
HW3 ME 739 Introduction to robotics

SPRING 2015
DEPARTMENT OF MECHANICAL ENGINEERING
UNIVERSITY OF WISCONSIN, MADISON

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MAY 3, 2022

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0.1 Problem 1

Problem 1 [20 points]

Following the steps outlined below, derive the equations of motion of the two degree-of-freedom manipulator depicted. The links have mass, m , and an inertia tensor, I , given as.

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}I_o & 0 & 0 \\ 0 & I_o & 0 \\ 0 & 0 & I_o \end{bmatrix}$$

The location of the center of mass is located in the middle of each link.

Note: you are strongly advised to use the Matlab symbolic toolbox (or equivalent) for your algebraic and differential operations.

- ▶ On each link we have attached frames at the joints and at the center of mass of each link. Calculate the homogeneous transforms that relate these frames to the inertial frame $\{0\}$. In other words, find, T_1^0 , T_2^0 , $T_{C_1}^0$, and $T_{C_2}^0$. Verify that your matrices are correct before proceeding to the next step (e.g. check the location and orientation of your frames for simple configurations such as $\theta_1 = \theta_2 = 90^\circ$)
- ▶ The evaluation of the mass matrix of the manipulator will require the computation of the linear Jacobian of the center of mass, $J_{v_{c_i}}$, for each link and the angular velocity Jacobian for each link, J_{ω_i} .
 - Find $J_{v_{c_1}}$ and $J_{v_{c_2}}$.
 - Find J_{ω_1} and J_{ω_2} .
 Verify that your matrices are correct before proceeding to the next step.
- ▶ Evaluate the mass matrix, $D(q)$, of the manipulator in terms of mass and geometric properties and its configuration.
- ▶ Evaluate the centrifugal and Coriolis inertial terms of the manipulator in terms of mass and geometric properties and its configuration.
 - Find $B(q)[\dot{q}\dot{q}]$
 - Find $C(q)[\dot{q}^2]$
- ▶ Evaluate the gravity vector, $G(q)$. In frame $\{0\}$ the gravity vector is given as: $\mathbf{g} = g[1 \ 0 \ 0]^T$.
- ▶ Form the complete equations of motion in the form

$$D(q)\ddot{q} + B(q)[\dot{q}\dot{q}] + C(q)[\dot{q}^2] + G(q) = \tau$$

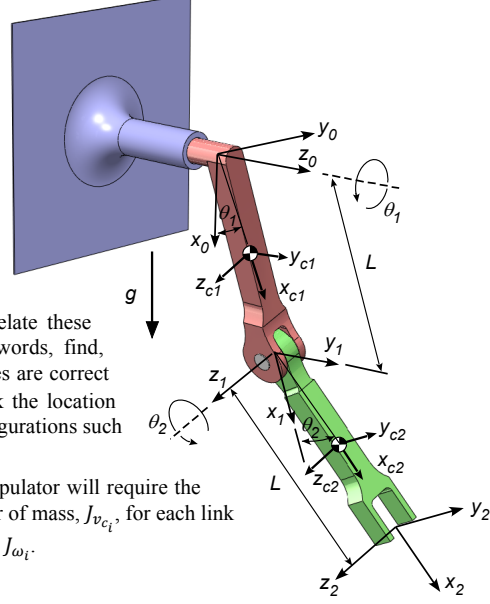


Figure 1: Problem 1 description

0.1.1 Part 1

The homogeneous transformation matrices T_i^{i-1} are found by direct inspection.

$$T_1^0 = \begin{pmatrix} [1.3] \cos \theta_1 & 0 & \sin \theta_1 & L \cos \theta_1 \\ \sin \theta_1 & 0 & -\cos \theta_1 & L \sin \theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$T_2^1 = \begin{pmatrix} [1.3] \cos \theta_2 & -\sin \theta_2 & 0 & L \cos \theta_2 \\ \sin \theta_2 & \cos \theta_2 & 0 & L \sin \theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Hence

$$\begin{aligned}
T_2^0 &= T_1^0 T_2^1 \\
&= \begin{pmatrix} [1.3] \cos \theta_1 & 0 & \sin \theta_1 & L \cos \theta_1 \\ \sin \theta_1 & 0 & -\cos \theta_1 & L \sin \theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} [1.3] \cos \theta_2 & -\sin \theta_2 & 0 & L \cos \theta_2 \\ \sin \theta_2 & \cos \theta_2 & 0 & L \sin \theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} [1.3] \cos \theta_1 \cos \theta_2 & -\cos \theta_1 \sin \theta_2 & \sin \theta_1 & L \cos \theta_1 + L \cos \theta_2 \cos \theta_1 \\ \cos \theta_2 \sin \theta_1 & -\sin \theta_1 \sin \theta_2 & -\cos \theta_1 & L \sin \theta_1 + L \cos \theta_2 \sin \theta_1 \\ \sin \theta_2 & \cos \theta_2 & 0 & L \sin \theta_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

$T_{c_i}^{i-1}$ are now found. These are the same as T_i^{i-1} but with the last column adjusted for the center of mass position which is given as being in the middle of the link, therefore

$$\begin{aligned}
T_{c_1}^0 &= \begin{pmatrix} [1.3] \cos \theta_1 & 0 & \sin \theta_1 & \frac{L}{2} \cos \theta_1 \\ \sin \theta_1 & 0 & -\cos \theta_1 & \frac{L}{2} \sin \theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
T_{c_2}^1 &= \begin{pmatrix} [1.3] \cos \theta_2 & -\sin \theta_2 & 0 & \frac{L}{2} \cos \theta_2 \\ \sin \theta_2 & \cos \theta_2 & 0 & \frac{L}{2} \sin \theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

Hence

$$\begin{aligned}
T_{c_2}^0 &= T_1^0 T_{c_2}^1 \\
&= \begin{pmatrix} [1.3] \cos \theta_1 & 0 & \sin \theta_1 & L \cos \theta_1 \\ \sin \theta_1 & 0 & -\cos \theta_1 & L \sin \theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} [1.3] \cos \theta_2 & -\sin \theta_2 & 0 & \frac{L}{2} \cos \theta_2 \\ \sin \theta_2 & \cos \theta_2 & 0 & \frac{L}{2} \sin \theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} [1.3] \cos \theta_1 \cos \theta_2 & -\cos \theta_1 \sin \theta_2 & \sin \theta_1 & L \cos \theta_1 + \frac{1}{2} L \cos \theta_1 \cos \theta_2 \\ \cos \theta_2 \sin \theta_1 & -\sin \theta_1 \sin \theta_2 & -\cos \theta_1 & L \sin \theta_1 + \frac{1}{2} L \cos \theta_2 \sin \theta_1 \\ \sin \theta_2 & \cos \theta_2 & 0 & \frac{1}{2} L \sin \theta_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

