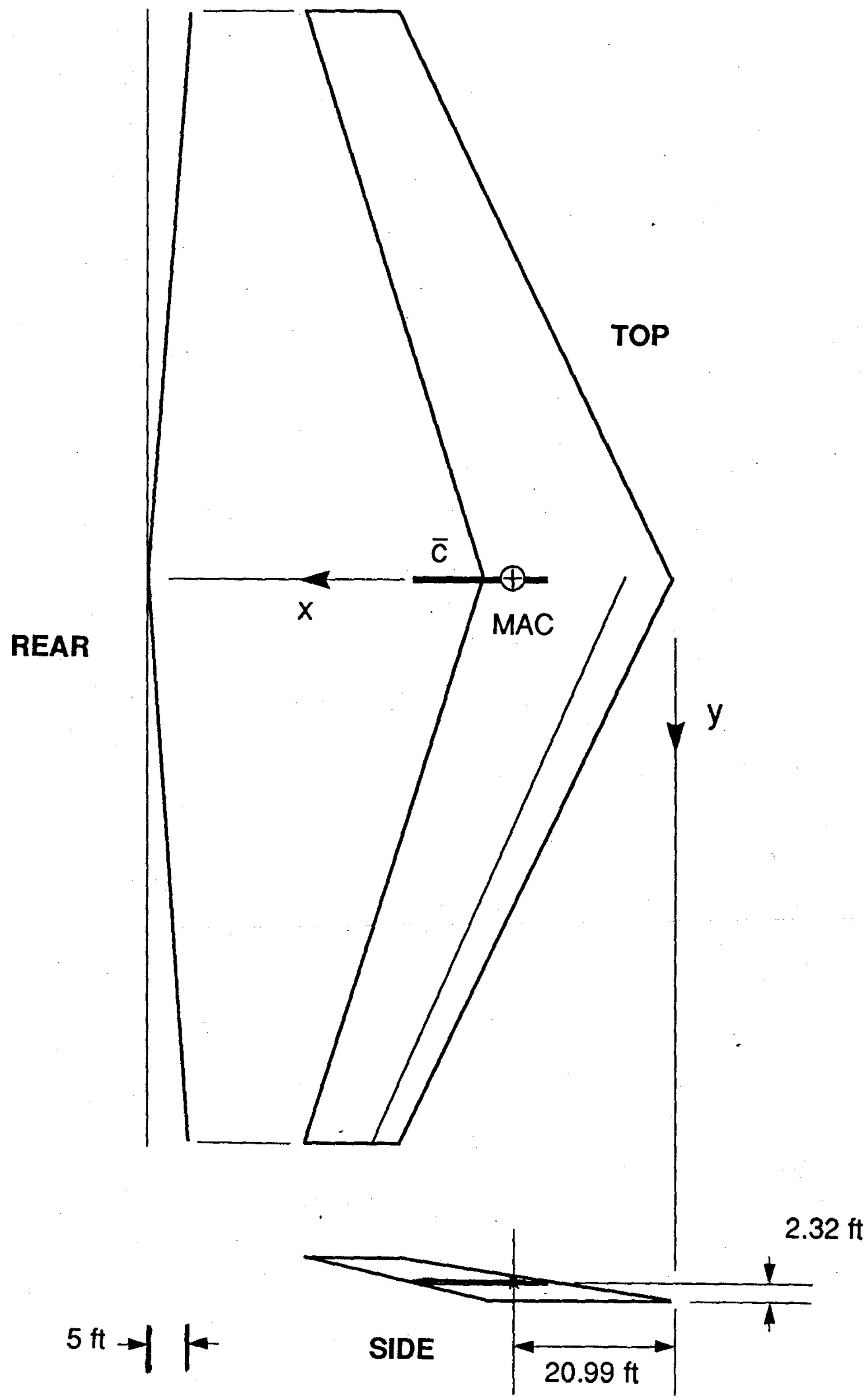
CHAPTER 2

2.1 (a)



