HW 5 (Second HW on ODE's), Math 601
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University Of Wisconsin, Milwaukee

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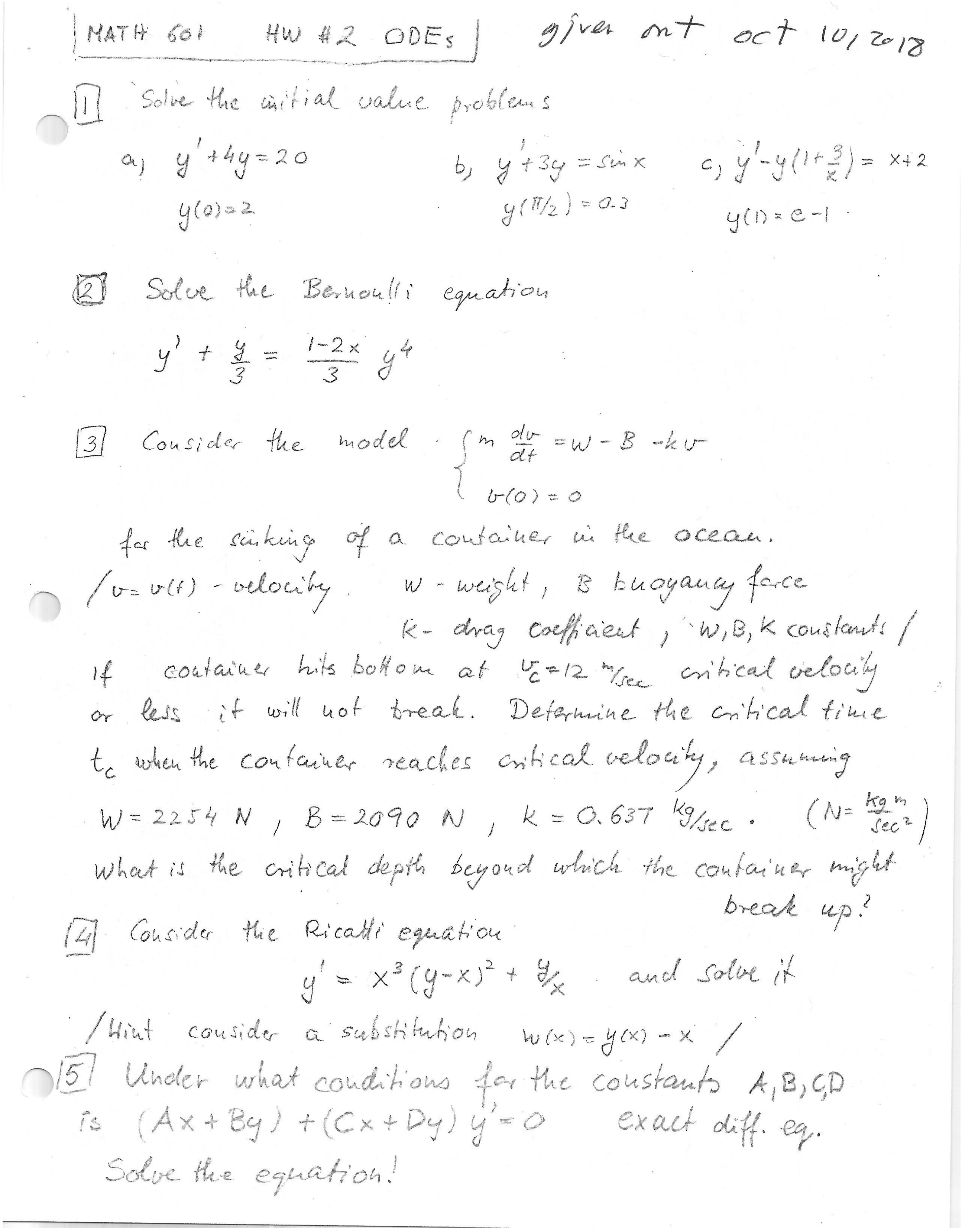
Contents

1 problems description



$\boxed{\text{MATH 601} \quad \text{HW \#2 ODEs}}$ given out oct 10, 2018

$\boxed{1}$ Solve the initial value problems

a) $y'+4y=20$, $y(0)=2$ b) $y'+3y=\sin x$, $y(\pi/2)=0.3$ c) $y'-y(1+\frac{3}{x})=x+2$, $y(1)=e-1$

$\boxed{2}$ Solve the Bernoulli equation

$$y'+\frac{y}{3}=\frac{1-2x}{3}y^4$$

$\boxed{3}$ Consider the model $\begin{cases} m\frac{dv}{dt}=W-B-kv \\ v(0)=0 \end{cases}$

for the sinking of a container in the ocean.

