

ME 440

Intermediate Vibrations

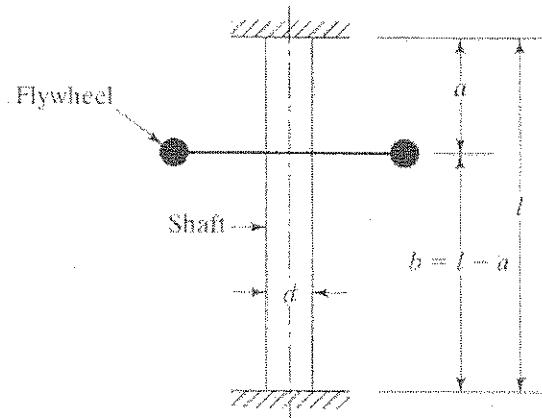
Homework #8 (3 problems)
due Thursday, November 9th, 2017

Problem 1

Download the ANSYS input file “*MODAL_pipe_flywheel.txt*” from HW7 on Canvas, run this input file in ANSYS and go through the file line by line to figure out what the system parameters are for this modal analysis. (Hint: When viewing the mode shapes within ANSYS, try plotting all 3 displacements and all 3 rotations (1 at a time) available under the “Nodal Solu” / “DOF Solution” option; this should be helpful in determining the type of displacement associated with each specific frequency.)

- A) Modify the “*MODAL_pipe_flywheel.txt*” file to use ANSYS to predict the natural frequencies and mode shapes for the problem listed below (NOTE: you should remember this problem from HW3).

A flywheel is mounted on a vertical shaft, as shown below. The shaft has a diameter d and length l and is fixed at both ends. The flywheel has a weight of W and a radius of gyration of r . Find the natural frequency of the longitudinal, the transverse, and the torsional vibration of the system. For the parameters, assume that $d = 1.2$ in, $a = 2$ ft, $b = 4$ ft, $W = 100$ lbs and $r = 16$ in. (Assume the shaft is massless and the flywheel is rigid.)



For this problem, submit a hard copy of your modified .txt file and also create a table comparing the analytical and finite element frequencies (including % error) for the first longitudinal, first transverse and first torsional mode. Which mode has the most error? Which mode SHOULD have the most error? And why?

Problem 2

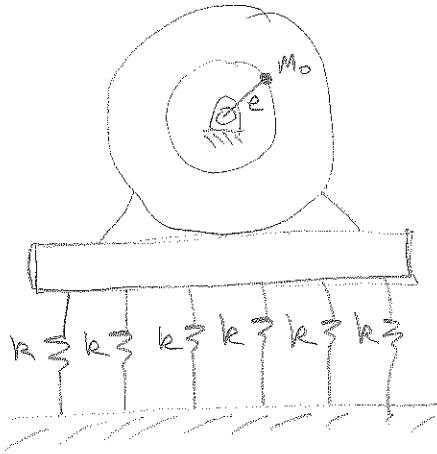
A centrifugal pump, weighing 700 N and operating at 1000 rpm, is mounted on six springs of stiffness 6000 N/m each. Find the maximum permissible unbalance in order to limit the steady-state deflection to 5.0 mm peak-to-peak.

CENTRIFUGAL PUMP $W = mg = 600N$

PUMP SPEED $= \omega = 1000 \text{ RPM}$

MOUNTED ON 6 SPRINGS, $K_{\text{ONE SPRING}} = 6000 \frac{N}{m}$

FIND $(M_o e)_{\text{MAX}}$ TO LIMIT STADY STATE DEFLECTION TO
5.0 mm PEAK $\sim 7.5 \text{ mm PEAK}$



$$M = \frac{700N}{9.81 \frac{m}{s^2}} = 71.36 \text{ kg}$$

$$\omega = 1000 \frac{\text{rad}}{\text{min}} \left(\frac{2\pi \text{ rad}}{60 \text{ sec}} \right) \left(\frac{2\pi \text{ rad}}{1 \text{ rev}} \right) = 104.72 \frac{\text{rad}}{\text{s}}$$

$$k = 6(6000 \frac{N}{m}) = 36,000 \frac{N}{m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{36,000 \frac{N}{m}}{71.36 \text{ kg}}} = 22.46 \frac{\text{rad}}{\text{s}}$$

$$r = \frac{\omega}{\omega_n} = \frac{104.72 \frac{\text{rad}}{\text{s}}}{22.46 \frac{\text{rad}}{\text{s}}} = 4.6622 \quad r^2 = 21.736$$

