Name: ______________________________

EMA 545 –Exam #2
Spring 2013 - Prof. M. S. Allen

Honor Pledge: On my honor, I pledge that I have neither given nor received inappropriate aid in the preparation of this exam.

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Signature

Formulas:

Stiff spring approx:
$\dot{\Delta} \approx \dot{u}_B - \dot{u}_A = \left(\vec{v}_B - \vec{v}_A\right) \bullet \vec{e}_{B/A}$

Newton's Laws (2D):
$\sum \vec{F} = m\vec{a}$
$\sum M_g = I_g \ddot{\theta}$ or
$\sum M_A = I_A \ddot{\theta}$ if $\vec{a}_A = 0$

moment of inertia of a thin rod about its center of mass:
$I_g = (1/12)mL^2$
about its end:
$I_{end} = (1/3)mL^2$

Appendix B from Ginsberg, Wiley, 2001: (Corrected)

$$\ddot{q} + 2\zeta\omega_{nat}\dot{q} + \omega_{nat}^2 q = \frac{F(t)}{M} \quad \zeta < 1, \quad \omega_d = \omega_{nat}\sqrt{1-\zeta^2}$$

- Free vibration: $F(t) = 0$

$$q = \exp(-\zeta\omega_{nat}t)\left[q(0)\cos(\omega_d t) + \frac{\dot{q}(0) + \zeta\omega_{nat}q(0)}{\omega_d}\sin(\omega_d t)\right]$$

- Impulse excitation: $F(t) = \delta(t)$

$$q = \frac{1}{M\omega_d}\exp(-\zeta\omega_{nat}t)\sin(\omega_d t)h(t)$$

- Step excitation: $F(t) = h(t)$

$$q = \frac{1}{M\omega_{nat}^2}\left\{1 - \exp(-\zeta\omega_{nat}t)\left[\cos(\omega_d t) + \frac{\zeta\omega_{nat}}{\omega_d}\sin(\omega_d t)\right]\right\}h(t)$$

- Ramp excitation: $F(t) = th(t)$

$$q = \frac{1}{M\omega_{nat}^3}\left\{(\omega_{nat}t) - 2\zeta + \exp(-\zeta\omega_{nat}t)\left[2\zeta\cos(\omega_d t) - (1-2\zeta^2)\frac{\omega_{nat}}{\omega_d}\sin(\omega_d t)\right]\right\}h(t)$$

- Exponential excitation:
$F(t) = \exp(-\beta t)\, h(t)$

$$q = \frac{1}{M(\omega_{nat}^2 - 2\zeta\omega_{nat}\beta + \beta^2)}\left\{\exp(-\beta t) - \exp(-\zeta\omega_{nat}t)\left[\cos(\omega_d t) + \frac{\zeta\omega_{nat} - \beta}{\omega_d}\sin(\omega_d t)\right]\right\}h(t)$$

- Transient sinusoidal excitation:
$F(t) = \sin(\omega t)h(t), \ \omega \neq \omega_{nat}$ if $\zeta \neq 0$

$$q = \frac{1}{M[(\omega_{nat}^2 - \omega^2)^2 + 4\zeta^2\omega_{nat}^2\omega^2]} \times \left\{(\omega_{nat}^2 - \omega^2)\sin(\omega t) - 2\zeta\omega_{nat}\omega\cos(\omega t) + \omega\exp(-\zeta\omega_{nat}t)\left[2\zeta\omega_{nat}\cos(\omega_d t) - \frac{(1-2\zeta^2)\omega_{nat}^2 - \omega^2}{\omega_d}\sin(\omega_d t)\right]\right\}h(t)$$

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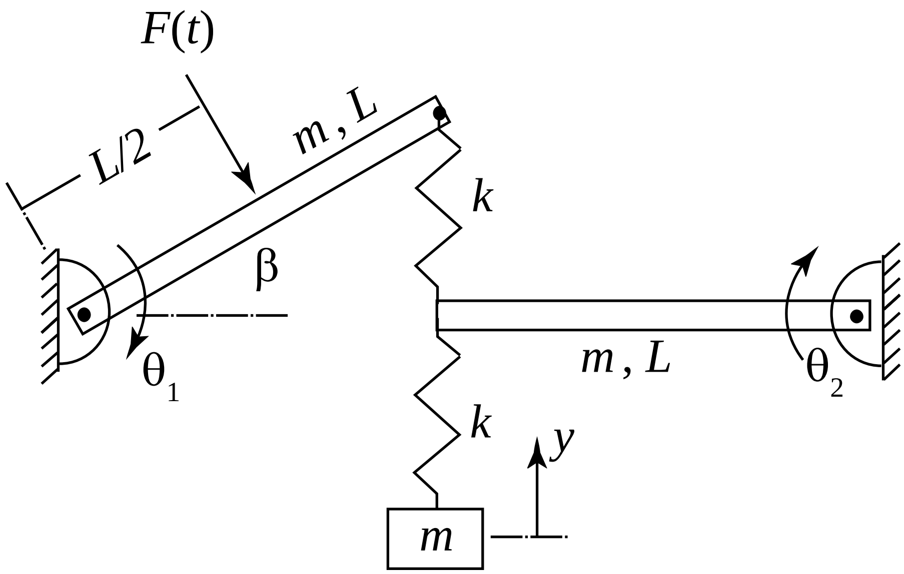
Problem #1 (20 pts)
The system pictured below consists of two identical bars of mass m and length L. The bars are connected as shown with stiff springs, so that the left bar is inclined at an angle β while the right bar is horizontal. This position is the static equilibrium position. Both springs have stiffnesses k. The generalized coordinates are the position of the mass, denoted y, and the angular deflections of the bars from static equilibrium, denoted θ_1 and θ_2. The angles θ_1 and θ_2 are positive in the directions shown.



A co-worker asserts that one of the equations of motion for this system is the following.

$$\frac{1}{3}mL\ddot{\theta}_1 + kL\cos\beta\left(L\theta_2 + L\theta_1\cos\beta\right) - kLy\cos\beta = \frac{FL}{2}$$

Consider the physics of the problem and check the **sign** and **units** of each term. Does each term produce the expected effect? Explain your reasoning.