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PrEPS Practice Exam 2
Fall 2017

1. At the given instant the wheel is rotating with the angular velocity and angular acceleration shown. Determine the velocity and acceleration of block B at this instant. The following given variables are provided due to lack of picture clarity.

$\omega = 2\, {rads}/{s}$

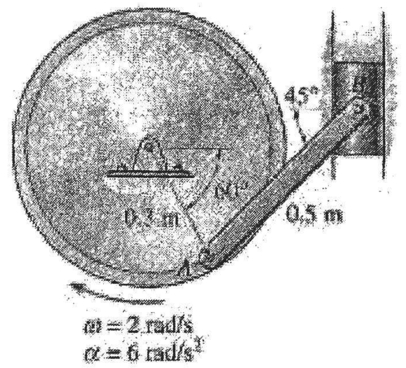
$\alpha = 6\, {rads}/{s^2}$

$r = 0.3\ m$

$L = 0.5\ m$

$\Theta = 60°$

$\Phi = 45°$



$$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A} = \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A}$$

<u>Start by finding $\vec{V}_A$ and $\vec{a}_A$</u>

$$\vec{r}_A = r\cos\theta\,\hat{x} - r\sin\theta\,\hat{y}$$

$$\vec{V}_A = \vec{\omega}_o \times \vec{r}_A = -\omega_o \hat{z} \times (r\cos\theta\,\hat{x} - r\sin\theta\,\hat{y})$$

$$\boxed{\vec{V}_A = -\omega_o r\cos\theta\,\hat{y} - \omega_o r\sin\theta\,\hat{x}}$$

$$\vec{a}_A = \vec{\alpha}_o \times \vec{r}_A - \omega_o^2 \vec{r}_A = -\alpha_o \hat{z} \times (r\cos\theta\,\hat{x} - r\sin\theta\,\hat{y}) - \omega_o^2 (r\cos\theta\,\hat{x} - r\sin\theta\,\hat{y})$$

$$= -\alpha_o r\cos\theta\,\hat{y} - \alpha_o r\sin\theta\,\hat{x} - \omega_o^2 r\cos\theta\,\hat{x} + \omega_o^2 r\sin\theta\,\hat{y}$$

$$\boxed{\vec{a}_A = (-\alpha_o r\sin\theta - \omega_o^2 r\cos\theta)\hat{x} + (-\alpha_o r\cos\theta + \omega_o^2 r\sin\theta)\hat{y}}$$

<u>now find $\vec{V}_B$ and $\vec{a}_B$</u>

$$\vec{r}_{B/A} = L\cos\varphi\,\hat{x} + L\sin\varphi\,\hat{y}$$

$$\vec{V}_B = \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A} = -\omega_o r\sin\theta\,\hat{x} - \omega_o r\cos\theta\,\hat{y} + \omega_{AB}\hat{z} \times (L\cos\varphi\,\hat{x} + L\sin\varphi\,\hat{y})$$

$$= -\omega_o r\sin\theta\,\hat{x} - \omega_o r\cos\theta\,\hat{y} + \omega_{AB} L\cos\varphi\,\hat{y} - \omega_{AB} L\sin\varphi\,\hat{x}$$

$$\vec{V}_B = (-\omega_o r\sin\theta - \omega_{AB} L\sin\varphi)\hat{x} + (-\omega_o r\cos\theta + \omega_{AB} L\cos\varphi)\hat{y}$$

$$V_{B,x} = 0 = -\omega_o r\sin\theta - \omega_{AB} L\sin\varphi$$

$$\omega_{AB} L\sin\varphi = -\omega_o r\sin\theta$$

$$\boxed{\omega_{AB} = \frac{-\omega_o r\sin\theta}{L\sin\varphi}}$$

$$\vec{V}_B = \left(-\omega_o r\cos\theta - \frac{\omega_o r\sin\theta}{L\sin\varphi} L\cos\varphi\right)\hat{y}$$

$$\boxed{\vec{V}_B = (-\omega_o r)\left(\cos\theta + \frac{\sin\theta}{\tan\varphi}\right)\hat{y}}$$

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A}$$

$$\vec{a}_B = -r(\alpha_0 \sin\theta + \omega_0^2 \cos\theta)\hat{x} + r(-\alpha_0 \cos\theta + \omega_0^2 \sin\theta)\hat{y}$$
$$+ \alpha_{AB}\hat{z} \times (L\cos\varphi\,\hat{x} + L\sin\varphi\,\hat{y}) - \frac{\omega_0^2 r^2 \sin^2\theta}{L^2 \sin^2\varphi}(L\cos\varphi\,\hat{x} + L\sin\varphi\,\hat{y})$$

$$= -r(\alpha_0 \sin\theta + \omega_0^2 \cos\theta)\hat{x} + r(-\alpha_0 \cos\theta + \omega_0^2 \sin\theta)\hat{y}$$
$$+ \alpha_{AB} L\cos\varphi\,\hat{y} - \alpha_{AB} L\sin\varphi\,\hat{x} - \frac{\omega_0^2 r^2 \sin^2\theta\, L\cos\varphi}{L^2\sin^2\varphi}\hat{x} - \frac{\omega_0^2 r^2 \sin^2\theta\, L\sin\varphi}{L^2\sin^2\varphi}\hat{y}$$