$$\begin{aligned} \bar{x} &= \frac{M_o e \omega^2}{\sqrt{(k - M_o \omega^2)^2 + (e \omega)^2}} = \frac{M_o e \omega^2}{\sqrt{(k - M_o \omega^2)^2}} = \frac{\frac{1}{m} (M_o e \omega^2)}{\sqrt{\left(\frac{k}{m} - \frac{M_o \omega^2}{m}\right)^2}} \\ &= \frac{\frac{1}{m} (M_o e \omega^2) \left(\frac{1}{\omega_n^2}\right)}{\sqrt{\left(\omega_n^2 - \omega^2\right)^2 \left(\frac{1}{\omega_n^2}\right)}} = \frac{\frac{1}{m \omega_n^2} (M_o e \omega^2)}{\sqrt{\left(\frac{\omega^2}{\omega_n^2} - \frac{\omega^2}{\omega_n^2}\right)^2}} = \frac{\frac{1}{m \omega_n^2} (M_o e \omega^2)}{\sqrt{(1 - r^2)^2}} \end{aligned}$$

$$\text{IF } r > 1, \text{ THEN } \sqrt{(1 - r^2)^2} = r^2 - 1$$

5.0mm PEAK-TO-PEAK EQUIVALENT TO AMPLITUDE OF 2.5mm
= 0.0025m

$$X = 0.0025m = \frac{\frac{1}{2}m\omega_n^2 (M_o e \omega^2)}{r^2 - 1} \quad \text{SOLVE FOR } M_o e$$

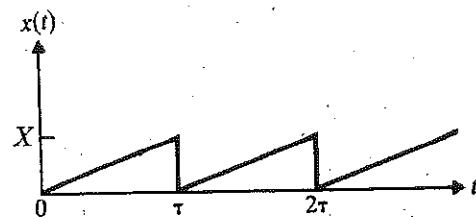
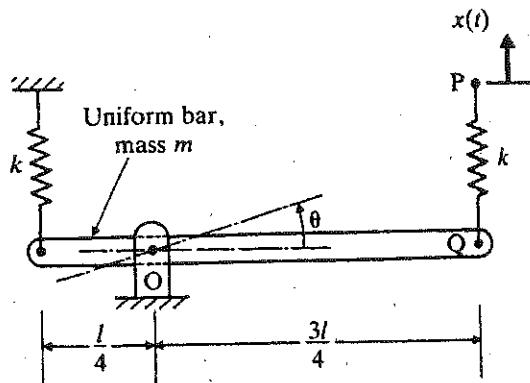
$$M_o e = \frac{X(r^2 - 1) m \omega_n^2}{\omega^2} = \frac{(0.0025m)(4.662^2 - 1)(71.36 \text{ kg})(22.46 \text{ rad/s})^2}{(104.72 \text{ rad/s})^2}$$

$$M_o e = 0.1702 \text{ kg} \cdot \text{m}$$

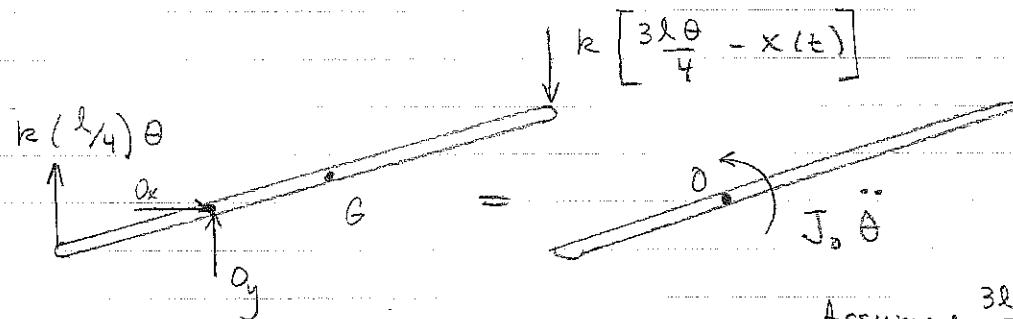
Homework Solution

Determine the steady-state response of the system $\theta(t)$ due to the input excitation shown, using the system parameters given in the figure. (Use a Fourier expansion of the input excitation.)

Assume small oscillations -



Draw F.B.D's and determine E.O.M of the system:



$$k \left[\frac{3l\theta}{4} - x(t) \right]$$

$$\text{Assume: } \frac{3l\theta}{4} > x(t)$$

$$\sum M_o = (\sum M_o)_{\text{eff}}$$

$$-k(\frac{l}{4})^2\theta - k \left[\frac{3l\theta}{4} - x(t) \right] \left(\frac{3l}{4} \right) = J_o \ddot{\theta}$$

$$J_o \ddot{\theta} + \frac{5}{8} k l^2 \theta = \frac{3kl}{4} x(t)$$

$$\text{But, } J_o = \frac{1}{12} m l^2 + m \left(\frac{l}{4} \right)^2 = \frac{7}{48} m l^2$$

$\frac{7}{48} m l^2 \ddot{\theta} + \frac{5}{8} k l^2 \theta = \frac{3kl}{4} x(t)$	E.O.M
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E.O.M

Finally,

$$x(t) = \frac{X}{2} - \frac{X}{\pi} \sum_{n=1,2,3..} b_n \sin \frac{2n\pi t}{\pi}$$

