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EMA 545 – Final Exam - Prof. M. S. Allen
Spring 2011

Honor Pledge: On my honor, I pledge that this exam represents my own work, and that I have neither given nor received inappropriate aid in the preparation of this exam.

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Signature

Problem 1 (20)	____________________
Problem 2 (20)	____________________
Problem 3 (30)	____________________
Problem 4 (10)	____________________
Problem 5 (10)	____________________
Problem 6 (10)	____________________
Total (100)	____________________

You are allowed one sheet of notes for this exam, front and back. Staple your note sheet to the back of your exam when you turn it in. Calculators are allowed, but you must show all of your work to receive credit.

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Appendix B from Ginsberg, Mechanical & Structural Vibration, Wiley, 2001: **(corrected)**

$$\ddot{q} + 2\zeta\omega_{\mathrm{nat}}\dot{q} + \omega_{\mathrm{nat}}^2 q = \frac{F(t)}{M} \quad \zeta < 1, \quad \omega_{\mathrm{d}} = \omega_{\mathrm{nat}}\sqrt{1-\zeta^2}$$

- Free vibration: $F(t) = 0$

$$q = \exp(-\zeta\omega_{\mathrm{nat}}t)\Big[q(0)\cos(\omega_{\mathrm{d}}t) + \frac{\dot{q}(0) + \zeta\omega_{\mathrm{nat}}q(0)}{\omega_{\mathrm{d}}}\sin(\omega_{\mathrm{d}}t)\Big]$$

- Impulse excitation: $F(t) = \delta(t)$

$$q = \frac{1}{M\omega_{\mathrm{d}}}\exp(-\zeta\omega_{\mathrm{nat}}t)\sin(\omega_{\mathrm{d}}t)h(t)$$

- Step excitation: $F(t) = h(t)$

$$q = \frac{1}{M\omega_{\mathrm{nat}}^2}\Big\{1 - \exp(-\zeta\omega_{\mathrm{nat}}t)\Big[\cos(\omega_{\mathrm{d}}t) + \frac{\zeta\omega_{\mathrm{nat}}}{\omega_{\mathrm{d}}}\sin(\omega_{\mathrm{d}}t)\Big]\Big\}h(t)$$

- Ramp excitation: $F(t) = th(t)$

$$q = \frac{1}{M\omega_{\mathrm{nat}}^3}\Big\{(\omega_{\mathrm{nat}}t) - 2\zeta + \exp(-\zeta\omega_{\mathrm{nat}}t)\Big[2\zeta\cos(\omega_{\mathrm{d}}t) - (1-2\zeta^2)\frac{\omega_{\mathrm{nat}}}{\omega_{\mathrm{d}}}\sin(\omega_{\mathrm{d}}t)\Big]\Big\}h(t)$$

- Quadratic excitation: $F(t) = t^2h(t)$

$$q = \frac{1}{M\omega_{\mathrm{nat}}^4}\Big\{(\omega_{\mathrm{nat}}t)^2 - 4\zeta(\omega_{\mathrm{nat}}t) - 2(1-4\zeta^2) + \exp(-\zeta\omega_{\mathrm{nat}}t) \times \Big[2(1-4\zeta^2)\cos(\omega_{\mathrm{d}}t) + (6\zeta - 8\zeta^3)\frac{\omega_{\mathrm{nat}}}{\omega_{\mathrm{d}}}\sin(\omega_{\mathrm{d}}t)\Big]\Big\}h(t)$$

- Exponential excitation:

$$F(t) = \exp(-\beta t)\,h(t)$$

$$q = \frac{1}{M(\omega_{\mathrm{nat}}^2 - 2\zeta\omega_{\mathrm{nat}}\beta + \beta^2)}\Big\{\exp(-\beta t) - \exp(-\zeta\omega_{\mathrm{nat}}t)\Big[\cos(\omega_{\mathrm{d}}t) + \frac{\zeta\omega_{\mathrm{nat}} - \beta}{\omega_{\mathrm{d}}}\sin(\omega_{\mathrm{d}}t)\Big]\Big\}h(t)$$

- Transient sinusoidal excitation:

$$F(t) = \sin(\omega t)h(t), \ \omega \neq \omega_{\mathrm{nat}} \text{ if } \zeta \neq 0$$

$$q = \frac{1}{M[(\omega_{\mathrm{nat}}^2 - \omega^2)^2 + 4\zeta^2\omega_{\mathrm{nat}}^2\omega^2]} \times \Big\{(\omega_{\mathrm{nat}}^2 - \omega^2)\sin(\omega t) - 2\zeta\omega_{\mathrm{nat}}\omega\cos(\omega t) + \omega\exp(-\zeta\omega_{\mathrm{nat}}t)\Big[2\zeta\omega_{\mathrm{nat}}\cos(\omega_{\mathrm{d}}t) - \frac{(1-2\zeta^2)\omega_{\mathrm{nat}}^2 - \omega^2}{\omega_{\mathrm{d}}}\sin(\omega_{\mathrm{d}}t)\Big]\Big\}h(t)$$

