec)	N/A	0.023	0.012	0.013	0.947	0.46	0.296	1.085

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	22	5	8
normalized size	1	1.	1.	1.17	1.33	3.67	0.83	1.33
time (sec)	N/A	0.018	0.013	0.027	0.946	0.433	0.414	1.076

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	36	12	8
normalized size	1	1.	1.	0.92	1.17	3.	1.	0.67
time (sec)	N/A	0.028	0.013	0.046	0.931	0.457	0.616	1.094

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	28	10	11
normalized size	1	1.	1.	0.9	1.1	2.8	1.	1.1
time (sec)	N/A	0.016	0.018	0.007	0.997	0.454	0.318	1.083

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	18	7	12
normalized size	1	1.	1.	1.11	1.33	2.	0.78	1.33
time (sec)	N/A	0.011	0.014	0.005	0.946	0.465	17.818	1.07

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	32	15	28	10	15
normalized size	1	1.	1.	2.13	1.	1.87	0.67	1.
time (sec)	N/A	0.003	0.003	0.006	0.96	0.419	0.249	1.072

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	16	13	20	49	34	20
normalized size	1	1.	0.7	0.57	0.87	2.13	1.48	0.87
time (sec)	N/A	0.004	0.004	0.001	0.955	0.419	0.883	1.065

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	12	53	8	12
normalized size	1	1.	1.	0.91	1.09	4.82	0.73	1.09
time (sec)	N/A	0.001	0.002	0.003	0.948	0.421	0.694	1.073

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	27	15	34	27	15
normalized size	1	1.	1.	1.8	1.	2.27	1.8	1.
time (sec)	N/A	0.003	0.003	0.005	0.95	0.424	0.349	1.094

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	12	15	41	12	15
normalized size	1	1.	1.	0.8	1.	2.73	0.8	1.
time (sec)	N/A	0.026	0.049	0.012	0.973	0.465	0.344	1.099

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	37	0	0	66	0	20
normalized size	1	1.	1.16	0.	0.	2.06	0.	0.62
time (sec)	N/A	0.101	0.048	0.056	0.	0.556	0.	1.113

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	36	14	18
normalized size	1	1.	1.	0.82	1.06	2.12	0.82	1.06
time (sec)	N/A	0.111	0.013	0.007	0.94	0.48	0.347	1.069

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	13	13	9	38	15	0
normalized size	1	1.	0.81	0.81	0.56	2.38	0.94	0.
time (sec)	N/A	0.008	0.006	0.003	0.967	0.418	1.219	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	9	11	27	7	11
normalized size	1	1.	1.	1.12	1.38	3.38	0.88	1.38
time (sec)	N/A	0.009	0.002	0.005	0.972	0.451	0.194	1.071

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	15	18	20	43	17	20
normalized size	1	1.	0.88	1.06	1.18	2.53	1.	1.18
time (sec)	N/A	0.021	0.013	0.005	0.958	0.452	0.321	1.068

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	19	24	27	55	26	27
normalized size	1	1.	0.83	1.04	1.17	2.39	1.13	1.17
time (sec)	N/A	0.036	0.013	0.006	0.952	0.451	0.554	1.063

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	20	25	28	57	26	28
normalized size	1	1.	0.83	1.04	1.17	2.38	1.08	1.17
time (sec)	N/A	0.036	0.012	0.004	0.96	0.452	0.57	1.07

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	20	5	8
normalized size	1	1.	1.	0.88	1.	2.5	0.62	1.
time (sec)	N/A	0.007	0.001	0.001	0.952	0.451	0.055	1.095

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	18	18	19	61	24	19
normalized size	1	1.	0.78	0.78	0.83	2.65	1.04	0.83
time (sec)	N/A	0.013	0.003	0.003	0.944	0.446	0.322	1.143

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	38	10	14
normalized size	1	1.	1.	0.79	1.	2.71	0.71	1.
time (sec)	N/A	0.005	0.002	0.005	0.942	0.456	0.058	1.079

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	15	11	15	31	8	15
normalized size	1	1.	1.15	0.85	1.15	2.38	0.62	1.15
time (sec)	N/A	0.006	0.001	0.	0.951	0.46	0.059	1.064

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	18	22	59	24	22
normalized size	1	1.	0.92	0.75	0.92	2.46	1.	0.92
time (sec)	N/A	0.009	0.002	0.117	0.936	0.475	0.057	1.08

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	23	17	23	53	17	23
normalized size	1	1.	1.1	0.81	1.1	2.52	0.81	1.1
time (sec)	N/A	0.007	0.002	0.03	0.956	0.481	0.06	1.071

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	24	32	84	36	30
normalized size	1	1.	0.88	0.71	0.94	2.47	1.06	0.88
time (sec)	N/A	0.016	0.002	0.033	0.937	0.467	0.059	1.09

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	26	63	36	26
normalized size	1	1.	1.	1.	1.04	2.52	1.44	1.04
time (sec)	N/A	0.014	0.011	0.005	0.952	0.455	0.325	1.102

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	31	23	31	74	39	31
normalized size	1	1.	0.94	0.7	0.94	2.24	1.18	0.94
time (sec)	N/A	0.022	0.007	0.03	0.964	0.453	0.565	1.121

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	29	37	35	89	56	35
normalized size	1	1.	0.71	0.9	0.85	2.17	1.37	0.85
time (sec)	N/A	0.031	0.032	0.02	0.966	0.456	0.665	1.078

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	11	14	36	10	14
normalized size	1	1.	1.	0.79	1.	2.57	0.71	1.
time (sec)	N/A	0.006	0.002	0.005	0.931	0.454	0.057	1.097

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	15	11	12	36	8	12
normalized size	1	1.	1.36	1.	1.09	3.27	0.73	1.09
time (sec)	N/A	0.006	0.001	0.026	0.95	0.454	0.058	1.125

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	18	22	59	24	22
normalized size	1	1.	0.92	0.75	0.92	2.46	1.	0.92
time (sec)	N/A	0.011	0.002	0.03	0.963	0.46	0.056	1.081

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	62	69	84	132	182	68
normalized size	1	1.	0.74	0.82	1.	1.57	2.17	0.81
time (sec)	N/A	0.015	0.067	0.006	1.437	0.441	3.843	1.106

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	25	22	46	58	39	38
normalized size	1	1.	0.66	0.58	1.21	1.53	1.03	1.
time (sec)	N/A	0.013	0.006	0.003	1.431	0.425	1.241	1.084

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	34	168	0	0	31	0
normalized size	1	1.	0.2	0.98	0.	0.	0.18	0.
time (sec)	N/A	0.029	0.005	0.098	0.	0.	0.61	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	8	18	7	8
normalized size	1	1.	1.	1.17	1.33	3.	1.17	1.33
time (sec)	N/A	0.004	0.005	0.004	1.442	0.446	0.058	1.077

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	18	13	16	36	19	16
normalized size	1	1.	1.29	0.93	1.14	2.57	1.36	1.14
time (sec)	N/A	0.009	0.003	0.003	1.445	0.452	0.067	1.097

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	12	14	53	8	24
normalized size	1	1.	1.	1.5	1.75	6.62	1.	3.
time (sec)	N/A	0.005	0.005	0.001	1.446	0.45	0.063	1.105

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	18	14	22	126	19	46
normalized size	1	1.	1.5	1.17	1.83	10.5	1.58	3.83
time (sec)	N/A	0.01	0.003	0.003	1.419	0.451	0.067	1.129

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	26	21	27	55	26	24
normalized size	1	1.	1.18	0.95	1.23	2.5	1.18	1.09
time (sec)	N/A	0.011	0.011	0.006	0.947	0.448	0.172	1.086

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	10	12	28	22	12
normalized size	1	1.	1.	0.77	0.92	2.15	1.69	0.92
time (sec)	N/A	0.002	0.002	0.002	0.946	0.423	0.174	1.09

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	11	52	12	142	58	12
normalized size	1	1.	0.85	4.	0.92	10.92	4.46	0.92
time (sec)	N/A	0.001	0.001	0.001	0.956	0.358	0.056	1.13

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	16	20	26	50	15	19
normalized size	1	1.	0.89	1.11	1.44	2.78	0.83	1.06
time (sec)	N/A	0.002	0.003	0.004	0.959	0.408	0.096	1.072

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	43	32	12	88	31	12
normalized size	1	1.	3.91	2.91	1.09	8.	2.82	1.09
time (sec)	N/A	0.002	0.001	0.001	0.949	0.357	0.051	1.093

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	140	107	143	373	131	143
normalized size	1	1.	2.5	1.91	2.55	6.66	2.34	2.55
time (sec)	N/A	0.025	0.002	0.002	0.938	0.352	0.072	1.098

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	14	3	5
normalized size	1	1.	1.	1.25	1.25	3.5	0.75	1.25
time (sec)	N/A	0.008	0.008	0.003	0.98	0.447	0.655	1.083

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	46	49	50	134	60	50
normalized size	1	1.	0.74	0.79	0.81	2.16	0.97	0.81
time (sec)	N/A	0.043	0.029	0.005	0.969	0.512	2.368	1.109

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	23	7	11
normalized size	1	1.	1.	0.9	1.1	2.3	0.7	1.1
time (sec)	N/A	0.007	0.002	0.003	0.96	0.517	0.301	1.109

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	16	39	15	248
normalized size	1	1.	1.	0.81	1.	2.44	0.94	15.5
time (sec)	N/A	0.031	0.011	0.018	0.95	0.52	2.322	1.116

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	9	11	24	7	12
normalized size	1	1.	1.	0.9	1.1	2.4	0.7	1.2
time (sec)	N/A	0.001	0.001	0.002	0.978	0.464	0.052	1.079

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	16	42	15	20
normalized size	1	1.	1.	1.07	1.07	2.8	1.	1.33
time (sec)	N/A	0.004	0.001	0.003	0.952	0.508	0.089	1.068

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	18	35	12	18
normalized size	1	1.	1.	0.82	1.06	2.06	0.71	1.06
time (sec)	N/A	0.003	0.001	0.002	0.953	0.511	0.085	1.087

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	23	61	22	30
normalized size	1	1.	1.	0.82	0.82	2.18	0.79	1.07
time (sec)	N/A	0.009	0.001	0.	0.961	0.538	0.095	1.09

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	5	5	16	3	7
normalized size	1	1.	1.	1.25	1.25	4.	0.75	1.75
time (sec)	N/A	0.	0.001	0.	0.947	0.48	0.055	1.102

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	41	3	15
normalized size	1	1.	1.	1.33	1.33	13.67	1.	5.
time (sec)	N/A	0.003	0.002	0.001	0.952	0.502	0.06	1.09

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	21	36	0	85	0	0
normalized size	1	1.	0.75	1.29	0.	3.04	0.	0.
time (sec)	N/A	0.01	0.005	0.013	0.	0.519	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	23	23	62	26	30
normalized size	1	1.	1.	0.82	0.82	2.21	0.93	1.07
time (sec)	N/A	0.017	0.001	0.002	0.947	0.534	0.097	1.087

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	3	3	3	4	4	18	3	5
normalized size	1	1.	1.	1.33	1.33	6.	1.	1.67
time (sec)	N/A	0.012	0.004	0.	1.014	0.494	0.087	1.104

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	27	8	14
normalized size	1	1.	1.	0.92	1.17	2.25	0.67	1.17
time (sec)	N/A	0.015	0.002	0.002	0.962	0.516	0.084	1.063

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	16	18	23	47	20	23
normalized size	1	1.	0.7	0.78	1.	2.04	0.87	1.
time (sec)	N/A	0.038	0.015	0.005	0.94	0.479	3.886	1.107

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	39	32	31	92	37	42
normalized size	1	1.	1.	0.82	0.79	2.36	0.95	1.08
time (sec)	N/A	0.03	0.001	0.002	0.926	0.501	0.126	1.102

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	7	8	18	5	8
normalized size	1	1.	1.	0.78	0.89	2.	0.56	0.89
time (sec)	N/A	0.012	0.001	0.002	0.952	0.49	0.082	1.102

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	12	16	27	10	15
normalized size	1	1.	1.	0.86	1.14	1.93	0.71	1.07
time (sec)	N/A	0.009	0.003	0.003	0.929	0.512	0.142	1.075

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	8	9	24	7	9
normalized size	1	1.	1.	0.8	0.9	2.4	0.7	0.9
time (sec)	N/A	0.009	0.007	0.005	0.936	0.51	0.293	1.069

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	14	14	15	46	15	15
normalized size	1	1.	0.74	0.74	0.79	2.42	0.79	0.79
time (sec)	N/A	0.007	0.012	0.003	0.954	0.567	0.301	1.106

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	12	14	12	45	15	12
normalized size	1	1.	0.63	0.74	0.63	2.37	0.79	0.63
time (sec)	N/A	0.007	0.008	0.005	1.047	0.497	0.286	1.099

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	12	12	24	7	12
normalized size	1	1.	1.	1.2	1.2	2.4	0.7	1.2
time (sec)	N/A	0.006	0.003	0.006	0.961	0.478	0.071	1.08

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	7	7	8	18	5	8
normalized size	1	1.	0.64	0.64	0.73	1.64	0.45	0.73
time (sec)	N/A	0.005	0.001	0.	0.967	0.525	0.074	1.097

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	11	10	12	23	7	12
normalized size	1	1.	0.69	0.62	0.75	1.44	0.44	0.75
time (sec)	N/A	0.007	0.004	0.001	0.952	0.516	0.081	1.089

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	12	12	15	28	10	15
normalized size	1	1.	0.63	0.63	0.79	1.47	0.53	0.79
time (sec)	N/A	0.015	0.004	0.002	0.957	0.513	0.075	1.083

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	19	19	22	45	17	22
normalized size	1	1.	0.59	0.59	0.69	1.41	0.53	0.69
time (sec)	N/A	0.018	0.006	0.002	0.962	0.487	0.081	1.081

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	16	17	15	36	20	15
normalized size	1	1.	0.67	0.71	0.62	1.5	0.83	0.62
time (sec)	N/A	0.008	0.005	0.003	0.95	0.493	0.174	1.08

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	16	14	18	34	12	18
normalized size	1	1.	0.62	0.54	0.69	1.31	0.46	0.69
time (sec)	N/A	0.02	0.002	0.002	0.943	0.506	0.083	1.09

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	28	40	36	74	139	49
normalized size	1	1.	0.68	0.98	0.88	1.8	3.39	1.2
time (sec)	N/A	0.011	0.025	0.01	0.967	0.559	1.323	1.089

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	29	41	39	76	136	51
normalized size	1	1.	0.69	0.98	0.93	1.81	3.24	1.21
time (sec)	N/A	0.011	0.024	0.003	0.936	0.507	1.365	1.065

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	43	12	20
normalized size	1	1.	1.	0.93	1.2	2.87	0.8	1.33
time (sec)	N/A	0.003	0.001	0.	0.931	0.514	0.186	1.082

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	64	22	47	103	0	34
normalized size	1	1.	3.37	1.16	2.47	5.42	0.	1.79
time (sec)	N/A	0.009	0.064	0.002	0.929	0.626	0.	1.105

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	64	20	47	103	0	35
normalized size	1	1.	3.76	1.18	2.76	6.06	0.	2.06
time (sec)	N/A	0.009	0.039	0.002	0.928	0.554	0.	1.099

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	24	31	68	22	31
normalized size	1	1.	1.	0.96	1.24	2.72	0.88	1.24
time (sec)	N/A	0.034	0.007	0.037	1.424	0.504	0.184	1.112

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	21	45	105	22	51
normalized size	1	1.	1.	0.95	2.05	4.77	1.	2.32
time (sec)	N/A	0.016	0.002	0.003	1.458	0.631	1.784	1.123

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	11	50	20	12
normalized size	1	1.	1.	0.94	0.69	3.12	1.25	0.75
time (sec)	N/A	0.002	0.002	0.003	1.426	0.509	1.01	1.119

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	14	10	15	55	0	12
normalized size	1	1.	1.4	1.	1.5	5.5	0.	1.2
time (sec)	N/A	0.009	0.005	0.003	1.432	0.51	0.	1.091

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	C	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	20	20	14
normalized size	1	1.	1.	1.1	1.4	2.	2.	1.4
time (sec)	N/A	0.002	0.002	0.002	1.444	0.495	0.106	1.075

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	151	53	20
normalized size	1	1.	1.	0.67	0.	6.29	2.21	0.83
time (sec)	N/A	0.004	0.005	0.002	0.	0.5	0.125	1.109

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	22	58	26	22
normalized size	1	1.	1.	0.89	1.16	3.05	1.37	1.16
time (sec)	N/A	0.009	0.005	0.001	1.439	0.479	0.121	1.083

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	20	45	15	20
normalized size	1	1.	1.	0.76	0.95	2.14	0.71	0.95
time (sec)	N/A	0.008	0.002	0.003	1.458	0.516	0.257	1.123

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	30	34	45	68	32	45
normalized size	1	1.	0.75	0.85	1.12	1.7	0.8	1.12
time (sec)	N/A	0.022	0.01	0.003	1.453	0.548	0.331	1.093

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	26	30	46	81	29	39
normalized size	1	1.	0.74	0.86	1.31	2.31	0.83	1.11
time (sec)	N/A	0.056	0.007	0.009	1.468	0.523	0.352	1.079

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	18	17	22	47	19	22
normalized size	1	1.	0.82	0.77	1.	2.14	0.86	1.
time (sec)	N/A	0.005	0.005	0.003	1.443	0.544	1.354	1.105

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	8	26	7	8
normalized size	1	1.	1.	0.88	1.	3.25	0.88	1.
time (sec)	N/A	0.03	0.006	0.004	0.937	0.52	2.047	1.075

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	18	23	74	15	23
normalized size	1	1.	0.87	0.78	1.	3.22	0.65	1.
time (sec)	N/A	0.003	0.005	0.003	1.454	0.514	0.193	1.147

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	A	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	22	22	37	16	0	53	31	0
normalized size	1	1.	1.68	0.73	0.	2.41	1.41	0.
time (sec)	N/A	0.041	0.01	0.003	0.	0.664	119.363	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	16	0	53	31	0
normalized size	1	1.	1.	0.8	0.	2.65	1.55	0.
time (sec)	N/A	0.023	0.005	0.003	0.	0.93	117.73	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	16	20	55	12	20
normalized size	1	1.	1.	0.84	1.05	2.89	0.63	1.05
time (sec)	N/A	0.004	0.004	0.007	1.443	0.632	0.094	1.123

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	4	4	18	15	4
normalized size	1	1.	1.	1.	1.	4.5	3.75	1.
time (sec)	N/A	0.016	0.003	0.003	1.434	0.746	0.107	1.07

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	25	26	84	19	28
normalized size	1	1.	1.	0.93	0.96	3.11	0.7	1.04
time (sec)	N/A	0.018	0.016	0.005	0.967	0.853	12.155	1.083

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	78	64	66	90	0	49
normalized size	1	1.	1.86	1.52	1.57	2.14	0.	1.17
time (sec)	N/A	0.013	0.037	0.013	1.411	0.832	0.	1.111

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	106	122	0	209	0	82
normalized size	1	1.	1.49	1.72	0.	2.94	0.	1.15
time (sec)	N/A	0.022	0.146	0.011	0.	0.887	0.	1.09

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	72	28	0	111	0	30
normalized size	1	1.	2.25	0.88	0.	3.47	0.	0.94
time (sec)	N/A	0.012	0.025	0.003	0.	0.746	0.	1.126

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	18	39	12	20
normalized size	1	1.	1.	0.93	1.2	2.6	0.8	1.33
time (sec)	N/A	0.004	0.003	0.005	0.929	0.673	0.093	1.074

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	14	18	45	14	20
normalized size	1	1.	1.	0.74	0.95	2.37	0.74	1.05
time (sec)	N/A	0.004	0.003	0.004	0.931	0.636	0.096	1.067

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	58	19	28
normalized size	1	1.	1.	0.87	1.13	2.52	0.83	1.22
time (sec)	N/A	0.027	0.004	0.004	0.944	0.727	0.103	1.073

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	61	17	27
normalized size	1	1.	1.	0.78	1.	2.65	0.74	1.17
time (sec)	N/A	0.033	0.005	0.007	0.929	0.655	0.127	1.097

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	22	19	24	84	19	32
normalized size	1	1.	0.92	0.79	1.	3.5	0.79	1.33
time (sec)	N/A	0.014	0.008	0.007	0.948	0.666	0.105	1.09

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	29	38	101	3	39
normalized size	1	1.	1.	1.04	1.36	3.61	0.11	1.39
time (sec)	N/A	0.028	0.01	0.005	1.443	0.789	0.116	1.093

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	61	37	49	154	14	50
normalized size	1	1.	1.24	0.76	1.	3.14	0.29	1.02
time (sec)	N/A	0.067	0.027	0.009	1.481	0.759	0.156	1.079

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	24	19	53	126	0	50
normalized size	1	1.	1.14	0.9	2.52	6.	0.	2.38
time (sec)	N/A	0.009	0.018	0.022	1.448	0.72	0.	1.149

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	266	19	144	44	19
normalized size	1	1.	1.	16.62	1.19	9.	2.75	1.19
time (sec)	N/A	0.045	0.027	0.043	1.015	0.824	2.642	1.109

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	9	9	12	28	8	15
normalized size	1	1.	0.82	0.82	1.09	2.55	0.73	1.36
time (sec)	N/A	0.004	0.003	0.001	0.99	0.798	0.081	1.067

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	20	26	66	20	30
normalized size	1	1.	1.	0.87	1.13	2.87	0.87	1.3
time (sec)	N/A	0.008	0.005	0.006	0.969	0.877	0.115	1.077

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	21	27	86	22	30
normalized size	1	1.	0.93	0.7	0.9	2.87	0.73	1.
time (sec)	N/A	0.017	0.008	0.007	0.967	0.78	0.098	1.065

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	31	68	20	35
normalized size	1	1.	1.	0.89	1.15	2.52	0.74	1.3
time (sec)	N/A	0.031	0.005	0.006	0.944	0.66	0.114	1.092

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	24	24	27	76	17	22
normalized size	1	1.	1.04	1.04	1.17	3.3	0.74	0.96
time (sec)	N/A	0.026	0.009	0.007	0.939	0.745	0.104	1.156

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	45	14	20
normalized size	1	1.	1.	0.94	1.19	2.81	0.88	1.25
time (sec)	N/A	0.017	0.004	0.003	1.406	0.748	0.085	1.089

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	13	16	53	14	16
normalized size	1	1.	1.	0.72	0.89	2.94	0.78	0.89
time (sec)	N/A	0.012	0.006	0.007	1.422	0.709	0.133	1.077

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	15	32	8	18
normalized size	1	1.	1.	1.09	1.36	2.91	0.73	1.64
time (sec)	N/A	0.005	0.002	0.005	0.933	0.717	0.087	1.097

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	21	32	89	19	39
normalized size	1	1.	1.	0.88	1.33	3.71	0.79	1.62
time (sec)	N/A	0.01	0.007	0.009	0.972	0.756	0.095	1.096

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	44	39	62	239	46	70
normalized size	1	1.	0.96	0.85	1.35	5.2	1.	1.52
time (sec)	N/A	0.013	0.014	0.01	0.937	0.79	0.18	1.122

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	46	8	15
normalized size	1	1.	1.	1.1	1.4	4.6	0.8	1.5
time (sec)	N/A	0.004	0.002	0.005	0.927	0.665	0.076	1.118

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	23	36	10	22
normalized size	1	1.	1.	1.06	1.35	2.12	0.59	1.29
time (sec)	N/A	0.007	0.002	0.006	0.929	0.877	0.083	1.078

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	15	19	50	17	22
normalized size	1	1.	1.	0.75	0.95	2.5	0.85	1.1
time (sec)	N/A	0.008	0.003	0.005	0.933	0.963	0.096	1.111

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	12	13	16	46	8	18
normalized size	1	1.	0.75	0.81	1.	2.88	0.5	1.12
time (sec)	N/A	0.005	0.003	0.003	0.962	1.147	0.075	1.077

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	19	51	10	19
normalized size	1	1.	1.	1.07	1.36	3.64	0.71	1.36
time (sec)	N/A	0.01	0.007	0.005	1.427	1.061	0.125	1.097

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	18	23	61	20	27
normalized size	1	1.	1.	0.86	1.1	2.9	0.95	1.29
time (sec)	N/A	0.018	0.004	0.005	0.957	1.156	0.123	1.106

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	27	28	31	90	20	34
normalized size	1	1.	1.29	1.33	1.48	4.29	0.95	1.62
time (sec)	N/A	0.003	0.006	0.007	0.969	1.248	0.097	1.094

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	17	22	54	17	23
normalized size	1	1.	1.	0.77	1.	2.45	0.77	1.05
time (sec)	N/A	0.011	0.003	0.004	1.43	0.974	0.086	1.116

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	19	46	8	19
normalized size	1	1.	1.	1.1	1.9	4.6	0.8	1.9
time (sec)	N/A	0.015	0.004	0.006	0.935	1.125	0.095	1.098

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	22	26	66	19	28
normalized size	1	1.	1.	0.71	0.84	2.13	0.61	0.9
time (sec)	N/A	0.009	0.002	0.006	0.937	1.13	0.101	1.068

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	59	15	23
normalized size	1	1.	1.	0.94	1.22	3.28	0.83	1.28
time (sec)	N/A	0.025	0.005	0.006	1.422	1.057	0.11	1.114

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	10	23	68	17	26
normalized size	1	1.	1.92	0.77	1.77	5.23	1.31	2.
time (sec)	N/A	0.003	0.003	0.001	1.437	0.99	0.118	1.114

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	64	58	97	302	73	97
normalized size	1	1.	0.75	0.68	1.14	3.55	0.86	1.14
time (sec)	N/A	0.039	0.017	0.003	1.422	1.207	0.137	1.085

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	15	16	20	72	14	20
normalized size	1	1.	0.65	0.7	0.87	3.13	0.61	0.87
time (sec)	N/A	0.007	0.007	0.005	1.412	1.011	0.104	1.086

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	41	15	24	8	15
normalized size	1	1.	1.	3.73	1.36	2.18	0.73	1.36
time (sec)	N/A	0.006	0.006	0.008	0.933	1.015	0.12	1.124

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	23	20	31	126	39	63
normalized size	1	1.	0.51	0.44	0.69	2.8	0.87	1.4
time (sec)	N/A	0.024	0.024	0.035	1.417	1.106	0.495	1.092

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	31	30	0	306	110	72
normalized size	1	1.	0.84	0.81	0.	8.27	2.97	1.95
time (sec)	N/A	0.018	0.023	0.012	0.	1.168	5.919	1.093

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	20	16	50	155	36	47
normalized size	1	1.	0.36	0.29	0.89	2.77	0.64	0.84
time (sec)	N/A	0.015	0.012	0.007	1.415	1.037	0.353	1.164

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	20	16	26	84	32	54
normalized size	1	1.	0.65	0.52	0.84	2.71	1.03	1.74
time (sec)	N/A	0.011	0.01	0.013	1.417	1.181	0.281	1.081

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	18	15	19	105	581	65
normalized size	1	1.	0.5	0.42	0.53	2.92	16.14	1.81
time (sec)	N/A	0.038	0.029	0.019	1.45	1.211	64.55	1.091

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	16	20	99	2866	35
normalized size	1	1.	1.	1.07	1.33	6.6	191.07	2.33
time (sec)	N/A	0.023	0.039	0.04	1.427	1.163	40.991	1.189

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	19	95	228	18
normalized size	1	1.	1.	0.82	1.12	5.59	13.41	1.06
time (sec)	N/A	0.012	0.028	0.077	0.952	1.004	123.877	1.105

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	22	25	55	39	22	34
normalized size	1	1.	0.73	0.83	1.83	1.3	0.73	1.13
time (sec)	N/A	0.029	0.034	0.048	1.418	1.028	0.297	1.104

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	23	30	72	24	30
normalized size	1	1.	1.	0.79	1.03	2.48	0.83	1.03
time (sec)	N/A	0.004	0.007	0.003	1.426	0.995	0.193	1.137

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	23	8	15
normalized size	1	1.	1.	0.92	1.15	1.77	0.62	1.15
time (sec)	N/A	0.002	0.001	0.002	0.928	1.062	0.143	1.097

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	30	55	104	88	63
normalized size	1	1.	0.89	0.81	1.49	2.81	2.38	1.7
time (sec)	N/A	0.02	0.007	0.004	1.424	1.129	1.379	1.091

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	31	22	34	72	0	35
normalized size	1	1.	1.41	1.	1.55	3.27	0.	1.59
time (sec)	N/A	0.007	0.014	0.004	0.947	1.079	0.	1.106

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	21	27	69	24	34
normalized size	1	1.	1.	0.78	1.	2.56	0.89	1.26
time (sec)	N/A	0.003	0.006	0.003	1.415	1.059	0.19	1.092

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	27	21	30	82	0	36
normalized size	1	1.	1.	0.78	1.11	3.04	0.	1.33
time (sec)	N/A	0.011	0.004	0.004	1.395	1.009	0.	1.089

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	29	12	20	46	0	24
normalized size	1	1.	2.07	0.86	1.43	3.29	0.	1.71
time (sec)	N/A	0.003	0.005	0.003	0.942	1.238	0.	1.114

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	68	88	80	232	0	227
normalized size	1	1.	1.	1.29	1.18	3.41	0.	3.34
time (sec)	N/A	0.037	0.027	0.005	1.452	1.127	0.	1.134

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	13	16	0	58	0
normalized size	1	1.	1.	1.	1.23	0.	4.46	0.
time (sec)	N/A	0.015	0.002	0.002	0.943	0.	2.348	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	15	3	11	2	3
normalized size	1	1.	1.	1.	0.2	0.73	0.13	0.2
time (sec)	N/A	0.01	0.009	0.012	0.928	1.074	0.183	1.121

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	8	3	9	2	3
normalized size	1	1.	1.	4.	1.5	4.5	1.	1.5
time (sec)	N/A	0.01	0.005	0.002	1.03	1.093	0.732	1.101

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	9	5	12	3	5
normalized size	1	1.	1.	2.25	1.25	3.	0.75	1.25
time (sec)	N/A	0.012	0.006	0.003	1.03	0.998	0.781	1.077

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	16	7	26	7	18
normalized size	1	1.	1.	1.45	0.64	2.36	0.64	1.64
time (sec)	N/A	0.02	0.007	0.002	1.022	1.043	0.965	1.102

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	12	28	10	0
normalized size	1	1.	1.	1.07	0.86	2.	0.71	0.
time (sec)	N/A	0.012	0.002	0.002	1.022	0.999	1.065	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	17	22	35	0	19
normalized size	1	1.	1.	1.13	1.47	2.33	0.	1.27
time (sec)	N/A	0.015	0.012	0.006	1.039	1.148	0.	1.083

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	32	0	14
normalized size	1	1.	1.	1.08	1.38	2.46	0.	1.08
time (sec)	N/A	0.065	0.018	0.006	1.035	1.122	0.	1.128

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	22	22	65	0	0
normalized size	1	1.	1.	1.16	1.16	3.42	0.	0.
time (sec)	N/A	0.023	0.019	0.003	1.032	1.1	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	24	58	0	22
normalized size	1	1.	1.	1.06	1.33	3.22	0.	1.22
time (sec)	N/A	0.02	0.008	0.02	1.04	1.169	0.	1.134

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	11	10	12	23	7	12
normalized size	1	1.	0.69	0.62	0.75	1.44	0.44	0.75
time (sec)	N/A	0.009	0.004	0.002	0.942	1.155	0.074	1.094

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	16	15	19	34	12	19
normalized size	1	1.	0.62	0.58	0.73	1.31	0.46	0.73
time (sec)	N/A	0.021	0.005	0.002	0.946	1.075	0.079	1.096

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	21	20	26	45	17	26
normalized size	1	1.	0.58	0.56	0.72	1.25	0.47	0.72
time (sec)	N/A	0.034	0.006	0.001	0.94	1.191	0.081	1.088

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	39	111	244	144	360	104
normalized size	1	1.	0.81	2.31	5.08	3.	7.5	2.17
time (sec)	N/A	0.034	0.095	0.079	1.442	1.284	2.493	1.118

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	9	4	23	2	4
normalized size	1	1.	1.	4.5	2.	11.5	1.	2.
time (sec)	N/A	0.002	0.017	0.003	1.031	1.037	0.461	1.082

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	17	8	50	7	15
normalized size	1	1.	1.	1.7	0.8	5.	0.7	1.5
time (sec)	N/A	0.004	0.001	0.002	1.023	1.324	0.472	1.086

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	0	30	47	24	0
normalized size	1	1.	1.	0.	1.36	2.14	1.09	0.
time (sec)	N/A	0.017	0.011	0.02	1.038	1.156	4.908	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	10	12	15	23	0	12
normalized size	1	1.	0.83	1.	1.25	1.92	0.	1.
time (sec)	N/A	0.015	0.013	0.003	1.036	1.127	0.	1.086

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	0	47	0	27
normalized size	1	1.	1.	1.05	0.	2.14	0.	1.23
time (sec)	N/A	0.056	0.033	0.006	0.	1.134	0.	1.095

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	17	116	0	0	27	0
normalized size	1	1.	0.17	1.13	0.	0.	0.26	0.
time (sec)	N/A	0.008	0.002	0.039	0.	0.	0.531	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [83] had the largest ratio of [2.]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.	9	0.111
2	A	2	1	1.	11	0.091
3	A	2	1	1.	11	0.091
4	A	2	1	1.	11	0.091
5	A	1	1	1.	14	0.071
6	A	2	1	1.	4	0.25
7	A	2	1	1.	9	0.111
8	A	2	2	1.	7	0.286
9	A	2	2	1.	17	0.118
10	A	2	2	1.	9	0.222
11	A	3	3	1.	11	0.273
12	A	3	3	1.	16	0.188
13	A	2	2	1.	10	0.2
14	A	1	1	1.	15	0.067
15	A	2	1	1.	9	0.111
16	A	1	1	1.	9	0.111
17	A	1	1	1.	15	0.067
18	A	1	1	1.	17	0.059
19	A	3	2	1.	20	0.1
20	A	1	1	1.	26	0.038

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	2	2	1.	20	0.1
22	A	2	2	1.	4	0.5
23	A	3	2	1.	6	0.333
24	A	4	2	1.	6	0.333
25	A	4	2	1.	6	0.333
26	A	2	2	1.	5	0.4
27	A	3	3	1.	6	0.5
28	A	2	2	1.	4	0.5
29	A	2	1	1.	4	0.25
30	A	3	2	1.	4	0.5
31	A	2	1	1.	4	0.25
32	A	4	2	1.	4	0.5
33	A	2	2	1.	6	0.333
34	A	3	3	1.	6	0.5
35	A	4	4	1.	8	0.5
36	A	2	2	1.	4	0.5
37	A	2	1	1.	4	0.25
38	A	3	2	1.	4	0.5
39	A	5	3	1.	13	0.231
40	A	3	2	1.	13	0.154
41	A	2	2	1.	13	0.154
42	A	2	2	1.	4	0.5
43	A	3	2	1.	4	0.5
44	A	2	2	1.	4	0.5
45	A	3	2	1.	4	0.5
46	A	2	2	1.	10	0.2
47	A	1	1	1.	11	0.091
48	A	1	1	1.	9	0.111
49	A	1	1	1.	13	0.077
50	A	1	1	1.	11	0.091
51	A	2	1	1.	11	0.091
52	A	2	2	1.	8	0.25
53	A	5	3	1.	8	0.375
54	A	1	1	1.	10	0.1
55	A	3	2	1.	17	0.118
56	A	1	1	1.	7	0.143
57	A	2	2	1.	4	0.5
58	A	1	1	1.	4	0.25
59	A	2	2	1.	6	0.333
60	A	1	1	1.	5	0.2
61	A	1	1	1.	2	0.5
62	A	1	1	1.	8	0.125
63	A	2	2	1.	8	0.25
64	A	2	2	1.	8	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
65	A	2	2	1.	14	0.143
66	A	3	2	1.	14	0.143
67	A	3	2	1.	8	0.25
68	A	1	1	1.	9	0.111
69	A	1	1	1.	13	0.077
70	A	2	2	1.	9	0.222
71	A	1	1	1.	6	0.167
72	A	1	1	1.	6	0.167
73	A	4	4	1.	7	0.571
74	A	2	2	1.	5	0.4
75	A	2	2	1.	7	0.286
76	A	3	2	1.	7	0.286
77	A	3	2	1.	9	0.222
78	A	3	3	1.	7	0.429
79	A	2	2	1.	11	0.182
80	A	1	1	1.	10	0.1
81	A	1	1	1.	10	0.1
82	A	2	2	1.	2	1.
83	A	4	4	1.	2	2.
84	A	4	4	1.	2	2.
85	A	3	3	1.	4	0.75
86	A	4	4	1.	6	0.667
87	A	2	2	1.	13	0.154
88	A	2	2	1.	14	0.143
89	A	1	1	1.	9	0.111
90	A	1	1	1.	9	0.111
91	A	2	2	1.	10	0.2
92	A	3	3	1.	4	0.75
93	A	4	3	1.	6	0.5
94	A	5	5	1.	6	0.833
95	A	4	4	1.	6	0.667
96	A	1	3	1.	17	0.176
97	A	2	2	1.	11	0.182
98	A	1	1	1.	15	0.067
99	A	1	1	1.	14	0.071
100	A	2	2	1.	11	0.182
101	A	2	2	1.	13	0.154
102	A	5	6	1.	10	0.6
103	A	3	3	1.	15	0.2
104	A	4	4	1.	15	0.267
105	A	3	3	1.	15	0.2
106	A	3	2	1.	16	0.125
107	A	3	2	1.	16	0.125
108	A	6	4	1.	18	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
109	A	3	2	1.	23	0.087
110	A	2	1	1.	19	0.053
111	A	5	5	1.	18	0.278
112	A	6	5	1.	31	0.161
113	A	2	2	1.	7	0.286
114	A	3	2	1.	22	0.091
115	A	2	1	1.	16	0.062
116	A	2	1	1.	17	0.059
117	A	2	1	1.	12	0.083
118	A	3	2	1.	21	0.095
119	A	2	1	1.	20	0.05
120	A	3	3	1.	16	0.188
121	A	4	3	1.	16	0.188
122	A	2	1	1.	11	0.091
123	A	3	2	1.	11	0.182
124	A	2	1	1.	16	0.062
125	A	2	1	1.	7	0.143
126	A	5	5	1.	9	0.556
127	A	4	3	1.	12	0.25
128	A	3	2	1.	14	0.143
129	A	4	4	1.	21	0.19
130	A	3	2	1.	18	0.111
131	A	2	2	1.	7	0.286
132	A	3	3	1.	11	0.273
133	A	3	2	1.	16	0.125
134	A	3	2	1.	11	0.182
135	A	5	4	1.	18	0.222
136	A	3	3	1.	7	0.429
137	A	9	6	1.	7	0.857
138	A	3	3	1.	14	0.214
139	A	1	1	1.	16	0.062
140	A	3	3	1.	12	0.25
141	A	2	2	1.	8	0.25
142	A	2	2	1.	8	0.25
143	A	1	1	1.	10	0.1
144	A	3	3	1.	13	0.231
145	A	2	1	1.	19	0.053
146	A	1	1	1.	11	0.091
147	A	3	3	1.	11	0.273
148	A	2	2	1.	11	0.182
149	A	1	1	1.	13	0.077
150	A	4	4	1.	15	0.267
151	A	3	3	1.	13	0.231
152	A	2	2	1.	9	0.222

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
153	A	3	3	1.	12	0.25
154	A	2	2	1.	9	0.222
155	A	6	6	1.	18	0.333
156	A	2	2	1.	8	0.25
157	A	3	3	1.	5	0.6
158	A	1	1	1.	7	0.143
159	A	1	1	1.	9	0.111
160	A	2	2	1.	7	0.286
161	A	2	2	1.	5	0.4
162	A	1	1	1.	14	0.071
163	A	2	2	1.	14	0.143
164	A	2	2	1.	9	0.222
165	A	2	3	1.	8	0.375
166	A	2	2	1.	7	0.286
167	A	3	2	1.	9	0.222
168	A	4	2	1.	9	0.222
169	A	1	1	1.	21	0.048
170	A	1	1	1.	4	0.25
171	A	2	2	1.	4	0.5
172	A	2	2	1.	8	0.25
173	A	1	1	1.	11	0.091
174	A	4	2	1.	16	0.125
175	A	1	1	1.	9	0.111

Chapter 3

Listing of integrals

3.1 $\int \sqrt{1+2x}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}(2x+1)^{3/2}$$

```
[Out] (1 + 2*x)^(3/2)/3
```

Rubi [A] time = 0.0009888, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {32}

$$\frac{1}{3}(2x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x],x]
```

```
[Out] (1 + 2*x)^(3/2)/3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{1+2x}\,dx = \frac{1}{3}(1+2x)^{3/2}$$

Mathematica [A] time = 0.0371224, size = 13, normalized size = 1.

$$\frac{1}{3}(2x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x],x]
```

```
[Out] (1 + 2*x)^(3/2)/3
```

Maple [A] time = 0.011, size = 10, normalized size = 0.8

$$\frac{1}{3}(1+2x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+2*x)^(1/2),x)
```

```
[Out] 1/3*(1+2*x)^(3/2)
```

Maxima [A] time = 0.944664, size = 12, normalized size = 0.92

$$\frac{1}{3}(2x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*x + 1)^(3/2)
```

Fricas [A] time = 0.416858, size = 28, normalized size = 2.15

$$\frac{1}{3}(2x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*x + 1)^(3/2)
```

Sympy [A] time = 0.051242, size = 8, normalized size = 0.62

$$\frac{(2x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)**(1/2),x)
```

```
[Out] (2*x + 1)**(3/2)/3
```

Giac [A] time = 1.06262, size = 12, normalized size = 0.92

$$\frac{1}{3}\,(2\,x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*x + 1)^(3/2)
```

3.2 $\int x\sqrt{1+3x}\,dx$

Optimal. Leaf size=27

$$\frac{2}{45}(3x+1)^{5/2}-\frac{2}{27}(3x+1)^{3/2}$$

```
[Out] (-2*(1 + 3*x)^(3/2))/27 + (2*(1 + 3*x)^(5/2))/45
```

Rubi [A] time = 0.0044375, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2}{45}(3x+1)^{5/2}-\frac{2}{27}(3x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + 3*x],x]
```

```
[Out] (-2*(1 + 3*x)^(3/2))/27 + (2*(1 + 3*x)^(5/2))/45
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{1+3x}\,dx &= \int\left(-\frac{1}{3}\sqrt{1+3x}+\frac{1}{3}(1+3x)^{3/2}\right)dx\\ &= -\frac{2}{27}(1+3x)^{3/2}+\frac{2}{45}(1+3x)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.005936, size = 18, normalized size = 0.67

$$\frac{2}{135}(3x+1)^{3/2}(9x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + 3*x],x]
```

```
[Out] (2*(1 + 3*x)^(3/2)*(-2 + 9*x))/135
```

Maple [A] time = 0.002, size = 15, normalized size = 0.6

$$\frac{18\,x-4}{135}\,(1+3\,x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(1+3*x)^(1/2),x)
```

```
[Out] 2/135*(1+3*x)^(3/2)*(9*x-2)
```

Maxima [A] time = 0.949593, size = 26, normalized size = 0.96

$$\frac{2}{45}\,(3\,x+1)^{\frac{5}{2}}-\frac{2}{27}\,(3\,x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/45*(3*x + 1)^(5/2) - 2/27*(3*x + 1)^(3/2)
```

Fricas [A] time = 0.412359, size = 54, normalized size = 2.

$$\frac{2}{135}\left(27\,x^2+3\,x-2\right)\sqrt{3\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/135*(27*x^2 + 3*x - 2)*sqrt(3*x + 1)
```

Sympy [A] time = 0.854789, size = 39, normalized size = 1.44

$$\frac{2x^2\sqrt{3x+1}}{5}+\frac{2x\sqrt{3x+1}}{45}-\frac{4\sqrt{3x+1}}{135}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+3*x)**(1/2),x)
```

```
[Out] 2*x**2*sqrt(3*x + 1)/5 + 2*x*sqrt(3*x + 1)/45 - 4*sqrt(3*x + 1)/135
```

Giac [A] time = 1.09895, size = 26, normalized size = 0.96

$$\frac{2}{45}\,(3\,x+1)^{\frac{5}{2}}-\frac{2}{27}\,(3\,x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+3*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/45*(3*x + 1)^(5/2) - 2/27*(3*x + 1)^(3/2)
```

3.3 $\int x^2\sqrt{1+x}\,dx$

Optimal. Leaf size=34

$$\frac{2}{7}(x+1)^{7/2}-\frac{4}{5}(x+1)^{5/2}+\frac{2}{3}(x+1)^{3/2}$$

```
[Out] (2*(1 + x)^(3/2))/3 - (4*(1 + x)^(5/2))/5 + (2*(1 + x)^(7/2))/7
```

Rubi [A] time = 0.0051121, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2}{7}(x+1)^{7/2}-\frac{4}{5}(x+1)^{5/2}+\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[1 + x],x]
```

```
[Out] (2*(1 + x)^(3/2))/3 - (4*(1 + x)^(5/2))/5 + (2*(1 + x)^(7/2))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int x^2\sqrt{1+x}\,dx &= \int\left(\sqrt{1+x}-2(1+x)^{3/2}+(1+x)^{5/2}\right)dx\\ &= \frac{2}{3}(1+x)^{3/2}-\frac{4}{5}(1+x)^{5/2}+\frac{2}{7}(1+x)^{7/2}\end{aligned}$$

Mathematica [A] time = 0.0056155, size = 21, normalized size = 0.62

$$\frac{2}{105}(x+1)^{3/2}\left(15x^2-12x+8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[1 + x],x]
```

```
[Out] (2*(1 + x)^(3/2)*(8 - 12*x + 15*x^2))/105
```

Maple [A] time = 0.003, size = 18, normalized size = 0.5

$$\frac{30\,x^2-24\,x+16}{105}\,(1+x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(1+x)^(1/2),x)
```

```
[Out] 2/105*(1+x)^(3/2)*(15*x^2-12*x+8)
```

Maxima [A] time = 0.93806, size = 30, normalized size = 0.88

$$\frac{2}{7}(x+1)^{\frac{7}{2}} - \frac{4}{5}(x+1)^{\frac{5}{2}} + \frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/7*(x + 1)^(7/2) - 4/5*(x + 1)^(5/2) + 2/3*(x + 1)^(3/2)
```

Fricas [A] time = 0.417573, size = 62, normalized size = 1.82

$$\frac{2}{105}\left(15x^3 + 3x^2 - 4x + 8\right)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2/105*(15*x^3 + 3*x^2 - 4*x + 8)*sqrt(x + 1)
```

Sympy [A] time = 1.19745, size = 48, normalized size = 1.41

$$\frac{2x^3\sqrt{x+1}}{7} + \frac{2x^2\sqrt{x+1}}{35} - \frac{8x\sqrt{x+1}}{105} + \frac{16\sqrt{x+1}}{105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(1+x)**(1/2),x)
```

```
[Out] 2*x**3*sqrt(x + 1)/7 + 2*x**2*sqrt(x + 1)/35 - 8*x*sqrt(x + 1)/105 + 16*sqr
t(x + 1)/105
```

Giac [A] time = 1.06609, size = 30, normalized size = 0.88

$$\frac{2}{7}(x+1)^{\frac{7}{2}} - \frac{4}{5}(x+1)^{\frac{5}{2}} + \frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/7*(x + 1)^(7/2) - 4/5*(x + 1)^(5/2) + 2/3*(x + 1)^(3/2)
```

3.4 $\int \frac{x}{\sqrt{2-3x}}\,dx$

Optimal. Leaf size=27

$$\frac{2}{27}(2-3x)^{3/2}-\frac{4}{9}\sqrt{2-3x}$$

```
[Out] (-4*Sqrt[2 - 3*x])/9 + (2*(2 - 3*x)^(3/2))/27
```

Rubi [A] time = 0.0046574, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$\frac{2}{27}(2-3x)^{3/2}-\frac{4}{9}\sqrt{2-3x}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[2 - 3*x],x]
```

```
[Out] (-4*Sqrt[2 - 3*x])/9 + (2*(2 - 3*x)^(3/2))/27
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{2-3x}}\,dx &= \int \left(\frac{2}{3\sqrt{2-3x}}-\frac{1}{3}\sqrt{2-3x}\right)dx\\ &= -\frac{4}{9}\sqrt{2-3x}+\frac{2}{27}(2-3x)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0049493, size = 18, normalized size = 0.67

$$-\frac{2}{27}\sqrt{2-3x}(3x+4)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[2 - 3*x],x]
```

```
[Out] (-2*Sqrt[2 - 3*x]*(4 + 3*x))/27
```

Maple [A] time = 0.004, size = 15, normalized size = 0.6

$$-\frac{6x+8}{27}\sqrt{2-3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(2-3*x)^(1/2),x)
```

```
[Out] -2/27*(3*x+4)*(2-3*x)^(1/2)
```

Maxima [A] time = 0.947613, size = 26, normalized size = 0.96

$$\frac{2}{27}\,(-3\,x+2)^{\frac{3}{2}}-\frac{4}{9}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-3*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/27*(-3*x + 2)^(3/2) - 4/9*sqrt(-3*x + 2)
```

Fricas [A] time = 0.416598, size = 43, normalized size = 1.59

$$-\frac{2}{27}\,(3\,x+4)\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-3*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/27*(3*x + 4)*sqrt(-3*x + 2)
```

Sympy [A] time = 0.82036, size = 61, normalized size = 2.26

$$\begin{cases} -\frac{2ix\sqrt{3x-2}}{9}-\frac{8i\sqrt{3x-2}}{27} & \text{for}\ \frac{3|x|}{2}>1 \\ -\frac{2x\sqrt{2-3x}}{9}-\frac{8\sqrt{2-3x}}{27} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-3*x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*x*sqrt(3*x - 2)/9 - 8*I*sqrt(3*x - 2)/27, 3*Abs(x)/2 > 1),
(-2*x*sqrt(2 - 3*x)/9 - 8*sqrt(2 - 3*x)/27, True))
```

Giac [A] time = 1.06409, size = 26, normalized size = 0.96

$$\frac{2}{27}\,(-3\,x+2)^{\frac{3}{2}}-\frac{4}{9}\,\sqrt{-3\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(2-3*x)^(1/2),x, algorithm="giac")
```

```
[Out] 2/27*(-3*x + 2)^(3/2) - 4/9*sqrt(-3*x + 2)
```

3.5 $\int \frac{1+x}{\left(2+2x+x^2\right)^3} \, dx$

Optimal. Leaf size=14

$$-\frac{1}{4\left(x^2+2x+2\right)^2}$$

```
[Out] -1/(4*(2 + 2*x + x^2)^2)
```

Rubi [A] time = 0.0028358, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {629}

$$-\frac{1}{4\left(x^2+2x+2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(2 + 2*x + x^2)^3,x]
```

```
[Out] -1/(4*(2 + 2*x + x^2)^2)
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{1+x}{\left(2+2x+x^2\right)^3} \, dx = -\frac{1}{4\left(2+2x+x^2\right)^2}$$

Mathematica [A] time = 0.0040282, size = 14, normalized size = 1.

$$-\frac{1}{4\left(x^2+2x+2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(2 + 2*x + x^2)^3,x]
```

```
[Out] -1/(4*(2 + 2*x + x^2)^2)
```

Maple [A] time = 0.007, size = 13, normalized size = 0.9

$$-\frac{1}{4\left(x^2+2x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^2+2*x+2)^3,x)
```

```
[Out] -1/4/(x^2+2*x+2)^2
```

Maxima [A] time = 0.983106, size = 16, normalized size = 1.14

$$-\frac{1}{4\left(x^2+2\,x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+2)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(x^2 + 2*x + 2)^2
```

Fricas [A] time = 0.404062, size = 51, normalized size = 3.64

$$-\frac{1}{4\left(x^4+4\,x^3+8\,x^2+8\,x+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+2)^3,x, algorithm="fricas")
```

```
[Out] -1/4/(x^4 + 4*x^3 + 8*x^2 + 8*x + 4)
```

Sympy [A] time = 0.105645, size = 22, normalized size = 1.57

$$-\frac{1}{4x^4+16x^3+32x^2+32x+16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**2+2*x+2)**3,x)
```

```
[Out] -1/(4*x**4 + 16*x**3 + 32*x**2 + 32*x + 16)
```

Giac [A] time = 1.0551, size = 16, normalized size = 1.14

$$-\frac{1}{4\left(x^2+2\,x+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^2+2*x+2)^3,x, algorithm="giac")
```

```
[Out] -1/4/(x^2 + 2*x + 2)^2
```

3.6 $\int \sin^3(x)\,dx$

Optimal. Leaf size=13

$$\frac{\cos^3(x)}{3}-\cos(x)$$

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rubi [A] time = 0.0063655, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2633}

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3,x]
```

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(x)\,dx &= -\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(x)\right)\\ &= -\cos(x)+\frac{\cos^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.0035208, size = 15, normalized size = 1.15

$$\frac{1}{12}\cos(3x)-\frac{3\cos(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3,x]
```

```
[Out] (-3*Cos[x])/4 + Cos[3*x]/12
```

Maple [A] time = 0.062, size = 11, normalized size = 0.9

$$-\frac{\left(2+(\sin(x))^2\right)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x)
```

```
[Out] -1/3*(2+sin(x)^2)*cos(x)
```

Maxima [A] time = 0.980011, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

Fricas [A] time = 0.455377, size = 31, normalized size = 2.38

$$\frac{1}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

Sympy [A] time = 0.058438, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3,x)
```

```
[Out] cos(x)**3/3 - cos(x)
```

Giac [A] time = 1.10449, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

3.7 $\int \sqrt[3]{-1+z}z\,dz$

Optimal. Leaf size=23

$$\frac{3}{7}(z-1)^{7/3}+\frac{3}{4}(z-1)^{4/3}$$

```
[Out] (3*(-1 + z)^(4/3))/4 + (3*(-1 + z)^(7/3))/7
```

Rubi [A] time = 0.0037208, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{3}{7}(z-1)^{7/3}+\frac{3}{4}(z-1)^{4/3}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + z)^(1/3)*z,z]
```

```
[Out] (3*(-1 + z)^(4/3))/4 + (3*(-1 + z)^(7/3))/7
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt[3]{-1+z}z\,dz &= \int \left(\sqrt[3]{-1+z}+(-1+z)^{4/3}\right)dz\\ &= \frac{3}{4}(-1+z)^{4/3}+\frac{3}{7}(-1+z)^{7/3}\end{aligned}$$

Mathematica [A] time = 0.0047318, size = 16, normalized size = 0.7

$$\frac{3}{28}(z-1)^{4/3}(4z+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + z)^(1/3)*z,z]
```

```
[Out] (3*(-1 + z)^(4/3)*(3 + 4*z))/28
```

Maple [A] time = 0.002, size = 13, normalized size = 0.6

$$\frac{12\,z+9}{28}\,(-1+z)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+z)^(1/3)*z,z)
```

```
[Out] 3/28*(-1+z)^(4/3)*(4*z+3)
```

Maxima [A] time = 0.932029, size = 20, normalized size = 0.87

$$\frac{3}{7} (z-1)^{\frac{7}{3}} + \frac{3}{4} (z-1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+z)^(1/3)*z,z, algorithm="maxima")
```

```
[Out] 3/7*(z - 1)^(7/3) + 3/4*(z - 1)^(4/3)
```

Fricas [A] time = 0.415033, size = 49, normalized size = 2.13

$$\frac{3}{28} \left(4 z^2 - z - 3\right)(z-1)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+z)^(1/3)*z,z, algorithm="fricas")
```

```
[Out] 3/28*(4*z^2 - z - 3)*(z - 1)^(1/3)
```

Sympy [A] time = 0.85472, size = 92, normalized size = 4.

$$\begin{cases} \frac{3z^2 \sqrt[3]{z-1}}{7} - \frac{3z \sqrt[3]{z-1}}{28} - \frac{9 \sqrt[3]{z-1}}{28} & \text{for } |z| > 1 \\ \frac{3z^2 \sqrt[3]{1-z} e^{\frac{i\pi}{3}}}{7} - \frac{3z \sqrt[3]{1-z} e^{\frac{i\pi}{3}}}{28} - \frac{9 \sqrt[3]{1-z} e^{\frac{i\pi}{3}}}{28} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+z)**(1/3)*z,z)
```

```
[Out] Piecewise((3*z**2*(z - 1)**(1/3)/7 - 3*z*(z - 1)**(1/3)/28 - 9*(z - 1)**(1/
3)/28, Abs(z) > 1), (3*z**2*(1 - z)**(1/3)*exp(I*pi/3)/7 - 3*z*(1 - z)**(1/
3)*exp(I*pi/3)/28 - 9*(1 - z)**(1/3)*exp(I*pi/3)/28, True))
```

Giac [A] time = 1.14098, size = 20, normalized size = 0.87

$$\frac{3}{7} (z-1)^{\frac{7}{3}} + \frac{3}{4} (z-1)^{\frac{4}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+z)^(1/3)*z,z, algorithm="giac")
```

```
[Out] 3/7*(z - 1)^(7/3) + 3/4*(z - 1)^(4/3)
```

3.8 $\int \cot(x)\csc^2(x)\,dx$

Optimal. Leaf size=8

$$-\frac{1}{2}\csc^2(x)$$

```
[Out] -Csc[x]^2/2
```

Rubi [A] time = 0.0130718, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2606, 30}

$$-\frac{1}{2}\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Csc[x]^2,x]
```

```
[Out] -Csc[x]^2/2
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cot(x)\csc^2(x)\,dx &= -\text{Subst}\left(\int x\,dx, x, \csc(x)\right)\\ &= -\frac{1}{2}\csc^2(x)\end{aligned}$$

Mathematica [A] time = 0.004811, size = 8, normalized size = 1.

$$-\frac{1}{2}\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Csc[x]^2,x]
```

```
[Out] -Csc[x]^2/2
```

Maple [A] time = 0.007, size = 7, normalized size = 0.9

$$-\frac{1}{2\left(\sin\left(x\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/sin(x)^3,x)
```

```
[Out] -1/2/sin(x)^2
```

Maxima [A] time = 0.942057, size = 8, normalized size = 1.

$$-\frac{1}{2\sin\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)^3,x, algorithm="maxima")
```

```
[Out] -1/2/sin(x)^2
```

Fricas [A] time = 0.43484, size = 27, normalized size = 3.38

$$\frac{1}{2\left(\cos\left(x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/2/(cos(x)^2 - 1)
```

Sympy [A] time = 0.059657, size = 8, normalized size = 1.

$$-\frac{1}{2\sin^{2}\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)**3,x)
```

```
[Out] -1/(2*sin(x)**2)
```

Giac [A] time = 1.08131, size = 14, normalized size = 1.75

$$\frac{1}{2\left(\cos\left(x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/2/(cos(x)^2 - 1)
```

3.9 $\int \cos(2x)\sqrt{4-\sin(2x)}\,dx$

Optimal. Leaf size=16

$$-\frac{1}{3}(4-\sin(2x))^{3/2}$$

```
[Out] -(4 - Sin[2*x])^(3/2)/3
```

Rubi [A] time = 0.0230421, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2668, 32}

$$-\frac{1}{3}(4-\sin(2x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[2*x]*Sqrt[4 - Sin[2*x]],x]
```

```
[Out] -(4 - Sin[2*x])^(3/2)/3
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(2x)\sqrt{4-\sin(2x)}\,dx &= -\left(\frac{1}{2}\text{Subst}\left(\int \sqrt{4+x}\,dx, x, -\sin(2x)\right)\right)\\ &= -\frac{1}{3}(4-\sin(2x))^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0119864, size = 16, normalized size = 1.

$$-\frac{1}{3}(4-\sin(2x))^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[2*x]*Sqrt[4 - Sin[2*x]],x]
```

```
[Out] -(4 - Sin[2*x])^(3/2)/3
```

Maple [A] time = 0.013, size = 13, normalized size = 0.8

$$-\frac{1}{3}(4-\sin(2x))^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(2*x)*(4-sin(2*x))^(1/2),x)
```

```
[Out] -1/3*(4-sin(2*x))^(3/2)
```

Maxima [A] time = 0.946658, size = 16, normalized size = 1.

$$-\frac{1}{3}(-\sin(2x)+4)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(4-sin(2*x))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-sin(2*x) + 4)^(3/2)
```

Fricas [A] time = 0.46015, size = 54, normalized size = 3.38

$$\frac{1}{3}(\sin(2x)-4)\sqrt{-\sin(2x)+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(4-sin(2*x))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(sin(2*x) - 4)*sqrt(-sin(2*x) + 4)
```

Sympy [B] time = 0.29622, size = 29, normalized size = 1.81

$$\frac{\sqrt{4-\sin(2x)}\sin(2x)}{3}-\frac{4\sqrt{4-\sin(2x)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(4-sin(2*x))**(1/2),x)
```

```
[Out] sqrt(4 - sin(2*x))*sin(2*x)/3 - 4*sqrt(4 - sin(2*x))/3
```

Giac [A] time = 1.08493, size = 16, normalized size = 1.

$$-\frac{1}{3}(-\sin(2x)+4)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(2*x)*(4-sin(2*x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*(-sin(2*x) + 4)^(3/2)
```

3.10 $\int \frac{\sin(x)}{(3+\cos(x))^2} dx$

Optimal. Leaf size=6

$$\frac{1}{\cos(x)+3}$$

```
[Out] (3 + Cos[x])^(-1)
```

Rubi [A] time = 0.0176402, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2668, 32}

$$\frac{1}{\cos(x)+3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(3 + Cos[x])^2,x]
```

```
[Out] (3 + Cos[x])^(-1)
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{(3+\cos(x))^2} dx &= -\text{Subst}\left(\int \frac{1}{(3+x)^2} dx, x, \cos(x)\right)\\ &= \frac{1}{3+\cos(x)}\end{aligned}$$

Mathematica [A] time = 0.0131301, size = 6, normalized size = 1.

$$\frac{1}{\cos(x)+3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(3 + Cos[x])^2,x]
```

```
[Out] (3 + Cos[x])^(-1)
```

Maple [A] time = 0.027, size = 7, normalized size = 1.2

$$(3+\cos(x))^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(3+cos(x))^2,x)
```

```
[Out] 1/(3+cos(x))
```

Maxima [A] time = 0.945644, size = 8, normalized size = 1.33

$$\frac{1}{\cos(x)+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(3+cos(x))^2,x, algorithm="maxima")
```

```
[Out] 1/(cos(x) + 3)
```

Fricas [A] time = 0.432652, size = 22, normalized size = 3.67

$$\frac{1}{\cos(x)+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(3+cos(x))^2,x, algorithm="fricas")
```

```
[Out] 1/(cos(x) + 3)
```

Sympy [A] time = 0.413677, size = 5, normalized size = 0.83

$$\frac{1}{\cos(x)+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(3+cos(x))**2,x)
```

```
[Out] 1/(cos(x) + 3)
```

Giac [A] time = 1.07614, size = 8, normalized size = 1.33

$$\frac{1}{\cos(x)+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(3+cos(x))^2,x, algorithm="giac")
```

```
[Out] 1/(cos(x) + 3)
```

3.11 $\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx$

Optimal. Leaf size=12

$$\frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

```
[Out] (2*Cos[x])/Sqrt[Cos[x]^3]
```

Rubi [A] time = 0.027797, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3207, 2565, 30}

$$\frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/Sqrt[Cos[x]^3],x]
```

```
[Out] (2*Cos[x])/Sqrt[Cos[x]^3]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{\sqrt{\cos^3(x)}}\,dx &= \frac{\cos^{\frac{3}{2}}(x)\int \frac{\sin(x)}{\cos^{\frac{3}{2}}(x)}\,dx}{\sqrt{\cos^3(x)}}\\ &= -\frac{\cos^{\frac{3}{2}}(x)\,\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, \cos(x)\right)}{\sqrt{\cos^3(x)}}\\ &= \frac{2\cos(x)}{\sqrt{\cos^3(x)}}\end{aligned}$$

Mathematica [A] time = 0.0129631, size = 12, normalized size = 1.

$$\frac{2\cos(x)}{\sqrt{\cos^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/Sqrt[Cos[x]^3],x]
```

```
[Out] (2*Cos[x])/Sqrt[Cos[x]^3]
```

Maple [A] time = 0.046, size = 11, normalized size = 0.9

$$2\,{\frac{\cos\left(x\right)}{\sqrt{\left(\cos\left(x\right)\right)^{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cos(x)^3)^(1/2),x)
```

```
[Out] 2*cos(x)/(cos(x)^3)^(1/2)
```

Maxima [A] time = 0.930996, size = 14, normalized size = 1.17

$$\frac{2\,\cos\left(x\right)}{\sqrt{\cos\left(x\right)^{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*cos(x)/sqrt(cos(x)^3)
```

Fricas [A] time = 0.456553, size = 36, normalized size = 3.

$$\frac{2\,\sqrt{\cos\left(x\right)^{3}}}{\cos\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(cos(x)^3)/cos(x)^2
```

Sympy [A] time = 0.615896, size = 12, normalized size = 1.

$$\frac{2\cos{(x)}}{\sqrt{\cos^3{(x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)**3)**(1/2),x)
```

```
[Out] 2*cos(x)/sqrt(cos(x)**3)
```

Giac [A] time = 1.09372, size = 8, normalized size = 0.67

$$\frac{2}{\sqrt{\cos{(x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cos(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/sqrt(cos(x))
```

3.12 $\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx$

Optimal. Leaf size=10

$$-2\cos\left(\sqrt{x+1}\right)$$

```
[Out] -2*Cos[Sqrt[1 + x]]
```

Rubi [A] time = 0.0162841, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3431, 15, 2638}

$$-2\cos\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] -2*Cos[Sqrt[1 + x]]
```

Rule 3431

```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f
_.)*(x_))^(n_)])^(p_.), x_Symbol] :> Dist[1/(n*f), Subst[Int[ExpandIntegran
d[(a + b*Sin[c + d*x])^p, x^(1/n - 1)*(g - (e*h)/f + (h*x^(1/n))/f)^m, x],
x], x, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x] && IGtQ[p,
 0] && IntegerQ[1/n]
```

Rule 15

```
Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^F
racPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x]
 && !IntegerQ[m]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin\left(\sqrt{1+x}\right)}{\sqrt{1+x}}\,dx &= 2\,\text{Subst}\left(\int \frac{x\sin(x)}{\sqrt{x^2}}\,dx, x, \sqrt{1+x}\right)\\ &= 2\,\text{Subst}\left(\int \sin(x)\,dx, x, \sqrt{1+x}\right)\\ &= -2\cos\left(\sqrt{1+x}\right)\end{aligned}$$

Mathematica [A] time = 0.0184512, size = 10, normalized size = 1.

$$-2\cos\left(\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[Sqrt[1 + x]]/Sqrt[1 + x],x]
```

```
[Out] -2*Cos[Sqrt[1 + x]]
```

Maple [A] time = 0.007, size = 9, normalized size = 0.9

$$-2\cos\left(\sqrt{1+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((1+x)^(1/2))/(1+x)^(1/2),x)
```

```
[Out] -2*cos((1+x)^(1/2))
```

Maxima [A] time = 0.997247, size = 11, normalized size = 1.1

$$-2\cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((1+x)^(1/2))/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*cos(sqrt(x + 1))
```

Fricas [A] time = 0.453937, size = 28, normalized size = 2.8

$$-2\cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((1+x)^(1/2))/(1+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*cos(sqrt(x + 1))
```

Sympy [A] time = 0.31801, size = 10, normalized size = 1.

$$-2\cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((1+x)**(1/2))/(1+x)**(1/2),x)
```

```
[Out] -2*cos(sqrt(x + 1))
```

Giac [A] time = 1.08254, size = 11, normalized size = 1.1

$$-2 \cos\left(\sqrt{x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((1+x)^(1/2))/(1+x)^(1/2),x, algorithm="giac")
```

```
[Out] -2*cos(sqrt(x + 1))
```

3.13 $\int x^{-1+n} \sin(x^n)\, dx$

Optimal. Leaf size=9

$$-\frac{\cos(x^n)}{n}$$

[Out] -(Cos[x^n]/n)

Rubi [A] time = 0.0111279, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3379, 2638}

$$-\frac{\cos(x^n)}{n}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n)*Sin[x^n],x]

[Out] -(Cos[x^n]/n)

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^{-1+n} \sin(x^n)\, dx &= \frac{\text{Subst}\left(\int \sin(x)\, dx, x, x^n\right)}{n}\\ &= -\frac{\cos(x^n)}{n}\end{aligned}$$

Mathematica [A] time = 0.0137676, size = 9, normalized size = 1.

$$-\frac{\cos(x^n)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + n)*Sin[x^n],x]

[Out] -(Cos[x^n]/n)

Maple [A] time = 0.005, size = 10, normalized size = 1.1

$$-\frac{\cos(x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+n)*sin(x^n),x)
```

```
[Out] -cos(x^n)/n
```

Maxima [A] time = 0.94586, size = 12, normalized size = 1.33

$$-\frac{\cos(x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*sin(x^n),x, algorithm="maxima")
```

```
[Out] -cos(x^n)/n
```

Fricas [A] time = 0.46481, size = 18, normalized size = 2.

$$-\frac{\cos(x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*sin(x^n),x, algorithm="fricas")
```

```
[Out] -cos(x^n)/n
```

Sympy [A] time = 17.818, size = 7, normalized size = 0.78

$$-\frac{\cos(x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+n)*sin(x**n),x)
```

```
[Out] -cos(x**n)/n
```

Giac [A] time = 1.07018, size = 12, normalized size = 1.33

$$-\frac{\cos(x^n)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+n)*sin(x^n),x, algorithm="giac")
```

```
[Out] -cos(x^n)/n
```

3.14 $\int \frac{x^5}{\sqrt{1-x^6}} \, dx$

Optimal. Leaf size=15

$$-\frac{1}{3}\sqrt{1-x^6}$$

```
[Out] -Sqrt[1 - x^6]/3
```

Rubi [A] time = 0.0027586, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {261}

$$-\frac{1}{3}\sqrt{1-x^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[1 - x^6],x]

[Out] -Sqrt[1 - x^6]/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{1-x^6}} \, dx = -\frac{1}{3}\sqrt{1-x^6}$$

Mathematica [A] time = 0.002819, size = 15, normalized size = 1.

$$-\frac{1}{3}\sqrt{1-x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[1 - x^6],x]

[Out] -Sqrt[1 - x^6]/3
```

Maple [B] time = 0.006, size = 32, normalized size = 2.1

$$\frac{(-1+x)(1+x)\left(x^2+x+1\right)\left(x^2-x+1\right)}{3} \frac{1}{\sqrt{-x^6+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(-x^6+1)^(1/2),x)
```

```
[Out] 1/3*(-1+x)*(1+x)*(x^2+x+1)*(x^2-x+1)/(-x^6+1)^(1/2)
```

Maxima [A] time = 0.959997, size = 15, normalized size = 1.

$$-\frac{1}{3}\sqrt{-x^6+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^6+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-x^6 + 1)
```

Fricas [A] time = 0.419237, size = 28, normalized size = 1.87

$$-\frac{1}{3}\sqrt{-x^6+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^6+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*sqrt(-x^6 + 1)
```

Sympy [A] time = 0.249065, size = 10, normalized size = 0.67

$$-\frac{\sqrt{1-x^6}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(-x**6+1)**(1/2),x)
```

```
[Out] -sqrt(1 - x**6)/3
```

Giac [A] time = 1.07187, size = 15, normalized size = 1.

$$-\frac{1}{3}\sqrt{-x^6+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(-x^6+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(-x^6 + 1)
```

3.15 $\int t\sqrt[4]{1+t}\,dt$

Optimal. Leaf size=23

$$\frac{4}{9}(t+1)^{9/4}-\frac{4}{5}(t+1)^{5/4}$$

```
[Out] (-4*(1 + t)^(5/4))/5 + (4*(1 + t)^(9/4))/9
```

Rubi [A] time = 0.0037507, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {43}

$$\frac{4}{9}(t+1)^{9/4}-\frac{4}{5}(t+1)^{5/4}$$

Antiderivative was successfully verified.

