Homework #5, EMA 545, Spring 2013

Due Thursday 2/28

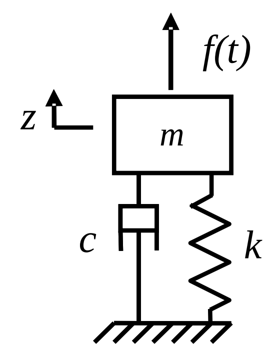
Comment: As I mentioned in class, I strongly encourage you to avoid hunting for formulas on forced response. All of these problems can be solved simply by knowing the differential equation and that the force and steady state response have the form:

$$f(t) = \mathrm{Re}\left(Fe^{i\omega t}\right) \qquad \rightarrow \qquad x(t) = \mathrm{Re}\left(Xe^{i\omega t}\right)$$

Problem 1: 3.9 from Ginsberg (Hint: assume that the motion of every component of the system is harmonic. Derive the equation(s) of motion and show the full derivation used to obtain the complex amplitude(s) from the equation(s) of motion.)

Problem 2: (3e1) (**20 points**)

A 450 kg generator, modeled as a rigid mass, must be installed on the same floor as some sensitive laboratory equipment. The operation of the generator results in a vertical force, $f(t)$, being applied to the generator (rigid mass) whose amplitude is 20kN and whose frequency is 1800 rpm. Use a damping ratio of $\zeta = 0.03$ for both (a) and (b) below.



a.) Find the stiffness of the support, k, such that the force transmitted to the ground is no more than 2kN.

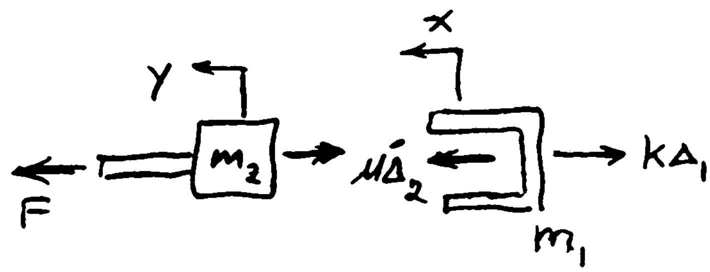
b.) Take your result from part (a) and compute the amplitude of the generator as the machine starts up. (As the machine starts up, assume that the force amplitude is constant at 20kN, but the frequency increases very slowly from zero to 1800 rpm. Do a worst-case analysis – just assure that the steady-state amplitude of the machine is less than 10mm for any forcing frequency between 0 and 1800 rpm.)

c.) Using your results from (a) and (b), suppose that the startup amplitude must not exceed 10mm. The startup amplitude can be decreased by adding mass to the generator while also increasing the stiffness of the support to keep the natural frequency of the system constant. How much mass must be added to keep the amplitude below 10mm?

Problem 3: 3.19 from Ginsberg

Problem 4: 3.23 from Ginsberg

Exercise 3.9



Given $y = A\sin\omega t$,
$m_1 = 0.5$ kg,
$m_2 = 1.0$ kg,
$k = 3.2(10^3)$ N/m,
$\mu = 40$ N-s/m,
$A = 0.02$ meter

Find amplitude & phase of F relative to y for $\omega = 150\pi$ & 170π.

Solution: Eqs of motion: $\Delta_1 = x$, $\Delta_2 = y - x$

Piston: $F - \mu(\dot{y} - \dot{x}) = m_2 \ddot{y}$

Tube: $\mu(\dot{y} - \dot{x}) - kx = m_1 \ddot{x}$

It is given that $y = A\sin(\omega t) = Re\left[\frac{A}{i} e^{i\omega t}\right]$

Harmonic variation ⇒ Assume $x = Re\left[\frac{X}{i} e^{i\omega t}\right]$, $F = Re\left[\frac{\hat{F}}{i} e^{i\omega t}\right]$

Note: Using $\frac{1}{i}$ as a factor for x & F means that the polar angles of $\hat{F}$ and X are phase angles relative to y.