$$\vec{a}_B = \left(-r\alpha_0\sin\theta - r\omega_0^2\cos\theta - \alpha_{AB} L\sin\varphi - \frac{\omega_0^2 r^2 \sin^2\theta\cos\varphi}{L\sin^2\varphi}\right)\hat{x}$$
$$+\left(-r\alpha_0\cos\theta + r\omega_0^2\sin\theta + \alpha_{AB} L\cos\varphi - \frac{\omega_0^2 r^2\sin^2\theta}{L\sin\varphi}\right)\hat{y}$$

$$a_{B,x} = 0 = r\alpha_0\sin\theta + r\omega_0^2\cos\theta + \alpha_{AB} L\sin\varphi + \frac{\omega_0^2 r^2\sin^2\theta\cos\varphi}{L\sin^2\varphi}$$

$$\alpha_{AB} L\sin\varphi = -\left(r\alpha_0\sin\theta + r\omega_0^2\cos\theta + \frac{\omega_0^2 r^2\sin^2\theta\cos\varphi}{L\sin^2\varphi}\right)$$

$$\alpha_{AB} = -\left(\frac{r\alpha_0\sin\theta + r\omega_0^2\cos\theta}{L\sin\varphi} + \frac{\omega_0^2 r^2\sin^2\theta\cos\varphi}{L^2\sin^3\varphi}\right)$$

$$\boxed{\vec{a}_B = \left(-r\alpha_0\cos\theta + r\omega_0^2\sin\theta - \frac{r\alpha_0\sin\theta\cos\varphi + r\omega_0^2\cos\theta\cos\varphi}{\sin\varphi} + \frac{\omega_0^2 r^2\sin^2\theta\cos^2\varphi}{L\sin^3\varphi} - \frac{\omega_0^2 r^2\sin^2\theta}{L\sin\varphi}\right)\hat{y}}$$

$$\boxed{\begin{aligned}\vec{v}_B &= -0.8196\ \mathrm{m/s}\\ \vec{a}_B &= -2.0196\ \mathrm{m/s^2}\end{aligned}}$$

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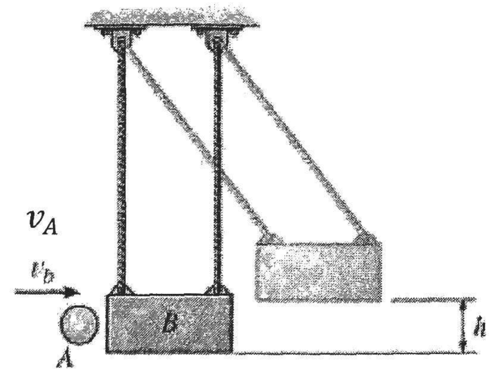
2. The ball of mass m_A is thrown at the suspended block of mass m_B with velocity v_A. If the coefficient of restitution between the ball and the block is e.



a) Determine the velocity of the ball and block after the impact.
b) Determine the maximum height h to which the block will swing before it momentarily stops.
c) If the time of impact between the ball and the block is Δt, determine the average normal force exerted on the block.

Knowns:

$$m_A = 2kg\,, m_B = 20kg\,, v_A = 4\frac{m}{s}\,, g = 9.81\,\frac{m}{s^2}\,, e = 0.8\,, \Delta t = 0.005s$$

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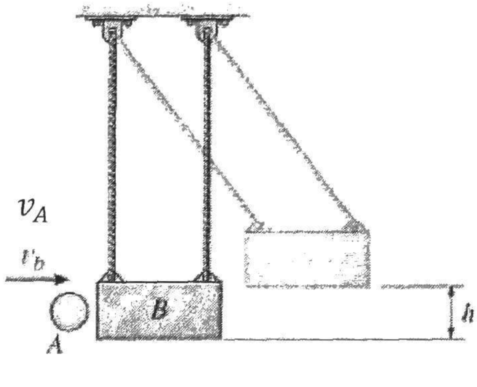

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Conservation of Momentum

$$m_A V_A^- + \cancel{m_B V_B^-} = m_A V_A^+ + m_B V_B^+$$

$$0$$

Coefficient of Restitution

$$e = \frac{V_A^+ - V_B^+}{V_B^- - V_A^-}$$

a)

$$e = \frac{V_A^+ - V_B^+}{-V_A^-} \quad \star\, V_B^- = 0$$

$$-e \cdot V_A^- = V_A^+ - V_B^+$$

$$V_B^+ - eV_A^- = \underline{V_A^+}$$

$$m_A V_A^- = m_A(V_B^+ - eV_A^-) + m_B V_B^+$$

$$m_A V_A^- = m_A V_B^+ - m_A e V_A^- + m_B V_B^+$$

$$m_A V_A^- + m_A e V_A^- = V_B^+(m_A + m_B)$$

$$V_B^+ = \frac{m_A V_A^- + m_A e V_A^-}{(m_A + m_B)} \Rightarrow V_B^+ = 0.655\frac{m}{s}$$

