HW 1, ME 440 Intermediate Vibration, Fall 2017

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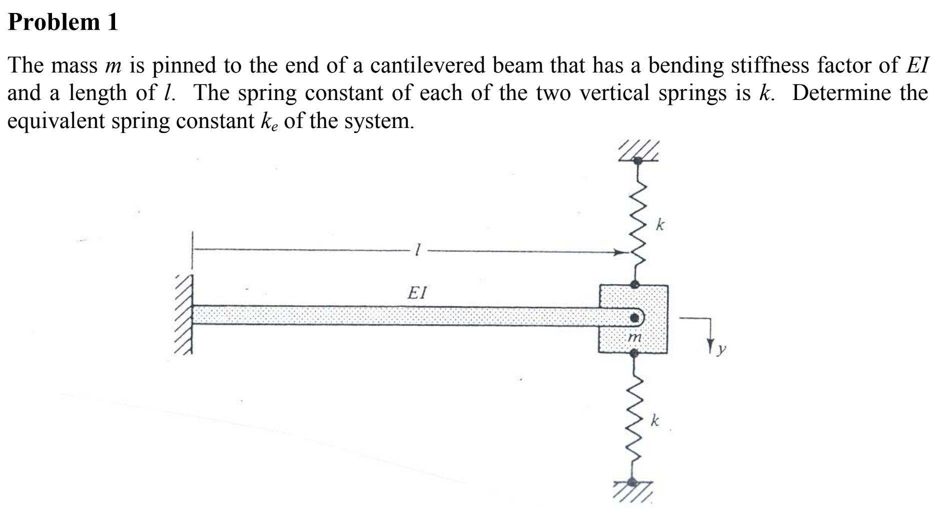
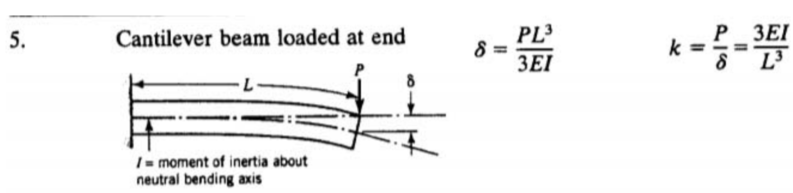
0.1 Problem 1



Problem 1

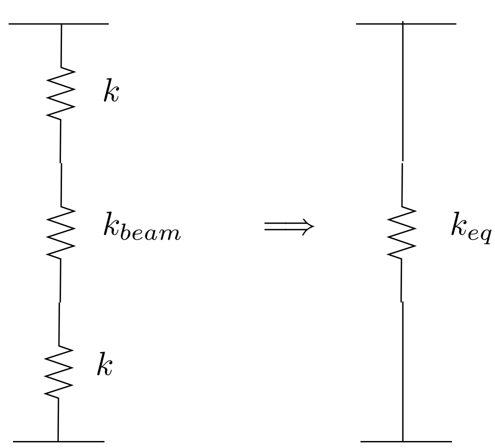
The mass m is pinned to the end of a cantilevered beam that has a bending stiffness factor of EI and a length of l. The spring constant of each of the two vertical springs is k. Determine the equivalent spring constant k_e of the system.

From tables we find that for cantilever beam loaded at end, the vertical deflection is $\delta = \frac{mL^3}{3EI}$, hence by definition $k_b = \frac{m}{\delta} = \frac{3EI}{L^3}$.



5. Cantilever beam loaded at end $\delta = \frac{PL^3}{3EI}$ $k = \frac{P}{\delta} = \frac{3EI}{L^3}$

Therefore, we can model the stiffness of the system as



springs in series

Therefore

$$
\begin{aligned}
k_{eq} &= k + k_{beam} + k \\
&= 2k + k_{beam}
\end{aligned}
$$

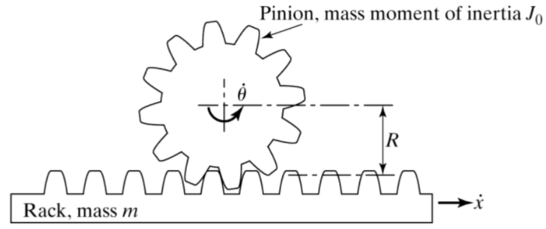
Since $k_b = \frac{3EI}{L^3}$ then the above becomes

$$k_{eq} = 2k + \frac{3EI}{L^3}$$

0.2 Problem 2

Problem 2

The pinion of the rack and pinion system shown below is free to rotate about its mass center but it can not translate in any direction. For this 1 degree-of-freedom system, find it's equivalent mass a) if the generalized coordinate that captures this degree of freedom is the angle θ, b) if the generalized coordinate that captures this degree of freedom is the horizontal displacement x of the rack.



