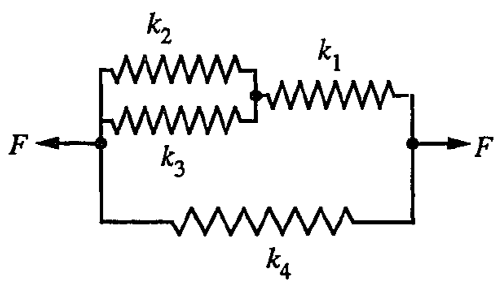
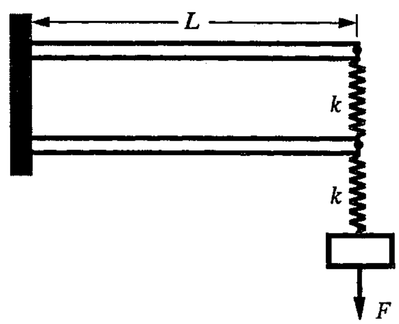
Homework #1
EMA 545, Spring 2013

Problem 1: 1.1 from Ginsberg:

1.1 Determine the spring stiffness that is equivalent to the action of the four springs in the sketch.



Problem 2: Find the equation of motion of the system pictured below. The mass of the block is *m* and the mass of the beams and springs is negligible. Assume that all of the displacements are very small. (Recall that the displacement of the tip of a cantilever beam, Δ_{tip}, is related to the force at the tip by: $F_{tip}=(3EI/L^3)\Delta_{tip}$)



USE COMPLEX EXPONENTIALS to derive the solution to problems 3-6 (i.e. do not simply look up a trig identity).

Problem 3: 2.3 from Ginsberg

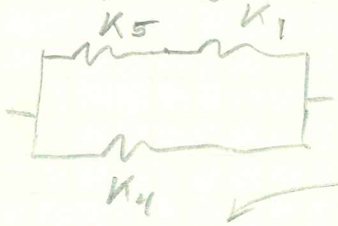
Problem 4: 2.5 from Ginsberg. Note that "this quantity" in the last sentence is referring to "the complex amplitude of d*v*/dt."

Problem 5: 2.8 from Ginsberg.

Problem 6: 2.10 from Ginsberg.

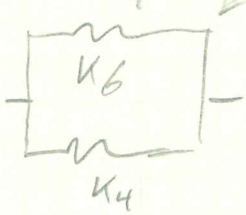
Extra: (this problem will not be graded) If you are not already familiar with Matlab, review the Matlab® tutorial on the EMA 545 course website (created by Prof. Negrut).

P1 (1.1) eq. to



$K_5 = K_2 + K_3$ (parallel)

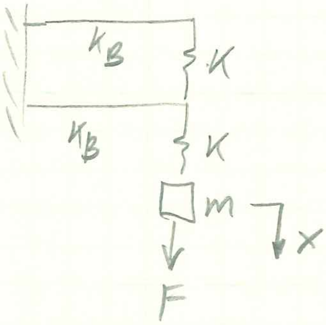
K_1, K_5 in series



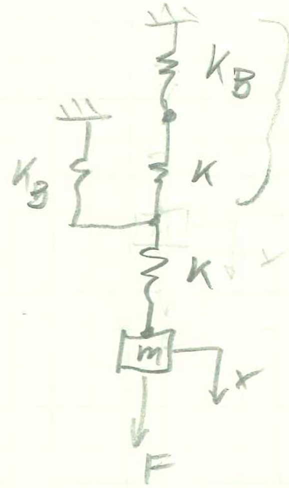
$$K_6 = \frac{K_5 K_1}{K_5 + K_1}$$

$$\Rightarrow \boxed{K_{eq} = \frac{(K_2+K_3)K_1}{K_1+K_2+K_3} + K_4}$$

P3 $F_{tip} = \frac{3EI}{L^3}\Delta_{tip} = K_b \Delta_{tip} \rightarrow K_b = \frac{3EI}{L^3}$

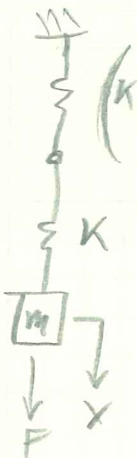


equivalent to:



