$$\mathbf{h}_B = \mathbf{I}_B \boldsymbol{\omega}_B + \mathbf{h}_B' \tag{8}$$

From (4.5,5)

$$\mathbf{G}_B = \dot{\mathbf{h}}_B + \tilde{\boldsymbol{\omega}}_B \mathbf{h}_B \tag{9}$$

Thus from (8) and (9) the additional terms in the moment equations due to spinning rotors are (if we assume $\dot{\mathbf{h}}_B' = 0$)

$$\tilde{\boldsymbol{\omega}} h_B' = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{bmatrix} h_x' \\ h_y' \\ h_z' \end{bmatrix}$$

$$= \begin{bmatrix} qh_z' - rh_y' \\ rh_x' - ph_z' \\ ph_y' - qh_x' \end{bmatrix} \tag{10}$$

as given by (4.6,2).

4.6 In (4.10,3) let $\pi = T_s \frac{u_o}{\ell}$ where T_s is the characteristic time of the spiral divergence. Since we are told to ignore M and RN, consider the nondimensional combinations $\frac{m}{\rho \ell^3}$ and $\frac{{u_o}^2}{\ell g}$.

For dynamic similarity, these groupings must be the same for the two aircraft.

(a) Let $(\ )_A$ be aircraft A values. Equating the above two groupings for the two aircraft leads to

$$\frac{m_A}{\rho_A {\ell_A}^3} = \frac{m_B}{\rho_B {\ell_B}^3} \tag{1}$$

and

$$\frac{u_{oA}^2}{\ell_A g} = \frac{u_{oB}^2}{\ell_B g} \tag{2}$$

From App. D, at 20,000 ft

$$\rho_B = 1.2673 \times 10^{-3} \text{ slug/ft}^3$$

Thus from (1)

$$\rho_A = \frac{m_A}{m_B} \cdot \left(\frac{\ell_B}{\ell_A}\right)^3 \cdot \rho_B \tag{3}$$

where

$$\frac{m_A}{m_B} = 100{,}000/225{,}000$$

$$= 0.444$$

$$\frac{\ell_B}{\ell_A} = \frac{150}{100}$$

$$= 1.5$$

Hence, from (3),

$$\rho_A = 0.444 \times (1.5)^3 \times 1.2673 \times 10^{-3}$$

$$= 1.899 \times 10^{-3} \text{ slug/ft}^3$$

From App. D the altitude corresponding to ρ_A is 7,500 ft.

From (2)

$$u_{oA}^2 = \frac{\ell_A}{\ell_B} u_{oB}^2 \tag{4}$$

Thus

$$u_{oA}^2 = \frac{1}{1.5} \times (400)^2$$

or

$$u_{oA} = 326.6 \text{ knots}$$

Thus A will be dynamically similar to B at 7,500 ft altitude and a speed of 326.6 knots.

(b) From $\pi = T_s \frac{u_o}{\ell}$, under conditions where the two aircraft are dynamically similar $\pi_A = \pi_B$.

Thus

$$T_{sA} \frac{u_{oA}}{\ell_A} = T_{sB} \frac{u_{oB}}{\ell_B} \qquad (5)$$

Hence

$$T_{sA} = T_{sB} \frac{u_{oB}}{u_{oA}} \cdot \frac{\ell_A}{\ell_B}$$

$$= 20 \times \frac{400}{326.6} \cdot \frac{100}{150}$$

$$= 16.33 \text{ seconds}$$

(c)

$$C_{LA} = \frac{W_A}{\frac{1}{2} \rho_A u_{oA}^2 S_A} \qquad (6)$$

$$C_{LB} = \frac{W_B}{\frac{1}{2} \rho_B u_{oB}^2 S_B} \qquad (7)$$

Thus

$$\frac{C_{L_A}}{C_{L_B}} = \frac{W_A}{W_B} \cdot \frac{\rho_B}{\rho_A} \cdot \frac{u_{oB}^2}{u_{oA}^2} \cdot \frac{S_B}{S_A}$$

$$= \frac{100{,}000}{225{,}000} \cdot \frac{1.2673 \times 10^{-3}}{1.899 \times 10^{-3}} \cdot \left(\frac{400}{326.6}\right)^2 \left(\frac{150}{100}\right)^2$$

$$= 1$$

as expected, since C_L is also a nondimensional combination, and as such must also be the same for dynamic similarity.

