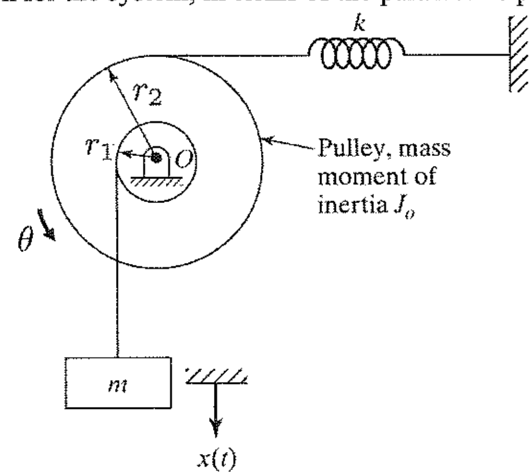
ME 440
Intermediate Vibrations

Homework #4 (2 problems)
due Friday, October 13, 2017

Problem 1

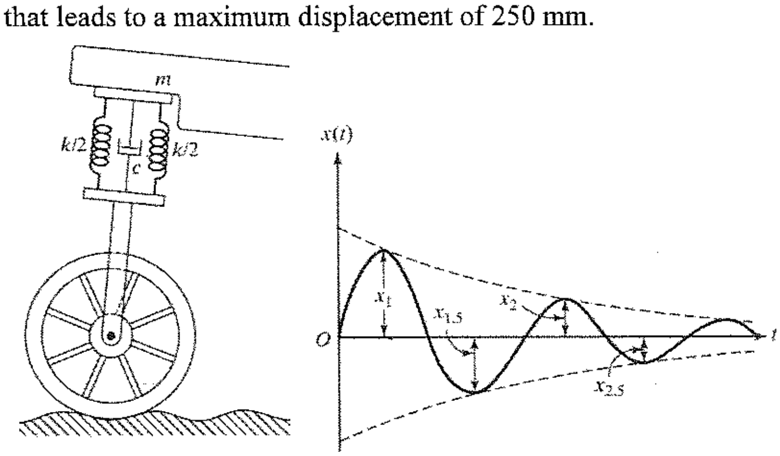
The pulley is in fixed axis rotation about Point O. Using energy concepts and θ as the generalized coordinate, determine

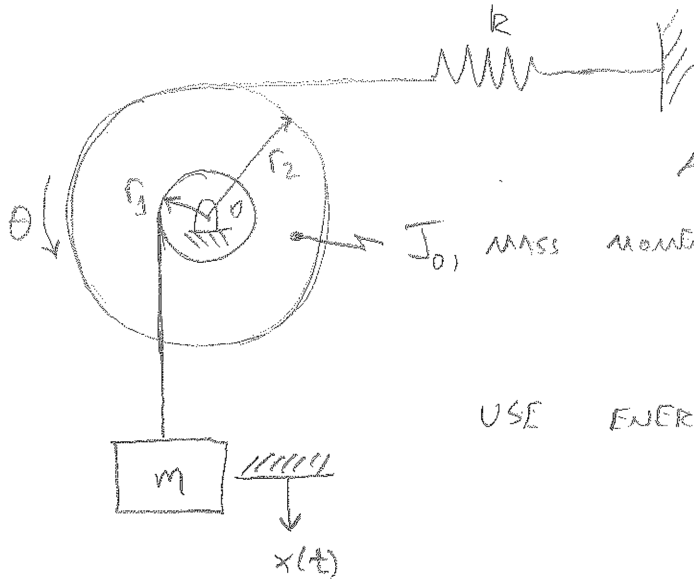
a) the natural frequency of the system shown below, and
b) the equation of motion for the system, in terms of the parameters provided.



Problem 2

An underdamped shock absorber is to be designed for motorcycle of mass 200 kg. When the shock absorber is subjected to an initial vertical velocity due to a road bump, the resulting displacement-time curve is to be as illustrated below. Determine the necessary stiffness and damping constants of the shock absorber if the damped period of vibration is to be 2 seconds and the amplitude x_1 is to be reduced to ¼ in one half cycle (i.e., $x_{1.5} = x_1/4$). Also find the minimum initial velocity that leads to a maximum displacement of 250 mm.





