HW 5

EMA 545
Mechanical Vibrations

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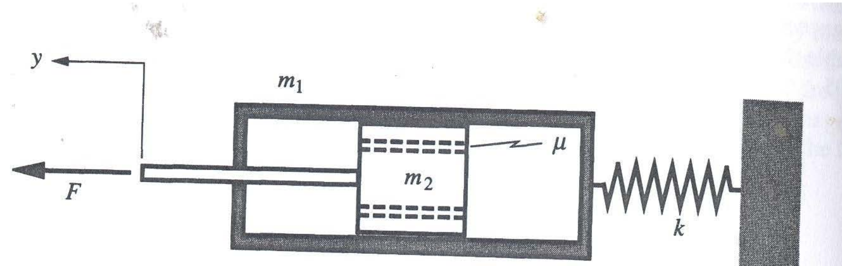
Contents

1 problem 1



EXERCISE 3.9

3.9 The shock absorber consists of an outer tube of mass m_1 that is restrained by spring k and an inner piston of mass m_2. Orifices in the piston permit passage of a viscous fluid that fills the tube; the coefficient of damping between the piston and the tube is μ. An unknown force $F(t)$ is applied to the piston, with the result that the absolute displacement of the piston is $y = A\sin(\omega t)$. The parameters of the system are: $m_1 = 0.5$ kg, $m_2 = 1.0$ kg, $k = 3.2$ kN/m, $\mu = 40$ N-s/m, and $A = 20$ mm. Determine the amplitude and phase of the force $F(t)$ relative to the displacement $y(t)$ when $\omega = 75$ rad/s and $\omega = 85$ rad/s.

Assuming the 2 masses move together (else we will have 2 systems and 2 equations of motions. Hence I assumed that they move together as one body).

$$(m_1 + m_2)y'' + y'\mu + ky = f(t)$$

Since $y(t) = A\sin(\omega t)$ hence

$$y(t) = \operatorname{Re}\left(\frac{A}{i}e^{i\omega t}\right)$$

Let

$$f(t) = \operatorname{Re}\left(\frac{\hat{F}}{i}e^{i(\omega t)}\right)$$

Where $\hat{F}$ is the complex amplitude of the force. Now we substitute all these in the differential equation above.

$$y' = \operatorname{Re}\left(\omega A e^{i\omega t}\right)$$
$$y'' = \operatorname{Re}\left(i\omega^2 A e^{i\omega t}\right)$$

$$(m_1 + m_2)y'' + y'\mu + ky = \operatorname{Re}\left(\frac{\hat{F}}{i}e^{i(\omega t)}\right)$$
$$\operatorname{Re}\left(i\omega^2 A e^{i\omega t}\right)(m_1 + m_2) + \operatorname{Re}\left(\omega A e^{i\omega t}\right)\mu + k\operatorname{Re}\left(\frac{A}{i}e^{i\omega t}\right) = \operatorname{Re}\left(\frac{\hat{F}}{i}e^{i(\omega t)}\right)$$
$$\operatorname{Re}\left[\left(i\omega^2(m_1 + m_2) + \omega\mu + \frac{1}{i}k\right)Ae^{i\omega t}\right] = \operatorname{Re}\left(\frac{\hat{F}}{i}e^{i(\omega t)}\right)$$
$$\left(i\omega^2(m_1 + m_2) + \omega\mu + \frac{1}{i}k\right)A = \frac{\hat{F}}{i}$$

Hence

$$\hat{F} = \left(-\omega^2(m_1 + m_2) + i\omega\mu + k\right)A$$

$k = 3.2 \times 10^3$ Nm, $\mu = 40$ Ns/m,$A = 0.02$ meter. When $\omega = 75$rad/sec the above becomes

$$\begin{aligned}\hat{F} &= \left(-75^2(1.5) + i75 \times 40 + 3.2 \times 10^3\right)0.02 \\ &= -104.75 + 60.0i\end{aligned}$$

Hence $\operatorname{Re}\left(\hat{F}\right) = -104.75$N and the phase is $\tan^{-1}\left(-\frac{60.0}{104.75}\right) = 2.62$ rad/sec.

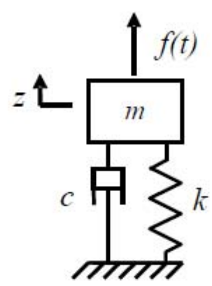
When $\omega = 85$

$$\begin{aligned}\hat{F} &= 1.5\left(-85^2 + i85\frac{40}{1.5} + 2133.3\right)0.02 \\ &= -152.75 + 68.0i\end{aligned}$$

$\operatorname{Re}\left(\hat{F}\right) = -152.75$ N and the phase is $\tan^{-1}\left(-\frac{68}{152.75}\right) = 2.722$ rad/sec.

