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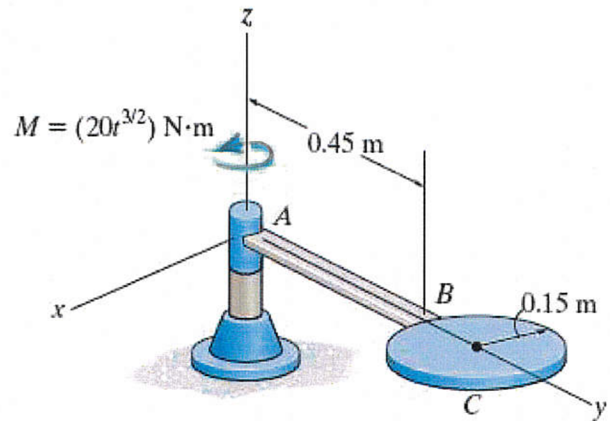
Final, closed book/notes, 120 min.

Question 1 (25 points)

The assembly shown consists of a 10-kg rod AB and a 20-kg circular disc C. If it is subject to a torque of $\mathbf{M} = (20t^{3/2})$ Nm, where t is in seconds, determine its angular velocity when t = 3sec. When $t = 0$ the assembly is rotating clockwise at 6 rad/s.



Angular momentum Impulse

$\hat{k})$ $$I_A \omega_1 + \int_0^3 M_z dt = I_A \omega_2$$

$$I_A = I_{rod_A} + I_{disc_A} = \frac{1}{3} m_{rod} L_{rod}^2 + \frac{1}{2} m_{disc} r_{disc}^2 + m_{disc}(L_{rod} + r_{disc})^2$$

$$I_A = \frac{1}{3}(10kg)(.45m)^2 + \frac{1}{2}(20kg)(.15m)^2 + 20kg(.45 + .15m)^2$$

$$\underline{I_A = 8.1 \text{ kg-m}^2}$$

$$\omega_1 = -6 \text{ r/s}$$

$$M_z = 20t^{3/2} \text{ N.m}$$

$$(8.1 \text{ kg-m}^2)(-6 \text{ r/s}) + \int_0^3 20t^{3/2} dt = (8.1 \text{ kg-m}^2)\omega_2$$

$$-48.6 \text{ Nm-s} + \frac{20t^{5/2}}{5/2}\Bigg|_0^3 = (8.1 \text{ kg-m}^2)\omega_2$$

$\omega_2 = 9.4$ r/s counter clockwise

$$\boxed{\vec{\omega}_2 = 9.4 \text{ r/s } \hat{k}}$$

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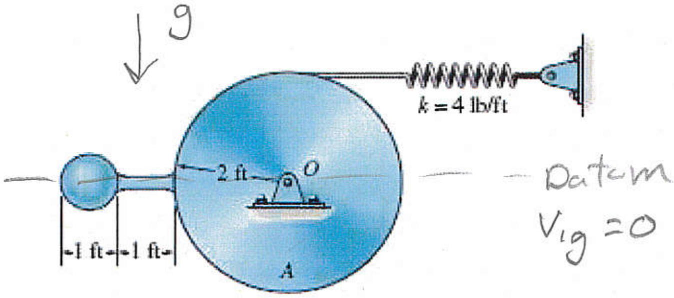
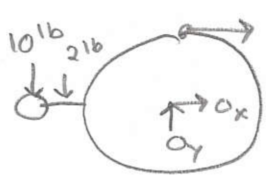
Question 2 (25 points)

The disc A is pinned at O and weighs 15lbs. A 1-ft rod weighing 2lbs and a 1-ft diameter sphere weighing 10lbs are welded to the disc, as shown. If the spring is originally stretched 1ft and the sphere is released from the position shown, determine the angular velocity of the disc when it has rotated 90°. Assume that the spring wraps around the disc.





