HW 5

EMA 550
Astrodynamics

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University of Wisconsin, Madison

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Contents

1 Problem 1

A spacecraft is initially in a 300 km altitude circular orbit about the Earth in the ecliptic plane. It is to be sent on a Hohmann transfer to Saturn, also in the ecliptic plane. Assume that Saturn is in the correct position in its orbit for a flyby to occur when the spacecraft gets there.

1.1 part(a)

Calculate the initial ΔV_1 required to start the trip to Saturn.

$$r_{b0} = r_E + alt$$

Where r_E is radius of earth and alt is spacecraft altitude. Hence

$$r_{b0} = 6378 + 300 = 6678 \text{ km}$$

The distance from earth to sun is $R_E = 1.496 \times 10^8$ km and the distance from saturn to sun is $R_s = 9.536 \times 1.496 \times 10^8 = 1.4266 \times 10^9$ km therefore $a = \frac{R_E + R_s}{2} = \frac{1.496\times 10^8 + 1.426\,6\times 10^9}{2} = 7.8815 \times 10^8$ km.

The earth speed around the sun is $V_e = \sqrt{\frac{\mu_s}{r_e}} = \sqrt{\frac{1.327\times 10^{11}}{1.496\times 10^8}} = 29.783$ km/sec. When the spacecraft escape the earth it has to be at speed

$$V_{perigee} = \sqrt{\mu_s \left(\frac{2}{R_E} - \frac{1}{a}\right)} = \sqrt{1.327 \times 10^{11} \left(\frac{2}{1.496 \times 10^8} - \frac{1}{7.8815 \times 10^8}\right)} = 40.07 \text{ km/sec}$$

Therefore, V_∞ is the escape speed found from

$$\begin{aligned} V_\infty &= V_{perigee} - V_e \\ &= 40.07 - 29.783 \\ &= 10.287 \text{ km/sec} \end{aligned}$$

Now the burn out speed is found

$$\frac{V_{bo}^2}{2} - \frac{\mu_E}{r_{b0}} = \frac{V_\infty^2}{2} - \frac{\mu_E}{r_{SOI}}$$

Where r_{SOI} is the earth sphere of influense given by 9.24×10^5 km. Solving for V_{bo}

$$\frac{V_{bo}^2}{2} - \frac{3.986 \times 10^5}{6678} = \frac{10.287^2}{2} - \frac{3.986 \times 10^5}{9.24 \times 10^5}$$
$$V_{bo} = 14.978 \text{ km/sec}$$

Hence

$$\begin{aligned}\Delta V_1 &= V_{bo} - \sqrt{\frac{\mu_E}{r_{bo}}} \\ &= 14.97 - \sqrt{\frac{3.986 \times 10^5}{6678}} \\ &= 7.2442\end{aligned}$$

1.2 part(b)

Calculate the angle past the Earth's dawn-dusk line where the ΔV should be applied.

$$\begin{aligned}e &= \sqrt{1 + \frac{V_\infty^2 V_{bo}^2 r_{bo}^2}{\mu_E^2}} \\ &= \sqrt{1 + \frac{\left(10.287^2\right)\left(14.978^2\right)\left(6678^2\right)}{\left(3.986 \times 10^5\right)^2}} \\ &= 2.7683\end{aligned}$$

Hence

$$\begin{aligned}\eta &= \arccos\left(\frac{-1}{e}\right) = \arccos\left(\frac{-1}{2.7683}\right) = 1.9404 \text{ radian} \\ &= 111.18^0\end{aligned}$$

Hence $\theta = 180 - 111.18 = 68.82^0$

1.3 part(c)

For how long is the spacecraft on the heliocentric Hohmann transfer between Earth and Saturn? (Note: you do not need to calculate the time within either planet's sphere of influence, as that will be small relative to the Hohmann transfer time, but you are welcome to do so and compare those values for yourself.)

The time is half the period of the elliptical orbit. Hence

$$T = \pi\sqrt{\frac{a^3}{u_s}} = \pi\sqrt{\frac{\left(7.8815\times 10^8\right)^3}{1.327\times 10^{11}}} = 1.9082\times 10^8 \text{ sec}$$
$$= \frac{1.908\,2\times 10^8}{60\times 60\times 24\times 365} = 6.051 \text{ year}$$

1.4 part(d)

After crossing into the sphere of influence of Saturn, the spacecraft is to be placed in a circular orbit about Saturn with an orbital radius of 150,000 km. Calculate the ΔV_2 required to place the spacecraft on this orbit.

Solution completed in the Mathematica solution. See above for links.

2 Problem 2

A spacecraft on an interplanetary mission in the same plane as Jupiter's orbit about the Sun enters Jupiter's sphere of influence. The spacecraft has a speed of 10 km/s relative to the Sun at this point, which you can estimate as the Jupiter's average orbital radius about the Sun. (See the Planetary Constants sheet in your notes for values.) Assume that Jupiter is in a circular orbit about the Sun.

