## **Homework #4** EMA 545, Spring 2013

Problem 1: Exercise 2.54 from Ginsberg.

Problem 2: (30 pts, each part below is worth 10 pts).

The read head on a Hard Disk Drive (HDD) can be modeled as a pinned bar with a torsional spring at its base as shown below with L = 2 cm, m = 3 grams and  $\kappa = 20$  N/rad. The damping ratio for the system is  $\zeta = 0.02$ . The equation of motion for this system is: (later we will discuss how to find the EOM for a system like this)

$$\frac{1}{3}mL^2\ddot{\theta} + c\dot{\theta} + \kappa\theta = \Gamma(t)$$

A certain read operation involves applying a step torque  $\Gamma(t) = F(t)$  with amplitude  $F_0$  and duration T as shown below, where  $F_0$  is the static torque required to displace the bar 30 degrees.



- (a) Find the response of the system numerically over the time interval  $0 < 1 < 10^{*}$ T, with  $T = 2.5^{*}T_{d}$ , where  $T_{d}$  is the damped period of the system. Use a numerical procedure, preferably Matlab's "ode45" function together with a suitably modified version of eom\_2\_12.m, which is available on the class website.
- (b) Assuming an underdamped response, write down a closed-form solution for the response in terms of Heaviside-step functions, and unit step responses, q<sub>s</sub>(t). Compare this with the response that you found numerically.
- (c) Plot the displacement as a function of time for the case where  $T = 3*T_d$  and  $T = 2.5*T_d$ . What do you observe? Why is the residual vibration larger in the latter case? (Hint –an undamped version of your analysis in (b) may make this easier to see.)

**Problem 3**: A SDOF system modeling a car bouncing on its suspension has m=1000kg, k=11 kN/m and c=660 N-s/m. The car is released from rest at t=0 with z(0)=-0.10m. It is possible to bring the car exactly to rest by exerting an impulsive force  $f(t)=F_0\delta(t-T)$  at some instant t=T. (e.g. hitting it with a very large hammer at just the right instant). Find the magnitude of the impulse and the instant, T, at which it should be applied such the bouncing of the car stops completely after at least 2.0 seconds have elapsed but before 5.0 seconds have elapsed. **Problem 4**: Suppose that the bridge over University Avenue (pictured below) can be modeled as a simply-supported beam with length L=50 m. To simplify the analysis, let's assume that the beam has rectangular cross section with height 18 inches, width 4 feet and that it is constructed from steel with  $\rho$ =7800 kg/m<sup>3</sup> and E=210 GPa. (Note that the stiffness for various beam configurations is given in Figure 1.1 in the text.) Model this bridge as a SDOF system with an effective mass that is one third of the total mass of the beam and a stiffness equal to the stiffness of the beam when a static force is applied at its center. The damping ratio of the system is observed to be  $\zeta$ =0.01.

Suppose that a single student jumping up and down on the bridge can exert a force  $f(t)=(1000 \text{ N})\cos(\omega t)$  where  $\omega$  can be between 0 and  $8\pi$  rad/s depending on how quickly he jumps up and down. How many students must jump on the bridge to cause a displacement amplitude of 50 cm? What frequency should they jump at to minimize the number of students required? (Don't worry, the actual bridge is stiffer and lighter than that given in the problem statement. Extra Credit: What would be more reasonable values for its mass and natural frequency? How does this change the solution?)



**Problem 5: 3.2** from Ginsberg. Note that you are approximating the radar display as a rigid mass ("mounted mass is 8 kg"), which is mounted on a spring and dashpot.

**Problem 6: 3.5** from Ginsberg. Also, please sketch the force and the response of the system (by hand) over one or two cycles, taking care to properly represent the amplitude and phase difference. Do this for both cases, 0.95 kHz and 1.05 kHz.