7.3 When the throttle is first opened, thrust will be greater than drag and the airspeed will increase. Thus you will progress up the drag polar towards P. When P is reached, thrust will equal drag and the airspeed will no longer change. P is a stable equilibrium point.

7.4 From (7.7,11b)

$$G_{\theta\delta_e} = \frac{b_1 s + b_o}{s(s^2 + c_1 s + c_o)} \tag{1}$$

(a) Expressing (1) in terms of $\lambda_{1,2} = n \pm i\omega$

$$G_{\theta\delta_e} = \frac{b_1 s + b_o}{s(s - \lambda_1)(s - \lambda_2)}$$

$$= \frac{b_1 s + b_o}{s[(s - n)^2 + \omega^2]} \tag{2}$$

For $\delta_e = \delta(t)$, it follows from Table A.1

$$\bar{\delta}_e = 1 \tag{3}$$

and

$$\bar{\theta} = G_{\theta\delta_e} \cdot \bar{\delta}_e$$

$$= G_{\theta\delta_e} \tag{4}$$

Expand $G_{\theta\delta_e}$ using partial fractions

$$\bar{\theta} = \frac{b_1}{\omega} \cdot \frac{\omega}{(s - n)^2 + \omega^2} + \frac{b_o}{s[(s - n)^2 + \omega^2]} \tag{5}$$

The second term in (5) can be expanded as follows:

$$\frac{b_o}{s[(s-n)^2+\omega^2]} = b_o\left[\frac{A}{s}+\frac{Bs+C}{(s-n)^2+\omega^2}\right] \tag{6}$$

The numerator of the part of (6) inside the square brackets is

$$As^2 - 2nAs + (n^2+\omega^2)A + Bs^2 + Cs \tag{7}$$

and equating this to 1 in order to satisfy (6) results in

$$A + B = 0 \tag{8}$$

$$C - 2nA = 0 \tag{9}$$

$$(n^2+\omega^2)A = 1 \tag{10}$$

From (10)

$$A = (n^2+\omega^2)^{-1} \tag{11}$$

From (8) and (11)

$$B = -(n^2+\omega^2)^{-1} \tag{12}$$

From (9) and (11)

$$C = 2n(n^2+\omega^2)^{-1} \tag{13}$$

From (5), (6), (11), (12) and (13)

$$\bar{\theta} = \frac{b_1}{\omega} \cdot \frac{\omega}{(s-n)^2 + \omega^2} + \frac{b_o}{(n^2+\omega^2)} \cdot \frac{1}{s}$$

$$+ b_o \left\{ \frac{B(s-n) + (Bn + C)}{(s-n)^2 + \omega^2} \right\}$$

$$= \frac{b_1}{\omega} \left\{ \frac{\omega}{(s-n)^2 + \omega^2} \right\} + \frac{b_o}{(n^2+\omega^2)} \cdot \frac{1}{s}$$

$$- \frac{b_o}{(n^2+\omega^2)} \left\{ \frac{(s-n)}{(s-n)^2 + \omega^2} \right\}$$

$$+ \frac{b_o \cdot n}{(n^2+\omega^2)\,\omega} \cdot \frac{\omega}{(s-n)^2 + \omega^2} \tag{14}$$

From (14) and Table A.1 (3, 13 and 14)

$$\theta(t) = e^{nt} \sin \omega t \left[\frac{b_1}{\omega} + \frac{b_o \cdot n}{(n^2+\omega^2)\,\omega} \right] + \frac{b_o}{(n^2+\omega^2)} \left[1 - e^{nt} \cos \omega t \right] \tag{15}$$

(b) b_o and b_1 can be found by using (15) at two different times t_1 and t_2 (since $\theta(t_1)$ and $\theta(t_2)$ are known) and then solving the two equations for the two unknowns b_o and b_1. c_o and c_1 can be found by using (1) and (2). By expanding and equating the two denominators it follows that

$$s^2 + c_1 s + c_o = s^2 - 2ns + (n^2 + \omega^2) \tag{16}$$

for all s. Thus equating the coefficients of the same powers in s

$$c_1 = -2n \tag{17}$$

$$c_o = n^2 + \omega^2 \tag{18}$$

where n and ω are known.

7.5 From Table 4.5

$$Y_v = \frac{1}{2}\rho u_o S C_{y\beta} \tag{1}$$

$$L_v = \frac{1}{2}\rho u_o b S C_{\ell\beta}$$

$$N_v = \frac{1}{2}\rho u_o b S C_{n\beta}$$

$$Y_{\delta_r} = \frac{1}{2}\rho u_o^2 S C_{y\delta_r}$$

$$L_{\delta_a} = \frac{1}{2}\rho u_o^2 b S C_{\ell\delta_a}$$

$$L_{\delta_r} = \frac{1}{2}\rho u_o^2 b S C_{\ell\delta_r}$$

$$N_{\delta_a} = \frac{1}{2}\rho u_o^2 b S C_{n\delta_a}$$

$$N_{\delta_r} = \frac{1}{2}\rho u_o^2 b S C_{n\delta_r}$$

Thus the condition that must be satisfied is

$$\det A \neq 0 \tag{6}$$

7.10 (a) From (6.7,2)

$$f(s) = s^4 + .6358s^3 + .9388s^2 + .5114s + .003682 \tag{1}$$

In (1) replace s by $i\omega$ with $\omega = 0$, thus

$$f(0) = .003682 \tag{2}$$

In (7.9,5) the static gain case is also given by $N_{ij}(0)$ which is the constant term in the expression. Thus the static gains $G_{ij}(0) = N_{ij}(0)/f(0)$ are given by

$$G_{v\delta_a}(0) = .6220/.003682 = 168.9 \tag{3}$$

$$G_{v\delta_r}(0) = -5.934/.003682 = -1612$$

$$G_{p\delta_a}(0) = 0$$

$$G_{p\delta_r}(0) = 0$$

$$G_{r\delta_a}(0) = .004539/.003682 = 1.233$$

$$G_{r\delta_r}(0) = -.05647/.003682 = -15.34$$

$$G_{\phi\delta_a}(0) = .1102/.003682 = 29.93$$

$$G_{\phi\delta_r}(0) = -1.368/.003682 = -371.5$$

(b) As $\omega \rightarrow \infty$, $G_{ij}(i\omega)$ can be simplified by keeping only the dominant high order terms in each of $N_{ij}(i\omega)$ and $f(i\omega)$. From (1)

$$\lim_{\omega \to \infty} f(i\omega) = \omega^4 \tag{4}$$

From (7.9,5) as $\omega \rightarrow \infty$ in $N_{ij}(i\omega)$ only the first term need be kept. Thus

$$\lim_{\omega \to \infty} |N_{ij}(i\omega)| = a_{ij}\, \omega^n \tag{5}$$

where n is the largest index of ω in $N_{ij}(i\omega)$. Thus

$$\lim_{\omega \to \infty} |G_{ij}(i\omega)| = a_{ij}\, \omega^{(n-4)} \tag{6}$$

and the slope of (6) in decades per decade can be determined by taking the $\log_{10}$ of the right-hand side

$$\log_{10} a_{ij} + (n - 4)\log_{10}\omega \tag{7}$$

The desired slope is given by (n - 4) decades/decade. From (7.9,5)

Transfer Function	Slope (decades/decade)
$G_{v\delta_a}$	-2
$G_{v\delta_r}$	-1
$G_{p\delta_a}$	-1
$G_{p\delta_r}$	-1
$G_{r\delta_a}$	-1
$G_{r\delta_r}$	-1
$G_{\phi\delta_a}$	-2
$G_{\phi\delta_r}$	-2

(c) Consider the case with δ_a deflection only. From part (a)

$$G_{\phi\delta_a}(0) = 29.93$$

$$= \phi/\delta_a \tag{8}$$

Thus for $\phi = 15°$

$$\delta_a = 15/29.93$$

$$= 0.501° \tag{9}$$

To determine β just find v from (a)

$$G_{v\delta_a}(0) = 168.9$$

$$= v/\delta_a \tag{10}$$

Hence

$$v = 168.9\ \delta_a$$

(with δ_a in rad)

But $\beta \approx v/u_o$ (u_o = 774 fps from Sec. 6.2) and thus for $\delta_a = 0.501°$

$$\beta = \frac{168.9}{774}\left(\frac{0.501}{57.3}\right)57.3$$

$$= 0.109° \tag{11}$$

The yaw rate can be found from

$$G_{r\delta_a}(0) = 1.233$$

$$= r/\delta_a \tag{12}$$

Hence for $\delta_a = 0.501°$

$$r = 1.233 \times \left(\frac{0.501}{57.3}\right)57.3$$

$$= 0.618 \text{ deg/s} \tag{13}$$

Consider the case with δ_r deflection only. From part (a)

$$G_{\phi\delta_r}(0) = -371.5$$

$$= \phi/\delta_r \tag{14}$$

Thus for $\phi = 15°$

$$\delta_r = -15/371.5$$

$$= -0.0404° \tag{15}$$

From (a)

$$G_{v\delta_r}(0) = -1612$$

$$= v/\delta_r \tag{16}$$

Hence

$$v = -1612\delta_r$$

and for $\delta_r = -0.0404°$

$$\beta = \frac{1612}{774}\left(\frac{0.0404}{57.3}\right)57.3$$

$$= 0.0841° \qquad (17)$$

From (a)

$$G_{r\delta_r}(0) = -15.34$$

$$= r/\delta_r \qquad (18)$$

Hence for $\delta_r = -0.0404°$

$$r = 15.34 \times \left(\frac{0.0404}{57.3}\right)57.3$$

$$= 0.620 \text{ deg/s} \qquad (19)$$

7.11 (a) From Fig. 7.18a it is found that at the frequency of the short-period mode

$$|G_{n_z\delta_e}| = 13.5 \qquad (1)$$

Thus for a sinusoidal δ_e of amplitude $|\delta_e|$ at that frequency, the amplitude $|\Delta n_z|$ of the response Δn_z (about $n_{z_0} = 1$) is (for $|\delta_e|$ in rad)

$$|\Delta n_z| = 13.5 \times |\delta_e|$$

$$= 13.5 \times \frac{2}{57.3}$$

$$= 0.471 \qquad (2)$$

and $$n_z = 1 + 0.471 \sin(\omega_{sp} t) \qquad (3)$$

(b) Since n_z is measured at the CG and must reach a value of $n_z = 0$ for a passenger at the CG to be lifted from his seat, it follows that Δn_z must reach a value of -1. This is achieved by (see (2))

$$|\delta_c| = (1/13.5) \times 57.3$$
$$= 4.24° \qquad (4)$$

(c) For n_z to reach a peak value of 2.5 would require Δn_z to reach a peak of 1.5. From (2)

$$|\delta_c| = (1.5/13.5) \times 57.3$$
$$= 6.37° \qquad (5)$$

PROB. 4

WE'RE LOOKING FOR

$$\underline{G} = \begin{Bmatrix} G_{u\delta_p} \\ G_{w\delta_p} \\ G_{q\delta_p} \\ G_{\theta\delta_p} \end{Bmatrix} = \begin{Bmatrix} \frac{N_{u\delta_p}(s)}{D(s)} \\ \frac{N_{w\delta_p}(s)}{D(s)} \\ \frac{N_{q\delta_p}(s)}{D(s)} \\ \frac{N_{\theta\delta_p}(s)}{D(s)} \end{Bmatrix}$$

RECALL $\underline{\underline{G}} = (s\underline{\underline{I}} - \underline{\underline{A}})^{-1}\underline{\underline{B}}$

w/ $\underline{\underline{B}} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ B_{31} & B_{32} \\ B_{41} & B_{42} \end{bmatrix}$