/$v=v(t)$ - velocity W - weight, B buoyancy force

k- drag coefficient, W,B,K constants /

If container hits bottom at $v_c=12$ m/sec critical velocity or less it will not break. Determine the critical time t_c when the container reaches critical velocity, assuming

$W=2254$ N, $B=2090$ N, $k=0.637$ kg/sec. $(N=\frac{\text{kg m}}{\text{sec}^2})$

what is the critical depth beyond which the container might break up?

$\boxed{4}$ Consider the Ricatti equation

$$y'=x^3(y-x)^2+y/x \quad \text{and solve it}$$

/Hint consider a substitution $w(x)=y(x)-x$ /

$\boxed{5}$ Under what conditions for the constants A,B,C,D is $(Ax+By)+(Cx+Dy)y'=0$ exact diff. eq.

Solve the equation!

2 problem 1

2.1 part a

The ODE to solve is

$$\frac{d}{dx}y(x)+4y(x)=20$$

with initial conditions $y(0)=2$.
Trying separable ODE.
In canonical form, the ODE is written as

$$\begin{aligned} y' &= F(x,y) \\ &= -4y+20 \end{aligned}$$

The ODE $\frac{dy}{dx} = -4\,y + 20$, is separable. It can be written as

$$\frac{dy}{dx} = f(x)g(y)$$

Where $f(x) = 1$ and $g(y) = -4\,y + 20$. Therefore

$$\frac{dy}{dx} = -4\,y + 20$$

Hence

$$\begin{aligned}(-4\,y + 20)^{-1}\,dy &= dx\\ \int (-4\,y + 20)^{-1}\,dy &= \int dx\\ -1/2\,\ln(2) - 1/4\,\ln(|y - 5|) &= x + C_1\end{aligned}$$

Solving for y gives

$$y = -1/4\,e^{-4\,x-4\,C_1} + 5$$

The solution above can be written as

$$\boxed{y = -1/4\,C_1 e^{-4\,x} + 5} \tag{1}$$

Initial conditions are now used to solve for C_1. Substituting $x = 0$ and $y = 2$ in the above solution gives an equation to solve for the constant of integration.

$$\begin{aligned}2 &= -1/4\,C_1 e^0 + 5\\ &= -1/4\,C_1 + 5\end{aligned}$$

Hence

$$C_1 = 12\,\left(e^0\right)^{-1}$$

Which is simplified to

$$C_1 = 12$$

Substituting C_1 found above back in the solution gives

$$y(x) = -3\,e^{-4\,x} + 5$$

2.2 part b

The ODE to solve is

$$\frac{d}{dx}y(x) + 3\,y(x) = \sin(x)$$

with initial conditions $y(\pi/2) = 3/10$.
Trying Linear ODE.
In canonical form, the ODE is written as

$$\begin{aligned}y' &= F(x, y)\\ &= -3\,y + \sin(x)\end{aligned}$$

The ODE is linear in y and has the form

$$y' = yf(x) + g(x)$$

Where $f(x) = -3$ and $g(x) = \sin(x)$.
Writing the ODE as

$$\begin{aligned}y' - (-3\,y) &= \sin(x)\\ y' + 3\,y &= \sin(x)\end{aligned}$$

Therefore the integrating factor μ is

$$\mu = e^{\int 3\,dx} = e^{3\,x}$$

The ode becomes

$$\begin{aligned}\frac{d}{dx}\mu y &= \mu(\sin(x))\\ \frac{d}{dx}\left(ye^{3\,x}\right) &= \sin(x)\,e^{3\,x}\\ d\left(ye^{3\,x}\right) &= \left(\sin(x)\,e^{3\,x}\right)\,dx\end{aligned}$$

Integrating both sides gives

$$ye^{3x} = -1/10\ \cos(x)\,e^{3x} + 3/10\ \sin(x)\,e^{3x} + C_1$$

Dividing both sides by the integrating factor $\mu = e^{3x}$ results in

$$y = \frac{-1/10\ \cos(x)\,e^{3x} + 3/10\ \sin(x)\,e^{3x}}{e^{3x}} + \frac{C_1}{e^{3x}}$$