```
[In] Int[t*(1 + t)^(1/4),t]
```

```
[Out] (-4*(1 + t)^(5/4))/5 + (4*(1 + t)^(9/4))/9
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int t\sqrt[4]{1+t}\,dt &= \int\left(-\sqrt[4]{1+t}+(1+t)^{5/4}\right)dt\\ &= -\frac{4}{5}(1+t)^{5/4}+\frac{4}{9}(1+t)^{9/4}\end{aligned}$$

Mathematica [A] time = 0.0040473, size = 16, normalized size = 0.7

$$\frac{4}{45}(t+1)^{5/4}(5t-4)$$

Antiderivative was successfully verified.

```
[In] Integrate[t*(1 + t)^(1/4),t]
```

```
[Out] (4*(1 + t)^(5/4)*(-4 + 5*t))/45
```

Maple [A] time = 0.001, size = 13, normalized size = 0.6

$$\frac{20\,t-16}{45}\,(1+t)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t*(1+t)^(1/4),t)
```

```
[Out] 4/45*(1+t)^(5/4)*(5*t-4)
```

Maxima [A] time = 0.955077, size = 20, normalized size = 0.87

$$\frac{4}{9}(t+1)^{\frac{9}{4}} - \frac{4}{5}(t+1)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*(1+t)^(1/4),t, algorithm="maxima")
```

```
[Out] 4/9*(t + 1)^(9/4) - 4/5*(t + 1)^(5/4)
```

Fricas [A] time = 0.419432, size = 49, normalized size = 2.13

$$\frac{4}{45}\left(5\,t^2+t-4\right)(t+1)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*(1+t)^(1/4),t, algorithm="fricas")
```

```
[Out] 4/45*(5*t^2 + t - 4)*(t + 1)^(1/4)
```

Sympy [A] time = 0.882601, size = 34, normalized size = 1.48

$$\frac{4t^2\sqrt[4]{t+1}}{9} + \frac{4t\sqrt[4]{t+1}}{45} - \frac{16\sqrt[4]{t+1}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*(1+t)**(1/4),t)
```

```
[Out] 4*t**2*(t + 1)**(1/4)/9 + 4*t*(t + 1)**(1/4)/45 - 16*(t + 1)**(1/4)/45
```

Giac [A] time = 1.06519, size = 20, normalized size = 0.87

$$\frac{4}{9}(t+1)^{\frac{9}{4}} - \frac{4}{5}(t+1)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t*(1+t)^(1/4),t, algorithm="giac")
```

```
[Out] 4/9*(t + 1)^(9/4) - 4/5*(t + 1)^(5/4)
```

3.16 $\int \frac{1}{(1+x^2)^{3/2}}\,dx$

Optimal. Leaf size=11

$$\frac{x}{\sqrt{x^2+1}}$$

```
[Out] x/Sqrt[1 + x^2]
```

Rubi [A] time = 0.0011778, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {191}

$$\frac{x}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)^(-3/2),x]
```

```
[Out] x/Sqrt[1 + x^2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{1}{\left(1+x^2\right)^{3/2}}\,dx = \frac{x}{\sqrt{1+x^2}}$$

Mathematica [A] time = 0.0020271, size = 11, normalized size = 1.

$$\frac{x}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)^(-3/2),x]
```

```
[Out] x/Sqrt[1 + x^2]
```

Maple [A] time = 0.003, size = 10, normalized size = 0.9

$$x\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+1)^(3/2),x)
```

```
[Out] x/(x^2+1)^(1/2)
```

Maxima [A] time = 0.948197, size = 12, normalized size = 1.09

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] x/sqrt(x^2 + 1)
```

Fricas [B] time = 0.420945, size = 53, normalized size = 4.82

$$\frac{x^2+\sqrt{x^2+1}x+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] (x^2 + sqrt(x^2 + 1)*x + 1)/(x^2 + 1)
```

Sympy [A] time = 0.694002, size = 8, normalized size = 0.73

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+1)**(3/2),x)
```

```
[Out] x/sqrt(x**2 + 1)
```

Giac [A] time = 1.07279, size = 12, normalized size = 1.09

$$\frac{x}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] x/sqrt(x^2 + 1)
```

3.17 $\int x^2 \left(27+8x^3\right)^{2/3} \, dx$

Optimal. Leaf size=15

$$\frac{1}{40}\left(8x^3+27\right)^{5/3}$$

[Out] (27 + 8*x^3)^(5/3)/40

Rubi [A] time = 0.0030355, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {261}

$$\frac{1}{40}\left(8x^3+27\right)^{5/3}$$

Antiderivative was successfully verified.

[In] Int[x^2*(27 + 8*x^3)^(2/3),x]

[Out] (27 + 8*x^3)^(5/3)/40

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^2 \left(27+8x^3\right)^{2/3} \, dx = \frac{1}{40}\left(27+8x^3\right)^{5/3}$$

Mathematica [A] time = 0.0033828, size = 15, normalized size = 1.

$$\frac{1}{40}\left(8x^3+27\right)^{5/3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(27 + 8*x^3)^(2/3),x]

[Out] (27 + 8*x^3)^(5/3)/40

Maple [B] time = 0.005, size = 27, normalized size = 1.8

$$\frac{(3+2x)\left(4x^2-6x+9\right)}{40}\left(8x^3+27\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(8*x^3+27)^(2/3),x)
```

```
[Out] 1/40*(3+2*x)*(4*x^2-6*x+9)*(8*x^3+27)^(2/3)
```

Maxima [A] time = 0.950064, size = 15, normalized size = 1.

$$\frac{1}{40}\left(8x^3+27\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(8*x^3+27)^(2/3),x, algorithm="maxima")
```

```
[Out] 1/40*(8*x^3 + 27)^(5/3)
```

Fricas [A] time = 0.42443, size = 34, normalized size = 2.27

$$\frac{1}{40}\left(8x^3+27\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(8*x^3+27)^(2/3),x, algorithm="fricas")
```

```
[Out] 1/40*(8*x^3 + 27)^(5/3)
```

Sympy [B] time = 0.349162, size = 27, normalized size = 1.8

$$\frac{x^3\left(8x^3+27\right)^{\frac{2}{3}}}{5}+\frac{27\left(8x^3+27\right)^{\frac{2}{3}}}{40}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(8*x**3+27)**(2/3),x)
```

```
[Out] x**3*(8*x**3 + 27)**(2/3)/5 + 27*(8*x**3 + 27)**(2/3)/40
```

Giac [A] time = 1.09399, size = 15, normalized size = 1.

$$\frac{1}{40}\left(8x^3+27\right)^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(8*x^3+27)^(2/3),x, algorithm="giac")
```

```
[Out] 1/40*(8*x^3 + 27)^(5/3)
```

3.18 $\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}} \, dx$

Optimal. Leaf size=15

$$\frac{3}{2}(\sin(x)-\cos(x))^{2/3}$$

```
[Out] (3*(-Cos[x] + Sin[x])^(2/3))/2
```

Rubi [A] time = 0.0264653, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {3145}

$$\frac{3}{2}(\sin(x)-\cos(x))^{2/3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Sin[x])/(-Cos[x] + Sin[x])^(1/3),x]
```

```
[Out] (3*(-Cos[x] + Sin[x])^(2/3))/2
```

Rule 3145

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_.)*(
cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :
> Simp[((c*B - b*C)*(b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1))/(e*(n + 1)*(
b^2 + c^2)), x] /; FreeQ[{b, c, d, e, B, C}, x] && NeQ[n, -1] && NeQ[b^2 +
c^2, 0] && EqQ[b*B + c*C, 0]
```

Rubi steps

$$\int \frac{\cos(x)+\sin(x)}{\sqrt[3]{-\cos(x)+\sin(x)}} \, dx = \frac{3}{2}(-\cos(x)+\sin(x))^{2/3}$$

Mathematica [A] time = 0.0494849, size = 15, normalized size = 1.

$$\frac{3}{2}(\sin(x)-\cos(x))^{2/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])/(-Cos[x] + Sin[x])^(1/3),x]
```

```
[Out] (3*(-Cos[x] + Sin[x])^(2/3))/2
```

Maple [A] time = 0.012, size = 12, normalized size = 0.8

$$\frac{3}{2}\left(-\cos\left(x\right)+\sin\left(x\right)\right)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x)
```

```
[Out] 3/2*(-cos(x)+sin(x))^(2/3)
```

Maxima [A] time = 0.973147, size = 15, normalized size = 1.

$$\frac{3}{2}\,(-\cos(x) + \sin(x))^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x, algorithm="maxima")
```

```
[Out] 3/2*(-cos(x) + sin(x))^(2/3)
```

Fricas [A] time = 0.464619, size = 41, normalized size = 2.73

$$\frac{3}{2}\,(-\cos(x) + \sin(x))^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x, algorithm="fricas")
```

```
[Out] 3/2*(-cos(x) + sin(x))^(2/3)
```

Sympy [A] time = 0.344041, size = 12, normalized size = 0.8

$$\frac{3\,(\sin(x) - \cos(x))^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(-cos(x)+sin(x))**(1/3),x)
```

```
[Out] 3*(sin(x) - cos(x))**(2/3)/2
```

Giac [A] time = 1.09935, size = 15, normalized size = 1.

$$\frac{3}{2}\,(-\cos(x) + \sin(x))^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((cos(x)+sin(x))/(-cos(x)+sin(x))^(1/3),x, algorithm="giac")
```

```
[Out] 3/2*(-cos(x) + sin(x))^(2/3)
```

3.19 $\int \frac{x}{\sqrt{1+x^2+\left(1+x^2\right)^{3/2}}} \, dx$

Optimal. Leaf size=32

$$\frac{2\sqrt{\left(x^2+1\right)\left(\sqrt{x^2+1}+1\right)}}{\sqrt{x^2+1}}$$

```
[Out] (2*Sqrt[(1 + x^2)*(1 + Sqrt[1 + x^2])])/Sqrt[1 + x^2]
```

Rubi [A] time = 0.100574, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {6715, 1588}

$$\frac{2\sqrt{\left(x^2+1\right)\left(\sqrt{x^2+1}+1\right)}}{\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 + x^2 + (1 + x^2)^(3/2)],x]
```

```
[Out] (2*Sqrt[(1 + x^2)*(1 + Sqrt[1 + x^2])])/Sqrt[1 + x^2]
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{1+x^2+\left(1+x^2\right)^{3/2}}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{\sqrt{1+x+(1+x)^{3/2}}} \, dx, x, x^2\right)\\
&= \text{Subst}\left(\int \frac{x}{\sqrt{x^2(1+x)}} \, dx, x, \sqrt{1+x^2}\right)\\
&= \frac{2\sqrt{\left(1+x^2\right)\left(1+\sqrt{1+x^2}\right)}}{\sqrt{1+x^2}}
\end{aligned}$$

Mathematica [A] time = 0.0484645, size = 37, normalized size = 1.16

$$\frac{2\left(x^2+\sqrt{x^2+1}+1\right)}{\sqrt{\left(x^2+1\right)\left(\sqrt{x^2+1}+1\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 + x^2 + (1 + x^2)^(3/2)],x]
```

```
[Out] (2*(1 + x^2 + Sqrt[1 + x^2]))/Sqrt[(1 + x^2)*(1 + Sqrt[1 + x^2])]
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int x\frac{1}{\sqrt{1+x^2+\left(x^2+1\right)^{\frac{3}{2}}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x)
```

```
[Out] int(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{x^2+\left(x^2+1\right)^{\frac{3}{2}}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/sqrt(x^2 + (x^2 + 1)^(3/2) + 1), x)
```

Fricas [A] time = 0.555716, size = 66, normalized size = 2.06

$$\frac{2\sqrt{x^2+\left(x^2+1\right)^{\frac{3}{2}}+1}}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(x^2 + (x^2 + 1)^(3/2) + 1)/sqrt(x^2 + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{\left(x^2+1\right)\left(\sqrt{x^2+1}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x**2+(x**2+1)**(3/2))**(1/2),x)
```

```
[Out] Integral(x/sqrt((x**2 + 1)*(sqrt(x**2 + 1) + 1)), x)
```

Giac [A] time = 1.11284, size = 20, normalized size = 0.62

$$2\sqrt{\sqrt{x^2+1}+1}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x^2+(x^2+1)^(3/2))^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(sqrt(x^2 + 1) + 1) - 2
```

3.20 $\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}} dx$

Optimal. Leaf size=17

$$2\sqrt{\sqrt{x^2+1}+1}$$

```
[Out] 2*Sqrt[1 + Sqrt[1 + x^2]]
```

Rubi [A] time = 0.111079, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {6686}

$$2\sqrt{\sqrt{x^2+1}+1}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[1 + x^2]]),x]
```

```
[Out] 2*Sqrt[1 + Sqrt[1 + x^2]]
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{1+x^2}\sqrt{1+\sqrt{1+x^2}}} dx = 2\sqrt{1+\sqrt{1+x^2}}$$

Mathematica [A] time = 0.0125079, size = 17, normalized size = 1.

$$2\sqrt{\sqrt{x^2+1}+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 + x^2]*Sqrt[1 + Sqrt[1 + x^2]]),x]
```

```
[Out] 2*Sqrt[1 + Sqrt[1 + x^2]]
```

Maple [A] time = 0.007, size = 14, normalized size = 0.8

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+1)^(1/2)/((x^2+1)^(1/2)+1)^(1/2),x)
```

```
[Out] 2*((x^2+1)^(1/2)+1)^(1/2)
```

Maxima [A] time = 0.939651, size = 18, normalized size = 1.06

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/2)/((x^2+1)^(1/2)+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(sqrt(x^2 + 1) + 1)
```

Fricas [A] time = 0.479772, size = 36, normalized size = 2.12

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/2)/((x^2+1)^(1/2)+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(sqrt(x^2 + 1) + 1)
```

Sympy [A] time = 0.346681, size = 14, normalized size = 0.82

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+1)**(1/2)/((x**2+1)**(1/2)+1)**(1/2),x)
```

```
[Out] 2*sqrt(sqrt(x**2 + 1) + 1)
```

Giac [A] time = 1.06909, size = 18, normalized size = 1.06

$$2\sqrt{\sqrt{x^2+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+1)^(1/2)/((x^2+1)^(1/2)+1)^(1/2),x, algorithm="giac")
```

```
[Out] 2*sqrt(sqrt(x^2 + 1) + 1)
```

3.21 $\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx$

Optimal. Leaf size=16

$$-\frac{5}{2}\sqrt[5]{x^2-2x+1}$$

```
[Out] (-5*(1 - 2*x + x^2)^(1/5))/2
```

Rubi [A] time = 0.0079128, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {643, 629}

$$-\frac{5}{2}\sqrt[5]{x^2-2x+1}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - 2*x + x^2)^(1/5)/(1 - x),x]
```

```
[Out] (-5*(1 - 2*x + x^2)^(1/5))/2
```

Rule 643

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sym
bol] :> Dist[e^(m - 1)/c^((m - 1)/2), Int[(d + e*x)*(a + b*x + c*x^2)^(p +
(m - 1)/2), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[p] && EqQ[2*c*d - b*e, 0] && IntegerQ[(m - 1)/2]
```

Rule 629

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt[5]{1-2x+x^2}}{1-x}\,dx &= \int \frac{1-x}{\left(1-2x+x^2\right)^{4/5}}\,dx\\ &= -\frac{5}{2}\sqrt[5]{1-2x+x^2}\end{aligned}$$

Mathematica [A] time = 0.0058255, size = 13, normalized size = 0.81

$$-\frac{5}{2}\sqrt[5]{(x-1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - 2*x + x^2)^(1/5)/(1 - x),x]
```

```
[Out] (-5*((-1 + x)^2)^(1/5))/2
```

Maple [A] time = 0.003, size = 13, normalized size = 0.8

$$-\frac{5}{2}\sqrt[5]{x^2-2x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2-2*x+1)^(1/5)/(1-x),x)
```

```
[Out] -5/2*(x^2-2*x+1)^(1/5)
```

Maxima [A] time = 0.967288, size = 9, normalized size = 0.56

$$-\frac{5}{2}(x-1)^{\frac{2}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x+1)^(1/5)/(1-x),x, algorithm="maxima")
```

```
[Out] -5/2*(x - 1)^(2/5)
```

Fricas [A] time = 0.418288, size = 38, normalized size = 2.38

$$-\frac{5}{2}\left(x^2-2x+1\right)^{\frac{1}{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x+1)^(1/5)/(1-x),x, algorithm="fricas")
```

```
[Out] -5/2*(x^2 - 2*x + 1)^(1/5)
```

Sympy [A] time = 1.21885, size = 15, normalized size = 0.94

$$-\frac{5\sqrt[5]{x^2-2x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-2*x+1)**(1/5)/(1-x),x)
```

```
[Out] -5*(x**2 - 2*x + 1)**(1/5)/2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{\left(x^2-2\,x+1\right)^{\frac{1}{5}}}{x-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-2*x+1)^(1/5)/(1-x),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 - 2*x + 1)^(1/5)/(x - 1), x)
```

3.22 $\int x\sin(x)\,dx$

Optimal. Leaf size=8

$$\sin(x) - x\cos(x)$$

```
[Out] -(x*Cos[x]) + Sin[x]
```

Rubi [A] time = 0.0089634, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3296, 2637}

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x],x]
```

```
[Out] -(x*Cos[x]) + Sin[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sin(x)\,dx &= -x\cos(x) + \int \cos(x)\,dx\\ &= -x\cos(x) + \sin(x)\end{aligned}$$

Mathematica [A] time = 0.0020035, size = 8, normalized size = 1.

$$\sin(x) - x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x],x]
```

```
[Out] -(x*Cos[x]) + Sin[x]
```

Maple [A] time = 0.005, size = 9, normalized size = 1.1

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x),x)
```

```
[Out] -x*cos(x)+sin(x)
```

Maxima [A] time = 0.972054, size = 11, normalized size = 1.38

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="maxima")
```

```
[Out] -x*cos(x) + sin(x)
```

Fricas [A] time = 0.45065, size = 27, normalized size = 3.38

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="fricas")
```

```
[Out] -x*cos(x) + sin(x)
```

Sympy [A] time = 0.193615, size = 7, normalized size = 0.88

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x)
```

```
[Out] -x*cos(x) + sin(x)
```

Giac [A] time = 1.07147, size = 11, normalized size = 1.38

$$-x\cos(x) + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x),x, algorithm="giac")
```

```
[Out] -x*cos(x) + sin(x)
```

3.23 $\int x^2 \sin(x)\, dx$

Optimal. Leaf size=17

$$x^2(-\cos(x)) + 2x\sin(x) + 2\cos(x)$$

```
[Out] 2*Cos[x] - x^2*Cos[x] + 2*x*Sin[x]
```

Rubi [A] time = 0.0207939, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2638}

$$x^2(-\cos(x)) + 2x\sin(x) + 2\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[x],x]
```

```
[Out] 2*Cos[x] - x^2*Cos[x] + 2*x*Sin[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x^2 \sin(x)\, dx &= -x^2\cos(x) + 2\int x\cos(x)\, dx\\ &= -x^2\cos(x) + 2x\sin(x) - 2\int \sin(x)\, dx\\ &= 2\cos(x) - x^2\cos(x) + 2x\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0131344, size = 15, normalized size = 0.88

$$2x\sin(x) - \left(x^2-2\right)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[x],x]
```

```
[Out] -((-2 + x^2)*Cos[x]) + 2*x*Sin[x]
```

Maple [A] time = 0.005, size = 18, normalized size = 1.1

$$2 \cos(x) - x^2 \cos(x) + 2x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x),x)
```

```
[Out] 2*cos(x)-x^2*cos(x)+2*x*sin(x)
```

Maxima [A] time = 0.958066, size = 20, normalized size = 1.18

$$-\left(x^2 - 2\right) \cos(x) + 2x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="maxima")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

Fricas [A] time = 0.451698, size = 43, normalized size = 2.53

$$-\left(x^2 - 2\right) \cos(x) + 2x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

Sympy [A] time = 0.320582, size = 17, normalized size = 1.

$$-x^2 \cos(x) + 2x \sin(x) + 2 \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x),x)
```

```
[Out] -x**2*cos(x) + 2*x*sin(x) + 2*cos(x)
```

Giac [A] time = 1.0679, size = 20, normalized size = 1.18

$$-\left(x^2 - 2\right) \cos(x) + 2x \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x),x, algorithm="giac")
```

```
[Out] -(x^2 - 2)*cos(x) + 2*x*sin(x)
```

3.24 $\int x^3 \cos(x)\, dx$

Optimal. Leaf size=23

$$x^3 \sin(x) + 3x^2 \cos(x) - 6x \sin(x) - 6 \cos(x)$$

```
[Out] -6*Cos[x] + 3*x^2*Cos[x] - 6*x*Sin[x] + x^3*Sin[x]
```

Rubi [A] time = 0.0355037, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2638}

$$x^3 \sin(x) + 3x^2 \cos(x) - 6x \sin(x) - 6 \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x],x]
```

```
[Out] -6*Cos[x] + 3*x^2*Cos[x] - 6*x*Sin[x] + x^3*Sin[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^3 \cos(x)\, dx &= x^3 \sin(x) - 3 \int x^2 \sin(x)\, dx \\
&= 3x^2 \cos(x) + x^3 \sin(x) - 6 \int x \cos(x)\, dx \\
&= 3x^2 \cos(x) - 6x \sin(x) + x^3 \sin(x) + 6 \int \sin(x)\, dx \\
&= -6 \cos(x) + 3x^2 \cos(x) - 6x \sin(x) + x^3 \sin(x)
\end{aligned}$$

Mathematica [A] time = 0.0129928, size = 19, normalized size = 0.83

$$x \left(x^2 - 6\right) \sin(x) + 3 \left(x^2 - 2\right) \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[x],x]
```

```
[Out] 3*(-2 + x^2)*Cos[x] + x*(-6 + x^2)*Sin[x]
```

Maple [A] time = 0.006, size = 24, normalized size = 1.

$$-6\cos(x) + 3x^2\cos(x) - 6x\sin(x) + x^3\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x),x)
```

```
[Out] -6*cos(x)+3*x^2*cos(x)-6*x*sin(x)+x^3*sin(x)
```

Maxima [A] time = 0.952307, size = 27, normalized size = 1.17

$$3\left(x^2 - 2\right)\cos(x) + \left(x^3 - 6x\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x),x, algorithm="maxima")
```

```
[Out] 3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

Fricas [A] time = 0.450515, size = 55, normalized size = 2.39

$$3\left(x^2 - 2\right)\cos(x) + \left(x^3 - 6x\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x),x, algorithm="fricas")
```

```
[Out] 3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

Sympy [A] time = 0.553506, size = 26, normalized size = 1.13

$$x^3\sin(x) + 3x^2\cos(x) - 6x\sin(x) - 6\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(x),x)
```

```
[Out] x**3*sin(x) + 3*x**2*cos(x) - 6*x*sin(x) - 6*cos(x)
```

Giac [A] time = 1.06311, size = 27, normalized size = 1.17

$$3\left(x^2 - 2\right)\cos(x) + \left(x^3 - 6x\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x),x, algorithm="giac")
```

```
[Out] 3*(x^2 - 2)*cos(x) + (x^3 - 6*x)*sin(x)
```

3.25 $\int x^3 \sin(x)\, dx$

Optimal. Leaf size=24

$$3x^2 \sin(x) + x^3(-\cos(x)) - 6\sin(x) + 6x\cos(x)$$

```
[Out] 6*x*Cos[x] - x^3*Cos[x] - 6*Sin[x] + 3*x^2*Sin[x]
```

Rubi [A] time = 0.0358987, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3296, 2637}

$$x^3(-\cos(x)) + 3x^2 \sin(x) - 6\sin(x) + 6x\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sin[x],x]
```

```
[Out] 6*x*Cos[x] - x^3*Cos[x] - 6*Sin[x] + 3*x^2*Sin[x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int x^3 \sin(x)\, dx &= -x^3\cos(x) + 3\int x^2 \cos(x)\, dx \\
&= -x^3\cos(x) + 3x^2\sin(x) - 6\int x\sin(x)\, dx \\
&= 6x\cos(x) - x^3\cos(x) + 3x^2\sin(x) - 6\int \cos(x)\, dx \\
&= 6x\cos(x) - x^3\cos(x) - 6\sin(x) + 3x^2\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.0124194, size = 20, normalized size = 0.83

$$3\left(x^2-2\right)\sin(x) - x\left(x^2-6\right)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sin[x],x]
```

```
[Out] -(x*(-6 + x^2)*Cos[x]) + 3*(-2 + x^2)*Sin[x]
```

Maple [A] time = 0.004, size = 25, normalized size = 1.

$$6\,x\cos\left(x\right)-x^{3}\cos\left(x\right)-6\,\sin\left(x\right)+3\,x^{2}\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*sin(x),x)
```

```
[Out] 6*x*cos(x)-x^3*cos(x)-6*sin(x)+3*x^2*sin(x)
```

Maxima [A] time = 0.959794, size = 28, normalized size = 1.17

$$-\left(x^{3}-6\,x\right)\cos\left(x\right)+3\,\left(x^{2}-2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x),x, algorithm="maxima")
```

```
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

Fricas [A] time = 0.451723, size = 57, normalized size = 2.38

$$-\left(x^{3}-6\,x\right)\cos\left(x\right)+3\,\left(x^{2}-2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x),x, algorithm="fricas")
```

```
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

Sympy [A] time = 0.570449, size = 26, normalized size = 1.08

$$-x^{3}\cos\left(x\right)+3x^{2}\sin\left(x\right)+6x\cos\left(x\right)-6\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*sin(x),x)
```

```
[Out] -x**3*cos(x) + 3*x**2*sin(x) + 6*x*cos(x) - 6*sin(x)
```

Giac [A] time = 1.07002, size = 28, normalized size = 1.17

$$-\left(x^{3}-6\,x\right)\cos\left(x\right)+3\,\left(x^{2}-2\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*sin(x),x, algorithm="giac")
```

```
[Out] -(x^3 - 6*x)*cos(x) + 3*(x^2 - 2)*sin(x)
```

3.26 $\int \cos(x)\sin(x)\,dx$

Optimal. Leaf size=8

$$\frac{\sin^2(x)}{2}$$

```
[Out] Sin[x]^2/2
```

Rubi [A] time = 0.006863, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2564, 30}

$$\frac{\sin^2(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]*Sin[x],x]

[Out] Sin[x]^2/2
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(x)\sin(x)\,dx &= \text{Subst}\left(\int x\,dx, x, \sin(x)\right)\\ &= \frac{\sin^2(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.001191, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]*Sin[x],x]

[Out] -Cos[x]^2/2
```

Maple [A] time = 0.001, size = 7, normalized size = 0.9

$$\frac{(\sin(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x),x)
```

```
[Out] 1/2*sin(x)^2
```

Maxima [A] time = 0.95214, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/2*cos(x)^2
```

Fricas [A] time = 0.451307, size = 20, normalized size = 2.5

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)^2
```

Sympy [A] time = 0.054588, size = 5, normalized size = 0.62

$$\frac{\sin^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x)
```

```
[Out] sin(x)**2/2
```

Giac [A] time = 1.0953, size = 8, normalized size = 1.

$$-\frac{1}{2}\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/2*cos(x)^2
```

3.27 $\int x\cos(x)\sin(x)\,dx$

Optimal. Leaf size=23

$$-\frac{x}{4}+\frac{1}{2}x\sin^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] -x/4 + (Cos[x]*Sin[x])/4 + (x*Sin[x]^2)/2
```

Rubi [A] time = 0.0127059, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3443, 2635, 8}

$$-\frac{x}{4}+\frac{1}{2}x\sin^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]*Sin[x],x]
```

```
[Out] -x/4 + (Cos[x]*Sin[x])/4 + (x*Sin[x]^2)/2
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int x\cos(x)\sin(x)\,dx &= \frac{1}{2}x\sin^2(x)-\frac{1}{2}\int \sin^2(x)\,dx\\ &= \frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)-\frac{\int 1\,dx}{4}\\ &= -\frac{x}{4}+\frac{1}{4}\cos(x)\sin(x)+\frac{1}{2}x\sin^2(x)\end{aligned}$$

Mathematica [A] time = 0.002663, size = 18, normalized size = 0.78

$$\frac{1}{8}\sin(2x)-\frac{1}{4}x\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]*Sin[x],x]
```

```
[Out] -(x*Cos[2*x])/4 + Sin[2*x]/8
```

Maple [A] time = 0.003, size = 18, normalized size = 0.8

$$-\frac{x\left(\cos\left(x\right)\right)^{2}}{2}+\frac{\cos\left(x\right)\sin\left(x\right)}{4}+\frac{x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)*sin(x),x)
```

```
[Out] -1/2*x*cos(x)^2+1/4*cos(x)*sin(x)+1/4*x
```

Maxima [A] time = 0.944388, size = 19, normalized size = 0.83

$$-\frac{1}{4}\,x\cos\left(2\,x\right)+\frac{1}{8}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*sin(x),x, algorithm="maxima")
```

```
[Out] -1/4*x*cos(2*x) + 1/8*sin(2*x)
```

Fricas [A] time = 0.446078, size = 61, normalized size = 2.65

$$-\frac{1}{2}\,x\cos\left(x\right)^{2}+\frac{1}{4}\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*sin(x),x, algorithm="fricas")
```

```
[Out] -1/2*x*cos(x)^2 + 1/4*cos(x)*sin(x) + 1/4*x
```

Sympy [A] time = 0.322426, size = 24, normalized size = 1.04

$$\frac{x\sin^{2}\left(x\right)}{4}-\frac{x\cos^{2}\left(x\right)}{4}+\frac{\sin\left(x\right)\cos\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*sin(x),x)
```

```
[Out] x*sin(x)**2/4 - x*cos(x)**2/4 + sin(x)*cos(x)/4
```

Giac [A] time = 1.14303, size = 19, normalized size = 0.83

$$-\frac{1}{4}\,x\cos\left(2\,x\right)+\frac{1}{8}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)*sin(x),x, algorithm="giac")
```

```
[Out] -1/4*x*cos(2*x) + 1/8*sin(2*x)
```

3.28 $\int \sin^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0051129, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{x}{2}-\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2,x]

[Out] x/2 - (Cos[x]*Sin[x])/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^2(x)\,dx &= -\frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}-\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0016107, size = 14, normalized size = 1.

$$\frac{x}{2}-\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2,x]

[Out] x/2 - Sin[2*x]/4
```

Maple [A] time = 0.005, size = 11, normalized size = 0.8

$$\frac{x}{2} - \frac{\cos(x)\sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2,x)
```

```
[Out] 1/2*x-1/2*cos(x)*sin(x)
```

Maxima [A] time = 0.942415, size = 14, normalized size = 1.

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x - 1/4*sin(2*x)
```

Fricas [A] time = 0.455659, size = 38, normalized size = 2.71

$$-\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.058097, size = 10, normalized size = 0.71

$$\frac{x}{2} - \frac{\sin(x)\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2,x)
```

```
[Out] x/2 - sin(x)*cos(x)/2
```

Giac [A] time = 1.07878, size = 14, normalized size = 1.

$$\frac{1}{2}x - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2,x, algorithm="giac")

[Out] 1/2*x - 1/4*sin(2*x)
```

3.29 $\int \sin^3(x)\,dx$

Optimal. Leaf size=13

$$\frac{\cos^3(x)}{3}-\cos(x)$$

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rubi [A] time = 0.0056728, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2633}

$$\frac{\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3,x]
```

```
[Out] -Cos[x] + Cos[x]^3/3
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(x)\,dx &= -\operatorname{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(x)\right)\\ &= -\cos(x)+\frac{\cos^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.0014732, size = 15, normalized size = 1.15

$$\frac{1}{12}\cos(3x)-\frac{3\cos(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3,x]
```

```
[Out] (-3*Cos[x])/4 + Cos[3*x]/12
```

Maple [A] time = 0., size = 11, normalized size = 0.9

$$-\frac{\left(2+(\sin(x))^2\right)\cos(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3,x)
```

```
[Out] -1/3*(2+sin(x)^2)*cos(x)
```

Maxima [A] time = 0.951273, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

Fricas [A] time = 0.460247, size = 31, normalized size = 2.38

$$\frac{1}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

Sympy [A] time = 0.05892, size = 8, normalized size = 0.62

$$\frac{\cos^3(x)}{3} - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3,x)
```

```
[Out] cos(x)**3/3 - cos(x)
```

Giac [A] time = 1.06398, size = 15, normalized size = 1.15

$$\frac{1}{3}\cos(x)^3 - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/3*cos(x)^3 - cos(x)
```

3.30 $\int \sin^4(x)\,dx$

Optimal. Leaf size=24

$$\frac{3x}{8}-\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{8}\sin(x)\cos(x)$$

```
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rubi [A] time = 0.0092784, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{3x}{8}-\frac{1}{4}\sin^3(x)\cos(x)-\frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - (3*Cos[x]*Sin[x])/8 - (Cos[x]*Sin[x]^3)/4
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^4(x)\,dx &= -\frac{1}{4}\cos(x)\sin^3(x)+\frac{3}{4}\int \sin^2(x)\,dx\\ &= -\frac{3}{8}\cos(x)\sin(x)-\frac{1}{4}\cos(x)\sin^3(x)+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}-\frac{3}{8}\cos(x)\sin(x)-\frac{1}{4}\cos(x)\sin^3(x)\end{aligned}$$

Mathematica [A] time = 0.0015549, size = 22, normalized size = 0.92

$$\frac{3x}{8}-\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^4,x]
```

```
[Out] (3*x)/8 - Sin[2*x]/4 + Sin[4*x]/32
```

Maple [A] time = 0.117, size = 18, normalized size = 0.8

$$-\frac{\cos(x)}{4}\left((\sin(x))^3 + \frac{3\sin(x)}{2}\right) + \frac{3x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^4,x)
```

```
[Out] -1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x
```

Maxima [A] time = 0.935796, size = 22, normalized size = 0.92

$$\frac{3}{8}x + \frac{1}{32}\sin(4x) - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

Fricas [A] time = 0.474605, size = 59, normalized size = 2.46

$$\frac{1}{8}\left(2\cos(x)^3 - 5\cos(x)\right)\sin(x) + \frac{3}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(2*cos(x)^3 - 5*cos(x))*sin(x) + 3/8*x
```

Sympy [A] time = 0.056754, size = 24, normalized size = 1.

$$\frac{3x}{8} - \frac{\sin^3(x)\cos(x)}{4} - \frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**4,x)
```

```
[Out] 3*x/8 - sin(x)**3*cos(x)/4 - 3*sin(x)*cos(x)/8
```

Giac [A] time = 1.08009, size = 22, normalized size = 0.92

$$\frac{3}{8}x + \frac{1}{32}\sin(4x) - \frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*x) - 1/4*sin(2*x)
```

3.31 $\int \sin^5(x)\,dx$

Optimal. Leaf size=21

$$-\frac{1}{5}\cos^5(x)+\frac{2\cos^3(x)}{3}-\cos(x)$$

```
[Out] -Cos[x] + (2*Cos[x]^3)/3 - Cos[x]^5/5
```

Rubi [A] time = 0.0066113, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2633}

$$-\frac{1}{5}\cos^5(x)+\frac{2\cos^3(x)}{3}-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^5,x]
```

```
[Out] -Cos[x] + (2*Cos[x]^3)/3 - Cos[x]^5/5
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^5(x)\,dx &= -\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,\cos(x)\right)\\ &= -\cos(x)+\frac{2\cos^3(x)}{3}-\frac{\cos^5(x)}{5}\end{aligned}$$

Mathematica [A] time = 0.0016591, size = 23, normalized size = 1.1

$$-\frac{5\cos(x)}{8}+\frac{5}{48}\cos(3x)-\frac{1}{80}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^5,x]
```

```
[Out] (-5*Cos[x])/8 + (5*Cos[3*x])/48 - Cos[5*x]/80
```

Maple [A] time = 0.03, size = 17, normalized size = 0.8

$$-\frac{\cos(x)}{5}\left(\frac{8}{3}+(\sin(x))^4+\frac{4\,(\sin(x))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^5,x)
```

```
[Out] -1/5*(8/3+sin(x)^4+4/3*sin(x)^2)*cos(x)
```

Maxima [A] time = 0.956211, size = 23, normalized size = 1.1

$$-\frac{1}{5}\cos(x)^{5}+\frac{2}{3}\cos(x)^{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5,x, algorithm="maxima")
```

```
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

Fricas [A] time = 0.480676, size = 53, normalized size = 2.52

$$-\frac{1}{5}\cos(x)^{5}+\frac{2}{3}\cos(x)^{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5,x, algorithm="fricas")
```

```
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

Sympy [A] time = 0.060354, size = 17, normalized size = 0.81

$$-\frac{\cos^{5}(x)}{5}+\frac{2\cos^{3}(x)}{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**5,x)
```

```
[Out] -cos(x)**5/5 + 2*cos(x)**3/3 - cos(x)
```

Giac [A] time = 1.07129, size = 23, normalized size = 1.1

$$-\frac{1}{5}\cos(x)^{5}+\frac{2}{3}\cos(x)^{3}-\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^5,x, algorithm="giac")
```

```
[Out] -1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)
```

3.32 $\int \sin^6(x)\,dx$

Optimal. Leaf size=34

$$\frac{5x}{16}-\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{24}\sin^3(x)\cos(x)-\frac{5}{16}\sin(x)\cos(x)$$

```
[Out] (5*x)/16 - (5*Cos[x]*Sin[x])/16 - (5*Cos[x]*Sin[x]^3)/24 - (Cos[x]*Sin[x]^5
)/6
```

Rubi [A] time = 0.0156523, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{5x}{16}-\frac{1}{6}\sin^5(x)\cos(x)-\frac{5}{24}\sin^3(x)\cos(x)-\frac{5}{16}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^6,x]
```

```
[Out] (5*x)/16 - (5*Cos[x]*Sin[x])/16 - (5*Cos[x]*Sin[x]^3)/24 - (Cos[x]*Sin[x]^5
)/6
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^6(x)\,dx &= -\frac{1}{6}\cos(x)\sin^5(x)+\frac{5}{6}\int \sin^4(x)\,dx\\ &= -\frac{5}{24}\cos(x)\sin^3(x)-\frac{1}{6}\cos(x)\sin^5(x)+\frac{5}{8}\int \sin^2(x)\,dx\\ &= -\frac{5}{16}\cos(x)\sin(x)-\frac{5}{24}\cos(x)\sin^3(x)-\frac{1}{6}\cos(x)\sin^5(x)+\frac{5\int 1\,dx}{16}\\ &= \frac{5x}{16}-\frac{5}{16}\cos(x)\sin(x)-\frac{5}{24}\cos(x)\sin^3(x)-\frac{1}{6}\cos(x)\sin^5(x)\end{aligned}$$

Mathematica [A] time = 0.0016544, size = 30, normalized size = 0.88

$$\frac{5x}{16}-\frac{15}{64}\sin(2x)+\frac{3}{64}\sin(4x)-\frac{1}{192}\sin(6x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^6,x]
```

```
[Out] (5*x)/16 - (15*Sin[2*x])/64 + (3*Sin[4*x])/64 - Sin[6*x]/192
```

Maple [A] time = 0.033, size = 24, normalized size = 0.7

$$-\frac{\cos(x)}{6}\left((\sin(x))^5+\frac{5(\sin(x))^3}{4}+\frac{15\sin(x)}{8}\right)+\frac{5x}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^6,x)
```

```
[Out] -1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x
```

Maxima [A] time = 0.936967, size = 32, normalized size = 0.94

$$\frac{1}{48}\sin(2x)^3+\frac{5}{16}x+\frac{3}{64}\sin(4x)-\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6,x, algorithm="maxima")
```

```
[Out] 1/48*sin(2*x)^3 + 5/16*x + 3/64*sin(4*x) - 1/4*sin(2*x)
```

Fricas [A] time = 0.467144, size = 84, normalized size = 2.47

$$-\frac{1}{48}\left(8\cos(x)^5-26\cos(x)^3+33\cos(x)\right)\sin(x)+\frac{5}{16}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6,x, algorithm="fricas")
```

```
[Out] -1/48*(8*cos(x)^5 - 26*cos(x)^3 + 33*cos(x))*sin(x) + 5/16*x
```

Sympy [A] time = 0.05916, size = 36, normalized size = 1.06

$$\frac{5x}{16}-\frac{\sin^5(x)\cos(x)}{6}-\frac{5\sin^3(x)\cos(x)}{24}-\frac{5\sin(x)\cos(x)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**6,x)
```

```
[Out] 5*x/16 - sin(x)**5*cos(x)/6 - 5*sin(x)**3*cos(x)/24 - 5*sin(x)*cos(x)/16
```

<u>Giac</u> [A] time = 1.0904, size = 30, normalized size = 0.88

$$\frac{5}{16}\,x-\frac{1}{192}\,\sin\left(6\,x\right)+\frac{3}{64}\,\sin\left(4\,x\right)-\frac{15}{64}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^6,x, algorithm="giac")
```

```
[Out] 5/16*x - 1/192*sin(6*x) + 3/64*sin(4*x) - 15/64*sin(2*x)
```

3.33 $\int x \sin^2(x)\,dx$

Optimal. Leaf size=25

$$\frac{x^2}{4}+\frac{\sin^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$

```
[Out] x^2/4 - (x*Cos[x]*Sin[x])/2 + Sin[x]^2/4
```

Rubi [A] time = 0.0139333, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3310, 30}

$$\frac{x^2}{4}+\frac{\sin^2(x)}{4}-\frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x]^2,x]
```

```
[Out] x^2/4 - (x*Cos[x]*Sin[x])/2 + Sin[x]^2/4
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\sin^2(x)\,dx&=-\frac{1}{2}x\cos(x)\sin(x)+\frac{\sin^2(x)}{4}+\frac{\int x\,dx}{2}\\&=\frac{x^2}{4}-\frac{1}{2}x\cos(x)\sin(x)+\frac{\sin^2(x)}{4}\end{aligned}$$

Mathematica [A] time = 0.0111651, size = 25, normalized size = 1.

$$\frac{x^2}{4}-\frac{1}{4}x\sin(2x)-\frac{1}{8}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x]^2,x]
```

```
[Out] x^2/4 - Cos[2*x]/8 - (x*Sin[2*x])/4
```

Maple [A] time = 0.005, size = 25, normalized size = 1.

$$x\left(\frac{x}{2}-\frac{\cos\left(x\right)\sin\left(x\right)}{2}\right)-\frac{x^{2}}{4}+\frac{\left(\sin\left(x\right)\right)^{2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^2,x)
```

```
[Out] x*(1/2*x-1/2*cos(x)*sin(x))-1/4*x^2+1/4*sin(x)^2
```

Maxima [A] time = 0.952459, size = 26, normalized size = 1.04

$$\frac{1}{4}\,x^{2}-\frac{1}{4}\,x\sin\left(2\,x\right)-\frac{1}{8}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

Fricas [A] time = 0.454715, size = 63, normalized size = 2.52

$$-\frac{1}{2}\,x\cos\left(x\right)\sin\left(x\right)+\frac{1}{4}\,x^{2}-\frac{1}{4}\,\cos\left(x\right)^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*x*cos(x)*sin(x) + 1/4*x^2 - 1/4*cos(x)^2
```

Sympy [A] time = 0.32495, size = 36, normalized size = 1.44

$$\frac{x^{2}\sin^{2}\left(x\right)}{4}+\frac{x^{2}\cos^{2}\left(x\right)}{4}-\frac{x\sin\left(x\right)\cos\left(x\right)}{2}+\frac{\sin^{2}\left(x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**2,x)
```

```
[Out] x**2*sin(x)**2/4 + x**2*cos(x)**2/4 - x*sin(x)*cos(x)/2 + sin(x)**2/4
```

Giac [A] time = 1.1022, size = 26, normalized size = 1.04

$$\frac{1}{4}\,x^{2}-\frac{1}{4}\,x\sin\left(2\,x\right)-\frac{1}{8}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^2 - 1/4*x*sin(2*x) - 1/8*cos(2*x)
```

3.34 $\int x\sin^3(x)\,dx$

Optimal. Leaf size=33

$$\frac{\sin^3(x)}{9}+\frac{2\sin(x)}{3}-\frac{2}{3}x\cos(x)-\frac{1}{3}x\sin^2(x)\cos(x)$$

```
[Out] (-2*x*Cos[x])/3 + (2*Sin[x])/3 - (x*Cos[x]*Sin[x]^2)/3 + Sin[x]^3/9
```

Rubi [A] time = 0.0219208, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3310, 3296, 2637}

$$\frac{\sin^3(x)}{9}+\frac{2\sin(x)}{3}-\frac{2}{3}x\cos(x)-\frac{1}{3}x\sin^2(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Sin[x]^3,x]
```

```
[Out] (-2*x*Cos[x])/3 + (2*Sin[x])/3 - (x*Cos[x]*Sin[x]^2)/3 + Sin[x]^3/9
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int x\sin^3(x)\,dx &= -\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}+\frac{2}{3}\int x\sin(x)\,dx\\&= -\frac{2}{3}x\cos(x)-\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}+\frac{2}{3}\int\cos(x)\,dx\\&= -\frac{2}{3}x\cos(x)+\frac{2\sin(x)}{3}-\frac{1}{3}x\cos(x)\sin^2(x)+\frac{\sin^3(x)}{9}\end{aligned}$$

Mathematica [A] time = 0.0070249, size = 31, normalized size = 0.94

$$\frac{3\sin(x)}{4}-\frac{1}{36}\sin(3x)-\frac{3}{4}x\cos(x)+\frac{1}{12}x\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sin[x]^3,x]
```

```
[Out] (-3*x*Cos[x])/4 + (x*Cos[3*x])/12 + (3*Sin[x])/4 - Sin[3*x]/36
```

Maple [A] time = 0.03, size = 23, normalized size = 0.7

$$-\frac{x\left(2+(\sin(x))^2\right)\cos(x)}{3}+\frac{(\sin(x))^3}{9}+\frac{2\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(x)^3,x)
```

```
[Out] -1/3*x*(2+sin(x)^2)*cos(x)+1/9*sin(x)^3+2/3*sin(x)
```

Maxima [A] time = 0.963632, size = 31, normalized size = 0.94

$$\frac{1}{12}x\cos(3x)-\frac{3}{4}x\cos(x)-\frac{1}{36}\sin(3x)+\frac{3}{4}\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="maxima")
```

```
[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

Fricas [A] time = 0.453009, size = 74, normalized size = 2.24

$$\frac{1}{3}x\cos(x)^3-x\cos(x)-\frac{1}{9}\left(\cos(x)^2-7\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*x*cos(x)^3 - x*cos(x) - 1/9*(cos(x)^2 - 7)*sin(x)
```

Sympy [A] time = 0.564931, size = 39, normalized size = 1.18

$$-x\sin^2(x)\cos(x)-\frac{2x\cos^3(x)}{3}+\frac{7\sin^3(x)}{9}+\frac{2\sin(x)\cos^2(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)**3,x)
```

```
[Out] -x*sin(x)**2*cos(x) - 2*x*cos(x)**3/3 + 7*sin(x)**3/9 + 2*sin(x)*cos(x)**2/
3
```

Giac [A] time = 1.12113, size = 31, normalized size = 0.94

$$\frac{1}{12}\,x\cos\left(3\,x\right) - \frac{3}{4}\,x\cos\left(x\right) - \frac{1}{36}\,\sin\left(3\,x\right) + \frac{3}{4}\,\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(x)^3,x, algorithm="giac")
```

```
[Out] 1/12*x*cos(3*x) - 3/4*x*cos(x) - 1/36*sin(3*x) + 3/4*sin(x)
```

3.35 $\int x^2 \sin^2(x) \, dx$

Optimal. Leaf size=41

$$\frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) - \frac{x}{4} + \frac{1}{2}x\sin^2(x) + \frac{1}{4}\sin(x)\cos(x)$$

```
[Out] -x/4 + x^3/6 + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin[x]^2)/2
```

Rubi [A] time = 0.0313716, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3311, 30, 2635, 8}

$$\frac{x^3}{6} - \frac{1}{2}x^2 \sin(x)\cos(x) - \frac{x}{4} + \frac{1}{2}x\sin^2(x) + \frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sin[x]^2,x]
```

```
[Out] -x/4 + x^3/6 + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2 + (x*Sin[x]^2)/2
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int x^2 \sin^2(x)\,dx &= -\frac{1}{2}x^2\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x) + \frac{\int x^2\,dx}{2} - \frac{1}{2}\int \sin^2(x)\,dx \\
&= \frac{x^3}{6} + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x) - \frac{\int 1\,dx}{4} \\
&= -\frac{x}{4} + \frac{x^3}{6} + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x)
\end{aligned}$$

Mathematica [A] time = 0.0322167, size = 29, normalized size = 0.71

$$\frac{1}{24}\left(4x^3+\left(3-6x^2\right)\sin(2x)-6x\cos(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sin[x]^2,x]
```

```
[Out] (4*x^3 - 6*x*Cos[2*x] + (3 - 6*x^2)*Sin[2*x])/24
```

Maple [A] time = 0.02, size = 37, normalized size = 0.9

$$x^2\left(\frac{x}{2}-\frac{\cos(x)\sin(x)}{2}\right)-\frac{x(\cos(x))^2}{2}+\frac{\cos(x)\sin(x)}{4}+\frac{x}{4}-\frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*sin(x)^2,x)
```

```
[Out] x^2*(1/2*x-1/2*cos(x)*sin(x))-1/2*x*cos(x)^2+1/4*cos(x)*sin(x)+1/4*x-1/3*x^
3
```

Maxima [A] time = 0.965939, size = 35, normalized size = 0.85

$$\frac{1}{6}x^3-\frac{1}{4}x\cos(2x)-\frac{1}{8}\left(2x^2-1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="maxima")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

Fricas [A] time = 0.455595, size = 89, normalized size = 2.17

$$\frac{1}{6}x^3-\frac{1}{2}x\cos(x)^2-\frac{1}{4}\left(2x^2-1\right)\cos(x)\sin(x)+\frac{1}{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/6*x^3 - 1/2*x*cos(x)^2 - 1/4*(2*x^2 - 1)*cos(x)*sin(x) + 1/4*x
```

Sympy [A] time = 0.665146, size = 56, normalized size = 1.37

$$\frac{x^3\sin^2(x)}{6}+\frac{x^3\cos^2(x)}{6}-\frac{x^2\sin(x)\cos(x)}{2}+\frac{x\sin^2(x)}{4}-\frac{x\cos^2(x)}{4}+\frac{\sin(x)\cos(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*sin(x)**2,x)
```

```
[Out] x**3*sin(x)**2/6 + x**3*cos(x)**2/6 - x**2*sin(x)*cos(x)/2 + x*sin(x)**2/4
- x*cos(x)**2/4 + sin(x)*cos(x)/4
```

Giac [A] time = 1.07779, size = 35, normalized size = 0.85

$$\frac{1}{6}x^3 - \frac{1}{4}x\cos(2x) - \frac{1}{8}\left(2x^2 - 1\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*sin(x)^2,x, algorithm="giac")
```

```
[Out] 1/6*x^3 - 1/4*x*cos(2*x) - 1/8*(2*x^2 - 1)*sin(2*x)
```

3.36 $\int \cos^2(x)\,dx$

Optimal. Leaf size=14

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rubi [A] time = 0.0060605, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{x}{2}+\frac{1}{2}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2,x]
```

```
[Out] x/2 + (Cos[x]*Sin[x])/2
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(x)\,dx &= \frac{1}{2}\cos(x)\sin(x)+\frac{\int 1\,dx}{2}\\ &= \frac{x}{2}+\frac{1}{2}\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0015343, size = 14, normalized size = 1.

$$\frac{x}{2}+\frac{1}{4}\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2,x]
```

```
[Out] x/2 + Sin[2*x]/4
```

Maple [A] time = 0.005, size = 11, normalized size = 0.8

$$\frac{x}{2}+\frac{\cos\left(x\right)\sin\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2,x)
```

```
[Out] 1/2*x+1/2*cos(x)*sin(x)
```

Maxima [A] time = 0.931024, size = 14, normalized size = 1.

$$\frac{1}{2}\,x+\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

Fricas [A] time = 0.454356, size = 36, normalized size = 2.57

$$\frac{1}{2}\,\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*sin(x) + 1/2*x
```

Sympy [A] time = 0.057039, size = 10, normalized size = 0.71

$$\frac{x}{2}+\frac{\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2,x)
```

```
[Out] x/2 + sin(x)*cos(x)/2
```

Giac [A] time = 1.09655, size = 14, normalized size = 1.

$$\frac{1}{2}\,x+\frac{1}{4}\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x + 1/4*sin(2*x)
```

3.37 $\int \cos^3(x)\,dx$

Optimal. Leaf size=11

$$\sin(x)-\frac{\sin^3(x)}{3}$$

```
[Out] Sin[x] - Sin[x]^3/3
```

Rubi [A] time = 0.0060146, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2633}

$$\sin(x)-\frac{\sin^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3,x]
```

```
[Out] Sin[x] - Sin[x]^3/3
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(x)\,dx &= -\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(x)\right)\\ &= \sin(x)-\frac{\sin^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.0014772, size = 15, normalized size = 1.36

$$\frac{3\sin(x)}{4}+\frac{1}{12}\sin(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3,x]
```

```
[Out] (3*Sin[x])/4 + Sin[3*x]/12
```

Maple [A] time = 0.026, size = 11, normalized size = 1.

$$\frac{\left(2+(\cos(x))^2\right)\sin(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3,x)
```

```
[Out] 1/3*(2+cos(x)^2)*sin(x)
```

Maxima [A] time = 0.949701, size = 12, normalized size = 1.09

$$-\frac{1}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="maxima")
```

```
[Out] -1/3*sin(x)^3 + sin(x)
```

Fricas [A] time = 0.453978, size = 36, normalized size = 3.27

$$\frac{1}{3}\left(\cos(x)^2 + 2\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + 2)*sin(x)
```

Sympy [A] time = 0.057907, size = 8, normalized size = 0.73

$$-\frac{\sin^3(x)}{3} + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3,x)
```

```
[Out] -sin(x)**3/3 + sin(x)
```

Giac [A] time = 1.1249, size = 12, normalized size = 1.09

$$-\frac{1}{3}\sin(x)^3 + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3,x, algorithm="giac")
```

```
[Out] -1/3*sin(x)^3 + sin(x)
```

3.38 $\int \cos^4(x)\,dx$

Optimal. Leaf size=24

$$\frac{3x}{8}+\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{8}\sin(x)\cos(x)$$

```
[Out] (3*x)/8 + (3*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
```

Rubi [A] time = 0.0112994, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2635, 8}

$$\frac{3x}{8}+\frac{1}{4}\sin(x)\cos^3(x)+\frac{3}{8}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4,x]
```

```
[Out] (3*x)/8 + (3*Cos[x]*Sin[x])/8 + (Cos[x]^3*Sin[x])/4
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(x)\,dx &= \frac{1}{4}\cos^3(x)\sin(x)+\frac{3}{4}\int \cos^2(x)\,dx\\ &= \frac{3}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)+\frac{3\int 1\,dx}{8}\\ &= \frac{3x}{8}+\frac{3}{8}\cos(x)\sin(x)+\frac{1}{4}\cos^3(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0015738, size = 22, normalized size = 0.92

$$\frac{3x}{8}+\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4,x]
```

```
[Out] (3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32
```

Maple [A] time = 0.03, size = 18, normalized size = 0.8

$$\frac{\sin(x)}{4}\left((\cos(x))^3+\frac{3\cos(x)}{2}\right)+\frac{3x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4,x)
```

```
[Out] 1/4*(cos(x)^3+3/2*cos(x))*sin(x)+3/8*x
```

Maxima [A] time = 0.962921, size = 22, normalized size = 0.92

$$\frac{3}{8}x+\frac{1}{32}\sin(4x)+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4,x, algorithm="maxima")
```

```
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

Fricas [A] time = 0.459668, size = 59, normalized size = 2.46

$$\frac{1}{8}\left(2\cos(x)^3+3\cos(x)\right)\sin(x)+\frac{3}{8}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(2*cos(x)^3 + 3*cos(x))*sin(x) + 3/8*x
```

Sympy [A] time = 0.056065, size = 24, normalized size = 1.

$$\frac{3x}{8}+\frac{\sin(x)\cos^3(x)}{4}+\frac{3\sin(x)\cos(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4,x)
```

```
[Out] 3*x/8 + sin(x)*cos(x)**3/4 + 3*sin(x)*cos(x)/8
```

Giac [A] time = 1.08064, size = 22, normalized size = 0.92

$$\frac{3}{8}x+\frac{1}{32}\sin(4x)+\frac{1}{4}\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4,x, algorithm="giac")
```

```
[Out] 3/8*x + 1/32*sin(4*x) + 1/4*sin(2*x)
```

3.39 $\int \left(a^2-x^2\right)^{5/2} \, dx$

Optimal. Leaf size=84

$$\frac{5}{16}a^4x\sqrt{a^2-x^2}+\frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{5}{16}a^6\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[Out] (5*a^4*x*Sqrt[a^2 - x^2])/16 + (5*a^2*x*(a^2 - x^2)^(3/2))/24 + (x*(a^2 - x
^2)^(5/2))/6 + (5*a^6*ArcTan[x/Sqrt[a^2 - x^2]])/16
```

Rubi [A] time = 0.0145512, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {195, 217, 203}

$$\frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{5}{16}a^6\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)+\frac{5}{16}a^4x\sqrt{a^2-x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - x^2)^(5/2),x]
```