VERIFICATION:

The end effector position (origin of frame (2)), relative to the inertial frame is given by

$$\begin{aligned}
 r_2^0 &= T_2^0 r_2^2 \\
 &= \begin{pmatrix} [1.3] \cos \theta_1 \cos \theta_2 & -\cos \theta_1 \sin \theta_2 & \sin \theta_1 & L \cos \theta_1 + L \cos \theta_2 \cos \theta_1 \\ \cos \theta_2 \sin \theta_1 & -\sin \theta_1 \sin \theta_2 & -\cos \theta_1 & L \sin \theta_1 + L \cos \theta_2 \sin \theta_1 \\ \sin \theta_2 & \cos \theta_2 & 0 & L \sin \theta_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} [1.3]0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \\
 &= \begin{pmatrix} [1.3]L \cos \theta_1 + L \cos \theta_1 \cos \theta_2 \\ L \sin \theta_1 + L \cos \theta_2 \sin \theta_1 \\ L \sin \theta_2 \\ 1 \end{pmatrix} \tag{1}
 \end{aligned}$$

When $\theta_1 = \theta_2 = 0$ the above becomes

$$r_2^0 = \begin{pmatrix} [1.3]2L \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Which is what expected. Now it is verified for $\theta_1 = 90^\circ, \theta_2 = 0$, Substituting these angle values in (1) gives

$$\begin{pmatrix} [1.3]0 \\ 2L \\ 0 \\ 1 \end{pmatrix}$$

as expected. At $\theta_1 = 0, \theta_2 = 90^\circ$ and substituting these angle values in (1) gives

$$\begin{pmatrix} [1.3]L \\ 0 \\ L \\ 1 \end{pmatrix}$$

as expected. Finally, at $\theta_1 = \theta_2 = 90^\circ$ and substituting these angle values in (1) gives

$$\begin{pmatrix} [1.3]0 \\ L \\ L \\ 1 \end{pmatrix}$$

as expected. The result of part (1) have been verified.

0.1.2 Part 2

To determine the linear Jacobian for the center of the mass, the following quantities needs to be determined

$$z_0^0, z_1^0, o_0^0, o_1^0, o_{c_1}^0, o_{c_2}^0$$

These are obtained from the result of part (1)

$$\begin{aligned} z_0^0 &= \begin{pmatrix} [1.3]0 \\ 0 \\ 1 \end{pmatrix} \\ z_1^0 &= \begin{pmatrix} [1.3] \sin \theta_1 \\ -\cos \theta_1 \\ 0 \end{pmatrix} \\ o_0^0 &= \begin{pmatrix} [1.3]0 \\ 0 \\ 0 \end{pmatrix} \\ o_1^0 &= \begin{pmatrix} [1.3]L \cos \theta_1 \\ L \sin \theta_1 \\ 0 \end{pmatrix} \\ o_{c_1}^0 &= \begin{pmatrix} [1.3] \frac{L}{2} \cos \theta_1 \\ \frac{L}{2} \sin \theta_1 \\ 0 \end{pmatrix} \\ o_{c_2}^0 &= \begin{pmatrix} [1.3]L \cos \theta_1 + \frac{1}{2}L \cos \theta_1 \cos \theta_2 \\ L \sin \theta_1 + \frac{1}{2}L \cos \theta_2 \sin \theta_1 \\ \frac{1}{2}L \sin \theta_2 \end{pmatrix} \end{aligned}$$

Therefore

$$\begin{aligned} J_{v_{c_1}} &= ([1.3]z_0^0 \times (o_{c_1}^0 - o_0^0) \quad \mathbf{0}) \\ &= \left([1.3] \begin{pmatrix} [1.3]0 \\ 0 \\ 1 \end{pmatrix} \times \left(\begin{pmatrix} [1.3] \frac{L}{2} \cos \theta_1 \\ \frac{L}{2} \sin \theta_1 \\ 0 \end{pmatrix} - \begin{pmatrix} [1.3]0 \\ 0 \\ 0 \end{pmatrix} \right) \quad \mathbf{0} \right) \\ &= \boxed{\begin{pmatrix} [1.3] - \frac{1}{2}L \sin \theta_1 & 0 \\ \frac{1}{2}L \cos \theta_1 & 0 \\ 0 & 0 \end{pmatrix}} \end{aligned}$$

$$\begin{aligned}
J_{v_{c_2}} &= \left([1.3]z_0^0 \times o_{c_2}^0 \quad z_1^0 \times (o_{c_2}^0 - o_1^0) \right) \\
&= \left([1.3] \begin{pmatrix} [1.3]0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} [1.3]L \cos \theta_1 + \frac{1}{2}L \cos \theta_1 \cos \theta_2 \\ L \sin \theta_1 + \frac{1}{2}L \cos \theta_2 \sin \theta_1 \\ \frac{1}{2}L \sin \theta_2 \end{pmatrix} \begin{pmatrix} [1.3] \sin \theta_1 \\ -\cos \theta_1 \\ 0 \end{pmatrix} \times \left(\begin{pmatrix} [1.3]L \cos \theta_1 + \frac{1}{2}L \cos \theta_1 \cos \theta_2 \\ L \sin \theta_1 + \frac{1}{2}L \cos \theta_2 \sin \theta_1 \\ \frac{1}{2}L \sin \theta_2 \end{pmatrix} \right) \right) \\
&= \boxed{\begin{pmatrix} [1.3] - L \sin \theta_1 - \frac{1}{2}L \cos \theta_2 \sin \theta_1 & -\frac{1}{2}L \cos \theta_1 \sin \theta_2 \\ L \cos \theta_1 + \frac{1}{2}L \cos \theta_1 \cos \theta_2 & -\frac{1}{2}L \sin \theta_1 \sin \theta_2 \\ 0 & L \cos \theta_2 \end{pmatrix}}
\end{aligned}$$

Now the angular velocity Jacobians are found

$$J_{\omega_1} = \left([1.3]\bar{\varepsilon}_1 z_0^0 \quad 0 \right) = \boxed{\begin{pmatrix} [1.3]0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}}$$

$$J_{\omega_2} = \left([1.3]\bar{\varepsilon}_1 z_0^0 \quad \bar{\varepsilon}_2 z_1^0 \right) = \boxed{\begin{pmatrix} [1.3]0 & \sin \theta_1 \\ 0 & -\cos \theta_1 \\ 1 & 0 \end{pmatrix}}$$

VERIFICATION:

To verify the linear velocity Jacobians the following relations are used

$$J_{v_{c_1}} = \left([1.3] \frac{\partial r_{c_1}}{\partial \theta_1} \quad \frac{\partial r_{c_1}}{\partial \theta_2} \right) \quad (2)$$

$$J_{v_{c_2}} = \left([1.3] \frac{\partial r_{c_2}}{\partial \theta_1} \quad \frac{\partial r_{c_2}}{\partial \theta_2} \right) \quad (3)$$

