EMA 202, Spring 2015 **Name:** ______________________________

Midterm #2, closed book/notes, 90 min.

Question 1 (10 points)

1A (5 points)

The expected damage from two types of perfectly plastic collisions are to be compared. In case 1, two identical cars traveling at the same speed impact each other head on. In case 2, the same car driving the same speed impacts a massive concrete wall. Would you expect the car to be more damaged in Case 1, Case 2, or equal damage in both cases? Use the impulse-momentum principle to justify your answer.

Look @ Impulse-momentum for left car in case1 & case 2

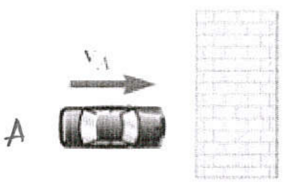
Case 1

car A) $m\vec{V}_{A1} + \vec{I}_A = m\vec{V}_{A2}^{\,\to 0}$

$\boxed{\vec{I}_A = -m\vec{V}_{A1}}$

$\vec{V}_{A1} = V_a \hat{\imath}$

$\vec{V}_{A2} = 0$ ← show this for case1



Case 2

case1 cons. of momentum

$m\vec{V}_{A1} + m\vec{V}_{B1} = m\vec{V}_{A2} + m\vec{V}_{B2}$ w/ $\vec{V}_{A2} = \vec{V}_{B2}$ perfectly plastic

$\hat{\imath})$ $mV_A - mV_A = 2mV_{A2}$

$0 = 2mV_{A2}$ $V_{A2} = 0$

→ case 1 & 2 have the same Impulse → same damage

1B (5 points)

Points A, B, and C are attached to the circular body shown below. Given the velocity at point A is 6 m/s in the vertical direction and the angular velocity of the circular body is clockwise at 2 rad/s. Locate the instantaneous center of velocity and draw the velocity vectors for points B and C in a clear and unambiguous fashion.

How does the magnitude of the velocity at point C compare to the magnitude of the velocity at point A?

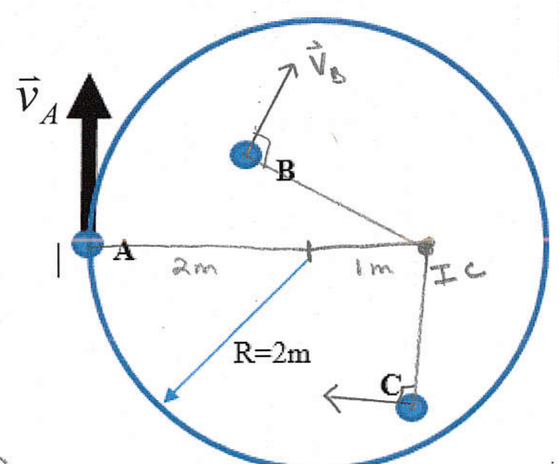
$|\vec{V}_c| < |\vec{V}_a|$, because point C is closer to the IC, ($|r_{C/IC}| < |r_{A/IC}|$) and they have the same $\vec{\omega}$.

$\vec{V}_A = \vec{\omega} \times \vec{r}_{A/IC}$

$\vec{V}_A = 6\,\text{m/s}\,\hat{\jmath}$

$\vec{\omega} = -2\,\text{r/s}\,\hat{k}$

$\vec{r}_{A/IC} = r_x\hat{\imath} + r_y\hat{\jmath}$



$$6\,\text{m/s}\,\hat{\jmath} = \begin{pmatrix} i & j & k \\ 0 & 0 & -2\,\text{r/s} \\ r_x & r_y & 0 \end{pmatrix}$$

$6\,\text{m/s}\,\hat{\jmath} = -2r_x\hat{\jmath} + 2r_y\hat{\imath} \Rightarrow \boxed{r_y = 0 \;\&\; r_x = -3m}$ $\boxed{r_{A/IC} = -3m\hat{\imath} + 0\hat{\jmath}}$

