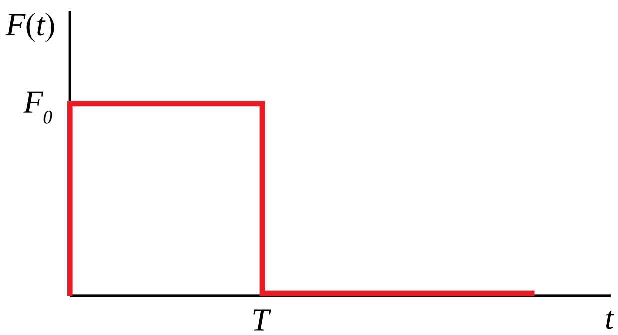
Homework #4
EMA 545, Spring 2013

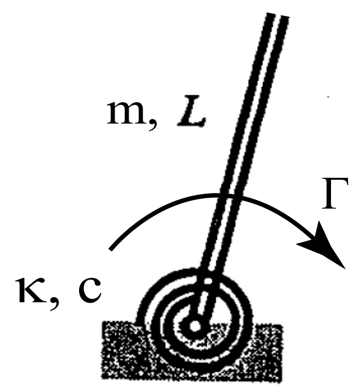
Problem 1: Exercise 2.54 from Ginsberg.

Problem 2: (**30 pts**, each part below is worth 10 pts).
The read head on a Hard Disk Drive (HDD) can be modeled as a pinned bar with a torsional spring at its base as shown below with L = 2 cm, m = 3 grams and κ = 20 N/rad. The damping ratio for the system is $\zeta = 0.02$. The equation of motion for this system is: (later we will discuss how to find the EOM for a system like this)

$$\frac{1}{3}mL^2\ddot{\theta} + c\dot{\theta} + \kappa\theta = \Gamma(t)$$

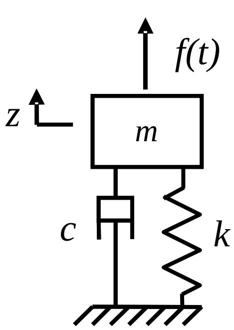
A certain read operation involves applying a step torque Γ(t) = F(t) with amplitude F_0 and duration T as shown below, where F_0 is the static torque required to displace the bar 30 degrees.





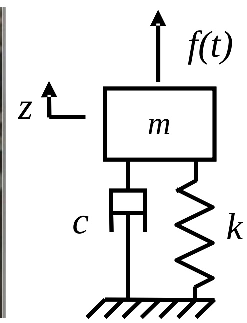
(a) Find the response of the system numerically over the time interval 0 < 1 < 10*T, with T = 2.5*T_d, where T_d is the damped period of the system. Use a numerical procedure, preferably Matlab's "ode45" function together with a suitably modified version of eom_2_12.m, which is available on the class website.
(b) Assuming an underdamped response, write down a closed-form solution for the response in terms of Heaviside-step functions, and unit step responses, $q_s(t)$. Compare this with the response that you found numerically.
(c) Plot the displacement as a function of time for the case where T = 3*T_d and T = 2.5*T_d. What do you observe? Why is the residual vibration larger in the latter case? (Hint –an undamped version of your analysis in (b) may make this easier to see.)

Problem 3: A SDOF system modeling a car bouncing on its suspension has *m*=1000kg, *k*=11 kN/m and *c*=660 N-s/m. The car is released from rest at t=0 with z(0)= –0.10m. It is possible to bring the car exactly to rest by exerting an impulsive force $f(t)=F_0\delta(t-T)$ at some instant $t=T$. (e.g. hitting it with a very large hammer at just the right instant). Find the magnitude of the impulse and the instant, *T*, at which it should be applied such the bouncing of the car stops completely after at least 2.0 seconds have elapsed but before 5.0 seconds have elapsed.



Problem 4: Suppose that the bridge over University Avenue (pictured below) can be modeled as a simply-supported beam with length L=50 m. To simplify the analysis, let's assume that the beam has rectangular cross section with height 18 inches, width 4 feet and that it is constructed from steel with ρ=7800 kg/m^3 and E=210 GPa. (Note that the stiffness for various beam configurations is given in Figure 1.1 in the text.) Model this bridge as a SDOF system with an effective mass that is one third of the total mass of the beam and a stiffness equal to the stiffness of the beam when a static force is applied at its center. The damping ratio of the system is observed to be ζ=0.01.

