EMA 545 – Practice Exam #1
Spring 2013 - Prof. M. S. Allen

Honor Pledge: On my honor, I pledge that I have neither given nor received inappropriate aid in the preparation of this exam.

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Signature

Calculators are allowed but not really needed. You may use one sheet of notes (one side).
Formulas:

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

Transient Response

The general solution to an underdamped SDOF system

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

is:

$$x(t) = \mathrm{Re}\left(Ae^{-\zeta\omega_n t}e^{i\omega_d t}\right)$$

where $\omega_d = \omega_n\sqrt{1-\zeta^2}$ and A is a complex constant.

Forced Transient Response:

$$x(t) = x_{IC}(t) + x_F(t)$$

where $x_F(t)$ can be found in Appendix B

Forced Steady-State Response:

$$f(t) = \mathrm{Re}\left(Fe^{i\omega t}\right) \qquad \rightarrow \qquad x(t) = \mathrm{Re}\left(Xe^{i\omega t}\right)$$

Appendix B from Ginsberg, Wiley, 2001:

$$\ddot{q} + 2\zeta\omega_{nat}\dot{q} + \omega_{nat}^2 q = \frac{F(t)}{M} \quad \zeta < 1, \quad \omega_d = \omega_{nat}\sqrt{1-\zeta^2}$$

- Free vibration: $F(t) = 0$

$$q = \exp(-\zeta\omega_{nat}t)[q(0)\cos(\omega_d t) + \frac{\dot{q}(0) + \zeta\omega_{nat}q(0)}{\omega_d}\sin(\omega_d t)]$$

- Impulse excitation: $F(t) = \delta(t)$

$$q = \frac{1}{M\omega_d}\exp(-\zeta\omega_{nat}t)\sin(\omega_d t)h(t)$$

- Step excitation: $F(t) = h(t)$

$$q = \frac{1}{M\omega_{nat}^2}\{1 - \exp(-\zeta\omega_{nat}t)[\cos(\omega_d t) + \frac{\zeta\omega_{nat}}{\omega_d}\sin(\omega_d t)]\}h(t)$$

- Ramp excitation: $F(t) = th(t)$

$$q = \frac{1}{M\omega_{nat}^3}\{(\omega_{nat}t) - 2\zeta + \exp(-\zeta\omega_{nat}t)[2\zeta\cos(\omega_d t) - (1-2\zeta^2)\frac{\omega_{nat}}{\omega_d}\sin(\omega_d t)]\}h(t)$$

- Quadratic excitation: $F(t) = t^2 h(t)$

$$q = \frac{1}{M\omega_{nat}^4}\{(\omega_{nat}t)^2 - 4\zeta(\omega_{nat}t)$$
$$-2(1-4\zeta^2) + \exp(-\zeta\omega_{nat}t)$$
$$\times [2(1-4\zeta^2)\cos(\omega_d t) + (6\zeta$$
$$-8\zeta^3)\frac{\omega_{nat}}{\omega_d}\sin(\omega_d t)]\}h(t)$$

- Exponential excitation:
$$F(t) = \exp(-\beta t)h(t)$$

$$q = \frac{1}{M(\omega_{nat}^2 - 2\zeta\omega_{nat}\beta + \beta^2)}\{\exp(-\beta t)$$
$$- \exp(-\zeta\omega_{nat}t)[\cos(\omega_d t)$$
$$+\frac{\zeta\omega_{nat} - \beta}{\omega_d}\sin(\omega_d t)]\}h(t)$$

