Name: KEY

PrEPS Practice Exam 2
Fall 2017

3. The pendulum consists of a disk of weight W_1 and a slender rod of weight W_2.

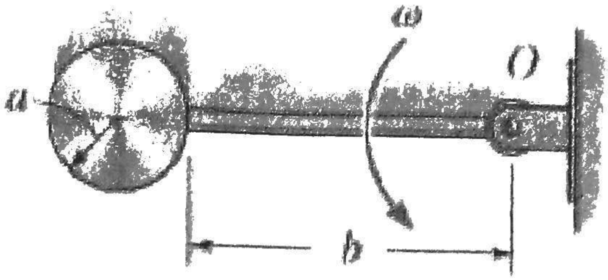
$W_1 = 15\ lb$

$W_2 = 10\ lb$

$a = 0.75\ ft$

$b = 3\ ft$

$\omega = 8\ \frac{rads}{s}$



Determine:

A) Knowing the disk and the rod are rigidly attached, find the mass moment of Inertia about point O.

B) Determine the horizontal and vertical components of reaction that the pin O exerts on the rod just as it passes the horizontal position, at which time its angular velocity is ω.

a) $I_O = I_{O,rod} + I_{O,disk}$

$$I_{O,rod} = I_{rod,end} = \frac{1}{3} m L^2 = \frac{1}{3g} W_2 b^2$$

$$I_{O,disk} = I_{disk} + md^2 = \frac{1}{2} m r^2 + md^2 = \frac{1}{2g} W_1 a^2 + \frac{W_1}{g}(a+b)^2$$

$$I_O = \frac{1}{g}\left[\frac{W_2 b^2}{3} + \frac{W_1 a^2}{2} + W_1 (a+b)^2\right]$$

$$\boxed{I_O = 7.613\ \text{slug}\cdot\text{ft}^2}$$

b) FBD KBD

Name: KEY

PrEPS Practice Exam 2

Fall 2017

(1) ΣF_x: $R_x = m a_{Gx}$

(2) ΣF_y: $R_y - W_1 - W_2 = m a_{Gy}$

(3) ΣM_O: $(W_1 + W_2)\bar{l} = I_O \alpha$

$$\bar{l} = \frac{\frac{W_2 b}{2} + W_1 (a+b)}{W_1 + W_2}$$

$\bar{l} = 2.85$ ft

3 equations, 5 unknowns ($a_{Gx}, a_{Gy}, R_y, R_x, \alpha$)

Kinematics for last two equations

$$\vec{a}_G = \cancel{\vec{a}_O}^{\,0} + \vec{\alpha} \times \vec{r}_{G/O} - \omega^2 \vec{r}_{G/O}$$

$$a_{Gx}\hat{i} + a_{Gy}\hat{j} = (\alpha\hat{k} \times -\bar{l}\hat{i}) - \omega^2(-\bar{l}\hat{i})$$

$$a_{Gx}\hat{i} + a_{Gy}\hat{j} = -\alpha\bar{l}\hat{j} + \omega^2\bar{l}\hat{i}$$

$\hat{i}$ $\hat{k}$ $\hat{j}$

(4) $\hat{i}$: $a_{Gx} = \omega^2 \bar{l}$

(5) $\hat{j}$: $a_{Gy} = -\alpha \bar{l}$

From (1) and (4)

$$R_x = m\omega^2 \bar{l} = \left[\frac{W_1 + W_2}{g}\right]\omega^2 \bar{l}$$

$$\boxed{R_x = 141.615 \text{ lbs}}$$

Put (3) into (5) and then (5) into (2)

$$\alpha = \frac{(W_1 + W_2)\bar{l}}{I_O} \qquad \alpha = 9.358 \text{ rad/s}^2$$

$$R_y = W_1 + W_2 + \left[\frac{W_1 + W_2}{g}\right]\left[-\alpha\bar{l}\right]$$