```
[Out] (5*a^4*x*Sqrt[a^2 - x^2])/16 + (5*a^2*x*(a^2 - x^2)^(3/2))/24 + (x*(a^2 - x
^2)^(5/2))/6 + (5*a^6*ArcTan[x/Sqrt[a^2 - x^2]])/16
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \left(a^2-x^2\right)^{5/2} \, dx &= \frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{1}{6}\left(5a^2\right)\int \left(a^2-x^2\right)^{3/2} \, dx\\
&= \frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{1}{8}\left(5a^4\right)\int \sqrt{a^2-x^2} \, dx\\
&= \frac{5}{16}a^4x\sqrt{a^2-x^2}+\frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{1}{16}\left(5a^6\right)\int \frac{1}{\sqrt{a^2-x^2}} \, dx\\
&= \frac{5}{16}a^4x\sqrt{a^2-x^2}+\frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{1}{16}\left(5a^6\right)\text{Subst}\left(\int \frac{1}{1+x^2} \, dx,x,\frac{x}{\sqrt{a^2-x^2}}\right)\\
&= \frac{5}{16}a^4x\sqrt{a^2-x^2}+\frac{5}{24}a^2x\left(a^2-x^2\right)^{3/2}+\frac{1}{6}x\left(a^2-x^2\right)^{5/2}+\frac{5}{16}a^6\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0665723, size = 62, normalized size = 0.74

$$\frac{1}{48}\sqrt{a^2-x^2}\left(-26a^2x^3+\frac{15a^5\sin^{-1}\left(\frac{x}{a}\right)}{\sqrt{1-\frac{x^2}{a^2}}}+33a^4x+8x^5\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 - x^2)^(5/2),x]
```

```
[Out] (Sqrt[a^2 - x^2]*(33*a^4*x - 26*a^2*x^3 + 8*x^5 + (15*a^5*ArcSin[x/a])/Sqrt
[1 - x^2/a^2]))/48
```

Maple [A] time = 0.006, size = 69, normalized size = 0.8

$$\frac{5\,a^2x}{24}\left(a^2-x^2\right)^{\frac{3}{2}}+\frac{x}{6}\left(a^2-x^2\right)^{\frac{5}{2}}+\frac{5\,a^6}{16}\arctan\left(x\frac{1}{\sqrt{a^2-x^2}}\right)+\frac{5\,a^4x}{16}\sqrt{a^2-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-x^2)^(5/2),x)
```

```
[Out] 5/24*a^2*x*(a^2-x^2)^(3/2)+1/6*x*(a^2-x^2)^(5/2)+5/16*a^6*arctan(x/(a^2-x^2
)^(1/2))+5/16*a^4*x*(a^2-x^2)^(1/2)
```

Maxima [A] time = 1.43738, size = 84, normalized size = 1.

$$\frac{5}{16}\,a^6\arcsin\left(\frac{x}{\sqrt{a^2}}\right)+\frac{5}{16}\,\sqrt{a^2-x^2}a^4x+\frac{5}{24}\left(a^2-x^2\right)^{\frac{3}{2}}a^2x+\frac{1}{6}\left(a^2-x^2\right)^{\frac{5}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^6*arcsin(x/sqrt(a^2)) + 5/16*sqrt(a^2 - x^2)*a^4*x + 5/24*(a^2 - x^2
)^(3/2)*a^2*x + 1/6*(a^2 - x^2)^(5/2)*x
```

Fricas [A] time = 0.441388, size = 132, normalized size = 1.57

$$-\frac{5}{8}\,a^6\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)+\frac{1}{48}\left(33\,a^4x-26\,a^2x^3+8\,x^5\right)\sqrt{a^2-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -5/8*a^6*arctan(-(a - sqrt(a^2 - x^2))/x) + 1/48*(33*a^4*x - 26*a^2*x^3 + 8
*x^5)*sqrt(a^2 - x^2)
```

Sympy [A] time = 3.84263, size = 182, normalized size = 2.17

$$\begin{cases} -\frac{5ia^6 \operatorname{acosh}\left(\frac{x}{a}\right)}{16} - \frac{11ia^5x}{16\sqrt{-1+\frac{x^2}{a^2}}} + \frac{59ia^3x^3}{48\sqrt{-1+\frac{x^2}{a^2}}} - \frac{17iax^5}{24\sqrt{-1+\frac{x^2}{a^2}}} + \frac{ix^7}{6a\sqrt{-1+\frac{x^2}{a^2}}} & \text{for } \frac{|x^2|}{|a^2|} > 1 \\ \frac{5a^6 \operatorname{asin}\left(\frac{x}{a}\right)}{16} + \frac{11a^5x\sqrt{1-\frac{x^2}{a^2}}}{16} - \frac{13a^3x^3\sqrt{1-\frac{x^2}{a^2}}}{24} + \frac{ax^5\sqrt{1-\frac{x^2}{a^2}}}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-x**2)**(5/2),x)
```

```
[Out] Piecewise((-5*I*a**6*acosh(x/a)/16 - 11*I*a**5*x/(16*sqrt(-1 + x**2/a**2))
+ 59*I*a**3*x**3/(48*sqrt(-1 + x**2/a**2)) - 17*I*a*x**5/(24*sqrt(-1 + x**2
/a**2)) + I*x**7/(6*a*sqrt(-1 + x**2/a**2)), Abs(x**2)/Abs(a**2) > 1), (5*a
**6*asin(x/a)/16 + 11*a**5*x*sqrt(1 - x**2/a**2)/16 - 13*a**3*x**3*sqrt(1 -
 x**2/a**2)/24 + a*x**5*sqrt(1 - x**2/a**2)/6, True))
```

Giac [A] time = 1.10558, size = 68, normalized size = 0.81

$$\frac{5}{16} a^6 \arcsin\left(\frac{x}{a}\right) \operatorname{sgn}(a) + \frac{1}{48} \left(33 a^4 - 2 \left(13 a^2 - 4 x^2\right) x^2\right) \sqrt{a^2 - x^2} x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(5/2),x, algorithm="giac")
```

```
[Out] 5/16*a^6*arcsin(x/a)*sgn(a) + 1/48*(33*a^4 - 2*(13*a^2 - 4*x^2)*x^2)*sqrt(a
^2 - x^2)*x
```

3.40 $\int \frac{x^5}{\sqrt{5+x^2}} \, dx$

Optimal. Leaf size=38

$$\frac{1}{5}\left(x^2+5\right)^{5/2}-\frac{10}{3}\left(x^2+5\right)^{3/2}+25\sqrt{x^2+5}$$

```
[Out] 25*Sqrt[5 + x^2] - (10*(5 + x^2)^(3/2))/3 + (5 + x^2)^(5/2)/5
```

Rubi [A] time = 0.0129593, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {266, 43}

$$\frac{1}{5}\left(x^2+5\right)^{5/2}-\frac{10}{3}\left(x^2+5\right)^{3/2}+25\sqrt{x^2+5}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/Sqrt[5 + x^2],x]
```

```
[Out] 25*Sqrt[5 + x^2] - (10*(5 + x^2)^(3/2))/3 + (5 + x^2)^(5/2)/5
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{5+x^2}} \, dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2}{\sqrt{5+x}} \, dx, x, x^2\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{25}{\sqrt{5+x}}-10\sqrt{5+x}+(5+x)^{3/2}\right) dx, x, x^2\right)\\
&= 25\sqrt{5+x^2}-\frac{10}{3}\left(5+x^2\right)^{3/2}+\frac{1}{5}\left(5+x^2\right)^{5/2}
\end{aligned}$$

Mathematica [A] time = 0.0063945, size = 25, normalized size = 0.66

$$\frac{1}{15}\sqrt{x^2+5}\left(3x^4-20x^2+200\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/Sqrt[5 + x^2],x]
```

```
[Out] (Sqrt[5 + x^2]*(200 - 20*x^2 + 3*x^4))/15
```

Maple [A] time = 0.003, size = 22, normalized size = 0.6

$$\frac{3x^4 - 20x^2 + 200}{15}\sqrt{x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(x^2+5)^(1/2),x)
```

```
[Out] 1/15*(x^2+5)^(1/2)*(3*x^4-20*x^2+200)
```

Maxima [A] time = 1.43064, size = 46, normalized size = 1.21

$$\frac{1}{5}\sqrt{x^2+5}x^4 - \frac{4}{3}\sqrt{x^2+5}x^2 + \frac{40}{3}\sqrt{x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(x^2 + 5)*x^4 - 4/3*sqrt(x^2 + 5)*x^2 + 40/3*sqrt(x^2 + 5)
```

Fricas [A] time = 0.425337, size = 58, normalized size = 1.53

$$\frac{1}{15}\left(3x^4 - 20x^2 + 200\right)\sqrt{x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*x^4 - 20*x^2 + 200)*sqrt(x^2 + 5)
```

Sympy [A] time = 1.24128, size = 39, normalized size = 1.03

$$\frac{x^4\sqrt{x^2+5}}{5} - \frac{4x^2\sqrt{x^2+5}}{3} + \frac{40\sqrt{x^2+5}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(x**2+5)**(1/2),x)
```

```
[Out] x**4*sqrt(x**2 + 5)/5 - 4*x**2*sqrt(x**2 + 5)/3 + 40*sqrt(x**2 + 5)/3
```

Giac [A] time = 1.08353, size = 38, normalized size = 1.

$$\frac{1}{5}\left(x^2+5\right)^{\frac{5}{2}} - \frac{10}{3}\left(x^2+5\right)^{\frac{3}{2}} + 25\sqrt{x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(x^2+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*(x^2 + 5)^(5/2) - 10/3*(x^2 + 5)^(3/2) + 25*sqrt(x^2 + 5)
```

3.41 $\int \frac{t^3}{\sqrt{4+t^3}}\,dt$

Optimal. Leaf size=172

$$\frac{2}{5}t\sqrt{t^3+4}-\frac{8\,2^{2/3}\sqrt{2+\sqrt{3}}\left(t+2^{2/3}\right)\sqrt{\frac{t^2-2^{2/3}t+2\sqrt[3]{2}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{t+2^{2/3}\left(1-\sqrt{3}\right)}{t+2^{2/3}\left(1+\sqrt{3}\right)}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{t+2^{2/3}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{t^3+4}}$$

```
[Out] (2*t*Sqrt[4 + t^3])/5 - (8*2^(2/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) + t)*Sqrt[(2*
2^(1/3) - 2^(2/3)*t + t^2)/(2^(2/3)*(1 + Sqrt[3]) + t)^2]*EllipticF[ArcSin[
(2^(2/3)*(1 - Sqrt[3]) + t)/(2^(2/3)*(1 + Sqrt[3]) + t)], -7 - 4*Sqrt[3]])/
(5*3^(1/4)*Sqrt[(2^(2/3) + t)/(2^(2/3)*(1 + Sqrt[3]) + t)^2]*Sqrt[4 + t^3])
```

Rubi [A] time = 0.0288655, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {321, 218}

$$\frac{2}{5}t\sqrt{t^3+4}-\frac{8\,2^{2/3}\sqrt{2+\sqrt{3}}\left(t+2^{2/3}\right)\sqrt{\frac{t^2-2^{2/3}t+2\sqrt[3]{2}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{t+2^{2/3}\left(1-\sqrt{3}\right)}{t+2^{2/3}\left(1+\sqrt{3}\right)}\right),-7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{t+2^{2/3}}{\left(t+2^{2/3}\left(1+\sqrt{3}\right)\right)^2}}\sqrt{t^3+4}}$$

Antiderivative was successfully verified.

```
[In] Int[t^3/Sqrt[4 + t^3],t]
```

```
[Out] (2*t*Sqrt[4 + t^3])/5 - (8*2^(2/3)*Sqrt[2 + Sqrt[3]]*(2^(2/3) + t)*Sqrt[(2*
2^(1/3) - 2^(2/3)*t + t^2)/(2^(2/3)*(1 + Sqrt[3]) + t)^2]*EllipticF[ArcSin[
(2^(2/3)*(1 - Sqrt[3]) + t)/(2^(2/3)*(1 + Sqrt[3]) + t)], -7 - 4*Sqrt[3]])/
(5*3^(1/4)*Sqrt[(2^(2/3) + t)/(2^(2/3)*(1 + Sqrt[3]) + t)^2]*Sqrt[4 + t^3])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{t^3}{\sqrt{4+t^3}}\,dt = \frac{2}{5}t\sqrt{4+t^3} - \frac{8}{5}\int \frac{1}{\sqrt{4+t^3}}\,dt$$

$$= \frac{2}{5}t\sqrt{4+t^3} - \frac{8\, 2^{2/3}\sqrt{2+\sqrt{3}}\left(2^{2/3}+t\right)\sqrt{\frac{2\sqrt[3]{2}-2^{2/3}t+t^2}{\left(2^{2/3}(1+\sqrt{3})+t\right)^2}}\,F\left(\sin^{-1}\left(\frac{2^{2/3}(1-\sqrt{3})+t}{2^{2/3}(1+\sqrt{3})+t}\right)\middle| -7-4\sqrt{3}\right)}{5\sqrt[4]{3}\sqrt{\frac{2^{2/3}+t}{\left(2^{2/3}(1+\sqrt{3})+t\right)^2}}\sqrt{4+t^3}}$$

Mathematica [C] time = 0.00516, size = 34, normalized size = 0.2

$$\frac{2}{5}t\left(\sqrt{t^3+4} - 2\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-\frac{t^3}{4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[t^3/Sqrt[4 + t^3],t]
```

```
[Out] (2*t*(Sqrt[4 + t^3] - 2*Hypergeometric2F1[1/3, 1/2, 4/3, -t^3/4]))/5
```

Maple [A] time = 0.098, size = 168, normalized size = 1.

$$\frac{2t}{5}\sqrt{t^3+4} + \frac{8i}{15}\sqrt{3}2^{\frac{2}{3}}\sqrt{i\left(t-\frac{2^{\frac{2}{3}}}{2}-\frac{i}{2}\sqrt{3}2^{\frac{2}{3}}\right)\sqrt{3}\sqrt[3]{2}}\sqrt{\frac{2^{\frac{2}{3}}+t}{\frac{3\,2^{2/3}}{2}+\frac{i}{2}\sqrt{3}2^{\frac{2}{3}}}}\sqrt{-i\left(t-\frac{2^{\frac{2}{3}}}{2}+\frac{i}{2}\sqrt{3}2^{\frac{2}{3}}\right)\sqrt{3}\sqrt[3]{2}}\text{EllipticF}\left(\frac{\sqrt{6}}{6}\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^3/(t^3+4)^(1/2),t)
```

```
[Out] 2/5*t*(t^3+4)^(1/2)+8/15*I*3^(1/2)*2^(2/3)*(I*(t-1/2*2^(2/3)-1/2*I*3^(1/2)*
2^(2/3))*3^(1/2)*2^(1/3))^(1/2)*((2^(2/3)+t)/(3/2*2^(2/3)+1/2*I*3^(1/2)*2^(
2/3)))^(1/2)*(-I*(t-1/2*2^(2/3)+1/2*I*3^(1/2)*2^(2/3))*3^(1/2)*2^(1/3))^(1/
2)/(t^3+4)^(1/2)*EllipticF(1/6*6^(1/2)*(I*(t-1/2*2^(2/3)-1/2*I*3^(1/2)*2^(2
/3))*3^(1/2)*2^(1/3))^(1/2),(I*3^(1/2)*2^(2/3)/(3/2*2^(2/3)+1/2*I*3^(1/2)*2
^(2/3)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{t^3}{\sqrt{t^3+4}}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/(t^3+4)^(1/2),t, algorithm="maxima")
```

```
[Out] integrate(t^3/sqrt(t^3 + 4), t)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{t^3}{\sqrt{t^3+4}}, t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/(t^3+4)^(1/2),t, algorithm="fricas")
```

```
[Out] integral(t^3/sqrt(t^3 + 4), t)
```

Sympy [A] time = 0.61008, size = 31, normalized size = 0.18

$$\frac{t^4\Gamma\left(\frac{4}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{4}{3}\\ \frac{7}{3}\end{matrix}\middle|\frac{t^3e^{i\pi}}{4}\right)}{6\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t**3/(t**3+4)**(1/2),t)
```

```
[Out] t**4*gamma(4/3)*hyper((1/2, 4/3), (7/3,), t**3*exp_polar(I*pi)/4)/(6*gamma(
7/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{t^3}{\sqrt{t^3+4}}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/(t^3+4)^(1/2),t, algorithm="giac")
```

```
[Out] integrate(t^3/sqrt(t^3 + 4), t)
```

3.42 $\int \tan^2(x)\, dx$

Optimal. Leaf size=6

$$\tan(x) - x$$

```
[Out] -x + Tan[x]
```

Rubi [A] time = 0.0043751, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 8}

$$\tan(x) - x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^2,x]
```

```
[Out] -x + Tan[x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tan^2(x)\, dx &= \tan(x) - \int 1\, dx\\ &= -x + \tan(x)\end{aligned}$$

Mathematica [A] time = 0.0048106, size = 6, normalized size = 1.

$$\tan(x) - x$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^2,x]
```

```
[Out] -x + Tan[x]
```

Maple [A] time = 0.004, size = 7, normalized size = 1.2

$$-x + \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^2,x)
```

```
[Out] -x+tan(x)
```

Maxima [A] time = 1.44163, size = 8, normalized size = 1.33

$$-x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="maxima")
```

```
[Out] -x + tan(x)
```

Fricas [A] time = 0.446145, size = 18, normalized size = 3.

$$-x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="fricas")
```

```
[Out] -x + tan(x)
```

Sympy [B] time = 0.058233, size = 7, normalized size = 1.17

$$-x+\frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**2,x)
```

```
[Out] -x + sin(x)/cos(x)
```

Giac [A] time = 1.07744, size = 8, normalized size = 1.33

$$-x+\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^2,x, algorithm="giac")
```

```
[Out] -x + tan(x)
```

3.43 $\int \tan^4(x)\,dx$

Optimal. Leaf size=14

$$x+\frac{\tan^3(x)}{3}-\tan(x)$$

```
[Out] x - Tan[x] + Tan[x]^3/3
```

Rubi [A] time = 0.0091532, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 8}

$$x+\frac{\tan^3(x)}{3}-\tan(x)$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]^4,x]

[Out] x - Tan[x] + Tan[x]^3/3
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \tan^4(x)\,dx &= \frac{\tan^3(x)}{3}-\int \tan^2(x)\,dx\\ &= -\tan(x)+\frac{\tan^3(x)}{3}+\int 1\,dx\\ &= x-\tan(x)+\frac{\tan^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.003065, size = 18, normalized size = 1.29

$$x-\frac{4\tan(x)}{3}+\frac{1}{3}\tan(x)\sec^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]^4,x]

[Out] x - (4*Tan[x])/3 + (Sec[x]^2*Tan[x])/3
```

Maple [A] time = 0.003, size = 13, normalized size = 0.9

$$x - \tan(x) + \frac{(\tan(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)^4,x)
```

```
[Out] x-tan(x)+1/3*tan(x)^3
```

Maxima [A] time = 1.44506, size = 16, normalized size = 1.14

$$\frac{1}{3}\tan(x)^3 + x - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Fricas [A] time = 0.452121, size = 36, normalized size = 2.57

$$\frac{1}{3}\tan(x)^3 + x - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

Sympy [A] time = 0.067089, size = 19, normalized size = 1.36

$$x + \frac{\sin^3(x)}{3\cos^3(x)} - \frac{\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)**4,x)
```

```
[Out] x + sin(x)**3/(3*cos(x)**3) - sin(x)/cos(x)
```

Giac [A] time = 1.09716, size = 16, normalized size = 1.14

$$\frac{1}{3}\tan(x)^3 + x - \tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)^4,x, algorithm="giac")
```

```
[Out] 1/3*tan(x)^3 + x - tan(x)
```

3.44 $\int \cot^2(x)\,dx$

Optimal. Leaf size=8

$$-x-\cot(x)$$

```
[Out] -x - Cot[x]
```

Rubi [A] time = 0.0054236, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 8}

$$-x-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^2,x]
```

```
[Out] -x - Cot[x]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cot^2(x)\,dx &= -\cot(x)-\int 1\,dx\\ &= -x-\cot(x)\end{aligned}$$

Mathematica [A] time = 0.0048237, size = 8, normalized size = 1.

$$-x-\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^2,x]
```

```
[Out] -x - Cot[x]
```

Maple [A] time = 0.001, size = 12, normalized size = 1.5

$$-\cot(x)+\frac{\pi}{2}-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^2,x)
```

```
[Out] -cot(x)+1/2*Pi-x
```

Maxima [A] time = 1.44599, size = 14, normalized size = 1.75

$$-x - \frac{1}{\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2,x, algorithm="maxima")
```

```
[Out] -x - 1/tan(x)
```

Fricas [B] time = 0.450081, size = 53, normalized size = 6.62

$$-\frac{x\sin(2x) + \cos(2x) + 1}{\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2,x, algorithm="fricas")
```

```
[Out] -(x*sin(2*x) + cos(2*x) + 1)/sin(2*x)
```

Sympy [A] time = 0.063287, size = 8, normalized size = 1.

$$-x - \frac{\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**2,x)
```

```
[Out] -x - cos(x)/sin(x)
```

Giac [B] time = 1.10455, size = 24, normalized size = 3.

$$-x - \frac{1}{2\tan\left(\frac{1}{2}x\right)} + \frac{1}{2}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^2,x, algorithm="giac")
```

```
[Out] -x - 1/2/tan(1/2*x) + 1/2*tan(1/2*x)
```

3.45 $\int \cot^4(x)\,dx$

Optimal. Leaf size=12

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

```
[Out] x + Cot[x] - Cot[x]^3/3
```

Rubi [A] time = 0.0102067, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3473, 8}

$$x-\frac{1}{3}\cot^3(x)+\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^4,x]
```

```
[Out] x + Cot[x] - Cot[x]^3/3
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \cot^4(x)\,dx &= -\frac{1}{3}\cot^3(x)-\int \cot^2(x)\,dx\\
&= \cot(x)-\frac{\cot^3(x)}{3}+\int 1\,dx\\
&= x+\cot(x)-\frac{\cot^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.0027488, size = 18, normalized size = 1.5

$$x+\frac{4\cot(x)}{3}-\frac{1}{3}\cot(x)\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^4,x]
```

```
[Out] x + (4*Cot[x])/3 - (Cot[x]*Csc[x]^2)/3
```

Maple [A] time = 0.003, size = 14, normalized size = 1.2

$$-\frac{(\cot(x))^3}{3} + \cot(x) - \frac{\pi}{2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^4,x)
```

```
[Out] -1/3*cot(x)^3+cot(x)-1/2*Pi+x
```

Maxima [A] time = 1.41867, size = 22, normalized size = 1.83

$$x + \frac{3\tan(x)^2 - 1}{3\tan(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="maxima")
```

```
[Out] x + 1/3*(3*tan(x)^2 - 1)/tan(x)^3
```

Fricas [B] time = 0.451499, size = 126, normalized size = 10.5

$$\frac{4\cos(2x)^2 + 3(x\cos(2x) - x)\sin(2x) + 2\cos(2x) - 2}{3(\cos(2x) - 1)\sin(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="fricas")
```

```
[Out] 1/3*(4*cos(2*x)^2 + 3*(x*cos(2*x) - x)*sin(2*x) + 2*cos(2*x) - 2)/((cos(2*x
) - 1)*sin(2*x))
```

Sympy [A] time = 0.067302, size = 19, normalized size = 1.58

$$x + \frac{\cos(x)}{\sin(x)} - \frac{\cos^3(x)}{3\sin^3(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**4,x)
```

```
[Out] x + cos(x)/sin(x) - cos(x)**3/(3*sin(x)**3)
```

Giac [B] time = 1.12875, size = 46, normalized size = 3.83

$$\frac{1}{24}\tan\left(\frac{1}{2}x\right)^3 + x + \frac{15\tan\left(\frac{1}{2}x\right)^2 - 1}{24\tan\left(\frac{1}{2}x\right)^3} - \frac{5}{8}\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^4,x, algorithm="giac")
```

```
[Out] 1/24*tan(1/2*x)^3 + x + 1/24*(15*tan(1/2*x)^2 - 1)/tan(1/2*x)^3 - 5/8*tan(1
/2*x)
```

3.46 $\int (2+3x)\sin(5x)\,dx$

Optimal. Leaf size=22

$$\frac{3}{25}\sin(5x)-\frac{1}{5}(3x+2)\cos(5x)$$

```
[Out] -((2 + 3*x)*Cos[5*x])/5 + (3*Sin[5*x])/25
```

Rubi [A] time = 0.0112121, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3296, 2637}

$$\frac{3}{25}\sin(5x)-\frac{1}{5}(3x+2)\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)*Sin[5*x],x]
```

```
[Out] -((2 + 3*x)*Cos[5*x])/5 + (3*Sin[5*x])/25
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (2+3x)\sin(5x)\,dx &= -\frac{1}{5}(2+3x)\cos(5x)+\frac{3}{5}\int \cos(5x)\,dx\\ &= -\frac{1}{5}(2+3x)\cos(5x)+\frac{3}{25}\sin(5x)\end{aligned}$$

Mathematica [A] time = 0.0105456, size = 26, normalized size = 1.18

$$\frac{3}{25}\sin(5x)-\frac{3}{5}x\cos(5x)-\frac{2}{5}\cos(5x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)*Sin[5*x],x]
```

```
[Out] (-2*Cos[5*x])/5 - (3*x*Cos[5*x])/5 + (3*Sin[5*x])/25
```

Maple [A] time = 0.006, size = 21, normalized size = 1.

$$-\frac{2\cos\left(5x\right)}{5}+\frac{3\sin\left(5x\right)}{25}-\frac{3\cos\left(5x\right)x}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+3*x)*sin(5*x),x)
```

```
[Out] -2/5*cos(5*x)+3/25*sin(5*x)-3/5*cos(5*x)*x
```

Maxima [A] time = 0.946682, size = 27, normalized size = 1.23

$$-\frac{3}{5}x\cos\left(5x\right)-\frac{2}{5}\cos\left(5x\right)+\frac{3}{25}\sin\left(5x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*sin(5*x),x, algorithm="maxima")
```

```
[Out] -3/5*x*cos(5*x) - 2/5*cos(5*x) + 3/25*sin(5*x)
```

Fricas [A] time = 0.447508, size = 55, normalized size = 2.5

$$-\frac{1}{5}\left(3x+2\right)\cos\left(5x\right)+\frac{3}{25}\sin\left(5x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*sin(5*x),x, algorithm="fricas")
```

```
[Out] -1/5*(3*x + 2)*cos(5*x) + 3/25*sin(5*x)
```

Sympy [A] time = 0.171743, size = 26, normalized size = 1.18

$$-\frac{3x\cos\left(5x\right)}{5}+\frac{3\sin\left(5x\right)}{25}-\frac{2\cos\left(5x\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*sin(5*x),x)
```

```
[Out] -3*x*cos(5*x)/5 + 3*sin(5*x)/25 - 2*cos(5*x)/5
```

Giac [A] time = 1.08561, size = 24, normalized size = 1.09

$$-\frac{1}{5}\left(3x+2\right)\cos\left(5x\right)+\frac{3}{25}\sin\left(5x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+3*x)*sin(5*x),x, algorithm="giac")

[Out] -1/5*(3*x + 2)*cos(5*x) + 3/25*sin(5*x)
```

3.47 $\int x\sqrt{1+x^2}\,dx$

Optimal. Leaf size=13

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

```
[Out] (1 + x^2)^(3/2)/3
```

Rubi [A] time = 0.0019292, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[1 + x^2],x]

[Out] (1 + x^2)^(3/2)/3
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\sqrt{1+x^2}\,dx=\frac{1}{3}\left(1+x^2\right)^{3/2}$$

Mathematica [A] time = 0.0015283, size = 13, normalized size = 1.

$$\frac{1}{3}\left(x^2+1\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[1 + x^2],x]

[Out] (1 + x^2)^(3/2)/3
```

Maple [A] time = 0.002, size = 10, normalized size = 0.8

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)^(1/2),x)

[Out] 1/3*(x^2+1)^(3/2)
```

Maxima [A] time = 0.946437, size = 12, normalized size = 0.92

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="maxima")

[Out] 1/3*(x^2 + 1)^(3/2)
```

Fricas [A] time = 0.423158, size = 28, normalized size = 2.15

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="fricas")

[Out] 1/3*(x^2 + 1)^(3/2)
```

Sympy [B] time = 0.174353, size = 22, normalized size = 1.69

$$\frac{x^2\sqrt{x^2+1}}{3}+\frac{\sqrt{x^2+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)**(1/2),x)

[Out] x**2*sqrt(x**2 + 1)/3 + sqrt(x**2 + 1)/3
```

Giac [A] time = 1.09012, size = 12, normalized size = 0.92

$$\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)^(1/2),x, algorithm="giac")

[Out] 1/3*(x^2 + 1)^(3/2)
```

3.48 $\int x\left(-1+x^2\right)^9\,dx$

Optimal. Leaf size=13

$$\frac{1}{20}\left(1-x^2\right)^{10}$$

```
[Out] (1 - x^2)^10/20
```

Rubi [A] time = 0.0014274, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {261}

$$\frac{1}{20}\left(1-x^2\right)^{10}$$

Antiderivative was successfully verified.

```
[In] Int[x*(-1 + x^2)^9,x]
```

```
[Out] (1 - x^2)^10/20
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x\left(-1+x^2\right)^9\,dx=\frac{1}{20}\left(1-x^2\right)^{10}$$

Mathematica [A] time = 0.0013549, size = 11, normalized size = 0.85

$$\frac{1}{20}\left(x^2-1\right)^{10}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(-1 + x^2)^9,x]
```

```
[Out] (-1 + x^2)^10/20
```

Maple [B] time = 0.001, size = 52, normalized size = 4.

$$\frac{x^{20}}{20}-\frac{x^{18}}{2}+\frac{9\,x^{16}}{4}-6\,x^{14}+\frac{21\,x^{12}}{2}-\frac{63\,x^{10}}{5}+\frac{21\,x^{8}}{2}-6\,x^{6}+\frac{9\,x^{4}}{4}-\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2-1)^9,x)
```

```
[Out] 1/20*x^20-1/2*x^18+9/4*x^16-6*x^14+21/2*x^12-63/5*x^10+21/2*x^8-6*x^6+9/4*x
^4-1/2*x^2
```

Maxima [A] time = 0.955849, size = 12, normalized size = 0.92

$$\frac{1}{20}\left(x^2-1\right)^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^9,x, algorithm="maxima")
```

```
[Out] 1/20*(x^2 - 1)^10
```

Fricas [B] time = 0.358493, size = 142, normalized size = 10.92

$$\frac{1}{20}x^{20}-\frac{1}{2}x^{18}+\frac{9}{4}x^{16}-6x^{14}+\frac{21}{2}x^{12}-\frac{63}{5}x^{10}+\frac{21}{2}x^{8}-6x^{6}+\frac{9}{4}x^{4}-\frac{1}{2}x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^9,x, algorithm="fricas")
```

```
[Out] 1/20*x^20 - 1/2*x^18 + 9/4*x^16 - 6*x^14 + 21/2*x^12 - 63/5*x^10 + 21/2*x^8
 - 6*x^6 + 9/4*x^4 - 1/2*x^2
```

Sympy [B] time = 0.056396, size = 58, normalized size = 4.46

$$\frac{x^{20}}{20}-\frac{x^{18}}{2}+\frac{9x^{16}}{4}-6x^{14}+\frac{21x^{12}}{2}-\frac{63x^{10}}{5}+\frac{21x^{8}}{2}-6x^{6}+\frac{9x^{4}}{4}-\frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2-1)**9,x)
```

```
[Out] x**20/20 - x**18/2 + 9*x**16/4 - 6*x**14 + 21*x**12/2 - 63*x**10/5 + 21*x**
8/2 - 6*x**6 + 9*x**4/4 - x**2/2
```

Giac [A] time = 1.12956, size = 12, normalized size = 0.92

$$\frac{1}{20}\left(x^2-1\right)^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2-1)^9,x, algorithm="giac")
```

```
[Out] 1/20*(x^2 - 1)^10
```

3.49 $\int \frac{3+2x}{(7+6x)^3}\,dx$

Optimal. Leaf size=18

$$-\frac{(2x+3)^2}{8(6x+7)^2}$$

```
[Out] -(3 + 2*x)^2/(8*(7 + 6*x)^2)
```

Rubi [A] time = 0.0015898, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {37}

$$-\frac{(2x+3)^2}{8(6x+7)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/(7 + 6*x)^3,x]
```

```
[Out] -(3 + 2*x)^2/(8*(7 + 6*x)^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{3+2x}{(7+6x)^3}\,dx = -\frac{(3+2x)^2}{8(7+6x)^2}$$

Mathematica [A] time = 0.003006, size = 16, normalized size = 0.89

$$-\frac{3x+4}{9(6x+7)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/(7 + 6*x)^3,x]
```

```
[Out] -(4 + 3*x)/(9*(7 + 6*x)^2)
```

Maple [A] time = 0.004, size = 20, normalized size = 1.1

$$-\frac{1}{126+108\,x}-\frac{1}{18\,(7+6\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)/(7+6*x)^3,x)
```

```
[Out] -1/18/(7+6*x)-1/18/(7+6*x)^2
```

Maxima [A] time = 0.958868, size = 26, normalized size = 1.44

$$-\frac{3x+4}{9\left(36x^2+84x+49\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(7+6*x)^3,x, algorithm="maxima")
```

```
[Out] -1/9*(3*x + 4)/(36*x^2 + 84*x + 49)
```

Fricas [A] time = 0.407813, size = 50, normalized size = 2.78

$$-\frac{3x+4}{9\left(36x^2+84x+49\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(7+6*x)^3,x, algorithm="fricas")
```

```
[Out] -1/9*(3*x + 4)/(36*x^2 + 84*x + 49)
```

Sympy [A] time = 0.095918, size = 15, normalized size = 0.83

$$-\frac{3x+4}{324x^2+756x+441}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(7+6*x)**3,x)
```

```
[Out] -(3*x + 4)/(324*x**2 + 756*x + 441)
```

Giac [A] time = 1.07244, size = 19, normalized size = 1.06

$$-\frac{3x+4}{9(6x+7)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(7+6*x)^3,x, algorithm="giac")
```

```
[Out] -1/9*(3*x + 4)/(6*x + 7)^2
```

3.50 $\int x^4 \left(1+x^5\right)^5 \, dx$

Optimal. Leaf size=11

$$\frac{1}{30} \left(x^5+1\right)^6$$

```
[Out] (1 + x^5)^6/30
```

Rubi [A] time = 0.0018053, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {261}

$$\frac{1}{30} \left(x^5+1\right)^6$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 + x^5)^5,x]
```

```
[Out] (1 + x^5)^6/30
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int x^4 \left(1+x^5\right)^5 \, dx = \frac{1}{30} \left(1+x^5\right)^6$$

Mathematica [B] time = 0.0012812, size = 43, normalized size = 3.91

$$\frac{x^{30}}{30}+\frac{x^{25}}{5}+\frac{x^{20}}{2}+\frac{2x^{15}}{3}+\frac{x^{10}}{2}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 + x^5)^5,x]
```

```
[Out] x^5/5 + x^10/2 + (2*x^15)/3 + x^20/2 + x^25/5 + x^30/30
```

Maple [B] time = 0.001, size = 32, normalized size = 2.9

$$\frac{x^{30}}{30}+\frac{x^{25}}{5}+\frac{x^{20}}{2}+\frac{2\,x^{15}}{3}+\frac{x^{10}}{2}+\frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^5+1)^5,x)
```

```
[Out] 1/30*x^30+1/5*x^25+1/2*x^20+2/3*x^15+1/2*x^10+1/5*x^5
```

Maxima [A] time = 0.94927, size = 12, normalized size = 1.09

$$\frac{1}{30}\left(x^5+1\right)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+1)^5,x, algorithm="maxima")
```

```
[Out] 1/30*(x^5 + 1)^6
```

Fricas [B] time = 0.357088, size = 88, normalized size = 8.

$$\frac{1}{30}x^{30}+\frac{1}{5}x^{25}+\frac{1}{2}x^{20}+\frac{2}{3}x^{15}+\frac{1}{2}x^{10}+\frac{1}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+1)^5,x, algorithm="fricas")
```

```
[Out] 1/30*x^30 + 1/5*x^25 + 1/2*x^20 + 2/3*x^15 + 1/2*x^10 + 1/5*x^5
```

Sympy [B] time = 0.05131, size = 31, normalized size = 2.82

$$\frac{x^{30}}{30}+\frac{x^{25}}{5}+\frac{x^{20}}{2}+\frac{2x^{15}}{3}+\frac{x^{10}}{2}+\frac{x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**5+1)**5,x)
```

```
[Out] x**30/30 + x**25/5 + x**20/2 + 2*x**15/3 + x**10/2 + x**5/5
```

Giac [A] time = 1.09281, size = 12, normalized size = 1.09

$$\frac{1}{30}\left(x^5+1\right)^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^5+1)^5,x, algorithm="giac")
```

```
[Out] 1/30*(x^5 + 1)^6
```

3.51 $\int (1-x)^{20} x^4 \, dx$

Optimal. Leaf size=56

$$-\frac{1}{25}(1-x)^{25}+\frac{1}{6}(1-x)^{24}-\frac{6}{23}(1-x)^{23}+\frac{2}{11}(1-x)^{22}-\frac{1}{21}(1-x)^{21}$$

```
[Out] -(1 - x)^21/21 + (2*(1 - x)^22)/11 - (6*(1 - x)^23)/23 + (1 - x)^24/6 - (1
- x)^25/25
```

Rubi [A] time = 0.0252189, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {43}

$$-\frac{1}{25}(1-x)^{25}+\frac{1}{6}(1-x)^{24}-\frac{6}{23}(1-x)^{23}+\frac{2}{11}(1-x)^{22}-\frac{1}{21}(1-x)^{21}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x)^20*x^4,x]
```

```
[Out] -(1 - x)^21/21 + (2*(1 - x)^22)/11 - (6*(1 - x)^23)/23 + (1 - x)^24/6 - (1
- x)^25/25
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int (1-x)^{20} x^4 \, dx &= \int \left((1-x)^{20}-4(1-x)^{21}+6(1-x)^{22}-4(1-x)^{23}+(1-x)^{24}\right) dx\\ &= -\frac{1}{21}(1-x)^{21}+\frac{2}{11}(1-x)^{22}-\frac{6}{23}(1-x)^{23}+\frac{1}{6}(1-x)^{24}-\frac{1}{25}(1-x)^{25}\end{aligned}$$

Mathematica [B] time = 0.0015064, size = 140, normalized size = 2.5

$$\frac{x^{25}}{25}-\frac{5x^{24}}{6}+\frac{190x^{23}}{23}-\frac{570x^{22}}{11}+\frac{1615x^{21}}{7}-\frac{3876x^{20}}{5}+2040x^{19}-\frac{12920x^{18}}{3}+7410x^{17}-\frac{20995x^{16}}{2}+\frac{184756x^{15}}{15}-\frac{839}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x)^20*x^4,x]
```

```
[Out] x^5/5 - (10*x^6)/3 + (190*x^7)/7 - (285*x^8)/2 + (1615*x^9)/3 - (7752*x^10)
/5 + (38760*x^11)/11 - 6460*x^12 + 9690*x^13 - (83980*x^14)/7 + (184756*x^1
5)/15 - (20995*x^16)/2 + 7410*x^17 - (12920*x^18)/3 + 2040*x^19 - (3876*x^2
0)/5 + (1615*x^21)/7 - (570*x^22)/11 + (190*x^23)/23 - (5*x^24)/6 + x^25/25
```

Maple [B] time = 0.002, size = 107, normalized size = 1.9

$$\frac{x^{25}}{25}-\frac{5\,x^{24}}{6}+\frac{190\,x^{23}}{23}-\frac{570\,x^{22}}{11}+\frac{1615\,x^{21}}{7}-\frac{3876\,x^{20}}{5}+2040\,x^{19}-\frac{12920\,x^{18}}{3}+7410\,x^{17}-\frac{20995\,x^{16}}{2}+\frac{184756\,x^{15}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x)^20*x^4,x)
```

```
[Out] 1/25*x^25-5/6*x^24+190/23*x^23-570/11*x^22+1615/7*x^21-3876/5*x^20+2040*x^1
9-12920/3*x^18+7410*x^17-20995/2*x^16+184756/15*x^15-83980/7*x^14+9690*x^13
-6460*x^12+38760/11*x^11-7752/5*x^10+1615/3*x^9-285/2*x^8+190/7*x^7-10/3*x^
6+1/5*x^5
```

Maxima [B] time = 0.93812, size = 143, normalized size = 2.55

$$\frac{1}{25}\,x^{25}-\frac{5}{6}\,x^{24}+\frac{190}{23}\,x^{23}-\frac{570}{11}\,x^{22}+\frac{1615}{7}\,x^{21}-\frac{3876}{5}\,x^{20}+2040\,x^{19}-\frac{12920}{3}\,x^{18}+7410\,x^{17}-\frac{20995}{2}\,x^{16}+\frac{184756}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^20*x^4,x, algorithm="maxima")
```

```
[Out] 1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 - 3876/5*x^2
0 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16 + 184756/15*x^15 -
83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/11*x^11 - 7752/5*x^10 + 1615/3
*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3*x^6 + 1/5*x^5
```

Fricas [B] time = 0.352231, size = 373, normalized size = 6.66

$$\frac{1}{25}x^{25}-\frac{5}{6}x^{24}+\frac{190}{23}x^{23}-\frac{570}{11}x^{22}+\frac{1615}{7}x^{21}-\frac{3876}{5}x^{20}+2040x^{19}-\frac{12920}{3}x^{18}+7410x^{17}-\frac{20995}{2}x^{16}+\frac{184756}{15}x^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^20*x^4,x, algorithm="fricas")
```

```
[Out] 1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 - 3876/5*x^2
0 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16 + 184756/15*x^15 -
83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/11*x^11 - 7752/5*x^10 + 1615/3
*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3*x^6 + 1/5*x^5
```

Sympy [B] time = 0.071957, size = 131, normalized size = 2.34

$$\frac{x^{25}}{25}-\frac{5x^{24}}{6}+\frac{190x^{23}}{23}-\frac{570x^{22}}{11}+\frac{1615x^{21}}{7}-\frac{3876x^{20}}{5}+2040x^{19}-\frac{12920x^{18}}{3}+7410x^{17}-\frac{20995x^{16}}{2}+\frac{184756x^{15}}{15}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)**20*x**4,x)
```

```
[Out] x**25/25 - 5*x**24/6 + 190*x**23/23 - 570*x**22/11 + 1615*x**21/7 - 3876*x*
*20/5 + 2040*x**19 - 12920*x**18/3 + 7410*x**17 - 20995*x**16/2 + 184756*x*
```

```
*15/15 - 83980*x**14/7 + 9690*x**13 - 6460*x**12 + 38760*x**11/11 - 7752*x*
*10/5 + 1615*x**9/3 - 285*x**8/2 + 190*x**7/7 - 10*x**6/3 + x**5/5
```

Giac [B] time = 1.09752, size = 143, normalized size = 2.55

$$\frac{1}{25}x^{25} - \frac{5}{6}x^{24} + \frac{190}{23}x^{23} - \frac{570}{11}x^{22} + \frac{1615}{7}x^{21} - \frac{3876}{5}x^{20} + 2040\,x^{19} - \frac{12920}{3}x^{18} + 7410\,x^{17} - \frac{20995}{2}x^{16} + \frac{184756}{15}x^{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)^20*x^4,x, algorithm="giac")
```

```
[Out] 1/25*x^25 - 5/6*x^24 + 190/23*x^23 - 570/11*x^22 + 1615/7*x^21 - 3876/5*x^2
0 + 2040*x^19 - 12920/3*x^18 + 7410*x^17 - 20995/2*x^16 + 184756/15*x^15 -
83980/7*x^14 + 9690*x^13 - 6460*x^12 + 38760/11*x^11 - 7752/5*x^10 + 1615/3
*x^9 - 285/2*x^8 + 190/7*x^7 - 10/3*x^6 + 1/5*x^5
```

3.52 $\int \frac{\sin\left(\frac{1}{x}\right)}{x^2} dx$

Optimal. Leaf size=4

$$\cos\left(\frac{1}{x}\right)$$

```
[Out] Cos[x^(-1)]
```

Rubi [A] time = 0.0082888, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3379, 2638}

$$\cos\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x^(-1)]/x^2,x]
```

```
[Out] Cos[x^(-1)]
```

Rule 3379

```
Int[(x_)^(m_.)*((a_.) + (b_.)*Sin[(c_.) + (d_.)*(x_)^(n_)])^(p_.), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Sin[c + d*x])^p
, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(
m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(
m + 1)/n], 0]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\sin\left(\frac{1}{x}\right)}{x^2} dx &= -\text{Subst}\left(\int \sin(x)\,dx, x, \frac{1}{x}\right) \\ &= \cos\left(\frac{1}{x}\right) \end{aligned}$$

Mathematica [A] time = 0.0075307, size = 4, normalized size = 1.

$$\cos\left(\frac{1}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x^(-1)]/x^2,x]
```

```
[Out] Cos[x^(-1)]
```

Maple [A] time = 0.003, size = 5, normalized size = 1.3

$$\cos\left(x^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(1/x)/x^2,x)
```

```
[Out] cos(1/x)
```

Maxima [A] time = 0.979889, size = 5, normalized size = 1.25

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/x)/x^2,x, algorithm="maxima")
```

```
[Out] cos(1/x)
```

Fricas [A] time = 0.44653, size = 14, normalized size = 3.5

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/x)/x^2,x, algorithm="fricas")
```

```
[Out] cos(1/x)
```

Sympy [A] time = 0.654624, size = 3, normalized size = 0.75

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/x)/x**2,x)
```

```
[Out] cos(1/x)
```

Giac [A] time = 1.08327, size = 5, normalized size = 1.25

$$\cos\left(\frac{1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(1/x)/x^2,x, algorithm="giac")
```

```
[Out] cos(1/x)
```

3.53 $\int \sin\left(\sqrt[4]{-1+x}\right) dx$

Optimal. Leaf size=62

$$12\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right)-24\sin\left(\sqrt[4]{x-1}\right)-4(x-1)^{3/4}\cos\left(\sqrt[4]{x-1}\right)+24\sqrt[4]{x-1}\cos\left(\sqrt[4]{x-1}\right)$$

```
[Out] 24*(-1 + x)^(1/4)*Cos[(-1 + x)^(1/4)] - 4*(-1 + x)^(3/4)*Cos[(-1 + x)^(1/4)
] - 24*Sin[(-1 + x)^(1/4)] + 12*Sqrt[-1 + x]*Sin[(-1 + x)^(1/4)]
```

Rubi [A] time = 0.0426793, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3361, 3296, 2637}

$$12\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right)-24\sin\left(\sqrt[4]{x-1}\right)-4(x-1)^{3/4}\cos\left(\sqrt[4]{x-1}\right)+24\sqrt[4]{x-1}\cos\left(\sqrt[4]{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[(-1 + x)^(1/4)],x]
```

```
[Out] 24*(-1 + x)^(1/4)*Cos[(-1 + x)^(1/4)] - 4*(-1 + x)^(3/4)*Cos[(-1 + x)^(1/4)
] - 24*Sin[(-1 + x)^(1/4)] + 12*Sqrt[-1 + x]*Sin[(-1 + x)^(1/4)]
```

Rule 3361

```
Int[((a_.) + (b_.)*Sin[(c_.) + (d_.)*((e_.) + (f_.)*(x_))^(n_)])^(p_.), x_S
ymbol] :> Dist[1/(n*f), Subst[Int[x^(1/n - 1)*(a + b*Sin[c + d*x])^p, x], x
, (e + f*x)^n], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && Integer
Q[1/n]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sin\left(\sqrt[4]{-1+x}\right) dx &= 4\,\text{Subst}\left(\int x^3\sin(x)\,dx, x, \sqrt[4]{-1+x}\right)\\
&= -4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right)+12\,\text{Subst}\left(\int x^2\cos(x)\,dx, x, \sqrt[4]{-1+x}\right)\\
&= -4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right)+12\sqrt{-1+x}\sin\left(\sqrt[4]{-1+x}\right)-24\,\text{Subst}\left(\int x\sin(x)\,dx, x, \sqrt[4]{-1+x}\right)\\
&= 24\sqrt[4]{-1+x}\cos\left(\sqrt[4]{-1+x}\right)-4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right)+12\sqrt{-1+x}\sin\left(\sqrt[4]{-1+x}\right)-24\,\text{Subst}\left(\right.\\
&= 24\sqrt[4]{-1+x}\cos\left(\sqrt[4]{-1+x}\right)-4(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right)-24\sin\left(\sqrt[4]{-1+x}\right)+12\sqrt{-1+x}\sin\left(\sqrt[4]{-\right.
\end{aligned}$$

Mathematica [A] time = 0.0286381, size = 46, normalized size = 0.74

$$12\left(\sqrt{x-1}-2\right)\sin\left(\sqrt[4]{x-1}\right)-4\left(\sqrt{x-1}-6\right)\sqrt[4]{x-1}\cos\left(\sqrt[4]{x-1}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[(-1 + x)^(1/4)],x]
```

```
[Out] -4*(-6 + Sqrt[-1 + x])*(-1 + x)^(1/4)*Cos[(-1 + x)^(1/4)] + 12*(-2 + Sqrt[-
1 + x])*Sin[(-1 + x)^(1/4)]
```

Maple [A] time = 0.005, size = 49, normalized size = 0.8

$$24\sqrt[4]{-1+x}\cos\left(\sqrt[4]{-1+x}\right)-4\,(-1+x)^{3/4}\cos\left(\sqrt[4]{-1+x}\right)-24\,\sin\left(\sqrt[4]{-1+x}\right)+12\,\sin\left(\sqrt[4]{-1+x}\right)\sqrt{-1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin((-1+x)^(1/4)),x)
```

```
[Out] 24*(-1+x)^(1/4)*cos((-1+x)^(1/4))-4*(-1+x)^(3/4)*cos((-1+x)^(1/4))-24*sin((
-1+x)^(1/4))+12*sin((-1+x)^(1/4))*(-1+x)^(1/2)
```

Maxima [A] time = 0.968972, size = 50, normalized size = 0.81

$$-4\left((x-1)^{\frac{3}{4}}-6\,(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right)+12\left(\sqrt{x-1}-2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-1+x)^(1/4)),x, algorithm="maxima")
```

```
[Out] -4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqrt(x - 1) -
 2)*sin((x - 1)^(1/4))
```

Fricas [A] time = 0.512283, size = 134, normalized size = 2.16

$$-4\left((x-1)^{\frac{3}{4}}-6\,(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right)+12\left(\sqrt{x-1}-2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-1+x)^(1/4)),x, algorithm="fricas")
```

```
[Out] -4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqrt(x - 1) -
 2)*sin((x - 1)^(1/4))
```

Sympy [A] time = 2.36792, size = 60, normalized size = 0.97

$$-4\,(x-1)^{\frac{3}{4}}\cos\left(\sqrt[4]{x-1}\right)+24\sqrt[4]{x-1}\cos\left(\sqrt[4]{x-1}\right)+12\sqrt{x-1}\sin\left(\sqrt[4]{x-1}\right)-24\sin\left(\sqrt[4]{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-1+x)**(1/4)),x)
```

```
[Out] -4*(x - 1)**(3/4)*cos((x - 1)**(1/4)) + 24*(x - 1)**(1/4)*cos((x - 1)**(1/4
)) + 12*sqrt(x - 1)*sin((x - 1)**(1/4)) - 24*sin((x - 1)**(1/4))
```

Giac [A] time = 1.10905, size = 50, normalized size = 0.81

$$-4\left((x-1)^{\frac{3}{4}}-6\,(x-1)^{\frac{1}{4}}\right)\cos\left((x-1)^{\frac{1}{4}}\right)+12\left(\sqrt{x-1}-2\right)\sin\left((x-1)^{\frac{1}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin((-1+x)^(1/4)),x, algorithm="giac")
```

```
[Out] -4*((x - 1)^(3/4) - 6*(x - 1)^(1/4))*cos((x - 1)^(1/4)) + 12*(sqrt(x - 1) -
 2)*sin((x - 1)^(1/4))
```

3.54 $\int x \cos(x^2) \sin(x^2) \, dx$

Optimal. Leaf size=10

$$\frac{1}{4} \sin^2(x^2)$$

```
[Out] Sin[x^2]^2/4
```

Rubi [A] time = 0.0071152, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {3441}

$$\frac{1}{4} \sin^2(x^2)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x^2]*Sin[x^2],x]
```

```
[Out] Sin[x^2]^2/4
```

Rule 3441

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[Sin[a + b*x^n]^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[
{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rubi steps

$$\int x \cos(x^2) \sin(x^2) \, dx = \frac{1}{4} \sin^2(x^2)$$

Mathematica [A] time = 0.0024186, size = 10, normalized size = 1.

$$-\frac{1}{4} \cos^2(x^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x^2]*Sin[x^2],x]
```

```
[Out] -Cos[x^2]^2/4
```

Maple [A] time = 0.003, size = 9, normalized size = 0.9

$$-\frac{(\cos(x^2))^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x^2)*sin(x^2),x)
```

```
[Out] -1/4*cos(x^2)^2
```

Maxima [A] time = 0.959578, size = 11, normalized size = 1.1

$$-\frac{1}{4}\cos\left(x^2\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)*sin(x^2),x, algorithm="maxima")
```

```
[Out] -1/4*cos(x^2)^2
```

Fricas [A] time = 0.517447, size = 23, normalized size = 2.3

$$-\frac{1}{4}\cos\left(x^2\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)*sin(x^2),x, algorithm="fricas")
```

```
[Out] -1/4*cos(x^2)^2
```

Sympy [A] time = 0.301366, size = 7, normalized size = 0.7

$$\frac{\sin^2\left(x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x**2)*sin(x**2),x)
```

```
[Out] sin(x**2)**2/4
```

Giac [A] time = 1.10867, size = 11, normalized size = 1.1

$$-\frac{1}{4}\cos\left(x^2\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x^2)*sin(x^2),x, algorithm="giac")
```

```
[Out] -1/4*cos(x^2)^2
```

3.55 $\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx$

Optimal. Leaf size=16

$$-\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}$$

```
[Out] (-2*(4 - 3*Sin[x]^2)^(3/2))/9
```

Rubi [A] time = 0.0305212, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {12, 261}

$$-\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 3*Cos[x]^2]*Sin[2*x],x]
```

```
[Out] (-2*(4 - 3*Sin[x]^2)^(3/2))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+3\cos^2(x)}\sin(2x)\,dx &= \text{Subst}\left(\int 2x\sqrt{4-3x^2}\,dx, x, \sin(x)\right)\\ &= 2\,\text{Subst}\left(\int x\sqrt{4-3x^2}\,dx, x, \sin(x)\right)\\ &= -\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}\end{aligned}$$

Mathematica [A] time = 0.0111561, size = 16, normalized size = 1.

$$-\frac{2}{9}\left(4-3\sin^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 3*Cos[x]^2]*Sin[2*x],x]
```

```
[Out] (-2*(4 - 3*Sin[x]^2)^(3/2))/9
```

Maple [A] time = 0.018, size = 13, normalized size = 0.8

$$-\frac{2}{9}\left(1+3\left(\cos\left(x\right)\right)^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(2*x)*(1+3*cos(x)^2)^(1/2),x)
```

```
[Out] -2/9*(1+3*cos(x)^2)^(3/2)
```

Maxima [A] time = 0.949571, size = 16, normalized size = 1.

$$-\frac{2}{9}\left(3\cos\left(x\right)^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*(1+3*cos(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*(3*cos(x)^2 + 1)^(3/2)
```

Fricas [A] time = 0.5205, size = 39, normalized size = 2.44

$$-\frac{2}{9}\left(3\cos\left(x\right)^{2}+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*(1+3*cos(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/9*(3*cos(x)^2 + 1)^(3/2)
```

Sympy [A] time = 2.32179, size = 15, normalized size = 0.94

$$-\frac{2\left(3\cos^{2}\left(x\right)+1\right)^{\frac{3}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*(1+3*cos(x)**2)**(1/2),x)
```

```
[Out] -2*(3*cos(x)**2 + 1)**(3/2)/9
```

Giac [B] time = 1.11583, size = 248, normalized size = 15.5

$$-\frac{16\left(\left(\tan\left(\frac{1}{2}x\right)^{2}-\sqrt{\tan\left(\frac{1}{2}x\right)^{4}-\tan\left(\frac{1}{2}x\right)^{2}+1}\right)^{5}-\left(\tan\left(\frac{1}{2}x\right)^{2}-\sqrt{\tan\left(\frac{1}{2}x\right)^{4}-\tan\left(\frac{1}{2}x\right)^{2}+1}\right)^{3}-2\left(\tan\left(\frac{1}{2}x\right)^{2}-\cdots\right.\right.}{\left(\left(\tan\left(\frac{1}{2}x\right)^{2}-\sqrt{\tan\left(\frac{1}{2}x\right)^{4}-\tan\left(\frac{1}{2}x\right)^{2}+1}\right)^{2}+2\tan\left(\frac{1}{2}x\right)^{2}-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(2*x)*(1+3*cos(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -16*((tan(1/2*x)^2 - sqrt(tan(1/2*x)^4 - tan(1/2*x)^2 + 1))^5 - (tan(1/2*x)
^2 - sqrt(tan(1/2*x)^4 - tan(1/2*x)^2 + 1))^3 - 2*(tan(1/2*x)^2 - sqrt(tan(
1/2*x)^4 - tan(1/2*x)^2 + 1))^2 + 3*tan(1/2*x)^2 - 3*sqrt(tan(1/2*x)^4 - ta
n(1/2*x)^2 + 1) - 1)/((tan(1/2*x)^2 - sqrt(tan(1/2*x)^4 - tan(1/2*x)^2 + 1)
)^2 + 2*tan(1/2*x)^2 - 2*sqrt(tan(1/2*x)^4 - tan(1/2*x)^2 + 1) - 2)^3
```

3.56 $\int \frac{1}{2+3x} \, dx$

Optimal. Leaf size=10

$$\frac{1}{3}\log(3x+2)$$

```
[Out] Log[2 + 3*x]/3
```

Rubi [A] time = 0.0008861, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {31}

$$\frac{1}{3}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{2+3x} \, dx = \frac{1}{3}\log(2+3x)$$

Mathematica [A] time = 0.00067, size = 10, normalized size = 1.

$$\frac{1}{3}\log(3x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x)^(-1),x]
```

```
[Out] Log[2 + 3*x]/3
```

Maple [A] time = 0.002, size = 9, normalized size = 0.9

$$\frac{\ln(2+3x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2+3*x),x)
```

```
[Out] 1/3*ln(2+3*x)
```

Maxima [A] time = 0.977641, size = 11, normalized size = 1.1

$$\frac{1}{3}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="maxima")
```

```
[Out] 1/3*log(3*x + 2)
```

Fricas [A] time = 0.464186, size = 24, normalized size = 2.4

$$\frac{1}{3}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="fricas")
```

```
[Out] 1/3*log(3*x + 2)
```

Sympy [A] time = 0.051624, size = 7, normalized size = 0.7

$$\frac{\log(3x+2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x)
```

```
[Out] log(3*x + 2)/3
```

Giac [A] time = 1.07932, size = 12, normalized size = 1.2

$$\frac{1}{3}\log(|3x+2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2+3*x),x, algorithm="giac")
```

```
[Out] 1/3*log(abs(3*x + 2))
```

3.57 $\int \log^2(x)\,dx$

Optimal. Leaf size=15

$$2x + x\log^2(x) - 2x\log(x)$$

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Rubi [A] time = 0.0035862, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2296, 2295}

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]^2,x]
```

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Rule 2296

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Dist[b*n*p, Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 2295

```
Int[Log[(c_.)*(x_)^(n_.)], x_Symbol] :> Simp[x*Log[c*x^n], x] - Simp[n*x, x
] /; FreeQ[{c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \log^2(x)\,dx &= x\log^2(x) - 2\int \log(x)\,dx\\ &= 2x - 2x\log(x) + x\log^2(x)\end{aligned}$$

Mathematica [A] time = 0.0007935, size = 15, normalized size = 1.

$$2x + x\log^2(x) - 2x\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]^2,x]
```

```
[Out] 2*x - 2*x*Log[x] + x*Log[x]^2
```

Maple [A] time = 0.003, size = 16, normalized size = 1.1

$$2\,x - 2\,x\ln(x) + x(\ln(x))^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)^2,x)
```

```
[Out] 2*x-2*x*ln(x)+x*ln(x)^2
```

Maxima [A] time = 0.952412, size = 16, normalized size = 1.07

$$(\log(x)^2 - 2\log(x) + 2)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="maxima")
```

```
[Out] (log(x)^2 - 2*log(x) + 2)*x
```

Fricas [A] time = 0.507845, size = 42, normalized size = 2.8

$$x\log(x)^2 - 2\,x\log(x) + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="fricas")
```

```
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

Sympy [A] time = 0.088745, size = 15, normalized size = 1.

$$x\log(x)^2 - 2x\log(x) + 2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)**2,x)
```

```
[Out] x*log(x)**2 - 2*x*log(x) + 2*x
```

Giac [A] time = 1.06839, size = 20, normalized size = 1.33

$$x\log(x)^2 - 2\,x\log(x) + 2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)^2,x, algorithm="giac")
```

```
[Out] x*log(x)^2 - 2*x*log(x) + 2*x
```

3.58 $\int x \log(x)\, dx$

Optimal. Leaf size=17

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

```
[Out] -x^2/4 + (x^2*Log[x])/2
```

Rubi [A] time = 0.003479, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2304}

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x],x]

[Out] -x^2/4 + (x^2*Log[x])/2
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x \log(x)\, dx = -\frac{x^2}{4} + \frac{1}{2}x^2 \log(x)$$

Mathematica [A] time = 0.0005487, size = 17, normalized size = 1.

$$\frac{1}{2}x^2 \log(x) - \frac{x^2}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x],x]

[Out] -x^2/4 + (x^2*Log[x])/2
```

Maple [A] time = 0.002, size = 14, normalized size = 0.8

$$-\frac{x^2}{4} + \frac{x^2 \ln(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x),x)
```

```
[Out] -1/4*x^2+1/2*x^2*ln(x)
```

Maxima [A] time = 0.953067, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Fricas [A] time = 0.511466, size = 35, normalized size = 2.06

$$\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

Sympy [A] time = 0.085044, size = 12, normalized size = 0.71

$$\frac{x^2\log\left(x\right)}{2}-\frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x),x)
```

```
[Out] x**2*log(x)/2 - x**2/4
```

Giac [A] time = 1.08706, size = 18, normalized size = 1.06

$$\frac{1}{2}\,x^2\log\left(x\right)-\frac{1}{4}\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x) - 1/4*x^2
```

3.59 $\int x \log^2(x)\, dx$

Optimal. Leaf size=28

$$\frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rubi [A] time = 0.0093492, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {2305, 2304}

$$\frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int x \log^2(x)\, dx &= \frac{1}{2}x^2 \log^2(x) - \int x \log(x)\, dx \\
&= \frac{x^2}{4} - \frac{1}{2}x^2 \log(x) + \frac{1}{2}x^2 \log^2(x)
\end{aligned}$$

Mathematica [A] time = 0.0008118, size = 28, normalized size = 1.

$$\frac{x^2}{4} + \frac{1}{2}x^2 \log^2(x) - \frac{1}{2}x^2 \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Log[x]^2,x]
```

```
[Out] x^2/4 - (x^2*Log[x])/2 + (x^2*Log[x]^2)/2
```

Maple [A] time = 0., size = 23, normalized size = 0.8

$$\frac{x^2}{4} - \frac{x^2 \ln(x)}{2} + \frac{x^2 (\ln(x))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*ln(x)^2,x)
```

```
[Out] 1/4*x^2-1/2*x^2*ln(x)+1/2*x^2*ln(x)^2
```

Maxima [A] time = 0.961432, size = 23, normalized size = 0.82

$$\frac{1}{4}\left(2 \log(x)^2 - 2 \log(x) + 1\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*log(x)^2 - 2*log(x) + 1)*x^2
```

Fricas [A] time = 0.537681, size = 61, normalized size = 2.18

$$\frac{1}{2}x^2 \log(x)^2 - \frac{1}{2}x^2 \log(x) + \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

Sympy [A] time = 0.09522, size = 22, normalized size = 0.79

$$\frac{x^2 \log(x)^2}{2} - \frac{x^2 \log(x)}{2} + \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*ln(x)**2,x)
```

```
[Out] x**2*log(x)**2/2 - x**2*log(x)/2 + x**2/4
```

Giac [A] time = 1.08971, size = 30, normalized size = 1.07

$$\frac{1}{2}x^2 \log(x)^2 - \frac{1}{2}x^2 \log(x) + \frac{1}{4}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*log(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2*log(x)^2 - 1/2*x^2*log(x) + 1/4*x^2
```