2.1 part(a)

The largest possible value for the impact parameter, b, that will still result in a hyperbolic orbit about Jupiter in the patched conic method is Jupiter's SOI radius. Find that value on the Planetary Constants sheet in the course notes and enter it here for reference.

$b_{\max} = R_{SOI,Jupitor}$ = Answer km

For parts (b) through (g), assume that, relative to the Sun, the spacecraft is moving in the same direction as Jupiter when it enters Jupiter's SOI.

2.2 part(b)

What is the speed of the satellite relative to Jupiter when it enters Jupiter's SOI?

V_∞ = Answer km/s

2.3 part(c)

What is the smallest possible value for the impact parameter b? This value of impact parameter will result in a burnout radius that just grazes the surface of Jupiter, $r_{bo} = r_{Jupiter}$

$b_{min} = km$

2.4 part(d)

Select as your impact parameter the value halfway between b_{min} and b_{max}. Note that value here for reference and use it as your impact parameter for the rest of the problem.

b = Answer km

2.5 part(e)

Given the impact parameter from part (d), calculate the turning angle of the spacecraft relative to Jupiter during the flyby.

θ = Answer degrees

2.6 part(f)

What is the spacecraft's heliocentric speed following the flyby?

$V_D = km/s$

2.7 part(g)

What is the spacecraft's heliocentric flight path angle following the flyby?

$\gamma_D = deg$

For the remaining parts, assume that, relative to the Sun, the spacecraft DOES NOT arrive at Jupiter's SOI moving in the same direction at Jupiter. The spacecraft still has a heliocentric speed of 10 km/s at the distance of Jupiter's orbit from the Sun. But now it has a heliocentric eccentricity of 0.5. (What was the heliocentric eccentricity when the spacecraft arrived in the same direction as Jupiter, assuming that point was aphelion?)

2.8 part(h)

What is the spacecraft's heliocentric flight path angle when it arrives at Jupiter's SOI?

$\gamma_A = deg$

2.9 part(i)

What is the spacecraft's speed relative to Jupiter?

V_∞ = km/s

part(j)

Using the same impact parameter as in part (d), calculate the turning angle of the spacecraft relative to Jupiter.

$\theta = deg$

part(k)

Assuming that the spacecraft flies behind Jupiter, what is the spacecraft's heliocentric speed following the flyby?

V_D =km/s

2.10 part(L)

Assuming that the spacecraft flies behind Jupiter, what is the spacecraft's heliocentric flight path angle following the flyby?

$\gamma_D = deg$

3 Appendix

3.1 solution in Maple

HW5 by Nasser M. Abbasi, EMA 550

Problem 1

A spacecraft is initially in a 300 km altitude circular orbit about the Earth in the ecliptic plane. It is to be sent on a Hohmann transfer to Saturn, also in the ecliptic plane. Assume that Saturn is in the correct position in its orbit for a flyby to occur when the spacecraft gets there.

```
> local `~`:= proc(f::uneval, `$`::identical(` $`), expr::uneval)
> local x, opr:= op(procname);
>      if opr <> `<` then  return :-`~`[opr](args)  end if;
>      x:= eval(expr);
>      print(op(1,
>           subs(
>                _F_= nprintf("%a", f), _X_= x,
>                proc(_F_:= expr=_X_) end proc
>           )
>      ));
>      assign(f,x)
> end proc:
```

part(a)

These below are from tables

```
> AU  := 1.496*10^8;
  saturn_sun_distance := 9.537*1.496*10^8;
  sun_mu         := 1.327*10^11;
  earth_mu       := 3.986*10^5;
  earth_soi      := 9.24*10^5;
  satellite_earth_altitude := 300;
  earth_radius   := 6378;
```

$$AU := 1.496000000\ 10^{8}$$

$$saturn\_sun\_distance := 1.426735200\ 10^{9}$$

$$sun\_mu := 1.327000000\ 10^{11}$$

$$earth\_mu := 3.98600000\ 10^{5}$$

$$earth\_soi := 9.2400000\ 10^{5}$$

$$satellite\_earth\_altitude := 300$$

$$earth\_radius := 6378$$

Find burn out radius

```
> rb0_earth <~ satellite_earth_altitude+earth_radius;
```

$$rb0\_earth := satellite\_earth\_altitude + earth\_radius = 6678$$

find "a" for the Hohmann ellipse in sun centric space

```
> a <~ (AU+saturn_sun_distance)/2;
```

$$a := \frac{1}{2}\ AU + \frac{1}{2}\ saturn\_sun\_distance = 7.881676000\ 10^{8}$$

