# my solution to practice exam 2, EMA 545, Spring 2013

## Spring 2013
## University of Wisconsin, Madison

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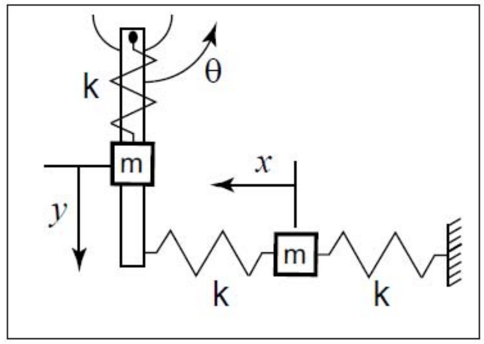
# Contents

# 1 Problem 1

**Problem #1 (10 pts)**
a.) A colleague asserts that the linearized equations of motion for this system are as given below, where ×'s denote terms that are not given to you, which may be zero or constant. $[]|_{springs}$ denotes the portion of the stiffness matrix due to the springs and $[]|_{gravity}$ denotes that portion due to gravity. Check the ***units*** and the ***sign*** on the $K_{12}|_{springs}$ term. If incorrect, please provide the corrected term and ***explain your reasoning***. (The left mass is constrained so that it slides along the bar as the bar rotates.)




$$\begin{bmatrix} I & 0 & \times \\ 0 & m & \times \\ \times & \times & \times \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} kL^2 & k & \times \\ k & \times & \times \\ \times & \times & \times \end{bmatrix}_{springs} \begin{Bmatrix} \theta \\ x \\ y \end{Bmatrix} + \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{bmatrix}_{gravity} \begin{Bmatrix} \theta \\ x \\ y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Taking $x$ as positive as shown, and $y$ as positive as shown, then the middle spring is in compression with change of length $\Delta = (x + L\theta)$ and the right most spring is in tension with change of length $\Delta = x$, hence

$$\begin{aligned} V_{spring} &= \frac{1}{2}ky^2 + \frac{1}{2}k(x + L\theta)^2 + \frac{1}{2}kx^2 \\ &= \frac{1}{2}ky^2 + \frac{1}{2}k\left(x^2 + L^2\theta^2 + 2xL\theta\right) + \frac{1}{2}kx^2 \\ &= \theta^2\left(\frac{1}{2}kL^2\right) + x^2(k) + y^2\left(\frac{1}{2}k\right) + x\theta(kL) \end{aligned}$$

Compare to quadratic form

$$V_{spring} = \frac{1}{2}K_{11}\theta^2 + \frac{1}{2}K_{22}x^2 + \frac{1}{2}K_{33}y^2 + K_{12}x\theta + K_{13}\theta y + K_{23}xy$$

Then

$$\begin{aligned} K_{11} &= kL^2 \\ K_{22} &= k \\ K_{33} &= k \\ K_{12} &= kL \\ K_{13} &= 0 \\ K_{23} &= 0 \end{aligned}$$

Hence the $K$ matrix due to stiffness is

$$\begin{pmatrix} kL^2 & kL & 0 \\ kL & k & 0 \\ 0 & 0 & k \end{pmatrix} \begin{pmatrix} \theta \\ x \\ y \end{pmatrix}$$

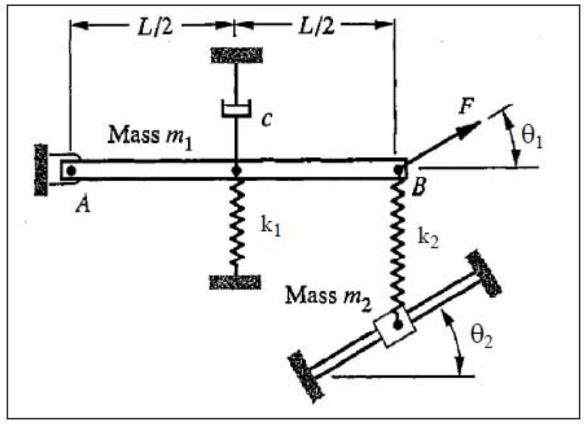
Therefore, $K_{12}$ had the wrong units. This reason is as follows: result of multiplying the first row of the $K_{spring}$ matrix with the column $\begin{pmatrix} \theta \\ x \\ y \end{pmatrix}$ should have units of torque. Therefore the units should be $force \times meter$ and hence $K_{12}x$ should come out as $Nm$ units. But as given in the problem, it has units $N$ only, ie. units of force. But now, the units will come out to be $Nm$.