$$\boxed{R_y = 4.291 \text{ lbs}}$$

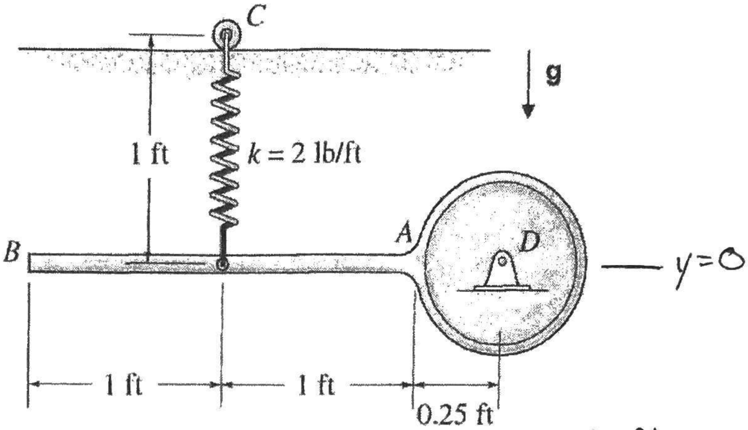
Name: KEY

PrEPS Practice Exam 2
Fall 2017

2. The pendulum consists of 2-lb rod *BA* and a 6 lb disk. The spring is stretched 0.3 ft when the rod is horizontal as shown. When the pendulum is released, it rotates about bearing *D*. The roller at *C* allows the spring to remain vertical as the rod falls.

(a) If released from rest, determine the pendulum's angular velocity at the instant the rod becomes vertical.

(b) Suppose we now add a torsional spring of strength k_t to the bearing at *D*. The torsional spring provides a restoring moment in much the same way as a linear spring provides a restoring force. It has an associated potential function of $V_t = \frac{1}{2} k_t \theta^2$, where θ is the angular displacement of the rod from its horizontal position. What value should k_t have so that the pendulum comes to rest again after displacing 45°?



$W_{rod} = 2$ lb $\quad L_o = 0.7$ ft $\quad R = 0.25$ ft

$W_{disk} = 6$ lb $\quad L_{AB} = 2$ ft

$$T_1 + V_1 = T_2 + V_2$$

$$T_1 = 0 \qquad V_1 = \frac{1}{2} k \delta_1^2$$

$$T_2 = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2 = \frac{1}{2} I_D \omega^2$$

$$I_D = I_{disk,g} + I_{rod,g} + m_{rod} d_{g/D}^2$$

$$I_D = \frac{1}{2} m_{disk} R^2 + \frac{1}{12} m_{rod} L_{AB}^2 + m_{rod}\left(\frac{L_{AB}}{2} + R\right)^2$$

$$V_2 = \frac{1}{2} k \delta_2^2 + m_{rod}\, g\, \Delta H_{rod,g}$$

$$\frac{1}{2} k \delta_1^2 = \frac{1}{2}\left[\frac{1}{2} \cdot \frac{W_{disk}}{g} \cdot R^2 + \frac{W_{rod}}{g}\left(\frac{1}{12} L_{AB}^2 + \left(\frac{L_{AB}}{2} + R\right)^2\right)\right] \omega^2 + \frac{1}{2} k \delta_2^2 + \frac{W_{rod}}{g} g\, \Delta H_{rod,g}$$

$$\delta_1 = 0.3 \text{ ft}$$

$$\delta_2 = \delta_1 + \frac{L_{AB}}{2} + R = 1.55 \text{ ft}$$

$$\Delta H_{rod} = -\left(\frac{L_{AB}}{2} + R\right) = 1.25 \text{ ft}$$

$$\frac{1}{2}\left(2 \frac{\text{lb}}{\text{ft}}\right)\left((.3\text{ft})^2 - (1.55\text{ft})^2\right) + (2\text{lb})(1.25\text{ft}) = \frac{1}{2}\left[\frac{1}{2} \cdot \frac{6\text{ lb}}{32.2\frac{\text{ft}}{s^2}}(.25\text{ft})^2 + \frac{2\text{ lb}}{32.2\frac{\text{ft}}{s^2}}\left(\frac{(2\text{ft})^2}{12} + \left(\frac{2\text{ft}}{2} + .25\text{ft}\right)^2\right)\right]\omega^2$$

$$.1875 = .0618 \cdot \omega^2$$

$$\boxed{\omega = 1.742 \frac{\text{rad}}{s}}$$

Name: KEY

PrEPS Practice Exam 2
Fall 2017

$$T_1 + V_1 = T_2 + V_2$$

$$T_1 = 0 \qquad T_2 = 0$$

$$V_1 = \frac{1}{2} k \delta_1^2 \qquad V_2 = \frac{1}{2} k \delta_2^2 + \frac{1}{2} k_t \theta^2 + m_{rod}\, g\, \Delta H_{rod,G}$$