(b) Each wing panel is a trapezoid, thus

$$S = 2\left[\frac{1}{2}(c_t + c_r)\,\frac{b}{2}\right]$$

$$= \frac{1}{2}\,[(12 + 25)\;150]$$

$$= 2{,}775 \text{ ft}^2 \;(257.81 \text{ m}^2)$$

From App. C, Sec. C.2

$$A = b^2/S$$

$$= (150)^2/2775$$

$$= 8.11$$

$$\lambda = c_t/c_r$$

$$= 12/25$$

$$= 0.48$$

From Table C.1

$$\bar{c} = \frac{2c_r}{3}\;\frac{1 + \lambda + \lambda^2}{1 + \lambda}$$

$$= \frac{2 \times 25}{3}\;\frac{1 + .48 + .48^2}{1 + .48}$$

$$\bar{c} = 19.26 \text{ ft } (5.87 \text{ m})$$

(c) From Table C.1 for uniform C_{ℓ_a}

$$\bar{y} = \frac{b}{2}\,\frac{1 + 2\lambda}{3(1 + \lambda)}$$

$$= \frac{150}{2}\,\frac{1 + 2 \times .48}{3(1 + .48)}$$

$$= 33.11 \text{ ft } (10.09 \text{ m})$$

for the right half wing. [$\bar{y} = 0$ for the complete wing (see Sec. C.1)].

From (C.1,5) for $n = \frac{1}{4}$

$$\bar{x} = \frac{c_r}{4} + \bar{y} \tan \Lambda_{1/4}$$

Use $\bar{y}$ = for the right half wing. From diagram 2.1 $\Lambda_{1/4} = 24°$. Thus

$$\bar{x} = \frac{25}{4} + 33.11 \tan 24°$$

$$= 20.99 \text{ ft } (6.40 \text{ m})$$

From (C.1,4)

$$\bar{z} = \frac{2}{C_L S} \int_0^{b/2} C_{\ell_a} c z \, dy$$

but $C_{\ell_a} = C_L$ = constant at given α, thus

$$\bar{z} = \frac{2}{S}\int_0^{b/2} czdy$$

Now for the right half wing

$$c(y) = 25 - \frac{13}{75}y = 25 - 0.173y$$

$$z(y) = y \tan 4^\circ = 0.0699y$$

$$\int czdy = \int (1.748y - 0.01209y^2)dy$$

$$= \frac{1.748y^2}{2} - \frac{0.01209y^3}{3} + K$$

$$\therefore \quad \bar{z} = \frac{2}{2775}\left[\frac{1.748}{2}y^2 - \frac{0.01209}{3}y^3\right]_0^{75}$$

$$= 2.32 \text{ ft } (0.707 \text{ m})$$

The 1/4-chord point of $\bar{c}$ is placed at $(\bar{x}, \bar{y}, \bar{z})$ as shown in diagram 2.1.

(d) From (2.3,6) the control-fixed static margin is

$$K_n = (h_n - h) \tag{1}$$

From (2.3,23) (ignoring propulsion effects)

$$h_n = h_{n_{wb}} + \frac{a_t}{a}\bar{V}_H\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right) \qquad (2)$$

From (2.2,10)

$$\bar{V}_H = \frac{\bar{\ell}_t S_t}{\bar{c}S} \qquad (3)$$

From (2.3,18)

$$a = a_{wb}\left[1 + \frac{a_t}{a_{wb}}\frac{S_t}{S}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right)\right] \qquad (4)$$

But we are given $a_t = a_{wb}$, thus (4) becomes

$$\frac{a}{a_t} = 1 + \frac{S_t}{S}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right) \qquad (5)$$

From the given fact that $h_{n_{wb}} = h_{n_w}$, diagram 2.1, Fig. 2.12 and the results $\bar{x}$ of part (c), at the aft CG location

NOTE: $X_{CG} \neq \bar{c}h$
$\bar{X} \neq \bar{c}h_{n_{wb}}$
BUT $(X_{CG} - \bar{X}) = \bar{c}(h - h_{n_{wb}})$

$$(X_{CG} - \bar{X}) = (\bar{c}h - \bar{c}h_{n_{wb}}) = 25 - 20.99$$

or

$$(h - h_{n_{wb}}) = \frac{4.01}{19.26} = 0.208 \qquad (6)$$

Since $K_n = 0.05$ at the aft CG location (where K_n is a minimum), therefore from (1) + (6)

$$h_n - h_{n_{wb}} = 0.05 + 0.208$$
$$= 0.258 \qquad (7)$$

From (2) and (7)

$$0.258 = \frac{a_t}{a}\bar{V}_H\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right) \qquad (8)$$