- Transient co-sinusoidal excitation:

$$F(t) = \cos(\omega t)h(t), \ \omega \neq \omega_{\mathrm{nat}} \text{ if } \zeta \neq 0$$

$$q = \frac{1}{M[(\omega_{\mathrm{nat}}^2 - \omega^2)^2 + 4\zeta^2\omega_{\mathrm{nat}}^2\omega^2]} \times \Big\{(\omega_{\mathrm{nat}}^2 - \omega^2)\cos(\omega t) + 2\zeta\omega_{\mathrm{nat}}\omega\sin(\omega t) - \exp(-\zeta\omega_{\mathrm{nat}}t)\Big[(\omega_{\mathrm{nat}}^2 - \omega^2)\cos\omega_{\mathrm{d}}t + \frac{\zeta\omega_{\mathrm{nat}}(\omega_{\mathrm{nat}}^2 + \omega^2)}{\omega_{\mathrm{d}}}\sin(\omega_{\mathrm{d}}t)\Big]\Big\}h(t)$$

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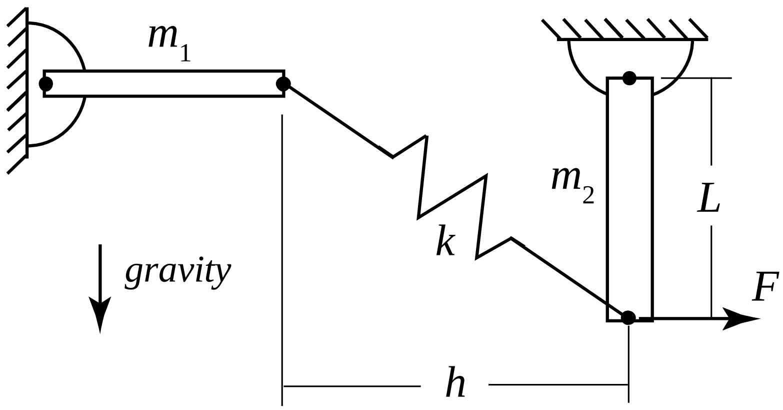
Problem #1 (20 pts)

Two rigid beams are pinned at their ends and arranged as shown below with a stiff spring connecting their tips. Gravity acts in the direction indicated. The position shown corresponds to the static equilibrium position. The masses of the two beams are m_1 and m_2 and they both have the same length, L. They are separated by a distance h. A dynamic force is applied to the tip of the right beam as shown. The moment of inertia of a bar is $I_g=(1/12)mL^2$ about its center and $I_{end}= (1/3)mL^2$ about its end.

Find the linearized equation(s) of motion for this system and <u>check that your equation(s) are physically reasonable</u>.



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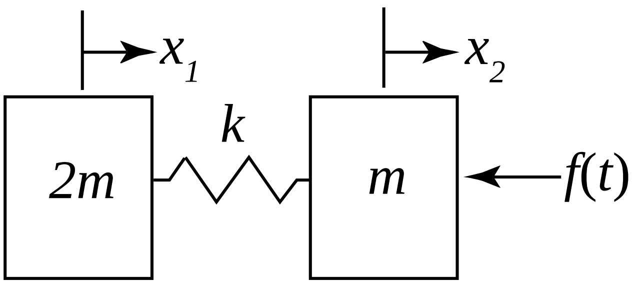
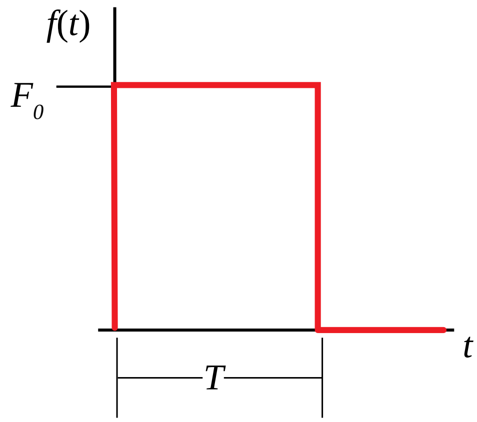
Problem #2 (20 pts)
The impact of a tennis ball with a racquet can be modeled using the two degree-of-freedom system shown below to represent the ball (the masses are only permitted to move in the horizontal direction). A ball is initially traveling to the right at speed v_0, (i.e. with $\dot{x}_1 = \dot{x}_2 = v_0$) when it strikes a racquet. Suppose that the impact force is known and is modeled as a square pulse whose duration is T. Damping is negligible.