Similarly, the second row of the $K$ matrix when multiplied by $\begin{pmatrix} \theta \\ x \\ y \end{pmatrix}$ should have units of force only (not torque). We can see this this is the case with this correction. So the sign was correct, but the units did not match before.

# 2 Problem 2

**Problem #2 (45 pts)**

Gravity acts downward (along dashpot c), and the initial lengths of the springs are such that the position shown corresponds to the static equilibrium when the applied dynamic force $F(t)$ is not present. The moment of inertia of a rod about its mass center is $I_g = (1/12)mL^2$ and about its end is $I_{end} = (1/3)mL^2$.

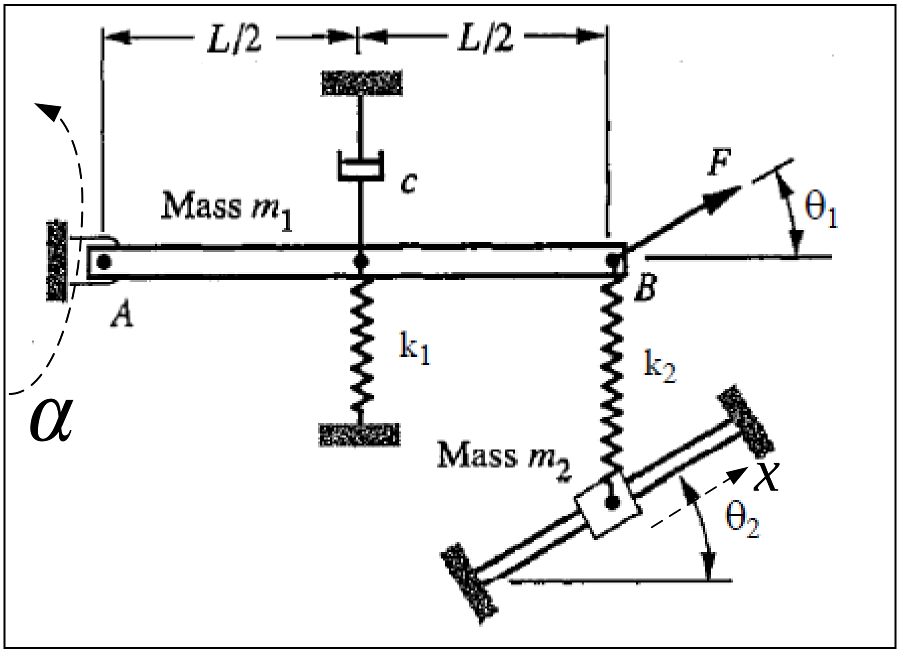



a.) Identify generalized coordinates and derive the corresponding equations of motion. Employ the stiff-spring approximation to simplify your analysis. Friction is negligible in the pin joint A and the friction force between the guide and $m_2$ is equal to $f=c_2 v$, where $v$ is the speed of the mass. (30 pts)

b.) Check that your answers make sense. Explain each check that you perform and why it shows that your EOM are/are not correct. (15 pts)

## 2.1 Part a

This is a 2 degrees of freedom system. The first generalized coordinate is taken as $\alpha$ which the angle of rotation of the top bar around joint $A$. The second degree of freedom is taken as $x$ which is the sliding distance that mass $m_2$ moves as it slides over the lower bar




Static equilibrium is at $\alpha = 0$ and $x = 0$.

We start by finding the kinetic energy. Since bar $m_1$ is fixed at one point to inertial space, then only its rotational kinetic energy is added to the system kinetic energy

$$T = \frac{1}{2}\left(\frac{1}{12}m_1L^2\right)(\alpha')^2 + \frac{1}{2}m_2(x')^2$$

Now we find the potential energy, assuming springs remain straight. Spring $k_1$ will extend by amount

$$\Delta_1 = \frac{L}{2}\alpha$$

and spring $k_2$ will extend by amount

$$\Delta_2 = L\alpha - x\sin\theta_2$$

Hence potential energy of the system is

$$V = \frac{1}{2}k_1(\Delta_1)^2 + \frac{1}{2}k_2(\Delta_2)^2 + m_1g\frac{L}{2}\sin\alpha + m_2gx\sin\theta_2$$