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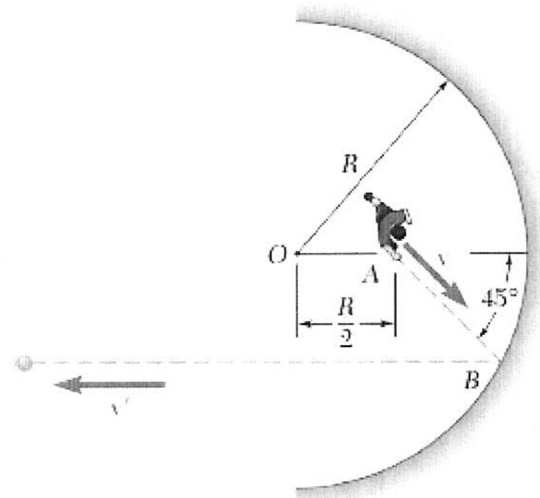
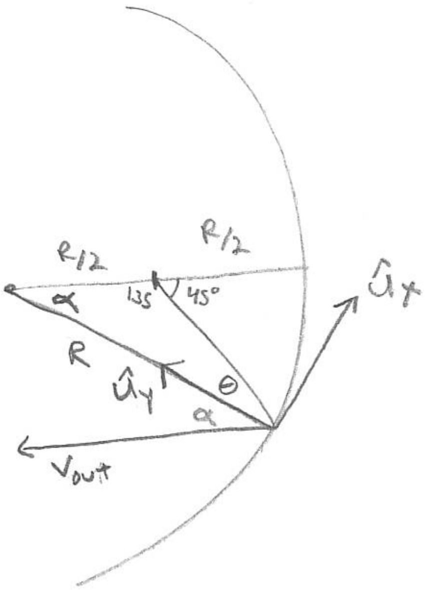
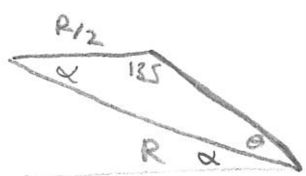
Question 2 (30 points)

A boy located at point A throws a ball at the wall in a direction forming an angle of 45° with the line OA. Knowing that after hitting the wall the ball rebounds in a direction parallel to OA, determine the coefficient of restitution between the ball and the wall. (hint: the law of sines may be helpful to figure out the geometry)





Law of sines

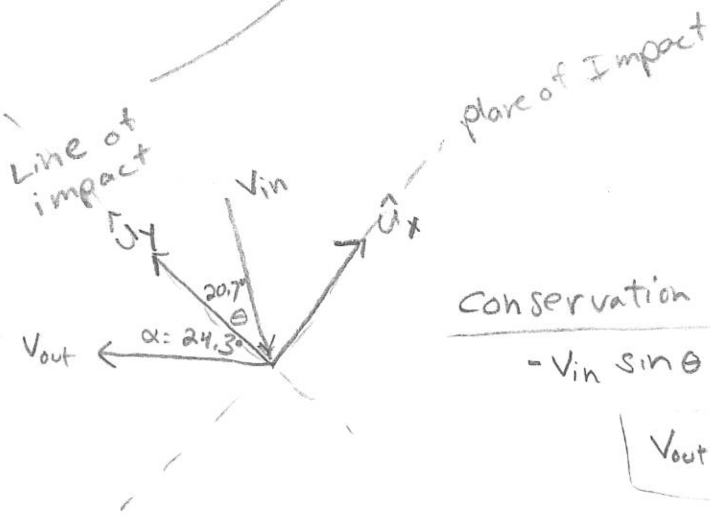


$$\frac{\sin\theta}{R/2} = \frac{\sin 135}{R}$$

$$\sin\theta = \frac{R}{2}\left(\frac{\sin 135°}{R}\right)$$

$$\boxed{\theta = 20.7°}$$

$$\alpha = 180 - 135 - \theta = \boxed{24.3° = \alpha}$$



Conservation of momentum in $\hat{u}_x$: $V_{in_x} = V_{out_x}$

$$-V_{in}\sin\theta = -V_{out}\sin\alpha$$

$$\boxed{V_{out} = \frac{\sin\theta}{\sin\alpha} V_{in}}$$

coefficient of restitution in $\hat{u}_y$ (LOI)

