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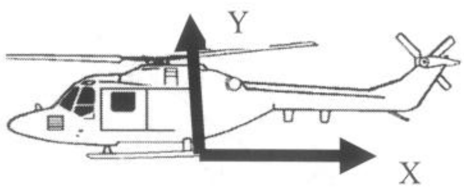
Midterm #2, closed book/notes, 90 min.

Question 1 (25 points)

Conceptual questions: please include a few sentences or equations to justify your answers

1A (7 points) A 1000kg helicopter starts from rest at t = 0 sec. Given the force components below, find the velocity vector of the helicopter after 10 seconds.



$$\sum F_x = 720t$$
$$\sum F_y = 2160 - 360t$$
$$\sum F_z = 0$$

$m\vec{v}_1 + \int \Sigma \vec{F} dt = m\vec{v}_2$ w/ $\vec{v}_1 = 0$

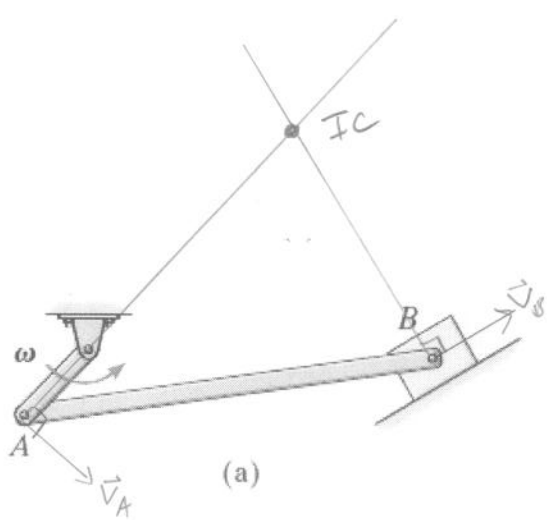
$\int_0^{10} [F_x \hat{\imath} + F_y \hat{\jmath} + F_z \hat{k}] dt = 1000\text{kg}\ \vec{v}_2$

$\int_0^{10} (720t\,\hat{\imath} + (2160 - 360t)\hat{\jmath}\, dt = 1000\,\vec{v}_2$

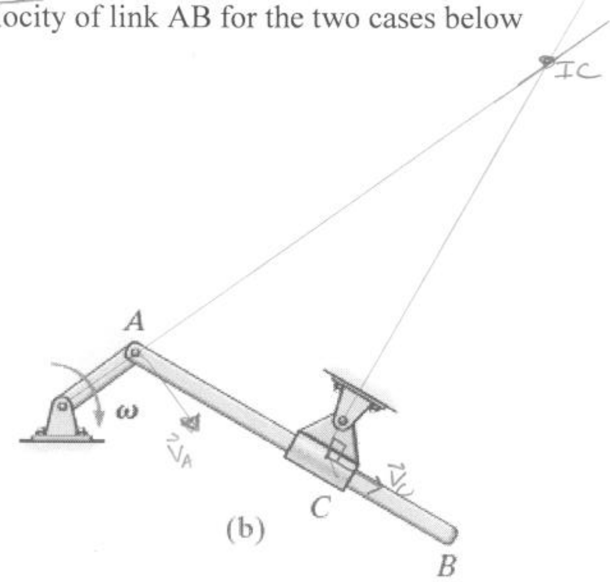
$360t^2\,\hat{\imath} + (2160t - 180t^2)\hat{\jmath}\,\Big|_0^{10} = 1000\,\vec{v}_2$

$\boxed{\vec{v}_2 = 36\,\hat{\imath} + 3.6\,\hat{\jmath}\ \text{m/s}}$

1B (6 points) Draw the instantaneous center of velocity of link AB for the two cases below



(a)



(b)

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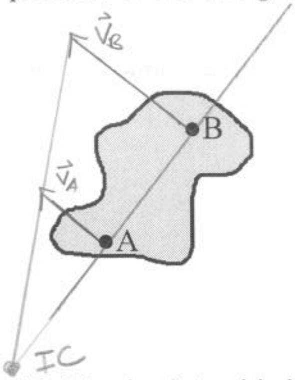
1C (6 points) For a given instant, a rigid body has velocity at point A parallel to the velocity at point B. Does this guarantee that the angular velocity of body is zero?