$F_s = 4\,\frac{lb}{ft}\cdot(1ft)$

$\circlearrowleft + \sum M_O = 10\text{ lbs}(3.5\text{ ft}) + 2\text{ lb}(2.5\text{ ft}) - 4\text{ lb}(2\text{ft}) = I_O\alpha$

$32\text{ ft-lb} = I_O\alpha$

α is in + k-direction

Work-energy eqn

$T_1 + V_1 + U_{12} = T_2 + V_2$

$T_1 = 0$ system released from rest

$\boxed{V_1 = \frac{1}{2}k\delta_1^2 = \frac{1}{2}(4\text{ lb/ft})(1\text{ft})^2 = 2\text{ ft-lb}}$

$U_{12} = 0$ no external N.C. work

$T_2 = T_{Disc_2} + T_{rod_2} + T_{sphere_2}$

$T_{Disc_2} = \frac{1}{2}I_{Disc_O}\omega_2^2 = \frac{1}{2}\left(\frac{1}{2}m_{Disc}r^2\right)\omega_2^2 = \frac{1}{2}\left(\frac{1}{2}\left(\frac{15\text{ lb}}{32.2\text{ ft/s}^2}\right)(2\text{ft})^2\right)\omega_2^2$

$T_{Disc_2} = .466\,\omega_2^2$

$T_{rod_2} = \frac{1}{2}I_{rod_O}\omega_2^2 = \frac{1}{2}\left(\frac{1}{12}m_{rod}L^2 + m_{rod}\left(\frac{L}{2}+r\right)^2\right)\omega_2^2$

$T_{rod_2} = \frac{1}{2}\left[\frac{1}{12}\left(\frac{2\text{ lbs}}{32.2\text{ ft/s}^2}\right)(1\text{ft})^2 + \left(\frac{2\text{ lbs}}{32.2\text{ ft/s}^2}\right)\left(\frac{1\text{ ft}}{2} + 2\text{ ft}\right)^2\right]\omega_2^2 = .197\,\omega_2^2$

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(Additional workspace for Question 2)

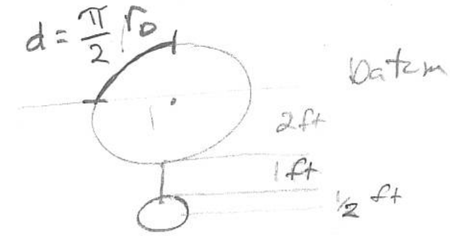
$$T_{sphere_2} = \frac{1}{2} I_{sphere_O} \omega_2^2 = \frac{1}{2}\left(\frac{2}{5} m_s r_s^2 + m_s (r_{disc} + L_{rod} + r_s)^2\right)\omega_2^2$$

$$T_{sphere_2} = \frac{1}{2}\left(\frac{2}{5}\left(\frac{10\,lb}{32.2\,ft/s^2}\right)\left(\frac{1}{2} ft\right)^2 + \left(\frac{10\,lb}{32.2\,ft/s^2}\right)\left(2\,ft + 1\,ft + \tfrac{1}{2}\,ft\right)^2\right)\omega_2^2$$

$$T_{sphere_2} = 1.918\,\omega_2^2$$

$$T_2 = (.466 + .197 + 1.918)\,\omega_2^2$$

$$\boxed{T_2 = 2.58\,(lb\text{-}ft\text{-}s^2)\,\omega_2^2}$$



$$V_2 = V_{2g,sphere} + V_{2g,rod} + V_{2\,spring}$$

$$V_{2g,sphere} = m_s g h_s = 10\,lbs\,(-3.5\,ft) = \underline{-35\,ft\text{-}lb}$$

$$V_{2g,rod} = m_r g h_r = 2\,lb\,(-2.5\,ft) = \underline{-5\,ft\text{-}lb}$$

$$V_{2\,spring} = \frac{1}{2} k \delta_2^2 = \frac{1}{2} k \left(\frac{\pi}{2} r_o + 1\,ft\right)^2$$

$$V_{2\,spring} = \frac{1}{2}(4\,lb/ft)\left(\frac{\pi}{2}(2\,ft) + 1\,ft\right)^2 = \underline{34.3\,ft\text{-}lb}$$