3.60 $\int \frac{1}{1+t} \, dt$

Optimal. Leaf size=4

$$\log(t+1)$$

```
[Out] Log[1 + t]
```

Rubi [A] time = 0.0004936, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {31}

$$\log(t+1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + t)^(-1),t]
```

```
[Out] Log[1 + t]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{1+t} \, dt = \log(1+t)$$

Mathematica [A] time = 0.0005086, size = 4, normalized size = 1.

$$\log(t+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + t)^(-1),t]
```

```
[Out] Log[1 + t]
```

Maple [A] time = 0., size = 5, normalized size = 1.3

$$\ln(1+t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+t),t)
```

```
[Out] ln(1+t)
```

Maxima [A] time = 0.9472, size = 5, normalized size = 1.25

$$\log(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+t),t, algorithm="maxima")
```

```
[Out] log(t + 1)
```

Fricas [A] time = 0.479768, size = 16, normalized size = 4.

$$\log(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+t),t, algorithm="fricas")
```

```
[Out] log(t + 1)
```

Sympy [A] time = 0.054785, size = 3, normalized size = 0.75

$$\log(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+t),t)
```

```
[Out] log(t + 1)
```

Giac [A] time = 1.10235, size = 7, normalized size = 1.75

$$\log(|t+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+t),t, algorithm="giac")
```

```
[Out] log(abs(t + 1))
```

3.61 $\int \cot(x)\,dx$

Optimal. Leaf size=3

$$\log(\sin(x))$$

```
[Out] Log[Sin[x]]
```

Rubi [A] time = 0.0025934, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3475}

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cot(x)\,dx = \log(\sin(x))$$

Mathematica [A] time = 0.0017423, size = 3, normalized size = 1.

$$\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x],x]
```

```
[Out] Log[Sin[x]]
```

Maple [A] time = 0.001, size = 4, normalized size = 1.3

$$\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x),x)
```

```
[Out] ln(sin(x))
```

Maxima [A] time = 0.952364, size = 4, normalized size = 1.33

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="maxima")
```

```
[Out] log(sin(x))
```

Fricas [B] time = 0.50159, size = 41, normalized size = 13.67

$$\frac{1}{2}\log\left(-\frac{1}{2}\cos(2x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="fricas")
```

```
[Out] 1/2*log(-1/2*cos(2*x) + 1/2)
```

Sympy [A] time = 0.059941, size = 3, normalized size = 1.

$$\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x)
```

```
[Out] log(sin(x))
```

Giac [B] time = 1.09009, size = 15, normalized size = 5.

$$\frac{1}{2}\log\left(-\cos(x)^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x),x, algorithm="giac")
```

```
[Out] 1/2*log(-cos(x)^2 + 1)
```

3.62 $\int x^n \log(ax)\,dx$

Optimal. Leaf size=28

$$\frac{x^{n+1}\log(ax)}{n+1}-\frac{x^{n+1}}{(n+1)^2}$$

```
[Out] -(x^(1 + n)/(1 + n)^2) + (x^(1 + n)*Log[a*x])/(1 + n)
```

Rubi [A] time = 0.0098877, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2304}

$$\frac{x^{n+1}\log(ax)}{n+1}-\frac{x^{n+1}}{(n+1)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^n*Log[a*x],x]
```

```
[Out] -(x^(1 + n)/(1 + n)^2) + (x^(1 + n)*Log[a*x])/(1 + n)
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^n \log(ax)\,dx=-\frac{x^{1+n}}{(1+n)^2}+\frac{x^{1+n}\log(ax)}{1+n}$$

Mathematica [A] time = 0.0052956, size = 21, normalized size = 0.75

$$\frac{x^{n+1}((n+1)\log(ax)-1)}{(n+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^n*Log[a*x],x]
```

```
[Out] (x^(1 + n)*(-1 + (1 + n)*Log[a*x]))/(1 + n)^2
```

Maple [A] time = 0.013, size = 36, normalized size = 1.3

$$\frac{x\ln(ax)\,e^{n\ln(x)}}{1+n}-\frac{xe^{n\ln(x)}}{n^2+2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^n*ln(a*x),x)
```

```
[Out] 1/(1+n)*x*ln(a*x)*exp(n*ln(x))-1/(n^2+2*n+1)*x*exp(n*ln(x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*log(a*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.519222, size = 85, normalized size = 3.04

$$\frac{((n+1)x\log(a) + (n+1)x\log(x) - x)x^n}{n^2 + 2n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*log(a*x),x, algorithm="fricas")
```

```
[Out] ((n + 1)*x*log(a) + (n + 1)*x*log(x) - x)*x^n/(n^2 + 2*n + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**n*ln(a*x),x)
```

```
[Out] Exception raised: TypeError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^n \log(ax)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^n*log(a*x),x, algorithm="giac")
```

```
[Out] integrate(x^n*log(a*x), x)
```

3.63 $\int x^2 \log^2(x)\,dx$

Optimal. Leaf size=28

$$\frac{2x^3}{27}+\frac{1}{3}x^3\log^2(x)-\frac{2}{9}x^3\log(x)$$

```
[Out] (2*x^3)/27 - (2*x^3*Log[x])/9 + (x^3*Log[x]^2)/3
```

Rubi [A] time = 0.0173121, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2305, 2304}

$$\frac{2x^3}{27}+\frac{1}{3}x^3\log^2(x)-\frac{2}{9}x^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Log[x]^2,x]

[Out] (2*x^3)/27 - (2*x^3*Log[x])/9 + (x^3*Log[x]^2)/3
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^2\log^2(x)\,dx &= \frac{1}{3}x^3\log^2(x)-\frac{2}{3}\int x^2\log(x)\,dx\\ &= \frac{2x^3}{27}-\frac{2}{9}x^3\log(x)+\frac{1}{3}x^3\log^2(x)\end{aligned}$$

Mathematica [A] time = 0.001087, size = 28, normalized size = 1.

$$\frac{2x^3}{27}+\frac{1}{3}x^3\log^2(x)-\frac{2}{9}x^3\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Log[x]^2,x]

[Out] (2*x^3)/27 - (2*x^3*Log[x])/9 + (x^3*Log[x]^2)/3
```

Maple [A] time = 0.002, size = 23, normalized size = 0.8

$$\frac{2x^3}{27} - \frac{2x^3\ln(x)}{9} + \frac{x^3(\ln(x))^2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*ln(x)^2,x)
```

```
[Out] 2/27*x^3-2/9*x^3*ln(x)+1/3*x^3*ln(x)^2
```

Maxima [A] time = 0.946643, size = 23, normalized size = 0.82

$$\frac{1}{27}\left(9\log(x)^2 - 6\log(x) + 2\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x)^2,x, algorithm="maxima")
```

```
[Out] 1/27*(9*log(x)^2 - 6*log(x) + 2)*x^3
```

Fricas [A] time = 0.53429, size = 62, normalized size = 2.21

$$\frac{1}{3}x^3\log(x)^2 - \frac{2}{9}x^3\log(x) + \frac{2}{27}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x)^2,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*log(x)^2 - 2/9*x^3*log(x) + 2/27*x^3
```

Sympy [A] time = 0.097307, size = 26, normalized size = 0.93

$$\frac{x^3\log(x)^2}{3} - \frac{2x^3\log(x)}{9} + \frac{2x^3}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*ln(x)**2,x)
```

```
[Out] x**3*log(x)**2/3 - 2*x**3*log(x)/9 + 2*x**3/27
```

Giac [A] time = 1.08727, size = 30, normalized size = 1.07

$$\frac{1}{3}x^3\log(x)^2 - \frac{2}{9}x^3\log(x) + \frac{2}{27}x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*log(x)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3*log(x)^2 - 2/9*x^3*log(x) + 2/27*x^3
```

3.64 $\int \frac{1}{x\log(x)}\,dx$

Optimal. Leaf size=3

$$\log(\log(x))$$

```
[Out] Log[Log[x]]
```

Rubi [A] time = 0.0118875, antiderivative size = 3, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2302, 29}

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Log[x]),x]

[Out] Log[Log[x]]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
 x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\log(x)}\,dx &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, \log(x)\right)\\ &= \log(\log(x))\end{aligned}$$

Mathematica [A] time = 0.003848, size = 3, normalized size = 1.

$$\log(\log(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*Log[x]),x]

[Out] Log[Log[x]]
```

Maple [A] time = 0., size = 4, normalized size = 1.3

$$\ln(\ln(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/ln(x),x)
```

```
[Out] ln(ln(x))
```

Maxima [A] time = 1.01406, size = 4, normalized size = 1.33

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="maxima")
```

```
[Out] log(log(x))
```

Fricas [A] time = 0.493761, size = 18, normalized size = 6.

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="fricas")
```

```
[Out] log(log(x))
```

Sympy [A] time = 0.086848, size = 3, normalized size = 1.

$$\log(\log(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/ln(x),x)
```

```
[Out] log(log(x))
```

Giac [A] time = 1.10441, size = 5, normalized size = 1.67

$$\log(|\log(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/log(x),x, algorithm="giac")
```

```
[Out] log(abs(log(x)))
```

3.65 $\int \frac{\log(1-t)}{1-t}\,dt$

Optimal. Leaf size=12

$$-\frac{1}{2}\log^2(1-t)$$

```
[Out] -Log[1 - t]^2/2
```

Rubi [A] time = 0.0154553, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2390, 2301}

$$-\frac{1}{2}\log^2(1-t)$$

Antiderivative was successfully verified.

```
[In] Int[Log[1 - t]/(1 - t),t]
```

```
[Out] -Log[1 - t]^2/2
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(1-t)}{1-t}\,dt &= -\text{Subst}\left(\int \frac{\log(t)}{t}\,dt, t, 1-t\right)\\ &= -\frac{1}{2}\log^2(1-t)\end{aligned}$$

Mathematica [A] time = 0.0017971, size = 12, normalized size = 1.

$$-\frac{1}{2}\log^2(1-t)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[1 - t]/(1 - t),t]
```

```
[Out] -Log[1 - t]^2/2
```

Maple [A] time = 0.002, size = 11, normalized size = 0.9

$$-\frac{(\ln(1-t))^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(1-t)/(1-t),t)
```

```
[Out] -1/2*ln(1-t)^2
```

Maxima [A] time = 0.961706, size = 14, normalized size = 1.17

$$-\frac{1}{2}\log(-t+1)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-t)/(1-t),t, algorithm="maxima")
```

```
[Out] -1/2*log(-t + 1)^2
```

Fricas [A] time = 0.516226, size = 27, normalized size = 2.25

$$-\frac{1}{2}\log(-t+1)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-t)/(1-t),t, algorithm="fricas")
```

```
[Out] -1/2*log(-t + 1)^2
```

Sympy [A] time = 0.084218, size = 8, normalized size = 0.67

$$-\frac{\log(1-t)^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(1-t)/(1-t),t)
```

```
[Out] -log(1 - t)**2/2
```

Giac [A] time = 1.06302, size = 14, normalized size = 1.17

$$-\frac{1}{2}\log(-t+1)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-t)/(1-t),t, algorithm="giac")

[Out] -1/2*log(-t + 1)^2
```

3.66 $\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx$

Optimal. Leaf size=23

$$\frac{2}{3}(\log(x)+1)^{3/2}-2\sqrt{\log(x)+1}$$

```
[Out] -2*Sqrt[1 + Log[x]] + (2*(1 + Log[x])^(3/2))/3
```

Rubi [A] time = 0.0383046, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2365, 43}

$$\frac{2}{3}(\log(x)+1)^{3/2}-2\sqrt{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Log[x]/(x*Sqrt[1 + Log[x]]),x]
```

```
[Out] -2*Sqrt[1 + Log[x]] + (2*(1 + Log[x])^(3/2))/3
```

Rule 2365

```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.) + Log[(c_.)*(x_)^(n
_.)]*(e_.))^(q_.))/(x_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(d +
e*x)^q, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\log(x)}{x\sqrt{1+\log(x)}}\,dx &= \text{Subst}\left(\int \frac{x}{\sqrt{1+x}}\,dx, x, \log(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{1}{\sqrt{1+x}}+\sqrt{1+x}\right)dx, x, \log(x)\right)\\
&= -2\sqrt{1+\log(x)}+\frac{2}{3}(1+\log(x))^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.0147389, size = 16, normalized size = 0.7

$$\frac{2}{3}(\log(x)-2)\sqrt{\log(x)+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[x]/(x*Sqrt[1 + Log[x]]),x]
```

```
[Out] (2*(-2 + Log[x])*Sqrt[1 + Log[x]])/3
```

Maple [A] time = 0.005, size = 18, normalized size = 0.8

$$\frac{2}{3}(1+\ln(x))^{\frac{3}{2}} - 2\sqrt{1+\ln(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(x)/x/(1+ln(x))^(1/2),x)
```

```
[Out] 2/3*(1+ln(x))^(3/2)-2*(1+ln(x))^(1/2)
```

Maxima [A] time = 0.939542, size = 23, normalized size = 1.

$$\frac{2}{3}(\log(x)+1)^{\frac{3}{2}} - 2\sqrt{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

Fricas [A] time = 0.479247, size = 47, normalized size = 2.04

$$\frac{2}{3}\sqrt{\log(x)+1}(\log(x)-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(log(x) + 1)*(log(x) - 2)
```

Sympy [A] time = 3.8864, size = 20, normalized size = 0.87

$$\frac{2(\log(x)+1)^{\frac{3}{2}}}{3} - 2\sqrt{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(x)/x/(1+ln(x))**(1/2),x)
```

```
[Out] 2*(log(x) + 1)**(3/2)/3 - 2*sqrt(log(x) + 1)
```

Giac [A] time = 1.10739, size = 23, normalized size = 1.

$$\frac{2}{3}(\log(x)+1)^{\frac{3}{2}} - 2\sqrt{\log(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(x)/x/(1+log(x))^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*(log(x) + 1)^(3/2) - 2*sqrt(log(x) + 1)
```

3.67 $\int x^3 \log^3(x)\,dx$

Optimal. Leaf size=39

$$-\frac{3x^4}{128}+\frac{1}{4}x^4\log^3(x)-\frac{3}{16}x^4\log^2(x)+\frac{3}{32}x^4\log(x)$$

```
[Out] (-3*x^4)/128 + (3*x^4*Log[x])/32 - (3*x^4*Log[x]^2)/16 + (x^4*Log[x]^3)/4
```

Rubi [A] time = 0.0303477, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2305, 2304}

$$-\frac{3x^4}{128}+\frac{1}{4}x^4\log^3(x)-\frac{3}{16}x^4\log^2(x)+\frac{3}{32}x^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Log[x]^3,x]
```

```
[Out] (-3*x^4)/128 + (3*x^4*Log[x])/32 - (3*x^4*Log[x]^2)/16 + (x^4*Log[x]^3)/4
```

Rule 2305

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n])^p)/(d*(m + 1)), x] - Dist[(b*n
*p)/(m + 1), Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b,
 c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

Rule 2304

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[((d*x)^(m + 1)*(a + b*Log[c*x^n]))/(d*(m + 1)), x] - Simp[(b*n*(d*x)^(
m + 1))/(d*(m + 1)^2), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x^3\log^3(x)\,dx &= \frac{1}{4}x^4\log^3(x)-\frac{3}{4}\int x^3\log^2(x)\,dx\\ &= -\frac{3}{16}x^4\log^2(x)+\frac{1}{4}x^4\log^3(x)+\frac{3}{8}\int x^3\log(x)\,dx\\ &= -\frac{3x^4}{128}+\frac{3}{32}x^4\log(x)-\frac{3}{16}x^4\log^2(x)+\frac{1}{4}x^4\log^3(x)\end{aligned}$$

Mathematica [A] time = 0.0012574, size = 39, normalized size = 1.

$$-\frac{3x^4}{128}+\frac{1}{4}x^4\log^3(x)-\frac{3}{16}x^4\log^2(x)+\frac{3}{32}x^4\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Log[x]^3,x]
```

```
[Out] (-3*x^4)/128 + (3*x^4*Log[x])/32 - (3*x^4*Log[x]^2)/16 + (x^4*Log[x]^3)/4
```

Maple [A] time = 0.002, size = 32, normalized size = 0.8

$$-\frac{3x^4}{128}+\frac{3x^4\ln(x)}{32}-\frac{3x^4(\ln(x))^2}{16}+\frac{x^4(\ln(x))^3}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*ln(x)^3,x)
```

```
[Out] -3/128*x^4+3/32*x^4*ln(x)-3/16*x^4*ln(x)^2+1/4*x^4*ln(x)^3
```

Maxima [A] time = 0.926354, size = 31, normalized size = 0.79

$$\frac{1}{128}\left(32\log(x)^3-24\log(x)^2+12\log(x)-3\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(x)^3,x, algorithm="maxima")
```

```
[Out] 1/128*(32*log(x)^3 - 24*log(x)^2 + 12*log(x) - 3)*x^4
```

Fricas [A] time = 0.501457, size = 92, normalized size = 2.36

$$\frac{1}{4}x^4\log(x)^3-\frac{3}{16}x^4\log(x)^2+\frac{3}{32}x^4\log(x)-\frac{3}{128}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*x^4*log(x)^3 - 3/16*x^4*log(x)^2 + 3/32*x^4*log(x) - 3/128*x^4
```

Sympy [A] time = 0.125523, size = 37, normalized size = 0.95

$$\frac{x^4\log(x)^3}{4}-\frac{3x^4\log(x)^2}{16}+\frac{3x^4\log(x)}{32}-\frac{3x^4}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*ln(x)**3,x)
```

```
[Out] x**4*log(x)**3/4 - 3*x**4*log(x)**2/16 + 3*x**4*log(x)/32 - 3*x**4/128
```

Giac [A] time = 1.10217, size = 42, normalized size = 1.08

$$\frac{1}{4}x^4\log(x)^3-\frac{3}{16}x^4\log(x)^2+\frac{3}{32}x^4\log(x)-\frac{3}{128}x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*log(x)^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4*log(x)^3 - 3/16*x^4*log(x)^2 + 3/32*x^4*log(x) - 3/128*x^4
```

3.68 $\int e^{x^3} x^2 \, dx$

Optimal. Leaf size=9

$$\frac{e^{x^3}}{3}$$

```
[Out] E^x^3/3
```

Rubi [A] time = 0.0124797, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2209}

$$\frac{e^{x^3}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[E^x^3*x^2,x]
```

```
[Out] E^x^3/3
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int e^{x^3} x^2 \, dx = \frac{e^{x^3}}{3}$$

Mathematica [A] time = 0.0011421, size = 9, normalized size = 1.

$$\frac{e^{x^3}}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x^3*x^2,x]
```

```
[Out] E^x^3/3
```

Maple [A] time = 0.002, size = 7, normalized size = 0.8

$$\frac{e^{x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^3)*x^2,x)
```

```
[Out] 1/3*exp(x^3)
```

Maxima [A] time = 0.952204, size = 8, normalized size = 0.89

$$\frac{1}{3}e^{(x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^3)*x^2,x, algorithm="maxima")
```

```
[Out] 1/3*e^(x^3)
```

Fricas [A] time = 0.490013, size = 18, normalized size = 2.

$$\frac{1}{3}e^{(x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^3)*x^2,x, algorithm="fricas")
```

```
[Out] 1/3*e^(x^3)
```

Sympy [A] time = 0.082264, size = 5, normalized size = 0.56

$$\frac{e^{x^3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**3)*x**2,x)
```

```
[Out] exp(x**3)/3
```

Giac [A] time = 1.1016, size = 8, normalized size = 0.89

$$\frac{1}{3}e^{(x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^3)*x^2,x, algorithm="giac")
```

```
[Out] 1/3*e^(x^3)
```

3.69 $\int \frac{2^{\sqrt{x}}}{\sqrt{x}} \, dx$

Optimal. Leaf size=14

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

```
[Out] 2^(1 + Sqrt[x])/Log[2]
```

Rubi [A] time = 0.0087419, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2209}

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Int[2^Sqrt[x]/Sqrt[x],x]

[Out] 2^(1 + Sqrt[x])/Log[2]
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{2^{\sqrt{x}}}{\sqrt{x}} \, dx = \frac{2^{1+\sqrt{x}}}{\log(2)}$$

Mathematica [A] time = 0.002692, size = 14, normalized size = 1.

$$\frac{2^{\sqrt{x}+1}}{\log(2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[2^Sqrt[x]/Sqrt[x],x]

[Out] 2^(1 + Sqrt[x])/Log[2]
```

Maple [A] time = 0.003, size = 12, normalized size = 0.9

$$2\,\frac{2^{\sqrt{x}}}{\ln(2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(2^(x^(1/2))/x^(1/2),x)
```

```
[Out] 2/ln(2)*2^(x^(1/2))
```

Maxima [A] time = 0.928614, size = 16, normalized size = 1.14

$$\frac{2^{\sqrt{x}+1}}{\log\left(2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

```
[Out] 2^(sqrt(x) + 1)/log(2)
```

Fricas [A] time = 0.511826, size = 27, normalized size = 1.93

$$\frac{2\cdot 2^{\left(\sqrt{x}\right)}}{\log\left(2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

```
[Out] 2*2^sqrt(x)/log(2)
```

Sympy [A] time = 0.141842, size = 10, normalized size = 0.71

$$\frac{2\cdot 2^{\sqrt{x}}}{\log\left(2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2**(x**(1/2))/x**(1/2),x)
```

```
[Out] 2*2**(sqrt(x))/log(2)
```

Giac [A] time = 1.07512, size = 15, normalized size = 1.07

$$\frac{2\cdot 2^{\left(\sqrt{x}\right)}}{\log\left(2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2^(x^(1/2))/x^(1/2),x, algorithm="giac")
```

```
[Out] 2*2^sqrt(x)/log(2)
```

3.70 $\int e^{2\sin(x)}\cos(x)\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}e^{2\sin(x)}$$

```
[Out] E^(2*Sin[x])/2
```

Rubi [A] time = 0.0089844, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4334, 2194}

$$\frac{1}{2}e^{2\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*Sin[x])*Cos[x],x]
```

```
[Out] E^(2*Sin[x])/2
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{2\sin(x)}\cos(x)\,dx &= \text{Subst}\left(\int e^{2x}\,dx, x, \sin(x)\right)\\ &= \frac{1}{2}e^{2\sin(x)}\end{aligned}$$

Mathematica [A] time = 0.0065793, size = 10, normalized size = 1.

$$\frac{1}{2}e^{2\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*Sin[x])*Cos[x],x]
```

```
[Out] E^(2*Sin[x])/2
```

Maple [A] time = 0.005, size = 8, normalized size = 0.8

$$\frac{e^{2\sin(x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*sin(x))*cos(x),x)
```

```
[Out] 1/2*exp(2*sin(x))
```

Maxima [A] time = 0.935649, size = 9, normalized size = 0.9

$$\frac{1}{2}\,e^{(2\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*sin(x))*cos(x),x, algorithm="maxima")
```

```
[Out] 1/2*e^(2*sin(x))
```

Fricas [A] time = 0.510337, size = 24, normalized size = 2.4

$$\frac{1}{2}\,e^{(2\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*sin(x))*cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*e^(2*sin(x))
```

Sympy [A] time = 0.293039, size = 7, normalized size = 0.7

$$\frac{e^{2\sin(x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*sin(x))*cos(x),x)
```

```
[Out] exp(2*sin(x))/2
```

Giac [A] time = 1.06855, size = 9, normalized size = 0.9

$$\frac{1}{2}\,e^{(2\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*sin(x))*cos(x),x, algorithm="giac")

[Out] 1/2*e^(2*sin(x))
```

3.71 $\int e^x \sin(x)\,dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

```
[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rubi [A] time = 0.0069675, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4432}

$$\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Sin[x],x]

[Out] -(E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x \sin(x)\,dx = -\frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)$$

Mathematica [A] time = 0.0122587, size = 14, normalized size = 0.74

$$\frac{1}{2}e^x(\sin(x) - \cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Sin[x],x]

[Out] (E^x*(-Cos[x] + Sin[x]))/2
```

Maple [A] time = 0.003, size = 14, normalized size = 0.7

$$-\frac{e^x \cos(x)}{2} + \frac{e^x \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*sin(x),x)

[Out] -1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Maxima [A] time = 0.953906, size = 15, normalized size = 0.79

$$-\frac{1}{2}(\cos(x) - \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="maxima")

[Out] -1/2*(cos(x) - sin(x))*e^x
```

Fricas [A] time = 0.567027, size = 46, normalized size = 2.42

$$-\frac{1}{2}\cos(x)e^x + \frac{1}{2}e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="fricas")

[Out] -1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A] time = 0.301108, size = 15, normalized size = 0.79

$$\frac{e^x\sin(x)}{2} - \frac{e^x\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x)

[Out] exp(x)*sin(x)/2 - exp(x)*cos(x)/2
```

Giac [A] time = 1.10604, size = 15, normalized size = 0.79

$$-\frac{1}{2}(\cos(x) - \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*sin(x),x, algorithm="giac")

[Out] -1/2*(cos(x) - sin(x))*e^x
```

3.72 $\int e^x \cos(x)\, dx$

Optimal. Leaf size=19

$$\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x \cos(x)$$

```
[Out] (E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rubi [A] time = 0.0070147, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4433}

$$\frac{1}{2}e^x \sin(x) + \frac{1}{2}e^x \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[E^x*Cos[x],x]
```

```
[Out] (E^x*Cos[x])/2 + (E^x*Sin[x])/2
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^x \cos(x)\, dx = \frac{1}{2}e^x \cos(x) + \frac{1}{2}e^x \sin(x)$$

Mathematica [A] time = 0.0081464, size = 12, normalized size = 0.63

$$\frac{1}{2}e^x(\sin(x) + \cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*Cos[x],x]
```

```
[Out] (E^x*(Cos[x] + Sin[x]))/2
```

Maple [A] time = 0.005, size = 14, normalized size = 0.7

$$\frac{e^x \cos(x)}{2} + \frac{e^x \sin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*cos(x),x)
```

```
[Out] 1/2*exp(x)*cos(x)+1/2*exp(x)*sin(x)
```

Maxima [A] time = 1.0471, size = 12, normalized size = 0.63

$$\frac{1}{2}(\cos(x) + \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x),x, algorithm="maxima")
```

```
[Out] 1/2*(cos(x) + sin(x))*e^x
```

Fricas [A] time = 0.496677, size = 45, normalized size = 2.37

$$\frac{1}{2}\cos(x)e^x + \frac{1}{2}e^x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x),x, algorithm="fricas")
```

```
[Out] 1/2*cos(x)*e^x + 1/2*e^x*sin(x)
```

Sympy [A] time = 0.285548, size = 15, normalized size = 0.79

$$\frac{e^x\sin(x)}{2} + \frac{e^x\cos(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x),x)
```

```
[Out] exp(x)*sin(x)/2 + exp(x)*cos(x)/2
```

Giac [A] time = 1.09889, size = 12, normalized size = 0.63

$$\frac{1}{2}(\cos(x) + \sin(x))e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*cos(x),x, algorithm="giac")
```

```
[Out] 1/2*(cos(x) + sin(x))*e^x
```

3.73 $\int \frac{1}{1+e^x} dx$

Optimal. Leaf size=10

$$x - \log(e^x + 1)$$

```
[Out] x - Log[1 + E^x]
```

Rubi [A] time = 0.0061695, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {2282, 36, 29, 31}

$$x - \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + E^x)^(-1),x]
```

```
[Out] x - Log[1 + E^x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+e^x} dx &= \text{Subst}\left(\int \frac{1}{x(1+x)} dx, x, e^x\right) \\
&= \text{Subst}\left(\int \frac{1}{x} dx, x, e^x\right) - \text{Subst}\left(\int \frac{1}{1+x} dx, x, e^x\right) \\
&= x - \log(1+e^x)
\end{aligned}$$

Mathematica [A] time = 0.0025529, size = 10, normalized size = 1.

$$x - \log(e^x + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + E^x)^(-1),x]
```

```
[Out] x - Log[1 + E^x]
```

Maple [A] time = 0.006, size = 12, normalized size = 1.2

$$\ln(e^x) - \ln(1 + e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+exp(x)),x)
```

```
[Out] ln(exp(x))-ln(1+exp(x))
```

Maxima [A] time = 0.960611, size = 12, normalized size = 1.2

$$x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)),x, algorithm="maxima")
```

```
[Out] x - log(e^x + 1)
```

Fricas [A] time = 0.478029, size = 24, normalized size = 2.4

$$x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)),x, algorithm="fricas")
```

```
[Out] x - log(e^x + 1)
```

Sympy [A] time = 0.070752, size = 7, normalized size = 0.7

$$x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)),x)
```

```
[Out] x - log(exp(x) + 1)
```

Giac [A] time = 1.07975, size = 12, normalized size = 1.2

$$x - \log(e^x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+exp(x)),x, algorithm="giac")
```

```
[Out] x - log(e^x + 1)
```

3.74 $\int e^x x\,dx$

Optimal. Leaf size=11

$$e^x x - e^x$$

```
[Out] -E^x + E^x*x
```

Rubi [A] time = 0.005418, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2176, 2194}

$$e^x x - e^x$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x,x]
```

```
[Out] -E^x + E^x*x
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int e^x x\,dx &= e^x x - \int e^x\,dx \\ &= -e^x + e^x x \end{aligned}$$

Mathematica [A] time = 0.0008766, size = 7, normalized size = 0.64

$$e^x (x-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x,x]
```

```
[Out] E^x*(-1 + x)
```

Maple [A] time = 0., size = 7, normalized size = 0.6

$$(-1+x)\,\mathrm{e}^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x,x)
```

```
[Out] (-1+x)*exp(x)
```

Maxima [A] time = 0.966979, size = 8, normalized size = 0.73

$$(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x, algorithm="maxima")
```

```
[Out] (x - 1)*e^x
```

Fricas [A] time = 0.525354, size = 18, normalized size = 1.64

$$(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x, algorithm="fricas")
```

```
[Out] (x - 1)*e^x
```

Sympy [A] time = 0.074152, size = 5, normalized size = 0.45

$$(x-1)\,e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x)
```

```
[Out] (x - 1)*exp(x)
```

Giac [A] time = 1.09732, size = 8, normalized size = 0.73

$$(x-1)e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x,x, algorithm="giac")
```

```
[Out] (x - 1)*e^x
```

3.75 $\int e^{-x}x\,dx$

Optimal. Leaf size=16

$$-e^{-x}x-e^{-x}$$

```
[Out] -E^(-x) - x/E^x
```

Rubi [A] time = 0.0068846, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2176, 2194}

$$-e^{-x}x-e^{-x}$$

Antiderivative was successfully verified.

```
[In] Int[x/E^x,x]

[Out] -E^(-x) - x/E^x
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-x}x\,dx &= -e^{-x}x+\int e^{-x}\,dx\\ &= -e^{-x}-e^{-x}x\end{aligned}$$

Mathematica [A] time = 0.0041336, size = 11, normalized size = 0.69

$$e^{-x}(-x-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/E^x,x]

[Out] (-1 - x)/E^x
```

Maple [A] time = 0.001, size = 10, normalized size = 0.6

$$-\frac{1+x}{e^x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/exp(x),x)
```

```
[Out] -(1+x)/exp(x)
```

Maxima [A] time = 0.951513, size = 12, normalized size = 0.75

$$-(x+1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/exp(x),x, algorithm="maxima")
```

```
[Out] -(x + 1)*e^(-x)
```

Fricas [A] time = 0.516459, size = 23, normalized size = 1.44

$$-(x+1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/exp(x),x, algorithm="fricas")
```

```
[Out] -(x + 1)*e^(-x)
```

Sympy [A] time = 0.080846, size = 7, normalized size = 0.44

$$(-x-1)\,e^{-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/exp(x),x)
```

```
[Out] (-x - 1)*exp(-x)
```

Giac [A] time = 1.08864, size = 12, normalized size = 0.75

$$-(x+1)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/exp(x),x, algorithm="giac")
```

```
[Out] -(x + 1)*e^(-x)
```

3.76 $\int e^x x^2 \, dx$

Optimal. Leaf size=19

$$e^x x^2 - 2e^x x + 2e^x$$

```
[Out] 2*E^x - 2*E^x*x + E^x*x^2
```

Rubi [A] time = 0.0145115, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2176, 2194}

$$e^x x^2 - 2e^x x + 2e^x$$

Antiderivative was successfully verified.

```
[In] Int[E^x*x^2,x]
```

```
[Out] 2*E^x - 2*E^x*x + E^x*x^2
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] && !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^x x^2 \, dx &= e^x x^2 - 2 \int e^x x \, dx \\ &= -2e^x x + e^x x^2 + 2 \int e^x \, dx \\ &= 2e^x - 2e^x x + e^x x^2\end{aligned}$$

Mathematica [A] time = 0.0039502, size = 12, normalized size = 0.63

$$e^x \left(x^2 - 2x + 2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x*x^2,x]
```

```
[Out] E^x*(2 - 2*x + x^2)
```

Maple [A] time = 0.002, size = 12, normalized size = 0.6

$$\left(x^2-2x+2\right)\mathrm{e}^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)*x^2,x)
```

```
[Out] (x^2-2*x+2)*exp(x)
```

Maxima [A] time = 0.957435, size = 15, normalized size = 0.79

$$\left(x^2-2x+2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2,x, algorithm="maxima")
```

```
[Out] (x^2 - 2*x + 2)*e^x
```

Fricas [A] time = 0.512646, size = 28, normalized size = 1.47

$$\left(x^2-2x+2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2,x, algorithm="fricas")
```

```
[Out] (x^2 - 2*x + 2)*e^x
```

Sympy [A] time = 0.075385, size = 10, normalized size = 0.53

$$\left(x^2-2x+2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x**2,x)
```

```
[Out] (x**2 - 2*x + 2)*exp(x)
```

Giac [A] time = 1.08273, size = 15, normalized size = 0.79

$$\left(x^2-2x+2\right)e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)*x^2,x, algorithm="giac")
```

```
[Out] (x^2 - 2*x + 2)*e^x
```

3.77 $\int e^{-2x}x^2\,dx$

Optimal. Leaf size=32

$$-\frac{1}{2}e^{-2x}x^2-\frac{1}{2}e^{-2x}x-\frac{e^{-2x}}{4}$$

```
[Out] -1/(4*E^(2*x)) - x/(2*E^(2*x)) - x^2/(2*E^(2*x))
```

Rubi [A] time = 0.0176593, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$-\frac{1}{2}e^{-2x}x^2-\frac{1}{2}e^{-2x}x-\frac{e^{-2x}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/E^(2*x),x]
```

```
[Out] -1/(4*E^(2*x)) - x/(2*E^(2*x)) - x^2/(2*E^(2*x))
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-2x}x^2\,dx&=-\frac{1}{2}e^{-2x}x^2+\int e^{-2x}x\,dx\\&=-\frac{1}{2}e^{-2x}x-\frac{1}{2}e^{-2x}x^2+\frac{1}{2}\int e^{-2x}\,dx\\&=-\frac{1}{4}e^{-2x}-\frac{1}{2}e^{-2x}x-\frac{1}{2}e^{-2x}x^2\end{aligned}$$

Mathematica [A] time = 0.0063322, size = 19, normalized size = 0.59

$$-\frac{1}{4}e^{-2x}\left(2x^2+2x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/E^(2*x),x]
```

```
[Out] -(1 + 2*x + 2*x^2)/(4*E^(2*x))
```

Maple [A] time = 0.002, size = 19, normalized size = 0.6

$$-\frac{2x^2+2x+1}{4\,\mathrm{e}^{2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/exp(2*x),x)
```

```
[Out] -1/4*(2*x^2+2*x+1)/exp(2*x)
```

Maxima [A] time = 0.96247, size = 22, normalized size = 0.69

$$-\frac{1}{4}\left(2x^2+2x+1\right)e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/exp(2*x),x, algorithm="maxima")
```

```
[Out] -1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

Fricas [A] time = 0.486861, size = 45, normalized size = 1.41

$$-\frac{1}{4}\left(2x^2+2x+1\right)e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/exp(2*x),x, algorithm="fricas")
```

```
[Out] -1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

Sympy [A] time = 0.081091, size = 17, normalized size = 0.53

$$\frac{\left(-2x^2-2x-1\right)e^{-2x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/exp(2*x),x)
```

```
[Out] (-2*x**2 - 2*x - 1)*exp(-2*x)/4
```

Giac [A] time = 1.08076, size = 22, normalized size = 0.69

$$-\frac{1}{4}\left(2x^2+2x+1\right)e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/exp(2*x),x, algorithm="giac")
```

```
[Out] -1/4*(2*x^2 + 2*x + 1)*e^(-2*x)
```

3.78 $\int e^{\sqrt{x}}\,dx$

Optimal. Leaf size=24

$$2e^{\sqrt{x}}\sqrt{x}-2e^{\sqrt{x}}$$

```
[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rubi [A] time = 0.0077448, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {2207, 2176, 2194}

$$2e^{\sqrt{x}}\sqrt{x}-2e^{\sqrt{x}}$$

Antiderivative was successfully verified.

```
[In] Int[E^Sqrt[x],x]
```

```
[Out] -2*E^Sqrt[x] + 2*E^Sqrt[x]*Sqrt[x]
```

Rule 2207

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> With[{k =
Denominator[n]}, Dist[k/d, Subst[Int[x^(k - 1)*F^(a + b*x^(k*n)), x], x, (c
 + d*x)^(1/k)], x]] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] &&  !Inte
gerQ[n]
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{\sqrt{x}}\,dx &= 2\,\text{Subst}\left(\int e^x x\,dx,x,\sqrt{x}\right)\\ &= 2e^{\sqrt{x}}\sqrt{x}-2\,\text{Subst}\left(\int e^x\,dx,x,\sqrt{x}\right)\\ &= -2e^{\sqrt{x}}+2e^{\sqrt{x}}\sqrt{x}\end{aligned}$$

Mathematica [A] time = 0.0050372, size = 16, normalized size = 0.67

$$2e^{\sqrt{x}}\left(\sqrt{x}-1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^Sqrt[x],x]
```

```
[Out] 2*E^Sqrt[x]*(-1 + Sqrt[x])
```

Maple [A] time = 0.003, size = 17, normalized size = 0.7

$$-2\,\mathrm{e}^{\sqrt{x}}+2\,\mathrm{e}^{\sqrt{x}}\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x^(1/2)),x)
```

```
[Out] -2*exp(x^(1/2))+2*exp(x^(1/2))*x^(1/2)
```

Maxima [A] time = 0.949928, size = 15, normalized size = 0.62

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Fricas [A] time = 0.493306, size = 36, normalized size = 1.5

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="fricas")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

Sympy [A] time = 0.173511, size = 20, normalized size = 0.83

$$2\sqrt{x}e^{\sqrt{x}}-2e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*exp(sqrt(x)) - 2*exp(sqrt(x))
```

Giac [A] time = 1.07999, size = 15, normalized size = 0.62

$$2\left(\sqrt{x}-1\right)e^{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x^(1/2)),x, algorithm="giac")
```

```
[Out] 2*(sqrt(x) - 1)*e^sqrt(x)
```

3.79 $\int e^{-x^2} x^3 \, dx$

Optimal. Leaf size=26

$$-\frac{1}{2}e^{-x^2}x^2 - \frac{e^{-x^2}}{2}$$

```
[Out] -1/(2*E^x^2) - x^2/(2*E^x^2)
```

Rubi [A] time = 0.0199832, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {2212, 2209}

$$-\frac{1}{2}e^{-x^2}x^2 - \frac{e^{-x^2}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/E^x^2,x]
```

```
[Out] -1/(2*E^x^2) - x^2/(2*E^x^2)
```

Rule 2212

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((c_.) + (d_.)*(x_))^(m_
.), x_Symbol] :> Simp[((c + d*x)^(m - n + 1)*F^(a + b*(c + d*x)^n))/(b*d*n*
Log[F]), x] - Dist[(m - n + 1)/(b*n*Log[F]), Int[(c + d*x)^(m - n)*F^(a + b
*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[(2*(m + 1))/
n] && LtQ[0, (m + 1)/n, 5] && IntegerQ[n] && (LtQ[0, n, m + 1] || LtQ[m, n,
 0])
```

Rule 2209

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))*((e_.) + (f_.)*(x_))^(m_
.), x_Symbol] :> Simp[((e + f*x)^n*F^(a + b*(c + d*x)^n))/(b*f*n*(c + d*x)^
n*Log[F]), x] /; FreeQ[{F, a, b, c, d, e, f, n}, x] && EqQ[m, n - 1] && EqQ
[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int e^{-x^2}x^3 \, dx &= -\frac{1}{2}e^{-x^2}x^2 + \int e^{-x^2}x \, dx\\ &= -\frac{e^{-x^2}}{2} - \frac{1}{2}e^{-x^2}x^2\end{aligned}$$

Mathematica [A] time = 0.0023307, size = 16, normalized size = 0.62

$$-\frac{1}{2}e^{-x^2}\left(x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/E^x^2,x]
```

```
[Out] -(1 + x^2)/(2*E^x^2)
```

Maple [A] time = 0.002, size = 14, normalized size = 0.5

$$-\frac{x^2+1}{2\,e^{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/exp(x^2),x)
```

```
[Out] -1/2*(x^2+1)/exp(x^2)
```

Maxima [A] time = 0.942501, size = 18, normalized size = 0.69

$$-\frac{1}{2}\left(x^2+1\right)e^{\left(-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/exp(x^2),x, algorithm="maxima")
```

```
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

Fricas [A] time = 0.505699, size = 34, normalized size = 1.31

$$-\frac{1}{2}\left(x^2+1\right)e^{\left(-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/exp(x^2),x, algorithm="fricas")
```

```
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

Sympy [A] time = 0.083491, size = 12, normalized size = 0.46

$$\frac{\left(-x^2-1\right)e^{-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/exp(x**2),x)
```

```
[Out] (-x**2 - 1)*exp(-x**2)/2
```

Giac [A] time = 1.08975, size = 18, normalized size = 0.69

$$-\frac{1}{2}\left(x^2+1\right)e^{\left(-x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/exp(x^2),x, algorithm="giac")
```

```
[Out] -1/2*(x^2 + 1)*e^(-x^2)
```

3.80 $\int e^{ax} \cos(bx)\,dx$

Optimal. Leaf size=41

$$\frac{be^{ax}\sin(bx)}{a^2+b^2}+\frac{ae^{ax}\cos(bx)}{a^2+b^2}$$

```
[Out] (a*E^(a*x)*Cos[b*x])/(a^2 + b^2) + (b*E^(a*x)*Sin[b*x])/(a^2 + b^2)
```

Rubi [A] time = 0.011306, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4433}

$$\frac{be^{ax}\sin(bx)}{a^2+b^2}+\frac{ae^{ax}\cos(bx)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a*x)*Cos[b*x],x]

[Out] (a*E^(a*x)*Cos[b*x])/(a^2 + b^2) + (b*E^(a*x)*Sin[b*x])/(a^2 + b^2)
```

Rule 4433

```
Int[Cos[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] + Simp[(e*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{ax}\cos(bx)\,dx=\frac{ae^{ax}\cos(bx)}{a^2+b^2}+\frac{be^{ax}\sin(bx)}{a^2+b^2}$$

Mathematica [A] time = 0.025097, size = 28, normalized size = 0.68

$$\frac{e^{ax}(a\cos(bx)+b\sin(bx))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a*x)*Cos[b*x],x]

[Out] (E^(a*x)*(a*Cos[b*x] + b*Sin[b*x]))/(a^2 + b^2)
```

Maple [A] time = 0.01, size = 40, normalized size = 1.

$$\frac{ae^{ax}\cos(bx)}{a^2+b^2}+\frac{e^{ax}b\sin(bx)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)*cos(b*x),x)
```

```
[Out] a*exp(a*x)*cos(b*x)/(a^2+b^2)+b*exp(a*x)*sin(b*x)/(a^2+b^2)
```

Maxima [A] time = 0.966976, size = 36, normalized size = 0.88

$$\frac{(a\cos(bx) + b\sin(bx))e^{(ax)}}{a^2 + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*cos(b*x),x, algorithm="maxima")
```

```
[Out] (a*cos(b*x) + b*sin(b*x))*e^(a*x)/(a^2 + b^2)
```

Fricas [A] time = 0.559013, size = 74, normalized size = 1.8

$$\frac{a\cos(bx)\,e^{(ax)} + be^{(ax)}\sin(bx)}{a^2 + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*cos(b*x),x, algorithm="fricas")
```

```
[Out] (a*cos(b*x)*e^(a*x) + b*e^(a*x)*sin(b*x))/(a^2 + b^2)
```

Sympy [A] time = 1.32268, size = 139, normalized size = 3.39

$$\begin{cases} x & \text{for } a = 0 \wedge b = 0 \\ \frac{ixe^{-ibx}\sin(bx)}{2} + \frac{xe^{-ibx}\cos(bx)}{2} + \frac{ie^{-ibx}\cos(bx)}{2b} & \text{for } a = -ib \\ -\frac{ixe^{ibx}\sin(bx)}{2} + \frac{xe^{ibx}\cos(bx)}{2} - \frac{ie^{ibx}\cos(bx)}{2b} & \text{for } a = ib \\ \frac{ae^{ax}\cos(bx)}{a^2+b^2} + \frac{be^{ax}\sin(bx)}{a^2+b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*cos(b*x),x)
```

```
[Out] Piecewise((x, Eq(a, 0) & Eq(b, 0)), (I*x*exp(-I*b*x)*sin(b*x)/2 + x*exp(-I*
b*x)*cos(b*x)/2 + I*exp(-I*b*x)*cos(b*x)/(2*b), Eq(a, -I*b)), (-I*x*exp(I*b
*x)*sin(b*x)/2 + x*exp(I*b*x)*cos(b*x)/2 - I*exp(I*b*x)*cos(b*x)/(2*b), Eq(
a, I*b)), (a*exp(a*x)*cos(b*x)/(a**2 + b**2) + b*exp(a*x)*sin(b*x)/(a**2 +
b**2), True))
```

Giac [A] time = 1.08904, size = 49, normalized size = 1.2

$$\left(\frac{a\cos(bx)}{a^2 + b^2} + \frac{b\sin(bx)}{a^2 + b^2}\right)e^{(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*cos(b*x),x, algorithm="giac")
```

```
[Out] (a*cos(b*x)/(a^2 + b^2) + b*sin(b*x)/(a^2 + b^2))*e^(a*x)
```

3.81 $\int e^{ax} \sin(bx)\, dx$

Optimal. Leaf size=42

$$\frac{ae^{ax}\sin(bx)}{a^2+b^2}-\frac{be^{ax}\cos(bx)}{a^2+b^2}$$

```
[Out] -((b*E^(a*x)*Cos[b*x])/(a^2 + b^2)) + (a*E^(a*x)*Sin[b*x])/(a^2 + b^2)
```

Rubi [A] time = 0.0113484, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {4432}

$$\frac{ae^{ax}\sin(bx)}{a^2+b^2}-\frac{be^{ax}\cos(bx)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[E^(a*x)*Sin[b*x],x]
```

```
[Out] -((b*E^(a*x)*Cos[b*x])/(a^2 + b^2)) + (a*E^(a*x)*Sin[b*x])/(a^2 + b^2)
```

Rule 4432

```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sin[(d_.) + (e_.)*(x_)], x_Symbol] :>
 Simp[(b*c*Log[F]*F^(c*(a + b*x))*Sin[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x
] - Simp[(e*F^(c*(a + b*x))*Cos[d + e*x])/(e^2 + b^2*c^2*Log[F]^2), x] /; F
reeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 + b^2*c^2*Log[F]^2, 0]
```

Rubi steps

$$\int e^{ax}\sin(bx)\,dx=-\frac{be^{ax}\cos(bx)}{a^2+b^2}+\frac{ae^{ax}\sin(bx)}{a^2+b^2}$$

Mathematica [A] time = 0.0241032, size = 29, normalized size = 0.69

$$\frac{e^{ax}(a\sin(bx)-b\cos(bx))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a*x)*Sin[b*x],x]
```

```
[Out] (E^(a*x)*(-(b*Cos[b*x]) + a*Sin[b*x]))/(a^2 + b^2)
```

Maple [A] time = 0.003, size = 41, normalized size = 1.

$$-\frac{e^{ax}b\cos(bx)}{a^2+b^2}+\frac{ae^{ax}\sin(bx)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*x)*sin(b*x),x)
```

```
[Out] -b*exp(a*x)*cos(b*x)/(a^2+b^2)+a*exp(a*x)*sin(b*x)/(a^2+b^2)
```

Maxima [A] time = 0.9364, size = 39, normalized size = 0.93

$$-\frac{(b\cos(bx) - a\sin(bx))e^{(ax)}}{a^2 + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*sin(b*x),x, algorithm="maxima")
```

```
[Out] -(b*cos(b*x) - a*sin(b*x))*e^(a*x)/(a^2 + b^2)
```

Fricas [A] time = 0.506773, size = 76, normalized size = 1.81

$$-\frac{b\cos(bx)\,e^{(ax)} - ae^{(ax)}\sin(bx)}{a^2 + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*sin(b*x),x, algorithm="fricas")
```

```
[Out] -(b*cos(b*x)*e^(a*x) - a*e^(a*x)*sin(b*x))/(a^2 + b^2)
```

Sympy [A] time = 1.36466, size = 136, normalized size = 3.24

$$\begin{cases} 0 & \text{for}\: a = 0 \wedge b = 0 \\ \frac{xe^{-ibx}\sin(bx)}{2} - \frac{ixe^{-ibx}\cos(bx)}{2} - \frac{e^{-ibx}\cos(bx)}{2b} & \text{for}\: a = -ib \\ \frac{xe^{ibx}\sin(bx)}{2} + \frac{ixe^{ibx}\cos(bx)}{2} - \frac{e^{ibx}\cos(bx)}{2b} & \text{for}\: a = ib \\ \frac{ae^{ax}\sin(bx)}{a^2+b^2} - \frac{be^{ax}\cos(bx)}{a^2+b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*sin(b*x),x)
```

```
[Out] Piecewise((0, Eq(a, 0) & Eq(b, 0)), (x*exp(-I*b*x)*sin(b*x)/2 - I*x*exp(-I*
b*x)*cos(b*x)/2 - exp(-I*b*x)*cos(b*x)/(2*b), Eq(a, -I*b)), (x*exp(I*b*x)*s
in(b*x)/2 + I*x*exp(I*b*x)*cos(b*x)/2 - exp(I*b*x)*cos(b*x)/(2*b), Eq(a, I*
b)), (a*exp(a*x)*sin(b*x)/(a**2 + b**2) - b*exp(a*x)*cos(b*x)/(a**2 + b**2)
, True))
```

Giac [A] time = 1.06469, size = 51, normalized size = 1.21

$$-\left(\frac{b\cos(bx)}{a^2 + b^2} - \frac{a\sin(bx)}{a^2 + b^2}\right)e^{(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*x)*sin(b*x),x, algorithm="giac")
```

```
[Out] -(b*cos(b*x)/(a^2 + b^2) - a*sin(b*x)/(a^2 + b^2))*e^(a*x)
```

3.82 $\int \cot^{-1}(x)\,dx$

Optimal. Leaf size=15

$$\frac{1}{2}\log\left(x^2+1\right)+x\cot^{-1}(x)$$

```
[Out] x*ArcCot[x] + Log[1 + x^2]/2
```

Rubi [A] time = 0.0033438, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {4847, 260}

$$\frac{1}{2}\log\left(x^2+1\right)+x\cot^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[x],x]
```

```
[Out] x*ArcCot[x] + Log[1 + x^2]/2
```

Rule 4847

```
Int[((a_.) + ArcCot[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cCot[c*x])^p, x] + Dist[b*c*p, Int[(x*(a + b*ArcCot[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \cot^{-1}(x)\,dx &= x\cot^{-1}(x)+\int \frac{x}{1+x^2}\,dx\\ &= x\cot^{-1}(x)+\frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0014764, size = 15, normalized size = 1.

$$\frac{1}{2}\log\left(x^2+1\right)+x\cot^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[x],x]
```

```
[Out] x*ArcCot[x] + Log[1 + x^2]/2
```

Maple [A] time = 0., size = 14, normalized size = 0.9

$$x\text{arccot}\left(x\right)+\frac{\ln\left(x^{2}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(x),x)
```

```
[Out] x*arccot(x)+1/2*ln(x^2+1)
```

Maxima [A] time = 0.931492, size = 18, normalized size = 1.2

$$x\,\text{arccot}\left(x\right)+\frac{1}{2}\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x),x, algorithm="maxima")
```

```
[Out] x*arccot(x) + 1/2*log(x^2 + 1)
```

Fricas [A] time = 0.513946, size = 43, normalized size = 2.87

$$x\,\text{arccot}\left(x\right)+\frac{1}{2}\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x),x, algorithm="fricas")
```

```
[Out] x*arccot(x) + 1/2*log(x^2 + 1)
```

Sympy [A] time = 0.18563, size = 12, normalized size = 0.8

$$x\,\text{acot}\left(x\right)+\frac{\log\left(x^{2}+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(x),x)
```

```
[Out] x*acot(x) + log(x**2 + 1)/2
```

Giac [A] time = 1.08174, size = 20, normalized size = 1.33

$$x\,\arctan\left(\frac{1}{x}\right)+\frac{1}{2}\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(x),x, algorithm="giac")

[Out] x*arctan(1/x) + 1/2*log(x^2 + 1)
```

3.83 $\int \sec^{-1}(x)\, dx$

Optimal. Leaf size=19

$$x\sec^{-1}(x) - \tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

```
[Out] x*ArcSec[x] - ArcTanh[Sqrt[1 - x^(-2)]]
```

Rubi [A] time = 0.0091662, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2., Rules used = {5214, 266, 63, 206}

$$x\sec^{-1}(x) - \tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSec[x],x]

[Out] x*ArcSec[x] - ArcTanh[Sqrt[1 - x^(-2)]]
```

Rule 5214

```
Int[ArcSec[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcSec[c*x], x] - Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sec^{-1}(x)\,dx &= x\sec^{-1}(x) - \int \frac{1}{\sqrt{1-\frac{1}{x^2}}x}\,dx \\
&= x\sec^{-1}(x) + \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\,dx, x, \frac{1}{x^2}\right) \\
&= x\sec^{-1}(x) - \mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right) \\
&= x\sec^{-1}(x) - \tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}$$

Mathematica [B] time = 0.0637212, size = 64, normalized size = 3.37

$$x\sec^{-1}(x) - \frac{\sqrt{x^2-1}\left(\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)\right)}{2\sqrt{1-\frac{1}{x^2}}x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSec[x],x]
```

```
[Out] x*ArcSec[x] - (Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 + x/Sqrt[
-1 + x^2]]))/(2*Sqrt[1 - x^(-2)]*x)
```

Maple [A] time = 0.002, size = 22, normalized size = 1.2

$$x\mathrm{arcsec}\,(x) - \ln\left(x + x\sqrt{1-x^{-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsec(x),x)
```

```
[Out] x*arcsec(x)-ln(x+x*(1-1/x^2)^(1/2))
```

Maxima [B] time = 0.928872, size = 47, normalized size = 2.47

$$x\,\mathrm{arcsec}\,(x) - \frac{1}{2}\log\left(\sqrt{-\frac{1}{x^2}+1}+1\right) + \frac{1}{2}\log\left(-\sqrt{-\frac{1}{x^2}+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x),x, algorithm="maxima")
```

```
[Out] x*arcsec(x) - 1/2*log(sqrt(-1/x^2 + 1) + 1) + 1/2*log(-sqrt(-1/x^2 + 1) + 1
)
```

Fricas [A] time = 0.626027, size = 103, normalized size = 5.42

$$(x-2)\operatorname{arcsec}(x)+4\arctan\left(-x+\sqrt{x^2-1}\right)+\log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x),x, algorithm="fricas")
```

```
[Out] (x - 2)*arcsec(x) + 4*arctan(-x + sqrt(x^2 - 1)) + log(-x + sqrt(x^2 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{asec}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asec(x),x)
```

```
[Out] Integral(asec(x), x)
```

Giac [A] time = 1.10526, size = 34, normalized size = 1.79

$$x\arccos\left(\frac{1}{x}\right)+\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsec(x),x, algorithm="giac")
```

```
[Out] x*arccos(1/x) + log(abs(-x + sqrt(x^2 - 1)))/sgn(x)
```

3.84 $\int \csc^{-1}(x)\,dx$

Optimal. Leaf size=17

$$\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)+x\csc^{-1}(x)$$

```
[Out] x*ArcCsc[x] + ArcTanh[Sqrt[1 - x^(-2)]]
```

Rubi [A] time = 0.0090909, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 2., Rules used = {5215, 266, 63, 206}

$$\tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)+x\csc^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCsc[x],x]
```

```
[Out] x*ArcCsc[x] + ArcTanh[Sqrt[1 - x^(-2)]]
```

Rule 5215

```
Int[ArcCsc[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsc[c*x], x] + Dist[1/c, Int
[1/(x*Sqrt[1 - 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \csc^{-1}(x)\,dx &= x\csc^{-1}(x) + \int \frac{1}{\sqrt{1-\frac{1}{x^2}}x}\,dx \\
&= x\csc^{-1}(x) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\,dx, x, \frac{1}{x^2}\right) \\
&= x\csc^{-1}(x) + \text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{1-\frac{1}{x^2}}\right) \\
&= x\csc^{-1}(x) + \tanh^{-1}\left(\sqrt{1-\frac{1}{x^2}}\right)
\end{aligned}
$$

Mathematica [B] time = 0.0386257, size = 64, normalized size = 3.76

$$
\frac{\sqrt{x^2-1}\left(\log\left(\frac{x}{\sqrt{x^2-1}}+1\right)-\log\left(1-\frac{x}{\sqrt{x^2-1}}\right)\right)}{2\sqrt{1-\frac{1}{x^2}}x} + x\csc^{-1}(x)
$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCsc[x],x]
```

```
[Out] x*ArcCsc[x] + (Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 + x/Sqrt[
-1 + x^2]]))/(2*Sqrt[1 - x^(-2)]*x)
```

Maple [A] time = 0.002, size = 20, normalized size = 1.2

$$
x\text{arccsc}\,(x) + \ln\left(x + x\sqrt{1-x^{-2}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccsc(x),x)
```

```
[Out] x*arccsc(x)+ln(x+x*(1-1/x^2)^(1/2))
```

Maxima [B] time = 0.928191, size = 47, normalized size = 2.76

$$
x\,\text{arccsc}\,(x) + \frac{1}{2}\log\left(\sqrt{-\frac{1}{x^2}+1}+1\right) - \frac{1}{2}\log\left(-\sqrt{-\frac{1}{x^2}+1}+1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x),x, algorithm="maxima")
```

```
[Out] x*arccsc(x) + 1/2*log(sqrt(-1/x^2 + 1) + 1) - 1/2*log(-sqrt(-1/x^2 + 1) + 1
)
```

Fricas [B] time = 0.553965, size = 103, normalized size = 6.06

$$(x-2)\operatorname{arccsc}(x)-4\arctan\left(-x+\sqrt{x^2-1}\right)-\log\left(-x+\sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x),x, algorithm="fricas")
```

```
[Out] (x - 2)*arccsc(x) - 4*arctan(-x + sqrt(x^2 - 1)) - log(-x + sqrt(x^2 - 1))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{acsc}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acsc(x),x)
```

```
[Out] Integral(acsc(x), x)
```

Giac [A] time = 1.09854, size = 35, normalized size = 2.06

$$x\arcsin\left(\frac{1}{x}\right)-\frac{\log\left(\left|-x+\sqrt{x^2-1}\right|\right)}{\operatorname{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccsc(x),x, algorithm="giac")
```

```
[Out] x*arcsin(1/x) - log(abs(-x + sqrt(x^2 - 1)))/sgn(x)
```

3.85 $\int \sin^{-1}(x)^2\,dx$

Optimal. Leaf size=25

$$2\sqrt{1-x^2}\sin^{-1}(x)-2x+x\sin^{-1}(x)^2$$

```
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Rubi [A] time = 0.0337108, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {4619, 4677, 8}

$$2\sqrt{1-x^2}\sin^{-1}(x)-2x+x\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]^2,x]
```

```
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Rule 4619

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*ArcSin[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4677

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSin[c*x])^n)/(2*e*(p + 1)), x] + Dist[(b*n*d^IntPart[p]*(d + e*x^2)^FracPart[p])/(2*c*(p + 1)*(1 - c^2*x^2)^FracPart[p]), Int[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcSin[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin^{-1}(x)^2\,dx &= x\sin^{-1}(x)^2-2\int \frac{x\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx\\ &= 2\sqrt{1-x^2}\sin^{-1}(x)+x\sin^{-1}(x)^2-2\int 1\,dx\\ &= -2x+2\sqrt{1-x^2}\sin^{-1}(x)+x\sin^{-1}(x)^2\end{aligned}$$

Mathematica [A] time = 0.006573, size = 25, normalized size = 1.

$$2\sqrt{1-x^2}\sin^{-1}(x)-2x+x\sin^{-1}(x)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]^2,x]
```

```
[Out] -2*x + 2*Sqrt[1 - x^2]*ArcSin[x] + x*ArcSin[x]^2
```

Maple [A] time = 0.037, size = 24, normalized size = 1.

$$-2\,x+x\left(\arcsin\left(x\right)\right)^{2}+2\,\arcsin\left(x\right)\sqrt{-x^{2}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)^2,x)
```

```
[Out] -2*x+x*arcsin(x)^2+2*arcsin(x)*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.42398, size = 31, normalized size = 1.24

$$x\arcsin\left(x\right)^{2}+2\,\sqrt{-x^{2}+1}\arcsin\left(x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="maxima")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Fricas [A] time = 0.503629, size = 68, normalized size = 2.72

$$x\arcsin\left(x\right)^{2}+2\,\sqrt{-x^{2}+1}\arcsin\left(x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="fricas")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

Sympy [A] time = 0.184303, size = 22, normalized size = 0.88

$$x\operatorname{asin}^{2}\left(x\right)-2x+2\sqrt{1-x^{2}}\operatorname{asin}\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)**2,x)
```

```
[Out] x*asin(x)**2 - 2*x + 2*sqrt(1 - x**2)*asin(x)
```

Giac [A] time = 1.11245, size = 31, normalized size = 1.24

$$x\arcsin\left(x\right)^{2}+2\,\sqrt{-x^{2}+1}\arcsin\left(x\right)-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)^2,x, algorithm="giac")
```

```
[Out] x*arcsin(x)^2 + 2*sqrt(-x^2 + 1)*arcsin(x) - 2*x
```

3.86 $\int \frac{\sin^{-1}(x)}{x^2}\,dx$

Optimal. Leaf size=22

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sin^{-1}(x)}{x}$$

```
[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

Rubi [A] time = 0.0155042, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4627, 266, 63, 206}

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sin^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSin[x]/x^2,x]

[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

Rule 4627

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcSin[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^{-1}(x)}{x^2}\,dx &= -\frac{\sin^{-1}(x)}{x}+\int \frac{1}{x\sqrt{1-x^2}}\,dx\\
&= -\frac{\sin^{-1}(x)}{x}+\frac{1}{2}\,\text{Subst}\left(\int \frac{1}{\sqrt{1-xx}}\,dx,x,x^2\right)\\
&= -\frac{\sin^{-1}(x)}{x}-\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{1-x^2}\right)\\
&= -\frac{\sin^{-1}(x)}{x}-\tanh^{-1}\left(\sqrt{1-x^2}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0018583, size = 22, normalized size = 1.

$$-\tanh^{-1}\left(\sqrt{1-x^2}\right)-\frac{\sin^{-1}(x)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSin[x]/x^2,x]
```

```
[Out] -(ArcSin[x]/x) - ArcTanh[Sqrt[1 - x^2]]
```

Maple [A] time = 0.003, size = 21, normalized size = 1.

$$-\frac{\arcsin(x)}{x}-\text{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsin(x)/x^2,x)
```

```
[Out] -arcsin(x)/x-arctanh(1/(-x^2+1)^(1/2))
```

Maxima [A] time = 1.45801, size = 45, normalized size = 2.05

$$-\frac{\arcsin(x)}{x}-\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2,x, algorithm="maxima")
```

```
[Out] -arcsin(x)/x - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))
```

Fricas [A] time = 0.631367, size = 105, normalized size = 4.77

$$-\frac{x\log\left(\sqrt{-x^2+1}+1\right)-x\log\left(\sqrt{-x^2+1}-1\right)+2\arcsin(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2,x, algorithm="fricas")
```

```
[Out] -1/2*(x*log(sqrt(-x^2 + 1) + 1) - x*log(sqrt(-x^2 + 1) - 1) + 2*arcsin(x))/
x
```

Sympy [A] time = 1.78431, size = 22, normalized size = 1.

$$\begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for}\ \frac{1}{|x^2|} > 1 \\ i\operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases} - \frac{\operatorname{asin}(x)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asin(x)/x**2,x)
```