Therefore the Lagrangian $\Phi$ is

$$
\begin{aligned}
\Phi &= T - V\\
&= \frac{1}{2}\left(\frac{1}{12}m_1L^2\right)(\alpha')^2 + \frac{1}{2}m_2(x')^2 - \left(\frac{1}{2}k_1(\Delta_1)^2 + \frac{1}{2}k_2(\Delta_2)^2 + m_1g\frac{L}{2}\sin\alpha + m_2gx\sin\theta_2\right)\\
&= \frac{1}{2}\left(\frac{1}{12}m_1L^2\right)(\alpha')^2 + \frac{1}{2}m_2(x')^2 - \left(\frac{1}{2}k_1\left(\frac{L}{2}\alpha\right)^2 + \frac{1}{2}k_2(L\alpha - x\sin\theta_2)^2 + m_1g\frac{L}{2}\sin\alpha + m_2gx\sin\theta_2\right)\\
&= \frac{1}{24}m_1L^2(\alpha')^2 + \frac{1}{2}m_2(x')^2 - k_1\frac{L^2}{8}\alpha^2 - \frac{1}{2}k_2\left(L^2\alpha^2 + x^2\sin^2\theta_2 - 2L\alpha x\sin\theta_2\right) - m_1g\frac{L}{2}\sin\alpha - m_2gx\sin\theta_2
\end{aligned}
$$

EOM for $x$ is

$$\frac{d}{dt}\left(\frac{\partial\Phi}{\partial x'}\right) - \frac{\partial\Phi}{\partial x} = Q_x$$

where $Q_x$ is the generalized for for the $x$ coordinate. To find $Q_x$ we make virtual displacement $\delta x$ while fixing all other coordinates and obtain virtual work done by non-conservative forces. Only non-conservative force acting on $m_2$ is the friction force $f = c_2v$ where $v$ is the speed of the mass $m_2$. The speed of the mass $m_2$ is the vertical direction is $v = x'\sin\theta_2$, hence the non-conservative force acting on $m_2$ is $c_2(\dot{x}\sin\theta_2)$ and is acting in negative direction. Hence taking projection of this force along $x$ gives

$$\delta W = -c_2(x'\sin\theta_2)\sin\theta_2\delta x$$

Therefore

$$Q_x = -c_2x'\sin^2\theta_2$$

Hence

$$
\begin{aligned}
\frac{d}{dt}\left(\frac{\partial\Phi}{\partial x'}\right) - \frac{\partial\Phi}{\partial x} &= -c_2x'\sin^2\theta_2\\
\frac{d}{dt}(m_2x') - \left(-k_2x\sin^2\theta_2 + 2k_2L\alpha\sin\theta_2 - m_2g\sin\theta_2\right) &= -c_2x'\sin^2\theta_2\\
m_2x'' + c_2x'\sin^2\theta_2 + k_2x\sin^2\theta_2 - 2k_2L\alpha\sin\theta_2 &= -m_2g\sin\theta_2
\end{aligned}
$$

EOM for $\alpha$ is

$$\frac{d}{dt}\left(\frac{\partial\Phi}{\partial\alpha'}\right) - \frac{\partial\Phi}{\partial\alpha} = Q_\alpha$$

where $Q_\alpha$ is the generalized for for the $\alpha$ coordinate. To find $Q_\alpha$ we make virtual displacement $\delta\alpha$ while fixing all other coordinates and obtain virtual work done by non-conservative forces. We see that the work is

$$
\begin{aligned}
\delta W &= -c(L\alpha')\frac{L}{2}\delta\alpha + (F\sin\theta_1)L\delta\alpha\\
&= \left(FL\sin\theta_1 - \frac{cL^2}{2}\alpha'\right)\delta\alpha
\end{aligned}
$$

