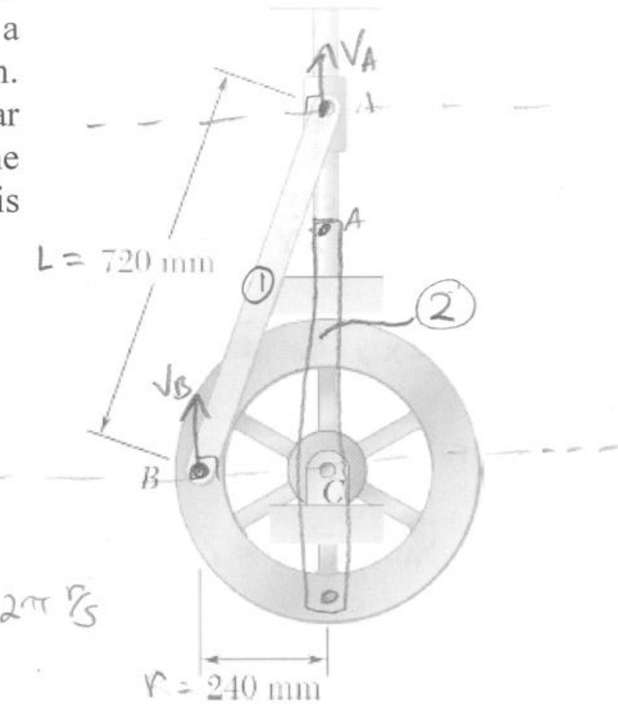
EMA 202, Fall 2014 **Name:** ______________________________

Final, closed book/notes, 120 min.

Question 1 (25 points)

The 4-kg rod AB is attached to a collar of negligible mass at A and to a flywheel at B. The flywheel has a mass of 16 kg and a radius of gyration of 180 mm. Knowing that in the position shown the angular velocity of the flywheel is 60 rpm clockwise, determine the angular velocity of the flywheel when Point B is directly below C.



Find ω_2 given

$m_A = 0$

$m_{AB} = 4\,kg$

$m_C = 16\,kg$

$k_C = .18\,m$

$\omega_1 = 60\,rpm\left(\frac{2\pi}{60}\right) = 2\pi\ r/s$

energy method

$$T_1 + \underbrace{\cancel{V_1}}_{0,\ \text{by choice of datum}} + \underbrace{\cancel{U_{12}}}_{0,\ \text{no external } F, M} = T_2 + V_2$$

state1

$$T_1 = \underbrace{T_{1C}}_{\text{disk}} + \underbrace{T_{1AB}}_{\text{rod}} + \underbrace{T_{1A}}_{\text{collar}}$$

$$T_1 = \tfrac{1}{2} I_C \omega_{C1}^2 + \tfrac{1}{2} m_{AB} V_{G/AB1}^2 + \tfrac{1}{2} I_{G/AB} \overbrace{\cancel{\omega_{AB1}^2}}^{0,\ \text{from IC}} + \tfrac{1}{2} \overbrace{\cancel{m_A}}^{0} V_{A1}^2$$

from IC of AB $\Rightarrow \omega_{AB1} = 0$

$$\vec{V}_{G,AB1} = \vec{V}_{B1} + \underbrace{\cancel{\vec{\omega}_{AB1}}}_{0} \times \vec{r}_{G_{AB}/A} \Rightarrow \vec{V}_{GAB1} = \vec{V}_{B1} = \vec{\omega}_{C1} \times \vec{r}_{B/C}$$

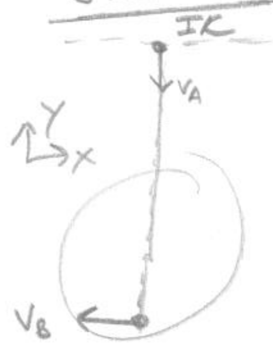
$$\vec{V}_{GAB1} = \omega_{C1} r\ \hat{j}$$

$$T_1 = \tfrac{1}{2}(m_C k_C^2)\omega_{C1}^2 + \tfrac{1}{2} m_{AB} (\omega_{C1} r)^2$$

$$T_1 = \tfrac{1}{2}(16\,kg \cdot .18^2\,m^2)(2\pi)^2 + \tfrac{1}{2}(4\,kg)(2\pi(.24\,m))^2 \qquad \boxed{T_1 = 14.78\ N\text{-}m}$$