```
[Out] Piecewise((-acosh(1/x), 1/Abs(x**2) > 1), (I*asin(1/x), True)) - asin(x)/x
```

Giac [A] time = 1.12322, size = 51, normalized size = 2.32

$$-\frac{\arcsin(x)}{x} - \frac{1}{2}\log\left(\sqrt{-x^2+1}+1\right) + \frac{1}{2}\log\left(-\sqrt{-x^2+1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsin(x)/x^2,x, algorithm="giac")
```

```
[Out] -arcsin(x)/x - 1/2*log(sqrt(-x^2 + 1) + 1) + 1/2*log(-sqrt(-x^2 + 1) + 1)
```

3.87 $\int \frac{1}{\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=16

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

```
[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Rubi [A] time = 0.0022237, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {217, 203}

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a^2 - x^2],x]
```

```
[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a^2-x^2}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \frac{x}{\sqrt{a^2-x^2}}\right)\\ &= \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0024595, size = 16, normalized size = 1.

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a^2 - x^2],x]
```

```
[Out] ArcTan[x/Sqrt[a^2 - x^2]]
```

Maple [A] time = 0.003, size = 15, normalized size = 0.9

$$\arctan\left(x\frac{1}{\sqrt{a^2-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2-x^2)^(1/2),x)
```

```
[Out] arctan(x/(a^2-x^2)^(1/2))
```

Maxima [A] time = 1.42585, size = 11, normalized size = 0.69

$$\arcsin\left(\frac{x}{\sqrt{a^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsin(x/sqrt(a^2))
```

Fricas [A] time = 0.508721, size = 50, normalized size = 3.12

$$-2\arctan\left(-\frac{a-\sqrt{a^2-x^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan(-(a - sqrt(a^2 - x^2))/x)
```

Sympy [A] time = 1.01036, size = 20, normalized size = 1.25

$$\begin{cases} -i\operatorname{acosh}\left(\frac{x}{a}\right) & \text{for}\ \frac{|x^2|}{|a^2|} > 1 \\ \operatorname{asin}\left(\frac{x}{a}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2-x**2)**(1/2),x)
```

```
[Out] Piecewise((-I*acosh(x/a), Abs(x**2)/Abs(a**2) > 1), (asin(x/a), True))
```

Giac [A] time = 1.11871, size = 12, normalized size = 0.75

$$\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(x/a)*sgn(a)
```

3.88 $\int \frac{1}{\sqrt{1-2x-x^2}}\,dx$

Optimal. Leaf size=10

$$\sin^{-1}\left(\frac{x+1}{\sqrt{2}}\right)$$

```
[Out] ArcSin[(1 + x)/Sqrt[2]]
```

Rubi [A] time = 0.0086627, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {619, 216}

$$\sin^{-1}\left(\frac{x+1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - 2*x - x^2],x]
```

```
[Out] ArcSin[(1 + x)/Sqrt[2]]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-2x-x^2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{8}}}\,dx, x, -2-2x\right)}{2\sqrt{2}}\\ &= \sin^{-1}\left(\frac{1+x}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.0050393, size = 14, normalized size = 1.4

$$-\sin^{-1}\left(\frac{-x-1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - 2*x - x^2],x]
```

```
[Out] -ArcSin[(-1 - x)/Sqrt[2]]
```

Maple [A] time = 0.003, size = 10, normalized size = 1.

$$\arcsin\left(\frac{(1+x)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-x^2-2*x+1)^(1/2),x)
```

```
[Out] arcsin(1/2*(1+x)*2^(1/2))
```

Maxima [A] time = 1.43247, size = 15, normalized size = 1.5

$$-\arcsin\left(-\frac{1}{2}\sqrt{2}(x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/2*sqrt(2)*(x + 1))
```

Fricas [B] time = 0.510158, size = 55, normalized size = 5.5

$$-2\arctan\left(\frac{\sqrt{-x^2-2x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*arctan((sqrt(-x^2 - 2*x + 1) - 1)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2-2x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x**2-2*x+1)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 - 2*x + 1), x)
```

<u>Giac</u> [A] time = 1.09148, size = 12, normalized size = 1.2

$$\arcsin\left(\frac{1}{2}\sqrt{2}(x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-x^2-2*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] arcsin(1/2*sqrt(2)*(x + 1))
```

3.89 $\int \frac{1}{a^2+x^2} \, dx$

Optimal. Leaf size=10

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

```
[Out] ArcTan[x/a]/a
```

Rubi [A] time = 0.0018177, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {203}

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/a]/a
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a^2+x^2} \, dx = \frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Mathematica [A] time = 0.0019178, size = 10, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{x}{a}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2 + x^2)^(-1),x]
```

```
[Out] ArcTan[x/a]/a
```

Maple [A] time = 0.002, size = 11, normalized size = 1.1

$$\frac{1}{a}\arctan\left(\frac{x}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a^2+x^2),x)
```

```
[Out] arctan(x/a)/a
```

Maxima [A] time = 1.44409, size = 14, normalized size = 1.4

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="maxima")
```

```
[Out] arctan(x/a)/a
```

Fricas [A] time = 0.495091, size = 20, normalized size = 2.

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="fricas")
```

```
[Out] arctan(x/a)/a
```

Sympy [C] time = 0.105524, size = 20, normalized size = 2.

$$\frac{-\frac{i\log\left(-ia+x\right)}{2}+\frac{i\log\left(ia+x\right)}{2}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a**2+x**2),x)
```

```
[Out] (-I*log(-I*a + x)/2 + I*log(I*a + x)/2)/a
```

Giac [A] time = 1.07547, size = 14, normalized size = 1.4

$$\frac{\arctan\left(\frac{x}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a^2+x^2),x, algorithm="giac")
```

```
[Out] arctan(x/a)/a
```

3.90 $\int \frac{1}{a+bx^2} dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rubi [A] time = 0.0042678, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{a+bx^2} dx = \frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Mathematica [A] time = 0.0046844, size = 24, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[b]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Maple [A] time = 0.002, size = 16, normalized size = 0.7

$$\arctan\left(bx\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x^2+a),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.500243, size = 151, normalized size = 6.29

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a))/(a*b), sqrt(
a*b)*arctan(sqrt(a*b)*x/a)/(a*b)]
```

Sympy [B] time = 0.125203, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ab}}\log\left(-a\sqrt{-\frac{1}{ab}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ab}}\log\left(a\sqrt{-\frac{1}{ab}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*b))*log(-a*sqrt(-1/(a*b)) + x)/2 + sqrt(-1/(a*b))*log(a*sqrt(-1
/(a*b)) + x)/2
```

Giac [A] time = 1.10855, size = 20, normalized size = 0.83

$$\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x^2+a),x, algorithm="giac")
```

```
[Out] arctan(b*x/sqrt(a*b))/sqrt(a*b)
```

3.91 $\int \frac{1}{2-x+x^2} \, dx$

Optimal. Leaf size=19

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[7]])/Sqrt[7]
```

Rubi [A] time = 0.0087962, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {618, 204}

$$-\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + x^2)^(-1),x]
```

```
[Out] (-2*ArcTan[(1 - 2*x)/Sqrt[7]])/Sqrt[7]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{2-x+x^2} \, dx &= -\left(2\,\text{Subst}\left(\int \frac{1}{-7-x^2} \, dx, x, -1+2x\right)\right)\\ &= -\frac{2\tan^{-1}\left(\frac{1-2x}{\sqrt{7}}\right)}{\sqrt{7}}\end{aligned}$$

Mathematica [A] time = 0.0052163, size = 19, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2x-1}{\sqrt{7}}\right)}{\sqrt{7}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(-1 + 2*x)/Sqrt[7]])/Sqrt[7]
```

Maple [A] time = 0.001, size = 17, normalized size = 0.9

$$\frac{2\sqrt{7}}{7}\arctan\left(\frac{(2x-1)\sqrt{7}}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-x+2),x)
```

```
[Out] 2/7*7^(1/2)*arctan(1/7*(2*x-1)*7^(1/2))
```

Maxima [A] time = 1.43877, size = 22, normalized size = 1.16

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="maxima")
```

```
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Fricas [A] time = 0.479371, size = 58, normalized size = 3.05

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="fricas")
```

```
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

Sympy [A] time = 0.120541, size = 26, normalized size = 1.37

$$\frac{2\sqrt{7}\,\mathrm{atan}\left(\frac{2\sqrt{7}x}{7}-\frac{\sqrt{7}}{7}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-x+2),x)
```

```
[Out] 2*sqrt(7)*atan(2*sqrt(7)*x/7 - sqrt(7)/7)/7
```

Giac [A] time = 1.08285, size = 22, normalized size = 1.16

$$\frac{2}{7}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(2x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-x+2),x, algorithm="giac")
```

```
[Out] 2/7*sqrt(7)*arctan(1/7*sqrt(7)*(2*x - 1))
```

3.92 $\int x \tan^{-1}(x)\,dx$

Optimal. Leaf size=21

$$\frac{1}{2}x^2\tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rubi [A] time = 0.0075216, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {4852, 321, 203}

$$\frac{1}{2}x^2\tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x],x]
```

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x\tan^{-1}(x)\,dx &= \frac{1}{2}x^2\tan^{-1}(x) - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx \\
&= -\frac{x}{2} + \frac{1}{2}x^2\tan^{-1}(x) + \frac{1}{2}\int \frac{1}{1+x^2}\,dx \\
&= -\frac{x}{2} + \frac{1}{2}\tan^{-1}(x) + \frac{1}{2}x^2\tan^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.002, size = 21, normalized size = 1.

$$\frac{1}{2}x^2\tan^{-1}(x)-\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x],x]
```

```
[Out] -x/2 + ArcTan[x]/2 + (x^2*ArcTan[x])/2
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$-\frac{x}{2}+\frac{\arctan(x)}{2}+\frac{x^2\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x),x)
```

```
[Out] -1/2*x+1/2*arctan(x)+1/2*x^2*arctan(x)
```

Maxima [A] time = 1.45803, size = 20, normalized size = 0.95

$$\frac{1}{2}x^2\arctan(x)-\frac{1}{2}x+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

Fricas [A] time = 0.515886, size = 45, normalized size = 2.14

$$\frac{1}{2}\left(x^2+1\right)\arctan(x)-\frac{1}{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 1)*arctan(x) - 1/2*x
```

Sympy [A] time = 0.257261, size = 15, normalized size = 0.71

$$\frac{x^2\operatorname{atan}(x)}{2}-\frac{x}{2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x),x)
```

```
[Out] x**2*atan(x)/2 - x/2 + atan(x)/2
```

Giac [A] time = 1.12349, size = 20, normalized size = 0.95

$$\frac{1}{2}\,x^2\arctan\left(x\right)-\frac{1}{2}\,x+\frac{1}{2}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x),x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(x) - 1/2*x + 1/2*arctan(x)
```

3.93 $\int x^2 \cos^{-1}(x)\, dx$

Optimal. Leaf size=40

$$\frac{1}{9}\left(1-x^2\right)^{3/2}-\frac{\sqrt{1-x^2}}{3}+\frac{1}{3}x^3\cos^{-1}(x)$$

```
[Out] -Sqrt[1 - x^2]/3 + (1 - x^2)^(3/2)/9 + (x^3*ArcCos[x])/3
```

Rubi [A] time = 0.0215891, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4628, 266, 43}

$$\frac{1}{3}x^3\cos^{-1}(x)+\frac{1}{9}\left(1-x^2\right)^{3/2}-\frac{\sqrt{1-x^2}}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcCos[x],x]
```

```
[Out] -Sqrt[1 - x^2]/3 + (1 - x^2)^(3/2)/9 + (x^3*ArcCos[x])/3
```

Rule 4628

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^n)/(d*(m + 1)), x] + Dist[(b*c*n
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCos[c*x])^(n - 1))/Sqrt[1 - c^2
*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int x^2\cos^{-1}(x)\,dx &= \frac{1}{3}x^3\cos^{-1}(x)+\frac{1}{3}\int\frac{x^3}{\sqrt{1-x^2}}\,dx\\
&= \frac{1}{3}x^3\cos^{-1}(x)+\frac{1}{6}\operatorname{Subst}\left(\int\frac{x}{\sqrt{1-x}}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\cos^{-1}(x)+\frac{1}{6}\operatorname{Subst}\left(\int\left(\frac{1}{\sqrt{1-x}}-\sqrt{1-x}\right)dx,x,x^2\right)\\
&= -\frac{1}{3}\sqrt{1-x^2}+\frac{1}{9}\left(1-x^2\right)^{3/2}+\frac{1}{3}x^3\cos^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.0103854, size = 30, normalized size = 0.75

$$\frac{1}{3}x^3\cos^{-1}(x)-\frac{1}{9}\sqrt{1-x^2}\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*ArcCos[x],x]
```

```
[Out] -(Sqrt[1 - x^2]*(2 + x^2))/9 + (x^3*ArcCos[x])/3
```

Maple [A] time = 0.003, size = 34, normalized size = 0.9

$$\frac{x^3\arccos(x)}{3}-\frac{x^2}{9}\sqrt{-x^2+1}-\frac{2}{9}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccos(x),x)
```

```
[Out] 1/3*x^3*arccos(x)-1/9*x^2*(-x^2+1)^(1/2)-2/9*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.45287, size = 45, normalized size = 1.12

$$\frac{1}{3}x^3\arccos(x)-\frac{1}{9}\sqrt{-x^2+1}x^2-\frac{2}{9}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(x),x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccos(x) - 1/9*sqrt(-x^2 + 1)*x^2 - 2/9*sqrt(-x^2 + 1)
```

Fricas [A] time = 0.548079, size = 68, normalized size = 1.7

$$\frac{1}{3}x^3\arccos(x)-\frac{1}{9}\left(x^2+2\right)\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(x),x, algorithm="fricas")
```

```
[Out] 1/3*x^3*arccos(x) - 1/9*(x^2 + 2)*sqrt(-x^2 + 1)
```

Sympy [A] time = 0.330808, size = 32, normalized size = 0.8

$$\frac{x^3\operatorname{acos}(x)}{3}-\frac{x^2\sqrt{1-x^2}}{9}-\frac{2\sqrt{1-x^2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acos(x),x)
```

```
[Out] x**3*acos(x)/3 - x**2*sqrt(1 - x**2)/9 - 2*sqrt(1 - x**2)/9
```

Giac [A] time = 1.09278, size = 45, normalized size = 1.12

$$\frac{1}{3}\, x^3 \arccos\left(x\right) - \frac{1}{9}\, \sqrt{-x^2 + 1}x^2 - \frac{2}{9}\, \sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccos(x),x, algorithm="giac")
```

```
[Out] 1/3*x^3*arccos(x) - 1/9*sqrt(-x^2 + 1)*x^2 - 2/9*sqrt(-x^2 + 1)
```

3.94 $\int x \tan^{-1}(x)^2 \, dx$

Optimal. Leaf size=35

$$\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}x^2\tan^{-1}(x)^2+\frac{1}{2}\tan^{-1}(x)^2-x\tan^{-1}(x)$$

```
[Out] -(x*ArcTan[x]) + ArcTan[x]^2/2 + (x^2*ArcTan[x]^2)/2 + Log[1 + x^2]/2
```

Rubi [A] time = 0.0557313, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {4852, 4916, 4846, 260, 4884}

$$\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}x^2\tan^{-1}(x)^2+\frac{1}{2}\tan^{-1}(x)^2-x\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*ArcTan[x]^2,x]
```

```
[Out] -(x*ArcTan[x]) + ArcTan[x]^2/2 + (x^2*ArcTan[x]^2)/2 + Log[1 + x^2]/2
```

Rule 4852

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^p)/(d*(m + 1)), x] - Dist[(b*c*p
)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2*x^2
), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && (EqQ[p, 1] || Integ
erQ[m]) && NeQ[m, -1]
```

Rule 4916

```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Dist[f^2/e, Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x])
^p, x], x] - Dist[(d*f^2)/e, Int[((f*x)^(m - 2)*(a + b*ArcTan[c*x])^p)/(d +
 e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

Rule 4846

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b*Ar
cTan[c*x])^p, x] - Dist[b*c*p, Int[(x*(a + b*ArcTan[c*x])^(p - 1))/(1 + c^2
*x^2), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4884

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\tan^{-1}(x)^2\,dx &= \frac{1}{2}x^2\tan^{-1}(x)^2 - \int \frac{x^2\tan^{-1}(x)}{1+x^2}\,dx \\ &= \frac{1}{2}x^2\tan^{-1}(x)^2 - \int \tan^{-1}(x)\,dx + \int \frac{\tan^{-1}(x)}{1+x^2}\,dx \\ &= -x\tan^{-1}(x) + \frac{1}{2}\tan^{-1}(x)^2 + \frac{1}{2}x^2\tan^{-1}(x)^2 + \int \frac{x}{1+x^2}\,dx \\ &= -x\tan^{-1}(x) + \frac{1}{2}\tan^{-1}(x)^2 + \frac{1}{2}x^2\tan^{-1}(x)^2 + \frac{1}{2}\log\left(1+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0070251, size = 26, normalized size = 0.74

$$\frac{1}{2}\left(\log\left(x^2+1\right)+\left(x^2+1\right)\tan^{-1}(x)^2-2x\tan^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*ArcTan[x]^2,x]
```

```
[Out] (-2*x*ArcTan[x] + (1 + x^2)*ArcTan[x]^2 + Log[1 + x^2])/2
```

Maple [A] time = 0.009, size = 30, normalized size = 0.9

$$-x\arctan(x)+\frac{(\arctan(x))^2}{2}+\frac{x^2(\arctan(x))^2}{2}+\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arctan(x)^2,x)
```

```
[Out] -x*arctan(x)+1/2*arctan(x)^2+1/2*x^2*arctan(x)^2+1/2*ln(x^2+1)
```

Maxima [A] time = 1.46837, size = 46, normalized size = 1.31

$$\frac{1}{2}x^2\arctan(x)^2-(x-\arctan(x))\arctan(x)-\frac{1}{2}\arctan(x)^2+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan(x)^2 - (x - arctan(x))*arctan(x) - 1/2*arctan(x)^2 + 1/2*log
(x^2 + 1)
```

Fricas [A] time = 0.523172, size = 81, normalized size = 2.31

$$\frac{1}{2}\left(x^2+1\right)\arctan(x)^2-x\arctan(x)+\frac{1}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(x^2 + 1)*arctan(x)^2 - x*arctan(x) + 1/2*log(x^2 + 1)
```

Sympy [A] time = 0.351926, size = 29, normalized size = 0.83

$$\frac{x^2 \operatorname{atan}^2(x)}{2} - x \operatorname{atan}(x) + \frac{\log(x^2+1)}{2} + \frac{\operatorname{atan}^2(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*atan(x)**2,x)
```

```
[Out] x**2*atan(x)**2/2 - x*atan(x) + log(x**2 + 1)/2 + atan(x)**2/2
```

Giac [A] time = 1.07895, size = 39, normalized size = 1.11

$$\frac{1}{2}\, x^2 \arctan(x)^2 - x \arctan(x) + \frac{1}{2}\, \arctan(x)^2 + \frac{1}{2}\, \log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arctan(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2*arctan(x)^2 - x*arctan(x) + 1/2*arctan(x)^2 + 1/2*log(x^2 + 1)
```

3.95 $\int \tan^{-1}\left(\sqrt{x}\right) dx$

Optimal. Leaf size=22

$$-\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\tan^{-1}\left(\sqrt{x}\right)$$

```
[Out] -Sqrt[x] + ArcTan[Sqrt[x]] + x*ArcTan[Sqrt[x]]
```

Rubi [A] time = 0.0051936, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5027, 50, 63, 203}

$$-\sqrt{x}+x\tan^{-1}\left(\sqrt{x}\right)+\tan^{-1}\left(\sqrt{x}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]],x]

[Out] -Sqrt[x] + ArcTan[Sqrt[x]] + x*ArcTan[Sqrt[x]]
```

Rule 5027

```
Int[ArcTan[(c_.)*(x_)^(n_)], x_Symbol] :> Simp[x*ArcTan[c*x^n], x] - Dist[c
*n, Int[x^n/(1 + c^2*x^(2*n)), x], x] /; FreeQ[{c, n}, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \tan^{-1}\left(\sqrt{x}\right) dx &= x\tan^{-1}\left(\sqrt{x}\right) - \frac{1}{2}\int \frac{\sqrt{x}}{1+x}\, dx \\
&= -\sqrt{x} + x\tan^{-1}\left(\sqrt{x}\right) + \frac{1}{2}\int \frac{1}{\sqrt{x}(1+x)}\, dx \\
&= -\sqrt{x} + x\tan^{-1}\left(\sqrt{x}\right) + \text{Subst}\left(\int \frac{1}{1+x^2}\, dx, x, \sqrt{x}\right) \\
&= -\sqrt{x} + \tan^{-1}\left(\sqrt{x}\right) + x\tan^{-1}\left(\sqrt{x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0051495, size = 18, normalized size = 0.82

$$(x+1)\tan^{-1}\left(\sqrt{x}\right) - \sqrt{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]],x]
```

```
[Out] -Sqrt[x] + (1 + x)*ArcTan[Sqrt[x]]
```

Maple [A] time = 0.003, size = 17, normalized size = 0.8

$$\arctan\left(\sqrt{x}\right) + x\arctan\left(\sqrt{x}\right) - \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2)),x)
```

```
[Out] arctan(x^(1/2))+x*arctan(x^(1/2))-x^(1/2)
```

Maxima [A] time = 1.44288, size = 22, normalized size = 1.

$$x\arctan\left(\sqrt{x}\right) - \sqrt{x} + \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="maxima")
```

```
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

Fricas [A] time = 0.544107, size = 47, normalized size = 2.14

$$(x+1)\arctan\left(\sqrt{x}\right) - \sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="fricas")
```

```
[Out] (x + 1)*arctan(sqrt(x)) - sqrt(x)
```

Sympy [A] time = 1.35363, size = 19, normalized size = 0.86

$$-\sqrt{x} + x \operatorname{atan}\left(\sqrt{x}\right) + \operatorname{atan}\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2)),x)
```

```
[Out] -sqrt(x) + x*atan(sqrt(x)) + atan(sqrt(x))
```

Giac [A] time = 1.10465, size = 22, normalized size = 1.

$$x \arctan\left(\sqrt{x}\right) - \sqrt{x} + \arctan\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2)),x, algorithm="giac")
```

```
[Out] x*arctan(sqrt(x)) - sqrt(x) + arctan(sqrt(x))
```

3.96 $\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}(1+x)}\,dx$

Optimal. Leaf size=8

$$\tan^{-1}\left(\sqrt{x}\right)^2$$

```
[Out] ArcTan[Sqrt[x]]^2
```

Rubi [A] time = 0.0304373, antiderivative size = 8, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {63, 203, 6686}

$$\tan^{-1}\left(\sqrt{x}\right)^2$$

Antiderivative was successfully verified.

```
[In] Int[ArcTan[Sqrt[x]]/(Sqrt[x]*(1 + x)),x]

[Out] ArcTan[Sqrt[x]]^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 6686

```
Int[(u_)*(y_)^(m_.), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Si
mp[(q*y^(m + 1))/(m + 1), x] /;  !FalseQ[q]] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tan^{-1}\left(\sqrt{x}\right)}{\sqrt{x}(1+x)}\,dx = \tan^{-1}\left(\sqrt{x}\right)^2$$

Mathematica [A] time = 0.0055862, size = 8, normalized size = 1.

$$\tan^{-1}\left(\sqrt{x}\right)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcTan[Sqrt[x]]/(Sqrt[x]*(1 + x)),x]
```

```
[Out] ArcTan[Sqrt[x]]^2
```

Maple [A] time = 0.004, size = 7, normalized size = 0.9

$$\left(\arctan\left(\sqrt{x}\right)\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arctan(x^(1/2))/(1+x)/x^(1/2),x)
```

```
[Out] arctan(x^(1/2))^2
```

Maxima [A] time = 0.936507, size = 8, normalized size = 1.

$$\arctan\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/(1+x)/x^(1/2),x, algorithm="maxima")
```

```
[Out] arctan(sqrt(x))^2
```

Fricas [A] time = 0.519548, size = 26, normalized size = 3.25

$$\arctan\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/(1+x)/x^(1/2),x, algorithm="fricas")
```

```
[Out] arctan(sqrt(x))^2
```

Sympy [A] time = 2.04667, size = 7, normalized size = 0.88

$$\operatorname{atan}^2\left(\sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(atan(x**(1/2))/(1+x)/x**(1/2),x)
```

```
[Out] atan(sqrt(x))**2
```

Giac [A] time = 1.07531, size = 8, normalized size = 1.

$$\arctan\left(\sqrt{x}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arctan(x^(1/2))/(1+x)/x^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(x))^2
```

3.97 $\int \sqrt{1-x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{2}\sqrt{1-x^2}x+\frac{1}{2}\sin^{-1}(x)$$

```
[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rubi [A] time = 0.0025152, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 216}

$$\frac{1}{2}\sqrt{1-x^2}x+\frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2],x]

[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1-x^2}\,dx &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{1-x^2}+\frac{1}{2}\sin^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0049745, size = 20, normalized size = 0.87

$$\frac{1}{2}\left(\sqrt{1-x^2}x+\sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2],x]

[Out] (x*Sqrt[1 - x^2] + ArcSin[x])/2
```

Maple [A] time = 0.003, size = 18, normalized size = 0.8

$$\frac{\arcsin(x)}{2} + \frac{x}{2}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2),x)
```

```
[Out] 1/2*arcsin(x)+1/2*x*(-x^2+1)^(1/2)
```

Maxima [A] time = 1.45404, size = 23, normalized size = 1.

$$\frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

Fricas [A] time = 0.514275, size = 74, normalized size = 3.22

$$\frac{1}{2}\sqrt{-x^2+1}x - \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x - arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [A] time = 0.193409, size = 15, normalized size = 0.65

$$\frac{x\sqrt{1-x^2}}{2} + \frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2),x)
```

```
[Out] x*sqrt(1 - x**2)/2 + asin(x)/2
```

Giac [A] time = 1.14678, size = 23, normalized size = 1.

$$\frac{1}{2}\sqrt{-x^2+1}x + \frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 1)*x + 1/2*arcsin(x)
```

3.98 $\int \frac{e^{\tan^{-1}(x)}x}{(1+x^2)^{3/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{(1-x)e^{\tan^{-1}(x)}}{2\sqrt{x^2+1}}$$

```
[Out] -(E^ArcTan[x]*(1 - x))/(2*Sqrt[1 + x^2])
```

Rubi [A] time = 0.0412641, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5077}

$$-\frac{(1-x)e^{\tan^{-1}(x)}}{2\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(E^ArcTan[x]*x)/(1 + x^2)^(3/2),x]
```

```
[Out] -(E^ArcTan[x]*(1 - x))/(2*Sqrt[1 + x^2])
```

Rule 5077

```
Int[(E^(ArcTan[(a_.)*(x_)]*(n_.))*(x_))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symb
ol] :> -Simp[((1 - a*n*x)*E^(n*ArcTan[a*x]))/(d*(n^2 + 1)*Sqrt[c + d*x^2]),
 x] /; FreeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] &&  !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(x)}x}{\left(1+x^2\right)^{3/2}}\,dx = -\frac{e^{\tan^{-1}(x)}(1-x)}{2\sqrt{1+x^2}}$$

Mathematica [C] time = 0.0099006, size = 37, normalized size = 1.68

$$\frac{1}{2}(1-ix)^{-\frac{1}{2}+\frac{i}{2}}(1+ix)^{-\frac{1}{2}-\frac{i}{2}}(x-1)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(E^ArcTan[x]*x)/(1 + x^2)^(3/2),x]
```

```
[Out] (-1 + x)/(2*(1 - I*x)^(1/2 - I/2)*(1 + I*x)^(1/2 + I/2))
```

Maple [A] time = 0.003, size = 16, normalized size = 0.7

$$\frac{(-1+x)\,e^{\arctan(x)}}{2}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(x))*x/(x^2+1)^(3/2),x)
```

```
[Out] 1/2*(-1+x)*exp(arctan(x))/(x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^{\arctan(x)}}{(x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))*x/(x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*e^arctan(x)/(x^2 + 1)^(3/2), x)
```

Fricas [A] time = 0.664137, size = 53, normalized size = 2.41

$$\frac{(x-1)e^{\arctan(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))*x/(x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x - 1)*e^arctan(x)/sqrt(x^2 + 1)
```

Sympy [A] time = 119.363, size = 31, normalized size = 1.41

$$\frac{xe^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}} - \frac{e^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(x))*x/(x**2+1)**(3/2),x)
```

```
[Out] x*exp(atan(x))/(2*sqrt(x**2 + 1)) - exp(atan(x))/(2*sqrt(x**2 + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{xe^{\arctan(x)}}{(x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))*x/(x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*e^arctan(x)/(x^2 + 1)^(3/2), x)
```

3.99 $\int \frac{e^{\tan^{-1}(x)}}{(1+x^2)^{3/2}} dx$

Optimal. Leaf size=20

$$\frac{(x+1)e^{\tan^{-1}(x)}}{2\sqrt{x^2+1}}$$

```
[Out] (E^ArcTan[x]*(1 + x))/(2*Sqrt[1 + x^2])
```

Rubi [A] time = 0.0230173, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {5069}

$$\frac{(x+1)e^{\tan^{-1}(x)}}{2\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[E^ArcTan[x]/(1 + x^2)^(3/2),x]

[Out] (E^ArcTan[x]*(1 + x))/(2*Sqrt[1 + x^2])
```

Rule 5069

```
Int[E^(ArcTan[(a_.)*(x_)]*(n_.))/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :>
Simp[((n + a*x)*E^(n*ArcTan[a*x]))/(a*c*(n^2 + 1)*Sqrt[c + d*x^2]), x] /; F
reeQ[{a, c, d, n}, x] && EqQ[d, a^2*c] && !IntegerQ[I*n]
```

Rubi steps

$$\int \frac{e^{\tan^{-1}(x)}}{(1+x^2)^{3/2}} dx = \frac{e^{\tan^{-1}(x)}(1+x)}{2\sqrt{1+x^2}}$$

Mathematica [A] time = 0.0054082, size = 20, normalized size = 1.

$$\frac{(x+1)e^{\tan^{-1}(x)}}{2\sqrt{x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^ArcTan[x]/(1 + x^2)^(3/2),x]

[Out] (E^ArcTan[x]*(1 + x))/(2*Sqrt[1 + x^2])
```

Maple [A] time = 0.003, size = 16, normalized size = 0.8

$$\frac{e^{\arctan(x)}(1+x)}{2}\frac{1}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(arctan(x))/(x^2+1)^(3/2),x)
```

```
[Out] 1/2*exp(arctan(x))*(1+x)/(x^2+1)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\arctan(x)}}{\left(x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))/(x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(e^arctan(x)/(x^2 + 1)^(3/2), x)
```

Fricas [A] time = 0.930216, size = 53, normalized size = 2.65

$$\frac{(x+1)e^{\arctan(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))/(x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x + 1)*e^arctan(x)/sqrt(x^2 + 1)
```

Sympy [A] time = 117.73, size = 31, normalized size = 1.55

$$\frac{xe^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}}+\frac{e^{\operatorname{atan}(x)}}{2\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(atan(x))/(x**2+1)**(3/2),x)
```

```
[Out] x*exp(atan(x))/(2*sqrt(x**2 + 1)) + exp(atan(x))/(2*sqrt(x**2 + 1))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{\arctan(x)}}{\left(x^2+1\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(arctan(x))/(x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(e^arctan(x)/(x^2 + 1)^(3/2), x)
```

3.100 $\int \frac{x^2}{\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=19

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2\left(x^2+1\right)}$$

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rubi [A] time = 0.0036853, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {288, 203}

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(1 + x^2)^2,x]

[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{\left(1+x^2\right)^2} \, dx &= -\frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\int \frac{1}{1+x^2} \, dx \\ &= -\frac{x}{2\left(1+x^2\right)} + \frac{1}{2}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0042312, size = 19, normalized size = 1.

$$\frac{1}{2}\tan^{-1}(x) - \frac{x}{2\left(x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(1 + x^2)^2,x]
```

```
[Out] -x/(2*(1 + x^2)) + ArcTan[x]/2
```

Maple [A] time = 0.007, size = 16, normalized size = 0.8

$$-\frac{x}{2\,x^2+2}+\frac{\arctan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)^2,x)
```

```
[Out] -1/2*x/(x^2+1)+1/2*arctan(x)
```

Maxima [A] time = 1.44258, size = 20, normalized size = 1.05

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

Fricas [A] time = 0.632103, size = 55, normalized size = 2.89

$$\frac{\left(x^2+1\right)\arctan(x)-x}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((x^2 + 1)*arctan(x) - x)/(x^2 + 1)
```

Sympy [A] time = 0.094428, size = 12, normalized size = 0.63

$$-\frac{x}{2x^2+2}+\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)**2,x)
```

```
[Out] -x/(2*x**2 + 2) + atan(x)/2
```

Giac [A] time = 1.12342, size = 20, normalized size = 1.05

$$-\frac{x}{2\left(x^2+1\right)}+\frac{1}{2}\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 + 1) + 1/2*arctan(x)
```

3.101 $\int \frac{e^x}{1+e^{2x}}\,dx$

Optimal. Leaf size=4

$$\tan^{-1}\left(e^x\right)$$

```
[Out] ArcTan[E^x]
```

Rubi [A] time = 0.0162207, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2249, 203}

$$\tan^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^x/(1 + E^(2*x)),x]
```

```
[Out] ArcTan[E^x]
```

Rule 2249

```
Int[((a_) + (b_.)*(F_)^((e_.)*((c_.) + (d_.)*(x_))))^(p_.)*(G_)^((h_.)*((f_
.) + (g_.)*(x_))), x_Symbol] :> With[{m = FullSimplify[(d*e*Log[F])/(g*h*Lo
g[G])]}, Dist[Denominator[m]/(g*h*Log[G]), Subst[Int[x^(Denominator[m] - 1)
*(a + b*F^(c*e - (d*e*f)/g)*x^Numerator[m])^p, x], x, G^((h*(f + g*x))/Deno
minator[m])], x] /; LtQ[m, -1] || GtQ[m, 1]] /; FreeQ[{F, G, a, b, c, d, e,
 f, g, h, p}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{e^x}{1+e^{2x}}\,dx &= \text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, e^x\right)\\ &= \tan^{-1}\left(e^x\right)\end{aligned}$$

Mathematica [A] time = 0.0025719, size = 4, normalized size = 1.

$$\tan^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^x/(1 + E^(2*x)),x]
```

```
[Out] ArcTan[E^x]
```

Maple [A] time = 0.003, size = 4, normalized size = 1.

$$\arctan\left(\mathrm{e}^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(x)/(1+exp(2*x)),x)
```

```
[Out] arctan(exp(x))
```

Maxima [A] time = 1.43357, size = 4, normalized size = 1.

$$\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(2*x)),x, algorithm="maxima")
```

```
[Out] arctan(e^x)
```

Fricas [A] time = 0.745567, size = 18, normalized size = 4.5

$$\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(2*x)),x, algorithm="fricas")
```

```
[Out] arctan(e^x)
```

Sympy [B] time = 0.106622, size = 15, normalized size = 3.75

$$\operatorname{RootSum}\left(4z^{2}+1,\left(i\mapsto i\log\left(2i+e^{x}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(2*x)),x)
```

```
[Out] RootSum(4*_z**2 + 1, Lambda(_i, _i*log(2*_i + exp(x))))
```

Giac [A] time = 1.06983, size = 4, normalized size = 1.

$$\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(x)/(1+exp(2*x)),x, algorithm="giac")
```

```
[Out] arctan(e^x)
```

3.102 $\int e^{-x} \cot^{-1}(e^x)\, dx$

Optimal. Leaf size=27

$$-x+\frac{1}{2}\log\left(e^{2x}+1\right)-e^{-x}\cot^{-1}\left(e^x\right)$$

```
[Out] -x - ArcCot[E^x]/E^x + Log[1 + E^(2*x)]/2
```

Rubi [A] time = 0.0180276, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {2194, 5208, 2282, 36, 29, 31}

$$-x+\frac{1}{2}\log\left(e^{2x}+1\right)-e^{-x}\cot^{-1}\left(e^x\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcCot[E^x]/E^x,x]
```

```
[Out] -x - ArcCot[E^x]/E^x + Log[1 + E^(2*x)]/2
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 5208

```
Int[((a_.) + ArcCot[u_]*(b_.))*(v_), x_Symbol] :> With[{w = IntHide[v, x]},
 Dist[a + b*ArcCot[u], w, x] + Dist[b, Int[SimplifyIntegrand[(w*D[u, x])/(1
 + u^2), x], x], x] /; InverseFunctionFreeQ[w, x]] /; FreeQ[{a, b}, x] && I
nverseFunctionFreeQ[u, x] && !MatchQ[v, ((c_.) + (d_.)*x)^(m_.) /; FreeQ[{
c, d, m}, x]] && FalseQ[FunctionOfLinear[v*(a + b*ArcCot[u]), x]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int e^{-x}\cot^{-1}(e^x)\,dx &= -e^{-x}\cot^{-1}(e^x)-\int \frac{1}{1+e^{2x}}\,dx\\
&= -e^{-x}\cot^{-1}(e^x)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx,x,e^{2x}\right)\\
&= -e^{-x}\cot^{-1}(e^x)-\frac{1}{2}\text{Subst}\left(\int \frac{1}{x}\,dx,x,e^{2x}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,e^{2x}\right)\\
&= -x-e^{-x}\cot^{-1}(e^x)+\frac{1}{2}\log\left(1+e^{2x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.016478, size = 27, normalized size = 1.

$$-x+\frac{1}{2}\log\left(e^{2x}+1\right)-e^{-x}\cot^{-1}(e^x)$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCot[E^x]/E^x,x]

[Out] -x - ArcCot[E^x]/E^x + Log[1 + E^(2*x)]/2
```

Maple [A] time = 0.005, size = 25, normalized size = 0.9

$$-\frac{\operatorname{arccot}(e^x)}{e^x}+\frac{\ln\left((e^x)^2+1\right)}{2}-\ln(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccot(exp(x))/exp(x),x)

[Out] -arccot(exp(x))/exp(x)+1/2*ln(exp(x)^2+1)-ln(exp(x))
```

Maxima [A] time = 0.967218, size = 26, normalized size = 0.96

$$-\operatorname{arccot}(e^x)e^{(-x)}+\frac{1}{2}\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="maxima")

[Out] -arccot(e^x)*e^(-x) + 1/2*log(e^(-2*x) + 1)
```

Fricas [A] time = 0.853065, size = 84, normalized size = 3.11

$$-\frac{1}{2}\left(2\,xe^x-e^x\log\left(e^{(2x)}+1\right)+2\,\operatorname{arccot}(e^x)\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*e^x - e^x*log(e^(2*x) + 1) + 2*arccot(e^x))*e^(-x)
```

Sympy [A] time = 12.1554, size = 19, normalized size = 0.7

$$-x+\frac{\log\left(e^{2x}+1\right)}{2}-e^{-x}\operatorname{acot}\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acot(exp(x))/exp(x),x)
```

```
[Out] -x + log(exp(2*x) + 1)/2 - exp(-x)*acot(exp(x))
```

Giac [A] time = 1.08302, size = 28, normalized size = 1.04

$$-\arctan\left(e^{(-x)}\right)e^{(-x)}+\frac{1}{2}\log\left(e^{(-2x)}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccot(exp(x))/exp(x),x, algorithm="giac")
```

```
[Out] -arctan(e^(-x))*e^(-x) + 1/2*log(e^(-2*x) + 1)
```

3.103 $\int \sqrt{\frac{a+x}{a-x}}\,dx$

Optimal. Leaf size=42

$$2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

```
[Out] -((a - x)*Sqrt[(a + x)/(a - x)]) + 2*a*ArcTan[Sqrt[(a + x)/(a - x)]]
```

Rubi [A] time = 0.0132306, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1959, 288, 203}

$$2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(a + x)/(a - x)],x]
```

```
[Out] -((a - x)*Sqrt[(a + x)/(a - x)]) + 2*a*ArcTan[Sqrt[(a + x)/(a - x)]]
```

Rule 1959

```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(
x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x],
x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x] &
& FractionQ[p] && IntegerQ[1/n]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sqrt{\frac{a+x}{a-x}}\,dx &= (4a)\,\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^2}\,dx, x, \sqrt{\frac{a+x}{a-x}}\right)\\ &= -(a-x)\sqrt{\frac{a+x}{a-x}}+(2a)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{\frac{a+x}{a-x}}\right)\\ &= -(a-x)\sqrt{\frac{a+x}{a-x}}+2a\tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)\end{aligned}$$

Mathematica [A] time = 0.0367801, size = 78, normalized size = 1.86

$$\frac{\sqrt{\frac{a+x}{a-x}}\left(-2a^{3/2}\sqrt{a-x}\sqrt{\frac{a+x}{a}}\sin^{-1}\left(\frac{\sqrt{a-x}}{\sqrt{2}\sqrt{a}}\right)-a^2+x^2\right)}{a+x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(a + x)/(a - x)],x]
```

```
[Out] (Sqrt[(a + x)/(a - x)]*(-a^2 + x^2 - 2*a^(3/2)*Sqrt[a - x]*Sqrt[(a + x)/a]*
ArcSin[Sqrt[a - x]/(Sqrt[2]*Sqrt[a])]))/(a + x)
```

Maple [A] time = 0.013, size = 64, normalized size = 1.5

$$-(-a+x)\sqrt{-\frac{a+x}{-a+x}}\left(a\arctan\left(x\frac{1}{\sqrt{a^2-x^2}}\right)-\sqrt{a^2-x^2}\right)\frac{1}{\sqrt{-(a+x)(-a+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a+x)/(a-x))^(1/2),x)
```

```
[Out] -(-(a+x)/(-a+x))^(1/2)*(-a+x)*(a*arctan(x/(a^2-x^2)^(1/2))-(a^2-x^2)^(1/2))
/(-(a+x)*(-a+x))^(1/2)
```

Maxima [A] time = 1.41068, size = 66, normalized size = 1.57

$$-2\,a\left(\frac{\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}-\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*(sqrt((a + x)/(a - x))/((a + x)/(a - x) + 1) - arctan(sqrt((a + x)/(a
- x))))
```

Fricas [A] time = 0.831605, size = 90, normalized size = 2.14

$$2\,a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)-(a-x)\sqrt{\frac{a+x}{a-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="fricas")
```

```
[Out] 2*a*arctan(sqrt((a + x)/(a - x))) - (a - x)*sqrt((a + x)/(a - x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\frac{a+x}{a-x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))**(1/2),x)
```

```
[Out] Integral(sqrt((a + x)/(a - x)), x)
```

Giac [A] time = 1.11132, size = 49, normalized size = 1.17

$$a \arcsin\left(\frac{x}{a}\right) \operatorname{sgn}(a-x) \operatorname{sgn}(a) - \sqrt{a^2 - x^2}\operatorname{sgn}(a-x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((a+x)/(a-x))^(1/2),x, algorithm="giac")
```

```
[Out] a*arcsin(x/a)*sgn(a - x)*sgn(a) - sqrt(a^2 - x^2)*sgn(a - x)
```

3.104 $\int \sqrt{(b-x)(-a+x)}\,dx$

Optimal. Leaf size=71

$$-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

```
[Out] -((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2])/4 - ((a - b)^2*ArcTan[(a +
b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```

Rubi [A] time = 0.0222152, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1981, 612, 621, 204}

$$-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2}-\frac{1}{8}(a-b)^2\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2])/4 - ((a - b)^2*ArcTan[(a +
b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])])/8
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{(b-x)(-a+x)}\,dx = \int \sqrt{-ab+(a+b)x-x^2}\,dx$$
$$= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2} + \frac{1}{8}(a-b)^2 \int \frac{1}{\sqrt{-ab+(a+b)x-x^2}}\,dx$$
$$= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2} + \frac{1}{4}(a-b)^2\,\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{a+b-2x}{\sqrt{-ab+(a+b)x}}\right.$$
$$= -\frac{1}{4}(a+b-2x)\sqrt{-ab+(a+b)x-x^2} - \frac{1}{8}(a-b)^2 \tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)$$

Mathematica [A] time = 0.146249, size = 106, normalized size = 1.49

$$\frac{(a-x)\left((a-b)^{5/2}\sqrt{b-x}\sqrt{\frac{a-x}{a-b}}\sinh^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{a-b}}\right)-(a-x)(b-x)(a+b-2x)\right)}{4(x-a)\sqrt{(a-x)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] ((a - x)*(-((a + b - 2*x)*(a - x)*(b - x)) + (a - b)^(5/2)*Sqrt[(a - x)/(a
- b)]*Sqrt[b - x]*ArcSinh[Sqrt[b - x]/Sqrt[a - b]]))/(4*(-a + x)*Sqrt[(a -
x)*(-b + x)])
```

Maple [A] time = 0.011, size = 122, normalized size = 1.7

$$-\frac{a+b-2\,x}{4}\sqrt{-ab+(a+b)\,x-x^2} - \frac{ab}{4}\arctan\left(\left(x-\frac{b}{2}-\frac{a}{2}\right)\frac{1}{\sqrt{-ab+(a+b)\,x-x^2}}\right) + \frac{a^2}{8}\arctan\left(\left(x-\frac{b}{2}-\frac{a}{2}\right)\frac{}{\sqrt{-a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b-x)*(-a+x))^(1/2),x)
```

```
[Out] -1/4*(a+b-2*x)*(-a*b+(a+b)*x-x^2)^(1/2)-1/4*arctan((x-1/2*b-1/2*a)/(-a*b+(a
+b)*x-x^2)^(1/2))*a*b+1/8*arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))*
a^2+1/8*arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.88736, size = 209, normalized size = 2.94

$$-\frac{1}{8}\left(a^{2}-2\,ab+b^{2}\right)\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^{2}}(a+b-2\,x)}{2\left(ab-(a+b)x+x^{2}\right)}\right)-\frac{1}{4}\,\sqrt{-ab+(a+b)x-x^{2}}(a+b-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -1/8*(a^2 - 2*a*b + b^2)*arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b -
2*x)/(a*b - (a + b)*x + x^2)) - 1/4*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2
*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{(-a+x)\,(b-x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Integral(sqrt((-a + x)*(b - x)), x)
```

Giac [A] time = 1.08998, size = 82, normalized size = 1.15

$$\frac{1}{8}\left(a^{2}-2\,ab+b^{2}\right)\arcsin\left(\frac{a+b-2\,x}{a-b}\right)\operatorname{sgn}\left(-a+b\right)-\frac{1}{4}\,\sqrt{-ab+ax+bx-x^{2}}(a+b-2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(a^2 - 2*a*b + b^2)*arcsin((a + b - 2*x)/(a - b))*sgn(-a + b) - 1/4*sqr
t(-a*b + a*x + b*x - x^2)*(a + b - 2*x)
```

3.105 $\int \frac{1}{\sqrt{(b-x)(-a+x)}} \, dx$

Optimal. Leaf size=32

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

```
[Out] -ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

Rubi [A] time = 0.0120996, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1981, 621, 204}

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] -ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]
```

Rule 1981

```
Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && Q
uadraticQ[u, x] &&  !QuadraticMatchQ[u, x]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{(b-x)(-a+x)}} \, dx &= \int \frac{1}{\sqrt{-ab+(a+b)x-x^2}} \, dx\\ &= 2\,\text{Subst}\left(\int \frac{1}{-4-x^2} \, dx, x, \frac{a+b-2x}{\sqrt{-ab+(a+b)x-x^2}}\right)\\ &= -\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab+(a+b)x-x^2}}\right)\end{aligned}$$

Mathematica [B] time = 0.0246584, size = 72, normalized size = 2.25

$$-\frac{2\sqrt{a-b}\sqrt{b-x}\sqrt{\frac{a-x}{a-b}}\sinh^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{a-b}}\right)}{\sqrt{(a-x)(x-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[(b - x)*(-a + x)],x]
```

```
[Out] (-2*Sqrt[a - b]*Sqrt[(a - x)/(a - b)]*Sqrt[b - x]*ArcSinh[Sqrt[b - x]/Sqrt[
a - b]])/Sqrt[(a - x)*(-b + x)]
```

Maple [A] time = 0.003, size = 28, normalized size = 0.9

$$\arctan\left(\left(x-\frac{b}{2}-\frac{a}{2}\right)\frac{1}{\sqrt{-ab+(a+b)\,x-x^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b-x)*(-a+x))^(1/2),x)
```

```
[Out] arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.745893, size = 111, normalized size = 3.47

$$-\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}(a+b-2\,x)}{2\left(ab-(a+b)x+x^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="fricas")
```

```
[Out] -arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2*x)/(a*b - (a + b)*x +
x^2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{(-a+x)\,(b-x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))**(1/2),x)
```

```
[Out] Integral(1/sqrt((-a + x)*(b - x)), x)
```

Giac [A] time = 1.12605, size = 30, normalized size = 0.94

$$\arcsin\left(\frac{a+b-2x}{a-b}\right)\operatorname{sgn}\left(-a+b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="giac")
```

```
[Out] arcsin((a + b - 2*x)/(a - b))*sgn(-a + b)
```

3.106 $\int \frac{3+5x}{-3+2x+x^2} dx$

Optimal. Leaf size=15

$$2\log(1-x)+3\log(x+3)$$

```
[Out] 2*Log[1 - x] + 3*Log[3 + x]
```

Rubi [A] time = 0.004191, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {632, 31}

$$2\log(1-x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 5*x)/(-3 + 2*x + x^2),x]
```

```
[Out] 2*Log[1 - x] + 3*Log[3 + x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{3+5x}{-3+2x+x^2}\,dx &= 2\int \frac{1}{-1+x}\,dx+3\int \frac{1}{3+x}\,dx\\ &= 2\log(1-x)+3\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.0034669, size = 15, normalized size = 1.

$$2\log(1-x)+3\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 5*x)/(-3 + 2*x + x^2),x]
```

```
[Out] 2*Log[1 - x] + 3*Log[3 + x]
```

Maple [A] time = 0.005, size = 14, normalized size = 0.9

$$2\ln(-1+x)+3\ln(3+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+5*x)/(x^2+2*x-3),x)
```

```
[Out] 2*ln(-1+x)+3*ln(3+x)
```

Maxima [A] time = 0.929475, size = 18, normalized size = 1.2

$$3\log(x+3)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*x)/(x^2+2*x-3),x, algorithm="maxima")
```

```
[Out] 3*log(x + 3) + 2*log(x - 1)
```

Fricas [A] time = 0.673259, size = 39, normalized size = 2.6

$$3\log(x+3)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*x)/(x^2+2*x-3),x, algorithm="fricas")
```

```
[Out] 3*log(x + 3) + 2*log(x - 1)
```

Sympy [A] time = 0.093474, size = 12, normalized size = 0.8

$$2\log(x-1)+3\log(x+3)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*x)/(x**2+2*x-3),x)
```

```
[Out] 2*log(x - 1) + 3*log(x + 3)
```

Giac [A] time = 1.07378, size = 20, normalized size = 1.33

$$3\log(|x+3|)+2\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+5*x)/(x^2+2*x-3),x, algorithm="giac")
```

```
[Out] 3*log(abs(x + 3)) + 2*log(abs(x - 1))
```

3.107 $\int \frac{5+2x}{-3+2x+x^2}\,dx$

Optimal. Leaf size=19

$$\frac{7}{4}\log(1-x)+\frac{1}{4}\log(x+3)$$

```
[Out] (7*Log[1 - x])/4 + Log[3 + x]/4
```

Rubi [A] time = 0.0044599, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {632, 31}

$$\frac{7}{4}\log(1-x)+\frac{1}{4}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[(5 + 2*x)/(-3 + 2*x + x^2),x]

[Out] (7*Log[1 - x])/4 + Log[3 + x]/4
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{5+2x}{-3+2x+x^2}\,dx &= \frac{1}{4}\int \frac{1}{3+x}\,dx+\frac{7}{4}\int \frac{1}{-1+x}\,dx\\ &= \frac{7}{4}\log(1-x)+\frac{1}{4}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.0032404, size = 19, normalized size = 1.

$$\frac{7}{4}\log(1-x)+\frac{1}{4}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 2*x)/(-3 + 2*x + x^2),x]

[Out] (7*Log[1 - x])/4 + Log[3 + x]/4
```

Maple [A] time = 0.004, size = 14, normalized size = 0.7

$$\frac{7\ln(-1+x)}{4}+\frac{\ln(3+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5+2*x)/(x^2+2*x-3),x)
```

```
[Out] 7/4*ln(-1+x)+1/4*ln(3+x)
```

Maxima [A] time = 0.931335, size = 18, normalized size = 0.95

$$\frac{1}{4}\log(x+3)+\frac{7}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(x^2+2*x-3),x, algorithm="maxima")
```

```
[Out] 1/4*log(x + 3) + 7/4*log(x - 1)
```

Fricas [A] time = 0.635538, size = 45, normalized size = 2.37

$$\frac{1}{4}\log(x+3)+\frac{7}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(x^2+2*x-3),x, algorithm="fricas")
```

```
[Out] 1/4*log(x + 3) + 7/4*log(x - 1)
```

Sympy [A] time = 0.096212, size = 14, normalized size = 0.74

$$\frac{7\log(x-1)}{4}+\frac{\log(x+3)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(x**2+2*x-3),x)
```

```
[Out] 7*log(x - 1)/4 + log(x + 3)/4
```

Giac [A] time = 1.0666, size = 20, normalized size = 1.05

$$\frac{1}{4}\log(|x+3|)+\frac{7}{4}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+2*x)/(x^2+2*x-3),x, algorithm="giac")

[Out] 1/4*log(abs(x + 3)) + 7/4*log(abs(x - 1))
```

3.108 $\int \frac{3x+x^3}{-3-2x+x^2}\,dx$

Optimal. Leaf size=23

$$\frac{x^2}{2}+2x+9\log(3-x)+\log(x+1)$$

```
[Out] 2*x + x^2/2 + 9*Log[3 - x] + Log[1 + x]
```

Rubi [A] time = 0.0267258, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1593, 1628, 632, 31}

$$\frac{x^2}{2}+2x+9\log(3-x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(3*x + x^3)/(-3 - 2*x + x^2),x]
```

```
[Out] 2*x + x^2/2 + 9*Log[3 - x] + Log[1 + x]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{3x+x^3}{-3-2x+x^2}\,dx &= \int \frac{x\left(3+x^2\right)}{-3-2x+x^2}\,dx \\
&= \int \left(2+x+\frac{2(3+5x)}{-3-2x+x^2}\right) dx \\
&= 2x+\frac{x^2}{2}+2\int \frac{3+5x}{-3-2x+x^2}\,dx \\
&= 2x+\frac{x^2}{2}+9\int \frac{1}{-3+x}\,dx+\int \frac{1}{1+x}\,dx \\
&= 2x+\frac{x^2}{2}+9\log(3-x)+\log(1+x)
\end{aligned}
$$

Mathematica [A] time = 0.0039116, size = 23, normalized size = 1.

$$\frac{x^2}{2}+2x+9\log(3-x)+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3*x + x^3)/(-3 - 2*x + x^2),x]
```

```
[Out] 2*x + x^2/2 + 9*Log[3 - x] + Log[1 + x]
```

Maple [A] time = 0.004, size = 20, normalized size = 0.9

$$\frac{x^2}{2}+2\,x+\ln\left(1+x\right)+9\,\ln\left(-3+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+3*x)/(x^2-2*x-3),x)
```

```
[Out] 1/2*x^2+2*x+ln(1+x)+9*ln(-3+x)
```

Maxima [A] time = 0.943727, size = 26, normalized size = 1.13

$$\frac{1}{2}\,x^2+2\,x+\log\left(x+1\right)+9\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x)/(x^2-2*x-3),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + 2*x + log(x + 1) + 9*log(x - 3)
```

Fricas [A] time = 0.727047, size = 58, normalized size = 2.52

$$\frac{1}{2}\,x^2+2\,x+\log\left(x+1\right)+9\,\log\left(x-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x)/(x^2-2*x-3),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 + 2*x + log(x + 1) + 9*log(x - 3)
```

Sympy [A] time = 0.102927, size = 19, normalized size = 0.83

$$\frac{x^2}{2} + 2x + 9\log(x - 3) + \log(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+3*x)/(x**2-2*x-3),x)
```

```
[Out] x**2/2 + 2*x + 9*log(x - 3) + log(x + 1)
```

Giac [A] time = 1.07347, size = 28, normalized size = 1.22

$$\frac{1}{2}x^2 + 2x + \log(|x + 1|) + 9\log(|x - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+3*x)/(x^2-2*x-3),x, algorithm="giac")
```

```
[Out] 1/2*x^2 + 2*x + log(abs(x + 1)) + 9*log(abs(x - 3))
```

3.109 $\int \frac{-1+5x+2x^2}{-2x+x^2+x^3} \, dx$

Optimal. Leaf size=23

$$2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(x+2)$$

```
[Out] 2*Log[1 - x] + Log[x]/2 - Log[2 + x]/2
```

Rubi [A] time = 0.033277, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {1594, 1628}

$$2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 5*x + 2*x^2)/(-2*x + x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x]/2 - Log[2 + x]/2
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+5x+2x^2}{-2x+x^2+x^3} \, dx &= \int \frac{-1+5x+2x^2}{x\left(-2+x+x^2\right)} \, dx \\
&= \int \left(\frac{2}{-1+x}+\frac{1}{2x}-\frac{1}{2(2+x)}\right) dx \\
&= 2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(2+x)
\end{aligned}$$

Mathematica [A] time = 0.0049563, size = 23, normalized size = 1.

$$2\log(1-x)+\frac{\log(x)}{2}-\frac{1}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 5*x + 2*x^2)/(-2*x + x^2 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[x]/2 - Log[2 + x]/2
```

Maple [A] time = 0.007, size = 18, normalized size = 0.8

$$-\frac{\ln(2+x)}{2}+\frac{\ln(x)}{2}+2\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2+5*x-1)/(x^3+x^2-2*x),x)
```

```
[Out] -1/2*ln(2+x)+1/2*ln(x)+2*ln(-1+x)
```

Maxima [A] time = 0.929451, size = 23, normalized size = 1.

$$-\frac{1}{2}\log(x+2)+2\log(x-1)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5*x-1)/(x^3+x^2-2*x),x, algorithm="maxima")
```

```
[Out] -1/2*log(x + 2) + 2*log(x - 1) + 1/2*log(x)
```

Fricas [A] time = 0.654882, size = 61, normalized size = 2.65

$$-\frac{1}{2}\log(x+2)+2\log(x-1)+\frac{1}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5*x-1)/(x^3+x^2-2*x),x, algorithm="fricas")
```

```
[Out] -1/2*log(x + 2) + 2*log(x - 1) + 1/2*log(x)
```

Sympy [A] time = 0.126741, size = 17, normalized size = 0.74

$$\frac{\log(x)}{2}+2\log(x-1)-\frac{\log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2+5*x-1)/(x**3+x**2-2*x),x)
```

```
[Out] log(x)/2 + 2*log(x - 1) - log(x + 2)/2
```

Giac [A] time = 1.09716, size = 27, normalized size = 1.17

$$-\frac{1}{2}\log(|x+2|)+2\log(|x-1|)+\frac{1}{2}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2+5*x-1)/(x^3+x^2-2*x),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(x + 2)) + 2*log(abs(x - 1)) + 1/2*log(abs(x))
```

3.110 $\int \frac{3+2x+x^2}{(-1+x)(1+x)^2}\,dx$

Optimal. Leaf size=24

$$\frac{1}{x+1}+\frac{3}{2}\log(1-x)-\frac{1}{2}\log(x+1)$$

```
[Out] (1 + x)^(-1) + (3*Log[1 - x])/2 - Log[1 + x]/2
```

Rubi [A] time = 0.0143797, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {893}

$$\frac{1}{x+1}+\frac{3}{2}\log(1-x)-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x + x^2)/((-1 + x)*(1 + x)^2),x]
```

```
[Out] (1 + x)^(-1) + (3*Log[1 - x])/2 - Log[1 + x]/2
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x+x^2}{(-1+x)(1+x)^2}\,dx &= \int \left(\frac{3}{2(-1+x)}-\frac{1}{(1+x)^2}-\frac{1}{2(1+x)}\right)dx\\ &= \frac{1}{1+x}+\frac{3}{2}\log(1-x)-\frac{1}{2}\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0083749, size = 22, normalized size = 0.92

$$\frac{1}{x+1}+\frac{3}{2}\log(x-1)-\frac{1}{2}\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x + x^2)/((-1 + x)*(1 + x)^2),x]
```

```
[Out] (1 + x)^(-1) + (3*Log[-1 + x])/2 - Log[1 + x]/2
```

Maple [A] time = 0.007, size = 19, normalized size = 0.8

$$(1+x)^{-1} - \frac{\ln(1+x)}{2} + \frac{3\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+2*x+3)/(-1+x)/(1+x)^2,x)
```

```
[Out] 1/(1+x)-1/2*ln(1+x)+3/2*ln(-1+x)
```

Maxima [A] time = 0.947586, size = 24, normalized size = 1.

$$\frac{1}{x+1} - \frac{1}{2}\log(x+1) + \frac{3}{2}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+3)/(-1+x)/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/(x + 1) - 1/2*log(x + 1) + 3/2*log(x - 1)
```

Fricas [A] time = 0.666378, size = 84, normalized size = 3.5

$$-\frac{(x+1)\log(x+1) - 3(x+1)\log(x-1) - 2}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+3)/(-1+x)/(1+x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((x + 1)*log(x + 1) - 3*(x + 1)*log(x - 1) - 2)/(x + 1)
```

Sympy [A] time = 0.104982, size = 19, normalized size = 0.79

$$\frac{3\log(x-1)}{2} - \frac{\log(x+1)}{2} + \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+2*x+3)/(-1+x)/(1+x)**2,x)
```

```
[Out] 3*log(x - 1)/2 - log(x + 1)/2 + 1/(x + 1)
```

Giac [A] time = 1.09003, size = 32, normalized size = 1.33

$$\frac{1}{x+1} + \log(|x+1|) + \frac{3}{2}\log\left(\left|-\frac{2}{x+1}+1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+2*x+3)/(-1+x)/(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/(x + 1) + log(abs(x + 1)) + 3/2*log(abs(-2/(x + 1) + 1))
```

3.111 $\int \frac{-2+2x+3x^2}{-1+x^3} dx$

Optimal. Leaf size=28

$$\log\left(1-x^3\right)+\frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

```
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 - x^3]
```

Rubi [A] time = 0.0276238, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {1871, 1586, 618, 204, 260}

$$\log\left(1-x^3\right)+\frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(-2 + 2*x + 3*x^2)/(-1 + x^3),x]

[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 - x^3]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1586

```
Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] :> Int[u*PolynomialQuotient[Px
, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] &&
 EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{-2+2x+3x^2}{-1+x^3}\,dx &= 3\int \frac{x^2}{-1+x^3}\,dx + \int \frac{-2+2x}{-1+x^3}\,dx \\
&= \log\left(1-x^3\right) + \int \frac{1}{\frac{1}{2}+\frac{x}{2}+\frac{x^2}{2}}\,dx \\
&= \log\left(1-x^3\right) - 2\,\text{Subst}\left(\int \frac{1}{-\frac{3}{4}-x^2}\,dx, x, \frac{1}{2}+x\right) \\
&= \frac{4\tan^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)}{\sqrt{3}} + \log\left(1-x^3\right)
\end{aligned}
$$

Mathematica [A] time = 0.0101806, size = 28, normalized size = 1.

$$
\log\left(1-x^3\right) + \frac{4\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)}{\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2 + 2*x + 3*x^2)/(-1 + x^3),x]
```

```
[Out] (4*ArcTan[(1 + 2*x)/Sqrt[3]])/Sqrt[3] + Log[1 - x^3]
```

Maple [A] time = 0.005, size = 29, normalized size = 1.

$$
\ln(-1+x) + \ln\left(x^2+x+1\right) + \frac{4\sqrt{3}}{3}\arctan\left(\frac{(1+2x)\sqrt{3}}{3}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2*x-2)/(x^3-1),x)
```

```
[Out] ln(-1+x)+ln(x^2+x+1)+4/3*arctan(1/3*(1+2*x)*3^(1/2))*3^(1/2)
```

Maxima [A] time = 1.44293, size = 38, normalized size = 1.36

$$
\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \log\left(x^2+x+1\right) + \log(x-1)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-2)/(x^3-1),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x - 1)
```

Fricas [A] time = 0.788545, size = 101, normalized size = 3.61

$$
\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right) + \log\left(x^2+x+1\right) + \log(x-1)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-2)/(x^3-1),x, algorithm="fricas")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(x - 1)
```

Sympy [A] time = 0.116302, size = 3, normalized size = 0.11

$$\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2*x-2)/(x**3-1),x)
```

```
[Out] log(x - 1)
```