Find velocity of earth relative to the sun

```
> earth_speed <~ sqrt(sun_mu/AU);
```

$$earth\_speed := \sqrt{\frac{sun\_mu}{AU}} = 29.78308388$$

Find velocity of spacecraft relative to earth

```
> satellite_speed_relative_to_earth <~ sqrt(earth_mu/rb0_earth);
```

$$satellite\_speed\_relative\_to\_earth := \sqrt{\frac{earth\_mu}{rb0\_earth}} = 7.725835198$$

find what the velocity of spacecraft should be at the perigee of the Hohmann orbit in sun centeric space

```
> velocity_perigee <~ sqrt(sun_mu*(2/AU - 1/a));
```

$$velocity\_perigee := \sqrt{sun\_mu\left(\frac{2}{AU} - \frac{1}{a}\right)} = 40.07117375$$

Find excess speed V infinity out, to escape earth

```
> velocity_infinity_entering_saturn <~  velocity_perigee-
  earth_speed;
```

$$velocity\_infinity\_entering\_saturn := velocity\_perigee - earth\_speed = 10.28808987$$

set up the energy equation and solve for V_b0

```
> saturn_vb0 := 'saturn_vb0';
  saturn_vb0 <~ sqrt(2 * ((velocity_infinity_entering_saturn^2/2
  -earth_mu/earth_soi)+  earth_mu/rb0_earth ));
```

$$saturn\_vb0 := saturn\_vb0$$

$$saturn\_vb0 := \sqrt{velocity\_infinity\_entering\_saturn^2 - \frac{2\ earth\_mu}{earth\_soi} + \frac{2\ earth\_mu}{rb0\_earth}} = 14.97862082$$

```
> delta_v1 <~  saturn_vb0 - satellite_speed_relative_to_earth ;
```

$$delta\_v1 := saturn\_vb0 - satellite\_speed\_relative\_to\_earth = 7.252785622$$

part(b)

Calculate the angle past the Earth's dawn-dusk line where the ΔV should be applied.
Find escape hyperbolic trajectory eccentricity

```
> e <~ sqrt(1+ (velocity_infinity_entering_saturn^2*saturn_vb0^2*
  rb0_earth^2)/earth_mu^2 );
```

$$e := \sqrt{1 + \frac{velocity\_infinity\_entering\_saturn^2\ saturn\_vb0^2\ rb0\_earth^2}{earth\_mu^2}} = 2.768660225$$

find angle eta

```
> eta <~ arccos(- 1/e);
```

$$\eta := \arccos\left(-\frac{1}{e}\right) = 1.940335258$$

```
> theta <~ evalf(180 - eta*180/Pi);
```

$$\theta := evalf\left(180 - \frac{180\ \eta}{\pi}\right) = 68.8269789$$

Part (c)

For how long is the spacecraft on the heliocentric Hohmann transfer between Earth and Saturn? (Note: you do not need to calculate the time within either planet's sphere of influence, as that will be

small relative to the Hohmann transfer time, but you are welcome to do so and compare those values for yourself.)
The time is half the period of the elliptical orbit. Hence

```
> T <~ evalf(Pi*sqrt(a^3/sun_mu));
```

$$T := evalf\left(\pi \sqrt{\frac{a^3}{sun\_mu}}\right) = 1.908280789\ 10^8$$

```
> T <~ T/(60*60*24*365);
```

$$T := \frac{1}{31536000}\ T = 6.051118687$$

Part (d)

After crossing into the sphere of influence of Saturn, the spacecraft is to be placed in a circular orbit about Saturn with an orbital radius of 150,000 km. Calculate the ΔV2 required to place the spacecraft on this orbit. When spacecraft reaches saturn is has speed relative to sun of

```
> saturn_vb0 := 'saturn_vb0';
  rb0_saturn := 150000;
  v_apogee   <~ sqrt(sun_mu*(2/saturn_sun_distance-1/a));
  satellite_speed_relative_to_earthurn <~ sqrt(sun_mu*
  (1/saturn_sun_distance));
  velocity_infinity_entering_jupitor <~
  satellite_speed_relative_to_earthurn - v_apogee;
  saturn_mu  := 37931187;
  saturn_SOI := 3.47*10^7;
  eq := saturn_vb0^2/2 - saturn_mu/rb0_saturn =
  velocity_infinity_entering_jupitor^2/2 - saturn_mu/saturn_SOI;
  saturn_vb0 := op(select( is, [solve(eq,saturn_vb0)], positive))
  ;
  satellite_speed_relative_to_earth <~ sqrt(saturn_mu/rb0_saturn)
  ;
  del_v2     <~ evalf(satellite_speed_relative_to_earth -
  saturn_vb0);
  total_delV <~ abs(delta_v1) + abs(del_v2);
```

$$saturn\_vb0 := saturn\_vb0$$

$$rb0\_saturn := 150000$$

$$v\_apogee := \sqrt{sun\_mu\left(\frac{2}{saturn\_sun\_distance} - \frac{1}{a}\right)} = 4.201653949$$