Simplifying the solution gives

$$\boxed{y = 3/10\ \sin(x) - 1/10\ \cos(x) + C_1 e^{-3x}}$$

Initial conditions are now used to solve for C_1. Substituting $x = \pi/2$ and $y = 3/10$ in the above solution gives an equation to solve for the constant of integration.

$$\begin{aligned}3/10 &= 3/10\ \sin(\pi/2) - 1/10\ \cos(\pi/2) + C_1 e^{-3/2\,\pi}\\ &= 3/10 + C_1 e^{-3/2\,\pi}\end{aligned}$$

Hence

$$C_1 = -1/10\,\frac{3\ \sin(\pi/2) - \cos(\pi/2) - 3}{e^{-3/2\,\pi}}$$

Which is simplified to

$$C_1 = 0$$

Substituting C_1 found above back in the solution gives

$$y(x) = 3/10\ \sin(x) - 1/10\ \cos(x)$$

2.3 part c

The ODE to solve is

$$\frac{d}{dx}y(x) - y(x)\left(1 + 3x^{-1}\right) = x + 2$$

with initial conditions $y(1) = e - 1$.
Trying Linear ODE.
In canonical form, the ODE is written as

$$\begin{aligned}y' &= F(x,y)\\ &= \frac{x^2 + xy + 2x + 3y}{x}\end{aligned}$$

The ODE is linear in y and has the form

$$y' = yf(x) + g(x)$$

Where $f(x) = \frac{x+3}{x}$ and $g(x) = \frac{x^2+2x}{x}$.
Writing the ODE as

$$\begin{aligned}y' - \left(\frac{(x+3)\,y}{x}\right) &= \frac{x^2 + 2x}{x}\\ y' - \frac{(x+3)\,y}{x} &= \frac{x^2 + 2x}{x}\end{aligned}$$

Therefore the integrating factor μ is

$$\mu = e^{\int -\frac{x+3}{x}\,dx} = e^{-x-3\,\ln(x)}$$

The ode becomes

$$\begin{aligned}\frac{d}{dx}\mu y &= \mu\left(\frac{x^2 + 2x}{x}\right)\\ \frac{d}{dx}\left(ye^{-x-3\,\ln(x)}\right) &= \frac{\left(x^2 + 2x\right)e^{-x-3\,\ln(x)}}{x}\\ d\left(ye^{-x-3\,\ln(x)}\right) &= \left(\frac{\left(x^2 + 2x\right)e^{-x-3\,\ln(x)}}{x}\right)dx\end{aligned}$$

Integrating both sides gives

$$y\mathrm{e}^{-x-3\,\ln(x)} = -\mathrm{e}^{-x-3\,\ln(x)}x + C_1$$

Dividing both sides by the integrating factor $\mu = \mathrm{e}^{-x-3\,\ln(x)}$ results in

$$y = -x + \frac{C_1}{\mathrm{e}^{-x-3\,\ln(x)}}$$

Simplifying the solution gives

$$\boxed{y = -x + C_1 x^3 \mathrm{e}^x}$$

Initial conditions are now used to solve for C_1. Substituting $x = 1$ and $y = \mathrm{e} - 1$ in the above solution gives an equation to solve for the constant of integration.

$$\mathrm{e} - 1 = -1 + C_1 \mathrm{e}$$

Hence

$$C_1 = 1$$

Substituting C_1 found above back in the solution gives

$$y(x) = -x + x^3 \mathrm{e}^x$$

3 problem 2

The ODE to solve is

$$\frac{\mathrm{d}}{\mathrm{d}x} y(x) + 1/3\, y(x) = 1/3\, (1 - 2\,x) (y(x))^4$$

Trying Bernoulli ODE.
In canonical form, the ODE is written as

$$\begin{aligned} y' &= F(x, y) \\ &= -y/3 - 2/3\, y^4 x + 1/3\, y^4 \end{aligned}$$

This is a Bernoulli ODE. Comparing the ODE to solve

$$y' = -y/3 - 2/3\, y^4 x + 1/3\, y^4$$

With Bernoulli ODE standard form

$$y' = f_0(x) y + f_1(x) y^n$$

Shows that $f_0(x) = -1/3$ and $f_1(x) = -2/3\,x + 1/3$ and $n = 4$.
Dividing the ODE by y^4 gives

$$y' y^{-4} = -1/3 y^{-3} + -2/3\,x + 1/3 \tag{1}$$

Let

$$v = y^{-3} \tag{2}$$

Taking derivative of (2) w.r.t x gives

$$\begin{aligned} v' &= -3\, y^{-4} y' \\ y^{-4} &= \frac{v'}{-3\, y'} \end{aligned} \tag{3}$$