2 problem 2

Problem 2: (3e1) (**20 points**)
A 450 kg generator, modeled as a rigid mass, must be installed on the same floor as some sensitive laboratory equipment. The operation of the generator results in a vertical force, *f*(*t*), being applied to the generator (rigid mass) whose amplitude is 20kN and whose frequency is 1800 rpm. Use a damping ratio of $\zeta = 0.03$ for both (a) and (b) below.



a.) Find the stiffness of the support, k, such that the force transmitted to the ground is no more than 2kN.
b.) Take your result from part (a) and compute the amplitude of the generator as the machine starts up. (As the machine starts up, assume that the force amplitude is constant at 20kN, but the frequency increases very slowly from zero to 1800 rpm. Do a worst-case analysis – just assure that the steady-state amplitude of the machine is less than 10mm for any forcing frequency between 0 and 1800 rpm.)
c.) Using your results from (a) and (b), suppose that the startup amplitude must not exceed 10mm. The startup amplitude can be decreased by adding mass to the generator while also increasing the stiffness of the support to keep the natural frequency of the system constant. How much mass must be added to keep the amplitude below 10mm?

2.1 Part(a)

Force transmitted to floor is given by

$$F_{tr} = cz' + kz$$

Let $f(t) = F\cos(\omega t) = \operatorname{Re}\left(Fe^{i\omega t}\right) = \operatorname{Re}\left(Fe^{i\omega t}\right)$ where we are given that $F = 20 \times 10^3$ N. $\omega = 2\pi\left(\frac{1800}{60}\right) = 60\pi = 188.50$ rad/sec or 30 Hz.

Let $z_{ss} = \operatorname{Re}\left(\frac{F}{k}|D|e^{i(\omega t-\phi)}\right)$ where $\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$ and $|D| = \frac{1}{\sqrt{\left(1-r^2\right)^2+(2\zeta r)^2}}$. and $r = \frac{\omega}{\omega_n}$

Hence $z' = \operatorname{Re}\left(i\omega\frac{F}{k}|D|e^{i(\omega t-\phi)}\right) = \operatorname{Re}\left(\omega\frac{F}{k}|D|e^{i\left(\omega t-\phi+\frac{\pi}{2}\right)}\right)$. Therefore

$$F_{tr} = c\operatorname{Re}\left(\omega\frac{F}{k}|D|e^{i\left(\omega t-\phi+\frac{\pi}{2}\right)}\right) + k\operatorname{Re}\left(\frac{F}{k}|D|e^{i(\omega t-\phi)}\right)$$

Where $c = 2\zeta\omega_n m$ and When $F_{tr} = 2 \times 10^3$N . We now solve for k from

$$2 \times 10^3 \geq 2\zeta\omega_n m\operatorname{Re}\left(\omega\frac{F}{k}|D|e^{i\left(\omega t-\phi+\frac{\pi}{2}\right)}\right) + k\operatorname{Re}\left(\frac{F}{k}|D|e^{i(\omega t-\phi)}\right)$$

Taking the maximum case for RHS where exponential are unity magnitude, hence

$$\begin{aligned} 2\times 10^3 &= 2\zeta\omega_n m\omega \frac{F}{k}|D| + F|D| \\ &= \left(2\zeta\omega_n m\omega\left(\frac{F}{k}\right) + F\right)|D| \\ &= \frac{F\left(1+2\zeta\omega_n \frac{m}{k}\omega\right)}{\sqrt{\left(1-r^2\right)^2 + (2\zeta r)^2}} \end{aligned}$$

Where $r = \frac{\omega}{\omega_n} = \frac{\omega}{\sqrt{\frac{k}{m}}}$. Hence the above becomes

$$2\times 10^3 = \frac{F\left(1+2\zeta\omega_n \frac{m}{k}\omega\right)}{\sqrt{\left(1-\frac{\omega^2}{\frac{k}{m}}\right)^2 + \left(2\zeta\frac{\omega}{\sqrt{\frac{k}{m}}}\right)^2}}$$

In the above everything is known except for k which we solve for. Plugging the numerical values given. $\omega = 2\pi\left(\frac{1800}{60}\right)$, $m = 450, F = 20\times 10^3, \zeta = 0.03$ hence

$$2\times 10^3 = \frac{20\times 10^3\left(1+2(0.03)\sqrt{\frac{k}{450}}\frac{450}{k}(60\pi)\right)}{\sqrt{\left(1-\frac{450(60\pi)^2}{k}\right)^2 + \left(2(0.03)\frac{60\pi}{\sqrt{\frac{k}{450}}}\right)^2}}$$

Hence $k = 1.2135\times 10^6$ N/m. Hence $\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1.2135\times 10^6}{450}} = 51.929$ rad/sec or 8.265 Hz.