Q1 page 2

State 2 KE



pt A = IC

collar A

$$T_2 = \underbrace{\tfrac{1}{2} I_C \omega_{C2}^2}_{\text{disc}} + \underbrace{\tfrac{1}{2} m_{AB} V_{G AB_2}^2 + \tfrac{1}{2} I_{G AB}\omega_{AB2}^2}_{\text{rod } AB} + \overbrace{\tfrac{1}{2} m_A \underbrace{V_{A2}^2}_{0}}^{\text{collar } A}$$

IC @ pt A, so rod AB is in pure rotation about A

from AB ① $\vec{V}_B = \underbrace{\vec{V}_A}_{0} + \vec{\omega}_{AB2} \times \vec{r}_{B/A}$ $\qquad \vec{\omega}_{AB2} = \omega_{AB2}\hat{k}$

$\vec{V}_B = \omega_{AB2} L \uparrow$ $\qquad \vec{r}_{B/A} = -L\hat{j}$

from Disc C ② $\vec{V}_B = \underbrace{\vec{V}_C}_{0} + \vec{\omega}_{C2} \times \vec{r}_{B/C}$ $\qquad \vec{\omega}_{C2} = \omega_{C2}\hat{k}$

$\vec{r}_{B/C} = -r\hat{j}$

$\vec{V}_B = \omega_{C2} r \uparrow$

set ①=② for $\vec{V}_B$

$$\omega_{AB2} L = \omega_{C2} r \rightarrow \boxed{\omega_{AB2} = \omega_{C2}\frac{r}{L}} \quad *$$

find: $\vec{V}_{G,AB2} = \underbrace{\vec{V}_A}_{0} + \vec{\omega}_{AB2} \times \vec{r}_{G/A}$ $\qquad \vec{r}_{G,AB/A} = -\frac{L}{2}\hat{j}$

$$\boxed{\vec{V}_{G,AB2} = \omega_{C2}\frac{r}{\cancel{L}}\left(\frac{\cancel{L}}{2}\right)\uparrow = \omega_{C2}\frac{r}{2}\uparrow} \quad **$$

Sub * & ** into T_2

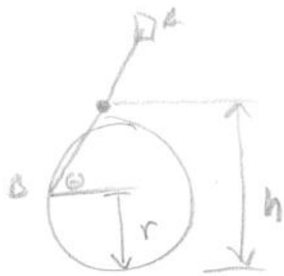
$$T_2 = \frac{1}{2}(m_C k_C^2)\omega_{C2}^2 + \frac{1}{2} m_{AB}\frac{r^2}{4}\omega_{C2}^2 + \frac{1}{2}\left(\frac{1}{12} m_{AB} \cancel{L^2}\right)\frac{r^2}{\cancel{L^2}}\omega_{C2}^2$$

$$T_2 = \frac{1}{2}(m_C k_C^2)\omega_{C2}^2 + \frac{1}{6} m_{AB} r^2 \omega_{C2}^2$$

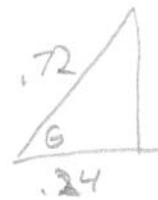
$$\boxed{T_2 = .298\,\omega_{C2}^2}$$

Q1 P.3

State 2: PE $\quad V_2 = m_{AB}\, g\, \Delta h_{AB_G}$



$h_2 = \frac{L}{2}$



$\cos\theta = \frac{.24}{.72}$

$\theta = 70.5°$

$h_1 = r + \frac{L}{2}\sin\theta$

$\Delta h_{AB} = h_2 - h_1 = \frac{L}{2} - \left(r + \frac{L}{2}\sin\theta\right)$