Substituting (3) into (1) gives

$$\begin{aligned} \frac{v'}{(-3)} &= (-1/3)\, v + -2/3\,x + 1/3 \\ v' &= (-3)(-1/3)\, v + (-3)(-2/3\,x + 1/3) \\ &= v + 2\,x - 1 \end{aligned}$$

The above now is a linear ODE in $v(x)$ which can be easily solved using an integrating factor.
In canonical form, the ODE is written as

$$\begin{aligned} v' &= F(x, v) \\ &= v + 2\,x - 1 \end{aligned}$$

The ODE is linear in v and has the form

$$v' = vf(x) + g(x)$$

Where $f(x) = 1$ and $g(x) = 2x - 1$.
Writing the ODE as

$$\begin{aligned} v' - (v) &= 2x - 1 \\ v' - v &= 2x - 1 \end{aligned}$$

Therefore the integrating factor μ is

$$\mu = e^{\int -1 \, dx} = e^{-x}$$

The ode becomes

$$\begin{aligned} \frac{d}{dx}\mu v &= \mu (2x - 1) \\ \frac{d}{dx}(ve^{-x}) &= (2x - 1) e^{-x} \\ d(ve^{-x}) &= ((2x - 1) e^{-x}) \, dx \end{aligned}$$

Integrating both sides gives

$$ve^{-x} = -(2x + 1) e^{-x} + C_1$$

Dividing both sides by the integrating factor $\mu = e^{-x}$ results in

$$v = -2x - 1 + \frac{C_1}{e^{-x}}$$

Simplifying the solution gives

$$\boxed{v = -2x - 1 + C_1 e^x}$$

Replacing v in the above by y^{-3} from equation (2), gives the final solution.

$$y^{-3} = -2x - 1 + C_1 e^x$$

Solving for y gives

$$\begin{aligned} y &= \frac{1}{\sqrt[3]{-2x - 1 + C_1 e^x}} \\ y &= -1/2 \frac{1}{\sqrt[3]{-2x - 1 + C_1 e^x}} + \frac{i/2\sqrt{3}}{\sqrt[3]{-2x - 1 + C_1 e^x}} \\ y &= -1/2 \frac{1}{\sqrt[3]{-2x - 1 + C_1 e^x}} - \frac{i/2\sqrt{3}}{\sqrt[3]{-2x - 1 + C_1 e^x}} \end{aligned}$$

4 problem 3

The ODE to solve is

$$m\frac{d}{dx}v(x) = w - B - kv(x)$$

with initial conditions $v(0) = 0$.
Trying separable ODE.
In canonical form, the ODE is written as

$$\begin{aligned} v' &= F(x, v) \\ &= -\frac{kv + B - w}{m} \end{aligned}$$

The ODE $\frac{dv}{dx} = -\frac{kv+B-w}{m}$, is separable. It can be written as

$$\frac{dv}{dx} = f(x)g(v)$$

Where $f(x) = 1$ and $g(v) = \frac{-kv-B+w}{m}$. Therefore

$$\frac{dv}{dx} = \frac{-kv - B + w}{m}$$

Hence

$$\begin{aligned}\left(\frac{m}{-kv-B+w}\right)\mathrm{d}v &= \mathrm{d}x\\ \int\left(\frac{m}{-kv-B+w}\right)\mathrm{d}v &= \int \mathrm{d}x\\ -\frac{m\ln\left(|kv+B-w|\right)}{k} &= x+C_1\end{aligned}$$

Solving for v gives

$$v=\frac{1}{k}\left(-\mathrm{e}^{-\frac{k(x+C_1)}{m}}-B+w\right)$$

Initial conditions are now used to solve for C_1. Substituting $x=0$ and $v=0$ in the above solution gives an equation to solve for the constant of integration.

$$0=\frac{1}{k}\left(-\mathrm{e}^{-\frac{kC_1}{m}}-B+w\right)$$

Hence

$$C_1=-\frac{m\ln\left(-B+w\right)}{k}$$

Substituting C_1 found above back in the solution gives

$$v\left(x\right)=\frac{1}{k}\left(-\mathrm{e}^{-\frac{k}{m}\left(x-\frac{m\ln(-B+w)}{k}\right)}-B+w\right)$$

The solution $\frac{1}{k}\left(-\mathrm{e}^{-\frac{k}{m}\left(x-\frac{m\ln(-B+w)}{k}\right)}-B+w\right)$ can be simplified to

$$\boxed{v\left(x\right)=\frac{1}{k}\left(-\mathrm{e}^{\frac{m\ln(-B+w)-xk}{m}}-B+w\right)}\tag{2}$$