The equations of motion of this system are:

$$\begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -f(t) \end{Bmatrix}$$

a.) (10 pts) Find the natural frequencies and mass-normalized mode shapes of the system.
b.) (10 pts) Find two uncoupled, second-order differential equations that could be solved to find the response of the tennis ball. Be sure to substitute all known quantities into each of the equations.
c.) (3 pts extra credit) Use the result from (b) to sketch the response of the first mass, $x_1(t)$, qualitatively for t > T, explaining any important features.

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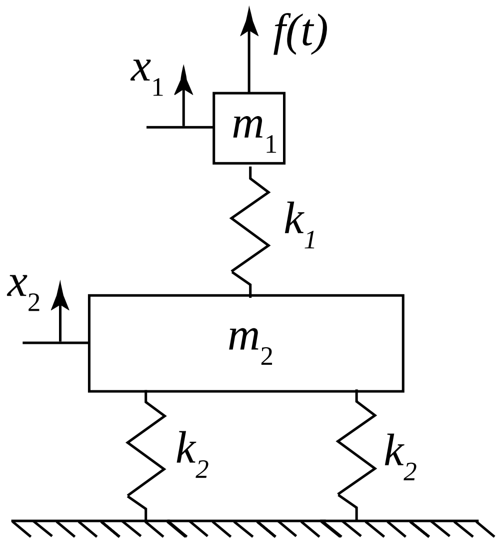
Problem #3 (30 pts)

The system below is a simplified model of an aircraft with an engine mounted on its tail.



The equations of motion for certain values of the k_1, m_1, etc…, are known except for the mass matrix,

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 0.04 & 0 \\ 0 & 0.05 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} 100 & -100 \\ -100 & 200 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f(t) \\ 0 \end{Bmatrix}$$

so M_{11}, M_{12} and M_{22} are unknown constants. The <u>mass normalized</u> modes are also known and are:

$$\{\Phi\}_1 = \begin{Bmatrix} 0.85 \\ 0.65 \end{Bmatrix} \quad \{\Phi\}_2 = \begin{Bmatrix} 1.1 \\ -0.5 \end{Bmatrix}$$

The second natural frequency is ω_2= 16.9 rad/s. Suppose the system is initially at rest when the engine starts exerting a force $f(t) = A\cos(\omega t)h(t)$ where $h(t)$ is the unit step function.

a.) (10 pts) What is the first natural frequency ω_1?

b.) (10 pts) <u>How long</u> will it take for the system's response to settle to within approximately 1% of its steady state value? (Think carefully about what is being asked here and only answer the question that was asked.)

c.) (10 pts) Find an expression for the steady state response of the first mass $x_1(t)$ in terms of the forcing frequency ω.

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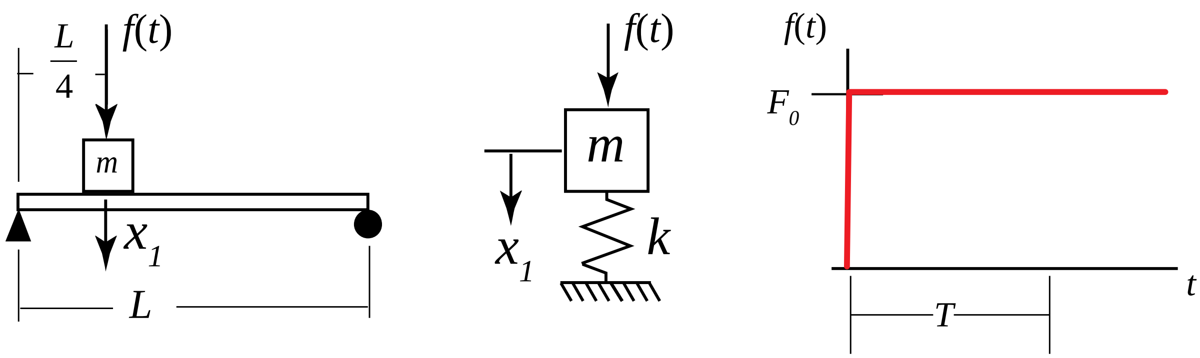
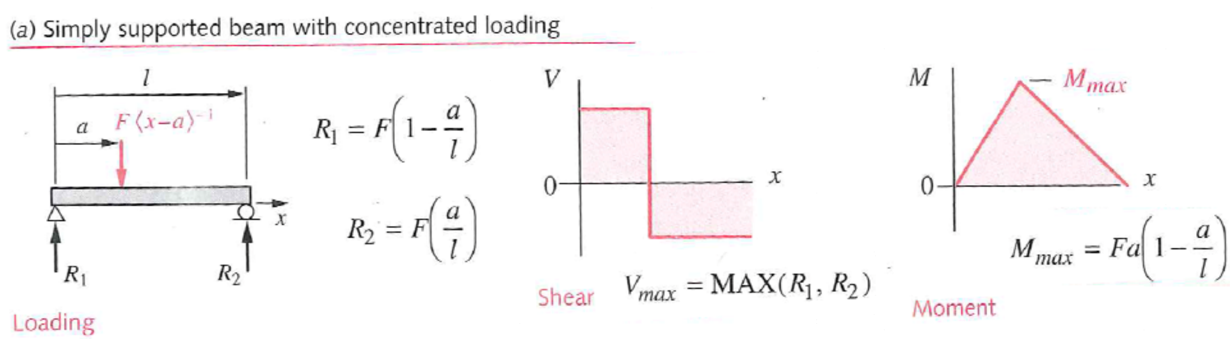
Problem #4 (10 pts)

