# quizz 6, ME 240 Dynamics, Fall 2017

Nasser M. Abbasi

December 30, 2019

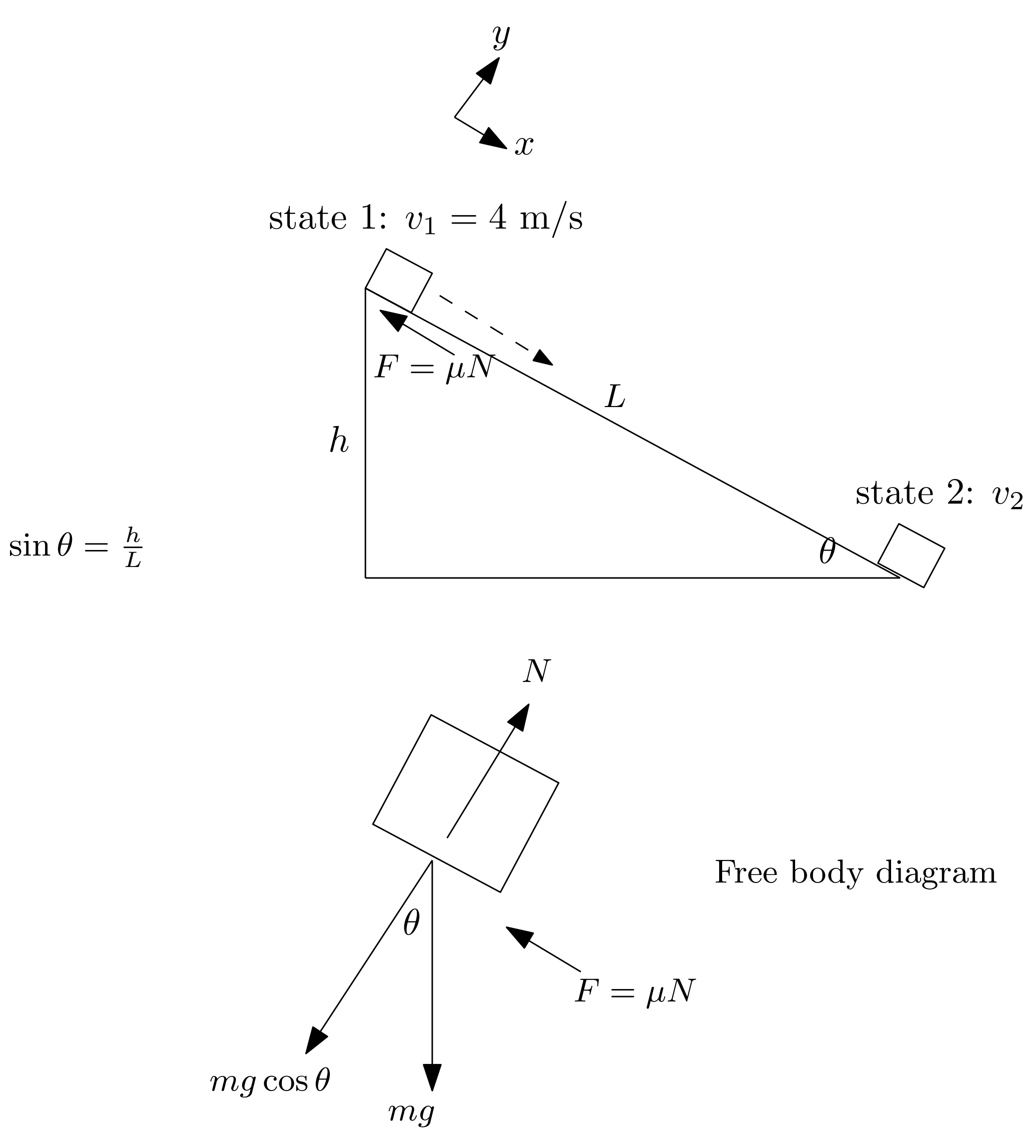
## 0.1 Problem 1

**Question 1** 4 pts

A 30 kg child runs 4.0 m/s and hops onto a sled at the top of a 15.0 meter tall hill. The hill has a 30 degree incline with respect to the horizontal axis, and the kinetic friction coefficient between the sled and the snow is 0.05.

Determine the velocity (m/s) of the child at the bottom of the hill. Neglect the mass of the sled.

This diagrams shows the setup.



Work-energy is used. There are two states. First state is at the top and the second state is when child reaches the bottom of the hill. Zero datum for gravity potential energy is taken at the bottom of the hill.

We now apply work-energy

$$T_1 + V_1 + U_{12}^{internal} + U_{12}^{external} = T_2 + V_2 \tag{1}$$

Where $U_{12}^{internal}$ is work due to internal non-conservative forces. In this case, this is the friction only. And $U_{12}^{external}$ is work due to external applied forces, which is zero in this case, as there are no external applied forces. Hence

$$U_{12}^{internal} = -\int_0^L F dx$$

Where $dx$ is taken as shown in the diagram. But $F$ which is friction force is $F = \mu N = \mu m g \cos\theta$. Therefore

$$\begin{aligned} U_{12}^{internal} &= -\int_0^L \mu m g \cos\theta dx \\ &= -\mu m g \cos\theta L \end{aligned}$$

But $L = \frac{h}{\sin\theta}$ therefore

$$U_{12}^{internal} = -\mu h m g \frac{\cos\theta}{\sin\theta}$$

Now, $T_1 = \frac{1}{2} m v_1^2$ and $V_1 = mgh$ and $V_2 = 0$ since we assume datum at bottom and $T_2 = \frac{1}{2} m v_2^2$ where $v_2$ is what we want to solve for. Putting all this in (1) gives

$$\begin{aligned} \frac{1}{2} m v_1^2 + mgh - \mu h m g \frac{\cos\theta}{\sin\theta} &= \frac{1}{2} m v_2^2 \\ v_2^2 &= \frac{2}{m}\left(\frac{1}{2} m v_1^2 + mgh - \mu h m g \frac{\cos\theta}{\sin\theta}\right) \\ v_2 &= \sqrt{v_1^2 + 2gh - 2\mu h g \frac{\cos\theta}{\sin\theta}} \end{aligned} \tag{2}$$

We notice something important here. The velocity at the bottom do not depend on mass $m$. This is the answer for problem 2. We now just plug-in the numerical values given to find $v_2$

$$\begin{aligned} v_2 &= \sqrt{4^2 + 2(9.81)(15) - 2(0.05)(15)(9.81)\frac{\cos\left(30\left(\frac{\pi}{180}\right)\right)}{\sin\left(30\left(\frac{\pi}{180}\right)\right)}} \\ &= 16.876 \text{ m/s} \end{aligned}$$

---

## 0.2 Problem 2

**Question 2** 1 pts

How would the final velocity of the child change if her mass was doubled?

- No change
- 2 times faster
- 4 times faster
- 1/2 as fast

Velocity at the bottom do not change if the mass doubles. We see from (2) in problem 1 that $v_2$ do not depend on mass.