Solving example, lecture Nov 30, 2017,ME 240 Dynamics, Fall 2017

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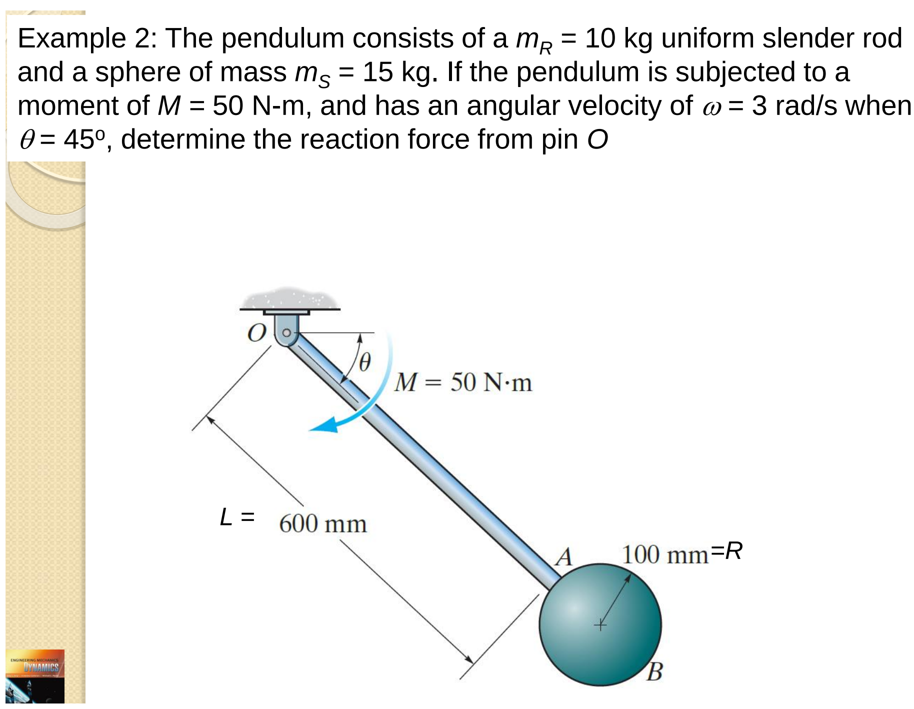
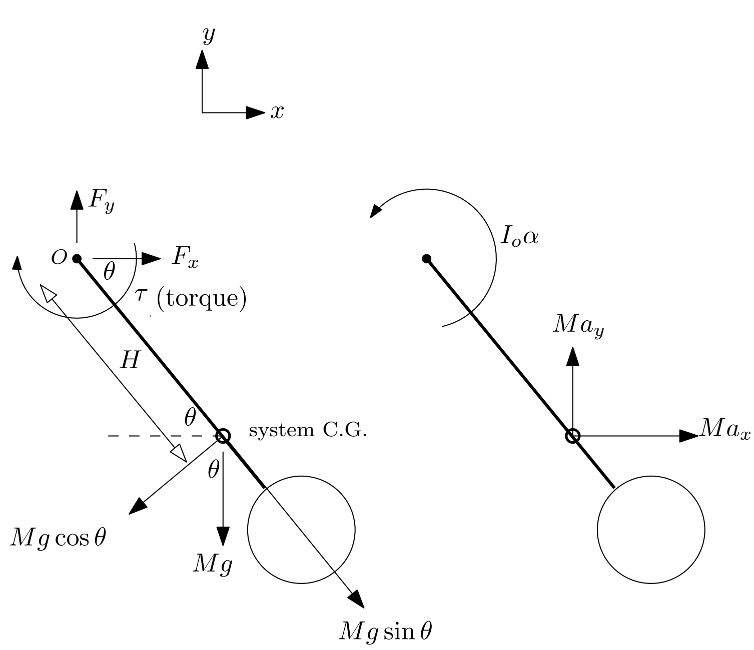
0.1 Problem 1



The FBD and inertia diagram is



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Where $M = m_{disk} + m_{bar}$ and H is location of system center of mass. Total mass is $M = 15 + 10 = 25$ kg.

$$
\begin{aligned}
H &= \frac{m_{sphere}(L+R) + m_{rod}\left(\frac{L}{2}\right)}{m_{sphere} + m_{rod}} \\
&= \frac{15(0.6+0.1) + 10(0.3)}{15+10} \\
&= 0.54 \text{ m}
\end{aligned}
$$

