Solving slide 362 example, ME 440 Intermediate Vibration, Fall 2017

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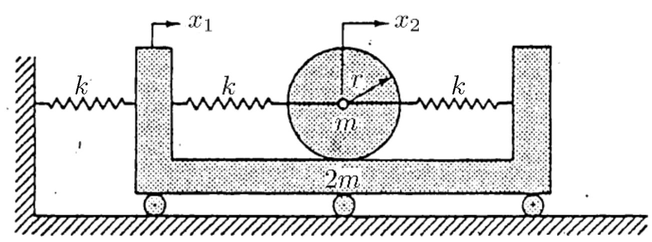
[AO]

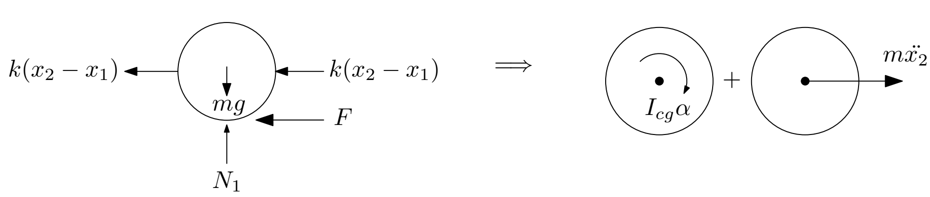
Example

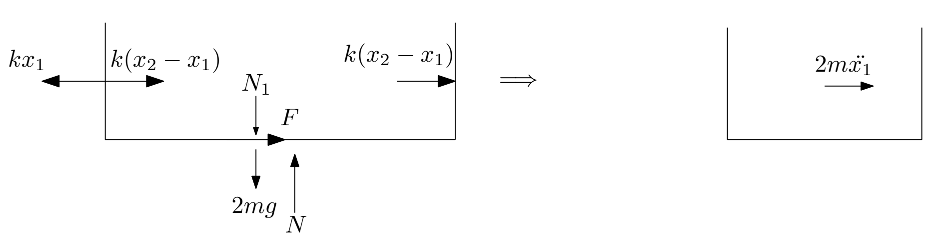


The cylinder of mass m, radius r, and centroidal mass moment of inertia $\bar{I} = mr^2/2$ rolls without slipping on the platform of mass $2m$ as shown in the figure. The generalized coordinates x_1 and x_2 of the system are the absolute displacements of the platform and the mass center of the cylinder, respectively. Note that the absolute angular displacement of the cylinder is $(x_2 - x_1)/r$.

- Derive the EOMs and indicate whether the EOMs are coupled
- Using MATLAB, determine the system's natural frequencies and modal matrix
- Determine the principal coordinates associated with this system and state the set of ODEs satisfied by these new generalized coordinates

Assuming $x_2 > x_1, \dot{x}_2 > \dot{x}_1, \ddot{x}_2 > \ddot{x}_1$ and all are positive, the free body diagram for the cylinder and the cart is





Equation of motion for cylinder. $\sum F_x$

$$-2k(x_2 - x_1) - F = m\ddot{x}_2 \tag{1}$$

And taking moment around C.G. of cylinder, using anti-clock wise as positive

$$-Fr = -I_{cg}\alpha$$

$$Fr = I_{cg}\alpha$$

Since we assumed no slip, then $(\ddot{x}_2 - \ddot{x}_1) = \alpha r$ and the above becomes

$$
\begin{aligned}
Fr &= I_{cg}\frac{(\ddot{x}_2 - \ddot{x}_1)}{r}\\
F &= I_{cg}\frac{(\ddot{x}_2 - \ddot{x}_1)}{r^2}\\
&= \frac{1}{2}mr^2\frac{(\ddot{x}_2 - \ddot{x}_1)}{r^2}\\
&= \frac{1}{2}m(\ddot{x}_2 - \ddot{x}_1) \qquad (2)
\end{aligned}
$$

Using (2) in (1) gives EQM for x_2

$$
\begin{aligned}
m\ddot{x}_2 + 2k(x_2 - x_1) + \frac{1}{2}m(\ddot{x}_2 - \ddot{x}_1) &= 0\\
\frac{3}{2}m\ddot{x}_2 - \frac{1}{2}m\ddot{x}_1 + 2kx_2 - 2kx_1 &= 0 \qquad (3)
\end{aligned}
$$

For EQM for x_1, resolving forces in x direction gives

$$
-kx_1 + 2k(x_2 - x_1) + F = 2m\ddot{x}_1
$$

Using F found in (2) into the above gives

$$
-kx_1 + 2k(x_2 - x_1) + \frac{1}{2}m(\ddot{x}_2 - \ddot{x}_1) = 2m\ddot{x}_1
$$

Simplifying

$$
\begin{aligned}
2m\ddot{x}_1 - \frac{1}{2}m(\ddot{x}_2 - \ddot{x}_1) + kx_1 - 2k(x_2 - x_1) &= 0\\
-\frac{1}{2}m\ddot{x}_2 + \frac{5}{2}m\ddot{x}_1 + 3kx_1 - 2kx_2 &= 0 \qquad (4)
\end{aligned}
$$

Writing (3,4) in matrix form gives (note. Using (4) for top row and then use (3) for second row)

$$
\begin{bmatrix} \frac{5}{2}m & -\frac{1}{2}m \\ -\frac{1}{2}m & \frac{3}{2}m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 3k & -2k \\ -2k & 2k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (5)
$$

If we had picked (3) for top row and then (4) for second row, the result will be

$$
\begin{bmatrix} -\frac{1}{2}m & \frac{3}{2}m \\ \frac{5}{2}m & -\frac{1}{2}m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} -2k & 2k \\ 3k & -2k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (6)
$$

Since So (5) and (6) are equivalent. To verify both (5) and (6) give the same eigenvalues, here is a check

```
In[49]:= (* eq 5*)
m = 1;
k = 1;
massMat = {{5 / 2 m, -1 / 2 m}, {-1 / 2 m, 3 / 2 m}};
kMat = {{3 k, -2 k}, {-2 k, 2 k}};
Amat = Inverse[massMat] . kMat;
Sqrt[Eigenvalues[Amat]] // N

Out[54]= {1.353042756497228, 0.5586881437327312}
```

```
In[63]:= (* eq 6*)

SetOptions[$FrontEndSession, PrintPrecision → 16]
m = 1;
k = 1;
massMat = {{-1 / 2 m, 3 / 2 m}, {5 / 2 m, -1 / 2 m}};
kMat = {{-2 k, 2 k}, {3 k, -2 k}};
inv = Inverse[massMat];
Amat = (inv. kMat);
Sqrt[Eigenvalues[Amat]] // N

Out[70]= {1.353042756497228, 0.5586881437327312}
```