$$(0.655) - eV_A^- = V_A^+$$

$$\boxed{V_B^+ = 0.655\frac{m}{s}} \quad \boxed{V_A^+ = -2.55\frac{m}{s}}$$

b) $T_1 + V_1 = T_2 + V_2$

$T_1 = \frac{1}{2} m_B V_B^{+2}$

$V_1 = mg(0) = 0$

$T_2 = \frac{1}{2} m_B (0)^2 = 0$

$V_2 = m_B g h$

$\frac{1}{2} \cancel{m_B} (V_B^+)^2 = \cancel{m_B} g h$

$\frac{V_B^{+2}}{2g} = h$

$\boxed{h = 0.02184\, m}$

c) $\sum m\vec{v}_1 + \sum \int_{t_1}^{t_2} \vec{F}\,dt = \sum m\vec{v}_2$

$\underbrace{\cancel{m_B V_B^-}}_{0} + \int_0^{\Delta t} F_{AVG}\,dt = m_B V_B^+$

$F_{AVG} \cdot \Delta t = m_B V_B^+$

$F_{AVG} = \frac{m_B V_B^+}{\Delta t}$

$\boxed{F_{AVG} = 2618.18\, N}$

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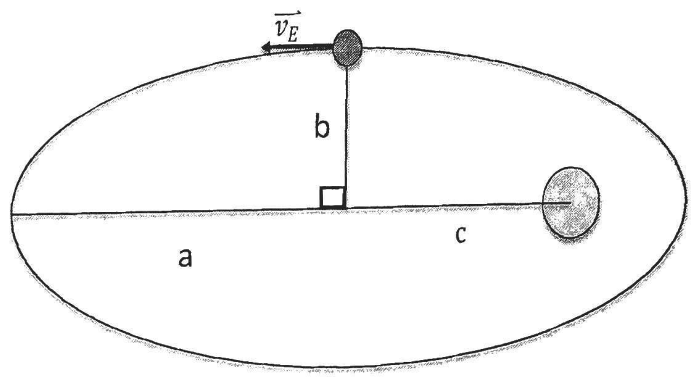
3. The Earth follows an elliptical path around the Sun as shown. Given the velocity of the Earth at it's current position is $30\frac{km}{s}\hat{u}_T$, find the maximum and minimum velocity of Earth as it travels around the Sun. The angular momentum of the Earth about the Sun is conserved. Note: a = 160 Gm , b = 140 Gm , and the equations for an ellipse have been included for your veiwing pleasure.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$a^2-b^2=c^2$$



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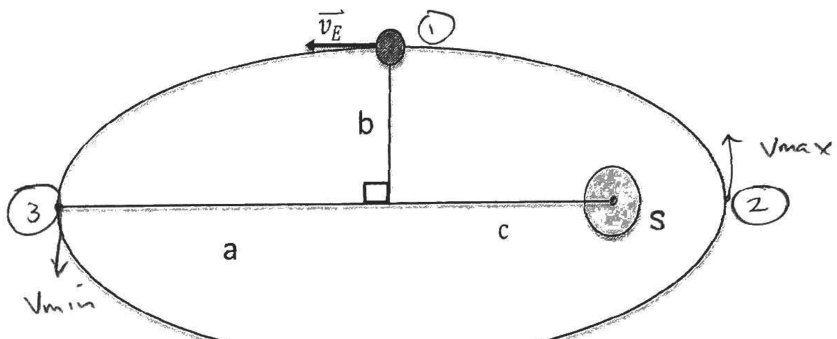
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$a^2-b^2=c^2$$

c = 77.45 Gm



Angular Momentum about Sun is Conserved at any point on the elipse

$\vec{h}_s = \vec{r} \times m\vec{v}$

$\hat{k}$ $\hat{\jmath}$ $\hat{\imath}$

1 $\vec{h}_s = b\hat{\jmath} \times -mV_E\hat{\imath}$

$\vec{h}_s = bmV_E\hat{k}$

proportional

If $h \propto rmv$ and h is const, than max v would occur at shortest r.

2 $\vec{h}_s = (a-c)\hat{\imath} \times mV_{max}\hat{\jmath}$

$\vec{h}_s = (a-c)mV_{max}\hat{k}$

1 = 2

$b\cancel{m}V_E = (a-c)\cancel{m}V_{max}$

$$V_{max} = \frac{bV_E}{(a-c)} = \frac{(140\times10^9\,m)(30\times10^3\,m/s)}{(160-77.45)10^9\,m}$$

$V_{max} = 50.88$ Km/s

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(3) $\vec{h_s} = -(a+c)\hat{i} \times -m V_{min} \hat{j}$

$\vec{h_s} = (a+c) m V_{min}$

(1) = (3)

$b\cancel{m} V_E = (a+c)\cancel{m} V_{min}$

$$\boxed{V_{min} = \frac{b V_E}{(a+c)}} = \frac{(140 \times 10^9)(30 \text{ Km/s})}{(160 + 77.45)}$$

$$\boxed{V_{min} = 17.69 \text{ Km/s}}$$