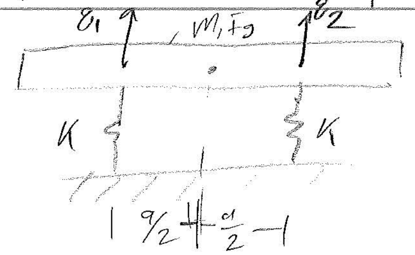


$$I_g = m r^2$$

$$[M]\{\ddot{q}\} + [K]\{q\} = 0$$

$$M = \begin{bmatrix} \left(\frac{ma}{4} + \frac{I_g}{a}\right) & \left(\frac{ma}{4} - \frac{I_g}{a}\right) \\ \left(\frac{ma}{4} - \frac{I_g}{a}\right) & \left(\frac{ma}{4} + \frac{I_g}{a}\right) \end{bmatrix}$$

$$M = ma \begin{bmatrix} \frac{1}{4} + \left(\frac{r}{a}\right)^2 & \frac{1}{4} - \left(\frac{r}{a}\right)^2 \\ \frac{1}{4} - \left(\frac{r}{a}\right)^2 & \frac{1}{4} + \left(\frac{r}{a}\right)^2 \end{bmatrix} \qquad K = Ka \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\omega_1^2 = \frac{2K}{m} \quad \phi_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \qquad \omega_2^2 = \frac{2K}{m}\left(\frac{a}{2r}\right)^2 \quad \phi_2 = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

if $(r/a) = 1/2$, $\omega_1 = \omega_2$

$M = \frac{ma}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ~ already diagonal (more complicated in general)

$[K - \omega^2 M]\phi =$

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\{\phi\} = 0$ ~ anything could be an eigenvector!

Choose arbitrary

$\phi_1 = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \quad \phi_2 = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$

$\{\phi_1\}^T[M]\{\phi_2\} = 0$

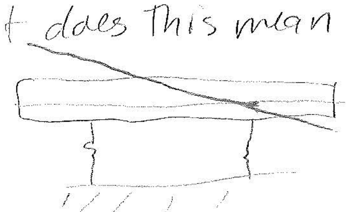
$\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = 1 \neq 0$ ~ we've lost orthogonality

Procedure: choose ϕ_1, find ϕ_2 to be orth $(\phi_1^T M \phi_2 = 0)$ (Graham Schmidt)

$\phi_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \phi_2 = \begin{bmatrix} a \\ b \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = 0 \rightarrow a = 0,$ b - anything, choose 1

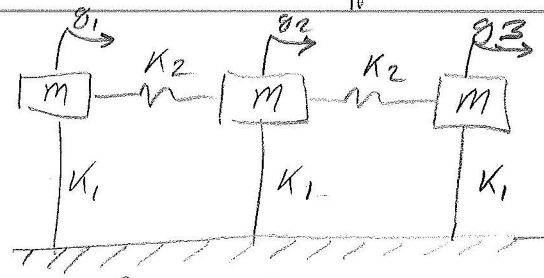
→ Mass normalize

What does this mean ~ Start the system in any initial pattern, that pattern persists for all time at $\omega_1 = \omega_2$!



$$q = \{\phi_1\}\eta_1 + \{\phi_2\}\eta_2$$

same frequency



K_1 represents a light beam upon which each mass is mounted

– Find nat. frq's and modes

EOM → Power Balance →

$$T = \frac{1}{2}\left(m\dot{g}_1^2 + m\dot{g}_2^2 + m\dot{g}_3^2\right)$$

$$V = \frac{1}{2}K_1\left(g_1^2 + g_2^2 + g_3^2\right) + \frac{1}{2}K_2(g_2 - g_1)^2 + \frac{1}{2}K_2(g_3 - g_1)^2$$

all connected to ground

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}\{\ddot{g}\} + \begin{bmatrix} (K_1+K_2) & -K_2 & 0 \\ -K_2 & (K_1+2K_2) & -K_2 \\ 0 & -K_2 & (K_1+K_2) \end{bmatrix}\{g\} = 0$$

– No stiffness coupling between $g_1 + g_3$ ✓

Solve EVP — $[K - w^2[M]]\{\phi\} = 0$ → (Matlab)

$$\left|\begin{bmatrix} (K_1+K_2-w^2m) & \boxed{-K_2} & 0 \\ -K_2 & (K_1+2K_2-w^2m & -K_2 \\ 0 & -K_2 & K_1+K_2-w^2m) \end{bmatrix}\right| = 0$$

