Solving slide 148 example, ME 440 Intermediate Vibration, Fall 2017

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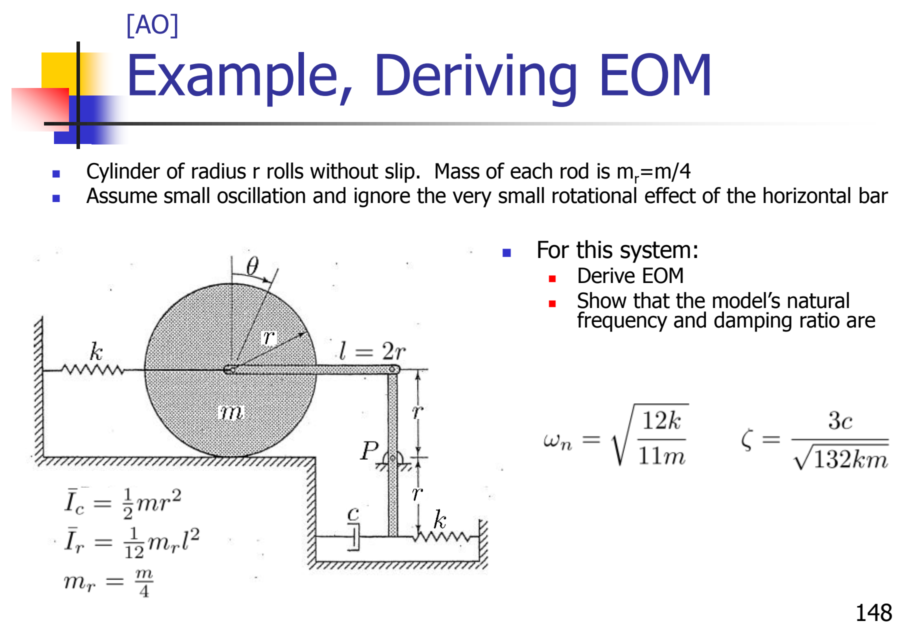
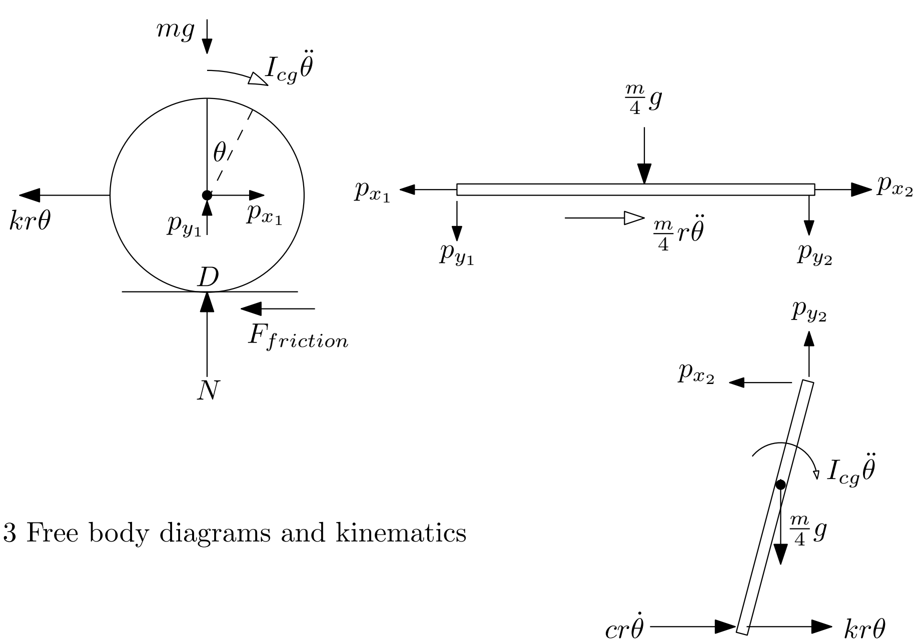


We will solve this using 3 separate bodies. So there are three free body diagrams as shown below



3 Free body diagrams and kinematics

In this diagram, it is assumed the horizontal bar only moves in the x direction and this is all for small angle θ. Now we apply Newton laws to each body.