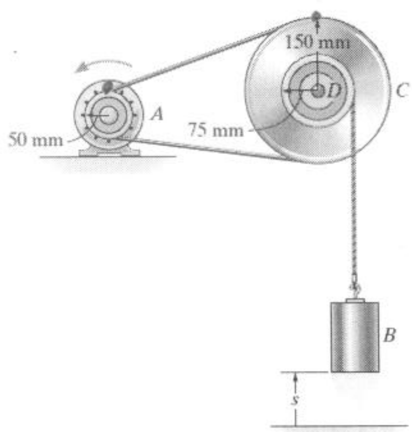


NO, $\vec{V}_B$ & $\vec{V}_a$ are parallel with coincident perpendicular lines,

$\vec{V}_B = \vec{V}_{IC} + \vec{\omega} \times \vec{r}_{B/IC}$, with $\vec{V}_{IC} = 0$

or $\vec{V}_A = \vec{\omega} \times \vec{r}_{A/IC}$

1D (6 points) At this instant, pulley A has an angular acceleration of 6 rad/s^2. What is the acceleration of block B?



Kinematic compatibility

1) $\alpha_A r_A = \alpha_C r_C$

& 2) $\alpha_C = \alpha_D$ because they are on the same shaft

3) $\alpha_D r_D = a_{By}$

From 1) $(6\ r/s^2)\ 50\text{mm} = \alpha_C\ (150\text{mm})$

$\alpha_C = 2\ r/s^2 = \alpha_D$

From 3) $(2\ r/s^2)(75\text{mm}) = a_{By}$

$\boxed{a_{By} = 150\ \text{mm}/s^2}$ ↑

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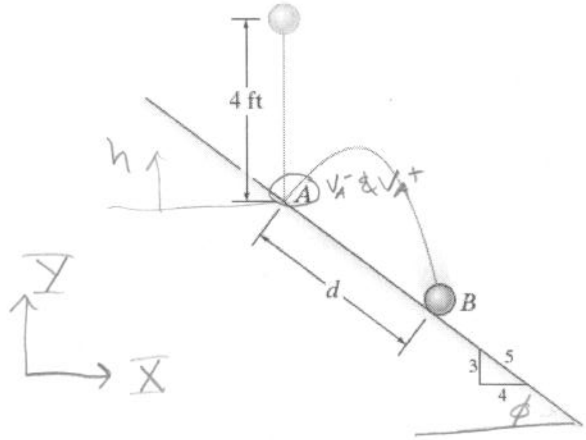
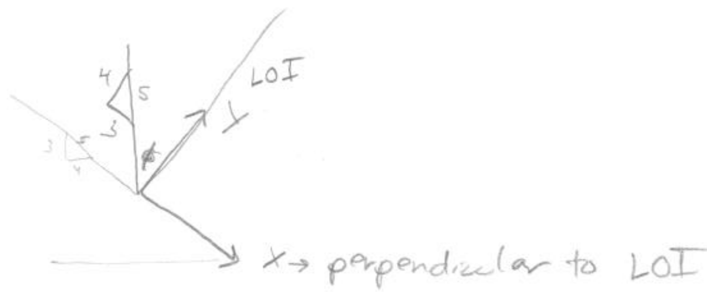
Question 2 (25 points)

A 1 lb ball is dropped from rest and falls a distance of 4ft before striking the smooth plane at A. If e = 0.8, determine the distance *d* to where the ball again strikes the plane at B.





1) find velocity b/c impact

$$T_1 + V_1 = T_2 + \cancel{V_2}$$

let $T_1 = 0 = \frac{1}{2} m v^2$

$V_2 = 0$ base on datum

$$\cancel{m} g h = \frac{1}{2} \cancel{m} (V_A^-)^2$$

$$32.2 \text{ ft/s}^2 (4 \text{ ft}) = \frac{1}{2} (V_A^-)^2$$

$$V_A^- = 16.05 \text{ ft/s}$$

In LOI direction

$$e = \frac{V_{Ay}^+ - \cancel{V_p^+}}{\cancel{V_p^-} - V_{Ay}^-}$$

let V_p = Velocity of plane $= 0$

$$.8 = \frac{V_{Ay}^+}{-V_{Ay}^-}$$

$$V_{Ay}^+ = (.8)(-V_{Ay}^-) = (.8)\left(\frac{4}{5}\right)(16.04 \text{ ft/s})$$

