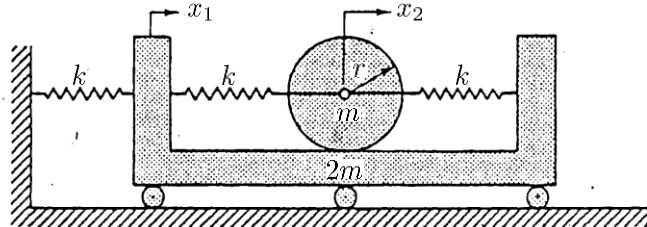


# Solving slide 362 example, ME 440 Intermediate Vibration, Fall 2017

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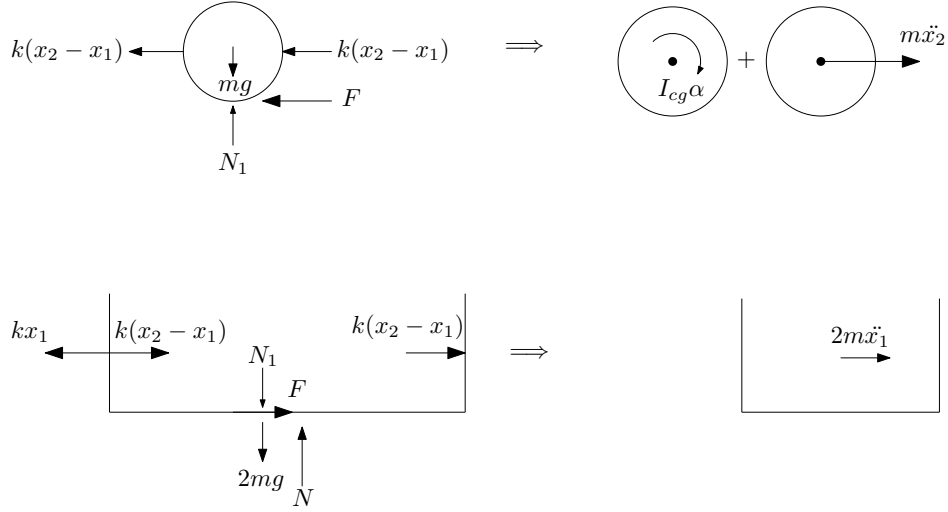
[AO]  
Example



The cylinder of mass  $m$ , radius  $r$ , and centroidal mass moment of inertia  $\bar{I} = mr^2/2$  rolls without slipping on the platform of mass  $2m$  as shown in the figure. The generalized coordinates  $x_1$  and  $x_2$  of the system are the absolute displacements of the platform and the mass center of the cylinder, respectively. Note that the absolute angular displacement of the cylinder is  $(x_2 - x_1)/r$ .

- Derive the EOMs and indicate whether the EOMs are coupled
- Using MATLAB, determine the system's natural frequencies and modal matrix
- Determine the principal coordinates associated with this system and state the set of ODEs satisfied by these new generalized coordinates

Assuming  $x_2 > x_1, \dot{x}_2 > \dot{x}_1, \ddot{x}_2 > \ddot{x}_1$  and all are positive, the free body diagram for the cylinder and the cart is



Equation of motion for cylinder.  $\Sigma F_x$

$$-2k(x_2 - x_1) - F = m\ddot{x}_2 \quad (1)$$

And taking moment around C.G. of cylinder, using anti-clock wise as positive

$$-Fr = -I_{cg}\alpha$$

$$Fr = I_{cg}\alpha$$

Since we assumed no slip, then  $(\ddot{x}_2 - \ddot{x}_1) = ar$  and the above becomes

$$Fr = I_{cg} \frac{(\ddot{x}_2 - \ddot{x}_1)}{r}$$

$$F = I_{cg} \frac{(\ddot{x}_2 - \ddot{x}_1)}{r^2}$$

$$= \frac{1}{2}mr^2 \frac{(\ddot{x}_2 - \ddot{x}_1)}{r^2}$$

$$= \frac{1}{2}m(\ddot{x}_2 - \ddot{x}_1) \quad (2)$$

Using (2) in (1) gives EQM for  $x_2$

$$m\ddot{x}_2 + 2k(x_2 - x_1) + \frac{1}{2}m(\ddot{x}_2 - \ddot{x}_1) = 0$$

$$\frac{3}{2}m\ddot{x}_2 - \frac{1}{2}m\ddot{x}_1 + 2kx_2 - 2kx_1 = 0 \quad (3)$$

For EQM for  $x_1$ , resolving forces in  $x$  direction gives

$$-kx_1 + 2k(x_2 - x_1) + F = 2m\ddot{x}_1$$

Using  $F$  found in (2) into the above gives

$$-kx_1 + 2k(x_2 - x_1) + \frac{1}{2}m(\ddot{x}_2 - \ddot{x}_1) = 2m\ddot{x}_1$$

Simplifying

$$\begin{aligned} 2m\ddot{x}_1 - \frac{1}{2}m(\ddot{x}_2 - \ddot{x}_1) + kx_1 - 2k(x_2 - x_1) &= 0 \\ -\frac{1}{2}m\ddot{x}_2 + \frac{5}{2}m\ddot{x}_1 + 3kx_1 - 2kx_2 &= 0 \end{aligned} \quad (4)$$

Writing (3,4) in matrix form gives (note. Using (4) for top row and then use (3) for second row)

$$\begin{bmatrix} \frac{5}{2}m & -\frac{1}{2}m \\ -\frac{1}{2}m & \frac{3}{2}m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 3k & -2k \\ -2k & 2k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (5)$$

If we had picked (3) for top row and then (4) for second row, the result will be

$$\begin{bmatrix} -\frac{1}{2}m & \frac{3}{2}m \\ \frac{5}{2}m & -\frac{1}{2}m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} -2k & 2k \\ 3k & -2k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (6)$$

Since So (5) and (6) are equivalent. To verify both (5) and (6) give the same eigenvalues, here is a check

```
In[49]:= (* eq 5*)
```

```
m = 1;
```

```
k = 1;
```

```
massMat = {{5/2 m, -1/2 m}, {-1/2 m, 3/2 m}};
```

```
kMat = {{3 k, -2 k}, {-2 k, 2 k}};
```

```
Amat = Inverse[massMat] . kMat;
```

```
Sqrt[Eigenvalues[Amat]] // N
```

```
Out[54]= {1.353042756497228, 0.5586881437327312}
```

```
In[63]:= (* eq 6*)
```

```
SetOptions[$FrontEndSession, PrintPrecision -> 16]
```

```
m = 1;
```

```
k = 1;
```

```
massMat = {{-1/2 m, 3/2 m}, {5/2 m, -1/2 m}};
```

```
kMat = {{-2 k, 2 k}, {3 k, -2 k}};
```

```
inv = Inverse[massMat];
```

```
Amat = (inv . kMat);
```

```
Sqrt[Eigenvalues[Amat]] // N
```

```
Out[70]= {1.353042756497228, 0.5586881437327312}
```