From (3), (5) and (8)

$$0.258\left[1 + \frac{S_t}{S}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right)\right] = \frac{\bar{\ell}_t S_t}{\bar{c}S}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right)$$

or

$$0.258\left[1 + \frac{S_t}{2775}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right)\right] = \frac{55S_t}{19.26 \times 2775}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right) \qquad (9)$$

For $\frac{\partial\varepsilon}{\partial\alpha} = 0.25$, (9) becomes

$$0.258 + 6.973 \times 10^{-5}S_t = 7.7180 \times 10^{-4}S_t$$

or

$$S_t = 367.5 \text{ ft}^2 \ (34.14 \text{ m}^2)$$

2.2 Assuming L = W

$$C_{L_w} = W \Big/ \left(\frac{1}{2}\rho V^2 S\right)$$

$$V = 180 \text{ m/s} = 590.6 \text{ ft/s}$$

2.

VERSION 1: $C_{m_\alpha} = a_{wb}(h - h_{n_{wb}}) - a_t V_H \left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) + \frac{\partial C_{m_p}}{\partial \alpha}$

VERSION 2: $C_{m_\alpha} = a(h - h_{n_{wb}}) - a_t \bar{V}_H \left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) + \frac{\partial C_{m_p}}{\partial \alpha}$

↑ NEGLECT

BY DEF.: $a \equiv a_{wb}\left[1 + \frac{a_t}{a_{wb}} \frac{S_t}{S}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right]$ ①

$\bar{V}_H \equiv V_H + (h - h_{n_{wb}}) \frac{S_t}{S}$ ②

PLUG ① & ② INTO VERSION 2:

$$C_{m_\alpha} = a_{wb}\left[1 + \frac{a_t}{a_{wb}} \frac{S_t}{S}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right](h - h_{n_{wb}}) +$$

$$- a_t \left[V_H + (h - h_{n_{wb}})\frac{S_t}{S}\right]\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) =$$

$$= a_{wb}(h - h_{n_{wb}}) - a_t V_H \left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right) = \text{VERSION 1}$$

$$0 = a(h_n - h_{n_{wb}}) - a_t\bar{V}_H\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right) + \frac{\partial C_{m_p}}{\partial\alpha}$$

or

$$h_n = h_{n_{wb}} + \frac{a_t}{a}\bar{V}_H\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right) - \frac{1}{a}\frac{\partial C_{m_p}}{\partial\alpha} \qquad (2.3,23)$$

2.5 (a) From (2.3,21a) ignoring propulsion effects

$$C_{m_\alpha} = a(h - h_{n_{wb}}) - a_t\bar{V}_H\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right) \qquad (1)$$

From (2.3,22a) ignoring propulsion effects

$$C_{m_0} = C_{m_{ac_{wb}}} + a_t\bar{V}_H(\varepsilon_0 + i_t)\left[1 - \frac{a_t}{a}\frac{S_t}{S}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right)\right] \qquad (2)$$

From (2.2,10)

$$\bar{V}_H = \frac{\bar{\ell}_t S_t}{\bar{c}S} \qquad (3)$$

From (2.3,18)

$$a = a_{wb}\left[1 + \frac{a_t}{a_{wb}}\frac{S_t}{S}\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right)\right] \qquad (4)$$

Evaluating (3)

$$\bar{V}_H = \frac{15.29 \times .368}{6.145 \times 1.50} = 0.6104$$

evaluating (4)

$$a = .077\left[1 + \frac{.064}{.077} \times \frac{.368}{1.50}(1 - .3)\right]$$

$$= .0880/\text{deg}$$

Setting $C_{m_\alpha} < 0$ and evaluating (1)

$$C_{m_\alpha} = .088(h - .25) - .064 \times .6104(1 - .3) < 0$$

or

$$h < 0.5607$$

This is the controls-fixed pitch stiffness boundary. The CG must be forward of the point represented by $h = 0.5607$.