$\Delta h_{AB} = -.219\ m$

$V_2 = m_{AB}\, g\, \Delta h_{AB} = 4(9.81)(-.219\ m) = -8.59\ Nm$

$T_1 = T_2 + V_2$

$14.78\ Nm = .298\, \omega_{C2}^2 - 8.59\ Nm$

$\boxed{\omega_{C2} = 8.86\ r/s\ \circlearrowright}$

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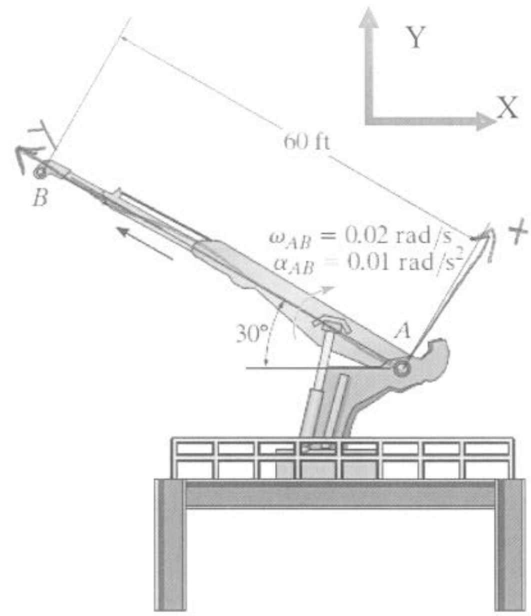
Question 2 (25 points)

The crane's telescopic boom rotates with the angular velocity and angular acceleration shown. At the same instant, the boom is extending with a constant speed of 0.5ft/s, measured relative to the boom. Determine the velocity and acceleration vectors of point B in the XY coordinate system given.



find $\vec{V}_{Bxy}$ & $\vec{a}_{Bxy}$

fix rotating CS @ pt A

$$\vec{V}_B = \cancel{\vec{V}_A}^{\,0} + \vec{\omega}_{AB} \times \vec{r}_{B/A} + \vec{V}_{Brel}$$

$\vec{V}_{Brel} = .5\,\text{ft/s}\ \hat{j}$

$\vec{\omega}_{AB} = -.02\ \text{r/s}\ \hat{k}$

$\vec{r}_{B/A} = 60\text{ft}\ \hat{j}$

$$\vec{V}_B = \omega_{AB}\, r_{B/A}\ \hat{i} + .5\hat{j}$$

$$\vec{V}_B = (.02)60\text{ft}\ \hat{i} + .5\ \text{ft/s}\ \hat{j}$$

xy $\underline{\vec{V}_B = 1.2\hat{i} + .5\hat{j}\ \ \text{ft/s}}$

Convert back to $\underline{XY}$ coords

$$\hat{i} = \sin 30\ \hat{I} + \cos 30\ \hat{J}$$

$$\hat{j} = -\cos 30\ \hat{I} + \sin 30\ \hat{J}$$

$\underline{XY}$ $\vec{V}_B = 1.2\left(\sin 30\hat{I} + \cos 30\hat{J}\right) + .5\left(-\cos 30\hat{I} + \sin 30\hat{J}\right)$ ft/s

$$\boxed{\vec{V}_B = .167\hat{I} + 1.289\hat{J}\ \ \text{ft/s}}$$

Q2 page 2

$$\vec{a}_B = \underbrace{\cancel{\vec{a}_A}}_{0,\text{ pinned}} + \vec{\alpha}_{AB} \times \vec{r}_{B/A} + \underbrace{\vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times \vec{r}_{B/A})}_{-\omega_{AB}^2 \vec{r}_{B/A}} + 2\vec{\omega}_{AB} \times \vec{v}_{B\,rel} + \underbrace{\cancel{\vec{a}_{B\,rel}}}_{0,\text{ constant } V_{B\,rel}}$$

$$\vec{\alpha}_{AB} = \alpha_{AB}\hat{k} = -.01\hat{k} \ \text{r/s}^2$$

$$\vec{a}_B = -.01\hat{k} \times 60\text{ft}\,\hat{j} - (.02)^2 60\hat{j} + 2(.02\hat{k}) \times .5\hat{j}$$

$$\vec{a}_B = 60(.01)\,\hat{\imath} - .02^2 \cdot 60\hat{j} + 2(.02)\cdot .5\,\hat{\imath}$$

