

HW 7 (4th HW on ODE's), Math 601
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Contents

1	problems description	1
2	Key solution	2

1 problems description

MATH 601 HW #4

HW# 7. 4th mode

[1] Find the general solution of

a) $16y'' - 8y' + 5y = 0$

b) $y'' + 4y' + (4 - \omega^2)y = 0$

[2] Find the solution of the ODE that satisfy the given conditions:

a) $y'' + 0.4y' + 0.29y = 0$
 $y(0) = 1, y'(0) = -1.2$

b) $y'' + 2y' + 2y = 0$
 $y(0) = 1, y(\frac{\pi}{2}) = 0$

[3] Can you make a harmonic oscillator

$$m y'' + ky = 0$$

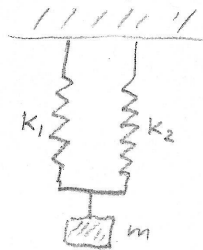
move faster by giving the body greater initial push?

[4] What are the frequencies of vibration of a 56g mass vibrating in vacuum

a) on a spring with $k_1 = 20 \frac{N}{m}$ spring constant

b) on a spring with $k_2 = 45 \frac{N}{m}$ spring constant

c) on the two springs connected parallelly?



2 Key solution

Math 601 HW #4 Solution

1 a) $16y'' - 8y' + 5y = 0$ characteristic eq. $16r^2 - 8r + 5 = 0$

$$r = \frac{8 \pm \sqrt{64 - 5 \cdot 64}}{2 \cdot 16} = \frac{1}{4} \pm \frac{i}{2}$$

$$y(x) = e^{x/4} (C_1 \cos \frac{x}{2} + C_2 \sin \frac{x}{2})$$

b) $y'' + 4y' + (4 - w^2)y = 0$ char eq $r^2 + 4r + 4 - w^2 = 0$

 $w > 0$

$$r = \frac{-4 \pm \sqrt{16 - 16 + 4w^2}}{2} = -2 \pm w$$

solution $y(x) = e^{-2x} (C_1 \cos wx + C_2 \sin wx)$

or

$$w = 0 \quad y(x) = C_1 e^{2x} + C_2 x e^{2x}$$

2 a) $y'' + 0.4y' + 0.29y = 0$ $r^2 + \frac{4}{10}r + \frac{29}{100} = 0$

$$\begin{cases} y(0) = 1, y'(0) = 1.2 \end{cases}$$

$$r = \frac{-0.4 \pm \sqrt{0.16 - 1.16}}{2} = -0.2 \pm i/2$$

$$y(x) = e^{-\frac{2x}{10}} (C_1 \cos \frac{x}{2} + C_2 \sin \frac{x}{2})$$

$$y'(x) = -\frac{2}{10} e^{-\frac{2x}{10}} (C_1 \cos \frac{x}{2} + C_2 \sin \frac{x}{2}) +$$

$$\frac{1}{2} e^{-\frac{2x}{10}} (-C_1 \sin \frac{x}{2} + C_2 \cos \frac{x}{2})$$

$$y(0) = C_1 = 1$$

$$y'(0) = -\frac{2}{10} C_1 + \frac{1}{2} C_2 = -\frac{2}{10} + \frac{C_2}{2} = -1.2 \Rightarrow C_2 = -2$$

$$y(x) = e^{-\frac{2x}{10}} (\cos \frac{x}{2} - 2 \sin \frac{x}{2})$$

b) $y'' + 2y' + 2y = 0$

$$r^2 + 2r + 2 = 0$$

$$r = \frac{-2 \pm \sqrt{4 - 8}}{2} = -1 \pm i$$

$$\begin{cases} y(0) = 1, y(\pi/2) = 0 \end{cases}$$

$$y(0) = C_1 = 1$$

$$y(x) = e^{-x} (C_1 \cos x + C_2 \sin x)$$

$$y(\pi/2) = e^{-\pi/2} (\cos \pi/2 + C_2 \sin \pi/2) = C_2 e^{-\pi/2} = 0 \Rightarrow C_2 = 0$$