Setting $C_{m_o} > 0$ and evaluating (2)

$$C_{m_o} = -.018 + .064 \times .6104(.72 + i_t)\left[1 - \frac{.064}{.088} \times \frac{.368}{1.50} \times (1 - .3)\right] > 0$$

or

$$i_t > -0.193°$$

The tail angle must be greater than -0.193° for the aircraft to be capable of trimmed flight with positive lift and positive pitch stiffness.

(b) For trimmed flight with $\delta_e = 0$, (2.3,20a) gives

$$C_m = C_{m_0} + C_{m_\alpha} \cdot \alpha = 0 \tag{5}$$

In level unaccelerated flight

$$L = W = a \cdot \alpha \cdot \frac{1}{2} \rho V^2 S$$

thus

$$\alpha = 2W/(\rho V^2 S a)$$

$$W = mg = 22{,}680 \times 9.81 = 222{,}491 \text{ N}$$

full scale

$$S = (25)^2 \times .139 \text{ m}^2$$
$$= 86.875 \text{ m}^2$$

thus

$$\alpha = \frac{2 \times 222{,}491}{1.225 \times (123)^2 \times 86.875 \times .088}$$
$$= 3.141°$$

Combining (1), (2) and (5) with the numerical data gives

$$-.018 + .0342\,(.72 + i_t) + [.088(h - .25) - .0273]\,3.141 = 0$$

or

$$.0342\, i_t + .276\, h = .1482$$

or

$$i_t = 4.33 - 8.07\, h \quad \text{deg.}$$

From diagram 2.5 it can be seen that as h moves rearward (h becomes larger) the plot of possible (h, i_t) hits the i_t boundary just at (0.560, -0.193°). Note that for this example C_{m_0} and C_{m_α} are both approximately zero at h = 0.560.

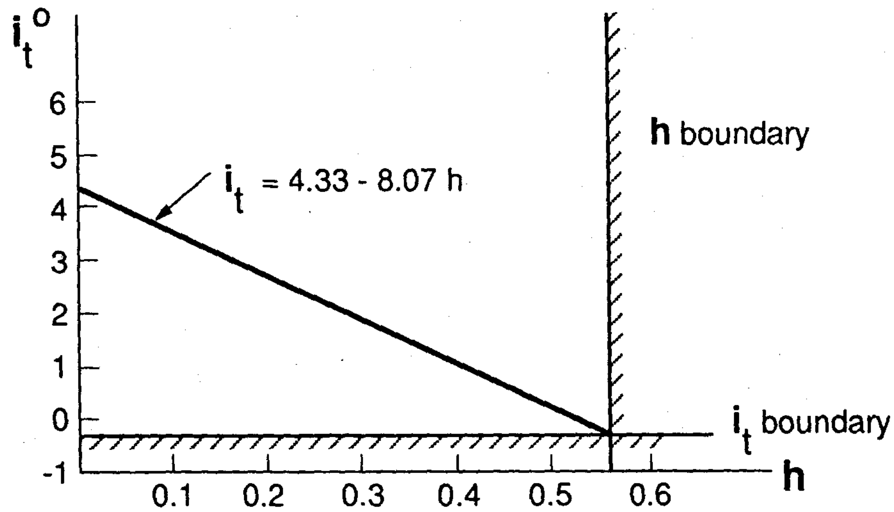


Diagram 2.5

2.6 (a) From App. C, Sec. C.2

$$A = b^2/S$$

$$= \frac{165^2}{3800} = 7.16$$

$$\lambda = c_t/c_r$$

$$= 8.8/37.3 = 0.236$$

From Table C.1

$$\bar{c} = \frac{2c_r}{3}\,\frac{1 + \lambda + \lambda^2}{1 + \lambda}$$

$$= \frac{2 \times 37.3}{3}\,\frac{1 + .236 + .236^2}{1 + .236}$$

$$= 25.99 \text{ ft } (7.92 \text{ m})$$

(b) Use the equation included on Fig. B.1,2.