Giac [A] time = 1.09295, size = 39, normalized size = 1.39

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\log\left(x^2+x+1\right)+\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2*x-2)/(x^3-1),x, algorithm="giac")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + log(x^2 + x + 1) + log(abs(x -
1))
```

3.112 $\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)(2+x^2)^2}\,dx$

Optimal. Leaf size=49

$$\frac{1}{2(x^2+2)}+\frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}}$$

```
[Out] 1/(2*(2 + x^2)) - ArcTan[x/Sqrt[2]]/(3*Sqrt[2]) + Log[1 - x]/3 + Log[2 + x^
2]/3
```

Rubi [A] time = 0.0672271, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1647, 1629, 635, 203, 260}

$$\frac{1}{2(x^2+2)}+\frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log(1-x)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 - x + 2*x^2 - x^3 + x^4)/((-1 + x)*(2 + x^2)^2),x]
```

```
[Out] 1/(2*(2 + x^2)) - ArcTan[x/Sqrt[2]]/(3*Sqrt[2]) + Log[1 - x]/3 + Log[2 + x^
2]/3
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2-x+2x^2-x^3+x^4}{(-1+x)\left(2+x^2\right)^2}\,dx &= \frac{1}{2\left(2+x^2\right)}-\frac{1}{4}\int \frac{-4+4x-4x^2}{(-1+x)\left(2+x^2\right)}\,dx\\
&= \frac{1}{2\left(2+x^2\right)}-\frac{1}{4}\int \left(-\frac{4}{3(-1+x)}-\frac{4(-1+2x)}{3\left(2+x^2\right)}\right)dx\\
&= \frac{1}{2\left(2+x^2\right)}+\frac{1}{3}\log(1-x)+\frac{1}{3}\int \frac{-1+2x}{2+x^2}\,dx\\
&= \frac{1}{2\left(2+x^2\right)}+\frac{1}{3}\log(1-x)-\frac{1}{3}\int \frac{1}{2+x^2}\,dx+\frac{2}{3}\int \frac{x}{2+x^2}\,dx\\
&= \frac{1}{2\left(2+x^2\right)}-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}}+\frac{1}{3}\log(1-x)+\frac{1}{3}\log\left(2+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.026986, size = 61, normalized size = 1.24

$$
\frac{1}{2\left((x-1)^2+2(x-1)+3\right)}+\frac{1}{3}\log\left((x-1)^2+2(x-1)+3\right)+\frac{1}{3}\log(x-1)-\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3\sqrt{2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 - x + 2*x^2 - x^3 + x^4)/((-1 + x)*(2 + x^2)^2),x]
```

```
[Out] 1/(2*(3 + 2*(-1 + x) + (-1 + x)^2)) - ArcTan[x/Sqrt[2]]/(3*Sqrt[2]) + Log[3
 + 2*(-1 + x) + (-1 + x)^2]/3 + Log[-1 + x]/3
```

Maple [A] time = 0.009, size = 37, normalized size = 0.8

$$
\frac{1}{2\,x^2+4}+\frac{\ln\left(x^2+2\right)}{3}-\frac{\sqrt{2}}{6}\arctan\left(\frac{x\sqrt{2}}{2}\right)+\frac{\ln\left(-1+x\right)}{3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x)
```

```
[Out] 1/2/(x^2+2)+1/3*ln(x^2+2)-1/6*arctan(1/2*x*2^(1/2))*2^(1/2)+1/3*ln(-1+x)
```

Maxima [A] time = 1.48141, size = 49, normalized size = 1.

$$
-\frac{1}{6}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}x\right)+\frac{1}{2\left(x^2+2\right)}+\frac{1}{3}\,\log\left(x^2+2\right)+\frac{1}{3}\,\log\left(x-1\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/2/(x^2 + 2) + 1/3*log(x^2 + 2) + 1/3
*log(x - 1)
```

Fricas [A] time = 0.7595, size = 154, normalized size = 3.14

$$-\frac{\sqrt{2}\left(x^2+2\right)\arctan\left(\frac{1}{2}\sqrt{2}x\right)-2\left(x^2+2\right)\log\left(x^2+2\right)-2\left(x^2+2\right)\log\left(x-1\right)-3}{6\left(x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(sqrt(2)*(x^2 + 2)*arctan(1/2*sqrt(2)*x) - 2*(x^2 + 2)*log(x^2 + 2) -
2*(x^2 + 2)*log(x - 1) - 3)/(x^2 + 2)
```

Sympy [A] time = 0.155543, size = 14, normalized size = 0.29

$$\frac{\log\left(x-1\right)}{3}+\frac{1}{2x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4-x**3+2*x**2-x+2)/(-1+x)/(x**2+2)**2,x)
```

```
[Out] log(x - 1)/3 + 1/(2*x**2 + 4)
```

Giac [A] time = 1.07925, size = 50, normalized size = 1.02

$$-\frac{1}{6}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{1}{2\left(x^2+2\right)}+\frac{1}{3}\log\left(x^2+2\right)+\frac{1}{3}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4-x^3+2*x^2-x+2)/(-1+x)/(x^2+2)^2,x, algorithm="giac")
```

```
[Out] -1/6*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/2/(x^2 + 2) + 1/3*log(x^2 + 2) + 1/3
*log(abs(x - 1))
```

3.113 $\int \frac{1}{\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=21

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -(ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2])
```

Rubi [A] time = 0.009178, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3074, 206}

$$-\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x] + Sin[x])^(-1),x]
```

```
[Out] -(ArcTanh[(Cos[x] - Sin[x])/Sqrt[2]]/Sqrt[2])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\cos(x)+\sin(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \cos(x)-\sin(x)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\cos(x)-\sin(x)}{\sqrt{2}}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [C] time = 0.0183429, size = 24, normalized size = 1.14

$$(-1-i)(-1)^{3/4}\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x] + Sin[x])^(-1),x]
```

```
[Out] (-1 - I)*(-1)^(3/4)*ArcTanh[(-1 + Tan[x/2])/Sqrt[2]]
```

Maple [A] time = 0.022, size = 19, normalized size = 0.9

$$\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{4}(2\tan(x/2)-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)+sin(x)),x)
```

```
[Out] 2^(1/2)*arctanh(1/4*(2*tan(1/2*x)-2)*2^(1/2))
```

Maxima [B] time = 1.44829, size = 53, normalized size = 2.52

$$-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-\frac{\sin(x)}{\cos(x)+1}+1}{\sqrt{2}+\frac{\sin(x)}{\cos(x)+1}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*log(-(sqrt(2) - sin(x)/(cos(x) + 1) + 1)/(sqrt(2) + sin(x)/(co
s(x) + 1) - 1))
```

Fricas [B] time = 0.720212, size = 126, normalized size = 6.

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}-\cos(x)\right)\sin(x)-2\sqrt{2}\cos(x)+3}{2\cos(x)\sin(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log((2*(sqrt(2) - cos(x))*sin(x) - 2*sqrt(2)*cos(x) + 3)/(2*cos
(x)*sin(x) + 1))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x)
```

```
[Out] Exception raised: TypeError
```

Giac [B] time = 1.14856, size = 50, normalized size = 2.38

$$-\frac{1}{2}\,\sqrt{2}\log\left(\frac{\left|-2\,\sqrt{2}+2\,\tan\left(\frac{1}{2}\,x\right)-2\right|}{\left|2\,\sqrt{2}+2\,\tan\left(\frac{1}{2}\,x\right)-2\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-2*sqrt(2) + 2*tan(1/2*x) - 2)/abs(2*sqrt(2) + 2*tan(1
/2*x) - 2))
```

3.114 $\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx$

Optimal. Leaf size=16

$$-\log\left(\sqrt{4-x^2}+1\right)$$

```
[Out] -Log[1 + Sqrt[4 - x^2]]
```

Rubi [A] time = 0.0450431, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2155, 31}

$$-\log\left(\sqrt{4-x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/(4 - x^2 + Sqrt[4 - x^2]),x]
```

```
[Out] -Log[1 + Sqrt[4 - x^2]]
```

Rule 2155

```
Int[(x_)^(m_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]
), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
 b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{4-x^2+\sqrt{4-x^2}}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{4+\sqrt{4-x}-x}\,dx, x, x^2\right)\\
&= -\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sqrt{4-x^2}\right)\\
&= -\log\left(1+\sqrt{4-x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.0267052, size = 16, normalized size = 1.

$$-\log\left(\sqrt{4-x^2}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(4 - x^2 + Sqrt[4 - x^2]),x]
```

```
[Out] -Log[1 + Sqrt[4 - x^2]]
```

Maple [B] time = 0.043, size = 266, normalized size = 16.6

$$-\frac{\ln\left(x^2-3\right)}{2}+\frac{1}{\left(4+2\sqrt{3}\right)\left(-2+\sqrt{3}\right)}\operatorname{Artanh}\left(\frac{2+2\sqrt{3}\left(x+\sqrt{3}\right)}{2}\frac{1}{\sqrt{-\left(x+\sqrt{3}\right)^2+2\sqrt{3}\left(x+\sqrt{3}\right)+1}}\right)-\frac{1}{\left(4+2\sqrt{3}\right)\left(-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(4-x^2+(-x^2+4)^(1/2)),x)
```

```
[Out] -1/2*ln(x^2-3)+1/2/(2+3^(1/2))/(-2+3^(1/2))*arctanh(1/2*(2+2*3^(1/2)*(x+3^(
1/2)))/(-(x+3^(1/2))^2+2*3^(1/2)*(x+3^(1/2))+1)^(1/2))-1/2/(2+3^(1/2))/(-2+
3^(1/2))*(-(x+3^(1/2))^2+2*3^(1/2)*(x+3^(1/2))+1)^(1/2)+1/2/(2+3^(1/2))/(-2
+3^(1/2))*(-(2+x)^2+4*x+8)^(1/2)+1/2/(2+3^(1/2))/(-2+3^(1/2))*arctanh(1/2*(
2-2*3^(1/2)*(x-3^(1/2)))/(-(x-3^(1/2))^2-2*3^(1/2)*(x-3^(1/2))+1)^(1/2))-1/
2/(2+3^(1/2))/(-2+3^(1/2))*(-(x-3^(1/2))^2-2*3^(1/2)*(x-3^(1/2))+1)^(1/2)+1
/2/(2+3^(1/2))/(-2+3^(1/2))*(-(-2+x)^2-4*x+8)^(1/2)
```

Maxima [A] time = 1.01516, size = 19, normalized size = 1.19

$$-\log\left(\sqrt{-x^2+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4-x^2+(-x^2+4)^(1/2)),x, algorithm="maxima")
```

```
[Out] -log(sqrt(-x^2 + 4) + 1)
```

Fricas [B] time = 0.824438, size = 144, normalized size = 9.

$$-\frac{1}{2}\log\left(x^2-3\right)+\frac{1}{2}\log\left(-\frac{x^2+3\sqrt{-x^2+4}-6}{x^2}\right)-\frac{1}{2}\log\left(-\frac{x^2+\sqrt{-x^2+4}-2}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4-x^2+(-x^2+4)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/2*log(x^2 - 3) + 1/2*log(-(x^2 + 3*sqrt(-x^2 + 4) - 6)/x^2) - 1/2*log(-(
x^2 + sqrt(-x^2 + 4) - 2)/x^2)
```

Sympy [B] time = 2.64185, size = 44, normalized size = 2.75

$$\frac{\log\left(2\sqrt{4-x^2}\right)}{2}-\frac{\log\left(2\sqrt{4-x^2}+2\right)}{2}-\frac{\log\left(x^2-\sqrt{4-x^2}-4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4-x**2+(-x**2+4)**(1/2)),x)
```

```
[Out] log(2*sqrt(4 - x**2))/2 - log(2*sqrt(4 - x**2) + 2)/2 - log(x**2 - sqrt(4 -
 x**2) - 4)/2
```

Giac [A] time = 1.10885, size = 19, normalized size = 1.19

$$-\log\left(\sqrt{-x^2+4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(4-x^2+(-x^2+4)^(1/2)),x, algorithm="giac")
```

```
[Out] -log(sqrt(-x^2 + 4) + 1)
```

3.115 $\int \frac{3+2x}{(-2+x)(5+x)} \, dx$

Optimal. Leaf size=11

$$\log(2-x)+\log(x+5)$$

```
[Out] Log[2 - x] + Log[5 + x]
```

Rubi [A] time = 0.004397, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {72}

$$\log(2-x)+\log(x+5)$$

Antiderivative was successfully verified.

```
[In] Int[(3 + 2*x)/((-2 + x)*(5 + x)),x]

[Out] Log[2 - x] + Log[5 + x]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{3+2x}{(-2+x)(5+x)} \, dx &= \int \left(\frac{1}{-2+x}+\frac{1}{5+x}\right) dx\\ &= \log(2-x)+\log(5+x)\end{aligned}$$

Mathematica [A] time = 0.0032597, size = 9, normalized size = 0.82

$$\log(x-2)+\log(x+5)$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + 2*x)/((-2 + x)*(5 + x)),x]

[Out] Log[-2 + x] + Log[5 + x]
```

Maple [A] time = 0.001, size = 9, normalized size = 0.8

$$\ln\left((-2+x)(5+x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+2*x)/(-2+x)/(5+x),x)
```

```
[Out] ln((-2+x)*(5+x))
```

Maxima [A] time = 0.989558, size = 12, normalized size = 1.09

$$\log(x+5)+\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="maxima")
```

```
[Out] log(x + 5) + log(x - 2)
```

Fricas [A] time = 0.797656, size = 28, normalized size = 2.55

$$\log\left(x^2+3\,x-10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="fricas")
```

```
[Out] log(x^2 + 3*x - 10)
```

Sympy [A] time = 0.081153, size = 8, normalized size = 0.73

$$\log\left(x^2+3x-10\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x)
```

```
[Out] log(x**2 + 3*x - 10)
```

Giac [A] time = 1.0666, size = 15, normalized size = 1.36

$$\log(|x+5|)+\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+2*x)/(-2+x)/(5+x),x, algorithm="giac")
```

```
[Out] log(abs(x + 5)) + log(abs(x - 2))
```

3.116 $\int \frac{x}{(1+x)(2+x)(3+x)}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

```
[Out] -Log[1 + x]/2 + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Rubi [A] time = 0.0076509, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {148}

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(2 + x)*(3 + x)),x]
```

```
[Out] -Log[1 + x]/2 + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+x)(2+x)(3+x)}\,dx &= \int \left(-\frac{1}{2(1+x)}+\frac{2}{2+x}-\frac{3}{2(3+x)}\right)dx\\ &= -\frac{1}{2}\log(1+x)+2\log(2+x)-\frac{3}{2}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.0052298, size = 23, normalized size = 1.

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*(2 + x)*(3 + x)),x]
```

```
[Out] -Log[1 + x]/2 + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Maple [A] time = 0.006, size = 20, normalized size = 0.9

$$-\frac{\ln(1+x)}{2}+2\ln(2+x)-\frac{3\ln(3+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)/(2+x)/(3+x),x)
```

```
[Out] -1/2*ln(1+x)+2*ln(2+x)-3/2*ln(3+x)
```

Maxima [A] time = 0.96862, size = 26, normalized size = 1.13

$$-\frac{3}{2}\,\log\left(x+3\right)+2\,\log\left(x+2\right)-\frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="maxima")
```

```
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

Fricas [A] time = 0.876651, size = 66, normalized size = 2.87

$$-\frac{3}{2}\,\log\left(x+3\right)+2\,\log\left(x+2\right)-\frac{1}{2}\,\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="fricas")
```

```
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

Sympy [A] time = 0.114764, size = 20, normalized size = 0.87

$$-\frac{\log\left(x+1\right)}{2}+2\log\left(x+2\right)-\frac{3\log\left(x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x)
```

```
[Out] -log(x + 1)/2 + 2*log(x + 2) - 3*log(x + 3)/2
```

Giac [A] time = 1.07683, size = 30, normalized size = 1.3

$$-\frac{3}{2}\,\log\left(|x+3|\right)+2\,\log\left(|x+2|\right)-\frac{1}{2}\,\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="giac")
```

```
[Out] -3/2*log(abs(x + 3)) + 2*log(abs(x + 2)) - 1/2*log(abs(x + 1))
```

3.117 $\int \frac{x}{2-3x+x^3} \, dx$

Optimal. Leaf size=30

$$\frac{1}{3(1-x)}+\frac{2}{9}\log(1-x)-\frac{2}{9}\log(x+2)$$

```
[Out] 1/(3*(1 - x)) + (2*Log[1 - x])/9 - (2*Log[2 + x])/9
```

Rubi [A] time = 0.0165808, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2074}

$$\frac{1}{3(1-x)}+\frac{2}{9}\log(1-x)-\frac{2}{9}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[x/(2 - 3*x + x^3),x]
```

```
[Out] 1/(3*(1 - x)) + (2*Log[1 - x])/9 - (2*Log[2 + x])/9
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{2-3x+x^3} \, dx &= \int \left(\frac{1}{3(-1+x)^2}+\frac{2}{9(-1+x)}-\frac{2}{9(2+x)}\right) dx\\ &= \frac{1}{3(1-x)}+\frac{2}{9}\log(1-x)-\frac{2}{9}\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.0075359, size = 28, normalized size = 0.93

$$-\frac{1}{3(x-1)}+\frac{2}{9}\log(1-x)-\frac{2}{9}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(2 - 3*x + x^3),x]
```

```
[Out] -1/(3*(-1 + x)) + (2*Log[1 - x])/9 - (2*Log[2 + x])/9
```

Maple [A] time = 0.007, size = 21, normalized size = 0.7

$$-\frac{2\ln(2+x)}{9}-\frac{1}{-3+3x}+\frac{2\ln(-1+x)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^3-3*x+2),x)
```

```
[Out] -2/9*ln(2+x)-1/3/(-1+x)+2/9*ln(-1+x)
```

Maxima [A] time = 0.966816, size = 27, normalized size = 0.9

$$-\frac{1}{3(x-1)} - \frac{2}{9}\log(x+2) + \frac{2}{9}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-3*x+2),x, algorithm="maxima")
```

```
[Out] -1/3/(x - 1) - 2/9*log(x + 2) + 2/9*log(x - 1)
```

Fricas [A] time = 0.779541, size = 86, normalized size = 2.87

$$-\frac{2(x-1)\log(x+2) - 2(x-1)\log(x-1) + 3}{9(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-3*x+2),x, algorithm="fricas")
```

```
[Out] -1/9*(2*(x - 1)*log(x + 2) - 2*(x - 1)*log(x - 1) + 3)/(x - 1)
```

Sympy [A] time = 0.098044, size = 22, normalized size = 0.73

$$\frac{2\log(x-1)}{9} - \frac{2\log(x+2)}{9} - \frac{1}{3x-3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**3-3*x+2),x)
```

```
[Out] 2*log(x - 1)/9 - 2*log(x + 2)/9 - 1/(3*x - 3)
```

Giac [A] time = 1.0652, size = 30, normalized size = 1.

$$-\frac{1}{3(x-1)} - \frac{2}{9}\log(|x+2|) + \frac{2}{9}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^3-3*x+2),x, algorithm="giac")
```

```
[Out] -1/3/(x - 1) - 2/9*log(abs(x + 2)) + 2/9*log(abs(x - 1))
```

3.118 $\int \frac{-6+2x+x^4}{-2x+x^2+x^3}\,dx$

Optimal. Leaf size=27

$$\frac{x^2}{2}-x-\log(1-x)+3\log(x)+\log(x+2)$$

```
[Out] -x + x^2/2 - Log[1 - x] + 3*Log[x] + Log[2 + x]
```

Rubi [A] time = 0.0307892, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1594, 1628}

$$\frac{x^2}{2}-x-\log(1-x)+3\log(x)+\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-6 + 2*x + x^4)/(-2*x + x^2 + x^3),x]
```

```
[Out] -x + x^2/2 - Log[1 - x] + 3*Log[x] + Log[2 + x]
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{-6+2x+x^4}{-2x+x^2+x^3}\,dx &= \int \frac{-6+2x+x^4}{x\left(-2+x+x^2\right)}\,dx\\ &= \int \left(-1+\frac{1}{1-x}+\frac{3}{x}+x+\frac{1}{2+x}\right)dx\\ &= -x+\frac{x^2}{2}-\log(1-x)+3\log(x)+\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.0051538, size = 27, normalized size = 1.

$$\frac{x^2}{2}-x-\log(1-x)+3\log(x)+\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-6 + 2*x + x^4)/(-2*x + x^2 + x^3),x]
```

```
[Out] -x + x^2/2 - Log[1 - x] + 3*Log[x] + Log[2 + x]
```

Maple [A] time = 0.006, size = 24, normalized size = 0.9

$$-x+\frac{x^2}{2}+\ln(2+x)+3\ln(x)-\ln(-1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+2*x-6)/(x^3+x^2-2*x),x)
```

```
[Out] -x+1/2*x^2+ln(2+x)+3*ln(x)-ln(-1+x)
```

Maxima [A] time = 0.944474, size = 31, normalized size = 1.15

$$\frac{1}{2}x^2-x+\log(x+2)-\log(x-1)+3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x-6)/(x^3+x^2-2*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 - x + log(x + 2) - log(x - 1) + 3*log(x)
```

Fricas [A] time = 0.660478, size = 68, normalized size = 2.52

$$\frac{1}{2}x^2-x+\log(x+2)-\log(x-1)+3\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x-6)/(x^3+x^2-2*x),x, algorithm="fricas")
```

```
[Out] 1/2*x^2 - x + log(x + 2) - log(x - 1) + 3*log(x)
```

Sympy [A] time = 0.114105, size = 20, normalized size = 0.74

$$\frac{x^2}{2}-x+3\log(x)-\log(x-1)+\log(x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+2*x-6)/(x**3+x**2-2*x),x)
```

```
[Out] x**2/2 - x + 3*log(x) - log(x - 1) + log(x + 2)
```

Giac [A] time = 1.09188, size = 35, normalized size = 1.3

$$\frac{1}{2}x^2-x+\log(|x+2|)-\log(|x-1|)+3\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+2*x-6)/(x^3+x^2-2*x),x, algorithm="giac")
```

```
[Out] 1/2*x^2 - x + log(abs(x + 2)) - log(abs(x - 1)) + 3*log(abs(x))
```

3.119 $\int \frac{7+8x^3}{(1+x)(1+2x)^3}\,dx$

Optimal. Leaf size=23

$$\frac{3}{2x+1}-\frac{3}{(2x+1)^2}+\log(x+1)$$

```
[Out] -3/(1 + 2*x)^2 + 3/(1 + 2*x) + Log[1 + x]
```

Rubi [A] time = 0.0263759, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05, Rules used = {1620}

$$\frac{3}{2x+1}-\frac{3}{(2x+1)^2}+\log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(7 + 8*x^3)/((1 + x)*(1 + 2*x)^3),x]
```

```
[Out] -3/(1 + 2*x)^2 + 3/(1 + 2*x) + Log[1 + x]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}\int \frac{7+8x^3}{(1+x)(1+2x)^3}\,dx &= \int \left(\frac{1}{1+x}+\frac{12}{(1+2x)^3}-\frac{6}{(1+2x)^2}\right)dx\\ &= -\frac{3}{(1+2x)^2}+\frac{3}{1+2x}+\log(1+x)\end{aligned}$$

Mathematica [A] time = 0.0092826, size = 24, normalized size = 1.04

$$\frac{6x+(2x+1)^2\log(x+1)}{(2x+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(7 + 8*x^3)/((1 + x)*(1 + 2*x)^3),x]
```

```
[Out] (6*x + (1 + 2*x)^2*Log[1 + x])/(1 + 2*x)^2
```

Maple [A] time = 0.007, size = 24, normalized size = 1.

$$-3\,(1+2\,x)^{-2}+3\,(1+2\,x)^{-1}+\ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((8*x^3+7)/(1+x)/(1+2*x)^3,x)
```

```
[Out] -3/(1+2*x)^2+3/(1+2*x)+ln(1+x)
```

Maxima [A] time = 0.938688, size = 27, normalized size = 1.17

$$\frac{6\,x}{4\,x^2+4\,x+1}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+7)/(1+x)/(1+2*x)^3,x, algorithm="maxima")
```

```
[Out] 6*x/(4*x^2 + 4*x + 1) + log(x + 1)
```

Fricas [A] time = 0.74517, size = 76, normalized size = 3.3

$$\frac{\left(4\,x^2+4\,x+1\right)\log\left(x+1\right)+6\,x}{4\,x^2+4\,x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+7)/(1+x)/(1+2*x)^3,x, algorithm="fricas")
```

```
[Out] ((4*x^2 + 4*x + 1)*log(x + 1) + 6*x)/(4*x^2 + 4*x + 1)
```

Sympy [A] time = 0.104271, size = 17, normalized size = 0.74

$$\frac{6x}{4x^2+4x+1}+\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x**3+7)/(1+x)/(1+2*x)**3,x)
```

```
[Out] 6*x/(4*x**2 + 4*x + 1) + log(x + 1)
```

Giac [A] time = 1.15593, size = 22, normalized size = 0.96

$$\frac{6\,x}{\left(2\,x+1\right)^2}+\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*x^3+7)/(1+x)/(1+2*x)^3,x, algorithm="giac")
```

```
[Out] 6*x/(2*x + 1)^2 + log(abs(x + 1))
```

3.120 $\int \frac{1+x+4x^2}{-1+x^3} \, dx$

Optimal. Leaf size=16

$$\log \left(x^2+x+1\right)+2 \log (1-x)$$

```
[Out] 2*Log[1 - x] + Log[1 + x + x^2]
```

Rubi [A] time = 0.0167353, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1875, 31, 628}

$$\log \left(x^2+x+1\right)+2 \log (1-x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x + 4*x^2)/(-1 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[1 + x + x^2]
```

Rule 1875

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (-(a/b))^(1/3)}, Dist[(q*(A + B
*q + C*q^2))/(3*a), Int[1/(q - x), x], x] + Dist[q/(3*a), Int[(q*(2*A - B*q
 - C*q^2) + (A + B*q - 2*C*q^2)*x)/(q^2 + q*x + x^2), x], x] /; NeQ[a*B^3 -
 b*A^3, 0] && NeQ[A + B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x,
2] && LtQ[a/b, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1+x+4 x^2}{-1+x^3} \, dx &= -\left(\frac{1}{3} \int \frac{-3-6 x}{1+x+x^2} \, dx\right)-2 \int \frac{1}{1-x} \, dx \\ &= 2 \log (1-x)+\log \left(1+x+x^2\right) \end{aligned}$$

Mathematica [A] time = 0.0037856, size = 16, normalized size = 1.

$$\log \left(x^2+x+1\right)+2 \log (1-x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x + 4*x^2)/(-1 + x^3),x]
```

```
[Out] 2*Log[1 - x] + Log[1 + x + x^2]
```

Maple [A] time = 0.003, size = 15, normalized size = 0.9

$$2\ln(-1+x)+\ln\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^2+x+1)/(x^3-1),x)
```

```
[Out] 2*ln(-1+x)+ln(x^2+x+1)
```

Maxima [A] time = 1.40583, size = 19, normalized size = 1.19

$$\log\left(x^2+x+1\right)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(x^3-1),x, algorithm="maxima")
```

```
[Out] log(x^2 + x + 1) + 2*log(x - 1)
```

Fricas [A] time = 0.747851, size = 45, normalized size = 2.81

$$\log\left(x^2+x+1\right)+2\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(x^3-1),x, algorithm="fricas")
```

```
[Out] log(x^2 + x + 1) + 2*log(x - 1)
```

Sympy [A] time = 0.085175, size = 14, normalized size = 0.88

$$2\log(x-1)+\log\left(x^2+x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**2+x+1)/(x**3-1),x)
```

```
[Out] 2*log(x - 1) + log(x**2 + x + 1)
```

Giac [A] time = 1.08864, size = 20, normalized size = 1.25

$$\log\left(x^2+x+1\right)+2\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2+x+1)/(x^3-1),x, algorithm="giac")
```

```
[Out] log(x^2 + x + 1) + 2*log(abs(x - 1))
```

3.121 $\int \frac{x^4}{4+5x^2+x^4}\,dx$

Optimal. Leaf size=18

$$x-\frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tan^{-1}(x)$$

```
[Out] x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

Rubi [A] time = 0.0120354, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1122, 1166, 203}

$$x-\frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(4 + 5*x^2 + x^4),x]

[Out] x - (8*ArcTan[x/2])/3 + ArcTan[x]/3
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4}{4+5x^2+x^4}\,dx &= x-\int \frac{4+5x^2}{4+5x^2+x^4}\,dx\\ &= x+\frac{1}{3}\int \frac{1}{1+x^2}\,dx-\frac{16}{3}\int \frac{1}{4+x^2}\,dx\\ &= x-\frac{8}{3}\tan^{-1}\left(\frac{x}{2}\right)+\frac{1}{3}\tan^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0064369, size = 18, normalized size = 1.

$$x+\frac{8}{3}\tan^{-1}\left(\frac{2}{x}\right)+\frac{1}{3}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(4 + 5*x^2 + x^4),x]
```

```
[Out] x + (8*ArcTan[2/x])/3 + ArcTan[x]/3
```

Maple [A] time = 0.007, size = 13, normalized size = 0.7

$$x-\frac{8}{3}\arctan\left(\frac{x}{2}\right)+\frac{\arctan(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(x^4+5*x^2+4),x)
```

```
[Out] x-8/3*arctan(1/2*x)+1/3*arctan(x)
```

Maxima [A] time = 1.42165, size = 16, normalized size = 0.89

$$x-\frac{8}{3}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="maxima")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Fricas [A] time = 0.708556, size = 53, normalized size = 2.94

$$x-\frac{8}{3}\arctan\left(\frac{1}{2}x\right)+\frac{1}{3}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="fricas")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

Sympy [A] time = 0.1328, size = 14, normalized size = 0.78

$$x-\frac{8\operatorname{atan}\left(\frac{x}{2}\right)}{3}+\frac{\operatorname{atan}(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(x**4+5*x**2+4),x)
```

```
[Out] x - 8*atan(x/2)/3 + atan(x)/3
```

Giac [A] time = 1.07655, size = 16, normalized size = 0.89

$$x - \frac{8}{3}\,\arctan\left(\frac{1}{2}\,x\right) + \frac{1}{3}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(x^4+5*x^2+4),x, algorithm="giac")
```

```
[Out] x - 8/3*arctan(1/2*x) + 1/3*arctan(x)
```

3.122 $\int \frac{2+x}{x+x^2} \, dx$

Optimal. Leaf size=11

$$2\log(x) - \log(x+1)$$

```
[Out] 2*Log[x] - Log[1 + x]
```

Rubi [A] time = 0.0051985, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {631}

$$2\log(x) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(x + x^2),x]

[Out] 2*Log[x] - Log[1 + x]
```

Rule 631

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x}{x+x^2} \, dx &= \int \left(\frac{1}{-1-x} + \frac{2}{x}\right) dx \\
&= 2\log(x) - \log(1+x)
\end{aligned}$$

Mathematica [A] time = 0.001957, size = 11, normalized size = 1.

$$2\log(x) - \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(x + x^2),x]

[Out] 2*Log[x] - Log[1 + x]
```

Maple [A] time = 0.005, size = 12, normalized size = 1.1

$$2\ln(x) - \ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(x^2+x),x)
```

```
[Out] 2*ln(x)-ln(1+x)
```

Maxima [A] time = 0.932655, size = 15, normalized size = 1.36

$$-\log(x+1) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x),x, algorithm="maxima")
```

```
[Out] -log(x + 1) + 2*log(x)
```

Fricas [A] time = 0.717003, size = 32, normalized size = 2.91

$$-\log(x+1) + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x),x, algorithm="fricas")
```

```
[Out] -log(x + 1) + 2*log(x)
```

Sympy [A] time = 0.086883, size = 8, normalized size = 0.73

$$2\log(x) - \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2+x),x)
```

```
[Out] 2*log(x) - log(x + 1)
```

Giac [A] time = 1.09685, size = 18, normalized size = 1.64

$$-\log(|x+1|) + 2\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2+x),x, algorithm="giac")
```

```
[Out] -log(abs(x + 1)) + 2*log(abs(x))
```

3.123 $\int \frac{1}{x(1+x^2)^2} \, dx$

Optimal. Leaf size=24

$$\frac{1}{2(x^2+1)} - \frac{1}{2}\log(x^2+1) + \log(x)$$

```
[Out] 1/(2*(1 + x^2)) + Log[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0095761, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {266, 44}

$$\frac{1}{2(x^2+1)} - \frac{1}{2}\log(x^2+1) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(1 + x^2)^2),x]

[Out] 1/(2*(1 + x^2)) + Log[x] - Log[1 + x^2]/2
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(1+x^2)^2} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(1+x)^2} \, dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{-1-x} + \frac{1}{x} - \frac{1}{(1+x)^2}\right) dx, x, x^2\right) \\
&= \frac{1}{2(1+x^2)} + \log(x) - \frac{1}{2}\log(1+x^2)
\end{aligned}$$

Mathematica [A] time = 0.0066393, size = 24, normalized size = 1.

$$\frac{1}{2(x^2+1)} - \frac{1}{2}\log(x^2+1) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(1 + x^2)^2),x]
```

```
[Out] 1/(2*(1 + x^2)) + Log[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.009, size = 21, normalized size = 0.9

$$\frac{1}{2\,x^2+2}+\ln\left(x\right)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(x^2+1)^2,x)
```

```
[Out] 1/2/(x^2+1)+ln(x)-1/2*ln(x^2+1)
```

Maxima [A] time = 0.972026, size = 32, normalized size = 1.33

$$\frac{1}{2\left(x^2+1\right)}-\frac{1}{2}\,\log\left(x^2+1\right)+\frac{1}{2}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/2/(x^2 + 1) - 1/2*log(x^2 + 1) + 1/2*log(x^2)
```

Fricas [A] time = 0.756444, size = 89, normalized size = 3.71

$$-\frac{\left(x^2+1\right)\log\left(x^2+1\right)-2\left(x^2+1\right)\log\left(x\right)-1}{2\left(x^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((x^2 + 1)*log(x^2 + 1) - 2*(x^2 + 1)*log(x) - 1)/(x^2 + 1)
```

Sympy [A] time = 0.095338, size = 19, normalized size = 0.79

$$\log\left(x\right)-\frac{\log\left(x^2+1\right)}{2}+\frac{1}{2x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+1)**2,x)
```

```
[Out] log(x) - log(x**2 + 1)/2 + 1/(2*x**2 + 2)
```

Giac [A] time = 1.09623, size = 39, normalized size = 1.62

$$\frac{x^2+2}{2\left(x^2+1\right)}-\frac{1}{2}\log\left(x^2+1\right)+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/2*(x^2 + 2)/(x^2 + 1) - 1/2*log(x^2 + 1) + 1/2*log(x^2)
```

3.124 $\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx$

Optimal. Leaf size=46

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

```
[Out] (2 + x)^(-1) + 1/(4*(3 + x)^2) + 5/(4*(3 + x)) + Log[1 + x]/8 + 2*Log[2 + x
] - (17*Log[3 + x])/8
```

Rubi [A] time = 0.0128362, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {88}

$$\frac{1}{x+2}+\frac{5}{4(x+3)}+\frac{1}{4(x+3)^2}+\frac{1}{8}\log(x+1)+2\log(x+2)-\frac{17}{8}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (2 + x)^(-1) + 1/(4*(3 + x)^2) + 5/(4*(3 + x)) + Log[1 + x]/8 + 2*Log[2 + x
] - (17*Log[3 + x])/8
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(1+x)(2+x)^2(3+x)^3}\,dx &= \int \left(\frac{1}{8(1+x)}-\frac{1}{(2+x)^2}+\frac{2}{2+x}-\frac{1}{2(3+x)^3}-\frac{5}{4(3+x)^2}-\frac{17}{8(3+x)}\right)dx\\ &= \frac{1}{2+x}+\frac{1}{4(3+x)^2}+\frac{5}{4(3+x)}+\frac{1}{8}\log(1+x)+2\log(2+x)-\frac{17}{8}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.0141594, size = 44, normalized size = 0.96

$$\frac{1}{8}\left(\frac{8}{x+2}+\frac{10}{x+3}+\frac{2}{(x+3)^2}+\log(-x-1)+16\log(x+2)-17\log(x+3)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x)*(2 + x)^2*(3 + x)^3),x]
```

```
[Out] (8/(2 + x) + 2/(3 + x)^2 + 10/(3 + x) + Log[-1 - x] + 16*Log[2 + x] - 17*Lo
g[3 + x])/8
```

Maple [A] time = 0.01, size = 39, normalized size = 0.9

$$(2+x)^{-1}+\frac{1}{4\,(3+x)^2}+\frac{5}{12+4\,x}+\frac{\ln(1+x)}{8}+2\,\ln(2+x)-\frac{17\,\ln(3+x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+x)/(2+x)^2/(3+x)^3,x)
```

```
[Out] 1/(2+x)+1/4/(3+x)^2+5/4/(3+x)+1/8*ln(1+x)+2*ln(2+x)-17/8*ln(3+x)
```

Maxima [A] time = 0.93655, size = 62, normalized size = 1.35

$$\frac{9\,x^2+50\,x+68}{4\left(x^3+8\,x^2+21\,x+18\right)}-\frac{17}{8}\,\log(x+3)+2\,\log(x+2)+\frac{1}{8}\,\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^2 + 50*x + 68)/(x^3 + 8*x^2 + 21*x + 18) - 17/8*log(x + 3) + 2*log
(x + 2) + 1/8*log(x + 1)
```

Fricas [B] time = 0.790046, size = 239, normalized size = 5.2

$$\frac{18\,x^2-17\left(x^3+8\,x^2+21\,x+18\right)\log(x+3)+16\left(x^3+8\,x^2+21\,x+18\right)\log(x+2)+\left(x^3+8\,x^2+21\,x+18\right)\log(x+}{8\left(x^3+8\,x^2+21\,x+18\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(18*x^2 - 17*(x^3 + 8*x^2 + 21*x + 18)*log(x + 3) + 16*(x^3 + 8*x^2 + 2
1*x + 18)*log(x + 2) + (x^3 + 8*x^2 + 21*x + 18)*log(x + 1) + 100*x + 136)/
(x^3 + 8*x^2 + 21*x + 18)
```

Sympy [A] time = 0.180191, size = 46, normalized size = 1.

$$\frac{9x^2+50x+68}{4x^3+32x^2+84x+72}+\frac{\log(x+1)}{8}+2\log(x+2)-\frac{17\log(x+3)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)**2/(3+x)**3,x)
```

```
[Out] (9*x**2 + 50*x + 68)/(4*x**3 + 32*x**2 + 84*x + 72) + log(x + 1)/8 + 2*log(
x + 2) - 17*log(x + 3)/8
```

Giac [A] time = 1.12156, size = 70, normalized size = 1.52

$$\frac{1}{x+2} - \frac{\frac{7}{x+2} + 6}{4\left(\frac{1}{x+2} + 1\right)^2} + \frac{1}{8}\log\left(\left|-\frac{1}{x+2} + 1\right|\right) - \frac{17}{8}\log\left(\left|-\frac{1}{x+2} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+x)/(2+x)^2/(3+x)^3,x, algorithm="giac")
```

```
[Out] 1/(x + 2) - 1/4*(7/(x + 2) + 6)/(1/(x + 2) + 1)^2 + 1/8*log(abs(-1/(x + 2)
+ 1)) - 17/8*log(abs(-1/(x + 2) - 1))
```

3.125 $\int \frac{x}{(1+x)^2} \, dx$

Optimal. Leaf size=10

$$\frac{1}{x+1} + \log(x+1)$$

```
[Out] (1 + x)^(-1) + Log[1 + x]
```

Rubi [A] time = 0.0036734, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {43}

$$\frac{1}{x+1} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Int[x/(1 + x)^2,x]

[Out] (1 + x)^(-1) + Log[1 + x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x}{(1+x)^2} \, dx &= \int \left(-\frac{1}{(1+x)^2} + \frac{1}{1+x}\right) dx \\ &= \frac{1}{1+x} + \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.0021451, size = 10, normalized size = 1.

$$\frac{1}{x+1} + \log(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(1 + x)^2,x]

[Out] (1 + x)^(-1) + Log[1 + x]
```

Maple [A] time = 0.005, size = 11, normalized size = 1.1

$$(1+x)^{-1} + \ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1+x)^2,x)
```

```
[Out] 1/(1+x)+ln(1+x)
```

Maxima [A] time = 0.927236, size = 14, normalized size = 1.4

$$\frac{1}{x+1} + \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="maxima")
```

```
[Out] 1/(x + 1) + log(x + 1)
```

Fricas [A] time = 0.664859, size = 46, normalized size = 4.6

$$\frac{(x+1)\log(x+1)+1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="fricas")
```

```
[Out] ((x + 1)*log(x + 1) + 1)/(x + 1)
```

Sympy [A] time = 0.075874, size = 8, normalized size = 0.8

$$\log(x+1) + \frac{1}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)**2,x)
```

```
[Out] log(x + 1) + 1/(x + 1)
```

Giac [A] time = 1.11752, size = 15, normalized size = 1.5

$$\frac{1}{x+1} + \log(|x+1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)^2,x, algorithm="giac")
```

```
[Out] 1/(x + 1) + log(abs(x + 1))
```

3.126 $\int \frac{1}{-x+x^3}\,dx$

Optimal. Leaf size=17

$$\frac{1}{2}\log\left(1-x^2\right)-\log(x)$$

```
[Out] -Log[x] + Log[1 - x^2]/2
```

Rubi [A] time = 0.007101, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {1593, 266, 36, 31, 29}

$$\frac{1}{2}\log\left(1-x^2\right)-\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-x + x^3)^(-1),x]

[Out] -Log[x] + Log[1 - x^2]/2
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{-x+x^3}\,dx &= \int \frac{1}{x\left(-1+x^2\right)}\,dx \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{(-1+x)x}\,dx, x, x^2\right) \\
&= \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, x^2\right) - \frac{1}{2}\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, x^2\right) \\
&= -\log(x) + \frac{1}{2}\log\left(1-x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.0020843, size = 17, normalized size = 1.

$$\frac{1}{2}\log\left(1-x^2\right) - \log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x + x^3)^(-1),x]

[Out] -Log[x] + Log[1 - x^2]/2
```

Maple [A] time = 0.006, size = 18, normalized size = 1.1

$$-\ln(x) + \frac{\ln(1+x)}{2} + \frac{\ln(-1+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3-x),x)

[Out] -ln(x)+1/2*ln(1+x)+1/2*ln(-1+x)
```

Maxima [A] time = 0.929392, size = 23, normalized size = 1.35

$$\frac{1}{2}\log(x+1) + \frac{1}{2}\log(x-1) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x),x, algorithm="maxima")

[Out] 1/2*log(x + 1) + 1/2*log(x - 1) - log(x)
```

Fricas [A] time = 0.877403, size = 36, normalized size = 2.12

$$\frac{1}{2}\log\left(x^2-1\right) - \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x),x, algorithm="fricas")
```

```
[Out] 1/2*log(x^2 - 1) - log(x)
```

Sympy [A] time = 0.083429, size = 10, normalized size = 0.59

$$-\log(x) + \frac{\log(x^2-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**3-x),x)
```

```
[Out] -log(x) + log(x**2 - 1)/2
```

Giac [A] time = 1.07797, size = 22, normalized size = 1.29

$$-\frac{1}{2}\log(x^2) + \frac{1}{2}\log(|x^2-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^3-x),x, algorithm="giac")
```

```
[Out] -1/2*log(x^2) + 1/2*log(abs(x^2 - 1))
```

3.127 $\int \frac{x^2}{-6+x+x^2} \, dx$

Optimal. Leaf size=20

$$x + \frac{4}{5}\log(2-x) - \frac{9}{5}\log(x+3)$$

```
[Out] x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

Rubi [A] time = 0.0075442, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {703, 632, 31}

$$x + \frac{4}{5}\log(2-x) - \frac{9}{5}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(-6 + x + x^2),x]
```

```
[Out] x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{-6+x+x^2} \, dx &= x + \int \frac{6-x}{-6+x+x^2} \, dx \\
&= x + \frac{4}{5}\int \frac{1}{-2+x} \, dx - \frac{9}{5}\int \frac{1}{3+x} \, dx \\
&= x + \frac{4}{5}\log(2-x) - \frac{9}{5}\log(3+x)
\end{aligned}$$

Mathematica [A] time = 0.0027724, size = 20, normalized size = 1.

$$x+\frac{4}{5}\log(2-x)-\frac{9}{5}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(-6 + x + x^2),x]
```

```
[Out] x + (4*Log[2 - x])/5 - (9*Log[3 + x])/5
```

Maple [A] time = 0.005, size = 15, normalized size = 0.8

$$x-\frac{9\ln(3+x)}{5}+\frac{4\ln(-2+x)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+x-6),x)
```

```
[Out] x-9/5*ln(3+x)+4/5*ln(-2+x)
```

Maxima [A] time = 0.933498, size = 19, normalized size = 0.95

$$x-\frac{9}{5}\log(x+3)+\frac{4}{5}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+x-6),x, algorithm="maxima")
```

```
[Out] x - 9/5*log(x + 3) + 4/5*log(x - 2)
```

Fricas [A] time = 0.963386, size = 50, normalized size = 2.5

$$x-\frac{9}{5}\log(x+3)+\frac{4}{5}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+x-6),x, algorithm="fricas")
```

```
[Out] x - 9/5*log(x + 3) + 4/5*log(x - 2)
```

Sympy [A] time = 0.096316, size = 17, normalized size = 0.85

$$x+\frac{4\log(x-2)}{5}-\frac{9\log(x+3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+x-6),x)
```

```
[Out] x + 4*log(x - 2)/5 - 9*log(x + 3)/5
```

Giac [A] time = 1.11125, size = 22, normalized size = 1.1

$$x - \frac{9}{5}\log(|x+3|) + \frac{4}{5}\log(|x-2|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+x-6),x, algorithm="giac")
```

```
[Out] x - 9/5*log(abs(x + 3)) + 4/5*log(abs(x - 2))
```

3.128 $\int \frac{2+x}{4-4x+x^2} \, dx$

Optimal. Leaf size=16

$$\frac{4}{2-x} + \log(2-x)$$

[Out] 4/(2 - x) + Log[2 - x]

Rubi [A] time = 0.0052333, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {27, 43}

$$\frac{4}{2-x} + \log(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + x)/(4 - 4*x + x^2),x]
```

```
[Out] 4/(2 - x) + Log[2 - x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{2+x}{4-4x+x^2} \, dx &= \int \frac{2+x}{(-2+x)^2} \, dx \\
&= \int \left(\frac{4}{(-2+x)^2} + \frac{1}{-2+x}\right) dx \\
&= \frac{4}{2-x} + \log(2-x)
\end{aligned}$$

Mathematica [A] time = 0.002953, size = 12, normalized size = 0.75

$$\log(x-2) - \frac{4}{x-2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + x)/(4 - 4*x + x^2),x]
```

```
[Out] -4/(-2 + x) + Log[-2 + x]
```

Maple [A] time = 0.003, size = 13, normalized size = 0.8

$$-4\left(-2+x\right)^{-1}+\ln\left(-2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2+x)/(x^2-4*x+4),x)
```

```
[Out] -4/(-2+x)+ln(-2+x)
```

Maxima [A] time = 0.961557, size = 16, normalized size = 1.

$$-\frac{4}{x-2}+\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2-4*x+4),x, algorithm="maxima")
```

```
[Out] -4/(x - 2) + log(x - 2)
```

Fricas [A] time = 1.14659, size = 46, normalized size = 2.88

$$\frac{(x-2)\log\left(x-2\right)-4}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2-4*x+4),x, algorithm="fricas")
```

```
[Out] ((x - 2)*log(x - 2) - 4)/(x - 2)
```

Sympy [A] time = 0.075429, size = 8, normalized size = 0.5

$$\log\left(x-2\right)-\frac{4}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x**2-4*x+4),x)
```

```
[Out] log(x - 2) - 4/(x - 2)
```

Giac [A] time = 1.07706, size = 18, normalized size = 1.12

$$-\frac{4}{x-2}+\log\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(x^2-4*x+4),x, algorithm="giac")
```

```
[Out] -4/(x - 2) + log(abs(x - 2))
```

3.129 $\int \frac{1}{(4-4x+x^2)(5-4x+x^2)} \, dx$

Optimal. Leaf size=14

$$\frac{1}{2-x} + \tan^{-1}(2-x)$$

```
[Out] (2 - x)^(-1) + ArcTan[2 - x]
```

Rubi [A] time = 0.0103845, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {27, 693, 618, 204}

$$\frac{1}{2-x} + \tan^{-1}(2-x)$$

Antiderivative was successfully verified.

```
[In] Int[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
```

```
[Out] (2 - x)^(-1) + ArcTan[2 - x]
```

Rule 27

```
Int[(u_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[u*Canc
el[(b/2 + c*x)^(2*p)/c^p], x] /; FreeQ[{a, b, c}, x] && EqQ[b^2 - 4*a*c, 0]
 && IntegerQ[p]
```

Rule 693

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(-2*b*d*(d + e*x)^(m + 1)*(a + b*x + c*x^2)^(p + 1))/(d^2*(m
 + 1)*(b^2 - 4*a*c)), x] + Dist[(b^2*(m + 2*p + 3))/(d^2*(m + 1)*(b^2 - 4*a
*c)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p +
 3, 0] && LtQ[m, -1] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) ||
IntegerQ[(m + 2*p + 3)/2])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(4-4x+x^2\right)\left(5-4x+x^2\right)}\,dx &= \int \frac{1}{(-2+x)^2\left(5-4x+x^2\right)}\,dx\\ &= \frac{1}{2-x}-\int \frac{1}{5-4x+x^2}\,dx\\ &= \frac{1}{2-x}+2\,\mathrm{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, -4+2x\right)\\ &= \frac{1}{2-x}+\tan^{-1}(2-x)\end{aligned}$$

Mathematica [A] time = 0.0069006, size = 14, normalized size = 1.

$$\tan^{-1}(2-x)-\frac{1}{x-2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((4 - 4*x + x^2)*(5 - 4*x + x^2)),x]
```

```
[Out] -(-2 + x)^(-1) + ArcTan[2 - x]
```

Maple [A] time = 0.005, size = 15, normalized size = 1.1

$$-\left(-2+x\right)^{-1}-\arctan\left(-2+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-4*x+4)/(x^2-4*x+5),x)
```

```
[Out] -1/(-2+x)-arctan(-2+x)
```

Maxima [A] time = 1.42693, size = 19, normalized size = 1.36

$$-\frac{1}{x-2}-\arctan\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="maxima")
```

```
[Out] -1/(x - 2) - arctan(x - 2)
```

Fricas [A] time = 1.06074, size = 51, normalized size = 3.64

$$-\frac{(x-2)\arctan\left(x-2\right)+1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="fricas")
```

```
[Out] -((x - 2)*arctan(x - 2) + 1)/(x - 2)
```

Sympy [A] time = 0.125062, size = 10, normalized size = 0.71

$$-\operatorname{atan}(x-2)-\frac{1}{x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-4*x+4)/(x**2-4*x+5),x)
```

```
[Out] -atan(x - 2) - 1/(x - 2)
```

Giac [A] time = 1.0966, size = 19, normalized size = 1.36

$$-\frac{1}{x-2}-\arctan(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-4*x+4)/(x^2-4*x+5),x, algorithm="giac")
```

```
[Out] -1/(x - 2) - arctan(x - 2)
```

3.130 $\int \frac{-3+x}{2x+3x^2+x^3}\,dx$

Optimal. Leaf size=21

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Rubi [A] time = 0.0179242, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1594, 800}

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{-3+x}{2x+3x^2+x^3}\,dx &= \int \frac{-3+x}{x\left(2+3x+x^2\right)}\,dx\\ &= \int \left(-\frac{3}{2x}+\frac{4}{1+x}-\frac{5}{2(2+x)}\right)dx\\ &= -\frac{3\log(x)}{2}+4\log(1+x)-\frac{5}{2}\log(2+x)\end{aligned}$$

Mathematica [A] time = 0.004419, size = 21, normalized size = 1.

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5}{2}\log(x+2)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-3 + x)/(2*x + 3*x^2 + x^3),x]
```

```
[Out] (-3*Log[x])/2 + 4*Log[1 + x] - (5*Log[2 + x])/2
```

Maple [A] time = 0.005, size = 18, normalized size = 0.9

$$-\frac{3\ln(x)}{2}+4\ln(1+x)-\frac{5\ln(2+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-3+x)/(x^3+3*x^2+2*x),x)
```

```
[Out] -3/2*ln(x)+4*ln(1+x)-5/2*ln(2+x)
```

Maxima [A] time = 0.957003, size = 23, normalized size = 1.1

$$-\frac{5}{2}\log(x+2)+4\log(x+1)-\frac{3}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="maxima")
```

```
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

Fricas [A] time = 1.15586, size = 61, normalized size = 2.9

$$-\frac{5}{2}\log(x+2)+4\log(x+1)-\frac{3}{2}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="fricas")
```

```
[Out] -5/2*log(x + 2) + 4*log(x + 1) - 3/2*log(x)
```

Sympy [A] time = 0.123433, size = 20, normalized size = 0.95

$$-\frac{3\log(x)}{2}+4\log(x+1)-\frac{5\log(x+2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x**3+3*x**2+2*x),x)
```

```
[Out] -3*log(x)/2 + 4*log(x + 1) - 5*log(x + 2)/2
```

Giac [A] time = 1.10645, size = 27, normalized size = 1.29

$$-\frac{5}{2}\log(|x+2|)+4\log(|x+1|)-\frac{3}{2}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-3+x)/(x^3+3*x^2+2*x),x, algorithm="giac")
```

```
[Out] -5/2*log(abs(x + 2)) + 4*log(abs(x + 1)) - 3/2*log(abs(x))
```

3.131 $\int \frac{1}{\left(-1+x^2\right)^2} \, dx$

Optimal. Leaf size=21

$$\frac{x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] x/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rubi [A] time = 0.0027293, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {199, 207}

$$\frac{x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^2)^(-2),x]
```

```
[Out] x/(2*(1 - x^2)) + ArcTanh[x]/2
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(-1+x^2\right)^2} \, dx &= \frac{x}{2\left(1-x^2\right)}-\frac{1}{2}\int \frac{1}{-1+x^2} \, dx\\ &= \frac{x}{2\left(1-x^2\right)}+\frac{1}{2}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0062248, size = 27, normalized size = 1.29

$$\frac{1}{4}\left(-\frac{2x}{x^2-1}-\log(1-x)+\log(x+1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^2)^(-2),x]

[Out] ((-2*x)/(-1 + x^2) - Log[1 - x] + Log[1 + x])/4
```

Maple [A] time = 0.007, size = 28, normalized size = 1.3

$$-\frac{1}{4+4x}+\frac{\ln(1+x)}{4}-\frac{1}{4x-4}-\frac{\ln(-1+x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^2,x)

[Out] -1/4/(1+x)+1/4*ln(1+x)-1/4/(-1+x)-1/4*ln(-1+x)
```

Maxima [A] time = 0.96877, size = 31, normalized size = 1.48

$$-\frac{x}{2\left(x^2-1\right)}+\frac{1}{4}\log(x+1)-\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="maxima")

[Out] -1/2*x/(x^2 - 1) + 1/4*log(x + 1) - 1/4*log(x - 1)
```

Fricas [B] time = 1.24772, size = 90, normalized size = 4.29

$$\frac{\left(x^2-1\right)\log(x+1)-\left(x^2-1\right)\log(x-1)-2x}{4\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="fricas")

[Out] 1/4*((x^2 - 1)*log(x + 1) - (x^2 - 1)*log(x - 1) - 2*x)/(x^2 - 1)
```

Sympy [A] time = 0.097466, size = 20, normalized size = 0.95

$$-\frac{x}{2x^2-2}-\frac{\log(x-1)}{4}+\frac{\log(x+1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**2,x)

[Out] -x/(2*x**2 - 2) - log(x - 1)/4 + log(x + 1)/4
```

Giac [A] time = 1.09415, size = 34, normalized size = 1.62

$$-\frac{x}{2\left(x^2-1\right)}+\frac{1}{4}\log\left(|x+1|\right)-\frac{1}{4}\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2,x, algorithm="giac")
```

```
[Out] -1/2*x/(x^2 - 1) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))
```

3.132 $\int \frac{1+x}{-1+x^3} \, dx$

Optimal. Leaf size=22

$$\frac{2}{3}\log(1-x)-\frac{1}{3}\log\left(x^2+x+1\right)$$

```
[Out] (2*Log[1 - x])/3 - Log[1 + x + x^2]/3
```

Rubi [A] time = 0.0106187, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1861, 31, 628}

$$\frac{2}{3}\log(1-x)-\frac{1}{3}\log\left(x^2+x+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x)/(-1 + x^3),x]
```

```
[Out] (2*Log[1 - x])/3 - Log[1 + x + x^2]/3
```

Rule 1861

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[-(a/b), 3]], s = Denominator[Rt[-(a/b), 3]]}, Dist[(r*(B*r + A*s))/
(3*a*s), Int[1/(r - s*x), x], x] - Dist[r/(3*a*s), Int[(r*(B*r - 2*A*s) - s
*(B*r + A*s)*x)/(r^2 + r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x]
&& NeQ[a*B^3 - b*A^3, 0] && NegQ[a/b]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1+x}{-1+x^3} \, dx &= \frac{1}{3}\int \frac{-1-2x}{1+x+x^2} \, dx-\frac{2}{3}\int \frac{1}{1-x} \, dx\\ &= \frac{2}{3}\log(1-x)-\frac{1}{3}\log\left(1+x+x^2\right)\end{aligned}$$

Mathematica [A] time = 0.0026044, size = 22, normalized size = 1.

$$\frac{2}{3}\log(1-x)-\frac{1}{3}\log\left(x^2+x+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x)/(-1 + x^3),x]
```

```
[Out] (2*Log[1 - x])/3 - Log[1 + x + x^2]/3
```

Maple [A] time = 0.004, size = 17, normalized size = 0.8

$$\frac{2\ln(-1+x)}{3}-\frac{\ln\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)/(x^3-1),x)
```

```
[Out] 2/3*ln(-1+x)-1/3*ln(x^2+x+1)
```

Maxima [A] time = 1.42956, size = 22, normalized size = 1.

$$-\frac{1}{3}\log\left(x^2+x+1\right)+\frac{2}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3-1),x, algorithm="maxima")
```

```
[Out] -1/3*log(x^2 + x + 1) + 2/3*log(x - 1)
```

Fricas [A] time = 0.973768, size = 54, normalized size = 2.45

$$-\frac{1}{3}\log\left(x^2+x+1\right)+\frac{2}{3}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3-1),x, algorithm="fricas")
```

```
[Out] -1/3*log(x^2 + x + 1) + 2/3*log(x - 1)
```

Sympy [A] time = 0.086469, size = 17, normalized size = 0.77

$$\frac{2\log(x-1)}{3}-\frac{\log\left(x^2+x+1\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x**3-1),x)
```

```
[Out] 2*log(x - 1)/3 - log(x**2 + x + 1)/3
```

Giac [A] time = 1.11627, size = 23, normalized size = 1.05

$$-\frac{1}{3}\,\log\left(x^2+x+1\right)+\frac{2}{3}\,\log\left(|x-1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)/(x^3-1),x, algorithm="giac")
```

```
[Out] -1/3*log(x^2 + x + 1) + 2/3*log(abs(x - 1))
```

3.133 $\int \frac{1+x^4}{x\left(1+x^2\right)^2} \, dx$

Optimal. Leaf size=10

$$\frac{1}{x^2+1}+\log(x)$$

```
[Out] (1 + x^2)^(-1) + Log[x]
```

Rubi [A] time = 0.0149757, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1252, 894}

$$\frac{1}{x^2+1}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)/(x*(1 + x^2)^2),x]

[Out] (1 + x^2)^(-1) + Log[x]
```

Rule 1252

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+x^4}{x\left(1+x^2\right)^2} \, dx &= \frac{1}{2} \text{Subst}\left(\int \frac{1+x^2}{x(1+x)^2} \, dx, x, x^2\right)\\
&= \frac{1}{2} \text{Subst}\left(\int \left(\frac{1}{x}-\frac{2}{(1+x)^2}\right) \, dx, x, x^2\right)\\
&= \frac{1}{1+x^2}+\log(x)
\end{aligned}$$

Mathematica [A] time = 0.0042527, size = 10, normalized size = 1.

$$\frac{1}{x^2+1}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)/(x*(1 + x^2)^2),x]
```

```
[Out] (1 + x^2)^(-1) + Log[x]
```

Maple [A] time = 0.006, size = 11, normalized size = 1.1

$$\left(x^2+1\right)^{-1}+\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4+1)/x/(x^2+1)^2,x)
```

```
[Out] 1/(x^2+1)+ln(x)
```

Maxima [A] time = 0.934839, size = 19, normalized size = 1.9

$$\frac{1}{x^2+1}+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="maxima")
```

```
[Out] 1/(x^2 + 1) + 1/2*log(x^2)
```

Fricas [A] time = 1.12522, size = 46, normalized size = 4.6

$$\frac{\left(x^2+1\right)\log\left(x\right)+1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="fricas")
```

```
[Out] ((x^2 + 1)*log(x) + 1)/(x^2 + 1)
```

Sympy [A] time = 0.094624, size = 8, normalized size = 0.8

$$\log\left(x\right)+\frac{1}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**4+1)/x/(x**2+1)**2,x)
```

```
[Out] log(x) + 1/(x**2 + 1)
```

Giac [A] time = 1.09801, size = 19, normalized size = 1.9

$$\frac{1}{x^2+1}+\frac{1}{2}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^4+1)/x/(x^2+1)^2,x, algorithm="giac")
```

```
[Out] 1/(x^2 + 1) + 1/2*log(x^2)
```

3.134 $\int \frac{1}{-2x^3+x^4}\,dx$

Optimal. Leaf size=31

$$\frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}$$

```
[Out] 1/(4*x^2) + 1/(4*x) + Log[2 - x]/8 - Log[x]/8
```

Rubi [A] time = 0.0090875, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1593, 44}

$$\frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(-2*x^3 + x^4)^(-1),x]
```

```
[Out] 1/(4*x^2) + 1/(4*x) + Log[2 - x]/8 - Log[x]/8
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-2x^3+x^4}\,dx &= \int \frac{1}{(-2+x)x^3}\,dx\\ &= \int \left(\frac{1}{8(-2+x)}-\frac{1}{2x^3}-\frac{1}{4x^2}-\frac{1}{8x}\right)dx\\ &= \frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}\end{aligned}$$

Mathematica [A] time = 0.0018443, size = 31, normalized size = 1.

$$\frac{1}{4x^2}+\frac{1}{4x}+\frac{1}{8}\log(2-x)-\frac{\log(x)}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-2*x^3 + x^4)^(-1),x]
```

```
[Out] 1/(4*x^2) + 1/(4*x) + Log[2 - x]/8 - Log[x]/8
```

Maple [A] time = 0.006, size = 22, normalized size = 0.7

$$\frac{1}{4x^2}+\frac{1}{4x}-\frac{\ln(x)}{8}+\frac{\ln(-2+x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-2*x^3),x)
```

```
[Out] 1/4/x^2+1/4/x-1/8*ln(x)+1/8*ln(-2+x)
```

Maxima [A] time = 0.937358, size = 26, normalized size = 0.84

$$\frac{x+1}{4x^2}+\frac{1}{8}\log(x-2)-\frac{1}{8}\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2*x^3),x, algorithm="maxima")
```

```
[Out] 1/4*(x + 1)/x^2 + 1/8*log(x - 2) - 1/8*log(x)
```

Fricas [A] time = 1.13007, size = 66, normalized size = 2.13

$$\frac{x^2\log(x-2)-x^2\log(x)+2x+2}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2*x^3),x, algorithm="fricas")
```

```
[Out] 1/8*(x^2*log(x - 2) - x^2*log(x) + 2*x + 2)/x^2
```

Sympy [A] time = 0.100686, size = 19, normalized size = 0.61

$$-\frac{\log(x)}{8}+\frac{\log(x-2)}{8}+\frac{x+1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-2*x**3),x)
```

```
[Out] -log(x)/8 + log(x - 2)/8 + (x + 1)/(4*x**2)
```

Giac [A] time = 1.06752, size = 28, normalized size = 0.9

$$\frac{x+1}{4x^2}+\frac{1}{8}\log(|x-2|)-\frac{1}{8}\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-2*x^3),x, algorithm="giac")
```

```
[Out] 1/4*(x + 1)/x^2 + 1/8*log(abs(x - 2)) - 1/8*log(abs(x))
```

3.135 $\int \frac{1-x^3}{x\left(1+x^2\right)} dx$

Optimal. Leaf size=18

$$-\frac{1}{2}\log\left(x^2+1\right)-x+\log(x)+\tan^{-1}(x)$$