0.2.1 Part (a)

Using energy method

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_0\dot{\theta}^2 = \frac{1}{2}J_{eq}\dot{\theta}_{eq}^2$$

But $\dot{\theta}_{eq} = \dot{\theta}$ for this part. And since $x = R\theta$ or $\dot{x} = R\dot{\theta}$, then the above becomes

$$\frac{1}{2}m\left(R\dot{\theta}\right)^2 + \frac{1}{2}J_0\dot{\theta}^2 = \frac{1}{2}J_{eq}\dot{\theta}^2$$

Simplifying gives

$$J_{eq} = mR^2 + J_0$$

0.2.2 Part (b)

Using energy method

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_0\dot{\theta}^2 = \frac{1}{2}m_{eq}\dot{x}_{eq}^2$$

But $\dot{x}_{eq} = \dot{x}$ for this part. And since $x = R\theta$ or $\dot{x} = R\dot{\theta}$, then $\dot{\theta} = \frac{\dot{x}}{R}$ and the above becomes

$$\frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_0\left(\frac{\dot{x}}{R}\right)^2 = \frac{1}{2}m_{eq}\dot{x}^2$$

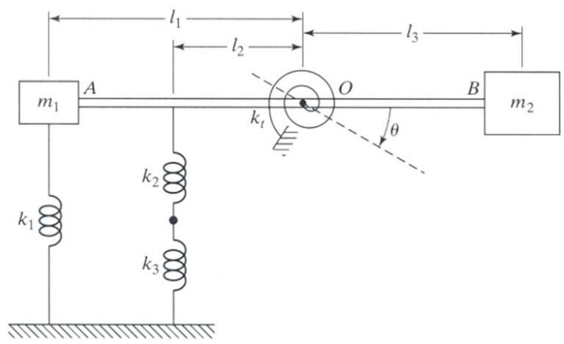
Simplifying gives

$$m_{eq} = m + \frac{J_0}{R^2}$$

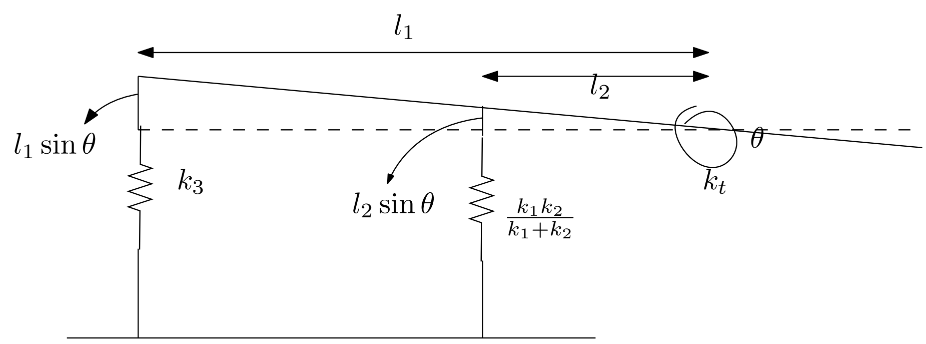
0.3 Problem 3

Problem 3

Find the equivalent spring constant and equivalent mass of the system shown below with regards to the θ degree of freedom shown in the figure. Assume that the bar *AOB* is rigid with negligible mass.



Assuming a small deflection as shown



0.3.1 Mass equivalent

The kinetic energy of the system is (assuming small angles)

$$\frac{1}{2}m_1\left(L_1\dot{\theta}\right)^2 + \frac{1}{2}m_2\left(L_3\dot{\theta}\right)^2 = \frac{1}{2}I_{eq}\dot{\theta}^2$$

Hence

$$m_1L_1^2 + m_2L_3^2 = I_{eq}$$

Where I_{eq} is the equivalent mass moment of inertia. The problem does not say where the equivalent mass should be located relative to the pivot point (where the torsional spring is located) so we can stop here. But assuming that distance was some $\bar{x}$, then we can write $I_{eq} = M_{eq}\bar{x}^2$ where equivalent mass is used as a point mass, and simplify the above more

$$\begin{aligned} m_1L_1^2 + m_2L_3^2 &= M_{eq}\bar{x}^2 \\ M_{eq} &= \frac{m_1L_1^2 + m_2L_3^2}{\bar{x}^2} \end{aligned}$$

0.3.2 Stiffness equivalent

Using potential energy method, where energy stored by a spring due to extension or compression is $\frac{1}{2}k\Delta^2$, then we see that the total energy using the above deformation is given by

$$\frac{1}{2}k_1\left(l_1\sin\theta\right)^2+\frac{1}{2}\left(\frac{k_3k_2}{k_3+k_2}\right)\left(l_2\sin\theta\right)^2+\frac{1}{2}k_t\theta^2=\frac{1}{2}k_{t,eq}\theta_{eq}^2$$

Where $\frac{k_3k_2}{k_3+k_2}$ is the equivalent stiffness of the springs k_2, k_3 since they are in series. The above assumes small angle θ, therefore we can simplify the above using $\sin\theta\approx\theta$, and obtain

$$\frac{1}{2}k_1\left(l_1\theta\right)^2+\frac{1}{2}\left(\frac{k_3k_2}{k_3+k_2}\right)\left(l_2\theta\right)^2+\frac{1}{2}k_t\theta^2=\frac{1}{2}k_{t,eq}\theta_{eq}^2$$

But here $\theta=\theta_{eq}$, therefore solving for $k_{t,eq}$ gives

$$k_{t,eq}=k_1l_1^2+\left(\frac{k_3k_2}{k_3+k_2}\right)l_2^2+k_t$$