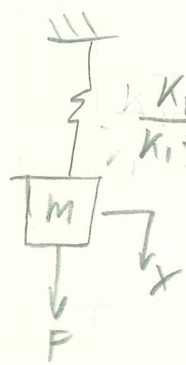
$\frac{K K_B}{K+K_B}$ series

Parallel



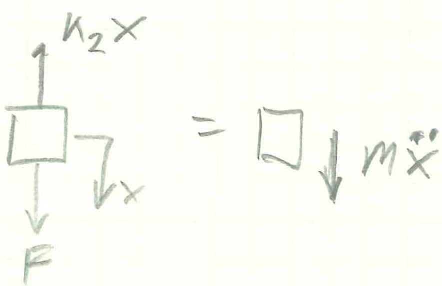
$\left(K_B + \frac{K K_B}{K+K_B}\right) = K_1$

series $\rightarrow$



$\frac{K_1 K}{K_1+K} = K_2$

FBD:



ΣF in x dir:

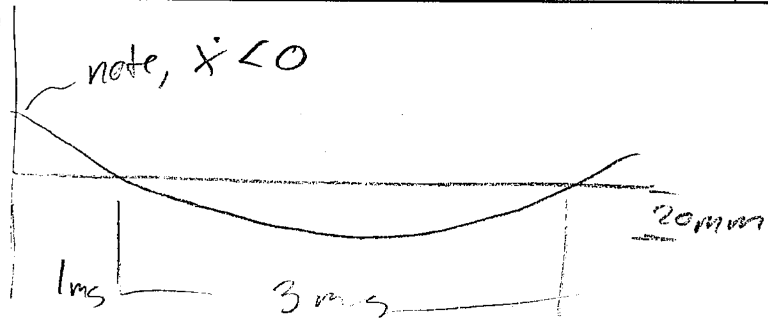
$m\ddot{x} = F - K_2 x$

$$\boxed{\begin{gathered} m\ddot{x} + \left(\frac{K_1 K}{K_1+K}\right)x = F \\ K_1 = K_B + \frac{K K_B}{K+K_B} \end{gathered}}$$

Notice that I have assumed gravity was negligible since it was not mentioned in the problem statement. If gravity was included, then I would have a static term on the right hand side and I would have to re-define x about the static equilibrium to eliminate it.

Another interesting exercise is to do this problem without the rules for series and parallel springs. You can write several equilibrium equations and use those to eliminate the unknown displacements at the ends of the beam.

(2.3)



$\dot{x}(0) < 0$,
$x(0) > 0$

a) find x, $\dot{x}$

Using complex exponentials, write $x(t)$ as $x(t) = Re(\hat{A} e^{i\omega t})$

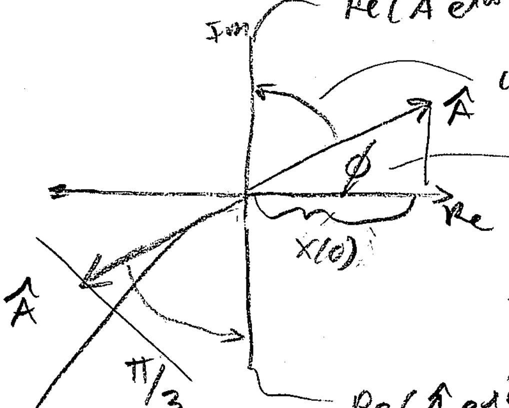
$\omega = ?$ $T = 6ms \rightarrow \omega = \frac{2\pi}{T} \rightarrow \boxed{\omega = \frac{2\pi}{0.006}}$

→ From plot $|\hat{A}| = 20mm$, so write $\hat{A} = Ae^{i\phi}$, $A = 20mm$

→ Harmonic function reaches first maximum 1ms after $t = 0$, and since $T = 6$, This is 1/6th a cycle or $2\pi/6$ rad,

→ so, $\hat{A}$ must be in the 1st or 3rd quadrants

$Re(\hat{A} e^{i\omega \cdot 0.001}) = 0$



$\omega t = \frac{2\pi}{0.006} \cdot 0.001 = \frac{\pi}{3}$

* 3rd quad. gives
$X(0) = Re(\hat{A}) < 0$ - Wrong

$\phi = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$
(from geometry)

