EMA 202, Spring 2015 **Name: ______________________________**

Midterm #2, closed book/notes, 90 min.

Instructions

- Write your name on every sheet.
- Show all of your work.
- To maximize opportunities for partial credit, solve problems symbolically to the extent possible and substitute numerical values at the end.
- Include free body diagrams for all equilibrium equations.

- This is a closed book examination.
- The only notes allowed are the equations provided with this exam.
- Calculators are allowed.
- The use of cell phones is prohibited.

- The instructors and the University of Wisconsin expect the highest standards of honesty and integrity in the academic performance of its students. It is important that the work submitted on this examination is yours and yours alone.
- Receiving or giving aid in an examination or using notes on a closed note exam will be considered cheating and will result in a grade of F and the case being reported to the Dean of Students Office.

Circle Your Discussion Section:

301 (Tu 1:20, Harsha)

302 (Tu 2:25, Harsha)

303 (W 3:30, David)

304 (Th 8:50, David)

305 (Th 12:05, David)

306 (Th 1:20, David)

Grading:

Q1	/10
Q2	/30
Q3	/30
Q4	/30
Total	/100

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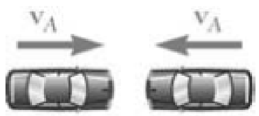
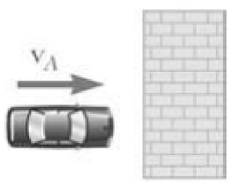
<u>Question 1 (10 points)</u>

<u>1A (5 points)</u>

The expected damage from two types of perfectly plastic collisions are to be compared. In case 1, two identical cars traveling at the same speed impact each other head on. In case 2, the same car driving the same speed impacts a massive concrete wall. Would you expect the car to be more damaged in Case 1, Case 2, or equal damage in both cases? Use the impulse-momentum principle to justify your answer.



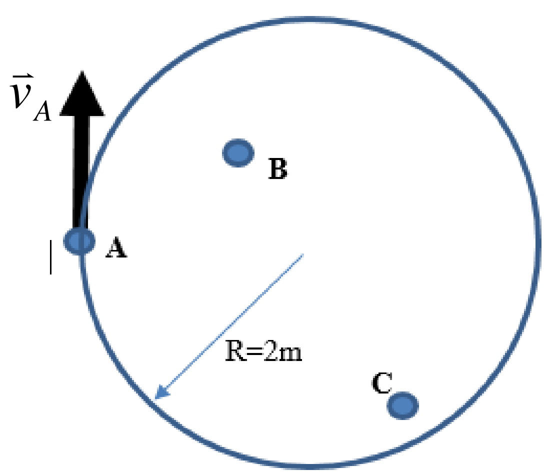
Case 1



Case 2

<u>1B (5 points)</u>
Points A, B, and C are attached to the circular body shown below. Given the velocity at point A is 6 m/s in the vertical direction and the angular velocity of the circular body is clockwise at 2 rad/s. Locate the instantaneous center of velocity and draw the velocity vectors for points B and C in a clear and unambiguous fashion.

How does the magnitude of the velocity at point C compare to the magnitude of the velocity at point A?



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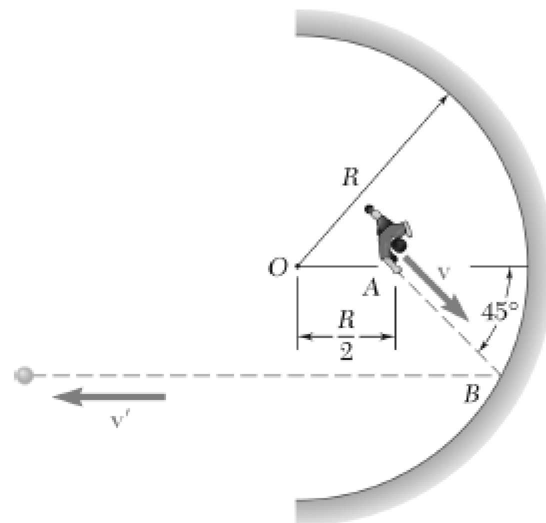
Question 2 (30 points)

A boy located at point A throws a ball at the wall in a direction forming an angle of 45^o with the line OA. Knowing that after hitting the wall the ball rebounds in a direction parallel to OA, determine the coefficient of restitution between the ball and the wall. (hint: the law of sines may be helpful to figure out the geometry)