Result obtained from the above is compared to part (2) result. In the above, r_{c_i} is the position of the center of mass of link i measured from the origin of the inertial frame. Therefore¹

$$\begin{aligned}
r_{c_1}^0 &= T_1^0 r_{c_1}^1 \\
&= \begin{pmatrix} [1.3] \cos \theta_1 & 0 & \sin \theta_1 & L \cos \theta_1 \\ \sin \theta_1 & 0 & -\cos \theta_1 & L \sin \theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} [1.3] - \frac{L}{2} \\ 0 \\ 0 \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} [1.3] \frac{1}{2} L \cos \theta_1 \\ \frac{1}{2} L \sin \theta_1 \\ 0 \\ 1 \end{pmatrix}
\end{aligned}$$

¹we can also use $r_{c_1}^0 = T_{c_1}^0 r_{c_1}^{c_1}$ where in this case $r_{c_1}^{c_1} = [0, 0, 0, 1]^T$

Therefore

$$r_{c_1}^0 = \begin{pmatrix} [1.3] \frac{1}{2} L \cos \theta_1 \\ \frac{1}{2} L \sin \theta_1 \\ 0 \end{pmatrix}$$

Using the above in (2) gives

$$\begin{aligned} J_{v_{c_1}} &= \left([1.3] \frac{\partial}{\partial \theta_1} \begin{pmatrix} [1.3] \frac{1}{2} L \cos \theta_1 \\ \frac{1}{2} L \sin \theta_1 \\ 0 \end{pmatrix} \quad \frac{\partial}{\partial \theta_2} \begin{pmatrix} [1.3] \frac{1}{2} L \cos \theta_1 \\ \frac{1}{2} L \sin \theta_1 \\ 0 \end{pmatrix} \right) \\ &= \boxed{\begin{pmatrix} [1.3] - \frac{1}{2} L \sin \theta_1 & 0 \\ \frac{1}{2} L \cos \theta_1 & 0 \\ 0 & 0 \end{pmatrix}} \end{aligned}$$

The above is the same result obtained in part (2). Similarly $J_{v_{c_2}}$ is verified

$$\begin{aligned} r_{c_2}^0 &= T_2^0 r_{c_2}^2 \\ &= \begin{pmatrix} [1.3] \cos \theta_1 \cos \theta_2 & -\cos \theta_1 \sin \theta_2 & \sin \theta_1 & L \cos \theta_1 + L \cos \theta_2 \cos \theta_1 \\ \cos \theta_2 \sin \theta_1 & -\sin \theta_1 \sin \theta_2 & -\cos \theta_1 & L \sin \theta_1 + L \cos \theta_2 \sin \theta_1 \\ \sin \theta_2 & \cos \theta_2 & 0 & L \sin \theta_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} [1.3] - \frac{L}{2} \\ 0 \\ 0 \\ 1 \end{pmatrix} \\ &= \begin{pmatrix} [1.3] L \cos \theta_1 + \frac{1}{2} L \cos \theta_1 \cos \theta_2 \\ L \sin \theta_1 + \frac{1}{2} L \cos \theta_2 \sin \theta_1 \\ \frac{1}{2} L \sin \theta_2 \\ 1 \end{pmatrix} \end{aligned}$$

Hence

$$r_{c_2}^0 = \begin{pmatrix} [1.3] L \cos \theta_1 + \frac{1}{2} L \cos \theta_1 \cos \theta_2 \\ L \sin \theta_1 + \frac{1}{2} L \cos \theta_2 \sin \theta_1 \\ \frac{1}{2} L \sin \theta_2 \end{pmatrix}$$

Using the above in (3) gives

$$\begin{aligned} J_{v_{c_2}} &= \left([1.3] \frac{\partial}{\partial \theta_1} \begin{pmatrix} [1.3] L \cos \theta_1 + \frac{1}{2} L \cos \theta_1 \cos \theta_2 \\ L \sin \theta_1 + \frac{1}{2} L \cos \theta_2 \sin \theta_1 \\ \frac{1}{2} L \sin \theta_2 \end{pmatrix} \quad \frac{\partial}{\partial \theta_2} \begin{pmatrix} [1.3] L \cos \theta_1 + \frac{1}{2} L \cos \theta_1 \cos \theta_2 \\ L \sin \theta_1 + \frac{1}{2} L \cos \theta_2 \sin \theta_1 \\ \frac{1}{2} L \sin \theta_2 \end{pmatrix} \right) \\ &= \boxed{\begin{pmatrix} [1.3] - L \sin \theta_1 - \frac{1}{2} L \sin \theta_1 \cos \theta_2 & -\frac{1}{2} L \cos \theta_1 \sin \theta_2 \\ L \cos \theta_1 + \frac{1}{2} L \cos \theta_2 \cos \theta_1 & -\frac{1}{2} L \sin \theta_2 \sin \theta_1 \\ 0 & \frac{1}{2} L \cos \theta_2 \end{pmatrix}} \end{aligned}$$

The above is the same as found in part(2).

0.2 Part 3

The mass matrix $D(q)$ is evaluated. By definition

$$D(q) = \sum_{i=1}^2 m_i J_{v_{c_i}}^T J_{v_{c_i}} + J_{\omega_i}^T R_i^0 I_{c_i}^i (R_i^0)^T J_{\omega_i} \quad (4)$$

The following is found in part (1)

$$R_1^0 = \begin{pmatrix} [1.3] \cos \theta_1 & 0 & \sin \theta_1 \\ \sin \theta_1 & 0 & -\cos \theta_1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$R_2^0 = \begin{pmatrix} [1.3] \cos \theta_1 \cos \theta_2 & -\cos \theta_1 \sin \theta_2 & \sin \theta_1 \\ \cos \theta_2 \sin \theta_1 & -\sin \theta_1 \sin \theta_2 & -\cos \theta_1 \\ \sin \theta_2 & \cos \theta_2 & 0 \end{pmatrix}$$