2.2 part(b)

The total displacement is given by

$$\begin{aligned} z(t) &= z_{transient}(t) + z_{ss}(t) \\ &= e^{-\zeta\omega_n t}(A\cos\omega_d t + B\sin\omega_d t) + \operatorname{Re}\left(\frac{F}{k}|D|e^{i(\omega t-\phi)}\right) \end{aligned}$$

Where

$$z_{transient}(t) = e^{-\zeta\omega_n t}(A\cos\omega_d t + B\sin\omega_d t)$$

Assuming at $t = 0$ the system is relaxed hence $z(0) = 0$ and $z'(0) = 0$ we can determine A, B from Eq **??**.

At $t = 0$,

$$\begin{aligned} z(0) &= 0 \\ &= A + \operatorname{Re}\left(\frac{F}{k}|D|e^{-i\phi}\right) \end{aligned}$$

Hence

$$A = -\operatorname{Re}\left(\frac{F}{k}|D|e^{-i\phi}\right)$$

and

$$z'(t) = -\zeta\omega_n e^{-\zeta\omega_n t}(A\cos\omega_d t + B\sin\omega_d t) + e^{-\zeta\omega_n t}(-\omega_d A\sin\omega_d t + \omega_d B\cos\omega_d t) + \operatorname{Re}\left(\omega\frac{F}{k}|D|e^{i\left(-\phi+\frac{\pi}{2}\right)}\right)$$

Hence at $t = 0$

$$\begin{aligned} z'(0) &= 0 \\ &= -\zeta\omega_n A + \omega_d B + \operatorname{Re}\left(\omega\frac{F}{k}|D|e^{i\left(-\phi+\frac{\pi}{2}\right)}\right) \end{aligned}$$

Hence

$$\begin{aligned} B &= \frac{\zeta\omega_n}{\omega_d}A - \frac{1}{\omega_d}\operatorname{Re}\left(\omega\frac{F}{k}|D|e^{i\left(-\phi+\frac{\pi}{2}\right)}\right) \\ &= -\frac{\zeta\omega_n}{\omega_d}\operatorname{Re}\left(\frac{F}{k}|D|e^{-i\phi}\right) - \frac{1}{\omega_d}\operatorname{Re}\left(\omega\frac{F}{k}|D|e^{i\left(-\phi+\frac{\pi}{2}\right)}\right) \end{aligned}$$

Therefore the displacement is

$$\begin{aligned} z(t) = e^{-\zeta\omega_n t}&\left[-\operatorname{Re}\left(\frac{F}{k}|D|e^{-i\phi}\right)\cos\omega_d t + \left\{-\frac{\zeta\omega_n}{\omega_d}\operatorname{Re}\left(\frac{F}{k}|D|e^{-i\phi}\right) - \frac{1}{\omega_d}\operatorname{Re}\left(\omega\frac{F}{k}|D|e^{i\left(-\phi+\frac{\pi}{2}\right)}\right)\right\}\sin\omega_d t\right] \\ &+ \operatorname{Re}\left(\frac{F}{k}|D|e^{i\left(\omega t-\frac{\pi}{2}-\phi\right)}\right) \end{aligned}$$

Hence expressed in sin and cos

$$\begin{aligned} z(t) &= -\left(\frac{F}{k}|D|\cos\phi\right)e^{-\zeta\omega_n t}\cos\omega_d t + e^{-\zeta\omega_n t}\left\{-\frac{\zeta\omega_n}{\omega_d}\left(\frac{F}{k}|D|\cos\phi\right) - \frac{1}{\omega_d}\left(\omega\frac{F}{k}|D|\sin\phi\right)\right\}\sin\omega_d t \\ &+ \frac{F}{k}|D|\cos\left(\omega t - \phi\right) \\ &= -\left(\frac{F}{k}|D|\cos\phi\right)e^{-\zeta\omega_n t}\cos\omega_d t + e^{-\zeta\omega_n t}\frac{F|D|}{\omega_d k}\left(-\zeta\omega_n\cos\phi - \omega\sin\phi\right)\sin\omega_d t + \frac{F}{k}|D|\cos\left(\omega t-\phi\right) \\ &= \frac{F}{k}|D|e^{-\zeta\omega_n t}\left[-\cos\phi\cos\omega_d t + \frac{1}{\omega_d}\left(-\zeta\omega_n\cos\phi - \omega\sin\phi\right)\sin\omega_d t\right] + \frac{F}{k}|D|\cos\left(\omega t-\phi\right) \end{aligned}$$