$$\boxed{V_2 = -35 - 5 + 34.3\,(ft\text{-}lb) = \underline{-5.7\,ft\text{-}lb}}$$

$$\cancel{T_1}^{\,0} + V_1 + \cancel{U_{12}}^{\,0} = T_2 + V_2$$

$$2\,ft\text{-}lb = 2.58\,\omega_2^2 - 5.7\,ft\text{-}lb$$

$$\boxed{\omega_2 = 1.73\ r/s}$$

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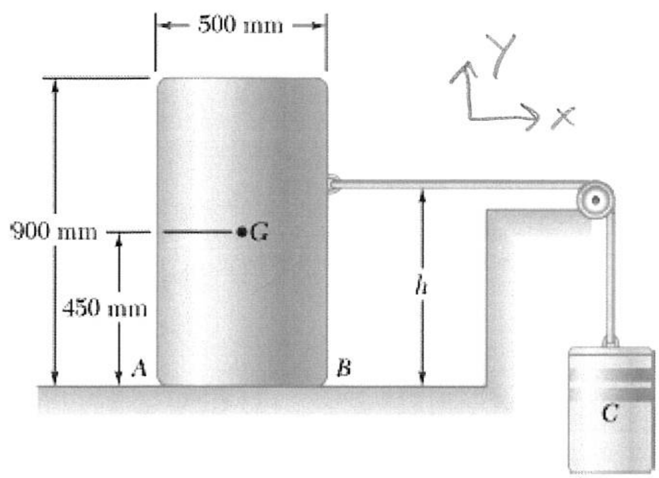
Question 3 (25 points)

A completely filled barrel and its contents have a combined mass of 90kg. A cylinder C is connected to the barrel at a height of h =550mm as shown. Knowing μ_k = 0.35, determine the maximum mass of C so the barrel will slide and not tip.

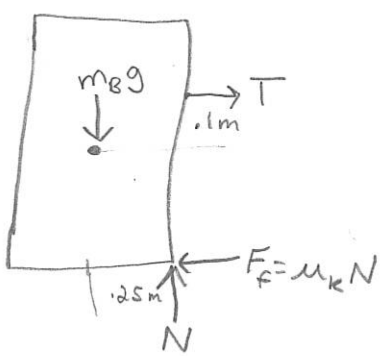


Not tipping $\vec{\alpha}_B = \vec{0}$

inextensible cable

$a_{Gx} = -a_{cy}$

FBD Barrel



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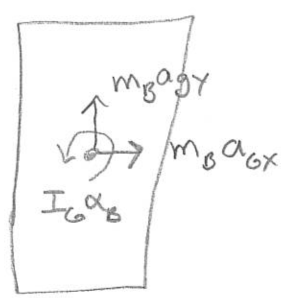
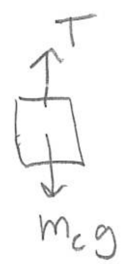
KD



w/ $a_{gY} = 0$

$\alpha_B = 0$

FBD mass C



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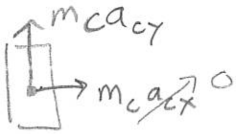

$$\Sigma F_y = T - m_c g = m_c \cancel{a_{cY}}^{\,-a_{Gx}}$$

$$T = m_c g - m_c a_{Gx} \overset{1}{\Rightarrow} \boxed{T = m_c(g - a_{Gx})}$$

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(Additional workspace for Question 3)

Barrel

$+\uparrow \Sigma F_y = N - m_B g = m_B \cancel{a_{BY}}^{0}$

$\boxed{N = m_B g}$

$\circlearrowleft + \; \Sigma M_G = N(.25m) - F_f(.45m) - T(.1m) = I_G \cancel{\alpha}^{0}$