$$e = \frac{\overset{0}{V^{+}_{wall}} - V_{out_y}}{V_{in_y} - \underset{0}{V^{-}_{wall}}} = \frac{-V_{out}\cos\alpha}{-V_{in}\cos\theta} = \frac{\left(\frac{\sin\theta}{\sin\alpha}V_{in}\right)\cos\alpha}{V_{in}\cos\theta} = \frac{\tan\theta}{\tan\alpha}$$

$$\boxed{e = \frac{\tan 20.7°}{\tan 24.3°} = .837}$$

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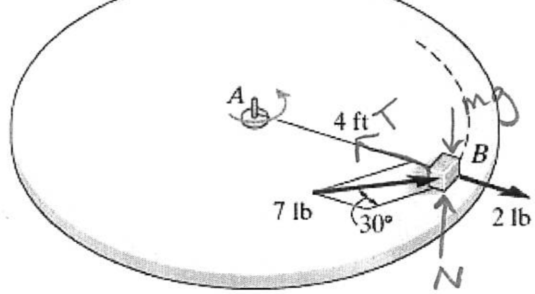
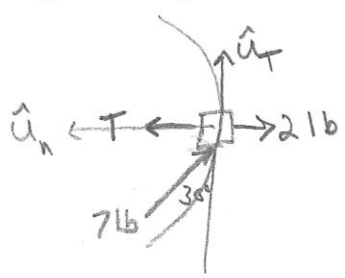
Question 3 (30 points)

The 10-lb block is originally at rest on the smooth surface. It is acted upon by a radial force of 2lbs and a horizontal force of 7lbs that is always directed at 30° from the line tangent to the path as shown. If the cord fails at a tension of 30lbs, determine the time required to break the cord. What is the speed of the block when this occurs? The block may be treated as a point mass.



FBD top view



use ΣF_n to find Velocity @ failure

$$\Sigma F_n = m a_n$$

$$T - 2\text{lbs} - 7\text{lb}\sin 30° = m\frac{v^2}{r}$$

$m = \dfrac{10 \text{ lbs}}{32.2 \text{ ft/s}^2}$

$$v^2 = \frac{4\text{ ft}}{10\text{lb}/32.2\text{ ft/s}^2}(30 - 2 - 7\sin 30°)(\text{lbs})$$

$r = 4\text{ ft}$

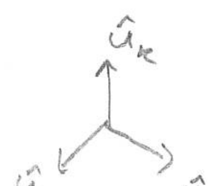
$T_{failure} = 30\text{ lbs}$

$$\boxed{V_{fail} = 17.76 \text{ ft/s}}$$

Impulse-momentum in z-dir. $(\hat{u}_K = \hat{u}_T \times \hat{u}_N)$

$$(H_{A1})_z + \int_0^t M_{Az}\,dt = (H_{A2})_z$$

$\vec{H}_{A1} = \vec{r} \times m\vec{v}_1 \rightarrow 0$, @ rest $\rightarrow (H_{A1})_z = 0$



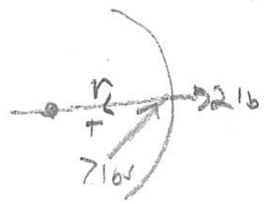
$\vec{H}_{A2} = \vec{r} \times m\vec{V}_2$ $\vec{r} = -4\,\hat{u}_n \text{ ft}$ $\vec{V}_2 = \vec{V}_{fail} = 17.76 \text{ ft/s}\ \hat{u}_T$