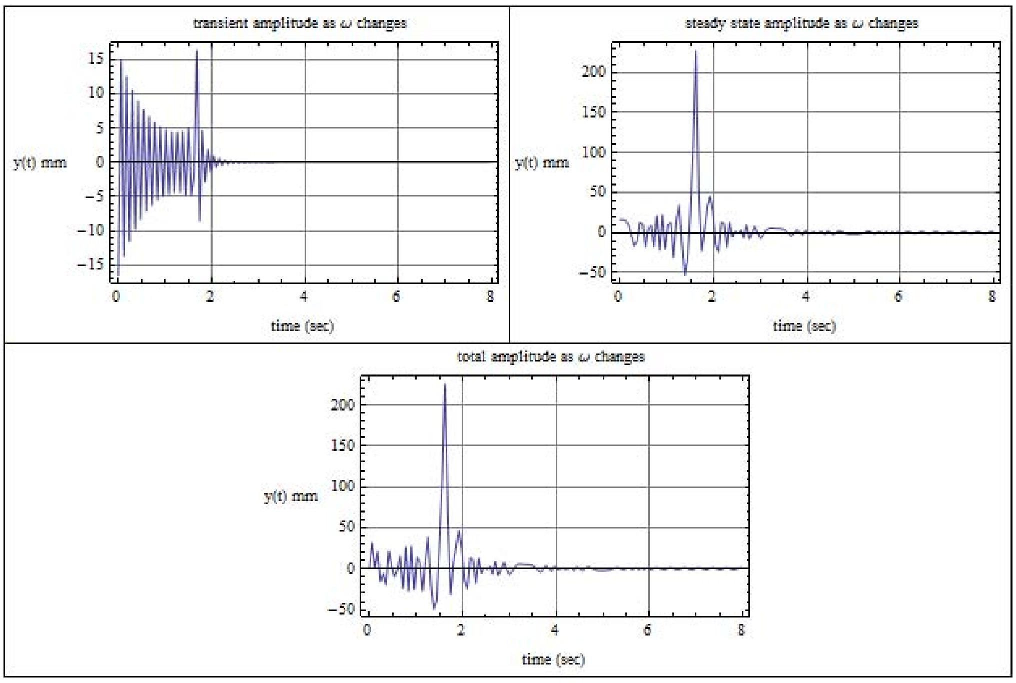
Since $\zeta = 0.03$ then $\omega_d = \omega_n\sqrt{1-\zeta^2} = \omega_n\sqrt{1-0.03^2} = 0.99955(\omega_n)$. Therefore in the above we can just replace ω_d by ω_n with very good approximation, hence we now obtain

$$\begin{aligned} z(t) &= \frac{F}{k}|D|e^{-\zeta\omega_n t}\left[-\cos\phi\cos\omega_n t + \frac{1}{\omega_n}\left(-\zeta\omega_n\cos\phi - \omega\sin\phi\right)\sin\omega_n t\right] + \frac{F}{k}|D|\cos\left(\omega t-\phi\right) \\ &= \frac{F}{k}|D|e^{-\zeta\omega_n t}\left[-\cos\phi\cos\omega_n t - \left(\zeta\cos\phi + \frac{\omega}{\omega_n}\sin\phi\right)\sin\omega_n t\right] + \frac{F}{k}|D|\cos\left(\omega t-\phi\right) \end{aligned}$$

This is the amplitude. In the above $|D| = \frac{1}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\left(\frac{\omega}{\omega_n}\right)\right)^2}}$, and $\phi = \tan^{-1}\frac{2\zeta\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}$. The transient solution usually goes away after 5 or 6 cycles. Hence let us assume that the start up time takes $6\times\frac{2\pi}{\omega_n} = 6\times\frac{2\pi}{\sqrt{\frac{k}{m}}} = 6\times\frac{2\pi}{\sqrt{\frac{1.2135\times10^6}{450}}} = 0.72597$ seconds. Or 1 second at worst.

