HW: MATH 601 | Sections 15.2 – 15.5 Eds 9, 10 | Sept 24, 201 Tuesday

[1] Consider the power series below. Give the center and find the radius of convergence for each.

a) $\sum_{n=1}^{\infty} n(z+i\sqrt{2})^n$ b) $\sum_{n=0}^{\infty} \left(\frac{a}{b}\right)^n (z-\pi i)^n$ c) $\sum_{n=0}^{\infty} \frac{(3n)!}{2^n (n!)^3} z^n$

d) $\sum_{n=0}^{\infty} \frac{1}{(1+i)^n} (z+2-i)^n$

[2] Find the radius of convergence using both

① The formula $R = \dfrac{1}{\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|}$ (Hadamard)

② The termwise differentiation / integration properties of power series

for the series: a) $\sum_{n=1}^{\infty} \frac{6^n}{n} (z-i)^n$; b) $\sum_{n=0}^{\infty} \frac{3^n (n+1)n}{5^n} z^{2n}$

[3] Show that $\dfrac{1}{(1-z)^2} = \sum_{n=0}^{\infty} (n+1) z^n$

a) using the Cauchy product
b) differentiating a suitable series

[4] If $f(z)$ is an even function (i.e. $f(-z) = f(z)$) where $f(z) = \sum_{n=0}^{\infty} a_n z^n$, show that $a_n = 0$ when n is odd.

If $f(z)$ is odd function (i.e $f(-z) = -f(z)$) show that $a_n = 0$ for n even.

[5] Develop the functions below in a Maclaurin series and determine the radius of convergence R for each

a) $\cos(2z^2)$ b) $\dfrac{z+2}{1-z^2}$

$\boxed{6}$ Develop

a) $f(z) = \frac{1}{z}$ in a Taylor series with center $z_0 = i$

b) $g(z) = e^z$ — " — $z_0 = a$.

What is the radius of convergence for each?

$\boxed{7}$ Show that $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} z^n$ converges uniformly in $|z| \leq 3$

$\boxed{8}$ Where does the series $\sum_{n=1}^{\infty} \left(\frac{n+2}{5n-3}\right)^n z^n$ converges uniformly?