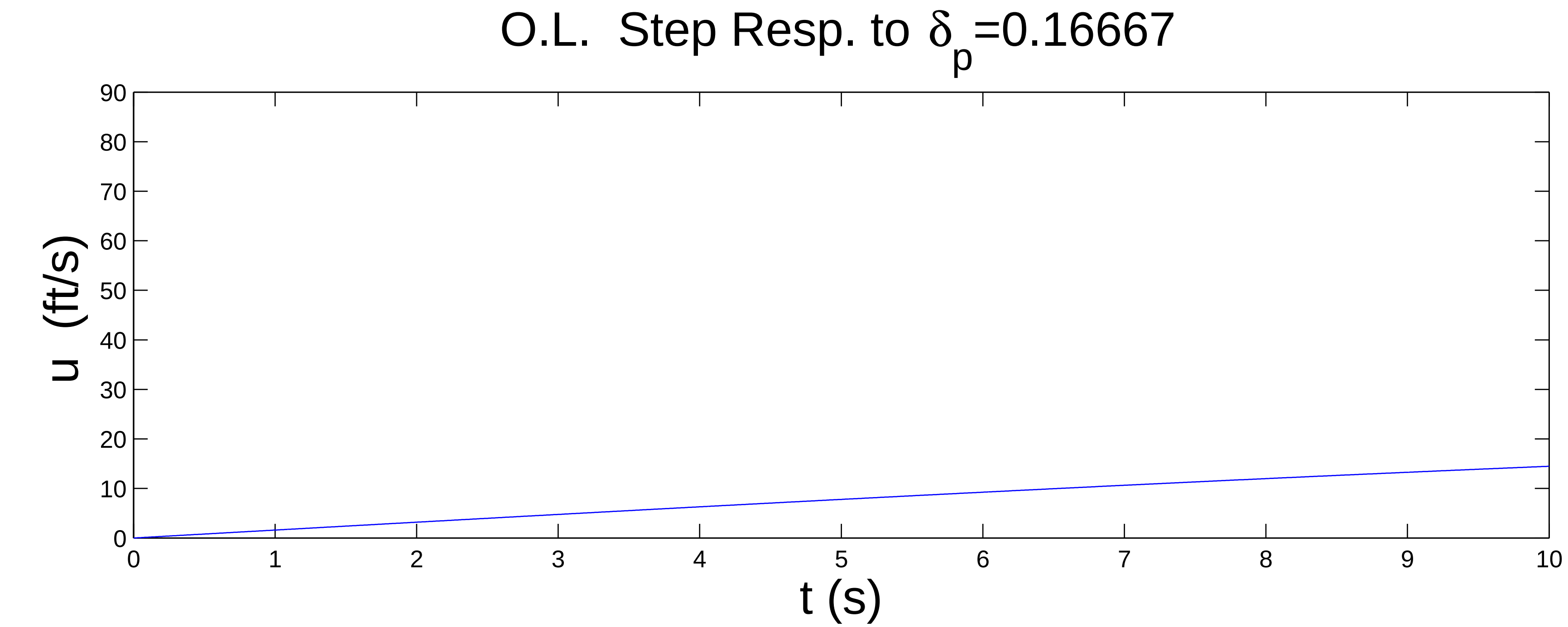
HERE ONLY USE $\underline{B} = \begin{Bmatrix} B_{12} \\ B_{22} \\ B_{32} \\ B_{42} \end{Bmatrix} = \begin{Bmatrix} 9.66 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$

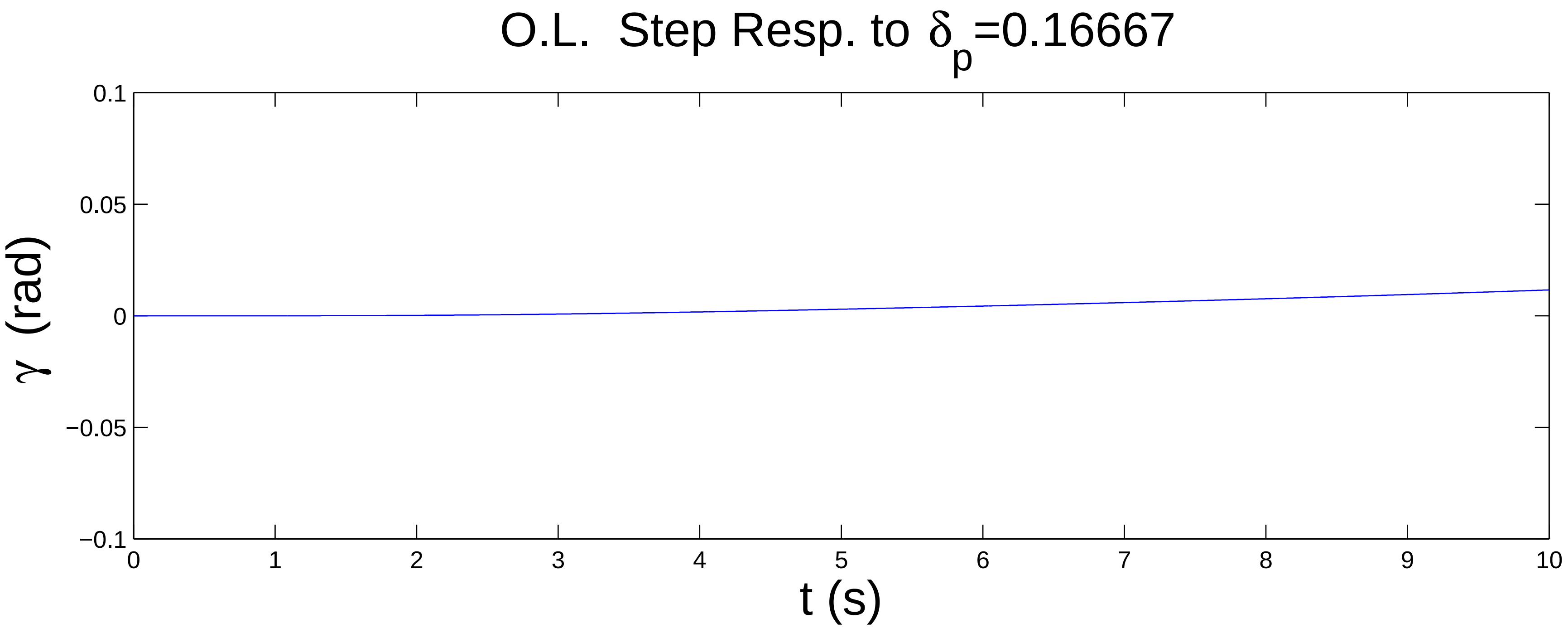
IN HANDOUTS, WE HAVE $\underline{G} = \begin{Bmatrix} G_{u\delta_e} \\ G_{w\delta_e} \\ G_{q\delta_e} \\ G_{\theta\delta_e} \end{Bmatrix} = \begin{Bmatrix} \frac{N_{u\delta_e}(s)}{D(s)} \\ \vdots \\ \vdots \\ \vdots \end{Bmatrix}$

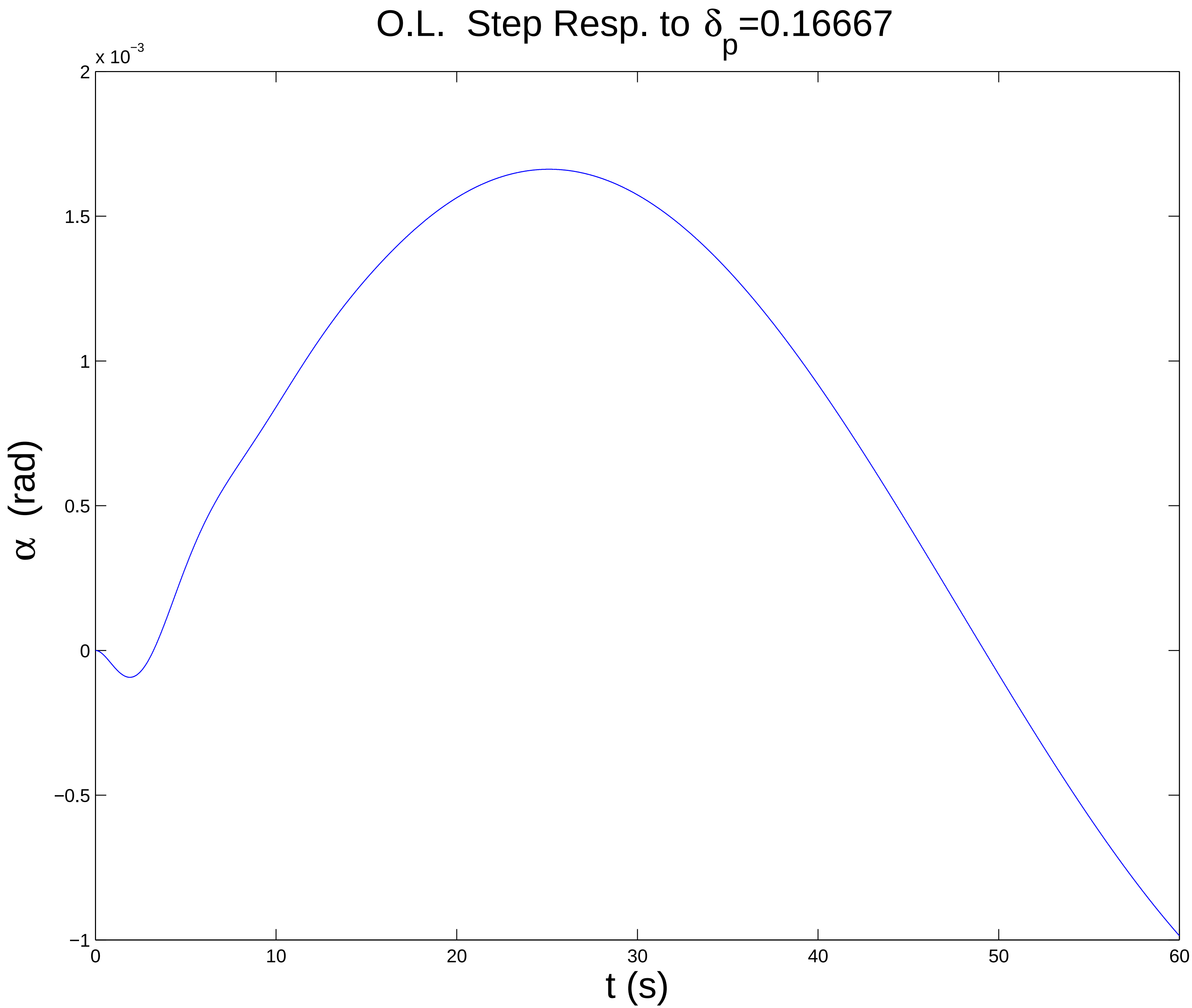
AND AT THE 2nd STEP OF THE EVALUATION WE READ (SEE p. 78 OF THE HANDOUT ON COURSE WEBSITE)