For disk, we apply $\tau = I_o\ddot{\theta}$ but using the point D on the figure to take moments around in order to get rid of the friction F and N terms. This gives (using counter clock wise as positive)

$$
\begin{aligned}
(kr\theta)\, r - p_{x_1} r &= -I_o\ddot{\theta}\\
kr^2\theta - p_{x_1} r &= -\left(I_{cg} + mr^2\right)\ddot{\theta}\\
&= -\left(\frac{1}{2}mr^2 + mr^2\right)\ddot{\theta}\\
&= -\frac{3}{2}mr^2\ddot{\theta} \qquad (1)
\end{aligned}
$$

We now move to the second body, which is the horizontal bar.

$$
\begin{aligned}
\sum F_x &= m_{bar}\ddot{x}\\
-p_{x_1} + p_{x_2} &= \frac{m}{4}r\ddot{\theta} \qquad (2)
\end{aligned}
$$

From (2) we solve for p_{x_1} and plug it into (1)

$$
p_{x_1} = p_{x_2} - \frac{m}{4}r\ddot{\theta}
$$

Hence (1) now becomes

$$
\begin{aligned}
kr^2\theta - \left(p_{x_2} - \frac{m}{4}r\ddot{\theta}\right) r &= -\frac{3}{2}mr^2\ddot{\theta}\\
kr^2\theta - p_{x_2} r &= -\left(\frac{3}{2}mr^2 + \frac{m}{4}r^2\right)\ddot{\theta}\\
&= -\frac{7}{4}mr^2\ddot{\theta} \qquad (3)
\end{aligned}
$$

To find p_{x_2}, we use the third body, the vertical bar. Taking moments about C.G. of bar using counter clock wise as positive gives

$$
\begin{aligned}
\tau &= -I_{cg}\ddot{\theta}\\
(kr\theta)\, r\cos\theta + \left(cr\dot{\theta}\right) r\cos\theta + p_{x_2} r\cos\theta + p_{y_2} r\sin\theta &= -\frac{1}{12}\left(\frac{m}{4}\right)(2r)^2\,\ddot{\theta}\\
&= -\frac{1}{12}mr^2\ddot{\theta}
\end{aligned}
$$

For small angle the above becomes

$$
kr^2\theta + cr^2\dot{\theta} + p_{x_2} r + p_{y_2} r\theta = -\frac{m}{12}r^2\ddot{\theta} \qquad (4)
$$

p_{y_2} is now found from vertical balance of horizontal bar. Since it does not move vertically and assumed to only move horizontally, then

$$
\begin{aligned}
\sum F_y &= 0\\
-p_{y_1} - p_{y_2} - \frac{m}{4}g &= 0
\end{aligned}
$$

Due to symmetry, $p_{y_1} = p_{y2}$ and the above becomes

$$
\begin{aligned}
-2p_{y_2} &= \frac{m}{4}g\\
p_{y_2} &= -\frac{m}{8}g
\end{aligned}
$$

Plugging this value for p_{y_2} into (4) and solving for p_{x_2} gives

$$
\begin{aligned}
kr^2\theta + cr^2\dot{\theta} + p_{x_2} r - \frac{m}{8}gr\theta &= -\frac{m}{12}r^2\ddot{\theta}\\
p_{x2} &= \frac{1}{r}\left(-\frac{m}{12}r^2\ddot{\theta} + \frac{m}{8}gr\theta - kr^2\theta - cr^2\dot{\theta}\right)
\end{aligned}
$$

Plugging the above into (3) gives the equation of motion for disk

$$
\begin{aligned}
kr^2\theta - \left(-\frac{m}{12}r^2\ddot{\theta} + \frac{m}{8}gr\theta - kr^2\theta - cr^2\dot{\theta}\right) &= -\frac{7}{4}mr^2\ddot{\theta} \\
kr^2\theta + \frac{m}{12}r^2\ddot{\theta} - \frac{m}{8}gr\theta + kr^2\theta + cr^2\dot{\theta} &= -\frac{7}{4}mr^2\ddot{\theta} \\
\theta\left(2kr^2 - \frac{m}{8}gr\right) + cr^2\dot{\theta} &= -\frac{7}{4}mr^2\ddot{\theta} - \frac{m}{12}r^2\ddot{\theta} \\
\frac{11}{6}mr^2\ddot{\theta} + cr^2\dot{\theta} + \theta\left(2kr^2 - \frac{m}{8}gr\right) &= 0
\end{aligned}
$$

Or

$$
\ddot{\theta} + \frac{6c}{11m}\dot{\theta} + \theta\left(\frac{12}{11}\frac{k}{m} - \frac{3}{44}\frac{g}{r}\right) = 0
$$

Writing the above in the standard form $\ddot{\theta} + 2\zeta\omega_n\dot{\theta} + \omega_n^2\theta = 0$ we see that

$$
\omega_n^2 = \sqrt{\frac{12}{11}\frac{k}{m} - \frac{3}{44}\frac{g}{r}}
$$

And

$$
\begin{aligned}
2\zeta\omega_n &= \frac{6c}{11m} \\
\zeta &= \frac{3c}{11m\omega_n} \\
&= \frac{3c}{11m\sqrt{\frac{12}{11}\frac{k}{m} - \frac{3}{44}\frac{g}{r}}} \\
&= \frac{3c}{\sqrt{132km - \frac{363}{44}\frac{gm^2}{r}}}
\end{aligned}
$$