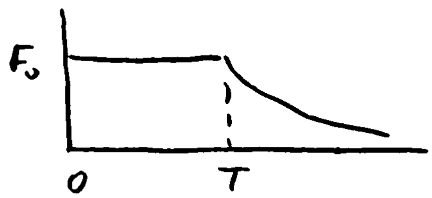
Suppose that a single student jumping up and down on the bridge can exert a force $f(t)$=(1000 N)cos(ωt) where ω can be between 0 and 8π rad/s depending on how quickly he jumps up and down. How many students must jump on the bridge to cause a displacement amplitude of 50 cm? What frequency should they jump at to minimize the number of students required? (Don't worry, the actual bridge is stiffer and lighter than that given in the problem statement. Extra Credit: What would be more reasonable values for its mass and natural frequency? How does this change the solution?)



Problem 5: 3.2 from Ginsberg. Note that you are approximating the radar display as a rigid mass ("mounted mass is 8 kg"), which is mounted on a spring and dashpot.

Problem 6: 3.5 from Ginsberg. Also, please sketch the force and the response of the system (by hand) over one or two cycles, taking care to properly represent the amplitude and phase difference. Do this for both cases, 0.95 kHz and 1.05 kHz.

Exercise 2.54



$$Q = F_0\left[h(t) - h(t-T)\right] + F_0 \exp\left[-\beta(t-T)\right] h(t-T)$$

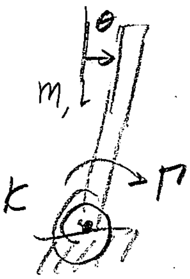
$m\ddot{q} + kq = Q$, Assume $q(0) = \dot{q}(0) = 0$

Let u be the unit step response, and x be the unit exponential response. Then

$$q = F_0\, u(t) - F_0\, u(t-T) + F_0\, x(t-T)$$

where $$u(t) = \frac{1}{M\omega_{nat}^2}\left[1 - \cos(\omega_{nat} t)\right] h(t)$$

$$x(t) = \frac{1}{M(\omega_{nat}^2 - 2\beta\omega_{nat} + \beta^2)}\left\{\exp(-\beta t) - \left[\cos(\omega_{nat} t) - \frac{\beta}{\omega_{nat}} \sin(\omega_{nat} t)\right]\right\} h(t)$$



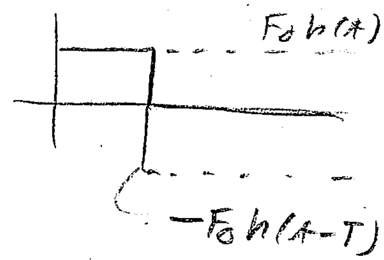
→ Eom $\frac{1}{3} m L^2 \ddot{\theta} + c\dot{\theta} + K\theta = \Gamma(t)$

$M = \frac{1}{3} m L^2$ ⟶

$\ddot{\theta} + 2\zeta \omega_n \dot{\theta} + \omega_n^2 \theta = \frac{\Gamma(t)}{M}$

$\zeta = 0.02, \quad \omega_n = \sqrt{\frac{K}{\frac{1}{3} m L^2}}$

$F(t) = F_0 h(t) - F_0 h(t-T)$



$g(t) = F_0 g_s(t) - F_0 g_s(t-T)$

$g_s(t)$ given in Appendix B
– has 1/M term included

→ Do the rest in matlab → a) b)

– time vector – sample 10 times per period T_d

c) for an undamped system,
$g_s(t) = \text{const.}\left(1 - \cos(\omega_n t)\right) h(t)$

The response is a sum of 2 step responses

$g(t) = c_1\left(1 - \cos(\omega_n t)\right) - c_1\left(1 - \cos(\omega_n(t-T))\right) h(t-T)$