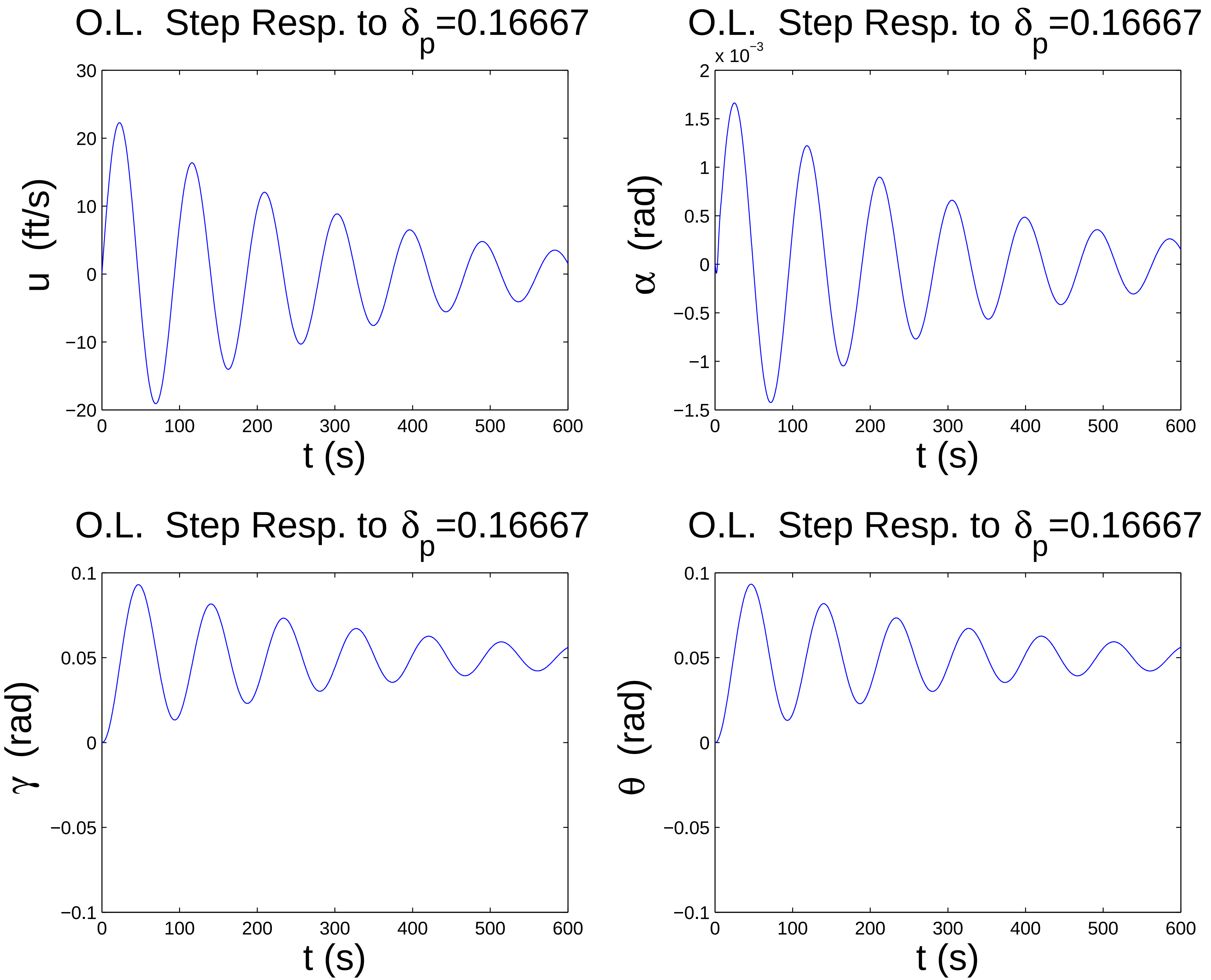
$$N_{u\delta e}(s) = -0.000187\, s\left(s^2 + 0.743\, s + 0.932\right) +$$
$$-17.8\left(0.0140\, s^2 + 0.00599\, s + 0.0332\right) +$$
$$+1.16\left(21.4\, s + 10.1\right)$$

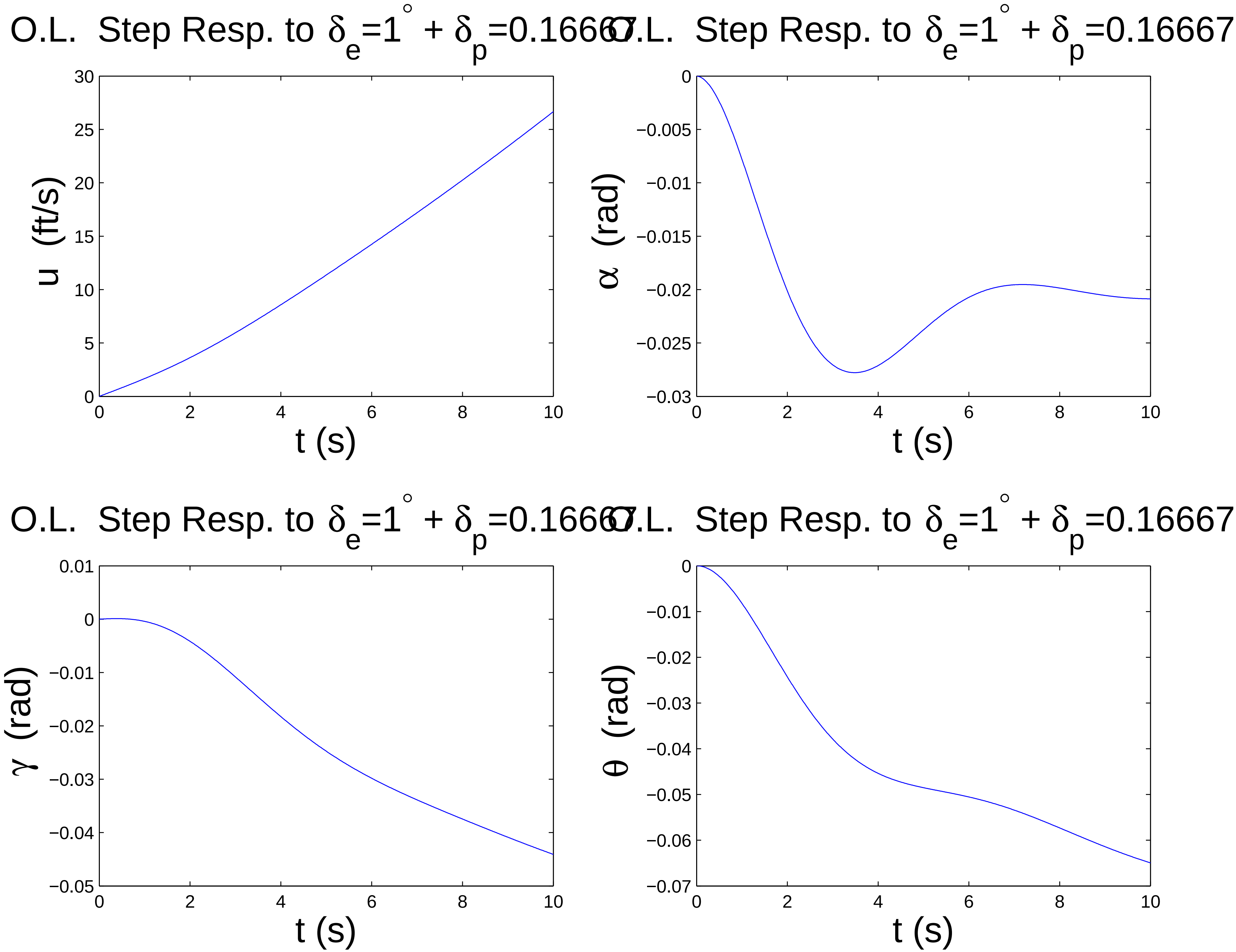
HERE, SINCE THE ONLY ELEMENT OF $\underline{B} \neq 0$ IS THE FIRST ONE, ALL WE HAVE TO DO IS USE THE FIRST TERM OF EACH OF THE $N_{u\delta e}$, $N_{w\delta e}$, $N_{q\delta e}$, $N_{\theta\delta e}$ AND REPLACE THE FACTOR -0.000187 w/ 9.66


O.L. Step Resp. to δp=0.16667
u (ft/s)
t (s)
0
10
20
30
40
50
60
70
80
90
1
2
3
4
5
6
7
8
9
10

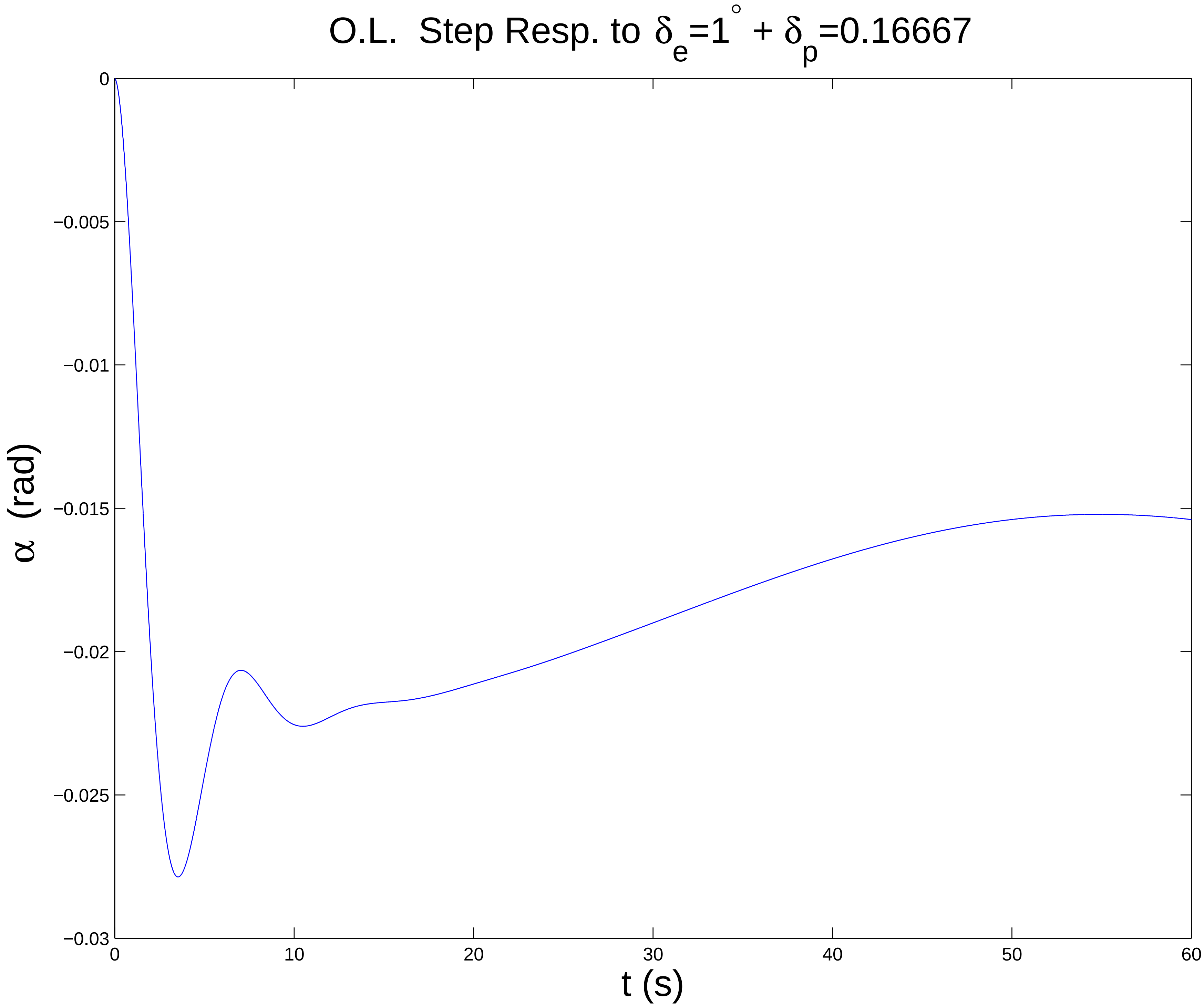

O.L. Step Resp. to δp=0.16667
γ (rad)
t (s)
0.1
0.05
0
−0.05
−0.1
1
2
3
4
5
6
7
8
9
10