4.7 From (4.9,17)

$$\Delta Z = Z_u \Delta u + Z_w w + Z_{\dot{w}} \dot{w} + Z_q q + \Delta Z_c \tag{1}$$

$$\Delta M = M_u \Delta u + M_w w + M_{\dot{w}} \dot{w} + M_q q + \Delta M_c \tag{2}$$

From (4.9,7c) and (4.9,8b)

$$\dot{w} = \frac{\Delta Z}{m} - g\Delta\theta \sin\theta_o + u_o q \tag{3}$$

$$\dot{q} = \frac{\Delta M}{I_y} \tag{4}$$

Substitute (1) and (2) into (3) and (4)

$$\dot{w} = \frac{Z_u}{m}\Delta u + \frac{Z_w w}{m} + \frac{Z_{\dot{w}}\dot{w}}{m} + \frac{Z_q q}{m} + \frac{\Delta Z_c}{m} - g\Delta\theta \sin\theta_o + u_o q \tag{5}$$

$$\dot{q} = \left[M_u \Delta u + M_w w + M_{\dot{w}} \dot{w} + M_q q + \Delta M_c\right] / I_y \tag{6}$$

From (5)

$$\dot{w}(m - Z_{\dot{w}}) = Z_u\Delta u + Z_w w + (Zq + mu_o)q - mg\Delta\theta \sin\theta_o + \Delta Z_c \tag{7}$$

and (7) is the second component of (4.9,18). Substitute (7) into (6) to eliminate $\dot{w}$

$$\dot{q}I_y = M_u\Delta u + \frac{M_{\dot{w}}Z_u\Delta u}{m - Z_{\dot{w}}} + M_w w + \frac{M_{\dot{w}}Z_w w}{m - Z_{\dot{w}}}$$

$$+ M_q q + M_{\dot{w}}\frac{(Zq + mu_o)q}{m - Z_{\dot{w}}} - \frac{M_{\dot{w}}mg\Delta\theta \sin\theta_o}{m - Z_{\dot{w}}}$$

$$+ \Delta M_c + M_{\dot{w}}\frac{\Delta Z_c}{m - Z_{\dot{w}}} \tag{8}$$

and (8) is the third component of (4.9,18).

4.8 $\underline{X_q}$

Follow the method used in the text (Sec. 4.11) to generate Z_q

$$X_q = \left(\frac{\partial X}{\partial q}\right)_o \tag{1}$$

where

$$X = C_x \frac{1}{2}\rho V^2 S \tag{2}$$

Thus

$$X_q = \frac{1}{2}\rho u_o^2 S \left(\frac{\partial C_x}{\partial q}\right)_o \tag{3}$$

since $V_o = u_o$.

4.10 Assume that $\phi_0 = \theta_0 = 0$. Thus making the usual assumptions

$$Z_0 = -mg + Z_z z_{E_0} \tag{1}$$

$$\Delta Z = Z_z \Delta z_E \tag{2}$$

$$\Delta X = \Delta Y = \Delta N = 0 \tag{3}$$

$$\Delta L = L_\phi \phi \tag{4}$$

$$\Delta M = M_\theta \theta \tag{5}$$

From the above and (4.9,7)

$$\Delta \dot{u} = -g\theta \tag{6}$$

$$\dot{v} = +g\phi - u_0 r \tag{7}$$

$$\dot{w} = \frac{Z_z}{m} \Delta z_E + u_0 q \tag{8}$$

From (4.9,10c)

$$\Delta \dot{z}_E = -u_0 \theta + w \tag{9}$$

From (4.9,9)