$m_B g(.25m) - \mu_k m_B g(.45m) - T(.1m) = 0$

$90kg(9.81 m/s^2)(.25m) - .35 \cdot 90kg \cdot 9.81 m/s^2(.45m) - .1T = 0$

$T = 816.68N$

$\overset{+}{\rightarrow} \Sigma F_x = T - F_f = m_B a_{Gx}$

$816.68N - \mu_k m_B g = m_B a_{Gx}$

$816.68N - .35(90)(9.81 m/s^2) = 90 kg\, a_{Gx}$

$\boxed{a_{Gx} = 5.64 \; m/s^2}$ plug into ① from mass C ΣF_y

① $T = m_c(g - a_{Gx})$

$816.68N = m_c(9.81 m/s^2 - 5.64 m/s^2)$

$\boxed{m_c = 195.9 \; kg}$

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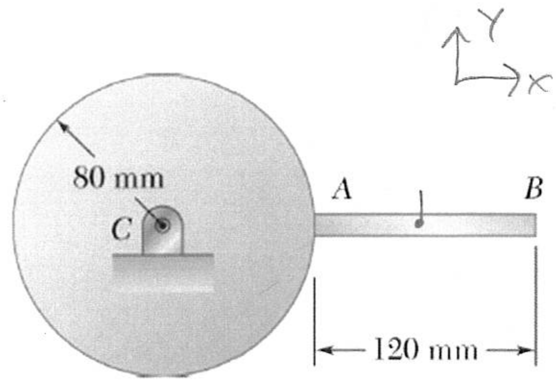
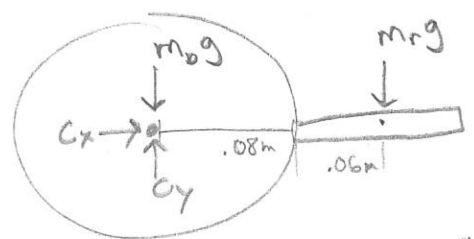
Question 4 (25 points)

A 1.5 kg slender rod is welded to a 5 kg uniform disc as shown. The assembly swings freely about C in a vertical plane. Knowing that in the position shown the assembly has an angular velocity of 10rad/s clockwise, determine

a) The angular acceleration of the assembly
b) The components of the reaction at C



FBD



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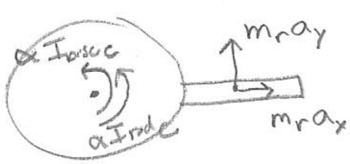

$$\text{A)}\ \circlearrowleft{+}\ \Sigma M_C = -m_r g\big(\overset{.08}{r_D} + \overset{.12/2}{\tfrac{L_r}{2}}\big) = \alpha I_{disc_C} + \alpha I_{rod_C}$$

$$-(1.5\,kg)(9.81\,m/s^2)(.14m) = \alpha\left(\tfrac{1}{2} m_D r_D^2 + \tfrac{1}{12} m_r L_r^2 + m_r\left(r_D + \tfrac{L_r}{2}\right)^2\right)$$

$$-2.06\,Nm = \alpha\underbrace{\left(\tfrac{1}{2}(5kg)(.08m)^2 + \tfrac{1}{12}(1.5kg)(.12m)^2 + 1.5kg(.14m)^2\right)}_{.0472(kg\text{-}m^2)}$$

$$\alpha = -43.64\,r/s^2$$

$$\boxed{\vec{\alpha} = -43.64\,r/s^2\ \hat{k}}$$

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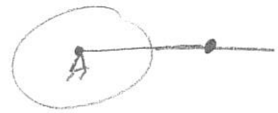
(Additional workspace for Question 4)

B) Find C_x & C_y



$\overset{+}{\rightarrow} \sum F_x = C_x = m_r a_x$ with $a_x = -a_n = -\omega^2 (r_D + \frac{L_r}{2})$

$C_x = 1.5\,kg\left(-(-10\,r/s)^2\right)(.08m + \frac{.12m}{2})$

$\boxed{C_x = -21\,N}$

$\uparrow \sum F_y = C_y - m_D g - m_r g = m_r a_y$ with $a_y = \alpha (r_D + L_r/2)$

$C_y - 9.81\,m/s^2 (5kg + 1.5kg) = 1.5kg(-43.64\,r/s^2)(.08m + .06m)$

$\boxed{C_y = 54.6\,N}$