Rule for determinant

det|[ ]| = 0 take ◯ · det( ) + ☐ · det(...

$$(K_1+K_2-w^2m)\left[(K_1+2K_2-w^2m)(K_1+K_2-w^2m) - K_2^2\right] +$$

$$-(-K_2)\left[(-K_2)(K_1+K_2-w^2m) - 0\right] + 0[\quad]$$

Sign!

factor out leaves only K_2^2

$$(K_1+K_2-w^2m)\left[(K_2+2K_2-w^2m)(K_1+K_2-w^2m) - K_2^2 - K_2^2\right] = 0$$

This gives one eigenvalue:

$$(K_1+K_2-w^2m) = 0 \rightarrow w^2 = \frac{K_1+K_2}{m}$$

solving part in [ ] gives w_1, w_3 →

$$w_1^2 = \frac{K_1}{m} \qquad w_2^2 = \frac{K_1+K_2}{m} \qquad w_3^2 = \frac{K_1+3K_2}{m}$$

for ϕ_2 $[\phi_1\ \phi_2\ \phi_3]$

$$\begin{bmatrix} 0 & -K_2 & 0 \\ -K_2 & K_2 & -K_2 \\ 0 & -K_2 & 0 \end{bmatrix} \begin{Bmatrix} \phi_{12} \\ \phi_{22} \\ \phi_{32} \end{Bmatrix} = 0 \rightarrow -K_2\phi_{22} = 0$$

$$\rightarrow -K_2\phi_{12} - K_2\phi_{32} = 0$$

$$\phi_{12} = -\phi_{32}$$

$$\rightarrow \phi_2 = \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix}$$

ϕ_1

$$\begin{bmatrix} K_2 & -K_2 & 0 \\ -K_2 & 2K_2 & -K_2 \\ 0 & -K_2 & K_2 \end{bmatrix} \begin{Bmatrix} \phi_{11} \\ \phi_{21} \\ \phi_{31} \end{Bmatrix} = 0 \rightarrow K_2\phi_{11} - K_2\phi_{21} = 0 \rightarrow \phi_{11} = \phi_{21}$$

$$\rightarrow \phi_{21} = \phi_{31} \rightarrow \phi_1 = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$$

ϕ_3 (skip + give answer)

$$\begin{bmatrix} -2K_2 & -K_2 & 0 \\ -K_2 & -K_2 & -K_2 \\ 0 & -K_2 & -2K_2 \end{bmatrix} \begin{Bmatrix} \phi_{13} \\ \phi_{23} \\ \phi_{33} \end{Bmatrix} = \rightarrow 2\phi_{13} = -\phi_{23}$$

$$\rightarrow \phi_{23} = -2\phi_{33} \rightarrow \phi_3 = \begin{Bmatrix} 1 \\ -2 \\ 1 \end{Bmatrix}$$

Mode 1 Mode 2 Mode 3

$\rightarrow$ Mass Normalize

In matlab, $\rightarrow$ Generalized eigenvalue prob – $[A]\{x\} = \lambda[B]\{x\}$

$\rightarrow$ [phi, λ] = eig(K,M)

```
m = 1;  K1 = 1;  K2 = 2;

M = [m, 0, 0; 0, m, 0; 0, 0, m];
K = [K1+K2, -K2, 0; -K2, K1+2K2, -K2; 0, -K2, K1+K2];
[phi, lam] = eig(K, M);
wnsq = diag(lam);
wn =   (wnsq).^(1/2)
[wnsort, Isort] = sort(wn);
phisort = phi(:, Isort);
```

mass normalizing:

```
for K = 1:length(Wn)
    mu = phi(:,K).'*M*phi(:,K);
    PHI(:,K) = phi(:,K)/sqrt(mu);
end
```

or phisort

or — a slick way to do it in one line

```
PHI = phi*diag(sqrt(diag(phi.'*M*phi)).^-1);
```

$$\begin{bmatrix} \phi_1 & \phi_2 & \cdots \end{bmatrix} \begin{bmatrix} \mu_1^{-1/2} & & \\ & \mu_2^{-1/2} & \\ & & \ddots \end{bmatrix} \quad \checkmark$$