$$V_{Ay}^+ = 10.27 \text{ ft/s}$$

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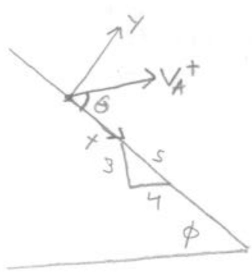
(Additional workspace for Question 2)

perpendicular to LOI

$$V_{Ax}^{-} = V_{Ax}^{+} = \frac{3}{5}(16.05 \text{ ft/s})$$

$$\underline{V_{Ax}^{+} = 9.63 \text{ ft/s}}$$



find angle

$$\tan\theta = \frac{V_{Ay}^{+}}{V_{Ax}^{+}} = \frac{10.27 \text{ ft/s}}{9.63 \text{ ft/s}} \qquad \theta = 46.85^\circ$$

$$\phi = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$$

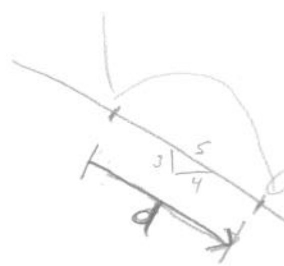
Back to XY global coordinates

let $\alpha = \theta - \phi = 46.85^\circ - 37.87^\circ = \underline{9.98^\circ}$

& $V_A^{+} = \sqrt{V_{Ay}^{+2} + V_{Ax}^{+2}} = \sqrt{10.27^2 + 9.63^2} = \underline{14.08 \text{ ft/s}}$

X projectile motion

$$X_f = \underset{0}{X_0} + V_{0x} t$$



$$X_f = 14.08 \cos(9.98)\, t \qquad \& \qquad X_f = d\left(\tfrac{4}{5}\right)$$

$$\frac{4}{5} d = 14.08 \cos(9.98)\, t \Rightarrow \boxed{t = \frac{\frac{4}{5} d}{14.08 \cos 9.98}} \; \star \qquad t = .0577 d$$

Y $\quad Y_f = \underset{0}{Y_0} + V_{0y} t - \frac{32.2 \text{ ft/s}^2}{2} t^2 \qquad \& \quad Y_f = d\left(-\tfrac{3}{5}\right)$

$$-\frac{3}{5} d = 14.08 \sin(9.98)\, t - \frac{32.2}{2} t^2 \quad \star\star$$

Sub $\star$ into $\star\star$ & solve for d

$$-\frac{3}{5} d = .141 d - .0536 d^2 \qquad \boxed{d = 13.8 \text{ ft}}$$

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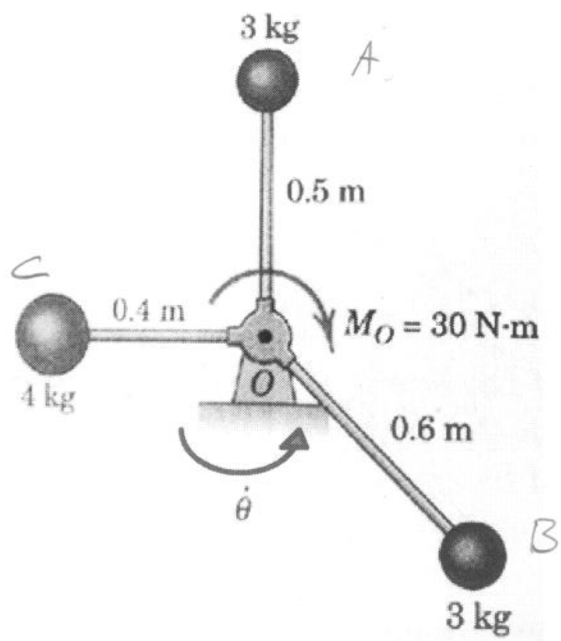
Question 3 (25 points)

Three small spheres are welded to the light rigid frame which is rotating in a horizontal plane about point O with an angular velocity of 20 rad/s in the counter clockwise direction. If a moment $M_o = 30$ Nm in the clockwise direction is applied to the frame for 5 seconds,

