Math 320 (Smith): Final Exam, Part I

Sunday May 7, 7:25-9:25 PM, Social Sciences 5206

YOUR NAME:

PLEASE WRITE YOUR NAME ON EVERY PAGE.

YOUR SECTION NUMBER:

Prob 1 /20	20
Prob 2 /20	15
Prob 3 /25	25
Prob 4 /20	20
Prob 5 /15	12
TOTAL /100	

1. Find the general solution:

$$y'' + 4y' + 4y = t^{-2} \exp(-2t), \quad t > 0.$$

2. Solve the initial value problem:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\,\mathbf{x}(t), \quad \mathbf{A} = \begin{bmatrix} 5 & -2 \\ 1/2 & 3 \end{bmatrix}, \quad \mathbf{x}(1) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

4. Given the solution $y_1(x) = \exp(x)$, use the method of Reduction of Order to find the solution to the following initial value problem, and state where the solution is defined.

$$xy'' - (1+x)y' + y = 0, \quad y(1) = 2, \ y'(1) = 5$$

Show your work! No work, no credit.

5. (a) Find the general solution:

$$2x^2y'' - 3xy' + 2y = 0$$

(b) Find the form of the general solution

$$y^{(v)} + 4y^{(iv)} + 4y''' + y'' + 4y' + 4y = x^2\exp(-2x) + \exp(-x)\sin(x)$$

given that the characteristic equation is

$$(r+2)^2(r^3+1) = 0.$$

You do not need to solve for the coefficients of the particular solution.

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15
25
20
12

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$-2e^4 c_2 = \frac{1}{2}$ ~~$c_2 = -e$~~

$-e^4 c_2 = \frac{1}{4}$ $c_2 = -\frac{e^{-4}}{4}$

2 ✓ **#3** $\dot{x} = Ax + g(t)$ $A = \begin{pmatrix} 2 & 1/2 \\ 2 & 2 \end{pmatrix}$, $g(t) = \begin{pmatrix} e^{-t} + 2 \\ 5 - e^{-t} \end{pmatrix}$

$$\underline{x} = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t} + c_2 \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{t} + \begin{pmatrix} -1/2 \\ -2 \end{pmatrix} + \begin{pmatrix} -7/16 \\ 5/8 \end{pmatrix} e^{-t}$$

✓ **#1** $y'' + 4y' + 4y = t^{-2} e^{-2t}$

$$y = c_1 e^{-2t} + c_2 t e^{-2t} - e^{-2t}(\ln t + 1)$$

½ **#2** $\dot{x} = Ax$, $A = \begin{pmatrix} 5 & -2 \\ 1/2 & 3 \end{pmatrix}$, $x(1) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$$y = \begin{pmatrix} 4 \\ 2 \end{pmatrix} e^{4t-4} - \frac{1}{4} \begin{pmatrix} 2 + 2t \\ t - 1 \end{pmatrix} e^{4t-4}$$

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$$x_1 = 4e^{4t-4} - \frac{1}{4}(2+2t)e^{4t-4}$$

$$= 4e^{4t-4} - \frac{1}{2}e^{4t-4} - \frac{1}{2}te^{4t-4}$$

$$= -te^{4t-4} + 4e^{4t-4}$$

$y(1)=2$ $\quad 2 = C_2 e^1 + 2C_2$ — ①

$y' = C_2 e^x + C_1$

$y'(1)=5 \Rightarrow 5 = C_2 e^1 + C_1$

$\underline{0} = A\underline{a} + \begin{pmatrix}1\\0\end{pmatrix}$

✓ #4 $8e^{x-1} - 3(1+x)$

$x_p = \underline{a} + \underline{b}e^{-t}$

$y'_p = -\underline{b}e^{-t}$

$\frac{5 \pm \sqrt{25-4(2)(2)}}{4} = \frac{5 \pm \sqrt{25-16}}{4} = \frac{5 \pm 3}{4} = 2, \frac{1}{2}$

$2 - \frac{3}{2} = \frac{1}{2}$

$c_1 x^2 + c_2 x^{\frac{1}{2}}$

$-\frac{1}{t} \quad (-t^{-1})$

$= - \quad (-t^{-2})$

#2 $y_h = 2e^{-4}\begin{pmatrix}2\\1\end{pmatrix}e^{4t} - \frac{e^{-4}}{4}\begin{pmatrix}2+2t\\t-1\end{pmatrix}e^{4t}$

$-1 - \frac{1}{2}(-2) =$

$0v_2 = 0$

$v_1 - 2v_2 = 2 \Rightarrow 1 = 2 + 2v_2$

$1 - 2 = 2v_2 \quad v_2 = -\frac{1}{2}$

~~A~~ $\begin{pmatrix}5 & -1\\ 3 & \end{pmatrix}$ 3

$\begin{pmatrix}2\\1\end{pmatrix}$ $(A\underline{b} - \underline{b})$

$\begin{pmatrix}1\\-\frac{1}{2}\end{pmatrix} = \begin{pmatrix}2\\-1\end{pmatrix}$ $(A - I)\underline{b}$

Con

$\underline{0} = A\underline{a} + \begin{pmatrix}1\\0\end{pmatrix}$

$\underline{e^{-t}} \quad -\underline{b} = A\underline{b} + \begin{pmatrix}0\\-1\end{pmatrix}$