Here $\tan \Lambda_{c/2} = \tan 22°$

$$= 0.404$$

The component

$$\sqrt{\frac{A^2\beta^2}{\kappa^2}\left((1 + \frac{\tan^2 \Lambda_{c/2}}{\beta^2})\right) + 4}$$

$$= \left(7.16^2 \times \frac{1}{1}\left(1 + \frac{0.404^2}{1}\right) + 4\right)^{1/2}$$

$$= 7.977$$

Thus

$$a_w = C_{L_\alpha} = \frac{2\pi A}{2 + 7.977}$$

$$= \frac{2\pi \times 7.16}{9.977}$$

$$= 4.51/\text{rad}$$

(c)

$$\frac{\partial \varepsilon}{\partial \alpha} = \frac{2a_w}{\pi A}$$

$$= \frac{2 \times 4.51}{\pi \times 7.16}$$

$$= 0.40$$

$$a_t = 0.068/\text{deg} = 3.90/\text{rad}$$

From (2.3,18)

$$a = a_{wb}\left[1 + \frac{a_t}{a_{wb}}\frac{S_t}{S}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)\right]$$

$$= 4.51\left[1 + \frac{3.90}{4.51} \times \frac{870}{3800} \times (1 - .40)\right]$$

$$= 5.05/\text{rad}$$

(d) If $\ell_t = \bar{\ell}_t$ then $V_H = \bar{V}_H$

From (2.2.11)

$$V_H = \bar{V}_H - \frac{S_t}{S}(h - h_{n_{wb}})$$

becomes for $V_H = \bar{V}_H$

$$h = h_{n_{wb}}$$

From (2.3,23) with $h_{n_{wb}} = h$

$$(h - h_n) = \frac{-a_t}{a}\bar{V}_H\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right) \qquad (1)$$

From (2.2,10)

$$\bar{V}_H = \frac{\bar{\ell}_t S_t}{\bar{c}S} \qquad (2)$$

From (2.3,25c), (1) and (2)

$$C_{m_\alpha} = a(h - h_n) \qquad (3)$$

$$= -a_t \bar{V}_H\left(1 - \frac{\partial\varepsilon}{\partial\alpha}\right)$$

$$= \frac{-a_t \bar{\ell}_t S_t}{\bar{c} S}\left(1 - \frac{\partial \varepsilon}{\partial \alpha}\right)$$

$$C_{m_\alpha} = \frac{-3.90 \times 92 \times 870 \times (1 - 0.40)}{25.99 \times 3800}$$

$$= -1.90/\text{rad}$$

(e) From the C_m vs C_L curves of Fig. 2.29 it can be seen that $C_{m_{\delta_c}}$ is independent of C_L and C_m; δ_c just shifts the lines by a constant $\times\ \delta_c$.

$$C_{m_{\delta_e}} = \frac{-0.56}{20}$$

$$= -0.028/\text{deg}$$

From the $\delta_{e_{trim}}$ vs $C_{L_{trim}}$ curves of Fig. 2.30

$$\frac{d\delta_{e_{trim}}}{dC_{L_{trim}}} = \frac{-22.8}{2} = -11.4^\circ \quad @\ h = .35$$

$$= \frac{-27.8}{1.85} = -15.03^\circ\ @\ h = .25$$

From (2.4,13c)

$$\frac{d\delta_{e_{trim}}}{dC_{L_{trim}}} = \frac{-a}{\det}(h - h_n)$$

or

$$-\frac{det}{a}\frac{d\delta_{e_{trim}}}{dC_{L_{trim}}} = (h - h_n)$$

Thus

$$(.35 - h_n) = 11.4\frac{det}{a}$$

$$(.25 - h_n) = 15.03\frac{det}{a}$$

or

$$.0877\,h_n = .0307 - \frac{det}{a} \qquad (4)$$

$$.0665\,h_n = .0166 - \frac{det}{a} \qquad (5)$$

Now from (4) and (5)

$$.0212\,h_n = .0141$$

or

$$h_n = 0.665$$

From (2.3,25c) and 'a' from part (c)

$$C_{m_\alpha} = a(h - h_n)$$

$$= 5.05(.3 - .665)$$

$$= -1.84/\text{rad}$$

Note that this value of C_{m_α} differs from that in part (d) because the CG location is different.