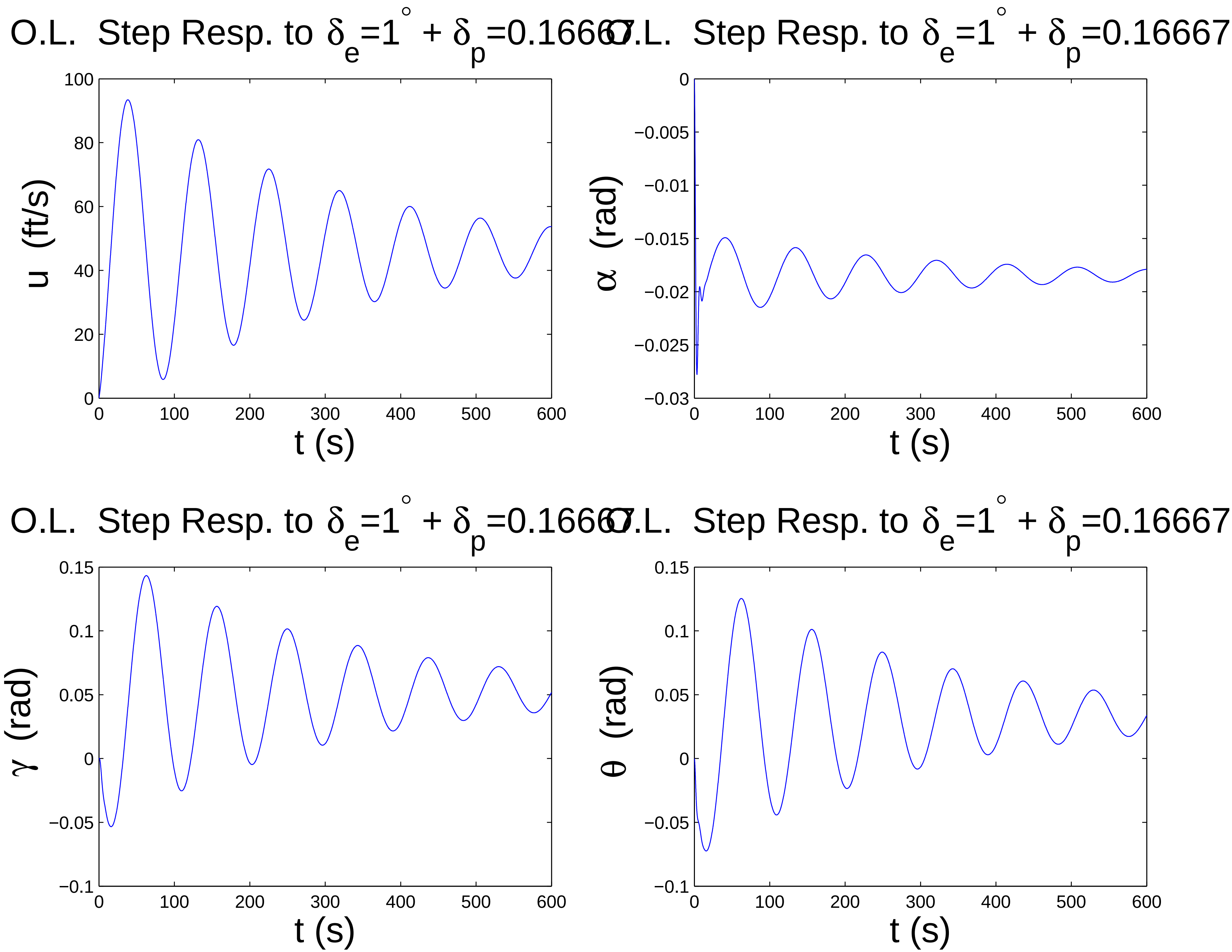

O.L. Step Resp. to δp=0.16667
x 10−3
2
1.5
1
0.5
0
−0.5
−1
α (rad)
0
10
20
30
40
50
60
t (s)

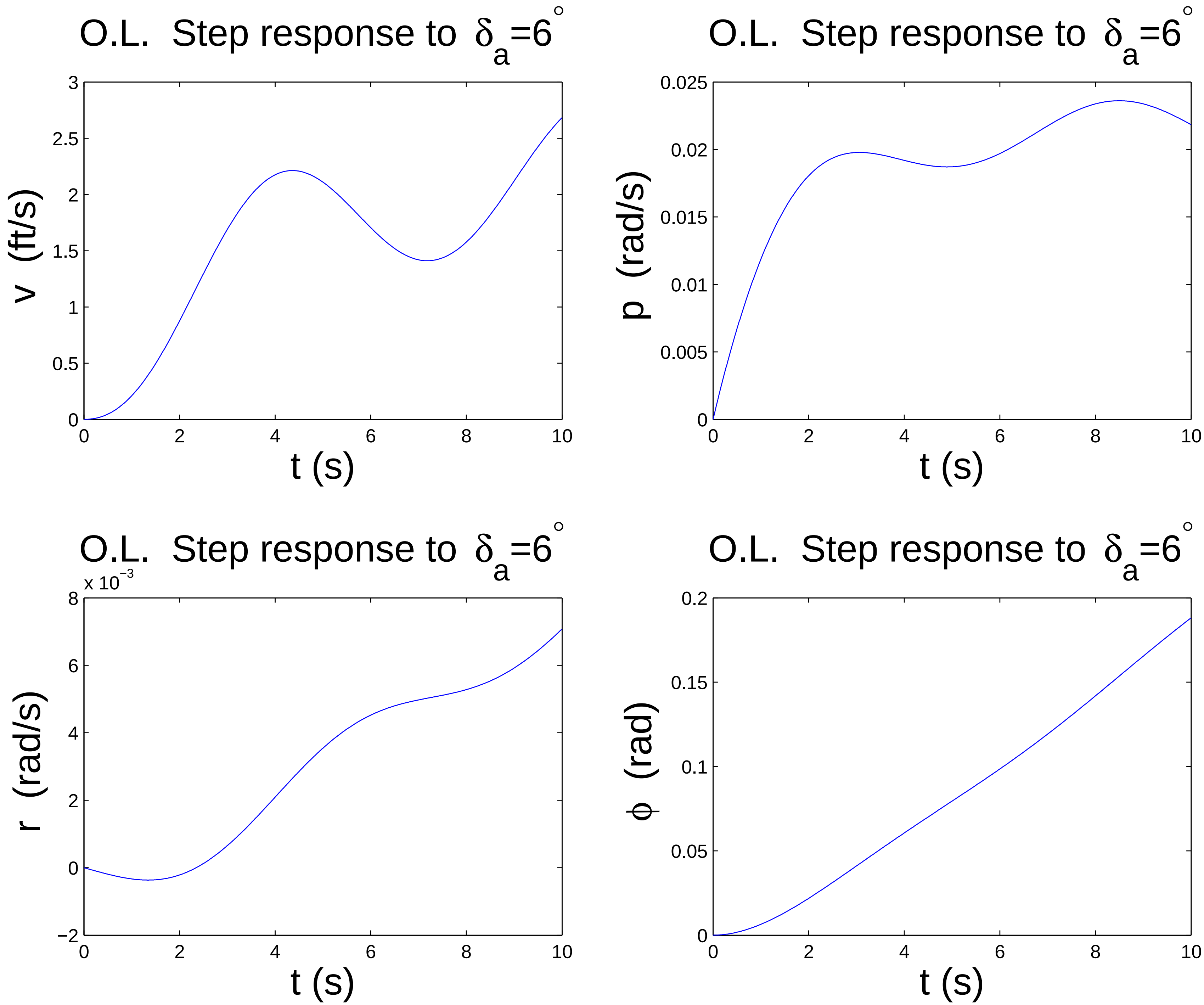

O.L. Step Resp. to δp=0.16667
u (ft/s)
t (s)
O.L. Step Resp. to δp=0.16667
x 10^-3
α (rad)
t (s)
O.L. Step Resp. to δp=0.16667
γ (rad)
t (s)
O.L. Step Resp. to δp=0.16667
θ (rad)
t (s)