Hence

$$Q_\alpha = FL\sin\theta_1 - \frac{cL^2}{2}\alpha'$$

Therefore

$$\frac{d}{dt}\left(\frac{\partial\Phi}{\partial\alpha'}\right) - \frac{\partial\Phi}{\partial\alpha} = FL\sin\theta_1 - \frac{cL^2}{2}\alpha'$$

$$\frac{d}{dt}\left(\frac{1}{12}m_1L^2\alpha'\right) - \left(-k_1\frac{L^2}{4}\alpha - k_2L^2\alpha + 2Lx\sin\theta_2 - m_1g\frac{L}{2}\cos\alpha\right) = FL\sin\theta_1 - \frac{cL^2}{2}\alpha'$$

$$\frac{1}{12}m_1L^2\alpha'' + k_1\frac{L^2}{4}\alpha + k_2L^2\alpha - 2k_2Lx\sin\theta_2 + m_1g\frac{L}{2}\cos\alpha = FL\sin\theta_1 - \frac{cL^2}{2}\alpha'$$

$$\frac{1}{12}m_1L^2\alpha'' + \frac{cL^2}{2}\alpha' + \left(k_1\frac{L^2}{4} + k_2L^2\right)\alpha - 2k_2Lx\sin\theta_2 = FL\sin\theta_1 - m_1g\frac{L}{2}\cos\alpha$$

Hence the 2 EOM are

$$m_2x'' + c_2x'\sin^2\theta_2 + k_2x\sin^2\theta_2 - 2k_2L\alpha\sin\theta_2 = -m_2g\sin\theta_2$$

$$\frac{1}{12}m_1L^2\alpha'' + \frac{cL^2}{2}\alpha' + \left(k_1\frac{L^2}{4} + k_2L^2\right)\alpha - 2k_2Lx\sin\theta_2 = FL\sin\theta_1 - m_1g\frac{L}{2}\cos\alpha$$

Linearize around static equilibrium, $\alpha = 0, x = 0$ then we obtain

$$m_2x'' + c_2x'\sin^2\theta_2 + k_2x\sin^2\theta_2 - 2k_2L\alpha\sin\theta_2 = -m_2g\sin\theta_2$$

$$\frac{1}{12}m_1L^2\alpha'' + \frac{cL^2}{2}\alpha' + \left(k_1\frac{L^2}{4} + k_2L^2\right)\alpha - 2k_2Lx\sin\theta_2 = FL\sin\theta_1 - m_1g\frac{L}{2}$$

In Matrix form

$$\begin{pmatrix} \frac{1}{12}m_1L^2 & 0 \\ 0 & m_2 \end{pmatrix}\begin{pmatrix} \alpha'' \\ x'' \end{pmatrix} + \begin{pmatrix} \frac{cL^2}{2} & 0 \\ 0 & c_2\sin^2\theta_2 \end{pmatrix}\begin{pmatrix} \alpha' \\ x' \end{pmatrix} + \begin{pmatrix} k_1\frac{L^2}{4} + k_2L^2 & -2k_2L\sin\theta_2 \\ -2k_2L\sin\theta_2 & k_2 \end{pmatrix}\begin{pmatrix} \alpha \\ x \end{pmatrix} = \begin{pmatrix} FL\sin\theta_1 - m_1g\frac{L}{2} \\ -m_2g\sin\theta_2 \end{pmatrix}$$

I think the weight contributions should be zero. So I need to look more into this, but I think the OEM should be as follows

$$\begin{pmatrix} \frac{1}{12}m_1L^2 & 0 \\ 0 & m_2 \end{pmatrix}\begin{pmatrix} \alpha'' \\ x'' \end{pmatrix} + \begin{pmatrix} \frac{cL^2}{2} & 0 \\ 0 & c_2\sin^2\theta_2 \end{pmatrix}\begin{pmatrix} \alpha' \\ x' \end{pmatrix} + \begin{pmatrix} k_1\frac{L^2}{4} + k_2L^2 & -2k_2L\sin\theta_2 \\ -2k_2L\sin\theta_2 & k_2 \end{pmatrix}\begin{pmatrix} \alpha \\ x \end{pmatrix} = \begin{pmatrix} FL\sin\theta_1 \\ 0 \end{pmatrix}$$

# 3 Part b

Checking the Damping matrix units. First row of $C\begin{pmatrix}\alpha' \\ x'\end{pmatrix}$ should give units of torque. looking at $\frac{cL^2}{2}\alpha'$ . viscous damping coefficient $c$ has units of $N\frac{T}{L}$, hence the units of the expression $\frac{cL^2}{2}\alpha'$ are $N\frac{T}{L}(L)^2\frac{1}{T} = NL$, in other words, a torque. (in here, $L$ stands for length units, $T$ stands for time units and $N$ stands for force units). Now to verify the second row of $C$. We see it is $c_2 \sin^2\theta_2 x'$ which has units of force (given in the problem). Since the second must have units of force, this is verified.