$$\alpha_w = 0.133 + \frac{a_w}{a} i_t \text{ rad} \qquad (9)$$

From (1)

$$\frac{a_w}{a} = \left(2 - \frac{\partial \varepsilon}{\partial \alpha}\right)^{-1}$$

$$= (2 - .2)^{-1} = 0.556$$

thus

$$\alpha_w = 0.133 + .556\, i_t \text{ rad} \qquad (10)$$

Substitute (10) into (8)

$$\frac{L_w}{L_t} = \frac{0.133 + .556\, i_t}{0.8(.133 + .556\, i_t) - i_t}$$

$$= \frac{0.133 + 0.556\, i_t}{0.106 - 0.555\, i_t}$$

2.8 (a) $C_L = L / \frac{1}{2} \rho V^2 S$

$= L / \frac{1}{2} \rho_o V_E{}^2 S$

where ρ_o is sea level atmospheric density.

In level flight L = W, thus

$$C_L = W / \frac{1}{2} \rho_o V_E{}^2 S$$

Use the Standard Atmosphere table from App. D to find $\rho_o = 2.3769 \times 10^{-3}$ slug/ft^3.

$$V_E \text{ (in mph)} \times 1.467 = V_E \text{ (in fps)}$$

$$S = 174.5 \text{ ft}^2$$

x_{CG}	altitude (ft)	V_E (fps)	W (lb)	$C_{L_{trim}}$	$i^o_{t\,trim}$
93.89	4540	133.50	3651	0.99	1.5
	4560	159.90	3639	0.69	0
	4700	184.84	3635	0.51	-1.0
	4580	227.39	3629	0.34	-2.0
86.82	5320	130.56	3233	0.91	4.5
	4620	154.04	3226	0.66	2.0
	4740	180.44	3220	0.48	0.3
	4900	221.52	3215	0.32	-1.0
80.43	4880	127.63	2850	0.84	7.2
	4820	151.10	2845	0.60	3.5
	4880	178.97	2840	0.43	1.5
	4740	222.98	2835	0.27	0

(see diagram 2.8a)

(b) As shown in Fig. 2.18 $\delta_{e_{trim}}$ (here $i_{t_{trim}}$) vs $C_{L_{trim}}$ are straight lines for a given CG location. All the lines intersect at a common point on the $C_{L_{trim}} = 0$ axis. This was true for the present data (see diagram 2.8a)

