$\boxed{3.6b}$ $y'' - 2xy' + 8y = 0$, $y(0) = 0$, $y'(0) = 4$

Let $y(x) = \sum_{n=0}^{\infty} a_n x^n$ where the initial conditions give $a_0 = 0$, $a_1 = 4$

Plug in the expansion to obtain

$$\sum_{n=2}^{\infty} a_n n(n-1)x^{n-2} - 2x \sum_{n=1}^{\infty} n a_n x^{n-1} + 8 \sum_{n=0}^{\infty} a_n x^n = 0$$

$\uparrow$

$m = n-2$

$$\sum_{m=0}^{\infty} a_{m+2}(m+2)(m+1)x^m + \sum_{n=0}^{\infty} (-2n+8) a_n x^n = 0$$

or equivalently

$$\sum_{n=0}^{\infty} (n+2)(n+1) a_{n+2} x^n + \sum_{n=0}^{\infty} (-2n+8) a_n x^n = 0$$

$$\Rightarrow \quad (n+2)(n+1)a_{n+2} - 2(n-4)a_n = 0 \qquad n \geq 0 \text{ integer}$$

Since $a_0 = 0 \Rightarrow$ all even a_n's are zero

For odd $n = 2m+1$ $m \geq 0$ integer

$$a_{n+2} = \frac{2(n-4)a_n}{(n+2)(n+1)} \Rightarrow$$

$$a_{2m+3} = \frac{2(2m+1-4)a_{2m+1}}{(2m+3)(2m+1)}$$

$$= \frac{2(2m-3)a_{2m+1}}{(2m+3)(2m+1)}$$

The series does not truncate because the numerator is not zero for any $m \geq 0$ integer

$$y(x) = 4x + \sum_{m=0}^{\infty} a_{2m+3}\, x^{2m+3}$$

a_{2m+3} given above.