A) compute the angular impulse

B) compute the new angular velocity after 5 seconds



A) Impulse

$$\vec{I}_M = \int \vec{M}\,dt = \int_0^5 -30\ \text{Nm}\,dt\ \hat{k}$$

$$\boxed{\vec{I}_M = -30t \Big|_0^{5\text{ sec}} \hat{k} = -150\ \text{Nm·s}\ \hat{k}}$$

B) Impulse - momentum in $\hat{k}$

$$1)\quad (h_{OA})_{z1} + (h_{OB})_{z1} + (h_{OC})_{z1} + I\hat{k} = (h_{OA})_{z2} + (h_{OB})_{z2} + (h_{OC})_{z2}$$

For any ball

$$(h_o)_z = \vec{r} \times m\vec{v} = \begin{pmatrix} \hat{u}_r & \hat{u}_\theta & \hat{u}_k \\ r & 0 & 0 \\ 0 & mv & 0 \end{pmatrix} = \begin{pmatrix} \hat{u}_r & \hat{u}_\theta & \hat{u}_k \\ r & 0 & 0 \\ 0 & mr\omega & 0 \end{pmatrix}$$

circular path

$v = r\omega$

$$2)\quad (h_o)_z = mr^2\omega\ \hat{k}$$

use 2) in 1)

$$m_A r_A^2 \omega_o + m_b r_b^2 \omega_o + m_c r_c^2 \omega_o + I_M = m_A r_A^2 \omega_f + m_b r_b^2 \omega_f + m_c r_c^2 \omega_f$$

$$\omega_o \left[m_a r_A^2 + m_b r_b^2 + m_c r_c^2\right] + I_m = \omega_f \left(m_a r_A^2 + m_b r_b^2 + m_c r_c^2\right)$$

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(Additional workspace for Question 3)

$$\left[m_a r_a^2 + m_b r_b^2 + m_c r_c^2\right] = \left[3\text{kg}(.5\text{m})^2 + 3\text{kg}(.6\text{m})^2 + 4\text{kg}(.4\text{m})^2\right]$$

$$= \underline{2.47}\ \text{kg m}^2$$

$$\omega_0 (2.47\ \text{kg m}^2) - 150\ \text{NmS} = \omega_f (2.47\ \text{kg-m}^2)$$

$20\ \text{r/s}$

$$\boxed{\omega_f = -40.73\ \text{r/s}}$$ Clock wise rotation

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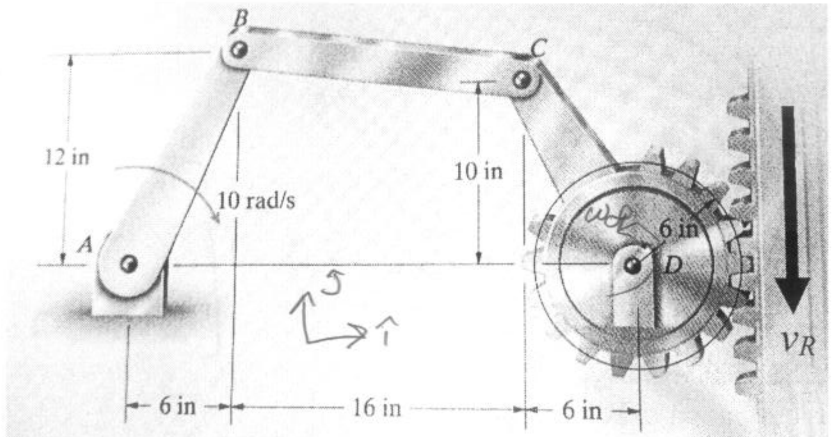
<u>Question 4 (25 points)</u>

Bar AB rotates with a clockwise angular velocity of 10 rad/sec.