```
[Out] -x + ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rubi [A] time = 0.0251952, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1802, 635, 203, 260}

$$-\frac{1}{2}\log\left(x^2+1\right)-x+\log(x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - x^3)/(x*(1 + x^2)),x]
```

```
[Out] -x + ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1-x^3}{x\left(1+x^2\right)} dx &= \int \left(-1+\frac{1}{x}+\frac{1-x}{1+x^2}\right) dx \\
&= -x+\log(x)+\int \frac{1-x}{1+x^2} dx \\
&= -x+\log(x)+\int \frac{1}{1+x^2} dx-\int \frac{x}{1+x^2} dx \\
&= -x+\tan^{-1}(x)+\log(x)-\frac{1}{2}\log\left(1+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.0047565, size = 18, normalized size = 1.

$$-\frac{1}{2}\log\left(x^2+1\right)-x+\log(x)+\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - x^3)/(x*(1 + x^2)),x]

[Out] -x + ArcTan[x] + Log[x] - Log[1 + x^2]/2
```

Maple [A] time = 0.006, size = 17, normalized size = 0.9

$$-x+\arctan(x)+\ln(x)-\frac{\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^3+1)/x/(x^2+1),x)

[Out] -x+arctan(x)+ln(x)-1/2*ln(x^2+1)
```

Maxima [A] time = 1.42209, size = 22, normalized size = 1.22

$$-x+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^2+1),x, algorithm="maxima")

[Out] -x + arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Fricas [A] time = 1.05672, size = 59, normalized size = 3.28

$$-x+\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)+\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^2+1),x, algorithm="fricas")

[Out] -x + arctan(x) - 1/2*log(x^2 + 1) + log(x)
```

Sympy [A] time = 0.110263, size = 15, normalized size = 0.83

$$-x+\log(x)-\frac{\log\left(x^2+1\right)}{2}+\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**3+1)/x/(x**2+1),x)
```

```
[Out] -x + log(x) - log(x**2 + 1)/2 + atan(x)
```

Giac [A] time = 1.11354, size = 23, normalized size = 1.28

$$-x + \arctan(x) - \frac{1}{2} \log\left(x^2 + 1\right) + \log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^3+1)/x/(x^2+1),x, algorithm="giac")
```

```
[Out] -x + arctan(x) - 1/2*log(x^2 + 1) + log(abs(x))
```

3.136 $\int \frac{1}{-1+x^4} \, dx$

Optimal. Leaf size=13

$$-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

```
[Out] -ArcTan[x]/2 - ArcTanh[x]/2
```

Rubi [A] time = 0.0030568, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {212, 206, 203}

$$-\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + x^4)^(-1),x]
```

```
[Out] -ArcTan[x]/2 - ArcTanh[x]/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{-1+x^4} \, dx &= -\left(\frac{1}{2}\int \frac{1}{1-x^2} \, dx\right)-\frac{1}{2}\int \frac{1}{1+x^2} \, dx\\ &= -\frac{1}{2}\tan^{-1}(x)-\frac{1}{2}\tanh^{-1}(x)\end{aligned}$$

Mathematica [A] time = 0.0032092, size = 25, normalized size = 1.92

$$\frac{1}{4}\log(1-x)-\frac{1}{4}\log(x+1)-\frac{1}{2}\tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x^4)^(-1),x]
```

```
[Out] -ArcTan[x]/2 + Log[1 - x]/4 - Log[1 + x]/4
```

Maple [A] time = 0.001, size = 10, normalized size = 0.8

$$-\frac{\arctan(x)}{2}-\frac{\operatorname{Artanh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4-1),x)
```

```
[Out] -1/2*arctan(x)-1/2*arctanh(x)
```

Maxima [A] time = 1.43694, size = 23, normalized size = 1.77

$$-\frac{1}{2}\arctan(x)-\frac{1}{4}\log(x+1)+\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="maxima")
```

```
[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Fricas [A] time = 0.98969, size = 68, normalized size = 5.23

$$-\frac{1}{2}\arctan(x)-\frac{1}{4}\log(x+1)+\frac{1}{4}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(x) - 1/4*log(x + 1) + 1/4*log(x - 1)
```

Sympy [A] time = 0.117836, size = 17, normalized size = 1.31

$$\frac{\log(x-1)}{4}-\frac{\log(x+1)}{4}-\frac{\operatorname{atan}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4-1),x)
```

```
[Out] log(x - 1)/4 - log(x + 1)/4 - atan(x)/2
```

<u>Giac</u> [B] time = 1.11441, size = 26, normalized size = 2.

$$-\frac{1}{2}\arctan(x) - \frac{1}{4}\log(|x+1|) + \frac{1}{4}\log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4-1),x, algorithm="giac")
```

```
[Out] -1/2*arctan(x) - 1/4*log(abs(x + 1)) + 1/4*log(abs(x - 1))
```

3.137 $\int \frac{1}{1+x^4} \, dx$

Optimal. Leaf size=85

$$-\frac{\log \left(x^2-\sqrt{2} x+1\right)}{4 \sqrt{2}}+\frac{\log \left(x^2+\sqrt{2} x+1\right)}{4 \sqrt{2}}-\frac{\tan ^{-1}\left(1-\sqrt{2} x\right)}{2 \sqrt{2}}+\frac{\tan ^{-1}\left(\sqrt{2} x+1\right)}{2 \sqrt{2}}$$

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) - Lo
g[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])
```

Rubi [A] time = 0.0394903, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {211, 1165, 628, 1162, 617, 204}

$$-\frac{\log \left(x^2-\sqrt{2} x+1\right)}{4 \sqrt{2}}+\frac{\log \left(x^2+\sqrt{2} x+1\right)}{4 \sqrt{2}}-\frac{\tan ^{-1}\left(1-\sqrt{2} x\right)}{2 \sqrt{2}}+\frac{\tan ^{-1}\left(\sqrt{2} x+1\right)}{2 \sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^4)^(-1),x]
```

```
[Out] -ArcTan[1 - Sqrt[2]*x]/(2*Sqrt[2]) + ArcTan[1 + Sqrt[2]*x]/(2*Sqrt[2]) - Lo
g[1 - Sqrt[2]*x + x^2]/(4*Sqrt[2]) + Log[1 + Sqrt[2]*x + x^2]/(4*Sqrt[2])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+x^4}\,dx &= \frac{1}{2}\int \frac{1-x^2}{1+x^4}\,dx+\frac{1}{2}\int \frac{1+x^2}{1+x^4}\,dx\\ &= \frac{1}{4}\int \frac{1}{1-\sqrt{2}x+x^2}\,dx+\frac{1}{4}\int \frac{1}{1+\sqrt{2}x+x^2}\,dx-\frac{\int \frac{\sqrt{2}+2x}{-1-\sqrt{2}x-x^2}\,dx}{4\sqrt{2}}-\frac{\int \frac{\sqrt{2}-2x}{-1+\sqrt{2}x-x^2}\,dx}{4\sqrt{2}}\\ &= -\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,1-\sqrt{2}x\right)}{2\sqrt{2}}-\frac{\text{Subst}\left(\int \frac{1}{-1-x^2}\,dx,x,\right.}{2\sqrt{2}}\\ &= -\frac{\tan^{-1}\left(1-\sqrt{2}x\right)}{2\sqrt{2}}+\frac{\tan^{-1}\left(1+\sqrt{2}x\right)}{2\sqrt{2}}-\frac{\log\left(1-\sqrt{2}x+x^2\right)}{4\sqrt{2}}+\frac{\log\left(1+\sqrt{2}x+x^2\right)}{4\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0170554, size = 64, normalized size = 0.75

$$\frac{-\log\left(x^2-\sqrt{2}x+1\right)+\log\left(x^2+\sqrt{2}x+1\right)-2\tan^{-1}\left(1-\sqrt{2}x\right)+2\tan^{-1}\left(\sqrt{2}x+1\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^4)^(-1),x]
```

```
[Out] (-2*ArcTan[1 - Sqrt[2]*x] + 2*ArcTan[1 + Sqrt[2]*x] - Log[1 - Sqrt[2]*x + x
^2] + Log[1 + Sqrt[2]*x + x^2])/(4*Sqrt[2])
```

Maple [A] time = 0.003, size = 58, normalized size = 0.7

$$\frac{\arctan\left(1+x\sqrt{2}\right)\sqrt{2}}{4}+\frac{\arctan\left(-1+x\sqrt{2}\right)\sqrt{2}}{4}+\frac{\sqrt{2}}{8}\ln\left(\frac{1+x^2+x\sqrt{2}}{1+x^2-x\sqrt{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4+1),x)
```

```
[Out] 1/4*arctan(1+x*2^(1/2))*2^(1/2)+1/4*arctan(-1+x*2^(1/2))*2^(1/2)+1/8*2^(1/2
)*ln((1+x^2+x*2^(1/2))/(1+x^2-x*2^(1/2)))
```

Maxima [A] time = 1.42245, size = 97, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\left(x^2-\sqrt{2}x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

Fricas [A] time = 1.20729, size = 302, normalized size = 3.55

$$-\frac{1}{2}\sqrt{2}\arctan\Big(-\sqrt{2}x+\sqrt{2}\sqrt{x^2+\sqrt{2}x+1}-1\Big)-\frac{1}{2}\sqrt{2}\arctan\Big(-\sqrt{2}x+\sqrt{2}\sqrt{x^2-\sqrt{2}x+1}+1\Big)+\frac{1}{8}\sqrt{2}\log\big(x^2+\sqrt{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 + sqrt(2)*x + 1) - 1) - 1
/2*sqrt(2)*arctan(-sqrt(2)*x + sqrt(2)*sqrt(x^2 - sqrt(2)*x + 1) + 1) + 1/8
*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)*log(x^2 - sqrt(2)*x + 1)
```

Sympy [A] time = 0.13663, size = 73, normalized size = 0.86

$$-\frac{\sqrt{2}\log\left(x^2-\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)}{8}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x-1\right)}{4}+\frac{\sqrt{2}\operatorname{atan}\left(\sqrt{2}x+1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**4+1),x)
```

```
[Out] -sqrt(2)*log(x**2 - sqrt(2)*x + 1)/8 + sqrt(2)*log(x**2 + sqrt(2)*x + 1)/8
+ sqrt(2)*atan(sqrt(2)*x - 1)/4 + sqrt(2)*atan(sqrt(2)*x + 1)/4
```

Giac [A] time = 1.08464, size = 97, normalized size = 1.14

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x+\sqrt{2}\right)\right)+\frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(2x-\sqrt{2}\right)\right)+\frac{1}{8}\sqrt{2}\log\left(x^2+\sqrt{2}x+1\right)-\frac{1}{8}\sqrt{2}\log\big(x^2-\sqrt{2}x+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^4+1),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2))) + 1/4*sqrt(2)*arctan(1/2*sq
rt(2)*(2*x - sqrt(2))) + 1/8*sqrt(2)*log(x^2 + sqrt(2)*x + 1) - 1/8*sqrt(2)
*log(x^2 - sqrt(2)*x + 1)
```

3.138 $\int \frac{x^2}{(2+2x+x^2)^2}\,dx$

Optimal. Leaf size=23

$$\tan^{-1}(x+1)-\frac{x(x+2)}{2\left(x^2+2x+2\right)}$$

```
[Out] -(x*(2 + x))/(2*(2 + 2*x + x^2)) + ArcTan[1 + x]
```

Rubi [A] time = 0.0072044, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {722, 617, 204}

$$\tan^{-1}(x+1)-\frac{x(x+2)}{2\left(x^2+2x+2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(2 + 2*x + x^2)^2,x]

[Out] -(x*(2 + x))/(2*(2 + 2*x + x^2)) + ArcTan[1 + x]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{\left(2+2x+x^2\right)^2}\,dx &= -\frac{x(2+x)}{2\left(2+2x+x^2\right)} + \int \frac{1}{2+2x+x^2}\,dx \\
&= -\frac{x(2+x)}{2\left(2+2x+x^2\right)} - \text{Subst}\left(\int \frac{1}{-1-x^2}\,dx, x, 1+x\right) \\
&= -\frac{x(2+x)}{2\left(2+2x+x^2\right)} + \tan^{-1}(1+x)
\end{aligned}
$$

Mathematica [A] time = 0.0073388, size = 15, normalized size = 0.65

$$\frac{1}{x^2+2x+2} + \tan^{-1}(x+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(2 + 2*x + x^2)^2,x]
```

```
[Out] (2 + 2*x + x^2)^(-1) + ArcTan[1 + x]
```

Maple [A] time = 0.005, size = 16, normalized size = 0.7

$$\left(x^2+2\,x+2\right)^{-1} + \arctan\left(1+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+2*x+2)^2,x)
```

```
[Out] 1/(x^2+2*x+2)+arctan(1+x)
```

Maxima [A] time = 1.41192, size = 20, normalized size = 0.87

$$\frac{1}{x^2+2\,x+2} + \arctan\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+2)^2,x, algorithm="maxima")
```

```
[Out] 1/(x^2 + 2*x + 2) + arctan(x + 1)
```

Fricas [A] time = 1.0115, size = 72, normalized size = 3.13

$$\frac{\left(x^2+2\,x+2\right)\arctan\left(x+1\right)+1}{x^2+2\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+2)^2,x, algorithm="fricas")
```

```
[Out] ((x^2 + 2*x + 2)*arctan(x + 1) + 1)/(x^2 + 2*x + 2)
```

Sympy [A] time = 0.104021, size = 14, normalized size = 0.61

$$\operatorname{atan}(x+1) + \frac{1}{x^2+2x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+2*x+2)**2,x)
```

```
[Out] atan(x + 1) + 1/(x**2 + 2*x + 2)
```

Giac [A] time = 1.0862, size = 20, normalized size = 0.87

$$\frac{1}{x^2+2x+2} + \arctan(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+2*x+2)^2,x, algorithm="giac")
```

```
[Out] 1/(x^2 + 2*x + 2) + arctan(x + 1)
```

3.139 $\int \frac{-1+4x^5}{(1+x+x^5)^2}\,dx$

Optimal. Leaf size=11

$$-\frac{x}{x^5+x+1}$$

[Out] -(x/(1 + x + x^5))

Rubi [A] time = 0.0064788, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1588}

$$-\frac{x}{x^5+x+1}$$

Antiderivative was successfully verified.

[In] Int[(-1 + 4*x^5)/(1 + x + x^5)^2,x]

[Out] -(x/(1 + x + x^5))

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{-1+4x^5}{\left(1+x+x^5\right)^2}\,dx=-\frac{x}{1+x+x^5}$$

Mathematica [A] time = 0.0056537, size = 11, normalized size = 1.

$$-\frac{x}{x^5+x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 4*x^5)/(1 + x + x^5)^2,x]

[Out] -(x/(1 + x + x^5))

Maple [B] time = 0.008, size = 41, normalized size = 3.7

$$-\frac{-3x^2+5x-1}{7x^3-7x^2+7}+\frac{-3x-1}{7x^2+7x+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^5-1)/(x^5+x+1)^2,x)
```

```
[Out] -1/7*(-3*x^2+5*x-1)/(x^3-x^2+1)+1/7*(-3*x-1)/(x^2+x+1)
```

Maxima [A] time = 0.933101, size = 15, normalized size = 1.36

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="maxima")
```

```
[Out] -x/(x^5 + x + 1)
```

Fricas [A] time = 1.01495, size = 24, normalized size = 2.18

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="fricas")
```

```
[Out] -x/(x^5 + x + 1)
```

Sympy [A] time = 0.119909, size = 8, normalized size = 0.73

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**5-1)/(x**5+x+1)**2,x)
```

```
[Out] -x/(x**5 + x + 1)
```

Giac [A] time = 1.12447, size = 15, normalized size = 1.36

$$-\frac{x}{x^5+x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^5-1)/(x^5+x+1)^2,x, algorithm="giac")
```

```
[Out] -x/(x^5 + x + 1)
```

3.140 $\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx$

Optimal. Leaf size=45

$$\frac{x}{2\sqrt{5}}+\frac{\tan^{-1}\left(\frac{\sin(x)+2\cos(x)}{2\sin(x)-\cos(x)+2\sqrt{5}+5}\right)}{\sqrt{5}}$$

```
[Out] x/(2*Sqrt[5]) + ArcTan[(2*Cos[x] + Sin[x])/(5 + 2*Sqrt[5] - Cos[x] + 2*Sin[
x])]/Sqrt[5]
```

Rubi [A] time = 0.0242, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3124, 618, 204}

$$\frac{x}{2\sqrt{5}}+\frac{\tan^{-1}\left(\frac{\sin(x)+2\cos(x)}{2\sin(x)-\cos(x)+2\sqrt{5}+5}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(5 - Cos[x] + 2*Sin[x])^(-1),x]
```

```
[Out] x/(2*Sqrt[5]) + ArcTan[(2*Cos[x] + Sin[x])/(5 + 2*Sqrt[5] - Cos[x] + 2*Sin[
x])]/Sqrt[5]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{5-\cos(x)+2\sin(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{4+4x+6x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\left(4\,\text{Subst}\left(\int \frac{1}{-80-x^2}\,dx, x, 4+12\tan\left(\frac{x}{2}\right)\right)\right)\\ &= \frac{x}{2\sqrt{5}}+\frac{\tan^{-1}\left(\frac{2\cos(x)+\sin(x)}{5+2\sqrt{5}-\cos(x)+2\sin(x)}\right)}{\sqrt{5}}\end{aligned}$$

Mathematica [A] time = 0.0238248, size = 23, normalized size = 0.51

$$\frac{\tan^{-1}\left(\frac{3\tan\left(\frac{x}{2}\right)+1}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 - Cos[x] + 2*Sin[x])^(-1),x]
```

```
[Out] ArcTan[(1 + 3*Tan[x/2])/Sqrt[5]]/Sqrt[5]
```

Maple [A] time = 0.035, size = 20, normalized size = 0.4

$$\frac{\sqrt{5}}{5}\arctan\left(\frac{\sqrt{5}}{10}(6\,\tan(x/2)+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5-cos(x)+2*sin(x)),x)
```

```
[Out] 1/5*5^(1/2)*arctan(1/10*(6*tan(1/2*x)+2)*5^(1/2))
```

Maxima [A] time = 1.41722, size = 31, normalized size = 0.69

$$\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{5}\sqrt{5}\left(\frac{3\,\sin(x)}{\cos(x)+1}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-cos(x)+2*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/5*sqrt(5)*arctan(1/5*sqrt(5)*(3*sin(x)/(cos(x) + 1) + 1))
```

Fricas [A] time = 1.10643, size = 126, normalized size = 2.8

$$\frac{1}{10}\sqrt{5}\arctan\left(-\frac{\sqrt{5}\cos(x)-2\sqrt{5}\sin(x)-\sqrt{5}}{2\,(2\,\cos(x)+\sin(x))}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-cos(x)+2*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/10*sqrt(5)*arctan(-1/2*(sqrt(5)*cos(x) - 2*sqrt(5)*sin(x) - sqrt(5))/(2*c
os(x) + sin(x)))
```

Sympy [A] time = 0.494684, size = 39, normalized size = 0.87

$$\frac{\sqrt{5}\left(\operatorname{atan}\left(\frac{3\sqrt{5}\tan\left(\frac{x}{2}\right)}{5}+\frac{\sqrt{5}}{5}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-cos(x)+2*sin(x)),x)
```

```
[Out] sqrt(5)*(atan(3*sqrt(5)*tan(x/2)/5 + sqrt(5)/5) + pi*floor((x/2 - pi/2)/pi)
)/5
```

Giac [A] time = 1.09243, size = 63, normalized size = 1.4

$$\frac{1}{10}\sqrt{5}\left(x+2\arctan\left(-\frac{\sqrt{5}\sin(x)-\cos(x)-3\sin(x)-1}{\sqrt{5}\cos(x)+\sqrt{5}-3\cos(x)+\sin(x)+3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5-cos(x)+2*sin(x)),x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5)*(x + 2*arctan(-(sqrt(5)*sin(x) - cos(x) - 3*sin(x) - 1)/(sqrt(
5)*cos(x) + sqrt(5) - 3*cos(x) + sin(x) + 3)))
```

3.141 $\int \frac{1}{1+a\cos(x)}\,dx$

Optimal. Leaf size=37

$$\frac{2\tan^{-1}\left(\frac{\sqrt{1-a}\tan\left(\frac{x}{2}\right)}{\sqrt{a+1}}\right)}{\sqrt{1-a^2}}$$

```
[Out] (2*ArcTan[(Sqrt[1 - a]*Tan[x/2])/Sqrt[1 + a]])/Sqrt[1 - a^2]
```

Rubi [A] time = 0.0183064, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2659, 205}

$$\frac{2\tan^{-1}\left(\frac{\sqrt{1-a}\tan\left(\frac{x}{2}\right)}{\sqrt{a+1}}\right)}{\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*Cos[x])^(-1),x]
```

```
[Out] (2*ArcTan[(Sqrt[1 - a]*Tan[x/2])/Sqrt[1 + a]])/Sqrt[1 - a^2]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+a\cos(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1+a+(1-a)x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= \frac{2\tan^{-1}\left(\frac{\sqrt{1-a}\tan\left(\frac{x}{2}\right)}{\sqrt{1+a}}\right)}{\sqrt{1-a^2}}\end{aligned}$$

Mathematica [A] time = 0.0234304, size = 31, normalized size = 0.84

$$\frac{2\tanh^{-1}\left(\frac{(a-1)\tan\left(\frac{x}{2}\right)}{\sqrt{a^2-1}}\right)}{\sqrt{a^2-1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*Cos[x])^(-1),x]

[Out] (2*ArcTanh[((-1 + a)*Tan[x/2])/Sqrt[-1 + a^2]])/Sqrt[-1 + a^2]
```

Maple [A] time = 0.012, size = 30, normalized size = 0.8

$$2\,\frac{1}{\sqrt{(1+a)\,(a-1)}}\operatorname{Artanh}\left(\frac{(a-1)\tan(x/2)}{\sqrt{(1+a)\,(a-1)}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+a*cos(x)),x)

[Out] 2/((1+a)*(a-1))^(1/2)*arctanh((a-1)*tan(1/2*x)/((1+a)*(a-1))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+a*cos(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 1.16777, size = 306, normalized size = 8.27

$$\left[\frac{\log\left(-\frac{(a^2-2)\cos(x)^2-2\sqrt{a^2-1}(a+\cos(x))\sin(x)-2a^2-2a\cos(x)+1}{a^2\cos(x)^2+2a\cos(x)+1}\right)}{2\sqrt{a^2-1}}, -\frac{\sqrt{-a^2+1}\arctan\left(\frac{\sqrt{-a^2+1}(a+\cos(x))}{(a^2-1)\sin(x)}\right)}{a^2-1}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+a*cos(x)),x, algorithm="fricas")

[Out] [1/2*log(-((a^2 - 2)*cos(x)^2 - 2*sqrt(a^2 - 1)*(a + cos(x))*sin(x) - 2*a^2
 - 2*a*cos(x) + 1)/(a^2*cos(x)^2 + 2*a*cos(x) + 1))/sqrt(a^2 - 1), -sqrt(-a
^2 + 1)*arctan(sqrt(-a^2 + 1)*(a + cos(x))/((a^2 - 1)*sin(x)))/(a^2 - 1)]
```

Sympy [A] time = 5.919, size = 110, normalized size = 2.97

$$\begin{cases} -\frac{1}{\tan\left(\frac{x}{2}\right)} & \text{for } a=-1 \\ \tan\left(\frac{x}{2}\right) & \text{for } a=1 \\ -\frac{\log\left(-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}} + \frac{\log\left(\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}+\tan\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}-\sqrt{\frac{a}{a-1}+\frac{1}{a-1}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+a*cos(x)),x)
```

```
[Out] Piecewise((-1/tan(x/2), Eq(a, -1)), (tan(x/2), Eq(a, 1)), (-log(-sqrt(a/(a
- 1) + 1/(a - 1)) + tan(x/2))/(a*sqrt(a/(a - 1) + 1/(a - 1)) - sqrt(a/(a -
1) + 1/(a - 1))) + log(sqrt(a/(a - 1) + 1/(a - 1)) + tan(x/2))/(a*sqrt(a/(a
 - 1) + 1/(a - 1)) - sqrt(a/(a - 1) + 1/(a - 1))), True))
```

Giac [A] time = 1.09308, size = 72, normalized size = 1.95

$$-\frac{2\left(\pi\left\lfloor\frac{x}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2)+\arctan\left(\frac{a\tan\left(\frac{1}{2}x\right)-\tan\left(\frac{1}{2}x\right)}{\sqrt{-a^2+1}}\right)\right)}{\sqrt{-a^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+a*cos(x)),x, algorithm="giac")
```

```
[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(2*a - 2) + arctan((a*tan(1/2*x) - tan(1/2*
x))/sqrt(-a^2 + 1)))/sqrt(-a^2 + 1)
```

3.142 $\int \frac{1}{1+2\cos(x)}\,dx$

Optimal. Leaf size=56

$$\frac{\log\left(\sin\left(\frac{x}{2}\right)+\sqrt{3}\cos\left(\frac{x}{2}\right)\right)}{\sqrt{3}}-\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}$$

```
[Out] -(Log[Sqrt[3]*Cos[x/2] - Sin[x/2]]/Sqrt[3]) + Log[Sqrt[3]*Cos[x/2] + Sin[x/
2]]/Sqrt[3]
```

Rubi [A] time = 0.0154401, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2659, 206}

$$\frac{\log\left(\sin\left(\frac{x}{2}\right)+\sqrt{3}\cos\left(\frac{x}{2}\right)\right)}{\sqrt{3}}-\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*Cos[x])^(-1),x]
```

```
[Out] -(Log[Sqrt[3]*Cos[x/2] - Sin[x/2]]/Sqrt[3]) + Log[Sqrt[3]*Cos[x/2] + Sin[x/
2]]/Sqrt[3]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+2\cos(x)}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{3-x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right)\\ &= -\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}+\frac{\log\left(\sqrt{3}\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{\sqrt{3}}\end{aligned}$$

Mathematica [A] time = 0.0124048, size = 20, normalized size = 0.36

$$\frac{2\tanh^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*Cos[x])^(-1),x]
```

```
[Out] (2*ArcTanh[Tan[x/2]/Sqrt[3]])/Sqrt[3]
```

Maple [A] time = 0.007, size = 16, normalized size = 0.3

$$\frac{2\sqrt{3}}{3}\mathrm{Artanh}\left(\frac{\sqrt{3}}{3}\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+2*cos(x)),x)
```

```
[Out] 2/3*3^(1/2)*arctanh(1/3*3^(1/2)*tan(1/2*x))
```

Maxima [A] time = 1.41467, size = 50, normalized size = 0.89

$$-\frac{1}{3}\sqrt{3}\log\left(-\frac{\sqrt{3}-\frac{\sin(x)}{\cos(x)+1}}{\sqrt{3}+\frac{\sin(x)}{\cos(x)+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*cos(x)),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*log(-(sqrt(3) - sin(x)/(cos(x) + 1))/(sqrt(3) + sin(x)/(cos(x)
 + 1)))
```

Fricas [A] time = 1.03689, size = 155, normalized size = 2.77

$$\frac{1}{6}\sqrt{3}\log\left(-\frac{2\cos(x)^2-2\left(\sqrt{3}\cos(x)+2\sqrt{3}\right)\sin(x)-4\cos(x)-7}{4\cos(x)^2+4\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*cos(x)),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-(2*cos(x)^2 - 2*(sqrt(3)*cos(x) + 2*sqrt(3))*sin(x) - 4*co
s(x) - 7)/(4*cos(x)^2 + 4*cos(x) + 1))
```

Sympy [A] time = 0.352938, size = 36, normalized size = 0.64

$$-\frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)-\sqrt{3}\right)}{3}+\frac{\sqrt{3}\log\left(\tan\left(\frac{x}{2}\right)+\sqrt{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*cos(x)),x)
```

```
[Out] -sqrt(3)*log(tan(x/2) - sqrt(3))/3 + sqrt(3)*log(tan(x/2) + sqrt(3))/3
```

Giac [A] time = 1.16382, size = 47, normalized size = 0.84

$$-\frac{1}{3}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2\tan\left(\frac{1}{2}x\right)\right|}{\left|2\sqrt{3}+2\tan\left(\frac{1}{2}x\right)\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*cos(x)),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*log(abs(-2*sqrt(3) + 2*tan(1/2*x))/abs(2*sqrt(3) + 2*tan(1/2*x
)))
```

3.143 $\int \frac{1}{1+\frac{\cos(x)}{2}}\,dx$

Optimal. Leaf size=31

$$\frac{2x}{\sqrt{3}}-\frac{4\tan^{-1}\left(\frac{\sin(x)}{\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

[Out] (2*x)/Sqrt[3] - (4*ArcTan[Sin[x]/(2 + Sqrt[3] + Cos[x])])/Sqrt[3]

Rubi [A] time = 0.0114407, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.1, Rules used = {2657}

$$\frac{2x}{\sqrt{3}}-\frac{4\tan^{-1}\left(\frac{\sin(x)}{\cos(x)+\sqrt{3}+2}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Cos[x]/2)^(-1),x]

[Out] (2*x)/Sqrt[3] - (4*ArcTan[Sin[x]/(2 + Sqrt[3] + Cos[x])])/Sqrt[3]

Rule 2657

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2*ArcTan[(b*Cos[c + d*x])/(a + q + b*S
in[c + d*x])])/(d*q), x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Rubi steps

$$\int \frac{1}{1+\frac{\cos(x)}{2}}\,dx=\frac{2x}{\sqrt{3}}-\frac{4\tan^{-1}\left(\frac{\sin(x)}{2+\sqrt{3}+\cos(x)}\right)}{\sqrt{3}}$$

Mathematica [A] time = 0.0099244, size = 20, normalized size = 0.65

$$\frac{4\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Cos[x]/2)^(-1),x]

[Out] (4*ArcTan[Tan[x/2]/Sqrt[3]])/Sqrt[3]

Maple [A] time = 0.013, size = 16, normalized size = 0.5

$$\frac{4\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+1/2*cos(x)),x)
```

```
[Out] 4/3*3^(1/2)*arctan(1/3*3^(1/2)*tan(1/2*x))
```

Maxima [A] time = 1.41696, size = 26, normalized size = 0.84

$$\frac{4}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sin(x)}{3(\cos(x)+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/2*cos(x)),x, algorithm="maxima")
```

```
[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*sin(x)/(cos(x) + 1))
```

Fricas [A] time = 1.18087, size = 84, normalized size = 2.71

$$-\frac{2}{3}\sqrt{3}\arctan\left(\frac{2\sqrt{3}\cos(x)+\sqrt{3}}{3\sin(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/2*cos(x)),x, algorithm="fricas")
```

```
[Out] -2/3*sqrt(3)*arctan(1/3*(2*sqrt(3)*cos(x) + sqrt(3))/sin(x))
```

Sympy [A] time = 0.281181, size = 32, normalized size = 1.03

$$\frac{4\sqrt{3}\left(\operatorname{atan}\left(\frac{\sqrt{3}\tan\left(\frac{x}{2}\right)}{3}\right)+\pi\left\lfloor\frac{\frac{x}{2}-\frac{\pi}{2}}{\pi}\right\rfloor\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/2*cos(x)),x)
```

```
[Out] 4*sqrt(3)*(atan(sqrt(3)*tan(x/2)/3) + pi*floor((x/2 - pi/2)/pi))/3
```

Giac [A] time = 1.08095, size = 54, normalized size = 1.74

$$\frac{2}{3}\sqrt{3}\left(x+2\arctan\left(-\frac{\sqrt{3}\sin(x)-\sin(x)}{\sqrt{3}\cos(x)+\sqrt{3}-\cos(x)+1}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+1/2*cos(x)),x, algorithm="giac")
```

```
[Out] 2/3*sqrt(3)*(x + 2*arctan(-(sqrt(3)*sin(x) - sin(x))/(sqrt(3)*cos(x) + sqrt
(3) - cos(x) + 1)))
```

3.144 $\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx$

Optimal. Leaf size=36

$$-\frac{x}{\sqrt{2}}+x-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

```
[Out] x - x/Sqrt[2] - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/Sqrt[2]
```

Rubi [A] time = 0.0377339, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3171, 3181, 203}

$$-\frac{x}{\sqrt{2}}+x-\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(1 + Sin[x]^2),x]
```

```
[Out] x - x/Sqrt[2] - ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/Sqrt[2]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(x)}{1+\sin^2(x)}\,dx &= x-\int \frac{1}{1+\sin^2(x)}\,dx\\ &= x-\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\tan(x)\right)\\ &= x-\frac{x}{\sqrt{2}}-\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [A] time = 0.0288295, size = 18, normalized size = 0.5

$$x-\frac{\tan^{-1}\left(\sqrt{2}\tan(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(1 + Sin[x]^2),x]
```

```
[Out] x - ArcTan[Sqrt[2]*Tan[x]]/Sqrt[2]
```

Maple [A] time = 0.019, size = 15, normalized size = 0.4

$$-\frac{\sqrt{2}\arctan\left(\tan\left(x\right)\sqrt{2}\right)}{2}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(1+sin(x)^2),x)
```

```
[Out] -1/2*2^(1/2)*arctan(tan(x)*2^(1/2))+x
```

Maxima [A] time = 1.44985, size = 19, normalized size = 0.53

$$-\frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\tan\left(x\right)\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1+sin(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(2)*arctan(sqrt(2)*tan(x)) + x
```

Fricas [A] time = 1.21058, size = 105, normalized size = 2.92

$$\frac{1}{4}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos\left(x\right)^{2}-2\sqrt{2}}{4\cos\left(x\right)\sin\left(x\right)}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1+sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x))) +
x
```

Sympy [B] time = 64.5499, size = 581, normalized size = 16.14

$$\frac{7\sqrt{2}x\sqrt{3-2\sqrt{2}}\sqrt{2\sqrt{2}+3}}{7\sqrt{2}\sqrt{3-2\sqrt{2}}\sqrt{2\sqrt{2}+3}+10\sqrt{3-2\sqrt{2}}\sqrt{2\sqrt{2}+3}}+\frac{10x\sqrt{3-2\sqrt{2}}\sqrt{2\sqrt{2}+3}}{7\sqrt{2}\sqrt{3-2\sqrt{2}}\sqrt{2\sqrt{2}+3}+10\sqrt{3-2\sqrt{2}}\sqrt{2\sqrt{2}+3}}-\frac{3\sqrt{\ldots}}{7\sqrt{2}\sqrt{3\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(1+sin(x)**2),x)
```

```
[Out] 7*sqrt(2)*x*sqrt(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3)/(7*sqrt(2)*sqrt(3 - 2*s
qrt(2))*sqrt(2*sqrt(2) + 3) + 10*sqrt(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3)) +
 10*x*sqrt(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3)/(7*sqrt(2)*sqrt(3 - 2*sqrt(2)
)*sqrt(2*sqrt(2) + 3) + 10*sqrt(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3)) - 3*sqr
t(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2
)/pi))/(7*sqrt(2)*sqrt(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3) + 10*sqrt(3 - 2*s
qrt(2))*sqrt(2*sqrt(2) + 3)) - 2*sqrt(2)*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)
/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(7*sqrt(2)*sqrt(3 - 2*sq
rt(2))*sqrt(2*sqrt(2) + 3) + 10*sqrt(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3)) -
17*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2
- pi/2)/pi))/(7*sqrt(2)*sqrt(3 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3) + 10*sqrt(3
 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3)) - 12*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(t
an(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(7*sqrt(2)*sqrt(3
 - 2*sqrt(2))*sqrt(2*sqrt(2) + 3) + 10*sqrt(3 - 2*sqrt(2))*sqrt(2*sqrt(2) +
 3))
```

Giac [A] time = 1.09124, size = 65, normalized size = 1.81

$$-\frac{1}{2}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin\left(2x\right)-2\sin\left(2x\right)}{\sqrt{2}\cos\left(2x\right)+\sqrt{2}-2\cos\left(2x\right)+2}\right)\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(1+sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x)
 + sqrt(2) - 2*cos(2*x) + 2))) + x
```

3.145 $\int \frac{1}{b^2 \cos^2(x)+a^2 \sin^2(x)} \, dx$

Optimal. Leaf size=15

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Rubi [A] time = 0.0230196, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {205}

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Int[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{b^2 \cos^2(x) + a^2 \sin^2(x)} \, dx &= \text{Subst}\left(\int \frac{1}{b^2 + a^2 x^2} \, dx, x, \tan(x)\right) \\
&= \frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}
\end{aligned}$$

Mathematica [A] time = 0.0388559, size = 15, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b^2*Cos[x]^2 + a^2*Sin[x]^2)^(-1),x]
```

```
[Out] ArcTan[(a*Tan[x])/b]/(a*b)
```

Maple [A] time = 0.04, size = 16, normalized size = 1.1

$$\frac{1}{ab}\arctan\left(\frac{a\tan(x)}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b^2*cos(x)^2+a^2*sin(x)^2),x)
```

```
[Out] arctan(a*tan(x)/b)/a/b
```

Maxima [A] time = 1.42691, size = 20, normalized size = 1.33

$$\frac{\arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(a*tan(x)/b)/(a*b)
```

Fricas [B] time = 1.16321, size = 99, normalized size = 6.6

$$-\frac{\arctan\left(\frac{(a^2+b^2)\cos(x)^2-a^2}{2\,ab\cos(x)\sin(x)}\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*arctan(1/2*((a^2 + b^2)*cos(x)^2 - a^2)/(a*b*cos(x)*sin(x)))/(a*b)
```

Sympy [A] time = 40.991, size = 2866, normalized size = 191.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b**2*cos(x)**2+a**2*sin(x)**2),x)
```

```
[Out] Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), ((tan(x/2)
/2 - 1/(2*tan(x/2)))/a**2, Eq(b, 0)), (-2*tan(x/2)/(b**2*(tan(x/2)**2 - 1))
, Eq(a, 0)), (-16*a**5*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*
log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a*
*5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(
a**2 - b**2) + 2*a*b**6) + 16*a**5*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x
/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2
*b**4*sqrt(a**2 - b**2) + 2*a*b**6) + 16*a**4*sqrt(a**2 - b**2)*sqrt(-2*a**
2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(
a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 -
 b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - 16*a**4
*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b*
```

```
*2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2
- b**2) + 2*a*b**6) + 20*a**3*b**2*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(
x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**
2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - 20*a**3*b**2*sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/
b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a
**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - 4*a**3*b**2*sqrt(-2*
a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sq
rt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**
2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) + 4*a*
*3*b**2*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**
2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**
4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2
*a*b**6) - 12*a**2*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2
 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)
 + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4
+ 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) + 12*a**2*b**2*sqrt(a**2 - b**2
)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2
 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2
*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**
6) + 4*a**2*b**2*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2
)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(
x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**
2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - 4*a**2*b**2*sqrt(a**2 - b**2)*sqrt(-
2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*s
qrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a*
*2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - 5*a
*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2
/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4
*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*
a*b**6) + 5*a*b**4*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(
sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b*
*2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2
- b**2) + 2*a*b**6) + 3*a*b**4*sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b*
*2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2)
)/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b*
*4*sqrt(a**2 - b**2) + 2*a*b**6) - 3*a*b**4*sqrt(-2*a**2/b**2 + 2*a*sqrt(a*
*2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1
) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4
 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) + b**4*sqrt(a**2 - b**2)*sqrt(
-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 - 2*a
*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(
a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) - b
**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1)*l
og(sqrt(-2*a**2/b**2 - 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5
*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a*
*2 - b**2) + 2*a*b**6) - b**4*sqrt(a**2 - b**2)*sqrt(-2*a**2/b**2 + 2*a*sqr
t(a**2 - b**2)/b**2 + 1)*log(-sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**
2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*sqrt(a**2 - b**2) - 16*a**3
*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6) + b**4*sqrt(a**2 - b**2)*
sqrt(-2*a**2/b**2 + 2*a*sqrt(a**2 - b**2)/b**2 + 1)*log(sqrt(-2*a**2/b**2 +
 2*a*sqrt(a**2 - b**2)/b**2 + 1) + tan(x/2))/(16*a**5*b**2 - 16*a**4*b**2*s
qrt(a**2 - b**2) - 16*a**3*b**4 + 8*a**2*b**4*sqrt(a**2 - b**2) + 2*a*b**6)
, True))
```

Giac [A] time = 1.18852, size = 35, normalized size = 2.33

$$\frac{\pi \left\lfloor \frac{x}{\pi} + \frac{1}{2} \right\rfloor + \arctan\left(\frac{a\tan(x)}{b}\right)}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*cos(x)^2+a^2*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2) + arctan(a*tan(x)/b))/(a*b)
```

3.146 $\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx$

Optimal. Leaf size=17

$$\frac{\sin(x)}{b(a\sin(x)+b\cos(x))}$$

```
[Out] Sin[x]/(b*(b*Cos[x] + a*Sin[x]))
```

Rubi [A] time = 0.0122878, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3075}

$$\frac{\sin(x)}{b(a\sin(x)+b\cos(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[x] + a*Sin[x])^(-2),x]
```

```
[Out] Sin[x]/(b*(b*Cos[x] + a*Sin[x]))
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(b\cos(x)+a\sin(x))^2}\,dx=\frac{\sin(x)}{b(b\cos(x)+a\sin(x))}$$

Mathematica [A] time = 0.02755, size = 17, normalized size = 1.

$$\frac{\sin(x)}{b(a\sin(x)+b\cos(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[x] + a*Sin[x])^(-2),x]
```

```
[Out] Sin[x]/(b*(b*Cos[x] + a*Sin[x]))
```

Maple [A] time = 0.077, size = 14, normalized size = 0.8

$$-\frac{1}{a\,(a\tan(x)+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*cos(x)+a*sin(x))^2,x)
```

```
[Out] -1/a/(a*tan(x)+b)
```

Maxima [A] time = 0.952206, size = 19, normalized size = 1.12

$$-\frac{1}{a^2\tan(x)+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -1/(a^2*tan(x) + a*b)
```

Fricas [B] time = 1.00399, size = 95, normalized size = 5.59

$$-\frac{a\cos(x)-b\sin(x)}{\left(a^2b+b^3\right)\cos(x)+\left(a^3+ab^2\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -(a*cos(x) - b*sin(x))/((a^2*b + b^3)*cos(x) + (a^3 + a*b^2)*sin(x))
```

Sympy [A] time = 123.877, size = 228, normalized size = 13.41

$$\begin{cases}\frac{\tilde{\infty}\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right)-1} \\ -\frac{2\tan\left(\frac{x}{2}\right)}{b^2\left(\tan^2\left(\frac{x}{2}\right)-1\right)} \\ \frac{x\tan^2(x)}{2a^2\sin^2(x)-4a^2\sin(x)\cos(x)\tan(x)+2a^2\cos^2(x)\tan^2(x)}+\frac{x}{2a^2\sin^2(x)-4a^2\sin(x)\cos(x)\tan(x)+2a^2\cos^2(x)\tan^2(x)}+\frac{\tan}{2a^2\sin^2(x)-4a^2\sin(x)\cos(x} \\ \frac{\tan^2\left(\frac{x}{2}\right)}{2a^2\tan\left(\frac{x}{2}\right)-ab\tan^2\left(\frac{x}{2}\right)+ab}-\frac{1}{2a^2\tan\left(\frac{x}{2}\right)-ab\tan^2\left(\frac{x}{2}\right)+ab}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x))**2,x)
```

```
[Out] Piecewise((zoo*tan(x/2)/(tan(x/2)**2 - 1), Eq(a, 0) & Eq(b, 0)), (-2*tan(x/
2)/(b**2*(tan(x/2)**2 - 1)), Eq(a, 0)), (x*tan(x)**2/(2*a**2*sin(x)**2 - 4*
a**2*sin(x)*cos(x)*tan(x) + 2*a**2*cos(x)**2*tan(x)**2) + x/(2*a**2*sin(x)*
*2 - 4*a**2*sin(x)*cos(x)*tan(x) + 2*a**2*cos(x)**2*tan(x)**2) + tan(x)/(2*
a**2*sin(x)**2 - 4*a**2*sin(x)*cos(x)*tan(x) + 2*a**2*cos(x)**2*tan(x)**2),
 Eq(b, -a*tan(x))), (tan(x/2)**2/(2*a**2*tan(x/2) - a*b*tan(x/2)**2 + a*b)
- 1/(2*a**2*tan(x/2) - a*b*tan(x/2)**2 + a*b), True))
```

<u>**Giac**</u> **[A]** time = 1.10501, size = 18, normalized size = 1.06

$$-\frac{1}{(a\tan(x)+b)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*cos(x)+a*sin(x))^2,x, algorithm="giac")
```

```
[Out] -1/((a*tan(x) + b)*a)
```

3.147 $\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx$

Optimal. Leaf size=30

$$\frac{x}{2}-\frac{1}{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

```
[Out] x/2 - Log[1 + Cos[x] + Sin[x]]/2 - Log[1 + Tan[x/2]]/2
```

Rubi [A] time = 0.0290074, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3137, 3124, 31}

$$\frac{x}{2}-\frac{1}{2}\log\left(\tan\left(\frac{x}{2}\right)+1\right)-\frac{1}{2}\log(\sin(x)+\cos(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(1 + Cos[x] + Sin[x]),x]

[Out] x/2 - Log[1 + Cos[x] + Sin[x]]/2 - Log[1 + Tan[x/2]]/2
```

Rule 3137

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x_Symbol] :> Simp[(c*C*(d + e*x))
/(e*(b^2 + c^2)), x] + (Dist[(A*(b^2 + c^2) - a*c*C)/(b^2 + c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] - Simp[(b*C*Log[a + b*Cos[d + e*
x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x]) /; FreeQ[{a, b, c, d, e, A, C},
x] && NeQ[b^2 + c^2, 0] && NeQ[A*(b^2 + c^2) - a*c*C, 0]
```

Rule 3124

```
Int[(cos[(d_.) + (e_.)*(x_)]*(b_.) + (a_) + (c_.)*sin[(d_.) + (e_.)*(x_)])^
(-1), x_Symbol] :> Module[{f = FreeFactors[Tan[(d + e*x)/2], x]}, Dist[(2*f
)/e, Subst[Int[1/(a + b + 2*c*f*x + (a - b)*f^2*x^2), x], x, Tan[(d + e*x)/
2]/f], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[a^2 - b^2 - c^2, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{1+\cos(x)+\sin(x)}\,dx &= \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\frac{1}{2}\int \frac{1}{1+\cos(x)+\sin(x)}\,dx\\ &= \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\text{Subst}\left(\int \frac{1}{2+2x}\,dx,x,\tan\left(\frac{x}{2}\right)\right)\\ &= \frac{x}{2}-\frac{1}{2}\log(1+\cos(x)+\sin(x))-\frac{1}{2}\log\left(1+\tan\left(\frac{x}{2}\right)\right)\end{aligned}$$

Mathematica [A] time = 0.0341823, size = 22, normalized size = 0.73

$$\frac{x}{2}-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(1 + Cos[x] + Sin[x]),x]
```

```
[Out] x/2 - Log[Cos[x/2] + Sin[x/2]]
```

Maple [A] time = 0.048, size = 25, normalized size = 0.8

$$\frac{1}{2}\ln\left(\left(\tan\left(\frac{x}{2}\right)\right)^{2}+1\right)-\ln\left(1+\tan\left(\frac{x}{2}\right)\right)+\frac{x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1+cos(x)+sin(x)),x)
```

```
[Out] 1/2*ln(tan(1/2*x)^2+1)-ln(1+tan(1/2*x))+1/2*x
```

Maxima [A] time = 1.4185, size = 55, normalized size = 1.83

$$\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)-\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)+\frac{1}{2}\log\left(\frac{\sin(x)^{2}}{(\cos(x)+1)^{2}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="maxima")
```

```
[Out] arctan(sin(x)/(cos(x) + 1)) - log(sin(x)/(cos(x) + 1) + 1) + 1/2*log(sin(x)
^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 1.02831, size = 39, normalized size = 1.3

$$\frac{1}{2}x-\frac{1}{2}\log\left(\sin(x)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*x - 1/2*log(sin(x) + 1)
```

Sympy [A] time = 0.296861, size = 22, normalized size = 0.73

$$\frac{x}{2}-\log\left(\tan\left(\frac{x}{2}\right)+1\right)+\frac{\log\left(\tan^{2}\left(\frac{x}{2}\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x)
```

```
[Out] x/2 - log(tan(x/2) + 1) + log(tan(x/2)**2 + 1)/2
```

Giac [A] time = 1.10418, size = 34, normalized size = 1.13

$$\frac{1}{2}x + \frac{1}{2}\log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) - \log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+cos(x)+sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*x + 1/2*log(tan(1/2*x)^2 + 1) - log(abs(tan(1/2*x) + 1))
```

3.148 $\int \sqrt{3-x^2}\,dx$

Optimal. Leaf size=29

$$\frac{1}{2}\sqrt{3-x^2}x+\frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

```
[Out] (x*Sqrt[3 - x^2])/2 + (3*ArcSin[x/Sqrt[3]])/2
```

Rubi [A] time = 0.003629, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {195, 216}

$$\frac{1}{2}\sqrt{3-x^2}x+\frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x^2],x]
```

```
[Out] (x*Sqrt[3 - x^2])/2 + (3*ArcSin[x/Sqrt[3]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{3-x^2}\,dx &= \frac{1}{2}x\sqrt{3-x^2}+\frac{3}{2}\int \frac{1}{\sqrt{3-x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{3-x^2}+\frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)\end{aligned}$$

Mathematica [A] time = 0.0069582, size = 29, normalized size = 1.

$$\frac{1}{2}\sqrt{3-x^2}x+\frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x^2],x]
```

```
[Out] (x*Sqrt[3 - x^2])/2 + (3*ArcSin[x/Sqrt[3]])/2
```

Maple [A] time = 0.003, size = 23, normalized size = 0.8

$$\frac{3}{2}\arcsin\left(\frac{x\sqrt{3}}{3}\right)+\frac{x}{2}\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)^(1/2),x)
```

```
[Out] 3/2*arcsin(1/3*x*3^(1/2))+1/2*x*(-x^2+3)^(1/2)
```

Maxima [A] time = 1.42589, size = 30, normalized size = 1.03

$$\frac{1}{2}\sqrt{-x^2+3}x+\frac{3}{2}\arcsin\left(\frac{1}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 3)*x + 3/2*arcsin(1/3*sqrt(3)*x)
```

Fricas [A] time = 0.9952, size = 72, normalized size = 2.48

$$\frac{1}{2}\sqrt{-x^2+3}x-\frac{3}{2}\arctan\left(\frac{\sqrt{-x^2+3}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 3)*x - 3/2*arctan(sqrt(-x^2 + 3)/x)
```

Sympy [A] time = 0.192871, size = 24, normalized size = 0.83

$$\frac{x\sqrt{3-x^2}}{2}+\frac{3\operatorname{asin}\left(\frac{\sqrt{3}x}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)**(1/2),x)
```

```
[Out] x*sqrt(3 - x**2)/2 + 3*asin(sqrt(3)*x/3)/2
```

Giac [A] time = 1.13689, size = 30, normalized size = 1.03

$$\frac{1}{2}\sqrt{-x^2+3}x+\frac{3}{2}\arcsin\left(\frac{1}{3}\sqrt{3}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-x^2 + 3)*x + 3/2*arcsin(1/3*sqrt(3)*x)
```

3.149 $\int \frac{x}{\sqrt{3-x^2}} \, dx$

Optimal. Leaf size=13

$$-\sqrt{3-x^2}$$

[Out] -Sqrt[3 - x^2]

Rubi [A] time = 0.0021519, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {261}

$$-\sqrt{3-x^2}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[3 - x^2],x]

[Out] -Sqrt[3 - x^2]

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\int \frac{x}{\sqrt{3-x^2}} \, dx = -\sqrt{3-x^2}$$

Mathematica [A] time = 0.0014776, size = 13, normalized size = 1.

$$-\sqrt{3-x^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[3 - x^2],x]

[Out] -Sqrt[3 - x^2]

Maple [A] time = 0.002, size = 12, normalized size = 0.9

$$-\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x^2+3)^(1/2),x)

```
[Out] -(-x^2+3)^(1/2)
```

Maxima [A] time = 0.927835, size = 15, normalized size = 1.15

$$-\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -sqrt(-x^2 + 3)
```

Fricas [A] time = 1.0625, size = 23, normalized size = 1.77

$$-\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-x^2 + 3)
```

Sympy [A] time = 0.142602, size = 8, normalized size = 0.62

$$-\sqrt{3-x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+3)**(1/2),x)
```

```
[Out] -sqrt(3 - x**2)
```

Giac [A] time = 1.09733, size = 15, normalized size = 1.15

$$-\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-x^2 + 3)
```

3.150 $\int \frac{\sqrt{3-x^2}}{x} dx$

Optimal. Leaf size=37

$$\sqrt{3-x^2}-\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{3}}\right)$$

```
[Out] Sqrt[3 - x^2] - Sqrt[3]*ArcTanh[Sqrt[3 - x^2]/Sqrt[3]]
```

Rubi [A] time = 0.0195136, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {266, 50, 63, 206}

$$\sqrt{3-x^2}-\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x^2]/x,x]
```

```
[Out] Sqrt[3 - x^2] - Sqrt[3]*ArcTanh[Sqrt[3 - x^2]/Sqrt[3]]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{3-x^2}}{x}\,dx &= \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{\sqrt{3-x}}{x}\,dx,x,x^2\right)\\
&= \sqrt{3-x^2}+\frac{3}{2}\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{3-x}x}\,dx,x,x^2\right)\\
&= \sqrt{3-x^2}-3\,\mathrm{Subst}\left(\int \frac{1}{3-x^2}\,dx,x,\sqrt{3-x^2}\right)\\
&= \sqrt{3-x^2}-\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3-x^2}}{\sqrt{3}}\right)
\end{aligned}
$$

Mathematica [A] time = 0.0068919, size = 33, normalized size = 0.89

$$
\sqrt{3-x^2}-\sqrt{3}\tanh^{-1}\left(\sqrt{1-\frac{x^2}{3}}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x^2]/x,x]
```

```
[Out] Sqrt[3 - x^2] - Sqrt[3]*ArcTanh[Sqrt[1 - x^2/3]]
```

Maple [A] time = 0.004, size = 30, normalized size = 0.8

$$
\sqrt{-x^2+3}-\sqrt{3}\mathrm{Artanh}\left(\sqrt{3}\frac{1}{\sqrt{-x^2+3}}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+3)^(1/2)/x,x)
```

```
[Out] (-x^2+3)^(1/2)-3^(1/2)*arctanh(3^(1/2)/(-x^2+3)^(1/2))
```

Maxima [A] time = 1.42444, size = 55, normalized size = 1.49

$$
-\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{-x^2+3}}{|x|}+\frac{6}{|x|}\right)+\sqrt{-x^2+3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -sqrt(3)*log(2*sqrt(3)*sqrt(-x^2 + 3)/abs(x) + 6/abs(x)) + sqrt(-x^2 + 3)
```

Fricas [A] time = 1.12917, size = 104, normalized size = 2.81

$$
\frac{1}{2}\sqrt{3}\log\left(-\frac{x^2+2\sqrt{3}\sqrt{-x^2+3}-6}{x^2}\right)+\sqrt{-x^2+3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(3)*log(-(x^2 + 2*sqrt(3)*sqrt(-x^2 + 3) - 6)/x^2) + sqrt(-x^2 + 3)
```

Sympy [A] time = 1.37867, size = 88, normalized size = 2.38

$$\begin{cases} i\sqrt{x^2-3}-\sqrt{3}\log(x)+\frac{\sqrt{3}\log(x^2)}{2}+\sqrt{3}i\operatorname{asin}\left(\frac{\sqrt{3}}{x}\right) & \text{for } \frac{|x^2|}{3}>1 \\ \sqrt{3-x^2}+\frac{\sqrt{3}\log(x^2)}{2}-\sqrt{3}\log\left(\sqrt{1-\frac{x^2}{3}}+1\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+3)**(1/2)/x,x)
```

```
[Out] Piecewise((I*sqrt(x**2 - 3) - sqrt(3)*log(x) + sqrt(3)*log(x**2)/2 + sqrt(3
)*I*asin(sqrt(3)/x), Abs(x**2)/3 > 1), (sqrt(3 - x**2) + sqrt(3)*log(x**2)/
2 - sqrt(3)*log(sqrt(1 - x**2/3) + 1), True))
```

Giac [A] time = 1.09109, size = 63, normalized size = 1.7

$$\frac{1}{2}\sqrt{3}\log\left(\frac{\sqrt{3}-\sqrt{-x^2+3}}{\sqrt{3}+\sqrt{-x^2+3}}\right)+\sqrt{-x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+3)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*log((sqrt(3) - sqrt(-x^2 + 3))/(sqrt(3) + sqrt(-x^2 + 3))) + sq
rt(-x^2 + 3)
```

3.151 $\int \frac{\sqrt{x+x^2}}{x}\,dx$

Optimal. Leaf size=22

$$\sqrt{x^2+x}+\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

```
[Out] Sqrt[x + x^2] + ArcTanh[x/Sqrt[x + x^2]]
```

Rubi [A] time = 0.0070825, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {664, 620, 206}

$$\sqrt{x^2+x}+\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x + x^2]/x,x]

[Out] Sqrt[x + x^2] + ArcTanh[x/Sqrt[x + x^2]]
```

Rule 664

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x
] - Dist[(p*(2*c*d - b*e))/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a +
b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*
c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || Eq
Q[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x+x^2}}{x}\,dx &= \sqrt{x+x^2}+\frac{1}{2}\int \frac{1}{\sqrt{x+x^2}}\,dx\\
&= \sqrt{x+x^2}+\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{x+x^2}}\right)\\
&= \sqrt{x+x^2}+\tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0137368, size = 31, normalized size = 1.41

$$\sqrt{x(x+1)}\left(\frac{\sinh^{-1}\left(\sqrt{x}\right)}{\sqrt{x}\sqrt{x+1}}+1\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[x + x^2]/x,x]
```

```
[Out] Sqrt[x*(1 + x)]*(1 + ArcSinh[Sqrt[x]]/(Sqrt[x]*Sqrt[1 + x]))
```

Maple [A] time = 0.004, size = 22, normalized size = 1.

$$\sqrt{x^2+x}+\frac{1}{2}\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+x)^(1/2)/x,x)
```

```
[Out] (x^2+x)^(1/2)+1/2*ln(1/2+x+(x^2+x)^(1/2))
```

Maxima [A] time = 0.94717, size = 34, normalized size = 1.55

$$\sqrt{x^2+x}+\frac{1}{2}\log\left(2x+2\sqrt{x^2+x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + x) + 1/2*log(2*x + 2*sqrt(x^2 + x) + 1)
```

Fricas [A] time = 1.07853, size = 72, normalized size = 3.27

$$\sqrt{x^2+x}-\frac{1}{2}\log\left(-2x+2\sqrt{x^2+x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] sqrt(x^2 + x) - 1/2*log(-2*x + 2*sqrt(x^2 + x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{x(x+1)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+x)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(x*(x + 1))/x, x)
```

Giac [A] time = 1.10562, size = 35, normalized size = 1.59

$$\sqrt{x^2 + x} - \frac{1}{2}\log\left(\left|-2x + 2\sqrt{x^2 + x} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x)^(1/2)/x,x, algorithm="giac")
```

```
[Out] sqrt(x^2 + x) - 1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

3.152 $\int \sqrt{5+x^2}\,dx$

Optimal. Leaf size=27

$$\frac{1}{2}\sqrt{x^2+5}x+\frac{5}{2}\sinh^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

```
[Out] (x*Sqrt[5 + x^2])/2 + (5*ArcSinh[x/Sqrt[5]])/2
```

Rubi [A] time = 0.0032337, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {195, 215}

$$\frac{1}{2}\sqrt{x^2+5}x+\frac{5}{2}\sinh^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[5 + x^2],x]
```

```
[Out] (x*Sqrt[5 + x^2])/2 + (5*ArcSinh[x/Sqrt[5]])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}\int \sqrt{5+x^2}\,dx &= \frac{1}{2}x\sqrt{5+x^2}+\frac{5}{2}\int \frac{1}{\sqrt{5+x^2}}\,dx\\ &= \frac{1}{2}x\sqrt{5+x^2}+\frac{5}{2}\sinh^{-1}\left(\frac{x}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A] time = 0.0063963, size = 27, normalized size = 1.

$$\frac{1}{2}\sqrt{x^2+5}x+\frac{5}{2}\sinh^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[5 + x^2],x]
```

```
[Out] (x*Sqrt[5 + x^2])/2 + (5*ArcSinh[x/Sqrt[5]])/2
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$\frac{5}{2}\mathrm{Arcsinh}\left(\frac{x\sqrt{5}}{5}\right)+\frac{x}{2}\sqrt{x^2+5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+5)^(1/2),x)
```

```
[Out] 5/2*arcsinh(1/5*x*5^(1/2))+1/2*x*(x^2+5)^(1/2)
```

Maxima [A] time = 1.41488, size = 27, normalized size = 1.

$$\frac{1}{2}\sqrt{x^2+5}x+\frac{5}{2}\,\mathrm{arsinh}\left(\frac{1}{5}\sqrt{5}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 + 5)*x + 5/2*arcsinh(1/5*sqrt(5)*x)
```

Fricas [A] time = 1.05863, size = 69, normalized size = 2.56

$$\frac{1}{2}\sqrt{x^2+5}x-\frac{5}{2}\log\left(-x+\sqrt{x^2+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^2 + 5)*x - 5/2*log(-x + sqrt(x^2 + 5))
```

Sympy [A] time = 0.190459, size = 24, normalized size = 0.89

$$\frac{x\sqrt{x^2+5}}{2}+\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}x}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+5)**(1/2),x)
```

```
[Out] x*sqrt(x**2 + 5)/2 + 5*asinh(sqrt(5)*x/5)/2
```

Giac [A] time = 1.09194, size = 34, normalized size = 1.26

$$\frac{1}{2}\sqrt{x^2+5}x - \frac{5}{2}\log\left(-x+\sqrt{x^2+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 + 5)*x - 5/2*log(-x + sqrt(x^2 + 5))
```

3.153 $\int \frac{x}{\sqrt{1+x+x^2}} \, dx$

Optimal. Leaf size=27

$$\sqrt{x^2+x+1}-\frac{1}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

```
[Out] Sqrt[1 + x + x^2] - ArcSinh[(1 + 2*x)/Sqrt[3]]/2
```

Rubi [A] time = 0.0109699, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {640, 619, 215}

$$\sqrt{x^2+x+1}-\frac{1}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[1 + x + x^2],x]
```

```
[Out] Sqrt[1 + x + x^2] - ArcSinh[(1 + 2*x)/Sqrt[3]]/2
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{1+x+x^2}}\,dx &= \sqrt{1+x+x^2}-\frac{1}{2}\int \frac{1}{\sqrt{1+x+x^2}}\,dx\\
&= \sqrt{1+x+x^2}-\frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}}\,dx,x,1+2x\right)}{2\sqrt{3}}\\
&= \sqrt{1+x+x^2}-\frac{1}{2}\sinh^{-1}\left(\frac{1+2x}{\sqrt{3}}\right)
\end{aligned}$$

Mathematica [A] time = 0.0038627, size = 27, normalized size = 1.

$$\sqrt{x^2+x+1}-\frac{1}{2}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[1 + x + x^2],x]
```

```
[Out] Sqrt[1 + x + x^2] - ArcSinh[(1 + 2*x)/Sqrt[3]]/2
```

Maple [A] time = 0.004, size = 21, normalized size = 0.8

$$\sqrt{x^2+x+1}-\frac{1}{2}\mathrm{Arcsinh}\left(\frac{2\sqrt{3}}{3}\left(x+\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x^2+x+1)^(1/2),x)
```

```
[Out] (x^2+x+1)^(1/2)-1/2*arcsinh(2/3*3^(1/2)*(x+1/2))
```

Maxima [A] time = 1.39499, size = 30, normalized size = 1.11

$$\sqrt{x^2+x+1}-\frac{1}{2}\,\mathrm{arsinh}\left(\frac{1}{3}\sqrt{3}(2x+1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2 + x + 1) - 1/2*arcsinh(1/3*sqrt(3)*(2*x + 1))
```

Fricas [A] time = 1.00863, size = 82, normalized size = 3.04

$$\sqrt{x^2+x+1}+\frac{1}{2}\log\left(-2x+2\sqrt{x^2+x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 + x + 1) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{x}{\sqrt{x^2+x+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x**2+x+1)**(1/2),x)

[Out] Integral(x/sqrt(x**2 + x + 1), x)
```

Giac [A] time = 1.08945, size = 36, normalized size = 1.33

$$\sqrt{x^2 + x + 1} + \frac{1}{2} \log \left(-2x + 2\sqrt{x^2 + x + 1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x^2+x+1)^(1/2),x, algorithm="giac")

[Out] sqrt(x^2 + x + 1) + 1/2*log(-2*x + 2*sqrt(x^2 + x + 1) - 1)
```

3.154 $\int \frac{1}{\sqrt{x+x^2}}\,dx$

Optimal. Leaf size=14

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

```
[Out] 2*ArcTanh[x/Sqrt[x + x^2]]
```

Rubi [A] time = 0.0031101, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {620, 206}

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2+x}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[x + x^2],x]
```

```
[Out] 2*ArcTanh[x/Sqrt[x + x^2]]
```

Rule 620

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1
- c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x+x^2}}\,dx &= 2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \frac{x}{\sqrt{x+x^2}}\right)\\
&= 2\tanh^{-1}\left(\frac{x}{\sqrt{x+x^2}}\right)
\end{aligned}$$

Mathematica [B] time = 0.0046658, size = 29, normalized size = 2.07

$$\frac{2\sqrt{x}\sqrt{x+1}\sinh^{-1}\left(\sqrt{x}\right)}{\sqrt{x(x+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[x + x^2],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 + x]*ArcSinh[Sqrt[x]])/Sqrt[x*(1 + x)]
```

Maple [A] time = 0.003, size = 12, normalized size = 0.9

$$\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+x)^(1/2),x)
```

```
[Out] ln(1/2+x+(x^2+x)^(1/2))
```

Maxima [A] time = 0.942062, size = 20, normalized size = 1.43

$$\log\left(2\,x+2\,\sqrt{x^2+x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x)^(1/2),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 + x) + 1)
```

Fricas [A] time = 1.23835, size = 46, normalized size = 3.29

$$-\log\left(-2\,x+2\,\sqrt{x^2+x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x)^(1/2),x, algorithm="fricas")
```

```
[Out] -log(-2*x + 2*sqrt(x^2 + x) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2 + x), x)
```

Giac [A] time = 1.11361, size = 24, normalized size = 1.71

$$-\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+x)^(1/2),x, algorithm="giac")
```

```
[Out] -log(abs(-2*x + 2*sqrt(x^2 + x) - 1))
```

3.155 $\int \frac{\sqrt{2-x-x^2}}{x^2} dx$

Optimal. Leaf size=68

$$-\frac{\sqrt{-x^2-x+2}}{x}+\frac{\tanh^{-1}\left(\frac{4-x}{2\sqrt{2}\sqrt{-x^2-x+2}}\right)}{2\sqrt{2}}+\sin^{-1}\left(\frac{1}{3}(-2x-1)\right)$$

```
[Out] -(Sqrt[2 - x - x^2]/x) + ArcSin[(-1 - 2*x)/3] + ArcTanh[(4 - x)/(2*Sqrt[2]*
Sqrt[2 - x - x^2])]/(2*Sqrt[2])
```

Rubi [A] time = 0.0367591, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {732, 843, 619, 216, 724, 206}

$$-\frac{\sqrt{-x^2-x+2}}{x}+\frac{\tanh^{-1}\left(\frac{4-x}{2\sqrt{2}\sqrt{-x^2-x+2}}\right)}{2\sqrt{2}}+\sin^{-1}\left(\frac{1}{3}(-2x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 - x - x^2]/x^2,x]
```

```
[Out] -(Sqrt[2 - x - x^2]/x) + ArcSin[(-1 - 2*x)/3] + ArcTanh[(4 - x)/(2*Sqrt[2]*
Sqrt[2 - x - x^2])]/(2*Sqrt[2])
```

Rule 732

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 1)), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 843

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2-x-x^2}}{x^2}\,dx &= -\frac{\sqrt{2-x-x^2}}{x}+\frac{1}{2}\int \frac{-1-2x}{x\sqrt{2-x-x^2}}\,dx\\
&= -\frac{\sqrt{2-x-x^2}}{x}-\frac{1}{2}\int \frac{1}{x\sqrt{2-x-x^2}}\,dx-\int \frac{1}{\sqrt{2-x-x^2}}\,dx\\
&= -\frac{\sqrt{2-x-x^2}}{x}+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{9}}}\,dx,x,-1-2x\right)+\operatorname{Subst}\left(\int \frac{1}{8-x^2}\,dx,x,\frac{4-x}{\sqrt{2-x-x^2}}\right)\\
&= -\frac{\sqrt{2-x-x^2}}{x}+\sin^{-1}\left(\frac{1}{3}(-1-2x)\right)+\frac{\tanh^{-1}\left(\frac{4-x}{2\sqrt{2}\sqrt{2-x-x^2}}\right)}{2\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.0274292, size = 68, normalized size = 1.

$$-\frac{\sqrt{-x^2-x+2}}{x}+\frac{\tanh^{-1}\left(\frac{4-x}{2\sqrt{2}\sqrt{-x^2-x+2}}\right)}{2\sqrt{2}}+\sin^{-1}\left(\frac{1}{3}(-2x-1)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[2 - x - x^2]/x^2,x]
```

```
[Out] -(Sqrt[2 - x - x^2]/x) + ArcSin[(-1 - 2*x)/3] + ArcTanh[(4 - x)/(2*Sqrt[2]*
Sqrt[2 - x - x^2])]/(2*Sqrt[2])
```

Maple [A] time = 0.005, size = 88, normalized size = 1.3

$$-\frac{1}{2x}\left(-x^2-x+2\right)^{\frac{3}{2}}-\frac{1}{4}\sqrt{-x^2-x+2}-\arcsin\left(\frac{1}{3}+\frac{2x}{3}\right)+\frac{\sqrt{2}}{4}\operatorname{Artanh}\left(\frac{(4-x)\sqrt{2}}{4}\frac{1}{\sqrt{-x^2-x+2}}\right)+\frac{-1-2x}{4}\sqrt{-x^2-x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-x+2)^(1/2)/x^2,x)
```

```
[Out] -1/2/x*(-x^2-x+2)^(3/2)-1/4*(-x^2-x+2)^(1/2)-arcsin(1/3+2/3*x)+1/4*arctanh(
1/4*(4-x)*2^(1/2)/(-x^2-x+2)^(1/2))*2^(1/2)+1/4*(-1-2*x)*(-x^2-x+2)^(1/2)
```

Maxima [A] time = 1.45172, size = 80, normalized size = 1.18

$$\frac{1}{4}\sqrt{2}\log\left(\frac{2\sqrt{2}\sqrt{-x^2-x+2}}{|x|}+\frac{4}{|x|}-1\right)-\frac{\sqrt{-x^2-x+2}}{x}+\arcsin\left(-\frac{2}{3}x-\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(2*sqrt(2)*sqrt(-x^2 - x + 2)/abs(x) + 4/abs(x) - 1) - sqrt(
-x^2 - x + 2)/x + arcsin(-2/3*x - 1/3)
```

Fricas [A] time = 1.12746, size = 232, normalized size = 3.41

$$\frac{\sqrt{2}x\log\left(-\frac{4\sqrt{2}\sqrt{-x^2-x+2}(x-4)+7x^2+16x-32}{x^2}\right)+8x\arctan\left(\frac{\sqrt{-x^2-x+2}(2x+1)}{2(x^2+x-2)}\right)-8\sqrt{-x^2-x+2}}{8x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*x*log(-(4*sqrt(2)*sqrt(-x^2 - x + 2)*(x - 4) + 7*x^2 + 16*x -
32)/x^2) + 8*x*arctan(1/2*sqrt(-x^2 - x + 2)*(2*x + 1)/(x^2 + x - 2)) - 8*s
qrt(-x^2 - x + 2))/x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{-(x-1)(x+2)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-x+2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 2))/x**2, x)
```

Giac [B] time = 1.13398, size = 227, normalized size = 3.34

$$-\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+\frac{2\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+6\right|}{\left|4\sqrt{2}+\frac{2\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+6\right|}\right)+\frac{6\left(\frac{3\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+1\right)}{\frac{6\left(2\sqrt{-x^2-x+2}-3\right)}{2x+1}+\frac{\left(2\sqrt{-x^2-x+2}-3\right)^2}{(2x+1)^2}+1}-\arcsin\left(\frac{2}{3}x+\frac{1}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-x+2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(abs(-4*sqrt(2) + 2*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) +
6)/abs(4*sqrt(2) + 2*(2*sqrt(-x^2 - x + 2) - 3)/(2*x + 1) + 6)) + 6*(3*(2*s
qrt(-x^2 - x + 2) - 3)/(2*x + 1) + 1)/(6*(2*sqrt(-x^2 - x + 2) - 3)/(2*x +
1) + (2*sqrt(-x^2 - x + 2) - 3)^2/(2*x + 1)^2 + 1) - arcsin(2/3*x + 1/3)
```

3.156 $\int \frac{\log(t)}{1+t} \, dt$

Optimal. Leaf size=13

$$\text{PolyLog}(2,-t)+\log(t)\log(t+1)$$

```
[Out] Log[t]*Log[1 + t] + PolyLog[2, -t]
```

Rubi [A] time = 0.0153927, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2317, 2391}

$$\text{PolyLog}(2,-t)+\log(t)\log(t+1)$$

Antiderivative was successfully verified.