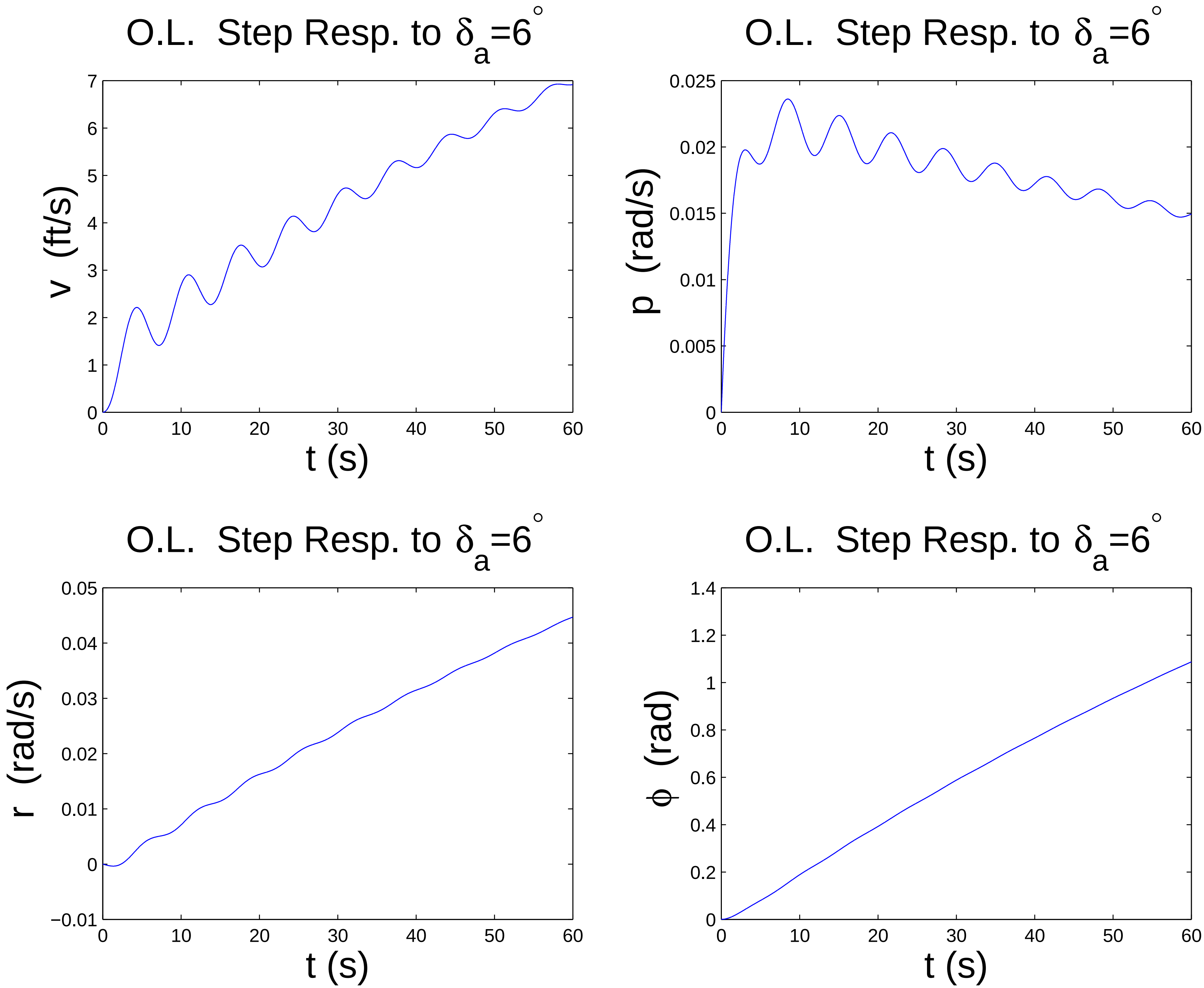


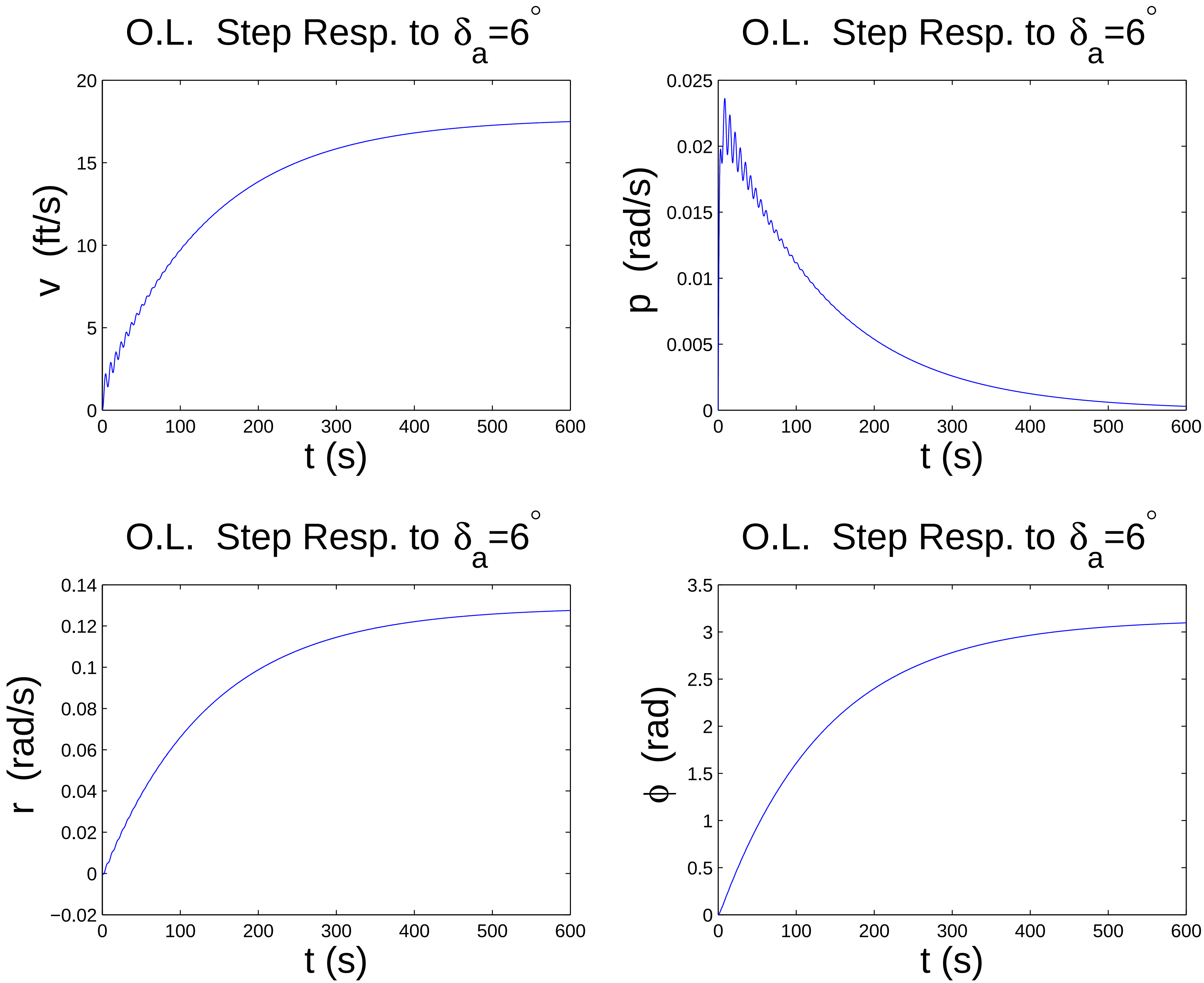
O.L. Step Resp. to δe=1° + δp=0.16667
u (ft/s)
30
25
20
15
10
5
0
t (s)
0
2
4
6
8
10
O.L. Step Resp. to δe=1° + δp=0.16667
α (rad)
0
−0.005
−0.01
−0.015
−0.02
−0.025
−0.03
t (s)
O.L. Step Resp. to δe=1° + δp=0.16667
γ (rad)
0.01
0
−0.01
−0.02
−0.03
−0.04
−0.05
t (s)
O.L. Step Resp. to δe=1° + δp=0.16667
θ (rad)
0
−0.01
−0.02
−0.03
−0.04
−0.05
−0.06
−0.07
t (s)


O.L. Step Resp. to δ_e=1° + δ_p=0.16667
α (rad)
t (s)
0
−0.005
−0.01
−0.015
−0.02
−0.025
−0.03
0
10
20
30
40
50
60

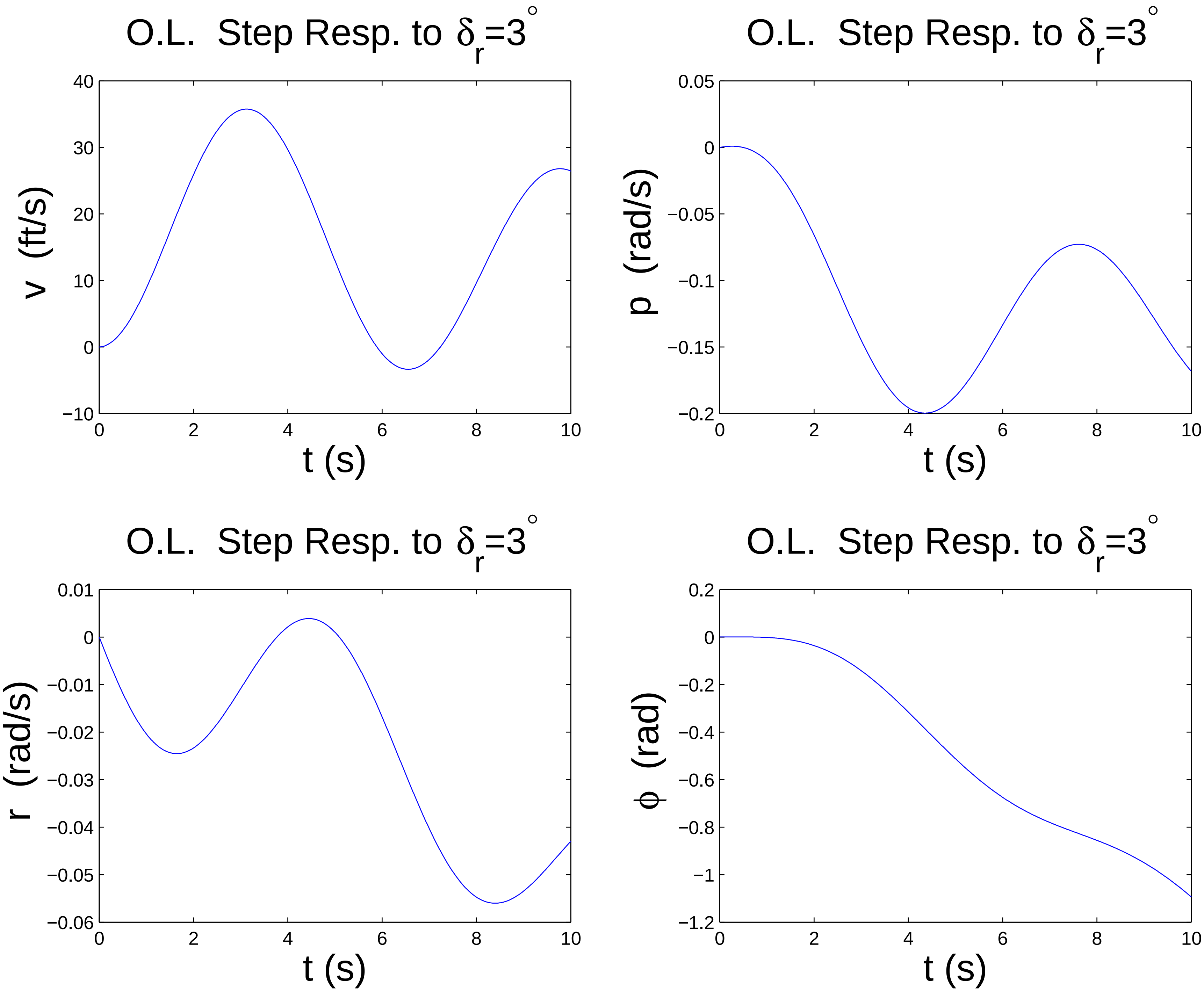

O.L. Step Resp. to δ_e=1° + δ_p=0.16667
100
80
60
40
20
0
u (ft/s)
0 100 200 300 400 500 600
t (s)
O.L. Step Resp. to δ_e=1° + δ_p=0.16667
0
−0.005
−0.01
−0.015
−0.02
−0.025
−0.03
α (rad)
0 100 200 300 400 500 600
t (s)
O.L. Step Resp. to δ_e=1° + δ_p=0.16667
0.15
0.1
0.05
0
−0.05
−0.1
γ (rad)
0 100 200 300 400 500 600
t (s)
O.L. Step Resp. to δ_e=1° + δ_p=0.16667
0.15
0.1
0.05
0
−0.05
−0.1
θ (rad)
0 100 200 300 400 500 600
t (s)


O.L. Step response to δ_a=6°
v (ft/s)
t (s)
O.L. Step response to δ_a=6°
p (rad/s)
t (s)
O.L. Step response to δ_a=6°
x 10^−3
r (rad/s)
t (s)
O.L. Step response to δ_a=6°
ϕ (rad)
t (s)