$$satellite\_speed\_relative\_to\_earthurn := \sqrt{\frac{sun\_mu}{saturn\_sun\_distance}} = 9.644145932$$

$$velocity\_infinity\_entering\_jupitor := satellite\_speed\_relative\_to\_earthurn - v\_apogee = 5.442491983$$

$$saturn\_mu := 37931187$$

$$saturn\_SOI := 3.470000000\ 10^7$$

$$eq := \frac{1}{2}\ saturn\_vb0^2 - \frac{12643729}{50000} = 13.71724171$$

$$saturn\_vb0 := 23.09076966$$

$$satellite\_speed\_relative\_to\_earth := \sqrt{\frac{saturn\_mu}{rb0\_saturn}} = \frac{1}{500}\sqrt{63218645}$$

$$del\_v2 := evalf(satellite\_speed\_relative\_to\_earth - saturn\_vb0) = -7.18873897$$

$$total\_delV := |delta\_v1| + |del\_v2| = 14.44152459$$

Problem 2

A spacecraft on an interplanetary mission in the same plane as Jupiter's orbit about the Sun enters Jupiter's sphere of influence. The spacecraft has a speed of 10 km/s relative to the Sun at this point, which you can estimate as the Jupiter's average orbital radius about the Sun. (See the Planetary Constants sheet in your notes for values.) Assume that Jupiter is in a circular orbit about the Sun.

part(a)

The largest possible value for the impact parameter, b, that will still result in a hyperbolic orbit about Jupiter in the patched conic method is Jupiter's SOI radius. Find that value on the Planetary Constants sheet in the course notes and enter it here for reference.

```
> jupitor_SOI  := 4.83*10^7;
  sun_mu       := 1.327*10^11;
  jupitor_mu   := 126686534;
  b_max        <~ jupitor_SOI;
```

$$jupitor\_SOI := 4.830000000\ 10^{7}$$

$$sun\_mu := 1.327000000\ 10^{11}$$

$$jupitor\_mu := 126686534$$

$$b\_max := jupitor\_SOI = 4.830000000\ 10^{7}$$

part(b)

For parts (b) through (g), assume that, relative to the Sun, the spacecraft is moving in the same direction as Jupiter when it enters Jupiter's SOI

What is the speed of the satellite relative to Jupiter when it enters Jupiter's SOI?

```
> satellite_speed_relative_to_sun :=10;
  jupitor_sun_distance := 5.203*1.495978*10^8;
  jupitor_speed <~ sqrt((sun_mu)/(jupitor_sun_distance));
  velocity_infinity_entering_jupitor <~ jupitor_speed -
  satellite_speed_relative_to_sun;
```

$$satellite\_speed\_relative\_to\_sun := 10$$

$$jupitor\_sun\_distance := 7.783573534\ 10^{8}$$

$$jupitor\_speed := \sqrt{\frac{sun\_mu}{jupitor\_sun\_distance}} = 13.05707640$$

$$velocity\_infinity\_entering\_jupitor := jupitor\_speed - satellite\_speed\_relative\_to\_sun = 3.05707640$$

part(c)

What is the smallest possible value for the impact parameter b? This value of impact parameter will result in a burnout radius that just grazes the surface of Jupiter

```
> jupitor_radius :=71492;
  jupitor_vb0_min <~ sqrt(jupitor_mu/jupitor_radius);
```

```
b_min <~ evalf(jupitor_radius*
jupitor_vb0_min/velocity_infinity_entering_jupitor);
```

$$jupitor\_radius := 71492$$

$$jupitor\_vb0\_min := \sqrt{\frac{jupitor\_mu}{jupitor\_radius}} = \frac{1}{35746}\sqrt{2264268422182}$$

$$b\_min := evalf\left(\frac{jupitor\_radius\ jupitor\_vb0\_min}{velocity\_infinity\_entering\_jupitor}\right) = 9.844363876\ 10^{5}$$

part(d)

Select as your impact parameter the value halfway between b_{min} and b_{max}. Note that value here for reference and use it as your impact parameter for the rest of the problem

```
> b <~ (b_max+b_min)/2;
```

$$b := \frac{1}{2}\ b\_max + \frac{1}{2}\ b\_min = 2.464221819\ 10^{7}$$

part(e)

