EMA 545 – Review Problems for Final Exam - Prof. M. S. Allen
Spring 2011

Problem #1:

4.34 The mass matrix, excitation, and modal properties of a two-degree-of-freedom system are known to be

$$[m] = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix} \text{kg}, \quad \{Q\} = \begin{Bmatrix} 0 \\ 20t \end{Bmatrix} N$$

$$\omega_1 = 5.42 \text{ rad/s}, \quad \omega_2 = 13.00 \text{ rad/s},$$

$$\{\phi_1\} = \begin{Bmatrix} 1 \\ 0.414 \end{Bmatrix}, \quad \{\phi_2\} = \begin{Bmatrix} 1 \\ \alpha \end{Bmatrix}$$

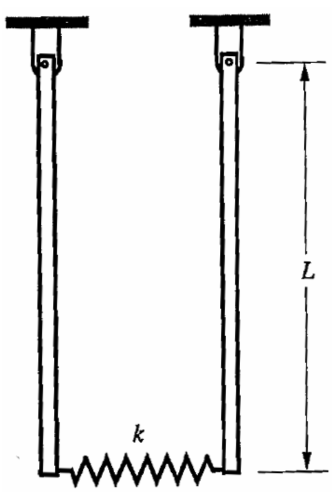
where α is an unspecified value. The system was initially at rest in the static equilibrium position. Determine the response.

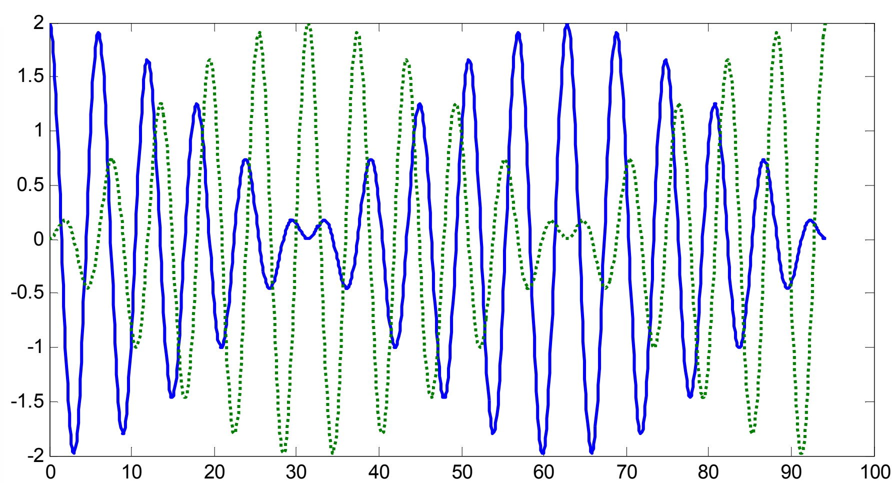
Problem #2:

The system below consists of two pendulums on frictionless pins, connected at their tips by a soft spring $k = \alpha mg(L/2)$ where α is a small constant. The equations of motion are the following, where θ_1 corresponds to the left bar.

$$\frac{1}{3}mL^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + mg\frac{L}{2} \begin{bmatrix} \alpha + 0.5 & -\alpha \\ -\alpha & \alpha + 0.5 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

The bar on the left is given an initial displacement of 2 degrees with the bar on the right vertical and the response is recorded and is shown below. The blue line corresponds to θ_1 while the dashed green line corresponds to θ_2. The amplitude of the motion of the left beam decays and then grows with time as the vibration energy transfers from one pendulum to the other and then back again. The beat period is 63 seconds. What is the value of α? Justify your answer, but you need only do those parts of the analysis that are critical to understand what is happening.





