quizz 11, ME 240 Dynamics, Fall 2017

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0.1 Problem 1

Question 1 2.5 pts

A bicycle wheel is spun up to an angular velocity of 13.3 rad/s, then the tire is pressed against the ground until the wheel stops rotating. Assume that the bicycle's horizontal and vertical positions remain fixed.

Knowing that the kinetic friction coefficient is 0.3 and the wheel rotated 3 complete revolutions before stopping, determine the average vertical force in (N) used to press the tire onto the ground. (assume the vertical force is a combination of the wheel's weight and the person's holding force)

The wheel can be modeled as a 1.1 kg ring with radius 0.3 m.

Applying work energy for rigid bodies

$$T_1 + V_1 + \int_0^{\theta_2} M d\theta = T_2 + V_2$$

But $V_1 = V_2$, and let $F = \mu_k P$, where P is the force pushing down and F is the friction force, then

$$\begin{aligned}
\frac{1}{2} I_{cg}\omega_1^2 + \int_0^{\theta_2} M d\theta &= 0 \\
\frac{1}{2}\left(mr^2\right)\omega_1^2 + \int_0^{\theta_2} -Fr d\theta &= 0 \\
\frac{1}{2}\left(mr^2\right)\omega_1^2 + P\mu_k r\theta_2 &= 0
\end{aligned}$$

But $\theta_2 = 6\pi$ since 3 revolutions, then

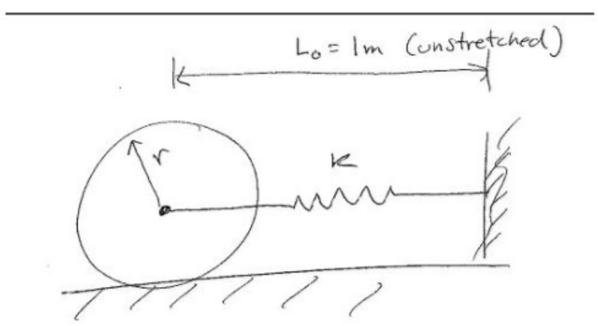
$$\frac{1}{2}\left(mr^2\right)\omega_1^2 - P\mu_k r\left(6\pi\right) = 0$$

Solving for P

$$\begin{aligned}
P &= \frac{\frac{1}{2}\left(mr^2\right)\omega_1^2}{\mu_k r\left(6\pi\right)} \\
&= \frac{\frac{1}{2}\left(\left(1.1\right)\left(0.3\right)^2\right)\left(13.3\right)^2}{\left(0.3\right)\left(0.3\right)\left(6\pi\right)} \\
&= 5.161 \text{ N}
\end{aligned}$$

0.2 Problem 2

Question 2 2.5 pts



A 14.0 kg thin disc of radius 0.5 m is attached to a spring with initial length 1m. The spring constant k = 20 N/m

The disc is rolled backwards until the spring is stretched to 2.7 m, then the disc is released from rest. Assuming roll without slip, determine the velocity in (m/s) at the center of the disc when the spring length reaches 1m.

When the disk is pulled, it gains potential energy $V_1 = \frac{1}{2}kx^2$ where x is amount of spring extension from equilibrium, which is 1.7 meter in this example. When released, all this energy will be converted to kinetic energy when the disk reaches its original equilibrium position. The final kinetic energy is $T_2 = \frac{1}{2}mv_g^2 + \frac{1}{2}I_{cg}\omega^2$. But since the disk rolls without slip, then $v_{cg} = r\omega$ and

$$T_2 = \frac{1}{2}mv_g^2 + \frac{1}{2}I_{cg}\frac{v_{cg}^2}{r^2}$$

But $I_{cg} = \frac{1}{2}mr^2$ and the above becomes

$$\begin{aligned} T_2 &= \frac{1}{2}mv_g^2 + \frac{1}{4}mr^2\frac{v_{cg}^2}{r^2} \\ &= \frac{1}{2}mv_g^2 + \frac{1}{4}mv_{cg}^2 \\ &= \frac{3}{4}mv_{cg}^2 \end{aligned}$$

Hence equating the initial potential energy to the final kinetic energy we obtain

$$\frac{1}{2}kx^2 = \frac{3}{4}mv_g^2$$

Solving for v_g gives

$$\begin{aligned} v_{cg}^2 &= \frac{\frac{1}{2}kx^2}{\frac{3}{4}m} \\ &= \frac{2}{3}\frac{kx^2}{m} \\ &= \frac{2}{3}\frac{(20)(1.7)^2}{14} \\ &= 2.7524 \end{aligned}$$

Therefore

$$\begin{aligned} v_{cg} &= \sqrt{2.7524} \\ &= 1.659 \text{ m/s} \end{aligned}$$