$$y(x) = e^{-x} \cdot \cos x$$

$$\boxed{3} \quad \begin{cases} my'' + ky = 0 \\ y(0) = y_0, y'(0) = v_0 \end{cases}$$

$$y(x) = y_0 \cos \sqrt{\frac{k}{m}} x + v_0 \sqrt{\frac{m}{k}} \sin \sqrt{\frac{k}{m}} x$$

$$y'(x) = \underbrace{-\sqrt{\frac{k}{m}} y_0 \sin \sqrt{\frac{k}{m}} x}_A + \underbrace{(v_0) \cos \sqrt{\frac{k}{m}} x}_B = \sqrt{\frac{k}{m} y_0^2 + v_0^2} \cos \left(\sqrt{\frac{k}{m}} x - \delta \right)$$

velocity

$$mr^2 = -k \quad r = \pm i \sqrt{\frac{k}{m}}$$

$$y(x) = c_1 \cos \sqrt{\frac{k}{m}} x + c_2 \sin \sqrt{\frac{k}{m}} x$$

$$y'(x) = \sqrt{\frac{k}{m}} \left[c_1 (-\sin \sqrt{\frac{k}{m}} x) + c_2 \cos \sqrt{\frac{k}{m}} x \right]$$

$$y(0) = c_1 = y_0$$

$$y'(0) = \sqrt{\frac{k}{m}} c_2 = v_0 \Rightarrow c_2 = v_0 \sqrt{\frac{m}{k}}$$

$$\delta = \tan^{-1} \left(-\frac{y_0}{v_0} \sqrt{\frac{k}{m}} \right)$$

The magnitude of the velocity $\sqrt{\frac{k}{m} y_0^2 + v_0^2}$ increases as v_0 increases.

The period of the oscillation is not affected by changing v_0 .

4

$$my'' = -ky - cy' \quad \begin{array}{l} k \text{ spring coefficient} \\ c = 0 \text{ damping coefficient} \\ \text{in vacuum no air resistance} \end{array}$$

$$my'' + ky = 0$$



Solution as in $\boxed{3}$ is

$$y(x) = c_1 \cos \sqrt{\frac{k}{m}} x + c_2 \sin \sqrt{\frac{k}{m}} x$$

a) $k_1 = 20 \frac{\text{kg}}{\text{sec}^2}$, $m = 5$, $y(x)$ has period

$$T = 2\pi \sqrt{\frac{m}{k_1}} = \frac{2\pi}{2} = \pi \text{ sec}$$

and frequency $\sqrt{\frac{k_1}{m}} \frac{1}{2\pi} = \frac{1}{\pi} \frac{1}{\text{sec}}$

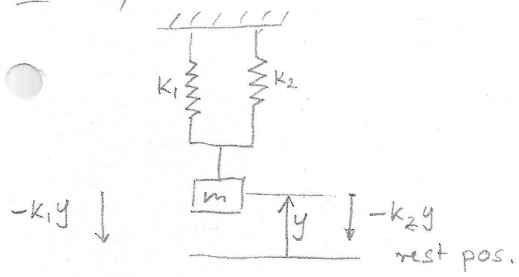
b) $k_2 = 45 \frac{\text{kg}}{\text{sec}^2}$, $m = 5$, $y(x)$ has period

$$T = 2\pi \sqrt{\frac{m}{k_2}} = \frac{2\pi}{3} \text{ sec}$$

and frequency $\frac{3}{2\pi} \frac{1}{\text{sec}}$

oscillations per second. \rightarrow

[4] c,



(Spring force)
 from spring 1 from spring 2

$$m y'' = -k_1 y - k_2 y = -(k_1 + k_2) y$$

$$m y'' + k y = 0 \quad k = k_1 + k_2$$

$$= 65 \frac{\text{kg}}{\text{sec}^2}$$

has period $2\pi \sqrt{\frac{m}{k}} = \frac{2\pi}{\sqrt{11}} \text{ sec}$,

frequency $\frac{\sqrt{11}}{2\pi} \frac{1}{\text{sec}}$