Therefore we can now plot the amplitude for $t = 0$ to $t = 1$ second in increments of 0.1 second, and each time advance, we can increment ω from 0 to 60π in linear fashion, hence each 0.1 second we update ω by an amount 6π. After 1 second has passed, the system is assumed to be in steady state, and then we keep ω fixed at 60π rad/sec. This is a plot showing $z(t)$ for $t = 0$ to 2 seconds given the above method of changing ω

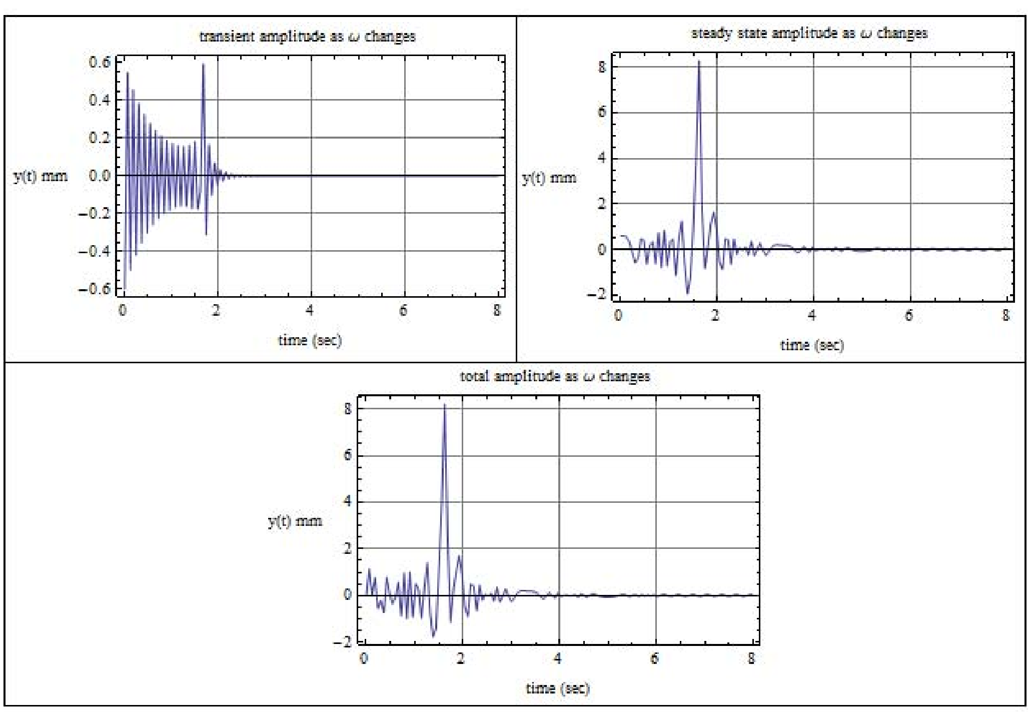
To avoid going over $10mm$, this means we have to avoid the case of $r = 1$ or $\omega = \omega_n$. When I first just incremented ω_n such that $r = 1$ was not avoided, resonance caused the amplitude to go over $10mm$ as given in this plot. The transient solution itself stayed just below 10mm but the steady state solution went over 10mm due to resonance



2.3 Part(c)

To insure that the amplitude does not go over 10mm, we need to add mass to the generator. Maximum amplitude is given by $\frac{F}{k}\frac{1}{2\zeta} = \frac{20\times10^3}{1.2135\times10^6}\frac{1}{2(0.03)} = 0.27469$ meter or 274mm

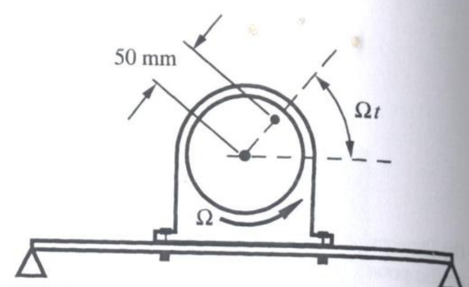
So to insure maximum does not exceed 10mm , solve for new k from $0.01 = \frac{20\times10^3}{k_n}\frac{1}{2(0.03)}$, hence $k_n = 3.3333\times10^7$. Since $\omega_n = 51.929 = \sqrt{\frac{k_n}{m_n}}$ then new mass is $m_n = \frac{3.3333\times10^7}{51.929^2} = \boxed{12361 \text{ kg}}$ using these values, the above plot now are redone. This is the result



We see that now the maximum displacement remained below 10 mm.

