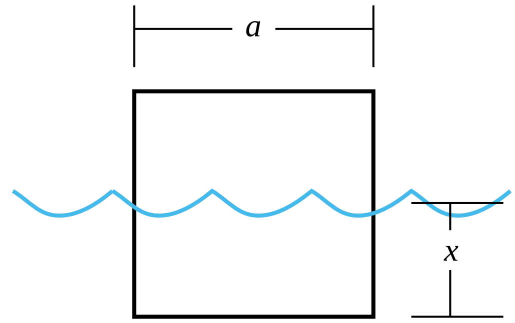
Homework #2
EMA 545, Spring 2013

Problem 1:



A cube with density ρ and side length *a* is floating freely in a pool of water.

a.) Find the equation of motion of the cube when it is displaced in the vertical direction. (Recall that the buoyant force on a floating object is equal to the weight of the water displaced.) If necessary, re-define your vertical coordinate to eliminate any static forces.

b.) Derive an expression for the natural frequency of the cube.

c.) If the block is pine (ρ=400 kg/m^3) with a side length of 10cm, what is the natural frequency in Hz?

(Note that, while an analysis like this would be important when designing a boat or ocean vessel, in reality the rotational motions of the vessel would usually be more important and those require a more complicated analysis.)

Problem 2: 2.17 from Ginsberg

Problem 3: 2.19 from Ginsberg

Problem 4: Show that $x(t)=Be^{\lambda t}$ is a solution to $\ddot{x}+2\zeta\omega_n\dot{x}+\omega_n^{\,2}x=0$ and find λ for the following cases: 1.) Underdamped system, 2.) Overdamped system. Write the solution $x(t)$ for both cases for an arbitrary set of initial conditions and draw a sketch to illustrate how each response $x(t)$ would look. Show that $x(t)$ can be written as $x(t)=\mathrm{Re}\left(Ae^{-\zeta\omega_n t}e^{\mathrm{i}\omega_d t}\right)$ in case (1).

Comment: I would encourage you to see if you can solve the following problems using only math and the fact that the general solution to an underdamped SDOF system

$$\ddot{x}+2\zeta\omega_n\dot{x}+\omega_n^{\,2}x=0$$

is:

$$x(t)=\mathrm{Re}\left(Ae^{-\zeta\omega_n t}e^{\mathrm{i}\omega_d t}\right)$$

where $\omega_d=\omega_n\sqrt{1-\zeta^2}$ and *A* is a complex constant. If you're hunting through the book for equations to solve these problems then you might be making them more difficult than they need to be and perhaps failing to connect the concepts.

Problem 5: 2.29 from Ginsberg

Problem 6: 2.32 from Ginsberg. **(part a=5pts, b=5pts, c=10pts)**