$= -c_1 \cos(\omega_n t) + c_1 \cos(\omega_n t - \omega_n T)$

if $T = nT_d$, $\omega_n T = n \cdot 2\pi$

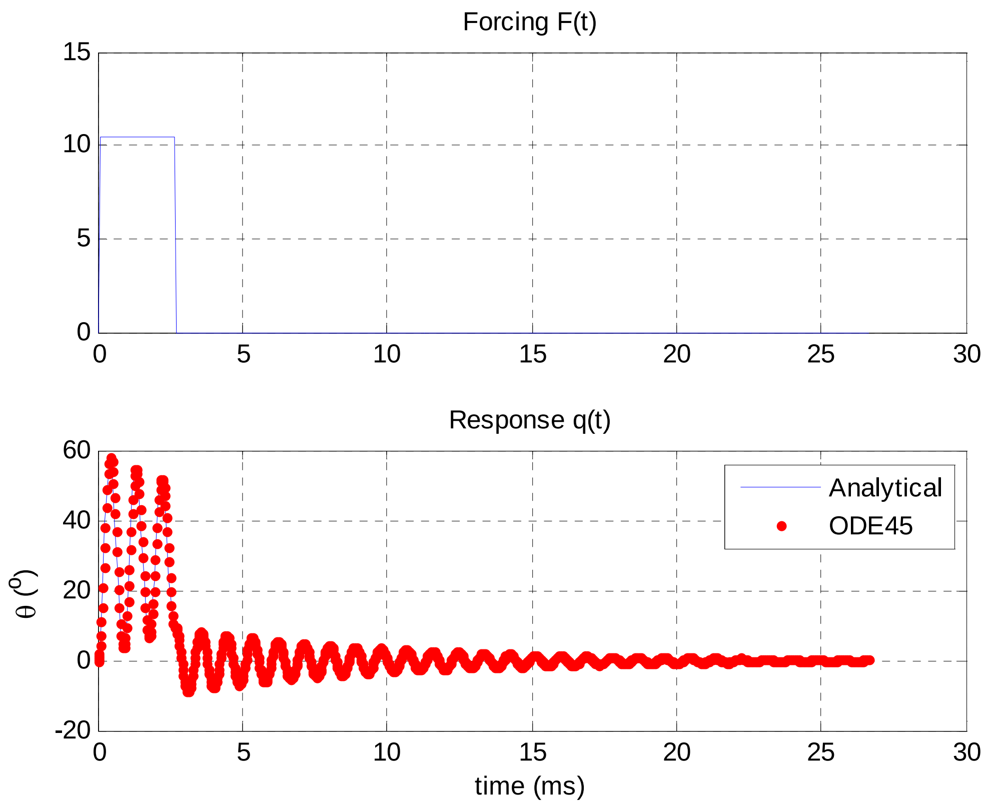
– and the two cosine terms are in phase and cancel completely

– if $T = (n + \frac{1}{2}) T_d$ the terms reinforce and the residual vibration is <u>largest</u>.

* In the damped case, the constants multiplying the two terms are slightly different, so there is some residual vibration, even when $T = nT_d$