Given the impact parameter from part (d), calculate the turning angle of the spacecraft relative to Jupiter during the flyby.

```
> saturn_vb0 := 'saturn_vb0': rb0_earth := 'rb0_earth':
rb0_jupitor <~ b*
velocity_infinity_entering_jupitor/jupitor_vb0;
eq <~ (jupitor_vb0^2/2 - jupitor_mu/rb0_jupitor =
velocity_infinity_entering_jupitor^2/2 -
jupitor_mu/jupitor_SOI);
sol <~ solve(eq,jupitor_vb0);
jupitor_vb0 <~ op(select( is, [sol], positive));
```

$$rb0\_jupitor := \frac{b\ velocity\_infinity\_entering\_jupitor}{jupitor\_vb0} = \frac{7.533314367\ 10^{7}}{jupitor\_vb0}$$

$$eq := \left(\frac{1}{2}\ jupitor\_vb0^{2} - \frac{jupitor\_mu}{rb0\_jupitor} = \frac{1}{2}\ velocity\_infinity\_entering\_jupitor^{2} - \frac{jupitor\_mu}{jupitor\_SOI}\right) = \left(\frac{1}{2}\ jupitor\_vb0^{2} - 1.681683889\ jupitor\_vb0 = 2.049948451\right)$$

$$sol := solve(eq, jupitor\_vb0) = (4.313785256,\ -0.9504174777)$$

$$jupitor\_vb0 := op(select(is, [sol], positive)) = 4.313785256$$

```
> rb0_jupitor;
```

$$1.746335045\ 10^{7}$$

```
> e <~ sqrt(1+(velocity_infinity_entering_jupitor^2*
jupitor_vb0^2*rb0_jupitor^2)/jupitor_mu^2 );
eta &= arccos(-1/e);
evalf(eta*180/Pi);
theta &= (2*eta-Pi);
evalf(theta*180/Pi);
```

$$e := \sqrt{1 + \frac{velocity\_infinity\_entering\_jupitor^{2}\ jupitor\_vb0^{2}\ rb0\_jupitor^{2}}{jupitor\_mu^{2}}} = 2.074762092$$

$$(1.940335258)\ \&= (2.073712835)$$

$$111.1730211$$
$$(68.8269789)\ \&= (3.880670516 - \pi)$$
$$3943.495406$$

part(f)

What is the spacecraft's heliocentric speed following the flyby? (11.73 is correct)

```
> vd <~ sqrt(jupitor_speed^2+
  velocity_infinity_entering_jupitor^2-2*jupitor_speed*abs
  (velocity_infinity_entering_jupitor)*cos(theta));
```

$$vd := \left(jupitor\_speed^2 + velocity\_infinity\_entering\_jupitor^2 - 2\, jupitor\_speed\, |velocity\_infinity\_entering\_jupitor|\, \cos(\theta)\right)^{1/2} = 10.16313731$$

part(g)

What is the spacecraft's heliocentric flight path angle following the flyby

```
> gamma_d <~ arcsin(velocity_infinity_entering_jupitor*sin
  (theta)/vd);
  evalf(gamma_d*180/Pi);
```

$$gamma\_d := \arcsin\left(\frac{velocity\_infinity\_entering\_jupitor \sin(\theta)}{vd}\right) = -0.08555941389$$
$$-4.902193312$$

Hohmann from earth to moon (for project)

```
> satellite_earth_altitude := 300;
  earth_radius   := 6378;
  r__p           <~ satellite_earth_altitude+earth_radius;
  r__a           <~ 384400;
  a              <~ ((r__p+r__a)/2);
  earth_mu      := 3.986*10^5;
  satellite_speed_relative_to_earth  <~ sqrt(earth_mu/r__p);
  velocity_perigee  <~ sqrt(earth_mu*(2/r__p - 1/a));
  del__V1 <~ velocity_perigee -
  satellite_speed_relative_to_earth;
  e <~ evalf((r__a-r__p)/(r__a+r__p));
```

$$satellite\_earth\_altitude := 300$$
$$earth\_radius := 6378$$
$$r_p := satellite\_earth\_altitude + earth\_radius = 6678$$
$$r_a := 384400 = 384400$$
$$a := \frac{1}{2}\, r_a + \frac{1}{2}\, r_p = 195539$$
$$earth\_mu := 3.98600000\ 10^5$$
$$satellite\_speed\_relative\_to\_earth := \sqrt{\frac{earth\_mu}{r_p}} = 7.725835198$$

$$velocity\_perigee := \sqrt{earth\_mu \left(\frac{2}{r_p} - \frac{1}{a}\right)} = 10.83229389$$

$$del_{V1} := velocity\_perigee - satellite\_speed\_relative\_to\_earth = 3.106458692$$

$$e := evalf\left(\frac{r_a - r_p}{r_a + r_p}\right) = 0.9658482451$$

```
> velocity_apogee <~  sqrt(earth_mu*(2/r__a - 1/a));
```

$$velocity\_apogee := \sqrt{earth\_mu \left(\frac{2}{r_a} - \frac{1}{a}\right)} = 0.1881843356$$

```
> v2 <~ sqrt(earth_mu/r__a);
```

$$v2 := \sqrt{\frac{earth\_mu}{r_a}} = 1.018302846$$

```
> delV2 <~ v2-velocity_apogee;
```

$$delV2 := v2 - velocity\_apogee = 0.8301185104$$

```
> totalDelV <~ abs(del__V1)+abs(delV2);
```