Substitute into eqs of motion & cancel $e^{i\omega t}$:

$$\hat{F} - \mu(i\omega)(A - X) = -m_2\omega^2 A \qquad (1)$$

$$\mu(i\omega)(A - X) - kX = -m_1\omega^2 X \qquad (2)$$

These are two simultaneous equations for F and X at any ω because $A = 0.02$:

From (2): $$X = \frac{i\omega\mu}{k + i\omega\mu - m_1\omega^2} A$$

From (1):

$$\hat{F} = \left[(i\omega\mu - m_2\omega^2) + \frac{\mu^2\omega^2}{k + i\omega\mu - m_1\omega^2}\right] A$$

For $\omega = 75$ rad/s:

$F = -104.88 + 0.99i = 104.88 e^{i3.132}$

or $|F| = 104.88$ N @ $\phi = 3.132 \frac{180}{\pi} = 179.46°$ ahead of y ⇐

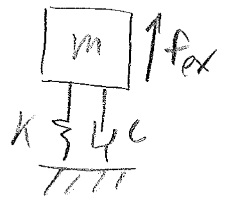
For $\omega = 85$ rad/s:

$F = -152.63 + 0.99i = 152.63 e^{3.135i}$

or $|F| = 152.63$ N @ $\phi = 3.135 \frac{180}{\pi} = 179.63°$ ahead of y ⇐

HW 7 Solution | V2 - new damping

P3e-01) Generator



$m = 450\,kg \qquad |f| = 20\,kN \quad @\ 1800\,rpm$

$\xi = 0.03$

EOM:

$$m\ddot{x} + c\dot{x} + Kx = f_{ex} \qquad (1)$$

$$f_{tr} = c\dot{x} + Kx \qquad (2)$$

from (1)

$$\boxed{X = \frac{F_{ex}/m}{w_n^2 - w^2 + i w \cdot 2\xi w_n}} \qquad (\text{since } f_{ex} = Re(F_{ex} e^{iwt}))$$

$$F_{tr} = (iwc + K)\,X$$

$$F_{tr} = \frac{(iwc + K)/m \cdot F_{ex}}{\cdots}$$

$$F_{tr} = \frac{(i 2\xi w_n w + w_n^2)}{(w_n^2 - w^2 + i 2\xi w w_n)} F_{ex}$$

define $r = w/w_n$

$$\boxed{F_{tr} = \frac{(i 2\xi r + 1)}{(1 - r^2 + i 2\xi r)} F_{ex}}$$

a) $|F_{tr}| = 0.1 |F_{ex}| \qquad 0.1 = \dfrac{\sqrt{(1 + 4\xi^2 r^2)}}{\sqrt{(1-r^2)^2 + 4\xi^2 r^2}}$

$$0.01\left[(1 - 2r^2 + r^4) + 4\xi^2 r^2\right] = 1 + 4\xi^2 r^2$$

$$0.01 r^4 + (0.01 \cdot 4\xi^2 - 0.02 - 4\xi^2) r^2 - 0.99 = 0$$

polynomial solver, or plot $\rightarrow r^2 = 11.198$

$\frac{w}{w_n}$ $\quad r \geq 3.3463$

$$w = 1800 \frac{rev}{min} \cdot \frac{1 min}{60 sec} \cdot \frac{2\pi\, rad}{rev} = 60\pi\ rad/s \approx 188.5\ rad/s$$

$$w_n \leq \frac{60\pi}{3.35} \rightarrow w_n \leq 56.33\ rad/s \qquad w_n = \sqrt{\frac{K}{m}}$$

$$K \leq w_n^2 m \rightarrow \boxed{K \leq 1.428 \times 10^6\ N/m}$$

b) $X = \dfrac{20{,}000\ N / 450\ kg}{w_n^2 - w^2 + i w 2\xi w_n} \Rightarrow$ Peak @ $w = w_n\sqrt{1 - 2\xi^2}$

let $w = 56.33\sqrt{1 - 2 \cdot 0.03^2} \rightarrow X = 0.007 - 0.233\hat{i}$

$\longrightarrow$

b) continued

$abs(X) = 0.234\ m$

exceeds 10mm → add mass

* assume Damping ratio unchanged,

$$X^{new} = \frac{F/m'}{w_n^2 - w^2 + i w \cdot 2\zeta w_n} = X \cdot \frac{m}{m'}$$

$$\frac{m}{m'} = \frac{10\,mm}{234\,mm} \rightarrow m' = \frac{234}{10} m$$

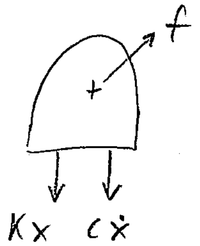
$$\boxed{m' = 10{,}510\ kg \sim \text{or add } 10{,}060\ kg}$$