$Re(\hat{A} e^{i\omega \cdot 0.001}) = 0$

so, $\hat{A} = 20mm\, e^{i\pi/6}$ → $\boxed{x(t) = Re(20mm\, e^{i\pi/6} e^{i\omega t})}$

a) $x(0) = Re(\hat{A}) = 20mm \cdot \cos(\pi/6) = 17.321$ $\boxed{x(0) = 17.3}$

$\dot{x}(t) = Re(i\omega \cdot 20mm\, e^{i\pi/6} e^{i\omega t})$

$\dot{x}(0) = Re(i \cdot \frac{2\pi}{0.006 s} \cdot 0.020m \cdot e^{i\pi/6} \cdot 1) \rightarrow \boxed{\dot{x}(0) = -10.47 \text{ m/s}}$

b) Minimum at $t = 1ms + 3ms/2 =$ $\boxed{\min(x) \text{ at } 2.5 ms}$

2.3 max $\dot{x}$ and when? ~ From plot, point where slope is maximum positive $\boxed{t = 4ms}$

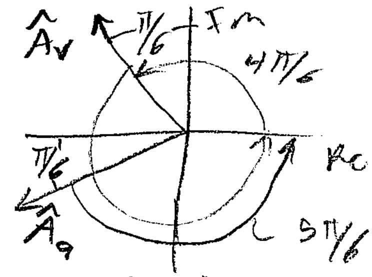
c)

$$\max \dot{x} = \left| i \cdot \frac{2\pi}{0.006s} \cdot 0.020m \cdot e^{i\pi/6} \right| \qquad \boxed{\max \dot{x} = 20.944}$$

or:

$$\dot{x} = Re\left(e^{i\pi/2} \cdot \frac{2\pi \cdot 20}{6} e^{i\pi/6} e^{i\omega t}\right)$$

$$\dot{x}(t) = Re\Big(\underbrace{\frac{2\pi \cdot 20}{6} \cdot e^{i4\pi/6}}_{\hat{A}_v} e^{i\omega t}\Big)$$



Velocity

max pos. when $\omega t = 2\pi - \frac{4\pi}{6}$

$$\omega t = \frac{12\pi - 4\pi}{6} = \frac{8\pi}{6}$$

$$t = \frac{4\cancel{8}\pi}{6} \cdot \frac{0.006}{2\pi} = \boxed{0.004 s} \checkmark \qquad \boxed{\max(\dot{x}) = |\hat{A}_v| = \frac{2\pi \cdot 20}{6}}$$

d)

$$\ddot{x} = Re\Big(\underbrace{i\left(\frac{2\pi}{0.006}\right)_s \cdot \frac{2\pi \cdot 20}{6}\left(\frac{m}{s}\right) e^{i4\pi/6}}_{\hat{A}_a} e^{i\omega t}\Big)$$

$$\boxed{\max(\ddot{x}) = |\hat{A}_a| = \frac{4\pi^2 \cdot 20}{0.036} \approx 21{,}932 \ m/s^2}$$

$\omega t = \frac{5\pi}{6}$ (from picture above) $\rightarrow$ $\boxed{t = \frac{5\cancel{\pi}}{6} \cdot \frac{0.006}{2\cancel{\pi}} = 2.5 ms}$

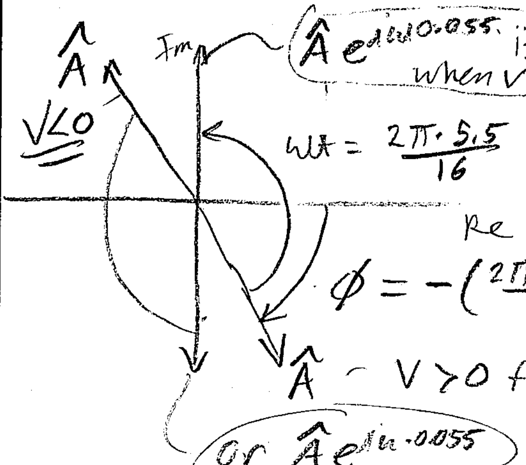
M. S. Allen

(25) Zeros $\sim T/2$, so $\boxed{\omega = \frac{2\pi}{0.016s} \approx 393 \text{ rad/s}}$

