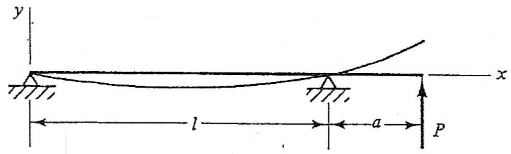
Pinned-pinned beam with overhang (P at $x = l + a$)*

$$k = \frac{P}{y|_{x=a+l}} = \frac{3EI}{a^2(a+l)}$$

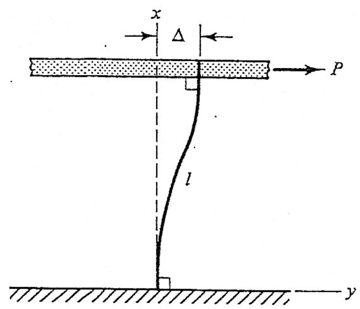


(g)

$$y = \frac{Pax}{6EIl}(x^2 - l^2) \qquad x \le l$$

$$y = \frac{P}{6EIl}[ax(x^2 - l^2) - (l+a)(x-l)^3] \qquad x \ge l$$

Fixed-fixed beam with lateral displacement



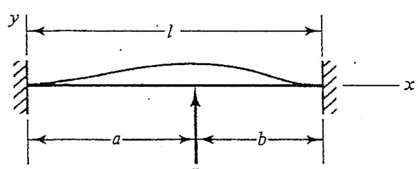
(h)

$$\Delta = \frac{Pl^3}{12EI}$$

$$k = \frac{12EI}{l^3}$$

$$y = \frac{P}{12EI}(3lx^2 - 2x^3)$$

Fixed-fixed beam*



(i)

$$k = \frac{P}{y|_{x=a}}$$

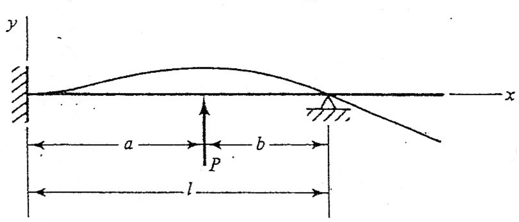
$$k|_{a=l/2} = \frac{192EI}{l^3}$$

$K = 3EI\left(\frac{L}{9b}\right)^3$

$$y = \frac{Pb^2}{6EIl^3}[(2b - 3l)x^3 + 3l(l-b)x^2] \qquad (x \le a)$$

$$y = \frac{Pb^2}{6EIl^3}\left[(2b - 3l)x^3 + 3l(l-b)x^2 + \frac{l^3}{b^2}(x-a)^3\right] \qquad (x \ge a)$$

Fixed-pinned beam with overhang*



(j)

$$k = \frac{P}{y|_{x=a}}$$

$$k|_{a=l/2} = \frac{768EI}{7l^3}$$

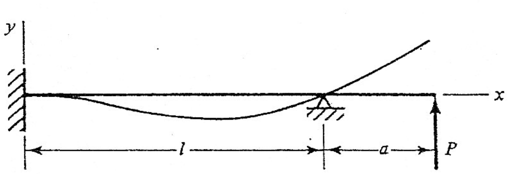
$$y = \frac{P}{12EI}\left[3b\left(1 - \frac{b^2}{l^2}\right)x^2 - \frac{b}{l}\left(3 - \frac{b^2}{l^2}\right)x^3\right] \qquad x \le a$$

$$y = \frac{P}{12EI}\left[3b\left(1 - \frac{b^2}{l^2}\right)x^2 - \frac{b}{l}\left(3 - \frac{b^2}{l^2}\right)x^3 + 2(x-a)^3\right] \qquad a \le x \le l$$

$$y = \frac{-pba^2}{4EIl}(x - l) \qquad x \ge l$$

Fixed-pinned beam with overhang (P at $x = l + a$)*

$$k = \frac{P}{y|_{x=l+a}} = \frac{12EI}{a^2(3l + 4a)}$$



(k)

$$y = \frac{Pa}{4EIl}(x^3 - lx^2) \qquad x \le l$$

$$y = \frac{Pa}{4EIl}\left[x^3 - lx^2 - \left(\frac{2l}{3a} + 1\right)(x - l)^3\right] \qquad x \ge l$$

* Axial extensions due to axial end constraints considered negligible.