3 Problem 3

3.19 The total mass of a motor is 80 kg. It is observed that gently placing the motor on a beam produces a static downward displacement of 40 mm. When the motor rotates at an angular speed of 145 rev/min, the steady-state amplitude of the beam is 10 mm. It also is observed that at a rotation rate of 145 rev/min, the radial line to the center of mass of each rotor is 75° above horizontal when the beam is at its static reference position (where $q = 0$).



EXERCISE 3.19

(a) Determine the damping ratio ζ for the system.
(b) Determine the imbalance εm.
(c) Determine the smallest possible amplitude of vibration of the beam if the motor turns at a rate that is much larger than the natural frequency of the system.

Let $\varepsilon = 50mm = 0.05m$ be the distance of the unbalance mass m. Let $M = 80kg$ be the mass of the motor. The equation of motion is given by

$$(M+m)y'' + cy' + ky = m\varepsilon\Omega^2 \sin(\Omega t)$$
$$y'' + 2\zeta\omega_n y' + \omega_n^2 y = \frac{m}{m+M}\varepsilon\Omega^2 \operatorname{Re}\left(\frac{1}{i}e^{i\Omega t}\right)$$

Where $\omega_n = \sqrt{\frac{k}{m+M}}$ and $\zeta = \frac{c}{2(M+m)\omega_n}$. Let $y = \operatorname{Re}\left(\frac{Y}{i}e^{i\Omega t}\right)$. This leads to

$$Y = \frac{m}{m+M}\frac{\varepsilon\Omega^2}{\omega_n^2 - \Omega^2 + 2i\zeta\omega_n\Omega}$$

Since static deflection is $40mm$, then

$$\frac{(M+m)g}{k} = 0.04$$
$$k = \frac{(M+m)g}{0.04}$$

But $\omega_n^2 = \frac{k}{m+M} = \frac{(M+m)g}{0.04(M+m)} = \frac{g}{0.04}$, hence $\omega_n = \sqrt{\frac{9.81}{0.04}} = 15.66$ rad/sec or 2.492 Hz.

3.1 part(a)

Since at steady state the displacement is 10 mm, then $\Omega = 2\pi\frac{145}{60} = 15.184$ or $\boxed{2.4167 \text{ Hz}}$ hence

$$y = \operatorname{Re}\left(\frac{Y}{i}e^{i\Omega t}\right) = \operatorname{Re}\left(\frac{\varepsilon m}{m+M}\frac{r^2}{\left(1-r^2+2i\zeta r\right)}e^{i\left(\Omega t - \frac{\pi}{2}\right)}\right)$$
$$= \frac{\varepsilon m r^2}{m+M}\frac{1}{\sqrt{\left(\left(1-r^2\right)^2 + (2\zeta r)^2\right)}}\operatorname{Re}\left(e^{-i\phi}e^{i\left(\Omega t - \frac{\pi}{2}\right)}\right)$$

Where $\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$. $r = \frac{\Omega}{\omega_n} = \frac{15.184}{15.66} = \boxed{0.9696}$ hence the above becomes, at steady state

$$0.01 = \frac{(0.05)m}{m+80}\frac{0.9696^2}{\sqrt{\left(1-0.9696^2\right)^2+(2\zeta 0.9696)^2}}\operatorname{Re}\left(e^{i\left(15.184\,t-\frac{\pi}{2}-\phi\right)}\right)$$
$$= \frac{(0.05)m}{m+80}\frac{0.9696^2}{\sqrt{\left(1-0.9696^2\right)^2+(2\zeta 0.9696)^2}}\sin\left(15.184\,t-\phi\right) \quad (1)$$

We are now told that at $\Omega = 15.184$ and when $\Omega t = 75^0$ then the displacement is zero, hence

$$0 = \frac{(0.05)m}{m+80}\frac{0.9696^2}{\sqrt{\left(1-0.9696^2\right)^2+(2\zeta 0.9696)^2}}\sin\left(75^0-\phi\right)$$

or

$$\sin\left(75^0-\phi\right) = 0$$
$$75^0-\phi = 0$$
$$\phi = 75^0$$

Since $\phi = \tan^{-1}\left(\frac{2\zeta r}{1-r^2}\right)$ then

$$75\left(\frac{\pi}{180}\right) = \tan^{-1}\left(\frac{2\zeta 0.9696}{1-0.9696^2}\right)$$