New stiffness:

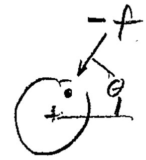
$$\boxed{K^{new} = w_n^2 \cdot m' = 3.33 \times 10^7\ N/m}$$

$$1/s^2 \cdot kg = kg/s^2 = \frac{kg\text{-}m}{s^2} \cdot 1/m = N/m \checkmark$$

$m = 80\,Kg$ ~ Motor + rotor $\qquad m_1$ = motor $\quad m_2$ = rotor



$= m_1 \ddot{x}$



$= m_2 \vec{a}_g$

$\vec{a}_g = \ddot{x} j - \varepsilon \Omega^2 (\cos\theta \hat{i} + \sin\theta \hat{j})$

$-f = m_2 \ddot{x} j - m_2 \varepsilon \Omega^2 (\cos\theta \hat{i} + \sin\theta \hat{j})$

$m_1 \ddot{x} + c\dot{x} + Kx = f \cdot \hat{j}$

$= -m_2 \ddot{x} + m_2 \varepsilon \Omega^2 \sin\theta \qquad \theta = \Omega t$

$\boxed{(m_1 + m_2)\ddot{x} + c\dot{x} + Kx = m_2 \varepsilon \Omega^2 \sin(\Omega t)}$ EOM

Note: pos of rotor ∴ let $F = -i m_2 \varepsilon \Omega^2 \rightarrow f(t) = Re(F e^{i\omega t})$
— $f(t) = 0$ when $t = 0$

- Steady state : $x(t) = Re(X e^{i\Omega t})$

$$X = \frac{-i m_2 \varepsilon \Omega^2}{-(m_1 + m_2)\Omega^2 + c(i\Omega) + K}$$

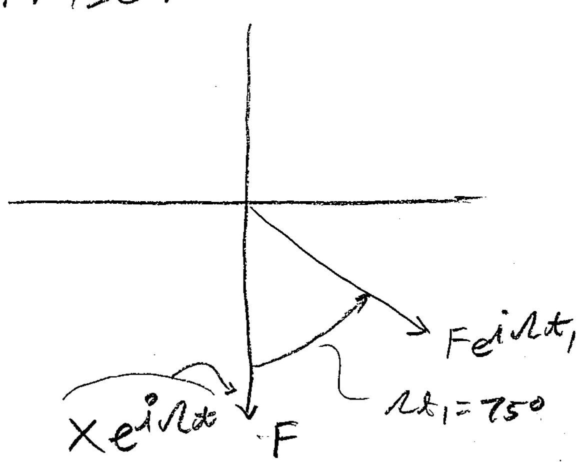
given → static deflection due to $F = 80\,kg \cdot 9.81\,m/s^2$ is 40 mm

$f = Kx \longrightarrow K = \frac{80\,kg \cdot 9.81\,m/s^2}{0.040\,m} = 19{,}620\ N/m$

$m_1 + m_2 = 80\,kg$

Unknowns : ζ and m_2

Phase :



Given @ t_1, $\theta = 75°$
Note: $\theta = 0$, $f(t) = 0$
$\theta = 90°$ $f(t) = +m_2 \varepsilon \Omega^2$

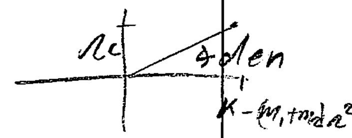
So, the angle below is 75°

Similarly, $x(t)$ is zero and increasing at t_1
$x(t) = Re(X e^{i\Omega t_1})$
so it must also be purely imaginary