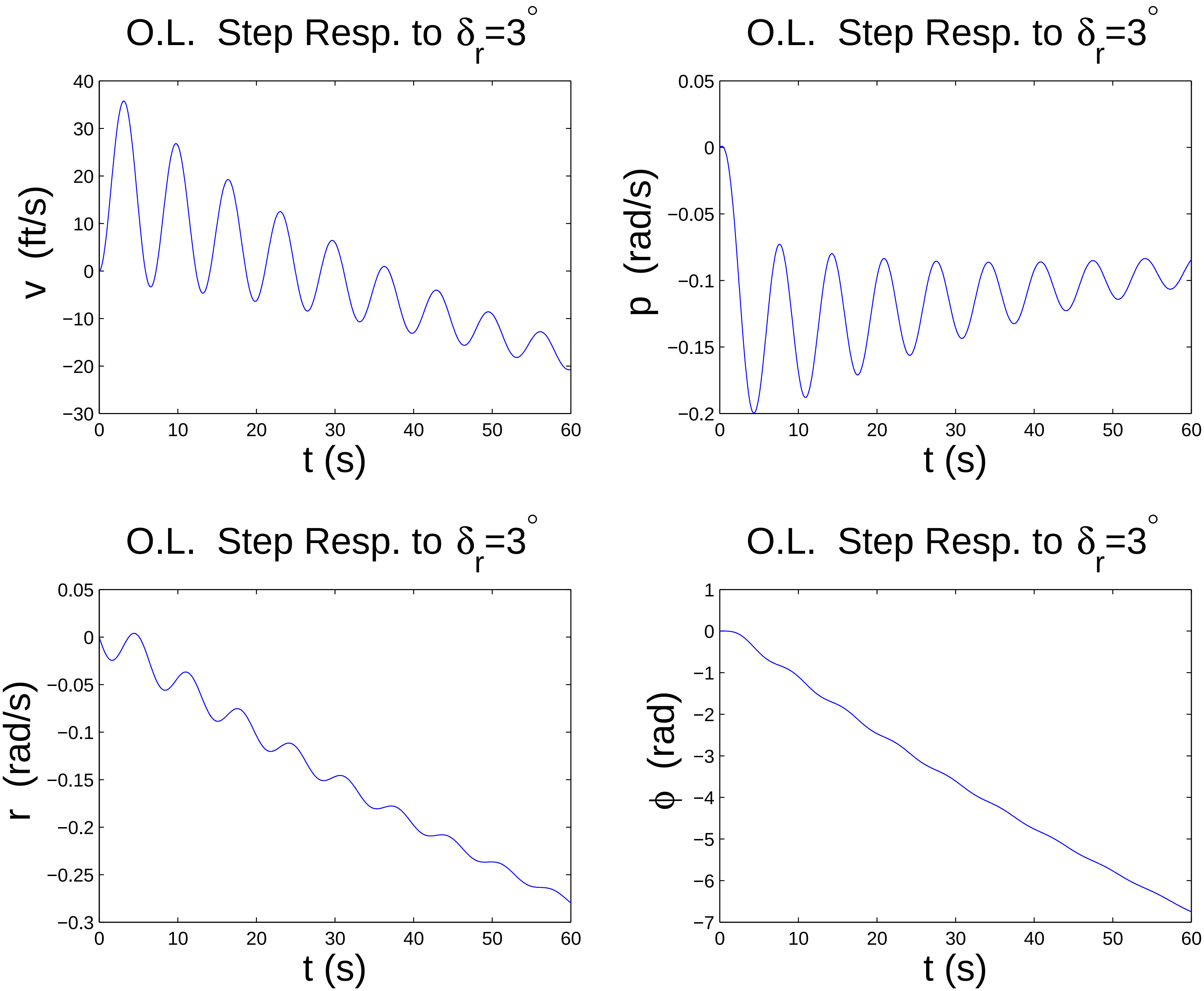

O.L. Step Resp. to δa=6°
v (ft/s)
t (s)
O.L. Step Resp. to δa=6°
p (rad/s)
t (s)
O.L. Step Resp. to δa=6°
r (rad/s)
t (s)
O.L. Step Resp. to δa=6°
φ (rad)
t (s)



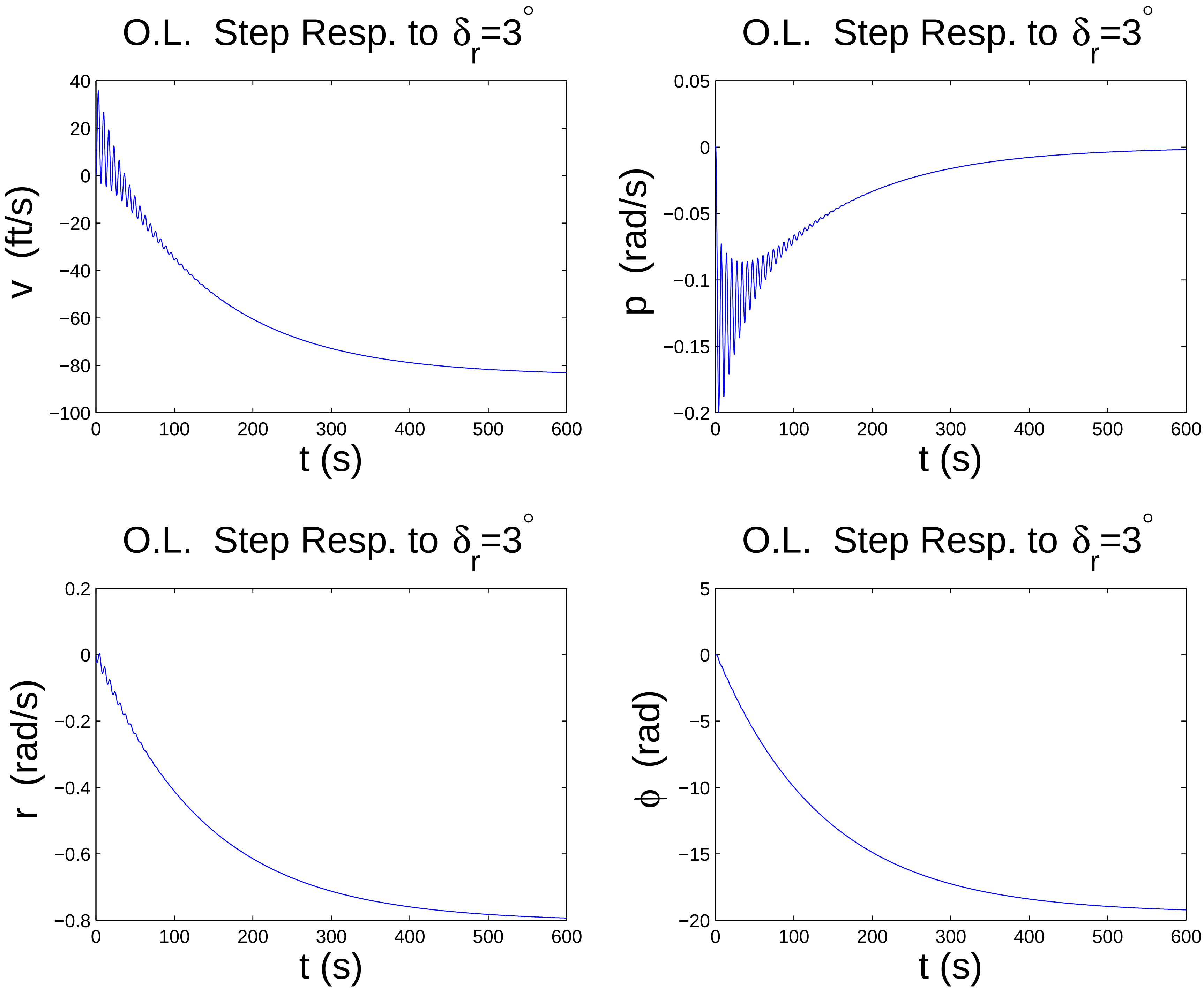
O.L. Step Resp. to $\delta_a=6^\circ$
v (ft/s)
t (s)
O.L. Step Resp. to $\delta_a=6^\circ$
p (rad/s)
t (s)
O.L. Step Resp. to $\delta_a=6^\circ$
r (rad/s)
t (s)
O.L. Step Resp. to $\delta_a=6^\circ$
ϕ (rad)
t (s)



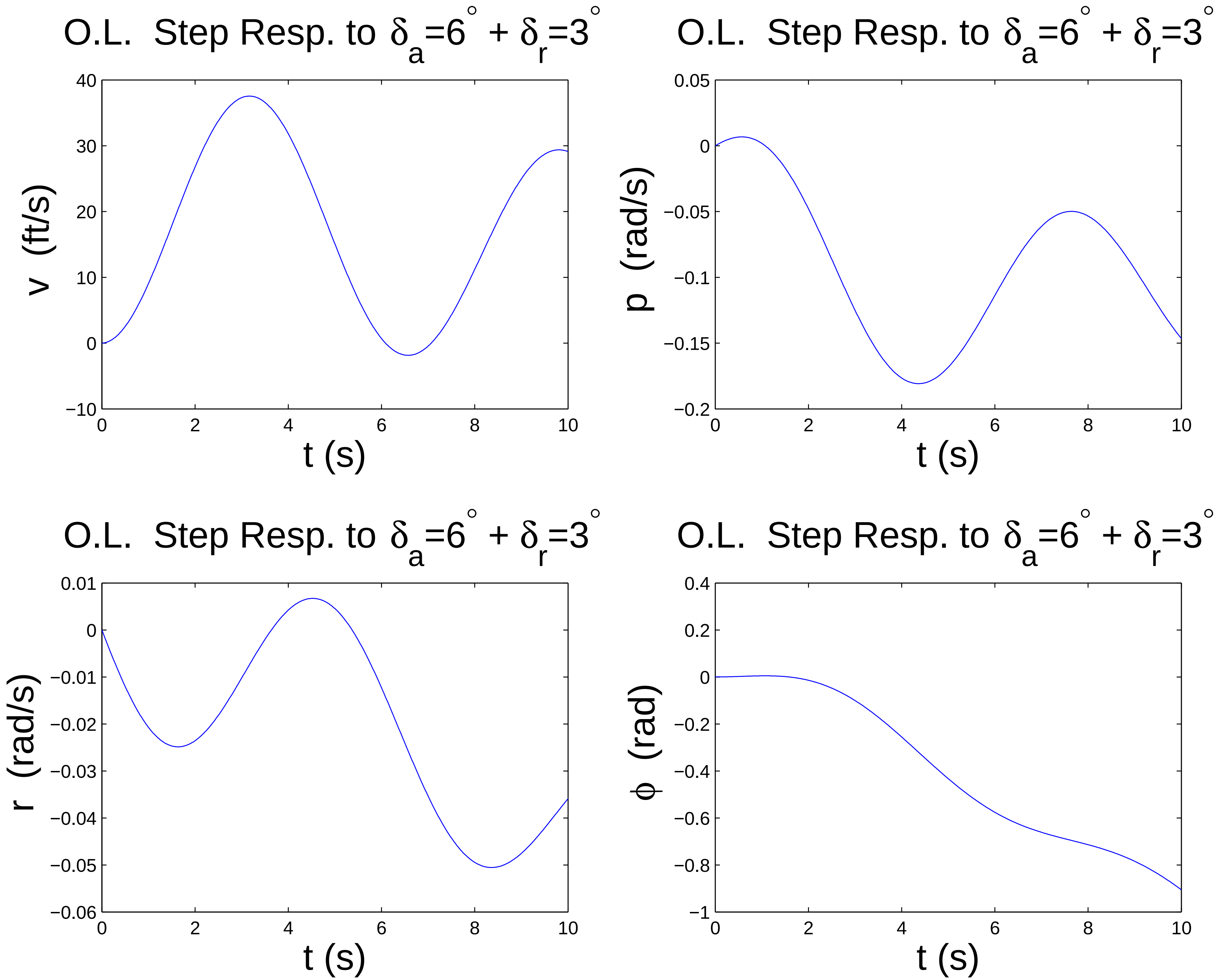
O.L. Step Resp. to δr=3°
v (ft/s)
40
30
20
10
0
−10
0
2
4
6
8
10
t (s)
O.L. Step Resp. to δr=3°
p (rad/s)
0.05
0
−0.05
−0.1
−0.15
−0.2
0
2
4
6
8
10
t (s)
O.L. Step Resp. to δr=3°
r (rad/s)
0.01
0
−0.01
−0.02
−0.03
−0.04
−0.05
−0.06
0
2
4
6
8
10
t (s)
O.L. Step Resp. to δr=3°
φ (rad)
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
−1.2
0
2
4
6
8
10
t (s)