$$\vec{H}_{A2} = -4\text{ ft}\,\hat{u}_n \times m(17.76)\,\hat{u}_T$$

$$\vec{H}_{A2} = (4\text{ ft})\left(\frac{10\text{ lbs}}{32.2\text{ ft/s}^2}\right)(17.76\text{ ft/s})\,\hat{u}_K = \underline{22.06}\ \text{ft-lb-s}$$

prob 3 continued

$$\vec{F} = 30\,\hat{u}_N - 2\hat{u}_N - 7\sin 30\,\hat{u}_N + 7\cos 30^\circ\,\hat{u}_T + (N - mg)\hat{u}_K$$



$$\vec{r} = -4\text{ ft}\ \hat{u}_N$$

$$\vec{M}_A = \vec{r} \times \vec{F}$$

$$= \begin{pmatrix} \hat{u}_T & \hat{u}_N & \hat{u}_K \\ 0 & -4 & 0 \\ F_T & F_N & F_K \end{pmatrix}$$

$$M_{Az} = 4F_T = (4\text{ ft})(7\text{ lbs})\cos 30^\circ$$

$$\cancel{(H_{A1})_z}^{\,0} + \int_0^t M_{Az}\,dt = (H_{Az})_2$$

$$\int_0^t (4\text{ ft})(7\text{ lbs})(\cos 30^\circ)\,dt = 22.06\text{ ft-lb-s}$$

$$(4\text{ ft})(7\text{ lbs})\cos 30^\circ\, t\Big|_0^t = 22.06\text{ ft lbs}$$

$$\boxed{t = 0.91\text{ sec}}$$

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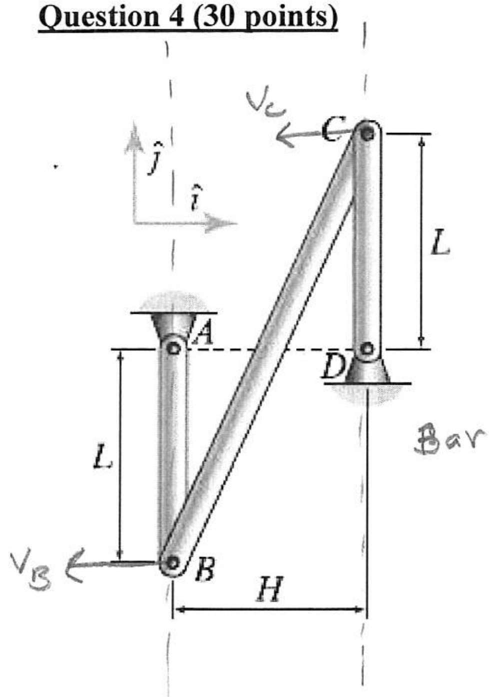
Question 4 (30 points)

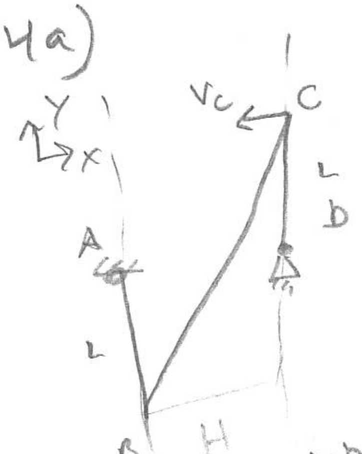


At the instant shown, bars AB and CD are vertical. In addition, point C is moving to the left with an increasing speed of 4m/s. The magnitude of the acceleration at point C is 55m/s^2. If L = 0.5m and H = 0.2m, determine the angular accelerations of bars AB and BC.