In other words, it is lagging the force by 75°

so,

$$\text{angle}(Xe^{i\Omega t_1}) = \text{angle}\left(\frac{-i m_2 \varepsilon \Omega^2 e^{i\Omega t_1}}{-(m_1+m_2)\Omega^2 + i\Omega c + K}\right)$$



$-i = e^{-\pi/2}$ $\qquad$ ∡num − ∡den

$$-\cancel{90^\circ} = -\cancel{90} + 75^\circ - \operatorname{atan}\left(\frac{\Omega c}{K-(m_1+m_2)\Omega^2}\right)$$

$$K-(m_1+m_2)\Omega^2 \tan(75^\circ) = \Omega c$$

$$(\omega_n^2 - \Omega^2)\tan 75 = \Omega \cdot 2\zeta\omega_n$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{19620}{80}}$$

$$\omega_n = 15.66 \text{ rad/s}$$

$$\zeta = \frac{\omega_n^2 - \Omega^2}{\Omega \cdot 2 \cdot \omega_n} \cdot \tan(75^\circ)$$

a) $\boxed{\zeta = 0.1152}$

$$|X| = 0.010 \quad @ \ \Omega = \frac{145 \cdot 2\pi}{60}$$

$$|X| = m_2\varepsilon \left|\frac{-i\Omega^2}{-(m_1+m_2)\Omega^2 + ic\Omega + K}\right| \qquad c = (m_1+m_2)2\zeta\omega_n$$

$$0.010 = m_2\varepsilon \cdot 0.0508$$

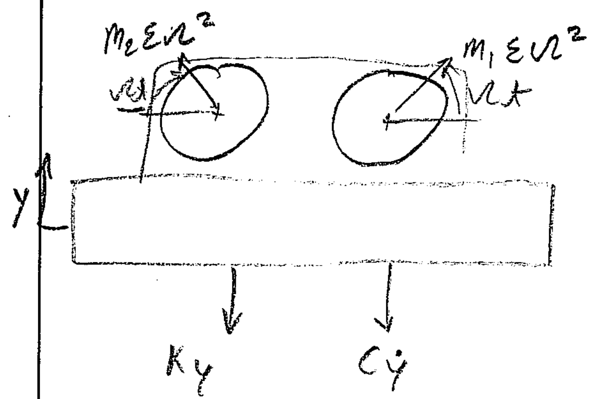
b) $\boxed{m_2\varepsilon \approx \frac{0.010}{0.0508} = 0.1969}$

c) as $\Omega \to \infty$ $\quad X$ on prev page becomes

$$X\big|_{\Omega\to\infty} = \frac{-i m_2 \varepsilon \cancel{\Omega^2}}{-(m_1+m_2)\cancel{\Omega^2}}$$

$$\boxed{|X|_{\Omega\to\infty} = \frac{m_2\varepsilon}{(m_1+m_2)} \approx \frac{0.1969}{80} \approx 0.00246 \text{ m}}$$

(Min of $|X|$ occurs at $\Omega = \infty$ if $\Omega > \omega_n$)



Given: $\Omega = 900\,rpm = \frac{900 \cdot 2\pi}{60}$ rad/s

- Block passes static eq. with $\dot{y} > 0$

$|Y| = 8.5\ mm$, $m_{tot} = 200\,kg$

$m_1 \varepsilon = 0.5\ kg\text{-}m$

a) find w_n b) find c

c) find $|Y|$ and angle of rotors at $\Omega = 1000\,rpm$.

Power Balance: (Beam mass negligible $\rightarrow$ spring)

$T = \frac{1}{2} m_{tot} \dot{y}^2 \quad V = \frac{1}{2} K y^2 \quad P_{dis} = C\dot{y}^2$

$P_{in} = m_1 \varepsilon \Omega^2 (\cos\Omega t\,\hat{i} + \sin\Omega t\,\hat{j}) \cdot \dot{y}\hat{j} +$