Now checking the stiffness matrix units. First row of $K\begin{pmatrix}\alpha \\ x\end{pmatrix}$ should have units of torque. But $\left(k_1\frac{L^2}{4} + k_2L^2\right)\alpha$ has units of torque since $k$ has units of force per unit length. and $2k_2L\sin\theta_2 x$ has units of torque also (note $\alpha$ has no units as it is an angle).

For the second row of $K$, it should have units of force, which it does, since $k_2x$ has units of force and $-2k_2L\sin\theta_2\alpha$ has units of force. Hence verified.

Check signs on the $x$ EOM:

$$m_2x'' + c_2x'\sin^2\theta_2 + k_2x\sin^2\theta_2 - 2k_2L\alpha\sin\theta_2 = 0$$
$$m_2x'' + c_2x'\sin^2\theta_2 + k_2x\sin^2\theta_2 = 2k_2L\alpha\sin\theta_2$$

$x'' > 0, x' > 0, x > 0$ then $\alpha > 0$,checks OK, since when $x > 0$ then the top bar will be rotating in the positive direction and $\alpha > 0$, i.e. the top bar will be above the horizontal.

Check signs on the $\alpha$ EOM:

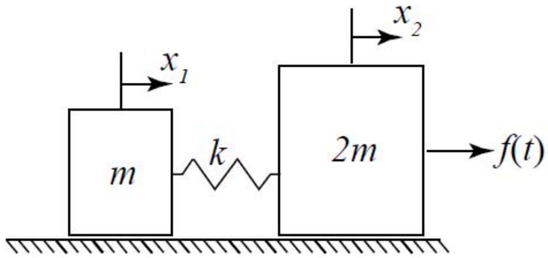
$$\frac{1}{12}m_1L^2\alpha'' + \frac{cL^2}{2}\alpha' + \left(k_1\frac{L^2}{4} + k_2L^2\right)\alpha - 2k_2Lx\sin\theta_2 = FL\sin\theta_1$$
$$\frac{1}{12}m_1L^2\alpha'' + \frac{cL^2}{2}\alpha' + \left(k_1\frac{L^2}{4} + k_2L^2\right)\alpha = FL\sin\theta_1 + 2k_2Lx\sin\theta_2$$

$\alpha'' > 0, \alpha' > 0, \alpha > 0$ then $x > 0$,checks OK, since when $\alpha > 0$ then the top bar will be rotating in the positive direction and $x > 0$, means the lower mass $m_2$ is moving upwards.

# 4 Problem 3

**Problem #3 (45 pts)**

The system pictured is initially at rest when an impulsive force $f(t) = F_0\delta(t\text{-}T)$ is applied to the mass on the right. The masses are constrained so that they only translate in the horizontal direction, and there is no friction between the masses and ground.




The equations of motion of this system are:

$$\begin{bmatrix} m & 0 \\ 0 & 2m \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix}+\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}=\begin{Bmatrix} 0 \\ f(t) \end{Bmatrix}$$

Find the response of the first mass, $x_1(t)$, as a function of time.

We solve this in modal coordinates so to de-couple the EOM's. First find the 2 natural frequencies

$$\left\| k\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} - \omega^2 m\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \right\| = 0$$

$$\left\| \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} - \omega^2 \frac{m}{k}\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \right\| = 0$$

Let $\omega^2 \frac{m}{k} = \eta^2$ then

$$\left\| \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} - \eta^2\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \right\| = 0$$

$$\left\| \begin{pmatrix} 1-\eta^2 & -1 \\ -1 & 1-2\eta \end{pmatrix} \right\| = 0$$

$$\left(1-\eta^2\right)\left(1-2\eta^2\right)-1 = 0$$

Hence taking positive roots $\boxed{\eta = 1.2247, \eta = 0}$. When $\eta = 0$

$$\begin{pmatrix} 1-\eta^2 & -1 \\ -1 & 1-2\eta^2 \end{pmatrix} \begin{Bmatrix} \varphi_{11} \\ \varphi_{12} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{Bmatrix} 1 \\ \varphi_{12} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Hence $1 - \varphi_{12} = 0$ or $\varphi_{12} = 1$, therefore $\varphi_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$