5 problem 4

The ODE to solve is

$$\frac{\mathrm{d}}{\mathrm{d}x}y\left(x\right)=x^3\left(y\left(x\right)-x\right)^2+\frac{y\left(x\right)}{x}$$

Trying Riccati ODE.
In canonical form, the ODE is written as

$$\begin{aligned}y' &= F(x,y)\\ &= \frac{x^6-2\,x^5y+x^4y^2+y}{x}\end{aligned}$$

This is a Riccati ODE. Comparing the ODE to solve

$$y'=x^5-2\,x^4y+x^3y^2+\frac{y}{x}$$

With Riccati ODE standard form

$$y'=f_0(x)+f_1(x)y+f_2(x)y^2$$

Shows that $f_0(x)=x^5$, $f_1(x)=\frac{-2\,x^5+1}{x}$ and $f_2(x)=x^3$.
Let

$$\begin{aligned}y &= \frac{-u'}{f_2u}\\ &= \frac{-u'}{ux^3}\end{aligned}\tag{1}$$

Using the above substitution in the given ODE results (after some simplification) in a second order ODE to solve for $u(x)$ which is

$$f_2u''(x)-\left(f_2'+f_1f_2\right)u'(x)+f_2^2f_0u(x)=0\tag{2}$$

But

$$
\begin{aligned}
f_2' &= 3\,x^2\\
f_1 f_2 &= \left(-2\,x^5+1\right)x^2\\
f_2^2 f_0 &= x^{11}
\end{aligned}
$$

Substituting the above terms back in (2) gives

$$x^3\frac{\mathrm{d}^2}{\mathrm{d}x^2}u\left(x\right)-\left(3\,x^2+\left(-2\,x^5+1\right)x^2\right)\frac{\mathrm{d}}{\mathrm{d}x}u\left(x\right)+x^{11}u\left(x\right)=0$$

Solving the above ODE gives

$$u\left(x\right)=\mathrm{e}^{-1/5\,x^5}\left(x^5C_2+C_1\right)$$

The above shows that

$$u'(x)=-x^4\mathrm{e}^{-1/5\,x^5}\left(x^5C_2+C_1-5\,C_2\right)$$

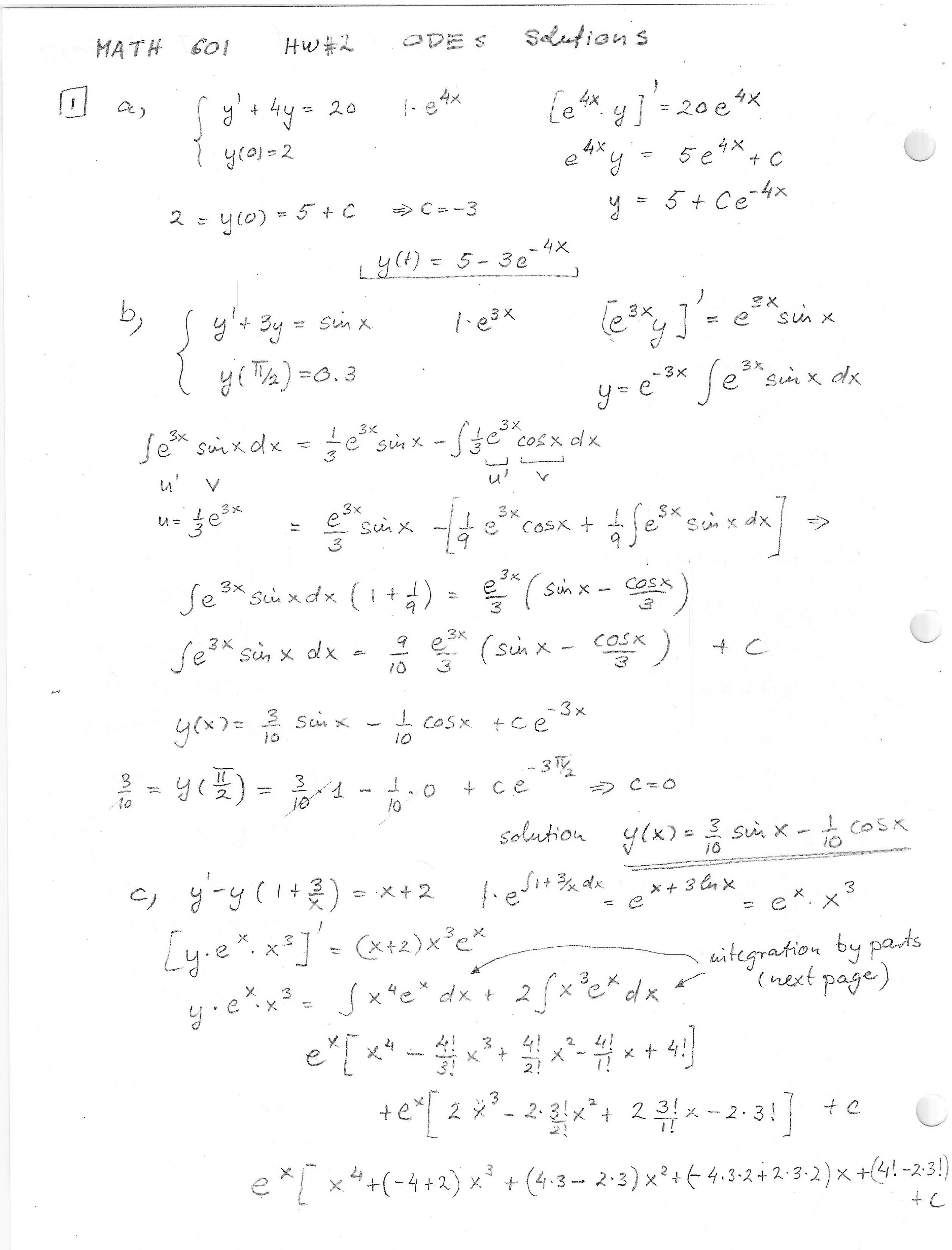
Hence, using the above in (1) gives the solution