Therefore (4) becomes

$$\begin{aligned} D_1(q) &= m_1 \begin{pmatrix} [1.3] - \frac{1}{2}L \sin \theta_1 & 0 \\ \frac{1}{2}L \cos \theta_1 & 0 \\ 0 & 0 \end{pmatrix}^T \begin{pmatrix} [1.3] - \frac{1}{2}L \sin \theta_1 & 0 \\ \frac{1}{2}L \cos \theta_1 & 0 \\ 0 & 0 \end{pmatrix} \\ &\quad + \begin{pmatrix} [1.3]0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix}^T \begin{pmatrix} [1.3] \cos \theta_1 & 0 & \sin \theta_1 \\ \sin \theta_1 & 0 & -\cos \theta_1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} [1.3]\frac{1}{2}I_a & 0 & 0 \\ 0 & I_a & 0 \\ 0 & 0 & I_a \end{pmatrix} \\ &\quad \begin{pmatrix} [1.3] \cos \theta_1 & 0 & \sin \theta_1 \\ \sin \theta_1 & 0 & -\cos \theta_1 \\ 0 & 1 & 0 \end{pmatrix}^T \begin{pmatrix} [1.3]0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix} \\ &= m_1 \begin{pmatrix} [1.3]\frac{1}{4}L^2 + I_a & 0 \\ 0 & 0 \end{pmatrix} \\ \\ D_2(q) &= m_2 \begin{pmatrix} [1.3] - L \sin \theta_1 - \frac{1}{2}L \sin \theta_1 \cos \theta_2 & -\frac{1}{2}L \cos \theta_1 \sin \theta_2 \\ L \cos \theta_1 + \frac{1}{2}L \cos \theta_2 \cos \theta_1 & -\frac{1}{2}L \sin \theta_2 \sin \theta_1 \\ 0 & \frac{1}{2}L \cos \theta_2 \end{pmatrix}^T \begin{pmatrix} [1.3] - L \sin \theta_1 - \frac{1}{2}L \sin \theta_1 \cos \theta_2 & -\frac{1}{2}L \cos \theta_1 \sin \theta_2 \\ L \cos \theta_1 + \frac{1}{2}L \cos \theta_2 \cos \theta_1 & -\frac{1}{2}L \sin \theta_2 \sin \theta_1 \\ 0 & \frac{1}{2}L \cos \theta_2 \end{pmatrix} \\ &\quad + \begin{pmatrix} [1.3]0 & \sin \theta_1 \\ 0 & -\cos \theta_1 \\ 1 & 0 \end{pmatrix}^T \begin{pmatrix} [1.3] \cos \theta_1 \cos \theta_2 & -\cos \theta_1 \sin \theta_2 & \sin \theta_1 \\ \cos \theta_2 \sin \theta_1 & -\sin \theta_1 \sin \theta_2 & -\cos \theta_1 \\ \sin \theta_2 & \cos \theta_2 & 0 \end{pmatrix} \begin{pmatrix} [1.3]\frac{1}{2}I_a & 0 & 0 \\ 0 & I_a & 0 \\ 0 & 0 & I_a \end{pmatrix} \\ &\quad \begin{pmatrix} [1.3] \cos \theta_1 \cos \theta_2 & -\cos \theta_1 \sin \theta_2 & \sin \theta_1 \\ \cos \theta_2 \sin \theta_1 & -\sin \theta_1 \sin \theta_2 & -\cos \theta_1 \\ \sin \theta_2 & \cos \theta_2 & 0 \end{pmatrix}^T \begin{pmatrix} [1.3]0 & \sin \theta_1 \\ 0 & -\cos \theta_1 \\ 1 & 0 \end{pmatrix} \\ &= m_2 \begin{pmatrix} [1.3]\frac{1}{8} (6I_a + 9L^2 + 8L^2 \cos \theta_2 + (2I_a + L^2) \cos(2\theta_2)) & 0 \\ 0 & I_a + \frac{L^2}{4} \end{pmatrix} \end{aligned}$$

Hence the $D(q)$ becomes

$$\begin{aligned}
 D(q) &= \begin{pmatrix} [1.3]m_1 \left(\frac{1}{4}L^2 + I_a\right) & 0 \\ 0 & 0 \end{pmatrix} + m_2 \begin{pmatrix} [1.3]\frac{1}{8} (6I_a + 9L^2 + 8L^2 \cos \theta_2 + (2I_a + L^2) \cos (2\theta_2)) & 0 \\ 0 & I_a + \frac{L^2}{4} \end{pmatrix} \\
 &= \boxed{\begin{pmatrix} \left([1.3]m_1 \left(I_a + \frac{1}{4}L^2\right) + \frac{1}{8}m_2 (6I_a + 9L^2 + 8L^2 \cos \theta_2 + (2I_a + L^2) \cos (2\theta_2))\right) & 0 \\ 0 & m_2 \left(I_a + \frac{L^2}{4}\right) \end{pmatrix}}
 \end{aligned}$$

0.2.1 Part 4

The Coriolis term $B(q) [\dot{q}\dot{q}]$ is now evaluated

$$B(q) [\dot{q}\dot{q}] = \begin{bmatrix} 2b_{1,12} \\ 2b_{2,12} \end{bmatrix} [\dot{\theta}_1 \dot{\theta}_2] \quad (1)$$

Where $b_{i,jk}$ is the Christoffel symbol of first kind defined as

$$b_{i,jk} = \frac{1}{2} (d_{ijk} + d_{ikj} - d_{jki})$$

Where $d_{ijk} = \frac{\partial d_{ij}}{\partial q_k}$. Using these in (1) gives

$$\begin{aligned}
 B(q) [\dot{q}\dot{q}] &= \begin{bmatrix} d_{112} + d_{121} - d_{121} \\ d_{212} + d_{221} - d_{122} \end{bmatrix} [\dot{\theta}_1 \dot{\theta}_2] \\
 &= \begin{bmatrix} d_{112} \\ d_{212} + d_{221} - d_{122} \end{bmatrix} [\dot{\theta}_1 \dot{\theta}_2] \\
 &= \begin{bmatrix} \frac{\partial d_{11}}{\partial q_2} \\ \frac{\partial d_{21}}{\partial q_2} + \frac{\partial d_{22}}{\partial q_1} - \frac{\partial d_{12}}{\partial q_2} \end{bmatrix} [\dot{\theta}_1 \dot{\theta}_2]
 \end{aligned}$$

The mass matrix was found in part (3) as

$$D(q) = \begin{bmatrix} m_1 \left(I_a + \frac{1}{4}L^2\right) + \frac{1}{8}m_2 (6I_a + 9L^2 + 8L^2 \cos \theta_2 + (2I_a + L^2) \cos (2\theta_2)) & 0 \\ 0 & m_2 \left(I_a + \frac{L^2}{4}\right) \end{bmatrix}$$

Hence

$$\begin{aligned}
 \frac{\partial d_{11}}{\partial q_2} &= \frac{\partial d_{11}}{\partial \theta_2} \\
 &= \frac{\partial}{\partial \theta_2} \left[m_1 \left(I_a + \frac{1}{4}L^2\right) + \frac{1}{8}m_2 (6I_a + 9L^2 + 8L^2 \cos \theta_2 + (2I_a + L^2) \cos (2\theta_2)) \right] \\
 &= \frac{1}{8}m_2 \left(-8L^2 \sin \theta_2 - 2(2I_a + L^2) \sin (2\theta_2)\right) \\
 &= -m_2 L^2 \sin \theta_2 - \frac{1}{4}m_2 (2I_a + L^2) \sin (2\theta_2)
 \end{aligned}$$