$$\dot{\phi} = p \tag{10}$$

$$\dot{\theta} = q \tag{11}$$

$$\dot{\psi} = r \tag{12}$$

When the moment equations of (4.7,2) are linearized to produce (4.9,3) the $\mathbf{h}'_B$ were dropped. In the present problem $h'_x = h'_y = 0$ but $h'_z = H$ and thus should be retained in (4.9,3). Also, since the body axes are principal axes it follows that $I_{zx} = 0$. Thus the linearized moment equations become (since $L_o = M_o = N_o = 0$)

$$L_\phi \phi = I_x \ddot{\phi} + qH \tag{13}$$

$$M_\theta \theta = I_y \ddot{\theta} - pH \tag{14}$$

$$0 = I_z \dot{r} \tag{15}$$

From (15) $\dot{r} = 0$, thus if we start up with $\psi = r = 0$ then they will remain equal to zero. Thus if this were true we could drop (12) and (15) and set $r = 0$ in (7).

4.11 Assume that **W** is the wind as seen at the CG of the aircraft. Next make the <u>point approximation</u>, that is, the aircraft is small compared with any spatial variations in the wind. This means that the wind at any value of time is uniform over the complete aircraft. Thus from (1.6,1)

$$\mathbf{V}^E = \mathbf{V} + \mathbf{W} \tag{1}$$

and the angular velocity $\boldsymbol{\omega}$ used in calculating aerodynamic forces and moments is unchanged from the text. Recall that the forces and moments also depend on **V**, the airspeed. From (4.2,15)

$$\mathbf{f}_E = m\dot{\mathbf{V}}_E^E \tag{2}$$

4.12 For this problem

(a) $\psi = \dot{\psi} \equiv 0$ since it is a vertical circle.

(b) $\dot{\theta} = \frac{V}{R}$ for circular loop of radius R.

(c) At the bottom of the loop we are given $\phi = \theta = \psi = 0$.

From (4.7,3) it follows from the above that at the bottom of the loop:

$$\begin{aligned} p &= \dot{\phi} - \dot{\psi} \sin\theta \\ &= \dot{\phi} = \frac{\pi}{2} \text{ rad/s} \end{aligned} \tag{1}$$

$$\begin{aligned} q &= \dot{\theta} \cos\phi + \dot{\psi} \cos\theta \sin\phi \\ &= \dot{\theta} = \frac{V}{R} \end{aligned} \tag{2}$$

$$\begin{aligned} r &= \dot{\psi} \cos\theta \cos\phi - \dot{\theta} \sin\phi \\ &= 0 \end{aligned} \tag{3}$$

To evaluate the moment equations we need $\dot{p}, \dot{q}, \dot{r}$. Obtain these by differentiating (4.7,3) and using (a), (b), (c), (1), (2) and (3)

$$\begin{aligned} \dot{p} &= \ddot{\phi} \\ &= 0 \end{aligned} \tag{4}$$

$$\dot{q} = \ddot{\theta} \cos\phi - \dot{\theta}\dot{\phi} \sin\phi = 0 \tag{5}$$

$$\dot{r} = -\ddot{\theta}\sin\phi - \dot{\theta}\dot{\phi}\cos\phi = -\dot{\theta}\dot{\phi} \qquad (6)$$

From (4.7,2) and the above, and making use of $I_{zx} = 0$ and $\mathbf{h}' = \mathbf{0}$:

$$L = 0 \qquad (7)$$

$$M = 0 \qquad (8)$$

$$N = I_z\dot{r} + pq(I_y - I_x) \qquad (9)$$

From (1), (2), (6) and (9)

$$N = \dot{\theta}\dot{\phi}[I_y - I_x - I_z]$$

$$= \frac{V}{R}\cdot\frac{\pi}{2}[I_y - I_x - I_z]$$

$$= \frac{500}{2000}\cdot\frac{\pi}{2}[300 - 500]$$

$$= -78.54 \quad \text{ft.lb} \qquad (10)$$