$$\delta_1 = 0.3 \text{ ft}$$

$$\delta_2 = 0.3 \text{ ft} + \left(\frac{L_{AB}}{2} + R\right) \sin 45° = 1.184 \text{ ft}$$

$$\Delta H_{rod,G} = -\left(\frac{L_{AB}}{2} + R\right) \sin 45° = -0.8838 \text{ ft}$$

$$\frac{1}{2} k \delta_1^2 = \frac{1}{2} k \delta_2^2 + \frac{1}{2} k_t \theta^2 + W_{rod}\, \Delta H_{rod,G}$$

$$\frac{\frac{1}{2} k (\delta_1^2 - \delta_2^2) - W_{rod}\, \Delta H_{rod,G}}{\frac{1}{2} \theta^2} = k_t$$

$$k_t = \frac{\left(2 \frac{\text{lb}}{\text{ft}}\right)\left((0.3\text{ft})^2 - (1.184\text{ft})^2\right) - 2(2\text{lb})(-0.8838\text{ft})}{(\pi/4 \text{ rad})^2}$$

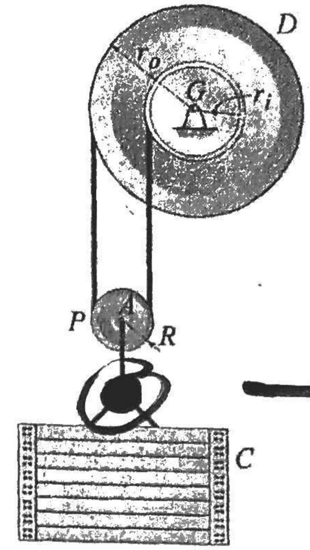
$$\boxed{k_t = 1.479 \text{ lb-ft}}$$

Name: KEY

PrEPS Practice Exam 2
Fall 2017

3. The double pulley D has a mass of 15kg, center of mass G coinciding with its geometric center, radius of gyration $k_G = 10\text{cm}$, outer radius $r_O = 15\text{cm}$, and inner radius $r_i = 7.5\text{cm}$. It is connected to the pulley P with radius R via a cord of negligible mass that unwinds from the inner and outer spools of the double pulley D. The crate C, which has a mass of 20kg, is released from rest.

Assuming the pulley P has a mass of 1.5 kg and a radius of gyration $k_G = 3.5\text{cm}$, determine the speed of the crate C and the angular velocity of pulley D after the crate has dropped a distance $h = 2\text{m}$



— y=0

$m_D = 15\text{ kg}$ $\quad r_o = 15\text{ cm}$ $\quad m_c = 20\text{ kg}$ $\quad K_{G,P} = 3.5\text{cm}$

$k_{G,D} = 10\text{cm}$ $\quad r_i = 7.5\text{ cm}$ $\quad m_P = 1.5\text{ kg}$ $\quad h = 2\text{m}$

$$T_1 + V_1 = T_2 + V_2$$

$T_1 = 0$ $\quad T_2 = \frac{1}{2} I_D \omega_D^2 + \frac{1}{2} m_P v_{G,P}^2 + \frac{1}{2} I_P \omega_P^2 + \frac{1}{2} m_c v_c^2$

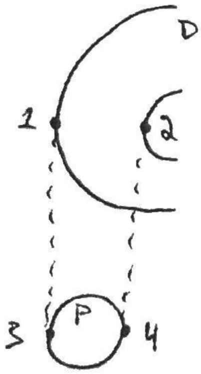
$V_1 = 0$ $\quad V_2 = -(m_P + m_c) h g$

$I_D = m_D k_{G,D}^2$

$I_P = m_P k_{G,P}^2$

find: $\omega_D \quad \omega_P \quad v_P \quad v_c$

$v_P = v_c$ ✓



$v_1 = \omega_D r_o = v_3 \qquad v_2 = \omega_D r_i = v_4$

$v_P = \frac{1}{2}(v_3 + v_4)$

$v_P = v_3 + \omega_P \times \vec{r}_{P/3} = v_4 + \omega_P \times \vec{r}_{P/4} \doteq \omega_D r_i + \omega_D \cdot \frac{1}{2}(r_o - r_i)$

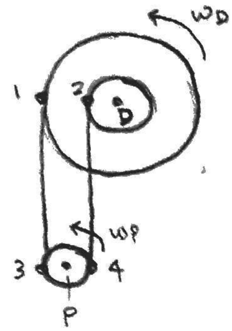
$= \omega_D (r_i + r_o) \cdot \frac{1}{2}$