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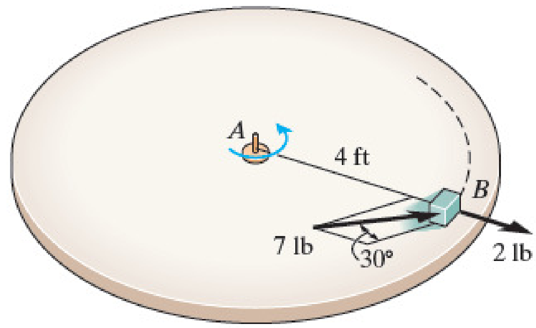
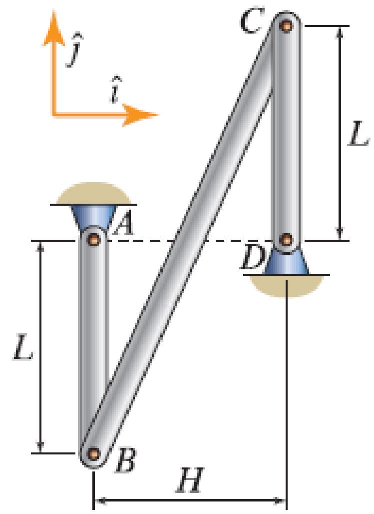
Question 3 (30 points)

The 10-lb block is originally at rest on the smooth surface. It is acted upon by a radial force of 2lbs and a horizontal force of 7lbs that is always directed at 30º from the line tangent to the path as shown. If the cord fails at a tension of 30lbs, determine the time required to break the cord. What is the speed of the block when this occurs? The block may be treated as a point mass.



Question 4 (30 points)



At the instant shown, bars AB and CD are vertical. In addition, point C is moving to the left with an increasing speed of 4m/s. The magnitude of the acceleration at point C is $55 m/s^2$. If L = 0.5m and H = 0.2m, determine the angular accelerations of bars AB and BC.

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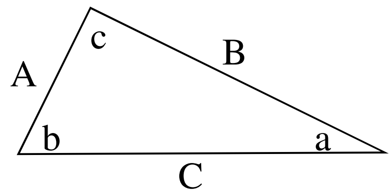
Geometry:

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

$$C^2 = A^2 + B^2 - 2AB\cos c$$

$$\cos^2\theta + \sin^2\theta = 1$$



Particle Rectilinear Motion:

$$a = \frac{dv}{dt} \qquad v = \frac{ds}{dt} \qquad ads = vdv$$

For the special case of constant acceleration (a_c), and assuming initial conditions are specified at $t = 0$:

$$v(t) = v_o + a_c t \qquad s(t) = s_o + v_o t + \frac{1}{2}a_c t^2 \qquad v^2 = v_o^2 + 2a_c(s - s_o)$$

Particle Curvilinear Motion:

Cartesian Results: $\vec{\mathbf{v}} = \dot{x}\hat{\mathbf{i}} + \dot{y}\hat{\mathbf{j}} + \dot{z}\hat{\mathbf{k}}$ $\qquad \vec{\mathbf{a}} = \ddot{x}\hat{\mathbf{i}} + \ddot{y}\hat{\mathbf{j}} + \ddot{z}\hat{\mathbf{k}}$

Normal/Tangential Results:

$$\vec{\mathbf{v}} = \dot{s}\hat{\mathbf{u}}_t$$

$$\vec{\mathbf{a}} = \dot{v}\hat{\mathbf{u}}_t + \frac{v^2}{\rho}\hat{\mathbf{u}}_n \qquad \text{where} \quad \rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|\, d^2y/dx^2 \,|}$$

Polar Results:

$$\vec{\mathbf{v}} = \dot{r}\,\hat{\mathbf{u}}_r + r\dot{\theta}\,\hat{\mathbf{u}}_\theta$$

$$\vec{\mathbf{a}} = (\ddot{r} - r\dot{\theta}^2)\,\hat{\mathbf{u}}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\mathbf{u}}_\theta$$

Spherical Results:

$$\vec{\mathbf{v}} = \dot{r}\,\hat{\mathbf{u}}_r + r\dot{\phi}\,\hat{\mathbf{u}}_\phi + r\dot{\theta}\sin\phi\,\hat{\mathbf{u}}_\theta$$

$$\vec{\mathbf{a}} = (\ddot{r} - r\dot{\phi}^2 - r\dot{\theta}^2\sin^2\phi)\,\hat{\mathbf{u}}_r + (r\ddot{\phi} + 2\dot{r}\dot{\phi} - r\dot{\theta}^2\sin\phi\cos\phi)\,\hat{\mathbf{u}}_\phi$$
$$+ (r\ddot{\theta}\sin\phi + 2\dot{r}\dot{\theta}\sin\phi + 2r\dot{\phi}\dot{\theta}\cos\phi)\hat{\mathbf{u}}_\theta$$