ASSUME SMALL θ

USE ENERGY $(T_{MAX} = U_{MAX})$

$x(t) = r_1[\theta(t)]$ $\quad \dot{x} = r_1\dot{\theta} = v$

$$T_{MAX} = \left[\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right]_{MAX}$$

$$= \frac{1}{2}m(r_1\dot{\theta}_{MAX})^2 + \frac{1}{2}J_0(\dot{\theta}_{MAX})^2 \qquad \underline{\text{EQN 1}}$$

$$U_{MAX} = \left[\frac{1}{2}k\Delta^2\right]_{MAX} \qquad \Delta = \theta r_2$$

$$= \frac{1}{2}k(\theta_{MAX}r_2)^2 \qquad \underline{\text{EQN 2}}$$

ASSUME

$\theta = \theta_{MAX}\sin\omega_n t$ THEN

$\dot{\theta} = \theta_{MAX}\omega_n\cos\omega_n t$ AND $\dot{\theta}_{MAX} = \theta_{MAX}\omega_n$ $\qquad \underline{\text{EQN 3}}$

EQUATE

$$\frac{1}{2}mr_1^2\dot{\theta}_{MAX}^2 + \frac{1}{2}J_0\dot{\theta}_{MAX}^2 = \frac{1}{2}kr_2^2\theta_{MAX}^2 \qquad \text{PLUG IN 3}$$

$$\cancel{\frac{1}{2}}mr_1^2(\cancel{\theta_{MAX}}\omega_n)^2 + \cancel{\frac{1}{2}}J_0(\cancel{\theta_{MAX}}\omega_n)^2 = \cancel{\frac{1}{2}}kr_2^2\cancel{\theta_{MAX}^2} \qquad \text{SOLVE FOR } \omega_n$$

$$\omega_n = \sqrt{\frac{kr_2^2}{J_0 + mr_1^2}}$$

PART b

$$\frac{d}{dt}(T+U) = 0$$

$$T + U = \frac{1}{2} m r_1^2 \dot{\theta}^2 + \frac{1}{2} J_0 \dot{\theta}^2 + \frac{1}{2} k r_2^2 \theta^2$$

$$\frac{d}{dt}(T+U) = \cancel{2}\left(\cancel{\tfrac{1}{2}}\right) m r_1^2 \cancel{\dot{\theta}} \underbrace{\boxed{\frac{d\dot{\theta}}{dt}}}_{\ddot{\theta}} + \cancel{2}\left(\cancel{\tfrac{1}{2}}\right) J_0 \cancel{\dot{\theta}} \underbrace{\boxed{\frac{d\dot{\theta}}{dt}}}_{\ddot{\theta}} + \cancel{2}\left(\cancel{\tfrac{1}{2}}\right) k r_2^2 \theta \underbrace{\boxed{\frac{d\theta}{dt}}}_{\dot{\theta}} = 0$$

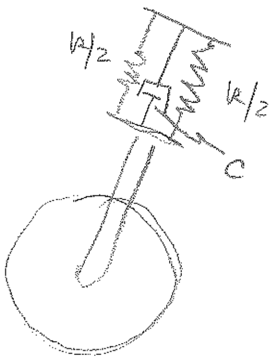
[2, 1/2, $\dot{\theta}$ ALL CANCEL]

$$m r_1^2 \ddot{\theta} + J_0 \ddot{\theta} + k r_2^2 \theta = 0$$

E.O.M.

$$\ddot{\theta}\left[m r_1^2 + J_0\right] + \theta\left[k r_2^2\right] = 0$$

UNDERDAMPED $m = 200\,kg$



– WANT $\tau_d = 2s$ AND AMPLITUDE TO BE REDUCED TO 1/4 IN ONE HALF CYCLE $\left(x_{1.5} = \frac{x_1}{4}\right)$