And

$$\begin{aligned}\frac{\partial d_{21}}{\partial q_2} &= \frac{\partial d_{21}}{\partial \theta_2} \\ &= 0\end{aligned}$$

And

$$\begin{aligned}\frac{\partial d_{22}}{\partial q_1} &= \frac{\partial d_{22}}{\partial \theta_1} \\ &= 0\end{aligned}$$

And

$$\begin{aligned}\frac{\partial d_{12}}{\partial q_2} &= \frac{\partial d_{12}}{\partial \theta_2} \\ &= 0\end{aligned}$$

Hence

$$\begin{aligned}B(q) [\dot{q}\dot{q}] &= \begin{bmatrix} -m_2 L^2 \sin \theta_2 - \frac{1}{4} m_2 (2I_a + L^2) \sin (2\theta_2) \\ 0 \end{bmatrix} [\dot{\theta}_1 \dot{\theta}_2] \\ &= \boxed{\begin{bmatrix} -m_2 \dot{\theta}_1 \dot{\theta}_2 L^2 \sin \theta_2 - \frac{1}{4} \dot{\theta}_1 \dot{\theta}_2 m_2 (2I_a + L^2) \sin (2\theta_2) \\ 0 \end{bmatrix}}\end{aligned}$$

The centrifugal term is now evaluated

$$C(q) [\dot{q}^2] = \begin{bmatrix} b_{1,11} & b_{1,22} \\ b_{2,11} & b_{2,22} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{bmatrix}$$

Where

$$\begin{aligned}b_{1,11} &= \frac{1}{2} (d_{111} + d_{111} - d_{111}) \\ &= \frac{1}{2} d_{111} \\ &= \frac{1}{2} \frac{\partial d_{11}}{\partial q_1} \\ &= 0\end{aligned}$$

And

$$\begin{aligned}b_{1,22} &= \frac{1}{2} (d_{122} + d_{122} - d_{221}) \\ &= d_{122} - \frac{1}{2} d_{221} \\ &= \frac{\partial d_{12}}{\partial \theta_2} - \frac{1}{2} \frac{\partial d_{22}}{\partial \theta_1} \\ &= 0\end{aligned}$$

And

$$\begin{aligned}
b_{2,11} &= \frac{1}{2} (d_{211} + d_{211} - d_{112}) \\
&= d_{211} - \frac{1}{2} d_{112} \\
&= \frac{\partial d_{21}}{\partial \theta_1} - \frac{1}{2} \frac{\partial d_{11}}{\partial \theta_2} \\
&= 0 - \frac{1}{2} \left[-m_2 L^2 \sin \theta_2 - \frac{1}{4} m_2 (2I_a + L^2) \sin (2\theta_2) \right] \\
&= \frac{1}{2} m_2 L^2 \sin \theta_2 + \frac{1}{8} m_2 (2I_a + L^2) \sin (2\theta_2)
\end{aligned}$$

And

$$\begin{aligned}
b_{2,22} &= \frac{1}{2} (d_{222} + d_{222} - d_{222}) \\
&= \frac{1}{2} d_{222} \\
&= \frac{1}{2} \frac{\partial d_{21}}{\partial \theta_2} \\
&= 0
\end{aligned}$$

Therefore

$$\begin{aligned}
C(q) [\dot{q}^2] &= \begin{bmatrix} 0 & 0 \\ \frac{1}{2} m_2 L^2 \sin \theta_2 + \frac{1}{8} m_2 (2I_a + L^2) \sin (2\theta_2) & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{bmatrix} \\
&= \boxed{\begin{bmatrix} 0 \\ \frac{1}{2} m_2 \dot{\theta}_1^2 L^2 \sin \theta_2 + \frac{1}{8} m_2 \dot{\theta}_1^2 (2I_a + L^2) \sin (2\theta_2) \end{bmatrix}}
\end{aligned}$$

0.2.2 Part (5)

The gravity vector $G(q)$ is now evaluated

$$G(q) = - \left[J_{v_{c_1}}^T m_1 \mathbf{g} + J_{v_{c_2}}^T m_2 \mathbf{g} \right] \quad (1)$$

From part(2) we found

$$\begin{aligned}
J_{v_{c_1}} &= \begin{bmatrix} -\frac{1}{2} L \sin \theta_1 & 0 \\ \frac{1}{2} L \cos \theta_1 & 0 \\ 0 & 0 \end{bmatrix} \\
J_{v_{c_2}} &= \begin{bmatrix} -L \sin \theta_1 - \frac{1}{2} L \sin \theta_1 \cos \theta_2 & -\frac{1}{2} L \cos \theta_1 \sin \theta_2 \\ L \cos \theta_1 + \frac{1}{2} L \cos \theta_2 \cos \theta_1 & -\frac{1}{2} L \sin \theta_2 \sin \theta_1 \\ 0 & \frac{1}{2} L \cos \theta_2 \end{bmatrix}
\end{aligned}$$

Hence (1) becomes

$$\begin{aligned}
G(q) &= -m_1g \begin{bmatrix} -\frac{1}{2}L \sin \theta_1 & 0 \\ \frac{1}{2}L \cos \theta_1 & 0 \\ 0 & 0 \end{bmatrix}^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - m_2g \begin{bmatrix} -L \sin \theta_1 - \frac{1}{2}L \sin \theta_1 \cos \theta_2 & -\frac{1}{2}L \cos \theta_1 \sin \theta_2 \\ L \cos \theta_1 + \frac{1}{2}L \cos \theta_2 \cos \theta_1 & -\frac{1}{2}L \sin \theta_2 \sin \theta_1 \\ 0 & \frac{1}{2}L \cos \theta_2 \end{bmatrix}^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{2}gm_1L \sin \theta_1 \\ 0 \end{bmatrix} - \begin{bmatrix} -Lgm_2 \sin \theta_1 - \frac{1}{2}Lgm_2 \cos \theta_2 \sin \theta_1 \\ -\frac{1}{2}gm_2L \cos \theta_1 \sin \theta_2 \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{2}Lgm_1 \sin \theta_1 + Lgm_2 \sin \theta_1 + \frac{1}{2}Lgm_2 \cos \theta_2 \sin \theta_1 \\ \frac{1}{2}Lgm_2 \cos \theta_1 \sin \theta_2 \end{bmatrix}
\end{aligned}$$

0.2.3 Part (6)

The equation of motion is now found using above results. Using the notation $[\]$ for a matrix and $\{ \}$ for a vector, it is written as

$$\begin{array}{ccccc}
\text{mass matrix} & \text{Coriolis} & \text{centrifugal} & \text{gravity} & \text{torques} \\
\boxed{[D] \{ \ddot{q} \} + [B] \{ \dot{q} \dot{q} \} + [C] \{ \dot{q}^2 \} + [G] = \{ \tau \}}
\end{array}$$