$$y\left(x\right)=\frac{x\left(x^5C_2+C_1-5\,C_2\right)}{x^5C_2+C_1}$$

Dividing both numerator and denominator by C_2 gives, after renaming the constant $\frac{C_1}{C_2}=C_0$ the following

$$\boxed{y\left(x\right)=\frac{x\left(x^5+C_0-5\right)}{x^5+C_0}}$$

6 Key solution



MATH 601 HW#2 ODEs Solutions

$\boxed{1}$ a)

$$\begin{cases} y' + 4y = 20 \\ y(0) = 2 \end{cases} \qquad \Big| \cdot e^{4x} \qquad [e^{4x} \cdot y]' = 20 e^{4x}$$

$$e^{4x} y = 5 e^{4x} + C$$

$$y = 5 + C e^{-4x}$$

$$2 = y(0) = 5 + C \Rightarrow C = -3$$

$$\underline{y(t) = 5 - 3e^{-4x}}$$

b)

$$\begin{cases} y' + 3y = \sin x \\ y(\pi/2) = 0.3 \end{cases} \qquad \Big| \cdot e^{3x} \qquad [e^{3x} y]' = e^{3x} \sin x$$

$$y = e^{-3x} \int e^{3x} \sin x \, dx$$

$$\int \underset{u'}{e^{3x}} \underset{v}{\sin x} \, dx = \frac{1}{3} e^{3x} \sin x - \int \underbrace{\frac{1}{3} e^{3x}}_{u'} \underbrace{\cos x}_{v} \, dx$$

$$u = \frac{1}{3} e^{3x} \qquad = \frac{e^{3x}}{3} \sin x - \left[\frac{1}{9} e^{3x} \cos x + \frac{1}{9} \int e^{3x} \sin x \, dx\right] \Rightarrow$$

$$\int e^{3x} \sin x \, dx \left(1 + \frac{1}{9}\right) = \frac{e^{3x}}{3} \left(\sin x - \frac{\cos x}{3}\right)$$

$$\int e^{3x} \sin x \, dx = \frac{9}{10} \frac{e^{3x}}{3} \left(\sin x - \frac{\cos x}{3}\right) + C$$

$$y(x) = \frac{3}{10} \sin x - \frac{1}{10} \cos x + C e^{-3x}$$

$$\frac{3}{\cancel{10}} = y\left(\frac{\pi}{2}\right) = \frac{3}{\cancel{10}} \cdot 1 - \frac{1}{\cancel{10}} \cdot 0 + C e^{-3\pi/2} \Rightarrow C = 0$$

solution $\underline{\underline{y(x) = \frac{3}{10} \sin x - \frac{1}{10} \cos x}}$

c)