Bar BC has IC @ ∞ $\rightarrow$ $\omega_{BC} = 0$

(Additional workspace for Question 4)

4a)



given

$\vec{V}_c = -4\ m/s \uparrow$

$|a_c| = 55\ m/s^2$

$IC \to \infty$ for BC

$\omega_{BC} \to 0$

find α_{AB} & α_{BC}

write 2eqns for acceleration @ pt. B

① $\to \vec{a}_B = \cancel{\vec{a}_A}^{0} + \vec{\alpha}_{AB} \times \vec{r}_{B/A} - \omega_{AB}^2 \vec{r}_{B/A}$ w/ $\vec{r}_{B/A} = -L\hat{j}$

② $\vec{a}_B = \vec{a}_c + \vec{\alpha}_{BC} \times \vec{r}_{B/C} - \omega_{BC}^2 r_{B/C}$

from IC of bar BC

$\omega_{BC} = 0$

$\boxed{\vec{V}_B = \vec{V}_c + \cancel{\vec{\omega}_{BC}}^{0} \times \vec{r}_{B/C}}$

$\vec{V}_B = \vec{\omega}_{AB} \times \vec{r}_{B/A} = \omega_{AB}\hat{k} \times -L\hat{j}$

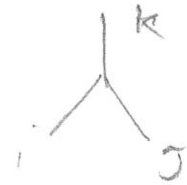
$-4\ m/s \uparrow = \begin{bmatrix} i & j & k \\ 0 & 0 & \omega_{AB} \\ 0 & -L & 0 \end{bmatrix} = L\omega_{AB} \uparrow$

$\omega_{AB} = \frac{-4\ m/s}{L} = \frac{-4\ m/s}{.5m} = \underline{-8\ r/s}$

$\boxed{\vec{\omega}_{AB} = -8\ r/s\ \hat{k}}$

4B

From ①



$$\vec{a}_B = \alpha_{AB}\hat{k} \times -L\hat{j} - (-8 \,\text{r/s})^2(-L\hat{j})$$

$$① \quad \boxed{\vec{a}_B = L\alpha_{AB}\hat{i} + 64L\hat{j}} = (.5\,\text{m})\,\alpha_{AB}\hat{i} + 32\,\text{m/s}^2\,\hat{j}$$

from ② $\vec{r}_{B/C} = -H\hat{i} - 2L\hat{j}$

$$\vec{a}_B = \underbrace{a_{cx}\hat{i} + a_{cy}\hat{j}}_{-44.7\hat{i} - 32\hat{j}} + \underbrace{\alpha_{BC}\hat{k} \times (-H\hat{i} - 2L\hat{j})} - \underbrace{\cancel{\omega_{BC}^2}}_{0 \text{ from IC}}(-H\hat{i} - 2L\hat{j})$$

$$\begin{array}{ccc} i & j & k \end{array}$$
$$\begin{pmatrix} 0 & 0 & \alpha_{BC} \\ -H & -2L & 0 \end{pmatrix} = -H\alpha_{BC}\hat{j} + 2L\alpha_{BC}\hat{i}$$

given $|a_c| = 55\,\text{m/s}^2$

$$a_c^2 = a_{cx}^2 + a_{cy}^2$$

$$a_{cy} = -\frac{V_c^2}{L} = -\frac{(-4\,\text{m/s})^2}{.5\,\text{m}} = -32\,\text{m/s}^2$$

$$a_{cx}^2 = (55\,\text{m/s}^2)^2 - (-32\,\text{m/s}^2)^2$$

$a_{cx} = \pm 44.7\,\text{m/s}^2$ → problem statement says $|V_c|$ increasing to the left.

$$\vec{a}_{cx} = -44.7\,\hat{i}\ (\text{m/s}^2)$$

set ①=② for $\vec{a}_B$

i) $.5\,\text{m}\,\alpha_{AB} = -44.7\,\text{m/s}^2 + 2L\alpha_{BC}$ $\boxed{\vec{\alpha}_{AB} = -729.4\,\text{r/s}^2\,\hat{k}}$

j) $32\,\text{m/s}^2 = -32\,\text{m/s} - H\alpha_{BC}$ $\alpha_{BC} = -320\,\text{r/s}^2$

$$\boxed{\vec{\alpha}_{BC} = -320\,\text{r/s}^2\,\hat{k}}$$