And

$$
\begin{aligned}
I_o &= I_{sphere_0} + I_{bar_0} \\
&= \left(\frac{2}{5} m_{sphere} R^2 + m_{sphere}(L+R)^2\right) + \frac{1}{3} m_{bar} L^2 \\
&= \left(\frac{2(15)(0.1)^2}{5} + 15(0.6+0.1)^2\right) + \frac{1}{3}(10)(0.6)^2 \\
&= 8.61 \text{ kg-m}^2
\end{aligned}
$$

From FBD we obtain 3 equations.

$$
\begin{aligned}
F_x &= M a_x \\
F_y - Mg &= M a_y \\
-\tau - \left(Mg\cos\theta\right) H &= I_o \alpha
\end{aligned}
$$

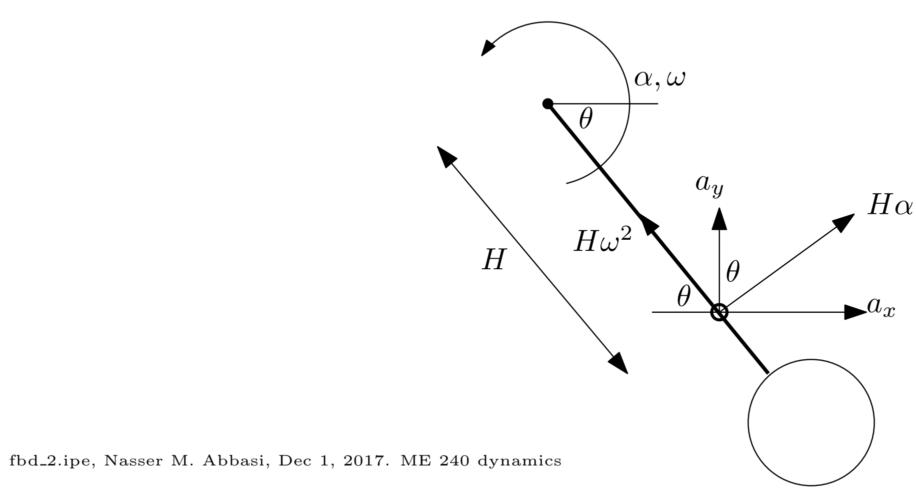
Or

$$
\begin{aligned}
F_x &= 25 a_x \\
F_y - (25)(9.81) &= 25 a_y \\
-50 - \left((25)(9.81)\cos\left(45\left(\frac{\pi}{180}\right)\right)\right)(0.54) &= (8.61)\alpha
\end{aligned}
$$

Or

$$
\begin{aligned}
F_x &= 25 a_x &\quad (1) \\
F_y - 245.25 &= 25 a_y &\quad (2) \\
-16.684 &= \alpha &\quad (3)
\end{aligned}
$$

3 equations with 4 unknowns: F_x, F_y, a_x, a_y. But looking at this diagram, which relates a_x, a_y to α.



We see that

$$\begin{aligned} a_y &= H\omega^2 \sin\theta + H\alpha\cos\theta \\ &= (0.54)(3)^2 \sin\left(45\left(\frac{\pi}{180}\right)\right) + (0.54)(-16.684)\cos\left(45\left(\frac{\pi}{180}\right)\right) \\ &= -2.934 \text{ m/s}^2 \end{aligned}$$

And

$$\begin{aligned} a_x &= H\alpha\sin\theta - H\omega^2\cos\theta \\ &= (0.54)(-16.684)\sin\left(45\left(\frac{\pi}{180}\right)\right) - (0.54)\left(3^2\right)\cos\left(45\left(\frac{\pi}{180}\right)\right) \\ &= -9.807 \text{ m/s}^2 \end{aligned}$$

Using these in (1,2), we find the reaction forces

$$\begin{aligned} F_x &= 25a_x \\ &= 25(-9.807) \\ &= -245.175 \text{ N} \end{aligned}$$

And

$$\begin{aligned} F_y - 245.25 &= 25a_y \\ F_y - 245.25 &= 25(-2.934) \\ F_y &= 171.9 \text{ N} \end{aligned}$$

Total reaction force is $\sqrt{F_y^2 + F_x^2} = \sqrt{(171.9)^2 + (-245.175)^2} = 299.4334$ N