$$totalDelV := |del_{V1}| + |delV2| = 3.936577202$$

```
> delT:=Pi* sqrt(a^3/earth_mu);
```

$$delT := 1.369561180\ 10^5\ \pi$$

```
> evalf(delT);
```

$$4.302603342\ 10^5$$

```
> evalf(delT/(60*60*24));
```

$$4.979864981$$

3.2 solution in Mathematica

HW5 EMA 550, University of Wisconsin, Madison

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March 11,2014

problem 1

A spacecraft is initially in a 300 km altitude circular orbit about the Earth in the ecliptic plane. It is to be sent on a Hohmann transfer to Saturn, also in the ecliptic plane. Assume that Saturn is in the correct position in its orbit for a flyby to occur when the spacecraft gets there.

Part (a)

Find ΔV_1 for Hohmann transfer

define constants to use

```
Clear["Global`*"];
AU = 1.495978 * 10^8;
r_earth = 6378;
μ_sun = 1.327 * 10^11;
μ_earth = 3.986 * 10^5;
R_earth = 1.496 * 10^8;
R_earth_SOI = 9.24 * 10^5;
R_saturn = 9.537 AU;
```

Velocity of earth relative to the sun

$$V_{earth} = \sqrt{\frac{\mu_{sun}}{R_{earth}}}$$

```
29.7831
```

spacecraft altitude over earth

```
alt = 300;
```

$$r_{b0} = r_{earth} + alt$$

6678

Find Hohmann paramters for trip to Saturn

$$a = \frac{R_{earth} + R_{saturn}}{2}$$

7.88157×10^{8}

Find V_p the velocity are perigee

$$V_{perigee} = \sqrt{\mu_{sun} \left(\frac{2}{R_{earth}} - \frac{1}{a}\right)}$$

40.0711

Find V_∞ the excess velocity to escape from Earth

$$V_{out} = V_{perigee} - V_{earth}$$

10.2881

Find V_{b0} at earth

$$V_{b0} = \sqrt{2\left(\left(\frac{V_{out}^{2}}{2} - \frac{\mu_{earth}}{R_{earth_{SOI}}}\right) + \frac{\mu_{earth}}{r_{b0}}\right)}$$

14.9786

Find V_{sat} the spacecraft speed around eath

$$V_{sat} = \sqrt{\frac{\mu_{earth}}{r_{b0}}}$$

7.72584

find ΔV_1

```
delV1 = Vb0 - Vsat
```

$delV_1 = V_{b0} - V_{sat}$

7.25277

Part (b) Angle calculation at departure

Calculate the angle past the Earth’s dawn-dusk line where the ΔV should be applied.

find *e* the eccentricty for the escape hyperbola

$$e = \sqrt{1 + \frac{V_{out}^2 V_{b0}^2 r_{b0}^2}{\mu_{earth}^2}}$$

2.76865

$$\eta = \text{ArcCos}\left[-\frac{1}{e}\right];$$

$$\text{Row}\left[\left\{\text{"}\eta \text{ Degree = ", } \eta * \frac{180}{\pi}\right\}\right]$$

η Degree = 111.173

$$\theta = \text{Pi} - \eta;$$

$$\text{Row}\left[\left\{\text{"}\theta \text{ Degree = ", } \theta * \frac{180}{\pi}\right\}\right]$$

θ Degree = 68.8269

Part (c)

For how long is the spacecraft on the heliocentric Hohmann transfer between Earth and Saturn? (Note: you do not need to calculate the time within either planet’s sphere of influence, as that will be small relative to the Hohmann transfer time, but you are welcome to do so and compare those values for yourself.)

find time to fly, which is half the period

```
T = 2 π Sqrt[a^3/μsun];
Row[{"time to fly in years = ", (1/2) T/(60*60*24*365)}]
```

$$T = 2\pi\sqrt{\frac{a^3}{\mu_{sun}}};$$

time to fly in years = 6.051

Part (d)

After crossing into the sphere of influence of Saturn, the spacecraft is to be placed in a circular orbit about Saturn with an orbital radius of 150,000 km. Calculate the ΔV2 required to place the spacecraft on this orbit. When spacecraft reaches saturn is has speed relative to sun of

Paramters to use

r_{b0} = 150000;
μ_{saturn} = 37931187;
$R_{saturn_{SOI}}$ = 3.47*10^7;

Find $V_{apegree}$ of the Hohmann transfer

$$V_{apegee} = \sqrt{\mu_{sun}\left(\frac{2}{R_{saturn}} - \frac{1}{a}\right)}$$

4.20171

find saturn speed relative to sun

$$V_{saturn} = \sqrt{\frac{\mu_{sun}}{R_{saturn}}}$$

9.64422

Find V_{in} the speed by which spacecraft enters saturn SOI

$$V_{in} = V_{saturn} - V_{apegee}$$

5.4425

Use energy equation to solve for V_{b0} at Saturn

$$V_{b0} = \sqrt{2\left(\left(\frac{V_{in}^{2}}{2} - \frac{\mu_{saturn}}{R_{saturn_{SOI}}}\right) + \frac{\mu_{saturn}}{r_{b0}}\right)}$$

```
23.0908
```