When $\eta = 1.2247$

$$\begin{pmatrix} 1-\eta^2 & -1 \\ -1 & 1-2\eta^2 \end{pmatrix} \begin{Bmatrix} \varphi_{12} \\ \varphi_{22} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{pmatrix} -0.5 & -1 \\ -1 & -2 \end{pmatrix} \begin{Bmatrix} 1 \\ \varphi_{22} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Hence $-0.5 - \varphi_{22} = 0$ or $\varphi_{22} = -0.5$, therefore $\varphi_2 = \begin{Bmatrix} 1 \\ -0.5 \end{Bmatrix}$. Now do mass normalization

$$\begin{aligned} \mu_1 &= \{\varphi\}_1^T [M] \{\varphi\}_1 \\ &= \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}^T \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \\ &= 3 \end{aligned}$$

and

$$\begin{aligned} \mu_2 &= \{\varphi\}_2^T [M] \{\varphi\}_2 \\ &= \begin{Bmatrix} 1 \\ -0.5 \end{Bmatrix}^T \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{Bmatrix} 1 \\ -0.5 \end{Bmatrix} \\ &= 1.5 \end{aligned}$$

Hence

$$\{\Phi\}_1 = \frac{\{\varphi\}_1}{\sqrt{\mu_1}} = \frac{\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}}{\sqrt{3}} = \begin{Bmatrix} 0.57735 \\ 0.57735 \end{Bmatrix}$$

$$\{\Phi\}_2 = \frac{\{\varphi\}_2}{\sqrt{\mu_2}} = \frac{\begin{Bmatrix} 1 \\ -0.5 \end{Bmatrix}}{\sqrt{1.5}} = \begin{Bmatrix} 0.81650 \\ -0.40825 \end{Bmatrix}$$

Hence

$$\boxed{[\Phi] = \begin{pmatrix} 0.57735 & 0.81650 \\ 0.57735 & -0.40825 \end{pmatrix}}$$

Then the modal EOM are

$$[\Phi]^T[M][\Phi] + [\Phi]^T[K][\Phi] = [\Phi]^T\{F\}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{Bmatrix} \ddot{\eta}_1 \\ \ddot{\eta}_2 \end{Bmatrix} + \begin{pmatrix} \eta_1^2 & 0 \\ 0 & \eta_2^2 \end{pmatrix}\begin{Bmatrix} \eta_1 \\ \eta_2 \end{Bmatrix} = \begin{pmatrix} 0.57735 & 0.57735 \\ 0.81650 & -0.40825 \end{pmatrix}\begin{Bmatrix} 0 \\ F_0\delta(t) \end{Bmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{Bmatrix} \ddot{\eta}_1 \\ \ddot{\eta}_2 \end{Bmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 1.5 \end{pmatrix}\begin{Bmatrix} \eta_1 \\ \eta_2 \end{Bmatrix} = \begin{Bmatrix} 0.57735F_0\delta(t) \\ -0.40825F_0\delta(t) \end{Bmatrix}$$

For the first mass, EOM is

$$\begin{aligned} \ddot{\eta}_1 &= 0.57735F_0\delta(t) \\ \dot{\eta}_1 &= \int_0^t 0.57735F_0\delta(t)dt + C_1 \\ &= 0.57735F_0\left(h(t) - \frac{1}{2}\right) + C_1 \\ \eta_1(t) &= \int_0^t \left(0.57735F_0\left(h(t) - \frac{1}{2}\right) + C_1\right)dt + C_2 \\ &= 0.57735F_0t\left(h(t) - \frac{1}{2}\right) + tC_1 + C_2 \end{aligned}$$

Now initial conditions are zero since $\begin{Bmatrix} x_1(0) \\ x_2(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$ and also $\begin{Bmatrix} x_1'(0) \\ x_2'(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$ then $\begin{Bmatrix} \eta_1(0) \\ \eta_2(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$ and also $\begin{Bmatrix} \dot{\eta}_1(0) \\ \dot{\eta}_2(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$