– FIND k AND c. AND $v_0 = \dot{x}_0$ SO MAX DISPLACEMENT IS 250 mm

WE WANT $x_{1.5} = \frac{x_1}{4}$ AND $\tau_d = 2\,sec$

IF $x_{1.5} = \frac{x_1}{4}$ THEN $x_2 = x_{1.5}/4 = x_1/16$

CONSEQUENTLY

$$\ln\left(\frac{x_1}{x_2}\right) = \zeta \omega_n \tau_d = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

$$\ln(16) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = 2.7726$$

$$\zeta^2 \pi^2 = 1.922(1-\zeta^2)$$

$$\zeta^2 = 0.16298$$

$$\zeta = 0.4037 \quad OR \quad \zeta = \sqrt{\frac{\{\ln(X_1/X_2)\}^2}{\{4\pi^2 + \ln(X_1/X_2)\}^2}}$$

– NOW FIND ω_n

$$\omega_n = \frac{2\pi}{\tau_d\sqrt{1-\zeta^2}} = \frac{2\pi}{2\sqrt{1-0.4037^2}} = 3.434 \frac{RAD}{SEC}$$

$$\omega_n = \sqrt{\frac{k}{m}} \Rightarrow k = \omega_n^2 m = \left(3.434 \frac{RAD}{SEC}\right)^2 200\,kg = 2358 \frac{kg}{SEC^2}$$

$$\boxed{k = 2358 \frac{N}{m}}$$

$C = 2\xi\omega_n m = 2(0.4037)(3.434\ \frac{RAD}{SEC})(200\ Kg)$ OR $C = 2\xi\sqrt{km}$

$C = 554.5\ \frac{Kg}{SEC} \Rightarrow \boxed{C = 554.5\ \frac{N \cdot s}{m}}$

UNDERDAMPED FREE VIBRATION RESPONSE (SLIDE 126)

$$x(t) = e^{-\xi\omega_n t}\left[B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t)\right]$$

$$B_1 = x_0 \qquad B_2 = \frac{\dot{x}_0 + \xi\omega_n x_0}{\omega_d}$$

$$\begin{aligned} x_0 &= x(0) = 0 \\ \dot{x}_0 &= \dot{x}(0) = v_0 \end{aligned} \Rightarrow \begin{aligned} B_1 &= 0 \\ B_2 &= \frac{\dot{x}_0}{\omega_d} = \frac{v_0}{\omega_d} \end{aligned}$$

$$x(t) = \frac{\dot{x}_0}{\omega_d} e^{-\xi\omega_n t} \sin \omega_d t$$

TO FIND WHEN $x(t)$ IS A MAX, TAKE DERIVATIVE, THEN = 0.

$$\frac{dx(t)}{dt} = \frac{\dot{x}_0}{\omega_d} e^{-\xi\omega_n t}\left[-\xi\omega_n \sin\omega_d t + \omega_d \cos\omega_d t\right] = 0$$

OR $-\xi\omega_n \sin\omega_d t + \omega_d \cos\omega_d t = 0$

$$\tan\omega_d t = \frac{\omega_d}{\xi\omega_n} = \frac{\cancel{\omega_n}\sqrt{1-\xi^2}}{\xi\cancel{\omega_n}} \quad *$$

$$\tan\omega_d t = \frac{\sqrt{1-\xi^2}}{\xi} \Rightarrow \tan 3.434\sqrt{1-0.4037^2}^{\,*}\, t = \frac{\sqrt{1-0.4037^2}}{0.4037}$$

$\left[\text{NOTE } \omega_d = \omega_n\sqrt{1-\xi^2}\right]$

$t = 0.3677$ SEC, PLUG BACK INTO $x(t)$

$$0.25\,m = \frac{\dot{x}_0\, e^{-0.4037(3.434)(0.3677)}}{(3.434)\sqrt{1-0.4037^2}} \sin 3.434\sqrt{1-0.4037^2}\ 0.3677$$

$\dot{x}_0 = 1.429\ m/s = 1429\ mm/sec$