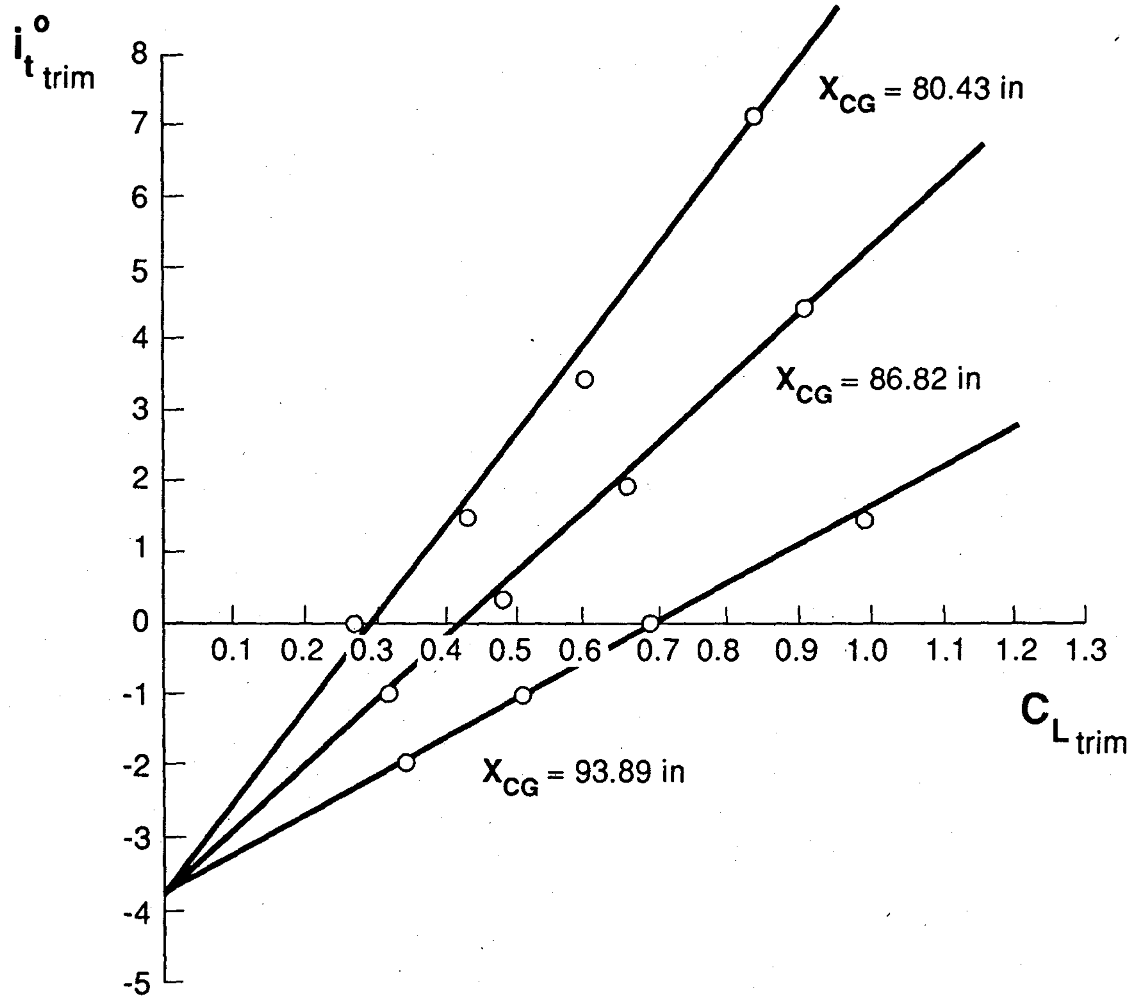


Diagram 2.8a

(c) From (2.4,29) and Fig. 2.21, when the CG is located at the neutral point then $\frac{\partial \delta_{e_{trim}}}{\partial C_{L_{trim}}} = 0$ $\left(\text{here we have } \frac{\partial i_{t_{trim}}}{\partial C_{L_{trim}}} = 0\right)$. From our graph find the slopes for the 3 CG locations and make a plot like Fig. 2.21.

x_{CG} (in)	$\frac{\partial i^o_{t\,trim}}{\partial C_{L_{trim}}}$
93.89	5.50
86.82	9.09
80.43	13.04