Since spacecrasft will end up in an orbit around saturn, find its parking speed

$$\left(V_{sat} = \sqrt{\frac{\mu_{saturn}}{r_{b0}}}\right) \text{ // N}$$

```
15.902
```

find ΔV_2

```
delV2 = Vsat - Vb0
```

```
-7.18874
```

Find total speed change needed

```
totalV = Abs[delV1] + Abs[delV2]
```

```
14.4415
```

Problem 2

A spacecraft on an interplanetary mission in the same plane as Jupiter's orbit about the Sun enters Jupiter's sphere of influence. The spacecraft has a speed of 10 km/s relative to the Sun at this point, which you can estimate as the Jupiter's average orbital radius about the Sun. (See the Planetary Constants sheet in your notes for values.) Assume that Jupiter is in a circular orbit about the Sun.

Part (a)

The largest possible value for the impact parameter, b, that will still result in a hyperbolic orbit about Jupiter in the patched conic method is Jupiter's SOI radius. Find that value on the Planetary Constants sheet in the course notes and enter it here for reference.

Paramters

```
ClearAll["Global`*"];
AU = 1.495978 * 10^8;
r_earth = 6378;
μ_sun = 1.327 * 10^11;
μ_earth = 3.986 * 10^5;
μ_jupitor = 126 686 534;
R_earth = 1.496 * 10^8;
R_earth_SOI = 9.24 * 10^5;
R_jupitor = 5.203 AU;
r_jupitor = 71 492;
jupitor_SOI = 4.83 * 10^7;
bmax = jupitor_SOI;
```

Part(b)

For parts (b) through (g), assume that, relative to the Sun, the spacecraft is moving in the same direction as Jupiter when it enters Jupiter's SOI
What is the speed of the satellite relative to Jupiter when it enters Jupiter's SOI?

```
Vin = 10;
```

find Jupitor speed relative to sun

$$V_{jupitor} = \sqrt{\frac{\mu_{sun}}{R_{jupitor}}}$$

```
13.0571
```

Find speed of spacecraft relative to Jupitor

```
VinRelative = V_jupitor - Vin
```

```
3.05708
```

Part(c)

What is the smallest possible value for the impact parameter b? This value of impact parameter will result in a burnout radius that just grazes the surface of Jupiter

```
eq = bmin VinRelative == r_jupiter Sqrt[μ_jupiter / r_jupiter];
bmin /. First@Solve[eq, bmin];
(bmin = %) // N
```

```
984 436.
```

Part(d)

Select as your impact parameter the value halfway between b_{min} and b_{max}. Note that value here for reference and use it as your impact parameter for the rest of the problem

```
b = Mean[{bmin, bmax}]
```

$$2.46422 \times 10^{7}$$

Part(e)

Given the impact parameter from part (d), calculate the turning angle of the spacecraft relative to Jupiter during the flyby.

```
eq1 = (rb0 ) (vb0) == (b) (VinRelative);
rb0 = ((b) (VinRelative)) / vb0
```

$$\frac{7.53331 \times 10^{7}}{vb0}$$

setup the energy equation at Jupitor

$$eq2 = \frac{vb0^2}{2} - \frac{\mu_{jupitor}}{rb0} == \frac{VinRelative^2}{2} - \frac{\mu_{jupitor}}{jupitor_{SOI}}$$

$$-1.68168\, vb0 + \frac{vb0^2}{2} == 2.04995$$

Solve for V_{b0}

```
sol = vb0 /. NSolve[eq2, vb0]
```

```
{-0.950417, 4.31379}
```

```
vb0 = First@Select[%, Positive]
```

```
4.31379
```

check the correspoding r_{b0}

```
rb0
```

1.74634×10^7

Find e at jupitor and find η and θ

$$e = \sqrt{1 + \frac{(\text{VinRelative})^2 \, (\text{vb0})^2 \, (\text{rb0})^2}{\mu_{\text{jupitor}}{}^2}}$$

```
2.07476
```

$$\eta = \text{ArcCos}\left[-\frac{1}{e}\right];$$

$$\text{Row}\left[\left\{\text{"}\eta \text{ Degree = ", } \eta * \frac{180}{\pi}\right\}\right]$$

η Degree = 118.815

$$\theta = 2\,\eta - \text{Pi};$$

$$\text{Row}\left[\left\{\text{"}\theta \text{ Degree = ", } \theta * \frac{180}{\pi}\right\}\right]$$