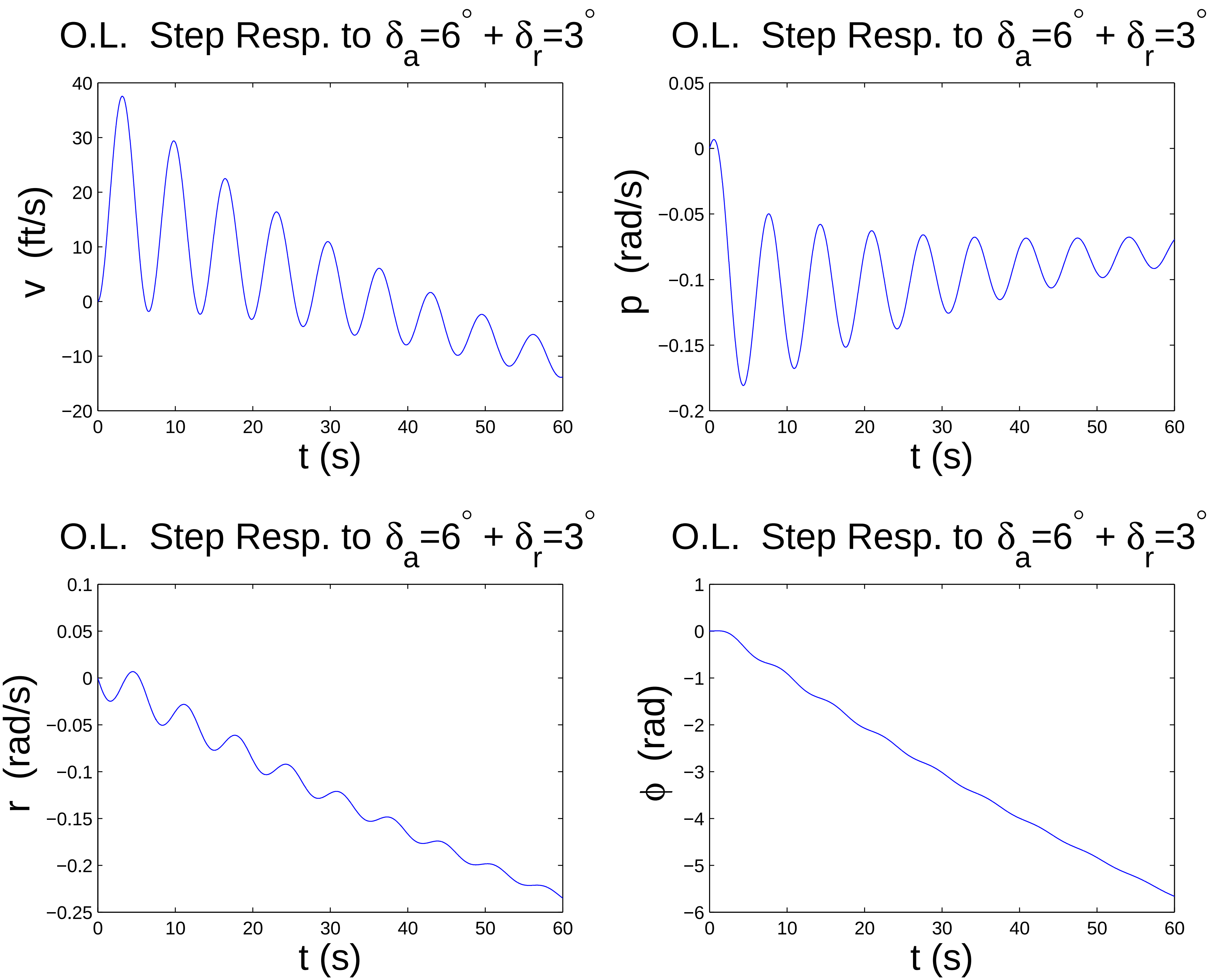


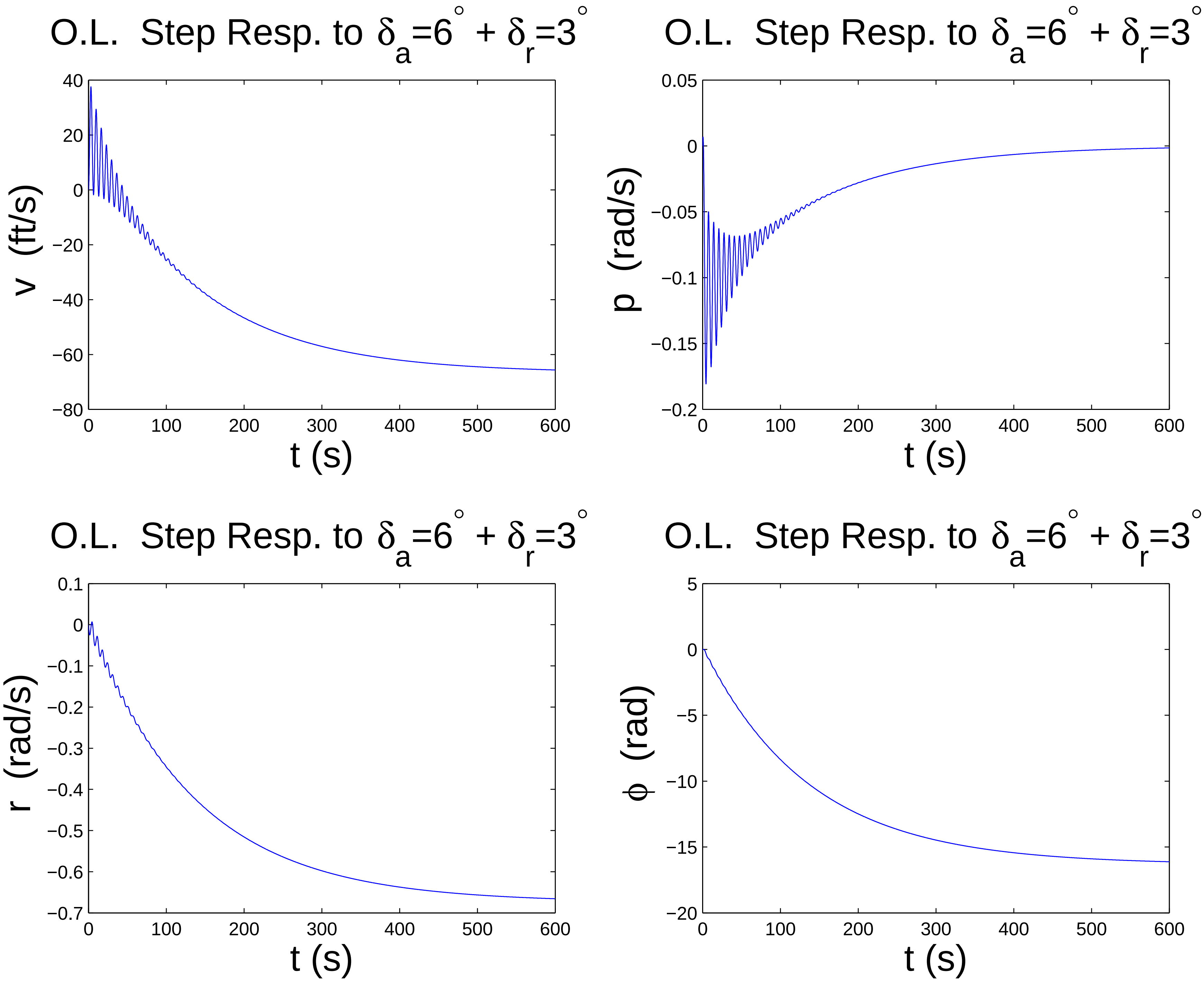
O.L. Step Resp. to δr=3°
v (ft/s)
t (s)
O.L. Step Resp. to δr=3°
p (rad/s)
t (s)
O.L. Step Resp. to δr=3°
r (rad/s)
t (s)
O.L. Step Resp. to δr=3°
φ (rad)
t (s)

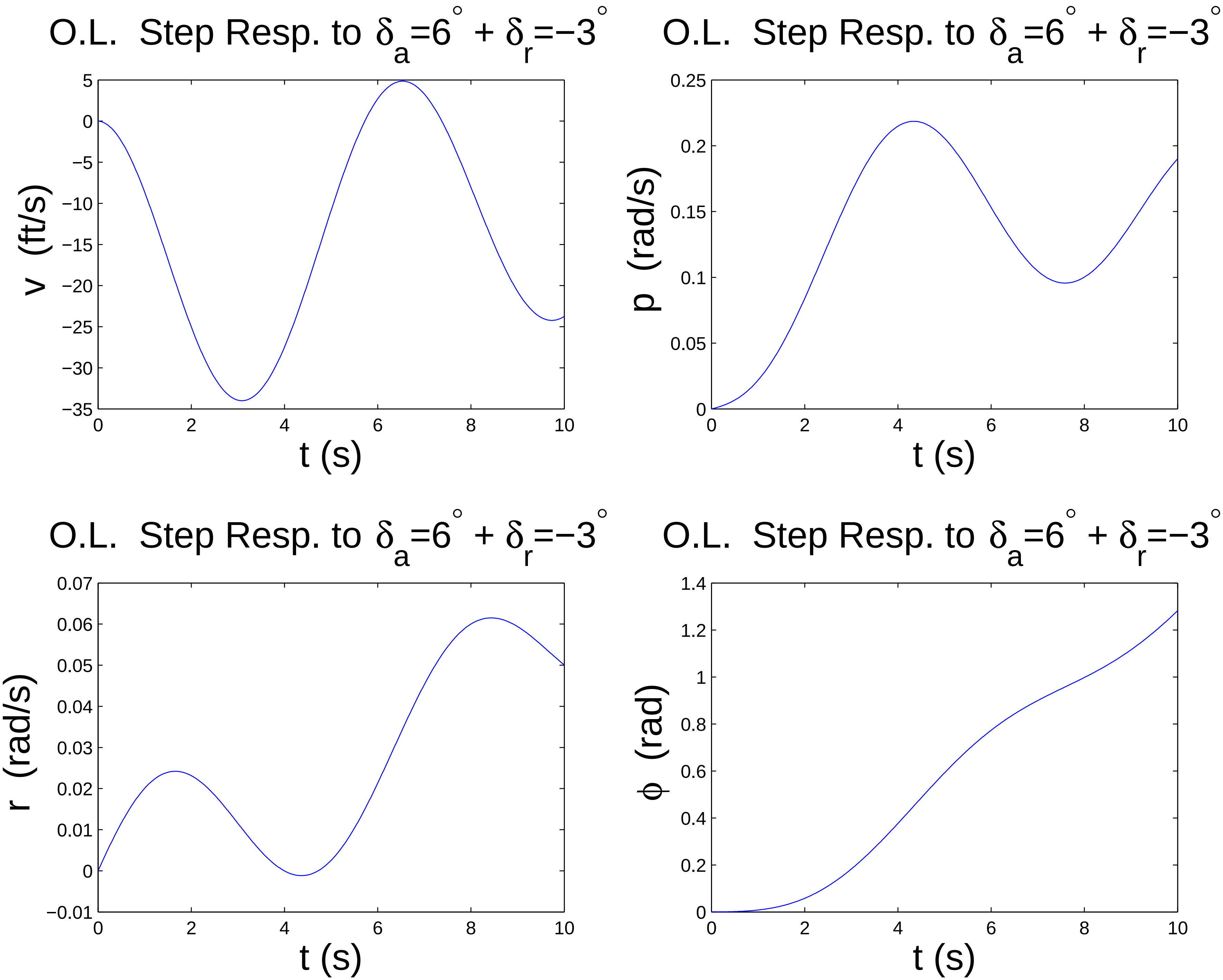


O.L. Step Resp. to δr=3°
v (ft/s)
t (s)
O.L. Step Resp. to δr=3°
p (rad/s)
t (s)
O.L. Step Resp. to δr=3°
r (rad/s)
t (s)
O.L. Step Resp. to δr=3°
φ (rad)
t (s)