- Transient sinusoidal excitation:
$$F(t) = \sin(\omega t)h(t), \quad \omega \neq \omega_{nat} \text{ if } \zeta \neq 0$$

$$q = \frac{1}{M[(\omega_{nat}^2 - \omega^2)^2 + 4\zeta^2\omega_{nat}^2\omega^2]}$$
$$\times \{(\omega_{nat}^2 - \omega^2)\sin(\omega t) - 2\zeta\omega_{nat}\omega\cos(\omega t)$$
$$+ \omega\exp(-\zeta\omega_{nat}t)[2\zeta\omega_{nat}\cos(\omega_d t)$$
$$-\frac{(1-2\zeta^2)\omega_{nat}^2 - \omega^2}{\omega_d}\sin(\omega_d t)]\}h(t)$$

- Transient co-sinusoidal excitation:
$$F(t) = \cos(\omega t)h(t), \quad \omega \neq \omega_{nat} \text{ if } \zeta \neq 0$$

$$q = \frac{1}{M[(\omega_{nat}^2 - \omega^2)^2 + 4\zeta^2\omega_{nat}^2\omega^2]}$$
$$\times \{(\omega_{nat}^2 - \omega^2)\cos(\omega t) + 2\zeta\omega_{nat}\omega\sin(\omega t)$$
$$- \exp(-\zeta\omega_{nat}t)[(\omega_{nat}^2 - \omega^2)\cos\omega_d t$$
$$+\frac{\zeta\omega_{nat}(\omega_{nat}^2 + \omega^2)}{\omega_d}\sin(\omega_d t)]\}h(t)$$

Short Answer Questions: (10 pts each)

1.) A very lightly damped single-degree-of-freedom system is observed to oscillate at 200 Hz after being excited by an impulsive force at $t = 0$. The oscillations are observed to fall below 1% of the initial amplitude after $t = 1.5$ seconds. What is the damping ratio, ζ? (***Show your work! An unjustified answer will not receive credit.***)

2.) A certain single-degree of freedom system is excited by a force $f(t) = -\cos(\omega t)-1.5\sin(\omega t)$, resulting in a response $x(t) = 1.2\cos(\omega t)+0.226\sin(\omega t)$. By what angle does the response lag the force? (***Show your work! An unjustified answer will not receive credit.***) Is this possible? If it is, would this require that the excitation frequency be smaller or larger than the natural frequency?

Problem #3 (40 pts)
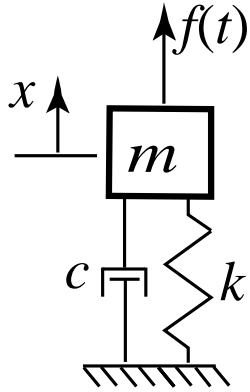
A single-degree-of-freedom system is initially at rest in its static equilibrium position when, at $t=0$, a force, $f(t) = \text{Re}(F_0 e^{i\omega t})$ is applied, where the drive frequency, ω, is one fourth of the natural frequency of the system and F_0 is a real constant. The stiffness of the system is 10 N/m, its mass is 0.1 kg, and the system can be modeled as undamped over the time interval of interest, so $c = 0$.

a.) Find an expression for the response of the system for $t > 0$.

b.) Sketch the response noting any important features.

Problem #4 (40 pts)
The schematic shows a simplified model for an automotive suspension, where the mass represents a wheel and the springs and dashpots represent the suspension and tire. The equations of motion for the system shown are:

$$\begin{bmatrix} m & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} c+c_g & -c \\ -c & c+c_2 \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} k_g & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 0 \\ f(t) \end{Bmatrix}$$

All of the system parameters, m, c, c_g, c_2, and k_g are known. A harmonic force $f(t)$ excites the system and steady state conditions have been reached so the motion of the massless point, $y(t)$, is harmonic, $y(t) = \mathrm{Re}(Ye^{i\omega t})$. This motion has been measured so the complex amplitude, Y, and frequency, ω, are known.

(a) (20 pts) Derive the transfer function relating the complex amplitude of $x(t)$ to that of $y(t)$.

(b) (10 pts) What would the **amplitude** of $x(t)$ be for very small and very large drive frequencies? You may express your answer in terms of m, c, c_g, c_2, and k_g, Y and ω.