$X(5.5) = 0$, $|\hat{A}| = 1.2V$, $V > 0$ @ $t = 0$ — use polar form $\hat{A} = 1.2V e^{i\phi}$

let $V(t) = Re(1.2V \cdot e^{i\phi} e^{i\omega t})$

$$V(5.5) = Re(1.2V \cdot e^{i\phi} \underbrace{e^{i \cdot \frac{2\pi}{0.016} \cdot 0.055}}_{\gamma = \frac{2\pi \cdot 5.5}{16} \approx 0.7\pi}) = 0$$



$$\phi = -\left(\frac{2\pi \cdot 5.5}{16} - \frac{\pi}{2}\right) \approx -0.589 \qquad \left(\phi = -\frac{3\pi}{16}\right)$$

Matlab A_hat = 1.2*exp(-i*0.589)

so, a.)

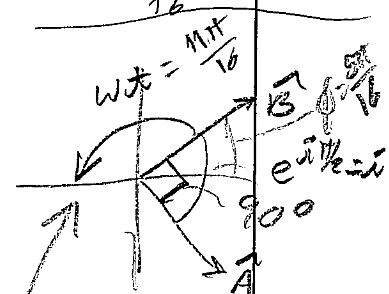
$$\boxed{\begin{aligned}\hat{A} &= 1.2V \cdot e^{-i\,0.589} = 1.2\cos 0.589 - i\,1.2\sin 0.589 \\ \hat{A} &= 0.998 - 0.667 i \qquad V(t) = Re(\hat{A} e^{i393t})\end{aligned}}$$

b.) $\dot{V}(t) = Re(\underbrace{i\omega \cdot \hat{A}}_{\hat{B}} e^{i\omega t}) = Re(\hat{B} e^{i\omega t})$

$$\hat{B} = e^{i\pi/2} \cdot \frac{2\pi}{0.016s} \cdot 1.2V\, e^{i\left(\frac{\pi}{2} - \frac{11\pi}{16}\right)} \sim \pi - \frac{11\pi}{16}$$

$$\boxed{\hat{B} = \frac{\pi \cdot 2.4}{0.016} (V/s) \cdot e^{i5\pi/16}}$$ polar

$$\boxed{\begin{aligned}\hat{B} &= \frac{\pi \cdot 2.4}{0.016}\left(\cos\left(\frac{5\pi}{16}\right) + i\sin\left(\frac{5\pi}{16}\right)\right) \\ \hat{B} &\approx 261.8 + 391.8 i\end{aligned}}$$ rect.



c.) $\boxed{\max(\dot{x}) = |\hat{B}| = \frac{\pi \cdot 2.4}{0.016} V/s \approx 471.2 \text{ V/s}}$

Occurs when $\omega t = \frac{11\pi}{16} + n\pi$ — considering $\max(|\dot{x}|)$

$$t = \left(\frac{11\cancel{\pi}}{16} + n\cancel{\pi}\right)\frac{0.016s}{2\cancel{\pi}}$$

