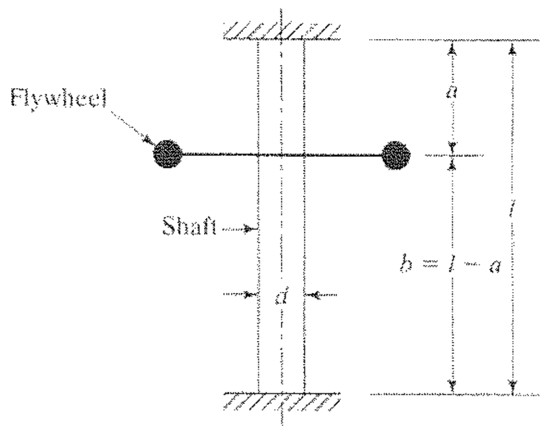
SOLUTIONS
ORIGINAL

ME 440
Intermediate Vibrations

Homework #3
due Thursday, October 5, 2017

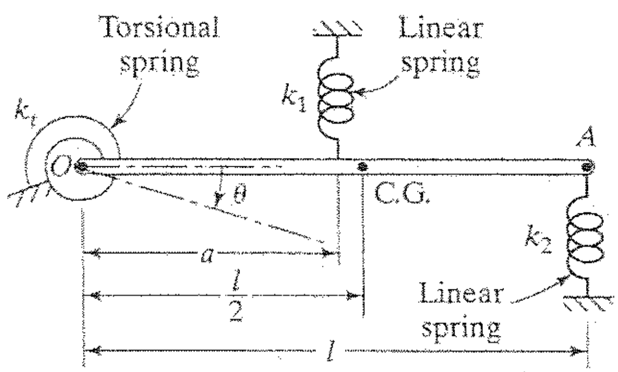
Problem 1

A flywheel is mounted on a vertical shaft, as shown below. The shaft has a diameter d and length l and is fixed at both ends. The flywheel has a weight of W and a radius of gyration of r. Find the natural frequency of the longitudinal, the transverse, and the torsional vibration of the system.



Problem 2

The uniform rigid bar OA of length L and mass m is pinned about point O. Using Newton's Second Law, find the equation of motion for the system using the generalized coordinate θ and also find the system's natural frequency.



TRANSVERSE (NO ROTATION) FIXED - FIXED BEAM WITH LATERAL DISPLACEMENT

(OPTION 1)

$$k = \frac{12EI}{\ell^3} \qquad k_a = \frac{12EI}{a^3} \qquad k_b = \frac{12EI}{b^3}$$

BOTH BEAMS HAVE THE SAME DEFLECTION/DEFORMATION

∴ PARALLEL

$$k_{eq} = k_a + k_b$$

$$= \frac{12EI}{a^3} + \frac{12EI}{b^3} = 12EI\left(\frac{1}{a^3} + \frac{1}{b^3}\right) = 12EI\left(\frac{a^3+b^3}{a^3 b^3}\right)$$

$$=$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{12EI\,(a^3+b^3)\,g}{W\,a^3 b^3}}$$

[ASSUMES NO ROTATION OF FLYWHEEL OCCURS]

FIXED - FIXED BEAM WITH NON CENTER TRANSVERSE POINT LOAD.

Solution -

Transverse (OPTION 2)

Deflection $y = \frac{Pb^2}{6EI\ell^3}\left[(2b-3\ell)a^3 + 3\ell(\ell-b)a^2\right]$

$$y = \frac{Pa^2b^2}{6EI\ell^3}\left[2ab - 3a\ell + 3\ell^2 - 3\ell b\right]$$

$$y = \frac{Pa^3b^3}{3EI(a+b)^3}$$

$$k_{eq} = \frac{P}{y} = \frac{3EI(a+b)^3}{a^3b^3} \qquad m_{eq} = \frac{W}{g}$$

$$\omega_n^* = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{3EI(a+b)^3 g}{a^3b^3 W}} \qquad \text{where} \quad I = \frac{\pi d^4}{64}$$