$v_3 + \omega_P R = v_4 + \omega_P(-R)$

$2\omega_P R = v_4 - v_3$

$$\omega_P = \frac{v_4 - v_3}{2R} = \frac{\omega_D r_o - \omega_D r_i}{2(\frac{1}{2})(r_o - r_i)} = \frac{\omega_D (r_o - r_i)}{r_o - r_i} = \omega_D$$

Problem 3 Kinematics



$$\vec{V}_1 = \cancel{\vec{V}_D}^{\,0} + \vec{\omega}_D \times \vec{r}_{1/D}$$

$$\vec{V}_1 = \omega_D \hat{k} \times -r_o \hat{\imath}$$

$$\vec{V}_1 = -\omega_D r_o \hat{j}$$

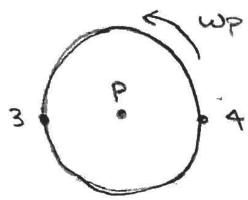
$\hat{k}$ $\hat{j}$

$$\vec{V}_2 = \cancel{\vec{V}_D}^{\,0} + \vec{\omega}_D \times \vec{r}_{2/D}$$

$$\vec{V}_2 = \omega_D \hat{k} \times -r_i \hat{\imath}$$

$$\vec{V}_2 = -\omega_D r_i \hat{j}$$

$\vec{V}_1 = \vec{V}_3$ and $\vec{V}_2 = \vec{V}_4$



Let the distance between 3 and 4 be $r_o - r_i$

$$\vec{V}_4 = \vec{V}_3 + \vec{\omega}_P \times \vec{r}_{4/3}$$

$$-\omega_D r_i \hat{j} = -\omega_D r_o \hat{j} + \omega_P \hat{k} \times (r_o - r_i)\hat{\imath}$$

$$-\omega_D r_i \hat{j} = -\omega_D r_o \hat{j} + \omega_P (r_o - r_i)\hat{j}$$

$$\omega_D (r_o - r_i) = \omega_P (r_o - r_i)$$

$$\therefore \boxed{\omega_D = \omega_P}$$

$$\vec{V}_P = \vec{V}_3 + \vec{\omega}_P \times \vec{r}_{P/3}$$

$$\vec{V}_P = -\omega_D r_o \hat{j} + \omega_D \hat{k} \times \frac{(r_o - r_i)}{2} \hat{\imath}$$

$$\vec{V}_P = \left[-\omega_D r_o + \omega_D \frac{(r_o - r_i)}{2}\right] \hat{j}$$

$$\vec{V}_P = \frac{-2\omega_D r_o + \omega_D r_o - \omega_D r_i}{2}$$

$$\boxed{\vec{V}_P = \frac{-\omega_D (r_o + r_i)}{2}}$$

$\vec{V}_P = \vec{V}_c$

Knowing $\omega_D = \omega_P$ and $V_P = \frac{\omega_D (r_o + r_i)}{2}$ and $V_c = V_P$ . .

$$0 = T_2 + V_2$$

$$0 = \frac{1}{2} I_D \omega_D^2 + \frac{1}{2} m_P V_P^2 + \frac{1}{2} I_P \omega_P^2 + \frac{1}{2} m_c V_c^2 - (m_P + m_c) h g$$

$$2(m_P + m_c) h g = m_D K_{G,D}^2 \omega_D^2 + m_P \frac{\omega_D^2 (r_o + r_i)^2}{4} + m_P K_{G,P}^2 \omega_D^2 + m_c \frac{\omega_D^2 (r_o + r_i)^2}{4}$$

$$2(m_P + m_c) h g = \omega_D^2 \left[m_D K_{G,D}^2 + m_P K_{G,P}^2 + \frac{(m_P + m_c)(r_o + r_i)^2}{4} \right]$$

$$\omega_D = \sqrt{\frac{2(m_P + m_c) h g}{m_D K_{GD}^2 + m_P K_{GP}^2 + \frac{(m_P + m_c)(r_o + r_i)^2}{4}}}$$

$$\boxed{\omega_D = 44.61 \ \frac{rad}{s}}$$

Since $V_c = V_P = \frac{\omega_D (r_o + r_i)}{2}$

$$\boxed{V_c = 5.019 \ m/s}$$