Hence

$$\tan^{-1}\left(\frac{2\zeta 0.9696}{1-0.9696^2}\right) = 1.3090$$
$$\frac{2\zeta 0.9696}{1-0.9696^2} = \tan(1.3090)$$
$$\frac{2\zeta 0.9696}{1-0.9696^2} = 3.7321$$

Hence $\boxed{\zeta = 0.11523}$

3.2 Part(b)

From Eq 1

$$0.01 = \frac{(0.05)m}{m+80}\frac{0.9696^2}{\sqrt{\left(1-0.9696^2\right)^2+(2\zeta 0.9696)^2}}\sin\left(15.184\,t-\phi\right)$$

The maximum amplitude is when

$$0.01 = \frac{(0.05)m}{m+80}\frac{0.9696^2}{\sqrt{\left(1-0.9696^2\right)^2+(2\zeta 0.9696)^2}}$$

But $\zeta = 0.11523$, hence we now solve for m

$$0.01 = \frac{(0.05)m}{m+80}\frac{0.9696^2}{\sqrt{\left(1-0.9696^2\right)^2+(2(0.11523)0.9696)^2}}$$

Hence

$$\boxed{m = 4.1\ kg}$$

Hence $\varepsilon m = (0.05)(4.1) = \boxed{0.20 \text{ kg meter}}$

3.3 Part(c)

since

$$y=\frac{\varepsilon m r^2}{m+M}\frac{1}{\sqrt{\left(\left(1-r^2\right)^2+(2\zeta r)^2\right)}}\operatorname{Re}\left(e^{i\left(\Omega t-\frac{\pi}{2}-\phi\right)}\right)$$

As Ω becomes much larger than ω_n then $\left(1-r^2\right)^2\rightarrow r^4$. Now dividing numerator and denominator by r^2 gives

$$\begin{aligned}y&=\frac{\varepsilon m}{m+M}\frac{1}{\sqrt{\frac{\left(r^4+(2\zeta r)^2\right)}{r^4}}}\sin\left(\Omega t-\phi\right)\\&=\frac{\varepsilon m}{m+M}\frac{1}{\sqrt{\left(1+\frac{4\zeta^2}{r^2}\right)}}\sin\left(\Omega t-\phi\right)\end{aligned}$$

as r becomes large then $\frac{4\zeta^2}{r^2}\rightarrow 0$ hence

$$y\simeq\frac{\varepsilon m}{m+M}\sin\left(\Omega t-\phi\right)$$

The smallest possible amplitude is

$$\begin{aligned}|y|&=\frac{0.20}{4.1+80}\\&=2.3781\times10^{-3}\text{ meter}\end{aligned}$$

or

$$|y|=2.38\text{ mm}$$

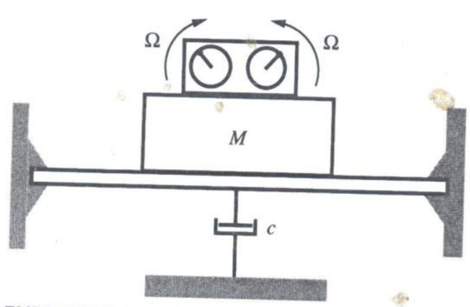
4 problem 4

3.23 A counter-rotating eccentric mass exciter is attached to a block, which is supported by a lightweight beam. Stroboscopic measurement at an angular speed $\omega=900$ rev/min indicates that the block passes its static equilibrium position with an upward velocity at the instant when the eccentric masses are at their highest position. The amplitude of vertical displacement at this speed is 8.5 mm. The total mass of the system is $m=200$ kg and the rotating imbalance of each rotor is 0.5 kg-m. Determine

(a) The natural frequency of the system.

(b) The damping constant c.

(c) For the case where $\Omega=1000$ rev/min, the amplitude of the vertical displacement of the mass and the angular position of the eccentric masses when the block passes its equilibrium position.



EXERCISES 3.23, 3.24

4.1 Part(a)

(note: total mass of system includes the small unbalanced masses) Since static deflection is $8.5mm$, then

$$\begin{aligned}\frac{Mg}{k}&=0.0085\\k&=\frac{Mg}{0.0085}\end{aligned}$$

But $\omega_n^2=\frac{k}{M}=\frac{Mg}{0.0085M}=\frac{g}{0.0085}$, hence $\omega_n=\sqrt{\frac{9.81}{0.0085}}=33.972$ rad/sec or 5.4068 Hz

