HW5 by Nasser M. Abbasi, EMA 550

Problem 1

A spacecraft is initially in a 300 km altitude circular orbit about the Earth in the ecliptic plane. It is to be sent on a Hohmann transfer to Saturn, also in the ecliptic plane. Assume that Saturn is in the correct position in its orbit for a flyby to occur when the spacecraft gets there.

```
> local `~`:= proc(f::uneval, `$`::identical(` $`), expr::uneval)
> local x, opr:= op(procname);
>      if opr <> `<` then  return :-`~`[opr](args)  end if;
>      x:= eval(expr);
>      print(op(1,
>           subs(
>                _F_= nprintf("%a", f), _X_= x,
>                proc(_F_:= expr=_X_) end proc
>           )
>      ));
>      assign(f,x)
> end proc:
```

part(a)

These below are from tables

```
> AU := 1.496*10^8;
  saturn_sun_distance := 9.537*1.496*10^8;
  sun_mu          := 1.327*10^11;
  earth_mu        := 3.986*10^5;
  earth_soi       := 9.24*10^5;
  satellite_earth_altitude := 300;
  earth_radius    := 6378;
```

$$AU := 1.496000000\ 10^{8}$$

$$saturn\_sun\_distance := 1.426735200\ 10^{9}$$

$$sun\_mu := 1.327000000\ 10^{11}$$

$$earth\_mu := 3.98600000\ 10^{5}$$

$$earth\_soi := 9.2400000\ 10^{5}$$

$$satellite\_earth\_altitude := 300$$

$$earth\_radius := 6378$$

Find burn out radius

```
> rb0_earth <~ satellite_earth_altitude+earth_radius;
```

$$rb0\_earth := satellite\_earth\_altitude + earth\_radius = 6678$$

find "a" for the Hohmann ellipse in sun centric space

```
> a <~ (AU+saturn_sun_distance)/2;
```

$$a := \frac{1}{2}\ AU + \frac{1}{2}\ saturn\_sun\_distance = 7.881676000\ 10^{8}$$

Find velocity of earth relative to the sun

```
> earth_speed <~ sqrt(sun_mu/AU);
```

$$earth\_speed := \sqrt{\frac{sun\_mu}{AU}} = 29.78308388$$

Find velocity of spacecraft relative to earth

```
> satellite_speed_relative_to_earth <~ sqrt(earth_mu/rb0_earth);
```

$$satellite\_speed\_relative\_to\_earth := \sqrt{\frac{earth\_mu}{rb0\_earth}} = 7.725835198$$

find what the velocity of spacecraft should be at the perigree of the Hohmann orbit in sun centeric space

```
> velocity_perigee <~ sqrt(sun_mu*(2/AU - 1/a));
```

$$velocity\_perigee := \sqrt{sun\_mu\left(\frac{2}{AU} - \frac{1}{a}\right)} = 40.07117375$$

Find excess speed V infinity out, to escape earth

```
> velocity_infinity_entering_saturn <~  velocity_perigee-
  earth_speed;
```

$$velocity\_infinity\_entering\_saturn := velocity\_perigee - earth\_speed = 10.28808987$$

set up the energy equation and solve for V_b0

```
> saturn_vb0 := 'saturn_vb0';
  saturn_vb0 <~ sqrt(2 * ((velocity_infinity_entering_saturn^2/2
  -earth_mu/earth_soi)+  earth_mu/rb0_earth ));
```

$$saturn\_vb0 := saturn\_vb0$$

$$saturn\_vb0 := \sqrt{velocity\_infinity\_entering\_saturn^2 - \frac{2\ earth\_mu}{earth\_soi} + \frac{2\ earth\_mu}{rb0\_earth}} = 14.97862082$$

```
> delta_v1 <~  saturn_vb0 - satellite_speed_relative_to_earth ;
```

$$delta\_v1 := saturn\_vb0 - satellite\_speed\_relative\_to\_earth = 7.252785622$$

part(b)