General Time Derivatives of a Vector:

$$\dot{\hat{\mathbf{u}}}(t) = \vec{\boldsymbol{\omega}}_u \times \hat{\mathbf{u}}$$

$$\dot{\vec{\mathbf{A}}}(t) = \dot{A}\hat{\mathbf{u}}_A + \vec{\boldsymbol{\omega}}_A \times \vec{\mathbf{A}} \qquad \ddot{\vec{\mathbf{A}}}(t) = \ddot{A}\hat{\mathbf{u}}_A + 2\vec{\boldsymbol{\omega}}_A \times \dot{A}\hat{\mathbf{u}}_A + \dot{\vec{\boldsymbol{\omega}}}_A \times \vec{\mathbf{A}} + \vec{\boldsymbol{\omega}}_A \times (\vec{\boldsymbol{\omega}}_A \times \vec{\mathbf{A}})$$

Principle of Work and Energy: $\Sigma U_{1\to2} = T_2 - T_1$ where $U_{1\to2} = \int_1^2 \vec{\mathrm{F}} \cdot d\vec{\mathrm{r}}$ and $T = \frac{1}{2}mv^2$

Conservation of Energy (assumes only conservative forces): $T_1 + V_1 = T_2 + V_2$

(For rigid bodies, these results are unchanged except for the fact that $T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$)

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V is potential energy and takes different forms depending on the source. For terrestrial gravity, $V = mgy$, (mgy_G for a rigid body) while for a linear elastic spring, $V = \frac{1}{2}k\delta^2$

Power: $P = \vec{F} \cdot \vec{v}$

Efficiency: $\varepsilon = \frac{\text{power output}}{\text{power input}}$

Linear Impulse and Momentum: $\sum \vec{\mathrm{I}}_{1\to2} = \vec{\mathrm{p}}_2 - \vec{\mathrm{p}}_1$ where $\vec{\mathrm{I}}_{1\to2} = \int_1^2 \vec{\mathrm{F}}dt$ and $\vec{\mathrm{p}} = m\vec{\mathrm{v}}$
(For rigid bodies, the linear impulse and momentum expressions are identical but with $\vec{\mathrm{v}} = \vec{\mathrm{v}}_G$)

Coefficient of restitution: $e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

Principle of Angular Impulse and Momentum: $\sum \vec{\mathrm{I}}_{M\,1\to2} = \vec{\mathrm{H}}_{O2} - \vec{\mathrm{H}}_{O1}$ where
$\vec{\mathrm{I}}_{M\,1\to2} = \int_1^2 \vec{\mathrm{M}}_O dt$ and $\vec{\mathrm{H}} = \vec{r} \times m\vec{\mathrm{v}}$

(For rigid bodies, this principle holds as long as O is either the center of gravity G or a point of fixed rotation. If the center of gravity, $H_G = I_G\omega$; if a fixed axis, $H_O = I_O\omega$.)

Relative General Plane Motion:

Translating Axes:
$$\vec{\mathbf{v}}_B = \vec{\mathbf{v}}_A + \vec{\mathbf{v}}_{B/A} = \vec{\mathbf{v}}_A + \vec{\boldsymbol{\omega}} \times \vec{\mathbf{r}}_{B/A}$$
$$\vec{\mathbf{a}}_B = \vec{\mathbf{a}}_A + \vec{\mathbf{a}}_{B/A} = \vec{\mathbf{a}}_A + \vec{\boldsymbol{\alpha}} \times \vec{\mathbf{r}}_{B/A} - \omega^2 \vec{\mathbf{r}}_{B/A}$$

Rotating Axes:
$$\vec{\mathbf{v}}_B = \vec{\mathbf{v}}_A + \left(\vec{\mathbf{v}}_{B/A}\right)_{xy} + \vec{\Omega} \times \vec{\mathbf{r}}_{B/A}$$
$$\vec{\mathbf{a}}_B = \vec{\mathbf{a}}_A + \left(\vec{\mathbf{a}}_{B/A}\right)_{xy} + \dot{\vec{\Omega}} \times \vec{\mathbf{r}}_{B/A} + \Omega \times \left(\Omega \times \vec{\mathbf{r}}_{B/A}\right) + 2\Omega \times \left(\vec{\mathbf{v}}_{B/A}\right)_{xy}$$

Equations of Motion: $\sum F_x = m(a_G)_x$; $\sum F_y = m(a_G)_y$
or $\sum F_t = m(a_G)_t$; $\sum F_n = m(a_G)_n$

$\sum M_G = I_G\alpha$ *or* $\sum M_O = I_O\alpha$ *or* ... if you must ...
$\sum M_P = -\bar{y}m(a_G)_x + \bar{x}m(a_G)_y + I_G\alpha$

Mass Moment of Inertia: Definition, and in terms of radius of gyration, k: $I = \int r^2 dm = k^2 m$

Parallel axis theorem: $I = I_G + md^2$