Since there is no applied external torques or forces, the right hand side is zero. The equation of motion becomes

$$\begin{aligned}
&\begin{bmatrix} m_1 \left(I_a + \frac{1}{4}L^2 \right) + \frac{1}{8}m_2 (6I_a + 9L^2 + 8L^2 \cos \theta_2 + (2I_a + L^2) \cos (2\theta_2)) & 0 \\ 0 & m_2 \left(I_a + \frac{L^2}{4} \right) \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} \\
&+ \begin{bmatrix} -m_2L^2 \sin \theta_2 - \frac{1}{4}m_2 (2I_a + L^2) \sin (2\theta_2) \\ 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{1}{2}m_2L^2 \sin \theta_2 + \frac{1}{8}m_2 (2I_a + L^2) \sin (2\theta_2) & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1^2 \\ \dot{\theta}_2^2 \end{bmatrix} \\
&+ \begin{bmatrix} \frac{1}{2}Lgm_1 \sin \theta_1 + Lgm_2 \sin \theta_1 + \frac{1}{2}Lgm_2 \cos \theta_2 \sin \theta_1 \\ \frac{1}{2}Lgm_2 \cos \theta_1 \sin \theta_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_1 \end{bmatrix}
\end{aligned}$$

Looking at each equation of motion on its own, for θ_1 the equation of motion is

$$\begin{aligned}
&\ddot{\theta}_1 \left[m_1 \left(I_a + \frac{1}{4}L^2 \right) + \frac{1}{8}m_2 \left(6I_a + (\cos(2\theta_2)) \left(L^2 + 2I_a \right) + 8L^2 \cos \theta_2 + 9L^2 \right) \right] \\
&- \left(m_2L^2 \sin \theta_2 + \frac{1}{4}m_2 (2I_a + L^2) \sin (2\theta_2) \right) \dot{\theta}_1 \dot{\theta}_2 + \frac{1}{2}Lgm_1 \sin \theta_1 + Lgm_2 \sin \theta_1 + \frac{1}{2}Lgm_2 \cos \theta_2 \sin \theta_1 = 0
\end{aligned}$$

And the equation of motion for θ_2 is

$$\ddot{\theta}_2 m_2 \left(\frac{1}{4}L^2 + I_a \right) + \left(\frac{1}{2}m_2L^2 \sin \theta_2 + \frac{1}{8}m_2 (2I_a + L^2) \sin (2\theta_2) \right) \dot{\theta}_1^2 + \frac{1}{2}Lgm_2 \cos \theta_1 \sin \theta_2 = 0$$

These are coupled differential equations since θ_2, θ_1 appears in both equations. They are also nonlinear due to the $\dot{\theta}_2^2, \dot{\theta}_1^2$ terms.

0.3 Problem 2

Problem 2. [20 points]

Using your results from the previous question, write a Matlab numerical simulation of the manipulator described in Question 2. Implement a 4th order Runge-Kutta numerical integration of your equations of motion. The model and simulation parameters are given below.

Geometric parameters: $L = 2$ meters

Mass properties: $m = 10$ kg

$I_a = 5$ kg/m²

$g = 9.81$ m/s²

Simulation parameters: $\Delta t = 0.01$ seconds (integration time step)

$T_{final} = 20$ seconds (simulated time duration)

Note, to help you develop your code, some example code has been posted to the course Learn@UW page. Please use these scripts as a starting point. I would advise you to review all of the posted examples before you start working on your code.

- ▶ Simulate (*and animate*) the release of the manipulator from a vertical position (i.e. $q_1 = 180^\circ, q_2 = 0$) with small initial joint velocity ($\dot{q}_1 = \dot{q}_2 = 0.1$ r/s). You may want to add a small amount of damping to the joint torque vector such that the system starts to settle after a few major oscillations (suggested $c_i = 1$ N-m/r/s where $\tau_i = -c_i \dot{q}_i$).
- ▶ To check that your simulation (and equations of motion) are correct, calculate and plot the system energy for the case where the damping coefficients are set equal to zero. Specifically, please plot the total system kinetic energy, T , the total system potential energy, V , and the total system energy ($T + V$) as a function of time. If your simulation and underlying equations of motion are correct, the total energy should stay constant.

Figure 2: Problem 2 description

0.4 Part 1

The equations of motions for the 2 link serial manipulator were simulated by modifying the learn UW Matlab code using results of problem 1.

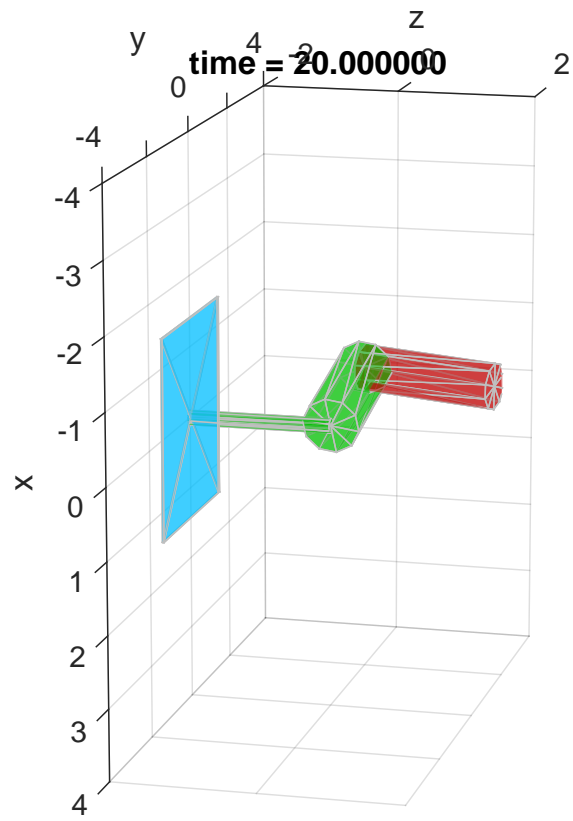


Figure 3: screen shot of simulation

A small amount of damping $c = 1$ N-m/r/s was added to both joints in the file `zDot2dof.m` and the simulation was run for 20 seconds.

When adding damping, the equation of motion becomes

$$\overbrace{[D]\{\ddot{q}\}}^{\text{mass matrix}} + \overbrace{c\{\dot{q}\}}^{\text{damping}} + \overbrace{[B]\{\dot{q}\dot{q}\}}^{\text{Coriolis}} + \overbrace{[C]\{q^2\}}^{\text{centrifugal}} + \overbrace{[G]}^{\text{gravity}} = \overbrace{\{\tau\}}^{\text{torques}}$$

Where c above is the damping constant used.

The diagram below shows that total energy decreased as would be expected.