Cantilevered Beam Slopes and Deflections

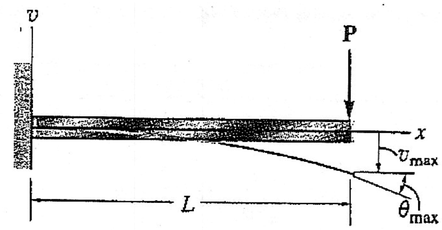
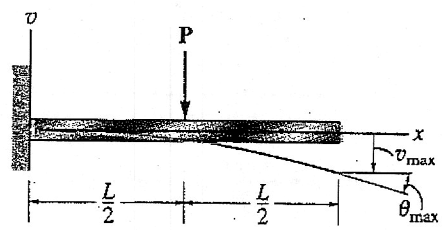
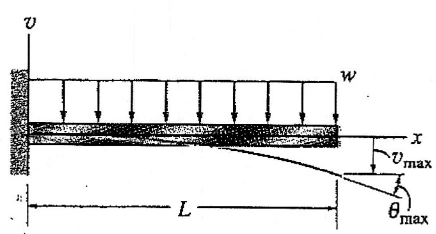
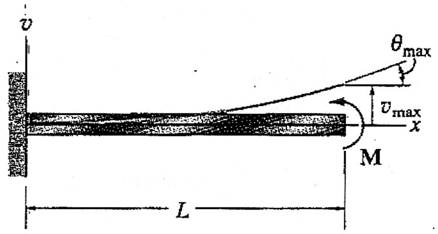
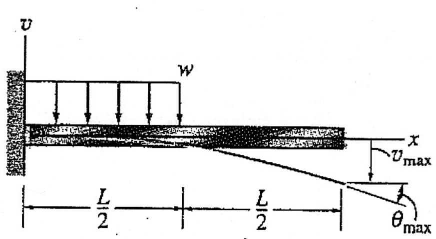
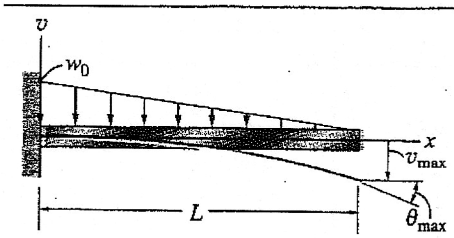
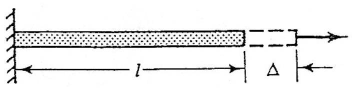
Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \dfrac{-PL^2}{2EI}$	$v_{max} = \dfrac{-PL^3}{3EI}$	$v = \dfrac{-Px^2}{6EI}(3L - x)$
	$\theta_{max} = \dfrac{-PL^2}{8EI}$	$v_{max} = \dfrac{-5PL^3}{48EI}$	$v = \dfrac{-Px^2}{6EI}(\tfrac{3}{2}L - x)$ $0 \le x \le L/2$ $v = \dfrac{-PL^2}{24EI}(3x - \tfrac{1}{2}L)$ $L/2 \le x \le L$
	$\theta_{max} = \dfrac{wL^3}{6EI}$	$v_{max} = \dfrac{-wL^4}{8EI}$	$v = \dfrac{-wx^2}{24EI}(x^2 - 4Lx + 6L^2)$
	$\theta_{max} = \dfrac{ML}{EI}$	$v_{max} = \dfrac{ML^2}{2EI}$	$v = \dfrac{Mx^2}{2EI}$
	$\theta_{max} = \dfrac{-wL^3}{48EI}$	$v_{max} = \dfrac{-7wL^4}{384EI}$	$v = \dfrac{-wx^2}{24EI}(x^2 - 2Lx + \tfrac{3}{2}L^2)$ $0 \le x \le L/2$ $v = \dfrac{-wL^3}{192EI}(4x - L/2)$ $L/2 \le x \le L$
	$\theta_{max} = \dfrac{-w_0L^3}{24EI}$	$v_{max} = \dfrac{-w_0L^4}{30EI}$	$v = \dfrac{-w_0x^2}{120EIL}(10L^3 - 10L^2x + 5Lx^2 - x^3)$

TABLE 2-1 SPRING CONSTANTS AND DEFLECTION EQUATIONS OF ELASTIC ELEMENTS

A = area of cross section
E = modulus of elasticity
I = area moment of inertia about neutral axis
G = modulus of rigidity
J = polar moment of inertia

Axial (rods, cables, etc.)