xy $\vec{a}_B = .62\,\hat{\imath} - .024\,\hat{j} \ \text{ft/s}^2$

in $\underline{XY}$

$$\vec{a}_B = .62(\sin 30\,\hat{I} + \cos 30\,\hat{J}) - .024(-\cos 30\,\hat{I} + \sin 30\,\hat{J}) \ \text{ft/s}^2$$

$$\boxed{\vec{a}_B = .331\,\hat{I} + .525\,\hat{J} \ \text{ft/s}^2}$$

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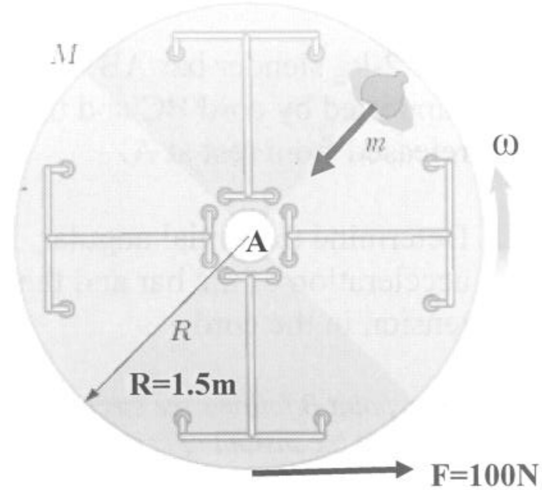
<u>Question 3 (25 points)</u>

Top view



A merry go round with a rider (m=20kg) is initially rotating at a rate of 5 rad/s about point A. Someone pushes the merry go round with an average force of F=100N for 5 seconds. During that time the rider moves from the edge of the merry go round to a position 0.5 meters from the center. The angular velocity after the push is 10 rad/s.

Determine the mass of the merry go round assuming it can be modeled as a circular disk.

<u>Given</u>

$\vec{\omega}_1 = 5 \text{ r/s } \hat{k}$

$\vec{\omega}_2 = 10 \text{ r/s } \hat{k}$

$m_r = 20 \text{ kg}$

$F_{ave} = 100 \text{ N}$ for 5 seconds

$R = 1.5 \text{ m}$

<u>find</u>

$m_{DISC} = ?$

<u>moment of inertia</u>

$I_{A1} = I_D + m_r R^2 \qquad I_{DISC} = \frac{1}{2} m_D R^2$

$I_{A1} = \frac{1}{2} m_D R^2 + m_r R^2$

$I_{A2} = \frac{1}{2} m_D R^2 + m_r (.5m)^2$

Angular momentum - Impulse

$I_{A1} \vec{\omega}_1 + \int \vec{M} dt = I_{A2} \vec{\omega}_2 \qquad \vec{M} = \vec{r} \times \vec{F} = 1.5m (100N) \hat{k}$

$\left(\frac{1}{2} m_D (1.5)^2 + 20kg(1.5^2)\right) 5 \text{ r/s } \hat{k} + \int_0^{5 sec} 150 Nm \, dt = \left(\frac{1}{2} m_D (1.5)^2 + 20(.5m)^2\right) 10 \text{ r/s}$

Q3 page 2

$$5.625\, m_D + 225\,\frac{kg\text{-}m^2}{s} + 150\,Nm(5\,sec) = 11.25\,\frac{\cancel{kg}m^2}{s}(m_D) + 50\,\frac{kg\,m}{s}$$