Calculate the angle past the Earth's dawn-dusk line where the ΔV should be applied.
Find escape hyperbolic trajectory eccentricity

```
> e <~ sqrt(1+ (velocity_infinity_entering_saturn^2*saturn_vb0^2*
  rb0_earth^2)/earth_mu^2 );
```

$$e := \sqrt{1 + \frac{velocity\_infinity\_entering\_saturn^2\ saturn\_vb0^2\ rb0\_earth^2}{earth\_mu^2}} = 2.768660225$$

find angle eta

```
> eta <~ arccos(- 1/e);
```

$$\eta := \arccos\left(-\frac{1}{e}\right) = 1.940335258$$

```
> theta <~ evalf(180 - eta*180/Pi);
```

$$\theta := evalf\left(180 - \frac{180\ \eta}{\pi}\right) = 68.8269789$$

Part (c)

For how long is the spacecraft on the heliocentric Hohmann transfer between Earth and Saturn? (Note: you do not need to calculate the time within either planet's sphere of influence, as that will be

small relative to the Hohmann transfer time, but you are welcome to do so and compare those values for yourself.)
The time is half the period of the elliptical orbit. Hence

```
> T <~ evalf(Pi*sqrt(a^3/sun_mu));
```

$$T := evalf\left(\pi \sqrt{\frac{a^3}{sun\_mu}}\right) = 1.908280789\ 10^8$$

```
> T <~ T/(60*60*24*365);
```

$$T := \frac{1}{31536000}\ T = 6.051118687$$

Part (d)

After crossing into the sphere of influence of Saturn, the spacecraft is to be placed in a circular orbit about Saturn with an orbital radius of 150,000 km. Calculate the ΔV2 required to place the spacecraft on this orbit. When spacecraft reaches saturn is has speed relative to sun of

```
> saturn_vb0 := 'saturn_vb0';
  rb0_saturn := 150000;
  v_apogee   <~ sqrt(sun_mu*(2/saturn_sun_distance-1/a));
  satellite_speed_relative_to_earthurn <~ sqrt(sun_mu*
  (1/saturn_sun_distance));
  velocity_infinity_entering_jupitor <~
  satellite_speed_relative_to_earthurn - v_apogee;
  saturn_mu  := 37931187;
  saturn_SOI := 3.47*10^7;
  eq := saturn_vb0^2/2 - saturn_mu/rb0_saturn =
  velocity_infinity_entering_jupitor^2/2 - saturn_mu/saturn_SOI;
  saturn_vb0 := op(select( is, [solve(eq,saturn_vb0)], positive))
  ;
  satellite_speed_relative_to_earth <~ sqrt(saturn_mu/rb0_saturn)
  ;
  del_v2     <~ evalf(satellite_speed_relative_to_earth -
  saturn_vb0);
  total_delV <~ abs(delta_v1) + abs(del_v2);
```

$$saturn\_vb0 := saturn\_vb0$$

$$rb0\_saturn := 150000$$

$$v\_apogee := \sqrt{sun\_mu\left(\frac{2}{saturn\_sun\_distance} - \frac{1}{a}\right)} = 4.201653949$$

$$satellite\_speed\_relative\_to\_earthurn := \sqrt{\frac{sun\_mu}{saturn\_sun\_distance}} = 9.644145932$$

$$velocity\_infinity\_entering\_jupitor := satellite\_speed\_relative\_to\_earthurn - v\_apogee = 5.442491983$$

$$saturn\_mu := 37931187$$

$$saturn\_SOI := 3.470000000\ 10^7$$

$$eq := \frac{1}{2}\ saturn\_vb0^2 - \frac{12643729}{50000} = 13.71724171$$

$$saturn\_vb0 := 23.09076966$$

$$satellite\_speed\_relative\_to\_earth := \sqrt{\frac{saturn\_mu}{rb0\_saturn}} = \frac{1}{500}\sqrt{63218645}$$

$$del\_v2 := evalf(satellite\_speed\_relative\_to\_earth - saturn\_vb0) = -7.18873897$$

$$total\_delV := |delta\_v1| + |del\_v2| = 14.44152459$$

Problem 2

A spacecraft on an interplanetary mission in the same plane as Jupiter's orbit about the Sun enters Jupiter's sphere of influence. The spacecraft has a speed of 10 km/s relative to the Sun at this point, which you can estimate as the Jupiter's average orbital radius about the Sun. (See the Planetary Constants sheet in your notes for values.) Assume that Jupiter is in a circular orbit about the Sun.

part(a)