The system shown consists of a beam with a large mass mounted one fourth of the distance from its end. This can be represented with the undamped spring-mass system shown to the right, with $k=85EI/L^3$. The system is initially in its static equilibrium position when a step force, $f(t)=F_0h(t)$, is applied to the mass.



The following information is available from a static analysis of the beam. When a static load, F, is applied to a beam, the maximum bending stress occurs in the outer fiber of the beam is given by $\sigma_{max}=-M_{max}c/I$, where M_{max} is the maximum bending moment in the beam, c is the (known) distance to the outer fiber and I is the area moment of inertia (also known). See the figure below for additional details regarding a static loading scenario.



<u>What is the amplitude of the load, F_0, that causes the beam to exceed its yield stress, σ_y?</u>

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Problem #5 (10 pts)

A single degree-of-freedom system's response is given by $x(t) = \mathrm{Re}\left(Xe^{i\omega t}\right)$, with $X=e^{i2\pi/3}$.

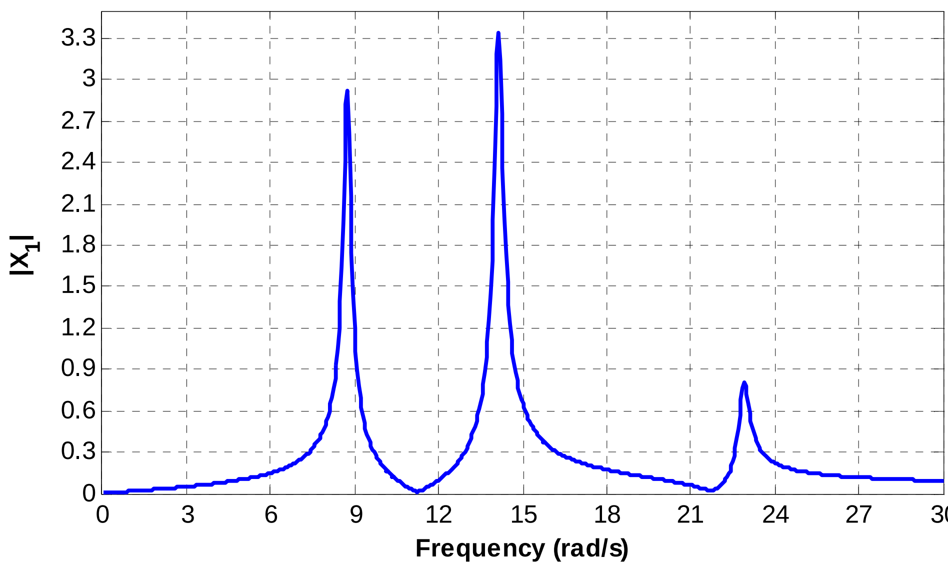
Sketch the complex amplitude, X, in the complex plane and sketch the corresponding time function $x(t)$ over at least one cycle.

Problem #6 (10 pts)

A three degree-of-freedom system is excited by a sinusoidal force, $f(t)=\cos(\omega t)$.

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{F\}\,f(t)$$

The frequency response was computed using $\{X\}=\left(-\omega^2[M]+i\omega[C]+[K]\right)^{-1}\{F\}$ and $|X_1|$ from that calculation is plotted below.



Suppose that the input, $f(t)$, is replaced with a periodic function that can be expressed as follows,

$$f(t)=\frac{1}{2}\sum_{n=-\infty}^{\infty}\left(\frac{100-n}{n}\right)e^{in\omega_1 t}$$

with ω_1 = 3.0 rad/s. What frequencies would be present in the steady-state response $x_1(t)$? Which of those would be dominant (i.e. have the largest amplitude)?