Find

a) velocity vector for pt B
b) velocity vector for pt C
c) angular velocity of bar BC
d) velocity of rack, v_r



A) Find $\vec{V}_B$

$\vec{V}_B = \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$

$\vec{V}_A = 0$ pinned

$\vec{\omega}_{AB} = -10 \text{ r/s } \hat{k}$

$\vec{r}_{B/A} = 6\hat{i} + 12\hat{j}$ (in)

$$\vec{V}_B = 0 + \begin{bmatrix} i & j & k \\ 0 & 0 & -10 \\ 6 & 12 & 0 \end{bmatrix} = (-10 \text{ r/s})(6 \text{ in})\hat{j} - (-10 \text{ r/s})(12 \text{ in})\hat{i}$$

$$\boxed{\vec{V}_B = 120\hat{i} - 60\hat{j} \text{ in/s} = 10\hat{i} - 5\hat{j} \text{ ft/s}}$$

B & C) need $\vec{V}_C$ & ω_{BC}, write 2 eqns for $\vec{V}_C$

rom eft 1) $\vec{V}_{CL} = \vec{V}_B + \vec{\omega}_{BC} \times \vec{r}_{C/B}$ (Next page)

rom right 2) $\vec{V}_{CR} = \underbrace{\cancel{\vec{V}_D}}_{0 \text{ pinned}} + \vec{\omega}_{CD} \times \vec{r}_{C/D}$

$$\vec{V}_{CR} = \begin{bmatrix} i & j & k \\ 0 & 0 & \omega_{CD} \\ -6 \text{ in} & 10 \text{ in} & 0 \end{bmatrix} = \underline{-10\omega_{CD}\hat{i} - 6\omega_{CD}\hat{j}} \text{ (in/s)}$$

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(Additional workspace for Question 4)

from 1) left side

$\vec{V}_{CL} = \vec{V}_B + \vec{\omega}_{BC} \times \vec{r}_{C/B}$ w/ $\vec{r}_{C/B} = 16 \text{ in } \hat{i} - 2 \text{ in } \hat{j}$

$$= \underbrace{120\hat{i} - 60\hat{j} \text{ (in/s)}}_{\vec{V}_B} + \begin{bmatrix} i & j & k \\ 0 & 0 & \omega_{BC} \\ 16 & -2 & 0 \end{bmatrix}$$

$\vec{V}_{CL} = 120\hat{i} - 60\hat{j} + \omega_{BC} 16 \hat{j} - (-2 \text{ in})(\omega_{BC})\hat{i}$

3) $\boxed{\vec{V}_{CL} = [120 + 2\omega_{BC}]\hat{i} + [-60 + 16\omega_{BC}]\hat{j}}$ (in/s)

set $\vec{V}_{CR} = \vec{V}_{CL}$

$\hat{i}$) $-10\,\omega_{CD} = 120 + 2\omega_{BC}$

$\hat{j}$) $-6\,\omega_{CD} = 16\omega_{BC} - 60$

2 eqns & 2 unknowns, Solve for ω_{CD} & ω_{BC}

* $\omega_{CD} = -\frac{16}{6}\omega_{BC} + 10$

sub * into $\hat{i}$

$-10\left(-\frac{16}{6}\omega_{BC} + 10\right) = 120 + 2\omega_{BC}$

$\frac{160}{6}\omega_{BC} - 100 = 120 + 2\omega_{BC}$

C) $\boxed{\omega_{BC} = 8.92 \text{ r/s}}$ rotates clock wise

$\boxed{\omega_{CD} = -13.78 \text{ r/s}}$ rotates counter clock wise

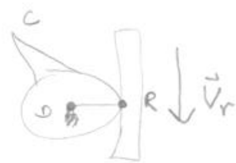
* sub ω_{BC} or ω_{CD} into $\vec{V}_{C\,left}$ or $\vec{V}_{C\,Right}$ to get $\vec{V}_C$

B)

$\vec{V}_{CL} = (120 + 2(8.92 \text{ r/s}))\hat{i} + (-60 + 16(8.92))\hat{j} \Rightarrow \boxed{\vec{V}_C = 137.84\hat{i} + 68.9\hat{j} \text{ in/s}}$

Q4 part D

Find $\vec{V}_R$



$$\vec{V}_R = \underbrace{\cancel{\vec{V}_D}}_{0} + \vec{\omega}_{CD} \times \vec{r}_{R/D} \quad \text{w/} \quad \vec{r}_{R/D} = 6 \text{ in } \hat{i}$$

$$\vec{V}_R = \begin{bmatrix} i & j & k \\ 0 & 0 & -13.78 \\ 6 & 0 & 0 \end{bmatrix} = (-13.78 \text{ r/s})(6 \text{ in})\,\hat{j}$$

D) $\boxed{\vec{V}_R = -82.68 \text{ in/s } \hat{j}}$