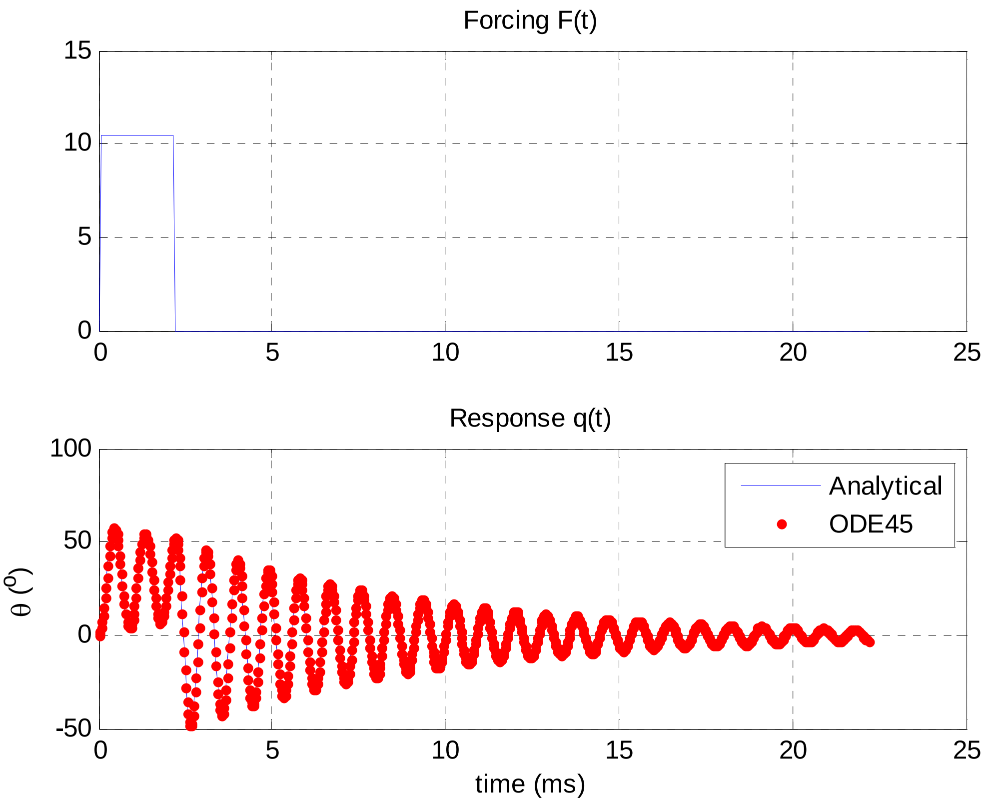
HW#6, Problem 2e-01, Hard Disk Drive
SOLUTION
MSA – Mar. 2009

Response with T = 3*Td



The transient response due to the step up is in phase with that of the transient response due to the step down, so the two almost cancel. They do not quite cancel because the response has damped somewhat, so the second step is larger than what would be needed to cancel the residual vibration from the first step.

Response with T = 2.5*Td



Matlab code:

```
% Solution to HW 2e-01 Hard Disk Drive Head

clear all; close all

% Parameter values
L = 0.02; % m
k = 20; % N/rad (torsional)
m = 0.003; %
M = (1/3)*m*L^2; K = k; % SDOF parameters
wn = sqrt(K/M)
zt = 0.02

F0 = 30*pi/180*k;
q_0 = 0; q_dot_0 = 0;
Td = 2*pi/wn;
Tc = 1/(zt*wn); % time constant of the system

T = 3*Td
% T = 2.5*Td;

global S
vns = whos; % put into a global variable
for k = 1:length(vns);
    eval(['S.',vns(k).name, ' = ',vns(k).name,';']);
end
```

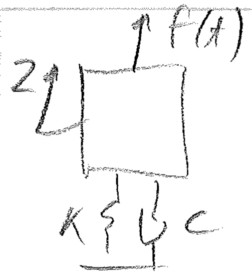
```
% Time Vector
ts = [0:Td/10:10*T]; % 4*Tctime vector, sample 10x per period and over
4 time constants.

% Forcing - sum of step and ramp
% note h(t) written as (t>0) in Matlab
F = F0*(ts>0) - F0.*(ts-T>0);

% Analytical Solution

% Unit step and ramp responses from Ginsberg - includes particular and
% complimentary solutions
qs = inline(['(1/(M*wn^2))*(1-exp(-zt*wn*t)*(cos(wn*sqrt(1-
zt^2)*t)+',...
    '(zt/sqrt(1-zt^2))*sin(wn*sqrt(1-
zt^2)*t)))*(t>0)'],'t','M','wn','zt');

% Response is a sum of step and ramp responses
q = zeros(size(ts));
for k = 1:length(ts)
    q(k) = F0*qs(ts(k),M,wn,zt) - F0*qs(ts(k)-T,M,wn,zt);
end

figure(1)
subplot(2,1,1)
plot(ts*1e3,F); grid on;
title('Forcing F(t)');
subplot(2,1,2);
plot(ts*1e3,q*180/pi); grid on;
title('Response q(t)');
xlabel('time (ms)'); ylabel('\theta (^o)');

% Solution using ODE45

% Define equations of motion in eom_2_12.m
% Note - ode45 requires only the time span, not the whole time vector
tic
[tout,yout] = ode45('eom_2e_1',[ts(1),ts(end)],[q_0; q_dot_0]);
t_ode = toc

q_ode = yout(:,1); % the first of the y variables is q(t), the second
is q_dot(t)

% Add red dots to plot above
hold on; plot(tout*1e3,q_ode*180/pi,'r.'); hold off;
legend('Analytical','ODE45');
```

