

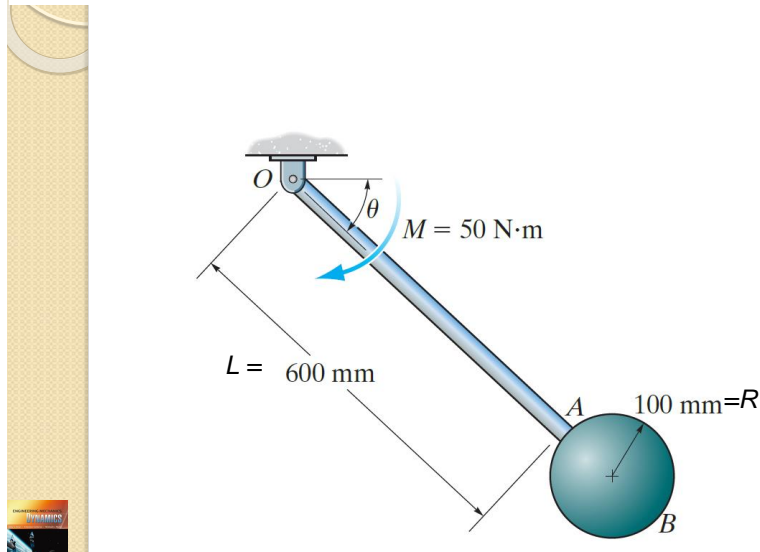
Solving example, lecture Nov 30, 2017, ME 240 Dynamics, Fall 2017

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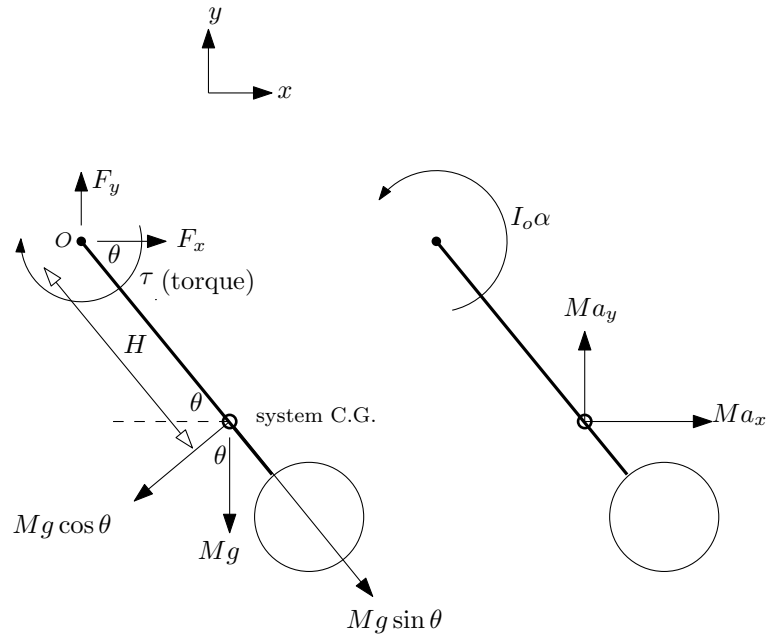
December 30, 2019

0.1 Problem 1

Example 2: The pendulum consists of a $m_R = 10$ kg uniform slender rod and a sphere of mass $m_S = 15$ kg. If the pendulum is subjected to a moment of $M = 50$ N·m, and has an angular velocity of $\omega = 3$ rad/s when $\theta = 45^\circ$, determine the reaction force from pin O



The FBD and inertia diagram is



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Where $M = m_{disk} + m_{bar}$ and H is location of system center of mass. Total mass is $M = 15 + 10 = 25$ kg.

$$\begin{aligned}
 H &= \frac{m_{sphere} (L + R) + m_{rod} \left(\frac{L}{2}\right)}{m_{sphere} + m_{rod}} \\
 &= \frac{15 (0.6 + 0.1) + 10 (0.3)}{15 + 10} \\
 &= 0.54 \text{ m}
 \end{aligned}$$

And

$$\begin{aligned}
 I_o &= I_{sphere0} + I_{bar0} \\
 &= \left(\frac{2}{5} m_{sphere} R^2 + m_{sphere} (L + R)^2\right) + \frac{1}{3} m_{bar} L^2 \\
 &= \left(\frac{2 (15) (0.1)^2}{5} + 15 (0.6 + 0.1)^2\right) + \frac{1}{3} (10) (0.6)^2 \\
 &= 8.61 \text{ kg}\cdot\text{m}^2
 \end{aligned}$$

From FBD we obtain 3 equations.

$$\begin{aligned} F_x &= Ma_x \\ F_y - Mg &= Ma_y \\ -\tau - (Mg \cos \theta) H &= I_o \alpha \end{aligned}$$

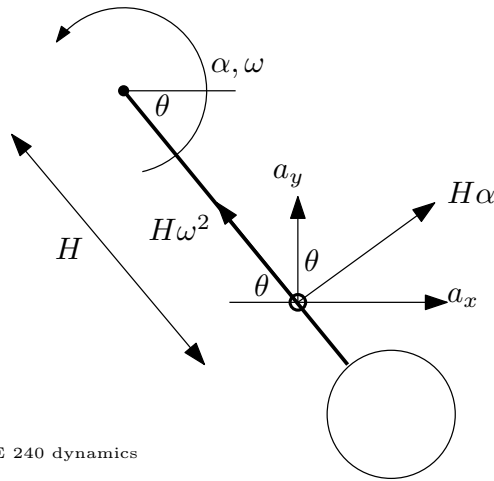
Or

$$\begin{aligned} F_x &= 25a_x \\ F_y - (25)(9.81) &= 25a_y \\ -50 - \left((25)(9.81) \cos \left(45 \left(\frac{\pi}{180} \right) \right) \right) (0.54) &= (8.61) \alpha \end{aligned}$$

Or

$$\begin{aligned} F_x &= 25a_x & (1) \\ F_y - 245.25 &= 25a_y & (2) \\ -16.684 &= \alpha & (3) \end{aligned}$$

3 equations with 4 unknowns: F_x, F_y, a_x, a_y . But looking at this diagram, which relates a_x, a_y to α .



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We see that

$$\begin{aligned} a_y &= H\omega^2 \sin \theta + H\alpha \cos \theta \\ &= (0.54)(3)^2 \sin \left(45 \left(\frac{\pi}{180} \right) \right) + (0.54)(-16.684) \cos \left(45 \left(\frac{\pi}{180} \right) \right) \\ &= -2.934 \text{ m/s}^2 \end{aligned}$$

And

$$\begin{aligned}
 a_x &= H\alpha \sin \theta - H\omega^2 \cos \theta \\
 &= (0.54)(-16.684) \sin\left(45\left(\frac{\pi}{180}\right)\right) - (0.54)(3^2) \cos\left(45\left(\frac{\pi}{180}\right)\right) \\
 &= -9.807 \text{ m/s}^2
 \end{aligned}$$

Using these in (1,2), we find the reaction forces

$$\begin{aligned}
 F_x &= 25a_x \\
 &= 25(-9.807) \\
 &= -245.175 \text{ N}
 \end{aligned}$$

And

$$\begin{aligned}
 F_y - 245.25 &= 25a_y \\
 F_y - 245.25 &= 25(-2.934) \\
 F_y &= 171.9 \text{ N}
 \end{aligned}$$

Total reaction force is $\sqrt{F_y^2 + F_x^2} = \sqrt{(171.9)^2 + (-245.175)^2} = 299.4334 \text{ N}$