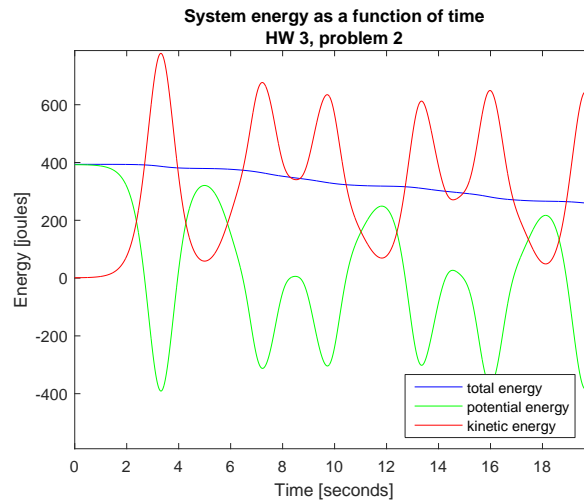


Figure 4: Damping $c = 1$ N-m/r/s for 20 seconds

The damping c was now increased to 5 N-m/r/s and the simulation time increased to 50 seconds. Now the total energy decreased to almost zero by the end of the simulation and the robot arm came close to being at rest. This was done as an additional verification to verify the equations of motion.

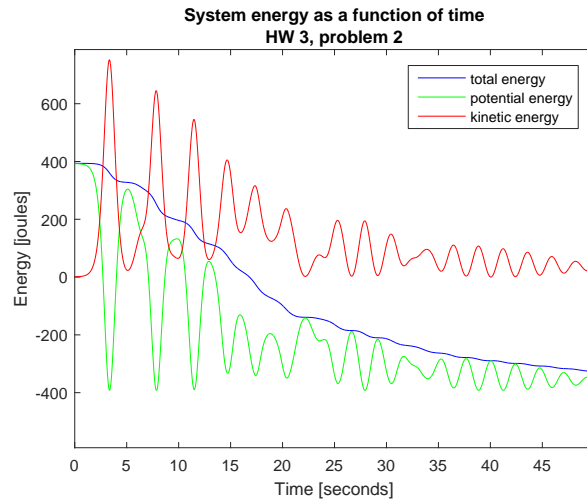


Figure 5: Damping $c = 5 \text{ N-m/r/s}$ for 50 seconds

0.4.1 Part 2

The damping constant was removed and the simulation run again for 20 seconds. The total energy remained constant as would be expected

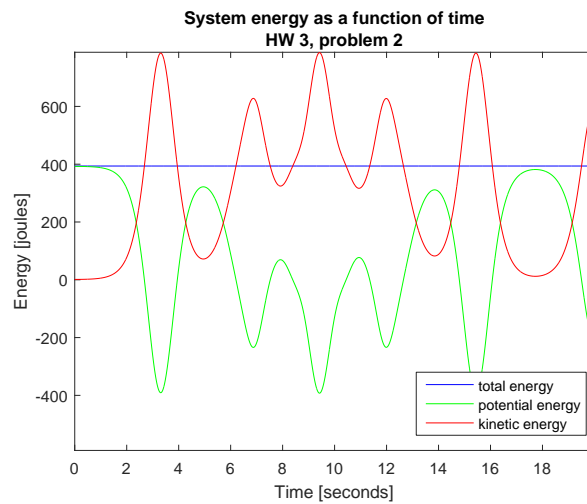


Figure 6: Damping $c = 0$ for 20 seconds

0.4.2 Animation

The following is a movie of the first 10 seconds of the simulation for the zero damping case.

0.4.3 source code

The following are the Matlab source code listings of all the files used used for the implementation of this problem.

```
%nma_HW3_problem.m
%This is modified version of the UW learn script used to solve
%problem 2, HW 3. ME 739, Univ. Wisconsin Madison
%Nasser M. Abbasi
%-----
%  NUMERICAL INTEGRATION OF DYNAMIC EQUATIONS
%-----
clear all; close all; clc;

DO_MOVIE = false; %make true to generate movie frames

% model parameters
modelParameters.g = 9.81; % gravitational constant [m/s^2]
modelParameters.m = 10; % link mass [kg]
modelParameters.L = 2; % link length [m]
modelParameters.Ia = 5; % inertia (about link's CL) [kg/m^2]

% initialize integration variables
dT = .01; %integration step size
tend = 20; %simulation run time
numPts = floor(tend/dT);
q = zeros(2,numPts); %pre-allocate array
dq = zeros(2,numPts);
t = zeros(1,numPts);
q(:,1) = [pi; 0]; %initial position
qd(:,1) = [0.1; 0.1]; %initial velocity
z = [q(:,1); qd(:,1)]; %initialize the state variables

% integrate equations of motion
for i = 1:numPts-1
    % Runge-Kutta 4th order
    k1 = zDot2dof(z,modelParameters);
    k2 = zDot2dof(z + 0.5*k1*dT,modelParameters);
    k3 = zDot2dof(z + 0.5*k2*dT,modelParameters);
    k4 = zDot2dof(z + k3*dT,modelParameters);
    z = z + (1/6)*(k1 + 2*k2 + 2*k3 + k4)*dT;

    % store joint position and velocity for post processing
    q(:,i+1) = z(1:2);
    qd(:,i+1) = z(3:4);
    t(1,i+1) = t(1,i) + dT;
end

%-----
%  RENDERING INITIALIZATION
```

```

%-----
%----set rendering window view parameters
L          = modelParameters.L;
L1         = L;
L2         = L;
f_handle   = 1;
axis_limits = L*[-2 2 -2 2 -1 1];
render_view = [-.4 -.8 .5]; view_up = [-1 0 0];
SetRenderingViewParameters(axis_limits,render_view,view_up,f_handle);
camproj perspective % turns on perspective

%----initialize rendering

% link 00 rendering initialization, this is the base on the wall
% fixed, used for illustration
r00 = L; sides00 = 4; axis00 = 3; norm_L00 = 1.0;
linkColor1 = [.6 0.75 0]; plotFrame00 = 0;
d00 = CreateLinkRendering(0.01*L,r00,sides00,axis00,norm_L00,...
    linkColor1,plotFrame00,f_handle);

% link 0 this is the rod holding the arm, fixed does not move
r0 = L/10; sides0 = 10; axis0 = 3; norm_L = 1.0;
linkColor0 = [0.75 1 1]; plotFrame0 = 0;
d0 = CreateLinkRendering(L,r0,sides0,axis0,norm_L,linkColor0,...
    plotFrame0,f_handle);

% link 1 rendering initialization
r1 = L1/5; sides1 = 10; axis1 = 1; norm_L1 = 1.0;
linkColor1 = [0 0.75 0]; plotFrame1 = 0;
d1 = CreateLinkRendering(L1,r1,sides1,axis1,norm_L1,linkColor1,...
    plotFrame1,f_handle);

% link 2 rendering initialization
r2 = L2/6; sides2 = 10; axis2 = 1; norm_L2 = 1.0;
linkColor2 = [0.75 0 0]; plotFrame2 = 0;
d2 = CreateLinkRendering(L2,r2,sides2,axis2,norm_L2,linkColor2,...
    plotFrame2,f_handle);

%-----
% DISPLAY INTERATION RESULTS
%-----
%since these links are fixed, they are set outside the loop
T00 = [1 0 0 0
       0 1 0 0
       0 0 1 -L
       0 0 0 1];

T0 = [1 0 0 0
      0 1 0 0
      0 0 1 0
      0 0 0 1];
UpdateLink(d00,T00);
UpdateLink(d0,T0);