%%% Equations of Motion:

```
function [xdot] = eom_2e_1(t,x)

global S % bring in parameters

% Forcing - sum of step and ramp
```

```
F = S.F0*(t>0) - S.F0.*(t-S.T>0);

% Equations of Motion
xdot(1,1) = x(2);
xdot(2,1) = -S.wn^2*x(1)-2*S.zt*S.wn*x(2) + F/S.M;
```

Solution Pe2-03 Car Vib.



m, K, c known $\longrightarrow$ $\omega_n = \sqrt{\frac{K}{m}}$ $\quad 2\zeta\omega_n = c/m$

$\zeta = \frac{c}{2m\omega_n}$ $\longrightarrow$ $\boxed{\omega_n = 3.32\ \text{rad/s}}$

$\boxed{\zeta = 0.099}$

$$m\ddot{z} + c\dot{z} + Kz = f(t)$$

$z(0) = -0.1\,m, \quad \dot{z}(0) = 0$

apply $f(t) = F_0\,\delta(t-T)$ to get to rest

Free response:

$z = Re(A e^{\lambda t})$

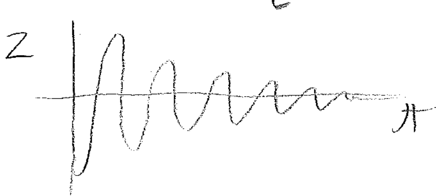
$z(0) = Re(A) = -0.01 \longrightarrow \boxed{A = -0.1 + ib}$

$\dot{z}(0) = Re(\lambda A) = 0$

$Re\left((-0.1 + ib)(-\zeta\omega_n + i\omega_d)\right) = (+0.1\zeta\omega_n - b\omega_d) = 0$

$\boxed{b = \frac{0.1\zeta\omega_n}{\omega_d}}$

$z(t) = Re\left[\left(-0.1 + i\,\frac{0.1\zeta\omega_n}{\omega_d}\right) e^{\lambda t}\right]$



(Note, could solve this with impulse - momentum from EMA 202.)

Here we'll use superposition

Notice that the impulse response from Appendix B

$z_F = \frac{1}{m\omega_d} e^{-\zeta\omega_n t} \sin(\omega_d t)\, h(t) \qquad \lambda = -\zeta\omega_n + i\omega_d$

has the exact same form! (Obvious in complex expon. form:)

$z_F = Re(B e^{\lambda t})\, h(t) \quad , \quad B = \frac{F_0}{m\omega_d}(-i)$

So it should be possible to cancel the response for $t > T$

$$z_{tot} = z + z_F(t-T) = Re\left(A e^{\lambda t} + B e^{\lambda(t-T)} h(t-T)\right)$$

for $t \geq T$, $\quad z_{tot} = Re\left[\left(A + B e^{-\lambda T}\right) e^{\lambda t}\right]$

Response zero for all $t > T$ iff

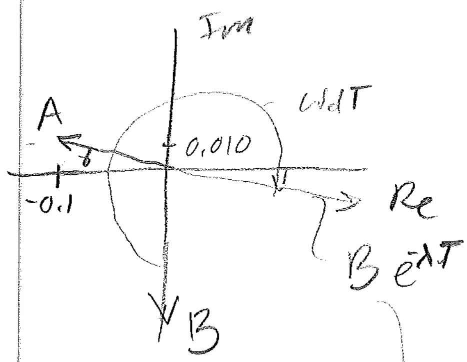
$$A + Be^{-\lambda T} = 0 \qquad \lambda = -\zeta\omega_n + i\omega_d$$