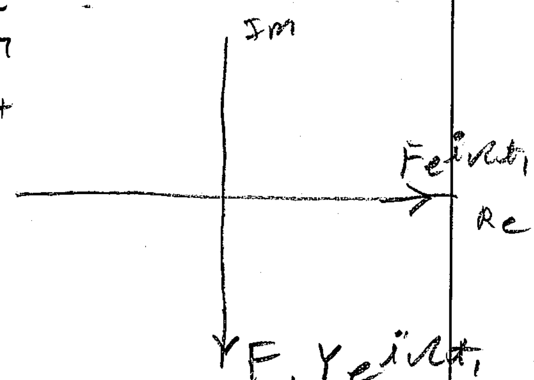
$+ m_1 \varepsilon \Omega^2 (-\cos\Omega t\,\hat{i} + \sin\Omega t\,\hat{j}) \cdot \dot{y}\hat{j}$

$= \underbrace{2 m_1 \varepsilon \Omega^2 \sin\Omega t}_{Q}\ \dot{y}$

$m_{tot}\ddot{y} + C\dot{y} + Ky = 2 m_2 \varepsilon \Omega^2 \sin\Omega t$

$y(t) = Re(Y e^{i\Omega t})$, $w_n = \sqrt{\frac{K}{m_{tot}}}$

$$Y = \frac{2 m_2 \varepsilon \Omega^2}{K + i\Omega C - \Omega^2 m_{tot}}$$



let $2 m_2 \varepsilon \Omega^2 \sin\Omega t = Re(F e^{i\Omega t})$

$F = -i 2 m_2 \varepsilon \Omega^2$

from description: @ t_1 masses at highest pos,

so $Fe^{i\Omega t_1}$ is real, $\Omega t_1 = 90° + n360°$, n = any integer

$y(t) = 0$ and increasing, so $Ye^{i\Omega t_1}$ = purely imaginary

from above — only possible if $\Omega^2 = w_n$

$$\left| Y|_{\Omega = 30\pi} \right| = \left| \frac{2 m_2 \varepsilon \Omega^2}{i\Omega c} \right|$$

$$c = \frac{2 m_2 \varepsilon \Omega}{|Y|} = \frac{2 \cdot 0.5\,kg\text{-}m \cdot 30\pi\ rad/s}{0.0085\,m}$$

b) $\boxed{c = 11{,}088\ kg/s}$ a) $\boxed{w_n = \Omega = 30\pi\ rad/s}$

$\zeta = 0.2941 \rightarrow$ heavily damped

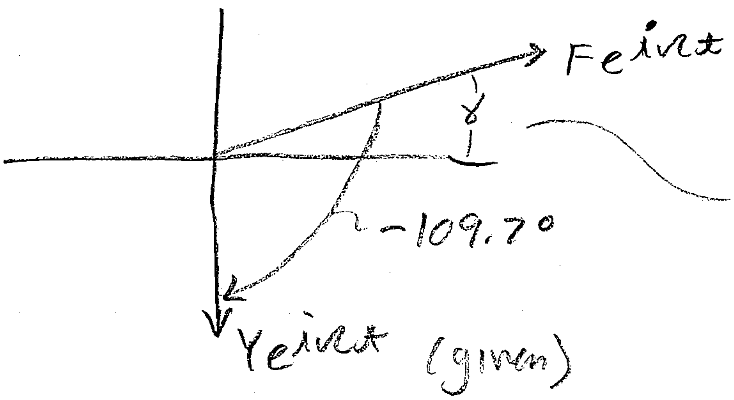
$\Omega = 1000 \text{ rpm} = \frac{1000}{60} \cdot 2\pi \text{ rad/s}$

$$Y = \frac{2 m_2 \varepsilon \Omega^2}{m_{tot}} \left(\frac{1}{\omega_n^2 + i\Omega \cdot 2\zeta\omega_n - \Omega^2} \right)$$

matlab

$$\boxed{Y = -0.00300 - 0.00837 = 0.00889 \angle -109.7^\circ}$$

Lags force by 109.7°,



γ = angle of masses past vertical

$\gamma = 29.7^\circ$

$$\boxed{|Y| = 8.89 \text{ mm}, \quad \Omega t = 109.7^\circ \text{ or } \gamma = 29.7^\circ}$$