Review for midterm exam Math 322, Tuesday, March 13, 2018

The midterm exam is on sections 10.1–10.8 of our textbook.

1. (Section 10.1) Find the solution (if it exists) of the boundary value problem

$$y'' - y = e^x, \quad y(0) = 1, y(1) = 0.$$

2. (Sections 10.2–10.4) Find the Fourier cosine series for the function

$$f(x) = \begin{cases} x & \text{if } 0 < x < 1, \\ 1 & \text{if } 1 < x < 2. \end{cases}$$

Choose $L = 2$. Apply the Fourier convergence theorem. What do we get at $x = 1$?

3. (Sections 10.2–10.4) Find the Fourier sine series for the function $f(x)$ of Problem 2. Choose $L = 2$. Apply the Fourier convergence theorem.

4. (Section 10.5) Solve the heat equation

$$u_t = u_{xx}$$

with boundary conditions

$$u_x(0, t) = 0, \quad u_x(2, t) = 0$$

and initial condition

$$u(x, 0) = f(x)$$

with $f(x)$ from Problem 2. Find the steady-state temperature.

5. (Section 10.6) Solve the heat equation

$$u_t = u_{xx}$$

with boundary conditions

$$u(0, t) = t, \quad u(\pi, t) = 0$$

and initial condition

$$u(x, 0) = 0.$$

6. (Section 10.7) Solve the wave equation

$$u_{tt} = 4u_{xx}, \quad 0 < x < \pi, t > 0$$

with boundary conditions

$$u(0, t) = 0, \quad u(\pi, t) = 0$$

and initial conditions

$$u(x, 0) = \sin^2 x, \quad u_t(x, 0) = 0.$$

Find the d'Alembert solution and the Fourier series solution.

7. (Section 10.7) Find d'Alembert's solution for the wave equation

$$u_{tt}(x, t) = 4u_{xx}(x, t), \quad -\infty < x < \infty, t > 0$$

with initial conditions

$$u(x,0) = \sin x, \quad u_t(x,0) = \cos x.$$

8. (Section 10.8) Solve the Dirichlet problem $u_{xx} + u_{yy} = 0$ in the disk $x^2 + y^2 < 1$ and

$$u(x,y) = \begin{cases} 20 & \text{if } y > 0 \\ 0 & \text{if } y < 0 \end{cases}$$

on the unit circle $x^2 + y^2 = 1$. Find $u(0,0)$ and $u(0, \frac{1}{2})$.

9. (Section 10.8) Find the solution $u(x,y)$ of Laplace's equation $u_{xx} + u_{yy} = 0$ in the semi-infinite strip $0 < x < a, y > 0$, that satisfies $u(0,y) = 0$, $u(a,y) = 0$ for $y > 0$ and $u(x,0) = F(x)$, $0 < x < a$ and the additional condition that $u(x,y) \to 0$ as $y \to \infty$.