✓Known — Don't Know F_0 or T $\longrightarrow$ first find T

$$A = -0.1 + i\,0.010 \qquad \text{(Matlab)}$$

$$\gamma = \tan^{-1}\left(\frac{0.010}{0.1}\right) = \pi - \text{angle}(A)$$

$$\gamma = 0.0997 \text{rad}$$



$$B = \frac{-iF_0}{m\omega_d} e^{+\zeta\omega_n T} e^{-i\omega_d T}$$

Choose T so these cancel

$$\omega_d T = \frac{3\pi}{2} + \gamma + n(2\pi)$$

could have gone through any number of cycles

$$T = \frac{\frac{3\pi}{2} + \gamma + n(2\pi)}{\omega_d} = 1.46, 3.36, 5.3, \ldots$$

$$T = 3.36 \text{ sec}$$

Match magnitude:

$$|A| = |Be^{-\lambda T}| = \frac{F_0}{m\omega_d} e^{\zeta\omega_n T}$$

$$F_0 = m\omega_d |A| e^{-\zeta\omega_n T} = 109.4 \text{ N-s}$$

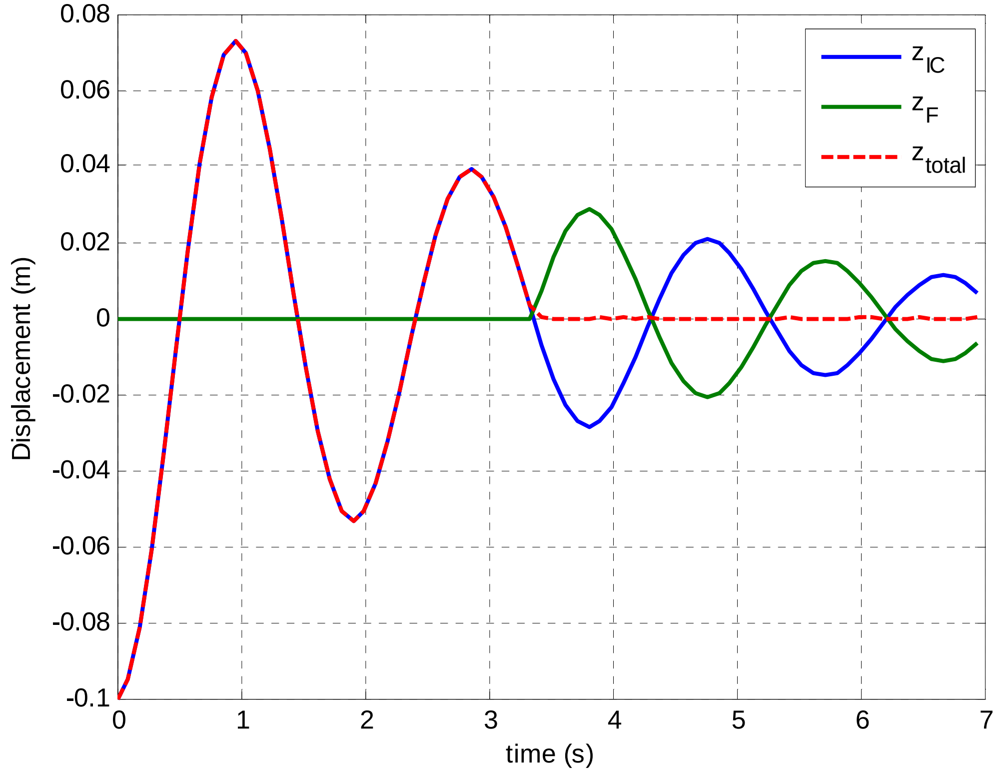
See Matlab for plot

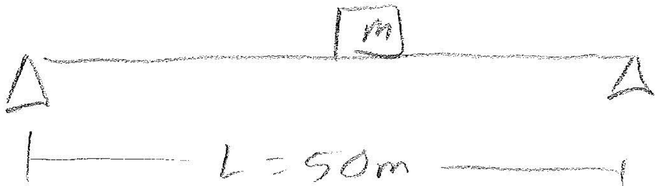
```
% Solution to HW e2-03, Car impulse

m=1000;
wn=sqrt(11000/1000)
zt=660/1000/2/wn
wd=wn*sqrt(1-zt^2);
lam=-zt*wn+1i*wd;

A=-0.1+1i*0.1*zt*wn/(wn*sqrt(1-zt^2))

gam=pi-angle(A)

T=(3*pi/2+gam+1*2*pi)/wd

F0=m*wd*abs(A)*exp(-zt*wn*T)
% Notice, the value of F0 seems rather small.  Remember that it is an
% impulse, so to get realistic units we need to integrate over the
impulse.
% For example, if the impulse were a constant force, Fc, that is
0.001sec long,
% then the integral of Fc*0.001 would equal F0, or in other words,

Fc=F0/0.001 % N
% Maybe that still seems a little small but it seems to be correct.

B=(-1i*F0/(m*wd))

% Check to see if this works:
dt=(2*pi/wd)/20; % 20 samples per period
ts=[0:dt:7];
z_IC=real(A*exp(lam*ts));
z_F=real(B*exp(lam*(ts-T))).*(ts>T);

figure(1);
plot(ts,z_IC, ts,z_F, ts, z_IC+z_F,'--'); grid on;
legend('z_{IC}','z_F','z_{total}');
set(get(gca,'Children'),'LineWidth',2);
xlabel('time (s)'); ylabel('Displacement (m)');
```