θ Degree = 57.63

Part(f)

What is the spacecraft's heliocentric speed following the flyby?

$$\text{vd} = \sqrt{V_{\text{jupitor}}{}^2 + \text{VinRelative}^2 - 2\,V_{\text{jupitor}}\,\text{VinRelative}\,\text{Cos}[\theta]}$$

```
11.7086
```

Part (g)

What is the spacecraft's heliocentric flight path angle following the flyby

```
γd = ArcSin[VinRelative Sin[θ] / vd];
Row["γd in degree ", γd 180 / Pi]
```

```
Row[γd in degree , 12.7398]
```

For the remaining parts, assume that, relative to the Sun, the spacecraft DOES NOT arrive at Jupiter's SOI moving in the same direction at Jupiter. The spacecraft still has a heliocentric speed of 10 km/s at the distance of Jupiter's orbit from the Sun. But now it has a heliocentric eccentricity of 0.5. (What was the heliocentric eccentricity when the spacecraft arrived in the same direction as Jupiter, assuming that point was aphelion?)

Part(h)

What is the spacecraft's heliocentric flight path angle when it arrives at Jupiter's SOI?

```
Clear[a];
e = 0.5;
```

$$eq = V_{in} == \sqrt{\mu_{sun} \left(\frac{2}{R_{jupiter}} - \frac{1}{a}\right)}$$

$$10 == 364280.\ \sqrt{2.56951 \times 10^{-9} - \frac{1}{a}}$$

```
a = a /. First@NSolve[eq, a]
```

$$5.50681 \times 10^{8}$$

$$\gamma = ArcCos\left[\sqrt{\frac{a^2 \left(1 - e^2\right)}{R_{jupiter} \left(2\,a - R_{jupiter}\right)}}\right];$$

```
Row[{"angle is ", γ 180 / Pi, " degree"}]
```

```
angle is 17.9875 degree
```

Part(i)

What is the spacecraft's speed relative to Jupiter

```
VinRelative = Sqrt[Vjupitor^2 + Vin^2 - 2 Vjupitor Vin Cos[γ]]
```

```
4.70206
```

part(j)

Using the same impact parameter as in part (d), calculate the turning angle of the spacecraft relative to Jupiter.

```
Clear[vb0];
eq1 = rb0 vb0 == b VinRelative;
rb0 = (b VinRelative)/vb0
```

$$\frac{1.15869\times 10^{8}}{\text{vb0}}$$

setup the energy equation at Jupitor

```
Clear[vb0];
eq2 = vb0^2/2 - μjupitor/rb0 == VinRelative^2/2 - μjupitor/jupitorSOI
```

$$-1.09336\,\text{vb0}+\frac{\text{vb0}^2}{2}==8.43177$$

Solve for V_{b0}

```
sol = vb0 /. NSolve[eq2, vb0]
```

```
{-3.15623, 5.34294}
```

```
vb0 = First@Select[%, Positive]
```

```
5.34294
```

check the correspoding r_{b0}

```
rb0
```

$$2.16864\times 10^{7}$$

Find *e* at jupitor and find η and θ

$$e = \sqrt{1 + \frac{(\text{VinRelative})^2\,(\text{vb0})^2\,(\text{rb0})^2}{\mu_{\text{jupitor}}{}^2}}$$

4.4153

$$\eta = \text{ArcCos}\left[-\frac{1}{e}\right];$$

$$\text{Row}\left[\left\{\text{"}\eta\text{ Degree = ", } \eta * \frac{180}{\pi}\right\}\right]$$

η Degree = 103.09

$$\theta = 2\,\eta - \text{Pi};$$

$$\text{Row}\left[\left\{\text{"}\theta\text{ Degree = ", } \theta * \frac{180}{\pi}\right\}\right]$$

θ Degree = 26.1805

Part(k)

Assuming that the spacecraft flies behind Jupiter, what is the spacecraft's heliocentric speed following the flyby?

V_{jupitor}

13.0571

VinRelative

4.70206

Vin

10

$$\beta = \text{ArcSin}\left[\frac{\text{Vin Sin}[\gamma]}{\text{VinRelative}}\right]$$

0.716508

```
vd = Sqrt[Vjupitor^2 + VinRelative^2 - 2 Vjupitor VinRelative Cos[β + θ]]
```

$$vd = \sqrt{V_{jupitor}^2 + VinRelative^2 - 2\, V_{jupitor}\, VinRelative\, Cos[\beta + \theta]}$$

```
12.0449
```

Part(L)

Assuming that the spacecraft flies behind Jupiter, what is the spacecraft's heliocentric flight path angle following the flyby?

$$\gamma_d = ArcSin\left[\frac{VinRelative\, Sin[\beta + \theta]}{vd}\right];$$

```
Row["γd in degree ", γd 180 / Pi]
```

```
Row[γd in degree , 21.0979]
```