FYI: You can see a video of a system such as this at:
http://www.youtube.com/watch?v=RoSYKPTdlxs

Problem #3:

The two degree-of-freedom system shown below is forced by means of ground excitation. The lower spring rests on the moving base, but is not attached to it. When y(t)=0, the system is in static equilibrium when $x_1 = x_2 = 0$. The equations of motion for the system are given by:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix}+\begin{bmatrix} c & -c \\ -c & c \end{bmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}+\begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}=\begin{Bmatrix} k_1 y \\ 0 \end{Bmatrix}$$

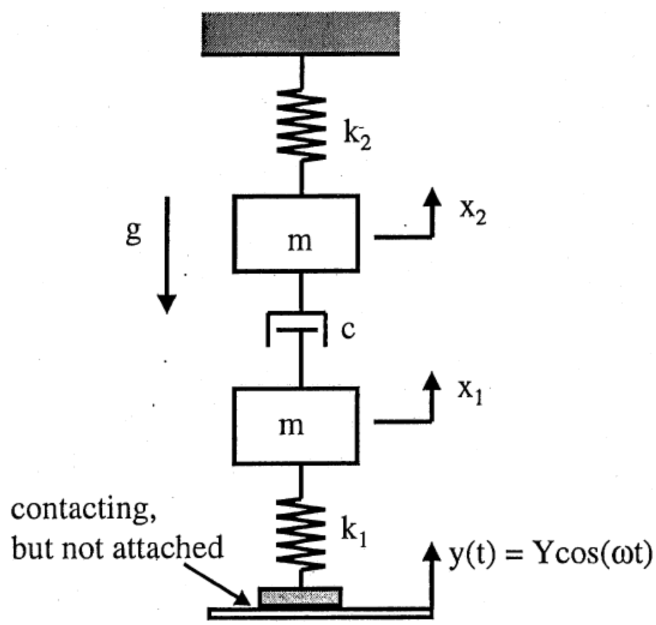
For all of the following you may assume that the base of spring k_1 never looses contact with the platform.

a.) Find the steady-state response $x_1(t)$ and $x_2(t)$ when ω is equal to the first natural frequency of the system. Use $k_1 = k$ and $k_2 = 2k$.

b.) Are there any frequencies of excitation for which the steady-state response amplitude for x_1 is identically zero? Use $k_1 = k$ and $k_2 = 2k$.

c.) If $k_1 = k_2 = k$, what is the steady-state response amplitude of each mass when the system is excited at a resonant frequency? Can you explain the result?

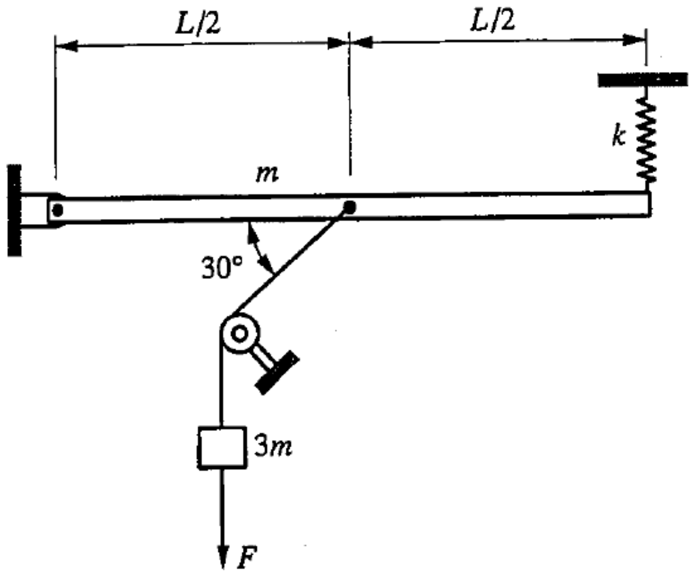
d.) For arbitrary k_1 and k_2, if y(t) is such that $x_1(t) = A\cos(\omega t)$, what is the response $x_2(t)$?



Extra credit – under what conditions does the base of the spring k_1 loose contact with the platform?

Problem #4

Find the linearized equations of motion for the system pictured below. Model the cable as a massless spring with stiffness *k*. The system is in static equilibrium in the position shown when the dynamic force F is not present.



Problem #5

Sorry, I didn't have time to write a 5^{th} problem. This final probably isn't quite long enough.