Command Window Output:

```
wn =
       3.3166
zt =
     0.099499
A =
         -0.1 +  0.0099995i
gam =
     0.099664
T =
        3.362
F0 =
       109.36
Fc =
```

```
    1.0936e+05
B =
              0 -    0.033138i
```



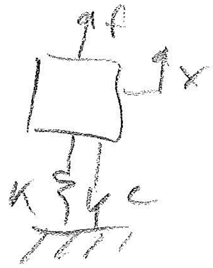


$\rho = 7800$

$E = 210\ GPa$

$\zeta = 0.01$

$$K = \frac{48EI}{L^3} \quad \text{(p.5 in book)}$$



$$m\ddot{x} + c\dot{x} + kx = f(t)$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{f(t)}{m}$$

$$\omega_n^2 = \sqrt{\frac{K}{m}} = ?$$

$$I = \frac{bh^3}{12} = \frac{(18')(12'')^3}{12} \rightarrow 9.7\times10^{-3}\ m^4$$

$$m = \underbrace{\left(\tfrac{1}{3}\right)}_{\text{given}} \rho \cdot bhL = 72.4\times10^3\ kg$$

$$\longrightarrow \boxed{\omega_n = 3.28\ rad/s \Rightarrow 0.52\ Hz}$$

Steady state response $x(t) = Re(X e^{i\omega t})$

$$X = \frac{(1000\ N/m)}{\omega_n^2 - \omega^2 + i\omega 2\zeta\omega_n}$$

Largest response when $\omega = \omega_n\sqrt{1-2\zeta}$ eq. 3.1.11

or $\omega \approx \omega_n$ for $\zeta \ll 1$

$$X = \frac{(1000\ N/m)}{i\,\omega_n^2\, 2\zeta} \longrightarrow \boxed{\begin{array}{c}|X| = \textbf{0.064 m} \\ \text{for 1 student}\end{array}}$$

$\longrightarrow$ **8** students required to get 50 cm amplitude

For reference, to get the same displacement by standing on the bridge,

$$X_{static} = \frac{F}{K} = 1.2\ mm \longrightarrow \underline{392\ \text{students!}}$$

Exercise 3.2

$$m\ddot{q} + c\dot{q} + kq = F\cos(\omega t)$$

$m = 8$ kg, $\zeta = 0.25$, $\omega_d = 10\pi$ rad/s, $F/k = 0.002$ meter

$$\omega_{nat} = \frac{\omega d}{(1-\zeta^2)^{1/2}} = 32.446 \text{ rad/s}$$

$$r = \frac{5.2(2\pi)}{\omega_{nat}} = 1.006976$$

$$q = \frac{F}{k}|D(r,\zeta)|\cos(\omega t - \phi)$$

$$|D(r,\zeta)| = \frac{1}{[(1-r^2)^2 + 4\zeta^2 r^2]^{1/2}} = 1.9854$$

$$\phi = \tan^{-1}\frac{2\zeta r}{(1-r^2)} = -1.5430 + \pi = 1.5986 \text{ rad} = 91.593°$$

Thus $q = 0.003971\cos(10.4\pi t - 1.5986)$ meter

3.5 $m = 10\,kg$, $f_n = 1\,KHz$ $\quad F(t) = 1.2\sin(\omega t)\ KN$

$\omega = 1\,KHz$, $|X| = 2.4\,mm$, find $|X|$ when $\omega = 0.95\,KHz$, $\omega = 1.05\,KHz$