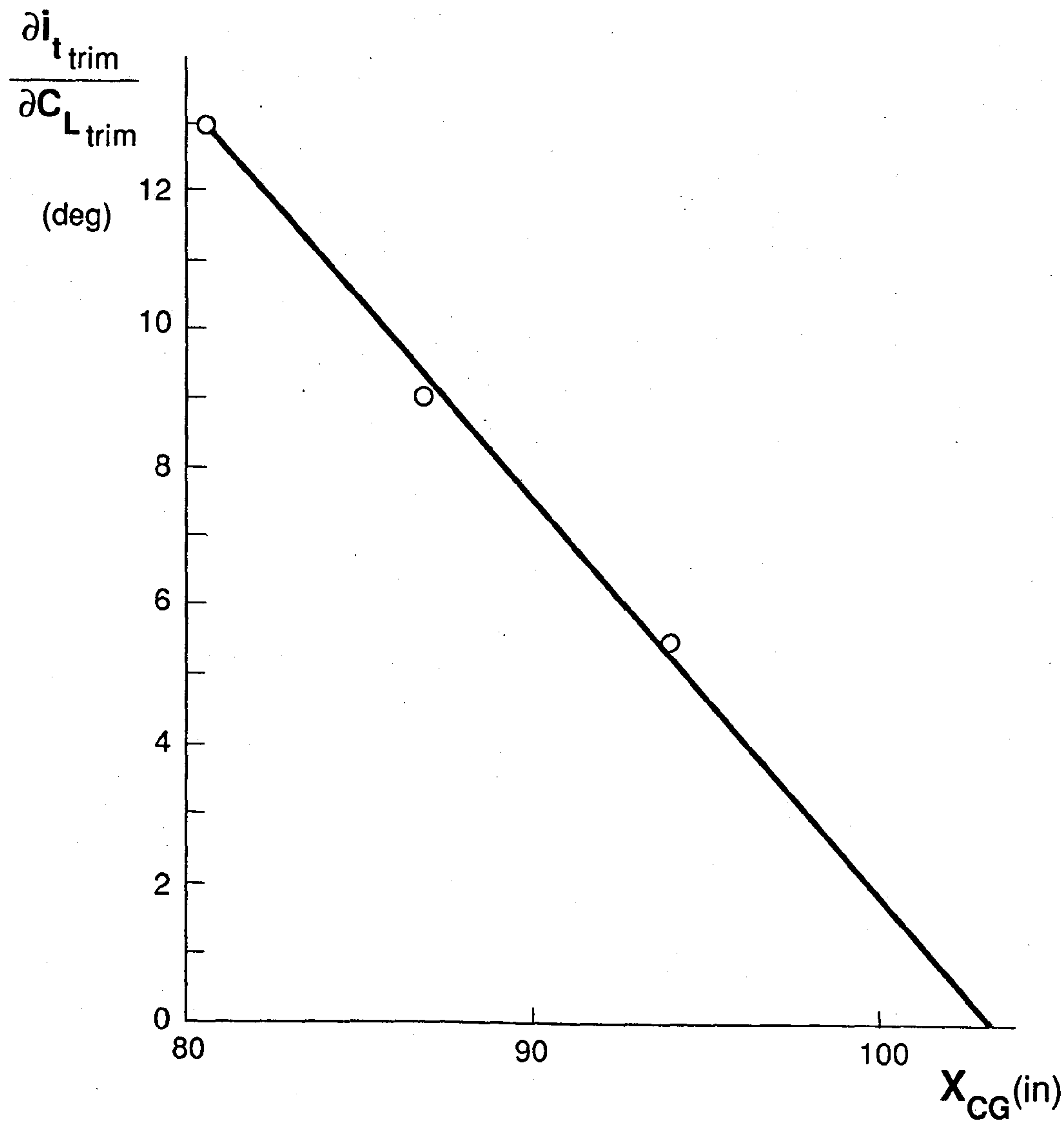


Diagram 2.8c

Since (2.4,29) indicates a linear variation with h a linear fit to the plot was made and the x intercept was (diagram 2.8c)

$$x_{CG} = 103.3 \text{ in}$$

as the location of the CG for $h = h_n$, i.e., the NP location.

2.9 (2.6,11b)

$$(h - h_n') = \frac{1}{a'}\left[a(h - h_n) - \frac{C_{m\delta_e} C_{he_\alpha}}{b_2}\right]$$

(2.4,8b)

$$C_{m\delta_e} = -a_e \bar{V}_H + C_{L\delta_e}(h - h_{n_{wb}})$$

Substitute (2.4,8b) into (2.6,11b)

$$(h - h_n') = \frac{a}{a'}(h - h_n) - \frac{C_{he_\alpha}}{a'b_2} C_{L\delta_e}(h - h_{n_{wb}}) + \frac{C_{he_\alpha}}{a'b_2} a_e \bar{V}_H \qquad (1)$$

(2.6,4b) gives

$$a' = a - \frac{C_{L\delta_e} C_{he_\alpha}}{b_2}$$

Use (2.6,4b) to replace "a" in "ah" on the right-hand side of (1) to obtain

$$(h - h_n') = h - \frac{1}{a'}\left(ah_n - \frac{C_{he_\alpha}}{b_2} C_{L\delta_e} h_{n_{wb}}\right) + \frac{C_{he_\alpha} a_e \bar{V}_H}{a'b_2} \qquad (2)$$