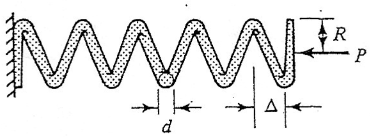


(a)

$$\Delta = \frac{Pl}{AE}$$

$$k = \frac{P}{\Delta} = \frac{AE}{l}$$

Coil spring



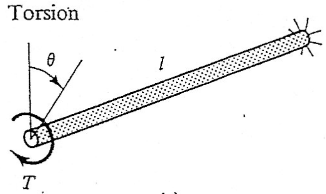
(b)

$$\Delta = \frac{64PnR^3}{Gd^4}$$

$$k = \frac{P}{\Delta} = \frac{Gd^4}{64nR^3}$$

n = number of active coils
R = mean helix radius

Torsion



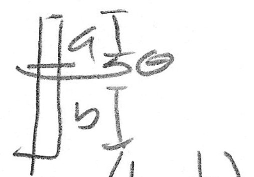
(c)

$$\theta = \frac{Tl}{GJ}$$

$$k = \frac{T}{\theta} = \frac{GJ}{l}$$

$$J = \frac{\pi d^4}{32} \quad (d = \text{dia.})$$

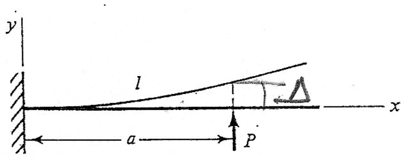
Polar area moment / Polar moment of inertia



$K = GJ\left(\frac{1}{a} + \frac{1}{b}\right)$

$F = K\Delta$

Cantilever beam



(d)

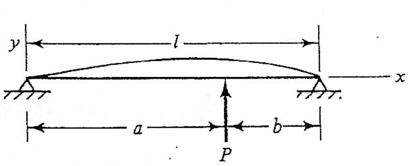
$$k = \frac{P}{y|_{x=a}}$$

$$k|_{a=l} = \frac{3EI}{l^3}$$

$$y = \frac{P}{6EI}(3ax^2 - x^3) \qquad x \le a$$

$$y = \frac{P}{6EI}(3a^2x - a^3) \qquad x \ge a$$

Simply supported beam (pinned-pinned)*



(e)

$$k = \frac{P}{y|_{x=a}}$$

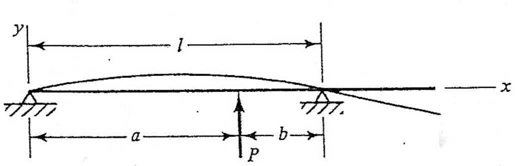
$$k|_{a=l/2} = \frac{48EI}{l^3}$$

$K = \frac{3EIL}{(ba)^2}$

$$y = \frac{Pbx}{6EIl}(l^2 - x^2 - b^2) \qquad x \le a$$

$$y = \frac{Pb}{6EIl}\left[(l^2 - b^2)x - x^3 + \frac{l}{b}(x - a)^3\right] \qquad x \ge a$$

Pinned-pinned beam with overhang*



(f)

$$y = \frac{Pa}{6EIl}(a^2 - l^2)(x - l) \qquad x \ge l$$

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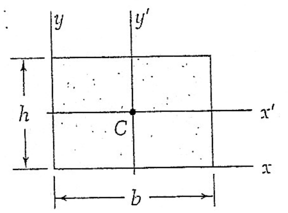
Moments of Inertia of Common Geometric Shapes

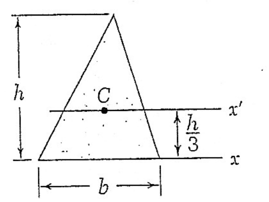
Rectangle

$\bar{I}_{x'} = \frac{1}{12}bh^3$
$\bar{I}_{y'} = \frac{1}{12}b^3h$
$I_x = \frac{1}{3}bh^3$
$I_y = \frac{1}{3}b^3h$
$J_C = \frac{1}{12}bh(b^2 + h^2)$