$m\ddot{x} + c\dot{x} + Kx = f(t) \rightarrow f(t) = Re(F e^{i\omega t}) \rightarrow x(t) = Re(X e^{i\omega t})$

$F = -i \cdot 1200\ N \quad (\text{sine})$

$$X = \frac{F/m}{\omega_n^2 - \omega^2 + i\omega \cdot 2\zeta\omega_n}$$

$$\omega = \omega_n,\quad |X| = \left|\frac{F/m}{i2\zeta\omega_n^2}\right| = \frac{|F|}{2m\omega_n^2 \cdot \zeta}$$

$$\zeta = \frac{|F|}{|X| \cdot 2 \cdot m \cdot \omega_n^2} = \frac{1200\ \cancel{Kg}\text{-}\cancel{m}/\cancel{s^2}}{0.0024\,\cancel{m} \cdot 2 \cdot 10\,\cancel{kg} \cdot (1000 \cdot 2 \cdot \pi)^2\ 1/\cancel{s^2}}$$

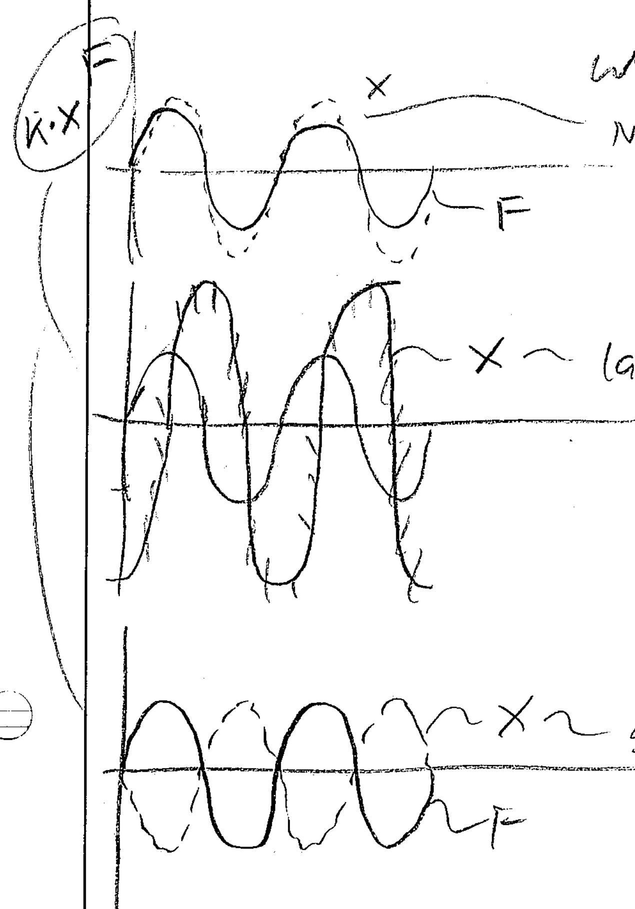
$$\boxed{\zeta = 6.33 \times 10^{-4}}$$

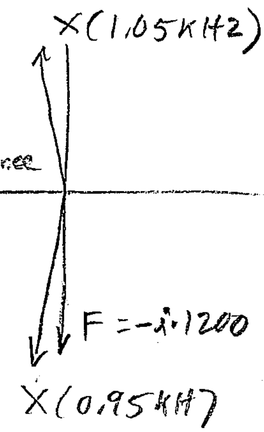
$\omega = 0.95\ KHz$

$$X = \frac{-i \cdot 1200/10}{(2\pi)^2(1000^2 - 950^2 + i \cdot 2 \cdot \zeta \cdot 1000 \cdot 950)} = 3.117 \times 10^{-5}\ \measuredangle -90.7^\circ$$

$\omega = 1.05\ KHz$

$$X = 2.965 \times 10^{-5}\ \measuredangle\ 90.7^\circ$$





$\omega = 0.95\ KHz$

Not much amplification X, resonance

$\omega = \omega_n$

large amplitude

$\omega = 1.05\ KHz$

small amplitude