$$y' - y\left(1 + \frac{3}{x}\right) = x + 2 \qquad \Big| \cdot e^{\int 1 + 3/x \, dx} = e^{x + 3 \ln x} = e^{x} \cdot x^3$$

$$[y \cdot e^{x} \cdot x^3]' = (x+2) x^3 e^{x}$$

$$y \cdot e^{x} \cdot x^3 = \int x^4 e^{x} \, dx + 2 \int x^3 e^{x} \, dx$$

integration by parts (next page)

$$e^{x}\left[x^4 - \frac{4!}{3!} x^3 + \frac{4!}{2!} x^2 - \frac{4!}{1!} x + 4!\right]$$

$$+ e^{x}\left[2 x^3 - 2 \cdot \frac{3!}{2!} x^2 + 2 \frac{3!}{1!} x - 2 \cdot 3!\right] + C$$

$$e^{x}\left[x^4 + (-4+2) x^3 + (4 \cdot 3 - 2 \cdot 3) x^2 + (-4 \cdot 3 \cdot 2 + 2 \cdot 3 \cdot 2) x + (4! - 2 \cdot 3!)\right.$$

$$+ C$$

$$\int \underset{v}{x^4}\underset{u'}{e^x}dx = e^x x^4 - 4\int x^3 e^x dx = \underline{x^4e^x - 4x^3e^x + 4\cdot 3x^2e^x - 4\cdot 3\cdot 2\,xe^x + 4\cdot 3\cdot 2\cdot 1\,e^x + c}$$

$$\int x^3e^x dx = x^3e^x - 3\int x^2e^x dx = x^3e^x - 3x^2e^x + 3!\,xe^x - 3!\,e^x + c$$

$$\int x^2e^x dx = x^2e^x - 2\int xe^x dx = x^2e^x - 2xe^x + 2e^x + c$$

$$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x + c$$

$$y = \frac{1}{\cancel{e^x}\cdot x^3}\cdot\left[\cancel{e^x}\left[x^4 - 2x^3 + 6x^2 - 12x + 12\right] + c\right] =$$

$$= x - 2 + \frac{6}{x} - \frac{12}{x^2} + \frac{12}{x^3} + \frac{c}{e^x x^3}$$

$$y(1) = 1 - 2 + 6 - \cancel{12} + \cancel{12} + \frac{c}{e} = e - 1 \Rightarrow c = e(e-6)$$

$$\boxed{y = x - 2 + 6/x - \frac{12}{x^2} + \frac{12}{x^3} + \frac{(e^2-6e)}{e^x x^3}}$$

$\boxed{2}$

$$y' + y/3 = \frac{1-2x}{3}y^4 \qquad u = \frac{1}{y^{4-1}} = y^{-3}\ ;\ u' = -3y^{-4}y' =$$

$$y = \frac{1}{\sqrt[3]{u}} \qquad = -\frac{3}{y^4}\left(\frac{1-2x}{3}y^4 - y/3\right) =$$

$$2x - 1 + y^{-3} = 2x - 1 + u$$

$$u' - u = 2x - 1$$

$$(ue^{-x})' = (2x-1)e^{-x}$$

$$ue^{-x} = 2\int xe^{-x}dx - \int e^{-x}dx$$

$$\boxed{\begin{aligned}\int \underset{u}{x}\underset{v'}{e^{-x}}dx &= -xe^{-x} + \int e^{-x}dx\\ v = -e^{-x} \quad &= -e^{-x}(x+1)\end{aligned}}$$

$$= -2e^{-x}(x+1) + e^{-x} + c =$$

$$= -e^{-x}(2x+2-1) + c \Rightarrow u = -\cancel{e^x}\cancel{e^{-x}}(2x+1) + ce^x$$

$$\underline{u = -1 - 2x + ce^x} \qquad y = \frac{1}{\sqrt[3]{ce^x - 1 - 2x}}$$

$\boxed{3}$

$$\begin{cases} m\,v' = W - B - kv \\ v(0) = 0 \end{cases}$$

$$v' + \frac{k}{m}v = \frac{W}{m} - \frac{B}{m} \quad \Big|\cdot e^{k/m\,x}$$

$$\left(ve^{k/m\,x}\right)' = \left(\frac{W}{m} - \frac{B}{m}\right)e^{k/m\,x}$$

$$v = \cancel{e^{-\frac{k}{m}x}}\left(\frac{W-B}{m}\,\frac{m}{k}\right)\cancel{e^{k/m\,x}} + c\,e^{-k/m\,x}$$