```

```

k = 0;

for i = 1:numPts
    % Update frame {1}
    c = cos(q(1,i)); s = sin(q(1,i)); L = L1;
    T10 = [c 0 s L*c
           s 0 -c L*s
           0 1 0 0
           0 0 0 1];

    % Update frame {2}
    c = cos(q(2,i)); s = sin(q(2,i)); L = L2;
    T21 = [c -s 0 L*c
           s c 0 L*s
           0 0 1 0
           0 0 0 1];
    T20 = T10*T21;

    UpdateLink(d1,T10);
    UpdateLink(d2,T20);

    title(sprintf('time = %f',i*dT));
    if i == 1; %pause at start of simulation rendering
        pause;
    end

    if DO_MOVIE
        k = k+1;
        I = getframe(gcf);
        imwrite(I.cdata, sprintf('frame%d.png',k));
    end

    pause(dT);
end

%-----
% ENERGY BALANCE - TO CHECK SIMULATION RESULTS
%-----

V = zeros(numPts,1);
T = V;
E = V;

for i = 1:numPts
    % kinetic energy
    Q = q(:,i);
    Qd = qd(:,i);
    D = Dmatrix2dof(Q,modelParameters);
    T(i) = (1/2)*Qd'*D*Qd;

    % potential energy
    % evaluate position of links center of mass
    % Link 1:
    c = cos(q(1,i));

```

```

s = sin(q(1,i)); L = L1;
T10 = [c 0 s L*c
       s 0 -c L*s
       0 1 0 0
       0 0 0 1];
L = L1/2;
T10c = [c 0 s L*c
        s 0 -c L*s
        0 1 0 0
        0 0 0 1];

% Link 2:
c = cos(q(2,i));
s = sin(q(2,i)); L = L2;
T21 = [c -s 0 L*c
       s c 0 L*s
       0 0 1 0
       0 0 0 1];
L = L2/2;
T21c = [c -s 0 L*c
        s c 0 L*s
        0 0 1 0
        0 0 0 1];

T20 = T10*T21;
T20c = T10*T21c;

% assign center of mass position vectors
rc1 = T10c(1:3,4);
rc2 = T20c(1:3,4);
g = modelParameters.g*[1; 0; 0];
m = modelParameters.m;

% calculate the gravitational potential energy
V(i) = -(m*g'*rc1 + m*g'*rc2 );

% calculate the total system energy
E(i) = V(i) + T(i);
end

% plot the energy terms as a function of time
figure; plot(t,E,'b',t, V,'g',t,T,'r');
legend('total energy', 'potential energy', 'kinetic energy')
xlabel('Time [seconds]');ylabel('Energy [joules]');
title({'System energy as a function of time','HW 3, problem 2'});
axis([0 max(t) -1.5*max(E) 2*max(E)])

```

```

%zDot2dof.m
%problem 2, HW 3. ME 739, Univ. Wisconsin Madison
%called from RK-4 numerical integration method
%Nasser M. Abbasi
function [zDot] = zDot2dof(z,modelParameters);

```

```

% assign joint displacements / velocities from state variables
q = [z(1); z(2)];
qd = [z(3); z(4)];

% calculate D, B, D, and G matrices
D = Dmatrix2dof(q,modelParameters);
B = Bmatrix2dof(q,modelParameters);
C = Cmatrix2dof(q,modelParameters);
G = Gvector2dof(q,modelParameters);

%viscous friction in joints
tau_friction = 0; %5*qd; %5*qd;

%acceleration
qdqd = [qd(1)*qd(2)];
qd2 = [qd(1)*qd(1); qd(2)*qd(2)];
qdd = D\(-B*qdq - C*qd2 - G - tau_friction);

% assign state variable derivatives
zDot = [qd; qdd];

end

```

```

function G = Gvector2dof(q,modelParameters)

% assign model parameters to local variables
g = modelParameters.g;
m = modelParameters.m;
L = modelParameters.L;

G=zeros(2,1);

G(1,1) = 1/2*L*m*g*sin(q(1))+L*g*m*sin(q(1))+1/2*L*g*m*cos(q(2))*sin(q(1));
G(2,1) = 1/2*L*g*m*cos(q(1))*sin(q(2));

end

```

```

%Dmatrix2dof.m
%problem 2, HW 3. ME 739, Univ. Wisconsin Madison
%build the mass matrix D
%Nasser M. Abbasi
function D = Dmatrix2dof(q,modelParameters)

% assign model parameters to local variables

m = modelParameters.m;
Ia = modelParameters.Ia;
L = modelParameters.L;

D=zeros(2,2);

```

```

D(1,1) = m*(Ia+1/4*L^2)+1/8*m*(6*Ia+9*L^2+8*L^2*cos(q(2))...
        +(2*Ia+L^2)*cos(2*q(2)));
D(1,2) = 0;
D(2,1) = D(1,2);
D(2,2) = m*(Ia+L^2/4);

```

```
end
```

```

%Cmatrix2dof.m
%problem 2, HW 3. ME 739, Univ. Wisconsin Madison
%build the C matrix
%Nasser M. Abbasi
function C = Cmatrix2dof(q,modelParameters)

    % assign model parameters to local variables
    m = modelParameters.m;
    Ia = modelParameters.Ia;
    L = modelParameters.L;

    C=zeros(2,2);

    C(1,1) = 0;
    C(1,2) = 0;
    C(2,1) = 1/2*m*L^2*sin(q(2))+1/8*m*(2*Ia+L^2)*sin(2*q(2));
    C(2,2) = 0;

```

```
end
```

```

%Bmatrix2dof.m
%problem 2, HW 3. ME 739, Univ. Wisconsin Madison
%build the B matrix
%Nasser M. Abbasi
function B = Bmatrix2dof(q,modelParameters)

    % assign model parameters to local variables

    m = modelParameters.m;
    Ia = modelParameters.Ia;
    L = modelParameters.L;

    B=zeros(2,1);

    B(1,1) = -m*L^2*sin(q(2))-1/4*m*(2*Ia+L^2)*sin(2*q(2));
    B(2,1) = 0;

```

```
end
```