And the EOM becomes:

$$\frac{7}{48} ml^2 \ddot{\theta} + \frac{5}{8} kl^2 \theta = \frac{3kl}{4} \left[\frac{X}{2} - \frac{X}{\pi} \sum_{n=1} b_n \sin \frac{2n\pi t}{\pi} \right]$$

From previous handouts

Given: $m_{eq}\ddot{x} + c_{eq}\dot{x} + k_x x = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)$

Response: $x_p(t) = \frac{a_0}{2k_{eq}} + \sum_n \frac{a_n \cos(n\omega t - \phi_n) + b_n \sin(n\omega t - \phi_n)}{k \sqrt{(1-n^2 r^2)^2 + (25\pi r)^2}}$

where: $\phi_n = \tan^{-1} \frac{25\pi r}{1-n^2 r^2}$ and $r = \frac{\omega}{\omega_n}$

Now, we need to compare the ODE of the system to the "form" previously established -

Comparing terms:

$$a_n = 0 \quad b_n = -\frac{3kl}{4} \left(\frac{X}{n\pi} \right) \quad \frac{a_0}{2k_{eq}} = \frac{3kl}{4} \left(\frac{X}{2} \right) \frac{8}{5kl^2} = \frac{3X}{5l}$$

$$\omega = \frac{2\pi}{T} \quad k_{eq} = \frac{5}{8} kl^2 \quad \omega_n = \sqrt{\frac{\frac{5}{8} kl^2}{\frac{7}{48} ml^2}} = \sqrt{\frac{30k}{7m}} \quad S = 0$$

Express $x(t)$ as a Fourier Series

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2n\pi t}{T} + b_n \sin \frac{2n\pi t}{T} \right)$$

(solve for a_n , b_n and a_0)

$$a_n = \frac{2}{T} \int_0^T x(t) \cos \frac{2n\pi t}{T} dt = \frac{2}{T} \int_0^T (X + t) \cos \frac{2n\pi t}{T} dt$$

$$a_n = \frac{2X}{T^2} \int_0^T t \cos \frac{2n\pi t}{T} dt = \frac{2X}{T^2} \left[\frac{T^2}{(2n\pi)^2} \sin \frac{2n\pi t}{T} \right]_0^T + \frac{2X}{T^2} \left[\frac{t}{2n\pi} \sin \frac{2n\pi t}{T} \right]_0^T$$

$$a_n = \frac{X}{2n^2\pi^2} \left[\cancel{\cos 2n\pi} - 1 \right] + \left(\frac{2X}{T^2} \right) \frac{T^2}{2n\pi} \cancel{\sin 2n\pi}$$

\Rightarrow

$$a_n = 0$$

$$b_n = \frac{2}{T} \int_0^T x(t) \sin \frac{2n\pi t}{T} dt = \frac{2}{T} \int_0^T \frac{X+t}{T} \sin \frac{2n\pi t}{T} dt$$

$$b_n = \frac{2X}{T^2} \int_0^T t \sin \frac{2n\pi t}{T} dt = \frac{2X}{T^2} \left[\frac{T^2}{(2n\pi)^2} \sin \frac{2n\pi t}{T} \right]_0^T - \frac{2X}{T^2} \left[\frac{t}{2n\pi} \cos \frac{2n\pi t}{T} \right]_0^T$$

$$b_n = \frac{X}{2n^2\pi^2} \left[\cancel{\sin 2n\pi} - 0 \right] - \frac{X}{n\pi} \left[\cancel{\cos 2n\pi} \right] = -\frac{X}{n\pi}$$

\Rightarrow

$$b_n = -\frac{X}{n\pi}$$

$$\frac{a_0}{2} = \frac{1}{T} \int_0^T x(t) dt = \frac{1}{T} \int_0^T \frac{X+t}{T} dt = \frac{X}{T^2} \int_0^T t dt = \frac{X}{T^2} \left[\frac{t^2}{2} \right]_0^T = \boxed{\frac{X}{2}}$$

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Consequently, the response is :

$$\theta_p(t) = \frac{3X}{5L} + \sum_{n=1,2,3\dots}^{\infty} \frac{-\frac{3kLX}{4n\pi} \sin(n\omega t - \phi_n)}{\frac{5}{8}kL^2 \sqrt{(1-n^2r^2)^2}}$$

$$\phi_n = \tan^{-1} \frac{2Snr}{1-n^2r^2} = 0 \quad r = \omega/\omega_n$$

and ω , ω_n and S are given above —

or

$$\theta_p(t) = \frac{3X}{5L} + \sum_{n=1,2,3\dots}^{\infty} \frac{-6X \sin n\omega t}{5n\pi L (1-n^2r^2)}$$

$$\text{where } r = \frac{\omega}{\omega_n} \quad \text{and} \quad \omega_n = \sqrt{\frac{30k}{7m}}$$

$$\omega = \frac{2\pi}{T}$$