TRANSIENT RESPONSES FOR UNDERDAMPED ONE-DEGREE-OF-FREEDOM SYSTEMS

$$\ddot{q} + 2\zeta\omega_{nat}\dot{q} + \omega_{nat}^2 q = \frac{F(t)}{M} \quad \zeta < 1, \quad \omega_d = \omega_{nat}\sqrt{1-\zeta^2}$$

- Free vibration: $F(t) = 0$

$$q = \exp(-\zeta\omega_{nat}t)\left[q(0)\cos(\omega_d t) \frac{\dot{q}(0) + \zeta\omega_{nat}q(0)}{\omega_d}\sin(\omega_d t)\right]$$

- Impulse excitation: $F(t) = \delta(t)$

$$q = \frac{1}{M\omega_d}\exp(-\zeta\omega_{nat}t)\sin(\omega_d t)h(t)$$

- Step excitation: $F(t) = h(t)$

$$q = \frac{1}{M\omega_{nat}^2}\left\{1 - \exp(-\zeta\omega_{nat}t)\left[\cos(\omega_d t) + \frac{\zeta\omega_{nat}}{\omega_d}\sin(\omega_d t)\right]\right\}h(t)$$

- Ramp excitation: $F(t) = th(t)$

$$q = \frac{1}{M\omega_{nat}^3}\left\{(\omega_{nat}t) - 2\zeta + \exp(-\zeta\omega_{nat}t)\left[2\zeta\cos(\omega_d t) - (1-2\zeta^2)\frac{\omega_{nat}}{\omega_d}\sin(\omega_d t)\right]\right\}h(t)$$

- Quadratic excitation: $F(t) = t^2h(t)$

$$q = \frac{1}{M\omega_{nat}^4}\left\{(\omega_{nat}t)^2 - 4\zeta(\omega_{nat}t) - 2(1-4\zeta^2) + \exp(-\zeta\omega_{nat}t) \times \left[2(1-4\zeta^2)\cos(\omega_d t) + (6\zeta - 8\zeta^3\frac{\omega_{nat}}{\omega_d}\sin(\omega_d t)\right]\right\}h(t)$$

- Exponential excitation:

$$F(t) = \exp(-\beta t)\,h(t)$$

$$q = \frac{1}{M(\omega_{nat}^2 - 2\zeta\omega_{nat}\beta + \beta^2)}\left\{\exp(-\beta t) - \exp(-\zeta\omega_{nat}t)\left[\cos(\omega_d t) + \frac{\zeta\omega_{nat} - \beta}{\omega_d}\sin(\omega_d t)\right]\right\}h(t)$$

- Transient sinusoidal excitation:

$$F(t) = \sin(\omega t)h(t), \quad \omega \neq \omega_{nat} \text{ if } \zeta \neq 0$$

$$q = \frac{1}{M[(\omega_{nat}^2 - \omega^2)^2 + 4\zeta^2\omega_{nat}^2\omega^2]} \times \left\{(\omega_{nat}^2 - \omega^2)\sin(\omega t) - 2\zeta\omega_{nat}\omega\cos(\omega t) + \omega\exp(-\zeta\omega_{nat}t)\left[2\zeta\omega_{nat}\cos(\omega_d t) - \frac{(1-2\zeta^2)\omega_{nat}^2 - \omega^2}{\omega_d}\sin(\omega_d t)\right]\right\}h(t)$$

- Transient co-sinusoidal excitation:

$$F(t) = \cos(\omega t)h(t), \quad \omega \neq \omega_{nat} \text{ if } \zeta \neq 0$$

$$q = \frac{1}{M[(\omega_{nat}^2 - \omega^2)^2 + 4\zeta^2\omega_{nat}^2\omega^2]} \times \left\{(\omega_{nat}^2 - \omega^2)\cos(\omega t) + 2\zeta\omega_{nat}\omega\sin(\omega t) - \exp(-\zeta\omega_{nat}t)\left[(\omega_{nat}^2 - \omega^2)\cos\omega_d t) + \frac{\zeta\omega_{nat}(\omega_{nat} + \omega^2)}{\omega_d}\sin(\omega_d t)\right]\right\}h(t)$$