The largest possible value for the impact parameter, b, that will still result in a hyperbolic orbit about Jupiter in the patched conic method is Jupiter's SOI radius. Find that value on the Planetary Constants sheet in the course notes and enter it here for reference.

```
> jupitor_SOI := 4.83*10^7;
  sun_mu      := 1.327*10^11;
  jupitor_mu  := 126686534;
  b_max       <~ jupitor_SOI;
```

$$jupitor\_SOI := 4.830000000\ 10^{7}$$

$$sun\_mu := 1.327000000\ 10^{11}$$

$$jupitor\_mu := 126686534$$

$$b\_max := jupitor\_SOI = 4.830000000\ 10^{7}$$

part(b)

For parts (b) through (g), assume that, relative to the Sun, the spacecraft is moving in the same direction as Jupiter when it enters Jupiter's SOI
What is the speed of the satellite relative to Jupiter when it enters Jupiter's SOI?

```
> satellite_speed_relative_to_sun :=10;
  jupitor_sun_distance := 5.203*1.495978*10^8;
  jupitor_speed <~ sqrt((sun_mu)/(jupitor_sun_distance));
  velocity_infinity_entering_jupitor <~ jupitor_speed -
  satellite_speed_relative_to_sun;
```

$$satellite\_speed\_relative\_to\_sun := 10$$

$$jupitor\_sun\_distance := 7.783573534\ 10^{8}$$

$$jupitor\_speed := \sqrt{\frac{sun\_mu}{jupitor\_sun\_distance}} = 13.05707640$$

$$velocity\_infinity\_entering\_jupitor := jupitor\_speed - satellite\_speed\_relative\_to\_sun = 3.05707640$$

part(c)

What is the smallest possible value for the impact parameter b? This value of impact parameter will result in a burnout radius that just grazes the surface of Jupiter

```
> jupitor_radius :=71492;
  jupitor_vb0_min <~ sqrt(jupitor_mu/jupitor_radius);
```

```
b_min <~ evalf(jupitor_radius*
jupitor_vb0_min/velocity_infinity_entering_jupitor);
```

$$jupitor\_radius := 71492$$

$$jupitor\_vb0\_min := \sqrt{\frac{jupitor\_mu}{jupitor\_radius}} = \frac{1}{35746}\sqrt{2264268422182}$$

$$b\_min := evalf\left(\frac{jupitor\_radius\, jupitor\_vb0\_min}{velocity\_infinity\_entering\_jupitor}\right) = 9.844363876\ 10^{5}$$

part(d)

Select as your impact parameter the value halfway between b_{min} and b_{max}. Note that value here for reference and use it as your impact parameter for the rest of the problem

```
> b <~ (b_max+b_min)/2;
```

$$b := \frac{1}{2}\, b\_max + \frac{1}{2}\, b\_min = 2.464221819\ 10^{7}$$

part(e)

Given the impact parameter from part (d), calculate the turning angle of the spacecraft relative to Jupiter during the flyby.

```
> saturn_vb0 := 'saturn_vb0': rb0_earth := 'rb0_earth':
  rb0_jupitor <~ b*
  velocity_infinity_entering_jupitor/jupitor_vb0;
  eq <~ (jupitor_vb0^2/2 - jupitor_mu/rb0_jupitor =
  velocity_infinity_entering_jupitor^2/2 -
  jupitor_mu/jupitor_SOI);
  sol <~ solve(eq,jupitor_vb0);
  jupitor_vb0 <~ op(select( is, [sol], positive));
```

$$rb0\_jupitor := \frac{b\, velocity\_infinity\_entering\_jupitor}{jupitor\_vb0} = \frac{7.533314367\ 10^{7}}{jupitor\_vb0}$$

$$eq := \left(\frac{1}{2}\, jupitor\_vb0^2 - \frac{jupitor\_mu}{rb0\_jupitor} = \frac{1}{2}\, velocity\_infinity\_entering\_jupitor^2 - \frac{jupitor\_mu}{jupitor\_SOI}\right) = \left(\frac{1}{2}\, jupitor\_vb0^2 - 1.681683889\, jupitor\_vb0 = 2.049948451\right)$$

$$sol := solve(eq, jupitor\_vb0) = (4.313785256, -0.9504174777)$$

$$jupitor\_vb0 := op(select(is, [sol], positive)) = 4.313785256$$

```
> rb0_jupitor;
```