* THIS APPROACH ALLOWS ROTATION IN k_{eq} CALCULATION BUT NOT IN m_{eq}

Torsional:

$$k_{eq} = (k_1)_t + (k_2)_t$$

$$k_{eq} = \frac{GJ}{a} + \frac{GJ}{b} = \frac{GJ(a+b)}{ab}$$

$$\text{but} \quad J = \frac{\pi d^4}{32}$$

$$k_{eq} = \frac{G\pi d^4}{32}\frac{(a+b)}{ab}$$

$$m_{eq} = J_o = mk^2 = mr^2 \qquad r = \text{radius of gyration}$$

$$m_{eq} = \frac{W}{g}r^2$$

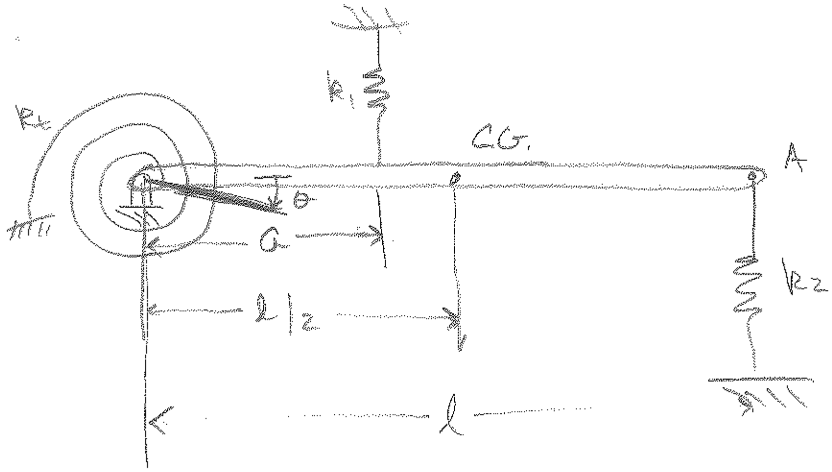
$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{G\pi d^4(a+b)g}{32\,ab\,Wr^2}}$$

Longitudinal:

$$k_{eq} = k_1 + k_2 = \frac{AE}{a} + \frac{AE}{b} = \frac{AE(a+b)}{ab}$$

$$m_{eq} = W/g$$

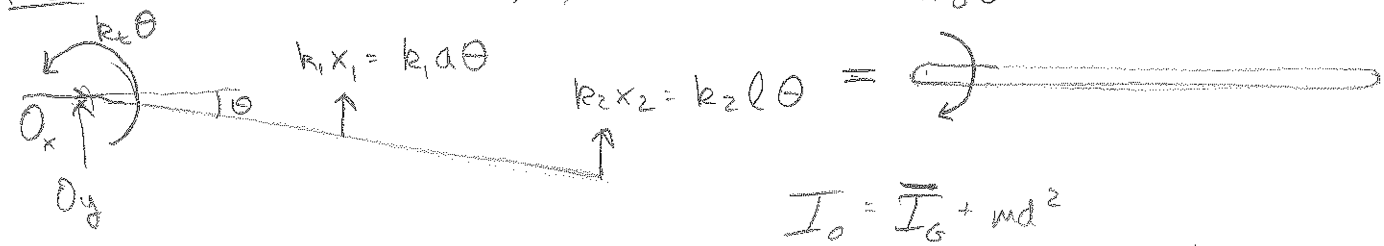
$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{AE(a+b)g}{abW}} \qquad \text{where } A = \frac{\pi d^2}{4}$$



MASS OA = m

USE NEWTON'S 2ND LAW TO FIND E.O.M. AND ω_n

FBD (ASSUME SMALL + $\theta, \dot{\theta}, \ddot{\theta}$)



$$I_0 = \bar{I}_G + md^2$$
$$= \frac{1}{12}ml^2 + m\left(\frac{l}{2}\right)^2 = \frac{1}{3}ml^2$$

$$+\circlearrowleft \sum M_0 = \sum M_{eff}$$

$$k_t\theta + (k_1a\theta)a + (k_2l\theta)l = -I_0\ddot{\theta}$$

E.O.M.

$$\boxed{\frac{1}{3}ml^2\ddot{\theta} + (k_2 + k_1a^2 + k_2l^2)\ddot{\theta} = 0}$$

$$\omega_n = \sqrt{\frac{3(k_t + k_1a^2 + k_2l^2)}{ml^2}}$$