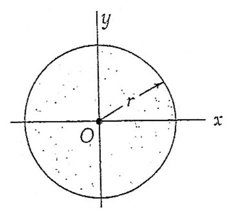
Triangle

$\bar{I}_{x'} = \frac{1}{36}bh^3$
$I_x = \frac{1}{12}bh^3$



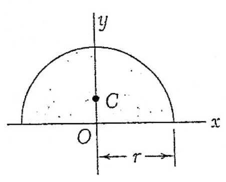
Circle

$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$
$J_O = \frac{1}{2}\pi r^4$



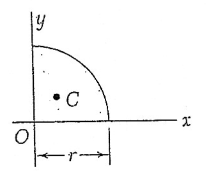
Semicircle

$I_x = I_y = \frac{1}{8}\pi r^4$
$J_O = \frac{1}{4}\pi r^4$



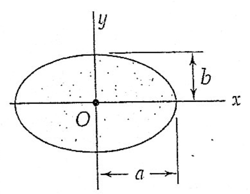
Quarter circle

$I_x = I_y = \frac{1}{16}\pi r^4$
$J_O = \frac{1}{8}\pi r^4$



Ellipse

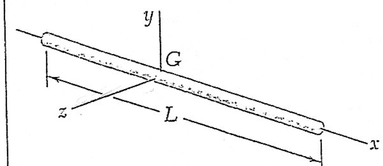
$\bar{I}_x = \frac{1}{4}\pi ab^3$
$\bar{I}_y = \frac{1}{4}\pi a^3b$
$J_O = \frac{1}{4}\pi ab(a^2 + b^2)$



Mass Moments of Inertia of Common Geometric Shapes

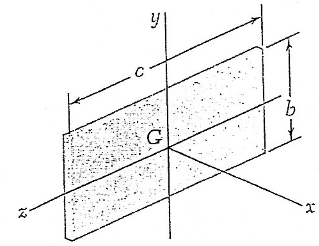
Slender rod

$I_y = I_z = \frac{1}{12}mL^2$



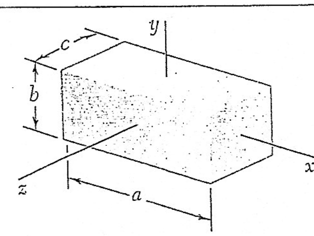
Thin rectangular plate

$I_x = \frac{1}{12}m(b^2 + c^2)$
$I_y = \frac{1}{12}mc^2$
$I_z = \frac{1}{12}mb^2$



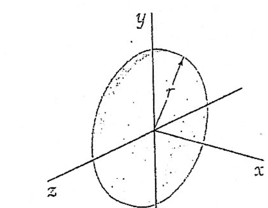
Rectangular prism

$I_x = \frac{1}{12}m(b^2 + c^2)$
$I_y = \frac{1}{12}m(c^2 + a^2)$
$I_z = \frac{1}{12}m(a^2 + b^2)$



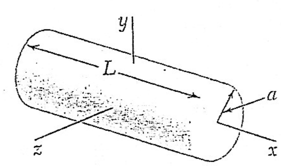
Thin disk

$I_x = \frac{1}{2}mr^2$
$I_y = I_z = \frac{1}{4}mr^2$



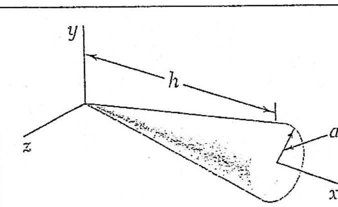
Circular cylinder

$I_x = \frac{1}{2}ma^2$
$I_y = I_z = \frac{1}{12}m(3a^2 + L^2)$



Circular cone

$I_x = \frac{3}{10}ma^2$
$I_y = I_z = \frac{3}{5}m(\frac{1}{4}a^2 + h^2)$



Sphere

$I_x = I_y = I_z = \frac{2}{5}ma^2$