Initial conditions $\eta_1(0) = 0$ implies

$$C_2 = 0$$

while and $\dot{\eta}_1(0) = 0$ implies

$$C_1 = -0.57735F_0\left(h(t) - \frac{1}{2}\right)$$

Hence the solution is

$$\begin{aligned}\eta_1(t) &= 0.57735F_0t\left(h(t) - \frac{1}{2}\right) + tC_1 + C_2\\ &= 0.57735F_0t\left(h(t) - \frac{1}{2}\right) - 0.57735F_0\left(h(t) - \frac{1}{2}\right)\\ &= 0.57735F_0\left(h(t) - \frac{1}{2}\right)(t-1)\end{aligned}$$

Now the second EOM is solved.

$$\ddot{\eta}_2 + 1.5\eta_2 = -0.40825F_0\delta(t)$$

Which has solution (using appendix B) and using $M = 1$ and $\omega_D = \omega_n = \sqrt{1.5} = 1.2247$ since $\zeta = 0$, hence

$$\eta_2(t) = \frac{-0.40825F_0}{1.2247}\sin(1.2247t)$$

Now to obtain the solution in normal coordinates

$$\begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} = [\Phi]\begin{Bmatrix} \eta_1(t) \\ \eta_2(t) \end{Bmatrix}$$

Then

$$\begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} = \begin{pmatrix} 0.57735 & 0.81650 \\ 0.57735 & -0.40825 \end{pmatrix}\begin{Bmatrix} 0.57735F_0\left(h(t) - \frac{1}{2}\right)(t-1) \\ \frac{-0.40825F_0}{1.2247}\sin(1.2247t) \end{Bmatrix}$$

So

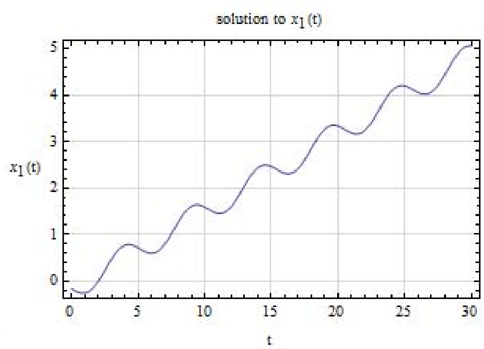
$$\begin{aligned}x_1(t) &= 0.57735\left[0.57735F_0\left(h(t) - \frac{1}{2}\right)(t-1)\right] - 0.81650\left[\frac{0.40825F_0}{1.2247}\sin(1.225t)\right]\\ x_2(t) &= 0.57735\left[0.57735F_0\left(h(t) - \frac{1}{2}\right)(t-1)\right] + 0.40825\left[\frac{0.40825F_0}{1.2247}\sin(1.225t)\right]\end{aligned}$$

For example, if $F_0 = 1$ then

$$x_1(t) = 0.57735\left[0.57735\left(h(t) - \frac{1}{2}\right)(t-1)\right] - 0.81650\left[\frac{0.40825}{1.2247}\sin(1.2247t)\right]$$
$$x_2(t) = 0.57735\left[0.57735\left(h(t) - \frac{1}{2}\right)(t-1)\right] + 0.40825\left[\frac{0.40825}{1.2247}\sin(1.2247t)\right]$$

Here is a plot of the solution $x_1(t)$ and $x_2(t)$. The 2 masses move to the right after the impulse, while in sinusoidal motion at the same frequency, but different amplitudes.

```
a0 = 0.57735;
b0 = 0.8165;
c0 = 0.40825;
wn = 1.2247;
x1[t_] := a0 (a0 (UnitStep[t] - 1/2) (t - 1)) - b0 (c0/wn Sin[wn t])
x2[t_] := a0 (a0 (UnitStep[t] - 1/2) (t - 1)) + c0 (c0/wn Sin[wn t])
Grid[
 {{Plot[x1[t], {t, 0, 30}, Frame → True, FrameLabel → {{"x1(t)", None}, {"t", "solution to x1(t)"}},
    GridLines → Automatic, GridLinesStyle → LightGray, RotateLabel → False, ImageSize → 300]},
  {Plot[x2[t], {t, 0, 30}, Frame → True, FrameLabel → {{"x2(t)", None}, {"t", "solution to x2(t)"}},
    GridLines → Automatic, GridLinesStyle → LightGray, RotateLabel → False, ImageSize → 300]}}]
```




ut[42]=