$$225 - 50 + 750 = (11.25 - 5.625)\, m_D$$

$$\boxed{m_D = 164.4\ kg}$$

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Find α_{AB} & T

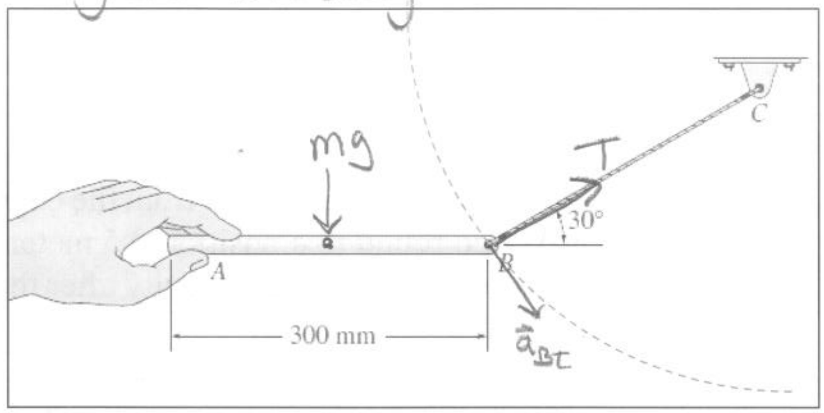
given $m = 2\,kg$

<u>Question 4 (25 points)</u>

The 2-kg slender bar AB is supported by cord BC and then released from rest at A.

Determine the initial angular acceleration of the bar and the tension in the cord.

Hint: point B follows the circular path shown at the instant of release



NT $\vec{a}_B = a_{BN}\hat{u}_N + a_{Bt}\hat{u}_t$

$a_{BN} = \frac{V_B^2}{\rho}$, with $V_B = 0$ @ release

$\vec{a}_B = a_{Bt}\hat{u}_T \rightarrow$ ① $\boxed{\vec{a}_B = a_B \sin\theta\,\hat{i} - a_B\cos\theta\,\hat{j}}$

$\overset{+}{\rightarrow} \Sigma F_x = T\cos 30° = m a_{Gx}$

$+\uparrow \Sigma F_y = -mg + T\sin 30° = m a_{Gy}$

$\circlearrowleft \Sigma M_G = T\sin 30° \left(\frac{L}{2}\right) = I_G \alpha_{AB}$

3 eqns 4 unknowns

$\rightarrow$ use what we know about $\vec{a}_B$ to eliminate a variable

accel @ B

$\vec{a}_B = \vec{a}_G + \vec{\alpha}_{AB} \times \vec{r}_{B/G}$ $\quad \vec{r}_{B/G} = \frac{.3}{2}\hat{i}$ (m)

② $\vec{a}_B = a_{Gx}\hat{i} + a_{Gy}\hat{j} + \frac{.3}{2}\alpha\,\hat{j}$

Set i & j components from ① = ②

i) $a_B \sin\theta = a_{Gx}$ $\quad$ j) $-a_B\cos\theta = a_{Gy} + \frac{.3}{2}\alpha$

Q4 page 2

i) $a_{Gx} = a_B \sin\theta$, $\theta = 30°$

j) $a_{Gy} = -a_B \cos\theta - .15\alpha$

back to ΣF & ΣM

X) $T\cos 30 = m(a_B \sin\theta)$ $\quad a_B = \dfrac{T}{m \cdot \tan 30°}$

Y) $T\sin 30 - mg = m(-a_B \cos\theta - .15\alpha)$

$$\frac{T\sin 30 - mg}{m} = -\frac{T}{m\tan 30}\cos 30 - .15\alpha$$

$$\frac{T}{m}\underbrace{\left(\sin 30 + \frac{\cos^2 30}{\sin 30}\right)} = g - .15\alpha$$

$$\frac{\sin^2 30 + \cos^2 30}{\sin 30} = \frac{1}{\sin 30} = 2$$

$$\boxed{T = g - .15\alpha}$$

M_G) $T\cancel{\sin 30}^{\,1/2}\left(\frac{L}{2}\right) = \frac{1}{12} m L^2 \alpha_{AB}$ $\quad$ w/ $I_G = \frac{1}{12} m L^2$

$$(g - .15\alpha)\frac{L}{4} = \frac{1}{12} m L^2 \alpha_{AB}$$

$$\frac{gL}{4} = \left(\frac{1}{12} m L^2 + \frac{.15L}{4}\right)\alpha_{AB}$$

$$\frac{(9.81\ m/s^2)(.3m)}{4} = \left(\frac{1}{12}(2kg)(.3m)^2 + \frac{.15(.3m)}{4}\right)\alpha_{AB}$$

$$\boxed{\alpha_{AB} = 28.03\ r/s^2}$$

$T = g - .15\alpha \Rightarrow \boxed{T = 5.61N}$