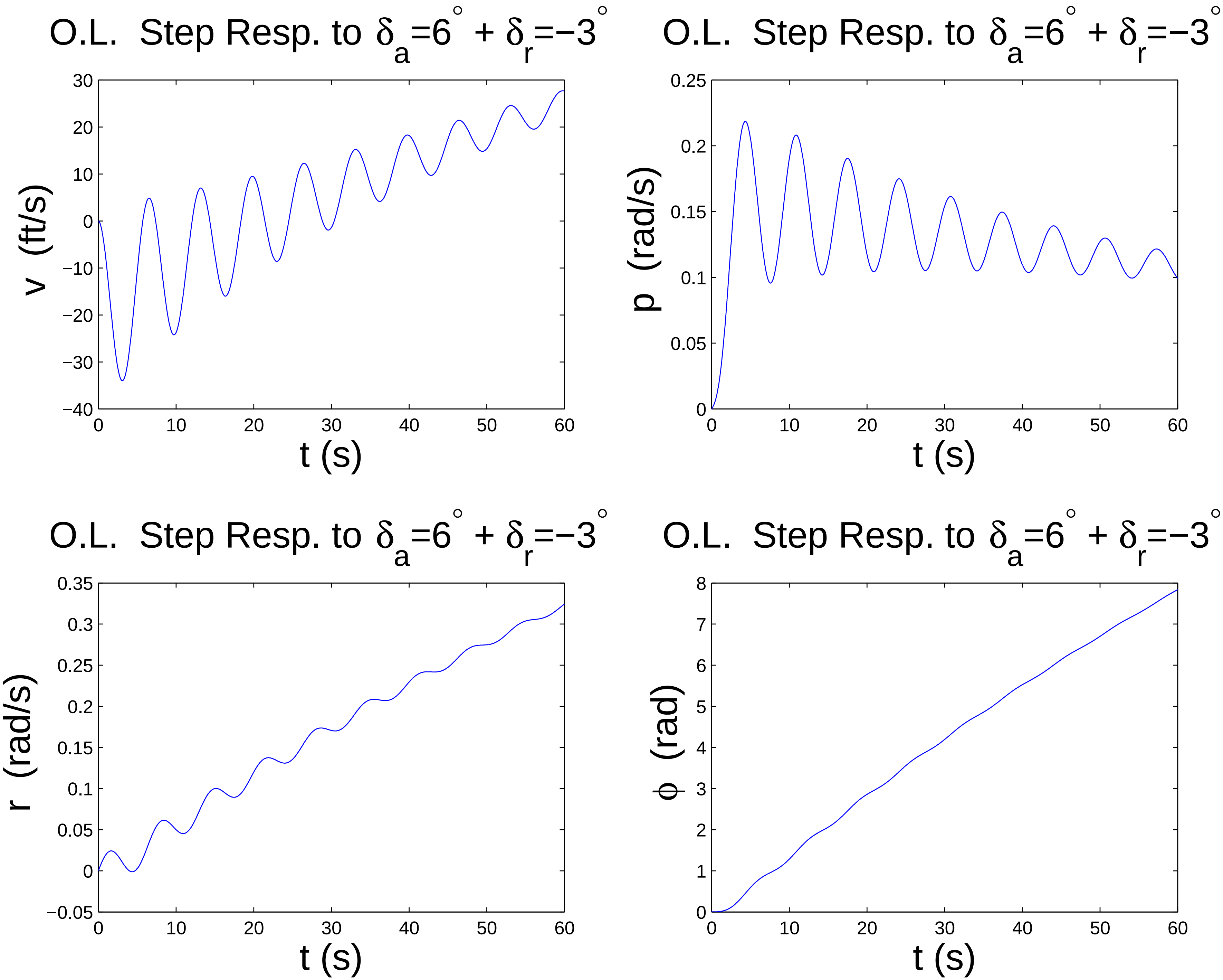

O.L. Step Resp. to δa=6° + δr=3°
v (ft/s)
t (s)
O.L. Step Resp. to δa=6° + δr=3°
p (rad/s)
t (s)
O.L. Step Resp. to δa=6° + δr=3°
r (rad/s)
t (s)
O.L. Step Resp. to δa=6° + δr=3°
φ (rad)
t (s)

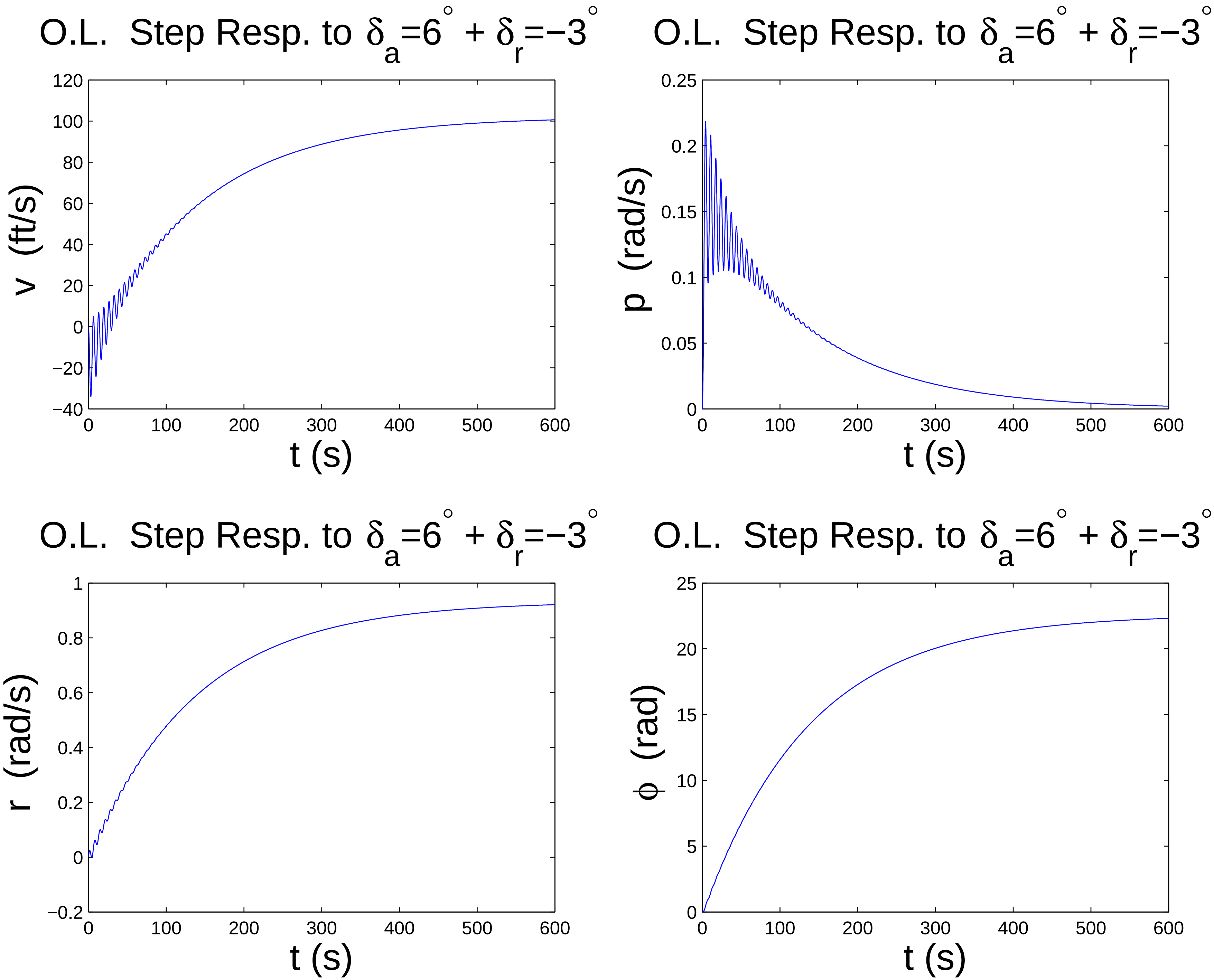

O.L. Step Resp. to δa=6° + δr=3°
v (ft/s)
40
30
20
10
0
−10
−20
t (s)
0
10
20
30
40
50
60
O.L. Step Resp. to δa=6° + δr=3°
p (rad/s)
0.05
0
−0.05
−0.1
−0.15
−0.2
t (s)
0
10
20
30
40
50
60
O.L. Step Resp. to δa=6° + δr=3°
r (rad/s)
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
−0.25
t (s)
0
10
20
30
40
50
60
O.L. Step Resp. to δa=6° + δr=3°
φ (rad)
1
0
−1
−2
−3
−4
−5
−6
t (s)
0
10
20
30
40
50
60


O.L. Step Resp. to δa=6° + δr=3°
v (ft/s)
40
20
0
−20
−40
−60
−80
0
100
200
300
400
500
600
t (s)
O.L. Step Resp. to δa=6° + δr=3°
p (rad/s)
0.05
0
−0.05
−0.1
−0.15
−0.2
0
100
200
300
400
500
600
t (s)
O.L. Step Resp. to δa=6° + δr=3°
r (rad/s)
0.1
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
0
100
200
300
400
500
600
t (s)
O.L. Step Resp. to δa=6° + δr=3°
ϕ (rad)
5
0
−5
−10
−15
−20
0
100
200
300
400
500
600
t (s)



O.L. Step Resp. to δa=6° + δr=−3°
v (ft/s)
5
0
−5
−10
−15
−20
−25
−30
−35
t (s)
0
2
4
6
8
10
O.L. Step Resp. to δa=6° + δr=−3°
p (rad/s)
0.25
0.2
0.15
0.1
0.05
0
t (s)
O.L. Step Resp. to δa=6° + δr=−3°
r (rad/s)
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
−0.01
t (s)
O.L. Step Resp. to δa=6° + δr=−3°
ϕ (rad)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
t (s)


O.L. Step Resp. to δa=6° + δr=−3°
v (ft/s)
t (s)
30
20
10
0
−10
−20
−30
−40
0
10
20
30
40
50
60
O.L. Step Resp. to δa=6° + δr=−3°
p (rad/s)
t (s)
0.25
0.2
0.15
0.1
0.05
0
O.L. Step Resp. to δa=6° + δr=−3°
r (rad/s)
t (s)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
−0.05
O.L. Step Resp. to δa=6° + δr=−3°
φ (rad)
t (s)
8
7
6
5
4
3
2
1
0



O.L. Step Resp. to δa=6° + δr=−3°
v (ft/s)
120
100
80
60
40
20
0
−20
−40
0 100 200 300 400 500 600
t (s)
O.L. Step Resp. to δa=6° + δr=−3°
p (rad/s)
0.25
0.2
0.15
0.1
0.05
0
0 100 200 300 400 500 600
t (s)
O.L. Step Resp. to δa=6° + δr=−3°
r (rad/s)
1
0.8
0.6
0.4
0.2
0
−0.2
0 100 200 300 400 500 600
t (s)
O.L. Step Resp. to δa=6° + δr=−3°
ϕ (rad)
25
20
15
10
5
0
0 100 200 300 400 500 600
t (s)