$$1.746335045\ 10^{7}$$

```
> e <~ sqrt(1+(velocity_infinity_entering_jupitor^2*
  jupitor_vb0^2*rb0_jupitor^2)/jupitor_mu^2 );
  eta &= arccos(-1/e);
  evalf(eta*180/Pi);
  theta &= (2*eta-Pi);
  evalf(theta*180/Pi);
```

$$e := \sqrt{1 + \frac{velocity\_infinity\_entering\_jupitor^2\, jupitor\_vb0^2\, rb0\_jupitor^2}{jupitor\_mu^2}} = 2.074762092$$

$$(1.940335258)\ \&=\ (2.073712835)$$

$$111.1730211$$
$$(68.8269789)\ \&= (3.880670516 - \pi)$$
$$3943.495406$$

part(f)

What is the spacecraft's heliocentric speed following the flyby? (11.73 is correct)

```
> vd <~ sqrt(jupitor_speed^2+
  velocity_infinity_entering_jupitor^2-2*jupitor_speed*abs
  (velocity_infinity_entering_jupitor)*cos(theta));
```

$$vd := \left(jupitor\_speed^2 + velocity\_infinity\_entering\_jupitor^2 - 2\, jupitor\_speed\, |velocity\_infinity\_entering\_jupitor|\, \cos(\theta)\right)^{1/2} = 10.16313731$$

part(g)

What is the spacecraft's heliocentric flight path angle following the flyby

```
> gamma_d <~ arcsin(velocity_infinity_entering_jupitor*sin
  (theta)/vd);
  evalf(gamma_d*180/Pi);
```

$$gamma\_d := \arcsin\left(\frac{velocity\_infinity\_entering\_jupitor \sin(\theta)}{vd}\right) = -0.08555941389$$
$$-4.902193312$$

Hohmann from earth to moon (for project)

```
> satellite_earth_altitude := 300;
  earth_radius   := 6378;
  r__p           <~ satellite_earth_altitude+earth_radius;
  r__a           <~ 384400;
  a              <~ ((r__p+r__a)/2);
  earth_mu      := 3.986*10^5;
  satellite_speed_relative_to_earth  <~ sqrt(earth_mu/r__p);
  velocity_perigee   <~ sqrt(earth_mu*(2/r__p - 1/a));
  del__V1 <~ velocity_perigee -
  satellite_speed_relative_to_earth;
  e <~ evalf((r__a-r__p)/(r__a+r__p));
```

$$satellite\_earth\_altitude := 300$$
$$earth\_radius := 6378$$
$$r_p := satellite\_earth\_altitude + earth\_radius = 6678$$
$$r_a := 384400 = 384400$$
$$a := \frac{1}{2}\, r_a + \frac{1}{2}\, r_p = 195539$$
$$earth\_mu := 3.98600000\ 10^5$$
$$satellite\_speed\_relative\_to\_earth := \sqrt{\frac{earth\_mu}{r_p}} = 7.725835198$$

$$velocity\_perigee := \sqrt{earth\_mu\left(\frac{2}{r_p} - \frac{1}{a}\right)} = 10.83229389$$

$$del_{V1} := velocity\_perigee - satellite\_speed\_relative\_to\_earth = 3.106458692$$

$$e := evalf\left(\frac{r_a - r_p}{r_a + r_p}\right) = 0.9658482451$$

```
> velocity_apogee <~  sqrt(earth_mu*(2/r__a - 1/a));
```

$$velocity\_apogee := \sqrt{earth\_mu\left(\frac{2}{r_a} - \frac{1}{a}\right)} = 0.1881843356$$

```
> v2 <~ sqrt(earth_mu/r__a);
```

$$v2 := \sqrt{\frac{earth\_mu}{r_a}} = 1.018302846$$

```
> delV2 <~ v2-velocity_apogee;
```

$$delV2 := v2 - velocity\_apogee = 0.8301185104$$

```
> totalDelV <~ abs(del__V1)+abs(delV2);
```

$$totalDelV := |del_{V1}| + |delV2| = 3.936577202$$

```
> delT:=Pi* sqrt(a^3/earth_mu);
```

$$delT := 1.369561180\ 10^5\ \pi$$

```
> evalf(delT);
```

$$4.302603342\ 10^5$$

```
> evalf(delT/(60*60*24));
```

$$4.979864981$$