```
[In] Int[Log[t]/(1 + t),t]
```

```
[Out] Log[t]*Log[1 + t] + PolyLog[2, -t]
```

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
 Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\log(t)}{1+t} \, dt &= \log(t)\log(1+t)-\int \frac{\log(1+t)}{t} \, dt\\ &= \log(t)\log(1+t)+\text{Li}_2(-t)\end{aligned}$$

Mathematica [A] time = 0.0015629, size = 13, normalized size = 1.

$$\text{PolyLog}(2,-t)+\log(t)\log(t+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[t]/(1 + t),t]
```

```
[Out] Log[t]*Log[1 + t] + PolyLog[2, -t]
```

Maple [A] time = 0.002, size = 13, normalized size = 1.

$$\text{dilog}\,(1+t)+\ln(t)\ln(1+t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(t)/(1+t),t)
```

```
[Out] dilog(1+t)+ln(t)*ln(1+t)
```

Maxima [A] time = 0.943187, size = 16, normalized size = 1.23

$$\log(t+1)\log(t) + \mathrm{Li}_2(-t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)/(1+t),t, algorithm="maxima")
```

```
[Out] log(t + 1)*log(t) + dilog(-t)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\log(t)}{t+1}, t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)/(1+t),t, algorithm="fricas")
```

```
[Out] integral(log(t)/(t + 1), t)
```

Sympy [C] time = 2.34793, size = 58, normalized size = 4.46

$$\begin{cases} i\pi\log(t+1) - \mathrm{Li}_2(t+1) & \text{for } |t+1| < 1 \\ -i\pi\log\left(\frac{1}{t+1}\right) - \mathrm{Li}_2(t+1) & \text{for } \frac{1}{|t+1|} < 1 \\ -i\pi G_{2,2}^{2,0}\left(\begin{matrix} & 1,1 \\ 0,0 & \end{matrix}\middle| t+1\right) + i\pi G_{2,2}^{0,2}\left(\begin{matrix} 1,1 & \\ & 0,0 \end{matrix}\middle| t+1\right) - \mathrm{Li}_2(t+1) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(t)/(1+t),t)
```

```
[Out] Piecewise((I*pi*log(t + 1) - polylog(2, t + 1), Abs(t + 1) < 1), (-I*pi*log
(1/(t + 1)) - polylog(2, t + 1), 1/Abs(t + 1) < 1), (-I*pi*meijerg(((), (1,
 1)), ((0, 0), ()), t + 1) + I*pi*meijerg(((1, 1), ()), ((), (0, 0)), t + 1
) - polylog(2, t + 1), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\log(t)}{t+1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)/(1+t),t, algorithm="giac")
```

```
[Out] integrate(log(t)/(t + 1), t)
```

3.157 $\int \log\left(e^{\cos(x)}\right)\,dx$

Optimal. Leaf size=15

$$\sin(x) - x\cos(x) + x\log\left(e^{\cos(x)}\right)$$

```
[Out] -(x*Cos[x]) + x*Log[E^Cos[x]] + Sin[x]
```

Rubi [A] time = 0.0098554, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {2548, 3296, 2637}

$$\sin(x) - x\cos(x) + x\log\left(e^{\cos(x)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Log[E^Cos[x]],x]
```

```
[Out] -(x*Cos[x]) + x*Log[E^Cos[x]] + Sin[x]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \log\left(e^{\cos(x)}\right)\,dx &= x\log\left(e^{\cos(x)}\right) + \int x\sin(x)\,dx\\ &= -x\cos(x) + x\log\left(e^{\cos(x)}\right) + \int \cos(x)\,dx\\ &= -x\cos(x) + x\log\left(e^{\cos(x)}\right) + \sin(x)\end{aligned}$$

Mathematica [A] time = 0.009336, size = 15, normalized size = 1.

$$\sin(x) + x\left(\log\left(e^{\cos(x)}\right) - \cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[E^Cos[x]],x]
```

```
[Out] x*(-Cos[x] + Log[E^Cos[x]]) + Sin[x]
```

Maple [A] time = 0.012, size = 15, normalized size = 1.

$$-x\cos(x)+x\ln\left(e^{\cos(x)}\right)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(exp(cos(x))),x)
```

```
[Out] -x*cos(x)+x*ln(exp(cos(x)))+sin(x)
```

Maxima [A] time = 0.92759, size = 3, normalized size = 0.2

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(cos(x))),x, algorithm="maxima")
```

```
[Out] sin(x)
```

Fricas [A] time = 1.07367, size = 11, normalized size = 0.73

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(cos(x))),x, algorithm="fricas")
```

```
[Out] sin(x)
```

Sympy [A] time = 0.183113, size = 2, normalized size = 0.13

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(exp(cos(x))),x)
```

```
[Out] sin(x)
```

Giac [A] time = 1.12051, size = 3, normalized size = 0.2

$$\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(exp(cos(x))),x, algorithm="giac")
```

```
[Out] sin(x)
```

3.158 $\int \frac{e^t}{t}\,dt$

Optimal. Leaf size=2

$$\text{ExpIntegralEi}(t)$$

```
[Out] ExpIntegralEi[t]
```

Rubi [A] time = 0.009748, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2178}

$$\text{ExpIntegralEi}(t)$$

Antiderivative was successfully verified.

```
[In] Int[E^t/t,t]
```

```
[Out] ExpIntegralEi[t]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\int \frac{e^t}{t}\,dt = \text{Ei}(t)$$

Mathematica [A] time = 0.0049735, size = 2, normalized size = 1.

$$\text{ExpIntegralEi}(t)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t/t,t]
```

```
[Out] ExpIntegralEi[t]
```

Maple [B] time = 0.002, size = 8, normalized size = 4.

$$-\text{Ei}\,(1,-t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)/t,t)
```

```
[Out] -Ei(1,-t)
```

Maxima [A] time = 1.03039, size = 3, normalized size = 1.5

$$\mathrm{Ei}\,(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t,t, algorithm="maxima")
```

```
[Out] Ei(t)
```

Fricas [A] time = 1.09348, size = 9, normalized size = 4.5

$$\mathrm{Ei}\,(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t,t, algorithm="fricas")
```

```
[Out] Ei(t)
```

Sympy [A] time = 0.732115, size = 2, normalized size = 1.

$$\mathrm{Ei}\,(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t,t)
```

```
[Out] Ei(t)
```

Giac [A] time = 1.10123, size = 3, normalized size = 1.5

$$\mathrm{Ei}\,(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t,t, algorithm="giac")
```

```
[Out] Ei(t)
```

3.159 $\int \frac{e^{at}}{t}\,dt$

Optimal. Leaf size=4

$$\text{ExpIntegralEi}(at)$$

```
[Out] ExpIntegralEi[a*t]
```

Rubi [A] time = 0.0120414, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2178}

$$\text{ExpIntegralEi}(at)$$

Antiderivative was successfully verified.

```
[In] Int[E^(a*t)/t,t]
```

```
[Out] ExpIntegralEi[a*t]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\int \frac{e^{at}}{t}\,dt = \text{Ei}(at)$$

Mathematica [A] time = 0.005839, size = 4, normalized size = 1.

$$\text{ExpIntegralEi}(at)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(a*t)/t,t]
```

```
[Out] ExpIntegralEi[a*t]
```

Maple [A] time = 0.003, size = 9, normalized size = 2.3

$$-\text{Ei}\,(1,-at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(a*t)/t,t)
```

```
[Out] -Ei(1,-a*t)
```

Maxima [A] time = 1.03016, size = 5, normalized size = 1.25

$$\mathrm{Ei}\,(at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*t)/t,t, algorithm="maxima")
```

```
[Out] Ei(a*t)
```

Fricas [A] time = 0.997875, size = 12, normalized size = 3.

$$\mathrm{Ei}\,(at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*t)/t,t, algorithm="fricas")
```

```
[Out] Ei(a*t)
```

Sympy [A] time = 0.78138, size = 3, normalized size = 0.75

$$\mathrm{Ei}\,(at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*t)/t,t)
```

```
[Out] Ei(a*t)
```

Giac [A] time = 1.07686, size = 5, normalized size = 1.25

$$\mathrm{Ei}\,(at)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(a*t)/t,t, algorithm="giac")
```

```
[Out] Ei(a*t)
```

3.160 $\int \frac{e^t}{t^2} dt$

Optimal. Leaf size=11

$$\text{ExpIntegralEi}(t) - \frac{e^t}{t}$$

```
[Out] -(E^t/t) + ExpIntegralEi[t]
```

Rubi [A] time = 0.0196118, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2177, 2178}

$$\text{ExpIntegralEi}(t) - \frac{e^t}{t}$$

Antiderivative was successfully verified.

```
[In] Int[E^t/t^2,t]
```

```
[Out] -(E^t/t) + ExpIntegralEi[t]
```

Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !$UseGamma === True
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned} \int \frac{e^t}{t^2} dt &= -\frac{e^t}{t} + \int \frac{e^t}{t} dt \\ &= -\frac{e^t}{t} + \text{Ei}(t) \end{aligned}$$

Mathematica [A] time = 0.0073473, size = 11, normalized size = 1.

$$\text{ExpIntegralEi}(t) - \frac{e^t}{t}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t/t^2,t]
```

```
[Out] -(E^t/t) + ExpIntegralEi[t]
```

Maple [A] time = 0.002, size = 16, normalized size = 1.5

$$-\frac{\mathrm{e}^{t}}{t}-\mathrm{Ei}\left(1,-t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)/t^2,t)
```

```
[Out] -exp(t)/t-Ei(1,-t)
```

Maxima [A] time = 1.02161, size = 7, normalized size = 0.64

$$\Gamma\left(-1,-t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t^2,t, algorithm="maxima")
```

```
[Out] gamma(-1, -t)
```

Fricas [A] time = 1.04303, size = 26, normalized size = 2.36

$$\frac{t\mathrm{Ei}\left(t\right)-e^{t}}{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t^2,t, algorithm="fricas")
```

```
[Out] (t*Ei(t) - e^t)/t
```

Sympy [A] time = 0.965294, size = 7, normalized size = 0.64

$$\mathrm{Ei}\left(t\right)-\frac{e^{t}}{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t**2,t)
```

```
[Out] Ei(t) - exp(t)/t
```

Giac [A] time = 1.10232, size = 18, normalized size = 1.64

$$\frac{t\mathrm{Ei}\left(t\right)-e^{t}}{t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/t^2,t, algorithm="giac")
```

```
[Out] (t*Ei(t) - e^t)/t
```

3.161 $\int e^{\frac{1}{t}}\,dt$

Optimal. Leaf size=14

$$e^{\frac{1}{t}}t - \text{ExpIntegralEi}\left(\frac{1}{t}\right)$$

```
[Out] E^t^(-1)*t - ExpIntegralEi[t^(-1)]
```

Rubi [A] time = 0.0120688, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {2206, 2210}

$$e^{\frac{1}{t}}t - \text{ExpIntegralEi}\left(\frac{1}{t}\right)$$

Antiderivative was successfully verified.

```
[In] Int[E^t^(-1),t]
```

```
[Out] E^t^(-1)*t - ExpIntegralEi[t^(-1)]
```

Rule 2206

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] :> Simp[((c +
 d*x)*F^(a + b*(c + d*x)^n))/d, x] - Dist[b*n*Log[F], Int[(c + d*x)^n*F^(a
+ b*(c + d*x)^n), x], x] /; FreeQ[{F, a, b, c, d}, x] && IntegerQ[2/n] && I
LtQ[n, 0]
```

Rule 2210

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_
Symbol] :> Simp[(F^a*ExpIntegralEi[b*(c + d*x)^n*Log[F]])/(f*n), x] /; Free
Q[{F, a, b, c, d, e, f, n}, x] && EqQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int e^{\frac{1}{t}}\,dt &= e^{\frac{1}{t}}t + \int \frac{e^{\frac{1}{t}}}{t}\,dt\\ &= e^{\frac{1}{t}}t - \text{Ei}\left(\frac{1}{t}\right)\end{aligned}$$

Mathematica [A] time = 0.0016972, size = 14, normalized size = 1.

$$e^{\frac{1}{t}}t - \text{ExpIntegralEi}\left(\frac{1}{t}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t^(-1),t]
```

```
[Out] E^t^(-1)*t - ExpIntegralEi[t^(-1)]
```

Maple [A] time = 0.002, size = 15, normalized size = 1.1

$$e^{t^{-1}}t + \mathrm{Ei}\left(1, -t^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(1/t),t)
```

```
[Out] exp(1/t)*t+Ei(1,-1/t)
```

Maxima [A] time = 1.0221, size = 12, normalized size = 0.86

$$-\Gamma\left(-1, -\frac{1}{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/t),t, algorithm="maxima")
```

```
[Out] -gamma(-1, -1/t)
```

Fricas [A] time = 0.998629, size = 28, normalized size = 2.

$$te^{\frac{1}{t}} - \mathrm{Ei}\left(\frac{1}{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/t),t, algorithm="fricas")
```

```
[Out] t*e^(1/t) - Ei(1/t)
```

Sympy [A] time = 1.06457, size = 10, normalized size = 0.71

$$te^{\frac{1}{t}} - \mathrm{Ei}\left(\frac{1}{t}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/t),t)
```

```
[Out] t*exp(1/t) - Ei(1/t)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int e^{\frac{1}{t}}\, dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(1/t),t, algorithm="giac")
```

```
[Out] integrate(e^(1/t), t)
```

3.162 $\int \frac{e^{-t}}{-1-a+t} dt$

Optimal. Leaf size=15

$$e^{-a-1}\text{ExpIntegralEi}(a-t+1)$$

```
[Out] E^(-1 - a)*ExpIntegralEi[1 + a - t]
```

Rubi [A] time = 0.0153825, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2178}

$$e^{-a-1}\text{ExpIntegralEi}(a-t+1)$$

Antiderivative was successfully verified.

```
[In] Int[1/(E^t*(-1 - a + t)),t]
```

```
[Out] E^(-1 - a)*ExpIntegralEi[1 + a - t]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\int \frac{e^{-t}}{-1-a+t} dt = e^{-1-a}\text{Ei}(1+a-t)$$

Mathematica [A] time = 0.0117154, size = 15, normalized size = 1.

$$e^{-a-1}\text{ExpIntegralEi}(a-t+1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(E^t*(-1 - a + t)),t]
```

```
[Out] E^(-1 - a)*ExpIntegralEi[1 + a - t]
```

Maple [A] time = 0.006, size = 17, normalized size = 1.1

$$-e^{-1-a}\text{Ei}(1,-1-a+t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/exp(t)/(-1-a+t),t)
```

```
[Out] -exp(-1-a)*Ei(1,-1-a+t)
```

Maxima [A] time = 1.03949, size = 22, normalized size = 1.47

$$-e^{(-a-1)}E_1(-a+t-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(t)/(-1-a+t),t, algorithm="maxima")
```

```
[Out] -e^(-a - 1)*exp_integral_e(1, -a + t - 1)
```

Fricas [A] time = 1.1479, size = 35, normalized size = 2.33

$$\mathrm{Ei}\,(a-t+1)\,e^{(-a-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(t)/(-1-a+t),t, algorithm="fricas")
```

```
[Out] Ei(a - t + 1)*e^(-a - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{-t}}{-a+t-1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(t)/(-1-a+t),t)
```

```
[Out] Integral(exp(-t)/(-a + t - 1), t)
```

Giac [A] time = 1.08339, size = 19, normalized size = 1.27

$$\mathrm{Ei}\,(a-t+1)\,e^{(-a-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/exp(t)/(-1-a+t),t, algorithm="giac")
```

```
[Out] Ei(a - t + 1)*e^(-a - 1)
```

3.163 $\int \frac{e^{t^2}t}{1+t^2}\,dt$

Optimal. Leaf size=13

$$\frac{\text{ExpIntegralEi}\left(t^2+1\right)}{2e}$$

```
[Out] ExpIntegralEi[1 + t^2]/(2*E)
```

Rubi [A] time = 0.0654647, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6715, 2178}

$$\frac{\text{ExpIntegralEi}\left(t^2+1\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(E^t^2*t)/(1 + t^2),t]
```

```
[Out] ExpIntegralEi[1 + t^2]/(2*E)
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int \frac{e^{t^2}t}{1+t^2}\,dt &= \frac{1}{2}\,\text{Subst}\left(\int \frac{e^t}{1+t}\,dt, t, t^2\right)\\ &= \frac{\text{Ei}\left(1+t^2\right)}{2e}\end{aligned}$$

Mathematica [A] time = 0.0183979, size = 13, normalized size = 1.

$$\frac{\text{ExpIntegralEi}\left(t^2+1\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(E^t^2*t)/(1 + t^2),t]
```

```
[Out] ExpIntegralEi[1 + t^2]/(2*E)
```

Maple [A] time = 0.006, size = 14, normalized size = 1.1

$$-\frac{\mathrm{e}^{-1}\mathrm{Ei}\left(1,-t^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t^2)*t/(t^2+1),t)
```

```
[Out] -1/2*exp(-1)*Ei(1,-t^2-1)
```

Maxima [A] time = 1.035, size = 18, normalized size = 1.38

$$-\frac{1}{2}\,e^{(-1)}E_1\left(-t^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t^2)*t/(t^2+1),t, algorithm="maxima")
```

```
[Out] -1/2*e^(-1)*exp_integral_e(1, -t^2 - 1)
```

Fricas [A] time = 1.12247, size = 32, normalized size = 2.46

$$\frac{1}{2}\,\mathrm{Ei}\left(t^2+1\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t^2)*t/(t^2+1),t, algorithm="fricas")
```

```
[Out] 1/2*Ei(t^2 + 1)*e^(-1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{te^{t^2}}{t^2+1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t**2)*t/(t**2+1),t)
```

```
[Out] Integral(t*exp(t**2)/(t**2 + 1), t)
```

Giac [A] time = 1.12824, size = 14, normalized size = 1.08

$$\frac{1}{2}\,\mathrm{Ei}\left(t^2+1\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t^2)*t/(t^2+1),t, algorithm="giac")
```

```
[Out] 1/2*Ei(t^2 + 1)*e^(-1)
```

3.164 $\int \frac{e^t}{(1+t)^2} \, dt$

Optimal. Leaf size=19

$$\frac{\text{ExpIntegralEi}(t+1)}{e} - \frac{e^t}{t+1}$$

```
[Out] -(E^t/(1 + t)) + ExpIntegralEi[1 + t]/E
```

Rubi [A] time = 0.0233803, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2177, 2178}

$$\frac{\text{ExpIntegralEi}(t+1)}{e} - \frac{e^t}{t+1}$$

Antiderivative was successfully verified.

```
[In] Int[E^t/(1 + t)^2,t]
```

```
[Out] -(E^t/(1 + t)) + ExpIntegralEi[1 + t]/E
```

Rule 2177

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_), x_Symbol] :> Simp[((c + d*x)^(m + 1)*(b*F^(g*(e + f*x)))^n)/(d*(m + 1))
, x] - Dist[(f*g*n*Log[F])/(d*(m + 1)), Int[(c + d*x)^(m + 1)*(b*F^(g*(e +
f*x)))^n, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && LtQ[m, -1] && Int
egerQ[2*m] &&  !$UseGamma === True
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned} \int \frac{e^t}{(1+t)^2} \, dt &= -\frac{e^t}{1+t} + \int \frac{e^t}{1+t} \, dt \\ &= -\frac{e^t}{1+t} + \frac{\text{Ei}(1+t)}{e} \end{aligned}$$

Mathematica [A] time = 0.019306, size = 19, normalized size = 1.

$$\frac{\text{ExpIntegralEi}(t+1)}{e} - \frac{e^t}{t+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t/(1 + t)^2,t]
```

```
[Out] -(E^t/(1 + t)) + ExpIntegralEi[1 + t]/E
```

Maple [A] time = 0.003, size = 22, normalized size = 1.2

$$-\frac{\mathrm{e}^{t}}{1+t}-\mathrm{e}^{-1}\mathrm{Ei}\left(1,-1-t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)/(1+t)^2,t)
```

```
[Out] -exp(t)/(1+t)-exp(-1)*Ei(1,-1-t)
```

Maxima [A] time = 1.03167, size = 22, normalized size = 1.16

$$-\frac{e^{(-1)}E_{2}\left(-t-1\right)}{t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/(1+t)^2,t, algorithm="maxima")
```

```
[Out] -e^(-1)*exp_integral_e(2, -t - 1)/(t + 1)
```

Fricas [A] time = 1.09991, size = 65, normalized size = 3.42

$$\frac{\left((t+1)\mathrm{Ei}\left(t+1\right)-e^{(t+1)}\right)e^{(-1)}}{t+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/(1+t)^2,t, algorithm="fricas")
```

```
[Out] ((t + 1)*Ei(t + 1) - e^(t + 1))*e^(-1)/(t + 1)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/(1+t)**2,t)
```

```
[Out] Exception raised: ValueError
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{t}}{(t+1)^{2}}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)/(1+t)^2,t, algorithm="giac")
```

```
[Out] integrate(e^t/(t + 1)^2, t)
```

3.165 $\int e^t \log(1+t)\,dt$

Optimal. Leaf size=18

$$e^t \log(t+1) - \frac{\text{ExpIntegralEi}(t+1)}{e}$$

```
[Out] -(ExpIntegralEi[1 + t]/E) + E^t*Log[1 + t]
```

Rubi [A] time = 0.0199313, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2194, 2554, 2178}

$$e^t \log(t+1) - \frac{\text{ExpIntegralEi}(t+1)}{e}$$

Antiderivative was successfully verified.

```
[In] Int[E^t*Log[1 + t],t]
```

```
[Out] -(ExpIntegralEi[1 + t]/E) + E^t*Log[1 + t]
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rule 2554

```
Int[Log[u_]*(v_), x_Symbol] :> With[{w = IntHide[v, x]}, Dist[Log[u], w, x]
 - Int[SimplifyIntegrand[(w*D[u, x])/u, x], x] /; InverseFunctionFreeQ[w, x
]] /; InverseFunctionFreeQ[u, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}\int e^t \log(1+t)\,dt &= e^t \log(1+t) - \int \frac{e^t}{1+t}\,dt\\ &= -\frac{\text{Ei}(1+t)}{e} + e^t \log(1+t)\end{aligned}$$

Mathematica [A] time = 0.0081084, size = 18, normalized size = 1.

$$e^t \log(t+1) - \frac{\text{ExpIntegralEi}(t+1)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[E^t*Log[1 + t],t]
```

```
[Out] -(ExpIntegralEi[1 + t]/E) + E^t*Log[1 + t]
```

Maple [A] time = 0.02, size = 19, normalized size = 1.1

$$e^{t}\ln(1+t)+e^{-1}\mathrm{Ei}(1,-1-t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(t)*ln(1+t),t)
```

```
[Out] exp(t)*ln(1+t)+exp(-1)*Ei(1,-1-t)
```

Maxima [A] time = 1.04043, size = 24, normalized size = 1.33

$$e^{(-1)}E_{1}(-t-1)+e^{t}\log(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*log(1+t),t, algorithm="maxima")
```

```
[Out] e^(-1)*exp_integral_e(1, -t - 1) + e^t*log(t + 1)
```

Fricas [A] time = 1.16872, size = 58, normalized size = 3.22

$$\left(e^{(t+1)}\log(t+1)-\mathrm{Ei}(t+1)\right)e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*log(1+t),t, algorithm="fricas")
```

```
[Out] (e^(t + 1)*log(t + 1) - Ei(t + 1))*e^(-1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int e^{t}\log(t+1)\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*ln(1+t),t)
```

```
[Out] Integral(exp(t)*log(t + 1), t)
```

Giac [A] time = 1.13445, size = 22, normalized size = 1.22

$$-\mathrm{Ei}(t+1)\,e^{(-1)}+e^{t}\log(t+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(t)*log(1+t),t, algorithm="giac")
```

```
[Out] -Ei(t + 1)*e^(-1) + e^t*log(t + 1)
```

3.166 $\int e^{-t} t \, dt$

Optimal. Leaf size=16

$$-e^{-t}t - e^{-t}$$

```
[Out] -E^(-t) - t/E^t
```

Rubi [A] time = 0.0086752, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2176, 2194}

$$-e^{-t}t - e^{-t}$$

Antiderivative was successfully verified.

```
[In] Int[t/E^t,t]
```

```
[Out] -E^(-t) - t/E^t
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int e^{-t} t \, dt &= -e^{-t}t + \int e^{-t} \, dt \\ &= -e^{-t} - e^{-t}t \end{aligned}$$

Mathematica [A] time = 0.0042621, size = 11, normalized size = 0.69

$$e^{-t}(-t - 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[t/E^t,t]
```

```
[Out] (-1 - t)/E^t
```

Maple [A] time = 0.002, size = 10, normalized size = 0.6

$$-\frac{1+t}{e^{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t/exp(t),t)
```

```
[Out] -(1+t)/exp(t)
```

Maxima [A] time = 0.942143, size = 12, normalized size = 0.75

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t, algorithm="maxima")
```

```
[Out] -(t + 1)*e^(-t)
```

Fricas [A] time = 1.15506, size = 23, normalized size = 1.44

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t, algorithm="fricas")
```

```
[Out] -(t + 1)*e^(-t)
```

Sympy [A] time = 0.073756, size = 7, normalized size = 0.44

$$(-t-1)\,e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t)
```

```
[Out] (-t - 1)*exp(-t)
```

Giac [A] time = 1.09395, size = 12, normalized size = 0.75

$$-(t+1)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t/exp(t),t, algorithm="giac")
```

```
[Out] -(t + 1)*e^(-t)
```

3.167 $\int e^{-t}t^2\,dt$

Optimal. Leaf size=26

$$-e^{-t}t^2-2e^{-t}t-2e^{-t}$$

[Out] -2/E^t - (2*t)/E^t - t^2/E^t

Rubi [A] time = 0.0205438, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$-e^{-t}t^2-2e^{-t}t-2e^{-t}$$

Antiderivative was successfully verified.

[In] Int[t^2/E^t,t]

[Out] -2/E^t - (2*t)/E^t - t^2/E^t

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-t}t^2\,dt&=-e^{-t}t^2+2\int e^{-t}t\,dt\\&=-2e^{-t}t-e^{-t}t^2+2\int e^{-t}\,dt\\&=-2e^{-t}-2e^{-t}t-e^{-t}t^2\end{aligned}$$

Mathematica [A] time = 0.0048568, size = 16, normalized size = 0.62

$$e^{-t}\left(-t^2-2t-2\right)$$

Antiderivative was successfully verified.

[In] Integrate[t^2/E^t,t]

[Out] (-2 - 2*t - t^2)/E^t

Maple [A] time = 0.002, size = 15, normalized size = 0.6

$$-\frac{t^2+2t+2}{e^t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^2/exp(t),t)
```

```
[Out] -(t^2+2*t+2)/exp(t)
```

Maxima [A] time = 0.94588, size = 19, normalized size = 0.73

$$-\left(t^2+2t+2\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^2/exp(t),t, algorithm="maxima")
```

```
[Out] -(t^2 + 2*t + 2)*e^(-t)
```

Fricas [A] time = 1.07538, size = 34, normalized size = 1.31

$$-\left(t^2+2t+2\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^2/exp(t),t, algorithm="fricas")
```

```
[Out] -(t^2 + 2*t + 2)*e^(-t)
```

Sympy [A] time = 0.079461, size = 12, normalized size = 0.46

$$\left(-t^2-2t-2\right)e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t**2/exp(t),t)
```

```
[Out] (-t**2 - 2*t - 2)*exp(-t)
```

Giac [A] time = 1.09597, size = 19, normalized size = 0.73

$$-\left(t^2+2t+2\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^2/exp(t),t, algorithm="giac")
```

```
[Out] -(t^2 + 2*t + 2)*e^(-t)
```

3.168 $\int e^{-t}t^3\,dt$

Optimal. Leaf size=36

$$-e^{-t}t^3-3e^{-t}t^2-6e^{-t}t-6e^{-t}$$

```
[Out] -6/E^t - (6*t)/E^t - (3*t^2)/E^t - t^3/E^t
```

Rubi [A] time = 0.0338808, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2176, 2194}

$$-e^{-t}t^3-3e^{-t}t^2-6e^{-t}t-6e^{-t}$$

Antiderivative was successfully verified.

```
[In] Int[t^3/E^t,t]
```

```
[Out] -6/E^t - (6*t)/E^t - (3*t^2)/E^t - t^3/E^t
```

Rule 2176

```
Int[((b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m
_.), x_Symbol] :> Simp[((c + d*x)^m*(b*F^(g*(e + f*x)))^n)/(f*g*n*Log[F]),
x] - Dist[(d*m)/(f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*(b*F^(g*(e + f*x)))^n
, x], x] /; FreeQ[{F, b, c, d, e, f, g, n}, x] && GtQ[m, 0] && IntegerQ[2*m
] &&  !$UseGamma === True
```

Rule 2194

```
Int[((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.), x_Symbol] :> Simp[(F^(c*(a +
 b*x)))^n/(b*c*n*Log[F]), x] /; FreeQ[{F, a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}\int e^{-t}t^3\,dt &= -e^{-t}t^3+3\int e^{-t}t^2\,dt\\ &= -3e^{-t}t^2-e^{-t}t^3+6\int e^{-t}t\,dt\\ &= -6e^{-t}t-3e^{-t}t^2-e^{-t}t^3+6\int e^{-t}\,dt\\ &= -6e^{-t}-6e^{-t}t-3e^{-t}t^2-e^{-t}t^3\end{aligned}$$

Mathematica [A] time = 0.0058054, size = 21, normalized size = 0.58

$$e^{-t}\left(-t^3-3t^2-6t-6\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[t^3/E^t,t]
```

```
[Out] (-6 - 6*t - 3*t^2 - t^3)/E^t
```

Maple [A] time = 0.001, size = 20, normalized size = 0.6

$$-\frac{t^{3}+3\,t^{2}+6\,t+6}{\mathrm{e}^{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(t^3/exp(t),t)
```

```
[Out] -(t^3+3*t^2+6*t+6)/exp(t)
```

Maxima [A] time = 0.940438, size = 26, normalized size = 0.72

$$-\left(t^{3}+3\,t^{2}+6\,t+6\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/exp(t),t, algorithm="maxima")
```

```
[Out] -(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

Fricas [A] time = 1.19131, size = 45, normalized size = 1.25

$$-\left(t^{3}+3\,t^{2}+6\,t+6\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/exp(t),t, algorithm="fricas")
```

```
[Out] -(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

Sympy [A] time = 0.080538, size = 17, normalized size = 0.47

$$\left(-t^{3}-3t^{2}-6t-6\right)e^{-t}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t**3/exp(t),t)
```

```
[Out] (-t**3 - 3*t**2 - 6*t - 6)*exp(-t)
```

Giac [A] time = 1.08778, size = 26, normalized size = 0.72

$$-\left(t^{3}+3\,t^{2}+6\,t+6\right)e^{(-t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(t^3/exp(t),t, algorithm="giac")
```

```
[Out] -(t^3 + 3*t^2 + 6*t + 6)*e^(-t)
```

3.169 $\int \frac{\mathbf{b1}\cos(x)+\mathbf{a1}\sin(x)}{b\cos(x)+a\sin(x)}\,dx$

Optimal. Leaf size=48

$$\frac{x(a\text{a1}+b\text{b1})}{a^2+b^2}-\frac{(\text{a1}b-a\text{b1})\log(a\sin(x)+b\cos(x))}{a^2+b^2}$$

```
[Out] ((a*a1 + b*b1)*x)/(a^2 + b^2) - ((a1*b - a*b1)*Log[b*Cos[x] + a*Sin[x]])/(a
^2 + b^2)
```

Rubi [A] time = 0.0343966, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3133}

$$\frac{x(a\text{a1}+b\text{b1})}{a^2+b^2}-\frac{(\text{a1}b-a\text{b1})\log(a\sin(x)+b\cos(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(b1*Cos[x] + a1*Sin[x])/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] ((a*a1 + b*b1)*x)/(a^2 + b^2) - ((a1*b - a*b1)*Log[b*Cos[x] + a*Sin[x]])/(a
^2 + b^2)
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\text{b1}\cos(x)+\text{a1}\sin(x)}{b\cos(x)+a\sin(x)}\,dx=\frac{(a\text{a1}+b\text{b1})x}{a^2+b^2}-\frac{(\text{a1}b-a\text{b1})\log(b\cos(x)+a\sin(x))}{a^2+b^2}$$

Mathematica [A] time = 0.0948063, size = 39, normalized size = 0.81

$$\frac{x(a\text{a1}+b\text{b1})+(a\text{b1}-\text{a1}b)\log(a\sin(x)+b\cos(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b1*Cos[x] + a1*Sin[x])/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] ((a*a1 + b*b1)*x + (-(a1*b) + a*b1)*Log[b*Cos[x] + a*Sin[x]])/(a^2 + b^2)
```

Maple [B] time = 0.079, size = 111, normalized size = 2.3

$$-\frac{\ln\left((\tan(x))^2+1\right)ab1}{2a^2+2b^2}+\frac{\ln\left((\tan(x))^2+1\right)a1\,b}{2a^2+2b^2}+\frac{\arctan(\tan(x))\,aa1}{a^2+b^2}+\frac{\arctan(\tan(x))\,bb1}{a^2+b^2}+\frac{\ln(a\tan(x)+b)\,a}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x)
```

```
[Out] -1/2/(a^2+b^2)*ln(tan(x)^2+1)*a*b1+1/2/(a^2+b^2)*ln(tan(x)^2+1)*a1*b+1/(a^2
+b^2)*arctan(tan(x))*a*a1+1/(a^2+b^2)*arctan(tan(x))*b*b1+1/(a^2+b^2)*ln(a*
tan(x)+b)*a*b1-1/(a^2+b^2)*ln(a*tan(x)+b)*a1*b
```

Maxima [B] time = 1.44219, size = 244, normalized size = 5.08

$$a_1\left(\frac{2a\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2+b^2}-\frac{b\log\left(-b-\frac{2a\sin(x)}{\cos(x)+1}+\frac{b\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^2+b^2}+\frac{b\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^2+b^2}\right)+b_1\left(\frac{2b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2+b^2}+\frac{a\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x, algorithm="maxima")
```

```
[Out] a1*(2*a*arctan(sin(x)/(cos(x) + 1))/(a^2 + b^2) - b*log(-b - 2*a*sin(x)/(co
s(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(a^2 + b^2) + b*log(sin(x)^2/(cos(x)
 + 1)^2 + 1)/(a^2 + b^2)) + b1*(2*b*arctan(sin(x)/(cos(x) + 1))/(a^2 + b^2)
 + a*log(-b - 2*a*sin(x)/(cos(x) + 1) + b*sin(x)^2/(cos(x) + 1)^2)/(a^2 + b
^2) - a*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^2 + b^2))
```

Fricas [A] time = 1.28448, size = 144, normalized size = 3.

$$\frac{2(aa_1+bb_1)x-(a_1b-ab_1)\log\left(2ab\cos(x)\sin(x)-\left(a^2-b^2\right)\cos(x)^2+a^2\right)}{2\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a*a1 + b*b1)*x - (a1*b - a*b1)*log(2*a*b*cos(x)*sin(x) - (a^2 - b^2
)*cos(x)^2 + a^2))/(a^2 + b^2)
```

Sympy [A] time = 2.49333, size = 360, normalized size = 7.5

$$\begin{cases}\tilde{\infty}\left(-a_1\log(\cos(x))+b_1x\right) & \text{for } a\\ -\frac{a_1x\sin(x)}{2ib\sin(x)-2b\cos(x)}-\frac{ia_1x\cos(x)}{2ib\sin(x)-2b\cos(x)}+\frac{ia_1\sin(x)}{2ib\sin(x)-2b\cos(x)}+\frac{ib_1x\sin(x)}{2ib\sin(x)-2b\cos(x)}-\frac{b_1x\cos(x)}{2ib\sin(x)-2b\cos(x)}-\frac{b_1\sin(x)}{2ib\sin(x)-2b\cos(x)} & \text{for } a\\ \frac{a_1x\sin(x)}{2ib\sin(x)+2b\cos(x)}-\frac{ia_1x\cos(x)}{2ib\sin(x)+2b\cos(x)}+\frac{ia_1\sin(x)}{2ib\sin(x)+2b\cos(x)}+\frac{ib_1x\sin(x)}{2ib\sin(x)+2b\cos(x)}+\frac{b_1x\cos(x)}{2ib\sin(x)+2b\cos(x)}+\frac{b_1\sin(x)}{2ib\sin(x)+2b\cos(x)} & \text{for } a\\ \frac{-a_1\log(\cos(x))+b_1x}{b} & \text{for } a\\ \frac{aa_1x}{a^2+b^2}+\frac{ab_1\log\left(\sin(x)+\frac{b\cos(x)}{a}\right)}{a^2+b^2}-\frac{a_1b\log\left(\sin(x)+\frac{b\cos(x)}{a}\right)}{a^2+b^2}+\frac{bb_1x}{a^2+b^2} & \text{othe}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x)
```

```
[Out] Piecewise((zoo*(-a1*log(cos(x)) + b1*x), Eq(a, 0) & Eq(b, 0)), (-a1*x*sin(x
)/(2*I*b*sin(x) - 2*b*cos(x)) - I*a1*x*cos(x)/(2*I*b*sin(x) - 2*b*cos(x)) +
 I*a1*sin(x)/(2*I*b*sin(x) - 2*b*cos(x)) + I*b1*x*sin(x)/(2*I*b*sin(x) - 2*
b*cos(x)) - b1*x*cos(x)/(2*I*b*sin(x) - 2*b*cos(x)) - b1*sin(x)/(2*I*b*sin(
x) - 2*b*cos(x)), Eq(a, -I*b)), (a1*x*sin(x)/(2*I*b*sin(x) + 2*b*cos(x)) -
I*a1*x*cos(x)/(2*I*b*sin(x) + 2*b*cos(x)) + I*a1*sin(x)/(2*I*b*sin(x) + 2*b
*cos(x)) + I*b1*x*sin(x)/(2*I*b*sin(x) + 2*b*cos(x)) + b1*x*cos(x)/(2*I*b*s
in(x) + 2*b*cos(x)) + b1*sin(x)/(2*I*b*sin(x) + 2*b*cos(x)), Eq(a, I*b)), (
(-a1*log(cos(x)) + b1*x)/b, Eq(a, 0)), (a*a1*x/(a**2 + b**2) + a*b1*log(sin
(x) + b*cos(x)/a)/(a**2 + b**2) - a1*b*log(sin(x) + b*cos(x)/a)/(a**2 + b**
2) + b*b1*x/(a**2 + b**2), True))
```

Giac [A] time = 1.11823, size = 104, normalized size = 2.17

$$\frac{(aa_1+bb_1)x}{a^2+b^2}+\frac{(a_1b-ab_1)\log\left(\tan(x)^2+1\right)}{2\left(a^2+b^2\right)}-\frac{\left(aa_1b-a^2b_1\right)\log\left(|a\tan(x)+b|\right)}{a^3+ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b1*cos(x)+a1*sin(x))/(b*cos(x)+a*sin(x)),x, algorithm="giac")
```

```
[Out] (a*a1 + b*b1)*x/(a^2 + b^2) + 1/2*(a1*b - a*b1)*log(tan(x)^2 + 1)/(a^2 + b^
2) - (a*a1*b - a^2*b1)*log(abs(a*tan(x) + b))/(a^3 + a*b^2)
```

3.170 $\int \frac{1}{\log(t)} \, dt$

Optimal. Leaf size=2

$$\text{LogIntegral}(t)$$

```
[Out] LogIntegral[t]
```

Rubi [A] time = 0.001934, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2298}

$$\text{LogIntegral}(t)$$

Antiderivative was successfully verified.

```
[In] Int[Log[t]^(-1),t]
```

```
[Out] LogIntegral[t]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\int \frac{1}{\log(t)} \, dt = \text{li}(t)$$

Mathematica [A] time = 0.0168389, size = 2, normalized size = 1.

$$\text{LogIntegral}(t)$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[t]^(-1),t]
```

```
[Out] LogIntegral[t]
```

Maple [B] time = 0.003, size = 9, normalized size = 4.5

$$-\text{Ei}\left(1, -\ln\left(t\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(t),t)
```

```
[Out] -Ei(1,-ln(t))
```

Maxima [A] time = 1.03128, size = 4, normalized size = 2.

$$\text{Ei}(\log(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t),t, algorithm="maxima")
```

```
[Out] Ei(log(t))
```

Fricas [A] time = 1.03717, size = 23, normalized size = 11.5

$$\text{log\_integral}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t),t, algorithm="fricas")
```

```
[Out] log_integral(t)
```

Sympy [A] time = 0.461404, size = 2, normalized size = 1.

$$\text{li}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(t),t)
```

```
[Out] li(t)
```

Giac [A] time = 1.08247, size = 4, normalized size = 2.

$$\text{Ei}(\log(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t),t, algorithm="giac")
```

```
[Out] Ei(log(t))
```

3.171 $\int \frac{1}{\log^2(t)} \, dt$

Optimal. Leaf size=10

$$\text{LogIntegral}(t) - \frac{t}{\log(t)}$$

```
[Out] -(t/Log[t]) + LogIntegral[t]
```

Rubi [A] time = 0.0041417, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {2297, 2298}

$$\text{LogIntegral}(t) - \frac{t}{\log(t)}$$

Antiderivative was successfully verified.

```
[In] Int[Log[t]^(-2),t]
```

```
[Out] -(t/Log[t]) + LogIntegral[t]
```

Rule 2297

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[(x*(a + b
*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[1/(b*n*(p + 1)), Int[(a + b*
Log[c*x^n])^(p + 1), x], x] /; FreeQ[{a, b, c, n}, x] && LtQ[p, -1] && Inte
gerQ[2*p]
```

Rule 2298

```
Int[Log[(c_.)*(x_)]^(-1), x_Symbol] :> Simp[LogIntegral[c*x]/c, x] /; FreeQ
[c, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\log^2(t)} \, dt &= -\frac{t}{\log(t)} + \int \frac{1}{\log(t)} \, dt \\ &= -\frac{t}{\log(t)} + \text{li}(t) \end{aligned}$$

Mathematica [A] time = 0.0012051, size = 10, normalized size = 1.

$$\text{LogIntegral}(t) - \frac{t}{\log(t)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[t]^(-2),t]
```

```
[Out] -(t/Log[t]) + LogIntegral[t]
```

Maple [A] time = 0.002, size = 17, normalized size = 1.7

$$-\frac{t}{\ln(t)} - \operatorname{Ei}(1, -\ln(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/ln(t)^2,t)
```

```
[Out] -t/ln(t)-Ei(1,-ln(t))
```

Maxima [A] time = 1.02299, size = 8, normalized size = 0.8

$$\Gamma(-1, -\log(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t)^2,t, algorithm="maxima")
```

```
[Out] gamma(-1, -log(t))
```

Fricas [A] time = 1.32385, size = 50, normalized size = 5.

$$\frac{\log(t)\,\text{log\_integral}(t) - t}{\log(t)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t)^2,t, algorithm="fricas")
```

```
[Out] (log(t)*log_integral(t) - t)/log(t)
```

Sympy [A] time = 0.471673, size = 7, normalized size = 0.7

$$-\frac{t}{\log(t)} + \operatorname{li}(t)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/ln(t)**2,t)
```

```
[Out] -t/log(t) + li(t)
```

Giac [A] time = 1.0858, size = 15, normalized size = 1.5

$$-\frac{t}{\log(t)} + \operatorname{Ei}(\log(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/log(t)^2,t, algorithm="giac")
```

```
[Out] -t/log(t) + Ei(log(t))
```

3.172 $\int \log^{-1-n}(t)\,dt$

Optimal. Leaf size=22

$$(-\log(t))^n \log^{-n}(t)(-\mathrm{Gamma}(-n,-\log(t)))$$

```
[Out] -((Gamma[-n, -Log[t]]*(-Log[t])^n)/Log[t]^n)
```

Rubi [A] time = 0.0174485, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {2299, 2181}

$$(-\log(t))^n \log^{-n}(t)(-\mathrm{Gamma}(-n,-\log(t)))$$

Antiderivative was successfully verified.

```
[In] Int[Log[t]^(-1 - n),t]
```

```
[Out] -((Gamma[-n, -Log[t]]*(-Log[t])^n)/Log[t]^n)
```

Rule 2299

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Dist[1/(n*c^(1
/n)), Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b,
 c, p}, x] && IntegerQ[1/n]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \log^{-1-n}(t)\,dt &= \mathrm{Subst}\left(\int e^t t^{-1-n}\,dt, t, \log(t)\right)\\ &= -\Gamma(-n,-\log(t))(-\log(t))^n \log^{-n}(t)\end{aligned}$$

Mathematica [A] time = 0.0106361, size = 22, normalized size = 1.

$$(-\log(t))^n \log^{-n}(t)(-\mathrm{Gamma}(-n,-\log(t)))$$

Antiderivative was successfully verified.

```
[In] Integrate[Log[t]^(-1 - n),t]
```

```
[Out] -((Gamma[-n, -Log[t]]*(-Log[t])^n)/Log[t]^n)
```

Maple [F] time = 0.02, size = 0, normalized size = 0.

$$\int (\ln(t))^{-1-n}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(ln(t)^(-1-n),t)
```

```
[Out] int(ln(t)^(-1-n),t)
```

Maxima [A] time = 1.03843, size = 30, normalized size = 1.36

$$-(-\log(t))^{n}\log(t)^{-n}\,\Gamma(-n,-\log(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)^(-1-n),t, algorithm="maxima")
```

```
[Out] -(-log(t))^n*log(t)^(-n)*gamma(-n, -log(t))
```

Fricas [A] time = 1.15611, size = 47, normalized size = 2.14

$$\cos(\pi+\pi n)\,\Gamma(-n,-\log(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)^(-1-n),t, algorithm="fricas")
```

```
[Out] cos(pi + pi*n)*gamma(-n, -log(t))
```

Sympy [A] time = 4.90796, size = 24, normalized size = 1.09

$$(-\log(t))^{n+1}\log(t)^{-n-1}\Gamma(-n,-\log(t))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln(t)**(-1-n),t)
```

```
[Out] (-log(t))**(n + 1)*log(t)**(-n - 1)*uppergamma(-n, -log(t))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \log(t)^{-n-1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(t)^(-1-n),t, algorithm="giac")
```

```
[Out] integrate(log(t)^(-n - 1), t)
```

3.173 $\int \frac{e^{2t}}{-1+t}\,dt$

Optimal. Leaf size=12

$$e^2\text{ExpIntegralEi}(-2(1-t))$$

```
[Out] E^2*ExpIntegralEi[-2*(1 - t)]
```

Rubi [A] time = 0.0148496, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2178}

$$e^2\text{ExpIntegralEi}(-2(1-t))$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*t)/(-1 + t),t]
```

```
[Out] E^2*ExpIntegralEi[-2*(1 - t)]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rubi steps

$$\int \frac{e^{2t}}{-1+t}\,dt = e^2\text{Ei}(-2(1-t))$$

Mathematica [A] time = 0.0131442, size = 10, normalized size = 0.83

$$e^2\text{ExpIntegralEi}(2(t-1))$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*t)/(-1 + t),t]
```

```
[Out] E^2*ExpIntegralEi[2*(-1 + t)]
```

Maple [A] time = 0.003, size = 12, normalized size = 1.

$$-e^2\text{Ei}\left(1,-2\,t+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*t)/(-1+t),t)
```

```
[Out] -exp(2)*Ei(1,-2*t+2)
```

Maxima [A] time = 1.03601, size = 15, normalized size = 1.25

$$-e^2 E_1(-2t+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)/(-1+t),t, algorithm="maxima")
```

```
[Out] -e^2*exp_integral_e(1, -2*t + 2)
```

Fricas [A] time = 1.12668, size = 23, normalized size = 1.92

$$\mathrm{Ei}(2t-2)e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)/(-1+t),t, algorithm="fricas")
```

```
[Out] Ei(2*t - 2)*e^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{2t}}{t-1}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)/(-1+t),t)
```

```
[Out] Integral(exp(2*t)/(t - 1), t)
```

Giac [A] time = 1.08619, size = 12, normalized size = 1.

$$\mathrm{Ei}(2t-2)e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*t)/(-1+t),t, algorithm="giac")
```

```
[Out] Ei(2*t - 2)*e^2
```

3.174 $\int \frac{e^{2x}}{2-3x+x^2}\,dx$

Optimal. Leaf size=22

$$e^4\text{ExpIntegralEi}(2x-4)-e^2\text{ExpIntegralEi}(2x-2)$$

```
[Out] E^4*ExpIntegralEi[-4 + 2*x] - E^2*ExpIntegralEi[-2 + 2*x]
```

Rubi [A] time = 0.0559034, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2268, 2178}

$$e^4\text{ExpIntegralEi}(2x-4)-e^2\text{ExpIntegralEi}(2x-2)$$

Antiderivative was successfully verified.

```
[In] Int[E^(2*x)/(2 - 3*x + x^2),x]
```

```
[Out] E^4*ExpIntegralEi[-4 + 2*x] - E^2*ExpIntegralEi[-2 + 2*x]
```

Rule 2268

```
Int[(F_)^((g_.)*((d_.) + (e_.)*(x_))^(n_.))/((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2), x_Symbol] :> Int[ExpandIntegrand[F^(g*(d + e*x)^n), 1/(a + b*x + c*x^
2), x], x] /; FreeQ[{F, a, b, c, d, e, g, n}, x]
```

Rule 2178

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - (c*f)/d))*ExpIntegralEi[(f*g*(c + d*x)*Log[F])/d])/d, x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rubi steps

$$\begin{aligned}
\int \frac{e^{2x}}{2-3x+x^2}\,dx &= \int \left(-\frac{2e^{2x}}{4-2x}-\frac{2e^{2x}}{-2+2x}\right)dx\\
&= -\left(2\int \frac{e^{2x}}{4-2x}\,dx\right)-2\int \frac{e^{2x}}{-2+2x}\,dx\\
&= e^4\text{Ei}(-4+2x)-e^2\text{Ei}(-2+2x)
\end{aligned}$$

Mathematica [A] time = 0.0325951, size = 22, normalized size = 1.

$$e^4\text{ExpIntegralEi}(2x-4)-e^2\text{ExpIntegralEi}(2x-2)$$

Antiderivative was successfully verified.

```
[In] Integrate[E^(2*x)/(2 - 3*x + x^2),x]
```

```
[Out] E^4*ExpIntegralEi[-4 + 2*x] - E^2*ExpIntegralEi[-2 + 2*x]
```

Maple [A] time = 0.006, size = 23, normalized size = 1.1

$$e^2\mathrm{Ei}(1, 2-2x) - e^4\mathrm{Ei}(1, 4-2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(exp(2*x)/(x^2-3*x+2),x)
```

```
[Out] exp(2)*Ei(1,2-2*x)-exp(4)*Ei(1,4-2*x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{(2x)}}{x^2 - 3x + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(x^2-3*x+2),x, algorithm="maxima")
```

```
[Out] integrate(e^(2*x)/(x^2 - 3*x + 2), x)
```

Fricas [A] time = 1.13443, size = 47, normalized size = 2.14

$$\mathrm{Ei}(2x-4)e^4 - \mathrm{Ei}(2x-2)e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(x^2-3*x+2),x, algorithm="fricas")
```

```
[Out] Ei(2*x - 4)*e^4 - Ei(2*x - 2)*e^2
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{e^{2x}}{(x-2)(x-1)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(x**2-3*x+2),x)
```

```
[Out] Integral(exp(2*x)/((x - 2)*(x - 1)), x)
```

Giac [A] time = 1.09543, size = 27, normalized size = 1.23

$$\mathrm{Ei}(2x-4)e^4 - \mathrm{Ei}(2x-2)e^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(exp(2*x)/(x^2-3*x+2),x, algorithm="giac")
```

```
[Out] Ei(2*x - 4)*e^4 - Ei(2*x - 2)*e^2
```

3.175 $\int \frac{1}{\sqrt{1+t^3}}\,dt$

Optimal. Leaf size=103

$$\frac{2\sqrt{2+\sqrt{3}}(t+1)\sqrt{\frac{t^2-t+1}{\left(t+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{t-\sqrt{3}+1}{t+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{t+1}{\left(t+\sqrt{3}+1\right)^2}}\sqrt{t^3+1}}$$

```
[Out] (2*Sqrt[2 + Sqrt[3]]*(1 + t)*Sqrt[(1 - t + t^2)/(1 + Sqrt[3] + t)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] + t)/(1 + Sqrt[3] + t)], -7 - 4*Sqrt[3]])/(3^(1/4)*
Sqrt[(1 + t)/(1 + Sqrt[3] + t)^2]*Sqrt[1 + t^3])
```

Rubi [A] time = 0.0084795, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {218}

$$\frac{2\sqrt{2+\sqrt{3}}(t+1)\sqrt{\frac{t^2-t+1}{\left(t+\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\sin^{-1}\left(\frac{t-\sqrt{3}+1}{t+\sqrt{3}+1}\right),-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{t+1}{\left(t+\sqrt{3}+1\right)^2}}\sqrt{t^3+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + t^3],t]
```

```
[Out] (2*Sqrt[2 + Sqrt[3]]*(1 + t)*Sqrt[(1 - t + t^2)/(1 + Sqrt[3] + t)^2]*Ellipt
icF[ArcSin[(1 - Sqrt[3] + t)/(1 + Sqrt[3] + t)], -7 - 4*Sqrt[3]])/(3^(1/4)*
Sqrt[(1 + t)/(1 + Sqrt[3] + t)^2]*Sqrt[1 + t^3])
```

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rubi steps

$$\int \frac{1}{\sqrt{1+t^3}}\,dt=\frac{2\sqrt{2+\sqrt{3}}(1+t)\sqrt{\frac{1-t+t^2}{\left(1+\sqrt{3}+t\right)^2}}F\left(\sin^{-1}\left(\frac{1-\sqrt{3}+t}{1+\sqrt{3}+t}\right)|-7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{1+t}{\left(1+\sqrt{3}+t\right)^2}}\sqrt{1+t^3}}$$

Mathematica [C] time = 0.0018868, size = 17, normalized size = 0.17

$$t\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{1}{2},\frac{4}{3},-t^3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + t^3],t]
```

```
[Out] t*Hypergeometric2F1[1/3, 1/2, 4/3, -t^3]
```

Maple [A] time = 0.039, size = 116, normalized size = 1.1

$$2\frac{3/2 - i/2\sqrt{3}}{\sqrt{t^3+1}}\sqrt{\frac{1+t}{3/2 - i/2\sqrt{3}}}\sqrt{\frac{t-1/2-i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\sqrt{\frac{t-1/2+i/2\sqrt{3}}{-3/2+i/2\sqrt{3}}}\text{EllipticF}\left(\sqrt{\frac{1+t}{3/2-i/2\sqrt{3}}},\sqrt{\frac{-3/2+i/2\sqrt{3}}{-3/2-i/2\sqrt{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(t^3+1)^(1/2),t)
```

```
[Out] 2*(3/2-1/2*I*3^(1/2))*((1+t)/(3/2-1/2*I*3^(1/2)))^(1/2)*((t-1/2-1/2*I*3^(1/
2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((t-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2))
)^(1/2)/(t^3+1)^(1/2)*EllipticF(((1+t)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/
2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{t^3+1}}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(t^3+1)^(1/2),t, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(t^3 + 1), t)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{t^3+1}}, t\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(t^3+1)^(1/2),t, algorithm="fricas")
```

```
[Out] integral(1/sqrt(t^3 + 1), t)
```

Sympy [A] time = 0.530859, size = 27, normalized size = 0.26

$$\frac{t\Gamma\left(\frac{1}{3}\right){}_2F_1\left(\begin{matrix}\frac{1}{3},\frac{1}{2}\\\frac{4}{3}\end{matrix}\middle| t^3e^{i\pi}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(t**3+1)**(1/2),t)
```

```
[Out] t*gamma(1/3)*hyper((1/3, 1/2), (4/3,), t**3*exp_polar(I*pi))/(3*gamma(4/3))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{t^3+1}}\,dt$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(t^3+1)^(1/2),t, algorithm="giac")
```

```
[Out] integrate(1/sqrt(t^3 + 1), t)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
    (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```