$$t = (5.5 ms, 13.5 ms, 21.5 ms, 29.5 ms)$$

positive $\dot{x}$ only

either answer OK if done correctly

2/17/2011

(2.8) $g = 0.01\sin(50t) - 0.02\cos(50t - 0.3\pi)$

a) $g = Re(-i \cdot 0.01 e^{i50t} - 0.02 e^{i(50t - 0.3\pi)})$

$g = Re(-0.01i\, e^{i50t} - 0.02 e^{-i0.3\pi} e^{i50t})$

$$\boxed{g = Re(A e^{i50t}) \qquad A = -0.01i - 0.02e^{-i0.3\pi}}$$

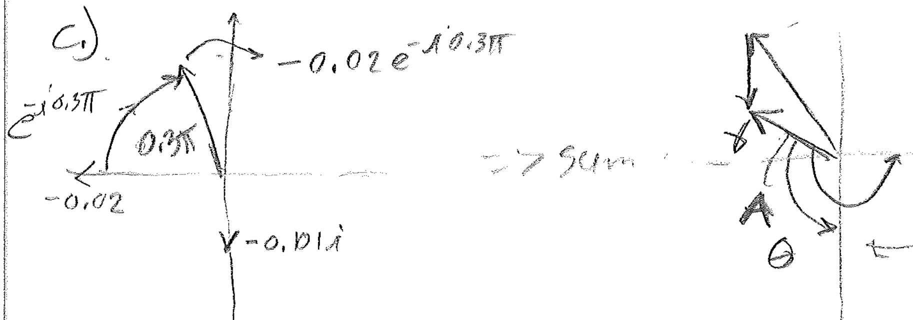
or could write $A = -0.01i - 0.02(\cos(-0.3\pi) + i\sin(-0.3\pi))$

$A = -0.02\cos(-0.3\pi) + i(-0.02\sin(-0.3\pi) - 0.01)$

b.) frequency $\omega = 50$, so $T = \frac{2\pi}{50}$ $\quad (e^{i50t} \to e^{i2\pi}: 1 \text{ cycle})$

zeros every half cycle, $\boxed{\Delta t = T/2 = \pi/50}$

c.)



$g = \max$ (part d)

$g = 0$ when the vector rotates to this position

when $50t = \theta$, $g = Re(Ae^{i\theta}) = 0$

$\theta = \frac{\pi}{2} + \gamma$

$\tan\gamma = \frac{|Im(A)|}{|Re(A)|}$

$$\boxed{\begin{gathered}\gamma = \tan^{-1}\left|\frac{-0.02\sin(-0.3\pi) - 0.01}{-0.02\cos(-0.3\pi)}\right| \\ t = \left(\frac{\frac{\pi}{2} + \gamma}{50}\right) \qquad t \approx 41.1\,\text{ms at first zero}\end{gathered}}$$

d.) $$\boxed{\begin{gathered}\max(g) = |A| = 0.0133 \\ \text{occurs when } 50t = \gamma + \pi \longrightarrow t \approx 72.5\,\text{ms}\end{gathered}}$$

see picture above

2/23/2011

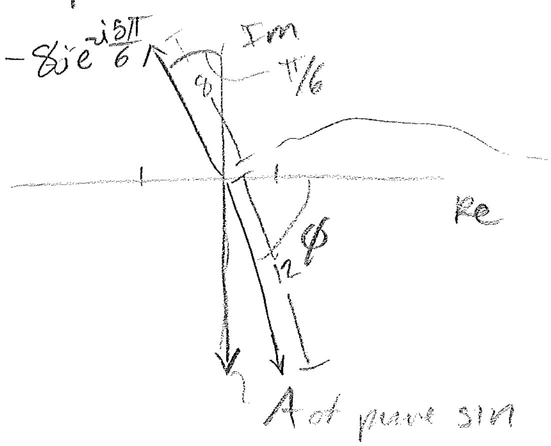
(2.10) $x = x_1 + x_2 = 8\sin(10t - 5\frac{\pi}{6}) + 12\cos(10t + \phi)$

Find ϕ s.t. x is a pure sine. I'll assume that the problem means "pure sine" = "positive sine", although it is, perhaps, a bit ambiguous.

Using definitions $\sin(\omega t) = Re(-iA e^{i\omega t})$, etc...

$$x = Re\Big[\underbrace{(-8ie^{-i5\pi/6} + 12e^{i\phi})}_{A} e^{i10t}\Big]$$

Need A to be purely imaginary and negative for a pure sin



So, ϕ must put the comp. Amp of the cos. term in the IV quad. to get A purely imaginary

real parts must cancel,

$$8\sin(\pi/6) = 12\cos|\phi|$$

$$|\phi| = \cos^{-1}\left(\frac{8}{12}\cos\left(\frac{\pi}{6}\right)\right)$$

$$\boxed{\phi = -1.231}$$

$$|x| = |A| = \left|(-8ie^{-i5\pi/6} + 12e^{i\phi})\right|$$

$\boxed{|A| = 4.386}$ $\quad A = -4.386\hat{i}$

✓ plotted in Matlab to verify that it gives a pure sine