$$0 = v(0) = \frac{W-B}{k} + C \Rightarrow C = \frac{B-W}{k}$$

$$v < V_0 = 12\ m/sec$$

$$\frac{W-B}{k}\left(1 - e^{-k/m\,x}\right) \leq 12$$

weight : $W = m \cdot g$

$m = g/W$

$\boxed{g = 9.89 \ m/sec^2}$

x – time in seconds

at $t \equiv x$

solve $\left(1 - e^{-\frac{kW}{g} t}\right) < \frac{12k}{W-B}$

$$e^{-\frac{kW}{g} t} > 1 - \frac{12k}{W-B}$$

$\left(\begin{matrix} k > 0 \\ W-B > 0 \end{matrix} \right)$

$$-\frac{kW}{g} t > \ln \frac{W-B-12k}{W-B}$$

$$t < -\frac{g}{kW} \ln \frac{W-B-12k}{W-B}$$

Thus $\boxed{t_{crit} = -\frac{g}{kW} \ln \frac{W-B-12k}{W-B}}$

$y(t)$ position (depth)

$$y = \int \frac{W-B}{k} \left(1 - e^{-k/m \, t}\right) dt =$$

$$= \frac{W-B}{k} t + \frac{W-B}{m} e^{-k/m \, t} + D$$

use $y(0) = 0$

start at surface

$$0 = \frac{W-B}{m} + D \quad \Rightarrow D = -\frac{(W-B)W}{g}$$

$$y = \frac{W-B}{k} \cdot t + \frac{(W-B)W}{g} e^{-\frac{kW}{g} t} - \frac{W-B}{g} W$$

critical depth

$$\boxed{y_{crit} = \frac{W-B}{k} t_{crit} + \frac{(W-B)W}{g} e^{-\frac{kW}{g} t_{crit}} - \frac{W-B}{g} W}$$

substitute data

$W = 2254 \ \frac{kg \, m}{sec^2}$

$g = 9.89 \ m/sec^2$

(in KMS) SI units.

$B = 2090 \ \frac{kg \, m}{sec^2}$

$K = 0.637 \ kg/sec$

$\boxed{4}$ $y' = x^3(y-x)^2 + y/x$

$w = y - x\,; \quad y' = w' + 1$

$\frac{w}{x} + 1 = y/x$

$w' \cancel{+1} = x^3 w^2 + \frac{w}{x} \cancel{+1}$

$w' - \frac{1}{x} w = x^3 w^2$

Bernoulli eq.

$u = \frac{1}{w^{2-1}} = \frac{1}{w} \qquad u' = -\frac{1}{w^2} w' =$

$= -\frac{1}{w^2}\left(\frac{1}{x} w + x^3 w^2\right) =$

$= -\frac{1}{x}\frac{1}{w} - x^3 = -\frac{1}{x} u - x^3$

$\swarrow$

$u' + \frac{1}{x} u = -x^3 \quad | \cdot (e^{\ln x} = x)$

$u'x + u = -x^4$

$(ux)' = -x^4$

$u = \frac{1}{x}\left(-\frac{x^5}{5} + C\right) = -\frac{x^4}{5} + \frac{C}{x}$

$W = \frac{1}{u} = \frac{5x}{5C - x^5} = \frac{5x}{D - x^5}$

$\underline{\underline{y(x) = w(x) + x = \frac{5x}{D - x^5} + x}}$

$\boxed{5}$

$\underbrace{(Ax + By)}_{M(x,y)} dx + \underset{N(x,y)}{(Cx + Dy)} dy = 0$ $\qquad M$

Equation exact when $M_y = N_x$ or $B = C$

Take $B = C$, $M(x,y) = \frac{\partial u(x,y)}{\partial x}$ $\quad N(x,y) = \frac{\partial u(x,y)}{\partial y}$

with $u(x,y) = r$ const implicit solution.

$u(x,y) = \int M(x,y)\,dx + k(y)$

$u_y = \left(\frac{Ax^2}{2} + Bxy + k(y)\right)_y = \cancel{B}x + k'(y) = N(x,y) = Cx + Dy$

$= \cancel{Bx} + Dy$

$\Rightarrow k'(y) = Dy$

Thus $u(x,y) = \frac{Ax^2}{2} + Bxy + \frac{Dy^2}{2} + q \qquad k(y) = \frac{Dy^2}{2} + q \leftarrow$ const.

implicit solution $\quad \frac{Ax^2}{2} + Bxy + \frac{Dy^2}{2} = \tilde{C} = r - q$ const.

You can use quadratic formula to obtain explicit expressions for y.