4.2 Part(b)

The equation of motion is (angle Ω is now measured from horizontal, anti-clock wise positive)

$$My'' + cy' = 2m\varepsilon\Omega^2 \sin(\Omega t) = \operatorname{Re}\left(\frac{1}{i}2m\varepsilon\Omega^2 e^{i(\Omega t)}\right)$$

Let $y(t) = \operatorname{Re}\left(\frac{1}{i}Ye^{i\Omega t}\right)$ hence $y'(t) = \operatorname{Re}\left(Y\Omega e^{i\Omega t}\right)$,$y''(t) = \operatorname{Re}\left(iY\Omega^2 e^{i\Omega t}\right)$, hence the above becomes

$$\begin{aligned}
\operatorname{Re}\left(iY\Omega^2 e^{i\Omega t}\right) + \frac{c}{M}\operatorname{Re}\left(Y\Omega e^{i\Omega t}\right) &= \operatorname{Re}\left(\frac{1}{i}\frac{2m\varepsilon\Omega^2}{M}e^{i\Omega t}\right)\\
\operatorname{Re}\left(\left(i\Omega^2 + \frac{c\Omega}{M}\right)Ye^{i\Omega t}\right) &= \operatorname{Re}\left(\frac{1}{i}\frac{2m\varepsilon\Omega^2}{M}e^{i\Omega t}\right)\\
\left[i\Omega^2 + \frac{c\Omega}{M}\right]Y &= \frac{1}{i}\frac{2m\varepsilon\Omega^2}{M}\\
Y &= \frac{1}{i}\frac{\frac{2m\varepsilon\Omega^2}{M}}{i\Omega^2 + \frac{c\Omega}{M}}\\
&= \frac{2m\varepsilon\Omega^2}{ic\Omega - M\Omega^2}
\end{aligned}$$

Hence

$$\begin{aligned}
y_{ss}(t) &= \operatorname{Re}\left(\frac{1}{i}Ye^{i(\Omega t)}\right)\\
&= \operatorname{Re}\left(\frac{1}{i}\frac{2m\varepsilon\Omega^2}{ic\Omega - M\Omega^2}e^{i(\Omega t)}\right)
\end{aligned}$$

Now we are told when $\Omega t = \frac{\pi}{2}$ (upright position) then $y = 0$(since it passes static equilibrium). At this moment $\Omega = 2\pi\frac{900}{60} = \boxed{94.248 \text{ rad/sec}}$, At this moment the centripetal forces equal the damping force downwards (since the mass was moving upwards). Hence

$$\boxed{m\varepsilon\Omega^2 = cy'(t)}$$

But from above we found that

$$\begin{aligned}
y'(t) &= \operatorname{Re}\left(\frac{2m\varepsilon\Omega^2}{ic\Omega - M\Omega^2}\Omega e^{i\Omega t}\right)\\
&= \operatorname{Re}\left(\frac{(0.5)94.248^2}{ic(94.248) - 200(94.248)^2}94.248e^{i\frac{\pi}{2}}\right)\\
&= \operatorname{Re}\left(\frac{8.3718\times10^5}{94.248ic - 1.7765\times10^6}e^{i\frac{\pi}{2}}\right)
\end{aligned}$$

Hence

$$\begin{aligned}
m\varepsilon\Omega^2 &= c\left|y'(t)\right|\\
(0.5)94.248^2 &= c\frac{8.3718\times10^5}{\sqrt{(94.248c)^2 + \left(1.7765\times10^6\right)^2}}\\
4441.3 &= 8.3718\times10^5\frac{c}{\sqrt{8882.7c^2 + 3.1560\times10^{12}}}
\end{aligned}$$

Solving numerically for c gives

$$c = 1.0882\times10^4 \text{ N second per meter}$$

4.3 Part(c)

When $\Omega = \left(2\pi \frac{1000}{60}\right) = 104.72$ rad/sec or 16.667 Hz. From

$$\begin{aligned} y &= \text{Re}\left(\frac{1}{i} \frac{2m\varepsilon\Omega^2}{ic\Omega - M\Omega^2} e^{i(\Omega t)}\right) \\ &= \text{Re}\left(\frac{2(0.5)(104.72)^2}{i\left(1.0882 \times 10^4\right)104.72 - 200(104.72)^2} e^{i\left(104.72t - \frac{\pi}{2}\right)}\right) \\ &= \text{Re}\left(\frac{10966.}{i1.1396 \times 10^6 - 2.1933 \times 10^6} e^{i\left(104.72t - \frac{\pi}{2}\right)}\right) \\ |y| &= 4.4 \text{ mm} \end{aligned}$$